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Steffen Ducheyne

The Main Business of Natural Philosophy

Isaac Newton's Natural-Philosophical Methodology

Springer

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Isaac Newton's Natural-Philosophical Methodology

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ISSN 1385-0180
ISBN 978-94-007-2125-8 e-ISBN 978-94-007-2126-5
DOI 10.1007/978-94-007-2126-5
Springer Dordrecht Heidelberg London New York

Library of Congress Control Number: 2011934265

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Of course, scientific activity can be understood fully only in relation to its context in the culture and society of its time. But "context" has no meaning without "text": the political economic, social, and cultural dimensions have little historical significance if their analysis neglects the precise claims to knowledge and epistemic goals that were the ostensible raison d'être of the scientific work.[1]

~~*↓[I]t is the nature of↓ this Philosophy ↓to↓ assent any ↓no↓thing more then can be proved by Experiments*~~.[2]

To explain all nature is too difficult a task for any one man or ↓even for↓ any one age. Tis much better to do a little with certainty [...] and leave the rest for others that come after you then to explain all things ↓by conjecture↓ without making sure of any thing.[3]

1 Rudwick, *Bursting the Limits of Time*, p. 4.

2 CUL Add. Ms. 3968, f. 586v.

3 CUL Add. Ms. 3970, f. 480v.

To my wonderful parents

Acknowledgements

While working on this monograph for several years, I have benefited from the support of many scholars. I have benefited from useful correspondence and discussion with Niccolò Guicciardini, William L. Harper, Andrew Janiak, Scott Mandelbrote, J. E. McGuire, Simon Schaffer, Eric Schliesser, Alan E. Shapiro, George E. Smith, and Stephen D. Snobelen. I am especially indebted to Ori Belkind, Niccolò Giucciardini, John Henry, Scott Mandelbrote, John P. McCaskey, J. E. McGuire, Bruce H. Pourciau, Alan E. Shapiro, Eric Schliesser, Stephen D. Snobelen, and Gábor Zemplén, who were willing to comment on earlier versions of parts of this book. That being said, I alone should take responsibility for any remaining mistakes. I explicitly want to thank Jed Z. Buchwald, Mordechai Feingold, Scott Mandelbrote, J. E. McGuire, and Stephen D. Snobelen for their unremitting advise and encouragement. I also would like to thank the anonymous referee who has offered many useful suggestions and comments, which allowed me to improve the text. I beg forgiveness to those whom I might have forgotten to name. I would also like to acknowledge the work and influence of the giants on whose shoulders I have been standing while working on this book: I. Bernard Cohen, B. J. T. Dobbs, J. E. McGuire, Alan E. Shapiro, George E. Smith, Richard S. Westfall, and D. T. Whiteside. I am also enormously indebted to the careful editing and transcription work of important primary Newtonian resources by scholars in the preceding and current century.[1] I have used their editions and transcriptions, almost literally, as daily instruments. Newton

[1] Obviously, with respect to the current century I refer to *The Newton Project* which seeks to make all of Newton's writings freely accessible online. *The Newton Project* is currently hosted by the University of Sussex in collaboration with institutions at Cambridge University (URL = <http://www.newtonproject.sussex.ac.uk>). Rob Iliffe (University of Sussex) and Scott Mandelbrote (Peterhouse, Cambridge University) act as the editorial directors of *The Newton Project*. The number of manuscripts which the directors and their collaborators have made available with such great editorial care is simply impressive on all accounts. The U.K.-based *Newton Project* collaborates with two sister projects: *The Newton Project Canada* based at King's College, Halifax, which is directed by Stephen D. Snobelen and which focuses on Newton's theology and prophecy (URL = <http://www.isaacnewton.ca>), and *The Chymistry of Isaac Newton* based at Indiana University, which is directed by William R. Newman (URL = <http://webapp1.dlib.indiana.edu/newton>).

scholarship will continue to progress, not so much by a literature which ad nauseam quotes and discusses the same *dicta*, but by continuously, vastly and systematically scrutinizing the entire bulk of Newton's *oeuvre*. As Newton himself might have suggested, a "justified principle" should be consistent will *all* evidence. In line with this *dictum*, I have spent considerable time studying Newton's manuscripts and I have tried to constrain my analyses in this tome as good as I could by their contents.

I am much obliged to the Provosts and Syndics of Cambridge University Library for permission to quote from the Portsmouth and Maclessfield Collection and to reproduce Figs. 3.1a–c, 3.3, 3.4 and 4.5 from the Portsmouth Collection and, especially, to Adam Perkins and his staff at the Manuscript Department at Cambridge University Library for their excellent assistance during my study of the Portsmouth and Macclesfield collection in 2006 and again in 2008–2009. Similar gratitude goes towards The Master and Fellows of Trinity College Cambridge for permission to quote from the material in their care and to reproduce Fig. 1.1 and especially to Jonathan Smith, chief-librarian at Wren Library, and his staff for their superb assistance during my work there in 2008–2009 and to Peter Jones and his staff at King's College Cambridge for their assistance during my study of the Keynes Collection in 2006. Sincere thanks also to Elisabeth Macheret-Van Daele for kind permission to study and to quote from the Bodmer Newton manuscript, *Of the church*, and to her staff at the Fondation Martin Bodmer for their assistance during a memorable research stay in Geneva in 2007. I also acknowledge my indebtedness to Rupert Baker and the staff at the Library and Archives of The Royal Society for their assistance during my study of the printer's copy of the first edition of the *Principia* (Ms. 69) and Gregory Ms. 247 in December 2008. I am much indebted also to Volkmar Schüller for sending me his carefully edited transcriptions of the Classical Scholia. I gladly acknowledge the financial support of the Fund of Scientific Research (FWO – Vlaanderen) for financing my pre- and postdoctoral research and my research stays at Cambridge. I am also indebted to my editor at Springer, Lucy Fleet, for a smooth transition from file to book.

I should indicate to the reader that some of the material on which this book is based has appeared earlier in scholarly journals.[2] However, a substantial amount

[2] Material from Ducheyne, "Newton's Training in the Aristotelian Textbook Tradition" and id., "Newton on Action at a Distance" has been used in Chapter 1. Material from id., "Mathematical Models in Newton's *Principia*"; id., "Newton's Notion and Practice of Unification"; id., "The Argument(s) for Universal Gravitation"; and, id., "Understanding (in) Newton's Argument of Universal Gravitation" has been used in Chapters 2 and 3. Id., "On Optical and Mechanical Models" is an early precursor of Chapter 4. Material from id., "The General Scholium: Some Notes on Newton's Published and Unpublished Endeavours"; id., "Anti-trintarianism in Newton's General Scholium to the *Principia*" and id., "Isaac Newton's 'Of the Church:' Manuscript Description and Analysis of Bodmer Ms. in Geneva" has been used in Chapter 5. I am obliged to the editors and publishers of *History of Science*, *International Studies in the Philosophy of Science*, *Studies in History and Philosophy of Science*, *Foundations of Science*, *The Journal for General Philosophy of Science*, *Logique et analyse*, *Lias*, and the *European Journal of Science and Theology* for their permission to base parts of this book on the papers which appeared in their journals.

of my claims have undergone major change and the arguments contained in "*The main Business of natural Philosophy*" have become more detailed and – I hope – more compelling.

On a more personal and private note, I dedicate this *libellum* to my close ones – impressed by force or at rest, nearby or at a distance. Thank you ever so much.

Introduction

In the book at hand, Newton's methodological ingenuity in natural philosophy is our main concern. The word ingenuity, notwithstanding, I shall not at all pursue a hagiographic narrative of Newton "the genius Lucasian Professor of Trinity College Cambridge" – which he surely was. Rather "*The main Business of natural Philosophy*"[1]: *Isaac Newton's Natural-philosophical Methodology* starts from the following premise, which is nicely phrased by Scott Mandelbrote:

> In order to appreciate Newton's originality and the success of his ideas, it becomes necessary to discover how he came to formulate his thoughts and how he provided proof to his contemporaries that his strange ideas were so often true. This involves removing him from the world of myth, into which his followers assiduously tried to introduce him and where he clearly felt at home by the end of his career. By placing Newton instead in the world of work, it is possible to see how he became a mathematician and natural philosopher through reading and practice, and how, once he had become one, he exceeded the bounds of all but a handful of his contemporaries, through the diversity of his interests as well as the determination with which he pursued them.[2]

Some past scholars have canvassed the story of Newton's natural philosophy as a heroic story of a solitary genius who changed the world of science or as the victory of mathematics and empiricism over hypothetical philosophy. Instead, this monograph will situate Newton's natural-philosophical methodology explicitly "in the world of [natural-philosophical] work." I shall point not only to Newton's successes, but also to the tensions and difficulties which he faced whilst trying to methodize natural philosophy.

"*The main Business of natural Philosophy:*" *Isaac Newton's Natural-philosophical Methodology*, furthermore, endorses the view that the History of Science requires both a historical study of the primary material at hand as well as a systematic study that focuses on conceptual, inferential, and methodological key-issues. Therefore, the aim of this monograph is to provide both a historically informed as well as systematically rich account of Newton's methodology. Although

[1] The first part of my monograph's title refers to an excerpt from Query 28 (Newton, *The Opticks*, p. 369).
[2] Mandelbrote, *Footprints of the Lion: Isaac Newton at Work*, pp. 9–10.

this work essentially deals with the method which Newton pursued to gain knowledge about the empirical world,[3] I will pay attention, not only to "mathematical-technical Newton," but also to "theological Newton." Obviously, Newton was both and, if one wishes to come to terms with Newton's œuvre, one should provide a balanced account of the "two Newtons."[4] However, that is not to say that we should not respect the disciplinary boundaries that Newton *himself* had imposed on his work.

Until the 1980s, the reception of Newton's methodological heritage by most twentieth-century philosophers of science has been rather disappointing – not to say straightforwardly embarrassing. In his *The Aim and Structure of Physical Theory* Pierre Duhem famously criticized Newton's methodology, as follows: "*The principle of universal gravity, very far from being derivable by generalization and induction from the observational laws of Kepler, formally contradicts these laws. If Newton's theory is correct, Kepler's laws are necessarily false.*"[5] This line of criticism, which charges Newton with methodological inconsistency, was taken over by many others. Karl R. Popper, for instance, ascribed to Newton the same inconsistent position: "it is impossible to derive Newton's theory from either Galileo's or Kepler's, or both, whether by deduction or by induction. For neither a deductive nor an inductive inference can ever proceed from consistent premises to a conclusion that formally contradicts these premises."[6] Similarly, Imre Lakatos has emphasized the discrepancy between Newton the methodologist, "who claimed that he *derived* his laws from Kepler's phaenomena," and Newton the scientist, "who claimed that knew very well that his laws *directly contradicted* these phenomena."[7] On closer scrutiny, the so-called contradiction, which has been brought to the fore by Duhem and which has been repeated by many others since then, is simply non-existent – as any reader of the first three propositions of Book I and Phenomena I–VI as stated in Book III of the *Principia* can testify. The particular criticism raised is beside the point, as Newton demonstrated that exact Keplerian motion occurs only in one-body systems and that, under specific configurations, Keplerian motion occurs

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[3] Here my aim is to provide an explication of Newton's methodology based on his methodological reflections *and* his scientific practice. E. A. Burtt once noted that "Newton never rose, in his conception of method, to any higher degree of generality than that revealed in his own practice" (Burtt, *The Metaphysical Foundations of Modern Physical Science*, p. 204). If we consider Newton's *Principia*, the number of methodological statements or sections is indeed rather slim. However, if we carefully study the *Principia* in its entirety it is possible to reconstruct a complex overarching methodological programme.

[4] Cf. Feingold, "Honor Thy Newton". In Chapter 6, the theological contents of Newton's General Scholium will be highlighted.

[5] Duhem, *The Aim and Structure of Physical Theory*, p. 193.

[6] Popper, *Realism and the Aim of Science*, p. 140.

[7] Lakatos, "Newton's Effects on Scientific Standards", p. 210. Lakatos also wrote that: "The schizophrenic combination of the mad Newtonian methodology, resting on the *credo quid absurdum* of 'experimental proof' and the wonderful Newtonian method strikes one now as a joke." (ibid., p. 212). Similar criticism was raised in Feyerabend, "Classical Empiricism".

as most closely as possible (*quam proxime*) in three- and many-body systems as well. In other words, since the physico-mathematical conditions in one-body are patently different from the physico-mathematical conditions under which Keplerian motion occurs *quam proxime*, there is no formal contradiction involved whatsoever.[8] Duhem's criticism (and that based upon it), has neglected the fact that Newton was approaching the celestial motions in a series of successive and increasingly complex approximations. Nothwithstanding, scholars have judged Newton's methodology to be flawed and they have left a more subtle account of Newton's methodology unexplored.[9]

All this was to change for the good in 1980 when I. Bernard Cohen's *The Newtonian Revolution* appeared in print. *The Newtonian Revolution* contained the first systematic and detailed exposition[10] of Newton's methodology in the *Principia*, which Cohen dubbed the "Newtonian Style."[11] From the 1990s several important papers on Newton's methodology began to appear.[12] It is therefore quite fair to say that serious scholarly research on Newton's methodology in the *Principia* has only emerged in the last three decades. The papers mentioned in the previous footnote correctly suggest that there is something profound about Newton's methodology. It is to this reassessment of Newton's methodology that this monograph seeks to contribute. To state matters clearly from the outset, *nowhere will it be argued or assumed that Newton practised a non-hypothetical methodology*; rather, my aim is to clarify in what sense Newton's methodology was more demanding and rich than a standard hypothetico-deductive methodology and to highlight that it encompassed procedures to minimize inductive risk. According to the hypothetico-deductive model of confirmation, a theoretical proposition is accepted if its empirically testable consequences are confirmed by experience.[13] In

[8] See Chapter 2 for a detailed treatment.

[9] Howard Stein was a notable exception to this trend. See his "Newtonian Space-Time" and his "On the Notion of Field in Newton, Maxwell, and Beyond".

[10] Earlier, but more modest, attempts notwithstanding (see, e.g., Strong, "Newton's Mathematical Way").

[11] Cohen, *The Newtonian Revolution*. Also see id., "The *Principia*, Universal Gravitation, and the "Newtonian Style"; id., "Newton's Method and Newton's Style"; and, id., "The *Principia*, the Newtonian Style, and the Newtonian Revolution".

[12] Especially the following papers come to mind: Harper, "Newton's Classic Deductions from Phenomena"; id., "Reasoning from Phenomena: Newton's Argument for Universal Gravitation and the Practice of Science"; id., "Isaac Newton on Empirical Success and Scientific Method"; id., "Measurement and Approximation"; id., "Newton's Argument for Universal Gravitation"; id., "Howard Stein on Isaac Newton: Beyond Hypotheses?"; id., "Newton's Methodology"; Harper and Smith, "Newton's New Way of Inquiry"; Smith, "From the Phenomena of the Ellipse to an Inverse-square Force: Why Not?"; id., "The Methodology of the *Principia*"; id., "The Newtonian Style in Book II of the *Principia*"; and, finally, Stein, "From the Phenomena of Motions to the Forces of Nature: Hypothesis or Deduction".

[13] Useful discussion of the hypothetico-deductive methodology is provided in Nola and Irzik, *Philosophy, Science, Education and Culture*, Chapter 8.

opposition to this model of confirmation, Newton raised the criticism that different hypotheses could be rendered consistent with the same experimental data.

Although more than half of this work will be directly addressing Newton's methodology proper in both his mechanical and optical work (Chapters 2–5), I shall also provide ample contextualisation for an understanding Newton's natural-philosophical methodology in Chapters 1 and 6.

Let me offer a prospectus of the tome at hand. In Chapter 1, I offer both a historical contextualisation of Newton's causal realism as well as an elucidation of the status of forces in the *Principia*. It will be argued that the seventeenth-century textbook tradition of method and logic in natural philosophy is relevant to bring *some* aspects of Newton's causal stance in natural philosophy into perspective. Newton drew from and was trained in a set of common texts and techniques, which were still the most important sources for university-trained natural philosophers at the mid-seventeenth century. Newton's causal outlook on scientific reasoning, both in the *Principia* as well as in *The Opticks*, has relevant parallels with this textbook-tradition, especially if one focuses on his views on analysis and synthesis in natural philosophy. Based on my study of the annotations and traces of dog-earing in his private copies of textbooks pertaining to this tradition, it is shown that Newton knew and studied some of these works intimately. Next, it is argued that Newton understood centripetal forces as true causes of motion and that there is no reason to interpret Newtonian forces as mere mathematical devices to describe motion. I shall furthermore argue that, although Newton radically reformed the notion of "cause" – for he introduced a notion of causation in terms of counterfactual-nomological dependency based on Law I, which enabled him to abstract from the cause of gravity, he formulated his views on natural philosophy on a more abstract level in traditional terminology. Finally, Newton's views on *actio in distans* will be discussed. Newton denied that matter could act at a distance – because this would imply that matter is innately self-acting, an option unacceptable for Newton; however, he endorsed action at a distance for the secondary mechanism producing gravity, because he stated that the "elastick" ether causing gravity consisted of repellent particles acting *at a distance*. Newton thus rejected action at a distance at a macro-level but accepted it at a micro-level. He had no a priori objections against *actio in distans*.

Chapters 2 and 3 correspond to two important and consecutive phases in Newton's methodology: a phase of model construction and a phase of model application, theory formation, and theory testing. It will be shown that both phases proceeded in a way more demanding than hypothetico-deductivism. As a stage-setting, I will highlight a potential source of confusion inherent in I. Bernard Cohen's influential account of Newton's methodology in the *Principia* in Chapter 3. The crux of what I call "the strong version" of Cohen's account is the successive adaptation of "mental constructs" (Cohen's terminology) through a series of comparisons with nature. Thus, the strong version Cohen's "Newtonian Style" suggests that in the phase of model construction there is a direct dynamics between the "mental constructs" and their corresponding physical systems. It is argued that, if Newton's method indeed involved such extra-theoretical dynamics, Cohen's account fails to be non-hypothetico-deductive.[14] Next, I present my own model-based account of Book I of the *Principia* and argue that Newton understood Book I as an exercise in studying the mathematical properties of – *in principle* – arbitrary centripetal forces. Nature did not enter the scene here. The growing complexity of Newton's models is then the result of exploring increasingly complex cases (*intra-theoretical dynamics*) rather than the direct result of a successive comparison with nature (*extra-theoretical dynamics*). Thereafter, I will point to the constituents of Newton's models and I shall distinguish between different sorts of propositions in Book I. By doing so, I shall at the end of Chapter 2 be able to clarify the extent to which Newton's methodology differs from a hypothetico-deductive method in the context of the phase of model construction. In Chapter 3, I then explicate how the physico-mathematical machinery, as developed in Book I of the *Principia*, is applied to phenomena and tested in the empirical world. I shall do so by focussing on Newton's argument for universal gravitation, especially the analytical part. First, I shall provide an overview of the development of Newton's *regulae philosophandi* by surveying the relevant manuscript material. Next, I shall discuss the final parts of Book II of the *Principia*, in which Newton set out several arguments against mechanical vortices. Finally, I will scrutinize both the analytic and synthetic part of Newton's argument for universal gravitation, which will result in a systematic non-hypothetico-deductive *exposé* of Newton's methodology in the context of the phase of model application, theory formation and theory testing.

In Chapter 4, I argue that Newton tried to apply a *Principia*-style methodology to optics. However, for reasons that will be spelled out, Newton's causal explanations of optical phenomena could not be constrained by theory, as was the case in the *Principia*. As I shall explain in the Chapters 2 and 3, an essential feature of the *Principia* is that Newton was able to generate "inference-tickets," which were *derived from the laws of motion* that allow one to derive the (proximate) cause from its effect. This is because in the *Principia* there are links between cause and effect via the laws of motion. In *The Opticks* such links would be possible only if one made assumptions about the nature and (non-observable) constituents of light, i.e. if one introduced a corpuscular account, which was unacceptable given Newton's anti-hypothetical stance. Finally, I shall explain why in Book III of the *Principia* transduction was less problematic than in *The Opticks*.

In "*The main Business of natural Philosophy*:*" Isaac Newton's Natural-Philosophical Methodology*, I shall approach these matters systematically, rather than strictly chronologically. In Chapter 5, however, I shall provide a brief chronology of Newton's methodological itinerary, which is thoroughly based on the material we have surveyed in Chapters 2–4.

In Chapter 6, I shall address the complex interplay between Newton's experimental philosophy and theology. I shall place the theology of the General Scholium

[14] In due fairness to Cohen, it should be noted that he became aware of the problems associated with his initial characterization of the "Newtonian Style" (Cohen, "The *Principia*, the Newtonian Style, and the Newtonian Revolution," pp. 92–93).

in its proper context by situating it in the broader content of Newton's theological manuscripts. Thereafter I shall focus on the anti-Cartesian dimensions of the General Scholium by taking into account some of Newton's scarcely studied manuscripts. Finally, I shall move on to a general discussion of the interaction between Newton's experimental philosophy and his theology. Before I shall do so, however, we will first turn to two representative case-studies. The outcome of my argumentation will be that, although he endorsed the theological significance of the results harvested by experimental philosophy, Newton considered experimental philosophy and theology as methodologically distinct.

The questions that will concern us in this monograph are the following: *What was distinctive about Newton's methodology?*, *In which sense is Newton's methodology different from a hypothetico-deductive approach?*, *What are the relevant traditions from which Newton drew his views of natural-philosophical explanation?*, *How did Newton's methodological ideas change over time?*, and, finally, *How did Newton's natural philosophy and his theology interrelate?* By the end of this monograph, I hope to have shown that there is indeed something significant to Newton's methodology. *Introeamus*.

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3.1 Newton's notes on Rule III (**a** CUL Adv.b.39.1, pp. 402–402A; **b** CUL Add. Mss. 3965, f. 419^{r}; **c** ibid., f. 419^{v}). Courtesy of the Cambridge University Library, Manuscript Department 110

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4.5 CUL Add. Ms. 3965, f. 422^r. Courtesy of the Cambridge University Library, Manuscript Department 224

Notes to the Reader

1. *Convention for my transcriptions*: Arrows pointing downwards (↓...↓) indicate that the text in between them was inserted from above Newton's original interlineation. Arrows pointing upwards (↑...↑) indicate that the text in between them was inserted under Newton's original interlineation. Unless otherwise indicated, *all text-editorial features are as in the original*.
2. *A note on footnotes*: Footnotes are commonly used to give the exact source on which the author draws. In "*The main Business of Natural Philosophy:*" *Isaac Newton's Natural-Philosophical Methodology* footnotes are frequently used in a wider sense and readers should be aware of that before they begin reading this monograph. Footnotes will also be used as a means to add more substance to the claims made in the main text, to nuance or corroborate them, or to counter possible objections – in these cases, the footnotes cannot be skipped and are to be considered as being part of the main text. The text-editorial separation between text and footnotes is therefore only virtual.
3. Unless otherwise indicated, all translations are mine.

Part I
Newton's Causal Methodology

Chapter 1
Newton and Causes: Something Borrowed and Something New

> *In mathematical analysis, we always set out from a hypothetical assumption, and our object is to arrive at some known truth, or some* datum*, by reasoning synthetically from which we may afterwards return, on our own footsteps, to the point where our investigation began. In all such cases, the synthesis is infallibly obtained by reversing the analytical process; and has as both of them have in view the demonstration of the same theorem, or the solution of the same problem, they form, in reality, but different parts of one and the same investigation. But in natural philosophy, a synthesis which merely reversed the analysis would be absurd. On the contrary, our analysis necessarily sets out from* known facts; *and after it has been conducted us to a general principle, the synthetical reasoning which follows, consists always of an application of this principle to phenomena,* different *from those comprehended in the original induction.*[1]

1.1 Introduction

In the above quote, the Scottish Common Sense philosopher Dugald Stewart (1753–1823) seems to have put his the finger on an important asymmetry between mathematical versus natural-philosophical analysis and synthesis. If the line taken by Stewart is correct – and this is indeed what I shall argue for in what follows, then the view that Newton's methodology, as spelled out in his well-known *exposé* on the Methods of Analysis and Synthesis in Query 31 of *The Opticks*, derives first and foremost from the mathematical tradition on analysis and synthesis becomes unsatisfactory. Furthermore, as a consequence, it needs to be shown for the historical record which traditions shaped Newton's views on natural-philosophical analysis and synthesis.

[1] Stewart, *Elements of the Philosophy of the Human Mind*, II, p. 370. This quotation comes from the section entitled "Critical Remarks on the vague Use, among Modern writers, of the Terms Analysis and Synthesis" (ibid., II, pp. 365–381).

S. Ducheyne, *The Main Business of Natural Philosophy*, Archimedes 29,
DOI 10.1007/978-94-007-2126-5_1,

In this chapter, I shall examine a neglected tradition that sheds light on Newton's views on natural-philosophical analysis and synthesis. I shall argue that certain works pertaining to the seventeenth-century "Aristotelian"[2] textbook tradition,[3] which were inspired by Aristotle's ideas on logic and science, are relevant to bring some basic features of Newton's conception of natural philosophy into its proper historical context. Allow me to stress an important issue from the outset: should future, badly needed, historical research come to revisit the "Aristotelian" features of these textbooks, the claims I shall make in this chapter will still stand. For my claims hinge, not so much on the term "Aristotelian," but rather on the premise that these textbooks were part of a causal tradition that dealt with *regressus* and natural-philosophical analysis and synthesis.

By situating the influence of the "Aristotelian" textbook tradition at the level of some "basic features" of Newton's conception of natural philosophy, I refer to his usage of certain terminology and his assimilation of some basic methodological premises. As I shall argue in what follows, the "Aristotelian" textbook tradition on logic and method helps to explain the following basic premises characteristic of Newton's views on methodology:

1. In the order of things (*ordo naturae*), *causes* are *ontologically primary* and *effects ontologically secondary*. In the order of knowing, *effects* are *epistemologically primary* and *causes epistemologically secondary*.[4]
2. The proper mode of demonstrating natural-philosophical knowledge, is *regressus demonstrativus*. *Regressus demonstrativus* is a dual method of demonstration which consists of two consecutive phases: in the *analysis* (also called *resolutio*), we proceed from effects to causes; in the *synthesis* (also called *compositio*), we proceed from causes to effects.

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[2] I must admit that I am somewhat suspicious of using the term "Aristotelian" as this seems to imply that it was a uniform tradition which was exclusively based on Aristotle's original ideas. Any tradition is modified as different authors interpret it. The Aristotelian tradition in the seventeenth century was definitely not a monolithic whole. However, what seems to be correct to claim is that these writers took "the principal concepts and divisions of their natural philosophy from Aristotle's books" (Reif, "The Textbook Tradition in Natural Philosophy", pp. 19–20). For a recent account of the tradition and its variety from antiquity to the seventeenth century see Leijenhorst, Lüthy and Thijssen, eds., *The Dynamics of Aristotelian Natural Philosophy from Antiquity to the Seventeenth Century*. Reif correctly points out that after 1650 the Aristotelian tradition was increasingly influenced by the emerging new science and philosophy (Reif, *Natural Philosophy in Some Early Seventeenth Century Scholastic Textbooks*, pp. 338–339), so that Aristotelian and new elements were mixed.

[3] Near the end of the seventeenth century, the textbook mode of exposition became more and more popular. Direct reading of the Aristotelian *corpus* correspondingly began to decline. During the seventeenth century philosophical textbooks began to dominate the teaching of the subject in most formal courses in institutions of higher learning (Schmitt, "The Rise of the Philosophical Textbook", p. 801).

[4] Cf. the discussion in Reif, *Natural Philosophy in Some Early Seventeenth Century Scholastic Textbooks*, pp. 270–272. Reif's study surveys some thirty Aristotelian textbooks from the first half of the seventeenth century and still contains valuable information.

3. There is a distinction between proximate causes, i.e. causes which produce their effects directly, and remote causes, i.e. causes which produce their effects by means of some intermediary. On the highest level of the causal hierarchy we have remote causes, next, the proximate causes which are produced by the remote causes, and, finally, the observed effects, which are directly produced by their proximate causes and indirectly by their remote causes.

The Paduan Aristotelians, exemplified by Jacopo Zabarella, are usually credited for elaborating Aristotle's logic of demonstration into a scientific method of demonstration in which one first reasons from the effects to the causes (resolution), and then from the causes to the effects (composition). Demonstrative *regressus* is a procedure which combines an inference from an observed effect to its proximate cause with an inference from the proximate cause to the observed effect.[5] Peter Dear notes:

> By no means wholly original with Zabarella but closely associated with his name throughout Europe in the later sixteenth and seventeenth centuries, the technique had developed from a commentary tradition that focused on Aristotle's *Posterior Analytics*, and in particular on Aristotle's distinction between two forms of demonstration: *apodeixis tou dioti* and *apodeixis tou hoti*, usually latinized as demonstration *propter quid* and demonstration *quia*.[6]

Aristotle's views on scientific inquiry, as embodied in his *Posterior Analytics*, revived when they were rediscovered in the twelfth century and they were further developed in the following centuries. *Regressus* was often identified with other methods and procedures: Aristotle's method of proceeding from the particular to the universal was frequently associated with the first movement of demonstrative *regressus*, the analysis, while the procedure of proceeding from the universal to the particular was frequently associated with the second movement of demonstrative *regressus*, the synthesis.[7]

In Section 1.5 I shall argue that Newton's causal outlook on scientific reasoning has relevant parallels with the Aristotelian textbook tradition. I will argue, more precisely, that, although Newton radically reinterpreted the very notion of "cause" in rational mechanics, which will be discussed in Section 1.6, he formulated his views on natural philosophy on a more abstract level in Aristotelian terminology. Accordingly, Newton recast his new notion of cause in a traditional language and veiled his innovative notion of causality in Aristotelian terminology. In other words, the syntax of Newton's natural philosophy was in part conservative, while its semantics was utterly innovative.

Our knowledge of the Aristotelian tradition(s) between roughly 1400 and 1650 remains rather poor compared to our knowledge of neo-Platonism.[8] If my claim

5 Jardine, "Epistemology of the Sciences", p. 686.

6 Dear, *Discipline and Experience*, p. 27. Also see Randall, "The Development of Scientific Method in the School of Padua and Wallace", pp. 166–167.

7 Jardine, "Epistemology of the Sciences", pp. 687–688.

8 Schmitt, *Aristotle and the Renaissance*, pp. 3, 108.

is correct, then this chapter offers a more balanced view on the contributions of Aristotelianism on Newton's natural philosophy – which, I repeat, were only situated at a rather basic level. Newton drew from and was trained in a set of common Aristotelian texts and techniques, which were still the most important sources for university-trained natural philosophers at the mid-seventeenth century. As a consequence, this chapter can also be read as a contextualisation of Newton's causal natural-philosophical stance.[9]

1.2 Stewart's Objection: The Logical Problem of Analysis and Synthesis

Many scholars consider the Greek geometers (Euclid and Pappus) as the primary sources of Newton's natural-philosophical conception of analysis and synthesis.[10] Alistair C. Crombie has claimed that more generally the seventeenth-century natural philosophers' model for scientific discovery and demonstration was none other than the ancient mathematical method of analysis and synthesis.[11]

Note that I wholeheartedly agree that this tradition was of utter importance in the context of Newton's mathematics.[12] In his latest book, Niccolò Guicciardini has forcefully shown how intimately the ancient method of synthesis was tied up with Newton's views on mathematical rigour: from the 1680s Newton became convinced that only the synthesis provides proper constructions and demonstrative certainty.[13] Therefore, Newton considered only the synthesis as "worthy of public utterance [publicae notitiae dignus]."[14] The young Newton had earlier enthusiastically endorsed the new, i.e. algebraic, analysis of the moderns.[15] In this period, he developed the method of infinite series and the analytical method of fluxions.[16]

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9 Newton's causal realism has been forcefully argued for in Janiak, *Newton as Philosopher*, Chapter 3, but no reference is made to the specific traditions which shaped Newton's causal stance. Neither has Janiak explicated Newton's notion of efficient causation in the *Principia*.

10 See Hintikka and Remes, *The Method of Analysis*, pp. 107–108; Westfall, *Never at Rest*, pp. 377–381; and, Dear, *Discipline and Experience*, p. 240.

11 See Crombie, *Styles of Scientific Thinking in the European Tradition*, I, p. 283. In an accompanying footnote Crombie refers to *The Opticks* but not to the *Principia* (ibid., I, p. 716). An overview of mathematical analysis and synthesis can be found in: Panza, "Classical Sources for the Concepts of Analysis and Synthesis".

12 For recent accounts and further references, see Guicciardini, "Analysis and Synthesis in Newton's Mathematical Work" and id., *Isaac Newton on Mathematical Certainty and Method*, Chapters 9 and 14.

13 Guicciardini, *Isaac Newton on Mathematical Certainty and Method*, pp. 230–232.

14 Whiteside, ed., *The Mathematical Papers of Isaac Newton*, III, pp. 278/279.

15 Guicciardini, "Analysis and Synthesis in Newton's Mathematical Work", p. 319; id., *Reading the Principia*, pp. 17–38; and, id., *Isaac Newton on Mathematical Certainty and Method*, Chapter 8.

16 See Chapters 7 and 8, respectively, in Guicciardini, *Isaac Newton on Mathematical Certainty and Method*. See, furthermore, Panza, *Newton et les origins de l'analyse: 1664–1666*.

Newton in fact saw his method of infinite series as an enrichment of the new finite analysis. Indeed, in a letter to Henry Oldenburg for Leibniz on June 13 1676, Newton wrote: "From all this it is to be seen how much the limits of analysis are enlarged by such infinite equations: in fact by their help analysis reaches, I might almost say, to all problems."[17] In his early mathematical works, however, Newton's method "was not yet a theory but rather a panoply of techniques ultimately justified by their success in resolving problems concerning curvilinear figures."[18] Most of these techniques had no firm foundation. In the 1680s Newton began to seek a firmer ground on which to establish his analytical methods of series and fluxions. Correspondingly, he turned to the synthetic method of fluxions and by doing so he came to distance himself from his previous endorsement of the new analysis of the moderns.[19] Newton had profound admiration for the geometrical writings of the ancients and especially for Pappus which led him to criticise the modern symbolic mathematics.[20] Several manuscripts, which were composed in the early 1690s, bear testimony of Newton's concern with the ancient analysis and synthesis in mathematics.[21]

Despite Newton's blending of natural-philosophical and mathematical analysis and synthesis, important differences exist between them.[22] That the *Principia* proceeds as a highly mathematical exposition has probably biased our views on Newton's conception of natural philosophy, which was not only mathematical but also causal. There is an important asymmetry between mathematical and natural-philosophical analysis and synthesis – I dub this asymmetry "the logical problem of analysis and synthesis". In the mathematical analysis one starts from what is sought – as if it has been achieved – and, by working backwards, one arrives at what is known; in the synthesis one works in the other direction: one starts with what is known and arrives at what is sought. From a conceptual point of view the mathematical account of analysis and synthesis is incompatible with Newton's conception of analysis as discovering causes and of synthesis as assuming these causes to explain other phenomena. In the mathematical tradition analysis consists of reasoning from

[17] Newton, *Correspondence*, II, p. 39. This quotation is further discussed in Guicciardini, *Isaac Newton on Mathematical Certainty and Method*, pp. 164–167.

[18] Guicciardini, *Isaac Newton on Mathematical Certainty and Method*, p. 212.

[19] Guicciardini, *Reading the Principia*, pp. 37–38.

[20] Guicciardini, "Analysis and Synthesis in Newton's Mathematical Work", p. 317 and id., *Isaac Newton on Mathematical Certainty and Method*, Chapter 9.

[21] The corresponding manuscript material is to be found on CUL Add. Ms. 3963, ff. 27^{r}-28^{r}, ff. 109^{r}-110^{r}, ff. 154^{r}-155^{v}, ff. 161^{r}-170^{v}, and ff. 159^{r}-160^{v}. Transcriptions and translations are to be found in Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VII, pp. 452–459. For the context of these manuscripts, see Whiteside's editorial introduction (ibid., VII, pp. 185–199).

[22] Guicciardini comments as follows: "It seems to me that Newton conflated these two different conceptions of analysis and synthesis (the Pappian and the Aristotelian) as a rhetorical move aimed at defending the certainty of his natural philosophy." (Guicciardini, *Newton on Mathematical Certainty and Method*, p. 324).

what is sought to what is known.[23] In Newton's natural-philosophical view, analysis consists in reasoning from what is known, the effect, to what is sought, the cause. Additionally, in the mathematical tradition the analysis is considered to be useful only at the level of heuristics and not at the level of demonstration. In Newton's natural-philosophical method, however, both the analysis and the synthesis are methods of demonstration. The mathematical synthesis is the reverse of the analysis: we proceed from what is known to what is sought. By contrast, the natural-philosophical synthesis is not simply the reverse direction of the analysis, since we proceed from the causal principle established in the analysis, which is based on particular phenomena, to its application to different phenomena for the purpose of explaining them. What seems, furthermore, to be absent from the mathematical tradition is that Newton's conception of natural philosophy is explicitly causal – a feature which is obviously lacking in the mathematical tradition of analysis and synthesis, since the relation between what is sought and what is known in either direction is purely deductive.[24]

We need to look elsewhere if we want to understand the origins of Newton's views on natural-philosophical analysis and synthesis. Given the logical problem of analysis and synthesis, it seems that a crucial element of Newton's conception of natural philosophy is left out and unexplained. The textbook tradition on logic and method, to which we shall turn in Section 1.4, adequately fills in this explanatory gap, or so I shall argue. As a stage-setting, I shall first contextualize Newton's familiarity with the Aristotelian literature.

1.3 Newton's Early Aristotelian Training

That Aristotelian works are required to understand Newton's intellectual trajectory is not a complete novelty. Maurizio Mamiani has recently argued that the tract on logic *Logicae Artis Compendium* (1615), written by Robert Sanderson, played an important role in the genesis of Isaac Newton's *regulae philosophandi*.[25] Sanderson's laws (such as the law of brevity) are as far as logic and rhetoric is

[23] See Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VII, pp. 249–251, 307–309 and CUL Add. Ms. 3968, f. 107^{r} [late 1710s] (= Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VIII, p. 449).

[24] Few authors have explicitly labelled Newton's reasoning abductive. See, however, Smith, "The Methodology of the *Principia*", pp. 160–161 and Stein, "On the Notion of Field in Newton, Maxwell, and Beyond", p. 272.

[25] See Mamiani, "To Twist the Meaning: Newton's *Regulae Philosophandi* Revisited". Newton owned the third edition of 1631 which he obtained in 1661 (Wren Library, Adv.e.1.15). Corrections are to be found on pp. 2, 19, 22, 53, 62, 65, 69, 89, 99, 134, 167, and 213, marginalia on pp. 2, 10–11, 18, 22, 28–29, 34–35, and 142, and signs of dog-earing on pp. 27, 29, 46, 60, 67–68, 71, 84, 117, 139, 147, 154, 157, 159, 161–163, 170, 175, 190, 199, 221, 223, and 232. There are no indications that Newton read the 124 pages long appendices.

concerned "the primary source of Newton's rules," Mamiani concludes.[26] More importantly, in their rich editorial introduction to Newton's Trinity Notebook J. E. McGuire and Martin Tamny have rightfully highlighted the relevance of Newton's early training in Aristotelian philosophy and they have pointed to Newton's notes from various Aristotelian authors.[27] When Newton was a student at Cambridge, study began with a heavy dose of Aristotle's logic, ethics, rhetoric and natural philosophy, as Aristotelianism was still the central system of thought in the educational system.[28] In 1661 Isaac Newton began to keep notes pertaining to his studies. Newton's Trinity Notebook (CUL Add. Ms. 3996) contains not only the famous *Questiones Quaedam Philosophicae*[29] but also copious notes from several Aristotelian authors, such as Joannes Magirus, Daniel Stahl and Gerardus Vossius and some *compendia* of Aristotle's *corpus*.[30] I shall briefly discuss what these works have to say on causal explanation. It will turn out that only Joannes Magirus and Daniel Stahl's work, which according to Richard S. Westfall introduced Newton to natural philosophy,[31] contained a thorough discussion of the Aristotelian doctrine of the causes. However, Stahl wrote nothing explicit on *regressus*, but there are some statements which are compatible with the works I will present in the following section of this chapter.

In CUL Add. Ms. 3996, Newton took notes from Gerardus Vossius' *Rhetorices contractae, sive partionum oratorium*, *libri V* (1631).[32] The material from which Newton took notes dealt with demonstration, deliberation, conjectural reasoning and the various states of the mind.[33] Vossius' book focused on rhetoric and does not engage in a discussion of causal explanation or *regressus* in natural philosophy. Joannes Magirus' Scholastic compendium on physiology *Physiologiae peripatetica, Libri sex cum commentaries* (1619) did contain a presentation of the Aristotelian doctrine of the causes, but it did not contain a discussion of *regressus* in natural philosophy.[34] Newton took various notes on such topics as:

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[26] Mamiani, "To Twist the Meaning: Newton's *Regulae Philosophandi* Revisited", p. 4.

[27] McGuire and Tamny, eds., *Certain Philosophical Questions* and Wallace, "Newton's Early Writings: Beginnings of a New Direction".

[28] See Westfall, *Never at Rest,* pp. 81–82. Students read Aristotle's *Physica*, *De Caelo*, and *De Anima*. See especially Allen, "Scientific Studies in the English Universities of the Seventeenth Century", p. 220. For a general survey see: Costello, *The Scholastic Curriculum at Early Seventeenth-century Cambridge*.

[29] These are on ff. 88^{v}-135^{v}.

[30] McGuire and Tamny, eds., *Certain Philosophical Questions* and Hall, "Sir Isaac Newton's Note-Book, 1661-65". On ff. 3^{r}-10^{v} Newton took notes on Pace[/Porphyry], *Aristotelis Stagiritæ Peripateticorum principis Organum*. Newton also composed notes on Aristotelian ethics (ff. 34^{r}-36^{r} and ff. 38^{r}-40^{r}).

[31] Westfall, *Never at Rest*, p. 84.

[32] CUL Add. Ms. 3996, ff. 77^{r}-81^{v}.

[33] McGuire and Tamny, eds., *Certain Philosophical Questions*, p. 19.

[34] Magirus' treatment of the doctrine of the causes can be found in *Librum I: De natura deque naturalium principiis, affectionibus & accidentibus* (Magirus, *Physiologiae peripatetica*, pp. 1–56). For Magirus' discussion on efficient and final causes, see ibid., pp. 21–25.

motion,[35] rest, infinity, place, vacuum, internal and external affections,[36] Aristotelian cosmology, and specific natural phenomena.[37] Magirus defined physics as "the science of natural bodies" ("*corporum naturalium scientia*"):

> Physics in fact investigates the causes, principles and the individual affections of natural bodies; and demonstrates the affections [of natural bodies] by their causes.[38]

Daniel Stahl's *Axiomata philosophica, sub titulis XX* (1645), a more advanced Aristotelian compendium, was of considerable interest to Newton. Stahl's work is clearly more philosophical in orientation than Magirus' and this is perhaps the reason why Newton's notes on Stahl's *Axiomata philosophica* surpass those of all works previously mentioned in quantitative terms.[39] Newton took notes on the nature of essence, actuality and potentiality, the theory of the causes, the appetites, the will, agency and patient, matter, form, the theory of predication, the theory of genus, species, and difference, the idea of definition, the distinction between

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[35] Ibid., pp. 26–56.

[36] Ibid., pp. 9, 21.

[37] McGuire and Tamny, eds., *Certain Philosophical Questions*, pp. 15–17; CUL Add. Ms. 3996, ff. 16^r-26^v. Newton made notes on all chapters of Book I, entitled *De Naturæ deque rerũ naturalium principijs, affectionibus & accidentibus* (namely, on Cap. 1 *Quid Phisiologia, quod item natura*, Cap. 2 *De principijs rerum naturalium Principijs intrinsecis*, Cap. 3 *De principijs rerum naturalium extrinsecis*, Cap. 4 *De Motu*, Cap. 5 *De Motûs Speciebus*, Cap. 6 *De Quiete*, Cap. 7 *De Finito & infinito*, Cap. 8 *De loco*, Cap 9. *De Vacuo sive inani*, and Cap. 10 *De tempore*), all chapters of Book II, entitled *De Mundo, et ejus Regione æthereâ* (namely, on Cap. 1 *De Mundo in genere ejusque causis & accidentibus*, Cap. 2 *Quid Cælum, & quæ ejus divisio*, Cap. 3 *De motu sphærorum recto & transverso*, Cap. 4 *De Stellarum Naturâ*, Cap. 5 *De Stellis fixis*, Cap. 6 *De Planetis*, Cap. 7 *De Eclipsi solis et lunæ*), all chapters of Book III, entitled *De Elementis et eorum qualitatibus, Mistione, et Temperamentis* (namely, on Cap. 1 *De Elementorum naturâ, in genere*, Cap. 2 *De Igne*, Cap. 3 *De Aëre*, Cap. 4 *De Aquâ*, Cap. 5 *De Terrâ*, Cap. 6 *De Primis Elementorum qualitatibus*, Cap. 7 *De Secundis qualitatibus*, Cap. 8 *De obscuris & occultas Qualitatibus*, Cap. 9 *De agentibus & Patientibus deque contractu*, Cap. 10 *De Mixtione*, Cap. 11 *De Temperamentis*, Cap. 12 *De Generatione simplici, & Putrefactione*) and some chapters of Book IV, entitled *De Corporibus imperfectè mixtis vel de Metereologiâ* (namely, on Cap. 1 *De Meteorum causis in genere*, Cap. 2 *De Meteoris ignitis puris*, Cap. 3 *De Meteoris ignitis puris Ac primùm de Cometis*, Cap. 4 *De reliquis ignitis puris Meteoris Mixtis fulmine tonitru & fulgure*, Cap. 5 *De Meteoris apparentibus ab Arist. Φάσματα nunoupantur*). At this point Newton's notes ended: he did not make further notes on the two remaining chapters of Book IV nor on Book V, entitled *De corporibus perfecte mixtis, tum inanimis, tum animatis*, or Book VI, entitled *De anima*. Immediately after these notes Newton made some notes on parallaxes, eclipses and the mean distances of the planets (CUL Add. Ms. 3996, ff. 27^r-30^v). One is tempted to speculate that Newton did not finish his notes on the rest of Magirus' *Physiologiae peripateticae* because his recent interest in the comet in 1664 prevented him from further doing so. On Newton's study of this comet, see McGuire and Tamny, "Newton's Astronomical Apprenticeship: Notes of 1664/5".

[38] Translation of: "Hæc [i.e., physica] enim corporum naturalium causas atque principia & proprias Affectiones inquirit, affectionesque causis suis demonstrat." (Magirus, *Physiologiae peripateticae*, p. 1).

[39] Bear in mind that these folios, which are quite compactly written, recto as well as verso, start on f. 43^r and end on f. 71^v of CUL Add. Ms. 3996).

subject and accident, and the problem of truth and falsity.[40] The third chapter in Stahl's *Axiomata philosophica* contains 23[41] rules concerning the doctrine of causes (*Titulus III continens Regulae XXI* [sic] *circa doctrinam causæ & causati*[42]).[43] According to Stahl, effects are the "first part of experience", only later do we know the causes of things.[44] Effects occur first in our experiences, the causes only latently (cf. "incurrunt enim effectus in sensus, causis nos latentibus").[45] For my present endeavour, this brief overview suffices to contextualize Newton's familiarity with the Aristotelian tradition during his student years at Cambridge.

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[40] McGuire and Tamny, eds., *Certain Philosophical Questions*, p. 18. Other interesting topics mentioned are: final causes (ff. 48^{r}-49^{v}), formal causes (ff. 59^{v}-60^{v}), matter (ff. 58^{v}-59^{v}), and the part-whole relation (ff. 61^{v}-63^{r}). Newton left notes on Chapters 1–21.

[41] This chapter contains 20 numbered *regulae* and three additional (unnumbered) ones.

[42] Stahl, *Axiomata philosophica*, pp. 60ff.

[43] Newton's notes are on CUL Add. Ms. 3996, ff. 44^{r}-47^{v}. The rules are: 1. Nihil est causa suiipsius (ibid., f. 44^{r}; Stahl, *Axiomata philosophica*, pp. 61–64), 2. Nihil fit sine causâ (CUL Add. Ms. 3996, f. 44^{v}; Stahl, *Axiomata philosophica*, pp. 64–66), 3. Positâ causâ ponitur causatum & vice versâ (CUL Add. Ms. 3996, ff. 44^{v}-45^{r}; Stahl, *Axiomata philosophica*, pp. 66–70), 4. Negatâ causâ negatur effectus, & vice versâ (CUL Add. Ms. 3996, f. 45^{r}; Stahl, *Axiomata philosophica*, pp. 70–73), 5. Causa est prior causato (CUL Add. Ms. 3996, f. 45^{r}; Stahl, *Axiomata philosophica*, pp. 73–75), 6. Causa est notior effectu (CUL Add. Ms. 3996, f. 45^{r}; Stahl, *Axiomata philosophica*, pp. 75–77), 7. Qualis causa, talis effectus (CUL Add. Ms. 3996, f. 45^{v}; Stahl, *Axiomata philosophica*, pp. 77–78), 8. Si affirmatio est causa affirmationis etiam negatio est causa negationis (CUL Add. Ms. 3996, f. 45^{v}; Stahl, *Axiomata philosophica*, pp. 78–79), 9. Causa non est deterior effectu (CUL Add. Ms. 3996, f. 45^{v}; Stahl, *Axiomata philosophica*, pp. 79–84), 10. Quidquid est in effectu præexistit in causâ (CUL Add. Ms. 3996, f. 45^{v}; Stahl, *Axiomata philosophica*, pp. 86–87), 11. Impossibile est, aliquem ejusdem causæ effectum æquivorum nobilionem esse effectu univoco (CUL Add. Ms. 3996, f. 45^{v}; Stahl, *Axiomata philosophica*, pp. 84–86), 12. Causa causæ est etiam causa causati (CUL Add. Ms. 3996, ff. 45^{v}-46^{r}; Stahl, *Axiomata philosophica*, pp. 87–94), 13. Idem quatenus idem semper facit idem (CUL Add. Ms. 3996, f. 46^{v}; Stahl, *Axiomata philosophica*, pp. 94–98), 14. Idem est causa contrariorum (CUL Add. Ms. 3996, f. 46^{v}; Stahl, *Axiomata philosophica*, pp. 98), 15. Principium latius patet quam causa (CUL Add. Ms. 3996, f. 46^{v}; Stahl, *Axiomata philosophica*, pp. 98–100), 16. Omnia causa agit (CUL Add. Ms. 3996, f. 46^{v}-47^{r}; Stahl, *Axiomata philosophica*, pp. 100–108), 17. Effectus non prædicatur de suâ causâ nota causa de effectu (CUL Add. Ms. 3996, f. 47^{r}; Stahl, *Axiomata philosophica*, pp. 108–113), 18. Causae sunt sibi invicem causae (CUL Add. Ms. 3996, f. 47^{r}; Stahl, *Axiomata philosophica*, pp. 113–117), 19. Causae possunt coïncidere (Add. Ms. 3996, f. 47^{r}; Stahl, *Axiomata philosophica*, pp. 117–119), 20. Cujus bonus est, id ipsum bonum: & cuijus causa mala idipsum malum (CUL Add. Ms. 3996, f. 47^{r}; Stahl, *Axiomata philosophica*, pp. 119–120), 21. Propter quod unumquodque est tale, illud est magis talo. Sit intelligo huisquid est talo, quia aliud est talo, id aliud est magis talo. Sit haec causa proxima, adaequata & propter quam. (CUL Add. Ms. 3996, f. 47^{r}; Stahl, *Axiomata philosophica*, pp. 120–130), 22. Cujus causæ omnes sunt universalis, illud ipsum est universale. Inter has causas includatur causa intrumentalis sine quâ aliae causae nihil agant. (CUL Add. Ms. 3996, f. 47^{r}; Stahl, *Axiomata philosophica*, pp. 130–133), 23. Omnis effectus per accidens reducitur ad causam per se (CUL Add. Ms. 3996, f. 47^{r}; Stahl, *Axiomata philosophica*, pp. 133–136).

[44] Stahl, *Axiomata philosophica*, p. 69. Also see Newton's copy of Sanderson, *Logicae Artis Compendium* (= Wren Library, Adv.e.1.15), pp. 196–197.

[45] Stahl, *Axiomata philosophica*, p. 76.

1.4 Textbooks on Logic and Method

J. E. McGuire and Martin Tamny have suggested that "Newton probably became familiar with the method of analysis and synthesis in a philosophical context" from Thomas Hobbes' *Elements of Philosophy* (1656).[46] Chapter IV of *Elements of Philosophy*, entitled "Of Method," contains a discussion of the "*Analyticall*" and "*Syntheticall*" method.[47] Hobbes characterized "method" as "*the shortest way of finding out Effects by their known Causes, or of Causes by their known Effects*."[48] The former part refers to the "*Compositive*" or "*Syntheticall* Method," or the "Method of Demonstration," whereby we try to ascertain "in what manner particular Causes ought to be compounded for the production of some certaine Effect." The latter part refers to the "*Resolutive*" or "*Analyticall* Method," or the "Method of Invention," whereby "we enquire into the Cause of some determined Appearance, or endeavour to find out the certainty of something in question, as what is the cause of *Light*, of *Heat*, of *Gravity*, of a *Figure* propounded, and the like."[49] In the concluding paragraph of Chapter IV, Hobbes distinguished between analysis in natural philosophy and analysis in mathematics: in mathematics method is threefold, namely first to establish an equation between known and unknown things, secondly, "to judge whether the Truth or Falsity of the Question may be deduced from it or no," and, finally, to ascertain how to resolve the equation.[50] Moreover, resolution in mathematics, contrary to resolution in natural philosophy, cannot be practiced unless one is well versed in the theorems of geometry.[51] Although McGuire and Tamny's suggestion is reasonable, it remains to be shown for the historical record that Hobbes was indeed a factual source of Newton's ideas on analysis and synthesis: it is unknown whether Newton read Hobbes' *Elements of Philosophy*.

In order to determine factual sources, it is useful to begin by considering those books that treated *regressus* and natural-philosophical analysis and synthesis and that were either part of Newton's library or which were written by natural philosophers who Newton knew well. On the basis of such works it is, furthermore, possible give a representative sample of Aristotelian textbooks available to Newton. My choice might appear somewhat subjective, but it points to the importance of this tradition. As Cees Leijenhorst pointed out in his recent study on Hobbes:

> Although our choice of Aristotelian authors thus remains somewhat arbitrary, nonetheless a fairly representative sample of the different kinds of Aristotelianism *en vogue* in Hobbes' days is possible.[52]

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[46] See McGuire and Tamny, eds., *Certain Philosophical Questions*, p. 24.

[47] Hobbes, *Elements of Philosophy*, pp. 48–66.

[48] Ibid., pp. 48–49. See, furthermore, Jesseph, "Hobbes and the Method of Natural Science", pp. 92–96.

[49] Hobbes, *Elements of Philosophy*, p. 50, cf. pp. 55–59.

[50] Ibid., pp. 65–66.

[51] Ibid., p. 65.

[52] See Leijenhorst, *The Mechanisation of Aristotelianism*, p. 8.

The same holds for Newton's Aristotelianism. For copies pertaining to Newton's private library it is possible to ascertain traces of Newton's reading on the basis of annotations, corrections, dog-earing, and the like. If it can be shown on the basis of material traces that Newton thoroughly studied one of these books, and if, it can be shown, furthermore, that their content is relevantly similar to Newton's views on the matter, then this counts a strong indication for considering the work at hand as a factual source. Let us now investigate which of the following works was important for Newton's views on natural-philosophical analysis and synthesis.

In 1687, the year in which Newton's first edition of the *Principia* appeared, John Wallis' published his *Institutio logicae*.[53] Newton did not own a private copy of it. Wallis' work was dedicated to the Officers and Fellows of the Royal Society and what Wallis said on method was primarily intended for young natural philosophers.[54] In the third part, in *Caput XV De Inductione & Exemplo*, Wallis discussed induction as a type of imperfect syllogism where one proceeds from particulars to a universal. Experimental philosophy (*Philosophia Experimentalis*) proceeds from effects to causes:

> For although in the order of nature the progression is from causes to effects, yet in the order of knowing the progression is from observed effects to the investigation of causes. And indeed in the magnetic effects which I have already observed before (and that originally in a concrete case, I believe), that a magnet attracts iron and points to the north, no one would know or indeed suspect such a thing from the nature of the magnet. And so equally in many other things.[55]

In the speculative sciences, one seeks:

> to begin with the cause (or what is first in regard to way of operating) and from there to proceed to the effect; or alternatively to begin with the subject (the name, nature, and species which are first investigated) and thence to proceed to accidents, adjuncts, properties, and relations, along with the principles and causes of these last; [. . .][56]

In the speculative sciences method is twofold: the first is a method of investigation (analysis); the second is a method of exposition or education (synthesis). The first proceeds from individuals to universals ("a Particularibus ad Universalia procedit");

[53] Howell, *Eighteenth-Century British Logic and Rhetoric*, p. 29. Wallis is of course primarily known as a mathematician – his *Arithmetica Infinitorum* (1655) being his most popular work.

[54] Ibid., p. 39.

[55] Howell's translation (ibid., p. 36) of: "Quando enim, in ordine Naturæ, processus sit a Causis ad Effecta: in ordine tamen cognitionis, proceditur ab effectis observatis, ad inquisitionem causarum, Et quidem, in Magnetis effectibus jam notis; nisi observatum ante fuisset (idque casu, creda, primitus,) quod Magnes ferrum alliceret, atque respiceret Septentrionem; nemo id, ex magnetis natura cognosceret, aut suspicaret quidem. Pariterque in multis aliis." (Wallis, *Institutio logicae*, p. 172). Cf. "Est igitur causa naturâ notior effectu. Effectus autem sunt notiores nobis." (Stahl, *Axiomata philosophica*, p. 76).

[56] Howell's translation (Howell, *Eighteenth-Century British Logic and Rhetoric*, p. 39) of: "à Causa ordiuntur (seu quod primum est in operando) indeque ad Effectus procedunt. Aut etiam à Subjecto; cujus Nomen, Naturam, & Species inquirunt; indeque ad Accidentia, Adjuncta, Proprietates & Affectiones procedunt; cum Principiis Causisque harum Affectionum;" (Wallis, *Institutio logicae*, p. 213).

the second proceeds in the opposite direction.[57] Finally, Wallis discussed method in mathematics, which proceeds from definitions, axioms and postulates to the proof of propositions.[58] Wallis' treatment of natural-philosophical synthesis as a method of education did not correspond to Newton's views on the matter.

One might also have reason to believe that Antoine Arnauld and Pierre Nicole's *Logica sive ars cogitandi*, an important source of the so-called *Port-Royal Logic*, which remained for well over a century a standard textbook in philosophy,[59] affected Newton's views on natural-philosophical analysis and synthesis and causal reasoning. This work was originally printed in French in 1662 as *La Logique ou l'art de penser*. It is generally known as a Cartesian work.[60] The insistence of the authors on syllogistic reasoning, however, gives away the Aristotelian concerns. Moreover, the authors grant that, although Aristotle's *Analytics* were somewhat confused, all they know concerning the rules of logic is taken from Aristotle's *Analytics*,[61] that when they suggest corrections to his works this does not affect the importance of his philosophy,[62] and that they sought to clarify Aristotle's ideas so that they are more clearly understandable and free of errors.[63] In other words, they sought to provide a Cartesian update of Aristotle's logic.[64] The first Latin edition was printed in 1674; the first English edition in 1685. Newton owned the Latin version of 1687.[65] In Chapter II of the fourth book entitled *De duabis Methodus, Analysi & Synthesi,* the authors Arnauld and Nicole discussed the nature of (scientific) method (*methodus*). The aim of this method is to guide the mind from a state of oblivion to a state of knowledge of truth. This method is twofold:

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57 Wallis, *Institutio logicae*, pp. 212–213.

58 See Howell, *Eighteenth-Century British Logic and Rhetoric*, pp. 40–41; Wallis, *Institutio logicae*, pp. 215–217.

59 For the basics on this work see Howell, *Eighteenth-Century British Logic and Rhetoric*, pp. 350–363.

60 One might entertain the hypothesis that Descartes' views on analysis and synthesis (e.g., Descartes, *Œuvres de Descartes*, VII, pp. 155–157) may have influenced Newton's. However, it must be noted that Descartes' statements on analysis and synthesis are quite puzzling in themselves and that Descartes, upon closer scrutiny, considered the synthesis as a redundant method of presentation of the results obtained in the analysis, as Raftopoulos has recently cogently argued in a thorough study of Descartes' *oeuvre* (Raftopoulos, "Cartesian analysis and synthesis"; cf. Timmermans, "The Originality of Descartes's Conception of Analysis as Discovery", p. 442; Descartes, *Œuvres de Descartes*, VII, p. 156). Moreover, Roger Ariew notes that: "There are numerous methods called analysis and synthesis in early philosophy, most of which have nothing to do with the various things Descartes called analysis and synthesis – resolution and composition within the method of the *Regulae*, the two modes or demonstrations of the *Second Replies*, or the analysis (and synthesis) of the ancients." (Ariew, "Descartes, the First Cartesians, and Logic", p. 253, footnote 31).

61 Arnauld and Nicole, *Logica sive ars cogitandi*, xxii, cf. xxiv.

62 Cf.: "aliunde clarum est quæ hic redarguuntur, non esse magni ponderis, nec Philosopiæ Aristotelicæ corpus concurere, quam impugnare, nunquam in animum induximus" (ibid., xxi).

63 Ibid.

64 This work was a mixture between Cartesian and Aristotelian elements, which is entirely consistent with Reif's claims on post-1650 textbooks which were mentioned previously.

65 See Harrison, *The Library of Isaac Newton*, p. 182 [item n° 980].

> the one to discover Truth, which is called *Analysis*, or *Method of Unfolding*, and which may also be called *Method of Invention*: And the other to make it understood by others when it is found out, which is called *Synthesis*, of the *Method of Composition*, and may also be called the *Method of Doctrine*.[66]

An important difference between analysis and synthesis is this: analysis proceeds from particular cases to general propositions; synthesis proceeds from general propositions to particular cases.[67] Consistent with Cartesian philosophy, Arnould and Nicole seem to associate the synthesis with a pedagogical context exclusively. Every scientific investigation is done analytically: it attempts to resolve a specific question.[68] Let us look at the two kinds of *quaestio rei*. The first kind of questioning of things is:

> The first, when we seek for the causes by the effects.[69]

The authors then give some examples. From the various effects of magnets we try to infer the causes. When we notice that *horror vacui* effects occur in nature, we try to determine the cause that produces such effects. From the flux and reflux of the sea we try to establish its true cause. The second kind concerns the reverse:

> The second is, when we seek to find out the Effect by the Causes.[70]

Suppose that we have determined that wind and water have a great force to move bodies. For practical purposes (e.g., technological ones), we will then try to manipulate this force in order to obtain desirable effects. The difference with the first strategy is that:

> So that it may be said, the first sort of Questions, whereby we seek the Causes by the Effects, includes the speculative part of Physics, and the second part that seeks for the Effects by the Causes, contains the Practical part.[71]

In Chapter XI, Arnauld and Nicole provided rules that are of use in science. These mainly concern the need for clarity of definitions and axioms.[72] Two features are worth mentioning: according to *Logica sive ars cogitandi*, analysis proceeds from the particular to the universal (and vice versa for synthesis) and in theoretical physics we infer causes from effects.

However, Newton's private copy (Wren Library, NQ.10.27) contains no notes, no signs of dog-earing and, in fact, no traces of reading at all. In other words, given the

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66 Arnauld and Nicole, *Logic: or, the Art of Thinking*, p. 368; Arnauld and Nicole, *Logica sive ars cogitandi*, p. 386.

67 Arnauld and Nicole, *Logica sive ars cogitandi*, p. 375.

68 Ibid., pp. 375–376. Arnauld and Nicole claim that the greatest part concerning questions was taken from a manuscript of the deceased Descartes (ibid., p. 375).

69 Ibid., p. 375; Arnauld and Nicole, *Logic: or, the Art of Thinking*, p. 368.

70 Arnauld and Nicole, *Logic: or, the Art of Thinking*, p. 369; Arnauld and Nicole, *Logica sive ars cogitandi*, p. 376.

71 Arnauld and Nicole, *Logic: or, the Art of Thinking*, p. 369; Arnauld and Nicole, *Logica sive ars cogitandi*, p. 376.

72 Arnauld and Nicole, *Logica sive ars cogitandi*, p. 416.

historical data available there is no evidence to suggest that *Logica sive ars cogitandi* ought to be considered as an actual source of inspiration on Newton's natural-philosophical analysis-synthesis. Furthermore, Arnauld and Nicole only treated analysis as a proper method of inquiry and ascribe a purely pedagogical function to the synthesis. Also, they associated the analysis with theoretical sciences and synthesis with practical sciences. These views were far removed from Newton's views on the matter.

There is one work which can quite safely be considered as an actual source for Newton's ideas on natural-philosophical analysis and synthesis: Samuel Smith's *Aditus ad logicam* (1613).[73] Newton owned the version of 1649 and there is serious evidence that Newton thoroughly studied this work (see Fig. 1.1).[74] I will discuss some relevant fragments from *Liber III*. Smith noted that science involves knowing things. To know something is to know the cause of it.[75] Although we first know the effect and only later their cause, in the order of things (*ordo naturae*) the cause takes place first.[76] In order to have causal knowledge, it is required that we know the cause, that this cause is a proximate cause, that the connection between cause and effect is known, and, finally, that our judgment of this connection is certain.[77] This entails that for Smith causal knowledge generally speaking refers to knowledge of the proximate cause (*causa proxima*).[78] In *Caput IV* and *V* Smith dealt with

[73] For the basics on this work see Howell, *Eighteenth-Century British Logic and Rhetoric*, pp. 292–298. At the time I published Ducheyne, "Newton's Training in the Aristotelian Textbook Tradition", I had not yet been able to study Newton's private copy of Smith's *Aditus ad logicam*.

[74] Newton's private copy is conserved at Wren Library, Trinity College, Cambridge, NQ.9.166[1] (Harrison, *The Library of Isaac Newton*, p. 110 [item n° 293]). It shows numerous signs of dog-earing (namely on pp. 1, 12, 14, 16, 18, 20, 24, 26, 28, 30, 32, 50, 56, 76, 81, 86, 93–94 [note that p. 94 is incorrectly numbered as p. 97], 95–96, 105, 107, 109, 121, 123, 126, 133, 136, 142, 144, 146, 147, 149, 151, 155, 163, 165, 166, 168, 169, 173, 178, 180, 181, 190, and 193) and contains several sections marked with an "X" in the margin (namely on pp. 21–22, 123, 125, 128, 132, 136–137, 139, 144, 146, 149, and 166), corrections (namely on pp. 123, 135, 136, 142, 170, and 182), and marginalia (namely on pp. 124–125, 142, 145, 161, 167, 170, 171, 173, and 187). Newton also underscored or accentuated sections (namely on pp. 122, 129, 142–143, 146, 148, 149). The *arbor Porphyriana*, on f. 2[r] of Add. Ms. 3996 is in fact based on Samuel Smith's division (NQ.9.166[1], p. 32[bis], cf. p. 14). Newton's copy of Smith's *Aditus ad logicam* is bound together with Brerewood's *Elementa logicae* (NQ.9.166[2]) (Harrison, *The Library of Isaac Newton*, p. 240, item n° 1537). Newton added some minor marginalia in the *præfatio* of Brerewood's *Elementa* and added underscores, but these are irrelevant to our current investigation.

[75] Smith, *Aditus ad logicam*, p. 97.

[76] Ibid., p. 154.

[77] Ibid., p. 97.

[78] In Newton's copy the following text is marked with an "X" in the margin: "Demonstratio primariò & præcipuè ex notioribus natura procedit quia conclusionis proximam causam ostendit, & ex consequenti à notioribus nobis: quia omnis demonstratio ob nostram cognitionem sit; quando enim causa semel nobis innotescit, meliùs apprehendimus effecta." (Wren Library, NQ.9.166[1], pp. 135–136).

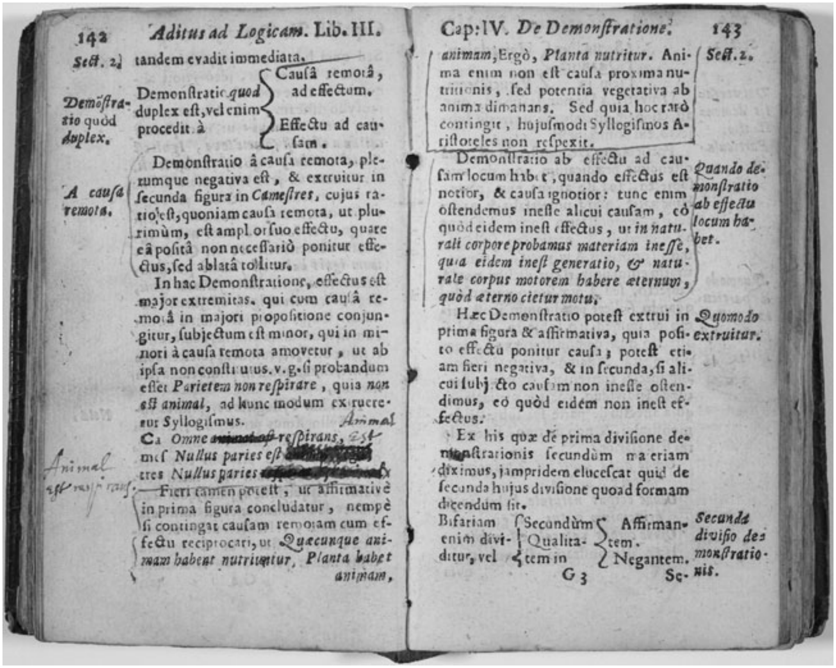

Fig. 1.1 Pages 142–143 of Newton's copy of Samuel Smith's *Aditus ad logicam* (1649). Courtesy of the Wren Library, Trinity College, University of Cambridge. Classmark: NQ.9.166[1]

demonstratio quod and *demonstratio propter quid*. *Demonstratio propter quid* proceeds from the proximate cause to the effect.[79] *Demonstratio quod* is twofold and proceeds either from the effect to the (proximate) cause or from the remote cause to the effect.[80] According to Smith, there are three forms of causal reasoning:

1. proximate causes → effects (*demonstratio propter quid*)
2. effects → proximate causes (*demonstratio quod*, type 1)
3. remote causes → effects (*demonstratio quod*, type 2)

[79] Cf. On the difference between the two types of demonstrations, Smith noted: "Hæ duæ species [...] differunt quòd *δίοτι* [i.e., *demonstratio propter quid*] procedit à prioribus secundùm naturam, & causis proximis conclusionis; at '*οτι* [i.e., *demonstratio quod*], vel ab effectu procedit, & sic non erit à causa; vel si à causa procedat, erit remota non proxima, & sic non constabit ex immediatiis." (Smith, *Aditus ad logicam*, p. 111).

[80] "Demonstratio *quod* duplex est, vel enim procedit a $\left\{\begin{array}{c}\text{Causâ remotâ ad effectum.}\\ \text{Effectu ad causam.}\end{array}\right.$" (ibid., p. 112).

On *demonstratio quod*, type 1, Smith wrote:

> The demonstration from the effect to the cause takes place when the effect is more known and the cause is less known: at that point we in fact show that the cause is in some [effect] [. . .].[81]

Demonstrative *regressus* is moreover treated explicitly:

> Regressus is that method of demonstration in which we collect a previously unknown cause from a more known effect. Thereafter, by working backwards, we truly demonstrate the same effect from the same cause.[82]

Note that Smith's conception of *regressus* is not entirely identical to Jardine's description which I have referred to above. According to Jardine, *regressus* is a procedure which combines an inference from an observed effect to its proximate cause with an inference from the proximate cause to the observed effect. Smith's account is different in that it also incorporates a distinction between proximate and remote causes. Smith's account retains the idea that science essentially involves reasoning from effects to causes (and vice versa) and places emphasis on the quest for proximate causes. However, this slight deviation from common *regressus* strategy as characterized by Jardine has the welcome effect that it is makes Smith's views closer to Newton's, as I shall argue in the following section. In what follows, I show that Newton endorsed a causal stance that is highly compatible with Smith's *Aditus ad logicam* in particular. I will also argue that Newton's conception of natural-philosophical analysis and synthesis derives to a significant extent from Smith's *Aditus ad logicam*, to which Newton was exposed during his formative years.

1.5 Newton on Natural-Philosophical Analysis and Synthesis

This section serves the purpose of rendering my causal reading of Newton more plausible by showing that Newton used causal terminology in a *cornucopia* of published and manuscript material. I also argue that Newton conceptualized natural-philosophical analysis and synthesis in line with the tenets of the "Aristotelian" textbook tradition on the matter. In Section 1.6, I will clarify the status of causation in the *Principia* proper.

In his optical research Newton sought and notoriously claimed to have established the cause of refraction. In light of the outcome of the *experimentum crucis*,

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[81] Translation of: "Demonstratio ab effectu ad causam locum habet, quando effectus est notior, & causa ignotior: tunc enim ostendimus inesse alicui causam [. . .]" (ibid., p. 113). It is worth mentioning that Newton accentuated this very text in Wren Library, NQ.9.166[1], p. 143: "Demonstratio ab effectu ad causam locum habet, quando effectus est notior, & causa ignotior: tunc enim ostendimus inesse alicui causam, eò quòd eidem inest effectus, ut *in naturali corpore probamus materiam inesse, quia eidem inest generatio, & naturale corpus motorem habere æternum, quòd æterno cietur motu.*"

[82] Translation of: "Regressus est illa demonstrandi ratio, quâ prius causam ignotam ex effectu notiore colligimus; postea verò regredientes ex eadem causâ eundem effectum demonstramus." (Smith, *Aditus ad logicam*, p. 116).

Newton asserted to have established the cause of the elongated shape of the (originally) circular light beam after passing through a prism.[83] The "*true cause*," Newton remarked, "of the length of that Image was detected to be no other, then that *Light* consists of Rays differently refrangible."[84] In his *Lectiones opticae* Newton stated that he intended to "describe individually the *particular and immediate causes of the effects* [causas particulares et immediatas] that I have not previously treated, not for the sake of the geometers (to whom, it will appear unnecessary) but for others"[85] and that he had "sufficiently amply revealed" the cause of prismatic effects.[86] In *The Opticks* Newton pointed out that "the main Business of natural Philosophy" is "to argue from Phaenomena without feigning Hypotheses, and to deduce Causes from Effects."[87] In Query 31, he wrote:

> As in Mathematics, so in Natural Philosophy, the Investigation of difficult Things by the method of Analysis, ought ever to precede the Method of Composition. This Analysis consists in making Experiments and Observations, and in drawing general Conclusions from them by Induction, and admitting of no Objections against the Conclusions, but such as are taken from Experiments, or other certain Truths. For hypotheses are not to be regarded in experimental Philosophy. [. . .] By this way of Analysis we may proceed from Compounds to Ingredients, and *from Motions to the Forces producing them*; *and in general, from Effects to their Causes*, and from particular Causes to more general ones, till the Argument end in the most general. This is the Method of Synthesis: And the method of Synthesis consists in assuming the Causes discover'd and establich'd as Principles, and by them explaining the Phænomena proceeding from them, and proving the Explanations.[88]

According to Newton, natural-philosophical inquiry consists of two directions, which constitute "a single procedure:"[89] first, from effects to causes (analysis) and, next, from causes to effects (synthesis). In draft material related to his famous *exposé* of natural-philosophical analysis and synthesis, Newton noted:

> As Mathematicians have two Methods of doing things w^{ch} they call Composition & Resolution & in all difficulties have recourse to their method of resolution ↓before they compound↓ so in explaining the Phænomena of nature the like methods are to be used & he

[83] Further contextualisation of Newton's optical work – including a detailed discussion of the *experimentum crucis* – will be provided in Chapter 4.

[84] Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, p. 51 [italics added].

[85] Shapiro, ed., *The Optical Papers of Isaac Newton*, I, pp. 522/523 [italics added].

[86] Ibid., I, pp. 524/525.

[87] See Newton, *The Opticks*, p. 369. It is fair to say that the *terminology* of deducing causes by their effects was in the air before Newton. In the conclusion of his *Experimental Philosophy* (1664) Henry Power used the expression "deducing the Causes of things" (Power, *Experimental Philosophy*, p. 192). Newton owned a copy of its first edition (Harrison, *The Library of Isaac Newton*, p. 221, item n° 1344). In *regula XIII* of his *Regulae ad directionem ingenii* Descartes wrote: "Sed insuper ut quaestio sit perfecta, volumus illam omnino determinari, adeo ut nihil amplius quaeratur, quam id quod *deduci potest ex datis*." (Descartes, *Œuvres de Descartes*, X, p. 431 [italics added]).

[88] Newton, *The Opticks*, pp. 404–405 [italics added]. See the Coda to this chapter for more on *explanare* versus *explicare*.

[89] Guerlac, *Essays and Papers in the History of Modern*, p. 206.

> that expects success must resolve before he compounds. *ffor the explications of Phænomena are Problems much harder then those in ~~abstracted~~ Mathematicks*.[90]

Newton explained that in *The Opticks* he had proceeded first analytically and next synthetically:

> In the two first Books of these Opticks, I proceeded by this Analysis to discover and prove the original Differences of the Rays of Light in respect of Refrangibility, Reflexibility, and Colour, and their alternate Fits and easy Reflexion and easy Transmission, and the Properties of Bodies, both opake and pellucid, on which their Reflexions and Colours depend. And these Discoveries being proved, may be assumed in the Method of Composition for explaining the Phænomena arising from them: An Instance of which Method I gave in the End of the first Book.[91] In this third Book I have only begun the Analysis of what remains to be discover'd about Light and its Effects upon the Frame of Nature, hinting several things about it, and leaving the Hints to be examin'd and improv'd by the farther Experiments and Observations of such are inquisitive.[92]

In draft material composed for the first edition of *The Opticks*, Newton commented on analysis and synthesis in Book II of *The Opticks* as follows:

> Most of the second Book was written some years ↓before↓ ~~after~~ the ffirst & so is not in so good a method. However it proceeds by Analysis to discover the fits of easy reflexion & easy transmission of the rays, & thence ↓~~it is easy to compound~~↓ the explication of the colours of ↓bubbles & other↓ transparent plates, & ↓~~those of~~↓ feathers & tinctures ↓~~are easily compounded~~↓.[93]

Note that in Query 31, Newton treated motions as a sub-class of effects and the forces producing them as a sub-class of causes. This seems to suggest that the *Principia* also contained an analytical and a synthetic part. Although Newton did not formally distinguish between an analytical and synthetic part of the argument for universal gravitation, several of his statements in Book III are perfectly consistent with such differentiation. In the *scholium* to the *Definitions*, Newton wrote:

> But in what follows, a fuller explanation will be given of how to determine true motions from their causes, effects, and apparent differences, and conversely, of how to determine from motions, whether true or apparent, their causes and effects.[94]

In the preface of the *Principia*, Newton indicated that the basic difficulty of natural philosophy is "to discover the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces [a phænomenis

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[90] CUL Add. Ms. 3970, f. 480^{v} [ca. 1704; italics added], cf. f. 242$^{r\text{-}v}$, f. 244^{v}, f. 288^{r}. This material comes from an intended preface to the first edition of *The Opticks*. See: McGuire, "Newton's 'Principles of Philosophy'".

[91] Once Newton had established the heterogeneity of white light, he used it to explain the phenomenon of the rainbow (Newton, *The Opticks*, pp. 168–178).

[92] Ibid., p. 405.

[93] CUL Add. Ms. 3970, f. 242^{v}, cf. f. 244^{r}, f. 292^{v} [ca. 1700–1704].

[94] Newton, *The Principia*, p. 415. "Motus autem veros ex eorum causis, effectibus & apparentibus differentiis colligere, & contra ex motibus sue veris seu apparentibus eorum causas & effectus, docebitur fusius in sequentibus." (Koyré, Cohen and Whitman, eds., *Principia mathematica*, I, p. 53).

motuum investigemus vires naturæ, deinde ab his viribus demonstremus phænomena reliqua]:"[95]

> It is to these ends that that the general propositions in books 1 and 2 are directed, while in book 3 our explanation of the system of the world [*explicationem systematis mundani*] illustrates these propositions. For in book 3, by means of propositions demonstrated mathematically in books 1 and 2, we derive from celestial phenomena the gravitational forces by which bodies tend toward the sun and toward the individual planets. Then the motions of the planets, the comets, the moon, and the sea are deduced from these forces by propositions that are also mathematical.[96]

In other words, the analysis in the *Principia* consists in deriving "from celestial phenomena the gravitational forces by which bodies tend toward the sun and the individual planets" and the synthesis in deducing "the motions of the planets, the comets, the moon, and the sea" from the forces derived from the theory of universal gravitation.[97] In the words of the editor of the second edition of the *Principia*, Roger Cotes:

> Therefore they [i.e., Newtonian philosophers] proceed by a twofold method, analytic and synthetic. From certain selected phenomena they deduce by analysis the forces of nature and the simpler laws of those forces, from which they then give the constitution of the rest of the phenomena by synthesis.[98]

The analytical part of Newton's argument for universal gravitation is given in Propositions I–VIII of Book III, where the theory of universal gravitation is derived. In this part Newton proceeded from motions to forces: from the Keplerian motions of the primary and secondary planets to the causes of these motions, i.e. inverse-square centripetal forces (see Propositions I–V, Book III). The synthetic part, in which Newton shows that the motion of the moon, the tides, and comets can be deduced from the causes proposed by the theory of universal gravitation, stretches out to the very end of Book III. He set out to demonstrate that other phenomena, which were not contained in the original analysis, could be explained by their causes as established by the theory of universal gravitation. In the *scholium* to Proposition XXXV of Book III, for instance, Newton wrote that he wished "to show by these computations of the lunar motions that the lunar motions can be computed from their causes by the theory of gravity [quod motus lunares per theoriam gravitatis a causis suis computari possint]."[99]

Once Gottfried W. Leibniz criticised Newton for introducing a *qualitas occulta*,[100] i.e. gravity, into natural philosophy, Newton became increasingly

[95] Newton, *The Principia,* p. 382.

[96] Ibid.

[97] The analytical and synthetic phase of the *Principia* will be treated in detail in Chapter 3.

[98] Newton, *The Principia,* p. 386.

[99] Ibid., p. 869.

[100] Westfall, *Never at Rest*, pp. 772–773. In an unpublished letter to the editor of *The Memoirs of Literature* from ca. May 1712 Newton defended himself as follows against Leibniz' criticism: "Because they do not explain gravity by a mechanical hypothesis, he charges them with making it a supernatural thing, a miracle and a fiction invented to support an ill-grounded opinion and

pressed to clarify the kind of explanation he had offered in the *Principia* and, more generally,[101] to clarify his method of philosophizing from a methodological point of view.[102] The crux of Newton's solution for meeting this public criticism lay in carefully distinguishing between different "levels of causation." In this context, he came to distinguish between proximate and remote causes. According to Newton, causal processes are structured hierarchically: phenomena derive from causes which in their turn are caused by more general causes. At the end of this causal chain, God is the ultimate cause of everything. Hence, Newton declared in *The Opticks* that "the main Business of natural Philosophy is to argue from Phaenomena without feigning Hypotheses, and to deduce Causes from Effects, *till we come to the very first Cause, which is certainly not mechanical*."[103] In manuscript material dating from around 1714–1716, Newton was more explicit: in natural philosophy one argues "from Phæ↓no↓mena & Experiments, ↓~~first~~↓ to the causes thereof, & ↓~~thence~~↓ to the causes of those causes, & so on till we come to the first cause."[104] In CUL Add. Ms 3968.9 (early 1710s), Newton explicitly articulated his views on causal explanation as follows:

> He who investigates the laws and effects of electric forces with the same success and certainty will greatly promote philosophy [i.e., natural philosophy], even if perhaps he does not know the cause of these forces. First, the phenomena should be observed, then their proximate causes — and afterward the causes of the causes — should be investigated, and finally it will be possible *to come down from the causes of the causes (established by phenomena) to their effects, by arguing a priori*. Natural philosophy should be founded not on metaphysical opinions, but on its own principles and [end of text][105]

compares their method of philosophy to that of Mr. de Roberval's Aristarchus, which is all one as to call it romantic [i.e., fictional]. They show that there is a universal gravity and that all phenomena of the heavens are the effect of it and with the cause of gravity they meddle not but leave it to be found out by them that can explain it, whether mechanical or otherwise. [. . .] And therefore if any man should say that bodies attract one another by a power whose cause is unknown to us, or by a power seated in the frame of nature by the will of God, or by a power seated in a substance in which bodies move and float without resistance and which has therefore no *vis inertiae* but acts by other laws than those that are mechanical: I know not why he should be said to introduce miracles and occult qualities and fictions into the world." (Janiak, ed., *Newton, Philosophical Writings*, pp. 115–116).

[101] Gjertsen, *The Newton Handbook*, p. 463 and Cohen in Newton, *The Principia*, p. 274.

[102] Shapiro, "Newton's 'Experimental Philosophy'".

[103] Newton, *The Opticks*, p. 369 [italics added].

[104] CUL Add. Ms. 3968.39, f. 586^{v}, cf. f. 27^{r}.

[105] Cohen-Whitman's translation (Newton, *The Principia*, pp. 53–54) of: "Qui leges et effectus Virium electricarum pari successu et certitudine eruerit, philosophiam multum promovebit, etsi ↓forte↓ causam harum Virium ignoraverit. ~~Nam~~ Phaenomena ↓observanda↓ primo ↓~~spectanda~~↓ ~~consideranda~~ ↓sunt↓, dein horum causae proximae, & postea causae causarum ~~eruenda~~ eruenda; ac tandem a causis ↓supremis causarum↓ per phaenomena stabilitis, ad ↓~~causas~~↓ ~~caus phaenomena~~ ↓eorum effectus↓, ↓~~eorum causas proximas~~↓ argumentando a priori, descendere licebit. ~~Et inter Phaenomena numerandae sunt actiones mentis quae nobis innotescunt quarum conseij sumus~~ Philosophia naturalis non in opinionibus Metaphysicis, sed in Principiis propijs fundanda est; & haec [end of text]" (CUL Add. Ms. 3970, f. 109^{v}). A complete transcription of f. 109$^{r\text{-}v}$ is provided in the Appendix to this chapter.

A priori here refers to what comes first in the order of nature. Here Newton was thinking along the lines of Samuel Smith's *demonstratio quod* type 2, i.e. reasoning from the remote cause to the effect as an desirable goal for future natural philosophy. One may object to Newton that unless the remote cause is unveiled, no proper explanation is provided.[106] Newton, however, considered explanations referring exclusively to the primary cause, while neglecting the remote cause causing the proximate cause, as fully legitimate. In CUL Add. Ms 9597.2.11 (ca. 1716–1718), Newton thought the consequences of not accepting such "partial explanations" through: this would imply – a view impossible for Newton to accept – that the only satisfactory explanations were "causally complete," i.e. that they fully explain all causal agents occurring in between the observed phenomena and the ultimate cause:

> Otherwise, altogether no phenomenon could be rightly explained by its cause, unless the cause of this cause and the cause of the prior cause were to be delivered and so successively [and] continuously as long as the primary cause were to be arrived at.[107]

Another indication of Newton's acceptance of such "partial explanations" can be found in a letter to the editor of the *Memoirs of Literature* in 1712:

> And to understand this without knowing the cause of gravity, is as good a progress in philosophy as to understand the frame of a clock & the dependence of y^e^ wheels upon one another without knowing the cause of the gravity of the weight which moves the machine is in the philosophy of clockwork, or the understanding the frame of the bones & muscles by the contracting or dilating of the muscles without knowing how the muscles are contracted or dilated by the power of y^e^ mind is [in] the philosophy of animal motion.[108]

Similarly, in CUL Add. Ms. 9597.2.11 (ca. 1716–1718), Newton wrote:

> And, although, for not all [of] philosophy lies steadily open [to us], it is nevertheless quite sufficient to apprehend something from day to day than to occupy human minds with the prejudices of hypotheses.[109]

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[106] One may also object that, unless the proximate cause is shown to be the full and only cause of its effect, we cannot properly proceed to a discussion of the remote cause. With respect to the motion of the moon, Newton declared that "[*a*]*ll the motions of the moon and all inequalities in its motions follow from the principles that have been set forth* [i.e., from the theory of universal gravitation]" (Newton, *The Principia*, p. 832). Interestingly, Newton could in fact not completely account for the motion of the moon's apsis by gravitational forces alone and considered the earth's magnetic field as a possible additional factor (ibid., p. 880). In other words, Newton allowed magnetic forces in the full explanation of the motion of the moon's apsis. Newton was, however, correct that gravitational forces are highly dominant in comparison to other, i.e. non-gravitational, forces. In this sense, Newton's discussion of the cause of gravity is in fact a discussion of the cause of a highly dominant proximate cause, gravity. I am indebted to the anonymous referee for triggering my thinking on this matter.

[107] Translation of: "Alias nullum omnino phaenomenon ↓per causam suam↓ recte explicari posset nisi causa ~~ejus~~ ↓hujus↓ causae, & causa ~~priori~~ causae prioris redderetur & sic deinceps usque donec ad causam primam deventum sit." (CUL Add. Ms. 9597.2.11, f. 3^{r}).

[108] Newton to the editor of the Memoirs of Literature, after 5 May 1712, Newton, *Correspondence*, V, p. 300; cf. CUL Add. Ms. 3968.17, f. 257^{v} [17 May 1712].

[109] Translation of: "Et quamvis tota philosophia non statim pateat, tamen satius est aliquid indies addiscere quam hypotheseωn praejudicijs mentes hominum preoccupare." (CUL Add. Ms. 9597.2.11, f. 2^{r}).

In the General Scholium Newton famously declared:

> Thus far I have explained the phenomena of the heavens and of our sea by the force of gravity, but I have not yet assigned a cause to gravity [Hactenus phænomena cælorum & maris nostri per vim gravitatis exposui, sed causam gravitatis nondum assignavi.]. Indeed, this force arises from some cause that penetrates as far as the centers of the sun and planets without diminution of its power to act, and that acts not in proportion to the quantity of the surfaces of the particles on which it acts (as mechanical causes are wont to do) but in proportion to the quantity of solid matter, and whose action is extended everywhere to immense distances, always decreasing as the squares of the distances. [. . .] And it is enough that gravity really exits and acts according to certain laws that we have set forth and is sufficient to explain all the motions of the heavenly bodies [Et satis est quod gravitas revera existat, & agat secundum leges a nobis expositas, & ad corporum cælestium & maris nostri motus omnes sufficiat.].[110]

Newton took this to mean that he had unveiled gravity as primary or proximate cause for the heavenly and terrestrial motions,[111] but that he did not succeed in discovering a further secondary or remote cause for gravity.[112] Newton had only provided explanations involving the proximate causes of orbital motion (centripetal forces), while he deliberately abstracted from the remote causes as not to engage in the act of feigning hypotheses. In the "Account of the Booke entituled *Commercium Epistolicum*" Newton emphasized that "The Philosophy which Mr. *Newton* in his *Principles* and *Optiques* has pursued is Experimental; and it is not the Business of Experimental Philosophy to teach *the Causes of things any further than they can by proved by Experiment*."[113] In manuscript material related to the Clarke-Leibniz correspondence, Newton similarly wrote that "Occult qualities have been exploded *not because their causes are unknown to us* but because by giving this name to the specific qualities of things, a stop has been put to all enquiry into the causes ↓of their qualities↓ *as if they could not be known*."[114] In this section I have argued that the causal explanations were a crucial part in Newton's natural philosophy.[115] However,

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[110] Newton, *The Principia*, p. 943; Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 764.

[111] Cf. Dobbs, *The Janus Faces of Genius*, pp. 36–37.

[112] Gerd Buchdahl has earlier emphasized that we should keep "the logical status of gravity itself, as a 'primary' cause" and "the modus operandi, if any, of a secondary explanatory mechanism for gravity" asunder (Buchdahl, "Gravity and Intelligibility: Newton to Kant", esp. p. 81).

[113] [Newton], "An account of the book entituled *Commercium Epistolicum*", p. 222 [italics added].

[114] Newton, Manuscript in Miracles, Lehigh University Libraries, Bethlehem (Pennsylvania), f. 1^{v} as quoted from: Dobbs, *The Janus Faces of Genius*, p. 230 [italics added].

[115] Also, in draft material related to Definition II of the *Principia*, Newton noted that experimental philosophy establishes efficient and final causes: "↓A phænomenis Philosophia ~~naturalis~~ incipit↓. In his tractandis Philosophia experimentalis consistit. ~~Ab hac~~ Philosophia ↓experimentali ad rerum↓ ~~ad~~ causas efficientes & finales, & ↓ab his omnibus ad naturam rerum insensibilium & ultimo↓ ad Philosophiam hypotheticam transeundum est:]" (CUL Add. Ms. 3965, f. 422^{r} [additions and corrections to the second edition of the *Principia*]). See Section 4.8 of Chapter 4, for more discussion of this passage.

the causal status of forces in the *Principia* requires further elaboration and this is the subject of the next section.

1.6 Centripetal Forces as Causes

Any reader of the *Principia* will notice that there is a "Janus-like ambiguity"[116] to one of the central concepts in the *Principia*, namely, that of "force." On the one hand, Newton's treatment of force appears to be purely mathematical, on the other hand, it appears to be causal and realist as well. How are we to make sense of this tension inherent to Newton's work? Can Newton be considered as a causal realist with respect to the status of forces? Despite Newton's causal talk, which we have discussed in Section 1.5, the *Principia prima facie* contains positivistic sounding statements. In a comment to Definition VIII, Newton warned his readers, that he is "not now considering the physical causes and sites of forces [virium causas & sedes physicas jam non expendo]:"[117]

> Moreover, I use interchangeably and indiscriminately words signifying attraction, impulse, or any sort of propensity toward a center, considering these forces not from a physical but only from a mathematical point of view [has vires non physice sed mathematice tantum considerando]. *Therefore let the reader beware of thinking that by words of this kind I am anywhere defining a species or mode of action of a physical cause or reason, or that I am attributing forces in a true and physical sense to centers* (which are mathematical points) if I happen to say that centers attract or that centers have forces [Unde caveat lector, ne per hujusmodi voces cogitet me speciem vel modum actionis causamve aut rationem physicam alicubi definire, vel centris (quæ sunt puncta mathematica) vires vere & physice tribuere, si forte aut centra trahere, aut vires centrorum esse dixero.].[118]

Similarly, in the *scholium* to Section XI of Book I, Newton wrote:

> I use the word "attraction" here in a general sense for every endeavor whatever of bodies to approach one another [pro corporum conatu quocunque accedendi ad invicem], whether that endeavour occurs as a result of the action of the bodies either drawn toward one another or acting on one another by means of spirits emitted or whether it arises from the action of aether or of air or of any medium whatsoever – whether corporeal or incorporeal – in any way impelling toward one another the bodies floating therein. I use the word "impulse" in the same sense, considering in this treatise not the species of forces and their physical qualities but their quantities and mathematical proportions, as I have explained in the definitions.[119]

In these passages, Newton forcefully dispensed with an agent-causal interpretation of attraction, according to which one attributes real causal agency to the centre of a

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[116] McMullin, "Conceptions of Science in the Scientific Revolution", p. 72.

[117] Newton, *The Principia*, p. 407.

[118] Ibid., p. 408 [italics added]; Koyré, Cohen and Whitman, eds., *Principia mathematica*, I, pp. 45–46.

[119] Newton, *The Principia*, pp. 588–589.

body. Locating the centripetal force at the centre of a body is a convenient mathematical technique to deal with its overall centripetal force, but it is not to be taken physically, i.e. one should not attribute force "in a true and physical sense" to centres. Newton also made it clear that he wished to remain neutral on the cause of gravity. This is, furthermore, confirmed by manuscript material, which Newton had prepared for his account of the *Commercium epistolicum*. There he remarked that he wanted to remain:

> silent about the cause of gravity, there occurring no experiments or phænomena by w^{ch} he might prove what was the cause thereof: And this he hath abundantly declared in his Principles neare the beginning thereof in these words; Virium causas et sedes Physicas jam non expendo. And a little after: Voces attractionis, impulsus vel propensionis cujusque in centrum indifferenter & pro se mutuo promiscue usurpo, has vires non physice sed mathematice tantum considerando. Unde caveat Lector ne per hujusmodi voces cogitet me speciem vel modum actionis, causamve aut rationem physicam alicubi definire, vel centris (quæ sunt puncta Mathematica) vires vere et physice tribuere, si forte aut centra trahere aut vires centrarum esse dixero.[120]

Correspondingly, these statements do not imply a refusal to treat of causes and real forces, they rather serve as a caveat not to consider the centres of bodies as being the real physical seat of the attracting force, and moreover, as a refusal to discourse about the cause of gravity.[121] What physically produces gravity, is not part of Newton's analysis in the *Principia*. In other words, while an account of the remote cause of celestial and terrestrial motions is explicitly absent from the *Principia*, it is nowhere implied thereby that Newton also avoided an account of their proximate cause. I would like to point out that I. Bernard Cohen's "Newtonian Style"[122] is compatible with a causal reading of the *Principia*. Cohen stated that, in commenting on Propositions I–III, Book I, Newton demonstrated that a mathematically descriptive law of motion was shown by mathematics to be equivalent to a set of *causal* conditions of forces and motions.[123] Given that the laws of motion are valid, Newton was able to deduce that the area law is caused by its necessary and sufficient causal condition: a centripetal force.[124] This perfectly allows for reasoning from effects to causes.

When Newton stated that he was "considering these forces not from a physical but only from a mathematical point of view," he was referring to his treatment

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[120] CUL Add. Ms. 3968, f. 584^r. Cf. Janiak, ed., *Newton, Philosophical Writings*, pp. 123–124.

[121] Cf. Newton, *The Opticks*, p. 376.

[122] In Chapter 2, I shall point to some difficulties for Cohen's 'Newtonian Style'. For the moment, let it suffice to indicate that the problems with Cohen's "Newtonian Style" lie not in the realm of ontology, but rather in the realm of methodology.

[123] Cohen, *The Newtonian Revolution*, pp. 28, 37. Also see his "Newton's Method and Newton's Style", in which Cohen emphasized that Newton was concerned with true causes in the *Principia* (p. 29).

[124] Cohen, *The Newtonian Revolution*, p. 63.

of force in the context of Book I.[125] In the opening section of Book III of the *Principia*, Newton noted that in Books I and II he had "presented principles of philosophy that are not, however, philosophical but strictly mathematical – that is, those on which the study of philosophy can be based" and that "[i]t still remains for us to exhibit the system of the world from these same principles."[126] In the General Scholium to the *Principia* Newton pointed out that "gravity really exists [gravitas revera existat],"[127] thereby implying that gravity is a real force. In Book III, Newton's mathematical treatment of (centripetal) forces turned into a physical account of the actual forces in the empirical world. The "Janus-like ambiguity" in Newton's treatment of force is therefore to be explained by his manner of proceeding from a (physico-)mathematical treatment of force in the context of Book I, which does not yet investigate the forces in our solar system, to a physical treatment of force in the context of Book III, which does exactly so. In the *scholium* to Section XI of Book I, to which I have already referred, Newton wrote that establishing the forces in nature proceeds along the following consecutive steps:

> 1. Mathematics requires an investigation of those quantities of forces and their proportions that follow from any conditions that may be supposed.
> 2. Then, coming down to physics, these proportions must be compared with the phenomena, so that it may be found out which conditions (or laws) of forces apply to each kind of attracting bodies.
> 3. And then, finally, it will be possible to argue more securely concerning the physical species, physical causes and physical proportions of these forces.
>
> Let us see, therefore, what the forces are by which spherical bodies, consisting of particles that attract in the way already set forth, must act upon one another, and what sorts of motions results from such forces.[128]

> In mathesi investigandæ sunt virium quantitates & rationes illæ, quæ ex conditionibus quibuscunque positis consequentur: deinde, ubi in physicam descenditur, conferendæ sunt hæ rationes cum phænomenis; ut innotescat quænam virium conditiones singulis corporum attractivorum generibus competant. Et tum demum de virium speciebus, causis & rationibus physicis tutius disputare licebit. Videamus igitur quibus viribus corpora sphærica, ex particulis modo jam exposito attractivis constantia, debeant in se mutuo agere; & quales motus inde consequantur.[129]

Book I is written for the purpose of demonstrating what is the case mathematically if certain physico-mathematical conditions, i.e. forces and laws which regulate the motion of bodies, hold (and vice versa) – thereby neglecting the actual physical

[125] In this paragraph, I will characterize Newton's methodology *in very broad lines*, referring the reader instead to Chapters 2 and 3 for a detailed account and for extensive justification of the claims made here.

[126] Newton, *The Principia*, p. 793.

[127] Ibid., p. 943.

[128] Ibid., pp. 588–589 [numbers added].

[129] Koyré, Cohen and Whitman, eds., *Principia mathematica*, I, p. 298.

conditions or forces in the empirical world. More specifically, step 1 is an investigation of what follows mathematically from the activity of certain (centripetal) forces (and vice versa), given the laws of motion. As we will see in Chapter 2, from the laws of motion, Newton was able to derive that inverse-square centripetal forces are the necessary and sufficient conditions for Keplerian motion to occur. Next, when one "comes down the physics" in step 2, one investigates the actual mathematical properties exhibited by the terrestrial and celestial motions *in rerum natura*, and on the basis of the systematic dependencies between cause and effect established in step 1,[130] one infers the forces producing these motions or, as Newton put it, "these proportions must be compared with the phenomena, so that it may be found out which conditions (or laws) of forces apply to each kind of attracting bodies." In Propositions I–V of Book III, Newton argued from effects – from Kepler's laws, or rules, as they were called at the time – to causes – to the inverse-square centripetal forces producing Keplerian motion.[131] Finally, step 3 results in a more secure way to discuss the physical species, causes and proportions of these forces. With respect to the physical species and causes Newton had established, respectively, that gravity is a universal property, i.e. a property that can be intended and remitted,[132] and that gravity acts non-mechanically, i.e. that gravity acts "not in proportion to the quantity of the *surfaces* of the particles on which it acts," "but in proportion to the quantity of *solid* matter."[133] Moreover, mechanical forces act at small distances, while gravity acts at great distances.[134] In other words, Newton had shown that mass is a causally salient variable in gravitational interactions, but he had not yet uncovered the *full* cause of gravity – note that Newton candidly admitted that in the *Principia* he had not touched upon the nature and characteristics of a gravitationally relevant, i.e. non-mechanical, ether. On the basis of his physico-mathematical demonstrations Newton introduced a new *explanans*, a universal attracting force, which was radically different from the commonly accepted mechanical explanations of his contemporaries, which were based on the direct contact between bodies. His contemporaries (Leibniz and Huygens, for instance), therefore, criticised Newton for not giving an intelligible, i.e. a mechanical, explanation of motion and accused him of introducing occult qualities in natural philosophy.[135] Louis Bertrand Castel criticised Newton because Newton's mathematical demonstrations did not provide physical explanations.[136] The novelty involved in Newton's explanations baffled

[130] Note that earlier on CUL Add. Ms. 3965, ff. 1^{r-v}, f. 64^{r}, f. 66^{r}/f. 67^{r}/f. 68^{r}/f. 69^{r}/f. 70^{r}, Newton only proved the sufficient direction. In the initial revise of *De motu* he demonstrated both directions (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 122–127).

[131] Newton, *The Principia*, pp. 802–806.

[132] The meaning of this will be explained in Section 3.1 of Chapter 3.

[133] Ibid., p. 943.

[134] On CUL Add. Ms. 3968, f. 260^{v} [ca. 1715–1720], Newton wrote: "Causæ mechanicæ agere solent ad parvas distantias: causa gravitatis agit ad maximas."

[135] Westfall, *Force in Newton's Physics*, pp. 376–400.

[136] Gingras, "What did Mathematics do to Physics?", pp. 399–404.

most of his contemporaries.[137] With respect to the proportions of the force of gravity, Newton had established that gravity is proportional to the quantity of matter and inversely proportional to the square of the distance.

That Newton conceived of forces as causes,[138] can furthermore be gathered from the following considerations. A key element in the analytical part of the derivation of the theory of universal gravitation was Newton's generalisation that the various inverse-square centripetal forces acting in our solar system are instances of the same cause, namely gravity (see the *scholium* to Proposition V, Book III). This particular step was licensed by Rule II, according to which, "the causes assigned to natural effects of the same kind must be, so far as possible, the same [*effectuum naturalium ejusdem generis eædem assignandæ sunt causæ, quatenus fieri potest*]."[139] Since Newton identified forces of the same kind by Rule II, this implies that he was treating inverse-square centripetal forces as causes. Moreover, among his drafts related to the second *regula philosophandi*, Newton wrote that the "proximate causes" assigned to effects of the same type should, so far as possible, be the same.[140] As we have seen in the previous section, in his famous exposition of natural-philosophical analysis and synthesis in Query 31 of *The Opticks*, Newton treated motions as a sub-class of effects and the forces producing them as a sub-class of causes. Elsewhere, Newton stressed that force is "the causal principle [causale principium] of motion and rest."[141] When discussing the centripetal forces acting in the solar system in the original tract *De motu*, Newton wrote that we cannot reasonably expect to develop a model that considers all *causes* – by which he meant forces – of motion at once

[137] Iliffe, "Abstract Considerations: Disciplines and the Coherence of Newton's Natural Philosophy", p. 439.

[138] A conclusion which is also reached in Bechler, "Newton's Ontology of the Force of Inertia", pp. 298–299 and Janiak, *Newton as Philosopher*, e.g., p. 73. See Guerlac, *Essays and Papers in the History of Modern*, pp. 211–213 for an earlier causal reading of Newton.

[139] Newton, *The Principia*, p. 795. Of course, a lot remains to be said about Newton's *regulae philosophandi*. I will, however, postpone further discussion of them to Chapter 3, Section 3.2. Here I only wish to indicate that Rule II is designed to identify causes of the same kind.

[140] "Reg. IV Ideoque Effectuum naturalium ejusdem generis eædem ~~assumendæ~~ ↓assignandæ↓ sunt *causæ* ↓[~~proximæ nisi [forte diversitas aliqua~~↓, ~~nisi quatenus diversitas ex phænomenis patefacta sit hæ causæ phænomenis explicandis sufficiunt.~~] ~~nisi diversitas~~ ↓~~aliqua~~↓ ex ph~~ænomenis patefacta sit.]~~ quatenus fieri potest." (CUL Add. Ms. 3965, f. 419^{r} [additions and corrections to the first edition of the *Principia*; italics added]).

[141] Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 148; CUL Add. Ms. 4003, p. 30. Note that Newton's understanding of Descartes' law of inertia (1665) was explicitly in causal terms ("Every thing doth naturally persevere in y^{t} state in w^{ch} it is unlesse it bee interrupted by some *external cause*." (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 32, footnote 9 [italics added]; Herivel, *The Background to Newton's Principia*, p. 153). In the original tract *De motu*, Newton pointed out that "~~Corpora nec medio impediri nec alijs causis externis quo minus viribus insitæ et centripetæ exquisite cedant.~~" (CUL Add. Ms 3965, f. 55^{r} [italics added]).

(cf. "Tot autem motuum causas simul considerare [. . .] superat ni fallor vim omnem humani ingenij."[142]).

In his *Newton as Philosopher*, Andrew Janiak has cogently argued that Newton's statement that gravity "really exists" is to be understood as the claim that "gravity" refers to "a physical quantity that non-mechanically causes various motions of bodies near the surface of the earth, of our oceans, and of the heavenly bodies, in such a way that distance and mass are the salient variables in their changes in states of motion."[143] Janiak's interpretation contains a number of important implications. First, it entails that gravity exists because it is a measurable physical quantity.[144] By contrast, mechanical ethers do not exist, because they cannot be measured. Gravity is thus not a mere calculating device, but refers to a real force in nature. Secondly, Janiak's argumentation implies that Newton conceived of centripetal forces as proper causes, i.e. as the causes of motion and rest.[145] Thirdly, it entails that a wide range of previously disparate phenomena have the same cause. Fourthly, it indicates that these phenomena are caused in such a manner such that mass and distance are the only salient variables in the causal chain that involves them. Finally, it implies that, given that mass is one of the salient variables, gravity is a non-mechanical cause, i.e. it does not act on the surfaces of bodies.[146]

In the posthumously published *A Treatise of the System of the World*/*De mundi systemate* (1728),[147] Newton explained, furthermore, that gravity is a universal interaction force between pairs of bodies. At the beginning of Book III of the

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[142] Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 78; CUL Add. Ms. 3965, f. 47^{r}.

[143] Janiak, *Newton as Philosopher*, Chapter 3, esp. pp. 76–77. Cf. Janiak, "Newton and the Reality of Force", p. 143.

[144] Janiak, *Newton as Philosopher*, p. 60.

[145] Ibid., p. 73.

[146] These last three implications are discussed in ibid., pp. 27, 74–75. Ernan V. McMullin's recent account in terms of "dynamical explanations" or dispositions is less satisfactory (McMullin, "The Impact of Newton's *Principia* on the Philosophy of Science", p. 298; id., "The Origins of the Field Concept in Physics"). Janiak has pointed out, by focussing on a revealing passages in McMullin, "The Origins of the Field Concept in Physics", p. 24, that the way in which McMullin characterizes how the dispositional treatment of force is to be supplemented by the physical treatment of force *ipso facto* rules out any potential medium, which is inconsistent with Newton's own desire to remain neutral with respect to the cause of gravity (Janiak, *Newton as Philosopher*, pp. 67–68). While I agree with Janiak's criticism of McMullin's account, I think the problem is more fundamental: it seems that no dispositional account per se will be able to do justice to Newton's causal stance, let alone explicate the meaning of efficient causation in the *Principia*.

[147] *De mundi systemate* was published by John Conduitt and it was based on CUL Add. Ms. 3990, ff. 1^{r}-56^{r}, i.e. *De motu corporum liber secundus* (Cohen, ed., *A Treatise of the System of the World*, p. xi; id., *Introduction to Newton's "*Principia*"*, pp. 327–335). CUL Add. Ms. 3990, ff. 1^{r}-56^{r} is written in the hand of Newton's amanuensis Humphrey Newton (not related) and it contains corrections which are written in Newton's hand. Its English version, *A Treatise of the System of the World*, is partially based on CUL 3990, ff. 1^{r}-56^{r}, but it also contains additional material, which means that the translator either used a version unavailable to us or that he interpolated some material of his own (Cohen, ed., *A Treatise of the System of the World*, p. i).

Principia, Newton explained why he had withheld its publication during his lifetime:

> On this subject I composed an earlier version of book 3 in popular form [methodo populari], so that it might be more widely read. *But those who have not sufficiently grasped the principles set down here will certainly not perceive the force of the conclusions* [vim consequentiarum minime percipient]*, nor will they lay aside the preconceptions* [præjudicia] *to which they have become accustomed over many years; and therefore, to avoid lengthy disputations* [& propetea ne res in disputationibus trahatur]*, I have translated the substance of the earlier version into propositions in a mathematical style* [more mathematico]*, so that they may be read only by those who have first mastered the principles* [qui principia prius evolverint].[148]

In CUL Add. Ms. 3990 (1685), Newton indeed blended the mathematical and the physical treatment of force and he devoted less attention to the physico-mathematical principles on which the physical treatment of force was based. In the relevant section on gravity as a single interaction force, which was originally contained in CUL Add. Ms. 3990, Newton wrote:

> For all action is mutual, and makes the bodies mutually to approach one to the other,[149] and therefore must be the same in both bodies. It is true that we may consider one body as attracting another as attracted. But this distinction is more mathematical than natural.[150] The attraction is really common of either to other, and therefore of the same kind in both.[151] [. . .] *And though the mutual actions of two Planets may be distinguished and considered as two, by which each attracts the other; yet as those actions are intermediate, they don't make two, but one operation between two terms*.[152] Two bodies may be mutually attracted, each to the other, by the contraction of a cord interposed. There is a double cause of action, to wit, the disposition of both bodies, as well as a double action in so far as the action is considered as upon two bodies.[153] But as betwixt two bodies it is but a single one. 'Tis not one action by which the Sun attracts Jupiter, and another by which Jupiter attracts the Sun. But it is one action by which the Sun and Jupiter mutually endeavour to approach each other. By the action with which the Sun attracts Jupiter, Jupiter and the Sun endeavour to come nearer

148 Newton, *The Principia*, p. 793 [italics added]; Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 549. Cf. Newton, *A Treatise of the System of the World*, xxiv.

149 At this point, the translator omitted a translation of "(per Motus Legem 3.)" (Newton, *De mundi systemate*, p. 25).

150 "Considerari potest corpus unum ut attrahens, alterum ut attractum, sed hæc distinctio magis mathematica est quàm naturalis." (CUL Add. Ms. 3990, f. 14^{r}; Newton, *De mundi systemate*, p. 25). As Eric Schliesser correctly remarks, here Newton was alerting his readers that "one cannot simply infer ontology from one's mathematical expression" (Schliesser, "Without God", p. 84).

151 "Attractio reverà est corporis utriusque in utrumque, atque adeo ejusdem generis in utroque." (CUL Add. Ms. 3990, f. 14^{r}; Newton, *De mundi systemate*, p. 25).

152 "Et quamvis binorum Planetarum actiones in se mutuò distingui possint ab invicem ut actiones binæ quibus uterque trahit alterum considerari: tamen [. . .] quatenus intermediæ sunt non sunt binæ sed operatio simplex inter binos terminos." (CUL Add. Ms. 3990, f. 14^{r}; Newton, *De mundi systemate*, p. 25).

153 "Causa actionis gemina est, nimirum dispositio utriusque corporis; actio item gemina quatenùs in bina corpora: at [. . .] quatenùs inter bina corpora simplex est et unica." (CUL Add. Ms. 3990, f. 14^{r}-15^{r}; Newton, *De mundi systemate*, p. 25).

> together [by the *third Law of Motion*][154] and by the action, with which Jupiter attracts the Sun, likewise Jupiter and the Sun endeavour to come nearer together. But the Sun is not attracted towards Jupiter by a two-fold action, nor Jupiter by a two-fold action towards the Sun: but 'tis one single intermediate action, by which both approach nearer together. [. . .] *In this sense it is that we are to conceive one single action to be exerted betwixt two Planets, arising from the conspiring natures of both*.[155] And this action standing in the same relation to both, if it is proportional to the quantity of matter in the one, it will be also proportional to the quantity of matter in the other.[156]

According to Newton, gravity consists of the interaction between pairs of bodies – an interaction of which the strength is dependent on the spatio-temporal features of those bodies. Mass, on the other hand, is an immutable or essential property of bodies, i.e. bodies are material in virtue of their having mass. Therefore, mass is a non-relational property, i.e. it is independent from a body's spatio-temporal features.[157] In *A Treatise of the System of the World*/*De mundi systemate* Newton was pointing out that gravity is – given Law III – an interaction force and, moreover, a *relational* property or force, as the strength of gravitational interactions depends of the spatio-temporal features of the bodies at hand.[158] There is no reason to suggest that Newton was considering "the conspiring natures of both" as the *full* cause of gravity. In the passage referred to above, he was singling out mass as a causally salient variable which was part of the full story about cause of gravity: *in virtue of having mass and given Law III all bodies attract and are attracted universally*. A treatment of the full cause of gravity will need to encompass a story about the non-mechanical medium through which gravitational interactions operate. Whilst Newton was making additions in preparation for the third edition of the *Principia*, he returned to a similar treatment of mass as a causally salient variable in his manuscripts. In this context Newton wrote down the following addition to the text of Law III:

> Every body that attracts another body, is attracted just as much by that other body in contrary directions. But the attraction, which depends on the whole attracting body, which always accompanies [the body] itself, and which cannot separately exist, must be understood *as if*

154 The reference to Law III was added in the 1737 edition of *A Treatise of the System of the World* and was originally contained in CUL Add. Ms. 3990, f. 15^r and Newton, *De mundi systemate*, p. 26.

155 "Ad hunc modum concipe simplicem exerceri inter binos Planetas ab utriusque conspirante naturâ oriundum operationem; & hæc eodem modo se habebit ad utrumque: adeò proportionalis existens materiæ in uno eorum, proportionalis eris materiæ in altero." (CUL Add. Ms. 3990, f. 15^r; Newton, *De mundi systemate*, p. 26).

156 Newton, *A Treatise of the System of the World*, pp. 38–40 [italics added].

157 This is crucial to understand the sorts of universal properties or forces (to wit: relational versus immutable ones) Newton presupposed in his treatment of Rule III. See Section 3.2 of Chapter 3 for further discussion.

158 See Howard Stein, "Newton's Metaphysics", pp. 287–288 and Schliesser, "Without God" for nice expositions of this strand in Newton's thinking to which I am indebted here.

it is its virtue and sphere of activity (in the same mode as magnetical attraction[159]). *Because if the attraction can be separated from the attracting body and can exist separately, it will not be the attraction by that body, which is the cause of gravity, but it will arise from another source and it will be mutual between the attracted body and some other thing*. Meanwhile, here I do not consider what the attraction is and how it operates be it by perpetually emitted insensible atmospheres of bodies or by different mediums of which the bodies mutually and forcefully propagate the secret actions.[160]

Note Newton's careful use of "tanquam" in the second sentence of this fragment: the attraction of the attracting body "must be understood *as if* it is its virtue and sphere of activity." It is clear from Newton's discussion that when he claims that "the attraction by that body" is "the cause of gravity," he is not discoursing about the *full* cause of gravity, since he reminds us explicitly in the final sentence that he has nothing to say about the medium through which bodies propagate their actions. Rather, Newton is discoursing about a causally salient variable involved in gravitational attractions: *mass*, without which the attraction "cannot separately exist." Newton's physico-mathematical investigation of gravity had indeed unravelled that mass is one of those causally salient variables. Given the application of Law III it also teaches us that gravity is an interaction force.[161] Newton's use of "tanquam" signals that he is inferring natural-philosophical conclusions on the basis of approximations. Note that all of Newton's physico-mathematical models – even the most complex ones – deliberately abstract from a gravitationally relevant medium. Given Rule IV, which will be discussed in Chapter 3, we may – until we have a more detailed picture of the full cause of gravity at our disposal – treat gravitational forces *as if* they reside in distant bodies. Note, furthermore, that Newton could only have accepted action at a distance *between gravitationally interacting bodies* had he interpreted the results of his natural-philosophical investigation of gravity not in approximative but in literal terms.[162] A literal interpretation of his physico-mathematical models would

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159 Although magnetism is an interaction force (Newton, *A Treatise of the System of the World*, pp. 39–40), it is not a *universal* interaction force like gravity, because it does not pertain to all bodies universally.

160 Translation of: "Corpus omne quod corpus alterum attrahit, tantundem ab illo altero in partes contrarias attrahitur. Sed hoc intelligendum est de attractione quæ ↓tota↓ a corpore attrahente dependet tanquam virtus ejus et sphæra activitatis, & ipsum ubique comitatur & seorsim existere non potest; cujusmodi est attractio magnetica. Nam si attractio a corpore attrahente separari potest & seorsim existere, hæc non erit corporis illius attractio sed aliunde orietur & mutua erit inter corpus attractum et rem illam quæ attractionis est causa. Interea quid sit attractio et quomodo ↓fiat↓ sive per corporum insensibiles atmosphæras perpetuo emissas sive per alia media quorum ope corpora propagant actiones secretas in se mutuò, hic non expendo." (CUL Add. Ms. 3965, f. 731^{r} [italics added]). I am indebted to Danny Praet's comments which allowed me to improve my translation. See, furthermore, Appendix 2 to Chapter 3.

161 Newton's often contested application of Law III to gravitationally interacting bodies is less problematic once it is predicated under Rule IV. See Section 3.4.3 in Chapter 3 for further discussion.

162 In the following section, I shall explain why Newton did not consider action at a distance between two gravitationally interacting material bodies plausible. As we will see, Newton did not reject this possibility because of the non-intelligibility of action at a distance, but rather because

make mass the full cause of gravity and it would make the quest for a gravitationally relevant medium superfluous. However, as we have seen, Newton was careful enough not to straightforwardly infer ontology from the physico-mathematics he had developed in the *Principia*.

As we have seen, Newton conceived of the *Principia* as containing causal explanations. But what can be said about the status of Newton's notion of efficient causation in the *Principia*? Newton never addressed its status in detail. Accordingly, we have to reconstruct the status of efficient causation from a thorough reading of the *Principia*. Note that Newton's inference that centripetal forces keep the primary and secondary planets in their orbits follows from the law of inertia. Since the primary and secondary planets are not at rest nor move uniformly along a straight line, an impressed force (in this case, a centripetal force[163]) is acting on them. Newton thus relied on Law I, i.e. the law of inertia, which states that if a body is *not* impressed by an external force (abbreviated as F_E), then it will *not* describe a non-inertial trajectory, i.e. it will preserve its state of motion or rest (= abbreviated as S), as a criterion to decide whether an external force acts on a body or not.[164] Correspondingly, Law I states that not-F_E implies S. By contraposition of Law I, not-S implies F_E. Since we know by observation that not-S is the case, it follows from the contraposed version of Law I that F_E is the case. Newton thus derived F_E by *modus ponens* from not-S implies F_E and not-S. From the perspective of Law I, non-inertial motion can thus be seen as evidence of the presence of a real force. Essentially, we are comparing the non-inertial motions, which celestial bodies actually describe, with the uniform rectilinear motions that these bodies *would* describe, if they were not acted upon by an external force.[165] Newton's notion

of the unwanted implications of action at a distance in the context of gravitationally interacting bodies. The translator seemed to have understood Newton's deliberate choice to remain silent on the cause of gravity. In his preface to *A Treatise of the System of the World*, he wrote: "*What the cause is of this force, we do not yet pretend to determine; our business is, since such a force is found to exist, to search into the properties and proportions of that force, before we think of enquiring into the cause of it.*" (ibid., xv, cf. xxiii: "*Some persons will probably be ready to enquire what is the cause of this hidden virtue of gravity which is here attributed to the heavenly bodies. To this the only answer is, that this cause is as yet one of Nature's secrets: and perhaps it will ever remain so.*").

163 Newton, *The Principia*, p. 405.

164 I. Bernard Cohen correctly statest that Law I was "a condition for the existence of certain insensible forces, not otherwise known to us," i.e. "our awareness of such a force is based on the first law and the observed fact that the planets do not follow a uniform rectilinear path" (Newton, *The Principia*, p. 110).

165 In Chapter 2, we will see that my counterfactual reading of Law I is compatible with Newton's own understanding of Laws I and II, as stated in Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 541/542. By the way, in *The Philosophy of the Inductive Sciences* (1840), William Whewell understood the counterfactual dimension of Law I very well: "*Force is any cause which has motion, or change of motion, for its effect*; and thus, all the exchange of velocity of a body which can be referred to extraneous bodies, – as the air which surrounds it, or the support on which it rests, – is considered as the effect of forces; and *this consideration is looked upon as explaining the difference between the motion which really takes place in the experiment, and that motion*

of efficient causation therefore involves *counterfactual dependence*, and, since it is furthermore based on Law I, it involves *counterfactual-nomological dependence*.[166] Note in order to back up causal claims in the counterfactual sense, we need some theoretical background principle that informs us what happens when the putative causal factor is absent. In the case of orbital motion, this information is provided by Law I: if a body is not being acted upon by an external force, it will conserve its state of motion or rest. My reading of Newton's notion of efficient causation has the welcome effect that it is consistent not only with Newton's causal talk, but also with his preoccupation of staying neutral with respect to the remote cause of gravitational effects.

In conclusion, let me add some words on final and formal causation. Newton clearly saw natural philosophy as providing room for final causes.[167] Although, the motion of the celestial bodies are governed by the law of universal gravitation, their regular positions cannot be explained by the law of attraction:

> The six primary planets revolve about the sun in circles concentric with the sun, with the same direction of motion, and very nearly in the same plane. Ten moons revolve about the earth, Jupiter, and Saturn in concentric circles, with the same direction of motion, very nearly in the planes of the orbits of the planets. And all these regular motions do not have their origin in mechanical causes, since comets go freely in very eccentric orbits and into all parts of the heavens. And with this kind of motion the comets pass very swiftly and very easily through the orbits of the planets; and in their aphelia, where they are slower and spend a longer time, they are at the greatest possible distance from one another, <u>so as to</u> attract one another as little as possible. This most elegant system of the sun, planets, and comets could not have arisen without the design and dominion of an intelligent and powerful being [Elegantissima hæcce solis, planetarum & cometarum compagnes non nisi consilio & dominio entis intelligentis & potentis oriri potuit.]. And if the fixed stars are the centers of similar systems, they will all be constructed according to a similar design and subject to the dominion of *One* [simili consilio constructa suberunt *Unius* dominio], especially since the light of the fixed stars is of the same nature as the light of the sun, and all the systems send light into all the others.[168] And <u>so that</u> the systems of the fixed stars will not fall upon one another as a result of their gravity, he has placed them at immense distances from one another.[169]

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which, as the law asserts, would take place if the body were not acted on by any forces." (Yeo, ed., *Collected Works of William Whewell*, IV, p. 217 [italics added]).

166 By definition, *C* is a counterfactual cause of *E*, if and only if, if *C* had not happened, then *E* would not have happened. Correspondingly, *C* is a counterfactual-nomological cause of *E*, if and only if, there is a law which stipulates that if *C* had not happened, then *E* would not have happened.

167 On the importance of final causation in seventeenth-century natural philosophy, see esp. Osler, "From Immanent Natures to Nature as Artifice".

168 See, furthermore, Hoskins, "Newton, Providence and the Universe of Stars".

169 Newton, *The Principia*, p. 941 [underscore added]; Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 760. In *The Opticks* Newton commented: "For while Comets move in very excentrick Orbs in all manner of Positions, *blind Fate could never make all the Planets move in one and the same way in Orbs concentrick*, some inconsiderable Irregularities excepted, which may have arisen from the mutual Actions of Comets and Planets upon one another, and which will be apt to increase, till this System wants a Reformation. Such a wonderful Uniformity in the Planetary System must be allowed the Effect of Choice. And so must the Uniformity in the Bodies of Animals, they having generally a right and a left side shaped alike, and on either side of

"We know him [i.e., God]," Newton wrote in the General Scholium, "only by his properties and attributes and *by the wisest and best construction of things and their final causes* [sapientissimas & optimas rerum structuras & causas finales]."[170] One could also interpret the laws of motion or the law of universal gravity as providing the formal causes of celestial and terrestrial motions.[171] Niccolò Guicciardini has recently pointed out that Newton and his acolytes saw the laws of motion as "expressing causal relations, the equivalent of artificial tracing mechanisms in organic geometry and the fluxional method, between forces and motions."[172] In line with his view that the genesis and subject-matter geometry is founded upon mechanics, Newton endorsed the view that to provide the mechanical description of a curve is the provide the *reason of its genesis*.[173]

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their Bodies two Legs behind, and either two Arms, or two Legs, or two Wings before upon their Shoulders, and between their Shoulders a Neck running down into a Back-bone, and a Head upon it; and in the Head two Ears, two Eyes, a Nose, a Mouth, and a Tongue, alike situated. Also the first Contrivance of those very artificial Parts of Animals, the Eyes, Ears, Brain, Muscles, Heart, Lungs, Midriff, Glands, Larynx, Hands, Wings, swimming Bladders, natural Spectacles, and other Organs of Sense and Motion; and the Instinct of Brutes and Insects, can be *the effect of nothing else than the Wisdom and Skill of a powerful ever-living Agent*; [. . .]" (Newton, *The Opticks*, pp. 402–403 [italics added]).

170 Newton, *The Principia*, p. 942 [italics added].

171 Joy, "Scientific Explanation: From Formal Causes to Laws of Nature".

172 Guicciardini, *Isaac Newton on Mathematical Certainty and Method*, p. 380, footnote 34. Isaac Barrow is well known for his defence of formal causation in natural philosophy. In contrast to Newton, Barrow, however, *only* accepted formal causes, such as those demonstrated by syllogism and mathematics, as true and necessary causes: "For there can be no such Connection of an *external*, *ex. gr. efficient Cause* with its *Effect*, (at least non such can be understood by us) through which, strictly speaking, the *Effect* is necessarily supposed by the Supposition of the *efficient Cause*; or any *determinate Cause* by the Supposition of the Effect. Nay there can be no efficient Cause in the Nature of Things of a Philosophical Consideration which is altogether necessary. For every Action of an *efficient Cause*, as well as its consequent *Effect*, depends on upon the *Free-Will* of *Almighty God*, who can hinder the Influx and Efficacy of any *Cause* at his Pleasure; neither is there any *Effect* so confined to one *Cause*, but it may be produced by perhaps innumerable others. Hence it is possible that there may be such a *Cause* without a *subsequent Effect*; or such an *Effect* and no peculiar *Cause* to afford any Thing its Existence." (Barrow, *The Usefulness of Mathematical Learning Explained and Demonstrated*, pp. 88–89). This observation is not in any way intended to cast doubt on the importance of the Barrovian programme (see Iliffe, "Abstract Considerations: Disciplines and the Coherence of Newton's Natural Philosophy", pp. 432–434 and Dear, *Discipline and Experience*, Chapter 8).

173 Guicciardini, *Isaac Newton on Mathematical Certainty and Method*, p. 319 and, furthermore, Chapter 13, esp. pp. 299–305. Although Newton never explicitly stated that the laws of motion are to be conceived as formal causes, Guicciardini's suggestion is based on one of the core assumption of Newton's philosophy of mathematics. I was able to locate the following excerpt, in which Newton came quite close to conceiving of the laws of universal gravitation as the formal cause of motion: "These principles [i.e., active principles of motions] I consider not as ~~the~~ occult Qualities resulting from the ↓specific↓ ~~particular~~ forms of things, but as *the general laws of Nature from whence the forms themselves result*. To tell us that every species of things is endowed with an occult quality by w[ch] it acts is to tell us nothing." (CUL Add. Ms. 3970, f. 242[v] [ca. 1700–1704; written upside down; italics added]). That being said, it is fair to say that formal causation remained rather undeveloped in Newton's work.

1.7 Newton on Action at a Distance

The *Principia* entailed the possibility of action at a distance. But did Newton accept this possibility? In the *Principia*, Newton had shown that gravity does not act mechanically, i.e. "not in proportion to quantity of the *surfaces* of the particles on which it acts (as mechanical causes are wont to do)," but "in proportion to the quantify of *solid* matter."[174] Later, in the General Scholium of 1713, this point was rendered more explicit: "All these regular motions do not have their origin in mechanical causes [originem non habent ex causis mechanicis], since comets go freely in very eccentric orbits and into all parts of the heavens."[175] In the Corollaries to Proposition VI of Book III and in the *Scholium* to Proposition LIII in Book II of the *Principia*, Newton concluded that the celestial regions contain large voids.[176] Instead of seeing the solar system filled with *vortices*, Newton saw it as a Boylian vacuum in which the celestial bodies could move freely.[177] Only an extremely rarified ether could be rendered consistent with the results established in the *Principia*. Recently, Andrew Janiak and Hylarie Kochiras have argued that Newton did not accept the possibility of action at a distance per se, *because* he endorsed the maxim that substance (or matter) "cannot act where it is not."[178] In their argumentation they have considered Newton's letter to Richard Bentley on 25 February 1692/3, which I shall analyse in what follows, as crucial evidence in support of their interpretation. In what follows, I argue that Janiak's and Kochiras's views are misguided and that Newton's views on action at a distance are more complex. As we will see in what follows, Newton rejected action at a distance *at the macro-level*, i.e. the view that material bodies directly attract one another gravitationally *in vacuo* without the mediation of a *tertium quid*, because matter is passive. In *The Opticks*, however, Newton seriously considered the possibility of action at a distance *at the micro-level* when he speculated on the inter-particular mutually repellent forces of the elastic ether, which he introduced as a possible cause of gravity. Before I begin my discussion of Newton's views on action at a distance, I shall provide some useful background, which has been left unmentioned by Janiak and Kochiras.

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[174] Newton, *The Principia*, p. 943. The complete sentence is: "Oritur utique hæc vis a causa aliqua, quæ penetrat ad usque centra solis & planetarum, sine virtutis diminutione, quæque agit non pro quantitate *superficierum* particularum, in quas agit (ut solent causæ mechanicæ) sed pro quantitate materiæ *solidæ*; & cujus actio in immensas distantias undique extenditur, decrescendo semper in duplicata ratione distantiarum." (Koyré, Cohen and Whitman, eds., *Principia Mathematica*, II, p. 764). In Query 28, Newton wrote that gravity is produced by "some other Cause than dense Matter" (Newton, *The Opticks*, p. 369).

[175] Newton, *The Principia*, p. 940.

[176] Ibid., pp. 809–810 and pp. 789–790, respectively. See, furthermore, Query 28 in Newton, *The Opticks*, pp. 365, 368.

[177] Newton, *The Principia*, p. 939.

[178] Janiak, *Newton as Philosopher*, p. 35 and Kochiras, "Gravity and Newton's Substance Counting Problem", p. 275. Direct confirmation of Janiak's and Kochiras' account cannot, to the best of my knowledge, be found in Newton's published or unpublished work.

Let me begin by pointing out that, when Newton started to question vortex cosmology, he was already in his forties. Previously, Newton was inclined to explain gravitation in mechanical terms.[179] In 1680–1681, when drafting up some propositions on cometary motion, Newton clearly thought along the lines of a vortex cosmology (see especially: "2. Materiam coelorum fluidam esse. 3. Materiam ↓coelorum↓ ~~illam~~ circa centrum systematis cosmici secundum cursum Planetarum gyrare.").[180] Consistent with this, in his 1680/1 correspondence with Thomas Burnet, Newton claimed that vortices offered a sensible explanation of gravity and he posited centrifugal forces in the explanation of the celestial motions.[181]

When Nicolas Fatio De Duillier was working on his *De la cause de la pesanteur* (1690), in which he introduced a mechanical ether to explain gravitation, he pointed out in a letter to Huygens on 24 February 1689/90, that "Je marquerai seulement en passant que Mr. Newton trouve que l'experience s'accorde avec cette pensée."[182] Later, in a memorandum by David Gregory on 28 December 1691, however, it is reported that "Mr Newton and Mr Hally [*sic*] laugh at Mr Fatios manner of explaining gravity."[183] In a letter to Leibniz on 30 March 1694, De Duillier wrote that "Monsr. Newton est encore indeterminé entre ces deux sentiments. Le premier que la cause de la de la Pesanteur soit inherente dans la matiére par une Loi immediate du Createur de l'Univers: et l'autre que la Pesanteur soit produite par la cause Mécanique que j'en ai trouvée."[184] These sources indicate that Newton between February 1689/90 and March 1694 wavered between a mechanical and non-mechanical explanation of gravity. We do know, however, with certainty that from May 1694 (and perhaps slightly earlier), i.e. the time when David Gregory saw

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[179] Dobbs, *The Janus Faces of Genius*, Chapter 4, esp. pp. 120–121.

[180] CUL Add. Ms. 3965, f. 613^{r}.

[181] Newton, *Correspondence*, II, pp. 329–335. Cf. Herivel, *The Background to Newton's Principia*, pp. 54–64, 194. See, furthermore, Whiteside, "The Prehistory of the *Principia* from 1664 to 1686", Meli, "The Relativization of Centrifugal Force", and Newton's *De aere et aethere* (Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, pp. 214–228).

[182] Ibid., III, p. 69. Cf. De Duillier, *De la cause de la pesanteur*, p. 117. On CUL Add. Ms. 4005.6, f. 28^{r} [ca. 1690–1693], Newton wrote: "Errant igitur qui corporum particulas minimas ~~corpo~~ ad modum particularum arenæ aut lapidum coacervatorum confertim jungunt. Si particulæ aliquæ tam dense constipentur, causa gravitans desinet esse proportionalis materiæ. Excogitandæ sunt aliæ particulatum texturæ quibus interstitia earum reddantur amplissima. Et hæ sunt necessariæ conditiones Hypotheseos per quam gravitas explicetur mechanicè. Hujus autem generis Hypothesis est unica per quam gravitas explicari potest, eamque Geometra ingeniosissimus D. N. Fatio primus excogitavit." By ca. 1716–18 Newton's tone had changed drastically: "Mechanicam gravitatis causam D. Fatio olim excogitavit, sed veram esse non probavit. Hypothesis erat, & in Philosophia experimentali hypotheses non considerantur. Argumenta hic desumuntur ab experimentis per Inductione. Et argumentum ab inductione ↓licet demonstratio perfecta non sit tamen↓ fortius est quam argumentum ab Hypothesi sola. Et quo plura sint experimenta vel Phaenomena a quibus deducitur eo fortius evadit. Hypothesis igitur in hoc Tractatu non fingimus neque argumenta inde desumimus, cum cedant argumentis ab inductione [end of text]" (CUL Add. Ms. 9597.2.11, f. 3^{r}). On Fatio, see further: Mandelbrote, "The Heterodox Career of Nicolas Fatio de Duillier".

[183] Newton, *Correspondence*, III, p. 191.

[184] Ibid., III, p. 309.

Newton's Classical Scholia, Newton no longer considered a mechanical agent as a plausible candidate to explain gravitation,[185] for in material related to the Classical Scholia he posited "some mediating active principle" as the cause of gravity.[186] In their 1966 joint paper, which was to become the *locus classicus* in the study of Newton's Classical Scholia, "Newton and the 'Pipes of Pan,'" J. E. McGuire and Piyo M. Rattansi first pointed to the importance of several of Newton's unpublished draft *scholia* to Propositions IV–IX of Book III of the *Principia*.[187] After having written his highly technical and innovative *Principia*, Newton sought to justify his concept of attraction by showing that the ancients had already discovered the law of universal gravitation.[188] B. J. T. Dobbs remarked similarly that, given Newton's

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[185] McGuire and Rattansi, "Newton and 'The Pipes of Pan'", p. 125.

[186] Cf. CUL Add. Ms. 3965, f. 269^{r} [miscellaneous additions and corrections to the first edition of the *Principia*; ca. 1693–1694], where Newton wrote: "Nam Planetæ [...] non [...] ↓petent se mutuo↓ vi ↓aliqua↓ gravitates neque ullo modo agent in se invicem nisi mediante principio aliquo activo quod utrumque intercedat, et per quod vis ab utroque in alterum propagetur." and "[Hoc medium ex mente veterum non erat corporeum cum corpora universa ex essentia sua gravia esse dicerent, atque atomos ↓ipsos vi æterna↓ naturæ suæ absque aliorum corporum impulse per spatia vacua in terram cadere.]."

[187] There are similar references to "God Pan & his Pipe" on CUL Add. Ms. 3970, f. 291^{r} [ca. 1700–1704] and f. 619^{r} [ca. 1706]. Only in 1984 did Newton's Classical Scholia become widely accessible, see Casini, "Newton: The Classical Scholia", pp. 25–38. It should be noted that although Casini has taken the CUL manuscripts into account (Casini, "Newton: The Classical Scholia", p. 18), his actual transcriptions are mainly based upon David Gregory's annotations of Newton's Classical Scholia (Royal Society of London, Ms. 247, ff. 6–14). For some corrections to Casini, see De Smet and Verhelst, "Newton's Scholium Generale: The Platonic and Stoic Legacy", p. 21 and, especially, Schüller, "Newton's *Scholia*", pp. 218–245, in which the definitive transcriptions are provided. Note that Schüller has also transcribed Gregory's introduction to his *Astronomiae physicae & geometricae elementa* (1702), in which Gregory drew from Newton's papers without making the source explicit (Schüller, "Newtons *Scholia* aus David Gregorys Nachlaβ"). Schüller's edition (ibid., pp. 89–117), furthermore, contains reproductions of Royal Society, Gregory Ms. 247, ff. 6–14. Gregory visited Newton on 4–7 May 1694 at Cambridge and Newton later entrusted the manuscript containing the Classical Scholia to him (see ibid., pp. 16–17 and Schüller, "Newton's *Scholia*", pp. 214–215 for the details). Schüller dates Gregory's memorandum back to July 1694 (ibid., p. 214).

Casini's paper contains several criticisms on McGuire and Rattansi's 1966 paper. Casini tried to temper their view that the Cambridge Platonists, Ralph Cudworth and Henry More, were a direct source of inspiration for Newton (Casini, "Newton: The Classical Scholia", pp. 4–5), and, on a more general level, he attacked their Hermetic-alchemist interpretation of Newton (ibid., pp. 10–15). With respect to Casini's first criticism, it should be noted that McGuire and Rattansi did not exactly claim that there was a direct influence of the Cambridge Platonists on Newton. Apart from referring to some general affinities between the Cambridge Platonists and Newton, they also emphasized that "Newton disagrees with the two other authors [i.e., Cudworth and More] on certain important points of interpretation" (McGuire and Rattansi, "Newton and 'The Pipes of Pan'", p. 135). With respect to Casini second criticism, it should be pointed out that McGuire underlined that "Hermeticism is too simple an answer to a complex problem of this sort: it gives only a single-valued account of how Newton liberalized the ontology of the mechanical philosophy to include various types of agents" (McGuire, "Neoplatonism and Active Principles: Newton and the Corpus Hermeticum", p. 126).

[188] There is no direct evidence that Newton's alchemy contributed in a significant way to Newton's theory and concept of universal gravitation (cf. Newman, "Newton, Isaac", p. 273). Richard S.

conviction that he was restoring the *prisca sapientia*, it was perfectly natural for him, "when the (for him, modern) mechanical explanation of gravity failed," to turn "to ancient sources in an attempt to recapture the truer explanation of gravity once known to the wise ancients."[189] Moreover, for Newton "they represented a deeper penetration into the *prisca sapientia*, possible only when the preliminary work has been accomplished through experience."[190] In a nutshell, McGuire and Rattansi's view is that:

> The central purpose of the "classical" scholia was to support the doctrine of universal gravitation as developed in these Propositions, and to enquire into its nature as a cosmic force. This doctrine is shown by Newton to be identifiable in the writings of the ancients. As will become clear, he is not using this historical evidence in a random fashion, or merely for literary ornamentation. Rather the evidence is used in a serious and systematic fashion, as support for, and justification of, the components of Newton's theory of matter, space and gravitation. The evidence is used to establish four basic theses, which correspond to the matter of Proposition IV to IX. These are, that there was an ancient knowledge of the truth of the following four principles: that matter is atomic in structure and moves by gravity through void space[191]; that gravitational force acts

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Westfall suggested that "[a]lchemy had led him [Newton] to consider concepts of activity and force that were susceptible to mathematical treatment in a way that aethereal mechanisms were not." (Westfall, *Never at Rest*, p. 407; cf. Cohen in Newton, *The Principia*, pp. 57–58). While it is plausible to suggest (but difficult to establish) that, since Newton's alchemical work familiarized him with non-mechanical active principles, alchemy may have facilitated his conceptual acceptation of gravity as a non-mechanical force, it needs to be stressed that such acceptation would not have occurred if Newton had not had the relevant empirical evidence at his disposal to back this claim up. Dobbs has emphasized that "Newton was forced to abandon his mechanical definition of gravitation through a combination of mathematical and observational-experimental evidence" (Dobbs, *The Janus Faces of Genius*, pp. 91–93). See, furthermore, ibid., pp. 89–121, 207–208. Dobbs' claim is a weakening of the position she defended in Dobbs, *The Foundations of Newton's Alchemy*, pp. 148–150, 210–213 – for an insightful commentary of this work, see Figala, "Newton as Alchemist". At present there is no positive evidence whatsoever for the strong claim that Newton's alchemy directly suggested to him the concept of a non-mechanical gravitational force. On Newton's alchemy, see, furthermore, Newman, "Newton's *Clavis* as Starkey's *Key*"; id., "The Background to Newton's Chymistry"; id., "Geochemical Concepts in Isaac Newton's Early Alchemy"; id., "Newton's Early Optical Theory and its Debt to Chymistry"; and, Principe, "Reflections on Newton's Alchemy in Light of the New Historiography of Alchemy".

189 Dobbs, *The Janus Faces of Genius*, p. 186.

190 McGuire and Rattansi, "Newton and the 'Pipes of Pan'", p. 137.

191 Royal Society, Gregory Ms. 247, f. $10^{r\text{-}v}$. See, furthermore: "That all bodies located around the earth, air and fire as well as others, are heavy toward the earth and that their gravity is proportional to the quantity of the matter of which they consist, was known to the ancients." and "Accordingly ~~it was the opinion of the earlier~~ it is an old view that gravity toward the entire earth originates from this gravity to its individual particles, just as the attractive force of an entire magnet is composed of the attractive forces of the individual particles of which the magnet ~~is composed~~ consists." (Schüller, "Newton's *Scholia*" p. 225 and p. 233). Schüller's translation of: "Corpora omnia quae circa terram sunt tam aerem et ignem quam relinqua ~~XXXX~~ esse gravia ~~esse~~ in Terram et eorum gravitatem proportionalem esse quantitati materiae ex qua constant Veteribus etiam innotuit." (ibid., p. 224; Royal Society, Gregory Ms. 247, f. 6^{v}, cf. f. 10^{v}) and "Igitur quemadmodum vis attractiva Magnetis totius componitur ex viribus attractivis particularum singularum ex quibus Magnes ~~componitur~~ constat sic ~~XXX~~ gravitatem in Terram totam ex gravitate in singulos ejus

universally[192]; that gravity diminishes in the ratio of the inverse square of the distances between bodies[193]; and that the true cause of gravity is the direct action of God.[194]

The latter sentence, "that the true cause of gravity is the direct action of God," should be disambiguated. It should be noted that claiming that gravity depends on the will of God is not the same as claiming that God directly causes gravity. That Newton made the latter claim is evident from manuscript material.[195] To the best of my knowledge, however, there is no positive evidence to suggest that Newton entertained the former. On Gregory Ms. 247, f. 14^{v}, Newton wrote:

> Up to this point I have explained the properties of gravity. I have not made the slightest consideration about its cause. However, I would like to relate what the ancients thought about this. ~~Doubtlessly a certain spiritus through the heavens~~ Quite apparently the heavens are nearly free of bodies, but nevertheless filled everywhere with a certain infinite *spiritus*, which they call God. The bodies, however, move around freely in this *spiritus*, as a consequence of its forces and natural efficiency ~~the bodies~~ they are ~~thrust~~ constantly thrust toward each other, more or less <strongly> in accordance with the harmonic ratio of the distances, and gravity consists in this impact. Some differentiated this spiritus from the highest God and called it the world soul.[196]

In this quotation, Newton reported that the ancients called "a certain infinite *spiritus*" God; he did not claim that God directly causes gravity.[197] The same caveat applies to Newton's later assertion (ca. 1706) that "matter depends upon a Deity for its ↓laws of↓ motion as well as for its existence:"[198] again, Newton did not explicitly state that gravitation is produced *directly* by God. B. J. T. Dobbs has, furthermore, noted that the claim that gravity is produced directly by God would

particulas oriri ~~sententia fuit Veterum~~ antiqua fuit opinio." (Schüller, "Newton's *Scholia*", p. 232; Royal Society, Gregory Ms. 247, f. 11^{r}). Note that Newton also thought that the ancients had discovered the law of inertia (Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, pp. 309–311).

192 Royal Society, Gregory Ms. 247, f. 11^{v}.

193 Ibid., f. 11^{v}, f. 12^{r}. Newton also stressed that the Ancients attributed "to every atom a gravity proportional to y^{e} quantity of matter, without assigning the cause of such gravity" (CUL. Add. Ms. 3970, f. 291^{r} [additions and corrections intended for the second edition of the *Principia*]).

194 McGuire and Rattansi, "Newton and the 'Pipes of Pan'", pp. 111–112.

195 On Royal Society, Gregory Ms. 247, f. 14^{r}, Newton wrote: "Solum enim ens intelligens vi voluntatis suæ ↓secundum intellectuales rerum ideas↓ propter causes finales agendo varietatem rerum introducere potuit."

196 Schüller's translation of: "Hactenus proprietates gravitatis explicui. Causas ejus minime expendo. Dicam tamen quid Veteres hac de re senserint. ~~XXXX nimirum spiritum quedam per caelos XXXX~~ nempe caelos esse corporis prope vacuos ~~XXXdemptis~~ sed spiritu tamen quodam infinito quem Deum nominabant ubique ~~XXXXX et plenos~~ impleri: ~~in quo astra infimaXXXXX~~ corpora autam ~~XXXX~~ in spiritu illo libereme moveri ~~XXXX~~ ejus vi et virtute ~~corpora~~ naturali ad invicem ~~impelli~~ perpetuo impelli, idque magis vel minus pro ratione harmonica distantiarum, & in hic im<pul>su gravitatem consistere. Hunc spiritum aliqui a Deo summo distinxerunt & animam mundi vocarunt.)" (Schüller, "Newton's *Scholia*", pp. 240, 241; Royal Society, Gregory Ms. 247, f. 14^{v}).

197 See Dobbs, *The Janus Faces of Genius*, pp. 36–37 for related criticism.

198 CUL Add. Ms. 3970, f. 619^{r} [ca. 1706].

have caused some theological uneasiness in Newton's thinking.[199] The later Newton emphasized that God made & governs the world "by his Agents."[200]

After 1694, Newton favoured a non-mechanical agent as explanation of gravitation. The data gathered above shows two things: first, before May 1694 Newton wavered between a mechanical and immaterial explanation for gravitation; secondly, from May 1694 and onwards Newton settled on an immaterial agent to account for gravitation. Therefore, in 1692/3, the time at which Newton sent his famous letter to Bentley,[201] he genuinely doubted whether the agent producing gravitation is mechanical or non-mechanical.[202] In his letter to Richard Bentley (on 25 February 1692/3), Newton stated:

> Tis inconceivable, that inanimate brute Matter, should (*without ye mediation of something else wch is not material*), operate upon & affect other matter wthout mutual contact; as it must if gravitation in the sense of Epicurus, be essential & inherent in it. And this is one reason why I desired you not to ascribe innate gravity to me. That gravity should be innate inherent & essential to matter so yt one body may act upon another at a distance through a vacuum wthout the mediation of any thing else & by & through wch their action and force may be conveyed from one to another is to me such an absurdity that I beleive no man who has in philosophical matters any competent faculty of thinking can ever fall into it. Gravity must be caused by an agent acting constantly according to certain laws, but whether this agent be material of immaterial is a question I left to ye consideration of my readers.[203]

There is contextual evidence suggesting that with "mediation," Newton was referring to God's interaction.[204] The sentence immediately preceding the quotation from Newton's fourth letter to Bentley states: "The last clause of your second Position I like very well."[205] The clause from Bentley's letter on 18 February to which Newton referred to goes as follows: "[Sir, I make account, yt your courteous suggestion by your Last, yt a Chaos is inconsistent with ye Hypothesis of innate Gravity, is included in this paragraph of mine.] and again, tis inconceivable, yt inanimate brute

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199 Dobbs, *The Janus Faces of Genius*, Chapter 7.

200 CUL Add. Ms. 3965, f. 368^v [post-1713].

201 Newton's four letters to Bentley were first published in 1756 ([Cumberland], ed., *Four Letters From Sir Isaac Newton to Doctor Bentley*).

202 To suggest, as Kochiras does in her recent paper (Kochiras, "Gravity and Newton's Substance Counting Problem", p. 268, footnote 5), that in his fourth letter to Bentley Newton was communicating his own (non-mechanical) candidate for the explanation of gravity, is thus highly implausible and contrary to the historical records. Newton was not contrasting his own candidate and "what he did in the *Principia*:" he was reporting on two options which he considered as equally plausible at the time and keeping the question of mediation distinct from the question of agency. Moreover, in his letter to Bentley on 17 January 1672/3, Newton clearly indicated that he did not know "ye cause of gravity" (Newton, *Correspondence*, III, p. 240). As we will see, the ethers Newton introduced in *The Opticks* (and related manuscript material) were non-mechanical: they required non-mechanical micro-forces.

203 Newton, *Correspondence*, III, pp. 253–254 [italics added].

204 Cf. Henry, "Isaac Newton y el Problema de la Acción a Distancia", p. 215.

205 Newton, *Correspondence*, III, p. 253.

matter should (*without a divine impression*) operate upon & affect other matter without mutual contact: as it must be, if gravitation be essential and inherent in it."[206] Newton was rejecting Epicurean atomism, according to which matter can affect matter *without the mediation of a* secundum quid.[207] On the Epicurean account, gravity is an essential and inherent property of bodies. In an earlier letter to Richard Bentley, Newton stated clearly: "You sometimes speak of gravity as essential & inherent to matter: pray do not ascribe that notion to me; for ye cause of gravity is what I pretend not to know, & therefore would take more time to consider of it."[208]

Such Epicurean view was untenable for Newton to accept since it would imply that matter is self-activating. This was *the* reason why Newton rejected Epicurean gravitation. On the contrary, Newton emphasized that matter itself is utterly passive and that it requires external causation. Post-1694-Newton emphasized that gravitational interaction requires the activity of certain non-mechanical "Active Principles." Bodies are passive and are moved by active principles, i.e. immaterial agents: "[f]or we meet very little Motion in the World, besides what is owing to these active Principles."[209] Newton sought to establish that Epicurean attraction would result in a chaotic world and, correspondingly, that the elegance and harmony of the solar system could only be guaranteed by "the design and dominion [consilio & dominio] of an intelligent and powerful being"[210] and that matter is dependent on God.[211] In an unpublished manuscript sheet prepared for the second edition of the *Principia*, Newton recorded the most wise order of things could not have arisen "from matter alone and motion or from the nature of things [a materia sola et motu aut a rerum Natura]."[212] In order to accomplish this, Newton argued that God regulates the natural world by means of certain activating principles which he had installed and maintains ever since. Correspondingly, in an unpublished draft version of the Queries intended for the first edition of *The Opticks*, Newton emphasized that:

> Qu 23. By what means do ~~they~~ bodies act on one another at a distance. The ancient Philosophers who held Atoms & Vacuum attributed gravity to Atoms without telling us the means unless perhaps in figures: as by calling God Harmony & comparing ↓representing↓ him & matter by the God Pan & his Pipe, or by calling the Sun the prison of Jupiter because he keeps the Planets in their orbs. Whence it seems to have been an ancient opinion that matter depends upon a Deity for its ↓laws of↓ motion as well as for its existence. *The*

206 Ibid., III, p. 249 [italics added].

207 John Henry is surely to be given the credit for emphasizing the Epicurean position to which Newton was reacting against (Henry, "'Pray do not Ascribe that Notion to Me'", and id., "Isaac Newton y el Problema de la Acción a Distancia"). See, furthermore, Schliesser, "Newton's Substance Monism, Distant Action, and the Nature of Newton's Empiricism", pp. 164–165.

208 Newton to Bentley, 17 January 1672/3, Newton, *Correspondence*, III, p. 240.

209 Newton, *The Opticks*, p. 399.

210 Newton, *The Principia*, p. 940.

211 Sharrock, *De finibus virtutis Christianæ*, Sermon I, pp. 4–10 contained a relentless criticism of the Epicurean atheist, according to whom the order of the World is produced by mere chance. Newton owned the 1673 edition of this work (Harrison, *The Library of Isaac Newton*, p. 238 [item n° 1505]).

212 CUL Add. Ms. 3965, f. 152v.

> *Cartesians make God the author of all motion & its as reasonable to make him the author of the laws of motion. Matter is a passive principle & cannot move it self. It continues in its state of moving or resting unless disturbed. It receives motion proportional to the force impressing it. And resists as much as it is resisted. These are passive laws & to affirm that there are no other is to speak against experience*.[213]

In line with his metaphysical and theological concerns, he saw his theory as providing room for the non-mechanical forces in nature. John Henry adequately notes that the ethers Newton introduced to account to explain gravitation were not mechanical since they "consisted of particles held apart from one another, and from particles to other matter, by repulsive forces operating between them"[214] and that "the aether theories were not intended to be a way of avoiding actions at a distance."[215] These points are well taken.[216] Moreover, in the context of his optical research Newton openly allowed the possibility of action at a distance.[217]

The subtle answer to the conundrum of action at a distance is then the following: Newton denied that matter could act at a distance according to its own nature (because this would imply that matter is innately self-acting, an option unacceptable for Newton); however, Newton endorsed action at a distance when speculating about the remote cause of planetary motion, since he *postulated* a very subtle "elastick" ether, i.e. an extremely rare medium endowed with strong non-mechanical inter-particular repulsive forces," as a possible explanation or cause of

213 CUL Add. Ms. 3970, f. 619^{r} [ca. 1700–1704; italics added]. A relevant variant is: "What is it *by means of w^{ch}* bodies act on one another at a distance. And *To what Agent* did the Ancients attribute the gravity of their atoms. Or what did they mean [. . .] by calling God an harmony & *comparing* him & matter [. . .] to y^{e} God Pan & his Pipe." (CUL Add. Ms. 3970, f. 291^{r} [ca. 1700–1704; italics added]).

214 Henry, "'Pray do not Ascribe that Notion to Me'", p. 123.

215 Ibid., p. 135. This observation is correct, for otherwise we would have to accept the conclusion that Newton tried to explain away action at a distance at the macro-level by reintroducing it at the micro-level.

216 In Janiak, "Newton and the Reality of Force", where it is claimed that Newton rejected action at a distance (see also Janiak, *Newton as Philosopher*, pp. 54, 172), Janiak did not refer to Henry's work. He does so in *Newton as Philosopher*, p. 53, footnote 53. There Henry's views are quickly dismissed on the basis of an excerpt wherein Henry (incorrectly, indeed) wrote that gravity is "a superadded *inherent* property" (Henry, "'Pray do not Ascribe that Notion to Me'", p. 141; note however, that in Henry, "Isaac Newton y el Problema de la Acción a Distancia", this mistake is corrected). Henry's slip should not detract us from the important points he made: that the ether theories did not originate from Newton's dissatisfaction with action at a distance per se, and that Newton accepted action at a distance in his optical work and in his work on the cause of gravity. In his subsequent discussion (Janiak, *Newton as Philosopher*, pp. 53–65), Janiak does not consider these two points. Moreover, by contending that "[t]here can be no doubt [. . .] that Newton himself connected his denial of action at a distance with his conception of God's spatiotemporal ubiquity and corresponding potential role as a medium for all gravitational interactions" (ibid., p. 40, cf. p. 39) and that God acts directly on all bodies (ibid., p. 39), he neglects Newton's theologically motivated avoidance of considering God as the direct cause of gravitation.

217 Newton, *The Opticks*, pp. 339, 370–371; CUL Add. Ms. 3970, ff. 252^{r}-254^{r}, f. 257^{r}, f. 273^{r}, f. 291^{r} [ca. 1700–1704]. Neither Janiak nor Kochiras refer to this material.

gravity.[218] This seems to be supported by the Advertisement to the 1717 edition of *The Opticks*: "*And to shew that I do not take Gravity for an essential Property of Bodies* [i.e., to show that "inanimate brute Matter" does not have the capacity of attracting other matter across a vacuum without the mediation of a *tertium quid*]*, I have added one Question concerning its Cause* [i.e., a hypothesis on the cause of gravity, which posits an elastic ether which supposes inter-particular repulsive forces between its constituting particles acting at a distance]*, chusing to propose it by way of a Question*."[219] Newton was, of course, clearly aware that the elastic ether was conjectural and not demonstrated, for in the Advertisement added to the 1717 edition of *The Opticks* he observed that he was "not yet satisfied about it [i.e. his attribution of the cause of gravity] for want of Experiments."[220] The point is that he was willing to entertain its possibility and even make it public. Therefore, Newton did not reject *actio in distans* per se.[221] Newton did indeed reject action at a distance at a macro-level, for that would have entailed his approval of Epicurean attraction. This, however, does not mean that he found the notion of *actio in distans* intrinsically problematic, for in *The Opticks* he postulated a non-mechanical intermediary acting at a distance to account for the explanation or cause of gravity. The reason why Newton introduced a non-mechanical cause of gravity acting at a distance was that he – despite the possible threat of an infinite regress – considered it as a viable means to avoid the self-propelling activity of matter *and* to account for the non-mechanical nature of gravitational effects as suggested by empirical evidence. This suggests a rather different interaction between Newton's metaphysical and empirical considerations than the ones suggested by Janiak and Kochiras.

1.8 Conclusion

The impact of the "Aristotelian" textbook tradition is, as I have emphasized in Section 1.1, limited to some basic features of Newton's views on natural philosophy. This tradition is representative for the seventeenth-century intellectual climate to which Newton was exposed and, as I have been able to show from my study of Newton's personal copy of Samuel Smith's *Aditus ad logicam*, some of these works were crucial in Newton's formative years as a student. In this tradition natural-philosophical inquiry was conceived of as a dual process: its first part is *analytical* and consists in unravelling the causes of the effects we observe; the second part is *synthetic* and proceeds from the causes, which were established in the analysis, to their effects. Although Newton was reforming the very notion of cause,

[218] On this account, gravitational "attraction" results from the repellent forces of the elastic medium in which the celestial bodies are situated (cf. ibid., p. 376).

[219] Ibid., cxxiii.

[220] Ibid.

[221] This conclusion was reached earlier in Heimann and McGuire, "Newtonian Forces and Lockean Powers," pp. 242–243 and in McMullin, *Newton on Matter and Activity*, p. 144, footnote 13 and p. 151, footnote 210.

he took over some ideas and terminology of the "Aristotelian" account of natural-philosophical analysis and synthesis. When we juxtapose Newton's work to these textbooks, Newton's causal outlook comes as no surprise.

My analysis also helps to explain why Newton did not endorse a probabilistic view of natural-philosophical knowledge. Newton's stress on causes and certain knowledge was at odds with the oftentimes probabilistic climate of the Royal Society.[222] Contrary to the spirit of Bacon, many natural philosophers of the Royal Society endorsed a probabilistic view on human knowledge, which included hypotheses.[223] In their preparedness to accept probability they "moved away not only from thinkers like Hobbes, but even from Bacon (and for that matter, from Aristotle)," as Michael Hunter has observed.[224] In their view, humans could only arrive at morally certain knowledge: they could not know nature "in their true immediate, necessary causes."[225] Newton notoriously favoured certain knowledge and disliked hypotheses.[226] Certain knowledge involves proper knowledge of the causes of things. Already in Newton's first scientific publication, i.e. in his first optical paper of 1671/2, it became apparent that he was deeply concerned with demonstrative certainty.[227] It is striking in this respect that in "A Scheme for establishing the R. Society" Newton wrote: "Natural Philosophy consists in discovering the frame & operations of Nature, reducing them (as far as may be) to general Rules or Laws, establishing those Rules by observations & experiments, & thence deducing the causes & effects of things."[228]

This case-study also serves as an incentive to stimulate further research of this textbook tradition.[229] As these books are only the tip of the iceberg, many questions remain. What lies beneath, i.e. the relations between these and other textbooks,

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[222] See Newton, *The Principia*, pp. 588–589. For discussion, see: Feingold, "Mathematicians and Naturalists: Sir Isaac Newton and the Royal Society". We will return to the Royal Society *milieu* in Chapter 2.

[223] See, e.g., Van Leeuwen, *The Problem of Certainty in English Thought 1630–1690*. For a general discussion on the emergence of probability in the seventeenth century, see Hacking, *The Emergence of Probability* and Popkin, *The History of Scepticism from Erasmus to Spinoza*. See also: Ducheyne, "The Status of Hypothesis and Theory". Notable exceptions to this probabilistic epistemology were Christopher Wren, John Wallis and Isaac Barrow (see Guicciardini, *Isaac Newton on Mathematical Certainty and Method*, pp. 27–28).

[224] See Hunter, *Science and Society in Restoration England*, p. 180. Hunter correctly points at the "significant methodological differences" between the members. See Michael, *Establishing the New Science, the Experience of the Early Royal Society*, pp. 207–208. The corresponding chapter (pp. 185–244) in Hunter's books is a reprint of the original paper: Wood and Hunter, "Towards Solomon's House: Rival Strategies for Reforming the Early Royal Society".

[225] Joseph Glanvill as quoted in Van Leeuwen, *The Problem of Certainty in English Thought*, p. 76.

[226] See, e.g., Shapiro, *Probability and Certainty in Seventeenth-Century England*, p. 58.

[227] See Chapter 4 for further discussion.

[228] CUL Add. Ms. 4005, f. 6^{v} [ca. 1703].

[229] Schmitt suggested that it could be argued that there was a significant Aristotelian component to Newton's thought. See Schmitt, *Aristotle and the Renaissance*, p. 28 and Schmitt, "The Rise of the Philosophical Textbook", p. 7. He did not, however, elaborate on this.

remains unclear. Who were the authors of these textbooks? Were they in tune with the latest developments in natural philosophy? How exactly was this textbook-tradition introduced and assimilated in seventeenth-century England[230] and other European countries? Were these works important to other natural philosophers? What was the exact *rôle* of these textbooks in the formation of natural philosophers?

Nowhere in this chapter have I claimed that Newton was an Aristotelian concerning his ideas on natural-philosophical methodology nor that these textbooks are the only relevant sources for his conception of natural philosophy. Newton clearly succeeded in establishing a new sort of physico-mathematics which was entirely novel and which obviously cannot be explained by these "Aristotelian" textbooks. Newton's terminology and conception of natural-philosophical analysis and synthesis was, nevertheless, to a significant extent derived from the textbook tradition on logic and method. While casting his methodological terminology in conservative terms, Newton at the same time proposed a radically new kind of causal entity: a non-mechanical entity inferred by means of abstract mathematics, which he understood in terms of counterfactual dependence. In the following two chapters, we shall deal with Newton's complex physico-mathematics in the *Principia*.

Coda: Did Newton Actually Mean "Explanations"?

In this coda, I address a specific issue: was Newton in favour of using "explanare" (meaning: "to give the reason for or the cause of")?[231] Or did he, being the careful word smith that he was, insist on using the less strong "exponere"[232] (meaning: "to give an account of") or "explicare" (meaning: "to give a detailed analysis of"), as a means to convey that he was avoiding (causal) explanations? This is a relevant question, which I shall try to answer on the basis of several contemporary dictionaries[233] (external proof) and the Newtonian corpus itself (internal proof).

First of all, it should be noted that the verbs "explanare," "explicare," and "exponere" were semantically not that fundamentally different at the time.[234] In Wase's *Dictionarium Minus*, for instance, "to explain," "an explaining," "to explicate," "an explication," and "to expound" were translated as "*Explico,*

[230] For a rough sketch on the preceding period see Schmitt, *Aristotle and the Renaissance*, esp. Chapter 1.

[231] It was George E. Smith who brought this issue to my attention in Leiden in 2006.

[232] Newton frequently used "exponere" in a mathematical context (Koyré, Cohen and Whitman, eds., *Principia Mathematica*, I, pp. 200, 371, 369, 441, 433).

[233] Here I have consulted – in chronological order: Wase, *Dictionarium Minus, A Compendious DICTIONARY, English-Latin and Latin-English*; Coles, *A Dictionary English-Latin and Latin English*; Hawkins, ed., *Cocker's English Dictionary*; and, finally, Phillips, *The New World of Words: Or a Universal English Dictionary*.

[234] By contrast, in the *Oxford Latin Dictionary* "to explain" is not mentioned as a possible translation of "explicare" (Glare, ed., *Oxford Latin Dictionary*, I, p. 650). However, "to explain" is given as a possible translation of "exponere" (ibid., I, p. 652).

explano," "*Explicatio, explanatio*," "*Expono*," "*Expositio*," and "*Expono*," respectively.[235] "Explanare," "explanatio," "explicare," "explicatio," "exponere" and "expositio" were translated as "*To make smooth*, *to explain*," "*A declaring*, *or expounding*," "*To unfold*,"[236] "*An unfolding, an exposition*," "*to set out*," and "*An exposition, or narration*," respectively.[237] In Coles' *A Dictionary English-Latin and Latin English*, which Newton owned,[238] "explanare" is translated as "*to explain, make smooth*," "explanatio" as "*an explaining*," "explicatio" as "*an explaining*, *unfolding*," "explicare" as "*to unfold*, *display*, *draw up*, *accomplish*, *disintangle*, *make plain and smooth*," and "exponere" as "*to set forth, expose, lay bare, expound and declare*."[239] "To explain" and "to explicate" were synonymously translated as "*explico, expono*," while "an explaining," "an explication," and "to expound" are translated as "*explicatio*," "*expositio*" and "*expono, enarro, explico, interpretor*," respectively.[240] In *Cocker's English Dictionary*, to "to explain" the meaning "1. to declare, demonstrate, unfold," is given, to "explanation" "1. making manifest, plain and clear," to "explication" "1. explaining, unfolding, opening," and, to "exposition" "1. interpreting, explaining, expounding."[241] In Phillips' *The New World of Words*, "explanation" the meaning "a making plain manifest," is given, to "explication" "an unfolding or explaining of any thing obscure or ambiguous," and to "an exposition" "an expounding or interpreting."[242] Moreover, as we shall see in Chapter 2, in the context of seventeenth-century British natural philosophy "an explication" was synonymously used to "an explanation."

Secondly, it should be observed that Newton himself suggested that natural philosophy provides (causal) explanations of phenomena (e.g., in Query 31 of *The Opticks*, he wrote: "And the method of Synthesis consists in assuming the Causes discover'd and establish'd as Principles, and by them explaining the phaenomena proceeding from them, and proving the Explanations."[243]), and that, when he used "explicare," he did not seem to be averse to causal talk. For instance, on CUL Add. Ms. 9597.2.11, he wrote:

235 Wase, *Dictionarium Minus,* English-Latin, under *E* [page-numbers lacking].

236 This is accompanied with the following example: "Causam alicujus rei explicare, *To explain the cause of something*." (ibid.).

237 Ibid., Latin-English, under *E* [page-numbers lacking].

238 Newton owned a copy of the fourth edition of Coles' *A Dictionary English-Latin and Latin English* (1699) (Harrison, *The Library of Isaac Newton*, p. 121 [item n° 409 (= Wren Library, NQ.9.47)]).

239 Coles, *A Dictionary English-Latin and Latin English*, Latin-English, under *E* [page-numbers lacking].

240 Ibid., English-Latin, under *E* [page-numbers lacking].

241 Hawkins, ed., *Cocker's English Dictionary*, under *E* [page-numbers are lacking].

242 Phillips, ed., *The New World of Words*, under *E* [page-numbers are lacking].

243 Newton, *The Opticks*, pp. 404–405.

> ~~Ideoque a Phaenomenis in omni Philosophia incipiendum est.~~ In omni Philosophia incipere debemus a Phaenomenis, & nulla admittere ↓rerum↓ principia nullas causas nullas explicationes nisi quæ per phaenomena stabiliuntur.[244]

> Alias nullum om[n]ino phaenomenon <per causam suam> recte explicari posset nisi causa <hujus> causae, & causa ~~priori~~ causae prioris redderetur & sic deinceps usque donec ad causam primam deventum sit.[245]

On CUL Add. Ms. 3965, he wrote:

> Reg. IV Ideoque Effectuum naturalium ejusdem generis eædem ~~assumendæ~~ ↓assignandæ↓ sunt causæ ↓[~~proximæ nisi [forte diversitas aliqua~~↓, ~~nisi quatenus diversitas ex phænomenis patefacta sit hæ causæ phænomenis explicandis sufficiunt.~~] ~~nisi diversitas~~ ↓~~aliqua~~↓ ex ph~~ænomenis patefacta sit.]~~ quatenus fieri potest.[246]

> Phaenomena voco ejusdem generis ↓quatenus↓ per easdem causas explicari possunt.[247]

Taking the above considerations into account, it is indeed highly reasonable to conclude that, when Newton used "exponere," he did not intend to convey that the *Principia* did not contain (causal) explanations.

244 CUL Add. Ms. 9597.2.11, f. 2^{r} [1713–1715].

245 CUL Add. Ms. 9597.2.11, f. 3^{r} [ca. 1716–1718].

246 CUL Add. Ms. 3965, f. 419^{r} [additions and corrections to the first edition of the *Principia*].

247 CUL Add. Ms. 3965, f. 423^{v} [additions and corrections to the second edition of the *Principia*].

Appendix: Transcription of CUL Add. Ms. 3968, f. $109^{r\text{-}v}$ [Early 1710s][248]

[f. 109^r][249] Geometria Veteres quaesita investigabant per Analysin, inventa demonstrabant per Synthesin, demonstrata edebant ↓ut↓ in Geometriam reciperentur. Resoluta non statim recipiebantur in Geometriam: opus erat solutione per compositionem demonstrationum. Nam Geometriae vis et laus omnis in certitudine rerum, certitudo in demonstrationibus luculenter compositis constabat. In hac scientia non tam breviati ~~quam~~ scribendi quam certitudini rerum consulendum est. Ideoque [~~illegible word~~] in sequenti Tractatu Propositiones per Analysis inventas demonstravi synthetice.

Geometria Veterum versabatur quidem circa magnitudines; sed Propositiones de magnitudinibus non[n]unquam demonstrabantur ~~per~~ ↓mediante↓ motu locali: ut cum triangulorum aequalitas in Propositione quarta libri primi Elementorum Euclidis demonstraretur transferendo tr[i]angulum alterutrum in locum alterius. Sed et genesis magnitidinum per motum continuum recepta fuit in Geometria: ut cum linea recta duceretur in lineam rectam ad generandam aream, & area recta duceretur in lineam rectam ad generandum solidum. Si recta quae in aliam ducitur datae sit longitudinis generabitur area parallelogramma. Si longitudo ejus lege aliqua certa continuo mutetur generabitur area curvilinea. ↓Si magnitudo areae in rectam ductae continuo mutetur generabitur solidum superficie curva terminatum.↓ Si tempora, vires, motus et velocitates motuum exponantur ↓~~per~~↓ ~~longitudines~~ lineas ~~vel~~ ↓~~per~~↓ ~~magnitudines~~ ↓~~angulorum~~↓ areas solida ↓vel angulos↓, tractari etiam possunt hae quantitates in Geometria.[250]

Quantitates continuo fluxu crescentes vocamus fluentes & velocitates crescendo vocamus fluxiones, & incrementa momentanea vocamus momenta, et methodum qua tractamus ejusmodi quantitates vocamus methodum fluxionum et momentorum: estque haec methodus vel synthetica vel analytica.[251]

Methodus Synthetica fluxionum et momentorum in Tractatu sequente passim occurrit, et ejus elementa posui in Lemmatibus undecim primis Libri primi & Lemmate secundo Libri secundi.

Methodus analyticae ↓specimina↓ occurrunt in Prop XLV & Schol Prop XCII Lib. I & Prop X & XIV Lib. II. ↓Et praeterea describitur in Scholio ad Lem. II

[248] A draft of this manuscript can be found at CUL Add. Ms. 9597.2.11, ff. 1^r-3^v.

[249] D. T. Whiteside has provided a partial transcription of this manuscript – unfortunately omitting the last 2 paragraphs (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VIII, pp. 452–459). I have chosen to reproduce the entire manuscript, since it has nowhere been reproduced in its entirety and since Whiteside's transcription includes some minor inaccuracies. A complete translation of this manuscript is provided by I. Bernard Cohen (Newton, *The Principia*, pp. 49–54).

[250] Cf. the Leibniz Scholium already included in the first edition (Newton, *The Principia*, pp. 649–650).

[251] Here we clearly notice the impact of the priority debate with Leibniz. In 1713, Roger Cotes wrote to Richard Bentley to ask him to persuade Newton to annul submitting these potentially polemic parts making such overt reference to the priority debate (Westfall, *Never at Rest*, p. 749).

Lib.↓ II. Sed et ex demonstrationibus compositis Analysis qua Propositiones inventae fuerunt,[252] addisci potest regrediendo. [Et praeterea ↓describitur in Scholio ad Lem. II Lib: II.↓ [Tractatum de hac Analysi ex chartis antea editis desumptam, Libro Principiorum subjunxi.]

Scopus Libri Principiorum non fuit ut methodos mathematicas edocerem, non ut difficilia omnia ad magnitudinis ~~figuras~~ motus & vires spectantia ~~tractarem~~ eruerem; sed ut ea tantum tractarem quae ad Philosophiam naturalem et apprime ad motus coelorum spectarent ideoque quae ad hunc finem parum conducerent, vel penibus omisi, vel leviter tantum attigi, omissis demonstrationibus.

In Libris duobus primis vires generaliter tractavi, easque si in centrum aliquod seu immotum seu mobile tendunt, centripetas ~~vocavi~~ (nomine generali) vocavi, non inquirendo in causas vel species virium, sed earum quantitates determinationes & effectus tantum considerando. In Libro tertio quam primum didici ~~Lunam in~~ vires – quibus Planeta in orbibus suis retinentur, recedendo a Planetis in quorum centra vires illae tendunt, decrescere in duplicata ratione [~~illegible letters~~] distantiarum a centris, & vim qua Luna retinetur in Orbe suo circum Terram, descendendo ad superficiem Terrae aequalem evadere vi gravitatis nostrae, ~~caepi gravitatem tractare ut vim quae corpora coelestia~~ adeoque vel gravitatem esse vim [f. 109^{v}] vim gravitatis duplicare: caepi gravitatem tractare ut vim qua corpora coelestia in orbibus suis retineantur. Et in eo versatur Liber iste ↓tertius↓ ~~tertius~~, ut Gravitatis propietates, vires, directiones & effectus edoceat.[253]

Planetas in orbibus fere concentricis & Cometas in orbibus valde excentricis circum Solem revolvi, Chaldaei olim crediderunt, Et hanc Philosophiam Phythgorei in Graeciam [introduxerunt] invexerunt.[254] Sed et Lunam gravem esse in Terram, ↓& stellas graves esse in se mutuo↓, et corpora omnia in vacuo aequali cum velocitate in Terram ~~descend~~ cadere, adeoque gravia esse pro quantitate materiae in singulis notum fuit Veteribus. Defectu demonstrationibus haec philosophia intermissa fuit eandemque non inveni sed vi demonstrationum in lucem tantum revocare conatus sunt. Sed et Praecessionem Æquinoxiorum, & fluxum & refluxum maris et motus inaequalis Luna ~~illegible word~~ et orbes Cometarum & perturbationem orbis Saturni per gravitatem ejus in Jovem ab ijsdem Principijs consequi, et quae ab his Principijs consequuntur cum Phaenomenis probe congruere, his ostensum est. Causam gravitatis ex phaenomenis nondum didici.[255]

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[252] Newton himself promoted the myth that he had used his analytical method of fluxions to arrive at his discoveries in the *Principia* in order to ensure his claim of priority over Leibniz. A. Rupert Hall's accurate assessment goes as follows: "the tool he was developing from the autumn of 1684 onwards and brought to fruition in the final text of the *Principia* was an idiosyncratic geometry in which infinitesimal increments of lines and areas perform the functions of first and second order differentials, a geometry intimately integrated with his dynamical principles." (Hall, *Isaac Newton, Adventurer in Thought*, p. 213).

[253] Newton, *The Principia*, pp. 382, 793.

[254] Cf. Newton, *The Opticks*, p. 369.

[255] On CUL Add. Ms. 3965, f. 358^{v}, Newton wrote: "Causam gravitatis ↓ex phenomenis↓ ~~investigavi~~ nondum ↓*investigavi*↓ ~~posui~~." [italics added].

Qui leges et effectus Virium electricarum pari successu et certitudine eruerit, philosophiam multum promovebit, etsi ↓forte↓ causam harum Virium ignoraverit. ~~Nam~~ Phaenomena ↓observanda↓ primo ↓~~spectanda~~↓ ~~consideranda~~ ↓sunt↓, dein horum causae proximae, & postea causae causarum ~~eruenda~~ eruenda; ac tandem a causis ↓supremis causarum↓ per phaenomena stabilitis, ad ↓~~causas~~↓ ~~caus phaenomena~~ ↓eorum effectus↓, ↓~~eorum causas proximas~~↓ argumentando a priori, descendere licebit. ~~Et inter Phaenomena numerandae sunt actiones mentis quae nobis innotescunt quarum conseij sumus~~ Philosophia naturalis non in opinionibus Metaphysicis, sed in Principiis propijs fundanda est; & haec [end of text][256]

[256] The draft material of this text can be found in CUL Add. Ms. 9597.2, ff. 1^r-3^r.

Part II
Newton's Methodology: "The Best Way of Arguing in Natural Philosophy"

Chapter 2
Uncovering the Methodology of the *Principia* (I): The Phase of Model Construction

2.1 Introduction

In the editorial preface to the second edition of the *Principia* (1713), Roger Cotes observed that:

> There are some who do not like all this [Newtonian] celestial physics just because it seems to be in conflict with the doctrines of Descartes and seems scarcely capable of being reconciled with these doctrines. They are free to enjoy their own opinion, but they ought to act fairly and not to deny to others the same liberty that they demand for themselves. Therefore, we should be allowed to adhere to the Newtonian philosophy, which we consider truer, and to prefer causes proved by phenomena to causes imagined and not yet proved [NEWTONIANAM itaque philosophiam, quæ nobis verior habetur, retinere & amplecti licebit, & causas sequi per phænomena comprobatas, potius quam fictas & nondum comprobatas]. It is the province of true philosophy to derive the natures of things from causes that truly exist [Ad veram philosophiam pertinet, rerum naturas ex causis vere existentibus derivare], and to seek those laws by which the supreme artificer willed to establish this most beautiful order of the world, not those laws by which he could have, had it so pleased him. [...] *For even if these philosophers could account for the phenomena with the greatest exactness on the basis of their hypotheses, still they cannot be said to have given us a true philosophy and to have found the true causes of the celestial motions until they have demonstrated either that these causes really do exist or at least that others do not exist* [Nam si phænomenis vel accuratissime satisfacere possent ex hypothesibus suis; veram tamen philosophiam tradidisse, & versa causas motuum cælestium invenisse nondum dicendi sunt; nisi vel has revera existere, vel saltem alias non existere demonstraverint].[1]

In the above quote, Cotes was conveying two related points. The first is that the empirical confirmation of conclusions derived from a theoretical principle does not by itself guarantee the truth of that principle; the second is that Newton's *Principia* testifies of a "truer philosophy," in which causes are established that "truly exist."[2] It seemed, therefore, that the theory outlined in the *Principia* had not only passed the test of empirical verification, but that it had also succeeded in unravelling causes

[1] Newton, *The Principia*, p. 393 [italics added]; Koyré, Cohen and Whitman, eds., *Principia Mathematica*, I, p. 28.

[2] Newton, *The Principia*, p. 386.

S. Ducheyne, *The Main Business of Natural Philosophy*, Archimedes 29,
DOI 10.1007/978-94-007-2126-5_2, © Springer Science+Business Media B.V. 2012

that truly exist. A natural question that arises then is the following: how had Newton established such "truer philosophy"?

Clearly, many of Newton's contemporaries were baffled by the technicality of the *Principia* (first edition: 1687; second edition: 1713; third edition: 1726).[3] Even today Newton scholars continue to discuss the specifics of the mathematical methods and arguments contained in it. Notwithstanding Newton's occasional clues on natural-philosophical methodology, the reader of the *Principia* no doubt initially faces the difficulty of ascertaining an overarching methodology in Newton's complex web of interconnected propositions, lemmas, corollaries, problems, *scholia*, phenomena, *regulae philosophandi*, and hypotheses.[4] Although Newton undoubtedly used a plethora of different inferential techniques and procedures in the *Principia*, upon closer scrutiny, it is possible to reconstruct an overarching methodological endeavour. Uncovering this overarching endeavour will occupy us in this and the following chapter.[5] In what follows, I deal with the *Principia* on a macro-level without doing harm to its micro-level, i.e. the level of close scrutiny of particular propositions.[6] The present and the following chapter correspond to two important and consecutive phases in Newton's methodology: (1) a phase of model construction and (2) a phase of model application *cum* theory formation *cum* theory

[3] Useful background on the *Principia* is to be found in Cohen, *Introduction to Newton's "Principia"*.

[4] On the *regulae* and the hypotheses in Book III, see Chapter 3, Section 3.2.

[5] The account I spell out in these two chapters is indebted to the writings of I. Bernard Cohen, William L. Harper, and George E. Smith.

[6] Much understanding on the details of Newton's propositions has been gained over recent years, as the following papers testify: Brackenridge, "Newton's Easy Quadratures 'Omitted for the Sake of Brevity'"; Dobson, "On Lemmas 1 and 2 to Proposition 39 of Book 3 of Newton's *Principia*"; Erlichson, "Newton's Solution of the Equiangular Spiral Problem and a New Solution using only the Equiangular Property"; id., "The Visualization of Quadratures in the Mystery of Corollary 3 to Proposition 41 of Newton's *Principia*"; id., "Passage to the Limit in Proposition I, Book I of Newton's *Principia*"; Guicciardini, "An Episode in the History of Dynamics: Jakob Hermann's Proof (1716–1717) of Proposition 1, Book 1, of Newton's *Principia*"; Pourciau, "On Newton's Proof that Inverse-square Orbits must be Conics"; id., "Newton's Solution of the One-body Problem"; id., "Radical *Principia*"; id., "Newton's Interpretation of Newton's Second Law"; id., "The Integrability of Ovals: Newton's Lemma 28 and its Counterexamples"; id., "Newton's Argument for Proposition 1 of the *Principia*"; id., "The Importance of Being Equivalent: Newton's Two Models of One-body Motion"; id., "From Centripetal Forces to Conic Orbits: A Path through the Early Sections of Newton's *Principia*"; id., "Force, Deflection, and Time: Proposition VI of Newton's *Principia*"; id., "Proposition II (Book I) of Newton's *Principia*"; Nauenberg, "Newton's Early Computational Method for Dynamics"; id., "Kepler's Area Law in the *Principia*: Filling in some Details in Newton's Proof of Proposition 1"; Pesic, "The Validity of Newton's Lemma 28"; Smeenk and Smith, "Newton on Constrained Motion: A Commentary on Book I Section 10 of the *Principia*"; Weinstock, "Newton's *Principia* and the External Gravitational Field of Spherically Symmetric Mass Distribution"; id., "Newton's *Principia* and Inverse-square Orbits: The Flaw Reexamined"; id., "Inverse-square Orbits in Newton's *Principia* and Twentieth-century Commentary Thereon"; Wilson, "Newton on the Equiangular Spiral: An Addendum to Erlichson's Account". These papers provide deep insights in Newton's *Principia* – no historian of science should refrain from them.

testing, respectively. It will be shown that both phases proceeded in a way more sophisticated than hypothetico-deductivism. To state matters clearly from the outset: the aim of this and the following chapter is, *not* to establish that Newton's method was intrinsically non-hypothetical, but rather to highlight that his methodology involved procedures designed to minimize inductive risk in a way that is more demanding than hypothetico-deductivism. On a hypothetico-deductive rendering, a theoretical statement is confirmed when the consequences drawn from it are verified by observation – and that is basically it. Newton refused to endorse this mode of inference in natural philosophy.

In Section 2.3, I will highlight a possible source of confusion inherent in I. Bernard Cohen's influential account of Newton's methodology in the *Principia*: the so-called "Newtonian Style".[7] The crux of the strong version of Cohen's account is that *the phase of model construction* consists in a process of a piecemeal adaptations of "mental constructs" (Cohen's terminology) through a successive series of comparisons with nature. Thus, in Cohen's *The Newtonian Revolution* it is suggested that, in the phase of model construction, there is a direct relationship between the "mental constructs" and their corresponding physical systems. I argue in what follows that, if Newton's method indeed involved such extra-theoretical dynamics, Cohen's account fails to be different from a hypothetico-deductive approach – which has the unfavourable consequence that the "Newtonian Style" is at odds with Newton's fierce rejection of the hypothetico-deductive method. If we take Newton's rejection of the method of hypothesis at face value, an adequate account of his methodology should explicate how Newton – both in the phase of model construction as well as in the consecutive phases – proceeded differently from hypothetico-deductivism.

In the *pars construens* (Sections 2.4–2.6), I present my own model-based account of the phase of model construction in the *Principia* and argue that Newton conceived of Book I as an autonomous mathematical study of – *in principle* – arbitrary centripetal forces – thereby making abstraction from the physical forces active in the empirical world. The growing complexity of Newton's models is then the result of exploring increasingly complex cases (*intra-theoretical dynamics*) rather than the result of a series of successive comparison with nature (*extra-theoretical dynamics*). Although, for understandable reasons, I shall not comment on all 98 propositions and 29 lemmas[8] of Book I *a capite ad calcem*, I shall, given our present goal, provide an overview of some crucial types of propositions in Book I. Before we address these matters, I begin by briefly documenting Newton's fierce rejection of hypotheses.[9]

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[7] Cohen, *The Newtonian Revolution*.

[8] At least, in the final edition of the *Principia*.

[9] I refer the reader to Chapter 4, especially Section 4.6, and Chapter 6, Section 6.3, for further discussion of Newton's discarding of the method of hypothesis in his early optical work (1670s) and for further contextualisation of the elements leading up to Newton's vehement rejection of hypotheses in the second edition of the *Principia* (1713), respectively.

2.2 Newton's Rejection of the Method of Hypothesis

Newton's rejection of the method of hypothesis *à la* René Descartes,[10] Christiaan Huygens, and Gottfried W. Leibniz[11] is a widely known feature of his natural philosophy. As I have explained in Section 2.1, the defining characteristic of the hypothetico-deductive model of confirmation, is that one accepts a theoretical proposition if its empirically testable consequences are confirmed by experience. In the preface of *Traité de la Lumière* (1690), which was jointly published with the *Discours de la cause de la pesanteur*, Huygens, for instance, embraced aspects of the hypothetico-deductive method:

> On y verra de ces sortes de demonstrations, *qui ne produisent pas une certitude aussi grande que celles de Geometrie*, & qui mesme en different beaucoup, puisque au lieu que les Geometre prouvent leurs Propositions par des Principes certains & incontestables, *icy les Principes se verifient par les conclusions qu'on en tire*; la nature de ces choses ne souffrant pas que cela se fasse autrement. Il est possible toutefois d'y arriver à un degré de vraisemblance, qui bien souvent ne cede guere à une evidence entiere. *Sçavoir lors que les choses, qu'on a demontrées par ces Principes supposez, se raportent parfaitement aux phenomenes que l'experience a fait remarquer; sur tout quand il y en a grand nombre, & encore principalement quand on se forme & prevoit des phenomenes nouveaux, qui doivent suivre des hypotheses qu'on employe, & qu'on trouve qu'en cela l'effet repond à nostre attente.*[12]

In the General Scholium, added to the second edition of the *Principia*, Newton famously declared that he did not feign hypotheses ("hypotheses non fingo").[13] "For," he continued, "whatever is not deduced from the phenomena must be called a hypothesis; and hypotheses, whether metaphysical of physical, or based on occult qualities, or mechanical, have no place in experimental philosophy."[14] According to Newton, a hypothesis is a proposition that is not a phenomenon, nor deduced from any phenomena but assumed or supposed without any experimental proof.[15] Instead, Newton founded his natural philosophy on phenomena, i.e. those things which appear to our external or internal senses.[16] In *The Opticks*, Newton stated

[10] Newton, *The Principia*, p. 939.

[11] For Leibniz' defence of a mechanical ether composed of *bullae* see his *Hypothesis Physica Nova* (1671) in: Leibniz, *Leibniz: Mathematische Schriften*, VI, pp. 17–59, his *Tentamen de motuum coelestium causæ* (1689), in: ibid., VI, pp. 144–187, and his *De causa gravitatis, et defensio sententiæ authoris de veris naturæ legibus contra Cartesianos* (1690), in ibid., VI, pp. 193–203.

[12] Huygens, *Oeuvres complètes de Christiaan Huygens*, XIX, p. 454 [italics added].

[13] Newton, *The Principia*, p. 943.

[14] Ibid.

[15] Newton to Cotes, 28 March 1712/13, Newton, *Correspondence*, V, p. 397. Cf. "Hypothesin voco opinionem quæ ↓~~ex~~↓ ~~Phænomenis~~ nec demonstra↓tur↓ ~~nec Phænomenon est~~ ↑neque↑ ↓ex Phænomenis↓ per Argumentum Inductionis deducitur." (CUL Add. Ms. 3965, f. 420^r; cf. ibid., f. 419^r [additions and corrections to the second edition of the *Principia*]). The expression "ex Phænomenis per Inductionem deduci" occurs on ibid., f. 437^v as well.

[16] Cf. "Phænomena voco ↓non solum↓ quæcunque apparet ~~vel~~ ↓sed etiam↓ (~~sensu laxiore~~) quæcunque sentiri possunt, sive sint res externæ quæ per sensus quinque innotescunt, sive internæ quas in mentibus nostris intuemur ↓cogitando↓. [cf. CUL Add. Ms. 3965, f. 419^r] Ut quod ignis calidus est, aqua humida est, aurum grave est, sol lucidus est, Ego sum et cogito [end of text]." (CUL Add.

that the main business of natural philosophy is "to argue from Phaenomena without feigning Hypotheses, and to deduce Causes from Effects."[17] Arguing from phenomena is, as Cotes declared in his preface to the second edition of the *Principia*, that "incomparably best way of philosophizing [philosophandi [. . .] longe optima]" of Newton.[18] Newton himself had indicated, in the scholium at the end of Section 11 of Book I, that he intended "to argue *more securely* concerning the physical species, physical causes, and physical proportions" of forces [de virium speciebus, causis & rationibus physicis tutius disputare licebit]."[19] Alan E. Shapiro has shown that Newton introduced and came to emphasize such terms as "deduction from phenomena", "the Method of Analysis and Synthesis", "Experimental Philosophy", and "the Method of Induction", when the *Principia* came under attack and when he was accused of introducing occult qualities into natural philosophy.[20] In response to Leibniz' criticism that he had introduced occult qualities in natural philosophy, Newton wrote:

> These Principles I consider not as occult Qualities ↓supposed to↓ ~~resulting from which~~ ↓~~are supposed to~~↓ result~~ing~~ from the specific forms of things but as general Laws of Nature ~~from whence the forms~~ ↓by which y^{e} things↓ themselves ~~result~~ ↓are formed: Their Truth appearing to us by phænomena, though their causes be n^{t} y^{t} explained.↓ To tell us that every species of things is endowed wth an occult Quality, by w^{ch} it acts is to tell us nothing; but *to derive two or three general Principles of motions from Phænomena, & afterwards to tell us how the properties and actions of all corporeal things follow from those ↓manifest↓ Principles, would be a very great step in Philosophy, tho the ↓~~occult~~↓ causes of those Principles were not yet discovered: & therefore I scruple not to propose the Principles of motion above mentioned, they being of very general extent*.[21]

Moreover, he stated:

> [Occult qualities are not manifest qualities but ↓are↓ specific qualities w^{ch} do not yet appear ~~to be in the Species~~ but are only supposed to be in the species for producing manifest effects whose causes are unk.][22]

From the above statements it is clear that Newton thought he had arrived at the theory of universal gravitation in a non-hypothetico-deductive way.

Ms. 3965, f. 421^{r} [additions and corrections to the second edition of the *Principia*]). In the following chapter, we will see that Newton also considered inductive generalizations (such as Kepler's rules) as "phenomena". Such generalisations are based on a large number of singular astronomical observations and their complex mathematical processing.

[17] Newton, *The Opticks*, p. 369.

[18] Newton, *The Principia*, p. 386.

[19] Ibid., p. 589 [italics added].

[20] Shapiro, "Newton's 'Experimental Philosophy'". It is typical of Newton that, when his scientific results were criticized, he stressed the certainty and non-hypothetical character of his scientific results.

[21] CUL Add. Ms. 3970, f. 285^{r} [ca. 1700–1704; italics added].

[22] Ibid., f. 621^{v} [ca. 1700–1704].

Although Newton's recalcitrant attitude towards Cartesian[23] and Leibnizian hypotheses is most eye-catching in the second edition of the *Principia*, it in fact dates back to his early optical work, which he presented and defended in the Royal Society *milieu* around the early 1670s. One tradition within Royal Society natural philosophy, exemplified by Robert Boyle and Robert Hooke, put emphasis on the hypothetical and probabilistic character of human knowledge.[24] In a letter answering one of the (many) objections against his *experimentum crucis*, Newton wrote: "For if the possibility of hypotheses is to be the test of the truth and reality of things [literally: si quis ex solâ *Hypothesium* possibilitate de veritate rerum conjecturam faciat], I see not how certainty can be obtained in any science; since numerous hypotheses may be devised [alias hypotheses semper liceat excogitari], which shall seem to overcome new difficulties."[25] A major drawback of hypothetical philosophy was that several arbitrary hypotheses can save the same phenomena.

In the generations after Bacon, natural philosophers came to ascribe a crucial role to hypotheses in physical inquiry, as the work of Robert Boyle (1627–1691) and Robert Hooke (1635–1703) particularly testifies.[26] Both Boyle and Hooke conceived of hypotheses, which they used synonymously to "theories," as causally sufficient and probable "explications" of natural phenomena that stand in an evidential relation to the matters of fact they serve to elucidate. A hypothesis, Boyle wrote, is "a *supposition* (whether true or fals) that men have pitchd upon, or devis'd, as a Principles, <by> whose help the phænomeno[n] wherto it is to be applyd may be *explicated*, that is <clearly deducd from causes> understood." Furthermore, a hypothesis "ought to be more clear & known than the *phænomena* it is to explain & if it be not intelligible when proposd, it cannot but be useless when applyd, <And> to <go about> to illustrate the obscure transactions of nature, by an obscure hypothesis, is as improper as to attempt to <shew> a man <his way> in the dark <with> an unlighted torch."[27] In similar vein, in his *Mechanical Origins of Qualities* (1675–1676), Boyle stated that the aim of a hypothesis is "to render an intelligible

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[23] The following sections from Newton's private copy of Descartes *Principia Philosophiae* (Wren Library, Trinity College, Cambridge, NQ.9.116 (= Descartes, *Principia Philosophiæ*) show considerable signs of dog-earing: *Pars Prima*: ¶¶ XXIV–XXVIII [God is "indefinite"], pp. 7–8 (= Descartes, *Œuvres de Descartes*, VIII, pp. 14–15), ¶¶ LXXV–LXXVI [body and space], p. 23 (= Descartes, *Œuvres de Descartes*, VIII, pp. 38–39), *Pars Secunda*: ¶¶ IX–X, p. 27 (= Descartes, *Œuvres de Descartes*, VIII, p. 45), *Pars Tertia*: ¶¶ XLVI–XLIX [vortices], p. 66 ff. (= Descartes, *Œuvres de Descartes*, VIII, pp. 100–104), and *Pars Quarta*: ¶¶ LV–LVII [tides], p. 161, (= Descartes, *Œuvres de Descartes*, VIII, pp. 237–239). Newton also owned a copy of Descartes' *Meditationes* (Harrison, *The Library of Isaac Newton*, p. 132 [item n° 508]), which shows traces of dog-earing on p. 74 and p. 209).

[24] Newton owned several of Boyle's works (Harrison, *The Library of Isaac Newton*, pp. 107–109) and several of Hooke's works (ibid., p. 162).

[25] Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, p. 106 (= Newton to Oldenburg for Pardies, 10 June 1672, Newton, *Correspondence*, I, pp. 163–168, p. 164).

[26] For further discussion, see: Ducheyne, "The Status of Hypothesis and Theory".

[27] Boyle, *Requisites of a Good Hypothesis* (late 1650s), in: Boyle, *The Works of Robert Boyle*, XIII, pp. 271–272.

account of the Causes and the Effects or Phænomena propos'd, without crossing the Laws of Nature or other Phænomena."[28] "[T]he more numerous," Boyle added, "and the more various the Particulars are, whereof some are *explicable* by the assign'd Hypothesis, and some are *agreeable* to it, or at least are not dissonant from it, the more valuable is the Hypothesis, and the more likely to be true."[29] Explications of natural phenomena cannot be established a priori and, rather than demonstrative certainty, they offer moral certainty:

> And though the Inferences, as such, may have a Demonstrable Certainty; yet the Premisses they are drawn from having but an Historical one, the/presumed Physico-Mathematical Demonstration can produce in a wary mind but a Moral Certainty, and not the greatest neither of that kind that is possible to be attain'd; as he will not scruple to acknowledge, that knows by experience, how much more difficult it is, then most men imagine, to make Observations about such nice Subjects, with the exactness that is requisite for the building of an undoubted Theory upon them.[30]

Moreover, there is no guarantee that "many things may be discover'd in After-times by Industry or Chance, which are not now so much as dream'd of, and which may yet overthrow Doctrines speciously enough accommodated to the Observations that have been hitherto made."[31] Boyle conceived of hypotheses as temporary "superstructures," which "though they may be preferr'd before any others, as being the least imperfect, or, if you please, the best in their kind that we yet have, yet are they not entirely to be acquiesced in, as absolutely perfect, or uncapable of improving Alternations."[32] Because of causal underdetermination, mechanical explanations offer only a sufficient and not a necessary and sufficient account of physical effects:

> And here let us further consider, That as confidently as many Atomists, and other Naturalists, presume to know the true and genuine Causes of the Things they attempt to explicate, yet very often the utmost that they can attain to in their Explications, is, That the explicated *Phænomena* May be produc'd after such a Manner as they deliver, but not that they really Are so: For as an Articifer can set all the Wheels of a Clock a going, as well with Springs as with Weights, and may with violence discharge a Bullet out of the Barrel of a Gun, not onely by means of Gunpower, but of compress'd Air, and even of a Spring. So the same Effects may be produc'd by divers Causes different from one another; and it will

[28] Ibid., XIII, p. 325.

[29] Ibid.

[30] Boyle, *Excellency of Theology* (1674), in: ibid., VIII, p. 66. Cf. Boyle, *Fragment* (late 1650s), in: ibid., XIII, p. 345: "Thô men be not arriv'd at such a pitch of Knowledge as to be able to discover and solemnely establish compleat & *Generall Hypotheses*; yet *subordinate* Axioms & *Hypotheses*, if they be of the more comprehensive ones, and warily settled, may be of vast use, both Philosophy and to Human life, nay if by a considerable number of Observations, or otherwise, we can arrive at *Axioms* that *for the most part* will hold, thô there be some unforeseen cases wherein they may faile us, and by which it may be discover'd that the reason is Erroneously assign'd, or at least Insufficient, and needs the helps of Limitations, and distinctions; yet even in this case, the Axiom or Observation being grounded upon a great number of Particulars, (and consequently applicable to Them and such others as are *eiusdem rationis*) may be of very great advantage."

[31] Boyle, *Excellency of Theology*, in: ibid., VIII, p. 89.

[32] Boyle, *Certain Physiological Essays* (1661), in: ibid., II, p. 14.

> oftentimes be very difficult, if not impossible for our dim Reasons to discern surely which of those several ways, whereby it is possible for Nature to produce the same *Phænomena* she really made use of to exhibit them.[33]

Hooke was equally embracive of the use of hypotheses in natural philosophy: hypotheses had clear pedagogical and heuristic value. The fact that hypotheses were used in the process of theory construction did not mean, for Hooke, that the final result would remain conjectural: by systematically exploring and testing hypotheses, including potentially false ones, true axioms are established eventually.[34] Hooke, however, remained unclear as to how the latter is to be accomplished. Boyle and Hooke also freely introduced micro-structural explanations of macro-scopic phenomena.[35] With his call for demonstrative knowledge, Newton distanced himself from this probabilistic tradition within the Royal Society *milieu*.

The real challenge now is to ascertain – leaving Newton's (often) polemical assertions aside – whether Newton, throughout the *Principia*, had indeed developed a methodology more stringent and demanding than standard hypothetico-deductivism and, if so, to explicate it. In this and the following two chapter, I shall correspondingly try to uncover what Newton meant with the words "arguing more securely."

2.3 The Strong Version of I. Bernard Cohen's "Newtonian Style" and Its Predicament

One of the most influential accounts of Newton's methodology is I. Bernard Cohen's "Newtonian Style". The crux of the "Newtonian Style" is that the models in Book I (and Book II) are piecemeal adapted through a series of successive comparisons with nature until a sufficient level of approximation is reached – I stress that Cohen situates the comparison between the "mental constructs" and the empirical world in Books I and II.[36] There is, however, a problem for Cohen's account – at least for its strong version, which I will characterize in what follows. As the "Newtonian Style" is widely known and as only one scholar has explicitly put the finger on a source of

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[33] Boyle, *Usefulness of Natural Philosophy, I* (1663), in: ibid., III, pp. 255–256. In similar vein, in the *Mechanical Origin of Qualities* (1675), Boyle wrote: "But since, in my Explications of Qualities, I pretend only, that they *may* be explicated by *Mechanical Principles*, without enquiring, whether they are explicable by any other; that which I need to prove is, / not that Mechanical Principles are the *necessary* and *onely* things whereby Qualities may be explain'd, but that probably they will be found *sufficient* for their explication." (ibid., VIII, p. 322).

[34] Oldroyd, "Some Writings of Robert Hooke", p. 162; Hooke, *A General Scheme, or Idea of the Present State of Natural Philosophy* (1666), in: Hooke, *The Posthumous Works of Robert Hooke*, pp. 3, 20, 39, 61; and, Hooke, *Micrographia*, [viii].

[35] See Chapter 4, Section 4.8, in which I shall discuss the problem of transduction in the context of Newton's optical work.

[36] Cohen, *The Newtonian Revolution*, p. 63.

confusion in, what I shall consider as the strong version of Cohen's account, I take it as a point worthy of developing.[37]

Before going into the details of Cohen's account, let me briefly mention the problem at stake. The strong version of Cohen's account does not preclude the introduction of hypothetical elements. More specifically, the "Newtonian Style" suggests that Newton in Book I ab initio assumed that centripetal forces are the true causes of planetary motion, since he decided to model the forces involved in celestial motion by means of a mental *analogue* (Cohen's terminology). If his is correct, Newton modelled certain forces, which he has already identified as the true causes of the celestial motions. Newton, however, only started discoursing about the centripetal forces in the empirical world in Book III. At the start of Book III, Newton stressed that he had previously discussed the "strictly mathematical" principles of philosophy of but that it remained "for us to exhibit the system of the world from these same principles [ut ex iisdem principiis doceamus constitutionem systematis mundani]."[38]

The shortcomings of the strong version of the "Newtonian Style" highlight what an adequate account of Newton's methodology with respect to the phase of model construction in the *Principia* should accommodate: Cohen did not sufficiently stress that the models in the *Principia* "live a life of their own" so to speak – in the sense that they are not as strongly data-driven as the strong version of Cohen's "Newtonian Style" suggests.[39] My claim is that, in the context of Book I of the *Principia*, Newton constructed increasingly complex physico-mathematical models on the basis of the simpler models, rather than that he piecemeal adapted the simpler models through a successive series of comparisons with nature to arrive at the more complex models. If my suggestion is correct, then Book I of the *Principia* testifies of a logic under which the demonstration of the more complex models requires the demonstration of the simpler models.

One thing should be emphasized from the outset: Cohen's "Newtonian Style" in the *Principia* is about Newton's methodology as a "mode of presentation" or a "manner of composing."[40] Therefore, my claims about Newton's methodology – like Cohen's – are restricted to the presentational sequence of Newton's theory (*the method of justification*) and do not pertain to the chronological sequence of the discovery of the theory of universal gravitation (*the method of discovery*). Accordingly, when I use "method(ology)" in what follows, I only refer to the former, for there

[37] See Bechler, "Introduction: Some Issues of Newtonian Historiography".

[38] Newton, *The Principia*, p. 793.

[39] The autonomy of models has recently gained much interest in the philosophical literature on models in science. See especially Morgan and Morrison, eds., *Models as Mediators, Perspectives on Natural and Social Science*. While the "models as mediators" programme is to be considered as a general epistemological claim on the relation between data and theory, here "autonomy" refers only to the method of justification. Moreover, the specific sense of autonomy I have in mind here is more specific than that of the "models as mediators" programme.

[40] Newton, *The Principia*, p. 60.

surely is no guarantee that the sequence presented in the *Principia* reflects Newton's original train of thought which led to the theory, as the published result often involves a re-structuring of the original discovery process.[41] The rigid deductive scheme Newton spelled out in the *Principia* very unlikely parallels the chronological sequence of Newton's actual discovery of universal gravitation. As Thomas Nickles has pointed out, we need to distinguish between the historical mode of generating an idea and the method of justification: "The initial introduction of the salient ideas may be as hypothetical as you please. [. . .] But the justificatory ideal remains to show that, given what we know at the end, the problem solution is logically derivable – and preferably derivable in a routine manner."[42]

In his seminal study *The Newtonian Revolution*, Cohen characterized Newton's method in the *Principia* as essentially reductive and mathematical: "As we shall see [. . .], Newton's success in analyzing the physics of motion depended to a large degree on his ability to reduce complex physical situations to a mathematical simplicity, in effect by the mathematical properties of an analogue of the reality that he eventually wished to understand."[43] Cohen dubbed Newton's programme of mathematizing physical reality the "Newtonian Style".[44] According to the "Newtonian Style", Newton proceeded along three successive phases:

1. Newton started with set of assumed physical entities and physico-mathematical conditions that are simpler than those in nature.[45] For instance: as a first approximation, the problem of planetary motion is reduced to a one-body

[41] I have no pretence whatsoever of uncovering how Newton *actually* made this discovery. See Smith, "How did Newton discover Universal Gravitation?" and Wilson, "From Kepler's Laws, So-Called, to Universal Gravitation: Empirical Factors" on this issue.

[42] Nickles, "Positive Science and Discoverability", pp. 19–20. Cf. id., "Reconstructing Science: Discovery and Experiment", p. 35. Nickles requirement of logical derivability is obviously too strong. Therefore I agree with Nickles' assessment, if and only if, logical derivability is replaced by inductively established in a methodized manner and empirically warranted.

[43] Cohen, *The Newtonian Revolution*, p. 55. In CUL Add. Ms. 4003, Newton noted that he had accommodated "these definitions not to physical things but to mathematical reasoning, after the manner of the geometers who do not accommodate their definitions of figures to the irregularities of physical bodies [non ad res physicas sed mathematica ratiocinia accommodavi, sicut Geometræ definitiones figurarum non accommodant ad irregularitates physicorum corporum]." He continued as follows: "And just as the dimensions of physical bodies are best determined by their geometry – as with the dimension of a field by plane geometry, although a field is not a true plane; and the dimensions of the earth by the doctrine of the sphere, even though the earth is not precisely spherical – so the properties of physical fluids and solids are best known from this mathematical doctrine [solidorumve physicorum proprietates optimè a doctrinâ hàcce Mathematicâ noscentur], even though they are not perhaps absolutely nor uniformly fluid or solid as I have defined them here." (Janiak, ed., *Newton, Philosophical Writings*, p. 39; CUL Add. Ms. 4003, f. 35^{r}).

[44] See furthermore Cohen, "Newton's Method and Newton's Style", pp. 30–44 and id., "The *Principia*, the Newtonian Style, and the Newtonian Revolution".

[45] Cohen, *The Newtonian Revolution*, p. 62.

system.[46] This mental construct is imaginatively conceived as "the parallel or analogue of the natural system."[47] Newton started with a set of simplified physical entities and conditions which can be translated in mathematical terms. To the degree that the physico-mathematical conditions of the system become mathematical rules or propositions, their consequences may be deduced by the application of mathematical techniques.[48]

2. The second phase is the transfer of the results Newton has obtained in mathematics to physical nature.[49] Because the mathematical system duplicates the idealised physical system, the rules or proportions arrived mathematically in the first phase may be "transferred back to the other and then compared and contrasted with the data of experiment and observation."[50] The models are thus rendered more complex by direct and successive comparisons with "experiential data and the laws or rules derived from such data."[51] For instance, an initial component of inertial movement in a central force field is shown to be a necessary and sufficient condition for the exact validity of the law of areas. Since the area law does not hold exactly in the physical world, the initial mental constructs needs to be modified: "Newton knew that his simple system for Kepler's laws was a construct that does not correspond to reality; accordingly, he introduced more complex conditions that brought it into conformity with the real world as revealed by experiments and observation."[52] As Cohen notes, "[t]he comparison of the mental construct of a one-body system and the world of physical nature leads Newton from consideration of a mass point and a central force to a two-body system, in which two bodies or two mass points mutually attract one another."[53] This leads to new deductions and a new phase two. In this way, there is an alternation between these successive stages which leads to an increasing complexity and hence to an equally increasing "*vraisemblance*." Newton does not carry out phase two in full: once it has been shown that sufficient close approximations occur, then the investigation could move on to the third phase.[54]
3. In the third phase, the principles obtained in phases one and two will no longer be purely mathematical but will be applied to natural philosophy so as to elaborate a system of the world.[55] In this phase, the mathematical conditions and entities are no longer considered as imagined mathematical constructs, but as duplicates of the realities of the external world. However, they are not identical equivalents of the conditions of the external world, but only approximations.[56]

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46 Ibid. Cohen notes that "the Newtonian "one-body-system" is a "system" to the extent that it is composed of two entities, even though these are not homologous, as in the case of a system of two bodies: these are a single body (or mass point) and a center of force" (ibid., p. 302, footnote 3).

47 Ibid., p. 63.

48 Ibid.

49 Ibid.

50 Ibid.

51 Ibid., p. xiii, cf. pp. 63, 77, 85, 99.

52 Ibid., p. 77.

53 Cohen, "The *Principia*, Universal Gravitation, and the 'Newtonian Style'", p. 50.

54 Cohen, *The Newtonian Revolution*, p. 102.

55 Ibid., xiii, cf. p. 64.

56 Ibid., p. 65.

This interpretation converges, according to Cohen, with the *scholium* to Section 11 of Book I[57]:

> (1) Mathematics requires an investigation of those quantities of forces and their proportions that follow from any conditions that may be supposed. (2) Then, coming down to physics, these proportions must be compared with the phenomena, so that it may be found out which conditions (or laws)[58] of forces apply to each kind of attracting bodies [In mathesi investigandæ sunt virium quantitates & rationes illæ, quæ ex conditionibus quibuscunque positis consequentur: deinde, ubi in physicam descenditur, conferendæ sunt hæ rationes cum phænomenis; ut innotescat quænam virium conditiones singulis corporum attractivorum generibus competant]. (3) And then, finally, it will be possible to argue more securely concerning the physical species, physical causes and physical proportions of these forces [Et tum demum de virium speciebus, causis & rationibus physicis tutius disputare licebit]. Let us see, therefore, what the forces are by which spherical bodies, consisting of particles that attract in the way already set forth, must act upon one another, and what sorts of motions results from such forces.[59]

Cohen interpreted the second clause as follows: a successive comparison with the empirical world, performed in Book I, usually leads to an alteration of the initial conditions, i.e. the physical conditions as assumed in our initial mental analogue.[60] In Section 2.6, I will offer a different reading of the first two steps of Newton's methodology as spelled out in the scholium to Section 11 of Book I.[61] According to the strong version of Cohen's "Newtonian Style", a successive series of comparisons between the mathematical consequences derived from the physico-mathematical conditions pertaining to the simpler models of Book I and the mathematical properties as revealed by actual astronomical observations is the source of the increasing complexity of the models in Book I. One could also identify a weaker variant of Cohen's account, which goes as follows: as Newton had an agenda in mind of constructing a *systema mundi*, he decided to give considerable attention to those physico-mathematical models that are relevant for the study of the system of the world.[62] According to this weaker, unproblematic, version, piecemeal adaption through a series of successive comparisons with nature is not the source of the increasing complexity of the models in Book I.

According to Zev Bechler, Cohen's strong interpretation faces some difficulties: Cohen's interpretation has affinities with hypothetico-deductive methodology, and,

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57 Ibid., p. 85.

58 This is an insertion to the text added by Cohen-Whitman.

59 Newton, *The Principia*, pp. 588–589 [numbers added]; Koyré, Cohen and Whitman, eds., *Principia Mathematica*, I, p. 298.

60 Cohen, *The Newtonian Revolution*, p. 99.

61 Namely, I shall argue that the second phase (the comparison of the models "with the phenomena, so that it may be found out which conditions (or laws) of forces apply to each kind of attracting bodies") as spelled out in Newton's *scholium* to Section 11 occurs not in Book I, but in Book III.

62 More on this weaker version is to follow in the final paragraph of this section.

as Bechler adds, it is not reality but theory that "dictates."[63] Although Bechler's analysis is far from complete and adequate, I do think that he did make a genuine point. Newton starts by reducing planetary motion to a one-body system, according to Cohen. Such a system then introduces a centripetal force, i.e. the force producing the motion of which we intend to construct an *analogue* for. Thus, in the strong version of Cohen's "Newtonian Style", a significant assumption would have been made by Newton in the context of Book I, namely that centripetal forces exist in the *systema mundi*. In Book I Newton does not start from the premise that centripetal forces exist *in rerum natura*, rather, on the basis of the models developed in Book I and Phenomena I–VI, he demonstrates in Book III that centripetal forces exist and that they produce the motion of the primary and secondary planets. It is only at that point that Newton commits himself to the existence of centripetal forces. It is therefore more adequate to endorse the view that Book I provides a mathematical study of centripetal forces – *thereby making abstraction from the forces in the empirical world*.

In due fairness to Cohen, it should be pointed out that he himself seems to have become aware of this very problem in the strong version of the "Newtonian Style", i.e. in later work he seemed at points to endorse what I have described as the weaker version of the "Newtonian Style". In a paper written in 1993, Cohen emphasized the autonomy of Book I:

> But the end product is a mathematical construct, a creation of the mind, in which *Newton is perfectly free whatever kinds of forces he pleases, subject to any force that he may imagine – because he is dealing with a mathematical construct and not with a physical situation*. [. . .] Of course, the construct in question has been designed by Newton to be applied eventually to a specific end-use in natural philosophy, and so the construct has certain elements similar to the situation of the world of physics, the realm of natural philosophy revealed to us by our senses, by experiment, and by observation.[64] And it is clear to any reader that Newton is directing his efforts to producing rules and laws that can eventually be applied in natural philosophy.[65]

However, as it stood Cohen was unable to eliminate the inherent tension between his strong version of the "Newtonian Style" and the emphasis he put on the autonomy of Book I in later work. It is important to stress that I have spent much time in analysing Cohen's "Newtonian Style", not for criticism's sake, but as a stage-setting for my own treatment of Newton's phase of model construction. In what follows, I shall argue that the increasing complexity of the physico-mathematical in Book I is the outcome, not of a direct and successive comparison with the empirical world, but of a theoretical exploration of increasingly complex physico-mathematical conditions.

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63 Bechler, "Introduction: Some Issues of Newtonian Historiography", p. 10. Bechler notes: "Since the laws of motion dictate that the central mass cannot be stationary *if it acts directly* on the revolving mass, and since Newton assumes it actually acts directly, it follows that the motion will not be strictly Keplerian. Hence Newton concludes that the motion of the planets is not strictly Keplerian. It is the physical theory which dictates this, not physical reality." (ibid.).

64 Cf. the Scholium to Proposition IV, Book I (Newton, *The Principia*, p. 454–455).

65 Cohen, "The *Principia*, the Newtonian Style, and the Newtonian Revolution", pp. 92–93.

2.4 The Constituents of Newton's Models in Book I

In Book I, Newton constructed physico-mathematical models which are deduced from the foundational principles he had introduced at the outset of the *Principia*, i.e. the definitions and laws of motion. In this section, we shall consider the definitions, the laws of motion, and the mathematical machinery which Newton introduced at the beginning of the *Principia*. Put differently, in this section, I will explicate the constituents of Newton's physico-mathematical models – with a strong focus on Book I. Additionally, I seek to argue that Newton's models were *not purely mathematical*, but *physico-mathematical instead* (see Section 2.4.4).

2.4.1 Newton's Definitions

Newton's definitions introduced the key technical terms and their *measures* that would be crucial throughout the *Principia*.[66] Definition I states that quantity of matter is a measure of matter that arises from its density and volume jointly ("*Quantitas Materiæ est mensura ejusdem orta ex illius Densitate & Magnitudine conjunctim.*").[67] It is important to stress that for Newton density referred to specific gravity, whereby the density of water is taken to be unity.[68] Quantity of matter is measured by weight.[69] In Definition II, quantity of motion is defined as a measure of motion that arises from the velocity – taken as a scalar quantity – and the quantity of matter jointly ("*Quantitas Motus est mensura ejusdem orta ex Velocitate & Quantitate Materiæ conjunctim.*").[70] Definition III deals with the inherent (and essential[71]) force of matter (*materiae vis insita*), i.e. the power of resisting by which every body, so far as it is able ("*quantum est in se*"[72]), perseveres in its state either of

[66] Cf. Smith, "The Methodology of the *Principia*", p. 145.

[67] Newton, *The Principia*, p. 403. In the initial revise of *De motu*, in which quantity of matter and quantity of motion were introduced, Newton stated that "*Quantitas materiæ* est quæ oritur ex ipsius densitate et magnitudine conjunctim." (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 92 [= CUL Add. Ms. 3965, f. 21^r]).

[68] Cf. Crew, *The Rise of Modern Physics*, p. 124. This observation counters Ernst Mach's criticism that Definition I is circular: "The concept of mass is not made clearer by describing mass as the product of the volume into the density, as density itself denotes simply the mass of unit of volume." (Mach, *The Science of Mechanics*, p. 241).

[69] Newton, *The Principia*, p. 404. Cf. "Æstimatur autem quantitas corporis ex copia materiæ corporæ quæ gravitati suæ proportionalis esse solet." (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 189, footnote 13 [= CUL Add. Ms. 3965, f. 26^r]). See Newton, *The Principia*, pp. 89–92 for useful discussion of Definition I. According to Descartes, by contrast, one could not determine a body's quantity of matter by weighing it, because he considered weight as unrelated to a body's extension (Janiak, *Newton as Philosopher*, pp. 103–104).

[70] Newton, *The Principia*, p. 406.

[71] Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 191.

[72] On this matter see Cohen, "'Quantum in Se Est': Newton's Concept of Inertia in Relation to Descartes and Lucretius". Newton never gave credit to Descartes for the first law of motion

resting or of moving uniformly straight forward.[73] Definition IV defines impressed force as the (external) action exerted on a body to change its state of resting or of moving uniformly straight forward ("*Vis Impressa est actio in corpus exercita, ad mutandum ejus statum vel quiescendi vel movendi uniformiter in directum.*").[74] Newton distinguished between three types of impressed forces: percussion, pressure and centripetal force.[75] Definition V defines centripetal force as the force by which bodies, from rest,[76] are drawn from all sides, are impelled, or in any way tend, towards some point as to a centre ("*Vis Centripeta est, qua corpora versus punctum aliquod tanquam ad Centrum undique trahuntur, impelluntur, vel utcunque tendunt.*").[77] Newton's mathematical treatment of centripetal forces was meant to guarantee neutrality with respect to the modus operandi of the medium producing gravity.[78] As Newton had clearly indicated:

> Moreover, I use interchangeably and indiscriminately words signifying attraction, impulse, or any sort of propensity toward a center, considering these forces not from a physical but only from a mathematical point of view. Therefore, let the reader beware of thinking that by the words if this kinds I am anywhere defining a species or mode of action or a physical sense to centers (which are mathematical points) if I happen to say that centers attract or that centers have forces.[79]

Definitions VI, VII and VIII, first introduced in the initial revise of *De motu*, explicate the three measures of centripetal force: the *absolute quantity of centripetal force*, i.e. the measure of this force that is greater or smaller in proportion to the efficacy of the cause propagating it from a centre through the surrounding regions,[80] the *accelerative quantity of centripetal force*, i.e. the measure of this force that is

(Newton, *The Principia*, p. 136). See furthermore Herivel, *The Background to Newton's Principia*, pp. 42–53.

73 Newton, *The Principia*, p. 406; Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 30 [= CUL Add. Ms. 3965, f. 55^r], pp. 92–93 [= CUL Add. Ms. 3965, f. 21^r].

74 Newton, *The Principia*, p. 405; Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 92–93 [= CUL Add. Ms. 3965, f. 21^r].

75 Newton, *The Principia*, p. 405.

76 This guarantees that the centripetal force is independent from the speed and direction of the body (Pourciau, "From Centripetal Forces to Conic Sections", p. 61, footnotes 6–7).

77 Newton, *The Principia*, p. 405; Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 30 [= CUL Add. Ms. 3965, f. 55^r] and ibid., VI, pp. 94, 96 [CUL Add. Ms. 3965, f. 21^r].

78 See Chapter 1, Section 6. Cf. Smith, "The Methodology of the *Principia*", pp. 141–142.

79 Newton, *The Principia*, p. 408.

80 Ibid., p. 406. In *A Treatise of the System of the World* Newton recorded that: "the absolute force of every globe is as the quantity of matter which the globe contains" (Newton, *A Treatise of the System of the World*, p. 45). In the initial revise of *De Motu*, Newton wrote: "*Quantitas absoluta* (quæ et *vis absoluta* dici potest) major est ad unum centrum minor ad aliud, nullo habito respectu ad distantias et magnitudiones attractorum corporum; uti virtus magnetica major in uno magnete minor in alio." (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 94 [= CUL Add. Ms. 3965, f. 21^r]). See, furthermore, Stein, "On the Notion of Field in Newton, Maxwell, and Beyond", p. 266. For useful corrections to Stein's account, see Schliesser, "Without God", pp. 92–94.

proportional to the velocity it produces in a body (from rest) in a given time,[81] and, the *motive quantity of a centripetal force*, i.e. the measure of this force that is proportional to the motion it produces in a body (from rest) in a given time,[82] respectively. The fact that Newton added the *scholium* on space and time at the end of his definitions is significant in itself: relative space and time are defined as the sensible *measures* of absolute space and time.[83]

2.4.2 Newton's Laws of Motion

Newton's first law is the law of rectilinear inertia: "[e]very body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed."[84] Law I thus licences inferences to the presence of an impressed force from non-inertial motion: namely, if a body is not at rest or does not move uniformly straight forward, this counts as an indication that this body is being acted upon by an impressed force. With respect to the second law it needs to be stressed that Newton intended, contrary to standard reading,[85] this law to accommodate both impulsive as well as continuous forces, as Bruce Pourciau has forcefully documented.[86] Newton's second law, stating that "change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed" ("*Mutationem motus proportionalem esse vi motrici impressæ, & fieri secundum lineam rectam qua vis illa imprimitur.*"),[87] was intended to measure the deflection from rectilinear and uniform inertial motion, which Newton indicated by a directed line segment, i.e. a vector quantity, generated in a given time (see Fig. 2.1). In Newton's own words:

> If the body *A* should, at its place *A* where a force is impressed upon it, have a motion by which, when uniformly continued, it would describe [describeret] the straight line *Aa*, but shall by the impressed force be deflected [deflectatur] from this line into another one *Ab*

[81] Newton, *The Principia*, p. 407.

[82] Ibid.

[83] In Chapter 6, I shall provide ample discussion of Newton's views on space and time.

[84] Newton, *The Principia*, p. 416. For further contextualisation, see Gabbey, "Force and Inertia in Seventeenth-century Dynamics" and esp. id., "Force and Inertia in the Seventeenth Century: Descartes and Newton", esp. pp. 272–297.

[85] Newton, *The Principia*, p. 110.

[86] Pourciau, "Newton's Interpretation of Newton's Second Law"; Smith, "Newton's *Philosophiae Naturalis Principia Mathematica*". Apart from questioning the impulse-only reading of the second law, Pourciau offers a myriad of positive evidence in support of this interpretation. All this culminates in the showing that the "compound second law" was actually Newton's own interpretation (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 540–543; CUL Add. Ms. 3965, f. $274^{\text{r-v}}$ [additions and corrections for the second edition of the *Principia*]). See, furthermore, Pourciau, "Force, Deflection and Time, Proposition VI of Newton's *Principia*".

[87] Newton, *The Principia*, p. 416. See, furthermore, Fraser, "The Third Law in Newton's *Waste Book*".

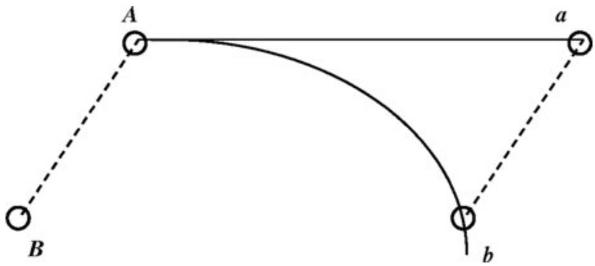

Fig. 2.1 Newton's second law of motion (after Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 540)

> and, when it ought to be located [reperiri deberet] at the place *a*, be found at the place *b*, then, because the body, free of the impressed force, would have occupied [occuparet] the place *a* and is thrust out from this place by that force and transferred therefrom to the place *b*, the translation of the body from the place *a* to the place *b* will, in the meaning of this Law, be proportional to this force and directed to the same goal towards which this force is impressed. Whence, if the same body deprived of all motion and impressed by the same force with the same direction, could in the same time be transported from the place *A* to the place *B*, the two straight lines *AB* and *ab* will be parallel and equal.[88]

Newton understood change in motion as the product of the quantity of matter and the deflection generated in a given time *from* the terminus *a*, which a body *would reach* when describing a rectilinear inertial path in a given amount of time, i.e. when unaffected by an external force, *to* the terminus *b*, which a body *reaches* when being urged by an impressed force in an equal amount of time.[89] Law II thus allows for conclusions about the magnitude *and* direction of an impressed force that produces non-inertial motion. Newton's conceptualisation of Law II squares nicely with the counterfactual-nomological view of causation which I have ascribed to Newton in the previous chapter.[90] W. W. Rouse Ball has noted that Law I "seems to be a consequence of the second law, and if so it is not clear why it was enunciated as a separate law."[91] Indeed, given our contemporary formulation of Law II, $F = ma$, it follows that, if $F = 0$, then $a = 0$ – since m is constant. Historically speaking, $F = ma$ was hardly Newton's own conceptualization of Law II, as we have just seen. The reason why Newton chose to enunciated Law I and II separately is the following. While Law I stipulates *what would happen* to a body *if no impressed force acts* on it (namely, it would persevere "in its state of being at rest or of moving uniformly straight forward"); Law II characterizes *how the impressed force*, considered as a motive force, *acts* (namely, it acts proportionally to the mass and proportionally to the deflection from the counterfactual inertial path to the actual path as explicated above). Newton's third law is the law of action–reaction which states that to any

88 Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 541/542.

89 Pourciau, "Newton's Interpretation of Newton's Second Law"; Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 540–541.

90 See Chapter 1, Section 1.6.

91 Rouse Ball, *An Essay on Newton's Principia*, p. 77.

action there is always an opposite and equal reaction.[92] Law III thus correlates the impressed force, by which a body is acted upon, to its corresponding reaction force.

In the *scholium* to the laws of motion, Newton pointed out that the "principles I have set forth are accepted by mathematicians and confirmed by experiments of many kinds."[93] In this sense, there was certainly a historical dimension in what Newton took as foundational principles. He claimed that Galileo had found that the descent of heavy bodies is in the squared ratio of the time and that projectiles describe parabolic trajectories by Laws I–II and Corollaries 1–2 – an assertion that is, however, historically incorrect for Galileo had obtained these results from purely kinematical considerations.[94] In a paragraph added in the third edition of the *Principia*, Newton rather showed that Laws I–II could accommodate Galileo's kinematical results by giving a dynamical reinterpretation of them.[95] Furthermore, from the same laws and corollaries and from Law III, "Sir Christopher Wren, Dr. John Wallis,[96] and Mr. Christiaan Huygens,[97] easily the foremost geometers of the previous generation [ætatis superioris geometrarum facile principes], independently found the rules of collisions and reflections of hard bodies."[98] When reporting on his own findings with ten-foot colliding pendulums Newton established that "[a]s a result of the meeting and collision of bodies, the quantity of motion – determined by adding the motions in the same direction and substracting the motions in the opposite directions – was never changed."[99] He found a discrepancy between measurements and theory of about "an inch or two" and attributed it to the difficulty in simultaneously releasing the pendulums and to the irregularities in the texture.[100] The results obtain equally well for elastic bodies, Newton added.[101] In further support of Law III, Newton indicated that if one of two mutually impelling bodies *A* and *B*, interposed by an obstacle, attracted the other more than the other, then the obstacle between them would not remain in equilibrium and the whole would be, contrary to Law I, set into an accelerated rectilinear motion.[102] He concluded the evidence for Law III by pointing to its application in machines and devices.[103] Newton wrote:

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[92] Newton, *The Principia*, p. 417.

[93] Ibid., p. 424.

[94] De Gandt, *Force and Geometry in Newton's* Principia, p. 86.

[95] Newton, *The Principia*, p. 424.

[96] E.g., Wallis and Wren, "A Summary Account of the General Laws of Motion".

[97] See Huygens, "A Summary Account of the Laws of Motion". See Smith, "Comments on Ernan McMullin's 'The Impact of the *Principia* on the Philosophy of Science'", p. 334, for useful discussion of Huygens' measurement of the strength of surface gravity which presupposes Law I and II. See, furthermore, Yoder, *Unrolling Time, Christian Huygens and the Mathematization of Nature*.

[98] Newton, *The Principia*, p. 424. For additional background, see Hall, "Mechanics and the Royal Society, 1668–1670".

[99] Newton, *The Principia*, p. 426.

[100] Ibid., pp. 426–427, 713.

[101] Ibid., p. 427.

[102] Ibid., pp. 427–428.

[103] Ibid., pp. 428–430.

> As bodies are equipollent in collisions and reflections if their velocities are inversely as their inherent forces [i.e., forces of inertia], so in the motions of machines those agents [i.e., acting bodies] whose velocities (reckoned in the direction of their forces) are inversely as their inherent forces are equipollent and sustain one another by their contrary endeavours.[104]

Newton indicated that the laws of motion were corroborated by the elaborate work of "the previous generation."[105] By implication, it seemed that the laws of motion had shown their usefulness and potential in the study of force and motion. Moreover, when Newton was working on *De motu*, his impression of their potential was strengthened: for the possibility to derive an increasingly complex set of models from them that could account for the motions in the empirical world began to dawn on him. Insofar as the physico-mathematical models of Book I are based on the laws of motion, the fruitfulness of the laws of motion can be measured by the fruitfulness of the physico-mathematical models derived from them – which counts as *indirect* support for the laws of motion. George E. Smith considers the laws of motions "working hypotheses," i.e. principles that are not testable in and of themselves but indispensable to a train of evidential reasoning.[106] Newton considered the laws of motion as "axioms:"

> I like your designe of adding something more particularly concerning the manner of Philosophizing made use of in the Principia & wherein it differs from the method of others, vizt *by deducing things mathematically from principles derived from Phænomena by Induction. These Principles are the 3 laws of motion. And these Laws in being deduced from Phænomena by Induction & backt with reason & the three general Rules of philosophizing are distinguished from Hypotheses & considered as Axioms*. Upon these are frounded [*sic*] all the Propositions in the first & second Book. And these Propositions are in the third Book applied to the motions of ye heavenly bodies.[107]

The laws of motion function as abstract principles that by themselves tell us little about the empirical world without the introduction of model-specific assumptions. The models derived from the laws of motion thus help to concretize abstract theory. If the models rigidly deduced from the definitions and laws of motion pass extensive testing in the empirical world, the initial premises are confirmed jointly – but indirectly – with their consequences. Newton's emphasis on the fact that the laws of motions were deduced from phenomena and rendered general by induction is indicative of his conviction of the privileged status of the laws of motion. Note also that the laws of motion remain silent on the mode of operation of the forces involved. When developing the propositions in Book I, Newton was in fact ascertaining the mathematical consequences of specific model-theoretical configurations

[104] Ibid., pp. 428–429.

[105] Perl, "Newton's Justification of the Laws of Motion".

[106] Smith, "Comments on Ernan McMullin's 'The Impact of the *Principia* on the Philosophy of Science'", p. 335.

[107] Draft of Newton to Cotes, 28 March 1712/13, Newton, *Correspondence*, V, pp. 396–399, p. 398 [italics added].

in which the idealized bodies were subject to the same physical laws as the bodies in the empirical world.

2.4.3 The Mathematical Machinery of the Principia

Section 1 of Book I (Lemmas I–XI), wherein first and ultimate ratios are introduced, bears the title "The method of first and ultimate ratios, for use in demonstrating what follows."[108] It should be conveyed that, despite frequent references to the lemmas on first and ultimate ratios, it is not the case that all propositions and lemmas of Books I–II were based on the method of ultimate ratios.[109] Although I shall not address the different mathematical techniques underlying the *Principia* – as it is not my current endeavour to contribute to the history of mathematics, I wish to point out that the *Principia* contained a variety of mathematical methods: infinite series expansions, quadratures of curvilinear figures, infinitesimals, classical theories of conic sections and higher curves, projective geometry, interpolation techniques, perturbation methods, algebraic equations, the famous method of ultimate ratios, etc.[110] Likely, Newton saw Section 1 as providing "a *potential* foundation for his later propositions."[111] A striking feature of Newton's method of first and ultimate ratios

[108] Newton, *The Principia*, p. 433. The myth that the demonstrations of the *Principia* were first proved by means of fluxional calculus and later reworked in the mathematical style in which it was eventually published has long been discarded as historically inadequate (see, e.g., Whiteside, "The Mathematical Principles Underlying Newton's *Principia Mathematica*"). Fluxional calculus was, however, used privately amongst Newton and his mathematical adherents (see: Guicciardini, "Did Newton use his Calculus in the *Principia*?" and id., *Newton on Mathematical Certainty and Method*, pp. 252–257).

[109] It is worth quoting D. T. Whiteside's words here: "In the sequel [of *De motu* and, by implication, of the *Principia*] this group of lemmas [on the method of first and last ratios] does not in fact everywhere play the auxiliary rôle which Newton here foresees, and they are only rarely invoked in the new propositions and lemmas which he subsequently introduced into his revised "De motu Corporum Liber primus" [. . .]. It would appear that his initial vision of presenting a logically tight exposition of the principles of motion under accelerative forces faded more and more when he came in detail to cast his arguments, and that he was happy after a while to lapse into the less rigorously justified mode of presentation which he largely exhibits in his published *Principia*. Whatever be the truth of the matter, these lemmas are undeniably a retrospective gloss on the arguments which they now collectively and generally justify, but in whose initial contrivance they play at best a subdued and unstated part." (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 107–108, footnote 39). For an exposition of these lemmas, see Densmore, *Newton's* Principia*: The Central Argument*, pp. 17–91 [it should be noted, however, that Densmore's discussion of limits (e.g., on pp. 17–18) is flawed (Nauenberg, "The Mathematical Principles Underlying the *Principia* Revisited")]; De Gandt, *Force and Geometry in Newton's* Principia, pp. 224–234; and, Pourciau, "The Preliminary Mathematical Lemmas of Newton's *Principia*".

[110] Guicciardini, "Conceptualism and Contextualism in the Recent Historiography of Newton's *Principia*", p. 407; id., *Reading the* Principia, Chapter 3; id., *Newton on Mathematical Certainty and Method*, esp. Chapters 9–12; and, Meli, *Thinking with Objects*, p. 266.

[111] Pourciau, "The Preliminary Mathematical Lemmas of Newton's *Principia*", p. 279. Cf. Guicciardini, *Isaac Newton on Mathematical Certainty and Method*, p. 222.

was the introduction of movement in geometry.[112] Geometrical quantities generated by continuous flow are the object of Newton's method of first and ultimate ratios.[113]

The first lemma, describing Newton's method of ultimate ratios, is that two quantities whose difference becomes arbitrarily small become "ultimately equal:"[114]

> *Quantities, and also ratios of quantities, which in any finite time constantly tend to equality [ad æqualitatem tempore quovis finito constanter tendunt], and which before the end of that time approach so close to one another that their difference is less than any given quantity [propriùs ad invicem accedunt quàm pro datâ quavis differentiâ], become ultimately equal [fiunt ultimò æquales].*[115]

The proof for Lemma I is based on a *reductio ad absurdum*: if one supposes that quantities (or ratios of quantities) become ultimately unequal, then they cannot approach so close to equality so that their difference is less than their ultimate difference – contrary to the hypothesis. Newton's proofs thereby involved not only classical geometrical ratios but also "ultimate" geometrical ratios. In Lemmas II–IV, Newton argued that, if any number of equal (or unequal) parallelograms inscribe or circumscribe a curvilinear figure and if "the width of these parallelograms is diminished and their number is increased indefinitely," the inscribed, circumscribed, and curvilinear figure become ultimately equal.[116] Lemma V then generalizes this result: "*All the mutually corresponding sides – curvilinear as well as rectilinear – of similar figures are proportional, and the areas of such figures are as the square of their sides*."[117] In Lemmas VI–VII, Newton showed that a vanishing angle contained by its chord and tangent will "be indefinitely diminished and will ultimately vanish," and that an evanescent arc and its corresponding chord and tangent become ultimately equal, respectively. In Corollary 3 to Lemma VII, Newton concluded that "therefore all these lines can be used for one another interchangeably in any argumentation concerning ultimate ratios."[118] In Lemma VIII, Newton stated that, when "*points* A *and* B *approach each other*," the triangle formed by the two straight lines *AR* and *BR* and the arc *ACB* becomes ultimately equal to the inscribed triangle formed by the straight lines *AR* and *BR* and the chord *AB* or to the circumscribed triangle formed by the straight lines *AR* and *BR* and the tangent *AD*, which lies on

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[112] As François De Gandt elegantly puts it: "La géometrie s'enrichit de tout l'apport de la cinématique: les points se déplacent sur les lignes, les courbes s'engendrent comme trajectoires de mobiles, les cercles sont en rotation ou en roulement, etc." (De Gandt, "Le style mathématique des *Principia* de Newton", p. 199).

[113] Guicciardini, *Newton on Mathematical Certainty and Method*, pp. 219–223, 241–242.

[114] Denoted in what follows as "$\approx$". In his "The Preliminary Mathematical Lemmas of Newton's *Principia*", Bruce H. Pourciau has shown that Newton's notion of limit was more rigorous than usually supposed. See, furthermore, Sellés, "Infinitesimals in the Foundations of Newton's Mechanics" and Guicciardini, *Reading the* Principia, p. 43 ff.

[115] Newton, *The Principia*, p. 433; Koyré, Cohen and Whitman, eds., *Principia Mathematica*, I, p. 72.

[116] Newton, *The Principia*, pp. 433–435.

[117] Ibid., p. 435.

[118] Ibid., p. 436.

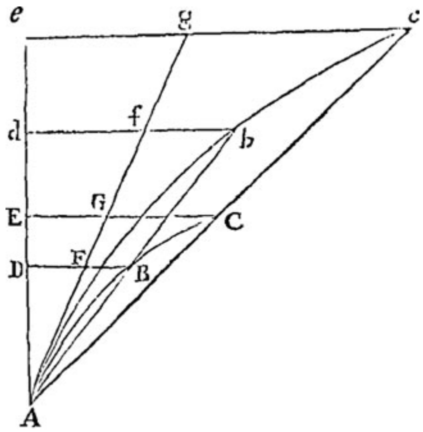

Fig. 2.2 Original figure accompanying Lemma IX and X, Book I. Reproduced from *PrE*3, p. 33

the meeting point of the tangent line and the extension of *BR* in the direction of the tangent line.[119] In the Corollary to Lemma VIII, Newton added that "hence those triangles can be used for one another interchangeably concerning ultimate ratios."[120] In Lemma IX, which itself is based on Lemma V, Newton had shown that the areas *ABD* and *ACE* are in the squared ratios of the times *AD* and *AE*, when "*points* B *and* C *simultaneously approach toward* A"[121] – Newton's trick to work back to the "very beginning of the motion" (see Fig. 2.2).[122] In Lemma X, Newton stated that "*[t]he spaces which a body describes when urged by any finite force, whether that force is determinate and immutable or is continually increased or continually decreased, are at the very beginning of the motion in the squared ratio of the times*."[123] For, if the times are represented by the lines *AD* and *AE* and the generated velocities by the ordinates *DB* and *EC*, the spaces described by these velocities will be given by the areas *ABD* and *ACE* described by the ordinates (by Lemma IX). The upshot of Lemma X is that, insofar as the velocity varies linearly with time, the displacement from inertial motion is proportional to the force times the square of time.[124]

In the scholium to Lemmas I–XI, Newton was explicit about his motivation to use the method of first and ultimate ratios[125]:

> In any case, I have presented these lemmas before the propositions in order to avoid the tedium of working out lengthy proofs by *reductio ad absurdum* in the manner of the ancient geometers. Indeed, proofs are rendered more concise by the method of indivisibles. But since the hypothesis of indivisibles is problematic [durior] and this method is therefore accounted less geometrical, I have preferred to make the proofs of what follows depend on

119 Ibid.

120 Ibid., p. 437.

121 Ibid., p. 437.

122 See, furthermore, De Gandt, *Force and Geometry in Newton's* Principia, pp. 230–233.

123 Newton, *The Principia*, pp. 437–438.

124 Guicciardini, *Reading the* Principia, p. 47.

125 Cf. De Gandt, *Force and Geometry in Newton's* Principia, p. 161.

> the ultimate sums and ratios of vanishing quantities and the first sums and ratios of nascent quantities [ad ultimas quantitatum evenescentium summas & rationes, primasque nascentium], that is, on the limits of such sums and ratios [ad limites summarum & rationum], and therefore to present proofs of those limits beforehand as briefly as I could. [. . .] Accordingly, whenever in what follows I consider quantities as consisting of particles [ex particulis] or whenever I use curved line-elements [lineolas curvas] in place of straight lines, I wish it always to be understood that I have in mind not indivisibles but evanescent divisibles [nolim indivisibilia, sed evanescentia divisibilia, non summas & rationes partium determinatarum, sed summarum & rationum limites], and not sums and ratios of definite parts but the limits of such sums and ratios, and that the force of such proofs always rests on the method of the preceding lemmas.[126]

In the *Commercium epistolicum* (1714/5), Newton noted that "the summing up of indivisibles to compose an area or solid was never yet admitted into Geometry."[127] Newton did not reject indivisibles because of his standards of mathematical rigour alone; he also endorsed the view that the language of mathematics,[128] insofar as it is applied to nature, should correspond to physical reality. Newton pointed out that his version "is more natural & geometrical because founded on *primæ quantitatum nascentium rationes* w[ch] have a being in Geometry, whilst *indivisibles* upon which the Differential method is founded have no being either in Geometry or in nature."[129] Likewise, in his *Quadrature of Curves* (1693), he emphasized the analogy between the generation of mathematical and physical motion in a continual flux of time:

> I don't consider Mathematical Quantities as consisting of *indivisibles*, whether least possible parts or infinitely small ones, but as *describ'd by a continual motion.* Lines are describ'd, and by describing are generated, not by any apposition of Parts, but by the continuous motion of Points, Surfaces by the motion of Lines, Solids by the motion of Surfaces, Angles by the Rotation of their Legs, Time by a continual flux, and so in the rest. These *Geneses* are founded in Nature, and are every Day enacted in the motions of Bodies, and paraded *before our eyes*.[130]

[126] Newton, *The Principia*, pp. 441–442; Koyré, Cohen and Whitman, eds., *Principia Mathematica*, I, pp. 86–87. Cf. the discussion to Lemma II in Book II, where Newton observed: "I here consider these quantities [i.e., "generated quantities"] as indeterminate and variable, and increasing and diminishing as if by a continual motion or flux; and it is their instantaneous increments or decrements that I mean by the word "moments," in such a way that increments are considered as added or positive moments, and decrements as subtracted or negative moments. But take care: do not consider them to be finite particles! Finite particles are not moments, but the very quantities generated from the moments. They must be understood to be the just-now nascent beginnings of finite magnitudes." (Newton, *The Principia*, pp. 645–647).

[127] Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VIII, p. 598.

[128] Newton divided mathematics as follows: "All things are numbered, magnitudes measured and bodies moved; and the arts of numbering, measuring and moving are called arithmetic, geometry and mechanics" (ibid., VIII, p. 175, cf. p. 179).

[129] Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VIII, p. 597 [underscore added].

[130] Ibid., III, p. 141, footnote 43 [underscore added]. Cf. ibid., VIII, pp. 107, 123.

In order to provide an adequate mathematical treatment of bodies in motion, *mathematics should mimic physics* or, more precisely, mathematical objects and operations should be analogous to physical bodies and their motions.[131]

Contrary to Descartes, who had excluded exactness from mechanics, Newton argued that mechanics is the exact science of motion. It is in exactly this context that, as Niccoló Guicciardini has beautifully shown,[132] we should understand Newton's claim that geometry is based ("fundatur") on mechanical practice ("in praxi mechanica") in the Preface to the *Principia*.[133] In this Preface, Newton wrote:

> The ancients divided *mechanics* into two parts: the *rational*, which proceeds rigorously through demonstrations, and the *practical*. *Practical mechanics* is the subject that comprises the manual arts, from which the subject of *mechanics* as a whole has adopted its name. But since those who practice an art do not generally work with a high degree of exactness, the whole subject of *mechanics* is distinguished from *geometry* by the attribution of exactness to *geometry* and of anything less than exactness to *mechanics*. Yet the errors do not come from the art but from those who practice the art. Anyone who works with less exactness is a more imperfect mechanic, and if anyone could work with the greatest exactness, he would be the most perfect mechanic of all. For the description of straight lines and circles, which is the foundation of *geometry* appertains to *mechanics* [*in quibus* geometria *fundatur, ad* mechanicam *pertinet*]. Geometry does not teach how to describe these straight lines and circles, but postulates such a description. For geometry postulates that a beginner has learned to describe lines and circles exactly before he approaches the threshold of *geometry*, and then it teaches how problems are solved by these operations. To describe straight lines and to describe circles are problems, but not problems in *geometry. Geometry* postulates the solution of these problems from *mechanics* and teaches the use of the problems thus solved. And *geometry* can boast that with so few principles obtainted from other fields, it can do so much. Therefore *geometry* is founded on mechanical practice[134] [*Fundatur igitur* geometria *in praxi mechanica*] and is nothing more than that part of *universal mechanics* which reduces the art

[131] Cf. Sepkoski, "Nominalism and Constructivism in Seventeenth-Century Mathematical Philosophy", p. 53, in which Sepkoski ascribes to Newton a "physicalist" philosophy of mathematics. See, furthermore, Sepkoski, *Nominalism and Constructivism in Seventeenth-Century Mathematical Philosophy*, pp. 107–123; Guicciardini, *Isaac Newton on Mathematical Certainty and Method*, pp. 313–315; and, id., "Conceptualism and Contextualism in the recent Historiography of Newton's *Principia*", pp. 413–418.

[132] Guicciardini, *Isaac Newton on Mathematical Certainty and Method*, Chapter 13, esp. pp. 293–305.

[133] Newton, *The Principia*, pp. 381–383.

[134] In the first book of *Geometria Libri Duo* (1690s), Newton wrote: "Both the genesis of the subject-matter of geometry, therefore, and the fabrication of its postulates pertain to mechanics. [Pertinet igitur ad Mechanicam tum genesis subjecti Geometrici tum Postulatorum effectio.] [...] Geometry does not posit modes of description: we are free to describe them by moving rulers around, using optical rays, taut threads, compasses, the angle given in a circumference, points separately ascertained, the unfettered motion of a careful hand, or finally any mechanical means whatsoever. Geometry makes the unique demand that they be described exactly. [Id solum postulat Geometria ut describantur exactè.] It has now, however, come to be usual to regard as geometrical everything which is exact, and as mechanical all that proves not to be of the kind, as though nothing could possibly be mechanical and at the same time exact. But this belief is a stupid one [Crassa verò est hæc vulgi opinio], and it has its origin in nothing else than that geometry postulates an exact mechanical practice in the description of a straight line and a circle, and moreover is

> of measuring to exact propositions and demonstrations. But since the manual arts applied especially to making bodies move, *geometry* is commonly used in reference [*vulgo referatur*] to magnitude, and mechanics in reference to motion. In this sense *rational mechanics* will be the science, expressed in exact propositions and demonstrations [*accurate proposita ac demonstrata*], of the motions that result from any forces whatever and of the forces that are required for any motions whatever.[135]

This passage has two important consequences.[136] First of all, since geometry is founded on mechanical practice, geometry can be rightly applied to the mathematical study of force and motion. Secondly, by subsuming geometry under mechanics, Newton could also defend the exactness of mechanics.

2.4.4 The Constituents of the Models in Books I–II

To conclude this section, let me give an overview of the constituents of the models in Books I–II of the *Principia*. On the basis of the above survey and the discussion in the following section, the following constitutive elements are relevant:

1. *model-specific elements*, i.e. elements pertaining to individual models, such as specific curves described and additional geometrical elements, mass points, bodies and the forces acting upon them,[137] resisting media,[138] etc.,
2. *nomological and definitional elements*, i.e. Laws I–III (and Corollaries I–VI) and Definitions I–VIII, respectively, and finally,
3. *mathematical elements*, i.e. a series of mathematical operations that license further deductions.[139]

exact in all its operations, while mechanics as it is commonly exercised is imperfect and without exact laws. [. . .] For assuredly the more mechanical – that is, skillfully wrought [artificiosum] – a thing is, the more exact it is, and the more perfect mechanic who works the more perfectly and exactly, while he alone is perfect who works exactly." (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VII, pp. 288/289). The *Preface* to the *Principia* is to be understood as a critical reaction to Descartes (see Guicciardini, *Isaac Newton on Mathematical Certainty and Method*, Chapter 13, esp. pp. 299–304; id., "Geometry and Mechanics in the Preface to Newton's *Principia*, A Criticism of Descartes" *Géométrie*'; and, Domski, "The Constructible and the Intelligible in Newton's Philosophy of Geometry".

135 Newton, *The Principia*, pp. 381–382 [underscore added]; Koyré, Cohen and Whitman, eds., *Principia Mathematica*, I, pp. 15–16.

136 Guicciardini, *Isaac Newton on Mathematical Certainty and Method*, pp. 297–299.

137 In a n-body-system there are $n \times (n - 1)$ forces directed to n centres.

138 Defined by Newton as "any body whose parts yield to any force applied to it and yielding are moved easily with respect to one another" (Newton, *The Principia*, p. 687).

139 Obviously, I do not claim that Newton himself distinguished between these constituents at a conscious level, rather I use them as a conceptual framework useful for understanding Newton's models.

Elements 2 and 3 constitute the interpretative and inferential toolbox that allowed Newton to draw conclusions from the model-specific elements as given. Elements 3 allowed Newton to establish deductions from the mathematical features of a given model. By elements 2, Newton was able to infer information about the abstract, i.e. non-referential, forces involved: (1) *by Law I* Newton was able to infer the activity of an impressed or centripetal force from non-interial motion, *by Law II* he was able to infer the magnitude and direction of an impressed or centripetal force, and *by Law III* he was able to relate the impressed or centripetal force to its corresponding reaction force; and, (2) *by the Definitions* Newton was able to relate specific *definienda*, i.e. technical concepts (quantity of matter, quantity of motion, etc.) to specific *definientes*, i.e. *measures* which are given in quasi-physical terms (specific weight, velocity, time, acceleration, etc.).[140]

Since the idealized "bodies" and motions in the models in Book I (and II) are iso-nomological to real-world bodies and motions, on the one hand, and since the real-world bodies and motions are analyzable by the same technical concepts, on the other hand, Newton is able to bridge the gap between mathematics and physics and to establish a physico-mathematics. Due to the inclusion of elements 2, Newton's models in Book I are not purely mathematical,[141] for they provide an abstract mathematical treatment of concepts with clear physical connotations (force, motion, density, volume, etc.). Also, because of elements 2, Newton's models have the *potential* of providing information about real physical forces once relevant empirical measurements are provided. At this point, one might object that I am contradicting Newton when he wrote that in Books I and II he had presented "principles of philosophy that are not, however, philosophical but *strictly mathematical*."[142] However, the remainder of the above sentence continues with "that is, those on which the study of philosophy *can be based* [ex quibus videlicet in rebus philosophicis disputari possit]."[143] Newton also added that "[i]t still remains for us to *exhibit the system of the world from these same principles* [ut ex iisdem principiis doceamus constitutionem systematis mundani]."[144] The difference that Newton seems to have in mind is the difference between models without empirical referents (*mathematical models*) and models with empirical referents (*philosophical models*).[145] While Book

[140] Obviously elements 2 and 3 can be used *in tandem*.

[141] Moreover, a rigid separation between mathematics (*in casu* geometry) and physics would be incompatible with Newton's views on geometry (see: Garrison, "Newton and the Relation of Mathematics to Natural Philosophy", esp. pp. 610–613, 618–619; Domski, "The Constructible and the Intelligible in Newton's Philosophy of Geometry"; Dunlop, "What Geometry Postulates: Newton and Barrow on the Relationship of Mathematics to Nature"; and, Sepkoski, "Nominalism and Constructivism in Seventeenth-Century Mathematical Philosophy").

[142] Newton, *The Principia*, p. 793 [emphasis added].

[143] Ibid. [emphasis added].

[144] Ibid. [emphasis added].

[145] Cf. Cohen, *The Newtonian Revolution*, p. 79. This differentiation has affinity with what Janiak calls Newton's "mathematical treatment" *versus* his "physical treatment" of force (Janiak, *Newton as Philosopher*, pp. 58–65; cf. Cohen, *The Newtonian Revolution*, p. 79).

I deals with abstract *measures* only, i.e. with abstract quasi-physical *definientes*, Book III deals with *measurements*, i.e. observed *definientes*, and, accordingly, with real-world forces.

2.5 Crucial Sorts of Propositions of Book I

In this section, I provide an overview of some relevantly different kinds of propositions in Book I that are significant to our present discussion of the phase of model construction.[146] Book I of the *Principia* contained at least five types of conditional propositions or "inference-tickets",[147] which were of use in the study of the forces active in the empirical world, i.e. in Book III:[148]

1. The first type, to which I shall refer to as *(exact) causal inference-tickets*, enabled Newton to bi-conditionally relate[149] certain physico-mathematical conditions, holding *exactly*, to specific motions that bodies describe according to a well-defined mathematical regularity that holds *exactly*. The bi-conditional relation is established by proving two directions: the first direction establishes that certain physico-mathematical conditions which hold exactly produce specific motions described by bodies according to a mathematical regularity that holds *exactly*

146 Hereby I seek to elaborate on George E. Smith's discussion of the sorts of propositions in Book I (Smith, "The Methodology of the *Principia*", pp. 144–147).

147 I borrow the term "inference-ticket" from George E. Smith, who in his turn borrowed it from Arthur Prior (Smith, "The Methodology of the *Principia*", p. 143).

148 Although I shall not further discuss Book II in this chapter (see, however, Chapter 3, Section 3.3), it is useful to point out that to a significant extent Book II resembles Book I. In Book II Newton also established "if-then" propositions and offered a generic theory of resistance forces – just as Book I offers a generic theory of centripetal forces. Sections 1, 2 and 3 of Book II, for instance, contain an investigation – entirely consistent with the first step of Newton's threefold methodology as spelled out in the scholium to Section 11 of Book I – of "what kind of motions arise" (Newton, *The Principia*, p. 641, cf. p. 699) from the law according to which the resistance is either proportional to the velocity, proportional to the square of the velocity, or proportional to the velocity partly and the square of the velocity partly, respectively. In this sense, Book II attempts to provide a generic account of different resisting forces. With respect to resisting forces, Newton considered the determining factors of the total resistance force of a medium as consisting of three distinct parts: the inertia of the fluid, its viscosity and its internal friction. However, he was unable to separate the inertial component from the total resistance force (Smith, "The Newtonian Style in Book II of the *Principia*", pp. 259, 263; id., "Fluid Resistance: Why did Newton Change his Mind?"; and id., "Was Wrong Newton Bad Newton?'). This resulted in the impossibility of inferring residual phenomena which define phenomena that can be converted into evidence (Smith, "The Newtonian Style in Book II of the *Principia*", p. 251). Hence, "his programme of deduction broke down" in Book II (Truesdell, "A Program toward Rediscovering the Rational Mechanics of the Age of Reason", p. 91 and id., "Reactions of Late Baroque Mechanics to Success, Conjecture, Error, and Failure in Newton's *Principia*", pp. 144–149). See Smith, "The Newtonian Style in Book II of the *Principia*", for useful discussion of the empirical data and discrepancies involved.

149 Cf. Harper, "Newton's Argument for Universal Gravitation", pp. 176–177; id., "Measurement and Approximation", p. 275.

(*sufficient direction*); the second, reverse, direction establishes that the motions, described by bodies according to a mathematical regularity that holds exactly, are produced by the physico-mathematical conditions, as stipulated in the sufficient direction (*necessary direction*).[150] Newton thus sought for the necessary *and* sufficient physico-mathematical conditions, *given the laws of motion*, under which bodies describe motions according to a mathematical regularity that holds exactly. As will be explained below, this requirement differs from a hypothetico-deductive way of inferring causes according to which causes are only shown to be sufficient for their effects. Examples of such causal inference-tickets are Propositions I–II, Book I.

2. A variant of the (exact) causal inference-tickets, discussed above, enabled Newton to bi-conditionally relate certain physico-mathematical conditions, which hold "as most closely as possible" ("quam proxime"[151] in Newton's terminology), to specific motions that bodies describe according to a mathematical regularity that holds as most closely as possible – I refer to such inference-tickets as *quam proxime causal inference-tickets*.[152] Newton incorporated causal inference-tickets of this type because he sought to anticipate the possibility that the mathematical regularities stipulated by the exact causal inference-tickets would not hold exactly in the empirical world – in fact, Section 2 of Book I entailed that exact Keplerian motion could not occur in the empirical world. In this way, Newton was able to infer more safely, i.e. more safely because of the systematic dependency he had deduced *from the laws of motion* between the *quam proxime* cause and its *quam proxime* effect, that certain physico-mathematical conditions that hold as most closely as possible entail their corresponding empirical effects which similarly hold as most closely as possible, and vice versa.[153] So, instead of neglecting such discrepancies or explaining

[150] In the original tract *De motu* (autumn 1684) Newton only proved the sufficient direction (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 34ff.). In the initial revise of *De motu* (winter/early spring 1684–1685) he demonstrated both directions (ibid., VI, pp. 122–127).

[151] In Newton, *The Principia*, "*quam proxime*" is consistently translated as "very nearly." However, its literal translation ("as most closely as possible" or "uttermost closely") is stronger than Cohen and Whitman's translation. The significance of inference-tickets of this type has been amply brought to the fore by George E. Smith (e.g., Smith, "The Methodology of the *Principia*", pp. 155–156).

[152] Newton first introduced the requirement of showing that *quam proxime* centripetal forces are the necessary and sufficient causes of *quam proxime* Keplerian motion and the micro-macro inference-tickets in the initial revise of *De motu* (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, Corollaries 2–3 to Proposition III, pp. 126–129 and pp. 180–187, respectively).

[153] In the words of George E. Smith: "The phenomena [. . .] are inductive generalisations from specific observations, and hence they hold at least *quam proxime* of these observations. But then, unless the laws of motion are fundamentally mistaken, the force law too is guaranteed to hold at least *quam proxime* of these observations. By way of contrast, the fact that a consequence deduced from a hypothesized force law holds *quam proxime* of specific observations need not provide any such guarantee." (Smith, "The Methodology of the *Principia*", p. 160).

them away by the introduction of an arbitrary – theoretically independent – disturbing factor, Newton deduced from the definitions and laws of motion that certain physico-mathematical conditions, which hold as most closely as possible, are the necessary and sufficient causes of the motions that bodies describe according to a mathematical regularity that holds as most closely as possible. Examples of the second type of causal inference-tickets are Corollaries 2–3 to Proposition III, Book I.

3. Propositions of the third type express so-called *systematic discrepancies*.[154] Systematic discrepancies allow for a systematic dependency between deviations from an exact mathematical solution and variations in the corresponding theoretical parameters. Their importance lies in the fact that they help to detect the physical sources of such discrepancies so as to be the starting point for further stages of natural-philosophical inquiry. Examples of such systematic discrepancies are Corollary 7 to Proposition IV and Corollary 1 to Proposition XLV, Book I.[155]
4. Ex hypothesi *inference-tickets*,[156] in general, establish further mathematical properties of certain physico-mathematical conditions that are more complex than those originally considered in the (exact or *quam proxime*) causal inference-tickets – in the sense that additional model-specific elements are involved or mathematically more sophisticated trajectories are considered. It is characteristic of *ex hypothesi* inference-tickets that they premise the activity of one or more forces – *of the type inferable by means of the causal inference-tickets* – in more complex configurations as a given, in order to investigate the mathematical properties of such more complex physico-mathematical conditions. However, once, in Book III, instances of such forces are inferred by means of the exact or *quam proxime* causal inference-tickets and once it is shown that model-specific assumptions are isomorphic to their physical targets, the antecedent of the *ex hypothesi* inference-tickets is shown to hold and hence their deductive conclusions follow by *modus ponens*. Examples of such inference-tickets are Propositions LXV and LXIX, Book I.
5. Propositions of the fifth type establish that the overall inverse-square centripetal force exerted by a body results from the individual inverse-square forces of each

[154] The importance of inference-tickets of this type have amply been brought to the fore by William L. Harper (esp. Harper, "Newton's Classic Deductions from Phenomena"; id., "Measurement and Approximation"; and, id., "Newton's Argument for Universal Gravitation").

[155] Newton introduced systematic discrepancies in *Liber I De motu corporum* (summer 1685–winter 1685/6) (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 369–383), and, in the first edition of the *Principia* (1687), he introduced the general dependency that the periodic time, T, varies as the n^{th} power of the radius R, if and only if, the centripetal force varies as R^{1-2n} as a corollary to Proposition IV, on the one hand, and the two-body correction for the harmonic law, on the other.

[156] Thus the term "*ex hypothesi* inference-ticket" refers to the format of a proposition taken in isolation. It does not imply that Newton's method was hypothetico-deductive, as is clear from the discussion that follows.

of the micro-particles composing that body, and vice versa.[157] I refer to propositions of this type as *micro-macro inference-tickets*, as they license conclusions about the inverse-square centripetal forces of the micro-particles that constitute a macroscopic body from the overall inverse-square centripetal force exerted by that body. Examples of such inference-tickets are Propositions LXXI–LXXVI, Book I.

I realize that, at this point, my discussion of the different types of propositions must appear quite abstract. Their significance and difference will be successively clarified in the discussion of specific propositions of Book I which follows below.

2.5.1 Inferring Inverse-Square Centripetal Forces from Exact or Quam Proxime Keplerian Motion[158]

Examples of inference-tickets of type 1 and type 2 can be found in Propositions I–II and Corollaries 2–3 to Proposition III, respectively. In Propositions I–III (Section 2, Book I) Newton dealt with the dynamical implications of Kepler's second "rule" – as this law was called and considered at the time.[159] In Propositions I–II, Newton argued that – given the laws of motion – a centripetal force is a necessary and sufficient causal condition for the planarity of the orbit and Kepler's area rule, i.e. he argued that the areas of a body described by radii drawn to an unmoving centre of force lie in a fixed plane and are proportional to the times (deductive direction 1; *sufficient cause*), and, conversely, that a body, which moves along a curved line described in a plane and by a radius drawn to a point describes areas about that point that are proportional to the times, is urged by a centripetal force

[157] Cf. Smith, "The Methodology of the *Principia*", pp. 143, 150.

[158] Here I shall not further discuss Sections 3–10 of Book I of the *Principia*. In Section 3 "The Motion of Bodies in Eccentric Conic Section" Newton argued that bodies moving in a conic section (Proposition XI: for ellipses; Proposition XII: for hyperbola; Proposition XIII: for parabola) under a centrally directed force to a focus will be drawn inversely as the square of the distance and a general solution to the inverse Kepler-problem is provided. The geometry of conic sections is further developed in Sections 4 and 5 (see, furthermore, Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 229–299). Section 6 deals with the determination of the position of an orbiting body at any given time. In Section 7 the ascent and descent of rectilinear motion under an inverse-square centripetal force is studied and it is shown that Galileo's theory of falling bodies is a limiting case of the theory of universal gravitation (esp. Newton, *The Principia*, p. 521). In Section 8 Newton seeks to determine the general problem of finding the orbits in which bodies revolve when acted upon by whatever centripetal forces. Section 9 addresses the motion of the (moon's) apsides. Finally, Section 10 deals with the motion of simple pendulums and it is shown that Huygens' account of pendulums under uniform gravity is a limiting case of the theory of universal gravitation (see Corollary 2 to Proposition LII, Book I in Newton, *The Principia*, p. 555; see, furthermore, Smeenk and Smith, "Newton on Constrained Motion: A Commentary on Book I Section 10 of the *Principia*").

[159] In the *scholium*, which follows, Newton notes that "the case of corol. 6 holds for our heavenly bodies (as our compatriots Wren, Hooke, and Halley have also found out independently)" (Newton, *The Principia*, p. 452).

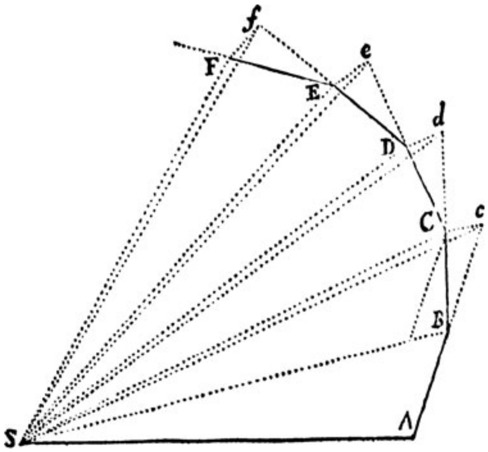

Fig. 2.3 Original figure accompanying Proposition I, Book I. Reproduced from *PrE*3, p. 40

tending toward that point (deductive direction 2; *necessary cause*). Mathematically integrating Kepler's rules with the idea of attraction was a highly innovative feature of the *Principia*, which others had failed to accomplish.[160] Proposition I states:

> *The areas which bodies made to move in orbits describe by radii drawn to an unmoving center of forces lie in unmoving planes and are proportional to the times.*[161]
>
> *Areas, quas corpora in gyros acta radiis ad immobile centrum virium ductis deseribunt, & in planis immobilibus consistere, & esse temporibus proportionales.*[162]

Its proof proceeds as follows (see Fig. 2.3). First, Newton divided the time into equal parts. Let, furthermore, *c*C be parallel to BS and meet BC at C. In the first part of time a body describes by its inherent force the straight line AB. In the second part of time the body would, by Law I, go straight on to *c* describing B*c*, but, as it orbits around S (by hypothesis), it is deflected from its original rectilinear inertial path by a centripetal force acting along BS (by Corollary 1[163] to the laws of motion) and describes BC. Then body will be found at C in the same plane as triangle ASB. Since SB and C*c* are parallel, triangle SBC will be equal to SB*c* (since both their height and base is equal) and thus to SAB. Hence the body describes an equal area in an equal amount of time. By similar argument this can be extended to all triangles of the polygon. Newton then argued that, if the number of triangles as well as

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160 Meli, *Thinking with Objects*, p. 259. See furthermore, Nauenberg, "Hooke's and Newton's Contributions to the Early Development of Orbital Dynamics and the Theory of Universal Gravitation"; Pugliese, "Robert Hooke and the Dynamics of Motion in a Curved Path"; and, Gal, *Meanest Foundations and Nobler Superstructures*, Chapters 1 and 3.

161 Newton, *The Principia*, p. 444.

162 Koyré, Cohen and Whitman, eds., *Principia Mathematica*, I, p. 88.

163 Which states: "A body acted on by [two] forces acting jointly describes the diagonal of a parallelogram in the same time in which it would describe the sides if the forces were acting separately." (Newton, *The Principia*, p. 417).

their width is increased infinitely ("augeatur jam numerus & minuatur latitudo triangulorum in infinitum") and, correspondingly, their ultimate perimeter ADF will be curved[164] (by Corollary 4 to Lemma III[165]), a body perpetually[166] drawn ("perpetuò tetrahitur") back from the tangent of the curve by a centripetal force towards S will describe equal areas in equal times. In this way, Newton reduced a discontinuous motion along the sides of a polygon is thus to a continuous motion along a smooth orbital path.[167] In a letter to Locke,[168] Newton commented that when the moments of time are diminished in length and increased in number *in infinitum* "the [discontinued[169]] impulses or impressions of the attraction may become continuall."[170]

Let us now turn to Proposition II:

> *Every body that moves in some curved line described in a plane and, by a radius drawn to a point,*[171] *either unmoving or moving uniformly forward with a rectilinear motion, describes*

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[164] On Newton's different measures of curvature, see Brackenridge, "Newton's Mature Dynamics: Revolutionary: Revolutionary or Reactionary"; id., "The Critical Role of Curvature in Newton's Developing Dynamics"; id., *The Key to Newton's Dynamics, The Kepler Problem and the* Principia; id., "The role of curvature in Newton's Dynamics"; and, id., "Newton's Mature Dynamics: A Crooked Path made Straight".

[165] Newton, *The Principia*, p. 434.

[166] Or, "incessantly" or "continually" (Newton, *Correspondence*, V, p. 398).

[167] Can, however, a continuous force be approximated by as a limit of discontinuous impulsive force as the time interval shrinks to zero? This point has often been debated. D. T. Whiteside dismissed its validity (Whiteside, "The Prehistory of the *Principia* from 1664 to 1686", p. 30; cf. Aiton, *The Vortex Theory of Planetary Motions*, pp. 103–104), while Nauenberg recently defended it (Nauenberg, "Kepler's Area Law in the *Principia*", esp. pp. 445–446, 451–452). However, an adequate assessment of the situation is not solely contingent on whether we grant Newton's assumption that a continuous force can be approximated as a limit of discontinuous impulsive force: for even if we grant Newton this step, his limiting procedure does not prove what he claimed it proved, namely that *all* centripetal forces produce orbits that lie in a fixed plane, as Pourciau has shown (Pourciau, "Newton's Argument for Proposition 1"). First of all, Newton could not independently prove the impulse assumption and the claim that areas involved lie on the same plain (the one is required to establish the other and this observations holds for any polygonal approximation) (ibid., esp. the reconstruction of Newton's proof on pp. 277–279, cf. id., "The Importance of Being Equivalent", pp. 314–316). Thus, Newton could only establish that the areas, which bodies made to move in orbits describe radii drawn to an unmoving center of forces *and* which lie in unmoving planes, are proportional to the times. Furthermore, and even more seriously, Newton's arguments for Proposition I clearly involves impulse motions and limits of impulse motions. However, the conclusion is supposed to be valid for *all* centripetal forces (impulse and continuous ones). Newton's proof of Proposition I can, however, be restored by introducing some additional conditions on the smoothness of a specific curve (see Pourciau, "Newton's Argument for Proposition 1", pp. 291–295).

[168] Newton to Locke, March 1689/90, Newton, *Correspondence*, III, pp. 71–77.

[169] Ibid., p. 71.

[170] Ibid., p. 72.

[171] I.e. every body that moves on a curved line and by a radius drawn to an unmoving central point lies on the same unmoving plane as that central point (Pourciau, "Proposition II (Book I) of Newton's *Principia*", pp. 14–15).

> *areas around that point proportional to the times, is urged by a centripetal force tending toward that same point.*[172]

> *Corpus omne, quod movetur in linea aliqua curva in plano descripta, & radio ducto ad punctum vel immobile, vel motu rectilineo uniformiter progrediens, describit areas circa punctum illud temporibus proportionales, urgetur a vi centripeta tendente ad idem punctum.*[173]

Proposition II, which Newton introduced for the first time in the initial revise of *De motu*,[174] basically amounts to stating that a body moving along a curved line described in a plane which describes equal areas in equal times requires a centripetal force exerted on that body.[175] Thus, it sets out to demonstrate the converse of Proposition I. The proof proceeds as follows (again see Fig. 2.3). By Law I we know that a body that moves in a curved line is deflected from a rectilinear course by some force acting on it. Then, by Law II, the force by which the body is deflected from rectilinear course and in equal times is made to describe around an immovable point S the equal minimally small triangles SAB, SBC, SCD, etc. acts at B along a line parallel to *c*C, i.e. along the line BS; at place C, parallel along the line *d*D, i.e. along CS, etc. Therefore, it always acts along lines tending towards S – here Newton again presupposed the equivalence between a continuous force and its corresponding limit of discontinuous impulsive forces.[176] By Corollary 5[177] this holds for a one-body system at rest or one describing uniform and rectilinear motion.

Propositions I–II thus jointly establish that:

> Centripetal force by which a body is drawn to an unmoving centre of force is directed *exactly* to this centre, *if and only if*, that body describes equal areas, which lie in a fixed plane, in equal times *exactly*.

In Corollaries 2–3 to Proposition III, Newton showed that an overall centripetal force directed *quam proxime*, i.e. as most closely as possible, to its attracting centre

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172 Newton, *The Principia*, p. 446.

173 Koyré, Cohen and Whitman, eds., *Principia Mathematica,* I, p. 92.

174 Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 124–127.

175 Thus a centripetal force requires whenever a body describes equal areas in equal times (cf. Pourciau's "existence theorem for centripetal motions" (Pourciau, "Proposition II (Book I) of Newton's *Principia*", p. 36)).

176 Proposition II also required the assumption that the resting deflections in the limit motion are directed toward the central point whenever each vertex of every polygonal motion has a resting deflection directed toward the central point (Pourciau, "Proposition II (Book I) of Newton's *Principia*", pp. 23–24). A restored proof of Proposition II can be found in ibid., pp. 26–27.

177 Which states "*When bodies are enclosed in a given space, their motions in relation to one another are the same whether the space is at rest or whether it is moving uniformly straight forward without circular motion.*" (Newton, *The Principia*, p. 423).

is a necessary and sufficient cause for Kepler's area rule to hold *quam proxime*.[178] In the scholium to Proposition II, Newton stated that:

> A body can be urged by a centripetal force compounded of several forces. In this case the meaning of the proposition is that the force which is compounded of all the forces tends towards point S. Further, if some force acts continually along a line perpendicular to the surface described, it will cause the body to deviate from the plane of its motion, but it will neither increase nor decrease the quantity of the surface-area described and is therefore to be ignored in the compounding of forces.[179]

> Urgeri potest corpus a vi centripeta composita ex pluribus viribus. In hoc casu sensus propositionis est, quod vis illa quæ ex omnibus componitur, tendit ad punctum *S*. Porro si vis aliqua agat perpetuo secundum lineam superficiei descriptæ perpendicularem; hæc faciet ut corpus deflectatur a plano sui motis: sed quantitatem superficiei descriptæ nec augebit nec minuet, & propterea in compositione virium negligenda est.[180]

In Proposition III, Newton argued that "*[e]very body that, by a radius drawn to the center of a second body moving in any way whatever, describes about that center areas that are proportional to the times is urged by a force compounded of the centripetal force tending toward that second body and of the whole accelerative force by which that second body is urged* [in the same direction along a parallel line[181]]."[182] Its proof proceeds as follows:

> Let the first body be L, and the second T; and (by corol. 6 of the laws[183]) if each of the two bodies is urged along parallel lines by a new force that is equal and opposite to the force by which body T is urged, body L will continue to describe about T the same areas as before; but the force by which body T was urged will now be annulled by an equal and opposite force, and therefore (by law 1) body T, now left to itself, either will be at rest or will move uniformly straight forward; and body L, since the difference of the forces [i.e., the remaining force] is urging it, will continue to describe areas proportional to the times

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[178] In fact, when working on *De motu*, Newton became aware that Kepler's area law holds only *quam proxime*, not because of imprecision of observation but because the celestial motions are intrinsically complex. Simultaneously taking in account all causes of planetary motion "exceeds the force of any human mind" (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 78; Smith, "The Methodology of the *Principia*", pp. 153–154).

[179] Newton, *The Principia*, pp. 447–448.

[180] Koyré, Cohen and Whitman, eds., *Principia Mathematica*, I, pp. 93–94.

[181] Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 126, footnote 77.

[182] Newton, *The Principia*, p. 448. Original: "*Corpus omne, quod radio ad centrum alterius utcunque moti ducto describit areas circa centrum illud temporibus proportionales, urgetur vi compositia ex vi centripeta tendente ad corpus illid alterum, & e vi omni acceleratrice qua corpus illud alterum urgetur.*" (Koyré, Cohen and Whitman, eds., *Principia Mathematica*, I, p. 94). For *quam proxime* elliptical orbits, the inverse-square law does not generally hold *quam proxime* (Smith, "From the Phenomena of the Ellipse to an Inverse-square Force: Why Not?", pp. 35–42)!

[183] Which states: "*If bodies are moving in any way whatsoever with respect to one another and are urged by equal accelerative forces along parallel lines, they will all continue to move with respect to one another in the same way as they would if they were not acted by such forces*. For those forces, by acting equally (in proportion to the quantities of the bodies to be moved) and along parallel lines, will (by law 2) move all the bodies equally (with respect to velocity), and so will never change their positions and motions with respect to one another." (Newton, *The Principia*, p. 423).

> about T. Therefore, the difference of the forces tends (by theor. 2) toward the second body T as the center.[184]

Hence, as Newton concluded in Corollary 1, "if a body L, by a radius drawn to another body T, describes areas proportional to the times, and from the total force (whether simple or compounded of several forces according to corol. 2 of the laws) by which body L is urged there is subtracted (according to the same corol. 2 of the laws) the total accelerative force by which body T is urged, the whole remaining force by which L is urged will tend toward body T as center."[185] On the basis of Corollary 6 to the laws of motion, Newton indicated that if a body is drawn equally and along parallel lines toward a third body, *S*, the area law would hold exactly.[186] Suppose that *S* is placed at a very large distance from *L* and *T*. In this case, the force by which *L* is drawn toward the centre of *S* can be considered as being nearly parallel to the force by which *T* is drawn toward *S*. If, furthermore, the force exerted by *S* is small in comparison to the force by which *L* is drawn toward *T*, *L* will not describe equal areas in equal times exactly but as most closely as possible, for *L* moves slightly more swiftly near "conjunction" or "opposition" and more slowly near the "quadratures." Corollaries 2–3 to Proposition III establish that, if the areas are as most closely as possible proportional to the times, the remaining force will tend toward body *T* as most closely as possible, and vice versa. For, if the additional force only slightly accelerates or slows down *L*'s exact description of equal times in equal periods around *T*, *L* will be drawn toward *T* as most closely as possible, and conversely. Thus, Corollaries 2–3 to Proposition III jointly establish:

> The overall centripetal force by which a body is drawn towards a second body is directed to this body *as most closely as possible*, *if and only if*, that body describes equal areas in equal times *as most closely as possible*.

At this point, Newton himself did not yet specify constellations of bodies satisfying Kepler's area rule *quam proxime* – this was actually done much later in Book I, namely in Proposition LXV (see Section 2.5.3). Corollaries 2–3, which are derived from theory, thus licence the inference of a centripetal force tending *quam proxime* towards its attractive centre from described areas that are *quam proxime* proportional to the times. In the scholium following Propositions I–III, Newton concluded:

> Since the uniform description of areas indicates the center towards which that force is directed by which a body is most affected and by which it is drawn away from rectilinear motion and kept in orbit, why should we not in what follows use uniform description of areas as a criterion for a center about which all orbital motion takes place in free spaces?[187]

> Quoniam æquabilis arearum descriptio index est centri, quod vis illa respicit, qua corpus maxime afficitur, quaque retrahitur a motu rectilineo, & in orbita sua retinetur; quidni

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[184] Ibid., p. 448; Chandrashekar, *Newton's Principia for the Common Reader*, p. 71.

[185] Newton, *The Principia*, p. 448.

[186] Cf. Case 1, Proposition LXVI (ibid., pp. 570–572).

[187] Ibid., p. 449.

> usurpemus in sequentibus æquabilem arearum descriptionem ut indicem centri, circum quod motus omnis circularis in spatiis liberis peragitur?[188]

In Propositions I–III, Newton related an astronomical phenomenon (the description of the area rule by celestial bodies) to a theoretical parameter (centripetal force): the area rule counted as a measure of the centripetal force. In the way outlined above, Newton argued that a centripetal force is a necessary (Proposition II) and sufficient (Proposition I) cause for the fixed plane property together with the area law. Similarly, in Corollaries 2–3 to Proposition III, i.e. the *quam proxime* counterparts of Propositions I–II, Newton argued that an overall centripetal force directed as most closely as possible towards a centre is a necessary and sufficient cause of the area law holding as most closely as possible.

The overarching question that Newton was addressing in Propositions I–III can be summarized as follows: *What are, given the definitions and the laws of motions, the necessary and sufficient causes of the area rule?* As a way of reducing the risk of wild speculation, Newton demanded that the causes of orbital motion ought to be derivable from the laws of motion, i.e. from principles that have already shown their merit in natural-philosophical inquiry. Newton, furthermore, demanded that the causes adduced in natural philosophy should not only be shown to entail their effects, but also that these effects are shown to be necessarily produced by those very causes. I admit that nowhere did Newton explicitly ascribe to the notion of necessity which I attribute to him. However, his logic of demonstration presupposed such notion. For instance, in Proposition II, Newton proved that a centripetal force is a necessary cause for Kepler's area rule, or, in other words, that Kepler's area law requires a centripetal force.[189]

Newton required that there be a systematic dependency between the theoretical parameters and the phenomena they serve to explain.[190] This requirement by itself surpasses a strict hypothetico-deductive methodology: the demand that a centripetal force (or an overall centripetal force directed as most closely as possible toward its centre of force) should be both a necessary and sufficient cause for bodies lying in a fixed plane and describing equal areas in equal times exactly (or as most closely as possible) is to guarantee that such motion is produced by (overall) centripetal forces and (overall) centripetal forces alone.

A further element surpassing hypothetico-deductivism lies in the fact that Newton's causal inference-tickets anticipate, accommodate and explain (small) deviations from exact time-area proportionality by means of the *quam proxime* variants, which are in their turn deduced from theory, i.e. from the laws and definitions

[188] Koyré, Cohen and Whitman, eds., *Principia Mathematica*, I, p. 96.

[189] Recall that proving that a cause, *C*, is a necessary cause of an effect, *E*, entails proving that *E* implies *C* (cf. the necessary direction proved in Proposition II of Book I). (Proving that a cause, *C*, is a sufficient cause of an effect, *E*, entails proving that *C* implies *E* (cf. the sufficient direction proved in Proposition I of Book I).) Since Newton is concerned with proving that *E* implies *C*, he is in principle concerned with proving that *C* is the necessary cause of *E*.

[190] Harper, "Isaac Newton on Empirical Success and Scientific Method", p. 55.

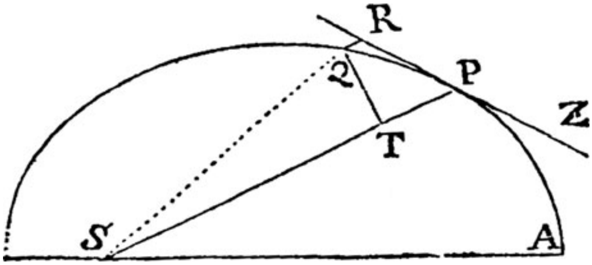

Fig. 2.4 Original figure accompanying Proposition VI, Book I. Reproduced from *PrE*3, p. 47

of motions. Let us contrast this with what would happen in a hypothetico-deductivist setting. A hypothetico-deductivist would begin by coming up with a theoretical proposition that is sufficient to explain certain phenomena. The problem is then that the consequences deduced from this theoretical proposition will not hold exactly in the physical world. In order to remedy this situation, a hypothetico-deductivist will either leave the situation as it is or introduce an ad hoc explanation that serves to explain the discrepancy between theory and observation (e.g., the introduction of some disturbing factor). Instead of proceeding in the hypothetico-deductive way outlined above, Newton was able to guarantee by the systematic dependency which he had established in Corollaries 2–3 to Proposition III, which are derived from the laws of motion, that the motions of bodies describing equal areas in equal times as most closely as possible are necessarily produced by an overall centripetal force urging these bodies as most closely as possible towards a centre of force. Instead of introducing ad hoc explanations to account for such deviations, Newton required that deviations, including small deviations, should be accounted for by theory as far as possible.

Proposition VI, furthermore, relates centripetal forces to the deflection from inertial motion in a given time and thus establishes a measure of a centripetal force (see Fig. 2.4):

> *If in a nonresisting space a body revolves in any orbit about an immobile center and describes any just-nascent arc in a minimally small time, and if the sagitta of the arc is understood to be drawn so as to bisect the chord and, when produced, to pass through the center of forces, the centripetal force in the middle of the arc* [not depicted on Fig. 2.4] *will be as the sagitta directly and as the time [i.e., as the square of the time] twice inversely.*[191]
>
> *Si corpus in spatio non resistente circa centrum immobile in orbe quocunque revolvatur, & arcum quemvis jamjam nascentem tempore quam minimo describat, & sagitta arcus duci intelligatur, quæ chordam bisecet, & producta transeat per centrum virium: erit vis centripeta in medio arcus, ut sagitta directe & tempus bis inverse.*[192]

[191] Newton, *The Principia*, pp. 453–454. See the useful discussion in Guicciardini, *Isaac Newton on Mathematical Certainty and Method*, pp. 244–245.

[192] Koyré, Cohen and Whitman, eds., *Principia Mathematica*, I, pp. 102/105.

As Q approaches P, it follows by Corollary 4 to Proposition I[193] that the nascent sagitta QR is proportional to the centripetal force – in this case, QR converges to the centre of force, i.e. tends to parallelism with SP – and by Corollaries 2 and 3 to Lemma 11[194] it follows that, under the same conditions, QR is proportional to the square of the time. Therefore the centripetal force will become as the sagitta directly and the square of the time inversely.[195] The proof requires a subtle limiting procedure which Newton did not explicitly develop.[196] As we know that the area law holds, the area SQP is proportional to the time. As, moreover, under the same conditions, the vanishing chord QP and arc QP become ultimately equal, the area SQP can be considered equal to the triangular area SQP. As the area of the triangular area SQP is given by (SP × QT) divided by 2, SQP is likewise given. Since we have previously shown that the centripetal force is as the sagitta directly and the square of the time inversely, the centripetal force is proportional to $\frac{\mathrm{QR}}{(\mathrm{SP}\times\mathrm{QT})^2}$.

2.5.2 The Harmonic Rule

Corollary 7 to Proposition IV, and Corollary 1 to Proposition XLV of Book I express "systematic discrepancies:"[197] they allow for a systematic dependency between deviations from an exact mathematical solution – in this case the harmonic rule – and variations in the corresponding theoretical parameters – in this case the power law of the distance to which the centripetal force is proportional. In Proposition IV Newton argued that the centripetal forces on bodies which describe uniform circular motion on different circles about the same central point are proportional to the "squares of the arcs" described and inversely proportional to the radius of that circle.[198] Two important corollaries to Proposition IV are:

> COROLLARY 6. If the periodic times are as the 3/2 powers of the radii, and therefore the velocities are inversely as the square roots of the radii, the centripetal forces will be inversely as the squares of the radii; and conversely.[199]

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[193] Which states: "The forces by which any bodies in nonresisting spaces are drawn back from rectilinear motions and are deflected into curved orbits are to one another as those sagittas of arcs described in equal times which converge to the center of forces and bisect the chords when the arcs are decreased indefinitely." (Newton, *The Principia*, p. 446).

[194] Ibid., p. 440.

[195] Ibid., p. 454.

[196] It is required to prove that the sagitta QR is ultimately equal to the sagitta "in the middle of the arc" (for its rigorous demonstration, see Pourciau, "Force, Deflection, and Time: Proposition VI of Newton's *Principia*", p. 157, cf. pp. 159–160).

[197] E.g., Harper, "Measurement and Approximation", p. 277.

[198] Newton, *The Principia*, p. 449–450, cf. pp. 452–453.

[199] This corollary is easily understood when using a slightly anachronistic proof. In Proposition IV, Newton basically established that $\mathrm{F} = \frac{k.v^2}{r}$. Since v equals $\frac{2.\pi.r}{t}$, $\mathrm{F} = \frac{k.4.\pi^2.r^2}{t^2.r}$. Multiplied by $\frac{r}{r}$, $\mathrm{F} = \frac{\mathrm{k}.4.\pi^2.r^3}{t^2.r^2}$. Since $\frac{r^3}{t^2}$ is a constant according to Kepler's third law, we can conclude: $\mathrm{F} = \frac{\text{constant}}{r^2}$. Newton also derived the harmonic law for ellipses in Proposition XV (ibid., p. 468).

COROLLARY 7. And universally, if the periodic time is as any power R^n of the radius R, and therefore the velocity is inversely as the power R^{n-1} of the radius, the centripetal force will be inversely as the power R^{2n-1} of the radius; and conversely.[200]

While Corollary 6 specifies the conditions under which Kepler's harmonic rule would hold exactly,[201] Corollary 7 offers a universal systematic dependency which makes the harmonic ratio a measure of the power law of the distance to which the centripetal forces, maintaining bodies in orbit, are proportional. Corollary 7 states that, if the periodic time T varies as the n^{th} power of the radius R, the centripetal force will vary as the $1–2n^{\text{th}}$ power of the R, and vice versa – note that, if T varies as the 3/2 power of R, the centripetal force varies as the 1–2(3/2) or –2 power of R. Thus:

(1) T varies as the n^{th} power of the radius R, *if and only if*, centripetal force varies as R^{1-2n}.

By consequence, if T varies to an n^{th} power of R smaller than 3/2, the centripetal force will fall off slower than the –2 power of R; if the n^{th} power of R is larger than 3/2, the centripetal force will fall off faster than the –2 power of R.[202] And, *in general*, T's varying to whatever power of R, becomes indicative of the power law of the distance to which the centripetal forces acting on bodies are proportional. Thus: T's varying to whatever power of R, becomes indicative of about the extent to which the centripetal force (acting on a body) deviates from or agrees to inverse-square proportionality.

Likewise, in Corollary 1 to Proposition XLV (Section 10), Newton provided a one-body system correction for apsidal motion which enabled him to relate the n^{th} power of the distance to which the centripetal force varies to the motion of

[200] Ibid., p. 451.

[201] Harper, "Newton's Argument for Universal Gravitation", p. 177. What Newton in fact had implicitly shown was that Kepler's harmonic rule cannot hold in the empirical world: "Up to this point I have been setting forth the motions of bodies attracted toward an immovable center, such as, however, hardly exist in the natural world [quale vix extat in rerum natura]. For attractions are always directed toward bodies, and – by the third law – the actions of attracting and attracted bodies are always mutual and equal; so that if there are two bodies, neither the attracting nor the attracted body can be at rest, but both (by corol. 4 of the laws) revolve about a common center of gravity as if by a mutual attraction; and if there are more than two bodies that either are all attracted by and attract a single body or all attract one another, these bodies must move with respect to one another in such a way that the common center of gravity either is at rest or moves uniformly straight forward." (Newton, *The Principia*, p. 561). Correspondingly, in Proposition LX Newton provided a two-body correction for the harmonic law distances given the masses of the two bodies for the elliptical one-body system he had developed earlier in Proposition XV (ibid., p. 468). If r' is the corrected distance for the two-body system of the principal axis of an ellipse, r is the harmonic distance for the one-body system of the principal axis of an ellipse, m_S is the mass of the greatest body S, and, finally, m_P is the mass of the smaller body P, then: $\frac{r'}{r} = \frac{m_S+m_P}{\left(m_S\times\left(m_S+m_P\right)^2\right)}1/3$ (ibid., p. 564).

[202] Cf. Harper, "Newton's Argument for Universal Gravitation", p. 177.

the apsides in degrees ($p°$) per revolution, as follows: $n = \left(\frac{360°}{360°+p°}\right)^2 - 3$, and conversely.[203] If $p°$ equals zero, then n equals –2. In other words, if a body has null procession the centripetal force by which it is drawn varies exactly as the inverse-power of the distance. Thus:

(2) $\frac{p°}{\text{revolution}}$, *if and only if*, centripetal force varies as $R^{(360°/(360°+p°))^2-3}$.

Thus, in general, degrees of precession become indicative of the extent to which the centripetal force (acting on this body) deviates from or agrees to inverse-square proportionality.

By means of the systematic dependencies expressed by (1) and (2) deviations from exact mathematical conditions can be detected in the empirical world.[204] In other words, in the context of Book III, they function as detecting devises for discrepancies from exact mathematical conditions. In the *Principia* deviations from exact conditions typically became the object of natural-philosophical inquiry.[205] Such systematic discrepancies, which are derived from theory, show yet again that Newton's methodology differed from a hypothetico-deductive methodology, for, not only do they stipulate the physico-mathematical conditions under which a centripetal force will vary exactly as R^{-2}, they also provide information about the force law characterizing the centripetal forces acting on orbiting bodies whose *T*'s do *not* vary as the 3/2 power of *R* or which do *not* have zero precession.[206]

2.5.3 Many-Body Systems

Propositions LXV and LXIX are examples of *ex hypothesi* inference tickets. As we have seen, in Corollaries 2–3 to Proposition III, Newton had shown that an overall centripetal force urging a body as most closely as possible to a centre of force is a necessary and sufficient cause for the area law holding as most closely as possible. Proposition LXV showed, moreover, that bodies which are drawn by inverse-square

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[203] Newton, *The Principia*, pp. 543–544; Harper, "Newton's Argument for Universal Gravitation", p. 180.

[204] I.e. they enable, what George E. Smith and William L. Harper call, "theory-mediated" measurements (Smith, "The Methodology of the *Principia*", p. 144; Harper, "Newton's Methodology", p. 44; and, id., "Isaac Newton on Empirical Success and Scientific Method", pp. 55, 57).

[205] Richard S. Westfall correctly observed that "Newton enlarged the definition of science to include those very perturbations by which material phenomena diverge from the ideal patterns that had represented the object of science to an earlier age. The *Principia* submitted the perturbations themselves to quantitative analysis, and it proposed the exact correlation of theory with material event as the ultimate criterion of scientific truth." (Westfall, "Newton and the Fudge Factor", p. 751; cf. Smith, "The Methodology of the *Principia*", p. 155).

[206] Cf. Smith, "The Methodology of the *Principia*", p. 144; Harper, "Measurement and Approximation", pp. 277–278; and, id., "Newton's Methodology", pp. 46–49.

centripetal forces are able to describe areas proportional to the times *quam proxime* under specific configurations:

> *More than two bodies whose forces decrease as the squares of the distances from their centers are able to move [moveri posse] with respect to one another in ellipses, by radii drawn to the foci, are able to describe areas proportional to the times very nearly.*[207]
>
> *Corpora plura, quorum vires decrescunt in duplicate ratione distantiarum ab eorundem centris, moveri posse inter se in ellipsibus; & radiis ad umbelicos ductis areas describere temporibus proportionales quam proxime.*[208]

Newton demonstrated that only slight perturbations will occur from Kepler's area rule in the following two configurations: (1) *case 1*, in which several lesser bodies revolve around a greater body at various distances from it (which in the context of Book III corresponds to the motions of the primary planets around the sun); and (2) *case 2*, which involves a system of several smaller bodies, or any other system of two bodies revolving around each other, that moves uniformly straight forward and at the same time is urged sideways by the force of another very much greater body situated at a great distance (which in the context of Book III corresponds to the motions described by the secondary planets around their corresponding planets, which in their turn revolve about the sun).[209] Such systems, as Newton added, can in principle be extended to analogous configurations indefinitely.[210] In the first case, the lesser bodies, which are so small in comparison to the greater body that it is never sensibly distant from their common centre of gravity, will describe areas proportional to the times insofar as errors introduced by the departure from the greater body from the common centre of gravity and the mutual interactions between the lesser bodies are neglected.[211] In the second case, the smaller bodies revolving around the greater body can be considered as one body because of the slight distance of those parts from one another. In this case, the smaller bodies, conceived as one body, describe areas proportional to the times except for slight perturbations produced by the inequality of the distances between those parts.[212] Proposition LXV is accompanied by three corollaries:

> COROLLARY 1. In case 2, the closer the greater body approaches to the system of two or more bodies, the more motions of the parts of the system with respect to one another will be perturbed, because the inclinations to one another of the lines drawn from this

[207] Newton, *The Principia*, p. 568.

[208] Koyré, Cohen and Whitman, eds., *Principia Mathematica*, I, p. 275.

[209] He, furthermore, wrote: "The more the law of force departs from the law there supposed, the more the bodies will perturb their mutual motions; nor can it happen that bodies will move exactly in ellipses while attracting one another according to the law here supposed, except by maintaining a fixed proportion of distances one from another. In the following cases, however, the orbits will not be very different from ellipses." (Newton, *The Principia*, p. 568).

[210] Ibid., p. 569.

[211] Ibid., p. 568.

[212] Ibid., p. 569.

great body of those parts are now greater, and the inequality of the proportion is likewise greater.

COROLLARY 2. But these perturbations will be the greatest if the accelerative attractions of the parts of the system toward the greater body are not to one another inversely as the squares of the distances, especially if the inequality of this proportion is greater than the inequality of the proportion of the distances from the greater body. [. . .]

COROLLARY 3. Hence, if the parts of this system – without any significant perturbation – move in ellipses or circles, it is manifest that either these parts are not urged at all (except to a very slight degree indeed) by accelerative forces tending toward other bodies, or are all urged equally and very nearly along parallel lines.[213]

In Proposition LXIX Newton established that in a many-body system, in which several bodies attract one another by accelerative forces that are inversely as the squares of the distances, the absolute forces of the attracting bodies will vary as the mass of those attracting bodies:

If in a system of several bodies A, B, C, D, . . ., *some body* A *attracts all the others,* B, C, D, . . ., *by accelerative forces that are inversely as the squares of the distances from the attracting body; and another body* B *also attracts the rest of the bodies* A, C, D, . . ., *by forces that are inversely as the squares of the distances from the attracting body; then the absolute forces of the attracting bodies* A *and* B *will be to each other in the same ratio as the bodies [i.e., the masses]* A *and* B *themselves to which those forces belong* [*erunt absolutæ corporum trahentium* A, B *vires ad invicem, ut sunt ipsa corpora* A, B, *quorum sunt vires*].[214]

In systemate corporum plurium A, B, C, D, *&c. si corpus aliquod* A *trahit cætera omnia* B, C, D *&c. viribus acceleratricibus quæ sunt reciproce ut quadrata distantiarum a trahente; & corpus aliud* B *trahit etiam cæterea* A, C, D, *&c. viribus quæ sunt reciproce ut quadrata distantiarum a trahente: erunt absolutæ corporum trahentium* A, B *vires ad invicem, ut sunt ipsa corpora* A, B, *quorum sunt vires.*[215]

By hypothesis, at equal distances, the accelerative attractions of all bodies *B*, *C*, *D*, . . . toward *A* are equal to one another, and similarly, at equal distance, the accelerative attractions of all bodies *A*, *C*, *D*, . . . toward *B* are equal to each other. Also, at equal distances, the absolute attractive force of *A* (i.e. the strength of the accelerative field toward *A*) is to the absolute attractive force of *B* (i.e. the strength of the accelerative field toward *B*) as the accelerative attraction of all the bodies toward *A* is to the accelerative attraction of all the bodies toward *B*. Moreover, the accelerative attraction of *B* toward *A* is in the same proportion to the accelerative attraction of *A* toward *B*. Thus, at equal distances, $\frac{\text{accelerative attraction of } B \text{ toward } A}{\text{accelerative attraction of } A \text{ toward } B} :: \frac{\text{absolute attractive force of } A}{\text{absolute attractive force of } B}$ (**1**). As the motive force of *B* on *A* equals the accelerative force of *B* toward *A* times the mass of B and the motive force of *A* on *B* equals the accelerative force of *A* toward *B* times the mass of *A*, it follows that $\frac{\text{accelerative force of } B \text{ toward } A}{\text{accelerative force of } A \text{ toward } B} :: \frac{\frac{\text{motive force of } B \text{ on } A}{\text{mass of B}}}{\frac{\text{motive force of } A \text{ on } B}{\text{mass of A}}}$. As these motive forces are equal by

[213] Ibid., pp. 569–570.

[214] Ibid., p. 587.

[215] Koyré, Cohen and Whitman, eds., *Principia Mathematica*, I, pp. 296–297.

Law III, it follows that $\frac{\text{accelerative force of } B \text{ on } A}{\text{accelerative force of } A \text{ on } B} :: \frac{\text{mass of } A}{\text{mass of } B}$ **(2)**. By combining **(1)** and **(2)**, we establish that $\frac{\text{absolute attractive force of } A}{\text{absolute attractive force of } B} :: \frac{\text{mass of } A}{\text{mass of } B}$, which was to be proven.

Proposition LXIX, Book I, played a crucial role in the argument for universal gravitation: once Newton had shown that the planets attract each other by inverse-square centripetal forces, he could infer that the gravity towards all planets is proportional to their quantity of matter (Proposition VII, Book III).

2.5.4 The Attractive Forces of Spherical Bodies

Newton's propositions on the attractive force of spherical bodies (Section XII of Book I) play a crucial role in the argument for universal gravitation.[216] Their importance lies in the fact that they demonstrate that the overall inverse-square centripetal force toward the centre of a sphere results from the summation of the individual inverse-square forces of each of the particles composing that sphere.[217] Such physico-mathematical decomposition provides a basis for drawing transductive inferences. More precisely, they show that, *if* spheres attract each other by overall inverse-square centripetal forces, these overall forces result from the individual inverse-square centripetal forces of each of the parts composing those spheres, and vice versa. Implicitly, Newton was constraining transductive inferences by imposing the requirement on them that they should be based on well-defined physico-mathematical decompositions.[218] In order to generalize this result in the context of Book III Newton additionally required Rule III.[219] These propositions, furthermore, show that the inverse-square law *would hold exactly* between spheres that have symmetrically distributed densities – by implication, they show that the inverse-square law will not hold exactly between bodies that are not perfectly spherical or that have asymmetrically distributed densities.[220] Newton established these results on the basis of a closely knit sequence of (de)compositive moves, which I shall now briefly survey.

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216 See Chapter 3, Section 3.4.5.

217 As Newton had announced in the *scholium* concluding Section XI, Book I: "By these propositions we are directed to the anology between centripetal forces and the central bodies toward which those forces tend. For it is reasonable that forces directed toward bodies depend on the nature and the quantity of matter of such bodies, as happens in the case of magnetic attraction. And whenever cases of this sort occur, *the attractions of the bodies must be reckoned by assigning proper forces to their individual particles and then taking the sums of these forces*." (Newton, *The Principia*, p. 588 [italics added]).

218 In Chapter 4, Section 4.8, we will see that no such thing was available in *The Opticks*.

219 See Chapter 3, Section 3.2.

220 Accordingly, each deviation from the ideal conditions stipulated by Newton's above model is considered as physically significant (Smith, "Was Wrong Newton Bad Newton?", p. 133).

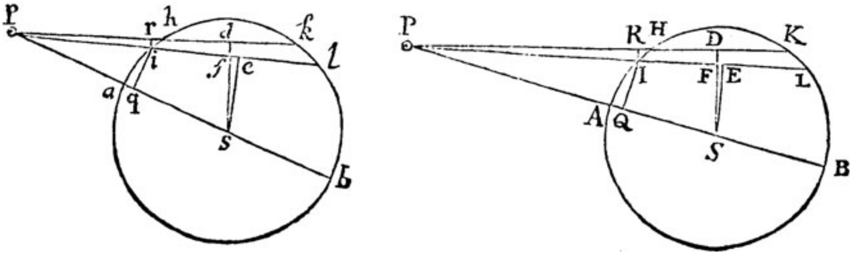

Fig. 2.5 Original figure accompanying Proposition LXXI, Book I. Reproduced from PrE_3, p. 190

Proposition LXXI[221] demonstrates that *a corpuscle placed outside a spherical surface* ("*extra sphæricam superficiem*"), when drawn toward each of the separate points of that spherical surface by equal inverse-square centripetal forces varying as the squares of the distance, is attracted to the centre of the sphere by a force inversely proportional to the square of its distance from that centre.[222] The following conditions are given (see Fig. 2.5):

> Let AHKB and *ahkb* be two equal spherical surfaces, described about centers S and *s* with diameters AB and *ab*, and let P and *p* be corpuscles located outside those spheres in those diameters produced. From the corpuscles draw lines PHK, PIL, *phk* and *pil*, so as to cut off from the great circles AHB and *ahb* the equal arcs HK and *hk*, And IL and *il*. And onto these lines drop perpendiculars SD and *sd*, SE and *se*, IR and *ir* of which SD and *sd* cut PL and *pl* at F and *f*. Also drop perpendiculars IQ and *iq* onto the diameters. Let angles DPE and *dpe* vanish; then, because DS and *ds*, ES and *es* are equal, lines PE, PF and *pe*, *pf* and the line-elements DF and *df* may be considered to be equal, inasmuch as their ultimate ratio, when angles DPE and *dpe* vanish simultaneously, is the ratio of equality [quippe quarum ratio ultima, angulis illis DPE, *dpe* simul evanescentibus, est æqualitatis].[223]

The first part of the proof consists in establishing some geometrical deductions.[224] From what is given, it follows that: $\frac{\mathrm{PI}}{\mathrm{PF}} :: \frac{\mathrm{RI}}{\mathrm{DF}}$ and $\frac{pf}{pi} :: \frac{df}{ri} :: \frac{DF}{ri}$. *Ex aequo*

[221] The proof for Proposition LXXI underwent only minor and, for our present purpose, irrelevant changes in the subsequent editions of the *Principia* (Koyré, Cohen and Whitman, eds., *Principia Mathematica*, I, pp. 299–301). Moreover, in the initial revise *De motu* (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 181–183) and in the printer's copy of the first edition of the *Principia* (Royal Society Ms. 69, ff. $172^r/173^r$) the proof was nearly identical. In all versions of the proof, the reference to Corollary 3 to Lemma VII is provided, which renders Weinstock's contention that the reference to Corollary 3 was possibly was a textual error highly unlikely (Weinstock, "Newton's *Principia* and the External Gravitational Field of a Spherically Symmetric Mass Distribution", p. 886).

[222] Newton, *The Principia*, p. 590.

[223] Ibid., pp. 590–591.

[224] Ibid., pp. 590–591. See furthermore Densmore, *Newton's* Principia*: The Central Argument*, pp. 358–372 and Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 183–185, footnote 184.

we get: $\frac{\mathrm{PI}\times pf}{\mathrm{PF}\times pi} :: \frac{\mathrm{RI}\times\mathrm{DF}}{\mathrm{DF}\times ri}\left(=\frac{\mathrm{RI}}{ri}\right)$. By Corollary 3 to Lemma VII,[225] it follows that: $\frac{\mathrm{PI}\times pf}{\mathrm{PF}\times pi} :: \frac{\text{arc IH}}{\text{arc } ih}$. From what is given it furthermore follows that $\frac{\mathrm{PI}}{\mathrm{PS}} :: \frac{\mathrm{IQ}}{\mathrm{SE}}$ and $\frac{ps}{pi} :: \frac{se}{iq} :: \frac{\mathrm{SE}}{\mathrm{iq}}$. From this, *ex aequo*, we get: **(2)** $\frac{\mathrm{PI}\times ps}{\mathrm{PS}\times pi} :: \frac{\mathrm{IQ}}{iq}\left(=\frac{\mathrm{IQ}\times\mathrm{SE}}{\mathrm{SE}\times iq}\right)$. *Ex aequo* **(1)** and **(2)**: $\frac{\mathrm{PI}^2\times pf\times ps}{pi^2\times\mathrm{PF}\times\mathrm{PS}} :: \frac{\text{arc IH}\times\mathrm{IQ}}{\text{arc } ih\times iq}$. As $\frac{\text{arc IH}\times\mathrm{IQ}}{\text{arc } ih\times iq} :: \frac{Surf.(\text{arc IH})}{Surf.(\text{arc } ih)}$, where *Surf.*(arc IH) and *Surf.*(arc *ih*) stand for the circular surface that the arc IH (or *ih*) will describe by the revolution of a semicircle AKB (or *akb*) around the diameter AB (or *ab*), we get $\frac{\mathrm{PI}^2\times pf\times ps}{pi^2\times\mathrm{PF}\times\mathrm{PS}} :: \frac{Surf.(\text{arc IH})}{Surf.(\text{arc } ih)}$. The surfaces by which *Surf.*(arc IH) and *Surf.*(arc *ih*) attract P and *p* are by hypothesis as the surfaces directly and the squares of the distances of these surfaces.[226] Thus, the force exerted by arc IH is to the force exerted by arc *ih* as $\frac{pf\times ps}{\mathrm{PF}\times\mathrm{PS}}\left(=\frac{\mathrm{PI}^2\times pf\times ps\times pi^2}{pi^2\times\mathrm{PF}\times\mathrm{PS}\times\mathrm{PI}^2}\right)$ **(3)**. Newton then decomposed the force exerted on the particle P (or *p*) in the direction PS (or *ps*) into two orthogonal components along the directions PQ (or *pq*) and QI (or *qi*). The force exerted in the direction PS is to the force exerted in the direction PS as their oblique parts, i.e. as PI to PQ and *pi* to *pq* or as PS to PF and *ps* to *pf* **(4)** (since angle PIQ $\approx$ angle PSF and angle *piq* $\approx$ angle *psf*).[227] Thus, *ex aequo* **(3)** and **(4)**, it follows that $\frac{F\mathrm{PS}(Surf.(\text{arc IH}))}{Fps(Surf.(\text{arc } ih))} :: \frac{pf\times ps\times\mathrm{PQ}(\approx\mathrm{PF})\times pi(\approx ps)}{\mathrm{PF}\times\mathrm{PS}\times pq(\approx pf)\times\mathrm{PI}(\approx\mathrm{PS})} :: \frac{ps^2}{\mathrm{PS}^2}$. By similar argument, the forces by which the surface described by the revolution of the arcs KL and *kl* attract the corpuscles will be as ps^2 to PS^2. The forces of all the spherical surfaces, which can be divided by taking *sd* always equal to SD and *se* to SE, will also do so. By composition ("per compositionem") the same ratio holds for the complete spherical surface.[228]

Proposition LXXII establishes that if toward each of the separate points of a sphere ("*ad sphæræ cujusvis punta*") there tend equal centripetal forces inversely proportional to the square of the distances from those points, and if both the density of the sphere and the ratio of the diameter to the distance of the corpuscle from the centre are given, the force by which *a corpuscle*, situated *at the surface of that sphere*, is attracted will be proportional to the semi-diameter of the sphere.[229] Suppose that two different particles are attracted separately by two

225 This corollary states that, if the ultimate ratios of arcs, chords and tangents are equal, these lines can be used interchangeably "in any argumentation concerning ultimate ratios" (Newton, *The Principia*, p. 436). *Pace* Weinstock, Erlichson has shown that Newton's derivation of this step can be licensed by Corollary 3 to Lemma VII (Erlichson, "Comment of 'Newton's *Principia* and the External Gravitational Field of a Spherically Symmetric Mass Distribution' by R. Weinstock", p. 276).

226 In the margin of one his copies of the second edition of the *Principia*, Newton corrected the text "ut ipsæ superficies applicatæ ad quadrata distantiarum suarum a corporibus" into "ut ipsæ superficies directe, et quadrata distantiarum superficierum a corporibus inverse" (Wren Library, NQ.16.196, p. 175).

227 Newton, *The Principia*, pp. 591–592.

228 Guicciardini, *Reading the* Principia, p. 69.

229 Newton, *The Principia*, p. 592.

different spheres, that their distances from the centres of these spheres are proportional to the respective diameters of these spheres, and that the two spheres are resolved into particles that are similar and similarly placed with respect to the corpuscles.[230] In this case, the attractions of the first corpuscle toward each of the separate particles of the first sphere will be to the attractions of the second corpuscle toward each of the separate analogous particles of the second sphere in a ratio compounded of the direct ratio of the number of particles (n) ("in ratione particularum directè") and the inverse-squared ratio of the distances (r^2). In other words, $\frac{F(\text{sphere1})}{F(\text{sphere2})} :: \frac{n(\text{sphere1}) \times r(\text{sphere2})^2}{n(\text{sphere2}) \times r(\text{sphere1})^2}$. As the numbers of particles are as the spheres, i.e. in the cubed ratio of the diameters: $\frac{n(\text{sphere1})}{n(\text{sphere2})} :: \frac{r(\text{sphere1})^3}{r(\text{sphere2})^3}$, and the distances are as the diameters, we obtain: $\frac{F(\text{sphere1})}{F(\text{sphere2})} :: \frac{r(\text{sphere1})^3 \times r(\text{sphere2})^2}{r(\text{sphere2})^3 \times r(\text{sphere1})^2} :: \frac{r(\text{sphere1})}{r(\text{sphere2})}$.[231] In this case, r equals the semi-diameter, which was to be proven.

On the basis of Propositions LXX and LXXII, Newton was able to establish in Proposition LXXIII that, if towards each of the separate points of any given sphere there tend equal centripetal forces decreasing in the squared ratio of the distances from those points, *a corpuscle* placed *inside the sphere* is attracted by a force proportional to the distance of the corpuscle from the centre of the sphere.[232] Suppose that a corpuscle placed inside the sphere *ABCD*, described about centre *S*, and that about the same centre *S* the inner sphere *PEQF* is described with radius *SP* (see Fig. 2.6). By Proposition LXX,[233] the spherical surface *ABCD* will not exert any influence on *P* since the equal and opposite attractions annul each other. Only the attraction of the inner sphere *PEQF* remains which is as the distance *PS* by Proposition LXXII.

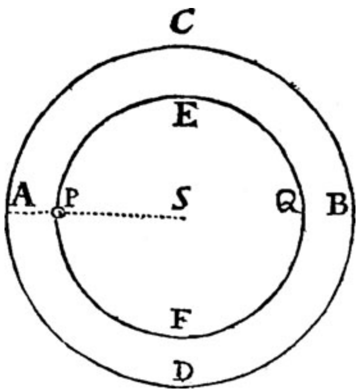

Fig. 2.6 Original figure accompanying Proposition LXXIII, Book I. Reproduced from *PrE*3, p. 192

230 Ibid.

231 See furthermore: Weinstock, "Newton's *Principia* and the External Gravitational Field of Spherically Symmetric Mass Distribution", pp. 887–888.

232 Newton, *The Principia*, p. 596.

233 Ibid., p. 590.

Proposition LXXIV demonstrates that, under the same conditions as in Proposition LXXIII, *a corpuscle* placed *outside a sphere* is attracted by a force inversely proportional to the square of the distance of the corpuscle from the centre of the sphere.[234] To prove this we *decompose* the sphere in innumerable concentric spherical surfaces ("distinguatur sphæra in superficies sphæricas innumeras concentricas").[235] By Proposition LXXI, we know that the attractions of each of the individual spherical surfaces will be inversely proportional to the square of the distance from the centre. By *composition* of these spherical shells, we obtain that the sum of all these individual attractions will come out in the same ratio, i.e. that a corpuscle placed outside a sphere is attracted by a total force that is inversely proportional to the square of the distance of the corpuscle from the centre of the sphere.

Proposition LXXV (concerning the attraction of "similar,"[236] i.e. homogeneous, spheres) demonstrates that, if all particles of $sphere_1$ are attracting all particles of a (homogeneous) $sphere_2$ with centripetal forces decreasing in the squared ratio of the distances from the points, $sphere_1$ will attract $sphere_2$ with a force inversely proportional to the square of the distance between the centres.[237] By Proposition LXXIV, it follows that the attraction of each particle of $sphere_2$ toward all the particles of $sphere_1$ varies inversely proportional to the square of the distance from the centre of the $sphere_1$; and, therefore, is the same "as if the total attracting force emanated from one single corpuscle situated in the centre of this sphere [i.e., the centre of $sphere_1$] [si vis tota attrahens manaret de corpusculo unico sito in centro hujus sphæræ]."[238] Moreover, the attraction of this imaginary centre of force, situated at the centre of $sphere_1$, is "as great as the attraction of the same corpuscle would be if, in turn, it were attracted by each of the individual particles of the attracted sphere [i.e., $sphere_2$] with the same force by which it attracts them [quanta foret vicissim attractio corpusculi ejusdem, si modo illud a singulis sphæræ attractæ particulis eadem vi traheretur, qua ipsa attrahit]."[239] By Proposition LXXIV, the attraction on that corpuscle would be inversely as the square of its distance from the centre of the $sphere_2$ and, therefore, $sphere_2$'s attraction, which is equal to the

[234] Ibid., p. 593.

[235] Newton commented: "The surfaces of which the solids are composed are here not purely mathematical, but orbs [or spherical shells] so extremely thin that their thickness is as null: namely, evanescent orbs of which the sphere ultimately consists when the number of those orbs is increased and their thickness diminished indefinitely. Similarly, when lines, surfaces, and solids are said to be composed of points, such points are to be understood as equal particles of a magnitude so small that it can be ignored." (ibid.).

[236] I.e. similar with respect to "the density of their matter and their attractive force [*quoad materiæ densitatem & vim attractivam*]," as Newton clarified in Proposition LXXVI (ibid., p. 595).

[237] Ibid., p. 594.

[238] Ibid.

[239] Ibid.

attraction of the corpuscle, is in the same ratio.[240] In Corollary 2 to Proposition LXXV, Newton added that this proportion will hold "when the attracted sphere also attracts," for "its individual points will attract the individual points of the other with the same force by which they are in turn attracted by them; and thus, since in every attraction the attracting point is as much urged (by law 3) as the attracted point, the force of the mutual attraction will be duplicated, the proportions remaining the same."[241]

In Proposition LXXVI Newton argued that, "*[i]f spheres are in any way non-homogeneous (as to the density of their matter and their attractive force) going from the center to the circumference, but are uniform throughout in every spherical shell at any given distance from the center, and the attractive force of each point decreases in the squared ratio of the distance of the attracted body*," the total force by which a sphere of this sort attracts another is inversely proportional to the square of the distance between their centres.[242] Now let there be a finite number of concentric homogeneous spheres *AB*, *CD*, *EF*, . . . and suppose that the addition of one or more inner hollow spheres to the outer ones composes a sphere more dense toward the centre, and, conversely, that their subtraction leaves it rarer ("sphæræ quotcunque concentricæ similares *AB*, *CD*, *EF*, &c. quarum interiores additæ exterioribus component materiam densiorem versus centrum, vel subductæ relinquant tenuiorem") (see Fig. 2.7). By Proposition LXXV, the homogeneous spheres *AB*, *CD*, *EF*, . . . attract any number of homogeneous spheres *GH*, *IK*, *LM*, . . . inversely proportional to the square of the distance *SP*. By adding up or subtracting these forces, "the sum of all those forces (or the excess of any one – or of some – of them above the others); that is, the force with which the whole sphere AB, composed of any concentric spheres (or the difference between some concentric spheres and others which have been taken away), attracts the whole sphere GH, composed of any

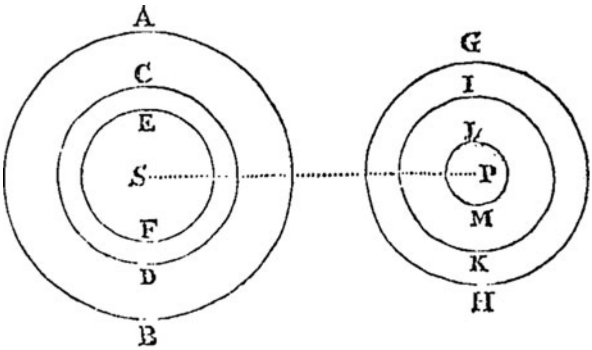

Fig. 2.7 Original figure accompanying Proposition LXXVI, Book I. Reproduced from *PrE*3, p. 195

[240] See Densmore, *Newton's* Principia*: The Central Argument*, pp. 374–378 for additional discussion.

[241] Newton, *The Principia*, p. 595.

[242] Ibid.

concentric spheres (or the differences between some such concentric spheres and others) – will be in the same inverse ratio of the square of the distance SP."[243] If we now let the number of concentric spheres increase indefinitely in such a way "that the density of matter, together with the force of attraction, may – on going from the circumference to the center – increase of decrease according to any law whatever [secundum legem quamcunque crescat vel decrescat"]; and by the addition of non-attracting matter, let the deficiencies in density be supplied wherever needed so that the spheres may acquire any form," the total force of the first sphere attracts the second varies inversely proportional to the distance *SP*.[244] An important corollary to Proposition LXXVI is Corollary 5, which states that "[t]hese results are valid when the attraction arises from each sphere's force of attraction being mutually exerted upon the other sphere. For the attraction is duplicated by both forces acting, the proportion remaining the same."[245]

It is useful to work oneself through these propositions, because they bring home the point that the level of complexity Newton increasingly established throughout Propositions LXXI–LXXVI did not result from a direct comparison with the empirical world, but from a mathematical logic that required that the demonstrations of the complex cases are based on the demonstrations of the simpler cases.

2.6 Newton's Methodology Part I: Book I as an "Autonomous Enterprise"

The ways in which Newton's methodology, in the phase of model construction, differs from a hypothetico-deductive method can now be gathered, as follows:

1. As a way of reducing the risk of arbitrary speculation and the introduction of feigned forces in natural philosophy, Newton demanded that the physical forces producing Keplerian motion should be derived from the laws of motion, i.e. a set of principles which have already undergone several independent empirical tests. Put differently, Newton's insistence that the causes introduced in natural philosophy should be derivable from the laws of motion, implies a prioritization of principles that have empirical support. The laws of motion on which Newton founded the *Principia* are also causally minimal in the sense that they remain silent on the *modus operandi* of the forces involved.
2. Moreover, not only did Newton require that the forces adduced in natural philosophy should be shown to be sufficient for their effects; he additionally required that these effects should be shown to be necessarily produced by those forces. In other words, Newton demanded that there be a systematic dependency between

[243] Ibid., p. 596. See, furthermore, Chandrashekar, *Newton's Principia for the Common Reader*, pp. 280–281.

[244] Newton, *The Principia*, p. 596.

[245] Ibid.

adduced forces and their effects. The question, then, that Newton is trying to answer in Section 2, Book I, is not so much *Which forces entail Keplerian motion?*, but rather *What are, given the laws of motion, the necessary and sufficient conditions for Keplerian motion?* Establishing that inverse-square centripetal forces are the necessary and sufficient conditions for Keplerian motion warrants that, *given the laws of motion*, Keplerian motion, in general, is produced by inverse-square centripetal forces, and inverse-square centripetal forces alone.

3. Newton also sought to respond to the following problem: *How can we infer physical forces from bodies that describe motions according to a mathematical regularity that does not hold exactly but only as most closely as possible?* Newton sought to overcome this difficulty by showing, by a deduction from the laws of motion, that an overall centripetal force directed to a centre of force *quam proxime* is a necessary and sufficient condition for *quam proxime* time-area proportionality. In this way, he was able to infer that, *given the laws of motion*, a body describing equal areas in equal times as most closely as possible is urged by a centripetal force tending as most closely as possible toward a centre of force.
4. In contrast to the hypothetico-deductivist's attitude towards deviations, according to which deviations are either discarded or explained away by the introduction of ad hoc factors, Newton made discrepancies between phenomena and the mathematical results derived from ideal conditions a focal point of natural-philosophical inquiry.[246] Newton began by establishing the physico-mathematical conditions under which, *according to the laws of motion*, exact Keplerian motion would occur, so that each deviation from exact Keplerian motion is an indication that there is an additional force to the one under which exact Keplerian motion would occur. In other words, *from the perspective of the laws of motion*, any deviation from exact time-area proportionality is seen as an indication that an additional force, not included in our ideal case, is affecting the situation.[247] Deviations thus become indicative of other forces not tracked in our initial approximation. By means of the propositions expressing systematic discrepancies, Newton was able to measure such additional forces and to trace, in Book III, additional physical sources that could account for these discrepancies.
5. In order to back-up his argument for *universal* gravitation Newton demonstrated that the overall inverse-square centripetal force exerted by a body results from the composition of each of the individual inverse-square centripetal forces of the particles constituting that body. Hereby, *Newton was constraining transductive inferences by imposing the requirement on them that they should be based on well-defined physico-mathematical decompositions*. The upshot of the propositions on the attractions of spherical surfaces is that the inverse-square law *would* hold exactly for perfect spheres with symmetrically distributed densities. By

[246] Cf. Smith, "The Methodology of the *Principia*", p. 155.

[247] At this point, Smith notes: "For any deviation of the actual motions from a given approximation will then be physically meaningful, and not just a reflection of the particular mathematical scheme employed in achieving the approximation, as in curve fitting." (ibid., p. 157; id., "Was Wrong Newton Bad Newton?", p. 133).

implication, if the inverse-square law does not hold exactly, then the body under consideration is not perfectly spherical or it has no symmetrically distributed density (or both). In other words, any deviation from the inverse-square law is seen as a deviation from perfect sphericity or from symmetrically distributed density. In this sense, the conditions under which an exact mathematical relation *would* hold exactly become informative about the physico-mathematical conditions that are producing deviations from a mathematical regularity which holds exactly[248]: if the areas described are not proportional to the times, it is not the case that a centripetal force is urging a body toward its centre of force exactly (or, put positively, an additional force is affecting the situation); if the inverse-square law does not hold exactly, it is not the case that the body under consideration is perfectly spherical or has a symmetrically distributed density.

Modelling the forces present in the empirical world is not the subject of Book I as it lacks empirical content.[249] The propositions of Book I are part of an autonomous enterprise: a physico-mathematical "investigation of those quantities of forces and their proportions that follow from any conditions that may be supposed" – i.e. a generic study of centripetal forces.[250] Newton's physico-mathematical treatment of motion refers to the fact that he related specific mathematical regularities to specific abstract and idealized physico-mathematical conditions on the basis of the laws of motion. What was distinctive about Newton's physico-mathematical approach was that it showed that, given the laws of motion, certain well-defined physico-mathematical conditions are necessary and sufficient conditions for the specific mathematical regularities they produce. Book I systematically correlated specific conditions, *in casu* inverse-square centripetal forces, to the motions that follow from them, *in casu* Keplerian motion – thereby neglecting at this stage whether Keplerian motion is to be found in the empirical world. Once it is established by astronomical observation that Keplerian motion occurs in the empirical world (see Phenomena I–VI, Book III), Newton was able to infer, given the systematic dependency between inverse-square centripetal forces and Keplerian motion which he had deduced from the laws of motion in Book I, more safely that inverse-square centripetal forces produce the motions as observed in our solar system. Or, in the words of Newton himself: "coming down to physics, these proportions must be compared with the phenomena, so that it may be found out which conditions of forces apply to each kind of attracting bodies." The autonomy entailed by the phase of model construction refers to the fact that Newton, although he clearly focussed on those force laws that are to be found in the *systema mundi* in the context of Book III, did not assume

248 This requirement is crucial. Smith notes: "The preferred starting point is a phenomenological regularity that *would* hold exactly in certain identifiable circumstances, for then observed deviations from it would indeed reflect specific physical factors, and not just imprecision in a description." (Smith, "From the Phenomenon of the Ellipse to an Inverse-square Force", p. 50).

249 Cf. De Gandt, *Force and Geometry in Newton's* Principia, p. 267.

250 Smith, "The Methodology of the *Principia*", p. 144 and Westfall, *Force in Newton's Physics*, p. 506.

the activity of inverse-square centripetal forces in the empirical world at that stage. Rather, he established the necessary and sufficient conditions under which (*quam proxime*) Keplerian motion occurs, given the laws of motion.

The models in Book I of the *Principia* do not have referential content. Book I deals with abstract *measures* only, i.e. with abstract quasi-physical *definientes* (e.g., specific weight, velocity, time). It is Book III that effectively deals with *measurements*, i.e. observed *definientes*, and, accordingly, with real-world forces. *Once measures are replaced by concrete measurements, Newton's mathematical models are turned into philosophical models, i.e. models with referential content*. At that point, the models in Book I become informative about the empirical world. In the following chapter, we will discuss Newton's study of the physical forces in the empirical world.

Chapter 3
Uncovering the Methodology of the *Principia* (II): The Phase of Model Application, Theory Formation and Theory Application

3.1 Introduction

At the start of Book III of the *Principia*, Newton noted that in Books I and II he had "presented [tradidi] principles of philosophy that are not, however, philosophical but strictly mathematical – that is, those on which the study of philosophy can be based [ex quibus videlicet in rebus philosophicis disputari possit]" and that "[i]t still remains for us to exhibit the system of the world from these same principles [ut ex iisdem principiis doceamus constitutionem systematis mundani]."[1] In Book III, Newton's physico-mathematical treatment of force turned into a physical account of the forces in the empirical world. Correspondingly, Newton implicitly offered a physical reinterpretation of quantity of matter in Book III, which explains why, in manuscript material prepared for the third edition, Newton set out to define "body" ("*corpus*") as any moveable and tangible thing that offers resistance to touch and of which the resistance can be sensed if it is big enough ("Corpus voco rem omnem ↓mobilem &↓ tangibilem qua tangentibus resistitur, & cujus resistentia, si satis magna sit, sentire potest."[2]).[3] By contrast, mathematical solids, he noted, do not yield resistance to touching and are not usually said to be physical bodies ("Solida mathematica non ~~sentiuntur~~ ↓agunt↓ tangendo ~~nec resistentiam creant~~, neque corpora dici solent."[4]). In these manuscripts, Newton stressed that his mathematical account of quantity of matter had turned into a physical account of real world bodies ("Initio Libri primi quantitatem materiæ definivi ut tracteretur ~~physice~~ mathematice: hic corpus ex tali materia constans definio ut tractetur physicè."[5]).

[1] Newton, *The Principia*, p. 793; Koyré, Cohen and Whitman, eds., *Principia Mathematica*, II, p. 549.

[2] CUL Add. Ms. 3965, f. 422^{r} [additions and corrections to the second edition of the *Principia*].

[3] For the full transcription of this definition, see the Appendix to Chapter 4. See, furthermore, J. E. McGuire's translations and transcription of this material (McGuire, *Tradition and Innovation*, pp. 113–119, 138–142).

[4] CUL Add. Ms. 3965, f. 422^{r}.

[5] Ibid. "Quantity of matter" was first introduced in Newton's initial revision of *De motu* (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 92; CUL Add. Ms. 3965, f. 21^{r}). It is significant to note that in the original tract of *De motu* Newton consistently used "gravitas" which

S. Ducheyne, *The Main Business of Natural Philosophy*, Archimedes 29,
DOI 10.1007/978-94-007-2126-5_3, © Springer Science+Business Media B.V. 2012

In Book I, Newton had carried out the first step of the threefold methodological scheme he had announced earlier in the *scholium* to Section 11 of Book I[6]: to inquire into "those quantities of forces and their proportions that follow from any conditions that may be supposed [virium quantitates & rationes illæ, quæ ex conditionibus quibuscunque positis consequentur]."[7] In fact, in the experimental parts of Book II, Newton had initiated the second step, i.e. the comparison of the mathematical properties of specific resisting forces (dealt with theoretically) with those of the resisting forces offered to actual bodies in various media.[8] In Book III, Newton determined the forces acting in the solar system by comparing the quantities and proportions of terrestrial and celestial motions to those quantities and proportions of the motions that, as he had shown in Book I, follow under the assumption of inverse-square centripetal forces, "so that it may be found out which conditions of forces apply to each kind of attracting bodies [quænam virium conditiones singulis corporum attractivorum generibus competant]."[9] From this moment on, Newton's treatment of forces ceased to involve merely abstract centripetal forces. In Book III the physico-mathematical models have empirical content, for they refer to real forces in the *systema mundi*.[10] Once the second step had been carried out, Newton set out to contribute to the final step in this process: "to argue more securely concerning [1] the physical species, [2] physical causes, and [3] physical proportions of these forces [de virium speciebus, causis & rationibus physicis tutius disputare]."[11] This would lead, as will be shown in what follows to the corresponding conclusions that (1) gravity is a quality pertaining to bodies universally (in contrast to magnetism which affects only metals),[12] that (2) whatever ultimately causes gravity does so in a non-mechanical way,[13] and that (3) gravity acts in proportion to the quantity of matter and inversely proportional to the square of the distance.

In this chapter, I shall address the confrontation of the physico-mathematical models established in Book I and the empirical world. I shall focus mainly on the analytic part of Newton's argument for universal gravitation, i.e. Propositions I–VIII of Book III, in which the theory of universal gravitation is derived from phenomena – see Section 3.4 in this chapter. The synthetic part, in which Newton shows that the motion of the moon, the tides, and comets can be deduced from the causes proposed

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he subsequently crossed out and replaced by "vis centripeta" (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 43, footnote 31 [see, e.g., CUL Add. Ms. 3965, f. 56^r, f. 57^r, f. 61^r]. In the augmented tract (ibid., ff. 40^r-51^r), in which Newton had more fully separated the mathematical from the physical treatment of force (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 97), he consistently used "vis centripeta."

6 See Chapter 2, Sections 2.3 and 2.6.

7 Newton, *The Principia*, p. 588.

8 See Section 3.3 of this chapter.

9 Newton, *The Principia*, p. 589.

10 Cf. McGuire and Rattansi, "Newton and the 'Pipes of Pan'", pp. 111–112.

11 Newton, *The Principia*, p. 589 [numbers added].

12 See the discussion of Corollary 5 to Proposition VI in Section 3.4.4 of this chapter.

13 See the interesting discussion in Janiak, *Newton as Philosopher*, pp. 27–28.

by the theory of universal gravitation, stretches out to the very end of Book III – see Section 3.5 in this chapter.[14] The analysis corresponds to the sub-phases of model application and theory formation; the synthesis corresponds to the sub-phase of theory application and testing. First we will turn to Newton's famous *regulae philosophandi*.

3.2 The Development and Meaning of Newton's *Regulae Philosophandi*

Newton's theory of universal gravitation as developed in Book III was not only established by means of the physico-mathematical machinery he had developed in Book I, which provided him with the different sorts of inference-tickets as discussed in the previous chapter, but also by means of the application of a set of methodological rules which were to justify and underwrite the inductive generalizations made in Book III. While the abductive inference-tickets – discussed in the previous chapter – offered a criterion for the inference of *instances* of centripetal forces, the rules of philosophizing regulated further inductive generalisations – once different instances of centripetal forces were inferred. Newton's *regulae philosophandi*[15] have a complex history which warrants a careful study of their development both in print as well as in corresponding manuscript material (see Fig. 3.1a–c). In its final form, the *Principia* contained four *regulae philosophandi*.[16] In the second edition of the *Principia*, Hypotheses I and II were relabelled Rule I and II, and Hypothesis III[17]

[14] Ducheyne, "Newton's Notion and Practice of Unification".

[15] At one point, when preparing the third edition of the *Principia*, Newton offered a rough definition of a rule of philosophizing: every proposition that agrees with phenomena – according to Newton's first attempt – or every proposition which is gathered from phenomena by the argument of induction and agrees with them – according to Newton's second attempt. The original text is: "Regulam voco Propositionem omnem quæ ↓~~cum~~↓ ?~~Phaen duabus~~? ~~vel cum pluribus~~ ↓ex↓ Phænomenis ~~respondet~~ congruit, ?↓~~seu~~↓ ~~ex ijsdem~~? per argumentum Inductionis stabilitur colligitur & cum ijsdem congruit." (CUL Add. Ms. 3965, f. 420^r [additions and corrections to the second edition of the *Principia*]; the words between question marks are hard to decipher). Note that Newton used to weaker "colligere," instead of "deducere" (McGuire, *Tradition and Innovation*, p. 70). To the best of my knowledge, nowhere else did Newton attempt to provide an explicit definition of a *regula philosophandi*.

[16] Cohen, "Hypotheses in Newton's Philosophy"; id., *Introduction to Newton's "Principia"*, pp. 23–26; Koyré, *Newtonian Studies*, Chapter 6; and, McDonald, "Properties and Causes: An Approach to the Problem of Hypothesis in the Scientific Methodology of Sir Isaac Newton".

[17] Hypothesis III stated that any body can be transformed into one another of whatever kind and assume all intermediate [and] successive degrees ("*Corpus omne in alterius cujuscunque generis corpus transformari posse & qualitatum gradus omnes intermediora successive induere.*" (CUL Adv.b.39.1, p. 402; CUL Add. Ms. 3965, f. 266^r [additions and corrections to the first edition of the *Principia*])). See furthermore: Cohen, "Hypotheses in Newton's Philosophy", p. 176 and Dobbs, "Newton's Alchemy and his Theory of Matter". See the discussion of Proposition VI of Book III in Section 3.4.4 of this chapter.

a)

b)

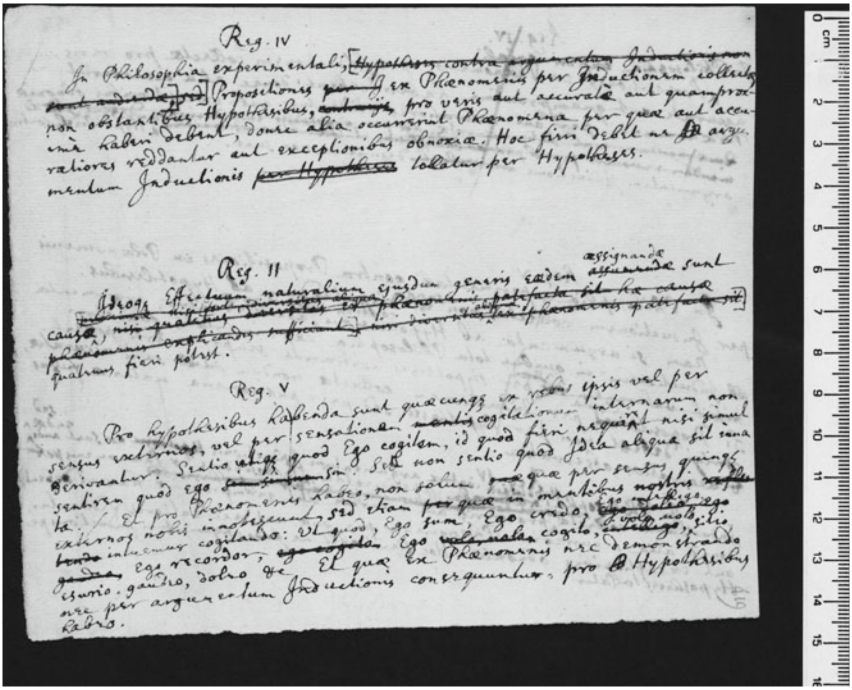
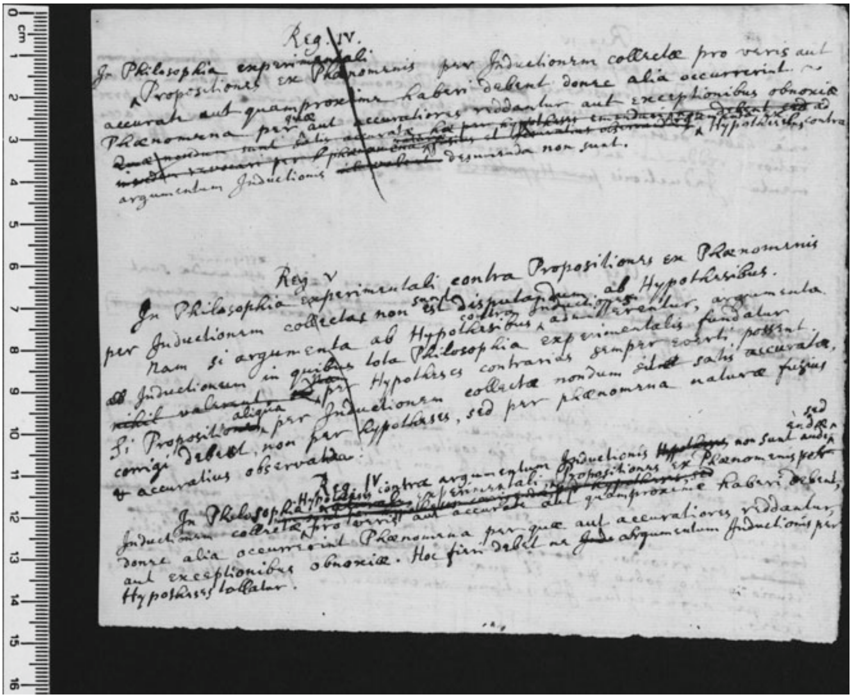


Fig. 3.1 Newton's notes on Rule III (**a** CUL Adv.b.39.1, pp. 402–402A; **b** CUL Add. Mss. 3965, f. 419[r]; **c** ibid., f. 419[v]). Courtesy of the Cambridge University Library, Manuscript Department

c)


Reg. IV.

Reg. V

Fig. 3.1 (continued)

was deleted and replaced by a new Rule III (see Fig. 3.1a).[18] In the third edition, Rules I–III were rephrased and Rule IV was introduced.[19]

The first of these rules states:

RULE I.
No more causes of natural things should be admitted than are both true and sufficient to explain[20] *their phenomena.*

[18] See Appendix 1 to this chapter in which an overview of the most important changes occurring in the second edition of the *Principia* is provided.

[19] See Appendix 2 to this chapter in which an overview of the most important changes occurring in the third edition of the *Principia* is provided.

[20] Quayshawn Spencer has recently questioned this translation. He notes: "Although Bernard Cohen and Anne Whitman translate Newton as using "explain" in rule 1, this translation is debatable. Newton's rule 1 in its original Latin is the following: "Regula I. Causas rerum naturalium non plures admitti debere, quam quae et vera sunt et earum Phenomenis explicandis sufficiunt" [. . .]. Notice that Newton uses "explicandis", a participle of "explicare", which in the period often has the sense of the English "explicate". Newton does not use "explanare", which in the period was often equivalent to the English "explain". This point is worth mentioning since "explicate" means "to give a detailed analysis of", while "explain" means "to give the reason for or cause of". From the wording of rule 1, Newton appears to be satisfied with detailed analyses of phenomena,

> *Causas rerum naturalium non plures admitti debere, quam quæ & veræ sint & earum phænomenis explicandis sufficiant.*[21]

> As the philosophers say: Nature does nothing in vain, and more causes are in vain when fewer suffice. For nature is simple and does not indulge in the luxury of superfluous causes.[22]

This rule *prima facie* expresses the idea of causal minimalism: the number of causes of phenomena should not be inflated beyond necessity because nature operates economically.[23] Upon closer consideration, and more importantly, Newton provided two desiderata that a proper cause in natural philosophy should meet: a cause should not merely be explanatory, it should also be true.[24] Put differently, a cause should not be a sufficient cause of its effect, but also a necessary one. Let me clarify how being a true cause and being a necessary cause are closely interrelated in Newton's demonstrative logic. In Proposition II, Newton proved that a centripetal force is a necessary cause for Kepler's area rule, or, in other words, that Kepler's area law requires a centripetal force. Since astronomical observation shows that the planets describe Kepler's area rule, it follows that centripetal forces necessarily produce these motions, or, in other words, that centripetal forces are true causes of Kepler's area rule – in the sense that are deducible from Newton's causal "inference-tickets", which are derived from the laws of motion. In other words, Rule I argues for the introduction of systematic dependencies between cause and effect.[25] On this

---

regardless of whether those phenomena have been assigned a cause. If this interpretation is accurate, then it would be consistent with what Newton says in the General Scholium of the *Principia* when he mentions that he has not deduced a mechanism for gravity even though he has established gravity as the force that maintains our planetary system [. . .]." (Spencer, "Do Newton's Rules of Reasoning guarantee Truth . . . must they?", p. 761, footnote 3). As we have seen in Chapter 1, Newton was not averse to causal talk and, moreover, he considered centripetal forces as proximate causes. Furthermore, in the Coda to Chapter 1 we have seen that at the time "explicare" and "explanare" were often used synonymously.

21 Koyré, Cohen and Whitman, eds., *Principia Mathematica*, II, p. 550.

22 Newton, *The Principia*, p. 794. In the second edition, Newton changed "sufficiunt" into "sufficiant" and added the sentence "Dicunt utique philosophi [. . .] potest per pauciora." (CUL Adv.b.39.2, p. 357; Koyré, Cohen and Whitman, eds., *Principia Mathematica*, II, pp. 550–551).

23 As is suggested in Mamiani, *Isaac Newton filosofo della natura*, p. 282.

24 Cf. Spencer, "Do Newton's Rules of Reasoning guarantee Truth . . . must they?", p. 768. In their edition of the *Principia*, Le Seur and Jacquier emphasized that: "Hæc regula duas habet partes; *prima est, ne philosophia in vana abeat opinionum commenta, causæ rerum naturalium non aliæ admitti debent quàm quæ reverâ existunt et quæ phænomenis explicandis sufficiunt*; [. . .]. *Altera pars regulæ, ea scilicet quæ præscribit non plures admittendas esse rerum naturalium causas quàm quæ eorum phænomenis explicandis sufficiunt, manifesta est*; nam cùm vera effectûs causa per experientiam semel inventa est, et matheseos ope præsertim demonstrandum est causæ illius eam esse vim quæ ad effectum producendum sufficiat, liquet aliam quamlibet causam esse inutilem." (Le Seur and Jacquier, *Philosophiæ naturalis principia mathematica*, III, p. 2, footnote 49 [italics added]).

25 In *On the Philosophy of Discovery* (1860), Whewell criticized Rule I because it applies to causes to which we are already familiar and, correspondingly, because it does not allow for the discovery of new causes (Yeo, ed., *Collected Works of William Whewell*, VII, pp. 186–192). Rule I, however,

reading, Rule I then asserts that causes shown to be necessary *and* sufficient of their effects, *and such causes alone*, are to be kept minimal and that hypothetical explanations ought to be rejected.

Rule II is basically a corollary to Rule I (cf. "*ideoque*"):

> RULE II.
> *Therefore, the causes assigned to natural effects of the same kind must be, so far as possible, the same.*[26]
>
> *Ideoque effectuum naturalium ejusdem generis eædem assignandæ sunt causæ, quatenus fieri potest.*[27]
>
> Examples are the cause of respiration in man and beast, or of the falling of stones in Europe and America, or of the light of a kitchen fire and the sun, or of the reflection of light on our earth and planets.[28]

The formulation of Rule II underwent significant change: whereas Newton originally wrote "*Ideoque effectuum naturalium ejusdem generis <u>eædem sunt causæ</u>*,"[29] in the second and third edition he changed this into: "*Ideoque effectuum naturalium ejusdem generis <u>eædem assignandæ sunt causæ, quatenus fieri potest</u>*."[30] The significance of this adjustment is that Newton moved from an ontological claim to an epistemological claim – thereby leaving room for future revision. Rule II licenses the identification of instances of causes of the same kind which have been shown to be true and sufficient to explain their phenomena. Obviously, Rule II requires a criterion to decide when effects or phenomena are "of the same kind," which Newton does not explicitly give. However, it can be reconstructed from Newton's actual applications of Rule II and will be discussed in the exposition of the analytical part of the argument for universal gravitation.[31] In an isolated note, which occurs in Newton's preparations for the third edition of the *Principia*, he recorded that phenomena are of the same kind insofar as they can be explained by the

instructs us to minimize the number of causes which have been shown to be necessary and sufficient of their causes, given a set of well-established principles. Rule I perfectly leaves open the possibility of establishing new systematic dependencies between causes and their effects.

[26] Whilst working on the corrections for the third edition of the *Principia* Newton wrote on CUL Add. Ms. 3965, f. 419^{r} [additions and corrections to the second edition of the *Principia*]: "Ideoque Effectuum naturalium ejusdem generis eædem ~~assumendæ~~ ↓assignandæ↓ sunt causæ ↓[~~proximæ nisi [forte diversitas aliqua~~↓, ~~nisi quatenus diversitas ex phænomenis patefacta sit hæ causæ phænomenis explicandis sufficiunt.~~] ~~nisi diversitas~~ ↓~~aliqua~~↓ ex ph~~ænomenis patefacta sit.~~] quatenus fieri potest."

[27] Koyré, Cohen and Whitman, eds., *Principia Mathematica*, II, p. 550.

[28] Newton, *The Principia*, p. 795.

[29] CUL Adv.b.39.1, p. 402 [underscore added].

[30] Koyré, Cohen and Whitman, *Principia Mathematica*, II, pp. 550–551 [underscore added]. Cf. Wren Library, NQ.16.196, p. 357.

[31] Cf. Spencer, "Do Newton's Rules of Reasoning guarantee Truth . . . must they?", pp. 761–762. This answers the criticism launched by Whewell against Rule II (Yeo, ed., *Collected Works of William Whewell*, VII, p. 193).

same causes ("Phaenomena voco ejusdem generis ↓quatenus↓ per easdem causas explicari possunt."[32]).

In the second edition of the *Principia*, Rule III, which has often baffled interpreters,[33] was introduced:

RULE III.
Those qualities of bodies that cannot be intended and remitted [i.e., qualities that cannot be increased and diminished] and that belong to all bodies on which experiments can be made should be taken[34] *as qualities of all bodies universally.*

Qualitates corporum quæ intendi & remitti nequeunt, quæque corporibus omnibus competunt in quibus experimenta instituere licet, pro qualitatibus corporum universorum habendæ sunt.[35]

For the qualities [qualitates] of bodies can be known only through experiments; and therefore qualities that square with experiments universally are to be regarded [statuendæ sunt] as universal [generales] qualities; and qualities that cannot be diminished cannot be taken away from bodies [minui non possunt, non possunt auferri]. Certainly idle fancies ought not to be fabricated [configenda] recklessly against evidence of experiments [contra experimentorum tenorem], nor should we depart from the analogy of nature, since nature is always simple and ever consonant with itself. The extension of bodies is known to us only though our senses, and yet there are bodies beyond the range of these senses; but because extension is found in all sensible bodies, it is ascribed to all bodies universally. We know by experience that some bodies are hard. Moreover, because the hardness[36] of the whole arises from the hardness of its parts, we justly infer from this not only the hardness of the undivided particles of bodies that are accessible to our senses, but also of all other bodies. That all bodies are impenetrable we gather not by reason but by our senses. We find those bodies

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[32] CUL Add. Ms. 3965, f. 423^v [additions and corrections to the second edition of the *Principia*].

[33] E.g., Finocchiaro, "Newton's Third Rule of Philosophizing".

[34] On the interleaved page between pp. 402–403 of CUL Adv.b.39.1, Newton wrote: "Qualitates corporum quæ intendi et remitti nequeunt, quæque corporibus omnibus competunt in quibus experimenta instituere licet, ~~sunt proprietates~~ ↓pro qualitatibus↓ corporum universorum habendæ sunt." An early precursor of Rule III can be found in the autograph endnotes of Wren Library, NQ.16.200. As a comment on "p. 402 l. 10," Newton wrote: "Hypoth III. Leges ↓et proprietates↓ corporum omnium in quibus experimenta instituere licet sunt leges ↓et proprietates↓ corporum universorum."

[35] Koyré, Cohen and Whitman, eds., *Principia Mathematica*, II, p. 552.

[36] Relevant variations are: "For all bodies so far as experience reaches are either hard or may be hardened ~~(. . .) or may be soft or fluid by the sliding of y^e particles amongst themselves. [For hard sands compose soft quicksands, & have small fragments of the hardest bodies compose soft.]~~ ↓& we have no other evidence of universal impenetrability besides a large experience↓ without exception. ~~& this Rule, that without experience exceptions are not to be made against the course of Nature."~~ (CUL Add. Ms. 3970, f. 242^r, cf. f. 242^v [ca. 1700–1704]) and "The whole tenour of experience & observation (w^{ch} is very large ↓~~& without any exceptions~~↓) makes for these qualities, ↓& without any exception↓ ~~without any exception that I know of~~ [& ↓all sound↓ ~~experimental~~ Philosophy ~~must be~~ ↓ought to be↓ bounded by experience & ~~& strict~~ ↓true↓ reasoning from phænomena (. . .) & y^e course of Nature] & if the whole course of a large experience is not a sufficient argument the ~~whole cour~~ ↓universal↓ impenetrability of matter may be.] ↓For↓ We have the whole course of a large experience for the universal gravity & (. . .) of matter & ↓for↓ y^e hardness of its particles ↓without any instance of the contrary↓ & we have nothing more for its universal impenetrability." (ibid., f. 243^v [ca. 1700–1704]).

> that we handle [tractamus] to be impenetrable, and hence we conclude that impenetrability is a property of all bodies universally. That all bodies are moveable and persevere in motion or in rest by means of certain forces (which we call forces of inertia) we infer from finding these properties in the bodies that we have seen [ex hisce corporum visorum proprietatibus colligimus]. The extension, hardness, impenetrability, mobility, and force of inertia of the whole arise [oritur] from the extension, hardness, impenetrability, mobility, and force of inertia of each of the parts; and thus we conclude that every one of the least parts of all bodies is extended, hard, impenetrable, movable, and endowed [præditas] with a force of inertia. And this is the foundation of all natural[37] philosophy. Further, from phenomena we know that the divided, contiguous parts of bodies can be separated form one another, and from mathematics it is certain that the undivided parts can be distinguished into smaller parts by our reason. But it is uncertain whether those parts which have been distinguished in this way and not yet divided can actually be divided and separated from one another by the forces of nature. But if it were established by even a single experiment that in the breaking of a hard and solid body, any undivided particle underwent division, we should conclude by the force of this third rule [concluderemus vi hujus Regulæ] not only that divided parts are separable but also that undivided parts can be divided indefinitely.
>
> Finally, if it is universally established by experiments and astronomical observations that all bodies on or near the earth gravitate toward the earth,[38] and do so in proportion to the quantity of matter in each body, and that the moon gravitates toward the earth in proportion to the quantity of motion of its matter, and that our sea in turn gravitates toward the moon, and that all planets gravitate toward one another, and that there is a similar gravity of comets toward the sun, it will have to be concluded by this third rule [dicendum erit per hanc Regulam] that all bodies gravitate toward one another. Indeed, the argument from phenomena will be even stronger [fortius] for universal gravity than for the impenetrability of bodies, for which, of course, we have not a single experiment, and not even an observation, in the case of the heavenly bodies. Yet I am no means affirming that gravity is essential to bodies.[39] By inherent force I mean only the force of inertia. This is immutable. Gravity is diminished [diminuitur] as bodies recede from the earth.[40]

Rule III instructs us to consider those qualities (or forces[41]) which "cannot be intended or remitted" and which pertain to all bodies within the reach of experimentation as universal qualities (or forces). But what exactly are qualities or forces that cannot be intended and remitted? Rule III confronts us with an interpretational conundrum. On the one hand, it is clear that Newton sets out to argue that gravity, in contrast to magnetism, is a universal force, i.e. a force which, according to

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[37] This word was added in the third edition of the *Principia*.

[38] In the crossed-out addition on the interleaved page next to Adv.b.39.1, p. 411, Newton noted: "~~Gravitas ↓in Terram↓ est qualitas corporum omnium ↓quæ circa Terram sunt &↓ in quibus experimenta instituere licet & quantitati materiæ in singulis proportionalis existens non potest intendi et remitti & propterea per Hypoth III proprietas corporum universorum.~~" [note that Hypothesis III corresponds to what later became Rule III (cf. ibid., interleaved page between pp. 402–403); in other words, Newton had composed the full text of Rule III (then called "Hypothesis III") before he changed its status from a hypothesis to a rule].

[39] Newton's caveat on universal versus essential qualities, i.e. the last three sentences, was added in the third edition (Wren Library, NQ.16.196, p. 359; CUL Add. Ms. 3965, f. 430^r [corrections and additions to the second edition of the *Principia*]).

[40] Newton, *The Principia*, pp. 795–796.

[41] Cf. McGuire, *Tradition and Innovation*, p. 247.

the Cohen-Whitman translation, cannot be "increased and diminished." Yet near the end of the text to Rule III, Newton noted that "gravity diminishes as bodies recede from the earth [[g]ravitas recendo a terra, diminuitur]." The implication is that, as Peter Achinstein succinctly puts it, "if a quality that can be "intended and remitted" is one that can be increased and diminished, as the translators suggest, then, contrary to what Newton wants, rule 3 cannot apply to gravity."[42] Correspondingly, I shall make it plausible that Newton did not equate the two. The very source of this problem is that in his discussion of Rule III Newton introduced two sorts of qualities or forces[43] without going into much detail as to they are related. He introduced: (1) *qualities or forces that can be intended and remitted* and which correspond to non-universal qualities or forces, and, (2) *qualities or forces that are universal*, which come in two kinds: universal-relational or universal-essential (or immutable).[44]

In order to understand what Newton meant with qualities that cannot be intended or remitted, it is useful to consider qualities that can be intended and remitted as a contrast-class. Unfortunately, in the text to Rule III, Newton did not provide any example of such qualities. However, a couple of pages later, namely in Corollary 5 to Proposition VI of Book III, Newton provided the required example. There he pointed out that "the magnetic force in one and the same body can be intended and remitted [vis magnetica in uno & eodem corpora intendi potest & remitti]."[45] That Newton considered magnetism as a force that can be intended and remitted, is corroborated by further evidence. In a memorandum composed on 5–7 May 1694, David Gregory reported:

> Magnetic virtue is destroyed by a flame, and by heat: a rod of iron, either by standing long in a perpendicular position, or by cooling in an erect position, acquires magnetic virtue from the Earth. But it gets magnetic virtue too with a strong blow of a hammer at either extremity. If it is struck hard at one or other end the poles of the iron rod are interchanged: if it is struck in the middle (say with hammering at an anvil) *it quite loses its magnetism*. And so this virtue seems to be produced by mechanical means [Unde Mechanice produci videtur hæc virtus.].[46]

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[42] Achinstein, *Science Rules: A Historical Introduction to Scientific Methods*, pp. 71–72.

[43] When discussing extension, impenetrability, etc. Newton was referring to qualities of matter. However, when discussing magnetism and gravity he is dealing with forces. Janiak has correctly signaled that "until we discover gravity's "physical cause," we are not in a position to say that gravity is a property of material bodies, for it may be a property of the ether or some other medium" (Janiak, *Newton as Philosopher*, p. 97).

[44] In his ground-breaking study "The Origin of Newton's Doctrine of Essential Qualities", J. E. McGuire presupposed that Newton closely identified essential or immutable qualities with qualities that cannot be intended or remitted (McGuire, *Tradition and Innovation*, pp. 252, 254, 256). In what follows, I will argue that Newton did not upheld this identification.

[45] Newton, *The Principia*, p. 810. See the discussion in Section 3.4.4 of this chapter.

[46] Newton, *Correspondence*, III, p. 335/p. 338 [italics added].

Related statements can be found amongst Newton's optical manuscripts. After having discussed the "vertue or disposition" of Island Crystal to produce double refraction, Newton noted:

> And as magnetism may be intended & remitted, & and is found only in the Magnet & in iron: so this vertue of refracting the perpendicular rays is greater in Island Crystal less in Crystal of the rock & is not yet found in other bodies.[47]

By contrast, when discussing gravity, he considered it as a force that cannot be intended and remitted. Newton's draft-versions of Rule III,[48] which he was preparing for the second edition of the *Principia*, contains the following variant:

> All bodies here below are heavy towards y^{e} Earth in proportion to the quantity of matter in ↓each of↓ them. Their gravity ↓in proportion to their matter↓ is not intended or remitted ↓in the same region of the earth by any variety of ↓fforms↓↓ & therefore *it cannot be taken away* ~~I speak of bodies equally distant from y^{e} centre of the earth~~ [. . .].[49]

Note that when Newton mentions qualities or forces that can be remitted, he refers to qualities or forces that can be lost or taken away – hence his choice to use the verb "auferri" in the published text to Rule III. Stating that *x* can be taken away has a stronger meaning than stating that *x* can be diminished: whereas the statement that *x* can be taken away implies that *x* can effectively be diminished to zero, stating that *x* can be diminished does not automatically imply that *x* can be taken away entirely. On this reading, Newton is not inconsistent in claiming that gravity can be diminished, but cannot be remitted, i.e. cannot be lost. It is obvious that a body's weight can be diminished: as Newton explained, as a body recedes further from the surface of the earth the gravitational force exerted on it diminishes. However, it cannot be the case that a body *in rerum natura* is not subject to gravitational force. By being material, i.e. by having mass, all bodies are subject to gravitational force. This contrasts significantly with magnetism: first of all, not all bodies are subjected to magnetic force; secondly, bodies of the sort that are subject to magnetism can lose their "magnetic virtue" (for instance, by heat or by severe hammering). Correspondingly, qualities or forces that cannot be lost or taken away are universal

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47 CUL Add. Ms. 3970, f. 258^{r} [ca. 1700–1704]. On CUL Add. Ms. 4005.15, f. 81^{r}, Newton gave some other examples of qualities that can be intended and remitted: "Calor et frigus, humiditas et siccitas, lux et tenebræ, color et nigredo, ↓vivacitas salus et ægritudo↓ aciditas amaritudo et dulcedo, volitatitas et fixitas considerandæ non veniunt."

48 See McGuire, *Innovation and Tradition*, Chapter 6.

49 CUL Add. Ms. 3970, f. 243^{v} [ca. 1700–1704; italics added], cf. f. 253^{r}. Note that in the accompanying text to the definition of the accelerative quantity of centripetal forces, Newton noted: "Another example is the force that produces gravity, which is greater in valleys and less on the peaks of high mountains and still less (as will be made clear below) at greater distances from the body of the earth, *but which is everywhere the same at equal distances* [in æqualibus autem distantiis eadem undique]*, because it equally accelerates all falling bodies* [æqualiter accelerat] (heavy or light, great or small), *provided that the resistance of the air is removed*." (Newton, *The Principia*, p. 407 [emphasis added]).

qualities or forces – irrespective of whether they can be increased or diminished. In the text to Rule III Newton, furthermore, distinguished between two different sorts of universal qualities or forces: those that are immutable and those that can be increased and diminished. While *mass* is an essential-universal property of bodies – in the sense that it is immutable and independent from a body's spatio-temporal features, *gravity* is a relational-universal property, i.e. it is dependent on a body's relation to other bodies.[50] Since all primary and secondary planets are subject to gravitation, Rule III instructs us to conclude that all bodies universally are subject to gravitation.

Rule IV was added in the third edition of the *Principia*:

> RULE IV.
> *In experimental philosophy, propositions gathered from phenomena by induction should be considered either exactly or very nearly true*[51] *notwithstanding any contrary hypotheses,*[52] *until yet other phenomena make such propositions either more exact or liable to exceptions*.[53]
>
> *In philosophia experimentali, propositiones ex phænomenis per inductionem collectæ, non obstantibus contrariis hypothesibus, pro veris aut accurate aut quamproxime haberi debent, donec alia occurrerint phænomena, per quæ aut accuratiores reddantur aut exceptionibus obnoxiæ*.[54]
>
> This rule should be followed so that arguments based on induction may not be nullified [tollatur[55]] by hypotheses.[56]

In manuscript material Newton is more explicit on the meaning of this final rule. "Because," Newton wrote in a crossed-out section on what was there called "Reg. V," "if arguments based on hypotheses were to be admitted against inductions, then inductive arguments, on which the whole of experimental philosophy is based, could always be overturned by contrary hypotheses"[57] ("Nam si argumenta ab

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[50] Funkenstein, *Theology and the Scientific Imagination*, p. 93. See, furthermore, the superb discussion of this distinction in Janiak, *Newton as Philosopher*, Chapter 4, esp. p. 115.

[51] As Newton used "*quamproxime*" here, it is better to translate this as "as most closely as possibly true".

[52] CUL Add. Ms. 3965, f. 428^r reads "~~Objectiones~~."

[53] Cf. ibid., f. 419^r, f. 428^r, f. 504^r, 519^r [additions and corrections to the second edition of the *Principia*] and CUL Adv.b.39.2, interleaved page between pp. 358–359.

[54] Koyré, Cohen and Whitman, eds., *Principia Mathematica*, II, p. 555.

[55] In one draft version Newton used "desumenda non sunt" (ibid., f. 419^v [additions and corrections to the second edition of the *Principia*]).

[56] Newton, *The Principia*, p. 796.

[57] Clifford A. Truesdell has remarked that Newton's Rule IV contained "an illogical defence" since different generalizations can always be consistent with the same observations (cf. "Newton is implying that since universal gravitation explains everything satisfactorily, no other hypotheses is to be admitted until universal gravitation is proved false. In other words, the *first* adequate has the right of predominance over equally adequate aftercomers." (Truesdell, "A Program toward Rediscovering the Rational Mechanics of the Age of Reason", p. 101, footnote 5)). However, Truesdell missed the following point: Newton was arguing for the predominance of propositions

Hypothesibus ↓contra Inductiones↓ admitterentur, argumenta ab Inductione↓um↓ in quibus tota Philosophia experimentalis fundatur ~~nihil valerent, sed~~ ↓~~Nam~~↓ per Hypotheses contrarias semper everti possent."[58]). If a proposition gathered by induction is not sufficiently accurate, then it should be corrected, not by introducing (ad hoc) hypotheses, but by more widely and accurately observed phenomena of nature ("Si Propositi~~ones~~ ↓aliqua↓ per Inductionem collect↓a↓~~æ~~ nondum s↓it↓~~unt~~ satis accurat↓a↓~~æ~~, corrigi deb~~en~~t, non per hypotheses, sed per phænomena naturæ fusius & accuratius observa↓t↓~~nd~~a."[59]). If this turns out impossible, however, then the proposition should be de-generalized (cf. "Argumenta ~~ab~~↓per↓ Inductione↓m↓ ~~non~~ ~~[fortiora sunt quam Hypotheses~~ non sunt Demonstrationes. ffortiora tamen sunt quam Hypotheses: & pro generalibus haberi debent nisi quatenus exceptiones ab experimentis desumptæ [~~illegible text~~] occurrant. Ideoque ubi nullæ occurrunt ejusmodi ~~ubi~~ ~~e~~↑ex↑ceptiones, generaliter ennunciandæ sunt."[60]). The latter quote reveals that Newton was perfectly aware of the risk involved in making inductive generalizations. Inductive-experimental arguments do not provide universal demonstrations, but they are stronger than arguments drawn from hypotheses:

> And although the arguing from Experiments and Observations by Induction be no Demonstration of general Conclusions; yet it is the best way of arguing which the Nature of Things admits of, and may be looked upon as so much stronger, by how much the Induction is more general. And if no Exception occur from Phænomena, the Conclusion may be pronounced generally. But if at any time afterwards any Exception shall occur from Experiments, it may then begin to be pronounced with such Exceptions as occur.[61]

Rule IV also brings home the point that Newton was approaching the empirical world in a sequence of approximations. For the moment, I pause here with my discussion of the *regulae philosophandi*.[62] Applications of Newton's *regulae*

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which are "deduced" from phenomena and rendered general by induction over propositions which are not inferred by that method. My criticism similarly applies to Paul K. Feyerabend's statement that the function of Rule IV is the following: "it discredits ideas which contradict the orthodox point of view." (Feyerabend, "On the Limited Validity of Methodological Rules", p. 139).

58 CUL Add. Ms. 3965, f. 419^{v} [additions and corrections to the second edition of the *Principia*].

59 Ibid.

60 Ibid., f. 428^{r} [additions and corrections to the second edition of the *Principia*].

61 Newton, *The Opticks*, p. 404.

62 Finally, it should be noted that Newton pondered on introducing a fifth *regula* in which statements are to be taken as hypotheses if they are not derived from things themselves ("ex rebus ipsis"), whether by the external senses ("per sensus externos"), or by the sensation of internal thoughts ("per sensationem cogitationum internarum") (CUL Add. Ms. 3965, f. 419^{r} [additions and corrections to the second edition of the *Principia*]). See, furthermore, Cohen, *Introduction to Newton's "Principia"*, pp. 30–31.

philosophandi will be discussed in Section 3.4. In order to set the stage for our treatment of Book III, I shall turn first to the experimental parts of Book II.

3.3 Justifying the Absence of a Resisting Medium

If the models established in Book I provide good approximations of the celestial motions, this implies that these motions are not resisted by a corpuscular ether. To many contemporaries of Newton, this came as quite a shock. Newton could not have been other than aware of the sense of uneasiness his fellow natural philosophers would face when their view of the system of the world as a *plenum* was called into question – after all, Newton himself came to reject this view only when he was in his forties. The *Principia* therefore gradually introduced its readers to the idea that the system of the world was not a *plenum*. In Book III Newton would, of course, offer empirical evidence in favour of a "Boylian vacuum" in the celestial regions. The experimental parts of Book II were in part intended to reduce the sudden shock caused by the very idea of a system of the world containing large voids. It is not a coincidence that, when Newton was preparing the third edition of the *Principia*, he considered introducing an explicit definition of a vacuum as that place "in which bodies move without resistance" ("DEFINITIO III. Vacuum voco locum omnem in quo corp~~us~~↓ora↓ sine resistentia move↓n↓tur."[63]).

In Book II of the *Principia*, Newton addressed fluid resistance.[64] Just as Book I provides a generic mathematical study of centripetal forces, Book II, in which Newton's program of mathematical deduction broke down,[65] develops a generic mathematical account of motion under resistance forces.[66] In Book II Newton considered the total fluid resistance, R_{total}, as consisting of three components: (1) the first component, which Newton reckoned to be independent of velocity,[67] arises

[63] CUL Add. Ms. 3965, f. 422^{r} [additions and corrections to the second edition of the *Principia*]. This definition was crossed-out in Newton's list of corrections and additions on ibid., f. 504.

[64] In the subsequent discussion of Book II, I draw heavily on a series of superb studies undertaken by George E. Smith, which have put Newton's study of fluid resistance in a totally new perspective. See – in reverse chronological order: Smith, "Was Wrong Newton Bad Newton?"; id., "The Newtonian Style in Book II of the *Principia*"; id., "Fluid Resistance: Why did Newton Change his Mind?"; and, id., "Newton's Study of Fluid Mechanics".

[65] Truesdell, "A Program toward Rediscovering the Rational Mechanics of the Age of Reason", p. 91; id., "Reactions of Late Baroque Mechanics to Success, Conjecture, Error, and Failure in Newton's *Principia*", p. 144. Clifford A. Truesdell has, moreover, pointed out that "Book II is almost entirely original, and much of it is false. New hypotheses start up at every block; concealed assumptions are employed freely, and the stated assumptions sometimes are not used at all." (ibid.).

[66] Note that Book II is not restricted to resistance forces that vary to first and second powers of velocity but includes resistance forces that vary as any power whatever.

[67] As in the remainder of this discussion, "velocity" refers to the relative velocity between the body and the fluid.

from the *tenacity* (or absence of slipperiness) of the fluid[68]; (2) the second component, which Newton took to be proportional to velocity, arises from the (internal) *friction* of the fluid – this term is now most commonly referred to as the "viscous" component; and, (3) the third component, which Newton took to be proportional to the square of the velocity, arises from the *inertia* of the fluid.[69] Accordingly, R_{total} can be expressed as $a + bv + cv^2$ – where a, b, and c are coefficients which need to be determined empirically.[70] This template may be considered as Newton's overall working hypothesis in Book II. Newton, furthermore, considered the inertial component, R_{inertia}, equal to $c' \times \rho_{\text{fluid}} \times A_{\text{front}} \times v^2$, where c' is a coefficient which may vary with shape, ρ_{fluid} is the density of the fluid, and A_{front} the frontal area of the body. Moreover, in the case of spheres, $R_{\text{inertia}} = c'' \times \rho_{\text{fluid}} \times d^2 \times v^2$, where c'' is now a strict constant and d the diameter of the sphere. Given this equality, R_{total} may be expressed as $a + bv + \left(c'' \times \rho_{\text{fluid}} \times d^2 \times v^2\right)$.

By decomposing fluid resistance in the above way, Newton set out to disaggregate and determine the contribution of each term.[71] Note that because gravitational forces are so dominant for celestial motion, the need of disaggregating gravity from different forces does not arise in the context of Book III. Accordingly, the empirical challenge of Book II was first to confirm the $\rho_{\text{fluid}} \times d^2$ constituents of the v^2-term and next to empirically characterize a, b, c'' – if not c'.[72] At least, this was Newton's hope.

With the benefit of hindsight, we are currently in a position to understand why Newton's attempt to separate each of the components of the total resistance was bound to fail, for the viscous (v) and inertial (v^2) components cannot be separated. Fluid resistance is never purely a function of the fluid's inertia, but always a combination of fluid inertia and its viscosity – a combination that depends on whether inertial or viscous forces dominate.[73]

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[68] Newton considered this term (or force) to be uniform, "or as the moment of the time" (Newton, *The Principia*, p. 678), i.e. as a special case of uniformly accelerated motion.

[69] Smith, "The Newtonian Style in Book II of the *Principia*", pp. 252–254. See, furthermore, Newton's discussion in the *scholium* to Section III, which was added in the third edition (Newton, *The Principia*, pp. 678–679).

[70] A, b and c may be understood as functions of properties of the body and properties of the fluid (Smith, "The Newtonian Style in Book II of the *Principia*", p. 253). Each of them will vary for specific combinations of bodies and mediums.

[71] Newton, *The Principia*, p. 749.

[72] Smith, "The Newtonian Style in Book II of the *Principia*", p. 254. Nowhere in the *Principia* would Newton mention how the coefficients a and b might vary from body to body and from fluid to fluid (ibid., pp. 253, 290, footnotes 13 and 15).

[73] See Smith's discussion of the empirically derived curve of the drag coefficient for spheres as a function of the Reynolds numbers (Smith, "The Newtonian Style in Book II of the *Principia*", pp. 255–257, 285; id., "Fluid Resistance: Why did Newton Change his Mind?", p. 125; and, id., "Was Wrong Newton Bad Newton?", p. 130).

In the first edition of the *Principia*, Newton had set his hope on pendulum-decay experiments to disaggregate the components of his working hypothesis.[74] The idea was to infer the resistance force from the rate of a pendulum's decay. In the General Scholium concluding Section 6, Newton reported on these experiments,[75] which were originally contained in the first edition of the *Principia* and of which their numerical results remained unchanged in all later editions.[76] Newton listed six different initial positions from which he set a "$57_{7/22}$ ounces avoirdupois" ball with a diameter of $6_{7/8}$ London inches to swing along a $10_{1/2}$ feet chord. The ball was consecutively released at an arc distance from the perpendicular of 2, 4, 8, 16, 32, and 64 in., respectively. Newton then counted the oscillations during which the ball would lose an eight of its motion,[77] i.e. 164, 121, 69, $35_{1/2}$, $18_{1/2}$ and $9_{2/3}$ oscillations, respectively. The differences between the arcs described in the first descent and the final ascent were: 1/4, 1/2, 1, 2, 4, 8 in., respectively. When we divide these by the numbers of oscillations in each case, then the difference of the arcs described in one mean oscillation will be 1/656, 1/242, 1/69, 4/71, 8/37, and 24/29, respectively. In the greater oscillations these differences are as most closely as possible in the squared ratio of the arcs described, while in the smaller oscillations they are a little greater than that ratio.[78] In Proposition XXXI Newton established that, if the resistance exerted by a medium on an oscillating body in each of the proportional parts of the arcs is increased (or decreased) in a given ratio, then the difference between the arc described in the descent and the arc described in the ascent will be increased (or decreased) in the same ratio.[79] The reason for this is that the difference, which arises from the retardation by the resistance of the medium, is as the whole retardation, Newton argued. Hence, Newton's conclusion: "the resistance of the ball when it moves more swiftly is as most closely as possible in the squared ratio of the velocity; when more slowly, a little greater in that ratio."[80] The results, however, proved to be disappointing: Newton had to introduce an arbitrary $v^{3/2}$-term to the v and v^2-terms to allow for a rather weak fit between data and theory.[81] Once Newton had performed the vertical-fall experiments, which were included in subsequent editions of the *Principia*, he came to explain this disappointing fit in the following way:

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74 See Smith, "The Newtonian Style in Book II of the *Principia*", pp. 257–264 for extensive commentary. See, furthermore, Gauld, "Newton's Use of the Pendulum to Investigate Fluid Resistance", pp. 391–395.

75 Note that Newton had previously reduced the motion of a pendulum to a cycloid (Newton, *The Principia*, pp. 701–703, 715).

76 In the first edition, however, this experiment appeared in the *scholium* concluding Section VII (*PrE₁*, pp. 339–340).

77 I.e. the number of oscillations required for the ball to reach an arc distance of 7/8 of the arc distance from which the ball was originally set in motion.

78 Newton, *The Principia*, p. 713.

79 Ibid., p. 711.

80 Ibid.

81 Ibid., p. 714.

he reasoned that, because of the pendulum's swing, a to-and-fro motion was generated in the fluid surrounding the bob. As a result of such motion, the relative velocity between the bob and the fluid could not be properly controlled.[82] George E. Smith summarizes Newton's pendulum-decay experiments, as follows:

> Newton concluded from them that the v^2 components of resistance is dominant at higher velocities in both water and air, and that once suitable allowances are made for shortcomings in the experiments,[83] this component varies as ρd^2, at least for spheres. Even this conclusion, however, has to be restated in a more qualified manner all too familiar to those who find themselves having to rely on simple hypothesis testing. The correct statement is that the pendulum experiments *did not clearly falsify* the claim that the dominant component of resistance on spheres at high velocities can be expressed in the form $c\rho d^2 v^2$. The results of the experiments failed to yield a stable value of the constant c. Nor did they yield any conclusions at all about the other component (or components) of resistance that become more prominent at low velocities.[84]

In the General Scholium to Section 6, Newton also reported on a test to ascertain whether the resistance of the ether to oscillating bodies in the air depends "wholly on their external surface [tota sit in eorum externâ superficie] or whether the internal parts also encounter a perceptible resistance on their own surface [partes etiam internæ in superficiebus propriis resistentiam notabilem sentiant]" – since the ether allegedly permeated the interior of bodies,[85] Newton compared the number of oscillations of an empty firwood box, which he suspended to a hook in such a way that the friction was minimal, with the number of oscillations of the same box filled with metals. The weight of the empty box was to that of the filled box as 1 to 78 – this included the weight of the air inside the box. Newton observed that the filled box returned to certain marked points, which he had determined earlier while experimenting with the empty box, at the completion of 77 oscillations instead of 78. Newton concluded that the whole resistance of the empty box is to the whole resistance of the full box as 77 to 78.[86] From this result Newton concluded:

> The resistance encountered by the empty box on its internal parts is therefore more than 5,000 times smaller than the similar resistance on the external surface. *This argument depends on the hypothesis that the greater resistance encountered by the full box does not arise from some other hidden cause but only from the action of some subtle fluid upon the enclosed metal* [non ab aliqua causa latente oriatur, sed ab actione sola fluidi alicujus subtilis in metallum inclusum].[87]

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82 Smith, "The Newtonian Style in Book II of the *Principia*", p. 263. Cf. Smith, "Was Wrong Newton Bad Newton?", p. 140 and Calero, *The Genesis of Fluid Mechanics, 1640–1780*, pp. 81–89, esp. pp. 88–89.

83 Read: *ad hoc* adjustments of the data.

84 Smith, "The Newtonian Style in Book II of the *Principia*", p. 263; id., "Was Wrong Newton Bad Newton?", p. 137.

85 Newton, *The Principia*, pp. 722–723.

86 Ibid., p. 723.

87 Ibid. [italics added].

This showed that the resistence of the ether on the internal parts was very small.[88] Newton pointed out that he "reported this experiment from memory," so that he was forced "to omit certain fractions of numbers."[89]

In the second and third edition, Newton introduced the afore mentioned vertical-fall experiments in water and air, which provided better data for the conclusions he set out to establish earlier.[90] In Experiment 13 (added to the second edition of the *Principia*) in the *scholium* concluding Section 7, Newton reported on free fall experiments performed on June 1710 by Francis Hauksbee at the balcony of St. Paul's Cathedral.[91] Two balls – one filled with quicksilver; the other with air – rested on a platform of which one side could be released by pulling a peg. Upon pulling the peg rapidly, the two balls fell simultaneously. Their fall did not show a significant difference compared to free fall in a vacuum, Newton reported.[92] In the third edition of the *Principia*, Newton added a discussion of J. T. Desaguliers' free fall experiments with hogs' bladders.[93] These were shown to be consistent with Hauksbee's results. In the penultimate paragraph of the concluding *scholium* to Section 7, Newton dropped a hint of what was to follow: "And therefore the celestial spaces, through which the globes of the planets and comets move continually in all directions very freely and without any sensible diminution of motion, are devoid of any corporeal fluid, except perhaps the very rarest vapors and rays of light transmitted through them."[94] "[T]herefore," Newton would note in Proposition X of Book III, "in the heavens, which are void of air and exhalations, the planets and comets, encountering no sensible resistance, will move through those spaces for a very long time."[95] The vertical-fall experiments established that, at least to a first approximation, the resistance force on spheres at high velocities in both water *and* air varies to a predominant extent as $c\rho d^2 v^2$, where c was quite close to Newton's theoretical value for continuous fluids.[96] This result therefore suggested to Newton that, given

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[88] The first edition of the *Principia* contained the extra lines: "At causam longe aliam esse opinor. Nam tempora oscillationum pyxidis plenæ minora sunt quam tempora oscillationum pyxidis vacuæ, & propterea resistentia pyxidis plenæ in externa superficie major est, pro ipsius velocitate & longitudine spatii oscillando descripti, quam ea pyxidis vacuæ. Quod cur ita sit, resistentia pyxidum in partibus internis aut nulla erit plane insensibilis." (PrE_1, p. 353).

[89] Newton, *The Principia*, p. 723.

[90] See Smith, "The Newtonian Style in Book II of the *Principia*", pp. 272–282 for extensive commentary.

[91] Newton initially pondered on doing such experiments at Trinity. In one of his memoranda (July 1694), David Gregory recorded "He is choosing the place for contriving his experiments from the top of Trinity College Chapel into his own garden on the right as one enters the College." (Newton, *Correspondence*, III, p. 384).

[92] See Newton's table in Newton, *The Principia*, p. 757 [with correction factor] and CUL Add. Ms. 3965, f. 101^r (ca. 1710).

[93] Newton, *The Principia*, pp. 758–759.

[94] Ibid., p. 761.

[95] Ibid., p. 816.

[96] Smith, "The Newtonian Style in Book II of the *Principia*", p. 279; id., "Was Wrong Newton Bad Newton?", pp. 143–145.

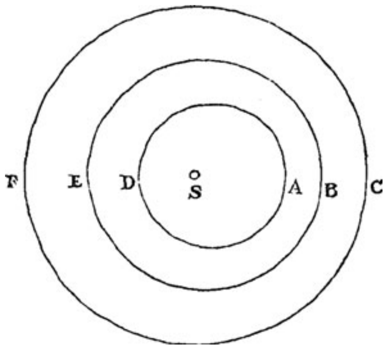

Fig. 3.2 Original figure accompanying the scholium to Proposition LIII, Book II. Reproduced from *PrE*3, p. 384

the closeness of c as determined theoretically and c as determined empirically, the $c\rho d^2v^2$-term was "something more than a mere approximation akin to a curve fit."[97] Additionally, these results suggested that both water and air behave as continuous fluids.[98] It seems therefore that Newton's continuous-fluid model should best be seen as a working hypothesis, i.e. as a "promising first idealization to the physics underlying fluid resistance."[99]

At the end of Book II, Newton set out to refute Cartesian planetary vortices. In the scholium to Proposition LIII, Newton argued that "the planets are not carried along by corporeal vortices [à Vortibus corporeis non deferri]."[100] Newton offered the following argument. Now, let AD, BE, and CF designate three orbits around the sun S and let A and B be the aphelia of the two inner ones, and D and E their perihelia (Fig. 3.2). Newton continued as follows:

> And a body that revolves in the orbit BE [or AD] will, according to the laws of astronomy, move more slowly in the aphelion B [or A] and more swiftly in the perihelion E [or D], although according to the laws of [vortex] mechanics, the matter of the vortex ought to move more swiftly in the narrower space between A and C than in the wider space between D and F, that is, more swiftly in the aphelion than in the perihelion.[101]

Thus, Newton concluded that "the hypothesis of the vortices can in no way be reconciled with astronomical phenomena and serves less to clarify the celestial motions that to obscure them"[102] for Cartesian vortices "would by their tenacity & stiffness communicate their motion to one another till they all rested among themselves."[103] This particular argument failed to convince the many "Patrons of a Plenum,"[104]

[97] Smith, "The Newtonian Style in Book II of the *Principia*", p. 281.
[98] See, furthermore, the discussion in the last paragraphs of Section 4.8 in Chapter 4.
[99] Smith, "The Newtonian Style in Book II of the *Principia*", p. 281.
[100] Newton, *The Principia*, p. 789.
[101] Ibid.
[102] Ibid. p. 790.
[103] CUL Add. Ms. 3970, f. 255r [ca. 1700–1704].
[104] Cotes' terminology in Cotes to Newton, 4 June 1711, Newton, *Correspondence*, V, p. 153.

since Newton treated the friction between the surfaces of the different layers of the fluid as if they were plane, i.e. as if the friction was acting in the same direction. For curved surfaces, however, the frictional force acts in different (tangential) directions all around the circumference.[105] Despite of this, Newton had good reasons to reject Cartesian vortices: his pendulum experiments indicated that the resistance of the subtle ether on the internal parts of bodies is very small, and, in Book III he showed, moreover, that no resisting medium should be taken into account in order to explain planetary or cometary motion.

Rejecting vortex cosmology was vital for the establishment of the theory of universal gravitation. It is, however, difficult to pinpoint an exact moment in time at which Newton made "the" discovery of universal gravitation. It required the combination of different and separately non-obvious elements (e.g., evidence that terrestrial and celestial bodies suffer little resistance from the surrounding medium, the new concept of centripetal force, a suitable mathematical apparatus that could adequately deal with motion, etc.). It was a gradual and complex process that started with his 1679 correspondence with Hooke, in which Hooke communicated his hypothesis that of "compounding the celestiall motions of the planetts of a direct motion by the tangent & an attractive motion towards the centrall body,"[106] and that was continued in his study of the motion of the comets of 1680 and 1682 (ca. 1681–1682 and later),[107] the versions of *De Motu* (1684–1685),[108] and in his correspondence with, the first Royal Astronomer, John Flamsteed.[109]

Nick Kollerstrom has recently argued that Newton only came to reject vortices once he had studied the retrograde comet of 1682, which in contrast to the comet of 1680 went in the reverse direction to the planetary orbits without showing any noticeable signs of retardation.[110] Although Newton was aware of John Flamsteed's

105 Dobson, "Newton's Errors with the Rotational Motion of Fluids", p. 245. See, furthermore, id., "Newton's Problems with Rigid Body Dynamics". Moreover, 2 years after the publication of the first edition of the *Principia*, Leibniz claimed in his *Tentamen de motuum coelestium causis* that his vortex theory was compatible with Kepler's rules.

106 Newton, *Correspondence*, II, p. 297. See furthermore: Nauenberg, "Robert Hooke's Seminal Contribution to Orbital Mechanics"; id., "Hooke's and Newton's Contributions to the Early Development of Orbital Dynamics and the Theory of Universal Gravitation", p. 519; and, Meli, "Who is Afraid of Centrifugal Force?", pp. 540–541 for an update on Hooke's significance for Newton's conceptualisation of orbital motion.

107 See Ruffner, *The Background and Early Development of Newton's Theory of Comets*, pp. 205–335. See id., "Newton's Propositions on Comets: Steps in Transition", 1681–84, for a discussion of Newton's 14 propositions on comets (early 1680s) (CUL Add. Ms. 3965, f. 613$^{r\text{-}v}$) [note that Propositions 10 and 11 are lacking in Newton's earlier version (ibid., ff. 564^{r}-565^{r})].

108 See Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI. For a facsimile edition of the three versions of *De motu*, see Whiteside, *The Preliminary Manuscripts for Isaac Newton's 1687 Principia: 1684–1685*.

109 For useful comments on the different versions of *De motu* and the transition to the *Principia*, see Smith, "How did Newton discover Universal Gravitation?", pp. 32–63 and Wilson, "From Kepler's Laws, So-Called, to Universal Gravitation: Empirical Factors".

110 Kollerstrom, "The Path of Halley's Comet, and Newton's Late Apprehension of the Law of Gravity", pp. 354–355. Cf. Newton, *The Principia*, pp. 895, 934–935. On Newton's study of comets, see furthermore: Genuth, "Comets, Teleology, and the Relationship of Chemistry to

claim that not two comets but only one comet had passed the sun in November and December 1680 (i.e., that the November-December comet was a permanent body describing curved motion), he had uttered several reservations against Flamsteed's view.[111] Newton only gradually abandoned the view that the system of the world was filled with vortices. We do not exactly know when Newton changed his mind. We do know that in the original tract *De motu* Newton stated that celestial bodies, including comets, move freely in space.[112] Newton's rejection of vortices resulted jointly from his study of the motion of comets and the pendulum experiments he performed, which showed that the gravity of oscillating bodies is proportional to their mass.

On the basis of these considerations, one can with a confident level of probability – I am aware that my proposal must remain somewhat conjectural – date the text of Newton's *De gravitatione et æquipondio fluidorum* (CUL Add. Ms. 4003), *as we have it*, to ca. 1683–1684, i.e. after his study of the retrograde comet of 1682 and before *De motu*.[113] *De gravitatione et æquipondio fluidorum* contains a

Cosmology in Newton's Thought"; Hughes, "The *Principia* and Comets; Kubrin, "Newton and the Cyclical Cosmos"; and, Schaffer, "Coments and Idols: Newton's Cosmology and Political Theology".

111 This is documented in detail in Ruffner, *The Background and Early Development of Newton's Theory of Comets*, pp. 239–301 and Wilson, "The Newtonian Achievement in Astronomy", pp. 247–253. Cf. Newton, *The Principia*, pp. 911, 915–916. See, furthermore, Newton's letter to Crompton for Flamsteed (28 February 1680/1) (Newton, *Correspondence*, II, p. 342) and Newton to Flamsteed (14 April 1680/1) (ibid., II, pp. 364–365). In a letter to Thomas Burnet in January 1680/1, Newton claimed that vortices offered a sensible explanation of gravity and he posited centrifugal forces in the explanation of the celestial motions.

112 CUL Add. Ms. 3965, f. 50^r (1684). Note that the sentence immediately following is: "Valide resistit argentum vivum, longè minùs aqua, aer verò longè adhuc minùs." (ibid.; this sentence is omitted in Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 79). No further empirical data is further mentioned, however.

113 Cf. Dobbs, *The Janus Faces of Genius*, p. 141; McGuire, "The Fate of the Date: The Theology of Newton's *Principia* Revisited"; and, Mandelbrote, *Footprints of the Lion: Isaac Newton at Work*, pp. 99–100). John Henry has recently argued for an earlier date for *De gravitatione et æquipondio fluidorum* (Henry, "Gravity and *De gravitatione*", pp. 23–26). In this context, Henry has emphasized two points: "Firstly, [. . .] there is no mention of an endeavour among the particles to recede from each other, much less any mention of repulsive forces [in *De gravitatione et æquipondio fluidorum*] (and yet, as we have seen, if this is written in 1685, such ideas had already been discussed by Newton). Secondly, in spite of Newton's scepticism about the aether as it appeared in Cartesian physics, he does not conclude that the aether does *not* exist." (ibid., p. 25). The key problem with Henry's arguments is that they presuppose that Newton's tackle on the cause of gravity was continuous through time. Correspondingly, Henry assumes that once Newton hit on an aether consisting of mutually repelling particles as an explanation for the cause of gravity, he never considered other possible explanations as candidates worthy of further investigation. As I have documented in Section 1.7 of Chapter 1, even after the first edition of the *Principia* Newton wavered between a non-mechanical and a mechanical explanation (*à la* Duillier) of gravity. Therefore, the absence of an ether consisting of mutually repelling particles in *De gravitatione et æquipondio fluidorum* does not necessarily entail that it must have been composed prior to the first time Newton introduced such non-mechanical ether. As I have explained in the same section, Newton's rejection of mechanical explanations of gravity occurred only a couple of years after the first edition of the

fierce rejection of Descartes' account of motion. At the earliest introduction of the material in *De motu* that later would become the *scholium* on space and time,[114] Newton explicitly criticized the Cartesian doctrine of motion as translation with respect to an adjacent body, a point also forcefully made in *De gravitatione*. In the past, *De gravitatione* has been dated much earlier.[115] Mordechai Feingold has recently offered an appealing *via media*: he suggests that the origins of *De gravitatione* date from around 1671, when Newton delivered at Cambridge a series of lectures against Descartes' mechanics and Henry More's hydrostatics.[116] Ten years or so later, Newton contemplated reworking the material he covered in his lectures "into a more sustained philosophical argument against Descartes."[117] In *De gravitatione*, Newton stated that he rejected the corporal nature of the ether and that he now presupposed the existence of a scattered vacuum (cf. "Quemadmodum si materia subtilis vi omni privaretur impediendi motus globulorum, non ampliùs crederem esse materiam subtilem sed vacuum disseminatum."[118]). (Note that, when Newton earlier in *De gravitatione* talked about vortices,[119] he was discussing the implications of Descartes' definition of motion as translation of $body_1$ with respect to its adjacent $body_2$ *within* Descartes' own cosmology and is thus not at all embracing vortices.[120]) In the next sentence, Newton also commented on the motion of comets ("Atque ita si spatium aëreum vel æthereum ejusmodi esset ut Cometarum vel corporum quorumlibet projectilium motibus sine aliquâ resistentia cederet crederem esse penitus inane."[121]), a comment he could only have made after his study of Halley's retrograde comet of 1682. In his vibrant criticism of Dobbs' dating, A. Rupert Hall unfortunately did not take into account the importance of Newton's

Principia. Furthermore, in *De gravitatione* Newton questioned the mechanical nature of the ether (Janiak, ed., *Newton Philosophical Writings*, p. 34).

114 Namely, in "Def. 10" of the initial revise of *De motu* (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 190; CUL Add. Ms. 3965, f. 26^r (1684–1685).

115 Hall and Hall, eds., *Unpublished Scientific Papers*, pp. 89–90 suggested between 1664–1668; Herivel, *The Background to Newton's Principia*, pp. 91–93 suggested between 1665–1669 (certainly before 1673); Westfall, *Never at Rest*, p. 301 suggested earliest 1668; Biarnais, *Isaac Newton, De la gravitation ou les fondements de la mécanique classique*, p. 13 suggested 1662–1665; Steinle, *Newtons Entwurf* "*Über die Gravitation*," p. 124 suggested late 1660s, and, D. T. Whiteside suggested between 1970–1973 (personal communication reported in Böhme, ed., *Über die Gravitation*, p. 10 [this German edition contains a facsimile of CUL Add. Ms. 4003]).

116 Feingold, *The Newtonian Moment*, p. 26.

117 Ibid.

118 CUL Add. Ms. 4003, p. 30. Translated by Janiak as: "In the same way, if the subtle matter were deprived of all forces of resistance to the motion of globules, I should no longer believe it to be subtle matter but a scattered vacuum." (Janiak, ed., *Newton, Philosophical Writings*, p. 34).

119 CUL Add. Ms. 4003, pp. 3–9, 11.

120 Cf. ibid., p. 31.

121 Ibid. Translated by Janiak as: "And so if there were any aerial or aetherial space of such a kind that it yielded without any resistance to the motions of the comets or any other projectiles, I should believe that it was utterly empty." (Janiak, ed., *Newton, Philosophical Writings*, p. 34).

study of cometary motion.[122] Moreover, Dobbs has emphasized that in *De gravitatione et æquipondio fluidorum* Newton referred to pendulum experiments, on the basis of which he concluded that the gravities of oscillating bodies are as their quantity of matter. This was a crucial insight, which helped to pave the way for the *Principia*. In *De gravitatione et æquipondio fluidorum*, Newton wrote:

> For if the aether were a corporeal fluid entirely without vacuous pores, however subtle its parts are made by division, it would be as dense as any other fluid, and it would yield to the motion of passing bodies with no less inertia; indeed with a much greater inertia if the projectile were porous, because then the aether would enter into its internal pores, and encounter and resist not only the whole of its external surface, but also the surface of all the internal parts. Since the resistance of the aether is on the contrary so small when compared with the resistance of quicksilver as to be over ten or a hundred thousand times less, there is all the more reason for thinking that by far the largest part of the aetherial space is empty, scattered between the aetherial particles. The same may also be conjectured from the various gravities of these fluids, for the descend of heavy bodies and the oscillations of pendulum show that these are in proportion to their densities, or as the quantities of matter contained in equal spaces. But this is not the place to go into this.[123]

In this very passage, Newton suggested that the resistance of the subtle ether is very small, namely 10,000 or 100,000 times less than quicksilver. Moreover, he noted that if extremely rarefied corporeal ether existed it would act on both the whole external surface and on each of the surfaces of its internal parts. However, Newton's experiments with pendulums had shown that the resistance exerted on the internal parts of oscillating bodies is negligible. Moreover, experiments with pendulums reveals that the gravities of oscillating bodies are as their densities or quantity of matter ("ut eorum densitates sive ut quantitates materiæ").[124] In the

[122] Hall, "Pitfalls of Editing Newton's Papers", pp. 415–421. In a recent article, Howard Stein also fails to take into account the importance of Newton's views on cometary motion and his experiments with pendula (Stein, "Newton's metaphysics", pp. 298–299, footnote 27, pp. 302–303, footnote 39).

[123] Janiak, ed., *Newton, Philosophical Writings*, p. 35. The original states: "Sed nequa supersit dubitatio, ex prædictis observandum venit quod inania spatia in rerum natura dantur. Nam si æther esset fluidum sine poris aliquibus vacuis penitus corporeum, illud, utcunque per divisionem partium subtiliatum, foret æquè densum atque aliud quodvis fluidum, et non minori inertiâ motibus trajectorum cederet, imò longè majori, si modò projectile foret porosum; propterea quod intimos ejus poros ingrederetur, et non modo totius externæ superficiei sed et omnium internarum partium superficiebus occurreret et impedimento esset. Sed cùm ætheris e contra tam parva est resistentia ut ad resistentiam argenti vivi collata videatur esse plusquam decies vel centies mille vicibus minor: sane spatij ætherei pars longè maxima pro vacuo inter ætherea corpuscula disseminato haberi debet. Quod idem præterea ex diversa gravitate horum fluidorum conjicere liceat, quam esse ut eorum densitates sive ut quantitates materiæ in æqualibus spatijs contentæ monstrant tum gravium descensus tum undulationes pendulorum. Sed his enucleandis jam non est locus." (CUL Add. Ms. 4003, pp. 30–31).

[124] Later, in Proposition XXIV of Book I of the *Principia*, Newton started with an application of the second law of motion: $v :: \frac{Fm \times t}{M}$. If the pendulums are of the same length, the motive forces are as the weights: $\frac{Fm1}{Fm2} :: \frac{W1}{W2}$. Then the velocities in the corresponding parts of the oscillations will be to one another as the motive forces and the whole times directly and the quantities of matter inversely: $\frac{v1}{v2} :: \frac{Fm1 \times t1 \times M2}{Fm2 \times t2 \times M1}$ or $\frac{M1}{M2} :: \frac{Fm1 \times t1 \times v2}{Fm2 \times t2 \times v1}$. Since the velocities are inversely as the squares of

original tract *De motu*, Newton pointed out that “the resistance of pure aether is either non-existent or extremely small.” Moreover, such mediums “resist according to their density, which is almost proportional to their weights and hence (I may almost say) according to the quantity of their solid matter.”[125] The latter parts are strong indications for dating *De gravitatione et æquipondio fluidorum, as we have it*, shortly before the composition of *De motu*, i.e. to ca. 1683–1684. Obviously, this dating does not exclude that Newton’s thoughts on space and time and his criticism on Descartes were conceived earlier, as is suggested by Feingold. Finally, it should be noted the handwriting does not differ significantly from that in *De Motu* (see Figs. 3.3 and 3.4).[126]

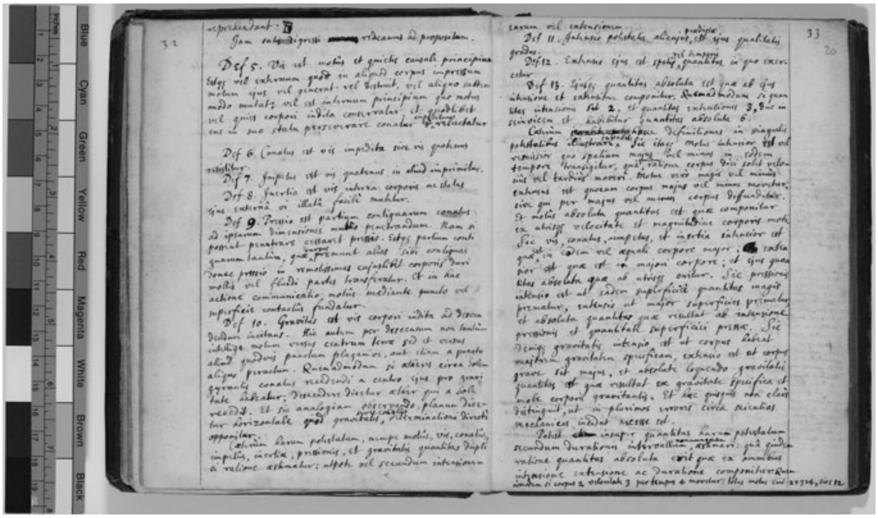

Fig. 3.3 CUL Add. Ms. 4003, pp. 32–33. Courtesy of the Cambridge University Library, Manuscript Department

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the times and we assume that the times are equal: $\frac{M1}{M2} :: \frac{Fm1\times t1^2}{Fm2\times t2^2} :: \frac{W1\times t1^2}{W2\times t2^2}$ or $\frac{M1}{M2} :: \frac{Fm1}{Fm2} :: \frac{W1}{W2}$. Hence, if the times are equal, the quantities of matter will be as weights.

125 Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 286. The original is on p. 261.

126 Cf. Dobbs’ assessment in Dobbs, *The Janus Faces of Genius*, p. 143. CUL Add. Ms. 4003 is consistently written in Newton’s handwriting. The corrections are written in a darker ink suggesting that Newton corrected the text afterwards. Given the fact that Newton’s corrections for the most part involve reformulations and minor changes, it seems plausible that the uncorrected text of *De gravitatione* was copied from an earlier draft version, which has gone lost. It is worth pointing out that Newton’s writing style is affected by the format of the paper on which he wrote. In contrast to the usual folios on which Newton composed *De motu*, CUL Add. Ms. 4003 is written on a small notebook with a leather cover, bound in Cambridge (ca. 15 × 19 cm; 191 folios *in toto*; written text in ink from ff. 1^r-23^v; Newton’s own numbering starts at f. 4^v and ends at f. 23^r; margin lines are drawn from f. 1^r to f. 31^r; watermark on f. 2: two lions holding a shield with three vertical x’s (indicating an Amsterdam origin); watermarks on f. 3 and f. 190: a crown).

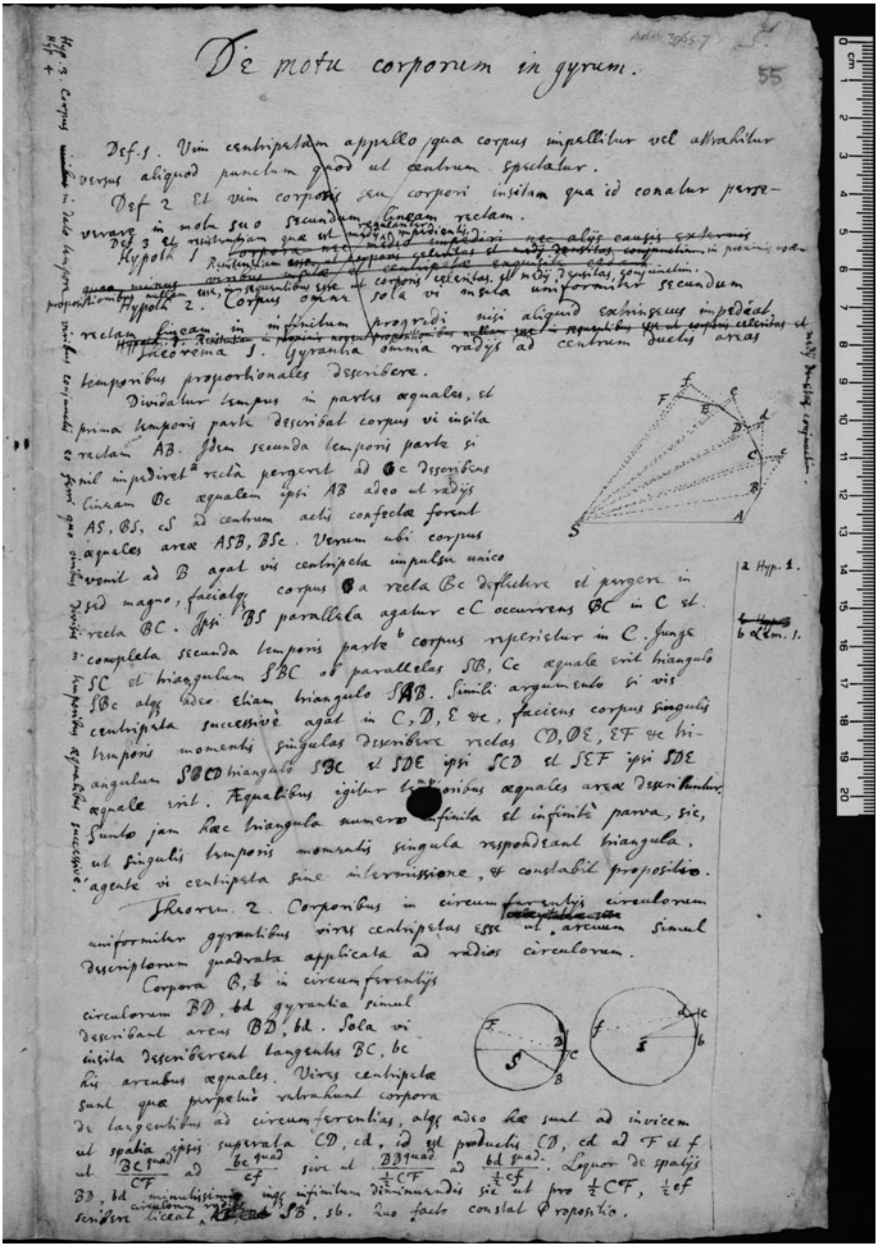

Fig. 3.4 CUL Add. Ms. 3965, f. 55r. Courtesy of the Cambridge University Library, Manuscript Department

3.4 The Arguments for Universal Gravitation: The Analysis

In the analytic part of Book III of the *Principia*, Newton set out to proceed "from Motions to the Forces producing them; and in general, from Effects to their Causes."[127] In Propositions I–V, Book III, Newton inferred the forces acting in the solar system, which corresponds to the phase of model application, and in Propositions VI–VIII, Book III he elaborates his theory of universal gravitation, which corresponds to the phase of theory formation.

3.4.1 Propositions I–II: The Inference of Inverse-Square Centripetal Forces Acting on the Primary and Secondary Planets

Propositions I and II of Book III underwent no significant changes in any of the *Principia*'s editions. In Proposition I, Newton inferred from Phenomenon I,[128] which states that "[1] *the circumjovial planets, by radii drawn to the center of Jupiter, describe areas proportional to the times*[129]*, and* [2] *their periodic times – the fixed stars being at rest*[130] *– are as the 3/2 powers* [i.e., sesquialteral powers] *of their distances from that center*,"[131] that "[1′] *[t]he forces by which the circumjovial planets are continually drawn away from rectilinear motions and are maintained in their respective orbits are directed to the center of Jupiter and* [2′] *are inversely as the squares of the distances of their places from that center*"[132] by Proposition II – in case Kepler's area law is taken exactly – or Proposition III – in case Kepler's area law is taken *quam proxime* – and Corollary 6 to Proposition IV[133] of Book I – which takes the harmonic law to hold exactly, respectively. The same procedure can be applied for the circumsaturnian planets by Phenomenon II and the same propositions of Book I.[134]

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[127] Newton, *The Opticks*, p. 404.

[128] Newton's "phenomena" are inductive generalizations based on a large number of singular astronomical observations and their complex mathematical processing. Here I shall not further discuss how these astronomical observations were obtained. Instead I refer the reader to Densmore's discussion of this in her *Newton's* Principia*: The Central Argument*, pp. 242–282.

[129] For "[t]he orbits of these planets do not differ sensibly from circles concentric with Jupiter, and their motions in these circles are found to be uniform" (Newton, *The Principia*, p. 797).

[130] I.e., here, as in the rest of the *Principia*, Newton considered relative motions.

[131] Newton, *The Principia*, p. 797.

[132] Ibid., p. 802 [numbers added].

[133] For the secondary planets Newton's application of Corollary 6 is no surprise, since he assumes that the orbits of the circumjovial planets, e.g., do "not differ sensibly from circles concentric with Jupiter" (ibid., p. 797).

[134] Although D. T. Whiteside's claim that the area rule "was seemingly firmly accepted by no one and even its formal enunciation but rarely stated in the period" (Whiteside, "Newton's Early

In Proposition II, Newton inferred from Phenomenon V, which states that "[1] *[t]he primary planets, by radii drawn to the earth, describe areas in no way proportional to the times but, by radii drawn to the sun, traverse areas proportional to the times*,"[135] and Phenomenon IV, which states that "[2] *[t]he periodic times of the primary planets and of either the sun about the earth or the earth about the sun*[136] *– the fixed stars being at rest – are as the 3/2 powers of their mean distances from the sun*,"[137] that "[1′] *[t]he forces by which the primary planets are continually drawn away from rectilinear motions and are maintained in their respective orbits are directed to the sun and* [2′] *are inversely as the squares of their distances from*

Thoughts on Planetary Motion", p. 121; cf. Russell, "Kepler's Laws of Planetary Motion: 1609–1666", p. 5) is somewhat of an overstatement (see: Thoren, "Kepler's Second Law in England"), there was no wide-spread consensus in pre-Newtonian astronomy about the adequacy of Kepler's area rule (see furthermore: Wilson, "Newton and Some Philosophers on Kepler's Laws", pp. 233–240). Although the early Newton must have been aware of Kepler's area rule, it is not discussed explicitly in his early work (Whiteside, "Newton's Early Thoughts on Planetary Motion", p. 124). Newton made annotations in his copy of Mercator, *Institutionum astronomicarum libri II* (Harrison, *The Library of Isaac Newton*, p. 191 [item n° 1072]; Wren Library, NQ.16.196), in which the area law is stated on p. 145. We are basically left in the dark on Newton's early thoughts on the area rule (cf. Whiteside, "Before the *Principia*: The Maturing of Newton's Thought on Dynamical Astronomy", 1666–1684, p. 9). Newton did not attach any physical significance to it, until shortly before he began composing *De motu*. A letter to Hooke suggests that Newton did not accept the area rule in 1679. In the very letter, Robert Hooke brought the area law and the uniform and concentric motion of the primary planets to Newton's consideration, and pointed to its connection with the "attractive motion towards the centrall body" (Hooke to Newton, 24 November 1679, Newton, *Correspondence*, II, pp. 297–300, esp. p. 297). Newton's response is telling: "But how ye Orbits of all ye primary Planets but ☿ can be reduced to so many concentric circles through each of wch ye primary Planet moves equal spaces in equal times (for that's ye Hypothesis if I mistake not your description) I do not yet understand. [. . .] I know no body in ye University addicted to making Astron. Observations: & my shortsightedness & tenderness of health makes me something unfit." (Newton to Hooke, 28 November 1679, Newton, *Correspondence*, II, pp. 300–304, p. 301). Newton only came to accept the area rule when John Flamsteed's astronomical observations indicated a fairly accurate confirmation of it (e.g., Flamsteed to Newton, 27 December 1684, Newton, *Correspondence*, II, pp. 403–406, pp. 404–405).

By contrast, from quite early on, Newton was in favour of the validity of Kepler's harmonic rule. In his Trinity Notebook, following Streete, *Astronomia Carolina, A New Theory of the Coelestial Motions*, Newton had affirmed the validity of Kepler's third rule (CUL Add. Ms. 3996, f. 29^{r}). Also, in his autograph endnotes to Wing, *Astronomia Britannica* (see Harrison, *The Library of Isaac Newton*, p. 263 [item n° 1734; Wren Library, NQ.18.36]), Newton judged that the periodic times and distances of the primary planets to the sun have a good fit with the harmonic rule ("accuratas esse juxta planetarum observationes judico") (see furthermore: Whiteside, "Newton's Early Thoughts on Planetary Motion", p. 125, footnote 30). In 1684 Newton obtained further accurate evidence from John Flamsteed that supported Kepler's harmonic rule *quam proxime* (Flamsteed to Newton, 27 December 1684, Newton, *Correspondence*, II, pp. 403–406, esp. p. 404).

135 Newton, *The Principia*, p. 801 [numbering added].

136 At this point, Newton leaves open the possibility of the Tychonic theory. It is only in Proposition XII of Book III that Newton established that the sun is the common centre of gravity of all planets (ibid., p. 817; Harper, "Newton's Argument for Universal Gravitation, p. 193). For Newton the question of the world systems was a dynamical problem.

137 Newton, *The Principia*, p. 800. See furthermore the discussion in Section 3.5.

its center,"[138] by Proposition II of Book I, which takes the area law to hold exactly, and Corollary 6 to Proposition IV, which takes the harmonic law to hold exactly, respectively. Moreover, in the case of the primary planets the inverse square law is proved "with the greatest exactness from the fact that the aphelia are at rest,"[139] since the slightest departure ("aberratio") from an inverse square law would entail motion in the aphelia (by Book I, Proposition XLV[140]).

3.4.2 Propositions III–IV: The Inference of an Inverse-Square Centripetal Force Acting on the Moon

In Proposition III Newton showed that "[t]he force by which the moon is maintained in its orbit is directed toward the earth and is inversely as the square of the distance of its places from the center of the earth."[141] The first part of this proposition is established by Phenomenon VI,[142] which states that the moon by a radius drawn to the centre of the earth describes areas proportional to the times,[143] and Proposition II or Proposition III of Book I. The area law for the moon holds exactly only in the absense of the sun's disturbing gravitational force of the sun on the moon. Because the moon is a solitary satellite, Newton cannot use the route via Corollary 6 to Proposition IV. The second part of Proposition III, however, follows from the very slow motion of the moon's apogee (which is caused by the sun's disturbing force): "[f]or that motion, which in each revolution is only three degrees and three minutes forward [or in consequentia, i.e., in an easterly direction] can be ignored."[144] From Corollary 1 to Proposition XLV, Book I, it follows that the centripetal force by which the moon is drawn to the earth is proportional to the n-th power of the distance, where n equals $\left(\frac{360°}{360° + p°}\right)^2 - 3$ and $p°$ is the moon's apsidal motion in degrees. In case of a $3°3'$ apsidal motion, n equals $\left(\frac{360°}{360° + 3°3'}\right)^2 - 3$, i.e. the centripetal force by which the moon is drawn to the earth is proportional to the ca. –2.01673 power of the distance (or, in Newton's phrasing, approximately as the inverse $2_{4/243}$

[138] Ibid., p. 802 [numbering added].

[139] Ibid.

[140] See Section 2.5.2 of Chapter 2. On Newton's apsidal precession theorem, see, furthermore, Valluri, Wilson and Harper, "Newton's Apsidal Precession Theorem and Eccentric Orbits."

[141] Newton, *The Principia*, p. 802.

[142] In commenting on Proposition VI, Newton noted: "Actually, the motion of the moon is somewhat perturbed by the force of the sun, but in these phenomena I pay no attention to minute errors that are neglegible." (ibid., p. 801).

[143] Ibid.

[144] Ibid., pp. 802–803. A translation more close to the original is: "which in each revolution is only three degrees and three minutes in consequentia [i.e., in an easterly direction forward]."

power[145] (≈ 2.01646) of the distance).[146] Thus: "the proportion of the force to the distance is inversely as a little greater than the second power of the distance, but is $59_{3/4}$ times closer to the square than to the cube."[147] In Corollary 2 to Proposition XLV, Book I, it is shown that, if the centripetal force by which a body (the moon) revolves in an ellipse varies inversely to the square of the distance from the centre of another body (the earth), the motion of the apsides (in degrees) that arises from the extraneous centripetal force of a third body (the sun) can be determined as follows – on the assumption that the extraneous force is 375.45 times less[148] than the inverse-square centripetal force by which the body describes an ellipse: $p^\circ = 180^\circ \times \sqrt{\frac{1-\frac{1}{357.45}}{1-\frac{4}{357.45}}} \approx 180.7623^\circ$ (i.e. $\approx 180^\circ 45' 44''$).[149] Since the moon when departing from the upper apsis will arrive at the lower apsis by an angular motion of $180^\circ 45' 44''$, it follows that in each revolution the upper apsis will move forward through $1^\circ 31' 28''$. Since Newton furthermore assumed that "[t]he [advance of the] apsis of the moon is about twice as swift,"[150] it follows that the extraneous force of the sun is to the centripetal of the moon as roughly $\frac{2}{357.45}$ or as $1/178_{29/40}$ and that in each revolution the upper apsis will move forward through $3^\circ 2' 56''$, a value that agreed nicely with the $3^\circ 3'$ derived from astronomical observation.[151] The latter step contained a serious lacuna, however, for the assumption that the moon's apsis moves twice as fast was not derived from theory.[152] Initially, Newton seemed to think that the sun's transverse radial component could account for the doubling

[145] This is the value Newton had calculated in Corollary 1 to Proposition XLV (ibid., p. 544).

[146] Ibid., p. 803.

[147] Ibid. This force differs $\frac{4}{243}$ from the inverse-square proportion and $\frac{239}{243}$ from the inverse-cube proportion. By dividing the difference from the cube proportion, $\frac{239}{243}$, by the difference from the inverse-square proportion, $\frac{4}{243}$, we arrive at ca. $59_{3/4}$.

[148] Newton, first of all, decomposed the sun's perturbing force on the moon into a radial and a transradial component. Given the mathematical properties of a three-body system based on Proposition LXVI of Book I, which takes the moon's orbit to be circular, and by an application of Corollary 17 to Proposition LXVI, Book I (cf. Proposition XXV, Book III (ibid., p. 840)), Newton calculated that the average value of the radial component of the sun's perturbing force that draws the moon away from the earth is to the acceleration of the moon to the earth as $1/2 \left(\frac{\mathrm{T}m}{\mathrm{T}e}\right)^2$, i.e. as $1/2 \left(\frac{27\mathrm{d}7\mathrm{h}43\mathrm{m} = 39{,}343 \text{ minutes}}{365\mathrm{d}6\mathrm{h}9\mathrm{m} = 525{,}969 \text{ minutes}}\right)^2$, which yields a ratio of ca. 1 to 357.45 (Wilson, "The Newtonian Achievement in Astronomy", p. 264).

[149] Newton, *The Principia*, pp. 545, 803. The computation based on Corollary 2 to Proposition XLV, Book I, was added in the second edition.

[150] Ibid., p. 545. The line "Apsis Lunæ est duplo velocior circiter" was added in the third edition and occurs in one of Newton's Wren copies of the second edition of the *Principia* (Wren Library, NQ 16.196, p. 131).

[151] Newton, *The Principia*, p. 803.

[152] Whiteside, "Newton's Lunar Theory", p. 320; Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 508–510, footnote 1, and pp. 518–519, footnote 26; Aoki, "The Moon-Test in Newton's *Principia*", p. 151, footnote 12; and Wilson, "Newton on the Moon's Variation and Apsidal Motion", pp. 155–172.

of the motion of the moon's apsis; later, for reasons unknown to us, he abandoned this explanation and perhaps even considered non-gravitational sources to account for the other half of the moon's apsidal motion factors (e.g., the earth's magnetic force).[153]

The inverse-square character of the centripetal force which draws the moon to the earth is, furthermore, established in Proposition IV,[154] which contains Newton's famous moon test. The moon test sets out to prove that the earth's gravity extends to the moon and varies inversely as the square of the distance from the centre of the earth. Required for the computation of the moon test are two basic elements: the mean distance of the moon from the earth and the moon's period, i.e. 27 days, 7 h, 43 min (or 39,343 min).[155] For the mean moon-earth distance different values were available at the time. Newton noted that according to Ptolemy[156] and most astronomers the mean distance of the moon from the earth is 59 earth semi-diameters (all editions), according to Vendelin (and Huygens[157]) 60 (all editions), according to Copernicus $60_{1/3}$ (all editions), according to Kircher $62_{1/2}$ (first edition only[158]), according to Tycho $56_{1/2}$[159] (all editions), and according to Street $60_{2/5}$ (third edition). In all editions, Newton took 60 terrestrial semi-diameters as the value for the mean moon-earth distance in the moon test.[160] Suppose that the sun is at rest and that the moon is deprived of all its motion and set to fall towards the earth with all that force by which it is normally kept in orbit, then the moon will fall a distance of $15_{1/12}$ Paris feet[161] ($\approx$ 15.083 Paris feet) or, as he added in the

153 Smith, "The Motion of the Lunar Apsis", in: Newton, *The Principia*, pp. 257–264, p. 261; Newton, *The Principia*, p. 880.

154 Several drafts of this proposition are in CUL Add. Ms. 3965, ff. 78^r-85^r.

155 This value remained unchanged in all editions.

156 Ptolemy's name was added in the third edition (PrE_3, p. 396).

157 Huygens' name was added in the third edition (PrE_3, p. 397).

158 PrE_1, p. 406.

159 With respect to Tycho's value, Newton observes: "But Tycho and all those who follow his table of refractions, by making the refractions of the sun and moon (entirely contrary to the nature of light) be greater than those of the fixed stars – in fact greater by about 4 or 5 min – have increased the parallax of the moon by that many minutes, that is, by about a twelfth or fifteenth of the whole parallax. Let that error by corrected, and the distance will come to be roughly $60_{1/2}$ terrestrial semidiameter, close to the value that has been assigned by others" (Newton, *The Principia*, pp. 803–804). In the first edition Newton corrected Tycho's value as to result in 61 terrestrial semi-diameters (PrE_1, p. 406); in the second edition he corrected Tycho's value as to result in $60_{1/2}$ terrestrial semidiameters (PrE_2, p. 364). On Newton and atmospheric/astronomical refraction, see furthermore Lehn, "Newton on Astronomical Refraction", and, Whiteside, "Kepler, Newton and Flamsteed on Refraction through a 'Regular Aire'".

160 Newton, *The Principia*, p. 804. The average of the five values Newton provided in the third edition is ca. 60.047. In the first edition, the average is ca. 60.567 terrestrial semi-diameters. In the second edition, it is ca. 59.958 terrestrial semi-diameters.

161 One Paris foot equals 1.066 English feet (Densmore, *Newton's* Principia*: The Central Argument*, p. 299).

third edition, "more exactly 15 feet, 1 inch, and $1_{4/9}$ lines"[162] (≈ 15.083 Paris feet) in one minute (by Corollary 9 to Proposition IV, Book I, or Proposition XXXVI, Book I).

From what is given, the period of the moon is 39,343 min $\left(T_M = 39{,}343'\right)$ and the circumference of the earth is 126,249,600 Paris feet ($C_E = 123{,}249{,}600$ Paris feet). The earth's diameter (D_E) can be determined as follows, $D_E = C_E/\pi$ or $D_E = \frac{123{,}249{,}600 \text{ Paris feet}}{\pi} \approx 39{,}231{,}566.1482$. Since the circumference of the moon equals 60 earth circumferences, $C_M = 60 \times 123{,}249{,}600$ Paris feet $= 7{,}394{,}976{,}000$ Paris feet. As $D_M = C_M/\pi$, $D_M = \frac{7{,}394{,}976{,}000 \text{ Paris feet}}{\pi} \approx 2{,}353{,}893{,}968.8919$ Paris feet. As the moon in orbit travels 7,394,976,000 Paris feet in a period of 39,343 min, it follows that it will travel through an orbital distance of 187,961.6704 Paris feet in 1 min. Since from Corollary 9 to Proposition IV of Book I, "the arc which a body, in revolving uniformly in a circle with a given centripetal force, describes in any time is a mean proportional between the diameter of the circle and the distance through which the body would fall under the action of the same given force and in the same time,"[163] it follows that the distance the moon would traverse in a 1 min fall to the earth equals: the square of the distance the moon traverses in orbit in a 1 min period divided by D_M, i.e. $\frac{(187{,}961.6704 \text{ Paris feet})^2}{2{,}353{,}893{,}968.8919 \text{ Paris feet}} \approx 15.009$ Paris feet.[164]

However, this is not yet the final value Newton obtained. Recall that Newton's initial approximation abstracts from the influence of the sun. Therefore, Newton corrects the above result with a factor that takes into account the moon's acceleration toward the sun. The obtained 15.009 Paris feet should be corrected by 1/178.725 of that value.[165] The corrected value (r') is then established as follows: $r' = \left(15.009 \text{ Paris feet} \times \frac{1}{178.725}\right) + 15.009$ Paris feet ≈ 15.093 Paris feet.[166]

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[162] Newton, *The Principia*, p. 804. In the first edition Newton wrote $15_{1/12}$ Paris feet *tout court* (*PrE*$_1$, p. 406; CUL Add. Ms. 3965, f. 84^r, f. 266^r); in the second edition he wrote "pedum Parisiensium $15_{1/12}$ circiter" (*PrE*$_2$, p. 364; cf. the corrections to the first edition on CUL Add. Ms. 3965, f. 87^r, f. 308$^{r\text{-}v}$). It is worth mentioning Shinko Aoki's conclusion on the accuracy of the moon test: "Newton believed he had shown the inverse-square law to be more exactly verified than was in fact the case. If in the Moon-test an accuracy of one part in 6000 was [implicitly] required, in Newton's opinion, to provide an empirical basis for the structure of the *Principia*, then Newton failed in his effort, because he mistook the calculations necessary for this purpose. He would have done better to remain content with the accuracy obtained in the first edition of Proposition IV of Book III; this was reasonably given because the observational data Newton used were poorly determined. It would not then have been necessary to consider sophisticated correction factors in verifying the inverse-square law; these were superfluous, or it was at least premature to take them into account." (Aoki, "The Moon-Test in Newton's *Principia*", p. 169).

[163] Newton, *The Principia*, p. 451.

[164] Here I have calculated this value from the route Newton suggested via Corollary 9 to Proposition IV of Book I. For the route via Proposition XXXVI of Book I using the versed sine, see Spencer, "Do Newton's Rules of Reasoning guarantee Truth … must they?", pp. 779–780.

[165] Newton, *The Principia*, pp. 803, 840.

[166] On the actual value given by Huygens and its derivation, see Aoki, "Corrections and Additions for 'The Moon-Test in Newton's *Principia*: Accuracy of Inverse-Square Law of Universal Gravitation", p. 394.

Accordingly, "since in approaching the earth that force is increased as the inverse square of the distance, and so at the surface of the earth is 60×60[167] times greater than at the moon, it follows that a body falling with that force, in our regions, ought in the space of one minute to describe $60 \times 60 \times 15_{1/12}$ Paris feet, or more exactly 15 feet, 1 in., and $1_{4/9}$ lines[168] [$\approx$ 15.093 Paris feet]."[169] Hence, in a period of one second a body at the surface of the earth falls 15.093 Paris feet.[170] Bodies falling near the earth traverse almost exactly the same distance in an equal amount of time. Huygens had measured that swinging bodies near the surface of the earth traverse 15 Paris feet, 1 in., $1_{7/9}$ lines or 15.0956 Paris feet in 1 s. Since the value calculated from the moon test and Huygens' value are very close, and since the moon would fall in the same direction as terrestrial bodies in free fall, Newton concluded:

> And therefore that force by which the moon is kept in orbit, in descending from the moon's orbit to the surface of the earth, comes out equal to the force of gravity here on earth [æqualis evadit vi gravitatis apud nos], and so (by rules 1 and 2) is that very force which we generally call gravity [est illa ipsa vis quam nos gravitatem dicere solemus].[171]

In other words: Newton, in his moon test, and Huygens, in his pendulum experiments, had measured the same force. As Howard Stein has shown, Newton's inductive conclusion does not simply establish that the accelerations of the terrestrial bodies and the moon vary according to the inverse-square law, but, more precisely, that the accelerations of the earth are everywhere directed to the earth while varying inversely proportional to the square of the distance, i.e. that terrestrial bodies and the moon are both subject to the accelerative force of the earth which extends equally in all directions.[172] In the *scholium* added to Proposition IV in third edition of the *Principia*, Newton noted that "the proof of this proposition can be treated more fully [fusius explicari potest]" by means of a thought-experiment.[173] Suppose that several moons revolve around the earth. Their periodic times will "by the argument

[167] In opting for 60 earth semi-diameters as the moon-earth distance Newton made the computation to the best advantage as to the numbers (cf. Westfall, "Newton and the Fudge Factor", p. 755). Nevertheless, the correlation Newton established was quite strong: William L. Harper has correctly indicated that if we neglect from Newton's $\frac{1}{178.725}$ correction and take each of the lunar distances cited in the third edition of the *Principia* separately, Huygens' value is still well within the error bounds of 14.612–15.47 Paris feet (Harper, "Newton's Argument for Universal Gravitation", p. 182; cf. Boulos, "Newton's Path to Universal Gravitation", p. 157).

[168] 1 "line" is a twelfth of an inch.

[169] Newton, *The Principia*, p. 804; Wren Library, NQ.16.196, p. 364.

[170] Since $(60 \times 60 \times 15.093 \text{ Paris feet})/(60\text{s})^2 = 15.093 \text{ Paris feet}/(1\text{s})^2$.

[171] Newton, *The Principia*, p. 804.

[172] Cf. Stein, "On the Notion of Field in Newton, Maxwell, and Beyond", pp. 267–268; id., "From the Phenomena of Motions to the Forces of Nature: Hypothesis or Deduction?", pp. 212–214; and id., "Newton's Metaphysics", pp. 286–287. (The same point holds for the accelerative force of the primary planets on their satellites, the accelerative force of the sun, and the accelerative force of all bodies universally.) See, furthermore, Harper, "Howard Stein on Isaac Newton: Beyond Hypotheses", pp. 88–91 for useful discussion.

[173] Newton, *The Principia*, p. 805.

of induction [per argumentum inductionis]" obey Kepler's law and therefore their centripetal forces will vary inversely as the square of the distance. Suppose further that the lowest of them would nearly touch the highest mountains. It follows, by the previous computation, that the gravities of this moon will be nearly equal to the gravities on the tops of the mountains. Now, if the force by which the lowest moon descends was different from gravity and the little moon was also heavy toward the earth, then it would, contrary to experience, either descend twice as fast by both forces acting together or not at all. Therefore, Newton repeated his conclusion:

> Since both forces – namely those of heavy bodies and those of the moons – are directed toward the center of the earth and similar to each other and equal [similes et æquales], they will (by rules 1 and 2) have the same cause [eandem habebunt causam].[174]

3.4.3 Proposition V: From Centripetal Force to "Gravity"

Proposition V established that the circumjovial planets, the circumsaturnian planets and the primary planets gravitate toward Jupiter, Saturn and the sun, respectively, and are "always drawn back from rectilinear motions and kept in curvilinear orbits [*retrahi semper a motibus rectilineis, & in orbibus curvilineis retineri*]."[175] The *revolutions of the circumjovial planets about Jupiter*, the *revolutions of the circumsaturnian planets about Saturn*, and the *revolutions of the primary planets about the sun* are phenomena of the same kind as the *revolution of the moon about the earth* and, therefore, by Rule II "depend on causes of the same kind [a causis ejusdem generis dependent], especially since it has been proved that the forces on which those revolutions depend are directed toward the centers of Jupiter, Saturn, and the sun, and decrease according to the same ratio and law (in receding from Jupiter, Saturn, and the sun) as the force of gravity (in receding from the earth)."[176] The latter counts as Newton's clarification as to why these various phenomena are phenomena of the same kind.

Since the primary and secondary planets are bodies of the same kind and since, by Law III, every attraction is mutual, Jupiter and Saturn will in their turn gravitate toward their satellites, the earth will gravitate toward the moon, and the sun will gravitate toward the primary planets (Corollary 1).[177] In Corollary 2, Newton argued that the gravity that is directed towards every planet is inversely as the square of the distance of places from the centre of the planet.[178] In Corollary 3, which was added in the second edition of the *Principia*, he noted "[a]nd hence Jupiter and

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[174] Ibid.

[175] Ibid.

[176] *Ibid*, p. 806.

[177] Corollary 1 was slightly different in the first edition (PrE_1, p. 408). The difference is not relevant to our present discussion.

[178] Note that Corollary 2 was identical in all editions (PrE_1, p. 408; PrE_2, p. 365; PrE_3, p. 399).

Saturn near conjunction, by attracting each other, sensibly[179] perturb each other's motions, the sun perturbs the lunar motions, and the sun and moon perturb our sea, as will be explained in what follows [ut in sequentibus explicabitur]."[180] In the *scholium* following Proposition V, which was added in the third edition of the *Principia*, Newton wrote:

> Hitherto we have called "centripetal" that force by which celestial bodies are kept in their orbits. It is now established that this force is gravity, and therefore we shall call it gravity from now on. For the cause of the centripetal force by which the moon is kept in orbit ought to be extended [extendi debet] to all the planets, by rules 1, 2, and 4.[181]

Let us now go back for a moment to a crucial step in Newton's argument for universal gravitation: the application of Law III in Corollary 1 to Proposition V. As we have seen in the previous chapter,[182] Newton had illustrated the law of action-reaction by means of two bodies which were interposed by a third body. The question now was whether Law III could be legitimately applied to two, spatially separated, gravitationally interacting celestial bodies. This concern was raised by Roger Cotes, who in a letter to Newton in 1712/13, wrote:

> But in the first Corollary of the 5th [proposition of Book III] I meet with a difficulty, it lyes in these words *Et cum Attractio omnis mutua sit* I am persuaded they are true when the Attraction may properly be so call'd, otherwise they may be false. You will understand my meaning by an Example. Suppose two Globes *A* & *B* placed at a distance from each other upon a Table, & that whilst *A* remains at rest *B* is moved towards it by an invisible Hand. A by-stander who observes this motion but not the cause of it, will say that *B* does certainly tend to the centre of *A*, & thereupon he may call the force of the invisible Hand

179 I. B. Cohen pointed out that the effect Newton was looking for was too small to be detected with the instruments available at the time (Newton, *The Principia*, p. 211). To Newton's query whether Flamsteed had observed "Saturn to err considerably from Keplers tables about ye time of his conjunction with Jupiter" (Newton to Flamsteed, 20 December 1684/5, Newton, *Correspondence*, II, pp. 406–408, p. 407), the Royal Astronomer replied he had found the motion of Saturn to be about 27′ slower and Jupiter's about 14′ or 15′ swifter. Furthermore, he remarked that although the errors in Jupiter's and Saturn's motion are not always the same "yet the differences in both are regular & may be easily answered by a small alternation in ye Numbers." Flamsteed had himself corrected Jupiter's motion but he admitted that he had not "beene strict enough to affirme that there is no such exorbitation [of Jupiter] as you suggest." Flamsteed also confessed that he could not "conceave that any impression made by ye one planet upon it can disturbe ye motion of the other" and suggested that Jupiter's and Saturn's motions should be amended and altered before it is inquired whether Jupiter's motion had any influence on Saturn's (Flamsteed to Newton, 5 January 1684/5, Newton, *Correspondence*, II, pp. 408–412, pp. 408–409). Cohen, furthermore, remarked that although Newton made significantly stronger claims about Jupiter's perturbation of Saturn near conjunction in Proposition XIII of the second and third edition of the *Principia* (Newton, *The Principia*, pp. 818–819), we have no documentary evidence that would explain Newton's more assertive stance (ibid., pp. 209–210; see Memoranda 37 by David Gregory, 4 May 1694, Newton, *Correspondence*, III, p. 314/p. 318). On the history of this elusive astronomical problem, see: Wilson's detailed study "The Great Inequality of Jupiter and Saturn: from Kepler to Laplace".

180 Newton, *The Principia*, p. 806. Corollary 3 was added in the second edition and remained unchanged in the third (PrE_2, p. 368; PrE_3, p. 399).

181 Newton, *The Principia*, p. 806.

182 See the subsection entitled "Newton's Laws of Motion," in Section 2.4.2 of Chapter 2.

> the Centripetal force of *B*, or the Attraction of *A* since ye effect appears the same as if it did truly proceed from a proper & real Attraction of *A*. But then I think he cannot by virtue of the Axiom [Attractio omnis mutua est] conclude contrary to his Sense and Observation, that the Globe *A* does also move towards Globe *B* & will meet it at the common centre of Gravity of both Bodies. [...] For 'till this Objection be cleared I would not undertake to answer any one who should assert You do *Hypothesim fingere* I think You seem tacitly to make this Supposition that he Attractive force resides in the Central Body.[183]

According to Howard Stein's diagnosis, Newton was tacitly introducing a hypothesis in Corollary 1 to Proposition V (and again in Proposition VII)[184]:

> The third law of motion does not tell us that whenever one body is urged by a force directed towards a second, the second body experiences an equal force towards the first; it tells us, rather, that whenever one body is acted upon *by* a second, the second body is subject to a force of equal magnitude and opposite direction. Therefore – putting the point in proper generality – what we may legitimately conclude, from the proposition that each body is a center of gravitational force acting upon all bodies, is that for each body *B* there must be some body (or system of bodies) *B'* which, exerting this force on *B*, is subject to the required equal and opposite reaction.[185]

Identifying the two bodies as the two terms involved in applying Law III to gravitational interaction is not in itself warranted by that law. Obviously, Newton's conception of gravity as a universal interaction force underlies his application of Law III in this particular instance. Newton's formal response to Cotes' worry is often quoted.[186] It is, however, the draft version of this letter, which varies significantly from the letter Cotes received, that sheds more light on Newton's tackle of the matter. The significance of the draft lies in the fact that it makes it clear that Newton did not consider the application as a straightforward deduction from Law

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[183] Cotes to Newton, 18 March 1712/13, Newton, *Correspondence*, V, pp. 391–394, p. 392.

[184] This has some truth to it, for Newton obviously not measure the equality of the active gravitational mass and the passive gravitational mass, a point forcefully made in Harper's paper "Howard Stein on Isaac Newton: Beyond Hypotheses", pp. 92–94). For an older analysis, see Koyré, "Les *regulae philosophandi*".

[185] Stein, "From the Phenomena of Motions to the Forces of Nature": Hypothesis or Deduction?", p. 217 [italics in original]. See also his "Newtonian Space-Time", pp. 263–264.

[186] For the reader's convenience I provide the relevant excerpt from the letter which Cotes received: "[T]he Difficulty you mention wch lies in these words [Et cum Attractio omnis mutua sit] is removed by considering that as in Geometry the word Hypothesis is not taken in so large a sense as to include Axiomes & Postulates, so in experimental Philosophy it is not to be taken in so large a sense as to include the first Principles or Axiomes wch I call the laws of motion. These Principles are deduced from Phænomena & made general by Induction: wch is the highest evidence that a Proposition can have in this philosophy. And the word Hypothesis is here used by me to signify only such a Proposition as is not a Phænomena nor deduced from any Phænomena but assumed or supposed wthout experimental proof. Now the mutual & mutually equal attraction of bodies is a branch of the third Law of motion & how this branch is deduced from Phænomena you may see in the end of the Corollaries of ye Laws of Motion, page 22. If a body attracts another body contiguous to it & is not mutually attracted by the other: the attracted body will drive the other before it & both will go away together wth an accelerated motion in infinitum, as it were by a self moving principle, contrary to ye first law of motion, whereas there is no such phænomena in all nature." (Newton to Cotes, 28 March 1713, Newton, *Correspondence*, V, pp. 396–399, 396–397).

III, but rather as an inductive generalization of Law III by Rule III, which was at the same time predicated under Rule IV:

> And when you come at the difficulty you mention in the first Corollary of the 5t Proposition of the third Book, wch lies in these words *Et cum Attractio omnis mutua sit*: the Objection you mention may be proposed & answered in this manner. 1 That it is but an Hypothesis not founded upon any one Observation. 2 That it is attended wth the absurd consequence described[187] p. 22, namely that a body attracted by another body without mutually attracting it would go to the other body and drive it away before it with an accelerated motion in infinitum, contrary to ye first law of Motion. And such an absurd Hypothesis wch would disturb all nature, is not to be admitted in opposition to the first & third Laws of motion wch are grownded upon Phænomena. For that all attraction is mutual & mutually equal follows from both those laws. One may suppose that bodies may by an unknown power be perpetually accelerated & so reject the impenetrability of matter. One may suppose that God can create a penetrable body & so reject the impenetrability of matter. But to admitt of such Hypotheses in opposition to rational[188] Propositions founded upon Phænomena by Induction is to destroy all arguments taken from Phænomena by Induction & all Principles founded upon such arguments. And therefore as I regard not Hypotheses in explaining the Phenomena of nature so I regard them not in opposition to arguments founded upon Phænomena by Induction or to Principles setled upun such arguments. In arguing for any Principle or Proposition from Phænomena by Induction, Hypotheses are not to be considered. The Argument holds good till some Phænomena can be produced against it. This Argument holds good by the third Rule of philosophizing. And if we break that Rule, we cannot affirm any one general law of nature: we cannot so much as affirm that all matter is impenetrable. Experimental Philosophy reduces Phænomena to general Rules & looks upon the Rules to be general when they hold generally in Phænomena. It is not enough to object that a contrary phænomenon may happen but to make a legitimate objection, a contrary phenomenon must be actually produced. Hypothetical Philosophy consists in imaginary explications of things & imaginary arguments for or against such explications, or against arguments of Experimental Philosophers founded upon Induction. The first sort Philosophy is followed by me, the latter too much by Cartes, Leibnitz & some others. And the mutual equality of Attraction (wch is a branch of the third Law of motion) is backt by this further argument that is if the attraction between two bodies was not mutual and mutually equall they would not stay in rerum natura. The body wch is most strongly attracted would go to the other & press upon it, & by the excess of its pressure both would go away together with a motion accelerated in infinitum. [. . .] Thus the Objection wch you mention is not only a Hypothesis & on that account to be excluded [from] experimental Philosophy, but also introduces a principle of self motion into bodies wch would disturbe the whole frame of nature, & in the general opinion of mankind is as remote from the nature of matter as impenetrability [*read*: penetrability] is recconed to be. Experimental philosophy argues only from phænomena, draws general conclusions from the consent of phænomena, & looks upon the conclusion as general when ye consent is general without exception, tho the generality cannot be demonstrated a priori. In Mathematicks all Propositions not demonstrated mathematically are Hypotheses, but some are admitted in as Principles under the name of Axioms or Postulates wthout being called Hypothesis. So in experimental Philosophy its proper to distinguish Propositions into Principles, Propositions & Hypotheses, calling those Propositions wch are deduced from Phænomena by proper Arguments & made general by Induction (the best way of arguing in Philosophy for a general Proposition) & those Hypotheses wch are not deduced from Phænomena by proper arguments. But if any man

[187] In the original "described" is preceded by "~~attende~~" (CUL Add. Ms. 3984.14, f. 1r).

[188] In the original "rational" is preceded by "~~Proposit~~" (ibid.).

> will take the word Hypothesis in a large sense, he may extend it, if he pleases to the impenetrability of matter the laws of motion & the Axioms of Geometer. For it is not worth the while to dispute about the signification of a word.[189]

Judging by the tone of the original draft, which differs greatly from the succint and more formal style of the letter as sent, Cotes' remark about *hypothesim fingere* must have struck a nerve with Newton. In the draft version of this letter, Newton basically gave three different arguments that were to justify the application of Law III in Corollary 1 to Proposition V, Book III. First of all, he provided an *ex negativo* argument – a point which he also made in the letter as sent to Cotes; namely, if gravitationally interacting bodies did not attract each other equally in opposite directions, "both would go away together with a motion accelerated in infinitum," which is an absurdity since it "introduces a principle of self motion into bodies" and "would disturbe the whole frame of nature." However, this argument offers no satisfactory answer to Cotes' worry. Newton's two remaining arguments are more to the point. Secondly, Newton pointed out that Law III was deduced from phenomena and that the application of this law in Corollary 1 was the result of a rendering general by induction by Rule III (cf. "Experimental philosophy argues only from phænomena, draws general conclusions from the consent of phænomena, & looks upon the conclusion as general when ye consent is general without exception, tho the generality cannot be demonstrated a priori."). Newton's third argument is the following: forces exerted by the invisible hand adduced by Cotes or forces exerted by the surrounding ether particles on the attracted body, are causes that are not deduced from phenomena and therefore cannot have any force against arguments that are based inductive generalizations of causes properly deduced from phenomena – here we find a striking anticipation of Rule IV. The point Newton was making is that proper causes, or *verae causae*, should have empirical support and be shown to be true and sufficient of their effects (cf. Rule I). Therefore, alternative causes not established along these criteria are merely hypothetical.[190] Hence, Newton's words: "It is not enough to object that a contrary phænomenon may happen *but to make a legitimate objection, a contrary phenomenon must be actually produced*."

Newton's tackle on the matter was then that, since no alternative forces had been established according to proper natural-philosophical standards[191] (for the evidence

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[189] Newton, *Correspondence*, V, pp. 398–399 (= CUL Add. Ms. 3984.14, f. 1$^{\text{r-v}}$).

[190] Cf. Newton to Cotes, 31 March 1713, ibid., V, p. 400.

[191] This was the point Newton made in a letter to Leibniz: "For since celestial motions are more regular than if they arose from vortices and observe other laws, so much so that vortices contribute not to the regulation but to the disturbance of the motions of planets and comets; and since all phenomena of the heavens and of the sea follow precisely, so far as I am aware, from nothing but gravity acting in accordance with the laws described by me; and since nature is very simple, *I have myself concluded that all other causes are to be rejected and that the heavens are to be stripped as far as may be of all matter*, lest the motions of planets and comets be hindered or rendered irregular [ipse causas alias omnes abdicandas judicavi et cælos materia omni quantum fieri licet privandos ne motus Planetarum et Cometarum impediantur out reddantur irregulares]." (Newton to Leibniz, 16 October 1693, ibid., III, pp. 285–289, p. 287 [italics added]).

for a mechanical ether surrounding the celestial bodies was rather slim), and since we do not have any idea about the cause of gravitation, *we may for the moment* (i.e., until we have a more detailed picture of the full cause of gravity at our disposal) *treat gravitational forces as if they reside in distant bodies*. The above maxim counts as an *approximation* of gravitational interaction, which abstracts, for the moment, from the cause of gravity. On CUL Add. Ms. 3965, f. 731^{r}, which I have discussed in Section 1.6 of Chapter 1, Newton implied that, until we have unravelled the cause of gravity, the attraction between distant bodies must be understood as if it is a virtue of the masses involved. Although Newton's application of Law III in Corollary 1 of Proposition V was not a rigid deduction from phenomena[192] (and Newton was clearly aware of that), it could be licensed by Rule III and be considered as an extension of Law III, which is justifiable, *if and only if*, it is at the same time predicated under Rule IV as well.[193]

3.4.4 Proposition VI: Weight-Mass Proportionality

The main text of Proposition VI, in which Newton established weight-mass proportionality, remained virtually unaltered in all editions – important additions and corrections occurred, however, in its corollaries. Newton first established weight-mass proportionality for terrestrial bodies. The falling of bodies toward the earth takes place in equal times, "at least on making an adjustment for the inequality of the retardation of the air."[194] By means of pendulums Newton was able to determine the equality of the times more accurately. He performed such pendulum tests using nine different materials (gold, silver, lead, glass, sand, common salt, wood, water and wheat):

> I got two wooden boxes, round and equal. I filled one of them with wood, and I suspended the same weight of gold (as exactly as I could) in the center of oscillation of the other. The boxes, hanging by equal eleven-foot cords, made pendulums exactly like each other with respect to their weight, shape and air resistance. Then, when placed so close to each other [and set into vibration], they kept swinging back and forth together with equal oscillations for a very long time. [. . .] And it was so for the rest of the materials. In these experiments, in bodies of the same weight, a difference of matter that would be even less than a thousandth part of the whole could been clearly noticed.[195]

[192] Smith, "From the Phenomena of the Ellipse to an Inverse-square Force: Why Not?", pp. 44–45; Harper, "Howard Stein on Isaac Newton: Beyond Hypotheses?", p. 93 ff., p. 101.

[193] See Will, "Experimental Gravitation from Newton to Einstein", pp. 96–98 for discussion of contemporary experiments on the matter.

[194] Newton, *The Principia*, p. 806.

[195] Ibid., p. 807.

From Corollary 1 to Proposition XXIV of Book II,[196] it then follows that $\frac{M1}{M2}$ :: $\frac{\mathrm{F}m1}{\mathrm{F}m2}$, which was to be demonstrated.[197] Newton then set out to establish mass-weight proportionality for planets, "for there is no doubt that the nature of gravity [natura gravitatis] toward the planets is the same as toward the earth" (cf. Rules II–IV).[198] Newton backed this claim up by several arguments. From the moon test in Proposition IV, it follows that terrestrial bodies, "raised as far as the orbit of the moon and, together with the moon, deprived of all motion, to be released so far as to fall to the earth simultaneously," will in equal times describe equal spaces as the moon, so that their quantity of matter is to the quantity of matter of the moon as their weights are to the weight of the moon.[199] As we know from Proposition I of Book III, the accelerative forces of the satellites of Jupiter/Saturn vary inversely as the square of the distance from the centre of Jupiter/Saturn. Accordingly, in falling from equal heights in equal times they would describe equal spaces, so that their quantity of matter is to the quantity of matter of the Jupiter/Saturn as their weights are to the weight of Jupiter/Saturn.[200] The same argument holds for the primary planets. Furthermore, that the weights of the primary and secondary planets are to their quantities of matter follows by Corollary 3 to Proposition LXV of Book I.[201] For, if they were more or less strongly attracted in proportion to their quantity of matter, then by Corollary 2, Proposition LXV their motions would be perturbed by the inequality of attraction, which is not the case.[202]

In its final edition, Proposition VI was followed by five corollaries. Corollaries 1–4 taken jointly can be seen as a polemic blow at Descartes' vortex cosmology – Corollary 2 is in fact the sole place in the *Principia* where Descartes was mentioned by name. There, Newton rebutted the Cartesian explanation of gravity (Corollaries 1–2) and the Cartesian *plenum* (Corollaries 2–4). In Corollary 1, which remained identical in all editions,[203] Newton concluded that "the weights of bodies do not depend on their forms and textures [pondera corporum non pendent ab eorum formis & texturis]."[204] "For," as Newton continued, "if the weights could be altered with the forms, they would be, in equal matter, greater or less according to the

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[196] Ibid., pp. 700–701. For a recent reconstruction of Newton's experiments, see Wilson, "Redoing Newton's Experiment for Establishing the Proportionality of Mass and Weight".

[197] Proposition XXIV of Book II established that $\frac{M1}{M2}$ :: $\frac{\mathrm{F}m1 \times \mathrm{t}1^2}{\mathrm{F}m2 \times \mathrm{t}2^2}$ obtains for swinging bodies. Since the times are equal, we derive $\frac{M1}{M2}$ :: $\frac{\mathrm{F}m1}{\mathrm{F}m2}$ (Corollary 1).

[198] Newton, *The Principia*, p. 807.

[199] Ibid.

[200] Ibid.

[201] Ibid., pp. 807–808.

[202] As Harper points out: "Absence of such orbital polarization counts as a phenomenon measuring the equality of ratios of mass to weight toward the Sun at equal distances." (Harper, "Newton's Argument for Universal Gravitation", p. 189). For more details on Newton's estimation of the direction and amount of polarization, see Harper, Valluri and Mann, "Jupiter's Moons as a Test of the Equivalence Principle".

[203] PrE_1, p. 410; PrE_2, p. 367; PrE_3, p. 402.

[204] Newton, *The Principia*, p. 809.

variety of forms, entirely contrary to experience."[205] As a body's gravity is proportional to its quantity of matter, it remains unchanged in case the same amount of mass is preserved – irrespective of the body's form. Gravity therefore does not act on a body's surface (Descartes' claim), but on its quantity of matter. In Corollary 2, Newton stated that all bodies universally that are on or near the earth gravitate toward the earth and the weights of all bodies that are equally distant from the centre of the earth are proportional to their quantity of matter. Moreover, gravity is "a quality of all bodies on which experiments can be made and therefore by rule 3 is to be affirmed of all bodies universally [Hæc est qualitas omnium in quibus experimenta instituere licet, & propterea per reg. III. de universis affirmanda est]."[206] If you deny this, then either bodies are devoid of gravity or gravitate less in proportion to their quantity of matter. Now,

> If the aether or any other body whatever either were entirely devoid of gravity or gravitated less in proportion to the quantity of matter, then, since (according to the opinion of Aristotle, Descartes, and others) it does not differ from other bodies except in the form of its matter, it could by a change of its form be transmuted[207] by degrees into a body of the same conditions as those that gravitate the most in proportion to the quantity of their matter; and, on the other hand, the heaviest bodies, through taking on by degrees the form of the other body, could by degrees lose their gravity. *And accordingly the weights would depend on the forms of bodies and could be altered with the forms, contrary to what has been proved in corol. 1*.[208]

In Corollary 3, Newton wrote that "[a]ll spaces are not equally full [Spatia omnia non sunt æqualiter plena.]."[209] For if there were a fluid "with which the region of

[205] Ibid.

[206] Ibid. The argument based on Rule III as well as the reference to Aristotle and Descartes were added in the second edition and remained unaltered in the third edition (*PrE*$_2$, p. 368; *PrE*$_3$, p. 402).

[207] In the first edition of the *Principia*, Newton used Hypothesis III in Corollary 2 to Proposition VI of Book III: "For if the aether or any other body whatever either were entirely devoid of gravity or gravitated less in proportion to its quantity of matter, then, since it does not differ from other bodies except in the form of its matter, it could by a change of its form be changed by degrees into a body of the same condition as those that gravitate the most in proportion to the quantity of their matter (by hyp. 3) [posset idem per mutationem formæ gradatim transmutari in corpus ejusdem conditionis cum iis quæ pro quantitate materiæ quam maximè gravitant, (per Hypoth. III) (*PrE*$_1$, p. 411)], and, on the other hand, the heaviest bodies, through taking on by degrees the form of the other body, could by degrees lose their gravity [& vicissim corpora maxime gravia, formam illius gradatim induendo, possent gravitatem suam gradatim amittere (ibid.)]." (Newton, *The Principia*, p. 809, footnote a). On CUL Add. Ms. 3965, f. 266^r, Newton suggested the following addition to Hypothesis III to be inserted "post induere:" "Peripateticorum et Cartesianorum est Hypothesis & contra eorum præjudicia solummodo dirigitur." In one of his memoranda, Gregory David commented as follows: "This the Cartesians will easily concede. But not the Peripatetics, who make a specific difference between celestial and terrestrial matter. Nor the followers of the Epicurean Philosophy, who make atoms and seeds of things immutable." (quoted from: Newton, *The Principia*, p. 203).

[208] Newton, *The Principia*, p. 809 [italics added].

[209] Ibid., p. 810. In the first edition, the first sentence of Corollary 3 was "Itaque Vacuum necessariò datur." (*PrE*$_1$, p. 411).

the earth would be filled, because of the extreme density of its matter," its specific gravity "would not be less than the specific gravity of quicksilver or of gold or of any other body with the greatest density, and therefore neither gold nor any other body could descend in air."[210] Newton then added the suggestion that the ether could be diminished indefinitely.[211] In Corollary 4, which was added in the second edition of the *Principia* and taken over in the third edition,[212] Newton provided yet another argument against the Cartesian *plenum*: "If all the solid particles of all bodies have the same density and cannot be rarefied without pores, there must be a vacuum [Si omnes omnium corporum particulæ solidæ sint ejusdem densitatis, neque absque poris rarefieri possint, vacuum datur.]."[213] In Corollary 5,[214] Newton differentiated gravitational forces from magnetic ones – put differently, he showed that gravity and magnetism are forces of a different *species*:

> The force of gravity is of a different kind from the magnetic force [diversi est generis a vi magnetica]. For magnetic attraction is not proportional to the [quantity of] matter attracted. Some bodies are attracted [by a magnet] more [than in proportion to their quantity of matter], and others less, while most bodies are not attracted [by a magnet at all]. And the magnetic force in one and the same body can be intended and remitted [i.e., increased and decreased] [vis magnetica in uno & eodem corpore intendi potest et remitti] and is sometimes far greater in proportion to the quantity of matter than the force of gravity; and this force, in receding from the magnet, decreases not as the square but almost as the cube of the distance, as far as I have been able to tell from certain rough observations [ex crassis quibusdam observationibus animadvertere potui].[215]

Magnetism is not a universal force, since it can intended and remitted.[216] This was an important point as Newton's contemporary Robert Hooke, for instance, thought that gravitational attraction is a non-universal force like magnetism.[217] Magnetic forces are, furthermore, not proportional to the quantity of matter attracted and they do not vary inversely proportional to the distance.

[210] Newton, *The Principia*, p. 810.

[211] This final sentence to Corollary 4 was added in the second edition and remained unchanged in the third edition (*PrE*$_2$, p. 368; *PrE*$_3$, p. 402).

[212] *PrE*$_2$, p. 368; *PrE*$_3$, pp. 402–403.

[213] Newton, *The Principia*, p. 810.

[214] Corollary 5 in the second and third edition (*PrE*$_2$, p. 368; *PrE*$_3$, p. 403) correspond to Corollary 4 in the first edition (*PrE*$_1$, p. 411).

[215] Newton, *The Principia*, p. 810. This corollary was identical in the second and third editions. In the first edition the final sentence read: "Estque vis magnetica longe major pro quantitate materiæ quam vis gravitatis: sed & in eodem corpore intendi potest & remitti; in recessu verò à magnete decrescit in ratione distantiæ plusquam duplicata; propterea quod vis longe fortior sit in contactu, quam cum attrahentia vel minimum separantur ab invicem." (*PrE*$_1$, p. 411).

[216] See the discussion of Rule III in Section 3.2 of this chapter.

[217] Aiton, *The Vortex Theory of Planetary Motions*, p. 95.

3.4.5 Proposition VII–VIII: Universal Gravitation

Proposition VII, in which Newton argued for the law of universal gravitation, is one of the few propositions of Book III that is identical in all editions of the *Principia*.[218] Thus far, Newton had demonstrated that all planets gravitate towards each other and that the gravity toward any planet (taken by itself) varies inversely as the square of the distance of the places from the centre of that planet. By Proposition LXIX of Book I, it follows that the gravity towards all planets is proportional to their quantity of matter.[219] At this point, Newton relied on what I have called an *ex hypothesi* inference-ticket – in Propositions I–IV he had shown that its required antecedent holds. Furthermore, since (1) all the parts ("partes") of a planet A are heavy towards planet B, (2) the gravity of each part is to the gravity of the whole as the quantity of matter of that part to the quantity of matter of the whole, and (3) to every action there is an equal reaction (by Law III), it follows that planet B will gravitate in turn toward all the parts of A, and its gravity to any one part will be to its gravity toward the whole of the planet as the quantity of matter of that part to the quantity of matter of the whole.[220] In other words, this conclusion was established by decomposing the overall inverse-square centripetal force of a body *A* on body *B*, which is proportional to the quantity of matter of *A*, into the inverse-square centripetal forces of each of the parts of *A* which individually act on *B* (and which are proportional to the quantities of matter of those parts). By applying Law III and by a similar decomposition of the overall reaction force of body *B* on *A*, it follows that each of the parts of *B* will act upon each of the parts of body *A* inversely proportional to the square of the distance. The above decomposition illustrates the pivotal role of the micro-macro propositions in the derivation of the law of universal gravitation. By Rule III the results of this decomposition are generalized to all bodies universally and, under Rule IV, considered as correct until exceptions occur. Thus, as Newton continued in Corollary 1, the gravity toward the whole planet arises from and is compounded of the gravity of the individual parts ("Oritur igitur & componitur gravitas in planetam totum ex gravitate in partes singulas.").[221] From Corollary 3 to Proposition LXXIV of Book I, which establishes that, "[i]f a corpuscle is placed outside a homogeneous sphere is attracted by a force inversely proportional to the square of the distance of the corpuscle from the center of the sphere, and the sphere consists of attracting particles, the force of each particle will decrease in the squared ratio of the distance from the particle,"[222] it furthermore follows that the gravity toward each of the

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[218] Newton, *The Principia*, pp. 810–811.

[219] See Section 2.5.3 of Chapter 2.

[220] Newton, *The Principia*, p. 811.

[221] Newton noted: "If anyone objects that by this law all bodies on our earth would have to gravitate toward one another, even though gravity of this kind is by no means detected by our senses, my answer is that gravity toward these bodies is far smaller than what our senses could detect, since such gravity is to the gravity toward the whole earth as [the quantity of matter in each of] these bodies to the [quantity of matter in the] whole earth." (ibid.).

[222] Ibid., p. 594. See Section 2.5.4 of Chapter 2.

individual particles of a body is inversely as the square of the distance of the places from those particles (Corollary 2).[223]

In Proposition VIII, Newton observed:

> I was still not certain whether that proportion of the inverse square obtained exactly in a total force compounded of a number of forces, or only nearly so [obtineret accurate in vi tota ex viribus pluribus composita, an vero quam proxime]. For it could happen that a proportion which holds exactly enough [satis accurate obtineret] at very great distances [in majoribus distantiis satis accurate obtineret] might be markedly in error [notabiliter erraret] near the surface of the planet, because there the distances of the particles may be unequal and their situations dissimilar [ob inæquales particularum distantias & situs dissimiles]. But at length, by means of book 1, props. 75 and 76 and their corollaries,[224] I discerned [intellexi] the truth of the proposition dealt with here.[225]

Recall that in Propositions LXXV and LXXVI Newton had shown that the inverse-square law *would hold* exactly for spheres that are homogeneous or for non-homogeneous spheres with symmetrically distributed densities – regardless of the distance at which these spheres are placed with respect to each other.[226] By implication, the inverse-square law will not hold exactly when the involved spheres are not perfectly spherical or have asymmetrically distributed densities. As it stood, Newton did not develop more complex models that could systematically relate non-spherical or asymmetrically distributed bodies to changes in the inverse-square law. However, by Rule IV, he could conclude that the inverse-square law is to be taken exactly or as most closely as possibly true notwithstanding any contrary hypotheses, until yet other phenomena make such propositions either more exact or liable to exceptions. Propositions VI–VIII jointly establish what we now consider Newton's law of universal gravitation: $F = \frac{g.m.m'}{r^2}$, an equation which was alien to Newton himself because his mathematical reasoning was based on proportions only. It was only in the second half of the nineteenth century that the gravitational constant was established, so that gravitational interaction could be measured in standard units henceforth.[227]

As a real *tour de force*, in Corollary 1 to Proposition VIII, Newton determined the proportion of the weight force on bodies of equal mass at equal distances from Jupiter, Saturn and the earth, respectively, to the weight force on bodies of equal mass at equal distances from the sun.[228] For the distance from Jupiter Newton took

[223] In 2001 at the University of Washington Newton's inverse-square law was tested down to 218 μm using a metal ring, suspended from a torsion pendulum, and containing ten equally spaced holes (Hoyle e.a., "Submillimeter Test of the Gravitational Inverse-Square Law"). No deviations from the inverse-square law occurred during this ingenious experiment.

[224] Newton, *The Principia*, pp. 594–596.

[225] Ibid., p. 811.

[226] Cf. Newton, *The Principia*, p. 815 [Proposition IX].

[227] See Ducheyne, "Testing Universal Gravitation in the Laboratory", for a detailed account.

[228] The significance of these calculations lies in the fact that they provide crucial evidence in favour of the Copernican system (Proposition XII, Book III, ibid., pp. 816–817). While Newton's argument for the Copernican system is based on empirical considerations, in Proposition XII he

the orbital radius of Callisto, for the distance from Saturn the orbital radius of Titan, for distance from the earth the orbital radius of the moon, and for the distance from the sun the orbital radius of Venus.[229] As was often the case, Newton did not bother to give the details of the computations – except for a reference to Corollary 2 to Proposition IV of Book I. As Newton here assumed that the orbits of Callisto, Titan, the moon and Venus are circular, we can indeed rely on Proposition IV of Book I. According to Corollary 2 to Proposition IV of Book I,[230] the (accelerative) centripetal forces by which bodies that uniformly describe different circles tend towards their centres are in a ratio compounded proportional to the ratio of the radii directly and the squared ratio of the of the periodic times inversely, i.e. $\frac{Fa1}{Fa2} :: \frac{R1}{R2} \times \frac{T2^2}{T1^2}$. As $\frac{Fg1}{Fg2} :: \frac{M1}{R1^2} \times \frac{R2^2}{M2}$[231] $\left(\text{and therefore:} \frac{M1}{M2} :: \frac{Fg1 \times R1^2}{Fg2 \times R2^2}\right)$ and, furthermore, $\frac{Fg1}{Fg2} = \frac{Fa1}{Fa2}$ (by Law III), then the ratio we seek to determine, i.e. $\frac{Fg1 \times R1^2}{Fg2 \times R2^2}$ or equivalently $\frac{M1}{M2}$, i.e. the weight forces at equal distances for identical unit masses, is proportional to $\left(\frac{R1}{R2}\right)^3 \times \left(\frac{T2}{T1}\right)^2$ – henceforth, I denote the proportion $\frac{M1}{M2} :: \left(\frac{R1}{R2}\right)^3 \times \left(\frac{T2}{T1}\right)^2$ by (*).

To determine the relative weight for equal unit masses drawn at equal distances toward Jupiter and Saturn, Newton compared the weight force on equal unit masses drawn toward Jupiter at a distance equal to the orbital radius of Callisto to the weight force on equal unit masses drawn toward the sun at a distance equal to the orbital radius of Venus (R_{Venus}), which was known to be equal to 0.724 of the distance of the earth to the sun (henceforth: 0.724 AU), and the weight force on equal unit masses drawn toward Saturn at a distance equal to the orbital radius of Titan to the weight force on equal unit masses drawn toward the sun at a distance equal to the orbital radius of Venus, respectively. Now, by application of (*), we basically need to solve $\left(\frac{R\text{Callisto}}{R\text{Venus}}\right)^3 \times \left(\frac{T\text{Venus}}{T\text{Callisto}}\right)^2$ (**) and $\left(\frac{R\text{Titan}}{R\text{Venus}}\right)^3 \times \left(\frac{T\text{Venus}}{T\text{Titan}}\right)^2$ (***). The periodic times of Venus, Callisto and Titan were listed among Newton's phenomena. If we can calculate R_{Callisto} and R_{Titan}, then the proportions we are in want of are established. Newton computed R_{Callisto} and R_{Titan} by determining the angle of maximum elongation as seen from the sun (θ), which he could determine on the

also concluded that the "common center of gravity of the earth, the sun, and all the planets is at rest," which follows from Hypothesis (!) I which states that "[t]he center of the system of the world is at rest" (ibid., p. 816).

[229] Ibid., pp. 812–813. Here I shall determine the relative masses by computing the data as given in the third edition of the *Principia*. See Garisto, "An Error in Isaac Newton's Determination of Planetary Properties", p. 44, table 1, for a summary of Newton's values in all three editions (CUL Adv.b.39.1-2 and NQ.16.200). See furthermore I. Bernard Cohen's guide to the *Principia* (Newton, *The Principia*, pp. 217–231), which contains an updated version of Cohen's paper "Newton's Determination of the Masses and Densities of the Sun, Jupiter, Saturn and the Earth".

[230] Newton, *The Principia*, p. 450.

[231] Since we consider identical unit masses, $F_g \sim \frac{m \times M}{r^2}$ becomes $F_g \sim \frac{M}{r^2}$, so that $M \sim \left(F_g \times r^2\right)$.

basis of the angle of maximum elongation as seen from the earth (θ'),[232] as follows: $R_{\text{Callisto}} = R_{\text{Jupiter}} \times \tan\left(\theta'_{\text{Jupiter}}\right)$ and $R_{\text{Titan}} = R_{\text{Saturn}} \times \tan\left(\theta'_{\text{Saturn}}\right)$.[233] As $R_{\text{Jupiter}} = 5.211$ AU and $\theta'_{\text{Jupiter}} = 8'16''$, $R_{\text{Callisto}} \approx 0.01253$ AU. Similarly, as $R_{\text{Saturn}} = 9.526$ AU and $\theta'_{\text{Saturn}} = 3'4''$, $R_{\text{Titan}} \approx 0.008498$ AU. When filling in (**), we get $\left(\frac{0.01253\text{ AU}}{0.724\text{ AU}}\right)^3 \times \left(\frac{224.6176\text{ days}}{16.6890\text{ days}}\right)^2 \approx 0,0009390$ or roughly as 1 to 1,064[234] (note that the current value is ca. 1 to 1,047[235]); for (***) we obtain: $\left(\frac{0.008498\text{ AU}}{0.724\text{ AU}}\right)^3 \times \left(\frac{224.6176\text{ days}}{15.9451\text{ days}}\right)^2 \approx 0,0003209$ or roughly as 1 to 3,116[236] (note that the present value is ca. 1 to 3,498[237]).

The relative weight of the earth on equal unit masses at equal distances from the centre of the earth can be found in the same way – albeit that there is a slight complication – by application of (*) we must solve $\left(\frac{R\text{moon}}{R\text{Venus}}\right)^3 \times \left(\frac{T\text{Venus}}{T\text{moon}}\right)^2$ (****). Now, T_{moon} equals ca. 27.3215 days. R_{moon} is equal to 1 AU $\times$ tan (θ), where θ is the (angle of) elongation of the moon from the earth (θ), which can be determined by multiplication of the radius of the moon (in earth radii) ($= 60_{1/5}$) and the solar parallax ($\theta' = 10.5''$). Unfortunately, the solar parallax was difficult to measure and there was no consensus at the time about its value. From hindsight we know that Newton overestimated the value for the solar parallax. Now, $\theta = 60_{1/5} \times 10.5'' = 10'32.1''$, so that R_{moon} is equal to 1 AU $\times \tan\left(10'32.1''\right) \approx 0,003065$ AU. Completing (****) gives: $\left(\frac{0.003065\text{ AU}}{0.724\text{ AU}}\right)^3 \times \left(\frac{224.6176\text{ days}}{27.3215\text{ days}}\right)^2 \approx 0.000005128$ or roughly 1 to 195,008.[238] Due to the wrong value for the solar parallax, Newton's result for the relative weight of the earth (for equal unit masses at equal distances from the centre of the earth) differs significantly from our modern value of ca. 1 to 332,946.[239]

232 As the angles are small, it can be assumed that $\theta = \theta'$.

233 Garisto, "An Error in Isaac Newton's Determination of Planetary Properties", p. 43.

234 Newton's value was 1/1067 (Newton, *The Principia*, p. 813). In order to keep these calculations manageable, Newton at some points of his calculations rounded some data, which explains the difference between the proportions I obtain here and those obtained in the *Principia* (Garisto, "An Error in Isaac Newton's Determination of Planetary Properties", p. 45).

235 Newton, *The Principia*, p. 225.

236 Newton's actual value was 1 to 3,021 (Newton, *The Principia*, p. 813). As Garisto has indicated, this value can be obtained exactly if Newton would have rounded off his data to 225 days and 9.53 AU (Garisto, "An Error in Isaac Newton's Determination of Planetary Properties", p. 46).

237 Newton, *The Principia*, p. 225.

238 Newton's value was 1/169,282 (Newton, *The Principia*, p. 813). Garisto has reconstructed that Newton obtained this ratio by rounding his data to 225 days and 27.32 days and making a copy error so that he mistakenly took the solar parallax to be $11''$ where he should have taken $10.5''$ (Garisto, "An Error in Isaac Newton's Determination of Planetary Properties", p. 47). I omit further discussion of determination of the (relative) surface gravities and the (relative) densities. Once the relative masses have been determined, the relative surface gravities and densities easily follow and are established by computing $\frac{M\text{planet}}{M\text{sun}} \times \left(\frac{r\text{sun}}{r\text{planet}}\right)^2$ and $\frac{M\text{planet}}{M\text{sun}} \times \left(\frac{r\text{sun}}{r\text{planet}}\right)^3$, respectively – where r here stands for the relative surface radius (Newton, *The Principia*, pp. 813–814).

239 Newton, *The Principia*, p. 225. Related to this, see Kollerstrom, "Newton's Lunar Mass Error and Hughes, Measuring the Moon's Mass".

3.5 The Argument for Universal Gravitation: The Synthesis or the Phase of Theory Application

If Newton would have ended the *Principia* after Proposition VIII, it would already have been a tremendous accomplishment. But Newton was not prepared to stop here. Recall that in *The Opticks* Newton described "the Method of Synthesis" as "assuming the Causes discover'd and establich'd as Principles, and by them explaining the phaenomena proceeding from them, and proving the Explanations."[240] In the synthetic part of the argument for universal gravitation, i.e. from roughly Proposition XIX to the very end of Book III, Newton accordingly set out to demonstrate that other phenomena, which were not contained in the original analysis, could be explained by the causes as established in the phase of theory formation. The synthesis corresponds to the phase of theory application and testing. Although Newton did not formally distinguish between an analytical and synthetic part of the argument for universal gravitation, several of his statements in Book III are perfectly consistent with such differentiation.[241] In what follows, I shall focus mainly on Newton's explanation of the figure of the earth, the tides, the motion of the moon, and cometary motion.

In order to explain why a seconds pendulum is shorter near the equator, i.e. to explain why in the same amount of time a pendulum traverses less space at the equator than elsewhere on the earth,[242] Newton stated that, on the assumption that the earth is an oblate sphere of homogeneous density, *surface* gravity at the equator results from the combination of two effects, namely the centrifugal forces (at the equator) *and* the gravitational forces arising from the inverse-square forces directed toward the individual parts on an oblate earth.[243] By contrast, Christiaan Huygens, who explained gravity in mechanical terms, claimed that the earth's centrifugal forces at the equator alone are sufficient to explain the different lengths of seconds-pendulums. Although Huygens was genuinely impressed by physico-mathematical results of Newton's *Principia* (e.g., the inverse-square law and the derivation of Keplerian motion, the moon test, etc.), he could not accept the idea that bodies in a Boylian vacuum attracted one other:

> Je n'ay donc rien contre la *Vis Centripeta*, comme Mr. Newton l'appelle, par la quelle il fait peser les Planetes vers le Soleil, & la Lune vers la Terre, mais j'en demeure d'accord sans difficulté: parce que non seulement on sçait par experience qu'il y a telle maniere d'attraction ou d'impulsion dans la nature, mais qu'aussi elle s'explique par les loix du mouvement, comme on a vû dans ce que j'ay écrit cy dessus de la pesanteur. Car rien

[240] Newton, *The Opticks*, pp. 404–405.

[241] Newton, *The Principia*, pp. 869, 888.

[242] A difference which was first observed by Jean Richer in 1672–1673 when he compared the length of a seconds pendulum at Cayenne to one at Paris.

[243] Newton, *The Principia*, pp. 830–831.

> n'empêche que la cause, de cette *Vis Centripeta* vers le Soleil, ne soit semblable à celle qui pousse les corps, qu'on appelle pesants, à descendre vers la Terre.[244]

According to Huygens, gravity is produced when a body, which is immersed in a rapidly moving fluid consisting of small particles contained in a spherical space so that each particle of that fluid matter has a tendency to move away from the centre, does not follow the circular motion of the surrounding fluid or moves more slowly than the surrounding fluid, it will be pushed by the surrounding fluid towards the centre.[245] By consequence, the variation of surface gravity with latitude is larger according to Newton's theory than according to Huygens'.[246] In other words, measuring the variations of surface gravity could provide an answer to which gravitational account was correct. Collecting trustworthy data and devising an adequate physico-mathematical treatment of the earth's oblateness turned out a difficult matter and it was only in the eighteenth century that the issue was settled in favour of universal gravitation.[247]

Newton established the primary cause of the tides as deduced from the theory of universal gravitation. His explanation of the tides is essentially contained in Proposition XXIV, Book III, in which he showed that the joint attractive pull of the sun and the moon on the earth will give rise to a tide-generating force.[248] Newton treated the tides as analogous to the motion of satellites under the influence of a disturbing body.[249] Moreover, he endorsed an equilibrium-theory according to which the attractive forces of the sun and moon cause the ocean to approach the shape of a spheroid with its major axis approximately aligned so that the greatest elevations of water occur one below the moon and the other on the opposite side of the earth.[250] In Corollary 19 to Proposition LXVI of Book I, Newton had considered the following case:

> Now imagine the globe T, which consists of nonfluid matter, to be so enlarged as to extend out to this ring, and to have a channel to contain water dug out around its whole circumference; and imagine this new globe to revolve uniformly about its axis with the same periodic

[244] Huygens, *Discours de la cause de la pesanteur* (1690), in Huygens, *Œuvres complètes de Christiaan Huygens*, XXI, p. 472.

[245] Ibid., XXI, pp. 452–462. See furthermore: Snelders, "Christiaan Huygens and Newton's Theory of Gravitation", pp. 212–215.

[246] In forthcoming material, Eric Schliesser and George E. Smith have reconstructed in detail the evidence at Huygens' disposal (Schliesser and Smith, "Huygens's 1688 Report to the Directors of the Dutch East Indian Company on the Measurement of Longitude at Sea and the Evidence it Offered Against Universal Gravity").

[247] See Greenberg, *The Problem of the Earth's Shape from Newton to Clairaut*. In Propositions XIX and XX, Newton introduced several assumptions and mathematical conclusions which he left unjustified (see Greenberg, *The Problem of the Earth's Shape from Newton to Clairaut*, pp. 1–14 and id., "Isaac Newton and the Problem of the Earth's Shape", pp. 372–382 for useful discussion of Newton's underlying presuppositions).

[248] Newton, *The Principia*, pp. 835–839.

[249] Aiton, "The Contributions of Newton, Bernouilli and Euler to the Theory of the Tides".

[250] Deacon, *Scientists and the Sea 1650–1900, A Study of Marine Science*, pp. 252–253.

> motion. This water, being alternately accelerated and retarded (as in the previous corollary) will be swifter in the syzygies and slower in the quadratures than the surface of the globe itself, and thus will ebb and flow in the channel just as the sea does.[251]

Under the influence of such tide-generating force, "the sea should twice rise and twice fall in every day, lunar as well as solar," and as Newton continued, "the greatest height of the water, in deep and open seas, should occur less than six hours after the appulse of the luminaries to the meridian of a place, as happens in the whole eastern section of the Atlantic Ocean and the Ethiopic [or South Atlantic] Sea between France and the Cape of Good Hope, and also on the Chilean and Peruvian shore of the Pacific Ocean."[252] Furthermore, "on all these shores the tide comes in at about the second, third, or fourth hour, except in cases when the motion has been propagated from the deep ocean through shallow places and is delayed until the fifth, sixth, or seventh hour, or later."[253] When the sun and the moon are aligned in conjunction or opposition, the effects of the solar and lunar gravitational forces are combined and this results in spring tide. By contrast, in the quadratures, where the lunar and solar gravitational forces counteract one another, neap tide occurs. Newton also considered other relevant parameters affecting the phenomena of the tides: the distance of the moon and the sun from the earth, the declination, the latitude of places, and the specific way the water is transported into a shore.[254] As an example of the latter, Newton turned to the anomalous case of the tidal currents at the harbour of Batsha, situated in the Gulf of Tonkin[255] (now Vietnam), where the water stays still on the day following the transit of the moon over the Equator. Newton suggested that the tides currents in some harbours are sometimes transported through different channels and pass more quickly through some than through others. Applying this idea to the tides at the harbour of Batsha, Newton suggested that the tidal currents coming from two different inlets – "one from the China Sea between the continent and island of Leuconia," the other "from the Indian Ocean between the continent and the island of Borneo" – annul each other.[256] Newton commented, as follows:

> Let us suppose that two equal tides come from different places to the same harbour and that the first precedes the second by a space of six hours after the appulse of the moon to the meridian of the harbour. If the moon is on the equator at the time of this appulse to the meridian, then every six hours there will be equal flood tides coming upon corresponding

[251] Newton, *The Principia*, p. 582.

[252] Ibid., p. 835.

[253] Ibid.

[254] Ibid., pp. 836–839.

[255] These were first reported on in Halley, "An Account of the Course of the Tides at Tonqueen". See, furthermore, Hughes and Wall, "Francis Davenport's Tonkin Tidal Report".

[256] Newton, *The Principia*, p. 839.

> equal ebb tides and causing those tides to be balanced by the flood tides, and thus during the course of that day they will cause the water to stay quiet and still.[257]

Implicit in Newton's explanation of the tides at Batsha was, what later was called, the principle of interference.[258] Note, however, that Newton stressed that he had left the issue whether there actually are such tides coming from the China Sea and the Indian Sea "to be determined by observation of the neighboring shores."[259] Newton's approach to the tides was qualitative and general, rather than quantitative and specific: as it stood, it had little predictive value use in tidal research.[260] Also, his treatment of the tides reflected the *status quaestionis* of tidal research at the time which was based on a relatively limited and not always equally reliable corpus of tidal observations – a situation which would only significantly change in the nineteenth century. Rather than offering a definite explanation of particular tidal phenomena, Newton used the tides "to exemplify his theory of universal gravitation," as Michael S. Reidy has adequately put it.[261]

The propositions on lunar and cometary motion are the most technical topics occurring in the synthetical part of the argument for universal gravitation. In Propositions XXII and XXV–XXXV Newton "wished to show [ostendere volui][262] by these computations of the lunar motions that the lunar motions can be computed from their causes by the theory of gravity [quod motus lunares per theoriam gravitatis a causis suis computari possint]."[263] In fact, Newton claimed that "*[a]ll the motions of the moon and all the inequalities in its motion follow from the principles that have been set forth* [*[m]otus omnes lunares, omnesque motuum inæqualitates ex allatis principiis consequi.*]."[264] The latter statement should be seen as referring to the promise of future empirical research based on universal gravity, rather than as an appraisal of the lunar theory[265] he himself had developed in the *Principia*.

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[257] Ibid., p. 838.

[258] Cohen's introduction in ibid., pp. 240–241 and Mollon, "The Origins of the Concept of Interference".

[259] Newton, *The Principia*, p. 839.

[260] Cf. Reidy, *Tides of History*, *Ocean Science and Her Majesty's Navy*, p. 31. In Propositions XXXVI and XXXVII Newton determined the relative contributions of the gravitational forces of the sun and the moon to the earth's tides, respectively (Newton, *The Principia*, pp. 874–878). In Proposition XXVII he calculated the contribution of the lunar force to the tides by comparing the highest and lowest tides as observed by Samuel Sturmy in the Bristol Channel (Sturmy, "An Account of Some Observations").

[261] Reidy, *Tides of History*, p. 31.

[262] Cf. CUL Add. Ms. 3966, f. 65^r. On f. 61^v Newton used "didici" instead.

[263] Newton, *The Principia*, p. 869. Cf. CUL Add. Ms. 3966, ff. 65^r-66^r, ff. 67^r-68^v, ff. 84^r-85^r, f. 86^v. See George E. Smith's "Newton and the Problem of the Moon's Motion", in Newton, *The Principia*, pp. 252–257, pp. 252–253, for an overview of the complexity involved.

[264] Newton, *The Principia*, p. 869, cf. p. 832.

[265] Much of the details on the development of Newton's lunar theory are still somewhat unclear (and will perhaps remain so) (see, furthermore, Waff, *Universal Gravitation and the Motion of the Moon's Apogee*). The most important resources are: Newton's early manuscripts drafts on the motions of the moon [ca. late-1686; transcribed and translated in Whiteside,

Newton was fully aware of the problems involved in lunar theory. In a famous letter to Flamsteed in 1695/6, Newton uttered:

> For I find this Theory [of the Moon] so very intricate & the Theory of Gravity so necessary to it, that I am satisfied it will be never perfected but by somebody who understands ye Theory of gravity as well or better then I do.[266]

Although Newton's lunar theory was far from complete,[267] it was at least theoretically significant for it turned the study of the motions of the moon "from an intricate celestial geometry into a branch of gravitational physics," as I. Bernard Cohen observed.[268] In Proposition LXVI of Book I and its corollaries, Newton had provided the basics for a qualitative treatment of the lunar inequalities by invoking a three-body system in which the motion of the moon is approximated as circular and uniform.[269] Newton's lunar Propositions (esp. Propositions XXV–XXXV) include a quantitative treatment of three of the most important lunar inequalities: (1) the lunar variation, i.e. the speeding up of the moon in the syzygies and its slowing down in the quadratures caused by the sun's transradial perturbing force (Propositions XXVI–XXIX),[270] (2) the inequalities of the inclination and the motion of the lunar nodes (Propositions XXX–XXXV), and (3) the annual inequality, i.e. the annual fluctuation in the moon's speed caused by the sun's radial perturbing force (analyzed

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ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 508–537], his *Theory of the Moon's Motion* [first published in Latin as an appendix to David Gregory's *Astronomiae Physicae & Geometricae Elementa*; see, furthermore, Cohen, ed., *Isaac Newton's Theory of the Moon* (1702) and, more recently, Kollerstrom, *Newton's Forgotten Lunar Theory*] of which the revised version was inserted in the *scholium* to Proposition XXXV in the second edition of the *Principia*, his manuscript *Theoria Lunae* [date unclear; Kollerstrom, *Newton's Forgotten Lunar Theory*, pp. 128–132] (Newton, *Correspondence*, IV, pp. 1–6), and, of course, the three editions of the *Principia*. More specifically, it remains unclear, in general, to what extent certain statements contained in Newton's lunar theory were derived from empirical or theoretical means (or a combination of both) and how Newton obtained certain parameters which he assumed in his lunar calculations (cf. Waff, "Newton and the Motion of the Moon, An Essay Review", p. 67). In this paragraph, I have no pretence of shedding new light on this intricate matter. For further discussion see Nauenberg, "Newton's Unpublished Perturbation Method for the Lunar Motion" and id., "Newton's Perturbation Methods for the Three-Body Problem and Their Application to Lunar Motion". Both papers contain a discussion of Newton's "Portsmouth method" (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 508–537). For potential worries, see: Palmieri, "Review of *Isaac Newton's Natural Philosophy*", pp. 115–117.

[266] Newton to Flamsteed, 16 February 1695/5, Newton, *Correspondence*, IV, p. 87.

[267] See especially Whiteside, "Newton's Lunar Theory: From High Hope to Disenchantment".

[268] Newton, *The Principia*, p. 246.

[269] See most notably Newton, *The Principia*, pp. 569–573, for Newton's decomposition of the three-body system.

[270] For a detailed technical treatment, see: Wilson, "Newton on the Moon's Variation and Apsidal Motion", pp. 141–155.

in Proposition XXII).[271] The results obtained in Propositions XXV–XXXV, which are based on Newton's reduction of the earth-moon-sun system to a "restricted three-body system,"[272] i.e. a three-body system in which the eccentricity is assumed to be insignificant, agree quite well with modern results insofar as the effects of eccentricity, i.e. changes in the distances between the focus and centre of an ellipse, are ignored.[273] Nicholas Kollerstrom has recently concluded that although, "Newton did not evidently deduce his new [lunar] equations from such principles [i.e., the principles of universal gravitation], "we found that these [lunar] equations were all valid and not far from their optimum amplitudes."[274] When working on the problem of the motion of the moon's apsides, of which no detailed quantitative analysis is given in the *Principia*,[275] Newton relied on a *purely kinematical* Horrocksian model of the lunar orbit wherein "it is assumed that the Moon's pristine path is a Keplerian ellipse traversed around the earth at a focus and that the effect of solar perturbation is slightly to alter its eccentricity while maintaining the length (but not the direction) of its major axis."[276] This clearly shows that not all elements of Newton's treatment of the motions of the moon were derived from the theory of universal gravitation.[277] In Proposition XXXIX, Newton showed that the precession of the equinoxes derives from the gravitational pull by the sun and the moon on earth's equatorial bulge[278] towards the ecliptic.[279] Although the basics of Newton's explanation were correct, his corresponding calculations were quite off-scale.[280]

In the final sections of Book III, Newton showed that comets describe extremely elongated closed curves, namely ellipses (Proposition XL), in the planetary regions.[281] Newton approximated the elliptical trajectories of comets by parabolic

[271] Smith, "Newton and the Problem of the Moon's Motion", in Newton, *The Principia*, p. 253. See furthermore: id., "The Motion of the Lunar Apsis", in ibid., pp. 257–264 and Wilson, "The Newtonian Achievement in Astronomy", pp. 263–268.

[272] Whiteside, "Newton's Lunar Theory: From High Hope to Disenchantment", pp. 319–320.

[273] Cf. Kollerstrom, *Newton's Forgotten Lunar Theory*, pp. 150–151, 169. Cf. Wilson, "Newton on the Moon's Variation and Apsidal Motion", pp. 162–163.

[274] Kollerstrom, *Newton's Forgotten Lunar Theory*, p. 229.

[275] Smith, 'The Motion of the Lunar Apsis', in Newton, *The Principia*, pp. 257–260.

[276] Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 509, footnote 1. Cf. Kollerstrom, *Newton's Forgotten Lunar Theory*, Chapter 7 and Wilson, "The Newtonian Achievement in Astronomy", pp. 265–266.

[277] Cf. Smith, "The Motion of the Lunar Apsis", in Newton, *The Principia*, pp. 259–260 and Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, p. 509, footnote 1.

[278] As deduced in Proposition XIX – on the assumption of uniformly dense matter and on the assumption that the shape of the earth would be identical if it was completely fluid – and confirmed in Proposition XX (Newton, *The Principia*, pp. 828, 832).

[279] Ibid., pp. 885–888.

[280] Wilson, "The Newtonian Achievement in Astronomy", pp. 269–270.

[281] Newton, *The Principia*, p. 895.

ones,[282] for as he noted: “these orbits will be so close to parabolas that parabolas can be substituted for them without sensible errors.”[283] Parabolic trajectories are more conveniently dealt with mathematically since all parabolas are geometrically similar and are determined by fewer conditions than ellipses or hyperbolas.[284] In Proposition XLI, Newton established a graphical interpolation procedure[285] to construct the cometary orbit from three observations made at nearly equal time intervals.[286] In Corollary 3 to Proposition XXXIX Newton observed:

> Hence it also is manifest that the heavens are lacking in resistance [quod cœli resistentia destituuntur]. For the comets, following paths that are oblique and sometimes contrary to the course of the planets [vias obliquas & nonnunquam cursui planetarum contrarias secuti], move in all directions very freely and preserve their motions for a very long time even when these are contrary to the course of the planets [moventur omnifariam liberrime, & motus suos, etiam contra cursum planetarum diutissime conservant].[287]

Newton frequently stressed that the quantitative conclusions made in the synthetical part of the argument for universal gravitation required further refinement.[288] Although some parts of the synthetical part for the argument for universal gravitation were incomplete and even contained off-track calculations, Newton had unveiled an overarching theoretical research programme by which his claims could be corrected, “rendered more accurate,” and tested in future empirical research.[289] A theory with such explanatory potential was simply unprecedented. Given what we have covered in the preceding sections, we are now in a position to provide . . .

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282 In the concluding *scholium* to Section 2 of Book I, Newton stated that, if the centre of an ellipse goes off to infinity, the ellipse turns into a parabola (ibid., p. 460).

283 Ibid., p. 895.

284 Wilson, “The Newtonian Achievement in Astronomy”, p. 270.

285 See Fraser, “Newton and Interpolation” on this matter. Furthermore, Aleksei Nikolaevich Krylov has shown that Newton’s graphical interpolation method can be reduced to an equivalent arithmetical method (id., “On Sir Isaac Newton’s Method of Determining the Parabolic Orbit of a Comet”).

286 Newton, *The Principia*, pp. 901–904. See Wilson, “The Newtonian Achievement in Astronomy”, pp. 270–272 for a brief, but insightful outline of Newton’s approach of constructing cometary orbits. According to his own testimony, Newton hit on this graphical method only after “[h]aving tried many approaches to this exceedingly difficult problem” (Newton, *The Principia*, p. 901).

287 Newton, *The Principia*, p. 895.

288 E.g., explicitly in Newton, *The Principia*, p. 839 [Proposition XXIV], p. 843 [Proposition XXVII], p. 848 [Proposition XXIX], p. 874 [Proposition XXXV], p. 880 [Proposition XXXIX].

289 John L. Greenberg once remarked: “Had the *Principia* truly been a completely self-contained “mechanics course,” consisting of nothing but straightforward demonstrations of whatever particular conclusions follow from them, without any gaps or limitations or conjectures or errors, would it have provoked mathematicians and, in this way, induced them to try to determine the range of NEWTON’s world system and in so doing enable some of them to show that its range could be enlarged?” (Greenberg, “Isaac Newton and the Problem of the Earth’s Shape”, p. 390). See Smith, “How Newton’s *Principia* Changed Physics”, for additional discussion.

3.6 An Outline of Newton's Methodology in Book III of the *Principia*

In Chapter 2, we have addressed what I have called the *phase of model construction*. Book III itself consists of several phases. In this chapter, we have seen that:

1. Newton applied the exact or *quam proxime* causal inference-tickets of Book I to infer the real-world forces in our solar system, which corresponds to the *phase of model application* of Newton's methodology, i.e. proceeding from effects to causes (Propositions I–IV, Book III).
2. In Propositions V–VIII of Book III, Newton passed over to the *phase of theory formation* and arrived step-by-step at the theory of universal gravitation.
3. In the remainder of the *Principia*, Newton initiated the synthetic phase (or, *phase of theory application and testing*) of his methodological programme, i.e. "assuming the Causes discover'd and establich'd as Principles [i.e., the causes as established in the preceding analysis], and by them explaining the phaenomena proceeding from them, and proving the Explanations."

Correspondingly, in this final section I shall highlight the non-hypothetico-deductive features of Newton's methodology in the context of Book III. I shall arrange these features systematically rather than proposition-by-proposition or phase-by-phase. Once we have dealt with Newton's optical work in Chapter 4, I shall provide a brief chronological overview of Newton's methodological itinerary in Chapter 5. The ways in which Newton's methodology, in the context of Book III, differs from a hypothetico-deductive method can now be gathered, as follows:

1. *Treating force physically*. In order to treat forces physically, one should first treat forces physico-mathematically. That is to say, one should first turn to a abstract physico-mathematical study of forces along the lines of the five points mentioned in Section 2.6 in Chapter 2. Crucial to this endeavour is the demand that the forces or causes which one sets out to infer should be constrained by a set of well-established premises. This involves a prioritization of principles, i.e. the laws of motion that have empirical warrant over arbitrary principles.

2. *Theory-mediated measurements*. In Section 2.5.1 in Chapter 2, we have seen that in Book I Newton had established a bi-conditional relation or systematic dependency between the presence of centripetal forces and Kepler's area rule (Propositions I–III, Book I), on the one hand, and between inverse-square centripetal forces and Kepler's harmonic rule (Corollary 6 to Proposition IV, Book I), on the other. Given these systematic dependencies, Newton could claim that a centripetal force is a necessary and sufficient condition for Kepler's area law and that an inverse-square centripetal force is a necessary and sufficient condition for Kepler's harmonic rule. More precisely, as these systematic dependencies (and their *quam proxime* counterparts) were deduced from a non-arbitrary set of already confirmed and promising principles, i.e. the laws of motion, Newton basically showed that, *given the laws of motion*, a centripetal force is a necessary and sufficient condition

for Kepler's area law and that an inverse-square centripetal force is a necessary and sufficient condition for Kepler's harmonic rule.

In the context of Book III, the area law measures the presence of a centripetal force and the harmonic ratio measures the inverse-square proportionality to the distance of a centripetal force. This agrees well with what Newton says in the Scholium to Propositions I–III of Book I on the relation between the area law and the presence of a centripetal force:

> Since the uniform description of areas indicates [index est[290]] the center towards which that force is directed by which a body is most affected and by which it is drawn away from rectilinear motion and kept in orbit, why should we not in what follows use uniform description of areas as a criterion for a center [ut indicem centri] about which all orbital motion takes place in free spaces?[291]

The laws of motion are the background knowledge that enables Newton to bi-conditionally relate a theoretical parameter, *in casu* an inverse-square centripetal force, to certain well-defined quantitative relations which characterize the motions of celestial bodies (Keplerian motion), which measure this very parameter.[292] Correspondingly, in order to provide empirical answers to theoretical questions, *Newton made theory-mediated measurements the focal point of scientific research*.[293] In this way, Newton could infer centripetal forces "more securely" from exact or *quam proxime* causal inference-tickets and actual astronomical measurements. Moreover, given the *bi-conditional* relations he had established, Newton could present his inferences of centripetal forces as straightforward deductions, which is exactly what he did.[294] Although, Newton could, *on an inferential level*, proceed from specific motions to the forces producing such motions by the systematic dependencies which he had established, *on a physical level* it is clear that there is an asymmetry involved: forces produce motions and not the other way around. To avoid possible confusion, I want to emphasize that Newton's proceeding from phenomena to theory, i.e. his presenting of certain inferences as deductions from phenomena, taken as such is *not* what makes his method essentially

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[290] A more literal translation of "index est" is "is an informer", "is a sign", or "is an indicator".

[291] Newton, *The Principia*, p. 449.

[292] Cf. Harper, "Measurement and Approximation", p. 274.

[293] Harper, "Newton's Methodology", p. 44; id., "Isaac Newton on Empirical Success and Scientific Method", pp. 55, 57; Smith, "The Methodology of the *Principia*", pp. 144, 147.

[294] Newton, *The Principia*, p. 943; Newton, *The Opticks*, p. 369; CUL Add. Ms. 4005.2, f. 6^v [ca. 1703]. In Propositions I–II of Book III, Newton concluded that the primary/secondary planets are drawn towards their centres (the sun/Jupiter or Saturn) by a centripetal force, which follows from Proposition II of Book I, i.e. the necessary direction proceeding from the area law to the presence of a centripetal force.

different from hypothetico-deductivism.[295] Rather, *proceeding from phenomena to theory is the by-product of what genuinely makes Newton's method distinctive from hypothetico-deductivism: the establishment of systematic dependencies backed-up by the laws of motion*.

3. *Systematic discrepancies as detection devices for residual forces*. As we have seen, Newton established a series of systematic discrepancies, which allow for a systematic dependency between deviations from a mathematical regularity which holds exactly and variations in the corresponding theoretical parameters. In Corollary 7 to Proposition IV and Corollary 1 to Proposition XLV in Book I, Newton established the following systematic dependencies, respectively: (1) T varies as the n^{th} power of the radius R, *if and only if*, the centripetal force varies as R^{1-2n} (correction for circular motion) and (2) $\frac{p^\circ}{\text{revolution}}$, *if and only if*, the centripetal force varies as $\mathrm{R}^{(360^\circ/(360^\circ+p^\circ))^2-3}$ (correction of apsidal motion). Given these systematic discrepancies, Newton's models are robust with respect to approximations.[296]

Once empirical data is provided, these *systematic discrepancies,* in the context of Book III, *function as theoretical detection devises that enable one to track residual forces*. Accordingly, a focal point of the *Principia* is the search for residual forces. In contrast to a hypothetico-deductive rendering of theory confirmation, in which confirmation of the consequences deduced from a theoretical proposition by itself occupies centre stage, *in Newton's methodology the attention shifts to a continuous exploration of residual forces and the establishment of their potential explanation*, which brings us to the next feature.

4. *Initiating a sequence of successive approximations*.[297] A striking feature of Newton's way of dealing with motion is that he did not approach the empirical world through a single theoretical model or equation, but rather through a series of successive approximations. *The starting point is to begin with a first approximation, i.e. an explication of the physico-mathematical conditions under which a mathematical regularity would hold exactly given the laws of motion* (see the discussion of Propositions I–II, Corollary 6 to Proposition IV and Proposition LXXVI of Book I in Chapter 2). *From the perspective of the laws of motion, any deviation from the first approximation*[298] *is seen as an indication that an additional force, not tracked in the first approximation, is affecting the situation, i.e. each deviation is seen as evidence*

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[295] Which seems, nevertheless, to be the view which John Worrall endorses (cf.: "Whereas H–D methods start with the theory and proceed by investigating the truth of observational consequences deduced from it, in Newton's method the theory is the *conclusion* of an argument that begins with observational premises." (Worrall, "The Scope, Limits, and Distinctiveness of the Method of 'Deduction from the Phenomena'", p. 64)).

[296] Harper, "Isaac Newton on Empirical Success and Scientific Method", pp. 57–58.

[297] This feature has been emphasized in Cohen, *The Newtonian Revolution*, and in several papers of George E. Smith (e.g., Smith, "From the Phenomenon of the Ellipse to an Inverse-Square Force; Why not?", pp. 46–49; id., "The Methodology of the *Principia*", pp. 155–158).

[298] Smith considers deviations "second-order" phenomena, for they do not straightforwardly appear in observation, but presuppose the theory and the initial approximation deduced from it (Smith, "From the Phenomenon of the Ellipse to an Inverse-Square Force: Why not?", p. 47; id., "The Methodology of the *Principia*", p. 157).

that an additional force is present and, consequently, as evidence for a refinement of the initial approximation. In the discussion of Propositions III–IV of Book III, we have seen that the moon's 0.01673 deviation from the inverse-square law, as calculated from Newton's apsidal procession theorem (Corollary 1 to Proposition XLV, Book I), became evidence for a refinement of a more complex model that takes into account the action of the sun. Also, the discrepancy from the inverse-square law, as revealed by the shortening of seconds pendulums near the equator, became evidence for refining Proposition LXXVI of Book I and, more precisely, for Newton's approximation of the earth as an oblate sphere as developed in Propositions XIX–XX of Book III. The significance of this requirement is that it prevents the introduction of ad hoc forces in order to save the law of universal gravitation.[299] Instead, *by focussing on discrepancy reduction and by adding the demand that each deviation should become evidence for a more refined model, the law of universal gravitation is continually put to the test*.[300]

5. *Inductive gradualism*. Newton did not immediately proceed from particulars to the theory of universal gravitation. Rather, he proceeded step-by-step from particulars to increasingly wider generalizations until a universal generalization is established.

6. *Accurate measurement and convergence of independently measured parameters*. A hypothetico-deductivist endorses the view that a theoretical proposition is confirmed when the deductions from that proposition are agreeable with the phenomena at hand. Newton demanded more of a theory than empirical adequacy: *in order to be accepted (provisionally), a theory should provide accurate measurements of its parameters from the phenomena they serve to explain*.[301] *Furthermore, it is required that independently measured parameters should converge.* In what follows, I will give one example of each of the two procedures.

Example 1: Propositions I–III, Book III. The correspondence between Newton's phenomena and the values computed from the harmonic rule was very convincing.[302] In what follows, I will show exactly how strong by computing Newton's actual data.[303] Let us begin with Newton's data for the satellites of Jupiter in the first edition of the *Principia*.[304] In the first edition of the *Principia*, the values given for the periodic times of Jupiter's four satellites were the following:

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299 Smith, "The Methodology of the *Principia*", p. 158.

300 This train of thought has been emphasized by George E. Smith in many of his papers.

301 Harper, "Newton's Methodology", p. 44; id., "Newton's Argument for Universal Gravitation", p. 185; id., "Measurement and Approximation", p. 278.

302 For the evidence supporting the area rule, see Densmore, *Newton's* Principia*: The Central Argument*, pp. 243–283, esp. pp. 249–251 [Jupiter], p. 257 [Saturn], pp. 280–281 [primary planets], pp. 282–283 [moon].

303 Values not originally provided by Newton, but computed from his phenomena, are preceded by an asterisk. I shall – somewhat arbitrary – compute Newton's data to four numbers behind the comma.

304 *PrE*$_1$, p. 403.

	Satellite 1	Satellite 2	Satellite 3	Satellite 4
Periodic times (= T)	$1^d18^h28'_{3/5}$	$3^d13^h17'_{9/10}$	$7^d3^h59'_{3/5}$	$16^d18^h5'_{1/5}$
*T in hours (= T')	≈ 42.4767	≈ 85.2983	≈ 171.9933	≈ 402.0867

The distances (D) provided for the satellites from the centre of Jupiter in semi-diameters of Jupiter were – note that Flamsteed's observations by the eclipses of the satellites provided the most accurate data:

	Satellite 1	Satellite 2	Satellite 3	Satellite 4
Cassini	5	8	13	23
Borelli	$5_{2/3}$	$8_{2/3}$	14	$24_{2/3}$
Townly by micrometer	5.51	8.78	13.47	24.72
Flamsteed by micrometer	5.31	8,85	13.98	24.23
Flamsteed by eclipses of the satellites (= D_{OFE})	5.578	8.876	14.159	24.903
*$(D_{OFE})^{3/2}$[305]	≈ 13.1740	≈ 26.4439	≈ 53.2781	≈ 124.2732
*$(D_{OFE})^{3/2}$/T'	≈ 0.3101	≈ 0.3100	≈ 0.3098	≈ 0.3091
From the periodic times[306] (= D_{PT})	5.578	8.878	14.168	24.968
*$(D_{PT})^{3/2}$	≈ 13.1740	≈ 26.4529	≈ 53.3289	≈ 124.7601
*$(D_{PT})^{3/2}$/T'	≈ 0.3101	≈ 0.3101	≈ 0.3101	≈ 0.3103
*Difference [=$(D_{OFE})^{3/2}$/T')–$(D_{PT})^{3/2}$/T']][307]	≈ 0	– 0.0001	– 0.0003	– 0.001
*Deviation (%)[308]	≈ 0 %	≈ 0.0003 %	≈ 0.0010 %	≈ 0.0032 %

In the second and third edition of the *Principia*,[309] the values provided for the periodic times of Jupiter's four satellites were:

	Satellite 1	Satellite 2	Satellite 3	Satellite 4
Periodic times (T)	$1^d18^h27'34''$	$3^d13^h13'42''$	$7^d3^h42'36''$	$16^d16^h32'9''$
*T in hours (T')	≈ 42.4594	≈ 85.2283	171.7100	400.5358

305 According to Kepler's harmonic rule, T is proportional to the 3/2 power of the distances.

306 These are the distances that correspond to the periodic times computed from Kepler's harmonic law holding *exactly* (cf. Newton, *The Principia*, p. 800) which can be obtained by calculating backwards from the observed times and distances of at least 2 satellites.

307 I.e. the discrepancy between observation and theory – assuming that the values for $\frac{(DPT)^{\frac{3}{2}}}{T'}$ tend to 0.3101.

308 Assuming idem.

309 *PrE*$_2$, p. 359; *PrE*$_3$, p. 390.

The distances (D) provided for the satellites from the centre of Jupiter in semi-diameters of Jupiter were – note that Cassini's observations by the eclipses of the satellites provided the most accurate data:

	Satellite 1	Satellite 2	Satellite 3	Satellite 4
Borelli	$5_{2/3}$	$8_{2/3}$	14	$24_{2/3}$
Townly by micrometer	5,52	8,78	13,47	24,72
Cassini by telescope	5	8	13	23
Cassini by eclipses of the satellites (D_{OCE})	$5_{2/3}$	9	$14_{23/60}$	$25_{3/10}$
*$(D_{OCE})^{3/2}$	≈ 13.4894	27	≈ 54.5493	≈ 127.2567
*$(D_{OCE})^{3/2}$/T'	≈ 0.3177	≈ 0.3168	≈ 0.3177	≈ 0.3177
D_{PT}	5.667	9.017	14.384	25.299
*$(D_{PT})^{3/2}$	≈ 13.4906	≈ 27.0765	≈ 54.5531	≈ 127.2492
*$(D_{PT})^{3/2}$/T'	≈ 0.3177	≈ 0.3177	≈ 0.3177	≈ 0.3177
*Difference [=$(D_{OCE})^{3/2}$/T'– $(D_{PT})^{3/2}$/T']	≈ 0	–0.0009	≈ 0	≈ 0
*Deviation (%)	≈ 0%	≈ 0.0028%	≈ 0%	≈ 0%

In the second edition of the *Principia*,[310] the values provided for the periodic times of Saturn's five satellites were:

	Satellite 1	Satellite 2	Satellite 3	Satellite 4	Satellite 5
Periodic times (T)	$1^{d}21^{h}19'$	$2^{d}17^{h}41'$	$4^{d}13^{h}47'$	$15^{d}22^{h}41'$	$79^{d}22^{h}4'$
*T in hours (T')	≈ 45.3167	≈ 65.6833	≈ 109.7833	≈ 382.6833	≈ 1,918.0667

The distances (D) of the satellites from the centre of Saturn in semi-diameters of Saturn's ring were:

	Satellite 1	Satellite 2	Satellite 3	Satellite 4	Satellite 5
From observations (D_O)	$1_{19/20}$	$2_{1/2}$	$3_{1/2}$	8	24
*$(D_O)^{3/2}$	≈ 2.7230	≈ 3.9528	≈ 6.5479	≈ 22.6274	≈ 117.5755
*$(D_O)^{3/2}$/T'	≈ 0.0601	≈ 0.0602	≈ 0.0596	≈ 0.0591	≈ 0,0613
From their periodic times (D_{PT})	1.95	2.5	3.52	8.09	23.71
*$(D_{PT})^{3/2}$	≈ 2.7230	≈ 3.9528	≈ 6.6041	≈ 23.0103	≈ 115.4509
*$(D_{PT})^{3/2}$/T'	≈ 0.0601	≈ 0.0602	≈ 0.0602	≈ 0.0601	≈ 0.0602
*Difference [=$(D_O)^{3/2}$/T'–$(D_{PT})^{3/2}$/T'][311]	–0.0001	≈ 0	–0.0006	–0.0011	+ 0.0011
*Deviation (%)[312]	≈ 0.0017 %	≈ 0%	≈ 0.0010%	≈ 0.0183 %	≈ 0.0183 %

310 *PrE*$_2$, p. 360. In the first edition such data was lacking.

311 Assuming that the values for $\frac{(DPT)^{\frac{3}{2}}}{T'}$ tend to 0.0602.

312 Assuming idem.

In the third edition of the *Principia*,[313] the values provided for the periodic times of Saturn's five satellites were:

	Satellite 1	Satellite 2	Satellite 3	Satellite 4	Satellite 5
Periodic times (T)	$1^d21^h18'27''$	$2^d17^h41'22''$	$4^d12^h25'12''$	$15^d22^h41'14''$	$79^d7^h48'0''$
*T in hours (T')	45.3075	≈ 65.6894	108.4200	380	1,903.8000

The distances (D) of the satellites from the centre of Saturn in semi-diameters of Saturn's ring were:

	Satellite 1	Satellite 2	Satellite 3	Satellite 4	Satellite 5
From observations (D_O) (Cassini)	$1_{19/20}$	$2_{1/2}$	$3_{1/2}$	8	24
*$(D_O)^{3/2}$	≈ 2.7230	≈ 3.9528	≈ 6.5479	≈ 22.6274	≈ 117.5755
*$(D_O)^{3/2}$/T'	≈ 0.0601	≈ 0.0602	≈ 0.0596	≈ 0.0591	≈ 0,0618
From their periodic times (D_{PT})	1.93	2.47	3.45	8	23.35
*$(D_{PT})^{3/2}$	≈ 2.6812	≈ 3.8819	≈ 6.4081	≈ 22.6274	≈ 112.8315
*$(D_{PT})^{3/2}$/T'	≈ 0.0591	≈ 0.0591	≈ 0.0591	≈ 0.0595	≈ 0.0593
*Difference [$(D_O)^{3/2}$/T'– $(D_{PT})^{3/2}$/T'][314]	+0.0010	+0.0011	+0.0005	≈ 0	+0,0027
*Deviation (%)[315]	≈ 0.0169%	≈ 0.0186%	≈ 0.0085%	≈ 0%	≈ 0.0457%

Let us now turn to Newton's data for the primary planets. In the first and second edition of the *Principia*,[316] the values provided for the periodic times of the primary planets were not mentioned.[317] The mean distances of the primary planets from the sun were:

	Saturn	Jupiter	Mars	Earth	Venus	Mercury
D according to Kepler	951,000	519,650	152,350	100,000	72,400	38,806
D according to Bouillau	954,198	522,520	152,350	100,000	72,398	38,585
D from their periodic times	953,806	520,116	152,399	100,000	72,333	38,710

313 *PrE*$_3$, pp. 391–392.

314 Assuming that the distances from the periodic times tend to 0.0591.

319 Assuming idem.

316 *PrE*$_1$, p. 404; *PrE*$_2$, p. 361.

317 However, Newton recorded that "Eadem utique sunt tempora periodica, eædemque orbium dimensiones, sive Sol circa Terram, sive Terra circa Solem revolvatur. Ac de mensura quidem temporum periodicorum convenit inter Astronomos universos." (*PrE*$_2$, p. 360, cf. *PrE*$_1$, p. 403).

In the third edition,[318] Newton added the exact values for the periodic times of the primary planets with respect to the fixed stars and corrected some of the mean distances.

	Saturn	Jupiter	Mars	Earth	Venus	Mercury
Periodic times (T)[319]	10,759.275	4,332.514	686.9785	365.2565	224.6176	87.9692
D according to Kepler (D_{OK})[320]	951,000	519,650	152,350	100,000	72,400	38,806
D according to Bouillau (D_{OB})	954,198	522,520	152,350	100,000	72,398	38,585

In the following table, I compute Bouillau's data:

*$(D_{OB})^{3/2}$	≈ 932,031,185	≈ 377,706,429	≈ 59,465,310	≈ 31,622,776	≈ 19,480,848	≈ 7,579,277
*$(D_{OB})^{3/2}/T$	≈ 86,626	≈ 87,180	≈ 86,561	≈ 86,577	≈ 86,729	≈ 86,158
From their periodic times (D_{PT})	954,006	520,096	152,369	100,000	72,333	38,710
*$(D_{PT})^{3/2}$	≈ 931,808,481	≈ 375,081,177	≈ 59,476,435	≈ 31,622,776	≈ 19,453,812	≈ 7,616,137
*$(D_{PT})^{3/2}/T$	≈ 86,605	≈ 86,574	≈ 86,577	≈ 86,577	≈ 86,609	≈ 86,577
*Difference[321]	+49	+603	–16	≈ 0	+152	–419
*Deviation (%)[322]	≈ 0.0006%	≈ 0.0070%	≈ 0.0002%	≈ 0%	≈ 0.0018%	≈ 0.0048%

Therefore, throughout all editions of the *Principia* Newton's empirical evidence for the harmonic rule was very good.

Example 2: Proposition IV. Newton's moon test provided a good case of consilience of independently measured parameters. We have shown in Section 3.4.2 in this chapter that by using some straightforward data and by applying Corollary 9 to Proposition IV of Book I Newton could establish that the distance that the moon would traverse – when deprived from its motion and while neglecting the sun's perturbation – a distance of 15.009 Paris feet in a one minute fall to the earth. Furthermore, while assuming the inverse-square law, Newton could establish that in the region of the earth the moon would fall 15.009 Paris feet in 1 s. With his adduced, but disputable, correction for the sun's influence, Newton argued that the initial calculation of 15.009 Paris feet per second should be corrected to 15.093 Paris feet per

318 *PrE3*, p. 393.

319 Given in days and decimal parts of a day.

320 There is no evidence indicating that Newton ever read Kepler's *Astronomia Nova*. These exact values are, however, given, for instance, in: Mercator, *Institutionum astronomicarum libri II*, p. 151. Newton owned this edition and inserted annotations (Wren Library, NQ.16.196; Harrison, *The Library of Isaac Newton*, p. 191 [item n° 1072], p.151). In one of his Wren copies of the second edition of the *Principia*, Newton corrected the first three periodic times and gave the following numbers: 954,006, 520,096 and152,369 (Wren Library, NQ.16.196, p. 361).

321 Assuming that the values for $\frac{(DPT)^{\frac{3}{2}}}{T}$ tend to 86,577.

322 Assuming idem.

second, thereby arriving at a number in very close agreement to Huygens' measurement of the distance a seconds pendulum traverses near the surface of the earth, i.e. 15.0956 Paris feet per second (namely, a difference of 0.0026 or a deviation of ca. 0.017% from Huygens' value). However, Newton's initial value of 15.009 Paris feet per second differed only by ca. 0.57% of Huygens' value, thereby still entailing a good fit.

7. *The synthesis*. In order to further test the law of universal gravitation, Newton provided a long synthetic argument for the law of universal gravitation based on different phenomena that were not originally included in the analytical argument for universal gravitation (most importantly: the figure of the earth, the tides, lunar and cometary motion). Newton aimed to show that these phenomena could be explained by the theory of universal gravitation and attempted to prove these explanations. Although Newton had succeeded in showing the fruitfulness of pursuing the theory of universal gravitation, he did not exactly succeed in "proving the Explanations." This would be a challenge for the generations after him.[323] By providing the synthesis Newton was inviting future generations to continually test and refine the theory of universal gravitation. The goal of the synthesis was *to test the validity and universality of the theory of gravitation by considering phenomena that are as remote as possible from those phenomena originally contained in the analysis*.[324]

8. *The* regulae philosophandi *and provisionalism*. As we have seen in Section 3.2 in this chapter, in the final edition of the *Principia* (1726) Newton relied on four rules of philosophizing in order to back up the inductive steps in the argument of universal gravitation. The first *regula*, which states that "*No more causes of natural things should be admitted than are both true and sufficient to explain their phenomena*. [*Causas rerum naturalium non plures admitti debere, quam quæ & veræ sint & earum phænomenis explicandis sufficiant*],"[325] expresses the idea that causes shown to be necessary and sufficient of their effects are to be kept minimal. This rule squares nicely with Newton's insistence on systematic dependencies between causes and effects. Rule II, which states that "*Therefore, the causes assigned to natural effects of the same kind must be, so far as possible, the same*. [*Ideoque effectuum naturalium ejusdem generis eædem assignandæ sunt causæ, quatenus fieri potest*],"[326] is a corollary to Rule I. Manuscript evidence suggests that when Newton talked about causes in the context of Rules I–II he was referring to proximate causes.[327] Rule III, which states that "*Those qualities of bodies that cannot*

[323] See Greenberg, *The Problem of the Earth's Shape from Newton to Clairaut*, and Wilson, "Newton and Celestial Mechanics".

[324] Cf. Stein, "Newtonian Space-Time", p. 263.

[325] Newton, *The Principia*, p. 794.

[326] Ibid.

[327] CUL Add. Ms. 3965, f. 419^r [additions and corrections to the first edition of the *Principia*; italics added].

be intended and remitted [i.e., qualities that cannot be increased and diminished] and that belong to all bodies on which experiments can be made should be taken as qualities of all bodies universally. [*Qualitates corporum quæ intendi & remitti nequeunt, quæque corporibus omnibus competunt in quibus experimenta instituere licet, pro qualitatibus corporum universorum habendæ sunt*.],"[328] was introduced in the second edition of the *Principia* and rephrased in the third edition. While the meaning of "intendi & remitti" has often been debated, manuscript evidence suggest that "remitti nequeunt" meant "cannot be taken away"[329] and, conversely, that "intendi nequeunt" meant "cannot be imposed/enforced."[330] Rules II–III underwent significant change in the third edition of the *Principia*. Rather than stating that the causes of natural effects of the same kind *are* the same as Newton did in the second edition (1713), in the third edition (1726) Newton stated that the causes *assigned to* natural effects of the same kind *must be, so far as possible, the same*. Similarly, rather than writing that the qualities of bodies that cannot be intended and remitted and that belong to all bodies on which experiments can be made *are qualities of all bodies universally* as he did in the second edition, in the third edition Newton wrote that the qualities of bodies that cannot be intended and remitted and that belong to all bodies on which experiments can be made *should be taken as qualities of all bodies universally.* In their reformulation in the third edition of the *Principia*, Rules II–III now involved an *epistemological* rather than an *ontological* claim and, furthermore, they anticipate possible revision. All this was, of course, connected with Newton's introduction of Rule IV, which states that "*In experimental philosophy, propositions gathered from phenomena by induction should be considered either exactly or very nearly true notwithstanding any contrary hypotheses, until yet other phenomena make such propositions either more exact or liable to exceptions*. [*In philosophia experimentali, propositiones ex phænomenis per inductionem collectæ, non obstantibus contrariis hypothesibus, pro veris aut accurate aut quamproxime haberi debent, donec alia occurerint phænomena, per quæ aut accuratiores reddantur out exceptionibus obnoxiæ*.],"[331] in the third edition of the *Principia*. Rule IV conveys essentially two points: (1) theoretical claims deduced from phenomena and rendered general by induction have epistemic authority over assumed (and possibly contrary) hypotheses, (2) theoretical claims established inductively should be considered as exactly or as closely as possibly true – thereby indicating that theoretical propositions are approximations – until they are either rendered more accurately by more widely and accurately observed phenomena[332] or until they are de-generalized as exceptions occur.[333]

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328 Newton, *The Principia*, p. 795.

329 CUL Add. Ms. 3970, f. 243^{v}.

330 Cf. Coles, *A Dictionary English-Latin and Latin English*, under "I" [page numbers lacking].

331 Newton, *The Principia*, p. 796.

332 CUL Add. Ms. 3965, f. 419^{v}. See Section 3.2 of this chapter.

333 Ibid., f. 428^{r}.

While Propositions I–IV of Book III mainly involved deductions from phenomena,[334] Propositions V–VIII explicitly contained several inductive steps. The inductive generalization to *universal* gravitation was indeed a bold leap.[335] Newton's awareness of this bold leap lead him ultimately to introduce Rule IV on the basis of which universal gravitation could be considered as exactly or as closely as possibly true, until more accurate observations render the law of universal gravitation more exactly of liable to exception. If exceptions occur to the law of universal gravitation then the law needs no longer to be considered as universal but as having a more restricted domain of applicability.[336] The provisionalism Newton envisioned did not apply to the inferences of inverse-square centripetal forces in Propositions I–IV of Book III, but rather to the identification of causes producing effects of the same kind in Proposition V and especially to the generalization to universal gravitation in Propositions VII–VIII.[337] This implies that Newton's provisionalism applied to the generality of the theory of universal gravitation but not to the inferences of instances of inverse-square centripetal forces. For this very reason, Shapiro's claim that "[o]nly in the last decades of his life did he [i.e., Newton] accept the probabilism of his contemporaries"[338] is somewhat misleading.

From the features mentioned above, it becomes obvious that the late-Newton saw scientific practice as an ongoing activity with room for future revision and, on that note, I am contented to end my analysis of Newton's methodology in the *Principia*. In the next chapter, I shall compare Newton's *Principia*-style methodology to the method he pursued in his optical work and in *The Opticks*, more particularly.

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[334] The moon test is a notable exception to this.

[335] He was aware for instance that the application of Law III to celestial bodies and bodies universally was an inductive generalization (see the discussion of Proposition V, Book III in Section 3.3 of this chapter). See also the discussion in Smith, "The Methodology of the *Principia*", pp. 163–164.

[336] Cf.: "But if at any time afterwards any Exception shall occur from Experiments, it may then begin to be pronounced with such Exceptions as occur." (Newton, *The Opticks*, p. 404).

[337] Moreover, Quayshawn Spencer has successfully argued (1) that Rule II can be uphold insofar as higher-order approximations succeed in providing evidence for the lower-level approximations generalized by Rule II and insofar as the theory of universal gravitation is relentlessly put to the test as soon as it is established by applying it to phenomena not originally dealt with in the analysis (Propositions I–VIII) (cf. Smith, "From the Phenomenon of the Ellipse to an Inverse-Square Force: Why not?", p. 55) and (2) that Rule IV can be uphold insofar as the theory can provide cross-checks, entailed by the systematic discrepancies, which can back up an inductive generalization (Spencer, "Do Newton's Rules of Reasoning guarantee Truth . . . must they?", pp. 776–777 [Rule II], pp. 774–775 [Rule IV]).

[338] Shapiro, *Fits, Passions, and Paroxysms*, p. 14.

Appendix 1: Relevant Additions and Changes Occurring in the Second Edition[339] of the *Principia* (1713)[340]

The first sentence of Definition I was changed[341]; more text was added to Definition V[342]; a clarification was added to Definition VI[343] and to Definition VII[344]; the formulation of Law III changed, a new concluding sentence to Law III was inserted[345];

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[339] For an early list of changes Newton planned for a revised edition of the *Principia*, see the memoranda by David Gregory, July 1694, Newton, *Correspondence*, III, pp. 384–389 (other memoranda by Gregory are to be found in letters 441, 443–448, and 450 in the same volume). Several corrections to the *Principia* were suggested to Newton by Roger Cotes who was in charge of the preparation of the third edition. Their rich and fruitful exchange of thought can be found in letters 765–766, 775, 777–779, 783–784, 786–789, 793–794, 800, 807, 809–811, 826, 829, 844, 846–847, 851, 854, 857, 860, 863, 871, 879, 889–891, 893–894, 896, 898–899, 901, 903, 908–917, 919–922, 931–933, 935, 937–938, 941–944, 946, 951, 953, 956, 958, 961–962, 977, 980, 985, and 988–989 in Newton, *Correspondence*, V and letter 1029 in ibid., VI. The second edition was long in the making: 19 years. Newton first declared his plans for a second edition to David Gregory in 1694. Many of the planned improvements were dependent on the astronomical observations made by the Royal Astronomer John Flamsteed who was preparing his *Historia Coelestis Britannica*. Newton himself admitted that "all ye world knows yt I make no observations my self" (Newton to Cotes, 16 February 1694/5, Newton, *Correspondence*, IV, p. 87). In a letter to John Colson Flamsteed wrote "Mr Newtons Theory when perfected must needs agree wth my Observations since tis built, as he freely owns[,] upon them and his doctrine of Gravitation: and the one wthout the other will not doe the buissness; but both altogether will, as he says himself" (Flamsteed to Colson, 10 October 1698, ibid., IV, p. 285). The publication *Historia Coelestis Britannica* was much delayed, until Flamsteed's incomplete catalogue of observations were edited and published by Edmund Halley in 1712 (see letters 700, 702, 737, 746 in ibid., IV). The Flamsteed-Newton correspondence can be consulted in letters 470, 473–490, 493–497, 499–501, 505–506, 515, 517, 520–527, 530–531, 543, 574, 576, 599–601, 604, 627, 681, 683–684, 686, 690, 694, 696, 699, 701, 707, 711, 716, and 739 in ibid., IV.

[340] In two appendices to this chapter, I shall obviously not attempt to give an overview of every change Newton introduced in subsequent editions of the *Principia*. Rather, my aim is to provide a nearly exhaustive catalogue of references to Newton's corresponding manuscripts and annotations which will, I hope, be of further use to those who wish to study the development of specific propositions in more detail. Since Newton's Wren copy of the second edition of the *Principia* (Wren Library, NQ.16.196) mostly contains straightforward textual corrections, I omit references to it here. References will be made, however, to annotations in Newton's Wren copy of the first edition of the *Principia* (Wren Library, NQ.16.200).

[341] NQ.16.200, p. 1.

[342] Namely: "Lapis, in funda circumactus, [...] cum velocitate egressum a data vi flectatur." (CUL Adv.b.39.2, pp. 2–3; CUL Adv.b.39.1, pp. 3–4 + interleaved page between pp. 2–3; NQ.16.200, p. 3; CUL Add. Ms. 3965, f. 182$^{\text{r-v}}$).

[343] Namely: "Ut vi Magnetica pro mole magnetis et intensione virtutis major in uno magnete, minor in alio." (CUL Adv.b.39.2, p. 4; CUL Adv.b.39.1, p. 3; NQ.16.200, p. 3). Newton originally planned to include additions to Law II (see CUL Add. Ms. 3965, f. 274$^{\text{r-v}}$, f. 731$^{\text{r-v}}$ [badly damaged by fire]), but these never appeared in print. For transcriptions of these folios, see Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. pp. 538–543. On the importance of these additions, see Pourciau, "Newton's Interpretation of Newton's Second Law".

[344] NQ.16.200, p. 3.

[345] Namely: "Obstinet etiam hæc Lex in Attractionibus, ut in Scholio proximo probabitur." (CUL Adv.b.39.2, 13, cf. CUL Adv.b.39.1, p. 13).

the text to its corresponding Corollary 1 was slightly altered and clarified[346]; and, the *scholium* to Law III contained some clarifications and an additional[347] paragraph and a figure.[348]

In Book I,[349] Lemmas I–XI underwent some revision: most notably, Lemma I was rewritten[350]; the texts to Lemmas VI–IX were altered[351]; three additional corollaries (Corollaries 3–5) to Lemma X, of which the formulation was revised, were added as well as a new *scholium*; two additional corollaries (Corollaries 2–3) were inserted between Corollaries 1–2 to Lemma XI, of which the formulation and the accompanying text were revised, and a new *scholium* was added[352]; the two original corollaries to Proposition I were deleted and replaced by six new corollaries[353]; to Proposition II two corollaries were added[354]; Proposition III was slightly changed[355]; Proposition IV was entirely reworked (its corresponding *scholium* remained untouched)[356]: the demonstration of Proposition IV was altered, Corollaries 1–6 were slightly rephrased, Corollary 7 was renumbered "Corollary 8," and a new Corollary 7 was introduced[357]; the text of Proposition V underwent some change[358]; Proposition VI was reworked: its proof changed, four additional corollaries were added and the original Corollary 1 was renumbered as Corollary 5[359];

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346 CUL Adv.b.39.1, pp. 13–14. A reference to the first and second law was inserted (CUL Adv.b.39.2, p. 13; cf. CUL Adv.b.39.1, p. 13–14, NQ.16.200, pp. 13–14).

347 Namely: "Sic etiam gravitas inter Terram & ejus partes, [. . .], & ab eo fugiendo abiret in infinitum." (CUL Adv.b.39.2, p. 22).

348 CUL Adv.b.39.1, pp. 20–25 + interleaved page between pp. 24–25; CUL Adv.b.39.2, pp. 41–42; CUL Add. Ms. 3965, f. 180^{r}. The texts to Corollaries 2–6 contained only minor changes which are irrelevant here (CUL Adv.b.39.1, pp. 14–20).

349 For further discussion of the changes made to Book II in the second edition of the *Principia*, see Hall, "Correcting the *Principia*", pp. 302–326.

350 CUL Adv.b.39.1, p. 26; NQ.16.200, p. 26; CUL Adv.b.39.2, p. 24.

351 CUL Adv.b.39.1, pp. 29–32; NQ.16.200, pp. 29–31; CUL Adv.b.39.2, pp. 27–29.

352 CUL Adv.b.39.1, pp. 32–36 + interleaved page between pp. 32–33; NQ.16.200, pp. 32–33; CUL Adv.b.39.2, pp. 29–34.

353 CUL Adv.b.39.1, p. 38 + interleaved page between pp. 38–39; CUL Adv.b.39.2, pp. 35–36; CUL Add. Ms. 3965, f. 179^{r}.

354 CUL Adv.b.39.1, interleaved page between pp. 38–39; NQ.16.200, pp. 38–39; CUL Adv.b.39.2, p. 36.

355 CUL Adv.b.39.1, pp. 39–40; NQ.16.200, pp. 39–40; CUL Adv.b.39.2, pp. 39–40.

356 CUL Adv.b.39.1, pp. 41–42 + interleaved page between pp. 41–42; NQ.16.200, pp. 41–42; CUL Adv.b.39.2, pp. 38–40; CUL Add. Ms. 3965, f. 179^{v}.

357 CUL Adv.b.39.1, pp. 41–42 + interleaved page between p. 41; NQ.16.200, pp. 41–42; CUL Adv.b.39.2, pp. 38–39; CUL Add. Ms. 3965, f. 179^{r}, f. 181^{r}.

358 CUL Adv.b.39.1, p. 44; NQ.16.200, p. 44; CUL Adv.b.39.2, p. 41; CUL Add. Ms. 3965, f. 179^{v}.

359 CUL Adv.b.39.1, pp. 44–45 + interleaved pages between pp. 40–41 and pp. 42–43; CUL Adv.b.39.2, pp. 41–42; CUL Add. Ms. 3965, f. 180$^{r\text{-}v}$/181$^{r\text{-}v}$/183$^{r\text{-}v}$.

Proposition VII was reworked[360]; the *scholium* after Proposition X was extended[361]; a *scholium* was added to Proposition XVII[362]; Lemma XXI was corrected[363]; some lines were added to Lemma XXVIII[364]; the *scholium* to Proposition XXXIII underwent major revision[365]; Proposition XLIV underwent revision[366]; a corollary was added to Proposition LI;[367] and, finally, a third corollary was added to Proposition LXXII.[368]

In Book II, two corollaries were added to Proposition IV[369]; Proposition X was reworked[370]; Corollaries 6–8 to Proposition XXXIII were deleted and Corollary 9 was renumbered "Corollary 6;"[371] Proposition XXXIV was deleted[372]; Proposition XXXV [Proposition XXXIV in the second edition] was slightly altered[373]; a new Proposition XXXV was inserted[374]; Proposition XXXVI was deleted[375]; Proposition XXXVII [Proposition XXXVI in the second edition] underwent major revision[376]; Proposition XXXVIII [Proposition XXXVII in the second edition] underwent major revision and a *scholium* was added[377]; Proposition XXXIX was deleted and two additional Propositions (XXXVIII–XXXIX) were included[378]; the

360 CUL Adv.b.39.1, interleaved page between pp. 42–43; CUL Adv.b.39.2, pp. 42–44; CUL Add. Ms. 3965, f. 183$^{\mathrm{v}}$, f. 188$^{\mathrm{r\text{-}v}}$.

361 An additional concluding sentence was added (CUL Adv.b.39.1, p. 49; CUL Adv.b.39.2, p. 47; CUL Add. Ms. 3965, ff. 190$^{\mathrm{r}}$-201$^{\mathrm{v}}$).

362 CUL Adv.b.39.1, p. 60; CUL Adv.b.39.2, p. 58.

363 NQ.16.200, p. 78.

364 CUL Adv.b.39.1, interleaved page between pp. 106–107; CUL Adv.b.39.2, p. 99.

365 CUL Adv.b.39.1, pp. 109–114; CUL Adv.b.39.2, pp. 101–104.

366 CUL Adv.b.39.1, pp. 133–137; NQ.16.200, pp. 133–135; CUL Adv.b.39.2, pp. 122–126.

367 CUL Adv.b.39.1, p. 152; NQ.16.200, p. 152; CUL Adv.b.39.2, p. 140.

368 CUL Adv.b.39.1, p. 196; NQ.16.200, p. 196; CUL Adv.b.39.2, p. 176.

369 CUL Adv.b.39.1, interleaved page between pp. 242–243; CUL Adv.b.39.2, p. 217.

370 CUL Adv.b.39.1, pp. 260–269; CUL Adv.b.39.2, pp. 232–244; CUL Add. Ms. 3695, ff. 190$^{\mathrm{r}}$-201$^{\mathrm{v}}$, ff. 219$^{\mathrm{r}}$-220$^{\mathrm{v}}$. Just before publication, Nicolas Bernouilli spotted an error in Proposition X and Newton corrected the proofs just before printing (Newton, *Correspondence*, VII, pp. 62–69).

371 CUL Adv.b.39.1, pp. 321–322; CUL Adv.b.39.2, pp. 297–298; CUL Add. Ms. 3965, f. 202$^{\mathrm{r\text{-}v}}$.

372 CUL Adv.b.39.1, pp. 323–324.

373 CUL Adv.b.39.1, pp. 324–327; CUL Adv.b.39.2, pp. 298–300; CUL Add. Ms. 3965, f. 207$^{\mathrm{r}}$.

374 CUL Adv.b.39.2, pp. 300–303; CUL Add. Ms. 3965, ff. 245$^{\mathrm{r}}$-246$^{\mathrm{r}}$.

375 CUL Adv.b.39.1, pp. 27–29.

376 CUL Adv.b.39.1, pp. 330–332; CUL Adv.b.39.2, pp. 303–309; CUL Add. Ms. 3965, ff. 205$^{\mathrm{r}}$-206$^{\mathrm{r}}$, f. 231$^{\mathrm{r\text{-}v}}$, ff. 237$^{\mathrm{r}}$-240$^{\mathrm{r}}$ [not in Newton's hand], ff. 247$^{\mathrm{r}}$-248$^{\mathrm{r}}$/249$^{\mathrm{r}}$; ff. 738$^{\mathrm{r}}$/739$^{\mathrm{r}}$ [badly damaged by fire].

377 CUL Adv.b.39.1, pp. 332–337; CUL Adv.b.39.2, pp. 310–314; CUL Add. Ms. 3965, ff. 240$^{\mathrm{r}}$-242$^{\mathrm{r}}$, ff. 247$^{\mathrm{r}}$/250$^{\mathrm{r}}$/251$^{\mathrm{r}}$; ff. 739$^{\mathrm{r}}$/740$^{\mathrm{r}}$ [badly damaged by fire], ff. 747$^{\mathrm{r}}$-748$^{\mathrm{v}}$ [badly damaged by fire].

378 CUL Adv.b.39.1, pp. 337–338; CUL Adv.b.39.2, pp. 316–317; CUL Add. Ms. 3965, f. 242$^{\mathrm{v}}$.

experimental data in Proposition XL was extended[379]; and, finally, Proposition L underwent major revision.[380]

In Book III, the *regulae philosophandi* (including a new Rule III) replaced Hypothesis I–III[381]; Hypothesis V became Phænomenon I [with corrected empirical data]; a new Phænomenon II was added [on the circumsaturnian planets]; Hypothesis VI became Phænomenon III; Hypothesis VII became Phænomenon IV; Hypothesis VIII became Phænomenon V; Hypothesis IX became Phænomenon VI[382]; Proposition IV underwent major revision[383]; to Proposition V a third corollary was added[384]; Proposition VI underwent revision[385]; Proposition VIII underwent major revision[386]; Hypothesis IV ("*Centrum Systematis Mundani quiescere*."[387]) became Hypothesis I; Proposition XIII underwent major revision[388]; adjustments of empirical data were made in Propositions X–XI[389]; a *scholium* was added to Proposition XIV[390]; Proposition XIX underwent major revision[391]; Proposition XX underwent major revision[392]; Proposition XXIV underwent revision[393]; Proposition XXIX underwent

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[379] *Pr.E*$_1$, pp. 338–354 [here I cannot refer to Newton's private copy, as the corresponding pages are badly damaged by fire and some of them are completely missing.]; CUL Adv.b.39.2, pp. 317–328; CUL Add. Ms. 3965, ff. 212^{r}-214^{r}, ff. 221^{r-}225^{r}, ff. 228^{r}-229^{v}, ff. 233^{r}-236^{v}, ff. 253^{v}-254^{r}/255^{r}/256^{r}/257^{r}/258^{r}/259^{r}, ff. 261^{r}-262^{v}.

[380] CUL Adv.b.39.1, pp. 369–372; NQ.16.200, pp. 369–372; CUL Adv.b.39.2, pp. 342–342; CUL Add. Ms. 3965, f. 740^{r}.

[381] CUL Adv.b.39.1, p. 402 + interleaved page between pp. 402–403; NQ.16.200, p. 402 + autograph endnotes on "p. 402 l. 10;" CUL Adv.b.39.2, pp. 357–358; CUL Add. Ms. 3965, f. 266^{r}.

[382] CUL Adv.b.39.1, pp. 402–404; CUL Adv.b.39.2, pp. 359–361.

[383] CUL Adv.b.39.1, pp. 406–407 + interleaved page between pp. 406–407; NQ.16.200, p. 407; CUL Adv.b.39.2, pp. 363–364; CUL Add. Ms. 3965, f. 306^{r-v}, ff. 308^{r-v}/309^{v}.

[384] CUL Adv.b.39.1, p. 408; NQ.16.200, p. 408; CUL Adv.b.39.2, p. 365.

[385] CUL Adv.b.39.1, pp. 408–411 + interleaved page between pp. 410–411; NQ.16.200, pp. 408–411; CUL Adv.b.39.2, pp. 363–364; CUL Add. Ms. 3965, ff. 310^{r}-311^{v}.

[386] CUL Adv.b.39.1, pp. 412–416 + interleaved pages between pp. 412–413 and pp. 414–415; NQ.16.200, pp. 413–415; CUL Adv.b.39.2, pp. 370–372; CUL Add. Ms. 3965, ff. 312^{r}/313^{r}.

[387] CUL Adv.b.39.1, p. 402; CUL Adv.b.39.2, p. 373.

[388] CUL Adv.b.39.1, pp. 419–420 + interleaved page between pp. 418–419; CUL Adv.b.39.2, pp. 375–376.

[389] CUL Adv.b.39.1, pp. 416–418; CUL Adv.b.39.2, pp. 372–374.

[390] CUL Adv.b.39.1, interleaved page between pp. 422–423; CUL Adv.b.39.2, p. 376.

[391] CUL Adv.b.39.1, pp. 422–424; CUL Adv.b.39.2, pp. 378–381; CUL Add. Ms. 3965, ff. 284^{r}/285^{r}/286^{r-v}, ff. 297^{r}-300^{r}/301^{r}/302^{r}.

[392] CUL Adv.b.39.1, pp. 424–426; CUL Adv.b.39.2, pp. 382–387; CUL Add. Ms. 3965, f. 292^{r-v}, f. 294^{r}.

[393] NQ.16.200, pp. 432–433.

revision[394]; the *scholium* to Proposition XXXV underwent revision[395]; Proposition XXXVI underwent revision[396]; Proposition XXXVII underwent major revision[397]; Lemma I to Proposition XXXVIII underwent revision[398]; Proposition XLI underwent major revision[399]; Proposition XLII underwent major revision[400]; and, finally, the General Scholium[401] was introduced.[402]

394 CUL Adv.b.39.1, pp. 442–443; NQ.16.200, pp. 442–443; CUL Adv.b.39.2, pp. 401–403.

395 CUL Adv.b.39.1, pp. 462–463; CUL Adv.b.39.2, pp. 421–425; CUL Add. Ms. 3965, f. 318^{r}, f. 319^{v}.

396 CUL Adv.b.39.1, pp. 463–464; CUL Adv.b.39.2, pp. 426–427; CUL Add. Ms. 3965, f. 320^{r-v}, f. 326^{r}, f. 327^{r}.

397 CUL Adv.b.39.1, pp. 464–467; CUL Adv.b.39.2, pp. 427–431; CUL Add. Ms. 3965, f. 320^{r-v}, ff. 324^{r}-325^{r}, ff. 326^{r}-327^{v}.

398 CUL Adv.b.39.1, pp. 467–469; CUL Adv.b.39.2, pp. 431–435; CUL Add. Ms. 3965, ff. 330^{r}-332^{r}, ff. 324^{r}-325^{r}, ff. 334^{r}/335^{r}, ff. 336^{r}-337^{v}.

399 CUL Adv.b.39.1, pp. 487–509; pp. 487–496; CUL Adv.b.39.2, pp. 451–475; CUL Add. Ms. 3965, ff. 338^{r-v}/339^{v}, ff. 340^{r}-342^{r}/343^{r}/344^{r}/345^{r}/346^{r-v}, f. 347^{r}.

400 CUL Adv.b.39.1, pp. 509–510; CUL Adv.b.39.2, pp. 475–481; CUL Add. Ms. 3965, ff. 324^{r}-325^{r}, ff. 348^{r}-353^{r}/354^{r}/355^{r}/356^{r}.

401 For more details see Chapter 6.

402 CUL Adv.b.39.2, pp. 481–484.

Appendix 2: Relevant Additions and Changes Occurring in the Third Edition[403] of the *Principia* (1726)

Newton's private copy of the second edition of the *Principia* contains two important additions to the Definitions which did not make it to the published text of the third edition. Commenting on Definition III he wrote "Non intelligo vim inertiæ Kepleri qua corpora ad quietem tendunt, sed vim manendi in eodem seu quiescendi seu movendi statu."[404] To Definition IV he added the disclaimer that impressed force "neque corpori essentialis est."[405]

The text to Law II was extended,[406] and the *scholium* to Law III was extended.[407]

In Book I, Proposition XVII was slightly extended.[408]

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[403] Henry Pemberton, who definitely was not a second Cotes, was in charge of the preparation of the third edition. The correspondence between Pemberton and Newton can be found in letters 1413–1416, 1420–1422, 1426, 1439, 1443, 1445, 1447–1448, 1451, 1453, 1457–1458, 1462, 1467–1468, 1470, 1472–1473 in Newton, *Correspondence*, VII, and 1486 in ibid., VII.

[404] CUL Adv.b.39.2, p. 2. Cf. Cohen, "Newton's Copy of Leibnitz' *Théodicée* [i.e., Wren Library, NQ.8.82]", pp. 411–412.

[405] CUL Adv.b.39.2, p. 2 + interleaved page between pp. 2–3.

[406] In his private copy of the second edition Newton wrote: "Ut si corporis cadentis gravitas sit uniformis, hæc singulis temporis particulis æqualibus æqualiter agendo, imprimet vires æquales in corpus illud, et velocitates æquales generabit; et propterea vis tota in corpus cadens impressa, et velocitas tota genita semper erunt ut tempus totum cadendi." (CUL Adv.b.39.2, interleaved page between pp. 12–13; CUL Add. Ms. 3965, f. 497^{r}, f. 500^{r}, f. 515^{r}; PrE_3, p. 13). Newton reiterated his plans to include revisions of Law II which were similar to the ones he had proposed when correcting the first edition of the *Principia* (CUL Add. Ms. 3965, f. 274$^{r\text{-}v}$). The revisions intended for the third edition are found on: ibid., f. 731$^{r\text{-}v}$ but once again they did not make it to print. This material is transcribed and translated in: Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 538–543. Note that Whiteside's reconstruction is based on a combination of f. 274$^{r\text{-}v}$ and f. 731$^{r\text{-}v}$ – the latter is damaged by fire. Whiteside did not reproduce the following corollary on f. 732^{r}: "Corol. 5. Eædem vires sunt etiam ut synchronæ corporum deflexiones quam minimæ ab orbium tangentibus in centra virium, id est ut altitudines quàm minimæ quas corpora ab Orbium tangentibus ad Orbes ipsos cadendo simul describunt, sive ut arcuum simul descriptorum sinus versi ad centra virium convergentes. Nam altitudines illæ vel sinus illi versi sunt sagittæ arcuum duplis temporibus descriptorum. Sed et per solam Motûs Legem II constat hoc Corollarium."

[407] The following text was inserted: "Corpore cadente, gravitas uniformis vires imprimit [. . .] et cujus Ordinata BC est ut ABq." (CUL Adv.b.39.2, interleaved pages between pp. 18–19 and pp. 20–21; CUL Add. Ms. 3965, f. 497^{r}, f. 500^{r}; PrE_3, pp. 21–22). On CUL Add. Ms. 3965, f. 731^{r}, Newton wrote down the following addition to the text of Law III which did not make it to print: "Corpus omne quod corpus alterum attrahit, tantundem ab illo altero in partes contrarias attrahitur. Sed hoc intelligendum est de attractione quæ ↓tota↓ a corpore attrahente dependet tanquam virtus ejus et sphæra activitatis, & ipsum ubique comitatur & seorsim existere non potest; cujusmodi est attractio magnetica. Nam si attractio a corpore attrahente separari potest & seorsim existere, hæc non erit corporis illius attractio sed aliunde orietur & mutua erit inter corpus attractum et rem illam quæ attractionis est causa. Interea quid sit attractio et quomodo ↓fiat↓ sive per corporum insensibiles atmosphæras perpetuo emissas sive per alia media quorum ope corpora propagant actiones secretas in se mutuò, hic non expendo." For discussion, see Chapter 1, Section 1.6.

[408] CUL Adv.b.39.2, pp. 56–57; CUL Add. Ms. 3965, f. 497^{v}; PrE_3, p. 64.

In Book II, the so-called "Leibniz Scholium" was revised[409]; Proposition IX underwent some rephrasing[410]; an additional last sentence was added to Proposition XXII[411]; Proposition XXXVI was slightly altered[412]; and, finally, Experiment 13 to the *scholium* to Proposition XL was corrected[413] and Experiment 14 was introduced.[414]

In Book III, Rule II was altered from "*Ideoque Effectuum naturalium ejusdem generis eædem sunt Causæ.*" into "*Ideoque effectuum naturalium ejusdem generis eædem assignandæ sunt causæ, quatenus fieri potest.*"[415]; two additional concluding sentences to Rule III were added[416]; Rule IV was introduced[417]; an additional paragraph to Phenomenon I was introduced[418]; the empirical data in Phenomenon II was altered and an additional paragraph was added[419]; Proposition III was slightly altered[420]; Proposition IV was corrected and a *scholium* was added[421]; the empirical data in Proposition VIII was corrected[422]; five additional sentences were added at the end of Proposition X[423]; the empirical data in Proposition XIV was corrected[424]; an additional observation was added to Proposition XVII[425]; Propositions XIX and

409 CUL Adv.b.39.2, interleaved folios between pp. 226–227; PrE_3, p. 246. Drafts of it can be found CUL Add. Ms. 3968, ff. 20^{r}-36^{v}.

410 CUL Adv.b.39.2, pp. 229–332; PrE_3, pp. 248–251; CUL Add. Ms. 3965, f. 501^{r}.

411 CUL Adv.b.39.2, p. 270; PrE_3, p. 292; CUL Add. Ms. 3965, f. 501^{r} [related draft material is to be found on ibid., f. 380$^{r\text{-}v}$].

412 CUL Adv.b.39.2, interleaved page between pp. 304–305; PrE_3, pp. 329–330; CUL Add. Ms. 3965, f. 501^{r} [related draft material is to be found on ibid., ff. 385$^{r\text{-}v}$/386^{r}/387^{v}/388^{r}-389^{r}/390^{r}-392^{v}].

413 CUL Adv.b.39.2, pp. 325–326; PrE_3, pp. 351–352; CUL Add. Ms. 3965, f. 501^{v}, ff. 517^{v}-518^{v} [related draft material is to be found on ibid., ff. 396^{r}-397^{r}, f. 398^{r}, f. 399$^{r\text{-}v}$, f. 401$^{r\text{-}v}$, f. 403^{r}, f. 406$^{r\text{-}v}$, f. 410^{r}-411^{v}].

414 PrE_3, pp. 353–354; CUL Add. Ms. 3965, ff. 412$^{r\text{-}v}$.

415 CUL Adv.b.39.2, pp. 357; PrE_3, p. 387.

416 CUL Adv.b.39.2, p. 358; PrE_3, p. 389.

417 CUL Adv.b.39.2, interleaved page between pp. 358–359; PrE_3, p. 389; CUL Add. Ms. 3965, f. 504^{r}, f. 519^{r}.

418 CUL Adv.b.39.2, pp. 359; PrE_3, p. 391; CUL Add. Ms. 3965, f. 504^{r}, f. 519^{r} [related draft material is to be found on ibid., ff. 425^{r}-426^{r}/427^{r}].

419 CUL Adv.b.39.2, pp. 360; PrE_3, pp. 391–392; CUL Add. Ms. 3965, f. 504^{v}, f. 519^{r}.

420 CUL Adv.b.39.2, p. 363; PrE_3, p. 396; CUL Add. Ms. 3965, f. 504^{v}.

421 CUL Adv.b.39.2, pp. 363–364 + interleaved page between pp. 364–365; PrE_3, pp. 396–398; CUL Add. Ms. 3965, ff. 504^{v}-505^{r}, f. 519^{v}.

422 CUL Adv.b.39.2, pp. 370–371; PrE_3, pp. 404–405; CUL Add. Ms. 3965, f. 505^{r}, f. 519^{v} [related draft material is to be found on ibid., f. 432^{r}, f. 433r-v, ff. 435^{r}-436^{r}].

423 CUL Adv.b.39.2, interleaved page between pp. 372–373; PrE_3, p. 407; CUL Add. Ms. 3965, f. 505^{v}, f. 520^{r}.

424 CUL Adv.b.39.2, pp. 376; PrE_3, p. 411; CUL Add. Ms. 3965, f. 505^{v}.

425 CUL Adv.b.39.2, p. 377; PrE_3, pp. 411–412; CUL Add. Ms. 3965, f. 446$^{r\text{-}v}$, f. 505^{v}, f. 520^{r}.

XX underwent major revision[426]; a *scholium* to Proposition XXXIII was added containing a paper by John Machin on the motion of the nodes of the Moon[427]; Corollaries 7–9 to Proposition XXXVII underwent revision and a new Corollary 10 was added[428]; the empirical data in Proposition XLI was thoroughly revised[429]; and, finally, Proposition XLII was reworked and new material was added.[430]

Additional text was added to the General Scholium: namely, the sentences “Omnis anima sentiens [. . .] deus semper & ubique.”,[431] “Colimus enim ut servi [. . .] fatum & natura.” and, “A cæca necessitate metaphysica [. . .] sed aliquam tamen.”[432] and footnote b.[433] Finally, the list of classical and biblical references was extended.[434]

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[426] CUL Adv.b.39.2, pp. 378–388; PrE_3, pp. 404–405; CUL Add. Ms. 3965, f. 502^{r}, ff. 505^{v}-506^{v}, f. 508$^{r\text{-}v}$, f. 510^{r}, f. 512^{r}, f. 513^{r}, f. 514$^{r\text{-}v}$, ff. 520^{v}-521^{v} [related draft material is to be found on ibid., f. 447^{r}, ff. 449^{r}-450^{r}/451^{r}, f. 456$^{r\text{-}v}$, f. 460^{r}].

[427] CUL Add. Ms. 3965, f. 507^{r}.

[428] CUL Adv.b.39.2, pp. 427–431; PrE_3, pp. 465–471; CUL Add. Ms. 3965, f. 459^{r}, f. 460$^{r\text{-}v}$, f. 462^{r}, f. 509^{r}, ff. 521^{v}-522^{r}.

[429] CUL Adv.b.39.2, pp. 456–475; PrE_3, pp. 492–518; CUL Add. Ms. 3965, f. 491^{v}, f. 494$^{r\text{-}v}$, f. 496^{r}, ff. 522^{r}-525^{v}, f. 527^{r}/f. 528^{r}/f. 529^{r}/f. 530$^{r\text{-}v}$/ff. 531^{r}-534^{r}, ff. 535^{r}-536^{r} [related draft material is to be found on ibid., f. 466^{r}, f. 467^{r}, f. 471^{r}, f. 472^{r}, f. 473^{r}, ff. 475^{r}-476^{r}, f. 477^{r}, ff. 479^{r}-486^{r}, f. 487^{r}/f. 488^{r}].

[430] CUL Adv.b.39.2, pp. 475–481; PrE_3, pp. 518–526; CUL Add. Ms. 3965, ff. 525^{v}-526^{r}, f. 537^{r}/f. 538^{r}. The insertion “Talis fuit stella [. . .] & partem obscuram per vices ostendere.” was added.

[431] CUL Adv.b.39.2, interleaved page between pp. 482–483; PrE_3, p. 528; CUL Add. Ms. 3965, f. 494^{v}, f. 526^{r}.

[432] PrE_3, p. 529; CUL Add. Ms. 3965, f. 494^{v}, f. 536^{v}, f. 539^{r}, f. 543$^{r\text{-}v}$.

[433] PrE_3, p. 482; CUL Add. Ms. 3965, f. 539^{v}.

[434] CUL Adv.b.39.2, p. 483 + interleaved page between pp. 482–483; PrE_3, p. 529; CUL Add. Ms. 3965, f. 539^{r}.

Chapter 4
Facing the Limits of Deductions from Phenomena: Newton's Quest for a Mathematical-Demonstrative Optics

4.1 Introduction

In this chapter, I shall elaborate on the claims made by the late I. Bernard Cohen according to which Newton's methodological ideal of "deducing causes from phenomena," on which we have elaborated on in Chapters 2 and 3, was not equally attainable in the study of optical phenomena. If his suggestion is correct, then in *The Opticks*,[1] the apex of his optical researches, which in fact contained a set of separate but interrelated theories, Newton failed to rigidly establish these theories in the same way as he had established the theory of universal gravitation in the *Principia*. By contrasting Newton's methodology in the *Principia*, as previously characterized, to the method by which theoretical and causal conclusions are established in *The Opticks*, I shall attempt to explain why Newton was less successful in accommodating optical phenomena *according to his own methodological desiderata* of deducing causes from phenomena. I wish to emphasize that, when I occasionally use the word "failure," I refer to Newton's inability to accommodate optical phenomena according to his *own* methodological ideals – nowhere, do I use the word in an evaluative sense. Ab initio, I would like to emphasize that this chapter in no way pretends to survey the entirety of Newton's optical research. Where relevant, I shall illustrate specific points by taking Newton's optical writings and manuscripts from 1664 and onwards into account, but my main focus will be on the comparison of the different methods by which causal conclusions were established in the *Principia* and in *The Opticks*.

In contrast to the first edition of the *Principia*, the first edition of *The Opticks* was written over a very wide time-span. Newton's first proper piece on optics was his *Of Colours*[2] which reported on a series of experiments undertaken in 1665/66.

[1] First English edition: 1704; first Latin edition: 1706; second English edition: 1717 (reissued in 1718); second Latin edition: 1719; third English edition: 1721; fourth posthumous English edition: 1730.

[2] CUL Add. Ms. 3975, ff. 1^{r}–22^{r}. This tract is transcribed in McGuire and Tamny, eds., *Certain Philosophical Questions*, pp. 466–489. Newton's earlier *Questiones quaedam philosophicae* (= CUL Add. Ms. 3996, ff. 122^{r}–124^{v} [ca. 1664]) contained a number of short sections related to optics, which were primarily based on material from Robert Boyle's *Experiments*

S. Ducheyne, *The Main Business of Natural Philosophy*, Archimedes 29,
DOI 10.1007/978-94-007-2126-5_4,

Newton's early thoughts on things optical, which he developed in this private work, would within a few years develop into a more elaborate theory on the structure of white light and the origin of colours. It was in his first optical paper in 1671/72, that Newton publically announced the perhaps surprising theory, according to which white light consists of coloured rays differently refrangible.[3] In doing so, he broke with his earlier endorsement of a modification-type theory of light.[4] In 1674, a few years after the publication of his first optical paper, Newton submitted his *Lectiones opticae*, which were based on a series of inaugural lectures he had delivered between 1670 and 1672 in order to fulfil his requirements as the new Lucasian Professor of Mathematics of Trinity College, at the University Library at Cambridge.[5] In 1675 Newton's second optical paper appeared in print: *An Hypothesis explaining the Properties of Light, discoursed of in my several Papers*.[6] According to Newton's own testimony, most of the material contained in *The Opticks* dated back from 1675 except for the end of Book II and the complete Book III.[7] The *Fundamentum opticae* (ca. 1687–88), of which the content is very similar to Book I of *The Opticks*, can be considered as the starting point of the composition of *The Opticks*. Shortly afterwards, Newton shifted from a Latin to an English text.[8] In a memorandum by David Gregory on 5–7 May 1694, Gregory testified that Newton had let him see three books on optics.[9] In 1704, a decade later, *The Opticks* finally appeared in print.[10]

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and Considerations Touching Colours (1664) (Lohne, "Isaac Newton: The Rise of a Scientist, 1661–1671").

[3] Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, pp. 47–59. The unedited version, which Newton sent to Henry Oldenburg on 6 February 1671/2, is to be found in Newton, *Correspondence*, I, pp. 92–107.

[4] McGuire and Tamny, eds., *Certain Philosophical Questions*, p. 389.

[5] When Newton became Lucasian Professor in 1669 he was required to submit a selection of 10 lectures he had given during the academic year for deposit in the University Library. The *Lectiones opticae* thus predate his first optical paper. Whilst Newton was working on his first optical paper in 1671/72, he had also begun on a revision (*Optica*) of the *Lectiones opticae*, which was published posthumously in 1729 (Newton, *Lectiones opticae*). The material related to Newton's optical lectures can be found in two manuscripts: the *Lectiones opticae* proper (CUL Add. Ms. 4002) and *Optica*, the later variant of the *Lectiones opticae* (CUL Ms. Dd.9.67). Both are transcribed in Shapiro, *The Optical Papers of Isaac Newton*, I.

[6] Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, pp. 178–199; Newton, *Correspondence*, I, pp. 362–392.

[7] *OEL_1*, [iii]. Cf. Cohen, "Versions of Isaac Newton's First Published Paper", p. 359. For details on the composition of *The Opticks*, see especially: Shapiro, "Beyond the Dating Game: Watermark Clusters and the Composition of Newton's *Opticks"*, pp. 198–226.

[8] Cohen, "The Case of the Missing Author: The Title Page of Newton's *Opticks* (1704)", pp. 18–19.

[9] Newton, *Correspondence*, III, p. 336.

[10] Apart from two corrected calculations in Part I, Book I of *The Opticks* remained unchanged in all editions. With the exception of the introduction of an extra paragraph, which remained unchanged in all later editions, at the end of Proposition VIII of Part III in the second edition (OE_2, pp. 242–244), Book II of *The Opticks* remained nearly unchanged in all editions. Book III remained the same in all editions, except for the concluding Queries. In the first edition, *The Opticks* contained

4.2 *The Opticks* as an Incomplete Treatise

It is quite reasonable to suggest that Newton conceived of *The Opticks* as an incomplete work.[11] In the Advertisement to the first edition of *The Opticks*, Newton wrote:

> *To avoid being engaged in Disputes about these Matters, I have hitherto delayed in printing, and should still have delayed it, had not the Importunity of Friends prevailed upon me. If any Papers writ on the Subject are got out of my Hands they are imperfect, and were perhaps written before I tried all the Experiments here set down, and fully satisfied my self about the Laws of Refraction and Composition of Colours. I have here publish'd what I think proper to come abroad, wishing that it may not be translated into another language without my consent.*[12]

Note that it was only with respect to Book I of *The Opticks*, which set out to establish that the sun's light consists of rays differently refrangible, that Newton was prepared to state that his design "in this Book is not to explain the Properties of Light by Hypotheses, but to propose and prove them by Reason and Experiments."[13] The late I. Bernard Cohen has pointed out that there were at least three reasons why Newton remained unsatisfied with *The Opticks*.[14] First of all, Newton had not been successful in his study of diffraction, which is dealt with in Book III of *The Opticks*.[15]

16 Queries. In the second edition, Newton inserted 15 additional Queries – while also extending Queries 8, 10, 11, and 16 (ibid., pp. 313–382). The Queries added in the second edition remained unchanged in all later editions.

[11] Cf. Westfall, *Never at Rest*, pp. 797–798. This is most certainly the case for the third book of which Newton himself admitted that the arguments contained in it were imperfect. In the introductory section to the Queries Newton, wrote: "And since I have not finish'd this part of my Design, I shall conclude with proposing only some Queries, in order to a farther search to be made by other." (Newton, *The Opticks*, p. 339). Colin MacLaurin pointed out: "He [i.e., Newton] knew where to stop when experiments were wanting, and when the subtility of nature carried things out of his reach: nor would he abuse the great authority and reputation he had acquired, by delivering his opinion concerning these, otherwise than as matter of question." (MacLaurin, *An Account of Sir Isaac Newton's Philosophical Discoveries*, p. 10; cf. Desaguliers, *A Course of Experimental Philosophy*, vol. II, p. 403).

[12] Newton, *The Opticks*, cxxi. For the reasons prodding Newton to publish *The Opticks*, see: Cohen, "Versions of Isaac Newton's First Published Paper", pp. 359–361. After the publication of the first edition *The Opticks*, Newton commissioned Samuel Clarke to make a Latin translation which appeared in 1706 as *Optice*. After having praised Newton for his experimental-mathematical method of demonstration in the *Principia* by which Newton had dispensed with "fictitious hypotheses" and "capricious conjectures" (*OEL₁*, [i]), Samuel Clarke pointed out in his *Præfatio Interpretis* that the Latin translation was begun by the order of the author and absolutely approved by the same person ("Id hic certior faciendus est Lector, hanc Versionem & Authoris jussu incoeptam, & eodem approbante absolutam" (ibid., [ii])).

[13] Newton, *The Opticks*, p. 1.

[14] Cohen, "The Case of the Missing Author", pp. 18–23.

[15] Shapiro, "Newton's Experimental Investigation of Diffraction for the Opticks: A Preliminary Study", pp. 63, 70.

In manuscript material, Newton commented as follows on his work on diffraction in Book III of *The Opticks*[16]:

> Many experiments are wanting for completing the Analysis of this part of Nature & coming to a clear ↓& distinct↓ knowledge of ↓all↓ y^{e} causes of these things, many or more for perfecting the Analysis of all Nature & ~~establishing the~~ making a full & clear discovery of all the first Principles of Natural Philosophy. ~~And yet~~ To compass this is a work w^{ch} requires many heads and hands & a long time & yet this ought to be done before we proceed from the first Principles by Composition to explain all Nature.[17]

Secondly, *The Opticks* ended with a considerable list of yet unanswered Queries, which were intended as "hints to be examined & improved."[18] Finally, Cohen emphasized that Newton failed to construct a mathematical theory on par with the physics in the *Principia*, i.e. "*The Opticks* does not proceed, in the same manner of the *Principia*, by proving its propositions by using mathematical techniques (algebra and geometry, theory of limits, infinite series, and fluxions of the calculus)."[19] In this chapter it is my aim to argue that in *The Opticks* Newton did not attain the same methodological rigour as he had done in the *Principia*.

The early I. Bernard Cohen claimed that the essential difference between *The Opticks* and the *Principia* is that the former proceeds analytically, i.e. by making experiments and observations and drawing general conclusions from them by induction, and the latter synthetically, i.e. by proceeding from the discovered causes.[20] Cohen's distinction, however, is clearly hampered by the fact that Newton himself pointed out that both the *Principia* and *The Opticks* contained an analytical as well as a synthetic part.[21] In Propositions I–VIII of Book III of the *Principia*, Newton proceeded analytically, i.e. from *phenomena* (the motions of the terrestrial bodies and the primary and secondary planets) to *theory* (universal gravitation); in the remainder of Book III, he proceeded synthetically, i.e. from *theory* to new *phenomena* (the irregular motion of the moon, the tides, the motion of comets, and the oblate form of the earth). Similarly, in *The Opticks* Newton proceeded from *phenomena* (prism-based experiments) to *theory* (the heterogeneity of white light)[22]; thereafter, he proceeded from *theory* to new *phenomena* (for instance, the rainbow).[23] In his

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[16] Ibid., pp. 317–338.

[17] CUL Add. Ma. 3970, f. 244^{v}, cf. f. 286^{r} [ca. 1700–1704].

[18] Ibid., f. 242^{r} [ca. 1700–1704]. Newton was not simply "propounding Hypotheses" but offering "Quaeres to be examined by experiments" (Newton, "Manuscript in Miracles", Lehigh University Libraries, Bethlehem (Pennsylvania), f. 1^{v}; quoted from: Dobbs, *The Janus Faces of Genius*, p. 230). Cf. [Newton], "An Account of the Book entitled *Commercium Epistolium*", p. 222. On the evolution of "queries" in Newton's thought, see Anstey, "The Methodological Origins of Newton's Queries".

[19] Cohen, "The Case of the Missing Author", pp. 22–23.

[20] Cohen, *Franklin and Newton*, p. 192.

[21] See Guerlac, *Essays and Papers in the History of Modern Science*, pp. 212–215 for related objections against Cohen's assessment.

[22] Newton, *The Opticks*, Book I, Part I, Proposition I, pp. 20–72.

[23] Ibid., p. 405.

The Newtonian Revolution, Cohen suggested that *The Opticks* is different from the *Principia* since the former does not exhibit the "Newtonian Style:"

> More significantly, the propositions [i.e., the propositions in *The Opticks*] are not proved by the application of mathematical techniques. Rather Newton must often proceed by giving 'Proof by Experiment', and he tends to refer back to previous experiments rather than to the preliminary axioms. Hence, although Newton uses numbers (as in the results of experiments), his *Opticks* can in no legitimate sense be considered a mathematical treatise. Another way of stating this conclusion is that in the *Opticks* Newton does not proceed by using what I have been calling the Newtonian Style.[24]

In one of his last papers, I. Bernard Cohen reiterated this point and argued that in *The Opticks* Newton had failed to establish an optics based on mathematical principles: "the omission of the author's name, like the choice of English rather than Latin as the language of the text, would seem to be a kind of admission by Newton of the imperfect or incomplete nature of the *Opticks* and was to some degree an echo of the failure to produce a mathematical treatise or at least on optics based on mathematical principles."[25]

Alan E. Shapiro has argued that Newton approached optics by means of the *phenomenological* "Newtonian Style".[26] According to Shapiro, in optics, just as in mechanics, Newton restricted himself to experimentally observed properties without any reference to causal explanations.[27] Although Shapiro's superb research has undoubtedly provided profound insight into Newton's optics, he supposes that the "Newtonian Style" is essentially non-causal, a view that, as we have seen in the preceding chapters, does not correspond to Newton's causal realism nor to Cohen's own exposition of the "Newtonian Style". Similarly, Caspar Hakfoort has argued that Newton attempted to proceed in *The Opticks* in the same way as he did in the *Principia*, i.e. by his descriptive and anti-causal "Newtonian style",[28] but that this style was less effective in optics.[29]

As I shall argue in what follows, *the difference between* The Opticks *and the* Principia *does not lie, respectively, in the absence or presence of causal explanations, but in the different ways in which theoretical and causal statements were established*. In the preceding chapters, I have highlighted how Newton's *Principia*-style methodology was more rich and complex than hypothetico-deductivism. Newton was especially occupied with the question of how to treat and define hypotheses in natural philosophy. Essential to the *Principia*'s methodology was a careful delineation between "deductions from phenomena," i.e. explanations which are based on systematic dependencies between cause and effect, and hypotheses,

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[24] Cohen, *The Newtonian Revolution*, pp. 134–135, cf. pp. 136, 141.

[25] Cohen, "The Case of the Missing Author", p. 41.

[26] Shapiro, *Fits, Passions, and Paroxysms*, pp. 22–23.

[27] Cf. "While he [= Newton] considered causal explanations to be desirable, they never play an essential or necessary role in his science." (Shapiro, "Newton's Optics and Atomism", p. 228).

[28] Hakfoort, "Newton's *Opticks* and the Incomplete Revolution", p. 103.

[29] Ibid., p. 109.

i.e. explanations which are not derived from phenomena. It was, however, in the context of his early optical work in the early 1670s that Newton had launched his anti-hypothetical programme of methodological reform. Therefore, in this chapter I additionally seek to illuminate Newton's lifelong interest in separating demonstrations from hypotheses. As I shall argue in what follows, although in his early optical work Newton had clearly formulated an ideal of establishing demonstrative causes, a clear formulation of the method by which to arrive at "deductions from phenomena" was still lacking.[30] It was only by the time of the *Principia* that Newton could answer this issue in a sufficiently detailed way. Shortly after the publication of the first edition of the *Principia* (1687), Newton returned to his optical work in 1687–1688. Accordingly, the question which will be addressed in this chapter is the following: *could Newton successfully methodize his optical work according to the methodological desiderata he had established in the* Principia? As in the *Principia*, in *The Opticks* Newton was highly concerned with providing demonstrations and eliminating hypotheses. As I shall argue in what follows, Newton was quite unsuccessful in banning hypothetical explanations from *The Opticks*.

4.3 The Corporality of Light as a Hypothesis

I will first turn to Newton's alleged defence of the corpuscular nature of light. From early on, Newton began to explore the potential of a corpuscular account of light.[31] Although his corpuscular stance with regard to the nature of light would remain vital to his optical research at the level of heuristics,[32] he never considered it as a demonstrated principle.[33] It should be noted that the corpuscular underpinnings of Newton's optics were far from being static, for later on Newton would modify his corpuscular account of light and incorporate additional *explanantes*. In the early 1670s, Newton began rejecting a strictly corpuscular explanation of light by introducing a subtle ether upon which *globuli* impinged.[34] Although Newton was

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[30] See Section 4.6 in this chapter.

[31] Shapiro, "Newton's Optics and Atomism" and Westfall, *Never at Rest*, pp. 159–163. For Newton's early corpuscular model see: Bechler, "Newton's Search for a Mechanistic Model of Colour Dispersion" and id., "Newton's Laws of Forces which are Inversely as Mass".

[32] E.g., McGuire and Tamny, eds., *Certain Philosophical Questions*, p. 585. On CUL Add. Ms. 3970, f. 289^r, Newton wrote: "I have therefore proposed the Question whether the rays of light may not be small bodies emitted by shining substances" (cf. ibid., f. 299^r). He added furthermore that the resemblance between rays and bodies is "very great:" "For ~~such~~ ↓such↓ bodies will pass through uniform Mediums in right lines without bending into the shadow w^{ch} is the property of the rays of light." (ibid., f. 289^r).

[33] Shapiro, *Fits, Passions, and Paroxysms*, pp. 12–40; Ihmig, "Newton's Program of Mathematizing Nature". In Query 31 Newton wrote: "Even the Rays of Light seem to be hard Bodies; for otherwise they would not retain different Properties in their different Sides." (Newton, *The Opticks*, p. 389).

[34] Shapiro, *Fits, Passions, and Paroxysms*, pp. 77–78. In the same period Newton came to accept non-mechanical "vegetable spirits" (Dobbs, "Newton's Alchemy and His Theory of Matter", p. 515).

clearly in favour of the corporality of light, he was fully aware that its veracity could not be derived from phenomena and, correspondingly, he never asserted it in the demonstrative part of optics.

It has been claimed that in Definition I of *The Opticks* Newton committed himself to an emission theory of light.[35] Note that this would have been a very strange move: for, on several occasions, Newton stressed that his optical theory was neutral with respect to an emission theory. In his second optical paper, for instance, Newton wrote in response to Hooke, who had accused him of introducing a corpuscular view into optics in his first optical paper: "*The hypothesis of light's being a body, had I propounded it, has much greater affinity with the objector's own hypothesis, than he seems to be aware of; the vibrations of the æther being as useful and necessary in this as in his.* [...] *I shall leave it to their consideration, who may think it worth their endeavour to apply this hypothesis to the solution of phænomena.*"[36] Also, when he was drafting up the first edition of *The Opticks*, Newton made it clear that both emission and wave theories were compatible with the rectilinear propagation of light: "~~whether y^e^ rays of light be small bodies or only motion or pression propagated from shining substances, they move in reight lines~~."[37] In the *scholium* to Proposition XCVI in Book I of the *Principia*, Newton noted that he was "not arguing at all about the nature of the rays (that is, whether they are bodies or not)."[38]

Newton's Definition I goes as follows:

> *By the Rays of Light I understand its least Parts, and those as well Successive in the same Lines, as contemporary in several Lines*. For it is manifest that Light consists of Parts, both Successive and Contemporary; because in the same place you may stop that which comes one moment, and let pass that which comes presently after; and in the same time you may stop it in anyone place, and let it pass in any other. For that part of Light which is stopp'd cannot be the same with that which is let pass. The least Light or part of Light, which may be stopp'd alone without the rest of the Light, or propagated alone, or do or suffer any thing alone, which the rest of the Light doth not or suffers not, I call a Ray of Light.[39]

Here Newton stated that light consists of discrete *parts*, *not* particles.[40] Alan E. Shapiro has shown that, while Definition I is free from the assumption of the corporality of light, it is incompatible with diffusion theories of light, which typically posit that individual rays are not physically independent. As is known, Hooke defended

[35] For instance, Abdelhamid I. Sabra has claimed that Newton's rays were always those of the corpuscular theory and that a wave interpretation was denied a priori (Sabra, *Theories of Light from Descartes to Newton*, p. 288, cf. p. 284).

[36] Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, p. 179; Newton, *Correspondence*, I, p. 363. Cf. Westfall, "The Development of Newton's Theory of Color", pp. 352–353.

[37] CUL Add. Ms. 3970, f. 296^{r} [ca. 1700–1704; italics added].

[38] Newton, *The Principia*, p. 626 [italics added].

[39] Newton, *The Opticks*, pp. 1–2.

[40] Laymon, "Newton's Experimentum Crucis and the Logic of Idealization and Theory Refutation", p. 61.

a modificationist theory of light and colours and stated that "the motion of Light in an uniform medium, in wch it is generated, is propagated by simple & uniform pulses or waves, which are at Right angles with the line of Direction; but falling obliquely on the Refracting medium, it Receives an other impression or motion, which disturbes the former motion."[41] According to Hooke, a ray of light is "Not a Mathematical Line, but a Physical one of some Latitude."[42] He reasoned that, when a ray interacts with a refractive surface, its pulses no longer stand at a right angle to their surrounding mathematical lines but at an oblique one. According to Hooke, such obliqueness is the very cause of the production of colours.[43] In uncoloured rays the motion is propagated by a pulse "at Right Angles with the Line of Direction."[44] An essential feature of Hooke's diffusion model of refraction is that the outcome of the refraction of a ray depends on whether or not it has a neighbouring ray with which it interacts; in other words, on whether or not rays of light interact.[45] Correspondingly, Shapiro concluded that:

> While Newton's definition is free of an hypothesis as to the nature of light, either an emission or wave theory, it is not free of an hypothesis as to the nature of white light and colors, for the definition and its applications *assume that "parts," or colors, of light are present before refractions*. Thus, just as Newton's definition has been cleared of the charge of "dogmatism" as a definition of a light corpuscle, its "dogmatism" reemerges in the assumption *that colours are present in white light before refraction*.[46]

Newton's Definition I did not presuppose a corpuscular view of light; it did imply, however, that rays are independent and physically discrete entities. Definition I is, however, compatible with wave theories.[47]

4.4 Newton's Argument for the Heterogeneity of White Light

Newton's first public enunciation of the heterogeneity of white light, i.e. the view that white light *consists* of rays differently refrangible, is to be found in his first optical paper to the Royal Society, *New Theory about Light and Colors* (19 February 1671/2).[48] In the first experiment, which sets the stage for the actual *experimentum*

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[41] Hooke to Oldenburg, 15 February 1671/2, Newton, *Correspondence*, I, p. 114; Hooke, *Micrographia*, p. 57.

[42] Hooke, *Lampas*, p. 39.

[43] Shapiro, "Kinematic Optics", p. 194.

[44] Hooke, *Lampas*, p. 39.

[45] Hooke to Oldenburg, 15 February 1671/2, Newton, *Correspondence*, I, p. 114. See Laymon, "Newton's *Experimentum Crucis*", pp. 64–65 for an opposing view.

[46] Shapiro, "The Evolving Structure of Newton's Theory of White Light and Color", p. 208 [italics added].

[47] Ibid., p. 196. Huygens' optical theory for instance was compatible with Newton's definition, for Huygens' principle is compatible with Newton's criteria of successiveness and contemporaneity.

[48] We know that in these early years Newton read Kepler's *Paralipomena* (1604), Descartes' optics (*La Dioptrique* and *Météores* (1637)), Isaac Vossius' *De lucis natura et proprietate* (1662), Robert

crucis, Newton reported on the dispersion pattern he had observed when darkening his chamber. Newton had made a small circular hole with a diameter of 0.25 inch in his window-shuts through which the sun's light entered the room and was refracted by the prism with a vertical angle of $63°12'$ degrees near the entrance of the sun's light from that hole – he also noted that without refraction the rays would reach the wall in an angle of $44°56'$ degrees (see Fig. 4.1). Newton had also made the angles of the incident and emergent rays as equal as he could, i.e. about $54°4'$.[49] The sun's light was refracted on the wall, which was located at a distance of 22 foot from the window shuts, and all the colours of the spectrum appeared in an oblong shape.[50] Studying refraction patterns at such distance was in itself significant. Descartes had previously observed dispersion patterns produced by a prism at only a couple of inches from the source of refraction, in which case the proper form of the refracted image cannot be detected.[51]

Closer examination revealed that the image on the wall was not circular – as it should have been according to the received view of refraction[52] – but in fact exhibited an oblong form with a length of 13.25 inches and a breadth of 2.625 inches. According to "the received laws of Refraction," we expect elongation, except for the position of minimal deviation.[53] Thus, when subtracting the circular surface, a

Boyle's *Considerations of Colours* (1664), Robert Hooke's *Micrographia* (1665), Francisco M. Grimaldi's *Physico-mathesis de lumine, coloribus, et iride* (1665) and Isaac Barrow's *Lectiones LVIII* and his *Lectiones Geometricae*. On 7 July 1670, Barrow sent Newton the published copies of his *Lectiones XVIII* and his *Lectiones Geometricae*, which Newton had previously proofread – they were bound together as one copy (Wren Library, NQ.16.181). The former contains signs of dog-earing on: pp. 24, 32, 40, 70, 72, 78, 80, 83, 88, 94, 96, 102, 104, 110, 112, 118–120 and one correction on p. 25. The latter contains signs of dog-earing on: [p. i], pp. 6, 8, 38, 40, 46, 48, 50, 51, 54, 56, 89, 91–92. Newton did not read Marcus Marci's *Thaumantias, Liber de arcu cœlesti deque colorum apparentium natura, ortu, & causis* (1648) which was not available to him. For a recent and interesting piece on Marci, who came close to endorsing a theory similar to Newton's, see: Garber, "Chymical Wonders of Light: J. Marcus Marci's Seventeenth-century Bohemian Optics". In his *Physiologia Epicuro-Gassendo-Charletoniana or A Fabrick of Science Natural, Upon the Hypothesis of Atoms*, which Newton had read, Walter Charleton defended a corpuscular account of light in the chapter entitled "The Nature of Light."

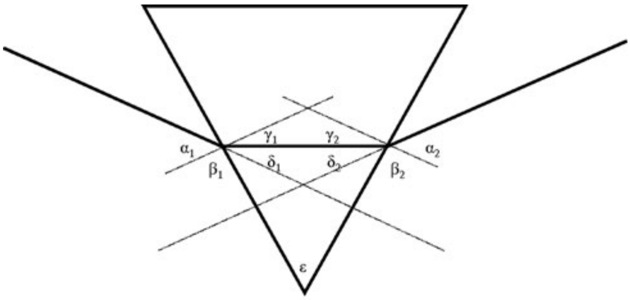
[49] Newton to Oldenburg, 6 February 1671/2, Newton, *Correspondence*, I, p. 93; Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, p. 49; cf. Newton, *The Opticks*, p. 28.

[50] On an interesting note, see Topper, "Newton on the Number of Colours in the Spectrum".

[51] E.g., Westfall, "The Development of Newton's Theory of Color", p. 341; Shapiro, "Kinematic Optics", p. 190. Shapiro notes that: "Descartes's refraction model, which was based solely on mechanical parameters – the velocities of the light corpuscles and "impulses" normal to the refracting surface – made a profound and enduring impression on Newton, both as a persuasive example of how all nature might be reduced to similar mechanical principles and also in its own right as a mathematical model of a general optical law, which Newton quickly incorporated into his own investigations of refraction and dispersion." (Shapiro, ed., *The Optical Papers of Isaac Newton*, I, pp. 7–8; Shapiro, *Fits, Passions, and Paroxysms*, p. 8). On Descartes' optical research, see Buchwald, "Descartes' Experimental Journey Past the Prism and through the Invisible World".

[52] Newton to Oldenburg, 6 February 1671/2, Newton, *Correspondence*, I, p. 92; Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, p. 48.

[53] Newton, *The Opticks*, p. 31.



$$\alpha_1 = \alpha_2 = 54°04';$$
$$\beta_1 = \beta_2 = 45°04';$$
$$\gamma_1 = \gamma_2 = 31°36';$$
$$\delta_1 = \delta_2 = 22°28';$$
$$\varepsilon = 62°12'$$

Fig. 4.1 The position of minimal deviation (not to scale)

remaining 13 inches of length and 2.375 inches of breadth were left unaccounted for by the received view of light. If one assumes that all rays are equally refrangible, then in the position of minimal deviation,[54] in which the angles of incidence and refraction are equal, i.e. at 54°4′ degrees, the refracted image must be geometrically similar to the shape of the source (see again Fig. 4.1). According to Newton, the oblong form was produced by innumerable and overlapping circular images of rays differently refrangible.[55] It should be noted, however, that the image which Newton described was certainly an idealized account, for the axial intensity of the spectrum is not constant thereby leaving the ends of the observed oblong appearance diffused rather than regular.[56]

Next, Newton eliminated several alternative explanations for the observed phenomenon.[57] He showed, for instance, that the prismatic effect could not be produced

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[54] In his *Lectiones Opticae*, which were inaccessible outside Cambridge at the time, Newton made this point more explicit, and offered a geometrical demonstration of equality of shape between the optical source and the refracted image (Shapiro, ed., *The Optical Papers of Isaac Newton*, I, pp. 53–61, pp. 285–293), which was accompanied by an illustration (ibid., I, p. 52 and 286 [for the illustration]). In the fourth and posthumous edition of *The Opticks* references to Newton's *Lectiones opticae* were added. In *The Opticks* Newton did not emphasize the certainty of the outcome of the *experimentum crucis* as he had done in his first optical paper (Shapiro, "The Evolving Structure of Newton's Theory of White Light and Color", pp. 216–217).

[55] Shapiro, ed., *The Optical Papers of Isaac Newton*, I, pp. 62–65; Newton, *The Opticks*, Proposition II, Book I, Part I, pp. 38–40.

[56] Lohne, "Newton's *Experimentum Crucis*", pp. 172–173; Kuhn, "Newton's Optical Papers", pp. 34–35.

[57] See, furthermore, Raftopoulos, "Newton's Experimental Proofs as Eliminative Reasoning".

by an irregularity in the prism itself. Newton took "another Prisme like the former, and so placed it, that the light, passing through them both, might be refracted contrary ways, and so by the latter returned into that course, from which the former had diverted it" to establish whether the irregular effect would be augmented "by the multiplicity of refractions."[58] However, it was shown that the second prism inversed the effect of the first prism resulting in "an *orbicular* one with much regularity."[59] Also, he showed that, when turning the prism at an angle of 4 or 5 degrees, the colours did not sensibly change their position on the wall which thus indicated that "the difference of the Incidence of Rays, flowing from divers parts from the Sun, could not make them after decussation diverge at a sensibly greater angle."[60] Newton also established that the length of the image and the diameter of the hole were proportional to the distance – which indicated that the trajectories of the rays after refraction were straight and not curved.[61]

In the actual *experimentum crucis*[62] (see Fig. 4.2)[63], Newton took two additional boards and one additional prism. He placed the first board *DE* behind the prism at the window-shuts, so that the refracted light might pass through a small hole. At a distance of 12 feet he installed a second board with a circular whole through which the light could pass a small hole and reach the prism placed behind it. Prism

58 Newton to Oldenburg, 6 February 1671/2, Newton, *Correspondence*, I, p. 93; Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, p. 48; Shapiro, ed., *The Optical Papers of Isaac Newton*, I, pp. 74–75, 304–306.

59 Newton to Oldenburg, 6 February 1671/2, Newton, *Correspondence*, I, p. 93; Cohen, *Isaac Newton's Papers and Letters on Natural Philosophy*, p. 48; cf. Shapiro, ed., *The Optical Papers of Isaac Newton*, I, pp. 74–75.

60 Newton to Oldenburg, 6 February 1671/2, Newton, *Correspondence*, I, p. 93; Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, pp. 49–50.

61 Newton to Oldenburg, 6 February 1671/2, Newton, *Correspondence*, I, p. 94; Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, p. 50.

62 See, e.g., Westfall, *Never at Rest*, pp. 213–214; Newton, *The Opticks*, pp. 46–48; Shapiro, "The Evolving Structure of Newton's Theory of White Light and Color"; and, esp. Mills, "Newton's Prisms and his Experiments on the Spectrum". Again, it should be stressed that the experiment is idealised. As Laymon has pointed out, Newton assumed that the rays refracted by the second prism were of a single colour (Laymon, "Newton's *Experimentum Crucis"*, pp. 51–77, 53, 56; cf. Newton to Oldenburg, 6 February 1671/2, Newton, *Correspondence*, I, p. 102; Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, p. 59). This will be the case only if the ray has an equal breadth as the hole in the second board. The conclusion of the *experimentum crucis* in support of a one-to-one correspondence between colour and a specific degree of refrangibility succeeds *on the assumption* that an idealised description of the resultant image is used (ibid., pp. 69–70). The term *experimentum crucis* was first used by Boyle (Robert Boyle, *Defence against Linus* (1662), in: Boyle, *The Works of Robert Boyle*, III, p. 50), who derived it from Bacon's *instantia crucis* (Bacon, *The Works of Francis Bacon*, I, *Instauratio magna*, *Novum Organon*., II.xxxvi, p. 436). Hooke had used the term *experimentum crucis* in his *An Attempt to prove the Motion of the Earth from Observation*, p. 2 and in his *Micrographia*, p. 54.

63 Newton's first optical paper did not contain a figure of the *experimentum crucis*, which added to difficulty of understanding the configuration of the experiment. In fact, even Henry Oldenburg and Samuel Horsley gave mistaken figures to illustrate the *experimentum crucis* (Lohne, "The Increasing Corruption of Newton's Diagrams", pp. 72–73).

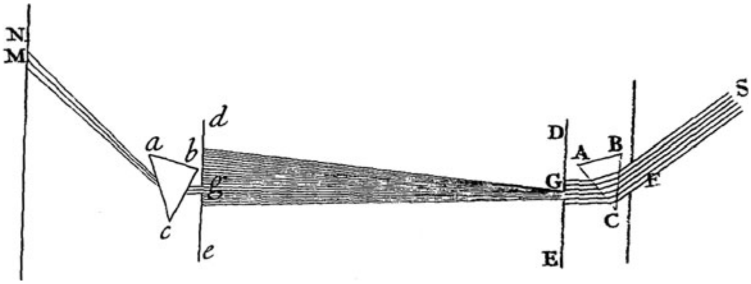

Fig. 4.2 Newton's *experimentum crucis* (Newton, *The Opticks*, p. 47). Reproduced from figure 18 among the figures inserted between OE_1, pp. 80–81

abc was kept fixed, while prism *ABC* was slowly moved about its axis. When the individual colours passed through the prism *abc*, they maintained their colour and each colour was refracted at an angle identical to the angle by which that colour had been refracted by prism *ABC*. Note that Newton based his theory of colours almost exclusively on prismatic colours. The mechanical philosophers before him (especially Descartes) had already dismissed the distinction between real and apparent, i.e. prismatic, colours.[64]

The *experimentum crucis* intended to show, first of all, that rays preserve their colour as well as their degrees of refrangibility upon being refracted through prism *abc* and that each colour had a specific degree of refrangibility. Rays emerging from prism *ABC* at the greatest angle of refraction are equally re-refracted by prism *abc*; similarly, rays emerging at the least angle of refraction are equally re-refracted by the second prism. And so on, for all intermediate rays of the spectrum. Thus, each part of the spectrum has its own inherent degree of refrangibility, Newton concluded.[65] This entails that, generalizing for different media, for each two different rays of light, when the first is refracted more than the second in one transparent medium, the first will always be refracted more in any other medium.[66] After describing the outcome of the *experimentum crucis*, Newton introduced the following explanation:

[64] Schaffer, "Glass Works: Newton's Prisms and the Uses of Experiment", p. 74; Westfall, "The Development of Newton's Theory of Color", p. 341; Shapiro, *Fits, Passions, and Paroxysms*, pp. 6–8. Newton's underlying premise is that radiant colours have a conceptual primacy over those of natural bodies (Shapiro, *Fits, Passions, and Paroxysms*, pp. 6–7).

[65] In his Trinity Notebook in the section entitled "On Colours," Newton had already pointed to the different refrangibility of light without proceeding further to the conclusion that white light *consists* of rays differently refrangible (McGuire and Tamny, eds., *Certain Philosophical Questions*, p. 468; CUL Add. Ms. 3996, f. 122^{v}). The evidence adduced there was based on observations made by looking through a prism. See the editorial introduction to Shapiro, ed., *The Optical Papers of Isaac Newton*, I, pp. 1–25 for Newton's early optical work.

[66] Worrall, "The Scope, Limits, and Distinctiveness of the Method of 'deduction from the phenomena'", pp. 56–57.

> And so the true cause of the length of that Image was detected to be no other, then that *Light* consists of *Rays differently refrangible*, which, without any respect to a difference in their incidence, were, according to their degree of refrangibility, transmitted towards divers parts of the wall.[67]

According to Newton, the *experimentum crucis*' most important implications were[68]:

1. As the Rays of light differ in degrees of Refrangibility, so they also differ in their disposition to exhibit this or that particular colour. Colours are not *Qualifications of Light*, derived from Refractions, or Reflections of natural Bodies (as 'tis generally believed,) but *Original* and *connate properties*, which in divers Rays are divers.[69] Some Rays are disposed to exhibit a red colour and no other; some a yellow and no other, some a green and no other and so for the rest. Nor are there only Rays proper and particular to the more eminent colours, but even to all their intermediate gradations.
2. To the same degree of Refrangibility ever belongs the same colour, and to the same colour ever belongs the same degree of Refrangibility. The *least Refrangible* Rays are so disposed to exhibit a *Red* colour, and contrarily those Rays, which are disposed to exhibit a *Red* colour, are all the least refrangible: So the *most refrangible* Rays are all disposed to exhibit a deep *Violet Colour*, and contrarily those which are apt to exhibit such a violet colour, are the most Refrangible. And so to all the intermediate colours in a continued series belong intermediate degrees of refrangibility. And this Analogy 'twixt colours, and refrangibility, is very precise and strict; The Rays always either exactly agreeing in both, or proportionally disagreeing in both.
3. The species of colour, and degree of Refrangibility proper to any particular sort of Rays, is not mutable by Refraction, nor by Reflection from natural bodies, nor by any other cause, that I could yet observe. When any one sort of Rays hath been well parted from those of other kinds, it hath afterwards obstinately retained its colour, notwithstanding my utmost endeavours to change it. [...] And therefore, if by refraction, or any other of the aforesaid causes, the difform Rays, latent in such a mixture, be separated, there shall emerge colours different from the colour of the composition. Which colours

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[67] Newton to Oldenburg, 6 February 1671/2, Newton, *Correspondence*, I, p. 95; Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, p. 51.

[68] Newton gave a total of 13 points (Newton to Oldenburg, 6 February 1671/2, Newton, *Correspondence*, I, pp. 97–100; Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, pp. 53–57).

[69] This sentence contains a criticism on the Aristotelian account of light according to which light is a quality (Shapiro, ed., *The Optical Papers of Isaac Newton*, I, pp. 78–85, 434–437; Cohen, "Versions of Isaac Newton's First Published Paper", pp. 364–366 and Mamiani, *Isaac Newton filosofo della natura*, pp. 39–67). Somewhat later in his paper Newton wrote: "These things being so, it can no longer be disputed, whether there be colours in the dark, nor whether they be the qualities of the objects we see, nor perhaps whether Light be a Body. For, since Colours are the *qualities* of Light, having its Rays for their intire and immediate subject, how can we think those Rays *qualities* also, unless one quality may be the subject of and sustain another; which in effect is to call it *Substance*. We should not know bodies for their substances, were it not for their sensible qualities, and the Principal of those being now found due to something else, we have good reason to believe that to be a Substance also." (Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, p. 57).

are not New generated, but only made Apparent by being parted; for if they be again mix't and blended together, they will again compose that colour, which they did before separation.[70] [. . .]
[. . .]

8. Hence therefore it comes to pass, that *Whiteness* is the usual colour of *Light*; for Light is a confused aggregate of Rays indued with all sorts of Colors, as they are promiscuously darted from the various parts of luminous bodies. And of such a confused aggregate, as I said, is generated Whiteness, if there be a due proportion of the Ingredients; but if any one predominate, the Light must incline to that colour; as it happens in the Blew flame of Brimstone; the yellow flame of a Candle; and the various colours of the Fixed stars.[71]

Newton also reported on several variations he had pursued: he had refracted homogeneous colours with prisms, reflected them with bodies, intercepted them with the coloured film of air interceding two compressed plates, and transmitted them through coloured mediums and also through mediums irradiated with other sorts of rays.[72] It was never the case that a new colour was produced. Afterwards Newton started enumerating some more "instances of nature" that can be explained by his new theory of light (e.g., the colours of the rainbow, the phenomena of infusion of *Lignum Nephriticum*, leaf gold, and fragments of coloured glass).[73]

[70] This sentence and the surrounding text were absent from the unedited version (6 February 1671/2, Newton, *Correspondence*, I, p. 97).

[71] Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, pp. 53–55; Newton to Oldenburg, 6 February 1671/2, Newton, *Correspondence*, I, p. 97–98.

[72] Newton to Oldenburg, 6 February 1671/2, Newton, *Correspondence*, I, p. 97; Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, p. 54. Newton's *ceteris paribus* clause was not entirely empty, contrary to what Worrall has claimed (Worrall, "The Scope, Limits, and Distinctiveness of the Method of 'deduction from the phenomena'", p. 57, footnote 7). Newton did, however, not take into account other factors such as the temperature of the air or the glass, atmospheric conditions, etc.

[73] Newton to Oldenburg, 6 February 1671/2, Newton, *Correspondence*, I, pp. 99–100; Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, pp. 55–56. This seems to imply that Newton was thinking in terms of analysis-synthesis here: once the true causes have been established, they can be applied to phenomena that were not included in the original analysis. Similarly, in his *Lectiones Opticae*, Newton stated: "Thus far I have erected the foundations whereby the phenomena of colors produced in any way can be explained, but now I will describe individually the particular and immediate causes of the effects that I have not previously treated, not for the sake of the geometers (to whom, it will appear unnecessary) but for others." (Shapiro, ed., *The Optical Papers of Isaac Newton*, I, p. 523; for Newton's explanation of the rainbow, see ibid., pp. 593–601). Newton, furthermore, noted that in the first two books of *The Opticks* he proceeded by analysis and that he had provided an instance of synthesis at the end of the first book (Newton, *The Opticks*, p. 405). On CUL Add. Ms. 3790, f. 242^{v} [ca. 1700–1704], Newton wrote: "Most of the second Book was written some years ↓before↓ ~~after~~ the ffirst & so is not in so good a method. However it proceeds by Analysis to discover the fits of easy reflexion & easy transmission of the rays, & thence ↓~~it is easy to compound~~↓ the explication of the colours of ↓bubbles & other↓ transparent plates, & ↓~~those of~~↓ feathers & tinctures ↓~~are easily compounded~~↓" (cf. ibid., f. 244^{r}, cf. 292^{v}).

In his 1671/2 paper on light and colour Newton conceived of the heterogeneity of light explicitly as a cause.[74] Newton had provided what I consider to be a *structural explanation* of the dispersion patterns he observed: white light is composed of, i.e. structured as, a heterogeneous mixture of different rays, which in their turn are refrangible at constant rates. Newton claimed that his optical theory had unravelled the structure of white light, i.e. the immediate or primary cause, but not the constitution or nature of the rays composing that mixture. In his response to Robert Hooke on 11 June 1672 Newton noted that he spoke a light "considering it abstractly [...] without determining what that Thing is."[75] Similarly, in a letter to Henry Oldenburg on 3 April 1673, Newton declared:

> But to examin how colours may be thus explained Hypothetically is besides my purpose. I never intended to show wherein consists the nature and difference of colours, but onely to show that *de facto* they are originall & immutable qualities of rays wch exhibit them, & to leave it to others to explicate by Mechanicall Hypotheses the nature of those qualities; wch I take to be no very difficult matter.[76]

As we have seen, Newton's new theory of light and colours contained basically two claims: one the one hand, it encompassed the claim that there is a one-on-one correspondence between refrangibility and colour (conclusion$_1$); on the other hand, it contained the claim that white light consists of a heterogeneous mixture of different colours (conclusion$_2$). The latter claim entails that the colours, which are originally contained in white light before refraction, become visible when separated by a prism. Prismatic colours are thus *not created* but *rendered visible by separation*.[77] The prism served as Newton's tool for decomposing light into its irreducible components.

4.5 Scrutinizing Newton's Two Conclusions

Newton could indeed understandingly claim that, given the experiments he had performed, to every colour there corresponds a specific degree of refrangibility.[78] However, this claim was based on experiments on two different refracting mediums

[74] Newton to Oldenburg, 6 February 1671/2, Newton, *Correspondence*, I, p. 97; Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, p. 51. Cf. Shapiro, ed., *The Optical Papers of Isaac Newton*, I, pp. 433, 525, 603. In *The Opticks*, immediately after his famous exposition of the methods of analysis and synthesis, Newton declared that he used the method of analysis to discover and prove the "original Differences of the Rays of Light in respect of their Refrangibility, Reflexibility, and Colour, and their alternate Fits of easy Reflection and easy Transmission, and the Properties of Bodies both opake and pellucid, on which their Reflexions and Colours depend." (Newton, *The Opticks*, p. 405; for other causal statements on this matter see, e.g., ibid., pp. 57, 113, 119, 244).

[75] Newton to Oldenburg, 11 June 1672, Newton, *Correspondence*, I, p. 174, cf. Newton's second optical paper (ibid., I, pp. 373–374).

[76] Ibid., I, p. 264.

[77] Shapiro, ed., *The Optical Papers of Isaac Newton*, I, p. 129.

[78] However it is useful to keep in mind Laymon's remark to which I referred to in footnote 62.

only: glass and water, which testifies of Newton's belief in the uniformity of nature. Nowadays, we know that there are mediums in which red colours are more refracted than violet ones (e.g., in dye fuchsine or iodine vapour), and that the refrangibility of a ray can be modified (e.g., in the Doppler-effect where it is reflected from a moving mirror). Nevertheless, given the media which Newton had studied, his claim on the one-on-one correspondence of colour and refrangibility was based on experiments – albeit on a limited range of experiments.

But what about the generalization that white light *consists* of a heterogeneous mixture of different colours? In his first optical paper, Newton did not dwell much on the specifics of how this generalization follows from the *experimentum crucis*. We do know that Newton was quite convinced of the correctness of his conclusion on the heterogeneity of white light. Much later, in a comment on Experiment 9 of Part I of Book I of *The Opticks* Newton made the accompanying supposition more explicit:

> Whence 'tis made manifest, that the Beam of Light reflected by the Base of the Prism, being augmented first by the more refrangible Rays, and afterwards by the less refrangible ones, is compounded of Rays differently refrangible. *And that all such reflected Light is of the same Nature with the Sun's Light before its Incidence on the Base of the Prism, no Man ever doubted; it being generally allowed, that Light by such Reflections suffers no Alteration in its Modifications and Properties*.[79] [. . .] So then*, the Sun's incident Light being of the same Temper and Constitution with his emergent Light, and the last being compounded of Rays differently refrangible, the first must be in like manner compounded*.[80]

Newton, who clearly believed in the immutability of light, reasoned as follows: if we do not accept that white light initially consists of rays differently refrangible, then we have to assume that a different causal process occurred at the first prism than at the second prism of the *experimentum crucis*: one for the *creation* of colours and one for the *common refraction* of colours.[81] On this assumption, the rays were created at the first prism, while at the second prism the created rays were

[79] Cf. Newton, *The Opticks*, Book I, Part II, pp. 113–191, cf. p. 244.

[80] Ibid., pp. 55–56 [italics added]; Shapiro, ed., *The Optical Papers of Isaac Newton*, I, pp. 142–142. This statement was identical in all versions of *The Opticks* (OE_1, p. 38; OE_2, p. 46; OE_3, p. 46; OE_4, p. 46). On Newton's non-modificationist account of light see furthermore Newton, *The Opticks*, Proposition II, Book I, Part II, p. 124, Proposition VII, Book I, Part II, pp. 158–161 and Book II, Part II, p. 244.

[81] Cf. Worrall, "The Scope, Limits, and Distinctiveness of the Method of 'deduction from the phenomena'", p. 59. According to modern wave optics, which is based on Fourier analysis, it makes no sense to claim that white light initially contains homogeneous colours, since it consists of *completely random* phase and frequency variations together with restricted randomness in amplitude. From the perspective of wave optics, what happens during the *experimentum crucis* is the following. The first prism contains oscillators which form sets with specific eigenfrequencies. These are stimulated by the passing white light so that each kind of oscillator emits radiation of its own eigenfrequency. The latter interferes with the stimulating radiation, which after sequential such shifts by successive oscillators produce a slowing down of the velocity in the prism, and, hence, refraction. The second prism is then struck by the specific frequency of the radiation generated by one such oscillator set in the first prism and the effect repeats. Therefore, according to wave optics,

refracted. If we suppose, on the other hand, that white light consists of rays differently refrangible, then the effects in both prisms can be explained by the same causal process, that is both are instances of common refraction.[82] It seems therefore that causal parsimony and belief in the uniformity of light are what motivated Newton's endorsement of the view that white light consists of a heterogeneous mixture of colours. According to Newton, colours are never created but only separated.[83] Unfortunately, the problem is that observing coloured rays in white light is empirically impossible, for coloured rays become visible only after refraction. My point is that *Newton's claim that white light is a heterogeneous mixture of colours therefore utterly depends on this specific argument from uniformity*.

Earlier on, Newton had attempted to experimentally demonstrate the heterogeneity of white light, but ultimately he gave up on these attempts. In the *Lectiones opticae* Newton tried to experimentally establish the innateness of white light by total reflection, an argument which was later dropped from *The Opticks*.[84] Newton also gave up his attempt to derive the heterogeneity of light from the independence of the rays.[85] In the *Fundamentum opticae* Newton offered several other arguments for establishing the heterogeneity of light, but these were also abandoned in *The Opticks*.[86] It is worth emphasizing that the common reason of their failure was the problematic transference of properties found in colours *after* refraction to the structure of light *before* refraction. Shapiro is entirely correct to point out that "[i]nnateness was not dropped from the *Opticks*, just the attempt to prove it."[87]

At this point, one may point out that Newton might have relied on Rules I and II of the *regulae philosophandi*.[88] However, given my interpretation of these rules in the previous chapter, this will not do. As we have seen, Rules I and II license the identification of instances of causes of the same kind which have been shown

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the causal process is exactly the same, but the object on which they operate is different in the two cases. I am indebted to Jed Z. Buchwald for illuminating discussion on this matter.

[82] In manuscript material Newton also entertained such line of reasoning in his discussion of the double refraction of island spar (CUL Add. Ms. 3970, f. 298^{r}).

[83] Hooke claimed that his pulse hypothesis could equally account for the experimental results without requiring the heterogeneity of white light (Sabra, *Theories of Light from Descartes to Newton*, pp. 233–234; Hooke to Oldenburg, 15 February 1671/2, Newton *Correspondence*, I, pp. 110–116). Ideas similar to Hooke's were later reintroduced in the nineteenth century by G. L. Gouy who demonstrated that white light can be represented as the superposition of an infinite number of waves (Sabra, *Theories of Light from Descartes to Newton*, pp. 280–281).

[84] Shapiro, ed., *The Optical Papers of Isaac Newton*, I, pp. 128–145; Shapiro, "The Evolving Structure of Newton's Theory of White Light and Color", pp. 231–234.

[85] Newton first introduced this particular argument in Proposition 6 of his reply to the criticisms raised by Huygens, which stated that the "rays of light do not act upon one another in passing through the same Medium" (Newton to Oldenburg, 23 June 1673, Newton, *Correspondence*, I, p. 293).

[86] Shapiro, "The Evolving Structure of Newton's Theory of White Light and Color", pp. 230–232.

[87] Ibid., p. 232; Newton, *The Opticks*, pp. 153–154.

[88] This is, nevertheless, what Shapiro has suggested (Shapiro, *Fits, Passions, and Paroxysms*, pp. 47–48, cf. p. 118).

to be necessary and sufficient to explain phenomena of the same kind. In other words, in the context of the *Principia*, this implied that, when we have deduced from phenomena that the motions of, for instance, the circumsaternian planets and the motion of the moon are produced by an inverse-square centripetal force, we may assume that these phenomena are produced by the same force: gravity. On this reading, Rules I and II assert that causes shown to be necessary and sufficient of their effects, *and only such causes*, are to be kept minimal and that hypothetical explanations ought to be rejected. Thus, on the basis of Rules I and II we identify *two instances of causes of the same kind, which have been separately derived from phenomena*. *The disanalogy involved is thus that in the* experimentum crucis *we use an argument of uniformity to establish a (single) causal claim, while Rules I and II licence the identification of similar causal parameters which were independently deduced from phenomena by means of the systematic dependencies in Book I of the* Principia*.*

What I see Newton doing in his comments on Experiment 9 of Part I of Book I of *The Opticks* is showing – at least on an implicit level – that the innateness of white light cannot be directly deduced from phenomena[89] and that the strongest argument adducible for its establishment is the aforementioned argument of causal parsimony. As I have also argued, this argument could not be licensed by *regulae philosophandi* I and II. By implication, since the heterogeneity of light cannot be derived from phenomena, it qualifies as being a hypothesis according to Newton's criteria. Ultimately, Newton had only provided a sufficient cause for prismatic dispersion, not a necessary and sufficient one.

4.6 Early Newton's Demonstrative Rhetoric

During the controversy following the publication of his first optical paper,[90] Newton defended his theory of white light as a theory derived from phenomena. In fact, in a section in the unedited first optical paper, which was deleted by Oldenburg, Newton wrote:

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[89] Cf. Shapiro, "The Evolving Structure of Newton's Theory of White Light and Color", p. 216.

[90] The criticism to which Newton was exposed must have really annoyed him and it is important to understand Newton's reluctance to go in print (see especially Westfall, "Newton and his Critics on the Nature of Colors", pp. 47–58). In a letter to Oldenburg on 6 July 1672 Newton noted that he wished that "all objections were suspended, taken from Hypotheses" – a remark that vaguely foreshadowed his fourth *regula philosophandi* (Newton, *Correspondence*, I, p. 210). In a letter to Henry Oldenburg on 8 March 1672/3, he wrote as follows: "Sr I desire that you will procure that I may be put out from being any longer fellow of ye R. Society. For though I honour that body, yet since I see I shall neither profit from them nor (by reason of this distance) can partake of the advantage of their Assemblies, I desire to withdraw." (ibid., I, p. 262–263). In *Optica* Newton had declared: "I seem to have lingered too long on these matters, and consequently I have now decided to turn to the more abstract parts of mathematics" (Shapiro, ed., *The Optical Papers of Isaac Newton*, I, p. 603). In a letter to Oldenburg on 5 December 1674, Newton pointed out that: "I am sorry you put yourself to ye trouble of transcribing Fr. Linus's conjecture, since (besides yt it needs no answer)

> A naturalist would scearce expect to see ye science of those [i.e., of colours] become mathematicall, & yet I dare affirm that there is as much certainty in it as in any other part of Opticks. For what I shall tell concerning them is not a Hypothesis but most rigid consequence, not conjectured by barely inferring 'tis thus because not otherwise or because

I have long since determined to concern myself no further about ye promotion of Philosophy" (Newton, Correspondence, I, p. 328; cf. Newton to Oldenburg, 13 November 1675, ibid., I, p. 358). On 18 August 1676 in a response to a letter from Anthony Lucas containing several "experimentall exceptions" to Newton's theory of light (sent on 17 May 1676), Newton put an end to all criticism: "seeing that I am well assured of ye truth & exactness of my own observation I shall be unwilling to be diverted by any other experiments from having a fair end made of this in ye first place" (ibid., II, p. 81; see, furthermore, Westfall, "Newton Defends his first Publication: The Newton Lucas Correspondence"). On 13 October 1676 Newton received another letter from Lucas (Newton, *Correspondence*, II, pp. 104–108). Newton wrote back to Oldenburg on 18 November that year as follows: "I promised you an answer to Mr Lucas this next Tuesday but I find I shall scarce finish what I have designed, so as to get a copy taken of it by that time, & therefore I beg your patience a week longer. I see I have made my self a slave to Philosophy, but if I get free of Mr Linus's buissiness I will resolutely bid adew to it eternally, excepting what I do for my privat satisfaction or leave to come after me. For I see a man must either resolve to put out nothing new or to become a slave to defend it." (ibid., II, pp. 182–183). On 28 November of that year, Newton responded that he "will not run into any other dispute till I see a full end of what relates to Mr Linus" (ibid., II, p. 183). When Lucas kept on sending further criticisms to Oldenburg, one to Hooke for Newton, and yet another one to Newton himself, Newton responded to Hooke that he wanted to rid himself from "this frivolous dispute & stop their clamouring against Oldenburg" and that he had answered Lucas' letters sufficiently (ibid., II, p. 253). On 5 March 1677/8 Newton finally replied in two letters to Lucas. In the second of these letters, Newton wrote: "I forbeare to explain these things further for I do not think this a fit Subject to dispute about, & therefore have given these hints only in a private Letter" (ibid., II, p. 263).

Prima facie one might wonder why Newton ever bothered to publish his second optical paper. In his second optical paper "An Hypothesis explaining the Properties of Light, discoursed in my several Papers" (1675), however, Newton made it clear that his findings were hypotheses so that he did not have to enter a new battle to defend his views against vehement criticism (Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, pp. 177–235; Newton to Oldenburg, 7 December 1675, Newton, *Correspondence*, I, pp. 362–389). In the opening section of the second optical paper, in which Newton introduced the hypothesis that "agitated parts of bodies according to their severall sizes, figure and motions, doe excite Vibrations in the Æther of various depths and bignesses", he asserted: "Sir, I had formerly purposed never to write any hypothesis of light and colours, fearing it might be a means to engage me in vain disputes: but I hope a declared resolution to answer nothing, that looks like a controversy, unless possibly at my own time upon some by-occasion, may defend me from that fear." (Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, p. 178). He emphasized that: "And though I shall not assume either this of any other hypothesis, not thinking it necessary to concern myself, whether the properties of light, discovered by me, be explained by this, of Mr. HOOKE's [which consisted in nothing more than in changing "DES CARTES's pressing or progressive motion of the medium to a vibrating one" (ibid., p. 209; cf. Shapiro, ed., *The Optical Papers of Isaac Newton*, I, p. 161)], or any other hypothesis compatible of explaining them; *yet while I am describing this, I shall sometimes, to avoid circumlocution, and to represent it more conveniently, speak of it, as if I assumed it, and propounded it to be believed*. This I thought fit to express, that no man may confound this with my other discourses, *or measure the certainty of one by the other*, or think me obliged to answer objections against this script: for I desire to decline being involved in such troublesome and insignificant disputes." (Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, p. 179 [italics added]; Newton, *Correspondence*, I, pp. 363–364).

> it satisfies all phænomena (the Philosophers universall Topick,) but evinced by ye mediation of experiments concluding directly & without any suspicion of doubt.[91] To continue the historical narration of these experiments would make a discourse too tedious & confused, & therefore I shall rather lay down the *Doctrine* first, and then, for its examination, give you an instance or two of the *Experiments*, as a specimen of the rest.[92]

Note, first of all, that Newton here rejected empirical adequacy in itself as a valid criterion of theory acceptance. The above quotation clearly contained a sneer at the natural philosophy as pursued by of Robert Boyle and Robert Hooke.[93] While Boyle was quite critical of establishing true causes, Hooke thought that true causes can be established eventually by systematically exploring and testing hypotheses (including potentially false ones). However, both agreed that establishing an excellent hypothesis (Boyle) or the true cause (Hooke) was the outcome of a long process of hypotheses testing and a large stock of natural and artificial histories. And there came Newton, who endeavoured to establish, on the basis of a few experiments, the true cause of refraction in a single paper!

In reply to a second letter to Ignace Gaston Pardies on 13 April 1672 Newton explained:

> [T]he best and safest method of philosophizing [[o]ptimus enim tutissimus philosophandi modus] seems to be, first to inquire diligently onto the properties of things, and establishing [stabiliamus] those properties by experiments and then to proceed more [contendamus] slowly to hypotheses for the explanation [pro earum explicatione] of them. For hypotheses should be subservient only in explaining the properties of things, but not assumed in determining them [Nam *Hypotheses* ad explicandas rerum proprietaties tantùm accommodari debent & non ad determinandas usurpari]; unless so far as they may furnish [subministrare] experiments. For if the possibility of hypotheses is to be to test the truth and reality of things, I see not how certainty can be obtained in any science; since numerous hypotheses may be devised [excogitare], which shall seem to overcome [suppeditare videbuntur] new difficulties.[94]

Similarly, in a letter to Henry Oldenburg on 6 July 1672 Newton wrote:

> You know the proper Method for inquiring after the properties of things is to deduce them from Experiments. And I told you that the Theory w[ch] I propounded was evinced to me, not by inferring tis thus because not otherwise, that is not by deducing it onely from a confutation of contrary suppositions, but by deriving it from Experiments concluding positively & directly.[95]

[91] In a letter to Oldenburg on 6 June 1672, Newton wrote *concluding positively & directly* (Newton, *Correspondence*, I, p. 209).

[92] Newton to Oldenburg, 6 February 1671/2, Newton, *Correspondence*, I, pp. 96–97 [underscore added]; cf. Newton to Oldenburg, 11 June 1672, Newton, *Correspondence*, I, pp. 187–188.

[93] See, furthermore, Section 4.8 in this chapter.

[94] Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, pp. 106, 109; Newton, *Correspondence*, I, p. 164.

[95] CUL Add. Ms. 9597.2.8.1.19, [f. 1[r]]; cf. Newton, *Correspondence*, I, p. 209. In correspondence Newton tempered his position: "And the absolute certainty of a Science cannot exceed the certainty of its Principles. Now the evidence by wch I asserted the Propositions of colours is in the

The above statements are characteristic of Newton's programme of methodizing optics which he had also announced in the *Lectiones opticae*. In the *Lectiones opticae* Newton launched a Barrovian program which set out to inject natural philosophy with the certainty of mathematics. I concur with Niccoló Guicciardini that Newton's "rather extremist methodological position" was at that point still lacking elaboration and justification.[96] At that time, Newton had clearly formulated an ideal of establishing demonstrative causes.[97] What was lacking, however, was a clear formulation of the method by which to arrive at such causes.[98] It was only by the time of the *Principia* that Newton could answer this very question in a detailed way. In his optical lectures, Newton stated that, by mathematizing optics, optics could to a significant extent partake in the certainty provided by mathematics[99]:

next words expressed to be from *Experiments* & so but *Physicall*: Whence the Propositions themselves can be esteemed no more than *Physicall Principles* of a Science. And if those Principles be such that on them a Mathematician may determin all Phænomena of colours that can de caused by refraction, & that by computing or demonstrating after what manner & how much those refractions doe separate or mingle the rays in wch severall colours are originally inherent; I suppose the *Science of Colours* will be granted *Mathematicall* & as certain as any part of *Optiques*. And that this may be done I have good reason to beleive because ever since I became first acquainted with these Principles, I have with constant successe in the events made use of them for this purpose." (Newton to Oldenburg, 11 June 1672, Newton, *Correspondence*, I, p. 187). See, furthermore, the conclusions reached in Zemplén and Demeter, "Being Charitable to Scientific Controversies: On the Demonstrativity of Newton's *Experimentum Crucis"* and Shapiro, *Fits, Passions, and Paroxysms*, pp. 37–38.

[96] Guicciardini, *Isaac Newton on Mathematical Certainty and Method*, p. 21.

[97] Because of his insistance on causes, Newton was seeking to innovate the mixed sciences (Dear, *Discipline and Experience*, p. 235).

[98] Note that in Newton's early optical work a general characterization of hypotheses or proper demonstrations was still absent.

[99] See: Guicciardini, *Newton on Mathematical Certainty and Method*, esp. Chapter 2 and Mamiani, *Isaac Newton filosofo della natura*, pp. 34, 46, 65–67. It appears that in his early optical work, Newton was somewhat overconfident that nature would easily yield to his methodological ideal of explaining refraction by relying on a priori derived refractive indices and some basics mathematical rules alone. The following statement is typical in that respect: "Although I have not yet derived the certainty of this proposition [Newton's a priori dispersion law] from experiments, nevertheless I do not doubt that it will satisfy all of them which it is possible to do with that respect to it." (Shapiro, ed., *Isaac Newton's Optical Papers*, I, p. 201). However, as Newton would learn early on, optical phenomena would not easily lend them to the deductive ideal he envisioned. In an unpublished manuscript letter (U.L.C. Add. 3970, f. 443^{r}–444^{r}), Newton tried to make a deductive model of refraction (Bechler, "'A Less Agreeable Matter:' The Disagreeable Case of Newton and Achromatic Refraction"). The experiment consisted in letting an uncoloured ray pass through a prismatic box ABC made of polished plates of glass cemented together at the edges, which is filled with water. In the box another prism DEF made of glass or crystal is placed upside down – so that the vertex of DEF points to the base of ABC. The bases of ABC and DEF are parallel to each other. The only relevant data were the refractive indices and dispersive powers of glass, water and air. Newton recorded that "for determining their refractions made in their passage out of any one into any other of these three medium glasse water & Air I made use of those proportions of the sines w^{ch} I have already mentioned" (CUL Add. Ms. 3970, f. 443^{r-v}). Once these are known, the rest follows without further experimentation. The model predicted that, given equal contrary refractive indexes, colours would appear (ibid., f. 443^{r}) (and that, given that the refractive index

> Thus although colors may belong to physics, the science of them must nevertheless be considered mathematical, insofar as they are treated by mathematical reasoning. Indeed, since an exact science of them seems to be one of the most difficult that philosophy is in need of, I hope to show – as it were, by my example – how valuable mathematics is in natural philosophy. I therefore urge geometers to investigate nature more rigorously, and those devoted to natural science to learn geometry first. Hence the former shall not entirely spend their time in speculations of no value to human life, nor shall the latter, while working assiduously with an absurd method [praeposterâ methodo], perpetually fail to reach their goal. But truly with the help of philosophical geometers and geometrical philosophers, instead of the conjectures and probabilities that are being blazoned about everywhere [pro conjecturis et probabilibus quae venditantur ubique], we shall finally achieve a natural science supported by the greatest evidence [scientiam Naturæ summis tandem evidentijs firmatam nanciscamur].[100]

As can be seen from the quotations just consulted, Newton, on the one hand, claimed that he had established the heterogeneity of white light "by deriving it from Experiments concluding positively and conclusively" and that is was a "most rigid consequence" established "without any suspicion of doubt;" on the other hand, he stated that the evidence in support of the heterogeneity of white light was dependent on "*Experiments* & so but *Physicall*."[101] This tension is, in my opinion, characteristic of Newton's early and still underdeveloped methodological thought.

In view of several criticisms on his first optical paper, Newton never denounced his thesis on the heterogeneity of light. In response to Huygens' criticism, Newton did, however, restrict the *universality* of the theory. Whereas in the original paper he had made claims on the innateness of white light in general, in a letter to Oldenburg on 23 June 1673 he now restricted his theory to the innateness of the sun's light.[102]

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of the interior prism is less than that of the exterior one, no colours will appear (ibid., f. 443^{v})). In *The Opticks* we encounter the same experiment (Experiment 8, Part II, Book I) with only one significant difference: the conclusion was the exact opposite to Newton's earlier model (Newton, *The Opticks*, pp. 129–130)! Newton now claimed that, given equal contrary refractive indexes, light continues "ever after to be white" (ibid., p. 129). For this reason, Bechler has concluded that "it might well have been wholly thought-experiment" (Bechler, "'A Less Agreeable Matter:' The Disagreeable Case of Newton and Achromatic Refraction", p. 114). On the basis of CUL Add. Ms. 3970, ff. 411^{r}–412^{r}, Alan E. Shapiro has tempered Bechler's claim that Newton never performed this experiment (Shapiro, "Newton's 'Achromatic' Dispersion Law: Theoretical Background and Experimental Evidence", pp. 113–114, p. 97). However, he grants that the outcome of Newton's experiment was contrary to the later published version, that the refractive indexes Newton used "were not experimentally derived, but calculated ones taken from the refraction rules" (ibid., pp. 105, 107, cf. p. 114), and that Newton had falsified his earlier claim (id., "Skating on the Edge: Newton's Investigations of Chromatic Dispersion and Achromatic Prisms and Lenses", p. 119).

100 Shapiro, ed., *The Optical Papers of Isaac Newton*, I, pp. 87/89, cf. p. 439.

101 For the references see footnote 95.

102 Ibid., I, p. 291, cf. Propositions 1, 4, 5, 10 on pp. 293–294; Newton, *The Opticks*, Proposition II, Book I, Part I, p. 26, Proposition III, Book I, Part I, p. 63, Proposition V, Book I, Part II, p. 134. Newton's acceptance of Huygens' criticism also lead him to accept two kinds of white: a natural and an artificial white. For further discussion see Shapiro, "The Evolving Structure of Newton's Theory of White Light and Color", pp. 223–225.

4.7 Further Problems in *The Opticks*

In this section, I shall focus on two additional problems which Newton encountered while methodizing optics. The rectilinear propagation of light had been an important assumption in Newton's optical research.[103] However, in the context of his researches on optical diffraction, or as Newton called it "inflexion," he came to learn that light could be bent, i.e. propagated along non-rectilinear trajectories. The difficulties involved in Newton's book on diffraction, i.e. Book III of *The Opticks*,[104] was one of the major reasons why Newton delayed so long with its publication.[105] Diffraction, as we currently understand, is an interference phenomenon produced by the obstruction of a wave.[106] The second problem Newton faced was that he could not provide a non-hypothetical derivation for the sine law of refraction.

In Observations 8–10 of Book III, Newton gave an account of a series of diffraction experiments.[107] In one of these experiments, he had installed two knives on a board so that they were in a v-like position and made an angle of $1°54'$. Furthermore, the two knives were placed at a distance of 10 or 15 feet behind a small hole with a diameter of 1/42 inch, through which the sun's light could enter, so that fringes would appear on a smooth white ruler which could be placed at a variable distance from the knives.[108] When Newton placed the knives at a distance of 8 feet and 5 inches from the aforementioned hole, he observed that the shadows formed by the knives' edges were bordered with a series of coloured fringes. Furthermore, by varying the distances between the knives, he established at what distances the paper should be placed, so that the first of the shadows between the fringes, coming from both sides, intersected. When the distances between the edges of the knives were 0.012, 0.020, 0.034, 0.057, 0.081 and 0.087 thousandth parts of an inch, respectively, the distances at which the first shadows intersected at the centre of the spectrum were $1_{1/2}$, $3_{1/3}$, $8_{3/5}$, 32, 96, and 131 inches, respectively.[109] From this, Newton concluded that "the Light which makes the Fringes upon the Paper is not the same Light at all distances of the Paper from the Knives, but when the Paper is held near

[103] In Proposition VIII of Part III of Book II of *The Opticks*, Newton wrote: "The Rays of Light, whether they be very small Bodies projected, or only Motion or Force propagated, are moved in right Lines; and whenever a Ray of Light is by any Obstacle turned out of its rectilinear way, it will never return into the same rectilinear way, unless perhaps by very great accident." (Newton, *The Opticks*, p. 268; cf. Newton's annotation in CUL Adv.b.39.3, p. 70).

[104] Newton, *The Opticks*, Book III, Part I, pp. 317–339.

[105] Shapiro, "Twenty-Nine Years in the Making: Newton's *Opticks*".

[106] For detailed treatments see: Shapiro, "Newton's Experimental Investigation of Diffraction for the *Opticks*: A Preliminary Study"; id., "Newton's Experiments on Diffraction"; and, Nauenberg, "Comparison of Newton's Diffraction Measurements with the Theory of Fresnel".

[107] Reconstructions of Newton's experiments are furthermore to be found in Stuewer, "A Critical Analysis of Newton's Work on Diffraction" and, more recently, in Silverman and Strange, "The Newton Two-Knife Experiment: Intricacies of Wedge Diffraction".

[108] Newton, *The Opticks*, pp. 329–330.

[109] Ibid., pp. 331–332.

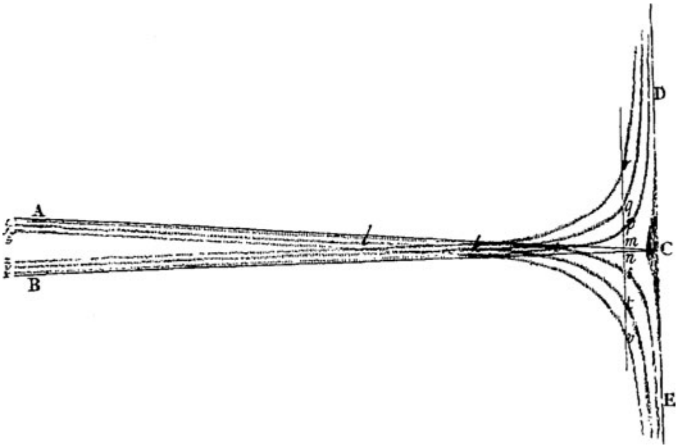

Fig. 4.3 Illustration to Observation 10, Book III, Part I (Newton, *The Opticks*, p. 333). Reproduced from figure 3 among the figures inserted between OE_1, pp. 137–138

the Knives, the Fringes are made by Light which passes by the edges of the Knives at a less distance, and is more bent than when the Paper is held at a greater distance from the Knives."[110] In other words, the fringes are not propagated rectilinearly – a conclusion that went against Newton's earlier attempts of treating diffraction by relying on a rectilinear model.[111] In Observation 10 Newton concluded, moreover, from observation that the fringes are propagated along hyperbolic trajectories (see Fig. 4.3). In Query 3, Newton was prepared to entertain the following possibility: "Are not the Rays of Light in passing by the edges and sides of Bodies, bent several times backwards and forwards, with a motion like that of an Eel? And do not the three Fringes of colour'd Light above-mention'd arise from three such bendings?"[112]

In Propositions XCIV–XCV of Book I of the *Principia* Newton succeeded in deriving the law of refraction *conditionally*, i.e. by assuming a uniform force attracting small bodies downwards along the normal.[113] In the *scholium* to Proposition

[110] Ibid., p. 332.

[111] Shapiro, "Newton's Experiments on Diffraction", p. 66. For a reconstruction of Newton's rectilinear model of diffraction, see: ibid., pp. 54–63.

[112] Newton, *The Opticks*, p. 339; CUL Add. Ms. 3970, f. 338^{v}. This has lead Ronchi to make the rather exaggerated statement that "no one could have worked better than Newton, not to build, but rather to demolish, the corpuscular theory" (Ronchi, *The Nature of Light, A Historical Survey*, p. 191).

[113] The title of Section XIV is "The motion of minimally small bodies that are acted on by centripetal forces tending toward each of the individual parts of some great body" [*De motu corporum minimorum, quæ viribus centripetis ad singulas magni alicujus corporis partes tendentibus agantur*] (Newton, *The Principia*, p. 622). In Proposition XCIV Newton stipulated these conditions: "*If two homogeneous mediums are separated from each other by a space terminated on the two sides by parallel planes, and a body passing through this space is attracted or impelled perpendicularly*

XCVI, Book I, Newton noted that: "[t]hese attractions are very similar to the reflections and refractions of light made according to a given ratio of secants, as Snel discovered, and consequently according to a given ratio of the sines, as Descartes set forth."[114] However, near the end of this *scholium* Newton made it clear, however, that "because of the analogy that exists between the propagation of rays of light and the motion of bodies, I decided to subjoin the following propositions [i.e., Propositions XCVII–XCVIII] for optical uses, *meanwhile not arguing at all about the nature of the rays (that is, whether they are bodies or not), determining the trajectories of bodies, which are very similar to the trajectories of rays* [interea de natura radiorum (utrum sint corpora necne) nihil omnino disputans, sed trajectorias corporum trajectoriis radiorum persimiles solummodo determinans]."[115] In Proposition VI, Book I, Part I of *The Opticks* Newton's discussion of the law of refraction is quite different in the sense that it was stripped down from the explicit physical account he introduced in the *Principia*. Newton's proof proceeded as follows (see Fig. 4.4):

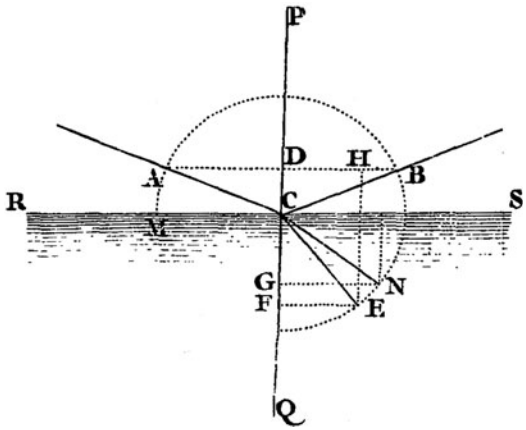

Fig. 4.4 Accompanying figure to Newton's derivation of the law of refraction in Proposition VI, Book I, Part I of *The Opticks* (Newton, *The Opticks*, p. 6). Reproduced from figure 1 among the figures inserted between OE_1, pp. 80–81

toward either medium [corpus in transitu per hoc spatium attrahatur vel impellatur perpendiculariter versus medium alterutrum] and is not acted on or impeded by any other force, and the attraction at equal distances from each plane (taken on the same side of that place) is the same everywhere; then I say that the sine of the angle of incidence onto either plane will be to the sine of the angle of emergence from the other plane in a given ratio." (Newton, *The Principia*, p. 622). In Proposition XCV, Newton established that under the same conditions as in Proposition CXIV, "*the velocity of the body before incidence is to its velocity after emergence as the sine of the angle of emergence to the sine of the angle of incidence*" (ibid., p. 623). For further details on Newton's proofs for the sine law of refraction, see Dijksterhuis, *Lenses and Waves*, pp. 196–200 and Bechler, "Newton's Search for a Mechanistic Model of Colour Dispersion", pp. 14–17.

[114] Newton, *The Principia*, p. 625. Descartes' discussion of the law of refraction can be found in *La Dioptrique* (Descartes, *Œuvres de Descartes*, VI, pp. 93–105, esp. pp. 101–102).

[115] Newton, *The Principia*, p. 626 [italics added].

> Suppose now that a Ray coming most obliquely[116] in the Line MC be refracted at C by the Plane RS into the Line CN, and if it be required to find the Line CE, into which any other Ray AC shall be refracted; let MC, AD, be the Sines of Incidence of the two Rays, and NG, EF, their Since of Refraction, and let the equal Motions of the incident Rays be represented by the equal Lines MC and AC, and the Motion MC being considered as parallel to the refracting Plane, let the other Motion AC be distinguished into two Motions AD and DC, one of which AD is parallel, and the other DC perpendicular to the refracting Surface. In like manner, let the Motions of the emerging Rays be distinguish'd into two, whereof the perpendicular ones are MC/NG × CG and AD/EF × CF. And if the force of the refracting Plane begins to act upon the Rays either in that Plane or at a certain distance from it on the other side, and ends at a certain distance from it on the other side, and in all places between those limits act upon the Rays in Lines perpendicular to that refracting Plane, and the Actions upon the Rays at equal distances from the refracting Plane be equal, and at unequal ones either equal or unequal according to any rate whatever; that Motion of the Ray which is parallel to the refracting Plane, will suffer no Alteration by that Force; and the Motion which is perpendicular to it will be altered according to the rule of the foregoing Proposition.[117] If therefore for the perpendicular velocity of the emerging Ray CN you write MC/NG × CG as above, then the perpendicular velocity of any other emerging Ray CE which was as AD/EF × CF, will be equal to the square Root of CD*q* + MC*q*/NG*q* × CG*q*. And by squaring these Equals, and by adding to them the Equals AD*q* and MC*q* – CD*q*, and dividing the Sums by the Equals CF*q* + EF*q* and CG*q* + NG*q*, you will have MC*q*/NG*q* equal to AD*q*/EF*q*.[118] Whence AD, the Sine of Incidence, is to EF the Sine of Refraction, as MC to NG, that is, in a given *ratio*.[119]

116 This detail is significant, for Newton considers those cases in which the incident ray is nearly parallel to the plane RS, so that the perpendicular velocity of the incident ray is infinitely little. See the following footnote.

117 The proposition to which Newton refers is in Newton, *The Opticks*, pp. 79–80 [italics added]: "If any Motion or moving thing whatsoever be incident with any Velocity on any broad and thin space terminated on both sides by two parallel Planes, and in its Passage through that space be urged perpendicularly towards the farther Plane is of given Quantities; the perpendicular velocity of that Motion of Thing, as its emerging out of that space, shall be always equal to the square Root of the sum of the square of the perpendicular velocity of that Motion or Thing as its Incidence on that space; and of the square of the perpendicular velocity which that Motion or Thing would have at its Emergence, *if at its Incidence its perpendicular velocity was infinitely little*."

118 In other words, Newton showed that $\frac{AD^2}{EF^2} \times CF^2 = CD^2 + \left(\frac{MC^2}{NG^2} \times CG^2\right)$. Since $AD^2 = MC^2 - CD^2$, we can add these to both sides of the equality sign, so that: $AD^2 + \left(\frac{AD^2}{EF^2} \times CF^2\right) = MC^2 - CD^2 + CD^2 + \left(\frac{MC^2}{NG^2} \times CG^2\right)$. Thus we obtain: $\frac{(EF^2 \times AD^2) + (AD^2 \times CF^2)}{EF^2} = \frac{(NG^2 \times MC^2) + (MC^2 \times CG^2)}{NG^2}$, from which $\frac{AD^2}{EF^2} \times \left(EF^2 + CF^2\right) = \frac{MC^2}{NG^2} \times \left(NG^2 + CG^2\right)$ follows. However, from what is given its follows that $\left(EF^2 + CF^2\right) = \left(NG^2 + CG^2\right)$, so that $\frac{AD^2}{EF^2} = \frac{MC^2}{NG^2}$ and therefore $\frac{AD}{EF} = \frac{MC}{NG}$ obtains. Furthermore, from what is given, AD represents the sine of the incident ray ($= \sin(R_i)$) and EF represents the sine of the emerging ray ($= \sin(R_e)$), so that $\frac{\sin(Ri)}{\sin(Re)} = \frac{MC}{NG}$.

119 Newton, *The Opticks*, pp. 80–81. See furthermore: Sabra, *Theories of Light: from Descartes to Newton*, pp. 305–308 and Bechler, "Newton's Search for a Mechanistic Model of Colour Dispersion", pp. 27–31. Newton's proof remained unchanged in all editions. It is worth quoting Whiteside's comment: "In this over-complicated restatement of the simple Cartesian emission

Newton concluded his proof as follows:

> And this Demonstration being general, *without determining what Light is, or by what kind of Force it is refracted, or assuming any thing further than that the refracting Body acts upon the Rays in Lines perpendicular to its Surface*; I take it to be a very convincing Argument of the full truth of this Proposition.[120]

It is true that the proof did not contain any explicit reference to small bodies. The proof did suppose, however, that the motion *AC* can be decomposed ("distinguished") into motions *AD* and *DC*. This step can only be licensed by Corollary 2 to the laws of motion, which implies that corporeal bodies are being acted upon.[121] While Newton tried to provide a strictly mathematical proof for the law of refraction, ultimately he could not remove all traces which showed that, on an implicit level, his demonstration was based on the supposition that light consists of *globuli*.

Similarly, in his study of the colours produced by thin films and thick plates, Newton relied heuristically on the hypothesis of light particles in a vibrating ether "to suggest and interpret experiments and to deduce the mathematical and physical properties of periodic colors."[122] The hypothetical version can be found in Newton's second optical paper (1675), in which he provided the following explanation:

> But it remains further to be explained, how rays alike incident on the same superficies (suppose of crystal, glass, or water) may be at the same time some refracted, others reflected. And for explaining this, I suppose, that the rays, when they impinge on the rigid resisting æthereal superficies, as they are acted upon by it, so they react upon it and cause vibrations in it, as stones thrown into water do in its surface; and that these vibrations are propagated every way into both the rarer and denser mediums; as the vibrations of air, which cause sound, are from a stroke, but yet continue strongest where they began, and alternately contract and dilate the æther in that physical superficies. For it is plain by the heat, which light

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model Newton thoroughly obscures the basic point that the sine-law of refraction – here produced much like a *deus ex machina* – derives from the invariance of the horizontal component of the speed of the light "ray" in its passage through the optical interface at which it receives its downwards impulse or refractive force, and his introduction of an undemonstrated rule for the increase in "orbital" speed thereby engendered serves only further to confuse it." (Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 430–431, footnote 28).

120 Newton, *The Opticks*, pp. 81–82 [italics added].

121 Zev Bechler has furthermore shown that Newton's model presupposed that rays *AC* and *MC* have equal velocity and that the constancy of the sine law depends on the velocity of the incident ray (Bechler, "Newton's Search for a Mechanistic Model of Colour Dispersion", pp. 27–31). In the proof of the sine law in *The Opticks*, Newton entirely obscured the physical meaning of the constancy of $\frac{MC}{NG}$.

122 Shapiro, *Fits, Passions, and Paroxysms*, p. 50, cf. pp. 65, 76. On Newton's rings, see Newton, *The Opticks*, pp. 193–244 [= Parts I and II of Book II] and Shapiro, *Fits, Passions, and Paroxysms*, pp. 79–89; on the colours of thick plates, see Newton, *The Opticks*, pp. 289–315 [= Part IV of Book II] and Shapiro, *Fits, Passions, and Paroxysms*, pp. 150–181. In Part IV of Book II Newton extended the model he used in his treatment of the colours of thin plates to the colours of thick plates (Newton, *The Opticks*, Observations 7–9, Book II, Part IV, pp. 297–307; Shapiro, *Fits, Passions, and Paroxysms*, Chapter 3).

> produces in bodies, that it is able to put their parts in motion, and much more to heat and put in motion the more tender æther; and it is more probable, that it communicates motion to the gross parts of bodies by the mediation of æther than immediately [. . .]. The shock of every single ray may generate many thousand vibrations, and by sending them all over the body, move all parts, and that perhaps with more motion than it could move one single part by an immediate stroke; for the vibrations, by shaking each particle backward and forward, may every time increase its motion, [. . .], and so at length move the particles to a very great degree of agitation, which neither the simple shock of a ray, nor any motion of the æther, besides a vibrating one could do. [. . .] Æthereal vibrations are therefore the best means by which such a subtile agent as light can shake the gross particles of solid bodies to heat them: and so supposing that light, impinging on a refracting or reflecting æthereal superficies, puts it into a vibrating motion, that physical superficies being by the perpetual appulse of rays always kept in a vibrating motion, and the æther therein continually expanded and compressed by turns; if a ray of light impinge upon it, while it is much compressed, I suppose it is then too dense and stiff to let the ray pass through, and so reflects it; but the rays, that impinge on it at other times, when it is either expanded by the interval of two vibrations, or not too much compressed and condensed, go through and are refracted.[123]

In *The Opticks*, by contrast, Newton's treatment of the colours produced by thin films and thick plates was presented in terms of the phenomenological and dispositional theory of fits of easy reflection or transmission and was, accordingly, cleansed from the hypothesis of "aethereal pulses."[124] Although the theory of fits was quite successful in terms of its predictions,[125] Newton never considered predictive success *alone* as a viable criterion for the truth or correctness of a model.

4.8 Looking with Unseeing Eyes into the Invisible Realm: The Problem of Transduction

Transduction refers to the reasoning process whereby inferences are made about the *microscopic components* of bodies from the observed laws and properties of the *macroscopic bodies*, which these microscopic components constitute (or are supposed to constitute).[126] Given their methodological outlook, Boyle, Hooke and Descartes could speculate and hypothesize freely on the micro-constituents involved

[123] Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, pp. 188–189; Newton *Correspondence*, I, pp. 373–374.

[124] Newton, *The Opticks*, pp. 281–288. In Part III, Book II of *The Opticks* Newton introduced the following definition: "*The returns of the disposition of any Ray to be reflected I will call its* Fits of easy Reflection, *and those of its disposition to be transmitted its* Fits of easy Transmission, *and the space it passes between every return and the next return, the* Interval of its Fits." (ibid., p. 281). For further discussion, see Shapiro, *Fits, Passions, and Paroxysms*, Chapter 4.

[125] Newton, *The Opticks*, pp. 289–315; Shapiro, *Fits, Passions, and Paroxysms*, pp. 150–179.

[126] See: Mandelbaum, *Philosophy, Science, and Sense Perception* and McGuire, "Atoms and the 'Analogy of Nature'". "Transduction" is also referred to as "transdiction." I prefer "transduction" because of the analogy with other forms of ampliative reasoning such as induction and abduction.

in optical phenomena, and that is exactly what they did.[127] With respect to unobservables Hooke suggested that "the best and utmost we can do towards the discovery of them, is only accurately to observe and examine all those Effects produced by them, which fall within the Power of our Senses, and comparing them with like Effects, produced by Causes that fall within the reach of our Senses, to examine, and so from Sensibles to argue the Similitude of the nature of Causes that are wholly insensible."[128] Boyle was equally embracive of transductive reasoning and stated: "And therefore to say, that, though in Natural Bodies, whose bulk is manifest and their structure visible, the Mechanical Principles may be usefully admitted, that are not to be extended to such portions of Matter, whose parts and Textures are invisible; may perhaps look to some, as if a man should allow, that the Laws of Mechanism may take place in a Town-Clock, but cannot in a Pocket-Watch."[129] In his *Principia philosopiae* (1644), Descartes pointed out:

> But I attribute determined figures, and sizes, and movements to the imperceptible particles of bodies, as if I had seen them; and yet I acknowledge that they are imperceptible. And on that account, some readers may perhaps ask how I therefore know what they are like. To which I reply: that I first generally considered, from the simplest and best known principles (the knowledge of which is imparted to our minds by nature), what the principle differences in the sizes, figures, and situations of bodies which are imperceptible solely on account of their smallness could be, and what perceptible effects would follow from their various encounters. And next, when I noticed some similar effects in perceptible things, I judged that these things had been created by similar encounters of such imperceptible bodies; especially when it seemed that no other way of explaining these things could be devised.[130]

However, Descartes cautioned that, although in this way it may be understood how all natural bodies *could have come into being*, it should not therefore be concluded that they *were in fact so made*[131]: "For just as the same artisan can make two clocks which indicate the hours equally well and are exactly similar externally, but are internally composed of an entirely dissimilar combination of small wheels: so there is no doubt that the greatest Artificer of things could have made

[127] See Section 2.1 in Chapter 2.

[128] Hooke, *A Discourse of Comets* in: Hooke, *The Posthumous Works of Robert Hooke*, p. 165.

[129] Boyle, *Excellency of Mechanical Hypothesis* (ca. 1674), in: Boyle, *The Works of Robert Boyle,* VIII, p. 108, cf. p. 16.

[130] Descartes, *René Descartes: Principles of Philosophy*, p. 285. For the original, see Descartes, *Œuvres de Descartes*, VIII, *Pars quarta*, § CCIII, pp. 325–326: "At insensilibus corporum particulis determinatas figuras & magnitudines & motus assigno, tanquam si eas vidissem, & tamen fateor esse insensiles; atque ideò quærent fortasse nonnuli, unde ergo quales sint agnoscam. Quibus respondeo: me primò quidem, ex simplicissimis & maximè notis principiis, quorum cognitio mentibus nostris à naturâ indita est, generaliter considerâsse, quænam præcipuæ differentiæ inter magnitudines & figuras & situs corporum, ob solam exiguitatem suam, insensilium esse possent, & quinam sensiles effectus ex variis eorum concursibus sequerentur. Ac deinde, cùm similes aliquos effectus in rebus sensibilibus animadverti, eas ex simili talium corporum concursu ortas existimâsse; praesertim cùm nullus alius ipsas explicandi modus excogitari posse videbatur."

[131] Cf.: "At quamvis fortè hoc pacto intelligatur quomodo res omnes naturales fieri potuerint, non tamen ideò concludi debet, ipsas revera sic factas esse." (ibid., *Pars quarta*, § CCIV, p. 327).

all those things which we see in many diverse ways."[132] Descartes' (microscopic) mechanical explanations were therefore only morally certain ("certus moraliter"), i.e. sufficiently certain for the needs of everyday life.[133]

Newton, however, who insisted in the General Scholium (1713) that scientific propositions should be based on phenomena directly or on deductions from phenomena (for "whatever is not deduced from the phenomena must be called a hypothesis"[134]), could not, as I shall explain in what follows, infer statements about the invisible realm of optical phenomena in the demonstrative part of optics without introducing hypotheses. In Book III of the *Principia* transduction was less problematic, as we will shortly see. Transductive inferences were a crucial feature of Newton's methodology, as can be gathered from the following passage:

> (Experimental) Philosophy starts from phenomena. Experimental Philosophy consists in treating of such things. One should pass from experimental Philosophy to the efficient and final causes of things and from all these *to the nature of imperceptible things* and, finally, to hypothetical Philosophy.[135]

In the context of transduction, the "Analogy of Nature" was a guiding principle in Newton's research, for without the uniformity of microscopic and macroscopic components, it would be impossible "to derive the qualities of imperceptible bodies from the qualities of perceptible ones,"[136] as Newton observed in manuscript material on Hypotheses III (later Rule III) in 1692. In draft material for the never to be

[132] Descartes, *René Descartes: Principles of Philosophy*, p. 286. For the original, see Descartes, *Œuvres de Descartes*, VIII, *Pars quarta*, § CCIII, p. 327: "Nam quemadmodum ab eodem artifice duo horologia fieri possunt, quæ, quamvis horas æquè bene indicent, & extrinsecus omnino similia sint, intus tames ex valde dissimili rotularum compage constant: ita non dubium est, quin summus rerum opifex omnia illa, quæ videmus, pluribus diversis modis potuerit efficere."

[133] Ibid., *Pars quarta*, § CCV, p. 327. Descartes' explanation of refraction in *Les Météores* (1637) is a notable example of transduction in his work (Descartes, *Œuvres de Descartes*, VI, Discours VIII, pp. 331–333). For a thorough discussion, see Buchwald, "Descartes' Experimental Journey Past the Prism and through the Invisible World", pp. 8–21.

[134] Newton, *The Principia*, p. 943.

[135] Translation of: "↓A phænomenis Philosophia ~~naturalis~~ incipit↓. In his tractandis Philosophia experimentalis consistit. A~~b hac~~ Philosophia ↓experimentali ad rerum↓ ~~ad~~ causas efficientes & finales, & ↓ab his omnibus ad naturam rerum insensibilium & ultimo↓ ad Philosophiam hypotheticam transeundum est:]" (CUL Add. Ms. 3965, f. 422r [additions and corrections to the second edition of the *Principia*]).

[136] Hypothesis III goes as follows: "Hypoth. III. Qualitates corporum quæ intendi et remitti nequeunt quæque corporibus omnibus competunt in quibus experimenta instituere licet sunt qualitates corporum universorum. Idem intelligendum est de qualitatibus corporum omnium ejusdem generis[.] Fundamentum ↓esse↓ videtur Philosophiæ totius. [This passage explains what Newton had in mind when, in the published text to Rule III, he wrote somewhat out of the blue "And this is the foundation of all natural philosophy." (Newton, *The Principia*, p. 796).] *Neque enim aliter ↓qualitates ↓corporum↓ insensibilium↓ a qualitatibus sensibilium ~~[illegible word] qualitates sensibilium~~ derivare licet*." (CUL 3965, f. 266r, cf. f. 267r [additions and corrections to the first edition of the *Principia*; italics added]). See further: McGuire, "Atoms and the 'Analogy of Nature'"; Cohen, *Introduction to Newton's Principia*, pp. 184–187; and, Shapiro, *Fits, Passions, and Paroxysms*, p. 45.

published fourth book of *The Opticks* “concerning the nature of light & y^{e} power of bodies to refract & reflect it,”[137] Newton formulated his belief in the analogy of nature as follows:

> ~~Prop 18~~ ↓Hypoth 2↓ As all the great motions in the world depend upon a certain kind of force ~~(vulgarly called gravity)~~ w^{ch} in these earth we call gravity) whereby great bodies attract one another at great distances: so all the ↓little↓ ~~minute~~ motions in y^{e} world depend upon certain kinds of forces whereby minute bodies attract or dispell one another at little distances. [. . .] *The truth of this Hypothesis I assert not because I cannot prove it, but I think it very probable because a great part of the phænomena of nature ~~w^{ch}~~ do easily flow from it w^{ch} seem otherwise inexplicable*.[138]

While the “Analogy of Nature” suggested to Newton that the micro-constituents of optical phenomena are similar or analogical to the bodies that fall within the reach of our observation,[139] i.e. corpuscular, these micro-constituents are in themselves unobservable and, moreover, there is no way to justify the “Analogy of Nature” in this particular context.[140] Therefore any characterization of them in corpuscular terms would remain utterly speculative. Alan E. Shapiro has rightly pointed out that the difficulties that Newton met in his optical research was due to the failure of the method of transduction *within* the domain of optics.[141] In manuscript material, which was composed around 1715, Newton treated transduction as a special case of induction:

> Here only sensible [bodies] and their parts are treated [and it is] for this reason that the argument of induction may have [its] place in them only. Distant [bodies] that cannot be perceived, but which are nevertheless hypothetically called bodies by some, should be more adequately treated in hypothetical Metaphysics and Philosophy.[142]

In this very passage, Newton was methodizing transduction: transduction can only be legitimately applied *to sensible bodies and their parts*. As we have seen in Section 2.5.4 in Chapter 2, Newton was constraining transductive inferences by imposing the requirement on them that they should be based on well-defined physico-mathematical decompositions. In other words, Newton came to equate methodologically sound transduction to the sort of transduction which he had carried out in Propositions VII–VIII of Book III of the *Principia*. If one were to assert

[137] This material is on CUL Add. Ms. 3970, ff. 337^{r}–338^{v} and ff. 342^{r}–346^{r} [ca. 1700–1704].

[138] CUL Add. Ms. 3970, f. 388$^{r–v}$ [late 1680s–early 1690s; italics added].

[139] Such analogy was invoked by Newton in his 1675 *An Hypothesis explaining y^{e} properties of Light* (Newton, *Correspondence*, I, pp. 374, 376; Cohen, ed., *Isaac Newton’s Papers and Letters on Natural Philosophy*, pp. 188, 192).

[140] Cf. the *scholium* to Proposition XCVI in Book I of the *Principia* already referred to in the previous section.

[141] Shapiro, *Fits, Passions, and Paroxysms*, pp. 45, 125, 134.

[142] Translation of: “De solis sensibilibus et eorum partibus hic agitur propterea quod argumentum Inductionis in ijs solis locum habeat. Reliqua quæ non sentituntur sed per hypothesin tamen a nonnulis corpora nominantur, in Metaphysica et Philosophia hypothetica rectius tractanda sunt.” (CUL Add. Ms. 3965, f. 422^{r} [ca. 1715]). A full transcription of this definition is given in the appendix to his chapter.

that the *explananda* in optics are corpuscular, by contrast, one would be feigning a hypothesis. When Newton increasingly began to emphasize "Deductions from Phenomena" shortly after 1700,[143] the tension between *Principia*-style transductions and the sort of transductions found in his optical work became all the more stronger.

Notwithstanding, in his optical research Newton *made* claims on the micro-constituents of optical phenomena. In Book II, Part III, which addressed *the permanent Colours of natural Bodies, and the Analogy between them and the Colours of thin transparent Plates*,[144] Newton stated that coloured bodies, consisting of absorbing primordial particles and pores, are produced by the highest order corpuscles in the same way as a fragment of a thin film, since similar effects are produced by the same cause.[145] On this matter, Newton wrote:

> I am now come to another part of this Design, which is to consider how the Phænomena of thin transparent Plates stand related to those of all other natural Bodies. Of these Bodies I have already told you that they appear of divers Colours, accordingly as they are disposed to reflect most copiously the Rays originally endued with those Colours. *But their Constitutions*, whereby they reflect some Rays more copiously than others, remain to be discover'd; *and these I shall endeavour to manifest in the following Propositions*.[146]

Closely related to this "Analogy," Newton also introduced a hierarchical account on the structure of matter: in a speculative moment, at the end of Proposition VIII, Book II, Part III ("The cause of Reflexion us not the impinging of Light on the solid or impervious parts of Bodies, as is commonly believed."), Newton introduced this hypothesis as follows:

> How Bodies *can have* sufficient quantity of Pores for producing these Effects is *very difficult to conceive, but perhaps not altogether impossible*. For the Colours of Bodies arise from the Magnitudes of the Particles which refract them as was explained above. Now *if we conceive* these Particles of Bodies to be so disposed amongst themselves, that the Intervals or empty Spaces between them *may* be equal in magnitude to them all; and that these Particles *may* be composed of other Particles much smaller, which have as much empty Space between them as equals all the Magnitudes of these small particles: And that in like manner these smaller Particles are again composed of others much smaller all which together are equal to all the Pores or empty spaces between them; and so on perpetually till you come to solid Particles, such as have no Pores or empty Spaces within them: And if in any gross Body there be, for instance, three small degrees of Particles the least of which are solid; this Body will have seven times more Pores then solid Parts. But if there be four such degrees of Particles the least of which are solid, the Body will have fifteen times more Pores then solid Parts. If there be five degrees, the Body will have one & thirty times more Pores then solid Parts. If six degrees, the Body will have sixty & three times more Pores then solid Parts. And so on

143 See Section 2.2 in Chapter 2.

144 Newton, *The Opticks*, p. 245 [underscore added]; cf. Proposition II, Book II, Part III, p. 248ff. and Proposition I, Book II, Part III, p. 251ff. Newton had dealt with the colours of thin plates in Parts I–II of Book III.

145 Newton, *The Opticks*, Book II, Part III, pp. 245–288; Shapiro, *Fits, Passions, and Paroxysms*, pp. 113, 118.

146 Newton, *The Opticks*, p. 245 [italics added].

> perpetually. *And there are other ways of conceiving how bodies may be exceeding porous. But what is really their inward Frame is not yet known to us*.[147]

This text was added in the second edition of *The Opticks* and can be found among Newton's annotations in his private copy of the first edition.[148] At the risk of going beyond what strictly falls within the scope of observation, Newton occasionally speculated on the invisible realm of optical phenomena.[149]

Up to this point, I have discussed the problems surrounding Newton's optical research, however, I should also add some observations on how transduction was less problematic in the *Principia*. Let us recall from the previous chapter how Newton arrived at *universal* gravitation in Proposition VII, Book III. In the preceding propositions in Book III, Newton has argued that all planets gravitate towards each other and that the gravity of each planet varies inversely as the square of the distance. It follows by proposition LXIX, Book I, that gravity towards all planets is proportional to their mass. Since all the parts of a planet A are heavy towards planet B, and since the gravity of each part to the gravity of the whole is as the matter of that part to the matter of the whole, and since to every action there is an equal reaction (by the third law of motion), it follows that planet B will gravitate in turn towards all the parts of A, and its gravity to any one part will be to its gravity toward the whole of the planet as the matter of that part to the matter of the whole. Hence, the gravity towards the whole planet arises from and is compounded of the gravity of the individual parts (Corollary 1). From Corollary 3 to Proposition LXXIV, Book I, it follows that the gravity toward each of the individual particles of a body is inversely as the squares of the distance of the places from those particles (Corollary 2). As we have seen, Newton's propositions on the attractive force of spherical bodies play a crucial role in the argument for universal gravitation.[150] On the basis of the propositions on the attraction of spherical bodies, Newton could argue that the overall inverse-square centripetal force of a sphere on an external particle (or sphere) results from the summation of the individual inverse-square forces of each of the particles composing that sphere. In mechanics, transduction is less problematic because the constituents of bodies share the same theoretically relevant

[147] Ibid., pp. 268–269 [italics added]; cf. CUL Add. Ms. 3970, f. 479r.

[148] CUL Adv.b.39.3, p. 70. On Newton's hierarchical hypothesis see furthermore: Figala, "Newton as an Alchemist"; id., "Die exakten Alchemie von Isaac Newton", pp. 162–173; Gregory, "The Newtonian Hierarchic System of Particles"; Kubbinga, "Newton's Theory of Matter"; Thackray, *Atoms and Powers – an Essay on Newtonian Matter-Theory and the Development of Chemistry*; and, Shapiro, "Newton's Optics and Atomism", esp. pp. 245–249. At the end of Proposition VII in Part III of Book I, Newton had spelled out the implication of his hierarchical account of the structure of matter: "However it will add much to our Satisfaction, if those Corpuscles can be discover'd with Microscopes; which if we shall at length attain to, I fear it will be the utmost improvement of this Sense. *For it seems impossible to see the more secret and noble Works of Nature within the Corpuscles by reason of their transparency*." (Newton, *The Opticks*, p. 262 [italics added]).

[149] Huygens' tackle on the problem of transduction was by reducing the propagation of light to a problem of velocity (Dijksterhuis, *Lenses and Waves*, p. 192).

[150] See Section 2.4.5 in Chapter 2 and Section 3.4.5 in Chapter 3.

property with the bodies they constitute: namely, mass. Allow me to explain this. Newton conceived of mass as an additive property: a body's (total) mass consists of the masses of its parts.[151] Since gravity is proportional to mass,[152] it follows that gravity is likewise additive: the gravity of the whole results from the gravity of its parts. Given this subtle process of decomposition, Newton could bridge the gap between the micro- and the macro-level in the *Principia*. In optics, by contrast, transduction is problematic because it amounts to endorsing a particular hypothesis on the nature of light.

To avoid giving the impression that transduction was always successful in the *Principia*, I shall refer to an unsuccessful case of transduction in Book II of the *Principia*. Section VII of Book II of the *Principia* contains a series of hypothetical models for the resistance force on a body arising from the fluid's inertia.[153] The hope was that by showing that *a theoretically defined type of fluid*, *which is microscopically constituted in a specific way*,[154] produces *a macroscopic inertial resistance that varies according to certain proportions*, conclusions could be reached about the micro-constituents underlying a fluid's inertial resistance. Apart from finding decisive evidence in favour of one of these microscopic models, the problem was that the microscopic models for inertial resistance which Newton developed were not derived from a set of stable and well-established principles akin to the laws of motion. The requirement that the mathematical models ought to be constrained by a set of non-arbitrary physical principles is, as we have seen, a notable feature of Newton's methodology. The microscopic models developed in Book II therefore only provided plausible models for the inertial component of fluid resistance lacking rigorous justification. All they established was that the microscopic conditions are explanatory sufficient for their macroscopic effects.

It should be noted, however, that Newton in the *Principia* never made any rash conclusions about the microscopic constituents of inertial resistance. As a further striking illustration of Newton's attitude, one might consider the case of his hypothetical model for Boyle's law, which was based on an elastic fluid composed of particles that repel one another inversely as the distances between their centres (Proposition XXIII in Section V of Book II).[155] Once Newton had outlined

151 Cf. Janiak, *Newton as Philosopher*, p. 108.

152 Newton, *The Principia*, p. 404.

153 Smith, "The Newtonian Style in Book II of the *Principia*", p. 266.

154 In Section VII Newton considered three types of fluids: *rarified*, *elastic*, and *continuous* ones. A rarified fluid consists of small bodies which are spread out evenly. An elastic fluid is basically a rarified fluid endowed with additional repulsive forces, or as Newton called them "centrifugal forces," between the impinging bodies and the moving body. Newton's results for these two types of non-continuous fluids are provided in Propositions XXXII–XXXV (Newton, *The Principia*, pp. 724–733). A continuous fluid is a fluid in which the particles are in contact (for Newton's results, see Propositions XXXVI–XL in: ibid., pp. 733–750). See Gauld, "Newton's Investigations of the Resistance to Moving Bodies in Continuous Fluids and the Nature of 'Frontier Science'" for discussion of some of the propositions from Section VII.

155 Newton, *The Principia*, pp. 697–698.

the model, he immediately pleaded ignorance: "Whether elastic fluids consist of particles that repel one another is, however, a question for physics. We have mathematically demonstrated a property of fluids consisting of particles of this sort so as to provide natural philosophers with the means with which to treat that question."[156]

4.9 Newton on Non-gravitational Forces

Newton ran into similar problems when he dealt with non-gravitational forces. It is abundantly clear from both published work as well as from manuscript material that Newton sought to experimentally demonstrate, i.e. to rigidly deduce from phenomena according to his own, highly developed, methodological standards, the forces of magnetism and electricity,[157] short-range attractive and repulsive forces,[158] and the causes producing fermentation, nutrition, corruption and generation of organisms,[159] putrescence, muscular movement and perception,[160] refraction, reflection and, finally, diffraction.[161] These rather "speculative" experiments were never included in the *Principia*, the work "which Newton was the most anxious to make immune from attack,"[162] and they are merely hinted at in the General Scholium. In the final paragraph of the General Scholium, Newton concluded the *Principia* as follows:

> A few things could now be added concerning a certain subtle spirit [spiritu quodam subtilissimo] pervading gross bodies and lying hidden in them; by its force and actions, the particles of bodies attract one another at very small distances and cohere when they become contiguous; and electrical bodies act at greater distances, repelling as well as attracting neighboring corpuscles; and light is emitted, reflected, refracted, inflected, and heats bodies; and all sensation is exited, and the limbs of animals move at command of the will, namely, by the

[156] Ibid., p. 699.

[157] See Howe, "Newton on Electricity and The Aether". It should be noted that Home's study is based on material from or related to *The Opticks*.

[158] In his manuscript *De natura acidorum* (March 1691/2), Newton had shown the predominance of attraction during certain chemical processes (Newton, *Correspondence*, III, pp. 205–214).

[159] See especially Newton's manuscript *De vita et morte vegetabili* (CUL Add. Ms. 3970, ff. 237^{r}–238^{v}; transcribed in Mamiani and Trucco, "Newton E I Fenomeni della Vita", pp. 78–79). According to Mamiani and Trucco, this piece was composed in the same period as the General Scholium.

[160] See especially Newton's unpublished manuscript, *De motu et sensatione animalium* which was also related to the composition of the General Scholium (CUL Add. Ms. 3970, f. 236^{r}, transcribed in Mamiani and Trucco, "Newton E I Fenomeni della Vita", pp. 78–79). See also Wes Wallace's interesting study "The vibrating nerve impulse in Newton, Willis and Gassendi".

[161] Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, pp. 333, 350–351, 355–359; Mamiani and Trucco, "Newton E I Fenomeni della Vita", pp. 69–96, 78–87 [for transcriptions of CUL Add. Ms. 3970, f. 236^{r}, f. 237^{r-v}, f. 238^{v-r}, f. 240^{r-v}]. Cf. Newton, *The Principia*, pp. 287–292, 943–944 and Newton, *The Opticks*, p. 340 [Query 8], pp. 353–354 [Query 25], pp. 375–400 [Query 31].

[162] Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 187.

> vibrations of this spirit being propagated through the solid fibers of the nerves from the external organs of the senses to the brain and from the brain to the muscles.[163]

The "spirit" Newton referred to was most certainly an "elastic and electric spirit."[164] Here I will provide a systematic survey of the subtle changes in the observations or experimental set-ups which Newton at some points between 1687 and 1713 intended to be included in the *Principia*, and by means of which he sought to establish and study the non-gravitational forces in nature. In order to understand Newton's various attempts in this area, it is necessary to consult the observations and experiments he referred to in the suppressed *Preface* and *Conclusion* to the first edition of the *Principia*, the draft versions of the General Scholium (especially the A-version and to lesser extent the C-version), and the text as it appeared in the second and third edition of the *Principia*. Newton's motivation to suppress most of this material can be traced from these manuscripts.

I shall start chronologically with the *Preface* written in 1687. There Newton expressed his hope that the other (non-gravitational) phenomena would be derived from mechanical principles by the same mode of reasoning (as the force of gravitation) [ex Principijs Mechanicis eodem argumentandi genere derivare liceret[165]]. He distinguished between three classes of fundamental forces: gravity, magnetism and the force producing attractive and repellent forces between particles at small distances.[166] In order to render the force of the latter type more plausible, Newton gave a cornucopia of phenomena that served the purpose of illustrating this force. Newton stated that attractive and repellent forces at small distances accounted for various chemical reactions and for the cohesion of bodies, and that they also explained why bodies are "hard, soft, fluid, elastic, malleable, dense, rare, volatile, fixt; [capable of] emitting, refracting, reflecting or stopping light."[167] In trying to justify his claim, he fiercely relied on the analogy of nature[168] and speculated that the motions of smaller bodies could be explained by forces "just as the motions of larger bodies are ruled by the greater force of gravity:"

> For if Nature be simple and pretty comfortable to herself, causes will operate in the same kind of way in all phenomena, so that the motions of smaller bodies depend upon certain smaller forces just as the motions of larger bodies are ruled by the greater force of gravity. It remains therefore that we inquire by means of fitting experiments whether there are forces of this kind in nature, then what are their properties, quantities and effects.[169] For if all natural

[163] Newton, *The Principia*, pp. 943–944.

[164] In the left margin of the concluding paragraph of the General Scholium on CUL Adv.b.39.2, p. 484, Newton added the words "electrici & elastici," which were to be inserted after "Spiritus." In a draft version of the final paragraph of the General Scholium on CUL Add. Ms. 3965, f. $152^{r\text{-}v}$, Newton wrote "spiritus electricus."

[165] Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 303.

[166] Ibid., p. 304.

[167] Ibid., p. 306.

[168] Cf. Newton, *The Opticks*, p. 397 and McGuire and Rattansi, "Newton and the 'Pipes of Pan'", p. 125.

[169] Cf. Newton, *The Principia*, pp. 411, 588–589.

> motions of great or small bodies can be explained through such forces, nothing more will remain to inquire the causes of gravity, magnetic attraction and the other forces.[170]

However, to account for these short-range forces is more problematic than to account for the motions of terrestrial and celestial bodies. This is more apparent from Newton's suppressed *Conclusion* intended for the first edition of the *Principia*. Here Newton's tone changed and he now hinted at some of the difficulties an experimental treatment of such short-range forces poses. Newton began the *Conclusion* by observing that there are plenty of other motions than those caused by the force of gravity:

> Hitherto I have explained the System of this *visible world* [Mundi aspectabilis], as far as concerns the *greater motions which can be easily detected* [facile sentiri possunt]. There are however innumerable other local motions which *on account of the minuteness of the moving particles cannot be detected* [ob parvitatem corpusculorum moventium, sentiri nequeunt], such as the motions of the moving particles in hot bodies, in fermenting bodies, in putrescent bodies, in growing bodies, in the organs of sensation and so forth. If any one shall have the good fortune to discover all these, I might almost say that he will have laid bare the whole nature of bodies so far as the mechanical causes of things are concerned. I have least of all undertaken the improvement of this part of philosophy. I may say briefly, however, that nature is exceedingly simple and conformable to herself [natura valde simplex est et sibi consona]. Whatever reasoning [Quam rationem] holds for the greater motions [in majoribus motibus], should hold for lesser ones as well [in minoribus]. The former depend upon the greater attractive forces of larger bodies, and I suspect that the latter depend upon the lesser forces, as yet unobserved, of *insensible particles* [particularum insensibilium].[171]

Newton began by reiterating his belief in the analogy of nature.[172] One obvious stumbling block for applying such mode of argumentation is that the "insensible particles" cannot be observed. Basically, Newton's sole justification was his appeal to Nature's consonance: "whatever reasoning holds for greater motions, should hold for lesser ones as well." As we have seen, Newton later began to see the difficulty with this sort of arguments. Next, Newton gave several illustrations of such short-range attractive forces. He listed several chemical reactions during the course of which we can observe that the particles involved attract each other, i.e. approach one another, and pointed to the activity of cohesive forces.[173] Subsequently, he gave several illustrations of short-range repellent forces (for instance, the force by which the particles of oil repel the particles of water when mixed together).[174] Newton merely provided these examples in order to render the existence of short-range attractive and repellent forces more *plausible* and to, at least, justify further research on these matters. Newton's claims on these matters certainly did not have

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[170] Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 307.

[171] Ibid., p. 333 [italics added], cf. p. 321.

[172] On CUL Add. Ms. 3965, f. 5^{v}, Newton wrote that it follows from observation that all bodies are hard, mobile and impenetrable, even insensible ones ("etiam insensibiles").

[173] Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, pp. 333–336, cf. pp. 321–323.

[174] Ibid., pp. 336–340, cf. pp. 324–327.

a demonstrative nature. Newton freely admitted that he had not demonstrated that such non-gravitational forces are *verae causae*:

> I have briefly set these matters out, not in order to make a rash assertion that there are attractive and repulsive forces in bodies, but so that I can give opportunity to imagine further experiments by which it can be ascertained more certainly whether or not they exist. For if it shall be settled that they are true [forces] it will remain for us to investigate their causes and properties diligently, as being the true principles from which, according to geometrical reasoning, all *the more secret motions of the least particles* are no less brought into being than are the motions of the greater bodies which we saw in the foregoing [books] derived from the laws of gravity[175] [tanquam vera principia a quibus omnes particularum minimarum secretiores motus secundum rationes Geometricas non minus oriantur quam motus majorum corporum ex legibus Gravitatis in praecedentibus derivari vidimus].[176]

In an additional paragraph, he wrote:

> I am far from affirming that my views are correct, and I acknowledge their great imperfection, nevertheless they are simple and easy to conceive, and of the same kind as the natural philosophy of the cosmic system which depends on the attractive forces of greater bodies.[177]

Now we will make a jump in time and turn to three experiments Newton intended to include in the General Scholium (see especially the A-version) concerning the electrical force (vis electrica) by which the particles of bodes are variously moved.[178] From 1707 onwards Newton was enormously fascinated by electricity as a direct consequence of a series of electrostatic experiments which were performed by Francis Hauksbee (1666–1713) at The Royal Society of London.[179] Initially Newton planned to add "about a quarter of a Sheet" on the attraction of the small particles of bodies to the General Scholium, but replaced it by the paragraph on "a certain subtle, electric spirit."[180] Ultimately these experiments turned out unsatisfactory to Newton, since in the General Scholium he did not mention them and he only declared that:

> But these things cannot be explained in a few words; furthermore, there is not a sufficient number of experiments to determine and demonstrate accurately the laws governing the

[175] On CUL Add. Ms. 3965, f. 266^{r}, Newton wrote: "Datur spiritus infinitus et omnepræsens in quo materia secundum leges mathematicas movetur."

[176] Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, pp. 340–341 [italics added].

[177] Ibid., p. 345.

[178] Westfall, *Never at Rest*, pp. 744–748.

[179] Ibid., pp. 684–686; Dobbs, *The Janus Faces of Genius*, p. 222; Newton, *The Principia*, p. 281; Guerlac, "Newton's Optical Aether, His Draft of a Porposed Addition to his *Opticks"*; and, Hawes, "Newton's Revival of the Aether Hypothesis and the Explanation of Gravitational Attraction".

[180] Cohen, *Introduction to Newton's Principia*, pp. 240–241. See Westfall, *Never at Rest*, p. 745, footnote 149 for a list of possibly similar drafts. Of course, Newton was not for the first time asserting his belief in such a spirit (cf. his second optical paper (1675) in Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, pp. 179–184). There he noted that these spirits were installed "at first by the immediate hand of the Creator; and ever since by the power of nature" (ibid., p. 180).

> actions of this spirit [Sed hæc paucis exponi non possunt; neque adest sufficiens copia experimentorum, quibus leges actionum hujus spiritus accurate determinari & montrari debent.].[181]

In the first experiment Newton observed that when two contiguously placed pieces of glass were immersed in still water "the water [by the attraction of the glass] ascends between the pieces of glass above the surface of the water, and the height of ascent will be inversely as the distances between the glasses."[182] He also added that the experiment "succeeds in the Boylian vacuum and so does not depend on the weight of the incumbent atmosphere."[183] A variant of this experiment consisted in placing a drop of orange oil[184] between two plates of glass (the first plate was placed horizontally and, at one end, met the second plate; the second plate was kept inclined and touched the drop of orange oil which lay at the other end of the first plate). As soon as the second plate touched the drop of orange oil, "the drop began to move towards the meeting-point of the glasses."[185] This also succeeded in vacuo. Therefore: "the origin of this motion lies in the attraction of the glasses."[186] Because of Newton's usage of screening-off procedures, these experimental set-ups were more sophisticated than those from the suppressed *Preface* and *Conclusion* to the first edition of the *Principia*, and, correspondingly, they added more substance to Newton's claims on these short-range attractive forces. The fact that these experiments also occurred in vacuo guaranteed that other forces (e.g., the pressure of the atmosphere[187]) can no longer be adduced as the causes of the phenomena under observation. But the sophistication was by no means limited to Newton's reliance on screening-off procedures: Newton also tried to quantify such short-range attractive forces. In a third experiment he described how, when the meeting point of the plates was raised so that the lower glass is now inclined to the horizon, the drop rose more slowly than previously and finally came to rest. In such a state of equilibrium, the weight is equal to the attraction of the glass:

> Thus from the inclination of the lower glass the weight of the drop is given, and from the weight of the drop the attraction of the glass is given. The inclinations of the lower glass by which the drop was maintained in equilibrium, and the distances of the drop from the meeting-point of the glasses, are shown in the following table [not given].[188]

In an ingenious way Newton tried to measure the short-range accelerative force of the glass by studying the weight in equilibrium of the drop at various inclinations. The missing table can be found in another draft related to the General Scholium

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[181] Newton, *The Principia*, p. 944.

[182] Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 354.

[183] Ibid.

[184] And not "orange juice" as Westfall notes (Westfall, *Never at Rest*, p. 746).

[185] Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton,* p. 345.

[186] Ibid.

[187] This was Newton's earlier explanation of the rising of water between glass plates (Westfall, *Never at Rest*, p. 746).

[188] Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 455.

(*De vi electrica* (= CUL Add. Ms. 3965, ff. 602–604)).[189] Since the experiments are described in more detail,[190] Newton's writing is relatively neat throughout the manuscript,[191] and the implications of the experiments are now discussed in a more straightforward manner, this draft is most likely written after the composition of the A-version to the General Scholium. As we have seen, in the A-version Newton had succeeded in providing more sophisticated proof for the existence of the "electrical spirit." Correspondingly, his 1687 agnosticism about the existence of such force disappeared and Newton's tone became more determined in *De vi electrica*: "It is *certain from phenomena* that electric and magnetic attractions also exist."[192] By these experiments, Newton noted, "it is *fully enough clear* that glass at small distances always abounds in electric force."[193] However, the data Newton obtained when comparing the varying inclinations of the plate and the distances, from the meeting point of the plates to the place where the drop of orange oil is in equilibrium, did not suffice to yield an accurate determination of the law governing such attraction.[194] Newton could therefore only provisionally conclude that this force is "very nearly inversely in the ratio of the square of the distance."[195] However, the existence of such forces could no longer be disputed.[196] The second half of this draft is devoted to showing how this "electric spirit" might also account for optical phenomena (refraction, reflection and inflexion), the state of aggregation of bodies, and, finally, fermentation and digestion.[197]

The problem of transduction holds *salva veritate* for the other forces causing "other local motions which on account of the minuteness of the moving particles cannot be detected, such as the motions of the moving particles in hot bodies, in

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[189] Newton, *The Principia*, pp. 287–292, 289. CUL Add. Ms. 3965, f. 351$^{r–v}$ is but irrelevantly different from the folios Cohen has transcribed. Related material is on CUL Add. Ms. 9597.2.18.92, f. 2^{r}.

[190] Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, pp. 288–290.

[191] Ibid., p. 283.

[192] Ibid., p. 287 [italics added].

[193] Ibid., p. 289 [italics added].

[194] The reader can understand Newton's dissatisfaction by comparing the values obtained when multiplying the inclination (which stood as a measure for the attractive force of the glass) with the square of the corresponding distance.

[195] Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 289. In CUL Add. Ms 3968, f. 586 [draft material pertaining to Newton's review of the *Principia* in *Acta Eruditorum*], Newton noted that "He [Newton] has told his friends that there are sufficient Phaenomena to ground an inquiry upon but not yet sufficient to determine the laws of attraction." (Cohen's translation in Newton, *The Principia*, p. 282).

[196] In CUL Add. Ms. 3970, f. 240^{r}, Newton wrote: "Et (attractiones elect) quemadmodum attractio gravitatis ad majores Planetarum (Cometarum) & maris nostri motus explicandos sufficit: sic vires electricae et magneticae (ad motus minores alios omnes particularum corporum motus exp[l]icando sufficere videntur) ad explicandas actiones et motus particularum (inter se) corporis cujuscumque inter se sufficere videntur." (Mamiani and Trucco, "Newton E I Fenomeni della Vita", p. 86).

[197] Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, pp. 290–292. Again there is much correspondence with material from Query 31.

fermenting bodies, in putrescent bodies, in growing bodies, in the organs of sensation and so forth."[198] However, it is clear that, as manuscript evidence testifies, demonstrating these (other) non-gravitational forces experimentally, according to the highly developed mathematical methodology he had spelled out in the *Principia*, was one of Newton's paramount endeavours as a natural philosopher.

4.10 The Asymmetry Between the *Principia* and *The Opticks*

Optical phenomena did not easily lend themselves to a *Principia*-style physico-mathematical treatment. As Newton experienced in his study of diffraction, optical phenomena can behave in odd ways, rendering their study and characterization particularly resilient. Nowadays, we have come to accept the wave-particle duality of light: in some cases it behaves particle-like (e.g., in the Compton and photoelectric effect); in others, it behaves wave-like (especially during interference processes such as diffraction). In mechanics, the affected entities, i.e. the *explananda*: *bodies* moving along specific trajectories, and their constituting elements, i.e. the *particles* constituting these very bodies, all have a theoretically salient property in common: *mass*. Because gravity is proportional to mass, and, because the latter is additive, gravity is likewise additive. The aim of the micro-macro inference-tickets is to show that an overall inverse-square centripetal force exerted by a sphere, physico-mathematically results from the individual inverse-square centripetal forces exerted by of each of the particles constituting that sphere. In optics, by contrast, we do not know – at least not without speculating on the matter – the constituting elements of the *explananda*. Therefore, the difference between transduction in the *Principia* versus *The Opticks* can – apart from Newton's views on proper transduction – also be formulated from a more neutral perspective: the epistemology of mechanical versus optical phenomena.

In order to get a grasp on the difference between *The Opticks* and the *Principia*, let us take a closer look at the definitions and especially the axioms of *The Opticks*, which remained unchanged in all editions. After Newton's definition of a ray of light (Definition I), which we have discussed in Section 4.3 of this chapter, the *refrangibility of the rays of light* (Definition II) and the *reflexivity of the rays of light* (Definition III) are defined as the rays' "Disposition to be refracted" and the rays' "Disposition to be reflected," respectively.[199] Thereafter, the definitions of the *angle of incidence*, *reflection* and *refraction* (Definitions IV–V) and *sines of incidence*, *reflection*, and *refraction* (Definition VI) are provided.[200] Next, "*Simple, Homogeneal and Similar*" light is defined as that light of which the rays are equally refrangible and "*Compound, Heterogeneal and Dissimilar*" light is defined as that

[198] Ibid., p. 333.

[199] Newton, *The Opticks*, pp. 2–3.

[200] Ibid., p. 3

light of which the rays are differently refrangible (Definition VII).[201] In the final definition, the colours of simple light are defined as simple colours and the colours of compound light are defined as compound colours (Definition VIII).[202] The eight axioms which follow these definitions deal with the geometrical properties of reflected or refracted light. Here I shall, however, not discuss all of them. Rather I shall provide some examples, which are required to make my point. The first axiom states that the "*Angles of Reflection and Refraction, lie in the same Plane with the Angle of Incidence*;"[203] the second that the angle of reflection is equal to the angle of incidence[204]; the third that if a refracted ray is returned to the point of incidence it will be refracted into the initial line by the incident ray[205]; the fifth axiom contains the sine law of refraction which, according to Newton, holds "either accurately or very nearly."[206]

In contrast to the laws of motion in the Principia*, the axioms in* The Opticks *characterize phenomenological relations described in geometrical terms and they do not carry information about the proximate causes producing these phenomena*. The laws of motion served as a theoretical toolbox that allows one to gather information about the forces acting upon bodies. By Law I, Newton could infer the presence of an impressed or centripetal force from non-inertial motion; by Law II, he could establish both the magnitude and direction of an impressed or centripetal force producing non-inertial motion; and, by Law III, he was able to relate the impressed or centripetal force to its counteracting reaction force.[207]

Another way of putting it is that *The Opticks* could not transcend the tradition of the *scientiae mixtae* and of geometrical optics more particularly – at least not without considerable abductive and inductive risk. In contrast to the physico-mechanical theory of the *Principia*, which provides a causal explanation of a given phenomenon from within the theory, a mixed science mathematically describes a given phenomenon mathematically, without an accompanying explanatory story.[208] Newton,

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201 Ibid., p. 4.

202 Ibid.

203 Ibid., p. 5.

204 Ibid.

205 Ibid.

206 Ibid.

207 See Section 2.4.4 in Chapter 2.

208 Cf. Schuster, "'Waterworlds': Descartes' Vortices and their Crafting as Explanations of Gravity", p. 37. A lot of additional historical work remains to be done on the status of the mixed sciences and relation to physico-mathematics in the early modern period. For the early modern period, see Dear, *Discipline and Experience*, esp. Chapter 6. On the mixed sciences, Laird, *The Scientiae Mediae in Medieval Commentaries on Aristotle's Posterior Analytics* remains central. A useful overview of the "mixed sciences" is provided in Brown, "The Evolution of the Term 'Mixed Mathematics'". In his *Discipline and Experience*, Peter Dear explains that the emergence of physico-mathematics is to be considered as a answer to some characteristic tensions within the mixed sciences, which in order to establish themselves as rigorous associated themselves with *mathematical* demonstrations, but to a lesser extent with *physical* explanations (Dear, *Discipline and Experience*, pp. 163, 168, 170).

however, wanted to do more than to simply establish the phenomenological laws regulating optical phenomena: he also wanted to provide a solid physical account of optical phenomena.[209] In order to be explanatory, a mixed science requires the addition of a causal story, which by definition falls outside of the mathematical description. Because of his insistence of causal explanations, Newton was seeking to innovate the mixed sciences in his *Lectiones opticae*. However, previous to the establishment of his *Principia*-style methodology it was unclear how the demonstration of causes was to be accomplished.

In the *Principia*, which stood in the tradition of physico-mathematics, such causal stories could be licensed by the systematic dependencies between cause and effect, which were derived from the laws of motion. This procedure of constraining the causes to be adduced was vital to Newton's *Principia*-style methodology for it remedied the introduction of speculative causes. In other words, the proximate causes to be adduced in Book III of the *Principia* are constrained by the laws of motion. This contrasts significantly with Newton's explanation of the *experimentum crucis*, viz. that white light consists of rays differently refrangible, which was established by an external consideration, i.e. a consideration which was independent of the mathematical *exposé* of the reported refraction patterns: Newton's argument from uniformity.[210] Given the absence of systematic dependencies, Newton could moreover offer only *sufficient causes* in *The Opticks*. In his letter to Newton on 13 October 1676, Anthony Lucas, the most acute observer of Newton's optical work at the time, pointed out that the heterogeneity of white light is only a sufficient cause of the effects described in the *experimentum crucis*: "Now as to this Syllogisme (the onely demonstrative proof Mr Newton can pretend to, from the *experimentum Crucis*) the maine difficulty lyes in the Minor, viz *that this unequall refraction necessarily implyes an unequall refrangibility in rays differently coloured*. Which Minor is neither evident in its selfe [...] nor evinced from the *experimentum Crucis*, this experiment at most prooving an unequall refraction at an equall incidence, but not evinceing that this inequality ariseth from an unequall *refrangibility*, *intrinsecall* to *rays differently coloured*, rather than from some extrinsecall and accidental cause [...]."[211] More generally, given the empirical and methodological problems which Newton encountered while trying to methodize optics in a *Principia*-style, he found out that establishing non-hypothetical causal interpretations of optical phenomena was quite a difficult matter. As we have documented in the preceding sections, Newton's ideal of deducing causes by their effects turned out to be unattainable in optics: first, the fundamental principles in optics remained uncertain, secondly, no methodological justification for the use of transduction within the realm of optics could be provided,[212] and, thirdly, the causal claims introduced in

[209] Dijksterhuis, "Once Snell Breaks Down", p. 185.

[210] See Section 4.5 in this chapter.

[211] Newton, *Correspondence*, II, p. 104.

[212] Cf. Shapiro, "Newton's 'Achromatic' Dispersion Law", pp. 123, 128.

The Opticks could not be constrained by theory. As a consequence, in *The Opticks* Newton did not succeed in refraining from hypotheses.

Hypothesizing that the constituents of optical phenomena are particles would surely have been interesting from an inferential perspective. If the rays of light are assumed to be corpuscular, it could be claimed that they are attracted by material bodies. If they have different masses it could be claimed that they are subjected to different deviations in function of their masses.[213] Assuming the corporality of light, it would also have been straightforward to state that refraction is caused by a centripetal force tending downwards along the normal.[214] At several occasions in *The Opticks*, Newton suppressed perfectly "tenable" – i.e. "tenable" from the methodological perspectives of Hooke and Boyle – working-hypotheses, because they were not rigidly deduced from phenomena.[215]

Newton's optical research taught him how difficult it was to adhere to one of his core methodological credos: "~~↓it is the nature of↓ this Philosophy ↓to↓ assent any ↓no↓thing more then can be proved by Experiments~~."[216] Transcending the strictly observational features of optical phenomena, an activity which other less methodologically rigid natural philosophers were happily pursuing, did not exactly fit hand in glove with Newton's anti-hypothetical standards. Ultimately, Newton's "failure" in *The Opticks* consisted in his failure to meet the very standards he had formulated himself. In his optical work, therefore, Newton was genuinely facing the limits of deductions from phenomena.

213 Ronchi, *The Nature of Light*, pp. 162–163; Westfall, "The Development of Newton's Theory of Color", p. 357 and CUL Add. Ms. 3996, f. 72^{v}, cf. CUL Add. Ms. 3970, f. 291^{r}.

214 Cf. "If refraction be performed by attraction of the rays, the sines of incidence must be to the sines of refraction in a given proportion as we shewed in o^{r} Principles of Philosophy; & this Rule is true by experience." (CUL Add. Ms. 3970, f. 289^{r}).

215 Cf. Shapiro who notes on Newton's periodic ethereal vibrations: "Newton, [. . .], was trapped by his own methodology, which compelled him to suppress his physical model and thereby rob his theory of much of its intelligibility" (Shapiro, "Huygens" *Traité de la Lumière* and Newton's *Opticks*", p. 224).

216 Crossed-out sentence on CUL Add. Ms. 3968, f. 586^{v} [ca. 1714].

Appendix: Transcription of CUL Add. Ms. 3965, f. 422r [ca. 1715]

Here follows my transcription of CUL Add. Ms. 3965, f. 422r (see Fig. 4.5).

DEFINITIO II[217]

Corpus voco rem omnem ↓mobilem &↓ tangibilem qua tangentibus resistitur, & cujus resistentia, si satis magna sit, sentire potest.

Hoc enim sensu vulgus vocem corporis semper accipit. Et hujus generis sunt metalla, lapides, arenæ, argilla, lutum, terra, salia, ligna, ossa, carnes, aqua, oleum, lac, sanguis, aer, ventus, fumus, exhalatio, flamma, & quicquid sub elementis quatuor comprehendi potest, vel ab his exhalando manare, & in hæc per condensationem redire. Addo corpora coelestia. Hæc emittunt et reflectunt lucem et inter phænomena numerantur, a partibus suis ~~premun~~ incumbentibus premuntur, figuram rotundam induunt & in motibus suis observant leges corporum. Vapores & exhalationes ob raritatem suam amittunt resistentiam prope omnem sensibilem, & apud vulgus sæpe amittunt etiam nomen corporum & spiritus vocantur. Corpora tamen vocari possunt quatenus sunt effluvia corporum & ↓vim habent↓ resistendi ~~vim habent~~ densitati proportionalem. Solida mathematica non ~~sentiuntur~~ ↓agunt↓ tangendo ~~nec resistentiam creant~~, neque corpora dici solent. [Quintessentia ab elementis quatuor diversa nullis est sensibus obnoxia nec inter phænomena numerari potest. Materia prima quæ nec quid nec quale nec quantum est, non est phænomenon. Orbes solidi in quibus Planetæ ~~innatent~~ inhæreant non sunt phænomena. Materia subtilis in qua ↓Planetæ↓ innatent et ~~in quibus~~ corpora sine resistentia moveantur, non est phænomenon. Et quæ phænomena non sunt nec ullis sensibus obnoxia, ea in Philosophia experimentali locum non habent. Argumentum Inductionis ab experimentis et sensibilium observationibus desumptum, in quo Philosophia experimentalis fundatur, ad entia vel hypothetica vel metaphysica quæ phænomena non sunt, applicari non potest nisi per hypothesin, ideoque quæ de corporibus vi Inductionis in hoc libro dicuntur, ad ejusmodi entia nil spectant. De solis sensibilibus et eorum partibus hic agitur propterea quod argumentum Inductionis in ijs solis locum habeat. Reliqua quæ non sentiuntur sed per hypothesin tamen a nonnullis corpora nominantur, in Metaphysica et Philosophia hypothetica rectius tractanda sunt. ↓A phænomenis Philosophia ~~naturalis~~ incipit↓. In his tractandis Philosophia experimentalis consistit. A~~b ?his?~~ Philosophia ↓experimentali ad rerum↓ ~~ad~~ causas efficientes & finales, & ↓ab his omnibus ad naturam rerum insensibilium & ultimo↓ ad Philosophiam hypotheticam transeundum est:] Initio Libri primi quantitatem materiæ definivi ut tractaretur ~~physice~~ mathematice: hic corpus ex tali materia constans definio ut tractetur physicè.[218]

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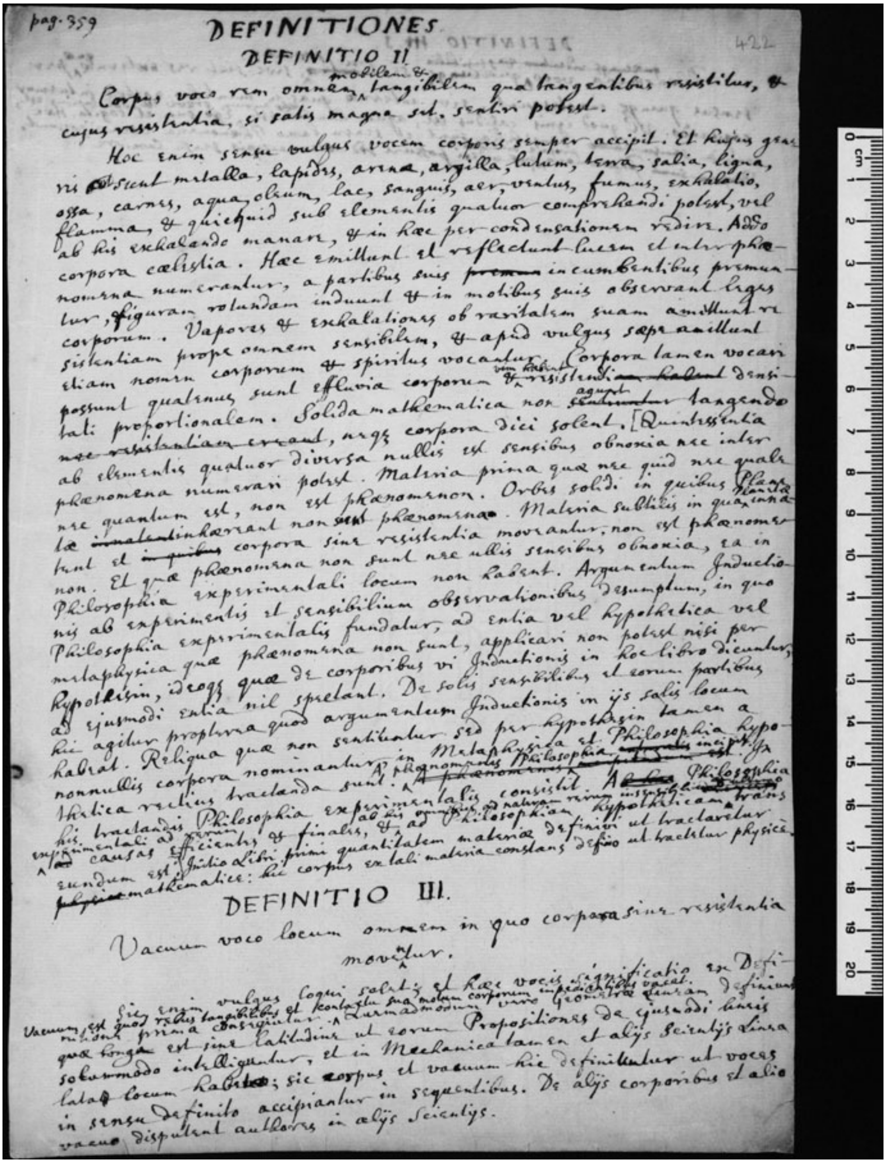
217 Cf. CUL Add. Ms. 3965, f. 504r. In Newton's final list of corrections for the second edition of the *Principia* these definitions were crossed out and did not make it into print. CUL Add. Ms. 3965, f. 437v seems to contain a previous version of this definition.

218 Cf. CUL Add. Ms. 3965, f. 430r, f. 544r; cf. CUL 9597.2.18.97, [ff. 1r–2r].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DEFINITIONES

DEFINITIO II

Corpus voco rem omnem mobilem & tangibilem qua tangentibus resistitur, & cujus resistentia, si satis magna sit, sentiri potest.

Hoc enim sensu vulgus vocem corporis semper accipit. Et hujus generis sicut metalla, lapides, arena, argilla, lutum, terra, salia, ligna, ossa, carnes, aqua, oleum, lac, sanguis, aer, ventus, fumus, exhalatio, flamma, & quicquid sub elementis quatuor comprehendi potest, vel ab his exhalando manare, & in hæc per condensationem redire. Addo corpora cœlestia. Hæc emittunt et reflectunt lucem et inter phænomena numerantur, a partibus suis incumbentibus premuntur, figuram rotundam induunt & in motibus suis observant leges corporum. [illegible]

DEFINITIO III.

Vacuum voco locum omnem in quo corpora sine resistentia moventur.

[illegible]

Fig. 4.5 CUL Add. Ms. 3965, f. 422r. Courtesy of the Cambridge University Library, Manuscript Department

Chapter 5
Uncovering the Methodology of the *Principia* (III): A Brief Chronology of Newton's Methodological Itinerary

5.1 Introduction

In the preceding chapters I have dealt with Newton's methodology in a systematic rather than strictly chronological fashion. In this brief chapter, I wish to provide a general chronological overview of Newton's methodological itinerary – thereby summoning together the material we have discussed previously. For this purpose, I shall divide the development of Newton's methodology in four phases.

5.2 The Early Period (ca. 1671–1675): Mathematizing Optics

The early Newton was influenced by two dominant traditions: the text-book tradition on logic and method and the Barrovian programme of mathematizing nature. Newton's early way of studying the natural world (ca. 1671–1675) can be seen as a synthesis of the causal tradition epitomized by the textbooks on logic and method[1] and the mathematically oriented natural-philosophical legacy of Isaac Barrow which emphasized the need for structural similarity between mathematical terms and operations and physical concepts and processes. Although Barrow embraced the idea that mathematical physics could establish causes, he restricted the sort of causation involved to formal causation. Newton enriched the Barrovian tradition with the notion of efficient causation. In the spirit of his early methodological programme, Newton asserted on the outcome of the *experimentum crucis* that the "*true cause* of the length of that Image was detected to be no other, then that *Light* consists of Rays differently refrangible."[2] Similarly, in his *Lectiones opticae* Newton stated that he intended to "describe individually the *particular and immediate causes of the effects* that I have not previously treated."[3] However, Newton was very careful not to introduce material causation in physics. With respect to the cause of prismatic dispersion, Newton provided a specific type of structural explanation of the

[1] See Sections 1.4–1.5 in Chapter 1.

[2] Cohen, ed., *Isaac Newton's Papers and Letters on Natural Philosophy*, p. 51 [italics added].

[3] Shapiro, ed., *The Optical Papers of Isaac Newton*, I, p. 523 [italics added].

S. Ducheyne, *The Main Business of Natural Philosophy*, Archimedes 29,
DOI 10.1007/978-94-007-2126-5_5, © Springer Science+Business Media B.V. 2012

dispersion patterns he observed: white light is composed of, i.e. structured as, a heterogeneous mixture of different rays, which in their turn are refrangible at constant rates.[4] Newton made a theoretical claim on the structure of white light, but not on the constitution or nature of the rays composing that mixture – an analogue case would appear later in Newton's discussion of the cause of gravity.[5] Accordingly, Newton stressed to have considered light "abstractly [. . .] without determining what that Thing is."[6] Moreover, he pointed out that he never intended "to show wherein consists the nature and difference of colours, but onely to show that de facto they are originall & immutable qualities of rays wch exhibit them, & to leave it to others to explicate by Mechanicall Hypotheses the nature of those qualities; wch I take to be no very difficult matter."[7]

As we have seen in Section 4.6 in Chapter 4, Newton maintained that, by injecting mathematics into natural philosophy, the latter could partake in the rigidity and certainty of the former. In a bold statement in his first optical paper, which did not make it to print, Newton asserted without reservation that:

> A naturalist would scearce expect to see ye science of those [i.e., of colours] become mathematicall, & yet I dare affirm that there is as much certainty in it as in any other part of Opticks. <u>For what I shall tell concerning them is not a Hypothesis but most rigid consequence, not conjectured by barely inferring 'tis thus because not otherwise or because it satisfies all phænomena (the Philosophers universall Topick,) but evinced by ye mediation of experiments concluding directly & without any suspicion of doubt</u>. To continue the historical narration of these experiments would make a discourse too tedious & confused, & therefore I shall rather lay down the *Doctrine* first, and then, for its examination, give you an instance or two of the *Experiments*, as a specimen of the rest.[8]

Correspondingly, Newton affirmed the possibility of deducing true causes from experiments, for he noted that "the Theory wch I propounded was evinced to me, not by inferring tis thus because not otherwise, that is not by deducing it onely from a confutation of contrary suppositions, but by *deriving it from Experiments* concluding positively & directly."[9] This view was diametrically opposed to the way of pursuing natural philosophy as envisioned by an important fraction within the Royal Society. Robert Boyle and Robert Hooke had stressed that a large stock of natural and artificial histories should be collected before proceeding to the "speculative part" of natural philosophy, i.e. the search for causes. They also emphasized that the speculative part of natural philosophy is predicated under probability. In a single paper, which was essentially based on a single demonstrative experiment, Newton claimed to have established the true cause of refraction beyond a shadow of a doubt.

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[4] See Section 4.4 in Chapter 4.

[5] However, as we have seen, in the *Principia* a different notion of causation was at play: counterfactual-nomological dependency.

[6] Newton to Oldenburg, 11 June 1672, Newton, *Correspondence*, I, p. 174.

[7] Newton to Oldenburg, 3 April 1673, ibid., I, p. 264. Cf. Newton's second optical paper (ibid., I, pp. 373–374).

[8] Newton to Oldenburg, 6 February 1671/2, ibid., I, pp. 96–97 [underscore added].

[9] Newton to Oldenburg, 6 July 1672, ibid., I, p. 209.

No wonder that Robert Hooke was ardently pressed to criticize Newton's approach and his claims on certainty! Newton's *entrée* in the *Philosophical Transactions of the Royal Society* in 1671/2 was therefore not only a scientific debut: Newton presented a new methodological ideal on how knowledge of the empirical world is to be established. After Hooke's criticism, Newton had to weaken his claims on the certainty harvested by natural philosophy – although the ideal of demonstrative certainty in natural philosophy never faded. In a letter to Oldenburg, Newton declared that:

> In the last place I should take notice of a casuall expression wch intimates a greater certainty in these things that I ever promised, viz: *The certainty of Mathematicall Demonstrations*. I said indeed that the *Science of Colours was Mathematicall & as certain as any other part of Optiques*; but who knows not that Optiques & many other Mathematicall Sciences depend as well on Physicall Principles as on Mathematicall Demonstrations: And the absolute certainty of a Science cannot exceed the certainty of its Principles. Now the evidence by wch I asserted the Propositions of colours is in the next words expressed to be from *Experiments* & so but *Physicall*: Whence the Propositions themselves can be esteemed no more then *Physicall Principles* of a Science. And if those Principles be such that on them a Mathematician may determin all the Phænomena of colours that can be caused by refractions, & that by computing or demonstrating after what manner & how much those refractions doe separate or mingly the rays in wch severall colours are originally inherent; I suppose the *Science of Colours* will be granted *Mathematicall* & as certain as any part of Optiques.[10]

While at the same time rejecting the "preposterous method" of conjectures and probabilities (and thereby implicitly assuming the idea of certain natural-philosophic knowledge), he insisted that by mathematizing nature we can establish a natural science "supported by the greatest evidence."[11] As we have seen in Section 2.2 in Chapter 2, the method of hypothesis, as promoted by Boyle and Hooke, put two important questions on the agenda of later generations of natural philosophers, the question of transduction and the question of causal sufficiency. Newton sought to overcome these very problems by developing a different natural-philosophical method. In order to evade the problem of transduction, Newton chose to treat light "abstractly." In order to provide explanations superior to causally sufficient ones, he relied on the mathematical nature of his demonstrations. Newton was highly critical of causally sufficient explanations. "For if," Newton recorded, "the possibility of hypotheses is to be to test the truth and reality of things, I see not how it may certainty can be obtained in any science; since numerous hypotheses may be devised, which shall seem to overcome new difficulties."[12] In his early optical work, Newton was confronted with an essential tension: on the one hand, he tried to concede to Hooke's criticism that the certainty of mathematized natural philosophy is "but *Physicall*" (and in doing so, he came close to defending causal sufficiency and farther from "concluding directly & without any suspicion of doubt"), while,

[10] Newton to Oldenburg, 11 June 1672, ibid., I, pp. 187–188 [underscore added].

[11] Shapiro, ed., *The Optical Papers of Isaac Newton*, I, pp. 87/89.

[12] Newton to Oldenburg for Pardies, 10 June, 1672, Newton, *Correspondence*, I, p. 164.

on the other hand, he wanted to go beyond the conjectural and probable status of causally sufficient explanations. In his early years as natural philosopher, Newton also came to systematically separate the demonstrative part of natural philosophy from the hypothetical part – a separation that is reflected by the titles of his two optical papers, *A New Theory about Light and Colors* (1671/2) and *An Hypothesis explaining the Properties of Light, discoursed of in my several Papers* (1675).

In the early phase of his methodological development, Newton's characterization of natural-philosophical method was more negative than positive. It was clear that Newton rejected the method of hypothesis which was vexed with the problem of causal sufficiency, the problem of transduction and the problem of under-determination. However, in terms of positively characterizing his method, Newton was less explicit. Undoubtedly, proper natural-philosophical methodology had something to do with mathematics and deriving causes from effects, but Newton's characterization of this remained on a fair level of generality during this period. Mathematizing nature surely enables one to establish accurate descriptions of physical quantities, but, in order to establish causal and theoretical claims, a certain level of theoretical interpretation, which is by definition absent from a strictly mathematical apparatus, is required. In other words, in order to establish the theoretical conclusions that Newton sought to make in the domain of optics, the results gathered from a mixed-mathematical treatment of optical phenomena needed to be supplanted by some sort of theoretical interpretation. Now what was the status of such theoretical interpretation, if it is not imbedded in our mathematical apparatus? It is hard to tell when exactly Newton became aware of this issue. What is certain, however, is that, when Newton had established his superior methodology of the *Principia*, the difference with his earlier mixed-mathematical treatment of optical phenomena would have been impossible for him to ignore – see Section 5.3 in this chapter.

5.3 The *Principia*-Period (ca. 1684–1687): The *Principia* and Its Methodology

In the *Principia* Newton succeeded in finding a solution to the problems of physical interpretation by integrating mathematics with the laws of motion and establishing a physico-mathematics. In the *Principia*, Newton established a physico-mathematics by which propositions could be derived which were backed-up by the laws of motion.[13] This was a procedure Newton followed in order to minimize the inductive risk involved in unravelling the causes of terrestrial and celestial motions. In Book I of the *Principia*, Newton established that, given the laws of motion, inverse-square centripetal forces are the necessary and sufficient of Keplerian motion – and,

[13] The distinctive features of Newton's methodology have been outlined in Section 2.6 in Chapter 2 and in Section 3.6 in Chapter 3.

moreover, that inverse-square centripetal forces *quam proxime* directed to a centre are the necessary and sufficient causes of *quam proxime* Keplerian motion.[14] The requirement for necessary and sufficient causes was Newton's answer to the problem of causal sufficiency. We can confidently pinpoint the moment of the introduction of Newton's requirement of providing necessary *and* sufficient causes. In the original tract *De motu* (autumn 1684) Newton proceeded from the assumption of a centripetal force being exerted to the mathematical properties entailed thereby,[15] and as a consequence, he only proved the sufficient direction, while in the initial revise of *De motu* (winter/early spring 1684–1685) he demonstrated both directions.[16] In the initial revise of *De motu*, Newton also introduced the *quam proxime* causal inference-tickets[17] and the micro-macro inference-tickets.[18] Shortly afterwards, Newton introduced the systematic discrepancies: in *Liber I De motu corporum* (summer 1685–winter 1685/6), he introduced Proposition XLV, Book I,[19] and, in the first edition of the *Principia* (1687), he introduced Corollary 7 to Proposition IV, Book I.[20] The methodological significance of these types of propositions has been highlighted in Section 2.4 of Chapter 2. At this point, Newton's claim that he could deduce causes from effects was no longer contentious: for Newton had established the necessary and sufficient causes of Keplerian motion, *given the laws of motion*, and, as a consequence of this, he could proceed from phenomena to causes by relying on the necessary direction of the bi-conditional dependency, which he had established between the presence of a centripetal force and Keplerian motion. In order not the feign hypotheses on the cause of gravity, Newton relied on a counterfactual-nomological notion of causation which was based on Law I.[21] We have seen in Section 3.2 of Chapter 3 that Newton's methodology also encompassed a set of rules which guided inductive generalization once instances of different inverse-square centripetal forces have been derived. The status of these rules was to change dramatically in Newton's later thought – see Section 5.5 in this chapter.

5.4 The Post-*Principia* Period (ca. 1690–1704): The Implications of the *Principia* Methodology for Newton's Optical Research

Once having established his *Principia* style methodology, Newton increasingly became aware of the impossibility of methodizing his optical work in line with his *Principia*-style methodology. As we have seen in Section 4.8 in Chapter 4, Newton

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14 See Section 2.5.1 in Chapter 2.

15 Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VI, pp. 34 ff.

16 Ibid., VI, pp. 122–127.

17 See Section 2.5.4 in Chapter 2.

18 See, respectively, ibid., Corollaries 2–3 to Proposition III, pp. 126–129 and pp. 180–187.

19 Proposition XLV in: ibid., VI, pp. 369–383.

20 See Section 2.5.2 in Chapter 2.

21 See Section 1.6 in Chapter 1.

became aware of the glaring problem of transduction within the realm of optics. In *The Opticks* Newton could not rely on a set of physical principles which could generate causal inference-tickets without making hypotheses about the nature of light. As Alan E. Shapiro's study *Fits, Passions, and Paroxysms* and Chapter 4 show, in order to respect Newton's rigid separation between the demonstrative and the speculative part of natural philosophy, in *The Opticks* Newton cleansed his optical theories from their physical content, which gave rise to a rather phenomenological and mixed-mathematical approach.[22] Closely related to the absence of causal inference-tickets was the fact that Newton could not infer the *explanans* for refraction in the same way as he had derived inverse-square centripetal forces. In the *Principia*, inferring causes could be done by the (*quam proxime*) causal inference-tickets. In order to establish that the inherent refrangibility of white light is the true cause of refraction, Newton had relied on an argument of uniformity in order to establish a single causal claim. Here he could not rely on Rule II, which licences the identification of two different and independently established instances of causal parameters of the same kind.[23] Newton could not have been other than aware of this asymmetry. The fact that Newton in *The Opticks* or in manuscript never referred to Rule II in the context of the argument for the *explanans* of refraction is highly significant in itself. Ironically, the discipline, which Newton so ardently tried to render certain by mathematics in his younger years, turned out to be the most resilient to the rigid methodization he had hoped for, once he had developed his *Principia*-style methodology.

5.5 The Later Period (ca. 1713–1726): Inductive Provisionalism

In the second edition, Newton radically re-interpreted the status of the *regulae philosophandi*. The formulation of Rule II underwent significant change: whereas in the first edition of the *Principia* Newton wrote that "*the causes assigned to natural effects of the same kind are the same*",[24] in the second and third edition he was more careful: "*the causes assigned to natural effects of the same kind must be, so far as possible, the same*."[25] In the second edition Newton also introduced Rule III which stated that "*Those qualities of bodies that cannot be intended and remitted [i.e., qualities that cannot be increased and diminished] and that belong [competent] to all bodies on which experiments can be made should be taken as qualities of all bodies universally*."[26] In the second edition of the *Principia*, Rules II and III clearly reflected Newton's idea of accepting established scientific propositions provisionally. Finally, in the third edition of the *Principia* Newton introduced Rule IV:

[22] See Section 4.10 in Chapter 4.

[23] See Section 4.5 in Chapter 4.

[24] *PrE*$_1$, p. 402.

[25] Newton, *The Principia*, p. 794.

[26] Ibid., p. 795.

"*In experimental philosophy, propositions gathered from phenomena by induction should be considered either exactly or very nearly true notwithstanding any contrary hypotheses, until yet other phenomena make such propositions either more exact or liable to exceptions*."[27] Newton was aware of the need of introducing something along the lines of Rule IV at least by March 1713, when Roger Cotes pointed out to Newton that by applying Law III to non-adjacent bodies he was *hypothesim fingere* – see Section 3.4.3 in Chapter 3.[28] In the draft version of Newton's formal reply to Cotes, Newton wrote that:

> It is not enough to object that a contrary phænomenon may happen but to make a legitimate objection, a contrary phenomenon must be actually produced. Hypothetical Philosophy consists in imaginary explications of things & imaginary arguments for or against such explications, or against arguments of Experimental Philosophers founded upon Induction. The first sort Philosophy is followed by me, the latter too much by Cartes, Leibnitz & some others.[29]

In Query 31, which was introduced in the second edition of *The Opticks* (1717), Newton pointed out that although inductive-experimental arguments do not provide universal demonstrations, they are "the best way of arguing which the Nature of Things admits of:"

> And although the arguing from Experiments and Observations by Induction be no Demonstration of general Conclusions; yet it is the best way of arguing which the Nature of Things admits of, and may be looked upon as so much stronger, by how much the Induction is more general. And if no Exception occur from Phænomena, the Conclusion may be pronounced generally. But if at any time afterwards any Exception shall occur from Experiments, it may then begin to be pronounced with such Exceptions as occur.[30]

In manuscript material Newton was more explicit on the meaning of Rule IV: for, "if arguments based on hypotheses were to be admitted against inductions, then inductive arguments, on which the whole of experimental philosophy is based, could always be overturned by contrary hypotheses."[31] If a proposition gathered by induction is not sufficiently accurate, then it should be corrected, not by (introducing ad hoc) hypotheses, but by more widely and accurately observed phenomena of nature.[32]

Newton was thus perfectly aware of the risk involved in inductively generalizing. As I have argued in Section 3.2 in Chapter 3, the provisionalism Newton envisioned did not apply to the inferences of instances of inverse-square centripetal forces in Propositions I–IV of Book III, but rather to the identification of causes producing effects of the same kind in Proposition V and especially to the generalization to universal gravitation in Propositions VII–VIII. The provisionalism embedded in

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[27] Ibid., p. 796.

[28] Cotes to Newton, 18 March 1712/13, Newton, *Correspondence*, V, pp. 391–394, p. 392.

[29] Draft of Newton to Cotes, 28 March 1713, Newton, *Correspondence*, V, pp. 398–399.

[30] Newton, *The Opticks*, p. 404; OE_2, p. 380.

[31] CUL Add. Ms. 3965, f. 419^v [additions and corrections to the second edition of the *Principia*].

[32] Ibid.

the *regulae philosophandi* also reflects Newton's characteristic tackle on problems of motion: to approach the matter by a series of successive approximations. As is clear from what we have surveyed in the preceding chapters, *Newton's methodology was far from static: it evolved and was elaborated and fine-tuned in the course of Newton's natural-philosophical career*.

We have now come to the end of my treatment of Newton's methodology in the *Principia* and *The Opticks*. Since Newton's natural-philosophical considerations were closely intertwined with theological ones, in the third and final part of this book I seek to elucidate the relationship between both.

Part III
Newton's Theology

Chapter 6
"To Treat of God from Phenomena"

6.1 Introduction

In Part II I have analysed Newton's method of acquiring knowledge about the empirical world. As his manuscripts testify and as research over the last decades has made abundantly clear, Newton did not, however, limit himself to the sort of knowledge obtained by a methodized study of the empirical world alone. He equally accepted that, by carefully studying the Scriptures and by taking into account the results harvested by natural philosophy, knowledge could be obtained about the divine creator, his providence, and his dominion over the world and his servants. In this chapter, my focus is on *the nature of the rapport between Newton's theology and natural philosophy*. Although it is not my current endeavour to offer a detailed chronological account of Newton's theological work, this chapter is thoroughly based on his theological manuscripts. In this chapter, special attention will be paid to the theology of the General Scholium, which provides an excellent *entrée* into Newton's views on the interaction between theology and natural philosophy. Although Newton had been privately studying matters theological before 1713, in the General Scholium,[1] which was added to the second edition of the *Principia*, he composed several paragraphs in which he publicly communicated some of his theological views.[2]

For the manuscripts not pertaining to the Portsmouth and Macclesfield Collection, I have relied on the transcriptions provided by *The Newton Project* (http://www.newtonproject.ic.ac.uk) – although I should note that I have converted *The Newton Project's* text-editorial conventions into my own. I have also followed *The Newton's Project's* dating of Newton's theological manuscripts. The material drawn from the *Of the church* at the Bodmer Foundation in Geneva results from my own inspection of the original. For Royal Society Gregory Ms. 247 I have compared Schüller, "Newton's *Scholia* from David Gregory's Estate" against the original and I will follow Schüller's excellent transcriptions of this notoriously difficult material.

[1] For the editorial details, see: Cohen, *Introduction to Newton's* "Principia", pp. 240–245, 249–251 and Westfall, *Never at Rest*, pp. 744–751. For a catalogue of extant and related manuscript material, see Appendix 1.

[2] While the 16 Queries added to the first edition of *The Opticks* (1704) did not contain any theological material (OE_1, pp. 132–137), *Questiones* 20 and 23 of the first Latin edition of *The Opticks*

DOI 10.1007/978-94-007-2126-5_6,

Newton was involved in theology long before the first edition of the *Principia* and his interest in theology remained a *basso continuo* throughout his intellectual life. As the historical records show, Newton became seriously interested in theological matters in the mid and especially the late 1670s.[3] In the mid and especially the late 1670s, Newton began working on prophecy[4] and in the late 1670s he made his first modest steps in Church history[5] and chronology.[6,7] In the 1680s, Newton began working on Gentile theology[8] and on the dimensions of the Temple of

(1706) did (*OEL*$_1$, pp. 314–315, 343–348, respectively). The Queries crystallized in their final form in the second edition of *The Opticks* (1717).

[3] To fulfil his requirements as Lucasian Professor, Newton was preparing to take up holy orders in the early 1670s. Due to the intervention of Isaac Barrow, Newton and all future Lucasian Professors were exempted from this obligation in April 1675 (Westfall, *Never at Rest*, p. 333). Newton was, however, not exempted from the requirement of preparing a divinity act, in which he was to consider "the orthodox case against Socinianism" (Mandelbrote, "'Than this noting can be plainer:' Isaac Newton reads the Fathers", p. 283). When preparing for this task, Newton got his first taste of scholarly theological discussions and at the same time he was exposed to heterodox theology. At the time, divinity acts were increasingly neglected and ultimately Newton never kept his (Westfall, *Never at Rest*, pp. 184–185). Traditionally, Newton's heterodoxy has been dated as early as 1673 (ibid., p. 332). Scott Mandelbrote has correctly pointed out, however, that it is quite difficult to date the start of Newton's own closet heterodoxy exactly (Mandelbrote, "'Than this noting can be plainer:' Isaac Newton reads the Fathers", p. 283). There is no clear-cut manuscript evidence that supports Westfall's early dating of Newton's heresy. See Buchwald and Feingold, *Newton and the Origin of Civilizations*, Chapter 4 for further elaboration. On the Lucasian statutes, see Stewart, "The Lucasian Statutes: Translation and Introduction".

[4] Yahuda Ms. 1 [Untitled treatise on Revelation; ca. 1670s–1680s], Yahuda Ms. 2.1 [Part of a treatise on interpreting the symbolism of Biblical prophecy; 1670s], Yahuda Ms. 2.2 [Incomplete chapter *Quod Bestia bicornis locata sit ut Draco*; late 1670s–early 1680s], and Keynes Ms. 1 [*Tuba Quarta*; ca. 1675–1680].

[5] Yahuda Ms. 2.3 [Various text on Revelation, Solomon's Temple and Church history; late 1670s–early 1680s], Yahuda Ms. 12 [Treatise on Church History; late 1670s], Ms. 28b [Fragments on the kingdoms of the European tribes, the Temple and the history of Jewish and Christian Churches; ca. 1675–1685], and Ms. 29 [Fragments on Church history, mainly concerning Athanasius; ca. 1675–1685].

[6] Yahuda Ms. 28a [Jottings on chronology; ca. 1675–1685].

[7] In the 1670s, Newton also made notes from the French Jesuit Petavius (1583–1652) on the Council of Nicaea (325) (Keynes Ms. 4 [1670s]) and he finished an exposition of 2 Kings, 17: 15–16 (to wit: "And they rejected his statutes, and his covenant that he made with their fathers, and his testimonies which he testified against them; and they followed vanity, and became vain, and went after the heathen that were round about them, concerning whom the LORD had charged them, that they should not do like them. And they left all the commandments of the LORD their God, and made them molten images, even two calves, and made a grove, and worshipped all the host of heaven, and served Baal."). See Yahuda Ms. 21 [Exposition on 2 Kings 17:15–16; 1670s], Babson Ms. 437 [Part of an Exposition of 2 Kings 17:25–26; 1670s], and Harry Ransom Humanities Research Center Ms. 130 [Exposition on 2 Kings 17:15–16; 1670s]. All scriptural references in this chapter are taken from Carroll and Prickett, eds., *The Bible, Authorized King James Version with Apochrypha*.

[8] Yahuda Ms. 16 [Rough draft portions of and notes for *Theologiæ Gentilis Origines Philosophicæ* and *The Original of Monarchies*; 1684–1690], Yahuda Ms. 17 [Three bundles of notes on the ancients' physico-theology, related to *Theologiæ Gentilis Origines Philosophicæ*; 1680s and early

Salomon,[9] whilst continuing his work on prophecy[10] and elaborating his work on Church history.[11] Between ca. 1684–1690, Newton kept a theological notebook (ca. 67,000 words).[12] In the 1690s, Newton composed *Paradoxical Questions concerning the morals and actions of Athanasius and his followers*,[13] *Two Notable Corruptions of Scripture*,[14] *Prolegomena ad Lexici Prophetici Partem secundam*,[15] the so-called "Classical Scholia"[16] (1694), several draft chapters for a treatise on *The Original of Religions*,[17] and *Variantes Lectiones Apocalypticae*,[18] while continuing his work on Gentile theology.[19] After 1700 Newton composed several

1690s], and Yahuda Ms. 33 [Notes on ancient Greek, Roman and Egyptian deities; 1680s]. Newton owned the 1641 edition of Vossius' *De theologia Gentili, et physiologia Christiana*, which shows signs of extensive dog-earing (Harrison, *The Library of Isaac Newton*, p. 258 [item n° 1697].

9 Yahuda Ms. 2.4 [1680s–1690s].

10 James White Library ASC Ms. N47 HER [*Prophesies concerning Christs 2nd coming*; early-1680s], Yahuda Ms. 9 [mid-late 1680s] and Keynes Ms. 5 [Two draft theological treatises; ca. mid-1680s and ca. 1705–1710].

11 Yahuda Ms. 2.5b. Newton's preoccupation with theological matters in the 1680s can also be seen from *De gravitatione et aequipondio fluidorum* (CUL Add. Ms. 4003; see, furthermore, Section 6.4.2 in this chapter and Section 3.3 in Chapter 3) and from his letter to Thomas Burnet on January 1680/1 (Newton, *Correspondence*, II, pp. 329–335; see furthermore, Mandelbrote, "Isaac Newton and Thomas Burnet").

12 Keynes Ms. 2.

13 Keynes Ms. 10 [ca. early 1690s]. See also Williams Andrews Clark Memorial Library Ms. **N563M3 P222 [ca. early 1690s]. See Iliffe, "Prosecuting Athanasius: Protestant Forensics and the Mirrors of Prosecution" and, especially, Delgado-Moreira, "Newton's 'Paradoxical Questions concerning the Morals & Actions of Athanasius and his Followers' and its Intellectual Origins", for ample discussion.

14 See Newton to Locke, 14 November 1690, Newton, *Correspondence*, III, pp. 83–129, Newton to Locke, ? November 1690, ibid., III, pp. 129–144 and New College Library Ms. 316(4) [Various drafts and copies of the *Two Notable Corruptions of Scripture* and related material; 1690–91]. In this material, Newton argued that several Trinitarian passages in the Bible, including 1 John 5:7 ["For there are three that bear record in heaven, the Father, the Word, and the Holy Ghost: *and these three are one*."] and 1 Timothy 3:16 ["And without controversy great is the mystery of godliness: *God was manifest in the flesh*, justified in the Spirit, seen of angels, preached unto the Gentiles, believed on in the world, received up into glory"], were corruptions inserted afterwards. See, furthermore, Champion, "Acceptable to inquisitive men".

15 Babson Ms. 434 [after 1690]. Cf. Harry Ransom Humanities Research Center Ms. 132 [Notes on the Temple of Solomon and a tabular comparison of measurement systems; probably after 1690]. Babson Ms. 434 has been transcribed in Morano and Sánchez Ron, *Isaac Newton, El Templo de Salomón* and Morano, *Isaac Newton, El Templo de Salomón*.

16 Cohen, *Introduction to Newton's* "Principia", pp. 188–189; Westfall, *Never at Rest,* pp. 510–511. See, furthermore, Appendix 2 to this chapter.

17 Yahuda Ms. 41 [Draft chapters of a treatise on the origin of religion and its corruption; ca. early 1690s].

18 Yahuda Ms. 4 [1693 and earlier].

19 New College Library Ms. 361(3) [Papers relating to chronology and *Theologicæ Gentilis Origines Philosophicæ*; after 1693]. For Newton's preoccupation with theological matters in the 1690s, see, furthermore, his letters to Richard Bentley, which we have discussed in Section 1.7 in Chapter 1.

shorter theological manuscripts,[20] he worked on a long treatise on the history of the Church,[21] and he started working on *The Chronology of Ancient Kingdoms Amended*[22] and *Observations upon the Prophecies of Daniel, and the Apocalypse of St. John*,[23] which were posthumously published in 1728 and 1733, respectively.[24]

In the first edition of *Principia*, which ended with Proposition XLII, "Trajectorum Cometæ graphicè inventam corrigere", and a list of errata, Newton did not provide a proper conclusion to his revolutionary physico-mathematical edifice. On 2 March 1712/1713 Newton declared to Roger Cotes, who was in charge of the editorial work for the preparation of the second edition of the *Principia*, that he wanted to add a *scholium* on the attraction of the small particles of bodies which would finish the book. Newton's initial intention was to say "much more about the attraction of the small particles of bodies," but, as he continued, "upon second thought I have chosen rather to add but one short Paragraph about that part of Philosophy."[25] Newton's intention of doing so dates back to the time when he was wrapping up the first edition of the *Principia* – however, the intended preface and conclusion to this edition, in which this matter would be addressed, were suppressed by him.[26] On 18 March 1712/1713, Cotes suggested to Newton that:

> I think it will be proper besides the account of the Book & its Improvements to add something more particularly concerning the manner of Philosophizing made use of & wherein it differs from that of De Cartes & others. I mean in first demonstrating the Principles it imploys. This I would not only assert but make evident by a short deduction of the Principle

[20] E.g., Keynes Ms. 11 [Twenty-three queries regarding the 'ομουσιος; early 1700s], Ms. 146 [*Chap. 1. The Original of Monarchies*; 1701–1702], Yahuda Ms. 6 [*The syncronisms of the three parts of the prophetick interpretation*; after 1700], Yahuda Ms. 8 [Notes on prophesies; post-1710], Babson Ms. 438 [Draft notes on early Church rites and the Creed; probably 1710 or later], Yahuda Ms. 435 [Fragmentary draft on the chief rulers of ancient synagogues; 1710 or later], Keynes Ms. 3 [*Irenicum, or Ecclesiastal Polyty tending to Peace*; post-1710], Ms. 6 [Seven statements on religion; post-1710], Ms. 7 [*A short Schem of the true Religion*; post-1710], Ms. 8 [Twelfe articles on religion; post-1710], and Ms. 9 [Three paragraphs on religion; post-1710].

[21] Yahuda Ms. 15 [Drafts on the history of the Church; 1710s], Bodmer Ms. [*Of the church*; ca. 1710], and CUL Add. Ms. 3989 [*Of the church*, partly in another hand].

[22] Yahuda Ms. 25 [Draft passages on chronology and biblical history; after 1710], Yahuda Ms. 26 [Draft chapters of *The Chronology of Ancient Kingdoms Amended*; post-1710], and Yahuda Ms. 27 [Seven drafts of Newton's defence of *The Chronology of Ancient Kingdoms Amended*; after 11 November 1725]. For related earlier material, see New College Library Ms. 361(1) [Drafts of the *Short Chronicle* and *Original of Monarchies*; 1701–1702] and Ms. 361(2) [Miscellaneous papers apparently comprising drafts of or notes for *The Chronology of Ancient Kingdoms Amended*].

[23] Yahuda Ms. 7 [Miscellaneous drafts and fragments on phrophesy, principally Daniel and Revelation; post-1700].

[24] Newton, *The Chronology of Ancient Kingdoms Amended* and id., *Observations upon the Prophecies of Daniel, and the Apocalypse of St. John*.

[25] Newton to Cotes, 2 March 1712/13, Newton, *Correspondence*, V, pp. 384–385.

[26] See Section 4.9 in Chapter 4.

> of Gravity from the Phænomena of Nature, in a popular way, that it may be understood by ordinary Readers & may serve at the same time as a Specimen to them of the Method of the whole Book.[27]

Furthermore, Cotes suggested to "add some things by which Your Book may be cleared from some prejudices which have been industriously laid against it. As that it deserts Mechanical causes, is built upon Miracles & recurrs to Occult qualitys."[28] Newton accepted these suggestions but ultimately decided to go beyond both Cotes' suggestions as well as his own initial intention to dedicate some words to short-range attractive forces, for Newton included theological material as well. The result would become Newton's immensely famous and tersely written General Scholium. The General Scholium highlights a diversity of themes which were central to Newton's natural philosophy in general: matters of experimentation (found in Newton's attempts to deduce non-gravitational forces from phenomena),[29] methodological issues (related to clarifying the explanatory status of gravitation), theological matters (culminating with Newton's attempt to show that naturalphilosophical inquiry has important theological implications), matters related to the instauration of *prisca sapientia* and *religio*,[30] epistemological claims central to Newton's empiricism, and, finally, metaphysical issues (Newton's treatment of space and time as logical consequences of God's existence). In other words, the General Scholium is in fact a micro-representation of Newton's various lifelong interests, which ranged from the empirical to the more abstract realms of God's attributes.

This chapter will, apart from its concluding section, consist of three main sections. In Section 6.2, I shall focus on the theology underlying the General Scholium. In Section 6.3, I will draw attention to the underlying anti-Cartesian features of the General Scholium, while basing myself on scarcely consulted portions from Newton's manuscripts. In Section 6.4, I will addres the complex interaction between Newton's theology and natural philosophy by focussing on two representative case-studies. In Section 6.5, I shall proceed to a general discussion of the matter.

6.2 The Theology of the General Scholium

Recently, Rudolf De Smet and Karin Verhelst have argued that the General Scholium highlights not only Newton's religious concerns but also his philosophical concerns – a claim that, as far as I know, has not been denied by any notable Newton

[27] Cotes to Newton, 18 March 1712/13, Newton, *Correspondence*, V, pp. 391–394. For Newton's reply on 28 March and on 31 March, see respectively: ibid., V, pp. 396–399 and pp. 400–401.

[28] Ibid., V, p. 392.

[29] See Section 4.9 in Chapter 4.

[30] See Knoespel, "Interpretative Strategies in 'Theologiae gentilis origines philosophiae'"; Markley, "Newton, Corruption, and the Tradition of Universal History"; and, Mandelbrote, "Isaac Newton and the Flood".

scholar[31] – and, correspondingly, they have attempted to explore, following B. J. T. Dobbs,[32] Newton's indebtedness to neo-Platonism and Stoicism by focussing on Philo Judaeus and Justus Lipsius.[33] Stephen D. Snobelen, on the other hand, has unravelled the heterodox underpinnings of the General Scholium, which he characterizes as a "theologically-charged appendix,"[34] and he has rendered the underlying anti-Trinitarian and, more specifically, Socinian strands explicit.[35] Snobelen has provided ample contextualisation of the theology in the General Scholium by shedding light on how Newton, in several theological manuscripts written around the same time as the General Scholium, frequently stressed that only the Father is truly "God of Gods."[36] Newton endorsed a heretic anti-Trinitarian

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31 Especially not after the pioneering work of J. E. McGuire on Newton's metaphysics (McGuire, *Tradition and Innovation*).

32 Dobbs, *The Janus Faces of Genius*, pp. 202–206. See, furthermore, Dobbs, "Newton's Alchemy and His 'Active Principle' of Gravitation".

33 Although De Smet and Verhelst provide convincing evidence for their claims on Philo (De Smet and Verhelst, "Newton's Scholium Generale", esp. p. 8), similar evidence seems to be lacking for their claims on the Cambridge Platonists and Justus Lipsius. They have merely succeeded in demonstrating vague parallelisms. Characteristic of this is their conjectural conclusion on Ralph Cudworth: "It is clear that despite the absence of explicit proof, there are sufficient similarities and parallels to suggest that Newton's debt to Cudworth was greater than one might be led to believe from his manuscript *Out of Cudworth*." (ibid., p. 13).

34 Snobelen, "God of Gods, and Lord of Lords", p. 170. Note that Snobelen's title refers to Deuteronomy 10: 17. On Newton's heterodoxy, see Iliffe, "'Those whose business it is to cavell:' Newton's Anti-Catholicism". On the background of seventeenth-century natural philosophy *cum* theology, see Mandelbrote, "The Uses of Theology in Seventeenth-Century England".

35 Snobelen, "God of Gods, and Lord of Lords", pp. 191–196. Notwithstanding parallels with Socinian authors, Newton was not a full-blown Socinian (Snobelen, "Isaac Newton, Socinianism, and 'The One Supreme God'", pp. 241–242, 255–265, 275–283). Newton owned Samuel Crell's *Initium evangelii S. Joannis Apostoli ex antiquitate ecclesiastica restitutum* (1726) (Harrison, *The Library of Isaac Newton*, p. 127 [item n° 459]), Georgius Eniedinus' (*né*: György Enyédi) *Explicationes locurum Veteris & Novi Testamenti, ex quibus Trinitatis dogma stabiliri solet* (1670) (ibid., p. 137 [item n° 557]), Jonas Schlichting's (*né*: Jonasz Szlichtyng) *Commentarius in Epistolam Hebræos* (1643) (ibid., p. 234 [item n° 1470]), and Faustus Socinus' (*né*: Fausto Sozzini) *De Iesu Christi filii Dei natura sive essentia* (1627) (ibid., p. 130 [item n° 495]). Newton and Samuel Crell actually met in 1726 (and perhaps on other occasions as well). During that meeting Crell requested Newton's help to fund the publication of his *Initium Evangelii S. Joannis Apostoli*, which appeared later that year (Snobelen, "Isaac Newton, Socinianism and 'The One Supreme God'", pp. 248–249). The letter which Crell send to Newton before this meeting is transcribed and translated in ibid., pp. 294–295.

36 Cf. Keynes Ms. 3 [*Irenicum*, post-1710], pp. 29, 35, 47–48; Keynes Ms. 7 [*A short Schem of the true Religion*; post-1710], f. 1^v; and, Yahuda Ms. 12 [*Treatise on Church History*; late 1670s], f. 1^r. In an entry "Deus pater" Newton wrote that "There is one Body, one spirit, even as ye are called in one hope of your calling One Lord, one Faith, one Baptisme, One God & Father of all, who is above all & through all & in you all. Eph 4.6." (Keynes Ms. 2 [*Theological Notebook*, ca. 1684–1690], part 1, p. XI). In this manuscript, Newton also sharply contrasted "the only true God" with "Iesus Christ whom thou hast sent" (ibid.). In a different manuscript, Newton called God's son "the *Man* Christ Jesus" (Keynes Ms. 8 [*Twelve articles on religion*; post-1710], f. 1^r [italics added]). Newton, furthermore, urged that a proper *mon*archy requires the dominion of only *one* God (cf. "Poterit autem et ad istum modum dici unum esse principium Deitatis, non duo, propria Monarchia unius

position that in part consisted of a complex mix of Arian and Socinian criticisms.[37] Numerous studies have aptly brought the importance of Newton's "God of Dominion" into perspective.[38] Correspondingly, in Newton's Christology, the

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dominatus haberi debet" (Keynes Ms. 2 [*Theological Notebook*, composed ca. 1684–1690], part 2, p. 14; see also the entry "De Deo uno" on p. 85)). Newton's radical subordinationist view of Christ is especially clear in the manuscript *Of the Church* (Bodmer Ms. [ca. 1710]; see Snobelen, "'God of Gods, and Lord of Lords'", esp. pp. 181–186). For further details on the Bodmer manuscript, see Ducheyne, "Isaac Newton's 'Of the church'".

[37] Cf. Iliffe, "Prosecuting Athanasius: Protestant Forensics and the Mirrors of Prosecution", p. 125. For an excellent study on Newton's heretical position, the tradition on which drew, the selected few to whom he privately entrusted his theological views, and his strategies of concealment, see Snobelen, "Isaac Newton, heretic: the Strategies of a Nicodemite". See, furthermore, Delgado-Moreira, "Newton's 'Paradoxical Questions concerning the Morals and Actions of Athanasius and his Followers' and its Intellectual Origins", pp. 35–45. Newton concealed his heretical position for obvious legal and social reasons, but also because of Newton's conviction that theology (the "strong meats") "should only be handled by the experienced and mature members of the remnant, and, even then, only in private" (ibid., p. 407). For proper contextualisation of Newton's rapport with Socinianism, see id., "Isaac Newton, Socinianism and "The One Supreme God", esp. 255–265, and, furthermore, id., "To us there is but one God, the Father". While it is definitely correct that Newton used Socinian terminology in the General Scholium, we should not neglect the possible significance of other anti-Trinitarian strands (for instance, to name but one: dynamic monarchianism) for the development of Newton's theology. Newton was, for instance, clearly aware of the work of Paul of Samosota and Theodotus of Byzantium. He refers to the later in Yahuda Ms. 15 [ca. 1710s], f. 105^{r}, f. 122^{r}, 126^{r} and to the former in Yahuda Ms. 15, f. 7^{r}, ff. $9^{r}/10^{r}$, f. 18^{v}–19^{r}, f. 26^{r}, ff. 37^{r}–38^{r}, f. 117^{r}, f. 126^{v}, f. $131^{r–v}$, f. 132^{r}, f. 136^{r}, f. 151^{r}, f. 161^{r}, f. 192^{v}, Keynes Ms. 4, ff. $25^{r}/26^{r}$, f. 28^{r}, and, Williams Andrews Clark Memorial Library Ms. **N563M3 P222 [ca. early 1690s], f. 39^{r}, f. 41^{r}, f. 82^{r}. In general, Newton was well in tune with the patristic literature. In similar vein, Scott Mandelbrote and Raquel Delgado-Moreira have recently argued that Newton's Socinian sympathy should be set within a more complex process of appropriation of texts for anti-Trinitarian purposes (Mandelbrote, "'Than this noting can be plainer:' Isaac Newton reads the Fathers" and Delgado-Moreira, "Newton's 'Paradoxical Question concerning the Morals and Actions of Athanasius and his Followers' and its Intellectual Origins", pp. 34, 38–40). See, furthermore, Manuel, *The Religion of Isaac Newton*, p. 58.

[38] See especially Manuel, *The Religion of Isaac Newton*, pp. 16–17, 20–22, 40, 74–76; Force, "Newton's God of Dominion: The Unity of Newton's Theological, Scientific and Political Thought", esp. pp. 78–83; id.., "The Nature of Newton's 'Holy Alliance' between Science and Religion"; id., "Natural Law, Miracles, and Newtonian Science"; Stewart, "Seeing Through the Scholium: Religion and Reading Newton in the Eighteenth Century", pp. 128–131; McGuire, "The Fate of the Date: The Theology of Newton's *Principia* Revisited"; and, Snobelen, "God of Gods, and Lord of Lords". In *Tempus et locus*, Newton listed a number of diverging views concerning God's agency in the physical world – he himself obviously endorsed the second of each disjunct: "Let them consider whether it is more agreeable to reason that God's eternity should be all at once (*totum simul*) or that his duration is more correctly designated by the names Jehovah and "he that was and is and is to come"; (1) that the substance of God is not present in all places, or that the Jews more correctly call God Place, that is the substance essential to all places in which we are placed and (as the Apostle says) in which we live [and move] and have our being; (2) that God is everywhere as regards power and nowhere [as regards] substance, or that God's power should subsist everywhere in the divine substance [as its] proper sub[strate], and exists [nowhere separately], and have [no medium] by which it be propagated from its proper substance [into external places]; (3) that place itself and thus the [omnipresence] of God was created in finite time, or that [God was everywhere from] eternity; (4) that all the properties created things [argue imperfections to the

unity of the Father and the Son is moral and to be considered, not in a metaphysical, but in a monarchical sense[39]:

> When therefore the father is called God & the son is called Lord (as is done in the Creed,) it signifies that the ffather is the highest Lord & the Son is Lord ↓next↓ under him ↓or that y^{e} son sits at y^{e} right hand of God↓. And when the Son is also called God it signifies that the ~~Son~~ ↓name of God is in him & that he↓ is Lord over all things ~~next~~ under the father. And yet they are not two Gods, because a king & his viceroy are not two kings, nor is the name God to be understood of both together. It allways signifies the father unless by any circumstance it be restrained to y^{e} son; ↓even↓ as the name King always signifies the ~~super raign~~ ↓superior King↓ unless by any circumstance it be restrained to the viceroy. And as a man may give Kings & Princes that ~~ho~~ worship w^{ch} is ~~due to them~~ suitable to their ↓dignity↓ power & dominion over us, wth out being guilty of idolatry, so we may give ↓some↓ ~~Christ~~ ↓Chris [sic] Iesus a much greater worsh^{p}↓ that worship w^{ch} is suitable to his ↓dignity, power &↓ dominion over us without being guilty of idolatry tho he be not the supreme God.[40]

Since anti-Trinitarianism was in fact illegal at the time, Newton had good reasons to hide his heterodoxy.[41] Traces of Newton's heretical views can, nevertheless, be found in the General Scholium.

Central to his theology was Newton's conception of a substantially omnipresent Lord of Dominion, to whom exclusively religious worship is to be preserved. Though, "Others may be called Gods, but thou shall not worship them as Gods," Newton insisted.[42] Christ was but the visible Prince, "who came in a mortal

extent] that they are absolutely removed from God, [or that creatures share so far as possible the attributes of God (as fruit] the nature [of the tree, and an image the likeness of man,) and by sharing tend towards perfection, and to that extent God be discerned in the more perfect creatures as in a mirror; (5) that the more perfect God is he who produces the more imperfect and fewer creatures, or he who produces more perfect and countless ones; (6) that the Creator's power is infinite, and the possibility of creating only finite, or that the power of God in no wise extends to that what is impossible; (7) that a dwarf-god should fill only a tiny part of infinite space with this visible world created by him, or that the best and greatest God willed everywhere what was good, <and> did everywhere what he willed." (McGuire, "Newton on Place, Time, and God: An Unpublished Source", pp. 121/123).

[39] See Snobelen, "Isaac Newton, heretic: the Strategies of a Nicodemite", p. 386, footnote 41 for further references to Newton's manuscripts. Cf. Bodmer Ms., f. 402^{r}: "Ego et pater unum sumus. Ego in patre & pater in me. Qui videt me videt patrem. Sed his non denotatur essentialis unitas sed moralis tantum. Joan 10.30 & 14.9, 10, 11, 20 & 17.18, 21, 22, 23." On Yahuda Ms. 15.5, f. 97^{r}, Newton remarked that corruptions originate by "turning of the scriptures from a moral ~~to a~~ ↓& monarchical to a physical &↓ metaphysical ↓~~& physical~~↓ sense."

[40] Yahuda 15.5, f. 98^{r}.

[41] See, especially, Snobelen, "Isaac Newton, Heresy Laws and the Presecution of Dissent". It is telling that when William Whiston went public with his own heretical views, at the danger of revealing Newton's theological inclinations, Newton kept his lips sealed and ultimately their friendship went cold (Snobelen, "William Whiston, Isaac Newton and the Crisis of Publicity").

[42] Bodmer Ms., f. 390^{v}. Cf. Yahuda Ms. 15.3 [early 1710s], f. 44^{r}: "We are forbidden to worship two Gods but are not forbidden to worship one God, & one Lord in our worship: one God for creating all things & one Lord for redeeming us with his blood."

body,"[43] begotten by the invisible "God of Gods;" he is only "*the image of the invisible God*."[44] In his manuscript *Irenicum* (post-1710), Newton wrote on idolatry in the following words:

> Idolatry is a breach of the first & greatest commandment. It is giving to Idols the love honour & worship w^{ch} is due to the true God alone. It is forsaking the true God to commit whoredome with other lovers. It makes a Church guilty of Apostasy from God ↓as an Adulteress forsakes her husband.↓. It makes her guilty of spiritual whoredome with other lovers. It makes her become the Church of the Idols, fals Gods, or Dæmons whom she worships, such a true Church as in Scripture is called a Synagogue of Satan.[45]

In Babson Ms. 438 (probably 1710 or later), Newton noted the following with respect to our worship of Christ:

> We are not commanded in scripture to worship him [i.e., Christ] as God Almighty (for by the ~~first the~~ first comm^{t} we are to have no other Gods in o^{r} [illegible word] worship then him who ~~in~~ ↓according to↓ the fourth Comm^{t} ~~is s~~ made heaven & earth & the Sea) but we are to worship him ↓& give him honour & glory↓ in respect of his humanity. Because he humbled himself to death even the death of the cross therefore God ↓hath highly exalted him &↓ ~~gave~~ given him a name above every name that at the name of Iesus every kne [sic] should bow, of things in heaven & earth & under the earth & that every tongue should confess that Iesus Christ is Lord to the glory of God the father. We are therefore to bow to him as the Lord Iesus Christ ↓the Prince of the kings of the earth↓.[46]

Newton then developed the consequences of this credo: "There is one body, one spirit, one hope, one Lord, one faith, one baptism,[47] one God & father of all [. . .]. [Eph. iv.1]."[48] Christ was only a servant of God and it is the latter that we should

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[43] See: "Hic est ille qui a Judaeis diu expectatus venit mortali corpore per aquam in baptismo et immortali dein corpore per sanguinem effusum, et subsecutam resurrectionem a mortuis [. . .]" (CUL Add. Ms. 3965, f. 244^{v} [additions and corrections intended for the second edition of *Principia*]).

[44] Bodmer Ms., f. 16^{r}. The differentiation between the visible Prince and the invisible God was a common theme in contemporary anti-Trinitarian literature (e.g., Sandius, *Nucleus historiæ ecclesiasticæ*, Liber I, p. 153: "Deum Patrem autem invisibilem nunquam visum, de quo dictum: Deum nemo videbit & vivet." and Crellius, *Initium Evangelii S. Joannis Apostoli*, Pars I, Caput III, p. 6). See, furthermore, Dobbs, *The Janus Faces of Genius*, p. 214.

[45] Keynes Ms. 3, p. 14. On Yahuda 15.4, f. 68^{r} [early 1710s], Newton wrote: "Idolatry is the worshipping of a fals God a God who is not what your worship supposes him to be, a ~~vanity~~ fictitious God, a Vanity." On Bodmer Ms, f. 29^{r}, Newton listed several other examples of idolatry, for instance: the worship of Ghosts or Deamons, the veneration of image or dead men, divination by Oracles, the sacrifice of animals or stars (or intelligences seated in them), charms, spells, enchantments or invocations of the dead, the attribution of supernatural powers or operations to substances, and, finally, submitting to "the carnal desires of the flesh" (unless for "the lawful procreation of children"). The worship of images, the Holy Ghost and Saints was considered idolatrous in Stillingfleet, *A discourse concerning the idolatry practised in the Church of Rome*, Chapters I and II. Newton owned the 1671 edition of this work (Harrison, *The Library of Isaac Newton*, p. 244 [item n° 1561]).

[46] Babson Ms. 438, f. 1^{r}.

[47] On baptism, see Yahuda Ms.15.5 [early 1710s], f. 86^{r}.

[48] Bodmer Ms., f. 32^{r}. Cf. Yahuda Ms. 15.6 [early 1710s], f. 109^{r}: "He is simple not compound. He is all like & equal to himself, all sense all spirit, all perception all Ennœa, all λόγος all ear, all

worship exclusively. Newton formulated his conception of the biblical Pantokrator, as follows:

> We must beleive that he is παντοκράτωρ Lord of all things with an [illegible word] irresistible & boundless ↓power &↓ dominion that we may not hope to escape if we rebell & set up other Gods or transgress the laws of his monarchy & ↓that we may↓ expect ↓great↓ rewards if we do his will. We we [sic] must beleive [sic] that he is the ↓is the [illegible word] is the God of the Iews who↓ created the heaven & earth all things therein as is exprest in the ten commandments that we may thank him for our being & for all the blessings of this life, & forbear to take his name in vain or worship images or other Gods. We are not forbidden to give the name of Gods to Angels & Kings, but we are forbidden to have them ~~as gods~~ ↓as Gods↓ in our worship.[49]

Newton's anti-Trinitarianism can also be gathered from the twelve articles of faith composed around the same period as the General Scholium:

> Artic. 1. There is one God the Father ~~eternal~~ ↓everliving,↓ omnipresent, omniscient, almighty, the maker of heaven & earth, & one Mediator between God & Man the Man Christ Iesus.
> Artic. 2. The father is the invisible God whom no eye hath seen or can see, all other beings are sometimes visible.
> Artic. 3. The Father hath life in himself & hath given the son to have life in himself.
> Artic. 4. The father is omniscient & hath all knowledge ~~of future things~~ originally in his own breast, & communicates knowledge ~~to the son~~ of future things to ~~the son~~ ↓Iesus Christ↓ & none in heaven or earth or under the earth is worthy to receive knowledge of future things immediately from the father except the Lamb. ↓And therefore the testimony of Iesus is the Spirit of Prophesy & Iesus is the Word or Prophet of God.↓
> Artic. 5. The father is immoveable ~~th~~ no place being cap~~l~~able of becoming emptier or fuller of him then it is by the eternal necessity of nature: all other being [sic] are moveable from place to place.
> Artic. 6. All the worship (whether of prayer praise or thanks giving w^ch was due to the father before the coming of Christ is still due to him. Christ came not to diminish the worship of his father.
> Artic. 7. ~~A~~ Prayers are most prevalent when directed to the father in the name of y^e son
> Artic. 8. We are to return thanks to y^e father alone for creating us & giving us food & raiment & other blessings of this life & whatsover we are to thank him for or desire that he would do for us we ask of him immediately in the name of Christ
> Artic. 9. We need not pray to Christ to intercede for us. If we pray the father aright he will intercede.
> Artic. 10. ~~T~~ It is not necessary to salvation to direct or prayers to any other then y^e father in y^e name of the son.
> Artic. 11. To give the name of God to Angels or Kings is not against y^e first commandment. To give the worship of the God of the Iews to Angels or Kings is against it. The meaning of the commandment is T~~o~~↓h↓|ou shalt worship no other Gods but me.

eye, all light. He is all sense w^ch cannot be separated from it self, nor is there any thing in him w^ch can be emitted from any thing else."

[49] Ibid., f. 16^r. Cf. Yahuda Ms. 15.3 [early 1710s], f. 65^r.

> Artic. 12. ~~To give glory Christ [sic] & the Holy Ghost~~ To us there is but one God y^{e} father of whom are all things ↓& we of him↓, & one Lord Iesus Christ by whom are all things & we by him. ~~And each have their proper~~ ~~worship We are~~ that is, we are to worship the father alone as God Almighty & the Lor Iesus alone as ~~the Lord~~ the Messiah the ~~king of kin~~ great King the [illegible word] Lamb of God who was slain & hath redeemed us wth his blood & ~~was~~ made us kings & Priests.[50]

The Father is not divisible into different personhoods: "[Existit [i.e., Deus] semper sine successione Personarum; adest ubique sine divisibilitate in personas [~~illegible word~~]]."[51] Newton, furthermore, recorded the following on the Holy Trinity: "Homousion unintelligible. Twas not understood in y^{e} Council of Nice [. . .] nor ever since. What cannot be understood is no object of belief."[52]

The General Scholium in its final published form[53] consists of almost 60% of theological material. In the first two paragraphs of the General Scholium, Newton set the stage for his treatment of God: the motion of the celestial bodies acts according to the law of universal gravitation, but their regular position (the primary planets, for instance, revolve in concentric circles around the sun, in the same direction and as most closely as possible on the same plane) can only be explained by "the design and dominion of an intelligent and powerful being."[54] God rules everything according to the mathematical laws which he constantly enacts ("constanter agitans") unless he chooses to violate those laws.[55] Newton then subtly added that the fixed stars are "constructed according to a similar design and subject to the dominion of *One* [*Unius* dominio]."[56] The predicate "unus" hints at Newton's anti-Trinitarian agenda.[57] This is, as we shall shortly see, further confirmed in the draft versions of the General Scholium. In the following paragraph, Newton clarified "God" is a relative term ("vox relativa") which refers to dominion, and that, while lesser "Gods" might have *some* dominion, there is *only one* "Lord of Lords," constituted by *supreme* dominion. In the fourth paragraph Newton began to expound his theological views:

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50 Keynes Ms. 8, f. 1^{r} [post-1710].

51 CUL. Add. Ms. 3965, f. 420^{r} [ca. 1715–1716]. On CUL Add. Ms. 3965, f. 542^{r} [ca. 1692–1693], Newton wrote that the most perfect idea of God is as one, simple and indivisible substance. See, furthermore, McGuire, "Newton on Place, Time, and God: An Unpublished Source", pp. 122/123.

52 Bodmer Ms., f. 402^{v}.

53 The final manuscript version of the theological portion of the General Scholium appears in Newton's list of corrections for the second edition (CUL Add. Ms. 3965, f. 526$^{r–v}$ and again on f. 539$^{r–v}$).

54 Newton, *The Principia,* p. 940.

55 CUL Add. Ms. 3965, f. 542^{r} [additions and corrections intended for the second edition of *Principia*].

56 Ibid.

57 See Snobelen, "God of Gods, and Lord of Lords", pp. 177–178.

> He rules all things, not as the world soul [anima mundi[58]] but as the lord of all. And because of his dominion he is called Lord God *Pantokrator*. For "god" is a relative word and has reference to servants, and godhood [deitas] is the lordship of God, not over his own body as is supposed by those for whom God is the world soul, but over his servants. The supreme God [Deus summus] is an eternal, infinite, and absolutely perfect being; but a being, however perfect, without dominion is not the Lord God. For we do say my God, your God, the God of Israel, the God of Gods [deus deorum], and Lord of Lords [dominus dominorum], but we do not say my eternal one, your eternal one, the eternal one of Israel, the eternal one of the gods; we do not say my infinite one, or my perfect one. These designations [appellationes] do not have reference to servants. The word "god" is used far and wide to mean "lord", but every lord is not a god. The lordship of a spiritual being constitutes a god, a true lordship constitutes a true god [vera [dominatio] verum [deum]], a supreme lordship a supreme god [summa [dominatio] summum [deum]], and imaginary lordship an imaginary god.[59] And from true lordship it follows that the true God is living, intelligent and powerful; from the other perfections that he is supreme, or supremely perfect. He is eternal and infinite, omnipotent and omniscient, that is, he endures from eternity to eternity [ab aeterno in aeternum], and he is present from infinity to infinity [ab infinito in infinitum]; he rules all things, and he knows all things that happen or can happen. He is not eternity and infinity, but eternal and infinite; he is not duration and space, but he endures and is present. He endures always and is present everywhere, and by existing always and everywhere he constitutes [constituit] duration and space. Since each and every particle of space is *always*, and each and every indivisible moment of duration is *everywhere*, certainly the maker and lord of all things will not be *never* or *nowhere*.[60]

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58 Royal Society Gregory Ms. 247, f. 7^{r-v} reads: "Therefore, the body of the celestial sphere, which the world-soul fashioned to participate in its immortality, in order that it should never cease living, is always in motion and does not know how to rest, since the soul itself, by which the sphere is impelled, is never at rest. And slightly later: Even when [Cicero] called the outermost sphere, which so revolves, the supreme God, this does not imply that he believed it to be the first cause and all-powerful God. For the sphere itself, which is the sky, is the creation of the soul, and soul proceeded from the mind, and mind from God, who is truly the supreme: indeed, he called it supreme with respect to the other spheres lying beneath, [...]" (Schüller, "Newton's *Scholia* from David Gregory's Estate", p. 245). Schüller's translation of "Igitur et caeleste corpus quod anima futurum sibi immortalitatis particeps fabricata est, ne unquam vivendo deficiat semper in motu est et stare nescit, quia nec ipsa stat anima qua impellitur. Et Paulo post: Quod autem [Cicero] extimum globum qui ita volvitur summum Deum vocavit, non ita accipiendum est, ut ipse prima causa et Deus ille omnipotentissimus existimetur, cum globus ipse quod caelum esse animæ sit fabricata; anima ex mente processerit, mens ex Deo, qui vere summus est procreata sit." (ibid., p. 244). In Query 31, Newton recorded: "And yet we are not to consider the World as the Body of God, or the several Parts thereof, as the Parts of God. He is an uniform Being, void of Organs, Members or Parts, and they are his Creatures subordinate to him, and subservient to his Will; and he is no more the Soul of them, than the Soul of Man is the Soul of the Species of Things carried through the Organs of Sense into the place of its Sensations, where it perceives them by means of its immediate Presence, without the Intervention of any third thing. The Organs of Sense are not for enabling the Soul to perceive the Species of Things in its Sensorium, but only for conveying them thither; and God has no need of such Organs, he being every where present to the Things themselves." (Newton, *The Opticks*, p. 403). See Vassányi's *Anima Mundi: The Rise of the World Sould Theory in Modern German Philosophy* for useful background on the *anima mundi* theory.

59 This is almost certainly a sneer at the Cartesians and Leibniz's *intelligentia supra-mundana*.

60 Newton, *The Principia*, pp. 940–941. In the third edition, Newton added that "God is one and the same God always and everywhere." For several biblical references that concur with Newton's theological stance, see Snobelen, "God of Gods, and Lord of Lords", p. 177.

God thus constitutes space and time, because "if space had not existed, God would have been nowhere, and hence he either created space later, where he himself was not, or else, which is no less absurd to reason, he created his own ubiquity."[61] This does not mean that space and time are independent entities, which exist autonomously from God. Space and time are the logical, rather than ontological or emanative, consequences of God's omnipresence. Hence, Newton's parlance of space and time "as if an emanative effect" ("tanquam Dei effectus emanativus"[62]) of God's existence.[63] Once God is posited, space and time are posited.[64] Newton continued by noting that, as "active power [virtus] cannot subsist without substance [substantia]," God is substantially omnipresent and that in him "all things are contained and move." Because God is a spiritual, incorporeal being bodies do not act on him, nor conversely. By adopting a relative notion of "God" in terms of dominion, Newton rejected absolute characterisations of "God:" we cannot define God by using predicates such as "eternal", "infinite", or "perfect", since such designations ("appellationes") "do not have reference to servants," and, hence, not to God's dominion.[65] Moreover, such absolute designations give the impression that we can

[61] Quoted from McGuire, "Space, Infinity and Indivisibility: Newton on the Creation of Matter", p. 147.

[62] CUL Add. Ms. 4003, p. 12 (= f. 9[v]). Here Newton was careful enough to add "tanquam" and not to commit himself to an ontological emanation or the view that God is dependent on space and time (*pace* Carriero, "Newton on Space and Time: Comments on J. E. McGuire"). Yahuda Ms. 15.1, f. 7[r], 15.5, f. 87[v], 15.6–7, f. 83[v], f. 88[r], f. 108[r], f. 111[v] and Bodmer Ms., f. 116[r], f. 147[r], f. 400[r], f. 410[r] contain fierce criticism of emanationist cosmogonies which would compromise God's unity. See Goldish, "Newton's *Of the Church*: Its content and Implications", p. 163; Castillejo, *The Expanding Force in Newton's Cosmos*, p. 65 ff. Were Newton omits "tanquam" on two occasions nothing suggests an ontological emanation but rather a logical relation (cf.: "Deus est ubiq, mentes creatæ sunt alicubi, et corpus in spatio quod implet, et quicquid nec ubiq nec ullibi est id non est. *Et hinc sequitur* [And hence it follows] quod spatium sit entis primariò existentis effectus emanativus." (ibid., ff. 17[r]/18[r] [italics added]; Janiak, ed., *Newton, Philosophical Writings*, p. 25) and "Deniq spatium est æternæ durationis et immutabilis naturæ, idq quòd sit æternis et immutabilis entis effectus emanativus. *Siquando non fuerit spatium* [If ever space had not existed], Deus tunc nullibi adfuerit, et proinde spatium creabat postea ubi ipse non aderat, vel quod non minùs absonum est, creabat suam ubiquitatem." (ibid., f. 19[r] [italics added]; Janiak, ed., *Newton, Philosophical Writings*, p. 26)).

[63] For discussion of the sources on which Newton drew, see: Arthur, "Newton's Fluxions and Equally Flowing Time", pp. 323–351, esp. pp. 330–333 and Ducheyne, "J. B. Van Helmont's De tempore as an Influence on Isaac Newton's Doctrine of Absolute Time". For an excellent overview of the intellectual context of Newton's views on space, see McGuire and Slowik, "Newton's Ontology of Omnipresence and Infinite Space". I shall address Newton's views on space and time in more detail in Section 6.4.2 to this chapter.

[64] CUL Add. Ms. 4003, p. 18 (= f. 12[v]).

[65] Newton, *The Principia,* p. 941. The Cohen-Whitman edition here translates "relationem non habent ad servos" as "do not have reference to servants." A translation more close to the original would be "do not have a relation to servants." Snobelen discusses several examples of compatible manuscript material (see Snobelen, "God of Gods, and Lord of Lords", pp. 180–186). For Descartes' idea of God, see Beyssade, "The Idea of God and the Proofs of His Existence".

define God's essence or substance, a view to which Newton fiercely protested. Newton drew close analogy here with our knowledge of the primary, i.e. substantial, properties of bodies:

> We see only the shapes and colors of bodies, we hear only their sounds, we touch only their external surfaces, we smell only their odors, and we taste only their flavors.[66] *But there is no direct sense and there are no indirect reflected actions by which we know innermost substances, much less do we have an idea of the substance of God. We know him only by his properties and attributes and by the wisest and best construction of things and their final causes*, and we admire him because of his perfections, but we venerate and worship him because of his dominion. For we worship him as servants, and a god without dominion, providence, and final causes is nothing other than fate and necessity.[67]

In this passage, Newton was arguing that we can only know God by his attributes and properties, on the one hand, and by "the wisest and best construction of things and their final causes," on the other hand. The former refers to our knowledge that God is omnipotent, infinite, eternal, and the like. The latter is to be unravelled by natural philosophy. Newton, furthermore, stressed that, when we utilize human-like expressions to discourse about God, such language is purely allegorical and not literally true.[68] In similar vein in Keynes Ms. 3, Newton wrote:

> We are to ~~believe~~ ↓conceive↓ him void of ~~shape~~ external shape, ↓or bounds, a being↓ intangible ~~incorporeall~~ ↓&↓ invisible ~~& therefore incorporeal~~ for whom no eye hath seen or can see, ~~a bein~~ & therefore also incorporeal.[5] A being immoveable ~~[because necessarily in all places so yt no place can be without him]~~ ↓~~& indivisible~~↓ & the first cause of motion in all other things ffor he is necesarily in all places alike so that no place can ~~be~~ ↓subsist↓ without him or be emptier or fuller of him then it is by the necessity of nature.[69]

God, "who is ~~being indivisibly~~ in all places, after some such manner as that w^ch^ thinks in us in all parts of our sensorium,"[70] perceives "all things accurately in their true solid dimensions by the immediate presence of the things themselves, while that w^ch^ thinks in us perceives only the superficial pictures of the things made in our sensorium by motion conveyed thither from the ~~things~~."[71] By the study of

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[66] Cf.: "Hypoth 5. The essential properties of bodies are not yet fully known to us. Explain this by y^e^ cause of gravity, & by y^e^ ↓metaphysical↓ power of bodies to cause sensation, imagination & memory & mutually to be moved by o^r^ thoughts." (CUL Add. Ms. 3970, f. 338^v^ [ca. 1700–1704]).

[67] Newton, *The Principia,* p. 942 [italics added].

[68] This material was added in the third edition of the *Principia* (cf. CUL Adv.b.39.2, interleaved page between pp. 482–483; CUL Add. Ms. 3965, f. 494^v^ [additions and corrections intended for the third edition of the *Principia*]).

[69] Keynes Ms. 3, p. 35 [post-1710].

[70] For Leibniz' famous criticism of the "sensorium" Newton ascribed to God, see Newton, *Correspondence*, VI, pp. 212–214.

[71] CUL Add. Ms. 3970, f. 286^r^ [ca. 1700–1704].

natural philosophy we can obtain knowledge of God's dominion, his providence and the final causes[72] he installed. Correspondingly, Newton concluded that "to treat of God from phenomena is certainly a part of natural philosophy."[73] In manuscript material related to the Queries in *The Opticks*, Newton recorded that: "The business of Experimental Philosophy is ~~only~~ to find out by experience & observation *not how things were created but what is the present frame of Nature*."[74] This experimental-philosophical quest is ultimately conducive to a theological-moral outlook on the natural world, according to Newton.[75]

Let us now probe into the five draft versions of the General Scholium. In the first edition of the *Principia* there is only one reference to God.[76] In the various consecutive drafts (A–E) of the General Scholium, Newton added more and more theology. In the A-version of drafts of the General Scholium, Newton's only explicit reference to God is the following:

> If the fixed stars are the centres of similar systems, all these are under the same one dominion [unius dominio]: This being rules all things not as the soul of the world but as the Lord of the Universe. He is omnipresent and in him all things are contained and move, and without resistance since this Being is not corporeal and is not resisted by body.[77]

The B-version already contained the essentials of Newton's Hebraic credo of God as a universal ruler ("Imperator universalis"[78]) or *Παντοκράτωρ*, albeit that the relevant paragraph is somewhat shorter near the end.[79] In this version, Newton is also more explicit on how the discourse of God from phenomena pertains to experimental philosophy properly:

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[72] This is another sneer at Descartes' view that only efficient and not final causes are desirable in natural philosophy. The page of Newton's 1656 copy of Descartes' *Principia Philosophiae* (Wren Library, NQ.9.116, p. 8) which contains this statement (*Pars prima*, ¶ XXVIII) is dog-eared.

[73] Newton, *The Principia*, p. 943. In the second edition Newton had written "experimental philosophy." Cf. CUL Add. Ms. 3965, f. 152^{v} [notes on comets; revisions intended for the second edition of the *Principia*].

[74] CUL Add. Ms. 3970, f. 242^{v}, cf. f. 243^{r} [italics added; ca. 1700–1704].

[75] Newton, *The Opticks*, pp. 405–406.

[76] Scilicet: "Collocavit igitur Deus Planetas in diversis distantiis a Sole, ut quilibet pro gradu densitatis calore Solis majore vel minore fruatur." (Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 582, footnote concerning lines 31–36 of page 405 of the third edition of the *Principia*; Newton, *The Principia*, p. 813). This statement occurred in Corollary 5 to Proposition VIII of Book III. The reference to God was deleted in all later editions.

[77] Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 352, cf. p. 349; CUL Add. Ms. 3965.12, f. 357^{r}.

[78] CUL Add. Ms. 3965, f. 366^{v}.

[79] Cf. Keynes Ms. 3 [*Irenicum*; post-1710], p. 43.

> And thus much concerning God, to discourse of whom from the phenomena undoubtedly pertains to experimental philosophy. The intermediate causes of things appear from the phaenomena, and from these the more profound causes, until one arrives at the highest cause.[80]

According to Newton, our scientific knowledge progresses from knowledge of "intermediate" causes, to knowledge of "more profound" causes, and, ultimately, to knowledge of the highest cause. In this version Newton began adding several scriptural references, which are also included in the published version: Acts 17: 27–28 ["That they should seek the Lord, if haply they might feel after him, and find him, though he be not far from every one of us: For in him we live, and move, and have our being; as certain also of your own poets have said, For we are also his offspring."], Deuteronomy 4: 39 ["Know therefore this day, and consider *it* in thine heart, that the LORD he *is* God in heaven above, and upon the earth beneath: *there* is none else."] and 10: 14 ["Behold, the heaven and the heaven of heavens *is* the LORD's thy God, the earth *also*, with all that therein *is*."], I Kings 8: 27 ["But will God indeed dwell on the earth? behold, the heaven and heaven of heavens cannot contain thee; how much less this house that I have builded?"], Job 22: 12 ["*Is* not God in the height of heaven? and behold the height of the stars, how high they are!"], Psalms 139: 7 ["Whither shall I go from thy spirit? or whither shall I flee from thy presence?"], and, Jeremiah 23: 23–24 ["*Am* I a God at hand, saith the LORD, and not a God afar off? Can any hide himself in secret places that I shall not see him? saith the LORD. Do not I fill heaven and earth? saith the LORD."]. These references give the necessary scriptural backing to Newton's views on God's omnipresence.

In the C-version Newton's list of scriptural references verses was extended.[81] In addition to the references of the B-version, Newton now added: John 1: 18 ["No man hath seen God at any time[82]; the only begotten Son, which is in the bosom of the Father, he hath declared *him*."[83]] and 5: 37 ["And the Father himself, which hath sent me, hath borne witness of me. Ye have neither heard his voice at any time, nor seen his shape."[84]], I John 4: 12 ["No man hath seen God at any time. If we love one another, God dwelleth in us, and his love is perfected in us."], I Timothy 1: 17 ["Now unto the King eternal, immortal, invisible, the only wise God, *be* honour and

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80 Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 348, footnote 1; CUL Add. Ms. 3965.12, f. 359^{r}. Newton's view of science as a progressive ascent to causes of increasing generality which ultimately reveals the highest and most general cause is also given in Query 31 of *The Opticks* (Newton, *The Opticks*, p. 404).

81 The references to the ancients (Pythagoras, Cicero, Thales, Anaxagoras, Virgil, Philo, and Aratus) were not included here (Newton, *The Principia*, pp. 941–942, footnote j; cf. CUL Adv.b.39.2, interleaved page between pp. 482–483).

82 Cf. Keynes Ms. 8 [*Twelve articles on religion*, post-1710], f. 1^{r}.

83 This reference is discussed in Crellius, *Initium Evangelii S. Joannis Apostoli*, Pars I, Caput III, pp. 6–7, Caput XXXIV, p. 174, and Pars II, Caput XXIII, p. 412 ff.

84 This reference is mentioned in: Biddle, *The Apostolical and True Opinion concerning the Holy Trinity* (= Wren Library, NQ.9.32^{1}; Harrison, *The Library of Isaac Newton*, p. 142 [item n° 604]), p. 15. John Biddle (1615–1662) was a(n in)famous Unitarian (Iliffe, "Prosecuting Athanasius: Protestant Forensics and the Mirrors of Prosecution", pp. 117–118).

glory for ever and ever. Amen."[85]], and 6: 16 ["Who only hath immortality, dwelling in the light which no man can approach unto; whom no man hath seen, nor can see: to whom *be* honour and power everlasting. Amen."], Colossians 1: 15 ["Who is the image of the invisible God, the firstborn of every creature:"[86]], Exodus 28: 4 ["And these *are* the garments which they shall make; a breastplate, and an ephod, and a robe, and a broidered coat, a mitre, and a girdle: and they shall make holy garments for Aaron thy brother, and his sons, that he may minister unto me in the priest's office."[87]], Deuteronomy 4: 12 ["And the LORD spake unto you out of the midst of the fire: ye heard the voice of the words, but saw no similitude; only *ye heard* a voice."] and 4: 15–16 ["Take ye therefore good heed unto yourselves; for ye saw no manner of similitude on the day *that* the LORD spake unto you in Horeb out of the midst of the fire: Lest ye corrupt *yourselves*, and make you a graven image, the similitude of any figure, the likeness of male or female,"], and, Isaiah 40: 18–19 ["To whom then will ye liken God? or what likeness will ye compare unto him? The workman melteth a graven image, and the goldsmith spreadeth it over with gold, and casteth silver chains."]. The references from the Old Testament support Newton's view that, whenever we apply human-like properties to God, our talk of God is purely allegorical. The content of the references from the New Testament is, however, striking: they point to Newton's anti-Trinitarian agenda. It should be kept in mind that these scriptural references were not chosen haphazardly: as a highly skilled Bible scholar,[88] Newton was well aware of the Unitarian significance of these references.[89] That Newton himself distinguished between orthodox and heretical references is further confirmed in the D-version.

In the D-version Newton dropped reference to Exodus 28: 4, Deuteronomy 4: 12, 14–15 and Isaiah 40: 18–19 and regrouped the biblical references into two groups: (1) CUL Add. Ms. 3965, f. 363^{r} contains the orthodox references: Acts 17: 27–28,

[85] This and the two following references are mentioned in: [Biddle], *A Confession of Faith, Touching the Holy Trinity* (= Wren Library, NQ.9.32^{2}; Harrison, *The Library of Isaac Newton*, p. 142 [item n° 604]), p. 16. 1 Timothy 1: 17 is also mentioned in Crellius, *Initium Evangelii S. Joannis Apostoli*, Pars I, Caput XLVI, p. 263 and Pars II, Caput XXVIII, p. 453, Caput XLVII, pp. 544–545, and, Dissertatio I, p. 556.

[86] Also mentioned in Keynes Ms. 2, f. 12^{r}: "Who is y^{e} Image of y^{e} invisible God, the first born of every creature. For by him ↓[God y^{e} Father]↓ were all things created that are in heaven & y^{t} are in earth visible & invisible, whether they be thrones or dominions, or principalities or powers, all things were created by him & for him. And he is before all things & by him all things consist. And he ↓[Christ]↓ is y^{e} head of y^{e} body the church, who is the beginning, y^{e} first born from y^{e} dead; that in all things he might have y^{e} preeminence. For it pleased y^{e} Father y^{t} in him should all fulness dwell Colos 1.15." This reference is also listed in Newton's collection of biblical references related to Trinity in *Of the church* (Bodmer Ms., f. 406^{r}).

[87] This prima facie strange reference relates to the fact that in the Bible mortal beings are sometimes called "gods." In the second edition Newton wrote: "And in this sense princes are called gods, Psalms 82.6 and John 10.35. And Moses is called a god of his brother Aaron and a god of king Pharaoh (Exod. 4.16 and 7.1)" (ibid., p. 941, footnote g).

[88] See Popkin, "Newton as a Bible Scholar".

[89] See footnotes 87–89.

Deuteronomy 4: 39 and 10: 14, I Kings 8: 27, Job 22: 12, Psalms 139: 7, Jeremiah 23: 23–24, while (2) CUL Add. Ms. 3965, f. 363^{v} contains the anti-Trinitarian ones: John 1: 18[90] and 5: 37, Colossians 1: 15,[91] I Timothy 1: 17 and 6: 16, and I John 4: 12. The former group relates to the omnipresence of God, the latter to Newton’s radical subordinationist view of Christ.[92]

In the E-version group (2) has completely disappeared and, like in the B-version, Newton, by way of possible compromise, underscored and capitalized the first letter of “unius.” Newton chose to withdraw these all but too revealing references and decided to hide his intention behind a more subtle typography. As the inclusion of group (2) would have made Newton’s anti-Trinitarianism explicit, Newton preferred to suppress them and write a more complex anti-Trinitarian hermeneutics[93] into the General Scholium. Some contemporaries brought Newton’s anti-Trinitarianism to the fore.[94]

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[90] Also referred to in Keynes Ms. 2 [*Theological Notebook*, ca. 1684–1690], part 1, p. 12^{v}.

[91] Also referred to in ibid., p. XII.

[92] In the second edition of the *Principia* Newton referred to Acts 17: 27–28, John 14: 2, Deuteronomy 4: 39 and 10: 14, Psalms 139: 7–9, I Kings 8: 27, Job 22: 12–14 and Jeremiah 23: 23–24. In the third edition, he omitted John 14: 2 (Newton, *The Principia*, p. 942, footnote j). The latter group is identical to the references given in the E-version.

[93] Cf. Snobelen, “God of Gods, and Lord of Lords”, p. 170, cf. p. 180, footnote 43.

[94] See, furthermore, Mandelbrote, “Eighteenth-Century Reactions to Newton’s Anti-Trinitarianism”. In the postscript (pp. 36–40) to *Some Brief Critical remarks on Dr. Clarke’s Last Papers* (1714), for instance, John Edwards revealed Newton’s anti-Trinitarian agenda. On p. 36 Edwards noted: “I HAD observ’d before that ’twas Dr. *Clarke*’s Notion that [God] *is a Relative Word*, *and a Word of Dominium and Power*. I have since found that this is borrow’d from *Crellius*, *De Deo ejusq*; *Attributis*, cap. xiii. who uses the like *Instances* to prove it that the Dr. doth. In the same Place, *Crellius* affirms, *Dei vox Potestatis imprimis & Imperij nomen est*. But further, I have found, that our Celebrated Philosopher and Matematician, Sir *Isaac Newton*, hath taken up these odd Notions at the end of his *Philosoph. Nat. Princip. Mathemat. Edit. ult.* pag. 482. *Deus est vox Relativa*: – *Deitas est Dominatio Dei*, saith he.” G. W. Leibniz also pointed to the Socinian underpinnings of Newton’s notion of God (Alexander, ed., *The Leibniz-Clarke Correspondence*, p. 19). On CUL Add. Ms. 3965, f. 547^{r} [additions and corrections intended for the third edition of *Principia*; italics added], Newton reacted to such allegations, as follows: “[5] In argumentis pro existentia Dei, Deus ita definiendus est ut a Natura ~~[two illegible words]~~ sapientissima potentissima et ~~summe~~ ↓absolute↓ perfecta distinguatur. [~~. . .~~] Id quod fit attribuendo ipsi vitam voluntatem & dominium. [~~. . .~~] Unde definiendo significationem ↓antiquam↓ vocis Dei, ↓scripsi↓ in Scholio sub finem Libri Principiorum ~~scripsi~~ in hæc verba. Vox Deus passim significat Dominum, sed omnis Dominus non est Deus. Dominatio Entis Spiritualis Deum constituit, vera verum, summa summum, ficta fictum. Et ex dominatione vera sequitur Deum ~~esse~~ verum esse vivum, intelligentem et potentem; e↓x↓~~t~~ reliquis perfectionibus summum esse vel summe perfectum. *Spectant hæc verba non ad* ~~[illegible word]~~ *doctrinam de Trinitate ut aliqui somniant, non ad* ~~*existentiam*~~ *↓cultum↓ Dei alicujus a summo diverso* ~~*non ad religione*~~ *non ad religionem, (quæ utique ↓in↓ Philosophia tractari non deb*~~*ent*~~*), sed* ~~*ad antiquam*~~ *significationem vocis Dei, quatenus a natura diversus est solummodo exhibent ↓ut existentia Dei in hoc sensu* ~~*demonstretur*~~ *doceatur↓*.” (On the same folio, Newton added: “[4] Doctrina de resurrectione hominum longe antiquissima est. Quaestiones ↓autem↓ de origine materiæ, de immortabilate animæ ac de resurrectione hominum locum non habent in Philosophia.”) However, in view of the biblical references which Newton deleted from the final edition of the General Scholium and in

6.3 Newton on the Dangers of Cartesian Philosophy

Richard S. Westfall has noted that "the *General Scholium* contained a vigorous reassertion of those principles Newton had adopted in his rebellion against the perceived dangers of Cartesian mechanical philosophy."[95] In similar vein, Alan E. Shapiro has recently argued that Newton "had consciously avoided using 'experimental philosophy' until the beginning of the eighteenth century, when he publicly introduced that venerable term in the second edition of the *Principia* in 1713 in order to defend his work, especially the theory of gravity, against the criticism of Cartesians and Leibnizians but, above all, Leibniz himself."[96] Newton himself considered Cartesian philosophy as "little better then a ↓Romance↓ ~~Philosophical Romance~~."[97] Newton was fully aware that the content of the *Principia*, which contained "~~such a convincing ↓mathematical↓ way of arguing as has given satisfaction procured the assent of ↓all the↓ ↓ablest↓ Mathematicians ↓who have had the leisure & skill to examine the book~~," was "very remote from the *conceptions* of Philosophers" – undoubtedly a sneer at Cartesian-inspired philosophers.[98] In this period, Newton used "experimental philosophy" as a means to distance himself from Cartesian-inspired "hypothetical philosophy,"[99] which introduced fictions into natural philosophy. Correspondingly, Newton began to introduce and to emphasize such terms as "the Method of Analysis and Synthesis" (1703–1704),[100] "the Method of Induction" (1717),[101] and "Deduction from Phenomena" (1713).[102] In this period, the priority dispute was seriously heating up and from 1715 to 1716 Samuel Clarke defended Newton's natural philosophy against Leibniz' attacks.[103] Since Leibniz attacked Newton on a physical, theological and mathematical level, Newton developed an extreme dislike of his adversary, as may be seen

view of his statement that God exists always without succession of persons ("sine successione Personarum") and is present everywhere without division in persons ("sine divisibilitate in personas"), Newton's account here is not exactly reliable.

95 Westfall, *Never at Rest*, p. 749.

96 Shapiro, "Newton's 'Experimental Philosophy'", p. 186. The earliest usage of "experimental philosophy" dates back to 1706 in a draft of a paragraph in Query 23 (CUL Add. Ms. 3970, f. 243^r; ibid., p. 189). Stephen D. Snobelen has also pointed to the impact of Leibniz' 1712 attack on Newton (Snobelen, "God of Gods, and Lord of Lords", p. 174; see, furthermore Westfall, *Never at Rest*, pp. 729–732).

97 CUL Add. Ms. 3970, f. 480^v. On CUL Add. Ms. 3968, f. 257^r [post-1710], Newton called Leibniz' method "a Romantic method of philosophy."

98 CUL Add. 3970, f. 338^r [ca. 1700–1704; italics added].

99 See the draft of Newton to Cotes, 28 March 1713, Newton, *Correspondence*, V, pp. 398–399.

100 Shapiro, "Newton's 'Experimental Philosophy'", p. 191.

101 Ibid., p. 197.

102 Ibid., pp. 211–215.

103 On the priority dispute see Hall, *Philosophers at War*, Whiteside's editorial introduction to Part 2 of Whiteside, ed., *The Mathematical Papers of Isaac Newton*, VIII, pp. 469–538, and Westfall, *Never at Rest*, Chapter 14. On the Clarke-Leibniz correspondence, see Koyré and Cohen, "Newton and the Leibniz-Clarke Correspondence".

from the following examples. In an *ad hominem* moment, Newton wrote: "And if he [= Leibniz] is happy in disciples (as he boasts) it is because he has spent all his life in corresponding with men of all nations for propagating his opinions whilst I have ↓rested &↓ left truth to lift for its self."[104] Newton also mocked Leibniz' experimental skills and pointed out that "M^{r} Leibniz never found but a new experiment in all his life."[105]

Although Leibniz transformed Descartes' qualitative account of the celestial motions into a quantitative one, and although he frequently criticisized the technical aspects of Cartesian vortical mechanics, in his *Tentamen de motuum caelestium causis* (1689)[106] he accepted the idea that "the cause of celestial motions should originate in the motions of the aether, or using astronomical term, in orbs which are deferent, yet fluid."[107] From this perspective, Newton saw Leibniz as the heir of Descartes' celestial mechanics. Newton also must have seen methodological continuity between Descartes and Leibniz: both natural philosophers proposed their explanations of the celestial motions on the basis of hypotheses. Newton left some notes on Leibniz' *Tentamen*, which date back to 1714.[108] In these notes, Newton noted several physical absurdities in Leibniz' *Tentamen*. Apart from these technical points, Newton also listed the absurd theological implication that "Deus non regit mundum proindeque non est Dominus Deus."[109] In *Essais de Theodicée* (1710), Leibniz had also criticized the idea of gravitational attraction.[110] Newton was, furthermore, infuriated by the appearance of a letter of Leibniz in *Mémoirs des Trévoux* on 5 May 1712, in which he launched his famous anti-Newtonian criticism that a perpetual miracle is needed to keep planets in their orbits[111]: "Because they do not explain gravity by a mechanical hypothesis, he [Leibniz] charges them wth making it a supernatural thing, a miracle & a fiction invented to support an ill grounded opinion & compares their method of philosophy to that of Mr de

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104 CUL Add. Ms. 3968, f. 587^{v} [crossed-out section; 1714].

105 CUL Add. Ms. 3968.39, f. 586^{v}, quoted from Shapiro, "Newton's 'Experimental Philosophy'", p. 205.

106 For the original Latin text of Leibniz' *Tentamen*, see Gerhardt, ed., *Leibnizens Mathematische Schriften*, VI, pp. 144–161; for its translation, see Meli, *Equivalence and Priority*, pp. 126–142.

107 Meli, *Equivalence and Priority*, p. 128, cf. p. 129. Cf. CUL Add. Ms. 3968, f. 74^{r} [ca. 1712]. See, furthermore, Meli, *Equivalence and Priority*, p. 42.

108 Newton, *Correspondence*, VI, pp. 116–122. See, furthermore, Koyré and Cohen, "Newton and the Leibniz-Clarke Correspondence", pp. 118–122.

109 Ibid., VI, p. 116. For additional background, see Meli, *Equivalence and Priority*, pp. 186–190.

110 Farrer and Huggard, eds., *Theodicy*, p. 85. On the *Théodicée*, see Shapiro, "Newton's 'Experimental Philosophy'", p. 200ff and Cohen, "Newton's Copy of Leibnitz' *Théodicée*". Newton owned original 1710 edition of the *Théodicée* (Harrison, *The Library of Isaac Newton*, p. 177 [item n° 935]).

111 Newton came across this letter only ten days before he sent his final changes for the second edition of the *Principia* to Cotes on 28 March 1713 (Shapiro, "Newton's 'Experimental Philosophy'", p. 201; Cotes to Newton 18 March 1712/13, Newton, *Correspondence*, V, pp. 392–393).

Robervals Aristarchus, wch is all one as to call it Romantic."[112] In his anonymously published account of the *Commercium epistolicum* (1715), Newton emphasized the differences between Leibniz and himself:

> The one proceeds upon the evidence arising from experiments and phenomena, and stops where such evidence is wanting; the other is taken up with hypotheses, and propounds them, not to be examined by experiments, but to be believed without examination. The one for want of experiments to decide the question doth not affirm whether the cause of gravity be mechanical or not mechanical: the other that it is a perpetual miracle if it be not mechanical. The one (by way of inquiry) attributes it to the power of the creator that the least particles of matter are hard: the other attributes the hardness of matter to conspiring motions, and calls it a perpetual miracle if the cause of this hardness be other than mechanical. The one doth not affirm that animal motion in man is purely mechanical: the other teaches that it is purely mechanical, the soul or mind (according to the hypothesis of a pre-established harmony) never acting upon the body so as to alter or influence its motions.[113] *The one teaches that God (the God in whom we live and move and have our being) is omnipresent, but not as the soul of the world: the other that he is not the soul of the world, but INTELLIGENTIA SUPRAMUNDANA, and intelligence above the bounds of the world; whence it seems to follow that he cannot do any thing within the bounds of the world, unless by an incredible miracle.* The one teaches philosophers are to argue from phenomena and experiments to the causes thereof, and thence to the causes of those causes, and so on till we come to the first cause: the other that all the actions of the first cause are miracles, and all the laws impressed on nature by the will of God are perpetual miracles and occult qualities, and therefore not to be considered in philosophy. But must the constant and universal laws of nature, if derived from the power of God or the action of a cause not yet known to us, be called miracles and occult qualities, that is to say, wonders and absurdities? Must all the arguments for a God taken from the phenomena of nature be exploded by new hard names? And must experimental philosophy be exploded as miraculous and absurd because it asserts nothing more than can be proved by experiments, and we cannot yet prove by experiments that all the phenomena in nature can be solved by mere mechanical causes?[114]

The first line of the General Scholium famously reads: "The hypothesis of vortices is beset with many problems."[115] Newton's objections against Cartesianism were, however, not limited to physical ones. Newton also had profound issues with

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[112] Newton, *Correspondence*, V, p. 299. See, furthermore, Dobbs, *The Janus Faces of Genius*, pp. 230–233.

[113] For a recent interpretation of Newton as a mind-body substance monist, see Dempsey, "Written in the Flesh: Newton on the Mind-Body Relation".

[114] Janiak, ed., *Newton, Philosophical Writings*, pp. 125–126 [italics added]. In his first reply to Leibniz, Clarke pointed out that: "The Notion of the World's being a great *Machine*, going on *without the Interposition of God*, as a Clock continues to go without the Assistance of a Clockmaker; is the Notion of *Materialism* and *Fate*, and tends, (under pretense of making God a *Supra-Mundane Intelligence*,) to exclude *Providence* and *God's Government* in reality out of the World. [...] If a *King* had a *Kingdom*, wherein all Things would continually go on *without* his Government or Interposition, or *without* his Attending to and Ordering what is done therein; It would be to *him*, merely a *Nominal* Kingdom; nor would he in reality deserve at all the Title of King or Governor." (Clarke, ed., *A Collection of Papers, which passed between the late Learned Mr. Leibniz and Dr. Clarke*, pp. 15/17).

[115] Newton, *The Principia*, p. 939. In his early years Newton favoured Descartes' vortex theory (McGuire and Tamny, eds., *Certain Philosophical Questions*, pp. 357, 363–365).

Descartes' mathematical method.[116] Newton pointed out that an algebraic analysis does not reveal how the geometrical synthesis is to be performed. Correspondingly, Newton argued that algebraic considerations have no bearing on the construction of geometrical curves. Newton also objected to Descartes' view that the degree of an (algebraic) equation determines the simplicity of a geometrical curve. Rather, it is determined by the "simplicity" or "elegance" – terms which remained vague in Newton's exposition – of a curve's mechanical description. Knowing a curve's mechanical description implies that one knows "the reason of its genesis."[117] Therefore, mechanical curves (e.g., the cycloid) were perfectly acceptable means of construction, according to Newton, on the condition that their description is exact.[118] It was in this anti-Cartesian context that Newton developed a profound admiration of the geometry of the ancients. Newton's sentiments on Descartes' tackle of indeterminate problems contained similar criticism: not only was the Cartesian synthesis unsatisfactory, the Cartesian analysis was also inferior to the analysis of the ancients.[119]

In manuscript material pertaining to the Macclesfield Collection (CUL Add. Ms. 9597.2.11), Newton added several other points criticism on Cartesianism, which have scarcely been noted. In these manuscripts, which I date post-1713 (most probably between 1713 and 1715), Newton also launched a philosophical criticism of Cartesianism. It is in this context that Newton rejected Cartesian innatism. Newton stressed that all our knowledge, including ideas, derives from phenomena. In the following passage, Newton endorsed an empiricist stance on on human knowledge:

> What is taught in metaphysics, and if it is deduced from [divine] revelation, is religion; if it is *deduced from phenomena by means of the five external senses*, it pertains to physics; if it [is *deduced*] *from knowledge of the internal actions of our mind through the faculty of reflection*, it is only philosophy concerning the human mind and its ideas – as if internal phenomena – and likewise pertains to physics. *To dispute about the objects of ideas, unless insofar as they are phenomena, is a dream*. In all philosophy we have to start from phenomena, and not to admit any principles, causes or explanations of things, unless they are established by phenomena.[120]

[116] Guicciardini, *Newton on Mathematical Certainty and Method*, Chapters 4–6. See the Section 2.4.3 in Chapter 2.

[117] Ibid., p. 72.

[118] Ibid., p. 67.

[119] Ibid., p. 79.

[120] Translation of: "Quod in Metafysica docetur ↓& si a relevatione [missing word: "divina"?] deducitur religio esse↓, si a Phaenomenis per sensus quinque externos, ~~deducitur~~ a Physicā pertinet, ~~si a revelatione divina, religio ↓est~~↓; si a cognitione actionum internarum mentis nostræ per sensum reflexionis, philosophia est de sola mente humana & ejus ideis ↓tanquam Phaenomenes internas↓ & ad Physicam ↓item↓ pertinet. De Idearum objectis disputare nisi quatenus sunt phaenomena ~~somniamus~~ ↓somnium est↓. ~~Ideoque a Phaenomenis in omni Philosophia incipiendum est.~~ In omni Philosophia incipere debemus a Phaenomenis, & nulla admittere ↓rerum↓ principia nullas causas nullas explicationes nisi quæ per phaenomena stabiliuntur." (CUL Add. Ms. 9597.2.11, f. 2^{r} [italics added]; cf. Cohen's translation in Newton, *The Principia*, p. 54). This text is to be found on a half-sheet folio which contains "2" in the right corner. It is blank on the verso side.

Here Newton argued that, since acts of reflection can "in a more loose sense"[121] ("sensu laxiore") be considered as internal phenomena, they are part of physics, i.e. the study of phenomena.[122] The distinction Newton made between (direct) perception by the senses and reflexion agrees to John Locke's distinction between "intrinsical" and "extrinsical" knowledge. Locke and Newton became well acquainted in 1689. As a token of their intimate relation, one might point out that Newton sent a copy of his anti-Trinitarian *Two notable corruptions* to Locke in 1690.[123] According to Locke, the materials of thinking, or as he refers to them, the "fountains of knowledge," stem either from *experience*, i.e. knowledge directly provided by the senses ("extrinsical knowledge"), or from the *inner sense* (also called "reflection"), i.e. knowledge which results from the cognitive process of reflecting on the inner operations of the human mind ("intrinsical knowledge").[124] Newton owned 13 works by Locke in his private library,[125] including the *Essay*[126] and its Latin edition *De intellectu humano*.[127] We also know that Newton read parts of the *Essay* by May 1693.[128] In Book I of his *Essay*, Locke argued that there are no innate principles originally imprinted in the human mind. Locke's epistemology provided Newton with a philosophical terminology with which he could attack Cartesian philosophy.

121 Newton interpreted the notion "phenomena" broadly as to include not only what can be known by the five senses but also "things internal which we contemplate in our minds by thinking" (McGuire, *Tradition and Innovation*, p. 132; cf. CUL Add. Ms. 3970, f. 621^{v}).

122 On CUL Add. Ms. 3965, f. 421^{r} [additions and corrections intended for the second edition of *Principia*] Newton wrote: "Phænomena voco ↓non solum↓ quæcunque apparet ~~vel~~ ↓sed etiam↓ (~~sensu laxiore~~) quæcunque sentiri possunt, sive sint res externæ quæ per sensus quinque innotescunt, sive internæ quas in mentibus nostris intuemur ↓cogitando↓. Ut quod ignis calidus est, aqua humida est, aurum grave est, sol lucidus est, Ego sum et cogito [end of text]". On ibid., f. 422^{v} he wrote: "Hæc omnia ~~sensu laxo~~ sensibilia sunt et sensu laxo Phænomena vocari possunt. ~~Proprie sunt~~ Phænomena ↓proprie dicuntur,↓ quæ videri possunt sed vocem accipio sensu laxiore." (cf. ibid., f. 420^{r}).

123 Newton to a Friend, 14 November 1690, Newton, *Correspondence*, III, pp. 83–129. Cf. Snobelen, "Isaac Newton, Heretic: the strategies of a Nicodemite," pp. 401–402. See, furthermore, Parker, "Newton, Locke and the Trinity."

124 Locke, *An Essay concerning Human Understanding*, II.i, §§ 3–4, pp. 104–105, § 24, pp. 117–118, II.vii, § 10, p. 131. See Ducheyne, "The Flow of Influence: From Newton to Locke … and Back"; Rogers, "The System of Locke and Newton"; and, id., "Locke's *Essay* and Newton's *Principia*", for further details.

125 Harrison, *The Library of Isaac Newton*, pp. 180–181 [items n° 966–978].

126 Newton owned the 1690 edition of the *Essay*.

127 Newton owned the 1701 fourth edition of Locke's *De intellectu humano*. Newton's only remaining private copy contains no signs of reading (Wren Library, NQ.11.27).

128 In Newton's famous letter to Locke, in which Newton apologized for accusing Locke of embroiling him "wth woeman" during his depressive breakdown, Newton wrote: "For I am now satisfied that what you have done is just and I beg your pardon for my having hard thoughts of you for it & for representing that you struck at ye root of morality in a principle you laid down in your book of Ideas & designed to pursue in another book & that I took you for a Hobbist." (Newton to Locke, 16 September 1693, Newton, *Correspondence*, III, p. 280).

In 1713 the second edition of Pierre Rémond de Montmort's (1678–1719) *Essai d'analyse sur les jeux de hazard* appeared in print. In this work, Monfort included a letter he had sent to Nikolaus I Bernoulli (1687–1759) on 20 August 1713, in which he defended Descartes' (meta)physics and recorded the following on the appearance of the second edition of the *Principia*:

> Enfin on m'apprend que ce sont toujours les mêmes dogmes, qu'on y suppose encore les vertus attractrices tant décriées en France & par tout, qu'on y prétend prouver & même démontrer le vuide, & qu'on y combat plus que jamis les tourbillons de M. Descartes: tout cela, je vous avoue, me met en pays perdu; je croirois avoir de bonnes démonstrations que le movement en rond des Planettes demande necessairement un fluide environnant qui entraîne la Planette, & que le vuide supposé, la lumiere, & generalement tous les autres phenomenes de la nature, sont inexplicable. [. . .] Dérangé comme je le suis par autorité de M. Newton, & d'un si grand nombre de sçavans Geometres Anglois, je serois presque tenté de renoncer pour jamais à l'étude de la Physique, & de remettre à sçavoir tout cela dans le Ciel; mais non, l'autorité des plus grands esprits ne doit point nous faire de loi dans les choses où la raison doit decider. Malgré la diversité d'opinions qui regnent ajourd'hui entre les Sçavans, je ne desespere pas qu'un jour ils ne soient tous d'accord; mais il faudra pour cela que *les Geometres fondent leurs hypotheses mathematiques sur les idées d'une saine Metaphysique* [. . .]. La Geometrie & la Mechanique ainsi fondée sur des principes vrais & sur des experiences certaines, produira des découvertes admirables, & fournira enfin un systême lié, dans lequel on trouvera l'explication des principaux phenomenes de la nature, sans être oblige de faire des suppositions pour chacun en particulier.[129]

De Montmort's letter to Bernouilli did not fail to draw Newton's immediate attention. Newton prepared a draft letter to address Montmort's criticism: "~~To the R^{L} Hon~~^{ble}[,] ~~There being~~ In a Letter of Mons^{r} Monmort to N. Bernouilli dated 20 Aug. 1713 ↓& printed in the second edition of his Analysis upon the play at Hazzard↓, there are ~~several~~ ↑some↑ passages w^{ch} deserve to be rectified & therefore a part of that Letter with animaversions thereupon hath been thought fit to be reprinted."[130] In the aforementioned draft, Newton launched several criticisms against the position which Montmort was defending. In defense of his own natural philosophy, Newton pointed out that he had only introduced the force of inertia, the force of gravity, and electricity, which are forces "that are not condemned in France,"[131] that he had demonstrated that all the motions of the planets, the comets and the sea have their origin in gravity alone in non-resisting media,[132] that he had shown that light does

[129] Rémond de Montmort, *Essai d'analyse sur les jeux de hazard*, pp. 396–398 [italics added]. Newton owned both the 1708 as well as the 1713 edition of this work (Harrison, *The Library of Isaac Newton*, p. 194 [item n° 1098]).

[130] CUL Add. Ms. 3968, f. 469^{r}. Newton's draft letter is 2.25 folios long and ends on f. 470^{r}.

[131] "Ad explicanda Phænomena nullas inducit vires præter vim inertiæ vim gravitatis & vim electricam quæ vires in Gallia non damnantur." (ibid., f. 469^{r}).

[132] "3. Demonstravit Newtonus motus ↓omnes↓ Planetarum Cometarum et Maris nostri ex gravitate sola in spatijs non resistentibus necessario consequi idque quam accuratissime; perturbari vero per hypotheses vorticum. Vide Opticem Quæst. 20. p. 313. Et Princip. Philos. p. 355 & 481. D. Leibnitius ~~vim gravitatis et~~ Hypothesin Vorticum ↓cum vi gravitatis↓ conciliare conatus est sed frustra." (ibid.).

not consist in pressure,[133] and, finally, that he had abstained from hypotheses.[134] Then Newton launched some criticisms of his own. "Does the author suppose with Leibniz," Newton wrote, "that there is no other force in nature except motion [. . .], i.e. that altogether nothing is given in nature except matter and motion, that there are no final causes, that all is governed by fate, that God is a supra-mundane intelligence, that all natural philosophy is engaged in this so that we explain by hypotheses how everything could be produced by matter and motion without providence and final causes?"[135] It is clear that Newton responded to what he saw as the dangers of Cartesian (natural) philosophy. Additionally, he formulated his objections against Cartesian epistemology and especially its doctrine of innate ideas, as follows:

> [. . .] the author [Montmort] hopes that the philosophy of Newton, [which] is founded on phenomena through mathematical demonstrations,[136] is rejected and that all finally unite in a philosophy which geometers will found on hypotheses adapted to the notions of a sane metaphysics. Metaphysics is based on innate ideas; the philosophy of Newton on phenomena through mathematical demonstrations. Innate ideas are hypotheses and our author wishes to found natural philosophy on phenomena and on demonstrations through metaphysical hypotheses; [. . .][137]

In the same period as he composed the above material, Newton also consulted Leibniz' *De primae philosophiae emendatione, et de notione substantiae* (1694).[138] As proper natural philosophy is based on experience alone, no room was left for the hypothesis of innate ideas. Elsewhere, Newton declared that "Even that celebrated Proposition *Ego cogito ergo sum* is known to us by experience. We know that we think by an inward sensation of o$^{\mathrm{r}}$ thoughts."[139] Newton then continued, as follows:

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133 Ibid.

134 Ibid., f. 469$^{\mathrm{v}}$.

135 Translation of: "Supponit Author cum D. Leibnitio aliam in rerum natura nullam esse vim præter motum [. . .], id est, nihil omnino in rerum natura dari præter materiam et motum, nullas esse causes finales, omnia fato regi, Deum esse intelligentiam supra-mundanam, totam philosophiam naturalem in eo versari ut per hypotheses explicemus quomodo omnia per materiam et motum absque providentia et causis finalibus produci potuerunt." (ibid., ff. 469$^{\mathrm{r}}$–469$^{\mathrm{v}}$).

136 It was precisely the lack of proper (mathematical) demonstrations that led to the downfall of proper natural philosophy: "Defectu demonstrationibus haec philosophia intermissa fuit eandemque non inveni sed vi demonstrationum in lucem tantum revocare conatus sunt." (CUL Add. Ms. 3968, f. 109$^{\mathrm{r}}$ [early 1710s]).

137 Translation of: "[. . .] sperat Author ut Philosophia Newtoni in Phaenomenis per Demonstrationes Mathematicas fundata rejiciatur & omnes tandem conveniant in Philosophia quam Geometræ in Hypothesibus ad notiones Metaphysicæ sanæ aptatis fundabunt. Metaphysica in ~~In Hypothesibus Idearum~~ Idaeis innatis, Philo↓so↓phia Newtoni in Phaenomenis ↓per mathematicis Demonstrationibus↓ fundatur. Idææ innatæ sunt hypotheses & vult author noster Philosophiam naturalem in ~~hypothesibus me~~[ta]~~physicis fundari. Et~~ phaenomenis ac demonstrationibis per hypotheses metaphysicas fundari; [. . .]" (CUL Add. Ms. 9597.2.14, f. 4$^{\mathrm{r}}$).

138 McGuire, *Tradition and Innovation*, p. 129; Gerhardt, ed., *Die Philosophischen Schriften von Gottfried Wilhelm Leibniz*, IV, pp. 468–470.

139 CUL Add. Ms. 3970, f. 621$^{\mathrm{v}}$.

> And under a sane metaphysics he [i.e., Montfort] understands a Cartesian one: where it is everywhere asserted that there exists an absolutely perfect Entity, and [where] Descartes approves it [i.e., the existence of God] from the idea of it [i.e., the idea of God], from the absolute necessity which is included in this idea, and from man having an author. [Descartes] nowhere demonstrates that that actual Entity is distinct from the most wise Nature which generates everything and does nothing in vain. Namely, in his metaphysics he states that the author is such and such in some manner or other and that this Author is God; in *Principles of Philosophy* (Part III, Section 47[140]) he states that matter, posited in whatever form, successively assumes all forms of which it is capable by the help of the laws of nature and that it eventually reaches that [form] which is that of this world. Descartes has thus nowhere shown that the force or faculty of thinking is *res cogitans* or that every extended thing is extension, or that [every] extension is something mobile, or that the motion of bodies consists only in relative translation without inherent force, or that *res cogitans* is nowhere present in space, or that God is not omnipresent through his own substance, or that we have ideas of substances. All these things are mere hypotheses.[141]

Newton rejected Descartes' account of true motion, extension, his extreme dualism, and his treatment of God. Note that Newton's argumentation on this matter is similar to some of the arguments given in *De Gravitatione*.[142] As is well known, Descartes distinguished between three different ontological levels: *res cogitans*, *res extensa*

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[140] Descartes's words are: "Quae pauca sufficere mihi videntur, ut ex iis tanquam causis omnes qui in hoc mundo apparent effectus secundum leges naturae supra expositas oriantur. Et non puto alia simpliciora, vel intellectu faciliora, vel etiam probabiliora rerum principia posse excogitari. Etsi enim forte etiam ex Chao per leges naturae idem ille ordo qui jam est in rebus deduci posset, idque olim susceperim explicandum; quia tamen confusio minus videtur convenire cum summa Dei rerum creatoris perfectione, quam proportio vel ordo, & minus distincte etiam a nobis percipi potest, nullaqua proportio, nullusve ordo simplicior est, & congenitu facilior, qua ille qui constat omnimoda aequalitate: idcirco hic suppono omnes materiae particulas initio fuisse, tam in magnitudine quam in motu, inter se aequales, & nullam in universo inaequalitatem relinquo, praeter illam quae est in situ Fixarum, & quae unicuique coelum noctu intuenti tam clare apparet, ut negari plane non possit. Atque omnino parum refert, quid hoc pacto supponatur, quia postea juxta leges naturae est mutantum. Et vix aliquid supponi potest, ex quo non idem effectus (quanquam fortasse operosius) per easdam naturae leges deduci possit: cum enim illarum ope materia formas omnies quarum est capax, successive assumat, si formas istas ordine consideremus, tandem ad illam quae est hujus mundi poterimus devenire: adeo ut hic nihil erroris ex falsa suppositione sit timendum." (Descartes, *Œuvres de Descartes*, VIII, pp. 101–103).

[141] Translation of: "Et Metaphysicam sanam intelligit Cartesianam: Qua ubique asseritur Ens absolute perfectum existere, idque ab ejus Idæa, ab existentia necessaria in Idæa illa inclusâ & ab homine authorem habente Cartesius probat. Verum Ens illud a Natura sapientissima omnia procreante & nihil frustra faciente, diversum esse nusquam demonstrat. Scilicet in Metaphysica sua se authorem habere istumque Authorem esse Deum; in Principijs Philosophiae (Part III sect 47) materiam in forma quacunque positam, ope legum naturæ formas omnes quarum est capax successive assumere, tandemque ad illam quæ est hujus mundi devenire statuit. Sic nec vim aut facultatem cogitandi rem cogitantem esse aut rem omnem extensam extentionem esse, aut existentionem rem mobilem esse, aut motum corporum in sola translatione relativa sine vi inertiae consistere, aut rem cogitantem nulli spatio praesentem esse, aut Deum non esse omnipræsentem per substantiam suam aut nos Idæas habere substantiarum Cartesius alicubi probavit. Hæc omnia sunt meræ hypotheses." (CUL Add. Ms. 9597.2.14, f. 4^{r}). An earlier variant is contained on the aforementioned CUL Add. Ms. 3968, f. 469^{v}.

[142] See Janiak, ed., *Newton*, *Philosophical Writings*, pp. 14–22.

and God. The *prima causa* of the physical world is God, who has created matter and who indirectly maintains the amount of motion by the conservation laws he has installed.[143] Newton's fear was that in the Cartesian *systema mundi* matter would be self-propelling and that God's substantial omnipresence could not be guaranteed. Newton was thereby reiterating the concerns of Cambridge Platonist Henry More. Although Henry More[144] was initially enthusiastic for Descartes's philosophy,[145] he later came to reject his philosophy as it could not provide place for the "Spirit(s)" in the physical realm.[146] Descartes derived these laws from the constancy of God.[147] From Newton's perspective, this meant that God, after having created matter and having installed the conservation laws, no longer acted in the world. If this were the case, then matter would be able to "regulate" its own motion, so to speak, and, by consequence, it would no longer depend on God. By contrast, Newton stressed that matter is essentially passive:

> Whence it seems to have been an ancient opinion that matter depends upon a Deity for its ↓laws of↓ motion as well as for its existence. The Cartesians make God the author of all motion & its as reasonable to make him the author of the laws of motion. Matter is a passive principle & cannot move it self. It continues in its state of moving or resting unless disturbed. It receives motion proportional to the force impressing it. And resists as much as it is resisted. These are passive laws & to affirm that there are no other is to speak against experience.[148]

In CUL Add. Ms. 9597.2.11, Newton continued his criticism and claimed that Cartesian philosophy was in fact an anti-Mosaic idolatry that ultimately derived from Gentile theogony:

> Metaphysics originates from the ancient Theogony of the Gentiles, where the Gentiles were everywhere feigning that the Sun, the Moon, the stars, the elements, intelligences, humane souls, animals, and everything which is in nature are either parts of the highest God or either His powers. From this it follows that nature herself is the highest God. In this philosophy, the Gentiles founded their idolatry. And Moses, by abolishing [this] cult of parts of the

[143] Descartes's ideas on motion can be found in *Le Monde* (1632), but are perhaps presented more clearly in the *Principia philosophiae* (1644). For a good exposition of Descartes's conservation laws, see, e.g., Garber, *Descartes' Metaphysical Physics*, esp. Chapters 6–9.

[144] For a recent biography on Henry More, see Crocker, *Henry More, 1614–1687*.

[145] More wrote: "*For that admirable Master of Mechanics* Des-Cartes *has improved this way to the highest, I dare say, that the wit of man can reach to in such* Phaenomena *as he has attempted to render the causes of*." (More, *The Immortality of the Soul*, Preface, p. 32). Also see More's utterance that "Moses has been aforehand with *Cartesius*" (More, *Conjectura Cabbalistica or, a Conjectural Essay of Interpreting the Minde of Moses According to a threefold Cabbala: viz. Literal, Philosophical, Mystical, or, Divinely Moral*, p. 151).

[146] In the Cartesian *systema mundi* God would be nowhere ("nullibi"). More therefore called René Descartes a "nullibist." See Cassirer, *The Platonic Renaissance in England*, p. 149 and Koyré, *From the Closed World to the Infinite Universe*, p. 139.

[147] Descartes, *Œuvres de Descartes*, VIII, pp. 62–65.

[148] CUL Add. Ms. 3970, f. 619^{r}.

world, condemned this philosophy and established Lord God as omnipresent and distinct from nature.[149]

Newton therefore ultimately considered Cartesianism as a heretical natural philosophy. He was equally convinced that properly methodized natural philosophy, would be conducive to the acceptance of the dominion of an all-powerful divine being,[150] as is clear from the following excerpt:

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149 Translation of: “~~Metaphysicæ~~ Metaphysica ↓~~ubique~~↓ ab antiqua Gentium Theogonia originem habuit qua ubique Gentes Solem Lunam stellas, ↓elementa↓ ~~Deos omnes,~~ ↓Intelligentias↓ animas humanas animalia & omnia ~~mundi [illegible word]~~ quæ in rerum natura sunt vel partes esse Dei summi vel ↓ejus↓ potentias ↓esse↓ fingebant. ~~adeoque naturam ipsam esse Deum~~ Unde consequens est quod ipsa rerum Natura sit Deus summus. In hac Philosophia[~~m~~] ↓Gentes↓ idolatriam suam fundebant. Et Moses ~~[illegible word]~~ ↓abrogando↓ cultum ~~partium [illegible word] a Dêo conditarum hanc Philosophiam damnavit stellarum~~ partium mundi, damnavit hanc philosophiam ac Dom. Deum omnipræsentem a ~~Natura~~ rerum natura diversum stabilivit.” (CUL Add. Ms. 9597.2.14, f. 4^{r} [1717]). Judaism occupied an important place in Newton’s reconstruction of the *Ur*-religion for by “reading the Law and Prophets in the Synagogues, those books have been kept freer from corruption than the *Hagiographia*” (Newton, *Observations upon the Prophecies of Daniel and the Apocalypse of St. John*, p. 13; cf. Bodmer Ms., f. ii^{r}). Furthermore, in Bodmer Ms., f. 1^{r}, cf. 21^{v}, Newton wrote: “The true religion was propagated by Noah to his posterity, & when they revolted to the worship of their dead kings & thereby denyed their God & ceased to be his people, it continued in Abraham & his posterity who revolted not.”

150 Newton often stressed God’s free will. For instance, Newton wrote “^{2} Materia non est æterna sed originem habuit a voluntate Dei” and “^{3} Animæ humanæ sunt immortales, non per causas naturales sed per voluntatem Dei.” (CUL Add. Ms. 3965, f. 547^{r}). On ibid., f. 542^{r} (ca. 1692–1694), in a crossed-out section, Newton commented on God as a substance (“substantia”), as follows: “quæ tota intelligit [~~. . .~~] quod in singulis ejus locis agitur, [~~. . .~~] cujus vis [~~. . .~~] tota in singulis locis viget, [~~. . .~~] ↓quæ↓ possibilia omnia semper et ubique pro arbitrio suo in actum deducere potest, ↓& libere agit↓ quæ optima ~~omnia~~ ↓sunt↓ et rationi maxime consentatea [~~. . .~~] et fato cæco [~~. . .~~] adduci non potest ut ~~aliquod~~ aliter agat.” On ibid., f. 542^{v}, he wrote: “Certe potentia creandi non major ↓esse↓ potest in Deo quam possibilitas in creaturis, ideoque aut hæc infinita est aut illa tantum finita. [Nam Dei potentia ad impossibilia non extenditur.]” On one occasion, Newton wrote that God had freely chosen the laws of nature (“~~Natura legibus ↓semper↓ obtemperat, Deus libere eligit.. et leges constituit.~~” (ibid., f. 496^{r})). Similarly in Dibner NMAHRB Ms. 1031 B, f. 4^{v}, Newton wrote: “The world might have been otherwise then it is (because there may be worlds otherwise framed then this) Twas therefore noe necessary but a voluntary & free determination y^{t} it should bee thus. And thus a voluntary [cause must bee a God]. determination implys a God. If it be said y^{t} y^{e} w^{ld} could bee noe otherwise y^{n} tis because tis determined by an eternall series of causes, y^{ts} to pervert not to anwser y^{e} 1st prop: ffor I meane not y^{t} y^{e} [world] might have been otherwise notwthstanding the precedent series of causes, but y^{t} y^{e} whole series of causes might from eternity have beene otherwise here, <because they as well as, *deleted*> ↑ because they may be otherwise ↓ in other places” (quoted from Dobbs, *The Janus Faces of Genius*, p. 266). Unfortunately, this material has not received proper attention by recent scholars writing on early modern theological voluntarism (e.g., Harrison, “Voluntarism and Early Modern Science”, esp. pp. 75–76, in which Harrison tries to temper Newton’s theological voluntarism on the basis of an unrelated excerpt on *moral* laws from Newton’s *Of the church* (cf. Harrison, “Was Newton a voluntarist?”, p. 42), and, Henry, “Voluntarist Theology at the Origins of Modern Science: A Response to Peter Harrison”, pp. 86, 88, 89, in which Henry seeks to emphasize Newton’s voluntarism – alas, without taking the manuscript material referred to above into consideration).

> – – – Moral Philosophy. ffor ~~when~~ ↓it↓ we ~~know~~ ↓see↓ clearly by the light of Nature that there is a God, we shall see ↓clearly↓ by the same light of Nature that he is to be ↓acknowledged feared ~~honoured~~ &↓ ~~feared & honoured praised thanked & worshipped with fear wth fit expressions of gratitude for the benefits we receive wth gratitude & supplication~~ & adored.[151]

Newton was convinced that the true faith had been corrupted in the course of history.[152] Newton conceived the instauration of a sound natural philosophy as a return to the Noachite *prisca sapientia* and *religio*.[153] Heresy was not merely a mistake of the past, it was also a danger lurking in the present. On the final pages of *The Opticks*,[154] Newton wrote:

> And no doubt, if the Worship of false Gods has not blinded the Heathen, their moral philosophy would have gone farther than to the four Cardinal Virtues; and instead of teaching the Transmigration of Souls, and to worship the Sun and Moon, and dead Heroes, they would have taught us to worship our true Author and Benefactor, as their Ancestors did under the Government of *Noah* and his Sons before they corrupted themselves.[155]

From the final excerpt of CUL Add. Ms. 9597.2.11, f. 4^{r}, it becomes clear that Newton saw himself as a modern day Moses, who sought to abolish the idolatry of Cartesian (meta)physics. As far as Newton was concerned, Leibniz was *Cartesius renovatus*. It is clear from his criticisms that Newton saw Descartes, Leibniz and de Montfort as part of one homogeneous movement.

6.4 The Interaction Between Newton's Natural Philosophy and His Theology (I): Case-Studies

How did Newton's theological doctrines bear on his experimental physico-mathematical work in the *Principia* (and *The Opticks*)? And what about the potential unity in Newton's natural-philosophical and theological work? In this section, I seek

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151 CUL Add. Ms. 3970, f. 244^{v} [ca. 1704–1706].

152 See Iliffe, "'Those whose business it is to cavell:' Newton's Anti-Catholicism'" and id., "Prosecuting Athanasius: Protestant Forensics and the Mirrors of Prosecution". With respect to the corruption of Trinity, the 1669 edition of Christoph(orus) Sand(ius), *Nucleus historiæ ecclesiasticæ* provided Newton with a lot of historical information (Harrison, *The Library of Isaac Newton*, p. 232 [item n° 1444]).

153 On Newton's belief in the *prisca sapientia*, see, e.g., Yahuda Ms. 41; McGuire and Rattansi, "Newton and the 'Pipes of Pan'"; Manuel, *The Religion of Isaac Newton*; Rattansi, "Newton and the Wisdom of the Ancients"; Dobbs, *The Janus Faces of Genius*; id., *The Foundations of Newton's Alchemy*; Goldish, *Judaism in the Theology of the Sir Isaac Newton*; Markley, "Newton, Corruption, and the Tradition of Universal History"; Haycock, "The Long-lost Truth: Sir Isaac Newton and the Newtonian Pursuit of Ancient Knowledge"; Snobelen, "'The true frame of Nature:' Isaac Newton, Heresy, and the Reformation of Natural Philosophy", esp. 234–242; and, Mandelbrote, "Isaac Newton and the Flood".

154 For the theological context of Query 31, see, furthermore, Snobelen, "'The Light of Nature:' God and Natural Philosophy in Isaac Newton's *Opticks*".

155 Newton, *The Opticks*, pp. 405–406. This excerpt was added to the 1706 edition of Newton's *Optice*.

to clarify the interaction between Newton's natural philosophy and his theology by focussing on two representative case-studies. In the next section, I will proceed to a general discussion of the interaction of Newton's theology and natural philosophy, as Newton understood it.

6.4.1 Case 1: Methodizing Prophecy

It has been observed that Newton's *Rules for interpreting y^e^ words & language in Scripture* (ca. 1670s–1680s)[156] are similar to Rules I–II of Newton's *regulae philosophandi*,[157] and, on the basis of this, it has been suggested that Newton thought that the two Books "should be approached with the same method"[158] and, moreover, that the *regulae philosophandi* have a theological origin.[159] In this context, particular significance is attributed to Newton's ninth rule for interpreting the Scriptures, which he introduced for the purpose of "methodising of y^e^ Apocalyps"[160]:

> 9. To ~~prefer~~ ↓choose↓ those ~~interpretations~~ ↓constructions↓w^ch^ without straining reduce things to the greatest simplicity. The reason of this is manifest by the precedent Rule. Truth is ever to be found in simplicity, & not in y^e^ multiplicity & confusion of things. As y^e^ world, w^ch^ to y^e^ naked eye exhibits the greatest variety of objects, appears very simple in its internall constitution when surveyed by a philosophic understanding, & so much y^e^ simpler by how much the better it is understood, so it is in these visions. It is y^e^ perfection of ~~all~~ God's works that they are all done w^th^ ye greatest simplicity. He is y^e^ God of order & not of confusion. And therefore as they that would understand y^e^ frame of y^e^ world must indeavour to reduce their knowledg to all possible simplicity, so it must be in seeking to

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156 Yahuda Ms. 1.1, ff. 12^r^–19^r^. See, furthermore, Newton, *Observations upon the Prophecies of Daniel and the Apocalypse of St. John*, pp. 16–23.

157 See Section 3.2 in Chapter 3.

158 Snobelen, "'God of Gods, and Lord of Lords'", p. 200; id., "To Discourse of God: Isaac Newton's Heterodox Theology and His natural Philosophy", pp. 46–47. In the latter article, Snobelen further strengthens his claim that "[T]he same methods [...] could be applied in the study of both Books" (ibid., p. 47) by referring to the following quotation from the *scholium* to the Definitions: "Accordingly those who there interpret these words as referring to the quantities being measured do violence to the Scriptures. And they no less corrupt mathematics and philosophy who confused true quantities with their relations and common measures." (Newton, *The Principia*, p. 414). Cf. Force, "Newton, the 'Ancients', and the 'Moderns'", p. 244..

159 Snobelen, "To Discourse of God: Isaac Newton's Heterodox Theology and His natural Philosophy", p. 46. It has also been suggested that "Newton's method of fluxions is inevitably connected with his theory of the continuous dominion of God since the creation" and that "Newton's calculus is based on the continuity of flow as supervised by the God of Dominion operating in his generally provident mode of creator and preserver of the current state of natural law" (Force, *Newton's God of Dominion*, p. 88). Force's suggestions have recently been further developed by Leshem, *Newton on Mathematics and Spiritual Purity*, Chapters 3 and 4 and Ramati, "The Hidden Truth of Creation: Newton's Method of Fluxions". All Leshem/Ramati establishes is that Newton *could have* attributed theological significance to the method of fluxions. Evidence that he did so is lacking.

160 Yahuda Ms. 1.1, f. 8^r^.

> understand these visions. And they that shall do otherwise do not onely make sure never to understand them, but derogate from y^e perfection of y^e prophesy; & ~~declare~~ ↓make it suspicious↓ also that their designe is not to understand it but to shuffle it of & confound y^e understandings of men by making it intricate & confused.[161]

In other words, Newton's ninth prophetic rule advises us to chose for those constructions with the greatests simplicity. Adequately methodizing the Scriptures was crucial, as Newton would declare later, for by doing so religious unity could be guaranteed:

> Contending for a language w^ch was not handed down from the Prophets & Apostles is a breach of this command & they that break it are also guilty of the disturbances & schisms occasioned thereby. It is not enough to say that an article of faith may be deduced from scripture. It must be exprest in the ↓very↓ form of sounds words in w^ch it was delivered by the Apostles. Otherwise there can be no lasting unity or peace of the Church catholick. ffor men are apt to ↓vary↓ dispute & run into parties about deductions. All the old Heresiers lay in deductions, the true faith was in the text.[162]

Before I shall discuss the plausibility of the above suggestions, I would like to shed some light on Newton's motivations for his study of the prophecies.[163] Newton considered the interpretation of the scriptures as a religious obligation – a duty which, as we will see in just a moment, he took very seriously. The opening paragraph of Yahuda Ms. 1, Newton addressed his putative readers as follows:

> Having searched ~~[& by the grace of God obtained~~ ↓after↓ knowleg in the prophetique scriptures, I have thought my self bound to communicate if ↓for the benefit↓ of others, remembring y^e judgment of him who hid his talent in a napkin. [. . .] And therefore the longer they have continued in obscurity, the more one hopes there is that y^e time is at hand in which they are to be made manifest. If they are never to be understood, to what end did God reveale them? Certainly he did it for y^e edification of y^e church; & if so, then it is as certain that y^e church shall at length attain to y^e understanding thereof. *I mean not all that*

161 Yahuda Ms. 1.1, f. 14^r. Similarly in Dibner NMAHRB Ms. 1031 B, Newton wrote: "Tis suitable w^th infinite wisdom [. . .] not to multiply causes w^thout necessity" (quoted from Dobbs, *The Janus Faces of Genius*, p. 265). Dobbs dates this manuscript to 1672 (ibid., p. 257); the dating provided by *The Newton Project* is 1670–1675.

162 Yahuda Ms. 15.1, f. 11^r [1710s]. On Keynes Ms. 3, p. 2 [post-1710], Newton added: "The first Principles of the Christian religion are founded, not on disputable ~~conjectures~~ ↓conclusions or humane sanctions, opinions or conjectures↓, but on the express words of Christ & his Apostles; & we are to hold fast the form of sound words. It is not enough that a Proposition be true or in the express words of scripture: it must also appear to have been taught from the ~~beginning~~ ↓days of the Apostles↓ in order to baptism & communion."

163 On Newton's work on prophecy, see Manuel, *The Religion of Isaac Newton*, Chapter 4; Castillejo, *The Expanding Force in Newton's Cosmos*, Chapter 2; Popkin, "Newton's Biblical Theology and His Theological Physics"; Hutton, "The Seven Trumpets and the Seven Seals: Apocalypticism and Christology in Newton's Theological Writings"; id., "More, Newton, and the Language of Biblical Prophecy"; Iliffe, "'Making a Shew'" Kochavi, "One Prophet Interprets Another: Sir Isaac Newton and Daniel"; Mandelbrote, "Isaac Newton and Thomas Burnet"; id., "A Duty of the greatest Moment;" and, id., "Isaac Newton and the Exegesis of the Book of Daniel". On Joseph Mede's (and Henry More's) influence on Newton's prophetical work, see Hutton, "More, Newton, and the Language of Biblical Prophecy" and Iliffe, "Making a Shew". Cf. Yahuda, 1.1, f. 8^r.

> *call themselves Christians, but a remnant, a few scattered persons which God hath chosen, such as without being ~~blinded~~ led by interest, education, of humane authorities, can set themselves sincerely & earnestly to search after truth. For as Daniel hath said that y^{e} wise shall understand, so he hath said also that none of y^{e} wicked shall understand.*[164]

Newton clearly thought to be part of that remnant. Interpreting the prophecies is a duty that the chosen ones should undertake in their private study:

> Let me therefore beg of thee not to trust to y^{e} opinion of any man concerning these things, for so it is great odds but thou shall be deceived. Much less oughtest thou to ~~keep to~~ rely upon the judgement of y^{e} multitude, for so thou shalt certainly be deceived. But search the scriptures thy self & that by frequent reading & constant meditation upon what thou readest, & earnest prayer to God to enlighten thine understanding if thou desirest to find the truth. [. . .] That the benefit w^{ch} may accrew by y^{e} understanding ↓the↓ sacred Prophesies & the danger by neglecting them is very great & that y^{e} obligation to study them is as great may appear by considering y^{e} ↓like↓ case of y^{e} Iews at y^{e} coming of Christ. [. . .] *If God was so angry with y^{e} Iews for not searching more diligently into y^{e} Prophesies w^{ch} he had given them to know Christ by: why should we think he will excuse us for not searching into y^{e} Prophesies w^{ch} he hath given us to know Antichrist by? Thou seest therefore that this is no idle speculation, no matters of indifferency but a duty of the greatest moment.* Wherefore it concerns thee to look about thee narrowly least thou shouldest in so degenerate an age be dangerously seduced & not know it. Antichrist was to seduce y^{e} whole Christian world and therefore he may easily seduce thee if thou beest not well prepared to discern him.[165]

Moreover, framing false interpretations is "a corruption equipollent to y^{e} adding or taking from it, since it equally deprives men of y^{e} use & benefit thereof."[166] Those who seek to properly understand the prophecies will, however, be rejected by the world, as Newton sullenly pointed out:

> [They will call thee it may be a ↓[illegible word]↓ ~~hot-headed fellow~~ a Bigot, a Fanatique, a Heretique &c: And tell thee of the uncertainty of these interpretations, & vanity of attending to them: Not considering that the prophesies concerning o^{r} Saviour's first coming were of more difficult interpretation, and yet God rejected y^{e} Iews for not attending better to them. And whither they will beleive it or not, there are greater judgments hang over the Christians for their remisness then ever the Iews yet felt. But y^{e} world loves to be deceived, they will not understand, they never consider equally, but are wholly led by prejudice, interest, the prais of men, and authority of y^{e} Church they live in: as is plain becaus all parties keep close to y^{e} Religion they have been brought up in, & ↓yet↓ in all parties there are wise & learned as well as fools & ignorant. There are but few that seek to understand the religion they profess, & those that study for understanding therein, do it rather for worldly ends, or that they may defend it, then ~~for worldl~~ to examin whither it be true wth a resolution to chose & profess that religion w^{ch} in their judgment appears the truest. [. . .] *This is the guise of y^{e} world, and therefore trust it not, nor value their censures & contempt. But rather consider y^{t} it is y^{e} wisdom of God that his Church should appear despicable to y^{e} world to try the faithfull. For this end he made it a curs under Law to hang upon a tree that the scandal of y^{e} Cross might be a tryall to the Iews; & for y^{e} like tryall of the Christians he hath suffered y^{e} Apostacy of the latter times, as is declared in calling it the hower of temptation w^{ch} should*

164 Yahuda Ms. 1.1, f. 1^{r} [italics added].

165 Ibid., ff. 1^{r}–2^{r} [italics added].

166 Ibid., f. 9^{r}.

> *come upon all y^e^ world to try them that dwell upon the earth Rev 3.10. Be not therefore scandalised at the reproaches of y^e^ world but rather looke upon them as a mark of y^e^ true church.*[167]

It is easy to show, as we have seen in Section 3.4.3 in Chapter 3, that the *regulae* are required to establish Newton's argument of universal gravitation. Some scholars have been tempted to ascribe a theological origin to Newton's first and second rule of philosophizing. Does their conclusion stand?

Maurizio Mamiani has recently suggested that Newton's views on simplicity derive from a common and explicitly non-theological source: Robert Sanderson's *Logicae Artis Compendium*, of which Newton possessed a copy of the 1631 edition which he bought in 1661.[168] As far as Mamiani is concerned, Sanderson's laws are the "conceptual source" of the *regulae philosophandi*.[169] This suggests that Newton's prophetic and natural-philosophical *regulae* were derived from a common source, which he appropriated differently in different areas of research, rather than that Newton's *regulae* were directly derived from his rules for methodizing the prophecies. If Mamiani is correct, the parallelism between Newton's prophetic and natural-philosophical rules could be accounted for by Sanderson's textbook. In the context of Rule I, Mamiani ascribes a crucial role to Sanderson's *law of brevity* ("Nothing should be left out or be superfluous in a discipline.") and his *law of harmony* ("The individual parts of each doctrine should agree among themselves.") and, in the context of Rule II, he ascribes a crucial role to Sanderson's *law of unity or homogeneity* ("No doctrine should be taught that is not homogeneous with subject or end.").[170] I do not wish to deny that Sanderson's emphasis on rules might have influenced Newton's decision to incorporate a number of *regulae* in the *Principia*. However, when it comes to explaining the methodological meaning of Newton's *regulae philosophandi*, the parallelism, which Mamiani emphasizes between Sanderson's laws and Newton's *regulae philosophandi* is rather uninformative. Similarly, the parallelism between Newton's ninth rule for interpreting the scriptures and Rules I and II remains rather vague. As I have argued in Section 3.2 of Chapter 3, Newton's first two *regulae philosophandi* cannot be straightforwardly reduced to a maxim of simplicity, brevity, harmony or homogeneity. In these *regulae philosophandi*, Newton provided two desiderata that a proper cause in natural philosophy should meet: a cause should not merely be explanatory, it should also be true. In other words, if we look at the technical meaning of Newton's *regulae*, then much was new in them.[171] And precisely, what was new in them cannot

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[167] Ibid., ff. 5^r–6^r [italics added].

[168] Mamiani, "To twist the meaning: Newton's *regulae philosophandi* revisited", esp. pp. 11–12.

[169] Ibid., p. 10.

[170] Ibid., p. 11.

[171] Mamiani's move to account for the content of Rules III and IV on the basis of the *law of connection* and Sanderson's characterisation of induction, respectively, is even less convincing (ibid., pp. 11–12).

be accounted for by Sanderson's laws or Newton's own rules for methodizing prophecy.[172]

Moreover, despite the parallelism between Newton's prophetic and natural-philosophical rules, it should also be emphasized that prophecy and natural philosophy both had their own separate epistemological status, as Raquel Delgado-Moreira has recently made forcefully clear.[173] She explains that, given Newton's avoidance of hypothetical elements in prophecy,[174] his use of prophetic syncronisms in a "mathematical way," his introduction "Definitions" and "Proofs," and his concern with founding prophesy on empirical considerations (*in casu* the world political),[175] it might *prima facie* be suggested that there is considerable methodological unity behind Newton's natural philosophical and prophetical work. However, detailed scrutiny reveals that "the mathematical demonstrations valid for one discipline were not valid for the other."[176] Newton himself cleary indicated that prophetical interpretation, unlike natural philosophy, was not a demonstrative science:

> *And hence I cannot but on this occasion reprove the blindness of a sort of ~~people~~ men who although they have neither better nor other grounds for their faith then y[e] Scribes & Pharisees had for their ~~religion~~ Traditions, yet are so pervers as to call upon other men for such a demonstration of y[e] certainty of faith in y[e] scriptures that a meer naturall man, how wicked soever, who will but read it, may judg of it & perceive y[e] strength of it w[th] as much perspicuity & certainty as he can a demonstration in Euclide*. Are not these men like y[e] Scribes & Pharisees who would not attend to y[e] law & y[e] Prophets but required a signe of Christ? Wherefore if Christ thought it just to deny a signe to that wicked & adulterate generation notwithstanding that they were God's own people, ~~even~~ ↓&↓ the Catholique Church; much more may God think it just that this generation should be permitted to dy in their sins, who do not onely like y[e] Scribes neglect but trample upon the law and y[e] Prophets, & endeavour by all possible means to destroy y[e] faith w[ch] men have in them, & ↓to↓ make them disregarded. [. . .] *I could wish they would consider how contrary it is to God's purpose y[t] y[e] truth of his religion should be as obvious & perspicuous to all men as a mathematical demonstration. Tis enough that it is able to move y[e] assent of those w[ch] he hath chosen; & for y[e] rest who are so incredulous, it is just that they should be permitted to dy in their sins.* Here then is y[e] wisdom of God, that he hath so framed y[e] Scriptures

[172] James E. Force has, furthermore, argued that in Rule IV Newton "noted the methodological impact for human knowledge of a physical nature subservient either to God's ordinary or to his extraordinary acts of will" (Force, "Newton's God of Dominion", p. 89). As we have seen in Section 3.2 in Chapter 3, Newton's fourth rule referred to the demand that objections should be drawn from phenomena and to the fact that in the *Principia* Newton proceeded along a series of approximations – Force, however, remains silent about this technical context. To the best of my knowledge, in none of Newton's manuscripts is Rule IV (or its adumbrations) discussed in a theological context.

[173] Delgado-Moreira, "Newton's Treatise on Revelation".

[174] Cf. Snobelen, "God of Gods, and Lord of Lords", pp. 200–202.

[175] Delgado-Moreira, "Newton's Treatise on Revelation: The use of mathematical discourse", p. 225.

[176] Ibid., p. 230, cf. p. 234. See, furthermore, her paper "Newton on the Civil and Religious Origins of Humanity". Delgado-Moreira has thereby tempered some of the claims made in Mamiani, "The Rhetoric of Certainty: Newton's Method in Science and the Interpretation of the Apocalypse".

as to discern between y^{e} good and y^{e} bad, that they should be demonstration to y^{e} one & foolishness to y^{e} other.[177]

It seems therefore entirely correct that "there was a fundamental epistemological difference between Newton's marshalling of evidence in both domains."[178]

6.4.2 Case 2: Newton on Space and Time

In *De gravitatione et aequipondio fluidorum* (CUL Add. Ms. 4003) and *Tempus et locus*[179] (CUL Add. Ms. 3965.13, ff. 541^{r}–542^{r} and ff. 545^{r}–546^{r}; early 1690s), Newton provided a detailed account of his views on space and time. Along the way, he attacked Descartes' theory of space, motion and matter and he offered several technical *cum* theological arguments to substantiate his own views on the matter. Descartes had defined the true motion of a body *x* in terms of change of position of *x* vis-à-vis an adjacent object *y*.[180] In *De gravitatione et aequipondio fluidorum* Newton began by defining place, body, rest, and motion – quantity, duration, and space are too well known to be defined, he added – as follows:

Definition 1. Place [locus] is a part of space [spatium] which something fills completely.[181]
Definition 2. Body [corpus] is that which fills place.
Definition 3. Rest is remaining in the same place.
Definition 4. Motion is change of place.[182]

In sharp contrast to Descartes, to whom extension and matter were identical, Newton distinguished between space ("spatium") and body ("corpus").[183] Newton's main objections in *De gravitatione et aequipondio fluidorum* to the Cartesian account of matter, motion and space can be summarized as follows[184]:

1. Descartes contradicts himself when he ascribes to all bodies one true, philosophical motion, but at the same time ascribes innumerable other motions to bodies through participation.[185]

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177 Yahuda 1.1, f. 18^{r}–19^{r} [italics added].
178 Delgado-Moreira, "Newton's Treatise on Revelation", p. 245.
179 Transcribed and translated in McGuire, "Newton on Place, Time, and God: An Unpublished Source".
180 Janiak, ed., *Newton, Philosophical Writings*, p. 14.
181 Cf. Newton, *The Principia*, p. 409.
182 Janiak, ed., *Newton, Philosophical Writings*, pp. 11–12.
183 Ibid., p. 14.
184 Compare with Palter, "Saving Newton's Text", p. 408 and Steinle, *Newtons Entwurf* "*Über die Gravitation*. . . .," pp. 18–19.
185 Janiak, ed., *Newton, Philosophical Writings*, pp. 14–16.

2. On this account, no motion can ever be said to be true, absolute and proper ("verus ↓absolutus↓ et proprius").[186]
3. Moreover, Descartes' account of motion cannot adequately account for motion by participation. According to Descartes' criterion of true motion, bodies which move by participation can never be said to be in motion, because they are at rest with respect to the bodies which contain them.[187]
4. It follows from Descartes' theory that motion can be generated when no force is acting.[188]
5. On Descartes' account, God could not generate motion in some bodies although he impelled them with great force.[189]
6. Moreover, on the same account, God could not define the past position of any moving body.[190]
7. From the Cartesian doctrine, it follows that a moving body has "no determinate velocity and no definite line in which it moves [dico quòd exinde sequitur nullam esse mobilis alicujus determinatam velocitatem nullámq definitam lineam in qua movetur]."[191]
8. Descartes offers a path to atheism by neglecting the intimate relation between space, time and matter, on the one hand, and God, on the other hand.[192]
9. Descartes' *systema mundi* does not allow that immaterial beings reside in space.[193] This entails, furthermore, that God cannot reside in space (and time).

Note that, while objections 5–6 and 8–9 are theological, objections 1–4 and 7 are of a more technical nature. With respect to 7, Newton noted, furthermore, that "And, what is worse, that the velocity of a body moving without resistance cannot be said to be uniform, nor the line said to be straight in which its motion is accomplished [corporis sine impedimentis moti velocitas non dici potest uniformis, neque linea recta in quâ motus perficitur]."[194] According to Newton, Descartes' account of motion is not only internally inconsistent (see 1), but also inconsistent with the law of inertia.[195] Given the above objections, Newton concluded that in order to provide "truer foundations of the mechanical sciences [ut veriora Mechanicarum scientiarum fundamenta substruantur[196]]" it is necessary "that the definition of places,

186 Ibid., p. 17; CUL Add. Ms. 4003, f. 6^{r}.
187 Janiak, ed., *Newton, Philosophical Writings*, pp. 16–17.
188 Ibid., p. 18.
189 Ibid.
190 Ibid., p. 20.
191 Ibid., p. 19; CUL Add. Ms. 4003, p. 8.
192 Janiak, ed., *Newton, Philosophical Writings*, p. 31.
193 Ibid.
194 Ibid., p. 19; CUL Add. Ms. 4003, p. 8.
195 See, furthermore, Stein, "Newton's Metaphysics", p. 265 and Palter, "Saving Newton's Text", pp. 411–412.
196 CUL Add. Ms. 4003, p. 11.

and hence of local motion, be referred to some motionless being [ad ens aliquod immobile referatur[197]] such as extension alone or space in so far as it is seen to be truly distinct from bodies."[198] At that point, Newton started expounding his doctrine of absolute space:

> Perhaps now it may be expected that I should define extension as substance, or accident, or else noting at all. But by no means, for it has its own manner of existing which is proper to it and which fits neither substances nor accidents. It is not a substance: on the one hand, because it is not absolute in itself, but is *as it were an emanative effect of God and an affection of every kind of being*; on the other hand, because it is not among the proper affections that denote substance, namely actions, such as thought in the mind and motions in body. [Non est substantia tum quia non absolutè per se, sed tanquam Dei effectus emanativus, et omnis entis affectio quædam subsistit; tum quia non substat ejusmodi proprijs affectionibus quæ subtantiam denominant, hoc est actionibus, quales sunt cogitationes in mente et motus in corpore.][199]

In the above quotation, Newton emphasized that space is not absolute in itself, but dependent on God, i.e. "as it were an emanative effect of God," and that space is "an affection of every kind of being." In the following quotation, Newton elaborated on the meaning of "an affection of every kind of being:"

> *Space is an affection of a being just as a being* [entis quatenus ens affectio].[200] *No being exists or can exist which is not related to space in some way. God is everywhere, created minds are somewhere, and body is in the space it occupies; and whatever is neither everywhere nor anywhere does not exist [nec ullibi est id non est]. And hence it follows that space is an emanative effect of the first existing being, for if any being whatsoever is posited, space is posited*. [Et hinc sequitur quod spatium sit entis primariò existentis effectus emanativus, ↓quia posito quolibet ente ponitur spatium↓.] And the same may be asserted of duration: for certainly both are affections of attributes of a being according to which the quantity of any things existence is individuated to the degree that the size of its presence and persistence is specified.[201]

Here Newton clarified that the statement that space and time are "affections of every kind of being" means that space and time are "affections of every kind of being just as a being" ("entis quatenus ens affectio"). In other words, "to exist" means "to exist in space and time". Since space and time are affections of every kind of being, it follows that they are also affections of God's existence. In another quotation, Newton explained the meaning of "as it were an emanative effect of God:"

> *Lastly, space is eternal in duration and immutable in nature because it is the emanative effect of an eternal and immutable being. If ever space had not existed, God at the time would have been nowhere; and hence he created space later (where he was not present himself), or else, which is no less repugnant to reason, he created his own ubiquity*. [Siquando

[197] Ibid., p. 10.

[198] Janiak, ed., *Newton, Philosophical Writings*, pp. 20–21.

[199] Ibid., p. 21 [italics added]; CUL Add. Ms. 4003, p. 12.

[200] This statement was later repeated in McGuire, "Newton on Place, Time, and God: An Unpublished Source", pp. 116/117 and Clarke, ed., *A Collection of Papers, which passed between the late Learned Mr. Leibniz and Dr. Clarke*, p. 73, p. 129.

[201] Janiak, ed., *Newton, Philosophical Writings*, p. 25 [italics added].

non fuerit spatium, Deus tunc nullibi adfuerit, et proinde spatium creabat postea ubi ipse non aderat, vel quod non minùs rationi absonum est, creabat suam ubiquitatem.][202]

From this quotation it becomes clear that the "first existing being" is God, i.e. "an eternal and immutable being." In Section 6.2 in this chapter, we have seen that space and time are the logical, rather than ontological or emanative, consequences of God's omnipresence. I will now develop this point further. Newton endorsed the view that God is a necessary being. That God is a necessary being means that there is no entity preceding God in space and time that caused his existence: God endures always and he is present everywhere.[203] (That God, moreover, has supreme dominion, furthermore, implies that God is *substantially* present always and everywhere.) God's omnipresence in space and time is the result of his intrinsic necessity,[204] or, in other words, given the existence of a necessary being (God), infinite space and time necessarily exist. In the General Scholium, Newton wrote on God: "He endures always and is present everywhere, and *by existing always and everywhere* [existendo semper and ubique] *he constitutes* [constituit] *duration and space*."[205] In addition to this, he recorded the following: "It is agreed that the supreme God necessarily exists, and by the same necessity [[e]t eadem necessitate] he is *always* and *everywhere*."[206] Immediately after the words "space is eternal in duration and immutable in nature because it is the emanative effect of an eternal and immutable being," the following explanation follows: "If ever space had not existed, God at the time would have been nowhere; and hence he created space later (where he was not present himself), or else, which is no less repugnant to reason, he created his own ubiquity." In other words, if we wish to avoid both absurdities, namely that God would have been nowhere or that God created his own ubiquity, we should endorse the view that space and time co-exist with God. More precisely, God's necessary existence constitutes infinite space and time. This is exactly what saying that space and time are "(as it were) emanative effects of God" amounts to. By arguing that space and time are affections "of a being just as a being," Newton sought to convey that space and time are not standard properties. Rather, space and time are external *propria*, i.e. abstract attributes which do not characterize or define something's essence, but which can be ascribed to something according to the way it is.[207] Correspondingly, space and time are not to be considered as (standard) properties of God, but rather as predications which characterize God's existence as an eternal and omnipresent

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202 Ibid. [italics added]; CUL Add. Ms. 4003, p. 19.

203 McGuire, *Tradition and Innovation*, p. 31.

204 Ibid.

205 Newton, *The Principia*, p. 941 [italics added]; Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 761.

206 Newton, *The Principia*, p. 942; Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 762.

207 Here I am indebted to McGuire and Slowik, "Newton's Ontology of Omnipresence and Infinite Space" for their characterization of space and time as external *propria* and their development of its theological implications. See, furthermore, McGuire, *Tradition and Innovation*, Chapter 1.

being. This corresponds to the following passage, which is a draft version of the *Avertissement au Lecteur* which accompanied Pierre Des Maizeaux' 1720 French edition of the Clarke-Leibniz correspondence[208]:

> The Reader is desired to observe, that wherever in the following papers through unavoidable narrowness of language, infinite space or Immensity & endless Duration or Eternity, are spoken of as Qualities or Properties of the substance w^ch is Immense or Eternal, the terms Quality or Property are not taken in that sense wherein they are vulgarly, by the writers of Logick & Metaphysicks applied to matter; but *in such a sense as only implies them to be modes of existence in all beings, & ~ unbounded modes & consequences of the existence of a substance which is really necessarily & substantially Omnipresent & Eternal: Which existence is neither a substance nor a quality, but the existence of a sublime will all ~~all~~ its attributes properties & qualities, & yet is so modified by place and duration that those modes cannot be rejected without rejecting the existence*. When we speak of things w^ch come not within the reach of our senses, it's difficult to speak without Tropes and Figures. ~~& danger of being misunderstood~~ In this sense the Schoolmen made a Nunc stans, ~~that is~~ to be eternity, ~~&~~ by consequence an attribute of God ~~that is one of Gods [two illegible words]~~ & eternal duration hath ~~as good a~~ a better title to that name, tho it be but a mode of his existence. For a Nunc stans is a moment w^ch always is & yet never was nor will be which is a contradiction in terms. And it is as much a contradiction to tell us that God is ever where by his vertue & no where by his substance, & yet some make this to be his Ubiquity & by consequence on of his Attributes.[209]

Here Newton made it explicit that space and time are "unbounded [i.e., infinite] modes & consequences of the existence of a substance which is really necessarily & substantially Omnipresent & Eternal." An important underlying premise here is that only an infinite "cause" (God) can produce an infinite "effect" (space and time). This passage establishes once again that the "first existing being" is indeed God.[210]

In addition to the discussion of space and time, Newton also addressed another key issue in *De gravitatione et aequipondio fluidorum*: the nature of bodies, which exist by divine will. Newton clarified that although the nature of bodies is unknown we can define bodies as "*determined quantities of extension which omnipresent God endows with certain conditions* [definire possemus esse Extensionis quantitates determinatas quas Dius ubique præsens conditionibus quibusdam aificit]."[211]

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208 Des Maizeaux, ed., *Recueil de diverses pièces*.

209 CUL Add. Ms. 9597.2.14.2, [f. 1^r] [period after the Clarke-Leibniz correspondence; italics added]. In a variant, Newton wrote that space and time are "modes of existence in all beings & ↓unbounded↓ consequents of the existence of a Being w^ch is really, ~~&~~ necessarily & substantially Omnipresent and Eternal; w^ch existence is neither ↓a↓ substance ~~attribute~~ nor a quality, but the existence of a substance with all its attributes properties & qualities, & accidents [S^r D^r Clarks 4^th Reply § 10.] & yet is so modified by place & ~~time~~ ↓duration↓ that ~~they~~ ↓these modes↓ cannot be rejected without rejecting ~~the~~ ↓the↓ existence." (C.U.L. Add. Ms. 3965, f. 270^r [period after the Clarke-Leibniz correspondence]).

210 I.e., *pace* Stein, "Newton's Metaphysics", p. 271. See Slowik, "Newton's Metaphysics of Space", p. 442 and McGuire and Slowik, "Newton's Ontology of Omnipresence and Infinite Space".

211 Janiak, ed., *Newton, Philosophical Writings*, p. 28; CUL Add. Ms. 4003, p. 22.

Bodies are mobile, cannot coincide and can excite various perceptions of the senses and imagination in our minds.[212] Additionally,

> For we cannot posit bodies of this kind without at the same time positing that God exists, and has created bodies in empty space out of nothing, and that they are beings distinct from created minds, but able to be united with minds. [. . .] If we say with Descartes that extension is body, do we not manifestly offer a path to atheism,[213] both because extension is not created but has existed eternally, and because we have an idea of it without any relation to God, and so in some circumstances it would be possible for us to conceive of extension while supposing God not to exist? [Si cùm Cartesio dicamus extensionem esse corpus, an non Athei~~sm~~↓æ↓ viam manifestè sternimus, tum quòd extensio non est creatura sed ab æterno fuit, tum quod Ideam ejus sine aliquà ad Deum relatione habemus absolutam, adeòque possumus ut existentem interea concipere dum Deum non esse fingimus.] [. . .] Moreover, if the distinction of substances between thinking and extended is legitimate and complete, God does not eminently contain extension within himself and therefore cannot create it; but God and extension would be separate, complete, absolute substances, and in the same sense. But on the contrary if extension is eminently contained in God, or the highest thinking being, certainly the idea of extension will be eminently contained within the idea of thinking, and hence the distinction between these ideas will not be such that both may fit the same created substance, that is, but that a body may think, and thinking being be extended. [Præterea si legitima et perfecta est distinctio substantiarum in cogitantes et extensas; tum Deus extensionem in se non continet eminenter et proinde creare nequit; sed Deus et extensio ↓duæ↓ erunt substantiæ seorsim completæ absolutæ et univocè dictæ. Aut contra si Extensio in Deo sive summo ente cogitante ↓eminenter↓ continetur, certè Idea Extensionis in Idea Cogitationis eminenter continebitur, et proinde distinctio Idearum non tanta erit quin ut am~~p~~↓b↓æ possint eidem creatæ substantiæ competere, hoc est corpora cogitare vel res cogitantes extendi.][214]

It is clear from the above discussion that in *De gravitatione et aequipondio fluidorum* both theological as well as technical arguments were presented side by side in Newton's first systematic exposition of space, time and matter. It is therefore undeniably the case that there is a significant theological dimension to Newton's views of space and time.[215] It is interesting to determine whether Newton maintained that dimension in the demonstrative part of natural philosophy, i.e. in the *scholium* on space and time which concludes the *Definitiones*. As I shall argue in what follows, in the *scholium* on space and time Newton consciously restricted himself to technical arguments.[216]

In the *scholium* on space and time, Newton announced that he wanted to eliminate "certain preconceptions" about space and time. There he defined absolute space and time, as follows:

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212 Janiak, ed., *Newton, Philosophical Writings*, pp. 27–28.

213 For Newton on atheism, see Keynes Ms. 7, f. 1^{r}.

214 Janiak, ed., *Newton, Philosophical Writings*, p. 31; CUL Add. Ms. 4003, p. 26.

215 See, furthermore, McGuire, *Tradition and Innovation*, Chapters 1 and 4, id., "Predicates of Pure Existence: Newton on God's Space and Time", and id., "The Fate of the Date: The Theology of Newton's *Principia* Revisited".

216 Cf. the conclusions reached in Janiak, *Newton as Philosopher*, Chapter 5.

> 1. Absolute, true, and mathematical time, in and of itself and of its own nature, without reference to anything external, flows uniformly and by another name is called duration. Relative time, apparent, and common time is any sensible and external measure of duration by means of motion; such a measure – for example a month an hour a day – is commonly used instead of true time. [Tempus absolutum, verum, & mathematicum, in se & natura sua sine relatione ad externum quodvis, æquabiliter fluit, alioque nomine dicitur duratio: Relativum, apparens, & vulgare est sensibilis & externa quævis durationis per motum mensura (seu accurata seu inæquabilis) qua vulgus vice veri temporis utitur; ut hora, dies, mensis, annus.]
>
> 2. Absolute space, of its own nature without reference to anything external, always remains homogeneous and immovable. Relative space is any movable measure or dimension of this absolute space; such a measure or dimension is determined by our senses from the situation of space with respect to bodies and is popularly used for immovable space, as in the case of space under the earth or in the air or in the heavens, where the dimension is determined from the situation of the space with respect to the earth. [Spatium absolutum, natura sua sine relatione ad externum quodvis, semper manet similare & immobile: Relativum est spatii hujus mensura seu dimensio quælibet mobilis, quæ a sensibus nostris per situm suum ad corpora definitur, & a vulgo pro spatio immobile usurpatur: uti dimensio spatii subterranei, aërii vel cœlestis definita per situm suum ad terram.] Absolute and relative space are the same in species and in magnitude, but they do not always remain the same numerically. For example, if the earth moves, the space of our air, which in a relative sense and with respect to the earth always remains the same, will now be one part of the absolute space into which the air passes, now another part of it, and thus will be changing continually in an absolute space.[217]

Correspondingly, he defined absolute motion – in opposition to Descartes[218] – as "the change of position of a body from one absolute place to another [translatio corporis de loco absoluto in locum absolutum]," while relative motion is defined as "change of position from one relative space to another [de relativo in relativum]."[219] Since we cannot see and distinguish the absolute parts of space we rely on their sensible measures instead, "which is not inappropriate in ordinary human affairs, although in philosophy abstraction from the senses is required."[220] "For it is possible that there is no body truly at rest to which places and motions may be referred," Newton added.[221] Absolute and relative rest and motion are distinguished by their properties, causes and effects. With respect to their properties, Newton pointed out the following:

> [1] It is a property of rest that bodies truly at rest are at rest in relation to one another. And therefore, since it is possible that some body in the regions of the fixed stars or far beyond is absolutely at rest, and yet it cannot be known from the position of bodies in relation to one another in our regions whether or not any of these maintains a given position with relation

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[217] Newton, *The Principia*, pp. 408–409; Koyré, Cohen and Whitman, eds., *Principia mathematica*, I, p. 46.

[218] Stein, "Newtonian Space-Time", p. 269 and Barbour, *Absolute or Relative Motion?*, I, p. 617 ff.

[219] Newton, *The Principia*, p. 409.

[220] Ibid., p. 411.

[221] Ibid. Only God can distinguish between true and apparent motion (Cohen, "Isaac Newton's *Principia*, the Scriptures, and the Divine Providence", p. 528).

> to that distant body, *true rest cannot be defined* [definiri nequit] *on the basis of the position of bodies in relation to one another*.
>
> [2] It is a property of motion that parts which keep given positions in relation to wholes participate in the motions of such wholes. [. . .] Therefore, when bodies containing others move, whatever is relatively at rest within them also moves. *And thus true and absolute motion cannot be determined* [definiri nequit] *by means of change of position from the vicinity of bodies that are regarded as being at rest*. [. . .]
>
> [3] A property akin to the preceding one is that when a place moves, whatever is placed in it moves along with it, and therefore a body moving away from a place that moves participates also in the motion of its place. [. . .] *Thus, whole and absolute motions can be determined* [definiri possunt] *only by means of unmoving places, and therefore in what has preceded I have referred such motions to unmoving places and relative motions to movable places*. Moreover, the only places that are unmoving are those that all keep given positions in relation to one another from infinity to infinity and therefore always remain immovable and constitute the space that I call immovable.[222]

With repect to their causes and effects, he remarked:

> The causes which distinguish true motions from relative motions are the forces impressed upon bodies to generate motion. True motion is neither generated nor changed except by forces impressed upon the moving body itself, but relative motion can be generated and changed without impression of forces upon this body. [. . .] The effects distinguishing absolute motion from relative motion are the forces of receding from the axis of circular motion. For in purely relative circular motion these forces are null, while in true and absolute circular motion they are larger or smaller in proportion to the quantity of motion. [. . .] Therefore that endeavour [i.e., the endeavour of the water in a rotating vessel to recede from the axis by rising up the sides of vessel] does not depend on the change of position of the water with respect to the surrounding bodies, and thus the true circular motion cannot be determined [definiri nequit] by means of such changes of positions.[223]

In the *scholium* on space and time, Newton offered indirect evidence for absolute space and time by highlighting the inadequacies of Descartes' account on the matter – accordingly, Newton's famous bucket experiment referred to above was designed to show the inadequacy of Descartes' account of true motion and to provide indirect evidence for absolute space.[224] The support for absolute space and time which Newton adduced was thus mainly established *ex via negativa*: i.e. by showing what was lacking in Descartes' account of space and time for an adequate dynamical study of motion.[225] It should be kept in mind that Newton was not so much concerned with proving that absolute space and time exist in the *scholium* of

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[222] Newton, *The Principia*, pp. 411–412 [numbers and italics added]; Koyré, Cohen and Whitman, eds., *Principia mathematica*, I, pp. 49–50.

[223] Newton, *The Principia*, pp. 412–413; Koyré, Cohen and Whitman, eds., *Principia mathematica*, I, pp. 50–51.

[224] Janiak, *Newton as Philosopher*, pp. 137–138. Consequently, Leibniz' relativism was not at all Newton's target.

[225] Rynasiewicz, "By their Properties, Causes and Effects: Newton's Scholium on Time, Space, Place and Motion – I", esp. p. 137; id., "By their Properties, Causes and Effects: Newton's Scholium on Time, Space, Place and Motion – II", pp. 295–306; and Barbour, *Absolute or Relative Motion?*, I, pp. 635–637.

space and time, but rather with *defining* – hence, Newton's repeated use of the verb "definire" – *the proper notions of space and time, which his dynamical study called for*. As we have seen above, in order to prove the existence of (absolute) space and time Newton turned to theological arguments.

Newton argued that true motion cannot be definied on the basis of "the position of bodies in relation to one another" or, more specifically, on the basis of "change of position from the vicinity of bodies that are regarded as being at rest."[226] In the *scholium* on space and time, Newton was concerned with defining "the conceptual framework that made relative motion physically intelligible within a conception of causal interaction," as Robert DiSalle has adequately put it.[227] If we define motion as relative motion, then motion can be generated or changed without impressed forces; if, on the other hand, we define motion with respect to absolute space, then the essential link between cause, i.e. impressed force, and effect, i.e. motion, is guaranteed. This is exactly what Newton was conveying in the following passage from the *scholium* on space and time: "True motion is neither generated nor changed except by forces impressed upon the moving body itself, but relative motion can be generated and changed without impression of forces upon this body. [Motus verus nec generatur nec mutatur, nisi per vires in ipsum corpus motum impressas: at motus relatives generari & mutari potest sine viribus impressis in hoc corpus.]"[228] In other words, without absolute space the essential tie between motion and force will be separated.[229] Newton's concept of absolute time, but not that of absolute space,[230] is implied by the laws of motion, which posit that there is a fundamental difference between inertial motions, which describe equal distances in equal times, and motions that are accelerated by impressed forces.[231] In Law I, "equally flowing" time intervals are implicitly defined as those time intervals in which an inertially moving body would traverse equal distances. Sensible measures of "equal" time, i.e. relative measures, are thus to be conceived as approximations open to further refinement of an ideal and uniform mathematical time,[232] which is indeed what Newton hints at:

> In astronomy, absolute time is distinguished from relative time by the equation of common time. For natural days, which are commonly considered equal for the purpose of measuring time, are actually unequal. Astronomers correct this inequality in order to measure celestial motions on the basis of a truer time. It is possible that there is no uniform motion by which

[226] Stein, "Newtonian Space-Time", p. 271.

[227] DiSalle, *Understanding Space-Time*, p. 13. See, furthermore, Kerszberg, "The Cosmological Question in Newton's Science", pp. 76–79.

[228] Newton, *The Principia*, p. 412; Koyré, Cohen and Whitman, eds., *Principia mathematica*, I, pp. 50–51

[229] Janiak, *Newton as Philosopher*, p. 136. See Belkind, "Newton's Conceptual Argument for Absolute Space", esp. 285–288 for an additional perspective.

[230] Bear in mind that Newton accepted Galilean relativity explicitly in Corollaries 5 and 6 to the laws of motion (Newton, *The Principia*, p. 423).

[231] DiSalle, "Newton's Philosophical Analysis of Space and Time", p. 39.

[232] DiSalle, *Understanding Space-Time*, pp. 21–22, 25; McGuire, *Tradition and Innovation*, p. 5.

> time may have an exact measure. All motions can be accelerated and retarded, but the flow of absolute time cannot be changed. The duration is rightly distinghuised from its sensible measures and is gathered from them by means of an astronomical equation. Moreover, the need for using this equation in determining when phenomena occur is proved by experience with a pendulum clock and also by the eclipses of the satellites of Jupiter.[233]

Newton's views on space and time as presented in the *scholium* on space and time are thus not based on his theological views.[234] The reason for this is that in the *scholium* on space and time Newton was not concerned with explicating God's bearing on space and time, which he was indeed explicitly addressing in *De gravitatione et aequipondio fluidorum*, *Tempus et Locus* and the General Scholium, but rather with defining the proper notions of space and time, which his dynamical study called for. The moral is that, although there is an undeniable theological dimension to Newton's views on space and time,[235] in the *scholium* on space and time Newton was addressing a different matter and, correspondingly, he offered a non-theological, i.e. technical, argumentation. True, in the General Scholium Newton claimed that "by existing always and everywhere" God "constitutes duration and space,"[236] but there he went beyond the scope of what *he* conceived of as the demonstrative part of natural philosophy.[237]

Having considered the above case-studies, I shall now turn to a general discussion of the interaction between Newton's theology and natural philosophy.

6.5 The Interaction Between Newton's Natural Philosophy and His Theology (II): General Discussion and Conclusion

A trend in recent Newton scholarship, has been to ascribe a unity to the totality of Newton's *oeuvre*. This approach has been exemplified by B. J. T. Dobbs, who in her *The Janus Faces of Genius* wrote:

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[233] Newton, *The Principia*, p. 410.

[234] DiSalle, "Newton's Philosophical Analysis of Space and Time", p. 49. Note that DiSalle's approach does thereby not deny Newton's theological motivations for endorsing absolute space and time: "Asking this question about Newton's theory does not deny its connection with his profound metaphysical convictions – not only about space and time, but about God and his relationship to the natural world. On the contrary, it illuminates the nature of those convictions and their relationship to Newton's physics." (ibid., p. 38; cf. id., *Understanding Space-Time*, p. 40).

[235] Funkenstein, *Theology and the Scientific Imagination*, pp. 90–96. A central thesis of Funkenstein's book is that seventeenth-century natural philosophers often defended their scientific ideas by means of theological arguments and vice versa. This idea will be taken up again in the following section.

[236] Newton, *The Principia*, p. 941.

[237] It is also significant that Newton corrected the statement, occurring in the second edition of the *Principia*, that "to treat of God from phenomena is certainly a part of experimental philosophy," into "to treat of God from phenomena is certaintly a part of natural philosophy" in the third edition (Newton, *The Principia*, p. 943).

> The ultimate unity of Newton's thought slowly centered itself for me on my growing realization that Newton supposed himself to be studying God's activity in every area – in micromatter, in cosmic order, in history. Although the connections between and among Newton's various pursuits are often far from obvious to the modern reader, I have attempted in this book to suggest the connections as it seems Newton saw them. [. . .] It was a system *not* just of "the mathematical principles of natural philosophy." On the contrary, it was to have been a grand unification of natural and divine principles and it included a vision of God's activity not only *in* this world as we know it but also at the world's beginning and at its apocalyptic end and renovation. It was a vision in which the Arian Christ, as God's "Agent" throughout time, always putting the will of the Supreme God into effect, kept God intimately connected both with the physical world and with humanity: that was Newton's ultimate answer to the twin specters of deism and atheism that had always haunted him. It was also a vision that forces one to the conviction that one must give a religious interpretation not only to Newton's alchemy but to *all* of Newton's work, including the *Principia* and the *Opticks*, since Newton himself was apparently motivate to study "the frame of nature" in order to learn God's activity.[238]

In similar vein, James E. Force has concluded: "Of Pythagoras, F. M. Cornford has written that "The vision of philosophic genius is a unitary vision. Such a man does not keep his thoughts in two separate compartments, one for weekdays, the other for Sundays." The same vision holds true for Newton."[239] Force has furthermore argued that Newton's theology "influenced his science every bit as much as his science influences the rigorous textual scholarship of his theology"[240] and that the "metaphysics of the *Principia* is absolutely pervaded by Newton's God."[241] Stephen D. Snobelen seems to endorse a similar position, for he has claimed that "interpenetration existed at a fundamental level between the cognitive content of the theological and the natural philosophical features of Newton's grand study"[242] and that "Newton's theological concerns [. . .] made a not insignificant impact on both the *methodological* and *cognitive* dimensions of his philosophy."[243]

Rob Iliffe has recently warned against the assumption that "the individual "Isaac Newton" was the undifferentiated author of a group of writings that were *all coherent or unified at some level*" and called attention to Newton's sensitivity to disciplinary compartmentalization.[244] Although Iliffe does not deal with the relation

[238] Dobbs, *The Janus Faces of Genius*, pp. 253–254 [underscore added]; cf. Dobbs, "'The unity of thought:' An integrated view of Newton's work". Cf. Force, "Newton's God of Dominion", p. 94.

[239] Force, "Newton, the 'Ancients,' and the 'Moderns'", p. 257.

[240] Force, "Newton's God of Dominion", p. 78. A different voice was raised by Richard S. Westfall, who noted that "even if we grant the influence [of Newton's theology on his experimental philosophy], we remain still on a plane of *high generality* from which it is difficult if not impossible to demonstrate an influence on some *concrete* element of his science." (Westfall, "Newton's Theological Manuscripts", pp. 139–140).

[241] Force, "Newton's God of Dominion", p. 87.

[242] Snobelen, "God of Gods, and Lord of Lords", p. 197.

[243] Ibid., p. 204. It should be noted that in his recent "The Theology of Isaac Newton's *Principia mathematica*" Snobelen has fine-tuned his tackle on the matter.

[244] Iliffe, "Abstract considerations: Disciplines and the Incoherence of Newton's Natural Philosophy", p. 428.

between Newton's theology and his natural philosophy in particular, "the recognition of disciplinary compartmentalization within his analyses of the natural world has ramifications for larger claims about the unity of his entire oeuvre, or for sorts of connection between different areas of his research."[245] What Iliffe proposes is essentially a bottom-up approach with respect to ascertaining an underlying unity in Newton's oeuvre: we should begin by taking Newton's disciplinary differentiation seriously and by studying the epistemological and methodological uniqueness of each of the discipline in which Newton was active. Once such detailed studies have been made, we can a posteriori begin searching for "conceptual links between different areas of Newton's work."[246] I am most sympathetic to Iliffe's proposal, because the ascription of an overall unity to Newton's work has the unwelcome effect that relations are seen where in fact there are none. There certainly is textual support in favour of Iliffe's interpretation. For instance, in *Seven Statements on Religion* Newton wrote: "That religion & Philosophy are to be preserved distinct. We are not to introduce divine revelations into Philosophy, nor philosophical opinions into religion."[247] At a different place, Newton pointed out that "The business of Experimental Philosophy is ~~only~~ to find out by experience & observation not how things were created but what is the present frame of Nature,"[248] thereby hinting at the differentiation between natural philosophy and cosmogonic matters. In the first edition of the *Principia* there is only one reference to God: "Therefore God placed the planets at different distances from the sun so that each one might, according to the degree of its density, enjoy a greater or smaller amount of heat from the sun."[249] In subsequent editions this reference was deleted. It seems that, in line with his disciplinary compartmentalization, Newton decided to treat such matters in the General Scholium.[250]

245 Ibid.

246 Ibid., p. 451.

247 Keynes Ms. 6, f. 1^r [post-1710]. Cf. CUL Add. Ms. 3965.13, f. 547^r [additions and corrections intended for the third edition of the *Principia*]: "In Philosophia ↓tractanda↓ abstinendum est a religione, in religione ↓tractanda↓ abstinenda est a philosophia." It is significant to note that this statement occurs in a draft scrap containing statements on "deus" as a "vox relativa". See, furthermore: "If you would know the meaning of the several names given to Christ in preaching the Gospel, you are ~~not~~ to have recourse ↓not↓ to Met~~h~~↓a↓physicks & Philosophy but to y^e scriptures of the old Testament. ffor Christ sent not his disciples to pre↓a↓ch Metaphysicks ~~& Philoso~~ to y^e common people & to their wives & children, but ~~instructed the~~ expounded to them ~~the scriptures of the~~ ↓out of Moses &↓ the Prophets ↓& Psalms↓ the things concerning himself & opened their understanding that they might understand the scriptures & then sent them to teach all nations what he had taught them. And the Apostle bids us beware of vain philosophy." (Keynes Ms. 3 [*Irenicum*], p. 32).

248 CUL Add. Ms. 3970, f. 242^v [post-1704].

249 Newton, *The Principia*, p. 814, footnote c.

250 In Cohen, "Isaac Newton's *Principia*, the Scriptures, and the Divine Providence", p. 530 my suggestion is found untenable on the gounds that "Newton eliminated the reference to God in this corollary in the interleaved copy of the *Principia* in his private library, apparently long before he had even contemplated a *General Scholium*" (cf. Snobelen, "The Theology of Isaac Newton's *Principia Mathematica*", p. 388). Cohen has, furthermore, suggested that "Newton might very

It is undeniable that the study of theology was of utter importance to Newton.[251] How else could we explain the sheer amount of theological papers which Newton composed throughout his life? As we have seen in Section 6.1 in this chapter, Newton was immersed in theological studies from the mid to the late 1670s, which renders the supposition that the theologically related Queries in the 1706 Latin edition of *The Opticks* and the General Scholium (1713) were Newton's *entrées* in matters theological historically inaccurate.[252] From *De gravitatione et aequipondio fluidorum*, Newton's 1692/1693 letters to Bentley, the Classical Scholia, and from the General Scholium it is equally undeniable that Newton saw theological significance in the results harvested by experimental philosophy. In this context, the theological significance of Newton's discovery of universal gravitation should not be underestimated.[253] While in the the pre-*Principia* period Newton had situated "active principles" exclusively at the microscopic level,[254] in the *Principia* he had established that such active principles were acting at a cosmic scale as well. Furthermore, in the General Scholium Newton clearly noted that we can only know God by his "properties and attributes and by the wisest and best construction of things and their final causes" and that "to treat of God from phenomena is certainly a part of natural philosophy."[255] As we have seen in Section 1.7 in Chapter 1, Newton's theological views are relevant for an understanding of Newton's tackle of the possibility of action at a distance between gravitationally interacting bodies.

In an excerpt from the *scholium* on space and time, Newton recorded:

> Relative quantities, therefore, are not the actual quantities whose names they bear but are those sensible measures of them (whether true or erroneous) that are commonly used instead of the quantities being measured. But if the meanings of words are to be definied by usage, then it is these sensible measures which should properly be understood by the terms 'time," "space," "place," and "motion," and the manner of expression will be out of the ordinary and purely mathematical if the quantities being measured are understood here. Accordingly those who there interpret these words as referring to the quantities being measured do

well have concluded that this topic either required a more considerable discussion, perhaps with further examples, or else should not be mentioned at all" (ibid.). I am not entirely convinced that Newton's deletion of this excerpt prior to the composition of the General Scholium rules out my interpretation: even without the plans for a General Scholium Newton might have decided that it is better not to brush on such theological topics in experimental philosophy.

[251] Westfall correctly noted the following on Newton's interest in theology: "Newton's interest in theology was not a private idiosyncrasy but a reflection of a general problem that occupied nearly every scientist of the late seventeenth century and every thinking person beyond the scientific community." (Westfall, "Newton's Scientific Personality", p. 56).

[252] Snobelen, "The Theology of Isaac Newton's *Principia Mathematica*", pp. 382–385.

[253] Dobbs, *The Janus Faces of Genius*, p. 211.

[254] Ibid., Chapter 2 and p. 147.

[255] Newton, *The Principia*, pp. 942–943.

> violence to the Scriptures. And they no less corrupt mathematics and philosophy who confuse true quantities with their relations and common measures.[256]

In the above excerpt, Newton implied that the Two Books do not contradict one another if the Scriptures are properly understood.[257] In the short manuscript entitled *An Account of the Systeme of the World* (ca. 1682–1694),[258] Newton endorsed the principle of accommodation:

> In determining the true systeme of the world the main Question is whether the earth do rest or be moved. For deciding this some bring text of scripture, but in my opinion misinterpreted, the Scriptures speaking not in the language of Astronomers (as they think) but in that of y[e] common people to whom they were written. So where tis said that [a]God hath made y[e] round world so fast that it cannot be moved, the Prophet intended not to teach Mathematicians the spherical figure ~~of the whole~~ & immoveableness of the whole earth & sea in the heavens but to tell the vulgar in their own dialect that God had made the great continent of Asia Europe & Africa so fast upon its foundations in the great Ocean that it cannot be moved therein after the manner of a flo↓a↓ting Island.[259]

Moreover:

> Now the Question being about motion is a mathematical one & therefore requires skill in Mathematics to decide it. And seeing it is difficulter to argue demonstratively about magnitude & motion together then about magnitude alone, there is greater skill required here then in pure Geometry so that none but able Mathematicians may pretend to be competent judges of this matter.[260]

In these passages, Newton was conveying that the Two Books address different questions and, accordingly, followed different methods, which resonates nicely with the conclusions I have reached in Section 6.4.1 in this chapter.

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256 Ibid., p. 414. Cohen has recorded several early precursors of this excerpt (Cohen, "Isaac Newton's *Principia*, the Scriptures, and the Divine Providence", pp. 527/535).

257 See the discussion in Snobelen, "The Theology of Isaac Newton's *Principia Mathematica*", pp. 399–401.

258 See Appendix 3 for its full transcription and Cohen, "Isaac Newton's *Principia*, the Scriptures, and the Divine Providence" for further background.

259 CUL Add. Ms. 4005, f. 39[r]. See, furthermore, Snobelen, "'Not in the Language of Astronomers:' Isaac Newton, the Scriptures and the Hermeneutics of Accommodation". In the following excerpt Newton again separated the literal-natural from the moral or allegorical sense: "But men of corrupt minds, not attending to the relation w[ch] the names of Christ have to the prophesies concerning him, [illegible] & w[ch] the several parts of scripture have to one another; but taking things in a litteral & ~~philosophical~~ ↓natural↓ sense w[ch] were spoken allegorically & ~~w[th]~~ ~~re~~ ↓morally↓ with relation to piety & virtue, & wresting the expressions of scripture to the opinions of philosophers, have brought into the Christian religion many philosophical opinions to w[ch] the first Christians were strangers. So where Christ saith This is my body, meaning a type of his body, the Romanists Catholicks understand it litterally as if the bread was ~~transubstantiated~~ changed into Christs body in a litteral sense. Where Christ saith, The father is greater then I, meaning in power, some have thence inferred that the Son is a part of the father." (Yahuda 15.3, f. 47[v]).

260 CUL Add. Ms. 4005, f. 41[r].

There is a myriad of examples of seventeenth-century natural philosophers who used theological arguments in order to justify certain conclusions about the empirical world – Descartes' derivation for his laws of motion is a striking illustration of this. However, when we look at Newton's natural-philosophical works proper,[261] i.e. the *Principia* minus the General Scholium and *The Opticks* minus the Queries, it is striking that Newton never resorts to theological arguments to reach conclusions about the empirical world. This explains why, as we have seen in Section 6.4.2 in this chapter, a theological argumentation in the *scholium* on space and time is absent. This has a striking implication: for Newton experimental philosophy had to stand on its own (empirical and (physico-)mathematical) footing. In the context of seventeenth and early-eighteenth-century natural philosophy, Newton was then not at all exceptional because he studied both experimental philosophy and theology; on my reading, he was, however, exceptional because he was convinced that, although the study of the Book of Nature contributes to unravelling God's plan of providence, the study of nature should be based on its own methodology: experiments and (physico-)mathematics. This suggests that Newton endorsed the idea that experimental philosophy and theology were mutually supportive, but nevertheless methodologically distinct from each other.

[261] By which I mean those parts of Newton's works in which he developed arguments based on (physico-)mathematics and empirical observation to reach conclusions about the empirical world or those parts in which he explicated the laws and concepts required for a physico-mathematical and empirical study of nature. I use 'natural philosophy proper' as synomymous with 'experimental philosophy' or 'the demonstrative part of natural philosophy'. 'Natural philosophy' refers to those parts in Newton's work in which he reaches conclusions about the empirical world without restricting himself to physico-mathematical or empirical arguments alone. Thus, while the Definitions in the *Principia* pertain to *experimental philosophy*, Newton's theological and ontological discussion of space and time pertains to *natural philosophy*.

Appendices

Appendix 1 is an inventory of the drafts of the General Scholium. In a paragraph-by-paragraph way, I shall summarize the content of the published version of the General Scholium. Subsequently, I provide a paragraph-by-paragraph overview of the draft-versions. In this way, all relevant differences between the drafts and the published version are gathered systematically.

Appendix 2 contains all manuscript material from the Portsmouth collection related to the Classical Scholia. Again, I document the relevant differences.

Appendix 3 provides a transcription of Newton's *An Account of the Systeme of the World*.

Appendix 1: Catalogue of the Manuscript Material Directly Related to the General Scholium

Before we proceed to a detailed comparison between the various draft versions and the published result, I shall give a paragraph-by-paragraph overview of the General Scholium based on Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 759–765[262]:

¶ 1: Newton stated that Cartesian vortices are incongruent with the observed celestial phenomena. The celestial phenomena cannot be accounted for unless vortices are eliminated ("nisi vortices tollantur"[263]).

¶ 2: Newton further explained that the celestial motions, above our atmosphere, occur in a Boylean vacuum ("in vacuo *Boyliano*"). He added, although that the celestial bodies persevere in their orbits according to the law of universal gravitation, they "could not originally have acquired the regular position of the orbits by these laws [of universal gravitation]."[264]

¶ 3: Newton illustrated the previous point by showing that the primary and secondary planets revolve in concentric circles, in the same direction and as most closely as possible on the same plane. The swiftness and ease by which comets pass in all parts of the heavens showed that these motions cannot be caused by "mechanical causes" as Descartes would want it ("originem non habent ex causis mechanicis"[265]). He continued that the construction of "this most elegant system of the sun [elegantissima haecce solis]" arises from the "design

[262] For the translation see Newton, *The Principia*, pp. 939–944. The numbering of the paragraphs is mine.

[263] Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 759.

[264] Newton, *The Principia*, p. 940.

[265] Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 760.

and dominion of an intelligent and powerful being [consilio & dominio entis intelligentis]", i.e. the "dominion of *One*" ("*Unius* dominio").[266]

¶ 4: Then Newton stated his famous Hebraic credo of God as a *Pantokrator*, that is, a universal ruler. God's godhood (deitas) lies in the lordship of God, "not over his own body as is supposed by those for whom God is the world soul,"[267] but over his servants. The designations (appellationes) "eternal," "infinite," "perfect," "omniscient," and "omnipotent" are subordinate to (and derive from) God's dominion. Newton clarified that God is not eternity, infinity, (absolute) space and time, but "by existing always and everywhere he constitutes duration and space."[268] Next, he added that God is omnipresent not only virtually but also *substantially* in his creation and that "the bodies feel no resistance from God's omnipresence."[269] Subsequently, he observed that we cannot know the inner substances of bodies: we can only know their external attributes and properties. Similarly, we cannot have any idea of the substance of God. We can only know God "by his properties and attributes and by the wisest and best construction of things and their final causes."[270] Correspondingly, Newton concluded this paragraph by claiming that "to treat of God from phenomena is certainly a part of natural philosophy."[271]

¶ 5: Here Newton stated he had deduced from phenomena the force of gravity (but not yet assigned a cause to it) and that he had shown that gravity acts in proportion to the quantity of matter (and not to the quantity of the surfaces of bodies, as the Cartesians claimed). In experimental philosophy, whatever is not deduced from phenomena is a hypothesis. As Newton had not succeeded in deducing from phenomena the cause of gravity, he preferred to remain silent on this matter (cf. Newton's famous *dictum* "hypotheses non fingo").

¶ 6: Newton concluded the General Scholium with some remarks on a certain subtle, electric[272] spirit "pervading gross bodies and lying hidden in them,"[273] which caused attractive and repellent forces at small distances, electricity, the emission, reflection, refraction and inflexion of light, heath, sensory perception, and muscular movement. The laws governing the actions of this spirit are yet unknown and in want of further experimental scrutiny.

We will use this division into paragraphs in what follows. Hereafter, follows my paragraph-by-paragraph description of Newton's drafts of the General Scholium.

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266 Newton, *The Principia*, p. 940.

267 Ibid.

268 Ibid., p. 941.

269 Ibid., pp. 941–942.

270 Ibid., p. 942. Further draft material related to this is on CUL 9597.2.18.97, [ff. 2^r].

271 Newton, *The Principia*, p. 943.

272 Ibid., p. 944, footnote p.

273 Ibid., p. 943.

[I] CUL Add. Ms. 3965, f. 357^{r-v} and f. 358^{r} [1712/1713] (= A-version)[274]

Transcribed and translated in Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, pp. 349–352, cf. pp. 352–355; transcribed on *The Newton Project*'s website.

¶ 1 corresponds[275] to ¶ 2 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 759.

¶ 2 corresponds roughly[276] to ¶ 3 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 759–760.

¶ 3 consists of two sentences[277] which are included in ¶ 5 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 764.

¶ 4 corresponds roughly[278] to ¶ 1 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 759.

¶ 5 corresponds roughly to ¶ 5 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 764.

¶ 6 corresponds roughly[279] to ¶ 6 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 764–765.

¶ 7–9 are unique paragraphs which were not included in the printed version of the *Principia*.[280] For the reader's convenience, I shall provide the original text of these last three paragraphs:

> [¶ 7] Si vitra duo plana & polita & quam proxime contigua soperficiebus parallelis in aquam stagnantem immergantur; aqua inter vitra ascendit supra superficiem aquae stagnantis & altitudo ascencus erit reciproce ut distantia vitrorum. Et hoc experimentum succeedit [sic] in vacuo Boyliano ideoque a gravitate atmospherae incumbentis non pendet. Partes vitri ad superficiem aquae ascenditis attrahi[un]t aquam ipsis proximam & inferiorem & ascendere faci[un]t. Attractio eadem est in variis distantiis vitrorum & idem pondus aquae

[274] All five holograph drafts to the *Principia* were written before January 1712/13 (Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 349). I have followed the Halls' division into paragraphs. In Cohen-Whitman's recent translation the division in paragraphs differs slightly.

[275] A relevant variation occurs near the end of the paragraph where Newton wrote, but later crossed out: "~~At motus illi~~ ↓~~sub initia~~↓ ~~ex causis mere mechanicis sub initia oriri non potuere.~~" (CUL Add. Ms. 3965, f. 357^{r}).

[276] It should be noted that this manuscript contains but a small portion on God's dominion: "Hic omnia regit non ut anima mundi sed ut ~~natura De~~ universorum Dominus. Omnipraesens est et in ipso ↓continentur &↓ moventur universa idque sine resistentia cum ↓sit Ens non corporeus neque↓ corpore restiatur." (CUL Add. Ms. 3965, f. 357^{r}).

[277] Namely, "Caeterum causam gravitatis nondum exposui neque exponendā suscepi siquidem ex phaenomenis colligere nondum potui ↓enim↓. Non oritur ex vi centrifuga vorticis alicujus siquidem ~~non~~ tendit non ad axem vorticis sed ad centrum Planetae." (CUL Add. Ms. 3965, f. 357^{r}).

[278] A noticable difference is: "Nam hypotheses seu physicas ~~seu mechanicas~~ seu qualitatum occultarum ~~fugiunt praejudica~~ fugio. Praejudicia sunt et scientiam pariunt." (CUL Add. Ms. 3965, f. 357^{v}; cf. Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 353).

[279] Here Newton pointed to the similarity between electricity and gravity as inter-particular forces.

[280] These experiments are not included in any later version. See Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, pp. 354–355. They occur, however, on CUL Add. Ms. 3968, f. 260^{v}.

attolit, ideoque aquam eo altius ascendere facit quo minor est distantia vitrorum Et simili de causa aqua ascendit in tubulis tenuibus vitreis idque eo altius quo tenuiores sunt tubulae, et liquores omnes ascendunt in substantiis spongiosis.[281]

[¶ 8] Vitra duo plana et polita longitudine viginti digitorum latitudine [ ] parabantur. Horum alterum horizonti parallelum jacebat, & ad unum ejus terminum gutta erat olei malorum citriorum. Alterum priori sic imponebatur ut vitra ad alterum eorum extremum se mutuo contingeret, ad alterum vero ubi gutta jacebat, a se invicem distarent intervallo quasi decimae sextae partis digiti, & vitrum superis contigeret guttam. Quo facto gutta statim incipiebat moveri versus concursum vitriorum eo velocius movebatur. Succesit etiam hoc experimentum in vacuo. Et ortus est hic motus ab attractione vitrorum.

[¶ 9] Si vitra ad concursum suum paululum attollerentur ut vitrum inferius inclinaretur ad horizontem gutta ascenderet, & vitrum superius positionem suam ad vitrum inferius servaret: gutta ascendendo tardius movebitur quam prius & quo major esset vitri inferioris inclinatio eo tardior erat motus guttae donec gutta quiesceret, pondere ejus attractionem vitrorum aequante. Sic ex inclinatione vitri inferioris dabatur pondus guttae et ex pondere guttae dabatur attractio vitrorum. Inclinationes autem vitri inferioris quibus gutta stabat in aequilibrio et distantiae guttae a concursu vitrorum exhibentur in Tabula sequente.[282]

[II] CUL Add. Ms. 3965, f. 359^{r-v} and f. 360^{r} [1712/1713] (= B-version)
Transcribed on *The Newton Project*'s website.

¶ 1 corresponds exactly to ¶ 1 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 759.
¶ 2 corresponds exactly to ¶ 2 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 759.
¶ 3 corresponds almost[283] exactly to ¶ 3 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 759–760.
¶ 4 corresponds almost[284] exactly to ¶ 4 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 760–764.

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[281] Cf. Newton, *The Opticks*, pp. 392–394.

[282] Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, pp. 350–351, cf. pp. 354–355.

[283] In the published version, the following sentence was added at the end of this paragraph: "Et ne fixarum systemata per gravitatem suam in se mutuo cadant, his eadem immensam ab invicem distantiam posuerit." (Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 760).

[284] It contains several sentences on the dominion and omnipresence of God. Newton wrote that "simili consilio constructa, suberunt <u>Unius</u> dominio" [Newton initially wrote "unius" and capitalized it afterwards] (f. 359^{r}). On f. 359^{v}, one relevant sentence was added: "Et haec de Deo, de quo utique ex Phaenomenis disserere, ad Philosophiam experimentalem pertinet. Ex Phaenomenis prodeunt proximae rerum causae: ex his causae superiores donec ad causam summā perveniatur." (ibid.; cf. Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 348). The penultimate sentence is somewhat different (but not relevantly different) from the published version and the paragraph is shorter than in the published version. Hence, in ¶ 4 between the penultimate and the last sentence of the B-version, the following text is omitted: "A caeca necessitate metaphysica, quae utique eadem est semper et ubique, nulla oritur rerum variatio. Tota rerum conditarum pro locis ac temporibus diversitas, ad ideis & voluntate entis necessario existentis solummodo oriri potuit. Dicitur autem deus per allegoriam videre, audere, loqui, ridere, amare, odio habere, cupere, dare, accipere, gaudere, irasci, pugnare, fabricare, condere, construere. Nam sermo omnis de deo a

¶ 5 corresponds exactly to ¶ 5 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 764.

¶ 6 corresponds exactly to ¶ 6 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 764–765.

[III] CUL Add. Ms. 3965, f. 361^{r-v} and f. 362^{r-v} [1712/1713] (C-version)

Transcribed and translated in Newton, *Unpublished Scientific Papers of Isaac Newton*, pp. 355–359, cf. 359–364; transcribed on *The Newton Project*'s Website.

¶ 1 corresponds exactly to ¶ 1 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 759.

¶ 2 corresponds exactly to ¶ 2 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 759.

¶ 3 corresponds roughly to ¶ 3 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 759–760.

¶ 4 corresponds roughly[285] to ¶ 5 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 764.

¶ 5 corresponds roughly[286] to ¶ 4 and ¶ 5 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 763, 764.

¶ 6 corresponds roughly[287] to ¶ 4 and ¶ 5 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 763, 764.

¶ 7 corresponds roughly[288] to ¶ 4 and ¶ 5 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 763, 764.

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rebus humanis per similitudinem aliquam desumitur, non perfectam quidem, sed aliqualem tamen." (Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 763–764; this corresponds to the Newton's additions in his private copy of the the second edition of the *Principia* (CUL Adv.b.39.2, inserted folio between pp. 482–483)). A scrap draft of the omitted text can be found on CUL 3965.13, f. 543^{r-v}. Newton refers to the following scriptural references: "Act. 17.27, 28, Deut 4.39. & 10.14. I King. 8.27. Job. 22.12. Psal. 139.7. Jer. 23.23, 24." (CUL Add. Ms. 3965, f. 359^{v}; cf. CUL Adv.b.39.2, interleaved folio between pp. 482–483).

285 This paragraph is shorter than the published version and continues on f. 361^{v}. The most notable sentences are: "Causam vero harum proprietatum ejus ex phaenomenis nondum potui invenire. Nam hypotheses seu mechanicas seu qualitatum occultarum fugio. Praejudicae sunt et scientiam pariunt. ~~Sufficiat~~ ↓Satis est↓ quod gravitas revera detur, & agat secundum leges a nobis expositas & ad ~~maris nostri~~ corporum coelestium et maris nostri ~~sufficiat~~ motus omnes sufficiat." (CUL Add. Ms. 3965, f. 361^{v}; cf. Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 356). On a separate scrap, Newton wrote: "Leges motuum ex phaenomenis & proprietates gravitatis ex alijs Phaenomenis his Legibus per Inductionem in haec philosophia & vero generalibus habentur cum nulla occurat Objectio ex Phaenomenis derivantur." (CUL Add. Ms. 3965, f. 544^{r}).

286 It is shorter and essentially makes two points: that we do not know the substances of things ("Substantias rerum non cognoscimus. Nullas habemus earum ideas." (CUL Add. Ms. 3965, f. 361^{r}; Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 356)) and that we only know the properties of things.

287 It adds nothing important to the previous paragraph.

288 In this paragraph Newton observed that we only see the figures and colours of things, hear but the sounds which objects produce, touch but the external surfaces of objects, smell but their odours, and taste but their tastes.

¶ 8 is redundant and recapitulates the previous paragraph.

¶ 9–15 contain several propositions on the electric force causing short-rang attractions between small particles.[289]

¶ 16 corresponds roughly[290] to ¶ 4 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 760–764.

¶ 17 corresponds roughly[291] to ¶ 2, ¶ 3 and ¶ 4 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 759, 759–760, 763.

¶ 18 corresponds to ¶ 4 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 762–763.[292]

¶ 19 contains one proposition on the vibration of light.[293]

¶ 20 contains one proposition on the electric spirit that causes animal motion.

¶ 21 contains one proposition stating that the vibrations of the electric spirit are faster than light itself.

¶ 22 contains one proposition on the emission, refraction, reflection and inflection of light caused by the electric spirit.

¶ 23 contains one proposition stating that homogeneous bodies are held together and heterogeneous bodies are separated by the electric spirit.

¶ 24 contains one proposition stating that nutrition is caused by electric attraction.

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[289] For the transcription of these propositions, i.e. ¶ 9–15, see Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 357. Newton gives only the propositions but not their proofs. After having shown that gravity exists and acts according to the inverse-square law, Newton also wished to establish the laws and effects of other attractive forces, viz. electricity and magnetism (cf. "superest ut vires reliquas attractivas, vis scilicet electrica et vis magnetica, examinentur, ut earum leges et effectus [*varias*] ad motus [*minimarum particularum materiae corporeae*] minimorum corporum in dissulatione, fermentatione, vegetatione, [*digestione, praecipitatione, separatione,*] & similibus operationibus [*observentur*] inveniantur" (Newton, *Correspondence*, V, p. 113)). Newton listed five experiments: "1. Vitrorum parallelorum. 2. Inclinatorum. 3. fistularum. 4. Spongiarum. 5. Olei malorum citriorum." (f. 361^{v}).

[290] This paragraph starts on f. 362^{r}. The biblical references which Newton included are: "Act. 17.27, 28, Psal. 139.7. Deut 4.39. & 10.14. I King. 8.27 Job. 22.12. Jer. 23.23, 24. + [VI] John 1.18 & 5.37 1 John 4.12. 1 Tim. 1.17 & 6.16. Col. 1.15" and, additionally, "Exod. 28.4", "Deut. 4.12, 15, 16", and "Isa 40.18, 19" (CUL Add. Ms. 3965, f. 362^{r}).

[291] The main point is that the motion of the celestial bodies can only be explained by postulating attraction over great distances. Newton noted in the middle of this paragraph: "certe causae finales in Philosophia naturali locum habent" (CUL Add. Ms. 3965, f. 362^{r}; cf. Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 358).

[292] This paragraph continues upside-down on f. 362^{r}.

[293] ¶ 19–24, which contain only the propositions but not their demonstrations, are mentioned in ¶ 6 of the published version (Koyré, Cohen and Whitman, *Principia mathematica*, II, pp. 764–765). For a transcription of these propositions, see Hall and Hall, eds., *Unpublished Scientific Papers of Isaac Newton*, p. 359.

[IV] CUL Add. Ms. 3965, f. 363^{r-v} [1712/1713] (= D-version)[294]
Transcribed on *The Newton Project*'s website.

¶ 1 corresponds exactly to ¶ 1 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 759.
¶ 2 corresponds exactly to ¶ 2 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 759.
¶ 3 corresponds[295] to ¶ 4 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 759–760.
¶ 4 corresponds[296] to ¶ 5 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 760–764.
¶ 5 corresponds roughly[297] to ¶ 5 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 764.
¶ 6 corresponds roughly[298] to ¶ 4 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 762–763.
¶ 7 corresponds roughly[299] to ¶ 4 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 761–762.

[V] CUL Add. Ms. 3965, f. 365^{r-v} [1712/1713] (= E-version)
Transcribed on *The Newton Project*'s website.

¶ 1 corresponds exactly to ¶ 1 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 759.
¶ 2 corresponds exactly to ¶ 2 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 759.

[294] Folio 364^{r-v} is blank.

[295] It is identical to the published text but the paragraph is left unfinished and ends with: "Et si stellae fixae sint centra similium systematum ~~suberunt~~ haec omnia ↓simili consilio constructa ~~suberunt~~ suberunt↓ unius dominio." (CUL Add. Ms. 3965, f. 363^{r}).

[296] Newton included the following biblical references: "Act. 17.27, 28 Deut 4.39, & 10.14. I King. 8.27. Job. 22.12. Psal. 139.7. Jer. 23. 23,24." (CUL Add. Ms. 3965.13: f. 363^{r}). On f. 363^{v}, Newton added: "6 John 1.18 & 5.37. Col. 1.15. 1 Tim. 1.17 & 6.16. 1 John 4.12."

[297] The content of this paragraph is, albeit identical to the published version, much shorter – especially, near the end of the paragraph. It did not yet contain Newton's famous line: "Rationem vero harum gravitatis proprietatum ex phaenomenis nondum potui deducere, & hypotheses non fingo." (Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 764).

[298] This paragraph is much shorter than in the published version.

[299] The text of this paragraph is almost identical to the final version, but it is much briefer. It starts with: "Nam Deus est vox relativa & ad servos referetur: & Deitas est dominio Dei in servos." and ends with "Æternus est & infinitus, omnipotens & omnisciens, id est [:] durat ab aeterno in aeternum, & adest ab infinito infinitum." (CUL Add. Ms. 3965, f. 363^{r}).

¶ 3 corresponds[300] to ¶ 3 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 759–760.
¶ 4 corresponds[301] to ¶ 4 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, pp. 760–764.
¶ 5 corresponds[302] to ¶ 5 in Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 764.

[IV] CUL Add. Ms. 3965, f. 539$^{r–v}$ [1712/1713]
Currently unpublished.

¶ 1 corresponds exactly[303] to ¶ 4 in Koyré, Cohen and Whitman, *Principia mathematica*, II, pp. 760–764

300 It is identical to the published text but the paragraph is left unfinished and ends with: "Et si stellae fixae sint centra similium systematum, haec omnia simili consilio constructa suberunt Unius dominio: praesertim & [end of text]" (CUL Add. Ms. 3965, f. 365^{r}; Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 760).

301 The text is identical to the published version, but ends with: "Hunc cognoscimus solummodo per ejus proprietates & attributa et per elegantes & opt[imas] rerum structuras & causas finales." (CUL Add. Ms. 3965, f. 365^{v}; cf. Koyré, Cohen and Whitman, eds., *Principia mathematica*, II, p. 763). Newton listed the following biblical references: "Act. 17.27, 28, Deut. 4.39, & 10.14. I King. 8.27. Job 22.12. Psal. 139.7. Jer. 23.23, 24" (ibid.).

302 It is somewhat shorter than the published version and the sole relevant difference is: "Causam vero harum gravitatis proprietatum ex phaenomenis nondum potui deducere, & hypotheses seu mechanicas seu qualitatum occultarum non sequor." (CUL Add. Ms. 3965, f. 365^{v}).

303 CUL Add. Ms. 3965, f. 539^{r} further contains two redundant sentences. An identical paragraph can be found in Newton's "Corrigenda et addenda in Lib. III. Princip." (ibid., f. 526$^{r–v}$).

Appendix 2: Manuscripts from the Portsmouth Collection Related to the Classical Scholia

[I] CUL Add. Ms. 3965, ff. 268^{r-v} and 269^{r-v} [1692/1693][304]
Currently unpublished.

¶ 1 corresponds exactly[305] to ¶ 1 in CUL Add. Ms. 3965, f. 270^{r}.
¶ 2 corresponds almost exactly[306] to ¶ 2 in CUL Add. Ms. 3965, f. 270^{v}–271^{v}.
¶ 3 correspond almost exactly[307] to ¶ 4 in CUL Add. Ms. 3965, f. 271^{v}.[308]
¶ 4 [309]corresponds roughly[310] to Royal Society, Gregory Ms. 247, ff. 11–12.[311]
¶ 5 is a unique paragraph.[312]
¶ 6 corresponds roughly to CUL Add. Ms. 3965, f. 271^{r}.
¶ 7 [313] is a draft of Royal Society, Gregory Ms. 247, f. 11^{r}.[314]
¶ 8 is a draft of Royal Society, Gregory Ms. 247, f. 12^{r}.[315]
¶ 9 is a draft to ¶ 1 of CUL Add. Ms. 3965, ff. 268^{v}.

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[304] This is clearly the draft of CUL Add. Ms. 3965, f. 270^{r}, f. 271^{r} and f. 272^{r}.

[305] Albeit that this paragraph it is shorter. The text stops at "Haec enim Lucretius ex mente veterum discuit Lib I vers 601." (CUL Add. Ms. 3965, f. 268^{r}).

[306] In the top of this folio Newton wrote: "see y^{e} backside". The paragraph is continued on CUL Add. Ms. 3965, f. 268^{v}. There is a slight variation at the end: "Et hic est motus declinationis quem Epicurus dedit atomos." (cf. Casini, "Newton: The Classical Scholia", p. 37).

[307] It is continued on CUL Add. Ms. 3965, f. 269^{r}.

[308] This paragraph is transcribed in Casini, "Newton: The Classical Scholia", p. 38.

[309] The following two paragraphs are on CUL Add. Ms. 3965, f. 268^{v}.

[310] It is most likely a draft of Royal Society, Gregory Ms. 247, ff. 11–12, since it omits much of the ancient references (for the translation of this piece, see McGuire and Rattansi, Newton and the "Pipes of Pan," pp. 115–117). A notable variation is: "Talis erat mystica illa Veterum Philosophia: estque hypothesis omnium simplicissima et eo nomine maxime philosophica. Sed et pia satis, si modo omnis huic Spiritui intelligendi et volendi potestas conedatur, astris autem nulla. Imò pientissima quatenus Deum a Philosophia naturali abesse non sinit." (CUL Add. Ms. 3965, f. 268^{v}).

[311] Schüller, "Newton's *Scholia*", pp. 230–238. Cf. Memoranda by David Gregory, 5–7 May 1694, Newton, *Correspondence*, III, pp. 334–340 [transcription of Royal Society, Gregory Ms. 247, ff. 68–69].

[312] It contains a note on the Egyptians and their symbolic use of the ouruborus in their rites (CUL Add. Ms. 3965, f. 268^{v}).

[313] The following three paragraphs are actually three separate notes (the last one being written upside down).

[314] Schüller, "Newton's *Scholia*", pp. 230–232.

[315] Ibid., pp. 234–236.

[II] CUL Add. Ms. 3965, f. 270^r, f. 271^r and f. 272^r [1692/1693][316]

All paragraphs of this manuscript has been fully transcribed and discussed, in Casini, "Newton: The Classical Scholia", pp. 36–38.

[III] CUL Add. Ms. 3965, f. 277$^{r–v}$ and 278$^{r–v}$ [1692/1693]

Currently unpublished.

¶ 1 on f. 277^r is a draft of Royal Society, Gregory Ms. 247, f. 6^r.[317]
¶ 1 on f. 277^v is an almost exact copy[318] of Royal Society, Gregory Ms. 247, f. 9^r.[319]
¶ 2 on f. 277^v is a draft[320] of f. 1^r of the additional leaves inserted between pages 412–413 of Newton's private first edition of the *Principia* (CUL Adv. b.39.1).
¶ 1 [321] on f. 278^r is a draft of Royal Society, Gregory Ms. 247, f. 11$^{r–v}$.[322]
¶ 2 on f. 278^r is a draft[323] of Royal Society, Gregory Ms. 247, f. 12$^{r–v}$.
¶ 3 [324] on f. 278^v is an almost exact copy of Royal Society, Gregory Ms. 247, f. 12$^{r–v}$.[325]

[IV] CUL Add. Ms. 3965, ff. 328$^{r–v}$–329^r, ff. 656$^{r–v}$/655$^{r–v}$/654^v

These folios contain variant material which seems to have been neglected by scholars of Newton's Classical Scholia. Folio 328 begins *in medias res* and here Newton documents how the ancient Egyptians had proper knowledge of heliocentric astronomy, how this knowledge was passed to the Greeks and how it became corrupted. The text ends with the line: "Neque mirum est hypothesin Copernicaeam

316 Note that folios CUL Add. Ms. 3965, f. 270^v, f. 271^v and f. 272^v are left blank.

317 For its transcription, see Casini, "Newton: The Classical Scholia", pp. 27–28 and Schüller, "Newton's *Scholia*", pp. 222–223.

318 The last paragraph ends abruptly and misses some of the final sentences of the corresponding first paragraph in Gregory Ms. 247, f. 9^r.

319 See Casini, Newton: "The Classical Scholia", pp. 25–26; Schüller, "Newton's *Scholia*", pp. 218–220.

320 The text has been shuffled around, but no relevant differences can be found. It is similar to Royal Society, London, Gregory Ms. 247, f. 8^r and f. 9^r. For the transcription, see Casini, "Newton: The Classical Scholia", pp. 25–27; Schüller, "Newton's *Scholia*", pp. 218, 220, 222.

321 The first paragraph on this folio contains Newton's remarks on Proposition VII of Book III of the *Principia*. The second paragraph contains Newton's remarks on Proposition VIII.

322 See Casini, "Newton: The Classical Scholia", pp. 30–31; Schüller, "Newton's *Scholia*", pp. 230–231.

323 It misses the references to Macrobius, Proclus, and Eusebius (Casini, "Newton: The Classical Scholia," p. 31) – however, these are given in a separate paragraph on CUL Add. Ms. 3965, f. 278^v.

324 As Newton used the envelope of a letter sent to him to take these notes, this paragraph continued in several directions on the folio: horizontally, vertically and upside-down.

325 See Casini, "Newton: The Classical Scholia", pp. 31–32; Schüller, "Newton's *Scholia*", pp. 234–235.

tum olire, ut nuper, in Tychonicam degererasse."[326] The second variant contains related material and is written on a double-folded folio.

[V] CUL Add. Ms. 3965, f. 640^{r-v} [1692/1693]:
Currently unpublished.

¶ 1 is a draft[327] of Royal Society, Gregory Ms. 247, f. 9^{r}.
¶ 2 [328] corresponds roughly[329] to Royal Society, Gregory Ms. 247, f. 6^{v}.
¶ 3–6 [330] discuss light and heavy elements.

[326] CUL Add. Ms. 3965, f. 328^{r}.
[327] This paragraph is shorter, but contains no relevant variations.
[328] This paragraph is continued on CUL Add. Ms. 3965, f. 640^{v}.
[329] It omits several quotations from Lucretius.
[330] The last two paragraphs have been crossed out.

Appendix 3: Manuscript Transcription of An Account of the Systeme of the World

On ff. 39^{r}–42^{r} of CUL Add. Ms. 4005, Newton left some notes on heliocentrism. This material, which nowhere made it to print, seems related to a brief and incomplete manuscript folio, entitled *Machinæ Mundanæ Descriptio Brevis*, which Newton composed ca. 1692–1694.[331] Newton wrote on himself in the third person, but the handwriting is definitely his.

[f. 39^{r}] An Account of the Systeme of the World described in M^{r} Newton's Mathematicall Principles of Philosophy.[332]

I. Scripture abused to prove the immoveableness of the ~~earth~~ globe of y^{e} ~~Eart~~ Earth.[333] In determining the true systeme of the world the main Question is whether the earth do rest or be moved. For deciding this some bring text of scripture, but in my opinion misinterpreted, the Scriptures speaking not in the language of Astronomers (as they think) but in that of y^{e} common people to whom they were written.[334] So where tis said that [a]God hath made y^{e} round world so fast that it cannot be moved, the Prophet intended not to teach Mathematicians the spherical figure ~~of the whole~~ & immoveableness of the whole earth & sea in the heavens but to tell the vulgar[335] in their own dialect that God had made the great continent of Asia Europe & Africa so fast upon its foundations in the great Ocean that it cannot be moved therein after the manner of a flo↓a↓ting Island. For this Continent was the whole habitable world anciently known & by y^{e} ancient eastern nations was accounted [b]round or circular as was also the [c]sea encompassing it. ↓& this earth & sea they accounted flat as if ye sun moon & stars ascended out of y^{e}↓ ↑ocean at their rising & went down into it again at their setting.↑ This continent is the world

[331] CUL Add. Ms. 3965, f. 542^{v}. This agrees with Cohen's dating of this manuscript to the early 1690s (Cohen, "Isaac Newton's *Principia*, the Scriptures, and the Divine Providence", p. 542). This brief and incomplete text is written on the backside of material related to *Tempus et locus* and is written with the same pen and handwriting. It contains a brief defence of heliocentrism (cf. "12. Nihil obstare quo minus Terra pro lege Planetarum circa solem moveatur. Diluuntur objectiones ex sacra litteris.").

[332] Some words are missing since the lower corners of these folios are damaged by fire. A transcription of this manuscript has previously appeared in: Cohen, "Isaac Newton's *Principia*, the Scriptures, and the Divine Providence", pp. 544–548. It is currently featured on *The Newton Project*'s website. I am indebted to Stephen D. Snobelen for allowing me to compare my own transcription with his and for discussion on the matter.

[333] The title of this section is written in the right margin of f. 39^{r}.

[334] Cf. Keynes 106, f. 6^{v} [1681/2]: "As to Moses I do not think his description of y^{e} creation either Philosophical or feigned, but that he described realities in a language artificially adapted to y^{e} sense of the vulgar. Thus where he speaks of two great lights I suppose he means their apparent, not real greatness. So when he tells us God placed those lights in y^{e} firmam[en]t, he speaks I suppose of their apparent not of their real place, his business being not to correct the vulgar notions in matters philosophical but to adapt a description of y^{e} creation as handsomly as he could to y^{e} sense and the capacity of y^{e} vulgar."

[335] Cf. Newton's accommodationism in his letter to Thomas Burnet on January 1680/1 (Newton, *Correspondence*, II, pp. 333–334).

or earth usually mentioned in scripture & there described to be ^{d}broad & to have ^{e}ends or ^{f}borders, ~~&~~ ↓that is ~~^{g}~~ circular ones↓ whose center some placed in Egypt others at Delphos, others at Jerusalem. And this world the Prophets consider as established in the Ocean upon sure & immoveable foundations at y^{e} first creation. The heavens were of old & the earth standing out of y^{e} water & in the water [that is in the midst of the Ocean like an Island] by the word of God. 2 Pet. 3.5. Thou Lord in the beginning hast laid the foundations of the earth & the heavens are the work of thine hands Psal 102.25. Prov. 8.29. Where wast thou when I laid the foundations of the earth. Declare if thou hast understanding who [word partially illegible: ?*hath*?] *laid the measures thereof or who hath stretched* [word missing: ?*the*?] *line over it.* ~~or~~ *Whereupon are the foundations thereof* [words partially illegible: ?*fix'd or*?] *who hath laid the corner stone thereof, when the starrs* [words missing: ?*of the morning*?] *praised me together*, &c. Job 38.4. ~~↓[missing word]~~ [word partially legible: ?~~When~~?] ~~he sat in a circle upon the face the deep [that is formed it circular~~[336] ~~about the earth] when he appointed the foundations of the earth, then was I by him. Prov. 8. 27,29.~~↓ *The earth* ↑*is*↑ [f. 40r] *is the Lord's & all that therein is the compas of the world & they that dwell therein. For he hath founded it upon the seas & established it upon the floods* Psal 24.1,2 & 136.6. ↓*Thou hast laid the foundation of the round world* Psal 89.12.†↓ [f. 39^{v}] *He laid the foundations of the earth that it never should move at any time: Thou encompassedst it wth the deep like as with a garment* Psal. 104.5. So then the round world spoken of in scriptures is such a word as hath foundations ↓~~&~~ is founded in the waters↓ & by consequence 'tis not the whole globe of the Earth & Sea but only the habitable dry land. ffor the whole Globe hath no foundations, but this ↓habitable↓ world is founded in the seas. And since this world by reason of the firmness of its foundations is said in scripture to be immoveable this immoveableness cannot be of y^{e} whole globe together, but only of its parts one amongst another it signifies nothing more then that those parts are firmly compacted together so that the dry land or Continent of Europe Asia & Africk cannot be moved upon the main body of y^{e} globe on w^{ch} 'tis founded. ffor this immoveableness of y^{e} earth is opposite to that it's motion spoken of in Job. *He removeth the mountains & they feel not when he overtroweth them in his wrath: He removeth the earth out of her place that the pillars thereof do shake* Job. 9.6.

II Mathematics abused to prove the Globe of the Earth immoveable[.][337] There is yet another sort of arguments against the motion of y^{e} whole earth taken from o^{e} senses, as if the earth could not be moved wthout o^{r} being many ways sensible of its motion. But this way of arguing proceeds from want of skill & judgement in Mathematical things, & therefore is insisted upon only by the common people & ~~some practical~~ ↓such↓ mathematicians ↓as understand not so much as the principles of Mechanics.↓ ~~who have skill enough only to write Collections~~. Were the earth moved uneavenly by joggs such motion would be easily perceived, but an eaven motion such as the earth's is supposed, ought to be imperceptible. ffor in

336 The remainder of the crossed-out text continues in the right margin in a 90° angle to the text.

337 The title of this section is written in the right margin of f. 40^{r}.

[word missing: ?any?] systeme of bodies the motions of y^e bodies one amongst [word partially illegible: ?↓anot↓her?] are the same whether the systeme rest or be [word missing: ?moved?] on uniformly, as is mathematically demonstrable. So [f. 41r] So the motions of all things in a ship are found the same whether the ship rest or be under sail. In both cases things fall perpendicularly down by the mast & projectiles fly alike towards all quarters. Nor can a blinded Marriner tell whether the ship move fast or slow or not at all. And there is the same reason of the System of the earth sea & air with the things therein. We cannot tell by o^r senses whether they all rest or move on eavenly together.

III Accurate skill in Geometry & Mechanics requisite to decide the Question.[338] Such arguments as these being insufficient to determin the Question, 'tis fit we should lay aside these & the like vulgar prejudices & have recourse to some strickt & proper way of reasoning. Now the Question being about motion is a mathematical one & therefore requires skill in Mathematics to decide it. And seeing it is difficulter to argue demonstratively about magnitude & motion together then about magnitude alone, there is greater skill required here then in pure Geometry so that none but able Mathematicians may pretend to be competent judges of this matter. The great difficulty of this part of Mathematics seems to be the reason that y^e Ancients made but little progress in it. In this last age since the revival & advancement of these studies, some able Mathematicians as Galileo & Hugenius have carried it further then y^e Ancients did. M^r Newton to advance it far enough for his purpose has spent to two first of his books in demonstrating new Propositions about force & motion before he begins to consider the systeme of the world. Then in his third Book he teaches that systeme from the Propositions demonstrated in the two first. The designe of this [missing word: ?↓pa↓per?] is to give you an account of this Systeme [words partially illegible: ?& ↓re↓fer?] you for the Demonstrations thereof to the [missing words: ?↓Book its↓elf?] or to the judgement of such Mathematicians as have [f. 42r] have perused it [end of text]

a Psal 93.2 & 96.10.
~~b Psal. 98.8.~~
b Strabo Geog. l.1. pp. 2, 4.
c Prov. 8.27. Job. 9.8.
d Job. 38.18. Psal. 50.1.
e Job. 28.24 & 37.3. Psal. 46.9. & 72.8
f Psal. 74.17
~~g. Prov. 8.27~~[339]

† *When he set a circle upon the face of the deep* [that is, formed it circular about the earth] – *when he gave to the sea his decree that y^e waters should not pass his commandmt, when he appointed the foundation of the earth, then was I by him*. Prov. 8.27,28.[340]

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338 The title of this section is written in the right margin of f. 41^r.

339 References a-g are in the right margin of f. 39^r.

340 This biblical reference is on f. 39^v.

Chapter 7
Conclusion

From what we have surveyed in the preceding chapters, it has become clear that the hypothesis that Newton was a bad or confused methodologist is beset with many difficulties. Newton was not a simplistic inductivist nor did he believe that causes could be derived *unconditionally* from phenomena, i.e. he did not believe in the *absolute deducibility* of theoretical propositions.

In Chapters 2 and 3, I have shown that Newton carefully distinguished between the (physico-)mathematical treatment of force and the physical treatment of force and that he emphasized that the former should always precede the latter in order to uncover the forces present *in rerum natura* "more safely." In the (physico-)mathematical treatment of force, Newton explicated the physico-mathematical conditions under which, given the laws of motion, certain motions would occur exactly or *quam proxime*. Of course, Newton clearly focused on those motions which would be relevant in the study of the *systema mundi*, i.e. Keplerian motions. However, in the context of Book I Newton did not assume ab initio the causes producing those motions, i.e. inverse-square centripetal forces, are acting in the empirical world. Rather, Book I proceeds *conditionally*, since it establishes which sorts of motions will occur, given the laws of motion, *if* certain physico-mathematical conditions hold and vice versa. This square nicely with what Newton said concerning the first phase of natural-philosophical investigation: "Mathematics requires an investigation of those quantities of forces and their proportions that follow from any conditions *that may be supposed* [ex conditionibus quibuscunque positis]."[1] I have also emphasized that the models of Book I are not purely mathematical, but physico-mathematical instead: the idealized "bodies" of Book I are subject to the same physical laws as real-world bodies and their forces and motions are analyzable by the same technical concepts, i.e. by Definitions I–VIII. Given these features, Newton could bridge the gap between mathematics and physics: the physico-mathematical conditions, which are relevantly similar to what would become their referents in the context of Book III, are predicated under the same laws that hold in the empirical world and, given the Definitions, one could relate certain technical terms to their quasi-physical measures. In Chapter 4 I have shown that one

[1] Newton, *The Principia*, p. 588 [italics added].

S. Ducheyne, *The Main Business of Natural Philosophy*, Archimedes 29,
DOI 10.1007/978-94-007-2126-5_7,

of the major problems which Newton encountered while methodizing optics was that Newton's causal explanations of optical phenomena could not be constrained by theory. What sets Newton's *Principia*-style approach apart from a hypothetico-deductive approach is the fact that he derived the physico-mathematical models presented in Book I *from the laws of motion*. This shows that Newton was clearly aware of the need to introduce certain theoretical assumptions. However, Newton carefully restricted these theoretical principles to those principles that have empirical support and that have already shown their potential in the study of force and motion, on the one hand, and to those principles that remain neutral with respect to the cause of gravity, on the other. Insofar as the physico-mathematical models of Book I are based on the laws of motion, the fruitfulness of the laws of motion can be measured by the fruitfulness of the physico-mathematical models derived from them – which counts as indirect support for the laws of motion. What mattered for Newton was the quality of his foundational principles. This process of theoretical prioritization served as a tool to reduce arbitrary speculation. At the same time, Newton's dependence on the laws of motion shows that he did not endorse the view that forces can be derived unconditionally from phenomena. A central purpose of Book I was to bi-conditionally relate certain physico-mathematical conditions to certain mathematical regularities, given the laws of motion. By means of the exact causal inference-tickets Newton explicated the physico-mathematical conditions under which Kepler's area rule *would be described* exactly, given the laws of motion. By means of the *quam proxime* causal inference-tickets, Newton anticipated the problem that the mathematical regularities as stipulated in the exact causal inference-tickets will not hold exactly in the empirical world, but only as most closely as possible. Newton sought to overcome this difficulty by showing, by a deduction from the laws of motion, that an overall centripetal force *quam proxime* directed to a centre of force is a necessary and sufficient condition for *quam proxime* time-area proportionality. In this way, he was able to infer that, *given the laws of motion*, a body describing equal areas in equal times as most closely as possible is urged by a centripetal force tending as most closely as possible toward a centre of force. The dynamics that drives Book I is, furthermore, inter-theoretical: it is predicated under a logic according to which the demonstration of the more complex models requires the demonstration of the more basic models.

Once Newton established that Keplerian motion occurs in the empirical world in Phenomena I–VI, he was no longer dealing with abstract quasi-physical measures but with concrete measurements. Correspondingly, in Book III Newton's physico-mathematical models turned into natural-philosophical models. On the basis of the exact or *quam proxime* causal inference-tickets established in Book I and on the basis of Phenomena I–VI, Newton inferred instances of inverse-square centripetal forces. This corresponds to the phase of natural-philosophical inquiry which Newton characterized, as follows: "Then, coming down to physics, these proportions must be compared with the phenomena, *so that it may be found out which conditions of forces apply to each kind of attracting bodies*."[2] Because of the bi-conditional

[2] Ibid., pp. 588–589 [italics added].

relations he had derived from the laws of motion, Newton could present his inferences of inverse-square centripetal forces as deduction, or as "deductions from phenomena." Typically, he did not stop at that point. Given the systematic discrepancies he had established in Book I, Newton moved on to the search for residual forces. In other words, in Newton's methodology the attention shifts to a continuous exploration of residual forces and the establishment of their potential explanation. Correspondingly, in the *Principia* Newton made the study of deviations from exact mathematical regularities a focal point of natural-philosophical investigation. From the perspective of the laws of motion, each deviation from a relevant mathematical regularity is seen as evidence that an additional force, not tracked in our initial approximation, is affecting the situation under consideration. Put differently, such deviations provide evidence for refining our initial approximation. In this sense, the mathematical regularities which hold exactly under certain physico-mathematical conditions become informative about other forces or relevant factors which are acting in the empirical world. Accordingly, Newton approached empirical questions in a sequence of successive approximations. In the context of confirmation, Newton demanded more than empirical confirmation of deduced consequences, since he insisted on accurate measurement of theoretical parameters and on convergence of independently measured parameters. Once Newton had inferred instances of inverse-square centripetal forces, he moved on to a series of gradual and increasingly "wider" inductive generalizations which would ultimately result in the law of universal gravitation. In order to back up these generalizations, Newton introduced a series of *regulae philosophandi*. In line with his demand that there should be systematic dependencies between effects and causes, Rule I (and its corollary, Rule II) licensed the identification of causes which have been shown necessary and sufficient for their effects, and only such causes. Rule III explicated the conditions under which we may assume that a quality or force is universal or not. As I have shown on the basis of Newton's unpublished manuscripts, properties that cannot be intended and remitted are those qualities that cannot be acquired at one point and taken away at another point. In later editions of the *Principia*, Rules II and III were reformulated in epistemological rather than ontological terms, which is consistent with the late Newton's endorsement of provisionalism. The inductive generalizations Newton established were all predicated under Rule IV according to which generalizations established by a methodized process of induction should – despite contrary hypotheses – be considered as exactly or as most closely as possible true, until further empirical investigation renders them more exact or liable to exceptions. It is important to emphasize that the provisionalism Newton envisioned did not extend to the "deductions from phenomena."

With respect to Newton's methodological originality, I have been unable to find precursors who endorsed equally sophisticated methodological views. It seems therefore that Newton's *Principia*-style methodology is to be considered as an original contribution to the History of Scientific Methodology. It needs no arguing that Newton established novel and previously unforeseen scientific results and insights. Science, however, not only encompasses the discovery of new phenomena or the unification of previously unrelated phenomena, on the one hand, and the establishment of new models and theories by means of which these phenomena can be explained or predicted, on the other: science equally encompasses a set of procedures, i.e. a methodology, which codifies *how we have learned to learn from the empirical world*[3] – a learning process which is characteristically never-ending. Correspondingly, the development of science can be analysed from at least three different levels: science's *empirical* development, science's *theoretical* development, and science's *methodological* development. Just as empirical findings and scientific models and theories change in the course of history, scientific methodologies change as scientists are facing new problems or as they are exploring new domains. As Dudley Shapere puts it:

> It is truly *all* aspects of science, not only what are considered its substantive beliefs about nature, but also its methods and aims, that are subject to change in ways that have continued to surprise us. The problems we face in our inquiry about nature, and the methods with which we attempt to deal with those problems, co-evolve with our beliefs about nature.[4]

Therefore there is no universal or fixed scientific methodology. In fact, in the context of Section 3.2 of Chapter 3, it was shown that Newton's method of starting from a first approximation, which explicates the physico-mathematical conditions under which an exact mathematical regularity *would* hold exactly, and proceeding to a determination of the residual causes, which are preventing the mathematical regularity as stipulated in our first approximation to hold exactly, was not at all suited to methodize fluid resistance, a complex phenomenon in which there is no single dominant cause, but several non-separable ones. Similarly, in Chapter 4 I have emphasized that Newton's *Principia*-style methodology was not up for the task of methodizing optics. Throughout this monograph, my primary aim has not been not so much to defend Newton, but rather to explicate his methodological ideas and practices by highlighting their successes as well as their failures.[5]

As a consequence, this monograph should not be seen as a contribution to Newton scholarship or Early Modern Science alone, but also as a contribution to the History of Scientific Methodology, which is a rich and promising field.[6] The study of the methodology of a particular natural philosopher or scientist will reveal a set of practices and convictions about how to obtain knowledge about the empirical world. It will refer inter alia to the inclusion of certain model-specific and mathematical assumptions, certain views on what qualifies as a successful experimental

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[3] My terminology is indebted to Dudley Shapere, who writes: "As long as we can be critical of what we take to be background information, what better basis could we have than what we have learned, *including what we have learned about how to learn*?" (Shapere, "Logic and the Philosophical Interpretation of Science", p. 52 [italics added]).

[4] Ibid., p. 5.

[5] Here I have focused on Newton's methodology proper and not so much on *if* and *how* later physical research was predicated on Newton's methodology. See Smith, "How Newton's *Principia* Changed Physics" on this matter.

[6] Some useful sources (mainly textbooks) are: Achinstein, ed., *Science Rules*; Féher, *Changing Tools*; Gower, *Scientific Methodology*; and, Losee, *A Historical Introduction to the Philosophy of Science*. In several papers I have provided a number of case-studies in the History of

outcome, a set of procedures that ensure the creation of a stable experimental phenomenon, a set a preferred inferences, and a set of inductive rules that underwrite specific inductive generalisations. In the course of *The main Business of natural Philosophy: Isaac Newton's Natural-Philosophical Methodology*, I have paid particular attention to how Newton conceived of and implemented such elements within the context of his natural-philosophical research. I have similarly highlighted that the so-called "Newtonian Revolution" was not restricted to the empirical and theoretical dimensions of science, but applied to the methodological dimension of science as well.

Scientific Methodology – in reverse chronological order: Ducheyne, "Testing Universal Gravitation in the Laboratory"; id., "Whewell's Tidal Researches: Scientific Practice and Philosophical Methodology"; id., "J. S. Mill's Canons of Induction: From True Causes to Provisional Ones"; id., "Galileo and Huygens on Free Fall: Mathematical and Methodological Differences"; id., "'Ignorance is Bliss:' On Bernard Nieuwentijt's *Doctrina Ignorantia* and His Contribution Our Understanding of Scientific Idealisation"; id., "Joan Baptiste Van Helmont and the Question of Experimental Modernism".

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Abbreviations

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PrE_1
First edition of the *Principia* (1687)

PrE_2
Second edition of the *Principia* (1713)

PrE_3
Third edition of the *Principia* (1726)

OE_1
First edition of *The Opticks* (1704)

OE_2
Second edition of *The Opticks* (1717)

OE_3
Third edition of *The Opticks* (1721)

S. Ducheyne, *The Main Business of Natural Philosophy*, Archimedes 29,
DOI 10.1007/978-94-007-2126-5, © Springer Science+Business Media B.V. 2012

OE_4
Fourth edition of *The Opticks* (1730)

OEL_1
First edition of *Optice* (1706)

OEL_2
Second edition of *Optice* (1719)

Manuscripts and Books from Newton's Library Consulted

1 Manuscripts (By Location)

1.1 Cambridge University Library

Portsmouth Collection

CUL Add. Ms. 3958
Early papers by Newton

CUL Add. Ms. 3963
Papers relating to geometry

CUL Add. Ms. 3965
Papers connected with the *Principia*

CUL Add. Ms. 3966
Papers connected with the *Principia* on lunar theory

CUL Add. Ms. 3968
Papers relating to the priority dispute

CUL Add. Ms. 3970
Papers on hydrostatics, optics, sound and heat (mostly optics)

CUL Add. Ms. 3975
Manuscript note-book, notes on precious stones, colours, temperatures, salts, medical matters, alchemy and other subjects

CUL Add. Ms. 3984
Rough drafts of some of Newton's letters to Cotes

CUL Add. Ms. 3989
Of the church

CUL Add. Ms. 3990
De motu corporum liber secundus

CUL Add. Ms. 3996
Trinity College Notebook

CUL Add. Ms. 4002
Lectiones opticae

CUL Add. Ms. 4003
De gravitatione

CUL Add. Ms. 4005
Miscellaneous papers

Macclesfield Collection

CUL Add. Ms. 9597.2
Papers of Isaac Newton

1.2 King's College, University of Cambridge

Keynes Ms. 1
Tuba Quarta

Keynes Ms. 2
Theological Notebook

Keynes Ms. 3
Irenicum, or Ecclesiastical Polyty tending to Peace

Keynes Ms. 4
Notes from Petau on the Nicene Council

Keynes Ms. 5
Two draft theological treatises

Keynes Ms. 6
Seven statements on religion

Keynes Ms. 7
A Short Schem of the true Religion

Keynes Ms. 8
Twelve articles on religion

Keynes Ms. 9
Three paragraphs on religion

Keynes Ms. 10
Paradoxical Questions concerning the morals and actions of Athanasius and his followers

Keynes Ms. 11
Twenty-three queries regarding the word ομοουσιος

Keynes MS. 16
Notes on Jan Baptista van Helmont's *Ortus medicinae*

Keynes Ms. 106
Correspondence with Thomas Burnet

Keynes Ms. 146
The Original of Monarchies

1.3 Royal Society's Library and Archives, London

Gregory Ms. 247
David Gregory's memoranda

Ms. 69
Printer's copy of the first edition of the *Principia*

1.4 Fondation Bodmer, Geneva

Newton Ms.
Of the church

1.5 New College Library, University of Oxford

Ms. 361(1)
Drafts of the *Short Chronicle* and *Original of Monarchies*

Ms. 361(3)
Miscellaneous papers apparently comprising drafts of or notes for the *Chronology of Ancient Kingdoms Amended*

Ms. 361(4)
Papers relating to chronology and *Theologiæ Gentilis Origines Philosophicæ*

1.6 The National Library of Israel

Yahuda Ms. 1
Untitled treatise on Revelation

Yahuda Ms. 2
Various texts on Revelation, Solomon's Temple and Church history

Yahuda Ms. 4
Variantes Lectiones Apocalypticæ

Yahuda Ms. 6
The synchronisms of the three parts of the prophetick Interpretation

Yahuda Ms. 7
Miscellaneous drafts and fragments on prophecy, principally Daniel and Revelation

Yahuda Ms. 8
Notes on prophecies

Yahuda Ms. 9
Treatise on Revelation

Yahuda Ms. 12
Treatise on Church history

Yahuda Ms. 15
Drafts on the history of the Church

Yahuda Ms. 16
Rough draft portions of and notes for *Theologiæ Gentilis Origines Philosophicæ* and *The Original of Monarchies*

Yahuda Ms. 17
Three bundles of notes for a work on the ancients' physico-theology, related to *Theologiæ Gentilis Origines Philosophicæ*

Yahuda Ms. 21
Exposition of 2 Kings 17:15–16

Yahuda Ms. 25
Draft passages on chronology and biblical history

Yahuda Ms. 28
Fragments on the kingdoms of the European tribes, the Temple and the history of Jewish and Christian Churches

Yahuda Ms. 29
Fragment on Church history, mainly concerning Athanasius

Yahuda Ms. 33
Notes on ancient Greek, Roman and Egyptian deities

Yahuda Ms. 41
Draft chapters of a treatise on the origin of religion and its corruption

1.7 The Grace K. Babson Collection of Newtonia, Babson College Archives, Wellesley, Massachusetts

Babson Ms. 434
Prolegomena ad Lexici Prophetici Partem secundam

Babson Ms. 435
Fragmentary draft on the chief rulers of ancient synagogues

Babson Ms. 437
Part of an exposition of 2 Kings, 17:15–16

Babson Ms. 438
Draft notes on early Church rites and the Creed

1.8 Williams Andrews Clark Memorial Library, University of California Los Angeles

**N563M3 P222
Paradoxical Questions concerning the morals and actions of Athanasius & his followers

1.9 Harry Ransom Humanities Research Center, The University of Texas at Austin

Ransom Ms. 130
Exposition of 2 Kings 17:15–16

Ransom Ms. 132
Notes on the Temple of Solomon and a tabular comparison of measurement systems

1.10 James White Library, Andrews University, Berrien Springs, Michigan

ASC Ms. N47 HER
Prophesies concerning Christs 2^d coming

2 Books from Newton's Private Library (By Location)

2.1 Cambridge University Library

CUL Adv. b.39.1
First edition of the *Principia*

CUL Adv. b.39.2
Second edition of the *Principia*

CUL Adv. b.39.3
First edition of *The Opticks*

CUL Adv. b.39.4
First edition of *The Opticks*

2.2 Wren Library, Trinity College, University of Cambridge

NQ.8.82
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