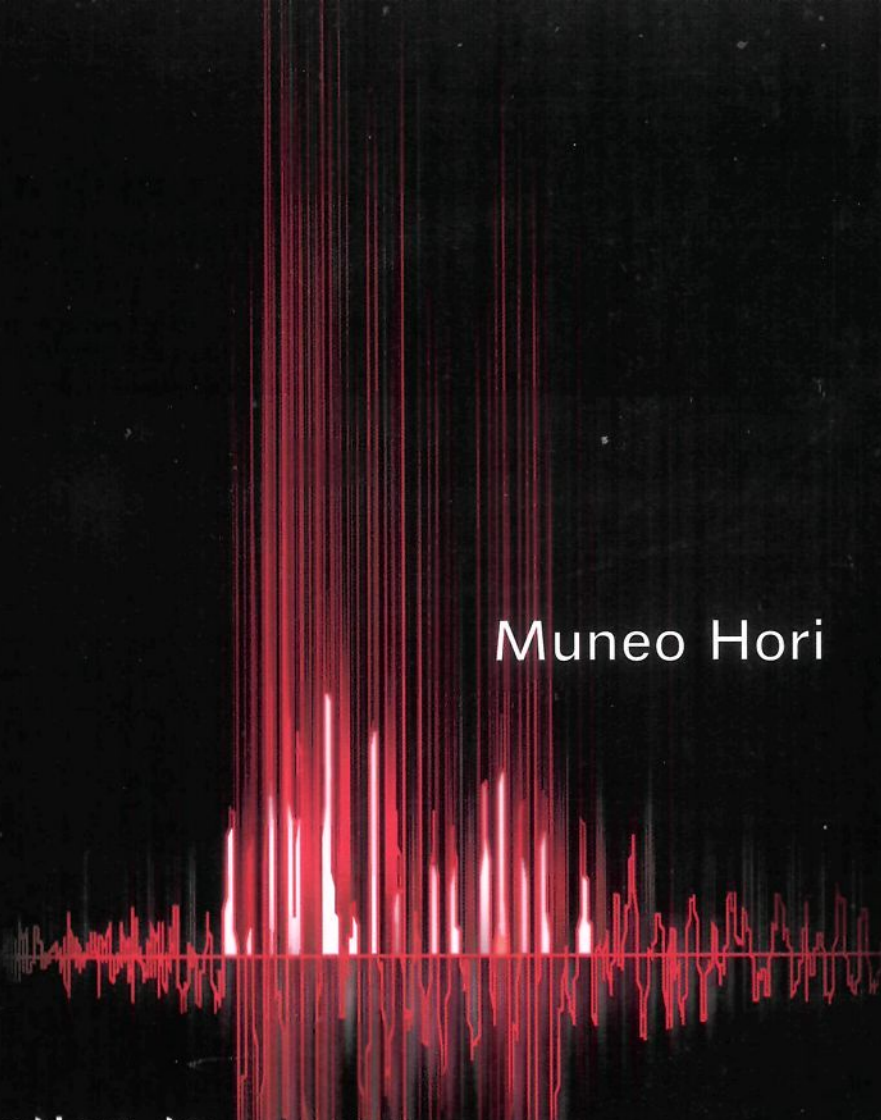




Muneo Hori

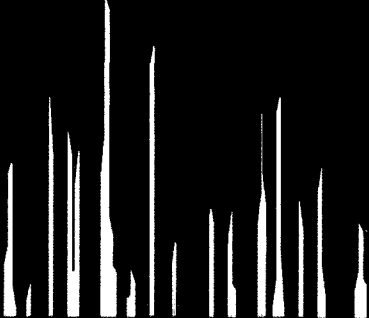
Introduction to
Computational

Earthquake Engineering

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ICP

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Preface

Computational mechanics has made a strong impact on classical continuum mechanics, which includes engineering mechanics and structure mechanics. Earthquake engineering has achieved significant progress with the aid of computational mechanics. This book covers some new aspects of earthquake engineering that are based on computational mechanics, i.e., computational earthquake engineering.

The book consists of four parts. The first part covers preliminaries, and provides the basics of continuum mechanics, the finite element method, and stochastic modeling. Only the key points of these subjects are briefly covered in this part. The first subject, however, is explained by separating physical principles and mathematical treatment to show that the basic structure of solid mechanics is as simple as a spring problem. The formulation and algorithms of the finite element method are included in the second subject; knowledge of existing numerical computation techniques is inevitably required to improve existing numerical analysis methods and to develop new ones. The third subject, stochastic modeling, is not necessarily familiar in earthquake engineering as it exists today; however, it is essential for later developments in the book and so is included in the first part.

The second and third parts make up the main content of this book. These two parts cover numerical analysis of strong ground motion and faulting respectively. The basics of the wave equation and failure analysis are briefly covered in the beginning of the second part. A stochastic model is constructed for underground structures, and a theory for wave propagation and fault rupture processes in this stochastic setting is explained. The theory leads to two analysis methods, namely, an analysis which provides bounds for the mean of the stochastic model response, and an analysis which fully calculates stochastically varying response of the stochastic model. Multi-scale analyses based on singular perturbation expansion and linear/non-linear failure analysis of the stochastic underground model are also provided for large-scale computation of strong ground motion, and for the formation of faults on the ground surface. Illustrative numerical examples involving actual strong ground motion and faulting are presented. The theory and analyses explained in these parts of the book are not standard subjects of conventional earthquake engineering. However, they do emerge in various fields of engineering and

science which are closely related to computational mechanics, and are applicable to various problems of earthquake engineering. With the aid of the analysis methods, high spatial and temporal resolution can be achieved in numerical simulations of strong ground motion for a given scenario of an earthquake. That is, the wave propagation processes from a fault to a target site are computable. The numerical analysis of the rupture processes in soil layers of uncertain composition leads to estimates of the probability that a surface earthquake fault is formed on the ground surface. The hazard of faulting is thus discussed based on quantitative information. For numerical analysis, the finite element method is mainly used to solve earthquake wave propagation problems and fault formation problems, although the boundary element method is used for the latter as well.

The last part of the book presents three advanced topics in computational earthquake engineering. The application of computer science and technology is not limited to numerical computation, and the following three topics are examples of such application. The topics are: seamless modeling and analysis of all phases of earthquake hazard and disaster by combining numerical analysis methods with Geographical Information Systems, the visualization of earthquake disaster simulation using various numerical methods integrated into one system with the aid of artificial intelligence, and an efficient object-oriented description of earthquake resistant design codes to broaden and disseminate knowledge of seismic structure response.

It should be emphasized that computational earthquake engineering is a rapidly developing field. For earthquake engineering, interaction with computational science and technology is essential since earthquakes are rare events and therefore simulation is a rational way to study earthquake hazard and disaster.

In closing this preface, the author expresses the most sincere appreciation to former graduate students of the University of Tokyo who have worked with him. They are Navaratnam Vaikunthan, H. M. Sunil Munashinge, Maciej Szymon Anders, Tsuyoshi Ichimura, and Hidenori Nakagawa. Without their enormous efforts, this book could not have been realized. The author is also grateful for comments, discussions and suggestions made by his colleagues in Earthquake Research Institute and Department of Civil Engineering, the University of Tokyo, namely, Kenji Oguni, Hiromichi Higashihara, Kazuo Konagai, Yozo Fujino, Yoshio Fukao, Takehiro Koyaguchi, and Yasuko Takei. Finally, he is thankful for the support from his family during the writing of this book.

Muneo Hori
July 2005

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PART 1

PRELIMINARIES

The advancement of computational mechanics enables us to solve numerous problems of solid mechanics which are described as a boundary value problem or an initial boundary value problem. In order to take advantage of computational mechanics, we need to build up a solid foundation, i.e., the comprehensive understanding of physics and mathematics that are the base of mechanics. Understanding algorithms and numerical techniques is important as well. Earthquake engineering, which has used numerical computation to reproduce or to predict responses of a structure subjected to an earthquake loading, is based on structure mechanics, structure dynamics or soil dynamics. Such engineering mechanics is a branch of solid continuum mechanics, and building the solid foundation of physics and mathematics helps us understand theories used in earthquake engineering.

As preliminary, we cover the foundation of computational earthquake engineering in this part. The foundation consists of three subjects, namely, solid continuum mechanics, finite element method, and stochastic modeling, and they are explained in Chapters 1, 2 and 3, respectively. Solid continuum mechanics is a subject related to physics and mathematics, and finite element method is a subject of numerical computation. Stochastic modeling is slightly different from these two subjects. We believe that stochastic modeling will be a key issue for computational earthquake engineering; when a well-developed method of numerical analysis is used, modeling of a target body determines the quality and reliability of the results of the numerical analysis. The configurations and the mechanical properties of underground structures are often uncertain, and the modeling of such uncertainty is a challenging problem. Stochastic modeling is a reasonable choice to tackle this problem.

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Chapter 1

Solid Continuum Mechanics

This chapter summarizes the essence of solid continuum mechanics, putting an emphasis on the separation of physical principles and mathematical treatment. The physical principles are deformation, equilibrium and material properties. These principles are simple. However, the mathematics which describes the principles can become complicated¹ when the object of study is a continuum which serves as a model of geological structures or a building structure. By separating the physical principles and the mathematical treatment, it is shown that there is a common framework for all mechanical problems, ranging from a simple spring problem to a general solid continuum problem. Only the mathematical treatment becomes more sophisticated as the problem becomes more complicated. The separation of the physical principles and the mathematical treatment is particularly helpful in studying earthquake wave propagation phenomena; see Appendix A for a brief summary on the earthquake mechanisms.

Three example problems are provided to summarize the essence of solid continuum mechanics in the next three sections. These problems are a spring problem, a pole problem and a continuum problem, and they are described as an algebraic equation, a boundary value problem with an ordinary differential equation, and a boundary value problem with a set of partial differential equations. While the form of the mathematical problems is different, these three problems are posed by suitably writing the three physical principles that govern the mechanics of solid continuum. This point should be emphasized. Also, it is shown that the problems of a different form can be recast into an *optimization problem* in which a solution is found by minimizing a suitable function or a suitable functional.

There have been numerous textbooks on continuum mechanics which are aimed at readers with various backgrounds. Only a few classical textbooks are mentioned in this book; the readers are recommended to find a text which matches their level of interest in continuum mechanics. As a basic textbook on continuum mechanics, [Chadwick (1976)] and [Spencer (1980)] are recommended; see also [Fung (1965)].

¹Solving a boundary value problem and using tensor for field variables are the two major difficulties among the mathematical treatments. Computational mechanics transforms a boundary value problem as a matrix equation and tensor quantities to vector quantities.

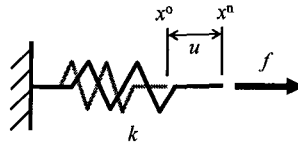


Fig. 1.1 A spring problem.

[Marsden *et al.* (1983)] is strongly recommended for readers who are interested in a more mathematical description of continuum mechanics. Although written in Japanese, [Yamamoto (1997)] is the best introduction for those who are willing to study a more physics-oriented description of classical mechanics that covers continuum mechanics.

1.1 Spring Problem

The spring problem is the simplest problem for solid mechanics. The problem setting is shown in Fig. 1.1. A spring with spring constant k is considered. The left end of the spring is fixed and the right end is subjected to external force f . The problem is to find the location of the right end. The solution is easily found, and f/k is the amount of movement of the right end. The framework of continuum mechanics, however, can be extracted even from this simple problem by carefully deriving the solution; the framework captures the three physical principles that govern the deformation of the spring. These principles provide necessary and sufficient conditions for the spring problem to have a unique solution.

First, how to express the deformation or elongation of the spring is considered. Since the left end is fixed, the amount of the elongation is represented by the movement of the right end. The elongation is written as $u = x^n - x^o$ with x^n and x^o being the location of the right end after and before the application of loading, respectively; x^n is unknown but x^o is known. Next, equilibrium is considered. Internal force acts in the spring; if the spring is cut in the middle, some force needs to be applied so that it is in equilibrium with the internal force which is acting. The amount of the applied force equals f . The internal force is denoted by s , and the equilibrium of the spring is thus expressed as $s = f$. Finally, the material property of the spring is considered. The property is simple. The internal force linearly increases with the elongation, and the spring constant is the ratio of the internal force and the elongation. Thus, the material property is expressed as $s = ku$.

Unknown quantities are the new right end location x^n , the elongation u , and the internal force s . There are a set of three equations for these three unknowns,

i.e.,

$$\begin{cases} u = x^n - x^o, \\ s = f, \\ s = ku. \end{cases} \quad (1.1)$$

It thus follows that a relation between f and x^n is $f=k(x^n-x^o)$, and hence the solution is given as $x^n=f/k+x^o$ by using² $u=f/k$. The key point is that

$$f = ku \quad (1.2)$$

is mathematically derived from the set of the three physical equations, Eq. (1.1). No physics is involved in deriving Eq. (1.2) from Eq. (1.1). The physical principles that govern the spring deformation are fully described in terms of the three equations of Eq. (1.1).

As will be shown in Sec. 1.3, all problems of solid continuum mechanics have the identical structure as the above spring problem. That is, there are a set of three equations for the physical principles, and an equation is derived from the set so that the solution of the problem is obtained. This structure is called a *framework* in this book. Also, the mathematically derived equation is called a *governing equation* and the three equations for the physical principles are called *field equations*. The three physical principles are mutually independent, in the sense that the three equations hold by themselves.

While Eq. (1.2) is readily solved, it is interesting to transform this problem to an equivalent optimization problem. First, Eq. (1.2) is rewritten as

$$\delta u(ku - f) = 0,$$

for arbitrary δu . Replacing δu by du and integrating the right side with respect to u , the following function for u is derived:

$$J(u) = \frac{1}{2}ku^2 - fu. \quad (1.3)$$

As is seen, u that minimizes this J is the solution of Eq. (1.2). It appears tricky, but what has been done is to define a function J so that the problem of minimizing J has the same solution as Eq. (1.2). That is, an optimization problem of minimizing J is equivalent with an algebraic equation of the spring problem.

1.2 Pole Problem

The next example is a pole problem, the simplest problem of structure mechanics. The problem setting is shown in Fig. 1.2. A pole with height H and cross section area A is considered. It consists of an elastic material of Young's modulus E . The bottom end is fixed and the top end is traction-free. The x -coordinate is taken

²As is seen, x^o plays a role similar to a boundary condition.

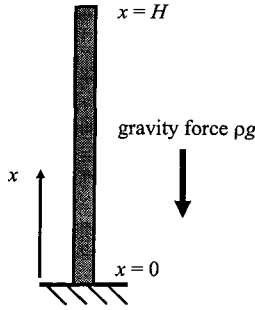


Fig. 1.2 A pole problem.

vertically, with $x=0$ and H corresponding to the bottom and the top of the pole, respectively. The pole problem is to find the deformation of the pole when it is subjected to gravity.

The three physical principles that govern the deformation of the pole are considered, and the corresponding field equations are described. First, denoting a displacement function by $u(x)$, strain $\epsilon(x)$ is derived as the derivative of $u(x)$. This strain is a measure³ of local deformation of a point at x . Next, denoting stress by $\sigma(x)$, the equilibrium for a thin portion at x is expressed as

$$A\sigma(x+dx) - A\sigma(x) = -\rho g(A dx)$$

with ρ and g being the density and the gravity constant and dx being the height of the portion; $\sigma(x)$ is introduced to define internal force which acts at a point of x . Finally, the material property is described as a linear relation between strain $\epsilon(x)$ and stress $\sigma(x)$. Since the pole consists of a uniform material, E gives the coefficient of this relation at any x .

Unknown field variables of the pole problem are displacement $u(x)$, strain $\epsilon(x)$ and stress $\sigma(x)$. The three physical principles are described as the following set of field equations that these field variables must satisfy:

$$\begin{cases} \epsilon(x) = u'(x), \\ A\sigma'(x) = -\rho g A, \\ \sigma(x) = E\epsilon(x). \end{cases} \quad (1.4)$$

The governing equation is readily derived from the set, and a differential equation for $u(x)$ is obtained. That is,

$$EA u''(x) = -\rho g A. \quad (1.5)$$

There are two boundary conditions, $u(0)=0$ and $\sigma(H)=0$. Thus, a well-posed

³In general, displacement is not a good measure of deformation since it can include rigid-body motion (translation or rotation).

boundary value problem for the displacement function $u(x)$ is described, i.e.,

$$\begin{cases} E u''(x) = -\rho g & 0 < x < H, \\ u(x) = 0 & x = 0, \\ u'(x) = 0 & x = H. \end{cases} \quad (1.6)$$

The solution is $u(x) = \rho g / 2E ((x-H)^2 - H^2)$, from which stress of the pole is found as $\sigma(x) = \rho g(x-H)$. Thus, it is seen that the top goes down by $u(H) = -\rho g H^2 / 2E$; the displacement increases linearly to the square of the height H . Also, the solution shows the distribution of internal forces acting in the pole. For instance, the pole bottom carries the cross section force of $A\sigma(0) = -\rho g AH$, which coincides with the total weight of the pole, as it should be.

While the boundary value problem of Eq. (1.6) is well-posed, it is often better to transform it to another problem of a different form in order to find a solution by means of numerical analysis. A variational problem is used to this end. Like the boundary value problem, the variational problem is to find a function as a solution. The solution is a function which makes a certain functional⁴ stationary. The boundary value problem of the pole is transformed to the variational problem that uses the following functional:

$$J(u) = \int_0^H \frac{1}{2} EA (u'(x))^2 - \rho g A u(x) dx, \quad (1.7)$$

for $u(x)$ satisfying $u(0)=0$; the boundary condition of $u'(H)=0$ is naturally satisfied in stationarizing⁵ J . The variational problem of Eq. (1.7) is equivalent with the boundary value problem of Eq. (1.6) in the sense that the solution of Eq. (1.7) coincides with that of Eq. (1.6). Due to this equivalence, a numerical solution of Eq. (1.7) gives an approximate solution of Eq. (1.6). The variational problem of J is an optimization problem since the solution of the variational problem minimizes J due to $E > 0$.

1.3 Continuum Problem

As the last example, a solid continuum problem is considered. Before studying this problem, the notation used in this book is explained. The Cartesian coordinate

⁴A functional is understood as a function for functions, in the sense that when a function is input to a functional, it outputs a value. Integration is often used to describe a functional.

⁵Stationarization means vanishing of the first variation, i.e.,

$$\delta J(u) = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} (J(u + \epsilon \delta u) - J(u))$$

for arbitrary δu satisfying $\delta u(0)=0$. The limit is readily calculated as

$$\delta J(u) = \int_0^H \delta u(x) (-E u''(x) - \rho g) A dx + [\delta u(x) (EA u'(x))]_0^H,$$

and hence the function that makes $\delta J=0$ coincides with the solution of Eq. (1.6).

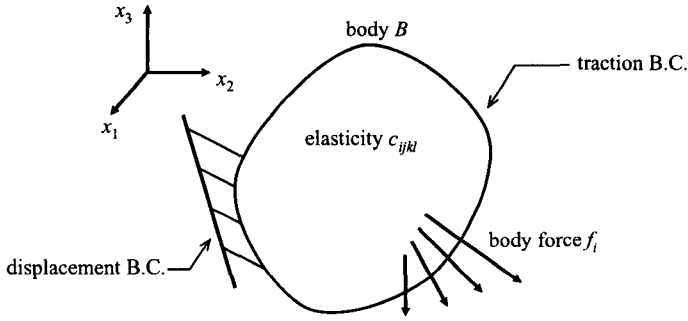


Fig. 1.3 A continuum problem.

system, $\{x_i\}$, is used, and vector and tensor quantities are designated by adding subscripts; for instance, u_i is a displacement vector, ϵ_{ij} and σ_{ij} are strain and stress tensors, and c_{ijkl} is an elasticity tensor. An argument of a function is omitted in the text; for instance, a displacement vector function is denoted by u_i instead of $u_i(\mathbf{x})$. Other rules are 1) subscript following a comma stands for the partial differentiation, i.e., $(\cdot)_{,i} = \partial(\cdot)/\partial x_i$; 2) summation convention is employed; and 3) δ_{ij} is Kronecker's delta, i.e., $\delta_{ij}=1$ for $i=j$ or 0 for $i \neq j$.

The problem setting is shown in Fig. 1.3. A deformable body with configuration B is the object of study. The body consists of a homogeneous and linear elastic material with elasticity c_{ijkl} . On some parts of the boundary denoted by ∂B_u , displacement u_i^o is prescribed, and on other parts denoted by ∂B_t , traction t_i^o is prescribed. External body force f_i is given. The continuum problem is to find the deformation of B . The major difference of the solid continuum problem from the previous two example problems is that the problem is stated in a two- or three-dimensional setting. This leads to the use of the following two mathematical tools:

- 1) a tensor⁶ is used to describe strain, stress and elasticity;
- 2) a partial differential equation⁷ is used to describe field equations.

These two mathematical tools are not an easy subject. As will be shown later, however, these tools are transformed to vector and matrix operation in numerical analysis of computational mechanics. Thus, the readers do not have to be an expert in using these mathematical tools although understanding these tools at a proper level is essential.

The three physical principles that govern the deformation of the body are considered. The first principle involves the *kinematics* that describe the state of deformation. It is assumed that deformation is infinitesimally small; this assumption is often

⁶Understanding tensor and tensor algebra is important. However, tensors are not essential for computational mechanics when fixed Cartesian coordinates are used.

⁷Analysis of partial differential equations is essential for classical physics on which earthquake engineering is based; see [Farlow (1982)] for the application of partial differential equations to a broad class of engineering problems.

made for solids. Denoting displacement by u_i , strain ϵ_{ij} is defined as the symmetric part of displacement gradient $u_{i,j}$; strain is a measure of deformation. The second principle accounts for *statics*⁸ that describes equilibrium. Assuming a quasi-static state, the equilibrium can be expressed as a balance between the gradient of stress and the external body forces, $\sigma_{ij,i} = f_j$, and the symmetry of stress, $\sigma_{ij} = \sigma_{ji}$; these two conditions are used to satisfy the equilibrium of force and moment, respectively. Stress is a measure of internal force that is generated by the deformation. The final principle designates the material property. Since B is linearly elastic, the property is described as a linear relation between σ_{ij} and ϵ_{ij} through c_{ijkl} .

Unknown field variables of the continuum problem are the displacement vector, u_i , and the strain and stress tensors, ϵ_{ij} and σ_{ij} . Like the previous problems, the physical principles that govern the continuum are expressed as the following set of field equations that the field variables must satisfy:

$$\begin{cases} \epsilon_{ij}(\mathbf{x}) = \frac{1}{2}(u_{i,j}(\mathbf{x}) + u_{j,i}(\mathbf{x})), \\ \sigma_{ij,i}(\mathbf{x}) + f_j(\mathbf{x}) = 0, \\ \sigma_{ij}(\mathbf{x}) = c_{ijkl}\epsilon_{kl}(\mathbf{x}). \end{cases} \quad (1.8)$$

Here, the symmetry of stress, $\sigma_{ij} = \sigma_{ji}$, is automatically satisfied by the symmetry of elasticity, $c_{ijkl} = c_{jikl}$; actually, elasticity is defined to satisfy this symmetry condition together with the other two symmetry conditions, $c_{ijkl} = c_{ijlk}$ and $c_{ijkl} = c_{klij}$. The governing equation⁹ for u_i is thus derived from Eq. (1.8), as

$$(c_{ijkl}u_{k,l}(\mathbf{x}))_{,i} + f_j(\mathbf{x}) = 0. \quad (1.9)$$

Displacement u_i^o is prescribed on some part of the boundary, ∂B_u , and traction t_i^o is prescribed on the other part of the boundary, ∂B_t . These two conditions serve as boundary conditions. Hence, the following boundary value problem is posed for the displacement function u_i :

$$\begin{cases} (c_{ijkl}u_{k,l}(\mathbf{x}))_{,i} + f_j(\mathbf{x}) = 0 & \text{in } B, \\ u_i(\mathbf{x}) = u_i^o(\mathbf{x}) & \text{on } \partial B_u, \\ n_i(\mathbf{x})c_{ijkl}u_{k,l}(\mathbf{x}) = u_j^o(\mathbf{x}) & \text{on } \partial B_t. \end{cases} \quad (1.10)$$

Here, n_i is the unit outer normal of the boundary, and $n_i\sigma_{ij}$ gives traction acting on the boundary. As is seen, the flow from the three field equations to the governing equation is common to the previous examples, and this is the framework of continuum mechanics.

Like the pole problem, it is often the case that the boundary value problem is transformed to an equivalent variational problem so that the solution is found by means of numerical analysis. The variational problem is for a vector function u_i

⁸Dynamics should be used instead of statics when inertia is included.

⁹There are cases where $(c_{ijkl}u_{k,l})_{,i} + f_j = 0$ is called the equilibrium equation. In this book, however, $\sigma_{ij,i} + f_j = 0$ is called the equilibrium equation as one of the three physical field equations; $(c_{ijkl}u_{k,l})_{,i} + f_j = 0$ is a governing equation which is mathematically derived from the set of the field equations.

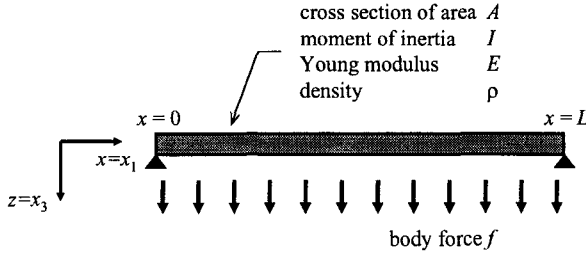


Fig. 1.4 A beam problem.

and uses the following functional:

$$J(u) = \int_B \frac{1}{2} c_{ijkl}(x) u_{i,j}(x) u_{k,l}(x) - f_i(x) u_i(x) dv_{\mathbf{x}} + \int_{\partial B_t} u_i(x) t_i^o(x) ds_{\mathbf{x}}, \quad (1.11)$$

for u_i satisfying $u_i = u_i^o$ on ∂B_u . The solution that stationarizes J , i.e., $\delta J(u) = 0$, coincides with the solution of the boundary value problem. This variational problem of Eq. (1.11) is the optimization problem that is equivalent with Eq. (1.10). The first term of the integrand, $\frac{1}{2} c_{ijkl} u_{i,j} u_{k,l}$, is called *strain energy density*, and the volume integral of this term gives the *strain energy* which is stored in B . The integral of the second term, $f_i u_i$, is usually called the *external work* done by the body force, and the surface integral is the external work done by the boundary traction.

It is interesting to note that a governing equation for some problems of structure mechanics is derived from that of an elastic continuum, just by assuming a particular form for a displacement vector function. For instance, the governing equation for the pole problem, which is presented in Sec. 1.2, is readily derived just by assuming that a non-zero component of a displacement vector function is u_3 only and that this u_3 does not depend on x_1 nor x_2 , i.e.,

$$u_1 = u_2 = 0 \quad \text{and} \quad u_3 = u(x) \quad (x = x_3).$$

Another example is the governing equation for a beam problem. As shown in Fig. 1.4, a horizontal beam with uniform cross section is considered. The x_1 -axis is taken along the longitudinal direction and body forces act in the x_3 -direction; the body force per unit volume is f . The following form is assumed for a displacement vector function:

$$u_1 = -zw'(x), \quad u_2 = 0 \quad \text{and} \quad u_3 = w(x) \quad (x = x_1, z = x_3)$$

with $z=0$ being the neutral axis. There is only one non-zero components of strain, $\epsilon_{11} = zw''$; ϵ_{13} vanishes since the assumed displacement vector components satisfy $u_{1,3} + u_{3,1} = 0$. Thus, only c_{1111} needs to be determined to calculate J of Eq. (1.11), and, for an isotropic elastic material, this component is given as $c_{1111} = E$. Substi-

tution of these strain and elasticity components into J yields

$$J(w) = \int_0^L \frac{1}{2} EI (w''(x))^2 + Af(x)w(x) dx,$$

where I and A are the moment of inertia and the cross section area, i.e., $I = \int z^2 ds$ and $A = \int ds$. The first variation, $\delta J = \lim_{\varepsilon \rightarrow 0} \frac{1}{\varepsilon} (J(w + \varepsilon \delta w) - J(w))$, is

$$\begin{aligned} \delta J = \int_0^L \delta w(x) (EI w''''(x) + Af(x)) dx \\ + \left[\delta w(x) EI w'''(x) - \delta w'(x) EI w''(x) \right]_0^L. \end{aligned}$$

A condition of $\delta J = 0$ leads to a boundary value problem for w ; a fourth-order differential equation of w ,

$$EI w''''(x) + Af(x) = 0,$$

is obtained, together with boundary conditions at $x=0$ and L . As is seen, the governing equation of a beam problem is derived from the functional of an elastic continuum, just by using a displacement vector function of a particular form.

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Chapter 2

Finite Element Method

A finite element method (FEM) is an efficient and powerful tool for numerically solving mathematical problems, which includes a boundary value problem of a structure or a continuum. There are numerous textbooks on FEM. In this book, [Belytschko *et al.* (2000)] and [Hughes (2000)] are recommended for readers who are interested in modern computational mechanics that takes advantage of FEM; see also [Washidu *et al.* (1983)], [Cheung *et al.* (1995)], [Logan (2000)] and [Reddy (2001); Reddy (2004)] as standard textbooks on FEM. A recent textbook [Brenner and Scott (2002)] presents mathematical foundations of FEM rigorously; see also [Liu (2002)] for a mesh free method, which is a new type of FEM, and [Bhatti (2005)] for more transparent programming of FEM which uses *mathematica* or *matlab*.

This chapter briefly explains FEM. The topics covered here are the overview, the formulation and the numerical techniques of FEM. In the formulation, discretization of a function in terms of computable functions is focused. FEM implements many numerical techniques and algorithm for efficient and fast computation, and some of the basic techniques and major algorithms are explained. Since large-scale numerical computation is needed for FEM, making use of an efficient algorithm is significant; the algorithm is used to solve a linear or non-linear equation for an unknown displacement function.

There has been a tremendous amount of research which makes use of FEM in earthquake engineering; see, for instance, [Petyt (1989)] and [Zienkiewicz and Taylor (1989)], for general methodologies of applying FEM to earthquake engineering problems. The utilization of FEM to dynamic structure response analysis is an ordinary practice. Although FEM is a mature tool, there remain some challenging problems¹ in applying FEM to engineering problems. Besides research which makes

¹[Leondes (1999)] provides a list of recent research related to the application of FEM to various structure engineering problems; see [Inoue *et al.* (1997)], [Ranzo and Petrangeli (1998)], [Ile and Reynouard (2000)], [Isobe and Tsuda (2003)] and [Huang *et al.* (2004)] for concrete structure problems; see [Abouseeda and Dakoulas (1996)] for dam problems; see [Pastor *et al.* (1995); Pastor *et al.* (1997)], [Asaoka *et al.* (1999)], [Komiya *et al.* (1999)], [Wakai *et al.* (1999)], [Li and Wu (2000)], [Matsuo *et al.* (2000); Matsuo *et al.* (2002)], [Vanzi (2000)], [Elgamel *et al.* (2002)], [Tejchman (2004)] and [Maeso *et al.* (2004)] for geotechnical problems; and see [Kalliontzis (1998)] and [Kanatani *et al.* (2001)] for marine structure problems.

Table 2.1 Characteristics of major numerical analysis methods.

		characteristics
discrete problem	CA	Cellular Automata: move agent sets in grids according to prescribed rule and random choice of next location
	DEM	Distinct Element Method: solve equation of motion for rigid-body elements which contact each other in linear/non-linear manner
continuous problem	FEM	Finite Difference Method: solve general class of differential equations by means of finite difference approximation
	FEM	Finite Element Method: solve linear/non-linear boundary value problem discretizing functions in a set of elements
	BEM	Boundary Element Method: solve integral equations making use of fundamental solution of problem and discretizing function on boundary

use of FEM, some new research² which is aimed at developing larger, faster and more accurate numerical analysis of FEM is being carried out in earthquake engineering. It is strongly recommended for readers to understand the basics of FEM in order to take advantage of the recent achievements.

While references to FEM are presented above, it is recommended that readers study the basics of numerical computation. As classic textbooks, [Isaacson and Keller (1966)], [Forsythe and Molder (1967); Forsythe *et al.* (1977)], [Ralston and Rabinowitz (1978)] and [Davis and Rabinowitz (1984)] provide a good foundation of numerical computation and analysis. Also, [Axelsson (1994)], [Hackbush (1994)], [Sugihara and Murota (1994)], [Higham (1996)], [Golub and Van Loan (1996)] and [Flannery *et al.* (2002)] are recommended as textbooks which cover more recent achievements of numerical computation. It should also be mentioned that the best source of numerical computation is on the internet, in which information related to numerical analysis as well as computer usages and programming is being stored although the level of reliability is not uniform as in a textbook.

2.1 Overview of FEM

First, it should be pointed out that while there are various methods of numerically solving mathematical problems, FEM is most popular in many fields of engineering. In Table 2.1, characteristics of major numerical analysis methods used in engineering and sciences are briefly summarized; the mathematical problems are categorized as a discrete problem of finding the movement of discrete particles and a continuous

²See, for instance, [Park *et al.* (1995); Park and Antin (2004)], [Qian *et al.* (1996)], [Shortreed *et al.* (2002)], [Wolf (2002)], [Honda *et al.* (2004)] and [Wong (2004)]; see also [Kawamura and Tanjung (2002)] for parallel and non-linear FEM analysis.

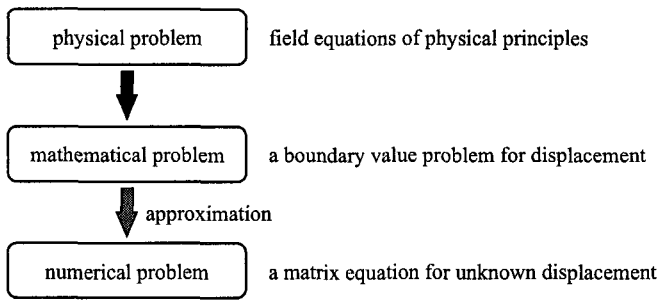


Fig. 2.1 Physical, mathematical and numerical problems.

problem of finding differentiable functions. Cellular automata (CA) and a distinct element method (DEM) are typical methods for the discrete problem³ although DEM is sometimes used⁴ to solve the continuous problem. As for a method for the continuous problem, a finite difference method (FDM) is more robust and applicable to wider class of problems, and a boundary element method (BEM) can produce a more accurate solution with similar amount of computational efforts. Still, as will be explained later, FEM is the most popular method to solve solid continuum problems which include structure mechanics problems.

It should be also pointed out that what FEM actually solves is a numerical problem which is transformed from a target mathematical problem. As the physical principles and the mathematical treatment are separated in Chapter 1, physical, mathematical and numerical problems are distinguished. For the continuum problem considered in Sec. 1.3, a physical problem is described as the three field equations, and a mathematical problem is a boundary value problem for displacement. As will be shown later, a numerical problem is a matrix equation for unknown displacement which is derived from the mathematical problem with suitable approximation; see Fig. 2.1. This numerical problem is solved by FEM. Distinguishing the three problems is important, in order to understand the accuracy and usefulness⁵ of FEM, which solves a numerical problem and provides an approximate solution of the mathematical problem or the physical problem.

The popularity of FEM is due to its two unique characteristics. From a mathematical viewpoint, FEM provides a scheme to obtain an approximate solution of a mathematical problem, the accuracy of which can be increased as desired. FEM is applicable even if the domain of analysis is of complicated configuration. From

³Some research is being undertaken to solve discontinuous problems by means of FEM; see [Arnold (1982)] and [Belytschko *et al.* (2001)]; see also [Larsson and Runesson (1993)], [Duarte *et al.* (2000)], [Oliver (2000)] and [Engel *et al.* (2002)].

⁴See [Kiyono (2004)] for the application of non-linear DEM to an earthquake engineering problem.

⁵The accuracy of FEM is evaluated as the difference from the exact solution of the mathematical problem. The usefulness of FEM is evaluated as the quality and quantity of the information that the FEM solution provides regarding to the physical problem of concern.

Table 2.2 Three basic procedures of FEM.

	procedure	objective
pre-processing	input data (material and configuration)	the domain of analysis is decomposed so that functions are discretized
computation	compute discretized function	a matrix equation, which is transformed from a boundary value problem, is solved
post-processing	retrieve solution from discretized function	the solution of a boundary value problem is presented (often graphically)

a computational viewpoint, FEM is efficient at solving a large-scale or non-linear problem which is posed for a structure or a continuum. Sophisticated numerical techniques and algorithms have been implemented in FEM. Parallel computing is a common practice to solve a large-scale problem of FEM, although there are some but not serious difficulties in applying parallel computing to FEM.

As mentioned above, the essence of FEM is to solve a matrix equation in order to obtain an approximate solution⁶ of a boundary value problem. Continuous functions and partial differential equations are transformed to form suitable vectors and matrices, respectively, and the matrix equation for an unknown vector which corresponds to displacement is derived. In general, FEM follows three basic procedures:

- i) Input data of a boundary value problem, decompose the domain of analysis into a set of elements, and define nodes so that a function is discretized.
- ii) Compute the discretized function, by solving a matrix equation which is transformed from the boundary value problem.
- iii) Output results of computation, retrieving the solution of the boundary value problem from the discretized functions.

These processes are called, pre-processing, computation and post-processing, respectively. Table 2.2 summarizes these three procedures.

In concluding this overview of FEM, it should be mentioned that from the viewpoint of analytical mechanics, FEM is aimed at solving the Lagrangean equation, not⁷ the canonical equation. The Lagrangean equation is equivalent to the equation of motion, and the canonical equation is derived from the Lagrangean equation; see Appendix B for a brief explanation of analytical mechanics. The Lagrangean equation is expressed in terms of a Lagrangean $\mathcal{L}(a, b)$ as

$$\frac{\partial \mathcal{L}}{\partial a}(u, \dot{u}) - \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial b}(u, \dot{u}) \right) = 0,$$

⁶An FEM solution is an approximate solution, and it is not easy to examine the accuracy unless the exact solution is known. However, there are several measures which can rigorously determine the accuracy of the FEM solution; see, for instance, [Belytschko *et al.* (2000)] and [Hughes (2000)].

⁷The Hamiltonian and the canonical equation are usually the object of study since they are directly related to the energy of a physical system.

where u and $\dot{u}$ are coordinate and its time derivative. FEM solves this equation although it uses $\mathcal{L}$ of a more complicated form; the functional J shown in Sec. 1.3 will be a Lagrangean if a dynamic state is considered. The canonical equation is expressed in terms of a Hamiltonian $\mathcal{H}(p, q)$ as

$$\dot{p} = -\frac{\partial \mathcal{H}}{\partial q}(p, q) \quad \text{and} \quad \dot{q} = \frac{\partial \mathcal{H}}{\partial p}(p, q),$$

where p and q are coordinate and momentum. The Lagrangean equation and the canonical equation are equivalent in the sense that the solutions are identical. The form of the canonical equation is much simpler; $\dot{p}$ and $\dot{q}$ are explicitly determined by substituting the current values of p and q into $\partial \mathcal{H}/\partial q$ and $\partial \mathcal{H}/\partial p$. And it is the Hamiltonian that provides energy stored in a body. However, the Lagrangean equation is more suitable for numerical analysis of a solid continuum problem; the Hamiltonian uses two functions and the number of unknown doubles in solving the canonical equation than the Lagrangean equation.

2.2 Discretization of Function

Numerical computation is aimed at solving a numerical problem; see Fig. 2.1. The numerical solution is an approximation of the exact solution of the original mathematical problem. This is because only a limited class of functions are treated in numerical computation. The class is related to discretization of function. That is, numerical computation makes use of *basis functions*⁸ of a finite number, and approximately expresses a function as a linear combination of these basis functions. For instance, a one-dimensional function $f(x)$ in a domain of $0 < x < 1$ is considered. In terms of a set of functions for this domain, $\{\phi^\alpha \mid \alpha=1, 2, \dots, A\}$, this function is approximated as

$$f(x) \approx \sum_{\alpha=1}^A f^\alpha \phi^\alpha(x) \quad 0 < x < 1. \quad (2.1)$$

Here, $\{f^\alpha\}$ are coefficients of the function f . The coefficients can be determined by, say, minimizing the error of $\int_0^1 (f - \sum f^\alpha \phi^\alpha)^2 dx$. When $\{\phi^\alpha\}$ are orthonormal, i.e.,

$$\int_0^1 \phi^\alpha(x) \phi^\beta(x) dx = \begin{cases} 1 & \text{for } \alpha = \beta, \\ 0 & \text{for } \alpha \neq \beta, \end{cases}$$

f^α is computed as

$$f^\alpha = \int_0^1 \phi^\alpha(x) f(x) dx. \quad (2.2)$$

⁸A simple example of bases are a set of trigonometric functions for the Fourier series expansion; a function of interest is approximated as a linear combination of these trigonometric functions.

As is seen, the error decreases as the number of basis functions increases. In this book, Eq. (2.1) is called *discretization* of function and the basis functions $\{\phi^\alpha\}$ are called the *bases*.

When the bases $\{\phi^\alpha\}$ are given, a discretized function is expressed as a linear combination of them, and hence a class of discretized functions are defined as $\{\sum f^\alpha \phi^\alpha\}$, where $\{f^\alpha\}$ are a set of arbitrary coefficients. In this class, a function is described by the set of the coefficients, and the coefficients are regarded as a representation of the function. This class⁹ of functions is called *function space*.

The advantage of using the function space is that a function f is regarded as an A -dimensional vector whose components are $\{f^\alpha\}$; recall that A is the number of the bases. In the function space, it is possible to treat some operations of a function as a linear transformation¹⁰ of its coefficients. As the simplest example, an operation of differentiation is considered. When the derivative of a basis, ϕ^α , is given in terms of other bases as $(\phi^\alpha)' = \sum_\beta D^{\alpha\beta} \phi^\beta$, the derivative of f becomes

$$f'(x) \approx \sum_\alpha (f^\alpha \phi^\alpha(x))' = \sum_\alpha f^\alpha (\phi^\alpha(x))' = \sum_{\alpha\beta} f^\alpha D^{\alpha\beta} \phi^\beta(x).$$

As is seen, the operation of differentiation is expressed as linear transformation of $\{f^\alpha\}$ to $\{\sum_\beta D^{\alpha\beta} f^\beta\}$. For a more complicated but linear differential operator, denoted by L , the operation becomes

$$L[f] \approx \sum_\alpha L[f^\alpha \phi^\alpha] = \sum_\alpha f^\alpha L[\phi^\alpha] = \sum_{\alpha,\beta} f^\alpha L^{\alpha\beta} \phi^\beta,$$

where $L^{\alpha\beta}$ is a coefficient of the transformed basis $L[\phi^\alpha]$, i.e., $L[\phi^\alpha] = \sum_\beta L^{\alpha\beta} \phi^\beta$. The operation L is regarded as linear transformation of $\{f^\alpha\}$ to $\{\sum_\beta L^{\alpha\beta} f^\beta\}$.

FEM uses special bases to express a solution of a boundary value problem which is posed for an arbitrary domain. The domain is decomposed into a set of elements which do not overlap each other, and the elements are formed by connecting nodes. The special bases are characterized by these nodes and elements. Each basis has one particular node at which it takes a value of 1 and vanishes at other nodes. Each basis also has several elements except which it vanishes. For instance, a two-dimensional setting as shown in Fig. 2.2 is considered. There is a node x^1 which is shared by four triangle elements Ω^1 , Ω^2 , Ω^3 and Ω^4 ; these elements are formed by other nodes x^2 , x^3 , x^4 and x^5 , i.e., $\{x^1, x^2, x^3\}$, $\{x^1, x^3, x^4\}$, $\{x^1, x^4, x^5\}$ and $\{x^1, x^5, x^2\}$ form Ω^1 , Ω^2 , Ω^3 and Ω^4 , respectively; see Fig. 2.2. A basis for x^1 ,

⁹To rigorously define function space, more mathematical tools need to be prepared. In this book, however, it is sufficient to regard the function space as a set of coefficients of a finite number. It is worth noting that function space defined here still has the following three properties: 1) if f is in the space, αf is in the space with α being an arbitrary number; 2) if f and g are in the space, $f+g$ is in the space; and 3) the norm of f and g is defined as, say, $|f|^2 = \int f^2 dx$.

¹⁰Transformation $\mathcal{L}$ is called linear if it satisfies $\mathcal{L}[\alpha f] = \alpha \mathcal{L}[f]$ with α being an arbitrary number and $\mathcal{L}[f+g] = \mathcal{L}[f] + \mathcal{L}[g]$.

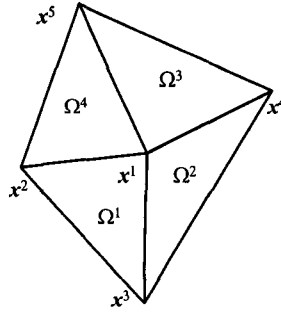


Fig. 2.2 Nodes and elements of FEM. A basis ϕ^1 is defined in $\Omega^1 \sim \Omega^4$ and satisfies Eq. (2.3).

denoted by ϕ^1 , vanishes outside of these four elements, and satisfies

$$\phi^1(x^\alpha) = \begin{cases} 1 & \text{for } \alpha = 1, \\ 0 & \text{for } \alpha = 2, 3, 4, 5. \end{cases} \quad (2.3)$$

In general, the basis ϕ^1 is expressed as a particular polynomial function within each element; some continuity across the element boundaries is satisfied.

It should be commented that there are functions which cannot be discretized accurately by using the bases of FEM. Indeed, some physical problems have a solution which is expressed in terms of functions which do not allow the Taylor expansion at some points. A crack problem is such a physical problem; displacement at a crack tip is singular, i.e., the displacement function vanishes as¹¹ $\mathcal{O}(r^{\frac{1}{2}})$ with r being the distance from the crack tip, and it accompanies strain or stress which diverges as $\mathcal{O}(r^{-\frac{1}{2}})$.

2.3 Formulation of FEM

This section presents two formulations of FEM, namely, the variational formulation and the weak form formulation. While these two formulations start from different target equations, they end up with the same discretized matrix equation that is solved by FEM. It should be pointed out that some boundary value problems do not have an equivalent variational problem, although it is always transformed to a weak form. When the boundary value problem has an equivalent variational problem¹² and when the stationary value of the functional is the global minimum, it enables us to choose a better approximate solution of the boundary value problem, which takes a smaller value of the functional.

¹¹Precisely speaking, $f(x) \sim \mathcal{O}(x^n)$ is used when $\lim_{x \rightarrow 0} f(x)/x^n$ becomes finite; $f(x) \sim o(x^n)$ is used when $\lim_{x \rightarrow 0} f(x)/x^n$ vanishes.

¹²Most of solid continuum problems are described as a boundary value problem or an initial boundary value problem, and it is possible to transform the problem to an equivalent variational problem. Lagrangean is used to describe a functional for such a variational problem.

Considering the most general case, a boundary value problem for a three-dimensional linearly elastic body is studied. The body, denoted by B , is subjected to body forces and displacement boundary conditions. The boundary value problem is posed as

$$\begin{cases} (c_{ijkl}(\mathbf{x})u_{k,l}(\mathbf{x}))_{,i} + f_i^o(\mathbf{x}) = 0 & \text{in } B, \\ u_i(\mathbf{x}) = u_i^o(\mathbf{x}) & \text{on } \partial B. \end{cases} \quad (2.4)$$

Here, c_{ijkl} is elasticity which is heterogeneous, and f_i^o and u_i^o are the body force and the boundary displacement prescribed in the body B and on the boundary ∂B , respectively.

As shown in Sec. 1.3, the variational problem that is equivalent with Eq. (2.4) uses the following functional:

$$J(\mathbf{u}) = \int_B \frac{1}{2} c_{ijkl}(\mathbf{x}) u_{i,j}(\mathbf{x}) u_{k,l}(\mathbf{x}) - f_i(\mathbf{x}) u_i(\mathbf{x}) dv_{\mathbf{x}}, \quad (2.5)$$

for u_i satisfying $u_i = u_i^o$ on ∂B . Compared with solving Eq. (2.4), finding u_i that stationarizes J of Eq. (2.5) has several advantages. First, J uses only the first derivatives of u_i even though the governing equation of Eq. (2.4) is a second-order differential equation. Second, J orders functions in the sense that a function which takes a smaller value of J is closer to the exact solution of Eq. (2.4); the exact solution minimizes J since c_{ijkl} is positive-definite, i.e., $c_{ijkl}e_{ij}e_{kl} > 0$ for non-zero but arbitrary e_{ij} .

In order to numerically solve Eq. (2.5), FEM discretizes u_i ; when a set of bases, denoted by $\{\phi^\alpha\}$, are given, u_i is expressed in terms of the coefficients, denoted by $\{u_i^\alpha\}$, as

$$u_i(\mathbf{x}) = \sum_{\alpha} u_i^\alpha \phi^\alpha(\mathbf{x}). \quad (2.6)$$

Here, by definition, u_i takes on the value of u_i^α at the node $\mathbf{x}^\alpha$. Thus, the coefficients are chosen so that boundary condition on ∂B is satisfied, i.e., $u_i = u_i^o(\mathbf{x}^\alpha)$ for $\mathbf{x}^\alpha$ on ∂B . By substituting Eq. (2.6) into Eq. (2.5), a functional J of u_i is reduced to a multi-variable function of $\{u_i^\alpha\}$,

$$J^*(\{u_i^\alpha\}) = \sum_{\alpha,\beta} \frac{1}{2} k_{ij}^{\alpha\beta} u_i^\alpha u_j^\beta - \sum_{\alpha} f_i^\alpha u_i^\alpha, \quad (2.7)$$

where

$$k_{ij}^{\alpha\beta} = c_{ikjl} \int_B \phi_{,k}^\alpha(\mathbf{x}) \phi_{,l}^\beta(\mathbf{x}) dv_{\mathbf{x}}; \quad (2.8)$$

the integration of $\int_B f_i u_i dv$ is calculated as $\sum f_i^\alpha u_i^\alpha$ with f_i^α being a concentrated nodal force at $\mathbf{x}^\alpha$; this f_i^α is calculated as $f_i^\alpha = \int f_i \phi^\alpha dv$ although the resultant of the body force in the elements are often used to calculate f_i^α . As is seen, this J^* is

a quadratic form with linear terms for $\{u_i^\alpha\}$, and can be expressed in the following simple form:

$$J^*([U]) = \frac{1}{2}[U]^T[K][U] - [F]^T[U],$$

where $[U]$ and $[F]$ are vectors¹³ corresponding to $\{u_i^\alpha\}$ and $\{f_i^\alpha\}$, and $[K]$ is a matrix corresponding to $\{k_{ij}^{\alpha\beta}\}$. Thus, the most suitable $[U]$ or the most suitable approximate solution of Eq. (2.5) that is expressed in the form of Eq. (2.6) must satisfy

$$[K][U] = [F]. \quad (2.9)$$

This is the target matrix equation that FEM seeks to solve in its numerical computation.

A question arises as to whether a suitable functional exists or not for a given boundary problem. The answer is no. Still, the boundary value problem can be transformed to an integral form so that FEM is applicable to obtain a numerical solution. The integral form leads to a *weak form* of the boundary value problem. The weak form is derived from the weighted integral of the governing equation, i.e.,

$$\int_B w_j(\mathbf{x}) ((c_{ijkl}(\mathbf{x}) u_{k,l}(\mathbf{x}))_{,i} + f_j^\circ(\mathbf{x})) dv_{\mathbf{x}} = 0,$$

where w_i is a smooth weight function; this w_i is arbitrary, but it needs to satisfy $w_i=0$ on ∂B so that an unnecessary boundary condition is not posed. Integration by part leads to

$$\int_B -c_{ijkl}(\mathbf{x}) w_{j,i}(\mathbf{x}) u_{k,l}(\mathbf{x}) + w_j(\mathbf{x}) f_j^\circ(\mathbf{x}) dv_{\mathbf{x}} = 0. \quad (2.10)$$

This is the weak form¹⁴ of Eq. (2.4). Note that only the first order derivatives of u_i are used in the weak form, Eq. (2.10), like the functional J of Eq. (2.5).

A set of equations are derived from Eq. (2.10) by substituting the discretized function u_i of Eq. (2.6). Indeed, if w_i is given as¹⁵

$$w_i(\mathbf{x}) = \delta u_i^\alpha \phi^\alpha(\mathbf{x}) \quad (\alpha \text{ not summed})$$

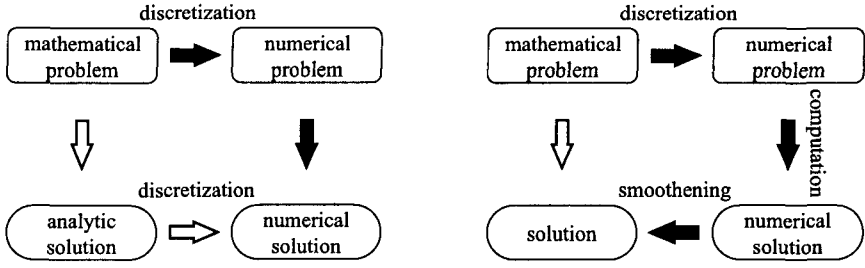
¹³More precisely speaking, the vector $[U]$ consists of only unknown u_i^α 's; known u_i^α 's which are determined from the boundary condition are included in $[F]$. Denoting the vectors consisting of unknown u_i^α 's and known u_i^α 's by $[U_1]$ and $[U_2]$, the vectors and matrix can be rewritten as

$$[U] = \begin{bmatrix} [U_1] \\ [U_2] \end{bmatrix}, \quad [F] = \begin{bmatrix} [F_1] \\ [F_2] \end{bmatrix} \quad \text{and} \quad [K] = \begin{bmatrix} [K_{11}] & [K_{12}] \\ [K_{21}] & [K_{22}] \end{bmatrix}$$

with $[K_{21}] = [K_{12}]^T$, and the unknown $[U_1]$ is obtained by solving $[K_{11}][U_1] = [F_1] - [K_{12}][U_2]$, which corresponds to Eq. (2.9).

¹⁴The solution of Eq. (2.4) always satisfies Eq. (2.10). However, there may be other functions which satisfy Eq. (2.10).

¹⁵This treatment of w_i is called the Galerkin method.



a) two paths to reach a numerical solution b) a solution obtained from numerical analysis

Fig. 2.3 The equivalence of a mathematical problem and a numerical problem.

with δu_i^α being an arbitrary vector, Eq. (2.10) leads to

$$\delta u_i^\alpha \left(- \sum_{\beta} k_{ij}^{\alpha\beta} u_j^\beta + f_i^\alpha \right).$$

Here, $k_{ij}^{\alpha\beta}$ is given as Eq. (2.8). Since δu_i^α is arbitrary, the terms in the parenthesis must vanish. Therefore, the matrix equation that is identical with Eq. (2.9) is derived.

In closing this section, it should be mentioned that the equivalence between a matrix equation of FEM and an original boundary value problem, which are called a numerical problem and a mathematical problem, respectively. First, a case when the mathematical problem has an analytic solution is considered. In Fig. 2.3a), black arrows show the path of FEM, i.e., it transforms the mathematical problem to a numerical problem applying discretization, and then obtains the numerical solution solving the numerical problem. If suitable discretization is used, the numerical solution coincides with the solution that is obtained by discretizing the analytic solution of the original mathematical problem, i.e., a solution obtained by following the path of white arrows. This is the *equivalence* of the numerical problem to the mathematical problem. Next, a case when the mathematical problem does not have an analytic solution is considered. It can be expected that an equivalent numerical problem is posed in this case, if the equivalence is guaranteed for simpler but similar mathematical problems. Thus, an FEM solution can be used as a solution of the mathematical problem. In Fig. 2.3b), three black arrows¹⁶ indicate a path to reach a solution of a mathematical problem by means of FEM, which transforms the mathematical problem to a numerical problem, computes the numerical solution, and retrieves the mathematical problem solution by smoothing the numerical solution.

¹⁶As is seen, the three black arrows correspond to pre-processing, computation and post-processing shown in Table 2.2.

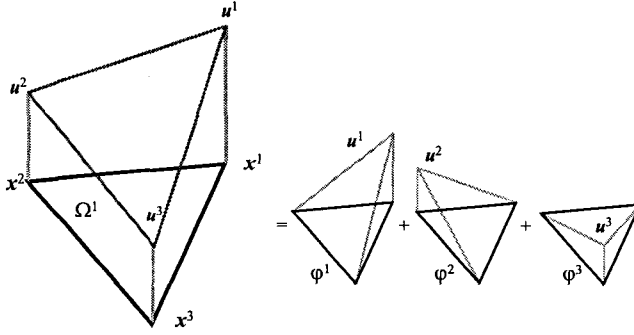


Fig. 2.4 Linear shape functions for a triangular element.

2.4 Major Numerical Techniques Used in FEM

Section 2.3 has shown that a boundary value problem for displacement function is transformed to a numerical problem which FEM solves. Some numerical techniques are used in the transformation, i.e., a function and a partial differential equation are transformed to a vector and a matrix equation, respectively. This section presents several key numerical techniques which are usually implemented in FEM.

2.4.1 Shape function

Bases used in FEM are called *shape functions* in this book. There is a trade-off between the accuracy and the simplicity of the shape function; if polynomials are used, the accuracy corresponds to the degree of the polynomial, and polynomials of higher degrees can express a smoother function. In general, the use of a smoother shape function results in a more accurate solution, although it requires more computational efforts. The choice of the shape function is thus important in applying FEM to solve a boundary value problem.

The simplest shape function¹⁷ is an element-wise linear function which has element-wise constant derivatives. For a two- or three-dimensional setting, such a shape function is used by a triangular element with three nodes or a tetrahedral element with four nodes, respectively; see Fig. 2.4 for shape functions of a triangular element with three nodes. Since the derivative is uniform in elements, it is easy to compute the matrix $[K]$ which appears in Eq. (2.9). Note that triangular elements or tetrahedral elements can cover a domain of complicated configuration easily.

The next most simple shape function is an element-wise quadratic function which has element-wise linear derivatives. A rectangular element with four nodes or a cubic element with eight nodes uses the element-wise quadratic functions in a two-

¹⁷Usually, a type of element is used to describe a shape function; for instance, in a two-dimensional setting, a three-node triangular or four-node rectangular element means an element-wise linear or element-wise quadratic shape function.

or three-dimensional setting, respectively. Compared with triangular or tetrahedral elements, rectangular or cubic elements have some limitation in covering a domain of complicated configuration.

The accuracy of using an element-wise linear shape function or an element-wise quadratic function should be commented. One rectangular domain in a two-dimensional setting is considered as the simplest example. There are two possibilities for discretization of functions in this domain; one possibility is that the domain is decomposed into two triangular elements, and the other possibility is that one rectangular element is used for the domain. Both possibilities use four nodes, and hence result in a matrix equation of the same dimension. However, the use of one rectangular element produces a more accurate solution than the use of two triangle elements, since the element-wise quadratic function expresses the derivative of the target function more smoothly; there is discontinuity in the derivative across the element boundary when the two element-wise linear functions are used. In general, using an element-wise quadratic function is more suitable than using an element-wise linear function.

2.4.2 Isoparametric element

Shape functions which are used for discretization need to be integrated over elements in order to transform a boundary value problem to a matrix equation; components of the matrix $[K]$ shown in Eq. (2.9) are computed by this integration. FEM takes advantage of an isoparametric element to efficiently compute the integration over numerous elements which have various configurations. The isoparametric element makes use of a linear mapping of an actual element to a common element of the simple configuration. For simplicity, the isoparametric element is explained using a rectangular element, ω , which has four nodes, x^α ($\alpha=1,2,3,4$), as an example. The mapped element is a square, $\Omega=\{\mathbf{X} \mid -1 < X_1, X_2 < +1\}$, and mapping is made through $x_i=F_i(\mathbf{X})$ with

$$F_i(\mathbf{X}) = \sum_{\alpha=1}^4 x^\alpha \frac{(X_1 - X_1^\alpha)(X_2 - X_2^\alpha)}{4}, \quad (2.11)$$

where $\mathbf{X}^{1,2,3,4}$ is $(X_1, X_2)=(-1, -1), (+1, -1), (+1, +1)$ and $(-1, +1)$. A function in ω is transformed to a function in Ω through this mapping function F_i . For instance, a quadratic shape function $\phi^\alpha(x)$ is transformed to $\Phi^\alpha(\mathbf{X})$ with $\Phi^\alpha(\mathbf{X})=\phi^\alpha(x)$ and $X_i=F_i^{-1}(x)$. Or, $\Phi^\alpha(\mathbf{X})$ is transformed to $\phi^\alpha(x)$ with $x_i=F_i(\mathbf{X})$. In using the isoparametric element, ϕ^α in ω does not have to be explicitly defined. Instead, ϕ^α is defined as a function which is transformed from a quadratic function Φ^α in Ω , i.e.,

$$\phi^\alpha = \frac{(X_1 - X_1^\alpha)(X_2 - X_2^\alpha)}{4}; \quad (2.12)$$

as is seen, say, Φ^1 takes on the value of 1 at $\mathbf{X}^1$ but vanishes at other three corners of the square. Thus, when a function U_i is defined in Ω , it is transformed to a function u_i in ω through $x_i = F_i(\mathbf{X})$, i.e., $U_i(\mathbf{X}) = \sum_{\alpha} u_i^{\alpha} \Phi^{\alpha}(\mathbf{X})$ is transformed to $u_i(\mathbf{x}) = \sum_{\alpha} u_i^{\alpha} \phi^{\alpha}(\mathbf{x})$. An isoparametric element is defined even for elements of a more complicated¹⁸ configuration. Note that the integration in an actual element is readily transformed to the integration in the corresponding isoparametric elements; for instance, the integration of $\phi_i^{\alpha} \phi_j^{\beta}$ becomes

$$\int_{\omega} \phi_i^{\alpha} \phi_j^{\beta} ds = \int_{\Omega} \left(\frac{\partial F_k^{-1}}{\partial x_i} \frac{\partial \Phi^{\alpha}}{\partial X_k} \right) \left(\frac{\partial F_l^{-1}}{\partial x_j} \frac{\partial \Phi^{\beta}}{\partial X_l} \right) \left| \frac{\partial F_p}{\partial X_q} \right| ds, \quad (2.13)$$

where $\frac{\partial F_p}{\partial X_q}$ is the Jacobian matrix of the mapping function F_i .

2.4.3 Gauss integral

The integration over Ω given by Eq. (2.13) is exactly computed since the integrand is a polynomial and the integration domain is a square. There is a smart method to compute such integration of a polynomial function in a square domain. This method is the Gauss integral, which is usually implemented in FEM. The formulae of the Gauss integral is stated as follows: for a polynomial f of the $2n-1$ -th order in a domain of $\{x | -1 < x < +1\}$, the integration $\int_{-1}^{+1} f(x) dx$ is evaluated as

$$\int_{-1}^{+1} f(x) dx = \sum_{k=1}^n w_k f(q_k), \quad (2.14)$$

where q_k 's are roots of the Legendre polynomial of the n -th order, denoted by P_n , and w_k is a weight given as $w_k = 2(1 - q_k^2) / (n P_{n-1}(q_k))^2$. In Eq. (2.14), the integration of f is evaluated by the values of f at n points, denoted by $\{q_k\}$. It is straightforward to compute the left side of Eq. (2.13) when FEM uses isoparametric elements.

The Gauss integral, Eq. (2.14), is based on the property of the Legendre polynomial. The Legendre polynomial of the n -th order is defined¹⁹ as

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} ((x+1)^n (x-1)^n). \quad (2.15)$$

By definition, for any polynomial Q of less than the n -th order, P_n satisfies²⁰ $\int_{-1}^{+1} P_n Q dx = 0$. Thus, if a function f , which is of the $2n-1$ -th order, is given as $f = Q P_n + R$ with Q and R being polynomials of less than equal the $n-1$ -th order,

¹⁸Even in this case, the transformed shape function ϕ^{α} is still regarded as a polynomial of the same order as Φ^{α} , and satisfies $\phi^{\alpha}(\mathbf{x}^{\beta}) = 1$ for $\alpha = \beta$ or 0 for $\alpha \neq \beta$.

¹⁹Usually, Eq. (2.15) for P_n is called the Rodriguez formulae of the Legendre polynomial.

²⁰This identity is easily proved by using integration by part and noting that the m -th derivative of $(x+1)^n (x-1)^n$ vanishes at $x = \pm 1$ for $m = 0, 1, \dots, n-1$.

then, it follows from the property of P_n that the integration of f is given as

$$\int_{-1}^{+1} f(x) dx = \int_{-1}^{+1} R(x) dx.$$

The remaining task is to determine R for a given Q . This is easily done by making use of the roots of P_n . When data of $\{(q_k, f(q_k))\}$ are given, a polynomial R is expressed as

$$R(x) = \sum_k f(q_k) \frac{P_n(x)}{(x - q_k)P'_n(q_k)};$$

the right side becomes $f(q_k)$ at $x=q_k$. Hence, the integration of R leads to Eq. (2.14) since, by definition, the integration of $\int_{-1}^{+1} P_n(x)/((x - q_k)P'_n(x)) dx$ is evaluated as $w_k = 2(1 - q_k^2)/(nP_{n-1}(q_k))^2$.

2.5 Algorithm Used to Solve A Matrix Equation of FEM

Once the matrix $[K]$ is formed, FEM solves the matrix equation, Eq. (2.9), for an unknown vector $[U]$. By putting subscripts to emphasize components of the matrix and vector, this matrix equation is rewritten as

$$[K_{ij}][U_j] = [F_i] \quad (i, j = 1, 2, \dots, n). \quad (2.16)$$

The dimension of $[K_{ij}]$ is large. Furthermore, there are cases when components of $[K_{ij}]$ are non-linear functions²¹ of components of $[U_i]$, i.e.,

$$[K_{ij}([U])][U_j] = [F_i]. \quad (2.17)$$

Thus, FEM makes use of the following two algorithms:

- 1) an algorithm used to solve a matrix equation of large dimension;
- 2) an algorithm used to solve a non-linear equation.

It should be noted that these algorithms are robust and applicable to other numerical analysis methods; see [Cheung *et al.* (1995)] and [Logan (2000)] for solving a matrix of large dimension, and [Simo and Taylor (1986)] and [de Borst (1993)] for solving a non-linear matrix equation; see also [Pietruszczak and Mroz (1981)], [Ortiz and Popov (1985); Ortiz and Simo (1986)], [Crisfield (1991)], and [Bonet and Wood (1997)] for the non-linear FEM analysis.

The choice of an algorithm used to solve a matrix equation is important for FEM. This is because the dimension of the matrix $[K_{ij}]$ is the total degree-of-freedom (DOF), or the number of unknown nodal displacement components, and the total DOF increases as the square or cubic of the number of discretization in one

²¹There is another formulation of non-linear FEM, which uses nodal forces without using a matrix $[K]$. Actually this formulation is more efficient for non-linear numerical analysis; an example of using this formulation will be explained in Sec. 8.2.

axis for a two- or three-dimensional setting, respectively. It should be emphasized that the inverse is not taken for the matrix $[K_{ij}]$ in numerically solving Eq. (2.16), since computing an inverse matrix requires relatively large computational efforts; the required computation processes increases as the cubic of the target matrix dimension. There are many algorithms which can efficiently solve a matrix equation, and an algorithm used in FEM is called a *solver* in this book. In general, the solver is categorized into two classes, a *direct* solver and an *iterative* solver. As the name suggests, the former directly solves the matrix equation while the latter solves the matrix equation in an iterative manner.

2.5.1 Direct solvers

The most standard direct solver is the *Gauss method*, which consists of the forward elimination and the backward substitution. The matrix equation of Eq. (2.16) is used as an example. The forward elimination transforms this matrix equation to

$$[K_{ij}^{(1)}][U_j] = [F_i^{(1)}],$$

where

$$K_{ij}^{(1)} = \begin{cases} K_{ij} & \text{for } i = 1, \\ K_{ij} - m_{i1}K_{1j} & \text{for } i \neq 1, \end{cases} \quad F_i^{(1)} = \begin{cases} F_i & \text{for } i = 1, \\ F_i - m_{i1}F_1 & \text{for } i \neq 1, \end{cases}$$

with $m_{i1} = K_{i1}/K_{11}$. By repeating this process n times, the matrix is triangularized; n is the dimension of $[K_{ij}]$. The backward substitution then solves the resulting matrix equation from the bottom. This equation is written as

$$[K_{ij}^{(n)}][U_j] = [F_i^{(n)}], \quad (2.18)$$

and U_n , the last component of $[U_i]$, can be calculated from the last row of this matrix equation, as $U_n = F_n^{(n)}/K_{nn}^{(n)}$. When U_n is obtained, U_{n-1} is determined by using the second last row. By repeating this process, i.e.,

$$U_i = \frac{1}{K_{ii}^{(n)}} \left(F_i^{(n)} - \sum_{j=i+1}^n K_{ij}^{(n)} U_j \right), \quad (2.19)$$

all components of $[U_i]$ are calculated.

The Gauss method is based on the *LU decomposition* of matrix. That is, $[K_{ij}]$ is decomposed into the product of two matrices,

$$[K_{ij}] = [L_{ik}][R_{kj}], \quad (2.20)$$

where $[L_{ij}]$ and $[R_{ij}]$ are lower and upper triangular matrices; non-zero components of $[L_{ij}]$ and $[R_{ij}]$ are only for $i \leq j$ and $i \geq j$, respectively. In view of Eq. (2.20), the forward elimination of the Gauss method, Eq. (2.18), corresponds

to $[R_{ij}][U_j] = [L_{ij}]^{-1}[F_j]$, and the backward substitution, Eq. (2.19), corresponds to $[U_i] = [R_{ij}]^{-1}([L_{jk}]^{-1}[F_k])$. Note that $[R_{ij}]$ and $[L_{ij}]^{-1}[F_j]$ are given as

$$[R_{ij}] = [K_{ij}^{(n)}] \quad \text{and} \quad [L_{ij}]^{-1}[F_j] = [F_i^{(n)}],$$

respectively.

The matrix $[K_{ij}]$ of FEM is symmetric and positive-definite, since it corresponds to the total strain energy when the matrix equation is formulated by using the variational problem. The LU decomposition of $[K_{ij}]$, Eq. (2.20), is then simplified as

$$[K_{ij}] = [S_{ij}]^T[S_{kj}], \quad (2.21)$$

where $[S_{ij}]$ is an upper triangular matrix. This decomposition is called the *Cholesky decomposition*. The matrix $[S_{ij}]$ in Eq. (2.21) is easily obtained if the LU decomposition of $[K_{ij}]$ is given. A diagonal matrix $[D_{ij}]$ is determined from $[R_{ij}]$ so that $[R_{ij}]$ is given as $[R_{ij}] = [D_{ik}][R'_{kj}]$ with all diagonal components of $[R'_{ij}]$ being 1. It can be proved that $[L_{ij}]$ is expressed in terms of $[R'_{ij}]$ as $[L_{ij}] = [R'_{ik}]^T[D'_{kj}]$ with $[D'_{ij}]$ being a diagonal matrix. Hence $[K_{ij}]$ is given as $[K_{ij}] = [R'_{ik}]^T[D'_{kl}][D_{lm}][R'_{mj}]$. Since $[D_{ij}]$ and $[D'_{ij}]$ have positive components, it follows from this equation that $[S_{ij}]$ is given as $[S_{ij}] = \sqrt{[D'_{ik}][D_{kl}][R'_{lj}]}$. When $[S_{ij}]$ of Eq. (2.21) is determined, the matrix equation, Eq. (2.16), is transformed into a pair of matrix equations with upper and lower triangular matrices, i.e.,

$$[S_{ij}][U_j] = [V_i] \quad \text{and} \quad [S_{ij}]^T[V_i] = [F_j]$$

Here, $[V_i]$ is a vector which is introduced to reduce calculations; this $[V_i]$ is easily found since $[S_{ij}]^T$ is a lower triangle matrix. Hence, $[U_i]$ is obtained by solving the matrix equation of the upper triangle matrix of $[S_{ij}]$ with this $[V_i]$.

2.5.2 Iterative solvers

An iterative solver is an algorithm that constructs a series of vectors the limit of which is the solution of the target matrix equation. Now subscript is omitted from the vector and matrix to simplify the expression. For the matrix equation of Eq. (2.16), the series is denoted by $\{[U^{(m)}] \mid m=0, 1, \dots\}$, and this series is converged if the difference of the m -th and $m+1$ -th vectors, $||[U^{(m+1)}] - [U^{(m)}]||$, becomes sufficiently small; $||\cdot||$ is, say, the square norm,

$$||\cdot|| = \sqrt{[\cdot]^T[\cdot]}.$$

For a matrix equation of larger dimension, an iterative solver often finds an accurate approximate solution more efficiently than a direct solver, if a suitable convergence criterion, which is related to the numerical precision, is chosen and if a suitable initial vector, $[U^{(0)}]$, is given.

The *Jacobi method* is a standard iterative solver. It decomposes the matrix $[K]$ into $[D] - [L] - [R]$, where $[D]$ consists of diagonal components of $[K]$, and $[L]$ and $[R]$ are the lower and upper triangularization of $[K] - [D]$. The matrix equation, Eq. (2.16), is then rewritten as

$$[U] = [D]^{-1} \left([F] + ([L] + [R])[U] \right).$$

Setting a suitable initial vector, $[U^{(0)}]$, the Jacobi method constructs the series as

$$[U^{(m+1)}] = [D]^{-1} \left([F] + ([L] + [R])[U^{(m)}] \right). \quad (2.22)$$

As is seen, $[U^{(m+1)}]$ is calculated just by multiplying $[D]^{-1}$ to a known vector $[F] + ([L] + [R])[U^{(m)}]$.

The *Gauss-Zeidel method* modifies the Jacobi method, substituting an updated vector into the right side of Eq. (2.22). Indeed, it uses

$$[U^{(m+1)}] = [D]^{-1} \left([F] + [L][U^{(m+1)}] + [R][U^{(m)}] \right),$$

which leads to

$$[U^{(m+1)}] = ([D] + [L])^{-1} [F] + ([D] + [L])^{-1} [R][U^{(m)}]. \quad (2.23)$$

The convergence of this series is fast if the matrix $[K]$ is diagonally dominant, i.e., $|K_{ii}| > \sum_{j \neq i} |K_{ij}|$ for $i=1, 2, \dots, n$. The computation of $([D] + [L])^{-1}$, however, is not trivial when the dimension of $[K]$ is large.

The *successive over-relaxation method* (SOR) applies acceleration²² to the Gauss-Zeidel method. Acceleration can lead to faster convergence since the solution of Eq. (2.16) exists and the series of vectors is convergent. Besides vector series $\{[U^{(m)}]\}$, SOR introduces another vector series, $\{[V^{(m)}]\}$, and seeks to obtain the solution. The recursive formulas of these two series are given as follows:

$$\begin{cases} [V^{(m+1)}] = [D]^{-1} \left([F] + [L][U^{(m+1)}] + [R][U^{(m)}] \right), \\ [U^{(m+1)}] = [U^{(m)}] + \omega ([V^{(m+1)}] - [U^{(m)}]). \end{cases} \quad (2.24)$$

Here, ω is called an acceleration parameter; Eq. (2.24) coincides with Eq. (2.23) if ω is set as $\omega=1$.

There is an iterative solver²³ of a slightly different type, called the *conjugate gradient method* (CG). This method regards the matrix equation of Eq. (2.16) as the equation which is used to solve a non-linear optimization problem, which seeks an unknown vector $[U]$ that minimizes²⁴ the following function:

$$f([U]) = \frac{1}{2} [U]^T [K] [U] - [F]^T [U]; \quad (2.25)$$

²²Acceleration is a general terminology used for a numerical algorithm.

²³As is seen, CG is different from the previous iterative methods; see, for instance, [Vorst (1992)] for a more detailed explanation of the nature of CG.

²⁴As is seen, f coincides with the functional for the total potential energy when discretized displacement is used.

this f is a non-linear function of $[U]$. While CG is one of the most robust algorithms used to solve a non-linear equation, it is also shown that CG is the most efficient algorithm which minimizes f of Eq. (2.25) among a certain class of iterative solvers. The basic algorithm of CG is the calculation of a vector series, $\{[U^{(m)}]\}$, by making use of two auxiliary vector series, $\{[R^{(m)}]\}$ and $\{[P^{(m)}]\}$; the recursive formulae for these series are

$$\begin{cases} [U^{(m+1)}] = [U^{(m)}] + \alpha^{(m)} [P^{(m)}], \\ [R^{(m+1)}] = [R^{(m)}] - \alpha^{(m)} [K] [P^{(m)}], \\ [P^{(m+1)}] = [R^{(m+1)}] - \beta^{(m)} [P^{(m)}], \end{cases} \quad (2.26)$$

where

$$\alpha^{(m)} = \frac{[R^{(m)}]^T [P^{(m)}]}{[P^{(m)}]^T [K] [P^{(m)}]}, \quad \beta^{(m)} = \frac{[R^{(m+1)}]^T [K] [P^{(m)}]}{[P^{(m)}]^T [K] [P^{(m)}]},$$

the initial vectors $[U^{(0)}]$ and $[P^{(0)}]$ are suitably chosen and $[R^{(0)}]$ is determined as $[R^{(0)}] = [F] - [K][U^{(0)}]$. In FEM, however, the matrix $[K]$ is more or less sparse since non-zero components are only located near the diagonal terms. Hence, CG is not usually applied to FEM; recently CG has been applied when huge discretization²⁵ is made for the three-dimensional setting.

2.5.3 Algorithms used to solve a non-linear equation

As mentioned above, the matrix equation of FEM becomes non-linear when the matrix $[K]$ is a function of an unknown vector $[U]$; see Eq. (2.17). Such non-linearity comes from the material properties or from finite or large deformation; these non-linearities are called material non-linearity and kinematic non-linearity, respectively. Solving a non-linear equation is much more laborious than solving a linear equation, and hence a variety of algorithms have been proposed to solve non-linear equations. Some algorithms are robust in the sense that they are applicable to a wider class of non-linear equations. Also, there are algorithms which are tuned in order to solve a particular class of non-linear equations. The nature of non-linearity should be clarified in choosing an algorithm.

The *Newton-Raphson method* is a robust method of solving a general class of non-linear equations. The key to this method is the successive iteration and the linearization. That is, the Newton-Raphson method finds a series of approximate solutions to linear equations which are obtained by expanding the non-linear equations at the previous approximate solution and by reducing the non-linear equations to a linear equation.

For simplicity, a scalar equation, $R(U)=0$, is studied, where R is a non-linear function of a scalar variable, U . If R admits the Taylor expansion at U , the value

²⁵Pre-processing of the matrix $[K]$ is often used in applying CG to FEM, in order to make the convergence of $\{[U^{(m)}]\}$ faster.

of R at a small perturbation from U , denoted by dU , is

$$R(U + dU) = R(U) + R'(U) dU + \mathcal{O}(dU^2).$$

Hence, for $R(U + dU)$ to vanish, the value of dU should be $dU = -R(U)/R'(U)$. In this manner, the Newton-Raphson method constructs the following series $\{U^{(m)}\}$:

$$U^{(m+1)} = U^{(m)} + \frac{R(U^{(m)})}{R'(U^{(m)})}. \quad (2.27)$$

As is seen, if the series $\{U^{(m)}\}$ converges, it satisfies $\lim_{m \rightarrow \infty} R(U^{(m)}) = 0$, and hence $\lim_{m \rightarrow \infty} U^{(m)}$ is the solution of $R(U) = 0$. A convergence criterion is either $|R(U^{(m)})| \ll 1$ or $|U^{(m+1)} - U^{(m)}|/|U^{(m)}| \ll 1$.

It is straightforward to extend this method to a case of vector-valued equations for a vector variable, i.e., $[R([U])] = [0]$ or $[R_i(U_j)] = 0$ with subscripts emphasizing that $[R_i]$ and $[U_i]$ are vectors. The series of $\{[U_i^{(m)}]\}$ is constructed as

$$[U_i^{(m+1)}] = [U_i^{(m)}] + \left[\frac{\partial R_i}{\partial U_j}([U^{(m)})] \right]^{-1} [U_j^{(m)}]. \quad (2.28)$$

The convergence criterion is either the norm of $[R_i]$ being sufficiently small or the norm of $[U_i^{(m+1)}] - [U_i^{(m)}]$ being sufficiently small.

As an extension of the Newton-Raphson method, there are two numerical algorithms which solve non-linear equations, namely, the *modified Newton-Raphson method* and the *arc length method*. The modified Newton-Raphson method is aimed at efficiently computing the vector series by reducing the calculation required for computing derivatives. The arc length method finds a suitable solution which is located within a certain *range* from the current solution. Both methods are often implemented in FEM.

In the New-Raphson method, calculating derivative R' or $\partial R_i / \partial U_j$ is laborious, in particular, for a case when R is a complicated function of U and R' is computed numerically or a case when the dimension of R_i and U_i is large. The modified Newton-Raphson method is developed to tackle this problem. For a case of a scalar non-linear equation, this method seeks a more suitable increment instead of $dU = R'(U)/K(U)$, just by updating error of $R(U)$. That is,

$$U^{(m+1)} = U^{(m)} + \frac{R(U^{(m)})}{R'(U^{(m^*)})}, \quad (2.29)$$

where m^* is an integer smaller than m ; R' has been evaluated at the m^* -th step which is prior to the current m -th step. In this manner, $U^{(m)}$ is found so that the target equation, $R=0$, is satisfied, without calculating R' every time when $U^{(m)}$ is updated; R' is recalculated when $|U^{(m+1)} - U^{(m^*)}|$ becomes large or after $m - m^*$ becomes large.

The arc length method is powerful for solving a non-linear equation which has varying parameters in it; a typical example of such a non-linear equation is an equation with terms for an external load which may decrease depending on the

deformation condition. As an example, a scalar non-linear equation $R(U) - F = 0$ with F being a parameter for the external load, is studied. The arc length method introduces a series of U and F , denoted by $\{(U^{(m)}, F^{(m)})\}$, and sets the following restriction for the series:

$$\alpha_1(U^{(m+1)} - U^{(m)})^2 + \alpha_2(F^{(m+1)} - F^{(m)})^2 = r^2, \quad (2.30)$$

where r is a fixed radius of an arc, and α_1 and α_2 are suitable constants. As is seen, the distance from $(U^{(m)}, F^{(m)})$ to $(U^{(m+1)}, F^{(m+1)})$ is fixed, and hence this method is called the arc length method. Expanding $R - F = 0$ at $U^{(m)}$, the arc length method uses the following equation for the next approximate solution and parameter:

$$R(U^{(m)}) + R'(U^{(m)})(U^{(m+1)} - U^{(m)}) - F^{(m+1)} = 0. \quad (2.31)$$

The series of $\{(U^{(m)}, F^{(m)})\}$ are thus constructed from Eqs. (2.30) and (2.31). The arc length method can determine a solution even when F starts to decrease; this corresponds to the situation when the load decreases even though the deformation proceeds.

Chapter 3

Stochastic Modeling

Modeling is essentially important for computational mechanics. Poor modeling results in poor reproduction or prediction, even though a numerical analysis method with advanced numerical techniques and sophisticated algorithms is employed. For the computation of earthquake phenomena, modeling underground structures thus plays a key role. It is certainly true that due to the recent progress of seismology and solid geophysics, more quantitative reproductions or predictions of earthquake phenomena are possible by using a more detailed model. However, constructing a detailed model for underground structures is difficult since it is not an easy task to accurately measure the underground structures, i.e., geological layers of a few kilometer thickness in the resolution of 100[m] or surface deposit layers of thickness less than 100[m] in the resolution of 0.1[m]. In this sense, the difficulty of underground structure modeling is an obstacle in quantitatively studying earthquake phenomena by means of numerical simulation.

An alternative to ordinary deterministic modeling is the construction of a stochastic model. A stochastic model¹ is a model which has mean, variance or covariance for parameters of the material properties and the configuration. Variance or covariance accounts for the uncertainty. Analyzing such a stochastic model is more laborious than a deterministic model which has material properties and configuration in a deterministic manner. This is because the behavior of the stochastic model becomes stochastic as well. The key issue in analyzing the stochastic model is the fact that

the variability in behavior is not necessarily proportional to the variability of the stochastic model.

In general, the variability in behavior will be magnified if non-linearity is included in the stochastic model. In dynamic state, the variability in velocity or acceleration

¹Probabilistic methods which are related to reliability and natural forces have been extensively studied in structural mechanics and engineering; see, for instance, [Shinozuka and Yao (1981)] and [Augusti *et al.* (1984)]; see also [Zendagui *et al.* (1999)] and [Sigbjornsson and Ambraseys (2003)] for stochastic problems in geotechnical engineering. These studies are on the same line as stochastic modeling. [Goff and Jordan (1988)] provides the foundation of stochastic modeling in a more general setting.

is drastically changed since the effects of the spatial material heterogeneities on frequency components are different. Monte-Carlo simulation is usually employed to analyze a stochastic model by generating numerous test samples as a realization of the stochastic model. However, Monte-Carlo simulation is not suitable if simulation of one test sample requires huge numerical computation.

This chapter briefly presents the theory and methods to analyze a stochastic model. The theory is an extension of a theory for a body with spatial distribution of heterogeneities, and takes into consideration the probabilistic distribution of heterogeneities. The key issue of the theory is to pose a *stochastic variational problem* for a random function which is defined in a physical field and a probabilistic space. Two analysis methods of solving this stochastic variational problem are then explained. The first method is used for evaluating the mean behavior of the stochastic model, and the second method is for fully computing the variable behavior of the stochastic model.

Some knowledge of probability theory is needed to understand the theory and methods of analyzing a stochastic model. The level of understanding, however, does not have to be high, and it will be sufficient if the basics of random function in probability space are grasped. [Adler (1981)] is recommended as a general textbook to understand the basics² of probability theory.

3.1 Formulation of A Stochastic Variational Problem

For simplicity, a two-dimensional anti-plane shear problem is considered as an example which explains the theory that is constructed to analyze a stochastic model; the formulation presented here is applicable to a more complicated setting or even to problems of structural mechanics. An isotropic but heterogeneous elastic body, B , is the object of study. Out-of-plane displacement and elastic modulus of B are denoted by u and c ($u=u_3$ and $2c=c_{1313}=c_{2323}$). When, say, displacement u^0 is prescribed on the boundary ∂B , the following boundary value problem is posed for B :

$$\begin{cases} (c(\mathbf{x})u_{,i}(\mathbf{x}))_{,i} = 0 & \text{in } B, \\ u(\mathbf{x}) = u^0(\mathbf{x}) & \text{on } \partial B. \end{cases} \quad (3.1)$$

This boundary value problem is transformed to a variational problem, which will be more suitable for the analysis of a stochastic model, as will be shown later. The variational problem uses the following functional:

$$J(u, c) = \int_B \frac{1}{2} c(\mathbf{x}) u_{,i}(\mathbf{x}) u_{,i}(\mathbf{x}) d\mathbf{s}_{\mathbf{x}}, \quad (3.2)$$

²There are a variety of textbooks written about probability theory. [Liu *et al.* (1987)], [Liu and Kiureghian (1991)] and [Li and Kiureghian (1993)] are recommended as a textbook which focuses on the application of probability theory to engineering problems.

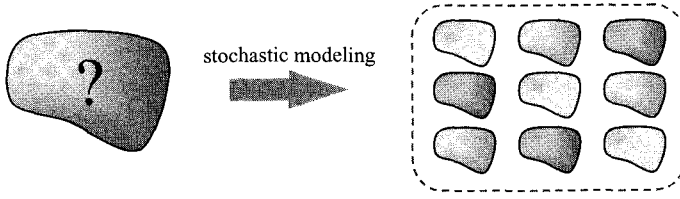


Fig. 3.1 A stochastic model for a body with uncertain material property.

for u satisfying $u=u^0$ on ∂B . The solution of Eq. (3.1), denoted by u^{exact} , minimizes J and the minimum value coincides with the total strain energy stored in the body, denoted by $\mathcal{E}$.

Now, it is supposed that the heterogeneity of B is uncertain and a function c cannot be determined. The uncertainty of B is described using stochastic c , i.e., the value of c at each $\mathbf{x}$ is not given deterministically, but the mean, variance or correlation is provided; see Fig. 3.1. To be specific, denoting a probabilistic event by ω , this c is regarded as a random function in $(\Omega, \mathcal{F}, P)$, where Ω is a set of all ω 's, $\mathcal{F}$ is Borel set³ of Ω and P is the probabilistic measure. Argument ω is put to emphasize that c is a random function, i.e., $c(\omega)$ or $c(\mathbf{x}, \omega)$. The displacement that satisfies Eq. (3.1) for such a stochastic $c(\omega)$ becomes a random function as well, i.e., u or $u(\mathbf{x})$ is replaced by $u(\omega)$ or $u(\mathbf{x}, \omega)$, respectively.

When $c(\omega)$ and $u(\omega)$ are random functions, Eq. (3.1) still holds for each realization, ω . That is, the following boundary value problem holds:

$$\begin{cases} (c(\mathbf{x}, \omega)u, i(\mathbf{x}, \omega)), i = 0 & \mathbf{x} \text{ in } B, \\ u(\mathbf{x}, \omega) = u^0(\mathbf{x}) & \mathbf{x} \text{ on } \partial B. \end{cases} \quad (3.3)$$

This is a stochastic boundary value problem. By definition, $u(\omega)$ is a behavior of B with one realization $c(\omega)$. The object of analysis is not a particular behavior but stochastic properties of all possible behaviors; the stochastic properties include the mean behavior and the variability in behavior. For instance, the mean of displacement at one point $\mathbf{x}$ needs to be estimated, i.e.,

$$\langle u(\mathbf{x}) \rangle = \int u(\mathbf{x}, \omega) P(d\omega). \quad (3.4)$$

Here, $\langle (\cdot) \rangle$ stands for the mean of $(\cdot)$, i.e.,

$$\langle (\cdot) \rangle = \int_{\Omega} (\cdot)(\omega) P(d\omega).$$

In Monte-Carlo simulation, the mean of displacement is obtained by generating numerous test samples of $c(\omega)$, solving the resulting boundary value problem of $u(\omega)$ for each sample, and taking the average of the values of $u(\omega)$ at $\mathbf{x}$. The probabilistic measure P is used to generate the set of these test samples; in this manner, the right

³Naively speaking, the Borel set is the smallest set which consists of subsets of Ω .

side of Eq. (3.4) which includes P in it does not have to be explicitly integrated. The computation cost of this Monte-Carlo simulation is expensive if solving Eq. (3.3) for each realization $c(\omega)$ is laborious. It should be also noted that even if the stochastic distribution of $c(\omega)$ at one point $\mathbf{x}$ is simple, such as a Gaussian distribution, the stochastic distribution $u(\omega)$ at $\mathbf{x}$ does not have to be simple; $u(\omega)$ is surely influenced by the elasticity near $\mathbf{x}$ and hence the stochastic distribution of $u(\omega)$ depends on the joint probability of $c(\omega)$ at $\mathbf{x}$ and the elasticity of the other points. In this case, the number of test samples which are used to compute Eq. (3.4) is increased, and the computational cost of carrying out Monte-Carlo simulation becomes more expensive.

An alternative to Monte-Carlo simulation needs to be developed in order to solve the stochastic boundary value problem. The difficulty in solving the stochastic boundary value problem lies in the fact that *conditional probability*⁴ must be accounted for. When one realization $c(\omega)$ is given, the corresponding $u(\omega)$ is obtained by solving Eq. (3.3), and hence this $u(\omega)$ is the solution under the condition that one particular $c(\omega)$ is realized. Actually, the conditional probability of $u(\omega)$ related to $c(\omega)$ does not have to be explicitly estimated, even if it were possible, since, for a given $c(\omega)$, the stochastic boundary value problem is only to find the random function, $u(\omega)$. Therefore, an alternative to Monte-Carlo simulation should not require consideration of the conditional probability.

Instead of directly solving Eq. (3.3), a weak form of the stochastic boundary value problem is considered. The weak form is expressed as the integration of Eq. (3.3) weighted by an arbitrary random function, $\delta u(\omega)$, i.e.,

$$\int_B \int_{\Omega} \delta u(\mathbf{x}, \omega) (c(\mathbf{x}, \omega) u_{,i}(\mathbf{x}, \omega))_{,i} ds_{\mathbf{x}} P(d\omega) = 0,$$

where $\delta u(\omega)$ satisfies $\delta u(\omega) = 0$ on ∂B for all ω 's. The probabilistic integration must be made since $u(\omega)$ is a random function. Applying integration by part, the weak form can be transformed to the following stochastic functional for $u(\omega)$ satisfying $u(\omega) = u^0$ on ∂B for all ω 's:

$$J^{\Omega}(u(\omega), c(\omega)) = \int_{B \times \Omega} \frac{1}{2} c(\mathbf{x}, \omega) u_{,i}(\mathbf{x}, \omega) u_{,i}(\mathbf{x}, \omega) ds_{\mathbf{x}} P(d\omega). \quad (3.5)$$

This J^{Ω} gives a stochastic variational problem for a stochastic model of B . Note that J^{Ω} is expressed in terms of J of Eq. (3.2) as

$$J^{\Omega}(u(\omega), c(\omega)) = \langle J(u(\omega), c(\omega)) \rangle, \quad (3.6)$$

i.e., the probabilistic mean of J . It immediately follows from $c(\omega) > 0$ that the random function that minimizes⁵ J^{Ω} is the solution of the stochastic boundary

⁴In this sense, Monte-Carlo simulation is a method of solving Eq. (3.3) by fully accounting for the conditional probability, although it is computationally expensive.

⁵For each ω , the realization of the exact solution $u^{\text{exact}}(\omega)$ minimizes the surface integration taken over the body B , and hence the integration of that over the probabilistic space Ω becomes the minimum. This is easily seen if J^{Ω} is rewritten as $J^{\Omega}(u(\omega), c(\omega)) = \langle J(u(\omega), c(\omega)) \rangle$; since $c > 0$, $J(u^{\text{exact}}(\omega), c(\omega))$ is the minimum of $J(u(\omega), c(\omega))$.

value problem, Eq. (3.3), and that a random function which makes J^Ω smaller is a better approximate solution. In this stochastic variational problem, there is no need to consider the conditional probability for $c(\omega)$ and $u(\omega)$; the exact solution, $u^{\text{exact}}(\omega)$, that minimizes J^Ω automatically accounts for the conditional probability since $u^{\text{exact}}(\omega)$ for each ω is the solution of Eq. (3.3) when the corresponding $c(\omega)$ is realized. A better approximate solution of $u(\omega)$, which gives the conditional probability more accurately, is automatically obtained just by minimizing J^Ω . This is the advantage of solving Eq. (3.5) instead of Eq. (3.3) in order to evaluate $u(\omega)$.

3.2 Analysis Methods of A Stochastic Variational Problem

While more detailed explanation will be presented in Chapters 5 and 8, this section briefly presents the two analysis methods of solving the stochastic variational problem of J^Ω given by Eq. (3.5). These methods are called the bounding medium analysis and the spectral method.

3.2.1 Bounding medium analysis

The first object of analysis is the mean or the expectation of stochastic displacement, which corresponds to the mean behavior of the stochastic model. It should be emphasized that the mean of $u(\omega)$ given as Eq. (3.4) is not the displacement when the mean of $c(\omega)$ is used in Eq. (3.1). Computing the mean behavior is difficult since the joint probability of $u(\omega)$ and $c(\omega)$ which satisfy Eq. (3.1) needs to be evaluated. Instead of finding the mean behavior directly, certain displacement fields which bound the mean behavior are sought by taking advantage of the *bounding medium theory*. This theory determines two fictitious but deterministic media for the stochastic model, which provide such bounding displacement fields; see Fig. 3.2.

The functional J of Eq. (3.2) is used to construct one bounding medium. For one realization $c(\omega)$, due to $c(\omega) > 0$, the following inequality holds for the strain energy $\mathcal{E}$:

$$\mathcal{E}(\omega) = J(u^{\text{exact}}(\omega), c(\omega)) \leq J(u(\omega), c(\omega)).$$

Here, $u^{\text{exact}}(\omega)$ is the exact solution of the boundary value problem of $c(\omega)$, and argument ω of $\mathcal{E}$ emphasizes that $\mathcal{E}$ is a random variable. The inequality of $\mathcal{E}(\omega) \leq J(u(\omega), c(\omega))$ holds even when deterministic u which satisfies $u = u^0$ on ∂B is used instead of $u(\omega)$. As shown in Eq. (3.6), the stochastic functional J^Ω is defined as the stochastic mean of J , and hence the following inequality for any arbitrary but deterministic u can be derived:

$$\langle \mathcal{E} \rangle = \langle J(u(\omega), c(\omega)) \rangle = J^\Omega(u^{\text{exact}}(\omega), c(\omega)) \leq J^\Omega(u, c(\omega)).$$

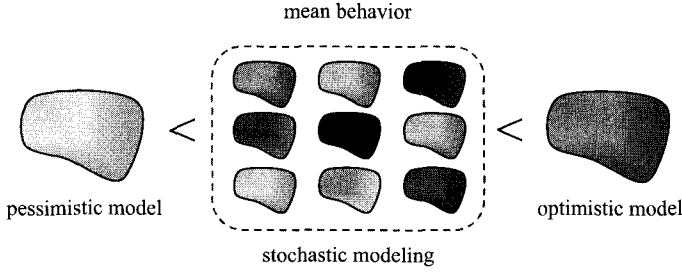


Fig. 3.2 Two bounding media for a stochastic model. The bounding media provide bounds for the mean of the stochastic model behavior.

By definition, J^Ω for the deterministic u is computed as $J^\Omega(u, c(\omega)) = J(u, \langle c \rangle)$. The integration in the stochastic space applies $c(\omega)$ only since u is deterministic; see Eqs. (3.2) and (3.5). Thus, the following inequality⁶ for $\langle \mathcal{E} \rangle$ is derived:

$$\langle \mathcal{E} \rangle \leq J(u, \langle c \rangle) \quad (3.7)$$

for u satisfying $u = u^\circ$ on ∂B . The sharpest bound for $\langle \mathcal{E} \rangle$ is obtained by solving of the variational problem of $J(u, \langle c \rangle)$, and the solution of this problem is displacement of a deterministic body which has the mean elasticity, denoted by $c^+ = \langle c \rangle$. Thus, in the sense that it gives an upper bound for the mean total strain energy, this fictitious but deterministic body is a bounding medium which overestimates the mean behavior of the stochastic body. This bounding medium is denoted by B^+ . It should be emphasized that the mean elastic modulus does not give the mean behavior; as shown in Eq. (3.7), the solution of B^+ with $c^+ = \langle c \rangle$ overestimates the mean total strain energy $\langle \mathcal{E} \rangle$.

Another bounding medium is constructed considering complementary strain energy. This energy is related to a function for stress σ_i ($\sigma_i = \sigma_{3i}$),

$$\begin{aligned} I(\sigma, 1/c) = & - \int_B \frac{1}{2c(\mathbf{x})} \sigma_i(\mathbf{x}) \sigma_i(\mathbf{x}) + \lambda(\mathbf{x}) \sigma_{i,i}(\mathbf{x}) ds_{\mathbf{x}} \\ & + \int_{\partial B} n_i(\mathbf{x}) \sigma_i(\mathbf{x}) u^\circ(\mathbf{x}) d\ell_{\mathbf{x}}, \end{aligned} \quad (3.8)$$

where λ is a Lagrange multiplier which enforces the equilibrium to stress, $\sigma_{i,i} = 0$, and n_i is the outer unit normal of the boundary ∂B . The integrand of the first integral, $\frac{1}{2c} \sigma_i \sigma_i$, is a complementary strain energy density, and the integral of this density is complementary strain energy. The first variation of I is

$$\delta I = \int_B \delta \sigma_i (\sigma_i / c - \lambda_{,i}) ds_{\mathbf{x}} + \int_{\partial B} n_i \delta \sigma_i (\lambda - u^\circ) d\ell_{\mathbf{x}}.$$

Hence, it is required that strain given by σ_i / c is⁷ compatible, and λ coincides with

⁶The spatial integration and the probabilistic integration are commuted in deriving Eq. (3.7).

⁷Compatibility of strain means that strain must be associated with displacement. In the present case, u generates strain $[\epsilon_1, \epsilon_2]^T = [u_{,1}, u_{,2}]^T$ ($\epsilon_i = 2\epsilon_{3i}$). For $[\epsilon_1, \epsilon_2]^T$ to be related to one function

the exact displacement u^{exact} . Thus, the maximum value of I coincides with $\mathcal{E}$. In the same manner as shown in the above, the following inequality is derived for $\langle \mathcal{E} \rangle$ with the aid of $I^\Omega(\sigma(\omega), 1/c(\omega)) = I(\sigma(\omega), 1/c(\omega))$ and $I^\Omega(\sigma(\omega), 1/c(\omega)) \leq \langle \mathcal{E} \rangle$:

$$\langle \mathcal{E} \rangle \geq I(\sigma, \langle 1/c \rangle); \quad (3.9)$$

the left term is derived from $I^\Omega(\sigma, 1/c(\omega)) = I(\sigma, \langle 1/c \rangle)$ with deterministic σ_i . The sharpest lower bound for $\langle \mathcal{E} \rangle$ is obtained using the stress that minimizes $I(\sigma, \langle 1/c \rangle)$. Thus, Eq. (3.9) shows that a fictitious but deterministic body with elasticity $c^- = 1/\langle 1/c \rangle$ is another bounding medium, denoted by B^- , in the sense that the mean total strain energy is underestimated.

In summary, the bounding medium analysis of a stochastic model is an analysis method which uses the two bounding media in order to estimate the mean behavior, and the bounding media $B^\pm$ are constructed as a deterministic body with elasticity $c^+ = \langle c \rangle$ or $1/c^- = \langle 1/c \rangle$. The difference in the bounding medium behavior corresponds to the uncertainty of the stochastic model, i.e., as the degree of the uncertainty becomes larger, the two bounding media behave more differently. It should be mentioned that the bounding medium analysis is an extension of analyzing heterogeneous materials; the elastic moduli of the bounding media, $c^+ = \langle c \rangle$ and $1/c^- = \langle 1/c \rangle$, correspond to the Voigt and Ruess bounds for the effective moduli if the probabilistic mean is replaced by the volume average. Sharper bounding media can be constructed if other analysis methods of the heterogeneous materials, such as Hashin-Shtrikman variational principle, are applied; see Sec. 8.1 for a detailed explanation of the principle.

3.2.2 Spectral method

The spectral method of Ghanem and Spanos is applicable to solve a variational problem of J^Ω ; see [Ghanem and Spanos (1991a)]; see also [Ghanem and Spanos (1990); Spanos and Ghanem (1989)]. The advantage of this method is that a random function is expanded in a probabilistic space, just like the Fourier series expansion; see [Loeve (1977)]. Unlike the Fourier series expansion, however, the expansion in the probabilistic space is abstract and does not have physical values, except that probabilistic moments such as mean and variance are computable; computing the probabilistic moments is actually sufficient to solve a stochastic model problem. The spectral method is formulated by using⁸ the stochastic functional. For simplicity, it is assumed that $c(\omega)$ is a Gaussian random distribution and the covariance function, $R(\mathbf{x}, \mathbf{y}) = \langle (c(\mathbf{x}, \omega) - \langle c \rangle)(c(\mathbf{y}, \omega) - \langle c \rangle) \rangle$, is given where $\langle c \rangle$ is the mean of $c(\omega)$. It is

⁸ u , they must satisfy $\epsilon_{1,2} = \epsilon_{2,1}$. This is the compatibility condition.

⁸The present formulation of the spectral method is slightly different from the original formulation; see [Ghanem and Spanos (1991a)].

shown that $c(\omega)$ admits the following *Karhunen-Loeve* (KL) expansion:

$$c(\mathbf{x}, \omega) = \sum_{n=0} \lambda^n \phi^n(\mathbf{x}) \xi^n(\omega), \quad (3.10)$$

where $\{(\lambda^n)^2, \phi^n\}$ are the eigen-values and eigen-functions of R , i.e.,

$$(\lambda^n)^2 \phi^n(\mathbf{x}) = \int_B R(\mathbf{x}, \mathbf{y}) \phi^n(\mathbf{y}) d\mathbf{s}_{\mathbf{y}},$$

and $\xi^n(\omega)$ is a Gauss distribution which is formally expressed in terms of $c(\omega)$ as

$$\xi^n(\omega) = \frac{1}{\lambda^n} \int_B c(\mathbf{x}, \omega) \phi^n(\mathbf{x}) d\mathbf{s}_{\mathbf{x}}.$$

In Eq. (3.10), ϕ^0 is uniform in B and ξ^0 is not random variable ($\xi^0=1$). By definition, the mean of $\xi^n(\omega)$ for $n=1, 2, \dots$ is zero, and they satisfy

$$\langle \xi^n \xi^m \rangle = \begin{cases} 1 & \text{for } n = m, \\ 0 & \text{for } n \neq m. \end{cases}$$

Thus, $\{\xi^n\}$ form an orthonormal complete system of Gaussian distributions in Ω .

Even when $c(\omega)$ admits the KL expansion, $u(\omega)$ does not; this is because the stochastic behavior of $u(\omega)$ is more complicated and is not fully expressed in terms of the Gaussian random functions which are used to expand $c(\omega)$. The bases for non-Gaussian random functions in Ω are constructed⁹ by using the polynomial chaos, or the multi-dimensional Hermit polynomials, of $\{\xi^n(\omega)\}$. That is, for a k -th dimensional vector of $(\xi^{m_1}, \xi^{m_2}, \dots, \xi^{m_k})$, a random function is defined as a polynomial of $\{\xi^{m_p}(\omega)\}$,

$$\Gamma^k(\omega) = (-1)^k \frac{1}{\gamma^k(\{\xi^{m_p}\})} \frac{\partial^k \gamma^k(\{\xi^{m_p}\})}{\partial \xi^{m_1} \dots \partial \xi^{m_k}},$$

with $\gamma^k(\omega) = \exp(-\frac{1}{2} \sum_p (\xi^{m_p})^2)$. A group of $\Gamma^k(\omega)$'s which are computed by using various $\xi^n(\omega)$'s with various order k 's form the bases for non-Gaussian distributions. This group is denoted by $\{\Psi^m\}$; for instance, Ψ^0 is deterministic ($\Psi^0=1$) and $\Psi^n(\omega)$ coincides with $\gamma^1(\xi^n) = \xi^n(\omega)$. Thus, the random function $u(\omega)$ is expanded in the following *polynomial chaos* (PC) expansion:

$$u(\mathbf{x}, \omega) = \sum_{m=0} u^m(\mathbf{x}) \Psi^m(\omega). \quad (3.11)$$

The number of Ψ^m 's is

$$M = 1 + \sum_{n=1}^K \frac{1}{n!} \Pi_{p=0}^{n-1} (N + p),$$

⁹As a reference, [Cameron and Martin (1947)] is recommended for interested readers; see also [Zhang and Ellingwood (1994)].

when the number of $\{\xi^n(\omega)\}$ is N and the K -th order polynomials is taken. Note that $\{\Psi^m\}$ are orthonormal bases for non-Gaussian distribution in Ω .

As is seen, the method of separation of variable is applied to the two random functions, $c(\omega)$ and $u(\omega)$. By substituting Eqs. (3.10) and (3.11) into J^Ω of Eq. (3.5), the following functional for the PC expansion coefficients, $\{u^m\}$, which are non-stochastic functions:

$$\bar{J}^\Omega(\{u^m\}) = \sum_{m,m'} \int_B \frac{1}{2} c^{mm'}(\mathbf{x}) u_{,i}^m(\mathbf{x}) u_{,i}^{m'}(\mathbf{x}) ds_{\mathbf{x}}, \quad (3.12)$$

where

$$c^{mm'}(\mathbf{x}) = \sum_n \lambda^n \phi^n(\mathbf{x}) \langle \xi^n \Psi^m \Psi^{m'} \rangle; \quad (3.13)$$

the mean of $\xi^n(\omega) \Psi^m(\omega) \Psi^{m'}(\omega)$ is computable since $\Psi^m(\omega)$'s are given by computable $\xi^n(\omega)$'s. The boundary condition of u^m 's is readily derived from $u(\omega) = u^o$; the orthonormality of $\{\Psi^m(\omega)\}$ leads to $u^m = u^o \langle \Psi^m \rangle$ on ∂B .

All terms in Eq. (3.12) are computable spatial functions. Thus, the variational problem¹⁰ of $\bar{J}^\Omega$ is numerically solved by spatially discretizing $\{c^{mm'}\}$ and $\{u^m\}$. This is *spectral* stochastic finite element method (SSFEM). When $\{u^m\}$ are obtained, the joint probability between $c(\omega)$ and $u(\omega)$ can be computed since they are expanded in $\{\xi^n(\omega)\}$ and $\{\Psi^m(\omega)\}$. It should be mentioned that SSFEM is different from an ordinary stochastic finite element method (SFEM) which takes perturbation for stochastic parameters assuming that they are smaller than deterministic parts; see [Hisada and Nakagiri (1981)], [Yamazaki and Shinozuka (1988)] and [Der Kiureghian and Ke (1988)]. The comparison of SSFEM with the ordinary SFEM is presented in Fig. 3.3.

In closing this subsection, it should be emphasized that the bounding medium analysis and the spectral method are developed to solve the stochastic variational problem of J^Ω of Eq. (3.5). The bounding medium analysis constructs two bounding media substituting a deterministic u into J^Ω , and obtain their displacement which provides bounds for the mean behavior of the stochastic model. The spectral method solves the stochastic variational problem by substituting an expansion of random functions, and obtains the random displacement of the stochastic model. Therefore, these methods are summarized as follows:

- 1) the bounding medium analysis is a method to estimate the bounds for the mean of the stochastic model behavior.

¹⁰It is interesting to note that Eq. (3.5) leads to the following boundary value problem for $\{u^m\}$.

$$\begin{cases} \sum_{m'=0}^M (c^{mm'}(\mathbf{x}) u_{,i}^{m'}(\mathbf{x})),_{,i} = 0 & \text{in } B, \\ u^m(\mathbf{x}) = u^o(\mathbf{x}) \langle \Psi^m \rangle & \text{on } \partial B, \end{cases}$$

This is a coupled boundary value problem for $\{u^m\}$; u^m 's are not decoupled unless $c^{mm'}$ given by Eq. (3.13) vanishes for $m \neq m'$.

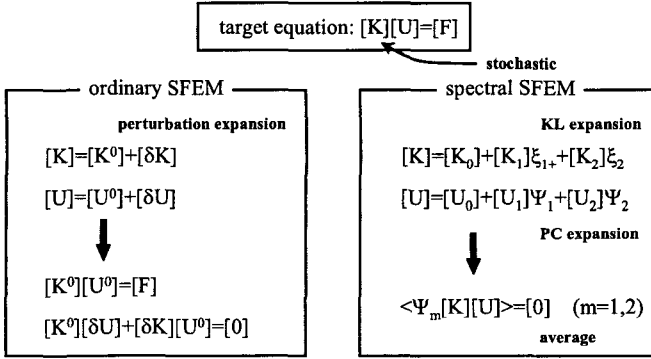


Fig. 3.3 The comparison of SSFEM (spectral stochastic FEM) with an ordinary SFEM (stochastic FEM). For a stochastic matrix equation $[K(\omega)][U(\omega)]=[F]$, the ordinary SFEM takes the regular perturbation expansion of $[K(\omega)]$ and $[U(\omega)]$, while SSFEM applies the KL and PC expansions to $[K(\omega)]$ and $[U(\omega)]$. The computation of SFEM is much simpler than that of SSFEM, although SFEM is not applicable to a case when the variability of $[K(\omega)]$ is large or complicated. SSFEM is applicable to this case.

- 2) the spectral method is a method to approximately but fully obtain the stochastic characteristics of the stochastic model behavior.

PART 2

STRONG GROUND MOTION

In this part, we demonstrate the application of computational mechanics to analysis of earthquake wave propagation phenomena. The goal of numerical analysis is the prediction of strong ground motion distribution with high spatial and temporal resolution. As will be soon explained, such prediction of strong ground motion distribution is not a trivial task. Some theoretical studies are needed to realize the prediction, so that an advanced numerical analysis method of computing wave propagation phenomena is developed.

We first present the basics of the wave equation which serves as the governing equation for wave propagation phenomena in Chapter 4. Stochastic modeling is applied to uncertain underground structures in which earthquake wave propagates, and strong ground motion distribution is estimated high spatial and temporal resolution by means of an advanced numerical analysis method. We explain this numerical analysis method in Chapter 5, and present some examples of applying this method to reproduce actual strong ground motion distribution in Chapter 6. The validity of the method is examined by comparing the simulation results with the observed data, and the usefulness of computing strong ground motion distribution with high resolution is discussed.

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Chapter 4

The Wave Equation for Solids

Understanding the wave equation for solids is essential to numerically compute the wave propagation in the crust and the wave amplification in surface layers. This chapter presents the formulation of the wave equation for solids as well as analytic solutions of the wave equation for some illustrative problems. First, the wave equation in a one-dimensional setting is studied. The characteristics of the wave equation are summarized using this simplest case. Then, analytic solutions are obtained in two- and three-dimensional settings. In particular, analytic and closed-form solutions for body waves and surface waves are derived. Finally, several key techniques which are commonly used in the numerical analysis of the wave equation are presented.

It should be pointed out that some topics presented in this chapter are out of the scope of conventional earthquake engineering. The topics related to mechanisms of earthquakes, which include more advanced contents related to seismology and geophysics, are briefly summarized in Appendix A. Also, the topics related to numerical techniques used to solve the wave equations, some of which are applicable to numerical analysis of other problems, are summarized in Appendix C. These appendices are recommended reading for all interested readers.

Like continuum mechanics, there have been a variety of textbooks available for readers with various backgrounds to study the formulation and analysis of the wave equation. Here, [Achenbach (1980)] is recommended for readers who are interested in theoretical study of linear problems of the wave equation. Appendix A presents a list of references which are related to seismology and engineering seismology. It should be pointed out that the recent trend of research related to the wave propagation, especially by means of numerical computation, is aimed at analyzing non-linear phenomena; see, for instance, [Nagolnykh and Ostrovsky (1998)] and [Blackstock (2000)]. It is necessary to take advantage of the achievement of this research in order to make further progress in computational earthquake engineering.

4.1 Basics of the Wave Equation

As the simplest problem of wave propagation phenomenon, a one dimensional problem of an elastic rod is considered. Young's modulus and density of the rod are E and ρ , respectively, and the cross section area is 1. According to the framework of solid continuum mechanics, the first step is to write three field equations that govern the wave propagation phenomenon in the rod, i.e.,

$$\begin{cases} \epsilon(x, t) = u'(x, t), \\ \sigma'(x, t) = \rho \ddot{u}(x, t), \\ \sigma(x, t) = E\epsilon(x, t). \end{cases} \quad (4.1)$$

Here, u , ϵ and σ are displacement, strain and stress which are functions of place and time, x and t , and prime and dot stand for derivative with respect to x and t , respectively. The absence of body force and the homogeneity of the rod are assumed in Eq. (4.1). The second step is to derive the following governing equation for u from Eq. (4.1):

$$\ddot{u}(x, t) - v^2 u''(x, t) = 0, \quad (4.2)$$

where v is the wave velocity given by

$$v = \sqrt{\frac{E}{\rho}}. \quad (4.3)$$

This equation is called the wave equation of the one-dimensional rod. Note that v given by Eq. (4.3) is regarded as a material¹ parameter; the wave velocity increases as E increases and ρ decreases.

The wave equation, Eq. (4.2), is a partial differential equation for a two variable function. A general solution of this equation can be expressed in terms of two single variable functions, i.e.,

$$u(x, t) = f(x - vt) + g(x + vt). \quad (4.4)$$

Here, f and g are² arbitrary functions of a single variable, $\xi = x - vt$ and $\eta = x + vt$, respectively. A function $f(\xi)$ represents a wave propagating to the positive x -direction, since $f(\xi)$ takes the same value if ξ does not change or if x and t are along a line of $x - vt = \xi$. The wave given by $f(\xi)$ propagates at the speed of v to the positive x -direction. Similarly, $g(\eta)$ represents a wave propagating to the negative x -direction.

¹It should be mentioned that some materials do not have constant wave velocity. Wave velocity changes depending on the frequency or on the wavelength. In this case, the wave changes its form as it propagates, and this is called *dispersion*. Non-linear materials have such wave dispersion, and linear but heterogeneous materials show apparent wave dispersion due to the scattering of local heterogeneities.

²Substituting Eq. (4.4) into Eq. (4.2) shows that $f(\xi)$ and $g(\eta)$ with $\xi = x - vt$ and $\eta = x + vt$ are the solution. Note that Eq. (4.2) is rewritten as $(\frac{\partial}{\partial t} - v \frac{\partial}{\partial x})(\frac{\partial}{\partial t} + v \frac{\partial}{\partial x})u = 0$, and $f(\xi)$ and $g(\eta)$ satisfy $(\frac{\partial}{\partial t} + v \frac{\partial}{\partial x})f = 0$ and $(\frac{\partial}{\partial t} - v \frac{\partial}{\partial x})g = 0$, respectively.

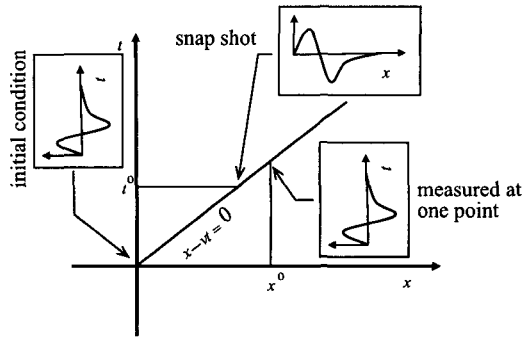


Fig. 4.1 A wave propagating in a one-dimensional rod. The waveform does not change when a snapshot is taken at $t=t^0$ or when the wave is measured at $x=x^0$.

In order to determine functions $f(\xi)$ and $g(\eta)$, initial and boundary conditions need to be prescribed. It is supposed that the rod lays from $x=0$ to $x=\infty$ and an input motion is given at $x=0$ for $t>0$, i.e.,

$$u(0, t) = \begin{cases} 0 & \text{for } t < 0, \\ u^0(t) & \text{for } t > 0. \end{cases} \quad (4.5)$$

Together with this condition, Eq. (4.2) forms an initial boundary value problem. Since the wave propagates only in the positive x -direction, $g(\eta)$ is dropped in Eq. (4.4), and $f(\xi)$ is determined from Eq. (4.5) using

$$f(-vt) = u(0, t).$$

Since f is expressed in terms of u^0 , displacement u is finally given as

$$u(x, t) = \begin{cases} u^0(t - \frac{x}{v}) & \text{for } t - \frac{x}{v} > 0, \\ 0 & \text{for } t - \frac{x}{v} < 0. \end{cases} \quad (4.6)$$

This is the solution of the initial boundary value problem, Eqs. (4.2) and (4.5). It is seen that at a certain instance $t=t^0$, the deformation of the rod can be expressed in terms of the input wave as $u^0(t^0 - x/v)$ for $x>0$, or that at a fixed point, $x=x^0$, the wave arrives at $t=x^0/v$ and the displacement at the point changes as $u^0(t - x^0/v)$ with respect to t ; see Fig. 4.1.

Now, a simple case when u^0 is expressed in terms of a trigonometric function is considered. That is,

$$u^0(t - \frac{x}{v}) = A \cos(k(x - vt)).$$

Here, A and k are arbitrary numbers; A and k are called amplitude and wave number, respectively. In terms of k , the following quantities are defined:

$$\text{wavelength} \quad \ell = 2\frac{\pi}{k},$$

$$\text{period} \quad T = \frac{2\pi}{kv},$$

$$\text{frequency} \quad \omega = kv;$$

note that ω is sometimes called *angular frequency*, to be distinguished from $\omega/2\pi$, which is also called frequency.

The wave propagating in the homogeneous rod does not change³ its form. However, when it passes through a different rod, the wave changes its form by reflection and refraction at the interface. To study this, a case when the rod studied above is connected to another rod at $x=0$ which lies in a domain of $x<0$ is considered. A quantity for the right or left rod is denoted by putting superscript $+$ or $-$, respectively, i.e., E^+ and ρ^+ for the right rod ($x>0$), or E^- and ρ^- for the left rod ($x<0$). The wave equation of these rods becomes

$$\begin{cases} \ddot{u}(x,t) - (v^+)^2 u''(x,t) = 0 & x > 0, \\ \ddot{u}(x,t) - (v^-)^2 u''(x,t) = 0 & x < 0, \end{cases}$$

where $v^\pm = \sqrt{E^\pm/\rho^\pm}$. There holds a continuity condition at the interface of the two rods, and velocity⁴ and traction are continuous across the interface. It is supposed that an incident wave which goes to the positive x -direction, $u^o(x-v^-t)$, is given to the left rod. Due to reflection and refraction at the interface, the left rod has a reflected wave, $g(x+v^-t)$, and the right rod has a transmitted wave, $f(x-v^+t)$; an argument of g or f is $x+v^-t$ or $x-v^+t$, respectively, to satisfy the wave equation for the left and right rods. Two unknown functions, g and f , can be determined using the continuity condition at $x=0$. That is, in view of $u=u^o+g$ for $x<0$ and $=f$ for $x>0$, velocity continuity leads to

$$-v^- u^{o'}(-v^-t) + v^- g'(v^-t) = -v^+ f'(-v^+t)$$

and traction continuity leads to

$$E^- u^{o'}(-v^-t) + E^- g'(v^-t) = E^+ f'(-v^+t),$$

³Precisely speaking, a plane wave does not change its form. There are other waves which change their form in a two- or three-dimensional setting. For instance, a radial or spherical wave is emitted from one point and propagating in a radial direction with reducing amplitude.

⁴Velocity continuity is usually used instead of displacement continuity, since it simplifies algebraic works.

where prime on u^o , f and g stands for the derivative with respect to their argument. Thus, g and f are determined as

$$g(\xi) = \frac{z^- - z^+}{z^- + z^+} u^o(-\xi) \quad \text{and} \quad f\left(\frac{v^+}{v^-} \xi\right) = \frac{v^-}{v^+} \frac{2z^-}{z^- + z^+} u^o(\xi),$$

where $z^\pm = \sqrt{E^\pm \rho^\pm}$. This $z = \sqrt{E\rho}$ is called *impedance*. As is seen from the form of g , no reflected wave is generated if the two rods have the same impedance, $z^- = z^+$, even if Young's modulus and density are different. The amplitude of wave is changed when it is transmitted to a different rod; the ratio of the amplitude is $\frac{v^-}{v^+} \frac{2z^-}{z^- + z^+}$. Also, since the argument of f is $\frac{v^+}{v^-} \xi$ or f is rewritten as

$$f(x - v^+ t) = \frac{v^-}{v^+} \frac{2z^-}{z^- + z^+} u^o\left(\frac{v^-}{v^+} x - v^- t\right),$$

the waveform is stretched or compressed, depending on the ratio of v^+ and v^- .

While waves propagating in two- or three-dimensional solids are expressed in terms of vector-valued displacement, there are some scalar-valued quantities which are used to characterize the waves. These quantities are related to energy carried by the waves, and are used as an indicator of wave strength or power. Using the wave in the one-dimensional rod, the scalar-valued quantities, namely, *kinetic energy density*, *strain energy density* and *power*, are defined as follows:

$$\text{kinetic energy density} \quad k = \frac{1}{2} \rho (\dot{u})^2,$$

$$\text{strain energy density} \quad e = \frac{1}{2} E (u')^2,$$

$$\text{power} \quad p = -E u' \dot{u}.$$

As is seen, p is power made by traction, i.e., the rate of work done by traction of $-\sigma = -Eu'$ with velocity $\dot{u}$. By definition, it is easily shown that total energy, $E = k + e$, satisfies

$$\bar{E} = \frac{1}{v} \bar{p}, \tag{4.7}$$

where $\bar{(\cdot)}$ stands for the average taken over a suitable period. For instance, when u is given as $u = A \cos(k(x - vt))$, these quantities are

$$k = \frac{1}{2} E A k^2 \sin^2(k(x - vt)),$$

$$e = \frac{1}{2} E A k^2 \sin^2(k(x - vt)),$$

$$p = E A^2 k^2 \sin^2(k(x - vt)).$$

Thus, Eq. (4.7) is satisfied if the average is taken for the time of $T = 2\pi/kv$. It should be emphasized that this relation holds when k , e and p are defined for vector-valued displacement.

The velocity at which power propagates is called the *group velocity*. In terms of $\mathbf{p}$ and $\mathbf{E}$, the group velocity is defined as

$$v_{\text{group}} = \frac{\bar{\mathbf{p}}}{\bar{\mathbf{E}}}; \quad (4.8)$$

see Eq. (4.7). In this sense, the group velocity is regarded as the speed of energy. The group velocity is not the same as the wave velocity or the *phase velocity*, denoted by v_{phase} , that is the speed of wave itself. The difference between the group velocity and the phase velocity is clearly seen if the frequency of the wave is given as a function of the wavelength, i.e., $\omega(k)$; the group velocity and the phase velocity are

$$v_{\text{group}} = \frac{d\omega}{dk}(k) \quad \text{and} \quad v_{\text{phase}} = \frac{\omega(k)}{k}.$$

Thus, unless ω is a linear function of k , the group velocity does not coincide with the phase velocity; waves in a homogeneous and linearly elastic material satisfy this condition although the function of $\omega(k)$ may changes depending on the direction.

4.2 Analytic Solutions of Particular Wave Problems

Now, this section studies wave propagation in a linearly elastic homogeneous⁵ solid for two- and three-dimensional settings. Considering the most general three-dimensional setting, the first step of study is to write the following field equations:

$$\begin{cases} \epsilon_{ij}(\mathbf{x}, t) = \frac{1}{2}(u_{i,j}(\mathbf{x}, t) + u_{j,i}(\mathbf{x}, t)), \\ \sigma_{ij,i}(\mathbf{x}, t) = \rho \ddot{u}_j(\mathbf{x}, t), \\ \sigma_{ij}(\mathbf{x}, t) = c_{ijkl} \epsilon_{kl}(\mathbf{x}, t). \end{cases} \quad (4.9)$$

Isotropy⁶ is usually assumed for elasticity. Then, c_{ijkl} is expressed in terms of Lamé constants, λ and μ , as $c_{ijkl} = \lambda \delta_{ij} \delta_{kl} + 2\mu I_{ijkl}$ with $I_{ijkl} = \frac{1}{2}(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk})$.

Two-dimensional field equations are derived from Eq. (4.9), just by assuming that u_i , ϵ_{ij} and σ_{ij} do not depend on x_3 . This assumption decouples Eq. (4.9) into the following two sets:

$$\begin{cases} \epsilon_{3i}(\mathbf{x}', t) = \frac{1}{2}u_{3,i}(\mathbf{x}', t), \\ \sigma_{3i,i}(\mathbf{x}', t) = \rho \ddot{u}_3(\mathbf{x}', t), \\ \sigma_{3i}(\mathbf{x}', t) = c_{3i3j} \epsilon_{3j}(\mathbf{x}', t), \end{cases} \quad (4.10)$$

⁵It is possible to analytically or numerically solve a linear wave equation with constant coefficients. When a wave equation for a two- or multi-phase solid is considered, reflection and refraction at interfaces between different phases need to be considered; reflected and transmitted waves are computed to satisfy continuity of velocity and traction.

⁶When a body is not isotropic, it is still possible to analytically solve the wave equation. A form of the solution becomes complicated. For instance, the wave velocity, the phase or group velocity, changes depending on the wave direction.

and

$$\begin{cases} \epsilon_{ij}(\mathbf{x}', t) = \frac{1}{2}(u_{i,j}(\mathbf{x}', t) + u_{j,i}(\mathbf{x}', t)), \\ \sigma_{ij,i}(\mathbf{x}', t) = \rho \ddot{u}_j(\mathbf{x}', t), \\ \sigma_{ij}(\mathbf{x}', t) = c_{ijkl} \epsilon_{kl}(\mathbf{x}', t), \end{cases} \quad (4.11)$$

for $i, j, k, l=1,2$. Here, $\mathbf{x}'$ stands for two-dimensional position vector, (x_1, x_2) . These sets are called *out-of-plane shear* and *in-plane⁷ deformation*, respectively.

This section first studies the out-of-plane shear wave. A general solution is obtained by applying the Fourier transform to the wave equation, and reflection and refraction at the interface of two media are analyzed. A Love wave is derived as a surface wave for this setting. Next, the in-plane wave is studied. The same procedures as the out-of-plane shear wave are taken, although manipulation becomes much more laborious. A Rayleigh wave is derived as a surface wave. Finally, the wave equation for a general three-dimensional setting is studied. A closed-form analytic solution of the wave equation is presented for an infinite body.

4.2.1 Out-of-plane shear wave

In a two-dimensional state of the out-of-plane shear deformation, displacement of the interest is u_3 , and the governing equation is derived from the three field equations, Eq. (4.10), as

$$\mu(u_{3,11}(\mathbf{x}, t) + u_{3,22}(\mathbf{x}, t)) - \rho \ddot{u}_3(\mathbf{x}, t) = 0, \quad (4.12)$$

where $c_{3ij} = \mu \delta_{ij}$ is used. For simplicity, prime on $\mathbf{x}$ is omitted, and $\mathbf{x}$ stands for the two-dimensional position vector. This is the wave equation for the out-of-plane shear deformation.

To solve Eq. (4.12), the Fourier transform is applied to u_3 by replacing x_i and t to k_i and ω , respectively. The transformed function is

$$\mathcal{F}u_3(\mathbf{k}, \omega) = \frac{1}{(\sqrt{2\pi})^3} \iiint u_3(\mathbf{x}, t) \exp(-i(k_i x_i + \omega t)) d\mathbf{x} dt,$$

where subscript i ranges from 1 to 2 and $d\mathbf{x}$ stands for $dx_1 dx_2$, and the inverse Fourier transform is

$$u_3(\mathbf{x}, t) = \frac{1}{(\sqrt{2\pi})^3} \iiint \mathcal{F}u_3(\mathbf{k}, \omega) \exp(i(k_i x_i + \omega t)) d\mathbf{k} d\omega.$$

Substitution of this equation into Eq. (4.12) yields

$$(\mu(k_i k_i) - \rho \omega^2) \mathcal{F}u_3(\mathbf{k}, \omega) = 0.$$

⁷Precisely speaking, Eq. (4.11) is based on the assumption of a state of plane strain; it is necessary to add $c_{ij33}\epsilon_{33}$ to the left side of the last equation when uniform ϵ_{33} is assumed. The other assumption of a two-dimensional state, i.e., a state of plane stress, leads to similar field equations, although c_{ijkl} is modified so that σ_{33} always vanishes.

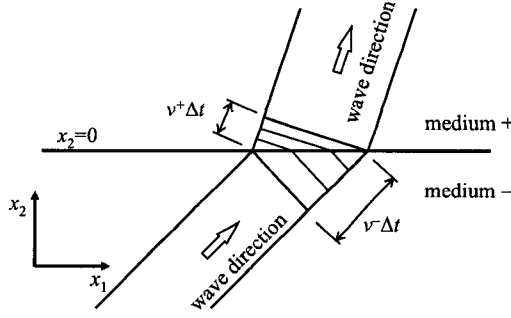


Fig. 4.2 The direction of the incident wave and the transmitted wave. The transmitted wave is bent upward if v^+ is smaller than v^- .

A non-trivial solution of $\mathcal{F}u_3$ exists if the Fourier variables, k_i and ω , satisfy $\mu|\mathbf{k}|^2 - \rho\omega^2 = 0$ with $|\mathbf{k}|^2 = k_i k_i$. Here, $|\mathbf{k}|$ is the wave number which gives the inverse of the wavelength, and μ/ρ is the speed of wave; indeed,

$$v = \sqrt{\frac{\mu}{\rho}} \quad (4.13)$$

is the velocity of the out-of-plane shear wave. Thus, a non-trivial solution of the wave equation is

$$u_3(\mathbf{x}, t) = \exp(i(k_i x_i - \omega t)), \quad (4.14)$$

for arbitrary ω , where k_i must satisfy $|\mathbf{k}| = \sqrt{\omega/v}$ but its direction, denoted by $n_i = k_i/|\mathbf{k}|$, is arbitrary. This is a *body wave* which goes to the $\mathbf{n}$ -direction with frequency ω and wave number $k = \omega/v$. This wave is called a *plane wave* as well, since it does not change its form in the direction which is normal to the $\mathbf{n}$ -direction.

Reflection and refraction of the out-of-plane shear wave are studied. For simplicity, an infinite domain which consists of two different media is considered; see Fig. 4.2. The media occupy an upper half plane of $x_2 > 0$ and a lower half plane $x_2 < 0$. The material parameters are $(\mu, \rho) = (\mu^+, \rho^+)$ for $x_2 > 0$ or (μ^-, ρ^-) for $x_2 < 0$, where superscript $+$ or $-$ designates a quantity for the upper or lower medium, respectively. Across the interface, $x_2 = 0$, continuity of velocity and traction is satisfied, i.e.,

$$\begin{cases} \dot{u}_3(x_1, 0^+, t) = \dot{u}_3^-(x_1, 0^-, t), \\ \mu^+ u_{3,2}(x_1, 0^+, t) = \mu^- u_{3,2}(x_1, 0^-, t), \end{cases} \quad (4.15)$$

where 0^+ or 0^- means that function is evaluated at the limit as x_2 goes to 0 from a positive or negative value, respectively. It is supposed that the lower medium has an incident wave, $A \exp(i(k_i x_i - \omega t))$ with $|\mathbf{k}| = \sqrt{\omega/v^-}$. It is reflected at the interface but some is transmitted to the upper medium. A reflected wave in $x_2 < 0$ and a transmitted wave in $x_2 > 0$ are expressed in a form similar to that of the incident

wave, as

$$B \exp(i(k_i^- x_i - \omega t)) \quad \text{and} \quad C \exp(i(k_i^+ x_i - \omega t)).$$

Due to continuity, the frequency ω must be shared and these waves have a term $\exp(-i\omega t)$. However, the wave number, $k_i^\pm$, is different from k_i . From the wave equation of the lower and upper media, the wave number $k_i^\pm$ is expressed in terms of unknown unit vector $n_i^\pm$ as $k_i^\pm = (\omega/v^\pm)n_i^\pm$; this n_i^+ or n_i^- gives the direction of the transmitted or reflected wave, respectively. Thus, unknowns are the amplitudes, B and C , and the unit vectors, n_i^- and n_i^+ . These unknowns can be determined using the continuity condition, Eq. (4.15). For instance, the unit vector of the wave direction is

$$\begin{bmatrix} n_1^- \\ n_2^- \end{bmatrix} = \begin{bmatrix} n_1 \\ -n_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} n_1^+ \\ n_2^+ \end{bmatrix} = \begin{bmatrix} \frac{v^+}{v^-} n_1 \\ \sqrt{1 - (\frac{v^+}{v^-} n_1)^2} \end{bmatrix},$$

where $n_i = \frac{\omega}{v^-} k_i$. The amplitude of the wave is

$$\begin{bmatrix} B \\ C \end{bmatrix} = \frac{1}{1 + \frac{z^+}{z^-} \frac{n_2^+}{n_2^-}} \begin{bmatrix} 1 - \frac{z^+}{z^-} \frac{n_2^+}{n_2^-} \\ 2 \end{bmatrix} A,$$

where $z^\pm = \sqrt{\mu^\pm \rho^\pm}$ is impedance. As is seen above, if the wave velocity of the upper medium is slower⁸ than that of the lower medium, i.e., $v^+/v^- < 1$, then, the path of the transmitted wave is bent upwards; see Fig. 4.2. In this case, the relative amplitude of the transmitted wave increases when the incident wave goes vertically ($[n_1, n_2]^T = [0, 1]^T$), i.e., $C/A = 2/(1 + \frac{z^+}{z^-}) > 1$.

It is rare to solve the wave equation without considering a suitable domain. For earthquake problems, a half plane is usually considered, and a traction-free condition is prescribed for the surface of the half plane. There exists a non-trivial homogeneous solution for this half plane problem; the solution satisfies the traction-free condition at the surface and vanishes at the far-field, but has non-zero amplitude near the surface. This solution is called a *surface wave*.

Now, the surface wave for the out-of-plane shear deformation is studied. It is known that the surface wave, called a *Love wave*, is generated when a half plane consists⁹ of two stratified layers, i.e., $(\mu, \rho) = (\mu^+, \rho^+)$ for $0 < x_2 < h$ or (μ^-, ρ^-) for $0 > x_2$; see Fig. 4.3. The Love wave satisfies a traction-free condition at $x_2 = h$ and

⁸This is the usual case of underground structures; a shallower layer tends to have slower wave velocity.

⁹In earthquake engineering, a case when the lower medium is much stiffer than the upper medium is often considered. Vanishing of the wave at the far-field is replaced by zero displacement at $x_2 = 0$. A non-trivial solution for a vertically propagating wave is readily obtained as

$$u_3(x_2) = A \sin\left(\frac{\omega_n x_2}{v}\right) \exp(i\omega_n t) \quad \text{and} \quad \omega_n = \frac{(n - \frac{1}{2})\pi v}{h},$$

for $n=1, 2, \dots$. This surface wave corresponds to vibration of the upper medium.

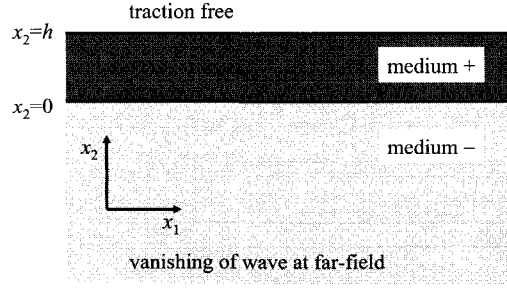


Fig. 4.3 A Love wave.

vanishing of the wave at far-field (the limit as x_2 goes to $-\infty$) as boundary conditions. In order to have a non-trivial solution, the following form is assumed for u_3 in the two layers:

$$u_3 = \begin{cases} A \exp(i(kx_1 - \omega t)) \cos(\nu^+(x_2 - h)) & 0 < x_2 < h, \\ B \exp(i(kx_1 - \omega t)) \exp(\nu^- x_2) & 0 > x_2, \end{cases}$$

where k and $\nu^\pm$ are unknown positive numbers, and A and B are unknown displacement amplitude; a frequency ω is assumed to be given. Conditions for $\nu^\pm$ are derived from the wave equation of the two media, as

$$\nu^+ = \sqrt{\left(\frac{\omega}{v^+}\right)^2 - k^2} \quad \text{and} \quad \nu^- = \sqrt{k^2 - \left(\frac{\omega}{v^-}\right)^2},$$

with $v^\pm = \sqrt{\mu^\pm/\rho^\pm}$. For $\nu^\pm$ to be a real number, the wave number in the x_1 -direction, k , must satisfy $(\omega/v^+)^2 - k^2 > 0$ and $k^2 - (\omega/v^-)^2 > 0$. Thus, an admissible range of k is

$$\frac{\omega}{v^-} < k < \frac{\omega}{v^+}. \quad (4.16)$$

Note that $v^- = \sqrt{\mu^-/\rho^-}$ must be larger¹⁰ than $v^+ = \sqrt{\mu^+/\rho^+}$ for Eq. (4.16) to hold. Two homogeneous equations are derived for A and B from continuity of velocity and traction at the interface. These equations are expressed in the following matrix form:

$$\begin{bmatrix} \cos(\nu^+ h) & -1 \\ \mu^+ \nu^+ \sin(\nu^+ h) & \mu^- \nu^- \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

For a given ω , there is a case when non-trivial $[A, B]^T$ exist for the above homogeneous matrix equation; the determinant of the two-by-two matrix, which is a function of k , must vanish. This condition is explicitly expressed as

$$\frac{\mu^+ \nu^+}{\mu^- \nu^-} \tan(\nu^+ h) + 1 = 0. \quad (4.17)$$

¹⁰As mentioned, this is the usual case for underground structures.

Finally, the Love wave of the following form is obtained:

$$u_3 = \begin{cases} 2A \exp(i(kx_1 - \omega t)) \cos(\nu^+(x_2 - h)) & 0 < x_2 < h, \\ A \cos(\nu^+ h) \exp(i(kx_1 - \omega t)) \exp(\nu^- x_2) & 0 > x_2. \end{cases} \quad (4.18)$$

Here, k satisfies Eq. (4.17) for a given ω , as well as Eq. (4.17). Note that the wave velocity of the Love wave in the lower medium is evaluated as $\omega/\sqrt{k^2 + (\nu^-)^2}$, which changes depending on the frequency ω , although the wave velocity in the upper medium is $v^+ = \sqrt{\mu^+/\rho^+}$.

4.2.2 In-plane wave

The wave equation for the in-plane deformation is slightly complicated, since it is a pair of partial differential equations for two components of a displacement vector. By expressing isotropic elasticity in terms of Lamé constants λ and μ , i.e., $c_{ijkl} = \lambda \delta_{ij} \delta_{kl} + 2\mu I_{ijkl}$, the following equation is derived from Eq. (4.11):

$$(\lambda \delta_{ij} \delta_{kl} + 2\mu I_{ijkl}) u_{k,li}(\mathbf{x}, t) = \rho \ddot{u}_j(\mathbf{x}, t),$$

for $i, j, k, l = 1, 2$. After some manipulation, it becomes

$$(\lambda + \mu) u_{i,ij}(\mathbf{x}, t) + \mu u_{j,ii}(\mathbf{x}, t) = \rho \ddot{u}_j(\mathbf{x}, t). \quad (4.19)$$

This is the wave equation for the in-plane deformation.

Now, Eq. (4.19) is solved by applying the Fourier transform to u_i , which replaces (x_1, x_2, t) with (k_1, k_2, ω) . The following equation for the Fourier transform of displacement, $\mathcal{F}u_i$, can be derived from Eq. (4.19):

$$\left((\lambda + \mu) k_i k_j + (\mu |\mathbf{k}|^2 - \rho \omega^2) \delta_{ij} \right) \mathcal{F}u_j(\mathbf{k}, \omega) = 0.$$

A non-trivial solution of $\mathcal{F}u_i$ exists when the Fourier variables make the determinant of the two-by-two matrix given by $(\lambda + \mu) k_i k_j + (\mu |\mathbf{k}|^2 - \rho \omega^2) \delta_{ij}$ vanishes, i.e.,

$$\det \begin{bmatrix} (\lambda + \mu) k_1^2 + (\mu |\mathbf{k}|^2 - \rho \omega^2) & (\lambda + \mu) k_1 k_2 \\ (\lambda + \mu) k_1 k_2 & (\lambda + \mu) k_2^2 + (\mu |\mathbf{k}|^2 - \rho \omega^2) \end{bmatrix} = 0.$$

This equation leads to a condition for k_i and ω , as $v_P^2 |\mathbf{k}|^2 - \omega^2 = 0$ or $v_S^2 |\mathbf{k}|^2 - \omega^2 = 0$ with

$$\begin{cases} v_P = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \\ v_S = \sqrt{\frac{\mu}{\rho}}. \end{cases} \quad (4.20)$$

These v_P and v_S are called the velocity of the primary wave (P-wave) and the secondary waves (S-wave), respectively. An eigen-vector of the matrix corresponds to the displacement vector. In terms of v_P and v_S , a non-trivial solution of Eq. (4.19)

can be expressed as

$$\begin{cases} u_i(\mathbf{x}, t) = n_i \exp(i(k_P(n_j x_j) - \omega t)), \\ u_i(\mathbf{x}, t) = s_i \exp(i(k_S(n_j x_j) - \omega t)), \end{cases} \quad (4.21)$$

for arbitrary ω , with $k_{P,S} = \sqrt{\omega/v_{P,S}}$. The first solution is P-wave, and the second solution is S-wave. In both cases, n_i in Eq. (4.21) is an arbitrary unit vector along which the wave propagates, and s_i is a unit vector normal to n_i , i.e., $n_i s_i = 0$. While these waves are body or plane waves going to the same $\mathbf{n}$ -direction, the direction of displacement vector is parallel or normal to the $\mathbf{n}$ -direction for P-wave or for S-wave, respectively.

In the same manner as for the out-of-plane shear wave, reflection and refraction at an interface between two different media are analyzed, by making use of continuity of velocity and traction. Since velocity and traction are two-dimensional vectors, the continuity condition leads to four equations. If the interface is $x_2=0$ and the material parameters of the media in $x_2>0$ and $x_2<0$ are denoted by putting superscript + and -, respectively, these equations are

$$\begin{cases} \dot{u}_i(x_1, 0^+, t) = \dot{u}_i(x_1, 0^-, t), \\ c_{2jkl}^+ u_{k,l}(x_1, 0^+, t) = c_{2jkl}^- u_{k,l}(x_1, 0^-, t). \end{cases} \quad (4.22)$$

with $c_{ijkl}^\pm = \lambda^\pm \delta_{ij} \delta_{kl} + 2\mu^\pm I_{ijkl}$. Thus, for one incident wave in $x_2<0$, which is P- or S-wave, both P- and S-waves are reflected or transmitted in $x_2<0$ or $x_2>0$, respectively. These four waves are fully determined by the four equations of Eq. (4.22), and the form of these waves are

$$\begin{aligned} \begin{bmatrix} u_1^\pm \\ u_2^\pm \end{bmatrix} &= \begin{bmatrix} k_1 \\ k_P^\pm \end{bmatrix} f_P^\pm(k_1 x_1 + k_P^\pm x_2) + \begin{bmatrix} k_1 \\ -k_P^\pm \end{bmatrix} g_P^\pm(k_1 x_1 - k_P^\pm x_2) \\ &+ \begin{bmatrix} k_S^\pm \\ -k_1 \end{bmatrix} f_S^\pm(k_1 x_1 + k_S^\pm x_2) + \begin{bmatrix} k_S^\pm \\ k_1 \end{bmatrix} g_S^\pm(k_1 x_1 - k_S^\pm x_2), \end{aligned}$$

where $\exp(i\omega t)$ is omitted for simplicity, and k_1 determines $k_{P,S}^\pm$ as

$$k_P^\pm = \sqrt{(\omega/v_P^\pm)^2 - k_1^2} \quad \text{and} \quad k_S^\pm = \sqrt{(\omega/v_S^\pm)^2 - k_1^2},$$

so that the wave equations for the upper and lower media are satisfied. Traction on $x_2=0$, which is given as $t_j^\pm = c_{2jkl}^\pm u_{k,l}$, is then expressed as

$$\begin{aligned} \begin{bmatrix} t_1^\pm \\ t_2^\pm \end{bmatrix} &= \begin{bmatrix} 2\mu^\pm k_1 k_P^\pm \\ \lambda^\pm k_1^2 + (\lambda^\pm + 2\mu^\pm) k_P^{\pm 2} \end{bmatrix} f_P^{\pm'} + \begin{bmatrix} -2\mu^\pm k_1 k_P^\pm \\ \lambda^\pm k_1^2 + (\lambda^\pm + 2\mu^\pm) k_P^{\pm 2} \end{bmatrix} g_P^{\pm'} \\ &+ \begin{bmatrix} -\mu^\pm (k_1^2 - k_S^{\pm 2}) \\ -2\mu^\pm k_1 k_S^\pm \end{bmatrix} f_S^{\pm'} + \begin{bmatrix} \mu^\pm (k_1^2 - k_S^{\pm 2}) \\ -2\mu^\pm k_1 k_S^\pm \end{bmatrix} g_S^{\pm'}. \end{aligned}$$

Here, prime on $f_{P,S}^\pm$ and $g_{P,S}^\pm$ stands for derivative and the arguments of functions are dropped. Since $f_{P,S}^\pm$ or $g_{P,S}^\pm$ represents the wave going to the positive or negative x_2 -direction, respectively, $f_{P,S}^+$ and $g_{P,S}^-$ are unknown functions which

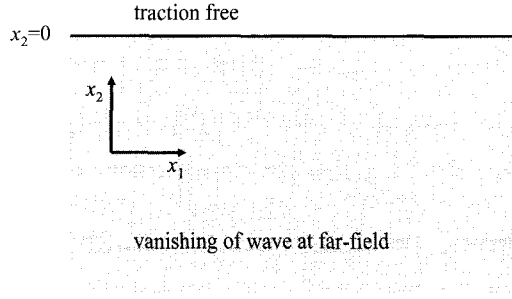


Fig. 4.4 A Rayleigh wave.

express transmitted and reflected waves, when these equations are substituted into Eq. (4.22).

To complete this subsection, the derivation of a *Rayleigh wave*, a surface wave for the in-plane deformation, is presented briefly. Unlike the Love wave, this surface wave exists for a homogeneous half plane ($x_2 < 0$); see Fig. 4.4. The Rayleigh wave corresponds to a non-trivial solution for a homogeneous problem of a half plane. A traction-free condition at the top surface and vanishing of the wave at the far-field are prescribed as boundary conditions. To satisfy the vanishing of the wave at the far-field, the Rayleigh wave must be of the following form:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \exp(ikx_1 + bx_2) \exp(-i\omega t) \quad x_2 < 0,$$

where k and b are unknown positive constants, and A_i is unknown displacement amplitude. Substituting the above $[u_1, u_2]^T$ into the wave equation, for a given ω , the following equation is obtained for the wave numbers k and b :

$$\begin{bmatrix} v_P^2 k^2 - v_S^2 b^2 - \omega^2 & i(v_P^2 - v_S^2)kb \\ i(v_P^2 - v_S^2)kb & v_S^2 k^2 - v_P^2 b^2 - \omega^2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

For nontrivial $[A_1, A_2]^T$ to exist, the determinant of the above two-by-two matrix must vanish, i.e.,

$$\omega^4 - (v_P^2 k^2 - v_S^2 b^2)(v_S^2 k^2 - v_P^2 b^2) + v_P^2 v_S^2 (k^2 - b^2) = 0. \quad (4.23)$$

From Eq. (4.23), it follows that b is determined as $b^2 = k^2(1 - (\frac{k\omega}{v_P})^2)$ or $k^2(1 - (\frac{k\omega}{v_S})^2)$ for given ω and k . These b 's are denoted by b_1 and b_2 , and the form of $[u_1, u_2]^T$ is replaced as

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \exp(-b_1 x_2) & \exp(-b_2 x_2) \\ \frac{b_1}{ik} \exp(-b_1 x_2) & \frac{ik}{b_2} \exp(-b_2 x_2) \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \exp(i(kx_1 - \omega t)),$$

where B_1 and B_2 are arbitrary constants. For the wave of this form, another characteristic equation for k and ω can be derived from the traction-free condition

on the top surface, $\sigma_{2i}=0$ at $x_2=0$. After some manipulation, the following equation is obtained:

$$\begin{bmatrix} 2b_1 & (2 - (\frac{v_S}{v_P})^2) \frac{k^2}{b_2} \\ (2 - \frac{v_S}{v_P})^2 & 2 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

For given ω and k , non-trivial $[B_1, B_2]^T$ can be determined from the above homogeneous matrix equation. Together with ω and k which determines b_i 's, the amplitudes $[B_1, B_2]^T$ describe the Rayleigh wave. Note that for the above matrix equation to have a non-trivial solution, the wave velocity of the Rayleigh wave, $v_R = \omega/k$, must satisfy

$$\left(2 - \left(\frac{v_R}{v_S}\right)^2\right)^2 - 4\sqrt{1 - \left(\frac{v_R}{v_P}\right)^2}\sqrt{1 - \left(\frac{v_R}{v_S}\right)^2} = 0. \quad (4.24)$$

It is easily seen from Eq. (4.24) that the Rayleigh wave velocity v_R satisfies $v_S < v_R < v_P$.

4.2.3 Plane wave in three-dimensional setting

Now, a general three-dimensional setting is considered. The governing equation for u_i is readily derived from Eq. (4.9), as

$$c_{ijkl}u_{k,li}(\mathbf{x}, t) - \rho\ddot{u}_j(\mathbf{x}, t) = 0. \quad (4.25)$$

for $i, j, k, l=1,2,3$, where $\mathbf{x}$ is a three-dimensional position vector. This is the wave equation for the three-dimensional setting. While Eq. (4.25) appears complicated, a class of solutions which satisfy this wave equation can be found. Indeed, the Fourier transform leads to a solution of

$$u_i(\mathbf{x}, t) = A_i \exp(i(k_i x_i - \omega t)), \quad (4.26)$$

for arbitrary ω , where A_i and k_i satisfy $(c_{ijkl}k_i k_l - \rho\omega^2 \delta_{jk})A_k = 0$. This class of solutions are called a body wave or a plane wave.

A solution of the wave equation, Eq. (4.25), is first sought assuming isotropy for elasticity. Substitution of isotropic elasticity tensor, $c_{ijkl} = \lambda\delta_{ij}\delta_{kl} + 2\mu I_{ijkl}$, into Eq. (4.25) leads to the following wave equation:

$$(\lambda + \mu)u_{i,ij}(\mathbf{x}, t) + \mu u_{j,ii}(\mathbf{x}, t) = \rho\ddot{u}_j(\mathbf{x}, t). \quad (4.27)$$

Here, λ and μ are Lamé constants.

A general solution of Eq. (4.27) has been studied by a number of researchers; see, for instance, [Achenbach (1980)]. One of the most fundamental solutions is a plane wave, i.e., a wave propagating in a fixed direction without changing its form. A plane wave is obtained without applying the Fourier transform to displacement. The form of plane wave is

$$u_i(\mathbf{x}, t) = U_i f(k_j x_j - vt), \quad (4.28)$$

where U_i is a constant vector for displacement amplitude and f is a single-variable function; k_i is wave number and v is velocity. Direct substitution of Eq. (4.28) into Eq. (4.27) yields

$$\left((\lambda + \mu)k_i k_j + (\mu|k|^2 - \rho\omega^2)\delta_{ij} \right) U_j f'' = 0.$$

For a non-trivial solution to exist, the determinant of the three-by-three matrix which is given by terms in the parenthesis must vanish. It follows from this condition that there are two cases when non-trivial solutions for v and U_i exist for given k_i , i.e.,

$$\begin{cases} \lambda + 2\mu - \rho v^2 = 0 & \text{for } U_i \text{ parallel to } k_i, \\ \mu - \rho v^2 = 0 & \text{for } U_i \text{ normal to } k_i \text{ } (U_i k_i = 0). \end{cases}$$

For the first case, the wave propagates with the speed of $v_P = \sqrt{(\lambda + 2\mu)/\rho}$ and the direction of the wave propagation coincides with the direction of wave displacement vector. For the second case, the speed is $v_S = \sqrt{\mu/\rho}$ and the wave propagation direction is normal to the wave displacement direction. As is shown in the two-dimensional in-plane wave, these two cases are for P-wave and S-wave, respectively. Note that P-wave is rotation-free, $\varepsilon_{ijk} u_{j,k} = 0$, and S-wave is divergence-free, $u_{i,i} = 0$. They are also called a longitudinal or pressure wave and a transverse or shear wave, respectively.

It is no wonder that there exist two types of plane waves as shown in Eq. (4.28). This is because Eq. (4.27) can be reduced to the two independent wave equations of the following form:

$$\Delta(\cdot) - \frac{1}{v^2}(\ddot{\cdot}) = 0, \quad (4.29)$$

where v is the wave velocity, v_P or v_S , and Δ is the Laplacian, i.e., $\Delta(\cdot) = (\cdot)_{,ii}$. To reduce Eq. (4.27) to Eq. (4.29), the following procedures are taken:

- i) Making use of the vector identity, $u_{i,jj} = u_{j,ji} - \varepsilon_{ijk}\varepsilon_{klm}u_{m,jl}$, with ε_{ijk} being the permutation symbol, rewrite Eq. (4.27) as

$$\ddot{u}_i = v_P^2 u_{j,ji} - v_S^2 \varepsilon_{ijk}\varepsilon_{klm} u_{m,jl}.$$

- ii) Taking the divergence and the rotation of the above equation, derive the following two equations:

$$\Delta(u_{i,i}) - \frac{1}{v_P^2} \frac{\partial^2(u_{i,i})}{\partial t^2} = 0 \quad \text{and} \quad \Delta(\varepsilon_{ijk} u_{k,j}) - \frac{1}{v_S^2} \frac{\partial^2(\varepsilon_{ijk} u_{k,j})}{\partial t^2} = 0,$$

where $\varepsilon_{ijk}(\cdot)_{,jk} = 0$ is used.

- iii) Introducing a scalar potential ϕ and a vector potential ψ_i with $\psi_{i,i} = 0$ for $u_{i,i}$ and $\varepsilon_{ijk} u_{k,j}$, rewrite the above equations as

$$\Delta\phi - \frac{1}{v_P^2} \ddot{\phi} = 0 \quad \text{and} \quad \Delta\psi_i - \frac{1}{v_S^2} \ddot{\psi}_i = 0.$$

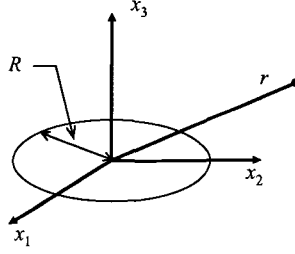


Fig. 4.5 A circular fault. A closed form expression of an analytic solution is available for a wave which is emitted from this fault.

As is seen, P-wave corresponds to the scalar potential ϕ , and S-wave corresponds to the vector potential ψ_i . It should be noted that ϕ and ψ_i have the velocities v_P and v_S , respectively. The wave equation which uses these potentials has been extensively studied in many branches of physics and mathematics.

An analytic solution of the above wave equation is obtained when a target is an infinitely extended body and an instantaneous point force acts at the origin. This solution is called a *fundamental solution*. For earthquake problems, the fundamental solution is used for theoretical studies of wave propagation, and several formulae have been derived from the fundamental solution; see, for instance, [Aki (1988)]. The following formulae are useful in studying an earthquake wave which propagates from a fault of circular cross section of radius R :

1) near field

$$u_i = -\frac{1}{2\pi} \frac{\partial^3 r^{-1}}{\partial x_i \partial x_1 \partial x_3} \int_{\frac{R}{v_P}}^{\frac{R}{v_S}} s M(t-s) ds$$

2) far-field

$$u_i = \frac{1}{4\pi\rho v_P^3 r} \frac{2x_i x_1 x_3}{r^3} \dot{M}\left(t - \frac{r}{v_P}\right) + \frac{1}{4\pi\rho v_S^3 r} \left(\delta_{i1} \frac{x_3}{r} + \delta_{i3} \frac{x_1}{r} - 2 \frac{x_i x_1 x_3}{r^3} \right) \dot{M}\left(t - \frac{r}{v_S}\right).$$

Here, M gives a time series of moment which corresponds to displacement gap across the fault plane and r is distance from the origin, i.e., $r = \sqrt{x_i x_i}$; see Fig. 4.5.

4.3 Numerical Analysis of the Wave Equation

This section presents several major numerical analysis methods of solving the wave equation for solids; see Appendix C for more detailed and technical explanations of numerical techniques of solving the wave equations. First of all, it is worth noting that the method of separation of variable is used in discretizing displacement, i.e.,

displacement is given as the product of shape functions which depend on the space variables only and coefficients which are functions of the time variable only, i.e.,

$$u_i(\mathbf{x}, t) = \sum_{\alpha} u_i^{\alpha}(t) \phi^{\alpha}(\mathbf{x}). \quad (4.30)$$

The matrix equation of FEM for a static case is thus transformed to an ordinary differential equation for nodal displacement. Solving the ordinary differential equation with respect to time is called *time integration*. Various algorithms and numerical techniques are available for time integration.

The ordinary differential equation of nodal displacement is readily derived from the weak form of the wave equation. While the wave equation is a partial differential equation with respect to space and time, in FEM, the weak form is taken only with respect to space, i.e.,

$$\int_V \delta u_j(\mathbf{x}) \left((c_{ijkl} u_{k,l}(\mathbf{x}, t))_{,i} - \rho \ddot{u}_j(\mathbf{x}, t) \right) dv_{\mathbf{x}} = 0; \quad (4.31)$$

note that δu_i is a function of space only. Applying integration by part and substituting $\delta u_i = \delta u_i^{\alpha} \phi^{\alpha}$ with δu_i^{α} being an arbitrary number, FEM uses the following ordinary equation for u_i^{α} which is transformed to vector $[U]$:

$$[M][\ddot{U}(t)] + [K][U(t)] = [0], \quad (4.32)$$

where $[M]$ is called a mass matrix whose components are defined as

$$\rho_{ij}^{\alpha\beta} = \int_V \rho \delta_{ij} \phi^{\alpha}(\mathbf{x}) \phi^{\beta}(\mathbf{x}) dv_{\mathbf{x}}, \quad (4.33)$$

and $[K]$ is stiffness matrix; see Sec. 2.3.

The mass matrix given by Eq. (4.33) is called a *consistent* mass matrix. While this matrix is rigorously derived, FEM uses another mass matrix. When it is assumed that each node separately has its own weight, a mass matrix is given as a simple diagonal matrix, i.e.,

$$\rho_{ij}^{\alpha\beta} = \begin{cases} m^{\alpha} & \text{for } \alpha = \beta \text{ and } i = j, \\ 0 & \text{otherwise.} \end{cases} \quad (4.34)$$

Here, m^{α} is the weight concentrated on a node $\mathbf{x}^{\alpha}$. The mass matrix given by Eq. (4.34) is¹¹ called *concentrated* or *lumped* mass matrix.

4.3.1 Algorithms used for time integration

First, the basic algorithm of time integration is explained using the simple example of an ordinary differential equation for a scalar-valued function u . The following

¹¹ While the consistent mass matrix is rigorously derived from discretization of u_i , i.e., Eq. (4.31), the lumped mass matrix is simple and makes numerical computation easier. Thus, the lumped mass matrix is usually used in FEM; it is a good approximation of the consistent mass matrix in the sense that error caused by using the lumped mass matrix is often negligible.

initial value problem for u is considered:

$$\begin{cases} \dot{u}(t) = f(u(t)) & t > 0, \\ u(t) = u^0 & t = 0. \end{cases} \quad (4.35)$$

Here, f is a possibly non-linear function of u . Formally, the solution of Eq. (4.35) can be expressed in integral form, as

$$u(t) = u^0 + \int_0^t f(u(t')) dt'. \quad (4.36)$$

It is not straightforward to numerically compute the integration in the left side without losing accuracy. This is because some errors may enter during the discretization process and they grow in quite a fast manner.

To show the algorithm of time integration in a transparent manner, a target function, $u(t)$, is first discretized as a series $\{u^n\}$ where $u^n = u(t^n)$ and t^n is a sequence of equally spaced time; $t^{n+1} - t^n$ does not depend on n and is given as Δt . The Taylor expansion of $u(t + \Delta t)$ at t is

$$u(t + \Delta t) = u(t) + \Delta t \dot{u}(t) + \dots,$$

and the left side of Eq. (4.35) is approximated as $(u^{n+1} - u^n)/\Delta t$. The following recursive equation for $\{u^n\}$ is derived:

$$u^{n+1} = u^n + \Delta t f(u^n). \quad (4.37)$$

Here, as an initial condition, the first term, which corresponds to $u(t^0)$ or $u(0)$, is given as $u^0 = u^0$. This is called the *forward Euler method*. It is called *explicit* since u^{n+1} is directly determined by Eq. (4.37).

While Eq. (4.37) is simple, the numerical solution is not always *stable* in the sense that there are cases when numerical errors included in the solution increase drastically. The following *backward Euler method* is proposed as an alternative to the forward Euler method:

$$u^{n+1} = u^n + \Delta t f(u^{n+1}). \quad (4.38)$$

Since unknown u^{n+1} is used for f in the right side, there are cases when Eq. (4.38) becomes a non-linear equation for u^{n+1} . Thus, the backward Euler method is called *implicit*, and it is much more laborious to solve Eq. (4.38) compared with Eq. (4.37).

The key issue of the forward and backward Euler methods is the trade-off between simplicity and stability; Eq. (4.37) is explicit while Eq. (4.38) is the most stable. As a compromise, the following modified Euler method is proposed:

$$u^{n+1} = u^n + \frac{\Delta t}{2} (u^n + \tilde{u}^{n+1}), \quad (4.39)$$

where $\tilde{u}^{n+1}$ is a trial value of u at $t^n = (n+1)\Delta t$ and given as $\tilde{u}^{n+1} = u^n + \Delta t f(u^n)$. Also, there are other types of compromises of the forward and backward Euler

methods, such as the *Runge-Kutta method* and the *multi-step method*; the multi-step method is a method which uses a set of several previous data. The following Runge-Kutta method is known as an accurate¹² method of time integration:

$$u^{n+1} = u^n + \frac{\Delta t}{6} \left(f^{(1)} + 2f^{(2)} + 2f^{(3)} + f^{(4)} \right), \quad (4.40)$$

where

$$\begin{aligned} f^{(1)} &= f(u^n), \quad f^{(2)} = f\left(u^n + \frac{\Delta t}{2} f^{(1)}\right), \quad f^{(3)} = f\left(u^n + \frac{\Delta t}{2} f^{(2)}\right), \\ f^{(4)} &= f(u^n + \Delta t f^{(3)}). \end{aligned}$$

As is seen, the Ruge-Kutta method is explicit, even though some computation is needed to calculate from $f^{(1)}$ to $f^{(4)}$.

4.3.2 Stability of time integration

The algorithm shown in the preceding subsection can be applied to solve the discretized wave equation, Eq. (4.32), for nodal displacement $[U]$, even though it is the second order differential equation. The forward Euler method is the simplest, i.e.,

$$[U^{n+2}] = 2[U^{n+1}] - [U^n] + (\Delta t)^2 [K][U^{n+1}], \quad (4.41)$$

where $[U^n] = [U(n\Delta t)]$.

Some attention must be paid to the stability of numerically solving Eq. (4.41). There exist a set of eigen-values and eigen-vectors for the homogeneous equation, $(\lambda[M] - [K])[\Phi] = [0]$; the L^{th} eigen-values and eigen-vectors, denoted by λ_L and $[\Phi_L]$, satisfy

$$\lambda_L [M][\Phi_L] - [K][\Phi_L] = [0]. \quad (4.42)$$

When $[M]$ and $[K]$ are obtained by suitable discretization, every eigen-value is a real and positive number. In terms of $\{\lambda_L, [\Phi_L]\}$, therefore, a homogeneous solution of Eq. (4.32) is expressed as

$$[U(t)] = \sum_L (A_L \exp(\imath \sqrt{\lambda_L} t) + B_L \exp(-\imath \sqrt{\lambda_L} t)) [\Phi_L],$$

where A_L and B_L are arbitrary coefficients. When all λ_L 's are positive real numbers, the right side of the above equation is periodic, and the numerical computation cannot be unstable, i.e., a numerical error which happens to be included in A_L 's and B_L 's does not grow. However, an unstable solution comes up when one of λ_L 's

¹²The accuracy is defined as the difference between the numerical solution and the exact solution, and the difference decreases as the order of the time increment Δt decreases. For instance, the Euler method given by Eq. (4.37) has the difference of $\mathcal{O}(\Delta t)$, while the modified Euler method given by Eq. (4.39) has the difference of $\mathcal{O}(\Delta t^2)$. These differences (or errors) are called order 1 and 2, respectively.

becomes a negative number or a complex-number; for instance, if λ_L is a negative number, the term corresponds to it becomes

$$\begin{aligned} & (A_L \exp(i\sqrt{\lambda_L}t) + B_L \exp(-i\sqrt{\lambda_L}t)) [\Phi_L] \\ & = (A_L \exp(-\sqrt{-\lambda_L}t) + B_L \exp(\sqrt{-\lambda_L}t)) [\Phi_L]. \end{aligned}$$

As is seen, the second term associated with B_L increases in an exponential manner; see Appendix C.1 for a more detailed explanation.

The eigen-values λ_L 's correspond to the natural frequencies of the target. Thus, if the eigen-values are ordered as $\lambda_1 < \lambda_2 < \dots$, the first eigen-value, λ_1 , is the natural frequency of the fundamental vibration mode. Principally, λ_L 's cannot be negative if $[K]$ and $[M]$ are obtained by suitable discretization. It might be more likely to happen that numerical errors included in $[K]$ and $[M]$ make one or a few of λ_L 's of a complex-number which causes the instability shown above.

Chapter 5

Analysis of Strong Ground Motion

The prediction of strong ground motion distribution is essential for earthquake engineering. It serves to provide an input wave at a specific site for earthquake resistant design even though past records of large strong ground motion are usually used for the purpose of design. It also serves to predict earthquake hazards from which possible earthquake disasters will be estimated; such an estimate is used as basic data in planning disaster mitigation against earthquake disasters. Higher spatial and temporal resolution¹ is desirable for the prediction of strong ground motion. Numerical simulation of the whole wave propagation processes from a fault to a target site is a possible tool which is used to achieve such high resolution.

There are two difficulties in numerically simulating wave propagation processes, namely, the requirement of huge computer resources and the uncertainty of underground structures; see [Ichimura and Hori (2000)]. The first difficulty is easily understood if the wave propagation in a cube of the order of $10 \times 10 \times 10$ [km] for a time duration of 40 [sec] is simulated. Table 5.1 shows a rough estimate² of the amount of memory which is required for a finite difference method (FDM), a finite element method (FEM) and a boundary element method (BEM) to solve this problem with the spatial and temporal resolution of 10 [m] and 0.1 [sec]. Although BEM appears acceptable, it must run a large number of computation steps which increases almost proportionally to the square of the number of elements which discretize unknown functions. The second difficulty may be more serious, since accurate modeling is essential for a reliable numerical simulation. Available information, however, is limited for underground structures, i.e., geological structure and ground structures. Because of these two difficulties, the latest numerical analysis is able to assure the precision of the computation with the time resolution up to slightly less than 1 [sec] only.

A new analysis method has been proposed for numerically simulating the whole wave propagation processes of an earthquake, in order to achieve highest spatial

¹For practical purposes, the time resolution up to the order of 0.1 [sec] is needed, and hence, the spatial resolution up to the order of 1 [m] is needed as well.

²More detailed explanation is presented in the Ph.D. dissertation of [Ichimura (2001)]; see also [Ichimura and Hori (1998); Ichimura and Hori (2000)].

Table 5.1 An estimate of memory required for numerical simulation of an earthquake wave.

	FEM	FDM	BEM
required memory [MB]	5×10^3	5×10^3	1

and temporal resolution in estimating strong motion distribution. This method is called a *macro-micro analysis method*. In the macro-micro analysis, the above two difficulties are solved by taking advantage of the singular perturbation expansion which leads to efficient multi-scale analysis and the bounding medium theory which provides optimistic and pessimistic models for underground structures by taking into consideration their uncertainty.

This chapter presents the macro-micro analysis method. Clarifying a problem setting which uses stochastic modeling of underground structures, the bounding medium theory and the singular perturbation expansion are first explained. A simple linear elastic problem at a quasi-static state is used as an example. The macro-micro analysis method is then constructed. Several numerical simulations are carried out to validate the macro-micro analysis method together with the bounding medium theory and the singular perturbation expansion, and the results of the simulations are discussed.

5.1 Stochastic Modeling of Underground Structures

For simplicity, it is assumed that the target underground structure, B , which include geological structures and surface layers, is linearly elastic and isotropic; Young's moduli and Poisson's ratio, denoted by E and ν , are parameters for elasticity and the density is denoted by ρ . This B consists of a set of subdomains, denoted by $\{B^\alpha\}$; each B^α is heterogeneous, but the configuration of the subdomain and the distribution of material properties are not precisely given. There remains some uncertainty about B . Furthermore, all B^α 's have a more or less common value for the length scale of heterogeneity, which is much smaller than the length scale of the entire B .

Now, a stochastic model is constructed for this uncertain B ; see Fig. 5.1. As explained in Sec. 3.1, a random function is used for the parameters of material properties. For simplicity, ν is homogeneous in B and only E varies in each B^α according to a suitable probability distribution function. The density changes for each B^α but is given in a deterministic manner. A non-dimensional parameter, $\varepsilon (\ll 1)$, is introduced to define the relative size of heterogeneity with respect to the dimension of B . The elasticity tensor field and the density field of B are thus denoted by

$$c_{ijkl}^\varepsilon(\mathbf{x}, \omega) = E^\varepsilon(\mathbf{x}, \omega) h_{ijkl} \quad \text{and} \quad \rho^\varepsilon(\mathbf{x}),$$

where $\mathbf{x}$ and ω are a position vector and a probabilistic event, and h_{ijkl} is a fourth-

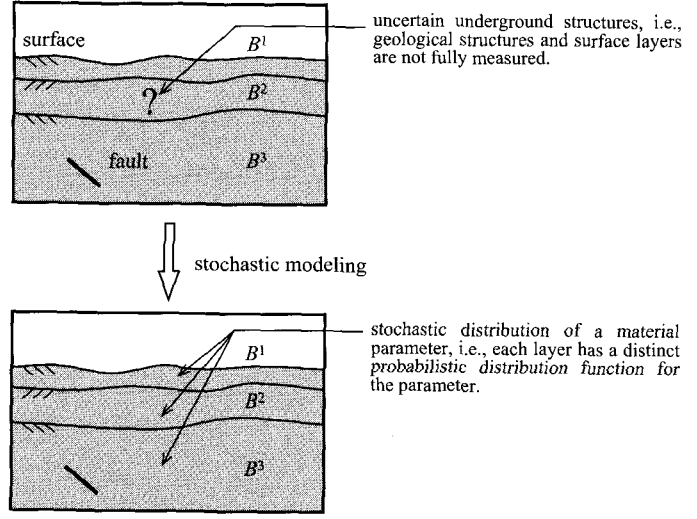


Fig. 5.1 A stochastic model of the uncertain underground structure B .

order tensor given by ν as

$$h_{ijkl} = \frac{\nu}{(1+\nu)(1-2\nu)} \delta_{ij} \delta_{kl} + \frac{1}{2(1+\nu)} I_{ijkl}$$

with $I_{ijkl} = \frac{1}{2}(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk})$ being the fourth-order symmetric identity tensor. Superscript ε is put to emphasize the relative length scale. The governing equation for displacement, denoted by u_i^ε , is the following wave equation:

$$d_i(c_{ijkl}^\varepsilon(\mathbf{x}, \omega) d_l u_k^\varepsilon(\mathbf{x}, t, \omega)) - \rho^\varepsilon(\mathbf{x}) \ddot{u}_j^\varepsilon(\mathbf{x}, t, \omega) = 0. \quad (5.1)$$

Here, d_i stands for the derivative with respect to x_i , i.e., $d_i = \partial / \partial x_i$.

5.2 Bounding Medium Theory

The bounding medium theory is briefly explained in Sec. 3.2 for a two-dimensional anti-plane shear problem. This section presents a more sophisticated bounding medium theory which analyzes a stochastic variational problem in a full three-dimensional setting. That is, the following governing equation for B at a quasi-static state is considered:

$$d_i(c_{ijkl}^\varepsilon(\mathbf{x}) d_l u_k^\varepsilon(\mathbf{x})) = 0 \quad \text{in } B. \quad (5.2)$$

Here, c_{ijkl}^ε is set as a deterministic but spatially heterogeneous elasticity tensor for a while. It is assumed that displacement boundary condition is prescribed.

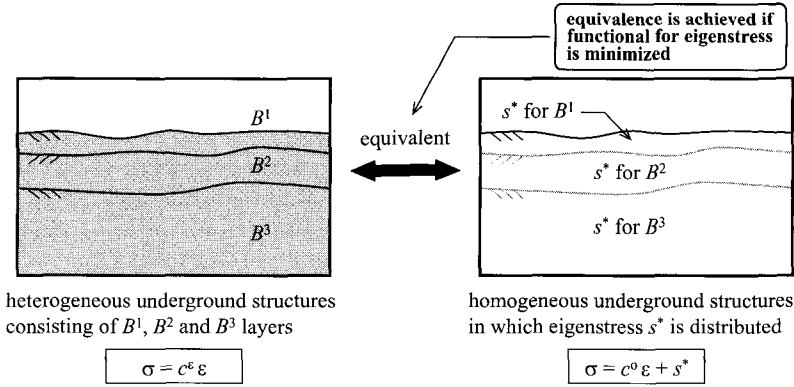


Fig. 5.2 The summary of the Hashin-Shtrikman variational principle.

Two approximate solutions are found for Eq. (5.2) so that the total strain energy stored in B is bounded. These bounds³ are derived from the *Hashin-Shtrikman variational principle*; see [Hashin and Shtrikman (1962a)]. In this principle, an ideally homogeneous body with the same configuration as B is introduced. When subjected to the same displacement boundary condition, the strain and stress of this body are different from those of the original body. By distributing eigenstress in the homogeneous body, however, it is possible to make the stain and stress coincide with those of the original body; eigenstress, denoted by σ_{ij}^* , disturbs the constitutive relations of the homogeneous body as

$$\sigma_{ij}(\mathbf{x}) = c_{ijkl}^o \epsilon_{kl}(\mathbf{x}) + \sigma_{ij}^*(\mathbf{x})$$

where c_{ijkl}^o is the elasticity of the homogeneous body. The presence of σ_{ij}^* changes stress as well as strain, and there exists a suitable distribution which makes stress and strain in the homogeneous body apparently related through the heterogeneous elasticity of B ; see Fig. 5.2. The Hashin-Shtrikman principle is used to find this eigenstress, with the aid of the following functional for σ_{ij}^* :

$$J(\sigma^*; c^e; c^o) = \int_B \frac{1}{2} \sigma_{ij}^* ((c_{ijkl}^e - c_{ijkl}^o)^{-1} \sigma_{kl}^* - \epsilon_{ij}^d(\sigma^*) - 2\epsilon_{ij}^h) dv. \quad (5.3)$$

Here, ϵ_{ij}^d is strain due to σ_{ij}^* with zero displacement boundary conditions, and ϵ_{ij}^h is strain caused by the prescribed boundary conditions in the absence of σ_{ij}^* . Indeed, the variation of J is

$$\delta J = \int_B \delta \sigma_{ij}^* ((c_{ijkl}^e - c_{ijkl}^o)^{-1} \sigma_{kl}^* - (\epsilon_{ij}^d + \epsilon_{ij}^h)) dv,$$

³This principle is a little bit complicated, as it uses a fictitious homogeneous body to solve a heterogeneous body problem. The key point is that some analytic solutions are obtained for the heterogeneous body problem by solving the homogeneous body problem; see [Hashin and Shtrikman (1962b); Hashin and Shtrikman (1963)]; see also [Willis (1977); Willis (1991); Willis (1992)] and [Hori (1994); Nemat-Nasser and Hori (1998); Hori and Munashinge (1999)].

from which $c_{ijkl}^o \epsilon_{kl} + \sigma_{ij}^* = c_{ijkl}^e \epsilon_{kl}$ with $\epsilon_{ij} = \epsilon_{ij}^d + \epsilon_{ij}^h$ is derived. This is the condition in which $\sigma_{ij} = c_{ijkl}^o \epsilon_{kl} + \sigma_{ij}^*$ and ϵ_{ij} are apparently related through c_{ijkl}^e . Denoting the strain energy stored in B by $\mathcal{E}(c^e)$, this functional gives upper and lower bounds for $\mathcal{E}(c^e)$ as follows:

$$\mathcal{E}^{o-} - J(\sigma^*; c^e; c^{o-}) < \mathcal{E}(c^e) < \mathcal{E}^{o+} - J(\sigma^*; c^e; c^{o+}). \quad (5.4)$$

Here, $\mathcal{E}^{o\pm}$ is the total strain energy stored in the homogeneous body in the absence of σ_{ij}^* , and the elasticity $c_{ijkl}^{o\pm}$ is chosen so that $c_{ijkl}^e - c_{ijkl}^{o+}$ becomes negative-definite and $c_{ijkl}^e - c_{ijkl}^{o-}$ becomes positive-definite.

When c_{ijkl}^e varies stochastically, the Hashin-Shtrikman variational principle can be extended to define two fictitious but deterministic media so that the media provide upper and lower bounds for the mean of the stochastic model behavior in the sense that the mean of the total strain energy is bounded. The following functional is used for this purpose:

$$J^\Omega(\sigma^*(\omega); c^e(\omega); c^o) = \int_\Omega J(\sigma^*(\omega); c^e(\omega); c^o) P(d\omega); \quad (5.5)$$

see Eq. (3.5). Inequalities similar to Eq. (5.6) hold for this J^Ω when suitable $c_{ijkl}^{o\pm}$ is chosen. That is,

$$\mathcal{E}^{o-} - J^\Omega(\sigma^*(\omega); c^e(\omega); c^{o-}) < \langle \mathcal{E} \rangle < \mathcal{E}^{o+} - J^\Omega(\sigma^*(\omega); c^e(\omega); c^{o+}). \quad (5.6)$$

Note that $\mathcal{E}^{o\pm}$ is deterministic since $c_{ijkl}^{o\pm}$ is deterministic.

Two fictitious but deterministic elasticity tensor fields are derived from Eq. (5.6) so that the bounding media are constructed. By assuming a sectionally uniform distribution of σ_{ij}^* , the above J^Ω is analytically⁴ evaluated, if the distribution of Young's modulus is statistically⁵ isotropic. The evaluated value provides the following functions for Young's modulus and Poisson's ratio:

$$E^{\epsilon\pm} = \left(\frac{3}{1-\nu} \frac{\langle \frac{E^e}{1+s_2 r^{o\pm}} \rangle}{\langle \frac{1}{1+s_2 r^{o\pm}} \rangle} \right) \left(3 + \frac{\langle \frac{E^e}{1+s_2 r^{o\pm}} \rangle \langle \frac{1}{1+s_1 r^{o\pm}} \rangle}{\langle \frac{E^e}{1+s_1 r^{o\pm}} \rangle \langle \frac{1}{1+s_2 r^{o\pm}} \rangle} \right)^{-1},$$

$$\nu^{\epsilon\pm} = \left(3 - 2 \frac{\langle \frac{E^e}{1+s_2 r^{o\pm}} \rangle \langle \frac{1}{1+s_1 r^{o\pm}} \rangle}{\langle \frac{E^e}{1+s_1 r^{o\pm}} \rangle \langle \frac{1}{1+s_2 r^{o\pm}} \rangle} \right) \left(6 + 3 \frac{\langle \frac{E^e}{1+s_2 r^{o\pm}} \rangle \langle \frac{1}{1+s_1 r^{o\pm}} \rangle}{\langle \frac{E^e}{1+s_1 r^{o\pm}} \rangle \langle \frac{1}{1+s_2 r^{o\pm}} \rangle} \right)^{-1},$$

where $r^{o\pm} = \frac{E^e}{E^{o\pm} - 1}$, $s_1 = \frac{1+\nu}{3(1-\nu)}$ and $s_2 = \frac{2(4-5\nu)}{15(1-\nu)}$. The two fictitious but deterministic bounding media, denoted by $B^\pm$, are thus determined by using the above $E^{\epsilon\pm}$ and $\nu^{\epsilon\pm}$. Indeed, the isotropic elasticity field of $B^\pm$ is given as

$$c_{ijkl}^{\epsilon\pm}(x) = E^{\epsilon\pm}(x) h_{ijkl}^{\epsilon\pm}(x), \quad (5.7)$$

⁴This analysis is slightly difficult, since the distribution of σ_{ij}^* must be computed for each realization of c_{ijkl}^e .

⁵Statistical isotropy means that a probabilistic characteristics such as the correlation function do not depend on the direction.

with

$$h_{ijkl}^{\varepsilon\pm} = \frac{\nu^{\varepsilon\pm}}{(1 + \nu^{\varepsilon\pm})(1 - 2\nu^{\varepsilon\pm})} \delta_{ij} \delta_{kl} + \frac{1}{2(1 + \nu^{\varepsilon\pm})} I_{ijkl}.$$

The elasticity field given by Eq. (5.7) provides sharper bounds for the mean behavior, compared with Young's modulus which is derived in Sec. 3.2. Indeed, these bounds are obtained from J^Ω by using a looser condition for σ_{ij}^* . For instance, when deterministic σ_{ij}^* is used, J^Ω becomes

$$J^\Omega(\sigma^*; c^\varepsilon(\omega); c) = \int_B \frac{1}{2} \sigma_{ij}^* (\langle (c_{ijkl}^\varepsilon(\omega) - c_{ijkl}^o)^{-1} \rangle \sigma_{kl}^* - \epsilon_{ij}^d(\sigma^*) - 2\epsilon_{ij}^h) dv.$$

The probabilistic mean is applied only to $(c_{ijkl}^\varepsilon - c_{ijkl}^o)^{-1}$. When common ν is used for c_{ijkl}^ε and c_{ijkl}^o , this mean becomes

$$\langle (c_{ijkl}^\varepsilon - c_{ijkl}^o)^{-1} \rangle = \langle \frac{1}{E^\varepsilon - E^o} \rangle h_{ijkl}^{-1}.$$

The mean of $1/(E^\varepsilon - E^{o\pm})$ is analytically computed when unboundedly increasing E^{o+} or vanishing E^{o-} is used, i.e.,

$$\lim_{E^{o+} \rightarrow \infty} \langle \frac{E^\varepsilon E^{o+}}{E^\varepsilon - E^{o+}} \rangle = -E^{\varepsilon+} \quad \text{and} \quad \lim_{E^{o-} \rightarrow 0} \langle \frac{1}{E^\varepsilon - E^{o-}} \rangle = \frac{1}{E^{\varepsilon-}},$$

where

$$E^{\varepsilon+} = \langle E^\varepsilon \rangle \quad \text{and} \quad \frac{1}{E^{\varepsilon-}} = \langle 1/E^\varepsilon \rangle.$$

These are the Voigt and Reuss bounds, and determine the elasticity tensor field of $B^\pm$ as

$$c_{ijkl}^{\varepsilon\pm}(x) = E^{\varepsilon\pm}(x) h_{ijkl}. \quad (5.8)$$

Note that while $c_{ijkl}^{\varepsilon\pm}$ of Eq. (5.8) is easily computed, it provides less sharper bounds for $\langle \mathcal{E} \rangle$ compared with $c_{ijkl}^{\varepsilon\pm}$ of Eq. (5.7).

5.3 Singular Perturbation Expansion

Now, it is supposed that two bounding media are determined; the bounding media are characterized with elasticity fields of $c_{ijkl}^{\varepsilon\pm}$. When the bounding media require a huge number of discretizations, the numerical computation becomes tremendous. For a smarter computation, the singular⁶ perturbation expansion is applied. To be specific, the following governing equation is considered as the object of study:

$$d_i(c_{ijkl}^\varepsilon(x) d_l u_k^\varepsilon(x)) = 0 \quad \text{in } B. \quad (5.9)$$

⁶The singular perturbation expansion is a mathematical tool which is often applied to some class of differential equations and initial or boundary value problems; see [Sanchez-Palencia (1981)], [Oleinik *et al.* (1992)], [Nemat-Nasser and Hori (1998)] and [Yuge and Kikuchi (1998)].

Here, for simplicity, superscript $\pm$ is dropped from $c_{ijkl}^{\pm}$. The singular perturbation expansion introduces a *slow* spatial variable, $\mathbf{X} = \varepsilon \mathbf{x}$, so that the spatial differentiation is replaced by

$$u_i^\varepsilon(\mathbf{x}) \approx u_i^{(0)}(\mathbf{X}, \mathbf{x}) + \varepsilon u_i^{(1)}(\mathbf{X}, \mathbf{x}) + \cdots; \quad (5.10)$$

see [Hori and Nemat-Nasser (1999)] for the singular perturbation expansion applied to mechanical problems. The introduced slow spatial variable is regarded as being taken on the geological length scale ($10^2 \sim 3$ [m]). Since B is a model for the underground structures, the slow spatial variable $\mathbf{X}$ varies in the entire B , while the ordinary spatial variable $\mathbf{x}$ varies in a small domain around $\mathbf{X}$, denoted by $B_{\mathbf{X}}$; the length scale of heterogeneity is $10^{-1} \sim 1$ [m], and hence ε is in a range of $10^{-3} \sim -1$. The derivative with respect to x_i is replaced by $d_i + \varepsilon D_i$ with $D_i = \partial/\partial X_i$. In view of this relation, Eq. (5.9) leads to the following expansion:

$$\begin{aligned} & \varepsilon^0 (d_i (c_{ijkl}^\varepsilon d_l u_k^{(0)})) \\ & + \varepsilon^1 (d_i (c_{ijkl}^\varepsilon (d_l u_k^{(1)} + D_l u_k^{(0)})) + D_i (c_{ijkl}^\varepsilon d_l u_k^{(0)})) \\ & + \varepsilon^2 (D_i (c_{ijkl}^\varepsilon (d_l u_k^{(1)} + D_l u_k^{(0)})) + d_i (c_{ijkl}^\varepsilon (d_l u_k^{(2)} + D_l u_k^{(1)}))) + \mathcal{O}(\varepsilon^3) = 0. \end{aligned}$$

In order to make the coefficients of ε^0 and ε^1 always vanish, it is set that $u_i^{(0)}$ is a function of $\mathbf{X}$ only and that $u_i^{(1)}$ is of the form of

$$u_i^{(1)}(\mathbf{X}, \mathbf{x}) = \chi_{ipq}(\mathbf{X}, \mathbf{x}) D_q u_p^{(0)}(\mathbf{X}), \quad (5.11)$$

where χ_{ipq} satisfies

$$d_i (c_{ijkl}^\varepsilon(\mathbf{X}, \mathbf{x}) (d_l \chi_{kpq}(\mathbf{X}, \mathbf{x}, t) + I_{klpq})) = 0; \quad (5.12)$$

recall that $I_{ijkl} = \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$ is the fourth-order symmetric identity tensor. The volume average over $B_{\mathbf{X}}$ is taken for terms which include $\mathbf{x}$ in the coefficient of ε^2 , i.e.,

$$\overline{D_i c_{ijpq}^\varepsilon (d_q \chi_{pkl} + I_{pqkl}) D_l u_k^{(0)}}(\mathbf{X}) = 0, \quad (5.13)$$

where $\overline{(\cdot)}$ stands for the volume average over $B_{\mathbf{X}}$, i.e.,

$$\overline{(\cdot)} = \frac{1}{B_{\mathbf{X}}} \int_{B_{\mathbf{X}}} (\cdot)(\mathbf{x}) \, dv_{\mathbf{x}}.$$

These equations, Eqs. (5.12) and (5.13), are the governing equation for the zeroth- and first-order terms in the singular perturbation expansion, $u_i^{(0)}$ and $u_i^{(1)}$, respectively.

A boundary condition is required to determine χ_{pij} in $B_{\mathbf{X}}$. However, the boundary condition cannot be exactly determined unless the solution u_i^ε is obtained. Instead of the exact boundary condition, the homogeneous strain and stress boundary conditions, $u_i = x_j E_{ji}$ and $t_i = n_j \Sigma_{ji}$ with constant E_{ij} and Σ_{ij} , respectively, are

used; these boundary conditions⁷ bound the strain energy which is stored in $B_{\mathbf{X}}$ when it is subjected to various boundary conditions, i.e.,

$$\overline{\frac{1}{2}\epsilon_{ij}^E c_{ijkl}^E \epsilon_{kl}^E} \leq \overline{\frac{1}{2}\epsilon_{ij}^G c_{ijkl}^E \epsilon_{kl}^G} \leq \overline{\frac{1}{2}\epsilon_{ij}^\Sigma c_{ijkl}^E \epsilon_{kl}^\Sigma}, \quad (5.14)$$

where superscripts E , G , and Σ stand for strains due to homogeneous strain, general, and homogeneous stress boundary conditions, respectively, and $\epsilon_{ij}^{E,G,\Sigma}$ satisfies

$$\overline{\epsilon_{ij}^E} = \overline{\epsilon_{ij}^G} = \overline{\epsilon_{ij}^\Sigma}.$$

Since the bounding media B^+ and B^- provide upper and lower bounds for the mean of the total strain energy, respectively, B^+ ought to use the homogeneous stress boundary condition to calculate χ_{pij} in $B_{\mathbf{X}}$, while B^- the homogeneous strain boundary condition. In this manner, the effective elasticity for $u_i^{(0)}$ can be defined as

$$\bar{c}_{ijkl}^\pm(\mathbf{X}) = \overline{c_{ijpq}^{\pm} (d_q \chi_{pkl}^{\Sigma,E} + I_{pqkl})}, \quad (5.15)$$

where χ_{pij}^Σ and χ_{pij}^E are computed for $c_{ijkl}^{\epsilon+}$ and $c_{ijkl}^{\epsilon-}$ with the homogeneous stress and strain boundary conditions, respectively. Finally, the following equation is derived for $u_i^{(0)}$:

$$D_i(\bar{c}_{ijkl}^\pm(\mathbf{X}) D_l u_k^{(0)}(\mathbf{X})) = 0. \quad (5.16)$$

Note that Eq. (5.16) replaces the governing equation of $u_i^{(0)}$, Eq. (5.13), since $\bar{c}_{ijkl}^\pm$ is approximately evaluated by using $\chi_{pij}^{\Sigma,E}$ which is determined by assuming the homogeneous stress and strain boundary conditions.

5.4 Formulation of Macro-Micro Analysis Method

The bounding medium theory and the singular perturbation expansion are now applied to wave propagation processes in the stochastic underground structure B . If relatively smaller frequencies are⁸ dominant in waves, the two bounding media, which are constructed for the stochastic model under the assumption of quasi-static state, can be used to solve this dynamic problem. Similarly, the effective moduli, which are determined by the multi-scale analysis of quasi-static state, can be used; the effective moduli of composites, which are measured at the quasi-static state, are used to analyze deformation at the dynamic state, when the frequency of interest is low so that the corresponding wavelength is larger than the size of composite constituents.

⁷See [Nemat-Nasser *et al.* (1978)] for the proof of the inequality for the total strain energy.

⁸It is certainly possible to define bounding media by considering inertia effects rigorously. The formulation of the bounding media at a dynamic state is essentially the same as that at a quasi-static state.

First, the elasticity tensor field for the bounding media, $B^\pm$, is determined. When the stochastic properties of Young's modulus are prescribed, the isotropic elasticity field of $B^\pm$ is given as Eq. (5.7) or (5.8). For simplicity, the latter bounding media are used, i.e., the Young's modulus is

$$E^{\varepsilon+}(\mathbf{x}) = \langle E^\varepsilon(\mathbf{x}) \rangle \quad \text{or} \quad \frac{1}{E^{\varepsilon-}(\mathbf{x})} = \left\langle \frac{1}{E^\varepsilon(\mathbf{x})} \right\rangle,$$

and the elasticity tensor field is $c_{ijkl}^{\varepsilon\pm} = E^{\varepsilon\pm} h_{ijkl}$, where h_{ijkl} is determined by Poisson's ratio ν which is not a random function. Next, the target of analysis is considered. The following singular perturbation expansion is taken for a displacement function of $B^\pm$:

$$u_i^\varepsilon(\mathbf{x}, t) = u_i^{(0)}(\mathbf{X}, t) + \varepsilon u_i^{(1)}(\mathbf{X}, \mathbf{x}, t),$$

with $\mathbf{X} = \varepsilon \mathbf{x}$ being the slow spatial variable.

The governing equation for the zero-th-order term, $u_i^{(0)}$, with respect to ε is now derived. When a suitable boundary condition is used, the effective elasticity on the length scale of the slow spatial variable, $\bar{c}_{ijkl}^\pm$, can be computed by using Eq. (5.15). The following equation is now derived from Eq. (5.13):

$$D_i(\bar{c}_{ijkl}^\pm(\mathbf{X}) D_l u_k^{(0)}(\mathbf{X}, t)) - \bar{\rho}(\mathbf{X}) \ddot{u}_j^{(0)}(\mathbf{X}, t) = 0 \quad (\text{for } \mathbf{X} \text{ in } B^\pm), \quad (5.17)$$

where $\bar{\rho}$ is the effective density, which is given as $\bar{\rho} = \rho^\varepsilon$ since the density is homogeneous in each layer.

Using $u_i^{(0)}$, the governing equation for the first-order term, $u_i^{(1)}$, is derived. The governing equation is similar to Eq. (5.12) and given as

$$d_i(c_{ijkl}^{\varepsilon\pm}(\mathbf{X}, \mathbf{x}) (d_l u_k^{(1)}(\mathbf{X}, \mathbf{x}, t) + D_l u_k^{(0)}(\mathbf{X}, t))) - \rho^\varepsilon(\mathbf{x}) \ddot{u}_j^{(1)}(\mathbf{X}, \mathbf{x}, t) = 0 \\ (\text{for } \mathbf{x} \text{ in } B_X^\pm). \quad (5.18)$$

The differential equation given by Eq. (5.17) or (5.18) is the governing equation for the *macro-analysis* or the *micro-analysis*, respectively. These equations form a pair of coupled equations for $u_i^{(0)}$ and $u_i^{(1)}$. As is seen, Eq. (5.18) appears more complicated than Eq. (5.17). However, Eq. (5.18) means⁹ that a small region $B_X^\pm$, which has varying elasticity $c_{ijkl}^{\varepsilon\pm}$ on the engineering length scale, is shaken by displacement on the geological length scale which accompanies spatially uniform strain in the region but changes in time.

It is interesting to take the perturbation expansion in a time domain. Since the slow spatial variable X_i is used, a slow time variable, $T = \sqrt{\varepsilon} t$, is introduced, and the singular perturbation expansion with respect to time as well as space is taken for displacement u_i^ε , i.e.,

$$u_i^\varepsilon(\mathbf{x}, t) \approx u_i^{(0)}(\mathbf{X}, T) + \varepsilon u_i^{(1)}(\mathbf{X}, \mathbf{x}, T, t) + \dots \quad (5.19)$$

⁹Although displacement is used as the input, Eq. (5.18) is similar to the governing equation of the dynamic response of a structure, to which ground motion is input.

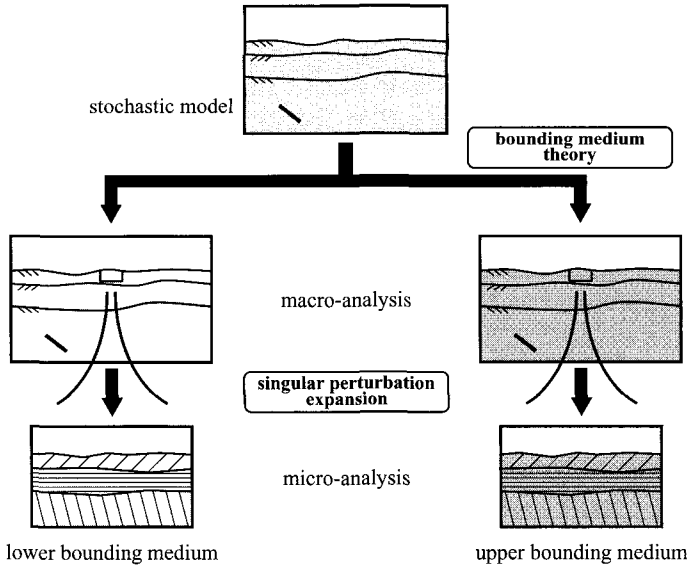


Fig. 5.3 A schematic view of the macro-micro analysis method.

Note that the square root for the time variable is used for T , in order to make interaction between $u_i^{(0)}$ and $u_i^{(1)}$ through inertia terms. Substitution of Eq. (5.19) into Eq. (5.1) leads to an equation similar to the one at a quasi-static state, and the governing equation for the first-order term with respect to ε can be derived, as

$$d_i \left(\bar{c}_{ijkl}^{\pm}(\mathbf{X}, \mathbf{x}) (d_l u_k^{(1)}(\mathbf{X}, \mathbf{x}, T, t) + D_l u_k^{(0)}(\mathbf{X}, T)) \right) - \bar{\rho}(\mathbf{x}) \left(d_t^2 u_j^{(1)}(\mathbf{X}, \mathbf{x}, T, t) + D_T^2 u_j^{(0)}(\mathbf{X}, T) \right) = 0, \quad (5.20)$$

where $D_T = \partial/\partial T$ and $d_t = \partial/\partial t$. It should be pointed out that these equations are essentially the same¹⁰ as the equation for scattering due to local heterogeneities; the scattered wave is represented by $u_i^{(1)}$ and an incident wave which is input to the heterogeneous body is given as $u_i^{(0)}$.

As is seen, the macro-micro analysis method produces two approximate solutions for wave propagation processes on different length scales when a stochastic model is given. These solutions are for the two fictitious bounding media and are computed by using the multi-scale analysis that takes advantage of the singular perturbation expansion; see Fig. 5.3 for the schematic view of the macro-micro analysis method. By definition, the approximate solutions given by the bounding media provide bounds only for the mean of the total energy which is stored in the stochastic model. Local responses may not be bounded. However, it is naively expected that the bounding media can bound local quantities such as the veloc-

¹⁰If another time scale is used, the resulting equations are different; for instance, $T' = \varepsilon t$ naturally leads to the damping on the larger spatial and slower time scales.

ity norm since it is related to the local energy density. At least, the macro-micro analysis method provides *optimistic* and *pessimistic* estimates for the mean of local behavior. Therefore, B^+ and B^- are called optimistic and pessimistic bounding media, respectively. Note that the optimistic and pessimistic estimates provided by $B^\pm$ have wider bounds for the mean of the stochastic response, if $B^\pm$ is constructed for a stochastic model with larger variability.

5.5 Verification of Macro-Micro Analysis Method

This section presents the results of numerical simulations which are carried out in order to validate the bounding medium theory and the singular perturbation expansion. The bounding medium theory itself is rigorous, but the validation is needed because the theory is used beyond the assumed condition of the medium being at a quasi-static state. The singular perturbation expansion is purely a mathematical technique, and hence the validation is aimed at examining the usefulness of applying the technique to the dynamic problem of wave propagation processes. Since the macro-micro analysis method is the combination of the bounding medium theory and the singular perturbation expansion, the limitation of these two influences the validity of the method. The validity of the macro-micro analysis method is discussed by making detailed investigations into the results of the numerical simulations that use the method.

5.5.1 Validation of bounding medium theory

The bounding medium theory assumes a quasi-static state for a target stochastic model. There should be some limitation in applying the theory to the stochastic model at a dynamic state. Indeed, it is questioned whether the bounding medium can bound the mean behavior at the dynamic state. To examine the applicability of the bounding medium theory to the dynamic state, Monte-Carlo simulation for a stochastic model of a simple spring is carried out as a reference, and the prediction of the bounding medium theory is compared with the Monte-Carlo simulation results.

The problem setting is as follows: a mass point with mass m is attached to a spring of spring constant k . While m is fixed, the value of k is uncertain; a normal distribution of mean μ and standard deviation (SD) σ is assumed as a probabilistic density function for this uncertain k . When deterministic loading f is given, the displacement of mass, denoted by u , satisfies

$$m \ddot{u}(t, \omega) + k(\omega)u(t, \omega) = f(t). \quad (5.21)$$

Here, argument ω emphasizes that k and u vary stochastically. An impulsive loading is used for f . The waveform of f is shown in Fig. 5.4; a) and b) are for the time domain and the frequency domain, respectively. The stochastic differential equation, Eq. (5.21), is numerically solved with the initial condition in which the

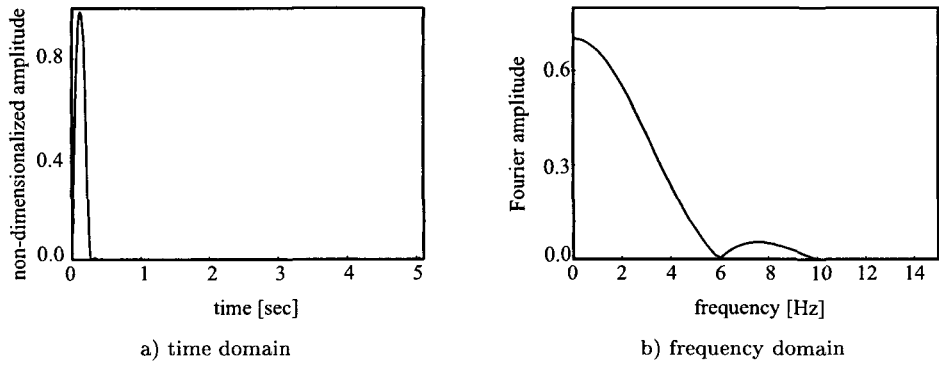


Fig. 5.4 An input wave used for the stochastic spring model.

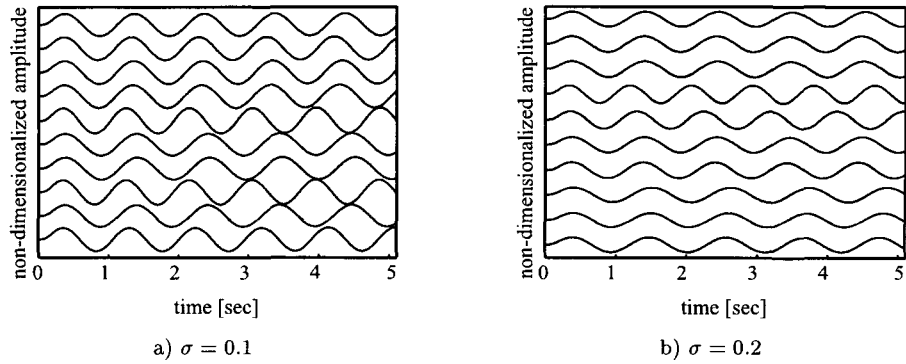


Fig. 5.5 Examples of displacement waveform of the stochastic spring model.

Table 5.2 The mean and standard deviation (SD) of the maximum acceleration obtained by the Monte-Carlo simulation of the stochastic spring model.

	mean	SD
$\sigma=0.1$	0.9424	0.04194
$\sigma=0.2$	0.9376	0.08535

spring stands still, i.e., the following stochastic initial value condition is solved:

$$\begin{cases} m \ddot{u}(t, \omega) + k(\omega)u(t, \omega) = f(t) & t > 0, \\ u(t) = 0 \text{ and } \dot{u}(t) = 0 & t = 0. \end{cases} \tag{5.22}$$

In the simulation, all quantities except for time are non-dimensionalized; the mean and SD of k are $\mu=1$ and $\sigma=0.1$ or 0.2 , the value of mass is $m=1$, and the maximum value of f is set as 1 . The frequency components over $10[\text{Hz}]$ are cut since the time step used in the numerical computation is $0.01[\text{sec}]$.

In Monte-Carlo simulation, $50,000$ samples are generated, and Eq. (5.22) is

Table 5.3 The values of a spring constant of bounding springs. The spring constants are computed as $k^+ = \langle k \rangle$ and $\frac{1}{k^-} = \langle \frac{1}{k} \rangle$.

	upper	lower
$\sigma=0.1$	1.000	0.9898
$\sigma=0.2$	1.000	0.9556

Table 5.4 Bounds for the maximum acceleration. The bounds are computed as the maximum acceleration of the two bounding springs.

	upper	lower
$\sigma=0.1$	0.9436	0.9397
$\sigma=0.2$	0.9436	0.9253

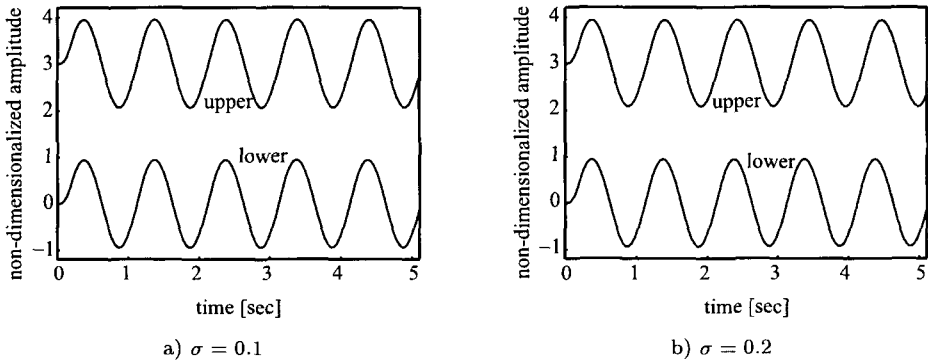


Fig. 5.6 Displacement waveform of the bounding springs.

solved for each of them. As an example of stochastic response of the spring, displacement waveform is presented in Fig. 5.5; a) and b) are for $\sigma=0.1$ and 0.2, and displacement waveforms of 10 samples are illustrated. While a vibration at the spring's natural frequency is dominant in the waveform, a vibration caused by the impulsive loading is observed. The results of this Monte-Carlo simulation are summarized in Table 5.2; the mean and SD of the maximum acceleration¹¹ are presented. The mean of $\sigma=0.2$ is smaller than that of $\sigma=0.1$, but the difference is small; the mean of $\sigma=0.2$ is decreased by around 2% of the mean of $\sigma=0.1$. As expected, the SD of the maximum acceleration of $\sigma=0.2$ is larger than that of $\sigma=0.1$. The SD is increased almost twice. It is seen that the response of the spring becomes more variable as the variability in the spring constant increases.

The two bounding springs, upper and lower, or optimistic and pessimistic, are

¹¹ Acceleration is more sensitive to input of higher frequency; if the Fourier transform of $u(t)$ is $\mathcal{F}u(\omega)$ with ω being frequency, the Fourier transform of acceleration, $\ddot{u}(t)$, is $-\omega^2 \mathcal{F}u(\omega)$, i.e., the increase of the transformed acceleration is linear to the square of the frequency.

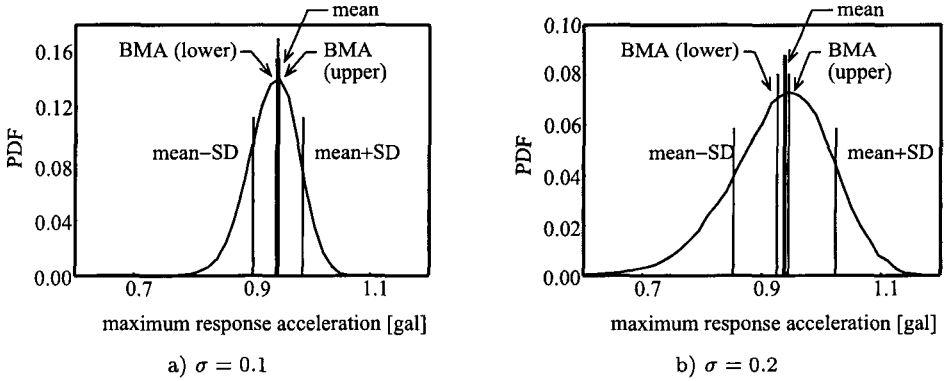


Fig. 5.7 A probability density function of the maximum acceleration of the stochastic spring model. Bounds obtained by the bounding springs are plotted for comparison.

easily constructed by applying the bounding medium theory. The spring constant of the bounding springs is

$$k^+ = \langle k \rangle \quad \text{and} \quad \frac{1}{k^-} = \left\langle \frac{1}{k} \right\rangle;$$

see Table 5.3 for the value of $k^\pm$. The displacement waveform of these two bounding springs is plotted in Fig. 5.6; the amplitude of displacement is magnified in this figure, compared with Fig. 5.5. The maximum acceleration computed by these springs is shown in Table 5.4. From the comparison with Table 5.2, it is shown that the bounding springs bound the mean of the stochastic model for the maximum acceleration.

Now, the prediction of the bounding medium theory is compared with the Monte-Carlo simulation results in more detail. To this end, the probability distribution of the maximum acceleration is investigated. A probability density function of the maximum acceleration is constructed from the Monte-Carlo simulation results, and it is presented in Fig. 5.7; a) and b) are for $\sigma=0.1$ and 0.2 , respectively. The mean and SD of the distribution are pointed out by vertical lines as well as the maximum acceleration of the bounding springs. As is seen, the distribution of the maximum acceleration appears slightly complicated than the normal distribution that is assumed for the distribution of the spring constant. The bounding springs bound the mean of this probabilistic distribution. While the bounding medium theory is based on the assumption that the stochastic spring is at the quasi-static state, the two spring models that are determined by the theory are applicable to the estimation of the mean of the maximum acceleration that is computed at the dynamic state.

As mentioned above, frequency components over $10[\text{Hz}]$ are cut since the time step of $0.01[\text{sec}]$ is used to discretize the displacement function. Hence, the results shown in Fig. 5.7 do not mean that there is no limitation in applying the bounding medium theory to dynamic problems. In the present problem, the vibration at the

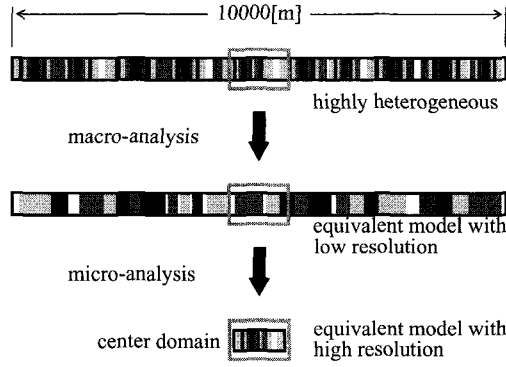


Fig. 5.8 A model of a heterogeneous bar with wildly varying Young's modulus.

natural frequency is dominant in the present stochastic spring since the input wave does not have much higher frequency components; see Fig. 5.4b). It is expected that if the input wave has more components of higher frequencies, the bounding springs do not bound a quantity such as the maximum acceleration.

5.5.2 Validation of singular perturbation expansion

The singular perturbation expansion is a purely mathematical technique of solving a differential equation with wildly varying coefficients. By nature of the perturbation expansion, it is not always guaranteed that the singular perturbation expansion yields a good approximate solution. In order to examine that the singular perturbation expansion is applicable to continuum mechanics problems, a simple example problem of a one-dimensional heterogeneous bar, denoted by B , is solved assuming a quasi-static state. The length of the bar is ℓ and heterogeneous Young modulus is denoted by E . Displacement, denoted by u , satisfies the governing equation,

$$\frac{d}{dx}(E(x) \frac{d}{dx} u(x)) = 0. \quad (5.23)$$

Together with the boundary condition of $u(x)=u^o(x)$ at $x=0, L$ with $u^o(0)=0$ and $u^o(L)=1$, Eq. (5.23) poses a boundary value problem for u . The key assumption is that E changes rapidly, i.e., on a much smaller length scale than L ; see Fig. 5.8.

It should be pointed out that the problem considered is different from a case when the change in E is small. In this case, the regular perturbation expansion is applicable to u . If E is rewritten as $E^0 + \varepsilon E(x)$ with constant E^0 and $\varepsilon \ll 1$, then, u is expanded as

$$u(x) = u^0(x) + \varepsilon u^1(x) + \varepsilon^2 u^2(x) + \cdots = \sum_{n=0} \varepsilon^n u^n(x).$$

It follows from Eq. (5.23) that u^0 satisfies the following boundary value problem:

$$\begin{cases} E^0 \frac{d^2}{dx^2} u^0(x) = 0 & 0 < x < L, \\ u^0(x) = u^o(x) & x = 0, L. \end{cases}$$

And u^n for $n=1, 2, \dots$ satisfies

$$\begin{cases} E^0 \frac{d^2}{dx^2} u^n(x) + \frac{d}{dx} (E(x) \frac{d}{dx} u^{n-1}(x)) = 0 & 0 < x < L, \\ u^n(x) = 0 & x = 0, L. \end{cases}$$

As is seen, a few terms of the expansion provide a good approximate solution for a smaller ε . The regular perturbation expansion, however, will need higher order terms to obtain a good approximate solution when ε is not small.

The singular perturbation expansion is aimed at obtaining a good approximate solution with a fewer terms when E changes on a small length scale and the magnitude of the change does not have to be small. A spatial slow variable, $X=\varepsilon x$, is introduced where ε is the relative length scale of heterogeneity with respect to L . The first two terms of the singular perturbation expansion is taken for u , i.e.,

$$u(x) \approx u^{(0)}(X) + \varepsilon u^{(1)}(X, x). \quad (5.24)$$

The zero-th-order term $u^{(0)}$ represents the overall displacement field, and the first-order term $\varepsilon u^{(1)}$ is the correction of $u^{(1)}$ that accounts for the local change in E . Note that although $u^{(0)} + \varepsilon u^{(1)}$ is of the order of ε^0 , it produces strain of order ε^1 , i.e.,

$$\begin{aligned} \frac{d}{dx} u(x) &\approx (\varepsilon D_X + d_x)(u^{(0)}(X) + \varepsilon u^{(1)}(X, x)) \\ &= \varepsilon (D_X u^{(0)}(X) + d_x u^{(1)}(X, x)) + \varepsilon^2 D_X u^{(1)}(X, x), \end{aligned}$$

where $d_x = \partial/\partial x$ and $D_X = \partial/\partial X$. As is seen, $\varepsilon u^{(1)}$ produces strain and hence stress which is of the same order as strain and stress produced by $u^{(0)}$.

Substitution of Eq. (5.24) into Eq. (5.23) results in the governing equations for $u^{(0)}$ and $u^{(1)}$. A suitable region B_X is taken around a point X ; this B_X is smaller than B but large enough for the change in the heterogeneity. By assuming the uniform strain or stress boundary condition¹² in B_X , the following boundary value problem is posed for $u^{(1)}$ in B_X :

$$\begin{cases} d_x (E(x)(d_x u^{(1)}(X, x) + D_X u^{(0)}(X))) = 0 & \text{in } B_X, \\ u^{(1)}(X, x) = 0 \quad \text{or} \quad d_x u^{(1)}(X, x) = 0 & \text{on } \partial B_X. \end{cases} \quad (5.25)$$

Since $D_X u^{(0)}$ is constant in B_X , the first-order term $u^{(1)}$ gives a local response due to the strain which is uniformly distributed there. The solution of Eq. (5.25) is thus

¹²Some attention must be paid to prescribing a boundary condition since the exact boundary condition is not known. According to the theory shown in Sec. 5.3, uniform strain and stress, which provide the minimum and maximum values to the strain energy among all admissible boundary conditions, are used for Eq. (5.25). A periodic boundary condition is usually used in analyzing composite materials.

expressed as

$$u^{(1)}(X, x) = \chi(X, x) D_X u^{(0)}(X),$$

where χ is computed by setting $D_X u^{(0)} = 1$ in Eq. (5.25). The effective Young modulus of B_X is then calculated; for instance, when the uniform strain boundary condition is prescribed, the average strain and stress of B_X are $D_X u^{(0)}$ and $\overline{E}(d_x \chi + 1) D_X u^{(0)}$, respectively, and hence the effective modulus is

$$\overline{E}(X) = \frac{1}{B_X} \int_{B_X} E(x) (d_x \chi(X, x) + 1) dx;$$

the same symbol B_X is used to denote the length and region. When $\overline{E}$ is determined, the governing equation for $u^{(0)}$ is derived from Eq. (5.23), and the following boundary value problem is posed for $u^{(0)}$:

$$\begin{cases} D_X(\overline{E}(X) D_X u^{(0)}(X)) = 0 & 0 < X < \varepsilon L, \\ u^{(0)}(X) = u^0(\frac{1}{\varepsilon} X) & X = 0, \varepsilon L. \end{cases} \quad (5.26)$$

As is seen, Eqs. (5.25) and (5.26) correspond to the micro-analysis and the macro-analysis, respectively.

In order to validate the applicability of the singular perturbation expansion, deformation of the rapidly varying heterogeneous bar is computed. The length is $L=10,000$ [m], and the bar consists of 10,000 segments of equal length 1[m]. Each segment has distinct Young's modulus, which is randomly distributed according to a normal distribution of the mean 1.0[MPa] and the standard deviation 0.2[MPa]; see Fig. 5.8. The value of ε is set to 10^{-1} , and X varies from 0 to 1000. A part near $X=500$ or $x=5,000$ [m] is used for B_X , i.e., $B_{500} = \{x | 4,950 < x < 5,050\}$. An exact displacement field, denoted by u^{exact} , is computed applying the finite difference method which uses the first-order linear element of length 1[m]. This u^{exact} is used as a reference, and compare displacement computed by means of the singular perturbation expansion with u^{exact} . The procedures of numerical analysis to compute this displacement are summarized as follows:

- i) Apply coarse discretization to the whole bar, which is divided into 1000 elements of equal length 10[m].
- ii) Using effective Young's modulus $\overline{E}$, solve the boundary value problem of the zero-th-order term $u^{(0)}$.
- iii) Apply finer discretization of the domain B_{500} of interest, which is divided into 100 elements of length 1[m].
- iv) Solve the boundary value problem of the first-order term $u^{(1)}$ in B_{500} .

First, the zero-th-order term of the singular perturbation expansion, $u^{(0)}$, is compared with u^{exact} . Computed $u^{(0)}$ and u^{exact} for B_{500} or $4,950 < x < 5,050$ [m] is plotted in Fig. 5.9; as shown in Eq. (5.25), the boundary condition for χ is either the uniform stress or strain, and macro (upper) or macro (lower) designates $u^{(0)}$

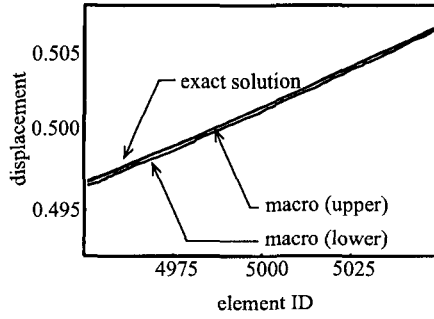
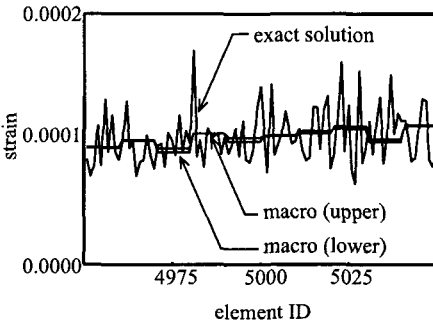
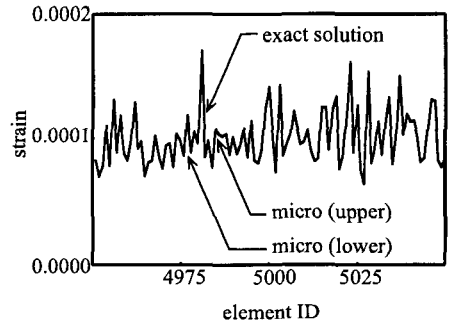


Fig. 5.9 The comparison of displacement obtained by singular perturbation expansion with the exact solution. The exact solution u^{exact} is numerically computed.



a) $\varepsilon D_X u^{(0)}$



b) $\varepsilon(D_X u^{(0)} + d_X u^{(1)})$

Fig. 5.10 The comparison of strain obtained by the singular perturbation expansion with the exact solution. The exact solution ϵ^{exact} is numerically computed.

which uses $\bar{E}$ obtained by χ of the uniform stress or strain boundary condition, respectively. As is seen, the agreement is satisfactory, and the relative difference of $u^{(0)}$ with respect to u^{exact} is less than 1%. This agreement is due to an accurate estimate of the effective moduli, $\bar{E}$, which is computed by considering the first-order term of the singular perturbation expansion, $u^{(1)}$.

Next, computed strain is compared; see Fig. 5.10 for strain distribution in B_{500} . The exact strain, denoted by $\epsilon^{\text{exact}} = d_x u^{\text{exact}}$, and strain associated with the zeroth-order term, $\varepsilon D_X u^{(0)}$, are plotted in Fig. 5.10a); macro (upper) or macro (lower) designates $\varepsilon D_X u^{(0)}$ which is computed by using the uniform stress or strain boundary condition. While $u^{(0)}$ agrees with u^{exact} very well, $\varepsilon D_X u^{(0)}$ of both bounding media does not reproduce the rapid change of ϵ^{exact} . In Fig. 5.10b), strain produced by the first-order-term of the singular perturbation expansion, $\varepsilon d_x u^{(1)}$, is added to $\varepsilon D_X u^{(0)}$; micro (upper) or micro (lower) designates $\varepsilon(D_X u^{(0)} + d_x u^{(1)})$ which is computed by using the uniform stress or strain boundary condition. The first-order-term, $\varepsilon u^{(1)}$, is negligibly small compared with $u^{(0)}$, and displacement of

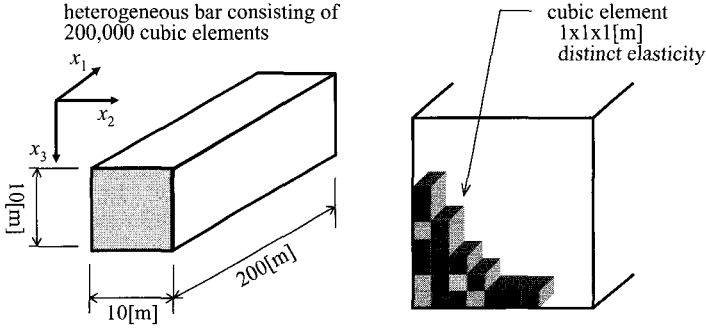


Fig. 5.11 A model used to validate the macro-micro analysis.

$u^{(0)} + \varepsilon u^{(1)}$ is almost the same as $u^{(0)}$. However, $\varepsilon d_x u^{(1)}$ is not negligible and producing rapidly changing strain. The agreement of $\varepsilon(D_X u^{(0)} + d_x u^{(1)})$ with ϵ^{exact} is almost perfect, and the relative error of $\varepsilon(D_X u^{(0)} + d_x u^{(1)})$ with respect to ϵ^{exact} is less than 0.5%.

It is thus concluded that the zero-th-order term $u^{(0)}$ of the singular perturbation expansion provides an accurate estimate of u^{exact} , although it fails in computing ϵ^{exact} ; see Fig. 5.10a). The first-order term $\varepsilon u^{(1)}$, which does not change displacement distribution, contributes to more accurate computation of strain; see Fig. 5.10b). The computation of strain at high spatial resolution needs the first-order term since strain associated with the zero-th-order term changes slowly. The multi-scale analysis that takes advantage of the singular perturbation expansion is useful to solve the mechanical problem of a body with wildly changing heterogeneities. Although the simple one-dimensional bar at a quasi-static state is studied, the results shown here suggest that the singular perturbation expansion is useful to analyze heterogeneous bodies with wildly changing material properties.

5.5.3 Validation of macro-micro analysis method

In order to validate the macro-micro analysis method, a numerical simulation is carried out to solve a simple dynamic problem of a stochastic model. A model is a rectangular bar, and the dimension is $200 \times 10 \times 10$ [m]. The bar is highly heterogeneous, consisting of 200,000 elements of $1 \times 1 \times 1$ [m] with distinct elasticity. While Poisson's ratio is common, the shear wave velocity v_s of an element obeys a normal distribution; see Fig. 5.11. When a plane wave passes, the wave is scattered due to the presence of the heterogeneities. Since a stochastic model is used, the scattered wave becomes stochastic as well. The present simulation is aimed at examining whether the macro-micro analysis method is capable of computing the effects of such scattered waves on the wave characteristic, say, on the maximum velocity. The maximum velocity at the center of the bar, $(x_1, x_2, x_3) = (100, 5, 5)$, is used to this end.

The probability distribution function of v_S is slightly complicated. The value is chosen from $\{120, 125, \dots, 165\}$ [m/sec] to make sharper contrast of the elements; the distribution of these ten values is similar to the bell shape of normal¹³ distribution, and the mean and SD are $\mu=140$ [m/sec] and $\sigma=14$ [m/sec]. The primary wave velocity is set as $v_P=\sqrt{2}v_S$.

At the left end of the bar, $x_1=0$, the half of a sinusoidal wave of period 2.0[sec] and amplitude 1[m] is prescribed. The paraxial boundary condition, which cuts the wave reflected at the right end ($x_1=200$), is prescribed, so that the wave put in the left end is forced to pass through the rectangular bar. This boundary condition is used to analyze wave propagation processes in a finite domain by removing unnecessary reflection at a boundary surface; see Appendix C.3 for more detailed explanation of the paraxial boundary condition; see also [Wolf (1988); Wolf (2002)]. Other boundary surfaces are traction-free, and some reflection is generated for waves which reach these surfaces.

To solve this stochastic bar problem, Monte-Carlo simulation is carried out to obtain the exact solution of the stochastic response. The number of test samples is 20,000, which are generated by assigning $v_P(=\sqrt{2}v_S)$ and v_S to each element according to the prescribed probability density function. In numerically computing one test sample, FEM is employed together with the central finite difference method for time integration. The model is spatially discretized by eight node cubic elements of $1 \times 1 \times 1$ [m], and the time increment is 0.002[sec]. With these spatial and temporal discretization, the accuracy of numerical computation is ensured up to 1[m] or 10[Hz]. The total duration of the computation time is 4.096[sec] (2048 steps). Note that a low pass filter of 10[Hz] is applied to time series of computed displacement, in order to cut higher frequency components.

The macro-micro analysis method is applied to obtain two bounding solutions of the stochastic problem of the highly heterogeneous bar. In view of the cross section of the bar and the size of the element, ε is set as $\varepsilon=10^{-1}$, and use a domain of $\{(x_1, x_2, x_3) | 80 < x_1 < 120, 0 < x_2, x_3 < 10\}$ for $B_{\mathbf{X}}$, with $\mathbf{X}$ corresponding to $\mathbf{x}=(100, 5, 5)$; the dimension of $B_{\mathbf{X}}$ is $40 \times 10 \times 10$ [m]. The procedures of the macro-micro analysis method are then summarized as follows:

- i) Construct two bounding media, by computing the bounding elasticity according to the bounding medium theory.
- ii) Carry out the macro-analysis, employing FEM to compute $u_i^{(0)}$ of the bar; 8-node cubic elements of the dimension $10 \times 10 \times 10$ [m] are used.
- iii) Carry out the micro-analysis, employing FEM to compute $u_i^{(1)}$ in $B_{\mathbf{X}}$; 8-node cubic elements of the dimension $1 \times 1 \times 1$ [m] are used.

The central finite difference method is applied for time integration; see Appendix C.1 for advanced techniques of time integration. Note that the effective velocities of $B_{\mathbf{X}}$

¹³Since 200,000 elements are used, assuming the normal distribution to v_S is not admissible; elements with negative shear wave velocity are generated.

Table 5.5 The comparison of the maximum velocity.

	MMAM -	MS	MMAM +
max. velocity [kine]	3.764	3.802	3.823
MMAM macro-micro analysis method +/- upper and lower bounding medium MS Monte-Carlo simulation (SD=0.01467[kine])			

are computed by using the effective elastic properties of B_X that are obtained for the uniform stress and uniform strain boundary conditions.

Attention must be paid to the accuracy of numerical computation of the macro-micro analysis method. Like FEM which is used for the Monte-Carlo simulation, the micro-analysis is capable to compute wave components up to 10[Hz]; this accuracy is due to the fine spatial discretization. The macro-analysis, however, does not ensure the accuracy over 1[Hz], because the spatial discretization is low. The frequency components of the macro-analysis have to be modified up to 10[Hz], in order to input these high frequency components to the micro-analysis. This is a shortcoming of the numerical computation of the macro-micro analysis method that uses different length scales. For a dynamic problem, the spatial resolution influences the time resolution as well. Since the spatial resolution determines the minimum value of the wavelength which can be calculated, a frequency component with wavelength shorter than this minimum value cannot be expressed in terms of shape functions which are used in FEM. However, it does not cause a fatal error in numerical computation to extrapolate the frequency components of the ensured accuracy to some extent, if relationship between low frequency components and high frequency components is known. In the present problem, this relationship is found by using the Monte-Carlo simulation results. The mean of the displacement spectra are computed by the Monte-Carlo simulation results, and this mean is used as an extrapolation function which links the macro-analysis outputs to the micro-analysis inputs.

Using the macro-micro analysis method, the maximum velocity in the x_1 -direction at the observation point (i.e., the center of the bar) is computed. The velocity obtained from the lower and upper bounding media are 3.764[kine] and 3.823[kine], respectively. The mean of the maximum velocity that is computed by the Monte-Carlo simulation is 3.802[kine], with the SD of 0.01467[kine]. Table 5.5 presents the value of the computed maximum velocity; MMAM- and MMAM+ are the maximum acceleration obtained by the macro-micro analysis method for the lower and upper bounding medium, respectively, and MM is the mean of the maximum acceleration obtained by Monte-Carlo simulation. It is clearly seen that the macro-micro analysis method provides upper and lower bounds for the mean of the maximum velocity.

In Fig. 5.12, the distribution of the maximum velocity which is obtained by the Monte-Carlo simulation is presented; the mean and SD are designated as vertical bars. The two estimates given by the macro-micro analysis method are plotted

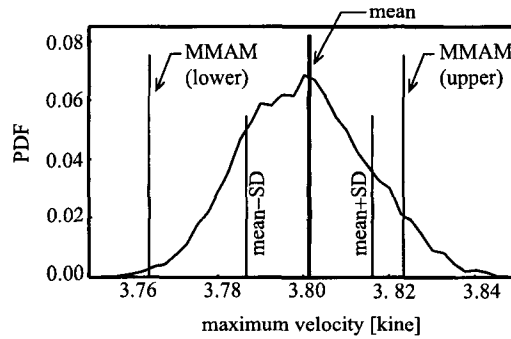


Fig. 5.12 The distribution of the maximum velocity computed by the Monte-Carlo simulation. Bounds obtained by the macro-micro analysis method (MMAM) are plotted for comparison.

in this figure. As is seen, the bound range that is obtained by the macro-micro analysis method (i.e., the difference of the maximum acceleration of the upper and lower bounding media) is almost three times larger than the SD of this distribution. The present bound range is much larger, compared with the bound range that is computed for the stochastic spring problem; see Figs. 5.12 and 5.7a). Note that the shear wave velocity distribution has the coefficient of variance¹⁴ (COV) $\sigma/\mu=10\%$, which is the same as the COV of k of the spring model. The COV of the maximum velocity is around 0.4%, while the COV of the maximum acceleration of the spring is around 2%. The reason for this small COV or small SD is not clear. However, the macro-micro analysis method is able to provide bounds for the mean, with the bound range being 1.5% of the mean.

The comparison with the Monte-Carlo simulation validates the macro-micro analysis method which is applied to the stochastic model of the wildly heterogeneous bar. While this does not fully guarantee that the validity of the macro-micro analysis method, it is expected¹⁵ that the macro-micro analysis method is applicable in estimating the mean behavior for dynamic problems of a stochastic model, if higher frequency components are not dominant in the response. It should be pointed out that the extrapolation of higher frequency components in linking the macro-analysis to the micro-analysis has a significant effect on the validity of the macro-micro analysis method. For an actual earthquake, such high frequency components are generated by the fault mechanism. Observation of actual data shows that each fault has a particular pattern for the Fourier spectral of emitted waves and that there

¹⁴The COV becomes 20% if the heterogeneity is measured as Young's modulus instead of the shear wave velocity.

¹⁵The present macro-micro analysis method involves the following two limitations: 1) the limitation of the bounding medium theory when it is applied to a stochastic model at a dynamic static state; and 2) the limitation of the singular perturbation expansion which uses the effective moduli for the zero-th-order term that is computed for the quasi-static state. Therefore, for frequency components over a certain frequency, the computation of the macro-micro analysis method cannot be accurate. This critical frequency needs to be identified.

is some relationship between lower frequency components and higher frequency components. Thus, the extrapolation can be made without causing fatal errors if observed data are used; a similar extrapolation has been made in conventional methods of predicting strong ground motion; see Appendix A.2.

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Chapter 6

Simulation of Strong Ground Motion

Establishing a reliable method of predicting strong ground motion distribution is one goal of earthquake engineering. With the aid of a more reliable method, a more rational estimate of earthquake hazards and a more efficient mitigation plan against earthquake disasters could be made. The improvement of earthquake resistant design codes is expected as well. The prediction method ought to predict distribution in a target site, area, or city with high accuracy and with high spatial and temporal resolution. Developing such a prediction method, however, is not an easy task; the method must take into full consideration the three major characteristics of earthquake, namely, the fault mechanism characteristics, the path characteristics along which wave propagates in the crust, and the amplification characteristics for ground structures; see Appendix A for the explanation of these three characteristics. In particular, three-dimensional topographical effects on the strong motion amplification, such as effects of complicated basin structures on the geological length scale or effects of unstratified surface layers on the engineering length scale, should be evaluated.

Among the three major characteristics mentioned above, the fault mechanism is not fully understood, since it involves dynamic rupture processes¹ of a fault; complicated temporal changes in non-uniform slip distribution are observed for actual faults. It is still difficult to predict a possible fault mechanism accurately. The other two characteristics are related to the wave propagation and amplification that are regarded as purely mechanical processes in which earthquake waves travel through the heterogeneous crust and ground structures. These processes can be numerically simulated if a suitable model is provided for the underground structures. When several scenarios of the failure mechanism are given, numerical simulation of the wave propagation and amplification processes with high accuracy and high resolution leads to an accurate estimate of possible strong ground motion distribution. Such simulation, which takes advantage of computational mechanics, will be a good candidate for a reliable method of predicting strong ground motion distribution.

As mentioned in Chapter 5, there are two serious difficulties in numerical com-

¹In seismology, it is pointed out that environmental effects on the failure, which includes underground water stored near the fault, make the fault mechanism more complicated.

putation of the wave propagation and amplification processes. The two difficulties are summarized as follows:

- 1) the difficulty in modeling underground structures;
- 2) the difficulty in carrying out large-scale computation.

The first difficulty is due to the fact that data which describe geological and ground structures are limited since in-site measuring is not easy. With limited data, modeling geological and ground structures as well as determining material properties of each layer will involve some uncertainty. The second difficulty is easily understood if the size of a target site is of the order of $10^3 \sim 4$ [m] and the spatial resolution required for the purpose of earthquake disaster mitigation is of the order of $10^{-1} \sim 0$ [m]. Huge discretization will be needed, resulting in extraordinary laborious computational efforts. However, the first difficulty is more serious than the second difficulty, since it cannot be expected that the layer configuration and material properties are accurately identified for actual geological and ground structures. Constructing an analysis model with high resolution is almost impossible. Due to these two difficulties, the wave propagation phenomena from a fault to the ground surface can be computed at most at the spatial resolution of 10^2 [m] when conventional numerical analysis methods are applied.

The bounding medium theory and the singular perturbation expansion explained in Chapter 5 have been invented in order to tackle the two difficulties mentioned above. When a stochastic model is constructed for underground structures as an alternative to a deterministic model, the bounding medium theory determines two models which provide optimistic and pessimistic estimates of the mean behavior of the stochastic model. The singular perturbation expansion leads to rigorous multi-scale analysis so that huge numerical computation of calculating earthquake waves with higher accuracy and resolution is separated into two coupled numerical computations. The method of predicting strong ground motion distribution that takes advantage of the bounding medium theory and the singular perturbation expansion is the macro-micro analysis method. A computer code of the macro-micro analysis method has been developed, and it is able to compute strong ground motion distribution at the spatial resolution of 10^0 [m].

In this chapter, the basic validity of the macro-micro analysis method is examined by reproducing measured data of strong ground motion at higher spatial and hence higher temporal resolution. To this end, first, the overview of the macro-micro analysis method is presented. Next, the current computer code of the macro-micro analysis method is explained. The code is based on FEM with voxel element; this FEM uses elements of common size and properties and is capable of carrying out more efficient numerical computation than an ordinary FEM. Since rheology of soil near ground surface should be taken into consideration in order to ensure high accuracy in analyzing the wave amplification process, a simple but non-elastic constitutive model is implemented to the current computer code. Two actual earthquakes

are synthesized by using the computer code of the macro-micro analysis method. The comparison of the synthesized waves with measured data is made, in order to validate the developed macro-micro analysis method. Special attention is paid to the topographical effects of three-dimensional ground structures on strong ground motion distribution. The topographical effects change depending on the wave frequency, and hence simulation with high spatial and temporal resolution is needed for quantitative estimate of the topographical effects for higher frequency components, say, at most 10[Hz].

6.1 Summary of Macro-Micro Analysis Method

The object of the macro-micro analysis method is a stochastic model. This model is constructed as an alternative to a deterministic model, for a body whose configuration or material properties are not fully identified. The stochasticity of the model accounts for the uncertainty of the body. The analysis of the stochastic model is difficult since its response becomes stochastic. Thus, the macro-micro analysis method has the following two key features:

- 1) the bounding medium theory which provides two fictitious but deterministic bounding media for a stochastic model;
- 2) the singular perturbation expansion which efficiently solves differential equations with wildly changing coefficients.

The bounding medium theory is aimed at estimating the mean of the stochastic response. The two bounding media constructed by the theory provide upper and lower bounds for the mean of stochastic total strain energy stored in the stochastic model. In this sense, the bounding media give optimistic and pessimistic estimates of the mean response. The underground structures of the bounding media are complicated, and the singular perturbation expansion is used to compute wave propagation processes in each bounding medium. This expansion is a mathematical technique which leads to multi-scale analysis, i.e., analysis on long and short length scales. The analysis on long or short length scale, which corresponds to the geological or engineering length scale, is called macro-analysis or micro-analysis, respectively. The macro-analysis provides waves propagating in the entire bounding medium with lower spatial resolution, and the micro-analysis computes waves with higher spatial resolution for each small subdomain of the bounding medium by refining the waves of the macro-analysis.

Input data of the macro-micro analysis method is a scenario of a target earthquake, i.e., a fault mechanism. The macro-micro analysis method outputs optimistic and pessimistic estimates of possible strong ground motion distribution with high spatial resolution. In the stochastic model of the underground structures, strong ground motion distribution varies stochastically, and the possible strong ground motion distribution is the mean of such stochastic strong ground motion distribu-

tion. High spatial resolution² is achieved by applying the multi-scale analysis which first obtains a low-spatial-resolution solution for the whole underground structures and then refines the solution by considering local underground structures for every small site of interest.

The procedures of the macro-micro analysis method are thus summarized as follows:

- i) Construct a stochastic model for the crust and ground structures, by prescribing the mean and variance for the configuration, such as the location of the interface between neighboring layers, and the material properties, such as elasticity and density, for each layer.
- ii) Applying the bounding medium theory, determine two fictitious but deterministic models that provide optimistic and pessimistic estimates of the mean response of the stochastic model.
- iii) Applying the singular perturbation expansion, compute the optimistic and pessimistic estimates of the possible strong ground motion distribution in a target area. The singular perturbation expansion leads to the multi-scale analysis that consists of the macro-analysis with low spatial resolution and the micro-analysis with high spatial resolution. That is,
 - iii.i) the macro-analysis which obtains the zero-th-order term in the perturbation expansion for the whole underground with the spatial resolution of 100[m].
 - iii.ii) the micro-analysis: which obtains the first-order term in the perturbation expansion for each small site near the ground surface with the spatial resolution of 1[m].

In the final procedure, some care must be taken of the link from the macro-analysis to the micro-analysis; see Sec. 5.5. This is because the micro-analysis needs frequency components higher than the macro-analysis outputs as input data; this is a drawback of the multi-scale analysis when it is applied to dynamic problems.

6.2 VFEM for Macro-Analysis and Micro-Analysis

Conditions of computational environment which are required for the macro-analysis and the micro-analysis are summarized in Table 6.1. Three-dimensional underground structures of complicated configuration are the object of analysis, and non-linearity of surface soil layers needs to be considered. Thus, the macro-micro analysis method employs FEM to satisfy these conditions. FEM is suitable to

²Roughly speaking, the macro-analysis is similar to seismological earthquake simulation, and the micro-analysis corresponds to strong ground motion analysis above engineering bedrock mass. However, modeling is different since the macro-micro analysis method uses bounding media by constructing a stochastic model for underground, and linking of the two simulations with different length scales is made with the aid of the singular perturbation expansion.

Table 6.1 Conditions of computational environment required for macro-micro analysis method.

	macro-analysis	micro-analysis
dimension	3D	3D
layers	non-parallel	non-parallel
material	linearly elastic	non-linearly visco-elastic

numerically compute waves³ propagating in three-dimensional unstratified layers with some impedance contrast. Implementing numerical analysis for material non-linearity into FEM is not difficult, although such non-linear analysis is not used in the current version of the macro-micro analysis method.

6.2.1 VFEM

The major advantage of FEM is the flexibility in modeling a body with complicated geometry; see Sec. 2.1. The flexibility is due to the discretization scheme, which, in view of local geometry or configuration, expresses a displacement function by using small elements which are deformed to fit the local geometry. Such a discretization scheme transforms the local wave equation to the matrix equation with element stiffness matrix, and the wave equation for the whole body to the matrix equation of global stiffness matrix which is obtained by just assembling element stiffness matrices. As the drawback of this simple but effective discretization scheme, FEM needs a larger amount of computer resources to store the global stiffness matrix, the dimension of which reaches $10^6 \sim 8$. The storage of this large matrix is a drawback for large-scale computation.

To overcome this drawback of FEM, a voxel element⁴ has been developed. The concept of a voxel element is simple; it discretizes a target domain by using tiny cubic elements of identical configurations and properties. This cubic element is called a voxel element or voxel. The number of required voxel elements will be large. However, the voxel elements share a common element stiffness matrix, and hence it is not necessary to store a global stiffness matrix in memory space of a computer. This is because the global stiffness matrix is always and easily reproduced by using a common element stiffness matrix of the voxel elements. FEM with such voxel elements is called a voxel finite element method (VFEM); see Fig. 6.1 for the schematic view of VFEM. It should be emphasized that the global stiffness matrix of VFEM is the assemblage of a single element stiffness matrix of a voxel element when the body is homogeneous. When the body consists of several distinct layers, VFEM prepares a voxel element for each layer. It is still easy to form a global stiffness matrix using several element stiffness matrices of the voxel elements.

When a target body is linearly elastic and homogenous or sectionally homoge-

³In particular, a traction-free top surface with varying elevation level is a tough target of numerical computation. FEM is most suitable for this numerical computation.

⁴See [Ichimura (2001); Ichimura and Hori (1998)] for a list of references related to VFEM.

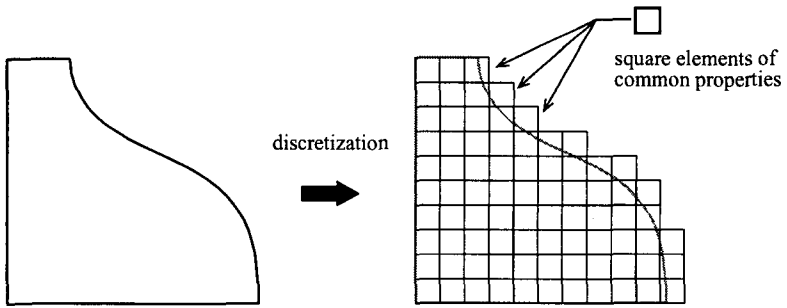


Fig. 6.1 The schematic view of VFEM.

neous, VFEM is an efficient tool for solving the wave equation for the body by means of large-scale computation, since the global stiffness matrix is not explicitly assembled and not stored in computer memory space. In addition, the highly efficient parallelization is achieved for VFEM. Such parallelization is due to the two algorithms that are implemented in VFEM, namely, the fictitious assemblage of the global matrix and the application of an iterative solver to solve the matrix equation; see Sec. 2.5 for the iterative solver.

In order to apply VFEM to the macro-analysis and the micro-analysis, a suitable length scale for discretization should be selected, so that the size of voxel element is determined. This is essential since there is a limitation in expressing a wave of shorter wavelength using cubic elements⁵ of the identical size. It is easily seen that VFEM is applicable to a body of irregular configuration just by employing suitably small voxel elements. In this case, the examination of the voxel size is important; it must be small enough to model the irregular configuration by means of the assembly of cubic elements. Also, it is important to check the efficiency of the paraxial boundary condition which is assigned on the artificial boundary of the voxel assembly in order to remove unnecessary reflection at the boundary; see Appendix C.3 for the paraxial boundary condition; see also [Wolf (2003)].

6.2.2 VFEM for macro-analysis

In the macro-analysis, VFEM is applied to solve the three-dimensional wave equation for linear elastic geological layers when a suitable fault mechanism is prescribed for a target fault. The accuracy of the macro-analysis that uses VFEM is examined. To this end, a simple problem which has an analytic solution is considered, and the numerical solution of the VFEM simulation is compared with the analytic solution. The problem is the propagation of a wave which is caused by a point explosion source. The domain is an infinitely extended body, and the solution satisfies the spherical symmetry; see Fig. 6.2. The following analytic solution is available for the

⁵While using common cubic elements is a key point of VFEM, it results in less accurate computation of solving a boundary value problem for a body of complicated configuration.

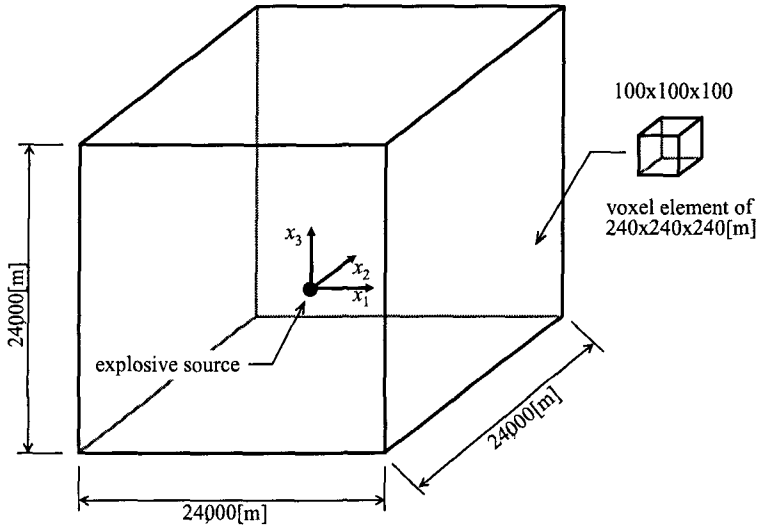


Fig. 6.2 A model computed by VFEM for the macro-analysis.

radial component u of displacement vector:

$$u^{\text{exact}}(r, t) = \frac{1}{r^2} F\left(t - \frac{r}{v_P}\right) + \frac{1}{rv_P} \dot{F}\left(t - \frac{r}{v_P}\right), \quad (6.1)$$

where r is the distance from the source, v_P is the primary wave velocity, and F is the reduced displacement potential; see [Lay and Wallace (1995)].

In numerical computation, a finite but large target domain is used; the domain is a cubic of $24,000 \times 24,000 \times 24,000$ [m], and is decomposed into an assembly of $100 \times 100 \times 100$ voxel elements. The dimension of a voxel element is $240 \times 240 \times 240$ [m], and the element has eight nodes and uses the shape function of the first-order, i.e., strain is expressed in linear function. As for the material property, $v_P = 5000$ [m/sec] is used, and a lumped mass matrix and a lumped damping matrix are used for simplicity. An impulsive wave of one sinusoidal of period 0.5 [sec] is used for the input wave F . The wave propagation is solved in the time domain, and time integration uses the Wilson θ method with time increment 0.01 [sec]; see Sec. 4.3. The paraxial boundary condition is applied on all surfaces of the cubic domain to remove unnecessary reflection; see Appendix C.3.

In Fig. 6.3, the time series of the radial displacement computed by VFEM is presented; a) and b) are for the radial displacement at $r = 3360$ [m] and 5760 [m], respectively. The analytic solution, u^{exact} , given by Eq. (6.1) is presented for the comparison. The value of displacement is normalized by the maximum value of the analytic solution at the point of $r = 3360$ [m]. No filter is applied in the time series of the computed displacement. As is seen, the agreement of the computed wave with u^{exact} is satisfactory. There are some discrepancies⁶ between the computed

⁶In Fig. 6.3b), the maximum displacement is overestimated by the VFEM simulation.

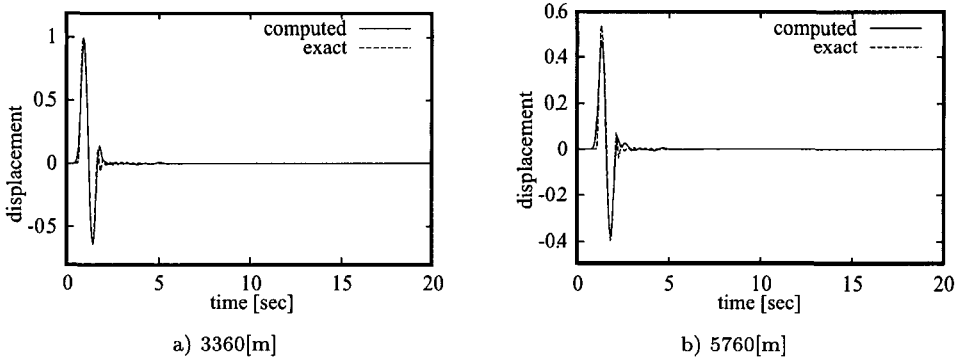


Fig. 6.3 The comparison of the numerical solution with the analytic solution in the time domain.

wave and u^{exact} in Fig. 6.3 at around $t=2[\text{sec}]$, which corresponds to passing of the incident wave. Such discrepancies, however, could be due to the numerical error which produces unrealistic high frequency waves. This numerical error is removed by means of a suitable low pass filter.

In numerical computation of wave propagation, it is known that the paraxial boundary condition could be less effective to remove reflected waves, when the direction of wave departs from the right angle with respect to the surface of the boundary. It is thus necessary to examine the ability for the paraxial boundary condition that is implemented in VFEM to remove reflected waves of low angles. In Fig. 6.3b), which presents the time series of the radial displacement at the measured point close to the boundary, there is no clear evidence that reflected wave arrives; only the incident wave is visible and no waves for $t>5[\text{sec}]$. The time series of the radial displacement shows that a very low amplitude reflected wave is coming from the boundary. Therefore, the ability of the paraxial boundary condition which is implemented in the present VFEM is satisfactory. Note that the agreement of the numerical simulation with the analytic solution, together with the good performance of the paraxial boundary condition, is not guaranteed if a more impulsive wave is used and higher frequency components become dominant. With the current setting, however, the present VFEM is able to accurately compute the wave propagation up to a few hertz.

The Fourier spectrum of both solutions is compared in order to examine the agreement of the numerical simulation with the analytic solution in more detail. For the numerical solution,

$$\mathcal{F}u(r, \omega) = \frac{1}{\sqrt{2\pi}} \int_0^T u(r, t) \exp(i\omega t) dt,$$

is used with $T=20[\text{sec}]$, and

$$\mathcal{F}u^{\text{exact}}(r, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} u^{\text{exact}}(r, t) \exp(i\omega t) dt$$

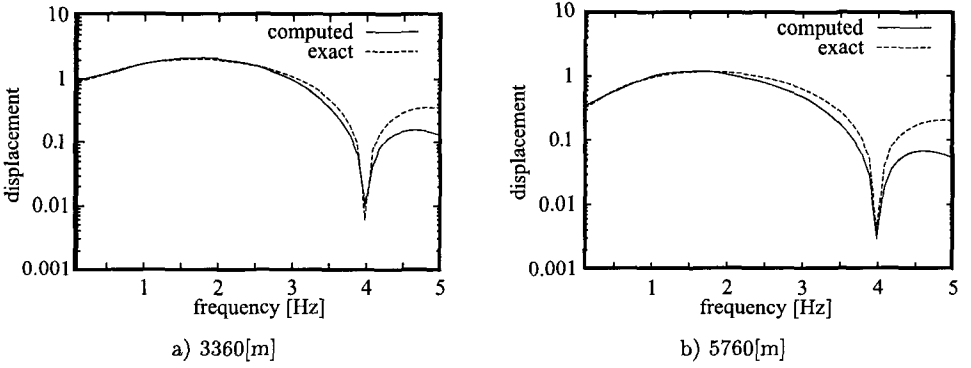


Fig. 6.4 The comparison of the numerical solution with the analytic solution in the frequency domain.

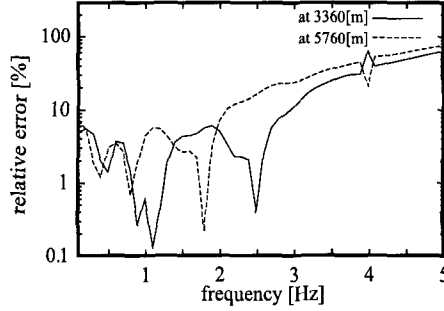


Fig. 6.5 A relative error of numerical solution, $|\mathcal{F}u - \mathcal{F}u^{\text{exact}}|/|\mathcal{F}u^{\text{exact}}|$.

is analytically computed for the analytic solution. The comparison in this frequency domain is shown in Fig. 6.3; again, a) and b) are for $r=3360[\text{m}]$ and $5760[\text{m}]$, respectively. It is seen that in a low frequency domain, $0 \sim 2[\text{Hz}]$, VFEM succeeds to compute $\mathcal{F}u^{\text{exact}}$ accurately. As expected, however, the agreement of $\mathcal{F}u$ with $\mathcal{F}u^{\text{exact}}$ becomes poorer for a higher frequency domain; for instance, the relative difference is few ten percent in a domain of $2 \sim 4[\text{Hz}]$, but it becomes approximately 100% in a domain of $4 \sim 5[\text{Hz}]$. To clarify the difference between the numerical solution and the analytic solution, a relative error, $|\mathcal{F}u - \mathcal{F}u^{\text{exact}}|/|\mathcal{F}u^{\text{exact}}|$, is computed, and it is plotted in Fig. 6.5. It is clearly seen that $\mathcal{F}u$ is accurate up to $2[\text{Hz}]$, since $|\mathcal{F}u - \mathcal{F}u^{\text{exact}}|/|\mathcal{F}u^{\text{exact}}|$ is a few percent at most. However, the relative error exceeds 10% for frequency higher than $2[\text{Hz}]$ or $3[\text{Hz}]$, for the case of $r=5760[\text{m}]$ or $3360[\text{m}]$, respectively. The wavelength of frequency component of $2[\text{Hz}]$ is $2500[\text{m}]$, which is approximately ten times larger than the size of the voxel element, $240[\text{m}]$. Hence, the source of the relative error shown in Fig. 6.5 is the spatial discretization of VFEM; the length scale of discretization becomes relatively coarse for higher

frequency components with a shorter wavelength. This result suggests⁷ that in the VFEM computation, the accuracy is guaranteed up to the frequency which has the wavelength ten times larger than the voxel element size.

6.2.3 VFEM for micro-analysis

In the micro-analysis, a key issue is the accurate computation of the wave amplification process that are caused by three-dimensional ground structures near the ground surface. At the high accuracy that the micro-analysis is aimed at achieving, the non-linear material properties of soil must be taken into consideration. This is because such non-linearity sometimes amplifies the strong ground motion much more than linear materials.

Although it is generally modeled as continuum, soil is actually a mixture of soil particles, water and air. The material properties of soil thus change drastically depending on various conditions, such as applied confining pressure, water contents or void ratio. Although soil can be modeled as elastic under sufficiently large confining pressure, its properties become quite complicated as the confining pressure decreases. In particular, the dynamic properties of soil exhibit large dependence on strain. Subjected to loads at ordinary strain rate with the dominant frequency being less than, say, 10^2 [Hz], soil is more or less elastic when strain remains up to the level of 10^{-4} . However, as strain increases from 10^{-4} to 10^{-2} , soil exhibits visco-elastic properties with non-negligible attenuation. Finally, soil tends to show large in-elastic deformation when strain becomes larger than 10^{-2} ; see [Ishihara (1996)].

In the micro-analysis, a hysteresis attenuation (HA) model is used as a constitutive relation for soil under dynamic state; see, for instance, [Kramer (1996)] for a list of related works. Although the HA model is the most primitive, it can express the key dynamic property of soil under the middle range strain ($\sim 10^{-2}$), i.e., the non-dependence of an attenuation constant on the frequency. The attenuation constant is the ratio between the work done by an external force and the strain energy stored during one cycle of loading. The HA model is expressed as linear constitutive relations in the frequency domain, and can accurately reproduce the complicated soil behavior under dynamic loading which includes cyclic loading with varying frequency.

The HA model is implemented into VFEM for the micro-analysis. The computation is made in the frequency domain, not in the time domain, in order to fully account for the viscosity or damping of the HA model. By making use of the Fast

⁷The lowest frequency that can be accurately computed by means of a numerical analysis method depends on the spatial and temporal discretization. In general, it is known that the lowest frequency is the one with the wavelength eight times larger than the discretization length and that the lowest frequency is the one with the period eight times larger than the time increment; this limitation of the frequency is due to the fact that discretizing a sinusoidal function in one period needs at least eight sections.

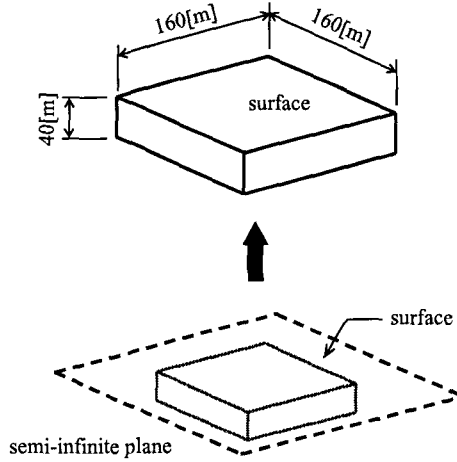


Fig. 6.6 A model computed by VFEM for the micro-analysis.

Fourier Transform, the computed response in the frequency domain is transformed into the time domain. To implement the HA model into VFEM, the formulation of the wave equation in the time domain is first considered. The three-dimensional wave equation that accounts for the HA model is formulated, as follows:

$$(c_{ijkl}(\mathbf{x})u_{k,l}(\mathbf{x},t)),_i + f_j(\epsilon(\mathbf{x})) - \rho(\mathbf{x})\ddot{u}_j(\mathbf{x},t) = 0. \quad (6.2)$$

Here, it is recalled that u_i is displacement, subscript following comma and dot stand for spatial and time differentiation ($(\cdot)_{,i} = \partial(\cdot)/\partial x_i$ and $\dot{(\cdot)} = \partial(\cdot)/\partial t$), and c_{ijkl} and ρ are elasticity and density. The second term, f_j , is an apparent body force which represents the attenuation of the HA model. In the time domain, f_j is a complicated non-linear function of strain rate. In the frequency domain, however, the amplitude of f_j is linear to the amplitude of displacement, $\mathcal{F}u_j = 1/\sqrt{2\pi} \int u_j \exp(i\omega t) dt$. The coefficient between f_j and $\mathcal{F}u_j$ does not depend on the frequency; recall that the coefficient between the viscous force and displacement is linear to the frequency in the Voigt model. In the frequency domain, Eq. (6.2) is thus expressed as follows:

$$(c_{ijkl}(\mathbf{x})\mathcal{F}u_{k,l}(\mathbf{x},\omega)),_i + i\bar{\eta}\mathcal{F}u_j(\mathbf{x},\omega) + \rho(\mathbf{x})\omega^2\mathcal{F}u_j(\mathbf{x},\omega) = 0. \quad (6.3)$$

As is seen, f_j is transformed to $i\bar{\eta}\mathcal{F}u_j$, and $\bar{\eta}$ is the modified attenuation constant of the HA model. The discretization of Eq. (6.3) is straightforward, i.e.,

$$[K][\mathcal{F}U] + i[C][\mathcal{F}U] - \omega^2[M][\mathcal{F}U] = [\mathcal{F}F], \quad (6.4)$$

where $[\mathcal{F}U]$ and $[\mathcal{F}F]$ are vectors for the transformed displacement and external force, respectively, and $[K]$, $[C]$ and $[M]$ are matrices for the stiffness, the modified attenuation, and the mass, respectively.

In order to examine whether VFEM is capable of accurately solving the wave equation with the HA model in the frequency domain, Eq. (6.3), a simple numerical

simulation is carried out using the discretized equation, Eq. (6.4). The object of analysis is a homogeneous semi-infinite domain, and it is assumed that a plane shear wave travels upwards. Even if the HA model is used, the analytic solution of displacement is obtained in the frequency domain, since Eq. (6.3) becomes a one-dimensional problem. Indeed, when the amplitude of the shear wave is set 1 and the direction is parallel to the x_3 -axis, the analytic solution is

$$\mathcal{F}u_i^{\text{exact}}(\mathbf{x}, \omega) = \begin{cases} \cos(kx_3) \exp(i\omega) & \text{for } i = 3, \\ 0 & \text{for } i = 1, 2, \end{cases}$$

where k is a complex-valued wave number, given as $k = \sqrt{(\rho\omega^2 - i\bar{\eta})/2\mu}$. In the numerical simulation, a part of the semi-infinite domain, with the dimension is $160 \times 160 \times 40$ [m], is used; see Fig. 6.6. As for the material properties, the primary wave velocity, the secondary wave velocity and the density are $v_P = 1040$ [m/sec], $v_S = 600$ [m/sec] and $\rho = 1800$ [kg/m³], respectively, and the modified attenuation constant is $\bar{\eta} = 0.02$ [tf/m²]. The domain of analysis is discretized by eight-node voxel elements of $2 \times 2 \times 2$ [m], and each voxel element has the first-order shape function. The paraxial boundary condition is used for the side and bottom faces of the domain; see Appendix C.3 for the paraxial boundary condition. A lumped mass matrix, which is explained in Sec. 4.3, is used, and $[C]$ is also set as a diagonal matrix; see Sec. 4.3. In this setting, displacement in frequency range between 0[Hz] and 2.5[Hz] with the resolution of 0.098[Hz] is computed.

In Fig. 6.7, the distribution of relative error of the numerical solution with respect to the exact solution, $(|\mathcal{F}u| - |\mathcal{F}u^{\text{exact}}|)/|\mathcal{F}u^{\text{exact}}|$, is plotted; a) and b) are for a lower frequency of $\omega = 1.27$ [Hz] and a higher frequency of $\omega = 2.44$ [Hz], respectively. The relative error is computed for the solution of the top surface, $x_3 = 0$. In general, the relative error for the higher frequency, $\omega = 2.44$ [Hz], is larger than that for the lower frequency, $\omega = 1.27$ [Hz]. However, the relative error is less than 0.1% even for the case of $\omega = 2.44$ [Hz], except for narrow zones close to the boundary. The relative error tends to increase near the surface edges, probably due to the paraxial boundary condition which fails to fully get rid of reflected waves. These two graphs show good agreement of the numerical solution with the analytic solution. Similar comparison is made for other frequencies, and similarly good agreement is obtained for frequencies less than 2[Hz].

It is shown that for the micro-analysis of the present setting of numerical computation, the accuracy of VFEM with the HA model is guaranteed up to 2[Hz]. The maximum relative error is at most 0.1%, if regions close to the boundary are omitted. In view of Fig. 6.7, it is suggested that a region around the center should be used for the micro-analysis; the dimension of the region is, say, 100×100 or 120×120 [m] at most, if the whole surface is 160×160 [m].

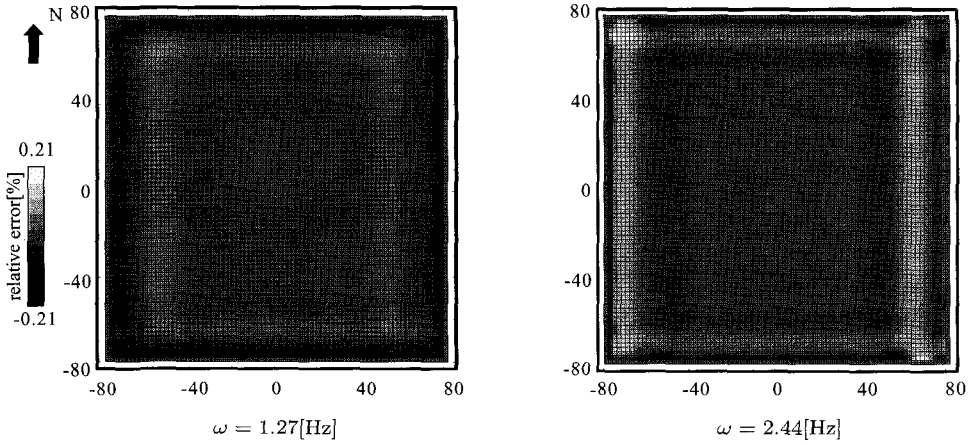


Fig. 6.7 Distribution of relative error, $(|\mathcal{F}u| - |\mathcal{F}u^{\text{exact}}|)/|\mathcal{F}u^{\text{exact}}|$, on the top surface.

6.2.4 Link from macro-analysis to micro-analysis

In the present macro-analysis method, it is possible to accurately compute frequency components up to 1.2[Hz] in the macro-analysis. The micro-analysis is aimed at computing frequency components up to 2.5[Hz]. Therefore, frequency components from 1.2[Hz] to 2.5[Hz] need to be extrapolated when the results of the macro-analysis is input to the micro-analysis. This extrapolation is essential for the macro-micro analysis method, in order to achieve the prediction of strong ground motion distribution up to 2.5[Hz]. As mentioned in Sec. 5.5, the link⁸ from the macro-analysis to the micro-analysis is an obstacle of the macro-micro analysis method that uses two different length scales for spatial discretization.

The extrapolation of the macro-analysis is made by taking advantage of past earthquake records. Most reliable extrapolation is the statistical mean of Fourier spectra which are taken for data of strong ground motion measured at a target site. This mean is called a *statistical spectrum*; see Appendix A.2; see also [Boore (1983)]. The procedures of making the statistical spectrum are summarized as follows:

- i) For accelerogram records measured near the ground surface, compute the Fourier spectrum up to the target frequency (2.5[Hz]).
- ii) Pull back the Fourier spectrum to the bedrock, assuming the stratified ground structure.
- iii) Take the mean of all pull back Fourier spectra, and determine the statistical spectrum.

From the statistical viewpoint, the statistical spectrum that is constructed in the

⁸The link from the macro-analysis to the micro-analysis does not mean that the macro-analysis is not able to compute higher frequency components. Even though the macro-analysis uses an underground model of low spatial resolution, it can compute higher frequency components if suitably fine discretization is made and if the fault mechanism with higher frequency is given.

above manner is the best source that can provide the most reliable extrapolation of higher frequency components for a given incident of lower frequency. However, it does not mean that the extrapolation based on the statistical spectrum is accurate. Attention must be paid to the variability of the Fourier spectra of the past records, in order to examine the accuracy, not the reliability, of the extrapolation based on the statistical spectrum.

Once the statistical spectrum is determined, linking from the macro-analysis to the micro-analysis is straightforward, although some calculation is required; see [Kamae *et al.* (1998)]. The procedures of linking are summarized as follows:

- i) Select points at which the macro-analysis results are input to the micro-analysis.
- ii) Calculate the accelerograms at the bedrock using the macro-analysis results.
- iii) Applying the statistical spectrum to extrapolate the accelerograms of ii), obtain new accelerograms for the micro-analysis.
- iv) Linearly interpolating the new accelerograms to other points, determine strong ground motion distribution which is input to the micro-analysis.

It should be emphasized that in the current macro-micro analysis method, the vertical distribution of strong ground motion is approximately computed by assuming the stratified or parallel layers. Thus, pulling back⁹ the macro-analysis results is done by using one-dimensional analysis, not the three-dimensional analysis which uses the model of the micro-analysis. This is because the spatial resolution of the macro-analysis is much coarser than that of the micro-analysis and the one-dimensional analysis does not make significant differences from the three-dimensional analysis.

6.3 Simulation of Actual Earthquakes

In order to verify the validity of the macro-micro analysis method, a numerical simulation of reproducing strong ground motion distribution which is caused by actual earthquakes is carried out. The target area is Yokohama City, and the domain of analysis is $40 \times 30 \times 70$ [km] (in the EW, NS, and UD directions), and two earthquakes, denoted by the case 1 and the case 2, are simulated; see Table 6.2 for the characteristics of these two earthquakes, which are provided by FREESIA in NIED (National Research Institute for Earth Science and Disaster Prevention). The results of the simulation are compared with the data which are observed by twelve seismographs located in the city; these seismographs belong to the Yokohama City strong motion network that has been operating since 1996. In Fig. 6.8, the epicenters of the two earthquakes and the location of the twelve observation sites are presented; the epicenter of the case 1 is in Tokyo Bay and that of the case 2 is

⁹Theoretically, the interpolation of the macro-analysis results should use the shape function which is employed for the numerical analysis of the macro-analysis. This does not produce a much more accurate estimate of earthquake waves propagating at the surface of the bedrock, compared with the present procedures of linking the macro-analysis to the micro-analysis.

Table 6.2 Characteristics of two target earthquakes.

	date	lat.	long.	depth [km]	strike [deg]	dip [deg]	rake [deg]	mag. [Mw]
case 1	08/11/1999	35.4N	129.8E	53	62	85	73	4.0
case 2	05/28/1999	35.5N	139.5E	38	283	70	112	3.5

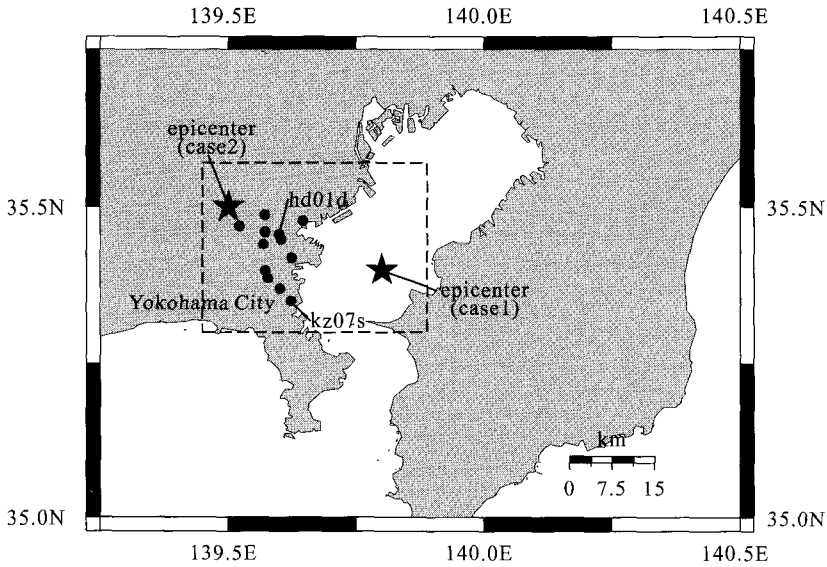


Fig. 6.8 The top view of Yokohama City. The epicenter of two target earthquakes and the location of twelve seismographs are indicated.

in inside of Kanto Plane, and the data measured at two sites of hd01d and kz07s are mainly used in the present study.

6.3.1 Modeling

A stochastic model is constructed for the underground structures beneath the city. The data used in constructing the model are as follows: 1) the geological structures presented by [Yamanaka *et al.* (1999)] for the southwestern part of Tokyo metropolitan area; and 2) the borehole data for ground structures provided by Yokohama City. The stochastic model of the geological structures is made for the whole city with the spatial resolution 1[km]. The model is a half space consisting of four distinct layers. The configuration of the interface between the layers is fixed, and a normal distribution is assumed for uncertain material properties, i.e., the primary and secondary wave velocities and the density; the mean is set to the value estimated by [Yamanaka *et al.* (1999)] and the coefficient of variance, the ratio of the

standard deviation to the mean, is set as 0.1 for all the parameters. The stochastic model of ground structures is made at twelve observation sites using the borehole data, which are available at grid of 50[m] distances in the EW and NS directions; the depth of boring hole is at most 40[m] and soil layers are discretized at 1[m] pitch. Since the configuration of soil layer interfaces is uncertain, an algorithm¹⁰ of minimizing curvature is used to construct the stochastic model. This algorithm interpolates the borehole data so that the interface between any two layers has the minimum curvature at every point. The uncertainty of the interpolated layer is prescribed by assuming disturbances to the interface, i.e., the depth of the interface is given as

$$d(x_1, x_2) + \delta d(x_1, x_2),$$

where d is the depth of the interface between two interpolated layers and δd is the disturbance which obeys a normal distribution of the mean and the coefficient of variance being 0[m] and 0.1, respectively. The material properties are fixed for each layer.

According to the bounding medium theory, two deterministic bounding media are constructed for the stochastic model, so that the optimistic and pessimistic estimates are provided for the mean of the stochastic model response. The bounding media are discretized for the numerical computation by using voxel elements. The spatial discretization is at most 40[m] for the macro-analysis. In this discretization, the uncertainty of the geological structures which results in the irregular interfaces of the geological layers is wiped out, and the difference of the two bounding media becomes negligibly small. Thus, only the optimistic medium is used for the macro-analysis. Figure 6.9 shows a model of the optimistic medium; the configuration of layer interfaces is presented and, as for the material properties, the primary and secondary wave velocities, v_P and v_S , and the density, ρ , of each layer are tabulated. For the micro-analysis, the spatial discretization is 2[m]; this length scale gives the spatial resolution of the micro-analysis. Hence, the location of the layers differs in this discretization, and two computer models are made for the bounding media of a stochastic ground structure model. As an example, two models for the micro-analysis of the bounding media are presented in Fig. 6.10; a) and b) are for optimistic and pessimistic media, respectively. The target is an area of 160×160[m] around the site hd01d, and the three-dimensional structures of the secondary wave velocity are shown.

Table 6.3 shows the computational characteristics for both the macro-analysis and the micro-analysis. As is seen, the scale of the macro-analysis is of relatively large, and the scale of the micro-analysis is much smaller than that of the macro-analysis; for instance, DOF of the micro-analysis is 1% of the macro-analysis. The number of the micro-analysis, however, is twelve; the micro-analysis is applied to

¹⁰See Sec. 11.3 for a more detailed explanation of constructing an underground model by using a set of limited data.

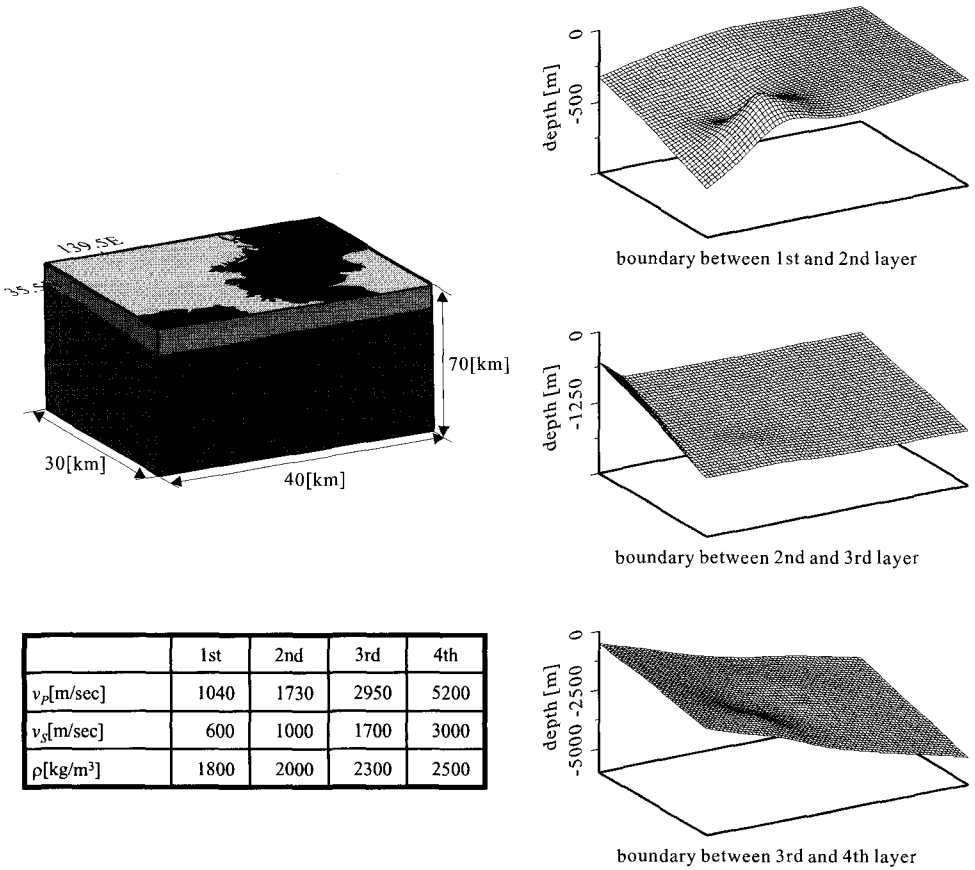


Fig. 6.9 A model used for the macro-analysis.

Table 6.3 Computational characteristics of the macro- and micro-analysis.

	macro-analysis	micro-analysis
element	40x40x40[m]~240x240x240[m]	2x2x2[m]
DOF	57,012,396	413,343
steps	6,000 (0.01[sec])	6,200 (0.0098[sec])
memory	4.4[Gb]	200[Mb]

DOF: total degree-of-freedom

a site for each observation site. Also, the HA model is implemented in the micro-analysis so that the material non-linearity in soil layers is taken into consideration. The material parameter of the modified attenuation constant is set as $\bar{\eta}=0.02[\text{tf/m}^2]$.

In this simulation, a point source model with a ramp function is used as the source of the earthquake waves, because the fault is located at a sufficiently large

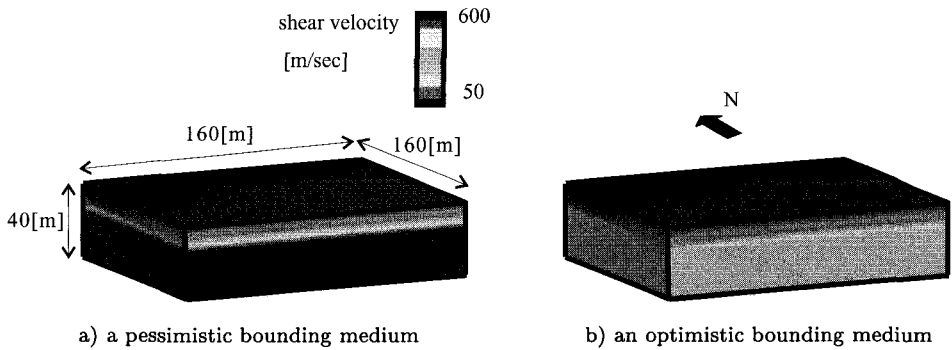


Fig. 6.10 Models near observation site hd01d for the micro-analysis.

depth and the magnitude is not large. Referring to [Kikuchi and Ishida (1993)], the rise time¹¹ is set to 0.35[sec] and 0.21[sec] for the case 1 and 2, respectively. It should be pointed out that this simple source model cannot produce earthquake waves of higher frequency components. The waves emitted from the source are only approximately calculated, and some higher frequency components are dropped during the calculation; see later discussions in this section.

As mentioned several times, special attention must be paid to the link between the macro-analysis and the micro-analysis. While the spatial discretization of the macro-analysis is sufficiently small to compute frequency components up to 1.2[Hz], due to the simple fault mechanism and huge amount of computation, there is a limitation of high frequency components that can be accurately simulated by the macro-analysis. To deliver high frequency components from the macro-analysis to the micro-analysis, the current macro-micro analysis method makes the following two approximations:

- 1) high frequency components produced by the source fault are extrapolated by using the statistical spectrum;
- 2) the propagation of high frequency components from the bedrock to the ground surface is approximately computed by using a one-dimensional model of parallel layers.

Based on these two assumptions, high frequency components of the macro-analysis are extrapolated and put into the micro-analysis. It should be emphasized that near the ground surface, approximated high frequency components of the macro-analysis will be amplified significantly in the micro-analysis, due to the three-dimensional topographical effects and the non-linear material properties of soil.

¹¹ Rise time is the time during which rupture processes on the source fault finish and a permanent displacement gap is created. An empirical relation between the rise time and the amplitude of earthquake is established, and hence the rise time can be estimated when the amplitude of an earthquake is known.

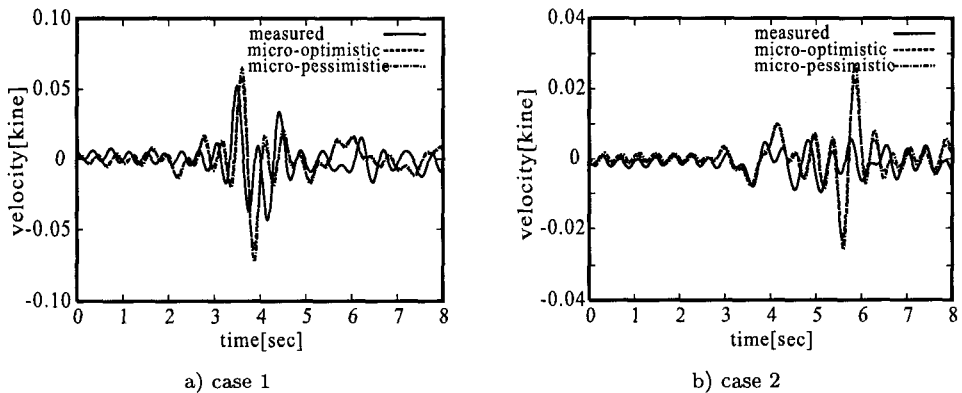


Fig. 6.11 EW velocity component computed at observation site hd01d.

6.3.2 Comparison of synthesized waveform with observed waveform

The waveform computed by the macro-micro analysis method is compared with the data which are measured at twelve observation sites. The numerical computation is stable, and strong ground motion at all sites is computed without causing any numerical problem. As a typical example, the EW component of the velocity at the site hd01d is plotted in Fig. 6.11; the duration time is 8 seconds, which correspond to the main shock. The accuracy of numerical computation is guaranteed up to 2.5[Hz], and the low pass filter is applied to the synthesized and observed waveforms.

As is seen, for the two earthquakes, the macro-micro analysis method succeeds in reproducing the measured waveform. It is certainly true that the two waveforms of the bounding media do not provide bounds for the measured waveform and that there are some differences between the synthesized waveform and the measured waveform. However, the agreement of the synthesized waveform and the measured waveform is satisfactory and the synthesized wave can be used for the prediction of strong ground motion if frequency components higher than 2.5[Hz] are filtered out. The reproduction of the observed data is similarly successful for most of other observation sites. It should be noted that the simplest fault mechanism that is used in the present macro-micro analysis method is the major source of error in reproducing the measured waves, although high frequency components emitted from the fault are only approximately included in the computation of the macro-micro analysis method. The agreement of the synthesized wave is better for the earthquake of the case 1, which has deeper epicenter and hence emits a lesser amount of higher frequency components.

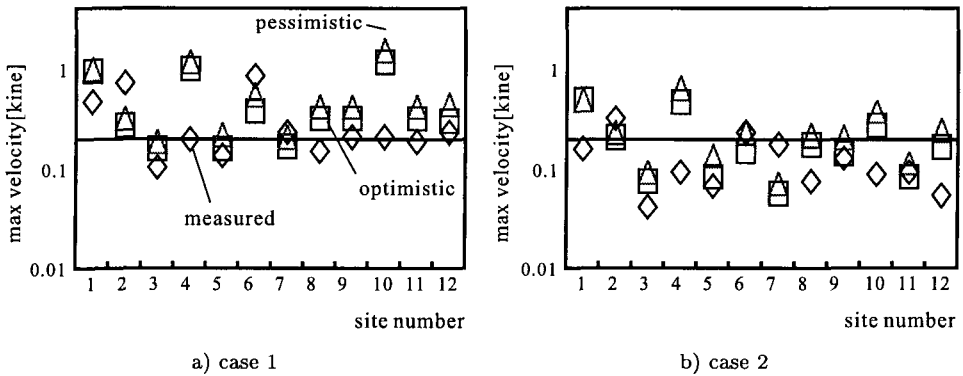


Fig. 6.12 The comparison of PGV at twelve sites.

6.3.3 Distribution of simulated strong ground motion

Since the basic validity of the macro-micro analysis method is verified, characteristics of strong ground motion distribution are computed at the observation sites, in order to examine the usefulness of the macro-micro analysis method; the comparison with the measured data is made for the characteristics which are usually used in earthquake engineering. It should be recalled that the low pass filter up to 2.5[Hz] is applied to the synthesized and observed waves, since the accuracy of the numerical computation is not guaranteed for frequency components higher than 2.5[Hz].

First, the comparison of the peak ground velocity (PGV) of the synthesized wave computed by the macro-micro analysis method with the observed data is presented in Fig. 6.12; a) and b) are for the case 1 and 2, respectively, and the two PGV's which are computed by using the optimistic and pessimistic media are plotted for each site. It is seen that the computed PGV is in a good agreement with the observed value except for the sites of number 4 and 10, even though the observed PGV is distributed in a wider range. The three-dimensional local topographical effects and possibly the material non-linearity contribute to such a wide range of the PGV distribution. Also, some contributions are made by the difference in the amplification near the ground surface at the same site when the source earthquake is different. As mentioned above, the agreement of the computed and observed PGV's is poor for the observation sites of number 4 and 10. When the low pass filter which is applied to the synthesized and observed waves is extended from 2.5[Hz] to 1.0[Hz], the agreement is improved, and no significant difference is seen between the computed and observed PGV's. This means that the difference is caused by higher frequency components, and hence it is suggested that the underground models constructed for these sites are not suitable. Indeed, compared with other sites, the ground structures at these sites change more wildly; these observation sites are located at the top and foot of small hills, and hence the topographical effects are more significant, especially for higher frequency components with the wavelength

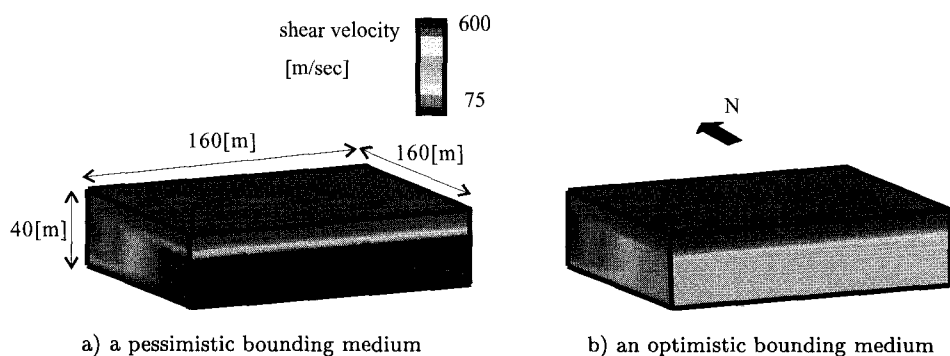


Fig. 6.13 Models near observation site kz07s for the micro-analysis.

being of the order of 10^0 [m].

Since PGV is satisfactorily computed at most of the observation sites, the distribution of PGV is computed within the model of the micro-analysis, i.e., an area of 100×100 [m] (the EW and NS directions) around the observation site; recall that the accuracy of the present micro-analysis method is ensured with 2[m] spatial resolution. As an illustrative example, the PGV distribution is computed for the micro-analysis models of the site kz07s, which has quite complicated three-dimensional ground structures; see Fig. 6.13 for the models of this observation site, where a steep valley of the bedrock is seen in the north west part. The PGV distribution is presented in Fig. 6.14; a) or b) is the distribution in the optimistic or pessimistic bounding medium for the case 1, and c) or d) is that for the case 2. It is seen that the strong ground motion distribution varies even in such a small area; the maximum and minimum values of PGV are 0.22 and 0.11[kine] for the case 1 and 0.14 and 0.06[kine] for the case 2. Since the strain for such velocity is large enough to induce the material non-linearity, the HA model surely contributes to this spatial variability of PGV. It should also be noted that a larger PGV is concentrated at the north-west corner of the surface. This concentration is mainly due to the topographical effects of the three-dimensional ground structures. At this observation site, a layer with a shear wave velocity larger than 600[m/sec] is regarded as the bedrock. Even within the depth of 40[m] and in the area of 100×100 [m], there is a steep valley of this bedrock around the west-north corner; see Fig. 6.13. In Fig. 6.15, the contour map of the bedrock depth is plotted to clarify this point, and the presence of valley is clearly observed. It is difficult to quantitatively verify such a concentration of strong ground motion unless a dense network of seismographs is operated to measure strong ground motion distribution. The present simulation of the micro-analysis method suggests that there is a concentration of PGV, which may lead to the localization of structural damage caused by strong ground motion. The numerical computation with the high spatial resolution is able to provide information of a possible concentration of strong ground motion on this length scale, which will be important for engineering purposes, such as the estimation of local

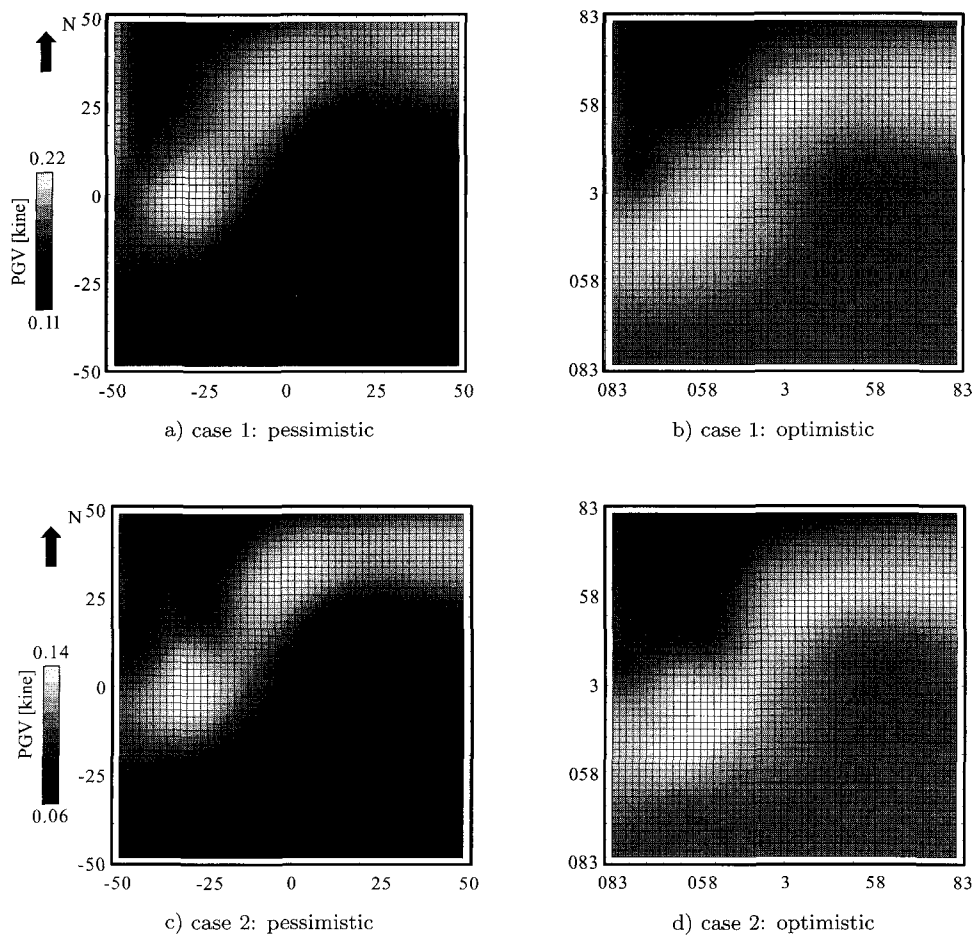


Fig. 6.14 Distribution of PGV near observation site kz07s.

earthquake hazards and the prediction of resulting earthquake disasters.

For such engineering purposes, it is also important to know the variability of strong ground motion depending on the source earthquake. In Fig. 6.14, the west-north corner concentration is commonly observed for the two earthquakes, although the variability is not clearly seen. The relative difference in the PGV distribution between the case 1 and the case 2 at the site kz07s is plotted in Fig. 6.16. The relative difference is computed for the optimistic model in the following manner:

- i) Standardize the distribution by using the value of the center point.
- ii) Take the ratio of the standardized velocity of the case 2 to that of the case 1.
- iii) Subtract 1 from the ratio and determine the relative difference.

Thus, at the center point, the relative difference takes on a value of 0; a positive or negative value of the relative difference means that the strong ground motion of the

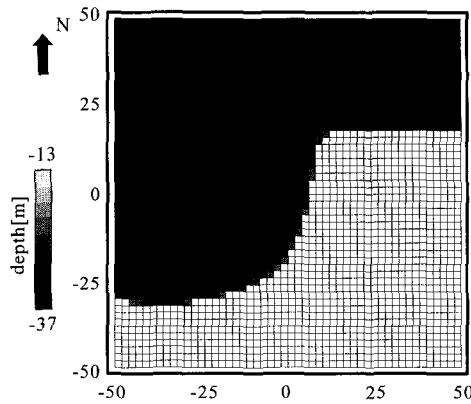


Fig. 6.15 Configuration of the bedrock at observation site kz07s.

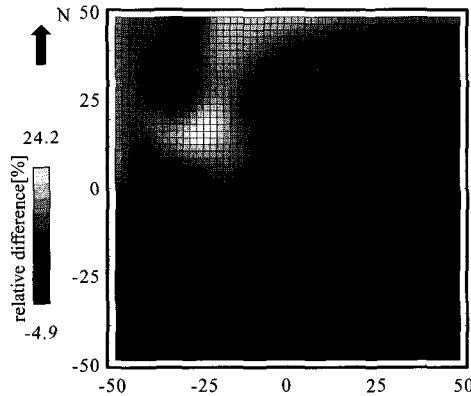


Fig. 6.16 Relative difference in the PGV distribution at observation site kz07s.

case 2 is amplified more or less than that of the case 1, respectively. As is seen in Fig. 6.16, the relative difference varies in a wide range of positive and negative values, from +24% to -4.9%. This means that, depending on the source earthquake, the PGV distribution changes from place to place, with the relative difference in strong ground motion being in the range of 20%. Therefore, the incident wave characteristics must be accounted for in considering the variability of a possible strong ground motion distribution. It should be pointed out that when a larger incident wave is used, the strain of the ground surface is increased so that the material non-linearity of soil is increased and possibly contributes to larger variability of the resulting strong ground motion distribution.

It is interesting to examine which frequency component principally contributes to the concentration of strong ground motion. The change in the PGV distribution

that is computed by using four low pass filters is plotted in Fig. 6.17; a), b), c) and d) are for the low pass filters of <1.0, <1.5, <2.0, and <2.5[Hz], respectively, and the optimistic model for the observation site kz07s and the earthquake of the case 1 are used. The change in PGV is computed as follows:

$$(\text{change in PGV}) = \frac{R_{PGV1} - R_{PGV2}}{R_{PGV1}}, \quad (6.5)$$

where

$$R_{PGV1,2} = (\text{ratio of PGV to the reference value for the case 1,2});$$

the reference value is PGV at the center when the low pass filter of <2.5[Hz]. As expected, the concentration of strong ground motion does not take place for lower components, say, up to 1.5[Hz]. For frequency components around 2.0[Hz], the concentration as well as the spatial variability is clearly observed; see the north-west corner of Fig. 6.17c) and d). Since the soil deposit over the bedrock has a very low shear wave velocity of 100[m/sec], the frequency components of around 2.0[Hz] can pick up the three-dimensional ground structures on the length scale of 10[m], which is around one-fifth of the wavelength. Such high frequency components are the cause of the self-excitation of shorter buildings which have a natural vibration frequency of around 5[Hz]. For the engineering purposes, therefore, computing strong ground motion up to a sufficiently high frequency is useful, even though the wave propagation from the source fault and the wave amplification near the ground surface are numerically computed; it must be taken for granted that there is some uncertainty in modeling which results inaccurate computation for this numerical simulation.

To demonstrate the effects of the spatial variability in strong ground motion upon the behavior of buildings, the response spectrum which is usually used for the purpose of structure design is computed; the response spectrum is obtained by modeling a building as¹² a one-degree-of-freedom system, computing its dynamic response subjected to a given strong ground motion, and plotting the maximum value of acceleration, velocity or displacement with respect to the natural frequency of the system. In Fig. 6.18, the response velocity spectra computed at seven points, denoted by a~g, are plotted; a~g are aligned in the NS direction near the west side, a) is the spectra for EW component, and b) for the depth of the bedrock at the location of these seven points. As is seen, even in such a small area, the magnitude as well as the shape of the spectrum differs for the seven points. For instance, the response spectrum has sharp peaks around 0.5[time] for the point e~g, while the spectrum is flat for the point a; the response velocity at the point e is larger

¹²The governing equation of a one-degree-of-freedom system with mass m and stiffness k is given as $m\ddot{u} + ku = -m\ddot{z} - kz$ where u is displacement and z is an input strong ground motion. From the solution of this equation, the maximum value of $\ddot{u}$, $\dot{u}$ or u is found. The plot of the maximum value with respect to $\omega = \sqrt{k/m}$ is a response spectrum. A damping term, $c\dot{u}$, is often added in the governing equation.

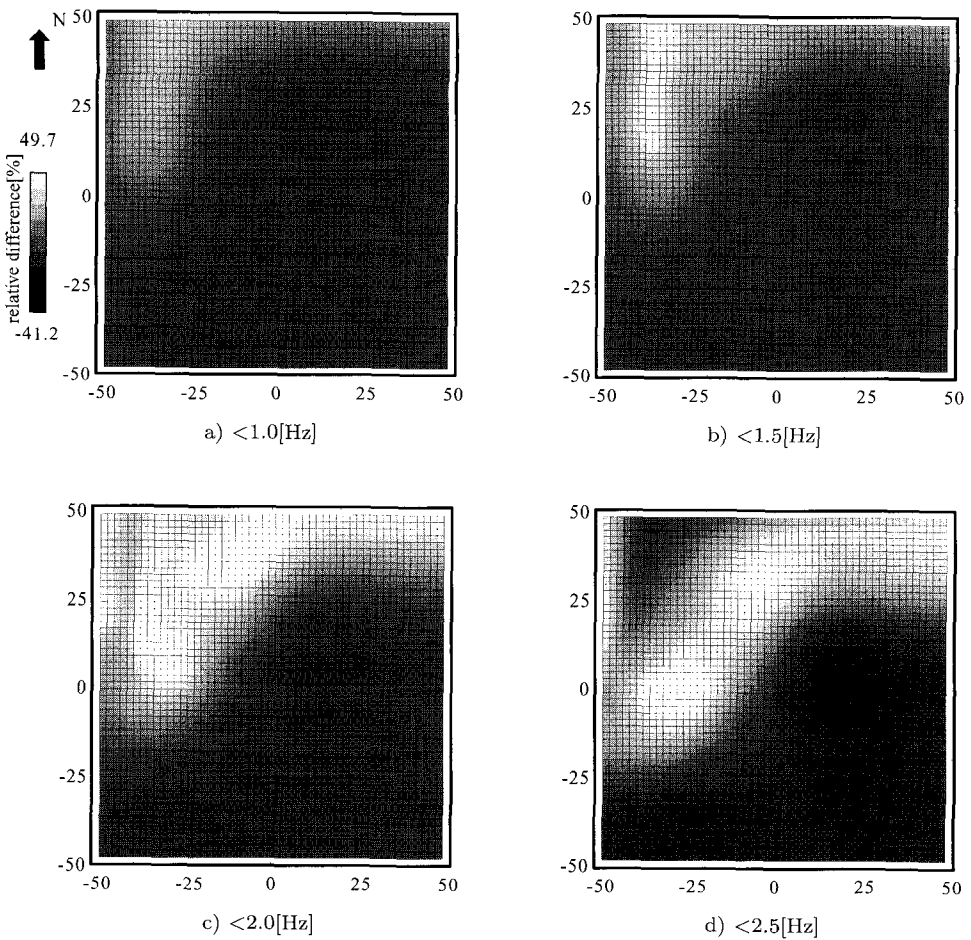


Fig. 6.17 Change in the PGV distribution in prescribed frequency components at observation site kz07s. PGV is computed for a waveform to which a low pass filter of the prescribed frequency range is applied.

almost eight times than that at the point a. This is due to the three-dimensional topographical effects that influence high frequency components of wave.

6.3.4 *The comparison of three-dimensional analysis and one-dimensional analysis*

Finally, the necessity of the three-dimensional computation which is carried out by the macro-micro analysis method is demonstrated. The strong ground motion amplification is larger as the bedrock is located deeper or as the thickness of soft soil deposit is larger. It is certainly true that such amplification can be explained by means of an ordinary one-dimensional analysis of wave propagation in ground structures; see Fig. 6.14 and 6.15. However, the three-dimensional ground structures

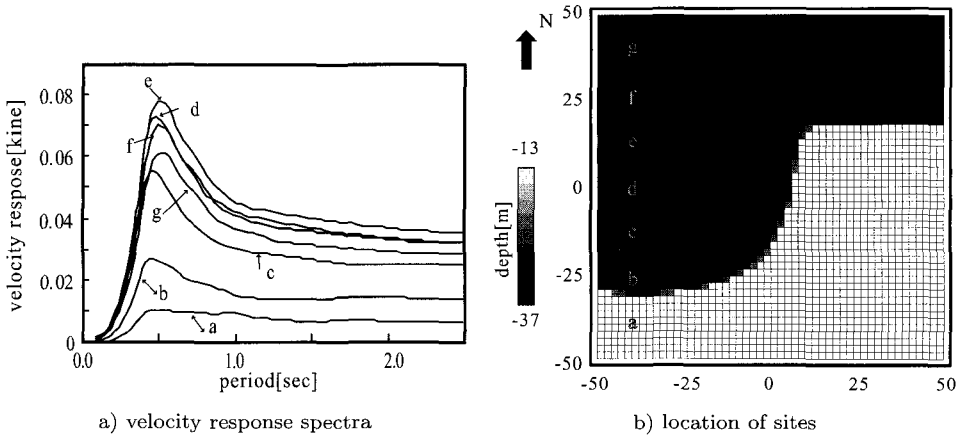


Fig. 6.18 Velocity response spectra at seven sites near observation site kz07s.

contribute to a negligible amount of the strong ground motion amplification. To clarify this, a one-dimensional analysis is carried out for the micro-analysis model of the site kz07s; parallel stratified layers are constructed from the optimistic bounding medium, and the response at the top surface is computed for a given input¹³ of the vertical wave at the bedrock. The relative difference between PGV of the three-dimensional micro-analysis and that of the one-dimensional analysis is computed, i.e.,

$$(\text{relative difference}) = \frac{v_{\max 3D} - v_{\max 1D}}{v_{\max 1D}}, \quad (6.6)$$

where $v_{\max 1D}$ or $v_{\max 3D}$ is PGV of waves which are computed by the one-dimensional analysis or the three-dimensional micro-analysis, respectively. The distribution of the relative difference is plotted in Fig. 6.19; a) and b) are for the case 1 and 2, respectively. The relative difference takes on a large positive value at the north-west corner and a larger negative value along the edge of the valley which is shown in Fig. 6.15. In these places, the ground structures are more complicated than other places. The result shown in Fig. 6.19 appears natural, since the one-dimensional analysis uses the horizontal stratification structure and cannot compute waves which are refracted at the three-dimensionally curved interfaces. Thus, the limitation of the one-dimensional analysis in computing the wave amplification is observed, as it underestimates the variability and concentration of possible strong ground motion distribution. In the present example, the underestimation of evaluating PGV reaches 60% at most. Therefore, it can be concluded that the three-dimensional analysis which is made in the present numerical analysis is necessary for predicting strong ground motion distribution when three-dimensional ground

¹³In the one-dimensional analysis, the incident wave is only in the vertical direction, and hence it cannot compute the refracted wave which changes its direction depending on the incident wave direction.

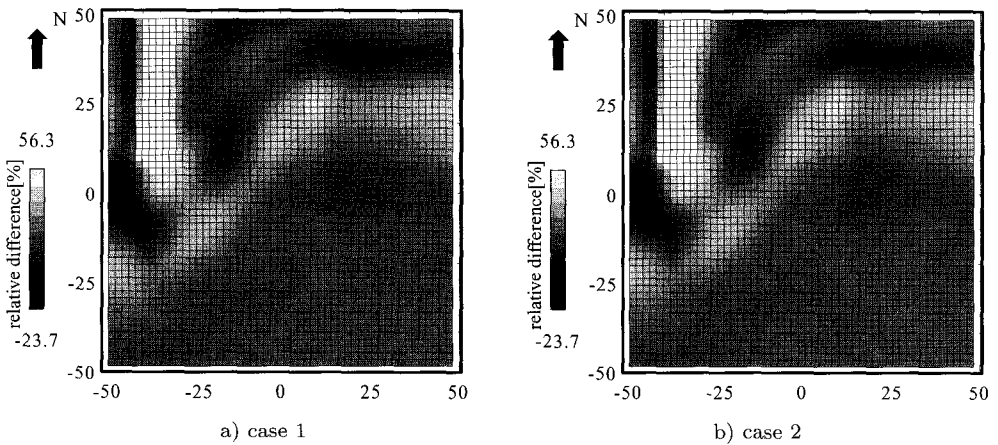


Fig. 6.19 The comparison of the PGV distribution computed by the three-dimensional analysis with PGV distribution computed by the one-dimensional analysis. The ground structures near observation site kz07 are analyzed.

structures are complicated.

It is expected that the difference between the three-dimensional analysis and the one-dimensional analyses becomes larger for computing higher frequency components since, in general, greater three-dimensional topographical effects are observed for higher frequency components. Figure 6.20 presents the relative difference of the PGV distribution when the three low pass filters are applied to the waves computed by these two analyses; a), b) and c) use the low pass filter of <1.5 , <2.0 , and <2.5 [Hz], respectively; and the optimistic bounding medium for the site kz07s and the earthquake of the case 1 are used. The relative difference is computed in the same form as Eq. (6.5); the PGV distribution is standardized with respect to the value at the center. The dependence of the relative difference on the frequency is clearly seen by comparing Fig. 6.20c), which uses the low pass filter of <2.5 [Hz] and has contributions of frequency components between 1.5 [Hz] and 2.5 [Hz], with Fig. 6.20a) and b). Since such high frequency components are strongly influenced by the ground structures, these results support the need for the three-dimensional analysis that is used in predicting strong ground motion distribution. Also, the results demonstrate the importance of an accurate model of the three-dimensional ground structures even if measuring the structure is not trivial.

As shown above, the usefulness of the macro-micro analysis method, to which VFEM and the HA model are implemented, has been examined. At this stage, the comparison of the numerical simulation with the observed data shows that there is some limitation in accurately reproducing the observed data by means of the macro-micro analysis method. The primary cause of this limitation is probably simple modeling of the fault mechanism. However, for the prediction of strong ground motion distribution, the high spatial resolution of the macro-micro analysis method is useful since it is able to clarify the variability and concentration of

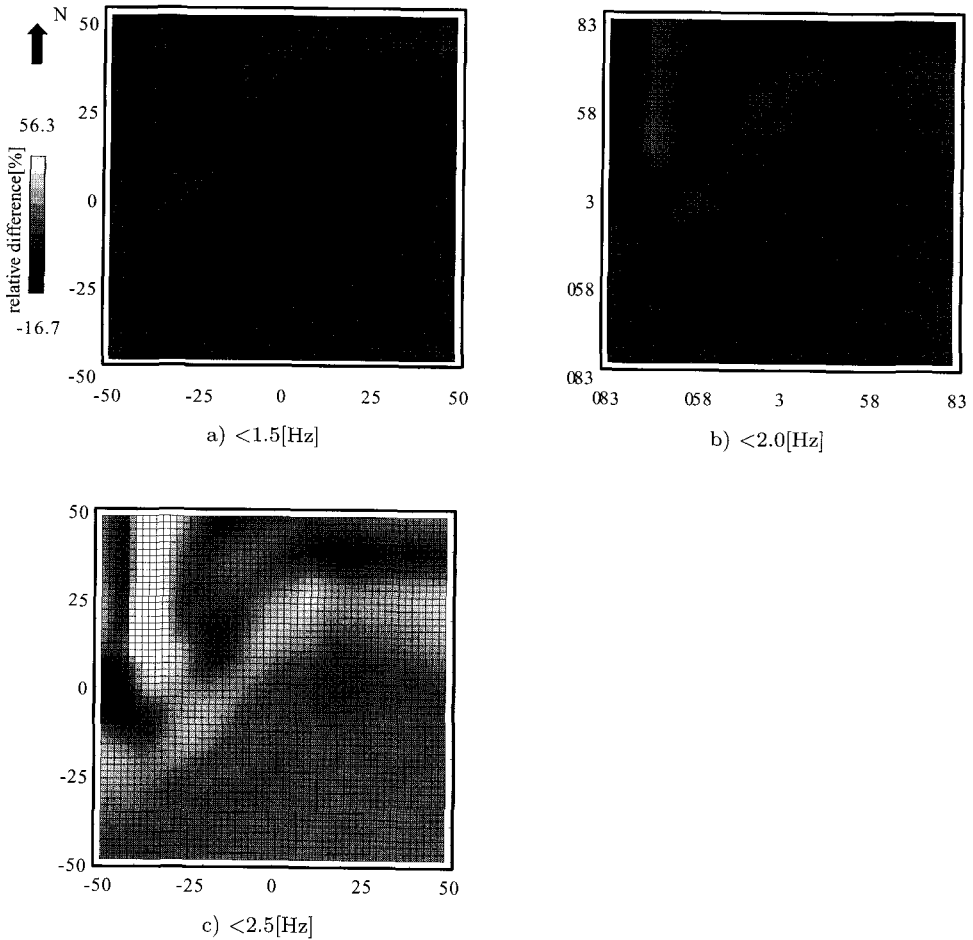


Fig. 6.20 The comparison of the PGV distribution in prescribed frequency components obtained by three-dimensional analysis with that obtained by the one-dimensional analysis. PGV is computed for a waveform to which a low pass filter of a prescribed frequency range is applied.

strong ground motion distribution provided that three-dimensional bounding media are constructed. In conclusion, the following three remarks are made on the performance of the present macro-micro analysis method as a candidate for predicting strong ground motion distribution with high accuracy and high spatial and temporal resolution:

- i) Even within a small area of $100 \times 100[\text{m}]$, the local concentration of strong ground motion is obtained for the three-dimensional model. The implementation of non-linear material properties such as the HA model is definitely required, since the strain reaches more than 10^{-2} at zones where the concentration takes place.
- ii) The variability of strong ground motion is not negligible. The macro-micro

analysis method succeeds in reproducing the variability of PGV observed at twelve sites to some extent. The simulation suggests that the variability changes up to 150% in PGV within the area of 100×100 [m].

- iii) For the engineering purposes of the estimation of local earthquake hazards and disasters, the three-dimensional analysis that takes the material non-linearity into consideration is needed to accurately predict higher frequency components. In the present example, the amplification is underestimated up to 80% by applying the conventional analysis which uses the one-dimensional model and the linear elasticity.

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PART 3

FAULTING

In this part, we demonstrate the application of computational mechanics to analysis of faulting. In this book, faulting means the formation of a surface earthquake fault, a fault which is formed on the ground surface and induces large ground deformation accompanying displacement gap on the order of $10^{0\sim 1}$ [m]. Such ground deformation is fatal to structures. However, hazards due to surface earthquake faulting are much less recognized compared with hazards due to strong ground motion; the probability of the fault formation is extremely low because the fault appears only for a large earthquake in the shallow crust and it is formed along a line.

Due to its low probability, surface earthquake fault formation has not been a major issue in earthquake engineering; a fault hazard is not considered except when constructing an important structure or in a region where the fault activity is high. However, two earthquakes in 1999 have changed the situation, since serious damage to large-scale structures such as highways and dams were caused by surface earthquake faults which appeared in these two earthquakes. Also, in Japan, public attention to active faults has increased since the Great Hanshin Awaji Earthquake hit Kobe City in 1995; this earthquake was caused by the Nojima Fault and formed surface earthquake faults on Awajishima Island.

After briefly summarizing elasto-plasticity and fracture mechanics in Chapter 7, we present two numerical analyses of faulting. The first analysis is based on elasto-plasticity. Stochastic modeling is applied to surface soil layers and stochastic and non-linear failure of these layers is simulated by means of an advanced numerical analysis method. We explain the method in Chapter 8, and present some results of reproducing model experiments of faulting and two actual surface earthquake faults in Chapter 9. The second analysis of faulting is based on fracture mechanics, and models faulting as a crack growing in underground structures. Introducing a boundary element method (BEM), we present numerical simulations of this crack modeling in Chapter 10. The two analyses of faulting presented in this chapter are different from those used in structure mechanics. Readers who are not familiar with the analysis methods are requested to read Chapter 7.

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Chapter 7

Elasto-Plasticity and Fracture Mechanics

Numerical analysis is a key tool for evaluating surface earthquake fault¹ hazards; the quantity and quality of past data about surface earthquake faults is low compared with that about strong ground motion, and hence more theoretical prediction is required. Numerical analysis of the surface earthquake fault is categorized as a failure analysis of solids, which is divided into two² groups. The first group is non-linear elasto-plastic analysis which models failure as the concentration of plastic deformation. The second group is failure analysis based on fracture mechanics, in which failure is modeled as a crack or a discontinuity of displacement.

This chapter presents a brief summary on the failure analyses. First, the non-linear elasto-plastic analysis is explained. This analysis is used for ductile failure which takes place in an elasto-plastic material. The key mechanism of ductile failure is the formation of a thin shear band in which large plastic strain is concentrated. Next, the fracture mechanics is explained. Fracture mechanics is developed for brittle failure. Brittle failure occurs in a material with cracks, and the crack growth is the key mechanism. Fracture mechanics analyzes this crack growth

7.1 Numerical Analysis of Failure

This section briefly surveys the literature on the analysis of ductile and brittle failures. These failures have been a major target of computational mechanics for solids. It should be pointed out that numerical analysis method for failure has recently made significant progress. These methods are based on FEM with the

¹Fault is originally a geological term. It means discontinuity in geological layers, which is observable on the vertical cross section of geological layers. The same term is used in seismology. Fault designates a displacement gap in the crust and emits earthquake waves. This book calls a fault in the crust a source fault and a fault appearing on the ground surface a surface earthquake fault.

²Failure analysis based on fracture mechanics is often used by seismologic or geophysics researches, but it is not popular in earthquake engineering. Failure analysis based on elasto-plasticity has been developed by the engineering community; many techniques are shared by structure mechanics and earthquake engineering. Earthquake and geotechnical engineering research has applied elasto-plasticity analysis to faulting.

implementation of special³ techniques. Meshless and related methods are representative techniques; see, for instance, [Belytschko *et al.* (2000)]. New numerical methods are proposed with the aid of theoretical studies on modeling the discontinuity in field variables; see [Oliver (2000)]. Originally developed for a broader class of mathematical problems, the discontinuous finite element method should be mentioned. The method uses non-continuous shape functions which are suitable to describe failure; see [Engel *et al.* (2002)] for a list of related references; see also [Duarte *et al.* (2000)].

A tremendous amount of research has been done on ductile failure since many materials are often broken due to ductile failure. The collapse of building structures caused by strong ground motion is regarded as ductile failure. Ductile failure induces finite or large deformation, which is modeled as non-linear relations⁴ between strain and displacement, and material properties for plastic deformation are usually non-linear. Thus, the analysis of ductile failure is non-linear in nature and has been an interesting subject of computational mechanics. [Simo and Hughes (1998)] is recommended as one of the most comprehensive textbooks for elasto-plasticity and related subjects of computational mechanics; see also [Chen and Han (1988)] for a more engineering oriented perspective of elasto-plasticity.

Fracture mechanics⁵ has been developed for brittle failure analysis, although the amount of research relating to fracture mechanics is limited in earthquake engineering or even in structure mechanics. This is mainly because cracks which cause brittle failure are found on the microscopic length scale of materials or on the length scale of structure members, and they are smeared on the macroscopic length scale of a structure so that inelastic or plastic deformation is used to represent them. For seismology or solid geophysics, fracture mechanics plays a primary role in analyzing the failure of a source fault from which earthquake waves are emitted. [Scholz (1990)] is recommended on the basics of the fracture mechanics that is related to earthquake phenomena. Also, [Freund (1990)] and [Broek (1991)] are recommended as general textbooks on fracture mechanics; the former covers mainly dynamic fracture and the latter applies fracture mechanics to some practical problems.

³A conventional and standard technique for modeling a failure surface is the implementation of a joint element, which represents the surface as a thin layer which transmits suitable forces caused by the relative displacement of the layer; see [Arnold (1982)].

⁴Finite deformation theory is complicated since it is expressed in terms of special tensors and their relations. For instance, the Green-Lagrange strain tensor is used as a measure of deformation, and the Piola-Kirchhoff stress is often used as a measure of stress. These tensors are mathematically defined, rather than physically described. Knowledge of *differential geometry* helps readers who are interested in the finite deformation theory understand the essence of the theory.

⁵Fracture mechanics was initiated by analyzing the collapse of ships caused by the cyclic loading of ocean waves; see [Griffith (1921)]. Since then, fracture mechanics has been applied to practical problems such as fatigue or dynamic fracture due to cracking; see [Horii and Nemat-Nasser (1983)] for the application of fracture mechanics to rock mechanics. Fracture mechanics is often applied to earthquake problems; see, for instance, [Williams (1957)].

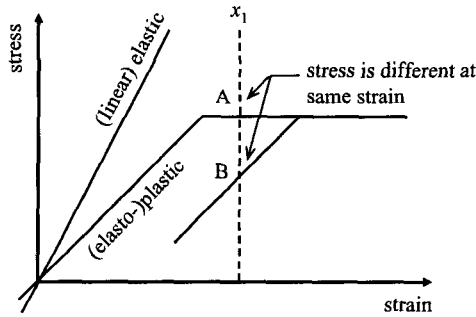


Fig. 7.1 Stress state depending on strain history.

7.2 Elasto-Plasticity

This section summarizes the basic formulation of elasto-plasticity which is used to analyze ductile failure. Before explaining elasto-plasticity, the definition of the two material properties, elasticity and plasticity, should be clarified. Elasticity means that stress is a function of strain only. That is, for given strain, the state of stress is uniquely determined. Plasticity means that stress changes depending on the history of strain. When subjected to a different loadings, the history of strain is different and the state of stress does not become the same even if the strain reaches the same state; see Fig. 7.1, where a stress-strain relation of elasto-plasticity shows that there are two stress points, designated by A and B, for one strain. It is thus said that plasticity is history-dependent while elasticity is history-independent.

Other terms for material properties are clarified. Viscosity means that stress depends on strain rate, the derivative of strain with respect to time or the temporal change in strain. Linear elasticity is one kind of elasticity in which stress is linearly related to strain. Isotropy means that relations among vector- or tensor-valued-quantities do not depend on their direction. Thus, isotropic elasticity means that the identical relations hold between strain components and stress components which are measured in any orthonormal Cartesian coordinate system; in particular, linearly isotropic elasticity means that the coefficients⁶ between strain and stress components do not depend on the choice of Cartesian coordinate system.

Metals, such as steel and aluminum, exhibit large elongation when they are pulled. The source of such elongation is plastic deformation. Plastic strain is generated when the materials yield. In the microscopic view, plastic strain represents slip or *dislocation*, which takes place in metal crystals and crystal boundaries. Concrete shows plastic deformation under compressive loading. Concrete is a composite material which consists of mortar and aggregate, and has a complicated mechanism of plastic deformation; subjected to large compressive stress, numerous micro-cracks in

⁶There are only two independent parameters to express the coefficients of linear isotropic elasticity.

mortar start to grow and some bonding between mortar and aggregates are forced to break. These cracks and debonding lead to irreversible deformation which is regarded as plastic deformation. Soil, such as sand and clay, shows plastic deformation under compression like concrete. Microscopically, however, soil is a granular material which is made of the assembly of granules. The rearrangement of the granule assembly is the main source of plastic deformation.

While the mechanism of plasticity varies depending on a material, there is a common mathematical model which describes plastic deformation. Elasto-plasticity is a typical model. In this model, plastic deformation is separated from elastic deformation; plastic deformation is induced only when stress reaches a certain critical state, and it is added to elastic deformation. The description of elasto-plasticity consists of three elements, namely, the definition of plastic deformation, the relation between plastic deformation and stress, and the consistency of plastic deformation.

In elasto-plasticity, plastic deformation is described⁷ in terms of plastic strain rate, and the rate of strain is decomposed into an elastic part and a plastic part, i.e.,

$$d\epsilon_{ij} = d\epsilon_{ij}^e + d\epsilon_{ij}^p. \quad (7.1)$$

Here, $d\epsilon_{ij}^e$ is elastic strain rate which is related to stress rate; for instance, if linear elasticity is assumed, stress rate $d\sigma_{ij}$ is related to $d\epsilon_{ij}^e$ through $d\sigma_{ij} = c_{ijkl}d\epsilon_{kl}^e$ with c_{ijkl} being an elasticity tensor.

Plastic deformation is induced when a material yields, i.e., stress reaches a certain critical state. Thus, plastic strain rate is related to stress. The relation is expressed in terms of a function which is called a *yield function*. The yield function, denoted by f , becomes zero when yielding takes place. Plastic strain is given as the gradient of f , i.e., a component of plastic strain rate is given as the derivative of f with respect to the corresponding component of stress,

$$d\epsilon_{ij}^p = d\lambda \frac{\partial f}{\partial \sigma_{ij}}, \quad (7.2)$$

where $d\lambda$ is called a plastic strain rate multiplier. The relation between f and $d\epsilon_{ij}^p$, Eq. (7.2), is called a *flow rule*. For simplicity, the gradient is denoted by $\nabla f_{ij} = \partial f / \partial \sigma_{ij}$. As is seen, $d\epsilon_{ij}^p$ is linear to ∇f_{ij} , but the amplitude remains unknown since $d\lambda$ in Eq. (7.2) is not determined. This means that plastic deformation is like flow in the sense that plastic strain keeps increasing as long as the material yields and $d\lambda$ does not vanish.

The two equations, Eqs. (7.1) and (7.2), are not sufficient to describe the constitutive relations of elasto-plasticity, since there is no condition for the plastic strain rate multiplier. To determine the multiplier and to establish relations between strain rate and stress rate, the fact that the yield function remains zero during the

⁷It should be emphasized that plastic deformation is not expressed in terms of displacement or displacement rate, i.e., elastic or plastic displacement rate is not defined. This means that elastic and plastic strain rates do not have to be compatible.

plastic deformation is used. That is, the rate of f must vanish to generate plastic strain rate,

$$df = 0. \quad (7.3)$$

This condition is called a *consistency condition* of plasticity. Usually, f is chosen so that it takes a negative value for elastic state. Thus, when f is a function of σ_{ij} only, in view of Eqs. (7.1) and (7.2), df is computed as

$$df = \nabla f_{ij} d\sigma_{ij} = \nabla f_{ij} c_{ijkl} (d\epsilon_{kl} - d\lambda \nabla f_{kl}),$$

where $d\sigma_{ij} = c_{ijkl} d\epsilon_{kl}^e$ is used. Hence, it follows from Eq. (7.3) that the plastic strain rate multiplier $d\lambda$ is related to strain rate as

$$d\lambda = \frac{\nabla f_{ij} c_{ijkl}}{\nabla f_{pq} c_{pqrs} \nabla f_{rs}} d\epsilon_{kl}.$$

Note that since ∇f_{ij} is known, $d\lambda$ and hence $d\epsilon_{ij}^p$ are expressed in terms of strain rate $d\epsilon_{ij}$. Thus, an apparent relation between stress rate and strain rate is derived as $d\sigma_{ij} = c_{ijkl}^{\text{ep}} d\epsilon_{kl}$, where

$$c_{ijkl}^{\text{ep}} = c_{ijkl} - \frac{(c_{ijtu} \nabla f_{tu})(c_{klvw} \nabla f_{vw})}{\nabla f_{pq} c_{pqrs} \nabla f_{rs}}. \quad (7.4)$$

This tensor c_{ijkl}^{ep} is called an elasto-plasticity tensor and its components are called instantaneous moduli. In deriving the elasto-plastic tensor given by Eq. (7.4), it is implicitly assumed⁸ that one yield function determines plastic strain rate and describes the consistency condition. This assumption is called an *associated flow rule*. It is certainly possible to use a different function, which is usually called a plastic potential, to describe the flow rule, i.e., Eq. (7.2), and this assumption is called a *non-associated flow rule*.

It should be emphasized that c_{ijkl}^{ep} relates $d\epsilon_{ij}$ to $d\sigma_{ij}$ only when the yield function f remains 0. Even at $f=0$, elasto-plasticity tensor c_{ijkl}^{ep} should not be used if f starts to decrease, i.e., if df is negative. This is summarized as

$$d\sigma_{ij} = \begin{cases} c_{ijkl} d\epsilon_{kl} & \text{for } f < 0 \text{ or } df < 0, \\ c_{ijkl}^{\text{ep}} d\epsilon_{kl} & \text{for } f = 0 \text{ and } df = 0. \end{cases}$$

Note that when f becomes 0, $df=0$ and $df<0$ corresponds to plastic loading and elastic unloading, respectively.

⁸A hardening rule is also an important element of elasto-plasticity. This rule describes the change in a yield function due to plastic deformation. There are several types of hardening rule which are inherent to material.

7.3 Fracture Mechanics

This section briefly explains the basics of fracture mechanics. As mentioned in the introduction of this chapter, the target of fracture mechanics is the analysis of brittle failure, i.e., failure due to the growth of a crack or cracks. Fracture mechanics analyzes field variables near the tip of a crack, and describes a condition in which the crack starts to propagate. This condition is related to the energy which is consumed to create new surfaces ahead of the crack tip. Some properties of the field variables are used to compute the energy stored at the crack tip.

Fracture mechanics assumes the presence of an initial crack and analyzes its growth when loading or environment conditions are changed. In general, the size of the crack is small and it may be invisible. Under certain conditions, the crack starts to grow in an unstable manner so that fracture due to cracking takes place. Thus, if a material does not have an initial crack, brittle fracture does not occur until an extremely large load⁹ is applied. In reality, however, there are initial cracks in a material. Most of metals have micro-cracks which are invisible. Concrete has micro-cracks in mortar; in the microscopic view, mortar is regarded as a porous material and its porosity includes cavities and cracks. Intact rock has micro-cracks of millimeters or less. In general, the number and size of the micro-cracks tend to increase as the material is aged or damaged.

The crack growth is governed by the energy that is stored at the crack tip. That is, if sufficient energy is stored, the tip is broken and new surfaces are created to consume the energy. In this manner, the crack starts to propagate. The crack growth leads to the redistribution of stress since the crack surfaces are regarded as new boundaries, which often become traction-free. There are cases when the crack cannot stop growing as sufficient energy is being supplied to the crack tip by the stress redistribution. This is the unstable crack growth which leads to the failure. There are other cases when the stress redistribution releases the energy at the crack tip so that the crack growth stops. When external loading conditions are changed, the crack grows again, and the crack gradually propagates in the material. This is the basic mechanism of fatigue from the viewpoint of fracture mechanics.

Based on the above mechanism, fracture mechanics focuses on two issues, the evaluation of the energy that is stored at the crack tip and the instability of the crack growth due to stress redistribution. It is interesting to note that, if linear elasticity and a two-dimensional state of plain strain are assumed, a finite amount of energy is stored at the crack tip. The strain energy density, $e = \frac{1}{2} \epsilon_{ij} c_{ijkl} \epsilon_{kl}$ with c_{ijkl} being linear elasticity tensor, is unbounded at the crack tip but $\int e ds$ takes a finite value even if the domain of integration includes the crack tip. This is because strain at the crack tip is singular; it diverges as $\mathcal{O}(r^{-\frac{1}{2}})$ with r being the distance

⁹For metals, the energy required to cause fracture is theoretically computed by considering the separation of neighboring atoms. This theoretical energy is larger by an order of magnitude than the energy which is actually needed. The presence of small micro-cracks is the cause of the discrepancy between the theoretical energy and the actual energy.

from the crack tip. The strain energy density diverges as $\mathcal{O}(r^{-1})$, and hence the integration is bounded; ds , which is given as $dx_1 dx_2$ in the Cartesian coordinate system, is expressed as $ds=r dr d\theta$ in the polar coordinate system, and hence $\int e ds$ is bounded.

In order to show the singularity of strain at the crack tip, a simple example problem for the two-dimensional out-of-plane deformation with isotropic linear elasticity is solved. Since the object of analysis is a crack tip, a region which includes a crack is modeled as an infinitely extended domain, B , and a crack as an semi-infinite slit, Ω , lying on the negative x_1 -axis; see Fig. 7.2. The out-of-plane displacement u_3 accompanies two shear strain components, $\epsilon_{3i}=\frac{1}{2}u_{3,i}$ for $i=1$ or 2 , and they produce shear stress components, $\sigma_{3i}=2\mu\epsilon_{3i}$, with μ being the shear modulus. In the absence of body forces, the equation of equilibrium is $\sigma_{3i,i}=0$. Thus, the governing equation for u_3 is

$$u_{3,ii}(x) = 0 \quad \text{in } B. \quad (7.5)$$

The upper and lower surfaces of the crack, denoted by $\Omega^\pm=\{(x_1, x_2)|x_1<0, x_2=0^\pm\}$, are traction-free. Hence, the boundary condition is

$$u_{3,2}(x) = 0 \quad \text{on } \Omega^\pm. \quad (7.6)$$

As is seen, Eqs. (7.5) and (7.6) form a homogeneous boundary value problem for u_3 . This problem, however, has a non-trivial¹⁰ solution of the following form:

$$u_3(x_1, x_2) = \frac{K_{III}}{\mu} \sqrt{\frac{2r}{\pi}} \sin\left(\frac{\theta}{2}\right), \quad (7.7)$$

with $r=\sqrt{x_1^2+x_2^2}$ and $\theta=\tan^{-1}(x_2/x_1)$; see, for instance, [Meguid (1989)]. Here, K_{III} is an arbitrary number. This u_3 produces the following strain, stress and strain energy density:

$$\text{strain} \quad \begin{bmatrix} \epsilon_{31} \\ \epsilon_{32} \end{bmatrix} = \frac{K_{III}}{\mu\sqrt{2\pi r}} \begin{bmatrix} \sin(\frac{\theta}{2}) \\ \cos(\frac{\theta}{2}) \end{bmatrix},$$

$$\text{strain energy density} \quad e = \frac{K_{III}^2}{2\pi\mu r}.$$

As is seen, strain is singular at the crack tip, $r=0$. The strain energy density is unbounded but integrable; if it is integrated in a circle of radius R , $\int e ds$ is calculated as $\int e ds=(K_{III}^2/\mu)R$.

¹⁰A non-trivial solution is found by transforming the Cartesian coordinates, (x_1, x_2) , to the polar coordinates, (r, θ) . The form of u_3 is assumed as $u_3(r, \theta)=r^\lambda f(\theta)$, and the characteristic equation of λ is derived from the boundary condition. Indeed, Eq. (7.5) is transformed to an ordinary differential equation for f by substituting the solution into Eq. (7.6), and the characteristic equation of λ is derived. Roots of the characteristic equation are $\lambda=\pm\frac{1}{2}, \pm\frac{3}{2}, \dots$, and the smallest root that is physically admissible is $\lambda=\frac{1}{2}$; the strain energy stored at the crack tip is unbounded for $\lambda<\frac{1}{2}$.

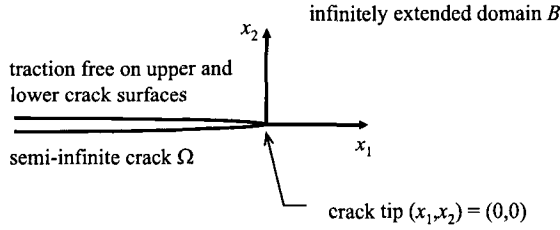


Fig. 7.2 A semi-infinite crack.

Assuming a two-dimensional state of plane strain or plane stress, two other non-trivial solutions for the semi-infinite crack can be derived; they are similar to Eq. (7.7). These solutions correspond to opening and shearing of the crack. In view of the three non-trivial solutions, there are the following three modes of crack propagation:

- Mode I: opening of a crack in plane strain or stress state,
- Mode II: shearing of a crack in plane strain or stress state,
- Mode III: out-of-plane shearing of a crack:

A coefficient of the singularity at a crack tip is called a *stress intensity factor*, and each mode has its own stress intensity factors. The stress intensity factor of the Mode I, II or III is denoted by K_I , K_{II} or K_{III} , respectively.

The stress intensity factors describe field variables near the crack tip. The strain energy that is stored at the crack tip is explicitly¹¹ expressed in terms of the three stress intensity factors, as follows:

$$\mathcal{G} = \frac{1}{E} K_I^2 + \frac{1}{E} K_{II}^2 + \frac{1+\nu}{E} K_{III}^2. \quad (7.8)$$

This energy is called the *energy release rate*. As a load increases, the energy release rate increases. There is a critical value for $\mathcal{G}$, denoted by $\mathcal{G}_c$, at which the crack starts to propagate. This critical value is called the *fracture toughness* and the condition of

$$\mathcal{G} = \mathcal{G}_c \quad (7.9)$$

is called the *fracture criterion*. It is easily seen that if $\mathcal{G}$ decreases with the propagation of crack, it leads to $\mathcal{G} < \mathcal{G}_c$ and the crack growth stops. This is called stable crack growth since the crack growth requires the increase of the loading. On the other hand, if $\mathcal{G}$ does not decrease as the crack propagates, the crack cannot stop growing. This is unstable crack growth. If $\mathcal{G}$ is regarded as a function of the crack

¹¹Note that Eq. (7.8) is for the state of plane strain.

length a , the stable or unstable crack growth is expressed as

$$\begin{cases} \frac{\partial \mathcal{G}}{\partial a} < 0 & \text{stable growth,} \\ \frac{\partial \mathcal{G}}{\partial a} > 0 & \text{unstable growth.} \end{cases}$$

Since $\mathcal{G}$ depends¹² on various parameters, it needs some analysis¹³ to evaluate whether a possible crack growth is stable or unstable.

¹²In general, the crack growth become stable or unstable, respectively, when traction boundary conditions are prescribed or when displacement boundary conditions are prescribed.

¹³Theoretically, $\mathcal{G}$ cannot exceed $\mathcal{G}_c$. Hence, in the case of $\partial \mathcal{G} / \partial a > 0$, external load needs to be decreased or some energy needs to be consumed as kinetic energy which results in dynamic fracture.

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Chapter 8

Analysis of Faulting

An earthquake is caused by the rupture of a source fault in the crust. When the scale of the rupture is large, i.e., a large fault area and a large displacement gap, and when the fault is located in the shallow crust, the rupture propagates through the crust and reaches the ground surface. A fault, or a surface earthquake fault, is thus formed. Such a fault produces large ground deformation, in particular, a displacement gap on the order of $10^0 \sim 1$ [m]. In designing a large-scale structure, therefore, the following three quantities need to be estimated:

- 1) the possibility that a surface earthquake fault will be formed;
- 2) the location at which the surface earthquake fault will appear;
- 3) the ground motion and deformation caused by the surface earthquake fault.

From the mechanical viewpoint, the rupture processes that lead to the formation of a surface earthquake fault are dynamic, probably unstable, fracture phenomena which take place in the heterogeneous crust, involving bifurcation and branching. To clarify the above three points, therefore, mechanical analysis of the rupture processes needs to be made based on comprehensive understanding of such quite complicated phenomena.

The propagation of fault ruptures through a rock mass has been extensively investigated in the past three decades. A number of approaches have been made, such as historical case studies on a particular fault, geological surveys by means of boring and trenching, and model experiments and numerical simulations for the rupture processes. [Scholz (1990)] is recommended to understand the basics of faulting, and it provides an extensive list¹ of related references; see [Plotnikov (1994)] for the shear band formulation related to faulting.

It was probably [Scott and Schoustra (1974)] who first performed a systematic numerical analysis of an earthquake fault. An 800[m] deep alluvial deposit subjected

¹See [Lawson (1908)] and [Riedel (1929)] as the pioneering work of the observation and experiment of faulting; see also [Roth *et al.* (1982)], [Walters and Thomas (1982)], [Cole and Lade (1984)], [Tatsuoka *et al.* (1990)], [Tani and Ueta (1991)] and [Holliger and Levander (1992)] for the fault research based on geotechnical engineering. For recent studies on faulting, see [Braun (1994)], [Ben-Zion and Rice (1995)], [Benson and Hash (1998)], [Greco (1999)], [Lehner and Urai (2000)] and [El-Engebawy *et al.* (2004)], which are more based on solid geophysics.

to a vertical bedrock displacement gap was analyzed by means of FEM. A two-dimensional state of plane strain was assumed, and the deposit was modeled as a linearly elastic and perfectly-plastic material which obeyed the modified von Mises yield criterion. Concentration of shear strain was regarded as a surface earthquake fault. Several researchers employed similar elasto-plastic FEM analyses. They used certain material strength criteria of the rupture processes and could not account for the *size effects*, i.e., the dependence of failure stress and rupture paths on the size of a target body. Recently, [Bray *et al.* (1994a)] performed a two-dimensional FEM analysis² employing a hyperbolic non-linear stress-strain relation, and observed a clear zone of strain localization; see [Bray *et al.* (1993); Bray *et al.* (1994b)] and [Lazarte (1996); Lazarte and Bray (1996)]; see also [Toki and Miura (1985)].

Besides these approaches using FEM, [Cundall (1971)] implemented a distinct element method (DEM) to simulate the fault formation; see Sec. 2.1 for a brief explanation of DEM. DEM appeared intuitively promising, since a fault plane could be modeled as a displacement discontinuity surface with particular frictional properties. In DEM, however, it was not easy to accurately determine the propagation path. As a hybrid of these two methods, [Tani (1994)] implemented joint elements as a model of fault for a two-dimensional FEM analysis, and tried to reproduce model experiment results for reverse faulting. While relatively simple strength-based criteria were used, he was able to show the formation of a conjugate shear plane.

In these numerical studies, no agreement has been reached as to the way to model the fault rupture processes. A strain localization model is easily implemented in FEM, although it ignores the frictional and cohesive properties³ of fault planes. A displacement discontinuity model which is adapted in DEM is more appealing, and analysis based on fracture mechanics is desirable since the rupture processes are dynamic fracture phenomena. In such analysis, attention must be paid to the discontinuity configuration; it is often observed that earthquake faults branch while they run through a soft rock mass, and conjugate sub-faults are formed near the main fault; see [Oguni *et al.* (1996)] for the pattern formation of surface earthquake faults; see also [Hori and Oguni (1998); Hori and Oguni (2000)]. Therefore, it is necessary to analyze⁴ one or several displacement discontinuities which induce bifurcation and branching; see, for instance, [Rice (1968a); Rice (1968b)]; see also [Marcher and Vermeer (2001)] for a list of recent research.

²[Munasinghe (1997)] provides an extensive list of references related to the FEM analysis of surface earthquake fault problems; see also [Munasinghe *et al.* (1996)].

³It is not easy to estimate the frictional and cohesive properties for an actual fault. *Constitutive relations* of sliding planes of rock samples are measured only in laboratory experiments in solid geophysics.

⁴See, for instance, [Sih (1972); Sih (1991)], [Lo (1978)], [Cotterell and Rice (1980)], [Karihaloo *et al.* (1981)] and [Gao and Chiu (1992)] for analysis of bifurcation and branching based on fracture mechanics; see also [Nemat-Nasser *et al.* (1978); Horii and Nemat-Nasser (1983)] and [Sumi *et al.* (1980); Sumi (1986)]; see also [Leryo and Molinari (1992)], [Ikeda *et al.* (1994)].

As mentioned in the introduction of this part, faults and surface earthquake faults have not been a major research topic in earthquake engineering. The primary threat of an earthquake is strong ground motion. Tsunami, liquefaction or large ground deformation has been studied to a considerable extent. While strong ground motion damages broader areas, a surface earthquake fault and induced ground deformation produce damage to a limited area close to the fault line. Furthermore, a fault is less likely to appear on the ground surface. Even in seismology or engineering seismology, surface earthquake faults are a secondary subject, compared with source faults in the crust. Geology⁵ has been studying surface earthquake faults. Two earthquakes in 1999, the ZhiZhi Earthquake in Taiwan and the Cocheli Earthquake in Turkey, changed⁶ this situation, and earthquake engineering research on surface earthquake faults has been activated. This is because large-scale civil structures such as high-rise piers of highways and large dams are severely damaged. [Konagai and Hori (2001); Konagai and Hori (2002)] provide a comprehensive list of recent earthquake engineering studies⁷ on structure damages induced by faulting, numerical analysis methods of the fault formation, and countermeasures against surface earthquake faults.

In Japan, the identification of faults and the clarification of their configuration and past events are intensively made by using aerial photographs and trench surveys. Since the 1995 Great Hanshin Awaji Earthquake, geological surveys on active faults which cause an in-land earthquake or an intra-plate earthquake have been accelerated; see, for instance, [Mavroeidis *et al.* (2004)].

It should not be overlooked that surface earthquake fault damage is spatially limited and a surface earthquake fault is less likely to occur compared with strong ground motion. While strong ground motion causes acceleration or velocity to structures, a surface earthquake fault induces large ground deformation or displacement discontinuity to nearby structures. Thus, if established, a fault resistant design will be completely different from conventional earthquake resistant designs which are based on the evaluation of dynamic structure responses. In Japan, the design philosophy against faulting is established only for extremely important structures such as nuclear power plants or large-scale dams, and it is to avoid a possible surface earthquake fault, by making sure of the non-existence of faults through extensive geological surveys. In the USA and New Zealand, a particular structure system has been applied to gas pipeline systems along fault lines (the Alaska gas pipeline) and a large-scale dam located in an active fault zone (the Clyde Dam). These are lim-

⁵See [Nakata and Imaizumi (2002)]; see also [Nakata *et al.* (1996)], [Ikeda (2002)] and [Ulusay *et al.* (2002); Aydan *et al.* (2002)]

⁶Although most of the surface earthquake fault analyses are based on the assumption of a quasi-static state, some attempts have been made to develop dynamic analysis of faulting; see, for instance, [Todorovska *et al.* (2001)] and [Ramancharla and Meguro (2002)].

⁷See also [Taniyama and Watanabe (2002)] and [Hori *et al.* (2002); Oniduka *et al.* (2002)] for geotechnical engineering studies on faulting, and [Kawashima (2002)] and [Kosa *et al.* (2002)] for structural engineering studies on faulting.

ited cases. An effective measure against faulting is to locate structures away from faults; in California, USA, there has been a regulation which is aimed at locating residential houses substantially far from a fault line.

Today, it is not inevitable⁸ that preparedness for a surface earthquake fault will be enhanced since its hazards are being recognized. The first step will be to select the most dangerous faults, by taking into consideration the following two facts: 1) the interval period of intra-plate earthquakes is of the order of 1,000 years; and 2) geological surveys over the Japanese Islands reveal several highly active faults and their past events are clarified to some extent. The second step will be the prediction of fault behavior. When a fault moves more or less regularly, it is highly possible to predict the next behavior, such as the location, direction and amount of displacement discontinuity, based on geological investigation of past events. Some consideration will be needed for so-called hidden faults in which rupture processes have not reached the ground surface or for cases where rupture processes happen to take a different path forming a new surface earthquake fault in a virgin area. Data obtained from geological surveys are not suitable for the prediction of such cases. Numerical simulation of the rupture processes is a candidate for a tool which can be used for the prediction. The simulation computes the rupture processes for a given earthquake scenario, in order to evaluate the possibility of the surface earthquake fault formation and to estimate the fault location and the magnitude of induced ground deformation.

Problem setting is of primary importance for the prediction of fault behavior by means of numerical simulation. First, the domain of analysis must be considered. Constructing a detailed model for underground structures is difficult. It is rare that the geological structures of thickness of 10 and more kilometers are fully identified. Thus, unconsolidated soil deposits or alluvium layers will be suitable as a domain of analysis; geological or geotechnical measurements are applicable and some model can be constructed. Next, suitable boundary conditions must be chosen. Load input to the model should not be underestimated, since only the last process of the fault rupture is simulated. Faulting hazards can be ignored if the simulation which overestimates the input load shows the dissipation of the rupture processes within the model; such dissipation actually occurs since the rupture process turns out to be ductile within soft layers even though a fault propagates in geological layers and bedrocks in a brittle manner. In a case when the simulation shows a possibility of the fault formation, more extensive study will be needed to clarify a zone which is influenced by faulting or to estimate the amount of induced ground deformation.

There is significant variability in fault behavior. On the engineering length scale of 1~10[m], a surface earthquake fault appears only in limited zones, and displace-

⁸In Japan, habitable land is 30% of the whole nation, and there are around 100 very active faults; the Japanese Islands are located near boundaries between three or four plates. Thus, locating all structures far from fault lines is not realistic. Also, ensuring safety by carrying out intensive geological surveys at every construction site is not feasible. Suitable countermeasures against fault hazards are surely needed.

ment discontinuity varies from place to place. On this length scale, heterogeneity of the surface layers which form complicated ground structures is one source of such variability. The identification of surface layer structures of 10~100 meters is not easy, even though advanced physical surveys⁹ are applicable. Bifurcation which leads to branching of the rupture surface is also a cause of variability. In model experiments, it is observed that several bifurcations take place during the rupture processes; [Oguni *et al.* (1996)] shows the existence of a bifurcated solution for the rupture processes of faulting. Taking these two causes into consideration, a stochastic model, in which bifurcation phenomena can be reproduced, needs to be constructed in order to numerically simulate the rupture processes of a surface earthquake fault.

8.1 NL-SSFEM

A stochastic model used here is an elasto-plastic body; see Sec. 3.1 for a stochastic model of a linearly elastic body. The body is spatially and stochastically heterogeneous, and the material parameters are given as a random functions. In such a stochastic model, field variables become random vectors or tensors which satisfy non-linear field equations. Due to the material non-linearity, the variability of field variables may become complicated. The nature of the variability in the non-linear plastic state will be quite different from that in the linear elastic state; see [Gutierrez and de Borst (1999)].

This section extends a spectral stochastic finite element method (SSFEM) to solve a non-linear problem of an elasto-plastic stochastic model. The extension is called a non-linear spectral stochastic finite element method¹⁰ (NL-SSFEM). In NL-SSFEM, by applying the spectral decomposition and related expansion, probabilistic characteristics such as mean and variance are calculated for the non-linear problem. Attention must be paid to the approximate computation of random vectors and tensors for field variables which are mutually related through non-linear relations; see [Anders and Hori (1999); Anders and Hori (2001)].

8.1.1 SSFEM

For simplicity, considered is a case when a body is isotropic and only Young's modulus is given as a random function, $E(\mathbf{x}, \omega)$, which varies spatially and stochastically; ω represents an event in a probability space Ω . Field variable such as displacement,

⁹Physical surveys range from a mechanical survey which uses wave propagation and reflection to find ground layers or measures gravity change to identify the density distribution, to an electromagnetic survey which seeks to identify underground structures by means of the electromagnetic conductivity.

¹⁰In summary, SSFEM of [Ghanem and Spanos (1990)] combines spectral decomposition with FEM, and NL-SSFEM is an extension of SSFEM which is aimed at solving non-linear problems; see [Anders (2000)].

strain and stress become stochastic, $u_i(\mathbf{x}, \omega)$, $\epsilon_{ij}(\mathbf{x}, \omega)$ and $\sigma_{ij}(\mathbf{x}, \omega)$. From the viewpoint of probability theory, a set of stochastic quantities ought to be described by the ensemble probability distribution. While the field equations establish direct relations which must be satisfied by field variables of each realization, the explicit dependence of stochastic quantities on elementary events is irrelevant. The only purpose of the field equations is to provide a relation between the given probability distribution of the material properties and the unknown joint distribution of the field variables. Therefore, the stochastic problem at hand should be stated as follows:

For a given probability distribution of $E(\omega)$, find the conditional probability distribution of $u_i(\omega)$, $\epsilon_{ij}(\omega)$ and $\sigma_{ij}(\omega)$.

As explained in Sec. 3.2, it is quite difficult to solve the above problem of finding the conditional probability. However, the conditional probability does not have to be explicitly considered by transforming the stochastic boundary value problem which is derived from the field equations to the equivalent stochastic variational problem.

Now, SSFEM which solves a linear stochastic variational problem is briefly summarized. The following two assumptions are made for $E(\omega)$:

- 1) the correlation function is given for E , i.e., $R(\mathbf{x}, \mathbf{y}) = \langle (E(\mathbf{x}, \omega) - \bar{E})(E(\mathbf{y}, \omega) - \bar{E}) \rangle$ with $\bar{E} = \langle E \rangle$, and this R admits the spectral decomposition,

$$R(\mathbf{x}, \mathbf{y}) = \sum_{n=1} r^n \phi^n(\mathbf{x}) \phi^n(\mathbf{y}),$$

where $\{\lambda^n, \phi^n\}$ are a set of eigen-values and eigen-functions, satisfying $(\lambda^n)^2 \phi^n(\mathbf{x}) = \int_B R(\mathbf{x}, \mathbf{y}) \phi^n(\mathbf{y}) dv_{\mathbf{y}}$ and $\int_B \phi^n(\mathbf{x}) \phi^m(\mathbf{x}) dv_{\mathbf{x}} = 1$ for $n=m$ and $=0$ for $n \neq m$;

- 2) a random variable defined as $\xi^n(\omega) = \int_B E(\mathbf{x}, \omega) \phi^n(\mathbf{x}) dv_{\mathbf{x}}$ obeys a Gauss distribution of zero mean and standard deviation σ^n .

With the aid of the two assumptions, the random function E admits the following the Karhunen-Loeve (KL) expansion:

$$E(\mathbf{x}, \omega) = \bar{E} \left(1 + \sum_{n=1} \lambda^n \phi^n(\mathbf{x}) \xi^n(\omega) \right), \quad (8.1)$$

where $\xi^n(\omega)$ and λ^n are redefined as $\xi^n(\omega)/\sigma^n$ and $\sqrt{\lambda^n}/\bar{E}$. The bases for non-Gaussian random distributions are constructed by using the polynomial chaos of $\{\xi^n(\omega)\}$. That is, for a k -th dimensional vector of $\{\xi^{m_p}(\omega)\}$,

$$\Gamma^k = (-1)^k \frac{1}{\gamma^k(\{\xi^{m_p}\})} \frac{\partial^k \gamma^k(\{\xi^{m_p}\})}{\partial \xi^{m_1} \dots \partial \xi^{m_k}} \left(\gamma^k = \exp(-\frac{1}{2} \sum_p (\xi^{m_p})^2) \right),$$

is computed. By changing k and $\{\xi^{m_p}(\omega)\}$, similar Γ^k 's are generated, and the group of such Γ^k , denoted by $\{\Psi^m\}$, form the bases for non-Gaussian random

distributions. A random vector u_i is now expanded in the following polynomial chaos (PC) expansion:

$$u_i(\mathbf{x}, \omega) = \sum_{m=0} u_i^m(\mathbf{x}) \Psi^m(\omega). \quad (8.2)$$

Note that $\Psi^0=1$ and u_i^0 gives the mean behavior. With the aid of these two expansions, Eqs. (8.1) and (8.2), SSFEM can be formulated by taking the following procedures.

- i) Consider a stochastic functional for u_i satisfying $u_i=u_i^o$ on ∂B ,

$$J^\Omega(\mathbf{u}; \mathbf{c}) = \int_B \int_\Omega \frac{1}{2} c_{ijkl}(\omega) u_{i,j}(\omega) u_{k,l}(\omega) dv P(d\omega). \quad (8.3)$$

- ii) Substitute Eqs. (8.1) and (8.2) into Eq. (8.3).

- iii) Integration of $\xi^n(\omega)$'s and $\Psi^m(\omega)$'s over Ω yields a functional for the coefficients $\{u_i^m\}$ satisfying the boundary condition of $u_i^m=\langle \Psi^m \rangle u_i^o$ on ∂B ,

$$J^\Omega(\{\mathbf{u}^m\}) = \sum_{m,m'} \int_B \frac{1}{2} E^{mm'}(\mathbf{x}) h_{ijkl} u_{i,j}^m(\mathbf{x}) u_{k,l}^{m'}(\mathbf{x}) dv_{\mathbf{x}}. \quad (8.4)$$

- iv) Discretization of $\{u_i^m\}$ in Eq. (8.4) by means of shape functions leads to SSFEM.

Here, $c_{ijkl}(\omega)$ is an stochastic elasticity tensor given as $c_{ijkl}(\omega)=E(\omega)h_{ijkl}$ with $h_{ijkl}=\frac{\nu}{(1+\nu)(1-2\nu)}\delta_{ij}\delta_{kl}+\frac{1}{2(1+\nu)}I_{ijkl}$ and $E^{mm'}$ is deterministic and defined as $E^{mm'}=\sum_n \lambda^n \phi^n \langle \xi^n \Psi^m \Psi^{m'} \rangle$. Note that the stochastic variational problem given by Eq. (8.3) is equivalent with the stochastic boundary value problem of the stochastic model, and the variational problem given by Eq. (8.4) leads to a deterministic boundary value problem for $\{u_i^m\}$; see Sec. 3.2.

8.1.2 NL-SSFEM

As briefly explained in Sec. 7.2, elasto-plasticity leads to the apparent constitutive relation between strain rate and stress rate. The apparent constitutive relation is applicable to the current non-linear stochastic model. The governing equation for random displacement rate vector is given as

$$(c_{ijkl}^{\text{ep}}(\boldsymbol{\sigma}(\omega), \omega) du_{k,l}(\mathbf{x}, \omega)),_i = 0, \quad (8.5)$$

with

$$c_{ijkl}^{\text{ep}}(\boldsymbol{\sigma}(\omega), \omega) = \begin{cases} E(\omega) \left(h_{ijkl} - \frac{h_{ijpq} \nabla f_{pq}(\boldsymbol{\sigma}(\omega)) h_{klrs} \nabla f_{rs}(\boldsymbol{\sigma}(\omega))}{\nabla f_{tu}(\boldsymbol{\sigma}(\omega)) h_{tuvw} \nabla f_{vw}(\boldsymbol{\sigma}(\omega))} \right) & \text{for } f = 0 \text{ and } df = 0, \\ E(\omega) h_{ijkl} & \text{for } f < 0 \text{ or } df < 0. \end{cases} \quad (8.6)$$

Here, f is a yield function of stress and $\nabla f_{ij} = \partial f / \partial \sigma_{ij}$ is the gradient of f . It should be emphasized that the elasto-plasticity c_{ijkl}^{ep} has two sources for stochasticity, $E(\omega)$ and $\nabla f_{ij}(\sigma(\omega))$.

When suitable boundary conditions are prescribed on the boundary ∂B , Eq. (8.5) forms a stochastic boundary value problem for displacement rate $du_i(\omega)$. The stochastic variational problem that is equivalent with the stochastic boundary value problem is readily derived, by defining the stochastic functional as

$$J^\Omega(du(\omega); c^{\text{ep}}(\omega)) = \int_B \int_\Omega \frac{1}{2} c_{ijkl}^{\text{ep}}(\sigma(\omega), \omega) du_{i,j}(\omega) du_{k,l}(\omega) dv P(d\omega). \quad (8.7)$$

The boundary conditions for displacement or traction rate are determined from the boundary conditions for displacement or traction. Like a stochastic functional for linear elasticity, the following inequality holds for J^Ω of non-linear elasto-plasticity:

$$\langle d^2 \mathcal{E} \rangle < J^\Omega(du(\omega); c^{\text{ep}}(\omega)), \quad (8.8)$$

where¹¹ $\langle d^2 \mathcal{E} \rangle$ is the mean of the apparent elasto-plastic strain energy, given as $J^\Omega(du^{\text{exact}}(\omega); c^{\text{ep}}(\omega))$ with du_i^{exact} being the solution of the boundary value problem. The proof is straightforward; since $c_{ijkl}^{\text{ep}}(\omega)$ is positive definite for each event, $du_i^{\text{exact}}(\omega)$ minimizes the functional,

$$J(du(\omega), c^{\text{ep}}(\omega)) = \int_B \int_\Omega \frac{1}{2} c_{ijkl}^{\text{ep}}(\omega) du_{i,j}(\omega) du_{k,l}(\omega) dv,$$

and hence $du_i^{\text{exact}}(\omega)$ minimizes $J^\Omega = \langle J \rangle$.

8.1.3 Bounding medium approximation

The gradient of the yield function, $\nabla f_{ij}(\sigma(\omega))$, is used in computing $c_{ijkl}^{\text{ep}}(\sigma(\omega), \omega)$ of the functional J^Ω . As its argument shows, this $\nabla f_{ij}(\sigma(\omega))$ depends on the random tensor $\sigma_{ij}(\omega)$, and hence it becomes a random tensor. Whether ∇f_{ij} is a non-linear function of σ_{ij} or not, the dependence of $c_{ijkl}^{\text{ep}}(\sigma(\omega), \omega)$ on $\nabla f_{ij}(\sigma(\omega))$ is non-linear, as shown in Eq. (8.6). Dealing with this non-linearity is the key issue in solving the stochastic variational problem of J^Ω given by Eq. (8.7).

In order to deal with the non-linear dependence of $c_{ijkl}^{\text{ep}}(\sigma(\omega), \omega)$ on $\sigma_{ij}(\omega)$, the approximation that the mean stress, $\langle \sigma_{ij} \rangle$, is used to evaluate $\nabla f_{ij}(\sigma(\omega))$ instead of stochastically varying $\sigma_{ij}(\omega)$ is made. This approximation does not produce a significant error, if the variance of E is sufficiently small; this is easily seen in the expansion of $\nabla f_{ij}(\sigma(\omega))$ at the mean stress, i.e.,

$$\nabla f_{ij}(\sigma(\omega)) = \nabla f_{ij}(\langle \sigma \rangle) + \mathcal{O}(|\sigma_{ij}(\omega) - \langle \sigma_{ij} \rangle|). \quad (8.9)$$

¹¹ As is seen, $d^2 \mathcal{E}$ is defined as $d^2 \mathcal{E} = J(du^{\text{exact}}, c^{\text{ep}})$. The form of $d^2 \mathcal{E}$ is identical with the total strain energy, $\mathcal{E}$.

In terms of f and $d\lambda$, the mean stress is expressed as

$$\langle \sigma_{ij} \rangle = \left\langle \int c_{ijkl} (d\epsilon_{kl} - d\lambda \nabla f_{kl}(\sigma)) \right\rangle.$$

As is seen, the computation of $\langle \sigma_{ij} \rangle$ is tedious because it includes the triple product of random variables, $c_{ijkl}(\omega)$, $d\lambda(\omega)$ and $\nabla f_{ij}(\sigma(\omega))$. Now, the following two assumptions are made to compute the mean stress by making use of the bounding media: 1) stress in the bounding media is used to evaluate ∇f_{ij} in computing stress rate in terms of strain rate; and 2) stress in the bounding media is related through the mean of strain and plastic coefficient. Denoting the bounding media with $E^+ = \langle E \rangle$ and $1/E^- = \langle 1/E \rangle$ by $B^\pm$ and stress in $B^\pm$ by $\sigma_{ij}^\pm$, the first assumption is written as

$$d\sigma_{ij}(\mathbf{x}, \omega) = E(\mathbf{x}, \omega) h_{ijkl} (d\epsilon_{kl}(\mathbf{x}, \omega) - d\lambda^\pm \nabla f_{kl}^\pm(\mathbf{x})), \quad (8.10)$$

where $\nabla f_{ij}^\pm = \nabla f_{ij}(\sigma^\pm)$ and $d\lambda^\pm$ is determined from the yield condition of $B^\pm$. The second assumption means that for the equality $\langle d\sigma_{ij} \rangle = \langle E h_{ijkl} (d\epsilon_{kl} - d\lambda \nabla f_{kl}) \rangle$ to hold, the left side is approximated by $d\sigma_{ij}^\pm$ and the right side is evaluated by using field variables in $B^\pm$, i.e.,

$$d\sigma_{ij}^\pm(\mathbf{x}) = E^\pm(\mathbf{x}) h_{ijkl} (\langle d\epsilon_{kl}(\mathbf{x}) \rangle - \langle d\lambda(\mathbf{x}) \rangle \nabla f_{kl}^\pm(\mathbf{x})). \quad (8.11)$$

As is seen, Eq. (8.10) is the replacement of the constitutive relation of elasto-plasticity, and Eq. (8.11) guarantees that the $\sigma_{ij}^\pm$ is used as the approximation of $\langle \sigma_{ij} \rangle$.

It is much easier to use Eq. (8.10) instead of $d\sigma_{ij}(\omega) = c_{ijkl}^{\text{ep}}(\sigma(\omega), \omega) d\epsilon_{kl}(\omega)$, even if Eq. (8.11) needs to be satisfied. This is because the non-linear dependence of $c_{ijkl}^{\text{ep}}(\sigma(\omega), \omega)$ on stochastic $\sigma_{ij}(\omega)$ is removed and Eqs. (8.10) and (8.11) do not have triple products of random field variables. This approximation is the cornerstone of NL-SSFEM, and is called *bounding medium approximation*. Using $\sigma_{ij}^\pm$ to evaluate f and ∇f_{ij} and using $\langle d\epsilon_{ij} \rangle$ to compute $\sigma_{ij}^\pm$ means that the conditions for the field equations to hold for each realization and the bounding media are relaxed. The relaxed conditions, however, produce no error in the deterministic limit, i.e., in the limit as the variance of $E(\omega)$ approaches zero. The validity of this approximation will be discussed with numerical computation results for an example problem.

Stress in the bounding medium, $\sigma_{ij}^\pm$, is used as the approximation of the mean stress, $\langle \sigma_{ij} \rangle$, and it is necessary to enforce $\sigma_{ij}^\pm$ or $\langle \sigma_{ij} \rangle$ to satisfy the equation of equilibrium. In NL-SSFEM, this condition is assigned for the mean stress, i.e., $\langle d\sigma_{ij} \rangle_{,i} = 0$ or $\langle \sigma_{ij} \rangle_{,i} = 0$. On the other hand, the yield condition of the bounding medium must be satisfied to determine $d\lambda^\pm$; see Eq. (8.10). This leads to the evaluation of the elasto-plasticity tensor by using the field in the bounding medium, i.e.,

$$c_{ijkl}^{\text{ep}}(\sigma(\omega), \omega) \approx c_{ijkl}^{\text{ep}\pm}(\omega)$$

where

$$c_{ijkl}^{\text{ep}\pm}(\omega) = E(\omega) \left(h_{ijkl} - \frac{h_{ijpq} \nabla f_{pq}^{\pm} h_{klrs} \nabla f_{rs}^{\pm}}{\nabla f_{tu}^{\pm} h_{tuvw} \nabla f_{vw}^{\pm}} \right). \quad (8.12)$$

As mentioned, $\nabla f_{ij}^{\pm}$ is the gradient of the yield stress evaluated by using $\sigma_{ij}^{\pm}$. Substituting Eq. (8.12) into Eq. (8.7) yields NL-SSFEM, together with the requirement that the mean stress satisfy the equation of equilibrium.

It is certainly true that the bounding medium approximation¹² is an approximation the validity of which cannot be mathematically verified; this approximation may excessively relax the conditions that are required for the random field variables. As mentioned several times, however, the bounding medium approximation can yield a more accurate solution when the variance of the material properties is smaller. Some physical insight can be provided as well. When subjected to displacement loading, yielding occurs at earlier state of loading, as realized Young's modulus takes on a larger value. Hence, both soft B^- and stiff B^+ satisfying $E^- < E^+$ provide some bounds for stochastic behavior of yielding; B^+ yields at earlier stage of loading and B^- at a later stage.

8.1.4 Formulation of NL-SSFEM

Now, the formulation of NL-SSFEM is summarized. The functional of the stochastic variational problem is the object of NL-SSFEM. For numerical computation, this non-linear problem is solved incrementally, i.e., $du_i(\omega)$ is computed for given increment of an external load. The solution of $du_i(\omega)$ is obtained in an iterative manner. To emphasize the difference between the increment and the iteration, the stochastic functional given by Eq. (8.7) is rewritten as

$$J^{\Omega}(du^{n,m}, c^{\text{ep } n,m}, u^{\circ n}),$$

where superscripts n and m indicate quantities of the n^{th} increment of loading and the m^{th} iteration during the n^{th} increment, respectively; $u_i^{\circ n}$ is boundary displacement prescribed at the n^{th} increment. According to the bounding medium approximation, $c_{ijkl}^{\text{ep } n,m}$ is evaluated using Eq. (8.12), as

$$c_{ijkl}^{\text{ep } n,m} = E(\omega) \left(h_{ijkl} - \frac{h_{ijpq} \nabla f_{pq}^{\pm n,m-1} h_{klrs} \nabla f_{rs}^{\pm n,m-1}}{\nabla f_{tu}^{\pm n,m-1} h_{tuvw} \nabla f_{vw}^{\pm n,m-1}} \right),$$

¹² Attention must be paid to the difference between the bounding media and the *limit theorems*, both of which provide some bounds; the bounding media are defined for a body with stochastically varying material properties and provide bounds for the total strain energy, while the limit theorems give limit loads for a deterministic body. For a stochastic body, the limit theorems are applicable to each realization and the probabilistic characteristics of the limit loads are obtained, although it may require huge numerical computation. It cannot be proved whether the limit theorems are applicable to the fictitious bounding media, since the stress used in the limit theorems may lie outside a yield surface of some realizations; the stress is required to remain within the yield surface of the bounding medium.

where $\nabla f^{\pm n,m-1}$ is evaluated for $\sigma_{ij}^{\pm n,m-1}$, the stress of the previous iteration in the bounding medium $B^{\pm}$. Denoting the terms in the parenthesis of the above equation by $h_{ijkl}^{\pm n,m-1}$, the functional is approximated as

$$J^{\Omega}(du^{n,m}, c^{ep n,m}; u^{\circ n}) \approx J^{\Omega}(du^{n,m}, Eh^{\pm n,m-1}; u^{\circ n}). \quad (8.13)$$

Note that the stochastic quantities in the right side of Eq. (8.13) are $du_i^{n,m}$ and E , which are expanded in the PC and KL expansions, respectively. Also, the mean stress $\langle \sigma_{ij}^{n,m} \rangle$ is required to satisfy the equilibrium and the yield condition is evaluated for the bounding media.

Through the discretization of random functions in the stochastic and physical spaces, the following matrix equation for unknown displacement rate can be formally obtained:

$$[K^{\pm n,m-1}] [dU^{n,m}] = [dF^{\text{int } n}] + [dF^{\text{ext } n,m}]. \quad (8.14)$$

Here, $[K^{\pm n,m}]$ is a stiffness matrix that corresponds to $c_{ijkl}^{ep n,m} = Eh_{ijkl}^{\pm n,m}$, and $[dF^{\text{int } n}]$ and $[dF^{\text{ext } n,m}]$ are external and internal force vectors; they are deterministic and stochastic parts which are derived from the boundary displacement.

In Eq. (8.14), the computation of the internal force vector $[dF^{\text{int } n,m}]$ is complicated¹³ since it corresponds to the equilibrium of stress, not stress rate. The discretization of the equation of equilibrium is expressed as $[B]^T[\Sigma]$ where $[B]$ is determined by the spatial discretization that uses shape functions and $[\Sigma]$ is a vector for stress components which is computed by using stress at Gauss points of all elements. Then, $[dF^{\text{int } n,m}]$ is

$$[dF^{\text{int } n,m}] = [B]^T[\Sigma^{n,m}]. \quad (8.15)$$

It should be noted that this estimation of $[dF^{\text{int } n,m}]$ is not essential, as it can be expanded in the same PC expansion as $[dU^{n,m}]$. However, ordinary non-linear analysis of elasto-plasticity uses an estimation of internal force vector similar to Eq. (8.15) since it simplifies numerical computation. Furthermore, in order to guarantee that the mean stress is approximated by stress in the bounding media, NL-SSFEM uses the backward Euler method to satisfy the following two equations for the mean stress:

$$\begin{cases} \langle \sigma_{ij}^{n,m} \rangle - \langle \sigma_{ij}^{n,m-1} \rangle = E^{\pm} h_{ijkl}^{n,m-1} (\langle \epsilon_{kl}^{n,m} \rangle - \langle \epsilon_{kl}^{n,m-1} \rangle - \langle d\lambda^{\pm n,m} \rangle \nabla f_{kl}^{\pm n,m}), \\ f^{\pm}(\langle \sigma^{\pm n,m} \rangle) \leq 0. \end{cases} \quad (8.16)$$

Here, $f^{\pm}(\langle \sigma^{\pm n,m} \rangle) \leq 0$ emphasizes that the yield condition of the bounding medium is satisfied by the mean stress $\langle \sigma_{ij}^{n,m} \rangle$. The flowchart of NL-SSFEM is shown in

¹³ Actually, discretization of $\delta J^{\Omega}(du^{n,m}, Eh^{\pm n,m-1}; u^{\circ n}) = 0$ leads to $[dF^{\text{int } n,m}] = [0]$. This discretization is not accurate in numerical computation; there is a small amount of error in satisfying the equilibrium for stress rate; and error in satisfying the equilibrium of stress, which is given as the sum of stress rate, is accumulated as the load increases.

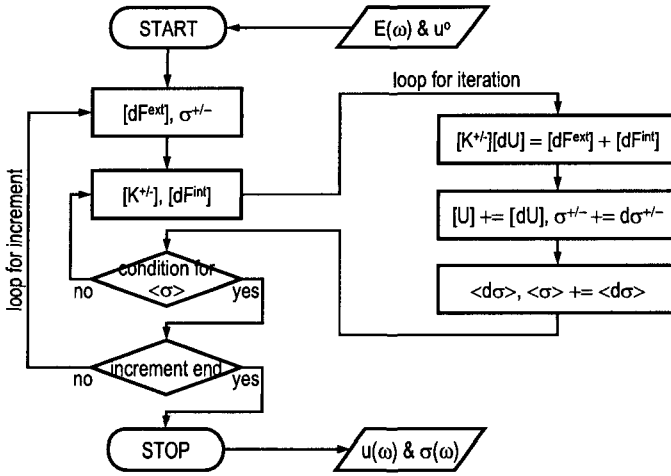


Fig. 8.1 A flow chart of NL-SSFEM.

Fig. 8.1; the two loops for the iteration and the increment are presented.

8.2 Numerical Algorithms of NL-SSFEM

As shown in Eq. (8.2), the degree-of-freedom of SSFEM is increased by M times when the PC expansion is applied to displacement. It causes numerical difficulty in solving¹⁴ the resulting matrix equation. For instance, if the algorithm of the Cholesky decomposition is used, the amount of calculation required to solve the matrix equation increases in proportion to the square of the matrix dimension; see Sec. 2.5 for the Cholesky decomposition. That is, if $M=6$, the required computation increases by 36 times. Furthermore, the nature of the matrix used in SSFEM is different from that of the matrix used in ordinary FEM, which is sparse with many off-diagonal components being zero. The matrix of SSFEM is not sparse, since it is weighted by the bases in the probabilistic space. In summary, SSFEM needs much more computational effort compared with FEM, due to the stiffness matrix being of large dimensions and not sparse.

8.2.1 Matrix Jacobi method

Parallel computation is presumed for NL-SSFEM, in particular, when a three-dimensional problem is solved. Some cares must be taken in solving the matrix equation for a less sparse matrix of large dimensions. First, a computer node is separately assigned to each term of the PC expansion, $u^m = \langle \Psi^m \rangle$; a parallel computer of $M+1$ nodes is suitable when the PC expansion uses M terms. An efficient

¹⁴This is related to *solvability of numerical problem*.

algorithm, called a *matrix Jacobi method*, is developed to enhance the computational efficiency of NL-SSFEM. As explained in Sec. 2.5, the original Jacobi method is an algorithm used to solve a matrix equation for a matrix which has larger diagonal components; by computing the power of the matrix, diagonal components increase and others decrease, and a solution of the matrix equation can be found in an iterative manner.

The matrix Jacobi method is an extension of the Jacobi method; see [Anders and Hori (1999)]. A stiffness matrix of SSFEM which consists of M^2 matrices of dimension L is written in the following manner:

$$\begin{bmatrix} [\langle \Psi^1 \Psi^1 K \rangle] & [\langle \Psi^1 \Psi^2 K \rangle] & \dots & [\langle \Psi^1 \Psi^M K \rangle] \\ [\langle \Psi^2 \Psi^1 K \rangle] & [\langle \Psi^2 \Psi^2 K \rangle] & \dots & [\langle \Psi^2 \Psi^M K \rangle] \\ \vdots & \vdots & \ddots & \vdots \\ [\langle \Psi^M \Psi^1 K \rangle] & [\langle \Psi^M \Psi^2 K \rangle] & \dots & [\langle \Psi^M \Psi^M K \rangle] \end{bmatrix}$$

Here, $[\langle \Psi^i \Psi^j K \rangle]$ is an $L \times L$ submatrix which is computed by an original stiffness matrix $[K(\omega)]$ and two weight functions of the PC expansions, $\Psi^i(\omega)$ and $\Psi^j(\omega)$. Due to the orthonormality of the bases, components of submatrices become smaller as the difference of i and j increases. Computing the power of the $ML \times ML$ stiffness matrix leads to smaller off-diagonal submatrices. The computational efficiency is increased by computing the power for each submatrix, since, by definition, all submatrices have a distribution of components which is similar to that of the matrix $[K(\omega)]$; the distribution of components is sparse with many zero off-diagonal components.

The outline of the matrix Jacobi method adopted in NL-SSFEM is summarized as follows.

- i) Assign to each computer node a set of submatrices in a common row, $\{[\langle \Psi^i \Psi^1 K \rangle], [\langle \Psi^i \Psi^2 K \rangle], \dots, [\langle \Psi^i \Psi^M K \rangle]\}$.
- ii) Take the inverse of each diagonal submatrix, $[\langle \Psi^i \Psi^i K \rangle]$ for $i=1, 2, \dots, M$.
- iii) Multiply inverse matrix to submatrices in the same row, so that diagonal submatrix becomes unit matrix of dimension L and off-diagonal matrices have components smaller than 1.
- iv) Find PC expansion coefficients, u^m for Ψ^m , by applying iterative algorithm to the $ML \times ML$ stiffness matrix.

In this algorithm, it surely requires large computation to take the inverse of M diagonal submatrices of dimension L . However, it is computationally more efficient compared with directly solving a matrix equation of dimensions ML .

8.2.2 Standardized KL expansion

SSFEM uses two discretization schemes of displacement which is given as a random vector-valued function. Spatial discretization has the same accuracy as ordinary

FEM. The accuracy of probabilistic discretization that is based on the KL expansion depend on the number of expanded terms and on the validity of presumed Gauss distribution to serve as the bases of the KL expansion.

As explained in Sec. 3.2, the KL expansion is the spectral decomposition of a random function in the probabilistic space. It is monotonic in nature; as more terms are used in the expansion, the accuracy increases. For instance, if the correlation for Young's modulus $E(\omega)$ is computed by using the KL expansion, the computed value monotonically increases as more terms are used, and it eventually converges to the exact value. This monotonicity is due to the choice of orthonormal bases in the probabilistic space which are used to construct the KL expansion.

In other words, the correlation of random functions is always underestimated in SSFEM which uses the truncated KL expansion for $E(\omega)$; the variability of displacement, strain or stress is underestimated as well. To solve this problem, the KL expansion is modified by increasing the eigen-values for each base used in the KL expansion so that the correlation of the expanded $E(\omega)$ remains the same. This modification is called *standardization* of the KL expansion; see [Anders and Hori (1999)]. The standardized KL expansion is easily formulated, when $E(\omega)$ is given as

$$E(\mathbf{x}, \omega) = \bar{E} \left(1 + \sum_{n=1} \lambda^n \phi^n(\mathbf{x}) \xi^n(\omega) \right);$$

see Eq. (8.1). The standard deviation of this $E(\omega)$ is $\bar{E} \sqrt{\sum (\lambda^n)^2}$, which is always smaller than the given σ unless infinitely many terms are used in the expansion. The modification is made only for eigen-values, $\{\lambda^n\}$, as follows:

$$E(\mathbf{x}, \omega) = \bar{E} \left(1 + \sum_{n=1} \frac{\lambda^n}{\sqrt{\theta_N}} \phi^n(\mathbf{x}) \xi^n(\omega) \right), \quad (8.17)$$

where

$$\theta_N = \frac{1}{\sigma^2} \sum_{n=0} (\lambda^n)^2. \quad (8.18)$$

As is seen, $\{\phi^n\}$ and $\{\xi^n(\omega)\}$ remain unchanged, and $\{\lambda^n/\sqrt{\theta_N}\}$ are the modified eigen-values so that the standard deviation of the expanded $E(\omega)$ takes on the same value as σ exactly. It is shown that SSFEM to which the standardized KL expansion is implemented is able to estimate the variability of displacement more accurately; see [Anders and Hori (1999)].

8.2.3 Numerical perturbation during analysis of stochastic model

NL-SSFEM uses an associate flow rule as a non-linear elasto-plastic constitutive relation. Strain localization that takes place only on a shear band is the key mechanism of failure of soil layers. Near the occurrence of strain localization, instantaneous elasto-plastic moduli change abruptly; the moduli of a plastic zone are smaller

by the orders of magnitude compared with those of the surrounding elastic zone. As strain localization proceeds, such a discontinuous change in the instantaneous elasto-plastic moduli propagates along the shear band. In numerical computation, stress state is computed only at Gauss points within each element, and hence finer meshing is needed to express a narrow shear band. However, such finer meshing may not be enough to produce strain localization, since it is difficult to numerically solve a differential equation with coefficients which change in a discontinuous manner. Furthermore, determining plastic loading or elastic unloading at each Gauss point sometimes requires additional computation if an iterative algorithm is used to solve this differential equation.

It should be emphasized that solving a non-linear differential equation with discontinuous coefficients is difficult even with the aid of sophisticated numerical algorithms used to solve a broad class of non-linear equations. For instance, the Newton-Raphson method or the Runge-Kutta method of higher order is a robust algorithm used to solve non-linear equations, but such an algorithm may not be able to efficiently find a solution of the non-linear differential equation which has discontinuous¹⁵ coefficients; see Sec. 2.5 and 4.3 for the explanation of the Newton-Raphson method and the Runge-Kutta method. The discontinuous change in the coefficients takes place spatially, and some coefficients change discontinuously as load increases. Strain localization is thus a phenomenon which is difficult to reproduce in numerical simulation.

When a stochastic model is used for an elasto-plastic body, the discontinuity in the coefficients of the governing differential equation does not happen. This is because plastic loading or elastic unloading takes place stochastically. That is, at one point, the probability that the state is in an elastic regime gradually decreases as stress there increases, since the stress varies stochastically. The state cannot be discontinuously changed from elastic to plastic. Therefore, NL-SSFEM solves a non-linear matrix equation which has continuously changing coefficients; the change may not be smooth when elastic unloading takes place, but it remains continuous.

Non-linear equations often have bifurcation, i.e., there occurs bifurcation from a unique solution to multiple solutions. From the mathematical viewpoint, the fault problem has this bifurcation. There happen to be multiple solutions when the base movement takes a certain value. While multiple solutions mathematically exist, in reality, realized deformation is unique. This realized deformation corresponds to one particular solution among the multiple solutions. It is known that such a solution is the most unstable among all solutions, and can be found in numerical analysis by giving some perturbation to a solution. The non-linear equation of stochastic elasto-plasticity is more suitable for finding the most unstable solution compared with a non-linear equation of deterministic elasto-plasticity, since the

¹⁵ As is seen, the discontinuity in the coefficients corresponds to the case where a region at plastic state is surrounded by regions at elastic state, or the case when a certain region is transformed to plastic state from elastic state or vice versa.

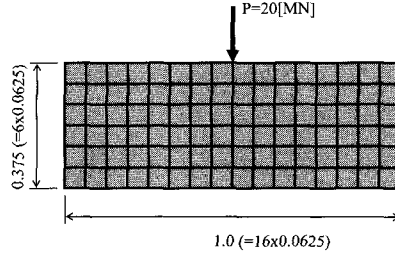


Fig. 8.2 A stochastic model of an elasto-plastic rectangular plate.

coefficients change continuously; see [Hori *et al.* (2002); Hori and Oguni (1998); Hori and Oguni (2000)]. Furthermore, the probabilistically expanded terms play a role of perturbation in numerical computation. It is expected that the most unstable solution is automatically found during the iteration process of solving the non-linear equation in NL-SSFEM.

8.3 Validation of NL-SSFEM Simulation

In order to validate NL-SSFEM, this section solves¹⁶ an example problem of an elasto-plastic stochastic model. The model is a rectangular plate in a two-dimensional state of plane strain, the dimension is 1×0.375 [m], and a line force load of 20 [MN/m] is applied at the center of the top surface; see Fig. 8.2. Young's modulus E is given as a random function; the mean is $\bar{E}=200$ [GPa] and the covariance function is

$$R(\mathbf{x}, \mathbf{y}) = \sigma^2 \exp\left(-\frac{|\mathbf{y}_1 - \mathbf{x}_1|}{\ell} - \frac{|\mathbf{y}_2 - \mathbf{x}_2|}{\ell}\right).$$

with $\ell=1$ [m] and the coefficient of variance (COV), $\frac{\sigma}{\bar{E}}$, is changed from 0.05 to 0.30. For this setting, the first four eigen-values and eigen functions of $\frac{R}{\bar{E}}$ are calculated as

$$\begin{aligned} \phi^1(\mathbf{x}) &= 1.80326 \cos(1.30654x_1) \cos(2.23966x_2), \lambda^1 = 0.245612, \\ \phi^2(\mathbf{x}) &= 2.22901 \sin(3.67319x_1) \cos(2.23966x_2), \lambda^2 = 0.045878, \\ \phi^3(\mathbf{x}) &= 2.39948 \cos(1.30654x_1) \sin(8.96973x_2), \lambda^3 = 0.018140, \\ \phi^4(\mathbf{x}) &= 2.32599 \cos(6.58462x_1) \cos(2.23966x_2), \lambda^4 = 0.014989. \end{aligned}$$

The yield function is the von Mises type, $f=\tau-\tau_y$ where τ is the maximum shear stress and τ_y is the yield stress with $\tau_y=200$ [MPa]. Monte-Carlo simulation is carried out for this non-linear stochastic problem, and the results are used as a reference. The procedures of generating test samples in the Monte-Carlo simulation are summarized as follows.

¹⁶More detailed explanation is presented in the Ph.D. dissertation of [Anders (2000)]; see also [Anders and Hori (1999); Anders and Hori (2001)].

- i) Generate a sample using the standard normal distribution.
- ii) Apply the truncating procedure to cut off the tail distribution simulations of negligible¹⁷ probability mass.
- iii) Normalize this truncated quasi-Gaussian sample so as to make the mean and variance estimator precisely equal to $\bar{E}$ and σ^2 , respectively.

In the present simulation, 95,000 test samples are generated, and their non-linear responses are computed to obtain the probabilistic characteristics of the field variables.

Since the primary objective of the present study is the comparison of NL-SSFEM with the Monte-Carlo simulation, rather coarse discretization is made for the rectangular body. A square element of 0.0625×0.0625 [m] is used, and the body is decomposed into 6×16 elements. Even for this coarse meshing, nodal displacement does not change much compared with a case of finer meshes, except for the nodal displacement at which the load is applied. The numerical solution of NL-SSFEM is stable, owing to the robust implicit backward Euler method which is applied to solve the non-linear equation for unknown coefficients of a random displacement rate vector.

Now, the results of the NL-SSFEM simulation are compared with the Monte-Carlo simulation results. First, the relation between the probabilistic characteristics of the response and the variability of Young's modulus is compared. In Fig. 8.3, the mean of the vertical displacement, denoted by $\bar{u} = \langle u_2 \rangle$, is plotted with respect to the standard deviation of Young's modulus σ ; the displacement is measured at the node at which the load is applied, and the mean which is computed by using the bounding medium, B^+ of $E^+ = \langle E \rangle$ or B^- of $\frac{1}{E^-} = \langle \frac{1}{E} \rangle$, is designated by NL-SSFEM+ or NL-SSFEM-, respectively. As is seen, the mean computed by B^- agrees with the exact relation computed by the Monte-Carlo simulation; the increase of $\bar{u}$ is well reproduced for B^- . The mean computed by B^+ does not change at all as σ increases; this is probably because $E^+ = \langle E \rangle$ is insensitive to σ and the dependence of $\bar{u}$ on σ becomes very small. The standard deviation of the vertical component, denoted by $\sigma_u = \sqrt{\langle (u_2 - \bar{u})^2 \rangle}$, is plotted in Fig. 8.4; a) and b) are for the results of B^+ and B^- , respectively. Four kinds of the PC expansion are used; the number of the KL expansion and the order of the PC expansion, denoted by K and N , are $(K, N) = (2, 2), (2, 3), (2, 4)$ and $(4, 2)$. All the four cases shown in the two figures are in fairly good agreement with the exact standard deviation of the Monte-Carlo simulation, i.e., the increase of σ_u with respect to increasing σ is well reproduced. For a larger value of σ , however, the discrepancy between

¹⁷Theoretically, Gaussian distribution allows an arbitrary realization to occur with positive probability, although this probability drops negligibly outside the neighborhood of three standard deviations from the mean. Simulations which fall outside the interval, the probability mass of which is below 0.003, are rejected, since they cause destabilizing effects on the convergence in solving a non-linear equation, especially in the case of large variances.

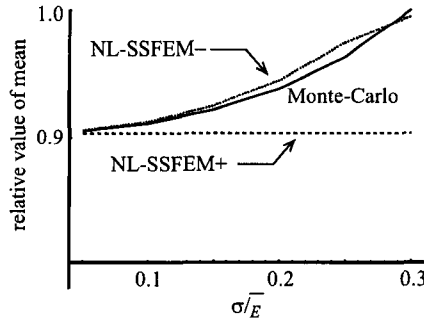
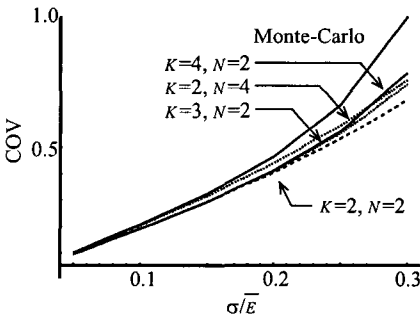
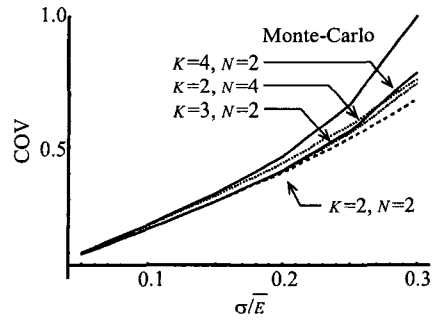


Fig. 8.3 Relation between the mean of displacement and the standard deviation of Young's modulus. The mean is computed for a vertical displacement component at the loading node.



a) NL-SSFEM+



b) NL-SSFEM-

Fig. 8.4 Relation between the standard deviation of displacement and the standard deviation of Young's modulus. The standard deviation of displacement is computed for a vertical displacement component at the loading node.

σ_u of the NL-SSFEM simulation and the exact σ_u increases; the discrepancy¹⁸ reaches around 10% at $\frac{\sigma}{\sqrt{E}}=0.3$. This is the limitation of NL-SSFEM in which the KL and PC expansions of random field variables are truncated at a fewer terms. The discrepancy becomes smaller when more terms are used in the PC expansion; σ_u for the case of $(K, N)=(4, 2)$ is closest to the exact σ_u . Although this is a naive explanation, it is expected that since the applied load is fixed in the present non-linear problem, the variability in strain tends to increase as the variability in Young's modulus increases. Thus, the variability in displacement increases as shown in Fig. 8.4. The NL-SSFEM simulation is able to capture this change in σ_u except for a case when σ becomes large.

In order to examine the probabilistic characteristic, a probability density function (PDF) is computed for the vertical and horizontal components at the loading

¹⁸For $\frac{\sigma}{\sqrt{E}} > 0.30$, negative values of E acquire considerable probability mass, and can be no longer neglected. Thus, $\frac{\sigma}{\sqrt{E}}=0.3$ is an extreme case of the variability in E .

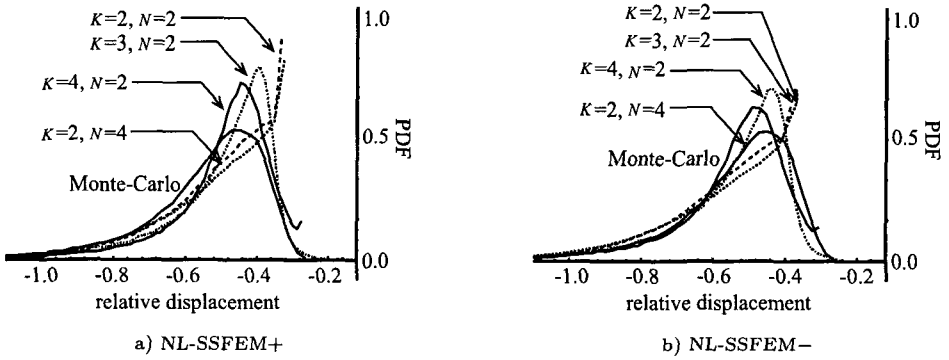


Fig. 8.5 A probability density function of the vertical displacement component at the loading node.

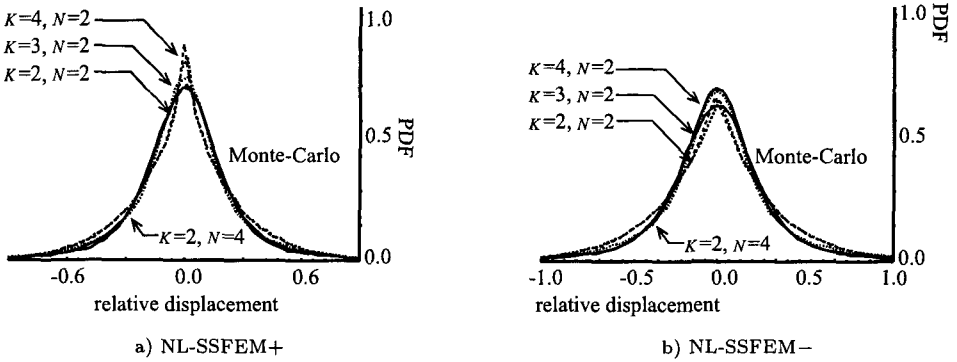


Fig. 8.6 A probability density function of the horizontal displacement component at the loading node.

node for the case of $\frac{\sigma}{E}=0.05$. The PDF of the vertical and horizontal components, u_2 and u_1 , is plotted in Fig. 8.5 and 8.6, respectively; a) and b) are for B^+ and B^- , respectively. Like Fig. 8.4, the four cases of the PC expansion are used, and the resulting PDF's are plotted. The PDF computed by the Monte-Carlo simulation is used as the exact PDF. The agreement of NL-SSFEM's PDF with the exact PDF becomes better as more terms are used in the PC expansion; the PDF for the case of $(K,N)=(4,2)$ is closest among other PDF's for both the vertical and horizontal components. There is large discrepancy between the NL-SSFEM's PDF and the exact PDF for the case of $(K,N)=(2,2)$ and $(2,3)$; the decrease of the exact PDF but the increase of NL-SSFEM's PDF for smaller values of u_2 are shown in Fig. 8.5. This is natural since the exact PDF is different from a normal distribution and more terms are required for the PC expansion to express such a distribution. If sufficiently many terms are used, like the case of $(K,N)=(4,2)$ or $(2,4)$, the PC expansion can surely express the distribution which is different from the normal distribution; re-

call that ξ_n^2 for $n=1, \dots, N$ are the bases for a non-Gaussian distribution when the order of the PC expansion is 2.

The present numerical simulation supports the basic validity of NL-SSFEM to solve a stochastic variational problem for a non-linear elasto-plastic model. The case of the PC expansion with $(K, N)=(4, 2)$ provides the most accurate solution which is satisfactorily close to the exact solution that is obtained by the Monte-Carlo simulation; for random displacement vector, a relative error of evaluating the mean or the standard deviation is less than 5%, and a non-Gaussian PDF is reproduced with the maximum relative error being 10%. It should be mentioned that the iterative calculation implemented in NL-SSFEM is well converged for the present simulation. The backward Euler method, which is used to enforce the mean stress to satisfy the equilibrium equation and the yield condition in the bounding medium, is able to find a solution less than 20 times of iteration at which the relative error of satisfying these conditions becomes less than 0.1%. Although such a good convergence of the iteration process is not always guaranteed, NL-SSFEM does not cause any numerical problem in solving the present non-linear problem.

8.4 Example of Fault Simulation of NL-SSFEM

Compared with metals, soil such as sand or clay has a more complicated constitutive relation. This is because soil consists of solid, liquid and airy phases. Interaction between solid and liquid phases plays a key role. For instance, it causes liquefaction. Subjected to some vibration, soil tends to behave as if it is liquid; see [Ishihara (1996)]. Dilatancy and confining pressure dependence are complicated phenomena even for dry geomaterials. To express such complicated properties of soil, numerous models and formulas have been proposed for its constitutive relation. Even when an elasto-plastic flow rule is assumed, it requires many material parameters in yield function, the number of which often exceeds ten.

Even near the ground surface, underground structures consist of several soil layers with distinct material properties. The constituents of layers sometimes drastically change. When stochastic modeling is applied, it is not necessary to use a sophisticated constitutive relation for each layer. This is because a stochastic model is used to evaluate the variability of fault behavior, such as whether a fault does appear or not. Variable fault behavior can be evaluated without using a sophisticated constitutive relation; such a sophisticated constitutive relation is needed to analyze local deformation with high accuracy. Similarly, a stochastic model does not have to include layers of complicated configurations. Therefore, it will be sufficient if a simple stochastic model is used in numerical simulation. The elasto-plastic constitutive relation that uses a yield function of Mohr-Coulumb type is used in the present code of NL-SSFEM; this constitutive relation requires only two parameters, cohesive force and friction angle. These two parameters can be easily measured with simple tests, or can be estimated from the type of a ground layer; see Table 8.1.

Table 8.1 Material parameters used for three types of ground.

	cohesive stress	friction angle
sand	0	27 – 34 [deg]
sandy gravel	0	35 – 50 [deg]
clay	5 – 50 [kN/m ²]	0

Table 8.2 Material parameters used for the preliminary simulation.

mean Young modulus [kN/m ²]	1225
Poisson ratio	0.25
density [g/cm ³]	1.6
friction angle [deg]	51
cohesion [kN/m ²]	38
initial compressive strength [kN/m ²]	214.6
initial tensile strength [kN/m ²]	26.9
COV of Young modulus [%]	30
correlation length of Young modulus [m]	0.5

Also, stratified layers are used in the simple stochastic model.

In order to validate NL-SSFEM as a tool for analyzing surface earthquake fault behavior, a simple numerical simulation¹⁹ is carried out to reproduce results of a three-dimensional model experiment made for lateral faulting. The validation is aimed at reproducing the rupture processes that accompany the evolution of shear bands with complicated configuration. In particular, the reproduction of Riedel shears, a periodic array of oblique shear bands produced by uniform shearing, is a key point of the validation, since it corresponds to the solution that is numerically most unstable among multiple solutions of the fault problem. Un-biased meshing is used and no special setting is prepared for the formation of Riedel shears. Only quantitative comparison with experimental observation is made in this section; quantitative comparison of the NL-SSFEM simulation with model experiments and actual fault data will be made in the next chapter.

A rectangular parallelepiped body is used as a stochastic model. The dimension of parallelepiped is $(D_1, D_2, D_3) = (25, 11, 5)$ [cm], and the material is elasto-plastic with a Mohr-Coulumb type yield function; see Table 8.2 for the material parameters used in the simulation. The boundary condition of this body is slightly complicated, and condition is stated as follows:

- 1) the top surface ($x_3=5$) is traction-free;
- 2) on the bottom surface ($x_3=0$), $(u_1, u_2, u_3) = (-\frac{U}{2}, 0, 0)$ for $x_1 > 0$ or $(\frac{U}{2}, 0, 0)$ for $x_1 < 0$ is prescribed as a displacement boundary condition, with U being the base

¹⁹More detailed explanation is presented in the Ph.D. dissertation of [Nakagawa (2002)]; see also [Nakagawa *et al.* (2001); Nakagawa and Hori (2002)].

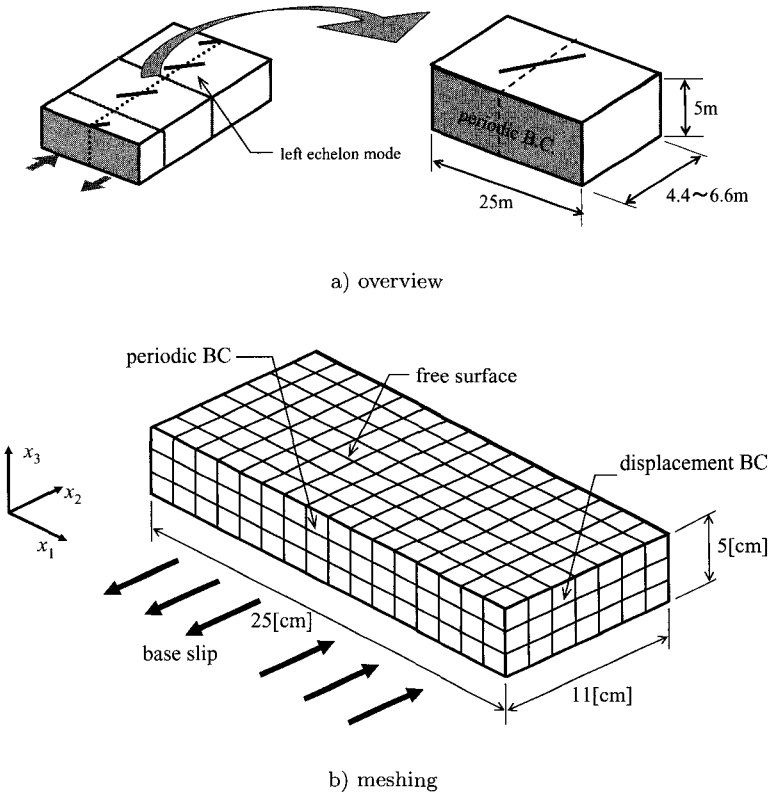


Fig. 8.7 A model used for the preliminary simulation.

movement;

- 3) on the longitudinal side surfaces ($x_2 = \pm 5.5$), the same displacement boundary condition as on the bottom surface is prescribed;
- 4) on the end surfaces ($x_1 = 0, 25$), periodic boundary condition that displacement and traction are continuous from $x_1 = 25$ to $x_1 = 0$ is prescribed;

see Fig. 8.7. The base movement, denoted by U , gradually increases from 0. The body is in equilibrium with gravity and an initial stress state is pre-computed at $U=0$. The periodic boundary condition in the longitudinal x_2 -direction is used for the reproduction of periodic Riedel shears. Since the uniform displacement of the same amount but the opposite direction is given as the boundary condition, the formation of one uniform shear band on the plane of $x_1=0$ is one solution; this uniform solution satisfies the periodic boundary condition as well as other boundary conditions. The formation of Riedel shears is a solution which is bifurcated from this uniform solution. As explained later, it will be examined whether the uniform deformation or the Riedel shear formation is more likely to happen, by comparing the work done by the base movement to cause these two solutions. When the

formation of Riedel shears is more likely to occur, the length of the Riedel shears, D_2 , can be determined. The external work changes depending on the value of D_2 , and the most suitable length is found as the one at which the external work is minimized.

First, it is examined whether NL-SSFEM can simulate strain localization. As a measure of strain localization, the maximum shear strain on the horizontal plane is used. This strain is defined as

$$\gamma = \sqrt{\frac{1}{4}(\epsilon_{11} - \epsilon_{22})^2 + \epsilon_{12}^2}$$

The mean and standard deviation (SD) of γ is plotted in Fig. 8.8; the distribution²⁰ of the mean, $\langle\gamma\rangle$, at $x_3=0, 1.7, 3.3$, and 5.0 [cm] are plotted for $U=2, 4, 6, 8, 10, 12$ [mm]. On the base plane of $x_3=0$, the large value of the mean is uniformly distributed along the vertical centerline which corresponds to the displacement gap on the base surface. On upper planes, $\langle\gamma\rangle$ is not distributed uniformly along the extension of the centerline; actually, the formation of twisted planes of periodic shear bands is observed. In NL-SSFEM, this solution is chosen instead of the solution which has uniform distribution of $\langle\gamma\rangle$ on the vertical plane that is extended from the centerline. As expected, NL-SSFEM is suitable for simulating the surface earthquake fault behavior that accompanies bifurcation. The SD of the maximum shear strain, σ_γ , is also plotted in Fig. 8.8; the value of σ_γ is almost 0 for $U<4$ [mm] and hence the distribution of σ_γ is plotted only for $U>4$ [mm]. The maximum value of σ_γ reaches around 3.0, which suggests large variability in the shear band formation. However, the distribution of σ_γ is similar in shape to that of $\langle\gamma\rangle$, and hence the variability of the Riedel shear configuration is not so large.

In order to examine the possibility that a bifurcated solution is realized, the external work which is required to make shear bands reach the top surface is calculated. If the probabilistic mean of traction in the x_2 -direction is denoted by $\langle t_2 \rangle$, the external work²¹ is defined as

$$W(U) = \frac{1}{L} \int_0^U \left(\int_S \langle t_2 \rangle ds \right) dU. \quad (8.19)$$

Here, S and L are the area and the length of the bottom surface, i.e., $S=D_1 \times D_2$ and $L=D_1$. The critical base movement is found by plotting the mean maximum shear strain at the center of the top surface, $(x_1, x_2, x_3)=(0, 0, D_3)$, with respect to U . The plot is shown in Fig. 8.9. It is seen that $\langle\gamma\rangle$ tends to increase rapidly at $U=7.6$ [mm], which suggests that plastic deformation reaches the top surface. Thus, the critical base movement is $U_c=7.6$ [mm]. For comparison, the SD, σ_γ , is plotted with respect to U in Fig. 8.9, which also shows a sudden increase at $U=7.6$ [mm]. For the solution which is forced to have uniform shear band, W is computed by using Eq. (8.19), and the relation between U and W is compared. The result is

²⁰The distribution shown in Fig. 8.8 is made by putting two identical distributions vertically, in order to emphasize the periodicity of the distribution.

²¹As is seen, W defined by Eq. (8.19) is regarded as the probabilistic mean of the external work.

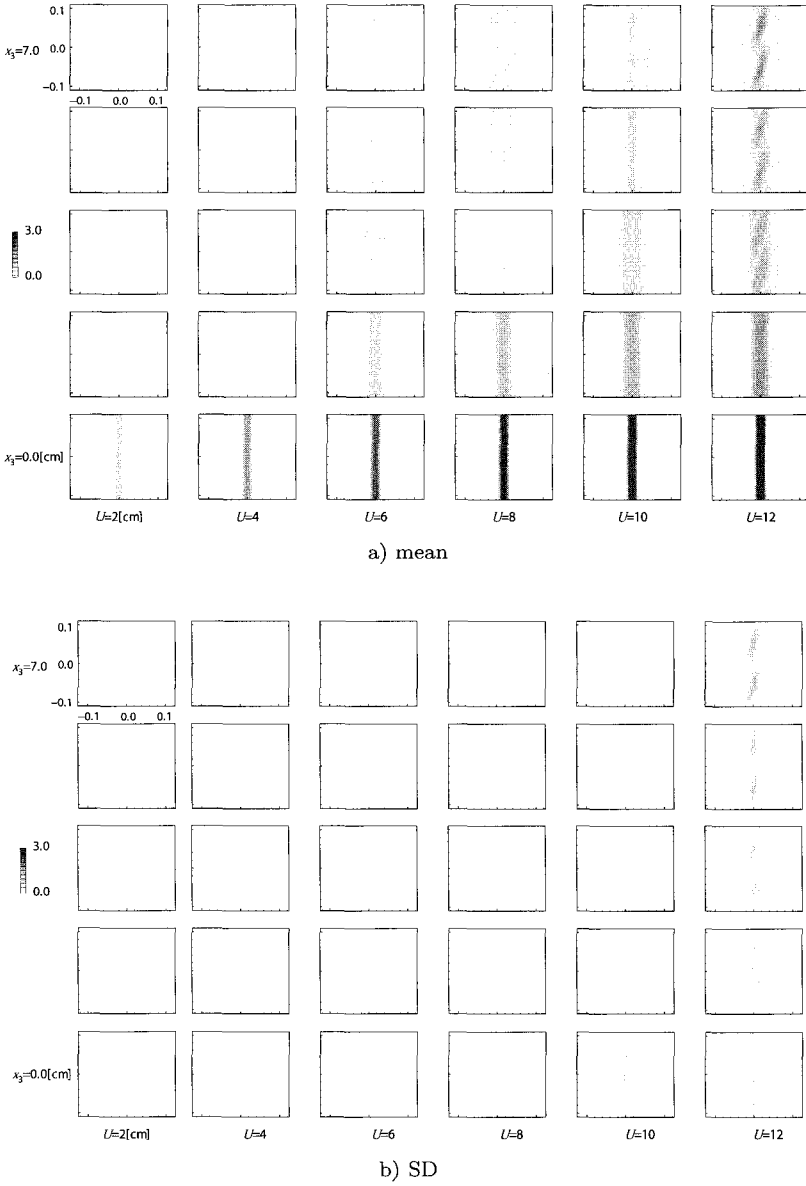


Fig. 8.8 Distribution of the maximum shear strain on the horizontal planes.

shown in Fig. 8.10. As is seen, W of the solution with Riedel shears is smaller than W of the solution with a uniform shear band; the value of W is smaller by 20% at $U=U_c$. This result shows that periodic shear bands are more likely to appear than the uniform shear band.

The results shown in Figs. 8.8~8.10 are obtained for the model which has peri-

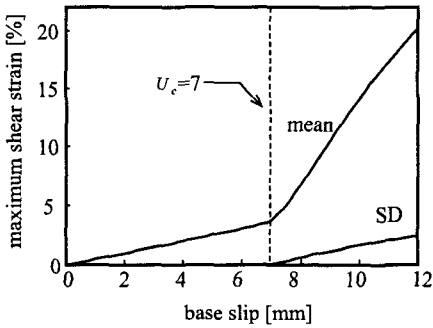


Fig. 8.9 Relation between base movement U and $(\langle\gamma\rangle, \sigma_\gamma)$ at center of top surface.

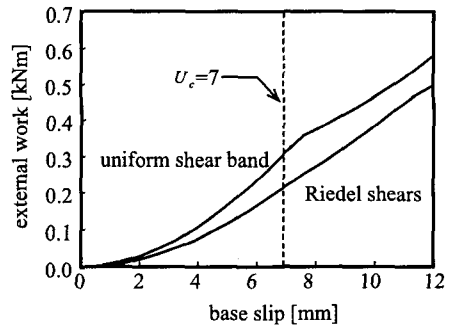


Fig. 8.10 Relation between base movement U and external work W .

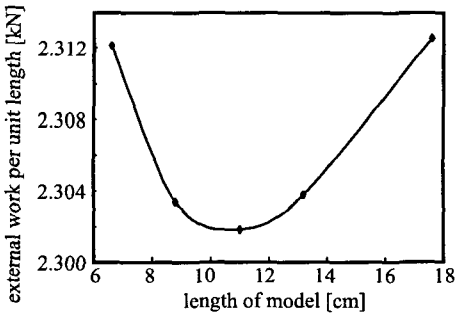


Fig. 8.11 Relation between periodicity D_2 (the length of mode) and critical external work W_c .

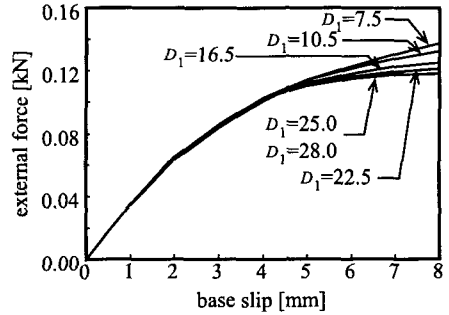


Fig. 8.12 Relation between base movement U and external work per unit length F for different values of D_1 .

odicity of $D_2=11$ [cm]. The periodicity or the Riedel shear length that is most likely to occur can be found by changing the value of D_2 and comparing the value of the external work. To this end, four models of $D_2=9, 10, 11$ and 12 [cm] are prepared. The work at the critical base movement, $W_c=W(U_c)$, is plotted with respect to D_2 in Fig. 8.11. It seems that W_c is minimized between $D_2=10$ [cm] and $D_2=11$ [cm]. Thus, the most possible length of Riedel shear is around $10\sim 11$ [cm].

While the width of the model is fixed as $D_1=25$ [cm], it is better to make D_1 larger so that the effect of the artificial boundary on the shear band evolution is minimized. In order to examine whether $D_2=25$ [cm] is sufficiently large, the model width is changed from 7.5 [cm] to 28.0 [cm], and NL-SSFEM simulations are carried out for these models. The results of the Riedel shear appearance, the distribution of $\langle\gamma\rangle$ or σ_γ , are similar when D_1 is larger than 20 [cm]. As a typical example, the external work per unit length, $F=\frac{W_c}{D_1}$, is plotted with respect to the base movement U in Fig. 8.12. It is shown that F is insensitive to D_1 when D_1 becomes larger than 22.0 [cm]. Therefore, it is concluded that the width four times larger than the

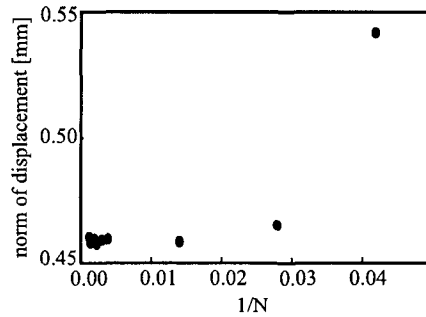


Fig. 8.13 The convergence of the norm of displacement mean, $\sqrt{\langle u_i \rangle \langle u_i \rangle}$.

thickness is sufficient to make the effects of the artificial boundary negligibly small.

It is known that in simulating strain localization phenomena, the numerical solution often changes depending on the discretization of displacement. This is because concentration of strain takes the shape of a plane and it is difficult to express such a plane in terms of smooth shape function which discretizes displacement or displacement rate. Thus, as finer meshes are used, strain concentration tends to be larger and the zone of strain concentration becomes thinner. Analyzing the evolution of shear bands is also required in numerical simulation of strain localization. Such mesh dependency of the solution is overcome for metals, which have relatively simple elasto-plastic constitutive relation, by assuming finite deformation instead of infinitesimal small deformation. For other materials such as concrete or soil, there have been proposed various theories and methods of analyzing the shear band evolution to obtain solutions with less mesh dependency. In the current version of NL-SSFEM, an *equivalent energy method* is adopted; see [Feenstra and de Borst (1995); Feenstra and de Borst (1996)] for a detailed explanation of this method. The equivalent energy method has been originally proposed for concrete which shows softening behavior, and gives a restriction to the element size that is determined by the energy required for failure; the equivalent energy is used to this end, which is defined as the strain energy per unit volume at the instant of failure. The equivalent energy method is simple, and needs only one material parameter which can be estimated from the fracture toughness.

In order to examine the mesh dependency of the NL-SSFEM solution, nine models of different meshing are prepared. The convergence of solutions with respect to meshing is examined. The norm of mean displacement, $\sqrt{\langle u_i \rangle \langle u_i \rangle}$, is computed at the center of model $(x_1, x_2, x_3) = (0, 0, D_3)$, and is plotted with respect to the inverse of the element number N in Fig. 8.13. As is seen, the norm takes almost the value for $\frac{1}{N} < 0.005$. However, the element size is larger than the actual shear band width, and hence the result of Fig. 8.13 does not fully show²² that solution is free from meshing. It is obvious that the finest meshing is needed in order to accurately

²²Strain or stress should be checked for the convergence of solutions.

reproduce or predict strain localization phenomena by using numerical simulation. The primary objective of the NL-SSFEM simulation is not aimed at such accurate computation. NL-SSFEM is used for the estimation of surface earthquake fault behavior with large variability. It is thus concluded that meshing with $N > 200$ is sufficient for the NL-SSFEM simulation of this purpose.

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Chapter 9

Simulation of Faulting

This chapter examines the validity and usefulness of NL-SSFEM which is developed to numerically analyze the behavior of a surface earthquake fault. As explained in the preceding chapter, hazards and disasters of a surface earthquake fault have the following two characteristics: 1) large variability in the fault behavior which includes the appearance on the ground surface; and 2) the requirement of special countermeasures against large ground deformation caused by the fault. The evaluation of the variability in fault behavior is a primary concern for the numerical simulation since the formation of a surface earthquake fault is an event of extremely low probability and it requires quite expensive countermeasures.

Due to the material non-linearity, the stochastic responses of the model become more complicated than those of a linear elastic stochastic model; in particular, the variability of the stochastic responses becomes larger when large plastic deformation is induced. It is thus difficult to obtain the stochastic response. NL-SSFEM solves this difficulty by

- 1) transforming a stochastic boundary value problem to an equivalent stochastic variational problem;
- 2) expanding a random material parameter and a random displacement vector in the probability space;
- 3) finding the most suitable approximate solution so that the stochastic functional is minimized with the aid of an efficient solver.

Recall that the KL expansion and the associated PC expansion are taken for Young's modulus and displacement rate vector, respectively.

Stochastic modeling plays a primary role in evaluating the variability in surface earthquake fault behavior. There are two key mechanisms that induce the variability, namely, the difference in amount of bedrock movement which is input to the ground layers and the complicated structures of the ground layers in which rupture processes propagate. Stochastic modeling accounts for the second mechanism; the uncertainty of the ground layers is expressed as a stochastic model. In the NL-SSFEM simulation, the bedrock movement is used as an input, i.e., a displacement boundary condition. The variability due to the first mechanism is thus evaluated by

analyzing the stochastic response at various values of the input bedrock movement.

In this chapter, the results of two NL-SSFEM simulations are presented. The first simulation is the reproduction of model experiments which use a sand box. The basic validity of the NL-SSFEM simulation of analyzing a surface earthquake fault is verified by comparing the simulation results with the observed experiment data. Model experiments are made for two-dimensional normal and reverse faults and for a three-dimensional lateral fault. The second simulation is the reproduction of actual surface earthquake faults. The targets of analysis are the Nojima Fault for the Great Hanshin Awaji Earthquake in 1995, and the Chelungpu Fault of the Chi-Chi Earthquake in 1999; the Nojima fault is in a mixed mode of reverse and lateral faulting and the Chelungpu Fault is a reverse fault. A non-linear elastoplastic stochastic model is constructed in view of reported geological data, and the simulation results are compared with the observed data of these faults.

9.1 Problem Setting for Fault Simulation

In the NL-SSFEM simulation, input data are the configuration of unconsolidated ground layers, material parameters, and bedrock movement. Output results are stochastic responses such as displacement and stress; these responses are given as a random function which changes depending on the bedrock movement. The configuration parameters of a surface earthquake fault are output as well. Also, NL-SSFEM computes a failure probability, which is the most important for the evaluation of surface earthquake fault hazards. This probability is given as a function of the bedrock movement, too.

It should be pointed out the definition of unconsolidated layers is not rigorous. Geologically, it means¹ Holocene layers. In earthquake engineering, unconsolidated layers correspond to surface ground layers which consist of relatively soft deposits upon a bedrock mass. The thickness of unconsolidated layers ranges from 10 to 100[m], with S-wave velocity $v_s=200\sim300$ [m/sec].

9.1.1 Input data

In relatively steep regions, unconsolidated layers rarely form stratified structures; the thickness and even the order of layers differ from place to place, as often observed by borehole surveys. Trench excavation surveys show irregular configuration of even thin layers with $10^0\sim1$ [m] thickness. Therefore, it is not realistic to construct a detailed three-dimensional model for unconsolidated layers. As an alternative, unconsolidated layers are modeled as a single but heterogeneous layer with stochastically varying material properties. When an interface is found for clearly distinct layers, it is necessary to construct a multi-layer model, with each layer being modeled stochastically; the interface between layers is modeled stochastically

¹These layers used to be called Quaternary layers occasionally.

by varying its depth.

The use of an elasto-plastic constitutive relation is a common practice in analyzing failure of ground and ground structures. The confining pressure dependence is a key property of soil, and it is included in terms of a yield function of Mohr-Coulumb or Dracker-Prager type. Since unconsolidated layers are heterogeneous, it is advantageous to use a simpler constitutive relation with a few parameters. The present NL-SSFEM code adapts the associate flow rule with a Mohr-Coulumb type yield function. Thus, the input material parameters are as follows: for elasticity, Young's modulus and Poisson's ratio, E and ν ; and for plasticity, cohesive stress and friction angle, c and ϕ . Both E and ν are estimated from N -value or elastic wave velocities. The plastic parameters are roughly evaluated from soil types such as sand or clay. NL-SSFEM uses a hardening rule to get rid of the mesh dependency for strain localization. This rule uses only one parameter which is related to the fracture energy and can be determined in an empirical manner.

In the present stochastic model of elasto-plasticity, only Young's modulus E is assumed as a random function; other material parameters are deterministic and spatially uniform. The Gauss distribution is assumed for E , and the covariance function, $R(\mathbf{x}, \mathbf{y})$, is given. It is assumed that form of $R(\mathbf{x}, \mathbf{y})$ is

$$R(\mathbf{x}, \mathbf{y}) = \sigma \exp(-\frac{r}{\ell}) \quad (9.1)$$

with σ and ℓ being the standard deviation and the correlation length of E ; r represents a distance of the two points, $r = \sum |x_i - y_i|$. It should be emphasized that determining σ and ℓ from actual data is not easy and that the form of R given by Eq. (9.1) is merely an assumption which is not verified. As is seen, the covariance function decreases exponentially with respect to r ; at $r = \ell$ the correlation becomes one third of σ . Thus, the simple setting of this R is

$$\text{relatively homogeneous} \quad \frac{\sigma}{\bar{E}} = 0.1 \text{ \& } \ell = 10[\text{m}],$$

$$\text{relatively heterogeneous} \quad \frac{\sigma}{\bar{E}} = 0.3 \text{ \& } \ell = 1[\text{m}],$$

where $\bar{E}$ is the mean of Young's modulus. Parametric studies for σ and ℓ will be carried out to examine the sensitivity of fault behavior upon these parameters.

The value of the maximum bedrock movement does not have to be set in the NL-SSFEM simulation. Since seismology shows that the displacement gap in a source fault has varying amount and direction, it is reasonably expected that the bedrock movement which is input to unconsolidated layers changes its amount and direction depending on the location. On the other hand, the amount of displacement gap on the source fault remains constant during the rupture processes through the crust. Hence, the displacement gap on the source fault² will be the maximum value of the bedrock movement. Also, the direction will be similar to the major direction of the

²Measuring stress in the crust helps determine the direction and amount of a displacement gap on a source fault more precisely, although it is extremely difficult.

source fault displacement gap when other information is not available. The amount and direction of the source fault are determined from the plate movement or from past earthquake records of the target source fault.

9.1.2 Output results

For normal and reverse faults, the primary concern is the location of a line on which a surface earthquake fault is formed; the rupture processes do not run in a straight manner, and hence a fault may appear in an unexpected place. For lateral sliding, deformation of the unconsolidated layers takes place in a three-dimensional manner, and the rupture processes become more complicated than those for normal and reverse faults. Indeed, lateral sliding produces an echelon fault, a periodic array of oblique fault line segments. The length, width and orientation need to be predicted, in order to estimate a zone which a surface earthquake fault influences. An output of the NL-SSFEM simulation is the configuration of a two- or three-dimensional surface fault.

The major output of the NL-SSFEM simulation is failure probability. This probability, denoted by P_f , is the probability³ that a surface earthquake fault is formed on the ground surface for a given bedrock movement U . The bedrock movement is used as a boundary condition since pre-determining a possible amount of the bedrock movement is quite difficult. Thus, it is recommended that the failure probability obtained from the NL-SSFEM simulation be combined with a probabilistic estimate of a possible bedrock movement. Indeed, the combination is formulated as a joint probability of the failure and the bedrock movement. If the probability density function of U is given as p_U , the joint probability is

$$P_F(U) = \int_0^U \frac{dP_f(U')}{dU} p_U(U') dU'. \quad (9.2)$$

Here, P_f is the failure probability computed by NL-SSFEM. As mentioned, determining p_U is more difficult than computing P_f .

In closing this subsection, some comments are made on the limitations of NL-SSFEM as a numerical analysis method for a non-linear elasto-plastic problem of underground structures. The simulation is applicable⁴ only to quasi-static state and infinitesimal small deformation. Effects of underground water on the rupture processes are completely ignored; water content strongly changes mechanical properties of soil. Thus, further improvement will be required to get over these limitations. At this moment, however, a more sophisticated numerical analysis is not needed. It

³In the NL-SSFEM simulation, the uncertainty of the underground structures is the source of variability in surface earthquake fault behavior. Thus, the computed failure probability is understood as a measure of the uncertainty of the fault formation which is due to not-fully-identified underground structures.

⁴While the rupture processes are in dynamic state, the inertia effect may have secondary effects in soft layers where rupture processes propagates, and the non-linear elasto-plasticity assumed for soil usually accounts for the effects of finite or large deformation.

will be sufficient if the fault formation, which is an event of extremely low probability, can be evaluated with reasonable⁵ accuracy by means of a simple but reliable numerical simulation.

9.2 Reproduction of Model Experiments

In this section, the NL-SSFEM simulation is carried out to reproduce two-dimensional and three-dimensional model experiments. The comparison of the simulated results with the experiment data is made, in order to verify the validity for NL-SSFEM as a tool used to simulate surface earthquake fault behavior.

9.2.1 Simulation of two-dimensional model experiment

A model experiment of surface earthquake fault uses a sand box, which is assumed in a two-dimensional state of plane strain. Base movement is applied to the sand box; normal or reverse faulting is simulated by applying uplift or downward displacement to one half of the bottom surface. The dip angle is fixed, and the base displacement is gradually increased. The propagation of a shear band running in sand is observed from the side surface of the box, and the following two quantities are measured: 1) the critical base movement at which a shear band reaches the top surface; and 2) the location on the top surface at which a shear band appears. The location is measured as the relative horizontal distance from the point at which the base movement is provided.

In the current simulation, experiments made by [Tani (1994)] are referred to; see also [Tani and Ueta (1991)]. The conditions of experiment and the major observations are summarized as follows:

experiment conditions

- **apparatus:** The length, width, and height of the sand box is $165 \times 40 \times 50$ [cm]. The base of the sand box consists of two blocks, and one block is lifted up or down. Toyoura sand is used, and sand is prepared by an air drop method to form a homogeneous sand layer; dyed sand is put in at every 2.5[cm] depth. The side face of the box uses a thick glass plate through which the deformation of dyed layers is observed. No treatment is made between the glass plate and sand to reduce friction. The base movement is in a range of 0.5~2.0[mm], and photos are taken to record the sand layer deformation.
- **material properties:** Dry standard Toyoura sand (gravity $G_S=2.635$, average radius $D_{50}=0.17$ [mm]) is used, and the sand box is made by densely compacting sand (void ratio $e=0.7$). The height of sand layers is $H=40$ [cm]. Since the exper-

⁵Since the amount of the bedrock movement is not predictable, there is an inherent limitation of accurately predicting surface earthquake fault behavior. However, it is probably sufficient that a quite large amount of bedrock movement is required in the NL-SSFEM simulation to make a fault reach the ground surface, in order to show that surface earthquake fault hazards can be ignored.

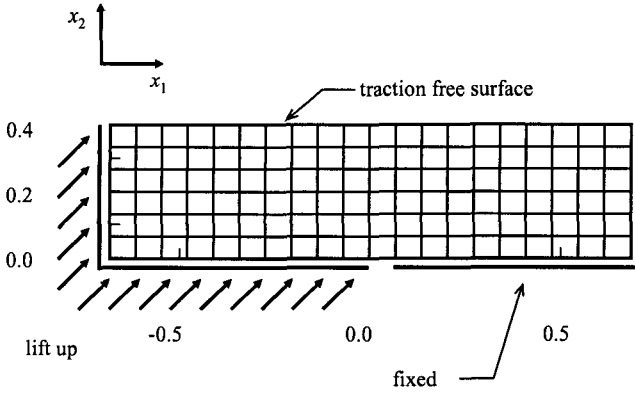


Fig. 9.1 Meshing of a model used for the two-dimensional simulation.

iment is carried out under 1G environment, the material properties of sand uses approximate values of Toyoura sands of dense compaction; see Table 9.1.

results of experiment

- propagation of a shear band: A shear band propagates smoothly, but some bifurcation takes place and plural shear bands are observed. The major shear band is an extension of the base movement, accompanying secondary shear bands which are initiated at the base. Some minor shear bands are formed in the middle of the major shear band.
- critical base movement: A critical base movement, U_c , at which the shear band reaches the top surface changes depending on the dip angle, denoted by α . However, the relative vertical movement, given by $\frac{D}{H}$ with $D=U_c \sin \alpha$ being the vertical component of the base movement, takes on a similar value; $\frac{D}{H}=1.0\sim 2.5\%$ for a normal fault and $2.5\sim 5.0\%$ for a reverse fault.
- Location of shear band appearance: For a reverse fault, the location of shear band appearance, W , depends⁶ on α , and its ratio with respect to the thickness is approximately given as $\frac{W}{H}=\frac{1}{\tan \alpha}$, which means the shear band propagates more or less along a straight line. For a normal fault, the fault reaches the right top, i.e., $\frac{W}{H}=0$.

The material parameters used in the NL-SSFEM simulation are summarized in Table 9.1; the parameters of primary importance are Young's modulus, Poisson's ratio, friction angle and cohesion, and other parameters such as initial compressive or tensile strength are used for more stable numerical computation. The coefficient of variance and the correlation length of Young's moduli are 15% and 0.5[m]. As shown in Fig. 9.1, unbiased meshes are used and no assumption is made for the

⁶It is reported by other researchers that $\frac{W}{H}$ has a little dependence on a material or the sand layer thickness, but strongly changes depending on α .

Table 9.1 Material parameters used for the two-dimensional simulation.

mean Young modulus [kN/m ²]	20000
Poisson ratio	0.30
density [g/cm ³]	1.6
friction angle [deg]	40
cohesion [kN/m ²]	38
initial compressive strength [kN/m ²]	150.0
initial tensile strength [kN/m ²]	33.0
COV of Young modulus [%]	15
correlation length of Young modulus [m]	0.5

path or the direction of a shear band.

In Figs. 9.2 and 9.3, the processes of the major shear band formation are presented for normal and reverse faults with the dip angle $\alpha=60[\text{deg}]$, respectively; these graphs are snapshots of the distribution of the mean and the standard deviation (SD) of the maximum shear strain,

$$\gamma = \sqrt{\frac{1}{4}(\epsilon_{11} - \epsilon_{33})^2 + \epsilon_{13}^2},$$

and snapshots are taken for $U=0.0\sim6.0[\text{mm}]$. Due to coarse meshing, the shear band is not expressed as a thin line but as a wide zone where plastic strain is accumulated; these zones are observed for the distribution of the mean and the SD. The propagation direction of the zone is clearly different from the normal and reverse faults; the normal fault grows vertically while the reverse fault is in the direction of the base movement. While the formation of the major shear band is seen, it does not accompany secondary shear bands or branching shear bands which are observed in the experiment. This is the limitation of the current NL-SSFEM simulation.

For the reverse fault, the failure probability is computed. Failure is defined in terms of the maximum strain. In Fig. 9.4, the mean of the maximum shear strain, $\langle\gamma\rangle$, at a point which the shear band reaches is plotted with respect to the base movement, U . At $U=7.6[\text{mm}]$, the relation between U and $\langle\gamma\rangle$ is changed; this is due to the generation of plastic strain, which means that the shear band reaches the top surface. The change at $U=9.4[\text{mm}]$ corresponds to the full formation of shear band, and no further plastic strain⁷ is induced; the left part of the sand layer moves like a rigid body. In view of the value of $\langle\gamma\rangle$ at $U=7.6[\text{mm}]$, the failure state is defined as $\gamma=0.65\%$. The failure probability is thus computed as

$$P_f(U) = \int_{\gamma_c} p_\gamma(\gamma, U) d\gamma, \quad (9.3)$$

⁷Due to the boundary condition prescribed for this simulation, elastic strain increases in a linear manner with respect to U .

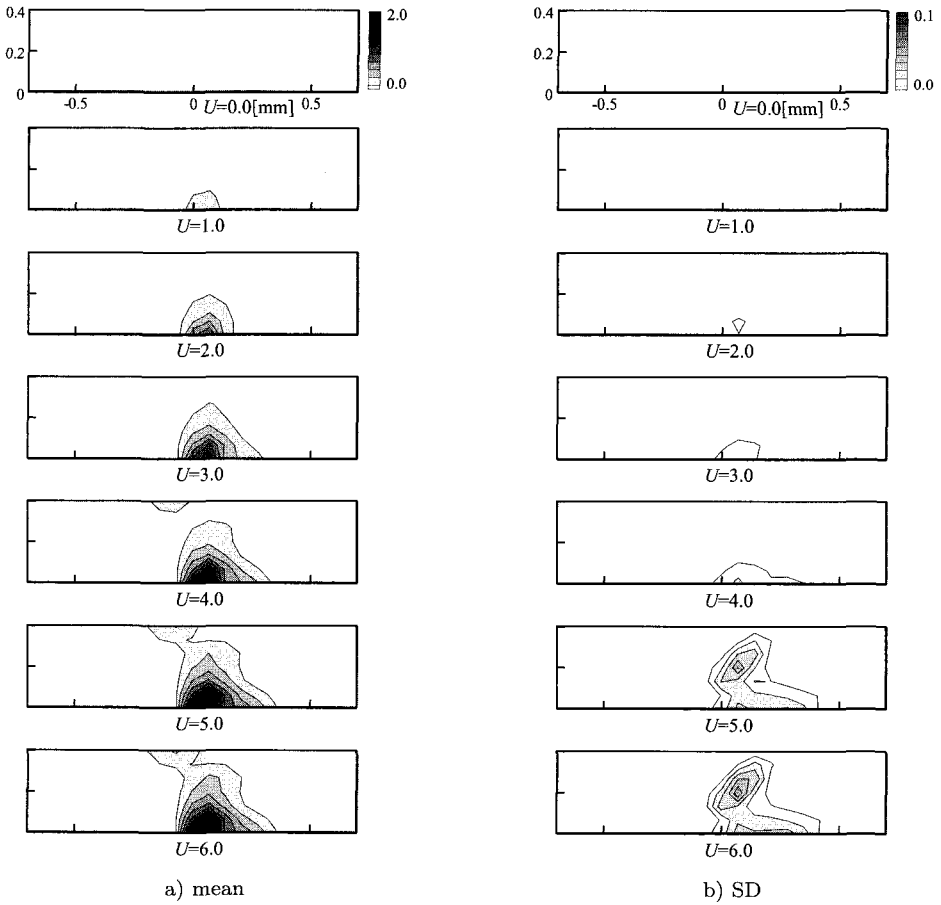


Fig. 9.2 Distribution of the maximum shear strain for the two-dimensional simulation: a normal fault.

where p_γ is the probability density function of γ and γ_c is the critical value of the maximum shear strain, $\gamma_c = 0.65\%$. This cumulative probability function and the probability density function (PDF), P_f and $\frac{dP_f}{dU}$, are plotted in Fig. 9.5. As is seen, the PDF has a sharp peak at $U = 9.0$ [cm]. This means that for the current model experiment, the variability in failure with respect to the base movement is small, and failure takes place at $U = 9.0 \pm 0.2$ [cm].

Since the failure state is defined, the simulation results are compared with the experimental data for the critical base movement and the location of the shear band appearance, denoted by U_c and W , respectively. In Fig. 9.6a), the relative vertical component, $\frac{D}{H}$ with $D = U_c \sin \alpha$, is plotted with respect to the dip angle, α ; for the NL-SSFEM simulation, U_c is chosen as the value at which $\frac{dP_f}{dU}$ is maximized. While some discrepancy is observed, different tendency of the $\frac{D}{H} - \alpha$ relation for the normal fault and the reverse fault is well reproduced in the NL-SSFEM simulation.

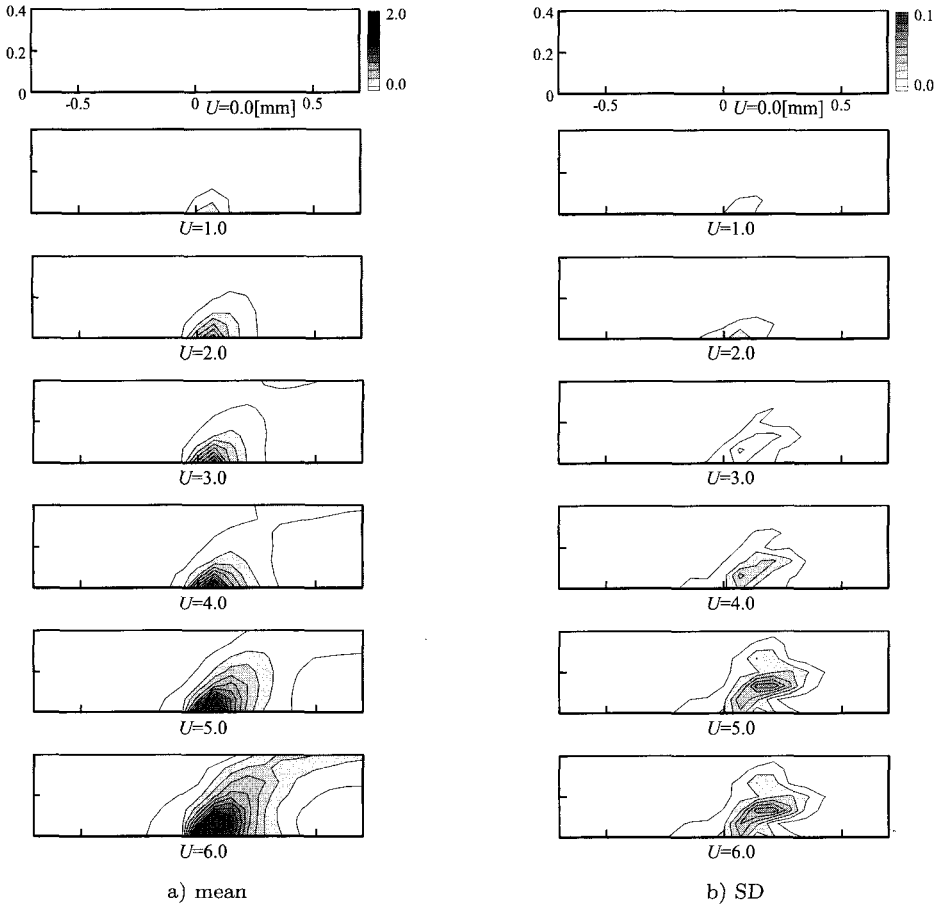


Fig. 9.3 Distribution of the maximum shear strain for the two-dimensional simulation: a reverse fault.

The agreement is better for the normal fault, and the simulation overestimates $\frac{D}{H}$ for the reverse fault. For instance, for the case of $\alpha=60[\text{deg}]$, to which P_f and $\frac{dP_f}{dU}$ are computed as shown in Fig. 9.5, the observed value of $\frac{D}{H}$ is just 60% of the simulated value. The degree of error of reproducing $\frac{D}{H}$ or U_c is around 40%. In Fig. 9.6b), W is plotted with respect to α ; in the simulation, W is chosen as the point at which γ takes the maximum value on the top surface. Like Fig. 9.6a), the NL-SSFEM simulation succeeds in reproducing a different relation between W and α for the normal and reverse faults. The agreement of the simulation results with the experimental data is better than Fig. 9.6a). Relative error is at most 15%.

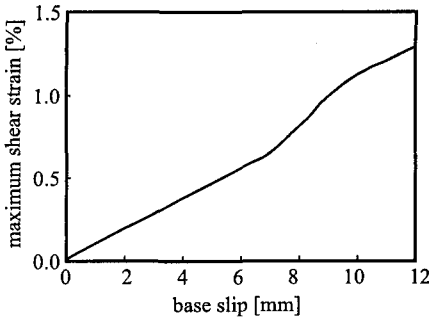


Fig. 9.4 Relation between the mean of maximum shear strain $\langle \gamma \rangle$ at the center and the base movement U .

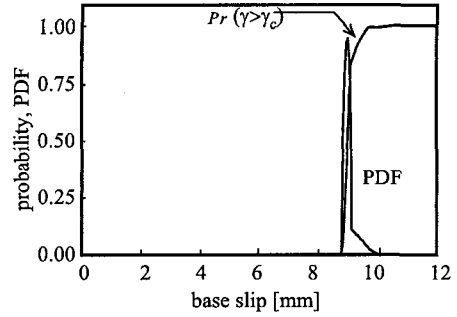
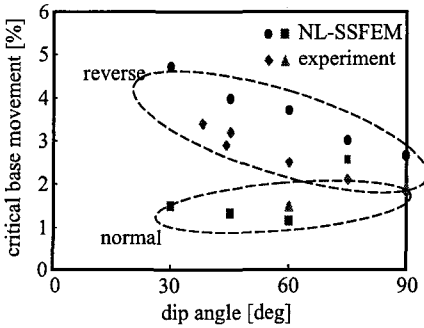
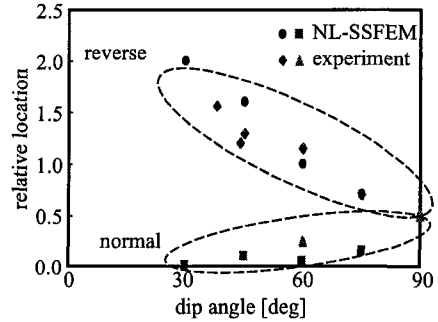


Fig. 9.5 Failure probability obtained from the two-dimensional simulation. P_f and $\frac{dP_f}{dU}$ are plotted.



a) relation between α and $\frac{D}{H}$



b) relation between α and $\frac{W}{H}$

Fig. 9.6 The comparison of the configuration parameters obtained by NL-SSFEM simulation with the experiment data.

9.2.2 Simulation of three-dimensional model experiment

A three-dimensional model experiment of surface earthquake fault is made for lateral sliding, i.e., the sand box is subjected to horizontal shear displacement gap. The rupture processes of this three-dimensional setting are more complicated than those of a two-dimensional setting. On the top surface, Riedel shears, a periodic array of shear bands, are formed when the shear bands reach there. As the base moves further, one shear band which forms a straight line becomes dominant and separates the surface into two. In the experiment, the measured quantities are the critical displacement at which the shear bands reach the top surface and the configurations of Riedel shears, such as orientation, interval and length.

In the current simulation, experiments made by [Tani and Ueta (1991)] are referred to. The conditions of experiment and the major observations are summarized as follows:

experiment conditions

- apparatus: The length, width and height of the sand box are $170 \times 25 \times 15$ [cm]. Sand paper is attached on the bottom to increase friction. The height of a sand layer, H , is changed from 3 to 7 [cm]. The bottom face consists of two blocks, and one moves horizontally. At the both ends of the block, membranes and cushions are put in to prevent disturbance⁸ caused by the block ends. The movement of the block is controlled with accuracy of 0.01 [m]. Photographs are taken to record the fault formation and evolution processes at the top surface.
- material properties: Toyoura sand, air dried, is used; the characteristics of sand are gravity $G_S=1.59$, the mean radius $D_{50}=0.17$ [mm], and water ratio $w=0.105 \sim 0.150\%$. Sand specimen is made with dense compaction (void ratio $e=0.648$). Each layer of around 1 [cm] thickness is put in the sand box and compacted by a vibrator.

results of experiment

- configuration of Riedel shears: The configuration of Riedel shears is characterized by the orientation with respect to the shear direction, θ_R , the interval, S_R , and the width, R_R . While θ_R varies for each Riedel shear, it does not change depending on H ; the value of θ_R almost coincides with the theoretical value which is derived from the assumption that the surface is in a state of plane stress, i.e., $\theta_R=20.5 \sim 30.5$ [deg]. The value of S_R is 11 [cm], although it increases a little as H increases. The value of R_R varies in a relatively large range for each Riedel shear, but R_R tends to increase as H increases. Thus, the configuration barely depends on H and is similar in shape.
- critical base movement: The value of the critical base movement, U_c , increases as H increases. Unlike the two-dimensional model experiment, $\frac{U_c}{H}$ changes depending on H ; it tends to take on a smaller value for a larger H . Similar size effects are reported by other researchers and the dependence of $\frac{U_c}{H}$ on H is more evident for a smaller H .
- shear band formation processes: The shear bands are formed on the top surfaces through the following four processes: 1) shear bands evolve continuously and symmetrically from the bottom with the inclination angle being $30 \sim 45$ [deg]; 2) shear bands have higher inclination angle, and the uniform evolution along the longitudinal direction is broken so that a three-dimensional structure starts to be formed. Right and left shear bands are separated into periodic pieces, and they tend to overlap each other as they are folded in the direction of shearing⁹; 3) the overlapped shear bands are joined, and joined shear bands are periodically aligned, grow upwards in forming an S-letter shape and form Riedel shears on the top surface, which are similar in shape to echelon faults caused by lateral faulting; and 4) shear bands growing upward further evolve, inducing the formation of

⁸The initiation of echelon faults from the end of the sand box is thus prevented.

⁹This is called an *imbolic structure*.

Table 9.2 Material parameters used for the three-dimensional simulation.

mean Young modulus [kN/m ²]	1225
Poisson ratio	0.25
density [g/cm ³]	1.6
friction angle [deg]	51
cohesion [kN/m ²]	38
initial compressive strength [kN/m ²]	214.6
initial tensile strength [kN/m ²]	26.9
COV of Young modulus [%]	30
correlation length of Young modulus [m]	0.5

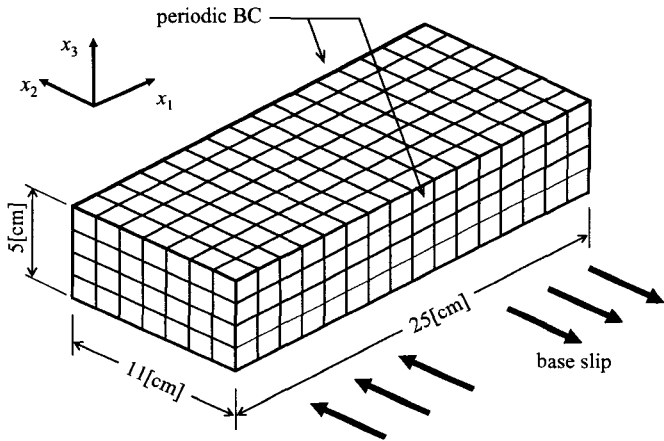
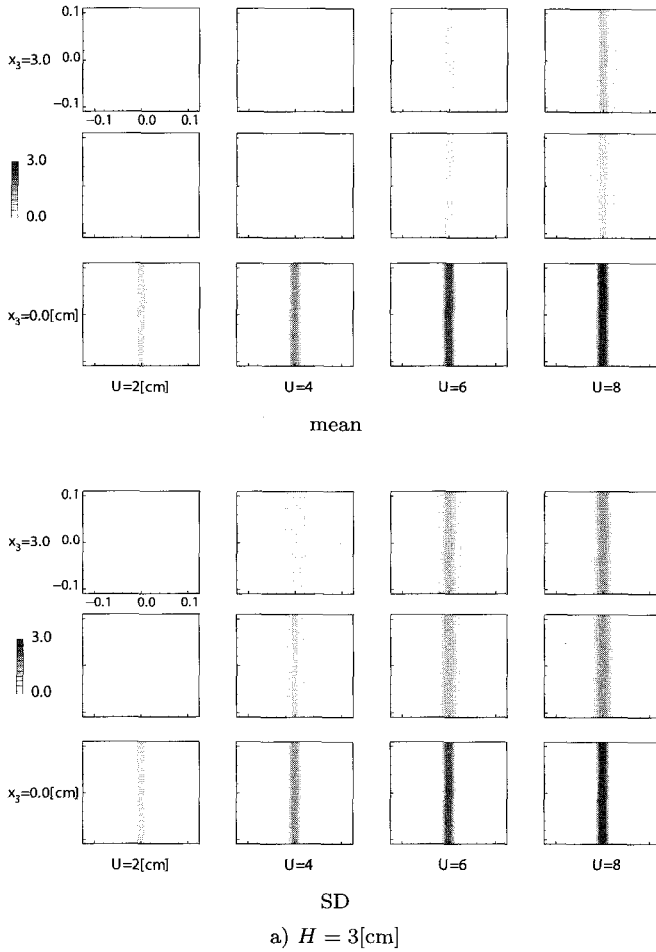


Fig. 9.7 Meshing of a model used for the three-dimensional simulation.

bifurcated shear bands. When the bifurcated shear band reach the top surface, a Riedel shear in a small dip angle, or a so-called R-shear with more complicated structures, is formed.

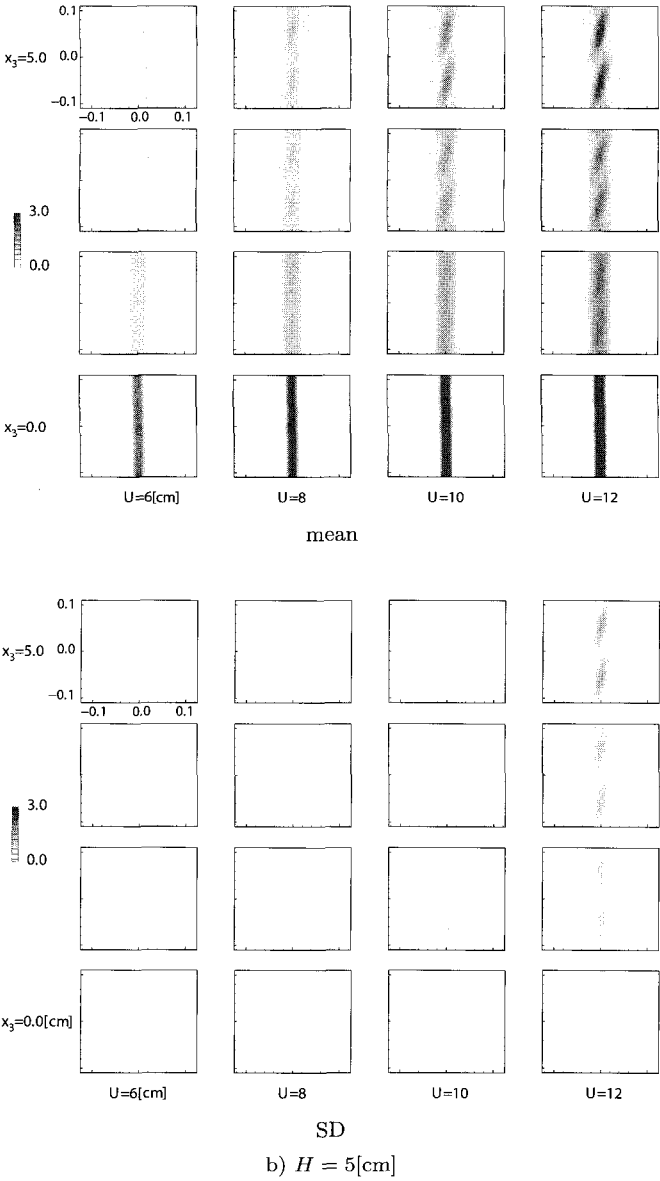
The three-dimensional computational model shown in Sec. 8.4 is used in this simulation; see Fig. 8.7. The dimension of $D_1=25[\text{cm}]$ and $D_2=11[\text{cm}]$ is common, but $D_3=H$ is changed to simulate the three samples with different thickness, $H=3, 5, 7[\text{cm}]$. The value of D_2 is determined from the experimental observation that the Riedel shear interval does not depend on the thickness if it is less than $12[\text{cm}]$ and the interval is about $11[\text{cm}]$. The maximum value of the base movement is $8[\text{mm}]$. The material parameters used in the simulation are summarized in Table 9.2. Since the sample is thin, it is assumed that the confining pressure in the horizontal direction is only due to the static sand weight. Based on this assumption, the material parameters of sand under low confining pressure are used, and Young's modulus shown in Table 9.2, which is measured for dry Gifu sand under very low confining



pressure, is used. The friction angle is the value obtained for a simple shear test which is carried out under low confining pressure of $0.05[\text{kgf}/\text{cm}^2]$; although the friction anlagen can be measured in this lateral shear experiment, the value is not used in the simulation since induced shear planes are not parallel. The maximum compressive and tensile strength are determined so that the base movement which makes the Riedel shear reach the top surface coincides with the observed value for a sample of $H=3[\text{cm}]$; this value is used for other cases of H . The fracture toughness, G_c , is determined according to the following empirical equation:

$$h_{\max} = \frac{EG_c}{k_{10}^2} \quad \text{and} \quad k_{10} = \frac{6c \cos \phi}{\sqrt{3}(3 - \sin \phi)},$$

where $h_{\max}$ is the size of the element; this equation holds when the element number is larger than 50. Meshing of the model is shown in Fig. 9.7; eight-node cubic elements of dimension $1.50 \times 1.56 \times 0.25[\text{cm}]$ are used. As is seen, meshing is unbiased



so that no assumption is made regarding to the configuration of evolving shear bands.

The distribution of the mean and the SD of the horizontal maximum shear strain, γ , is plotted in Fig. 9.8; a), b) and c) are for the case of $H=3, 5$ and $7[\text{cm}]$, respectively. In these figures, the mean and SD distribution on several horizontal planes, which are equally spaced, are arranged vertically for designated values of the base movement U . The formation of periodic Riedel shears is not clearly seen

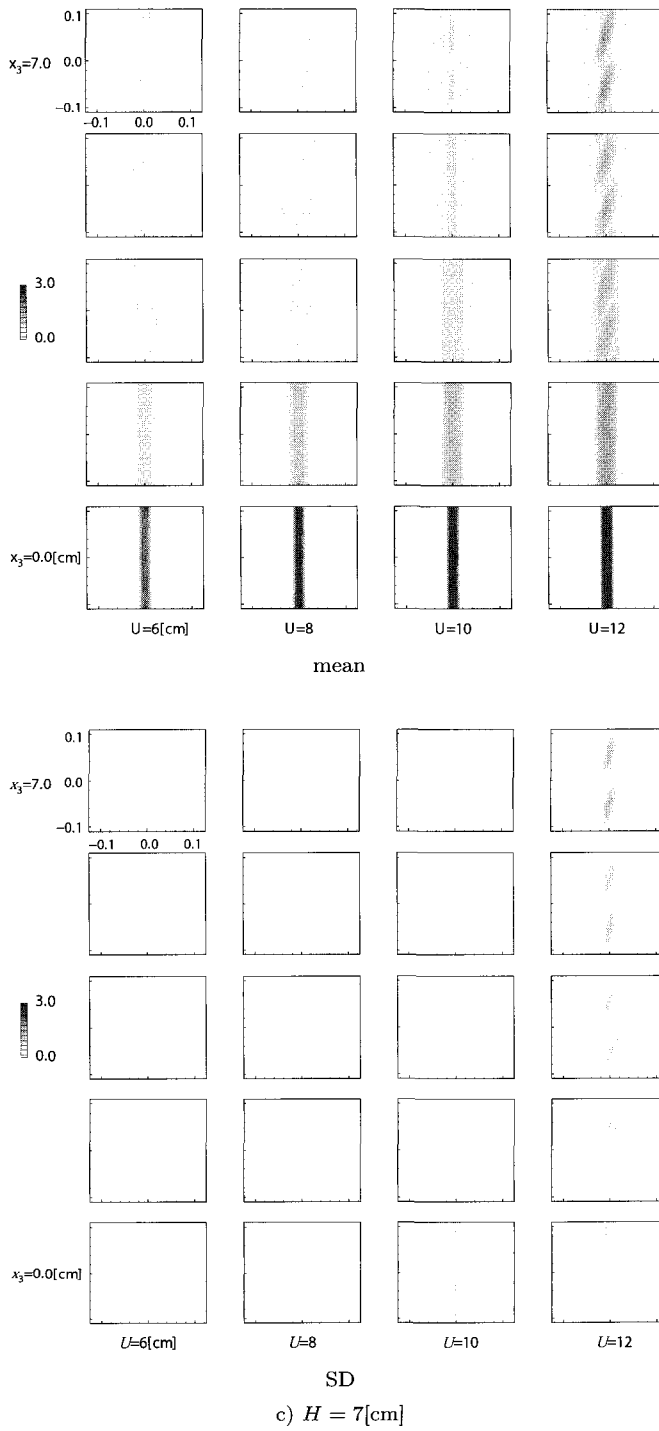


Fig. 9.8 Distribution of the maximum shear strain for the three-dimensional simulation.

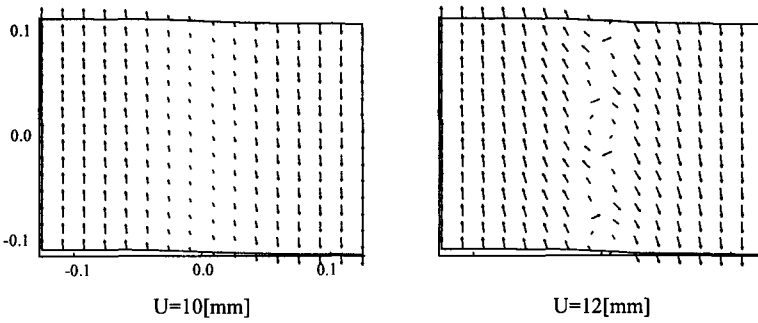


Fig. 9.9 Distribution of displacement vector.

for the case of $H=3[\text{cm}]$; the mean and SD distribution appears uniform in the x_2 -direction. However, the formation is clearly observed for the cases of $H=5$ and $7[\text{cm}]$; two Riedel shears¹⁰ are formed on the top surfaces. For the case of $H=5[\text{cm}]$, the mean of γ is distributed uniformly in the x_2 -direction until $U=8[\text{mm}]$, and the uniform distribution is broken; see Fig. 9.8b). The SD of γ has a visible distribution at $U=10[\text{mm}]$. Similarly, for the case of $H=7[\text{cm}]$, a periodic distribution of the mean and SD is visible at $U=10[\text{mm}]$; see Fig. 9.8c). A sudden change in the distribution pattern suggests that the NL-SSFEM simulation picks up a solution which is bifurcated from the uniform distribution at a certain value of U . Compared with the mean, the SD takes on a smaller value, and the zone in which the SD becomes larger is smaller than the zone with a larger mean. This indicates that the less variability in the configuration of the Riedel shear.

In Fig. 9.9, the distribution of the displacement on the top surface is plotted for the case of $H=7[\text{cm}]$; the base movement is $U=10$ and $12[\text{mm}]$ and the displacement is magnified by 100 times. As is seen, there is a visible difference between the displacement distribution at $U=10$ and $12[\text{mm}]$, like the maximum shear strain distribution shown in Fig. 9.8c). While the displacement along the center line is smaller at $U=10[\text{mm}]$, large displacement is observed at $U=12[\text{mm}]$. This displacement distribution corresponds to the displacement gap which is caused by large shear strain; the displacement gap is roughly estimated as

$$(\text{displacement gap}) = \gamma \times t_R$$

where t_R is the Riedel shear thickness; in the current simulation, t_R is, say, $2\sim 4[\text{mm}]$, which is much larger than actual thickness of the Riedel shears. While experimental data are not available, this displacement gap is a key parameter to consider large ground deformation due to faulting.

In order to validate the NL-SSFEM simulation, the configuration parameters of the computed Riedel shears are compared with the observed ones. The following

¹⁰As explained in Sec. 8.4, the distribution shown in Fig. 9.8 is made by putting the two same distribution of the mean or SD vertically, in order to emphasize the periodicity of Riedel shears.

Table 9.3 The comparison of the configuration parameters obtained by the NL-SSFEM simulation with the experiment data.

a) comparison of θ_R

depth [cm]	3	5	7
experiment [deg]	26	31	27
simulation [deg]	23	29	31

b) comparison of S_R

depth [cm]	3	5	7
experiment [cm]	10.5	11.0	11.0
simulation [cm]	10.8	11.6	11.0

c) comparison of R_R

depth [cm]	3	5	7
experiment [%]	2.5	7.0	8.0
simulation [%]	1.8	3.0	3.0

three parameters are compared: 1) the orientation angle, θ_R ; 2) the interval length, S_R ; and 3) the band width, R_R . In Table 9.3, the comparison of these parameters is summarized for the three cases of H ; a), b) and c) are for θ_R , S_R and R_R , respectively, and R_R is expressed as the relative value with respect to the height, $\frac{R_R}{H}$. Note that S_R is determined as the value of D_2 at which the external work that is required for the appearance of the Riedel shear on the top surface¹¹ is minimized. The value of R_R is the length of the zone where the maximum shear strain exceeds 5%. It is seen in Table 9.3 that fairly good agreement is achieved for the NL-SSFEM simulation. For instance, the range of θ_R is $26 < \theta_R < 31$ [deg] for the experiment and $23 < \theta_R < 31$ [deg] for the simulation. The agreement of S_R is satisfactory, although R_R is not accurately computed. The NL-SSFEM simulation tends to underestimate R_R by 50%.

Next, the computation results are compared with the observed values for the critical base movement, U_c , at which the shear bands appear on the top surface. In Fig. 9.10, the relation between the maximum shear strain, γ , at the center of shear band and the base movement, U , is plotted for the three cases of H , in order to determine U_c ; a) and b) are for the relation between U and the mean, $\langle \gamma \rangle$, and the relation between U and the SD, σ_γ , respectively. As is seen in Fig. 9.10a), at $\langle \gamma \rangle = 4.2\%$, there is a common abrupt change in the relation between $U - \langle \gamma \rangle$ for the three graphs. This indicates the arrival of shear bands on the top surface. Indeed, the inclination of $\langle \gamma \rangle$ is increased in a domain of $\langle \gamma \rangle > 4.2\%$, and this is due to the increase of plastic strain. The $U - \sigma_\gamma$ relation, plotted in Fig. 9.10b), has an abrupt change at $\sigma_\gamma = 0.08\%$ like the $U - \langle \gamma \rangle$ relation. The value of U which corresponds to

¹¹ The standard deviation of S_R is not computed according to this manner of calculation.

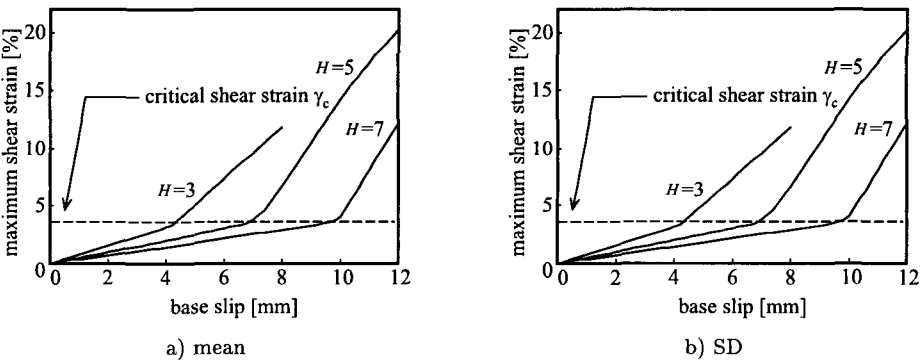


Fig. 9.10 Relation between U and γ for the three-dimensional simulation.

Table 9.4 The comparison of the critical base movement U_c for the three-dimensional simulation.

	mean[mm]		SD[mm]		SD/mean	
	exper.	simu.	exper.	simu.	exper.	simu.
3[cm]	5	5.3	1	0.80	20	15.2
5[cm]	7	8.6	1	0.21	14	2.5
7[cm]	8	11.1	—	0.16	—	1.5

the abrupt changes in σ_γ coincides with that of the $U-\langle\gamma\rangle$ relation, i.e., $U=4, 7, 10$ [mm] for $H=3, 5, 7$ [cm], respectively. Therefore, it is concluded that these U 's are the critical base movement for the three samples.

In view of Fig. 9.10a), made is the assumption that $\gamma_c=4.2\%$ is a critical value of the horizontal maximum shear strain at which the failure appears on the top surface. The failure probability is thus determined by taking the following procedures:

- Compute a probability density function, $p_\gamma(\gamma, U)$, for the maximum shear strain at a target point.
- Compute a cumulative probability density, $Pr(\gamma' > \gamma|U)$, as the integration of $\int_\gamma p_\gamma(\gamma', U) d\gamma'$.
- Determine the failure probability as $P_f(U)=P_\gamma(\gamma_c, U)$.

The probability density function of γ , i.e., p_γ , is plotted for various U 's in Fig. 9.11. In Fig. 9.12, the probability of failure is plotted as a function of the base movement U for $H=3, 5, 7$ [cm]; a) and b) are for P_f and $\frac{dP_f}{dU}$. From these figures, the mean and SD of the critical base movement, U_c , are calculated. Table 9.4 summarizes these probabilistic characteristics of U_c . The mean value of U_c computed by the NL-SSFEM simulation is in fair agreement with those observed in the experiment, although it underestimates the SD of U_c . This is the limitation of the current NL-SSFEM simulation; as mentioned several times, the simulation results show a small SD for the maximum shear strain; for instance, as shown in Fig. 9.10, the coefficient

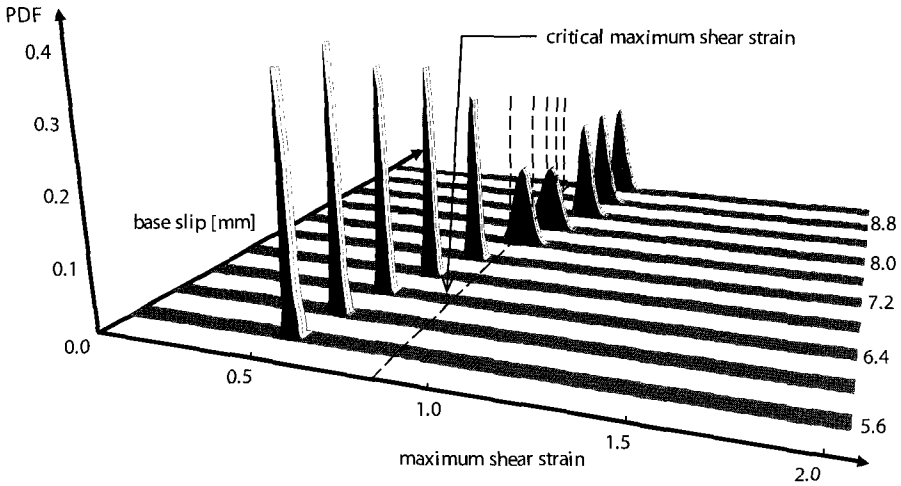


Fig. 9.11 A probability density function p_γ of maximum shear strain. p_γ changes as the base movement U increases.

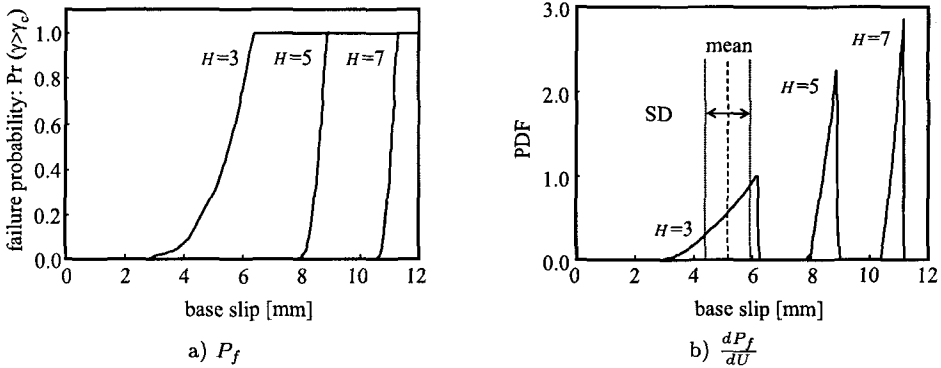


Fig. 9.12 Failure probability of the three-dimensional simulation.

of variance, $\frac{\sigma_\gamma}{\langle \gamma \rangle}$, is less than 10%. Also, it should be mentioned that the number of experiments for each H is limited to 2 or 1, and hence there are not sufficient data to evaluate the standard deviation of U_c .

The distribution of γ on the horizontal planes shows that the NL-SSFEM simulation is capable of reproducing the periodic Riedel shear formation even though the displacement gap is uniformly given at the base. The configuration parameters of the Riedel shears at the top surface are in agreement with the observed ones. However, the three-dimensional configuration of the simulated Riedel shears is different from those which are reported in the model experiment. To examine the three-dimensional configuration in detail, the vertical distribution of the horizontal

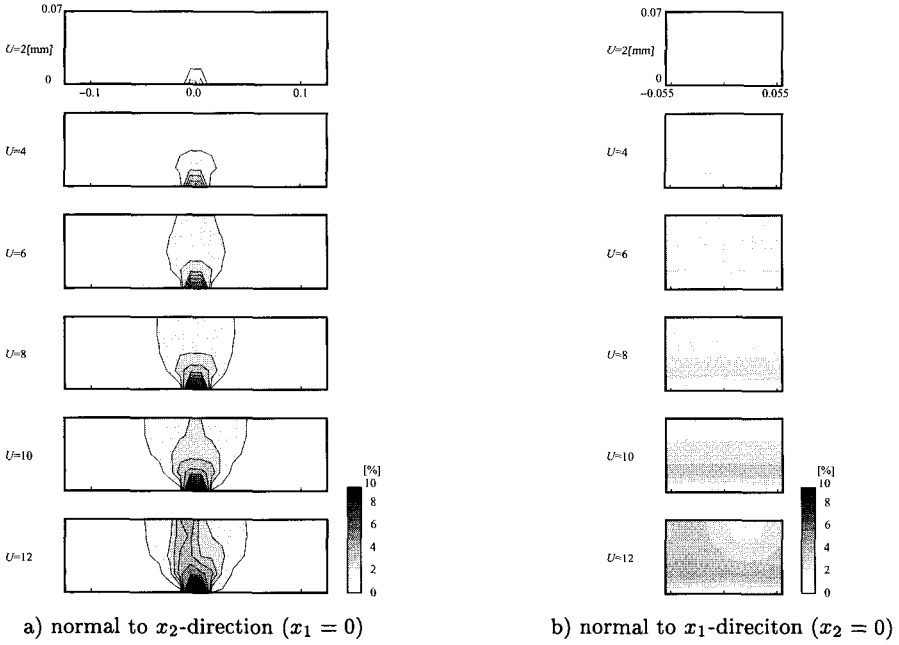


Fig. 9.13 Distribution of the maximum shear strain on vertical cross sections.

maximum shear strain¹² is plotted for $H=3[\text{cm}]$ in Fig. 9.13; a) and b) are for the distribution of $\langle \gamma \rangle$ on the vertical planes of $x_1=0$ and $x_2=0$, respectively, and these planes are the cross section normal to the x_2 - and x_1 -axis. It is seen in Fig. 9.13a) that the shear bands stand vertically from the bottom and then grow twisting to the left direction in the middle of the sample. This evolution pattern is different from the observed pattern, the initiation of Riedel shears from the bottom surface. In Fig. 9.13b), no imbricate structures are generated; although the bifurcation in shear bands is captured, complicated overlapping of shear bands are not simulated at all. Again, the fact that this is the limitation of the current NL-SSFEM simulation has to be admitted. There are several possible reasons for this limitation, such as the assumption of infinitesimal deformation and quasi-static state or the use of simple elasto-plastic constitutive relations. The major reason is probably insufficient discretization. However, finer discretization is difficult since NL-SSFEM requires larger degree-of-freedom than ordinary FEM in taking the PC expansion of a random function of displacement vector.

¹²Since the horizontal deformation is dominant, the horizontal maximum shear strain is almost the same as the three-dimensional maximum shear strain.

9.3 Simulation of Actual Faults

In this section, two examples of actual surface earthquake faults, namely, the Nojima Fault and the Chelungpu Fault, are simulated by means of NL-SSFEM. The parameters of simulated fault configuration are compared with the observed data to show the usefulness of the NL-SSFEM simulation. A key issue of the simulation is modeling. Some explanation is given of modeling which takes advantage of geological and ground data. Parameters for the stochastic characteristics such as the standard deviation and correlation length of Young's modulus are determined without any background data. Sensitivity analysis of the simulation results on these parameters is made to check the validity of these parameters.

9.3.1 Simulation of the Nojima Fault

The Great Hanshin Awaji Earthquake was caused by the 1995 Hyogoken Nanbu Earthquake, an intra-plate or in-land earthquake of $M=7.2$ on the Richter scale. This earthquake produced enormous damage to the mega-city of Kobe. Also, surface earthquake faults appeared, causing damage to nearby buildings and structures. The source fault of the 1995 Hyogoken Nanbu Earthquake was the Nojima Fault, and surface earthquake faults of left echelon type were observed along the Nojima Fault within Nojima Island.

According to the geological surveys, Nojima Island consists of the Osaka layer and a granite layer; see [Nakata *et al.* (1996)]. The Osaka layer tends to be thicker near the coast and the granite layer tends to be thicker inside the island. The target of the NL-SSFEM simulation is Nashimoto District of Nojima Island. This district lies along small rivers and is covered by the Osaka layer. In particular, the surface layer is the consistent of soft deposits of clay and gravel with thickness being 4~6[m]. This layer is regarded as an uncultivated layer. The in-site trench survey clarifies that the Osaka layer in Nasihimoto district is thicker than that in other parts of the island. While the surface layer is a mixture of clay and gravel, clear boundaries between the two layers are hardly found, and the simulation uses a single layer model which corresponds to the Osaka layer.

In view of these geological data, a three-dimensional stochastic model is constructed for the unconsolidated layer. The thickness of the model is 5[m]. The periodicity is assumed for the strike direction to reproduce echelon faults, and the length of the model is determined by computing the external work, like the previous cases shown in Sec. 8.4. The width of the model is set as 25[m] to remove the effects of the artificial boundary.

The elastic properties of the unconsolidated layer are evaluated by means of an N -value. The Osaka layer usually has $N<40$; N is around 10 for cases where clay is dominant and does not exceed 30 even for cases where gravel is dominant. In the simulation, N is set 9, from which Young's modulus is $E=6.1[\text{kPa}]$ according to the

Table 9.5 Material parameters used for the Nojima Fault simulation.

mean Young modulus [kN/m ²]	6125
Poisson ratio	0.25
density [g/cm ³]	2.1
friction angle [deg]	51
cohesion [kN/m ²]	38
initial compressive strength [kN/m ²]	215
initial tensile strength [kN/m ²]	27
COV of Young modulus [%]	30
correlation length of Young modulus [m]	10

empirical relation of $E=6.8 \times N[\text{kgf/cm}^2]$; a case of $E=12.2[\text{kPa}]$ will be used in the later simulation, which corresponds to a hard layer of $N=18$. The plastic properties of the unconsolidated layer are estimated from the soil type, and cohesive strength and friction angle are set as $c=38[\text{kPa}]$ and $\phi=50[\text{deg}]$ in view of the layer being a mixture of sand and gavel. Other plastic parameters are determined from these c and ϕ . The material parameters used in the NL-SSFEM simulation are summarized in Table 9.5.

The bedrock movement input to the stochastic model is determined by the amount and the direction of the source fault movement, which correspond to the displacement gap on the source fault and the direction of the displacement gap, respectively. The dip angle of the bedrock movement is thus determined as $60[\text{deg}]$, and the bedrock movement is increased up to $0.6[\text{m}]$. Since the displacement gap of the source fault has both reverse and lateral components, the strike angle of the bedrock movement is set as $45[\text{deg}]$, i.e., the ratio of vertical component and horizontal component is 1. These characteristics of the source fault are determined by the seismic tomography or the earthquake inversion, and several researchers report results of their own inversion. These results agree with each other, regarding the estimate that the maximum value of the displacement gap is approximately $1.5[\text{m}]$. It is true that the seismic inversion has limited accuracy due to the low spatial resolution of the fault plane which is discretized by meshing of $500[\text{m}]$ length, and the low temporal resolution of the measured acceleration which is filtered by cutting higher frequency components. However, the inversion results are useful to determine the bedrock movement of the NL-SSFEM simulation, as shown in the present case.

NL-SSFEM is applied to a stochastic model which is constructed by using the geological data. In order to investigate the usefulness of NL-SSFEM as a simulation tool used to evaluate surface earthquake fault behavior, the following three are set as the major objectives of the simulation:

- 1) the computation of echelon fault formation processes: NL-SSFEM must reproduce the bifurcation phenomenon of periodic echelon faults being formed even

- if the model is subjected to uniform bedrock movement;
- 2) the reproduction of fault configuration: for a fault computed by NL-SSFEM, configuration parameters such as orientation angle, interval and width of fault must be in agreement with observed values,
 - 3) the estimation of failure probability: the critical bedrock movement that makes a fault reach the ground surface must be evaluated from the failure probability which is computed by NL-SSFEM as a function of bedrock movement.

The validity of the constructed stochastic model needs to be verified separately. Some parametric studies on the stochastic quantities will be made in the next subsection.

First, it is examined whether the echelon fault formation is simulated by means of NL-SSFEM. The dimensions of the target unconsolidated layer are 100 times larger than the model experiment which is studied in Sec. 9.2, and hence the success of reproducing Riedel shears in numerical simulation does not guarantee the reproduction of the bifurcation phenomenon of echelon faults rather than a uniform fault being realized. Furthermore, the mixed mode of the bedrock movement, which consists of horizontal and vertical movements, may prevent the occurrence of bifurcation. In numerical simulation, depending on the model length, echelon faults are not produced. However, if the model length is set as around 6[m], plastic strain forms a periodic array on the surface, which requires less external work than a uniform distribution of plastic strain. As a typical example, Fig. 9.14 shows the distribution of the maximum shear strain γ ; the mean and SD, denoted by $\langle\gamma\rangle$ and σ_γ , are plotted on horizontal planes of $x_3=0.0\sim 4.8$ [m] for the bedrock movement $U=0.4\sim 69.2$ [cm]. It seems that localized strain is similar in shape to echelon faults. At $U=34.6$ [cm], oblique shear bands appear on the top surface as shown in the distribution of $\langle\gamma\rangle$, and the distribution of σ_γ is not uniform on the plane of $x_3=0.0, 1.6$ and 3.2 [m]. However, imbricate structures which are most likely to be formed in underground structures are not reproduced. Like the simulation of the three-dimensional model experiment, this is mainly due to insufficient spatial discretization. It should be pointed out that in comparison of the mean and the SD shown in Fig. 9.14, σ_γ is concentrated near the line along which $\langle\gamma\rangle$ takes a larger value and the maximum value of σ_γ is almost three times larger than the values of σ_γ in the surrounding zone. This suggests that while the shape of faults appearing on ground surface is more or less that of a line segment, the displacement gap varies from place to place. These two results are in agreement with observed data of the fault configuration and displacement gap.

Next, the configuration parameters (i.e., the orientation angle θ , the interval length S , and the width R) are compared with the values that are actually measured for the echelon faults. First, S is determined by carrying out the NL-SSFEM simulation with three models of different length, $D_2=4.4, 5.5$ and 6.6 [m]. The external work, denoted by W , which is required to make the rupture reach the ground surface is calculated for these models, and the value of S is chosen as the

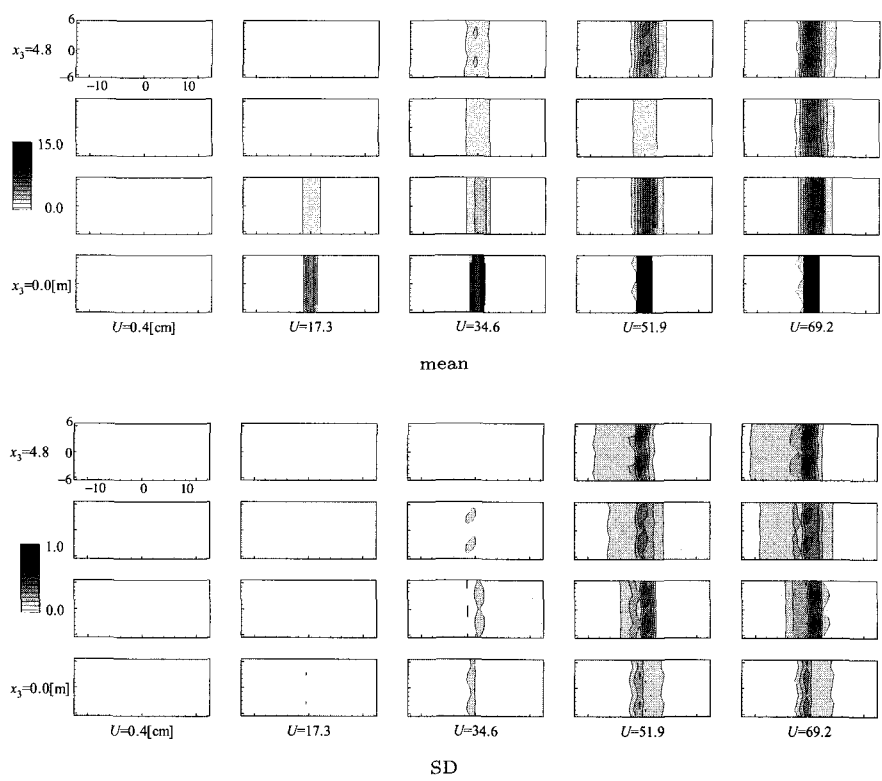


Fig. 9.14 Distribution of the maximum shear strain for the Nojima Fault simulation.

Table 9.6 The comparison of the configuration parameters for the Nojima Fault simulation.

	observed	simulation
configuration	left echelon	left echelon
θ [deg]	22 – 35	25
S [m]	4 – 6	6.6
W [m]	0.5 – 1.5	1.4

length of the model that minimizes W . The relation between the bedrock movement and the external work per unit length for the three models is plotted in Fig. 9.15. It is seen that the work is minimized for the model of $D_2=6.6$ [m]. Thus, the value of S is set a $S=6.6$ [m]. The value of θ and R is determined by using the simulation results of this model. In view of the distribution of the maximum shear strain which is shown in Fig. 9.14, $\theta=25$ [deg] and $R=1.4$ [m] are estimated; the estimation is made for the distribution of the mean $\langle \gamma \rangle$ at $U=0.6$ [m] and the echelon fault is regarded as a zone in which $\langle \gamma \rangle$ takes a larger value, i.e., $\langle \gamma \rangle > \gamma_c$ with $\gamma_c=0.85$. Table 9.6 presents the comparison of θ , S and R with the measured values. As is

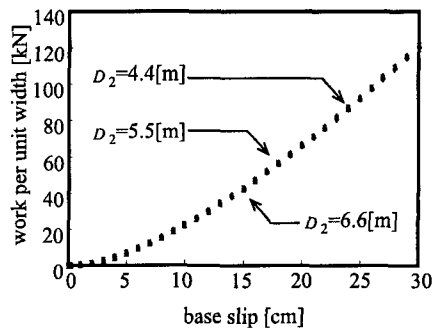


Fig. 9.15 Relation between the bedrock movement U and the external work per unit length W for the Nojima Fault simulation.

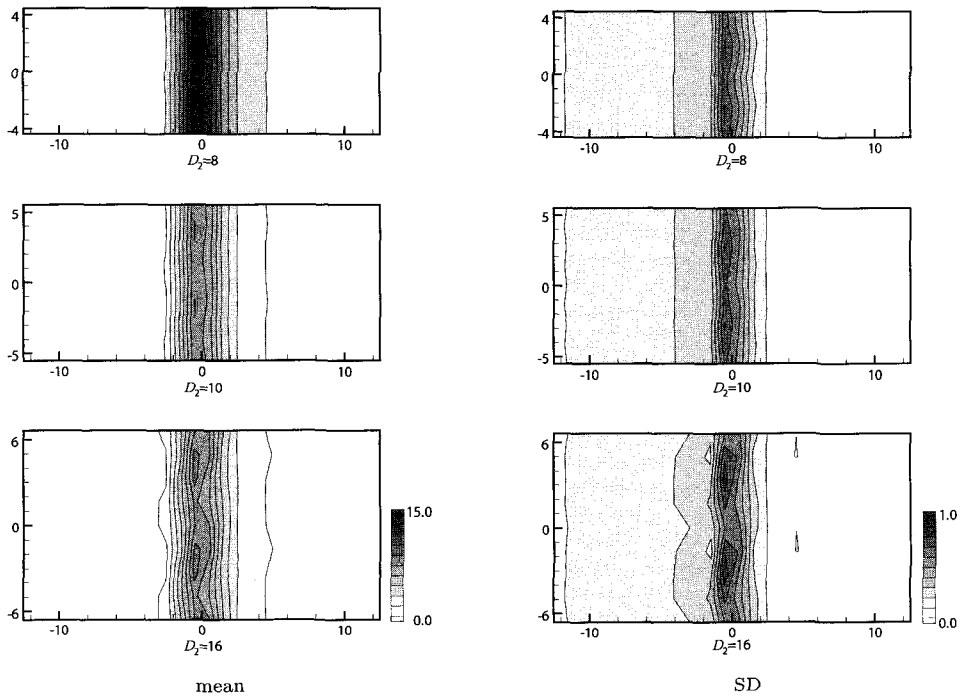


Fig. 9.16 Distribution of the maximum shear strain on the surface for different values of D_2 .

seen, the agreement is satisfactory. It should be commented that the distribution of the maximum shear strain changes drastically, depending on the value of D_2 . In Fig. 9.16, the distribution of the mean and SD, $\langle \gamma \rangle$ and σ_γ , is plotted for the case of $D_2=8, 10$ and 16 [m]. The distribution is more or less uniform for the case of $D_2=8$ [m], and the two oblique echelon faults tend to be seen more clearly as D_2 increases. Also, the variability of γ is increased for a larger D_2 .

Displacement gap, denoted by D , of the echelon fault is investigated. In Fig. 9.14, it is seen that the plastic strain appears after $U > 0.6[\text{m}]$ and the strain is around $80 \pm 20\%$. Thus, the displacement gap at the center of the echelon fault is evaluated as

$$D = (0.8 \pm 0.2)(U - 0.6)[\text{m}] \quad \text{for } U > 0.6[\text{m}].$$

For instance, $U = 1.0[\text{m}]$ gives $D = 0.3 \pm 0.1[\text{m}]$, which means that 50% of the bedrock mass movement is dissipated within the unconsolidated layers, and the remaining 50% appears as the fault displacement gap. This dissipation is an overestimation of the actual value, and the estimated D is smaller than the observed value of $D = 0.5 \pm 0.1[\text{m}]$. The elastic strain, which is dissipated in the stochastic model of the unconsolidated layers, is overestimated in the current simulation since the infinitesimally small deformation is assumed. This is the limitation of the current NL-SSFEM.

Finally, the failure probability which is computed by NL-SSFEM is validated. Since the simulation is made for an actual fault, the failure must be determined by accounting for the size of the fault; it does not mean that failure occurs if the maximum shear strain at one point reaches a critical value. Therefore, an appropriate definition of failure is when the maximum shear strain exceeds a critical value along every point along a fault line segment. Calculating the maximum shear strain along the line segment, however, is time consuming. Thus, this definition is simplified as being when the maximum shear strain reaches the critical value at both the center and the end of the echelon fault. Since these maximum shear strains are given as a random variable, the procedures of calculating the failure probability are summarized as follows:

- i) Calculate the maximum shear strain, γ^1 and γ^2 , at the center and the end point, $\mathbf{x}^1$ and $\mathbf{x}^2$, respectively.
- ii) Calculate a joint probability $P_J(\gamma^1, \gamma^2, U) = Pr(\gamma^1 > \gamma^1 \& \gamma^2 > \gamma^2 | U)$ as a function of the bedrock movement U .
- iii) Set a critical value γ_c , for the maximum shear strain which corresponds to failure.
- iv) Calculate the failure probability $P_f(U) = P_J(\gamma_c, \gamma_c, U)$ using the joint probability P_J and the critical value γ_c .

Note that the PC expansion is given to γ^α , i.e.,

$$\gamma^\alpha(\omega) = \sum \gamma^{\alpha m} \Psi^m(\omega),$$

where the bases $\{\Psi^m(\omega)\}$ are polynomials of $\xi^n(\omega)$'s which are computable Gauss distributions. Thus, it is possible to calculate $Pr(\gamma^1 > \gamma^1 \& \gamma^2 > \gamma^2 | U)$ using random numbers which are generated to determine these Ψ^m 's, and $\gamma_c = 0.6$ is used in computing the probability $P_f(U)$. In Fig. 9.17, the failure probability is plotted; the cumulative probability density and the probability density function, P_f and

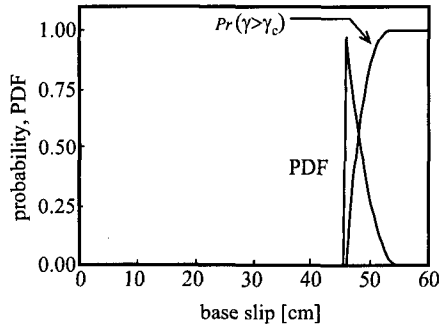


Fig. 9.17 Failure probability of the Nojima Fault simulation. An accumulate probability function P_f and a probability density function $\frac{dP_f}{dU}$ are plotted.

$\frac{dP_f}{dU}$, are plotted. A sharp peak of $\frac{dP_f}{dU}$ around $U=0.6$ [m] shows the possibility that the rupture processes reach the ground surface at this bedrock movement. It is seen from the distribution of P_f that the surface earthquake faults are likely to have been formed before U increases up to 1.0 [m]. There are no data available to examine the validity of this estimate which is made by the NL-SSFEM simulation, since the earthquake occurred only once. However, it seems that the appearance of a fault at the bedrock movement $U=0.6$ [m] is reasonable.

The limitation in accurately reproducing actual rupture processes by means of numerical simulation should be recognized, since a detailed computational model cannot be constructed due to the lack of data for geological structures and material properties of unconsolidated layers. However, the configuration parameters obtained by NL-SSFEM are in fair agreement with the observed parameters, even though coarse spatial discretization is used and a simple constitutive relation is assumed; see Table 9.6. A major reason for this agreement is probably the quality of data for the layer thickness and the material parameters as well as data for the bedrock movement. Other reasons are 1) the assumed elasto-plastic constitutive relation captures the mechanism of strain localization which leads to the formation of echelon faults; and 2) the most unstable solution among bifurcated solutions is picked up since the stochastic functions play a role of suitable perturbation from the uniform solution. High accuracy is not needed for the prediction of surface earthquake fault behavior. The current state of predicting surface earthquake fault formation is based on some empirical relations and is far from quantitative prediction. The agreement of the fault configuration parameters suggests that the use of advanced numerical simulation such as NL-SSFEM can change the state and that more quantitative prediction will be possible.

It should be emphasized the failure probability P_f shown in Fig. 9.17 is not verified. However, some reliability is expected since P_f is computed by a numerical simulation that accounts for the mechanism of the rupture processes within the unconsolidated layers and by using the stochastic model which accounts for the

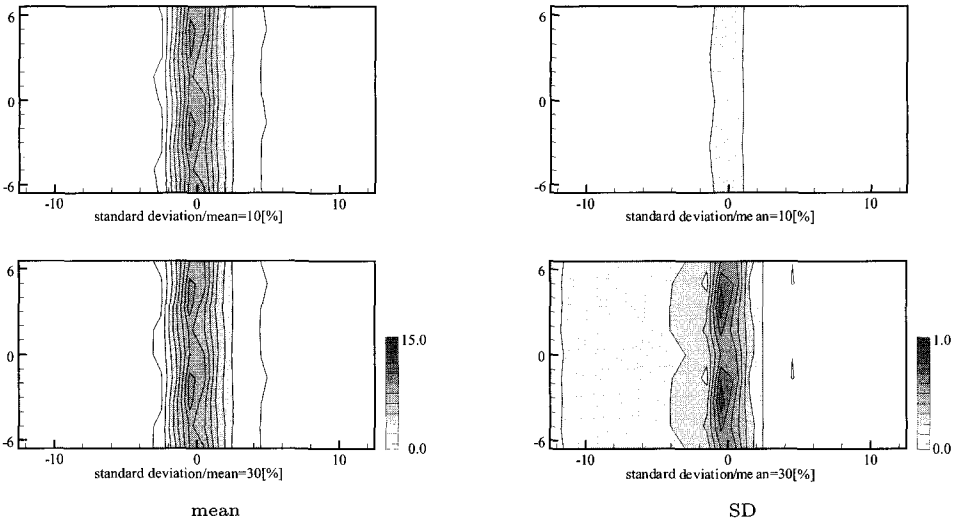


Fig. 9.18 Distribution of the maximum shear strain on the ground surface for different values of Young's modulus standard deviation σ .

uncertainty of underground structures. The value of P_f gives a quantitative measure for hazards of a surface earthquake faults. For instance, in view of P_f shown in Fig. 9.17, the hazards of faulting can be evaluated in the following three categories of U :

$$\begin{cases} U < 0.6 & \text{no faulting,} \\ 0.6 < U < 1.0 & \text{some possibility of faulting,} \\ 1.0 < U & \text{high possibility of faulting.} \end{cases}$$

When the maximum value of the bedrock movement is predicted or when the maximum displacement gap on the source fault is given, the value of U in the above equation is determined, and the hazards of faulting are estimated. When the probability distribution of U is given, a probabilistic estimate of faulting can be made. That is, if the probability of $U > 1.0[\text{m}]$ is 50%, then, the possibility of faulting is around 50%. Or, if the maximum value of the displacement gap on the source fault is less than $0.6[\text{m}]$ and the probability of $U > 0.6[\text{m}]$ is zero, then, the probability of faulting is 0.

9.3.2 Parametric study of stochastic parameters

In the above simulation, the stochastic parameters of Young's modulus are determined without using any background data. It is important to examine the sensibility of the simulation results upon these parameters. This subsection carries out parametric studies for them, namely, the standard deviation (SD) σ , the correlation length ℓ and the mean $\bar{E} = \langle E \rangle$ of Young's modulus E ; unless stated otherwise, the values shown in Table 9.5 are used for these parameters.

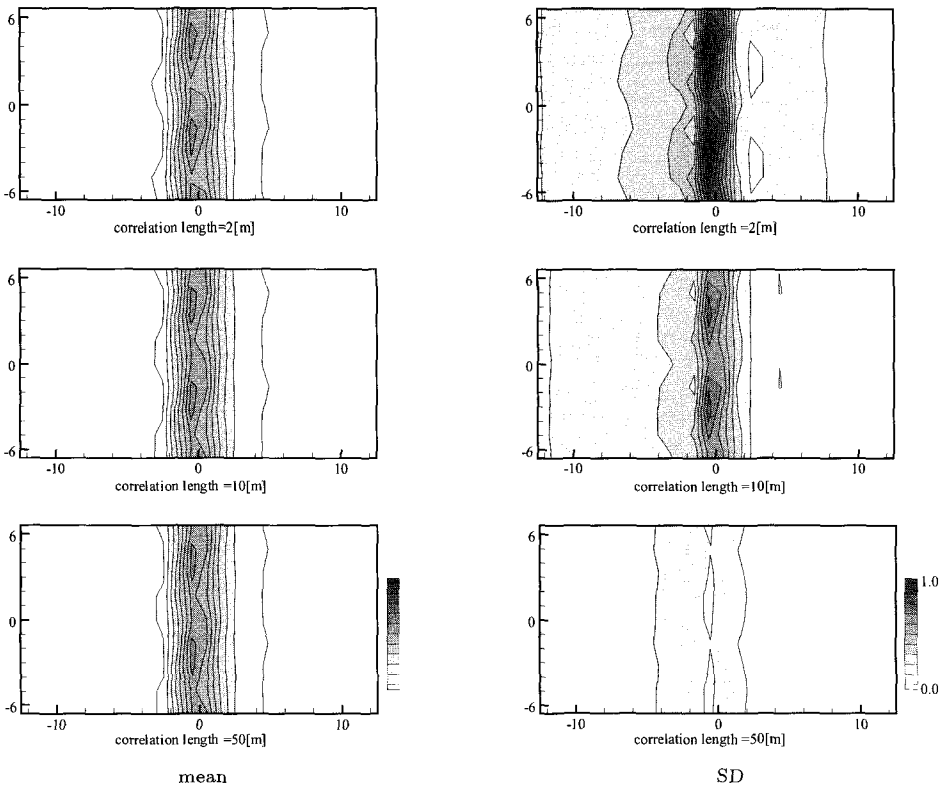


Fig. 9.19 Distribution of the maximum shear strain on the ground surface for different values of Young's modulus correlation length ℓ .

First, the effects of the SD of Young's moduls, σ , are studied by carrying out simulations which use different values of σ . Two values of σ are used; the coefficient of variance is $\frac{\sigma}{\bar{E}}=10\%$ and 30% . In Fig. 9.18, the distribution of the mean and the SD, $\langle\gamma\rangle$ and σ_γ , on the top surface, $x_3=4.8[\text{m}]$, at the bedrock movement, $U=0.6[\text{m}]$, is presented. As is seen, the distribution of $\langle\gamma\rangle$ is less sensitive to σ , but the distribution of σ_γ is strongly influenced. Indeed, $\frac{\sigma}{\bar{E}}=10\%$ produces negligibly small σ_γ while $\frac{\sigma}{\bar{E}}=30\%$ makes the maximum values of σ_γ 100%.

Next, the sensitivity of γ on the correlation length, ℓ , which is probably the most difficult to determine for actual ground structures, is examined. Three cases of $\ell=2, 10, 50[\text{m}]$ are examined. As typical examples, the distribution of $\langle\gamma\rangle$ and σ_γ on $x_3=4.8[\text{m}]$ at $U=0.6[\text{m}]$ is shown in Fig. 9.19. As is seen, the sensitivity of $\langle\gamma\rangle$ on ℓ is less than that of σ_γ ; the distribution of σ_γ is quite different near the zone of a larger $\langle\gamma\rangle$. While a larger value of ℓ means less variation in Young's modulus, the value of ℓ can be set on a logarithmic scale, i.e., $\ell=1[\text{m}]$ or $10[\text{m}]$ may be sufficient.

Finally, the effects of the mean, $\bar{E}$, on the distribution of γ is examined. The three cases of $\bar{E}=4900, 6125$ and $12250[\text{kPa}]$ are simulated. The distribution of $\langle\gamma\rangle$ and σ_γ on $x_3=4.8[\text{m}]$ at $U=0.6[\text{m}]$ is plotted in Fig. 9.20. For a smaller mean,

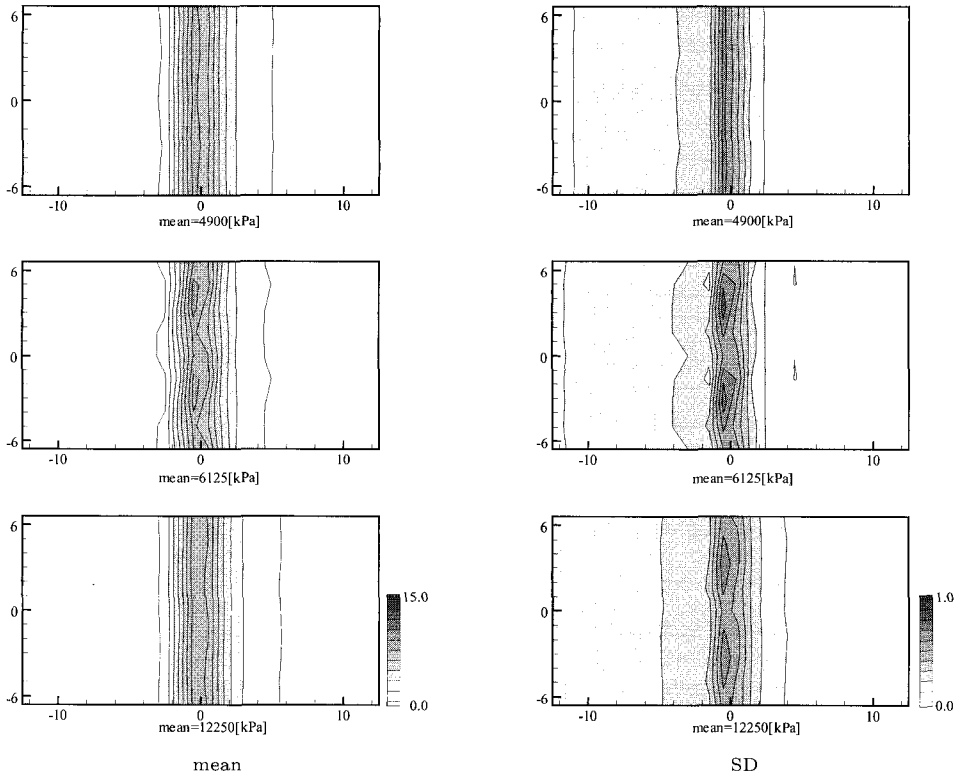


Fig. 9.20 Distribution of the maximum shear strain on the ground surface for different values of Young's modulus mean $\bar{E}$.

$\bar{E}=4900[\text{kPa}]$, the shear strain distribution becomes more or less uniform although some faint periodic pattern is observed. The configuration of echelon faults is different even for a larger value, $\bar{E}=6125$ and $12250[\text{kPa}]$. The echelon faults are more vivid for the case of $\bar{E}=6125[\text{kPa}]$ and the orientation angle of the faults becomes larger. Also, σ_γ tends to take on larger values for the case of $\bar{E}=6125[\text{kPa}]$. As is seen, quite different behavior is simulated if $\bar{E}$ is changed by $\pm 50\%$ from the value used in the Nojima Fault simulation.

As shown in Fig. 9.18 to Fig. 9.20, the sensibility of the simulation results on the stochastic material parameters, ℓ , σ and $\bar{E}$, is summarized as follows:

- 1) while σ and ℓ only influence the variability in fault behavior, $\bar{E}$ changes the mean of fault behavior;
- 2) although it is difficult to set a proper value for σ and ℓ , they are roughly set as $\frac{\sigma}{\bar{E}}=10$ or 30% and $\ell=1, 10$ or $100[\text{m}]$;
- 3) even if it is estimated from in-site data, some accuracy is required for $\bar{E}$ since the simulation results are sensitive to this parameter.

In closing this subsection, a comment is made on the effects of loading path upon

Table 9.7 Material parameters used for the Chelungpu Fault simulation.

mean Young modulus [kN/m ²]	20000
Poisson ratio	0.25
density [g/cm ³]	1.6
friction angle [deg]	40
cohesion [kN/m ²]	35
initial compressive strength [kN/m ²]	150
initial tensile strength [kN/m ²]	32
COV of Young modulus [%]	30
correlation length of Young modulus [m]	1.0

the simulation results. In the current example of the Nojima Fault, the direction of bedrock movement is fixed. In reality, however, the direction may change during the rupture processes. For instance, at an early stage, the horizontal movement is dominant and then vertical movement takes place, or vice versa. Since non-linear elasto-plasticity is assumed, the deformation of the unconsolidated layers will be different depending on the loading path even if the same amount and direction of the bedrock movement is given. The effect of loading path, however, is negligible for the current simulation of the Nojima Fault; once plastic strain is accumulated, the direction of the maximum plastic shear strain is determined by the final bedrock movement, since the layers behave as if they do not have any resistance to the bedrock movement.

9.3.3 Simulation of the Chelungpu Fault

In 1999, the Chelungpu Fault was moved during the Taiwan Chi-Chi Earthquake. The Chelungpu Fault is a typical reverse fault of a shallow dip angle. The NL-SSFEM simulation is carried out to reproduce this surface earthquake fault. The target area is South Tsao-Tun Nantou Prefecture of Central Taiwan, and the geological data obtained by the Japanese Geological Survey are used as well as the record for past displacement gap for surrounding river terraces. The Japanese Geological Survey has made a trench survey; see [Sugiyama *et al.* (2001)]. The dimensions of the trench are length 20[m], width 7[m], depth 2.5[m], and dip angle 60[deg]. The site is a cliff on alluvial terrace which is located around 1[km] in the southeast of Nantou City. The terrace is 2[m] height, and the trench survey cuts the cliff from the northwest to southeast in an oblique manner from the east to the west.

The trench survey shows that surface layers consist of a softer layer of sandy silts, which can be regarded as an unconsolidated layer, and a stiffer gravel layer. A stochastic model is constructed by regarding the sandy layer as a heterogeneous stratified layer. The thickness and width of the layer are 1.2[m] and 4.8[m], respectively. The material properties are summarized in Table 9.7. A two-dimensional state of plane strain is assumed, and the bedrock movement corresponds to reverse

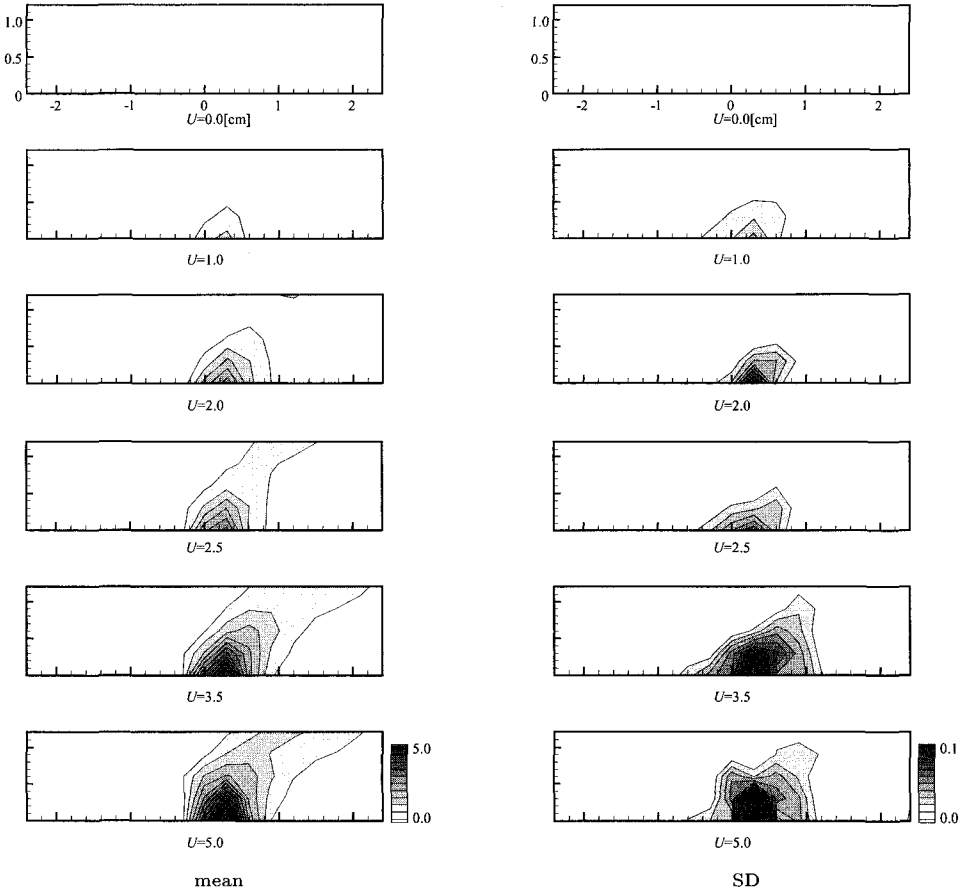


Fig. 9.21 Distribution of the maximum shear strain for the Chelungpu Fault simulation.

faulting of dip angle 60[deg]. Since the vertical displacement gap of the interface between the sandy layer and the gravel layer is around 1.3[m], the maximum value of the bedrock movement is set as this value. On the ground surface, the vertical displacement gap is 1.3~1.4[m], indicating that there is no dissipation of the rupture process within the thin soft layer of 1.2[m]. However, it is observed that surface layers are collapsed due to faulting, and some displacement is added to the ground surface. The trench survey cannot fully identify the fault displacement gap in the top surface.

First, the rupture processes which are computed by the NL-SSFEM simulation are presented. In Fig. 9.21, the distribution of the maximum shear strain,

$$\gamma = \sqrt{\frac{1}{4}(\epsilon_{11} - \epsilon_{33})^2 + \epsilon_{13}^2},$$

on the vertical cross section is plotted; the distribution of the mean and standard deviation (SD) of the maximum shear strain, $\langle \gamma \rangle$ and σ_γ , is arranged vertically for

Table 9.8 The comparison of the configuration parameters for the Chelungpu Fault simulation.

	D [m]	W [m]
Chelungpu fault	(1.3)	1.4
simulation	0.045	1.2

※ (·) shows the maximum dislocation of real fault on ground surface.

the bedrock movement $U=0.0\sim5.0$ [cm]. Like the simulation of the two-dimensional model experiment, the zone where $\langle\gamma\rangle$ and σ_γ take on relatively larger values is wide, and a plane of a reverse fault cannot be reproduced. This is the limitation of the NL-SSFEM simulation with coarse meshing. In view of the change in the distribution of $\langle\gamma\rangle$, it is seen that the rupture reaches the ground surface at $U=3.0$ [cm], and a shear band appears there at $U=4.5$ [cm]. Since the layer thickness is $H=1.2$ [m], the bedrock movement of this amount corresponds to more than 3% shear strain, if the effects of the dip angle are neglected. The change in the distribution of σ_γ is similar to that of $\langle\gamma\rangle$, although σ_γ does not take on a larger value near the surface even if $\langle\gamma\rangle$ does; see the distribution of $\langle\gamma\rangle$ and σ_γ at $U=5.0$ [cm]. However, there is a zone in which σ_γ takes on a larger value, to the left of the fault zone where σ_γ is concentrated. While a zone to the right of the fault corresponds to the major rupture processes, the left zone is probably for the conjugate rupture processes; note that the left zone appears only for the distribution of σ_γ .

Next, the fault configuration parameters which are obtained by the simulation are compared with the observed data. Table 9.8 summarizes the comparison of two configuration parameters, the displacement gap on the ground surface D and the location of fault appearance W with respect to the point at which the bedrock movement is input. The parameters are obtained for the distribution of $\langle\gamma\rangle$ at $U=1.2$ [m]. As is seen, D is underestimated significantly; the simulated value of D is less than 5% of the observed value. As mentioned several times, this is mainly because infinitesimal deformation is assumed and large elastic deformation is included in the computation. Thus, D is significantly¹³ underestimated. The agreement of W is satisfactory, since the fault runs through the ground layers in an almost straight line.

Finally, the failure probability is evaluated. Theoretically, the fault becomes a line segment since the two-dimensional model is used. However, as shown in Fig. 9.21, the simulated fault has some width. This width needs to be taken into consideration in evaluating the failure probability; the width corresponds to a zone of possible faulting. In the current simulation, it is assumed that the fault is defined as a zone where

$$\langle\gamma\rangle + \sigma_\gamma > 0.6.$$

The width of a possible shear band is calculated according to this assumption, and

¹³The observed displacement gap may include additional deformation which is caused by land collapse.

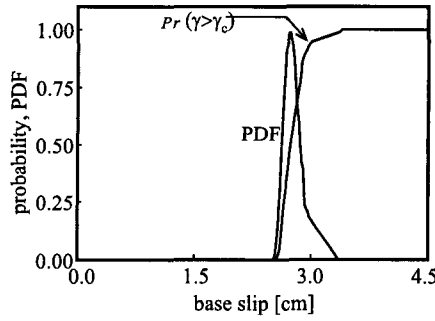


Fig. 9.22 Failure probability of the Chelungpu Fault simulation. An accumulate probability function P_f and a probability density function $\frac{dP_f}{dU}$ are plotted.

the variability in fault behavior is estimated. Like the Nojima Fault simulation, the maximum shear strain at the center and the end of this zone are denoted by γ^1 and γ^2 , respectively, and the joint probability $P_J(\gamma^1, \gamma^2, U)$ is defined¹⁴ as

$$P_J(\gamma_1, \gamma_2, U) = Pr(\gamma^{1'} > \gamma^1 \text{ or } \gamma^{2'} > \gamma^2 | U).$$

The failure probability is then computed as $P_f(U) = P_J(\gamma_c, \gamma_c, U)$ with γ_c being the critical value of the maximum shear strain. This probability P_f is plotted in Fig. 9.22, together with the probability density function, $\frac{dP_f}{dU}$. As is seen, P_f shows the appearance of a fault on the ground surface even at $U = 0.05$ [m]. A fault invariably appears at $U = 0.1$ [m]. The validity of these estimates cannot be examined since no observation data are available, even though the estimates appear intuitively acceptable.

Since the layer is thin, the reverse fault propagates without bending. Thus, the growth of the fault is well simulated in the current NL-SSFEM simulation, besides the fact that the displacement gap is significantly underestimated as shown in Table 9.8. It is actually observed that the presence of layer interfaces or large gravels prevent this smooth growth and the rupture is bent. The presence of such obstacles is ignored in the simulation. Due to the stochastic heterogeneity, however, there is ± 5 [deg] variation in the direction of propagation, which corresponds to a thick zone where shear strain is concentrated. The agreement of the location of fault appearance is thus probably due to the thinness of the layer. A simulation of thicker layers is needed to verify the validity of NL-SSFEM for reverse or normal faulting.

Like the Nojima Fault simulation, the validity of the failure probability P_f shown in Fig. 9.22 cannot be verified by using observed data. Provided that P_f is correct, it is possible to make the following three categories of the bedrock movement U

¹⁴Unlike the Nojima Fault simulation, P_J is computed for either γ^1 or γ^2 exceeds the critical value, $\gamma_c = 0.6$.

which cause surface earthquake fault.

$$\begin{cases} U < 0.4 & \text{no faulting,} \\ 0.4 < U < 0.5 & \text{some possibility of faulting,} \\ 0.5 < U & \text{high possibility of faulting.} \end{cases}$$

As is seen, the difference between U for no faulting and high possibility of faulting is small since the model is thin. It should be recalled that the Chi-Chi Earthquake was an inter-plate earthquake on the plate boundary between the Eurasia Plate and the Philippine Sea Plate. The maximum displacement gap on the source fault is estimated from the relative movement of these two plates. Indeed, the Chi-Chi Earthquake compensates for the displacement gap due to the difference in the plate velocity, 1[cm/year]. Thus, it is estimated¹⁵ that fifty years is sufficient to cause bedrock movement of ten centimeters which is large enough to make the rupture processes reach the ground surface. While a more quantitative estimation is needed to determine the bedrock movement for the formation of a surface earthquake fault, linking the earthquake surface fault problem to the geophysical research will be important. This link will provide a more reliable estimate of the bedrock movement, which is used to improve the accuracy and the reliability of the numerical simulation of surface earthquake fault behavior.

¹⁵This estimate is based on the assumption that the displacement gap of the two plates directly causes the bedrock movement under ground layers; the absorption of the displacement gap within the crust is fully ignored.

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Chapter 10

BEM Simulation of Faulting

In the preceding two chapters, a surface earthquake fault is modeled as a shear band in which plastic deformation is concentrated and strain localization takes place. This modeling is based on the assumption that failure of unconsolidated ground layers over bedrock mass, which consist of an elasto-plastic material, is ductile. The assumption of elasto-plasticity is a common practice for geotechnical engineering, and hence it is sensible to use the elasto-plastic shear band model for a surface earthquake fault.

A surface earthquake fault is an extension of the rupture which propagates from the source fault through the crust. The rupture processes in the crust occur in a brittle manner since the crust consists of intact rock which carries high pressure. On the geological length scale, therefore, a fault is modeled as a crack across which displacement is discontinuous. Indeed, in analyzing earthquake wave propagation, a source fault is usually modeled as a crack; actually, there are two kinds of modeling for the source fault, kinematic modeling in which the time series of displacement gap¹ distribution is presumed, and dynamic modeling in which relations between displacement gap and traction² on the crack surface is prescribed.

This chapter presents the numerical analysis of a surface earthquake fault by modeling the fault as a growing crack. The analysis is based on fracture mechanics. While it is possible, an FEM analysis of a crack requires the finest discretization at the crack tip due to the singularity of strain and stress there; see Sec. 7.3. Techniques related to the mesh refinement are definitely needed in analyzing the crack growth. Thus, a boundary element method (BEM) is employed as an alternative to FEM. It is advantageous to apply BEM to solve a singular problem like a growing crack; the singularity of field variables is more accurately calculated by BEM, and the discretization of field variables which changes as the crack grows is made in a much simpler manner.

¹The amount of total displacement gap is called *asperity* in seismology. The inversion of seismology usually means the prediction of the asperity distribution using the data measured of earthquake waves.

²Related to dynamic modeling, a *barrier* model is sometimes used in seismology. A barrier is a domain on the fault surface in which the strength is higher than the surroundings, and the rupture stops there.

Besides calculating singular field variables, the numerical analysis of a growing crack involves the difficulty of determining the direction and length of a crack extension for a given increment of loading. As explained in the preceding chapters, a surface earthquake fault propagates smoothly in the ground layers, and the FEM simulation is able to automatically find a smooth path along which the fault evolves. However, the BEM simulation is not able to determine the crack path. BEM is a method which solves a boundary value problem for a given crack, and a trial-and-error route³ is used in the BEM simulation to find the geometry of an infinitesimally small crack extension that satisfies the fracture criterion when an increment of loading is given. This route results in tremendous computational efforts. As an alternative to such a trial-and-error route, the rigorous formulation of a mathematical problem of finding a crack extension for a given load increment is presented. This problem is solved by means of BEM. The validity of solving this problem to find the crack extension geometry is shown, and some examples of the BEM simulation of a surface earthquake fault are presented.

It should be emphasized that the target of the present BEM simulation is the determination of a path along which a crack propagates as an external load increases, provided that the rupture processes in unconsolidated deposits are brittle and a surface earthquake fault is modeled as a crack. This problem is called a *crack path problem* in this book; see⁴ [Rice (1968a); Rice (1968b)], [Movchan and Willis (1995); Willis and Movchan (1995)] for the analysis of the growing crack problem with the aid of an elegant mathematical treatment of a growing crack. As will be shown later, the crack path problem is solved by considering an artificial loading at the crack tip that is equivalent with an infinitesimally small extension of the crack; see [Hori and Vaikunthan (1997)]. The artificial loading leads to a boundary value problem, and its solution corresponds to the change in field variables that is caused by the infinitesimal small crack extension. This boundary value problem ought to be solved by BEM, since the solution has higher singularity; for instance, the change in strain and stress has the singularity of $\mathcal{O}(r^{-\frac{3}{2}})$ at the crack tip with r being the distance from the crack tip, and the change in displacement has the singularity of $\mathcal{O}(r^{-\frac{1}{2}})$. It is difficult to calculate field variables of such high singularity by means of FEM.

10.1 Problem Setting for Fault Simulation

To focus our attention on the analysis of a growing crack, assumed are small deformation, a quasi-static state, and a linearly elastic and isotropic homogeneous medium at a two-dimensional state of plane strain. The target is a crack which grows smoothly in an infinite plate of thickness H and elasticity c_{ijkl} . The plate

³See [Vaikunthan (1996)] for a cosine list of related references.

⁴See also [Palaniswamy and Knauss (1978)], [Cotterell and Rice (1980)] and [Gao and Chiu (1992)].

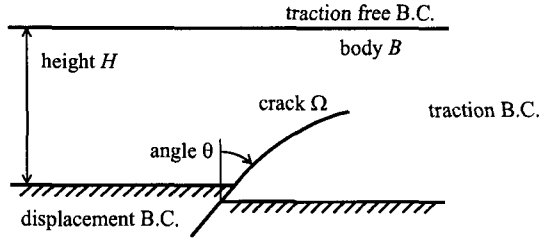


Fig. 10.1 A model used for a crack path problem.

and the crack are denoted by B and Ω ; see Fig. 10.1. Prescribed boundary conditions are as follows: 1) the top surface ∂B_t is traction free; 2) displacement u_i^o is prescribed on the bottom surface ∂B_b ; and 3) normal and shear tractions on the crack surfaces are given as t_n^o and t_s^o . Here, u_i^o corresponds to the bedrock movement and it is expressed in terms of the dip angle θ as $[u_1^o, u_2^o]^T = u^o [\sin \theta, \cos \theta]^T$ for $x_1 > 0$ and $[0, 0]^T$ for $x_1 < 0$, where u^o is a monotonically increasing function of time, t . Traction t_n^o and t_s^o represent friction acting on the fault plane, and take on constant values. Hence, the boundary value problem for displacement u_i is

$$\begin{cases} c_{ijkl} u_{k,li}(x) = 0 & \text{in } B \setminus \Omega, \\ t_i(x) = 0 & \text{on } \partial B_t, \\ u_i(x) = u_i^o(x) & \text{on } \partial B_b, \\ t_n(x) = t_n^o \text{ \& } t_s(x) = t_s^o & \text{on } \Omega, \end{cases} \quad (10.1)$$

where $t_j = n_i \sigma_{ij}$ is traction, and subscript n or s stands for a normal or tangential component on the crack surface. The argument t on u_i and t_i is dropped to prevent the wrong impression that Eq. (10.1) is a problem in dynamic state. As for the fracture criterion, a constant fracture toughness, $\mathcal{G}_c$, for the energy release rate, $\mathcal{G}$, is assumed, i.e.,

$$\mathcal{G} = \mathcal{G}_c. \quad (10.2)$$

Since B is linear elastic, $\mathcal{G}$ is expressed in terms of the Mode I and II stress intensity factors, denoted by K_I and K_{II} , as

$$\mathcal{G} = \frac{(1 - \nu)^2}{E} (K_I^2 + K_{II}^2), \quad (10.3)$$

where E and ν are Young's modulus and Poisson's ratio, and $K_{I,II}$ is computed from

$$\begin{bmatrix} K_I \\ K_{II} \end{bmatrix} = \lim_{r \rightarrow 0} \frac{E\sqrt{2\pi}}{8(1 - \nu^2)} r^{-\frac{1}{2}} \begin{bmatrix} [u_n] \\ [u_s] \end{bmatrix}, \quad (10.4)$$

where $[u_n]$ and $[u_s]$ are opening and sliding components of displacement discontinuity which has the singularity of $\mathcal{O}(r^{\frac{1}{2}})$ at the crack tip.

10.1.1 Perturbation expansion of field variables with respect to crack extension

The perturbation expansion⁵ of field variables with respect to an infinitesimally small extension, da , of Ω , is taken assuming that u_i^0 is fixed. The extension and the extended crack are denoted by $d\Omega$ and $\Omega+d\Omega$; see Fig. 10.2. For simplicity, the perturbation expansion of u_i is expressed as follows:

$$u_i(\mathbf{x}, da) = u_i(\mathbf{x}) + u'_i(\mathbf{x})da + \frac{1}{2}u''_i(\mathbf{x})da^2 + \cdots \quad (10.5)$$

Here, argument da of u_i on the left side emphasizes the dependence of u_i on $d\Omega$, and prime stands for formal derivative with respect to the crack extension da . In [Hori and Vaikunthan (1997)] and [Hori and Vaikunthan (1998a)], u'_i is called *displacement rate*⁶ with respect to the crack extension.

The perturbation expansion of the stress and unit normal on the crack extension are needed in order to take the perturbation expansion of traction. The stress associated with u'_i and u''_i is denoted by σ'_{ij} and σ''_{ij} , respectively, i.e.,

$$\sigma_{ij}(\mathbf{x}, da) = \sigma_{ij}(\mathbf{x}) + \sigma'_{ij}(\mathbf{x})da + \frac{1}{2}\sigma''_{ij}(\mathbf{x})da^2 + \cdots \quad (10.6)$$

The unit normal is considered in a fixed local coordinate system near $d\Omega$; the origin coincides with Ω 's tip, and the x_1 -axis is parallel to Ω 's direction. Then, $d\Omega$ is expressed⁷ as

$$d\Omega = \{(\phi_1(s), \phi_2(s)) = (s, \phi(s))^T \mid 0 < s < da, \phi(0) = \phi'(0) = 0\},$$

and the unit normal and tangent on $d\Omega$ are

$$\begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \frac{1}{\sqrt{1 + (\phi')^2}} \begin{bmatrix} -\phi' \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \frac{1}{\sqrt{1 + (\phi')^2}} \begin{bmatrix} 1 \\ \phi' \end{bmatrix};$$

prime on ϕ stands for derivative with respect to s . At $s=0$, these ϕ_i , n_i , s_i and their derivatives are calculated as

$$\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \phi'_1 \\ \phi'_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \phi''_1 \\ \phi''_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \kappa \end{bmatrix},$$

$$\begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} n'_1 \\ n'_2 \end{bmatrix} = \begin{bmatrix} -\kappa \\ 0 \end{bmatrix}, \quad \begin{bmatrix} n''_1 \\ n''_2 \end{bmatrix} = \begin{bmatrix} -\kappa^2 \\ -\rho \end{bmatrix},$$

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} s'_1 \\ s'_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \kappa \end{bmatrix}, \quad \begin{bmatrix} s''_1 \\ s''_2 \end{bmatrix} = \begin{bmatrix} -\rho \\ \kappa^2 \end{bmatrix},$$

⁵See [Hori and Vaikunthan (1997)] for a more detailed explanation on the present perturbation expansion; see also [Xu and Keer (1992)].

⁶See [Wu (1979)], [Sumi (1986)] [Xu and Keer (1992)] for other perturbation expansion techniques and asymptotic expansions which are applied to fields near the crack tip; see also [Hori and Vaikunthan (1998b)] and [Azhdari and Nemat-Nasser (1996)].

⁷As is seen, da is the length of the crack extension in the x_1 -direction.

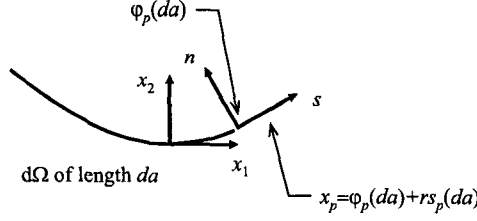


Fig. 10.2 Virtual extension of a crack.

where⁸ $\kappa = \phi''(0^+)$ and $\rho = \phi'''(0^+)$; see Fig. 10.2.

Since the coordinate of the crack tip for $\Omega + d\Omega$ is $\phi_i(da)$, traction acting on the outer crack tip is given as $n_i(da)\sigma_{ij}(x, da)$, where $x_i = \phi_i(da) + rs_i(da)$ with r being the distance from the tip. In view of Eq. (10.6), if the unit normal is expanded with respect to da , the perturbation expansion of normal and tangential traction components are computed as follows:

$$\begin{bmatrix} t_n \\ t_s \end{bmatrix} = \begin{bmatrix} \sigma_{22,2} \\ \sigma_{21,2} \end{bmatrix} + \begin{bmatrix} t'_n \\ t'_s \end{bmatrix} da + \frac{1}{2} \begin{bmatrix} t''_n \\ t''_s \end{bmatrix} da^2 + \dots, \quad (10.7)$$

where $t'_{n,s}$ and $t''_{n,s}$ are

$$\begin{bmatrix} t'_n \\ t'_s \end{bmatrix} = \kappa \begin{bmatrix} -2\sigma_{21} + r\sigma_{22,2} \\ \sigma_{22} - \sigma_{11} + r\sigma_{21,2} \end{bmatrix} + \begin{bmatrix} \sigma_{22,1} + \sigma'_{22} \\ \sigma_{21,1} + \sigma'_{21} \end{bmatrix} \quad (10.8)$$

and

$$\begin{aligned} \begin{bmatrix} t''_n \\ t''_s \end{bmatrix} &= \kappa^2 \begin{bmatrix} 2(\sigma_{11} - \sigma_{22}) + r^2\sigma_{22,22} - r(\sigma_{22,1} + 4\sigma_{21,2}) \\ \sigma_{21} + r^2(\sigma_{21,22} + r(2(\sigma_{22} - \sigma_{11}),_2 - \sigma_{21,1})) \end{bmatrix} \\ &+ \kappa \begin{bmatrix} \sigma_{22,2} - 4(\sigma_{21,1} + \sigma'_{21}) + 2r(\sigma_{22,1} + \sigma'_{22}),_2 \\ \sigma_{21,2} + 2(\sigma_{22,1} + \sigma'_{22}) - 2(\sigma_{21,1} + \sigma'_{11}) + 2r(\sigma_{21,1} + \sigma'_{21}),_2 \end{bmatrix} \\ &+ \begin{bmatrix} \sigma_{22,11} + 2\sigma'_{22,1} + \sigma''_{22} + \rho(-2\sigma_{21} + r\sigma_{22,2}) \\ \sigma_{21,11} + 2\sigma'_{21,1} + \sigma''_{21} + \rho(-2\sigma_{21} + r\sigma_{22,2}) \end{bmatrix}. \end{aligned} \quad (10.9)$$

Stress and its derivatives are evaluated at a point of $x_i = \phi_i(0) + rs_i(0) = rs_i(0)$. It should be noted that t'_i and t''_i include κ and ρ , and hence they are *not* given as σ'_{2i} and σ''_{2i} , i.e., $t''_n \neq \sigma'_{22}$ and $t''_s \neq \sigma'_{21}$.

10.1.2 Crack driving forces

As shown in Eq. (10.8), t'_n or t'_s includes the gradient of stress, $\sigma_{22,1}$ or $\sigma_{21,2}$, which has the singularity of $\mathcal{O}(r^{-\frac{3}{2}})$; see [Williams (1957)] for the fundamental solution that has the same singularity. This high singularity must be canceled by σ'_{2j} so that the strain energy stored at the tip of $d\Omega$ remains bounded. This condition for

⁸Superscript + emphasizes that quantities are measured at the outer tip, C^+ , e.g., $\phi(0^+)$ is the limit of $\phi(s)$ as s goes from a positive value to 0.

σ'_{ij} is equivalent to the limit of the boundary condition as a pair of concentrated forces acting on both crack surfaces approach the crack tip. The concentrated force, denoted by P_i , needs to satisfy

$$\begin{bmatrix} P_2 \\ P_1 \end{bmatrix} = - \lim_{b \rightarrow 0} \sqrt{\frac{\pi}{8b}} \begin{bmatrix} K_I \\ K_{II} \end{bmatrix},$$

where b is the distance between the crack tip and the point at which P_i acts; see [Hori and Vaikunthan (1997)]. The pair of concentrated forces are called⁹ *crack driving forces*.

The crack driving forces serve as a non-trivial boundary condition for u'_i , and a boundary value problem for u'_i is derived from Eq. (10.1), i.e.,

$$\begin{cases} c_{ijkl} u'_{k,li}(\mathbf{x}) = 0 & \text{in } B \setminus \Omega, \\ u'_i(\mathbf{x}) = 0 & \text{on } \partial B_t, \\ u'_i(\mathbf{x}) = 0 & \text{on } \partial B_b, \\ t'_i(\mathbf{x}) = 0 & \text{on } \Omega \setminus C^-, \\ t'_i(\mathbf{x}) = P_i & \text{at } C^-. \end{cases} \quad (10.10)$$

Here, C stands for a point at the crack tip and C^- is given as the end of the crack surface Ω . In solving Eq. (10.10), Westergaard's potentials that satisfy the boundary condition of diverging crack driving forces are used, i.e.,

$$\begin{bmatrix} Z_I^c \\ Z_{II}^c \end{bmatrix} = \frac{1}{\sqrt{8\pi}} z^{-\frac{1}{2}} \begin{bmatrix} K_I \\ K_{II} \end{bmatrix},$$

where¹⁰ $z = x_1 + ix_2$. By subtracting the fields caused by these potentials from Eq. (10.10), the boundary condition of the crack driving forces is excluded, and an ordinary boundary value problem which does not have a condition at the crack tip is obtained. In this manner, u'_i is expressed in terms of K_I and K_{II} as

$$u'_i = K_I(u_{Ii}^c + u_{Ii}^*) + K_{II}(u_{IIi}^c + u_{IIi}^*). \quad (10.11)$$

where $u_{I,IIi}^c$ is the displacement caused by $Z_{I,II}^c$, and $u_{I,IIi}^*$ is the displacement that is obtained from the following two boundary conditions: 1) $t_{I,IIi}^c + t_{I,IIi}^* = 0$ on ∂B_t and Ω ; and 2) $u_{I,IIi}^c + u_{I,IIi}^* = 0$ on ∂B_b .

Similarly, the condition for σ''_{ij} at the outer crack tip is identified so that the high singularity of $\kappa\sigma_{2i,2} + \sigma_{2i,11} + 2\sigma'_{2i,1}$ in t''_n and t''_s is canceled. Like the crack driving force, this condition is equivalent to the limit of a pair of certain concentrated forces, and a boundary value problem similar to Eq. (10.10) is posed for u''_i . As will be shown later, however, only terms which are associated with κ are required to

⁹The crack driving force is different from the *configurational force* that is considered by [Gurthin and Podio-Guidugli (1996)].

¹⁰See [Broek (1991)] for Westergaard's potential; see also [Westergaard (1939)]. Note that the branch cut of $z^{-\frac{1}{2}}$ is chosen so that it coincides with Ω .

evaluate u_i'' . It is necessary to evaluate the effects of $\kappa\sigma_{2i,2}$, which has the singularity of $\mathcal{O}(r^{-\frac{3}{2}})$, on u_i'' . It follows from

$$\begin{bmatrix} \sigma_{22,2} \\ \sigma_{21,2} \end{bmatrix} = \frac{r^{-\frac{3}{2}}}{\sqrt{8\pi}} \begin{bmatrix} K_{II} \\ K_I \end{bmatrix} + \dots$$

that the displacement associated with $\kappa\sigma_{2i,2}$ is computed in the same manner as u_i' and is given as $\kappa(K_I(u_{IIi}^c + u_{IIi}^*) + K_{II}(u_{Ii}^c + u_{Ii}^*))$.

In the same manner as shown in Eqs. (10.5) and (10.6), the perturbation expansion of K_I and K_{II} is taken, as

$$\begin{bmatrix} K_I(da) \\ K_{II}(da) \end{bmatrix} = \begin{bmatrix} K_I \\ K_{II} \end{bmatrix} + \begin{bmatrix} K_I' \\ K_{II}' \end{bmatrix} da + \frac{1}{2} \begin{bmatrix} K_I'' \\ K_{II}'' \end{bmatrix} da^2 + \dots, \quad (10.12)$$

where $K'_{I,II}$ is called a *stress intensity factor rate* in [Hori and Vaikunthan (1997)]. The value of $K_{I,II}(da)$ is given as $\lim_{r \rightarrow 0} \sqrt{2\pi r} t_{n,s}$ for $x_i = \phi_i + r s_i$. In view of Eq. (10.7), therefore, the coefficients for da and da^2 , i.e., $K'_{I,II}$ and $K''_{I,II}$, are given as

$$\begin{bmatrix} K_I' \\ K_{II}' \end{bmatrix} = \lim_{r \rightarrow 0} \sqrt{2\pi r} \begin{bmatrix} u_n' \\ u_s' \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} K_I'' \\ K_{II}'' \end{bmatrix} = \lim_{r \rightarrow 0} \sqrt{2\pi r} \begin{bmatrix} t_n'' \\ t_s'' \end{bmatrix};$$

recall that $t_{n,s}'$ and $t_{n,s}''$ have the singularity of $\mathcal{O}(r^{-\frac{1}{2}})$.

By taking advantage of Eqs. (10.8) and (10.9), the limit of $t_{n,s}'$ and $t_{n,s}''$ can be computed from an asymptotic expansion of the singular¹¹ stress on the positive x_1 -axis. For instance, the asymptotic expansion of the singular $\sigma_{22,1} + \sigma'_{22}$ is

$$\begin{aligned} & \left(-\frac{K_I r^{-\frac{3}{2}}}{\sqrt{8\pi}} + \frac{L_I r^{-\frac{1}{2}}}{\sqrt{2\pi}} + \dots \right) + \left(\frac{K_I r^{-\frac{3}{2}}}{\sqrt{8\pi}} + \frac{M_I r^{-\frac{1}{2}}}{\sqrt{2\pi}} + \dots \right) \\ &= \frac{L_I + M_I}{\sqrt{2\pi}} r^{-\frac{1}{2}} + \dots, \end{aligned}$$

where $L_{I,II}$ and $M_{I,II}$ are computed in essentially the same manner as $K_{I,II}$; $L_{I,II}$ is computed from u_i and $K_{I,II}$ as

$$\begin{bmatrix} L_I \\ L_{II} \end{bmatrix} = \lim_{r \rightarrow 0} \frac{3(1-\nu^2)}{4E} r^{-\frac{3}{2}} \begin{bmatrix} \sqrt{2\pi}[u_n] - K_I \sqrt{r} \\ \sqrt{2\pi}[u_s] - K_{II} \sqrt{r} \end{bmatrix}, \quad (10.13)$$

and $M_{I,II}$ are computed by replacing u_i in Eq. (10.4) with $K_I u_{Ii}^* + K_{II} u_{IIi}^*$, since σ'_{ij} is decomposed into

$$\sigma'_{ij} = K_I(\sigma_{Iij}^c + \sigma_{Iij}^*) + K_{II}(\sigma_{IIij}^c + \sigma_{IIij}^*);$$

$\sigma_{I,IIij}^c$ is produced by $Z_{I,II}^c$ and hence the singularity at the crack tip is $\mathcal{O}(r^{-\frac{3}{2}})$. It should be noted that in numerical computation, the accuracy of computing $M_{I,II}$ is

¹¹Regular stress, which is expanded with r^n for $n=0, 1, \dots$ is automatically excluded by considering crack-opening-displacements.

the same as the accuracy of computing $K_{I,II}$, but the accuracy of computing $L_{I,II}$ could be worse; this accuracy will be examined in the next section.

Once $L_{I,II}$ and $M_{I,II}$ are determined, $K'_{I,II}$ and $K''_{I,II}$ are given as

$$\begin{bmatrix} K'_{I} \\ K'_{II} \end{bmatrix} = \begin{bmatrix} -\frac{3}{2}\kappa K_{II} + L_I + M_{II} \\ \frac{1}{2}\kappa K_I + L_{II} + M_{II} \end{bmatrix}, \quad (10.14)$$

and

$$\begin{bmatrix} K''_{I} \\ K''_{II} \end{bmatrix} = \begin{bmatrix} -\frac{1}{4}\kappa^2 K_I + \kappa(-4L_{II} + 3M_{II} + N_I) \\ -\frac{21}{4}\kappa^2 K_{II} + \kappa(M_I + N_{II}) \end{bmatrix} + \dots, \quad (10.15)$$

where $\dots$ stands for terms which are not associated with κ and $N_{I,II}$ is computed from $K_I u_{I,i}^* + K_{II} u_{II,i}^*$ in the same manner as Eq. (10.4). As is seen, κ , the curvature of the crack extension, naturally appears in these coefficients.

10.1.3 Solution of crack path problem

This subsection presents the procedures of determining the configuration of the crack extension for a given increment of the bedrock displacement. It should be noted that the change in the stress intensity factors is easily computed when u_i^* increases by $\dot{u}_i^* dt$ but Ω remains unchanged; the associated displacement change, denoted by $\dot{u}_i dt$, satisfies a boundary value problem which is derived by taking the derivative of Eq. (10.1) with respect to t , i.e.,

$$\begin{cases} c_{ijkl} \dot{u}_{k,li}(\mathbf{x}) = 0 & \text{in } B \setminus \Omega, \\ \dot{t}_i(\mathbf{x}) = 0 & \text{on } \partial B_t, \\ \dot{u}_i(\mathbf{x}) = \dot{u}_i^o & \text{on } \partial B_b, \\ \dot{t}_i(\mathbf{x}) = 0 & \text{on } \Omega, \end{cases} \quad (10.16)$$

and the stress intensity factor associated with $\dot{u}_i$, denoted by $\dot{K}_{I,II}$, is computed from $\dot{u}_i$ in the same manner as in Eq. (10.4). Here, dot stands for derivative with respect to t .

Assuming that the crack length is a function of t , the change in the field variables due to the increase of the bedrock displacement and the associated extension of the crack is considered. This change is formally regarded as a total derivative with respect to t . For instance, in terms of $\dot{a}=da/dt$, as the rate of the crack growth, the change in the stress intensity factor caused by the bedrock displacement increment and the resulting crack growth is expressed as

$$\frac{dK_{I,II}}{dt} = \dot{K}_{I,II} + \dot{a}K'_{I,II}. \quad (10.17)$$

As shown in Eq. (10.14), κ is included in $K'_{I,II}$, and the geometrical parameters of $d\Omega$, namely, κ and $\dot{a}$, can be determined so that the fracture criterion of Eq. (10.2) is satisfied during the crack growth.

In terms of the stress intensity factors, the total derivative of the energy release rate is expressed as

$$\frac{d\mathcal{G}}{dt} = \frac{2(1-\nu^2)}{E} \left(K_I(\dot{K}_I + \dot{a}K'_I) + K_{II}(\dot{K}_{II} + \dot{a}K'_{II}) \right). \quad (10.18)$$

Since Eq. (10.2) leads to $d\mathcal{G}/dt=0$, the curvature and growth rate of the crack extension must satisfy

$$(\kappa K_I K_{II} - (K_I(L_I + M_I) + K_{II}(L_{II} + M_{II})) \dot{a} = K_I \dot{K}_I + K_{II} \dot{K}_{II}. \quad (10.19)$$

It should be noted that $\kappa \dot{a}$ corresponds to the change in the crack orientation after the growth.

In order to determine κ and $\dot{a}$, another criterion, the *maximum energy release rate*, is assumed, i.e., Ω grows in the direction that maximizes $\mathcal{G}$; see, for instance, [Wu (1979)] and [Hayashi and Nemat-Nasser (1981)]. In view of Eqs. (10.14) and (10.15), it is seen that while $\dot{\mathcal{G}}$ is a first-order polynomial of κ , $\ddot{\mathcal{G}}$ is a second-order polynomial and the coefficient of $\ddot{\mathcal{G}}$ for κ^2 is $-12(1-\nu^2)K_{II}^2/E < 0$. Hence, κ that maximizes $\mathcal{G}$ is determined from the condition of $\partial \ddot{\mathcal{G}}/\partial \kappa = 0$, i.e.,

$$\kappa = \frac{K_I(-3L_{II} + 4M_{II} + N_I) + K_{II}(-3L_I - 2M_I + N_{II})}{6K_{II}^2}. \quad (10.20)$$

All terms in Eqs. (10.19) and (10.20) are computed by solving two boundary value problems, Eqs. (10.10) and (10.16). It does not require any trial values of the geometrical parameters, $\dot{a}$ and κ . This is one advantage of solving the crack path problem. It should be emphasized that a condition similar to Eq. (10.20) can be derived if another fracture criterion is assumed rather than the maximum energy release rate, or even if there exist many growing cracks which interact with each other.

The presence of a *seed* of the crack on the boundary ∂B_b must be assumed in the numerical analysis; the length of the seed is set as $0.02H$, and its direction is set parallel to the dip angle of the source fault. The seed corresponds to a small displacement discontinuity which is caused by the infinitesimally small gap of the bedrock. As the bedrock displacement increases, a small crack extension is incrementally determined in the following manner.

- i) For a given Ω , solve Eq. (10.1) and evaluate $K_{I,II}$.
- ii) Solve Eq. (10.10) using the crack driving forces determined from $K_{I,II}$, and evaluate $L_{I,II}$, $M_{I,II}$, and $N_{I,II}$.
- iii) Solve Eq. (10.16), and evaluate $\dot{K}_{I,II}$.
- iv) Compute κ and $\dot{a}$ from Eqs. (10.19) and (10.20), and determine $da = \dot{a} dt$ using a suitably small dt .
- v) Update the configuration of the crack Ω using κ and da .

These procedures are repeated until Ω reaches the top surface ∂B_t .

10.2 Formulation of Boundary Element Method

BEM is employed¹² to solve the three boundary value problems given by Eqs. (10.1), (10.10) and (10.16). Green's function for an infinite body¹³ is used, and the traction and the displacement jump are distributed on ∂B_t , ∂B_b , and Ω so that boundary conditions are satisfied. Remeshing of Ω is made as it propagates, and sufficiently finer meshes are used near the crack tip to accurately compute the stress intensity factors and their derivatives.

In formulating BEM, Green's function for an infinite body and dual boundary integral equations are used. [Brebbia (1984)] and [Nishimura and Kobayashi (1991); Kobayashi *et al.* (2000)] are recommended as a comprehensive reference on BEM; see [Erdogan *et al.* (1973)] for the application of BEM to crack problems and [Nakagawa *et al.* (1984)] for efficient¹⁴ computation of BEM. The dual boundary integral equations use both a displacement boundary integral equation and a traction boundary integral equation on the crack surface. The BEM code which is used to solve the crack path problem has the following four characteristics:

- 1) piece-wise constant elements are used on ∂B_b and ∂B_t ;
- 2) on Ω , crack opening displacement ($[u_i]=u_i^+-u_i^-$) is weighted with $\sqrt{r}$ near the tip ([Saez *et al.* (1995)]);
- 3) sufficiently finer elements are used near the tip of Ω to account for the singularities there;
- 4) boundary conditions on ∂B_b and ∂B_t are approximately solved by considering a part of the boundary of length $6H$.

There are several formulations to derive BEM. Somiglina's formulation¹⁵ is em-

¹²The accuracy of computing the stress intensity factor rate could not be worse than that of the accuracy of computing the stress intensity factor, since the same numerical analysis method is applied to solve the boundary value problem of displacement rate and displacement. The stress intensity factor rate and the stress intensity factor are calculated in the same manner by using the displacement rate and displacement.

¹³Precisely speaking, it is called a fundamental solution. Green's function should be used for a boundary value problem.

¹⁴See also [Guiggiani and Gigante (1990)] and [Watanabe and Hayami (1994); Yamada and Hayami (1995)].

¹⁵Somiglina's formulation makes use of Betti's reciprocal theory. This theory is stated as follows: when a linear elastic body B is subjected to two different loadings which are designated by superscripts (1) and (2), the resulting field variables satisfy

$$\int_B \sigma_{ij}^{(1)} \epsilon_{ij}^{(2)} ds = \int_B \sigma_{ij}^{(2)} \epsilon_{ij}^{(1)} ds$$

or

$$\int_{\partial B} t_i^{(1)} u_i^{(2)} d\ell + \int_B b_i^{(1)} u_i^{(2)} ds = \int_{\partial B} t_i^{(2)} u_i^{(1)} d\ell + \int_B b_i^{(2)} u_i^{(1)} ds;$$

this theory means that the work computed by using force of the first loading system and deformation of the second system equals the work computed by using force of the second loading system and deformation of the first loading system.

ployed here, and Green's function for an infinite body is used, i.e.,

$$c_{ijkl}G_{kp,li}(\mathbf{x}) + \delta_{jp}\delta(\mathbf{x}) = 0. \quad (10.21)$$

Here, c_{ijkl} is uniform elasticity tensor and $\delta(\mathbf{x})$ is a delta function which gives 1 when integrated in a domain which includes the origin in it. Now, a uniform but finite body B with elasticity c_{ijkl} is considered. A general mixed boundary value problem is posed for displacement in B , i.e.,

$$\begin{cases} c_{ijkl}u_{k,li}(\mathbf{x}) + f_j^\circ(\mathbf{x}) = 0 & \mathbf{x} \text{ in } B, \\ u_i(\mathbf{x}) = u_i^\circ(\mathbf{x}) & \mathbf{x} \text{ on } \partial B_u, \\ n_i(\mathbf{x})c_{ijkl}u_{k,l}(\mathbf{x}) = t_i^\circ(\mathbf{x}) & \mathbf{x} \text{ on } \partial B_t, \end{cases} \quad (10.22)$$

where f_j° is body force prescribed in B , and u_i° and t_i° are displacement and traction prescribed on ∂B_u and ∂B_t , respectively; the boundary ∂B is divided into ∂B_u and ∂B_t . By applying integration by part to $\int_B c_{ijkl}u_{i,j}G_{kp,l}ds$, the following identity is derived for G_{ip} and u_i which satisfy Eqs. (10.21) and (10.22):

$$\begin{aligned} \int_{\partial B} n_i(\mathbf{y})H_{ijp}(\mathbf{y}-\mathbf{x})u_j(\mathbf{y})d\ell_{\mathbf{y}} + \int_B (\delta_{ip}\delta(\mathbf{y}-\mathbf{x}))u_i(\mathbf{y})ds_{\mathbf{y}} \\ = \int_{\partial B} t_i(\mathbf{y})G_{jp}(\mathbf{y}-\mathbf{x})d\ell_{\mathbf{y}} + \int_B f_i^\circ(\mathbf{y})G_{ip}(\mathbf{y}-\mathbf{x})ds_{\mathbf{y}}, \end{aligned}$$

where $H_{ijp}(\mathbf{x})=c_{ijkl}G_{kp,l}(\mathbf{x})$ and $t_j(\mathbf{y})=n_i(\mathbf{x})c_{ijkl}u_{k,l}(\mathbf{x})$. It follows from the definition of the delta function δ that u_i in B and on ∂B is

$$\begin{aligned} C u_p(\mathbf{x}) = \int_B G_{ip}(\mathbf{y}-\mathbf{x})f_i^\circ(\mathbf{y})ds_{\mathbf{y}} \\ + \int_{\partial B} G_{ip}(\mathbf{y}-\mathbf{x})t_i(\mathbf{y}) - n_i(\mathbf{y})H_{ijp}(\mathbf{y}-\mathbf{x})u_j(\mathbf{y})d\ell_{\mathbf{y}}, \end{aligned}$$

where $C=1$ for $\mathbf{x}$ within B and $=\frac{1}{2}$ for $\mathbf{x}$ on ∂B which is assumed to be smooth.

While displacement and traction are prescribed on some parts of the boundary ∂B , there are unknown displacement and traction on ∂B , i.e., u_i on ∂B_t and t_i on ∂B_u . The Somigliana integral equation for them is thus derived from the above equation, as follows:

$$\int_{\partial B_u} G_{ip}(\mathbf{y}-\mathbf{x})t_p(\mathbf{y})d\ell_{\mathbf{y}} + \int_{\partial B_t} n_i(\mathbf{y})H_{ijp}(\mathbf{y}-\mathbf{x})u_p(\mathbf{y})d\ell_{\mathbf{y}} = \hat{u}_i^\circ(\mathbf{x}), \quad (10.23)$$

for $\mathbf{x}$ on ∂B_u , where $\hat{u}_i^\circ$ is

$$\begin{aligned} \hat{u}_i^\circ(\mathbf{x}) = \frac{1}{2}u_i^\circ(\mathbf{x}) - \int_B G_{ip}(\mathbf{x}-\mathbf{y})f_i^\circ(\mathbf{y})ds_{\mathbf{y}} \\ + \int_{\partial B_u} n_i(\mathbf{y})H_{ijp}(\mathbf{y}-\mathbf{x})u_j^\circ(\mathbf{y})d\ell_{\mathbf{y}} - \int_{\partial B_t} G_{ip}(\mathbf{y}-\mathbf{x})t_p^\circ(\mathbf{y})d\ell_{\mathbf{y}}. \end{aligned}$$

Also, an integral equation¹⁶ similar to Eq. (10.23) is derived for traction on the boundary ∂B_u . By discretizing¹⁷ the unknown functions, u_i on ∂B_t and t_i on ∂B_u , this part of the integral equations leads to a matrix equation for the coefficients of the discretized functions. This matrix equation is solved by BEM.

In closing this subsection, a fundamental solution for an isotropic body at quasi-static state is presented. For a two-dimensional setting, the fundamental solution is called Kelvin's solution and given as

$$G_{ij} = \frac{1}{8\pi(1-\nu)\mu} \left(-(3-4\nu) \log(r) \delta_{ij} + r_{,i} r_{,j} \right),$$

$$n_p T_{pij} = -\frac{1}{4\pi(1-\nu)r} \left(((1-2\nu)\delta_{ij} + 2r_{,i} r_{,j}) n_i r_{,i} - (1-2\nu)(r_{,i} n_j - r_{,j} n_i) \right),$$

with $r = \sqrt{x_i x_i}$; no sum is taken for i or j . For a three-dimensional setting, the fundamental solution is given by [Brebbia (1984)], as

$$G_{ij} = \frac{1}{16\pi(1-\nu)\mu r} \left((3-4\nu)\delta_{ij} + r_{,i} r_{,j} \right),$$

$$n_p T_{pij} = -\frac{1}{8\pi(1-\nu)r^2} \left(((1-2\nu)\delta_{ij} + 3r_{,i} r_{,j}) n_i r_{,i} - (1-2\nu)(r_{,i} n_j - r_{,j} n_i) \right).$$

10.3 Verification of Analysis Method

The validity of solving the boundary value problem of the rate field, Eq.(10.10), to obtain the solution of the crack path problem is verified by [Hori and Vaikunthan (1997); Hori and Vaikunthan (1998a)]. They first consider a case of $\kappa=0$, a straight extension, using an analytic solution for simple crack problems and then study a case of $\kappa \neq 0$, applying BEM to numerically solve the crack path problem. The accuracy of computing u'_i and $K'_{I,II}$, which are obtained as a solution of the boundary value problem of the rate field, is examined by comparing the solution of the rate field which is numerically obtained by means of the finite difference method. The results of [Hori and Vaikunthan (1997)] are briefly presented in this section.

10.3.1 Use of analytic solution

The crack path problem is posed in Sec. 10.1 to determine the growth rate and curvature, $\dot{a}$ and κ , of the crack extension for a given increment of loading. The key

¹⁶It follows from the integral representation of displacement that traction on the boundary is explicitly expressed as

$$t_q(\mathbf{x}) = n_p(\mathbf{x}) c_{pqrs} \left(\int_B -G_{ir,s}(\mathbf{y} - \mathbf{x}) f_i^o(\mathbf{y}) ds_{\mathbf{y}} \right. \\ \left. + \int_{\partial B} -G_{ir,s}(\mathbf{y} - \mathbf{x}) t_i(\mathbf{y}) + n_i(\mathbf{y}) H_{ijr,s}(\mathbf{y} - \mathbf{x}) u_j(\mathbf{y}) d\ell_{\mathbf{y}} \right).$$

¹⁷A *collocation method* or a weak form of the integral equation is used in BEM. The collocation method uses a set of values for displacement and traction at several points on the boundary. The weak form of the integral equation solves the weighted integral of the integral equations.

point is that the rate field, the change in field variables caused by an infinitesimal increment of the crack, is obtained as a solution of the boundary value problem, Eq.(10.10). In order to make sure that the solution of Eq. (10.10) is the rate field, simple crack problems which have an analytic solution are studied. The problem setting is as follows: an infinite body, denoted by B , has a slit of length $2a$ which represents a crack, denoted by Ω , and simple boundary conditions are prescribed for B and Ω .

As the simplest problem, a case when the surfaces of Ω are traction-free and B is subjected to the far-field stress σ_{ij}^∞ is considered; see Fig. 10.3. Westergaard's potentials for this problem are

$$\begin{bmatrix} Z_I \\ Z_{II} \end{bmatrix} = \frac{z}{\sqrt{z^2 - a^2}} \begin{bmatrix} \sigma_{22}^\infty \\ \sigma_{12}^\infty \end{bmatrix} + \frac{i}{2} \begin{bmatrix} 0 \\ \sigma_{11}^\infty - \sigma_{22}^\infty \end{bmatrix}.$$

The stress intensity factors at the crack tip are

$$\begin{bmatrix} K_I \\ K_{II} \end{bmatrix} = \sqrt{\pi a} \begin{bmatrix} \sigma_{22}^\infty \\ \sigma_{12}^\infty \end{bmatrix}.$$

Due to the symmetry, both tips of Ω propagate and the amount of the propagation is the same. The corresponding potential rate is obtained by taking the derivative with respect to a , i.e.,

$$\begin{bmatrix} Z'_I \\ Z'_{II} \end{bmatrix} = az(z^2 - a^2)^{-3/2} \begin{bmatrix} \sigma_{22}^\infty \\ \sigma_{12}^\infty \end{bmatrix}, \quad (10.24)$$

and the stress intensity factor rate is obtained in the same manner, as

$$\begin{bmatrix} K'_I \\ K'_{II} \end{bmatrix} = \frac{1}{2} \sqrt{\frac{\pi}{a}} \begin{bmatrix} \sigma_{22}^\infty \\ \sigma_{12}^\infty \end{bmatrix}. \quad (10.25)$$

It is examined whether the solutions of the boundary value problem, Eq.(10.10), which uses the crack driving forces, coincide with the analytic solutions, Eqs. (10.24) and (10.25). First, it is shown that the potentials for a crack which has crack driving forces acting at both tips are given as $[Z'_I, Z'_{II}]^T$ of Eq. (10.24). When Ω is subjected to a pair of concentrated forces P_i at b , Westergaard's potentials are

$$\begin{bmatrix} Z_I \\ Z_{II} \end{bmatrix} = \frac{\sqrt{a^2 - b^2}}{\pi(z - b)\sqrt{z^2 - a^2}} \begin{bmatrix} P_2 \\ P_1 \end{bmatrix}. \quad (10.26)$$

The magnitude of the crack driving forces is determined from Eq. (10.26) since the stress intensity factors are given as $K_I = \sqrt{\pi a} \sigma_{22}^\infty$ and $K_{II} = \sqrt{\pi a} \sigma_{12}^\infty$. Thus, the potentials for the crack driving forces are computed as

$$\begin{aligned} \lim_{b \rightarrow a} \left(\frac{\sqrt{a^2 - b^2}}{\pi(z - b)\sqrt{z^2 - a^2}} \sqrt{\frac{\pi}{8(a - b)}} + \frac{\sqrt{a^2 - b^2}}{\pi(z + b)\sqrt{z^2 - a^2}} \sqrt{\frac{\pi}{8(a + b)}} \right) \begin{bmatrix} K_I \\ K_{II} \end{bmatrix} \\ = az(z^2 - a^2)^{-3/2} \frac{1}{\sqrt{\pi a}} \begin{bmatrix} K_I \\ K_{II} \end{bmatrix}. \end{aligned}$$

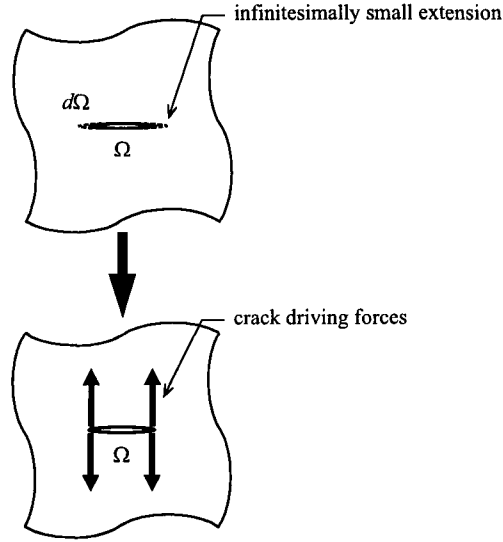


Fig. 10.3 An isolated crack in an infinite body.

In view of $K_{I,II}$, it is seen that the potentials in the right side coincide with Eq. (10.24).

Next, it is shown that the stress intensity factor rate of the crack path problem coincides with $[K'_I, K'_{II}]^T$ of Eq. (10.25). The stress intensity factor rate is computed by using stress fields near the crack tip. Indeed, on the x_1 -axis, $\sigma_{2i,1} + \sigma'_{2i}$ near the crack tip is

$$\sigma_{22,1} + \sigma'_{22} = \frac{K_I}{2a\sqrt{2\pi}} r^{-1/2} + \dots \quad \text{and} \quad \sigma_{21,1} + \sigma'_{21} = \frac{K_{II}}{2a\sqrt{2\pi}} r^{-1/2} + \dots$$

By definition, Eq. (10.14), the stress intensity factor rates are evaluated as

$$\begin{bmatrix} K'_I \\ K'_{II} \end{bmatrix} = \frac{1}{2a} \begin{bmatrix} K_I \\ K_{II} \end{bmatrix}.$$

In view of $K_{I,II}$, again, these rates coincide with Eq. (10.25).

As the next simple example, a crack with a pair of equilibrating concentrated forces acting on its surface is considered; see Fig. 10.4. The solution of this problem¹⁸ serves as Green's function for a body with one crack. If the magnitude and location of the forces are P_i and b , Westergaard's potentials are given by Eq. (10.26). When Ω propagates from, say, the right tip, the potential rates are obtained as

$$\begin{bmatrix} Z'_I \\ Z'_{II} \end{bmatrix} = \frac{1}{2\pi(z-a)\sqrt{z^2-a^2}} \sqrt{\frac{a+b}{a-b}} \begin{bmatrix} P_2 \\ P_1 \end{bmatrix}. \quad (10.27)$$

¹⁸A crack with distributed forces can be solved by superposition of these concentrated forces. Furthermore, a case when B has plural cracks can be solved by using a similar superposition; see [Horii and Nemat-Nasser (1983)].

The stress intensity factors at the right tip of Ω are

$$\begin{bmatrix} K_I \\ K_{II} \end{bmatrix} = \sqrt{\frac{a+b}{\pi a(a-b)}} \begin{bmatrix} P_2 \\ P_1 \end{bmatrix},$$

and hence the stress intensity factor rate is obtained as

$$\begin{bmatrix} K'_I \\ K'_{II} \end{bmatrix} = \frac{1}{8} a^{-\frac{3}{2}} \sqrt{\frac{a+b}{\pi(a-b)}} \begin{bmatrix} P_2 \\ P_1 \end{bmatrix}. \quad (10.28)$$

Note that if the crack has the extension da at the right tip, the potentials are given just by replacing a and b with $a+\frac{1}{2}da$ and $b-\frac{1}{2}da$, respectively, for $[Z_I, Z_{II}]^T$ of Eq. (10.26). Thus, Eq. (10.27) is computed as the derivative of this $[Z_I, Z_{II}]^T$ with respect to da at $da=0$. $[K'_I, K'_{II}]^T$ of Eq. (10.28) is computed in the same manner.

The crack path problem corresponding to the above problem is a case when the crack driving forces act at the right tip. By making use of Eqs. (10.24) and (10.26), the potentials for the crack driving forces are readily derived as

$$\lim_{c \rightarrow a} \frac{\sqrt{a^2 - c^2}}{\pi(z-c)\sqrt{z^2 - a^2}} \sqrt{\frac{\pi}{8(a-c)}} \begin{bmatrix} K_I \\ K_{II} \end{bmatrix} = \frac{1}{(z-a)\sqrt{z^2 - a^2}} \sqrt{\frac{a}{4\pi}} \begin{bmatrix} K_I \\ K_{II} \end{bmatrix}.$$

As is seen, these potentials coincide with Eq. (10.27). Furthermore, the stress intensity factor rates are computed from Eq. (10.14) as

$$\begin{bmatrix} K'_I \\ K'_{II} \end{bmatrix} = -\frac{1}{a} \begin{bmatrix} K_I \\ K_{II} \end{bmatrix},$$

which coincide with Eq. (10.28). Thus, it is shown that the solution of the boundary value problem coincides with the analytic solution of the rate field. Since the potential rates and the stress intensity factor rates are obtained, it is straightforward to compute the crack extension length for a given increment. The length is correctly computed by applying the solution of the boundary value problem, Eq.(10.10), since they coincide with the analytic solution.

The two crack path problems studied above are typical examples of unstable and stable crack growth; the stress intensity factor rates are positive in Eq. (10.25) and negative in Eq. (10.28). Hence, if a small far-field tensile stress is applied and produces $K'_I > 0$, the crack with the stress intensity factor rate of Eq. (10.25) grows in an unstable manner, while the crack with the stress intensity factor rate of Eq. (10.28) grows in a stable manner.

10.3.2 Use of numerical computation

The validity of using the boundary value problem for the rate field, Eq.(10.10), is examined by solving more complicated problems. It is possible to numerically compute the rate field by taking finite difference of field variables before and after a

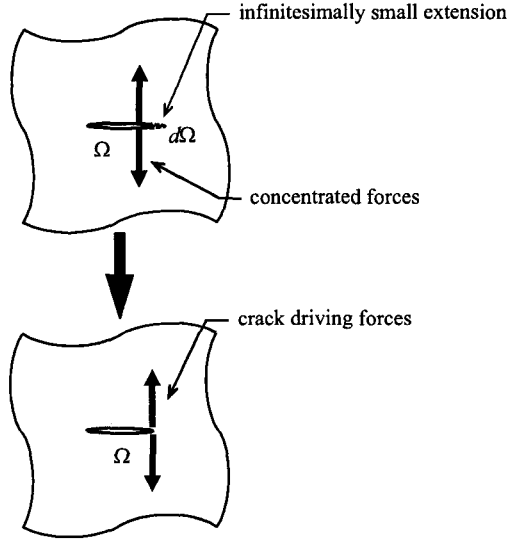


Fig. 10.4 An isolated crack subjected to a pair of concentrated forces.

small crack extension. For instance, u'_i , the first term in the perturbation expansion of displacement, is obtained by taking the following finite difference:

$$u_i'^N(\mathbf{x}) = \frac{u_i(\mathbf{x}, da) - u_i(\mathbf{x}, 0)}{da}, \quad (10.29)$$

where $u_i(da)$ and $u_i(0)$ are computed for $\Omega + d\Omega$ and Ω , respectively. Similarly, $K'_{I,II}$, the first term in the perturbation expansion of the stress intensity factor, is obtained as

$$K_{I,II}'^N = \frac{K_{I,II}(da) - K_{I,II}(0)}{da}. \quad (10.30)$$

Again, $K_{I,II}(da)$ and $K_{I,II}(0)$ are computed for $\Omega + d\Omega$ and Ω , respectively. Note that superscript N in Eqs. (10.29) and (10.30) emphasizes that the quantity is numerically computed. These $u_i'^N$ and $K_{I,II}'^N$ serve as a reference of the rate field, in examining the validity of the boundary value problem of the rate field. In this subsection, the solution of the boundary value problem is computed by means of BEM, and the displacement rate and the stress intensity factor rate obtained are denoted by u'_i and $K'_{I,II}$.

For simplicity, a case of $a \ll H$ is considered, and Ω is regarded as a small straight crack in a semi-infinite plane; see Fig. 10.5. The length and orientation of the crack are a and θ , the non-dimensionalized elastic moduli $E=1$ and $\nu=0.3$ are used, and the displacement vector on the right side is $[u_1^o, u_2^o]^T = [1.0 \times 10^{-2}, 0]^T$. Before examining the validity of the boundary value problem of the rate field, the accuracy of numerically computing the field of $\Omega + d\Omega$, which will be used to numerically calculate $u_i'^N$ and $K_{I,II}'^N$, is examined. In Fig. 10.6, the convergence of the numerical

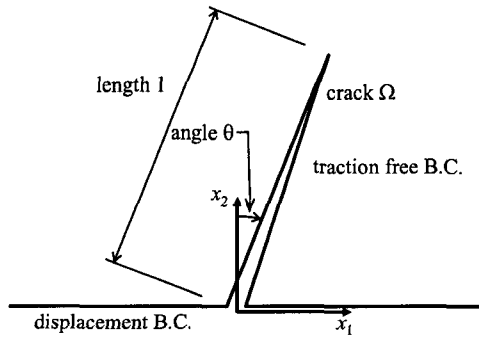


Fig. 10.5 A straight crack in a semi-infinite plane.

solution as the crack extension becomes zero is shown; a) and b) are for the tangential displacement gap and the Mode I stress intensity factor, respectively. The graphs show the change in $[u_2]$ and K_I as da goes to 0, and four cases of $\kappa=1, 2, 3, 4$ are plotted. As is seen, $[u_2]$ and K_I smoothly converge¹⁹ for all cases of κ 's. It is thus shown that the displacement rate and the stress intensity factor rates are numerically computed, and hence $u_i'^N$ and $K_{I,II}'^N$ of Eqs. (10.29) and (10.30) can be used as a reference solution with which u_i' and $K_{I,II}'$, the displacement field rate and the stress intensity factor rate obtained from the solution of Eq. (10.10), are compared.

Now, $[u_i']$ and K_I' which are obtained by solving the boundary value problem of the rate field, Eq.(10.10), are examined for the case of $\kappa \neq 0$. The boundary value problem accounts for the effects of curvature κ upon the displacement rate and the stress intensity factor rate explicitly. Thus, the correctness of this treatment is examined by comparing the rate fields which are numerically computed; see Eqs. (10.29) and (10.30). For simplicity, the orientation is set as $\theta=0$. Since κ does not appear in the boundary condition, the displacement rate should not depend on κ . In Fig. 10.7, the distribution of the normal component, $[u_1']$, is plotted, and that of numerically computed $[u_1'^N]$ is also plotted for comparison. As is seen, $[u_1']$ is in a good agreement with $[u_1'^N]$. To examine the dependency of $[u_i'^N]$ on κ , Fig. 10.8 plots $[u_1'^N]$ computed for $\kappa=0, 1, 2, 3, 4$. The relative differences of $[u_1'^N]$'s in these five curves are less than 1%, and it is seen that the dependence on $[u_1'^N]$ on κ is small. On the other hands, the stress intensity factor rate depends on κ ; it follows from $K_I \neq 0$ and $K_{II}=0$ that K_{II}' changes linearly with respect to κ , while K_I' does not depend on κ ; see Eq. (10.15). In Fig. 10.9, $K_{I,II}'$ as well as $K_{I,II}'^N$ is plotted for different values of κ ; a) and b) are for the Mode I and II stress intensity factor rates, respectively. The agreement of K_I' with $K_I'^N$ is less satisfactory; the relative difference is around 8% and K_I' overestimates the numerically computed $K_I'^N$. While the dependence of K_I' on κ is negligible, $K_{II}'^N$ tends to increase as κ increases.

¹⁹The convergence of $[u_2]$ and K_I supports the finite difference calculation of Eqs. (10.29) and (10.30); the accuracy of the calculation will be discussed later.

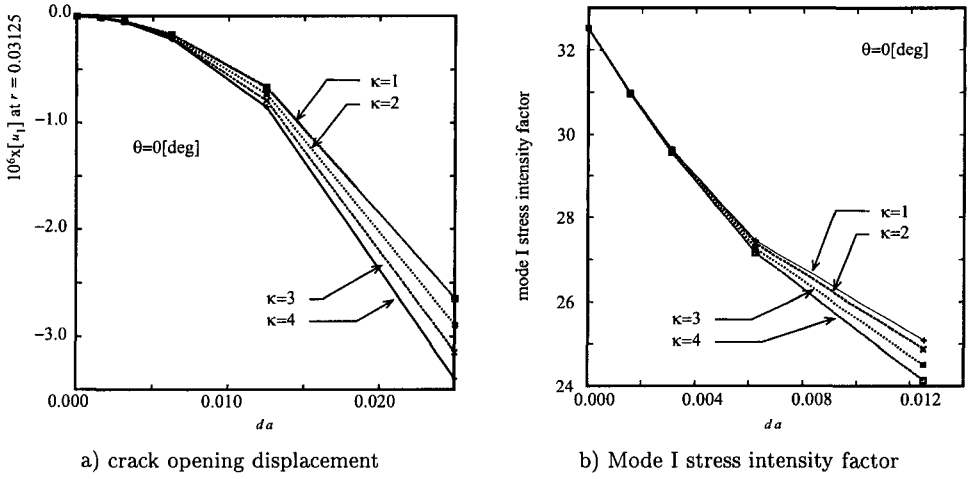


Fig. 10.6 Convergence of the numerical solution as the crack extension vanishes: $\theta = 0[\text{deg}]$.

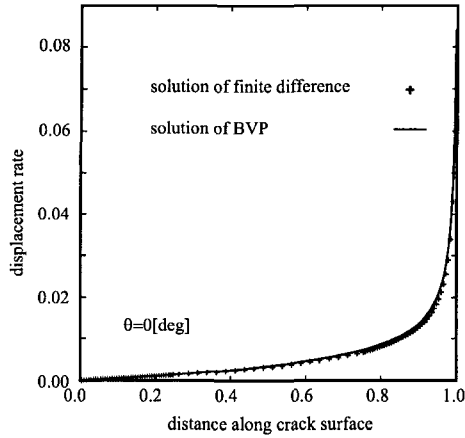


Fig. 10.7 The comparison of the displacement gap rates, $[u'_1]$ and $[u'_1]^N$: $\theta = 0[\text{deg}]$.

Hence, there are some errors in $K_I'^N$. From the comparison shown in Fig. 10.9, it is seen that the agreement between K_{II}' and $K_{II}'^N$ is better although the value of K_{II}' or $K_{II}'^N$ is much smaller than that of K_I' or $K_{II}'^N$; the relative error is 2% and K_{II}' underestimates $K_{II}'^N$. The independence of K_I' on κ and the linear dependence of K_{II}' on κ are clearly shown in Fig. 10.9, and K_I' and K_{II}' are in good agreement with the numerically computed values.

Next, the case of $\theta=45[\text{deg}]$ is considered, using the same numerical technique to compute the displacement rate and the stress intensity factor rate. Since the crack is inclined, the sliding displacement gap is generated. In Fig. 10.10 the profile of $[u'_i]$ and $[u'_i]^N$ for $\kappa=0$ is plotted; a) and b) are for $i=1$ and 2, respectively. The

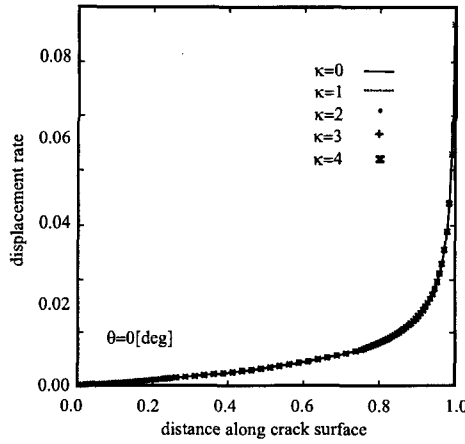


Fig. 10.8 Displacement gap rate $[u_1'^N]$ for various κ 's: $\theta = 0[\text{deg}]$.

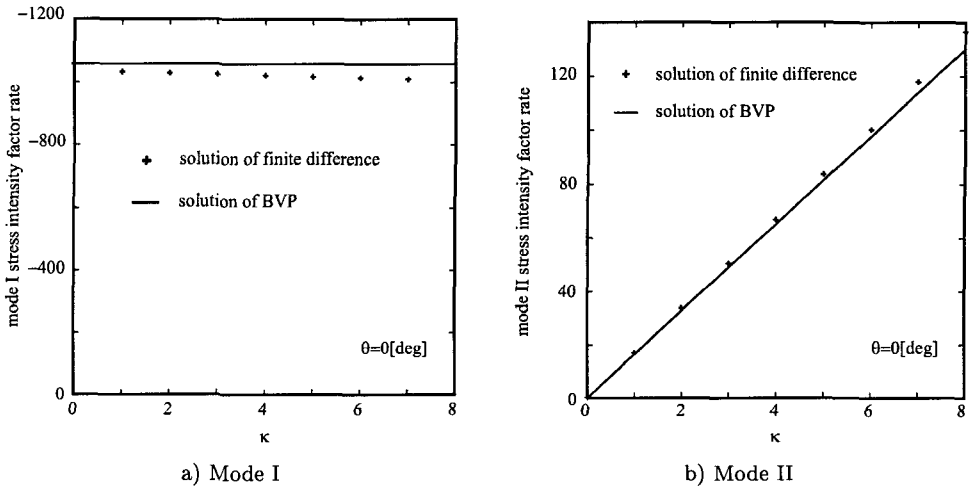


Fig. 10.9 The comparison of the stress intensity factor rates, $K'_{I,II}$ and $K'^N_{I,II}$: $\theta = 0[\text{deg}]$.

agreement of $[u'_1]$ and $[u'^N_1]$ becomes worse compared with the case of $\theta=0$, which is shown in Fig. 10.7); the relative differences remains within 5%. The agreement of $[u'_2]$ and $[u'^N_2]$ is similar to that of $[u'_1]$ and $[u'^N_1]$. It is seen that the rate of the normal and shear components of crack opening displacement gap, $[u'_n]$ and $[u'_s]$, is computed with the same accuracy as that of computing the rate by taking finite difference. In Fig. 10.11, the profile of $[u'^N_i]$ computed for $\kappa=0\sim 5$ is plotted; a) and b) are for $i=1$ and 2, respectively. Although it is not clearly seen in the figures, the agreement of $[u'_1]$ and $[u'^N_1]$ for different κ 's is worse compared with the case of $\theta=0$, which is shown in Fig. 10.8. The relative difference, however, is still within

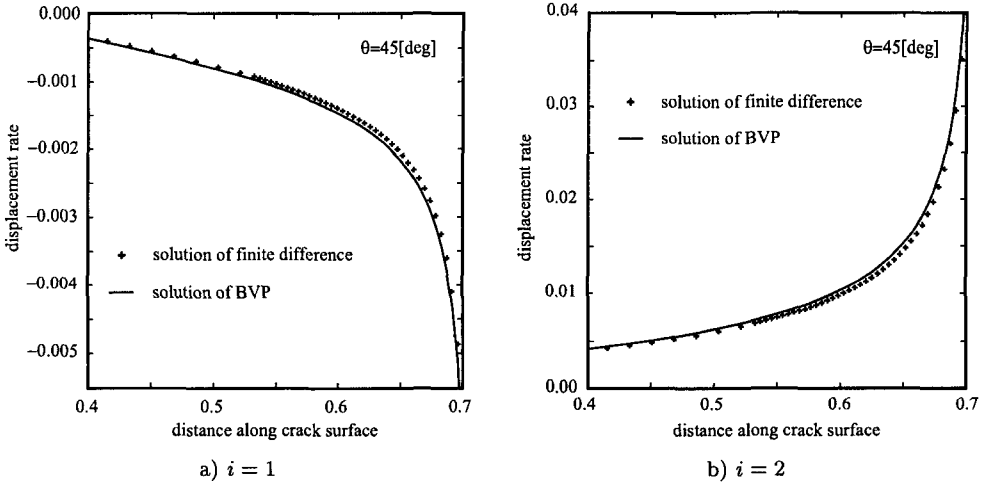


Fig. 10.10 The comparison of displacement gap rates, $[u'_i]$ and $[u_i'^N]$: $\theta = 45[\text{deg}]$.

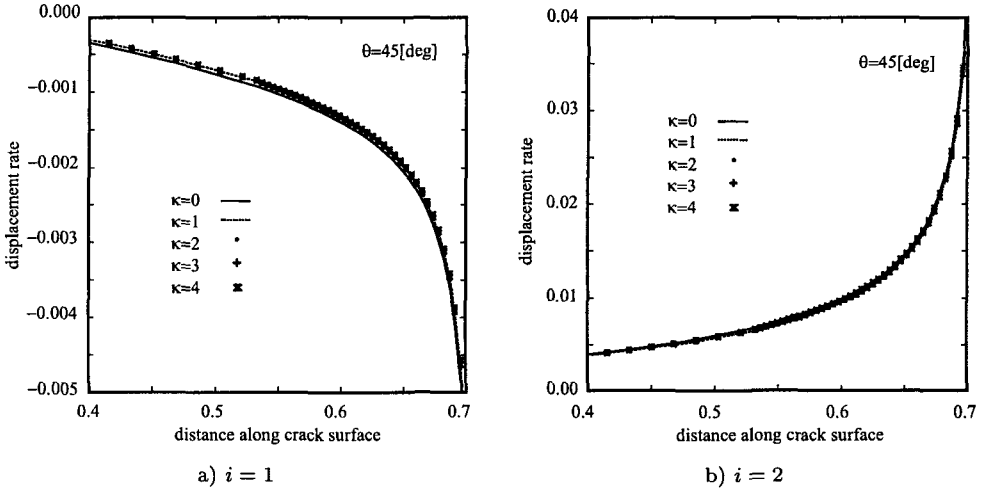


Fig. 10.11 Displacement gap rate $[u_i'^N]$ for various κ 's: $\theta = 45[\text{deg}]$.

1%. Due to the difference between u'_i and $u_i'^N$, the stress intensity factor rates, $K'_{I,II}$, does not agree well with $K'^N_{I,II}$. In Figs. 10.12, the comparison of $K'_{I,II}$ and $K'^N_{I,II}$ for several values of κ is plotted; a) and b) are for the Mode I and II stress intensity factor rates, respectively. The linear dependence of the rate on κ is seen for both $K'_{I,II}$ and $K'^N_{I,II}$. The slope with respect to κ , however, is different; $|dK'_I/d\kappa|$ is slightly smaller than $|dK'^N_I/d\kappa|$ while $|dK'_{II}/d\kappa|$ is larger than $|dK'^N_{II}/d\kappa|$.

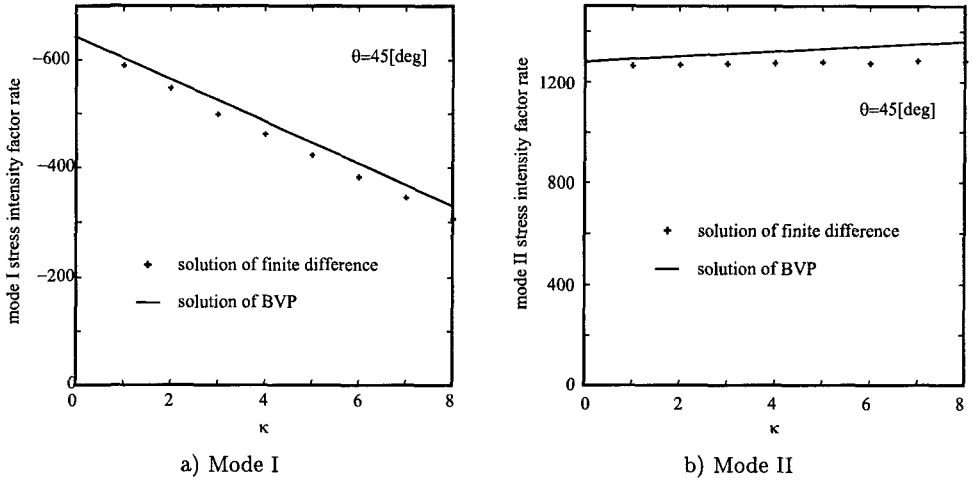


Fig. 10.12 The comparison of the stress intensity factor rates, $K'_{I,II}$ and $K'^N_{I,II}$: $\theta = 45^\circ$.

The agreement of $K'_{I,II}$ with $K'^N_{I,II}$ lies²⁰ in the range of the accuracy of the present numerical computation. Therefore, it is concluded that the boundary value problem for the rate field, Eq.(10.10), actually produces the rate field, the change in field variables caused by an infinitesimal extension of a crack. Even though Eq. (10.10) is a problem of higher singularity than an ordinary crack problem, it is possible to compute the solution by means of BEM. This supports the validity of solving the crack path problem by using the solution of the boundary value problem of the rate field.

10.4 Reproduction of Model Experiments

Applying the proposed analysis method of the crack path problem, the two model experiments of surface earthquake faults, which are carried out by [Bray *et al.* (1993)] and [Tani (1994)], are simulated. Both experiments are made for a two-dimensional reverse fault. As explained in Sec. 9.2, displacement is applied at the base of an experiment sample, and the growth of a shear band in the sample is observed. The material parameters used in the simulation are summarized in Table 10.1; the parameters are Young's modulus, E , Poisson's ratio, ν , and fracture toughness, G_c . It should be emphasized that these values are evaluated by a parametric study of reproducing the experiment results.

²⁰In these examples, finite difference of the first order is used to compute $[u'_i{}^N]$ or $K'^N_{I,II}$; see Eqs. (10.29) and (10.30). A more advanced technique will be required to compute $[u'_i{}^N]$ or $K'^N_{I,II}$ more accurately, although it is not needed for the present examination of the boundary value problem for the rate field.

Table 10.1 Material properties used for the simulation of the two experiments.

	$E[\text{kPa}]$	ν	$G_c[\text{Jm}^{-2}]$
Bray <i>et al.</i> (1994)	370	0.49	55
Tani (1994)	780	0.31	950

10.4.1 Simulation of model experiment of [Bray *et al.* (1994)]

First, [Bray *et al.* (1993)] is studied; [Bray *et al.* (1993)] examines reverse faults which appear in a plate of kaolinite-bentonite clay; the thickness of the plate is 30[cm]. In order to describe the boundary value problem for this experiment, t_n^o and t_s^o , normal and shear components of traction²¹ which uniformly acts on the crack surfaces, must be determined. The value of t_n^o and t_s^o is determined by carrying out a parametric study, i.e., the most suitable values are found by comparing the simulation results with the observed one. The determined values are tabulated in Table 10.2. As is seen, t_n^o is chosen to be much bigger than t_s^o , so that the displacement discontinuity slides rather than opens. As will be shown later, the propagation of the discontinuity can be reproduced in this simplified setting.

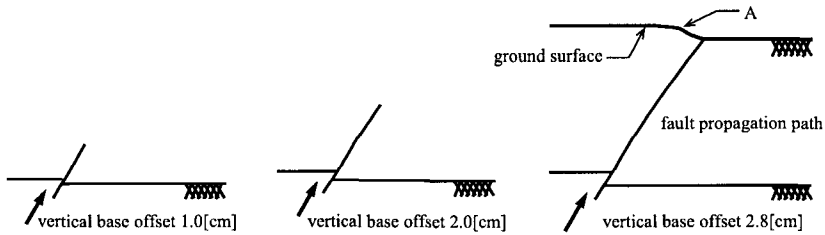
In Fig. 10.13, the configuration of the displacement discontinuity is shown for the case of $\theta=60[\text{deg}]$; three snapshots of the clay sample with a growing displacement discontinuity are presented. [Bray *et al.* (1993)] show contours of the maximum principal stress, to identify the configuration of a region with a large stress concentration. With the aid of suitably chosen $t_{n,s}^o$, the configuration of the numerically computed discontinuity is in good agreement with that of the shear band which is observed in the experiment. In Fig. 10.14, the height of the discontinuity tip with respect to the base movement is plotted. It is seen that as the discontinuity tip approaches the top surface, the dip angle tends to gradually decrease. This is also observed by [Bray *et al.* (1993)]. It is shown in their computational analysis that the deformation of the top surface increases significantly as the discontinuity reaches there. In Fig. 10.15, the relation between the vertical displacement on the top surface and the base movement is plotted; the vertical displacement is measured at a point A, which is pointed out in Fig. 10.13. The base displacement that makes the discontinuity appear on the top surface is around 3.0[cm], i.e., 10% of the sample thickness. This agrees well with the value of 2.7[cm] computed by [Bray *et al.* (1993)].

It should be emphasized that the agreement of the simulation results with the experimental observation data owes to the choice of $t_{n,s}^o$, which is determined by the parametric study, as well as other material parameters shown in Table 10.1. However, the agreement suggests that modeling a shear band as a displacement discontinuity which transmits some traction is acceptable. Since the configuration of

²¹The proposed analysis method can be applicable to a case when a relation between the traction and the displacement discontinuity is non-linear. In this book, however, it is assumed that traction acting on the crack surfaces are uniform, to reduce numerical computational efforts.

Table 10.2 The values of t_n^o and t_s^o used for the simulation of [Bray *et al.* (1994)].

normal comp. [Pa]	shear comp. [Pa]
123	14

Fig. 10.13 The configuration of displacement discontinuity. The comparison with [Bray *et al.* (1994)] is made.

the growing discontinuity is efficiently calculated by solving the crack path problem that uses the boundary value problem of the rate field, this modeling of a shear band is at least worth examining.

10.4.2 Simulation of model experiment of [Tani (1994)]

Next, model experiments done by [Tani (1994)] are examined; [Tani (1994)] studies reverse faults in sand specimens changing the dip angle. In his experiment, Toyoura sand is used. Like the previous example, $t_{n,c}^o$ cannot be determined for these model experiments. A parametric study is thus carried out to determine $t_{n,s}^o$ by simulating the experimental data. The value of $t_{n,s}^o$ thus depends on the dip angle, denoted by θ , and they are tabulated in Table 10.3 for $\theta=15, 45$ and 75 [deg].

The configuration of the displacement discontinuity, which is computed by solving the crack path problem, is shown in Fig. 10.16; a), b) and c) are for $\theta=15, 45$ and 75 [deg], respectively, and snapshots of the sand sample with growing displacement discontinuity are presented. These configurations agree well with the observed ones. In particular, for the case of $\theta=15$ [deg], the dip angle of the displacement discontinuity increases as it grows, and the discontinuity bends over the up-thrown block as it approaches the top surface. This typical configuration is pointed out by [Tani (1994)], since it is similar to an actual surface earthquake fault. For the case of $\theta=75$ [deg], the dip angle of the displacement discontinuity tends to decrease. It should be mentioned that in these examples, the growth of the discontinuity and the deformation of the top surface are accelerated as the discontinuity approaches the top surface; this tendency is similar to Figs. 10.14 and 10.15.

As mentioned several times, the agreement of the numerical simulation results with the experimental observation data is due to the choice of $t_{n,s}^o$ which is determined by the parametric study; the different values are used when the dip angle

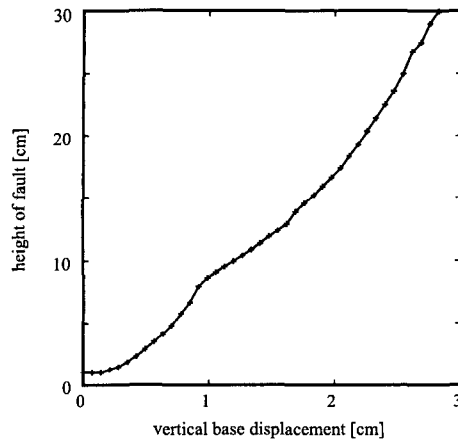


Fig. 10.14 Relation between the crack tip height and the base movement.

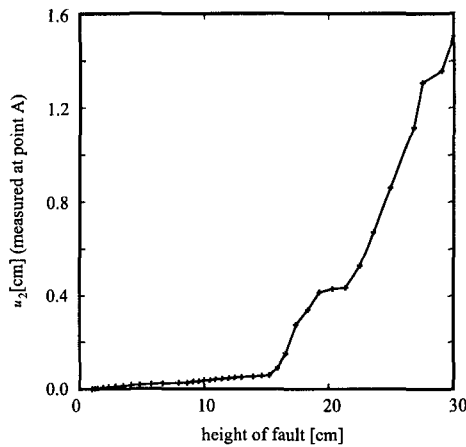


Fig. 10.15 Relation between vertical displacement on top surface and base movement.

θ is changed, as shown in Table 10.3. This is because the non-linearity of sand is represented by the traction that acts on the discontinuity surfaces, even though the traction is assumed to be uniform. Still, the agreement suggests the validity of modeling a shear band as a displacement discontinuity which transmits some traction. Since the shear band is caused by strain localization, the surrounding region is in elastic state. The effect of plasticity on strain localization appears only on the narrow shear band. Therefore, the displacement discontinuity model is able to capture strain localization, provided that the traction transmitted through the discontinuity surfaces is known. Sharp strain localization is easily simulated in this modeling; the displacement discontinuity actually has zero width, and smoothly curving growth is

Table 10.3 The value of t_n^o and t_s^o used for the simulation of [Tani (1994)].

θ [deg]	normal comp. [Pa]	shear comp. [Pa]
15	567	14
45	623	32
75	418	29

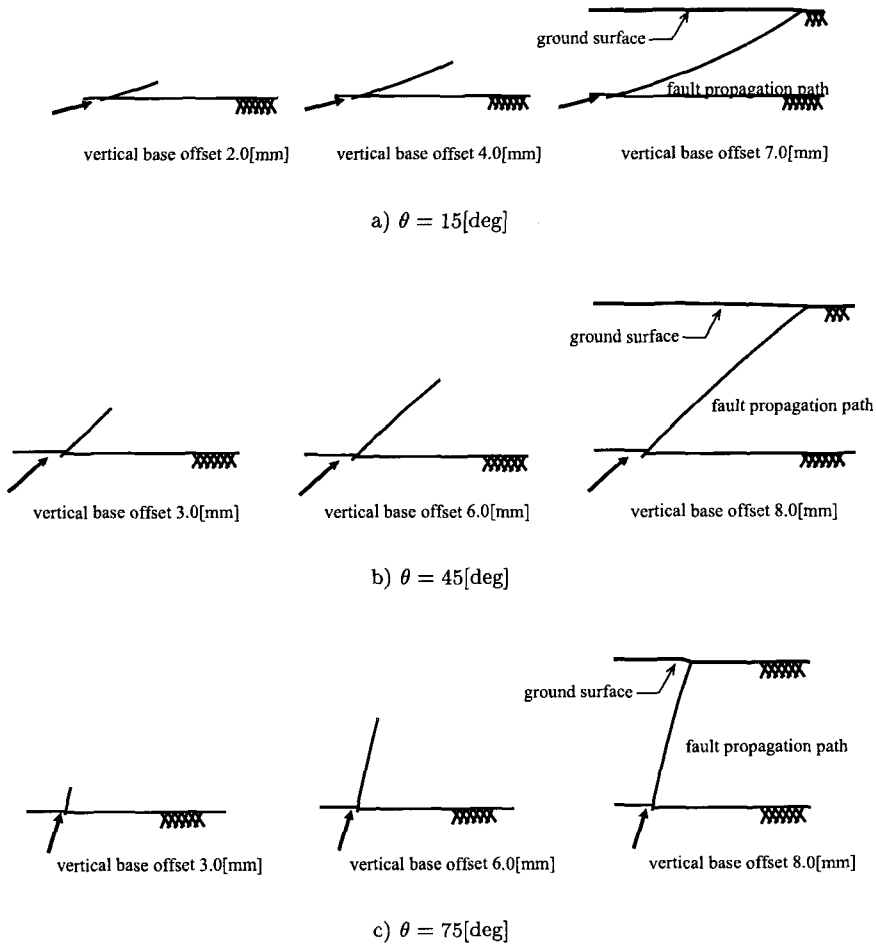


Fig. 10.16 The configuration of displacement discontinuity. The comparison with [Tani (1994)] is made.

computed by solving the crack path problem which uses the boundary value problem of the rate field. The BEM simulation of a growing displacement discontinuity is thus an alternative to the FEM simulation that uses non-linear elasto-plasticity, which includes the NL-SSFEM simulation shown in the preceding chapters. How-

ever, it must be emphasized that the traction²² on the discontinuity surfaces is pre-determined in order to apply the BEM simulation to actual surface earthquake fault problems.

²²Traction on the crack surfaces represents all effects of plastic deformation. It can be replaced as a certain relation between displacement gap and traction; if plastic strain is evaluated by the displacement gap, this relation will be derived from a non-linear elasto-plastic constitutive relation.

PART 4

ADVANCED TOPICS

In this part, we present advanced topics in computational earthquake engineering. These topics are related to computer science and technology, rather than solid continuum mechanics. They share a common target, the promotion of earthquake preparedness in urban areas.

The first topic is integrated earthquake simulation, i.e., a seamless simulation of all phases of an earthquake, namely, wave propagation, structure responses, and human actions against earthquake disasters. Such a simulation will be of primary importance for earthquake engineering; all elements of earthquake hazards and disasters are extensively studied, and integrating these elements into one system is realizable with the aid of advanced computer science and technology. We explain this integral earthquake simulation in Chapter 11.

The second and third topics are technologies for local government officials who are responsible for earthquake hazard mitigation in urban areas; knowledge of advanced earthquake engineering will help them make and enforce better mitigation plans. In Chapters 12 and 13, we explain the visualization of responses for all structures in a target area and the standardization of earthquake resistant design codes. The visualization provides a reliable prediction of earthquake hazards and the standardization is useful for understanding many codes.

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Chapter 11

Integrated Earthquake Simulation

Before discussing integrated earthquake simulation, the background of this simulation is briefly explained. Past huge earthquakes which have hit metropolises result in catastrophic disasters. The disasters range from human casualties to damage to personal estates or social infrastructures, and they may include direct and indirect losses in economic activities. To achieve higher safety against a future earthquake, it is essential to provide reliable predictions of earthquake disasters¹ which are likely to occur for a given earthquake scenario; see Appendix A.2. It is believed that such predictions contribute to earthquake preparedness, i.e., making more efficient disaster mitigation plans which include the development of an effective crisis management system.

A more detailed prediction of earthquake disasters contributes to a drastic decrease in *cascade effects* of the earthquake disasters, i.e., spatial and temporal propagation of damage and malfunctioning of mutually depending social systems. The cascade effects involve various elements of the social systems, such as structures, transportation systems, lifelines, energy systems, or communications. Thus, what is needed is an earthquake disaster prediction which is detailed as well as comprehensive and inclusive.

A candidate for a tool for predicting earthquake disasters is numerical simulation. The target of such numerical simulation is all phases of a possible earthquake, and it must achieve high spatial and time resolution to make a detailed prediction. At this moment, a hazard map is used as an estimation of strong ground motion distribution, and earthquake disasters can be predicted by applying results of statistical analysis² on past earthquake disasters. Numerical simu-

¹In this book, earthquake hazards (or seismic hazards) mean a possible distribution of strong ground motion, while earthquake disasters stand for a possible damage to structures; see, for instance, [Bommer (2002)] and [Bommer *et al.* (2003)], for a list of references to recent studies on earthquake hazard assessment. It should be emphasized that records of past earthquake disasters are essential for the earthquake hazard assessment; see, for instance, [Mahin (1998)] for records of Northridge Earthquake, 1994. Malfunctioning of structures and their facilities should be included in earthquake disasters, although they are omitted in this book.

²A *fragility curve* is a typical example of statistical analyses of past earthquake disaster records; see, for instance, [Gardoni *et al.* (2003)]. The curve gives the probability of damage or failure for a structure as a function of a certain index of strong ground motion.

lation ought to provide further predictions; for instance, an estimation of variability in earthquake disasters due to different earthquake scenarios or the rational evaluation of the cascade effects on mutually dependent social systems. Several research activities are carried out to develop a simulation system for the prediction and mitigation of earthquake hazards; for instance [Kitano *et al.* (1999); Tadokoro *et al.* (2000)] intends to establish a computer system in which various actions against earthquake hazards are simulated; see also [NIED-EDM (2005)].

It is possible to use the macro-micro analysis method explained in Chapters 5 and 6 for the prediction of strong ground motion distribution when an earthquake scenario is given. The prediction of high spatial and temporal resolution is achieved by means of the micro-analysis. It is also possible to use numerical analysis³ methods, which are developed for the purpose of earthquake resistant design, for the prediction of damage to each structure; structure responses are computed if a predicted strong motion is input to a suitable computer model of the structure.

These are the background of integrated earthquake simulation. This chapter presents an integrated earthquake simulator (IES), which serves as a platform where the integrated earthquake simulation is carried out. A key issue of IES is modeling, since numerical analysis methods are well developed. IES develops a methodology of constructing a computer model for ground structures and residential buildings, by using a Geographic Information System (GIS). An example of applying IES to an actual town is presented; earthquake hazards and disasters are simulated for a computer model of the whole town.

11.1 System of Integrated Earthquake Simulation

IES is a simulation system for the following three phases of an earthquake: 1) generation and propagation of an earthquake wave; 2) response and damage of a structure subjected to strong ground motion; and 3) human or social actions against earthquake disasters. Three numerical simulations are applied to each phase, i.e., the earthquake simulation, the structure response simulation and the action simulation. The basic characteristics of these numerical simulations are summarized as follows.

- i) earthquake simulation: An earthquake wave is synthesized according to a scenario of a possible fault mechanism. The propagation of the wave through the crust is computed, and the amplification of the wave near the ground surface is computed by accounting for the three-dimensional topographical effects and the non-linear properties of surface soil layers.
- ii) structure response simulation: Responses of all structures which are located in a target area are computed. Structures range from residential buildings, infras-

³Dynamic analysis becomes a common practice in designing a large-scale structure, and numerical analysis methods for various kinds of structures have been developed.

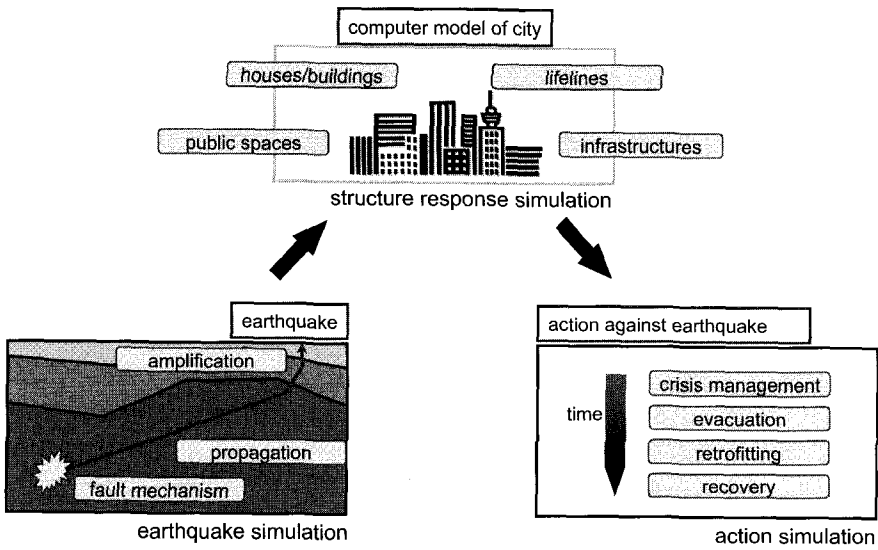


Fig. 11.1 The overview of IES.

structures which are categorized as concrete, steel or geotechnical structures, and lifelines of energy, transportation, communication or water supply. Since there are various kinds of structures, a suitable numerical analysis method should be chosen depending on the type of structure.

- iii) action simulation: Human behavior is simulated; included are evacuation from damaged structures, crisis management, and the analysis of recovery plans.

The overview of IES is presented in Fig. 11.1. While each simulation has its own purpose, the three simulations are related to each other, i.e., the earthquake simulation provides strong ground motion distribution to the structure response simulation; for each building, the strong ground motion at its site is used as an input wave. Structure damage which is computed by the structure response simulation provides an initial condition for the action simulation.

The basic structure of IES is shown in Fig. 11.2. It consists of GIS and the three groups of numerical simulations. GIS provides data to construct computer models, i.e., underground structure data for the earthquake simulation and structure data for the structure response simulation. Results of the numerical simulation are stored in GIS; the earthquake simulation and the structure response simulation provide predictions of earthquake hazards and disasters, respectively, for a given earthquake scenario.

The usage of IES is summarized in Fig. 11.3. For a given earthquake scenario, IES executes the sequence of the three simulations seamlessly, communicating with GIS on data gathering and storage. The results of the simulations are thus regarded as the consequence of the earthquake scenario. The consequence is *objective* in the

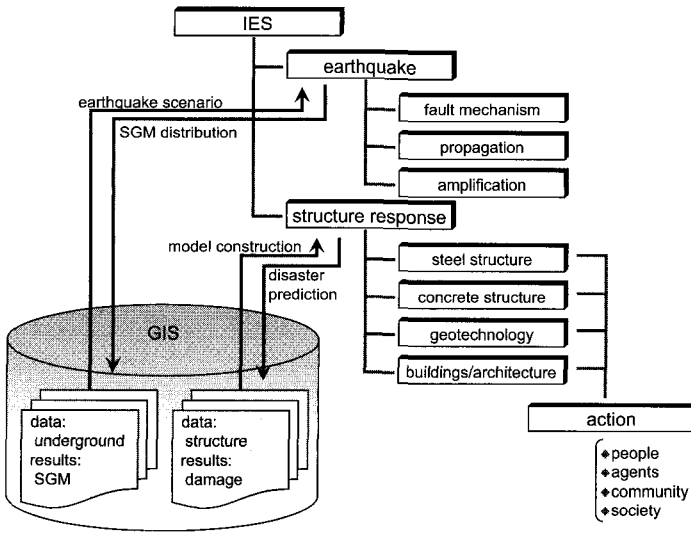


Fig. 11.2 The basic structure of IES.

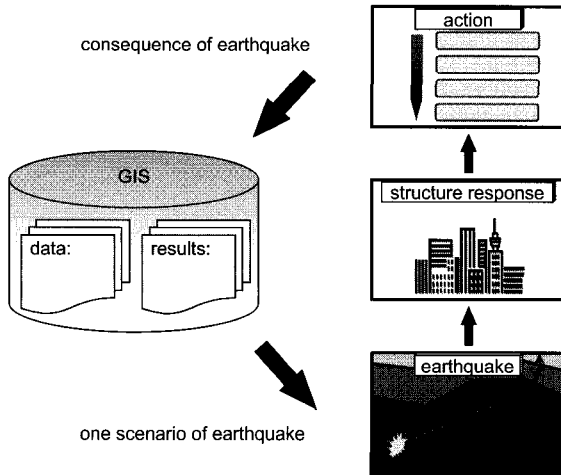


Fig. 11.3 The usage of IES.

sense that no additional consideration or evaluation is needed to predict earthquake hazards and disasters. This objectivity does not mean accuracy and reliability, although more quantitative data and more sophisticated numerical analysis will achieve more accurate and reliable simulation.

In summary, IES is a simulation system which inputs information of a target area and an earthquake scenario and which outputs predictions of strong ground motion distribution, structure damages, and human actions, as the consequence of

Table 11.1 Tasks of a kernel to control key elements of IES.

element	task
GIS/library	acquire data for ground layers and structures, and store simulation results such as strong ground motion distribution, structure damages and human actions
simulation programs	provide data for a computer model and input strong ground motion, executes simulations of structure responses, and receive simulation results
visualization tools	provide results of the structure response simulation, such as behavior and possible damage, of all structures located in a target city, and provide results of action simulation

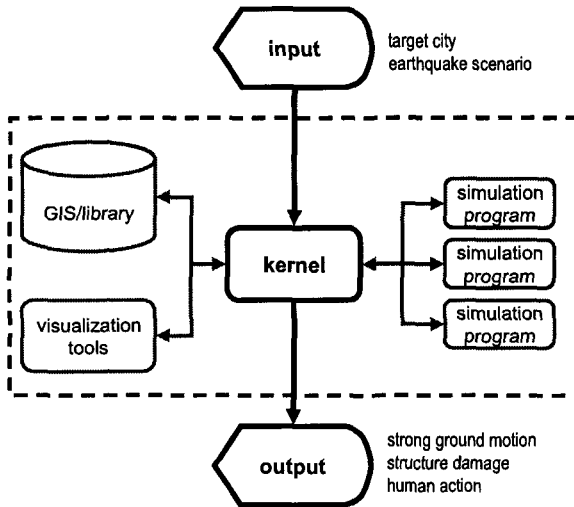


Fig. 11.4 A kernel of IES.

the input earthquake scenario; see Fig. 11.4. In this figure, IES is presented as a system which consists of a kernel, GIS, simulation⁴ programs, and visualization tools. The kernel is the key element of IES which controls IES itself. It is the kernel that actually communicates with GIS, which includes libraries, i.e., databases for structure members and material properties. The kernel executes simulation programs providing data and receiving results, and transforms the simulation results in a form which is applicable to various visualization tools. The visualization tools generate three-dimensional static images or animations; the length scales and view-points of the images and animations can be changed arbitrarily. The amount of the simulation results is huge, and such visualization tools are needed to present them

⁴Besides the fact that the earthquake simulation requires huge computer memory, parallel computing is suitable for IES since a simulation program for the structure response simulation is executed for each structure.

in an understandable manner. These tasks assigned to the kernel are summarized in Table 11.1.

11.2 GIS

For a metropolis, GIS's of many kinds⁵ have been developed. The utilization of GIS's is being accelerated⁶ for various purposes, as the quality and quantity of data stored in GIS's increase. Some GIS's have a set of borehole data which are measured at several sites, and others have data for buildings and structures such as the location, structure type and height. Thus, IES can construct a computer model for ground structures and a set of buildings and structures in a target area, with the aid of these GIS's, although it must be taken for granted that some uncertainty remains in modeling and that only a simple model is made for structures.

Table 11.2 summarizes the data which are required for constructing a computer model of numerically simulating earthquake hazards and disasters and the data which are currently available in GIS's. As is seen, there are discrepancies between the required data and the available data. A computer model for the structure response simulation needs data about structure properties, such as the dimensions and material properties of structure members and the configuration of the entire structure. These data are not available. Also, the data required to construct a ground structure model are quite limited; for instance, the number of borehole data is very small.

There is a trend that data which are used in designing a high-rise building or a larger-scale structure are stored in a common digital form⁷ and will be available, even if not freely, for the public. The development of an automobile navigation system is going much faster than expected, due to the strong demand of users, and more detailed data about structures with better quality are being stored in the GIS of this system. It is certainly true that the current GIS's have limited applicability to IES. However, GIS's which are suitable for the IES modeling will be available in the near future.

11.3 Construction of Computer Model

This section presents a methodology of constructing a computer model of ground structures and residential buildings by making use of data which are stored in

⁵A GIS is designed to fulfill its specific purpose. Thus, combining several GIS's is needed to construct computer models for a target city.

⁶For instance, in Japan, already developed are a GIS which stores grid data of ground elevation with 50[m] grid distance and a GIS which stores high resolution image data of buildings and structures in several cities.

⁷This is due to the development of a computer-aided-design (CAD) system or compute-aided-engineering (CAE).

Table 11.2 A summary of data required for IES and data currently available in GIS.

a) ground structure		
	required	available
configuration	boundary depth	elevation data
	soil type	borehole data
material properties	density & wave velocity non-linear properties	database for soil type- material parameter relation

b) structure		
	required	available
basic	location structure type	digitized perspective view
configuration	structure dimension member dimension	height data
material properties	elastic property non-elastic property	none

available GIS's. As mentioned in Sec. 11.2, the data stored in the GIS's are limited, and hence IES is aimed at constructing as reliable a model as possible, rather than a complete computer model.

11.3.1 Construction of ground structure model

This subsection focuses on modeling ground structures beneath urban areas since the target of IES is urban areas. A ground structure model is a set of non-stratified soil layers, and the configuration and material properties need to be determined for each layer. In general, however, the estimation of layer configuration is difficult mainly because borehole sites are sparse. Furthermore, the quality of borehole data is sometimes poor; a set of neighboring borehole data are not always consistent. The inconsistency means that the sequence, not the thickness, of distinct layers differs from site to site. A schematic view of inconsistent borehole data is shown in Fig. 11.5; each datum has its own sequence of soil layers, and the sequence is not the same even for neighboring borehole data.

First, developed is a methodology⁸ of estimating the sequence of soil layers for some domain where several borehole data, which may be inconsistent with each other, are available. The methodology must be robust so that inconsistency in the borehole data set is automatically corrected. Such robustness is achieved in *geoinformatics geology*, which studies geology by means of informatic technology. According to geoinformatics geology, the following procedures of estimating the soil layer sequence are developed:

⁸There are various methods to determine a consistent layer sequence. Most of them are applicable when a relatively few boring sites are available; see, for instance, [Journel (1989)]. These methods are not suitable for a set of many but inconsistent borehole data.

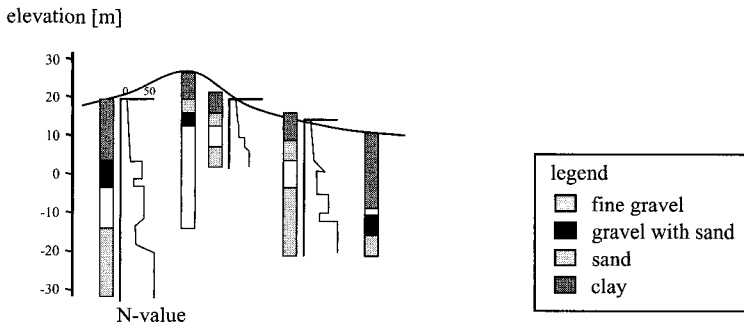


Fig. 11.5 Inconsistency of adjacent borehole data. The sequence of soil layers is different for five borehole data.

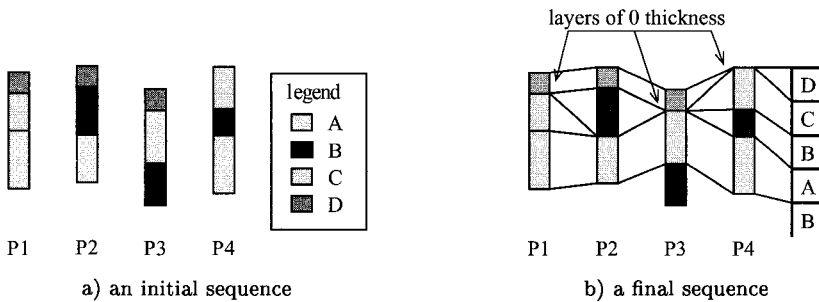


Fig. 11.6 An geoinformatics geology based method of determining a soil layer sequence.

- i) Pick up one borehole datum, and set a reference layer sequence from this datum.
- ii) Pick up another borehole datum, and compare the layer sequence with the reference.
 - ii.i) if two sequences match, go to another borehole datum;
 - ii.ii) if two sequences do not match, combine them to make a new reference layer sequence that is consistent with both of them.
- iii) Repeat the procedure ii) until a reference layer sequence which is consistent with all borehole data is constructed.
- iv) Finalize the reference layer sequence by picking up major layers and omitting other thinner layers.

A major layer means a layer whose average thickness is larger than a pre-determined value, say, 1[m]. Note that the thickness of some layers may be set to zero so that the layer consistency holds when a new reference layer sequence is made.

The above procedures are applied to four borehole data shown in Fig. 11.6 as an example. First, the borehole datum P1 is picked up, and the reference layer sequence is set as D-C-A; see Fig. 11.6a). By comparing this sequence with the borehole datum P2, the reference sequence is modified as D-C-B-A, and the thickness of the

B layer in P1 and the C layer in P2 is set to 0. By repeating this procedure, the reference layer sequence is determined as D-C-B-A-B, which is consistent with the four borehole data. The average thickness of the last B layer is small, and this layer is omitted. Thus, the reference sequence is finalized as D-C-B-A; see Fig. 11.6b).

When the reference layer sequence is determined, it is straightforward to construct a three dimensional ground structure by interpolating the borehole data even if they are mutually inconsistent. A *grid algorithm*, in which continuous distribution of thickness is replaced by discontinuous grid values, is adapted for interpolation. This algorithm is explained by using a distribution of, say, the ground elevation f in a domain D as an example. The elevation is measured at K points, denoted by $\{x^k\}$, and a set of data $\{f^k|k=1, 2, \dots, K\}$ are available. In order to interpolate $\{f^k\}$, the grid algorithm first introduces fictitious grid values of f , i.e., $f^{n_1 n_2}$ at a grid point of $(x_1^{n_1 n_2}, x_2^{n_1 n_2})$. In a square domain surrounded by four grid points, $(x_1^{n_1 n_2}, x_2^{n_1 n_2})$, $(x_1^{n_1+1 n_2}, x_2^{n_1+1 n_2})$, $(x_1^{n_1+1 n_2+1}, x_2^{n_1+1 n_2+1})$ and $(x_1^{n_1 n_2+1}, x_2^{n_1 n_2+1})$, a value at a point x is interpolated as

$$f(x) = w_1(x)f^{n_1 n_2} + w_2(x)f^{n_1+1 n_2} + w_3(x)f^{n_1+1 n_2+1} + w_4(x)f^{n_1 n_2+1},$$

where $w_{1 \sim 4}$ are bi-linear weight⁹ functions; for instance, w_1 is

$$w_1(x) = \frac{(x_1^{n_1+1 n_2+1} - x_1)(x_2^{n_1 n_2} - x_2)}{(x_1^{n_1+1 n_2+1} - x_1^{n_1 n_2})(x_2^{n_1 n_2} - x_2^{n_1 n_2+1})};$$

a graphical image of the weight functions used in the above equation is presented in Fig. 11.7. The grid algorithm finds a set of $\{f^{n_1 n_2}\}$ which minimizes

$$J(\{f^{n_1 n_2}\}) = \int_D |\nabla f|^2 ds + \rho \int_D |\nabla^2 f|^2 ds, \quad (11.1)$$

subject to constraints of

$$f(x^k) = f^k \quad \text{for } k = 1, 2, \dots, K. \quad (11.2)$$

Here, $|\nabla f|^2 = (f_{,1}^2 + f_{,2}^2)$ is the square of the gradient¹⁰ of f and $|\nabla^2 f|^2 = f_{,11}^2 + 2f_{,12}^2 + f_{,22}^2$ corresponds to the inverse of the curvature of f ; as $|\nabla f|^2$ and $|\nabla^2 f|^2$ become smaller, the graph of f becomes flatter. Also, ρ is a weight¹¹ of the curvature measure with respect to the gradient measure, and a suitable value is chosen. The grid algorithm applies a multi-grid method to determine $\{f^{n_1 n_2}\}$ efficiently; this method first uses a coarse grid to discretize f and then gradually increases the fineness of the grid. When the m -th grid values are denoted by $\{f^{(m)n_1 n_2}\}$ with n_i ranging from 1 to $N_i^{(m)}$, the solution of minimizing f of Eq. (11.1) subjected to Eq. (11.2) is given as the limit of $\{f^{(m)n_1 n_2}\}$ as m and $N_i^{(m)}$'s go to infinity. The

⁹Note that a weight w_i is the relative area of a rectangular which is determined by the point (x_1, x_2) with respect to the whole domain.

¹⁰These $|\nabla f|^2$ and $|\nabla^2 f|^2$ are invariant in the sense that the value does not change if another coordinate system is used.

¹¹The limit of $\rho \rightarrow \infty$ corresponds to the algorithm of minimizing curvature, which is used to determine an underground structure model which is explained in Sec. 6.3.

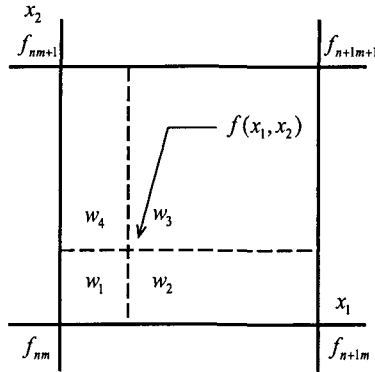


Fig. 11.7 A weight function used in the grid algorithm.

grid algorithm thus finds a good approximate solution of f using a suitably large m and $N_i^{(m)}$.

11.3.2 Construction of residential building model

A computer model which is used in dynamic structure response analysis ranges from a simple linear model such as a single-degree-of-freedom (SDOF) system to a sophisticated non-linear model¹² which is analyzed by mean of FEM. A linear model is enough to roughly examine the structure safety; the rough examination means that it checks whether the structure remains in an elastic response regime or not, by computing, say, the maximum deformation.

As mentioned in Sec. 11.2, some but limited data regarding to structure properties are stored in currently available GIS's; see Table 11.2. In view of the quality and quantity of these available data, it is realistic to construct a linear model for a residential building. However, numerical simulation will be made for all residential buildings which are located in a target area. Residential buildings include wooden houses, reinforced concrete buildings or steel reinforced concrete buildings. A method of constructing a linear multi-degree-of-freedom (MDOF) system for a residential building with the aid of available GIS data is developed.

First, the analysis¹³ of a linear MODF system is summarized to clarify the

¹²A non-linear model is required if a structure reaches a non-linear regime and has a possibility of local failure or total collapse. CAD data are useful in constructing such a non-linear model; the data include dimensions and material properties of all members, joints and connections.

¹³An SDOF or MDOF system is often used in earthquake engineering for dynamic response analysis of a structure; see [Craig (1981)], [Irvin (1986)] and [Chopra (1995)] as a general textbook on the application of these systems to dynamic structure response analysis; see also [Casolo (1998)], [O'Rourke and Liu (1999)] and [Moghaddam and Mohammadi (2001)] for recent studies. While an MDOF system appears simpler than an FEM model, the analysis of a well constructed MDOF system is capable of reproducing complicated structure responses; see [Mylonakis (2001)], [Pampanin *et al.* (2001); Pampanin *et al.* (2003)], [Faria *et al.* (2002)] and [Raheem and Hayashikawa (2003)]; see also [Ueng *et al.* (2000)] and [Usami *et al.* (2004)] for the application of the MDOF

requirement of a computer model. For a given time series of strong ground motion, z , the MDOF system has a response for each DOF, and all responses are represented by a vector $[U]$ which is a function of time. This $[U]$ is obtained by solving the following differential equation:

$$[M][\ddot{U}(t)] + [C][\dot{U}(t)] + [K][U(t)] = \ddot{z}(t)[M][1]; \quad (11.3)$$

$[M]$, $[C]$ and $[K]$ are mass, damping and stiffness matrices, respectively; $[1]$ stands for¹⁴ a vector $[1, 1, \dots, 1]^T$, and initial conditions of $[U]=[0]$ and $[\dot{U}]=[0]$ at $t=0$ are prescribed.

There are three matrices in Eq. (11.3), namely, $[M]$, $[C]$ and $[K]$, and they describe the mechanical properties of the MDOF system as a computer model of a residential building. It is impossible to fully determine these matrices using data stored in the current GIS. However, rewriting Eq. (11.3), $[U]$ can be approximately computed without determining these matrices rigorously. To this end, a *modal analysis* is applied. This method is briefly explained. A mode is given as $[U]=[\Phi] \exp(\omega t)$ where ω is a (natural) frequency and $[\Phi]$ is a mode vector. Substitution of this $[U]$ into Eq. (11.3) with $[C]$ and z being set $[C]=[0]$ and $z=0$ yields

$$(-\omega^2[M] + [K])[\Phi] = [0].$$

If the dimension of the matrix is N , there are N non-trivial ω 's and $[\Phi]$'s which satisfy this homogeneous equation; indeed, ω 's are the roots of $\det(\omega^2[M] - [K])=0$. The n -th root of ω and the associated¹⁵ $[\Phi]$ are denoted by ω_n and $[\Phi_n]$, respectively. Since $[\Phi_n]$'s form base vectors, $[U]$ can be expressed as a linear combination of them, i.e., $[U]=\sum q_n[\Phi_n]$. Furthermore, in view of

$$\omega_n^2[\Phi_n]^T[M][\Phi_n] + [\Phi_n]^T[K][\Phi_n] = 0,$$

substituting $[U]=q_n[\Phi_n]$ into Eq. (11.3) and multiplying $[\Phi_n]^T$ from the left, the following differential equation is obtained for q_n :

$$\ddot{q}_n(t) + 2\xi_n\omega_n\dot{q}_n(t) + \omega_n^2q_n(t) = \ddot{z}_n(t), \quad (11.4)$$

where

$$\xi_n = \frac{[\Phi_n]^T[C][\Phi_n]}{2[\Phi_n]^T[M][\Phi_n]} \quad \text{and} \quad z_n(t) = \frac{[\Phi_n]^T[M][1]}{2[\Phi_n]^T[M][\Phi_n]}z(t);$$

ξ_n is a damping ratio and z_n is an external inertia force for this mode. Solving Eq. (11.3) with initial conditions of $q_n=0$ and $\dot{q}_n=0$ at $t=0$ for $n=1, \dots, N$, displace-

system analysis to structure health monitoring and diagnosis.

¹⁴Input strong ground motion acts as if it is an inertia force to each DOF. This force are expressed in terms of z and $[1]$ as $\ddot{z}[M][1]$, the right side term of Eq. (11.3). It is possible to replace this term by $\ddot{z}[M][1] + \dot{z}[C][1] + z[K][1]$, which becomes $\ddot{z}[M][1] + \dot{z}[C][1]$ since $[K][1]$ vanishes.

¹⁵Usually, ω_n and $[\Phi_n]$ are called n -th natural frequency and mode of the MDOF system.

Table 11.3 The period and damping ratio of the first vibration mode.

building type	T_1 [sec]	ξ_1
WH	0.2 – 0.7	0.02
RC	$T_1 = 0.02H$	0.03
SRC	$T_1 = 0.03H$	0.02

ment $[U]$ is given as

$$[U(t)] = \sum_{n=1}^N q_n(t)[\Phi_n]. \quad (11.5)$$

The summation of Eq. (11.5) is truncated at a suitable n . This is the essence of the modal analysis.

The three matrices, $[K]$, $[C]$ and $[M]$, which describe Eq. (11.3), do not have to be rigorously determined in order to formulate Eq. (11.4); it is sufficient if the coefficients $\{\omega_n\}$ and $\{\xi_n\}$ are found. Furthermore, only some set of $\{[\Phi_n]\}$ are needed to use Eq. (11.5) when q_n 's are computed. Indeed, it is a common practice to express structure response in terms of the first few modes; for a shorter¹⁶ building, the effects of higher modes need not be considered, and it is enough to compute up to the third mode. According to a design code of residential buildings used in Japan, the period and the damping ratio of the fundamental mode, denoted by $T_1 = 2\pi/\omega_1$ and ξ_1 , are estimated from the building height, denoted by H . Table 11.3 gives the estimate of T_1 and ξ_1 for the three types of residential building; WH, RC and SRC stand for a wooden house, a reinforced concrete building and a steel reinforced concrete building, respectively. Furthermore, an empirical relation¹⁷ between the fundamental period T_1 and the second and third periods, T_2 and T_3 , has been established. The relation is

$$\begin{bmatrix} T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{5} \end{bmatrix} T_1. \quad (11.6)$$

Also, the following recursive formulae for the damping ratio hold:

$$\xi_{n+1} = \begin{cases} 1.4\xi_n & \text{for RC,} \\ 1.3\xi_n & \text{for SRC.} \end{cases} \quad (11.7)$$

Note that a wooden house is categorized as a shorter building, and it does not need to use the second and third modes for dynamic analysis.

Taking advantage of Eqs. (11.6) and (11.7), the natural frequency and the damping ratio for the first three modes can be determined, if the building height, H , is known. This H can be estimated from GIS which has data of a story number

¹⁶In general, if the period of the fundamental mode is shorter than 0.5[sec], a building is regarded as short.

¹⁷See, for instance, [Ju (2003)] for a list of the related studies.

for each building. Assuming that the average height of one floor is 3.5[m], H is estimated as

$$H = 3.5 \times K, \quad (11.8)$$

where K is a story number. There are no GIS data which can be used to determine a mode vector or a mode shape, $[\Phi_n]$. However, it is not difficult to estimate $[\Phi_n]$ for the first three modes, since they are dominant vibration modes; for instance, the fundamental mode is a one quarter of a sinusoidal function.

It should be pointed out that instead of the empirical relation, Eq. (11.6), the natural frequencies can be determined by assuming that an MDOF system uses lumped mass and uniform spring. That is, the form of the stiffness and mass matrices are

$$[K] = k \begin{bmatrix} 1 & -1 & \cdots & 0 \\ -1 & 2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \quad \text{and} \quad [M] = m \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

There are only two parameters, k and m , which describe $[K]$ and $[M]$. The natural frequency of the fundamental mode, which is the first root of $\det(\omega^2[K] - [M]) = 0$, is determined from the ratio of k and m . When the period of the fundamental mode T_1 is given, it follows from $T_1 = 2\pi/\omega_1$ that the natural frequency of the fundamental mode is

$$\omega_1 = \sqrt{\frac{[\Phi_1]^T [K] [\Phi_1]}{[\Phi_1]^T [M] [\Phi_1]}}. \quad (11.9)$$

When $[\Phi_1]$ is accurately or approximately given, this equation gives the ratio of k/m . It is then straightforward to determine $\{\omega_n, [\Phi_n]\}$ using the value of k/m .

11.4 Example of Integrated Earthquake Simulation

This section presets an example of applying IES to an actual town for the prediction of earthquake hazards and disasters. A *virtual town* is a computer model of a real town, which is constructed for an area near a previous Roppongi campus of the University of Tokyo. The target area is shown in Fig. 11.8; the center is at (139°63'54"E, 35°39'43"N) and the size is 300x300[m].

11.4.1 Modeling

The virtual town is constructed by using data stored in several GIS's which are available for the Roppongi area. The location of borehole sites and the constructed ground structure model are presented in Fig. 11.9. The number of borehole sites stored in GIS is limited, and there are only two sites within the target area of

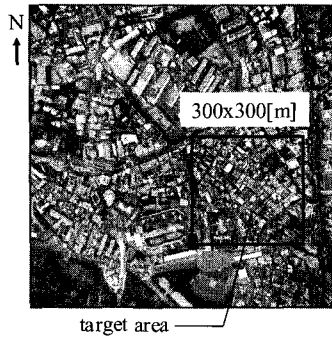


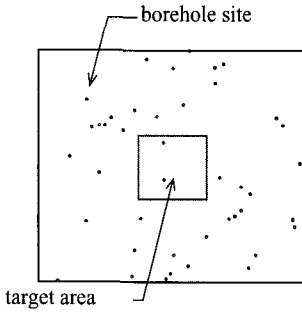
Fig. 11.8 A target area of a virtual town.

300×300[m]; see Fig. 11.9a), a bird's-eye view of the borehole site location. Thus, the domain of 1,000×1,000[m] is used to construct a ground structure model. The depth of the ground structure model is set as 60[m], and six major soil layers are found by applying the geoinformatics geology based method and the grid algorithm which are explained in Sec. 11.3. In Table 11.4, the density and elastic wave velocities are presented for the six soil layers which include the bedrock; these values are determined from the soil type and data stored in another GIS of velocity structures. As an example the layer configuration, Fig. 11.9b) shows the interface between the fifth layer and the bedrock. As is seen, the interface is expressed as a smoothly curved surface. A perspective of the ground structure model is presented in Fig. 11.9c). The layers are far from being stratified, and there is a *valley* in the east side which goes down toward the east direction.

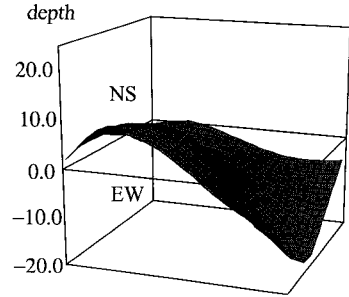
In the virtual town, there are 147 residential buildings, and an MDOF system is constructed for each of the buildings. Data for the structure type and the floor number are extracted from one GIS which has a list of the land usage, and the location of residential buildings is extracted from another GIS which contains the address of residential buildings; data in these GIS's are combined to construct an MDOF system. As mentioned above, there are three types of building, a wooden house (WH), a reinforced concrete building (RC) or a steel reinforced concrete building (SRC). Figure 11.10 shows a set of the constructed models of the buildings; a) is a bird's-eye view of the location of 147 buildings and b) is a perspective of the building set. Each building is expressed as a pole with a common square cross section but different height which corresponds to the floor number. Buildings in Fig. 11.10b) are magnified by 10 times in the vertical direction.

11.4.2 Strong ground motion simulation

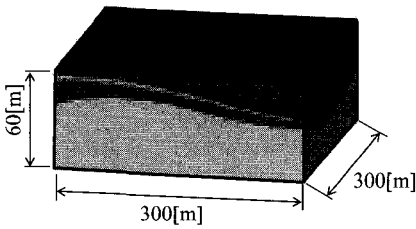
Strong ground motion which hits the virtual town is simulated using the micro-analysis; the spatial and temporal resolution of the micro-analysis are 2[m] and 0.01[sec], respectively, although the accuracy of numerical computation is guaran-



a) location of borehole



b) interface of bedrock and surface layers



c) whole layers

Fig. 11.9 A computer model of ground structures.

Table 11.4 Properties of soil layers.

layer	soil type	$\rho[\text{g/cm}^3]$	$v_s[\text{m/s}]$	$v_p[\text{m/s}]$
1	soil	1.625	120.0	204.0
2	loam	1.550	135.0	229.5
3	sand	1.800	400.0	680.0
4	clay	1.750	200.0	340.0
5	fine sand	1.900	425.0	722.5
bottom	rock	1.850	600.0	1020.0

teed up to 5[Hz] in the frequency range. As a simple example, a *Ricker wave*, a half period sinusoid wave, is used instead of a wave which is computed by the macro-analysis. The amplitude and frequency of the input wave is 1[cm] and 5[Hz], and the direction is east-west (EW), north-south (NS) or up-down (UD). Even such a simple wave produces complicated distribution of strong ground motion, which is influenced by the three-dimensional topographical effects that are inherent to the ground structures of the virtual town. The distribution changes depending on the direction of the input wave.

As an example of the strong ground motion distribution which is computed for

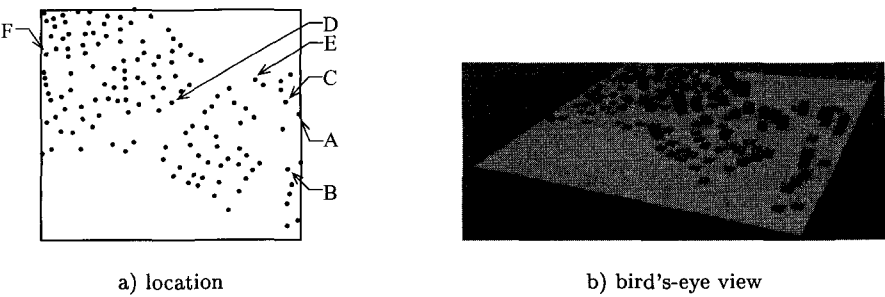


Fig. 11.10 Computer models of 147 residential buildings.

Table 11.5 The maximum values of PGD and PGV computed in the virtual town.

		EW	NS	UD
PGD [cm]	EW	3.54	0.86	0.71
	NS	0.85	4.30	0.56
	UD	0.73	0.65	2.23
PGV [kine]	EW	28.75	9.42	8.27
	NS	9.78	28.08	7.15
	UD	8.43	8.25	9.57

the virtual town, the distribution of the peak ground velocity (PGV) is plotted in Fig. 11.11; a), b) and c) are the results for the input wave in the EW, NS and UD directions, respectively, and the norm of velocity vector is presented on a gray scale. While a simple Ricker wave is input, the distribution of PGV is not uniform. For the input of horizontal¹⁸ waves, the distribution, shown in Fig. 11.11a) and b), appears rather complicated. There is some similarity between the two figures, although the distributions are locally different. Table 11.5 presents the maximum value of the peak ground displacement (PGD) and PGV on the ground surface; the maximum value of PGD is almost three times larger than the amplitude of the input earthquake (1[cm]). It should be noted that when the direction of the input wave is EW or NS, the maximum PGD and PGV is in the EW or NS direction, respectively. The amplification factor, however, is slightly different for PGD, 3.54 and 4.30 for the EW and NS directions, and for PGV, 28.75 and 28.08 for the EW and NS directions.

There are two mechanisms that cause the concentration of strong ground motion in the virtual town. The first mechanism is the amplification within surface soil layers which are above the bedrock. As the thickness of the surface layers increases, PGV tends to take on a larger value. In Fig. 11.12, the depth of the interface between the surface layers and the bedrock is presented; some similarity in pattern

¹⁸The distribution is more or less uniform when the wave in the UD direction is input.

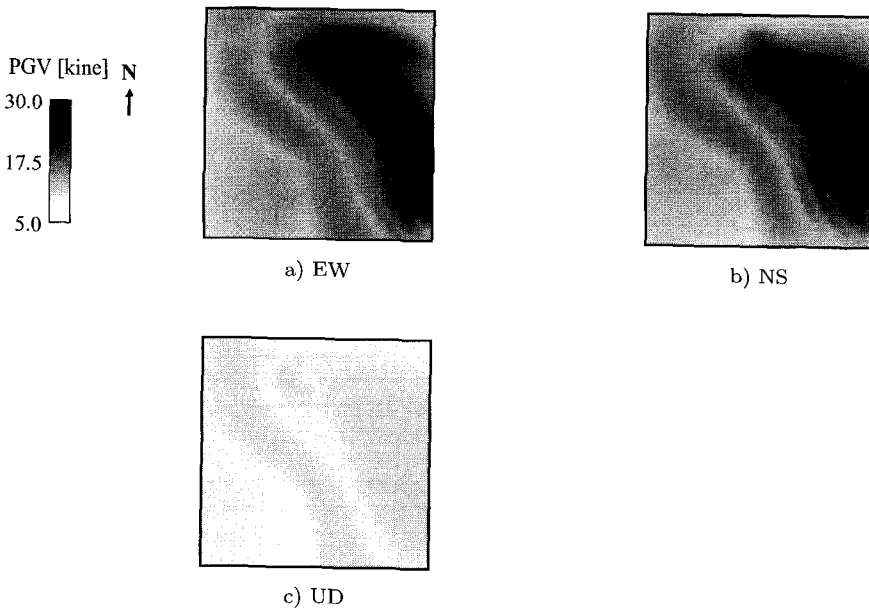


Fig. 11.11 The distribution of PGV. The direction of the input wave is EW, NS or UD.

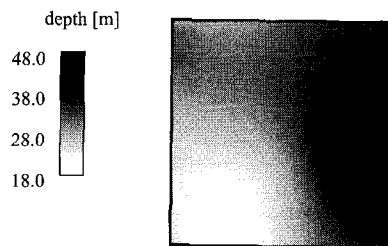


Fig. 11.12 The change in the thickness of softer surface layers.

is observed between Figs. 11.11 and 11.12. In the middle of the east side where the surface layers are thick, the strong ground motion concentration occurs for both the EW input and the NS input. The second mechanism is a purely three-dimensional topographical effect. Even if the soft layers are thin, depending on the input wave direction, some parts in the virtual town take on a larger value of PGV; see, for instance, the concentration of PGV in the north side of Fig. 11.11a). This implies that the location of sites where the strong ground motion is concentrated changes depending on the fault mechanism of an earthquake scenario. The amplification due to the first mechanism is analyzed by using a one-dimensional ground structure model of stratified structures or parallel soil layers. The amplification due to the second mechanism, the local amplification that is caused by the three-dimensional topographical effect, needs a full three-dimensional ground structure model and

Table 11.6 A summary of structure response. The input wave is in the EW direction.

a) WH				b) RC			
T_1 [sec]	floor number	min. MD[cm]	max. MD [cm]	T_1 [sec]	floor number	min. PD[cm]	max. PD[cm]
0.20	16	0.62	1.24	0.21	7	0.34	0.84
0.25	17	0.25	0.69	0.28	10	0.31	0.93
0.30	16	0.32	1.83	0.35	2	0.84	1.16
0.35	16	0.34	1.27	0.42	2	0.87	1.08
0.40	16	0.68	1.78	0.56	1	3.64	3.64
0.45	1	0.63	0.63	0.63	2	2.53	2.66
0.50	1	1.62	1.62	0.70	1	2.99	2.99
0.55	1	1.71	1.71	0.77	2	3.38	4.61
0.60	1	2.88	2.88				

c) SRC			
T_1 [sec]	floor number	min. MD[cm]	max. MD[cm]
0.315	7	0.66	1.58
0.420	5	0.91	1.83
0.525	5	1.74	2.34
0.630	1	1.85	1.85
0.735	4	2.63	3.34
0.840	4	3.18	9.81
0.945	1	2.57	2.57
1.155	1	11.01	11.01

three-dimensional wave propagation analysis. It is true that carrying out a three-dimensional wave propagation analysis is laborious. However, the need of such an analysis is clearly seen by comparing the two distributions of PGV, which are shown in Fig. 11.11a) and b).

11.4.3 Structure response simulation

The modal analysis is carried out for 147 residential buildings which are located in the virtual town, using the strong ground motion which is computed at the site of each building as an input wave. Each building is modeled as an isolated MDOF system which is subjected to distinct strong ground motion, and neither soil-structure interaction nor interaction among adjacent structures is considered. In Fig. 11.13, earthquake hazards and disasters which are predicted for the virtual town are presented as a snapshot of the perspective of the virtual town; a) is for the strong ground motion distribution in terms of PGD and b) is for the structure response of all residential buildings. These images are the output of the visualization tool that is implemented in the present IES.

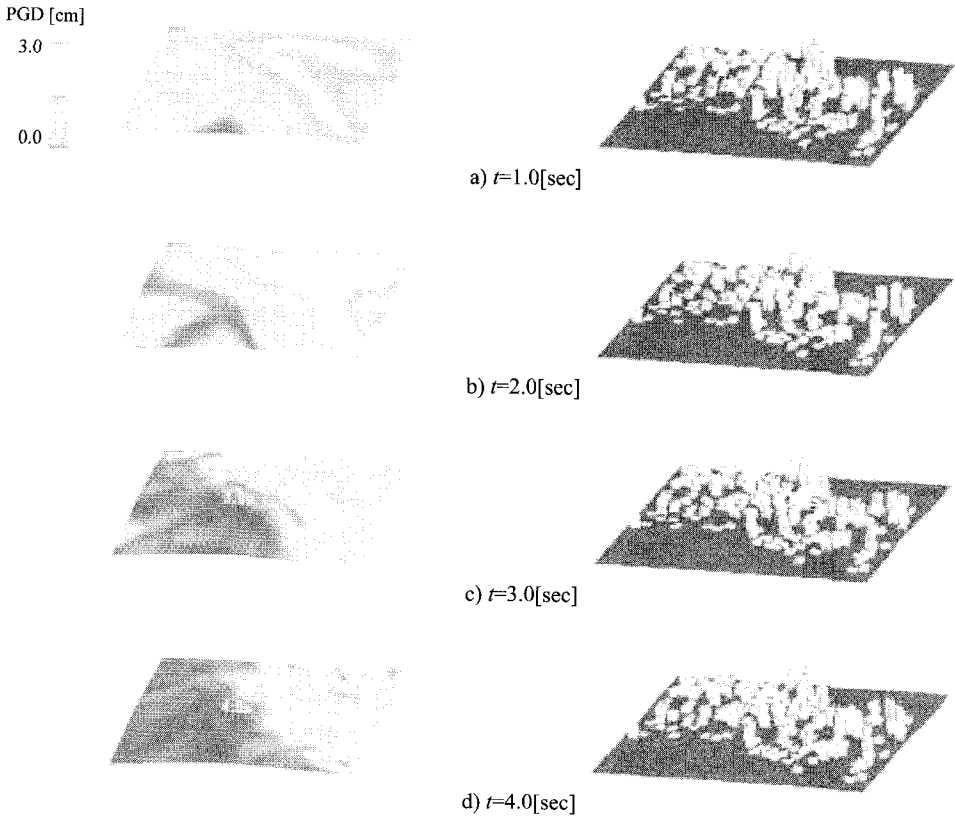


Fig. 11.13 Snapshots of strong ground motion distribution and building set behavior.

As an example of the wide distribution of the structure responses, Table 11.6 summarizes the minimum and maximum values of the maximum displacement (MD) of some buildings which have a similar fundamental period, T_1 , when an earthquake in the EW direction is input; a), b) and c) are for WH, RC and SRC, respectively. The ratio of the maximum value to the minimum value reaches almost 3. This is the consequence of the fact that the strong ground motion input to a building differs from place to place; see Fig. 11.11 for the distribution of PGV. Even for buildings of similar properties, i.e., the same structure type and similar fundamental periods, such difference is found in their responses. Thus, buildings of different properties have much larger differences. Indeed, the ratio of the maximum value of WH to the minimum value of SRC reaches around¹⁹ 20.

The wide distribution of the structure response is examined in detail. Three high-rise buildings in the virtual town, which have eleven stories, are picked up. In Fig. 11.10, they are marked as A, B and C; A is SRC, and B and C are RC.

¹⁹This number may be overestimated, since the dynamic analysis used in the present IES is linear analysis.

Table 11.7 The comparison of MD.

a) WH

building	type	T_1 [sec]	E-W	N-S	U-D
A	SRC	1.16	11.01	12.81	2.56
B	RC	0.77	4.61	4.38	1.70
C	RC	0.77	3.38	4.20	1.44

b) RC & SRC

building	T_1 [sec]	E-W	N-S	U-D
D	0.3	1.54	0.79	0.46
E	0.35	1.27	0.6	0.34
F	0.55	1.71	2.67	0.79

Three WH's, which are marked by D, E and F in Fig. 11.10, are picked up as well. Table 11.7 gives MD of these two groups for the three input earthquakes; a) is for A, B and C, and b) is for D, E and F, and the fundamental period T_1 of these buildings is presented. As is seen, A, B and C happen to be located at the area where the soft layers are the thickest as shown in Fig. 11.12. Hence, larger strong ground motion is input to these buildings for all the three input waves. Large difference in the maximum displacement is observed; A is shaken almost three times more than B and C. For D, E and F, which are WH, the results are more complicated. While the responses are more or less similar to the EW input, D and E are less shaken but F is shaken more for the NS input. This is because large strong ground motion is caused at the site of F for the EW input; such a concentration of strong ground motion is due to the second mechanism, the three-dimensional topographical effect, which is explained in the preceding subsection. Indeed, from the comparison of Fig. 11.10 with Fig. 11.11, it is seen that the concentration of strong ground motion takes place near F.

In closing this subsection, the use of IES, which outputs earthquake hazards and disasters as the consequence of an earthquake scenario, should be pointed out. Although a simple Ricker wave is used, three scenarios are provided for the virtual town. As shown in Fig. 11.11, the resulting earthquake hazards are locally different; the concentration of PGV changes depending on the input wave. The earthquake disasters are different as well. The response of each building changes, as the local strong ground motion which is input to the building is different. The maximum displacement doubles if the direction of the input wave changes from EW to NS; see the response of F in Table 11.7b). It should be emphasized that the reliability of the three earthquake hazards and disasters is limited, since the input wave is an artificial Ricker wave and only primitive models are constructed for residential buildings. However, the prediction of earthquake hazards and disasters in this spatial resolution which are realized by IES is appealing to the public.

Chapter 12

Unified Visualization of Earthquake Simulation

Local government officials play a crucial role in promoting *preparedness* against earthquake disasters since they make and enforce mitigation plans for a region which they are in charge of. However, their knowledge on earthquake engineering is not sufficient; they are legal administrators who do not have to understand seismology and earthquake engineering, and even technical officials do not have sufficient experiences on designing and constructing buildings or structures since their major concern is the authorization of building construction according to proper design codes and regulations. It is known that an earthquake has regional characteristics that are influenced by the local geological and ground structures and that regional structures have particular characteristics due to meteorological or historical reasons. In the near future, such regional characteristics of earthquakes and buildings should be taken into consideration in making mitigation plans against earthquake disasters. The local government officials will be required to make more technical decisions.

Improving the technical ability of the local government officials is an important issue in view of their current and future roles. Technical ability means the correct foreseeing¹ of possible damage to buildings and structures. Understanding the uncertainty of the prediction of structural damage which is made by experts is particularly important. Wide, not necessarily deep, knowledge on earthquake engineering will be required for the officials to acquire such technical ability.

Earthquake disasters include human casualties. Thus, technical ability must rely on a solid foundation. Understanding the response and failure mechanisms of a structure which is subjected to strong ground motion will be such a foundation since all processes are purely mechanical. An earthquake resistant design code² is a

¹From the viewpoint of risk management, an objective evaluation of risk, which includes the uncertainty or the probability of risk, is the first step to cope with risk. [Scawthorne (1996)] and [Dowrick (2003)] are recommended to study the basics of risk analysis and management related to earthquake hazards and disasters; see [Shinozuka and Yao (1981)], [Gurpinar (1997)], [Shibata (1998)], [Theodulidis *et al.* (1998)], [Lindt and Niedzwecki (2000)], [Ichii (2002)] and [Torregosa *et al.* (2002)]; see also [Huang and Leung (1999)] and [Lekidis and Dimitriu (2002)] for recent studies in this field.

good foundation for understanding the mechanisms. This is because the code is the essence of past knowledge and experiences on the mechanisms of structure response to strong ground motion.

Earthquake resistant design codes of various kinds of structures have been written for professional design engineers. Local government officials, whose specialty or expertise is not structure engineering, have some difficulty reading these codes; in particular, studying many codes of various structures will be a tough task. However, they do not have to understand the whole content of the codes. It is sufficient that local government officials understand the uncertainty or the range of possible structural damage which is predicted by experts. Understanding the codes at this level is not impossible, and the following two tools may help the local government officials understand many codes:

- 1) unified visualization of dynamic responses for a set of existing structures;
- 2) standardization of earthquake resistant design codes of various structures.

The unified visualization means the visualization of structure responses in a common manner so that the difference in responses of various structures can be understood intuitively. Structure responses are computed according to earthquake resistant codes, and the local government officials understand visualized structure responses as the consequence of the codes. A range of possible earthquake disasters is easily understood when the disasters are simulated by using different earthquake scenarios and the results are presented through the unified visualization. The standardization is aimed at describing design codes in a common framework, and will be explained in Chapter 13.

There are numerous numerical analysis methods for structures which are developed for the purpose of design. These methods are capable of predicting structure responses when a certain wave is input instead of a design earthquake wave. The target of the unified visualization is not one building but all structures which are located in a town, a city or an urban area. It is never easy to realize such visualization, even if it is taken for granted that there is limitation in constructing a computer model for all structures due to the lack in available data for them. The primary difficulty is the implementation of the numerical analysis methods of various structures into one system which does the unified visualization.

To tackle the above difficulty, this chapter presents the utilization of IES and a *mediator*. As explained in Chapter 11, IES is a simulation system of the whole phases of an earthquake. The mediator is a computer agent which implements a simulation program³ for the numerical analysis of a particular structure into IES.

²This book uses the term “earthquake resistant design” although other terms, such as seismic resistant design, earthquake proof design, or just seismic design can be used. The choice of terms mainly depends on the structure type.

³In general, numerical analysis methods which are used in earthquake resistant design are divided into two classes, the one used for design and the other used for performance evaluation. Either or both will be implemented in IES. They are thus called a simulation program without making any distinction.

The mediator is first explained, with focus on the construction of a mediator for a given simulation program. A prototype of IES which uses mediators is developed, and some examples of the unified visualization are presented. While the primary objective of mediators is the realization of the unified visualization, they also contribute to higher reliability of the simulation results made by IES since implemented simulation programs are actually used for the purpose of design.

12.1 System for Unified Visualization

The unified visualization presents three-dimensional static images or animations of structure response for a single structure or a set of structures which are located in a target area. It also shows possible damage to structures. Compared with a hazard map, the unified visualization delivers information about possible earthquake disasters with higher quality and larger quantity; for instance, structure damage simulated by IES changes depending on an earthquake scenario and the unified visualization shows a range of possible damage for one structure or for the whole area. Besides improving the technical ability of local government officials, it is expected that the unified visualization will provide realistic images of earthquake disasters to residents so that a common recognition of earthquake disasters is formed among them.

The reliability of the simulation is an important issue, in order for the unified visualization to be trustfully used by local government officials. As explained above, simulation programs which are developed for the purpose of design are used in IES. The reliability of the simulation results is at the same level as that of the design, since only a wave⁴ input to the program is different; the reliability of an input wave is a different issue, which is related to predictions of a fault mechanism of a possible earthquake.

It is not a trivial task to implement simulation programs of various structures into IES; ordinary plug-in is not realistic since the style and format of input/output statements⁵ must be modified for each simulation program. A mediator, a computer agent which plays a role of a translator between IES and each simulation program, is thus employed. Also, an artificial intelligence, called a *mediator maker*, is developed; the mediator maker automatically writes a source code of a mediator for a given simulation program.

⁴There are other alternatives to a design wave; see [Bommer and Acevedo (2004)] for the use of measured ground motion.

⁵Input data of a simulation program are data for structure and strong ground motion. These data are stored in GIS and libraries of IES or synthesized by the earthquake simulation of IES. Even though it uses data of similar kinds, each simulation program accepts only data which are written in its specific format. Thus, modifying the source code so that it accepts data written in a common format is required for the plug-in. Also, the source code needs to be modified so that its results are in a common format and can be used in IES.

12.1.1 Mediator

To explain a mediator, integration techniques which have been developed for a data base (DB) are briefly explained. The complexity of DB in computer structure as well as in data type and data number are much greater than the complexity of source codes of simulation programs which are written in one of a few programming languages. There are two major techniques which are used to integrate different DB's, the standardization⁶ of data structure format and the utilization of computer agents which handle data exchange between different DB's. The utilization of agents leads to the construction of a huge system which consists of numerous DB's. A new DB can be added to the system, just by creating a new agent which exchanges data of the new DB with existing DB's. This system is called a DB of a *federation type*. The major advantage of the federation type DB system is the independence of DB's, i.e., each DB can be changed or modified without considering the data exchange with other DB's. The corresponding agent must be updated so that the modified DB can be linked to other DB's.

In the federation type DB system, there is no hierarchy among DB's, and an agent for one DB exchanges data with all other DB's within the system. On the contrary, IES is controlled by a kernel, and each simulation program needs one particular agent which links it to the kernel; no communication is needed between different simulation programs. This computer agent of IES is a mediator. Therefore, the primary task of the mediator is summarized as follows:

put suitable input data into a simulation program for execution, and take simulation results for the unified visualization;

see Fig. 12.1 where each mediator plays a role of an interpreter between its supporting simulation program and the kernel in IES.

Seven functions are required for a mediator to do the above task, and they are summarized in Table 12.1. The first four functions, namely, the identification of a structure, the selection of a strong ground motion site, the acquisition of structure data and the acquisition of strong ground motion data, are for the data input. The fifth function, the execution of a simulation program, interprets the kernel's command of executing the simulation program. The sixth and seventh functions, the acquisition of simulation results and the visualization, are related to the unified visualization. The simulation results are transformed in a common form and sent to various visualization tools of IES.

A mediator is constructed as an *object*⁷ by describing each of the seven functions as a method. This will save efforts in making a mediator since mediators have a common program structure even though they serve different simulation programs. Advantage is taken of *inheritance* of object properties. The inheritance is explained using an example of constructing a mediator for an FEM-based simulation program,

⁶ As is seen, the standardization of DB corresponds to the plug-in of a simulation program.

⁷ See Sec. 13.1 for a more detailed explanation of objects.

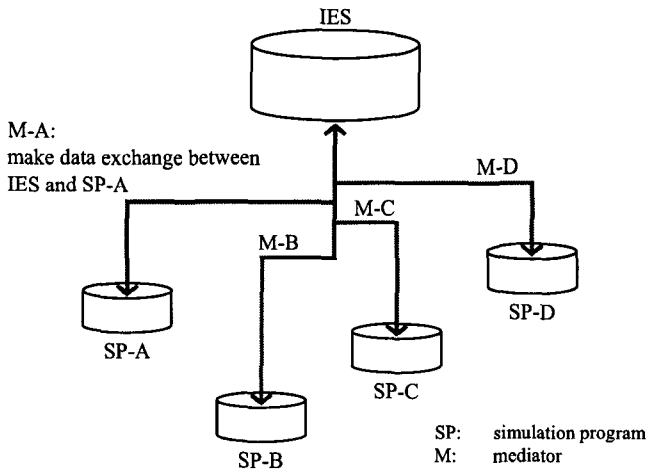


Fig. 12.1 A schematic view of IES in which simulation programs are connected to IES through mediators. Each simulation program has its own mediator which makes data exchange between IES and the program.

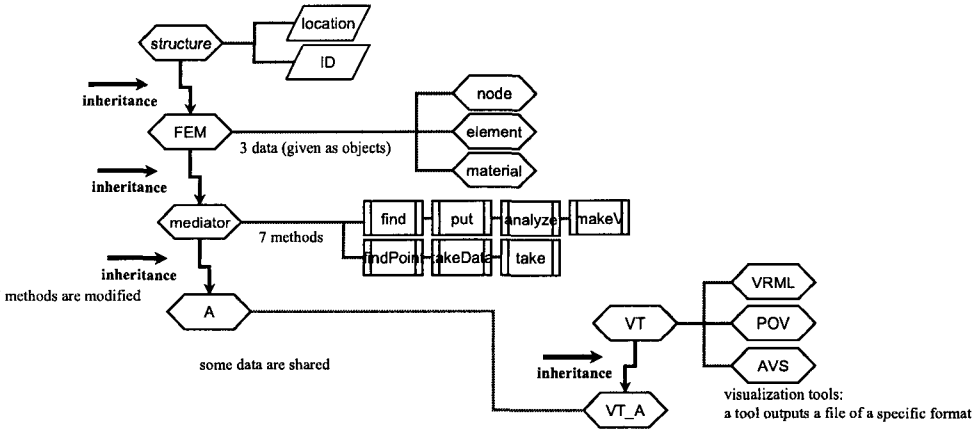
Table 12.1 Seven functions of a mediator.

basic function	content
identification of structure	identify structure and building in a target city from data extracted from GIS
selection of SGM site	select sites where SGM is computed and inform the site location to IES
acquisition of structure data	construct data files for input of SP, by extracting data from GIS and library
acquisition of SGM data	construct data files for SGM at sites, using results of earthquake simulator
execution of SP analysis	run SP using data files which are constructed by mediator
acquisition of results	take results of simulation from SP, and make result files
visualization	make visualization files from result files made by mediator

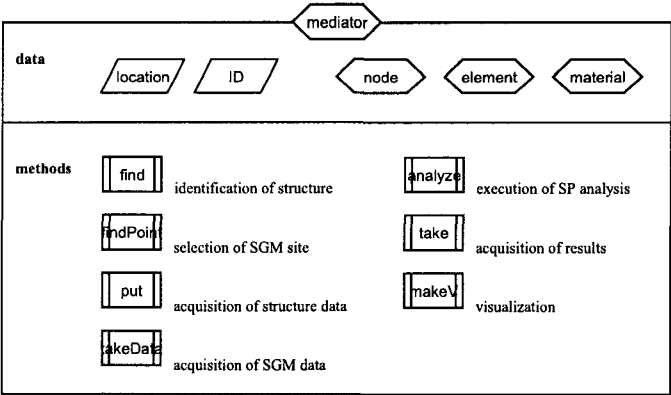
SGM: strong ground motion
SP: simulation program

named A; see Fig. 12.2a). An object **structure**, which consists of data of the structure ID and location, is first made. Then, objects **FEM** and **mediator** are made⁸ by applying the inheritance; **FEM** is made by adding data for nodes, elements and material properties to **structure**, and **mediator** is made by adding seven methods explained in Table 12.1 to **FEM**. Thus, the data of **mediator** are the same as those

⁸When c++ is used to write a mediator, a class for an object **structure** is first made. Regarding it as a superclass (or a base class), a subclass (or a derived class) for an object **FEM** is made.



a) flow of inheritance of objects



b) data and methods of object mediator

Fig. 12.2 Construction of class for mediator by taking advantage of inheritance of objects properties.

of FEM and **structure**; see Fig. 12.2b). Finally, an object **A** is made by modifying the seven methods of **mediator** so that **A** can be an interpreter between **A** and the kernel. The data and methods of **A** share the same name as those of **FEM** and **mediator**, and the kernel can use **A** without paying attention to the actual content of **A**. Another object **VT** is constructed for the visualization, and an object **VT_A** is made by applying the inheritance of **VT** and sharing some data and methods of **A**.

Table 12.2 Two functions of a mediator maker.

basic function	content
analysis of SP	identify and analyze input and loop parts for a given source code of FEM-based SP
construction of mediator	construct a class for SP, describing seven methods according to analysis results

SP: simulation program

12.1.2 Mediator maker

Making a mediator for a simulation program⁹ is essentially the same as modifying the source code of the simulation program for the plug-in. However, the task of making a mediator can be reduced since mediators share a common program structure. In principle, therefore, it is possible to automatically make a mediator for a given source code. What is needed is an artificial intelligence which does this task, i.e.,

analyze a source code for a given simulation program, and construct a mediator as a new program for the simulation program.

As mentioned, this artificial intelligence is called a *mediator maker*. Two functions are required for the mediator maker to do this task, and they are briefly explained in Table 12.2.

It should be emphasized that there is a common program structure for various simulation programs even if a target structure is different. This is mainly because the programs are usually based on FEM. Also, it should not be overlooked that earthquake resistant design codes according to which the programs are developed share several basic design procedures; Secs. 13.2 and 13.3 will explain the basic design procedures. They will save efforts in developing a mediator maker, besides the common program structure of mediators.

From the viewpoint of input and output procedures, an FEM-based simulation program is regarded as a combination of an input part and a loop part which further consists of an calculation subpart and an output subpart; see Fig. 12.3. Input and output are done by the input part and the output subpart, respectively. Thus, it will be the first task for the mediator maker to find the input part and the output subpart in a given source code and to extract input and output statements. In view of these part and subparts, the mediator maker actually makes a source code of a mediator.

There are two phases for the mediator maker to write a mediator code for a given simulation program; see Fig. 12.4. In the first phase, the simulation program is analyzed, and the mediator code is written in the second phase. An object of the mediator is made by taking advantage of inheritance. It should be emphasized

⁹One simulation program is used for many structures, and an entity of a mediator needs to be made for each structure. It is the kernel that makes such an entity.

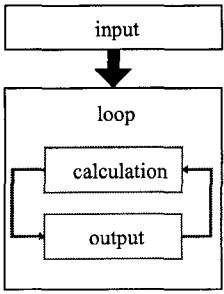


Fig. 12.3 Basic structure of FEM for dynamic response analysis.

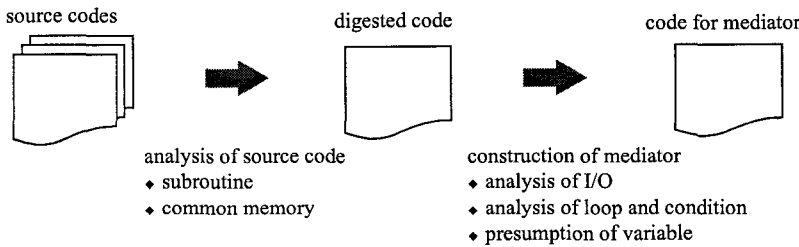


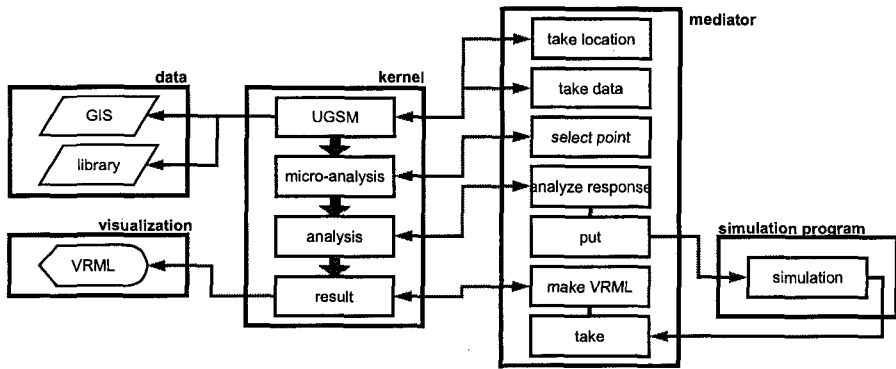
Fig. 12.4 Two phases of a mediator maker.

that constructing a robust artificial intelligence program for a mediator maker is extremely difficult. The current mediator maker is not complete, requiring some editing jobs in fully describing a mediator object. However, it saves manpower to some extent since it analyzes the program structure of a source code and identifies input/output variables.

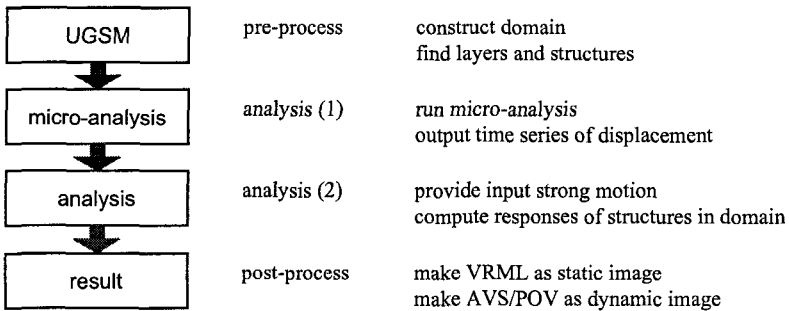
12.2 IES for Unified Visualization

Figure 12.5 shows a prototype of IES in which the kernel communicates a simulation program through a mediator; a) and b) are the overview of the prototype with some emphasis put on a mediator and the flow of four tasks which the kernel is assigned to do. Four simulation programs for gas pipelines, concrete piers, steel piers and ground embankments are implemented in the prototype. These simulation programs are made by an expert on each structure and written in FORTRAN. The programs are based on FEM and make non-linear dynamic response analysis of the structure, except for the program for gas pipelines. The four mediators are made with the help of the mediator maker.

The current state of the mediator maker is explained. As shown in Fig. 12.4 there are two phases for the mediator maker to construct a mediator. In the first



a) the overview



b) four tasks of the kernel

Fig. 12.5 The overview of the prototype of IES.

phase, the mediator maker generates a digested code from a source code, a simplified code which contains only input/output and loop statements. Examining all subroutines in the source code, the mediator maker finds input/output statements and determines their sequence. The determination of input/output statement sequence is not easy; the mediator maker must analyze all relations among subroutines including a main routine by investigating loop and condition statements which control these call statements. Also, the mediator maker checks variables¹⁰ which are used in input/output statement and file channels which are used as an argument of input/output statements. In Fig. 12.6, an example of a digested code which the mediator maker automatically generates is shown; the source code analyzed is for concrete piers. Each line starts from read/write, which corresponds to an input/output statement, or from ini/fin, which corresponds to the beginning/end

¹⁰Sometimes variables change their name in different subroutines. Analyzing the correspondence of variables transferred among subroutines, the mediator maker assigns a single name for each variable.

```

read:      10 IMEM[i] INODE[i] MX[i] KFCS[i] KSCS[i] KINDS[i]
ini:       I :      1 - INODE
read:      10 NO[i] (DDDO(6*I-L),L=5,0,-1)[d]
fin:       I :      1 - INODE
ini:       J :      1 - INODE
read:      10 NO[i] (NC(6*I-I),I=5,0,-1)[i] (NB(6*I-I),I=5,0,-1)[i]
fin:       J :      1 - INODE
ini:       I :      1 - IMEM
read:      10 MEM[i] NMEM(I,1)[i] NMEM(I,2)[i] ISHAP(I)[i] NFCS(I)[i] NSCS(I)[i]...
fin:       I :      1 - IMEM
read:      10 EC01[d] FT1[d] FC1[d] GFC1[d]
read:      10 EC02[d] FT2[d] FC2[d] GFC2[d]
read:      10 ARUFA[d] BETAC[d] C[d]
ini:       I :      1 - KFCS
read:      9 NOT[i] NFIB(I)[i]
ini:       JJ :     1 - NFIB(I)
read:      9 YYC[d] ZZC[d] ARE[d] YY1[d] YY2[d] ZZ1[d] ZZ2[d] HHY[d] BBZ[d]
fin:       JJ :     1 - NFIB(I)
fin:       I :      1 - KFCS
ini:       I :      1 - KSCS
read:      8 MNO[i] NS(I)[i]
ini:       J :      1 - NS(I)
read:      8 NO[i] SY(I,J)[d] SZ(I,J)[d] ITYPS(I,J)[i]
fin:       J :      1 - NS(I)
fin:       I :      1 - KSCS
ini:       I :      1 - KINDS
read:      8 NOT[i] AST(I)[d] ES0T(I)[d] ES1T(I)[d] FSYT(I)[d] FSYCT(I)[d]
fin:       I :      1 - KINDS
read:      10 NAXF[i]
ini:       I :      1 - NAXF
read:      10 NDAXL(I)[i] AXF(I)[d]
fin:       I :      1 - NAXF
read:      10 IDEFO[i]
read:      10 RAT[d] DIVER[d] IABAN[i]
read:      10 DT[d] BET[d] GANMA[d]
read:      10 IJK[i]
ini:       JJJ :    1 - IJK
read:      31 TIM[d] AOINP(JJJ)[d]
fin:       JJJ :    1 - IJK

```

Fig. 12.6 An example of a digested code generated by a mediator maker. The source code of concrete piers is used.

of the loop sentence. Arguments of **read/write** lines are the file channel number and variables, and those of **ini/fin** lines are a variable controlling the loop and the number of iteration.

In the second phase, the mediator maker writes a code for a mediator analyzing the digest code; a basic code, an object **FEM**, is given, and the mediator maker modifies seven methods to construct an object for the mediator as an inheritance of **FEM**. Since the digested code consists of the **read/write** lines and **ini/fin** lines, the major tasks are the following three: 1) the analysis of **read/write** lines; 2) the analysis of **ini/fin** lines; and 3) the presumption of variables which are used in the digest code. The presumption of variables is indispensable¹¹ to write the methods, **take_data**, **put** and **take**, which are used to input data and to output simulation results.

The second phase is much more difficult than the first phase. This is due to the difficulty in presuming variables in a target simulation program. While, say, the

¹¹ Unless the meaning of a variable is not known, the kernel cannot extract necessary information from GIS and libraries and send results to the visualization tools.

Table 12.3 The characteristics of key variables of FEM.

variable	characteristics
node number	appears at the beginning of the input part. controls the loop condition in the input part and in the output subpart.
element number	appears at the beginning of the input part. controls the loop condition in the input part and in the calculation subpart.
time step number	appears in the input part. controls the loop condition in the loop part.

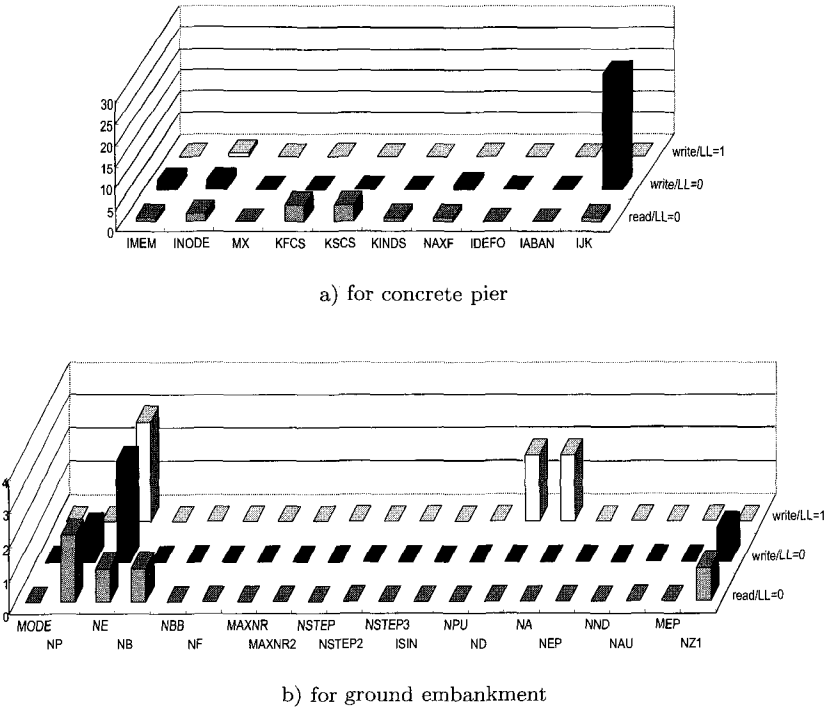


Fig. 12.7 Frequency of variables at different loop level. The frequency is automatically counted by a mediator maker.

data type can be automatically recognized for a variable by analyzing the format of input/output statements, guessing what a variable means is not easy at all. A sophisticated analysis similar to language analysis is surely needed, to presume variables which are used in a general source code. However, for a source code of an FEM-based simulation program, presuming the meaning of some variables is possible by taking advantage of the common program structures of the source code.

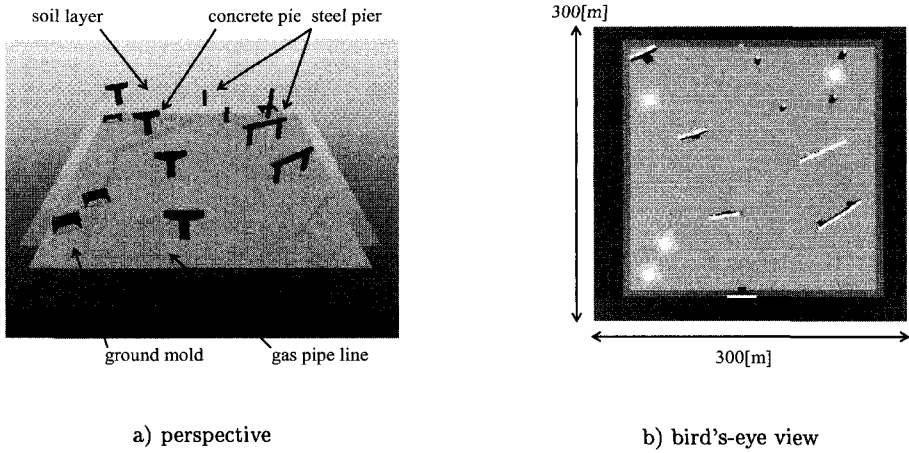


Fig. 12.8 Examples of three-dimensional view of VC.

Indeed, the current mediator maker is able to presume some key variables, the node number, the element number and the time increment number, which are generally used in FEM.

In FEM, the node number, the element number and the time increment number are input in the early stage of the input part and are used in the loop part; see Fig. 12.3. More specific characteristics of these variables are summarized in Table 12.3. In view of these characteristics, the mediator maker identifies these three variables as follows:

- 1) node number is a variable which controls the iteration of the input part and of the output subpart;
- 2) element number is a variable which controls the iteration of the output subpart;
- 3) time increment number is a variable which controls the iteration of the loop part.

As an example, the frequency of all variables appearing in the two digested codes is shown in Fig. 12.7; a) and b) are for the digested codes of concrete piers and ground embankments, respectively. The frequency is categorized for **read** or **write** and for a loop level (LL); LL is an order of nested loop sequences, i.e., LL=1 means the frequency of a variable appearing in the first nest of the loop. In Fig. 12.7a), variables INODE, IMEM and IJK are presumed as the node number, the element number and the time increment number, respectively. The mediator maker's presumption of the three variables is correct. Similarly, variables NP and NE in Fig. 12.7b) are presumed as the node number and the element number, respectively. The presumption is correct, although the mediator maker cannot identify the time increment number for the digested code of ground embankments.

At this moment, the mediator maker cannot presume variables except for the three key variables studied above. More study is surely needed to improve the me-

Table 12.4 Data about ground structures of the VC.

	v_p [m/s]	v_s [m/s]
1 st	229	135
2 nd	340	200
3 rd	425	250

Table 12.5 Data about structures in the VC.

a) concrete pier

steel	initial stiffness [MPa m ²]	17400
	second stiffness [MPa m ²]	3500
	yield stress [MPa]	3500
concr	maximum compressive strain	-0.002
	tensile strength [MPa]	20
	compressive strength [MPa]	-298
	compressive failure energy [MPa]	48.04
	model parameters α	-0.01
	model parameters β	0.02
	tension stiffening coefficient C	20.00
dead load 2000[ton]		
fiber element Newmark β method ($a=0.49$, $b=0.9$)		

b) steel pier

steel	Young's ratio [GPa]	200
	Poisson's ratio	0.3
	yield stress [MPa]	289.6
dead load 7000[ton]		
fiber element		

c) ground mold

mold	Young's ratio [MPa]	10
	Poisson's ratio	0.4
	cohesive strength [MPa]	10
	friction angle [deg]	15
	dilatancy angle [deg]	15
	density [kN/m ³]	16
hexagonal element with 20 nodes		
Rayleigh damping $a=0.171359$		
Rayleigh damping $b=0.001736$		

diator maker. However, the current mediator maker saves¹² efforts in constructing a mediator since it generates a digested code and identifies some key variables.

12.3 Example of Unified Visualization

With the aid of the mediators, the prototype of IES constructs a virtual city (VC) as a computer model for a target city. The prototype has a small GIS for borehole data and structures, and automatically constructs a computer model for ground structures and large-scale structures when a target area is specified. An example of a VC is shown in Fig. 12.8; the dimensions of the VC are 300×300 [m] in the NS and EW directions. The ground structures, up to depth of 40 [m], consist of three distinct layers, and there are four gas pipelines embedded. On the ground, there are five concrete piers, seven steel piers of two kinds and four ground embankments. The material properties of the soil layers and the structures, which are extracted from

¹²Construction of a mediator may become easier if an example of input data file is available; input variables are presumed by comparing the digested code and the input data file.

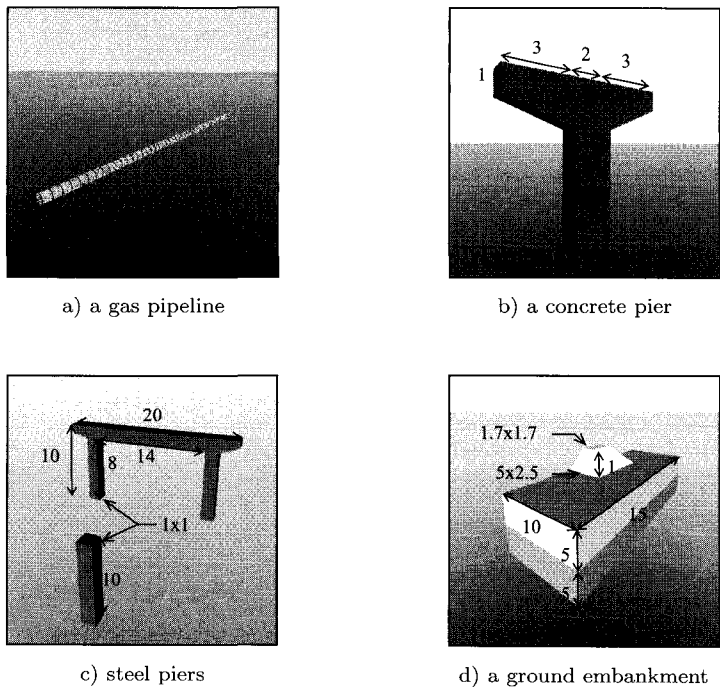


Fig. 12.9 Computer models automatically constructed by IES. The kernel transfers structure data from GIS and makes a computer model with aid of mediators. The dimension shown in the figures is in meters.

the library of the prototype, are summarized in Table 12.4 and 12.5. A schematic view of each structure in the VC is shown in Fig. 12.9; a)~d) are for a gas pipeline, a concrete pier, steel piers and a ground embankment, respectively. The figures are static images made by the unified visualization; the kernel automatically generates these images by transferring the structure configuration data to the visualization tool which makes a VRML file.

An example of dynamic images made by the unified visualization is shown in Fig. 12.10; a half sinusoidal wave of amplitude 10[cm] and period 1.0[sec] is input at the bedrock mass, and figures are snapshots of the perspective of the VC every 0.2[sec]. The displacement of structures is magnified by ten times, and the displacement norm is indicated on the gray scale. Difference in structure responses is clearly seen. For instance, the difference in the ground embankment responses is most significant; the numerical computation of the embankment at the right top becomes unstable at $t=0.2[\text{sec}]$ and yields unrealistically large displacement. Also, concrete and steel piers located at the center of the VC have a large contrast in the amplitude of displacement; the maximum displacement of the concrete pier is just 20% of that of the steel pier. It should be emphasized that the simulation programs implemented in the prototype make non-linear dynamic analysis of structure

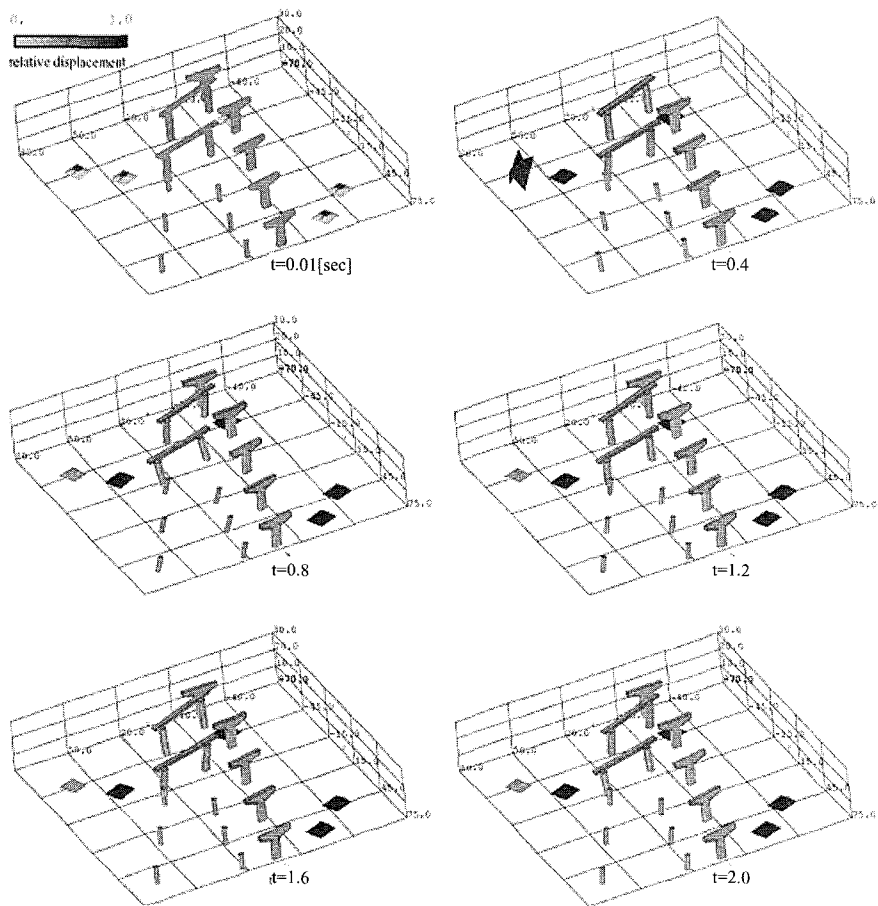


Fig. 12.10 Examples of unified visualization.

responses according to an actual earthquake resistance design code. As another example of the unified visualization, an animation is made for dynamic response of the whole structures within the VC. This animation provides vital images of possible earthquake disasters. It is expected that such unified visualization contributes to the improvement of the technical ability of local government officials, which results in more efficient mitigation plans against possible earthquakes.

The unified visualization is made to help local government officials understand many earthquake resistant design codes at a proper level. The validation of IES which generates the unified visualization is crucial; if IES does not attain¹³ full

¹³The simulation programs implemented in the prototype are made by experts on each structure. Thus, the reliability of IES strongly depends on an earthquake wave which is input to the VC. Since there are uncertainties about fault mechanism of a source fault, considering various scenarios of an earthquake, IES will provide some range of possible earthquake disasters which are computed based on these scenarios. Local government officials can understand the range easily if these disasters are visualized like Fig. 12.10.

reliability, the unified visualization does not help local government officials. Since there have been sufficient qualitative information about disasters, what is needed is quantitative information on which they can rely. The unified visualization made by IES is a tool for providing such information. From the viewpoint of computational mechanics, it should not be overlooked that each simulation program has its own accuracy to compute dynamic structure responses since the required accuracy depends on the target structure. The unified visualization needs to tell the audience this difference, in order for them to correctly understand the structure response as the consequence of earthquake resistant design codes.

Chapter 13

Standardization of Earthquake Resistant Design

As explained in Chapter 12, local government officials who are in charge of making and enforcing mitigation plans against earthquake disasters need to improve their technical ability. Technical ability means foreseeing possible damage to various structures which are located in the area of their responsibility, and the local government officials ought to understand earthquake resistant design codes of various structures which are a reliable foundation to develop technical ability. To help the local government officials understand many codes at a proper level, unified visualization and standardization of the codes are proposed. Chapter 12 presents the former, and this chapter explains the latter.

The standardization is aimed at describing earthquake resistant design codes¹ in a common style. A certain logical rule will be needed to establish such a common style. Thus, this chapter first general earthquake resistant designs, using an object-oriented description style. Actual earthquake resistant design codes are then studied, and the codes are reconstructed by using an expression in terms of objects. This expression is refined so that a common description style, which uses objects and their contents, is established. Finally, this chapter presents examples of the standardized design codes, which are reconstructed by applying the established description style to the existing codes.

The basic concepts and principles of earthquake resistant design are not covered in this chapter. [Dowrick (1989)], [Smith and Coull (1991)] and [Priestley *et al.* (1996)] are recommended for readers who are interested in the earthquake resistant design of general and particular structures; see² [Williasm (1995)] and [Penelis and Kappos (1997)]. Basic knowledge of computer languages³ is required to understand the object-oriented description style; for instance, [Elmasri and Navathe (2003)], [Pressman (2004)] and [Larman (2004)] are recommended as basic

¹While seismic isolation system is not covered in this book, design of the isolation system will be important in earthquake engineering; see [Skinner *et al.* (1993)], [Naeim and Kelly (1999)] and [Ryan and Chopra (2004)]; see also [Symans and Constantinou (1999)].

²See also [Krishna (1999)], [Ghobarah (2001)] and [Shimabuku and Takemiya (2002)].

³Readers who are interested in the most up-to-date computer language and related issues are strongly recommended to browse websites on the internet. Websites change very rapidly, and the readers need to search interesting sites by themselves.

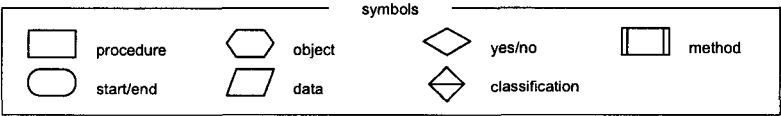
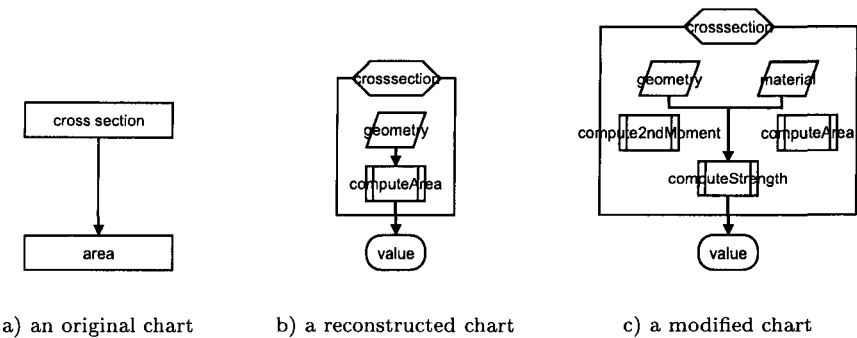


Fig. 13.1 An example of describing a flow chart in terms of objects. Symbols used in these charts are explained in the above box, and they are used in figures of the present chapter.

textbooks on computer languages.

13.1 Standardization of Description Style

There are several reasons for the difficulty in understanding various earthquake resistant design codes. A primary reason is the fact that each code has its own design procedures; variety of design procedures is inevitable since each structure has distinct mechanism of deformation and the design procedures must be made in a very specific manner that is inherent to the structure. Full understanding of a design code requires considerable preliminaries. An un-unified description style will be an obstacle to understanding many design codes. As mentioned before, the objective⁴ of the standardization is to help local government officials understand many earthquake resistant design codes. Therefore, these reasons, except for the description style, are not fatal for the standardization.

Writing a design codes in a common description style is called *standardization* in this book. A design code, including a guideline, is written for an experienced design engineer, and most of its contents are specialized. Such specialized parts of a design code need not to be understood by local government officials, and the major object of the standardization is a basic part of the design codes.

To realize the standardization, it is helpful to refer to programming languages, which have progressed to the level at which numerous program writers work without

⁴Although it is possible, the standardization studied here is not aimed at constructing a better design philosophy or concept which is applicable to various structures.

having long training in the computer language. It used to be the case that a program code became too specialized for numerical efficiency and the code could be hardly read by anyone except for the writer of the code. To solve this problem and to enhance the exchange of the whole or some part of well-written program codes, an *object-oriented programming language* has been developed. An *object* is an assembly of *data* and *methods*; data are a set of information relating to the object and methods are functions which use these data. The object-oriented⁵ programming language makes access to a program code much more transparent, since the code is expressed as a sequence of object operations and it is easy to implement objects into another program code. As an example, a flow chart of calculating an area of cross section is considered; see Fig. 13.1. This flow chart is reconstructed as one object **cross section** which has a datum **geometry** for geometrical parameters and a method **computeArea** of calculating the area. When the method **computeArea** is selected, this object extracts suitable parameters from the datum **geometry** and returns a value of the area. An object can be modified by adding data and methods; for instance, the object **cross section** can express a more complicated flow if a datum **material** and a method **computeStrength** are added; see Fig. 13.1c).

It should be mentioned that *unified modeling language* (UML) has been developed for designing and analyzing a computer system which consists of a set of numerous program codes or databases; see [Satzinger and Orvik (2001)], [Larman (2004)] and [Rumbaugh *et al.* (2004)]. UML is a computer language which uses only diagrams with rigorously defined description styles, and the style corresponds to the grammar of an oral language. The diagram expresses the function, the state and content of objects, and the relation between objects; see Appendix D for the brief explanation of UML. The standardization of earthquake resistant design codes is similar in concept to describing a computer system in terms of UML.

13.2 Description of Flow Chart in Terms of Object

In this section, flow charts for general earthquake resistant designs are rewritten in terms of objects, seeking to establish a rule of expressing design procedures in terms of objects. Design procedures at the same level of importance are extracted from a flow chart, expressed in terms of objects, and put into one or a few diagrams. The level of importance is only related to the importance of a design procedure for non-professionals to understand. Flow charts of actual earthquake resistant design codes are reconstructed according to the established rule, and the modification of the rule is considered by comparing the reconstructed flow charts.

⁵In general, a programming language is the sequence of procedures of certain subjects. An object-oriented programming language puts an emphasis on subjects rather than procedures; classical programming languages are procedure-oriented. The application of such object-oriented methodology is not restricted to programming language; for instance, it is used for the construction of a database.

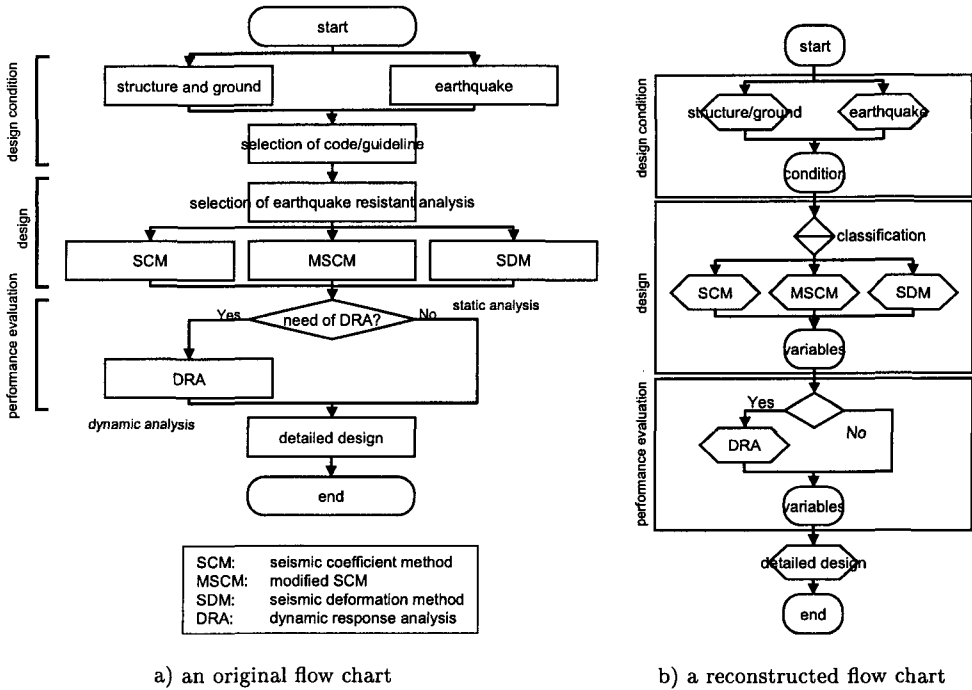
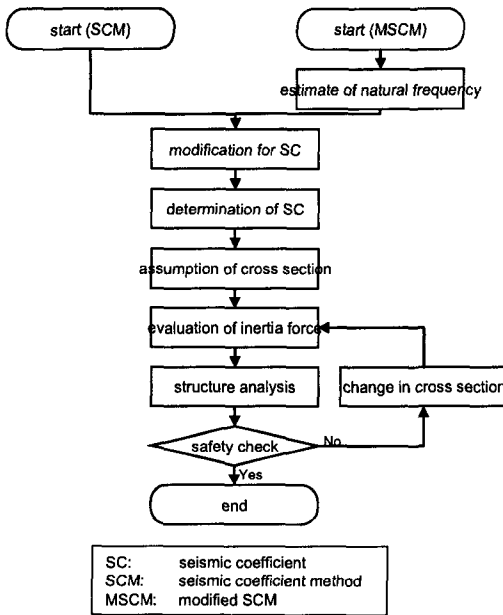


Fig. 13.2 A flow chart of general earthquake resistant design.

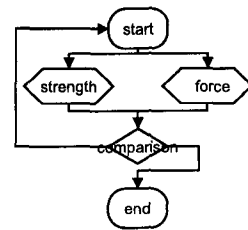
13.2.1 Reconstruction of a flow chart for general earthquake resistant designs

While each earthquake resistant design code has its own details, a basic flow of design procedures is common to most of them. Taking advantage of this fact, an attempt is made to rewrite a general flowchart of earthquake resistant design in terms of objects; see Fig. 13.2. The target is shown in Fig. 13.2a). The chart consists of three phases, i.e., a phase of considering design conditions, a phase of determining design variables, and a phase of the performance evaluation of the designed structure. Design conditions are made for a structure and a foundation ground and for an input earthquake wave. Design variables such as the dimensions and material properties of members are determined through a certain earthquake resistant analysis. The safety of using the determined design variables is checked by means of the performance evaluation which is based on dynamic response analysis.

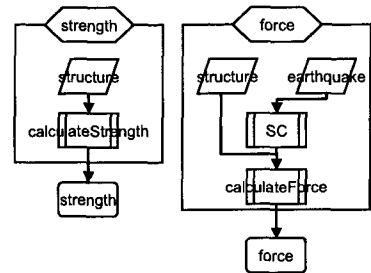
The flow chart is reconstructed as the sequence of three groups of objects, as shown in Fig. 13.2b). The first group consists of two objects (**structure/ground** and **earthquake**) and outputs the design conditions. The second group, which is the main part of the chart, has three objects for earthquake resistant analysis, namely, **SCM** (seismic coefficient method), **MSCM** (modified SCM) and **SDM** (seismic deformation method). One of these objects is chosen according to the structure type,



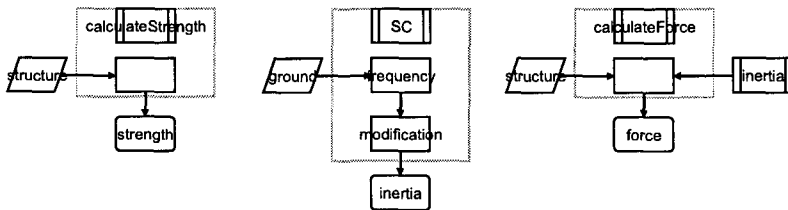
a) an original flow chart



b) a reconstructed flow chart



c) object contents



d) method structures

Fig. 13.3 A flow chart of (modified) seismic coefficient method.

and the chosen object outputs the design variables. The third group consists of one object DRA (dynamic response analysis), which is used only when it is necessary and outputs the design variables checked by the performance evaluation. In comparing Fig. 13.2b) with Fig. 13.2a), any significant difference cannot be found in the two charts. This is because all design procedures shown in the original flow chart are at the same level of importance. A minor difference is that the output for the three phases is clarified in Fig. 13.2b) since each object returns an output.

Next, typical earthquake resistant analyses which are used in design codes are studied. The most basic analysis is the seismic coefficient method or the modified seismic coefficient method; see Fig. 13.3. The flow chart of these methods is presented in Fig. 13.3a). It is easy to use this chart for the purpose of design, since the design procedures are sequentially arranged. However, this description style is

not suitable for non-professionals; it is difficult to see key design procedures or to grasp the perspective of the analysis. According to the flow chart, the essence of the seismic coefficient method is the comparison of the strength of a structure with the force caused by an earthquake. The original flow chart describes all design procedures to calculate the strength and the force by using data about the structure and the earthquake. Thus, the flow chart is divided into three diagrams. Each diagram extracts design procedures at the same level of importance. For instance, the basic procedures are put into a reconstructed flow chart shown in Fig. 13.3b); the chart shows that the comparison of the two objects **strength** and **force** is repeated until seismic safety is ensured. Design procedures to compute the strength and the force are put into two diagrams shown in Fig. 13.3c); the diagrams show the content for the objects **strength** and **force**, which output the value of strength and force through methods **calculateStrength** and **calculateForce** using data **structure** and **earthquake**, respectively. Finally, design procedures of detailed computation, which explain actual computation or formulae of strength and force, are put into two diagrams shown in Fig. 13.3d); the diagrams are the structure of the methods **calculateStrength** and **calculateForce**.

Next, the seismic response method is reconstructed; see Fig. 13.4. The original flow chart is presented in Fig. 13.4a), and the diagrams of the reconstructed flow chart, the object content and the method structure are shown in Fig. 13.4b), c) and d), respectively. The reconstructed flow chart, Fig. 13.4b), extracts the basic design procedures, the comparison of the objects **strength** and **force**, and it coincides with that of the seismic coefficient method shown in Fig. 13.3b). From the comparison of Fig. 13.4c) with Fig. 13.3c), however, it is seen that the object contents, which extract the design procedures to calculate strength and force, are different from those of the seismic coefficient method. While the object **strength** has the same contents for both cases, the object **force** is different; in **force** of the seismic response method, there are data **structure** and **ground**, and they are used in a simple sequence of the two methods **GRA** (ground response analysis) and **calculateForce**, while **force** of the seismic coefficient method has two data of **structure** and **earthquake** and the sequence of two methods **SC** (seismic coefficient) and **calculateForce** is slightly complicated. Accordingly, the method structures, Fig. 13.4d), are different from Fig. 13.3d). The three diagrams which rewrite the original flow chart in terms of objects clearly show the similarity and difference of the two earthquake resistant analyses.

As the last example of a typical earthquake resistant analysis, the dynamic analysis method is reconstructed; see Fig. 13.5. The original flow chart shown in Fig. 13.5a) does not have design procedures which explain actual computation; these procedures will be lengthy procedures of an FEM analysis. Thus, only the reconstructed flow chart and the object content are made; see Fig. 13.5b) and c). Compared with the seismic response method, the reconstructed flow chart of the dynamic analysis method includes a new object **modeling**, which indicates the

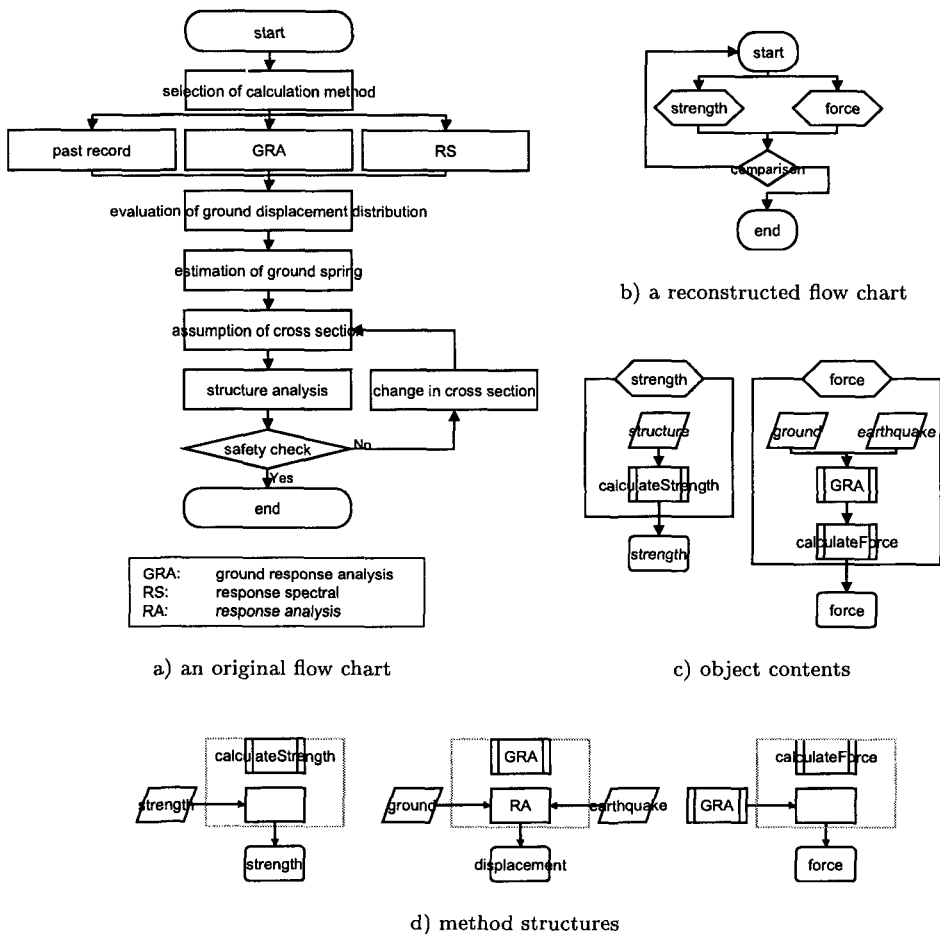
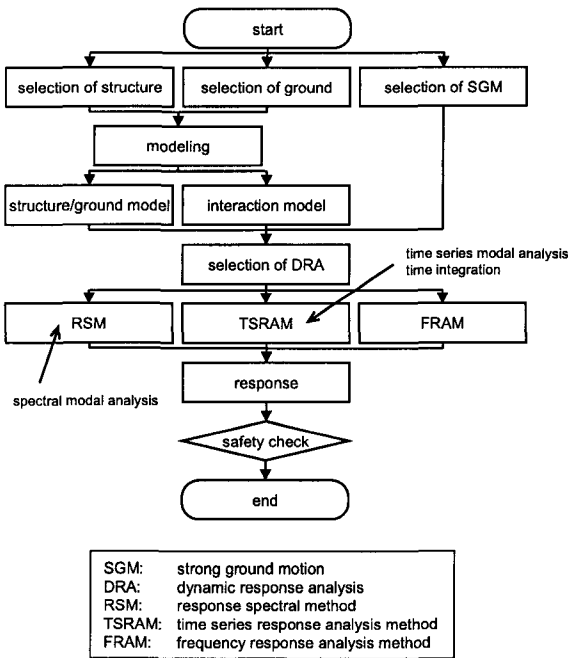


Fig. 13.4 A flow chart of seismic deformation method.

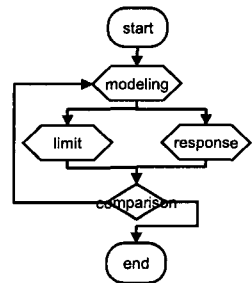
complexity of the dynamic analysis method. Also, the three objects used by the dynamic analysis method, namely, **modeling**, **limit** and **response**, have three data **structure**, **ground** and **SGM**⁶ (strong ground motion). The object contents of the dynamic analysis method appear much more complicated than those of the seismic coefficient method; as shown in Fig. 13.5c), the object **response** has four data and three methods, even though only one method is chosen to output the response. Thus, the common description style of the diagram in terms of objects shows the similarity and difference of the dynamic analysis method compared with the previous analyses.

It should be mentioned that symbols used in the three original flow charts are arranged to have the same meaning; for instance, a rectangular box is used for a design procedure and a diamond box for an evaluation. Such an arrangement

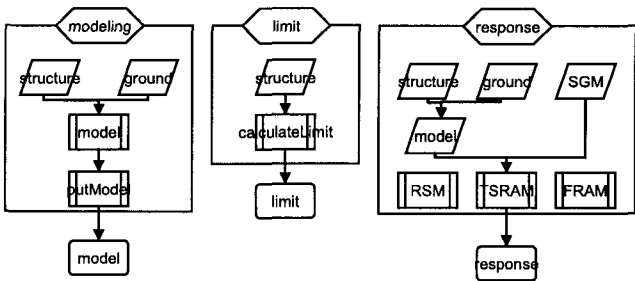
⁶The datum SGM represents time series of acceleration or displacement.



a) an original flow chart



b) a reconstructed flow chart



c) object contents

Fig. 13.5 A flow chart of dynamic analysis method.

is needed to avoid unnecessary confusion about symbols. The diagrams for the reconstructed flow chart, the object content and the method structure, use objects. Thus, no special arrangement is needed, since the style of expressing an object and its datum and method is pre-determined. Indeed, an object, a datum or a method is expressed as a pentagon, a parallelogram, or a box with two vertical edge lines, respectively, and a sequence of boxes with arrows indicates a method structure.

As mentioned several times, a design procedure written in a flow chart has a different level of importance. In the above examples, it is assumed that there are

three levels of importance for a design procedure, i.e.,

- Level 1: basic procedures which are essentially important to understand the concept of design.
- Level 2: procedures to compute key quantities which are used to determine design variables.
- Level 3: procedures for detailed computation which use actual calculation and design formulae.

A design procedure at Level 1 is put into the reconstructed flow chart, while a design procedure at Level 2 or 3 is used by the object content or the method structure, respectively. Thus, the rule of reconstructing the flow chart is summarized as follows:

put design procedures into a suitable diagram according to their level of importance, by expressing the procedures in terms of objects or the data and methods of an object.

Note that while the design procedures are classified into the three levels of importance, the procedures at Level 3 do not have to be understood by non-professionals.

In closing this subsection, it should be pointed out that there is some inconsistency in transforming design procedures to new diagrams. For instance, the earthquake resistant analysis is expressed as an object in Fig. 13.2, while it is expressed as an assembly of objects in Fig. 13.3~13.5. This inconsistent usage of objects is confusing. However, in rewriting the flow chart, it is advantageous to express one design procedure as an assembly of objects⁷ or an assembly of data and methods of one object. This is because expressing a complicated procedure as an object assembly intends to deliver the whole image of the procedure, while expressing it as an assembly of data and methods clarifies the details of the procedure. Although this point cannot be ignored, it is certainly true⁸ that the description style presented here needs improvement in order to remove the inconsistency.

13.2.2 Reconstruction of a flow chart for actual earthquake resistant design code

Now, the rule of rewriting a flow chart in terms objects is applied to actual earthquake resistant design codes. The first example is a simple code for high pressure gas pipelines, and the target is a flow chart of calculating seismic ground strain; see Fig. 13.6. The original flow chart is shown in Fig. 13.6a), which consists of design procedures at Level 2 and 3. Thus, the diagrams for the object content and the method structure are made from the chart; see Fig. 13.6b) and c), respectively. The object content shows that the object ground outputs seismic ground strain, using

⁷An object can use another object as data.

⁸More rigorous treatment has been achieved in UML; see Appendix D.

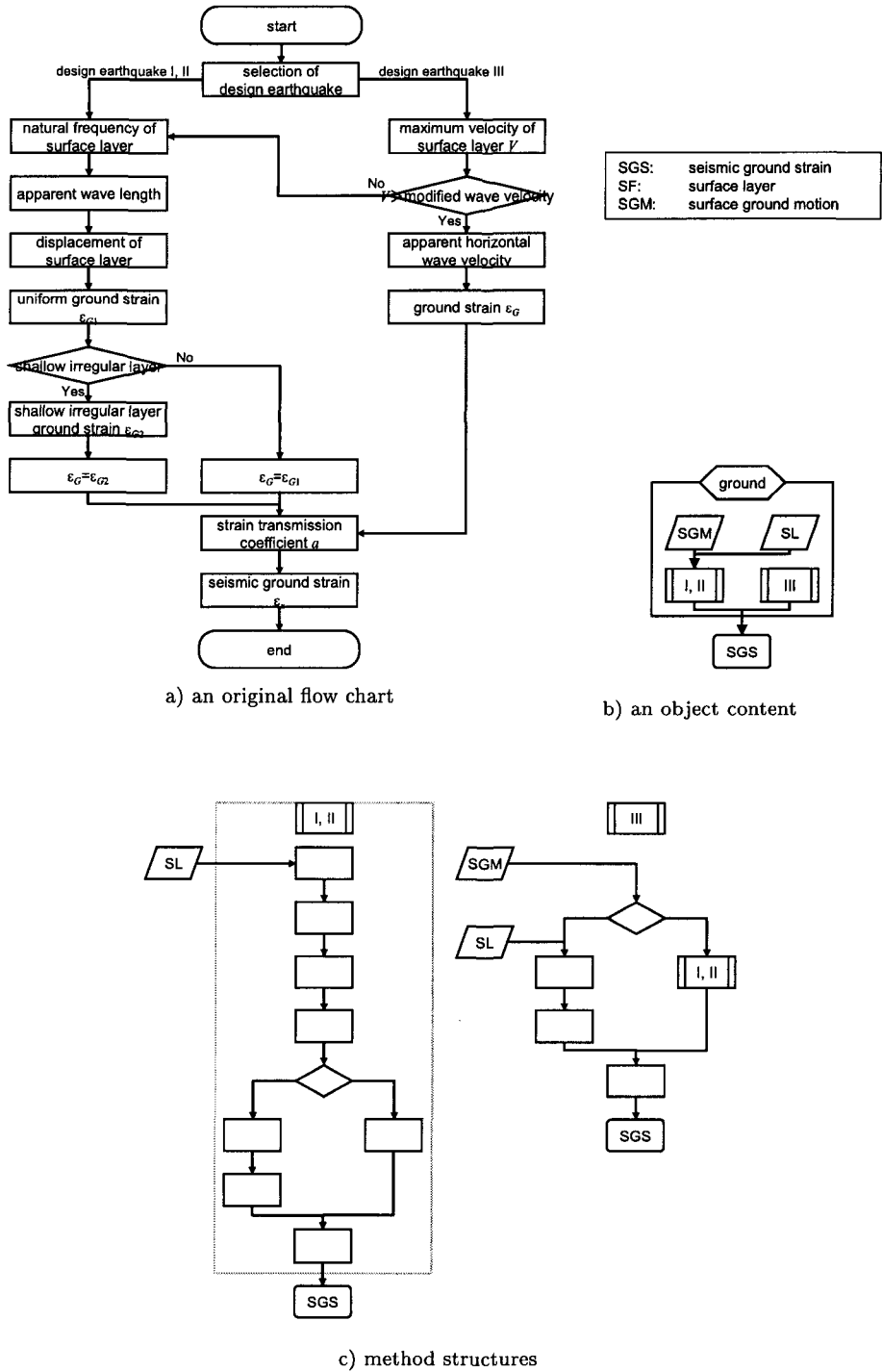


Fig. 13.6 A flow chart of calculating seismic ground strain of high pressure gas pipelines.

data SGM (strong ground motion) and SL (surface layer) by applying method I, II or III to calculate design ground strain; see Fig. 13.6b). The structure of method I, II or III is complicated since most of the design procedures written in the original flow charts are put into these diagrams; see Fig. 13.6c) where an empty box corresponds to a design procedure of the original flow chart. As mentioned several times, the original flow chart shown in Fig. 13.6a) is suitable for the purpose of design, i.e., calculating ground strain in designing a pipeline, since the procedures of calculating design variables are presented sequentially. However, most calculations are too detailed for non-professionals who may fail even to understand the fact that the seismic strain is calculated by using data of strong ground motion and surface layers through a method of design earthquake I, II or III; see Fig. 13.6b). Therefore, dividing the chart into the two kinds of diagrams will help non-professionals easily understand the design code.

Next, an earthquake resistant design code of concrete piers is studied; see Fig. 13.7. The original flow chart is shown in Fig. 13.7a). Since the chart has design procedures at Level 1 and 2, it is rewritten as two diagrams, the reconstructed flow chart and the object contents, as shown in Fig. 13.7b) and c), respectively. Unlike the general earthquake resistant design, the reconstructed flow chart includes an object `designBridgeRestrainer` which is inherent to the design of piers. Through the method DRA (dynamic linear elastic response analysis), this object checks the safety, which is expressed as an output of `checkSafety` in Fig. 13.7c). The object `design` includes⁹ two methods, SHSM (seismic horizontal strength method) and NL-DRA (non-linear dynamic response analysis), and one of them is chosen depending on the structure type.

Also, a flow chart of an earthquake resistant design code for steel piers is reconstructed; see Fig. 13.8. The original chart is shown in Fig. 13.8a). As is seen, the flow chart examines the safety depending on the three types of failure, bending, bending to shear, and shear failure, and either the ultimate horizontal strength (UHS) or the shear strength (SS) is used to compute the strength. In the reconstructed chart shown in Fig. 13.8b), three objects, B (bending), BtoS (bending to shear) and S (shear), are used to show the three analyses which are related to the three failure types, and two objects UHS and SS are used to determine the strength. This flow chart of steel piers appears more complicated compared with the reconstructed flow chart of concrete piers shown in Fig. 13.7b). In the steel pier's diagrams, however, each object uses only one datum, and the object content is much simpler than the content of objects which are used for the concrete pier's diagrams; compare Fig. 13.8c) with Fig. 13.7c). As is seen, a set of the two diagrams, the reconstructed flow chart and the object content, clarify the difference between the design codes of concrete piers and steel piers.

Finally, an earthquake resistant design code for residential buildings is studied; see Fig. 13.9. The original flow chart is shown in Fig. 13.9a). The chart starts from

⁹These methods, SHSM and L/NL-DRA, are commonly used for other structures.

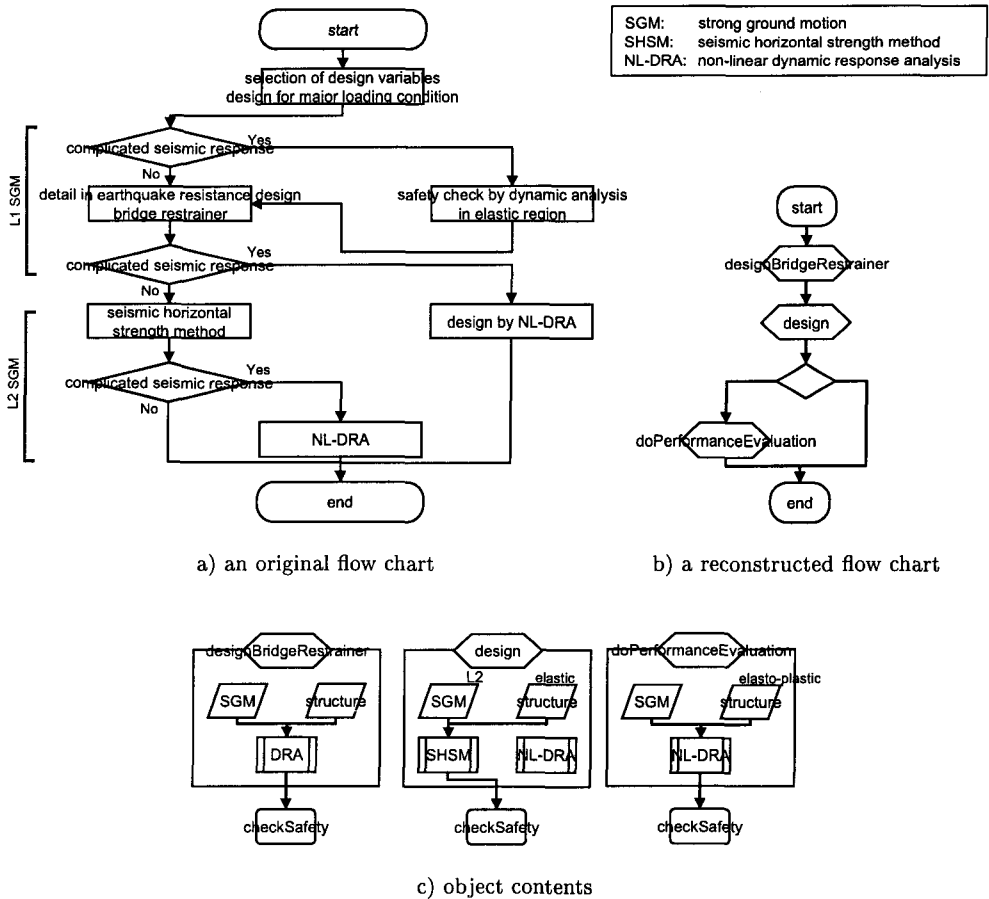


Fig. 13.7 A flow chart of earthquake resistant design of concrete piers.

the classification of a building according to its height, and goes on to modeling and calculation depending on the classification. The performance evaluation is made in all cases except for short buildings. The reconstructed flow chart shown in Fig. 13.9b) emphasizes this flow using only three objects, DSA (dynamic stress analysis), NL-DRA (non-linear dynamic response analysis) and QSEPSA (quasi-static elasto-plastic stress analysis). The contents of these objects are shown in Fig. 13.9c). As is seen, the contents are similar to each other except that the datum **structure** is elastic for DSA or elasto-plastic (EP) for NL-DRA and QSEPSA and that the objects use their own methods. Although they are not presented here, the structures of these methods are different from each other. Furthermore, for residential buildings, the seismic performance evaluation is studied in detail; see Fig. 13.10. The original flow chart is shown in Fig. 13.10a). This chart is slightly different from the previous flow chart, and the basic is the comparison of the cross section strength and the force which is determined by combining the results of static and dynamic analysis.

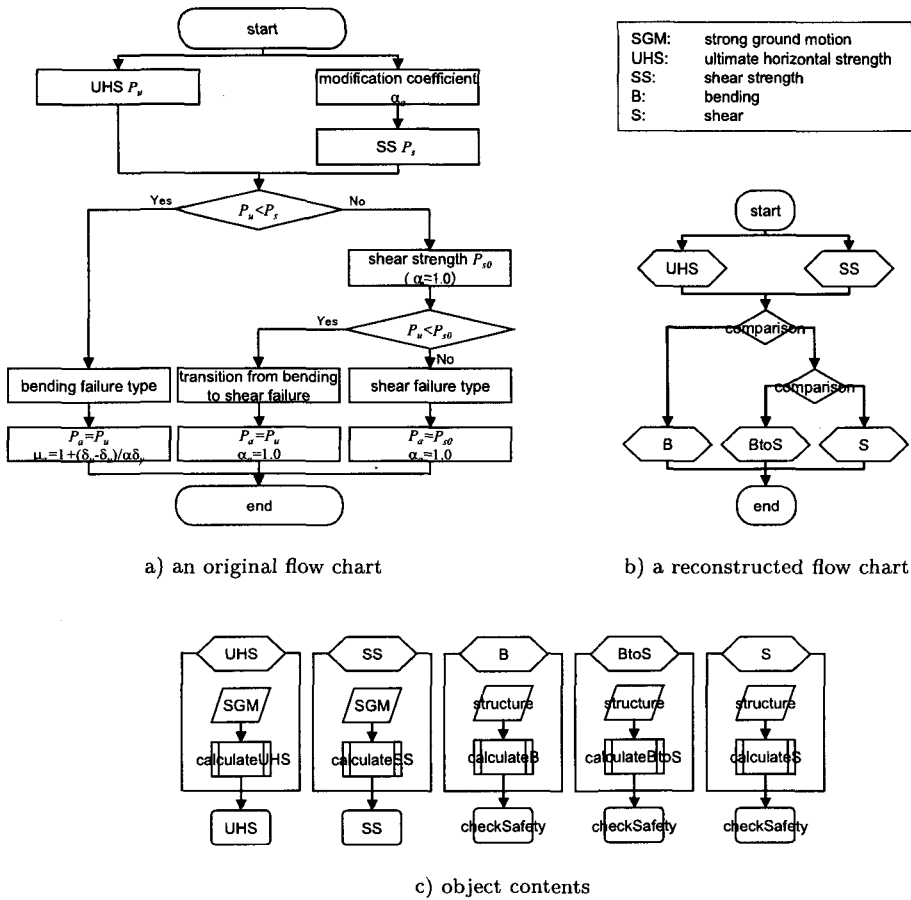
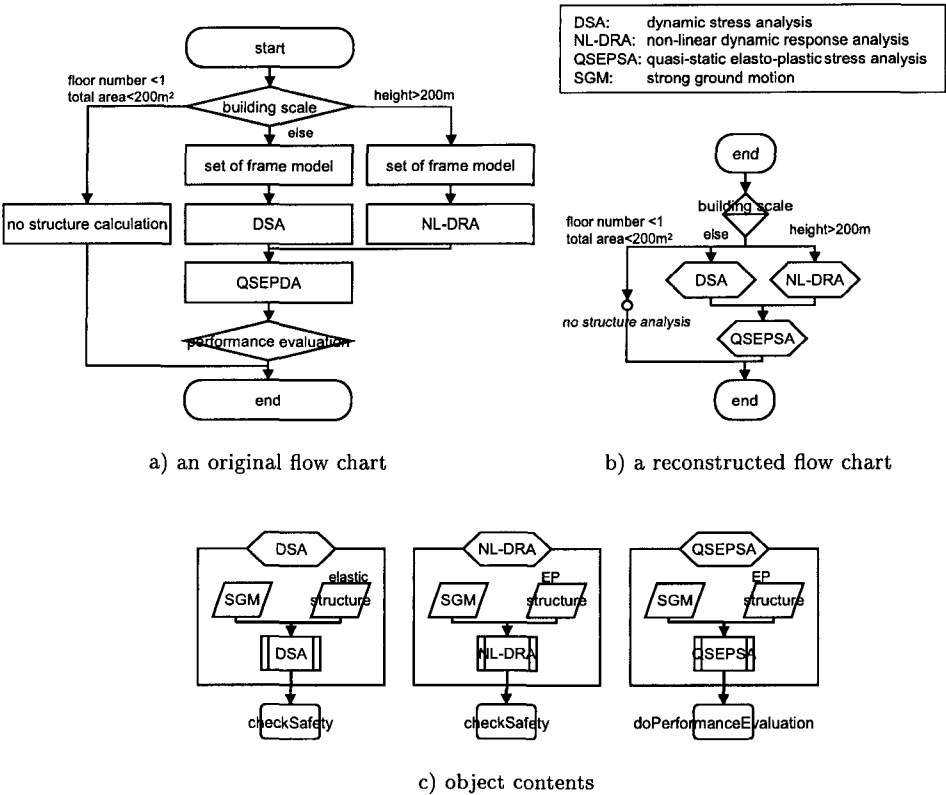


Fig. 13.8 A flow chart of earthquake resistant design of steel piers.

The reconstructed flow chart shown in Fig. 13.10b) clarifies this point, using four objects; the outputs of the objects SA (static analysis) and NL-DRA (non-linear dynamic response analysis) are used by the object FC (force combination), and the output of this object, which is the force, is compared with the output of the object CSS (cross section strength), the strength. The contents of these objects are presented in Fig. 13.9c); note that the outputs of the objects SA and NL-DRA are designated by static and dynamic cross section forces (CSF), and the object FC uses them as the data.

13.3 Example of Standardization

Now, an attempt is made to standardize the description style of diagrams, which are obtained by rewriting the flow chart of several earthquake resistant design codes. The standardization is aimed at rewriting codes in a common description style for



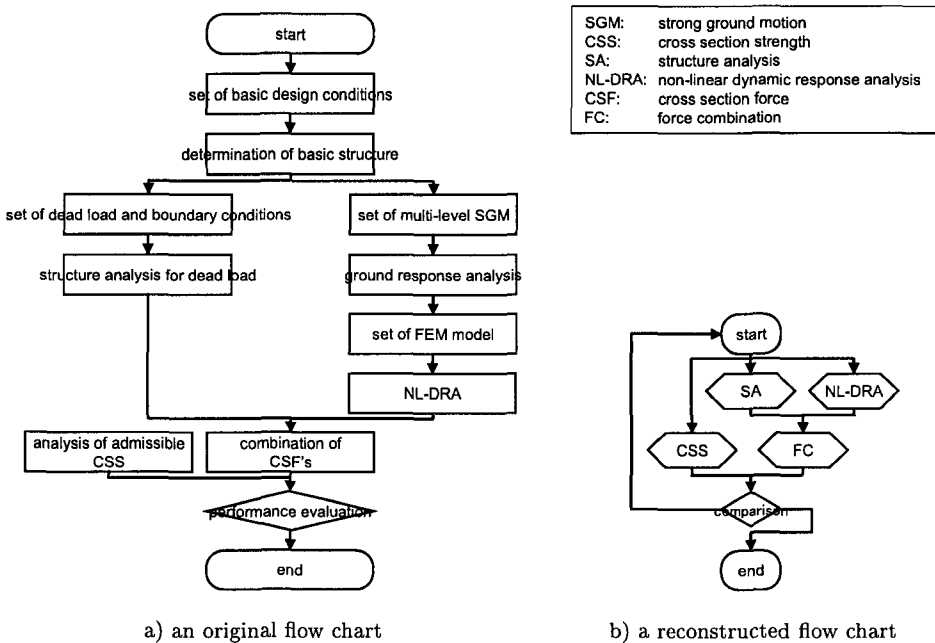
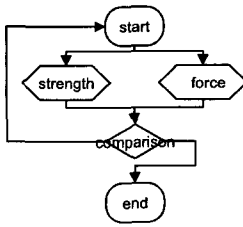


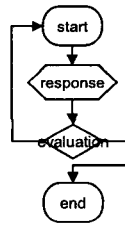
Fig. 13.10 A flow chart of seismic performance evaluation of buildings.

a method `calculateStrength`. Objects similar to force have two data about a structure and an earthquake, as well as a method of computing force. The object `force` thus must have two data `structure` and `earthquake` (which includes data `strong ground motion`) and a method `calculateForce`. The object `response` for the performance evaluation has the same content as the object `force` since both objects need to calculate the response of a structure. There is a difference in the method, and the object `response` has a method `analyzeResponse` which outputs a certain measure for the structure response.

In Fig. 13.11, presented are the two reconstructed flow charts, which will be used to describe standardized earthquake resistant design code; a) is for the design, and the objects `strength` and `force` are used; and b) is for the performance evaluation, and the object `response` is used. These charts correspond to the basic design

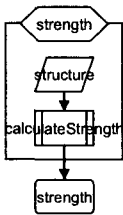


a) design

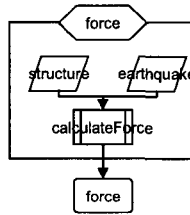


b) performance evaluation

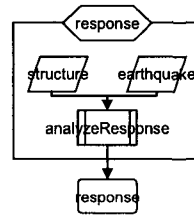
Fig. 13.11 A reconstructed flow chart for standardized earthquake resistant design code.



a) strength



b) force



c) response

Fig. 13.12 Object contents for standardized earthquake resistant design code.

procedures at Level 1 of the design and the performance evaluation, respectively. The contents of these three objects are shown in Fig. 13.12a)~c). The objects have data of **structure** and **earthquake**, but their methods, **calculateStrength**, **calculateForce** and **analyzeResponse**, are different.

It should be emphasized that the reconstructed flow charts and the object contents shown in Figs. 13.11 and 13.12 are the simplest diagrams; for instance, the object contents shown in Fig. 13.12 are described in terms of data and methods which are more or less commonly used in the earthquake resistant design codes which are analyzed in Sec. 13.1. Depending on an earthquake resistant design code, the object contents can be more complicated. By implementing special data or methods which are inherent to the code, the object becomes a special object which is particularly used to express the design code. Such implementation¹⁰ is called *inheritance*. As an example, Fig. 13.13 shows an object of **L2response**, which is made by implementing a datum **ground** and a method **analyzeEarthquake** to the original object **response**. Using the inheritance is helpful for non-professionals to understand the object content. One object consists of the following two kinds of data and methods; 1) data and methods which are commonly included in similar

¹⁰This is an advantage of using objects in programming. Note that in c++, an original object is called a superclass (or a basic class) and a new object is called a subclass (or a derived class). In applying the inheritance, common names are given to the data and methods of a subclass. In the main text, a new method, **analyzesEarthquake**, is introduced in **L2response**. This actually violates the inheritance although it distinguishes **L2response** from **response**.

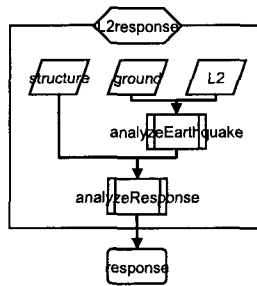


Fig. 13.13 The content of an object **L2response** which is made by inheritance of an object **response**.

objects; and 2) data and methods which are particularly used to describe the object. Therefore, the similarity and difference of codes are clearly seen when the object content is described in terms of these two kinds of data and methods.

According to the diagrams shown in Figs. 13.11 and 13.12, the four earthquake resistant design codes for gas pipelines (GP), concrete piers (CP), steel piers (SP) and residential buildings (AB), are standardized. These codes are shown in Fig. 13.6~13.9. The results are presented in Figs. 13.14 and 13.15. These four codes share the reconstructed flow chart as shown in Fig. 13.14. This chart is actually the one shown in Fig. 13.11a); although the design codes for AB has the performance evaluation, it is omitted in the figure for simplicity. The object content for the four design codes is different, and they are presented in Fig. 13.15; a), b), c) and d) are for GP, CP, SP and AB, respectively. As is seen, the contents include common data of **structure** and **earthquake** but each object has its own method; some methods such as L/NL-DRA (linear/non-linear dynamic response analysis) are commonly used. These objects are made by applying inheritance of the objects **strength** and **force** shown in Fig. 13.12; the method **calculateStrength** or **calculateForce** is omitted in Fig. 13.15 for simplicity.

The standardization leads to a common flow chart, Fig. 13.14, which corresponds to the basic design procedures at Level 1. Although the fact that the comparison of strength and force is the basic part of design is well understood by professional engineers, even non-professionals can understand this fact with the aid of a simple chart shown in Fig. 13.14. They also find out that the comparison of strength and force to find suitable design variables is a common task for various structures. At the same time, the difference in the object content, Fig. 13.15a)~d), clarifies the specialties of each earthquake resistant design code. As is seen, each code has a particular method¹¹ which uses data about a structure and an earthquake. It is expected that the present standardization is effective to help non-professionals understand various earthquake resistant design codes, by presenting design procedures at Level 1 or Level 2 as the flow chart of objects or the object content which explains

¹¹The difference will be more clearly seen if the method structure is presented.

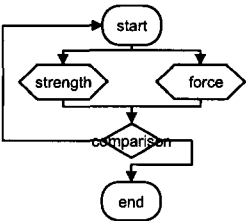


Fig. 13.14 A standardized flow chart for four earthquake resistant design codes.

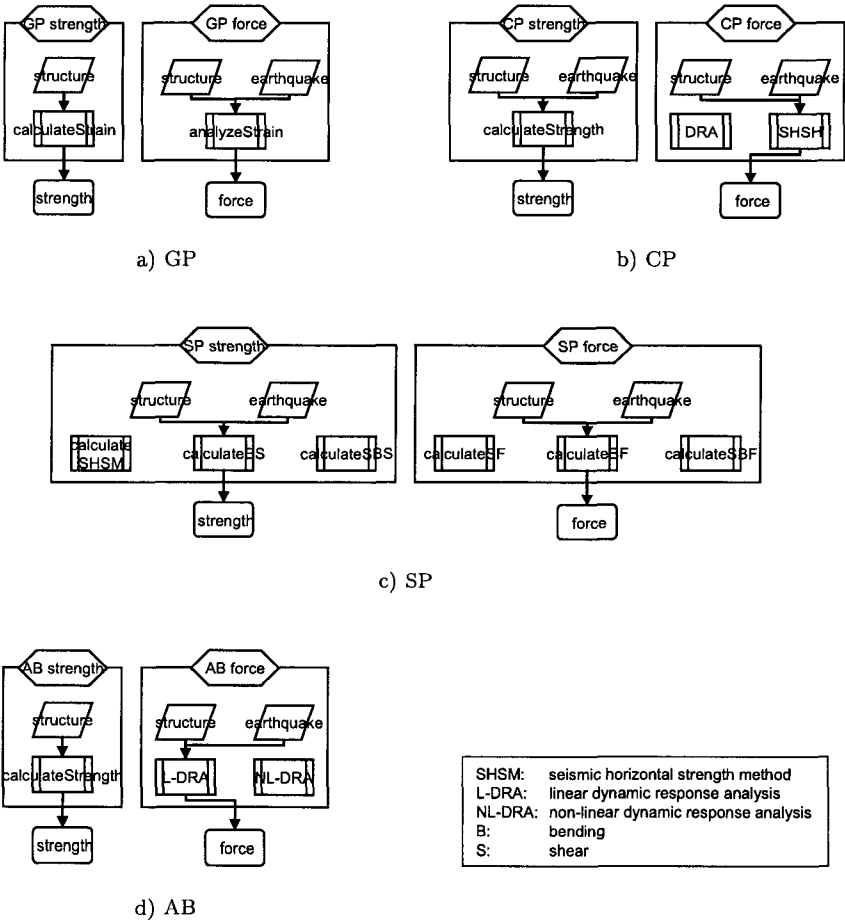


Fig. 13.15 Standardized object contents for four earthquake resistant design codes.

key calculation of the design codes, respectively.

It should be pointed out the description style of the standardized flow chart is different from that of the reconstructed flow chart which is obtained by rewriting specific earthquake resistant design codes in terms of objects. For instance, the flow chart shown in Fig. 13.9b) describes DRA (dynamic response analysis) as an object while DRA is described as a method in Fig. 13.15d). As mentioned, using an object or a method for one design procedure is confusing. Some improvement is surely needed to establish a better common description style. It depends on the objective of a diagram whether an object or a method is used to express a particular design procedure. From this viewpoint, the flow chart and the object content shown in Figs. 13.14 and 13.15 are more transparent in understanding the design codes than the original flow charts.

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Appendix A

Earthquake Mechanisms

In the context of seismic analysis and design, there are two important aspects of earthquake phenomena. The first aspect is the rupture of a fault which is located in the crust or between a plate boundary; an earthquake wave is emitted from the fault when it ruptures. The second aspect is the related ground quaking which shakes buildings and structures. Ground motion is a term which means this ground shaking, and strong ground motion is a ground motion with larger amplitude which can cause structural collapse in some cases. In order to achieve higher safety against earthquake hazards, it is desirable to accurately predict a possible distribution of strong ground motion. To this end, comprehensive understanding of the earthquake phenomena is important.

This appendix focuses on the first aspect of the earthquake phenomena; the main body of this book is related to the second aspect. As shown in Fig. A.1, the earthquake phenomena viewed on the geological length scale consist of the three processes, fault mechanism, wave propagation and wave amplification. The resulting strong ground motion is thus affected by the characteristics of these processes. Understanding the mechanisms of the three processes and the effects of their characteristics on strong ground motion is the basics of the comprehensive understanding of the earthquake phenomena.

A.1 Plate Tectonics and Active Faults

An earthquake¹² is produced by localized or widespread rupture of a fault within the Earth's crust. This fault is called a *source fault*, and waves are transmitted from the source fault. The waves propagate in the heterogeneous crust which consists of geological layers, and reflection and refraction occur when the waves pass through the interfaces between different layers. Near the ground surface, the crust structure drastically changes from hard rock layers to soft soil layers. The direction of wave propagation becomes more vertical and the amplitude is magnified when the waves

¹² Although they are often mixed, an earthquake (i.e., the failure of the crust) and strong ground motion (i.e., the ground shaking) should be distinguished.

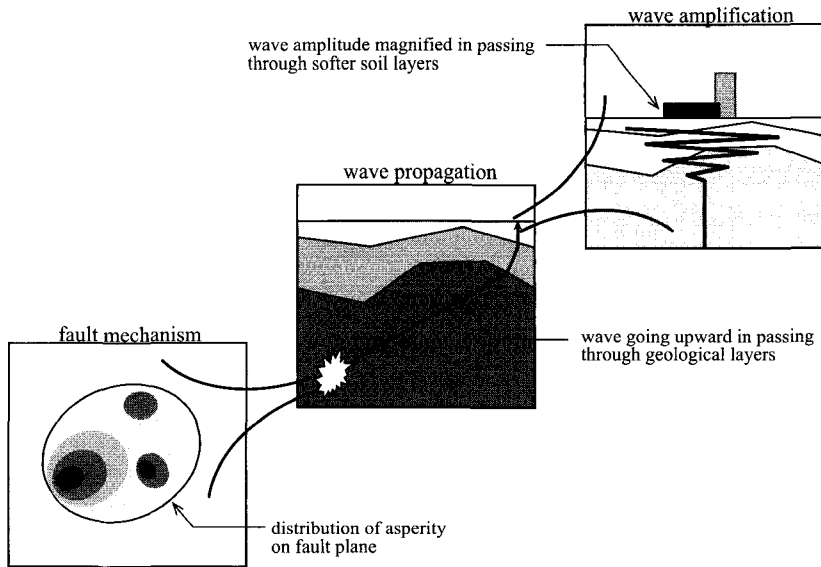


Fig. A.1 Three processes of an earthquake.

approach the ground surface. The resulting strong ground motion is influenced by the characteristics of the three processes explained above, namely, the fault mechanism characteristics, the path characteristics of the geological structures and the amplification characteristics of the ground structures. For instance, a *magnitude* of an earthquake (the magnitude on the Richter scale or the moment magnitude) is a characteristic of the fault mechanism. The path characteristics and the amplification characteristics can change the seismic intensity at a site of interest to a considerable extent.

By definition, seismology studies earthquakes. The current major concern is the fault mechanism; see, for instance, [Irikura (2002)]. The wave propagation or amplification process is not a major research topic, except for clarifying the underground structures on the geological length scale; see, for instance, [Magistrale *et al.* (1996)]. It is recommended to study basic textbooks on seismology; [Bolt (1999)] is a good introduction of seismology and related geology; and [Aki and Richards (2002)] is suitable for earthquake engineers since it is more mathematically oriented; see also [Lay and Wallace (1995)], [Rice (2001)] and [Lee *et al.* (2002)]. For the wave propagation and amplification processes, [Aki (1988)] gives a comprehensive overview; see [Bielak and Christiano (1984)], [Kawase and Aki (1989); Kawase (1996)], [Kikuchi and Ishida (1993)] and [Spudich and Iida (1993)]; see also [Bardet and Davis (1996)], [Toshinawa *et al.* (1999)] and [Luco (2004)] for recent works on these subjects. The textbook mentioned above, [Irikura (2002)], provides a comprehensive review of recent achievements of seismology related to strong ground motion; see also [Sekiguchi *et al.* (1996); Irikura and Miyake (2003)].

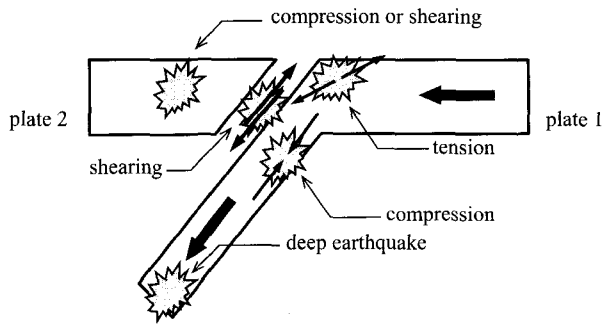


Fig. A.2 Subduction of a plate and induced earthquakes.

The primary driving force of an earthquake is the movement of *plates*; the plates form the Earth's crust and move at speed of $1\sim 10[\text{cm/year}]$ due to the flow of mantle beneath the plates. Each plate has its own velocity vector, and separation or conjunction takes place at the interfaces between two or three plates. This causes failure of the plates or failure at the interface between the plates; see Fig. A.2. There are two types of earthquake, an inter-plate earthquake and an intra-plate earthquake. The inter-plate earthquake is one which occurs on the plate boundary; its scale is relatively large and its period is in the order of $10^2[\text{year}]$. The intra-plate earthquake takes place within a plate near the interface, with relatively small scale and longer period of the order of $10^3[\text{year}]$. In Fig. A.2, it is shown that the inter-plate earthquake is caused by shearing, while intra-plate earthquakes are caused by shear, tensile or compressive stress acting within the plate; out-of-plane shearing also causes inter- or intra-plate earthquakes.

As mentioned above, failure takes place on a source fault, a plane segment on a plate boundary or a crack inside of a plate. The source fault is usually located below a depth of $10[\text{km}]$, and a large earthquake which generates strong ground motion has the fault size of $10\sim 100[\text{km}]$. The failure of the source fault generally follows a cycle of the three processes: 1) the stress accumulation over $100\sim 1000[\text{years}]$ in which the source fault gradually carries larger stress than the surrounding crust; 2) the formation of a failure nuclei in the time period of roughly several hours or a day; and 3) the rapid rupture of the fault from the nuclei in a few tens of seconds. The last rapid rupture process actually generates earthquake waves. There are cases when the duration of the rupture is in hours or a day, and such behavior can cause tsunami without generating earthquake waves of ordinary frequencies. This is called a *slow* or *silent* earthquake. Fault mechanism is characterized by parameters such as the location, direction and size of the fault plane. The size of the fault plane affects the magnitude of an earthquake. The size also determines the distribution of frequency components; as the size of the fault plane becomes larger, the generated earthquake waves tend to have larger components in lower frequencies. As will be explained later, earthquake waves emitted from the fault change depending on the

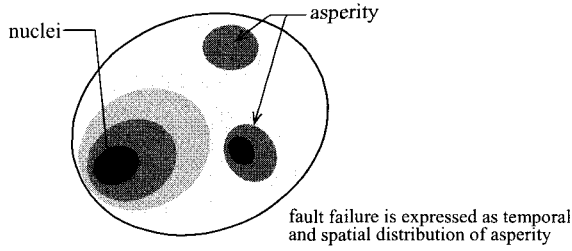


Fig. A.3 A schematic view of the asperity distribution on a source fault.

relative direction at which they propagate. Recently, it is found that failure is not uniform on the fault plane and some parts have a larger displacement gap after the failure. These parts are called *asperity*; see Fig. A.3. This physical model of fault mechanism has been used in earthquake resistant design, in order to synthesize an input earthquake wave for the analysis of an extremely important structure such as nuclear power plants.

Earthquake waves in the crust can be regarded as an elastic wave. The crust is isotropic but not homogeneous; it consists of several distinct geological layers, and the P-wave or S-wave velocity in these layers is of the order of kilometer per second, and the velocity decreases in shallower layers. The decrease in the wave velocity leads to refraction of the earthquake waves; the waves tend to propagate upwards as they approach the ground surface, as shown in Fig. A.4. Near the ground surface, therefore, most of the earthquake waves propagate vertically, not in the direction from the hypocenter of the earthquake. Since the amplitude of S-waves is larger than that of P-waves, horizontal components of strong ground motion are dominant and they are usually used in earthquake resistant design. It should be noted that due to the wave propagation process in the crust, which is modeled as a half-plane, surface waves, such as the Love wave or the Rayleigh wave, are generated in soft alluvium layers. The surface waves have an amplitude which exponentially increases near the ground surface; see Sec. 4.2. In general, however, the amplitude of the surface waves is smaller than that of S-waves, and the surface waves do not cause damage to structures unless they have longer periods of self-excitation.

While the fault mechanism of the source fault and the wave propagation in the crust are the processes on the geological length scale, the wave amplification in the ground structures is a process on the meter or sub-meter order length scale; the amplification is significant in soft layers of thickness¹³ 10~100[m]. Such layers, sometimes unconsolidated, have the S-wave speed around 200[m/sec]. Wave components of around 1[Hz] are magnified several times although the amplification depends on the layer thickness and the layer properties; see Fig. A.5. Besides small

¹³The wavelength of 3~10[Hz] becomes 100~30[m] when the wave velocity is 300[m/sec]. Thus, the fundamental model of ground vibration lies in this frequency range if the thickness of the layers and the shear wave velocity are 10~100[m] and around 300[m/sec]; a quarter sinusoidal function gives the fundamental mode since the ground surface is traction-free.

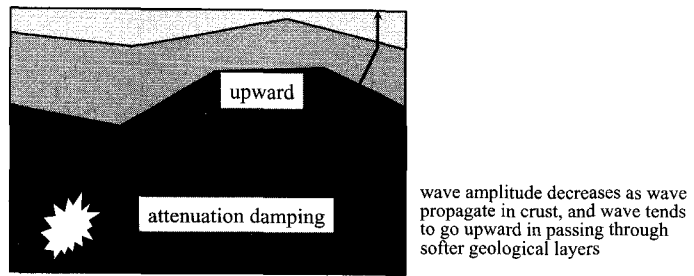


Fig. A.4 Refraction of an earthquake wave in geological structures.

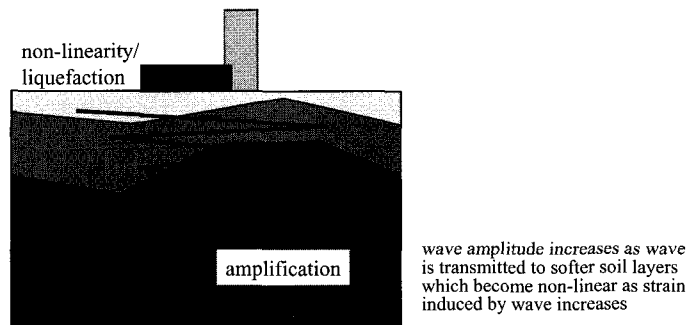


Fig. A.5 Amplification of an earthquake wave near the ground surface.

stiffness, soil in the surface ground layers behaves non-linearly¹⁴ when subjected to large strong ground motion. [Ishihara (1996)] is recommended for readers who are interested in the basics of non-linear soil properties subjected to dynamic loading; [Finn (2000)] is also¹⁵ recommended; see also [Tatsuoka *et al.* (1993)], [Mohammadioun (1997)], [Muravskii and Frydman (1998)] and [Erlingsson (1999)]. This is due to the nature of soil which consists of soil particles, water and air. If the water level is high, liquefaction can take place. The basic mechanism of liquefaction is as follows: subjected to repeated loading of strong ground motion, soil particles lose contacts with surrounding particles and water attains higher pressure, so that the soil particles are being floated in the water; see [Arango *et al.* (2000)] for a recent works on liquefaction.

The amplitude of strong ground motion is basically determined from the mag-

¹⁴Non-linear property of soil under static and dynamic state have been extensively studied; see, for instance, [Duncan and Chang (1970)], [Nishi and Esashi (1978)], [Ohuchi *et al.* (1994)] and [Semblat and Luong (1998)].

¹⁵See [Semblat and Luong (1998)], [Miles and Ho (1999)], [Glaser and Baise (2000)], [Rahhal and Lefebvre (2000)], [Stamatopoulos *et al.* (2000)], [Wang *et al.* (2000)], [Yang *et al.* (2000)], [Pecker *et al.* (2001)] and [Tabesh and Poulos (2001)] for recent experiment works on soil properties; see also [Yoshida *et al.* (2001)], [Cremer (2002)], [Zhang and Kimura (2002); Zhang *et al.* (2003)], [Hwang *et al.* (2003)] and [Tamari *et al.* (2003)].

nitude of an earthquake and the distance from the source fault. When strong ground motion is quantified in terms of PGV (peak ground velocity) or PGA (peak ground acceleration), such a seismic index can be estimated by means of an empirical formula, called an *attenuation damping equation*; see, for instance, [Wells and Coppersmith (1994)] and [Olafsson and Sigbjornsson (1999)] for a concise list of related references. In general, the index increases as for a larger earthquake and a shorter distance. There are a variety of the attenuation damping equations which use the *moment magnitude* instead of the *magnitude* on the Richter scale or the distance from the hypocenter instead of the epicenter. These equations are derived from statistical analysis of past records of strong ground motion, and are able to provide the *mean* or *average* behavior of attenuation damping. However, the accuracy is not high since it ignores the fault mechanism characteristics of the source fault and the amplification characteristics at a target site. It is possible to make a better equation by making suitable modifications to account for these characteristics. Still, the limitation in predicting the seismic index by using one single equation cannot be overlooked.

The radiation pattern and the directability are the two key characteristics of the earthquake propagation process. The radiation pattern reflects the failure of a source fault, which emits waves of larger amplitude in particular directions. As an example, the radiation pattern for a lateral sliding fault is presented in Fig. A.6; a P-wave in the direction of $\pm 45[\text{deg}]$ from the source fault has a larger amplitude while the amplitude of an S-wave is maximized in the normal and parallel directions of the source fault. Near the lateral fault, the direction of one maximum shear stress is parallel to the source fault and the direction of the two principle stresses (the maximum tensile and compressive stress) is $\pm 45[\text{deg}]$ with respect to the fault plane. Hence, the radiation pattern of S-wave and P-wave is consistent with the maximum shear stress and the principal¹⁶ stress, respectively. Due to this radiation pattern, the direction of S-waves is normal to the fault at the initial stage. The directability is another characteristic which causes differences in the distribution of strong ground motions. In general, the amplitude of a wave going in the direction that the rupture process on the fault plane propagates is larger than that of a wave going in the opposite direction. This is because waves generated by faults are being accumulated in the forward direction; see Fig. A.7. The directability is significant near the source fault, as clearly observed in the Great Hanshin Awaji Earthquake.

A.2 Earthquake as Wave Propagation

The determination of strong ground motion that is input to a structure is a key issue in earthquake resistant design; once the input motion is given, structure responses are computed by taking advantage of numerical methods which are developed in

¹⁶Pulling is in the direction of the maximum tensile stress, and pushing in the direction of the maximum compressive stress.

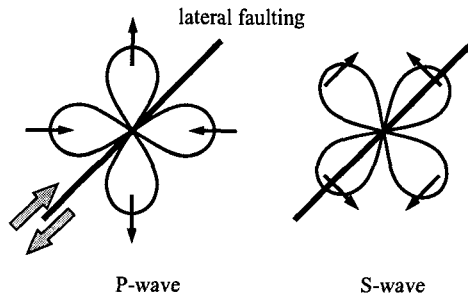


Fig. A.6 A radiation pattern of an earthquake wave.

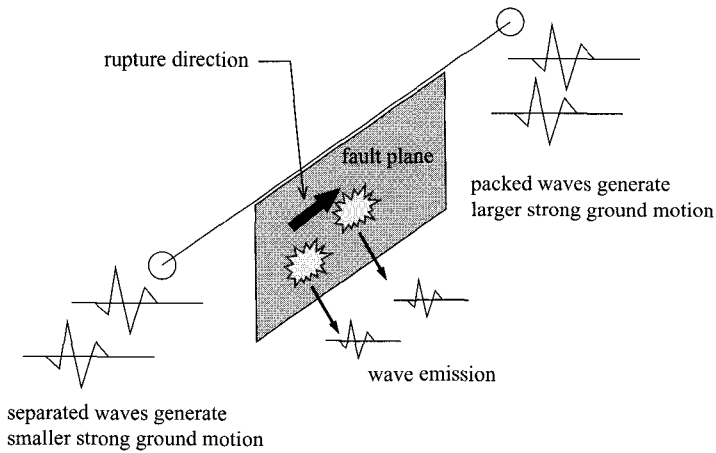


Fig. A.7 Directability of an earthquake wave.

structure mechanics and computational mechanics. In analyzing dynamic response of a structure, the effects of *soil-structure interaction* cannot be overlooked; the interaction effects between a structure and surrounding ground layers sometime cause larger external forces to the structure.

A.2.1 *Determination of input strong ground motion according to earthquake scenario*

As explained above, strong ground motion is induced as the consequences of the three earthquake processes, namely, the fault mechanism, the wave propagation and the wave amplification. When a suitable fault mechanism is assumed for a source fault of interest, it is possible to estimate a possible strong ground motion that takes place at a target cite, by evaluating the path characteristics and the amplification

characteristics. This is the basic procedure¹⁷ of synthesizing earthquake waves according to an *earthquake scenario*. Although it is not a tough task to find a reliable fault mechanism, the synthesized earthquake wave gives an accurate estimate of strong ground motion if the fault mechanism is correct. In this case, constructing a crust model on the geological length scale and a ground structure model on the engineering length scale is essentially important.

Even if the magnitude is small, earthquake waves, which are triggered near a fault of interest and which are observed at a target site, carry information related to the path characteristics and the amplification characteristics. This implies that a possible strong ground motion can be synthesized by making use of the observed data of ground motion which is caused by small earthquakes or tremors. Indeed, if a suitable fault mechanisms is assumed, the resulting strong ground motion is calculated as an assembly of the ground motion data which are suitably¹⁸ magnified. This method of synthesizing a possible strong ground motion is called *empirical Green's function method*; see [Irikura (2002)]. The data of ground motions which are caused by small earthquakes represents Green's function which provides a ground motion for a given fault mechanism; see [Hartzell (1978)].

Statistical Green's function method is proposed as an alternative to empirical Green's function method that requires observed data of earthquakes; see [Kamae *et al.* (1998)] and [Irikura (2002)]. This method constructs an artificial small earthquake considering a suitable fault mechanism. One common characteristic of the fault mechanism is the ω^{-2} -rule, i.e., the amplitude of a frequency component of displacement is linear¹⁹ to the inverse square of the frequency. Another common characteristic is that the displacement amplitude is constant for small frequencies; the critical frequency at which the amplitude becomes constant is called a *corner frequency*. Depending on the magnitude of an earthquake, the corner frequency and the value of displacement amplitude at the corner frequency change; the displacement amplitude at the corner frequency is inversely proportional to the cubic of the corner frequency. These two characteristics need to be taken into consideration in applying statistical Green's function method.

A *hybrid method* is quite different from empirical and statistical Green's function methods since it does not use measured data. This method combines a method of calculating longer frequency wave components and a method of calculating shorter frequency wave components. Longer frequency components are obtained by means of the three-dimensional analysis of wave propagation in the crust, and shorter frequency components are estimated by the one-dimensional analysis of wave am-

¹⁷A spatial and temporal distribution of asperity is usually used to describe the scenario.

¹⁸The Fourier spectrum of an earthquake wave changes depending on its magnitude. In general, longer period components are included and the amplitude of all period components becomes greater as the magnitude of an earthquake increases. See [Montaldo *et al.* (2003)] for recent works of synthesizing strong ground motion.

¹⁹This rule is found by statistical analysis of past data of strong ground motions that have been accurately measured.

plification in shallow ground structures; see Chapter 5 for the macro-micro analysis method which is similar to the hybrid method. This method is reliable provided that a model of fine quality is constructed for the crust and the ground structures.

A.2.2 Soil-structure interaction

While strong ground motion is input to a structure through ground layers, the reaction of the vibrating structure is absorbed in the ground layers. This causes dynamic interaction between the structure and the ground layers, which is called soil-structure interaction. In general, the interaction effects tend to increase²⁰ for softer ground layers; larger deformation is caused in the softer ground layers subjected to larger strong ground motion due to the smaller stiffness and sometimes due to the material non-linearity. It should be pointed out that while large displacement is provided to a structure by the ground deformation, reacting force cannot exceed the strength of soil which consists of the ground layers. Soil-structure interaction sometimes changes strong ground motion which is input to the structure. For instance, a pile foundation reduces ground shaking since stiff piles strengthen soft ground layers. Large-scale structures are thus put on pile foundations that reach the bedrock with shear wave velocity faster than 300[m/s]; reaction from the stiff bedrock is used instead of the reaction of soft ground layers. In this case, soil-structure interaction becomes more complicated, although the amplitude of waves input to the structure is decreased. The interaction between the piles and the ground layers needs to be considered; see [Kimura and Zhang (2000)] and [Konagai and Ahsan (2002)] for a cosine list of related references.

Interaction effects between neighboring structures should be distinguished from soil-structure interaction. It is certainly true that resonance can take place if nearby structures have similar natural frequencies; beating of two structures may lead to pounding of structures²¹ which causes serious damage to each other. At this moment, however, research on such structure-structure interaction effects is quite limited.

A spring model²² is used to express the interface between structure and soil, when a set of a structure and ground layers are simultaneously analyzed in order to evaluate the effects of soil-structure interaction; see, for instance, [Karin-ski *et al.* (2004)] for a list of related references. Due to the large difference in stiffness, the continuity of displacement is lost at the interface, and a proper spring model needs to be implemented. A dash pot model is used when viscosity is consid-

²⁰On the other hand, softer ground layers have a longer natural frequency, and resonance between the ground layers and a structure of shorter natural frequency happens less likely. Due to damping, such resonance is less critical to a structure, compared with larger acceleration caused by the larger amplification of softer ground layers.

²¹Pounding is a serious problem of bridge structure of longer span.

²²There are cases when ground layers are modeled as a few springs, called a sway-rocking model, or a set of springs which connect the structure and the bedrock mass. Non-linear springs are often used in this modeling.

ered; forces are generated due to the difference in velocity of the structure and the ground. While the spring model is simple, the determination of spring constants or spring properties is not trivial. This is because the accurate measurement²³ of soil behavior near the interface is difficult.

²³More advanced measurement techniques will be needed to accurately estimate the mechanical properties of the interface and to construct a reliable model for it.

Appendix B

Analytical Mechanics

The essence of analytical mechanics is the utilization of a *Lagrangian* and a *Hamiltonian*. The Lagrangian is a function for coordinates, and always satisfies the *Lagrangian equation* even if the coordinates are transformed to other coordinates. To explain this, a Lagrangian for two vectors, $\{a_i\}$ and $\{b_i\}$, is considered; this Lagrangian is denoted by $\mathcal{L}(\mathbf{a}, \mathbf{b})$. It is assumed that there are coordinates $\{x_i\}$ which satisfy

$$\frac{\partial \mathcal{L}}{\partial a_i}(\mathbf{x}, \dot{\mathbf{x}}) - \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial b_i}(\mathbf{x}, \dot{\mathbf{x}}) \right) = 0. \quad (\text{B.1})$$

Here, x_i is a function of time t , and $\dot{x}_i$ is derivative with respect to time. This equation is the Lagrangian equation, and it is equivalent with the equation of motion. For instance, if $\mathcal{L}$ is given as

$$\mathcal{L}(\mathbf{a}, \mathbf{b}) = \sum_{i,j} \frac{1}{2} k_{ij} (a_i - a_j)^2 - \sum_i \frac{1}{2} m b_i b_i,$$

Eq. (B.1) yields

$$-m_i \ddot{x}_i + \sum_j k_{ij} (x_j - x_i) = 0.$$

This is the equation of motion for a set of masses $\{m_i\}$ which are connected by linear springs $\{k_{ij}\}$. When coordinates $\{x_i\}$ are transformed to $\{x'_i\}$, another Lagrangian, $\mathcal{L}'$, is defined as $\mathcal{L}'(\mathbf{x}', \dot{\mathbf{x}}') = \mathcal{L}(\mathbf{x}, \dot{\mathbf{x}})$. This $\mathcal{L}'$ satisfies²⁴ the equation whose form is identical with that of Eq. (B.1), i.e.,

$$\frac{\partial \mathcal{L}'}{\partial a'_i}(\mathbf{x}', \dot{\mathbf{x}}') - \frac{d}{dt} \left(\frac{\partial \mathcal{L}'}{\partial b'_i}(\mathbf{x}', \dot{\mathbf{x}}') \right) = 0.$$

Solving the Lagrangian equation becomes simpler when some constraints are prescribed to the original coordinates. By suitably transforming the coordinates, complicated constraints are transformed to easier conditions for the new coordinates.

²⁴This is easily proved with the aid of the chain rule.

When the transformed Lagrangean equation is solved, its solution is transformed to a solution of the original coordinates.

If displacement function is regarded as coordinates, it is straightforward to define a Lagrangean for it. For simplicity, an elastic body, denoted by B , is considered. The Lagrangean is defined as the following functional:

$$\mathcal{L}(\mathbf{a}, \mathbf{b}) = \int_B \frac{1}{2} c_{ijkl} a_{i,j} a_{k,l} - \frac{1}{2} \rho b_i b_i dv. \quad (\text{B.2})$$

By substituting u_i and $\dot{u}_i$ into a_i and b_i , respectively, the following limit²⁵ leads to the wave equation:

$$\lim_{\varepsilon \rightarrow 0} \frac{1}{\varepsilon} \left(\mathcal{L}(\mathbf{u} + \varepsilon \delta \mathbf{u}, \dot{\mathbf{u}}) - \mathcal{L}(\mathbf{u}, \dot{\mathbf{u}}) + \frac{d}{dt} (\mathcal{L}(\mathbf{u}, \dot{\mathbf{u}} + \varepsilon \delta \dot{\mathbf{u}}) - \mathcal{L}(\mathbf{u}, \dot{\mathbf{u}})) \right) = 0.$$

The form of this equation is identical with Eq. (B.1). Indeed, the limit yields $\int_B \delta u_j (-c_{ijkl} u_{k,l})_{,i} + \rho \ddot{u}_j dv = 0$ for arbitrary δu_i .

While it is convenient to use a Lagrangean in solving the equation of motion, the Lagrangean equation is a second-order differential equation with respect to time. Using coordinates and inertias as independent variables, a Lagrangean can be transformed to a Hamiltonian so that the second-order differential equation is replaced by a first-order differential equation which explicitly gives the time derivative of the coordinates and inertias. To this end, new variables, $\{p_i, q_i\}$, are introduced to replace a_i and b_i , as follows:

$$p_i = a_i \quad \text{and} \quad q_i = \frac{\partial \mathcal{L}}{\partial b_i}(\mathbf{a}, \mathbf{b}).$$

Here, a_i and b_i are regarded as implicit functions of p_i and q_i . Next, the Lagrangean equation is rewritten in terms of $\{p_i, q_i\}$. With the aid of Legendre transform, the original variable b_i is implicitly given as

$$b_i = \frac{\partial}{\partial q_i} \left(\sum_j q_j b_j - \mathcal{L}(\mathbf{a}, \mathbf{b}) \right).$$

Thus, Eq. (B.1) is expressed as $\dot{q}_i = \frac{\partial \mathcal{L}}{\partial a_i}$ when b_i is given as $b_i = \dot{p}_i$, or, equivalently, when $\dot{p}_i$ is given as $\dot{p}_i = \frac{\partial}{\partial q_i} (q_i b_i - \mathcal{L})$. In summary, Hamiltonian, $\mathcal{H}$, is defined as

$$\mathcal{H}(\mathbf{p}, \mathbf{q}) = \sum_i q_i b_i - \mathcal{L}(\mathbf{a}, \mathbf{b}), \quad (\text{B.3})$$

and it satisfies

$$\frac{d}{dt} \begin{bmatrix} p_i \\ q_i \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathcal{H}}{\partial q_i}(\mathbf{p}, \mathbf{q}) \\ -\frac{\partial \mathcal{H}}{\partial p_i}(\mathbf{p}, \mathbf{q}) \end{bmatrix}. \quad (\text{B.4})$$

²⁵Certain boundary conditions are prescribed on the boundary, ∂B .

This is the *canonical equation*. Since the Hamiltonian is derived from the Lagrangean, the canonical equation always holds when the Lagrangean equation that is equivalent with the equation of motion holds.

Denoting inertia by $q_i = \rho \dot{u}_i$, a Hamiltonian for the elastic body B is defined as the following functional:

$$\mathcal{H}(\mathbf{p}, \mathbf{q}) = \int_B \frac{1}{2} c_{ijkl} p_{i,j} p_{k,l} - b_i p_i + \frac{1}{2\rho} q_i q_i dv. \quad (\text{B.5})$$

Here, p_i is displacement, i.e., $p_i = u_i$. While functional derivative is used to compute the right side of the canonical equation of Eq. (B.4), it leads to a pair of first-order differential equations for displacement and inertia. If these differential equations are discretized, a pair of vector-valued²⁶ first-order differential equations are derived.

²⁶If p_i and q_i are discretized in the same manner as FEM, then, for nodal displacement and inertia vectors, denoted by $[P]$ and $[Q]$, Eq. (B.5) leads to

$$\frac{d}{dt} \begin{bmatrix} [P] \\ [Q] \end{bmatrix} = \begin{bmatrix} [M]^{-1} [Q] \\ -[M]^{-1} [K] [P] \end{bmatrix},$$

where $[K]$ and $[M]$ are stiffness and mass matrices, respectively.

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Appendix C

Numerical Techniques of Solving Wave Equation

In order to solve the wave equation for solids, numerical analysis methods such as FDM, FEM and BEM are used. For structures, however, it is rare to solve the wave equation or to analyze the wave propagation phenomena; a common practice is the vibration analysis at a frequency of interest. This is mainly because the wave velocity of the structure member is fast and the wave propagation is not observed on ordinary time scale. On the other hand, a bulk body such as the crust or the soil layer which has relatively slower wave velocity is a good target for the wave propagation analysis which solves the wave equation. Indeed, many numerical techniques²⁷ are proposed in seismology; see, for instance, [Inoue and Miyake (1998)] and [Furumra and Koketsu (1998)].

The major numerical techniques of solving the wave equation are categorized as *time integration*, *damping*, *numerical dispersion*, and *artificial boundary*. As explained in Sec. 4.3, time integration is used to numerically solve an ordinary differential equation with respect to time. Damping, or a damping term, is needed to stabilize numerical computation of the wave equation; while some attempts have been made, the physical nature of the damping term which is used in the structure dynamics is not fully clarified. Numerical dispersion means the dependence of the computed wave velocity on its frequency or wavelength due to spatial and temporal

²⁷Traditionally, seismology has used FDM for numerical analysis of the wave equation, and only limited application of FEM has been made. However, FEM can take advantage of some numerical techniques which have been developed in seismology for time integration and temporal and spatial discretization. Typical numerical techniques, which are aimed at analyzing complicated wave propagation processes in the crust by means of FDM, are a *discontinuous grid method*, an unequally spaced grid, a *staggered grid method*, the use of different grid for displacement and stress, and a *pseudo-spectral method*, which uses the Fourier transform in spatial discretization to reduce error in the finite difference approximation; see [Graves (1996)], [Faccioli *et al.* (1997)], [Komatitsh and Vilotte (1998)], [Vidale and Helmberger (1988)], [Aoi and Fujiwara (1999)] and [Pitarka (1999)]; see also [McLaughlin and Day (1994)]. However, there is limitation for FDM to numerically solve the wave equation for a body with complicated configuration. Another key technique used in seismology is parallel computing. Extensive research has been recently made to develop a parallel computing code for FDM or FEM; see [Bao *et al.* (1996)]. Also, BEM with a *fast multi-pole expansion method*, an efficient computation of Green's function, has been studied in seismology; see [Fukui *et al.* (1997); Fukui and Inoue (1998)], [Ortiz-Aleman *et al.* (1998)], [Ichimura (1998)] and [Fujiwara (2000)].

discretization. An artificial boundary is used to get rid of unnecessarily reflected waves when a finite domain is used to analyze waves propagating in an infinite domain or a half plane; see Secs. 5.5, 6.2 and 6.3.

C.1 Explicit Method and Implicit Method

The basic numerical analysis method of solving the wave equation is a method of solving an ordinary differential equation. The target equation is formally expressed as follows:

$$\dot{x} = f(x). \quad (\text{C.1})$$

Here, x is a scalar-valued or vector-valued function of time and f is a function which depends on x only. Note that while the wave equation is the second-order differential equation with respect to time, it can be transformed to a first-order differential equation. For instance, if the wave equation is given as $k\ddot{u} + c\dot{u} + mu = 0$, then, $x = [x_1, x_2]^T$ is defined as $x = [u, \dot{u}]^T$ and f is set as $f = [x_2, -cx_2 - mx_1]^T$. For this setting, the wave equation is rewritten in the same form as Eq. (C.1),

$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -cx_2 - mx_1 \end{bmatrix}.$$

For simplicity, an equal time interval, Δt , is taken to temporally discretize x , and $x(n\Delta t)$ is denoted by x^n . Then, in Eq. (C.1), the derivative of x with respect to time is approximated as

$$\dot{x}((n+1)\Delta t) \approx \frac{x^{n+1} - x^n}{\Delta t}.$$

The error of this approximation is of the order of Δt^2 , i.e., $\mathcal{O}(\Delta t^2)$. When f is evaluated at x^n or x^{n+1} , the following two equations for x^{n+1} are derived as an approximation of Eq. (C.1):

$$x^{n+1} = x^n + \Delta t f(x^n), \quad (\text{C.2})$$

$$x^{n+1} = x^n + \Delta t f(x^{n+1}). \quad (\text{C.3})$$

Since x^n is known, both equations give x^{n+1} ; Eq. (C.2) explicitly determines x^{n+1} while Eq. (C.3) does implicitly. Thus, using Eq. (C.2) or (C.3) to determine x^{n+1} is called an explicit or implicit method, respectively. The explicit method results in simpler calculation although there are cases when numerical computation becomes unstable. The explicit method is stable although it requires a larger amount of calculation. Here, the numerical instability means that error in numerically computing $\{x^n\}$ increases as the time step increases.

The numerical instability is explained using the explicit method as an example. Considered is a case when x is a scalar-valued function and f is a linear function

of x , i.e., $f=\alpha x$ with α being a suitable scalar parameter. It follows from Eq. (C.2) that $\{x^n\}$ forms a geometric series with the following recursive formula:

$$x^{n+1} = (1 + \Delta t \alpha) x^n = (1 + \Delta t \alpha)^{n+1} x^0.$$

When it happens that an error ϵ is included at the m^{th} step, the error at the $m+k^{\text{th}}$ step is evaluated as $(1+\Delta t \alpha)^k \epsilon$. Since Δt is a small but positive number, it leads to an exponential increase of the error if α is a positive²⁸ number. This is the cause of the numerical instability. In this case, choosing a sufficiently small time interval Δt does not help avoiding the exponential increase of the numerical error.

When the wave equation is given as a partial differential equation, some additional cares must be taken in choosing the value of Δt for the numerical analysis. This care is explained using the wave equation for a one-dimensional bar as an example. The following equation, which is equivalent with the wave equation for the one-dimensional bar, Eq. (4.2), is considered:

$$\frac{\partial^2 u}{\partial t^2}(x, t) = v^2 \frac{\partial^2 u}{\partial x^2}(x, t). \quad (\text{C.4})$$

Here, v is a constant wave velocity given as $v=\sqrt{E/\rho}$ with E and ρ being Young's modulus and density. Displacement u is treated as a function in a two-dimensional plane of (x, t) , and discretized by using values at grid points which are equally spaced on the plane; the spacing in the x - or t -axis is Δx or Δt , respectively. The value of u at a grid point $(i\Delta x, n\Delta t)$ is denoted by u_i^n , i.e., a subscript and a superscript are used to denote x and t , respectively. An equation for the grid point values, $\{u_i^n\}$, is derived from Eq. (C.4), with the aid of the middle point finite difference approximation, i.e.,

$$\frac{u_i^{n+1} - 2u_i^n + u_i^{n-1}}{\Delta t^2} = v^2 \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2}.$$

When the values of u_i^n 's and u_i^{n-1} 's are known, the grid values at the next time step, u_i^{n+1} , is computed; this is the solution of an explicit method,

$$u_i^{n+1} = 2u_i^n - u_i^{n-1} + \frac{v^2 \Delta t^2}{\Delta x^2} (u_{i+1}^n - 2u_i^n + u_{i-1}^n). \quad (\text{C.5})$$

Now, the accuracy of this numerical solution is examined by using an analytic solution of Eq. (C.4); the solution is a complex sinusoidal function given as $u=\exp(-ik(x-vt))$ with an arbitrary wave number k and a unit amplitude. The discretized value of this solution, denoted by $\hat{u}_i^n$, is computed as $\hat{u}_i^n=\exp(-ik(i\Delta x-vn\Delta t))$. Substitution of $\hat{u}_i^n$ into Eq. (C.5) and the assumption of $k\Delta x \ll 1$ and $kv\Delta t \ll 1$ lead to

$$\sin \frac{kv\Delta t}{2} \approx \frac{v\Delta t}{\Delta x} \sin \frac{k\Delta x}{2}. \quad (\text{C.6})$$

²⁸If the coefficient α is a complex number, the numerical instability occurs when the real part of α is positive.

In order for Eq. (C.6) to hold, the grid spacings, Δx and Δt , are required to satisfy

$$\frac{\Delta x}{\Delta t} < v. \quad (\text{C.7})$$

This condition²⁹ is called the *Courant condition*. While Eq. (C.7) is derived for the one-dimensional bar problem, it can be extended to the grid spacing of higher dimensional problems.

In applying the explicit method to the wave equation, there are cases when some artificial dispersion appears, which leads to the loss of accuracy. This is called numerical dispersion. The cause of the numerical dispersion is the difference in the wave velocity for the exact solution and the numerical solution. The numerical dispersion that appears when the explicit method is used for time integration is explained using the one-dimensional bar problem as an example again. To this end, Eq. (C.6) is rewritten in terms of the wavelength $\ell = \frac{2\pi}{k}$ as follows:

$$\frac{\omega}{k} = \frac{\Delta x}{\Delta t} \frac{\ell}{\pi \Delta x} \sin^{-1} \left(v \frac{\Delta t}{\Delta x} \sin \frac{\pi \Delta x}{\ell} \right). \quad (\text{C.8})$$

As is seen, the left term differs from the exact wave velocity v depending on the value of $\frac{\ell}{\pi \Delta x}$; as $\frac{\ell}{\pi \Delta x}$ becomes smaller, the apparent phase velocity given by the right side of Eq. (C.8) becomes smaller. The numerical dispersion leads to slower phase velocity for waves of smaller wavelength. It is known that frequency component of the wavelength $\ell < 10\Delta x$ suffers the numerical dispersion. Also, it is known that numerical dispersion becomes larger for smaller $\frac{v\Delta t}{\Delta x}$. Attention should be paid to the fact that even if the grid spacing Δx and Δt satisfy the Courant condition, Eq. (C.7), the numerical dispersion appears for waves of shorter wavelength or higher frequency.

C.2 Analysis of Wave Equation Using FEM

FEM is the most popular numerical method that is used for dynamic analysis of structures. This is due to the applicability of FEM to solve a body with complicated configuration. Also, FEM is suitable to solve non-linear problems that account for the material and kinematic non-linearity or to solve coupled problems of ground and structures in order to evaluate soil-structure interaction. In dynamic analysis, FEM solves an ordinary differential equation with respect to time for nodal displacement vector, which is obtained by spatially discretizing field variables. Denoting nodal displacement vector by $[U]$ and mass, stiffness and damping matrices by $[M]$, $[K]$ and $[C]$, respectively, the object of FEM dynamic analysis is the following differential equation for $[U]$:

$$[M][\ddot{U}(t)] + [C][\dot{U}(t)] + [K][U(t)] = [F(t)], \quad (\text{C.9})$$

²⁹The Courant condition is a necessary condition to have an accurate solution. Even if this condition is satisfied, it is still required that k is sufficiently small so that Eq. (C.6) holds.

where $[F]$ is a nodal force vector corresponding to body forces or boundary conditions. The second term, $[C][\dot{U}]$, represents damping effects. The damping matrix $[C]$ is not rigorously derived from the wave equation, but is introduced to make the numerical time integration stable. Thus, $[C]$ is often given as a linear combination of the stiffness matrix and the mass matrix, i.e.,

$$[C] = a[K] + b[M], \quad (\text{C.10})$$

where a and b are suitably determined coefficients. This $[C]$ is called a *Rayleigh damping matrix*. In the frequency domain, Eq. (C.9) with Eq. (C.10) is rewritten as

$$(-\omega^2 + i\omega b)[M][\mathcal{F}U(\omega)] + (1 + i\omega a)[K][\mathcal{F}U(\omega)] = [\mathcal{F}F(\omega)],$$

where $[\mathcal{F}U]$ and $[\mathcal{F}F]$ are the Fourier transform of $[U]$ and $[F]$ and ω is frequency. As is seen, damping linear to $[K]$ is dominant for smaller ω while damping linear to $[M]$ is dominant for larger ω . Recall that there is no rigorous theory³⁰ to determine a and b ; they are empirically determined in view of natural frequencies³¹ of the structure, although $a=0.05$ or less is usually used.

It is known that the discretized equation that is solved by FEM is *stiff* compared with the original wave equation. This is because the discretized equation only gives an approximate solution which overestimates total strain energy. Also, while the natural frequency of the wave equation is a real number, there are cases when it appears that there are complex-valued natural frequencies due to numerical errors, which produce the numerical instability. The Rayleigh damping matrix is able to suppress such numerical instability.

There have been proposed various kinds of numerical analysis methods in solving a vector-valued ordinary differential equation, Eq. (C.9). As mentioned above, they are divided into two classes, an explicit method of simpler calculation or an implicit method with numerical stability. When Eq. (C.9) is linear to $[U]$, an explicit method is adapted with due consideration about the Courant condition, Eq. (C.7), and the numerical dispersion. Implicit methods are employed when the material or kinematic non-linearity is considered. For both implicit and explicit methods,

³⁰It is easily shown that the Rayleigh damping is not derived from the wave equation, by using the Lagrangean equation which is equivalent with the wave equation. If a discretized Lagrangean has a term like $[\dot{U}]^T[C][U]$, its variational is

$$\delta \left([\dot{U}(t)]^T [C] [U(t)] \right) = [\delta \dot{U}(t)]^T [C] [U(t)] + [\dot{U}(t)]^T [C] [\delta U(t)].$$

By applying integration by part, the terms in the left side are transformed to

$$[\delta U(t)]([C]^T - [C])[\dot{U}(t)] + \frac{d}{dt} \left([\delta U(t)]^T [C] [U(t)] \right).$$

Hence, only the anti-symmetric part of $[C]$, i.e., $[C] - [C]^T$, remains in the discretized equation. This means that $[C]$ is not symmetric. Thus, the Rayleigh damping matrix, a linear combination of two symmetric matrices $[M]$ and $[K]$, cannot be derived from the Lagrangean equation that is equivalent with the wave equation.

³¹The natural frequencies are roots of $\det(\omega^2[M] - [K]) = 0$.

there is a limitation in frequency up to which the numerical accuracy is guaranteed, depending on the fineness of spatial discretization. It should be mentioned that Eq. (C.9) can be solved analytically when it is scalar-valued or when it is decomposed to mutually independent modes³² which are obtained by applying the eigen-value decomposition of mass and stiffness matrices. This method is called the *direct integration*.

In earthquake engineering, the *Newmark β method* has been used as a reliable time integration algorithm which is used to solve Eq. (C.9). Introducing a set of coupled equations for displacement, velocity and acceleration vectors, this algorithm makes efficient and reliable computation. The Newmark β method starts from the Taylor expansion of velocity and acceleration vectors, i.e.,

$$[U(t+\Delta t)] = [U(t)] + [\dot{U}(t)]\Delta t + \frac{1}{2}[\ddot{U}(t)]\Delta t^2 + \frac{1}{3}\frac{d^3}{dt^3}[U(t)]\Delta t^3 + \dots,$$

$$[\dot{U}(t+\Delta t)] = [\dot{U}(t)] + [\ddot{U}(t)]\Delta t + \frac{1}{2}\frac{d^3}{dt^3}[U(t)]\Delta t^2 + \dots.$$

The essence of the Newmark β method is the evaluation of $\frac{d^3}{dt^3}[U]$. Denoting nodal displacement, velocity and acceleration at $t=n\Delta t$ by $[U^n]$, $[V^n]$ and $[A^n]$, it first assumes that acceleration linearly increase with respect to time and evaluates $\frac{d^3}{dt^3}[U]$ as

$$\frac{d^3}{dt^3}[U] = \frac{[A^{n+1}] - [A^n]}{\Delta t}.$$

To account for error due to the assumption of the linear increase of acceleration, the Newmark β method approximates³³ the right side by multiplying a suitable number. That is, nodal displacement and velocity vectors, $[U^{n+1}]$ and $[V^{n+1}]$, are approximated as

$$[U^{n+1}] = [U^n] + [V^n]\Delta t + \frac{1}{2}[A^n]\Delta t^2 + \beta([A^{n+1}] - [A^n])\Delta t^2, \quad (\text{C.11})$$

$$[V^{n+1}] = [V^n] + [A^n]\Delta t + \gamma([A^{n+1}] - [A^n])\Delta t^2. \quad (\text{C.12})$$

Here, β and γ are used to modify $\frac{d^3}{dt^3}[U]$; it is usually the case³⁴ that β is chosen as $\frac{1}{4}$ in order to avoid the numerical instability. Also γ is chosen as $\frac{1}{2}$; it is often observed that numerical solution for $\gamma > \frac{1}{2}$ or $< \frac{1}{2}$ overestimates or underestimates the exact solution, respectively. The target equation, Eq. (C.9), is expressed in terms of these nodal vectors, as

$$[M][A^{n+1}] + [C][V^{n+1}] + [K][U^{n+1}] = [F^{n+1}]. \quad (\text{C.13})$$

³²The independent mode corresponds to an eigen-vector of Eq. (C.9). For $[K][U] - \omega^2[M][U] = [0]$, an eigen-value of ω gives a natural frequency and the corresponding eigen-vector of $[U]$ determines the mode of displacement. These modes are linearly independent by definition.

³³In this sense, the Newmark β method is regarded as a modification of the numerical technique *acceleration*.

³⁴The choice of $\beta = \frac{1}{6}$ and $\gamma = \frac{1}{2}$ in Eqs. (C.11) and (C.12) leads to a standard numerical technique called (linear) *acceleration*.

Thus, Eqs. (C.11)~(C.13) form a set of linear equations for unknown $[U^{n+1}]$, $[V^{n+1}]$ and $[A^{n+1}]$. The Newmark β method solves these equations in an iterative manner. The procedures of the iterative computation are summarized as follows:

- i) Assume a suitable initial value for $[A^{n+1}]$.
- ii) Compute $[U^{n+1}]$ and $[V^{n+1}]$ using Eqs. (C.11) and (C.12).
- iii) Compute $[A^{n+1}]$ from Eq. (C.13) by substituting $[U^{n+1}]$ and $[V^{n+1}]$ into it.
- iv) Repeat the above procedures until $[A^{n+1}]$ converges.

The Wilson θ method is another representative algorithm which is used for time integration of Eq. (C.9). This method belongs to an implicit scheme, and it implements numerical acceleration. The basic formulation of Wilson θ method is similar to the Newmark β method. The key point is that it assumes a linear increase of acceleration in a small time domain between t and $t+\theta\Delta t$ with $\theta>1$ and computes displacement, velocity and acceleration at $t+\Delta t$. That is,

$$[U(t+\theta\Delta t)] = [U(t)] + [\dot{U}(t)](\theta\Delta t) + \frac{1}{4}([\ddot{U}(t)] + [\ddot{U}(t+\theta\Delta t)])(\theta\Delta t)^2, \quad (\text{C.14})$$

$$[\dot{U}(t+\theta\Delta t)] = [\dot{U}(t)] + \frac{1}{2}([\ddot{U}(t)] + [\ddot{U}(t+\theta\Delta t)])(\theta\Delta t), \quad (\text{C.15})$$

and $[\ddot{U}(t+\theta\Delta t)]$ satisfies Eq. (C.9) at $t=t+\theta\Delta t$. Three vectors at $t+\theta\Delta t$ are obtained by solving a set of these three equations, and acceleration at the next step, $t+\Delta t$ ($<t+\theta\Delta t$) is determined as

$$[\ddot{U}(t+\Delta t)] = \frac{(\theta-1)[\ddot{U}(t)] + [\ddot{U}(t+\theta\Delta t)]}{\theta}. \quad (\text{C.16})$$

Then, $[U(t+\Delta t)]$ is determined by solving Eq. (C.9) at $t=t+\Delta t$. While this method holds for any $\theta>1$, a recommended value is $\theta=1.4$

C.3 Absorption Boundary

When FEM is used to solve the wave equation for underground structures, some attention must be paid to a domain of analysis; see [Wolf (1988)] for exclusive studies on this problem and for an extensive list of related references. For instance, if a finite domain is used, some unnecessary waves may be generated as reflected waves at the boundary of the domain. The simplest method to avoid such reflected waves is the use of a special element which is infinitely extended and in which waves goes to the far-field. A hybrid of FEM with BEM is another method to get rid of the reflected waves. However, it is difficult to make suitable infinite elements, and combining FEM with BEM requires some efforts in programming. It is thus a usual practice to use a bounded body by introducing an *artificial boundary*; for a half plane which models underground structures, a suitable part is cut and is used as a domain of analysis. It is primarily important for the artificial boundary not to cause any reflective waves. In other words, waves hitting the boundary must be

absorbed there. This condition required for the artificial boundary is called an *absorption boundary condition*. When the absorption boundary condition is not properly satisfied, there happen several or many times of reflection at the artificial boundary, and numerical computation error is accumulated; it is easily seen that the energy stored in the domain increases monotonically since an external work done by input displacement or force is being kept within the domain. As mentioned above, [Wolf (2003)] is recommended for the readers who are interested in studying a variety of methods to satisfy the absorption boundary condition; see [Wolf (1988); Wolf (2002)] and [Zerfa and Loret (2004)].

While there are many methods to enforce the absorption boundary condition, they are divided into the following two classes: 1) a method which uses differential equations to exclude unnecessary reflection at the artificial boundary; and 2) a method which puts an artificial body next to the artificial boundary in which transmitted waves are rapidly damped. It should be noted that artificial boundary is used for an electro-magnetic wave problem as well. Some methods which have been developed for the electro-magnetic wave problems can be used for the mechanical wave problems; see, for instance, [Berenger (1994)].

A typical method of the first class is a method which transforms the wave equation so that reflected waves does not become a solution of the transformed equation. This method is called a *Clayton method*; see [Clayton and Engquist (1977)]. The Clayton method is explained using the one-dimensional bar problem as an example. By substituting a trigonometric function, $u=A \exp(-i(kx-\omega t))$, into the wave equation, $\ddot{u}+v^2u''=0$, the wave number k is obtained as $k=\pm\frac{\omega}{v}$ for a given ω . As explained in Sec. 4.2, $k=+\frac{\omega}{v}$ or $-\frac{\omega}{v}$ is the wave number for a wave going to the positive or negative x -direction, respectively. When an artificial boundary is set at $x=0$ for a body which is located in the domain of $x>0$, a wave which is reflected at $x=0$ and goes to the positive x -direction must be absorbed. Thus, the Clayton method transforms the wave equation which has two k 's so that only $k=-\frac{\omega}{v}$ is the wave number for a given ω . This is not a difficult task, and, in view of $\ddot{u}+v^2u''=(\frac{\partial}{\partial t}-v\frac{\partial}{\partial x})(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x})$, the transformed equation is

$$\frac{\partial u}{\partial t}(x,t) = -v \frac{\partial u}{\partial x}(x,t). \quad (\text{C.17})$$

As is seen, when $u=\exp(-i(kx-\omega t))$ is substituted into Eq. (C.17), only $k=+\frac{\omega}{v}$ is obtained. Thus, no wave is generated at the artificial boundary by using Eq. (C.17) near $x=0$. If finite difference is used, the absorption boundary condition which satisfies Eq. (C.17) is described as

$$\frac{u_0^{n+1} - u_0^n}{\Delta t} = -c \frac{u_1^n - u_0^n}{\Delta x},$$

where u_0^n and u_1^n are u at $x=0$ and Δx , respectively. The Clayton method achieves the perfect absorption for this one-dimensional setting. However, such perfect absorption is not expected for a two- or three-dimensional setting. This is

because the Clayton condition is able to absorb only a wave which goes to the artificial boundary at a right angle. As the angle of an input wave becomes shallower, a less amount of a reflected wave is absorbed by enforcing the Clayton method; the component of the input wave in the direction normal to the artificial boundary becomes smaller.

A typical method for the second class is a method of *perfect matched layer* (PML), which is complicated but very effective to absorb reflected waves at the artificial boundary. The method of PML introduces an artificial layer outside of the artificial boundary; the layer is heterogeneous but does not have any impedance contrast³⁵ in it. In general, it is difficult to make the impedance contrast vanish for all frequency. The method of PML solves this difficulty, and achieves zero impedance contrast in the layer. The key point is to design the artificial layer so that it has a smoothly changing density and elasticity with the impedance being fixed. For the previous example of the one-dimensional bar, the impedance is

$$z = \sqrt{\rho E}. \quad (\text{C.18})$$

The method of PML defines ρ and E as a function for x . If E increases in the artificial layer, waves transmitted in the layer is absorbed as their amplitude decreases. In the previous example of the one-dimensional bar, the domain of analysis ($x > 0$) is homogeneous, and Young's modulus and the density are denoted by E^0 and ρ^0 . Then, an artificial layer, which is put in $-a < x < 0$, has

$$\begin{bmatrix} E(x) \\ \rho(x) \end{bmatrix} = \begin{bmatrix} (1 + \frac{x}{a})^{-1} E^0 \\ (1 + \frac{x}{a}) \rho^0 \end{bmatrix}. \quad (\text{C.19})$$

Since E increases unboundedly near $x = -a$, the amplitude of waves decreases as they propagates through $-a < x < 0$. No reflection takes place since the impedance takes on the same value, $z^0 = \sqrt{E^0 \rho^0}$, in this layer. The method of PML is applicable to a two- or three-dimensional setting; it is easily seen that the impedance contrast is zero along any direction, not necessarily along the direction normal to the artificial boundary. While there are some limitations due to discretization, the method of PML satisfactorily achieves the absorbed boundary condition at the artificial boundary.

³⁵The condition for two layers to have the same impedance is called impedance matching.

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Appendix D

Unified Modeling Language

Unified modeling language (UML) is a computer language which describes a computer system taking advantage of the concepts of object-oriented design and analysis; see, for instance, [Satzinger and Orvik (2001)]. Several computer languages are developed for the system design and analysis, and UML has been used as a standard language that unifies these languages since 1997.

The key characteristic of UML is the utilization of well-defined diagrams. These diagrams are summarized as follows:

- i) use case diagram: a diagram used to model system requirements. It symbolizes communication between the system and an *actor* (or a user).
- ii) class diagram: a diagram used to model a class and a relation among classes
- iii) behavior diagrams:
 - iii.i) statechart diagram: a diagram used to model a state of an object. It explains state transformation.
 - iii.ii) activity diagram: a diagram used to model system behavior as a flow of actions. It consists of a flow chart or an object flow.
 - iii.iii) interaction diagram:
 - iii.iii.i) sequence diagram: a diagram used to model interaction among objects along a time axis.
 - iii.iii.ii) collaboration diagram: a diagram used to model interaction among objects. It explains structural relation of objects.
- iv) implementation diagram:
 - iv.i) component diagram: a diagram used to model a structure of a component and an interdependency of components
 - iv.ii) deployment diagram: a diagram used to model deployment of components

The description style of all diagrams is strictly specified. It is out of scope of this appendix to explain the description style in detail, and some diagrams which are able to describe figures presented in Chapters 10 and 11 are explained.

First, the use case diagram is explained. This diagram is suitable to describe how a designer uses a design code. In Sec. 13.3, the standardized earthquake resistant

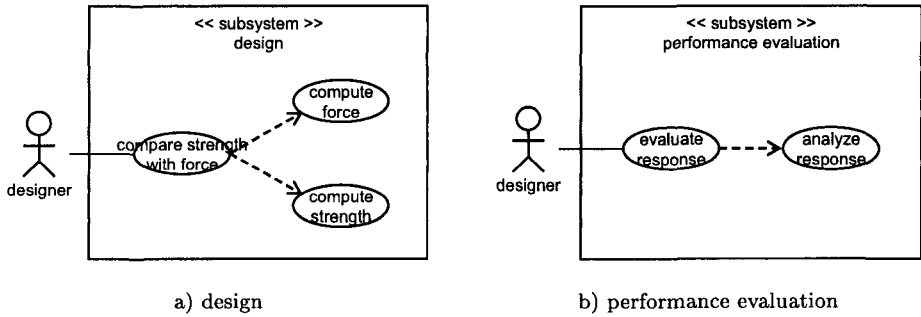


Fig. D.1 A use case diagram of earthquake resistant design. The use case diagram describes basic phases of the design.

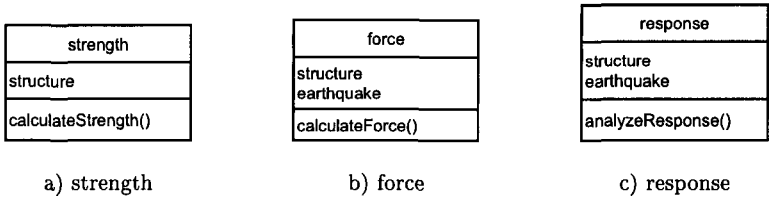


Fig. D.2 Class diagrams of earthquake resistant design. The class diagram describes the contents of an object.

design that is shown in Fig. 13.11 is presented; a) and b) are the reconstructed flow chart for design and performance evaluation, which are the two basic phases of designing. These flow charts are redrawn as a use case diagram of Fig. D.1; a) and b) are for design and performance evaluation, respectively. The diagram has a designer (or an actor) and a set of subsystems which are included in one box. A subsystem is expressed as an ellipse. A solid line represents a request of a designer to a subsystem, and an arrow in a dashed line is a request of one subsystem to another subsystem.

Next, the class diagram is explained. This diagram shows the basic content of an object, and hence is suitable to describe the object content that is presented in Sec. 13.3. As an example, the object contents shown in Fig. 13.12a)~c), which are made for the object **strength**, **force** and **response**, are redrawn. Class diagrams for these objects are presented in Fig. D.2. In these class diagrams, a name, data and methods of an object are put³⁶ into separate boxes; for instance, Fig. D.2a) is made for the object **strength**, and the name, datum and method are **strength**, **structure** and **compute_strength()**, respectively.

The class diagram is able to describe a relation between objects. For instance, in Sec. 12.1, the inheritance of objects **structure**, **FEM** and **A** for a certain structure of **structure_A** is studied; see Fig. 12.2a). This chart is rewritten as a class diagram

³⁶More specific definition is made in UML, for describing name, data and methods in the boxes.

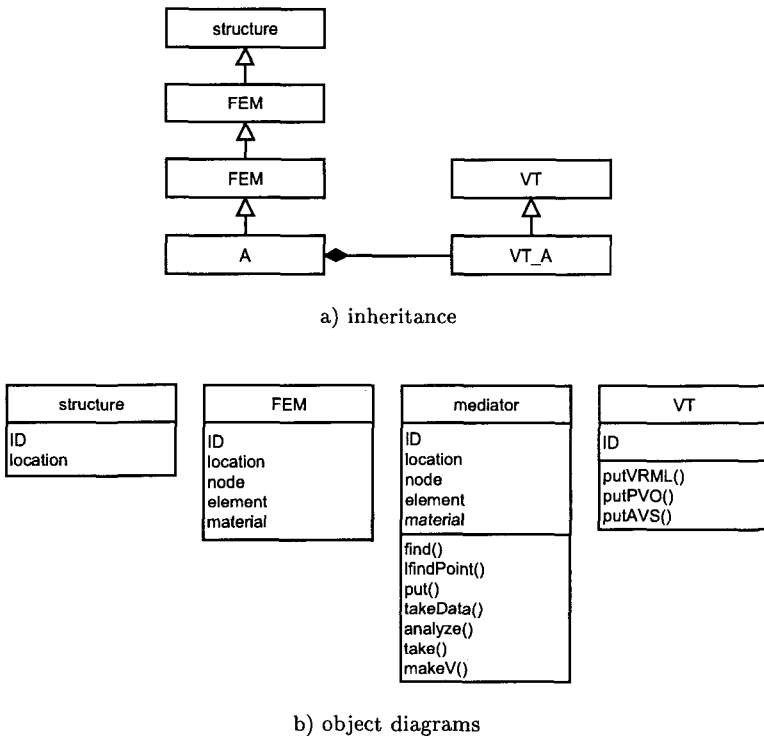


Fig. D.3 A class diagram to describe inheritance of objects.

shown in Fig. D.3; a) gives the relation among objects and b) is for the content of an object which is regarded as a superclass. An arrow in Fig. D.3a) represents inheritance; an arrow links from a subclass to a superclass, i.e., the arrow head indicates a superclass and the arrow tail a subclass. An arrow with a black diamond head represents a special relation, called *composition*; VRMLA is a part of A and it vanishes automatically when A is deleted. In Fig. D.3b), only data are given in a class diagram for **structure**, and a class diagram for **mediator** has seven methods. As explained in Sec. 12.1, these methods are key items of the mediator; the actual structures of the methods differ for each structure, but all mediators commonly have the seven methods, which are given by a superclass of **mediator**.

Finally, the sequence diagram which belongs to an interaction diagram of a behavior diagram is explained. The sequence diagram clarifies a flow of complicated procedures. As an example, the seven methods of a mediator, Fig. 12.5 shown in Sec. 12.2, are rewritten as a sequence diagram shown in Fig. D.4. An entity is created for a mediator, data, a simulation program or its result; they are denoted as A:mediator, DA:data, DA:result, SPA:simulation program or RA:result, respectively; other classes do not accompany an entity. An arrow with a black solid head and an arrow in a dashed line indicate an order and a response for the order,

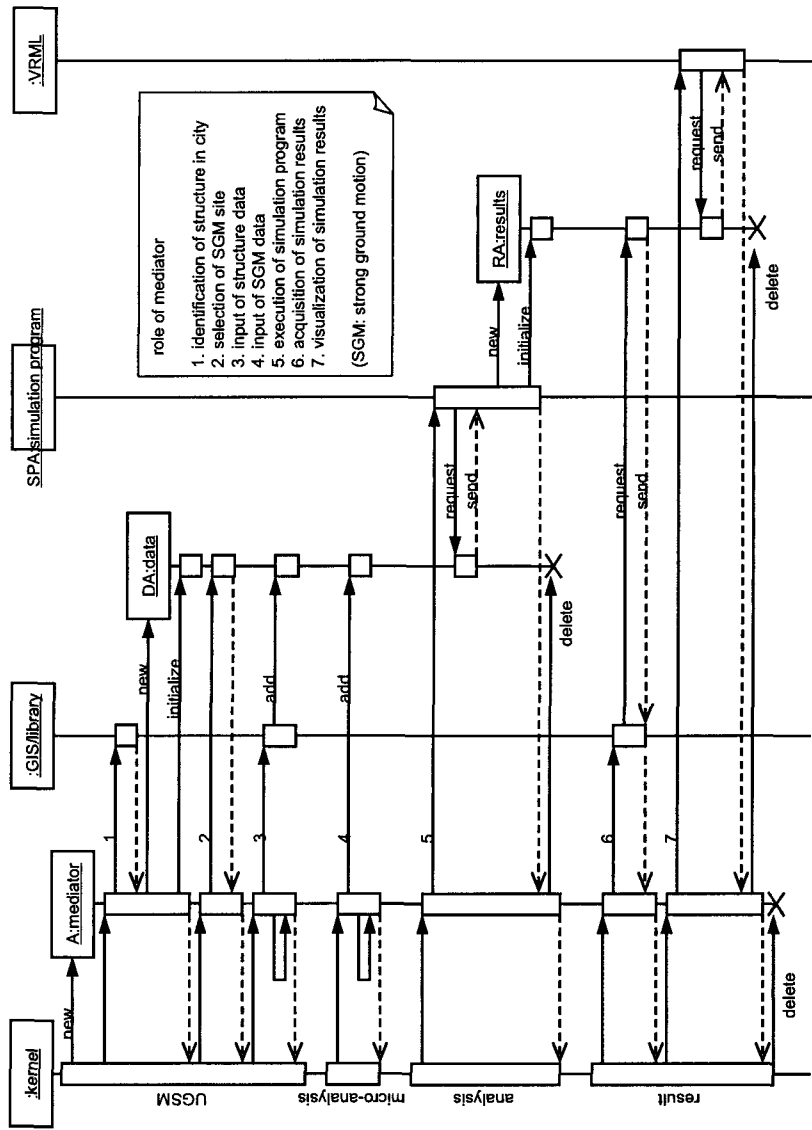


Fig. D.4 A sequence diagram for seven methods of a mediator.

respectively, and a square box shows that an entity is activated. Thus, the four major roles of kernel are indicated by a square box, and they activate other elements of IES.

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