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To my
beloved Parents
Amirtham–Uthariam

PREFACE

The objective of this book is to cover comprehensively the basics of turbo machines. Turbo machines has been an essential subject in the engineering curriculum and is often perceived as a tough subject mainly due to the unavailability of standard textbooks. I have made an attempt here to provide adequate material for the study of both compressible and incompressible flow turbo machines. The book adopts the SI system of units throughout. It includes numerous solved and unsolved problems in each chapter to assist students in understanding the subject.

Chapter 1 introduces the basics of turbo machines. The Euler's equation has been derived in this chapter. Chapter 2 deals with the geometries of blades and blade rows of different types of turbo machines. The blowers and fans are low-pressure compressible flow machines and are principally the same as the compressors. Therefore, these machines are combined with compressors. Centrifugal compressors, blowers and fans are discussed in chapter 3.

Axial flow compressors and fans are described in chapter 4. In the same chapter, multi-stage compressors are also covered. Axial flow gas and steam turbines are combined and discussed in chapter 5. Problems in axial flow turbines are solved by both analytical and graphical methods in this chapter. Chapter 6 deals with inward and outward flow radial turbines. Velocity triangles and enthalpy - entropy diagrams have been frequently used to explain the thermodynamic aspects of these compressible flow machines.

Chapter 7 includes dimensional and model analysis of turbo machines and types of similarities. A brief discussion on non-dimensional numbers is presented here.

Centrifugal and axial flow pumps are discussed in detail in chapter 8. It also includes the study of positive displacement pumps. Hydraulic turbines such as Pelton, Francis and Kaplan turbines are discussed in chapter 9. Governing of hydraulic turbines is also included here. Power transmitting turbo machines and their characteristics are discussed in detail in chapter 10.

The reference direction in velocity triangles for centrifugal machines is the tangential direction and that of axial machines is the axial direction. This is the convention adopted here. Computer software has been developed for some selected problems in turbo machines and is given in the Appendix.

This book will be useful to teachers and students of Mechanical Engineering, candidates of AMIE, competitive examinations like UPSC, TNPSC and GATE and practicing engineers.

I welcome comments and suggestions from readers.

The material in this book is based on the concepts already developed over the years by various authors, which can be readily gathered from the list of references given at the end of the book. I owe my gratitude to all of them.

I acknowledge with appreciation the encouragement and suggestions given by my colleagues as well as my students.

I am grateful to my family members for their moral support and sustained encouragement throughout the preparation of this book.

I place on record my sincere gratitude to my college Chairman Dr. Tmt. Radha Thiagarajan, the Correspondent Thiru. Karumuthu T. Kannan and the Principal Dr. V. Abhaikumar for their support.

I sincerely thank Mr. P.K. Madhavan, Vikas Publishing House Pvt Ltd, for publishing this book. I am indebted to my students Mr. H. Prasanna and Mr. A. Pradeep Chakkaravarthy for their help in proof reading.

A Valan Arasu

CONTENTS

1

BASIC CONCEPTS OF TURBO MACHINES

DEFINITION

Turbo machine is defined as a device that extracts energy or imparts energy to a continuously flowing stream of fluid by the dynamic action of one or more rotating blade rows. The prefix 'turbo' is a latin word meaning 'spin' or 'whirl' implying that turbo machines rotate in some way.

If the machine adds energy to the fluid, it is commonly called a **pump**. If it extracts energy, then it is called a **turbine**. A device which pumps liquids is simply called a pump, but if it pumps gases, then three different terms may be used depending upon the pressure rise achieved. Upto nearly 0.07 bar pressure rise, the device is called a **'fan'**, between 0.07 and 3 bar absolute pressure it is called a **'blower'**, and above 3 bar absolute pressure it is called a **'compressor'**.

The difference between the turbo machine and the positive displacement machine is that in the former, the fluid is moving continuously across the machine unlike in the latter, where the fluid enters a closed chamber, which is isolated from the inlet and outlet sections of the machine for a very short period of time within which work is done on or by the fluid.

CLASSIFICATION OF TURBO MACHINES

Turbo machines are broadly classified as Shrouded or Unshrouded turbo machines. If the rotating member is enclosed in a casing or shrouded in such a way that the working fluid cannot be diverted to flow around the edges of the impeller, it is called a **shrouded turbo machine**. Examples of this are turbines, pumps, etc. If the fluid flows around the edges of the impeller which is not shrouded, then it is called an **unshrouded turbo machine**. Examples of this are wind mill or aero-generator and aircraft propellers.

Turbo machines may fall into any one of the two classes depending on whether work is done by the fluid on the rotating member (examples: hydraulic turbine, gas turbine, etc.) or work is done by the rotating member on the fluid (examples: pump, compressor, etc.).

The turbo machines can also be classified by the energy transfer from or to the rotating blades, which are fixed onto a shaft. In the **work absorbing machines** the fluid pressure (or) head (in the case of hydraulic machines) (or) the enthalpy (for compressible flow machines) increases from inlet to outlet. But in **work delivering machines** the fluid pressure or enthalpy, decreases from the inlet to the outlet.

The product change in head or enthalpy, and the mass flow rate of the fluid through the machine, represents the energy absorbed by (or) extracted from the rotating blades. In turbo machines, the energy transfer is accomplished by changing the angular momentum of the fluid and so the shapes of the blades and rotating members differ from one type to another.

Turbo machines can also be classifed based on the direction of flow of fluid across the rotating member. If the flow is axial, the machine is called an **axial flow machine**. If the flow is only radial, it is known as *radial flow* or *centrifugal machine*. If the flow is partly axial and partly radial, the machine is known as *mixed flow machine*.

BASIC LAWS AND GOVERNING EQUATIONS

The basic laws of thermodynamics and fluid mechanics are used in turbo machines. The important laws and governing equations used in turbo machines are as follows:

1. The Principle of Conservation of Mass

The conservation of mass is one of the most fundamental principles in nature. Mass, like energy is a conserved property, and it cannot be created or destroyed. The conservation of mass principle for a controlled volume undergoing a steady flow process, requires that the mass flow rate(m) across the controlled volume remains constant. Mathematically,

$$\boxed{m = \rho_1 A_1 C_1 = \rho_2 A_2 C_2}$$

where subscripts 1 and 2 denote the inlet and outlet conditions respectively. The conservation of mass equation is often referred to as the **continuity equation** in fluid mechanics. In compressible flow machines, the mass flow rate (kg/s) is exclusively used while in hydraulic machines the volume flow rate (m^3/s) is preferred.

2. The First Law of Thermodynamics

The first law of thermodynamics which is also known as the conservation of energy principle states that energy can neither be created nor destroyed; it can only change from one form to another. The conservation of energy equation for a general steady flow system can be expressed verbally as

[Heat transferred] − [Shaft work] = (Mass flow rate) [(Change in enthalpy per unit mass) + (Change in kinetic energy per unit mass) + (Change in potential energy per unit mass)]

or

$$\boxed{Q - W = m[\Delta h + \Delta ke + \Delta pe]}$$

This equation is known as **steady flow energy equation**(SFEE). A turbo machine, being operated essentially under the same conditions for long periods of time, can be conveniently analysed as a steady flow device. This equation, when applied to a turbo machine, may be simplified pertaining to the type of turbo machine, because many of the terms are zero (or) get cancelled with others.

3. The Newton's Second Law of Motion

According to this law, the sum of all the forces acting on a controlled volume in a particular direction is equal to the rate of change of momentum of the fluid across the controlled volume in the same direction.

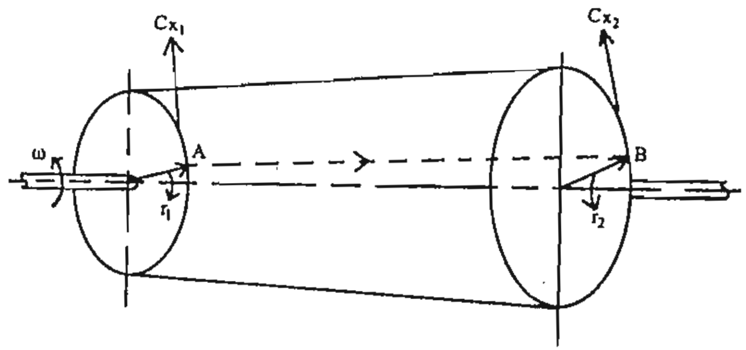

Figure 1.1 Movement of fluid particle across a controlled volume

In turbo machines, the impellers are rotating and the power output is expressed as the product of torque and angular velocity and so **angular momentum** is the prime parameter. Consider a fluid particle moving across a controlled volume as shown in Fig 1.1. The Fluid particle travels from point A to point B while simultaneously moving from a radius r_1 to radius r_2. If C_{x1} and C_{x2} are components of absolute velocities in the tangential direction, then the sum of all the torques acting on the system is equal to the rate of change of angular momentum. Mathematically,

$$\Sigma\tau = m(r_2 C_{x2} - r_1 C_{x1})$$

If the machine revolves with angular velocity ω, then the power (W) is

$$\Sigma\tau\omega = m(\omega r_2 C_{x2} - \omega r_1 C_{x1})$$

Since

$$\omega r = U$$

where U is the impeller tangential velocity

$$\therefore \boxed{W = m(U_2 C_{x2} - U_1 C_{x1})}$$

This equation is known as the general form of Euler's equation.
Euler's turbine equation is

$$\boxed{W = m(U_1 C_{x1} - U_2 C_{x2}) > 0}$$

Euler's pump equation is

$$\boxed{W = m(U_2 C_{x2} - U_1 C_{x1}) > 0}$$

4. The Second Law of Thermodynamics

The second law of thermodynamics leads to the definition of Entropy, and is defined as

$$\boxed{\delta Q_{rev} = TdS}$$

Entropy change is caused by heat transfer, mass flow, and irreversibilities. The entropy change during a process is **positive** for an irreversible process or zero for a reversible process. Thus, work producing devices such as turbines, deliver more work, and work consuming devices such as pumps and compressors consume less work when they operate reversibly.

The differential form of the conservation of energy equation for a closed stationary system (a fixed mass) can be expressed for a reversible process as

$$\delta Q_{rev} - \delta W_{rev} = dU$$

But

$$\delta Q_{rev} = TdS$$
$$\delta W_{rev} = PdV$$

Thus

$$TdS = dU + PdV$$

(or) on unit mass basis

$$\boxed{Tds = du + Pdv}$$

This equation is known as the First Tds equation or **Gibb's equation**. The second Tds equation is obtained by eliminating du from the first Tds equation by using the definition of enthalpy ($h = u + Pv$)

$$dh = du + Pdv + vdP$$

Thus

$$\boxed{Tds = dh - vdP}$$

The second Tds equation is extensively used in the study of compressible flow machines. In terms of stagnation properties

$$Tds = dh_0 - v_0 dP_0$$

For an incompressible fluid undergoing an isentropic process (i.e. $ds = 0$) as in fans the ideal change in stagnation enthalpy is

$$\boxed{(\Delta h_0)_s = v_0 \Delta P_0 = \Delta P_0/\rho_0 = \Delta P_0/\rho}$$

Since $v_o = 1/\rho_0$ and $\rho_0 = \rho$

EFFICIENCIES OF COMPRESSORS

Fig.1.2 shows the reversible and irreversible adiabatic compression processes on the enthalpy-entropy diagram. The initial condition of the fluid is represented by state-1. The stagnation point corresponding to this state is 01. The final condition of the fluid is denoted by state-2 and the corresponding stagnation point is 02. If the process were reversible, the final fluid static and stagnation conditions would be 2S and 02S respectively.

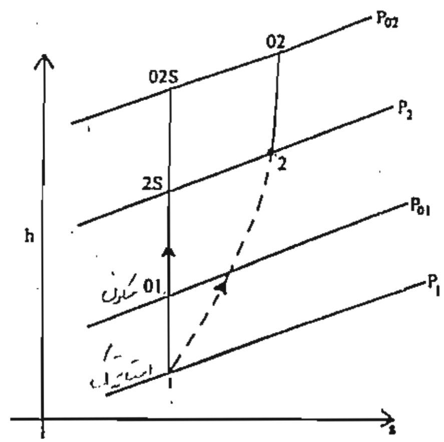

Figure 1.2 Reversible and irreversible compression processes

Process 1-2 is the actual compression process and is accompanied by an increase in entropy. Process 1-2S is the ideal compression process. The efficiencies of compressors may be defined in terms of either stagnation or static properties of the fluid or even a combination of both. The following are the commonly used compressor efficiencies:

1. Total-to-Total Efficiency

It is an efficiency based on stagnation properties at entry and exit.

$$\eta_{t-t} = \frac{W_{ideal}}{W_{actual}} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}}$$

2. Static-to-Static Efficiency

It is an efficiency based on static properties at entry and exit.

$$\eta_{s-s} = \frac{W_{ideal}}{W_{actual}} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

3. Polytropic Efficiency

A compressor stage can be viewed as made up of an infinite number of small stages. To account for a compression in an infinitesimal stage, polytropic efficiency is defined for an elemental compression process. Consider a small compressor stage as shown in Fig. 1.3 between pressures P and $P+dP$.

The polytropic efficiency of a compressor stage is defined as

$$\eta_p = \frac{dT_s}{dT} \tag{1.1}$$

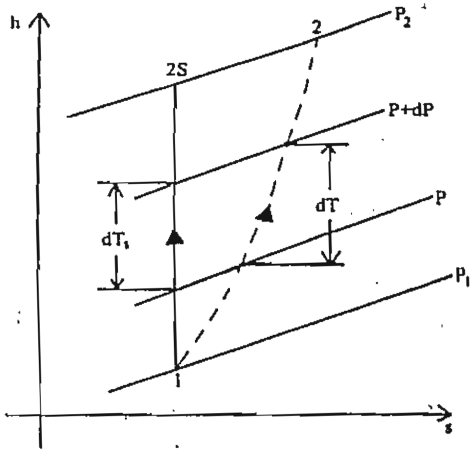

Figure 1.3 Compression process in infinitesimal and finite compressor stages

For an isentropic process, the relationship between pressure and temperature is given by

$$\frac{T}{P^{\left(\frac{r-1}{r}\right)}} = \text{constant} \tag{1.2}$$

Differentiating equation (1.2) and substituting equation (1.1), we get

$$dT \cdot \eta_p = \left[\frac{r-1}{r} \times P^{-\left(\frac{1}{r}\right)}\right] dp \times \text{constant}$$

Constant value is obtained from equation (1.2).
Therefore,

$$\frac{dT}{T} \cdot \eta_p = \left(\frac{r-1}{r}\right) \cdot \frac{dp}{P} \tag{1.3}$$

Integrating between the limits of the full compression from P_1 to P_2 we get

$$\ln\left(\frac{T_2}{T_1}\right) = \frac{r-1}{r} \cdot \frac{1}{\eta_p} \cdot \ln\left(\frac{P_2}{P_1}\right)$$

Rearranging,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{r-1}{\eta_p \cdot r}} \tag{1.4}$$

If the irreversible adiabatic compression process is assumed to be equivalent to a polytropic process with polytropic index, n, the following relationship between temperature and pressure will exist.

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \tag{1.5}$$

Equating eqns (1.4) and (1.5),

$$\left(\frac{P_2}{P_1}\right)^{\frac{r-1}{\eta_p r}} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

Comparing the power,

$$\frac{r-1}{\eta_p \cdot r} = \frac{n-1}{n}$$

or

$$\eta_P = \frac{r-1}{r} \cdot \frac{n}{n-1}$$

[The polytropic efficiency is also called as small stage or infinitesimal stage efficiency]

A typical value of polytropic efficiency for a compressor is 0.88 and in the initial design calculation it is often assumed that $\eta_p = \eta_s$, where η_s is the stage efficiency.

Alternatively, polytropic index of compression in the actual process is

$$n = \frac{r\eta_p}{1 - r(1 - \eta_p)}$$

4. Finite Stage Efficiency

A stage with a finite pressure drop is a finite stage. Taking static values of temperature and pressure (Fig. 1.2) and assuming perfect gas stage efficiency is defined as

$$\eta_s = \frac{T_{2s} - T_1}{T_2 - T_1}$$

The finite stage or stage efficiency can be expressed in terms of the small stage or polytropic efficiency

$$T_{2s} - T_1 = T_1\left(\frac{T_{2s}}{T_1} - 1\right) = T_1\left(\left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}} - 1\right)$$

and

$$T_2 - T_1 = T_1\left(\frac{T_2}{T_1} - 1\right) = T_1\left(\left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r\cdot\eta_p}} - 1\right)$$

Therefore,

$$\eta_s = \frac{\left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}} - 1}{\left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r\eta_p}} - 1}$$

For a multistage compressor with a constant stage pressure ratio, the overall pressure ratio is given by

$$\frac{P_{N+1}}{P_1} = \left(\frac{P_2}{P_1}\right)^N$$

where N is the number of stages and P_{N+1} is the pressure at the end of the N^{th} stage.

Therefore, the overall efficiency is

$$\eta_c = \frac{\left(\frac{P_2}{P_1}\right)^{N\left(\frac{r-1}{r}\right)} - 1}{\left(\frac{P_2}{P_1}\right)^{N\left(\frac{r-1}{r\eta_p}\right)} - 1}$$

The overall efficiency in terms of overall pressure ratio is

$$\eta_c = \frac{\left(\frac{P_{N+1}}{P_1}\right)^{\frac{r-1}{r}} - 1}{\left(\frac{P_{N+1}}{P_1}\right)^{\frac{r-1}{r\eta_p}} - 1}$$

PREHEAT FACTOR IN COMPRESSORS

Consider a two stage compressor working between P_{01} and P_{03} an shown in Fig. 1.4. In isentropic flow, the outlet conditions of the gas for the first stage and second stage are at 02_s and 03_s respectively, where as the actual outlet

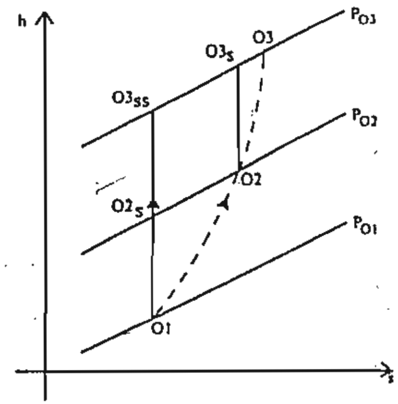

Figure 1.4 Preheat effect in a multistage compressor

conditions are at 02 and 03. Corresponding to first stage and second stage respectively. The subscript 's' refers to constant entropy and '0' refers to total conditions of the fluid. If the stage efficiencies were the same, the total actual work input to the different individual stages would be

$$W = \frac{1}{\eta_s}\left[W_{s_1} + W_{s_2}\right] = \frac{1}{\eta_s}\left[(h_{o2s} - h_{o1}) + (h_{o3s} - h_{o2})\right]$$

$$W = 1/\eta_s \sum_{i=1}^{2} W_{si} \qquad (1.6)$$

where $\sum_{i=1}^{2} W_{si}$ is the isentropic work input to the two stage compressor and is the sum of the stage isentropic works. For a compressor with 'n' stages,

$$W = \frac{1}{\eta_s}\sum_{i=1}^{N} W_{si} \qquad (1.7)$$

Consider now, a single stage compressor raising the fluid pressure from P_{01} to P_{03}. The actual work input that would be supplied is

$$W = \frac{1}{\eta_c} \cdot W_s \qquad (1.7a)$$

where η_c is the overall compressor efficiency and W_s is the isentropic work.

$$W_s = h_{03_{ss}} - h_{01}$$

The actual work input is the same for both single stage and multistage compression processes.

$$W = h_{03} - h_{01}$$

Then from equ's 1.7 and 1.7a,

$$\frac{\eta_s}{\eta_c} = \frac{\sum_{i=1}^{N} W_{si}}{W_s} \qquad (1.8)$$

(Since the constant pressure lines diverge in the direction of increasing entropy on h-s diagram, the isentropic enthalpy rise across each stage increases even for a constant stagnation pressure rise ΔP_0 across each stage. Then, the sum of the stage isentropic enthalpy rises is greater than the isentropic enthalpy rise in a single stage compression.

For a two stage compressor

$$(h_{02_s} - h_{01}) + (h_{03_s} - h_{02}) > (h_{03_{ss}} - h_{01})$$

i.e. $\sum_{i=1}^{2} W_{s_i} > W_s$

For N stages,

$$\frac{\sum_{i=1}^{N} W_{si}}{W_s} > 1$$

Equation (1.8) can be written as

$$\frac{\eta_c}{\eta_s} = \frac{W_s}{\sum_{i=1}^{N} W_{s_i}} = P.F \qquad (1.8a)$$

That is, the ratio of W_s to $\sum_{i=1}^{N} W_{s_i}$ is known as the Preheat factor (P.F)

$$\boxed{P.F. = \frac{W_s}{\sum_{i=1}^{N} W_{s_i}} < 1}$$

The preheat factor is less than unity. Then, equation (1.8a) becomes

$$\frac{\eta_c}{\eta_s} < 1$$

or

$$\boxed{\eta_c < \eta_s}$$

i.e., the overall compressor efficiency η_c is less than the compressor stage efficiencies η_s.

Consider again Fig. (1.4) for a first stage compression, state 02 may be obtained after an ideal compression from 01 to 02_s followed by "preheatin" of the fluid from state 02_s to 02 at constant pressure ($T_{02} > T_{02s}$).

(This inherent thermodynamic effect that reduces the efficiency of a multistage compressor is called the *preheat effect.*)

EFFICIENCIES OF TURBINES

The enthalpy-entropy diagram for flow both reversible and irreversible through a turbine is shown in Fig. 1.5. The static condition of the fluid at inlet is determined by state 1, with state 01, as the corresponding stagnation state. The final static properties are determined by the state 2, with 02, as the

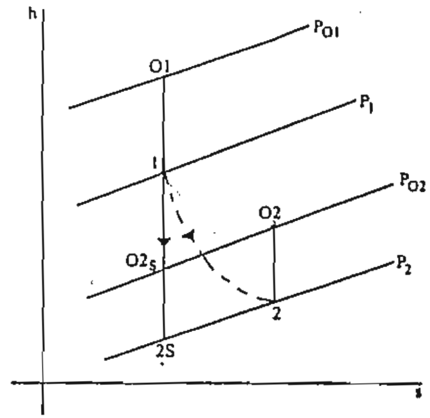

Figure 1.5 Reversible and irreversible expansion processes

corresponding stagnation state. If the process were reversible, the final fluid static state would be $2s$ and the stagnation state would be 02_s.

Process $1 - 2$ is the actual expansion process and process $1 - 2s$ is the isentropic or ideal expansion process. In turbines, the efficiencies may be defined using either the static or the stagnation properties of the fluid or even a combination of both. The commonly used turbine efficiencies are

1. Total-to-Total Efficiency

It is an efficiency based on stagnation properties at inlet and outlet.

$$\eta_{t-t} = \frac{W_{actual}}{W_{ideal}}$$

$$\boxed{\eta_{t-t} = \frac{h_{01} - h_{02}}{h_{01} - h_{02s}}}$$

2. Total-to-Static Efficiency

It is an efficiency in which the ideal work is based on stagnation property at inlet and static property at outlet.

$$W_{ideal} = h_{01} - h_{2s}$$

$$\boxed{\eta_{t-s} = \frac{W_{actual}}{W_{ideal}} = \frac{h_{01} - h_{02}}{h_{01} - h_{2s}}}$$

3. Polytropic Efficiency

A turbine stage can be considered as made up of an infinite number of small or infinitesimal stages. Then to account for expansion in an infinitesimal turbine

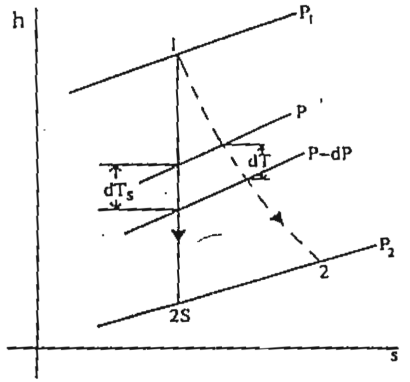

Figure 1.6 Expansion process in infinitesimal and finite turbine stages

stage, a small stage or infinitesimal stage or polytropic efficiency is defined. Consider a small stage (Fig. 1.6) between pressures P and $P - dp$. The efficiency of this turbine stage is defined as

$$\boxed{\eta_p = \frac{dT}{dT_s}} \tag{1.9}$$

For an isentropic process

$$\frac{T}{p^{\left(\frac{r-1}{r}\right)}} = \text{Constant} \tag{1.10}$$

Differentiating eqn. (1.10), we get

$$dT_s = \text{Constant}\left[p^{-(1/r)} \cdot \frac{(r-1)}{r}\right] dp$$

But $dT_s = \dfrac{dT}{\eta_p}$ and constant $= \dfrac{T}{p^{\left(\frac{r-1}{r}\right)}}$

then

$$\frac{dT}{\eta_p} = \frac{T}{p^{\left(\frac{r-1}{r}\right)}}\left[p^{-(1/r)} \cdot \frac{(r-1)}{r}\right] dp$$

$$\frac{dT}{T} = \eta_p \cdot \frac{r-1}{r} \cdot \frac{dP}{P} \tag{1.11}$$

Integrating between the limits of the overall expansion between P_1 and P_2

$$\ln\left(\frac{T_2}{T_1}\right) = \eta_p \cdot \frac{r-1}{r} \cdot \ln\left(\frac{P_2}{P_1}\right)$$

Rearranging,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\eta_p\left(\frac{r-1}{r}\right)} \tag{1.12}$$

Assuming the irreversible adiabatic expansion (1–2) as equivalent to a polytropic process with index n, the temperature and pressure are related by

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \tag{1.13}$$

Equating eqns. (1.12) and (1.13),

$$\left(\frac{P_2}{P_1}\right)^{\eta_p\left(\frac{r-1}{r}\right)} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

Comparing the powers,

$$\eta_p\left(\frac{r-1}{r}\right) = \frac{n-1}{n}$$

or

$$\eta_p = \frac{r}{(r-1)} \cdot \frac{n-1}{n}$$

Alternatively, the index of expansion in the actual process is expressed as

$$n = \frac{r}{r - (r-1)\eta_p}$$

When $\eta_p = 1$, $n = r$. The actual expansion of process curve $(1-2)$ coincides with the isentropic expansion line $(1-2s)$.

4. Finite Stage Efficiency

The stage efficiency, considering static value of temperature and pressure (Fig. 1.6.), is defined as

$$\eta_s = \frac{T_1 - T_2}{T_1 - T_{2s}}$$

The stage efficiency can now be expressed in terms of polytropic efficiency

$$T_1 - T_{2s} = T_1\left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right)$$

and

$$T_1 - T_2 = T_1\left(1 - \frac{T_2}{T_1}\right) = T_1\left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{\eta_p(\gamma-1)}{\gamma}}\right)$$

Therefore,

$$\eta_s = \frac{1 - \left(\frac{P_2}{P_1}\right)^{\frac{\eta_p(\gamma-1)}{\gamma}}}{1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}}$$

The same equation can be used to determine the overall efficiency of a multistage turbine, except that the stage pressure ratio is replaced by the overall pressure ratio. The overall efficiency, for a N-stage turbine with a constant stage pressure ratio, can be expressed as

$$\frac{P_{N+1}}{P_1} = \left(\frac{P_2}{P_1}\right)^N$$

$$\eta_t = \frac{1 - \left(\frac{P_2}{P_1}\right)^{N\left(\frac{\eta_p(\gamma-1)}{\gamma}\right)}}{1 - \left(\frac{P_2}{P_1}\right)^{N\left(\frac{\gamma-1}{\gamma}\right)}}$$

REHEAT FACTOR IN TURBINES

Consider a turbine with two stages where the fluid (perfect gas) expands from P_{01} to P_{03} as shown on the $h - s$ diagram, Fig. 1.7. State-01 is the initial condition at the entry of the first stage and $02s$ is the condition that would be reached at the first stage exit if the expansion process had been isentropic. The actual expansion leads to a final state-02 which has higher entropy than that of state $02s$. The corresponding exit conditions for the second stage are $03s$ and 03 respectively.

The isentropic work done by the two stage turbine is the sum of the stage isentropic works.

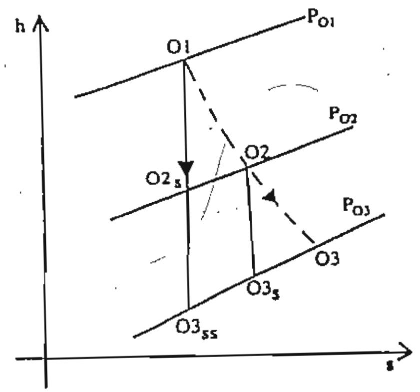

Figure 1.7 Reheat effect in a multistage turbine

$$\sum_{i=1}^{2} W_{s_i} = W_{s_1} + W_{s_2} \tag{1.14}$$
$$= (h_{01} - h_{02s}) + (h_{02} - h_{03s})$$

Consider now a turbine in which the fluid expands from P_{01} to P_{03} in one stage. The isentropic work is

$$W_s = h_{01} - h_{03ss} \tag{1.15}$$

In both the cases, the actual work done is

$$W = h_{01} - h_{03}$$

The constant pressure line diverges in the $h - s$ diagram as the entropy increases. Therefore, the isentropic enthalpy drop across a stage increases for a constant stagnation pressure drop ΔP_0 across each stage. Consequently, the sum of the stage isentropic enthalpy drops is greater than the isentropic enthalpy drop in a single stage expansion. That is

$$(h_{01} - h_{02s}) + (h_{02} - h_{03s}) > (h_{01} - h_{03ss})$$

or

$$\sum_{i=1}^{2} W_{s_i} > W_s$$

For a turbine with N-stages, we have

$$\frac{\sum_{i=1}^{N} W_{s_i}}{W_s} > 1 \tag{1.16}$$

The ratio of $\sum_{i=1}^{N} W_{s_i}$ to W_s is called the reheat factor (R.F).

$$\boxed{\text{R.F} = \frac{\sum_{i=1}^{N} W_{s_i}}{W_s} > 1} \tag{1.17}$$

The magnitude of the reheat factor in multistage turbines is about 1.03 or 1.04.

If the stage efficiencies were the same, the total actual work output from the various individual stages would be

$$W = \eta_s \sum_{i=1}^{N} W_{s_i} \tag{1.18}$$

The actual work-output that would be obtained from a single stage expansion is

$$W = \eta_t W_s \tag{1.19}$$

where η_t is the overall turbine efficiency. Combining equations (1.18) and (1.19), we get

$$\frac{\eta_t}{\eta_s} = \frac{\sum_{i=1}^{N} W_{s_i}}{W_s} \tag{1.20}$$

From eqn. (1.16), we find that

$$\eta_t > \eta_s$$

That is, the overall turbine efficiency η_t is greater than the turbine stage efficiencies η_s.

Combining eqns. (1.17) and (1.20), we have the following relation.

$$\boxed{\text{R.F} = \eta_t/\eta_s} \tag{1.21}$$

Consider again, Fig.(1.7) for a first stage expansion. It is seen that the final state -02 may be obtained after an ideal expansion from 01 – 02_s followed by a 'reheating' of the fluid from state 02_s to 02 at constant pressure ($T_{0_2} > T_{02_s}$)

Hence, the fluid at 02 has a greater availability than the fluid at 02_s ($h_{0_2} > h_{02_s}$). An expansion from state 02 to a lower pressure must necessarily result in a larger output than that obtainable from state 02_s. This effect that makes RF > 1 is called the *reheat effect*.

The reheat factor for the expansion of a perfect gas in an N-stage turbine, assuming that the stage efficiencies η_s and the pressure ratios $P_{0_i}/P_{0_{i+1}}$, where $i = 1, 2, \cdots, n$ for all the stages are equal, is expressed in terms of stage pressure ratio as follows. For the first stage,

$$T_{02_s} = T_{01}(P_{02}/P_{01})^{\frac{\gamma-1}{\gamma}}$$

so that

$$T_{01} - T_{02_s} = T_{01}\left[1 - (P_{02}/P_{01})^{\frac{\gamma-1}{\gamma}}\right] \tag{1.22}$$

Let

$$\mu = 1 - (P_{02}/P_{01})^{\frac{\gamma-1}{\gamma}}$$

Then

$$T_{01} - T_{02_s} = T_{01}\mu$$

and

$$T_{01} - T_{02} = \eta_s T_{01}\mu$$

or

$$\boxed{T_{02} = T_{01}(1 - \mu\eta_s)} \tag{1.23}$$

For the second stage

$$T_{02} - T_{03_s} = T_{02}\left[1 - (P_{03}/P_{02})^{\frac{\gamma-1}{\gamma}}\right]$$

but

$$\frac{P_{03}}{P_{02}} = \frac{P_{02}}{P_{01}}$$

Therefore

$$T_{02} - T_{03_s} = T_{02}\mu$$

and

$$T_{02} - T_{03} = \eta_s T_{02}\mu$$

or

$$T_{03} = T_{02}(1 - \eta_s\mu) = T_{01}(1 - \mu\eta_s)^2 \tag{1.24}$$

For the Nth stage

$$\boxed{T_{0_{N+1}} = T_{01}(1 - \mu\eta_s)^{N-1}}$$

The actual work output is

$$W = C_P \eta_s \mu [T_{01} + T_{02} + \cdots + T_{0_{N+1}}]$$
$$= C_P \eta_s \mu T_{01} \left[1 + (1-\mu\eta_s) + (1-\mu\eta_s)^2 + \cdots + (1-\mu\eta_s)^{N-1}\right]$$

The terms within the brackets are of the form $1 + r + r^2 + \cdots + r^{n-1}$ which is a geometric series with common ratio r. The solution is $\dfrac{1-r^n}{1-r}$.
In this case, $r = (1 - \mu\eta_s)$. Then, the equation reduces to

$$= C_P T_{01} \left[1 - (1-\mu\eta_s)^N\right]$$
$$= \eta_s \sum_{i=1}^{N} W_{s_i} \qquad (1.25)$$

The isentropic work output from a single stage expansion from P_{01} to $P_{0_{(N+1)}}$ may be obtained from eqn. (1.25) by setting $\eta_s = 1$.

$$W_s = C_P T_{01} \left[1 - (1-\mu)^N\right]$$
$$= C_P T_{01} \left[1 - R_0^{\frac{r-1}{r}}\right] \qquad (1.26)$$

where, $R_0 = P_{0_{(N+1)}} / P_{01}$ represents the overall pressure ratio. Since

$$\text{R.F} = \sum_{i=1}^{N} W_{s_i} / W_s$$
$$= \frac{C_P T_{01}}{\eta_s} \left[1 - (1-\mu\eta_s)^N\right] \Big/ C_P T_{01} \left[1 - R_0^{\frac{r-1}{r}}\right]$$

$$\boxed{\text{R.F} = \frac{\left[1 - (1-\mu\eta_s)^N\right]}{\eta_s \left[1 - R_0^{r-1/r}\right]}}$$

SOLVED PROBLEMS

Example 1.1 The initial and final total pressures of a fluid are 1 bar and 10 bar respectively. The initial total temperature is 10°C. What is the work of compression for adiabatic steady flow with a total-to-total efficiency of 75% if (a) the fluid is liquid water and (b) the fluid is air as a perfect gas.

Solution

$P_{01} = 1$ bar $\quad P_{02} = 10$ bar $\quad T_{01} = 283$ K. $\quad \eta_{tt} = 0.75$

(a) If the fluid is liquid water

Since the fluid is incompressible $\vartheta_0 =$ constant $= \dfrac{1}{\rho}$, $\rho =$ density of water.

For an isentropic compression,

$$\Delta h_{0_s} = \vartheta_0 \Delta P_0 = \frac{1}{\rho}(\Delta P_0)$$
$$= \frac{1}{1000}(10-1) \times 10^2$$
$$= 0.9 \text{ kJ/kg}$$

By definition

$$\eta_{tt} = \frac{\Delta h_{0_s}}{\Delta h_0}$$
$$\therefore \Delta h_0 = \frac{\Delta h_{0_s}}{\eta_{tt}} = \frac{0.9}{0.75}$$
$$= 1.2 \text{ kJ/kg}$$

This is the work of compression.

(b) If the fluid is air as a perfect gas

$$\Delta h_{0_s} = C_P (T_{02_s} - T_{01})$$

From isentropic relation

$$T_{02_s} = T_{01} \left(\frac{P_{02}}{P_{01}}\right)^{\frac{r-1}{r}}$$
$$= 283(10)^{\frac{1.4-1}{1.4}}$$
$$= 546.4 \text{ K}$$

Therefore,

$$\Delta h_{0_s} = 1.005(546.4 - 283)$$
$$= 264.70 \text{ kJ/kg}$$

and

$$\Delta h_0 = \frac{\Delta h_{0_s}}{\eta_{tt}}$$
$$= \frac{264.70}{0.75}$$
$$= 352.94 \text{ kJ/kg}$$

Hence, the adiabatic work of compression per kg of fluid is 352.94 kJ.

Example 1.2 Gases from a combustion chamber enter a gas turbine at a total pressure of 7 bar and a total temperature of 1100 K. The total pressure and total temperature at the turbine exit are 1.5 bar and 830 K. Take $r = 1.3$ and molecular weight of gases = 28.7. Evaluate total-to-total efficiency and the total-to-static efficiency if the exit velocity is 250 m/s. Assume adiabatic steady flow.

Solution

$P_{01} = 7$ bar $T_{01} = 1100$ K $P_{02} = 1.5$ bar
$T_{02} = 830$ K $C_2 = 250$ m/s $r = 1.3$

(a) Total-to-Total efficiency

$$\eta_{tt} = \frac{h_{01} - h_{02}}{h_{01} - h_{02s}} = \frac{T_{01} - T_{02}}{T_{01} - T_{02s}}$$

Using isentropic relation

$$\begin{aligned} T_{02s} &= T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{r-1}{r}} \\ &= 1100\left(\frac{1.5}{7}\right)^{\frac{1.3-1}{1.3}} \\ &= 771.1\ \text{K} \end{aligned}$$

Therefore,

$$\begin{aligned} \eta_{tt} &= \frac{1100 - 830}{1100 - 771.1} = 0.821 \\ &= 82.1\% \end{aligned}$$

(b) Total-to-Static efficiency

$$\begin{aligned} \eta_{t-s} &= \frac{h_{01} - h_{02}}{h_{01} - h_{2s}} = \frac{T_{01} - T_{02}}{T_{01} - T_{2s}} \\ T_{2s} &= T_{02s} - \frac{C_2^2}{2C_P} \\ C_P &= \frac{rR}{r-1} \text{ and } R = \frac{\bar{R}}{M} = \frac{8.314}{28.7} = 0.2897\,\text{kJ/kg}-\text{K} \\ \therefore C_P &= \frac{1.3 \times 0.2897}{0.3} = 1.255\,\text{kJ/kg}-\text{K} \\ T_{2s} &= 771.1 - \frac{250^2}{2 \times 1255} \\ &= 746.19\ \text{K} \\ \eta_{t-s} &= \frac{1100 - 830}{1100 - 746.19} = 0.763 \\ &= 76.3\% \end{aligned}$$

Example 1.3 Suppose a turbo machine is operated such that the change in total enthalpy is 6 kJ/kg of fluid when the inlet total temperature is 30°C and the inlet total pressure is 1 bar. (a) What general type of turbo machine would this be? (b) What is the exit total temperature if the fluid is air? (c) What is the total pressure ratio across the machine if the adiabatic total-to-total efficiency is 75% (i) if the fluid is air and (ii) if the fluid is liquid water.

Solution

$$\begin{aligned} \Delta h_0 &= h_{02} - h_{01} = 6\,\text{kJ/kg} \\ T_{01} &= 303\ \text{K} \quad P_{01} = 1\ \text{bar} \end{aligned}$$

(a) Finding the type of turbo machine

Since the change in enthalpy is positive (6 kJ/kg) this turbo machine would be a work absorbing machine.

(b) Exit total temperature

For air as a perfect gas, $\Delta h_0 = C_P \Delta T_0$.

$$\begin{aligned} \therefore T_{02} - T_{01} &= \frac{\Delta h_0}{C_P} \\ T_{02} &= T_{01} + \frac{\Delta h_0}{C_P} \\ &= 303 + \frac{6}{1.005} \\ &= 308.97\ \text{K or } 35.97^\circ\text{C} \end{aligned}$$

(c) Total pressure ratio

(i) If the fluid is air

$$\begin{aligned} \eta_{t-t} &= \frac{h_{02s} - h_{01}}{h_{02} - h_{01}} = \frac{C_P(T_{02s} - T_{01})}{C_P(T_{02} - T_{01})} \\ &= \frac{C_P T_{01}\left[\left(\frac{P_{02}}{P_{01}}\right)^{\frac{r-1}{r}} - 1\right]}{C_P(T_{02} - T_{01})} \\ \frac{P_{02}}{P_{01}} &= \left[1 + \frac{\eta_{t-t}(\Delta h_0)}{C_P T_{01}}\right]^{r/r-1} \\ &= \left[1 + \frac{0.75 \times 6}{1.005 \times 303}\right]^{\frac{1.4}{0.4}} \\ &= 1.053 \end{aligned}$$

(ii) If the fluid is liquid water

$$\Delta P_0 = \rho \Delta h_0,$$

where

$$\begin{aligned} \Delta h_{0s} &= \eta_{t-t}(\Delta h_0) = 0.75 \times 6 \\ &= 4.5\,\text{kJ/kg} \\ \therefore \Delta P_0 &= 4.5 \times 1000 \\ &= 45\ \text{bar} \\ P_{02} &= 45 + 1 = 46\ \text{bar} \end{aligned}$$

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The pressure ratio is

$$\frac{P_{02}}{P_{01}} = 46$$

Example 1.4 A power generating turbo machine develops 100 kW output when the flow through the device is 0.1 m^3/s of oil having a density of 800 kg/m^3. The total-to-total efficiency is 75%. Evaluate (a) the change in total pressure of the oil, and (b) the change in static pressure of the oil if the inlet and exit flow velocities are 3 and 10 m/s respectively.

Solution

$W = 100$ kW $\quad Q = 0.1\ m^3/s \quad \rho = 800\ kg/m^3 \quad \eta_{t-t} = 0.75$

Mass flow rate of oil $m = \rho Q$

$$= 800 \times 0.1 = 80 \text{ kg/s}$$

The change in total enthalpy

$$\begin{aligned} \Delta h_0 &= -W/m = 100/80 \\ &= -1.25 \text{ kJ/kg} \end{aligned}$$

The isentropic change in total enthalpy

$$\begin{aligned} (\Delta h_{0_s}) &= \frac{\Delta h_0}{\eta_{t-t}} = \frac{-1.25}{0.75} \\ &= -1.67 \text{ kJ/kg} \end{aligned}$$

(a) The change in total pressure of the oil

$$\begin{aligned} \Delta P_0 = \rho(\Delta h_{0_s}) &= \frac{800}{100} \times (-1.67) \\ &= -13.4 \text{ bar} \end{aligned}$$

(b) The change in static pressure

$$\begin{aligned} \Delta P &= \Delta P_0 - \rho\frac{(C_2^2 - C_1^2)}{2000} \\ &= -13.4 - \frac{800(10^2 - 3^2)}{2000 \times 100} \\ &= -13.4 - 0.364 \\ &= -13.8 \text{ bar} \end{aligned}$$

The negative sign implies that the pressure decreases during an expansion process.

Example 1.5 In a four stage turbine handling air, the stagnation pressure ratio between the exit and the inlet of each stage is 0.4. The stage efficiencies of the first two stages are 86% each, while those of the last two stages are 84% each. Find the overall efficiency of the turbine.

Solution

$N = 4 \quad P_{02}/P_{01} = 0.4$

For the first two stages, $\eta_s = 0.86$

For the last two stages, $\eta_s = 0.84$

(Refer Reheat Factor Section.)

$$\begin{aligned} \mu &= 1 - \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 1 - (0.4)^{\frac{0.4}{1.4}} \\ &= 0.23 \end{aligned}$$

The actual work output from the first two stages is

$$\begin{aligned} W_{1,2} &= C_P \mu T_{01}\left[1 + (1 - \mu\eta_s)\right] \times \eta_s \\ &= C_P T_{01}(0.23)\left[1 + (1 - (0.23 \times 0.86))\right] \times 0.86 \\ &= 0.356\, C_P T_{01} \end{aligned}$$

The actual work output from the last two stages is

$$W_{3,4} = C_P T_{03}\mu\left[1 + (1 - \mu\eta_s)\right] \times \eta_s$$

T_{03} – the temperature after the end of the first two stages is

$$\begin{aligned} T_{03} &= T_{01}(1 - \mu\eta_s)^2 = T_{01}(1 - (0.23)(0.86))^2 \\ &= 0.644\, T_{01} \\ W_{3,4} &= C_P T_{01}(0.644)(0.23)\left[1 + (1 - (0.23)(0.84))\right] 0.84 \\ &= 0.225\, C_P T_{01} \end{aligned}$$

Total actual work output from the turbine

$$W = W_{1,2} + W_{3,4} = 0.581\, C_P T_{01}$$

The total isentropic work due to a single stage compression is

$$\begin{aligned} W_s &= C_P T_{01}\left[1 - (1 - \mu)^N\right] \\ &= C_P T_{01}\left[1 - (1 - 0.23)^4\right] \\ &= 0.649\, C_P T_{01} \end{aligned}$$

Overall turbine efficiency

$$\begin{aligned} \eta_t &= W_s/W = 0.581/0.649 \\ &= 89.5\% \end{aligned}$$

Example 1.6 A low pressure air compressor develops a pressure of 1400 mm W.G. If the initial and final states of air are $P_1 = 1.01$ bar, $T_1 = 305$ K, $T_2 = 320$ K, determine compressor and the infinitesimal stage efficiencies. *[MKU – April'99]*

Solution

$$\begin{aligned}\Delta P &= 1400\,\text{mm W.G.}\\ P_1 &= 1.01\text{bar},\ T_1 = 305\text{K},\quad T_2 = 320\text{K}\\ \Delta P &= 1.4 \times 10^3 \times 9.81 = 13734\ \text{Pa}\\ &= 0.13734\ \text{bar}\end{aligned}$$

$$P_2 = P_1 + \Delta P = 1.01 + 0.13734 = 1.14734\ \text{bar}$$

From isentropic relation

$$\begin{aligned}\frac{T_{2s}}{T_1} &= \left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}}\\ &= \left(\frac{1.14734}{1.01}\right)^{\frac{0.4}{1.4}}\\ &= 1.037\\ T_{2s} &= 1.037 \times T_1 = 1.037 \times 305\\ &= 316.285\text{K}\end{aligned}$$

(a) Compressor efficiency

$$\begin{aligned}\eta_c &= \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{316.28 - 305}{320 - 305}\\ &= 75.23\%\end{aligned}$$

(b) Infinitesimal stage efficiency

$$\begin{aligned}\eta_p &= \frac{r-1}{r} \cdot \frac{\ln(P_2/P_1)}{\ln(T_2/T_1)}\\ &= \left(\frac{1.4-1}{1.4}\right) \cdot \frac{\ln(1.14734/1.01)}{\ln(320/305)}\\ &= 0.7588\\ &= 75.88\%\end{aligned}$$

Since the pressure rise in the compressor is low, the two efficiencies are close to each other.

Alternative method:

$$\begin{aligned}\eta_c &= \frac{\left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}} - 1}{\left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r\eta_p}} - 1} = \frac{\left(\frac{1.14734}{1.01}\right)^{\frac{0.4}{1.4}} - 1}{\left(\frac{1.14734}{1.01}\right)^{\frac{0.4}{1.4\eta_p}} - 1}\\ \eta_p &= 75.67\%\end{aligned}$$

Example 1.7 A high pressure compressor changes the state of air from $P_1 = 1.01$ bar, $T_1 = 305$ K to $P_2 = 3$ bar. The compressor efficiency is 75%. Determine the infinitesimal efficiency of the compressor.

Solution

$P_1 = 1.01$ bar $\quad T_1 = 305$ K $\quad P_2 = 3$ bar

$$\begin{aligned}P_2/P_1 &= 3/1.01 = 2.97\ \text{bar}\\ T_{2s} &= T_1\left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}} = 305(2.97)^{\frac{0.4}{1.4}}\\ &= 416.27\ \text{K}\end{aligned}$$

$$\begin{aligned}\eta_c &= \frac{T_{2s} - T_1}{T_2 - T_1}\\ T_2 - T_1 &= \frac{416.27 - 305}{0.75}\\ &= 148.36\end{aligned}$$

$$\begin{aligned}T_2 &= 148.36 + 305 = 453.36\text{K}\\ \eta_p &= \left(\frac{r-1}{r}\right)\frac{\ln(P_2/P_1)}{\ln(T_2/T_1)}\\ &= \left(\frac{0.4}{1.4}\right)\frac{\ln(3/1.01)}{\ln(453.36/305)}\\ &= 78.5\%\end{aligned}$$

Note that the infinitesimal stage efficiency is greater than the compressor efficiency. This difference is due to preheating.

Example 1.8 The pressure ratio across a gas turbine is 2.2 and efficiency is 88%. The temperature of gas at inlet is 1500 K determine polytropic efficiency.

Solution

$$\frac{P_1}{P_2} = 2.2 \quad \eta_T = 0.88 \quad T_1 = 1500\text{ K}$$

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}} = \left(\frac{1}{2.2}\right)^{\frac{0.4}{1.4}} = 0.798$$

$$T_{2s} = 0.798 \times 1500 = 1197.45\text{ K}$$

$$\eta_T = \frac{T_1 - T_2}{T_1 - T_{2s}} = \frac{1500 - T_2}{1500 - 1197.45}$$

$$1500 - T_2 = 0.88(1500 - 1197.45)$$

$$T_2 = 1233.76\text{ K}$$

$$\eta_P = \left(\frac{r}{r-1}\right)\frac{\ln(T_1/T_2)}{\ln(P_1/P_2)} = \left(\frac{1.4}{0.4}\right)\frac{\ln(1500/1233.76)}{\ln(2.2)}$$

$$\eta_P = 86.7\%$$

Note that the polytropic efficiency is less than the turbine efficiency. This is on account of reheating.

Example 1.9 An air compressor has eight stages of equal pressure ratio 1.3. The flow rate through the compressor and its overall efficiency are 45 kg/s and 80% respectively. If the conditions of air at entry are 1 bar and 35°C, determine

(a) state of air at compressor exit,
(b) polytropic efficiency, and
(c) efficiency of each stage.

[MKU – Nov '98]

Solution

$$\frac{P_2}{P_1} = 1.3 \quad m = 45\text{kg/s} \quad \eta_c = 0.8$$

$$P_1 = 1\text{bar} \quad T_1 = 273 + 35 = 308\text{ K}$$

(a) State of air at compressor exit

Overall pressure ratio,

$$\frac{P_{N+1}}{P_1} = \left(\frac{P_2}{P_1}\right)^8 = (1.3)^8 = 8.16$$

$$\frac{T_{N+1s}}{T_1} = \left(\frac{P_{N+1}}{P_1}\right)^{\frac{r-1}{r}} = (8.16)^{\frac{0.4}{1.4}} = 1.82$$

Ideal exit temperature

$$T_{N+1s} = 1.82 \times T_1 = 1.82 \times 308 = 560.56\text{ K}$$

Actual exit temperature is determined from the overall compressor efficiency expression.

$$\eta_c = \frac{T_{N+1s} - T_1}{T_{N+1} - T_1}$$

$$T_{N+1} = \frac{560.56 - 308}{0.8} + 308$$

$$T_{N+1} = 623.7\text{ K}$$

and

$$P_{N+1} = 8.16 \times P_1 = 8.16\text{ bar}$$

(b) Polytropic efficiency

$$\eta_p = \left(\frac{r-1}{r}\right)\frac{\ln(P_{N+1}/P_1)}{\ln(T_{N+1}/T_1)} = \left(\frac{0.4}{1.4}\right)\frac{\ln(8.16)}{\ln(623.7/308)}$$

$$\eta_p = 85\%$$

(c) Stage efficiency

$$\eta_s = \frac{\left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}} - 1}{\left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r\eta_p}} - 1} = \frac{(1.3)^{\frac{0.4}{1.4}} - 1}{(1.3)^{\frac{0.4}{1.4 \times 0.85}} - 1}$$

$$\eta_s = 84.43\%$$

Since the pressure ratio in each stage is the same, the stage efficiencies are also the same.

Example 1.10 The overall pressure ratio through a three stage gas turbine is 11.0 and efficiency is 88%. The temperature at inlet is 1500 K. If the temperature rise in each stage is the same, determine for each stage (a) pressure ratio and (b) stage efficiency

Solution

$$\frac{P_{N+1}}{P_1} = \frac{1}{11} \quad \eta_T = 0.88 \quad T_1 = 1500\text{ K}$$

$$
\begin{aligned}
T_1 - T_{N+1} &= \eta_T (T_1 - T_{N+1s}) \\
&= \eta_T T_1 \left(1 - \frac{T_{N+1s}}{T_1}\right) \\
&= \eta_T T_1 \left(1 - \left(\frac{P_{N+1}}{P_1}\right)^{\frac{r-1}{r}}\right) \\
&= 0.88 \times 1500 \left(1 - \left(\tfrac{1}{11}\right)^{\frac{0.4}{1.4}}\right) \\
&= 654.68 \text{ K} \\
(\Delta T)_{overall} &= 655 \text{ K} \\
T_{N+1} &= 1500 - 655 = 845 \text{ K} \\
\eta_P &= \frac{r}{r-1} \ln \frac{T_1}{T_{N+1}} \Big/ \ln\left(\frac{P_1}{P_{N+1}}\right) \\
&= \left(\frac{1.4}{0.4}\right) \frac{\ln(1500/845)}{\ln(11)} \\
\eta_P &= 0.837
\end{aligned}
$$

(a) For the first stage

$$
\begin{aligned}
(\Delta T)_{stage} &= \frac{(\Delta T)_{overall}}{3} \\
&= \frac{655}{3} = 218.33 \\
T_2 &= T_1 - 218.33 = 1500 - 218.33 \\
&= 1281.67 \text{ K}
\end{aligned}
$$

Pressure ratio,

$$
\begin{aligned}
\frac{P_1}{P_2} &= \left(\frac{T_1}{T_2}\right)^{\frac{r}{\eta_P (r-1)}} = \left(\frac{1500}{1281.67}\right)^{\frac{1.4}{0.4 \times 0.837}} \\
\frac{P_1}{P_2} &= 1.93
\end{aligned}
$$

Stage efficiency

$$
\begin{aligned}
\eta_{s,1} &= \frac{1 - \left(\frac{P_2}{P_1}\right)^{\frac{\eta_P (r-1)}{r}}}{1 - \left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}}} \\
&= \frac{1 - \left(\frac{1}{1.93}\right)^{\frac{0.837 \times 0.4}{1.4}}}{1 - \left(\frac{1}{1.93}\right)^{\frac{0.4}{1.4}}} \\
\eta_{s,1} &= 84.95\%
\end{aligned}
$$

(b) For the second stage

$$
\begin{aligned}
T_3 &= T_2 - (\Delta T)_{stage} \\
&= 1281.67 - 218.33 \\
&= 1063.34 \text{ K}
\end{aligned}
$$

Pressure ratio

$$
\begin{aligned}
\frac{P_2}{P_3} &= \left(\frac{T_2}{T_3}\right)^{\frac{r}{\eta_P (r-1)}} = \left(\frac{1281.67}{1063.34}\right)^{\frac{1.4}{0.4 \times 0.837}} \\
\frac{P_2}{P_3} &= 2.183
\end{aligned}
$$

Stage efficiency

$$
\begin{aligned}
\eta_{s,2} &= \frac{1 - \left(\frac{1}{2.183}\right)^{\frac{0.837 \times 0.4}{1.4}}}{1 - \left(\frac{1}{2.183}\right)^{\frac{0.4}{1.4}}} \\
\eta_{s,2} &= 85.2\%
\end{aligned}
$$

(c) For the third stage

$$
\begin{aligned}
T_4 &= T_3 - (\Delta T)_{stage} \\
&= 1063.34 - 218.33 \\
&= 845 \text{K}
\end{aligned}
$$

Pressure ratio

$$
\begin{aligned}
\frac{P_3}{P_4} &= \left(\frac{T_3}{T_4}\right)^{\frac{r}{\eta_P (r-1)}} = \left(\frac{1063.34}{845}\right)^{\frac{1.4}{0.4 \times 0.837}} \\
\frac{P_3}{P_4} &= 2.61
\end{aligned}
$$

Stage efficiency

$$
\begin{aligned}
\eta_{s,3} &= \frac{1 - \left(\frac{1}{2.61}\right)^{\frac{0.837 \times 0.4}{1.4}}}{1 - \left(\frac{1}{2.61}\right)^{\frac{0.4}{1.4}}} \\
\eta_{s,3} &= 85.51\%
\end{aligned}
$$

✱ **Example 1.11** Each stage of a 4 stage air compressor delivering 45 kg of air per second operates at a pressure ratio of 1.2, with a stage efficiency of 65%. Calculate overall efficiency and pressure ratio. Calculate power required to drive the compressor if air temperature at inlet is 20°C.

[MU, April '96 & Oct. '97]

Solution

$N = 4 \qquad m = 45\text{kg/s} \qquad \frac{P_2}{P_1} = 1.2 \qquad \eta_s = 0.65$

(a) Overall pressure ratio

$$\frac{P_{N+1}}{P_1} = \left(\frac{P_2}{P_1}\right)^N = (1.2)^4$$
$$= 2.1$$

(b) Overall efficiency

$$\eta_c = \frac{\left(\frac{P_2}{P_1}\right)^{N\left(\frac{\gamma-1}{\gamma}\right)} - 1}{\left(\frac{P_2}{P_1}\right)^{N\left(\frac{\gamma-1}{\eta_p \gamma}\right)} - 1}$$

η_p is obtained from the following equation:

$$\eta_s = \frac{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\eta_p \gamma}} - 1}$$

$$0.65 = \frac{(1.2)^{\frac{0.4}{1.4}} - 1}{(1.2)^{\frac{0.4}{\eta_p \times 1.4}} - 1}$$

$$(1.2)^{\frac{0.4}{\eta_p \times 1.4}} = 1.0823$$

Taking log on both sides, we get

$$\eta_p = 0.6589$$

$$\therefore \eta_c = \frac{(1.2)^{4\left(\frac{0.4}{1.4}\right)} - 1}{(1.2)^{\frac{4 \times 0.4}{0.6589 \times 1.4}} - 1}$$

$$\eta_c = 62.28\%$$

(c) Power required

$$W = mC_p(T_{N+1} - T_1)$$

$$T_{N+1} = T_1 \left(\frac{P_{N+1}}{P_1}\right)^{\frac{\gamma-1}{\gamma \eta_p}}$$
$$= 293(2.1)^{\frac{0.4}{1.4 \times 0.6589}}$$
$$= 404 \text{ K}$$

$$W = 45 \times 1.005(404 - 293)$$
$$W = 5019.98 \text{ kW}$$

Example 1.12 Air flows through a blower where in its total pressure is increased by 20 cm W.G. The inlet total pressure and total temperature of air are 1.04 bar and 18°C respectively. The total-to-total efficiency is 72%. Evaluate (a) the exit total pressure and total temperature (b) isentropic and actual changes in total enthalpy.

Solution

$\Delta P_0 = 0.2$ m W.G. $\quad P_{01} = 1.04$ bar

$T_{01} = 273 + 18 = 291$ K $\quad \eta_{tt} = 0.72$

(a) Exit total pressure and temperature

$$\Delta P_0 = 10^3 \times 9.81 \times 0.2 = 1962 \text{ N/m}^2$$
$$P_2 = \Delta P_0 + P_1 = 0.01962 + 1.04$$
$$= 1.0596 \text{ bar}$$

$$\eta_{tt} = \frac{\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] T_{01}}{T_{02} - T_{01}}$$

$$T_{02} - T_{01} = \frac{\left[\left(\frac{1.0596}{1.04}\right)^{\frac{0.4}{1.4}} - 1\right] 291}{0.72}$$

$$T_{02} = 293.16 \text{ K}$$

(b) Isentropic and actual changes in total enthalpy

$$\Delta h_0 = C_p(\Delta T_0) = C_p(T_{02} - T_{01})$$
$$= 1.005(293.16 - 291)$$
$$\Delta h_0 = 2.171 \text{ kJ/kg}$$

and isentropic change in total enthalpy is

$$\Delta h_{0,s} = (\Delta h_0) \times \eta_{tt}$$
$$= 2.171 \times 0.72$$
$$\Delta h_{0,s} = 1.563 \text{ kJ/kg}$$

Example 1.13 Air flows through an air turbine where its stagnation pressure is decreased in the ratio 5:1. The total-to-total efficiency is 0.8 and the air flow rate is 5 kg/s. If the total power output is 500 kW, find (a) inlet total temperature (b) the actual exit total temperature (c) the actual exit static temperature if the flow velocity is 100 m/s and (d) the total-to-static efficiency of the device. *[MSU, Nov. '96]*

Solution

$\frac{P_1}{P_2} = 5 \quad \eta_{tt} = 0.8 \quad m = 5 \text{ kg/s} \quad W = 500 \text{ kW}$

(a) Inlet total temperature

$$W = mC_p(T_{01} - T_{02})$$

Taking C_p of air as 1.005 kJ/kgK.

$$T_{01} - T_{02} = \frac{W}{mC_p} = \frac{500 \times 10^3}{5 \times 1005} = 99.5 \text{ K}$$

The turbine total-to-total efficiency is

$$\eta_{tt} = \frac{T_{01} - T_{02}}{T_{01} - T_{02s}}$$

$$T_{02s} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{5}\right)^{\frac{0.4}{1.4}} T_{01} = 0.631 T_{01}$$

$$\therefore T_{01} - T_{02s} = \frac{T_{01} - T_{02}}{\eta_{tt}}$$

$$T_{01} - 0.631 T_{01} = \frac{99.5}{0.8}$$

$$T_{01} = 337 \text{ K}$$

(b) The actual exit total temperature

$$T_{01} - T_{02} = 99.5$$

$$\therefore T_{02} = T_{01} - 99.5 = 337 - 99.5$$

$$T_{02} = 237.5 \text{ K}$$

(c) The actual exit static temperature

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 237.5 - \frac{100^2}{2 \times 1005}$$

$$T_2 = 232.5 \text{ K}$$

(d) The total-to-static efficiency

$$\eta_{t-s} = \frac{T_{01} - T_{02}}{T_{01} - T_{2s}}$$

$$T_{2s} = T_{02s} - \frac{C_2^2}{2C_p}$$

$$T_{02s} = 0.631 T_{01} = 0.631 \times 337 = 212.65 \text{ K}$$

$$T_{2s} = 212.65 - \frac{100^2}{2 \times 1005} = 207\ 68 \text{ K}$$

$$\therefore \eta_{t-s} = \frac{99.5}{337 - 207.68}$$

$$\eta_{t-s} = 76.94\%$$

Example 1.14 In a three stage turbine the pressure ratio of each stage is 2 and the stage efficiency is 75%. Calculate the overall efficiency and the power developed if air initially at a temperature of 600°C flows through it at the rate of 25 kg/s. Find reheat factor. *[MU, Oct. '96 & Apr. '97]*

Solution

$N = 3 \quad P_1/P_2 = 2 \quad \eta_s = 0.75 \quad T_1 = 600 + 273 = 873 \text{ K} \quad m = 25 \text{ kg/s}$

(a) Overall efficiency

$$\eta_T = \frac{T_1 - T_2}{T_1 - T_{2s}} = \frac{1 - \left(\frac{P_2}{P_1}\right)^{N\left(\frac{\eta_p(\gamma-1)}{\gamma}\right)}}{1 - \left(\frac{P_2}{P_1}\right)^{N\left(\frac{\gamma-1}{\gamma}\right)}}$$

η_p is determined using η_s expression.

$$\eta_s = \frac{1 - \left(\frac{P_2}{P_1}\right)^{\eta_p\left(\frac{\gamma-1}{\gamma}\right)}}{1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$0.75 = \frac{1 - \left(\frac{1}{2}\right)^{\eta_p\left(\frac{0.4}{1.4}\right)}}{1 - \left(\frac{1}{2}\right)^{\frac{0.4}{1.4}}}$$

$$\left(\frac{1}{2}\right)^{\eta_p\left(\frac{0.4}{1.4}\right)} = 0.865$$

Taking log on either side,

$$\eta_p \left(\frac{0.4}{1.4}\right) \ln (1/2) = \ln 0.865$$

$$\therefore \eta_p = 73.23\%$$

Then

$$\eta_T = \frac{1 - \left(\frac{1}{2}\right)^{3\left(\frac{0.7323 \times 0.4}{1.4}\right)}}{1 - \left(\frac{1}{2}\right)^{3\left(\frac{0.4}{1.4}\right)}}$$

$$\eta_T = 78.76\%$$

(b) Power developed

$$W = mC_p(T_1 - T_{N+1})$$

$$= mC_pT_1\left[1 - \left(\frac{P_{N+1}}{P_1}\right)^{\frac{\eta_p(\gamma-1)}{\gamma}}\right]$$

where

$$\frac{P_{N+1}}{P_1} = \left(\frac{P_2}{P_1}\right)^3 = \frac{1}{8}$$

$$= 25 \times 1.005 \times 873\left[1 - \left(\frac{1}{8}\right)^{\frac{0.7323 \times 0.4}{1.4}}\right]$$

$$= 7738 \text{ kW}$$

(c) Reheat factor

$$\text{R.F} = \frac{\eta_T}{\eta_s} = \frac{0.7876}{0.75}$$

$$= 1.05$$

EXERCISES

1.1. Define Turbo machine.
1.2. How are devices pumping gases classified?
1.3. What are shrouded and unshrouded turbo machines?
1.4. Classify turbo machines on the basis of work transfer.
1.5. Define the types of turbo machines based on fluid movement through the machine.
1.6. Derive the general Euler's expression for a turbo machine.
1.7. Define the following efficiencies of power absorbing turbo machines.
(a) Total-to-Total Efficiency
(b) Static-to-Static Efficiency

1.8. Define the following efficiencies of power generating turbo machines.
(a) Total-to-Total Efficiency
(b) Total-to-Static Efficiency
1.9. Draw the h–s diagram with static and stagnation states for the compression and expansion processes for a gas.
1.10. What is preheat factor in a multistage compressor? Prove that preheat factor is less than unity.
1.11. Prove that the compressor stage efficiency is greater than the compressor overall efficiency.
1.12. What is reheat factor in a multistage turbine? Prove that R.F is greater than unity.
1.13. Prove that the turbine overall efficiency is greater than the turbine stage efficiency.
1.14. Define polytropic efficiency of a compressor.
1.15. Derive the polytropic compression efficiency through an infinitesimal compression stage.
1.16. Define polytropic expansion efficiency.
1.17. Derive the polytropic expansion efficiency through an infinitesimal turbine stage.
1.18. Derive the reheat factor in terms of the stage and overall pressure ratios.
1.19. Air flows through a blower wherein its total pressure is increased by 15 cm of liquid water. The inlet total pressure and the temperature of the air are 1.05 bar and 15°C respectively. The total-to-total efficiency is 70%. Evaluate (a) the exit total pressure, (b) the exit isentropic total temperature, and (c) the isentropic and actual changes in total enthalpy. [*MU–April '96*]
[**Ans.** (a) 1.065 bar, (b) 289.2 K, and (c) 1.206 kJ/kg, 1.723 kJ/kg]
1.20. A compressor has a total-to-total efficiency of 80% and an overall total pressure ratio of 5:1. Calculate the small stage efficiency of the compressor.
[**Ans.** 83.9%]
1.21. Air flows through an air turbine where its stagnation pressure is decreased in the ratio 5:1. The total-to-total efficiency is 0.8 and the air flow rate is 5 kg/s. If the total power output is 405 kW. Find (a) the inlet total temperature, (b) the actual exit total temperature, (c) the actual exit static temperature if the exit flow velocity is 100 m/s. and (a) the total-to-static efficiency of the turbine.
[**Ans.** (a) 273 K, (b) 192.4 K, (c) 187.4 K, and (d) 76%]
1.22. A turbine has a small stage efficiency of 84% and an overall total pressure ratio of 4.5:1. Calculate the total-to-total efficiency of the turbine.
[**Ans.** 86.74%]
1.23. A gas turbine is required to develop 7360 kW with an air flow rate of 50 kg/s. If the turbine inlet temperature and pressure are 1000°C and 8 bar respectively. Calculate the exit temperature and pressure if the isentropic efficiency of the turbine is 90%. [**Ans.** (a) 1127 K, and (b) 5 bar]
1.24. A low pressure air compressor increases the air pressure by 1500 mm W.G. If the initial and final conditions of air are P_1 = 1.02 bar, T_1 = 300 K and T_2 = 315 K, determine and compare the compressor and the infinitesimal stage efficiencies. [**Ans:** (a) 78% and (b) 78.8 %]

1.25. The initial state of air flowing through a compressor is P_1 = 1.02 bar, T_1 = 300 K. The exit pressure is 2.5 bar and the compressor efficiency is 75%. Determine the infinitesimal efficiency of the compressor. Comment on the deviation in the efficiency. [Ans: 78%]

1.26. The overall pressure ratio across a three stage gas turbine is 11 and its efficiency is 88%. If the pressure ratio of each stage is the same and the inlet temperature is 1500 K determine (a) pressure ratio of each stage, (b) polytropic efficiency and (c) stage efficiency.
[Ans: (a) 2.22 (b) 83.7% and (c) 85.2%]

1.27. For an index of expansion of n = 1.3 and C_p/C_v = 1.4, calculate the polytropic efficiency. *[MU, Oct.'97]*
[Ans. 80.77%]

1.28. Air enters a compressor at a static condition of 150 kPa and 15°C and a velocity of 50 m/s. At the exit the static conditions are 0.3 MPa and 100°C and a velocity of 100 m/s. Evaluate (a) isentropic and actual changes in enthalpy and (b) total efficiency. *[MU, Oct. '96]*
[Ans. (a) 63.39 kJ/kg and 85.43 kJ/kg (b) 75.29%]

1.29. In a centrifugal compressor the air is compressed to double the pressure. The inlet temperature is 27°C and the final temperature is 107°C. Calculate the efficiency of the compressor and the power required to drive it if 30 kg/min of air is compressed. *[MU, Apr. '98]*
[Ans. (a) 82.13% and (b) 40.2 kW]

1.30. A centrifugal compressor takes in air at 101 kPa and 25°C and compresses it through a pressure ratio of 3.5:1. The index of compression n = 1.65 because of frictional heating. The mass flow rate of air handled by the compressor is 29 kg/s. Find (a) overall efficiency of compressor and (b) power supplied by the motor with mechanical efficiency of 95%
[Ans. (a) 67.4% and (b) 5809.2 kW]

1.31. Hot air enters a 3 stage turbine with total head properties of 750 kPa and 900°C at the rate of 25 kg/s. The final exit pressure is 105 kPa. The pressure at exit of I and II stages are 500 kPa and 250 kPa respectively. The individual stage efficiencies for the 3 stages are each 75%. Find (a) reheat factor (b) overall efficiency and (c) power developed.
[Ans. (a) 1.05 (b) 78.75% and (c) 9980 kW]

2

BLADE THEORY

The energy transfer in turbomachines is effected by changing the angular momentum of the fluid. The change in angular momentum is caused by the dynamic action of one or more rotating blade rows. The dynamic action of the rotating blade rows, sets up forces between the blade row and the fluid, while the components of these forces in the direction of blade motion give rise to the energy transfer between the blades and fluid. The theory of compressor and turbine blades is discussed in this chapter.

AERO-FOIL SECTION

An aero-foil may be defined as a streamlined form, bounded principally by two flattened curves whose length and width are very large, relative to its thickness. Aero-foil is classified as symmetrical aero-foil and non-symmetrical aero-foil.

Symmetrical Aero-foil

The aero-foil whose axis of symmetry is parallel to the direction of undisturbed velocity of approach is called *symmetrical aero-foil*. The flow pattern around a symmetrical aero-foil placed in a stream of gas is shown in Fig. 2.1. The flow divides around the aero-foil at the leading edge and then rejoins at the trailing edge. Though there is some local disturbance, there is no permanent deflection of the main stream. The forces exerted in this case are only due to friction and the local disturbance.

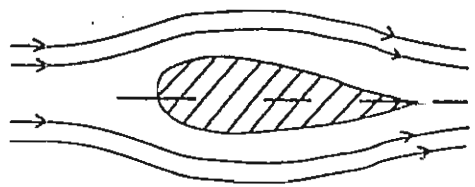


Figure 2.1 Flow pattern around a symmetrical aero-foil

Non-symmetrical Aero-foil

If the aero-foil axis is inclined at an angle 'i', called as the *angle of attack*, to the direction of the undisturbed approaching flow, then it is called as *non-symmetrical* *aero-foil*. Fig. 2.2 shows a non-symmetrical aero-foil placed in a stream of gas. Unlike the symmetrical aero-foil, in this case, there is a pronounced disturbance which results in greater local deflection of flow. To introduce such a high deflection over the gas stream, the aero-foil must exert a force on it, and hence an equal and opposite force of reaction is exerted by the gas on the aero-foil. The components of the resultant force are discussed in the following section.

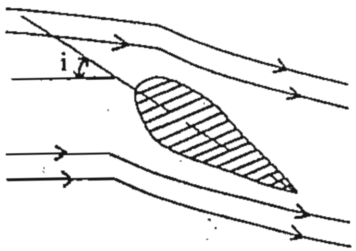


Figure 2.2 Flow pattern around a non-symmetrical aero-foil

DRAG AND LIFT

The resultant force normal to the aero-foil consists of two components, namely lift and drag. The forces of lift and drag on a blade section are shown in Fig. 2.3 with lift normal to the direction of the approach velocity and the drag parallel to it. The lift is due to an unbalanced force (pressure distribution) over the aero-foil surface and is denoted as L. The drag denoted as D, is due to the shearing stress at the surface and the consequent boundary layer. The drag force is made up of a friction drag, due to the pure skin friction effects, and a pressure drag, due to an unbalanced pressure distribution around the blade.

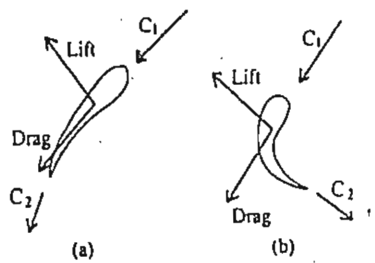


Figure 2.3 The forces of lift and drag on a) Compressor blade b) Turbine blade

The boundary layer is usually laminar for a short distance downstream of the leading edge, then it becomes turbulent. The drag due to a laminar boundary is less than a turbulent layer. Thus for low pure friction drag, it is important to maintain a laminar boundary layer over as much of the surface as possible.

If the pressure gradient is severe, that is if the rate of change of aero-foil profile is too rapid, then the fluid in the boundary layer is brought to rest and leaves the surface in confused eddies. This phenomenon is called *separation, break-away*, or *flow reversal* Fig. 2.4 and manifests itself in several other ways as well as in the simple case of diffusion in a straight duct. Flow separation due to adverse pressure gradient decreases the lift and increases the drag. In practice, the adverse pressure gradient near the tail causes a thick boundary layer and possibly separation.

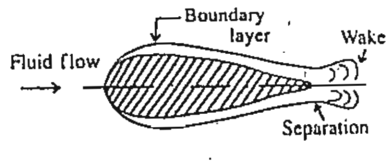


Figure 2.4 Separation and Wake

and the pressure has a much lower value, with the fluid near the surface leaving in a low energy region called the *wake* (Fig. 2.4). Measurements of pressure and velocity in the wake can give the loss of energy due to the presence of the body and the drag can be calculated. Alternatively, the drag can be measured as an actual force by means of a balance from the wind tunnel test.

LIFT AND DRAG COEFFICIENTS

Lift coefficient is a measure of the ability of a given section to support a weight when caused to move through a fluid, as in the case of aeroplane wing, or, alternatively, to transfer energy to a fluid, as in a pump or compressor, or to transfer energy to a rotor when a fluid is caused to flow over it, as in turbines. It is defined as

$$C_L = \frac{L}{(0.5\rho W_m^2 A)} \tag{2.1}$$

Drag coefficient is a measure of the loss of energy associated with the useful task of producing lift. It is defined as

$$C_D = \frac{D}{(0.5\rho W_m^2 A)} \tag{2.2}$$

where W_m is the mean relative velocity, A is the area of the body and the factor of 0.5 is inserted for convenience as $0.5\rho W_m^2$ is defined as *dynamic pressure*. Some care is needed in the evaluation of area associated with a given value of C_D. For bodies of revolution which are symmetrical about an axis and parallel to the flow, A is taken as the projected area normal to the direction of the flow (e.g. spheres, cylinders, etc.). For other bodies (e.g. blades, aeroplane wings, etc.) which are normally either unsymmetrical or not aligned parallel to the flow, or both, the area is evaluated in terms specifically defined as required. It may be noted that C_D as given in equation (2.2) is the ratio of the actual drag force to the force which would be exerted if the representative area of the body were acted upon by the dynamic pressure.

It is apparent that the maximum energy transfer implies the largest possible fluid deflection or lift coefficient, while maximum efficiency requires the lowest possible loss of pressure or drag coefficient. The conditions for a blade section should attempt to approach those for laminar flow over a flat plate, as this gives the lowest possible drag coefficient. But it is difficult to achieve this in practice, because

(1) blades must have the curvature to change the direction of the fluid, introducing a pressure gradient and a tendency for flow separation,
(2) blades must have a finite thickness from considerations of strength, and
(3) the fluid has a high turbulence level.

The best conditions are

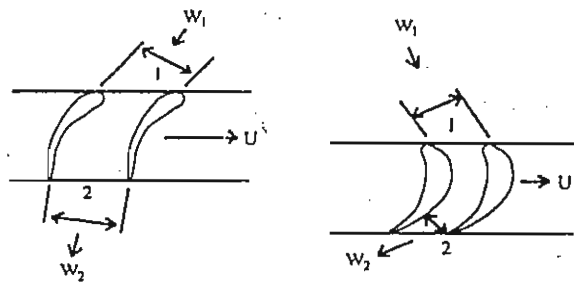
a) the blade should have a shape such that the separation is minimised and
b) for the flow to be above the critical Reynolds number (2×10^5). When Reynolds number reaches the critical value and the boundary layer becomes turbulent, C_D

drops abruptly because the separation point moves back to give a smaller wake on the downstream side of blade.

In compressors, the blades form a diverging passage i.e the area at inlet is less than at outlet and therefore the fluid is decelerated in the passage. In turbines, the blades form a converging passage, i.e. the area at inlet is greater than that at outlet. Thus, the fluid is accelerated in the passage.

A fluid can be accelerated over a wide range with high efficiency, but the process of diffusion cannot be carried out so rapidly due to the onset of separation on the suction side of the blades, and consequent stalling. This is similar to the included angle of a diffuser being too great and separation taking place along the diffuser walls. The maximum rate of efficient diffusion within the blade rows is equivalent to a cone angle of about 7° or 8°.

The curvature of compressor blades is less when compared to the curvature of turbine blades. Because, if the rate of change of compressor blade profile is high, flow separation will occur due to adverse pressure gradient. Whereas in a turbine, the pressure gradient is favourable and with a very large curvature, i.e. 90° or even more, can be employed without severe losses. As a result, the angular turning of the relative velocity vector is much greater in the turbine than in the compressor. Typical blade sections are shown in Figs. 2.4(a) and 2.4(b).



(a) Compressor blade passage (b) Turbine blade passage

Figure 2.4

ENERGY TRANSFER IN TERMS OF LIFT AND DRAG COEFFICIENTS

Consider a rotor blade shown in Fig. 2.5. with relative velocity vectors W_1 and W_2 at angles β_1 and β_2. This system is similar to flow over an aero-foil, so that lift and drag forces will be set up on the blade. The drag force is acting in the line of the mean velocity vector W_m at angle β_m to the axial direction and the lift force acts perpendicular to this.

The forces on the air will act in the opposite direction as shown in the Fig. 2.6. The resultant force experienced by the air is therefore given by the vector R in the figure.

Force acting in the direction of the blade rotation (x direction) is given by

$$F_x = L\cos\beta_m + D\sin\beta_m$$
$$= L\cos\beta_m\{1 + (C_D/C_L)\tan\beta_m\}$$

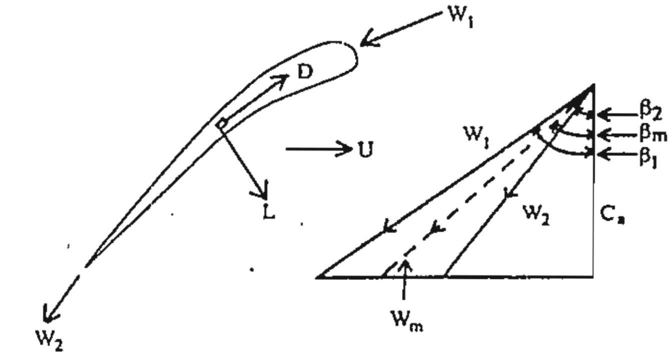


Figure 2.5 Lift and drag forces on a compressor rotor blade

where

$$C_D = \frac{D}{(0.5\rho W_m^2 A)}$$

and

$$C_L = \frac{L}{(0.5\rho W_m^2 A)}$$

where the blade area A is the product of the chord 'c' and the span 'l' (blade height) and putting

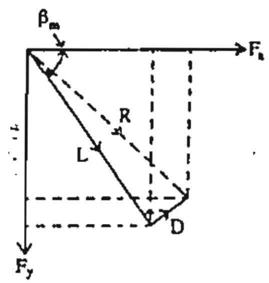


Figure 2.6 Resolving blade forces into the direction of rotation

$$W_m = \frac{C_a}{\cos\beta_m}$$

we have

$$F_x = \frac{\rho C_a^2 (cl) C_L \sec\beta_m [1 + (C_D/C_L)\tan\beta_m]}{2}$$

The power delivered to the air is given by

$$F_x U = m(\Delta h_0)$$

where Δh_0 is the change in total enthalpy across the rotor and $m = \rho C_a ls$

$$\therefore F_x U = \rho C_a ls(\Delta h_0)$$

where the flow through one blade passage of width 's' has been considered. The blade loading factor is given by

$$\psi_l = \frac{\text{Power delivered}}{mU^2}$$
$$= (\Delta h_0)/U^2$$
$$= F_x/(\rho C_a l s U)$$
$$= \frac{C_a(c/s)\sec\beta[C_L + C_D\tan\beta_m]}{2U}$$
$$= \frac{\phi(c/s)\sec\beta_m[C_L + C_D\tan\beta_m]}{2}$$

where ϕ is called the flow coefficient and is defined as

$$\boxed{\phi = \frac{\text{Axial velocity } (C_a)}{\text{Blade speed } (U)}}$$

For maximum efficiency the mean flow angle β_m is usually about 45° and, substituting this into the blade loading factor equation, the expression for optimum blade loading factor ψ_{lopt} is obtained.

$$\psi_{lopt} = \frac{\phi(c/s)[C_L + C_D]}{\sqrt{2}}$$

If C_D is much smaller than C_L, which usually occurs in the case of a well-designed blade, then

$$\psi_{lopt} = \frac{\phi(c/s)C_L}{\sqrt{2}}$$

BLADE TERMINOLOGY

Blade profiles are usually of aero-foil shape for optimum performance. But, simple geometrical shapes composed of circular arcs and straight lines are used when cost is more important than the efficiency. Many blade profiles are formed by bending a symmetrical aero-foil section on a curved mean line. The parameters used in describing blade shapes and configurations of blades (Fig. 2.7) are as follows

1. Base profile It is defined by dividing the major axis into equally spaced stations designated as a percentage of the blade length and specifying the height from axis to profile at each station.

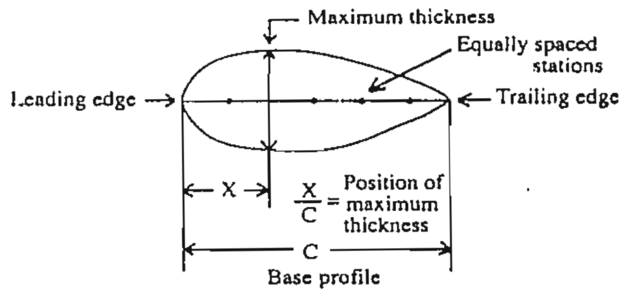


Figure 2.7 Parameters used in describing blade shapes and configurations of blades

2. Maximum thickness It is a useful parameter for describing an aero-foil and it is expressed as a percentage of the blade length.

3. Position of maximum thickness It is an another useful parameter which is specified as a percentage of the blade length.

4. Leading edge (Nose) It is usually a circular arc blended into the main profile and specified by its radius as a percentage of the maximum thickness.

5. Trailing edge It is ideally sharp, i.e. of zero radius, but as this is impossible from strength considerations, it is also a circular arc specified as a percentage of the maximum thickness.

6. Camber line If the axis of the linear profile is given some predetermined curvature, then it is called *camber line*. The base profile is fitted on to the curved camber line. This camber line is formed either by one or more circular arcs or one or two parabolic arcs. A single circular or parabolic arc is quite common as it is geometrically simple [Fig. 2.7(a)]

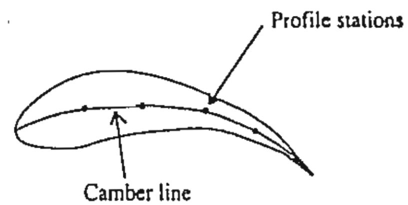


Figure 2.7 (a) Blade nomenclature

CASCADE NOMENCLATURE

A cascade is a row of geometrically similar blades arranged at equal distances from each other and aligned to the flow direction. The various important nomenclatures (Fig. 2.8) of a compressor cascade are

a) Camber angle The angle between the camber line and the axial direction is called *camber angle*, denoted by α'. The camber angles at inlet and outlet are α'_1 and α'_2 respectively. It is also called as *cascade blade angle*.

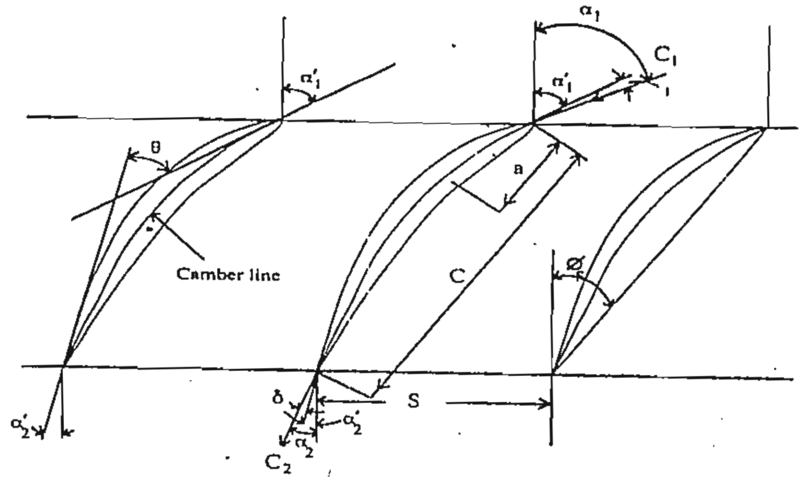


Figure 2.8 Compressor cascade nomenclatures

b) Blade camber angle The difference between the inlet and outlet camber angles is called the *blade camber angle*, denoted by θ. Mathematically

$$\theta = (\alpha'_1 - \alpha'_2)$$

c) Chord The chord is the maximum width of the blade profile in a direction parallel to the chord line, i.e. the distance between the blade leading and trailing edges, and is denoted by c.

d) Pitch The pitch or blade spacing, designated s, is the distance between the corresponding points of adjacent blades and is expressed either by the pitch to chord ratio (s/c) or alternatively the solidity (c/s). When the blades are evenly spaced around a rotor, the pitch is the circumference at any radius divided by the number of blades.

e) Air angle The angle between the direction of velocity relative to the blade and the axis of the blade row is the *air angle*, and is denoted by α. The air angles at the inlet and outlet are α_1 and α_2 respectively. The cascade air angle is equal to the compressor or turbine relative air angle.

f) Angle of incidence The difference between the inlet air angle and the inlet camber angle is known as the *angle of incidence*, denoted as i.

$$i = \alpha_1 - \alpha'_1$$

g) Deviation angle The difference between the outlet air angle and the outlet camber angle is known as the *deviation angle*, denoted as δ. Mathematically,

$$\delta = \alpha_2 - \alpha'_2$$

The deviation angle δ is caused by the air not remaining attached to the blade over its total curvature. δ is given by the empirical relationship

$$\delta = m\theta(s/c)^{1/2}$$

where

$$m = 0.23(2a/c)^2 + 0.1(\alpha_2/50)$$

and

'a' is the distance along the chord to the point of maximum camber. For a circular arc camber line, $(2a/c) = 1$, and this blade form is often chosen.

h) Air deflection angle The difference between the inlet air angle and the outlet air angle is called as the *air deflection angle*, denoted as ε. Mathematically,

$$\varepsilon = \alpha_1 - \alpha_2$$

From the definitions of different angles, it can be seen that they are related by the following expression.

$$\epsilon = \theta + i - \delta$$

i) Stagger angle The angle between the axial direction and the chord is known as stagger angle, denoted by ϕ_s, and it represents the angle at which the blade is set in the cascade.

TURBINE CASCADE NOMENCLATURE

The setting of blades in a turbine cascade is invariably at a stagger angle (ϕ_s). i.e. the chord lines of the trubine blades are tilted towards the blade curvature as shown in Fig. 2.8(a).

Camber angle The tangents to the camber line at the entry and exit make the camber angles α'_1 and α'_2 with the axial direction.

In contrast to the compressor cascade, the blade camber angle for the turbine cascade is defined as the sum of the inlet and outlet camber angles.

$$\theta = \alpha'_1 + \alpha'_2$$

The air angles α_1 and α_2 are different from the blade angles α'_1 and α'_2

Angle of incidence The difference between the air and the blade angle at the entry is known as the angle of incidence.

$$i = \alpha_1 - \alpha'_1$$

As in the compressor cascade, the incidence angle can be positive or negative. Flow at a large positive incidence is associated with positive stall, i.e. flow separation on the suction side of the blade; conversely, a large negative incidence is associated with negative stall, i.e. flow separation on the pressure side of the blade.

Deviation angle The difference between the air angle and blade angle at exit is referred to as deviation.

$$\delta = \alpha_2' - \alpha_2$$

It is different from the deviation angle for compressor cascade and the difference is due to the different convention used to define the stagger angle and the exit angles in turbine cascades.

Air deflection angle The deflection angle for the turbine cascade is defined as the sum of the air angles at the entry and exit.

$$\varepsilon = \alpha_1 + \alpha_2$$

This is again different from that of the compressor cascade. It is also expressed in terms of other angles as

$$\varepsilon = \theta + i - \delta$$

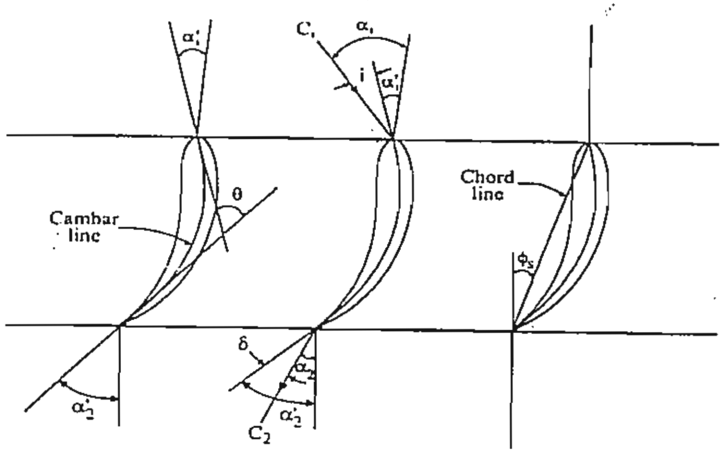


Figure 2.8 (a) Turbine cascade nomenclature

CASCADE TESTING AND CURVES

The flow around blades in compressors and turbines is different from flow around isolated aero-foils, because of the effect of adjacent blades, i.e. the gas flow around a blade is affected by the flow around an adjacent blade. This effect increases as the solidity ratio (c/s) increases. The solidity of axial flow compressor and gas turbine blades is high, while the blades of axial flow pumps and hydraulic turbines are of low solidity. In order to determine the performance characteristics of a blade section, groups of blades of constant profile are mounted in parallel fashion at the end of a wind tunnel as shown in the Fig. 2.9. The number of blades comprising the cascade has to be sufficient (usually 8 or 10 with an aspect ratio of 3 or above) to eliminate

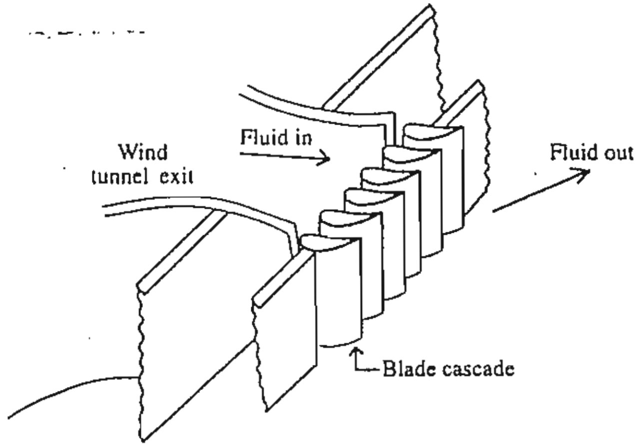


Figure 2.9 Cascade test

any wind tunnel boundary layer effects, and suction slots or porous metal inserts are often provided in the tunnel walls to control the boundary layer. The pressure, velocity and air flow angles are measured at the inlet and outlet of the cascade.

The results obtained from the cascade testing requires corrections because of the differences between the flow in the actual machine and flow through the cascade. These differences are as follows:

In the actual machine,

a) Annulus wall boundary layers exist at the blade hub and tip.
b) Adjacent blade rows interfere with the flow pattern around the blade row.
c) The solidity decreases from hub to tip.
d) Blade velocity varies from hub to tip and this affects the blade inlet angle.

From the last two points, it is evident that a cascade test only applies for one radius and inlet angle, and therefore it may be necessary to carry out a number of tests to obtain a reliable picture of the flow in the blades.

Fig. 2.9 is known as a linear cascade and can be imagined as a row of compressor or turbine blades unwound from the rotor to form the cascade.

The air deflection angle and the stagnation pressures at the inlet (P_{01}) and (P_{02}) are measured in the traverse along s. The results are usually presented as in Fig. 2.10. The stagnation pressure loss is plotted as a dimensionless number given by stagnation pressure loss coefficient = $(P_{01} - P_{02})/(0.5\rho C_1^2)$. The pressure defect corresponding to the regions close to the trailing edges of the blade represents the loss due to the blade wanes, caused by the boundary layer and possibly separated flow. In between the blades the fluid undergoes almost no loss of energy. The plot of air deflection angle shows that the air is not deflected uniformly, but with the maximum effect near the trailing edge. The air between the blades again is relatively unaffected.

A number of such curves are obtained for different incidence angles and the mean deflection and pressure loss coefficient for each curve, ε_m and $[(P_{01} - P_{02})/(0.5\rho C_1^2)]_m$ are plotted against incidence angle Fig. 2.11. The deflection increases with angle of incidence upto a maximum deflection angle (ε_{max}). This is the stall point where separation occurs on the suction surface of the blade. But this angle may not be well

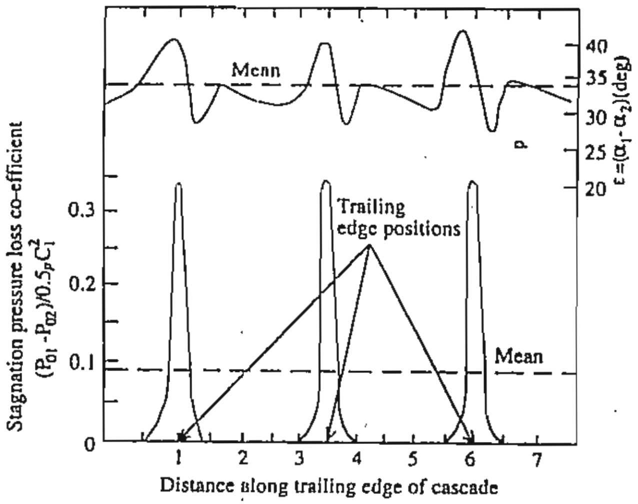


Figure 2.10 Cascade deflection and pressure loss curves at one angle of incidence

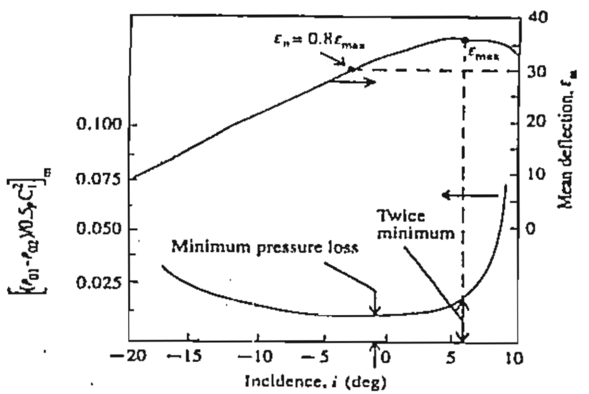


Figure 2.11 Cascade mean deflection and pressure loss curves

defined in some designs and it is taken as the angle of incidence, where the mean pressure loss is twice the minimum. The plot shows that, for a wide range of incidence, the pressure loss is fairly constant, and it is possible to select an angle of deflection ε_n (called *nominal deflection angle*) which is also compatible with low pressure loss as representative by the particular design. The nominal deflection angle is generally taken as 0.8 times the maximum deflection angle (i.e. $\varepsilon_n = 0.8\varepsilon_{max}$). It has been determined from large number of cascade tests that the nominal deflection angle is

dependent mainly on the pitch/chord ratio (s/c) and α_2. A plot between ε_n and α_2 for different values of (s/c) is shown in Fig. 2.12. These curves are particularly useful to the designer when any two of the three variables are fixed.

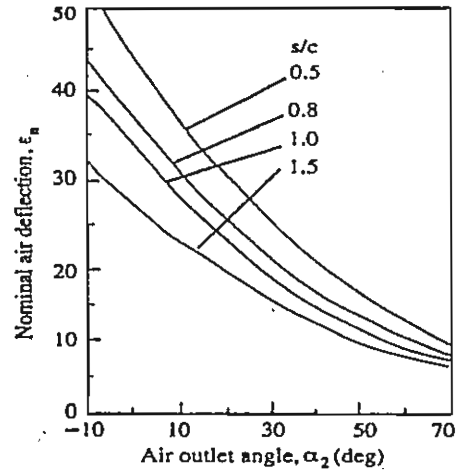


Figure 2.12 Cascade nominal deflection angle versus air outlet angle

CASCADE LIFT AND DRAG COEFFICIENTS

Fig. 2.13 shows two blades of a cascade having chord, c and pitch, s. At sections 1 and 2, the total pressures are P_{01} and P_{02} respectively with corresponding velocities of C_1 and C_2. The density change across the cascade is assumed to be negligible. The static pressure change across the cascade is given by

$$(P_2 - P_1) = \rho(C_1^2 - C_2^2)/2 - (P_{01} - P_{02})_m$$

where the difference ($P_{01} - P_{02}$) is obtained from the cascade test. It should be noted that $P_{01} > P_{02}$, because no work is in the cascade and the flow is proceeded irreversible. Hence, the above equation will be written as

$$\Delta P = \rho(C_1^2 - C_2^2)/2 - P_{0m}$$

where $\Delta P = (P_2 - P_1)$ and $P_{0m} = (P_{01} - P_{02})_m$

The summation of all forces acting on the air in the control volume x and y directions must equal to the rate of change of momentum of the air towards these directions. Considering the forces and the changes in velocity in the directions x and y, the following relations are obtained for drag and lift.

$$\text{Drag } D = s P_{0m} \cos\alpha_m$$

Dividing the drag by $0.5\rho C_m^2 c$ gives the **drag coefficient** (C_D)

$$C_D = 2((s/c)(P_{0m}/\rho C_m^2)\cos\alpha_m$$

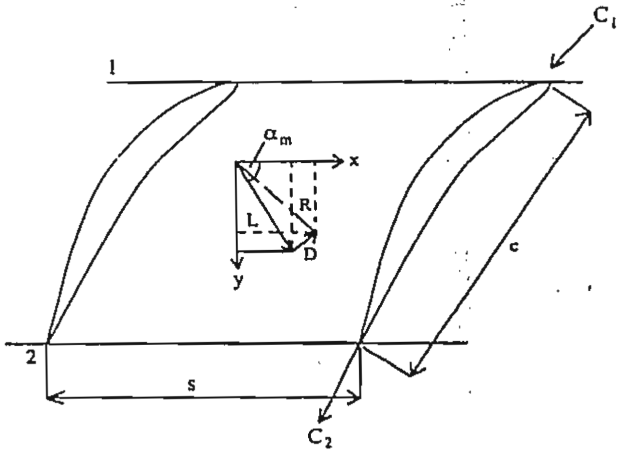


Figure 2.13 Velocities and forces in a cascade

since $C_a = C_1 \cos\alpha_1$ and $C_m = C_a/\cos\alpha_m$

$$\therefore C_D = 2(s/c)(P_{0m}/\rho C_1^2)(\cos^3\alpha_m/\cos^2\alpha_1)$$

The lift $L = [\rho C_a^2 s \cos\alpha_m)(\tan\alpha_1 - \tan\alpha_2)/\cos^2\alpha_m] - (s P_{0m}\cos\alpha_m)\tan\alpha_m$
Then the lift coefficient $C_L = L/0.5\rho C_m^2 c$

$$= 2(s/c)\cos\alpha_m(\tan\alpha_1 - \tan\alpha_2) - C_D\tan\alpha_m$$

The air inlet velocity C_1, the incidence angle i and the air inlet angle α_1 are also known. The deviation angle ε is read from the graph between ε_m and i for the given angle of incidence and

$$\alpha_m = \tan^{-1}[(\tan\alpha_1 + \tan\alpha_2)]/2$$ Comp

where $\alpha_m = \tan^{-1}[(\tan\alpha_2 - \tan\alpha_1)]/2$

$$\alpha_2 = (\alpha_1 - \varepsilon)$$

Knowing (s/c), values of $P_{0m}/0.5\rho C_1^2$ can be read from the same graph (ε vs i) for various incidence angles and substitution of these variables into equations for C_D and C_L, curves of C_L and C_D may be plotted against the incidence angle as shown in Fig. 2.14. Finally, the lift coefficient can be plotted against the air outlet angle α_2 for the nominal value of ε_n for a whole series of different geometry cascades, to give the variation of C_L with air outlet angle for a particular (s/c) ratio (Fig. 2.15).

The drag coefficient is very small in comparison with C_L and is therefore often ignored. So that equation for C_L becomes

$$C_L = 2(s/c)\cos\alpha_m(\tan\alpha_1 - \tan\alpha_2)$$

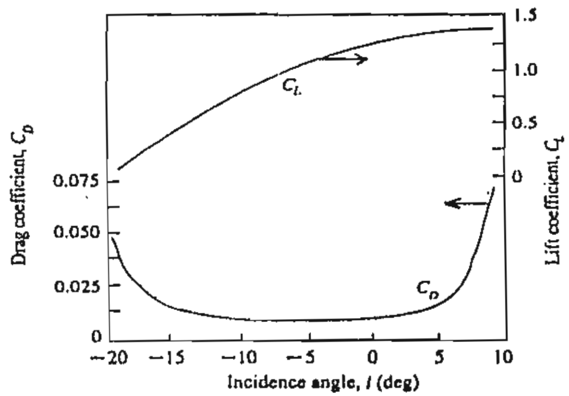


Figure 2.14 Lift and drag coefficients versus incidence angle

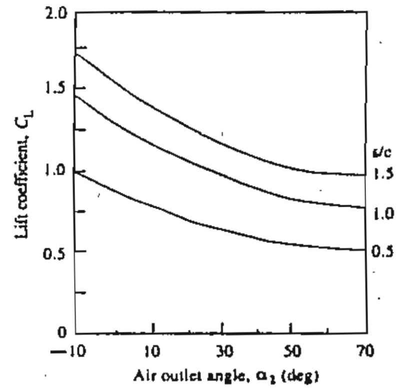


Figure 2.15 Cascade lift coefficient versus air outlet angle

To the profile drag as given by C_D, two further drags must be added to take the various cascades losses into account. There are drag effects due to the walls of the compressor, called the annulus drag and the secondary losses caused by trailing whirls (vortices) at the blade tips. Empirical relationships exist for these drags as follows:

Annulus drag coefficient is given by

$$C_{DA} = 0.002(s/l)$$

where l is the blade height, and

Secondary losses by

$$C_{DS} = 0.018C_L^2$$

The total drag coefficient is given by

$$C_{DT} = C_D + C_{DA} + C_{DS}$$

CASCADE LIFT AND DRAG COEFFICIENTS FOR TURBINE BLADES

Turbine rotor blades are designed based on the cascade data which is similar to the compressor rotor blades. The lift and drag coefficients are obtained from the cascade data curves. The drag coefficient is expressed as

$$C_D = 2(s/c)(P_{0m}/\rho C_1^2)(\cos^3\alpha_m/\cos^2\alpha_2)$$

and the lift coefficient

$$C_L = 2(s/c)\cos\alpha_m(\tan\alpha_1 + \tan\alpha_2) + C_D\tan\alpha_m$$

where $\alpha_m = \tan^{-1}[(\tan\alpha_2 - \tan\alpha_1)/2]$

Pressure losses can then be determined and an estimation of the efficiency be made. The drag coefficient must again be modified due to the blades actually being in annular form. Real boundaries exist at the hub and tip while the ideal flow pattern is disturbed by the preceding and succeeding blades. The drag coefficient is modified by tip clearance loss C_{DC} and secondary flow loss given by

$$C_{DC} = nC_L^2(kc/ls)$$

and

$$C_{DS} = C_L^2\lambda c/s$$

where k is the clearance between the casing and blade tip while n is taken as 0.25 for tip-shrouded blades and 0.5 for unshrouded blades. The parameter λ is estimated from a functional relationship of the form

$$\lambda = f\left[\left(\frac{\text{Blade outlet area normal to flow}}{\text{Blades inlet area normal to flow}}\right)^2 \bigg/ \left(1 + \frac{\text{Hub radius}}{\text{Tip radius}}\right)\right]$$

Hence for a turbine,

$$\boxed{C_{DT} = C_D + C_{DC} + C_{DS}}$$

where C_D is obtained from cascade data. Thus the linear cascade data may be effectively used to determine the lift and drag coefficients for the cascade and then be modified by the addition of annulus drag and secondary losses in order to approximate the drag coefficient for an annular cascade.

Losses in a Cascade

The following losses occur in a cascade.

1. Profile loss This occurs due to the boundary layer growth on the blade surface. This loss increases when the boundary layer separates from the blade surface and is governed by blade profile for given flow conditions.

2. Annulus loss This occurs due to the boundary layer growth on the floors and ceilings of the blade passages.

3. Secondary loss This occurs at the hub and tip due to the three dimensional nature of the flow and the blade curvature.

4. Tip clearance loss Loss due to tip clearance is caused due to the leakage of the flow from the pressure side to the suction side of the blades through the tip clearance. This loss is sometimes considered as a secondary loss.

Losses in a blade cascade of compressor and turbine are principally the same. The magnitude and mechanism of these losses differ from compressor to turbine.

SOLVED PROBLEMS

Example 2.1 An aerofoil having a chord length of 2.25 m and a span of 13.5 m moves at a velocity of 125 m/s through standard atmosphere at an elevation of 2500 m. The angle of attack being 5°25′. Calculate the weight which the wing carries and the power required to drive the aerofoil. Take corresponding to $i = 5°25'$, $C_L = 0.465$ and $C_D = 0.022$. Density of air = 1.25 kg/m³.

Solution

$C = 2.25$ m $l = 13.5$ m
$c = 125$ m/s $C_L = 0.465$
$C_D = 0.022$ $\rho = 1.25$ kg/m³

weight carried by the aerofoil should be equal to the lift force.

$$\therefore W = L = C_L\cdot\rho\cdot\frac{c^2}{2}\cdot A$$

$$= 0.465 \times 1.25 \times \frac{125^2}{2} \times (2.25 \times 13.5)$$

$$= 137.93 \text{ kN}$$

Drag force

$$D = C_D\cdot\rho\cdot\frac{c^2}{2}\cdot A$$

$$= 0.022 \times 1.25 \times \frac{125^2}{2} \times (2.25 \times 13.5)$$

$$= 6525.88 \text{ N}$$

Power required $= D \times c$

$$= 6525.88 \times 125$$

$$= 815.735 \text{ kW}$$

Example 2.2 Calculate the diameter of a parachute to be used for dropping an object weighing 980 N so that the maximum terminal velocity of dropping is 5 m/s. The drag coefficient for the parachute, which may be treated as hemispherical, is 1.3. The density of air is 1.22 kg/m³

Solution

$W = 980\text{ N} \quad C = 5\text{ m/s} \quad C_D = 1.3 \quad \rho = 1.22$

Drag force

$$D = C_D \cdot \rho \cdot \frac{C^2}{2} \cdot A$$

$$\text{but } D = W = 980\text{ N}$$

$$\therefore A = 980 \Big/ \left(1.3 \times 1.22 \times \frac{5^2}{2}\right)$$

$$= 49.43\text{ m}^2$$

Projected area of the hemispherical parachute

$$A = \frac{\pi}{4} D^2$$

$$\therefore D = \sqrt{\frac{49.43 \times 4}{\pi}} = 7.93\text{ m}$$

Example 2.3 A wing of a small airplane is rectangular in plan (10 m × 1.2 m). The total aerodynamic force acting on the wing, moving at 240 km/h, is 20 kN. If the lift-drag ratio is 10, calculate the coefficient of lift and the total weight the plane can carry. Take ρ of air $= 1.2\text{ kg/m}^3$.

Solution

$A = 10 \times 1.2 = 12\text{ m}^2 \quad C = \frac{240 \times 10^3}{3600} = 66.67\text{ m/s} \quad F = 20\text{ kN} \quad L/D = 10$

The total force,

$$F = \sqrt{D^2 + L^2}$$

$$= \sqrt{(0.1L)^2 + L^2}$$

$$F^2 = 1.01\, L^2$$

The weight that the plane can carry is the lift force.

$$\therefore L = \frac{20 \times 10^3}{(1.01)^{1/2}} = 19.9\text{ kN}$$

and

$$C_L = \frac{L}{\rho A C^2/2} = \frac{19.9 \times 10^3}{1.2 \times 12 \times \frac{66.67^2}{2}}$$

$$= 0.622$$

Example 2.4 An axial flow compressor has the following design data: mass flow rate of air – 25 kg/s, density – 1.1 kg/m³, axial velocity – 157 m/s, rotational speed – 150 rev/s. Mean blade speed – 200 m/s, rotor blade aspect ratio – 3, pitch chord ratio – 0.8.

Determine (a) the mean radius, (b) the blade height, (c) the pitch and chord and (d) the number of blades.

Solution

$m = 25\text{ kg/s} \quad \rho = 1.1\text{ kg/m}^3 \quad C_a = 157\text{ m/s}$

$N = 150\text{ rev/s} \quad U = 200\text{ m/s} \quad l/c = 3 \quad s/c = 0.8$

(a) Mean radius

$$r_m = \frac{U}{2\pi N}$$

$$= \frac{200}{2\pi \times 150}$$

$$= 0.212\text{ m}$$

(b) Blade height

The blade height is found from the annulus area of flow.

$$A = \frac{m}{\rho C_a}$$

$$= \frac{25}{1.1 \times 157}$$

$$= 0.145\text{ m}^2$$

∴ Blade height

$$l = \frac{A}{2\pi r_m}$$

$$= \frac{0.145}{2\pi \times 0.212}$$

$$= 0.11\text{ m}$$

(c) The chord and pitch

$$\text{Blade aspect ratio} = \frac{\text{span } (l)}{\text{chord } (C)}$$

$$\therefore C = \frac{l}{3}$$

$$= \frac{0.11}{3}$$

$$= 0.037\text{ m}$$

Blade pitch

$$S = \text{Pitch chord ratio} \times \text{chord}$$

$$= 0.8 \times 0.037$$

$$= 0.0296\text{ m}$$

(d) Number of blades

$$= \frac{\text{Circumference at mean radius}}{\text{Pitch at mean radius}}$$
$$= \frac{2\pi r_m}{S}$$
$$= \frac{2\pi \times 0.212}{0.0296}$$
$$= 45$$

Example 2.5 Determine for a compressor blade with a circular arc camber line and the following data. Pitch-chord ratio –0.8, relative air angle at inlet –45°, relative air angle at outlet – 15°. Assume zero incidence.
(a) nominal deflection angle
(b) the blade camber angle
(c) the deviation angle
(d) the blade stagger

Solution

$s/c = 0.8 \quad \beta_1 = 45° \quad \beta_2 = 15°$

Note that cascade air angle is equal to the compressor relative air angle. That is

$$\alpha_1 = \beta_1 \text{ and } \alpha_2 = \beta_2$$

(a) Nominal deflection angle

$$\epsilon_n = \alpha_1 - \alpha_2 = \beta_1 - \beta_2$$
$$= 45 - 15$$
$$= 30°$$

(b) Deviation angle

$$\delta = m\theta(s/c)^{1/2}$$
$$m = 0.23(2a/c)^2 + 0.1(\alpha_2/50)$$

For a circular arc camber $2a/c = 1$

$$\therefore m = 0.23(1) + 0.1 \times \left(\frac{15}{50}\right)$$
$$= 0.26$$
$$\delta = 0.26 \times \theta(0.8)^{1/2}$$
$$\delta = 0.233\theta$$

(c) Blade camber angle

$$\theta = \alpha_1' - \alpha_2'$$
$$\text{but } \alpha_2' = \alpha_2 - \delta$$
$$\therefore \theta = \alpha_1' - \alpha_2 + \delta$$
$$= \alpha_1' - \alpha_2 + 0.233\theta$$

Since there is no incidence on the blade $\alpha_1' = \alpha_1$

$$\theta = \alpha_1 - \alpha_2 + 0.233\theta$$
$$= 45 - 15 + 0.233\theta$$
$$\theta = 39.11°$$

Alternatively

$$\epsilon = \theta + i - \delta$$

Since $i = 0$

$$\epsilon = \theta - \delta = \theta - 0.233\theta$$
$$\theta = \frac{\epsilon}{1 - 0.233}$$
$$= \frac{30}{0.767}$$
$$\theta = 39.11°$$

Therefore, derivation angle is

$$\delta = 0.233\,(39.11)$$
$$\delta = 9.11°$$

(d) The blade stagger

For a circular arc cascade the blade stagger is given by

$$\phi_s = \alpha_1' - \frac{\theta}{2}$$
$$= \alpha_1 - \frac{\theta}{2}$$
$$= 45 - \frac{39.11}{2}$$
$$= 25.45°$$

Example 2.6 A compressor cascade is constructed from circular arc aero-foil blades (camber angle = 25°) set at a stagger angle of 30° with a pitch-chord ratio of 1.0. The momentum thickness chord ratio is 0.031. The nominal value of incidence is 5°. Determine the cascade blade angles and the nominal air angles. (*MKU–April'95*)

Solution

$\theta = 25° \quad \phi_s = 30° \quad s/c = 1 \quad i_n = 5°$

(a) Cascade blade angles

$$\theta = \alpha_1' - \alpha_2' = 25°$$
$$\phi_s = \alpha_1' - \frac{\theta}{2} = 30°$$

$$\alpha_1' = 30 + \frac{25}{2}$$
$$\alpha_1' = 42.5°$$

and

$$\alpha_2' = \alpha_1' - \theta$$
$$= 42.5 - 25$$
$$\alpha_2' = 17.5°$$

(b) Nominal air angles

$$i_n = \alpha_{1,n} - \alpha_1'$$
$$\therefore \alpha_{1,n} = i_n + \alpha_1'$$
$$= 5 + 42.5$$
$$= 47.5°$$

Nominal exit air angle is determined from the following empirical relation:

$$\tan\alpha_{1,n} - \tan\alpha_{2,n} = \frac{1.55}{1.0 + 1.5\left(\frac{s}{c}\right)}$$

$$\tan\alpha_{2,n} = \tan\alpha_{1,n} - \frac{1.55}{1.0 + 1.5\left(\frac{s}{c}\right)}$$
$$= \tan 47.5 - \frac{1.55}{1.0 + 1.5(1)}$$
$$\tan\alpha_{2,n} = 0.471$$
$$\alpha_{2,n} = 25.22°$$

Example 2.7 A compressor cascade has the following data: velocity of air at entry = 75 m/s, air angle at entry = 48°, air angle at exit = 25°, chord-pitch ratio = 0.91, stagnation pressure loss = 11 mm W.G, density of air = 1.25 kg/m³. Determine loss coefficient, drag and lift coefficients.

Solution

$C_1 = 75$ m/s $\quad \alpha_1 = 48° \quad \alpha_2 = 25°$
$c/s = 0.91 \quad P_{0,m} = 11$ mm W.G. $\quad \rho = 1.25$ kg/m³

(a) Pressure loss coefficient

$$= \frac{P_{0,m}}{0.5\rho C_1^2}$$

$$P_{0,m} = 10^3 \times 9.81 \times \frac{11}{10^3}$$

$$= 107.9 \text{ N/m}^2$$
$$= \frac{107.9}{0.5 \times 1.25 \times 75^2}$$
$$= 0.0307$$

(b) Drag coefficient

$$C_D = 2(s/c)(P_{0m}/\rho C_1^2)(\cos^3\alpha_m/\cos^2\alpha_1)$$
$$\alpha_m = \tan^{-1}\left[(\tan\alpha_1 + \tan\alpha_2)/2\right]$$
$$= \tan^{-1}\left[(\tan 48° + \tan 25°)/2\right]$$
$$\alpha_m = 38.25°$$
$$\therefore C_D = 2(1/0.91)\left(\frac{107.9}{1.25 \times 75^2}\right)\left(\frac{\cos^3 38.25}{\cos^2 48}\right)$$
$$= 0.0365$$

(c) Lift coefficient

$$C_L = 2(s/c)\cos\alpha_m(\tan\alpha_1 - \tan\alpha_2) - C_D\tan\alpha_m$$
$$= 2(1/0.91)\cos 38.25°(\tan 48° - \tan 25°) - 0.0365\tan 38.25°$$
$$= 1.083$$

Example 2.8 Air enters the test section of a turbine blade ($\alpha_1 = 40°, \alpha_2 = 65°$) cascade tunnel at 100 m/s ($\rho = 1.25$ kg/m³). The pitch-chord ratio of the cascade is 0.91. The average loss in the stagnation pressure across the cascade is equivalent to 17.5 mm W.G. Determine for this cascade (a) the pressure loss coefficient, (b) the drag coefficient and (c) the lift coefficient.

Solution

$\alpha_1 = 40° \quad \alpha_2 = 65° \quad C_1 = 100$ m/s $\quad \rho = 1.25$ kg/m³
$s/c = 0.91 \quad P_{0,m} = 17.5$ mm W.G.

(a) Pressure loss coefficient

$$= \frac{P_{0,m}}{0.5\rho C_1^2}$$
$$= \frac{\frac{17.5}{10^3} \times 9.81 \times 10^3}{0.5 \times 1.25 \times 100^2}$$
$$= 0.0275$$

(b) Drag coefficient

$$C_D = 2(s/c)(P_{0,m}/\rho C_1^2)(\cos^3\alpha_m/\cos^2\alpha_2)$$
$$\alpha_m = \tan^{-1}\left[(\tan\alpha_2 - \tan\alpha_1)/2\right]$$

$$= \tan^{-1}\left[(\tan 65^\circ - \tan 40^\circ)/2\right]$$

$$= 33.13^\circ$$

$$\therefore C_D = 2(0.91)\left(\frac{\frac{17.5}{10^3} \times 9.81 \times 10^3}{1.25 \times 100^2}\right)\left(\frac{\cos^3 33.13}{\cos^2 65}\right)$$

$$\simeq 0.0823$$

(c) Lift coefficient turbin

$$C_L = 2(s/c)\cos\alpha_m(\tan\alpha_1 + \tan\alpha_2) + C_D \tan\alpha_m$$

$$= 2(0.91)\cos 33.13(\tan 40 + \tan 65) + 0.0823 \tan 33.13$$

$$C_L = 4.601$$

Example 2.9 A jet plane which weighs 30,000 N and has a wing area of 20 m^2 flies at a velocity of 250 km/h when the engine delivers 750 kW. 65% of the power is used to overcome the drag resistance of the wing. Calculate the coefficients of lift and drag for the wing. Take density of air equal to 1.21 kg/m^3.

Solution

$W = 30,000$ N $A = 20$ m^2 $c = 250$ km/h $= 69.44$ m/s

Power required to overcome drag resistance

$$= 0.65 \times 750 = 487.5 \text{ kW}$$

Power required to overcome drag resistance in terms of drag force is given by

$$= D \times c$$

$$\therefore D = \frac{487.5}{69.44} \times 10^3 = 7020.5 \text{ N}$$

But

$$C_D = \frac{D}{0.5\rho c^2 A}$$

$$= \frac{7020.5}{0.5 \times 1.21 \times 69.44^2 \times 20}$$

$$C_D = 0.120$$

The lift force should be equal to the weight of the plane.

$$L = W = 30,000 \text{ N}$$

$$\therefore C_L = \frac{L}{0.5\rho c^2 A}$$

$$= \frac{30,000}{0.5 \times 1.21 \times 69.44^2 \times 20}$$

$$C_L = 0.514$$

SHORT QUESTIONS

2.1. What is the function of blades in a turbomachine?
2.2. Define an aero-foil section.
2.3. Classify the aero-foil sections.
2.4. What is a symmetrical aero-foil?
2.5. What is a non-symmetrical aero-foil?
2.6. Define the terms lift and drag.
2.7. Describe the flow pattern around an aero-foil.
2.8. The drag is made up of ________ drag and ________ drag.
2.9. Flow separation due to adverse pressure gradient
   (a) decreases lift and increases drag
   (b) increases lift and decreases drag
   (c) decreases lift and decreases drag
2.10. The lift is ________ to the drag.
2.11. Define (a) lift coefficient and (b) drag coefficient.
2.12. Lift coefficient should be as high as possible for maximum energy transfer. (True/False)
2.13. For maximum efficiency, the drag coefficient should as low as possible. (True/False)
2.14. Define blade loading factor and flow coefficient.
2.15. For a well-designed blade, the optimum blade loading factor is
   (a) $\phi(c/s)C_l$
   (b) $\phi(c/s)C_l/\sqrt{2}$
   (c) $\phi(c/s)C_l/2$
2.16. List the important blade terminologies.
2.17. Define (a) base profile, (b) camber line.
2.18. What is a cascade?
2.19. What is blade camber angle?
2.20. What are chord, span and pitch for a blade cascade?
2.21. Differentiate between angle of incidence and deviation angle.
2.22. Air deflection angle is the difference between the ________ and the ________.
2.23. What is stagger angle?
2.24. ________ angle represents the angle at which the blade is set in the cascade.
2.25. What is cascade testing?
2.26. Why do the results obtained from cascade testing need corrections?
2.27. What is pressure loss coefficient?
2.28. The value of nominal deflection angle in terms of maximum deflection angle is
   (a) 0.88 ϵ_{max}
   (b) 0.9 ϵ_{max}
   (c) 0.8 ϵ_{max}
2.29. What are the cascade losses?
2.30. What are profile, annulus and tip clearance losses?

EXERCISES

2.1. Derive the expression for energy transfer in terms of blade lift and drag coefficients.

2.2. Explain the blade terminologies with a neat sketch.

2.3. Explain the various cascade nomenclatures with a neat and illustrative sketch prove that $\epsilon = \theta + i - \delta$.

2.4. Write notes on cascade testing. Draw the cascade curves. How is the nominal value of deflection obtained?

2.5. Enumerate and explain briefly the different cascade losses.

2.6. A 16 m diameter rotor is required to lift and propel a 2500 kg helicopter at a speed of 15 m/s. Calculate power required by the helicopter assuming a drag coefficient of 0.0056 based on the rotor area. Ambient condition is 1 bar and 22°C.

(MKU-April '96 & MU-April '99)

[Ans: 2.24 kW]

3

CENTRIFUGAL COMPRESSORS AND FANS

INTRODUCTION

Compressors as well as pumps and fans are the devices used to increase the pressure of a fluid. But, they differ in the tasks they perform. A fan increases the pressure of a gas slightly and it is mainly used to move a gas around. A compressor is capable of compressing the gas to very high pressures. Pumps work very much like compressors except that they handle liquids instead of gases.

Centrifugal compressors and fans are turbo machines employing centrifugal effects to increase the pressure of the fluid. Single stage centrifugal compressors have the pressure ratio of 4:1. The best efficiencies are generally 3 to 4 per cent below those obtained from an axial flow compressor designed for the same duty. However, at very low mass flow rates, the axial flow compressor efficiency drops down rapidly.

The advantages of centrifugal compressor over the axial flow compressor are (1) smaller length, (2) wide range of mass flow rate of gas. Although the centrifugal compressor has been superseded by the axial flow compressor in jet aircraft engines, it is useful where a short overall engine length is required and where it is likely that deposits will be formed in the air passages. Because of the relatively short passage length, loss of performance due to build-up deposits will not be as great as the axial compressors. Therefore, the working fluid may even be a contaminated gas, like exhaust gas. The disadvantages are—larger frontal area and lower maximum efficiency. If the density ratio across the compressor is less than about 1.05, the term *'fan'* is used to describe the machine. In that case the fluid is treated as being incompressible; otherwise compressible flow equations must be used. The term *'blower'* is often used in place of 'fan'. In this chapter, the centrifugal compressor and fan are considered together as the theory applied to both machines is the same. The centrifugal compressor is mainly found in turbo chargers.

COMPONENTS AND DESCRIPTION

Fig. 3.1 shows a typical centrifugal compressor.

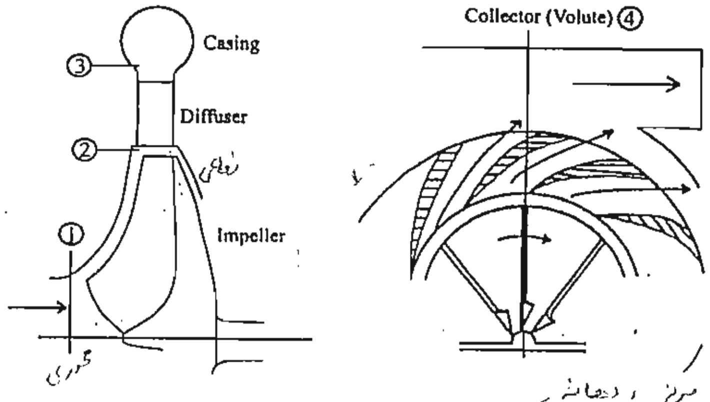


Figure 3.1 Typical centrifugal compressor

The principal components are the *impeller* and the *diffuser*. When the impeller is rotating at high speed, air is drawn in through the eye of the impeller. The absolute velocity of the inflow air is axial. The magnitude and direction of the entering relative velocity depends upon the linear velocity of the impeller at the radial position of the eye as well as the magnitude and direction of the entering absolute velocity. The impeller vanes at the eye are bent so as to provide shockless entry for the entering flow at its relative entry angle. The air then flows radially through the impeller passages due to centrifugal force. The total mechanical energy driving the compressor is transmitted to the fluid stream in the impeller where it is converted into kinetic energy, pressure and heat due to friction. The function of the diffuser is to convert the kinetic energy, of air that leaving the impeller, into pressure. The air leaving the diffuser is collected in a spiral passage (scroll or volute) from which it is discharged from the compressor. The pressure and velocity variation across the compressor is shown in Fig. 3.2.

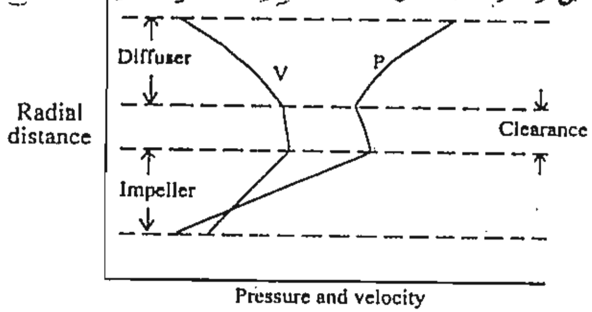
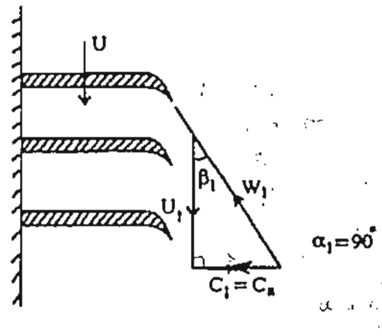


Figure 3.2 Pressure and velocity diagram for centrifugal compressor

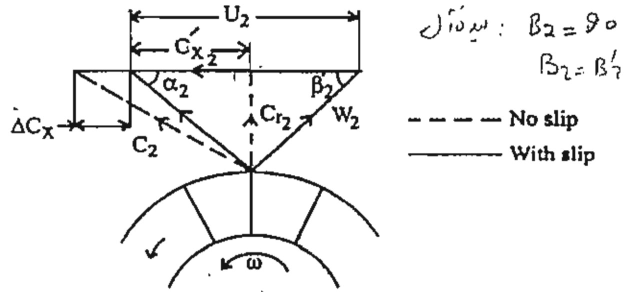
VELOCITY DIAGRAMS FOR A CENTRIFUGAL COMPRESSOR

The gas enters the compressor at the eye, in an axial direction with an absolute velocity C_1 and moves into the inducer section, which can be in a separate form or be a part of the blades. The inducer section transfers the gas onto the blades and enables it to move smoothly into the radial direction. Energy is imparted to the gas by the rotating blades, thereby increasing its static pressure; as it moves from radius r_1 to r_2, and the gas moves the blade with absolute velocity C_2.

It should be noted that the blades are radial i.e. the blade angle β_2 is 90° while the relative velocity vector W_2 is at angle β_2' because of slip. Ideally, the component C_{x_2} equals U_2. But it is reduced due to slip. The relative velocity vector W_1 is obtained by subtracting U_1 from C_1. The fluid enters the blade passages with an absolute velocity C_1; here $C_1 = C_a$. So, the impeller tangential velocity vector U_1 is at right angle to C_1. Where $U_1 = \omega r_1$, 'ω' being the angular velocity of the impeller. The resultant relative velocity vector at the inlet is W_1 as shown in Fig. 3.3(a)



(a) Inlet velocity triangle



(b) Outlet velocity triangle

Figure 3.3 Velocity triangles for a radial impeller

(refer Fig. 3.3(b) and Fig. 3.4). For zero slip situation, $\beta_2 = 90°$ and so $C_{x_2} = U_2$ and $C_{r_2} = W_2$ where C_r is the radial component of the absolute velocity and is perpendicular to the tangent at inlet and outlet. C_{x_1} is the component of the inlet absolute velocity vector resolved into the tangential direction. W_x and C_x are often called as the relative and absolute whirl components, respectively. All angles are measured from the tangential direction.

When $\beta_1 = \beta_1'$, this is referred to as the '*no shock condition*' at entry. In this case, the fluid moves tangentially onto the blade.

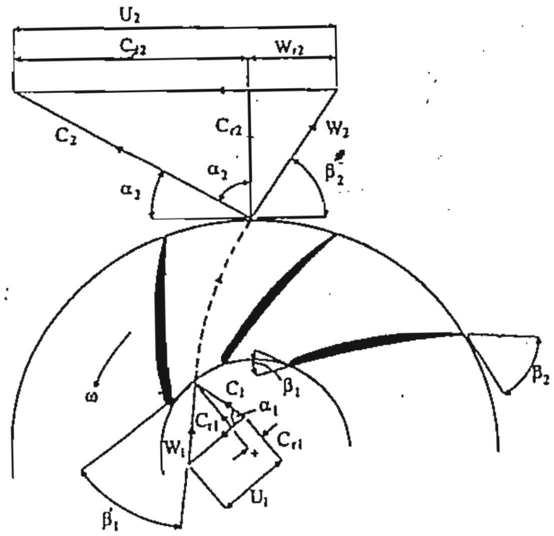


Figure 3.4 Velocity triangles for a backward curved impeller

When $\beta_2 = \beta_2'$, there is no fluid slip at the exit where β' refers to the angle of relative velocity vector.

SLIP FACTOR

The fluid leaves the impeller at an angle β_2' other than the actual blade angle β_2. This is due to '*fluid slip*'. Angle β_2' is less than angle β_2.

In centrifugal compressors, the air trapped between the impeller vanes is reluctant to move round with the impeller, and this results in a higher static pressure on the leading face of the vane than on the trailing face of the vane. This problem is due to the inertia of the air. Then the air tends to flow round the edges of the vanes in the clearance space between impeller and casing. One explanation for this is that of the relative eddy hypothesis. Fig. 3.5 shows the pressure distribution built up in the impeller passages due to the motion of the blades. On the leading side of the blade there is a high pressure region while on the trailing side of the blade there is a low pressure region; the pressure thus changes across the blade passage. This pressure distribution is similar to that of an aerofoil in a free stream and is like-wise associated with the existence of circulation around the blade, so that on the low pressure side the fluid velocity is increased while on the high pressure side it is decreased, and a non-uniform velocity distribution results at any radius. Indeed, the flow may separate from the suction surface of the blade. So, the mean direction of the flow leaving the impeller is β_2' and not β_2 as is assumed in the zero-slip condition.

Slip can be reduced by increasing the number of impeller vanes and reducing the clearance space. Thus C_{x_2} is reduced to C_{x_2}' and the difference ΔC_x is defined as the slip.

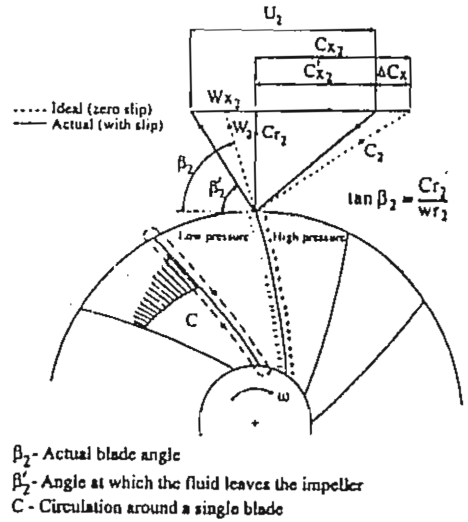


Figure 3.5 Slip and velocity distribution in centrifugal pump impeller blades

Slip factor is defined as

$$\boxed{\sigma_s = \frac{C_{x2}'}{C_{x2}}}$$

Referring to the above figure, for the no-slip condition,

$$C_{x2} = U_2 - W_{x2}$$

and

$$W_{x2} = \cot\beta_2 \cdot C_{r2}$$
$$C_{x2} = U_2 - C_{r2}\cot\beta_2$$

Slip factor

$$\sigma_s = \frac{C_{x2}'}{C_{x2}} = \frac{(C_{x2} - \Delta C_x)}{C_{x2}}$$

$$\boxed{\sigma_s = 1 - \frac{\Delta C_x}{C_{x2}}} \qquad (3.1)$$

Stodola proposed the existence of a relative eddy within the blade passages as shown in Fig. (3.6). By definition, a frictionless fluid which passes through the blade passages have no rotation. Therefore at the outlet of the passage the rotation should be

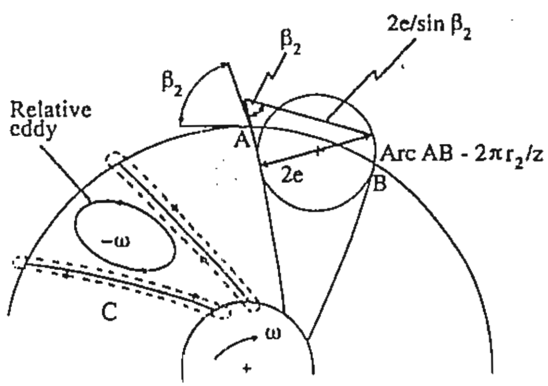


Figure 3.6 The relative eddy between impeller blades

zero. Now, the impeller has an angular velocity 'ω', so that, relative to the impeller, the fluid must have an angular velocity '$-\omega$' to match with the zero-rotation condition. If the radius of a circle that may be inscribed between two successive blades at outlet and at a tangent to the surfaces of both blades is 'e', then the slip is given by

$$\Delta C_x = \omega e$$

The impeller circumference is $2\pi r_2$ and therefore the distance between the blades is $2\pi r_2/z$ if we have 'z' blades of negligible thickness. This may be approximated to $2e/sin\beta_2$ and upon rearrangement

$$\begin{aligned} e &= (\pi r_2/z)\sin\beta_2 \\ \Delta C_x &= (U_2/zr_2)(\pi r_2 \sin\beta_2) \\ &= (U_2\pi \sin\beta_2)/z \end{aligned}$$

equation (3.1) becomes

$$\sigma_s = 1 - \frac{U_2\pi \sin\beta_2}{z(U_2 - Cr_2\cot\beta_2)}$$

For purely radial blades, which are often found in a centrifugal compressor, β_2 will be 90° and the Stodola slip factor becomes

$$\boxed{\sigma_s = 1 - \pi/z(\text{since } \cot 90° = 0)}$$

With radial vanes, a very high pressure rise can be obtained, and are suitable for high-speed machines.

The Stodola slip factor equation gives best results for the blade angle in the range $20° < \beta_2 < 30°$. For the range $30° < \beta_2 < 80°$, **Buseman slip factor equation** may be employed.

$$\sigma_s = \frac{[A - B(C_{r2}/U_2)\cot\beta_2]}{[1 - (C_{r2}/U_2)\cot\beta_2]} \qquad (3.2)$$

where A and B are functions of β_2, z and r_2/r_1.

The ***Stanitz slip factor*** is given by

$$\boxed{\sigma_s = 1 - 0.63\pi/\{z[1 - (C_{r2}/U_2)\cot\beta_2]\}}$$

is best used in the range $80° < \beta_2 < 90°$.

If $\beta_2 = 90°$, then $\sigma_s = 1 - (0.63\pi/z)$. Typically, slip factors lie in the region of 0.9, while slip occurs even if the fluid is ideal.

ENERGY TRANSFER

By Euler's pump equation, without slip

$$\boxed{E = W/mg}$$
$$E = (U_2C_{x2} - U_1C_{x1})/g$$

From inlet velocity triangle (Fig. 3.3(a))

$$C_{x1} = 0$$

For ideal condition, $U_2 = C_{x_2}$, from outlet velocity triangle (Fig. 3.3(b))

$$E = \frac{U_2C_{x2}}{g} = \frac{U_2^2}{g} \qquad (3.3)$$

and with slip, the theoretical work is

$$\boxed{E = \frac{\sigma_s U_2^2}{g}} \qquad (3.4)$$

Although equation (3.3) has been modified by the slip factor to give equation (3.4), $\sigma_s U_2^2/g$ is still the **'theoretical work'** done on the air, since slip will be present even if the fluid is friction-less (ideal fluid).

In a real fluid some of the power supplied by the impeller is used in overcoming losses that have a braking effect on the air conveyed by the vanes, and these include windage, disc friction and casing friction. The total power per unit weight of flow is therefore modified by a power input factor.

((POWER INPUT FACTOR))

The power input factor (or) the work factor

$$\boxed{\psi = \frac{\text{Actual work supplied}}{\text{Theoretical work supplied}}}$$

'ψ' typically takes values from 1.035 to 1.041.

So, the actual energy transfer becomes

$$E = \psi\sigma_s U_2^2/g$$

'ψ' is also known as *'stage loading coefficient'*. Upon leaving the impeller the gas enters a vaneless space where it moves in a spiral path before entering the diffuser, in which the static pressure is further increased. The clearance between the impeller blades and inner walls of the casing must be kept as small as possible to reduce leakage and in some cases the blades themselves are shrouded.

MOLLIER CHART

Since we are dealing with a gas and since the rise in temperature and pressure causes the density to change, it will be convenient to examine the performance of the machine in terms of the thermodynamic properties of the gas and this is done through the Mollier Chart. The $h-s$ diagram for the compression process across the centrifugal compressor is shown in Fig. 3.7.

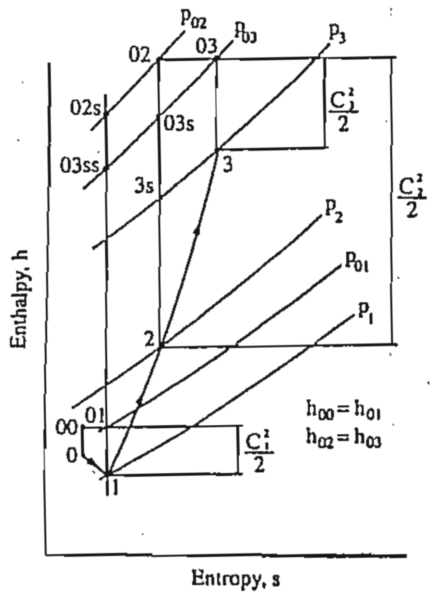


Figure 3.7 Mollier chart for a centrifugal compressor

1. Inlet Casing

The energy equation along a streamline may be written as

Total enthalpy, $h_0 = h + \frac{C^2}{2}$ = constant

Therefore, for the fluid drawn from the atmosphere into the inducer section, the total enthalpy is

$$h_{00} = h_0 + \frac{C_0^2}{2}$$

Total enthalpy at section-1, i.e. inlet of the impeller, is

$$h_{01} = h_1 + \frac{C_1^2}{2}$$

and since no shaft work has been done and assuming that adiabatic steady flow occurs

$$h_{00} = h_{01} \text{ [from S.F.E.E]}$$

Thus,

$$\boxed{h_0 + \frac{C_0^2}{2} = h_1 + \frac{C_1^2}{2}} \quad (3.5)$$

2. Impeller

Work is done on the fluid across the impeller and the static pressure is increased from P_1 to P_2. Writing the work done per unit mass on the fluid in terms of enthalpy, we get

$$w/m = h_{02} - h_{01}$$

From Euler's pump equation

$$w/m = U_2 C_{x2} - U_1 C_{x1}$$

Equating the two equations and after substituting for h_0

$$I = h_1 + \frac{C_1^2}{2} - U_1 C_{x_1} = h_2 + \frac{C_2^2}{2} - U_2 C_{x2}$$

where 'I' is the impeller constant.
In general

$$\begin{aligned}
I &= h + C^2/2 - UC_x \\
&= h + (C_r^2 + C_x^2/2) - UC_x \\
&= h + (W^2 - W_x^2 + C_x^2)/2 - UC_x \\
&= h + [W^2 - (U - C_x)^2 + C_x^2]/2 - UC_x \\
&= h + \frac{W^2}{2} - \frac{U^2}{2} - \frac{C_x^2}{2} + UC_x - UC_x + \frac{C_x^2}{2} \\
&= h + W^2/2 - U^2/2
\end{aligned}$$

or

$$\boxed{I = h_0.rel - U^2/2}$$

where h_0.rel is the total enthalpy based on the relative velocity of the fluid. Thus

$$h_2 - h_1 = ((U_2^2 - U_1^2)/2) + ((W_1^2 - W_2^2)/2) \quad (3.6)$$

Since $I_1 = I_2$ in equation (3.6), the chief contribution to the static enthalpy rise is from the term $(U_2^2 - U_1^2)/2$.

Usually, $C_{x_1} = 0$ is assumed in preliminary design calculations. Although, this is not always the case, from the actual energy transfer equation, the work done on the fluid per unit mass becomes

$$(w/m) = E \times g$$
$$h_{02} - h_{01} = \psi \sigma_s U_2^2$$

Substituting $h_0 = C_P T_0$ and rearranging the eqn., we get

$$T_{02} - T_{01} = \psi \sigma_s U_2^2 / C_P$$

where, C_P is the mean specific heat over this temperature range.

Since, no work is done in the diffuser, $h_{02} = h_{03}$ and so

$$T_{03} - T_{01} = \psi \sigma_s U_2^2 / C_P \qquad (3.7)$$

With reference to the h–s diagram and equation (3.7), a compressor's overall total-to-total isentropic efficiency 'η_c' is defined as

$$\eta_c = \frac{\text{Total isentropic enthalpy rise between inlet and outlet}}{\text{Actual enthalpy rise between same total pressure limits}}$$

$$\frac{h_{03_{ss}} - h_{01}}{h_{03} - h_{01}}$$

where the subscript 'ss' represents the end state on the total pressure line P_{03} when the process is isentropic.

$$\eta_c = \frac{(T_{03_{ss}} - T_{01})}{(T_{03} - T_{01})}$$
$$= T_{01} \frac{((T_{03_{ss}})/T_{01} - 1)}{(T_{03} - T_{01})}$$

But,

$$P_{03}/P_{01} = (T_{03_{ss}}/T_{01})^{r/r-1}$$
$$= [1 + \eta_c (T_{03} - T_{01})/T_{01}]^{r/(r-1)}$$
$$= [1 + \eta_c \psi \sigma_s U_2^2 / (C_P T_{01})]^{r/r-1}$$

The slip factor should be as high as possible, since it limits the energy transfer to the fluid even under isentropic conditions, and it is seen from the velocity diagrams that C_{x_2} approaches U_2 as the slip factor is increased. The slip factor may be increased by increasing the number of vanes but this increases the *'solidity'* at the impeller eye, resulting in decrease in the flow area at the inlet. To have a same mass flow rate, the flow velocity C_a at inlet must therefore be increased and this increases the loss due to friction. A compromise is usually made; slip factors of about 0.9 are being used for a compressor with 19-21 vanes.

It may seem that increase in 'ψ' increases the energy transfer, but the rate of decrease of isentropic efficiency with increase in ψ negates (nullifies) any apparent advantage. So, the ideal condition is to have a power input factor of unity ($\psi = 1$).

The pressure ratio increases with the impeller tip speed, but material strength should be more as centrifugal stresses are proportional to the square of the tip speed; and for a light alloy impeller, tip speeds are limited to about 460 m/sec. This gives a pressure ratio of 4:1. Pressure ratio of 7:1 is possible with titanium impellers. Equation for pressure ratio can be written in terms of fluid properties and flow angles.

Since $a_{0_1}^2 = rRT_{01}$ and $C_P = rR/(r-1)$, then $P_{03}/P_{01} = [1 + \eta_c \psi \sigma_s (r-1) U_2^2 / a_{01}^2]^{\frac{r}{r-1}}$

The change of pressure ratio with blade tip speed for various 'η_c' is shown in Fig. 3.8.

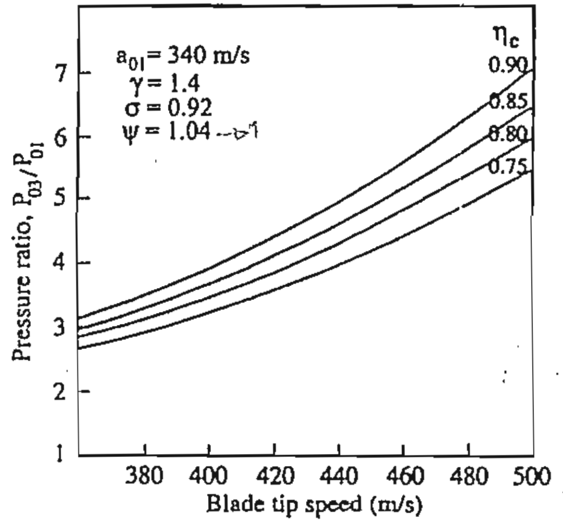


Figure 3.8 Overall pressure ratio versus impeller tip speed

STAGE PRESSURE RISE AND LOADING COEFFICIENT

The static pressure rise in centrifugal stage occurs in the impeller, diffuser and the volute.

No change in stagnation enthalpy occurs in the diffuser and volute. In this section, the pressure rise or pressure ratio across the stage for an isenstropic process is determined.

Work supplied

$$= h_{02_s} - h_{01}$$

$$= C_P(T_{02_s} - T_{01})$$

$$= C_P T_{01}\left(\frac{T_{02_s}}{T_{01}} - 1\right)$$

$$= C_P T_{01}\left[\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$= C_P T_{01}\left[R_0^{\frac{\gamma-1}{\gamma}} - 1\right] \qquad (3.8)$$

where R_0—stagnation pressure ratio

From Euler's equation,

Work supplied

$$= U_2 C_{x_2}$$

$$= U_2(U_2 - C_{r2}\cot\beta_2)$$

$$= U_2^2\left(1 - \frac{C_{r2}}{U_2}\cot\beta_2\right)$$

$$= U_2^2(1 - \phi_2\cot\beta_2) \qquad (3.9)$$

Equating (3.8) & (3.9),

$$C_P T_{01}(R_0^{\frac{\gamma-1}{\gamma}} - 1) = U_2^2(1 - \phi_2\cot\beta_2)$$

where 'ϕ_2' is the flow coefficient at the impeller exit.

$$\phi_2 = \frac{C_{r2}}{U_2}$$

$$R_0 = \left[1 + \frac{(1 - \phi_2\cot\beta_2)U_2^2}{C_P T_{01}}\right]^{\frac{\gamma}{\gamma-1}} \qquad (3.10)$$

The loading or pressure coefficient is defined as

$$\boxed{\psi_P = \frac{\text{Work done/kg}}{U_2^2}}$$

From the outlet velocity triangle [Fig. 3.8(a)],

$$C_{x2} = U_2 - C_{r2}\cot\beta_2$$

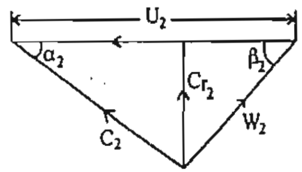


Figure 3.8 (a) Velocity triangle

From Euler's equation

$$\frac{W}{m} = U_2 \times C_{x2} = U_2(U_2 - C_{r_2}\cot\beta_2)$$

$$\frac{W}{m} = U_2^2(1 - \phi_2\cot\beta_2)$$

$$\therefore \psi_P = \frac{U_2^2(1 - \phi_2\cot\beta_2)}{U_2^2}$$

$$\psi_P = (1 - \phi_2\cot\beta_2) \qquad (3.11)$$

Substituting equation (3.11) in equation (3.10), we get

$$R_0 = \frac{P_{02}}{P_{01}} = \left[1 + \frac{\psi_P U_2^2}{C_P T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$$

In terms of static pressures, following the same procedure, we get

$$\boxed{R = \frac{P_2}{P_1} = \left[1 + \frac{\psi_P U_2^2}{C_P T_1}\right]^{\frac{\gamma}{\gamma-1}}}$$

These two ratios are known as stage pressure ratios.

Pressure Coefficient

The pressure or loading coefficient is also defined as the ratio of isentropic work to Euler's work.

$$\boxed{\text{Pressure coefficient}, \psi_P = \frac{\text{Isentropic work}}{\text{Euler's work}}}$$

$$\psi_P = \frac{C_P(T_{02s} - T_{01})}{U_2 C_{x2}}$$

For a radial vaned impeller,

$$C_{x2} = U_2$$

$$\psi_P = \frac{C_P(T_{02s} - T_{01})}{U_2^2}$$

Now, isentropic work = actual work × isentropic efficiency

$$= C_P(T_{02} - T_{01}) \times \eta_c$$

Then,

$$\psi_P = \eta_c \frac{C_P(T_{02} - T_{01})}{U_2^2}$$

But

$$C_P(T_{02} - T_{01}) = \psi \sigma_s U_2^2$$

Thus the pressure coefficient may be written as

$$\psi_P = \eta_c \frac{\psi \sigma_s U_2^2}{U_2^2}$$

$$\boxed{\psi_P = \psi \eta_c \sigma_s}$$

Another definition for ψ_P is

$$\psi_P = \frac{\text{Work done/kg}}{U_2^2/2}$$

$$= \frac{(\Delta h_0)_s}{U_2^2/2}$$

where Δh_0 is the ideal stagnation enthalpy change across the stage.

If the stage pressure rise is so small then the fluid can be treated as incompressible.

$$(\Delta h_0)_s = \Delta P_0/\rho$$

so that

$$\psi_P = \frac{\Delta P_0}{(\rho U_2^2/2)}$$

This definition for ψ_P has a numerical value twice that of the equation

$$\psi_P = (1 - \phi_2 \cot\beta_2)$$

DIFFUSER

It plays an important role in the overall compression process of a centrifugal compressor. The impeller imparts energy to the air by increasing its velocity. The diffuser converts this imparted kinetic energy into pressure rise. For a radial bladed impeller, the diffuser does compress and increase the pressure equal to 50 per cent of the over all static pressure rise.

(a) Volute or Scroll Collector

A simple volute or scroll collector is shown in Fig. 3.9 and consists of a circular passage of increasing cross-sectional area. The feature of the simple volute is its low cost. The cross-sectional area increases as the increment of discharge increases around the periphery of the impeller and it is found that a constant average velocity around the volute results in equal pressures around the compressor casing, and hence no radial thrust on the shaft.

Any deviation in flow rate from the design condition will result in a radial thrust, which ultimately results in shaft bending. Of the available kinetic energy at impeller outlet, 25–30 per cent may be recovered in a simple volute.

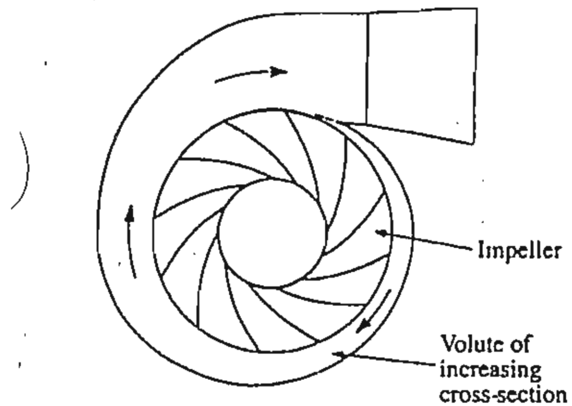


Figure 3.9 *Volute or scroll collector*

(b) Vaneless Diffuser

Here diffusion takes place in a parallel-sided passage and is governed by the principle of conservation of angular momentum of the fluid.

The radial component of absolute velocity is controlled by the radial cross-sectional area of flow 'b'. A vaneless diffuser passage is shown in Fig. 3.10.

Mass flow rate '$\dot{m}$' at any radius r is given by

$$\boxed{\dot{m} = \rho A C_r = \rho(2\pi r b) C_r}$$

where 'b' is the width of the diffuser passage perpendicular to the peripheral area of the impeller and is usually the same as the impeller width. Let the subscripted variables represent conditions at the impeller outlet and the unsubscripted variables represent conditions at any radius 'r' in the vaneless diffuser, then from continuity equation

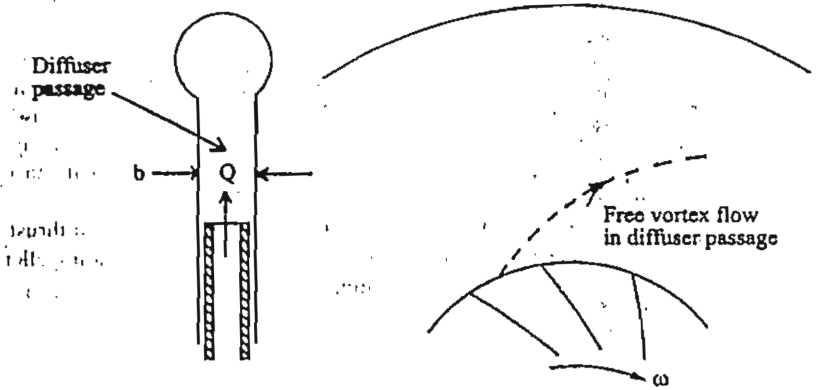


Figure 3.10 Vaneless diffuser

$$\rho r b C_r = \rho_2 r_2 b_2 C_{r_2}$$
$$C_r = \rho_2 r_2 b_2 C_{r_2} / \rho r b$$

If frictionless flow is assumed, then by conservation of angular momentum ($mC_x r$)

$$C_x r = C_{x_2} r_2$$

and $C_x = C_{x2} r_2 / r$

But $C_x \gg C_r$ (usually) and so the absolute velocity 'C' is approximately equal to C_x or

$$C = C_{x2} r_2 / r$$

i.e. $C_x r = C_{x_2} r_2 = \text{constant}$ (or) $C = \dfrac{\text{constant}}{r}$

Our aim is to reduce 'C'. To achieve this, 'r' must be large and therefore, for a large reduction in the outlet kinetic energy, a diffuser with a large radius is required.

A vaneless diffuser has wide range of mass flow rate. But because of long flow path with this type of diffuser, friction effects are important and the efficiency is low.

(c) Vaned Diffuser

In the vaned diffuser as shown in Fig. 3.11, the vanes are used to diffuse the outlet kinetic energy at a much higher rate, in a shorter length and with a higher efficiency (length of flow path and diameter are reduced) than the vaneless diffuser.

A ring of diffuser vanes surrounds the impeller at the outlet, and after leaving the impeller, the air moves in logarithmic spiral motion across a short vaneless space before entering the diffuser vanes. Once the fluid has entered the diffuser passage, the controlling variable on the rate of diffusion is the divergence angle of the diffuser passage, which is in the order of 8 – 10°, and there should be no separation of boundary layer on the passage walls.

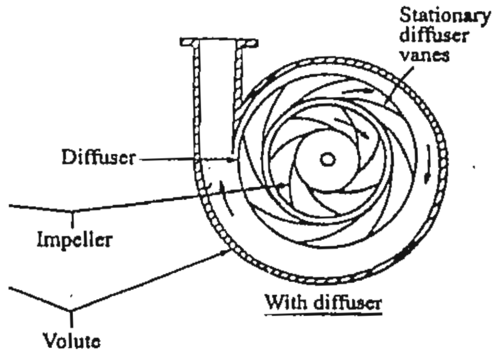


Figure 3.11 Vaned diffuser

The points to be considered to fix up the number of vanes on the diffuser ring are:

1. Diffusion increases with the increase in the vane number. But increasing the vane number increases the friction loss.
2. The number of diffuser vanes has no common factor with the number of impeller vanes. But, when the number of diffuser passages is less than the number of impeller passages, a more uniform total flow occurs.
3. The cross-section of the diffuser channel should be squared to give a maximum hydraulic radius (cross-sectional area/channel perimeter).

Change from the design mass flow rate and pressure ratio will change the smooth flow direction into the diffuser passage and will therefore result in lesser efficiency. This may be rectified by utilising variable angle diffuser vanes. The velocity of air leaving the diffuser should be as small as possible as this eases the problem of combustion chamber. The diffuser outlet velocity is usually designed at about 90 m/sec.

The diffuser efficiency

$$\eta D = \frac{\text{Ideal enthalpy drop}}{\text{Actual enthalpy drop}}$$

From the $h - s$ diagram (Fig. 3.7),

$$\eta_D = \frac{h_{3s} - h_2}{h_3 - h_2}$$
$$= \frac{T_2(T_{3s}/T_2 - 1)}{(T_3 - T_2)}$$
$$= \frac{T_2\left[\left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{(T_3 - T_2)}$$

For adiabatic deceleration of the fluid from absolute velocity C_2 to C_3 with a corresponding increase of static pressure from P_2 to P_3,

$$\begin{aligned} h_{02} &= h_{03} \quad \text{(or)} \\ h_2 + C_2^2/2 &= h_3 + C_3^2/2 \end{aligned}$$

DEGREE OF REACTION

The degree of reaction of a centrifugal compressor stage is given by

$$R = \frac{\text{Change in static enthalpy in the impeller}}{\text{Change in stagnation enthalpy in the stage}}$$

$$= \frac{h_2 - h_1}{h_{02} - h_{01}} \quad [h_{03} - h_{01} = h_{02} - h_{01} \text{ as } h_{02} = h_{03}]$$

If the velocity of the gas approaching the compressor inlet is negligible ($C_1 \simeq 0$), then $h_1 \simeq h_{01}$ and $\quad h_2 = h_{02} - C_2^2/2$

$$\therefore R = \frac{(h_{02} - h_{01}) - C_2^2/2}{(h_{02} - h_{01})}$$

$$\begin{aligned} h_{02} - h_{01} &= U_2 C_{x2} = U_2(U_2 - W_{x2}) \\ &= (U_2^2 - U_2 C_{r2} \cot\beta_2) \\ &= U_2^2 \left(1 - \left(\frac{C_{r_2}}{U_2}\right) \cot\beta_2\right) \qquad (3.12) \end{aligned}$$

$h_{02} - h_{01} = U_2^2(1 - \phi_2 \cot\beta_2)$ where 'ϕ_2' is the flow coefficient.
From the outlet velocity triangle [Refer Fig. 3.8(a)],

$$\begin{aligned} C_2^2 &= C_{x_2}^2 + C_{r_2}^2 \\ \text{and} \quad C_{x_2} &= U_2 - W_{x_2} \\ \therefore \quad C_2^2 &= C_{r_2}^2 + (U_2 - W_{x_2})^2 \\ \text{and} \quad W_{x_2} &= C_{r_2} \cot\beta_2 \\ C_2^2 &= C_{r_2}^2 + (U_2 - C_{r_2}\cot\beta_2)^2 \\ &= C_{r_2}^2 + U_2^2(1 - C_{r_2}/U_2)\cot\beta_2)^2 \\ &= C_{r_2}^2 + U_2^2(1 - \phi_2\cot\beta_2)^2 \\ \text{(or)} \quad C_2^2 &= U_2^2\left[(C_r^2/U_2^2) + (1 - \phi_2\cot\beta_2)^2\right] \\ &= U_2^2\left[\phi_2^2 + (1 - \phi_2\cot\beta_2)^2\right] \qquad (3.13) \end{aligned}$$

From the stage pressure rise expression (eqn. 3.9),

$$h_{02} - h_{01} = U_2^2(1 - \phi_2\cot\beta_2) \qquad (3.14)$$

Substituting equation (3.13) and equation (3.14) in the equation for degree of reaction, we get

$$\begin{aligned} R &= \frac{U_2^2(1 - \phi_2\cot\beta_2) - \frac{1}{2}U_2^2\left[\phi_2^2 + (1 - \phi_2\cot\beta_2)^2\right]}{U_2^2(1 - \phi_2\cot\beta_2)} \\ &= 1 - \frac{\phi_2^2 + (1 - \phi_2\cot\beta_2)^2}{2(1 - \phi_2\cot\beta_2)} \\ &= 1 - \frac{\phi_2^2 + 1 + \phi_2^2\cot^2\beta_2 - 2\phi_2\cot\beta_2}{2(1 - \phi_2\cot\beta_2)} \\ &= 1 - \frac{\phi_2^2(1 + \cot^2\beta_2) + 1 - 2\phi_2\cot\beta_2}{2(1 - \phi_2\cot\beta_2)} \\ &= 1 - \frac{\phi_2^2\operatorname{cosec}^2\beta_2 + 1 - 2\phi_2\cot\beta_2}{2(1 - \phi_2\cot\beta_2)} \\ &= \frac{2 - 2\phi_2\cot\beta_2 - \phi_2^2\operatorname{cosec}^2\beta_2 - 1 + 2\phi_2\cot\beta_2}{2(1 - \phi_2\cot\beta_2)} \end{aligned}$$

$$\boxed{R = \frac{1 - \phi_2^2\operatorname{cosec}^2\beta_2}{2(1 - \phi_2\cot\beta_2)}}$$

For radial vanes ($\beta_2 = 90°$),

$$\begin{aligned} R &= \frac{1}{2}(1 - \phi_2^2) \\ \text{and } \psi_P &= 1 \end{aligned}$$

because $\psi_p = (1 - \phi_2\cot\beta_2)$

Effect of Impeller Blade Shape on Performance

The different blade shapes utilised in impellers of centrifugal compressors can be classified as (refer Fig. 3.14)

(i) Backward-facing blades (ii) Radial blades (iii) Forward-facing blades

(i) Backward-curved blades

$$\beta_2 < 90°$$

We know from the outlet velocity triangle, $C_{x_2} = U_2 - C_{r_2}\cot\beta_2$
The energy transfer $E = U_2 C_{x_2}/g$
Then, $E = U_2(U_2 - C_{r_2}\cot\beta_2)/g$
(or),

$$\boxed{E = (U_2^2/g) - (mU_2\cot\beta_2/\rho g A)}$$

where $\dfrac{m}{\rho A} = C_{r_2}$

The above equation is in the form $E = a - bm$, where $a = U_2^2/g$ and $b = U_2 \cot \beta_2/\rho g A$.

As 'm' increases, E decreases. The characteristic is therefore falling.

(ii) Radial blades

$$\beta_2 = 90^\circ$$
$$\cot 90^\circ = 0$$
$$\boxed{E = a}$$

The energy transferred is constant at all flow rates and hence the characteristic is neutral.

(iii) Forward-curved blades

$$\beta_2 > 90^\circ$$
$$\boxed{E = a + bm}$$

When 'm' increases, E is increased. The characteristic will then be raising. β_2 would be typically 140° for a multi-bladed centrifugal fan.

These equations are plotted in Fig. 3.12.

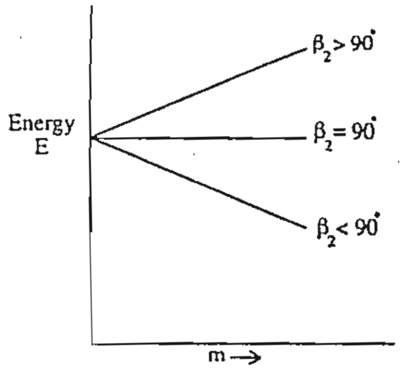


Figure 3.12 Theoretical characteristics for varying outlet blade angle

Actual characteristics for various blade outlet angles are shown in Fig. 3.13.

For both radial and forward facing blades, the power is rising continuously as the flow rate is increased. In the case of backward-facing vanes the maximum efficiency occurs in the region of maximum power. If m increases beyond 'designed m' (m_D), it will result in a power decrease, and therefore the motor used to drive the compressor may be safely rated at the maximum power. This is said to be a '*self-limiting characteristic*'

In case of the radial and forward-facing vanes, if the compressor motor is rated for maximum power, then it will be under-utilised most of the time, and extra cost will have to be incurred for the extra rating. Whereas, if a smaller motor is employed, rated at the design point, then, if m increases above m_D the motor will be overloaded and may fail. So, it is more difficult to decide on a choice of motor for these vanes.

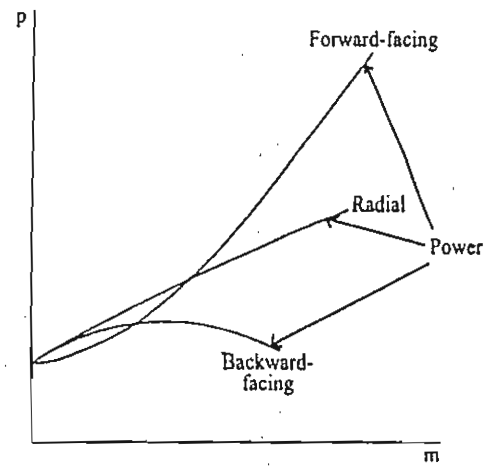


Figure 3.13 Actual characteristics for varying blade outlet angle

Forward-bent blades have higher pressure ratios. But the following disadvantages are the hurdles for its wide range of applications.

1. Low efficiency owing to large slip factor (between 1 and 2).
2. Operating range is closer to the surge line even under normal running conditions thus narrowing the stable operating range.

Better efficiencies can be obtained from backward-bent blades than with radial vanes, but the pressure ratio is lower. So, when a high compressor efficiency is desired, machines with backward curved vanes are used.

The radial-blade impellers are usually preferred because

1. Ease of manufacturing
2. Lowest unit blade stress for a given diameter and rotational speed (hence lightest weight.)
3. Equal energy conversion in impeller and diffuser giving higher pressure ratios with good efficiency.

Due to these advantages the radial blade impellers are used in aircraft centrifugal compressors. Experimental results show that the slip factor value for radial blade impellers is about 0.9. Hence, where a large pressure rise is required for a machine of small size, radial blades are used. The reason for the decrease in efficiency in forward-bent blades is that, as the slip factor increases (C'_{x_2} increases), the energy conversion required in the diffuser increases as a result of which diffuser inlet velocity is higher and the diffuser efficiency rapidly falls. Therefore it is very rare to find machines with forward curved vanes.

It should be noted that the *exit kinetic energy* $C_2^2/2$ increases quite rapidly as β_2 increases. But machines with large exit angles (β_2) will be less efficient than machines with small exit angles.

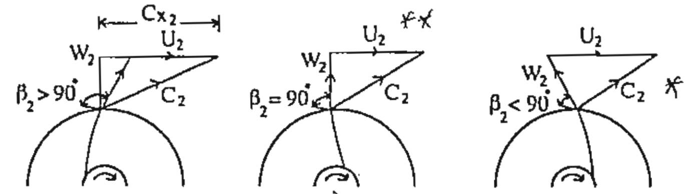


Figure 3.14 Centrifugal compressor outlet velocity triangles for varying blade outlet angle

PRE-WHIRL AND INLET GUIDE VANES

To restrict the Mach number at inlet to an acceptable value, pre-whirl should be imparted on the air entering the eye. This can be done by placing guide vanes at the inlet.

Fig. 3.15 clearly shows that the inlet guide vanes impart a whirl component C_{x_1} to the fluid, thus reducing W_1 to an acceptable value. However, the work capacity is reduced since C_{x_1} is no longer zero.

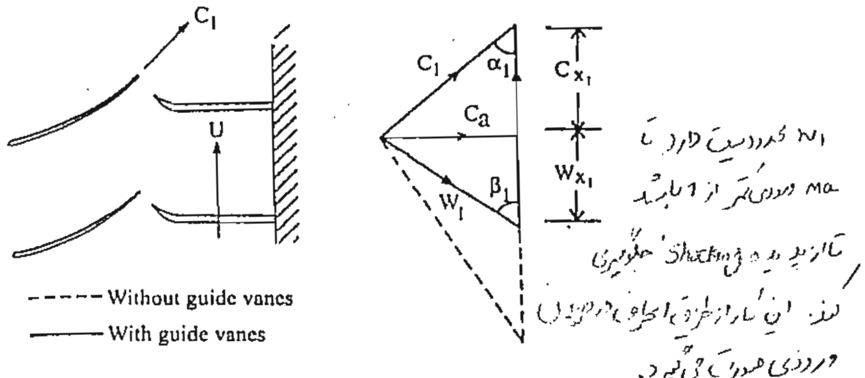


Figure 3.15 Effect of inlet guide vanes on the inlet relative velocity

It is not necessary to impart pre-whirl down to the hub, as in this region, the fluid is nowhere near sonic conditions due to the lower blade speed. The pre-whirl is therefore gradually reduced to zero by twisting the inlet guide vanes.

Apart from reducing the Mach number, the pre-whirl has another advantage of reduced curvature of the impeller vanes at inlet. Pre-whirl vanes have the disadvantage of introducing additional parts and additional weights, which should be an important parameter to be controlled in jet airplanes. Also there is a danger of possible icing in the vanes under unfavourable operating conditions i.e. at higher altitudes.

Limiting values of Mach number are usually kept between 0.7–0.8, for flow over the blades.

INLET VELOCITY LIMITATIONS

Controlling the Mach number at the eye of a centrifugal compressor affects the inlet relative velocity W_1. Two cases may be examined for the same mass flow rate having uniform absolute velocity C_1, with zero whirl velocity ($C_{x_1} = 0$) at the entry to a centrifugal compressor.

Case 1: Large eye tip diameter From continuity equation the axial velocity C_1 should be low ($m = \rho_1 A_1 C_1$). Blade speed is high. These are shown in the velocity triangle. (Fig 3.16(a))

Case 2: Small eye tip diameter The axial velocity is large, but the blade speed is small. It is shown in the velocity triangle diagram (Fig. 3.16(b))

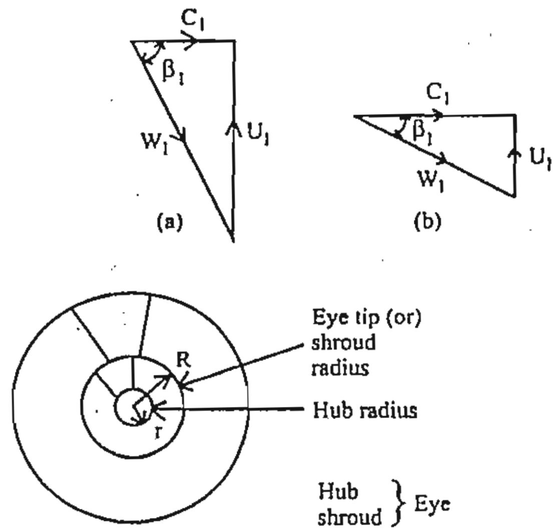


Figure 3.16 Velocity triangle for a) large and b) small inlet area

For both of these extreme cases, the relative velocity vector W_1 is high, but it must reach a minimum value when moving from one extreme to another. After determining this relative velocity by proper design, the Mach number effects can be avoided.

Flow into the eye takes place through the annulus formed by the shroud radius 'R' and the hub radius 'r'.

For uniform axial flow into the eye,

$$m = \rho_1 A_1 C_1$$

From the velocity triangles (Fig. 3.16), we get

$$C_1 = W_1 \cos \beta_1 \text{ and } U_1 = W_1 \sin \beta_1$$

The flow area is

$$
\begin{aligned}
A_1 &= \pi(R^2 - r^2)\\
&= \pi R^2(1 - r^2/R^2)\\
&= \pi R^2 k\\
\therefore m &= \rho_1(\pi R^2 k)C_1\\
\text{and } \omega &= U_1/R\\
\therefore m &= \rho_1\left(\frac{\pi U_1^2 k}{\omega^2}\right)C_1
\end{aligned}
$$

where 'U_1' is the inlet tangential velocity of the impeller at the shroud radius and 'ω' is the angular velocity.

$$
\begin{aligned}
m\omega^2/\rho_1\pi k &= U_1^2 C_1\\
&= W_1^3(\sin^2\beta_1)\cdot(\cos\beta_1) \qquad (3.15)
\end{aligned}
$$

For isentropic relationship

$$
\boxed{\frac{P_1}{T_1} = \frac{P_1}{P_{01}} \times \frac{T_{01}}{T_1} \times \frac{P_{01}}{T_{01}}}
$$

$$
= \frac{P_{01}}{T_{01}}\left[1 + \frac{r-1}{2}M_1^2\right]^{\frac{-r}{r-1}}\left[1 + \frac{r-1}{2}M_1^2\right]
$$

$$
= \frac{P_{01}}{T_{01}}\left[1 + \frac{r-1}{2}M_1^2\right]^{\frac{-1}{(r-1)}}
$$

Now $\rho_1 = P_1/RT_1$

$$
\rho_1 = \left(\frac{P_{01}}{RT_{01}}\right)\left[1 + \frac{r-1}{2}M_1^2\right]^{\frac{-1}{r-1}} \qquad (3.16)
$$

Substituting for ρ_1 from equation (3.16) in equation (3.15),

$$
m\omega^2 RT_{01}/\pi k P_{01} = \frac{W_1^3(\sin^2\beta_1)(\cos\beta_1)}{\left[1 + \frac{r-1}{2}M_1^2\right]^{\frac{1}{r-1}}} \qquad (3.17)
$$

Writing the relative Mach number based on the relative velocity W_1,

$$
\begin{aligned}
W_1 &= M_{1rel}\cdot a_1\\
\frac{m\omega^2 RT_{01}}{\pi k P_{01}} &= \frac{M_{1\cdot rel}^3 a_1^3(\sin^2\beta_1)(\cos\beta_1)}{\left[1 + \frac{r-1}{2}M_1^2\right]^{\frac{1}{r-1}}} \qquad (3.18)
\end{aligned}
$$

We know

$$
\begin{aligned}
a_1 &= (rRT_1)^{\frac{1}{2}} \quad \text{and}\\
a_{01} &= (rRT_{01})^{\frac{1}{2}}
\end{aligned}
$$

Therefore,

$$
\frac{a_{01}}{a_1} = \left(\frac{T_{01}}{T_1}\right)^{\frac{1}{2}} = \left[1 + \frac{(r-1)M_1^2}{2}\right]^{\frac{1}{2}} \qquad (3.19)
$$

$$
\begin{aligned}
M_1 &= \frac{C_1}{a_1} = \frac{W_1\cos\beta_1}{a_1} \text{ and } M_{1.rel} = \frac{W_1}{a_1}\\
\therefore \frac{W_1}{a_1} &= \frac{M_1}{\cos\beta_1} = M_{1.rel}\\
\text{(or) } M_1 &= M_{1.rel}\cos\beta_1 \qquad (3.20)
\end{aligned}
$$

Substituting for a_1 and M_1 from equations (3.19) and (3.20) respectively in equation (3.18),

$$
\frac{m\omega^2}{\pi k r P_{01}(rRT_{01})^{1/2}} = \frac{M_{1.rel}^3(\sin^2\beta_1)(\cos\beta_1)}{\left[1 + \frac{r-1}{2}M_{1.rel}^2(\cos^2\beta_1)\right]^{3r-1/2(r-1)}} \qquad (3.21)
$$

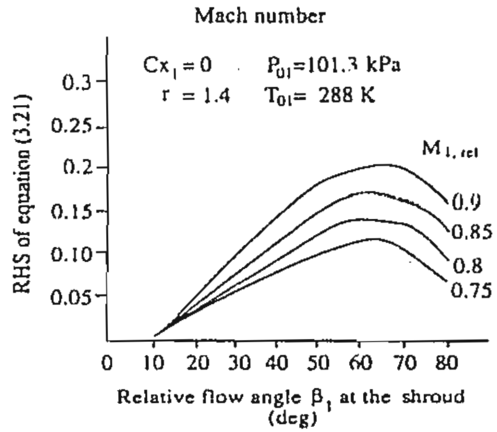


Figure 3.17 Optimisation of the mass flow function

It should be remembered that equation 3.21 is applied at the shroud radius 'R'. The relative blade angle at radius 'R' is β_1. At this radius 'R', the maximum value of relative velocity onto the blade occurs.

So, for a gas of known inlet stagnation conditions (e.g. the atmosphere), the right hand side of equation 3.21 may be plotted with the change of blade angle β_1. From which, the optimum value of both can be determined. This maximum value is then equated to the left hand side of equation 3.21 and the maximum mass flow rate is determined.

Fig. 3.17 shows that the blade angle is almost constant at 60° for maximum mass flow. So, by specifying the relative Mach number (M_{rel}), the maximum value of mass flow may be calculated.

Relative Mach numbers are usually restricted to about 0.8 to ensure that there is no shock-wave formation at the impeller inlet.

MACH NUMBER IN THE DIFFUSER

The absolute Mach number of the fluid leaving the impeller may exceed unity. There is no loss in efficiency caused by the formation of shock waves as long as the radial flow velocity C_{r_2} is subsonic. When the constant angular momentum with vortex motion is maintained in the vaneless space between impeller tip and diffuser, the supersonic diffusion can take place in the vaneless space. This reduces the Mach number at inlet to the diffuser vanes to about 0.8.

High Mach numbers at inlet to the diffuser vanes will cause high pressures at the stagnation points on the diffuser vane tips, which leads to a variation of static pressure around the circumference of the diffuser. This pressure variation is transmitted radially across the vaneless space and can cause cyclic loading of the impeller which may lead to early fatigue failure.

CENTRIFUGAL COMPRESSOR CHARACTERISTICS

Using the groups of variables, the characteristics of compressible flow machines are usually described. The characteristics are generally given as a series of curves of P_{03}/P_{01} plotted against the mass flow parameter $\dfrac{m\sqrt{T_{01}}}{P_{01}}$ for fixed speed intervals of $N/\sqrt{T_{01}}$.

An idealised fixed-speed characteristic is shown in Fig. 3.18.

Consider a centrifugal compressor delivering through a flow control valve situated after the diffuser. There is a certain pressure ratio P_{03}/P_{01}, even if the valve is fully closed, and is indicated by point 1. This pressure ratio is solely due to the vanes moving the air about in the impeller. The pressure head so developed is called "shut off" head. As the flow control valve is opened, the air starts flowing and the diffuser contributes to the pressure ratio. Thus, at point 2, the maximum pressure ratio is reached but the efficiency is just below the maximum efficiency.

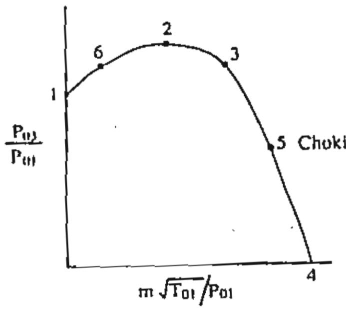


Figure 3.18 Idealised fixed speed characteristic of centrifugal compressor

Further increase in mass flow reduces the pressure ratio to state-3. But at this point, the efficiency is maximum compared with state-2. Thus the value corresponding to point-3 is said to be design mass flow rate and the pressure ratio.

Further increase in mass flow decreases the pressure ratio and reaches zero pressure ratio at point-4. Corresponding to this point, all the power absorbed by the compressor is used to overcome the internal friction and thus the compression efficiency is zero. Point-4 could be reached only theoretically. So, the curve just described is not obtainable practically.

But the actual curve is differing from the ideal curve due to the following reasons.

Surging

The phenomenon of a momentary increase in the delivery pressure resulting in unsteady, periodic and reversal of flow through the compressor is called surging. Consider a compressor operating at point-3 on P_{03}/P_{01} – vs $m\sqrt{T_{01}}/P_{01}$ curve, i.e. on the negative slope of the curve. A reduction in mass flow rate (due to momentary blockage) makes the point to move on to the left. Further reduction in mass flow rate increases the pressure ratio until it reaches the maximum value. Operating the compressor on the negative slope region (1–2) establishes 'stable operation'. Because, the delivery pressure 'P_{03}' increases, which in turn will control the further reduction of flow rate. It is self-correcting.

Now, the compressor is operating at point 6 on the positive slope (2–4) of the curve. Upon mass flow reduction the pressure ratio decreases, until it reaches the P_{03}/P_{01} axis i.e. zero mass flow rate. The mass flow even becomes negative through the compressor. When the back-pressure P_{03} has reduced itself further sufficiently due to the reduced flow rate, the positive flow becomes established once again and the compressor picks up until the "restricted mass flow rate" is reached again. when pressure reduction takes place once again. The compressor operates in an unstable fashion.

The pressure therefore surges back and forth, if the downstream conditions are unchanged. This phenomenon is known as 'surging' or 'pumping'. Thus, when the compressor has to operate at reduced mass flow rates, the air surges and pulsates throughout the compressor and the compressor does not give a steady flow of air. Surging, if severe enough, could lead to failure of the compressor parts. Surging occurence can be reduced by making the number of diffuser vanes an odd-number multiple of the impeller vanes. In this way, a pair of diffuser passages will be supplied with air from an odd number of vanes and pressure fluctuations are more likely to be evened out around the circumference than if exact multiples of diffuser vanes are employed.

Rotating Stall

The phenomenon of a reduction in mass flow rate through the blade passage at higher angles of incidence is known as rotating stall. It is a separate stall phenomenon, which may lead to surging but can exist on its own in a stable operating condition. Figure 3.19 illustrates the air flow directions in a number of blade passages.

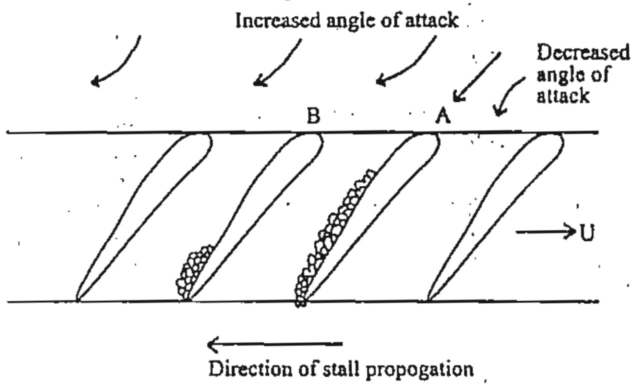


Figure 3.19 Air flow direction in rotating stall phenomenon

If the air angle of incidence onto blade A is excessive, perhaps due to a partial blockage (or) uneven flow in the diffuser the blade may stall. Because of this, the mass flow decreases which in turn increases the angle of incidence to the left of blade A, [due to the low mass flow rate through the passage, the entering air gets deflected, resulting in large angle of incidence] whereas angle of incidence decreases to the right of blade A. Thus blade B will be the next to stall while blade A will be unstalled and the process is repeated about the periphery of the disc.

Prolonged cyclic loading and unloading of the rotor blades can lead to fatigue failure or even immediate catastrophic failure. The stall propagates in the opposite direction to the blade motion at a frequency related to shaft speed. In compressor tests, rotating stall may be audibly recognised as a high frequency '*screech*'.

At low speed and starting the front stages are more likely to stall. But at high speeds, the stall occurs in the last stages. Low speed and starting stall may be eliminated by variable inlet guide vane rows.

Choking

When the mass flow is increased to the right of point-3 on the negative slope of the characteristic curve (as in Fig. 3.18) a point-5 is reached where no further increase in mass flow is possible no matter how wide open the flow control valve is. This indicates that the flow velocity in the passage reaches the speed of sound at some point within the compressor and the flow chokes. Choking means fixed mass flow rate regardless of pressure ratio. Choking may take place at the inlet, within the impeller, or in the diffuser section. It will occur in the inlet if stationary guide vanes are fitted.

In stationary passages like nozzles, the velocity that is choked is the absolute velocity. In the rotating impeller, it is the relative velocity 'W' that is the choked velocity.

Now

$$h_{01} = h_1 + C_1^2/2$$
$$= h_1 + (W_1^2 - V_1^2)/2$$

If choking occurs when the relative velocity equals the acoustic velocity (i.e $W_1^2 = a_1^2 = rRT_1$), the above equation becomes

$$T_{01} = T_1 + (rRT_1 - U_1^2)/2C_p$$

and dividing by T_{01} gives

$$1 = \frac{T_1}{T_{01}} + \frac{rRT_1}{2C_pT_{01}} - \frac{U_1^2}{2C_pT_{01}}$$

$$1 + \frac{U_1^2}{2C_pT_{01}} = \frac{T_1}{T_{01}}\left(1 + \frac{rR}{2C_p}\right)$$

Since $$C_p = \frac{rR}{r-1}$$

$$1 + \frac{U_1^2}{2C_pT_{01}} = \frac{T_1}{T_{01}}\left(1 + \frac{r-1}{2}\right)$$

$$\frac{T_1}{T_{01}} = \frac{2}{r+1}\left[1 + \frac{U_1^2}{2C_pT_{01}}\right] \tag{3.22}$$

For isentropic flow,

$$\boxed{\rho_1/\rho_{01} = (T_1/T_{01})^{1/r-1}}$$

From the continuity equation,

$$m = \rho A a$$
$$m/A = \rho a = \rho_{01}a_{01}(T_1/T_{01})^{(r+1)/2(r-1)} \tag{3.23}$$

$[a = a_{01} \times (T_1/T_{01})^{1/2}]$

Since $a \alpha T^{1/2}$. Substituting from equation (3.22) and rearranging,

$$m/A = \left\{rP_{01}\rho_{01}[2(1 + U_1^2/2C_pT_{01})/r+1]\right\}^{r+1/2(r-1)} \tag{3.24}$$

$$\left[a_{01} = \sqrt{rRT_{01}} = \sqrt{rP_{01}/\rho_{01}}\right]$$

Equation (3.24) implies that the choking mass flow rate increases with impeller speed.

Maximum mass flow rate equation for isentropic flow at the throat of a coverging nozzle

$$m/A = \left\{rP_{00}\rho_{00}[2/(r+1)]^{(r+1)/r-1}\right\}^{1/2} \tag{3.25}$$

In the diffuser passages, equation (3.25) is also valid with the subscripts changed to the impeller outlet conditions.

$$m/A = \left[rP_{02}\rho_{02}[2/(r+1)]^{(r+1)/r-1}\right]^{1/2} \tag{3.26}$$

The areas in equations (3.24), (3.25) and (3.26) refer to the flow areas at the respective locations.

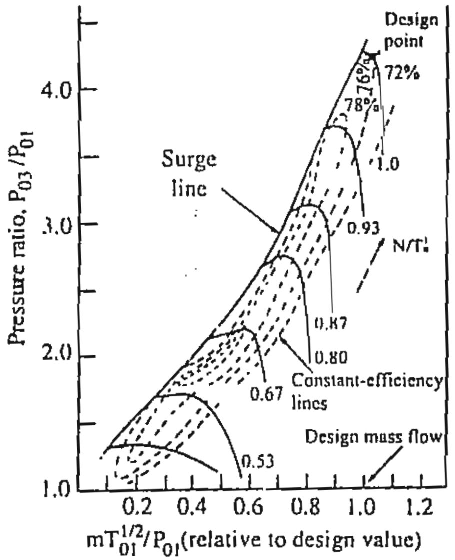


Figure 3.20 Centrifugal compressor characteristic curves

CHARACTERISTIC CURVE

Fig. 3.20 shows the overall pressure ratio and efficiency plotted against $mT_{01}^{1/2}/P_{01}$ at fixed speed intervals of $N/T_{01}^{1/2}$. It is usual to transfer constant efficiency points onto the corresponding constant speed curves of the pressure ratio characteristics and then join those points together to form constant efficiency curves. The following salient features can be observed from the graph.

1. At all speeds the range of mass flow over which the centrifugal compressor will operate before surging or choking occurs is quite wide, but this range decreases as the speed increases.
2. Maximum efficiency (dotted line) occurs well within the surge point, making this type of compressor relatively insensitive to mass flow changes.
3. For a particular speed, the mass flow rate decreases as the pressure ratio increases.
4. For a given pressure ratio, the mass flow rate increases with increase in speed. Under such conditions, the efficiency however falls rapidly.

The onset of surge occuring at increasingly high mass flows, as the speed increases, while the locus of the limit of stability is called the surge line. The limit of maximum flow is usually set by choking in the impeller, while the surge limit of minimum mass flow is set by stalling of the flow into the diffuser vanes.

SOLVED PROBLEMS

Example 3.1 10 kg of air per second is to be compressed in an uncooled centrifugal compressor of the single sided impeller type. The ambient air conditions are 1 bar and 20°C. The compressor runs at 20,000 rev/min, has isentropic efficiency of 80%, and compresses the air from 1 bar static pressure to 4.5 bar total pressure. The air enters the impeller eye with a velocity 150 m/s with no prewhirl. Assuming that the ratio of whirl speed to tip speed is 0.95, calculate: i) rise in total temperature during compression, if the change in kinetic energy is negligible, ii) the impeller tip speed and tip diameter, iii) power required to drive the compressor, iv) the external diameter of the eye, for which the internal diameter is 15 cm.

Solution

i) Rise in total temperature of the compressor

$m = 10$ Kg/s $\quad P_1 = 1$ bar $\quad T_1 = 293$ K $\quad N = 20{,}000$ rpm $\quad \eta_c = 0.8$
$P_{02} = 4.5$ bar $\quad C_1 = 150$ m/s $\quad C_{x1} = 0.$ $\quad C_{x2}/U_2 = 0.95$

Stagnation temperature at inlet

$$T_{01} = T_1 + C^2/2C_p = 293 + \frac{150^2}{2 \times 1005} = 304.19 \text{ K}$$

Stagnation pressure at inlet

$$P_{01} = P\left(\frac{T_{01}}{T_1}\right)^{r/r-1} = 1\left(\frac{304.19}{293}\right)^{1.4/0.4} = 1.14 \text{ bar}$$

The temperature after isentropic compression from P_{01} to P_{02} is

$$T_{02s} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{r-1/r} = 304.19\left(\frac{4.5}{1.14}\right)^{0.4/1.4} = 450.32 \text{ K}$$

Actual rise in total temperature is determined from the definition of isentropic efficiency

$$\eta_c = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

or

$$T_{02}-T_{01} = \frac{T_{02s}-T_{01}}{\eta_c}$$

$$= (450.32 - 304.19)/0.8$$

$$= 182.7 \text{ K}$$

ii) Impeller tip speed and impeller tip diameter

From Euler's equation

$$W/m = C_{x2}U_2$$

but

$$W/m = C_p \times \text{actual rise in total temperature}$$

$$= C_p(T_{02}-T_{01})$$

$$= 1.005 \times 182.7$$

$$= 183.61 \text{ kJ/kg}$$

and

$$\frac{C_{x2}}{U_2} = 0.95$$

$$183.61 \times 10^3 = 0.95U_2 \times U_2$$

$$U_2 = 439.62 \text{ m/s}$$

Thus the impeller tip speed is 439.62 m/s

If D_t is the tip diameter, then

$$U_2 = \frac{\pi D_t N}{60}$$

$$D_t = \frac{U_2 \times 60}{\pi N}$$

$$= \frac{439.62 \times 60}{\pi \times 20,000}$$

$$= 0.42 \text{ m}$$

∴ Tip diameter = 0.42 m or 42 cm

iii) Power required to drive the compressor

$$\text{Power required} = m \times \text{Work done/kg}$$

$$= 10 \times 183.61$$

$$= 1836.1 \text{ kW}$$

iv) Eye external diameter, (D_e)

Density of air at entry,

$$\rho_1 = \frac{P_1}{RT_1} = \frac{1 \times 10^5}{287 \times 293} = 1.189 \text{ kg/m}^3$$

Eye annulus $A_1 = \frac{\pi}{4}(D_e^2 - D_h^2)$

Now,

$$m = \rho_1 A_1 C_1$$

$$10 = 1.189 \times \frac{\pi}{4}(D_e^2 - 0.15^2) \times 150$$

$$D_e = 0.306 \text{ m}$$

∴ The eye external diameter $D_e = 30.6$ cm

Example 3.2 20 m³ of air per second at 1 bar and 15°C is to be compressed in a centrifugal compressor through a pressure ratio 1.5:1. The compression follows the law $PV^{1.5}$ = constant. The velocity of flow at inlet and outlet remains constant and is equal to 60m/s. If the inlet and outlet impeller diameters are respectively 0.6 m and 1.2 m and speed of rotation is 5000 rpm. Find (a) the blade angles at inlet and outlet of the impeller, and the angle at which the air from the impeller enters the casing; (b) breadth of impeller blade at inlet and outlet. It may be assumed no diffuser is fitted and the whole pressure increase occurs in the impeller and that the blades have a negligible thickness.

Solution

$Q_1 = 20 \text{ m}^3/\text{s}$ $\quad P_1 = 1$ bar $\quad T_1 = 288$ K

$\frac{P_2}{P_1} = 1.5$ $\quad C_1 = C_{r2} = 60$ m/s $\quad D_h = 0.6$ m

$D_t = 1.2$ m $\quad$ N = 5000 rpm

(a) Blade angles and flow angle

Peripheral velocity of impeller at inlet

$$U_1 = \frac{\pi D_h N}{60}$$

$$= \frac{\pi \times 0.6 \times 5000}{60}$$

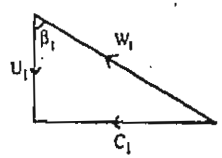
$$= 157.1 \text{ m/s}$$

From the inlet velocity vector diagram,

$$\tan\beta_1 = \frac{C_1}{U_1}$$

$$or\ \beta_1 = \tan^{-1}\left(\frac{60}{157.1}\right)$$

$$\beta_1 = 20.9°$$



$\therefore$ The blade angle at impeller inlet is 20.9°.
Peripheral velocity of impeller top at outlet

$$U_2 = \frac{\pi D_2 N}{60} = (\pi \times 1.2 \times 5000)/60$$
$$= 314.16 \text{ m/s}$$

Whirl component of absolute velocity C_{x_2} is obtained from

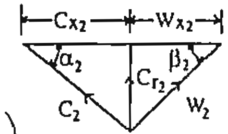
$$C_P(T_2 - T_1) = U_2 C_{x_2}$$
$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 288(1.5)^{\frac{0.5}{1.3}}$$
$$= 329.68 \text{ K}$$
$$\therefore C_{x_2} = \frac{1005(329.68 - 288)}{314.16}$$
$$= 133.33 \text{ m/s}$$

Now

$$W_{x2} = U_2 - C_{x2}$$
$$= 314.16 - 133.33$$
$$= 180.83 \text{ m/s}$$

From the outlet velocity vector diagram,

$$\tan\alpha_2 = \frac{C_{r_2}}{C_{x_2}}$$



or

$$\alpha_2 = \tan^{-1}\left(\frac{60}{133.33}\right)$$
$$\alpha_2 = 24.2°$$

$\therefore$ The blade angle at inlet to casing is 24.2°.

$$\tan\beta_2 = \frac{C_{r_2}}{W_{x_2}}$$

or

$$\beta_2 = \tan^{-1}\left(\frac{60}{180.83}\right)$$
$$\beta_2 = 18.36°.$$

$\therefore$ The blade angle at impeller outlet is 18.36°

b) Breadth of impeller blade at inlet and outlet

If Q_1 is the discharge in m^3/s then

$$Q_1 = 2\pi r_1 b_1 c_1$$

or

$$b_1 = \frac{20}{2 \times \pi \times 0.3 \times 60}$$
$$b_1 = 0.177 \text{ m}$$

$\therefore$ The breadth of impeller blade at inlet is 17.7 cm.
Now

$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1}$$
$$\therefore V_2 = \frac{P_1 V_1}{T_1} \times \frac{T_2}{P_2}$$
$$= \frac{1 \times 10^5 \times 20}{288} \times \frac{329.68}{1.5 \times 10^5}$$
$$V_2 = 15.26 \text{ m}^3/\text{s}$$

Then

$$Q_2 = V_2 = 2\pi r_2 b_2 C_{r2}$$
$$b_2 = \frac{15.26}{2 \times \pi \times 0.6 \times 60}$$
$$b_2 = 0.0675 \text{ m}$$

$\therefore$ The breadth of impeller blade at outlet is 6.75 cm.

Example 3.3 A single sided centrifugal compressor is to deliver 14 kg/s of air when operating at a stagnation pressure ratio of 4:1 and a speed of 200 revolution/sec. The inlet stagnation conditions may be taken as 288 K and 1.0 bar. Assuming a slip factor of 0.9, a power input factor of 1.04 and an overall isentropic efficiency of 0.8, estimate the overall diameter of the impeller. *(MKU Nov-1991)*

Solution

Given

$m = 14$ kg/s	$P_{01} = 1$ bar		
$P_{02}/P_{01} = 4$	$T_{01} = 288$ K		
$N = 200$ rps	$\sigma_s = 0.9$	$\psi = 1.04$	$\eta_{tt} = 0.8$

The pressure coefficient may be written as

$$\psi_p = \sigma_s \cdot \psi \cdot \eta_{tt}$$
$$= 0.9 \times 1.04 \times 0.8$$
$$= 0.749$$

$$\psi_p = \frac{C_p T_{01}\left[\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{U_2^2}$$

$$\therefore U_2^2 = \frac{1005 \times 288 \times \left[(4)^{\frac{0.4}{1.4}} - 1\right]}{0.749}$$

$$U_2 = 433.6 \text{ m/s}$$

$$U_2 = \pi D_2 N$$

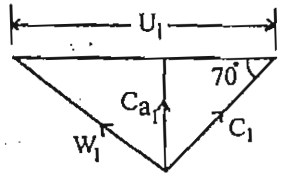
$$\therefore D_2 = \frac{U_2}{\pi \times N} = \frac{433.6}{\pi \times 200} = 0.69 \text{ m}$$

$$D = 69 \text{ cm}$$

Example 3.4 In a centrifugal compressor with inlet guide vanes, air leaving the guide vanes has a velocity of 91.5 m/s at 70 deg. to the tangential direction. Determine the inlet relative Mach number, assuming frictionless flow through the guide vanes and impeller total head isentropic efficiency. The other operating conditions are

a) Impeller diameter at inlet – 457 mm
b) Impeller diameter at exit – 762 mm
c) Radial component of velocity at impeller exit – 53.4 m/s
d) Slip factor – 0.9
e) Impeller speed – 11,000 rpm
d) Static pressure at impeller exit – 223 kPa (abs)
g) Take $T_{01} = 288$ K and $P_{01} = 1.013$ bar.

Solution



$$C_1 = 91.5 \text{ m/s}$$

$$C_{x_1} = C_1 \cdot \cos 70^\circ = 31.29 \text{ m/s}$$

$$C_{a1} = C_{x1} \cdot \tan 70^\circ = 85.98 \text{ m/s}$$

$$W_{x1} = U_1 - C_{x_1}$$

$$\therefore U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.457 \times 11,000}{60} = 263.21 \text{ m/s}$$

$$W_{x1} = 263.21 - 31.29 = 231.92 \text{ m/s}$$

$$W_1^2 = W_{x_1}^2 + C_{a_1}^2 = (231.92)^2 + (85.98)^2$$

$$\therefore W_1 = 247.34 \text{ m/s}$$

$$T_1 = T_{01} - C_1^2/2C_p = 288 - \frac{91.5^2}{2 \times 1005}$$

$$T_1 = 283.83 \text{ K}$$

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 283.83}$$

$$a_1 = 337.7 \text{ m/s}$$

Inlet relative Mach number

$$M_{1,r} = \frac{W_1}{a_1} = \frac{247.34}{337.7}$$

$$M_{1,r} = 0.732$$

Work done $= \sigma_s U_2^2 - U_1 C_{x_1}$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.762 \times 11,000}{60} = 438.88 \text{ m/s}$$

and

$$C_p(T_{02} - T_{01}) = 0.9(438.88)^2 - (263.21 \times 31.29)$$

$$T_{02} - T_{01} = 164.297$$

$$T_{02} = 164.297 + 288 = 452.29 \text{ K}$$

$$C_{r_2} = 53.4 \text{ m/s}$$

$$C'_{x_2} = \sigma_s \cdot U_2 = 0.9 \times 438.88 = 394.92$$

$$\therefore C_2^2 = C_{x_2}'^2 + C_{r_2}^2 = (394.92)^2 + (53.4)^2$$

$$C_2 = 398.51 \text{ m/s}$$

$$\therefore T_2 = T_{02} - \frac{C_2^2}{2C_p} = 452.29 - \frac{(398.51)^2}{2 \times 1005} = 373.28 \text{ K}$$

and

$$P_{02} = P_2\left(\frac{T_{02}}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= 2.23\left(\frac{452.29}{373.28}\right)^{3.5}$$

$$P_{02} = 4.37 \text{ bar}$$

Total head isentropic efficiency

$$\eta_c = \frac{T_{01}\left(\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)}{T_{02} - T_{01}}$$

$$= \frac{288\left(\left(\frac{4.37}{1.013}\right)^{0.286} - 1\right)}{164.297}$$

$$= \frac{149.39}{164.297}$$

$$\eta_c = 90.9\%$$

Example 3.5 In a radial blade centrifugal compressor running at 16,500 rpm, the total pressure ratio is 4:1 when the atmospheric pressure and temperature are 1 atm and 25° C. The diameter of the hub at impeller eye is 16 cm. The axial velocity at inlet and the absolute velocity at the diffuser exit are both 120 m/s. The mass flow rate is 8.3 kg/s. If the adiabatic total-to-total efficiency is 78% and pressure coefficient is 0.7, find the main dimensions of the impeller, static conditions at exit and required power to drive the compressor. *(MKU-April' 94.)*

Solution

$N = 16,500$ rpm $\frac{P_{03}}{P_{01}} = 4$ $P_{01} = 1$ bar $T_{01} = 298$ K

$D_h = 0.16$ m $C_a = C_1 = 120$ m/s $\Psi_p = 0.7$ $C_3 = 120$ m/s

$m = 8.3$ kg/s $\eta_c = 0.78$

(a) Main dimensions of impellers

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 298 - \frac{120^2}{2 \times 1005}$$

$$= 290.84 \text{ K}$$

$$P_1 = P_{01}\left(\frac{T_1}{T_{01}}\right)^{\gamma/\gamma-1} = 1 \times \left(\frac{290.84}{298}\right)^{\frac{1.4}{0.4}}$$

$$= 0.918 \text{ bar}$$

$$\therefore \rho_1 = \frac{P_1}{RT_1} = \frac{0.918 \times 10^5}{287 \times 290.84} = 1.0998 \text{ kg/m}^3$$

(i) Eye tip diameter (D_t)

$$m = \frac{\rho_1 \pi}{4}(D_t^2 - D_h^2)C_a$$

$$D_t^2 - D_h^2 = \frac{4m}{\pi \rho_1 C_a}$$

$$D_t^2 = \frac{4 \times 8.3}{\pi \times 1.0998 \times 120} + 0.16^2$$

$$D_t = 0.325 \text{ m}$$

(ii) Impeller tip diameter (D_2)

$$\eta_c = \frac{T_{01}\left[\left(\frac{P_{03}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_{03} - T_{01}}$$

$$T_{03} - T_{01} = \frac{298\left[(4)^{\frac{0.4}{1.4}} - 1\right]}{0.78}$$

$$= 185.67 \text{ K}$$

Also

$$T_{03} - T_{01} = \Psi\sigma_s U_2^2/C_P$$

$$\Psi\sigma_s = \Psi_p/\eta_c = 0.7/0.78$$

$$= 0.897$$

$$\therefore U_2 = \left[\frac{1005(185.67)}{0.897}\right]^{1/2}$$

$$= 456.1 \text{ m/s}$$

$$D_2 = \frac{U_2 \times 60}{\pi \times N} = \frac{456.1 \times 60}{\pi \times 16,500}$$

$$= 0.528 \text{ m}$$

(iii) Blade angles at eye hub and eye tip

$$U_h = \frac{\pi D_h N}{60} = \frac{\pi \times 0.16 \times 16500}{60}$$

$$= 138.23 \text{ m/s}$$

From inlet velocity triangle (Refer Fig. 3.3(a)).

$$\beta_h = \tan^{-1}\left(\frac{C_1}{U_h}\right)$$

$$= \tan^{-1}\left(\frac{120}{138.23}\right)$$

$$\beta_h = 40.96°$$

$$U_t = \frac{\pi D_t N}{60} = \frac{\pi \times 0.325 \times 16,500}{60}$$
$$= 280.78 \text{ m/s}$$
$$\beta_t = \tan^{-1}\left(\frac{C_1}{U_t}\right)$$
$$= \tan^{-1}\left(\frac{120}{280.78}\right)$$
$$\beta_t = 23.14°$$

(b) Static conditions at exit

$$T_3 = T_{03} - \frac{C_3^2}{2C_p}$$
$$T_{03} = T_{01} + 185.67 = 298 + 185.67$$
$$= 483.67 \text{ K}$$
$$\therefore T_3 = 483.67 - \frac{120^2}{2 \times 1005}$$
$$T_3 = 476.5 \text{ K}$$

Exit static pressure

$$P_3 = P_{03}\left(\frac{T_3}{T_{03}}\right)^{r/r-1}$$
$$= 4\left(\frac{476.5}{483.67}\right)^{\frac{1.4}{0.4}}$$
$$P_3 = 3.796 \text{ bar}$$

(c) Power required

$$W = mC_p(T_{03} - T_{01})$$
$$= 8.3 \times 1.005(185.67)$$
$$W = 1.549 \text{ mW}$$

Example 3.6 The following data refers to a centrifugal compressor: tip diameter of the eye-250 mm, hub diameter of the eye-100 mm, speed-120 rps. Mass of air handled-5 kg/s. Inlet stagnation pressure-102 kPa, inlet total temperature-335 K. Determine the air angle at inlet of the inducer blade and inlet relative Mach number. If IGV is used then determine air angle and relative mach number at the exit of IGV.

[MU-Oct '96]

Solution

$D_t = 0.25$ m $\quad D_h = 0.1$ m $\quad N = 120$ rps
$m = 5$ kg/s $\quad P_{01} = 102$ kPa $\quad T_{01} = 335$ K

Since the density and axial velocity component at the inlet of the inducer section are unknown, their values are determined by trial and error method.

Let

$$\rho_1 \simeq \rho_{01} = \frac{P_{01}}{RT_{01}}$$
$$= \frac{102 \times 10^3}{287 \times 335}$$
$$= 1.061 \text{ kg/m}^3$$
$$C_1 = \frac{m}{\rho_1(\pi D_m b)}$$

where D_m is impeller mean diameter and b is the impeller blade height.

$$D_m = \frac{D_h + D_t}{2} = \frac{0.1 + 0.25}{2} = 0.175 \text{ m}$$

and

$$b = \frac{D_t - D_h}{2} = \frac{0.25 - 0.1}{2} = 0.075 \text{ m}$$
$$\therefore C_1 = \frac{5}{1.061 \times (\pi \times 0.175 \times 0.075)}$$
$$= 114.3 \text{ m/s}$$

Now,

$$\rho_1 = \frac{P_1}{RT_1}$$
$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 335 - \frac{114.3^2}{2 \times 1005}$$
$$= 328.5 \text{ K}$$
$$P_1 = P_{01}\left(\frac{T_1}{T_{01}}\right)^{\frac{r}{r-1}} = 102\left(\frac{328.5}{335}\right)^{\frac{1.4}{0.4}}$$
$$= 95.24 \text{ kPa}$$
$$\therefore \rho_1 = \frac{95.24 \times 10^3}{287 \times 328.5} = 1.01 \text{ kg/m}^3$$

and

$$C_1 = \frac{m}{\rho_1(\pi D_m b)} = \frac{5}{1.01 \times (\pi \times 0.175 \times 0.075)}$$
$$= 120.1 \text{ m/s}$$

Then

$$\rho_1 = \frac{P_1}{RT_1}$$

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 335 - \frac{120.1^2}{2 \times 1005}$$
$$= 327.8 \text{ K}$$

$$P_1 = P_{01}\left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = 102\left(\frac{327.8}{335}\right)^{\frac{1.4}{0.4}}$$
$$= 94.53 \text{ kPa}$$

$$\rho_1 = \frac{94.53 \times 10^3}{287 \times 327.8} = 1.005 \text{ kg/m}^3$$

and

$$C_1 = \frac{m}{\rho_1(\pi D_m b)} = \frac{5}{1.005 \times (\pi \times 0.175 \times 0.075)}$$
$$= 120.66 \text{ m/s}$$

Since the values of ρ_1 and C_1 are approximately equal to the last iteration values, the iteration can now be stopped. Thus,

$$\rho_1 = 1.005 \text{ kg/m}^3$$
$$C_1 = 120.66 \text{ m/s}$$

$$U_1 = \pi D_m N = \pi \times 0.175 \times 120$$
$$= 65.97 \text{ m/s}$$

From the inlet velocity triangle (Refer Fig. 3.3(a)),

$$\tan \beta_1 = \frac{C_1}{U_1}$$

$$\therefore \beta_1 = \tan^{-1}\left(\frac{120.66}{65.97}\right)$$
$$= 61.33°$$

This is the air angle at entry to the inducer blade and $\sin \beta_1 = \frac{C_1}{W_1}$.

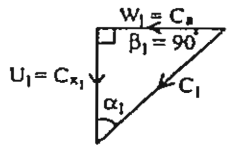
$$W_1 = C_1/\sin \beta_1$$
$$= 120.66/\sin 61.33$$
$$= 137.52 \text{ m/s}$$

$$M_{r,1} = \frac{W_1}{a_1} = \frac{W_1}{\sqrt{\gamma R T_1}}$$
$$= \frac{137.52}{\sqrt{1.4 \times 287 \times 327.8}}$$
$$M_{r,1} = 0.379$$

With IGV

Air angle at IGV exit

$$\tan \alpha_1 = \frac{W_1}{U_1} = \frac{C_a}{U_1}$$
$$\alpha_1 = \tan^{-1}\left(\frac{120.66}{65.97}\right)$$
$$\alpha_1 = 61.33$$



Now,

$$C_1 = C_a/\sin \alpha_1 = 120.66/\sin 61.33$$
$$= 137.52 \text{ m/s}$$

$$\therefore T_1 = T_{01} - \frac{C_1^2}{2C_p}$$
$$= 335 - \frac{137.52^2}{2 \times 1005}$$
$$= 325.59 \text{ K}$$

Then,

$$M_{r,1} = \frac{W_1}{a_1} = \frac{C_a}{a_1}$$
$$= \frac{120.66}{\sqrt{1.4 \times 287 \times 325.59}}$$
$$M_{r,1} = 0.334$$

Note that the inlet relative Mach number is reduced when IGVs are used.

Example 3.7 Determine the absolute Mach number of the flow at the exit of a radial vaned impeller of a centrifugal compressor when the radial component of the velocity at the impeller exit is 28 m/s and the slip factor is 0.9. The impeller tip speed is 350 m/s. If the impeller area is 0.08 m^2 and the total head isentropic efficiency is 90%, determine the mass flow rate. Take $T_{01} = 288$ K, $P_{01} = 1$ bar.

(MKU-Nov '95)

Solution

$C_{r_2} = 28$ m/s $\quad \sigma_s = 0.9 \quad U_2 = 350$ m/s $\quad A = 0.08$ m^2 $\quad \eta_c = 0.9$
$T_{01} = 288$ K

(a) Exit absolute Mach number

$$M_2 = \frac{C_2}{a_2}$$

From outlet velocity triangle (Refer Fig. 3.3(b)),

$$C_2 = \sqrt{C_{x_2}^2 + C_{r_2}^2}$$
$$C_{x_2} = \sigma_s \cdot U_2 = 0.9 \times 350 = 315 \text{ m/s}$$
$$\therefore C_2 = \sqrt{(315)^2 + (28)^2} = 316.24 \text{ m/s}$$

Since

$$T_{02} - T_{01} = \sigma_s U_2^2 / C_p = \frac{0.9 \times 350^2}{1005} = 109.7 \text{ K}$$
$$\therefore T_{02} = 109.7 + T_{01} = 109.7 + 288 = 397.7 \text{ K}$$

Now,

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 397.7 - \frac{316.24^2}{2 \times 1005} = 347.95 \text{ K}$$
$$\therefore M_2 = \frac{316.24}{\sqrt{1.4 \times 287 \times 347.95}}$$
$$M_2 = 0.8458$$

(b) Mass flow rate

$$m = \rho_2 A_2 C_{r_2}$$
$$\rho_2 = \frac{P_2}{RT_2}$$
$$P_2 = P_{02}\left(\frac{T_2}{T_{02}}\right)^{\frac{\gamma}{\gamma-1}}$$

P_{02} is determined using isentropic efficiency value.

$$\eta_c = \frac{T_{01}\left[\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_{02} - T_{01}}$$
$$\frac{P_{02}}{P_{01}} = \left[1 + \frac{0.9(109.7)}{288}\right]^{\frac{1.4}{0.4}} = 2.806$$
$$P_{02} = 2.806 \times 1 = 2.806 \text{ bar}$$

From the isentropic relation,

$$P_2 = 2.806 \times \left(\frac{347.95}{397.7}\right)^{\frac{1.4}{0.4}} = 1.758 \text{ bar}$$

Therefore,

$$\rho_2 = \frac{1.758 \times 10^5}{287 \times 347.95} = 1.76 \text{ kg/m}^3$$
$$m = 1.76 \times 0.08 \times 28$$
$$m = 3.9424 \text{ kg/s}$$

Example 3.8 A double-sided centrifugal compressor has impeller eye-root and tip diameters of 175 mm and 312.5 mm and is to deliver 20 kg of air per second at 16,000 rpm. The design ambient conditions are 288 K and 100 kPa. Calculate suitable values for the impeller value angles at the root and tip of eye if the air is given 20 deg. of pre-whirl at all radii. The axial component of inlet velocity is constant over the eye and is about 152 m/s. Also compute the maximum Mach number at the eye.

Solution

In double-sided centrifugal compressor impeller, there is an eye on either side of the impeller and the air is taken in on both sides. The double-sided compressor has the advantage that the impeller is subjected to approximately equal stresses in the axial direction.

$D_h = 0.175$ m $D_t = 0.3125$ m $m = 20$ kg/s
$N = 16,000$ rpm $T_{01} = 288$ K $P_{01} = 100$ kPa $C_a = 152$ m/s

Annulus area of flow at the impeller eye

$$A = \frac{\pi}{4}(D_t^2 - D_h^2) = \frac{\pi}{4}(0.3125^2 - 0.175^2) = 0.0527 \text{ m}^2$$

(a) (i) Impeller eye tip speed

$$U_t = \frac{\pi D_t N}{60} = \frac{\pi \times 0.3125 \times 16,000}{60} = 261.79 \text{ m/s}$$

(a) (ii) Impeller eye hub speed

$$U_h = \frac{\pi D_h N}{60}$$

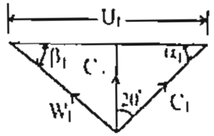
$$= \frac{\pi \times 0.175 \times 16{,}000}{60}$$

$$= 146.61 \text{ m/s}$$

From the velocity triangle at inlet,

$$C_1 = \frac{C_a}{\sin \alpha_1} \quad [\alpha_1 = 90 - 20^\circ = 70^\circ]$$

$$= \frac{152}{\sin 70^\circ} = 161.76 \text{ m/s}$$



$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 288 - \frac{161.76^2}{2 \times 1005}$$

$$= 274.98 \text{ K}$$

$$P_1 = P_{01}\left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = 100\left(\frac{274.98}{288}\right)^{\frac{1.4}{0.4}}$$

$$= 85.05 \text{ kPa}$$

$$\therefore \rho_1 = \frac{P_1}{RT_1} = \frac{85.05 \times 10^3}{287 \times 274.98} = 1.078 \text{ kg/m}^3$$

$$\tan \beta_{1,h} = \frac{C_a}{U_h - C_x} = \frac{C_a}{U_h - (C_a/\tan \alpha_1)}$$

$$\tan \beta_{1,h} = \frac{152}{146.61 - (152/\tan 70^\circ)} = 1.665$$

$$\beta_{1,h} = 59^\circ$$

and

$$\tan \beta_{1,t} = \frac{C_a}{U_t - (C_a/\tan \alpha_1)}$$

$$= \frac{152}{261.79 - (152/\tan 70^\circ)}$$

$$= 0.7362$$

$$\therefore \beta_{1,t} = 36.36^\circ$$

(b) Maximum Mach number at the eye

Maximum Mach number occurs at the eye tip

$$M_{r,1} = \frac{W_1}{\sqrt{\gamma R T_1}}$$

$$W_{x_1} = U_t - C_{x_1}$$

$$= 261.79 - (152/\tan 70^\circ)$$

$$= 206.47 \text{ m/s}$$

$$\therefore W_1 = \sqrt{W_{x_1}^2 + C_a^2} = \sqrt{206.47^2 + 152^2}$$

$$= 256.39 \text{ m/s}$$

$$M_{r,1} = \frac{256.39}{\sqrt{1.4 \times 287 \times 274.98}}$$

$$M_{r,1} = 0.77$$

Example 3.9 A centrifugal blower takes in air at 100 kPa and 309 K. It develops a pressure head of 750 mm W.G., while consuming a power of 33 kW. If the blower efficiency (η_B) is 79% and mechanical efficiency is 83%, determine the mass rate and volume rate and exit properties of air.

Solution

$P_1 = 100$ kPa $T_1 = 309$ K $\Delta H = 0.750$ m W.G.
input power = 33 kW $\eta_B = 0.79$ $\eta_m = 0.83$

(a) Mass rate

$$\rho = \frac{P_1}{RT_1} = \frac{100 \times 10^3}{287 \times 309} = 1.128 \text{ kg/m}^3$$

$$\Delta P = \rho g \Delta H = 10^3 \times 9.81 \times 0.750$$

$$= 7357.5 \text{ N/m}^2$$

$$\text{Ideal work done/kg} = \Delta P/\rho$$

$$= 7357.5/1.128$$

$$= 6522.61 \text{ J/kg}$$

$$\text{Actual work done/kg} = \frac{\text{Ideal work done/kg}}{\eta_B}$$

$$= \frac{6522.61}{0.79}$$

$$W/m = 8256.47 \text{ J/kg}$$

$$\text{Actual power input} = \text{Motor power input} \times \eta_m$$

$$= 33 \times 0.83$$

$$W = 27.39 \text{ kW}$$

Therefore, the mass flow rate

$$m = \frac{W}{\text{Actual work done/kg}}$$

$$= \frac{27.39 \times 10^3}{8256.47}$$

$$= 3.317 \text{ kg/s}$$

(b) Volume rate

$$Q = m/\rho = 3.317/1.128 = 2.94\ \text{m}^3/\text{s}$$

(c) Exit properties of air

$$P_2 - P_1 = \Delta P$$
$$P_2 = P_1 + \Delta P = 100 + \frac{7357.5}{10^3}$$
$$P_2 = 107.36\ \text{kPa}$$
$$W/m = C_p(T_2 - T_1)$$
$$T_2 = T_1 + \frac{W}{mC_p} = 309 + \frac{8256.47}{1005}$$
$$T_2 = 317.22\ \text{K}$$

Example 3.10 A backward-swept centrifugal fan develops a pressure of 75 mm W.G. It has an impeller diameter of 89 cm and runs at 720 rpm. The blade air angle at tip is 39° and the width of the impeller is 10 cm. Assuming a constant radial velocity of 9.15 m/s and density of 1.2 kg/m³, determine the fan efficiency, discharge, power required, stage reaction, and pressure coefficient.

Solution

$\Delta H = 0.075$ m W.G. $\quad D_2 = 0.89$m
$N = 720$ rpm $\quad \beta_2 = 39°$ $\quad b_2 = 0.1$ m $\quad C_r = 9.15$m/s
$\rho = 1.2\ \text{kg/m}^3$

(a) Fan efficiency

$$\text{Ideal work done/kg} = \Delta P/\rho = (\rho_w g \Delta H)/\rho = (10^3 \times 9.81 \times 0.075)/1.2 = 613.13\ \text{J/kg}$$

Actual work done/kg

$$W/m = U_2 C_{x_2}$$

From outlet velocity triangle (Refer Fig. 3.3(b)),

$$C_{x_2} = U_2 - W_{x_2} = U_2 - (C_r/\tan\beta_2)$$
$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.89 \times 720}{60} = 33.55\ \text{m/s}$$

$$\therefore C_{x_2} = 33.55 - (9.15/\tan 39°) = 22.26\ \text{m/s}$$
$$W/m = 33.55 \times 22.26 = 746.823\ \text{J/kg}$$

$$\text{Fan efficiency } \eta_f = \frac{\text{Ideal work done/kg}}{\text{Actual work done/kg}} = \frac{613.13}{746.823} = 82.1\%$$

(b) Discharge

$$Q = \pi D_2 b_2 C_r = \pi(0.89)(0.1)(9.15) = 2.558\ \text{m}^3/\text{s}$$

(c) Power required

$$m = \rho Q = 1.2(2.558) = 3.07\ \text{kg/s}$$
$$\therefore W = m(W/m) = 3.07(746.823) = 2292.7\ \text{W} = 2.2927\ \text{kW}$$

(d) Stage reaction

$$R = 1 - \frac{1}{2}\frac{C_{x_2}}{U_2} = 1 - \frac{1}{2}\left(\frac{22.26}{33.55}\right) = 0.6683 \text{ or } = 66.83\%$$

(e) Pressure coefficient

$$\psi_p = 2\frac{C_{x_2}}{U_2} \qquad \text{since} \quad \psi_p = \frac{U_2 C_{x_2}}{U_2^2/2}$$
$$= 2\left(\frac{22.26}{33.55}\right) = 1.327$$

Example 3.11 A backward-swept ($\beta_2 = 30°$) centrifugal fan with impeller diameter of 46.6 cm is required to deliver 3.82 m³/s (4.29 kg/s) of air at a total pressure of 63 mm W.G. The flow coefficient at the impeller exit is 0.25 and the power supplied to the impeller is 3 kW. Determine the fan efficiency, pressure coefficient, degree of reaction, rotational speed, and impeller width at exit.

Solution

$\beta_2 = 30°$ $\quad D_2 = 0.466$ m $\quad Q = 3.82\ m^3/s$

$m = 4.29$ kg/s $\quad \Delta H = 0.063$ mW.G.

$\phi_2 = \dfrac{C_{r_2}}{U_2} = 0.25 \quad W = 3$ kW

(a) Fan efficiency

$$\begin{aligned} \text{Ideal work done} &= m\left(\frac{\Delta P_0}{P}\right) \\ &= Q\Delta P_0 = Q(\rho g \Delta H) \\ &= 3.82(10^3 \times 9.81 \times 0.063) \\ &= 2.36 \text{ kW} \\ \text{Actual work done} &= 3 \text{ kW} \\ \therefore \text{Fan efficiency} &= \frac{2.36}{3} \\ \eta_f &= 78.7\% \end{aligned}$$

(b) Pressure coefficient

$$\psi_p = \frac{2C_{x_2}}{U_2}$$

From outlet velocity triangle (Refer Fig. 3.3(b)),

$$\begin{aligned} C_{x_2} &= U_2 - W_{x_2} = U_2 - (C_{r_2}/\tan\beta_2) \\ U_2C_{x_2} &= U_2\left[U_2 - (C_{r_2}/\tan\beta_2)\right] = W/m = \frac{3\times10^3}{4.29} \end{aligned}$$

Since $C_{r_2} = 0.25U_2$ and $\beta_2 = 30°$

$$\begin{aligned} U_2\left[U_2 - (0.25U_2/\tan 30°)\right] &= \frac{3\times10^3}{4.29} \\ 0.57U_2^2 &= 699.93 \\ U_2 &= 35.03 \text{ m/s} \\ C_{r_2} &= 0.25U_2 = 0.25\times35.03 = 8.76 \text{ m/s} \\ \therefore C_{x_2} &= U_2 - (C_{r_2}/\tan\beta_2) \\ &= 35.03 - (8.76/\tan 30°) \\ &= 19.86 \text{ m/s} \\ \psi_p &= \frac{2\times19.86}{35.03} \\ &= 1.134 \end{aligned}$$

(c) Degree of reaction

$$\begin{aligned} R &= 1 - \frac{1}{2}\frac{C_{x_2}}{U_2} \\ &= 1 - \frac{1}{2}\left(\frac{19.86}{35.03}\right) \\ &= 71.7\% \end{aligned}$$

(d) Rotational speed

$$\begin{aligned} N &= \frac{U_2\times60}{\pi D_2} = \frac{35.03\times60}{\pi\times0.466} \\ &= 1435.7 \text{ rpm} \end{aligned}$$

(e) Impeller width at exit

$$\begin{aligned} b_2 &= \frac{Q}{\pi D_2 C_{r_2}} = \frac{3.82}{\pi\times0.466\times8.76} \\ &= 0.298 \text{ m} \\ &= 29.8 \text{ cm} \end{aligned}$$

Example 3.12 A centrifugal blower runs at a speed of 3000 rpm. The impeller outer diameter is 75 cm and the blades of the impeller are designed for a constant radial velocity of 57 m/s. There are no inlet guide vanes so that the absolute velocity at the inlet is axial. If the degree of reaction is 0.58, compute the exit blade angle. Also determine the power input to the blower (total-to-total efficiency of 0.75) and the exit stagnation pressure. Take the total pressure and temperature at the inlet as 1 atm and 25°c respectively. *(MU-April '98)*

Solution

$N = 3000$ rpm $\quad D_2 = 0.75$ m $\quad C_{r_2} = 57$ m/s

$C_{x_1} = 0 \quad C_1 = C_a \quad R = 0.58 \quad \eta_c = 0.75$

(a) Exit blade angle

$$\begin{aligned} U_2 &= \frac{\pi D_2 N}{60} = \frac{\pi\times0.75\times3000}{60} \\ &= 117.81 \text{ m/s} \end{aligned}$$

The degree of reaction is

$$\begin{aligned} R &= 1 - \frac{C_{x_2}}{2U_2} \\ C_{x_2} &= 2(1-R)U_2 = 2(1-0.58)117.81 \\ &= 98.96 \text{ m/s} \end{aligned}$$

From the outlet velocity triangle (Refer Fig. 3.3(b)).

$$W_{x_2} = U_2 - C_{x_2}$$
$$= 117.81 - 98.96$$
$$= 18.85 \text{ m/s}$$

and

$$\tan\beta_2 = \frac{C_{r_2}}{W_{x_2}}$$
$$\beta_2 = \tan^{-1}\left(\frac{57}{18.85}\right)$$
$$\beta_2 = 71.7°$$

(b) Power input

$$\frac{W}{m} = U_2 C_{x_2} \quad \because C_{x_1} = 0$$
$$= (117.81 \times 98.96)/10^3$$
$$= 11.658 \text{ kW/(kg/s)}$$

(c) Exit stagnation pressure

$$\eta_c = \frac{T_{01}\left[\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_{02} - T_{01}}$$

$$T_{02} - T_{01} = W/mC_p = 11.658/1.005$$
$$= 11.6 \text{ K}$$

$$\therefore \frac{P_{02}}{P_{01}} = \left[1 + \frac{\eta_c(T_{02} - T_{01})}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$$

$$= \left[1 + \frac{0.75 \times 11.6}{298}\right]^{\frac{1.4}{0.4}}$$
$$= 1.106$$

$$\therefore P_{02} = 1.106 \times 101.325$$
$$P_{02} = 112.07 \text{ kPa}$$

Example 3.13 A centrifugal fan has the following data: inner diameter of the impeller–18 cm, outer diameter of the impeller–20 cm, the absolute velocity at entry is 21 m/s and at exit is 25 m/s, relative velocity at inlet and exit are 20 m/s and 17 m/s respectively, speed –1450 rpm, flow rate –0.5 kg/s, and motor efficiency 78%. Determine (a) stage total pressure rise (b) degree of reaction and (c) power required to drive the fan. Assume density of air as 1.25 kg/m³. [*MU-Oct '98*]

Solution

$D_1 = 0.18$ m $D_2 = 0.2$ m $C_1 = 21$ m/s $C_2 = 25$ m/s
$W_1 = 20$ m/s $W_2 = 17$ m/s $N = 1450$ rpm
$m = 0.5$ kg/s $\eta_m = 0.78$ $\rho = 1.25$ kg/m³

(a) Stage total pressure rise

$$\Delta P_0 = \rho(h_{02} - h_{01})$$

but

$$h_{02} - h_{01} = (h_2 - h_1) + \frac{(C_2^2 - C_1^2)}{2}$$

and

$$h_2 - h_1 = \frac{(U_2^2 - U_1^2)}{2} + \frac{(W_1^2 - W_2^2)}{2}$$

Therefore, the stage total pressure rise is

$$\Delta P_0 = \rho\left[\frac{U_2^2 - U_1^2}{2} + \frac{W_1^2 - W_2^2}{2} + \frac{C_2^2 - C_1^2}{2}\right]$$

$$U_1 = \frac{\pi \times 0.18 \times 1450}{60} = 13.67 \text{ m/s}$$

$$U_2 = \frac{\pi \times 0.2 \times 1450}{60} = 15.18 \text{ m/s}$$

$$\Delta P_0 = \left[\frac{15.18^2 - 13.67^2}{2} + \frac{20^2 - 17^2}{2} + \frac{25^2 - 21^2}{2}\right] \times 1.25$$
$$= [21.78 + 55.5 + 92] \times 1.25$$
$$\Delta P_0 = 211.6 \text{ N/m}^2$$

(b) Degree of reaction

$$R = \frac{(\Delta P)_{rotor}}{(\Delta P_0)_{stage}}$$

$$\Delta P = \rho(h_2 - h_1)$$
$$= \rho\left[\frac{U_2^2 - U_1^2}{2} + \frac{W_1^2 - W_2^2}{2}\right]$$
$$= 1.25[21.78 + 55.5]$$
$$= 96.6 \text{ N/m}^2$$

$$\therefore R = \frac{96.6}{211.6}$$
$$R = 0.457$$

(c) Power input

$$= \frac{\text{Work done/sec}}{\eta_m}$$

$$\text{Work done/sec} = m[h_{02} - h_{01}]$$

$$= m\left[\frac{U_2^2 - U_1^2}{2} + \frac{W_1^2 - W_2^2}{2} + \frac{C_2^2 - C_1^2}{2}\right]$$

$$= 0.5[21.78 + 55.5 + 92]$$

$$= 84.64 \text{ W}$$

$$\therefore \text{Power input} = \frac{84.64}{0.78}$$

$$= 108.5 \text{ W}$$

Example 3.14 A centrifugal fan rises the static pressure of air by 14 cm of water, while running at a speed of 650 rpm and consuming 85 metric HP as power. The static pressure and temperature of the air at the fan intake are respectively 75 cm Hg and 25°C, while the mass flow rate of air is 260 kg/min. Find the exit static pressure and the volume flow rate in m^3/min. *(MKU - April '94)*

Solution

$\Delta H = 0.14$ m of H_2O	$N = 650$ rpm
Power = 85 MHP	$H_1 = 0.75$ m of Hg
$T_1 = 273 + 25 = 298$ K	$m = 260$ kg/ min
1metric H.P = 0.735 kW	

(a) Exit static pressure

$$P_2 = P_1 + \Delta P$$

$$P_1 = \rho_{Hg} g H_1 = 13,590 \times 9.81 \times 0.75/10^3$$

$$= 99.988 \text{ kPa}$$

and

$$\Delta P = \rho_w g \Delta H = 1000 \times 9.81 \times 0.14/10^3$$

$$= 1.3734 \text{ kPa}$$

$$\therefore P_2 = 99.988 + 1.3734$$

$$P_2 = 101.36 \text{ kPa}$$

(b) Volume flow rate

$$Q = m/\rho_1$$

$$= 260/\rho_1$$

$$\rho_1 = P_1/RT_1 = 99.988 \times 10^3/(287 \times 298)$$

$$= 1.169 \text{ kg/m}^3$$

$$\therefore Q = 260/1.169$$

$$= 222.4 \text{ m}^3/\text{min}$$

SHORT QUESTIONS

3.1. What is centrifugal compressor?
3.2. Centrifugal compressors and fans are employing ———— effects to increase the pressure of the gas.
3.3. Centrifugal compressors can handle contaminated gas. (True/False)
√ 3.4. What are the advangates and disadvantages of centrifugal compressors?
3.5. What are the applications of centrifugal compressors?
3.6. Differentiate between the following terms.
 (a) Compressor
 (b) Blower
 (c) Fan
3.7. The impeller vanes at the eye are bent why?
3.8. The direction of flow of gas in the impeller of a centrifugal compressor is
 (a) Axial
 (b) Radial
 (c) Tangential
3.9. Impeller of a centrifugal compressor converts all the mechanical energy supplied into
 (a) pressure energy only
 (b) kinetic energy only
 (c) kinetic and pressure energies
3.10. What is the function of a diffuser in a centrifugal compressor?
3.11. Draw the pressure and velocity variation across a centrifugal compressor
3.12. It is conventional to measure blade angles from ——— direction in centrifugal compressors
3.13. The air-flow angle at inlet, measured from the radial direction is
 (a) 45°
 (b) 90°
 (c) 0°
3.14. Draw the inlet and outlet velocity triangles for a centrifugal compressor with radial/backward-curved/forward-curved blade impeller.
3.15. What is no-shock condition?
3.16. What is no-slip condition?
3.17. Angle β_2' is ——— than angle β_2 due to slip.
3.18. Define fluid slip in centrifugal compressor.
3.19. How can the fluid slip be reduced in centrifugal compressors?
3.20. Define slip factor.
3.21. Write the Stodola slip factor equation.
3.22. The Stanitz slip factor equation for radial vanes is
 (a) $\sigma_s = 1 - 0.63\pi/z$
 (b) $\sigma_s = 1 - 0.63z/\pi$
 (c) $\sigma_s = (1 - 0.63\pi)/z$

3.23. The theoretical work done on the air in a centrifugal compressor is
(a) $\sigma_s U_1^2$
(b) $U_2^2/2$
(c) $\sigma_s U_2^2$

3.24. Slip occurs even if the fluid is frictionless. (True/False)

3.25. Define power input factor (or) work factor (or) stage loading coefficient.

3.26. The impeller constant is
(a) $I = h_{0,rel} - U^2/2$
(b) $I = h_0 - U^2/2$
(c) $I = h_{0,rel} - W^2/2$

3.27. Define overall total-to-total isentropic efficiency.

3.28. Define pressure coefficient.

3.29. The pressure coefficient and overall isentropic efficiency are related by the following equation.
(a) $\varphi_P = \eta_c \cdot \sigma_s / \varphi$
(b) $\varphi_P = \eta_c \cdot \sigma_s$
(c) $\eta_c = \phi_P / \phi\sigma_s$

3.30. A simple volute recovers 25–30 per cent of the available kinetic energy at impeller exit. (True/False)

3.31. The diffusion efficiency of a vaneless diffuser is higher than that of a vaned diffuser. (True/False)

3.32. In a vaned diffuser, a more uniform total flow occurs when the number of diffuser passage is less than the number of impeller passages. (True/False)

3.33. The divergence angle of the diffusion passage is in the order of
(a) $8 - 10°$
(b) $20 - 40°$
(c) $30 - 45°$

3.34. Define diffuser efficiency.

3.35. Define degree of reaction.

3.36. The degree of reaction for radial vanes is
(a) $R = (1 + \phi_2^2)/2$
(b) $R = (1 - \phi_2^2)/2$
(c) $R = (1 - \phi_2^2)$

3.37. What are the different blade shapes of centrifugal impellers?

3.38. It is more difficult to decide on a choice of motor for radial and forward facing vanes impellers. Explain.

3.39. Forward facing vanes have higher pressure ratios. (True/False)

3.40. Better efficiencies can be obtained from backward facing vanes. (True/False)

3.41. Vane impellers are usually preferred in centrifugal compressors. Explain.

3.42. What are the advantages of inlet guide vanes?

3.43. What is surging?

3.44. How can the surging occurance be reduced?

3.45. What is rotating stall?

3.46. Rotating stall is a result of reduced mass flow rate. (True/False)

3.47. The stall propagates in *opposite* direction to blade motion and its speed is *low* compared to the compressor speed. (True/False)

EXERCISES

3.1. Draw the sketch of a centrifugal compressor stage indicating the principal parts.

3.2. Draw sketches of the three types of impellers and the velocity triangles at their entries and exits.

3.3. Draw an enthalpy-entropy diagram for a centrifugal compressor stage showing static and stagnation values of pressure and enthalpy at various sections.

3.4. Prove that

$$h_{01,rel} - \frac{U_1^2}{2} = h_{02,rel} - \frac{U_2^2}{2}$$

3.5. What is fluid slip? Define slip factor. Give three formulae to calculate the slip factor.

3.6. Derive Stodola's relation for the slip factor?

3.7. What is pressure coefficient of a centrifugal compressor stage? Derive

$$\varphi_p = 1 - \phi_2 \cot \beta_2$$

3.8. Define power input factor and compressor overall isentropic efficiency.

3.9. Prove that

$$\varphi_p = \eta_c \varphi \sigma_s$$

where φ_p–pressure coefficient, η_c is compressor efficiency, σ_s – slip factor and φ – power input factor.

3.10. Write short notes on
(a) Volute or scroll collector
(b) Vaneless diffuser
(c) Vaned diffuser

3.11. How is the degree of reaction of a centrifugal compressor stage defined. Prove that

$$R = \frac{1 - \phi_2^2 \operatorname{cosec}^2 \beta_2}{2(1 - \phi_2 \cot \beta_2)}$$

where ϕ is flow coefficient and β_2 is blade outlet angle.

3.12. Deduce that the degree of raction R for a centrifugal compressor with radial impeller vanes is given by

$$R = (1 - \phi_2^2)/2$$

3.13. Briefly explain the effect of each impeller vane on the performance of centrifugal compressor. Why is the radial tipped impeller most commonly used in centrifugal compressor stages?

3.14. Explain briefly the purpose of inlet guide vanes.

3.15. How do the Mach numbers at the entries of the impeller and diffuser affect the flow and efficiency of a centrifugal compressor stage? On what considerations are the limiting values of these Mach numbers decided?

3.16. Explain the phenomena of surging, stalling and choking in centrifugal compressor stage? What is their effect on the performance? How to minimise or prevent them.

3.17. A 580 kW motor drives a centrifugal compressor of 480 mm outer diameter at a speed of 20,000 rpm. At the impeller outlet the blade angle is 26.5° measured from the radial direction and the flow velocity at exit from the impeller is 122 m/s. If a mechanical efficiency of 95 per cent is assumed, what air flow is to be expected? Assume there is no slip. What are the eye tip and hub diameters if a radius ratio of 0.3 is chosen for the impeller eye and if the velocity at inlet is 95 m/s with zero whirl? What will be the overall total-to- total isentropic efficiency if an overall total pressure ratio of 5.5 is required? Assume that the flow inlet is incompressible and ambient air conditions are 101.3 kPa and 288 K.

[**Ans:** (a) 2.47 kg/s, (b) 172 mm and 51.6 mm (c) 81.8%]

3.18. A centrifugal compressor impeller has 17 radial vanes of tip diameter 165 mm. It rotates at 46,000 rpm and the air mass flow rate is 0.6 kg/s with no whirl at inlet. (a) Calculate the theoretical power transferred to the air. At inlet to the impeller, the mean diameter of the eye is 63.5 mm while the annulus height at the eye is 25 mm. The static pressure and temperature at the impeller inlet are 93 kPa and 293 K respectively. Determine (b) the blade angle at the mean diameter at impeller inlet (c) the stagnation temperature at impeller exit, and (d) the stagnation pressure at impeller exit if the total-to-total efficiency of the impeller is 90 per cent.

[**Ans:** (a) 83.76 kW, (b) 35.4°, (c) 437.8 K, and (d) 338.67 kPa]

3.19. A centrifugal compressor is desired to have a total pressure ratio 4:1. The inlet eye to the compressor impeller is 30 cm in diameter. The axial velocity at inlet is 130 m/s and the mass flow rate is 10 kg/s. The velocity in the delivery duct is 115 m/s. The tip speed of the impeller is 450 m/s, runs at 16,000 rpm with total head isentropic efficiency of 78% and pressure coefficient is 0.72. The ambient conditions are 1 bar and 15°C. Calculate (a) static pressure and temperature at inlet and exit of compressor (b) static pressure ratio (c) work done/kg of air (d) theoretical power required to drive the compressor.

(MKU-April 1993 April 1995)

[**Ans:** (a) 279.6 K, 0.9 bar and 3.89 bar, 460.86 K (b) 4.32 (c) 180.34 kJ/kg and (d) 1803.4 kW]

3.20. A single-sided centrifugal compressor delivers 8.15 kg/s of air with a pressure ratio of 4.4 to 1 at 18,000 rpm. The entry to the eye for which the internal diameter is 12.7 cm is axial and the mean veloicty at the eye section is 148 m/s with no prewhirl. Static conditions at the eye section are 15°C and 1 bar. The slip factor is 0.94 and the isentropic efficiency of compression is 78%. Neglecting losses calculate (a) the rise in temperature during compression if the kinetic energy is negligible (b) the tip speed of impeller eye and tip speed of impeller outlet and (c) outer diameter of impeller eye.

(MKU-Nov.'92)

[**Ans:** (a) 202.2 K (b) 257 m/s and 464.95 m/s, and (c) 49 cm]

3.21. The data of a centrifugal compressor are given below.
Outer diameter of the impeller = 50 cm
Tip diameter of the eye = 28 cm
Hub diameter to the eye = 14 cm
Speed 16,000 rpm
Mass of air handled = 10 kg/s
Inlet total pressure = 1.15 bar
Inlet total temperature = 20°C
Slip factor = 0.91
Total-to-total efficiency = 75% for zero whirl at entry. Determine the total pressure ratio developed and the power required to drive the compressor.

(MU-Oct. '97)

[**Ans:** (a) 3.3 and (b) 1596.7 kW]

3.22. A centrifugal compressor has inlet guide vanes at the eye such that free vortex flow is achieved at entry to the blades. At the tip radius of the eye the inlet relative Mach number is not to exceed 0.75 and an impeller total-to-total efficiency of 0.9 is required. The air leaves the tip of the inlet guide vanes with a velocity of 90 m/s, the impeller tip diameter is 0.45 m and the outlet diameter is 0.76 m. The radial component of velocity at exit from the impeller is 50 m/s and the impeller rotates at 12,000 rpm. If a slip factor of 0.9 is assumed, find the guide vane inlet angle at the tip and the static pressure at impeller outlet. Assume T_{01} = 288 K and P_{01} = 101.3 kPa.

[**Ans:** (a) 62.1° and (b) 247.2 kPa]

3.23. A centrifugal compressor compresses air at ambient temperature and pressure of 288 K and 101.3 kPa respectively. The impeller runs at a tipspeed of 365 m/s, the radial velocity at exit from the impeller is 30 m/s and the slip factor is 0.9. Calculate the Mach number of the flow at the impeller tip. If the impeller total-to-total efficiency is 90 per cent and the flow area from the impeller is 0.093 m^2. Calculate the mass flow rate of air. Assume zero whirl at inlet and radial blades.

[**Ans:** (a) 0.876 (b) 5.13 kg/s]

✓ 3.24. A compressor operating at a pressure ratio of 3.8 and a speed of 12,000 rpm delivers 8 kg/s of air. The slip factor is assumed to be 0.92, the power factor 1.04 and the overall isentropic efficiency 0.82. Calculate the impeller outlet diameter. Assume zero whirl. The Mach number of the air leaving the impeller vanes is to be unity so as to ensure that no shocks occur. If the losses in the impeller and the diffuser are the same, what must be the axial depth of the impeller. At inlet P_{01} = 101.3 kPa and T_{01} = 288 K.

[**Ans:** (a) 0.659 m (b) 23.5 mm]

3.25. A centrifugal compressor with an overall pressure ratio of 4 has an impeller speed of 320 m/s and the flow area at impeller outlet is 0.12 m^2. The radial component of the velocity at impeller exit is 30 m/s and the slip factor is 0.9. Calculate the absolute Mach number at the exit and the flow rate if the total-to-total efficiency is 90%. Take T_{01} = 288 K and P_{01} = 1 bar.

[**Ans:** (a) 0.786 and (b) 9.87 kg/s]

3.26. A centrifugal compressor works with no whirl at entry and has radial exit. The slip factor is 0.91. The rotor tip speed is 625 m/s. The mass flow rate is 15 kg/s. The ambient conditions are 98 kPa and 23°C. The pressure ratio is 7. Calculate (a) isentropic efficiency (b) work required per kg of air (c) power supplied for a mechanical efficiency of 97%.

[Ans: (a) 62% (b) 355.47 kJ/kg and (c) 5496.93 kW]

3.27. The following data refer to a centrifugal compressor. Impeller tip diameter = 100 cm, speed = 5950 rpm, mass flow rate of air = 30 kg/s. Static pressure ratio $P_2/P_1 = 2.125$: atmospheric pressure and temperature are 1 bar and 25°C. Slip factor = 0.90 and mechanical efficiency = 0.97. Find (a) the adiabatic efficiency of the impeller, (b) the temperature of the air at the exit, (c) the shaft power input and (d) the pressure coefficient.

(MKU-April-1994)

[Ans: (a) 82.3% (b) 385.3 K (c) 2709.3 kJ/kg and (d) 0.742]

3.28. A double-sided centrifugal compressor has impeller eye root and tip diameters 180 mm and 300 mm and is to deliver 16 kg of air per second at 16,0000 rpm. The design ambient conditions are 17°C and 1 bar. Calculate suitable values for the impeller vane angles at root and tip of the eye if air is given 20° measured from the radial direction of prewhirl at all radii. The axial component of inlet velocity is constant over the eye and is equal to 150 m/s.

[Ans: (a) 57.33° and (b) 37.33°]

3.29. A centrifugal blower takes in 180 m^3/min of air at $P_1 = 1.013$ bar and $t_1 = 43$° C and delivers it at 750mm W.G. Taking the efficiencies of the blower and drive as 80% and 82% respectively, determine the power required to drive the blower and the state of air at exit.

(MU-April '97 & Oct'99)

[Ans: (a) 33.65 kW (b) 1.0866 bar and 324.2 K]

3.30. A centrifugal fan with a radial impeller produces a pressure equivalent to 100 cm column of water. The pressure and temperature at its entry are 0.98 bar and 310 K. The electric motor driving the blower runs at 3000 rpm. The efficiencies of the fan and drive are 82% and 88%, respectively. The radial velocity remains constant and has a value of $0.2\,U_2$. The velocity at the inlet eye is $0.4\,U_2$. If the blower handles 200 m^3/min of air at the entry conditions, determine (a) power required by the electric motor (b) impeller diameter (c) inner diameter of the blade ring (d) blade air angle at entry and (e) impeller widths at entry and exit.

[Ans: (a) 45.3 kW (b) 66.4 cm (c) 31.9 cm (d) 22.6° and (e) 15.95 cm and 7.66 cm]

3.31. A fan running at 1480 rpm takes in 6 m^3/min of air at inlet conditions of $P_1 = 950$ m bar and $t_1 = 15$°C. If the fan impeller diameter is 40 cm and the blade tip air angle is 20°, determine the total pressure developed by the fan and the impeller width at exit. The radial velocity at the exit is 0.2 times the impeller tip speed. State the assumptions used.

[Ans: (a) 50.67 mm W.G. and (b) 1.28 cm]

3.32. A centrifugal compressor runs at 15,000 rpm and produces a stagnation pressure ratio of 4 between the impeller inlet and outlet. The stagnation conditions of air

at the compressor intake are 1.01325 bar and 25°C respectively. The absolute velocity at the compressor inlet is axial. If the compressor has radial blades at the exit such that the exit relative velocity is 135 m/s and the compressor total-to-total efficiency is 0.78. Compute the slip, slip coefficient and absolute velocity at the compressor exit if the rotor diameter at the outlet is 580 mm.

[Ans: (a) 45.9 m/s (b) 0.8993 and (c) 431.3 m/s]

3.33. An aircraft engine is fitted with a single-sided centrifugal compressor. The aircraft flies with a speed of 850 km/h at an altitude where the pressure is 0.23 bar and the temperature 217 K. The inlet duct of the impeller eye contains fixed vanes which give the air pre-whirl of 25° at all radii. The inner and outer diameter of the eye are 180 and 330 mm respectively. The diameter of the impeller tip is 540 mm and the rotational speed 16,000 rpm. Estimate the stagnation pressure at the compressor outlet when the mass flow is 216 kg/min. Assume the isentropic efficiency to be 0.8. Take the slip factor as 0.9 and the power input factor as 1.04.

[Ans: 1.63 bar]

3.34. A centrifugal fan with an efficiency of 80% runs at 720 rpm. Its impeller diameter is 1 m. The impeller tip angle is backward curved to 51° tangent to the wheel. The density of the air is 1.25 kg/m^3 and mass flow rate is 3 kg/s. The impeller width at the exit is 10 cm. Determine the power required, pressure coefficient, stage reaction, pressure head developed, and flow coefficient at exit. Assume zero whirl at inlet and the mechanical efficiency is 82%.

[Ans: (a) 4.35 kW (b) 1.34 (c) 0.58 (d) 1.188 kN/m^2 and (e) 0.203]

3.35. Air flows through a blower where its total pressure is increased by 15 cm of water head. The inlet total pressure and temperature are 105 kPa and 15°C. The total-to-total efficiency is 75%. Estimate

(a) exit total pressure and temperature and

(b) isentropic and actual change in total head enthalpy.

(MU-April '96)

[Ans: (a) 106.5 kPa, 289.5 K and (b) 1.152 kJ/kg, 1.536 kJ/kg]

3.36. A centrifugal compressor compresses 30 kg of air per sec. It runs at 15,000 rpm. The air enters the compressor axially. The radius at exit of blade is 300 mm. The relative velocity of air at exit tip is 100 m/s. The relative air angle at exit is 80°. Find the power and ideal head developed.

(MU-April '96)

[Ans: (a) 6416.5 kW and (b) 213.9 kJ/kg]

4

AXIAL FLOW COMPRESSORS AND FANS

INTRODUCTION

Axial flow compressors and fans are turbo machines that increase the pressure of the gas flowing continuously in the axial direction. Due to lack of knowledge of the aerodynamic behaviour, a reversed reaction turbine was used as an axial flow compressor, in early days. The efficiency was less than 40 per cent. Then, study of aerodynamic behaviour helped in designing the blades for axial flow compressors. Nowadays, the efficiency of the axial flow compressors surpass the maximum centrifugal compressor efficiency by about 4 per cent. But the efficiency of the axial flow compressor is very sensitive to the mass flow rate. Any deviation from the design condition causes the efficiency to drop off drastically. Thus the axial flow compressor is ideal for constant load applications such as in aircraft gas turbine engines. They are also used in fossil fuel power stations, where gas turbines are used to meet the load exceeding the normal peak load.

Advantages of Axial Flow Compressors

1. Axial flow compressor has higher efficiency than radial flow compressor.
2. Axial flow compressor gives higher pressure ratio on a single shaft with relatively high efficiencies.
3. Pressure ratio of 8:1 or even higher can be achieved using multistage axial flow compressors.
4. The greatest advantage of the axial flow compressor is its high thrust per unit frontal area.
5. It can handle large amount of air, inspite of small frontal area.

The main disadvantages are its complexity and cost.

DESCRIPTION OF AN AXIAL FLOW COMPRESSOR

An axial flow compressor consists of fixed and movable set of blades in alternating sequence. Moving blades are attached to the periphery of a rotor hub followed by fixed blades attached to the walls of the outer casing (Fig. 4.1) At the inlet of the compressor, an extra row of fixed vanes called *inlet guide vanes* are fitted. These do

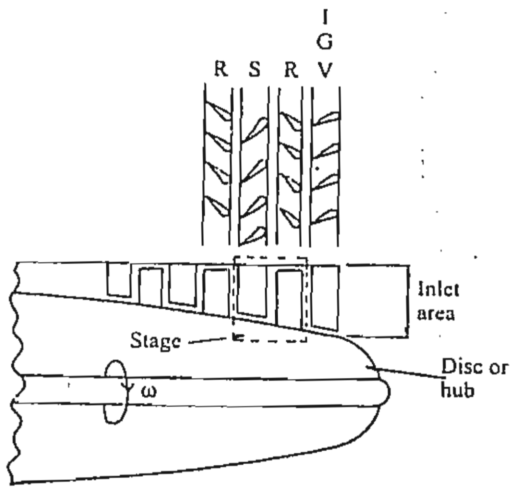


Figure 4.1 An axial compressor stage

not form part of the compressor stage but are solely to guide the air at the correct angle onto the first row of moving blades. The blades height is decreased as the fluid moves through the compressor. This is to maintain a constant axial velocity through the compressor as the density increases from the low to high pressure regions. A constant axial velocity is convenient from the point of view of design, but this by no means is a requirement. The flow through the stage is assumed to take place at a mean blade height, where the blade peripheral velocities at inlet and outlet are the same. There is no flow in the radial direction. Whirl components of velocity exist in the direction of blade motion.

WORKING PRINCIPLE

The kinetic energy is imparted to the air by the rotating blades, which is then converted into a pressure rise. So, the basic principle of working is similar to that of the centrifugal compressor.

Referring to Fig. 4.1, the air enters axially from the right into the inlet guide vanes, where it is deflected by a certain angle to impinge on the first row of rotating blades with the proper angle of attack. The rotating vanes add kinetic energy to the air. There is a slight pressure rise to the air. The air is then discharged at the proper angle to the first row of stator blades, where the pressure is further increased by diffusion. The air is then directed to the second row of moving blades and the same process is repeated through the remaining compressor stages. In most of the compressors, one to three rows of diffuser or straightener blades are installed at the end of the last stage to straighten and slow down the air before it enters into the combustion chamber.

VELOCITY TRIANGLES FOR AN AXIAL FLOW COMPRESSOR STAGE

In studying the flow of the fluid through an axial compressor, it is usual to consider the changes taking place through a compressor stage. The analysis for flow through the stage is assumed to be two dimensional and to take place at a mean blade height.

The rotor and stator rows of a stage are shown in Fig. 4.2.

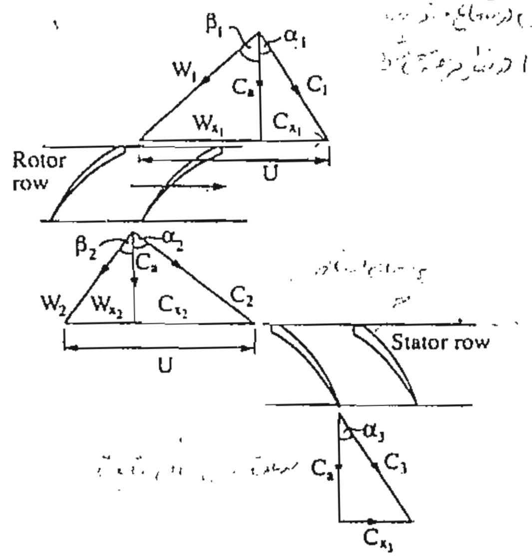


Figure 4.2 Velocity triangles for an axial flow compressor stage

A stage consists of a row of moving blades attached to the periphery of a rotor hub followed by a row of fixed blades attached to the walls of the outer casing.

The compressor is made up of a number of such stages to give an overall pressure ratio from the inlet to outlet.

Air exits from the previous row of stator bladers at an angle α_1 with absolute velocity C_1. The rotor row has tangential velocity U, and combining the two velocity vectors gives the relative velocity vector W_1 at an angle β_1. At rotor row outlet, the absolute velocity vector C_2 moves into the stator row where the flow direction is changed from α_2 to α_3 with absolute velocity C_3. If the following stage is same as the preceding one, then the stage is said to be normal. For a normal stage

$$C_1 = C_3 \text{ and } \alpha_1 = \alpha_3$$

W_2 is less than W_1, showing that diffusion of relative velocity has taken place with some static pressure rise across the rotor blades.

The air is turned towards the axial direction by the blade camber and the effective flow area is increased from inlet to outlet, thus causing diffusion to take place. Similar diffusion of the absolute velocity takes place in the stator, where the absolute velocity vector is again turned towards the axial direction and a further static pressure rise occurs.

Note that all angles are referred to the axial velocity vector C_a. The diagrams are drawn with a large gap (for clarity) between rotor and stator blades. But in practice, there is only a small clearance between them.

ENERGY TRANSFER OR STAGE WORK

The energy given to the air per unit mass flow rate is given by Euler's equation.

$$W/m = U_2 C_{x_2} - U_1 C_{x_1}$$

or

$$E = (U_2 C_{x_2} - U_1 C_{x_1})/g \tag{4.1}$$

From the velocity triangles, C_a is constant through the stage and $U_1 = U_2 = U$.

$$C_{x_2} = U - C_a \tan\beta_2$$

and

$$C_{x_1} = U - C_a \tan\beta_1$$
$$C_{x_2} - C_{x_1} = C_a(\tan\beta_1 - \tan\beta_2)$$

Therefore

$$E = UC_a(\tan\beta_1 - \tan\beta_2)/g \tag{4.2}$$

The energy transfer may also be written in terms of the absolute velocity flow angles.

$$E = UC_a(\tan\alpha_2 - \tan\alpha_1)/g \tag{4.3}$$

Equation (4.2) or (4.3) may be used depending upon the information available.

MOLLIER CHART

The flow through the axial flow compressor stage is shown thermodynamically on the Mollier chart (Fig. 4.3) and is similar to that of a centrifugal compressor.

Assuming adiabatic flow through the stage, $h_{03} = h_{02}$, and so equation for work supplied is

$$W/m = h_{02} - h_{01} \tag{4.4}$$

Writing

$$h_0 = h + \frac{C^2}{2}$$
$$= h + (C_a^2 + C_x^2)/2$$

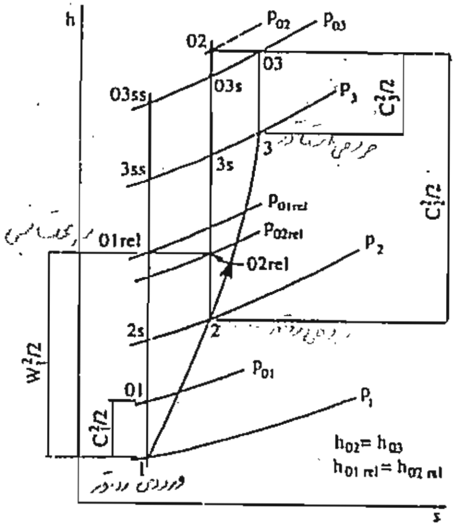


Figure 4.3 Mollier chart for an axial flow compressor stage

Then $h_{02} - h_{01} = (h_2 - h_1) + (C_{x_2}^2 - C_{x_1}^2)/2 = U(C_{x_2} - C_{x_1})$
or $(h_2 - h_1) - (C_{x_2} - C_{x_1})[2U - (C_{x_2} + C_{x_1})]/2 = 0$
Rearranging

$$(h_2 - h_1) - (C_{x_2} - C_{x_1})[(U - C_{x_2}) + (U - C_{x_1})]/2 = 0$$
$$(h_2 - h_1) + (W_{x_2} - W_{x_1})(W_{x_2} + W_{x_1})/2 = 0$$
$$(h_2 - h_1) + (W_{x_2}^2 - W_{x_1}^2)/2 = 0$$

But
$(W_{x_2}^2 - W_{x_1}^2) = (W_2^2 - W_1^2)$, since C_a is constant. Therefore

$$h_2 + \frac{W_2^2}{2} = h_1 + \frac{W_1^2}{2} \tag{4.5}$$

(or) $h_{02,rel} = h_{01,rel}$
where the relative total enthalpy is based on the relative velocity.

Equation (4.5) shows that the total enthalpy based on relative velocities in the rotor is constant across the rotor and this result is also valid for the '*axial flow gas turbine rotor*'. It is already proved that the change in enthalpy for a centrifugal compressor is $[I = h_{0,rel} - U^2/2]$.

$$h_2 - h_1 = (U_2^2 - U_1^2)/2 + (W_1^2 - W_2^2)/2 \tag{4.6}$$

A comparison of equation (4.6) with equation (4.5) indicates why the enthalpy change in a single stage axial flow compressor is so low compared to the centrifugal compressor. The relative velocities may be of the same order of magnitude, but the axial flow compressor receives no contribution from the change in tangential velocity (U).

The isentropic or overall total-to-total efficiency is written as

$$\eta_c = \frac{\text{Ideal isentropic work input}}{\text{Actual work input}}$$

$$= \frac{\text{Total isentropic enthalpy rise in the stage}}{\text{Actual enthalpy rise between the same total pressure limits}}$$

$$= \frac{h_{03ss} - h_{01}}{h_{03} - h_{01}}$$

which reduces to

$$\eta_c = T_{01}(T_{03ss}/T_{01} - 1)/(T_{03} - T_{01}) \tag{4.7}$$

Putting $P_{03}/P_{01} = (T_{03ss}/T_{01})^{\gamma/\gamma-1}$
the pressure ratio becomes

$$P_{03}/P_{01} = [1 + \eta_c(T_{03} - T_{01})/T_{01}]^{\gamma/(\gamma-1)} \tag{4.8}$$

The energy input to the fluid will be absorbed in raising the pressure and velocity of the air and some will be wasted in overcoming various frictional losses.

However, the whole of the work input will appear as a stagnation temperature rise in the air regardless of the isentropic efficiency.

Equating the work in terms of temperature and air angles,

$$(T_{03} - T_{01}) = UC_a(\tan\beta_1 - \tan\beta_2)/C_p \tag{4.9}$$

WORK DONE FACTOR

In practice 'C_a' is not constant along the length of the blade and, to account for this, a work done factor is introduced. It is defined as

$$\text{Work done factor, } \lambda = \frac{\text{Actual work absorbing capacity}}{\text{Ideal work absorbing capacity}}$$

Hence,

$$(T_{03} - T_{01}) = \lambda UC_a(\tan\beta_1 - \tan\beta_2)/C_p \tag{4.10}$$

A graph is drawn between the percentage of blade length and $\dfrac{C_a}{C_{a,\text{mean}}}$ (Fig. 4.4). It illustrates that it is only at the inlet of the machine that the velocity profile, i.e. C_a, is fairly constant. As the air moves through the compressor the change in C_a is more. This is due to the influence of the solid boundaries of the rotor and stator.

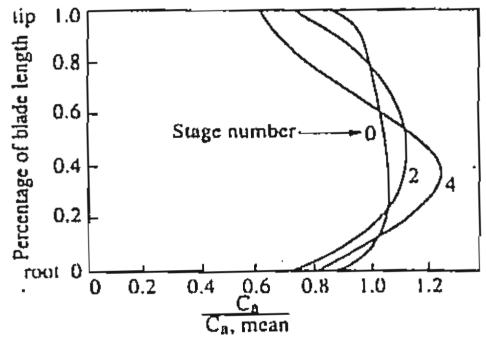


Figure 4.4 Variation of axial velocity along a blade

The variation in work done factor (λ) with stage number is shown in the Fig. 4.5. It shows that λ decreases as the number of compressor stage increases.

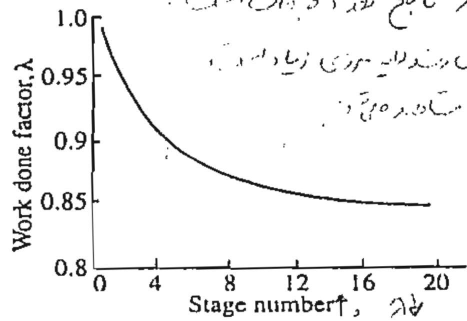


Figure 4.5 Variation of work done factor with number of stages

STAGE LOADING (OR) PRESSURE COEFFICIENT

Stage loading $(\psi)_P$ is defined as the ratio of the power input to the term mU^2.

$$\psi_p = \frac{\text{Work input}}{mU^2}$$

$$\psi_p = W/mU^2 = (h_{03} - h_{01})/U^2$$

$$= \lambda(C_{x_2} - C_{x_1})/U \tag{4.11}$$

$$= \lambda(C_a/U)(\tan\alpha_2 - \tan\alpha_1)$$

$$\psi_p = \lambda\phi(\tan\alpha_2 - \tan\alpha_1) \tag{4.12}$$

where ϕ is the flow coefficient.

REACTION RATIO

The *reaction ratio* is a measure of the static enthalpy rise that occurs in the rotor expressed as a percentage of the total static enthalpy rise across the stage. It is defined as

$$R = \frac{\text{Static enthalpy rise in rotor}}{\text{Static enthalpy rise in stage}}$$

$$= \frac{h_2 - h_1}{h_3 - h_1} \tag{4.13}$$

Since $h_{01rel} = h_{02rel}$ then, $h_2 - h_1 = (W_1^2 - W_2^2)/2$,
Also if $C_1 = C_3$ then, $h_3 - h_1 = (h_{03} - h_{01}) = U(C_{x_2} - C_{x_1})$
and substituting for $(h_2 - h_1)$ and $(h_3 - h_1)$ in equation 4.13.

$$R = (W_1^2 - W_2^2)/[2U(C_{x_2} - C_{x_1})] \tag{4.14}$$

$$= \frac{(C_a^2 + W_{x_1}^2) - (C_a^2 + W_{x_2}^2)]}{[2U(C_{x_2} - C_{x_1})]}$$

$$= \frac{(W_{x_1} + W_{x_2})(W_{x_1} - W_{x_2})}{[2U(C_{x_2} - C_{x_1})]}$$

But $C_{x_2} = U - W_{x_2}$ and $C_{x_1} = U - W_{x_1}$. Therefore,

$$(C_{x_2} - C_{x_1}) = W_{x_1} - W_{x_2}$$

Hence,

$$R = (W_{x_1} + W_{x_2})/2U \tag{4.15}$$

$$= C_a(\tan\beta_1 + \tan\beta_2)/2U$$

$$= \frac{C_a}{U}\tan\beta_m$$

$$= \phi\tan\beta_m \tag{4.16}$$

where, $\tan\beta_m = (\tan\beta_1 + \tan\beta_2)/2$
β_m – mean relative velocity vector angle and flow co-efficient

$$\phi = \frac{C_a}{U} = \frac{\text{axial velocity}}{\text{blade velocity}}$$

Similarly, the reaction ratio can be expressed in different forms as follows:
Substituting for W_{x_1} in equation 4.15

$$W_{x_1} = U - C_{x_1}$$

$$R = \frac{U - C_{x_1} + W_{x_2}}{2U}$$

$$= \frac{1}{2} + \frac{W_{x_2} - C_{x_1}}{2U}$$

$$= \frac{1}{2} + \frac{C_a\tan\beta_2 - C_a\tan\alpha_1}{2U}$$

$$= \frac{1}{2} + \frac{C_a}{U}\left(\frac{\tan\beta_2 - \tan\alpha_1}{2}\right)$$

$$R = [1 + \phi(\tan\beta_2 - \tan\alpha_1)]/2 \tag{4.17}$$

Similarly, substituting for W_{x_2} in equation (4.15).

$$W_{x_2} = U - C_{x_2}$$

$$R = \frac{W_{x_1} + U - C_{x_2}}{2U}$$

$$= \frac{1}{2} + \frac{W_{x_1} - C_{x_2}}{2U}$$

$$= \frac{1}{2} + \frac{C_a\tan\beta_1 - C_a\tan\alpha_2}{2U}$$

$$= \frac{1}{2} + \frac{C_a}{U}\left(\frac{\tan\beta_1 - \tan\alpha_2}{2}\right)$$

$$= \frac{1}{2} + \phi\left(\frac{\tan\beta_1 - \tan\alpha_2}{2}\right)$$

$$R = [1 + \phi(\tan\beta_1 - \tan\alpha_2)]/2 \tag{4.18}$$

For the case of incompressible and reversible flow, the expression used for reaction ratio is of static pressure rise in the rotor to the static pressure rise in the stage

$$R = (P_2 - P_1)/(P_3 - P_1)$$

In the case of compressible and irreversible flow, a more general definition of R is in terms of static enthalpies.

$$R = (h_2 - h_1)/(h_3 - h_1)$$

EFFECT OF REACTION RATIO ON THE VELOCITY TRIANGLES

Case – 1 When $R = 0.5$.
The reaction ratio R is

$$R = \frac{h_2 - h_1}{(h_3 - h_2) + (h_2 - h_1)} \tag{4.18(a)}$$

when R is 0.5

$$(h_2 - h_1) = (h_3 - h_2)$$

For a reaction ratio of 50 per cent, the static enthalpy and temperature increase in the stator and rotor are equal.
Also from the equation (4.17),

$$R = [1 + \phi(\tan\beta_2 - \tan\alpha_1)]/2$$

When $R = 0.5$,

$$\beta_2 = \alpha_1$$

So, when the outlet and inlet velocity triangles are superimposed, the resulting velocity diagram is symmetrical (Fig. 4.6a).

Case – 2 When $R > 0.5$.

From the equation of reaction ratio (4.17), it is seen that $\beta_2 > \alpha_1$, and from equation 4.18(a), the static enthalpy rise in the rotor is greater than in the stator.

Since $\beta_2 > \alpha_1$, the superimposed velocity triangle is skewed to the right (Fig. 4.6b)

Case – 3 When $R < 0.5$.

From the equation for reaction ratio 4.18(a), it is found that the static enthalpy rise and pressure rise are greater in the stator than in the rotor and from eqn. 4.17, we get $\beta_2 < \alpha_1$ So, the superimposed velocity triangle is skewed to the left (Fig. 4.6c).

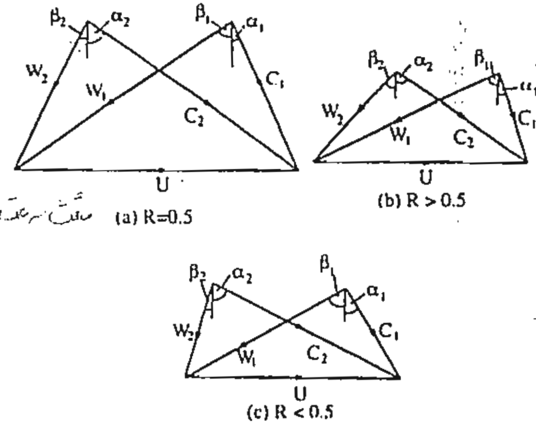


Figure 4.6 Effect of reaction ratio on the velocity triangles

A reaction ratio of 50 per cent is widely used as the adverse pressure gradient over the stage is shared equally by the stator and rotor. This choice of reaction minimises the tendency of the blade boundary layers to separate from the solid surfaces, thus avoiding large stagnation pressure losses. A reaction ratio of 50 per cent is the condition for maximum temperature rise and efficiency.

STATIC PRESSURE RISE

The main function of a compressor is to raise the static pressure of air. The ideal compressor stage is considered first, which has no stagnation pressure losses. Across the rotor row, $P_{0_{rel}}$ is constant and the equation is

$$P_2 - P_1 = \frac{1}{2}\rho(W_1^2 - W_2^2) \tag{4.19}$$

Across the stator row, P_0 is constant and the equation is

$$P_3 - P_2 = \frac{1}{2}\rho(C_2^2 - C_3^2) \tag{4.20}$$

Adding the equation (4.19) and (4.20), the pressure rise in each row and considering a normal stage ($C_3 = C_1$), gives

$$(P_3 - P_1) \times \frac{2}{\rho} = (C_2^2 - W_2^2) + (W_1^2 - C_1^2) \tag{4.21}$$

$$(\Delta P)_{Stage} = (\Delta P)_{Rotor} + (\Delta P)_{Stator}$$

From the velocity triangles, the cosine rule gives

$$C^2 = U^2 + W^2 - 2UW\cos\left(\frac{\pi}{2} - \beta\right)$$

and $\quad W\sin\beta = W_x$

or

$$C^2 - W^2 = U^2 - 2UW_x \tag{4.22}$$

Substituting equation 4.22 in equation 4.21

$$\begin{aligned} 2(P_3 - P_1)/\rho &= (U^2 - 2UW_{x_2}) - (U^2 - 2UW_{x_1}) \\ &= 2U(W_{x_1} - W_{x_2}) \end{aligned} \tag{4.23}$$

From the velocity diagram, we get

$$\begin{aligned} U_1 &= U_2 = U \\ \therefore W_{x_1} + C_{x_1} &= W_{x_2} + C_{x_2} \\ \text{or } W_{x_1} - W_{x_2} &= C_{x_2} - C_{x_1} \\ \therefore (P_3 - P_1)/\rho &= U(C_{x_2} - C_{x_1}) = h_3 - h_1 \end{aligned} \tag{4.24}$$

Since, for an isentropic process,

$$Tds = 0 = dh - (dP/\rho) \text{ and therefore } (\Delta h)_s = \Delta P/\rho.$$

The pressure rise in a real stage (involving irreversible process) can be determined, if the stage efficiency is known.

STAGE EFFICIENCY

Stage efficiency is defined as the ratio of the isentropic enthalpy rise to the actual enthalpy rise corresponding to the same finite pressure change.

$$\eta_s = \frac{\Delta h_s}{(\Delta h)_{act}} = \frac{(\Delta P/\rho)}{(\Delta h)_{act}} \tag{4.25}$$

Thus

$$(\Delta P/\rho) = \eta_s(\Delta h)_{act}$$

$$\therefore (\Delta P)_{act}/\rho = \eta_s U.\Delta C_x \quad (4.26)$$

If $C_1 = C_3$, then η_s is a very close approximation of the total-to-total efficiency η_{tt}.

$$\begin{cases} \eta_s = \dfrac{h_{3_{ss}} - h_1}{h_3 - h_1} \\ \eta_{tt} = \dfrac{h_{03_{ss}} - h_{01}}{h_{03} - h_{01}} \end{cases}$$

If $C_1 = C_3$

$$\eta_s = \frac{h_{3_{ss}} - h_1}{h_3 - h_1} \quad (4.27)$$

Although the above expressions are derived for incompressible flow they are nevertheless, a valid approximation for compressible flow, if the stage temperature (and pressure) rise is small.

PRESSURE COEFFICIENT (ψ_p)

It is defined as the ratio of the actual stagnation enthalpy change to the kinetic energy of a fluid, which has the same speed as the blades. Thus

$$\boxed{\psi_p = \frac{\Delta h_0}{U^2/2}} \quad (4.28)$$

$$= \frac{2\lambda C_a(\tan\beta_1 - \tan\beta_2)}{U}$$

$$= 2\lambda\phi(\tan\beta_1 - \tan\beta_2) \quad (4.29)$$

If the stage pressure change is so small then the fluid can be considered as incompressible,

$$\Delta h_0 = (\Delta P_0)/\rho \quad (4.30)$$

So that

$$\boxed{\psi_p = \frac{(\Delta P_0)}{\rho U^2/2}} \quad (4.31)$$

The pressure coefficient usually ranges from *0.4* to *0.7*.
This definition for ψ_p has a numerical value twice the numerical value of the equation

$$\psi_p = \lambda\phi(\tan\alpha_2 - \tan\alpha_1)$$

COMPRESSOR STALL AND SURGE

The most significant phenomenon affecting the performance of axial flow compressors is the *stall*. The blade stall is mainly due to the reduced flow rate (or reduced C_a) as

this results in positive incidence. At starting and low speed operation, little pressure rise takes place. Where as at the design point (i.e. at higher rpm) of the compressor, large pressure rise occurs from first to the last stage. Thus in a low speed operation, the absence of pressure in the last stages leads to higher values of C_a than the designed one, and this leads to choking. Choking limits the flow rate and this in turn increases the positive incidence in the first stage and hence there is a possibility of stall. Low speed stall occurs in front stages. At high speeds, the last stages are more likely to stall because their characteristic is steep and a small reduction in flow rate produces a considerable reduction of velocity and thus a large change of incidence.

All the blades are not stalled at the same time, but a stall is propagated from blade to blade in a direction opposite to the direction of rotation. This propagation speed is low compared to the rpm (compressor). Thus the stall appears to be in the direction of rotation. This phenomenon is called *'propagation'* or *'rotating stall'*.

Consider a portion of a blade row shown in Fig. 4.7. Let the blade A be stalled. This increases the incidence to the left of blade A, and reduces the incidence of the flow on to the blades on the right. The blade B will be the next to stall.

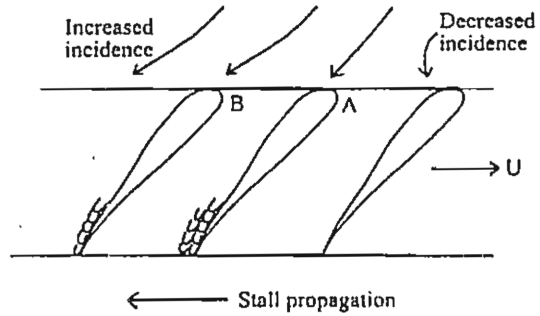


Figure 4.7 Compressor stall

Sufficiently extensive stall results in a sufficient reduction of total flow and surging takes place. So, stalling leads to surging.

RADIAL EQUILIBRIUM METHOD

The assumption of two dimensional flow is no longer valid when the ratio of hub/tip is less than 0.8. In the case of air craft compressors, mass flow is high and frontal area is small. To meet this requirement, longer blades on a smaller hub is used. So, radial velocities set up in the blade row changes the mass flow and the outlet velocity distribution. In such cases, the blades should not be designed on two dimensional principle. Radial flow is caused by a temporary imbalance between the centrifugal and radial pressure forces acting on the fluid. When these forces balance each other, there is no radial velocity and the fluid is said to be in radial equilibrium. This method of analysis is known as the *'radial equilibrium method'*. This analysis examines the pressure forces acting on an element of fluid and equates these forces to the centrifugal forces acting on the fluid.

RADIAL EQUILIBRIUM THEORY تعادل شعاعی

If the absolute velocity C is resolved into three components in the tangential, axial and radial directions,

$$\boxed{C^2 = C_x^2 + C_a^2 + C_r^2} \tag{4.32}$$

The stagnation pressure may be written as

$$P_0 = P + \rho C^2/2$$

or

$$P_0 = P + \rho\frac{C_x^2}{2} + \rho\frac{C_a^2}{2} + \rho\frac{C_r^2}{2} \tag{4.33}$$

For steady flow conditions along concentric stream lines, the velocity, static pressure and density are constant with time and $dC_r = 0$. Let 'r' be the radius of any concentric streamline. Then differentiating with respect to r, we get

$$\frac{dP_0}{dr} = \frac{dP}{dr} + \rho C_x\frac{dC_x}{dr} + \rho C_a\frac{dC_a}{dr} \tag{4.34}$$

But from fluid mechanics, for plane circulatory flow and for radial equilibrium,

$$\frac{dP}{dr} = \rho\frac{C_x^2}{r}$$

$$\therefore \frac{dP_0}{dr} = \rho\frac{C_x^2}{r} + \rho C_x\frac{dC_x}{dr} + \rho C_a\frac{dC_a}{dr} \tag{4.35}$$

The energy transferred to the blades is at the expense of the energy in the fluid and may be expressed in terms of the stagnation enthalpy change dh_o.

Writing the Tds equation using stagnation properties, we have

$$\boxed{Tds = dho - \frac{dP_0}{\rho}}$$

Differentiating w.r.t. 'r'

$$\frac{Tds}{dr} = \frac{dho}{dr} - \frac{dP_0}{\rho dr}$$

$$\text{or}\ \frac{dP_0}{\rho dr} = \frac{dh_o}{dr} - \frac{Tds}{dr} \tag{4.36}$$

Substituting equation 4.36 in equation 4.35

$$\frac{dh_o}{dr} - \frac{Tds}{dr} = \frac{C_x^2}{r} + C_x\frac{dC_x}{dr} + C_a\frac{dC_a}{dr}$$

To simplify this equation the following assumptions are made

$$\frac{dh_o}{dr} = 0 \text{ (Since energy transfer is constant at all radii)}$$

$$\text{and } \frac{Tds}{dr} = 0$$

$$\therefore \frac{C_x^2}{r} + C_x\frac{dC_x}{dr} + C_a\frac{dC_a}{dr} = 0 \tag{4.36a}$$

This equation is equivalent to

$$\frac{1}{r^2}\frac{d}{dr}(rC_x)^2 + \frac{d}{dr}(C_a)^2 = 0 \tag{4.37}$$

Because, differentiating eqn. 4.37, we get eqn. 4.36a.

$$\frac{1}{r^2}\left[2rC_x \cdot C_x\frac{dr}{dr} + 2rC_x r \cdot \frac{dC_x}{dr} + 2C_a \cdot \frac{dC_a}{dr} = 0\right]$$

$$2\frac{C_x^2}{r} + 2C_x\frac{dC_x}{dr} + 2C_a\frac{dC_a}{dr} = 0$$

(or)

$$\frac{C_x^2}{r} + C_x\frac{dC_x}{dr} + C_a\frac{dC_a}{dr} = 0$$

TYPES OF BLADES

(a) Free Vortex Blade

In this case the tangential velocity distribution has to be such that

$$\boxed{C_x r = \text{constant}}$$

With this condition, the equation 4.37 becomes

$$\frac{dC_a}{dr} = 0$$

$$\therefore C_a = \text{constant}$$

So, axial velocity C_a is constant along the blade height i.e. from rotor tip to stator hub. And reaction ratio R varies with radius, i.e. it decreases from tip to hub.

Work and reaction ratio in free vortex blade From Euler's equation.

$$\boxed{W = U(C_{x_2} - C_{x_1}) = \omega r(C_{x_2} - C_{x_1})}$$

For free vortex blade, the condition to be satisfied is

$$C_x r = \text{constant}$$

i.e. the whirl velocity is inversely proportional to the radius. Applying the conditions between two sections,

$$C_{x_1} \cdot r = C_{x_{1,m}} \cdot r_m = x_1 \text{ constant}$$

$$C_{x_2} \cdot r = C_{x_{2,m}} \cdot r_m = x_2 \text{ constant}$$

where subscript m refers to values at the mean radius r_m.

Then,

$$W = \omega_r(C_{x_2} - C_{x_1}) = \omega r\left(\frac{C_{x2,m} r_m}{r} - \frac{C_{x1,m} r_m}{r}\right)$$
$$= \omega r_m (C_{x2,m} - C_{x1,m}) = \omega(x_2 - x_1)$$
$$= \text{constant}$$

Thus, a machine with free vortex blades is a constant-work machine.
Reaction Ratio

$$R = \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2)$$
$$= \frac{1}{2U}(W_{x_1} + W_{x_2})$$
$$= \frac{1}{2U}[(U - C_{x_1}) + (U - C_{x_2})]$$
$$= \frac{2U - (C_{x_1} + C_{x_2})}{2U}$$

and for free vortex condition

$$\boxed{R = 1 - \frac{x_1 + x_2}{2Ur} = 1 - \frac{x_1 + x_2}{2\omega r^2}}$$

The reaction ratio varies with the radius. Therefore, at one designed point only the design reaction ratio can be obtained. This blade design method has the disadvantage that the highest Mach numbers occur at the rotor tip and at the stator hub.

(b) Forced Vortex or Solid Rotation Blades

For this type of flow the condition is

$$\boxed{\frac{C_{x_1}}{r_1} = \frac{C_{x_2}}{r_2} = x\,(\text{constant})}$$

To achieve the radial equilibrium the axial velocity decreases from root to tip. Since C_a is not constant along the blade height, it is determined at a particular section, using the following expressions.

At inlet:

$$C_{a1}^2 = K_1 - 2\left(\frac{C_{x_1}}{r}\right)^2 \cdot r^2$$

where r = section radius and $K_1 = C_{a1,m}^2 + 2C_{x1,m}^2$

At outlet:

$$C_{a2}^2 = K_2 - 2\left(\frac{C_{x_2}}{r}\right)^2 \cdot r^2 + 2\left(\frac{C_{x_2}}{r} - \frac{C_{x_1}}{r}\right)\omega r^2$$

where $\omega = \dfrac{2\pi N}{60}$ and

$$K_2 = C_{a2,m}^2 + 2C_{x2,m}^2 - 2\left(\frac{C_{x2,m}}{r_m} - \frac{C_{x1,m}}{r_m}\right)\omega r_m^2$$

For forced vortex design, $C_{x_2} = rx_2$ and $C_{x_1} = rx_1$

$$\therefore W = Ur(x_2 - x_1)$$

In this type of blade the energy transfer increases from root to tip (Fig. 4.8(a) & (b)).

There is a limit to hub/tip ratio in order to provide a practical minimum value of axial velocity C_a.

(a) At hub (or root) (b) At tip

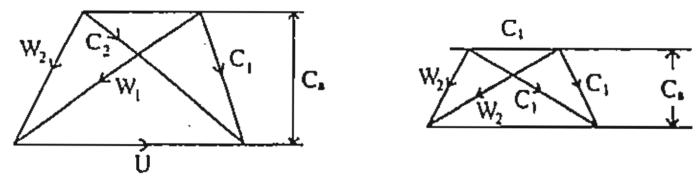


Figure 4.8 Work done in a forced vortex blade

(c) Constant Reaction Blade

For true radial equlibrium, C_a should vary with radius, but constant reaction blades have been commonly used with constant C_a. As W_1 and C_2 decrease only slightly from tip to root, almost constant Mach number occurs at different radii.

So, in constant reaction blade, the axial velocity C_a and the reaction ratio 'R' are constant at all radii i.e from tip to root. 'R' is equal to 0.5.

MULTISTAGE COMPRESSION

The total pressure ratio across a single stage is dependent upon the total temperature rise across the stage, whereas, in multistage compression, the rise in pressure across each stage for the same temperature rise per stage is not equal. For example, if 'P_{or}' is the pressure ratio for one stage, then the total pressure ratio is given by $(P_{or})^N$, where 'N' is the total number of stages. This condition is not true.

The difference is seen in the Mollier chart (Fig. 4.9). For the same temperature rise per stage, as the entropy increases, the pressure rise decreases.

If we compress in a single compression from 1 to 5, the isentropic work done is

$$\boxed{W/m = (h_{(N+1)_s} - h_1)}$$

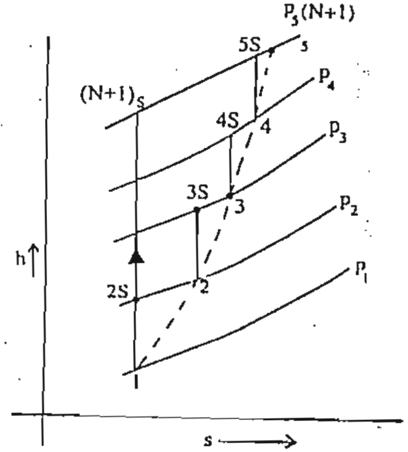


Figure 4.9 Compression process in a multistage compressor

and the isentropic efficiency of the compression is

$$\eta_c = (h_{(N+1)_s} - h_1)/(h_{N+1} - h_1)$$

If we now compress from 1 to 5 in a number of small finite stages, the isentropic work done is

$$W_s/m = (h_{2s} - h_1) + (h_{3s} - h_2) + (h_{4s} - h_3) + (h_{5s} - h_4)$$

and for similarly designed stages, the efficiency 'η_s' is the same. Hence

$$\eta_s = (W_s/m)/(h_{N+1} - h_1)$$

where the numerator consists of a number of isentropic enthalpy increases. But, as the entropy increases through the compression, the constant pressure lines diverge and

$$(h_{2s} - h_1) + (h_{3s} - h_2) + (h_{4s} - h_3) + (h_{5s} - h_4) > (h_{(N+1)_s} - h_1)$$

and thus

$$\boxed{\eta_s > \eta_c}$$

That is, the overall single isentropic compression efficiency is less than the stage efficiency. The difference also increases with pressure ratio and with the number of stages.

The overall static pressure ratio of a multistage compressor can be expressed in terms of small stage efficiency as

$$\boxed{P_{N+1}/P_1 = (T_{N+1}/T_1)^{\eta_p \cdot r/(r-1)}}$$

If it is assumed that we have equal total temperature rise in each stage and denoting the inlet conditions by 01 and outlet conditions at the last stage as 0N+1, then for 'N' stages

$$P_{0N+1}/P_{01} = (T_{0N+1}/T_{01})^{\eta_p \cdot r/(r-1)}$$

Also

$$T_{0N+1}/T_{01} = (T_{01} + N\Delta T_0)/T_{01}$$

where ΔT_0 is the stage total temperature rise. It is also usual to assume that the polytropic and total-to-total stage isentropic efficiencies are equal at a value of about 0.88.

Using this method, a very rapid calculation of pressure rise through the compressor can be made.

COMPARISON BETWEEN CENTRIFUGAL COMPRESSOR AND AXIAL FLOW COMPRESSOR

The comparison between the main features of centrifugal and axial flow compressors are given below

(1) Direction of Flow across the Compressor

In centrifugal compressors, the flow through the compressor impeller takes place largely in a plane which is perpendicular to the axis of the compressor.

In axial flow compressors, the flow proceeds throughout the compressor in a direction essentially parallel to the axis of the compressor.

(2) Pressure Rise per Stage

Centrifugal compressor has a high pressure rise per stage (4:1). The axial flow compressor however gives a pressure ratio of only 1.2:1 per stage. To achieve the desired pressure rise, the axial flow unit has to be provided with a large number of stages which makes the axial flow compressor less compact when compared with the equivalent centrifugal unit.

(3) Isentropic Efficiency

The isentropic efficency of centrifugal compressors is as high as 80%. Earlier axial flow machines had a low isentropic efficiency, but with aerofoil blading, a multistage axial flow compressor surpasses the maximum centrifugal compressor efficiency by about 4 per cent. The efficiency in both cases however declines at high pressure ratios.

(4) Range of Operation

Centrifugal compressors have a wide range of operation between surging and choking limits. A greater flexibility in operation can be achieved by the use of adjustable prewhirl and diffuser vanes. High efficiency for axial flow compressor is attained only within a narrow range of speed at the design pressure.

(5) Frontal Area

Centrifugal compressors have a larger frontal area than that of an axial flow compressor for the same rating. This makes the axial flow compressor more suitable for aircraft work.

(6) Working Fluid

The performance of centrifugal compressor is not much affected by the accumulation of deposits on the surface of flow passage when working with contaminating fluids. Under such conditions, the performance of axial flow compressor is generally impaired.

(7) Starting Torque

Centrifugal compressors need a lower starting torque than axial flow compressors for the same capacity.

(8) Construction

Centrifugal compressors have a simple, rigid and relatively cheap construction and also less prone to icing troubles at high altitudes.

(9) Multistaging

Multistaging is more suitable for an axial flow compressor where it gives an increase in pressure with negligible losses. The number of stages for the axial flow compressor varies from 5 to 14.

(10) Application

Centrifugal compressors have been successfully used as blowing machines in steel mills, compressors for low pressure refrigeration and industrial gases and turbochargers and superchargers in internal combustion engines. They have also been employed as compressors for small gas turbine aircrafts.

Axial flow compressors are mostly used in gas turbines and high pressure unit for industrial and large marine gas turbine plants.

CHARACTERISTIC CURVE

Fig. 4.10 shows the characteristic curve of a multistage axial flow compressor. Comparing this curve with that of the centrifugal compressor, it is observed that the pressure ratio of the centrifugal compressor is less sensitive to mass flow variations at a given speed than in the axial compressor. The characteristic curves have the following salient points.

1. The design mass flow and pressure ratio are at point 1. It is seen that the design point is very close to the surge line (point 5) and if the mass flow is only slightly reduced, the pressure ratio and density in the rear stages will both increase. Since $C_a = m/\rho A$, the axial velocity will decrease and hence the incidence angle α_1 will increase sharply in the rear stages, thereby causing stalling in these stages.

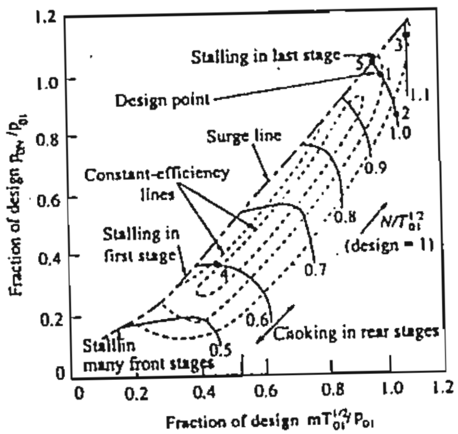


Figure 4.10 Axial flow compressor characteristic curves

2. A small increase in mass flow will lead to a sharp drop in pressure ratio to point 2. The density also drops sharply so that C_a increases. This results in the large decrease of the incidence angle in the rear stages, thereby causing stalling in the rear stages with negative incidence.
3. The operating point moves to point 4 if the speed of the compressor is reduced. Both m and C_a fall faster than the blade speed U resulting in an increased incidence angle and possible stalling in the first stage.
4. When the design speed is increased to point 3, the characteristic eventually becomes almost vertical. This will increase both the density and pressure ratio as the increased speed allows more air to be passed through the compressor. But at the inlet, the mass flow increases faster than the density and choking at the inlet is usually the first to occur.

All the limiting conditions discussed above should be avoided at all times since they lead to unstable or inefficient operation.

SOLVED PROBLEMS

Example 4.1 The following data refers to an axial-flow compressor: $\beta_1 = 60°$, turning angle = 30° and $\Delta C_x = 100$ m/s, degree of reaction 50%, rpm 36,000, mean blade diameter 140 mm, inlet pressure and temperature 2 bar and 57°C respectively. Find α_1, the pressure rise, the amount of air handled and power if the blade height is 20 mm.

Solution

$\beta_1 = 60°$	$\beta_1 - \beta_2 = 30°$	$\Delta C_x = 100$ m/s
$R = 0.5$	$N = 36,000$	$D = 0.140$ m
$P_1 = 2$ bar	$T_1 = 273 + 57 = 330$ K	$b = 0.02$ m

(a) Air flow angle (α_1)

Since $R = 0.5$, $\alpha_1 = \beta_2$,

$$\text{but, } \beta_2 = 60^\circ - 30^\circ = 30^\circ$$

$$\therefore \alpha_1 = 30^\circ$$

(b) The pressure rise

$$W/m = u\Delta C_x = C_p(T_2 - T_1)$$

$$\therefore T_2 = \frac{U\Delta C_x}{C_p} + T_1$$

Blade mean speed

$$U = \frac{\pi DN}{60} = \frac{\pi \times 0.14 \times 36,000}{60}$$

$$= 263.89 \text{ m/s}$$

$$\therefore T_2 = \frac{263.89 \times 100}{1005} + 330$$

$$= 356.26 \text{ K}$$

Assuming that there are no losses in the compressor, the pressure ratio can be determined from

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{r/r-1} = \left(\frac{356.26}{330}\right)^{\frac{1.4}{0.4}}$$

$$P_2 = 1.31 \times P_1 = 2.62 \text{ bar}$$

Pressure rise

$$\Delta P = P_2 - P_1 = 2.62 - 2 = 0.62 \text{ bar}$$

(c) The amount of air handled

$$m = \rho_1 A_1 C_a$$

Axial velocity, C_a, is given by

$$C_a = \frac{2U.R}{\tan\beta_2 + \tan\beta_1}$$

$$= \frac{2 \times 263.89 \times 0.5}{\tan 30^\circ + \tan 60^\circ}$$

$$= 114.27 \text{ m/s}$$

Flow area, $A_1 = \pi Db$

$$= \pi \times 0.14 \times 0.02$$

$$= 8.797 \times 10^{-3} \text{ m}^2$$

and

$$\rho_1 = \frac{P_1}{RT_1} = \frac{2 \times 10^5}{287 \times 330} = 2.11 \text{ kg/m}^3$$

$$\therefore m = 2.11 \times 8.797 \times 10^{-3} \times 114.27$$

$$m = 2.64 \text{ kg/s}$$

(d) Power

$$W = mC_p(T_2 - T_1)$$

$$= 2.64 \times 1.005(356.26 - 330)$$

$$W = 69.7 \text{ kW}$$

Example 4.2 The following data refers to a test on an axial flow compressor. Atmospheric temperature and pressure at inlet are 18°C and 1 bar. Total head temperature in delivery pipe is 165°C. Total head pressure in delivery pipe is 3.5 bar. Static pressure in delivery pipe is 3 bar. Calculate (a) total head isentropic efficiency, (b) polytropic eficiency, and (c) air Velocity in delivery pipe. *(MKU–April '91)*

Solution

$P_{01} = 1$ bar $\quad T_{01} = 291$ K

$T_{02} = 273 + 165 = 438$ K $\quad P_{02} = 3.5$ bar $\quad P_2 = 3$ bar

(a) Total head isentropic efficiency

$$\eta_c = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

$$T_{02s} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{r-1}{r}}$$

$$= 291\left(\frac{3.5}{1}\right)^{\frac{0.4}{1.4}}$$

$$= 416.24 \text{ K}$$

$$\therefore \eta_c = \frac{416.24 - 291}{438 - 291}$$

$$= 85.2\%$$

(b) Polytropic efficiency

$$\eta_p = \frac{\ln\left(\frac{P_{02}}{P_{01}}\right)}{\frac{r}{r-1}\ln\left(\frac{T_{02}}{T_{01}}\right)}$$

$$= \frac{\ln 3.5}{\frac{1.4}{0.4}\ln\left(\frac{438}{291}\right)}$$

$$= 87.5\%$$

(c) Air velocity in delivery pipe

$$T_2 = T_{02}\left(\frac{P_2}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}}$$
$$= 438\left(\frac{3}{35}\right)^{\frac{0.4}{1.4}}$$
$$= 419.13 \text{ K}$$

Now,

$$T_{02} = T_2 + \frac{C_2^2}{2C_P}$$
$$\therefore C_2 = \sqrt{2C_P(T_{02} - T_2)}$$
$$= \sqrt{2 \times 1005 \times (438 - 419.13)}$$
$$= 194.75 \text{ m/s}$$

Example 4.3 An eight stage axial flow compressor provides an overall pressure ratio of 6:1 with an overall isentropic efficiency 90%, when the temperature of air at inlet is 20°C. The work is divided equally between the stages. A 50 % reaction is used with a mean blade speed 188 m/s and a constant axial velocity 100 m/s through the compressor. Estimate the power required and blade angles. Assume air to be a perfect gas.

Solution

$N = 8$, $\frac{P_{0N}}{P_{01}} = 6$, $\eta_c = 0.9$, $T_{01} = 293$ K, $R = 0.5$, $U = 188$ m/s, $C_a = 100$ m/s.

For 50% reaction turbine, the blades are symmetrical and so the velocity diagrams are identical. $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$.

If the compression process were isentropic then the temperature of air leaving the compressor stage would be

$$T_{0(N+1)_s} = T_{01}\left(\frac{P_{0_{N+1}}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$
$$= 293(6)^{\frac{0.4}{1.4}}$$
$$= 488.9 \text{ K}$$

The overall isentropic efficiency is given by

$$\boxed{\eta_c = \frac{T_{0(N+1)_s} - T_{01}}{T_{0_{N+1}} - T_{01}}}$$

$$\therefore T_{0_{N+1}} = \frac{T_{0(N+1)_s} - T_{01}}{\eta_c} + T_{01}$$

$$= \frac{488.9 - 293}{0.9} + 293$$
$$= 510.67 \text{ K}$$

$$\text{Work done/kg} = C_p(T_{0_{N+1}} - T_{01}) = N \times UC_a(\tan\alpha_2 - \tan\alpha_1)$$
$$\tan\alpha_2 - \tan\alpha_1 = \frac{1.005 \times 10^3(510.67 - 293)}{(8 \times 188 \times 100)}$$
$$= 1.45$$

Since $\alpha_2 = \beta_1$, then

$$\tan\beta_1 - \tan\alpha_1 = 1.45 \quad (1)$$

From inlet velocity configuration (Fig. 4.2),

$$U = C_{x_1} + W_{x_1} = C_a(\tan\alpha_1 + \tan\beta_1)$$
$$\tan\alpha_1 + \tan\beta_1 = \frac{U}{C_a} = \frac{188}{100} = 1.88 \quad (2)$$

From (1) and (2),

$$\tan\beta_1 = \frac{1.45 + 1.88}{2} = 1.665$$
$$\beta_1 = 59° = \alpha_2$$

Substituting the value of $\tan\beta_1 = 1.665$ in (1), we get

$$\tan\alpha_1 = \tan\beta_1 - 1.45$$
$$= 1.665 - 1.45 = 0.215$$
$$\therefore \alpha_1 = 12° = \beta_2$$

Power required per kg of air/s.

$$= mC_P(T_{0_{N+1}} - T_{01})$$
$$= 1 \times 1.005(510.67 - 293)$$
$$= 218.76 \text{ kW}$$

Example 4.4 A multistage axial flow compressor absorbs 4.5 mW when delivering 20 kg/s of air from stagnation condition of 1 bar and 288 K. If polytropic efficiency of compression is 0.9 and if the stage stagnation pressure ratio is constant, calculate (a) pressure at compressor outlet, (b) the number of stages, (c) overall isentropic efficiency of compressor. Temperature rise in the first stage may be taken as 20°C.

Solution

$W = 4.5$ mW $\quad P_{01} = 1$ bar $\quad T_{01} = 288$ K

$\eta_P = 0.9 \quad \Delta T_0 = 20°$C $\quad T_{02} = T_{01} + \Delta T_0 = 288 + 20 = 308$ K

The compressor power

$$W = mC_P(T_{0_{N+1}} - T_{01})$$

$$\therefore T_{0_{N+1}} = \frac{W}{mC_P} + T_{01}$$

$$= \frac{4.5 \times 10^3}{20 \times 1.005} + 288$$

$$= 223.9 + 288$$

$$= 511.9 \text{ K}$$

(a) Pressure at compressor outlet

Pressure at compressor outlet is determined from the following relation.

$$\frac{P_{0_{N+1}}}{P_{01}} = \left(\frac{T_{0_{N+1}}}{T_{01}}\right)^{\eta_p \cdot r/r-1}$$

$$= \left(\frac{511.9}{288}\right)^{\frac{0.9\times1.4}{0.4}}$$

$$= 6.12$$

$$\therefore P_{0_{N+1}} = 6.12 \times 1 = 6.12 \text{ bar}$$

(b) Number of stages

Since pressure ratio for each stage is same,

$$\boxed{\frac{P_{02}}{P_{01}} = \frac{P_{03}}{P_{02}} = \frac{P_{04}}{P_{03}} = \frac{P_{0_{N+1}}}{P_{0_N}}}$$

where N is the number of stages.

$$\therefore \frac{P_{0_{N+1}}}{P_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^N$$

$$\text{or } N = \frac{\ln\left(\frac{P_{0_{N+1}}}{P_{01}}\right)}{\ln\left(\frac{P_{02}}{P_{01}}\right)}$$

where

$$\frac{P_{02}}{P_{01}} = \left(\frac{T_{02}}{T_{01}}\right)^{\frac{\eta_p r}{r-1}} = \left(\frac{308}{288}\right)^{\frac{0.9\times1.4}{0.4}}$$

$$= 1.24$$

Then

$$N = \frac{\ln\left(\frac{6.12}{1}\right)}{\ln(1.24)}$$

$$= 8.42$$

$\therefore$ Number of stages is 9.

(c) Overall Isentropic Efficiency

$$\boxed{\eta_c = \frac{T_{0_{(N+1)_s}} - T_{01}}{T_{0_{N+1}} - T_{01}}}$$

$$= \left(\frac{T_{0_{(N+1)_s}}}{T_{01}} - 1\right) \Big/ \left(\frac{T_{0_{N+1}}}{T_{01}} - 1\right)$$

but from isentropic relation

$$\frac{T_{0_{(N+1)_s}}}{T_{01}} = \left(\frac{P_{0_{N+1}}}{P_{01}}\right)^{r-1/r}$$

and

$$\frac{T_{0_{N+1}}}{T_{01}} = \left(\frac{P_{0_{N+1}}}{P_{01}}\right)^{\frac{r-1}{r\eta_p}} = \frac{511.9}{288} = 1.78$$

$$\frac{T_{0_{(N+1)_s}}}{T_{01}} = (6.12)^{\frac{0.4}{1.4}} = 1.68$$

$$\therefore \eta_c = \frac{1.68 - 1}{1.78 - 1} = 0.872$$

$$= 87.2\%$$

Example 4.5 An axial flow compressor of 50 % reaction design has blades with inlet and outlet angles of 44° and 13° respectively. The compressor is to produce a pressure ratio of 5:1 with an overall isentropic efficiency of 87 % when the inlet temperature is 290 K. The mean blade speed and axial velocity are constant throughout the compressor. Assuming a blade velocity is 180 m/s, and work input factor is 0.85. Find the number of stages required and the change of entropy.

Solution

a) Number of stages

$$R = 0.5 \qquad \beta_1 = \alpha_2 = 44° \qquad \beta_2 = \alpha_1 = 13°$$

$$\frac{P_{0_{N+1}}}{P_{01}} = 5 \qquad \eta_c = 0.87 \qquad T_{01} = 290 \text{ K}$$

$$U = C_u = 180 \text{ m/s} \qquad \lambda = 0.85$$

$$T_{0_{(N+1)_s}} = T_{01}\left(\frac{P_{0_{N+1}}}{P_{01}}\right)^{\frac{r-1}{r}}$$

$$= 290(5)^{\frac{0.4}{1.4}}$$

$$= 459.3 \text{ K}$$

$$T_{0_{N+1}} = \frac{T_{0_{(N+1)_s}} - T_{01}}{\eta_c} + T_{01}$$

$$= \frac{459.3 - 290}{0.87} + 290 = 484.6 \text{ K}$$

$$N \lambda U C_a(\tan \alpha_2 - \tan \alpha_1) = C_p(T_{0_{N+1}} - T_{01})$$

$$N = \frac{1005(484.6 - 290)}{0.85 \times 180 \times C_a \times (\tan 44^\circ - \tan 13^\circ)}$$

The axial velocity is

$$\frac{U}{C_a} = \tan \alpha_1 + \tan \beta_1$$

$$C_a = \frac{U}{\tan \beta_2 + \tan \beta_1} = \frac{180}{\tan 13^\circ + \tan 44^\circ}$$

$$= 150.4 \text{ m/s}$$

$$N = \frac{195573}{16909.1}$$

$$= 11.6$$

$$N \simeq 12$$

The number of stages is 12.

(b) Change of entropy

$$\Delta_s = C_p \ln \frac{T_{0_{N+1}}}{T_{01}} - R \ln \frac{P_{0_{N+1}}}{P_{01}}$$

$$= 1.005 \ln \frac{484.6}{290} - 0.287 \ln 5$$

$$= 0.054 \text{ kJ/kg} - \text{K}$$

Example 4.6 An axial compressor has a mean diameter of 60 cm and runs at 15,000 rpm. If the actual temperature rise and pressure ratio developed are 30°C and 1.3 respectively, determine (a) power required to drive the compressor while delivering 57 kg/s of air, assuming mechanical efficiency 86% and initial temperature of 35°C (b) the stage efficiency and (c) the degree of reaction if the temperature at the rotor exit is 55°C

(MKU-April '97)

Solution

$D = 0.6$ m $\qquad N = 15,000$ rpm

$\Delta T = T_3 - T_1 = 30°C \qquad \frac{P_3}{P_1} = 1.3 \qquad m = 57$ kg/s

$\eta_m = 0.86 \quad T_1 = 35°C \qquad T_2 = 55°C$

(a) Power required

$$\text{Work done} \quad W = m C_p \Delta T$$

$$= 57 \times 1.005 \times 30$$

$$= 1718.55 \text{ kW}$$

$$\text{Power input} = W/\eta_m$$

$$= 1718.55/0.86$$

$$= 1998.314 \text{ kW}$$

(b) Stage efficiency

$$\eta_s = \frac{T_{3s} - T_1}{T_3 - T_1}$$

$$= \frac{T_1\left[\left(\frac{P_3}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_3 - T_1}$$

$$= \frac{308\left[(1.3)^{\frac{0.4}{1.4}} - 1\right]}{30}$$

$$\eta_s = 79.92\%$$

(c) Reaction ratio

$$R = \frac{(\Delta h)_{\text{rotor}}}{(\Delta h)_{\text{stage}}} = \frac{(\Delta T)_{\text{rotor}}}{(\Delta T)_{\text{stage}}}$$

$$= \frac{T_2 - T_1}{T_3 - T_1} = \frac{55 - 35}{30}$$

$$R = 0.67$$

Example 4.7 The first stage of an axial flow compressor develops a pressure ratio of 1:2. The inlet pressure and temperature are 1.01 bar and 30°C respectively. The overall efficiency of compressor is 83%. The flow coefficient is 0.47. The velocity diagram is symmetrical and at the mean radius the ratio of change of whirl velocity to axial velocity is 0.5. Determine the compressor speed if the mean diameter is 50 cm. Also find the absolute velocity of the air leaving the stationary inlet guide vanes.

(MKU-May '97)

Solution

$\frac{P_3}{P_1} = 2 \qquad P_1 = 1.01$ bar $\qquad T_1 = 303$ K $\qquad \eta_c = 0.83$

$\phi = 0.47 \qquad \Delta C_x / C_a = 0.5 \qquad D = 0.5$ m

(a) Compressor speed

$$\Delta C_x = C_{x_2} - C_{x_1} = 0.5\, C_a$$

$$\phi = \frac{C_a}{U} = 0.47$$

$$C_a = 0.47\, U$$

$$\Delta C_x = 0.5\,(0.47\, U)$$

$$= 0.235\, U$$

$$\eta_c = \frac{T_{3s} - T_1}{T_3 - T_1}$$

$$T_3 - T_1 = \frac{T_1\left[\left(\frac{P_3}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{\eta_c}$$

$$= \frac{303\left[(2)^{\frac{0.4}{1.4}} - 1\right]}{0.83}$$

$$= 79.95\ \text{K}$$

The work done per stage is

$$\boxed{\frac{W}{m} = C_p(T_3 - T_1) = U(C_{x_2} - C_{x_1})}$$

$$\text{or}\quad (T_3 - T_1) = \frac{U(C_{x_2} - C_{x_1})}{C_p} = \frac{U\Delta C_x}{C_p}$$

$$79.95 = \frac{U(0.235\,U)}{1005}$$

$$U = 584.7\ \text{m/s}$$

$$\text{Speed,}\quad N = \frac{U \times 60}{\pi \times D} = \frac{584.7 \times 60}{\pi \times 0.5}$$

$$= 22334\ \text{rpm}$$

(b) Absolute velocity of air Since, the velocity diagram is symmetrical, $R = 0.5$.

$$R = \frac{2U - (C_{x_1} + C_{x_2})}{2U}$$

$$C_{x_1} + C_{x_2} = -0.5(2U) + 2U = U$$

$$C_{x_1} + C_{x_2} = U = 584.7\ \text{m/s}$$

$$\text{and}\quad C_{x_2} - C_{x_1} = 0.235\,U = 137.4\ \text{m/s}$$

Solving for C_{x_1} and C_{x_2}, we get

$$C_{x_2} = 361.05\ \text{m/s}$$

$$C_{x_1} = 223.65\ \text{m/s}$$

From inlet velocity triangle [Refer Fig. (4.2)],

$$C_1 = \sqrt{C_a^2 + C_{x_1}^2} \qquad C_a = 0.47 \times 584.7 = 274.81\ \text{m/s}$$

$$= \sqrt{274.81^2 + 223.65^2}$$

$$= 354.32\ \text{m/s}$$

Example 4.8 The first stage of an axial flow compressor is designed for free vortex condition, with no inlet guide vanes. The rotational speed is 9000 rpm, and stagnation temperature rise is 20°C. The hub-tip ratio is 0.6, the work done factor is 0.94 and isentropic efficiency of the stage is 0.90. Assuming an inlet velocity of 150 m/s and ambient conditions of 1 bar and 300 K, compute (a) the tip radius and corresponding rotor angles, if the Mach number relative to the tip is limited to 0.92, (b) mass flow entering the stage (c) stage stagnation pressure ratio and power required and (d) the rotor air angles at the root section.

Solution

$N = 9000$ rpm $\Delta T_0 = 20°\text{C}$ $\frac{D_h}{D_t} = 0.6$ $\lambda = 0.94$ $\eta_s = 0.9$

$C_1 = 150$ m/s $P_{01} = 1$ bar $T_{01} = 300$ K $M_{r,1} = 0.92$ (at tip)

Assuming axial inlet (since no inlet guide vanes), the velocity triangle at inlet can be drawn as shown in Fig. 4.12.

$$T_1 = T_{01} - \frac{C_1^2}{2C_p}$$

$$= 300 - \frac{150^2}{2 \times 1005}$$

$$= 288.81\ \text{K}$$

$$W_1 = M_{r,1}\left(\sqrt{\gamma R T_1}\right)$$

$$= 0.92\left(\sqrt{1.4 \times 287 \times 288.81}\right)$$

$$W_1 = 313.39\ \text{m/s}$$

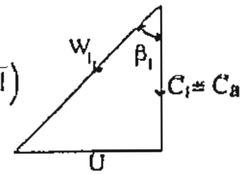


Figure 4.12

From the velocity triangle

$$\cos\beta_1 = \frac{C_1}{W_1} = \frac{150}{313.39}$$

$$\therefore \beta_1 = 61.4°$$

and

$$U_t = W_1 \sin\beta_1$$

$$= 313.39 \times \sin 61.4°$$

$$= 275.15\ \text{m/s}$$

(a) Tip radius

$$r_t = \frac{U_t \times 60}{2\pi N}$$

$$r_t = \frac{275.15 \times 60}{2\pi \times 9000} = 0.292\ \text{m}$$

$$r_t = 0.292\ \text{m}$$

Now,

$$(\tan\beta_1 - \tan\beta_2) = \frac{C_p \Delta T_0}{\lambda U_t C_a}$$

$$\tan\beta_2 = \tan\beta_1 - \frac{C_p \Delta T_0}{\lambda U_t C_a}$$

$$= \tan 61.4 - \frac{1005 \times 20}{0.94 \times 275.15 \times 150}$$
$$= 1.316$$
$$\therefore \beta_2 = 52.77°$$

(b) Mass flow

$$m = \rho_1 A_1 C_a$$
$$\rho_1 = \frac{P_1}{RT_1}$$
$$P_1 = P_{01}\left(\frac{T_1}{T_{01}}\right)^{r/r-1}$$
$$= 1 \times \left(\frac{288.81}{300}\right)^{\frac{1.4}{0.4}}$$
$$= 0.87542 \text{ bar}$$
$$\rho_1 = \frac{0.87542 \times 10^5}{287 \times 288.81} = 1.056 \text{ kg/m}^3$$
$$A_1 = \frac{\pi}{4}\left(D_t^2 - D_h^2\right)$$
$$D_h = 0.6\, D_t = 0.6(2 \times 0.292) = 0.6(0.584)$$
$$= 0.35 \text{ m}$$
$$A_1 = \frac{\pi}{4}\left(0.584^2 - 0.35^2\right)$$
$$= 0.1717 \text{ m}^2$$

(or) Flow area

$$A = 2\pi r_m h$$
$$r_m = \frac{r_t + r_h}{2} = \frac{\left(\frac{0.584}{2} + \frac{0.35}{2}\right)}{2}$$
$$= 0.2335 \text{ m}$$
$$h = r_t - r_h = \left(\frac{0.584}{2} - \frac{0.35}{2}\right)$$
$$= 0.117 \text{ m}$$
$$A = 2\pi \times 0.2335 \times 0.117$$
$$= 0.1717 \text{ m}^2$$
$$\dot m = 1.056 \times 0.1717 \times 150$$
$$\dot m = 27.197 \text{ kg/s}$$

(c) Stagnation pressure ratio

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_s \Delta T_0}{T_{01}}\right]^{\frac{r}{r-1}}$$
$$= \left[1 + \frac{0.9 \times 20}{300}\right]^{\frac{1.4}{0.4}}$$
$$= 1.226$$

$$\text{Power required} = \dot m C_p \Delta T_0$$
$$= 27.197 \times 1.005 \times 20$$
$$= 546.66 \text{ kW}$$

(d) Rotor air angles

$$U_h = \frac{\pi D_h N}{60} = \frac{\pi \times 0.35 \times 9000}{60}$$
$$= 164.93 \text{ m/s}$$
$$\tan \beta_1 = \frac{U_h}{C_1} = \frac{164.93}{150}$$
$$\therefore \beta_1 = 47.71°$$

and

$$\tan \beta_2 = \tan \beta_1 - \frac{C_p \Delta T_0}{\lambda U_h C_a}$$
$$= \tan 47.71 - \frac{1005 \times 20}{0.94 \times 164.93 \times 150}$$
$$= 0.235$$
$$\therefore \beta_2 = 13.23°$$

Example 4.9 An axial flow compressor stage has blade root, mean and tip velocities 150, 200 and 250 m/s respectively. The stage is to be designed for a stagnation temperature rise of 20°C and an axial velocity of 150 m/s which is constant throughout. The work done factor is 0.93. Assuming 50% reaction at mean radius, calculate the stage air and blade angles at mean, root and hub and degree of reaction for a free vortex design.

Solution

$U_r = 150$ m/s $U_m = 200$ m/s $U_t = 250$ m/s
$\Delta T_0 = 20$°C $C_a = 150$ m/s $\lambda = 0.93$ $R_m = 0.5$

(a) Stage air and blade angle at mean From the work done per stage expression, we have

$$\tan \beta_{1,m} - \tan \beta_{2,m} = \frac{C_p \Delta T_0}{\lambda U_m C_a} = \frac{1005 \times 20}{0.93 \times 200 \times 150}$$
$$\tan \beta_{1,m} - \tan \beta_{2,m} = 0.72 \qquad (1)$$

and from the reaction ratio

$$\tan\beta_{1,m} + \tan\beta_{2,m} = \frac{2RU_m}{C_a} = \frac{2\times 0.5\times 200}{150}$$

$$\tan\beta_{1,m} + \tan\beta_{2,m} = 1.33 \qquad (2)$$

Solving for $\beta_{1,m}$ and $\beta_{2,m}$

$$\tan\beta_{1,m} = \frac{1.33+0.72}{2} = 1.025$$

$$\therefore \beta_{1,m} = 45.71°$$

Since

$$R_m = 0.5$$

$$\alpha_{2,m} = \beta_{1,m} = 45.71°$$

$$\alpha_{2,m} = 45.71°$$

Substituting $\beta_{1,m}$ in equation (1), we get

$$\tan\beta_{2,m} = 1.025 - 0.72$$

$$\beta_{2,m} = 16.96$$

Since $R_m = 0.5$, $\alpha_{1,m} = \beta_{2,m}$
$\alpha_{1,m} = 16.96°$

(b) Stage air and blade angle at root (or) hub For a free vortex design,

$$\boxed{C_{x1,h}\cdot r_h = C_{x1,m}\cdot r_m}$$

(or)

$$C_a\tan\alpha_{1,h}\cdot r_h = C_a\tan\alpha_{1,m}\cdot r_m$$

$$\tan\alpha_{1,h} = \tan\alpha_{1,m}\cdot\frac{r_m}{r_h}$$

$$\frac{r_m}{r_h} = \frac{U_m}{U_h} = \frac{200}{150} = 1.33$$

$$\alpha_{1,h} = 22.1°$$

From velocity triangle [Refer Fig. 4.2],

$$U_h = C_a(\tan\alpha_{1,h} + \tan\beta_{1,h})$$

$$\tan\beta_{1,h} = \frac{150}{150} - \tan 22.1 = 0.594$$

$$\beta_{1,h} = 30.71°$$

Similarly,

$$C_{x2,h}\cdot r_h = C_{x2,m}\cdot r_m$$

or

$$\tan\alpha_{2,h} = \tan\alpha_{2,m}\cdot\frac{r_m}{r_h}$$

$$= \tan 45.71°\times 1.33$$

$$= 1.363$$

$$\alpha_{2,h} = 53.73°$$

and

$$\tan\beta_{2,h} = \frac{U_h}{C_a} - \tan\alpha_{2,h}$$

$$= \frac{150}{150} - \tan 53.73°$$

$$= -0.363$$

$$\beta_{2,h} = -19.95°$$

Degree of reaction

$$R_h = \frac{C_a(\tan\beta_{1,h} + \tan\beta_{2,h})}{2U}$$

$$= \frac{150(\tan 30.71° + \tan(-19.95°))}{2\times 150}$$

$$R_h = 0.116$$

(c) Stage air and blade angle at tip For free vortex condition,

$$C_{x1,t}\cdot r_t = C_{x1,m}\cdot r_m$$

or

$$C_a\cdot\tan\alpha_{1,t}\cdot r_t = C_a\cdot\tan\alpha_{1,m}\cdot r_m$$

$$\tan\alpha_{1,t} = \tan\alpha_{1,m}\cdot\frac{r_m}{r_t}$$

$$\frac{r_m}{r_t} = \frac{U_m}{U_t} = \frac{200}{250} = 0.8$$

$$\tan\alpha_{1,t} = \tan 16.96°\times 0.8$$

$$\therefore\alpha_{1,t} = 13.71°$$

and

$$\tan\beta_{1,t} = \frac{U_t}{C_a} - \tan\alpha_{1,t}$$

$$= \frac{250}{150} - \tan 13.71° = 1.423$$

$$\therefore\beta_{1,t} = 54.9°$$

Similarly,

$$\tan\alpha_{2,t} = \tan\alpha_{2,m}\cdot\frac{r_m}{r_t}$$
$$= \tan 45.71 \times 0.8 = 0.82$$
$$\therefore \alpha_{2,t} = 39.4^\circ$$

and

$$\tan\beta_{2,t} = \frac{U_t}{C_a} - \tan\alpha_{2,t}$$
$$= \frac{250}{150} - \tan 39.4^\circ = 0.85$$
$$\therefore \beta_{2,t} = 40.4^\circ$$

Degree of reaction

$$R_t = \frac{C_a(\tan\beta_{1,t} + \tan\beta_{2,t})}{2U_t}$$
$$= \frac{150(\tan 54.9^\circ + \tan(40.4^\circ))}{2\times 250}$$
$$R_t = 0.682$$

Example 4.10 An alternative design proposal to that in example 4.9, is to have forced vortex blade design. What then will the air and blade angles and degree of reaction be. Take rotational speed as 9000 rpm.

Solution

(1) Mean section

The parameters are calculated in the same manner as in example 4.9. The values are

$\alpha_{1,m} = \beta_{2,m} = 16.96^\circ$
$\alpha_{2,m} = \beta_{1,m} = 45.71^\circ$ and
$R = 0.5$

(2) Hub section

$$\text{Hub diameter, } D_h = \frac{U_h \times 60}{\pi \times N} = \frac{150\times 60}{\pi\times 9000}$$
$$= 0.318 \text{ m}$$
$$\text{Mean diameter, } D_m = \frac{U_m \times 60}{\pi \times N} = \frac{200\times 60}{\pi\times 9000}$$
$$= 0.424 \text{ m}$$

$$C_{x1,m} = C_a\cdot\tan\alpha_{1,m} = 150\times\tan 16.96^\circ = 45.75 \text{ m/s}$$
$$C_{x2,m} = C_a\cdot\tan\alpha_{2,m} = 150\times\tan 45.71^\circ = 153.76 \text{ m/s}$$

For forced vortex design

$$C_{x1,h} = \frac{C_{x1,m}}{r_m}\times r_h = \frac{45.75\times 0.159}{0.212} = 34.31 \text{ m/s}$$
$$C_{x2,h} = \frac{C_{x2,m}}{r_m}\times r_h = \frac{153.76\times 0.159}{0.212} = 115.32 \text{ m/s}$$

Axial velocity at hub inlet

$$C_{a1,h}^2 = K_1 - 2C_{x1,h}^2$$

where

$$K_1 = C_{a1,m}^2 + 2C_{x1,m}^2$$
$$= 150^2 + 2\times 45.75^2 = 26686.125$$
$$\therefore C_{a1,h}^2 = 26686.125 - 2\times 34.31^2 = 24331.77$$
$$C_{a1,h} = 155.99 \text{ m/s}$$

Axial velocity at hub outlet

$$C_{a2,h}^2 = K_2 - 2C_{x2,h}^2 + 2\left(\frac{C_{x2,h}}{r_h} - \frac{C_{x1,h}}{r_h}\right)\omega r_h^2$$

where

$$K_2 = C_{a2,m}^2 + 2C_{x2,m}^2 - 2\left(\frac{C_{x2,m}}{r_m} - \frac{C_{x1,m}}{r_m}\right)\omega r_m^2$$
$$= 150^2 + 2\times 153.76^2 - 2\left(\frac{153.76}{0.212} - \frac{45.75}{0.212}\right)\times\left(\frac{2\pi\times 9000}{60}\right)\times 0.212^2$$
$$= 26621.32$$
$$\therefore C_{a2,h}^2 = 26621.32 - 2\times 115.32^2 + 2\left(\frac{115.32}{0.159} - \frac{34.31}{0.159}\right)\times 942.5\times 0.159^2$$
$$= 24303.8$$
$$C_{a2,h} = 155.89 \text{ m/s}$$

(a) The air and blade angles

$$\tan\alpha_{1,h} = \frac{C_{x1,h}}{C_{a1,h}} = \frac{34.31}{155.99}$$
$$\alpha_{1,h} = 12.4^\circ$$
$$\tan\beta_{1,h} = \frac{U_h - C_{x1,h}}{C_{a1,h}} = \frac{150 - 34.31}{155.99}$$
$$\beta_{1,h} = 36.56^\circ$$
$$\tan\alpha_{2,h} = \frac{C_{x2,h}}{C_{a2,h}} = \frac{115.32}{155.89}$$
$$\alpha_{2,h} = 36.5^\circ$$

$$\tan\beta_{2,h} = \frac{U_h - C_{x2,h}}{C_{a2,h}} = \frac{150 - 115.32}{155.89}$$

$$\beta_{2,h} = 12.5^\circ$$

(b) Degree of reaction

$$R = \frac{W_1^2 - W_2^2}{2U(C_{x2} - C_{x1})}$$

$$R_h = \frac{\left(\frac{C_{a1,h}}{\cos\beta_{1,h}}\right)^2 - \left(\frac{C_{a2,h}}{\cos\beta_{2,h}}\right)^2}{2U_h(C_{x2,h} - C_{x1,h})}$$

$$= \frac{\left(\frac{155.99}{\cos 36.56^\circ}\right)^2 - \left(\frac{155.89}{\cos 12.5^\circ}\right)^2}{2 \times 150 \times (115.32 - 34.31)}$$

$$= 0.5$$

(3) Tip section

$$\text{Tip diameter, } D_t = \frac{U_t \times 60}{\pi \times 9000} = \frac{250 \times 60}{\pi \times 9000}$$

$$= 0.530 \text{ m}$$

$$C_{x1,t} = \frac{C_{x1,m}}{r_m} \times r_t = \frac{45.75 \times 0.265}{0.212} = 57.18 \text{ m/s}$$

$$C_{x2,t} = \frac{C_{x2,m}}{r_m} \times r_t = \frac{153.76 \times 0.265}{0.212} = 192.2 \text{ m/s}$$

Axial velocity at tip inlet

$$C_{a1,t}^2 = K_1 - 2C_{x1,t}^2$$

$$= 26686.125 - 2 \times 57.18^2$$

$$= 20147$$

$$\therefore C_{a1,t} = 141.9 \text{ m/s}$$

Axial velocity at tip outlet

$$C_{a2,t}^2 = K_2 - 2C_{x2,t}^2 + 2\left(\frac{C_{x2,t}}{r_t} - \frac{C_{x1,t}}{r_t}\right)\omega r_t^2$$

$$= 26621.32 - 2 \times 192.2^2 + 2\left(\frac{192.2}{0.265} - \frac{57.18}{0.265}\right) \times 942.5 \times 0.265^2$$

$$= 20185.51$$

$$\therefore C_{a2,t} = 142 \text{ m/s}$$

(a) Air and blade angles

$$\tan\alpha_{1,t} = \frac{C_{x1,t}}{C_{a1,t}} = \frac{57.18}{141.9}$$

$$\alpha_{1,t} = 21.95^\circ$$

$$\tan\beta_{1,t} = \frac{U_t - C_{x1,t}}{C_{a1,t}} = \frac{250 - 57.18}{141.9}$$

$$\beta_{1,t} = 53.65^\circ$$

$$\tan\alpha_{2,t} = \frac{C_{x2,t}}{C_{a2,t}} = \frac{192.2}{142}$$

$$\alpha_{2,t} = 53.54^\circ$$

$$\tan\beta_{2,t} = \frac{U_t - C_{x2,t}}{C_{a2,t}} = \frac{250 - 192.2}{142}$$

$$\beta_{2,t} = 22.15^\circ$$

(b) Degree of reaction

$$R = \frac{\left(\frac{C_{a1,t}}{\cos\beta_{1,t}}\right)^2 - \left(\frac{C_{a2,t}}{\cos\beta_{2,t}}\right)^2}{2U_t(C_{x2,t} - C_{x1,t})}$$

$$= \frac{\left(\frac{141.9}{\cos 53.65^\circ}\right)^2 - \left(\frac{142}{\cos 22.15^\circ}\right)^2}{2 \times 250 \times (192.2 - 57.18)}$$

$$= 0.5$$

Example 4.11 A single-stage axial flow blower with no inlet guide vanes runs at 3600 rpm. The rotor tip and hub diameter are 20 and 12.5 cm, respectively. The mass flow rate of air is 0.5 kg/s. The turning angle of the rotor is 20° towards the axial direction during air flow over the blade. The blade angle at inlet is 55° If the atmospheric temperature and pressure are at 1 atm and 25°C, respectively, assuming constant axial velocity through the machine, find (a) the total pressure of the air at the exit of the rotor (the rotor total-to-total efficiency being 90% and the total pressure drop across the intake is 0.25 cm of water), (b) the static pressure rise across the rotor, (c) the static pressure rise across the stator, if the stator efficiency is 75%, (d) the change in total pressure across the stator, (e) the overall total-to-total efficiency and (f) the degree of reaction for the stage.

Solution

(a) Total pressure of air exit of rotor

$N = 3600$ rpm $\quad D_t = 0.2$ m $\quad D_h = 0.125$ m $\quad P_1 = 1.013$ bar $\quad T_1 = 298$ K

$m = 0.5 \quad \beta_1 - \beta_2 = 20^\circ \quad \beta_1 = 55^\circ$

The pressure changes involved are small so that the flow may be treated as incompressible. At the inlet, the density of air is

$$\rho_0 = \frac{P_0}{RT_0} = \frac{1.013 \times 10^5}{287 \times 298} = 1.184 \text{ kg/m}^3 = \rho$$

Area of flow

$$A = \frac{\pi}{4}\left(D_t^2 - D_h^2\right) = \frac{\pi}{4}(0.2^2 - 0.125^2)$$
$$= 0.01914 \text{ m}^2$$

The axial velocity

$$C_a = \frac{m}{\rho A} = \frac{0.5}{1.184 \times 0.01914}$$
$$= 22.1 \text{ m/s}$$

Mean rotor blade velocity

$$U = \frac{\pi(D_h + D_t)N}{2 \times 60}$$
$$= \frac{\pi \times (0.125 + 0.2) \times 3600}{120}$$
$$= 30.6 \text{ m/s}$$

The actual total enthalpy rise across the rotor is

$$(\Delta h_0)_{rotor} = U \Delta C_X = U(C_{X_2} - C_{X_1})$$

Since the flow at inlet is axial, $C_{X_1} = 0$ and from velocity triangle at outlet (Fig. 4.2),

$$C_{X_2} = U - W_{X_2} = U - C_a \tan\beta_2$$
$$= 30.6 - 22.1 \times \tan(\beta_1 - 20)$$
$$= 30.6 - 22.1 \times \tan 35°$$
$$= 15.13 \text{ m/s}$$
$$(\Delta h_0)_{rotor} = 30.6 \times (15.13 - 0) = 462.98 \text{ J/kg}$$

The isentropic total enthalpy rise across the rotor

$$(\Delta h_{0_s})_{rotor} = \eta_c \times (\Delta h_0)_{rotor}$$
$$= 0.9 \times [462.98]$$
$$= 416.7 \text{ J/kg}$$

The total pressure rise across the rotor is

$$(\Delta P_0)_{rotor} = \rho(\Delta h_{0_s})_{rotor}$$
$$= 1.184 \times 416.7$$
$$= 493.35 \text{ N/m}^2$$
$$= 5.03 \text{ cm of } H_2O$$

Stagnation pressure at the rotor exit

$$= (\Delta P_0)_{rotor} - \text{Pressure drop at intake}$$
$$= 5.03 - 0.25$$
$$= 4.78 \text{ cm of } H_2O$$

(b) The static pressure rise across the rotor

$$(\Delta P)_{rotor} = (\Delta P_0)_{rotor} - \rho\left(C_2^2 - C_1^2\right)/2$$
$$C_1 = C_a = 22.1 \text{ m/s}$$
$$C_2 = \sqrt{C_a^2 + C_{X_2}^2}$$
$$= \sqrt{22.1^2 + 15.13^2}$$
$$= 26.78 \text{ m/s}$$
$$\therefore (\Delta P)_{rotor} = 493.35 - 1.184(26.78^2 - 22.1^2)/2$$
$$= 493.35 - 135.43$$
$$= 357.93 \text{ N/m}^2$$
$$(\Delta P)_{rotor} = 3.65 \text{ cm of water}$$

(c) The static pressure rise across the stator

The actual static enthalpy change across the stator is

$$(\Delta h)_{stator} = \left(C_2^2 - C_1^2\right)/2$$
$$= (26.78^2 - 22.1^2)/2$$
$$= 114.38 \text{ J/kg}$$

The theoretical static enthalpy change across the stator is

$$(\Delta h_s)_{stator} = \eta_{stator} \times (\Delta h)_{stator}$$
$$0.75 \times 114.38$$
$$= 85.79 \text{ J/kg}$$

The static pressure rise across the stator is

$$(\Delta P)_{stator} = \rho(\Delta h_s)_{stator}$$
$$= 71.184 \times 85.79$$
$$= 101.58 \text{ N/m}^2$$
$$= 1.04 \text{ cm of water}$$

(d) The change in total pressure across the stator

$$(\Delta P_0)_{stator} = (\Delta P)_{stator} + \rho\left(C_1^2 - C_2^2\right)/2$$
$$= 101.58 + 1.184 \times (-114.38)$$
$$= -33.85 \text{ N/m}^2$$

That is, the total pressure across the stator drops by an amount = 33.85 N/m^2 or = 0.35 cm of water.

(e) The overall total-to-total efficiency

Total pressure at stator exit

$$P_{03} = \text{Total pressure at stator inlet} - (\Delta P_0)_{stage} \text{ (or at rotor exit)}$$
$$= 4.78 - 0.35$$
$$= 4.43 \text{ cm of water}$$

Theoretical total enthalpy change across the stage

$$(\Delta h_{0s})_{stage} = \frac{(\Delta P_0)_{stage}}{\rho}$$
$$= \frac{\rho_w g h_w}{\rho} = \frac{1000 \times 9.81 \times \left(\frac{4.43}{100}\right)}{1.184}$$
$$= 367 \text{ J/kg}$$

Then,

$$\eta_{t-t} = \frac{(\Delta h_{0s})_{stage}}{(\Delta h_0)_{stage}} = \frac{(\Delta h_{0s})_{stage}}{(\Delta h_0)_{rotor}}$$
$$= 367462.98$$
$$= 79.3\%$$

(f) The degree of reaction for the stage

$$R = \frac{(\Delta P)_{rotor}}{(\Delta P)_{stage}}$$
$$= \frac{357.93}{357.93 + 101.58}$$
$$= 77.9\%$$

Example 4.12 An axial flow fan takes in 2.5 m^3/s of air at 1.02 bar and 42°C and delivers it at 75 cm W.G. and 52°C. Determine the mass flow rate through the fan, the power required to drive the fan and the static fan efficiency. *(MU Oct. '97)*

Solution

$Q = 2.5\,\text{m}^3/\text{s}$ $P_1 = 1.02\,\text{bar}$ $T_1 = 315\,\text{K}$ $\Delta H = 0.75\,\text{m W.G.}$ $T_2 = 325\,\text{K}$

(a) Mass flow rate

$$m = \rho Q$$
$$\rho = \frac{P_1}{RT_1} = \frac{1.02 \times 10^5}{287 \times 315} = 1.128 \text{ kg/m}^3$$
$$m = 1.128 \times 2.5$$
$$= 2.82 \text{ kg/s}$$

(b) Power required to drive the fan

$$W = mC_P(T_2 - T_1)$$
$$= 2.82 \times 1.005(325 - 315)$$
$$W = 28.34 \text{ kW}$$

(c) Static fan Efficiency

$$\eta_t = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{T_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_2 - T_1}$$
$$P_2 = P_1 + \Delta P$$
$$= 1.02 + \left(\frac{10^3 \times 9.81 \times 0.75}{10^5}\right)$$
$$= 1.094 \text{ bar}$$
$$\therefore \eta_t = \frac{315\left[\left(\frac{1.094}{1.02}\right)^{\frac{0.4}{1.4}} - 1\right]}{325 - 315}$$
$$= 63.66\%$$

Example 4.13 An axial fan stage consisting of only one rotor has the following data:

rotor blade air angle at exit: 10°; tip dia: 60 cm; hub dia: 30 cm; speed: 960 rpm; power required: 1 kW; flow coefficient: 0.245; inlet flow conditions: 1.02 bar and 316 K.

Determine the flow rate, the static pressure rise and the overall efficiency.

(MKU-April '95)

Solution

$\beta_2 = 10°$ $D_t = 0.6$ m $D_h = 0.3$ m $N = 960$ rpm Power = 1 kW

$\phi = \frac{C_a}{U} = 0.245$ $P_1 = 1.02$ bar $T_1 = 316$ K

(a) Flow rate

$$Q = AC_a$$
$$A = \frac{\pi}{4}\left(D_t^2 - D_h^2\right)$$
$$= \frac{\pi}{4}\left(0.6^2 - 0.3^2\right)$$
$$= 0.212 \text{ m}^2 C_a = \phi U$$
$$U = \frac{\pi D_m N}{60}$$

where D_m is the mean rotor diameter

$$D_m = \frac{D_t + D_h}{2} = \frac{0.6 + 0.3}{2} = 0.45$$

$$U_m = \frac{\pi \times 0.45 \times 960}{60} = 22.62 \text{ m/s}$$

$$C_a = 0.245 \times 22.62 = 5.542 \text{ m/s}$$

The flow rate, $Q = 0.212 \times 5.542$

$$Q = 1.175 \text{ m}^3/\text{s}$$

(b) Static pressure rise across the stage

$$(\Delta P)_{st} = \frac{1}{2}\rho(W_1^2 - W_2^2)$$

but

$$W_1^2 = U^2 + C_a^2$$

and

$$W_2^2 = C_a^2 + W_{X_2}^2 = C_a^2 + C_a^2 \tan^2 \beta_2$$

Then

$$\begin{aligned}(\Delta P)_{st} &= \frac{1}{2}\rho[U^2 + C_a^2 - C_a^2 - C_a^2 \tan^2 \beta_2] \\ &= \frac{1}{2}\rho[U^2 - C_a^2 \tan^2 \beta_2] \\ &= \frac{1}{2}\rho U^2[1 - \frac{C_a^2}{U^2} \tan^2 \beta_2] \\ &= \frac{1}{2}\rho U^2[1 - \phi^2 \tan^2 \beta_2]\end{aligned}$$

$$\rho = P/RT = \frac{1.02 \times 10^5}{287 \times 316} = 1.125 \text{ kg/m}^3$$

$$\begin{aligned}\therefore (\Delta P)_{st} &= \frac{1}{2} \times 1.125 \times 22.62^2[1 - 0.245^2 \times \tan^2 10^\circ] \\ &= 287.27 \text{ N/m}^2 \\ &= 0.029 \text{ m W.G.}\end{aligned}$$

(c) The overall efficiency

$$W/m = U(C_{x_2} - C_{x_1})$$

Since $C_{x_1} = 0$,

$$\begin{aligned}W/m = UC_{x_2} &= U(U - W_{x_2}) \\ &= U(U - C_a \tan \beta_2)\end{aligned}$$

$$\begin{aligned}&= 22.62(22.62 - 5.542 \tan 10^\circ) \\ &= 489.56 \text{ J/kg}\end{aligned}$$

$$m = \rho Q = 1.1125 \times 1.175 = 1.322 \text{ kg/s}$$

$$\begin{aligned}\therefore \text{Work done} &= m(W/m) \\ &= 1.322 \times 489.56 \\ &= 647.198 \text{ W}\end{aligned}$$

Overall efficiency

$$\begin{aligned}\eta_0 &= \frac{\text{Work done}}{\text{Power required}} \\ &= \frac{647.198}{1000} \\ &= 0.6472 \\ &= 64.72\%\end{aligned}$$

Example 4.14 Determine for the fan stage in problem no. 4.13
(a) rotor blade angle at the entry (b) degree of reaction

Solution

(a) Rotor blade angle at the entry From the inlet velocity triangle for fan stage without any guide vanes (Fig. 4.11),

$$\tan \beta_1 = \frac{U}{C_a}$$

$$\beta_1 = \tan^{-1}\left(\frac{22.62}{5.542}\right)$$

$$\beta_1 = 76.23^\circ$$

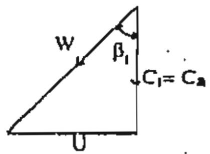


Figure 4.11

(b) Degree of reaction

$$R = (\Delta P)_{rotor}/(\Delta P_0)_{stage}$$

For an axial fan stage consisting of only a rotor,

$$\begin{aligned}(\Delta P)_{rotor} &= (\Delta P)_{stage} \\ &= 287.27 \text{ N/m}^2 \\ (\Delta P_0)_{stage} &= \rho(W/m) \\ &= 1.125 \times 489.56 \\ &= 550.76 \text{ N/m}^2 \\ \therefore R &= \frac{287.27}{550.76} = 0.522 \\ &= 52.2\%\end{aligned}$$

The degree of reaction can also be found from

$$R = \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2)$$
$$= \frac{\phi}{2}(\tan\beta_1 + \tan\beta_2)$$
$$= \frac{0.245}{2}(\tan 76.23° + \tan 10°)$$
$$= 0.522 \text{ or } 52.2\%$$

Example 4.15 An axial blower supplies air to a furnace at the rate of 3 kg/s. The atmospheric conditions being 100 kPa and 310 K. The blower efficiency is 80% and mechanical efficiency is 85%. The power supplied is 30 kW. Estimate the overall efficiency and pressure developed in mm W.G.

Solution

$m = 3$ kg/s $P_1 = 100$ kPa $T_1 = 310$ K
$\eta_b = 0.8$ $\eta_m = 0.85$ Power input = 30 kW

(a) Overall efficiency

$$\eta_0 = \eta_b \times \eta_m = 0.8 \times 0.85$$
$$= 0.68$$

(b) Pressure developed

$$\eta_0 = \frac{\dot{m}(\Delta P/\rho)}{\text{Power input}}$$
$$\rho = P/RT = 100 \times 10^3/287 \times 310 = 1.124 \text{ kg/m}^3$$
$$\Delta P = \frac{0.68 \times 30 \times 10^3}{3} \times 1.124$$
$$= 7643.2 \text{ N/m}^2$$
$$\Delta H = 779.12 \text{ mm W.G.}$$

Example 4.16 An axial fan without guide vanes has a pressure coefficient of 0.4 and delivers 3.5 kg/s of air at 750 rpm. Its hub diameter is 260 mm and hub to tip ratio is 1/3. The static properties at entry 98.4 kPa and 35°C, determine (a) overall efficiency, if $\eta_m = 0.9$, (b) power required, (c) flow coefficient (d) rotor inlet and exit angle, (e) ΔP in mm of WG if $\eta_f = 0.79$.

Solution

$\psi = 0.4$ $\dot{m} = 3.5$ kg/s $N = 750$ rpm $T_1 = 35$°C
$D_h = 0.26$ m $\frac{D_h}{D_t} = \frac{1}{3}$ $P_1 = 98.4$ kPa
$\eta_m = 0.9$ $\eta_f = 0.79$

(a) Overall efficiency

$$\eta_0 = \eta_m \times \eta_f = 0.9 \times 0.79$$
$$= 71.1\%$$

(b) Power required

Ideal work, $$W = \dot{m}\left(\frac{\Delta P}{\rho}\right)$$
$$\Delta P/\rho = \frac{U^2}{2} \times \psi$$
Mean velocity, $$U = \frac{\pi D_m N}{60}$$
$$D_m = \frac{D_t + D_h}{2}$$
$$D_t = 3 \times 0.26 = 0.78 \text{ m}$$
$$D_m = \frac{0.78 + 0.26}{2} = 0.52 \text{ m}$$

and $U = \frac{\pi \times 0.52 \times 750}{60} = 20.42$ m/s

Now

$$\frac{\Delta P}{\rho} = \frac{20.42^2}{2} \times 0.4$$
$$= 83.395 \text{ J/kg}$$
$$W_{ideal} = \frac{\Delta P}{\rho} \times m$$
$$= 83.395 \times 3.5$$
$$= 291.88 \text{ W}$$

Power required $$= \frac{W_{ideal}}{\eta_m}$$
$$= \frac{291.88}{0.9}$$
$$= 324.32 \text{ W}$$

(c) Flow coefficient

$$C_a = \frac{m}{\rho A}$$
$$\rho = \frac{P_1}{RT_1} = \frac{98.4 \times 10^3}{287 \times 308} = 1.113 \text{ kg/m}^3$$
$$A = \frac{\pi}{4}(D_t^2 - D_h^2) = \frac{\pi}{4}(0.78^2 - 0.26^2)$$
$$= 0.425 \text{ m}^2$$

$$C_a = \frac{3.5}{1.113 \times 0.425} = 7.399 \text{ m/s}$$

$$\phi = \frac{C_a}{U} = \frac{7.399}{20.42} = 0.36$$

$$\phi = 0.36$$

(d) Rotor inlet and exit angle

$$\psi = 2\phi(\tan\beta_1 - \tan\beta_2)$$

$$\tan\beta_1 - \tan\beta_2 = \frac{\psi}{2\phi} = \frac{0.4}{2 \times 0.36} = 0.56$$

$$W/m = U(Cx_2 - Cx_1)$$

$Cx_1 = 0$. No inlet guide vanes

$$W/m = UCx_2 = U^2(1 - \phi \tan\beta_2)$$

$$83.395 = 20.42^2(1 - 0.36 \tan\beta_2)$$

$$\beta_2 = 65.77'$$

and

$$\tan\beta_1 = \tan\beta_2 + \frac{\psi}{2\phi}$$

$$= \tan 65.77 + 0.56$$

$$\beta_1 = 70.23°$$

(e) Pressure developed

$$\Delta P = \rho \times 83.395$$

$$= 1.113 \times 83.395$$

$$= 92.82 \text{ N/m}^2$$

$$\Delta H = \Delta P/\rho g$$

$$= 92.82/10^3 \times 9.81$$

$$= 9.46 \times 10^{-3} \text{ m}$$

$$= 9.46 \text{ mm of W.G.}$$

SHORT QUESTIONS

4.1. What is an axial flow compressor?

4.2. What are the advantages and disadvantages of an axial flow compressor?

4.3. The axial flow compressors are ideal for constant load applications. Why?

4.4. The efficiency of the axial flow compressor is very sensitive to the mass flow rate. (True or False)

4.5. In the compressors, the blades form a diverging passage and the fluid is decelerated. (True or False)

4.6. The advantages of an axial flow compressor are
   (a) higher pressure ratio and mass flow rate
   (b) high thrust per unit frontal area
   (c) both (a) and (b)

4.7. The blade height is decreased as the fluid moves through the axial flow compressor. Why?

4.8. In axial flow compressor, there is no flow in the radial direction.(True/False)

4.9. What is the basic principle of working of an axial flow compressor?

4.10. What are the functions of the diffuser (or) straightener blades that are installed at the end of the last stage of an axial flow compressor?

4.11. What is an axial flow compressor stage?

4.12. Draw the inlet and outlet velocity diagrams for an axial flow compressor stage.

4.13. What is a normal axial flow compressor stage?

4.14. Inlet relative velocity is greater than the outlet relative velocity. (True/False)

4.15. In the rotor of an axial flow compressor,
   (a) $h_{01rel} = h_{02,rel}$
   (b) $h_{01rel} < h_{02,rel}$
   (c) $h_{01rel} > h_{02,rel}$

4.16. Draw the Mollier chart for an axial flow compressor stage.

4.17. The axial flow compressor receives no contribution from the change in tangential velocity. (True/False)

4.18. Define and express the total-to-total axial flow compressor efficiency.

4.19. What is work done factor?

4.20. Work done factor —— as the number of axial flow compressor stage increases.

4.21. Define stage loading.

4.22. Define reaction ratio.

4.23. Reaction ratio of a axial flow compressor stage is
   (a) $R = \phi \tan\beta_m/2$
   (b) $R = \phi \tan\beta_m$
   (c) $R = 2\phi \tan\beta_m$

4.24. Define flow coefficient.

4.25. Define reaction ratio for incompressible flow machines.

4.26. For a reaction ratio of 50 per cent
   (a) the static enthalpy and temperature increase in the stator and rotor are not equal.
   (b) the superimposed velocity diagram is not symmetrical.
   (c) none of the above.

4.27. For a reaction ratio of more than 50% the static enthalpy rise in the rotor is greater than in the stator. (True/False)

4.28. When reaction ratio is less than 50%, the static pressure and enthalpy rise are greater in the stator than in the rotor. (True/False)

4.29. A reaction ratio of 50% is widely used. Why?

4.30. A reaction ratio of 50% is the condition for maximum temperature rise and efficiency. (True/False)

4.31. Define compressor stage efficiency.
4.32. What is compressor stall?
4.33. What is rotating compressor stall?
4.34. What is compressor surging?
4.35. Why are longer blades used in aircraft compressors?
4.36. Define radial equilibrium in compressors.
4.37. What are free vortex blades?
4.38. In free vortex blade degree of reaction decreases from tip to hub. (True/False)
4.39. What are forced vortex blades?
4.40. In multistage compression, the stage pressure rise for the same temperature rise is equal. (True/False)
4.41. Overall total pressure ratio of a multistage axial flow compressors is equal to

(a) $(\Delta T_{0,overall})^{\frac{(\gamma-1)}{\eta_p \gamma}}$

(b) $(\Delta T_{0,overall})^{\frac{1}{\eta_p}\frac{\gamma}{(\gamma-1)}}$

(c) $(\Delta T_{0,overall})^{\frac{\eta_p \gamma}{\gamma-1}}$

EXERCISES

4.1. Draw a sketch of an axial flow compressor with inlet guide vanes and explain the working principle of the compressor.
4.2. Draw velocity triangle at the entry and exit for the axial compressor stage.
4.3. Draw the h–s diagram for a complete axial flow compressor stage.
4.4. (a) What is the work done factor for an axial compressor stage? How does it vary with the number of stages?
(b) Define: (a) flow coefficient (b) stage loading and (c) pressure coefficient for an axial compressor stage.
4.5. Define the degree of reaction for an axial flow compressor stage. Prove the following relations.

(a) $R = \frac{1}{2}\phi(\tan\beta_1 + \tan\beta_2)$

(b) $R = [1 + \phi(\tan\beta_2 - \tan\alpha_1)]/2$

(c) $R = [1 + \phi(\tan\beta_1 - \tan\alpha_2)]/2$

4.6. Draw superimposed velocity triangles for the following axial compressor stages.
(a) $R = 0.5$ (b) $R < 0.5$ (c) $R > 0.5$
4.7. Prove that

(a) $\frac{P_3 - P_1}{\rho U^2} = \phi(\tan\alpha_2 - \tan\alpha_1)$

(b) $\eta_s = (\Delta P)_{stage}/\rho U C_a(\tan\alpha_2 - \tan\alpha_1)$

4.8. What is surging in axial-flow compressors? What are its effects? Describe briefly.
4.9. What is stalling in an axial compressor stage? How is it developed? Why is it called rotating stall?
4.10. What is radial equilibrium method? Explain briefly.

4.11. Prove that the condition for radial equilibrium is

$$\frac{1}{r^2}\frac{d}{dr}(rC_x)^2 + \frac{d}{dr}(C_a)^2 = 0$$

4.12. What is free vortex blade? Derive the work done and reaction ratio for a free vortex blade.
4.13. What is a constant reaction blade?
4.14. What is a forced vortex blade? Draw the velocity triangles at the root and tip of a forced vortex blade.
✓ 4.15. Prove the following for a multistage compressor.

(a) $\eta_{stage} > \eta_{compressor}$

(b) $\frac{P_{0_{N+1}}}{P_{01}} = \left(\frac{T_{0_{N+1}}}{T_{01}}\right)^{\eta_p \gamma/(\gamma-1)}$

4.16. Compare a centrifugal compressor with an axial flow compressor.
4.17. Describe the fields of application of centrifugal and axial flow compressors. Explain why nowadays axial flow compressors are largely used for aviation gas turbines.
4.18. An axial flow compressor stage with 50% reaction has the following data. Air inlet stagnation temperature –290 K, relative flow angle at rotor outlet measured from the axial direction –32°, flow coefficient –0.55, relative machine number on to the rotor –0.75 If the stage is normal, what is the stagnation temperature rise in the first stage of the compressor? **[Ans: 26.1 K]**
4.19. An axial flow compressor stage draws air from with the stagnation conditions of 1 bar and 35°C. Assuming a 50% reaction stage with a flow coefficient of 0.52 and the ratio $\Delta C_x/U = 0.25$. Find the rotor blade angles at the inlet and the exit as well as the mean rotor speed. The total-to-total efficiency of the stage is 0.87 when the stage produces a total-to-total pressure ratio of 1.23. Find also the pressure coefficient and the power input to the system, assuming the work input factor to be 0.86. The mass flow rate is 12 kg/s. *(B.D.U April '96)*
[Ans: (a) 50.24°, 35.79° (b) 317.5 m/s (c) 0.43 and (d) 260kW**]**
4.20. An axial flow compressor stage is to be designed for a stagnation temperature rise of 20 K. The work done factor is 0.92 and the blade velocities at the root, mean radius and tip are 157.5, 210 and 262.5 m/s respectively. The axial velocity is constant from root to tip and is 157.5 m/s. If the reaction ratio at the mean radius is 0.5. What are the inlet and outlet air and blade angles at the root, mean radius and tip for a free vortex design? Calculate also the reaction at the root and tip.
[Ans: (a) $\alpha_{1,m} = \beta_{2,m} = 18.78°$ $\alpha_{2,m} = \beta_{1,m} = 44.8°$ $\alpha_{1,t} = 15.21°$ $\beta_{1,t} = 54.37°$, $\alpha_{2,t} = 38.45°$ $\beta_{2,t} = 40.9°$, $\alpha_{1,r} = 24.38°$ $\beta_{1,r} = 28.68°$ $\alpha_{2,r} = 52.94°$ $\beta_{2,r} = -17.95°$. (b) $R_t = 0.68$ and $R_r = 0.112$**]**
4.21. An alternative design proposal to that in the above problem is to have 50% reaction along the whole blade. What, then will the air and blade angles be?
[Ans: $\alpha_{1,t} = \beta_{2,t} = 29.64°$ $\alpha_{2,t} = \beta_{1,t} = 47.64°$ $\alpha_{1,r} = \beta_{2,r} = 3.43°$ $\alpha_{2,r} = \beta_{1,r} = 43.23°$**]**
4.22. An axial flow compressor under test in a laboratory exhibits a stage loading of 0.4 for a reaction ratio of 0.65 and flow coefficient 0.55. It is decided to reduce

the mass flow by 7 per cent while the blade speed is kept constant and it is assumed under this new condition that the relative flow exit angles for both the rotor and stator remain unchanged. What is the stage loading and reaction at the new condition? Assume the work done factor is 0.9.

[Ans: 0.435 and 0.64]

4.23. An axial flow compressor delivers a total pressure ratio of 6, the total head pressure and temperature at entry being 0.408 mPa and 300 K respectively, and the overall isentropic efficiency being 82 per cent. The degree of reaction is 50 per cent and all stages contribute an equal amount of work. At a particular stage, the blade speed at the mean height is 203 m/s, and the axial velocity is 171 m/s. If the absolute air angle entering the rotor at this stage is 15° and the work done factor is 0.92, determine, (a) the rotor air inlet angle, (b) the number of stages required, (c) the static temperature of the air at entry to the rotor and (d) the rotor inlet relative machine number.

[Ans: (a) 42.6° (b) 12 (c) 284.4 k and (d) 0.687]

4.24. A multistage axial compressor is required for compressing air at 293 K through a pressure ratio of 5 to 1. Each stage is to be 50% reaction and the mean blade speed 275 m/s, flow coefficient 0.5, and the stage loading factor 0.3 are taken as constant for all stages. Determine the flow angles and the number of stages required if the stage efficiency is 88.8%. Take $C_P = 1.005$ kJ/kg and $r = 1.4$ for air. Also, find the overall efficiency of the compressor.

[Ans: (a) $\alpha_1 = \beta_2 = 35°$ and $\alpha_2 = \beta_1 = 52.45°$ (b) 9 (c) 86.3%]

4.25. An axial flow compressor has 10 stages and the following data apply to each stage at the mean diameter. Blade speed –200 m/s, reaction –0.5, polytropic efficiency –0.88, stage efficiency –0.84. Angle of absolute air velocity at rotor inlet – 13°, and at rotor outlet –45°, work done factor –0.86. Stagnation pressure and temperature at inlet are 99.3 kPa and 15°C respectively. Determine the total pressure ratio of the first stage and the overall static pressure ratio.

[Ans: (a) 1.24 (b) 5.46]

√4.26. Each stage of an axial flow compressor of 50% reaction, has the same mean blade speed and same flow outlet angle of 30° relative to blades. The mean flow coefficient is 0.5 and remains constant. At entry to first stage the stagnation condition of air is 101.3 kPa and 278 K and static pressure is 87.3 kPa and flow area is 0.372 m^2. Using compressible flow analysis, find mass flow rate and flow velocity. Find the shaft power when there are 6 such stages when mechanical efficiency is 0.9. *(MU–April '97)*

[Ans: (a) 56.1 kg/s (b) 132.1 m/s and (c) 11021 kW]

4.27. An axial flow compressor has constant axial velocity throughout the compressor of 160 m/s, a mean blade speed of 244 m/s and delivers a pressure ratio of 5:1. Each stage is of 50% reaction and the relative outlet air angles are the same, 30°, for each stage. If a polytropic efficiency of 88 per cent is assumed, determine the number of stages in the compressor. [Ans: 14]

4.28. A helicopter gas turbine plant consists of a four stage axial flow compressor. The axial compressor has stage temperature rise of 30°C, using symmetrical stages with a stator outlet angle of 20° If the mean diameter of each stage is 250 mm and each stage is identical. The polytropic efficiency is 92 per cent. Calculate the required rotational speed. Assume a workdone factor of 0.86 and a constant axial velocity of 150 m/s. Estimate the total pressure rise across the compressor. [Ans: (a) 19,070 rpm (b) 3.07]

4.29. A multistage axial flow compressor is to have constant axial velocity of 150 m/s and 50% reaction. The pressure ratio developed is 4 and the infinitesimal stage efficiency is 85%. The temperature at the entry is 20°C. The mean diameter of the blade ring is 35 cm and speed is 15,000 rpm. The exit angle of the blades in each row is 27°. Calculate the blade angle at inlet, the number of stages and pressure ratio of each stage. *(MKU–April '96)*

[Ans: (a) 52.92° (b) 6 and (c) 1.38]

√ 4.30. A ten stage axial flow compressor has a pressure ratio of 6.6 and isentropic efficiency 90%. The compressor has symmetrical stages and the compression process is adiabatic. The axial velocity is uniform across the stage at 125 m/s and the mean blade speed of each stage is 200 m/s. If the air at 27°C enters the compressor at the rate of 3 kg/s, determine the direction of air at entry and exit from the rotor and stator blades. Also compute the power supplied to air. *(MKU–April '92)*

[Ans: (a) $\alpha_1 = \beta_2 = 17.8°$ and $\alpha_2 = \beta_1 = 51.97°$ (b) 718.17 kW]

4.31. An axial compressor stage has a mean diameter 55 cm and runs at 15,000 rpm. If the actual temperature rise and pressure ratio developed are 32°C and 1.4 respectively, determine (a) the power required to drive the compressor while delivering 57 kg/s of air. Assume mechanical efficiency as 85 per cent and an initial temperature of 35°C (b) the stage efficiency and (c) the degree of reaction if the temperature at the rotor exit is 55°C. *(MKU–April '98)*

[Ans: (a) 2156.6 kW, (b) 96.8% (c) 0.625]

4.32. An air compressor has eight stages of equal pressure ratio 1.35. The flow rate through the compressor and its overall efficiency are 50 kg/s and 82% respectively. If the condition of air at entry are 1 bar and 40°C, determine (a) the state of air at the compressor exit, (b) polytropic efficiency, (c) efficiency of each stage and (d) the power input assuming overall efficiency of the drive as 90%. *(MKU–Nov. '96)*

[Ans: (a) 11.03 bar and 689.8 K, (b) 87.1% (c) 86.5% and (d) 21.03 mW]

4.33. A fan takes in 2.5m^3/s of air at 1.02 bar and 42°C and delivers it at 70 cm W.G. and 52°C. Determine the mass flow rate through the fan, the power required to drive the fan and the static fan efficiency. *(MKU–Nov. '96)*

[Ans: (a) 2.82 kg/s, (b) 28.34 kW and (c) 59.5%]

√ 4.34. An axial ducted fan without any guide vanes has a pressure coefficient of 0.38 and delivers 3kg/s of air at 750 rpm. Its hub and tip diameters are 25 cm and 75 cm respectively. If the conditions at the entry are 1.0 bar and 38°C, determine (a) air angles at the entry and exit, (b) pressure developed in mm WG, (c) fan efficiency and (d) power required to drive the fan if the overall efficiency of the drive is 85%.

[Ans: (a) 70.85°, 66°, (b) 8.36 mm of WG, (c) 86.3% and (d) 299.36 W]

4.35. An axial fan consisting of rotor only has the following data. Hub and tip diameters are 30 and 60 cm respectively. Speed 1000 rpm, relative air exit angle 12°, axial velocity 6 m/s. Inlet static properties: 101 kPa and 315 K. Determine

(a) rotor blade angle inlet, (b) static pressure rise, (c) overall efficiency for a power input of 1.15 kW, (d) degree of reaction and (e) pressure coefficient.

[Ans: (a) 75.7° (b) 59.7 mm WG, (c) 65% (d) 52.6% and (e) 1.89]

4.36. In an axial flow fan the rotor and inlet guide vanes are symmetrical and arranged for 50% reaction. The hub and tip diameters are 45 cm and 75 cm respectively. Speed is 960 rpm. The motor power is 6 kW. The static properties of air at inlet are 100 kPa and 305 K. If the fan efficiency is 82% and the mechanical efficiency of the drive is 87%, find, the rotor angles at inlet and exit and pressure coefficient, if the quantity of air handled is 6 m^3/s.

[Ans: (a) $\beta_1 = \alpha_2 = 52.5°$, $\beta_2 = \alpha_1 = 6.6°$ and (b) 1.37]

4.37. The first stage of an axial flow compressor develops a pressure ratio of 1.2:1. The inlet pressure and temperature are 1 bar and 31° C respectively. The overall efficiency of the compressor is 83%. The flow coefficient is 0.47. The velocity diagram is symmetrical and at mean radius, the ratio of change of whirl velocity to axial velocity is 0.5. Determine the compressor speed if the mean diameter is 50 cm. *(MKU–April '93)*

[Ans: 11056 rpm]

4.38. The condition of air at the inlet of an axial air compressor is $P_1 = 768$ mm of Hg, $T_1 = 41°C$. At the mean blade section, the diameter and peripheral velocity are 500 mm and 100 m/s respectively. $\beta_1 = 51°$, $\alpha_1 = 7°$, $\beta_2 = 9°$, mass flow rate is 25 kg/s. Work done factor is 0.95 and mechanical efficiency is 92% stage efficiency is 88%. Determine (a) air angle at the stator entry (b) blade height at entry (c) hub to tip ratio (d) stage loading coefficient, (e) stage pressure ratio (f) power input and (g) relative machine number at the rotor inlet.

[Ans: (a) 50.2° (b) 0.19 m (c) 0.45, (d) 0.75 (e) 1.08 (f) 204.7 kW and (g) 0.33]

4.39. An axial compressor stage has mean diameter 600 mm and runs at 250 rps. The actual temperature rise is 30°C and the pressure ratio developed is 1.35. Inlet temperature is 35°C and the temperature rise in the rotor is 20°C. Mass flow rate is 50 kg/s and the mechanical efficiency is 85%, determine, (a) power required to drive the compressor (b) degree of reaction (c) loading coefficient (d) stage efficiency. *(MU–Oct. '99)*

[Ans: (a) 1773.5 kW (b) 0.67 (c) 0.136 and (d) 91.9%]

4.40. The conditions of air at the entry of an axial compressor stage are $P_1 = 768$ mm of Hg and $T_1 = 314$ K. The angles at the mean blade sections are $\alpha_1 = 7°$, $\beta_1 = 51°$, $\beta_2 = 9°$. The mean diameter and the peripheral speed are 500 mm and 100m/s respectively. Mass flow rate through the stage is 25 kg/s, the work done factor is 0.95, mechanical efficiency is 92% and stage efficiency is 88%. Assuming free vortex flow, determine (a) air and blade angles of rotor, (b) flow co-efficient, (c) loading co-efficient at the hub, mean and tip sections.

[Ans: (a) $\alpha_{1,m} = 7°$ $\alpha_{2,m} = 50.2°$, $\beta_{1,m} = 51°$, $\beta_{2,m} = 9°$,
$\alpha_{1,t} = 5.1°$ $\alpha_{2,t} = 40.99°$, $\beta_{1,t} = 60.7°$, $\beta_{2,t} = 45.2°$,
$\alpha_{1,r} = 11.21°$, $\alpha_{2,r} = 62.68°$, $\beta_{1,r} = 32.75°$, $\beta_{2,r} = -47.58°$
(b) 0.74, 0.53, 1.18 and (c) 0.75, 0.395, 1.96]

4.41. An axial flow compressor comprises a number of similar stages with equal work done per stage. The axial velocity remains constant throughout the compressor.

Overall total pressure ratio –3.5, total inlet temperature –333 K, relative air angle at rotor inlet and outlet are respectively 40° and 10°, blade velocity –185 m/s, overall total-head isentropic efficiency –87%, degree of reaction –0.5. Compute (a) Total outlet temperature and (b) no. of stages.

[Ans: (a) 497.7 K and (b) 8]

4.42. Find the polytropic efficiency of an axial flow compressor, with symmetrical stages from the following data, Total head pressure ratio –4, overall total head isentropic efficiency –85%, total head inlet temperature –290 K, The inlet and outlet air angles from the rotor blades are respectively 45° and 10°, the mean blade speed is 220 m/s, and the work done factor is 0.86. The axial velocity remains constant throughout the compressor. Find the number of stages required. Also find the inlet machine number relative to rotor at the mean blade height of the first stage. [Ans: (a) 87.6%, (b) 6 and (c) 0.8]

4.43. The velocities for upstream and downstream of an open propeller fan are 5 and 25 m/s respectively. The propeller diameter is 50 cm. If the ambient conditions are 1.02 bar and 37°C, Determine for the mean flow velocity through the propeller, (a) flow rate (b) total pressure developed and (c) the power required if the overall efficiency of the fan is 40% *(MU–Oct. '98)*

[Ans: (a) 3.37 kg/s, (b) 35 mm of W.G and (c) 2.52 kW]

4.44. An axial fan consisting of rotor (no IGV) has the following data: hub dia = 280 mm hub-tip ratio = 1/2, speed = 1000 rpm, relative air exit angle = 10°, axial velocity = 5.5 m/s, inlet static properties, 102 kPa and 310 K. Determine (a) flow coefficient (b) rotor blade angle at inlet (c) static pressure rise (d) overall efficiency (e) degree of reaction. The power input is 1.2 kW.

4.45. An axial flow compressor stage is designed on forced vortex principle. The following data refer to the stage at mean radius: mean diameter–0.5 m, peripheral speed–100 m/s, mass flow rate–25 kg/s, $\alpha_1 = \alpha_3 = 7°$, $\alpha_2 = 50.18\%$, $\beta_1 = 51°$ and $\beta_2 = 90°$. Calculate (a) air and blade angles (b) specific work (c) loading coefficient and (d) degree of reaction at hub, mean and tip sections. The air enters the stage at $P_1 = 768$ mm of Hg and $T_1 = 314$ K.

[Ans: (a) $\alpha_{1,m} = 7°$ $\alpha_{2,m} = 50.18°$ $\beta_{1,m} = 51°$ $\beta_{2,m} = 9°$
$\alpha_{1,h} = 4.31°$ $\alpha_{2,h} = 37.04°$ $\beta_{1,h} = 37.19°$ $\beta_{2,h} = 5.69°$
$\alpha_{1,t} = 9.75°$ $\alpha_{2,t} = 58.33°$ $\beta_{1,t} = 59.94°$ $\beta_{2,t} = 12.09°$
(b) 7929 J/kg, 3048.2 J/kg, 15099.27 J/kg, (c) 0.7929, 0.7929, 0.7929,
(d) 0.513, 0.555, 0.5]

4.46. An axial compressor stage has the following data: pressure and temperature at entry are 1.0 bar and 20°C, mean blade ring diameter–36 cm, speed–18,000 rpm, blade height at entry–6 cm, degree of reaction–50%, axial velocity–180 m/s. Air angles at rotor entry and stator exit–25°. Assume forced vortex flow. Determine (a) rotor blade air angles (b) degree of reaction (c) specific work (d) flow coefficient and (e) loading coefficient at the hub, mean and tip sections.

[Ans: (a) $\alpha_{1,m} = \beta_{2,m} = 25°$, $\alpha_{2,m} = \beta_{1,m} = 54.82°$, $\alpha_{1,h} = \beta_{2,h} = 20.05°$,
$\alpha_{2,h} = \beta_{1,h} = 47.99°$, $\alpha_{1,t} = \beta_{2,t} = 30.64°$, $\alpha_{2,t} = \beta_{1,t} = 60.98°$
(b) 58161 J/kg, 40389.4 J/kg, 79164 J/kg,
(c) 0.531, 0.678, 0.418, 0.7929, (d) 0.505, 0.505, 0.505,
(e) 0.5, 0.5, 0.5]

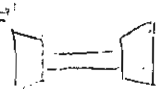
5

AXIAL FLOW STEAM AND GAS TURBINES

INTRODUCTION

Axial flow steam and gas turbines are turbomachines that expand a continuously flowing fluid essentially in the axial direction.

Development of the axial flow steam and gas turbine was hindered by the need to obtain both a high flow rate and compression ratio from a compressor to maintain the air requirement for the combustion process and subsequent expansion of the exhaust gases. Initially, the air was provided by centrifugal compressors and later the axial flow compressors were developed and used in turbojet engines, in which the power developed by the turbine is partly used to run the axial flow compressor.



Axial flow turbine has very wide applications ranging from aircraft propulsion to industrial and marine plants. In this chapter, steam and gas turbines are considered together, with the assumption that the same theory applies to both types of machines. This is only valid when the nature of the steam used in steam turbine exists in the superheated state which is assumed to behave as an ideal gas. Nowadays, the power output of steam turbines varies from few kilowatts to 660 kW. To have a high power output, superheated steam using superheater is made to expand in the turbine to below atmospheric pressure in the condenser, to extract the maximum energy from the steam. Gas turbines are used as the power unit for large jet aircraft propulsion, because they have a very high power to weight ratio. In the case of air-craft jet propulsion, to have enough jet thrust, high axial velocities are desirable. Usually one, or at the most two rows of nozzles and blades are required. But, gas turbines which are used in industrial or marine plants require large number of stages. This is required to reduce the carry-over loss, and to reduce the load on the blades to give the turbine a long working life. Steam turbines are used in fossil fuel power stations and for steam driven propulsion in ships, although gas turbine propulsion units are often fitted in the smaller class of naval vessels.

DESCRIPTION

The principle of energy extraction from the gas is gradually reducing the high pressure energy by converting it into kinetic energy. This is accomplished by passing the gas alternately through rows of fixed and moving blades. The kinetic energy of the gas is reduced in the moving blades, which are attached to the turbine hub and recovered in the fixed stationary blades attached to the casing (Fig. 5.1).

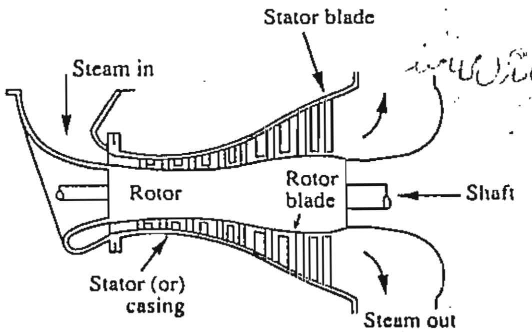


Figure 5.1 Axial flow turbine

The density of the gas gradually decreases as the gas moves through the turbine, and to maintain a constant axial flow velocity, the blade height is increased towards the low pressure end.

The stator row is often termed as the nozzle row and in certain types of steam turbine the nozzle row consists of a set of converging nozzles spaced around the drum.

While examining the flow through the stage, the following assumptions will be made.

1. Flow conditions are evaluated at the mean radius, unless otherwise stated.
2. Blade height/mean radius is small, allowing two-dimensional flow theory to be used.
3. Radial velocities are zero.

VELOCITY TRIANGLES FOR AN AXIAL FLOW TURBINE

A single turbine stage and velocity triangles are shown in the Fig. 5.2.

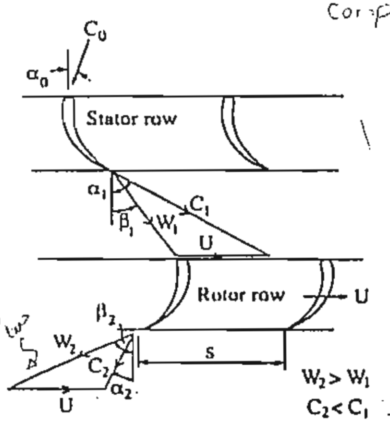


Figure 5.2 Velocity triangles for an axial flow gas (or steam) turbine stage

'A row of stator blades followed by a set of rotor blades is considered to be a stage'. The inlet to the stator blades is designated by the subscript 0. Inlet to the rotor section is referred by subscript-1 and outlet from the rotor section is indicated with subscript-2. All flow angles are measured from the axial direction and care must be taken when the flow angles are measured from the direction of blade motion i.e. tangential direction. The gas leaves the stator blades with absolute velocity C_1 at angle α_1 and by subtracting the blade velocity vector U, the relative velocity vector at entry to the rotor, W_1 is determined. In moving across the rotor blade, the flow direction is changed and the pressure reduced while the absolute velocity is decreased and the relative velocity increases. The gas leaves the blade tangentially at angle β_2 with relative velocity W_2. Vectorially subtracting the blade speed results in the absolute velocity C_2. This is now the inlet velocity to the next stator row at angle α_2, which for a normal stage equals C_0 at α_0.

For a normal stage

$$C_2 = C_0 \text{ and } \alpha_2 = \alpha_0$$

STAGE WORK AND DIAGRAM EFFICIENCY

The two velocity triangles are superimposed upon each other in Fig. 5.3. The energy transfer is given by the Euler's turbine equation.

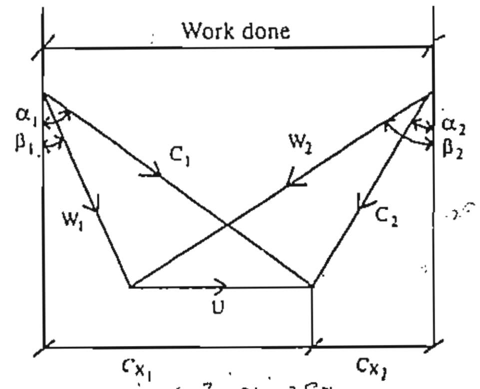


Figure 5.3 Superimposed velocity traingles

$$E = U(C_{x_1} - C_{x_2})/g \tag{5.1}$$

and since C_{x_2} is in the negative x direction, the work done per unit mass flow is given by

$$Eg = W/m = U(C_{x_1} + C_{x_2}) \tag{5.2}$$

$$= U(W_{x_1} + W_{x_2}) \tag{5.3}$$

If $C_{a_1} \neq C_{a_2}$ there will be an axial thrust in the flow direction. However, it is assumed that the axial velocity C_a is constant and therefore

$$W/m = UC_a(\tan\alpha_1 + \tan\alpha_2) \quad (5.4)$$

or

$$W/m = UC_a(\tan\beta_1 + \tan\beta_2) \quad (5.5)$$

Equation (5.5) is often referred to as the diagram work per unit mass flow and associated with this is the diagram efficiency. **Diagram efficiency (or)**

$$\textbf{Blade efficiency} = \frac{\text{Diagram work done per unit mass flow}}{\text{Work available per unit mass flow}}$$

$$= U(W_{x_1} + W_{x_2})/(C_1^2/2) \quad (5.6)$$

The diagram efficiency is also called the **'utilisation factor'**. The utilisation factor is high in most modern turbines and has a value between 90% and 95%.

h-s DIAGRAM FOR AN AXIAL FLOW TURBINE

The Mollier or h-s diagram for an axial flow turbine is shown in the Fig. 5.4. Total pressure P_{00} and total enthalpy h_{00} refer to the stator inlet conditions. For adiabatic flow through the stator row or nozzle ring $h_{00} = h_{01}$ but owing to irreversibilities, the total pressure drops to P_{01} at stator outlet (or rotor inlet). Expansion to P_{02} and total enthalpy h_{02} takes place in the rotor similar to the axial flow compressor. In this case also

$$h_{01,rel} = h_{02,rel}$$

The work done per unit mass flow by the gas is given by

$$W/m = h_{00} - h_{02} = h_{01} - h_{02} \quad (5.7)$$

or

$$W/m = C_p(T_{01} - T_{02}) \quad (5.8)$$

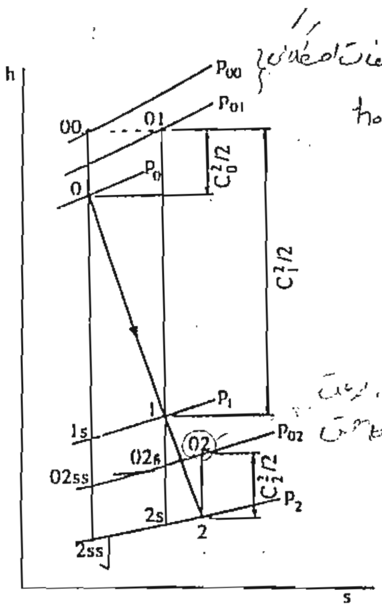


Figure 5.4 Mollier chart for expansion through an axial flow steam or gas turbine stage

Substituting for W/m from equation (5.8) in equation (5.5)

$$C_p(T_{01} - T_{02}) = UC_a(\tan\beta_1 + \tan\beta_2) \quad (5.9)$$

It should be noted that the work done factor(λ) is not used in equation (5.9). This is because, in a gas or steam turbine, flow through the blade passage is accelerating as opposed to decelerating. The effect of boundary – layer growth in the turbine is negligible. For a normal stage in which $C_0 = C_2$, the static temperature drop across the stage equals the total temperature drop.

i.e. $T_0 - T_2 = T_{00} - T_{02}$ [$C_0 = C_2$]

The turbine stage total-to-total isentropic efficiency is defined as

$$\eta_{t-t} = \frac{\text{Actual work done by the gas}}{\text{Isentropic work done}}$$

$$\eta_{t-t} = (T_{00} - T_{02})/(T_{00} - T_{02ss}) \quad (5.10)$$

Thus

$$T_{00} - T_{02} = \eta_{t-t} T_{00}(1 - T_{02ss}/T_{00})$$

$$T_{00} - T_{02} = \eta_{t-t} T_{00}\left[1 - (P_{02}/P_{00})^{\frac{\gamma-1}{\gamma}}\right] \quad (5.11)$$

To prove that, across the turbine rotor $h_{01,rel} = h_{02,rel}$, we proceed as follows

The work done per unit mass flow rate is

$$W/m = h_{01} - h_{02} \; [h_{00} = h_{01}]$$

Writing

$$h_0 = h + \frac{C^2}{2} = h + (C_a^2 + C_x^2)/2$$

$$\text{Then } h_{01} - h_{02} = (h_1 - h_2) + \left(\frac{C_{x_1}^2 - C_{x_2}^2}{2}\right) = U(C_{x_2} + C_{x_1})$$

$$h_1 - h_2 = \frac{C_{x_2} + C_{x_1}}{2}[2U - (C_{x_1} - C_{x_2})]$$

$$C_{x_1} - U = W_{x_1}$$

$$C_{x_2} + U = W_{x_2}$$

$$h_1 - h_2 = \frac{W_{x_2} + W_{x_1}}{2}[(U - C_{x_1}) + (U + C_{x_2})]$$

$$= \frac{W_{x_2} + W_{x_1}}{2}[W_{x_2} - W_{x_1}]$$

$$h_1 - h_2 = \frac{W_{x_2}^2 - W_{x_1}^2}{2} = \frac{W_2^2 - W_1^2}{2} \quad \because C_a = \text{constant}$$

Hence,

$$h_{01,rel} = h_{02,rel}$$

STATOR (NOZZLE) AND ROTOR LOSSES

Two isentropic efficiencies commonly used in axial flow turbine work are

1. Total-to-total efficiency
2. Total-to-static efficiency

1. Total-to-total Efficiency (η_{tt})

This is used when the kinetic energy at the outlet of the stage is utilised for producing work. Examples for such cases are the propelling nozzle of a turbojet exhaust, and any intermediate stage of a multi-stage turbine where the leaving kinetic energy is used in the following stage. In defining the efficiency, the temperature limits are taken between total temperatures and hence it is referred to as total-to-total efficiency.

$$\boxed{\eta_{tt} = \frac{h_{00} - h_{02}}{h_{00} - h_{02ss}}}$$

$$\eta_{tt} = \frac{h_0 - h_2}{h_0 - h_{2ss}} \text{ [for a normal stage } C_0 = C_2\text{]} \tag{5.12}$$

2. Total-to-static Efficiency (η_{ts})

This is used when the leaving kinetic energy is wasted, and hence it is not utilised to generate work. It is defined as

$$\boxed{\eta_{ts} = \frac{h_{00} - h_{02}}{h_{00} - h_{2ss}}} \tag{5.13}$$

For a normal stage

$$C_0 = C_2 \quad \text{and} \quad \alpha_0 = \alpha_2 \text{ and}$$

Upon rearranging

$$\eta_{tt} = (h_0 - h_2)/[(h_0 - h_2) + (h_2 - h_{2s}) + (h_{2s} - h_{2ss})] \tag{5.14}$$

But considering, the slope of a constant pressure line on the h-s diagram given by

$$(\partial h/\partial s)_P = T$$

$$T ds = dh - \vartheta dp$$

when p = constant

$$T = (\partial h/\partial s)_P$$

For a finite change of enthalpy Δh at constant pressure

$$\Delta h \simeq T \Delta s$$

Therefore, $(h_{2s} - h_{2ss}) \simeq T_2(s_{2s} - s_{2ss})$

And $(h_1 - h_{1s}) \simeq T_1(s_1 - s_{1s})$

By examining the Mollier chart, it is clear that

$$(s_{2s} - s_{2ss}) = (s_1 - s_{1s})$$

So, substituting this in the above equation,

$$(h_{2s} - h_{2ss}) = (T_2/T_1)(h_1 - h_{1s}) \tag{5.15}$$

The dimensionless loss coefficients may be defined in two ways.

(i) For the nozzle

The enthalpy loss coefficient

$$\boxed{L_N = (h_1 - h_{1s})/(0.5)C_1^2} \tag{5.16}$$

(or) the pressure loss coefficient

$$Y_N = (P_{00} - P_{01})/(P_{01} - P_1)$$
$$= (P_{00} - P_{01}) \Big/ \left(\frac{1}{2}\rho C_1^2\right)$$

(ii) For the rotor

The enthalpy loss coefficient

$$\boxed{L_R = (h_2 - h_{2s})/0.5W_2^2} \tag{5.17}$$

(or) the pressure loss coefficient

$$Y_R = (P_{01_{rel}} - P_{02_{rel}})/(P_{02_{rel}} - P_2)$$
$$= (P_{01_{rel}} - P_{02_{rel}}) \Big/ \left(\frac{1}{2}\rho W_2^2\right)$$

The value of L (or) Y in the stator and rotor gives the percentage drop of energy due to friction in the blades, which results in a total pressure and static enthalpy drop across the blades. The losses are usually in the order of 10–15 per cent and in proportion with flow coefficient. Losses can be lower for very low values of flow coefficient.

Total-to-total efficiency is calculated in terms of blade loss coefficient, equations (5.15), (5.16) and (5.17) being substituted in (5.14)

$$\eta_{tt} = (h_0 - h_2)\Big/\left[(h_0 - h_2) + \frac{L_R W_2^2}{2} + \left(\frac{T_2}{T_1} L_N \frac{C_1^2}{2}\right)\right] \tag{5.18}$$

If the exit velocity is not used, the total to static efficiency in terms of blade loss coefficients is given by

$$\eta_{ts} = \frac{h_{00} - h_{02}}{h_{00} - h_{2ss}}$$

For a normal stage $C_0 = C_2$. Hence,

$$\eta_{ts} = \frac{h_0 - h_2}{h_0 + \frac{C_0^2}{2} - h_{2ss}}$$

Upon rearranging

$$\eta_{ts} = \frac{h_0 - h_2}{(h_0 - h_2) + (h_2 - h_{2s}) + (h_{2s} - h_{2ss})} + \frac{C_0^2}{2}$$

From equations 5.15, 5.16, 5.17 and the above equation, we get

$$\eta_{ts} = (h_0 - h_2)/[(h_0 - h_2) + \frac{L_R W_2^2}{2} + \frac{T_2}{T_1} L_N \frac{C_1^2}{2} + C_0^2/2] \quad (5.19)$$

BLADE LOADING COEFFICIENT

The work capacity of the stage is expressed in terms of a temperature drop coefficient (or) 'blade loading coefficient'

$$\psi_l = \frac{W}{mU^2} = \frac{C_P(T_{01} - T_{02})}{U^2} \quad (5.20)$$

or

$$\psi_l = C_a(\tan\beta_1 + \tan\beta_2)/U$$

$$\Psi_l = \phi(\tan\beta_1 + \tan\beta_2) \quad (5.21)$$

where ϕ is the flow coefficient.

The implication of a low flow coefficient is that the frictional losses are reduced in the stage, since, C_a is low, ψ_l decreases with the decrease in ϕ. This implies that only a small amount of work is done per stage. Hence, for a required overall power output, a large number of stages are required.

In stationary industrial power plants where the specific fuel consumption is of prime importance, a large diameter, relatively long turbine of low flow coefficient and low blade loading, giving a high efficiency, would probably be accepted. Whereas, gas turbine used in aircraft propulsion have minimum weight and a small frontal area as chief factors. This means using higher values of flow coefficient and blade loading factor to give a shorter compact turbine. But as a consequence, the efficiency is lower. The foregoing discussion is summarized in the following table.

Table 5.1

S.No	Category	Prime factor	ϕ and ψ_l	Turbine size and η
1.	Stationary industrial turbine	Specific fuel consumption	Low	Large and high
2.	Aircraft gas turbine	Minimum weight and a small frontal area	High	Compact and low

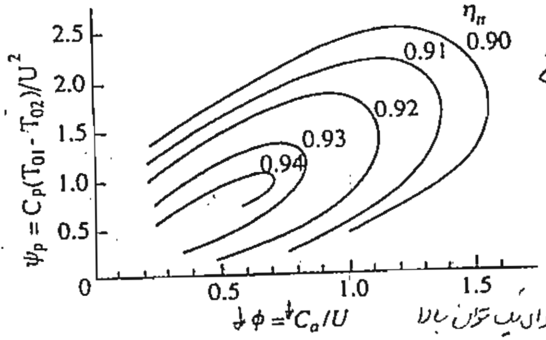


Figure 5.5 Blade loading coefficient versus flow coefficient

From the graph of blade loading coefficient versus flow coefficient (Fig. 5.5), it is seen that, for a high total-to-total efficiency, the blade loading factor should be as low as possible. If blade loading factor is low, the flow coefficient will be low, resulting in a high blade speed ($U = C_a/\phi$), which is consistent with allowable blade stresses. However, the variation of 'η_{tt}' with slight variation in the 'ψ_l' is very small. This is true for wide choice of reaction ratios. In contrast, the total-to-static efficiency is heavily dependent upon the reaction ratio and 'η_{ts}' can be optimised at a given 'ψ_l' by choosing a suitable value of reaction.

BLADE TYPES

Steam turbines are usually impulse or a mixture of impulse and reaction stages whereas gas turbines tend to be always of the reaction type.

The pressure ratio of steam turbines can be of the order of 1000 : 1 but for a gas turbine it is usually within the order of 10 : 1. So it is obvious that a very long steam turbine with many reaction stages would be required to reduce the pressure by a ratio of 1000:1. If the pressure drop per stage is made large to reduce the number of stages, the blade leakage loss would increase and lead to reduced efficiency. Therefore reaction stages are used where the pressure drop per stage is low and also where the overall pressure ratio of the turbine is relatively low (as in an aero-engine in which three (or) four reaction stages of (or) near 50 per cent reaction at the mean radius are employed). So, the shape of the blade varies with the different types of stages.

IMPULSE BLADING

The entire pressure drop of the gas occurs in the stator. In the rotor the gas velocity changes the direction but not magnitude.

Impulse blading is employed successfully at the high-pressure end of steam turbines. The velocity of the steam is increased in the convergent nozzle row to perhaps 800 m/sec before entering the rotor blades and passing through them at constant pressure. A simple impulse turbine is shown in Fig. 5.6.

Diagram efficiency is given by equation 5.6.

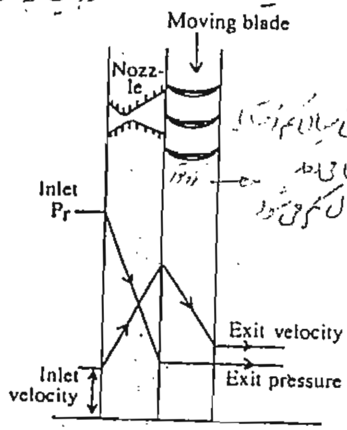


Figure 5.6 Pressure velocity diagram for a simple impulse turbine

$$\eta_{Diagram} = \frac{U(W_{x1} + W_{x2})}{\frac{1}{2}C_1^2}$$

$W_{x1} + W_{x2}$ can be rearranged as

$W_{x1} + W_{x2} = W_1 \sin \beta_1 + W_2 \sin \beta_2$

putting $\beta_1 = \beta_2$ for impulse or zero reaction

$W_{x1} + W_{x2} = W_1 \sin \beta_1 (1 + W_R)$

Where 'W_R' is the relative velocity ratio

$W_R = W_2/W_1$

$W_{x1} + W_{x2} = (C_1 \sin \alpha_1 - U)(1 + W_R)$

Therefore,

$$\eta_{Diagram} = \frac{2U(C_1 \sin \alpha_1 - U)(1 + W_R)}{C_1^2}$$

$$= 2\left[(U/C_1)\sin \alpha_1 - (U/C_1)^2\right](1 + W_R) \quad (5.22)$$

For maximum diagram efficiency, differentiate the above equation with respect to (U/C_1) and equate to zero. Then $\sin \alpha_1 - 2U/C_1 = 0$ or

$$\sigma = U/C_1 = \sin \alpha_1/2 \quad (5.23)$$

where σ is blade to gas speed ratio. Thus,

$$\eta_{max.diag} = \left[\sin^2 \alpha_1/2\right] \times (1 + W_R)$$

For an ideal turbine, $W_R = 1$

$$\therefore \eta_{max.diag} = \sin^2 \alpha_1 \quad (5.24)$$

This equation indicates that the nozzle angle α_1 should be as high as possible, the ideal being 90°. But α_1 is limited by C_a. Large α_1 means smaller 'C_a' resulting in longer blades to accommodate the required mass flow rate. Typical nozzle angles are between 65° and 78°. The rotor blade passages are usually of constant-area symmetrical cross section (Fig. 5.7) with inlet angle (β_1) and outlet angle (β_2) of 45°. Delaval turbine is an example of impulse turbine.

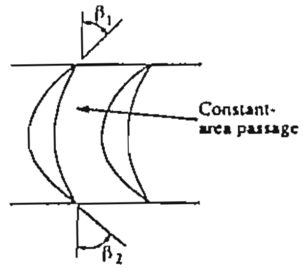


Figure 5.7 Impulse turbine blades

COMPOUNDING (OR) STAGING

The velocity of steam entering the turbine blades is very high, i.e. in the order of 1500 m/s, if the entire pressure drop from boiler pressure to condenser pressure is carried out in a single stage nozzle as in simple impulse turbine (Delaval turbine). As the turbine speed is directly proportional to the steam velocity, the turbine speed is increased as a result of high steam velocity. Speed may be in the order of 30,000 rpm. Such high rpm of the turbine rotors are not useful for practical purposes and a reduction gearing is necessary between the turbine and the generator driven by the turbine. There is also a danger of structural failure of the blades due to excessive centrifugal stress. Therefore the velocity of the blades is limited to about 400 m/s. The velocity of the steam at the exit of the turbine is also high, when a single stage of blades are used. This gives rise to a considerable loss of kinetic energy of about 10 to 12%.

These difficulties associated with the use of single stage turbine for large pressure drop and high speed is solved by compounding i.e. the use of more than one stage.

The types of compounding or staging are

(a) Pressure compounding: The total pressure drop of steam is divided into stages.

(b) Velocity compounding: The total enthalpy drop is converted into kinetic energy in one stage but the conversion of kinetic energy of steam into mechanical energy is divided into stages.

(c) Pressure and velocity compounding: The total pressure drop of steam is divided into stages and the velocity obtained in each stage is also compounded. This has the advantage of allowing a bigger pressure drop in each stage. Consequently less stages are necessary and a compact turbine will suffice for a given pressure drop.

TWO STAGE PRESSURE COMPOUNDED IMPULSE TURBINE

A number of simple impulse turbines stages, arranged in series is known as *pressure compounding*. In this, two stage pressure compounded impulse turbine, two impulse turbine (simple) are placed in series. The turbine is provided with one row of fixed blades which work as nozzles at the entry of each row of moving blades. The total pressure drop of the steam takes place in all the rows of fixed blades since they all work as nozzles.

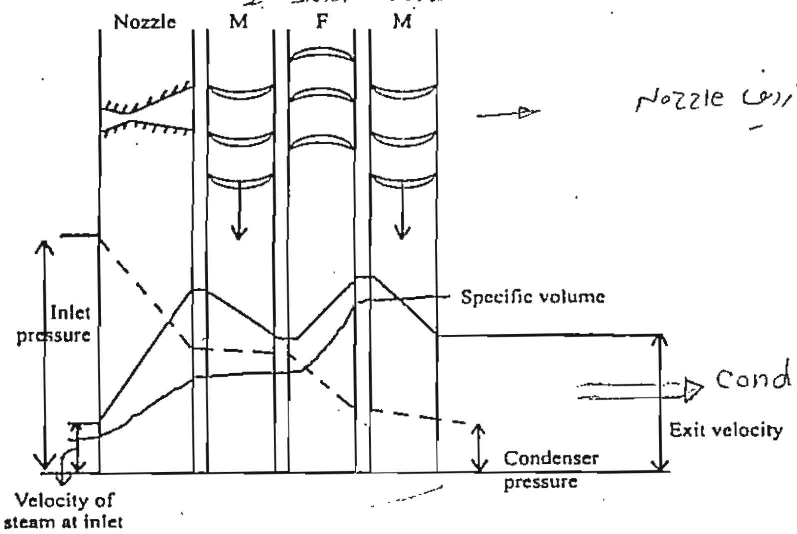


Figure 5.8 Pressure, velocity and specific volume diagram for a two stage pressure compounded impulse turbine

As the pressure of the steam gradually decreases, the specific volume of the steam gradually increases. Therefore, the blade height has to be increased towards the low pressure side.

The pressure and velocity variations across the turbine are shown in Fig. 5.8.

Examples for this type of turbines are 'RATEAU TURBINE' and 'ZOELLY TURBINE'

TWO STAGE VELOCITY COMPOUNDED IMPULSE TURBINE

The arrangement of a two stage velocity compounded impulse turbine is shown in the Fig. 5.9. There is only one set of nozzles and two or more rows of moving blades (in this case two rows of moving blades). There is a row of fixed blades in between the moving blades as shown in Fig. 5.9. The function of the fixed blades is only to direct the steam coming from the first moving row to the next moving row. So, these are also known as '*Guide blades*'. The enthalpy drop takes place only in the nozzle at the first stage and it is converted into kinetic energy. The kinetic energy of the steam gained in the nozzles is successively absorbed by the rows of moving blades and finally the steam is exhausted from the last row of the blades.

The variation of pressure and velocity of the steam along the axis are also shown in the Fig. 5.9.

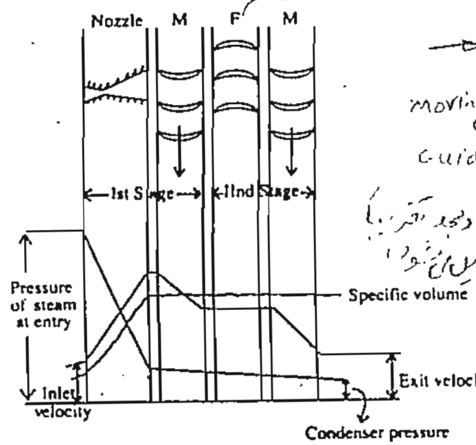


Figure 5.9 Pressure, velocity and specific volume diagram of two stage velocity compounded impulse turbine

Example for velocity compounded impulse turbine is 'CURTIS TURBINE'.

The specific volume of the steam remains constant as the steam flows along the axis of the turbine. Hence, the blade height is same in all rows.

The velocity compounded turbines are mainly used as drives for centrifugal compressors, pumps, small generators and for driving feed pumps in big power units.

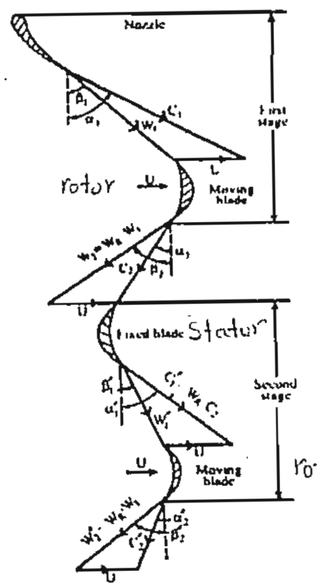


Figure 5.10 Two stage velocity compounded impulse turbine

VELOCITY TRIANGLES OF THE TWO STAGE IMPULSE TURBINE

The velocity triangles for a two stage impulse turbine are shown in Fig. 5.10. Fig. 5.10(a) and (b) show the combined velocity triangles of each stage.

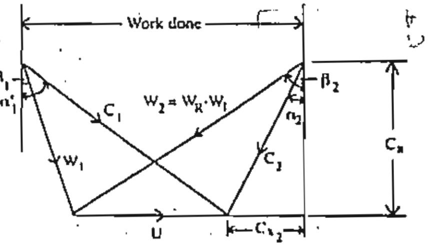


Figure 5.10(a) Superimposed velocity triangles for first stage

There is always a certain loss of velocity during the flow of steam over the blade and this loss is taken into account by introducing a factor called *blade velocity coefficient*.

Blade velocity coefficient (W_R) is given by $W_R = W_2/W_1$.

(The relative velocity of steam in the impulse turbine blade remains constant) as the steam glides over the blades (or) is reduced slightly due to friction.

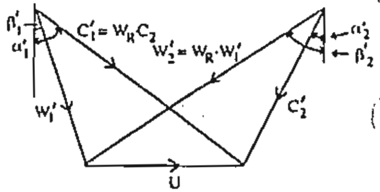


Figure 5.10(b) Superimposed velocity triangles for second stage

In reaction turbine blades the steam expands as it flows over the moving blades. This increases the relative velocity of steam.

$W_2 > W_1$ (Reaction turbines)

$W_2 \leq W_1$ (for Impulse turbines)

Power for the two rows of blades $= mU(\Delta W_{x_I} + \Delta W_{x_{II}})$

where

ΔW_{x_I} = change of velocity of whirl of blade row 1 $(W_2 \sim W_1)$

$\Delta W_{x_{II}}$ = change of velocity of whirl of blade row 2 $(W_2' \sim W_1')$

$$\text{Blade (or) diagram efficiency} = \frac{2U(\Delta W_{x_I} + \Delta W_{x_{II}})}{C_1^2}$$

Note that the reference here is still to the *kinetic energy* of the input steam $(C_1^2/2)$

$$\text{End thrust} = m(\Delta C_{a_I} + \Delta C_{a_{II}})$$

where

ΔC_{a_I} = change in axial velocity of blade row 1.

$\Delta C_{a_{II}}$ = change in axial velocity of blade row 2.

Note that either C_{a_I} (or) $C_{a_{II}}$ can be positive (or) negative.

DIAGRAM EFFICIENCY OF A TWO STAGE CURTIS TURBINE

Work done per unit mass flow from the first row of moving blades is given by

$$\begin{aligned}(W/m)_1 &= U(C_{x_1} + C_{x_2}) \\ &= U(C_1 \sin\alpha_1 + C_2 \sin\alpha_2) \\ &= U[(W_1 \sin\beta_1 + U) + (W_2 \sin\beta_2 - U)] \qquad (5.25)\end{aligned}$$

Assuming symmetrical blades with no friction loss, $W_1 = W_2$ and $\beta_1 = \beta_2$

$$\begin{aligned}(W/m)_1 &= 2UW_1 \sin\beta_1 \\ &= 2U[C_1 \sin\alpha_1 - U] \qquad (5.26)\end{aligned}$$

Work done from the second row of moving blades,

$$\begin{aligned}(W/m)_2 &= U(C_{x_1}' + C_{x_2}') \\ &= U(C_1' \sin\alpha_1' + C_2' \sin\alpha_2') \qquad (5.27)\end{aligned}$$

Assuming that $C_1' = C_2$, $\alpha_1' = \alpha_2$ and $\alpha_2' = 90°$,

$$= U(C_2 \sin\alpha_2) \qquad (5.28)$$

Equation (5.27) can be written as

$$= U[(W_1' \sin\beta_1' + U) + (W_2' \sin\beta_2' - U)]$$

If $\beta_1' = \beta_2'$ (symmetrical blades) and $W_1' = W_2'$ (no blade friction), then

$$(W/m)_2 = 2UW_1' \sin\beta_1'$$

$$W_1' \sin\beta_1' = C_1' \sin\alpha_1' - U$$

But from equation 5.28 $C_1' \sin\alpha_1' = C_2 \sin\alpha_2$

And

$$\begin{aligned}C_2 \sin\alpha_2 &= W_2 \sin\beta_2 - U \\ &= W_1 \sin\beta_1 - U \\ &= (C_1 \sin\alpha_1 - U) - U \\ &= C_1 \sin\alpha_1 - 2U\end{aligned}$$

Therefore,

$$(W/m)_2 = 2UW_1' \sin\beta_1' = 2U[C_1 \sin\alpha_1 - 3U] \qquad (5.29)$$

The total work done per unit mass flow

$$\begin{aligned}(W/m)_{total} &= (W/m)_1 + (W/m)_2 \\ &= 2U[C_1 \sin\alpha_1 - U] + 2U[C_1 \sin\alpha_1 - 3U] \qquad (5.30) \\ &= 2U[2C_1 \sin\alpha_1 - 4U]\end{aligned}$$

For a two stage turbine,

$$\eta_{dia} = \frac{2U[2C_1 \sin\alpha_1 - 4U]}{C_1^2/2}$$

$$= 8\left(\frac{U}{C_1}\right)\sin\alpha_1 - 16\left(\frac{U}{C_1}\right)^2 \quad (5.31)$$

$$= 8\sigma \sin\alpha_1 - 16\sigma^2$$

where $\sigma = \dfrac{U}{C_1}$, is the blade speed ratio.

For maximum efficiency,

$$\frac{d\eta_{dia}}{d\sigma} = 0$$

$$\therefore 8\sin\alpha_1 - 32\sigma = 0 \quad (5.32)$$

$$\sigma_{opt} = \frac{\sin\alpha_1}{4}$$

The maximum diagram efficiency is

$$\eta_{dia,max} = \frac{8\sin^2\alpha_1}{4} - 16\frac{\sin^2\alpha_1}{16} \quad (5.33)$$

$$= \sin^2\alpha_1$$

The maximum work done is

$$W/m = 2U\left[2C_1\left(\frac{4U}{C_1}\right) - 4U\right]$$

$$= 2U[4U] = 8U^2 \quad (5.34)$$

From the foregoing analysis, we can write, in general, for 'n' rows of blades the optimum blade speed ratio

$$\boxed{\sigma_{opt} = \frac{\sin\alpha_1}{2n}} \quad (5.34a)$$

Work done in the last row $= \dfrac{1}{2^n}$ of total work.

Thus for a three row velocity compounded turbine, the ideal $\sigma = \dfrac{\sin\alpha_1}{6}$ and the last row of blades would do only 1/8th of the total work.

Comparing the relations $\sigma = \dfrac{\sin\alpha_1}{2}$ for a single stage impulse turbine and $\sigma = \dfrac{\sin\alpha_1}{4}$ for a two row impulse turbine, we find that for the same blade speed and the same nozzle angle, the steam velocity at the nozzle exit (C_1) of two rows velocity compounded impulse turbine is twice that for a simple impulse turbine. Since blade work is proportional to kinetic energy, theoretically the work of a two row CURTIS stage is four times that of a simple stage, for the same blade speed.

REACTION BLADING

The pressure reduces through succeeding stator and rotor rows, the velocity is increased at the expense of pressure drop and this necessitates a blade passage that is convergent towards the outlet.

For 50 per cent reaction the stator and rotor blades will be the same, whereas zero reaction implies impulse rotor blades with constant cross-sectional area passages and no change in flow velocity. Reaction of 100 per cent implies that the stator blades are of the constant area impulse type. The inlet angle β_1 for the reaction blade is almost zero ($\beta_1 \simeq 0$) while the profile of the back of the blade is almost linear.

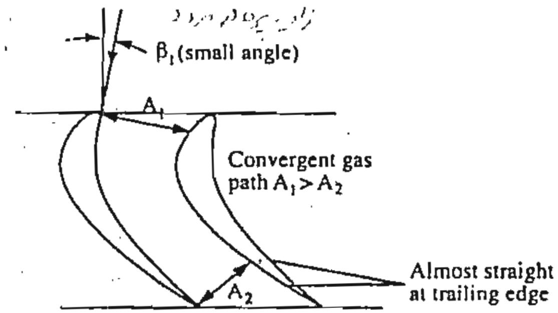


Figure 5.11 Reaction blading

Reaction blading is often shrouded at the tips, especially if the blades are long. This aids in preventing excessive vibration by tying the blades together and thus changing them from cantilevers to blades fixed at both ends (Fig. 5.11).

THE REACTION TURBINE

Construction of the reaction turbine is somewhat different from that of the impulse turbine.

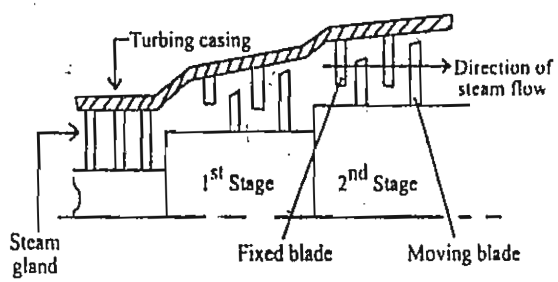


Figure 5.12 Reaction turbine

Essentially, the reaction turbine consists of rows of blades mounted on a drum. These drum blades are separated by rows of fixed blades mounted in the casing.

Unlike the impulse turbine, no nozzles as such, are mounted in a reaction turbine. The fixed blades act as nozzles in which the velocity of the steam is increased and they also direct the steam correctly onto the moving blades.

The diameter of both rotor and casing varies (increases) towards the low pressure side. This is mainly due to increase in specific volume, as the pressure of the steam decreases. The steam velocity in a reaction turbine is not very high and hence the speed of the turbine is relatively low.

A reaction turbine is illustrated diagrammatically in the Fig. 5.12.

In this turbine, the power is obtained mainly by an impulsive force of the incoming steam and small reactive force of the outgoing steam.

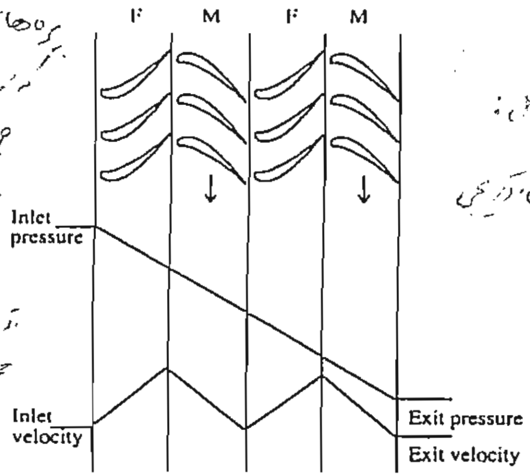


Figure 5.13 Pressure and velocity variations across a reaction turbine

The shape of the moving blades is so designed to have the reactive force of the leaving steam. To accomplish this area of the outlet between the two moving blades will be reduced than that at the inlet Fig . 5.13(a).

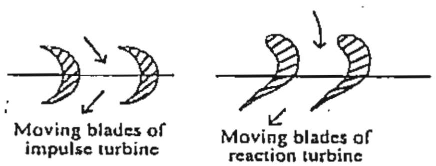


Figure 5.13(a)

This pressure and velocity variations across a reaction turbine is shown in the Fig. 5.13.

(The steam also expands in the moving blades) of a reaction turbine with consequent pressure drop and velocity increase in these moving blades. This is unlike the impulse turbine where the pressure drop takes place in the nozzles only and not in the turbine.

This, expansion in the moving blades of a reaction turbine gives an extra reaction to the moving blades over that which would be obtained if the blades were impulse. This extra reaction gives its name to the turbine, the '*reaction turbine*'.

(In a reaction turbine a stage is made up of a row of fixed blades followed by a row of moving blades. Steam acceleration usually occurs in both the fixed and moving blade rows and hence the steam passage between the blades are nozzle shaped. So, the blades of reaction turbine differ from that of the impulse turbine.

There is an enthalpy drop in the steam during its passage through the blades which produces the acceleration. The extent to which the enthalpy drop occurs in the moving blades is called the '*degree of reaction*'.

If 50% of the enthalpy drop occurs in the moving blades, the stage will be said 50% reaction stage. A 50% reaction stage is more common in a reaction turbine. But in an impulse turbine, the entire enthalpy drop occurs in the fixed blades.

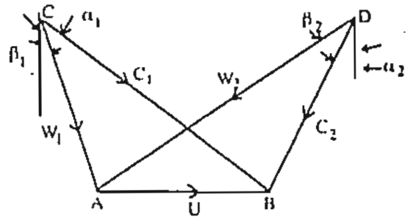


Figure 5.14 Superimposed velocity triangles for a reaction turbine

VELOCITY DIAGRAM FOR REACTION TURBINE STAGE

The velocity diagram for a reaction turbine stage is illustrated in the Fig. 5.14. The diagram illustrated is symmetrical, showing equal accelerations in both fixed and moving blades and hence this diagram illustrates the condition of 50% reaction.

Due to acceleration in the moving blades $W_2 > W_1$ and thus, other things being equal, there is a greater change in velocity of whirl over that, which would be obtained with impulse blading. Velocities are however, lower than that, which would be found in impulse turbines.

(With the symmetrical diagram illustrated, there is no change in velocity of flow and hence no end thrust due to this phenomenon)

However in a reaction turbine, there is a pressure drop across each stage and due to this, together with the blade annulus area presented, an end thrust will result.

As with the impulse turbine

$$\text{Power} = mU(\text{change in velocity of whirl})$$

End thrust (due to velocity change) = m(change in velocity of flow)

However the stage efficiency must be related to the energy available to the stage.

In this case energy available to the stage is given by Δh = Sp. enthalpy drop in stage.

Stage efficiency

$$\eta = \frac{\text{Work done in stage}}{\text{Enthalpy drop in stage}}$$

$$= \frac{mU(W_{x_1} + W_{x_2})}{m(\Delta h)}$$

$$\boxed{\text{Stage efficiency} = \frac{U(W_{x_1} + W_{x_2})}{\Delta h}}$$

STAGE EFFICIENCY OF A REACTION TURBINE

The stage efficiency of a reaction turbine is the ratio of work done to the energy input

$$\eta_{stage} = \frac{U(W_{x_1} + W_{x_2})}{\frac{C_1^2}{2} + \left(\frac{W_2^2 - W_1^2}{2}\right)} \tag{5.35}$$

Rewriting,

$$\eta_{stage} = \frac{U(W_{x_1} + W_{x_2})}{\frac{C_1^2 - C_2^2}{2} + \left(\frac{W_2^2 - W_1^2}{2}\right) + \frac{C_2^2}{2}} \tag{5.36}$$

For a turbine of 0.5 reaction,

$$C_1 = W_2 \text{ and } C_2 = W_1$$
$$\alpha_1 = \beta_2 \text{ and } \alpha_2 = \beta_1$$

Considering the numerator of equation 5.36 $U(W_{x_1} + W_{x_2})$,

$$W_{x_1} + W_{x_2} = (C_{x_1} - U) + W_{x_2}$$
$$= C_1 \sin\alpha_1 - U + W_2 \sin\beta_2$$

(or)

$$W_{x_1} + W_{x_2} = C_1 \sin\alpha_1 - U + C_1 \sin\alpha_1$$
$$= 2C_1 \sin\alpha_1 - U$$
$$\therefore\ U(W_{x_1} + W_{x_2}) = 2UC_1 \sin\alpha_1 - U^2 \tag{5.37}$$

Considering the denominator of equation 5.36,

$$\frac{C_1^2 - C_2^2}{2} + \frac{W_2^2 - W_1^2}{2} + \frac{C_2^2}{2} = \frac{W_2^2 - W_1^2}{2} + \frac{W_2^2 - W_1^2}{2} + \frac{W_1^2}{2}$$
$$= W_2^2 - W_1^2 + \left(\frac{W_1^2}{2}\right)$$
$$= W_2^2 - \left(\frac{W_1^2}{2}\right)$$

(or)

$$= \frac{2W_2^2 - W_1^2}{2} \tag{5.38}$$

Substituting equations (5.37) & (5.38) in equation (5.36).

$$\eta_{stage} = \frac{2[2UC_1 \sin\alpha_1 - U^2]}{2W_2^2 - W_1^2} \tag{5.39}$$

Using cosine rule,

$$W_1^2 = C_1^2 + U^2 - 2UC_1 \sin\alpha_1$$
$$\text{and}\quad W_2 = C_1$$

Equation (5.39) becomes

$$\eta_{stage} = \frac{2[2UC_1 \sin\alpha_1 - U^2]}{2C_1^2 - (C_1^2 + U^2 - 2UC_1 \sin\alpha_1)}$$
$$= \frac{2[2UC_1 \sin\alpha_1 - U^2]}{(C_1^2 - U^2 + 2UC_1 \sin\alpha_1)}$$
$$= \frac{2[2UC_1 \sin\alpha_1 - U^2 + C_1^2 - C_1^2]}{(C_1^2 - U^2 + 2UC_1 \sin\alpha_1)}$$
$$= 2 - \frac{2C_1^2}{(C_1^2 - U^2 + 2UC_1 \sin\alpha_1)}$$

or

$$\eta_s = 2 - \frac{2}{1 - \left(\frac{U}{C_1}\right)^2 + 2\frac{U}{C_1}\sin\alpha_1}$$
$$= 2 - \frac{2}{1 - \sigma^2 + 2\sigma \sin\alpha_1} \tag{5.40}$$

where $\sigma = \frac{U}{C_1}$ called the blade to gas speed ratio.

Maximum Stage Efficiency

This is found from the term $1 - \sigma^2 - 2\sigma \sin\alpha_1$.

This term should be maximum for the η_s to be maximum.

$$\therefore\ \frac{d\eta_s}{d\sigma} = -2\sigma - 2\sin\alpha_1 = 0.$$
$$\therefore\ \sigma = \sin\alpha_1 \tag{5.41}$$

So,

$$\eta_{max.stage} = 2 - \frac{2}{1 - \sin^2\alpha_1 + 2\sin^2\alpha_1}$$

$$= 2 - \frac{2}{1+\sin^2\alpha_1}$$

$$\eta_{max,s} = \frac{2\sin^2\alpha_1}{1+\sin^2\alpha_1} \quad (5.42)$$

REACTION RATIO

The reaction ratio of an axial flow turbine varies widely from 0 to 100 per cent where as the reaction ratio of an axial flow compressor is usually set at 50 per cent for the stage.

The reaction ratio R is given by

$$R = \frac{\text{Static enthalpy drop across rotor}}{\text{Static enthalpy drop across stage}}$$

$$= \frac{h_1 - h_2}{h_0 - h_2} \quad (5.43)$$

$$= \frac{h_1 - h_2}{\left(h_{00} - \frac{C_0^2}{2}\right) - \left(h_{02} - \frac{C_2^2}{2}\right)}$$

But for a normal stage $C_0 = C_2$ and since, $h_{00} = h_{01}$ in the nozzle

$$R = \frac{h_1 - h_2}{h_{01} - h_{02}} \quad (5.44)$$

Remembering that $h_{01,rel} = h_{02,rel}$ then $h_{01,rel} - h_{02,rel} = (h_1 - h_2) + \frac{(W_1^2 - W_2^2)}{2} = 0$ and substituting for $h_1 - h_2$ in equation (5.44),

$$R = \frac{W_2^2 - W_1^2}{[2(h_{01} - h_{02})]}$$

$$= \frac{W_2^2 - W_1^2}{2U(C_{x1} + C_{x2})]} \quad (5.45)$$

If 'C_a' is assumed to be constant through the stage, then

$$W_2^2 = C_a^2 + W_{x_2}^2$$

$$W_1^2 = C_a^2 + W_{x_1}^2$$

$$R = \frac{W_{x_2}^2 - W_{x_1}^2}{2U(U + W_{x_1} + W_{x_2} - U)}$$

$$= \frac{(W_{x_2} - W_{x_1})(W_{x_2} + W_{x_1})}{2U(W_{x_1} + W_{x_2})} = \frac{W_{x_2} - W_{x_1}}{2U}$$

$$= \frac{C_a(\tan\beta_2 - \tan\beta_1)}{2U}$$

$$= \frac{\phi(\tan\beta_2 - \tan\beta_1)}{2} \quad (5.46)$$

Equation (5.46) can be rearranged into a second form

$$R = \frac{W_{x_2} - W_{x_1}}{2U}$$

Put $W_{x_1} = (C_{x_1} - U)$

$$\text{Then, } R = \frac{(C_a\tan\beta_2)}{2U} - \left[\frac{C_a\tan\alpha_1}{2U} - \frac{U}{2U}\right]$$

$$= 0.5 + \left[\frac{C_a(\tan\beta_2 - \tan\alpha_1)}{2U}\right] \quad (5.47)$$

And a third form is given by substituting for

$$\tan\beta_2 = \frac{(U + C_{x_2})}{C_a} = (\tan\alpha_2 + U/C_a)$$

Equation (5.47) then becomes

$$R = 0.5 + \frac{C_a\left(\tan\alpha_2 + \frac{U}{C_a} - \tan\alpha_1\right)}{2U}$$

$$= 1 + \left[\frac{C_a(\tan\alpha_2 - \tan\alpha_1)}{2U}\right] \quad (5.48)$$

(a) Zero reaction stage If $R = 0$, from equation (5.46), $\beta_2 = \beta_1$ and from Equation (5.45) $W_2 = W_1$

The conditions of gas through the stage, and the accompanying velocity triangles are shown in the figure 5.15 (a).

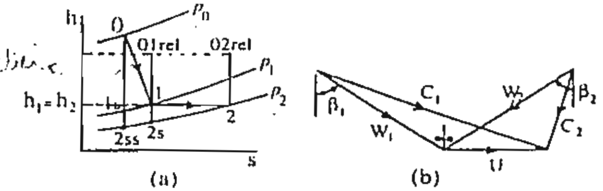


Figure 5.15(a) Zero reaction axial gas turbine

The velocity triangle is inclined more towards the left.

Ideally, for reversible adiabatic flow, the points 1,2 and 2s on the Mollier chart should coincide, and in that case no pressure drop occurs in the rotor.

(b) Pure impulse stage In this stage by definition there is no pressure drop in the rotor. For reversible adiabatic flow, the points 1,2 and 2s will coincide (Fig 5.15 (b)).

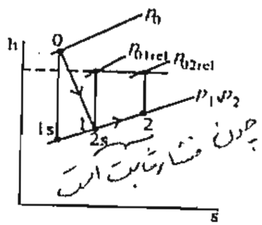


Figure 5.15(b) Pure impulse stage in an axial gas turbine

(c) Negative reaction stage Ideally, for reversible adiabatic flow, the points 1,2 and 2s on the Mollier chart coincide in the zero reaction stage. Therefore, with isentropic flow conditions prevailing the zero reaction stage is exactly the same as the impulse stage. However when the flow is irreversible, they are not same and in fact an increase in enthalpy occurs in the rotor of the impluse stage (Fig. 5.15 (c)). This stage is referred to as a negative reaction stage.

For a negative reaction stage, $W_2 < W_1$ (from equation (5.45)) thereby causing diffusion of the the relative velocity vector in the rotor and a subsequent rise in pressure. This condition should be avoided, since adverse pressure gradients causing flow separation on the blade surfaces results in poor efficiency.

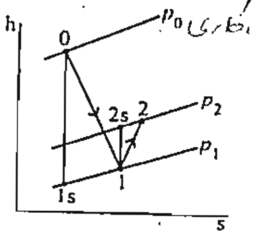


Figure 5.15(c) Negative reaction

(d) 50 per cent reaction stage When $R = 0.5$ from equation (5.47) $\beta_2 = \alpha_1$, $\beta_1 = \alpha_2$. Also $C_1 = W_2$ and $C_2 = W_1$. This results in a symmetrical velocity diagram. The drop in enthalpy in the stator and rotor are equal (Fig. 5.15 (d)).

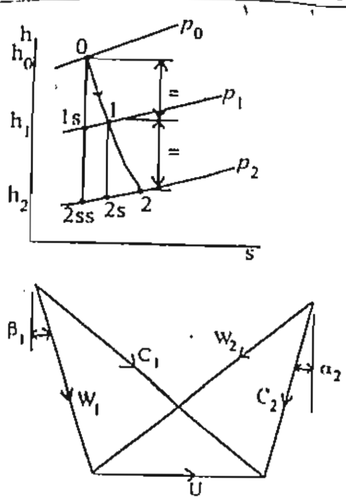


Figure 5.15(d)

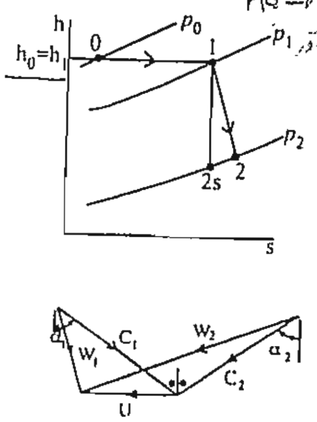


Figure 5.15(e)

Figure 5.15(d) A 50 per cent reaction stage in an axial gas turbine
Figure 5.15(e) A 100 per cent reaction stage in an axial gas turbine

(e) 100 per cent reaction stage When $R = 1$ (equation 5.48) gives $\alpha_1 = \alpha_2$ and $C_1 = C_2$. The velocity diagram is inclined to the right. There is no static enthalpy drop in the stator (Fig. 5.15 (e)).

(f) Reaction more than 100% Increasing the reaction ratio to greater than 1 gives rise to diffusion in the stator passages or nozzles with $C_1 < C_0$ (Fig. 5.15 (f)). This situation should also be avoided because of the likelihood of flow separation on the stator blade surfaces.

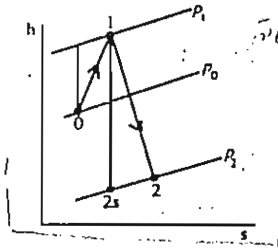


Figure 5.15(f) Stage expansion with reaction more than 100 per cent in an axial gas turbine

STAGE EFFICIENCY, TURBINE EFFICIENCY AND REHEAT FACTOR

Fig. 5.16 shows the expansion of steam through a number of turbine stages. A_1B_1 represents the isentropic expansion in the first stage. The actual state of steam with frictional reheating is shown by point A_2. So the actual heat drop is A_1C_1. Similarly, the isentropic and actual stages of heat drop for the succeeding stages are shown in Fig. 5.16 by A_2B_2, A_3B_3 and A_2C_2, A_3C_3 and so on.

The drop A_1D represents the overall isentropic heat drop (or) Rankine heat drop between the inlet and outlet state of steam.

The sum of the isentropic drops in all stages of the turbine ($A_1B_1 + A_2B_2 + A_3B_3 + \ldots$) is called the '*Cumulative enthalpy drop*'. The cumulative enthalpy drop is always greater than Rankine enthalpy drop (A_1D) as the constant pressure lines diverge from left to right on the Mollier chart.

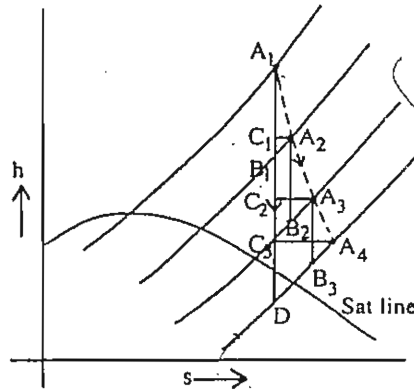


Figure 5.16 Expansion process in a multistage turbine

The stage efficiency for the given stage is

$$\boxed{\eta_s = \frac{\text{Actual enthalpy drop}}{\text{Isentropic enthalpy drop}}}$$

$$\eta_{s_1} = \frac{A_1C_1}{A_1B_1}$$

and $\eta_{s_2} = \dfrac{A_2C_2}{A_2B_2}$ and so on.

The reheat factor is defined as the ratio of the cumulative enthalpy drop to the Rankine enthalpy drop and it is given by

$$\boxed{R.F = \frac{A_1B_1 + A_2B_2 + \cdots}{A_1D}}$$

$$\text{For 3 stages} = \frac{\sum_{n=1}^{n=3} A_nB_n}{A_1D} \tag{5.49}$$

The turbine internal efficiency is defined as the ratio of the total actual turbine work to the Rankine work.

$$\boxed{\eta_t = \frac{A_1C_1 + A_2C_2 + \cdots}{A_1D}}$$

$$= \frac{\sum_{n=1}^{n=3} A_nC_n}{A_1D} \tag{5.50}$$

If $\eta_{s_1} = \eta_{s_2} = \eta_s$ is assumed

$$\eta_s = \frac{A_1C_1}{A_1B_1} = \frac{A_2C_2}{A_2B_2} = \frac{A_3C_3}{A_3B_3}$$

$$= \frac{A_1C_1 + A_2C_2 + A_3C_3}{A_1B_1 + A_2B_2 + A_3B_3}$$

$$\eta_s = \frac{\sum_{n=1}^{n=3} A_nC_n}{\sum_{n=1}^{n=3} A_nB_n} \tag{5.51}$$

From equations (5.50) and (5.51),

$$\boxed{\eta_s = \frac{\eta_t \times A_1D}{\sum_{n=1}^{n=3} A_nB_n}}$$

$$\eta_s = \frac{\eta_t}{R.F}$$

$$\text{or } \eta_t = \eta_s \times R.F$$

FREE VORTEX DESIGN

Free vortex principle is used for the design of long blades.

For constant stagnation enthalpy across the annulus $\left(\dfrac{dh_0}{dr} = 0\right)$ and constant axial velocity $\left(\dfrac{dC_a}{dr} = 0\right)$, the whirl component of velocity C_x is inversely proportional to the radius. Radial equilibrium is achieved in free vortex design only when

$$\boxed{C_x r = \text{constant}} \tag{5.52}$$

along the blade height.

CONSTANT NOZZLE ANGLE STAGE

Constant nozzle angle blades (or) straight blades are simple and less expensive, compared to twisted nozzle blades. For a constant absolute air angle (α_1) stage and radial equilibrium, the relationship between mean section and any section at radius, r, is

$$\boxed{\frac{C_1}{C_{1,m}} = \frac{C_{x_1}}{C_{x_{1,m}}} = \frac{C_{a_1}}{C_{a_{1,m}}} = \left(\frac{r_m}{r}\right)^{\sin^2\alpha_1}}$$

IMPULSE TURBINES VERSUS REACTION TURBINES

The salient differences between an impluse turbine and reaction turbine are stated below.

(1) In impulse turbine, the fluid is expanded completely in the nozzle and it remains at constant pressure during its passage through the moving blades.
In reaction turbine the fluid is only partially expanded in the nozzle and the remaining expansions take place in the rotor blades.

(2) In impulse turbines when the fluid glides over the moving blades, the relative velocity of fluid either remains constant or reduces slightly due to friction (i.e. $W_2 \leq W_1$).
In reaction turbine, since the fluid is continuously expanding, relative velocity does increase ($W_2 > W_1$).

(3) Impulse blades are of the plate or profile types and are symmetrical as shown in Fig. 5.17. Reaction turbine blades have aerofoil section and are asymmetrical. The blade is thicker at one end (Fig. 5.17) and this provides a suitable shaped passage for the fluid to expand.

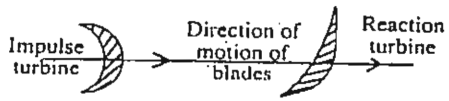


Figure 5.17

(4) The blades of an impluse turbine are only in action when they are in front of the nozzles, whereas blades of the reaction turbine are in action all the time.

(5) Impulse turbines have the same pressure on the two sides of the rotor blades, whereas different pressures exist on the two sides of the moving blade of a reaction turbine.

(6) Because of small pressure drop in each stage, the number of stages required for a reaction turbine are much greater than those for an impulse turbine of the same power.

(7) The fluid velocity and the blade speed for a reaction turbine are low as compared with those of an impulse turbine.

(8) The variation of diagram efficiency with blade speed ratio is shown in Fig. 5.18. On comparing both impulse turbine and reaction turbine, it is clear from the graph that, for a reaction turbine the efficiency curve is reasonably flat in the region of maximun diagram efficiency. This point is of great significance as the small variations in the blade speed ratio $\left(\frac{U}{C_1}\right)$, can be accepted without having much variation in the value of diagram efficiency.

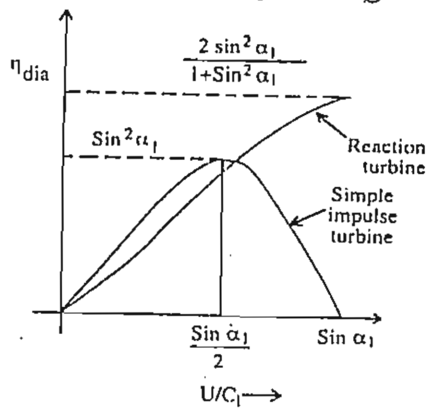


Figure 5.18 Variation of diagram efficiency with blade speed ratio

INTERNAL LOSSES IN TURBINES

An ideal turbine would develop work equivalent to the isentropic heat drop of the steam. But, in practice, the actual work obtained from a turbine is much less than the theoretical work (or) isentropic work. This difference is due to the various losses enumerated as follows,

(a) Governor valve losses Usually all turbines are fitted with governors for the purpose of speed regulation. The first loss occurs here in the form of *'throttling'* at the main stop valve. This loss may be of the order of 5 to 10%

(b) Nozzle friction losses The friction and eddies cause some pressure drop in the nozzle but the most significant loss occurs in the velocity of the jet at the exit of the nozzle. This loss may amount to another 5 to 10%

(c) Blade friction losses Due to this loss, the relative velocity of steam at the inlet to the blade is correspondingly reduced at the exit of the blade

$$W_2 = W_R W_1$$

where W_{R}, a coefficient which varies from 0.7 to 0.9, which takes into account the loss in velocity due to friction.

(d) Disc friction losses When the disc (or) the turbine wheel rotates in a dense (or) viscous medium like steam, certain fluid resistance is experienced by the wheel. As a result, the moving steam creates a drag on the steam which sets it in motion. Also, a certain definite *'circulation'* of steam within the wheel openings is developed, thereby increasing the frictional losses. It is difficult to reduce the losses due to such drag forces, known as *'disc friction losses'*, in turbines. This loss is about 10%.

(e) Partial admission losses In the first stage of a high pressure turbine, owing to comparatively small area required for the nozzles, the latter extend over the whole periphery of the stage and thus a few blades remain partially filled with steam, in which the flow also gets disturbed considerably. There will be certain eddies produced in the channels of the idle blades. Since the casing is full of steam, even these blades which are not under the direct influence of jets will churn (shake) the steam eddies and thus produce *'fan losses'* (or) *'windage losses'* which are known as partial admission losses. Attempts may be made to reduce these losses by fitting stationary shields around the moving blades which are not receiving the steam.

(f) Gland leakage losses There is a small loss of energy in each stage of the turbine, owing to the leakage of steam from one wheel chamber to the next through the glands. Here some space between the diaphram and the shaft may be existing and lead to leakage. Actually, the function of glands at high pressure end is to check the leakage of steam to the atmosphere, while that at the low pressure end is to prevent in-leakage of air to the turbine. However, even a best type of gland is susceptible to leaks and causes such type of losses. Two types of glands are in general use.

(i) The carbon ring glands
(ii) The labyrinth packing glands, which minimise the gland leakage losses.

(g) Residual velocity loss In the final stage of a turbine, the kinetic energy corresponding to the final absolute velocity of the steam as it leaves the wheel is lost wholly (or) partially. Arrangement may be made to recover a part of the energy by reducing the velocity between the last stage and the exhaust branch. This type of loss is known as the residual velocity loss, and may be reduced also by providing guide vanes in the exhaust hood to perform some diffuser action. This loss is equal to the

$C_2^2/2$, where C_2 is the absolute velocity of the steam at the blade exit. When this kinetic energy is passed over to the next stage, it is termed *"carry over"*.

In addition to these losses, there may be some loss of heat energy due to *'radiation'* to the ambient surroundings. In all, the total internal losses in a turbine may be 20 to 30% or so.

(h) Loss due to moisture In the lower stages of the turbine, the steam may become wet as the velocity of water particles is lower than that of the steam. So a part of the kinetic energy of steam is lost to drag the water particles along with it.

GOVERNING OF TURBINES

In a normal turbine driving an alternater, the energy output will vary in accordance with the load. The objective of a governor is to maintain the speed of the turbine constant irrespective of the load. The performance of the turbine itself depends, to a large extent, on the particular method employed for controlling the supply of steam to the turbine so that the *'speed of rotation'* will remain constant. The chief governing methods are

(a) Throttle governing The principle of this method basically requires *'throttling'* of the steam, so as to reduce the steam flow whenever there is a reduction of load on the turbine. In addition to a stop valve, the turbine has a *'double beat valve'* having seats of equal or nearly equal areas, so shaped that the forces on the valve due to static pressure and dynamic action are balanced. This double beat valve actually throttles the steam (Fig. 5.19). It is operated by a servo-motor controlled by a centrifugal governor, which is driven by a worm gear at a speed less than that of a turbine.

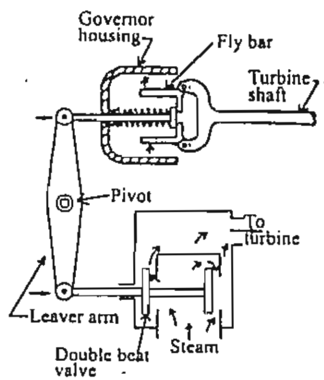


Figure 5.19 Throttle governing with double beat valve

(b) Nozzle control governing In this method the nozzles of the turbines are grouped in two, three (or) more groups and each group of the nozzles is fed with the steam supply controlled by valves (Fig. 5.20). Different types of arrangements of valves and groups of nozzles may be employed. But the nozzle control is necessarily

restricted to the first stage of the turbine, the nozzle areas in the other stages remaining constant. It follows that, provided the condition of the steam at the inlet to the second stage is not materially effected by the changed condition of the first stage, the absolute pressure of steam in front of the second stage nozzle will be directly proportional to the rate of steam flow through the turbine.

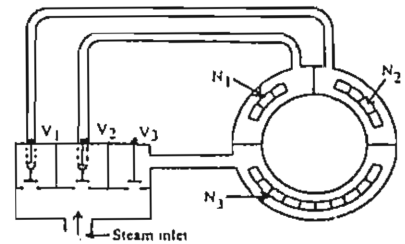


Figure 5.20 Nozzle control governing

(c) Bypass governing The modern high pressure turbines consist of a number of stages of comparatively small mean diameters of wheel. Owing to small heat drop in the first stage, employing nozzle control governing is not advisable. Further, in case of higher loads the extra steam required cannot be admitted through additional nozzles in the first stage, due to various reasons. These difficulties of regulation are overcome by the use of by pass governing (Fig. 5.21) In this method, the steam enters the turbine chest through a vave controlled by a speed governor. For higher loads a *'bypass line'* is provided, in such a way that steam passes from the first stage nozzle box directly into that of, say, the fourth stage. Such 'bypass' of steam is automatically regulated by a valve controlled by the steam of the first stage which senses the load variation.

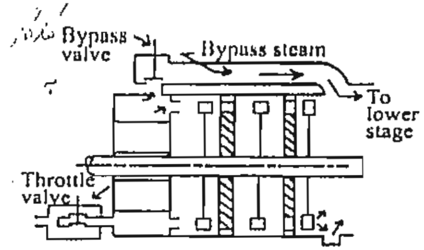


Figure 5.21 Bypass governing

SOLVED PROBLEMS

Example 5.1 The data pertaining to an impulse turbine is as follows:

Steam velocity = 500 m/s, blade speed=200 m/s. Exit angle of moving blade =25° measured from tangential direction, nozzle angle = 20°. Neglecting the effect of friction, when passing through blade passages, calculate: (a) inlet angle of moving blade (b) exit velocity and direction (c) work done per kg of steam (d) axial thrust and power developed for a steam flow rate of 5 kg/s and (e) diagram or blade efficiency.

Solution

$$C_1 = 500\ \text{m/s} \quad U = 200\ \text{m/s}$$
$$\beta_2 = 90 - 25^\circ = 65^\circ \alpha_1 = 90 - 20^\circ = 70^\circ$$

The velocity diagram is constructed as follows (Fig. 5.22).

(a) Select a suitable scale (mostly reduced scale).
(b) Draw AB equal to blade speed U.
(c) Draw BC with its inclination with AB equal to nozzle angle (α_1) and set off BC equal to the steam velocity (C_1) on the same scale.
(d) Join AC to complete the inlet velocity triangle ABC. AC = W_1.
(e) Set off AD at moving blade exit angle to AB. With A as centre and radius equal to the percentage of AC, draw an arc to cut the line AD at D. AD = W_2. In this problem since the effect of friction is neglected, AD=AC.
(f) Join BD to complete the outlet velocity triangle ABD. BD = C_2.

The velocity diagram is shown in Fig. 5.22.

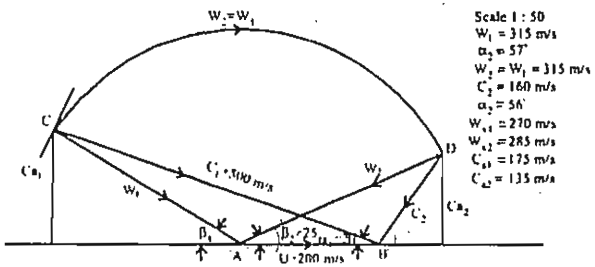


Figure 5.22

The following results are obtained from the velocity vector diagram.

(a) Moving blade inlet angle

$$\beta_1 = 33^\circ$$

(b) Exit velocity

$$C_2 = 160\ \text{m/s}$$

and direction of steam at exit

$$\alpha_2 = 56^\circ$$

(c) Work done per kg of steam

$$\begin{aligned} W/m &= U(W_{x1} + W_{x2}) \\ &= 200(270 + 285) \\ &= 111\text{kW/kg} \end{aligned}$$

(d) Axial thrust

$$= m(C_{a1} - C_{a2})$$

$$\begin{aligned} \text{Change in flow velocity} &= C_{a1} - C_{a2} \\ &= 175 - 135 = 45\ \text{m/s} \\ \text{Axial thrust} &= 5 \times 45 \\ &= 225\ \text{N} \end{aligned}$$

and power developed

$$\begin{aligned} W &= m \times (\text{Work done/kg}) \\ &= 5 \times 111 \\ &= 555\ \text{kW} \end{aligned}$$

(e) Diagram (or) blade efficiency

$$\eta_{dia} = \frac{U(W_{x1} + W_{x2})}{C_1^2/2}$$

Kinetic energy supplied to the blade

$$\begin{aligned} &= \frac{C_1^2}{2} = \frac{500^2}{2} \\ \text{Work done/kg} &= U(W_{x1} + W_{x2}) \\ &= 111 \times 10^3\ \text{W/kg} \\ \eta_{dia} &= \frac{111 \times 10^3}{500^2/2} \\ &= 88.8\% \end{aligned}$$

Example 5.2 The blade speed of a single ring of impulse blading is 300 m/s and the nozzle angle is 20°. The isentropic heat drop is 473 kJ/kg and the nozzle efficiency is 0.85. Given that the blade velocity coefficient is 0.7 and the blades are symmetrical, draw the velocity diagrams and calculate for a unit mass flow of steam (a) Axial thrust on the blading (b) steam consumption per kW if the mechanical efficiency is 90% (c) Blade or diagram efficiency, stage efficiency and maximum blade efficiency (d) energy loss in blade friction.

Solution

U=300 m/s, $\alpha = 20^\circ$, $(\Delta h)_s = 473$ kJ/kg, $\eta_N = 0.85$, $\frac{W_2}{W_1} = 0.7$, $\beta_2 = \beta_1$ (blades being symmetrical)

Useful heat drop which is converted into kinetic energy

$$\begin{aligned} \Delta h &= \eta_N\, \Delta h_s \\ &= 0.85 \times 473 \\ &= 402\ \text{kJ/kg} \end{aligned}$$

Velocity of steam at exit from the nozzle

$$\frac{C_1^2}{2} = 402 \times 10^3$$
$$C_1 = \sqrt{2000 \times 402}$$
$$= 896.7 \text{ m/s}$$

From this data, the velocity diagram (Fig. 5.23) can be drawn and the following results are obtained. Draw AB = U. Set off BC = C_1 at α_1 to AB. Join AC. Measure $\beta_1(\angle CAE)$. Then set off AD = 0.7 AC ($W_2 = 0.7\, W_1$) at $\beta_2 = \beta_1$ to AB. Join BD.

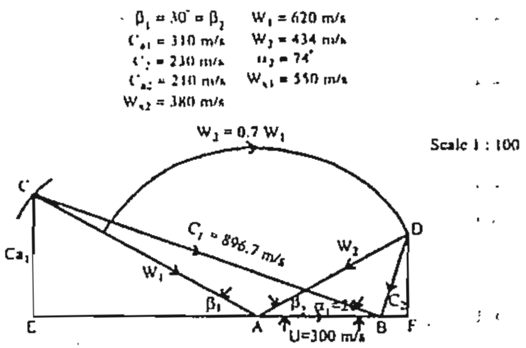


Figure 5.23

a) Axial thrust per kg

$$= C_{a1} - C_{a2}$$
$$= 310 - 210$$
$$= 100 \text{ N/kg}$$

(b) Power developed per kg of steam/sec.

Work done/kg of steam/s.

$$= U(W_{X1} + W_{X2})$$
$$= 300(550 + 380)$$
$$= 279 \text{ kW/(kg/s)}$$

Power developed per kg of steam per sec.

$$= 279 \times 0.9$$
$$= 251.1 \text{ kW/kg/s}$$

$$\text{Steam rate /kw.hr} = \frac{3600}{251.1}$$
$$= 14.3 \text{ kg}$$

(c) Blade efficiency

$$\eta_{dia} = \frac{U(W_{X1} + W_{X2})}{C_1^2/2}$$
$$= \frac{279 \times 10^3}{896.7^2/2}$$
$$= 69.4\%$$

Maximum blade efficiency under optimum condition

α₁ با α₁ نسبت به افق فرق دارد
نسبت به قائم در فرمول

$$= \sin^2 \alpha_1, \quad \alpha_1 = 90° - 20°$$
$$= \sin^2 70° \quad = 70°$$
$$= 88.3\%$$

$$\boxed{\text{Stage efficiency} = \frac{\text{Work done on blade}}{\text{Total energy supplied to blade}}}$$
$$= \frac{279}{473}$$
$$= 58.98\%$$

It can also be found from

$$\eta_{stage} = \eta_{dia} \times \eta_{Nozzle}$$
$$= 0.694 \times 0.85$$
$$= 58.99\%$$

(d) Energy loss in blade friction

$$= \frac{W_1^2 - W_2^2}{2}$$
$$= \frac{620^2 - 434^2}{2}$$
$$= 98022 \text{ J/kg}$$
$$= 98.022 \text{ kJ/kg}$$

Example 5.3 In a single row impulse turbine stage, steam is supplied dry and saturated at 5 bar and the exhaust pressure is 2.8 bar. There is carry over velocity of 75 m/s. from the previous stage and the kinetic energy at exit from the nozzle is only 90% of the theoretical available energy. The nozzle is inclined at 20° with the direction of blade rotation and blade speed ratio is 0.4. The blade exit angle is also 20°. For a steam flow rate of 2.5 kg/s the output of the stage is 206 kW. Estimate (a) velocity of steam at exit from the nozzle (b) diagram efficiency (c) the relative velocity ratio, (d) stage efficiency.

Solution

$P_1 = 5$ bar $\quad P_2 = 3$ bar $\quad C_0 = 75$ m/s. $\quad \alpha_1 = 20°$ $\quad U/C_1 = 0.4$ $\quad \beta_2 = 20°$
m =2.5 kg/s $\quad$ W = 206 kW

(a) Steam velocity at exit from the nozzle

Since the turbine is impulse, the total pressure drop will occur only in the nozzles. Assuming isentropic expansion, the enthalpy drop can be found from the steam table.

At $P_1 = 5$ bar $\quad h_1 = h_g = 2747.5$ kJ/kg

$s_1 = s_g = 6.819$ kJ/kg − K

For isentropic expansion, $s_2 = s_1$

At $P_2 = 2.8$ bar, $sg_{p_2} < s_2$; therefore, the steam is wet

$$x_2 = \frac{s_2 - s_{f_{p_2}}}{s_{fg_{p_2}}} = \frac{6.819 - 1.647}{5.367}$$
$$= 0.964$$
$$h_{2s} = h_f + x_2 h_{fg}$$
$$= 551.5 + 0.964(2170.1)$$
$$= 2643.5 \text{ kJ/kg}$$

Isentropic heat drop $(\Delta h_s) = h_1 - h_{2s}$

$$= 2747.5 - 2643.5$$
$$= 104 \text{ kJ/kg}$$

Now,

Gain in kinetic energy = Useful heat drop

$$\frac{C_1^2 - C_0^2}{2000} = \eta_N(\Delta h_s)$$

or

$$C_1 = \sqrt{2000 \times 0.9 \times 104 + 75^2}$$
$$= 439.12 \text{ m/s}$$

(b) Diagram efficiency

$$\eta_{dia} = \frac{U(W_{X_1} + W_{X_2})}{C_1^2/2}$$
$$U = 0.4\,C_1 = 0.4 \times 439.12$$
$$= 175.65 \text{ m/s}$$

and

$$W_{X_1} + W_{X_2} = \frac{W}{mU} = \frac{206 \times 10^3}{2.5 \times 175.65}$$
$$= 469.12 \text{ m/s}$$
$$\eta_{dia} = \frac{175.65 \times 469.12}{(439.12)^2/2}$$
$$= 85.47\%$$

(c) The relative velocity ratio

From the given and calculated data, we can draw the vector diagram as shown in Fig. 5.24. After drawing the inlet velocity triangle ABC, locate point E by drawing the perpendicular CE and then set off $EF = (W_{X_1} + W_{X_2})$ to locate point F. Set off AD at β_2 to AB and let AD intersect the perpendicular FD at D.

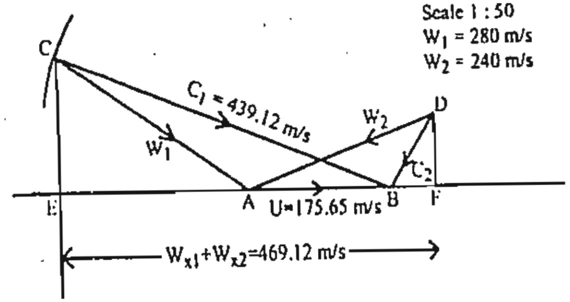


Figure 5.24

By measurement from the velocity diagram

Relative Velocity at inlet,

$$W_1 = 280 \text{ m/s}$$

Relative Velocity at outlet,

$$W_2 = 240 \text{ m/s}$$

∴ Relative Velocity ratio $= \dfrac{W_2}{W_1} = \dfrac{240}{280} = 0.857.$

(d) Stage efficiency

$$\eta_{stage} = \frac{\text{Work done/kg}}{\text{Energy Supplied to nozzle}}$$

$$\text{Energy supplied/kg} = (\Delta h_s) + \frac{C_0^2}{2000}$$
$$= 104 + \frac{75^2}{2000}$$
$$= 106.81 \text{ kJ/kg}$$
$$\eta_{stage} = \frac{175.65 \times 469.12}{(106.81 \times 10^3)}$$
$$= 77.15\%$$

(or)

$$\eta_{stage} = \eta_{dia} \times \eta_{nozzle} = 0.8547 \times 0.90$$
$$= 76.92\%$$

Example 5.4 The steam in a two row curtis stage leaves the nozzles at 600 m/s and the blade speed is 120 m/s. Before leaving the stage, it passes through a ring of moving blades, a ring of fixed blades and another ring of moving blades. The nozzle angle is 16°, while the discharge angles are 18° for the first moving ring, 21°, for the fixed ring, and 35° for the second moving ring, all measured relative to the plane of rotation. Assuming 10% drop in velocity during passage through each ring of blades, draw the velocity triangles and determine (a) blade inlet angle for each row (b) driving force and axial thrust for each row of moving blade (c) diagram power per kg/s steam flow and (d) diagram efficiency. What would be the maximum possible diagram efficiency.

Solution

$C_1 = 600$ m/s $\quad U = 120$ m/s $\quad \alpha_1 = 16°\quad \beta_2 = 18°$

$\alpha_1' = 21°$ and $\beta_2' = 35° \quad W_R = 0.9$

The velocity diagrams are drawn as shown in Fig. 5.25 and are constructed as follows.

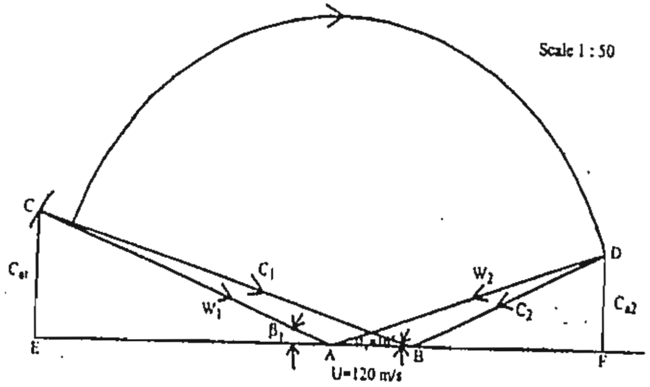


Figure 5.25

(a) Blade inlet angle for each row

Draw a horizontal line AB representing U set off BC at α_1 to AB and equal to C_1. Join AC which represents W_1 and caluculate W_2. $W_2 = W_R\, W_1$. Then set off AD equal to W_2 at β_2 to AB. Join D with B. This completes the velocity diagram for the first stage of the two row curtis impulse turbine (Fig. 5.25).

$$\boxed{W_2 = 0.9\, W_1}$$

By measurement from the velocity diagram

$$W_1 = 485 \text{ m/s}$$
$$W_2 = 0.9 \times 485 = 436.5 \text{ m/s}$$

$$W_{x_1} = 460 \text{ m/s} \quad C_{a_1} = 170 \text{ m/s}$$
$$W_{x_2} = 410 \text{ m/s} \quad C_{a_2} = 135 \text{ m/s}$$
$$\beta_1 = 20° \quad C_2 = 325 \text{ m/s}$$

That is, the blade inlet angle for first row of moving blade is 20°.

Measure BD representing C_2 and calculate C_1'. C_1' is the steam velocity at the inlet to the second row of moving blades.

$$C_1' = W_R \cdot C_2 = 0.9 \times 325 = 292.5 \text{ m/s}$$

The blade speed for each moving blade is same and so take $A'B' = U$. Set off $B'C' = C_1'$ at α_1' to $A'B'$. Join $A'C'$. Measure $A'C'$ representing W_1' and calculate W_2'.

$$\boxed{W_2' = W_R \cdot W_1'}$$

Set off $A'D' = W_2'$ at β_2' to $A'B'$ and join B' with D'. This completes the velocity diagram for the second stage of two row curtis impulse turbine (Fig. 5.26).

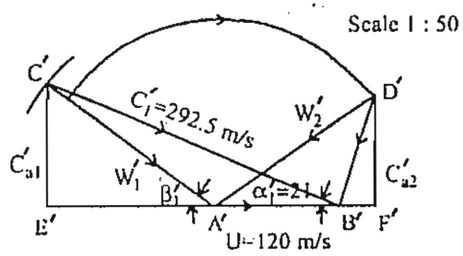


Figure 5.26

By measurement, $W_1' = 190$ m/s

$$W_2' = 0.9 \times 190 = 171 \text{ m/s}$$

$$W_{x_1}' = 155 \text{ m/s} \quad C_{a_1}' = 110 \text{ m/s}$$
$$W_{x_2}' = 140 \text{ m/s} \quad C_{a_2}' = 100 \text{ m/s}$$
$$\beta_1' = 35°$$

The blade inlet angle for the second row of moving blades is 35°.

(b) Driving force and axial thrust

Driving force for first stage is given by

$$\Delta W_{x_I} = W_{x_1} + W_{x_2} = 460 + 410 = 870 \text{ m/s}$$

Driving force for second stage is given by

$$\Delta W_{x_{II}} = W_{x_1}' + W_{x_2}' = 155 + 140 = 295 \text{ m/s}$$

For unit mass flow rate of steam.

Total driving force = (870 + 295) × 1 = 1165 N

Axial thrust for first stage = $(C_{a_1} - C_{a_2}) = 170 - 135 = 35$ m/s

Axial thrust for second stage

$$(C'_{a_1} - C'_{a_2}) = 110 - 100 = 10 \text{ m/s}$$

Axial thrust per unit mass flow rate = (35 + 10) × 1 = 45 N

(c) Diagram power

$$= mu(\Delta W_{x_I} + \Delta W_{x_{II}})$$
$$= 1 \times 120(1165)/10^3$$
$$= 139.8 \text{ kW}$$

(d) Diagram efficiency

$$= \frac{u(\Delta W_{x_I} + \Delta W_{x_{II}})}{C_1^2/2}$$
$$= \frac{120(1165)}{(600)^2/2}$$
$$= 77.7\%$$

(e) Maximum diagram efficiency

$$\eta_{dia} = \sin^2 \alpha_1$$
$$= \sin^2(90 - 16)$$
$$= 92.4\%$$

Example 5.5 A parson's reaction turbine having identical blading delivers dry saturated steam at 3 bar. The velocity of steam is 100 m/s. The mean blade height is 4 cm and the exit angle of the moving blade is 20°. At the mean radius the axial flow velocity equals 3/4 of the blade speed. For a steam flow rate of 10,000 kg/hr, calculate, (a) the rotor speed, in rev/min (b) the power output of stage (c) the diagram efficiency (d) the percentage increase in relative velocity in the moving blades due to expansion in these blades (e) the enthalpy drop of the steam in the stage.

Solution

$C_1 = 100$ m/s $h = 0.04$ m $\beta_2 = 20°$

$C_a = \frac{3}{4}U$ $m = 1000$ kg/hr

(a) Rotor speed

The flow velocity

$$C_a = C_1 \cos\alpha_1$$
$$= 100\cos(90 - 20)$$
$$= 34.2 \text{ m/s}$$

∴ The mean blade speed $(U) = \frac{4}{3} \times C_a$

$$= 45.6 \text{ m/s}$$

Mass flow of steam, $m = \frac{A \times C_a}{\vartheta}$

where A is the annulus area and ϑ is the specific volume of steam from the steam table. At 3 bar and with dry saturated steam

$$\vartheta = 0.60553 \text{ m}^3/\text{kg}$$

∴ Annulus area $= \frac{m\vartheta}{C_a}$

$$= \frac{(10.000/3600) \times 0.60553}{34.2}$$
$$= 0.04918 \text{ m}^2$$

Now, annulus area, $A = \pi Dh$.

where 'D' is the mean blade diameter and 'h' is the mean blade height

$$\therefore D = A/\pi h = \frac{0.04918}{\pi \times 0.04}$$
$$= 0.39 \text{ m}$$

$$\text{Rotor Speed, } N = \frac{U \times 60}{\pi D}$$
$$= \frac{45.6 \times 60}{\pi \times 0.39}$$
$$= 2233 \text{ rpm}$$

(b) Diagram power

Since the blades are identical, the inlet and outlet velocity triangles will also be identlcal. From the data

$U = 45.6$ m/s $\alpha_1 = 20°$ $C_1 = 100$ m/s the velocity diagram may be drawn as shown in Fig. 5.27. Draw AB = U. Set off BC at α_1 to AB and equal to C_1. Join AC. Then set off AD = $W_2 = C_1$ (identical blades) at β_2 to AB. Join D with B.

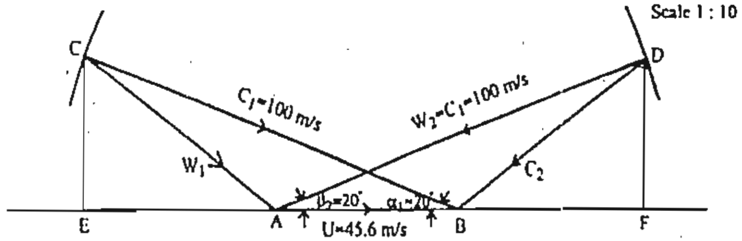


Figure 5.27

By measurement from the velocity diagram $W_1 = 59$ m/s $W_{x_1} + W_{x_2} = 142$ m/s.

$$\text{Diagram power} = mu(W_{x_1} + W_{x_2}) = \frac{10,000}{3600} \times 45.6 \times 142 = 17.99 \text{ kW}$$

(c) Diagram efficiency

$$\eta_{dia} = \frac{\text{Work done/kg}}{\text{Energy input/kg}}$$

$$\text{Work done/kg} = U(W_{x_1} + W_{x_2})$$

and

$$\text{Energy input/kg} = \frac{C_1^2}{2} + \frac{W_2^2 - W_1^2}{2}$$

Since $W_2 = C_1$, then

$$= \frac{2C_1^2 - W_1^2}{2}$$

$$\therefore \eta_{dia} = \frac{45.6 \times 142}{\left(\frac{2(100)^2 - 59^2}{2}\right)} = 78.4\%$$

(d) Percentage increase in relative velocity

$$= \frac{W_2 - W_1}{W_1} = \frac{100 - 59}{59}$$

$$= 69.5\%$$

(e) Enthalpy drop in the moving blades

$$= \frac{W_2^2 - W_1^2}{2} = \frac{100^2 - 59^2}{2}$$

$$= 3.259 \text{ kJ/kg}$$

Total enthalpy drop per stage $= 2 \times 3.259 = 6.518$ kJ/kg

Example 5.6 Steam enters a 0.5 degree of reaction turbine at 14 bar and 315°C and is expanded to a pressure of 0.14 bar. The turbine has a stage efficiency of 75% for each stage and the reheat factor is 1.04. The turbine has 20 successive stages and the total power output is 11,770 kW. Assuming that all stages develop equal work calculate the steam flow rate. At a certain place in the turbine, the steam has a pressure of 1.05 bar and was dry and saturated. The exit angle of the blade is 20° and the blade speed ratio is 0.4. Estimate the blade speed mean diameter of the annulus at this point in the turbine, and the rotor speed if the blade height is 1/12 of the blade mean diameter.

Solution

R	= 0.5	P_1	= 14 bar	T_1	= 315°C	P_2	= 0.14 bar
η_{stage}	= 0.75	RF	= 1.04	N	= 20	W	= 11,770 kW

(a) Steam flow rate

From Mollier chart the isentropic enthalpy drop when steam expands from 14 bar and 315°C to 0.14 bar is

$$\Delta hs = 3080 - 2270 = 810 \text{ kJ/kg}$$

We know that

Overall efficiency (η_t) = Stage efficiency (η_{stage}) × reheat factor (R.F)

$$\eta_f = 0.75 \times 1.04 = 0.78$$

Thus, actual enthalpy drop

$$\Delta h = \eta_t \times \Delta hs = 0.78 \times 810 = 631.8 \text{ kJ/kg}$$

$$\text{Enthalpy drop per stage} = \frac{\Delta h}{N} = \frac{631.8}{20} = 31.59 \text{ kJ/kg}$$

Diagram power (W) = $m\Delta h$ the mass flow rate of steam

$$m = \frac{W}{\Delta h} = \frac{11,770}{631.8} = 18.63 \text{ kg/s}$$

(b) Blade speed

$$\text{Work done/kg} = U(W_{x_1} + W_{x_2}) = U\left[(C_{x_1} - U) + W_{x_2}\right] = U\left[C_1 \sin\alpha_1 - U + W_2 \sin\beta_2\right]$$

since $\alpha_1 = \beta_2$ and $C_1 = W_2$ for a 50% reaction turbine

$$= U[2C_1 \sin\alpha_1 - U]$$

Now $U/C_1 = 0.7$ or $C_1 = 1.43\,U$

$$\text{Work done/kg} = U[2 \times (1.43U)\sin(90 - 20) - U] = 1.69\,U^2$$

This work done equals the enthalpy drop per stage. Therefore,

$$1.69\,U^2 = 31.59 \times 10^3$$

$$\text{or} \quad U = 136.7 \text{ m/s}$$

(c) Blade mean diameter

$$\text{Flow velocity} \quad C_a = C_1 \cos\alpha_1$$

$$C_1 = 1.43 \times U = 1.43 \times 136.7$$

$$C_1 = 195.5 \text{ m/s}$$

$$C_a = 195.5 \times \cos(90 - 20°)$$

$$= 66.9 \text{ m/s}$$

From steam table, at 1.05 bar, the specific volume of dry steam is given by

$$\vartheta_g = 1.618 \text{ m}^3/\text{kg}$$

Now,

$$\text{Mass flow rate} = \frac{\text{Volume flow rate}}{\text{Specific volume}}$$

$$\text{Volume flow rate} = (\pi Dh)C_a$$

where D - blade mean diameter and h - blade height

$$h = \frac{D}{12}$$

$$18.63 = \frac{\pi D\left(\frac{D}{12}\right) \times 66.9}{1.618}$$

(or)

$$D = 1.312$$

(d) Rotor speed

$$N = \frac{U \times 60}{\pi D}$$

$$= \frac{136.7 \times 60}{\pi \times 1.312}$$

$$= 1990 \text{ rpm}$$

Example 5.7 The data for a free vortex turbine blade are given below.

Blade tip diameter - 75 cm, blade roof diameter - 45 cm, inlet angle of the rotor blade at mid height - 45°, out let angle of the nozzle blade at mid height - 76°, outlet angle of the rotor blade at mid height - 75°, speed - 6000 rpm.

Axial velocity remains constant across the rotor. Determine for the hub and tip (a) nozzle exit angle (b) rotor blade angles (c) the degree of reaction.

Solution

$r_h = 0.225$ m, $r_t = 0.375$ m

$\beta_{1,m} = 45°$, $\alpha_{1,m} = 76°$, $\beta_{2,m} = 75°$, $N = 6000$ rpm

C_a = constant

The mean radius,

$$r_m = \frac{r_h + r_t}{2} = \frac{0.225 + 0.375}{2} = 0.3 \text{ m}$$

The mean blade speed

$$U_m = \frac{2\pi r_m N}{60}$$

$$= \frac{2\pi(0.3)\,6000}{60}$$

$$= 188.5 \text{ m/s}$$

From inlet velocity triangle, at the mean radius, refer Fig. 5.2

$$C_a = \frac{U_m}{\tan\alpha_{1,m} - \tan\beta_{1,m}}$$

$$= \frac{188.5}{\tan 76° - \tan 45°} = 62.61 \text{ m/s}$$

$$C_{X_{1,m}} = C_a \tan\alpha_{1,m} = 62.61 \times \tan 76°$$

$$= 251.12 \text{ m/s}$$

and from outlet velocity triangle at the mean radius, refer Fig. 5.2

$$C_{X_{2,m}} = W_{X_{2,m}} - U_m = C_a \tan\beta_{2,m} - U_m$$

$$= 62.61 \tan 75° - 188.5$$

$$= 45.16 \text{ m/s}$$

For the hub

For a free vortex design, $C_X \cdot r$ = const. at all radii. Therefore, between the mean radius and the hub

$$\boxed{C_{X_m} \cdot r_m = C_{X_h} \cdot r_h}$$

At the hub inlet

$$C_{X_{1,m}} \cdot r_m = C_{X_{1,h}} \cdot r_h$$

$$C_{X_{1,h}} = \frac{C_{X_{1,m}} \cdot r_m}{r_h}$$

$$= \frac{251.12 \times 0.325}{0.225}$$

$$= 334.83 \text{ m/s}$$

From inlet velocity triangle at the hub, refer Fig. 5.2

$$\tan\alpha_{1,h} = \frac{Cx_{1,h}}{C_a} = \frac{334.83}{62.61}$$

$$\alpha_{1,h} = 79.4°$$

and $\tan\alpha_{1,h} - \tan\beta_{1,h} = \dfrac{U_h}{C_a}$

$$U_h = \frac{2\pi r_h N}{60} = \frac{2\pi(0.225)6000}{60} = 141.37 \text{ m/s}$$

$$\tan\beta_{1,h} = \tan\alpha_{1,h} - \frac{U_h}{C_a} = \tan 79.4° - \frac{141.37}{62.61}$$

$$\beta_{1,h} = 72°$$

From the outlet velocity triangle, refer Fig. 5.2.

$$\boxed{\frac{U_h}{C_a} = \tan\beta_{2,h} - \tan\alpha_{2,h}}$$

But $\tan\alpha_{2,h} = \dfrac{Cx_{2,h}}{C_a}$

At outlet

$$Cx_{2,h}\cdot r_h = Cx_{2,m}\cdot r_m$$

$$\therefore Cx_{2,h} = \frac{45.16\times 0.3}{0.225} = 60.21 \text{ m/s}$$

$$\therefore \tan\beta_{2,h} = \frac{U_h}{C_a} + \frac{Cx_{2,h}}{C_a} = \frac{141.37 + 60.21}{62.61}$$

$$\beta_{2,h} = 72.75°$$

For the tip

Similar equations as above are used. At inlet

$$Cx_{1,t}\cdot r_t = Cx_{1,m}\cdot r_m$$

$$\therefore Cx_{1,t} = \frac{251.12\times 0.3}{0.375} = 200.896 \text{ m/s}$$

From inlet velocity triangle at the tip, refer Fig. 5.2.

$$\tan\alpha_{1,t} = \frac{Cx_{1,t}}{C_a} = \frac{200.896}{62.61}$$

$$\alpha_{1,t} = 72.69°$$

and $\dfrac{U_t}{C_a} = \tan\alpha_{1,t} - \tan\beta_{1,t}$

or

$$\tan\beta_{1,t} = \tan\alpha_{1,t} - \frac{U_t}{C_a}$$

$$U_t = \frac{2\pi r_t N}{60} = \frac{2\pi(0.375)6000}{60} = 235.62 \text{ m/s}$$

$$\therefore \tan\beta_{1,t} = \tan 72.69° - \frac{235.62}{62.61}$$

$$\beta_{1,t} = -29°$$

At outlet

$$Cx_{2,t}\cdot r_t = Cx_{2,m}\cdot r_m$$

$$Cx_{2,t} = \frac{45.16\times 0.3}{0.375} = 36.13 \text{ m/s}$$

$$\tan\beta_{2,t} = \frac{U_t}{C_a} + \frac{Cx_{2,t}}{C_a} = \frac{235.6 + 36.13}{62.61}$$

$$\beta_{2,t} = 77°$$

The degree of reaction at the hub and tip are

$$R_h = \frac{C_a}{2U_h}(\tan\beta_{2,h} - \tan\beta_{1,h}) = \frac{62.61}{2(141.37)}(\tan 72.75° - \tan 72°) = 3.16\%$$

and

$$R_t = \frac{C_a}{2U_t}(\tan\beta_{2,t} - \tan\beta_{1,t}) = \frac{62.61}{2(235.62)}(\tan 77° - \tan(-29°)) = 64.9\%$$

Example 5.8 The axial component of the air velocity at the exit of the nozzle of an axial flow reaction stage is 180 m/s. The nozzle inclination to the direction of rotation is 27°. Find the rotor blade angles at the inlet and outlet, if the degree of reaction should be 50% and the blade speed 180 m/s.

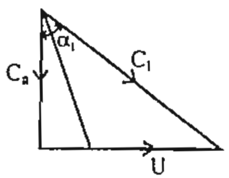
Also for the same blade speed, axial velocity and nozzle angle, find the degree of reaction, if the absolute velocity at the rotor outlet should be axial and equal to the axial velocity at the inlet.
MKU–April '94

Solution

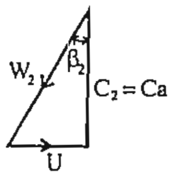
$$C_a = 180\text{ m/s} \quad \alpha_1 = 90 - 27 = 63^\circ \quad R = 0.5$$
$$U = 180\text{ m/s}$$

(a) Blade angles

$$C_{x1} = C_a \tan\alpha_1$$
$$= 180 \times \tan 63^\circ$$
$$= 353.27\text{ m/s}$$
$$\tan\beta_1 = \frac{W_{x1}}{C_a} = \frac{C_{x1} - U}{C_a} = \frac{353.27 - 180}{180}$$
$$\therefore \beta_1 = 43.9^\circ$$



(a)



(b)

Figure 5.28(a) Inlet velocity triangle *Figure 5.28(b) Exit velocity triangle*

Degree of reaction,

$$R = \phi\frac{(\tan\beta_2 - \tan\beta_1)}{2}$$
$$\phi = \frac{C_a}{U} = \frac{180}{180} = 1$$
$$\therefore \tan\beta_2 = \frac{2R}{\phi} + \tan\beta_1$$
$$= \frac{2 \times 0.5}{1} + \tan 43.9^\circ$$
$$= 1.962$$
$$\beta_2 = 62.99^\circ \simeq 63^\circ$$

Since $R = 0.5$, $\beta_2 = \alpha_1 = 63^\circ$ which is given in the problem.

(b) $C_2 = C_a = 180$ m/s.

C_2 is axial. Therefore, the outlet velocity triangle will be as shown in Fig. 5.28(b)

$$\tan\beta_2 = \frac{U}{C_2} = \frac{180}{180} = 1$$
$$\therefore \beta_2 = 45^\circ$$

As there is no change in the conditions at the rotor inlet, β_1 is the same.

$$\beta_1 = 43.9^\circ$$

Thus,
$$R = \phi\frac{(\tan\beta_2 - \tan\beta_1)}{2} = \frac{1 \times (\tan 45^\circ - \tan 43.9^\circ)}{2}$$
$$= 0.0188 \text{ (or)}$$
$$= 1.88\%$$

Example 5.9 The blade speed of an axial flow turbine is 300 m/s. The mass flow rate is 2.5 kg/s. The gas temperature at turbine inlet and outlet are 500°C and 300°C respectively. The fixed blade outlet angle is 70°C. Axial velocity remains constant at 200 m/s. Determine the power developed, degree of reaction and blade efficiency.
[MU – April '96, Oct. '97, Oct. '99 and April '99]

Solution

$$U = 300\text{ m/s} \quad \dot m = 2.5\text{ kg/s} \quad T_0 = 500^\circ\text{C}$$
$$T_2 = 300^\circ\text{C} \quad \alpha_1 = 70^\circ \quad C_a = 200\text{ m/s}$$

(a) Power developed

$$W = \dot m C_p(T_0 - T_2)$$
$$= 2.5 \times 1.005 \times (500 - 300)$$
$$= 502.5\text{ kW}$$

(b) Degree of reaction

$R = \dfrac{W_{x2} - W_{x1}}{2U}$. The stage work done/unit mass flow is

$$W/\dot m\, U(W_{x2} + W_{x1}) = C_p(T_0 - T_2)$$
$$= 1.005(500 - 300)$$
$$= 201\text{ kJ/kg}$$
$$\therefore W_{x2} + W_{x1} = \frac{201 \times 10^3}{300} = 670\text{ m/s}$$

From inlet velocity triangle, refer Fig. 5.2

$$W_{x1} = C_{x1} - U = C_a\tan\alpha_1 - U$$
$$= 200\tan 70^\circ - 300$$
$$= 549.5 - 300$$
$$= 249.5\text{ m/s}$$

$$\therefore W_{x_2} = 670 - 249.5 = 420.5 \text{ m/s}$$

Therefore

$$R = \frac{420.5 - 249.5}{2 \times 300} = 0.285$$
$$R = 28.5$$

(c) Blade efficiency

$$\eta_b = \frac{\text{Work done}}{\text{Energy input}}$$
$$= \frac{U(W_{x_1} + W_{x_2})}{\frac{C_1^2}{2} + \frac{W_2^2 - W_1^2}{2}}$$

Since

$$R = \frac{W_2^2 - W_1^2}{2U(W_{x_1} + W_{x_2})}$$
$$\frac{W_2^2 - W_1^2}{2} = 2U(W_{x_1} + W_{x_2})R$$
$$= 201 \times 10^3 \times 0.285$$
$$= 57285$$

and
$C_1 = C_a/\cos\alpha_1 = 200/\cos 70° = 584.76$ m/s

$$\therefore \eta_b = \frac{201 \times 10^3}{\frac{584.76^2}{2} + 57285}$$
$$= 0.8806 \text{ or}$$
$$\eta_b = 88.06$$

Example 5.10 Steam enters a 50% reaction stage at a pressure of 2.2 bar and 170°C of temperature. The rotor runs at 2400 rpm. The rotor mean diameter is 0.5 m and the symmetric rotor and stator blades have inlet and exit angles respectively of 36° and 19°. Find the actual stage power output. If the stage efficiency is 88% find also the enthalpy drop at the stage. *(MKU–May '97)*

Solution

$$R = 0.5 \quad P_0 = 2.2 \text{ bar} \quad T_0 = 170°\text{C}$$
$$N = 2400 \text{ rpm} \quad D_m = 0.5 \text{ m} \quad \alpha_1 = 36° \quad \alpha_2 = 19°.$$

Since $R = 0.5$ $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$

(a) Power output

$$W = mUC_a(\tan\alpha_1 + \tan\alpha_2)$$
$$U = \frac{\pi D_m N}{60} = \frac{\pi \times 0.5 \times 2400}{60}$$
$$= 62.83 \text{ m/s}$$
$$R = \frac{C_a(\tan\beta_2 - \tan\beta_1)}{2U}$$
$$C_a = \frac{2UR}{(\tan\beta_2 - \tan\beta_1)} = \frac{2 \times 62.83 \times 0.5}{\tan 36° - \tan 19°}$$
$$= 164.38 \text{ m/s}$$

∴ Power output for 1 kg of steam per sec

$$W = 1 \times 62.83 \times 164.38(\tan 36° + \tan 19°)$$
$$W = 110.59 \text{ kW}$$

(b) Stage enthalpy drop

$$\Delta h = \frac{W/m}{\eta_s}$$
$$= \frac{110.59}{0.88}$$
$$\Delta h = 125.67 \text{ kJ/kg}$$

Example 5.11 Hot gas at 800 kPa and 700°C enters a simple impulse turbine nozzle and expands adiabatically to 100 kPa with an efficiency of 90%. The nozzle angle is 73° to the flow direction. Assuming optimum conditions, find the rotor blade angles, flow coefficient, blade loading coefficient and power developed for a mass flow rate of 35 kg/s.

Solution

$$P_0 = 800 \text{ kPa} \quad T_1 = 973 \text{ K}$$
$$P_2 = 100 \text{ kPa} \quad \alpha_1 = 73°$$

For optimum condition, $\eta_{max.dia} = \sin^2\alpha_1$ and $\sigma_{opt} = \frac{U}{C_1} = \frac{\sin\alpha_1}{2}$. Velocity triangles for a single impulse stage with maximum diagram efficiency (or) utilization factor is shown in Fig. 5.29.

$$2U = C_1\sin\alpha_1 = C_{x_1}$$
$$W_{x_1} = C_{x_1} - U = 2U - U = U$$
$$W_1\sin\beta_1 = W_{x_1} = U$$

Since $W_1 = W_2$ and $\beta_1 = \beta_2$ for an ideal impulse stage,

$$W_2\sin\beta_2 = W_1\sin\beta_1 = U$$
$$Cx_2 = 0, \text{ i.e. } C_2 = C_a$$

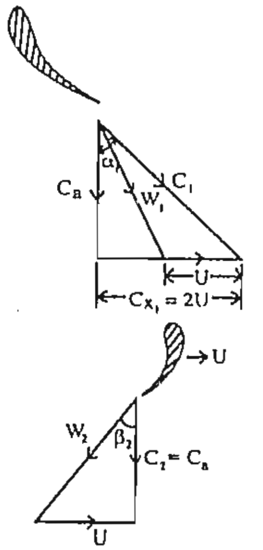


Figure5.29 Velocity triangles for a single impulse stage with maximum utilization factor

(a) Rotor blade angles

$$\tan\beta_1 = \frac{W_{x_1}}{C_a} = \frac{U}{C_a} = \frac{C_1\sin\alpha_1}{2C_1\cos\alpha_1} = \frac{\tan\alpha_1}{2}$$

$$= \frac{\tan 73°}{2} = 1.635.$$

$$\beta_1 = \tan^{-1}(1.635)$$

$$= 58.55°$$

$$\beta_1 = \beta_2 = 58.55°$$

(b) Flow coefficient

$$\phi = \frac{C_a}{U} = \frac{C_1\cos\alpha_1}{\frac{C_1\sin\alpha_1}{2}} = \frac{2}{\tan\alpha_1}$$

$$\phi = \frac{2}{\tan 73°}$$

$$\phi = 0.612$$

(c) Blade loading coefficient

$$\psi_l = \phi(\tan\beta_1 + \tan\beta_2)$$

$$= 0.612(\tan 58.55 + \tan 58.55)$$

$$= 2$$

(d) Power developed

$$W = \dot{m}(h_0 - h_2)$$

$$\eta_s = \frac{h_0 - h_2}{h_0 - h_{2s}} = \frac{(h_0 - h_2)}{C_pT_0\left(1 - \frac{T_{2s}}{T_0}\right)}$$

$$h_1 - h_2 = \eta_s C_p T_0\left[1 - \left(\frac{P_2}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right] = 0.9 \times 1.005 \times 973\left[1 - \left(\frac{100}{800}\right)^{\frac{0.4}{1.4}}\right]$$

$$= 394.24 \text{ kJ/kg}$$

$$W = 35 \times 394.24 = 3.8 \text{ mW}$$

Example 5.12 The initial pressure and temperature of steam entering a multi-stage turbine are 100 bar and 500°C respectively. The turbine diameter is 1 m and speed is 3000 rpm. The exit angle of the first stage nozzle is 70°. The steam flow rate is 100 kg/s. Assuming maximum blade efficiency and equal stage efficiencies of 78%, determine the rotor blade angles, power developed, final state of steam and the blade height if the turbine is two stage Rateau turbine. State the assumptions used.

Solution

$P_0 = 100$ bar $T_0 = 500°C$ $D = 1$m $N = 3000$ rpm
$\dot{m} = 100$ kg/s $\alpha_1 = 70°$ $\eta_{S1} = \eta_{S2} = 0.78$

(a) Rotor blade angles blade angles are assumed to be equal. The blade gas speed ratio for maximum blade efficiency is

$$\sigma_{opt} = \frac{U}{C_1} = \frac{\sin\alpha_1}{2}$$

$$C_1\sin\alpha_1 = 2U$$

$$U = \frac{\pi DN}{60} = \frac{\pi \times 1 \times 3000}{60} = 157.08 \text{ m/s}$$

$$\therefore C_1 = \frac{2 \times 157.08}{\sin 70} = 334.32 \text{ m/s}$$

The velocity triangles for a two stage Rateau turbine with maximum blade efficiency and assuming axial exit is shown in Fig. 5.30. For the first stage.

$$C_1\sin\alpha_1 = C_{x_1} = 2U$$

$$W_{x_1} = C_{x_1} - U = 2U - U = U$$

$$\text{and } C_{x_2} = 0.\ C_2 = C_a$$

$$\tan\beta_1 = \frac{W_{x_1}}{C_a} = \frac{U}{C_a} = \frac{\tan\alpha_1}{2}$$

$$\beta_1 = \tan^{-1}\left(\frac{\tan 70°}{2}\right)$$

$$\beta_1 = 53.95° = \beta_2$$

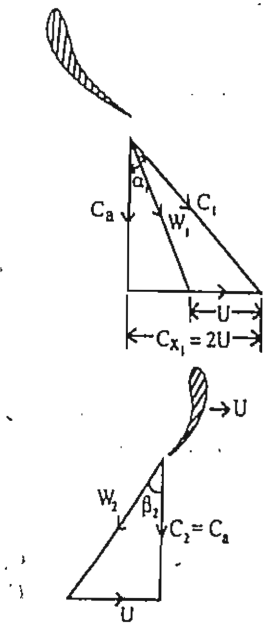


Figure 5.30 Velocity triangles for a rateau turbine with maximum utilization factor

Similarly for the second stage $\beta_1' = \beta_2' = 53.95°$.

(b) Power developed

$$\text{Work done in first stage} = \dot{m}U(C_{x_1} + C_{x_2})$$
$$W_I = m2U^2$$

$$\text{Work done in second stage} = \dot{m}U(C'_{x_1} + C'_{x_2})$$
$$= \dot{m}U(2U + 0)$$
$$W_{II} = \dot{m}2U^2$$

Therefore,

$$\text{Total work done} = W_I + W_{II}$$
$$= \dot{m}(2U^2 + 2U^2)$$
$$= 4\dot{m}U^2$$
$$= 4 \times 100 \times 157.08^2$$
$$= 9.87 \text{ mW}$$

(c) Final state of steam The expansion process is shown on $h - s$ diagram (Fig. 5.31).

For first stage

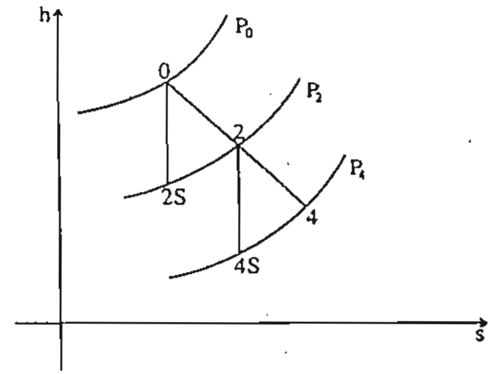


Figure 5.31

$$\eta_{s_I} = \frac{h_0 - h_2}{h_0 - h_{2s}}$$
$$h_0 - h_2 = W_I/\dot{m} = 2U^2 = 49.35 \text{ kJ/kg}$$
$$h_0 - h_{2s} = \frac{h_0 - h_2}{\eta_{s_I}} = \frac{49.35}{0.78}$$
$$= 63.269 \text{ kJ/kg}$$

From Mollier chart, $P_0 = 100$ bar $T_0 = 500°C$

$$h_0 = 3370 \text{ kJ/kg}$$
$$h_2 = 3370 - 49.35 = 3320.65 \text{ kJ/kg}$$
$$h_{2s} = 3370 - 63.269 = 3306.73 \text{ kJ/kg}$$

From chart state of steam at first stage exit is $P_2 = 82$ bar $T_2 = 470°C$
$\vartheta_2 = 0.041 \text{ m}^3/\text{kg}$

For second stage

$$\eta_{s_{II}} = \frac{h_2 - h_4}{h_2 - h_{4S}}$$
$$h_2 - h_4 = W_{II}/\dot{m} = 2U^2 = 49.35 \text{ kJ/kg}$$
$$h_2 - h_{4s} = \frac{h_2 - h_4}{\eta_{sII}} = \frac{49.35}{0.78} = 63.269 \text{ kJ/kg}$$
$$h_4 = 3320.65 - 49.35 = 3271.3 \text{ kJ/kg}$$
$$h_{4s} = 3320.64 - 63.269 = 3257.38 \text{ kJ/kg}$$

From Mollier chart $P_4 = 65$ bar $T_4 = 445°C$, $\vartheta_4 = 0.05 \text{ m}^3/\text{kg}$.
This is the final state of the steam at the turbine exit.

(d) Blade height

$$= \frac{\dot{m} \times \vartheta}{\pi D C_a}$$

At the first stage rotor exit

$$h_I = \frac{\dot{m} \times \vartheta_2}{\pi D C_a}$$

$$C_a = C_1 \cos\alpha_1 = 334.32 \times \cos 70^\circ$$

$$= 114.34 \text{ m/s}$$

$$h_I = \frac{100 \times 0.041}{\pi \times 1 \times 114.34} = 0.0114 \text{ m.}$$

All the second stage rotor exit

$$h_{II} = \frac{\dot{m} \times \vartheta_4}{\pi D C_a} = \frac{100 \times 0.05}{\pi \times 1 \times 114.34}$$

$$= 0.0139 \text{ m} .$$

Note that the blade height increaes through the turbine stages.

Example 5.13 For the above problem, if the turbine is two stage curtis turbine with stage efficiency of 65%, determine the parameters.

Solution

(a) Rotor blade angles

For maximum blade efficiency, the blade-gas speed ratio is

$$\sigma_{opt} = \frac{\sin\alpha_1}{4} = \frac{U}{C_1}$$

or $C_1 \sin\alpha_1 = C_{x_1} = 4U$

$$C_1 = \frac{4 \times 157.08}{\sin 70^\circ} = 668.64 \text{ m/s}$$

$$C_a = C_1 \cos\alpha_1 = 668.64 \text{ m/s} \times \cos 70^\circ$$

$$= 228.69 \text{ m/s}$$

This velocity triangles for the first stage of a two stage curtis impulse turbine with maximum blade efficiency is shown in Fig. 5.32.

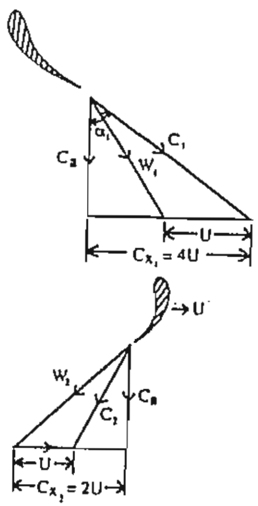


Figure 5.32 Velocity triangles for a curtis turbine with maximum utilization factor

$$W_{x_1} = C_{x_1} - U = 4U - U = 3U$$

$$\tan\beta_1 = \frac{W_{x_1}}{C_a} = \frac{3U}{C_a} = \frac{3 \times 157.08}{228.69}$$

$$\therefore \beta_1 = 64.11^\circ$$

Since, the rotor blade angles are assumed to be equal.

$$\beta_2 = \beta_1 = 64.11^\circ$$

Assuming frictionless flow over the blades, we have

$$W_2 = W_1;\ C_1' = C_2 \text{ and } W_2' = W_1'$$

From the first stage velocity triangle at inlet

$$W_1 \sin\beta_1 = 3U$$

Since $W_1 = W_2$ and $\beta_1 = \beta_2$

$$W_1 \sin\beta_1 = W_2 \sin\beta_2 = 3U$$

From first stage velocity triangle at exit,

$$W_2 \sin\beta_2 = C_{x_2} + U = 3U$$

$$\therefore C_{x_2} = 2U = C_2 \sin\alpha_2$$

The velocity triangles for the second stage is shown in Fig. 5.33.

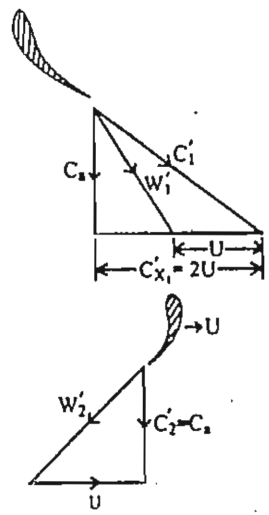


Figure 5.33

Since $C_1' = C_2$ and $\alpha_2 = \alpha_1'$ (assumed)

$$C_2 \sin\alpha_2 = C_1' \sin\alpha_1' = 2U = C_{x1}'$$

$$C_{x1}' = 2U = W_1' \sin\beta_1' + U$$

$$W_1' \sin\beta_1' = U$$

Since $W_1' = W_2'$ and $\beta_1' = \beta_2'$

$$W_1' \sin\beta_1' = W_2' \sin\beta_2' = U.$$

$$i.e.\ W_2' = UC_{x2}' = W_{x2}'U = U - U = 0.$$

$$\therefore C_{x2}' = C_2' = C_a$$

$$\tan\beta_2' = \frac{U}{C_2'} = \frac{U}{C_a} = \frac{157.08}{228.69}$$

$$\therefore \beta_2' = 34.48^\circ$$

Since, $\beta_2' = \beta_1'$

$$\beta_2' = \beta_1' = 34.48^\circ$$

(b) Power developed

$$W = W_I + W_{II}$$

$$W_I = \dot{m}U(C_{x1} + C_{x2}) = \dot{m}U(4U + 2U)$$

$$= \dot{m}(6U^2)$$

$$W_{II} = \dot{m}U(C_{x1}' + C_{x2}') = \dot{m}U(2U + 0)$$

$$= \dot{m}(2U^2)$$

$$\therefore W = \dot{m}(8U^2) = 100 \times 8 \times 1578.08^2$$
$$W = 19.74 \text{ mW}$$

(c) Final state of steam The expansion process is shown on $h - s$ diagram (Fig. 5.34).

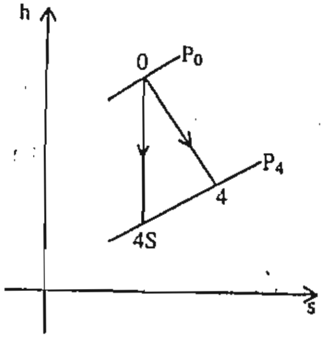


Figure 5.34

$$\eta_s = \frac{h_0 - h_4}{h_0 - h_{4s}}$$
$$h_0 - h_4 = \frac{19.74 \times 10^3}{100} = 197.4 \text{ kJ/kg}$$
$$h_0 - h_{4s} = \frac{197.4}{0.65} = 303.69 \text{ kJ/kg}$$
$$\therefore h_4 = h_0 - 197.4 = 3370 - 197.4 = 3172.6 \text{ kJ/kg}$$
$$h_{4s} = 3370 - 303.69 = 3066.31 \text{ kJ/kg}$$

From Mollier chart
$P_4 = 27$ bar, $T_4 = 365°$C and $\vartheta_4 = 0.105$ m^3/kg

(d) Rotor blade height

$$h = \frac{\dot{m}\vartheta_4}{\pi D C_a} = \frac{100 \times 0.105}{\pi \times 1 \times 228.69} = 0.0146 \text{ m}$$

Example 5.14 Air leaves the nozzle of an axial flow turbine stage at an angle of 30°, with an axial velocity of 180 m/s. If the rotor blade speed is 280 m/s. Find the rotor blade angle for the reaction of the stage to be 50%. *(MU-Oct. '98)*

Solution

The nozzle angle (α_1) measured from axial direction
$\alpha_1 = 90° - 30° = 60°$ $C_a = 180$ m/s $U = 280$ m/s $R = 0.5$

(a) Blade angle at inlet

$$\tan\beta_1 = \frac{C_{x_1} - U}{C_a}$$
$$C_{x_1} = C_a \tan\alpha_1 = 180 \times \tan 60° = 311.77 \text{ m/s}$$
$$\therefore \tan\beta_1 = \frac{311.77 - 280}{180}$$
$$\beta_1 \doteq 10°$$

(b) Blade angle at exit

$$\text{Since } R = 0.5$$
$$\beta_2 = \alpha_1 = 60°$$

Example 5.15 The conditions of hot gas at the inlet to a 50% reaction turbine with a stage efficiency of 85% are 800 kPa and 900 K. The blade speed is 160 m/s. The mass flow rate is 75 kg/s and the absolute air angle at first stage nozzle exit is 70°. Assuming maximum utilization factor, Determine the rotor blade angles, Power developed and isentropic enthalpy drop across the stage.

Solution

$R = 0.5$ $\eta_s = 0.85$ $P_0 = 800$ kPa
$T_0 = 900$ K $U = 160$ m/s $\dot{m} = 75$ kg/s
$\alpha_1 = 70°$

For maximum utilisation factor condition, blade to gas speed ratio is

$$\sigma_{opt} = \frac{U}{C_1} = \sin\alpha_1$$
$$\therefore C_1 = \frac{U}{\sin\alpha_1} = \frac{160}{\sin 70°} = 170.27 \text{ m/s}$$
$$C_a = C_1\cos\alpha_1 = 170.27 \times \cos 70°$$
$$C_a = 58.24 \text{ m/s}$$

The velocity triangles for a 50% reaction stage at maximum utilization factor are given in Fig. 5.35.
$C_{x_1} = C_1 \sin\alpha_1 = U$, $\therefore W_{x_1} = 0$ *i.e.* $W_1 = C_a$ and $\beta_1 = 0°$.
Since $C_1 = W_2$ and $\alpha_1 = \beta_2$

$$C_1 \sin\alpha_1 = W_2 \sin\beta_2 = U = W_{x2}$$
$$\therefore C_{x_2} = 0 \text{ i.e. } C_2 = C_a$$

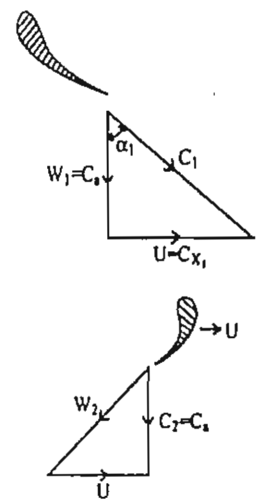


Figure 5.35 Velocity triangles for a 50% reaction stage with maximum utilization factor

(a) Rotor blade angles

$$\alpha_1 = \beta_2 = 70° \text{ and}$$
$$\alpha_2 = \beta_1 = 0°$$

(b) Power developed

$$W = \dot{m}U(C_{x1} + C_{x2}) = \dot{m}U(U + 0) = 75 \times (160)^2 = 1.92 \text{ mW}$$

(c) Isentropic Enthalpy drop

$$\eta_s = \frac{\Delta h}{\Delta h_s} = \frac{h_0 - h_2}{h_0 - h_{2s}}$$

$$\Delta h_s = \frac{\Delta h}{\eta_s} = \frac{W/m}{\eta_s} = \frac{1.92 \times 10^3/75}{0.85} = 30.12 \text{ kJ/kg}$$

Example 5.16 The data at mean radius of an axial turbine stage are as follows. Rotor blade angles at entry and exit are 46° and 75° respectively. Nozzle angle at exit is 75°. The hub to tip ratio is 0.6. Mean rotor speed is 7,500 rpm. Assuming free vortex flow conditions determine at mean and root section (a) degree of reaction and blade loading coefficient. Take hub diameter as 450 mm.

Solution

$\beta_{1,m} = 46°$ $\beta_{2,m} = 75°$ $\alpha_{1,m} = 75°$ $\frac{D_h}{D_t} = 0.6$

$N = 7,500$ rpm $D_h = 0.450$ m.

At mean section:

(a) Degree of reaction since

$\alpha_{1,m} = \beta_{2,m} = 75° R_m = 0.5$

(b) Blade loading coefficient

$$U_m = \frac{\pi D_m N}{60}$$

$$D_m = \frac{D_h + D_t}{2} = \frac{Dh + Dh/0.6}{2} = \frac{0.45 + 0.45/0.6}{2} = 0.6 \text{ m}$$

$$\therefore U_m = \frac{\pi \times 0.6 \times 7500}{60} = 235.62 \text{ m/s}$$

Since

$$U_m = C_a(\tan\alpha_{1,m} - \tan\beta_{1,m})$$

$$C_a = \frac{U_m}{\tan\alpha_{1,m} - \tan\beta_{1,m}} = \frac{235.62}{\tan 75° - \tan 46°} = 87.38 \text{m/s}$$

Flow coefficient, $\phi = \frac{C_a}{U} = \frac{87.38}{235.62} = 0.37$

$$\therefore \psi_l = \phi(\tan\beta_1 + \tan\beta_2) = 0.37(\tan 46° + \tan 75°)$$

$$\psi_l = 1.76$$

At root section, for free vortex flow,

$$C_{x1} \cdot r_m = C_{x1,h} \cdot r_h$$

$$C_a \cdot \tan\alpha_{1,m} \cdot r_m = C_a \cdot \tan\alpha_{1,h} \cdot r_h$$

$$\therefore \tan\alpha_{1,}h = \tan\alpha_{1,m}\frac{r_m}{r_h} = \tan 75° \times \frac{0.6/2}{0.45/2} = 4.976$$

$$\alpha_{1,}h = 78.64°$$

$$U_h = \frac{\pi D_h N}{60} = \frac{\pi \times 0.45 \times 7500}{60} = 176.72 \text{ m/s}$$

Since $U_h = C_a(\tan\alpha_{1,h} - \tan\beta_{1,h})$

$$\tan\beta_{1,h} = \tan\alpha_{1,h} - \frac{U_h}{C_a} = \tan 78.64° - \frac{176.72}{87.38} = 2.96$$

$$\beta_{1,h} = 71.33°$$

Similarly,

$$\tan\alpha_{2,h} = \tan\alpha_{2,m}\frac{r_m}{r_h} = \tan 46° \frac{0.6/2}{0.45/2} = 1.381$$

$$\alpha_{2,h} = 54.1°$$

And

$$\tan\beta_{2,h} - \tan\alpha_{2,h} = \frac{U_h}{C_a}$$

$$\tan\beta_{2,h} - \tan 54.1 = \frac{176.72}{87.38}$$

$$\tan\beta_{2,h} = \frac{176.72}{87.38} + \tan 54.1$$

$$= 3.4$$

$$\therefore \beta_{2,h} = 73.63^\circ$$

(a) Degree of reaction

$$R_h = \phi_h \frac{(\tan\beta_{2,h} - \tan\beta_{1,h})}{2}$$

$$\phi_h = \frac{C_a}{U} = \frac{87.38}{176.72} = 0.495$$

$$\therefore R_h = 0.495\frac{(\tan 73.63^\circ - \tan 71.33^\circ)}{2}$$

$$R_h = 0.11$$

(b) Blade loading coefficient

$$\psi_{l,h} = \phi_h(\tan\beta_{1,h} + \tan\beta_{2,h})$$

$$= 0.495(\tan 71.33^\circ + \tan 73.63^\circ)$$

$$\psi_{l,h} = 3.15$$

Example 5.17 The following particulars relate to a single stage turbine of free vortex design.

Total head inlet temperature – 973 K, total head inlet pressure – 4.5 bar, static head outlet pressure – 1.6 bar, gas flow rate – 20 kg/s. Nozzle outlet angle – 28° (measured from blade velocity), mean blade diameter to blade height ratio – 10, Nozzle loss coefficient – 0.1. Determine the gas velocities, temperatures and discharge angle at the blade mid, root and tip radii. Assume $C_p = 1.1556$ kg/kg-K and $r = 1.333$.

Solution

$T_{00} = 973$ K $P_{00} = 4.5$ bar $P_2 = 1.6$ bar $\dot{m} = 20$kg/s

$\alpha_1 = 90 - 28 = 62^\circ$ $D_m/h = 10$ $\dfrac{T_1 - T_{1s}}{C_1^2/2C_p} = 0.1$

(a) At mid section

$$T_{2_{ss}} = T_{00}\left(\frac{P_2}{P_{00}}\right)^{\frac{r-1}{r}} = 973\left(\frac{1.6}{4.5}\right)^{\frac{0.333}{1.333}}$$

$$= 751.49 \text{ K}$$

Assuming that there is no pressure drop in the moving blade, ($T_{2_{ss}} = T_{1_s}$) from Mollier chart.

$$T_{01} - T_{1s} = (T_{01} - T_1) + (T_1 - T_{1s})$$

$$973 - 751.49 = \frac{C_1^2}{2C_p} + 0.1\frac{C_1^2}{2C_p}$$

$$1.1C_1^2 = 2 \times 1155.6 \times (221.51)$$

$$C_{1,m} = 682.2 \text{ m/s}$$

Gas temperature

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 973 - \frac{682.2^2}{2 \times 1155.6}$$

$$T_1 = 771.6 \text{ K}$$

(b) At the hub

Hub diameter is determined as follows

$$\dot{m} = \rho A C_a$$

$$\rho = \frac{P_1}{RT_1} = \frac{1.6 \times 10^5}{289 \times 771.6} = 0.718\frac{\text{kg}}{m^3}$$

$$R = \frac{C_p(r-1)}{r} = 0.289 \text{ kJ/kg}$$

$$C_a = C_1\cos\alpha_1 = 682.2\cos 62 = 320.27 \text{ m/s}$$

$$A = \pi D_m h = \pi(10h)h$$

$$A = 10\pi h^2 = \frac{\dot{m}}{\rho C_a}$$

$$h^2 = \frac{20}{0.718 \times 320.27} \times \frac{1}{10\pi}$$

$$h = 0.053 \text{ m and}$$

$$D_m = 0.53 \text{ m}$$

Hub diameter $D_h = D_m - h = 0.53 - 0.053 = 0.477m$

For free vortex design

$C_{x1,m} \cdot r_m = C_{x1,h} \cdot r_h$

or

$$\tan\alpha_{1,h} = \frac{r_m}{r_h}\cdot\tan\alpha_{1,m} = \frac{0.53}{0.477}\tan 62^\circ$$

$$\alpha_{1,h} = 64.43^\circ$$

Gas velocity,

$$C_1 = \frac{C_a}{\cos\alpha_{1,h}} = \frac{320.27}{\cos 64.43}$$

$$C_1 = 742 \text{ m/s}$$

Gas temperature,

$$T_1 = T_{01} - \frac{C_1^2}{2C_p}$$

$$= 973 - \frac{742^2}{2 \times 1155.6} = 734.78 \text{ K}$$

(c) At the tip

$D_t = D_m + h = 0.53 + 0.053 = 0.583$ m

Gas discharge angle,

$$\tan\alpha_{1,t} = \frac{r_m}{r_t}\tan\alpha_{1,m}$$

$$= \frac{0.53}{0.583}\tan 62°$$

$$\therefore \alpha_{1,t} = 59.68°$$

$$C_1 = \frac{C_a}{\cos\alpha_{1,t}} = \frac{320.27}{\cos 59.68°} = 634.4 \text{ m/s}$$

and

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 973 - \frac{634.4^2}{2 \times 1155.6}$$

$$= 798.86 \text{ K}$$

Example 5.18 An axial turbine with constant nozzle air angle (75°) and zero reaction at the hub runs at 6000 rpm. Its hub and tip diameters are 45 and 75 cm respectively. All sections are designed for maximum utilisation factor. Assuming radial equilibrium conditions, determine for the hub, mean and tip sections (a) absolute and relative air angles (b) blade to gas speed ratio (c) degree of reaction. Assume axial exit from the stage at all sections.

Solution

$\alpha_1 = 75°$, $R_h = 0$, $N = 6000$, $D_h = 0.45$ m, $D_f = 0.75$ m

(1) Hub Section

Hub speed,

$$U_h = \frac{\pi D_h N}{60} = \frac{\pi \times 0.45 \times 6000}{60}$$

$$= 141.37 \text{ m/s}$$

Since $R_h = 0$ and for maximum utilisation factor

$$\sigma_{opt} = \frac{U_h}{C_{1,h}} = \frac{\sin\alpha_1}{2}$$

$$\therefore C_{1,h} = \frac{U_h}{(\sin\alpha_1/2)} = \frac{141.37}{(\sin 75°/2)}$$

$$= 292.71 \text{ m/s}$$

(a) Absolute and relative air angles

Nozzle exit angle, $\alpha_{1,h} = 75°$

$$C_{a,h} = C_{1,h}\cdot\cos\alpha_1 = 292.71 \times \cos 75°$$

$$= 75.76 \text{ m/s}$$

$$C_{x1,h} = C_{1h}\cdot\sin\alpha_1 = 292.71 \times \sin 75°$$

$$= 282.74 \text{ m/s}$$

From inlet velocity triangle

$$\tan\beta_{1,h} = \frac{W_{x1,h}}{C_{a,h}} = \frac{C_{x1,h} - U_h}{C_{a,h}}$$

$$= \frac{282.74 - 141.37}{75.76}$$

$$\therefore \beta_{1,h} = 61.81°$$

For zero reaction section,

$$\beta_{2,h} = \beta_{1,h}$$

$$\therefore \beta_{2,h} = 61.81°$$

(b) Blade to gas speed ratio

$$\sigma_{opt} = \frac{U_h}{C_{1,h}} = \frac{\text{Sin}\alpha_1}{2} = \frac{\sin 75°}{2}$$

$$= 0.483$$

(c) Degree of reaction

$$R_h = 0$$

(2) Mean Section

For a constant nozzle air angle,

$$\frac{C_{1,h}}{C_{1,m}} = \frac{C_{x1,h}}{C_{x1,m}} = \frac{C_{a1,h}}{C_{a1,m}} = \left(\frac{r_m}{r_h}\right)^{\sin^2\alpha_1}$$

$$r_m = \frac{r_h + r_t}{2} = \frac{\left(\frac{0.45}{2} + \frac{0.75}{2}\right)}{2} = 0.3 \text{ m}$$

$$\left(\frac{r_m}{r_h}\right)^{\sin^2 75°} = \left(\frac{0.3}{0.225}\right)^{0.933} = 1.308$$

$$\therefore C_{1,m} = \frac{C_{1,h}}{1.308} = \frac{292.71}{1.308} = 223.78 \text{ m/s}$$

$$C_{x1,m} = \frac{C_{x1,h}}{1.308} = \frac{282.74}{1.308} = 216.16 \text{ m/s}$$

$$C_{a1,m} = \frac{C_{a1,h}}{1.308} = \frac{75.76}{1.308} = 57.92 \text{ m/s}$$

(a) Absolute and relative air angles

Nozzle exit angle, $\alpha_{1,m} = \alpha_{1,h} = 75°$.

$$\tan\beta_{1,m} = \frac{W_{x1,m}}{C_{a1,m}} = \frac{C_{x1,m} - U_m}{C_{a1,m}}$$

$$U_m = \frac{\pi D_m N}{60} = \frac{\pi \times (2 \times 0.3) \times 6000}{60}$$

$$= 188.49 \text{ m/s}$$

$$\therefore \tan\beta_{1,m} = \frac{216.16 - 188.49}{57.92}$$

$$\beta_{1,m} = 25.54°$$

For an axial exit, $C_{x2} = 0$.

$$\therefore \tan\beta_{2,m} = \frac{U_m}{C_{a1,m}} = \frac{188.49}{57.92}$$

$$\beta_{2,m} = 72.92°$$

(b) Blade-to-gas speed ratio

$$\sigma = \frac{U_m}{C_{1,m}} = \frac{188.49}{223.78}$$

$$= 0.842$$

(c) Degree of reaction

$$R_m = \frac{C_{a1,m}(\tan\beta_{2,m} - \tan\beta_{1,m})}{2U_m}$$

$$= \frac{57.92 \times (\tan 72.92° - \tan 25.54)}{2 \times 188.49}$$

$$= 0.427$$

(3) Tip Section

$$\frac{C_{1,t}}{C_{1,m}} = \frac{C_{x1,t}}{C_{x1,m}} = \frac{C_{a1,t}}{C_{a1,m}} = \left(\frac{r_m}{r_t}\right)^{\sin^2\alpha_1}$$

$$\left(\frac{r_m}{r_t}\right)^{\sin^2 75°} = \left(\frac{0.3}{0.375}\right)^{0.933} = 0.812$$

$$\therefore C_{1,t} = C_{1,m} \times 0.812 = 223.78 \times 0.812 = 181.71 \text{ m/s}$$

$$C_{x1,t} = C_{x1,m} \times 0.812 = 216.16 \times 0.812 = 175.52 \text{ m/s}$$

$$C_{a1,t} = C_{a1,m} \times 0.812 = 57.92 \times 0.812 = 47.03 \text{ m/s}$$

(a) Absolute and relative air angles

Nozzle exit angle, $\alpha_{1,t} = \alpha_{1,m} = \alpha_{1,h} = 75°$

$$\tan\beta_{1,t} = \frac{C_{x1,t} - U_t}{C_{a1,t}}$$

$$U_t = \frac{\pi D_t N}{60} = \frac{\pi \times 0.75 \times 6000}{60} = 235.62 \text{ m/s}$$

$$\therefore \tan\beta_{1,t} = \frac{175.52 - 235.62}{47.03}$$

$$\beta_{1,t} = -51.96°$$

Since $C_{x2} = 0$

$$\tan\beta_{2,t} = \frac{U_t}{C_{a1,t}} = \frac{235.62}{47.03}$$

$$\beta_{2,t} = 78.71°$$

(b) Blade-to-gas Speed ratio

$$\sigma = \frac{U_t}{C_{1,t}} = \frac{235.62}{181.71}$$

$$= 1.297$$

(c) Degree of reaction

$$R_f = \frac{C_{a1,t}(\tan\beta_{2,t} - \tan\beta_{1,t})}{2U_t}$$

$$= \frac{47.03 \times (\tan 78.71 - \tan(-51.96))}{2 \times 235.62}$$

$$= 0.627$$

SHORT QUESTIONS

5.1. What is an axial flow steam (or) gas turbine?
5.2. What are the advantages of an axial flow turbine?
5.3. Why is single or two stage axial flow compressor preferred in aircraft jet propulsion?
5.4. The blade height is increased towards the low pressure end why?
5.5. Draw the velocity triangles for an axial flow turbine stage?
5.6. The diagram work per unit mass flow is given by
(a) $UC_a(\tan\beta_1 + \tan\beta_2)$
(b) $UC_a(\tan\beta_1 - \tan\beta_2)$
(c) $\frac{C_a}{U}(\tan\beta_1 + \tan\beta_2)$.
5.7. Define Diagram or blade efficiency (or) Utilisation factor.
5.8. Draw the Mollier diagram for expansion through an axial flow turbine stage.

5.9. In the rotor of the axial flow turbine
(a) $h_1 - h_2 = \text{constant}$.
(b) $h_1 - \frac{W_2^2}{2} = h_2 - \frac{W_1^2}{2}$
(c) $h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$

5.10. For a normal stage, the static temperature drop across the stage *equals* the total temperature drop. [True/False]

5.11. Define Turbine stage total-to-total isentropic efficiency.

5.12. When is the total-to-total effiiency used?

5.13. Define Blade loading coefficient.

5.14. The size of a stationary industrial turbine is *larger* than that of the aircraft gas turbine. [True/False]

5.15. Rateau turbine is an example of
(a) Two stage velocity compounded impulse turbine.
(b) Simple impulse turbine
(c) Two stage pressure compounded impulse turbine.

5.16. In pressure compounded impulse turbine, the blade height has to be increased towards the low pressure side. Why?

5.17. In velocity compounded impulse turbine, the blade height is same in all rows. (True/False)

5.18. Define Blade velocity coefficient.

5.19. The relative velocity of fluid increases across a reaction turbine. (True/False)

5.20. What is compounding or staging?

5.21. Why is compounding necessary?

5.22. The reaction turbine has
(a) no diffusers.
(b) no nozzles.
(c) no fixed blades.

5.23. The steam velocity in a reaction turbine is low and hence the speed is low relative to the impulse turbine. (True/False)

5.24. Differentiate between impulse and reaction turbines.

5.25. The steam expands in the movings fixed blades of a reaction turbine. (True/False)

5.26. In impulse turbine the pressure drop, unlike the reaction turbine, takes place in the nozzles only. (True/False)

5.27. Why is the term reaction used in reaction turbine?

5.28. Define Degree of reaction.

5.29. What is zero reaction stage?

5.30. With isentropic flow conditions prevailing, the zero reaction stage is exactly the same as the impulse stage. (True/False)

5.31. What is an impulse stage?

5.32. The following stages should be avoided due to the adverse pressure gradients causing flow separation
(a) Negative reactive stage only
(b) Reaction more than 100% only.
(c) both (a) and (b)

5.33. Gas turbines tend to be always of the reaction type. Why?

5.34. Steam turbines are usually impulse (or) a mixture of impulse and reaction stages becuase of
(a) High pressure ratio.
(b) Low overall pressure ratio.
(c) Type of working fluid.

5.35. The maximum diagram efficiency of an impulse turbine is
(a) $\cot^2\alpha_1$
(b) $\sin^2\alpha_1$
(c) $\sin\alpha_1$

5.36. The blade to gas speed ratio for maximum diagram efficiency is
(a) $\sin\alpha_1$
(b) $2\sin\alpha_1$
(c) $\sin\alpha_1/2$

5.37. The blade to gas speed ratio for maximum diagram efficiency of a reaction turbine is
(a) $\sin\alpha_1$
(b) $\sin\alpha_1/2$
(c) $2\sin\alpha_1$

5.38. The maximum stage efficiency of a reaction turbine is
(a) $1 + \sin^2\alpha_1$
(b) $1 + \sin^2\alpha_1/2sin^2\alpha_1$
(c) $2\sin^2\alpha_1/1 + sin^2\alpha_1$

5.39. The stage efficiency, turbine efficiency and reheat factor are related as
(a) $\eta_t \quad = \eta_s \times R.F$
(b) $\eta_s \quad = R.F \times \eta_t$
(c) $R.F = \eta_t \times \eta_s$

5.40. Annulus flow area interms of hub diameter and blade height is
(a) $\pi(D_h - h)h$
(b) $\pi h(D_h + h)$
(c) $\pi(D_h + h)^2$

5.41. What is Governing of turbines? How is it done?

EXERCISES

5.1. Describe the working principle of an axial flow turbine with a neat sketch.

5.2. What is an axial flow turbine stage?

5.3. Draw the inlet and outlet velocity triangles for an axial flow turbine stage.

5.4. Prove that the axial flow turbine stage work

$$W/m = UC_a(\tan\beta_1 + \tan\beta_2)$$

5.5. Define Diagram efficiency. What are the other names of it?

5.6. Draw the h–s diagram for expansion through an axial flow turbine stage.

5.7. The work done factor is not used in axial flow turbine but it is used in axial compressor why?

5.8. Prove that $h_{01,rel} = h_{02,rel}$ across the turbine rotor.
5.9. Define
(a) total-to-total efficiency
(b) total-to-static efficiency.
5.10. Define
(a) Nozzle loss coefficient and
(b) Rotor loss coefficient
5.11. Define blade loading coefficient? What is it's significance?
5.12. Prove that
$\psi_t = \phi(\tan\beta_1 + \tan\beta_2)$
5.13. Why are simple impulse turbines not so common?
5.14. What is compounding or staging?
5.15. Explain briefly a two stage pressure compounded impulse turbine and show the pressure and velocity variations across the turbine.
5.16. What is a velocity compounded turbine? Draw a two stage curtis turbine indicating the pressure and velocity variations across it.
5.17. Draw the inlet and outlet velocity triangles of a two stage impulse turbine.
5.18. Define blade velocity coefficient. How does it vary between an impulse and reaction turbine.
5.19. Explain with a neat diagram the operations of a reaction turbine.
5.20. Define: Reaction ratio for an axial flow turbine stage. Compare the degree of reaction for axial flow compressor and turbine.
√5.21. Derive the following relations.
(a) $R = \frac{1}{2}\phi(\tan\beta_2 - \tan\beta_1)$
(b) $R = \frac{1}{2} + \frac{C_a(\tan\beta_2 - \tan\alpha_1)}{2U}$
(c) $R = 1 + \frac{C_a(\tan\alpha_2 - \tan\alpha_1)}{2U}$
5.22. Draw the velocity triangles and h–s diagram for the following axial flow turbine stages.
(a) $R = 0$ (b) $R < 0$
(c) $R = 0.5$ (d) $R = 1.0$ (e) $R > 1.0$.
5.23. Prove that for an impulse turbine
(a) $\sigma = \sin\alpha_1/2$
(b) $\eta_{max\text{-}dia} = \sin^2\alpha_1$.
5.24. Prove that for a reaction turbine
(a) $\sigma = \sin\alpha_1$
(b) $\eta_{max\text{-}s} = \frac{2\sin^2\alpha_1}{1+\sin^2\alpha_1}$
5.25. Prove the following
$\eta_t = \eta_s \times R.F.$
5.26. Explain the various internal losses in axial flow turbines.
5.27. What is turbine governing? What are the different methods of governing of turbine? Explain briefly.

5.28. A single wheel impulse steam turbine has equi-angular rotor blades that develop 3.75 kW and produce a torque in the disc of 1.62 Nm at a mean radius of 132.5 mm. The rotor receives 0.014 kg/s of steam from nozzles inclined at 70° to the axial direction and steam discharges from the wheel chamber in an axial direction. Find (a) the blade angles (b) the diagram efficiency (c) the end thrust on the shaft (d) the tangential thrust on the blades.
[Ans: (a) 60.7 (b) 0.62 (c) 2.04 N and (d) 12.23 N]
√ 5.29. A 50 percent reaction steam turbine, running at 450 rpm develops 5 MW and has a steam mass flow rate of 6.5 kg/kW.hr. At a particular stage in the expansion the absolute pressure is 85 kPa at a steam dryness fraction of 0.94. If the exit angle of the blade is 70° measured from the axial flow direction, and the outlet relative velocity of the steam is 1.3 times the mean blade speed, find the blade height if the ratio of rotor hub diameter to blade height is 14.
[Ans: 0.131 m]
5.30. In a zero reaction gas turbine, the blade speed at the mean diameter is 290 m/s. Gas leaves the nozzle ring at an angle of 65° to the axial direction while the stage inlet stagnation temperature is 1100 K. The stagnation pressure at nozzle entry and exit are 400 kPa and 390 kPa respectively. Static pressure at nozzle exit is 200 kPa and that at rotor exit is 188 KPa. Assuming that the magnitude and direction of velocities at entry and exit of the stage are the same, determine the stage total-to-total efficiency. Take $C_P = 1148$ J/kg – K. [Ans: 87.3%]
5.31. Show that for a free vortex turbine blade with negligible degree of reaction at the hub, the degree of reaction R at any radius 'r' is related to the hub radius r_h by

$$R = 1 - \left(\frac{r_h}{r}\right)^2$$

5.32. Steam with a velocity of 600 m/s enters an impulse turbine row of blades at an angle of 25° to the plane of rotation of the blades. The mean blade speed is 255 m/s. The exit angle from the blades is 30°. There is a 10% loss in relative velocity due to friction in the blades. Determine (a) the entry angle of the blades measured from the axial direction (b) the work done per kg of steam/sec (c) the diagram efficiency (d) the end thrust per kg of steam/sec.
[Ans: (a) 48.5° (b) 150.71 kW/(kg/sec) (c) 83.7% and (d) 78 N/(kg/s)]
5.33. The nozzles of a simple impulse turbine are inclined at an angle of 20° to the direction of the path of the moving blades and the steam leaves the nozzles at 375 m/s. The blade speed is 165 m/s. Find suitable inlet and outlet angles for the blades in order that there shall be no axial thrust on the blades, allowing for the velocity of the steam in passing over the blades being reduced by 15%. Determine also the power developed for a steam flow of one kg/s at the blades, and the kinetic energy of the steam finally leaving the wheel.
[Ans: (a) 56°, 49° (b) 52.8 kW/(kg/s) and (c) 8.778 kJ/(kg/s)]
5.34. At a stage in a reaction turbine the mean blade ring diameter is 1m and the turbine runs at a speed of 50 rev/sec. The blades are designed for 50% reaction with exit angles 60° and inlet angles 40°. The turbine is supplied with steam at the rate of 6,00,000 kg/hr and the stage efficiency is 85%. Determine, (a) the

power output of the stage, (b) the specific enthalpy drop in the stage in kJ/kg and (c) the percentage increase in relative velocity.

[**Ans:** (a) 11.33 mW, (b) 79.98 kJ/kg and (c) 50.89%]

5.35. An axial flow gas turbine has a degree of reaction of 30%. The blade speed at the mean diameter is 300 mps and the mass flow is 2.5 kg/s. The gas temperature at the turbine inlet and outlet are 500°C and 300°C respectively. The fixed blade outlet angle is 20° measured in the same direction of blade speed. The axial velocity remains constant at 200 mps. Draw velocity diagram and determine the relative velocities and power developed. Take $C_P = 1.005$ kJ/kg K.

(MKU–Nov. '96)

[**Ans:** (a) 348 and 472 m/s, (b) 502.5 kW]

5.36. Two rows of a velocity compounded impulse turbine have a mean blade speed of 150 m/s and a nozzle velocity of 675 m/s. The nozzle angle is 20°. The exit angles of the first moving row, fixed and second row of moving blades are 25°, 25° and 30° respectively. There is a 10% loss of velocity due to friction in all blades. The steam flow is 4.5 kg/s. Draw the velocity diagram and determine (a) the power output and (b) the diagram efficiency.

[**Ans:** (a) 796.5 kW and (b) 77.7%]

5.37. The nozzle angle in a velocity compounded impulse turbine is 20° to the mean blade speed which is 100 m/s for each moving blades row. There are two rows of moving blades with exit angles 26° and 30°. Between these rows there is a row of fixed blades with exit angle 28°. The relative velocity of steam drops by 10% during passage through ring of blades and the final discharge is axial. Determine the velocity of steam leaving the nozzles and the blade efficiency. If the nozzle efficiency is 95% and the kinetic energy of steam leaving the stage is available as heat energy, estimate the state of steam entering the next stage. The conditions of the steam at nozzle inlet is 8.5 bar and dry saturated.

[**Ans:** (a) 450 m/s (b) 62% and (c) 5 bar, X = 0.96]

5.38. In an impulse turbine designed for free vortex flow at the rotor inlet, the blade root radius is 25 cm and the blade height is 6.3 cm. The absolute steam velocity at the rotor inlet is 450 m/s, the fluid being directed so as to make an angle of 15° with the wheel tangent at the blade root. If the speed ratio is 0.4 at the tip, the blade velocity coefficient is 0.97 and the difference between the rotor inlet and outlet angles is 3° all over the rotor, draw the velocity triangles for the blade tip and find the degree of reaction at the same position.

(MKU–May '97)

[**Ans:** −4.9%]

5.39. What is the percentage of reaction of a single stage turbine operating with a total pressure ratio of 3 and top temperature of 1100 K, if the velocity out of the stator blade is 550 m/s, and velocity out of the turbine is 300 m/s.

(MKU–April '97)

[**Ans:** 0.64]

5.40. The blades of a free vortex turbine rotor have inlet and outlet angles of 60 and 65 degrees at a mean diameter of 100 cm. The corresponding nozzle angle is 70°. The hub to tip ratio is 0.6 and the turbine runs at 3600 rpm. Calculate for

the hub, mean and tip sections of the blades (a) the blade angles, (b) degree of reaction (c) blade to gas speed ratio.

[**Ans:** mean section $\beta_1 = 60°$ $\beta_2 = 65°$ $\alpha_1 = 70°$
$R = 20.4\%$ and $\sigma = 0.35$.
hub section $\beta_1 = 71.0°$, $\beta_2 = 66.3°$, $\alpha_1 = 74.8°$
$R = 41.4\%$ and $\sigma = 0.2$.
tip section $\beta_1 = 43.1°$, $\beta_2 = 65.4°$, $\alpha_1 = 65.6°$
$R = 49.2\%$ and $\sigma = 0.54$]

5.41. An axial flow gas turbine has root and tip diameters of 600 mm and 750 mm. Rotor speed is 7500 rpm. Rotor blade design is based on free vortex principle and absolute velocity is axial at exit. The actual change in total temperature at mean section is 110°C. Calculate the air and blade angles, reaction ratio and blade loading coefficient at all sections. Take R = 347.2 J/kg-k and r = 1.3.

[**Ans:** $\alpha_{1,m} = 72.25°$, $\beta_{1,m} = 60.93°$, $\beta_{2,m} = 52.96°$, $R_m = -17.88\%$
$\psi_{l,m} = 2.36$ $\alpha_{1,h} = 74.12°$, $\beta_{1,h} = 66.83°$, $\beta_{2,h} = 49.67°$,
$R_h = -49.17\%$ $\psi_{l,h} = 2.98$ $\alpha_{1,t} = 70.42°$, $\beta_{1,t} = 53.24°$,
$\beta_{2,t} = 55.82°$, $R_t = 4.55\%$ $\psi_{l,t} = 1.91$]

5.42. A gas turbine stage has an initial absolute pressure of 350 kPa and temperature 565°C and negligible initial velocity. At the mean radius 0.36 m, the conditions are nozzle exit absolute static pressure is 207 kPa, nozzle exit flow angle is 68° and stage reaction is 0.2. Determine the flow coefficient, stage loading factor, stage reaction and air and blade angles at the mean, hub and tip. The hub radius and speed are respectively 0.31 m and 8000 rpm. Assume that the stage is to have a free vortex swirl at this speed. Take $C_p = 1.148$ kJ/kg-K and r = 1.33. Neglect the losses.

(MKU–Nov. '96)
(MU–Oct. '96)

[**Ans:** $\alpha_{1,m} = 68°$, $\alpha_{2,m} = -10.3°$, $\beta_{1,m} = 39.2°$, $\beta_{2,m} = 55.93°$
$\phi_m = 0.602$, $R_m = -0.2$ $\psi_{l,m} = 1.38$ $\alpha_{1,h} = 70.82°$.
$\alpha_{2,h} = -11.9°$, $\beta_{1,h} = 55.31°$, $\beta_{2,h} = 50.63°$ $\phi_h = 0.7$.
$R_h = -0.079$ $\psi_{l,h} = 1.76$ $\alpha_{1,t} = 65.29°$, $\alpha_{2,t} = -9.1°$.
$\beta_{1,t} = 15.75°$, $\beta_{2,t} = 59.9°$ $\phi_t = 0.528$, $R_t = 0.38$
$\psi_{l,t} = 1.06$]

5.43. The data for an axial turbine stage are: hub diameter = 450 mm, tip diameter = 750 mm, rotor speed = 600 rpm. At the mean section, $\alpha_1 = \beta_2 = 75°$, $\beta_1 = \alpha_2 = 0$, $R = 0.5$. Assuming radial equlibrium and constant nozzle angle, determine for the hub, mean and tip sections (a) the absolute and relative air angles (b) blade-to-gas speed ratio (c) degree of reaction. Assume axial exit from the stage at all sections.

[**Ans:** $\alpha_{1,h} = 75°$ $\beta_{1,h} = 57.87°$ $\beta_{2,h} = 65°$, 0.554, 0.218
$\alpha_{1,m} = 75°$ $\beta_{1,m} = 0°$ $\beta_{2,m} = 75°$, 0.966, 0.5
$\alpha_{1,t} = 75°$ $\beta_{1,t} = -63.58°$ $\beta_{2,t} = 80.13°$, 1.487, 0.675]

6

RADIAL FLOW GAS AND STEAM TURBINES

INTRODUCTION

The inward flow radial gas turbines are used for applications where the flow rate is very low, for example, turbochargers for commercial (diesel) engines and fire pumps. The radial gas turbines are very compact, the maximum diameter being about 0.2 m and speeds are high, ranging from 40,000 to 1,80,000 rpm. They are usually of the 90° type, the blades being ⊥r to the tangent at the periphery of the rotor outer inlet. The gas enters the turbines in the radial direction and leaves axially at the outlet.

DESCRIPTION

A 90° inward flow radial turbine is very similar to the centrifugal compressor and the only difference being that the gas flow is in the opposite direction (Figs. 6.1(a) and 6.1(b)). The gas enters the scroll casing, whose cross-sectional area is decreasing as the gas passes through it. This keeps the velocity at the entry to the nozzle vanes constant as the gas is gradually drawn off on its circumferential path. The nozzle vanes are converging to increase the kinetic energy of the gas and they set the gas angle for entry into the rotor. This angle, measured from the radial direction is usually

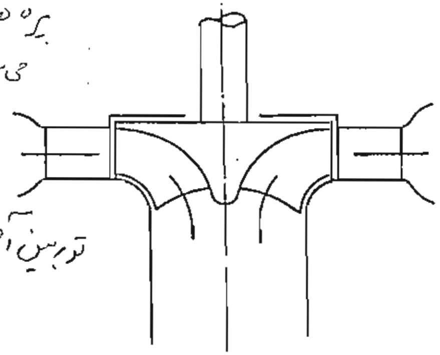


Figure 6.1(a) Inward flow radial (IFR) turbine

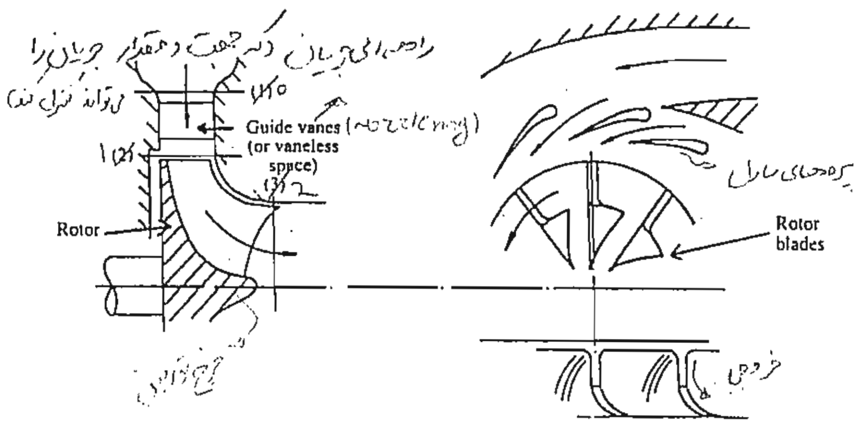


Figure 6.1(b) Radial inflow turbine

about 70°, but the vanes can be pivoted to allow for adjustment of the flow angle as the load changes. In some turbines, there may be no vanes at all, but a passage similar to that of the vaneless diffuser (discussed already) is fitted. A vaneless space exists between the outlet tip of the vanes and the rotor. This space is being utilised by the gas for futher flow adjustment and helping in the reduction of vibratory disturbances within the turbine.

Some of the common applications of the radial turbines are in the fields of turbocharging, aircraft and missile auxiliary drives, cryogenics, and gas liquefaction. Francis (Inward flow radial) turbine has been in use since a long time for hydropower generation. The Ljungstrom (outward flow radial) turbine is used in steam power generation.

The rotor which is usually manufactured with cast nickel alloy, has blades that are curved to change the flow from the radial to the axial direction. The shrouding for the blades is formed by the casing, and a diffuser can be fitted at the outlet to further reduce the high kinetic energy at that point and thereby increase the enthalpy drop across the rotor.

VELOCITY DIAGRAMS

The velocity triangles for the 90°inward flow radial gas turbines are shown in Fig. 6.2. Section 1-2 refers the rotor and 'o' subscript indicates the point of entry to the nozzle vanes and section 2-3 indicates the diffuser outlet section.

The work done per unit mass flow in the rotor is given by Euler's turbine equation

$$W/m = (U_1C_{x_1} - U_2C_{x_2})$$

If whirl velocity is zero at the exit ($C_{x_2} = 0$)then

$$W/m = U_1C_{x_1}$$

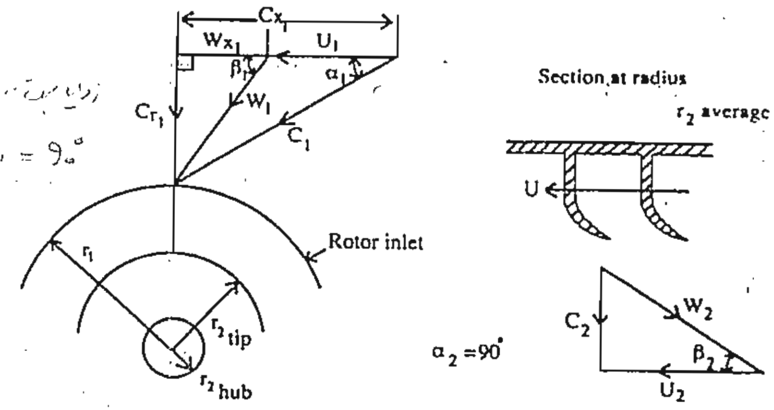


Figure 6.2 Velocity triangles for an inward flow radial turbine

and for radial relative velocity at entry i.e. for a 90° *IFR* turbine. $\beta_1 = 90°$

$$\therefore W/m = U_1^2 \text{ [since } C_{x_1} = U_1] \qquad (6.1)$$

The head or stage loading coefficient is defined by

$$\psi_I = \frac{\text{workdone/kg}}{U_1^2} = \frac{U_1C_{x_1}}{U_1^2} = \frac{C_{x_1}}{U_1}$$

From the velocity triangle, $C_{x_1} = C_{r_1} \cot\alpha_1$

$$\therefore \psi_I = \frac{C_{r_1}\cot\alpha_1}{U_1} = \phi_1 \cot\alpha \qquad (6.2)$$

where ϕ_1 is known as the *flow coefficient*. For a 90° *IFR* turbine, $U_1 = C_{x_1}$. Therefore,

$$\psi_I = \frac{U_1^2}{U_1^2} = 1 \qquad (6.3)$$

THE THERMODYNAMICS OF FLOW

The thermodynamic path followed by the gas is shown on the Mollier chart (Fig. 6.3). In the nozzle, no work is done. Therefore, $h_{00} = h_{01}$, (although the total pressure drops from p_{00} to p_{01} because of irreversibilities) Thus,

$$h_0 - h_1 = (C_1^2 - C_0^2)/2 \qquad (6.4)$$

The work done is given by

$$W/m = h_{01} - h_{02} \qquad (6.5)$$

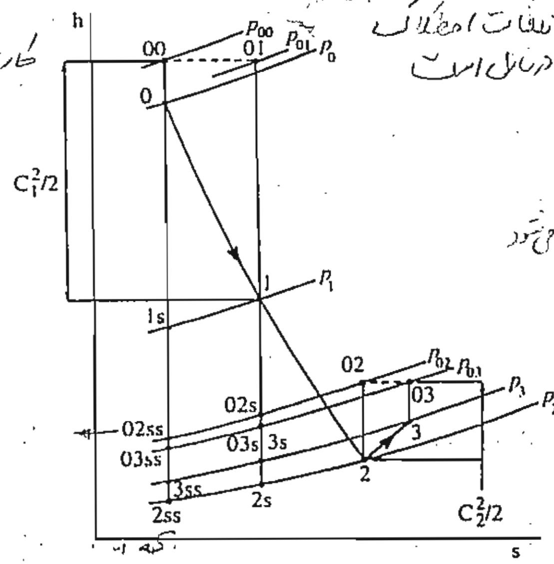


Figure 6.3 Mollier chart for expansion in a 90° inward flow radial gas turbine

Equating the two expressions

$$\boxed{h_{01} - h_{02} = U_1 C_{x_1} - U_2 C_{x_2}} \tag{6.6}$$

It is already shown that the quantity 'I' for a centrifugal compressor is given by

$$I = h_{0rel} - U^2/2$$

where 'I' is a constant

Therefore,

$$h_1 - h_2 = \frac{[(U_1^2 - U_2^2) - (W_1^2 - W_2^2)]}{2} \tag{6.6a}$$

If $C_{x_2} = 0$, then $(W_2^2 - U_2^2) = C_2^2$ and

$$h_1 - h_2 = \frac{(U_1^2 - W_1^2 + C_2^2)}{2} \tag{6.7}$$

In the diffuser $h_{02} = h_{03}$. Thus,

$$h_3 - h_2 = \frac{(C_2^2 - C_3^2)}{2} \tag{6.8}$$

If the losses in the diffuser are neglected, then

$$T_{03_{ss}} = T_{02_{ss}} \text{ and}$$

Total-to-total isentropic efficiency is given by

$$\eta_{tt} = \frac{(T_{00} - T_{02})}{(T_{00} - T_{02_{ss}})} \tag{6.9}$$

η_{tt} is in the range of 80–90 per cent.

The losses in an *IFR* turbine can be expressed in terms of the nozzle and rotor loss coefficients as defined for the axial stages.

From the $h - s$ diagram, these coefficients are

$$\boxed{L_N = \frac{h_1 - h_{1s}}{C_1^2/2}}$$

and

$$\boxed{L_R = \frac{h_2 - h_{2s}}{W_2^2/2}}$$

STAGE EFFICIENCIES

The actual work output of the stage is

$$\begin{aligned}\frac{W}{m} &= h_{00} - h_{02} = h_{01} - h_{02}\\ &= U_1 C_{x_1} = U_1(U_1 + W_{x_1})\\ &= U_1^2(1 + \phi_1 cot\beta_1)\end{aligned}$$

From the velocity triangle (Fig. 6.2)

$$C_{x_1} = U_1 + C_{r_1} \cot\beta_1$$

or

$$\frac{C_{x_1}}{U_1} = 1 + \frac{C_{r_1}}{U_1}\cot\beta_1$$

But

$$\begin{aligned}\frac{C_{x_1}}{U_1} &= \psi_I \text{ and } \frac{C_{r_1}}{U_1} = \phi_1\\ \therefore \psi_I &= 1 + \phi_1 \cot\beta_1\end{aligned}$$

Then $W/m = \psi_I U_1^2$.

For a perfect gas

$$W/m = C_p(T_{01} - T_{02}) = U_1^2(1 + \phi_1 \cot\beta_1) = \psi_I U_1^2 \tag{6.10}$$

The isentropic work output between the total conditions at the entry and exit of the stage is

$$\begin{aligned}W_s &= h_{00} - h_{02_{ss}} = C_p(T_{01} - T_{02_{ss}})\\ &= C_p T_{00}\left[1 - \left(\frac{P_{02}}{P_{00}}\right)^{\frac{\gamma-1}{\gamma}}\right]\end{aligned} \tag{6.11}$$

The total-to-total efficiency

$$\eta_{tt} = \frac{W}{W_s} = \frac{h_{00} - h_{02}}{h_{00} - h_{02_{ss}}}$$

$$= \frac{U_1^2(1 + \phi_1 \cot \beta_1)}{C_p T_{00}\left[1 - \left(\frac{P_{02}}{P_{00}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \frac{\psi_i U_1^2}{C_p T_{00}\left[1 - \left(\frac{P_{02}}{P_{00}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \quad (6.12)$$

The isentropic work between the total conditions at the entry and static conditions at the exit of the stage is given by

$$W_s = h_{00} - h_{2_{ss}} = C_p(T_{00} - T_{2_{ss}})$$

$$= C_p T_{00}\left[1 - \left(\frac{P_2}{P_{00}}\right)^{\frac{\gamma-1}{\gamma}}\right] \quad (6.13)$$

The total-to-static efficiency is given by

$$\eta_{t-s} = \frac{W}{W_s} = \frac{h_{00} - h_{02}}{h_{00} - h_{2_{ss}}}$$

$$= \frac{U_1^2(1 + \phi_1 \cot \beta_1)}{C_p T_{00}\left[1 - \left(\frac{P_2}{P_{00}}\right)^{\frac{\gamma-1}{\gamma}}\right]} = \frac{\psi_i U_1^2}{C_p T_{00}\left[1 - \left(\frac{P_2}{P_{00}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \quad (6.14)$$

SPOUTING VELOCITY

The isentropic total enthalpy drop in the turbine is given by

$$\boxed{(h_{00} - h_{02_{ss}})}$$

when the diffuser is not fitted. The expression becomes $(h_{00} - h_{03_{ss}})$ when the diffuser is fitted.

Conditions	Total-to-total	Total-to-static
with diffuser	$\frac{C_s^2}{2} = (h_{00} - h_{03_{ss}})$	$\frac{C_s^2}{2} = (h_{00} - h_{3_{ss}})$
without diffuser	$\frac{C_s^2}{2} = (h_{00} - h_{02_{ss}})$	$\frac{C_s^2}{2} = (h_{00} - h_{2_{ss}})$

The energy change may be related to kinetic energy where the associated velocity term is known as the *spouting velocity* designated as C_s. Thus four spouting velocities, with and without a diffuser and for total-to-total or total-to-static conditions may be defined as follows

The appropriate definition would be used depending upon the efficiency being determined.

For isentropic flow across the turbine,

$$W/m = U_1^2 = C_s^2/2$$

or

$$\boxed{U_1/C_s = 0.707}$$

In practice, U_1/C_s lies in the range 0.68 to 0.7. This blade to gas speed ratio is denoted as σ_s.∴

DEGREE OF REACTION

Degree of reaction for a radial flow turbine is defined by

$$\boxed{R = \frac{\text{Static enthalpy drop in the rotor}}{\text{Stagnation enthalpy drop in the stage}}}$$

$$= \frac{h_1 - h_2}{h_{00} - h_{03}}$$

$$= \frac{h_1 - h_2}{h_{01} - h_{02}} \quad (6.15)$$

$$= \frac{(h_{01} - h_{02}) - \frac{1}{2}(C_1^2 - C_2^2)}{h_{01} - h_{02}}$$

$$= 1 - \frac{C_1^2 - C_2^2}{2(h_{01} - h_{02})}$$

$$= 1 - \frac{C_1^2 - C_2^2}{2(U_1 C_{x_1} - U_2 C_{x_2})}$$

$$= 1 - \frac{C_1^2 - C_2^2}{2U_1 C_{x_1}} \quad (6.16)$$

[If the whirl velocity C_{x_2} is zero at exit and $C_2 = C_{r_2}$]

Assuming constant radial velocity

$$C_{r_2} = C_{r_1}$$

$$\therefore C_1^2 - C_2^2 = C_1^2 - C_{r_2}^2 = C_1^2 - C_{r_1}^2$$

$$= C_{x_1}^2$$

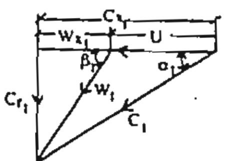


Figure 6.3(a)

$$\therefore R = 1 - \frac{C_{x_1}^2}{2U_1 C_{x_1}}$$

$$R = 1 - \frac{C_{x_1}}{2U_1} = 1 - \frac{\psi_l}{2} \qquad (6.17)$$

From the inlet velocity triangle Fig. 6.3(a)

$$C_{x_1} = U_1 + W_{x_1}$$

$$= U_1 + C_{r_1}\cot\beta_1$$

$$\therefore R = 1 - \left[\frac{U_1 + C_{r_1}\cot\beta_1}{2U_1}\right] \qquad (6.18)$$

$$= 1 - \left[\frac{1 + \phi_1\cot\beta_1}{2}\right]$$

$$R = \frac{1}{2}\left[1 - \phi_1\cot\beta_1\right]$$

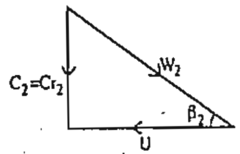


Figure 6.3(b)

When $\beta_1 = 90°$(angle measured from the plane of rotation) i.e., for radial vanes.

$$\cot 90° = 0$$

$$\therefore R = \frac{1}{2} \qquad (6.19)$$

Consider the expression (6.17)

$$R = 1 - \frac{C_{x_1}}{2U_1}$$

When $R = 0$ i.e. for an impulse stage $C_{x_1} = 2U_1$ and hence $\psi_l = 2$ and $R = 1/2$ i.e. for the fifty per cent reaction stage $C_{x_1} = U_1$ and therefore, $\psi_l = 1$

Fig. 6.4 shows the variation of the degree of reaction (at various values of the flow coefficient) with the rotor blade inlet angle.

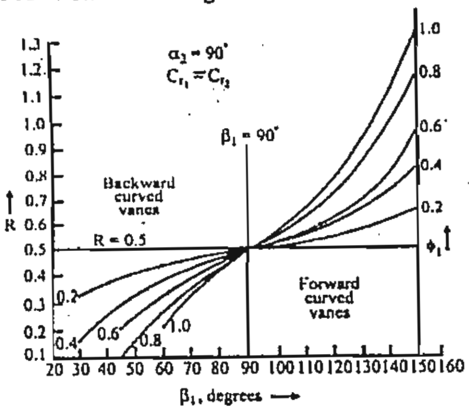


Figure 6.4 Variation of degree of reaction with rotor blade inlet angles

The half-degree reaction (R=0.5) has the advantage of having a constant value of the reaction at all values of the flow coefficient.

The degree of reaction increases with the increasing values of the inlet blade angle (β_1). Thus for a given value of the flow coefficient (ϕ)the forward curved vanes give a higher degree of reaction compared to the backward curved vanes.

The degree of reaction of the backward curved vanes decreases with increasing values of the flow coefficient. The converse is true for the forward curved vanes.

AN OUTWARD FLOW RADIAL TURBINE (LJUNGSTROM TURBINE)

Ljungstrom turbine is an outward flow radial turbine (Fig. 6.5(a)) in which the fluid enters the turbine in the axial direction and leaves radially at the exit. The area of flow increases towards the exit which accommodates the expanding gas. The most commonly employed Ljungstrom turbine consists of rings of cantilever blades mounted on two discs rotating in opposite directions. The counter rotating blade rings of a Ljungstrom turbine is shown in Fig. 6.5(b). Each row of blades forms a stage.

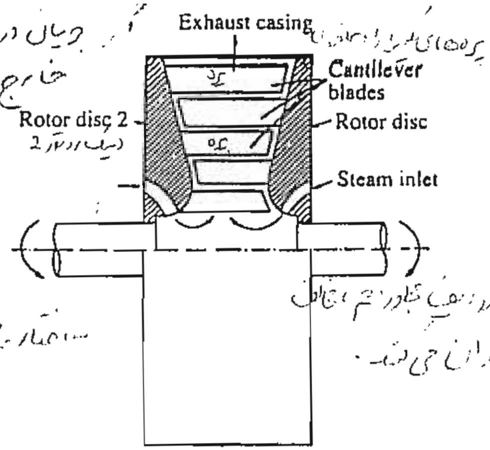


Figure 6.5(a) An outward flow radial turbine (Ljungstrom turbine)

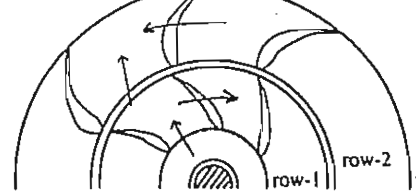


Figure 6.5(b) Counter rotating blade rings of Ljungstrom turbine

VELOCITY TRIANGLES AND STAGE WORK

The velocity triangles for the first two stages are shown in Fig. 6.6. Various velocities at the inlet of the first stage are designated by the suffix o.

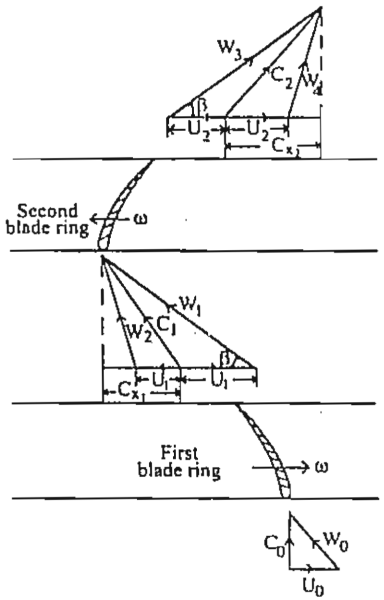


Figure 6.6 Velocity triangles for the Ljungstrom turbine stages

The relative velocity at the exit of the first stage is W_1 which along with the peripheral velocity U_1 gives the absolute velocity C_1. The relative velocity W_2 at the entry of the second stage is obtained by subtracting U_1 from C_1. Thus the exit velocity triangle of the first stage and the entry velocity triangle of the second stage are superimposed on each other. The same velocity triangle applies to the following stages. The relative velocity at the exit from the second stage is W_3 and at the entry to the third stage is W_4. The common absolute velocity at this section is C_2. The air angles (β) of all stages are assumed to be the same. The peripheral velocities of the two rows at a given section are also assumed to be equal.

Stage Work

The first blade row does not form a general stage. Therefore, any other stage, can be considered for the purpose of analysis. For the second stage, the work done/kg is given by

$$\boxed{W/m = h_{01} - h_{02} = U_1 C_{x1} + U_2 C_{x2}} \qquad (6.20)$$

Since C_{x_2} is in the opposite direction.

From the velocity triangles,

$$W/m = U_1(W_1 \cos\beta - U_1) + U_2(W_3 \cos\beta - U_2)$$

Assuming $U_1 \approx U_2$ and $W_1 \approx W_3$, then

$$\begin{aligned} W/m &= 2U_2(W_3 \cos\beta - U_2) \\ &= 2(U_2 W_3 \cos\beta - U_2^2) \end{aligned}$$

Multiplying and dividing by W_3, we get

$$= 2W_3^2\left(\frac{U_2}{W_3}\cos\beta - \frac{U_2^2}{W_3^2}\right) \qquad (6.21)$$

$$= 2W_3^2(\sigma_r \cos\beta - \sigma_r^2)$$

where σ_r is the ratio of the peripheral velocity of blades to the relative velocity at the exit and is assumed to be constant. i.e.

$$\sigma_r = \frac{U_1}{W_1} = \frac{U_2}{W_3} = \text{constant.}$$

For maximum work,

$$\begin{aligned} \frac{\partial W}{\partial \sigma_r} &= 0 \\ \cos\beta - 2\sigma_r &= 0. \end{aligned}$$

(or)

$$\sigma_{r_{opt}} = \frac{\cos\beta}{2} \qquad (6.22)$$

This is the optimum value of σ_r for the maximum work. Then

$$\begin{aligned} W_1 \cos\beta &= 2U_1 \\ W_3 \cos\beta &= 2U_2 \end{aligned}$$

The maximum work is obtained by substituting the value of $\sigma_{r_{opt}}$ in equation (6.21)

$$W_{max}/m = \frac{1}{2}W_3^2 \cos^2\beta$$

But

Therefore,

$$\left\{\begin{aligned} W_3 \cos\beta &= 2U_2 \\ W_{max}/m &= 2U_2^2 \end{aligned}\right. \qquad (6.23)$$

The above two equations show that the outward flow counter rotating radial stages behave like an impulse stage of the axial type. But this impression is not true. Because, the true or equivalent blade velocity is $2U_2$ on account of counter rotation. Therefore, the actual blade to gas relative velocity ratio should be

$$\sigma_{r_{opt}} = \frac{2U_2}{W_3} = \cos\beta$$

This is same as in the fifty per cent reaction stages of the axial type. In this sense, the outward flow counter rotating radial turbine behaves as a fifty per cent reaction stage of the axial type.

For ideal flow, from equation (6.20)

$$h_1 - h_2 = U_1 C_{x_1} + U_2 C_{x_2} - \frac{1}{2}(C_1^2 - C_2^2) \qquad (6.24)$$

From the velocity trianlges at the exits of the first and second rings

$$C_1^2 = U_1^2 + W_1^2 - 2U_1 W_1 \cos\beta \text{ and } C_{x_1} = W_1 \cos\beta - U_1$$
$$C_2^2 = U_2^2 + W_3^2 - 2U_2 W_3 \cos\beta \text{ and } C_{x_2} = W_3 \cos\beta - U_2$$

Substituting these relations in equ. 6.24 and assuming $U_1 \approx U_2$ and $W_1 \approx W_3$, we get

$$h_1 - h_2 = \frac{1}{2}(4U_1 W_1 \cos\beta - 3U_1^2 - U_2^2 - W_1^2 + W_3^2)$$

From the combined velocity triangles, at the exit of the first ring

$$W_2^2 = W_1^2 + (2U_1)^2 - 2W_1(2U_1)\cos\beta$$
$$4U_1 W_1 \cos\beta = W_1^2 + 4U_1^2 - W_2^2$$

Therefore,

$$h_1 - h_2 = \frac{1}{2}\left(W_1^2 + 4U_1^2 - W_2^2 - 3U_1^2 - U_2^2 - W_1^2 + W_3^2\right)$$
$$h_1 - h_2 = \left[\left(U_1^2 - U_2^2\right) - \left(W_2^2 - W_3^2\right)\right]/2 \qquad (6.25)$$

It should be noted that in inward flow radial turbines $U_1 > U_2$. So, the enthalpy drop $(h_1 - h_2)$ increases by an amount $(U_1^2 - U_2^2)/2$ (equ. 6.6a). But, in outward flow radial turbines $U_1 < U_2$, therefore enthalpy drop $(h_1 - h_2)$ decreases by a quantity $(U_1^2 - U_2^2)/2$ (equ. 6.25). Therefore, for the same size and speed, inward flow radial turbines develop higher power compared to the outward flow type.

SOLVED PROBLEMS

Example 6.1 A cantilever blade type IFR turbine receives air at $P_{00} = 3$ bar, $T_{00} = 373$ K. Other data for this turbine are rotor tip diameter = 50 cm, rotor exit diameter = 30 cm, speed = 7200 rpm, rotor blade width at entry = 3 cm, air angle at rotor entry = 60°, air angle at nozzle exit = 25°, nozzle efficiency = 97%, stage pressure ratio (P_{00}/P_2) = 2.0. The radial velocity is constant and the swirl at the rotor exit is zero.

Determine (a) the flow and loading coefficients, (b) the degree of reaction and stage efficiency, (c) the air angle and width at the rotor exit, and (d) the mass flow rate and power developed.

Solution

P_{00} = 3 bar	T_{00} = 373 K				
r_t = 0.5 m	r_h = 0.3 m	N = 7200 rpm			
b = 0.03 m	β_1 = 60°,	α_1 = 25°			
P_{00}/P_2 = 2.0	$C_{r_1} = C_{r_2} = C_r$	η_N = 0.97			

(a) Flow coefficient and stage loading coefficient

Flow coefficient $(\phi) = \frac{C_r}{U}$

$$U_1 = \frac{\pi d_t N}{60} = \frac{\pi \times 0.5 \times 7200}{60}$$
$$= 188.5 \text{ m/s}$$

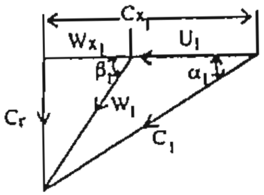


Figure 6.7(a)

From the inlet velocity triangle

$$U_1 = C_{x_1} - W_{x_1}$$
$$= C_r(\cot\alpha_1 - \cot\beta_1)$$
$$\therefore C_r = \frac{U_1}{\cot\alpha_1 - \cot\beta} = \frac{188.5}{\cot 25° - \cot 60°}$$
$$= 120.3 \text{ m/s}$$

$$\text{Flow coefficient } \phi_1 = \frac{C_r}{U_1} = \frac{120.3}{188.5}$$
$$= 0.638$$

Loading coefficient
$$\psi_l = 1 + \phi_1 \cot\beta_1$$
$$= 1 + 0.638 \cot 60°$$
$$= 1.368$$

(b) Degree of reaction and stage efficiency

$$R = \frac{1 - \phi_1 \cot\beta_1}{2}$$
$$= \frac{1 - 0.638 \cot 60°}{2}$$
$$= 31.58\%$$

$$\text{Stage efficiency}\ \eta_{ts} = \frac{\psi U_1^2}{C_p T_{00}\left[1-\left(\frac{P_2}{P_{00}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \frac{1.368(188.5)^2}{1005 \times 373\left[1-\left(\frac{1}{2}\right)^{\frac{0.4}{1.4}}\right]}$$

$$= 72.17\%$$

(c) The air angle and width at the rotor exit

$$C_2 = C_r = 120.3\ \text{m/s}$$

$$U_2 = \frac{\pi D_h N}{60}$$

$$= \frac{\pi \times 0.3 \times 7200}{60}$$

$$= 113.1\ \text{m/s}$$

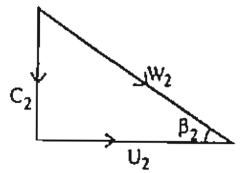


Figure 6.7(b)

From the outlet velocity triangle ($C_{x_2} = 0$) (Fig. 6.7(b))

$$\tan\beta_2 = \frac{C_2}{U_2}$$

$$\beta_2 = \tan^{-1}\left(\frac{120.3}{113.1}\right)$$

$$= 46.77^\circ$$

Rotor width at exit

$$R = \frac{h_1 - h_2}{U_1 C_{x_1}} = 0.3158$$

$$C_p(T_1 - T_2) = 0.3158 \times 188.5 \times (C_r \cot\alpha_1)$$

$$= 0.3158 \times 188.5 \times 120.3(\cot 25^\circ)$$

$$T_1 - T_2 = 15.3\ \text{K}$$

From the workdone/kg

$$W/m = C_p(T_{00} - T_{02}) = U_1 C_{x_1}$$

$$T_{00} - T_{02} = \frac{188.5 \times 120.3 \times \cot 25^\circ}{1005}$$

$$= 48.39\ \text{K}$$

$$\therefore T_{02} = 373 - 48.39 = 324.61\ \text{K}$$

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = T_{02} - \frac{C_r^2}{2C_p}$$

$$= 324.61 - \frac{120.3^2}{2 \times 1005}$$

$$= 317.4\ \text{K}$$

$$T_1 = 15.3 + 317.4 = 332.7\ \text{K}$$

The nozzle efficiency is given by

$$\eta_N = \frac{C_1^2/2}{C_p(T_{00} - T_{1s})}$$

$$T_{00} - T_{1s} = \frac{C_1^2/2}{C_p \times \eta_N}$$

$$C_1 = C_{x_1}/\cos\alpha_1$$

$$C_{x_1} = C_r \cot\alpha_1 = 120.3 \cot 25^\circ$$

$$= 257.98\ \text{m/s}$$

$$\therefore C_1 = 257.98/\cos 25^\circ$$

$$= 284.65\ \text{m/s}$$

$$\therefore T_{00} - T_{1s} = \frac{284.65^2/2}{1005 \times 0.97}$$

$$= 41.56\ \text{K}$$

$$T_{00} - T_{1s} = T_{00}\left[1-\left(\frac{P_1}{P_{00}}\right)^{\frac{\gamma-1}{\gamma}}\right] = 41.56$$

$$\frac{P_1}{P_{00}} = \left[1 - \frac{41.56}{T_{00}}\right]^{\frac{\gamma}{\gamma-1}} = (0.89)^{\frac{1.4}{0.4}} = 0.67$$

$$\therefore P_1 = 0.67 \times 3 = 2.01\ \text{bar}$$

Density of air at rotor entry

$$\rho_1 = P_1/RT_1 = 2.01 \times 10^5/287 \times 332.7$$

$$= 2.105\ \text{kg/m}^3$$

The mass flow rate through the rotor is

$$m = \rho_1 A_1 C_{r_1}$$

$$= 2.105 \times (\pi \times D_t \times b_t) \times 120.3$$

$$= 2.105 \times (\pi \times 0.5 \times 0.03) \times 120.3$$

$$m = 11.93\ \text{kg/s}$$

At rotor exit

$$m = \rho_2 A_2 C_2$$

$$\rho_2 = P_2/RT_2$$

$$P_2 = P_{00}/2 = 3/2 = 1.5\ \text{bar}$$

$$T_2 = 317.4\ \text{K}$$

$$\therefore \rho_2 = 1.5 \times 10^5/287 \times 317.4$$

$$= 1.65\ \text{kg/m}^3$$

$$\therefore 11.93 = 1.65(\pi \times D_h \times b_h) \times 120.3$$

or

$$b_h = \frac{11.93}{1.65 \times \pi \times 0.3 \times 120.3}$$
$$= 0.0638 \text{ m}$$
$$b_h = 6.38 \text{ cm}$$

(d) The mass flow rate and power developed

$$m = 11.93 \text{ kg/s}$$
$$\text{Power } W = mU_1C_{x_1} = 11.93 \times 188.5 \times 257.98$$
$$= 580.15 \text{ kW}$$

Example 6.2 A single stage 90° *IFR* turbine fitted with an exhaust diffuser has the following data.

Overall stage pressure ratio = 4.0, temperature at entry = 557 K, diffuser exit pressure = 1 bar, mass flow rate of air = 6.5 kg/s, flow coefficient = 0.3, rotor tip diameter = 42 cm, mean diameter at rotor exit = 21 cm, speed = 18000 rpm.

Enthalpy losses in the nozzle and the rest of the stage are equal. Assuming negligible velocities at the nozzle entry and diffuser exit, determine (a) the nozzle exit air angle, (b) the rotor width at entry, (c) the power developed, (d) the stage efficiency, (e) the rotor blade height at the exit, (f) Mach numbers at nozzle and rotor exits and (g) the nozzle and rotor loss coefficients.

Solution

$$P_{00}/P_3 = 4 \qquad T_{00} = 557 \text{ K} \qquad P_3 = 1 \text{ bar}$$
$$m = 6.5 \text{ kg/s} \qquad \phi_1 = \frac{Cr_1}{U_1} = 0.3 \qquad N = 18,000 \text{ rpm}$$
$$D_t = 0.42 \text{ m} \qquad D_{2,m} = 0.21 \text{ m}$$

(a) The nozzle exit air angle

$$U_1 = \frac{\pi D_t N}{60} = \frac{\pi \times 0.42 \times 18,000}{60}$$
$$= 395.84 \text{ m/s}$$
$$C_{r_1} = \phi_1 U_1 = 0.3 \times 395.84 = 118.75 \text{ m/s}$$

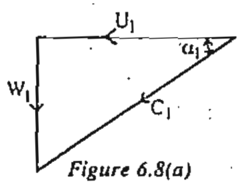


Figure 6.8(a)

From the inlet velocity triangle

$$\tan\alpha_1 = \frac{W_1}{U_1} = \frac{C_{r_1}}{U_1}$$
$$\alpha_1 = \tan^{-1}\left(\frac{118.75}{395.84}\right)$$
$$\alpha_1 = 16.69°$$

(b) Power developed

$$W = mU_1^2 = 6.5 \times (395.84)^2$$
$$W = 1018.5 \text{ kW}$$

(c) Stage efficiency

$$\eta_s = \frac{h_{00} - h_{03}}{h_{00} - h_{3ss}}$$

Since, $h_{03} = h_{02}$

$$\eta_s = \frac{h_{00} - h_{02}}{h_{00} - h_{3ss}}$$

From isentropic relation

$$\frac{T_{3ss}}{T_{00}} = \left(\frac{P_3}{P_{00}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{4}\right)^{\frac{0.4}{1.4}}$$
$$\therefore T_{3ss} = 0.673 \times 557 = 374.7 \text{ K}$$

and

$$W/m = h_{00} - h_{02} = \frac{1018.5}{6.5}$$
$$= 156.69 \text{ kJ/kg}$$

Therefore,

$$\eta_s = \frac{156.69}{C_p(T_{00} - T_{3ss})}$$
$$= \frac{156.69}{1.005(557 - 374.7)}$$
$$= 0.8552$$
$$\eta_s = 85.52\%$$

(d) Rotor width at the entry

Total enthalpy loss

$h_{03} - h_{3ss} = C_p(T_{03} - T_{3ss})$

$$T_{03} = T_{02} = T_{01} - \frac{W}{mC_p}$$
$$= 557 - \frac{1018.5}{6.5 \times 1.005}$$
$$= 401.1 \text{ K}$$

Then,

$$C_p(T_{02} - T_{3ss}) = 1.005(401.1 - 374.7)$$
$$= 26.53 \text{ kJ/kg}$$

Given enthalpy losses in the nozzle and the rest of the stage are considered equal, enthalpy loss in the nozzle is given by

$$h_1 - h_{1s} = \frac{26.53}{2} = 13.265 \text{ kJ/kg}$$

we can write,

$$h_{00} - h_{1s} = h_{01} - h_{1s} = (h_{01} - h_1) + (h_1 - h_{1s})$$

$$= \frac{C_1^2}{2C_p} + 13.265$$

$$C_1 = \frac{W_1}{\sin\alpha_1} = \frac{118.75}{\sin 16.69°} = 413.48 \text{ m/s}$$

Then,

$$= \frac{413.48^2}{2000} + 13.265$$

$$h_{01} - h_{1s} = 98.75 \text{ kJ/kg}$$

$$T_{01} - T_{1s} = \frac{98.75}{1.005} = 98.26 \text{ K}$$

Since,

$$T_{00} = T_{01}$$

$$T_{1s} = T_{01} - 98.26 = 557 - 98.26$$

$$= 458.74 \text{ K}$$

Now,

$$\frac{P_1}{P_{00}} = \left(\frac{T_{1s}}{T_{00}}\right)^{\frac{r}{r-1}}$$

$$\therefore P_1 = 4\left(\frac{458.74}{557}\right)^{\frac{1.4}{0.4}}$$

$$= 2.03 \text{ bar}$$

and

$$T_1 = T_{01} - \frac{C_1^2}{2C_p}$$

$$= 557 - \frac{413.48^2}{2 \times 1005}$$

$$= 471.94 \text{ K}$$

Then,

$$\rho_1 = \frac{P_1}{RT_1} = \frac{2.03 \times 10^5}{287 \times 471.94}$$

$$= 1.499 \text{ kg/m}^3$$

The width of the rotor at inlet

$$b_1 = \frac{m}{\rho_1 W_1 (\pi D_t)}$$

$$= \frac{6.5}{1.499 \times 118.75 \times \pi \times 0.42}$$

$$= 0.02767 \text{ m}$$

$$b_1 = 2.767 \text{ cm}$$

(e) The rotor blade height at the exit

$$h_{02} = h_2 + \frac{C_2^2}{2000}$$

or

$$T_{02} = T_2 + \frac{C_2^2}{2000 \times C_p}$$

$$C_2 = C_{r_2} = C_{r_1} = 118.75 \text{ m/s}$$

$$T_{02} = 401.1 \text{ K}$$

$$\therefore T_2 = 401.1 - \frac{118.75^2}{2 \times 1005}$$

$$= 394.08 \text{ K}$$

$$\frac{P_2}{P_{03}} = \left(\frac{T_2}{T_{03}}\right)^{r/r-1} = \left(\frac{394.08}{401.1}\right)^{\frac{1.4}{0.4}}$$

$$= 0.94$$

Since $P_{03} = P_3$ (negligible diffuser exit velocity), $P_{03} = 1$ bar

$$\therefore P_2 = 0.94 \times 1 = 0.94 \text{ bar}$$

The flow area at rotor exit is given by

$$A_2 = \frac{m}{\rho_2 C_2} = \frac{6.5}{\rho_2 C_2}$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{0.94 \times 10^5}{287 \times 394.08}$$

$$= 0.831 \text{ kg/m}^3$$

$$\therefore A_2 = \frac{6.5}{0.831 \times 118.75} = 0.0658 \text{ m}^2$$

$$h_2 = \frac{A_1}{\pi D_{2,m}} = \frac{0.0658}{\pi \times 0.21}$$

$$= 0.0997 \text{ m}$$

$$h_2 = 9.97 \text{ cm}$$

(f) Mach numbers at nozzle and rotor exit

$$M_1 = \frac{C_1}{a_1} = \frac{C_1}{\sqrt{rRT_1}} = \frac{413.48}{\sqrt{1.4 \times 287 \times 471.94}}$$

$$M_1 = 0.9495$$

and

$$M_{2,rel} = \frac{W_2}{a_2} = \frac{\sqrt{C_2^2 + U_2^2}}{\sqrt{rRT_2}}$$

$$U_2 = \frac{\pi D_{2m} . N}{60} = \frac{\pi \times 0.21 \times 18,000}{60}$$

$$= 197.92 \text{ m/s}$$

$$\therefore M_{2,rel} = \frac{\sqrt{118.75^2 + 197.92^2}}{\sqrt{1.4 \times 287 \times 394.08}}$$

$$= \frac{230.8}{397.92}$$

$$M_{2,rel} = 0.58$$

g) The nozzle and rotor loss coefficients

$$L_N = \frac{h_1 - h_{1s}}{C_1^2/2} = \frac{13.265 \times 10^3}{413.48^2/2}$$

$$L_N = 0.1552$$

and

$$L_R = \frac{h_2 - h_{2s}}{W_2^2/2} = \frac{13.265 \times 10^3}{230.8^2/2}$$

$$L_R = 0.498$$

Example 6.3 An IFR gas turbine operates with a total-to-total efficiency of 0.9. At entry to the nozzles, the pressure and temperature of the gas are 300 kPa and 1150 K respectively. At the outlet of the nozzle the static temperature of gas is 740°C and at the outlet of the diffuser the pressure is 100 kPa. The gas has negligible velocity at the diffuser exit. Find the impeller tip speed, flow angle at the nozzle outlet and Mach number at nozzle exit. Assume that the gas enters the impeller radially and there is no whirl at the exit. Take $C_p = 1147$ J/kg – K, $r = 1.33$ and R = 284.55/kg – K.

Solution

$\eta_{tt} = 0.9$ $\quad P_{00} = 300$ kPa $\quad T_{00} = 1150$ K

$T_1 = 740°C = 1013$ K $\quad P_{03} = 100$ kPa

(a) The impeller tip speed

$$\left[\eta_{tt} = \frac{U_1^2}{C_p T_{00} \left[1 - \left(\frac{P_{03}}{P_{00}} \right)^{r-1/r} \right]} \right]$$

$$\therefore U_1^2 = 0.9 \left[1147 \times 1150 \times \left(1 - \left(\frac{100}{300} \right)^{\frac{0.33}{1.33}} \right) \right]$$

$$U_1 = 532.2 \text{ m/s}$$

(b) Flow angle at nozzle outlet

Since $T_{00} = T_{01}$ and

$$T_{01} = T_1 + \frac{C_1^2}{2C_p}$$

$$\therefore C_1 = \sqrt{2000 \times C_p \times (T_{01} - T_1)}$$

$$= \sqrt{2000 \times 1.147 \times (1150 - 1013)}$$

$$= 560.61 \text{ m/s}$$

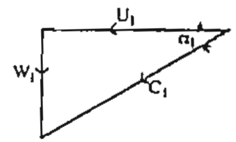


Figure 6.8(b)

From the inlet velocity triangle, Fig. 6.8(b)

$$\cos\alpha_1 = \frac{U_1}{C_1}$$

$$\text{or} \quad \alpha_1 = \cos^{-1}\left(\frac{532.2}{560.61}\right)$$

$$\alpha_1 = 18.31°$$

(c) Mach number at nozzle exit

$$M_1 = \frac{C_1}{a_1} = \frac{C_1}{\sqrt{rRT_1}}$$

$$= \frac{560.61}{\sqrt{1.33 \times 284.5 \times 1013}}$$

$$M_1 = 0.91$$

Example 6.4 A small *IFR* turbine run by exhaust gas has the following design data. Rotor inlet tip diameter = 9 cm, rotor outlet tip diameter = 6.2 cm, rotor outlet hub diameter = 2.5 cm.

$C_2/C_s = 0.447$ and $U_1/C_s = 0.707$. Blade speed = 30,000 rpm. Determine (a) volume flow rate at impeller outlet (b) the power developed and take density of exhaust gas at impellers exit as 1.8 kg/m^3

Solution

$$D_1 = 0.09 \text{ m} \quad D_{2,t} = 0.062 \text{ m}$$
$$D_{2,h} = 0.025 \text{ m} \quad N = 30,000 \text{ rpm} \quad \rho_1 = 1.8 \text{ kg/m}^3$$

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.09 \times 30,000}{60}$$
$$U_l = 141.37 \text{ m/s}$$
$$\therefore C_s = 141.37/0.707$$
$$= 199.96$$
$$C_2 = 0.447 \times C_s$$
$$= 0.447 \times 199.96$$
$$= 89.38 \text{ m/s}$$

Power developed $= mu_1^2$, where $m = \rho_2 Q_2$

(a) Volume flow rate at impeller exit

$$Q_2 = A_2 C_2$$

Flow area at impeller exit

$$A_2 = \frac{\pi}{4}(D_{2,t}^2 - D_{2,h}^2)$$
$$= \frac{\pi}{4}(0.062^2 - 0.025^2)$$
$$= 2.528 \times 10^{-3} \text{ m}^2$$
$$Q_2 = 2.528 \times 10^{-3} \times 89.38$$
$$Q_2 = 0.226 \text{ m}^3/\text{s}$$

The rate of mass flow is

$$M = \rho_2 Q_2 = 1.8 \times 0.226$$
$$= 0.407 \text{ kg/s}$$

(b) Power developed is given by

$$W = 0.407 \times 141.37^2$$
$$= 8134 \text{ W}$$
$$= 8.134 \text{ kW}$$

Example 6.5 A 90° inward flow radial turbine has the following data. Total-to-static pressure ratio (P_{00}/P_2) = 3.5, exit pressure P_2 = 1 bar, inlet total temperature (T_{00} = 923 K, blade to isentropic speed ratio = 0.66, rotor diameter ratio = 0.45, Speed = 16,000 rpm, nozzle exit angle 20°, nozzle efficiency = 0.95 and rotor width at inlet = 5cm. Assuming constant meridional velocity, determine (a) rotor diameters (b) rotor blade angle at exit (c) mass flow rate (d) power developed (e) hub and tip diameters at exit (f) total-to-static stage efficiency (g) Nozzle enthalpy loss coefficient and (h) rotor enthalpy loss coefficient.

Solution

$$\frac{P_{00}}{P_2} = 3.5 \quad P_2 = 1 \text{ bar} \quad T_{00} = 923 \text{ K} \quad \frac{U_1}{C_s} = 0.66$$
$$\frac{D_2}{D_1} = 0.45 \quad N = 16,000 \text{ rpm} \quad \alpha_1 = 20° \quad \eta_n = 0.95$$
$$b_1 = 0.05 \text{ m} \quad Cr_1 = Cr_2 = Cr$$

(a) Rotor diameter

$$U_1 = 0.66 C_s$$
$$C_s = \sqrt{C_p(T_{00} - T_{2ss})}$$
$$T_{2ss} = T_{00}\left(\frac{P_2}{P_{00}}\right)^{\frac{\gamma-1}{\gamma}} = 923\left(\frac{1}{3.5}\right)^{\frac{0.4}{1.4}}$$
$$= 645.28 \text{ K}$$
$$\therefore C_s = \sqrt{2 \times 1005 \times (923 - 645.28)}$$
$$= 747.14 \text{ m/s}$$
$$\therefore U_1 = 0.66 \times 747.14 = 493.11 \text{ m/s}$$

Rotor inlet diameter

$$D_1 = \frac{U_1 \times 60}{\pi N} = \frac{493.11 \times 60}{\pi \times 16,000}$$
$$D_1 = 0.59 \text{ m}$$

Rotor outlet diameter

$$D_2 = 0.45 \times 0.59$$
$$D_2 = 0.266 \text{ m}$$

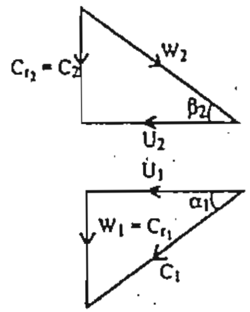


Figure 6.9(a) & (b)

(b) Rotor blade angle at exit

$$\tan\beta_2 = \frac{Cr_2}{U_2}$$
$$Cr_2 = Cr_1 = U_1 \tan\alpha_1$$
$$= 493.11 \times \tan 20°$$
$$Cr_2 = 179.48 \text{m/s}$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.266 \times 16,000}{60} = 222.84 \text{ m/s}$$

$$\tan\beta_2 = \frac{179.48}{222.84}$$
$$\beta_2 = 38.85°$$

(c) Mass flow rate for a 90° IFR turbine, R = 0.5

$$R = \frac{h_1 - h_2}{h_0 - h_2} = 0.5$$

$$C_p(T_1 - T_2) = 0.5C_p(T_0 - T_2)$$

but

$$C_p(T_0 - T_2) = W/m = U_1^2$$

$$\therefore T_2 = T_1 - \frac{0.5U_1^2}{C_p}$$

and

$$W/m = C_p(T_{00} - T_{02}) = U_1^2$$

$$\therefore T_{02} = T - 01 - \frac{U_1^2}{C_p} = 923 - \frac{493.11^2}{1005}$$

$$= 681.05 \text{ K}$$

$$\therefore T_2 = T_{02} - \frac{C_2^2}{2C_p} = 681.05 - \frac{179.48^2}{2 \times 1005}$$

$$= 665.02 \text{ K}$$

Now,

$$T_1 = T_2 + 0.5\frac{U_1^2}{C_p} = 665.02 + \frac{0.5 \times 493.11^2}{1005}$$

$$T_1 = 785.99 \text{ K}$$

Nozzle efficiency is given by

$$\eta_N = \frac{T_{00} - T_1}{T_{00} - T_{1s}} = 0.95$$

$$\therefore T_{1s} = T_{00} - \left(\frac{T_{00} - T_1}{0.95}\right)$$

$$= 923 - \left(\frac{923 - 785.99}{0.95}\right)$$

$$= 778.78 \text{ K}$$

From isentropic relation,

$$\frac{T_{00}}{T_{1s}} = \left(\frac{P_{00}}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$P_1 = P_{00}\left(\frac{T_{1s}}{T_{00}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= 3.5\left(\frac{778.78}{923}\right)^{\frac{1.4}{0.4}}$$

$$= 1.931 \text{ bar}$$

$$\therefore \rho_1 = \frac{P_1}{RT_1} = \frac{1.931 \times 10^5}{287 \times 785.99} = 0.856 \text{ kg/m}^3$$

Mass flow rate

$$\dot{m} = \rho_1 A_1 Cr_1$$

$$= 0.856 \times (\pi \times 0.59 \times 0.05) \times 179.48$$

$$\dot{m} = 14.24 \text{ kg/s}$$

(d) Power developed

$$W = \dot{m}U_1^2 = 14.24 \times (493.11^2)$$

$$W = 3.463 \text{ mW}$$

(e) Hub and tip diameters at exit

Rotor width at exit

$$b_2 = \frac{\dot{m}}{\rho_2(\pi D_2)Cr_2}$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{1 \times 10^5}{287 \times 665.02} = 0.524 \text{ kg/m}^3$$

$$\therefore b_2 = \frac{14.24}{0.524 \times \pi \times 0.266 \times 179.48}$$

$$= 0.1812 \text{ m}$$

Now, hub diameter at exit

$$D_{2,h} = D_2 - b_2 = 0.266 - 0.1812$$

$$= 0.0848 \text{ m}$$

and tip diameter at exit

$$D_{2,t} = D_2 + b_2 = 0.266 + 0.1812$$

$$= 0.4472 \text{ m}$$

(f) Total-to-static efficiency

$$\eta_{t-s} = \frac{W}{mC_p(T_{00} - T_{2ss})} = \frac{3.463 \times 10^6}{14.24 \times 1005(923 - 645.28)}$$

$$= 87.13\%$$

(g) Nozzle enthalpy loss coefficient

$$L_N = \frac{h_1 - h_{1s}}{C_1^2/2}$$

$$C_1 = U_1/\cos\alpha_1 = 493.11/\cos 20^\circ$$

$$= 524.76 \text{ m/s}$$

$$L_N = \frac{C_p(T_1 - T_{1s})}{C_1^2/2} = \frac{1005(785.99 - 778.78)}{524.76^2/2}$$

$$L_N = 0.0526$$

(h) Rotor enthalpy loss coefficient

$$L_R = \frac{h_2 - h_{2s}}{W_2^2/2}$$

$$W_2^2 = U_2^2 + Cr_2 = 179.48^2 + 222.84^2$$

$$\therefore W_2 = 286.13 \text{ m/s}$$

$$L_R = \frac{C_p(T_2 - T_{2s})}{W_2^2/2}$$

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}} = 785.99\left(\frac{1}{1.931}\right)^{\frac{0.4}{1.4}}$$

$$= 651.28 \text{ K}$$

$$\therefore L_R = \frac{1005(665.02 - 651.28)}{286.13^2/2}$$

$$L_R = 0.3373$$

Example 6.6 For a radial turbine stage the pressure and temperature through the stage at design conditions are: Total pressure and temperature upstream of nozzle are respectively 700 kPa and 1145 K, Static Pressure and temperature downstream of nozzle are respectively 527 kPa and 1029 K. At rotor exit, static pressure is 385 kPa, static temperature is 915 K and stagnation temperature is 925 K. The ratio of rotor exit mean diameter to rotor inlet diameter is 0.49 and speed is 24,000 rpm. Assuming relative flow at rotor inlet is radial and absolute flow at rotor exit axial determine (a) total-to-static efficiency (b) rotor diameters and (c) enthalpy loss coefficient for nozzle and rotor. Take $r = 1.67$ and molecular weight of gas as 39.94 (d) total-to-total efficiency.

Solution

$P_{00} = 700$ kPa	$T_{00} = 1145$ K	$P_1 = 527$ kPa
$T_1 = 1029$ K	$P_2 = 385$ kPa	$T_2 = 915$ K
$T_{02} = 925$ K	$D_{2,av}/D_1 = 0.49$	$N = 24,000$ rpm
$W_2 = C_{r_1}$	$C_2 = C_{r_2}$	$r = 1.67$

$$R = \frac{\tilde{R}}{m} = \frac{8.314}{39.94} = 0.208 \text{ kJ/kg-K}$$

$$C_p = \frac{rR}{r-1} = \frac{1.67 \times 0.208}{0.67} = 0.518 \text{ kJ/kg-K}$$

(a) Total-to-static efficiency

Referring to the Mollier Chart.

$$\eta_{t-s} = \frac{T_{00} - T_{02}}{T_{00} - T_{2ss}}$$

$$T_{2ss} = T_{00}\left(\frac{P_2}{P_{00}}\right)^{\frac{r-1}{r}}$$

$$= 1145\left(\frac{385}{700}\right)^{\frac{0.67}{1.67}}$$

$$= 900.8 \text{ K}$$

$$\therefore \eta_{t-s} = \frac{1145 - 925}{1145 - 900.8}$$

$$\eta_{t-s} = 90.1\%$$

(b) Rotor diameters

$$W/m = C_p(T_{00} - T_{02}) = U_1^2$$

$$\therefore U_1 = \sqrt{518 \times (1145 - 925)} = 337.58 \text{ m/s}$$

$$D_1 = \frac{U_1 \times 60}{\pi N} = \frac{337.58 \times 60}{\pi \times 24,000}$$

$$D_1 = 0.269 \text{ m}$$

$$D_{2,avg} = 0.49 \times 0.269$$

$$= 0.132 \text{ m}$$

(c) Enthalpy loss coefficients

For nozzle

$$L_N = \frac{C_p(T_1 - T_{1s})}{C_1^2/2}$$

$$C_1 = \sqrt{2C_p(T_{01} - T_1)}$$

$$= \sqrt{2 \times 518(1145 - 1029)}$$

$$= 346.66 \text{ m/s}$$

$$T_{1s} = T_{00}\left(\frac{P_1}{P_{00}}\right)^{\frac{r-1}{r}}$$

$$= 1145\left(\frac{527}{700}\right)^{\frac{0.67}{1.67}}$$

$$= 1021.75 \text{ K}$$

$$\therefore L_N = \frac{518(1029 - 1021.75)}{346.66^2/2}$$

$$L_N = 0.0625$$

For rotor

$$L_R = \frac{C_p(T_2 - T_{2s})}{W_1^2/2}$$

$$C_2 = \sqrt{2C_p(T_{02} - T_2)}$$

$$= \sqrt{2 \times 518 \times (925 - 915)} = 101.78 \text{ m/s}$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.132 \times 24,000}{60} = 165.88 \text{ m/s}$$

$$W_2 = \sqrt{C_2^2 + U_2^2} = \sqrt{101.78^2 + 165.88^2}$$

$$= 194.62 \text{ m/s}$$

$$T_{2s} = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 1029 \left(\frac{385}{527}\right)^{\frac{0.67}{1.67}}$$

$$= 907.22 \text{ K}$$

$$\therefore L_R = \frac{518(915 - 907.22)}{194.62^2/2}$$

$$L_R = 0.2128$$

(d) Total-to-total efficiency

$$\frac{1}{\eta_{t-t}} = \frac{1}{\eta_{t-s}} - \frac{C_2^2}{2U_1^2}$$

$$= \frac{1}{0.901} - \frac{101.78^2}{2 \times 337.58^2}$$

$$\eta_{t-t} = 93.95\%$$

SHORT QUESTIONS

6.1. What is a radial flow gas turbine?
6.2. What are the applications of radial flow gas turbines?
6.3. The flow rate in a radial flow gas turbine is very low. (True/False)
6.4. Can a 90° IFR turbine be used as a centrifugal compressor? How?
6.5. How are the load changes effected in radial flow turbines?
6.6. The rotor blades of a radial flow gas turbine are curved to change the flow from the *radial* to the *axial* direction. (True/False)
6.7. The workdone per unit mass flow in a 90° IFR turbine is equal to square of the blade velocity. (True/False)
6.8. The rotor constant of an IFR is equal to
(a) h_0, rel
(b) h_0, rel - $U^2/2$
(c) h_0
6.9. Draw the velocity diagrams for a 90° IFR turbine?

6.10. Draw the Mollier chart for expansion in a 90° IFR turbine?
6.11. Define degree of reaction
6.12. The degree of reaction of a radial flow gas turbine with radial vanes is
(a) 0.5
(b) 0.25
(c) 0.1
6.13. The degree of reaction increases with the *increasing* values of the inlet blade angle. (True/False)
6.14. For a given value of the flow coefficient, the forward curved vanes give *higher* degree of reaction compared to the backward curved vanes. (True/False)
6.15. As the flow coefficient increases, the degree of reaction of the backward curved vanes and the forward curved vanes,
(a) Increases and decreases respectively
(b) Decreases and Increases respectively
(c) remains constant
6.16. Ljungstrom turbine is a ———— turbine.
6.17. The optimum ratio of peripheral velocity of blade to the relative velocity at exit for maximum work is
(a) $\sigma_{r,opt} = 2\beta$
(b) $\sigma_{r,opt} = \sin\beta$
(c) $\sigma_{r,opt} = \cos\beta/2$
6.18. The maximum work output from a Ljungstrom outward flow reaction turbine is twice the square of the exit blade velocity. (True/False)
6.19. An outward flow radial turbines behaves as a ———— stage of the axial turbine.

EXERCISES

6.1. Draw the sketch of a 90° IFR turbine stage showing its main components.
6.2. What are the applications of IFR turbine?
6.3. Draw the entry and exit velocity triangles for a 90° IFR turbine
6.4. Prove that for a 90° IFR turbine
(a) $W/m = U_1^2$
(b) $\psi_l = 1$
6.5. Draw an enthalpy - entropy diagram for flow through an inward-flow radial turbine stage fitted with an exhaust diffuser.
6.6. Prove that
$h_{02,rel} - 1/2U_2^2 = h_{01,rel} - 1/2U_1^2$
6.7. (a) How is the degree of reaction of an IFR turbine stage defined?
(b) Prove that

$$R = 1 - \frac{Cx_1}{2U_1}$$

$$R = \frac{1}{2}(1 - \phi_1 \cot\beta_1)$$

$$R = 1 - \frac{\psi_1}{2}$$

6.8. (a) Define spouting velocity
(b) Prove that $\sigma_{sopt} = 0.707$

6.9. Show the sketch and describe the working principle of a double rotation outward flow radial steam turbine stage.

6.10. Draw the entry and exit velocity triangles for a Ljungstrom turbine.

6.11. Derive the following relations.

(a) $\sigma_{ropt} = \frac{\cos \beta}{2}$

(b) $W_{max/m} = 2U_2^2$

6.12. The design data of an inward flow exhaust gas turbine are as follows:
Stagnation pressure and temperature at nozzle inlet = 700 kPa and 1075 K.
Static pressure and temperature at exit from nozzle = 510 kPa and 995 K.
Static Pressure and temperature at rotor exit = 350 kPa and 918 K.
Stagnation temperature at rotor exit = 920 K.
Speed = 26,000 rpm.
Mean rotor exit radius to rotor tip radius = 0.5.
The flow into the rotor is purely radial and at exit the flow is axial. Calculate. (a) total-to-total efficiency (b) outer diameter of the rotor (c) the nozzle and rotor loss coefficients and (d) The blade outlet angle at the mean diameter (measured from the radial direction).

[Ans: (a) 80 % (b) 0.29 m (c) 0.1625 and 1.15 (d) 72.2°]

6.13. A 90° IFR turbine has the following data: Rotor diameter ratio (D_t/D_h) = 0.45, rotor speed = 16,000 rpm, nozzle exit air angle = 20°, nozzle efficiency = 0.95, rotor width at entry = 5 cm, blade to spouting velocity ratio = 0.66, total-to-static pressure ratio (P_{00}/P_2) = 3.5, exit pressure = 1 bar, stagnation temperature at entry = 650°C.
Assuming constant radial velocity and axial exit, determine (a) the rotor diameter (b) the rotor blade exit air angle (c) the mass flow rate (d) hub and tip diameter of tip rotor (e) the power developed (f) the total-to-total efficiency (g) nozzle and rotor enthalpy loss coefficients.

[Ans: (a) 59 cm (b) 38.9° (c) 14.2 kg/s (d) 8.4 cm and 44.6 cm (e) 3458 kW (f) 92.5% (g) 0.126 and 0.338]

6.14. An IFR turbine impulse stage with a flow coefficient of 0.4 develops 100 kW. The total-to-total efficiency is 90 % at 12000 rpm. If the flow rate of air is 2.0 kg/s at an entry temperature of 400 K, determine the rotor diameters and air angles at the entry and exit, the nozzle exit and angle and the stagnation pressure ratio across the stage. Assume zero exit swirl and constant radial velocity. Take rotor exit diameter is 0.8 times the rotor inlet diameter.

[Ans: (a) 35.6 cm and 28.5 cm. (b) $\beta_1 = 21.8°$, $\alpha_1 = 11.3°$ and $\beta_2 = 26.6°$. (c) 1.68]

6.15. A small IFR gas turbine, comprising a ring of nozzle blades, a radial vaned impeller and axial diffuser, operates with a total-to-total efficiency of 0.9. At inlet to the stage the stagnation pressure and temperature are 400 kPa and 1140 K respectively. The flow leaving the turbine is diffused to a pressure of 100 kPa and has negligible exit velocity. The nozzle angle at the exit is 16°. Determine the impeller peripheral speed spouting velocity and the Mach number at nozzle exit. *(MU–April '97)*

[Ans: (a) 580.7 m/s (b) 865.7 m/s (c) 0.973]

6.16. The design data of a Ljungstrom turbine are speed = 3600 rpm, inner diameter of the blade ring = 12 cm, blade width = 1cm, blade exit angle −20°, flow rate = 10 kg/s. Determine the power developed and the enthalpy drop in the blade ring for ideal flow and optimum conditions.

[Ans: (a) 12.08 kW and (b) 1.2 kJ/kg]

6.17. Determine the power developed by a 90° IFR turbine which has the following data. Impeller diameter at entry = 40 cm, impeller diameter (mean) at exit = 20 cm, mass flow rate = 5 kg/s, rpm = 18,000, isentropic efficiency = 85%, flow coefficient at entry = 0.3, static pressure ratio across the stage = 4, pressure at the impeller exit = 1 bar, temperature at the entry of the stage = 600°C. Assume half the static pressure ratio to occur in the nozzles and the volute. The discharge is axial. What is the nozzle angle and width of the impeller at entry?

[Ans: (a) 710.65 kW (b) 16.7° and (c) 3.7 cm]

7

DIMENSIONAL AND MODEL ANALYSIS

INTRODUCTION

Dimensional analysis is a mathematical technique used in research work for design and model testing. It deals with the dimensions of the physical quantities involved in the phenomenon. A ***dimension*** is the measure by which a physical variable is expressed quantitatively. For example, Length is a dimension associated with variables such as distance, displacement, width, height and deflection. A ***unit*** is a particular way of attaching a number to the quantitative dimension. Ex: Meter and centimeters are both numerical units for expressing length.

Dimensional analysis is a method for reducing the large number of variables involved in describing the performance characteristics of a turbomachine to a number of manageable dimensionless groups. For example if a characteristic depends upon 'x' dimensional variables, dimensional analysis will reduce the problem to only 'y' dimensionless variables. Generally $(x-y)$ equals the number of different fundamental dimensions which govern the problem.

Fundamental Dimensions

The three basic or primary or fundamental dimensions are length (L), time (T), and mass, (M). In compressible fluids, one more dimension, apart from the independent quantities M, L & T namely the temperature θ is also considered as a fundamental dimension. It is referred in short as the $MTL\theta$ system, when these quantities are used as the fundamental dimensions. There is no direct relationship between these dimensions.

Derived Dimensions

The dimensions which possess more than one fundamental dimension are called *derived* (or) *secondary dimensions*. For instance, velocity is denoted by two basic dimensions i.e. distance per unit time (L/T), density is denoted by mass per unit volume (M/L^3) and acceleration by distance per second square (L/T^2). The expressions (L/T), (M/L^3) and (L/T^2) are called dimensions of velocity, density and acceleration respectively. They are called secondary or derived dimensions.

The dimensions of mostly used physical quantities in turbo machines are given in Table 7.1

Table 7.1 *Physical Quantities used in turbo machines*

	Physical Quantity	*Symbol*	*Dimension*
1.	Area	A	L^2
2.	Volume	V	L^3
3.	Angular velocity	ω	T^{-1}
4.	Discharge	Q	L^3T^{-1}
5.	Acceleration due to Gravity	g	LT^{-2}
6.	Kinematic viscosity	$\nu = \frac{\mu}{\rho}$	L^2T^{-1}
7.	Force	F	MLT^{-2}
8.	Weight	W	MLT^{-2}
9.	Specific weight	w	$ML^{-2}T^{-2}$
10.	Dynamic viscosity	μ	$ML^{-1}T^{-1}$
11.	Work, Energy	W, E	ML^2T^{-2}
12.	Power	P	ML^2T^{-3}
13.	Torque	T	ML^2T^{-2}
14.	Momentum	M	MLT^{-1}
15.	Angle	θ	None
16.	Temperature	T	θ
17.	Specific heat	C_p, C_v	$L^2T^{-2}\theta^{-1}$
18.	Mass Flow rate	$\dot{m}$	MT^{-1}
19.	Efficiency	η	None

ADVANTAGES OF DIMENSIONAL ANALYSIS

The important advantages of dimensional analysis besides its main purpose to reduce variables and group them in dimensionless form, are

1. An enormous saving in time and money.

Suppose, force F on a particular body immersed in a stream of fluid moving with velocity C is a function of body length L, the fluid density ρ, the stream velocity C, and the fluid viscosity μ, it is expressed as

$$\boxed{F = f(L, C, \rho, \mu)} \tag{7.1}$$

where 'f' means 'a function of' and is to be determined experimentally. Generally, it takes about 10 experimental points to define a curve. To find the effect of body length in equation 7.1 we shall have to run the experiment for 10 lengths L. For each L we shall need 10 values of C, 10 values of ρ and 10 values of μ making a grand total of 10,000 experiments. At a rate of Rs 200 per experiment, the total expenditure would be in several lakhs. However, with dimensional analysis, we can immediately reduce the equation 7.1 to an equivalent form

$$\frac{F}{\rho C^2 L^2} = g\left(\frac{\rho C L}{\mu}\right) \tag{7.2}$$

or

$$C_f = g(Re) \tag{7.3}$$

that is the dimensionless force coefficient $F/\rho C^2 L^2$ and it is the only function of dimensionless Reynolds number $(\rho CL/\mu)$. We shall learn exactly how to make this reduction in the following sections.

The function 'g' is different mathematically from the original function 'f', but it contains all the same information. Nothing is lost in a dimensional analysis.

The function 'g' can be established by running the experiment for only 10 values of the single variable called the Reynolds number resulting in huge saving of time and money.

2. The prediction of a prototype performance from tests conducted on a scale model.

The dimensional analysis provides 'scaling laws' which can convert data from a cheap, small model into design information for an expensive large prototype. For example, one need not build a turbomachine and see whether it has the maximum desired efficiency. One can measure the efficiency on a small model and use a scaling law to predict the efficiency of a full- scale prototype turbomachine. For example, we don't build a million rupees air plane and see whether it has enough lift force. We measure the lift force on a small model and use a scaling law to predict the lift on the full scale prototype air plane.

3. The determination of the most suitable type of machine on the basis of maximum efficiency for a specified range of head, speed and flow rate.

DIMENSIONAL HOMOGENEITY

Dimensional homogeneity means that the dimensions of each term in an equation on both the sides are equal. Hence, if the dimensions of each term on both sides of an equation are the same, the equation is known as *dimensionally homogeneous equation*. The powers of fundamental dimensions (L, M, T) on both sides of the equation will be identical for a dimensionally homogeneous equation. Such equations are independent of the system of units (metric, English or $S.I$)

Consider the following equation

$$C = \sqrt{2gH}$$

where 'C' is velocity, H is height of any fluid column and g is acceleration due to gravity.

Dimension of $L.H.S$ of the above equation is given by

$$C = LT^{-1}$$

Dimension of R.H.S of the equation is

$$\sqrt{2gH} = \sqrt{(L/T^2).L} = \sqrt{(L^2/T^2)}$$
$$= LT^{-1}$$

Since, Dimension of $L.H.S$ = Dimension of $R.H.S = LT^{-1}$
equation $C = \sqrt{2gH}$ is dimensionally homogeneous and can be used in any system of units.

DIMENSIONAL ANALYSIS METHOD

Buckingham PI Theorem

The Buckingham PI theorem is one of the dimensional analysis methods of reducing a number of dimensional variables into a smaller number of dimensionless groups. The name pi comes from the mathematical notation π, meaning a product of variables.

If there are 'n' variables in a physical phenomenon and if these variables contain 'm' fundamental dimensions (M, L, T), then the variables are arranged into J(= n−m) dimension-less terms. Each term is called a *π-term*. The reduced J equals the maximum number of variables which do not form a pi among themselves and is always less than or equal to the number of dimensions describing the variables.

Typically, there are *six steps* involved,

1. List and count the 'n' variables involved in the problem. Dimensional analysis will fail, if any important variables are missing.
   For example. Let X_1 be the dependent variable and $X_2, X_3, \cdots, X_n$ are the independent variables on which X_1 depends in a physical problem. Then X_1 is a function of $X_2, X_3, \cdots, X_n$ and mathematically it is expressed as

$$X_1 = f(X_2, X_3, \cdots, X_n) \qquad (7.4)$$

   equation (7.4) is a dimensionally homogeneous equation. It contains 'n' variables (including dependent variable) equation (7.4) can also be written as

$$f_1(X_1, X_2, X_3, \cdots, X_n) = 0$$

2. List the dimensions of each variable according to $MLT\theta$. Determine the number of fundamental dimensions (say, there are 'm' fundamental dimensions).
3. Find J, according to Buckingham's π- theorem equation (7.4) can be written in terms of number of dimensionless groups or π terms in which number of π- terms is equal to (n−m) J. Hence, equation (7.4) becomes as

$$f_1(\pi_1, \pi_2, \cdots, \pi_{n-m}) = 0.$$

   (or)

$$\pi_1 = g(\pi_2, \pi_3, \cdots \pi_{n-m}) \qquad (7.5)$$

4. Select J variables which don't form a Pi product. Each π term contains $m+1$ variables, where 'm' is the number of fundamental dimensions and is also called the repeating variable.

For example, in the problem considered, X_2, X_3, and X_4 are repeating variables if the fundamental dimension $m(M, L, T) = 3$. Then each π term is written as

$$\left.\begin{aligned} \pi_1 &= X_2^{a_1} X_3^{b_1} X_4^{c_1} . X_1 \\ \pi_2 &= X_2^{a_2} X_3^{b_2} X_4^{c_2} X_5 \\ &\cdots\cdots\cdots\cdots \\ &\cdots\cdots\cdots\cdots \\ \pi_{n-m} &= X_2^{a_{n-m}} X_3^{b_{n-m}} X_4^{c_{n-m}} X_n \end{aligned}\right\} \qquad (7.6)$$

Method of Selecting Repeating Variables

The number of repeating variables are equal to the number of fundamental dimensions of the problem. The following points govern the choice of repeating variables.

a. As far as possible, the dependent variable should not be selected as repeating variable.
b. The repeating variable should be chosen in such a way that one variable is a geometric variable, other variable is a flow variable and the third variable is a fluid property variable
   Geometric variables– Length(l), Diameter (d), Height (h) etc.
   Flow variables – Velocity (c), Acceleration etc.
   Fluid property variables
   Kinematic viscosity (ϑ), Dynamic viscosity (μ), Density (ρ), etc.
c. The selected variables should not form a dimensionless group.
d. The repeating variables together must have the same number of fundamental dimensions.
e. No two repeating variables should have the same dimensions.

5. Add one additional variable to the selected J variables and form a power product. Each π term (as given in equation (7.6)) is solved by the principle of dimensional homogeneity.
6. Write the final dimensionless function by substituting the values of $\pi_1, \pi_2 \cdots \pi_{n-m}$ in equation (7.5) and check whether all pi groups are dimensionless.

MODEL ANALYSIS

To predict the performance of turbo machines such as turbines, compressors, pumps etc. Model Analysis is employed. That is, before manufacturing the real or actual machines, models of the machines are made and tests are performed on them to get the required information.

The model is the small scale replica of the actual machine. The actual machine is called *prototype*. The study of models of actual machines is called as *model analysis*. The model analysis is actually an experimental analysis of finding out solutions of complex problems using models of the actual system.

The *advantages* of the dimensional and model analysis are

1. The performance of the actual turbomachine can be easily predicted, in advance from its models. The reliability depends on the degree of similarity that exists between the model and the prototype.
2. With the help of dimensional analysis, a relationship between the variables influencing the problem in terms of the dimensionless parameters is obtained. This relationship helps in conducting tests on the model.
3. With the help of model testing, the most economical and safe design may be selected.

SIMILITUDE

Similitude is defined as the similarity between the model and its prototype in every respect. Three types of similarities must exist between the model and prototype. They are:

1. Geometric Similarity
2. Kinematic Similarity
3. Dynamic Similarity

1. Geometric Similarity

A model and prototype are geometrically similar if and only if all body dimensions in all the three coordinates have the same linear-scale ratio.

Let

L_m - Length of model
B_m - Breadth of model
D_m - Diameter of model
V_m - Volume of model
A_m - Area of model

and L_p, B_p, D_p, V_p and A_p are the corresponding values of the prototype. For geometric similarity between the model and prototype, the following relation must exist.

$$\frac{L_p}{L_m} = \frac{B_p}{B_m} = \frac{D_p}{D_m} = G_r$$

where G_r is called the *scale ratio*.

For area ratio and volume ratio, the relation should be as given below.

$$\frac{A_p}{A_m} = \frac{L_p \times B_p}{L_m \times B_m} = G_r.G_r = G_r^2$$

and

$$\frac{V_p}{V_m} = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{B_p}{B_m}\right)^3 = \left(\frac{D_p}{D_m}\right)^3 = G_r^3$$

1. All angles are preserved in geometric similarity.
2. All flow directions are preserved.
3. The orientation of the model and prototype, with respect to the surroundings must be identical.

4. The homologous points (The points which have the same relative location) must be related by the same linear-scale ratio.

Fig. 7.1. shows a prototype and a one-tenth scale model of a blade. The scale ratio is also applicable to the fasteners used.

The linear dimensions of the model are all one-tenth of the prototype blade. But its angle of attack with respect to the free stream is the same 10° not 1°. All physical detalis on the model must be scaled, i.e., the nose radius, the surface roughness etc. Any departure will lead to the violation of geometric similarity. Models which appear similar in shape but violate geometric similarity should not be compared.

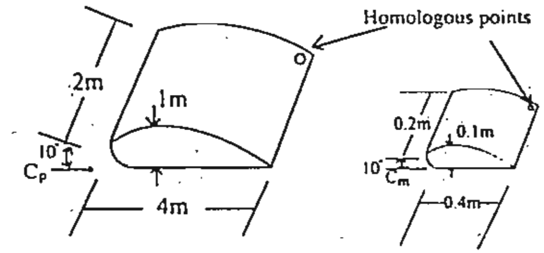


Figure 7.1 Prototype and its model with geometric similarity

2. Kinematic Similarity

A model and prototype are kinematically similar if and only if they have the same velocity scale ratio. i.e the motion of two systems are kinematically similar if homologous particles lie at homologous points and at homologous time (refer Fig. 7.2).

Let

C_{m_1} = Velocity of fluid at point 1 in model.
C_{m_2} = Velocity of fluid at point 2 in model.

and C_{p_1}, C_{p_2} are corresponding values at the corresponding points of fluid velocity in the prototype.

For kinematic similarity, to exist

$$\frac{C_{p_1}}{C_{m_1}} = \frac{C_{p_2}}{C_{m_2}} = C_r$$

Similarly for acceleration

$$\frac{a_{p_1}}{a_{m_1}} = \frac{a_{p_2}}{a_{m_2}} = a_r$$

where C_r is the velocity ratio and a_r is the accelartion ratio.

The directions of the velocity in the model and the prototype should be the same.

3. Dynamic Similarity

A model and prototype are dynamically similar if and only if they have the same force-scale (or mass-scale) ratio. Thus dynamic similarity is said to exist between the model and the prototype, if the ratios of the corresponding forces acting at the corresponding points are equal. Also the directions of the corresponding forces at the corresponding points should be the same (refer Fig. 7.2).

Let

$(F_i)_p$ = Inertia force at a point in the prototype.

$(F_g)_p$ = Gravity force at the point in the prototype.

and $(F_i)_m$, $(F_g)_m$ are corresponding forces at the corresponding points in the model.

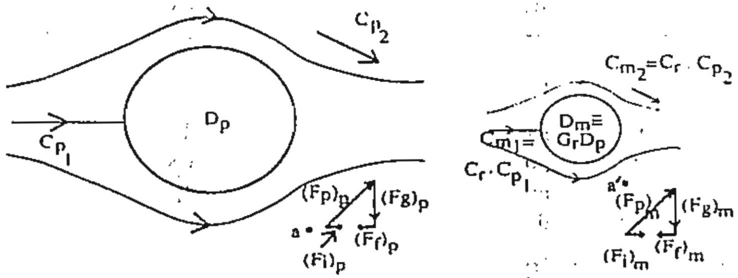


Figure 7.2 Three similarities–prototype and model of a disc

Then, for dynamic similarity, we have

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_g)_p}{(F_g)_m} = F_r$$

where F_r is the *force ratio.*

Mathematically, Newton's law for any fluid particle requires the sum of pressure force (F_p), gravity force (F_g), and friction force (F_f) equal to the acceleration term, or the inertia force (F_i)

i.e.

$$\boxed{F_p + F_g + F_f = F_i}$$

CLASSIFICATION OF HYDRAULIC MODELS

The hydraulic models, are classified, based on the scale ratio for the linear dimensions, as follows:

1) Undistorted Models

If the scale ratio for the linear dimensions of the model and its prototype are same, or if the model is geometrically similar to its prototype, the model is said to be an *undistorted model.*

2) Distorted Models

If the scale ratios for the linear dimensions of the model and its prototype are different or if the model is not geometrically similar to its prototype, then the model is called as *distorted model.* For distorted models two different scale ratios, one for the horizontal dimensions and the other for the vertical dimensions are adopted.

NON-DIMENSIONAL NUMBERS

Non-dimensional numbers are those numbers which are obtained by dividing the inertia force by the viscous force or the gravity force or the pressure force or surface tension force or the elastic force. As this is a ratio of one force to another force, it is a dimensionless number. These non-dimensional numbers are also called as *dimensionless numbers.* The following are the important non-dimensional numbers which are used as a criteria for the dynamic similarity between a model and its prototype.

1. Reynold's Number (Re)

It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid. It is named after Osborne Reynolds, a British engineer who first proposed it in 1883. It is expressed as

$$Re = \frac{\text{Inertia force } (F_i)}{\text{Viscous force } (F_v)} = \frac{\rho A C^2}{\mu\left(\frac{C}{L}\right) A}$$

$$\boxed{Re = \frac{\rho C L}{\mu} \text{ or } \frac{C L}{\vartheta}}$$

where $\vartheta = \mu / \rho$

In case of flow through pipes, L is taken as diameter D.

The law in which the models are based on Reynold's number is called Reynold's model law or similarity law. i.e. $(Re)_{model} = (Re)_{prototype}$.

2. Euler's Number (Eu)

It is defined as the square root of the ratio of the inertia force of a flowing fluid to the pressure force. This is named after Leonhard Euler. Mathematically, it is expressed as

$$Eu = \sqrt{\frac{\text{Inertia Force } (F_i)}{\text{Pressure Force } (F_p)}} = \sqrt{\frac{\rho A C^2}{P A}} = \frac{C}{\sqrt{P/\rho}}$$

The law in which the models are designed on Euler's number is known as Euler's model law i.e $(Eu)_{model} = (Eu)_{prototype}$

3. Froude's Number (F_e)

It is defined as the square root of the ratio of inertia force of a flowing fluid to the gravity force. It is named after William Froude, a British naval architect. It is expressed as

$$Fe = \sqrt{\frac{\text{Inertia Force } (F_i)}{\text{Gravity Force } (F_g)}}$$

$$= \sqrt{\frac{\rho A C^2}{\rho A L g}}$$

$$Fe = \frac{C}{\sqrt{Lg}}$$

The law in which models are based on Froude's number i.e. $(Fe)_{model} = (Fe)_{prototype}$ is known as the *Froude's model law*.

4. Weber's Number (W_e)

It is defined as the square root of the ratio of the inertia force of a flowing fluid to the surface tension force. It is named after Moritz Weber of the Polytechnic Institute of Berlin. It is given as

$$W_e = \sqrt{\frac{\text{Inertia Force } (F_i)}{\text{Surface tension Force } (F_s)}}$$

$$= \sqrt{\frac{\rho A C^2}{\sigma L}}$$

where σ = surface tension per unit length and $A = L^2$

$$W_e = \frac{C}{\sqrt{\sigma/\rho L}}$$

Weber's model law is the law in which models are based on Weber's number. According to this law there is dynamic similarity between the model and its prototype. Hence,

$$(W_e)_{model} = (We)_{prototype}$$

5. Mach's Number (M)

Mach number is defined as the square root of the ratio of the inertia force of a flowing fluid to the elastic force. Mathematically, it is defined as

$$M = \sqrt{\frac{\text{Inertia Force } (F_i)}{\text{Elastic Force } (F_e)}}$$

$$= \sqrt{\frac{\rho A C^2}{K A}}$$

where K = Elastic stress and A = L^2

$$M = \frac{C}{\sqrt{K/\rho}}$$

But $\sqrt{K/\rho}$ = a (velocity of sound in the fluid), $M = \frac{C}{a}$

The Mach number is named after Ernst Mach (1838 – 1916), an Austrian physicist. When the forces due to elastic compression predominates in addition to the inertial force, the dynamic similarity between the model and prototype is obtained by equating the Mach number of model and the prototype, i.e. $(M)_{model} = (M)_{prototype}$

SPECIFIC SPEED

The specific speed is the parameter which does not explicitly contain the diameter of the runner or impeller. It is denoted by the symbol N_s. The specific speed is used in comparing the different types of turbo machines as every type of turbomachine has different specific speed.

1. Pump Specific Speed

The specific speed of a centrifugal pump is defined as the speed at which the pump delivers one cubic metre of liquid per second against a head of one metre. It is expressed as

$$N_s = \frac{N\sqrt{Q}}{(H_m)^{3/4}}$$

where N is the speed in rpm, Q is the discharge in (m^3/sec) and H_m is the manometric head in metres. The dimensionless specific speed is given by

$$\frac{N Q^{1/2}}{(g H_m)^{3/4}}$$

Expression for pump specific speed

The discharge for a centrifugal pump is given by the relation,

$$Q = \pi D B C_r$$

where

D-Diameter of the pump impeller

B-Width of the impeller

We know that $D \alpha B$

$$Q \alpha D^2 C_r$$

The flow velocity, tangential velocity and manometric head (H_m) are related as

$$U \alpha C_r \alpha \sqrt{H_m} \tag{7.7}$$

We also know that

$$U \propto DN \tag{7.8}$$

From the above two equations 7.7 and 7.8 we have

$$D \propto \frac{\sqrt{H_m}}{N}$$

The discharge equation then becomes

$$Q \propto \frac{H_m}{N^2} \cdot (\sqrt{H_m})$$

or

$$Q = (\text{constant}) \frac{H_m^{3/2}}{N^2} \tag{7.9}$$

From the definition of specific speed
$N = N_s$ when $H_m = 1m$ and $Q = 1\ m^3/s$.
Substituting these values in the above equation 7.9, the value of constant of proportionality is determined.

$$= N_s^2$$

The discharge Q is

$$Q = N_s^2 \cdot \frac{H_m^{3/2}}{N^2}$$

or

$$N_s^2 = \frac{N^2 Q}{H_m^{3/2}}$$

The pump specific speed is

$$N_s = \frac{N\sqrt{Q}}{H_m^{3/4}} \tag{7.10}$$

2. Specific Speed of Turbine

The specific speed of a turbine is defined as the speed at which the turbine develops unit power when working under unit head. It is expressed as

$$\boxed{N_s = \frac{N\sqrt{P}}{(H)^{5/4}}}$$

where N is the speed in rpm, H is the head in (m) and P is the power in kilowatts. The dimensionless specific speed (or) the power specific speed is given by

$$N_{sp} = \frac{N P^{1/2}}{\rho^{1/2}(gH)^{5/4}}$$

The table 7.2 shows that each type of turbomachine works in a narrow range of specific speeds. The specific speed is the parameter expressing the variation of all the variables N, Q and H or N, P and H which cause similar flows in turbo machines that are geometrically similar.

Derivation of the turbine specific speed

The power developed by any turbine in terms of overall efficiency is given by

$$\boxed{P = \eta_0(\rho g Q H)}$$

or

$$P \propto (QH)(\text{as } \eta_0,\ \rho \text{ and } g \text{ are constants})$$

The absolute velocity, tangential velocity and head on the turbine are related as

$$C \propto \sqrt{H} \text{ and}$$
$$U \propto C \text{ or}$$
$$U \propto \sqrt{H} \tag{7.11}$$

But the tangential velocity 'U' is given by

$$U = \frac{\pi DN}{60} \text{ or}$$
$$U \propto DN \tag{7.12}$$

From equations (7.11) and (7.12), we have

$$D \propto \frac{\sqrt{H}}{N} \tag{7.13}$$

The discharge through the turbine is given by

$$Q = \text{Area} \times \text{Velocity}$$
$$= A \times C$$

But Area $\propto B \times D$ where B = width since $B \propto D$

$$\therefore A \propto D^2$$

or from equation 7.16

$$A \propto \frac{H}{N^2} \text{ and}$$
$$C \propto \sqrt{H}$$
$$\therefore Q \propto \frac{H^{3/2}}{N^2}$$

Substituting for Q in the power equation, we get

$$P \propto \left(\frac{H^{3/2}}{N^2}\right) \cdot H$$

(or)

$$P \alpha \frac{H^{5/2}}{N^2}$$

(or)

$$P = (\text{constant}) \frac{H^{5/2}}{N^2} \quad (7.14)$$

Constant is called the constant of proportionality.

According to the definition of specific speed.

$N = N_s$ when $P = 1$ kW and $H = 1$ m the above equation 7.14 reduces to

$$\text{constant} = N_s^2$$

$$\therefore P = N_s^2 \cdot \frac{H^{5/2}}{N^2}$$

or

$$N_s^2 = \frac{N^2 \cdot P}{H^{5/2}}$$

The turbine specific speed N_s is

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} \quad (7.15)$$

N is in rpm, P is in kW and H is in metres.

Table 7.2 *Ranges of specific speeds of turbo machines*

S. No.	*Turbo machine*	*Dimensionless specific speed*
1.	Pelton Wheel	0.02 – 0.39
	–Single jet	0.02 – 0.19
	–Twin jet	0.1 – 0.3
	–Four jet	0.14 – 0.39
2.	Francis turbine	0.39 – 2.3
	–Radial (slow speed)	0.39 – 0.65
	–Mixed flow (medium – express)	0.65 – 2.3
3.	Kaplan turbine	2.7 – 5.4
4.	Propeller turbine (axial)	1.6 – 3.6
5.	Centrifugal pumps (slow-high speed)	0.24 – 1.8
6.	Mixed flow pump	1.8 – 4.0
7.	Axial flow pump	3.2 – 5.7
8.	Radial flow compressors	0.4 – 1.4
9.	Axial flow steam and Gas turbines	0.35 – 1.9
10.	Axial compressors, blowers	1.4 – 20

MODEL TESTING OF HYDRAULIC TURBO MACHINES

Before manufacturing the large sized actual machines, their models which are in complete similarity with the actual machines (also called prototypes) are made. Tests will be conducted on the models and the performance of the prototypes will be predicted. The complete similarity between the actual machine and the model will exist if the following conditions are satisfied.

1. Specific speed of the Model = Specific speed of the prototype

$$\boxed{(N_s)_m = (N_s)_p}$$

or

$$\left(\frac{N\sqrt{Q}}{H^{3/4}}\right)_m = \left(\frac{N\sqrt{Q}}{H^{3/4}}\right)_p \quad (7.16)$$

2. Flow coefficient is the same for the model and the prototype

The discharge Q for a hydraulic machine is given by the relation

$$Q = \text{Area} \times \text{Velocity of flow}$$
$$\text{Area} = \pi DB$$

where

D - diameter of the turbo machine impeller

B - width of the impeller

and $B \alpha D$, hence

$$A = \pi D^2$$

Therefore,

$$Q = \pi D^2 C_r \quad (7.17)$$

We know that the tangential velocity is given by

$$U = \frac{\pi DN}{60}$$

or

$$U \alpha DN$$

The tangential velocity (U) and flow velocity are related to the diameter and speed as

$$U \alpha C_r \alpha DN$$

Substituting for C_r in equation 7.17 yields

$$Q = \pi D^2(DN)$$

(or)

$$Q \alpha D^3 N$$

$$\boxed{\frac{Q}{D^3 N} = \text{constant}}$$

and $\frac{Q}{ND^3}$ is known as the flow coefficient and denoted as ϕ

Applying the flow coefficient for the model and the prototype

$$\left(\frac{Q}{ND^3}\right)_m = \left(\frac{Q}{ND^3}\right)_p \tag{7.18}$$

3. Specific power of the model and prototype are same

Power of the hydraulic turbomachine is given by

$$P = \rho g Q H$$

(or)

$$P \alpha \rho Q H$$

where H is the head of the machine. We know

$$Q \alpha D^3 N \text{ and}$$
$$\sqrt{H} \alpha DN \; [\therefore U \alpha C_r \alpha \sqrt{H}]$$
$$P \alpha \rho D^3 N \, (D^2N^2)$$
$$P \alpha \rho D^5 N^3$$

or

$$\frac{P}{\rho N^3 D^5} = \text{constant}$$

$\frac{P}{\rho N^3 D^5}$ is known as the power coefficient or specific power and denoted by $\bar{P}$ for model testing

$$\left(\frac{P}{N^3D^5}\right)_m = \left(\frac{P}{N^3D^5}\right)_p \tag{7.19}$$

where $\rho_m = \rho_p$.

4. Specific head of the model and prototype are same

Tangential velocity (U) is given by

$$U = \frac{\pi DN}{60} \text{ also}$$

$U \alpha \sqrt{H}$ (g is dropped since it is a constant)

$$\sqrt{H} \alpha DN$$

or

$$\frac{H}{D^2N^2} = \text{constant}$$

$\frac{gH}{D^2N^2}$ is called the Head coefficient or specific head and represented by ψ for model testing

$$\left(\frac{\sqrt{H}}{DN}\right)_m = \left(\frac{\sqrt{H}}{DN}\right)_p \tag{7.20}$$

(or)

$$\left(\frac{H}{D^2N^2}\right)_m = \left(\frac{H}{D^2N^2}\right)_p \tag{7.21}$$

UNIT QUANTITIES

The quantities of a hydraulic turbomachine working under a unit head are called the *unit quantities*. The following are the three important unit quantities which must be studied under unit head.

1. Unit Speed

The speed of a turbomachine working under a unit head is termed as unit speed. It is denoted by (N_u).

The expression for unit speed (N_u) is obtained as follows.

Let, N–Speed of a turbomachine under a head H

H–Head under which the turbomachine is working.

U–Tangential velocity

The tangential velocity, absolute velocity of water and head on the turbomachine are related as

$U \alpha C$ where $C \alpha \sqrt{H}$

The tangential velocity (U) is given by

$$U = \frac{\pi DN}{60}$$

For a given turbomachine, the diameter (D) is constant.

$$\therefore U \alpha N \text{ or } N \alpha U$$

(or)

$$N \alpha \sqrt{H}$$

$$\therefore \quad N = K\sqrt{H} \tag{7.22}$$

where K is a constant of proportionality.

If head on the turbomachine becomes unity, the speed becomes unit speed .i.e. when H=1, N = N_u

Substituting these values in equation 7.22, we get $N_u = K$

Substituting the value of K in equation 7.22 and rearranging the equation, we obtain

$$N_u = \frac{N}{\sqrt{H}} \tag{7.23}$$

2. Unit Discharge

The discharge or flow through a turbomachine working under a unit head (i.e 1m) is termed as *unit flow or unit discharge*. It is denoted by the symbol (Q_u).

The expression for unit discharge is given as:

Let, H–Head of water on the turbomachine

Q–Discharge passing through the machine when head in it is 'H'

a–area of flow of water

The discharge passing through a given turbomachine under a head 'H' is given by.

Q= Area of flow × velocity

For a given turbomachine area of flow is constant and velocity is proportional to $\sqrt{H}$

$$\therefore \quad Q \,\alpha\, \text{velocity} \,\alpha \sqrt{H}$$

(or)

$$Q = K\sqrt{H} \tag{7.24}$$

When $H = 1$ m, $Q=Q_u$, the constant (K) value is $K = Q_u$

Substituting the value of K and rearranging equation 7.27, we get

$$Q_u = \frac{Q}{\sqrt{H}} \tag{7.25}$$

3. Unit Power

The power developed by a turbomachine working under a unit head is called the unit power. It is denoted by P_u.

The expression for P_u is obtained as follows.

Let, H–Head of water on the turbomachine

P–Power developed by the turbomachine under the head 'H'

Q–Discharge through turbomachine under the head 'H'

The overall efficiency is given as

$$\eta_0 = \frac{\text{Power developed}}{\text{Power input}}$$

$$= \frac{P}{\rho Q H}$$

$$\therefore P = \eta_0(\rho Q H)$$

(or)

$$P \,\alpha\, (Q \times H)$$

$$\text{Since } Q \,\alpha \sqrt{H}$$

$$P \,\alpha\, H^{3/2}$$

or

$$P = KH^{3/2} \tag{7.26}$$

When $H = 1$ m, $P = P_u \quad \therefore P_u = K$

Substituting the value of K in equation 7.29, we get

$$P_u = \frac{P}{H^{3/2}} \tag{7.27}$$

USE OF UNIT QUANTITIES

The behaviour of a hydraulic turbomachine working under different heads can be easily known from the values of the unit quantities.

Let

H_1, H_2 be the heads on the turbomachine,

N_1, N_2 are the corresponding speeds,

Q_1, Q_2 are the discharge, and

P_1, P_2 are the power developed by the turbomachine

Using the defining equations of unit quantities

$$N_u = \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}} \text{ and}$$

$$P_u = \frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

Hence, if the speed, discharge and power developed by a turbine under a particular head are known, then by using the above three relations, the speed, discharge, and power developed by the same turbomachine under a given head can be obtained.

SOLVED PROBLEMS

Example 7.1 Determine the dimensions of the following quantities in M-L-T system.

1. Force 2. Torque 3. Momentum 4. Power

Solution

1. Force = Mass × Acceleration

$$= M \times \frac{\text{Length}}{\text{Time}^2} = MLT^{-2}$$

2. *Torque* = Force × Distance

$$= MLT^{-2} \times L$$

$$= ML^2T^{-2}$$

3. Momentum = Mass × Velocity

$$= M \times \frac{\text{Length}}{\text{Time}}$$
$$= MLT^{-1}$$

4. Power $= \frac{\text{Workdone}}{\text{sec}}$

$$= \frac{\text{Force} \times \text{Distance}}{\text{Time}}$$
$$= \frac{(MLT^{-2}) \times L}{T}$$
$$= ML^2T^{-3}$$

Example 7.2 The efficiency of a turbomachine depends on density 'ρ', dynamic viscosity 'μ' of the fluid, angular velocity 'ω', diameter 'D' of the rotor and the discharge Q. Express 'η' in terms of the dimensionless parameters.

(MU–April '98)

Step-1

The efficiency of a turbomachine (η) depends on i) ρ ii) μ iii) ω iv) D and v) Q. Hence 'η' is a function of ρ, μ, ω, D, Q. Mathematically,

$$\eta = f(\rho, \mu, \omega, D, Q) \quad (7.28)$$

or it can be written as

$$f_1(\eta, \rho, \mu, \omega, D, Q) = 0 \quad (7.29)$$

Hence, the total number of variables (including dependent variable) n=6.

Step-2

The value of 'm' i.e. number of fundamental dimensions for the problem is obtained by writing dimensions of each variable. Dimensions of each variable are

$$\eta = \text{Dimensionless} \qquad \mu = ML^{-1}T^{-1}$$
$$\rho = ML^{-3}, \qquad Q = L^3T^{-1}$$
$$D = L \text{ and} \qquad \omega = T^{-1}$$

Hence, number of fundamental dimensions,

$$m = 3$$

Step-3

The number of dimensionless π-terms is given by

$$J = n - m = 6 - 3 = 3$$

Thus three π-terms say π_1, π_2 and π_3 are formed. Hence equation (7.29) is written as

$$f_1 \ (\pi_1, \pi_2, \pi_3) = 0 \quad (7.30)$$

Step-4

Each π-term contains $m+1$ variables, where m is equal to three and is also a repeating variable.

Out of six variables η, ρ, μ, ω, D and Q, three variables are to be selected as repeating variables. 'η' is a dependent variable and should not be selected as a repeating variable. Out of the five remaining variables, one variable should have geometric property, the second variable should have flow property and the third one have fluid property. These requirements are fulfilled by selecting D, ω, and ρ as repeating variables. The repeating variables themselves should not form a dimensionless term and should have themselves fundamental dimensions equal to m (i.e. 3 here).

Dimensions of D, ω and ρ are L, T^{-1}, ML^{-3} and hence the three fundamental dimensions exist in D, ω, and ρ and they themselves do not form a dimensionless group.

Step-5

Each π-term is written as

$$\pi_1 = D^{a_1} \cdot \omega^{b_1} \cdot \rho^{c_1} \cdot \eta$$
$$\pi_2 = D^{a_2} \cdot \omega^{b_2} \cdot \rho^{c_2} \cdot \mu$$
$$\pi_3 = D^{a_3} \cdot \omega^{b_3} \cdot \rho^{c_3} \cdot Q$$

Each π-term is then solved by the principle of dimensional homogeneity. For

First π-term

$$\pi_1 = D^{a_1} \cdot \omega^{b_1} \cdot \rho^{c_1} \cdot \eta$$

Substituting dimensions on both sides of π_1

$$M^\circ L^\circ T^\circ = L^{a_1} \cdot (T^{-1})^{b_1} \cdot (ML^{-3})^{c_1} \cdot M^\circ L^\circ T^\circ$$

Equating the powers of M, L, T on both sides.

Power of M,	$0 = c_1 + 0$	$\therefore c_1 = 0$
Powers of L,	$0 = a_1 + 0$	$\therefore a_1 = 0$
Powers of T,	$0 = -b_1 + 0$	$\therefore b_1 = 0$

Substituting the values of a_1, b_1 and c_1 in π_1, we get

$$\pi_1 = D^\circ \omega^\circ \rho^\circ . \eta = \eta$$

If a variable is dimensionless then, it itself is a π-term. Here the variable 'η' is a dimensionless and hence 'η' is a π-term. As it exists in first π-term, '$\pi_1 = \eta$', there is no need of equating the powers, the value can be obtained directly.

Second π-term

$$\pi_2 = D^{a_2} . \omega^{b_2} . \rho^{c_2} . \mu$$

Substituting the dimensions on both sides.

$$M^\circ L^\circ T^\circ = L^{a_2} . (T^{-1})^{b_2} . (ML^{-3})^{c_2} . ML^{-1}T^{-1}$$

Equating the powers of M, L, T on both sides.
Power of M, $0 = c_2 + 1 \quad \therefore c_2 = -1$
Power of L, $0 = a_2 - 3c_2 - 1 \quad \therefore a_2 = 3c_2 + 1 \quad a_2 = -3 + 1 = -2$
Power of T, $0 = -b_2 - 1 \quad \therefore b_2 = -1$
Substituting the values of a_2, b_2 and c_2 in π_2.

$$\pi_2 = D^{-2}.\omega^{-1}.\rho^{-1}.\mu = \frac{\mu}{D^2.\omega.\rho}$$

Third π-term

$$\pi_3 = D^{a_3}.\omega^{b_3}.\rho^{c_3}.Q$$

Substituting the dimensions on both sides.

$$M^\circ L^\circ T^\circ = L^{a_3}.(T^{-1})^{b_3}.(ML^{-3})^{c_3}.L^3T^{-1}$$

Equating the powers on M, L and T on both sides.
Power of M, $0 = c_3 \quad \therefore c_3 = 0$
Power of L, $0 = a_3 - c_3 + 3 \quad \therefore a_3 = 3c_3 - 3,\ a_3 = -3$
Power of T, $0 = -b_3 - 1 \quad \therefore b_3 = -1$
Substituting the values of a_3, b_3 and c_3 in π_3

$$\pi_3 = D^{-3}.\omega^{-1}.\rho^0.Q = \frac{Q}{D^3.\omega}$$

Step-6
Substituting the values of π_1, π_2 and π_3 in equation (7.30)

$$f_1\left(\eta, \frac{\mu}{D^2\omega\rho}, \frac{Q}{D^3\omega}\right) = 0$$

or

$$\eta = g\left[\frac{\mu}{D^2\omega\rho}, \frac{Q}{D^3\omega}\right]$$

Example 7.3 Friction coefficient (pressure drop per unit length) of a pipe depends on Average speed (V), pipe diameter (D), Viscosity (μ), density (ρ) and inside roughness ($\in$). Using Buckingham's π-theorem, express the friction coefficient of pipe as a function of dimensionless quantities. *(MKU–Nov. '98)*

Solution

Step-1
Friction coefficient C_F is a function of V, D, μ, ρ and $\in$.

$$\therefore C_F = f(V, D, \mu, \rho, \in)$$

$$\text{or} \quad f_1(C_F, V, D, \mu, \rho, \in) = 0 \tag{7.31}$$

Hence, total number of variables n = 6

Step-2
The dimensions of each variable

$$C_F = \text{dimensionless} \quad V = LT^{-1} \quad D = L$$
$$\mu = ML^{-1}T^{-1}, \rho = ML^{-3}, \text{ and } \in = L.$$

$\therefore$ Number of fundamental dimensions

$$m = 3$$

Then, number of π-terms = n – m = 6 – 3 = 3.
Now, equation (7.31) can be written as

$$f_1(\pi_1, \pi_2, \pi_3) = 0 \tag{7.32}$$

Step-3
Each π-term contains m + 1 or 3+1 = 4 variables, where m is equal to the number of repeating variable. Choosing D, V, ρ as the repeating variables, the three π-terms are

$$\pi_1 = D^{a_1}V^{b_1}\rho^{c_1}C_F$$
$$\pi_2 = D^{a_2}V^{b_2}\rho^{c_2}\mu$$
$$\pi_3 = D^{a_3}V^{b_3}\rho^{c_3}\in$$

Step-4
Each π-term is solved by the principle of dimensional homogeneity.
First π-term

$$\pi_1 = D^{a_1}V^{b_1}\rho^{c_1}C_F$$

Since the variable C_F is dimensionless, it itself is a π-term. Therefore,

$$\pi_1 = C_F$$

Second π-term

$$\pi_2 = D^{a_2}V^{b_2}\rho^{c_2}\mu$$

Substituting dimensions on either side

$$M^\circ L^\circ T^\circ = L^{a_2}(LT^{-1})^{b_2}(ML^{-3})^{c_2}ML^{-1}T^{-1}$$

Equating the powers of M, L, T on both sides

$$\begin{aligned}\text{Power of } M,\ 0 &= c_2 + 1 \quad \therefore c_2 = -1\\ \text{Power of } L,\ 0 &= a_2 + b_2 - 3c_2 - 1\\ & a_2 + b_2 = -2\\ \text{Power of } T,\ 0 &= -b_2 - 1 \quad \therefore b_2 = -1\\ \text{and} \quad & a_2 = -2 - b_2 = -1\end{aligned}$$

Substituting the values of a_2, b_2 and c_2 in π_2, we get

$$\pi_2 = D^{-1} V^{-1} \rho^{-1} \mu$$

(or)

$$\pi_2 = \frac{\mu}{DV\rho}$$

Third π-term

$$\pi_3 = D^{a_3} V^{b_3} \rho^{c_3} \in$$

(or)

$$M^{\circ} L^{\circ} T^{\circ} = L^{a_3}.(LT^{v})^{b_3}(ML^{-3})^{c_3} L$$

Equating the powers,

$$\begin{aligned}\text{For } M,\ 0 &= c_3 \quad \therefore c_3 = 0\\ \text{For } L,\ 0 &= a_3 + b_3 - 3c_3 + 1\\ & a_3 + b_3 = -1\\ \text{For } T,\ 0 &= -b_3 \quad \therefore b_3 = 0\\ \text{and} \quad & a_3 = -1\end{aligned}$$

Substituting the values of a_3, b_3 and c_3 in π_3

$$\pi_3 = D^{-1}.V^{\circ}.\rho^{\circ} \in$$

(or)

$$\pi_3 = \frac{\in}{D}$$

Step-5

Substituting the values of π_1, π_2, and π_3 in equation (7.32), we get

$$F_1\left(C_F, \frac{\mu}{DV\rho}, \frac{\in}{D}\right) = 0$$

(or)

$$C_F = g\left[\frac{\mu}{DV\rho}, \frac{\in}{D}\right]$$

Note that, $\dfrac{\mu}{DV\rho} = \dfrac{1}{Re}$ and $\dfrac{\in}{D}$ is called *roughness factor*. Then, Firiction coefficient of a pipe is a function of Reynold's number and roughness factor.

Example 7.4 The drag force exerted by a flowing fluid on a solid body depends upon the length of the body, L, Velocity of flow, V, density of fluid ρ and viscosity μ. Find an expression for drag force using Buckingham's theorem.

Solution

Step-1

The drag force F_D depends upon, L, V, ρ and μ. Therefore, D is a function of L, V, ρ and μ. Mathematically,

$$\boxed{F_D = f(L, V, \rho, \mu)}$$

(or)

$$f_1(F_D, L, V, \rho, \mu) = 0 \tag{7.33}$$

Total no. of variables, n = 5.

Step-2

Dimensions of each variable are

$F_D = MLT^{-2}$, $L = M$, $V = LT^{-1}$, $\rho = ML^{-3}$ and $\mu = ML^{-1}T^{-1}$

Hence, number of fundamental dimensions, m = 3.

∴ Number of π-terms = n − m = 5 − 3 = 2.

Step-3

Equation (7.33) can be written as

$$f_1(\pi_1, \pi_2) = 0 \tag{7.34}$$

Step-4

Each π-term contains m + 1 variables, where m is number of repeating variables and is equal to 3. Choosing L, V and ρ as repeating variables, we have

$$\begin{aligned}\pi_1 &= L^{a_1} V^{b_1} \rho^{c_1} F_D\\ \pi_2 &= L^{a_2} V^{b_2} \rho^{c_2} \mu\end{aligned}$$

Step-5

Each π-term is solved by the principle of dimensional homogeneity.

For the first π-term

$$\boxed{\pi_1 = L^{a_1} V^{b_1} \rho^{c_1} F_D}$$

substituting dimensions on both sides of π,

$M^{\circ}L^{\circ}T^{\circ} = L^{a_1}.(LT^{-1})^{b_1}(ML^{-3})^{c_1}.MLT^{-2}$

Equating the powers of M, L, T on both sides,

$$\begin{aligned}\text{Power of } M, 0 &= c_1 + 1 \quad \therefore c_1 = -1\\ \text{Power of } L, 0 &= a_1 + b_1 - 3c_1 + 1 \quad a_1 = -b_1 - 4\end{aligned}$$

$$\text{Power of } T, 0 = -b_1 - 2 \quad \therefore b_1 = -2$$
$$\text{and} \quad a_1 = 2 - 4 = -2$$

Substituting the values of a_1, b_1, and c_1 in π_1, we obtain

$$\pi_1 = L^{-2}V^{-2}\rho^{-1}.F_D$$

(or)

$$\pi_1 = \frac{F_D}{\rho L^2 V^2}$$

For the second π-term

$$\pi_2 = L^{a_2}V^{b_2}\rho^{c_2}\mu$$

$$M^o L^o T^o = L^{a_2}(LT^{-1})^{b_2}(ML^{-3})^{c_2}ML^{-1}T^{-1}$$

Equating the powers of M, L, T on both sides

$$\text{Power of } M, 0 = c_2 + 1 \quad \therefore c_2 = -1$$
$$\text{Power of } L, 0 = a_2 + b_2 - 3c_2 - 1 \quad \therefore a_2 = -b_2 - 2$$
$$\text{Power of } T, 0 = -b_2 - 1 \quad \therefore b_2 = -1 \text{ and } a_2 = -1$$

Substituting the values of a_2, b_2 and c_2 in π_2, we obtain

$$\pi_2 = L^{-1}V^{-1}\rho^{-1}\mu$$

(or)

$$\pi_2 = \frac{\mu}{\rho L V}$$

Step-6

Substituting the values of π_1, and π_2 in equation (7.34), we get

$$f_1\left(\frac{F_D}{\rho L^2 V^2}, \frac{\mu}{\rho L V}\right) = 0$$

(or)

$$\frac{F_D}{\rho L^2 V^2} = g\left(\frac{\mu}{\rho L V}\right)$$

(or)

$$F_D = \rho L^2 V^2 g\left(\frac{\mu}{\rho L V}\right)$$

Example 7.5 One-fifth scale model of a pump was tested in a laboratory at 1000 rpm. The head developed and the power input at the best efficiency point were found to be 8m and 30 kW respectively. If the prototype pump has to work against a head of 25 m, determine its working speed, the power required to drive it and the ratio of the flow rates handled by the two pumps.

Solution

One-fifth scale model means that the ratio of linear dimensions of a model and its prototype is equal to 1/5.

Speed of model N_m = 1000, Head of model H_m = 8 m, Power of model P_m = 30 kW, Head of prototype H_p = 25 m

(i) Speed of prototype (N_p)

Equating the head coefficient for the model and prototype (equation 7.20)

$$\left(\frac{\sqrt{H}}{DN}\right)_m = \left(\frac{\sqrt{H}}{DN}\right)_p$$

$$N_p = \frac{\sqrt{H_p}}{\sqrt{H_m}} \times \frac{D_m}{D_p} \times N_m$$
$$= \sqrt{\frac{25}{8}} \times \frac{1}{5} \times 1000$$
$$\left[\frac{D_m}{D_p} = \frac{1}{5}\right]$$
$$= 353.5 \text{ rpm}$$

(ii) Power developed by the prototype

Equating the power coefficient for the model and the prototype (equation 7.19)

$$\left(\frac{P}{D^5 N^3}\right)_m = \left(\frac{P}{D^5 N^3}\right)_p$$

$$P_p = P_m \times \left(\frac{D_p}{D_m}\right)^5 \times \left(\frac{N_p}{N_m}\right)^3$$
$$= 30 \times 5^3 \times \left(\frac{353.5}{1000}\right)^3$$
$$= 4143 \text{ kW}$$

(iii) Ratio of the flow rates of two pumps (i.e. model and prototype)

Equating the flow coefficient of the model and the prototype (equation 7.18)

$$\left(\frac{Q}{D^3 N}\right)_m = \left(\frac{Q}{D^3 N}\right)_p$$

or

$$\frac{Q_p}{Q_m} = \left(\frac{D_p}{D_m}\right)^3 \left(\frac{N_p}{N_m}\right)$$
$$= 5^3 \times \left(\frac{353.5}{1000}\right)$$
$$= 44.1875$$

Example 7.6 Specifications for an axial flow coolant pump for one loop of a pressurised water nuclear reactor are

Head = 85 m Flow rate = 20, 000 m^3/h Speed = 1490 rpm Diameter = 1200 mm
Water density = 714 kg/m^3 Power = 4 mW (electrical)

The manufacturer plans to build a model. Test conditions limit the available electric power to 500 kW and flow to 0.5 m^3/s of cold water. If the model and prototype efficiencies are assumed equal, find the head, speed and scale ratio of the model.

Solution

Equating the head, power and flow coefficients for the model and prototype

$$\frac{Q_p}{Q_m} = \left(\frac{N_p}{N_m}\right)\left(\frac{D_p}{D_m}\right)^3$$

or

$$\frac{N_p}{N_m} = \left(\frac{20,000}{0.5 \times 3600}\right)\left(\frac{D_m}{D_p}\right)^3 = 11.11\left(\frac{D_m}{D_p}\right)^3$$

Also

$$\frac{P_p}{P_m} = \left(\frac{N_p}{N_m}\right)^3\left(\frac{D_p}{D_m}\right)^5\left(\frac{\rho_p}{\rho_m}\right)$$

Substitute for N_p/N_m, then

$$\frac{4}{0.5} = (11.11)^3\left(\frac{D_m}{D_p}\right)^9\left(\frac{D_p}{D_m}\right)^5\left(\frac{714}{1000}\right)$$

$$= \left(\frac{D_m}{D_p}\right)^4 = \left(\frac{8}{(11.11)^3 \times 0.714}\right)$$

Scale ratio

$$\frac{D_m}{D_p} = 0.3$$

Then

$$N_p/N_m = 11.11 \times (0.3)^3$$

Speed ratio

$$N_m/N_p = 3.3$$

Also

$$\frac{H_m}{H_p} = \left(\frac{N_p}{N_m}\right)^2\left(\frac{D_p}{D_m}\right)^2 = \left(\frac{1}{3.3}\right)^2\left(\frac{1}{0.3}\right)^2$$

Head ratio

$$H_m/H_p = 1.0$$

Example 7.7 A geometrically similar model to scale 1 : 6 of a large centrifugal pump is tested. The prototype parameters are speed = 400 rpm, discharge = 1.7 m^3/s, head developed = 36.5 m and the power input 720 kW. If the model is tested under a head of 9 m, determine the speed, discharge at which it should be run and the power required to drive the pump model.

Solution

$$\frac{D_m}{D_p} = \frac{1}{6}, \quad N_p = 400 \text{ rpm}, \quad Q_p = 1.7 \text{ m}^3/\text{s}, \quad H_p = 36.5 \text{ m}$$
$$P_p = 720 \text{ kW}, \quad H_m = 9 \text{ m}$$

(a) Speed of the model Equating the head coefficient for the model and prototype

$$\left(\frac{\sqrt{H}}{DN}\right)_m = \left(\frac{\sqrt{H}}{DN}\right)_p$$

$$N_m = \frac{\sqrt{H_m}}{\sqrt{H_p}} \times \frac{D_p}{D_m} \times N_p = \sqrt{\frac{9}{36.5}} \times 6 \times 400$$

$$N_m = 1191.75 \text{ rpm}$$

(b) Discharge of the model Equating the flow coefficients of the model and prototype

$$\left(\frac{Q}{D^3N}\right)_m = \left(\frac{Q}{D^3N}\right)_p$$

$$Q_m = \left(\frac{D_m}{D_p}\right)^3 \times \frac{N_m}{N_p} \times Q_p = \left(\frac{1}{6}\right)^3 \times \frac{1191.75}{400} \times 1.7 = 0.0235 \text{ m}^3/\text{s}$$

(c) Power required by the model Equating the power coefficient for the model and prototype

$$\left(\frac{P}{D^5N^3}\right)_m = \left(\frac{P}{D^5N^3}\right)_p$$

$$P_m = \left(\frac{D_m}{D_p}\right)^5\left(\frac{N_m}{N_p}\right)^3 \times P_p = \left(\frac{1}{6}\right)^5\left(\frac{1191.75}{400}\right)^3 \times 720 = 2.45 \text{ kW}$$

Example 7.8 Two geometrically similar pumps are running at the same speed of 1000 rpm. One pump has an impeller diameter of 300 mm and lifts water at the rate of 0.02 m^3/s against a head of 15 m. Determine the head and impeller diameter of the other pump to deliver a discharge of 0.01 m^3/s.

Solution

For pump-1:

$N_1 = 1000$ rpm, $D_1 = 0.3$ m, $Q_1 = 0.02\ m^3/s$, $H_1 = 15$ m

For pump-2:

$N_2 = 1000$ rpm, $Q_2 = 0.01\ m^3/s$

(a) Impeller diameter of pump-2

$$\left(\frac{Q}{D^3N}\right)_1 = \left(\frac{Q}{D^3N}\right)_2$$

$$\left(\frac{D_2}{D_1}\right)^3 = \left(\frac{Q_2}{Q_1}\right)\left(\frac{N_1}{N_2}\right) = \left(\frac{0.01}{0.02}\right)\left(\frac{1000}{1000}\right) = 0.5$$

$$D_2 = (0.5)^{\frac{1}{3}} \times D_1 = (0.5)^{\frac{1}{3}} \times 0.3 = 0.238 \text{ m}$$

(b) Head developed by the Pump-2

$$\left(\frac{\sqrt{H}}{DN}\right)_1 = \left(\frac{\sqrt{H}}{DN}\right)_2$$

$$\left(\frac{H_2}{H_1}\right)^{\frac{1}{2}} = \left(\frac{D_2}{D_1}\right)\left(\frac{N_2}{N_1}\right)$$

$$H_2 = \left(\frac{0.238}{0.3}\right)^2 \times H_1 = \left(\frac{0.238}{0.3}\right)^2 \times 15 = 9.44 \text{ m}$$

Example 7.9 A turbine model of 1 : 10 develops 1.84 kW under a head of 5 m of water at 480 rpm. Find the power developed by the prototype under a head of 40 m. Also find the speed of the prototype. Assume efficiency of both the turbines to be same. Find and verify the specific speeds.

Solution

$\frac{D_m}{D_p} = \frac{1}{10}$ $P_m = 1.84$ kW $H_m = 5$ m $N_m = 480$ rpm $H_p = 40$ m

(a) Power developed by the prototype

$$P_p = \left(\frac{D_p}{D_m}\right)^5 \left(\frac{N_p}{N_m}\right)^3 P_m$$

(b) Speed of the prototype

$$N_p = \left(\frac{H_p}{H_m}\right)^{\frac{1}{2}} \left(\frac{D_m}{D_p}\right) N_m = \left(\frac{40}{5}\right)^{\frac{1}{2}} \left(\frac{1}{10}\right) 480 = 135.76 \text{ rpm}$$

$$P_p = (10)^5 \left(\frac{135.76}{480}\right)^3 1.84 = 4163 \text{ kW}$$

(c) Specific speed of the prototype

$$(N_s)_p = \frac{N_p\sqrt{P_p}}{H_p^{5/4}} = \frac{135.76 \times (4163)^{1/2}}{(40)^{5/4}} = 87.1$$

Specific speed of the model

$$(N_s)_m = \frac{N_m\sqrt{P_m}}{H_m^{5/4}} = \frac{480\sqrt{1.84}}{(5)^{5/4}} = 87.1$$

$$(N_s)_p = (N_s)_m = 87.1$$

The specific speed of the model is equal to the prototype end thus it is verified.

Example 7.10 A model of a Kaplan turbine, one tenth of the actual size is tested under a head of 5 m when actual head for proto turbine is 8.5 m. The power to be developed by prototype is 8000 kW. When running at 120 rpm at an overall efficiency of 85%, determine (a) speed (b) discharge and (c) power of model.

Solution

$\frac{D_m}{D_p} = \frac{1}{10}$ $H_m = 5$ m $H_p = 8.5$ m $P_P = 8000$ kW $N_p = 120$ rpm

(a) Speed of the model

$$N_m = \left(\frac{H_m}{H_p}\right)^{1/2}\left(\frac{D_p}{D_m}\right)N_p$$

$$= \left(\frac{5}{8.5}\right)^{1/2}(10)120$$

$$= 920.4 \text{ rpm}$$

(b) Discharge from the model

$$Q_m = \left(\frac{D_m}{D_p}\right)^3\left(\frac{N_m}{N_p}\right)Q_P$$

$$Q_P = \frac{P_P}{\rho g \eta_0 H_p} = \frac{8000 \times 10^3}{10^3 \times 9.81 \times 0.85 \times 8.5}$$

$$= 112.87 \text{ m}^3/\text{s}$$

$$Q_m = \left(\frac{1}{10}\right)^3\left(\frac{920.4}{120}\right) \times 112.87$$

$$= 0.866 \text{ m}^3/\text{s}$$

(c) Power of model

$$P_m = \left(\frac{D_m}{D_p}\right)^5\left(\frac{N_m}{N_p}\right)^3 P_p$$

$$= \left(\frac{1}{10}\right)^5\left(\frac{920.4}{120}\right)^3 8000$$

$$= 36.1 \text{ kW}$$

Example 7.11 A turbine develops 6600 kW, when running at 100 rpm. The head on the turbine is 30 m. If the head on the turbine is reduced to 18 m, determine the speed and power developed by the turbine.

Solution

Power developed $P_1 = 6600$ kW Speed $N_1 = 100$ rpm Head $H_1 = 30$ m

For the head $H_2 = 18$ m Speed $= N_2$ and Power $= P_2$

Using the unit speed equation

$$\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$N_2 = \frac{100 \times \sqrt{18}}{\sqrt{30}}$$

$$N_2 = 77.45 \text{ rpm}$$

Using the unit power equation

$$\frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

$$\therefore P_2 = \frac{6600 \times (18)^{3/2}}{(30)^{3/2}}$$

$$= 3067 \text{ kW}$$

Example 7.12 A turbine is to operate under a head of 25 m at 200 rpm. The discharge is 9 m³/s. If the efficiency is 90% determine the performance of the turbine under a head of 20 meters. *(BDU–Nov. '97)* *(MU–Oct. '99)*

Solution

Head on turbine $H_1 = 25$ m, Speed $N_1 = 200$ rpm, Discharge $Q_1 = 9$ m³/s

Overall efficiency $\eta_0 = 90\%$.

Performance of the turbine under the head $H_2 = 20$ m, means to find the speed, discharge and power developed by the turbine when working under the head of 20 m.

For the Head $H_2 = 20$ m, Speed $= N_2$, Discharge $= Q_2$ and Power $= P_2$

Using the unit speed equation

$$N_2 = \frac{N_1\sqrt{H_2}}{\sqrt{H_1}}$$

$$= \frac{200 \times \sqrt{20}}{\sqrt{25}}$$

$$= 178.88 \text{ rpm}$$

Using the unit discharge equation

$$Q_2 = \frac{Q_1\sqrt{H_2}}{\sqrt{H_1}}$$

$$= \frac{9 \times \sqrt{20}}{\sqrt{25}}$$

$$Q_2 = 8.05 \text{ m}^3/\text{s}$$

Using the unit power equation

$$P_2 = \frac{P_1(H_2)^{3/2}}{(H_1)^{3/2}}$$

Now,

$$\eta_0 = \frac{P_1}{\rho g Q_1 H_1}$$

$$\therefore P_1 = 0.9 \times 1000 \times 9.81 \times 9 \times 25$$

$$= 1986525 \text{ W}$$

$$= 1986.525 \text{ kW}$$

and

$$P_2 = \frac{1986.525(20)^{3/2}}{(25)^{3/2}}$$
$$= 1421.44 \text{ kW}$$

Example 7.13 A Pelton turbine produces 5000 kW under a head of 250 m and has speed of 210 rpm. Overall efficiency of turbine is 85%. Find the unit quantities. If the head falls to 160 m what are the new values of speed, discharge and power. Find also the specific speed.

Solution

$P_1 = 5000$ kW, $H_1 = 250$ m, $N_1 = 210$ rpm, $\eta_0 = 0.85$, $H_2 = 160$ m.

(a) Unit speed

$$N_u = \frac{N_1}{\sqrt{H_1}} = \frac{210}{\sqrt{250}} = 13.28$$

(b) Unit power

$$P_u = \frac{P_1}{H_1^{3/2}} = \frac{5000}{(250)^{3/2}} = 1.265$$

(c) Unit discharge

$$\eta_0 = \frac{P_1}{\rho g Q_1 H_1}$$

$$\therefore Q_1 = \frac{5000 \times 10^3}{0.85 \times 10^3 \times 9.81 \times 250}$$
$$= 2.399 \text{ m}^3/\text{s}$$

$$Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{2.399}{\sqrt{250}} = 0.152$$

when $H_2 = 160$ m

(d) Discharge

$$Q_2 = Q_u \times \sqrt{H_2}$$
$$= 0.152 \times \sqrt{160}$$
$$= 1.92 \text{ m}^3/\text{s}$$

(e) Speed

$$N_2 = N_u \times \sqrt{H_2}$$
$$= 13.28 \times \sqrt{160}$$
$$= 167.98 \text{ rpm}$$

(f) Power

$$P_2 = P_u (H_2)^{3/2}$$
$$= 1.265 \times (160)^{3/2}$$
$$= 2560.2 \text{ kW}$$

(g) Specific speed

$$N_s = \frac{N_2\sqrt{P_2}}{(H_2)^{5/4}} = \frac{167.98\sqrt{2560.2}}{(160)^{5/4}}$$
$$= 14.94$$

SHORT QUESTIONS

7.1. What is Dimensional analysis?
7.2. What is a Dimension?
7.3. What is Unit?
7.4. What are fundamental dimensions and derived dimensions? Give examples.
7.5. The dimension of power is
 (a) ML^2T^{-2}
 (b) ML^2T^{-3}
 (c) ML^3T^{-2}
7.6. The dimension of work is ——————
7.7. There is no dimension for efficiency (True/False)
7.8. Match the following
 (a) Force - MLT^{-1}
 (b) Dynamic viscosity - none
 (c) Angle - $ML^{-1}T^{-1}$
 (d) Momentum - MLT^{-2}
7.9. What are the important advantages of dimensional analyis?
7.10. What is dimensional homogeneity? Give an example.
7.11. Dimensionally homogeneous equations are —————— of the system of units.
7.12. What is Buckingham π-theorem?
7.13. Number of dimensionless terms in Buckingham's π-theorem is equal to
 (a) The sum of total no. of variables (n) and the no. of fundamental dimensions (m)
 (b) The difference between n and m
 (c) The product of n and m
7.14. Each π-term contains
 (a) m–1 variables
 (b) 1 + m variables
 (c) m variables
 where m is the number of fundamental dimensions.
7.15. Enumerate, the points governing the choice of repeating variables.

7.16. The number of repeating variables are equal to the number of fundamental dimensions of the problem. (True/False)
7.17. If a variable is dimensionless, it itself is a π −term. (True/False)
7.18. The product of two π −terms is dimensionless. (True/False)
7.19. What is model analysis?
7.20. What are the advantages of model analysis?
7.21. Define: Similitude
7.22. List the types of similarities.
7.23. What is Geometric similarity?
7.24. The following points should be considered in geometric similarity.
(a) Flow directions and all angles must be preserved.
(b) Orientation should be preserved
(c) both (a) and (b)
7.25. What is kinematic similarity?
7.26. What is dynamic similarity?
7.27. What are non-dimensional numbers?
7.28. Define Reynold's number.
7.29. Mach number is the square root of the
(a) ratio of inertia force to the pressure force
(b) ratio of inertia force to the gravity force
(c) ratio of inertia force to the elastic force
7.30. Froude's number is the square root of the ratio of ———— force to the ———— force.
7.31. Weber's number is the square root of the ratio of ———— force to the ———— force.
7.32. What are undistorted models?
7.33. When is a model called a distorted model?
7.34. Define: Specific speed of a pump and write the units of each quantity in the expression.
7.35. Define: Turbine specific speed and specify the units of each variables appearing in the expression.
7.36. The turbine specific speed is given by
(a) $\frac{N\sqrt{H}}{P^{5/4}}$
(b) $\frac{N\sqrt{P}}{H^{5/4}}$
(c) $\frac{N\sqrt{P}}{H^{3/4}}$
7.37. The pump specific speed is given by
(a) $\frac{\sqrt{NQ}}{(H_m)^{5/4}}$
(b) $\frac{N\sqrt{Q}}{(H_m)^{5/4}}$
(c) $\frac{N\sqrt{Q}}{(H_m)^{3/4}}$

7.38. What is model testing?
7.39. What are the conditions of complete similarity between a model and its prototype?
7.40. Specific speed of a model is ———— to the specific speed of its prototype.
7.41. What are unit quantities?
7.42. The unit speed is given by
(a) $N/\sqrt{H}$
(b) $\sqrt{N}/H$
(c) $\sqrt{N/H}$
7.43. What is the use of unit quantities?

EXERCISES

7.1. Discuss the advantages of dimensional analysis.
7.2. State Buckingham's π −theorem. Describe this theorem for dimensional analysis.
7.3. Show that the discharge of a centrifugal pump is given by

$$Q = ND^3 f\left[\frac{gH_m}{N^2D^2}, \frac{\mu}{ND^2\rho}\right]$$

where N is the speed of the pump in rpm, D is the diameter of the impeller, g acceleration due to gravity, H_m manometer head, μ viscocity of fluid and ρ the density of the fluid.
7.4. Show by dimensional analysis, the Power P developed by a hydraulic turbine is given by

$$P = \rho N^3 D^5 f\left[\frac{gh}{D^2N^2}\right]$$

where ρ is the density of liquid, N is the speed in rpm, D is the diameter of runner, H is the head and g is the gravitational accelaration.
7.5. What is model analysis? What are its advantages?
7.6. Explain the different types of hydraulic similarities that must exist between a prototype and its model.
7.7. What do you mean by dimensionless numbers. Define and explain four non-dimensional numbers.
7.8. Define and derive an expression for specific speed of a pump.
7.9. Derive an expression for specific speed of a turbine.
7.10. How is the model testing of a hydraulic turbomachine made?
7.11. A centrifugal pump was tested in a laboratory by a 1:8 model. It consumed 5 kW under a head of 5 m at 450 rpm. If the prototype is to work at 80 m head, determine its power, speed and discharge ratio.
[Ans. (a) 225 rpm (b) 20480 kW (c) 256]
7.12. A centrifugal pump discharges water at the rate of 0.167 m^3/s at 2000 rpm under a head of 100 m and consumes 300 kW power. A 1:5 scale model is to

run at 1500 rpm. Determine its power, discharge and head. Find and verify their specific speeds.

[Ans. (a) 0.001 m^3/s (b) 2.25 m (c) 0.0405 kW and (d) 25.8]

7.13. An axial flow pump with a rotor diameter of 30 cm handle liquid water at the rate of 2.7 m^3/min while operating at 1500 rpm. The corresponding energy input is 125 J/kg. If a geometrically similar pump with a rotor diameter of 20 cm operates at 3000 rpm what are its (a) flow rate (b) change in total pressure if the total to total efficiency is 0.75 and (c) input power. *(MKU–April '96)*

[Ans. (a) 1.6 m^3/min (b) 1.67 bar and (c) 5.93 kW]

7.14. A model of a Francis turbine of 1/5th of the actual size was tested in a laboratory under a head of 1.8 m. It develops 3 kW at 360 rpm. Determine the speed and power developed under 6 m head. Also, find the specific speed.

[Ans. (a) 131.5 rpm (b) 456.9 kW and (c) 299.3]

7.15. A model of a Kaplan turbine, 1/12th of the actual turbine size is tested under a head of 3 m. The head for the prototurbine is 7.5 m. The prototurbine is designed to produce 6000 kW at a speed of 150 rpm at an efficiency of 83%. Find speed, discharge, power and specific speed of the model.

[Ans. (a) 1138.42 rpm, (b) 0.432 m^3/s, (c) 10.54 kW and (d) 936]

7.16. What are unit quantities? Define the unit quantities for a turbine? Why are they important?

7.17. Obtain an expression for unit speed, unit discharge and unit power for a turbine.

7.18. A pelton turbine produces 10,000 kW of power while working under a head of 500 m. The speed is 300 rpm. Assuming the efficiency of the turbine to be 80%, find the values of unit quantities. If the head on the turbine falls to 350 m, find the new discharge, speed and power for the same efficiency. Verify the specific speed. [Ans. (a) 0.114 (b) 13.42 (c) 0.894 (d) 2.13 m^3/s (e) 251 rpm (f) 5853.8 kW (g) 12.69]

7.19. A Francis turbine works under a head of 5 m and produces 70 kW; the discharge through the turbine is 1.5 m^3/s and the speed is 180 rpm. Find the unit quantities and new values of speed, discharge and power, when the head increases to 15 m.

[Ans. (a) 0.671 (b) 80.5 (c) 6.261 (d) 2.6 m^3/s (e) 311.8 rpm (f) 363.7 kW]

7.20. A pelton wheel develops 5520 kW under a head of 240 m with an overall efficiency of 83% when revolving at a speed of 200 rpm. Find the unit quantities. If the head on the same turbine during off season falls to 150 m then, find the discharge, power and speed for this head. *(MU–Oct. '97)*

[Ans. (a) 13 rpm, 0.182 m^3/s, 1.5 kW (b) 158.1 rpm, 2.23 m^3/s, 2.73 mW]

8

HYDRAULIC PUMPS

CENTRIFUGAL PUMPS

INTRODUCTION

The hydraulic machines which convert the mechanical energy into hydraulic energy are called as *pumps*. The hydraulic energy is in the form of pressure energy. Two types of pumps commonly used are centrifugal and axial flow pumps. They are so named because of the general nature of the fluid flow through the impeller. The combination of centrifugal and axial flow pumps is called as *mixed flow pump* wherein part of the liquid flow in the impeller is axial and part is radial.

The hydraulic machines that convert mechanical energy into pressure energy, by means of centrifugal force acting on the fluid are called as *centrifugal pumps*. The centrifugal pump is similar in construction to the Francis turbine. But the difference is that the fluid flow is in a direction opposite to that in the turbine.

MAIN PARTS OF A CENTRIFUGAL PUMP

The parts of a centrifugal pump are similar to those of a centrifugal compressor. The three important parts of a centrifugal pump are (1) Impeller (2) Casing and (3) Suction and Delivery pipes (Fig. 8.1a).

1. Impeller

The rotating part of the centrifugal pump is called the '*Impeller*'. The impeller is a rotating solid disc with curved blades standing out vertically from the face of the disc. The tips of the blades in the impeller are sometimes covered by another flat disc to give shrouded blades, otherwise the blade tips are left open and the casing of the pump itself forms the solid outer wall of the blade passages. The advantage of the shrouded blade is that flow is prevented from leaking across the blade tips from one passage to another.

The impeller is mounted on a shaft connected to the shaft of an electric motor. As the impeller rotates, the fluid that is drawn into the blade passages at the impeller inlet is accelerated as it is forced radially outwards. In this way, the static pressure of fluid is raised.

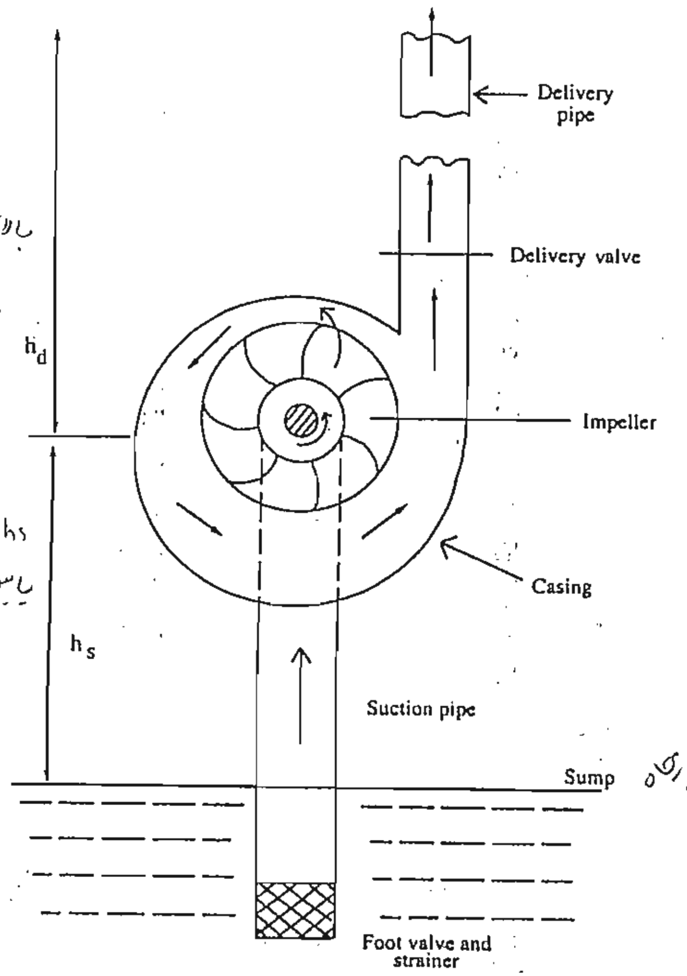


Figure 8.1(a) Centrifugal pump

2. Casing

It is an air-tight passage surrounding the impeller which converts the K.E of water leaving the impeller into pressure energy before the water leaves the casing and enters the delivery pipe. The three commonly used casings are

(a) Volute casing Casing that surrounds the impeller, is of spiral type in which flow area increases gradually. The increase in area of flow, decreases the velocity

of flow and thus increases the pressure of water. The efficiency of centrifugal pump having this casing is reduced due to the formation of eddies.

(b) Vortex casing (Fig 8.1 (b)) If a circular chamber is introduced between the casing and the impeller, then that casing is known as *vortex casing*. This considerably reduces the loss of energy due to the formation of eddies. Thus, the efficiency of the pump is more than the efficiency of volute casing centrifugal pump.

(c) Casing with guide blades (Fig. 8.1(c)) In this type of casing, the impeller is shrouded by a series of guide blades mounted on a ring which is known as '*diffuser*'. The guide vanes are designed in such a way that the water from the impeller enters the guide vanes without shock. The area of the guide vanes increases, thus, reducing the flow velocity through guide vanes and consequently increasing the pressure of water. The water from the guide vanes then passes through the surrounding casing, which is in most cases concentric with the impeller. The diffuser is optional and may not be present in a particular design depending upon the size and cost of the pump.

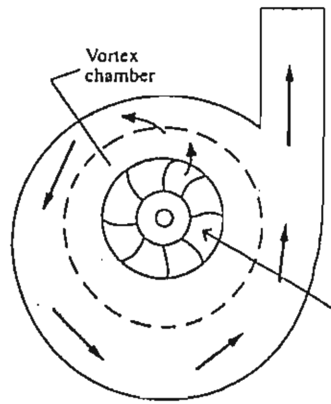


Figure 8.1(b) Vortex casing

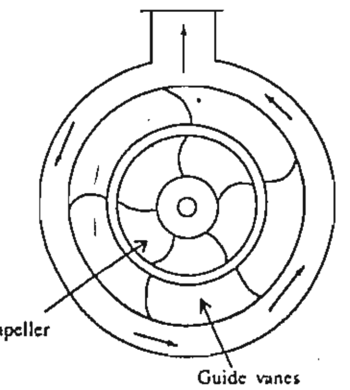


Figure 8.1(c) Casing with guide vanes

3. Suction Pipe and Delivery Pipe

A pipe whose one end is connected to the inlet of the pump and other end dipped into the water in a sump is known as the *suction pipe*. A foot valves, fitted at the lower end of the suction pipe, opens only in the upward direction. A strainer to remove debris from water, is also fitted at the lower end of the suction pipe. The pipe whose one end is connected to the outlet of the pump and the other end delivers the water at a required height is known as *delivery pipe*.

WORK DONE AND VELOCITY TRIANGLES

Figure 8.2 shows the velocity triangles at the inlet and outlet tips of the vane fixed to an impeller. The blades are curved between the inlet radius r_1 and the outlet radius

r_2. β_1 is the angle subtended by the blade at the inlet, measured from the tangent to the inlet radius, while β_2 is the blade angle measured from the tangent at the outlet.

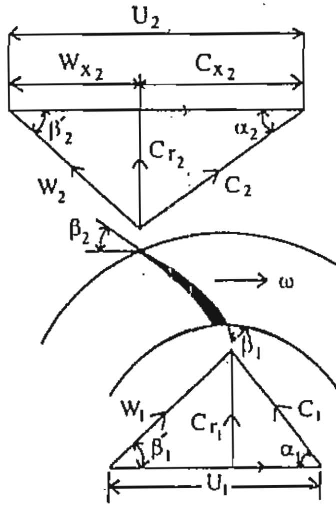


Figure 8.2 Velocity triangles for a centrifugal pump

The fluid enters the blade passage with an absolute velocity C_1 and at an angle α_1 to the impeller inlet the tangential velocity vector $U_1 = \omega r_1$, where ω is the angular velocity of the impeller. The resultant relative velocity of the flow into the blade passage is W_1 at an angle β_1', (and not β_1 due to the shock at the entry of blade) tangent to the inlet. Similarly, at outlet the relative velocity vector is W_2 at an angle β_2' (and not β_2 due to fluid slip at the exit) tangent to the blade. The absolute velocity vector C_2 is obtained by subtracting the impeller outlet tangential velocity vector U_2 from W_2 and C_2 is set at an angle α_2 from the tangent to the blade. As a general case unless otherwise stated, it will be assumed that the inlet and outlet blade angles are equal to their corresponding relative flow angles. (i.e) $\beta_1' = \beta_1$ (for No-shock condition) and $\beta_2' = \beta_2$ (for No-slip at the exit).

From Euler's pump equation the work done per second on the fluid per unit weight of fluid flowing is

$$E = W/mg = (U_2 C_{x_2} - U_1 C_{x_1})/g \qquad (8.1)$$

where, C_x is the component of absolute velocity in the tangential direction. 'E' is often referred to as the Euler head and represents the ideal (or) theoretical head developed by the impeller only.

'E' in terms of absolute velocity is obtained as follows:

From the velocity triangles

$$C_{x_1} = C_1 \cos \alpha_1$$

and

$$C_{x_2} = C_2 \cos \alpha_2$$

Thus

$$E = (U_2 C_2 \cos\alpha_2 - U_1 C_1 \cos\alpha_1)/g \qquad (8.2)$$

But by using the cosine rule,

$$W^2 = U^2 + C^2 - 2UC\cos\alpha$$

then $U_1 C_1 \cos\alpha_1 = (U_1^2 - W_1^2 + C_1^2)/2$

and $U_2 C_2 \cos\alpha_2 = (U_2^2 - W_2^2 + C_2^2)/2$.

Substituting the above relation in equation (8.2), gives

$$E = \frac{[(U_2^2 - U_1^2) + (C_2^2 - C_1^2) + (W_1^2 - W_2^2)]}{2g}$$

where, $(C_2^2 - C_1^2)/2g$ represents the increase of kinetic energy of the fluid across the impeller. $(U_2^2 - U_1^2)/2g$ represents the energy used in imparting circular motion about the impeller axis to the fluid and $(W_1^2 - W_2^2)/2g$ is the gain of static head due to reduction of the relative velocity within the impeller.

As the water enters the impeller radially, the absolute velocity of water at inlet is in the radial direction and hence $\alpha_1 = 90°$ and $C_{x_1} = 0$ because $C_1 = C_{r_1}$

So, equation (8.1) reduces to

$$E = U_2 C_{x_2}/g$$

The volume flow rate of water is

$$Q = 2\pi r_1 C_{r_1} b_1 = 2\pi r_2 C_{r_2} b_2$$

where,

C_r is the radial component of the absolute velocity and is perpendicular to the tangent at inlet and outlet while b is the width of the blade measured in the z direction.

When considering the slip factor σ_s, the Euler head equation becomes

$$E = (\sigma_s U_2 C_{x_2} - U_1 C_{x_1})/g$$

Typically, slip factors lie in the region of 0.9 where

$$\sigma_s = \frac{\text{Actual } C_x}{\text{Ideal } C_x} = \frac{C'_{x2}}{C_{x2}} \quad \text{(Refer Fig. 8.3)}$$

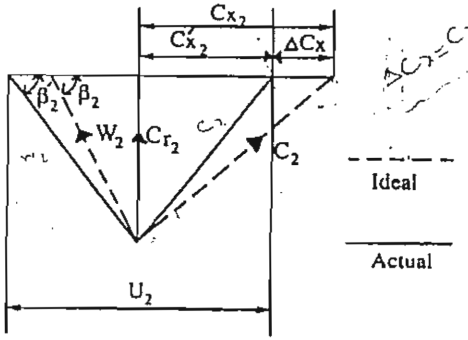


Figure 8.3 Exit velocity triangle with slip

HEAD DEVELOPED

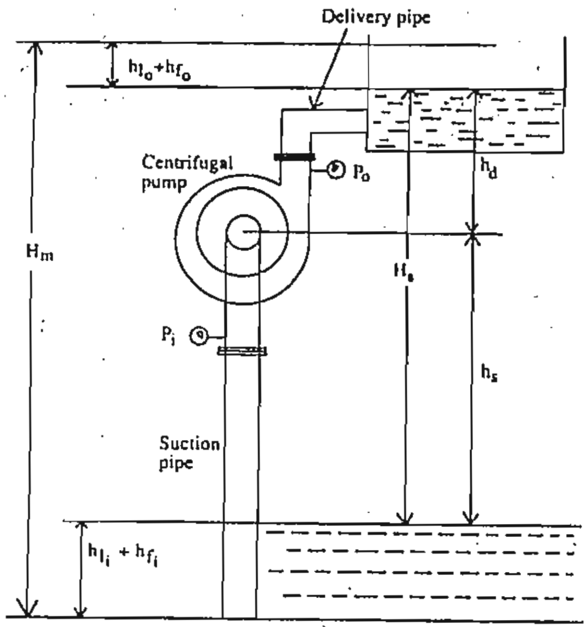


Figure 8.4 Heads associated with centrifugal pump

Static Head (H_s)

It is the vertical distance between the two levels in the reservoirs or $H_s = h_s + h_d$ i.e. Static head is the sum of the suction head and the delivery head.

Manometric Head (H_m)

It is a head against which a centrifugal pump has to work. It is denoted by H_m. It is given by the following expressions.

The pump total inlet and outlet heads are measured at the inlet and outlet flanges respectively and are given as

Pump total inlet head = $P_i/\rho g + V_i^2/2g + Z_i$

Pump total outlet head = $P_o/\rho g + V_o^2/2g + Z_o$

Total head developed by pump (or) manometric head (H_m)

$$H_m = \left[(P_o - P_i)/\rho g + (V_o^2 - V_i^2)/2g + (Z_o - Z_i)\right]$$

The type of pump is selected for a specific purpose based on the manometric head. H_m can also be expressed as

$$H_m = H_s + \Sigma H_{losses}$$
$$= H_s + h_{li} + h_{fi} + h_{lo} + h_{fo}$$

where h_f is the frictional head loss h_l is the other head losses and

H_m = Euler head − Losses in the impeller and casing

PUMP LOSSES AND EFFICIENCIES

The shaft power or energy P_s that is supplied to the pump by the prime mover is not the same as the energy received by the liquid. The difference is mainly due to the following losses:

1. **Mechanical friction power loss** (P_m) due to the friction between the fixed and the rotating parts in the bearing and the stuffing boxes.
2. **Disc friction power loss** (P_D) due to friction between the rotating faces of the impeller (or disc) and the liquid.
3. **Leakage and recirculation power loss** (P_L) due to the loss of liquid from the pump (or) recirculation of the liquid in the impeller and
4. **Casing power loss** (P_C)

Impeller Power Loss (Disc Friction Power Loss (P_D))

It is caused by an energy or head loss h_i in the impeller due to disc friction, flow separation and shock at the impeller entry. The flow rate through the impeller is Q_i, so the impeller power loss is expressed as

$$\boxed{P_D = \rho g Q_i h_i} \quad (8.3)$$

Leakage Power Loss

The pressure difference between the impeller eye and tip can cause a recirculation of a small volume of fluid q thus reducing the flow rate at the outlet to Q. Thus

$$\boxed{Q = Q_i - q} \quad (8.4)$$

If H_i is the total head across the impeller, then a leakage power loss can be defined as

$$P_L = \rho g H_i q \tag{8.5}$$

Equation (8.4) shows that when the discharge valve of the pump is closed, the leakage flow rate attains its highest value.

Casing Power Loss

If h_c is the head loss in the fluid between the impeller outlet and the pump outlet flange and the flow rate is Q, then P_c may be defined as

$$P_c = \rho g Q h_c \tag{8.6}$$

Summing up all the losses gives

$$\boxed{P_s = P_m + \rho g(h_i Q_i + h_c Q + H_i q + Q H_m)}$$

where 'H_m' is the manometric head.

(i) Mechanical efficiency

$$\eta_m = \frac{\text{Fluid power supplied to the impeller}}{\text{Power input to the shaft}} = \frac{\rho g Q_i(h_i + H_i)}{P_s}$$

H_i – Total head across the impeller
h_i – head loss in the impeller
(or)

$$\boxed{\eta_m = \frac{W}{P_s} = \frac{m(U_2 C_{x2})}{P_s}}$$

(ii) Manometric (or) hydraulic efficiency

$$\eta_H = \frac{\text{Actual Head developed by pump}}{\text{Theoretical head developed by impeller}} = H_m/(h_i + H_i)$$

(or)

$$E = U_2 C_{x2}/g = (H_i + h_i)$$

$$\boxed{\eta_H = \frac{H_m}{(U_2 C_{x2})/g}}$$

(iii) Overall efficiency

$$\eta_o = \frac{\text{Fluid power developed by pump}}{\text{Shaft power input}}$$

$$\boxed{\eta_o = \frac{\rho g Q H_m}{P_s}}$$

(iv) Volumetric efficiency

$$\eta_v = \frac{\text{Flow rate through pump}}{\text{Flow rate through impeller}}$$

$$\boxed{\eta_v = \frac{Q}{(Q+q)}}$$

(v) Impeller efficiency

$$\boxed{\eta_i = \frac{\text{Fluid power at impeller exit}}{\text{Fluid power supplied to impeller}}}$$

Fluid Power supplied to impeller = Fluid power developed by Impeller + Impeller loss

The Fluid power supplied to the impeller is also referred to as the power available at the impeller.

MINIMUM STARTING SPEED

The centrifugal pump will start delivering fluid if and only if the pressure rise in the impeller is more than or equal to manometric head (H_m). Otherwise the pump will not discharge fluid, although the impeller is rotating. When the impeller is rotating the water in contact with the impeller is also rotating. This is the case of forced vortex. In case of forced vortex, the centrifugal head or head due to pressure the rise in the impeller is

$$= \frac{\omega^2 r_2^2}{2g} - \frac{\omega^2 r_1^2}{2g}$$

where
ωr_2 – Tangential velocity of impeller at outlet = U_2 and
ωr_1 – Tangential velocity of impeller at inlet = U_1

∴ Head due to pressure rise in impeller

$$= \frac{U_2^2}{2g} - \frac{U_1^2}{2g}$$

The flow of water will commence, when

$$\frac{U_2^2 - U_1^2}{2g} \geq H_m$$

For minimum speed

$$\frac{U_2^2 - U_1^2}{2g} = H_m$$

H_m in terms of η_H

$$H_m = \eta_H \times \frac{U_2 C_{x_2}}{g}$$

Substituting the value of H_m in the above equation yields

$$\frac{U_2^2 - U_1^2}{2g} = \eta_H \times \frac{U_2 C_{x_2}}{g}$$

Now,

$$U_2 = \frac{\pi D_2 N}{60} \quad and \quad U_1 = \frac{\pi D_1 N}{60}$$

Therefore,

$$\frac{\left(\frac{\pi D_2 N}{60}\right)^2 - \left(\frac{\pi D_1 N}{60}\right)^2}{2g} = \eta_H \times \frac{C_{x_2}}{g} \times \frac{\pi D_2 N}{60}$$

Dividing by $\frac{\pi N}{g60}$, we get

$$\frac{\pi N D_2^2}{120} - \frac{\pi N D_1^2}{120} = \eta_H \times C_{x_2} \times D_2$$

(or)

$$\frac{\pi N}{120}\left[D_2^2 - D_1^2\right] = \eta_H \times C_{x_2} \times D_2$$

Hence the minimum starting speed is

$$N = \frac{120 \times \eta_H \times C_{x_2} \times D_2}{\pi [D_2^2 - D_1^2]}$$

and a centrifugal pump has to run at this speed atleast, to discharge liquid.

NET POSITIVE SUCTION HEAD

Net Positive Suction Head (*NPSH*) is the head required at the pump inlet to keep the liquid from cavitating or boiling. The pump inlet (or) suction side is the low-pressure point where cavitation will first occur.

The *NPSH* is defined as

$$NPSH = \frac{P_i}{\rho g} + \frac{V_i^2}{2g} - \frac{P_{vap}}{\rho g}$$

where P_i and V_i are the pressure and velocity at the pump inlet and P_{vap} is the vapour pressure of the liquid. All pressures are absolute pressures.

NPSH is also defined as a measure of the energy available on the suction side of the pump.

NPSH is a commercial term used by the pump manufacturers and indicates the suction head which the pump impeller can produce. In other words, it is the height of the pump axis from the water reservoir which can be permitted for installation.

PRIMING

Priming of a centrifugal pump is defined as the operation in which the suction pipe, casing of the pump and a portion of the delivery pipe upto the delivery valve is completely filled up with the liquid to be raised by the pump before starting the pump. Thus, the air from these parts of the pump is removed.

The head generated by the pump is given by

$$E = \frac{U_2 C_{x_2}}{g}$$

The above equation is independent of the density of the liquid. This means that when pump is running in air, the head generated is in terms of metre of air. If the pump is primed with water, the same head is generated but is expressed in metre of water. But as the density of air is very low, the generated head of air in terms of equivalent metre of water head is negligible. It is obvious, that if impeller is running in air, it will produce only a negligible pressure, which may not be sufficient to suck water from the sump. To avoid this difficulty, the pump is first primed, i.e. filled up with water.

PERFORMANCE CURVES OF CENTRIFUGAL PUMPS

Performance charts are always plotted for constant shaft-rotation speed N. The basic independent variable is taken to be the discharge Q. The dependent variables or output are taken to be head (H), power(P) and efficiency.

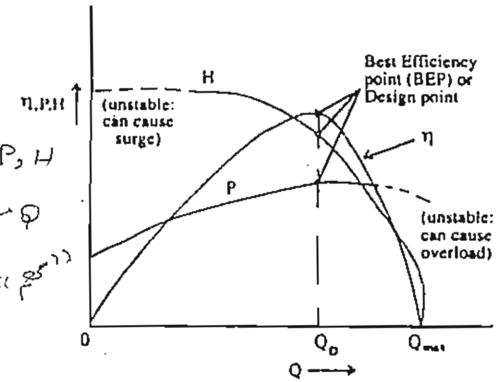


Figure 8.5 Typical centrifugal pump performance curves of constant impeller rotation speed

Figure 8.5 shows typical characteristic curves for a centrifugal pump. The head is approximately constant at low discharge and then drops to zero at $Q = Q_{max}$. At this speed and impeller size the pump cannot deliver any more fluid than Q_{max}. The positive slope part of the head is shown dashed as mentioned earlier, this region can be unstable and cause pump surge.

The power provided by the pump motor rises monotonically with discharge and then typically drops off slightly near Q_{max}. The drop off region is shown dashed because it is also potentially unstable and can cause motor overload during a transient.

The efficiency rises to a maximum at about 60 percent of Q_{max}. This is the design flow rate (Q_D) or Best efficiency point (BEP), $\eta = \eta_{max}$. The head and power at the BEP will be termed H_D and P_D respectively. Note that η is zero at the origin (no flow) and at Q_{max} (no head). Note also that the η curve is not independent but is simply calculated from the relation

$$\eta = \rho Q g H / P$$

CONSTANT EFFICIENCY OR MUSCHEL CURVES

The head versus discharge curve and efficiency versus discharge curve for various speeds are combined to get the constant efficiency curve. The Muschel curves are drawn as follows.

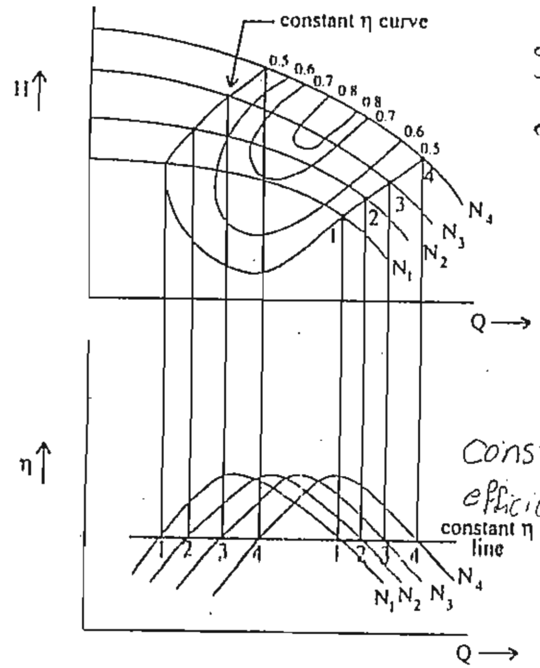


Figure 8.6 Muschel curves

The variation of pump efficiency (η) with discharge (Q) is plotted for different speeds N_1, N_2, N_3, ... etc. as shown in Fig. 8.5. A constant efficiency line is drawn on $\eta - Q$ plot to determine two discharge (Q) values for a particular speed. These points are marked as 1,2,3,... etc. and are projected on to the $H - Q$ plot for the same discharge values and speed for a particular efficiency. This procedure is repeated for other efficiency values. The points of same efficiency are joined by a curve as shown in Fig. 8.6. These curves are called the *constant efficiency or iso efficiency curves*. The point of maximum efficiency can be determined from this graph. The advantage of this plot is that a single point on this graph gives all the information regarding the speed, head, discharge, efficiency and power.

MULTISTAGE CENTRIFUGAL PUMPS

A Centrifugal pump that consists of two or more impellers mounted on the same shaft or on different shafts is called the *multistage centrifugal pump*. Multistage pumps are employed to accomplish the following two important functions.

1. To produce a high head.
2. To develop a high discharge.

PUMPS COMBINED IN PARALLEL

To develop high discharge, the pumps should be connected in parallel as shown in the figure 8.7.

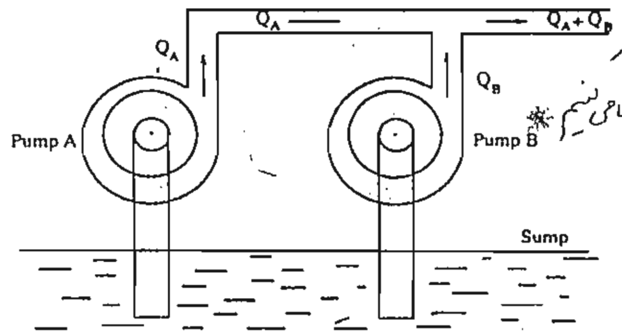


Figure 8.7 Pumps in parallel

Pump A and pump B lift the water from a common sump and discharge the water to a common pipe to which the delivery pipes of each pump is connected. Each of the pump works against the same head.

The two pumps A and B, which are in parallel need not be identical. But for the same head, the multistage pump discharge is the sum of the flow rates of each pump. i.e. $Q_A + Q_B$. (Fig. 8.9)

If pump A has more head than pump B, pump B cannot be added until the operating head is below the shut off head of pump B. Since, the system resistance curve rises with Q, the combined delivery Q_{A+B} will be less than the separate operating discharges

$Q_A + Q_B$ but certainly greater than either one. For a very flat (static) curve two similar pumps in parallel will deliver nearly twice the flow. (Fig. 8.8)

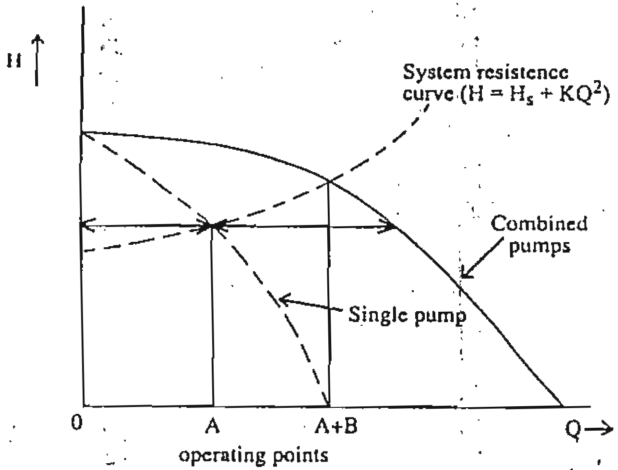


Figure 8.8 Two similar pumps connected in parallel

The combined brake power is found by adding the brake powers of pumps A and B at the same head as the operating point.

The combined efficiency equals

$$\eta = \frac{\rho g (Q_{A+B})(H_m)_{A+B}}{(P_s)_{A+B}}$$

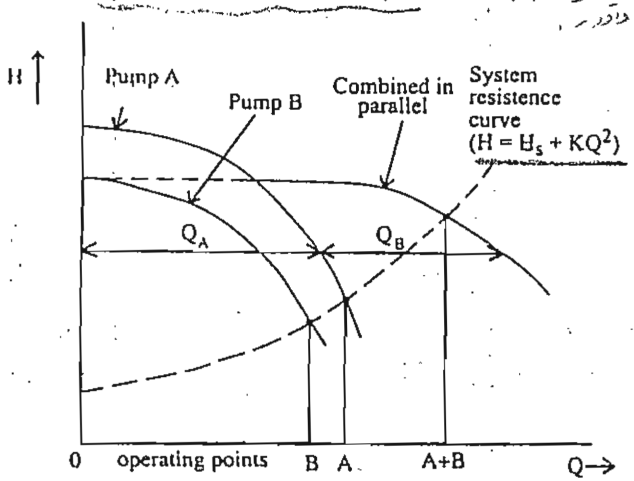


Figure 8.9 Two different pumps combined in parallel

If pumps A and B are not identical as shown in Fig. 8.9, then the pump B should not be run and cannot even be started up if the operating point is above its shutoff head.

PUMPS COMBINED IN SERIES

To develop a high head, the pumps should be connected in series as shown in figure 8.10. The water from the suction pipe enters the pump C at inlet and is discharged at the outlet with increased pressure. The water with increased pressure from the pump C is taken to the inlet of the pump D. The pressure of water at the pump D exit will be more than the pressure of water at the pump C exit.

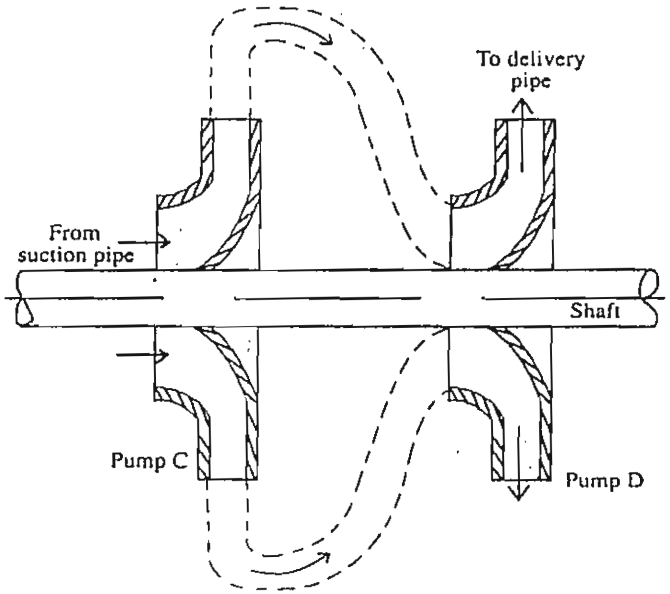


Figure 8.10 Pumps in series

The discharge through each pump is the same. The combined head is higher than either one. The pumps in series need not be identical at all, since they merely handle the same discharge. They may even have different speeds, although normally both are driven by the same shaft.

The combined operating point head for two different pumps in series will be more than either C or D separately, but not as great as their sum.

The combined pumps perfromance curves are shown in the figures 8.11 and 8.12.

The combined power is the sum of brake power for C and D at the operating point flow rate.

$$\text{The combined efficiency is} = \frac{\rho g (Q_{C+D})(H_m)_{C+D}}{(P_s)_{C+D}}$$

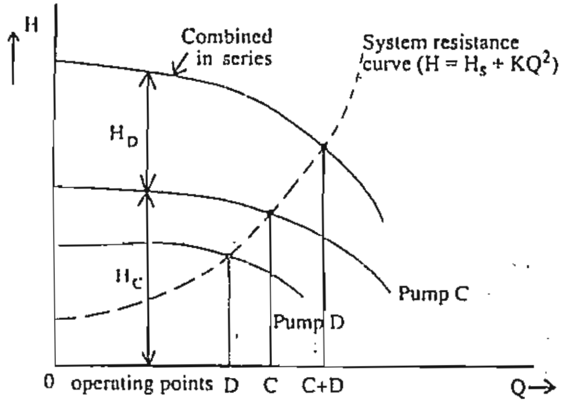


Figure 8.11 Two diffferent pumps connected in series

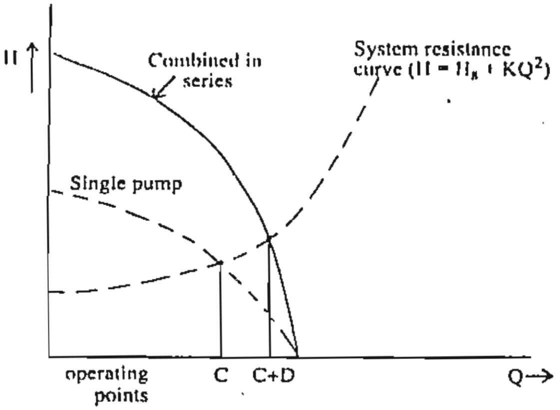


Figure 8.12 Two similar pumps connected in series

Pumps in Series

Let n = Number of identical impellers mounted on the same shaft.

H_m = Head developed by each impeller.

Then the total head developed = $n \times H_m$.

The discharge passing through each impeller is same.

Pumps in Parallel

Let n = number of identical pumps arranged in parallel.

Q = Discharge from one pump.

Total discharge = $n \times Q$.

Each pump works against the same head.

AXIAL FLOW PUMPS

DESCRIPTION

An axial flow pump consists of a propeller type impeller running in a casing with fine clearances between the blade tips and the casing inner walls. The fluid essentially passes almost axially through alternate rows of the fixed stator blades and moving rotor blades in a multistage axial flow pump. The figure (8.13) shows a single stage axial flow pump.

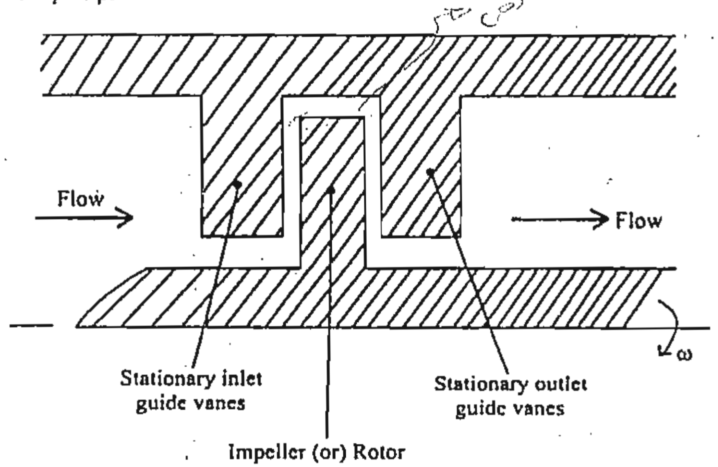


Figure 8.13 Single stage axial flow pump

The inlet guide vanes guide the fluid to enter the rotor with a purely axial velocity. The impeller blades, however impart a whirl component to the fluid. The outlet guide vanes eliminate the swirl on the outlet side and turn the flow towards the axis.

To ensure a smooth flow without shock at the design condition, the blades of the impeller must be twisted. The flow area is the same at inlet and outlet and the maximum head for this type of pump is of the order of 20 m. The usual number of blades lies between two and eight, with a hub diameter to impeller diameter ratio of 0.3 – 0.6. The advantage of an axial flow pump is its compact construction as well as its ability to run at extremely high speeds.

VELOCITY TRIANGLES

The inlet and outlet velocity triangles are drawn for axial flow pump (Fig. 8.14). It will be noticed that the impeller blade has an aerofoil section and that the inlet relative velocity vector W_1 does not impinge tangentially but rather the blade is inclined at an angle of incidence i to the relative velocity vector W_1. This is similar to the angle of attack of an aerofoil in a free stream. It is assumed that there is no shock at entry and that the fluid leaves the blade tangentially at exit.

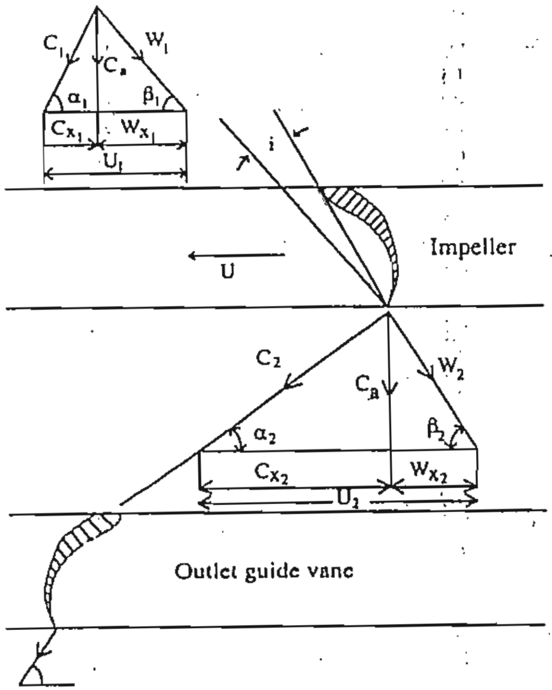


Figure 8.14 Velocity traingles of axial flow pumps

Since the stator is fixed, ideally the absolute velocity C_1 is parallel to the trailing edge of the blade (not shown in Fig. 8.14). The relative velocity W_1 is obtained by vectorially subtracting the impeller tangential velocity $U(= U_1 = U_2)$ from C_1. W_1 ideally should be parallel to the rotor leading edge of the rotor.

In the exit velocity diagram, the relative velocity vector W_2 is parallel to the blade trailing edge.

THE WORK DONE ON THE FLUID

From Euler's pump equation, the work done on the fluid

$$\boxed{W = m(U_2 C_{x_2} - U_1 C_{x_1})}$$

Changes in the condition of the fluid take place at a constant mean radius, therefore

$$U_1 = U_2 = U = \omega r$$

Also assuming a constant flow area from the inlet to outlet, we have

$$C_{r_1} = C_{r_2} = C_r = C_a$$

It should be noted that the flow area is the annulus formed between the hub and the blade tips. Then we may write

$$m = \rho C_a[\pi(R_t^2 - R_h^2)]$$

The work done by the pump is

$$\boxed{W = mU(C_{x_2} - C_{x_1})}$$

THE ENERGY TRANSFER OR HEAD

The energy transfer (E) is given by

$$E = \frac{W}{mg} = U(C_{x_2} - C_{x_1})/g$$

For maximum energy transfer the absolute flow velocity must be axial at the inlet i.e. $C_1 = C_a$. Hence,

$$\alpha_1 = 90^\circ \quad C_{x_1} = 0 \quad E = UC_{x_2}/g$$

E in terms of angle β_2 is obtained as

$$\cot\beta_2 = (U - C_{x_2})/C_a$$

or

$$C_{x_2} = U - C_a \cot\beta_2$$

Hence substituting for C_{x_2} in the above equation,

$$\boxed{E = U(U - C_a \cot\beta_2)/g}$$

This is the maximum energy transfer or head developed by an axial flow pump.

For E to be constant over the whole blade length, the blade angle β_2 should be increased because U increases with radius. It is seen from the above equation that there must be a corresponding increase in the term $C_a \cot\beta_2$. Since C_a is constant, β_2 should increase. Hence, the blade must be twisted as the radius increases.

Note: Since the density change across an axial flow fan is so low, it may be considered to be operating with an incompressible fluid and therefore the same equations as applied to axial flow pumps may be used.

AXIAL PUMP CHARACTERISTICS

Typical head-flow power and efficiency curves are shown in fig 8.15. A steep negative slope is evident on the head and power curves at low flow rates. This can be explained considering the maximum energy transfer equation. For a given blade design at fixed speed with axial flow at inlet

$$\boxed{E = U(U - C_a \cot\beta_2)/g}$$

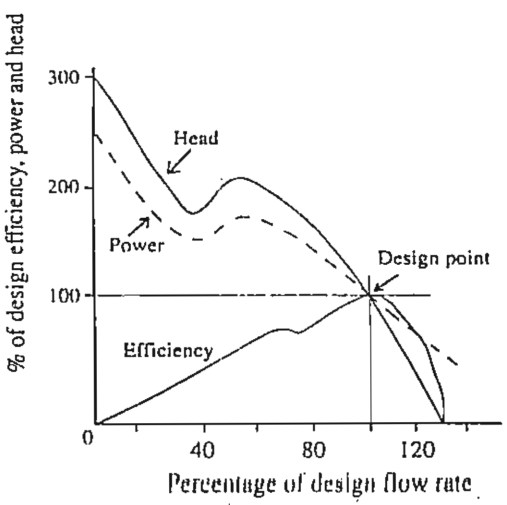


Figure 8.15 *Characteristic curves of axial flow pump*

Now Q is proportional to C_a and therefore

$$dE/dC_a \propto dE/dQ \propto -U \cot \beta_2$$

For axial flow at inlet, β_2 is relatively small, and thus for a given pump at a given speed the head-flow relationship has a steep negative slope. The power curve is similarly very steep, the power requirement at shut off being perhaps 2 – 2.5 times that required at the design point. This makes for a very expensive electric motor to cover the eventuality at the low flow rates and so the fixed blade axial flow pump is limited to operation at the fixed design point. In variable flow machines, the blade angle is adjusted so that the pump runs at its maximum efficiency at all loads and reduces the shut off power requirement.

In the figure 8.15, the power and head curves are seen to enter a region of instability at about 50 percent of the design flow rate. This is due to C_a becoming increasingly small and thereby increasing the angle of incidence of flow onto the blade until separation and stalling of the blade occurs. Further head rise at even lower flow rates and the consequent power rise is due to the recirculation of the fluid around the blade from the pressure side to the suction side and then up onto the pressure side of the next blade. An increased blade stagger angle will once again reduce this recirculation, thereby the power requirement.

CAVITATION

Cavitation is defined as a phenomenon of formation of vapour bubbles of a flowing liquid in the region where the local absolute static pressure of the liquid falls below the vapour pressure of the liquid and the sudden collapsing of these vapour bubbles in the high pressure region.

When the liquid flows through a centrifugal pump, the static pressure (suction pressure) at the eye of the impeller is reduced and the velocity increases. Therefore, there is a danger that cavitation bubbles may form at the inlet to the impeller. When the fluid moves into a higher pressure region, these bubbles collapse with tremendous force, giving rise to pressures as high as 3500 atm. Local pitting of the impeller can result when the bubbles collapse on a metallic surface, and serious damage can occur by this prolonged cavitation erosion. Noise is also generated in the form of sharp cracking sounds when cavitation takes place.

Cavitation is most likely to occur on the suction side of the pump between the lower reservoir surface and the pump inlet since it is in this region that the lowest pressure will occur. A cavitation parameter is defined as

$$\sigma = \frac{\text{Pump total inlet head above vapour pressure}}{\text{Head developed by pump}}$$

At the inlet flange

$$\sigma = \frac{P_i/\rho g + V_i^2/2g - P_{vap}/\rho g}{H_m}$$

where all pressures are absolute

(or)

$$\sigma = \frac{NPSH}{H_m}$$

Every pump has a critical cavitation number (σ_c), which can only be determined by testing to find the minimum values of *NPSH* before cavitation occurs. One method is to determine the normal head-flow characteristic of the pump and then to repeat the test with the inlet to the pump progressively throttled so as to increase the resistance to flow at the inlet (Fig. 8.16). It will be found that for different throttle valve settings the performance curve will fall away from the normal operating curve at various points

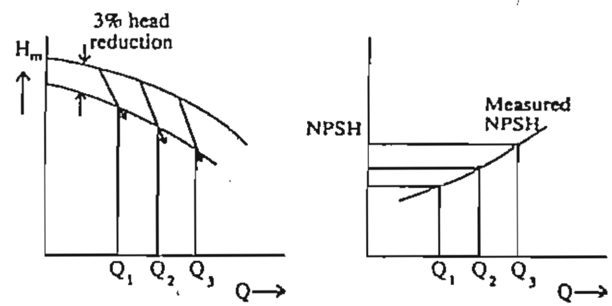


Figure 8.16 *H_m and NPSH versus Q*

and one definition of the occurence of minimum *NPSH* is the point at which the head *H* drops below the normal operating characteristic by some arbitrarily selected percentage, usually about 3 percentage. At this condition static inlet pressure P_i and inlet velocity V_i are measured, and σ_c is then calculated from the above equation. The minimum required *NPSH* or σ_c may then be plotted for the different degrees of inlet throttling to give a curve of σ_c vesus flow co-efficient.

From the steady flow energy equation, the energy loss between the free surface (A) and the inlet side of the pump can be determined. If the datum is placed at the lower reservoir surface, $V_A = 0$ and $Z_A = 0$. Then the equation interms of heads becomes

$$P_A/\rho g = P_i/\rho g + V_i^2/2g + H_{suction}$$

where $H_{suction} = Z_i + h_{fi} + h_{loss}$ at inlet. Substitution for $P_i/\rho g$ in the defining equation of σ, gives

$$\boxed{\sigma = \frac{(P_A/\rho g - P_{vap}/\rho g - H_{suction})}{H_m}}$$

If σ is above σ_c, cavitation will not occur. But in order to achieve this, it may be necessary to decrease $H_{suction}$ by decreasing Z_i and in some cases the pump may have to be placed below the reservoir or pump free surface. i.e. negative Z_i, especially if h_{fi} is particularly high due to a long inlet pipe.

METHOD OF PREVENTING CAVITATION

The following factors should be taken into account to prevent cavitation:

1. The pressure of the flowing liquid in any part of the hydraulic system should not be allowed to fall below its vapour pressure. For example, the absolute pressure head should not be below 2.5 m of water, if water is the working medium.
2. The special materials or coatings such as aluminium, bronze and stainless steel which are cavitation resistant materials should be used.

POSITIVE DISPLACEMENT PUMPS

The pumps are broadly classified into two as (a) Hydrodynamic (or) non- positive displacement pumps; ex: centrifugal and axial pumps and (b) Hydrostatic (or) positive displacement pumps; ex: Gear pumps and vane pumps. The former type is used for low-pressure, high-volume flow applications. These pumps are not self-priming. This is because there is too much clearance space between the rotating and stationary elements and to seal against atmospheric pressure and thus the displacement between the inlet and outlet is not a positive one. The latter type of pump ejects a fixed quantity of fluid per revolution of the pump shaft. A pressure relief valve is used to protect the pump against overpressure, because a positive displacement pump continues to eject fluid even though the outlet valve is fully closed causing an extremely rapid build up in pressure as the fluid is compressed. Where as, for non-positive displacement pumps, in such a case, there is no need for safety devices to prevent pump damage.

Positive displacement pumps can be classified by the type of motion of internal elements. The motion may be either rotary or reciprocating.

There are essentially of three basic types;

1. Gear pumps

   (a) External gear pumps
   (b) Internal gear pumps
   (c) Lobe pumps
   (d) Screw pumps
2. Vane pumps
3. Piston pumps

The details of the construction and operation of gear, vane and piston pumps are discussed in the following sections.

1. Gear Pumps

(a) External gear pump Fig. (8.17) illustrates the operation of an external gear pump which develops flow by carrying fluid between the teeth of two meshing gears.

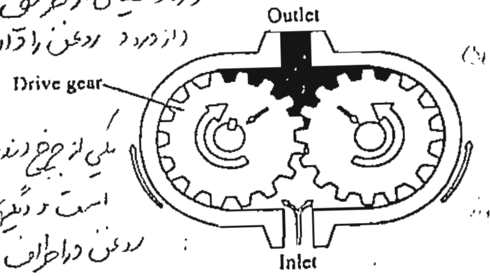


Figure 8.17 External gear pump

One of the gears is connected to a drive shaft connected to the primemover. The second gear is driven as it meshes with the driver gear. Oil is carried around housing in chambers formed between teeth, housing and side wear plates. The suction side is where teeth come out of mesh, and it is the place where the volume expands, bringing about a reduction in pressure below atmospheric pressure. Fluid is pushed into this void by atmospheric pressure because the oil supply tank is vented to the atmosphere. The discharge side is where teeth go into mesh, and it is here where the volume decreases between mating teeth.

The theoretical flow rate of a gear pump is given by

$$\boxed{Q_{th} = V_D \times N}$$

where, Q_{th} is the theoretical flow rate in m^3/min, V_D is the pump displacement volume in m^3/rev and N is the pump speed in rpm. The displacement volume V_D is

$$\boxed{V_D = \frac{\pi}{4}(D_o^2 - D_i^2)L}$$

where,

D_o - outside diameter of gear teeth, m
D_i - inside diameter of gear teeth, m
L - width of gear teeth, m

The external gear pump uses spur gears. (which are noisy at high speeds), helical gears (which are limited to low pressure application due to end thrust problems) and herringbone gears (which provide greater flow rates smoothy).

(b) Internal gear pump Fig. (8.18) shows the configuration of the internal gear pump. This pump consists of an internal gear, a regular spur gear, a crescent shaped seal and an external housing.

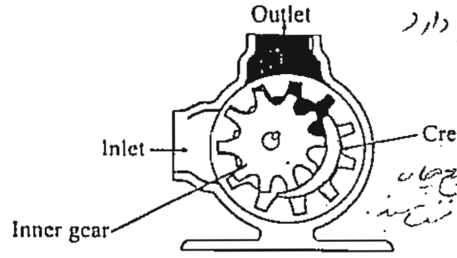


Figure 8.18 Internal gear pump

As power is applied to either gear, the motion of the gears draws fluid from the reservoir and forces it around both sides of the crescent seal, which acts as a seal between the suction and discharge ports. When the teeth mesh on the side opposite to the crescent seal, the fluid is forced to enter the discharge port of the pump.

(c) Lobe pump The lobe pump is illustrated in Fig. (8.19). This pump operates in a fashion similar to the external gear pump. But unlike the external gear pump, both lobes are driven externally so that they do not actually contact each other. Thus, they are quiter than other types of gear pumps. Due to the smaller number of mating elements, the lobe pump's output will have a greater amount of pulsation, although its volumetric displacement is generally greater than for other types of gear pumps.

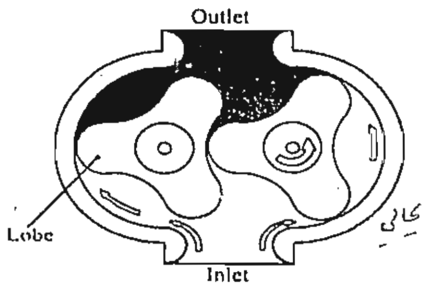


Figure 8.19 Lobe pump

(d) Screw pump The screw pump (Fig. 8.20) is an axial flow positive displacement unit. It consists of a rotor or screw to which the source of power i.e. electric motor is directly connected. The screw may be single helical or double helical. The advantage of double helical is that they are balanced axially. The fluid is carried forward to the discharge along the rotor in pockets formed between teeth and the casing.

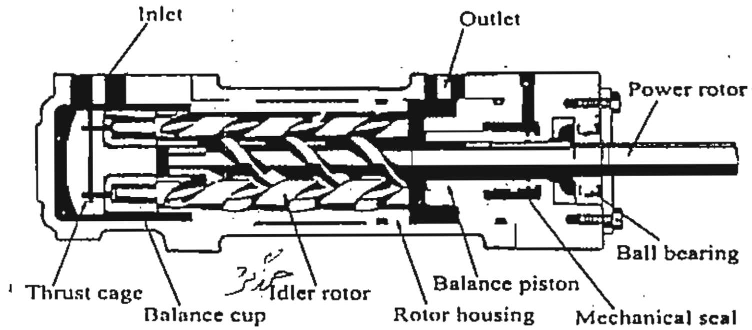


Figure 8.20 Screw pump

There may be one, two or three screws. In a 2-screw pump, one is rotor and the other is idler. In a 3-screw pump there are two idlers on either side of the rotor. Two idlers act as seals to the power rotor and are driven by fluid pressure and not by metallic contact with rotor. The liquid entering the inlet passage flows into the left end of the rotor where it is trapped in pockets formed by the threads and is carried towards the discharge. The movement of fluid is similar to that of a nut on the power screw.

The advantages of screw pump over the other types of pump are:

1. Screw pump operates upto a continuous working pressure of about 150 bar.
2. The screw pump is free from pulsation and vibration. So, the pump is silent while running.

2. Vane Pump

Vane pump (Fig. 8.21) consists of a rotor disc having a number of radial slots and is splined to the drive shaft. The rotor rotates inside a cam ring. Each slot has a vane designed to mate with the surface of the cam ring as the rotor turns. There is an eccentricity between the rotor and the cam ring. The vanes are free to slide radially with the help of springs and thus form the required seal between the suction and discharge connections. Centrifugal force keeps the vanes out against the surface of the cam ring.

mate :

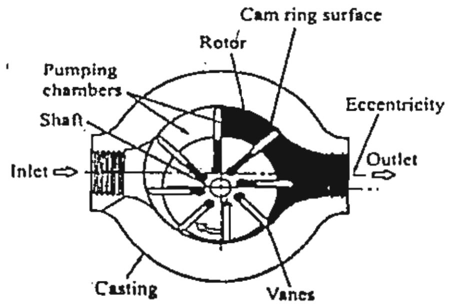


Figure 8.21 Vane pump

During one-half revolution of rotor rotation, the volume increases between the rotor and cam ring. The resulting volume expansion causes a reduction of pressure. This is the suction process which causes fluid to flow through the inlet port and fill the void. As the rotor rotates through the second half revolution, the surface of the cam ring pushes the vanes back into their slots, and the trapped volume is reduced resulting in increase of pressure. This positively ejects the trapped fluid through the discharge port.

3. Piston Pumps

A piston pump works on the principle that a reciprocating piston can draw in fluid when it retracts in a cylinder bore and discharge it at high pressures when it extends. There are two basic types of piston pumps (a) axial piston pump and (b) radial piston pump.

(a) Axial piston pumps Fig. (8.22) shows an axial piston pump. In axial piston pumps, the pistons are parallel to the axis of the cylinder block. It consists of a cylinder block rotating with the drive shaft. A universal link connects the block to the drive shaft to provide alignment and positive drive. The centre line of the cylinder block is set at an offset angle relative to the centre line of the drive shaft. The cylinder block contains a number of pistons arranged along a circle. The piston rods are connected to the drive shaft flange by ball and socket joints. The pistons are forced in and out of their bores as the distance between the drive shaft flange and cylinder block changes.

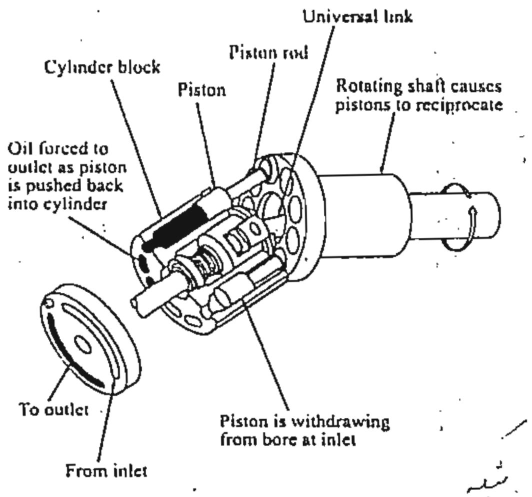


Figure 8.22 Axial piston pump

(b) Radial piston pump A radial pump is illustrated in Fig. (8.23) It consists of pintle to direct fluid in and out of the cylinders, a cylinder barrel with pistons, and a rotor containing a reaction ring. The pistons remain in constant contact with the reaction ring due to centrifugal force and back pressure on the pistons. For pumping action, the reaction ring is moved eccentrically with respect to the pintle or shaft axis.

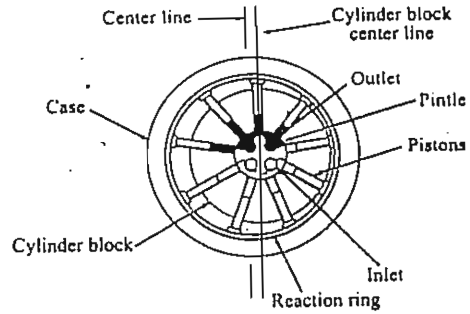


Figure 8.23 Radial piston pump

As the cylinder barrel rotates, the pistons on one side move outward. This draws in fluid, as each cylinder passes the suction ports of the pintle. When a piston passes the point of maximum eccentricity, it is forced inward by the reaction ring. This forces the fluid to enter the discharge port of the pintle.

MISCELLANEOUS TYPES OF PUMP

Jet Pump

A jet pump is shown in Fig. (8.24) Jet pumps are used to lift the water in a greater quantity when a small quantity of water is available deep down the earth. Steam (or) water at a high pressure is forced through a fine aperture nozzle, thereby converting most of the pressure energy into kinetic energy. It results in lowering of pressure causing suction to take place. A part of the suction is due probably to the drop of pressure resulting from condensation of the steam. Steam is generally used in jet pumps used to feed the water to a boiler.

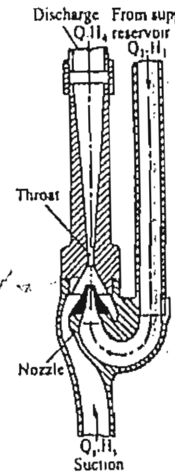


Figure 8.24 Jet pump

The nozzle can lower the pressure to three fourths of atmospheric pressure, i.e. about 2.5 m suction head can be obtained. Steam also serves to preheat the water fed to the boiler. With water at high pressure, a more perfect vaccum can be produced, so that a sucion lift of 5.5 to 6m can be obtained. The jet should be near the surface of water. Several jets may be employed if a large quantity of water is to be pumped. The capacity of a jet pump ranges upto about 50 l/s.

If the rate of flow from supply reservoir is Q_1 and the quantity of water sucked per second is Q_s, then the total discharge rate from the pump is $Q = Q_1 + Q_s$

If H_1 is the height of supply reservoir above the jet, H_s is the suction head and H_d is the delivery head, then efficiency of the jet pump is given by

$$\eta_j = \frac{\text{work done}}{\text{work consumed}}$$

$$\boxed{\eta_j = \frac{Q_s(H_s + H_d)}{Q_1(H_1 - H_d)}}$$

Air Lift Pump

The pumps that utilise compressed air to lift water are known as air lift pumps. The function of compressed air is to form a mixture of air and water and to reduce the mixture density. The density of a mixture of air and water is much lower than that of pure water. If such a mixture is balanced against a water column, the former will rise much higher than the latter.

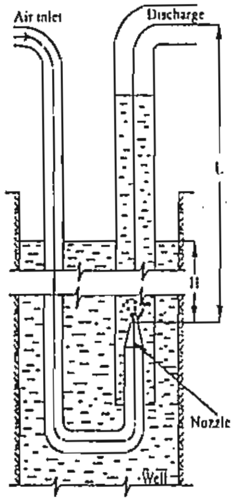


Figure 8.25 Air lift pump

Fig. (8.25) shows the air lift pump. Compressed air, supplied by a compressor installed on (or) above the ground level, is passed through one or more nozzles at the bottom into the water at a considerable depth, below the water surface. The rapid stream of air at the nozzle exit produce a jet and entraps the water in the immediate

vicinity and caries it upwards to the top of delivery line. The air-water mixture can rise above the water level because of its low density.

The air is introduced at a considerable depth below the water surface, so that the pressure due to the column of height L is less than that due to height H, height of water level above the nozzle, thus, causing the flow of water to the desired delivery level. The pump discharges water as long as there is a supply of air. For best results, the lift of the pump $(L - H)$ should be less than H.

Advantages

1. No moving parts, no valves. No damage due to solids in suspension in water.
2. Raises larger quantity of water for a given diameter than any other pump.
3. To drain out water from mines where the compressors are already available

Disadvantages

1. Very low efficiency. The volume of air in m^3 (V) at atmospheric pressure, required to lift one m^3 of water through a height $(L - H)$ depends on the efficiency.
2. Air leakage problem.

Submersible Pump

As the name implies, the pump and electric motor both are submerged in water. Submersible pumps are built together as single or multistage centrifugal pumps, directly coupled with the electric motor, which is totally enclosed. The motor and pump shafts are supported in water lubricated plain bearings.

These pumps are vertical centrifugal pumps with radial or mixed flow impellers. All the metallic bearings are water lubricated and protected against the sand. A non return valve is fitted to a flange at the top of the pump. The suction line of the pump is situated between the pump and the motor and provided with a perforated strainer. Motor of the submersible pumps are filled completely with water. The water cools the windings, insulated in a plastic impervious to water. The pump shaft is connected to the motor shaft by a muff coupling.

Submersible pumps are installed below the lowest expected water level in the well. These pumps can be usually employed in open wells where frequent changes of water level make the working of a horizontal pump difficult.

Before a submersible pump is installed, the motor must be filled with clean non-acid water free of sand for the purpose of priming. The pump must be started with the valve slightly open. If there is a considerable amount of sand in the water, then the pump must be operated with valve partly open, until the sand contents are reduced to acceptable quantity. The valve may then be opened gradually.

Advantages

1. High overall efficiency and therefore economical in operation.
2. No maintenance because of its glandless construction and water lubrication.
3. No priming is required due to submerged installations.
4. No suction problem, since foot valve is not required.
5. Low noise level.

SOLVED PROBLEMS

Example 8.1 A centrifugal pump of 1.3 m diameter delivers 3.5 m^3/min of water at a tip speed of 10 m/s and a flow velocity of 1.6 m/s. The outlet blade angle is at 30° to the tangent at the impeller periphery. Assuming zero whirl at inlet and zero slip, calculate the torque delivered by the impeller.

Solution There is no slip. Therefore,

$$\beta_2 = \beta_2'$$

The Euler head is given by

$$\boxed{E = \frac{W}{mg} = \frac{U_2 C_{x_2} - U_1 C_{x_1}}{g}}$$

Since, there is no inlet whirl component, $C_{x_1} = 0$

$$E = \frac{U_2 C_{x_2}}{g}$$
$$= \frac{U_2}{g}(U_2 - W_{x_2})$$

Given $\beta_2 = 30°$. From the outlet velocity triangle (Refer Fig. 8.2)

$$W_{x_2} = \frac{C_{r_2}}{\tan 30°}$$
$$= \frac{1.6}{\tan 30°}$$

$$E = \frac{10}{9.81}\left(10 - \frac{1.6}{\tan 30°}\right)$$
$$= 7.36 \text{ m or W/(N/S)}$$

$$\text{Power delivered } (W) = E \times (mg)$$
$$m = \rho \times Q$$
$$= 1000 \times 3.5/60$$
$$= 58.33 \text{ kg/s}$$

$$\therefore \text{Power delivered } (W) = 7.36 \times 58.33 \times 9.81$$
$$= 4211.8 \text{ W}$$

$$\text{Torque delivered} = \frac{\text{Power}}{\text{Angular velocity}}$$
$$= 4211.8 \times \frac{1}{(U/r)}$$

$$U = \omega r, \quad \text{tip speed } U = 10 \text{ m/s}$$
$$r = 1.3/2 = 0.65 \text{ m}$$

$$\text{Torque delivered} = \frac{4211.8 \times 0.65}{10}$$
$$= 273.7 \text{ Nm}$$

Example 8.2 The impeller of a centrifugal pump has backward-facing blades inclined at 30° to the tangent at impeller outlet. The blades are 20 mm in depth at the outlet, the impeller is 250 mm in diameter and it rotates at 1450 rpm. The flow rate through the pump is 0.028 m^3/s and a slip factor of 0.77 may be assumed. Determine the theoretical head developed by the impeller and the number of impeller blades.

Solution First consider the no-slip condition.

Assuming blades of infinitesimal thickness the flow area may be calculated as

$$\text{Flow area} = \text{Impeller periphery} \times \text{Blade depth}$$
$$= \pi \times 0.02 \times 0.25$$
$$= 15.7 \times 10^{-3} \text{ m}^2$$
$$\text{Flow velocity } C_{r_2} = Q/A$$
$$= 0.028/(15.7 \times 10^{-3})$$
$$= 1.78 \text{ m/s}$$

From the outlet velocity triangle (Fig. 8.2)

$$W_{x_2} = \frac{C_{r_2}}{\tan 30°}$$
$$= 1.78/\tan 30°$$
$$= 3.08 \text{ m/s}$$

Now,

$$U_2 = \pi DN/60$$
$$= \pi \times 0.25 \times 1450/60$$
$$= 19 \text{ m/s}$$

Absolute whirl component

$$C_{x_2} = U_2 - W_{x_2}$$
$$= 19 - 3.08$$
$$= 15.92 \text{ m/s}$$

The Euler head is

$$E = \frac{U_2 C_{x_2} - U_1 C_{x_1}}{g}$$

and assuming $C_{x_1} = 0$ (no whirl at inlet)

$$E = \frac{U_2 C_{x_2}}{g}$$
$$= \frac{19 \times 15.92}{9.81}$$
$$= 30.83 \text{ m}$$

The slip factor (σ) is given by

$$\sigma_s = \frac{C'_{x2}}{C_{x2}}$$

$$\therefore \quad C'_{x2} = 0.77 \times 15.92 = 12.26 \text{ m/s}$$

Therefore, the theoretical head with slip is

$$E_s = \frac{U_2 C'_{x2}}{g} = \sigma_s . E$$
$$= 0.77 \times 30.83$$
$$= 23.74 \text{ m/s}$$

The stodola slip factor is given by

$$\boxed{\sigma_s = 1 - \frac{\pi \sin \beta_2}{Z(1 - (C_{r_2}/U_2) \cot \beta_2)}}$$

Then,

$$0.77 = 1 - \frac{\pi \sin 30^\circ}{Z[1 - (1.78/19) \cot 30^\circ]}$$
$$Z = 8.15$$

Number of blades required = 8.

Example 8.3 The outer diameter of an impeller of a centrifugal pump is 40 cm and outlet width is 5 cm. The pump is running at 800 rpm and working against a head of 16 m. The vane angle at outlet is 40°. Assuming the manometric efficiency to be 75%. Determine the discharge. *(MKU-Nov. '89.)*

Solution

$D_2 = 0.4$ m, $b_2 = 0.05$ m, $N = 800$ rpm, $H_m = 16$ m, $\beta_2 = 40°$, $\eta_m = 0.75$.

The impeller tip speed

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 800}{60} = 16.76 \text{ m/s}$$

From the definition of manometric efficiency

$$\eta_m = \frac{H_m}{(U_2 C_{x2}/g)}$$

$$C_{x2} = \frac{g \times H_m}{U_2 \eta_m} = \frac{9.81 \times 16}{16.76 \times 0.75} = 12.46 \text{ m/s}$$

and from the outlet velocity triangle (Refer Fig. 8.2)

$$W_{x2} = U_2 - C_{x2} = 16.76 - 12.46 = 4.3 \text{ m/s}$$
$$C_{r2} = W_{x2} \tan \beta_2 = 4.3 \times \tan 40^\circ = 3.6 \text{ m/s}$$

Now,

$$Q = \text{area of flow} \times \text{Velocity}$$
$$= \pi D_2 b_2 \times C_{r2}$$
$$= \pi \times 0.4 \times 0.05 \times 3.6$$
$$= 0.2262 \text{ m}^3/\text{s}$$

Example 8.4 A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1200 rpm, works against a total head of 75 m. The velocity of flow through the impeller is constant and equal to 3 m/s. The vanes are set back at an angle of 30° at outlet. If the outlet diameter of the impeller is 60 cm and width at outlet is 5 cm, determine (a) vane angle at inlet (b) workdone per sec by impeller and (c) manometric efficiency. *(MKU-Nov. '96)*

Solution

$D_2 = 2D_1$, $N = 1200$ rpm, $H_m = 75$ m, $C_{r1} = C_{r2} = 3$ m/s, $\beta_2 = 30°$
$D_2 = 0.6$ m, $b_2 = 0.05$ m

(a) Vane angle at inlet The inlet velocity triangle for a centrifugal pump is shown in Fig. 8.26

$$\tan \beta_1' = \frac{C_{r1}}{U_1}$$

$$D_1 = \frac{D_2}{2} = \frac{0.6}{2} = 0.3 \text{ m}$$

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.3 \times 1200}{60} = 18.85 \text{ m/s}$$

$$\tan \beta_1 = \frac{3}{18.85}$$

$$\beta_1 = 9.043^\circ$$

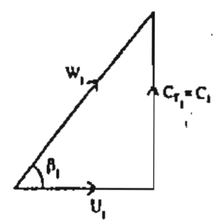


Figure 8.26

(b) Workdone per second

$$W = \dot{m} U_2 C_{x2}$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 1200}{60} = 37.7 \text{ m/s}$$
$$\dot{m} = \rho Q = \rho(\pi D_2 b_2 \times C_{r2})$$
$$= 10^3 \times \pi \times 0.6 \times 0.05 \times 3$$
$$= 282.7 \text{ kg/s}$$

From outlet velocity diagram,

$$C_{x2} = U_2 - W_{x2} = U_2 - \frac{C_{r2}}{\tan\beta_2}$$
$$= 37.7 - \frac{3}{\tan 30^\circ} = 37.7 - 5.196$$
$$= 32.504 \text{ m/s}$$
$$\therefore W = 282.7 \times 37.7 \times 32.504$$
$$W = 346.42 \text{ kW}$$

(c) Manometric efficiency

$$\eta_m = \frac{H_m}{(U_2 C_{x2}/g)}$$
$$= \frac{75 \times 9.81}{37.7 \times 32.504}$$
$$\therefore \eta_m = 60.04\%$$

Example 8.5 A radial, single stage, double suction, centrifugal pump is manufactured for the following data.

$Q = 75$ l/s	$D_1 = 100$ mm	$D_2 = 290$ mm
$H_m = 30$ m	$N = 1750$ rpm	$b_1 = 25$ mm per side
$b_2 = 23$ mm in total	$\alpha_1 = 90°$	$\eta_0 = 55\%$
Leakage loss = 2.25 l/s	Mechanical loss = 1.04 kW	$\beta_2 = 27°$

contraction factor due to vanes thickness = 0.87.

Determine (a) the inlet vane angle (b) the angle at which the water leaves the wheel (c) absolute velocity of water leaving impeller (d) the manometric efficiency and (e) the volumetric and mechanical efficiencies.

Solution

(a) Inlet vane angle

The blade speed at inlet,

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.1 \times 1750}{60} = 9.16 \text{ m/s}$$

Area of flow at inlet

$$A_1 = \pi D_1 b_1 \times \text{Contraction factor}$$
$$= \pi \times 0.1 \times 0.025 \times 0.87$$
$$= 6.83 \times 10^{-3} \text{ m}^2$$

$\therefore$ Velocity of flow at inlet

$$C_{r1} = \frac{Q}{A_1}$$

Total quantity of water handled by the pump

$$Q_t = Q + Q_{Loss} = 75 + 2.25$$
$$= 77.25 \text{ l/s}$$
$$Q_t \text{ per side} = \frac{77.25}{2} = 38.625 \text{ l/s}$$
$$\therefore C_{r1} = \frac{38.625 \times 10^{-3}}{6.83 \times 10^{-3}}$$
$$= 5.66 \text{ m/s}$$

Now,

$$\alpha_1 = 90°$$

From inlet velocity triangle, (Fig. 8.26(a))

$$\tan\beta_1 = \frac{C_{r1}}{U_1} = \frac{5.66}{9.16}$$
$$\beta_1 = 31.71°$$

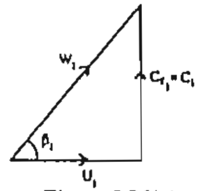


Figure 8.26(a)

(b) The absolute water angle

Area of flow at outlet = $\pi D_2 b_2 \times$ Contraction factor

where $b_2 = 23 \times \frac{1}{2} = 11.5$ mm per side

$$A_2 = \pi \times 0.29 \times 0.0115 \times 0.87$$
$$= 9.12 \times 10^{-3} \text{ m}^2$$

Velocity of flow at outlet

$$C_{r2} = \frac{Q}{A_2} = \frac{38.625 \times 10^{-3}}{0.00912}$$
$$= 4.24 \text{ m/s}$$

Peripheral speed at outlet

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.29 \times 1750}{60}$$
$$= 26.57 \text{ m/s}$$

Now, $\beta_2 = 27°$

$$\tan\alpha_2 = \frac{C_{r2}}{C_{x2}}$$

where

$$C_{x2} = U_2 - W_{x2} = U_2 - \frac{C_{r2}}{\tan\beta_2}$$
$$= 26.57 - \frac{4.24}{\tan 27°}$$
$$= 18.25 \text{ m/s}$$
$$\therefore \tan\alpha_2 = \frac{4.24}{18.25}$$
$$\alpha_2 = 13.08°$$

(c) Absolute velocity of water at exit

$$C_2 = \frac{C_{r2}}{\sin\alpha_2} = \frac{4.24}{\sin 13.08^\circ}$$
$$C_2 = 18.74 \text{ m/s}$$

(d) Manometric efficiency

$$\eta_H = \frac{H_m}{U_2 C_{x2}/g}$$
$$= \frac{30}{(26.57 \times 18.25)/9.81}$$
$$\eta_H = 60.7\%$$

(e) Volumetric efficiency

$$\eta_v = \frac{Q}{Qt} = \frac{75}{77.25}$$
$$\eta_v = 97.09\%$$

(f) Mechanical efficiency

$$\eta_0 = \frac{\text{Fluid power}}{\text{Shaft power}}$$

$$\therefore \text{Shaft power} = \frac{\rho g \left(\frac{Q}{2}\right) H_m}{\eta_0}$$
$$= \frac{1000 \times 9.81 \times \left(\frac{75 \times 10^{-3}}{2}\right) 30}{0.55}$$
$$= 20.07 \text{ kW}$$

$$\text{and } \eta_m = \frac{20.07 - 1.04}{20.07} \quad \left[\because \eta_m = \frac{\text{Shaft power} - \text{mechanical loss}}{\text{Shaft power}}\right]$$
$$\eta_m = 94.8\%$$

Example 8.6 During a test on the centrifugal pump, the following reading were obtained. Vaccum gauge reading = 25 cm of Hg, pressure gauge reading = 1.5 bar, effective height between guages = 0.5 m, power of electric motor = 22 kW, discharge of pump = 100 l/s, diameter of suction and delivery pipes is each 15 cm. Determine manometric head and overall efficiency of the pump.

Solution

$$H_i = 25 \text{ cm of Hg vaccum} \qquad P_0 = 1.5 \text{ bar}$$
$$Z_0 - Z_i = 0.5 \text{ m} \qquad P = 22 \text{ kW}$$
$$D = D_i = D_0 = 15 \text{ cm} \qquad Q = 0.1 \text{ m}^3/\text{s}$$

(a) Manometric head

$$H_m = (P_0 - P_i)/\rho g + \frac{V_0^2 - V_i^2}{2g} + (Z_0 - Z_i)$$
$$P_i = \rho g H_i = 13,600 \times 9.81 \times 0.25$$
$$= 33354 \text{ N/m}^2 \text{ (Vacuum)}$$
$$P_0 = 1.5 \times 10^5 \text{ N/m}^2$$

Velocity of water in both the suction and delivery pipes,

$$V_0 = V_i = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.1}{\frac{\pi}{4} \times 0.15^2}$$
$$= 5.66 \text{ m/s}$$

Therefore,

$$H_m = \left(\frac{1.5 \times 10^5 - (-33354)}{1000 \times 9.81}\right) + \left(\frac{5.66^2 - 5.66^2}{2g}\right) + 0.5$$
$$= 18.69 + 0 + 0.5$$
$$= 19.19 \text{ m}$$

(b) Overall efficiency

$$\eta_0 = \frac{\rho g Q H_m}{p}$$
$$= \frac{1000 \times 9.81 \times 0.1 \times 19.19}{22 \times 1000}$$
$$= \frac{18825.4}{22000}$$
$$= 85.6\%$$

Example 8.7 A centrifugal pump is working against a head of 20 m while rotating at the rate of 600 rpm. If the blades are curved back at outlet tip and velocity of flow remains constant at 2 m/s. Calculate the impeller diameter when 50% of the kinetic energy at impeller outlet is converted into pressure energy.

Solution

$H_m = 20$ m, $\beta_2 = 45^\circ$, $N = 600$ rpm, $C_{r_1} = C_{r_2} = 2\ m/s$
From outlet triangle for a centrifugal pump. (Refer Fig. 8.2)

$$C_{x2} = U_2 - W_{x2} = U_2 - \frac{C_{r_2}}{\tan\beta_2}$$
$$= U_2 - \frac{2}{\tan 45^\circ} = U_2 - 2$$

and

$$C_2 = \sqrt{C_{x2}^2 + C_{r2}^2} = \sqrt{(U_2 - 2)^2 + 2^2}$$

We know that,

$$H_m = \frac{U_2 C_{x2}}{g} - \text{Losses in the impeller and casing}$$

$$= \frac{U_2 C_{x2}}{g} - \text{Loss of kinetic energy}$$

$$\therefore H_m = \frac{U_2 C_{x2}}{g} - \frac{1}{2}\left(\frac{C_2^2}{2g}\right)$$

$$20 = \frac{U_2(U_2 - 2)}{g} - \frac{(U_2 - 2)^2 + 4}{4g}$$

$$3U_2^2 - 4U_2 - 792.8 = 0$$

Solving for U_2, we get

$$U_2 = 16.94 \text{ m/s}$$

but,

$$U_2 = \frac{\pi D_2 N}{60}$$

$$D_2 = \frac{60 \times 16.94}{\pi \times 600} = 0.5392 \text{ m}$$

$$(\text{or})\ D_2 = 53.92 \text{ cm}$$

Example 8.8 A centrifugal pump has an overall efficiency of 70 per cent, supplies 25 l/s of water to a height of 20 m through a pipe of 10 cm diameter and length of 100 m. Assume friction co-efficient, $f = 0.012$. Estimate the power required to drive the pump.

Solution

$\eta_o = 0.7$, $Q = 0.025 \text{ m}^3/\text{s}$, $H = 20$ m
$D = 0.1$ m, $L = 100$ m, $f = 0.012$

Loss of head due to friction in pipe

$$hf_o = \frac{4fLV_o^2}{2gD}$$

where V is the velocity of water in the pipe.

$$V_0 = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.025}{\frac{\pi}{4} \times 0.1^2} = 3.18 \text{ m/s}$$

$$\therefore hf_o = \frac{4 \times 0.012 \times 100 \times 3.18^2}{2 \times 9.81 \times 0.1} = 24.74 \text{ m}$$

Now, manometric head

$$H_m = (h_s + h_d) + hf_0 + \frac{V_o^2}{2g}$$

$$= H + hf_0 + \frac{V_0^2}{2g}$$

$$= 20 + 24.74 + \frac{3.18^2}{2 \times 9.81}$$

$$= 45.26 \text{ m}$$

Now using the relation

$$P = \frac{\rho g Q H_m}{\eta_o}$$

$$= \frac{1000 \times 9.81 \times 0.025 \times 45.26}{0.7}$$

$$= 15.86 \text{ kW}$$

Example 8.9 Find the rise in pressure in the impeller of a centrifugal pump through which water is flowing at the rate of 15 l/s. The internal and external diameters of the impeller are 20 cm and 40 cm respectively. The widths of impeller at inlet and outlet are 1.6 cm and 0.8 cm. The pump is running at 1200 rpm. The water enters the impeller radially at inlet and impeller vane angle at outlet is 30°. Neglect losses through the impeller.

Solution

$Q = 0.015 \text{ m}^3/\text{s}$, $D_1 = 0.2$ m, $D_2 = 0.4$ m, $b_1 = 0.016$ m
$b_2 = 0.008$ m, $N = 1200$ rpm, $C_1 = C_{r_1}$, $\beta_2 = 30°$

Applying Bernoulli's equation at the inlet and outlet of the impeller and neglecting losses from inlet to outlet.

Energy at inlet = Energy at outlet – workdone by impeller on water

$$\frac{P_1}{\rho g} + \frac{C_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{C_2^2}{2g} + Z_2 - \frac{C_{x2}U_2}{g}$$

If inlet and outlet of the impeller are at the same height ($Z_1 = Z_2$)

$$\frac{P_1}{\rho g} + \frac{C_1^2}{2g} = \frac{P_2}{\rho g} + \frac{C_2^2}{2g} - \frac{C_{x2}U_2}{g}$$

$$\therefore \frac{P_2}{\rho g} - \frac{P_1}{\rho g} = \frac{C_{x2}U_2}{g} - \frac{C_2^2}{2g} + \frac{C_1^2}{2g}$$

But,

$$\frac{P_2}{\rho g} - \frac{P_1}{\rho g} = \text{Pressure rise in impeller.}$$

Therefore,

$$\text{Pressure rise in impeller} = \frac{C_{x2}U_2}{g} - \frac{C_2^2}{2g} + \frac{C_1^2}{2g}$$

From inlet velocity triangle, (Fig. 8.27)

$$C_1 = C_{r1} \quad \alpha_1 = 90^\circ, \quad C_{x1} = 0$$

$$C_{r1} = \frac{Q}{\text{Area of flow at inlet}}$$

$$\frac{Q}{\pi D_1 b_1} = \frac{0.015}{\pi \times 0.2 \times 0.016} = 1.492 \text{ m/s}$$

$$\therefore C_1 = 1.492 \text{ m/s}$$

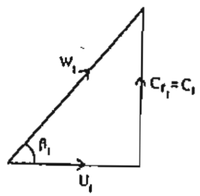


Figure 8.27

From outlet velocity triangle, (Fig. 8.28)

$$\tan\beta_2 = \frac{C_{r2}}{U_2 - C_{x2}}$$

$$U_2 - C_{x2} = \frac{C_{r2}}{\tan\beta_2}$$

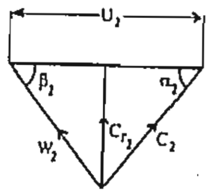


Figure 8.28

$$C_{r2} = \frac{Q}{\pi D_2 b_2} = \frac{0.015}{\pi \times 0.4 \times 0.008} = 1.492 \text{ m/s}$$

$$\therefore U_2 - C_{x2} = \frac{1.492}{\tan 30^\circ} = 2.584 \text{ m/s}$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1200}{60} = 25.133 \text{ m/s.}$$

$$\therefore C_{x2} = U_2 - 2.584 = 25.133 - 2.584$$

$$C_{x2} = 22.55 \text{ m/s}$$

Velocity at impeller exit

$$C_2 = \sqrt{C_{x2}^2 + C_{r2}^2} = \sqrt{22.55^2 + 1.492^2} = 22.6 \text{ m/s}$$

$$\therefore \text{Pressure rise} = \frac{25.133 \times 22.55}{9.81} - \frac{22.6^2}{2 \times 9.81} + \frac{1.492^2}{2 \times 9.81} = 31.853 \text{ m}$$

Example 8.10 In a centrifugal pump handling liquid water the head loss in the impeller is 3m, while the pressure gain in the casing is 4.7 m of water, which is 38.5% of the absolute kinetic energy at the impeller exit. If the velocity of flow at the exit of impeller is 4.64 m/s and the impeller tip speed is 30 m/s, when the difference in pressure gauge readings installed at the impeller inlet and outlet is 35.3 m of water find (a) the exit blade angle and the Euler work input (b) manometric efficiency (c) the actual pressure rise in the impeller. Assume slip coefficient of 0.85.

(MKU–April '96)

Solution

Impeller head loss - 3 m

$$\frac{C_2^2}{2g} = \frac{1}{0.385} \times 4.7 = 12.21 \text{ m/s} \quad C_{r2} = 4.64 \text{ m/s}$$

$$U_2 = 30 \text{ m/s} \quad (\Delta P)_{impeller} = 35.3 \text{ m}$$

(a) Exit blade angle and Euler work input

Kinetic energy at impeller exit

$$C_2 = \sqrt{2 \times 9.81 \times 12.21} = 15.48 \text{ m/s}$$

From the outlet velocity triangle with fluid slip at impeller exit (refer Fig. 8.3).

$$C'_{x2} = \sqrt{C_2^2 - C_{r2}^2} = \sqrt{15.48^2 - 4.64^2} = 14.77 \text{ m/s}$$

Ideal absolute whirl velocity

$$C_{x2} = \frac{C'_{x2}}{\sigma_s} = \frac{14.77}{0.85} = 17.38 \text{ m/s}$$

The blade angle at exit

$$\tan\beta_2 = \frac{C_{r2}}{U_2 - C_{x2}} = \frac{4.64}{30 - 17.38}$$

$$\beta_2 = 20.18^\circ$$

Euler work input

$$W/m = \sigma_s U_2 C_{x2} = 0.85 \times 30 \times 17.38 = 443.19 \text{ J/kg}$$

(b) Manometric efficiency

$$\eta_m = \frac{H_m}{(U_2 C'_{x2}/g)} = \frac{(\Delta P)_{impeller}}{(U_2 C'_{x2}/g)}$$

$$= \frac{35.3}{(30 \times 14.77/9.81)} = \frac{35.3}{45.168}$$

$$\eta_m = 78.15\%$$

(c) Pressure rise in the impeller

$$= \frac{U_2 C'_{x2}}{g} - \text{impeller loss} - \frac{C_2^2}{2g}$$

$$= 45.168 - 3 - 12.21$$

$$= 29.958 \text{ m}$$

Example 8.11 An impeller with an eye radius of 51 mm and an outside diameter of 406 mm rotates at 900 rpm. The inlet and outlet blade angles measured from the radial flow direction are 75° and 83° respectively, while the blade depth is 64 mm. Assuming zero inlet whirl, zero slip and an hydraulic efficiency of 89 percent. Calculate,

(a) the volume flow rate through the impeller
(b) the stagnation and static pressure rise across the impeller
(c) the power transferred to the fluid and
(d) the input power to the impeller

Solution Since there is no slip,

$$\beta'_2 = \beta_2$$

(a) Volume flow rate through the impeller

The outlet blade angle measured from the tangential direction

$$\beta_2 = 90° - 83° = 7°$$

and

$$\beta_1 = 90° - 75° = 15°$$

At inlet, tangential impeller velocity is

$$U_1 = \omega r_1$$

$$= \left(\frac{900 \times 2\pi}{60}\right) \times 0.051$$

$$= 4.81 \text{ m/s}$$

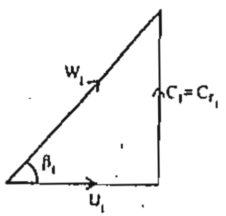


Figure 8.29

From the inlet velocity triangle (Refer Fig. 8.29)

$$\tan\beta_1 = \frac{C_{r_1}}{U_1} = \frac{C_1}{U_1} \quad \text{[Since zero whirl } C_{x_1} = 0\text{]}$$

$$C_1 = 4.81 \tan 15°$$

$$= 1.29 \text{ m/s}$$

Volume flow through the pump is

$$Q = \text{Area} \times \text{Velocity of flow}$$

$$= (2\pi r_1 b) \times C_{r_1}$$

$$= 2\pi \times 0.051 \times 0.064 \times 1.29$$

$$= 0.0265 \text{ m}^3/\text{s}$$

(b) Stagnation and static pressure rise across the impeller

Using the continuity equation $r_1 C_{r_1} = r_2 C_{r_2}$

$$C_{r_2} = \frac{0.051 \times 1.29}{0.203}$$

$$= 0.324 \text{ m/s}$$

At outlet, tangential impeller velocity is

$$U_2 = \omega r_2$$

$$= \left(\frac{900 \times 2\pi}{60}\right) \times 0.203$$

$$= 19.13 \text{ m/s.}$$

Hydraulic efficiency

$$\eta_H = \frac{\text{Total head developed by pump}}{\text{Theoretical head developed}}$$

$$= \frac{H_m}{E}$$

If the change in potential head across the pump is ignored, the total head developed by the pump is given by

$$H_m = \left(\frac{P_2 - P_1}{\rho g}\right) + \left(\frac{C_2^2 - C_1^2}{2g}\right)$$

and for an incompressible fluid, the total pressure head co-efficient is

$$\frac{P_{o2} - P_{o1}}{\rho g} = \left(\frac{P_2}{\rho g} + \frac{C_2^2}{2g}\right) - \left(\frac{P_1}{\rho g} + \frac{C_1^2}{2g}\right)$$

$$= H_m$$

Now,

$$E = \frac{U_2 C_{x_2}}{g} = \frac{U_2}{g}(U_2 - W_{x_2})$$

$$= \frac{19.13}{9.81}\left(19.13 - \frac{C_{r_2}}{\tan 7°}\right)$$

$$= \frac{19.13}{9.81}\left(19.13 - \frac{0.324}{\tan 7°}\right)$$

$$= 32.15 \text{ m}$$

Therefore,

$$H_m = 0.89 \times 31.91$$
$$= 28.6 \text{ m}$$

Hence,

$$P_{02} - P_{01} = 28.6 \times 10^3 \times 9.81$$
$$= 278.5 \text{ kPa}$$

Now,

$$C_2 = \left(C_{r_2}^2 + C_{x_2}^2\right)^{1/2}$$

and

$$C_{x_2} = U_2 - \frac{C_{r_2}}{\tan \beta_2}$$
$$= 19.13 - \frac{0.324}{\tan 7^\circ}$$
$$= 16.49 \text{ m/s}$$
$$C_2 = (0.324^2 + 16.49^2)^{1/2}$$
$$= 16.49 \text{ m/s}$$

Solving for the static pressure head

$$\frac{P_2 - P_1}{\rho g} = H_m - \left(\frac{C_2^2 - C_1^2}{2g}\right)$$
$$= 28.6 - \left(\frac{16.49^2 - 1.29^2}{2 \times 9.81}\right)$$
$$P_2 - P_1 = 143.5 \text{ kPa}$$

(c) Power given to fluid

$$= \rho g Q H_m$$
$$= 10^3 \times 9.81 \times 0.0265 \times 28.6$$
$$= 7.43 \text{ kW}$$

(d) Input power to impeller

$$P_s = \text{Power given to fluid}/\eta_{overall}$$
$$= 7.43/0.89$$
$$= 8.35 \text{ kW}$$

Example 8.12 The basic design of a centrifugal pump has a dimensionless specific speed of 0.075 rev. The blades are forward facing on the impeller and the outlet angle is 120° to the tangent, with an impeller passage width at outlet equal to one-tenth of the diameter. The pump is to be used to pump water from a vertical distance of 35 m at a flow rate of 0.04 m^3/s. The suction and delivery pipes are each of 150 mm diameter and have a combined length of 40 m with a friction factor of 0.005. Other losses at pipe entry, exit, bends etc are three times the velocity head in the pipes. If the blades occupy 6 percent of the circumferential area and the hydraulic efficiency (neglecting slip) is 76 percent, what must be the pump impeller diameter?

Solution From the continuity equation, the velocity in the pipes is

$$V = Q/A$$
$$= \frac{0.04}{\left(\frac{\pi}{4} \times 0.15^2\right)} = 2.26 \text{ m/s}$$

$$\text{Total losses} = \text{Pipe friction losses} + \text{other losses}$$
$$= \frac{4 f l V^2}{2gd} + 3\left(\frac{V^2}{2g}\right)$$
$$= \left(\frac{4 \times 0.005 \times 40}{0.015} + 3\right)\frac{V^2}{2g}$$
$$= \frac{8.333 \times 2.26^2}{2 \times 9.81}$$
$$= 2.16 \text{ m}$$

Total required head = 35 + 2.16 = 37.16 m

From the definition of the dimensionless specific speed, the speed of the pump is determined.

$$N_s = \frac{NQ^{1/2}}{(gH)^{3/4}}$$
$$N = \frac{0.075(9.81 \times 37.16)^{3/4}}{(0.04)^{1/2}}$$
$$= 31.28 \text{ rev/s}$$

Flow area perpendicular to impeller outlet periphery is

$$= \pi D \times \frac{D}{10} \times 0.94$$
$$= 0.295 D^2$$

Now,

$$C_{r_2} = \frac{Q}{\text{Flow area}}$$
$$= \frac{0.04}{0.295 D^2}$$
$$= \frac{0.136}{D^2} \text{ m/s}$$

Also,

$$U_2 = \pi D N$$
$$= \pi D \times 31.28$$
$$= 98.3 D \text{ m/s}$$

The hydraulic efficiency is

$$\eta_H = \frac{H_m}{E}$$

$$= \frac{\text{Fluid Power developed by pump}}{\text{Fluid power supplied to impeller}}$$

No slip, therefore,

$$E = \frac{U_2 C_{x_2}}{g}$$

and

$$C_{x_2} = \frac{g H_m}{U_2 \eta_H}$$

$$= \frac{9.81 \times 37.16}{98.3D \times 0.76}$$

$$= 4.87/D \text{ m/s}$$

The outlet velocity triangle (without slip) gives (Refer Fig. 8.2)

$$\tan 60^\circ = \frac{C_{r_2}}{C_{x_2} - U_2}$$

$$= \frac{0.136}{D^2(4.87/D - 98.3D)}$$

Hence,

$$D^3 = 0.0495D - 0.0008$$

∴ Impeller diameter(D)

$$= 0.214 \text{ m}$$

Example 8.13 A small centrifugal pump, when tested at $N = 2875$ rev/min with water, delivered $Q = 57.2$ m^3/h and $H = 42.1$ m at its best efficiency point ($\eta = 0.76$). Determine the specific speed of the pump at this test condition. Compute the required power input to the pump. *(MKU–Nov. '97)*

Solution

$$N = 2875 \text{ rpm}, \quad Q = \frac{57.2}{3600} = 0.016 \text{ m}^3/\text{s}, \quad H_m = 42.1 \text{ m}.$$

(a) Specific Speed of the pump

$$N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}$$

$$= \frac{2875\sqrt{0.016}}{(42.1)^{3/4}}$$

$$= 22$$

(b) Power input

$$P = \frac{\rho g Q H_m}{\eta}$$

$$= \frac{10^3 \times 9.81 \times 0.016 \times 42.1}{0.76 \times 10^3}$$

$$= 8.695 \text{ kW}$$

Example 8.14 A centrifugal pump with 1.2 m diameter runs at 200 rpm and pumps 1880 l/s, the average lift being 6 m. The angle which the vanes make at exit with the tangent to the impeller is 26° and the radial velocity of flow is 2.5 m/s. Determine the manometric efficiency and the least speed to start pumping against a head of 6 m, the inlet diameter of the impeller being 0.6 m.

Solution

Given $D_1 = 0.6$ m, $D_2 = 1.2$ m, $C_{r_2} = 2.5$ m/s

$N = 200$ rpm, $Q = 1880$ l/s $= 1.88$ m^3/s

$H_m = 6$ m, $\beta_2 = 26°$

(i) Manometric efficiency

$$\eta_m = \frac{H_m}{(U_2 C_{x_2}/g)}$$

$$U_2 = \frac{\pi D_2 N}{60}$$

$$= \frac{\pi \times 1.2 \times 200}{60}$$

$$= 12.57 \text{ m/s}$$

$$C_{x_2} = U_2 - W_{x_2}$$

$$W_{x_2} = \frac{C_{r_2}}{\tan \beta_2} = \frac{2.5}{\tan 26} = 5.13 \text{ m/s}$$

$$C_{x_2} = 12.57 - 5.13 = 7.44 \text{ m/s}$$

$$\therefore \quad \eta_m = \frac{6 \times 9.81}{12.5 \times 7.44} = 63.3\%$$

(ii) Least speed to start the pump It is given by

$$U_2^2 - U_1^2 = 2g\, H_m$$

$$\left(\frac{\pi \times 1.2 \times N}{60}\right)^2 - \left(\frac{\pi \times 0.6 \times N}{60}\right)^2 = 2 \times 9.81 \times 6$$

$$2.96\, N^2 = 117.72$$

$$N = 199.4 \text{ rpm}$$

$$N \simeq 200 \text{ rpm}$$

Example 8.15 A centrifugal pump having external and internal diameters of respectively 1.25 m and 0.5 m is discharging water at the rate of 2000 lit/s against a head of 16 m, when running at 300 rpm. The vanes are curved back at an angle of 30° with the tangent at the outlet and the velocity of flow is constant at 2.5 m/s. Find (a) hydraulic efficiency of the pump, (b) power required and (c) least speed at which athe pump commences to work. *(MU–Oct. '98)*

Solution

$$D_2 = 1.25 \text{ m} \quad D_1 = 0.5 \text{ m} \quad Q = 2 \text{ m}^3/\text{s}$$
$$H_m = 16 \text{ m} \quad N = 300 \text{ rpm} \quad \beta_2 = 30° \quad C_{r_1} = C_{r_2} = 2.5 \text{ m/s}$$

(a) Hydraulic (or) Manometric efficiency

$$\eta_m = \frac{H_m \times g}{U_2 C_{x_2}}$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.25 \times 300}{60} = 19.64 \text{ m/s}$$

$$C_{x_2} = U_2 - \frac{C_{r_2}}{\tan\beta_2} = 19.64 - \frac{2.5}{\tan 30°}$$

$$= 19.64 - 4.33 = 15.31 \text{ m/s}$$

$$\eta_m = \frac{16 \times 9.81}{19.64 \times 15.31}$$

$$\eta_m = 52.2\%$$

(b) Power required (P_S)

Neglecting the mechanical losses, the power required to run the pump is equal to the fluid power developed by the impeller

$$\text{i.e.,} \quad P_s = W = \dot{m} U_2 C_{x_2}$$

$$\dot{m} = \rho Q = 1000 \times 2$$

$$= 2000 \text{ kg/s}$$

$$\therefore P_s = 2000 \times 19.64 \times 15.31$$

$$= 601.38 \text{ kW}$$

(c) Minimum starting speed

$$U_2^2 - U_1^2 = 2gHm$$

$$\left(\frac{\pi D_2 N}{60}\right)^2 - \left(\frac{\pi D_1 N}{60}\right)^2 = 2gHm$$

$$N^2\left[\left(\frac{\pi D_2}{60}\right)^2 - \left(\frac{\pi D_1}{60}\right)^2\right] = 2gHm$$

$$\therefore N^2 = \frac{2gHm}{\left(\frac{\pi D_2}{60}\right)^2 - \left(\frac{\pi D_1}{60}\right)^2}$$

$$N^2 = \frac{2 \times 9.81 \times 16}{\left(\frac{\pi \times 1.25}{60}\right)^2 - \left(\frac{\pi \times 0.5}{60}\right)^2}$$

$$\therefore N = 295.4 \text{ rpm}$$

Example 8.16 A three stage centrifugal pump has impellers 40 cm in diameter and 2 cm wide at outlet. The vanes are curved back at the outlet at 45° and they reduce the circumferential area by 10%. The manometric efficiency is 90% and the overall efficiency is 80%. Determine the head generated by the pump, when running at 1000 rpm delivering 50 litres per second. What should be the shaft power? *(MKU–April '97)*

Solution

Given, Number of stage, $n = 3$

$$D_2 = 0.4 \text{ m}, \quad b_2 = 0.02 \text{ m}, \quad \beta_2 = 45°$$

Reduction in area at outlet = 10%

(i) Head generated by pump

$$\text{Area of flow at outlet} = 0.9(\pi D_2 b_2)$$

$$= 0.9 \times \pi \times 0.4 \times 0.02$$

$$A_2 = 0.023 \text{ m}^2$$

$$\eta_m = 0.9, \quad \eta_0 = 0.8, \quad N = 1000 \text{ rpm.}$$

$$Q = 0.05 \text{ m}^3/\text{s}$$

Velocity of flow at outlet

$$C_{r_2} = \frac{Q}{\pi D_2 b_2} = \frac{Q}{A_2}$$

$$= \frac{0.05}{0.0226}$$

$$= 2.21 \text{ m/s}$$

Outlet impeller tangential velocity

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1000}{60}$$

$$= 20.94 \text{ m/s}$$

From the outlet velocity triangles (Fig 8.2)

$$C_{x_2} = U_2 - W_{x_2}$$
$$W_{x_2} = C_{r_2} = 2.21 \text{ m/s}$$
$$[\because \tan 45^\circ = 1]$$
$$\therefore C_{x_2} = 20.94 - 2.21 = 18.73 \text{ m/s}$$

From η_m equation

$$H_m = \frac{\eta_m \times (U_2 C_{x_2})}{(g)}$$
$$= \frac{0.9 \times 20.94 \times 18.73}{9.81}$$
$$= 35.99 \text{ m}$$

Total head generated by the pump

$$= n \times H_m = 3 \times 35.99$$
$$= 107.97 \text{ m}$$

(ii) Shaft power input (P_S)

$$P_s = \frac{\rho g Q (H_m \times n)}{\eta_0}$$

Power output of the pump $= \rho g Q(H_m \times n)$

$$\therefore P_s = \frac{1000 \times 9.81 \times 0.05 \times 107.97}{0.8}$$
$$= \frac{52959.3}{0.8}$$
$$= 66199.1 \text{ W}$$
$$P_s = 66.199 \text{ kW}$$

Thus, Shaft power $= 66.199$ kW

Example 8.17 Find the number of pumps required to take water from a deep well under a total head of 156 m. The pumps are identical and run at 1000 rpm. The specific speed of each pump is given as 20 while the rated capacity of each pump is 150 l/s. *(MKU–Nov. '96)*

Solution

$H = 156$ m, $N = 1000$ rpm, $N_s = 20$, $Q = 0.150 \text{ m}^3/\text{sec}$

The head developed by each pump

$$H_m = \left[\frac{N\sqrt{Q}}{N_s}\right]^{4/3}$$
$$= \left[\frac{1000 \times \sqrt{0.150}}{20}\right]^{4/3}$$
$$= 52 \text{ m}$$

Total head developed $(H) = 156$ m.

$$\therefore \text{No. of pumps (n)} = \frac{H}{H_m}$$
$$= \frac{156}{52}$$
$$n = 3$$

Example 8.18 A centrifugal pump having three stages in parallel delivers 360 m³ of water per hour, against a head of 16 m when running at a speed of 1500 rpm. Diameter of its impeller being 150 mm.

A multi-stage pump, geometrically similar to the one given above, but having stages in series is to be designed to run at 1200 rpm and to deliver 450 m³/h of water against a head of 140 m. Find the impeller diameter and the number of stages required.

Solution

It should be noted that in the first pump, all the three stages are in parallel which means that head of each stage will be the same i.e 16 m. Where as the discharge of each stage will be $360/3 = 120 \text{ m}^3/\text{hr}$.

On the contrary, in the second pump the stages are in series which means that each stage would deliver a discharge of 450 m³/hr. Where as total head of 140*m* will be equally contributed by each stage.

So,

$$Q_1 = 120 \text{ m}^3/\text{hr}$$
$$Q_2 = 450 \text{ m}^3/\text{hr}$$
$$H_1 = 16 \text{ m}$$

H_2 = the head generated by each stage in the second pump.

Equating the specific speed of pump-1 and pump-2, we get

$$\frac{Q_1}{Q_2} = \left(\frac{N_2}{N_1}\right)^2 \left(\frac{H_1}{H_2}\right)^{6/4}$$
$$\frac{120}{450} = \left(\frac{1200}{1500}\right)^2 \left(\frac{H_1}{H_2}\right)^{6/4}$$

or

$$\left(\frac{H_2}{H_1}\right)^{6/4} = \frac{0.64}{0.27}$$
$$\frac{H_2}{H_1} = 1.78$$

and

$$H_2 = 1.78 \times 16 = 28.4 \text{ m}$$

Number of stages

$$n = \frac{140}{28.4} = 4.9$$

$$n = 5$$

Equating head co-efficient of pump-1 and pump-2

$$\frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2 \left(\frac{D_1}{D_2}\right)^2$$

$$\left(\frac{D_2}{D_1}\right)^2 = \left(\frac{N_1}{N_2}\right)^2 \left(\frac{H_2}{H_1}\right)$$

$$= \left(\frac{1500}{1200}\right)^2 \left(\frac{28.4}{16}\right)$$

$$= 2.77$$

$$D_2 = \sqrt{2.77} \times D_1$$

$$= \sqrt{2.77} \times 0.15$$

$$= 0.25 \text{ m}$$

or

$$D_2 = 250 \text{ mm}$$

Example 8.19 When a laboratory test was carried out on a pump, it was found that for a pump total head of 36 m at a discharge of 0.05 m^3/s, cavitation began when the sum of the static pressure plus the velocity head at inlet was reduced to 3.5 m. The atmospheric pressure 750 mm Hg and the vapour pressure of water is 1.8 kPa. If the pump is to operate at a location where atmospheric pressure is reduced to 620 mm Hg and the vapour pressure of water is 830 Pa, what is the value of the cavitation parameter when the pump develops the same total head and discharge? Is it necessary to reduce the height of the pump above the supply and if so by how much?

Solution Cavitation began, when

$$\frac{P_i}{\rho g} + \frac{V_i^2}{2g} = 3.5$$

and at this condition $P_i = P_{vap}$, the vapour pressure. Then

$$\left[NPSH = \frac{P_i}{\rho g} + \frac{V_i^2}{2g} - \frac{P_{vap}}{\rho g}\right]$$

$$\frac{V_i^2}{2g} = 3.5 - \frac{1.8 \times 10^3}{9.81 \times 10^3}$$

$$= 3.317 \text{ m} = NPSH$$

Now, the cavitation parameter

$$\sigma = \frac{NPSH}{H}$$

$$= \frac{3.317}{36}$$

$$= 0.0921$$

From the steady flow energy equation taking the reservoir level as datum ($Z_a = 0$) we get for case (1)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_a}{\rho g} - (\text{sum of head losses})$$

$$(Z_1 + h_{f_1}) = \frac{P_a}{\rho g} - (\sigma H) - \frac{P_1}{\rho g} \quad \left[\because NPSH = \frac{V_i^2}{2g}\right]$$

$$P_1 = 1.8 \text{ kPa}$$

$$= (0.75 \times 13.6) - 3.317 - 0.1835$$

$$= 6.7 \text{ m}$$

For case (2)

$$\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 = \frac{P_a}{\rho g} - h_{f_2} \quad P_2 = 830 \text{ Pa}$$

$$Z_2 + h_{f_2} = (0.62 \times 13.6) - 3.317 - 0.0846$$

$$= 5.03 \text{ m}$$

Since the flow rate is the same, $h_{f_1} = h_{f_2}$ and the pump must be lowered by a distance $(Z_1 - Z_2) = 1.67$ m at the new location.

Example 8.20 An axial flow pump has an impeller of outlet diameter 1.0 m. The dia of boss is 0.5 m. If specific speed of pump is 38 and velocity of flow is 2 m/s. Suggest a suitable speed of the pump to give a head of 6 m. Also determine vane angle at the entry of the pump, if the flow is axial at inlet.

Solution

$D_t = 1$ m, $D_h = 0.5$ m, $N_s = 38$, $C_a = 2$ m/s and $H = 6$ m

(a) Pump speed

$$\text{Discharge} = \text{Area of flow} \times \text{velocity of flow}$$

$$= \frac{\pi}{4}(D_t^2 - D_h^2) \times C_a$$

$$= \frac{\pi}{4}(1^2 - 0.5^2) \times 2$$

$$= 1.178 \text{ m}^3/\text{s}$$

Also,

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$\therefore \quad N = \frac{38 \times 6^{3/4}}{\sqrt{1.178}}$$

$$N = 134.22 \text{ rpm}$$

(b) From inlet velocity diagram

$$U_1 = \frac{\pi D_h N}{60} = \frac{\pi \times 0.5 \times 134.22}{60}$$

$$= 3.51 \text{ m/s}$$

$$\therefore \tan\beta_1 = \frac{C_a}{U_1} = \frac{2}{3.51}$$

$$\beta_1 = 29.67°$$

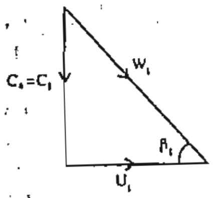


Figure 8.30

Example 8.21 An axial flow pump has the following particulars: discharge = 180 l/s, head developed = 2m, specific speed = 250, speed ratio = 2.4. Flow ratio = 0.5. Calculate (a) speed of the pump (b) the runner diameter (c) the boss diameter.

Solution

$Q = 0.180 \text{ m}^3/\text{s}$ $\quad H = 2 \text{ m}$ $\quad N_s = 250$

Speed ratio = 2.4 $\quad$ Flow ratio = 0.5

(a) Pump speed

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$N = \frac{N_s \times H^{3/4}}{\sqrt{Q}} = \frac{250 \times (2)^{3/4}}{\sqrt{0.180}}$$

$$N = 991 \text{ rpm}$$

(b) Runner diameter

Peripheral velocity

$$U = 2.4\sqrt{2gH}$$

$$= 2.4\sqrt{2 \times 9.81 \times 2}$$

$$= 15.03 \text{ m/s}$$

Since,

$$U = \frac{\pi DN}{60}$$

$$D = \frac{60 \times 15.03}{\pi \times 991}$$

$$D = 0.29 \text{ m}$$

(c) Boss diameter

Velocity of flow

$$C_a = 0.5\sqrt{2gH}$$

$$= 0.5\sqrt{2 \times 9.81 \times 2}$$

$$= 3.132 \text{ m/s}$$

Discharge through the pump

$$Q = \frac{\pi}{4}(D^2 - D_h^2)C_a$$

$$0.180 = \frac{\pi}{4}(0.29^2 - D_h^2) \times 3.132$$

$$D_h^2 = 0.0109$$

$$D_h = 0.11 \text{ m}$$

Example 8.22 A jet pump fitted 2.5 m above the suction reservoir and 18 m below the supply reservoir lifts water through a total height of 2.7 m. Determine the efficiency of the jet pump when it delivers 7.5 l/s of water while using 2.75 l/s from the supply reservoir.

Solution

$H_s = 2.5$ m $\quad H_1 = 18$ m $\quad H_s + H_d = 2.7$ m

$Q_1 = 2.75$ l/s $\quad Q_s + Q_1 = 7.5$ l/s

$H_d = 2.7 - H_s = 2.7 - 2.5 = 0.2$ m $\quad Q_s = 4.75$ l/s

$Q_s = 7.5 - Q_1 = 7.5 - 2.75$

Therefore,

Jet pump efficiency,

$$\eta_j = \frac{Q_s(H_s + H_d)}{Q_1(H_1 - H_d)}$$

$$= \frac{4.75(2.5 + 0.2)}{2.75(18 - 0.2)}$$

$$\eta_j = 26.2\%$$

SHORT QUESTIONS

8.1. What is a centrifugal pump?
8.2. The centrifugal pump is similar in construction to the francis turbine. (True/False)
8.3. The efficiency of the vortex casing centrifugal pump is ———— than the efficiency of volute casing centrifugal pump.
8.4. Draw the velocity triangles at inlet and exit of a centrifugal pump.
8.5. What is a slip factor? Write the expression for workdone per kg of water of a centrifugal pump with fluid slip.

8.6. Static head is the sum of
(a) suction head and manometric head.
(b) manometric head and delivery head.
(c) suction head and delivery head.
8.7. Define: manometric head.
8.8. Define the following for a centrifugal pump
(a) Manometric efficiency
(b) Mechanical efficiency
(c) Overall efficiency
8.9. What is meant by minimum starting speed of a centrifugal pump?
8.10. What is *NPSH*?
8.11. What is priming?
8.12. Pumps are connected in parallel to
(a) develop a high head
(b) develop a high discharge
(c) develop a high power
8.13. Pumps are connected in ——— to develop a high head.
8.14. Define the phenomenon cavitation
8.15. The cavitation parameter is defined as
(a) $\sigma = H_m/NPSH$
(b) $\sigma = (NPSH) \times H_m$
(c) $\sigma = NPSH/H_m$
8.16. Can cavitation be prevented in the centrifugal pumps? How?

EXERCISES

8.1. Explain the working of a single-stage centrifugal pump with sketches.
8.2. Differentiate between the volute casing and vortex casing for the centrifugal pump?
8.3. Obtain an expression for the workdone per kg of water, by the impeller of a centrifugal pump.
8.4. Define the terms: Suction head, delivery head, static head and manometric head.
8.5. Derive the expression for the minimum speed for starting a centrifugal pump.
8.6. Draw and discuss the performance curves of a centrifugal pump.
8.7. What is a multi-stage pump? Describe multistage pump with
(a) impellers in parallel and (b) impellers in series.
8.8. Explain the phenomenon of cavitation. What are the effects of cavitation? How can cavitation be prevented?
8.9. A centrifugal pump has external and internal impeller diameters as 60 cm and 30 cm respectively. The vane angle at inlet and outlet are 30° and 45° respectively. If the water enters the impeller at 2.5 meters/sec. Find
(a) speed of the impellers in rpm. (b) work done per kg of water.
[Ans: (a) 275.66 rpm and (b) 53.34 J/kg]
8.10. Calculate the vane angle at inlet of a centrifugal pump impeller having 300 mm diameter at inlet and 600 mm diameter at outlet. The impeller vanes are set back at an angle of 45° to the outer rim and the entry of the pump is radial. The pump runs at 1000 rpm and the velocity of flow through the impeller is constant at 3m/s. Also, calculate the workdone by the water per kg of water and the velocity and direction of water at outlet. *(MU–April '97)*
[Ans: (a) 10.8° (b) 892.9 J/kg and (c) 28.52 m/s and 6°]
8.11. A centrifugal pump delivers water against a net head of 14.5 meters and at a designed speed of 1000 rpm. The vanes are curved back at an angle of 30° with the periphery. The impeller diameter is 300 mm and the outlet width is 50 mm. Determine the discharge of the pump, if manometric efficiency is 95%.
[Ans: 0.168 m³/s]
8.12. A centrifugal pump delivers 30 litres of water per second to a height of 18 meters through a pipe of 90 m long and 100 mm diameter. If the overall efficiency of the pump is 75% find the power required to drive the pump. Take $f = 0.012$.
[Ans: 20 kW]
8.13. The diameters of an impeller of a centrifugal pump at inlet and outlet are 30 cm and 60 cm respectively. Determine the minimum starting speed of the pump if it works against a head of 30 m. **[Ans: 891.8 rpm]**
8.14. A four stage centrifugal pump has four identical impeller keyed to the same shaft. The shaft is running at 400 rpm and the total manometric head developed by the multistage pump is 40 m. The discharge through the pump is 0.2 m^3/s. The vanes of each impeller are having an outlet angle of 45°. If the width and diameter of each impeller at outlet is 5 cm and 60 cm respectively, find the manometric efficiency. **[Ans: 74.7%]**
8.15. The diameter of an impeller of a centrifugal pump at inlet and outlet are 30 cm and 60 cm respectively. The velocity of flow at outlet is 2.5 m/s and vanes are set back at an angle of 45° at outlet. Determine the minimum starting speed of the pump, if the manometric efficiency is 75%. **[Ans: 159.3 rpm]**
8.16. A three stage centrifugal pump has impeller 40 cm in diameter and 2.5 cm wide at outlet. The vanes are curved back at the outlet at 30° and reduce the circumference area by 15%. The manometric efficiency is 85% and overall efficiency is 75%. Determine the head generated by the pump when running at 12,000 rpm and discharging 0.06 m^3/s. Find also the shaft power.
[Ans: (a) 138.8 m and (b) 109 kW]
8.17. A centrifugal pump has an impeller of external diameter 60 cm and internal diameter 30 cm. The vane angles at inlet and outlet are 30° and 45° respectively. The velocity of flow is constant at 2.5 m/s and the velocity at inlet is radial. Find the pressure rise through the impeller, if the pump speed is 276 rpm.
[Ans: 5.45 m of water]
8.18. A centrifugal pump is working against a head of 20 m while rotating at the rate of 600 rpm. If the blades are curved back to an angle of 45° to the tangent at the outlet tip and velocity of flow remains constant at 2 m/s. Calculate the impeller diameter when all the kinetic energy at impeller outlet is wasted.
[Ans: 0.634 m]
8.19. A centrifugal pump handling water has backward curved vanes. The impeller tip diameter is 500 mm. The relative velocity at tip section is 45° to the tangent at exit. If the radial velocity at exit is 15 m/s, the flow at the inlet is radial and

the impeller develops a head of 68 m. Find (a) pump speed, if the impeller efficiency is 70% (b) the manometric head, assuming 50 percent of the kinetic energy at the impeller is wasted and the head loss in the impeller is 5 m, and (c) the lowest speed to start the pump if $U_1 = 0.5\ U_2$ *(MU–April '96)*
[Ans: (a) 1500 rpm (b) 42.3 m and (c) 1270 rpm]

8.20. Water is required to be lifted through 110 m height from a well. Number of identical pumps having speed 1000 rpm, specific speed 25 rpm, with a rated discharge of 6 kl/min are available. Determine how many pumps will be required and how they should be connected. [Ans: (a) 4 and (b) Series]

8.21. Centrifugal pumps delivering 10 m^3/min of liquid against a head of 20 m has 4 stages in parallel. Diameter of impeller is 20 cm and speed 1500 rpm. A geometrically similar pump is to be made up with number of stages in series to deliver 15 m^3/min against a head 200 m and to run at 1000 rpm. Calculate the impeller diameter and the number of stages required in the second case.
[Ans: (a) 6 (b) 0.41 m]

8.22. A single stage centrifugal pump with impeller diameter as 300 mm rotates at 2000 rpm and lifts 3 m^3 of water per sec to a height of 30 m with an efficiency of 75 per cent. Find the number of stages and diameter of each impeller of a similar multistage pump to lift 5 m^3 of water per sec to a height of 200 m when rotating at 500 rpm. [Ans: (a) 7 and (b) 0.39 m]

8.23. The impeller of an axial flow pump is 1.2 m in diameter while the boss is 0.6 m in diameter. Find the most suitable speed to provide a head of 2.5 m. The velocity of flow through the impeller is 4.5 m/s and the specific speed of the pump is 335 rpm. Find also the vane angle at inlet at the exterior tips and near the boss. Assume no whirl at inlet. [Ans: (a) 340.8 rpm (b) 11.5° and (c) 22.5°]

8.24. A jet pump is fitted at 3 m above the suction reservoir and 19 m below the supply reservoir lifts water through a total height of 4m. If the jet pump delivers 8 l/min of water while using 2.5 l/s from the supply reservoir, determine the efficiency of jet pump. [Ans: 48.9 %]

8.25. Describe the following with the help of a line diagram.
(a) Jet pump (b) Air lift pump

8.26. Enumerate the advanatages and disadvantages of air lift pump as compared with the centrifugal pump.

8.27. Describe the construction and working of a submersible pump.

8.28. Explain the construction and working of the following pumps with a neat sketch
(a) Gear pumps
(b) Vane pump
(c) Piston pumps

Reaction

9

HYDRAULIC TURBINES

INTRODUCTION

Hydraulic turbines convert the hydraulic energy into electrical energy. The main types of turbines used in these days are the impulse and reaction turbines. The predominant type of impulse turbine is the pelton wheel. Reaction turbines are of two types 1. Radial or Mixed flow 2. Axial flow. Two types of axial flow turbines exist, one is propeller turbine and the other one is Kaplan turbine. The former has fixed blades whereas the latter has adjustable blades. Francis turbine is an example for radial flow turbines. The following table summarizes the head, power and efficiency values for each type of turbine.

CLASSIFICATION OF HYDRAULIC TURBINES

The important classification of hydraulic turbines are

1. According to the type of energy at the Inlet

(a) Impulse turbine (Pelton wheel) Energy available at the turbine inlet is only kinetic energy and the pressure is atmospheric from inlet to the turbine outlet.

(b) Reaction turbine Energy available at the turbine inlet is both kinetic energy and pressure energy. Example: Francis, Kaplan and propeller turbines.

2. According to the direction of flow through the runner

(a) Tangential flow turbine Water flows along the tangent of the runner. Example Pelton wheel

(b) Radial flow turbine Water flows in the radial direction through the runner. If the water flows from outwards to inwards radially, the turbine is called the '*Inward radial flow turbine*'. On the other hand if water flows radially from inwards to outwards, the turbine is known as '*Outward radial flow turbine*.'

(c) Axial flow turbine Water flows through the runner along the direction parallel to the axis of rotation of the runner. Example Kaplan and propeller.

(d) Mixed flow turbine Water flows through the runner in the radial direction and leaves in the direction parallel to the axis of rotation of the runner.

Table 9.1

Parameter	*Pelton wheel*	*Francis turbine*	*Kaplan turbine*
Head (m)	100–1700	80–500	upto 400
Max. power (kW)	55	40	30
Best efficiency (%)	93	94	94

3. According to the head at the inlet of turbine

(a) High head turbine (250–1800 m) Example: Pelton wheel

(b) Medium head turbine (50–250 m) Example: Francis

(c) Low head turbine (less than 50 m) Example: Kaplan and propeller

4. According to the specific speed of the turbine

(a) Low specific speed turbine (< 50) Example: Pelton wheel

(b) Medium specific speed turbine. (50 < Ns < 25p) Example: Francis

(c) High specific speed turbine. (> 250) Example: Kaplan and propeller

PELTON WHEEL

The pelton wheel turbine is a pure impulse turbine in which a jet of fluid leaving the nozzle strikes the buckets fixed to the periphery of a rotating wheel. The energy available at the inlet of the turbine is only kinetic energy. The pressure at the inlet and outlet of the turbine is atmospheric. The turbine is used for high heads ranging from 150 to 2000 m. The turbine is named after

L. A. Pelton, an american engineer. The fluid flows in the tangential direction.

PARTS OF THE PELTON TURBINE

The main parts of the Pelton turbine are as shown in Fig. 9.1.

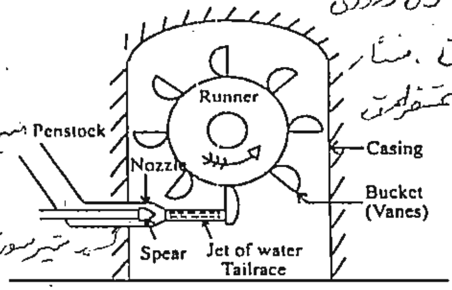


Figure 9.1 Pelton turbine

1. Nozzle and flow control arrangement (Spear) The water from the reservoir flows through the penstocks at the outlet of which a nozzle is fitted. The nozzle converts the total head at the inlet at the nozzle into kinetic energy. The amount of water striking the curved buckets of the runner is controlled by providing a spear in the nozzle. The spear is a conical needle which is operated either by a hand wheel or automatically in an axial direction, depending upon the size of the unit.

2. Runner and buckets The rotating wheel or circular disc is called the runner. On the periphery of the runner a number of buckets, evenly spaced, are fixed. The shape of the buckets is of a double hemispherical cup or bowl. Each bucket is divided into two symmetrical parts by a dividing wall which is known as the *splitter*. The jet of water strikes on the splitter. The splitter then divides the jet into two equal parts and the water comes out at the outer edge of the bucket. These buckets deflect the jet through an angle between 160 and 165° in the same plane as the jet. Due to this deflection of the jet, the momentum of the fluid is changed react ing on the buckets. A bucket is therefore pushed away by the jet.

3. Casing The casing prevents the splashing of the water and discharges the water to tail race. The spent water falls vertically into the lower reservoir or tailrace and the whole energy transfer from the nozzle outlet to tail race takes place at a constant pres sure. The casing is made of cast iron or fabricated steel plates.

4. Breaking jet To stop the runner within a short time, a small nozzle is provided which directs the jet of water on to the back of the vanes. The jet of water is called the *breaking jet*. If there is no breaking jet, the runner due to inertia goes on revolving for a long time.

VELOCITY TRIANGLES AND WORK DONE FOR PELTON WHEEL

The water supply is from a constant head reservoir at an *elevation* H_1, above the centre-line of the jet. The nozzle at the pen stock end, converts the total head at the inlet to the nozzle into a water jet with velocity C_1, at atmospheric pressure.

The velocity triangle for the flow of fluid onto and off a single bucket are shown in Fig. 9.2.

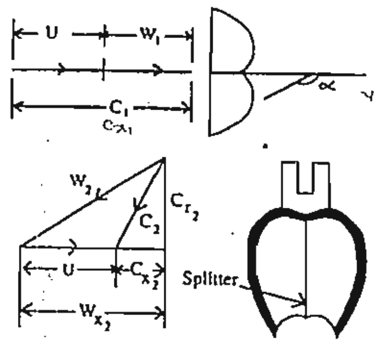


Figure 9.2 Velocity triangles for a pelton turbine

If the bucket is brought to rest, then subtracting the bucket speed U_1 from the jet velocity C_1 gives the relative fluid velocity W_1 onto the bucket. The angle turned through by the jet in the horizontal plane during its passage over the bucket surface is 'α' and the relative exit velocity is W_2. If the bucket speed vector U_2 is added to W_2 in the appropriate direction, the absolute velocity at exit C_2 will be obtained. It should be realized that the component C_{x_2} of C_2 can be in the positive or negative X direction depending on the magnitude of U.

From Euler's turbine equation

$$W/m = U_1 C_{x_1} - U_2 C_{x_2}$$

and since in this case C_{x_2} is in the negative x-direction

$$W/m = U[(U + W_1) + (W_2 cos(180° - \alpha) - U)]$$

Assuming no loss of relative velocity due to friction across the bucket surface ($W_1 = W_2$), then

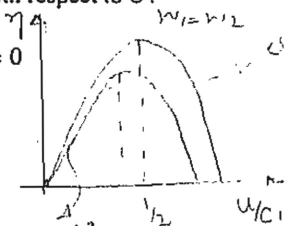
$$W/m = U(W_1 - W_1 cos\alpha)$$

Therefore,

$$E = U[(C_1 - U)(1 - \cos\alpha)]/g \quad \frac{W}{N.S} \qquad (9.1)$$

The above equation can be optimized by differentiating with respect to U.

$$\frac{dE}{dU} = (1 - \cos\alpha)(C_1 - 2U)/g = 0$$



For a maximum, and then

$$C_1 = 2U$$

(or)

$$U = C_1/2 \qquad (9.2)$$

But in practice, maximum energy is transferred when the wheel velocity is 0.46 times the velocity of jet.

Substituting equation 9.2 in equation 9.1, we have

$$E_{max} = C_1^2(1 - \cos\alpha)/4g$$

In practice surface friction of the bucket is present and $W_2 \neq W_1$. Then equation 9.1 becomes

$$E = U(C_1 - U)(1 - K cos\alpha)/g$$

where K is the relative velocity ratio W_2/W_1.

Hydraulic Efficiency

The hydraulic efficiency is defined as

$$\eta_H = \frac{\text{Energy transferred}}{\text{Energy available in jet}} = E/(C_1^2/2g)$$

if $\alpha = 180°$, the maximum hydraulic efficiency is 100 per cent. In practice, the deflection angle is in the order of $160° - 165°$ to avoid interference with the oncoming jet and η_H is accordingly reduced. The maximum hydraulic efficiency is

$$\eta_{H_{max}} = \frac{E_{max}}{(C_1^2/2g)} = \frac{1 - \cos\alpha}{2}$$

PELTON WHEEL LOSSES AND EFFICIENCIES

Head losses occur in the pipelines conveying the water to the nozzle and are composed of friction and bend losses. Losses also occur in the nozzle and these are expressed in terms of a velocity coefficient C_v (varies from 0.98 to 0.99). Finally, there are windage and friction losses in the wheel itself. The water supply is from a reservoir at a head H_1 above the nozzle. As the fluid moves through the pressure tunnel and the penstock upto the entry to the nozzle, a frictional head loss h_f occurs. A further head loss h_{in} due to losses in the nozzle takes place so that the head available for power generation at exit from the nozzle is H'. That is

$$H' = H_1 - (h_f + h_{in}) = C_1^2/2g$$

(a) Pipe-line Transmission Efficiency

$$\eta_{trans} = \frac{\text{Energy at end of pipeline}}{\text{Energy available at reservoir}} = \frac{(H_1 - h_f)}{H_1} = \frac{H}{H_1}$$

(b) Nozzle Efficiency

$$\eta_N = \frac{\text{Energy at nozzle outlet}}{\text{Energy at nozzle inlet}} = \frac{H'}{H_1 - h_f} = \frac{H'}{H} = \frac{C_1^2/2g}{H}$$

(or)

$$\eta_N = \frac{C_1^2}{2gH}$$

So, nozzle and pipeline transmission efficiency

$$= \left(\frac{H}{H_1}\right)\left(\frac{H'}{H}\right) = \left(\frac{H'}{H_1}\right)$$

$$= \frac{C_1^2}{2gH_1}$$

and

(c) Nozzle Velocity Coefficient

$$= \frac{\text{Actual jet velocity}}{\text{Theoretical jet velocity}}$$

$$C_v = \frac{C_1}{\sqrt{2gH}}$$

Therefore, the nozzle efficiency becomes

$$\eta_N = C_v^2$$

(d) Number of Buckets

Number of buckets on a runner is given by

$$N = 15 + \frac{D}{2d}$$

where 'D' is the pitch diameter of the pelton and d is the diameter of the jet. The ratio D/d is called the *jet ratio*.

(e) Overall Efficiency

$$\eta_o = \frac{\text{Power produced}}{\text{Actual energy supplied}}$$

$$= \frac{P_s}{\rho g Q H}$$

where H is the head at the nozzle inlet and

η_o is a measure of the performance of the turbine.

GOVERNING OF PELTON WHEEL TURBINE

Hydraulic turbines are usually coupled directly to an electrical generator and since the generator must run at a constant speed, the speed U of the turbine must remain constant when the load changes. It is also desirable to run at the maximum efficiency and therefore the ratio U/C_1, must remain the same. That is the jet velocity must not change. The only way left to adjust to the change in the turbine load is to change the input water power.

The input power is given by the product $\rho g Q H'$ where H' is constant (and hence C_1) and the only variable is Q. The change in flow rate is effected by noting that $Q = C_1 A$, where A is the nozzle area. Since C_1 is constant, the cross sectional area of the nozzle must change. This is accomplished by a spear valve. The position of the spear is controlled by a servo-mechanism that senses the load changes.

DOUBLE GOVERNING OF PELTON WHEEL

All modern pelton turbines use Double regulation method. This method of governing controls the turbine speed and pressure (i.e. water hammer) in the penstock by the combined spear and deflector control. Operated by the oil pressure governor (Fig. 9.2(a)).

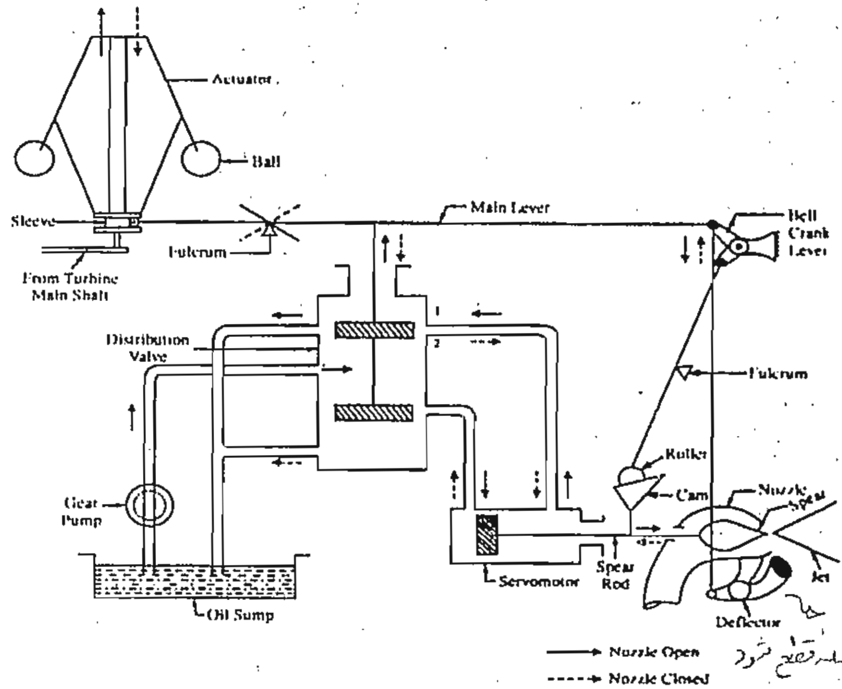


Figure 9.2(a) Double Regulation

The centrifugal governor (or) actuator is attached to the main shaft of the turbine. When the load on the turbine decreases, the speed of the turbine increases and the flyballs of the governor rotate at higher speed and move away from the axis by the centrifugal force. Therefore, the sleeve moves upward. The motion of the flyballs is transmitted to the bell crank lever and it rotates anticlockwise. The roller on the cam is raised and the deflector is brought between the nozzle and the buckets. At the same time, the double piston in the distribution value moves down. Now, *port-2* is open and port-1 is closed. High pressure oil from the sump enters the distributing value through the middle port and flows down to servo motor striking the left face of the piston. So the piston moves to the right, forcing the spear to more to the right i.e. into the nozzle.

The nozzle outlet area is reduced. Hence, the area of jet and the rate of flow of water striking the buckets is reduced. Consequently, the speed which was increasing with the decrease in load is brought under control and remains constant.

When the load on the turbine increases, the speed of the turbine decreases. This causes the fly balls of the actuator to come down and thus the sleeve moves downward, causing the bell crank lever to rotate clockwise. The distribution value rod moves upward. Now, port-1 is open and port-2 is closed. The high pressure oil enters the servo motor through port-1 to the right side of the piston. The piston moves to the left, causing the spear to move out of the nozzle. The nozzle outlet area is now increased. Hence the amount of water striking the buckets is increased. Thus the speed of the turbine is controlled and kept constant. Under this condition, the deflector is moved away from the jet and there is no obstruction to the jet.

When the turbine is running at normal load, piston in the distribution valve and the actuator occupy their normal positions as shown in figure. Both the ports 1 and 2 are closed. No flow of oil to the servo motor and the spear value is at its normal position.

CHARACTERISTICS OF AN IMPULSE TURBINE

The characteristic curves of an impulse turbine for a constant head are shown in figure 9.3(a) & (b). In Fig. 9.3(a) it is seen that the peak values of efficiency do not vary much for various gate openings. In Fig. 9.3(b), it is seen that the peak power occurs at the same speed irrespective of the nozzle settings. This is due to the nozzle velocity remaining constant in magnitude direction as the flow rate changes, giving an optimum value of U/C_1 at a fixed speed. Windage and mechanical losses and variations in loss coefficients cause the small variations.

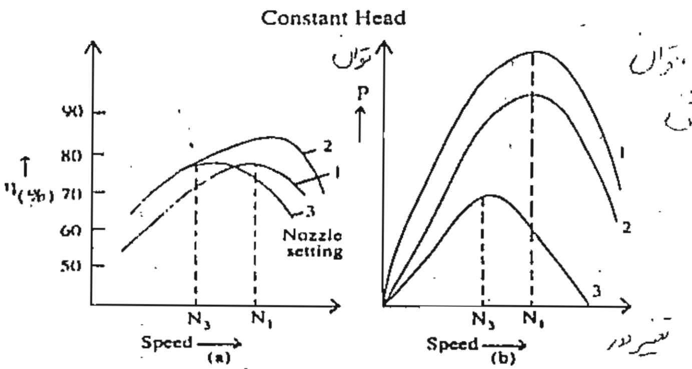


Figure 9.3 Efficiency and power output versus speed in an impulse turbine

In practice, one is usually more interested in the fixed speed condition since the generators run at constant speed. The Fig. 9.4 shows that the variation of efficiency with load is slight, except at low loads, where the decrease is due to changes in the nozzle efficiency, and at high loads the increased jet diameter gives rise to higher bucket losses.

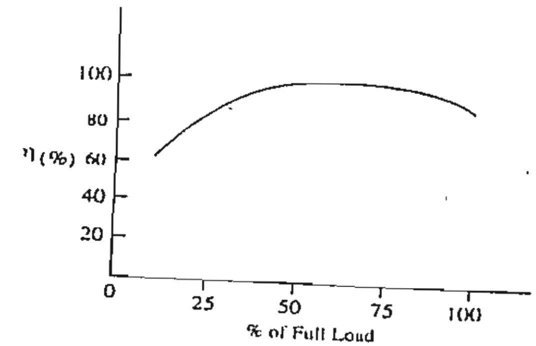


Figure 9.4 Variation of pelton turbine efficiency with load

RADIAL FLOW REACTION TURBINE

Reaction turbine means that the water at the inlet of the turbine possesses kinetic energy as well as pressure energy. As the water flows through the runner, a part of pressure energy goes on changing into kinetic energy. Thus the water through the runner is under pressure. The runner is completely enclosed in an air-tight casing and both the casing and the runner are always full of water. Radial flow turbine means that the water flows in the radial direction. Radial flow turbines are grouped as Inward radial flow turbines and Outward radial flow turbines. The total head of the radial flow turbines range from about 30 to 500 m.

MAIN PARTS OF A RADIAL FLOW REACTION TURBINE

The main components of a radial flow reaction turbine are (Fig. 9.5)

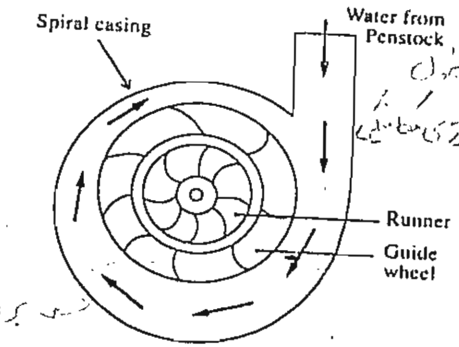


Figure 9.5 Radial flow turbine

1. Casing The water from the penstocks enter the casing, which is of spiral shape. The area of cross section of the casing goes on decreasing gradually. The casing completely surrounds the runner of the turbine. [illegible] to facilitate water flow at constant velocity throughout the circumference of the runner. The casing is usually made of concrete, cast steel or plate steel.

2. Guide vanes The stationary guide vanes are fixed on a stationary circular wheel which surrounds the runner. The guide vanes allow the water to strike the vanes fixed on the runner without shock at the inlet. This fixed guide vanes are followed by adjustable guide vanes. The cross sectional area between the adjustable vanes can be varied for flow control at part load.

3. Runner It is a circular wheel on which a series of radial curved vanes are fixed. The water passes into the rotor where it moves radial ly through the rotor vanes and leaves the rotor blades at a smaller diameter. Later, the water turns through 90° into the draft tube.

4. Draft tube The pressure at the exit of the rotor of a reaction turbine is generally less than the atmospheric pressure. The water at exit cannot be directly discharged to the tail race. A tube or pipe of gradually increasing area is used for discharging the water from the turbine exit to the tail race. In other words, the draft tube is a tube of increasing cross sectional area which converts the kinetic energy of water at the turbine exit into pressure energy.

VELOCITY TRIANGLES AND WORK DONE

The inlet and outlet velocity triangles for a runner are shown in figure 9.6.

Water enters the runner from the inlet guide vanes (at radius r_1), with absolute velocity, at an angle α_1 to the direction of rotation. The tangential velocity at the inlet is U_1. The relative velocity vector W_1 obtaind by substracting U_1 from C_1 at inlet, is at an angle β_1 to the direction of rotation. β_1 is also the inlet blade angle for shock-free entry.

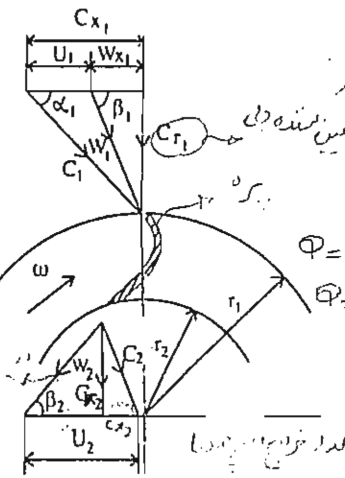


Figure 9.6 Velocity triangles for a Francis turbine

At the outlet, (radius r_2), the water leaves the blade at an angle β_2 to the tangential velocity vector. The absolute outlet velocity C_2 is the resultant of W_2 and U_2. The flow velocities C_{r_1} and C_{r_2} are directed towards the axis of rotation and are given by $Q/2\pi r_1 b_1$ and $Q/2\pi r_2 b_2$ respectively, where b is the height of the runner.

Euler's turbine equation gives

$E = W/mg = (U_1 Cx_1 - U_2 Cx_2)/g$

and E is a maximum when Cx_2 is zero, that is when the absolute and flow velocities are equal at the outlet or $\alpha_2 = 90°$

NET HEAD ACROSS REACTION TURBINE

The net head 'H' across the turbine is the difference in the total head between the inlet flange and the tail water level. The net head H is different from the gross head H_1. Thus

Turbine total inlet head =

$$\frac{P_o}{\rho g} + \frac{V_o^2}{2g} + Z_o$$

and turbine total outlet head =

$$\frac{P_3}{\rho g} + \frac{V_3^2}{2g} + Z_3$$

where the pressures are gauge pressure.

∴ Total head across turbine (H)

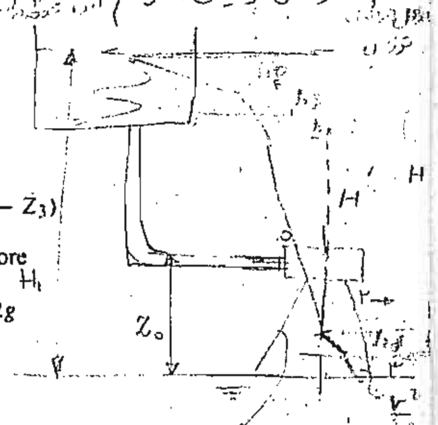
$$= (P_o - P_3)/\rho g + (V_0^2 - V_3^2)/2g + (Z_o - Z_3)$$

But in the tail-race P_3 is atmospheric and Z_3 is zero. Therefore

$$H = (P_o/\rho g + V_o^2/2g + Z_o) - V_3^2/2g$$

Also,

$$H = H_1 - h_{fp} - V_3^2/2g$$



where h_{fp} is the pipe friction head loss and the energy given to the runner by the water per unit weight of flow is

$$W/mg = H - h_d - h_g - h_r$$

If the water is discharged directly into the tail-race from the runner outlet, the kinetic energy lost would be high. By fitting a draft tube between the runner outlet and tailrace, a continuous stream of water is formed between the two. The tail-race velocity is reduced because of the increase in cross sectional area of the draft tube and, because the tailrace pressure is atmospheric the runner outlet pressure must now be below the atmospheric pressure. Applying the energy equation between the runner outlet and tail-race gives

$$\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + Z_3 + h_d$$

where h_d is the head loss in the draft tube. Putting P_3 and Z_3 equal to zero

$$\frac{P_2}{\rho g} = (V_3^2 - V_2^2)/2g - Z_2 + h_d$$

There is a limit for the extent of V_3 reduction because of the tube length. As the length increases h_d also increases. The draft tube could be very long since the angle of divergence should not exceed about 8° to ensure that separation of boundary layers does not occur. There is also an upper limit on the value of Z_2 due to the cavitation effect.

RADIAL FLOW TURBINE LOSSES

The losses in terms of energy balance through the turbine is given by

$$P = P_m + P_r + P_c + P_l + P_s \tag{9.3}$$

where P_s = shaft power output, P_m = mechanical power loss, P_r = runner power loss, P_c = casing and draft tube loss, P_l = leakage loss and P = water power available. $P_r + P_c + P_l$ together is the hydraulic power loss.

Runner power loss P_r is due to friction, shock at the impeller entry and flow separation. It results in a head loss h_r associated with the flow rate through the runner of Q_r.

$$P_r = \rho g Q_r h_r$$

Leakage power loss P_l is caused by a flow rate q leaking past the runner and therefore not being handled by the runner. Thus

$$Q = Q_r + q$$

and with a total head H_r across the runner, the leakage power loss becomes

$$P_l = \rho g q H_r$$

Casing power loss P_c is due to the friction eddy and flow separation losses in the casing and the draft tube.
If this head loss is h_c then

$$P_c = \rho g Q h_c$$

The total energy balance of equation 9.3 thus becomes

$$\rho g Q H = P_m + \rho g (h_r Q_r + h_c Q + H_r q + P_s)$$

Thus,

(a) Overall efficiency $= \dfrac{\text{Shaft output power}}{\text{Fluid power available at inlet flange}}$

$$\eta_o = P_s/(\rho g Q H)$$

(b) Hydraulic efficiency $(\eta_H) = \dfrac{\text{Power received by runner}}{\text{Fluid power available at inlet flange}}$

$$= (P_s + P_m)/\rho g Q H = \frac{E}{\rho g Q H}$$

The term $(P_s + P_m)/\rho g Q H$ is the theoretical energy transfer per unit weight of fluid flow. The maximum hydraulic efficiency is

$$\eta_H = \frac{U_1 C_{x_1}}{gH}$$

GOVERNING OF REACTION TURBINES

The quantity of water flowing to the reaction turbine is controlled by rotating the guide blades. These blades are pivoted and connected by levers and links to the regulating ring (Fig. 9.6(a)). The regulating ring and lever are connected by two regulating rods. This regulating lever in turn in connected with regulating shaft, which is operated by the piston of the servo motor.

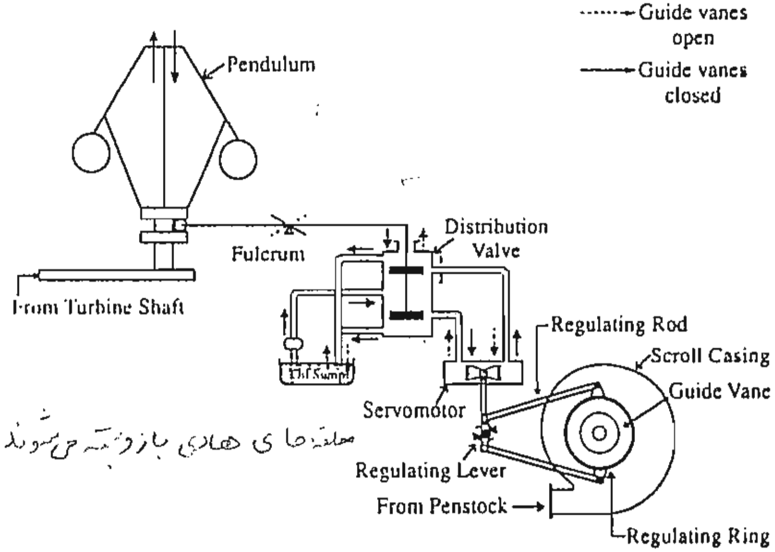


Figure 9.6(a) Francis Turbine Governing

When the load on the turbine increases, the speed increases. The fly balls of actuator move a way from the axis and the sleeve raises. The distribution valve rod is pushed down. Port-2 is open and port-1 is closed. The high pressure oil enters the servo motor through port-2 and pauses the piston to move towards right. The oil in the right side of the piston is pushed back into the oil sump through port-1 and upper past of the distributing valve. When the piston of the servo motor moves to the right, regulating ring is rotated to decrease the flow area between the guide vanes by changing guide vane angles. Thus the quantity of water reaching the runner blades is reduced. Therefore, the speed is brought to normal gradually, and then actuator, main lever and distribution valve attain their normal position. A relief valve is provided in the penstock to prevent water hammer which may arise due to the sudden reduction in the flow passage between the guide blades. Its function is similar to that of a deflector in pelton turbine. When there is a sudden decrease in load on the turbine, the relief valve opens and diverts the water to the tailrace. Thus double regulation (speed and pressure) is achieved in reaction turbines. Similarly when the load on the turbine increases i.e. when the speed has a tendency to increase, regulating ring is moved in the opposite direction so as to increase the passage area between the guide blades and allowing more water to strike the runner blade.

CHARACTERISTIC CURVES FOR A REACTION TURBINE

Curves of water power input torque exerted by the wheel, flow rate efficiency and brake power output for constant gate opening are shown in Fig. 9.7.

At full opening, the flow rate as shown in Fig. 9.7 varies with the runner speed. It is no longer independent of the wheel speed. The behaviour of the turbine at constant speed is of most interest since the generator runs at a fixed speed. As the electrical load changes, so the flow rate is changed by variation of the gate opening. It is seen from the Fig. 9.8 that the head increases slightly as the load decreases due to the friction head loss. This is proportional to Q^2 which is less at lighter loads. It will also be noted that the efficiency curve at constant speed is not as flat as in an impulse turbine.

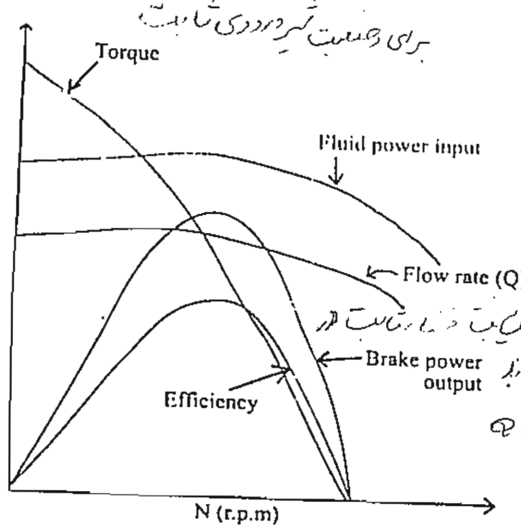


Figure 9.7 Reaction turbine characteristics at full opening

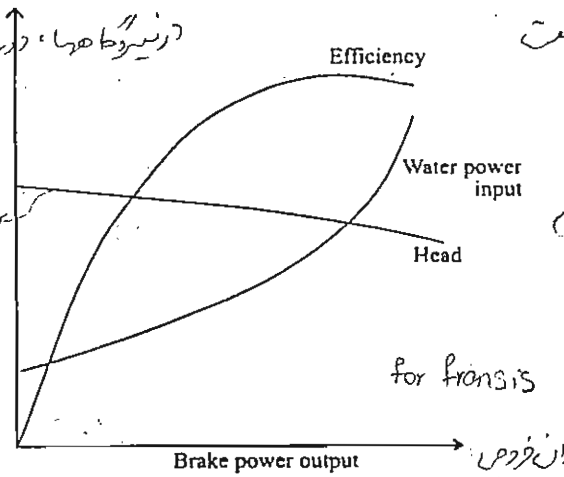


Figure 9.8 Reaction turbine characteristics at constant speed

When a reaction turbine operating at constant speed experiences a load decrease, the cross sectional area between the inlet vanes changes and angle α_1 decreases. The inlet runner area is constant and therefore to satisfy the continuity the relative velocity W_1 must decrease. The result is that the flow onto the runner is no longer shock free and at exit C_2 may increase. This gives a higher kinetic energy loss at the runner exit as well as an increase in the whirl component C_{X_2} down the draft tube. The flow is then spiral in nature, which decreases the draft tube efficiency. The efficiency of a reaction turbine at light loads therefore tends to be less than that of the pelton wheel, although the design maximum efficiency may be greater.

The advantage of an inward flow reaction turbine over an outward flow reaction turbine is that the former adjusts automatically according to the load on the turbine. Whenever the load on the turbine is decreased, it causes the shaft to rotate at a higher speed. The centrifugal force, which increases due to higher speed, tends to reduce the quantity of water flowing over the vanes, and thus the velocity of water at the entry is also re duced. It will ultimately tend to reduce the power produced by the turbine.

AXIAL FLOW REACTION TURBINE

In axial flow reaction turbine, water flows parallel to the axis of rotation of the shaft. In a reaction turbine, the head at the inlet of the turbine is the sum of pressure energy and kinetic energy and a part of the pressure energy is converted into kinetic energy as the water flows through the runner. For the axial flow reaction turbine, the shaft of the turbine is vertical. The lower end of the shaft which is made larger is known as '*hub*' or '*boss*'. The vanes are fixed on the hub and hence the hub acts as a runner for the axial flow reaction turbine. The two important axial flow reaction turbines are

1. Propeller Turbine and
2. Kaplan Turbine

(If the vanes are fixed to the hub and are not adjustable, then the turbine is known as propeller turbine on the other hand if the vanes on the hub are adjustable the turbine is known as a *Kaplan turbine*). It is named after V. Kaplan, an Austrian engineer. Kaplan turbine is suitable, where a large quantity of water at low heads (upto 400 m), is available.

The main parts of a Kaplan turbine are (Fig. 9.9)

1. Scroll casing
2. Guide vanes
3. Hub with vanes
4. Draft tube

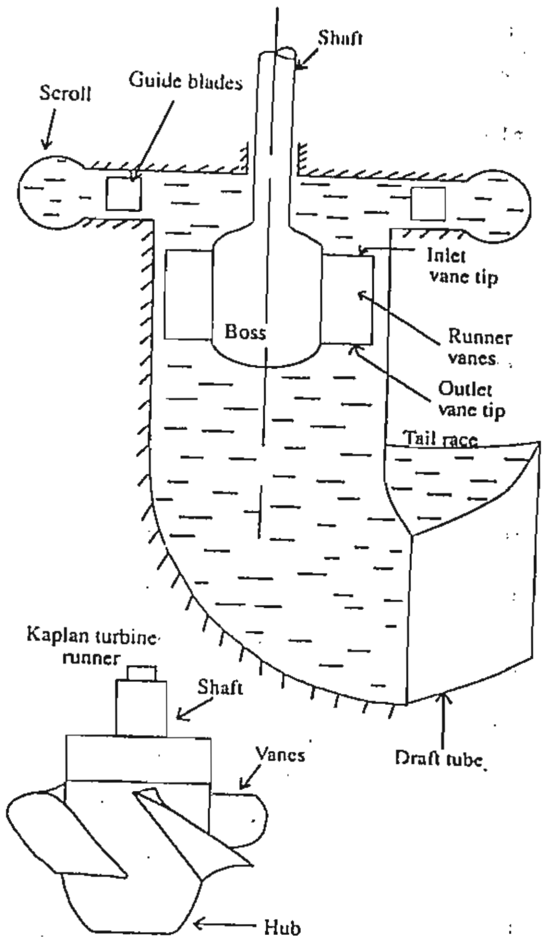


Figure 9.9 Kaplan turbine

Water from the penstock enters the guide vanes through scroll casing. The inlet guide vanes are fixed and are situated at a plane higher than the runner blades such that the fluid must turn through 90° to enter the runner in the axial direction. Load changes are effected by adjustment of the runner blade angle. The function of the guide vanes is to impart whirl to the fluid so that the radial distribution of velocity is the same as in a free vortex. Since this type of turbine is used for low heads and high flow rates, the blades must be long and have large chords so that they are strong enough to transmit the very high torques that arise. Pitch/Chord ratios of 1–1.5 are typical for axial flow turbines and this results in four, five or six-bladed runner.

VELOCITY TRIANGLES AND WORK DONE

The velocity triangles are usually drawn at the mean radius, since conditions change from hub to tip and are shown in figure 9.10. The flow velocity is axial at the inlet and outlet. Hence, $C_{r_1} = C_{r_2} = C_a$. The blade velocity vector U_1 is substracted from the absolute velocity vector. C_1 which is at angle α_1 to U_1 to yield the relative velocity vector W_1. For shock free entry onto the runner W_1 is at the blade angle β_1. For maximum efficiency the whirl component C_{x_2} is zero, in which case the absolute velocity at exit is axial, and then $C_2 = C_{r_2}$.

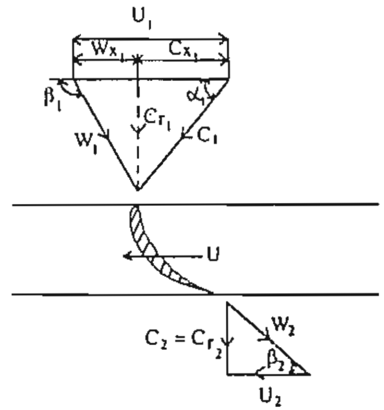


Figure 9.10 Velocity trianlges for an axial flow hydraulic turbine

Euler's turbine equation gives

$$E = U(C_{x_1} - C_{x_2})/g$$

and for zero whirl at exit

$$E = UC_{x_1}/g$$

Now,

$$C_{x_1} = U - Ca\cot(180° - \beta_1)$$
$$= U + Ca\cot\beta_1$$

and therefore,

$$E = (U^2 + UCa\cot\beta_1)/g$$

If E is constant along the blade radius, and Ca is constant over the cross-sectional area, then as U^2 increases from hub to tip, $UCa\cot\beta_1$ must decrease to keep E constant. Hence, β_1 must increase from hub to tip and the blade must therefore be twisted.

GOVERNING OF KAPLAN TURBINE

Kaplan turbine governing employs two servo motors one for operating guide vanes (as in Francis turbine) and the other for operating runner vanes (Figures 9.10(a) & (b)). Both the servo motor distribution valves are interconnected. There by, the runner vanes and guide vanes are simultaneously operated so that water passes through the blades without shock at all load conditions. This system of governing is known as 'Double regulation'.

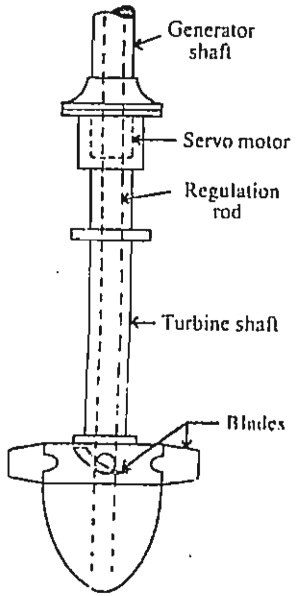


Figure 9.10(a) Governing of Kaplan turbine

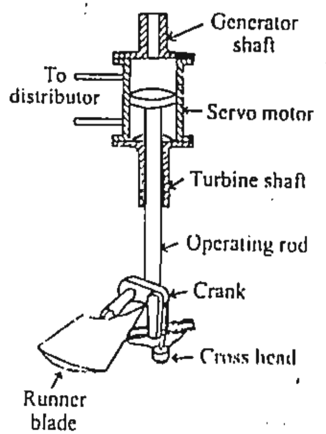


Figure 9.10(b) Servo motor mechanism operating runner vanes of kaplan turbine. Shown in section and on reduced scale

Servo motor for the runner vanes, consists of a cylinder with a piston operated by high pressure oil which is supplied by distribution value (not shown in figure). The servo motor piston is connected with an operating rod which moves up and down and passes through the turbine shaft which is made hollow for this purpose. The operating rod transmits the motion to the runner blades with an appropriate link mechanism enclosed in the runner hub.

COMPARISON OF HYDRAULIC TURBINE EFFICIENCIES

The characteristic curve (Fig. 9.11) for the axial flow Kaplan turbine is similar to that of the radial flow turbine. The maximum efficiency is lower and the efficiency curve is much flatter for the impulse turbine. The Francis turbine peaks at the highest efficiency but falls off rapidly at part load. The Kaplan turbine has a much flatter curve than the Francis turbine and exhibits a similar maximum efficiency of the two important axial flow turbines, and hence it has constant average maximum efficiency while the propeller turbine efficiency increases as load increases. After reaching a peak value, it falls off at the maximum load.

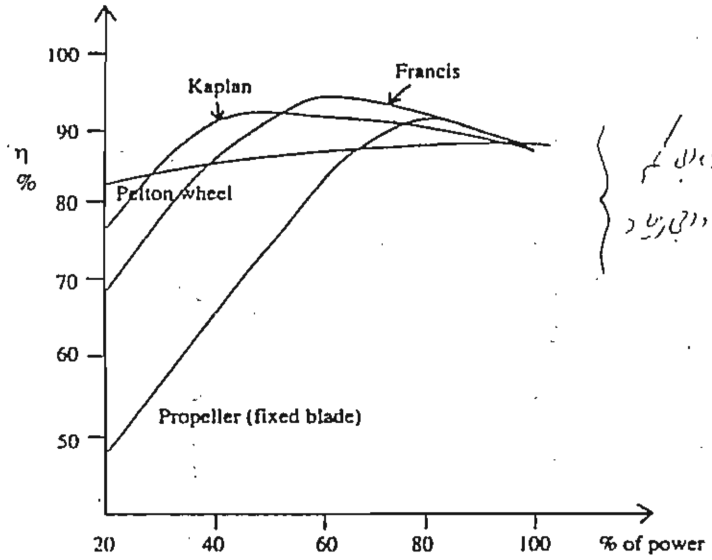


Figure 9.11 Characteristic curves for hydraulic turbine

SELECTION OF HYDRAULIC TURBINES

The type of turbine required for a specific purpose can be selected on the basis of the value of the specific speed. The type of turbine for different specific speeds is given below.

Specific Speed	*Type of Turbine*
5 – 35	Pelton wheel with single jet.
35 – 70	Pelton wheel with two (or) more jets.
70 – 450	Francis turbine.
450 – 1000	Kaplan turbine.

Selection of turbine is a highly technical job and requires great experience. Selection based on specific speed is a scientific method and gives a precise information. The turbine tyre is also selected based on head of water available at the turbine inlet. This method is based on experience and observational factors only.

Head of water in metres	*Type of Turbine*
0 – 25	Kaplan turbine
25 – 50	Kaplan or Francis (preferably Francis)
50 – 250	Francis turbine
250 and above	Pelton turbine.

DRAFT TUBE

The available head is high in an impulse turbine like the pelton wheel and there is not much loss in the overall turbine output although the turbine is placed a metre or two away from the level of the tail-race. Whereas, in the case of reaction turbines, the head available is low and a considerable fraction of this available head would be wasted, if the turbines are placed above the tail-race level and the water from the turbine just exhausted at the atmospheric pressure. But, both the output and the overall turbine efficiency may be considerably improved by placing the turbine above the tail-race level and leading the water from the turbine outlet to the tail race by a tube such that the water reaches atmospheric pressure only at the tail race. The pipe of gradually increasing area which is used for discharging water from the turbine exit to the tail race is called as '*draft tube*'.

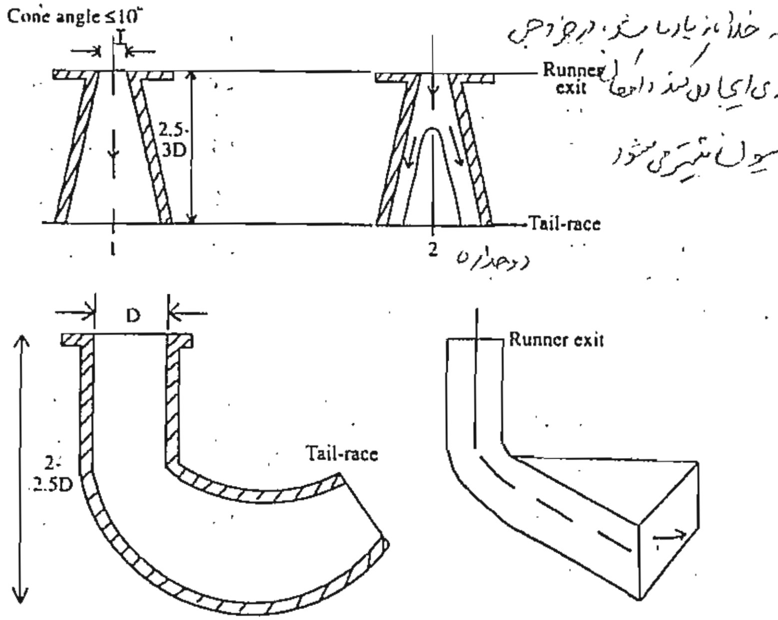


Figure 9.12 Types of draft tubes

The advantages of draft tube are

1. The pressure at the runner exit is below the atmospheric pressure and the turbine operates efficiently as if it is placed at the tail-race.
2. The kinetic energy ($C_2^2/2g$) of water at the turbine runner outlet is converted into useful pressure energy.
3. The turbine may be inspected easily and properly as it is placed above the tail-race.

TYPES OF DRAFT TUBES

There are four types of draft tubes, used in practice, depending upon the flow conditions and the height of the turbine above the tail-race and so on. They are namely,

1. Conical draft tube
2. Bell mouthed or moody spreading tube
3. Simple elbow or bent tube
4. Elbow-draft tube with circular inlet and rectangular outlet.

The first form is the straight concial type stretching from the turbine to the tail-race. The second type is also a straight draft tube except that it is bell-shaped. This type of draft tube has an advantage that it can allow flow with whirl component to occur with very small losses at the turbine exit. In any turbine, the exit absolute velocity usually has a whirl component especially at part load operation, the bell-shaped draft tube may be preferred where the operation is at part load for long periods of time.

The third form is the bent draft tube, is used when the turbine must be located very close to or below the tail-race level for some other considerations. However, the efficiency of bent draft-tube is usually not as great as that of the first two types. The fourth form of draft tube (Elbow draft tube) is similar to the third one except that the exit shape is square or rectangular instead of cylindrical as in the bent draft tube.

The height of the draft tube is governed by two factors. The first is cavitation, which requires that the pressure at the turbine exit or draft tube entry, should not be less than one-third of an atmosphere. The second factor is separation, which occurs if the draft tube has too large an angle of flare. In practice, the angle should not be more than 10° to prevent separation. Draft tube efficiencies range generally from 0.7 to 0.9 for the first two types while they are 0.6 to 0.85 for the bent tube types.

DRAFT TUBE THEORY

Consider the draft tube shown in Fig. 9.13. The turbine exit is at a height H_s above the tail-race level. Let subscript 1 denote the conditions at the rotor exit and subscript 2 denote the conditions at the outlet. Applying Bernoulli's equation to inlet and outlet of the draft tube taking the draft tube, exit as the datum line we get,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + (H_s + X) = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + O + h_d \tag{9.4}$$

where x is the distance of bottom of draft tube from tail-race and h_d is head loss in the draft tube.

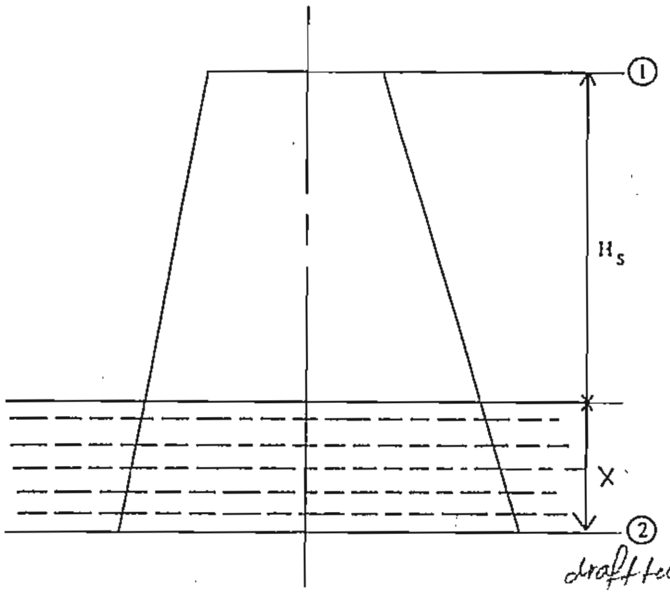


Figure 9.13 Draft Tube

and

$$\frac{P_2}{\rho g} = \text{Atmospheric Pressure Head} + X$$

$$= \frac{P_a}{\rho g} + X$$

Substituting this value of $\frac{P_2}{\rho g}$ in equation 9.4, we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + (H_s + X) = \left(\frac{P_a}{\rho g} + X\right) + \frac{V_2^2}{2g} + h_d$$

or

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + H_S = \frac{P_a}{\rho g} + \frac{V_2^2}{2g} + h_d$$

$$\therefore \frac{P_1}{\rho g} = \frac{p_a}{\rho g} - H_S - \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_d\right)$$

The above equation shows that the inlet pressure at the draft tube inlet is less than atmospheric pressure.

EFFICIENCY OF DRAFT TUBE

It is defined as the ratio the of actual conversion of kinetic energy into pressure energy in the draft tube to the kinetic energy available at the draft tube inlet.

$$\eta_{dt} = \frac{(V_1^2 - V_2^2/2g) - h_d}{(V_1^2/2g)}$$

or

$$\eta_{dt} = \frac{(V_1^2 - V_2^2) - 2gh_d}{V_1^2}$$

CAVITATION IN TURBINES

Turbine cavitation occurs on the suction surfaces of the blades, at the runner outlet, where the static pressure is a minimum and the absolute velocity high. It should be avoided although it has little effect on the performance of the turbine since it occurs after the runner.

Applying the energy equation between the runner outlet and tail-race gives

$$V_2^2 - V_3^2/2g = (P_3 - P_2)/\rho g - Z_2 + Z_3 + h_d$$

Putting $Z_3 = 0$, and as the outlet velocity V_2 increases, P_2 decreases and has its lowest value when the vapour P_r is reached. At this pressure, cavitation occurs (begins) and hence putting P_3 equal to P_{atm} and P_2 equal to P_{vap}, the above equation becomes

$$\left(V_2^2 - V_3^2\right)/2g - h_d = \left(P_{atm} - P_{vap}\right)/\rho g - Z_2$$

Dividing this equation by the net head across the turbine gives the '*Thoma cavitation parameter*' for the turbine

$$\sigma = \{(P_{atm} - P_{vap})/\rho g - Z_2\}/H$$

$$= (NPSH)/H$$

The critical value of *NPSH* at which cavitation occurs is determined from a test on a model or full size machine in which P_2 is decreased until the minimum value at which cavitation begins or the efficiency suddenly decreases is found. Knowing Z_2 and H it is easy to compute the critical value $sigma_c$, which is the value below σ, as given by the above equation, for any other similar machine of the same homologous series must not fall.

The above equation shows that the maximum elevation of the turbine above the tail-race is given by

$$Z_2 = (P_{atm} - P_{vap})/\rho g - \sigma_c H$$

This equation implies that, as the net head is increased, the turbine elevation above the tail-race must be decreased. For an excessive head, Z_2 might be negative, which implies that excavation would be needed to place the turbine below the level of the tail-race.

SOLVED PROBLEMS

Example 9.1 A generator is to be driven by a small Pelton wheel with a head of 91.5 m at the inlet to the nozzle and discharge of 0.04 m^3/s. The wheel rotates at 720 rpm and the velocity coefficient of the nozzle is 0.98. If the efficiency of the wheel, based on the energy available at entry to the nozzle is 80 per cent and the ratio of bucket speed to the jet speed is 0.46, determine the wheel-to-jet-diameter ratio at the centre-line of the buckets, and the speed of the wheel. What is the dimension less power specific speed of the wheel?

Solution

$$\text{Overall efficiency}_{\eta_0} = \frac{\text{Power developed}}{\text{Power available}}$$

$$P = \rho g Q H \eta_0$$

$$= 1000 \times 9.81 \times 0.04 \times 91.5 \times 0.8$$

Power developed = 28.72 kW

Velocity coefficient

$$C_V = \frac{C_1}{(2gH)^{1/2}}$$

$$C_1 = 0.98(2 \times 9.81 \times 91.5)^{1/2}$$

$$= 41.52 \text{ m/s}$$

Therefore,

$$U = 0.46C_1$$

$$= 0.46 \times 41.52$$

$$= 19.1 \text{ m/s}$$

Also,

$$U = \frac{\omega D}{2}$$

where D is wheel diameter

$$D = \frac{2 \times 19.1 \times 60}{720 \times 2\pi}$$

$$= 0.507 \text{ m}$$

Jet area

$$A = Q/C_1$$

$$= \frac{0.04}{41.52}$$

$$= 0.963 \times 10^{-3} \text{ m}^2$$

and jet diameter

$$d = \left(\frac{4A}{\pi}\right)^{1/2}$$

$$= \left(\frac{4 \times 0.963 \times 10^{-3}}{\pi}\right)^{1/2}$$

$$= 0.035 \text{ m}$$

Diameter ratio

$$\frac{D}{d} = \frac{0.507}{0.035}$$

$$= 14.5$$

Dimensionless power specific speed is given by equation

$$N_{sp} = \frac{N P^{1/2}}{\rho^{1/2}(gH)^{5/4}}$$

$$= \frac{720}{60} \times \left(\frac{28.72 \times 10^3}{10^3}\right)^{1/2} \times \left(\frac{1}{9.81 \times 91.5}\right)^{5/4}$$

$$= 0.0131 \text{ rev}$$

$$= 0.0131 \times 2\pi \text{ rad}$$

Hence, Power specific speed = 0.082 rad.

Example 9.2 A Pelton wheel working under a head of 500 meters, produces 13,000 kW at 430 rpm. If the efficiency of the wheel is 85% determine (a) discharge of the turbine (b) diameter of the wheel (c) diameter of the nozzle. Take $C_v = 0.98$ and speed ratio as 0.46. *(MU-Oct. '96)*

Solution

Given $H = 500$ m, $P = 13,000$ kW, $N = 430$ rpm, $\eta_0 = 0.85$

(a) Discharge of the turbine

$$Q = \frac{P}{\rho g H \times \eta_0}$$

$$= \frac{13,000 \times 10^3}{10^3 \times 9.81 \times 500 \times 0.85}$$

$$= 3.12 \text{ m}^3/\text{s}$$

(b) Diameter of the wheel We know the velocity of jet

$$C = C_v\sqrt{2gH}$$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 500}$$

$$= 97.06 \text{ m/s}$$

In actual practice, maximum efficiency takes place when the velocity of the wheel is 0.46 times the velocity of the jet.
i.e.

$$U = 0.46.C$$
$$= 0.46 \times 97.06$$
$$U = 44.65 \text{ m/s}$$

The wheel diameter is

$$D = \frac{U \times 60}{\pi \times N}$$
$$= \frac{44.65 \times 60}{\pi \times 430}$$
$$D = 1.98 \text{ m}$$

(c) Diameter of the nozzle (d)

The discharge through the nozzle must be equal to the discharge of the turbine. Therefore,

$$Q = C \times \left(\frac{\pi}{4}d^2\right)$$
$$d = \left(\frac{Q}{C} \times \frac{4}{\pi}\right)^{1/2}$$
$$= \left(\frac{3.12}{97.06} \times \frac{4}{\pi}\right)^{1/2}$$
$$d = 0.202 \text{ m or } 202 \text{ mm}$$

Example 9.3 A Pelton wheel is having a mean bucket diameter of 0.8 m and is running at 1000 rpm. The net head on the Pelton wheel is 400 m. If the side clearance angle is 15° and discharge through the nozzle is 150 lit/sec, find (a) Power available at the nozzle and (b) hydraulic efficiency of the turbine.

(MKU–Nov. 1996, April-1998)

Solution

Given $D = 0.8$, $N = 1000$ rpm, $H = 400$ m, $Q = 0.150 \text{ m}^3/\text{s}$

(a) Power available at the nozzle = $\frac{1}{2}mC_1^2$

$m = \rho Q = 10^3 \times 0.150 = 150$ kg/s
C_1 = Jet velocity
Assuming U = 0.46 C_1 and

$$U = \frac{\pi DN}{60} = \frac{\pi \times 0.8 \times 1000}{60}$$
$$= 41.89 \text{ m/s}$$
$$\therefore C_1 = 91.06 \text{ m/s}$$

Hence, Power available at the nozzle

$$= \frac{1}{2} \times 150 \times 91.06^2$$
$$= 621.903 \text{ kW}$$

Power available at the nozzle = 621.903 kW.

(b) Hydraulic efficiency

$$\eta_H = \frac{E}{C_1^2/2g}$$
$$\text{where } E = W/mg$$
$$W = U(C_{x_1} + C_{x_2})$$
$$C_{x_1} = C_1 = 91.06 \text{ m/s}$$

From the inlet velocity triangle [Refer Fig. 9.2]

$$W_1 = C_1 - U = 91.06 - 41.89$$
$$W_1 = 49.17 \text{ m/s}$$

Assuming no loss of relative velocity

$$W_2 = W_1 = 49.17 \text{ m/s}$$

From the outlet velocity triangle [Refer Fig. 9.2]

$$W_{x_2} = W_2 \cdot \cos 15°$$
$$= 49.17 \times (0.97)$$
$$= 47.49 \text{ m/s}$$

Then

$$C_{x_2} = W_{x_2} - U = 47.49 - 41.89$$
$$= 5.61 \text{ m/s}$$

$$W/m = U(C_{x_1} + C_{x_2})$$
$$= 41.89(91.06 + 5.61)$$
$$= 4049.51 \text{ W/kg/s}$$
$$\therefore \eta_H = \frac{(4049.51/9.81)}{(91.06^2/2 \times 9.81)}$$
$$= \frac{412.79}{422.63}$$
$$= 97.7\%$$

Example 9.4 Two jets strike at the buckets of a Pelton wheel, which is having shaft horse power as 20,000. The diameter of each jet is given as 15 cm. If the net head on the turbine is 500 m, find the overall efficiency of the turbine. Take $C_v = 1.0$.

(MKU–April '97)

Solution

Number of jets =2, Shaft power =20,000 HP, 1 HP =0.736 kW
hence, Shaft power =14,720 kW, $D = 0.15$ m, $H = 500$ m, $C_v = 1.0$.

$$\text{Velocity of each jet} = C_v\sqrt{2gH}$$
$$= 1.0\sqrt{2 \times 9.81 \times 500}$$
$$C_1 = 99.05 \text{ m/s}$$

$$\text{Area of each jet, } A = \frac{\pi}{4}D^2$$
$$= \frac{\pi}{4}(0.15)^2$$
$$= 0.0177 \text{ m}^2$$

∴ Discharge of each jet

$$= A \times C_1$$
$$= 0.0177 \times 99.05$$
$$= 1.75 \text{ m}^3/\text{s}$$

$$\therefore \text{Total discharge } Q = 2 \times 1.75$$
$$= 3.5 \text{ m}^3/\text{s}$$

Power at the turbine inlet

$$= \rho g Q H$$
$$= 10^3 \times 9.81 \times 3.5 \times 500/10^3$$
$$= 17198.75 \text{ kW}$$

Overall Efficiency

$$\eta_o = \frac{\text{ShaftPower}}{\rho g Q H} = \frac{14,720}{17198.75}$$
$$= 0.856$$
$$= 85.6\%$$

Example 9.5 The buckets of a Pelton wheel deflect the jet through an angle of 170° while the relative velocity of the water is reduced by 12 per cent due to the bucket friction. Calculate the theoretical hydraulic efficiency from the velocity triangles for a bucket/jet speed ratio of 0.47. Under a gross head of 600 m the wheel develops 1250 kW, when the loss of head due to pipe friction between the reservoir and the nozzle is 48 m. The bucket circle diameter of the wheel is 900 mm, and there are two jets. The nozzle velocity coefficient is 0.98. Find the speed of rotation of the wheel and the diameter of the nozzles if the actual hydraulic efficiency is 0.9 times at that calculated above.

Solution

The Fig. 9.14 illustrates the system with the velocity triangles.

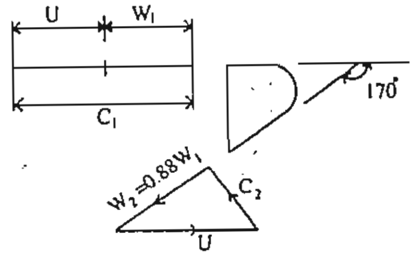


Figure 9.14

$$\text{Hydraulic efficiency} = \frac{\text{Power output}}{\text{Energy available in jet}}$$
$$= \frac{W}{(1/2mC_1^2)}$$

At entry to nozzle

$$H = 600 - 48$$
$$= 552 \text{ m}$$

Nozzle velocity coefficient

$$C_v = \frac{C_1}{\text{Theoretical velocity}}$$
$$= \frac{C_1}{(2gH)^{1/2}}$$

Thus,

$$C_1 = 0.98(2 \times 9.81 \times 552)^{1/2}$$
$$= 102 \text{ m/s}$$

Now,

$$W/m = U_1C_{x_1} - U_2C_{x_2}$$
$$= U[(U + W_1) - (U - W_2\cos(180° - \alpha))]$$
$$= U[(C_1 - U)(1 - K\cos\alpha)]$$

where $W_2 = KW_1$. Substituting the values,

$$W/m = 0.47C_1(C_1 - 0.47C_1)(1 - 0.88\cos 170°$$
$$W = 0.465\, mC_1^2$$

Therefore,
Theoretical Hydraulic Efficiency

$$= 0.465/0.5$$
$$= 0.93$$

Actual Hydrualic Efficiency is

$$= 0.9 \times 0.93$$
$$= 0.837$$

Wheel bucket speed

$$U = 0.47 \times 102$$
$$= 47.94 \text{ m/s}$$

Wheel rotational speed

$$N = \frac{U \times 60}{\pi D}$$
$$= \frac{47.94 \times 60}{\pi \times 0.9}$$
$$= 1017 \text{ rpm}$$

$$\text{Actual Hydraulic Efficiency} = \frac{\text{Actual power developed}}{\text{Energy available in jet}}$$

$$0.837 = \frac{1250 \times 10^3}{(1/2mC_1^2)}$$

Substituting for C_1 and solving for the mass flow rate

$$m = \frac{1250 \times 10^3}{0.837 \times 0.5 \times 102^2}$$
$$= 287 \text{ kg/s}$$

Hence for one nozzle,

$$m = 143.5 \text{ kg/s}$$

Also from continuity equation

$$m = \rho C_1 A$$

where A is the nozzle area

$$A = \frac{\pi}{4} d^2$$

where d is the nozzle diameter

$$\therefore m = \frac{\rho C_1 \pi d^2}{4}$$

and hence

$$d^2 = \frac{143.5 \times 4}{1000 \times 102 \times \pi}$$
$$= 1.792 \times 10^{-3} \text{ m}^2$$
$$d = 42.3 \text{ mm}$$

Example 9.6 Design a Pelton wheel for a head of 60 m and speed 200 rpm. The Pelton wheel develops 100 kW. Take $C_v = 0.98$, Speed ratio = 0.45 and overall efficiency = 0.85.

Solution

$H = 60\,\text{m}$ $N = 200\,\text{rpm}$ $P = 100\,\text{kW}$ $C_v = 0.98$ Speed ratio $= 0.45$ $\eta_0 = 0.85$.

(a) Diameter of wheel (D)

Velocity of jet

$$C_1 = C_v\sqrt{2gH}$$
$$= 0.98 \times \sqrt{2 \times 9.81 \times 60}$$
$$= 33.62 \text{ m/s}$$

Velocity of the buckets

$$U = \text{speed ratio} \times \sqrt{2gH}$$
$$= 0.45 \times \sqrt{2 \times 9.81 \times 60}$$
$$= 15.44 \text{ m/s}$$

But

$$U = \frac{\pi DN}{60}$$
$$\therefore D = \frac{60 \times 15.44}{\pi \times 200}$$
$$D = 1.47 \text{ m}$$

(b) Diameter of the jet (d)

Overall efficiency

$$\eta_0 = \frac{P}{\rho g Q H}$$
$$Q = \frac{P}{\rho g H \cdot \eta_0}$$
$$= \frac{100 \times 10^3}{10^3 \times 9.81 \times 60 \times 0.85}$$
$$= 0.1999 \text{ m}^3/\text{s}$$

But,

$$Q = \text{Area of jet} \times \text{velocity of jet}$$
$$= \frac{\pi}{4} d^2 \times c_1$$

$$\therefore d = \sqrt{\frac{4Q}{\pi c_1}} = \sqrt{\frac{4 \times 0.1999}{\pi \times 33.62}}$$

$$d = 0.087 \text{ m}$$

(c) Number of buckets

$$Z = 15 + \frac{D}{2d} = 15 + \frac{1.47}{2 \times 0.087}$$

$$= 23.45$$

$$Z = 24$$

(d) Size of buckets

$$\text{Width of buckets} = 5d = 5 \times 0.087$$

$$= 0.435\ m$$

$$\text{Depth of buckets} = 1.2d = 1.2 \times 0.087$$

$$= 0.104 \text{ m}$$

Example 9.7 A Pelton wheel is to be designed to run at 300 rpm under a head of 150 m. The nozzle diameter is not to exceed one-twelfth the wheel diameter. The overall efficiency is 0.84. Determine the diameter of the wheel, diameter of jet, quanitity of water required and power developed. Take $C_v = 0.98$ and speed ratio = 0.46.

Solution

$N = 300$ rpm $H = 150$ m $\frac{d}{D} = \frac{1}{12}$ $\eta_0 = 0.84$ $C_v = 0.98$ Speed ratio $= 0.46$.

(a) Diameter of the wheel (D)

$$\text{Velocity of jet } C_1 = C_v\sqrt{2gH}$$

$$= 0.98\sqrt{2 \times 9.81 \times 150}$$

$$= 53.16 \text{ m/s}$$

$$\text{Velocity of the wheel } (U) = \text{Speed ratio} \times \sqrt{2gH}$$

$$= 0.46\sqrt{2 \times 9.81 \times 150}$$

$$= 24.96 \text{ m/s}$$

But,

$$U = \frac{\pi DN}{60}$$

$$D = \frac{60 \times 24.96}{\pi \times 300}$$

$$D = 1.59 \text{ m}$$

(b) Diameter of the jet (d)

$$d = \frac{1}{12} \times D = \frac{1}{12} \times 1.59$$

$$d = 0.133 \text{ m}$$

(c) Quantity of water required

$$Q = \frac{\pi}{4} d^2 \times c_1$$

$$= \frac{\pi}{4} \times 0.133^2 \times 53.16$$

$$Q = 0.739 \text{ m}^3/\text{s}$$

(d) Power developed

$$P = \eta_0 \times \text{Power available at the nozzle}$$

$$= \eta_0 \times (\rho g Q H)$$

$$= 0.84 \times (1000 \times 9.81 \times 0.739 \times 150)$$

$$P = 913.5 \text{ kW}$$

Example 9.8 An electricity generating installation uses a Francis turbine with a rotational speed of 1260 rpm. The net head across the turbine is 124 m and the volume flow rate is 0.5 m^3/s. The radius of the runner is 0.6 m, the height of the runner vanes at inlet is 0.03 m and the angle of the inlet guide vanes is set at 72° from the radial direction. Assuming that the absolute flow velocity is radial at the exit, find the torque and power exerted by the water. Calculate the hydraulic efficiency.

Solution

From the angular momentum equation
Torque,

$$T = m(r_2 C_{x_2} - r_1 C_{x_1})$$

But $C_{x_2} = 0$, since the flow is radial at outlet, and therefore

$$T = -m r_1 C_{x_1}$$

$$= -\rho Q r_1 C_{x_1}$$

$$= -10^3 \times 0.5 \times 0.6 C_{x_1}$$

$$= -300 C_{x_1} \text{ Nm}$$

The inlet area

$$A = 2\pi r_1 b_1$$

where b_1 is the inlet runner height

$$= 2\pi \times 0.6 \times 0.03$$

$$= 0.113 \text{ m}^2$$

Now flow velocity C_{r_1}, is given by

$$C_{r_1} = Q/A$$
$$= 0.5/0.113$$
$$= 4.42 \text{ m/s}$$

From inlet velocity triangle

$$C_{x_1} = C_{r_1} \tan \beta_1$$
$$= 4.42 \times \tan(72^\circ)$$
$$= 13.6 \text{ m/s}$$

Substituting for C_{x_1} gives

$$T = -300 \times 13.6$$
$$= -4080 \text{ Nm}$$

This is the torque exerted on the fluid. The torque exerted by the fluid is +4080 Nm and is the torque exerted on the runner.

Torque exerted by water on runner = 4080 Nm

Power exerted

$$W = T\omega$$
$$= 4080 \times \left(\frac{2\pi \times 1260}{60}\right)$$
$$= 538 \text{ kW}$$

Hydraulic Efficiency, η_H

$$= \frac{\text{Power Exerted}}{\text{Power available}}$$
$$= \frac{538 \times 10^3}{\rho g Q H}$$
$$= \frac{538 \times 10^3}{1000 \times 9.81 \times 0.5 \times 124}$$
$$= 0.885$$
$$= 88.5\%$$

Example 9.9 An inward flow radial turbine has an overall efficiency of 74 per cent. The net head H across the turbine is 5.5 m and the required power output is 125 kW. The runner tangential velocity is $0.97(2gH)^{\frac{1}{2}}$ while the flow velocity is $0.4(2gH)^{\frac{1}{2}}$. If the speed of the runner is 230 rpm with hydraulic losses accounting for 18 per cent of the energy available, calculate the inlet guide vane exit angle, the inlet angle of the runner vane, the runner diameter at inlet and the height of the runner at inlet. Assume that the discharge is radial.

Solution

Hydraulic efficiency

$$\eta_H = \frac{\text{Power given to runner}}{\text{Power available}}$$
$$= \frac{m(U_1C_{x_1} - U_2C_{x_2})}{\rho g Q H}$$

But since the flow is radial at outlet, C_{x_2} is zero and 'm' equals ρQ. Therefore

$$\eta_H = \frac{U_1 C_{x_1}}{gH}$$
$$0.82 = \frac{0.97(2gH)^{\frac{1}{2}} C_{x_1}}{gH}$$

hence,

$$C_{x_1} = 0.423(2gH)^{\frac{1}{2}}$$

Now, from inlet velocity triangle [Refer Fig. 9.6]

$$\tan \alpha_1 = C_{r_1}/C_{x_1}$$
$$= 0.4/0.423$$

From which, inlet guide vane angle $\alpha_1 = 43.4°$

$$\tan \beta_1 = C_{r_1}/W_{x_1}$$
$$= \frac{C_{r_1}}{C_{x_1} - U_1}$$
$$= \frac{0.4}{0.423 - 0.97}$$
$$= -0.731$$

from which, $\beta_1 = -36.2°$ to give the blade angle β_1 as $(180° - 36.2°) = 143.8°$ with $U_1 > C_{x_1}$

Runner speed

$$U_1 = \frac{\pi D_1 N}{60}$$
$$D_1 = \frac{60 \times 0.97 \times (2 \times 9.81 \times 5.5)^{\frac{1}{2}}}{\pi \times 230}$$

Runner inlet diameter = 0.836 m.

Overall efficiency

$$\eta_o = \frac{\text{Power Output}}{\text{Power available}}$$

(or)

$$\rho g Q H = \frac{125 \times 10^3}{0.74}$$

Hence, flow rate

$$Q = \frac{125 \times 10^3}{0.74 \times 1000 \times 9.81 \times 5.5} = 3.13 \text{ m}^3/\text{s}$$

But also

$$Q = \pi D_1 b_1 C_{r_1}$$

Therefore,

$$b_1 = \frac{3.13}{\pi \times 0.836 \times 0.4(2 \times 9.81 \times 5.5)^{\frac{1}{2}}} = 0.287 \text{ m}$$

Hence, height of runner = 0.287 m.

Example 9.10 A Francis turbine has a diameter of 1.4 m and rotates at 430 rpm. Water enters the runner without shock with a flow velocity (C_{r_1}) of 9.5 m/s and leaves the runner without whirl with an absolute velocity of 7 m/s. The difference between the sum of the static and potential heads at entrance to the runner and at the exit from the runner is 62 m. If the turbine develops 12250 kW and has a flow rate of 12 m^3/s of water when the net head is 115 m, find

(a) the absolute velocity of the water at entry to the runner and the angle of the inlet guide vanes.
(b) the entry angle of the runner blades and
(c) the head lost in the runner

Solution

Runner tip speed

$$U_1 = \frac{\pi D N}{60} = \frac{\pi \times 430 \times 1.4}{60} = 31.5 \text{ m/s}$$

Power given to runner

$$W = m(U_1 C_{x_1} - U_2 C_{x_2})$$

But C_{x_2} is zero, since there is zero whirl at outlet. Hence

$$C_{x_1} = \frac{12250 \times 10^3 \times 60}{1000 \times 12 \times \pi \times 1.4 \times 430} = 32.4 \text{ m/s}$$

(a)(i) Guide vane angle

$$\alpha_1 = \tan^{-1}\left(\frac{C_{r_1}}{C_{x_1}}\right) = \tan^{-1}\left(\frac{9.5}{32.4}\right) = 16.3°$$

(ii) Inlet velocity

$$C_1 = (C_{r_1}^2 + C_{x_1}^2)^{\frac{1}{2}} = (9.5^2 + 32.4^2)^{\frac{1}{2}} = 33.8 \text{ m/s}$$

(b) Runner blade entry angle

$$\tan\beta_1 = \frac{C_{r_1}}{C_{x_1} - U_1} = \frac{9.5}{32.4 - 31.5} = 10.55$$

Hence,

$$\beta_1 = 84.6°$$

(c) Total head across runner

= Energy (head) transferred to runner + Head lost in runner

At inlet,

$$H_1 = \frac{P_1}{\rho g} + \frac{C_1^2}{2g} + Z_1$$

At outlet,

$$H_2 = \frac{P_2}{\rho g} + \frac{C_2^2}{2g} + Z_2$$

Now, for zero whirl at outlet

$$\frac{W}{mg} = \frac{U_1 C_{x_1}}{g}$$

Hence, loss of head in the runner

$$= \left(\frac{P_1 - P_2}{\rho g}\right) + \left(\frac{C_1^2 - C_2^2}{2g}\right) + (Z_1 - Z_2) - \frac{U_1 C_{x_1}}{g}$$

But,

$$\left(\frac{P_1 - P_2}{\rho g}\right) + (Z_1 - Z_2) = 62 \text{ m}$$

Head loss in runner

$$= 62 + \left(\frac{33.8^2 - 7^2}{2 \times 9.81}\right) - \left(\frac{31.5 \times 32.4}{9.81}\right)$$

$$= 13.69 \text{ m}$$

Example 9.11 An inward flow reaction turbine has external and internal diameters as 0.9 m and 0.45 m respectively. The turbine is running at 200 rpm and the width of turbine at inlet is 20 cm. The velocity of flow through the runner is constant and is equal to 1.8 m/s. The guide blades make an angle of 10° to the tangent of the wheel and the discharge at the outlet of the turbine is radial. Draw the inlet and outlet velocity triangles and determine

(i) The absolute velocity of water at the inlet of runner.
(ii) The velocity of whirl at the inlet
(iii) The relative velocity at the inlet
(iv) The runner blade angles
(v) Width of the runner at the outlet
(vi) Mass of water flowing through the runner per second
(vii) Head at the turbine inlet
(viii) Power developed and hydraulic efficiency of the turbine

Solution

Given $D_1 = 0.9$ m $\quad D_2 = 0.45$ m $\quad N = 200$ rpm $\quad b_1 = 0.2$ m

$C_{r_1} = C_{r_2} = 1.8$ m/s $\quad \alpha_1 = 10°$ $\quad \alpha_2 = 90°$

(a) The absolute velocity of water at inlet of runner (C_1)

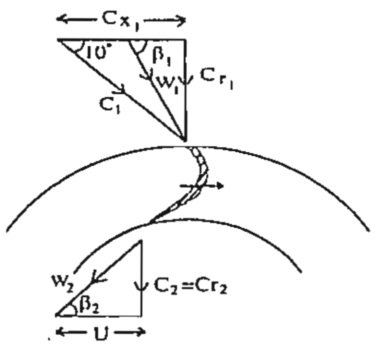


Figure 9.15

From inlet velocity triangle

$$\sin 10° = \frac{C_{r_1}}{C_1}$$

$$C_1 = \frac{C_{r_1}}{\sin 10°} = \frac{1.8}{0.1737}$$

$$C_1 = 10.366 \text{ m/s}$$

(b) The velocity of whirl at inlet

$$\tan 10° = \frac{C_{r_1}}{C_{x_1}}$$

$$C_{x_1} = \frac{1.8}{\tan 10°}$$

$$C_{x_1} = 10.208 \text{ m/s}$$

(iii) The relative velocity at inlet

$$W_1 = \sqrt{W_{x_1}^2 + C_{r_1}^2}$$

$$W_{x_1} = C_{x_1} - U_1$$

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.9 \times 200}{60}$$

$$U_1 = 9.425 \text{ m/s}$$

$$W_{x_1} = 10.208 - 9.425$$

$$= 0.783 \text{ m/s}$$

$$W_1 = (0.783^2 + 1.8^2)^{\frac{1}{2}}$$

$$W_1 = 1.963 \text{ m/s}$$

(iv) The runner blade angles

$$\beta_1 = \tan^{-1}\left(\frac{C_{r_1}}{W_{x_1}}\right)$$

$$= \tan^{-1}\left(\frac{1.8}{0.783}\right)$$

$$\beta_1 = 66.49°$$

$$\beta_2 = \tan^{-1}\left(\frac{C_{r_2}}{U_2}\right)$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.45 \times 200}{60}$$

$$U_2 = 4.712 \text{ m/s}$$

$$\beta_2 = \tan^{-1}\left(\frac{1.8}{4.712}\right)$$

$$\beta_2 = 20.91°$$

(v) Width of runner at outlet The discharge through a radial flow reaction turbine is given by

$$Q = \pi D_1 b_1 C_{r_1} = \pi D_2 b_2 C_{r_2}$$

(or)

$$D_1 b_1 = D_2 b_2$$
$$b_2 = \frac{D_1 b_1}{D_2}$$
$$= \frac{0.9 \times 0.2}{0.45}$$
$$b_2 = 0.4 \text{ m}$$

(vi) Mass of water flowing through the runner per second

$$m = \rho Q$$
$$Q = \pi D_1 b_1 C_{r_1} = \pi \times 0.9 \times 0.2 \times 1.8$$
$$= 1.018 \text{ m}^3/\text{s}$$
$$\dot{m} = 10^3 \times 1.018$$
$$\dot{m} = 1018 \text{ kg/s}$$

(vii) Head at the turbine inlet (H)

$$H = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} \text{ and}$$
$$H - \frac{V_2^2}{2g} = \frac{U_1 C_{x_1}}{g} \quad [\because C_{x_2} = 0]$$
$$H = \frac{U_1 C_{x_1}}{g} + \frac{V_2^2}{2g}$$
$$= \frac{9.425 \times 10.208}{9.81} + \frac{1.8^2}{2 \times 9.81}$$
$$H = 9.973 \text{ m} \quad [\because V_2 = C_2 = C_{r_2}]$$

(viii) Power developed

$$W = m(U_1 C_{x_1})$$
$$= 1018(9.425 \times 10.208)$$
$$W = 97.942 \text{ kW}$$

(ix) Hydraulic efficiency

$$\eta_H = \frac{U_1 C_{x_1}}{gH}$$
$$= \frac{9.425 \times 10.208}{9.81 \times 9.973}$$
$$= 98.34\%$$

Example 9.12 Design an inward flow Francis turbine whose power output is 330 kW under a head of 70 m running at 750 rpm. $\eta_H = 94\%$, $\eta_0 = 85\%$. The flow ratio at inlet is 0.15. The breadth ratio is 0.1. The outer diameter of the runner is twice the inner diameter of runner. The thickness of the vanes occupy 6% of the circumtial area of the runner. Flow velocity is constant and discharge is radial at outlet.

Solution

$P = 330$ kW $H = 70$ m $N = 750$ rpm $\eta_H = 0.94$ $\eta_0 = 0.85$
Flow ratio = 0.15 Breadth ratio = 0.1 $D_1 = 2D_2$.
Flow velocity

$$C_{r1} = \text{Flow ratio} \times \sqrt{2gH}$$
$$= 0.15 \times \sqrt{2 \times 9.81 \times 70}$$
$$= 5.56 \text{ m/s}$$

Discharge at outlet

$$Q = \frac{P}{\rho g H \times \eta_0}$$
$$= \frac{330 \times 10^3}{10^3 \times 9.81 \times 70 \times 0.85}$$
$$= 0.565 \text{ m}^3/\text{s}$$

But,

$$Q = \text{Actual area of flow} \times \text{Velocity of flow}$$
$$= 0.94\pi D_1 b_1 \times C_{r_1}$$

Since, breadth ratio

$$\frac{b_1}{D_1} = 0.1 \; b_1 = 0.1 D_1$$
$$Q = 0.94 \times \pi \times D_1 \times 0.1 D_1 \times C_{r_1}$$
$$D_1^2 = \frac{Q}{0.94 \times \pi \times 0.1 \times C_{r_1}} = \frac{0.565}{0.94 \times \pi \times 0.1 \times 5.56}$$
$$D_1 = 0.587 \text{ m}$$

and

$$b_1 = 0.1 \times D_1 = 0.0587 \text{ m}$$

Tangential speed of the runner at inlet

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.587 \times 750}{60}$$
$$= 23.05 \text{ m/s}$$

From hydrauic efficiency relation

$$C_{x_1} = \frac{\eta_H \times gH}{U_1} = \frac{0.94 \times 9.81 \times 70}{23.05} = 28 \text{m/s}$$

From inlet velocity triangle, Guide blade angle [Refer Fig. 9.6]

$$\tan\alpha_1 = \frac{C_{r_1}}{C_{x_1}}$$

$$\alpha_1 = \tan^{-1}\left(\frac{5.56}{28}\right)$$

$$\alpha_1 = 11.23^\circ$$

Runner vane angles at inlet and outlet

$$\tan\beta_1 = \frac{C_{r_1}}{C_{x_1} - U_1} = \frac{5.56}{28 - 23.05}$$

$$\beta_1 = 48.32^\circ$$

From outlet velocity triangle, [Refer Fig. 9.6]

$$\tan\beta_2 = \frac{C_{r_2}}{U_2}$$

$$\text{But } U_2 = \frac{\pi D_2 N}{60} \quad D_2 = \frac{1}{2}D_1 = 0.2935 \text{ m}$$

$$= \frac{\pi \times 0.2935 \times 750}{60} = 11.53 \text{ m/s}$$

$$\beta_2 = \tan^{-1}\left(\frac{5.56}{11.53}\right)$$

$$\beta_2 = 25.74^\circ$$

Width at outlet,

$$b_2 = \frac{D_1 b_1}{D_2} = \frac{0.587 \times 0.0587}{0.2935}$$

$$b_2 = 0.1174 \text{ m}$$

Example 9.13 A Francis turbine working under a head of 30 m has a wheel diameter of 1.2 m at the entrance and 0.6 m at the exit. The vane angle at the entrances is 90° and guide blade angle is 15°. The water at the exit leaves the vanes without any tangential velocity and the velocity of flow in the runner is constant. Neglecting the effect of draft tube, and losses in the guide and runner passages determine the speed of the wheel in rpm and vane angle at exit. *(BU–April '96)*

Solution

$H = 30$ m, $D_1 = 1.2$ m, $D_2 = 0.6$ m, $\beta_1 = 90^\circ$, $\alpha_1 = 15^\circ$, $C_{x_2} = 0$, $C_{r_1} = C_{r_2}$.

(a) Speed of the wheel If there is no loss of energy in the guide and runner vanes, the working head,

$$H = \text{Euler's head} + \frac{C_2^2}{2g} = E + \frac{C_2^2}{2g}$$

Since, $C_{x_2} = 0$, $C_2 = C_{r_2}$

$$E = \frac{U_1 C_{x_1}}{g}$$

$$\therefore H = \frac{U_1 C_{x_1}}{g} + \frac{C_2^2}{2g} = \frac{U_1 C_{x_1}}{g} + \frac{C_{r_2}^2}{2g}$$

Since $\beta_1 = 90^\circ$, $C_{x_1} = U_1$

$$H = \frac{U_1^2}{g} + \frac{C_{r_2}^2}{2g}$$

From inlet velocity triangle, (Refer Fig. 9.16)

$$\tan\alpha_1 = \frac{C_{r_1}}{U_1}$$

$$U_1 = C_{r1}/\tan\alpha_1 = C_{r1}/\tan 15^\circ$$

$$= 3.732 C_{r_1}$$

But $C_{r_1} = C_{r_2}$, and $U_1 = 3.732\, C_{r_1}$,

$$H = \frac{(3.732 C_{r_1})^2}{g} + \frac{C_{r_1}^2}{2g}.$$

$$30 = 1.42 C_{r_1}^2 + 0.051 C_{r_1}^2$$

$$C_{r_1} = 4.516 \text{ m/s}$$

Then $U_1 = 3.732 \times 4.516 = 16.854$ m/s

$$\text{Speed } N = \frac{60 \times U_1}{\pi D_1} = \frac{60 \times 16.854}{\pi \times 1.2}$$

$$N = 268.24 \text{ rpm}$$

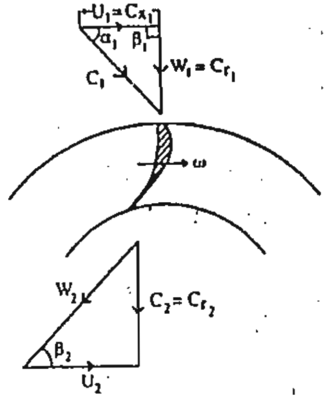


Figure 9.16

(b) Vane angle at exit

$$\tan\beta_2 = \frac{C_{r_2}}{U_2} = \frac{4.516}{U_2}$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 268.24}{60}$$

$$= 8.43 \text{ m/s}$$

$$\beta_2 = 28.18^\circ$$

Example 9.14 An outward flow reaction turbine has internal and external diameters of the runner as 0.6 m and 1.2 m respectively. The guide blade angle is 15° and velocity of flow through the runner is constant and equal to 4 m/s. If the speed of the turbine is 200 rpm. Head on the turbine is 10 m and discharge at outlet is radial.

Determine

(i) The runner vane angles at the inlet and outlet.
(ii) Work done by the water on the runner per kg of water and
(iii) Hydraulic efficiency

Solution

Given $D_1 = 0.6$ m, $D_2 = 1.2$ m, $\alpha_1 = 15°$, $C_{r_1} = C_{r_2} = 4$ m/s
$N = 200$ rpm, $H = 10$ m, $\alpha_2 = 90°$

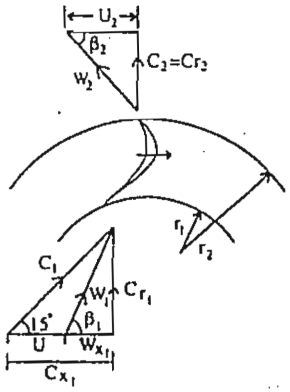


Figure 9.17

(i) Inlet and outlet vane angles

$$U_1 = \frac{\pi D_1 N}{60}$$

$$= \frac{\pi \times 0.6 \times 200}{60} = 6.28 \text{ m/s}$$

$$U_2 = \frac{\pi D_2 N}{60}$$

$$= \frac{\pi \times 1.2 \times 200}{60} = 12.57 \text{ m/s.}$$

$$W_{x_1} = C_{x_1} - U_1$$

$$C_{x_1} = \frac{C_{r_1}}{\tan 15°} = \frac{4}{\tan 15°}$$

$$= 14.93 \text{ m/s}$$

$$\therefore W_{x_1} = 14.93 - 6.28 = 8.65 \text{ m/s}$$

$$\tan \beta_1 = \frac{C_{r_1}}{W_{x_1}}$$

$$\beta_1 = \tan^{-1}\left(\frac{4}{8.65}\right)$$

Hence, $\beta_1 = 24.82°$

$$\tan \beta_2 = \frac{C_{r_2}}{U_2}$$

or

$$\beta_2 = \tan^{-1}\left(\frac{4}{12.57}\right)$$

Hence, $\beta_2 = 17.65°$

(ii) Work done

$$W/m = U_1 C_{x_1} \quad \because C_2 = C_{r2}$$

$$= 6.28 \times 14.93 \quad \because C_{x2} = 0$$

$$= 93.76 \text{ W/(kg/s)}$$

(iii) Hydraulic efficiency

$$\eta_H = \frac{U_1 C_{x_1}}{gH}$$

$$= \frac{93.76}{9.81 \times 10}$$

$$\eta_H = 95.58\%$$

Example 9.15 An axial flow hydraulic turbine has a net head of 23 m across it, and, when running at a speed of 150 rpm, develops 23 mW. The blade tip and hub diameters are 4.75 and 2.0 m respectively. If the hydraulic efficiency is 93 per cent and the overall efficiency is 85 per cent, calculate the inlet and outlet blade angles at the mean radius assuming axial flow at outlet.

Solution

Mean diameter

$$d_m = \frac{D + d}{2}$$

$$= \frac{4.75 + 2}{2}$$

$$= 3.375 \text{ m}$$

Overall efficiency

$$\eta_0 = \frac{\text{Power developed}}{\text{Power available}}$$

$$\text{Power available} = \frac{23 \times 10^6}{0.85}$$

$$= 27 \text{ mW}$$

Available power = $\rho g Q H$. Hence flow rate

$$Q = \frac{27 \times 10^6}{1000 \times 9.81 \times 23} = 119.7\ m^3/s$$

Rotor speed at mean diameter

$$U_m = \frac{\pi d_m N}{60} = \frac{\pi \times 3.375 \times 150}{60} = 26.5\ m/s$$

$$\text{Power given to runner} = \text{Power available} \times \eta_H = 27 \times 10^6 \times 0.93 = 25.11\ mW$$

But theoretical power given to runner is

$$W = \rho Q U_m C_{x_1} \quad (C_{x_2} = 0)$$

$$25.11 \times 10^6 = 1000 \times 119.7 \times 26.5 \times C_{x_1}$$

$$C_{x_1} = 7.9\ m/s \left[\eta_H = \frac{U_1 C_{x_1}}{gH}\right]$$

Axial velocity

$$C_a = \frac{Q}{\frac{\pi}{4}(D^2 - d^2)} = \frac{119.7 \times 4}{\pi \times (4.75^2 - 2^2)} = 8.21\ m/s$$

From the inlet velocity triangle [Refer Fig. 9.10], $C_a = C_{r_1}$

$$\tan(180^\circ - \beta_1) = \frac{C_a}{U_m - C_{x_1}} = \frac{8.21}{26.5 - 7.9}$$

Inlet blade angle

$$\beta_1 = 156.2^\circ$$

At outlet

$$\tan \beta_2 = C_a / W_{x_2}$$

But W_{x_2} equals U_m since C_{x_2} is zero. Hence

$$\tan \beta_2 = \frac{8.21}{26.5}$$

Outlet blade angle

$$\beta_2 = 17.2^\circ$$

Example 9.16 A Kaplan turbine runner is to be designed to develop 9100 kW. The net available head is 5.6 m. If the speed ratio = 2.09, flow ratio = 0.68, overall efficiency 86% and the diameter of the boss is 1/3 the diameter of the runner. Find the diameter of the runner, its speed and the specific speed of the turbine.

Solution

Given

P = 9100 kW $H = 5.6$ m Speed ratio = 2.09
Flow ratio = 0.68 Overall Efficiency $\eta_o = 0.86$

$$D_b = \frac{1}{3} D$$

D-Diameter of runner D_b-Diameter of boss.

Now, speed ratio = $\dfrac{U_1}{\sqrt{2gH}}$

$$U_1 = 2.09 \times (2 \times 9.81 \times 5.6)^{\frac{1}{2}} = 21.91\ m/s$$

Flow ratio = $\dfrac{C_{r_1}}{\sqrt{2gH}}$

$$\therefore C_{r_1} = 0.68 \times (2 \times 9.81 \times 5.6)^{\frac{1}{2}} = 7.13\ m/s$$

The Overall efficiency is given by

$$\eta_o = \frac{P}{\rho g Q H}$$

$$\therefore Q = \frac{9100 \times 10^3}{0.86 \times 9.81 \times 5.6 \times 10^3} = 192.6\ m^3/s$$

The discharge through a Kaplan turbine is given by

$$Q = \frac{\pi}{4}[D^2 - D_b^2] \times C_{r_1}$$

$$192.5 = \frac{\pi}{4}\left[D^2 - \left(\frac{D}{3}\right)^2\right] \times 7.13$$

$$0.88D^2 = 34.38$$

$$\therefore D = 6.22 \text{ m}$$

The speed of the turbine is given by

$$U_1 = \frac{\pi DN}{60}$$

$$\therefore N = \frac{60 \times 21.91}{\pi \times 6.22}$$

$$N = 67.3 \text{ rpm}$$

The Specific speed is given by

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$= \frac{67.3\sqrt{9100}}{(5.6)^{5/4}}$$

$$N_s = 745.25$$

Example 9.17 A Kaplan turbine working under a head of 20 m develops 11800 kW. The outer diameter of the runner is 3.5 m and hub diameter 1.75 m. The guide blade angle at the extreme edge of the runner is 35°. The hydraulic and overall efficiencies of the turbine are 88% and 84% respectively. If the velocity of whirl is zero at outlet determine.

(i) Runner inlet and outlet vane angles at the extreme edge of the runner and
(ii) Speed of the turbine.

Solution

Given $H = 20$ m, $P = 11800$ kW, $D = 3.5$ m, $D_b = 1.75$ m
Guide blade angle $\alpha_1 = 35°$, $\eta_H = 0.88$, $\eta_o = 0.84$ and, $C_{x_2} = 0$

The discharge through the runner is

$$\eta_o = \frac{P}{\rho g Q H}$$

$$Q = \frac{11800 \times 10^3}{10^3 \times 9.81 \times 20 \times 0.84}$$

$$Q = 71.598 \text{ m}^3/\text{s}$$

Q is given by

$$Q = \frac{\pi}{4}(D^2 - D_b^2) \times C_{r_1}$$

$$C_{r_1} = \frac{71.598}{\frac{\pi}{4} \times (3.5^2 - 1.75^2)}$$

$$= 9.92 \text{ m/s}$$

From inlet velocity triangle (Fig. 9.18(a))

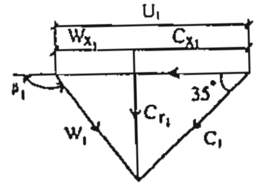


Figure 9.18(a)

$$C_{x_1} = \frac{C_{r_1}}{\tan 35°}$$

$$= \frac{9.92}{\tan 35°}$$

$$= 14.17 \text{ m/s}$$

$W_{x_1} = U_1 - C_{x_1}$
Using the relation for η_H

$$U_1 = \frac{\eta_H \times H \times g}{C_{x_1}}$$

$$= \frac{0.88 \times 20 \times 9.81}{14.17}$$

$$= 12.18 \text{ m/s}$$

$$W_{x_1} = 12.18 - 14.17 = -1.99 \text{ m/s}$$

The negative sign indicates that the W_{x_1} is in the negative X-axis direction.

(i) (a) Runner inlet angle,

$$\tan(180 - \beta_1) = \frac{C_{r_1}}{W_{x_1}}$$

$$180 - \beta_1 = 78.66$$

$$\beta_1 = 101.34°$$

(i) (b) Runner outlet angle,

From outlet velocity triangle (Fig. 9.18(b))
For a Kaplan turbine
$U_1 = U_2$ and $C_{r_1} = C_{r_2}$

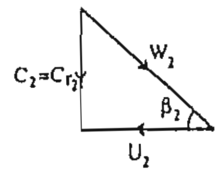


Figure 9.18(b)

$$\tan\beta_2 = \frac{C_{r_2}}{U_2}$$

$$\beta_2 = \tan^{-1}\left(\frac{9.92}{12.18}\right)$$

$$\beta_2 = 39.2°$$

(ii) The Speed of turbine

$$N = \frac{U_1 \times 60}{\pi D}$$

$$= \frac{12.18 \times 60}{\pi \times 3.5}$$

$$N = 66.46 \text{ rpm}$$

Example 9.18 A propeller turbine running at 50 rpm has a runner diameter as 6m and effective area of flow as 20 m². The angle of the runner blades at inlet and outlet are 150° and 20° respectively with the tangent to the wheel. Assuming constant velocity of flow calculate, (a) discharge, (b) theoretical power developed, and (c) hydraulic efficiency.

Solution

$N = 50$ rpm, $d = 6$ m $A_{effective} = 20\text{ m}^2$, $\beta_1 = 150°$, $\beta_2 = 20°$, $C_{r1} = C_{r2}$

(a) Discharge

$$U = \frac{\pi DN}{60} = \frac{\pi \times 6 \times 50}{60} = 15.7\text{ m/s}$$

As in Kaplan turbine, $U_1 = U_2 = U$.

$$\therefore U_2 = 15.7\text{ m/s}$$

From outlet velocity triangle [Refer Fig. 9.10]

$$\tan\beta_2 = \frac{C_{r_2}}{U_2} = \frac{C_{r_2}}{15.7}$$

$$\therefore C_{r_2} = 15.7 \times \tan 20° = 5.71\text{ m/s}$$

Given $C_{r_2} = C_{r_1} = C_r$
Discharge

$$Q = A_{effective} \times C_r$$
$$= 20 \times 5.71$$
$$Q = 114.2\text{ m}^3/\text{s}$$

(b) Theoretical power developed

From inlet velocity triangle [Refer Fig. 9.10]

$$\tan(180 - \beta_1) = \frac{C_{r_1}}{U - C_{x_1}}$$

$$\tan(180 - 150) = \frac{5.71}{15.7 - C_{x_1}}$$

$$C_{x_1} = 5.81\text{ m/s}$$

$$\text{Power developed} = \rho g Q\left(\frac{UC_{x_1}}{g}\right)$$

$$= 10^3 \times 9.81 \times 114.2\left(\frac{15.7 \times 5.81}{9.81}\right)$$

$$= 10416.98\text{ kW}$$

(c) Hydraulic efficiency

$$H = \frac{UC_{x_1}}{g} + \frac{C_2^2}{2g}$$

$$C_2 = C_{r_2} = 5.71\text{ m/s}$$

$$H = \frac{15.7 \times 5.81}{9.81} + \frac{(5.71)^2}{2 \times 9.81}$$

$$= 10.96\text{m}$$

Now,

$$\eta_H = \frac{UC_{x_1}/g}{H} = \frac{(15.7 \times 5.81)/9.81}{10.96}$$

$$\eta_H = 84.84\%$$

Example 9.19 A Kaplan turbine has an outer diameter of 8 m and inner diameter as 3 m and developing 30,000 kW at 80 rpm under a head of 12 m. The discharge through the runner is 300 m³/s. If hydraulic efficiency is 95%, determine (a) inlet and outlet blade angles, (b) mechanical efficiency and (c) overall efficiency.

Solution

$D = 8$ m, $D_b = 3$ m, $p = 30,000$ kW, $\eta_H = 0.95$
$N = 80$ rpm, $H = 12$ m, $Q = 300\text{ m}^3/\text{s}$

(a) Blade angles at inlet and outlet

In Kaplan turbines $U_1 = U_2$ as flow is axial and $C_{r_1} = C_{r_2}$
Now,

$$U = \frac{\pi DN}{60} = \frac{\pi \times 8 \times 80}{60} = 33.5\text{ m/s}$$

$$\text{Discharge, } Q = \frac{\pi}{4}(D^2 - D_b^2) \times C_r$$

$$C_r = \frac{300}{\frac{\pi}{4}(8^2 - 3^2)} = 6.95\text{m/s}$$

From outlet velocity triangle [Refer Fig. 9.10]

$$\tan\beta_2 = \frac{C_{r_2}}{U_2} = \frac{6.95}{33.5}$$

$$\beta_2 = 11.72°$$

Using the hydraulic efficiency relation

$$C_{x_1} = \frac{\eta_H \times gH}{U}$$

$$= \frac{0.95 \times 9.81 \times 12}{33.5}$$

$$= 3.34\text{ m/s}$$

Now, from inlet velocity triangle [Refer Fig. 9.10]

$$\tan(180 - \beta_1) = \frac{C_{r_1}}{U_1 - C_{x_1}}$$

$$180 - \beta_1 = \tan^{-1}\left(\frac{6.95}{33.5 - 3.34}\right)$$
$$= 12.98°$$
$$\beta_1 = 167.02°$$

(b) Mechanical efficiency

$$\eta_m = \frac{P}{\rho g Q\left(\frac{C_{x_1} U}{g}\right)} = \frac{30000 \times 10^3}{10^3 \times 9.81 \times 300}\left(\frac{3.34 \times 33.5}{9.81}\right)$$

$$\eta_m = 89.4\%$$

(c) Overall efficiency

$$\eta_0 = \eta_m \times \eta_H = 0.95 \times 0.894$$
$$\eta_0 = 84.9\%$$

Example 9.20 Find the operating speed and diameter of the runner of a Kaplan turbine having the following specifications.
Rated power = 11500 kW, Average head = 4.3 m.
Overall efficiency = 91%, Diameter of runner boss = 0.3 × diameter of runner, speed ratio = 2 Flow ratio = 0.65

Solution

P = 1150 kW	H = 4.3 m	η_0 = 0.91
D_h = 0.3 D	Speed ratio = 2	Flow ratio = 0.65

(a) Runner diameter

$$U = \text{Speed ratio} \times \sqrt{2gH}$$
$$= 2 \times \sqrt{2 \times 9.81 \times 4.3}$$
$$= 18.4 \text{ m/s}$$

Flow velocity,

$$C_r = \text{Flow ratio} \times \sqrt{2gH}$$
$$= 0.65 \times \sqrt{2 \times 9.81 \times 4.3}$$
$$= 5.97 \text{ m/s}$$

Now,

$$\eta_0 = \frac{P}{\rho g Q H}$$
$$Q = \frac{11500 \times 10^3}{0.91 \times 10^3 \times 9.81 \times 4.3}$$
$$= 299.58 \text{ m}^3/\text{s}$$

But,

$$Q = \frac{\pi}{4}(D^2 - D_b^2) \times C_r$$
$$= \frac{\pi}{4}(D^2 - (0.3D)^2) \times C_r$$
$$D^2 = \frac{4Q}{\pi(1 - 0.3^2) \times C_r}$$
$$= \frac{4 \times 299.58}{\pi \times 0.91 \times 5.97}$$
$$D = 8.38 \text{ m}$$

(b) Speed

$$U = \frac{\pi D N}{60}$$
$$N = \frac{60 \times 18.4}{\pi \times 8.38}$$
$$N = 41.9 \text{ rpm}$$

SHORT QUESTIONS

9.1. Define: Hydraulic turbine.
9.2. Differentiate between turbines and pumps
9.3. How are turbines classified?
9.4. Energy available at the impulse turbine inlet is only Kinetic Energy. (True/False)
9.5. What is a reaction turbine?
9.6. Classify hydraulic turbines according to the direction of flow through runner.
9.7. Pelton wheel is a high head turbine. (True/false)
9.8. Low head turbines are
 (a) Kaplan turbines
 (b) Propeller turbines
 (c) Both (a) and (b)
9.9. Francis turbine is a —— specific speed turbine.
9.10. The fluid flows in the —— direction through the Pelton turbine.
9.11. What is spear in a Pelton turbine?
9.12. Describe briefly the buckets of Pelton wheel.
9.13. The jet of water is deflected by the buckets through an angle of between
 (a) 100 and 120°
 (b) 160 and 165°
 (c) 130 and 140°
9.14. What is meant by breaking jet?
9.15. Draw the inlet and outlet velocity triangles for a single bucket of Pelton wheel.
9.16. For Maximum Energy transfer, the wheel velocity is
 (a) $2C_1$
 (b) C_1
 (c) $C_1/2$
 where C_1 is the velocity of jet at bucket inlet.

9.17. Define hydraulic efficiency.
9.18. Hydraulic efficiency is 100% when the angle turned through by the jet in the horizontal plane is ———.
9.19. Define the terms for a Pelton turbine,
(a) Nozzle Efficiency
(b) Nozzle velocity coefficient
9.20. The nozzle efficiency and velocity coefficient are related by
(a) $\eta_N / C_v^2 = 1$
(b) $\eta_N = C_v$
(c) $C_v = \eta_N^2$
9.21. Define overall efficiency of a Pelton turbine.
9.22. Classify radial flow turbines.
9.23. Draw the velocity triangles for an inward flow radial turbine.
9.24. What is a draft tube? Why is it used in a radial flow turbine?
9.25. Define the hydraulic efficiency of a radial flow turbine.
9.26. How does an inward flow radial turbine adjust automatically to the load variation?
9.27. What is an axial flow hydraulic turbine? Give Examples.
9.28. Differentiate between propeller turbine and Kaplan turbine.
9.29. Why must the blades of an axial flow turbines be long?
9.30. Draw the inlet and outlet velocity triangles for a Kaplan turbine?
9.31. How are the types of turbine selected for a particular application?
9.32. Give the range of specific speed values for the Pelton, Francis and Kaplan turbines.
9.33. What is a draft tube?
9.34. Why is a draft tube used in a reaction turbine?
9.35. List the types of draft tubes.
9.36. What are the advantages of a draft tube?
9.37. Define Efficiency of draft tube.
9.38. What is the phenomenon of cavitation in hydraulic turbines?
9.39. Compare the effect of cavitation in pumps and hydraulic turbines.
9.40. Define Thoma cavitation parameter.

EXERCISES

9.1. Draw a neat sketch of a Pelton turbine and describe the func tion of its main components.
9.2. Obtain an expression for the workdone per second by water on the runner of a Pelton wheel.
9.3. Draw inlet and outlet velocity triangles for a Pelton turbine and explain them briefly.
9.4. Derive an expression for maximum efficiency of the Pelton wheel giving the relationship between the jet speed and bucket speed.
9.5. Define the terms: speed ratio, jet ratio, and coefficient of velocity.

9.6. Design a Pelton wheel for a head of 80 m and speed 300 rpm. The Pelton wheel develops 140 kW. Take $C_v = 0.98$, speed ratio=0.45 and Overall efficiency=0.8.
[Ans: (a) D=1.112m, (b) d=8.55cm, (c) z=22]
9.7. A Pelton wheel is to be designed for the following specifications. Power = 1000 kW, head = 200 m, speed = 800 rpm, overall efficiency = 0.86 and jet diameter is not to exceed one-tenth the wheel diameter.
Determine: (a) Wheel diameter, (b) Diameter of the jet and (c) Number of jets required. [Ans: (a) 0.673 m, (b) 6.73 cm, (c) 2]
9.8. A Pelton wheel has a mean bucket speed of 10 m/s with a jet of water flowing at the rate of 700 litres/sec under a head of 30 m. The buckets deflect the jet through an angle of 160°. Calculate the power and efficiency of the turbine. Assume $C_v = 0.98$. *(MU–April '98)*
[Ans: (a) 187.110 kW, (b) 90.8%]
9.9. A Pelton wheel is having a mean bucket diameter of 1 m and is running at 1000 rpm. The net head on the Pelton wheel is 700 m. If the side clearance angle is 15° and discharge through nozzle is 0.1 m^3/s, find (a) Power available at the nozzle, and (b) hydraulic efficiency of the turbine.
[Ans: (a) 686.7 kW, (b) 97.2%]
9.10. The three jet Pelton wheel is required to generate 10,000 kW under a net head of 400 m. The blade angle at outlet is 15° and the reduction in relative velocity while passing over the blade is 5%. If the overall efficiency of the wheel is 80%, $C_v = 0.98$ and Speed ratio = 0.46, find (a) the diameter of the jet, (b) total flow in m^3/s and (c) the force exerted by a single jet on the buckets in the tangential direction. [Ans: (a) 12.5 cm (b) 3.18 m^3/s (c) 94.1 kN]
9.11. A Pelton turbine develops 8 MW under a net head of 130 m at a speed of 200 rpm. Assuming the coefficient of velocity for the nozzle as 0.98, hydraulic efficiency as 87% speed ratio as 0.46 and jet diameter to wheel diameter ratio as 1/9, determine, (a) the discharge required, (b) the diameter of the wheel, (c) the diameter and number of jets required, and (d) the specific speed.
[Ans: (a) 9.6 m^3/s, (b) 2.22 m, (c) 0.25 m and 4 and (d) 40.8]
9.12. A Pelton turbine develops 3 MW under a head of 300 m. The overall efficiency of the turbine is 83%. If speed ratio = 0.46, $C_v = 0.98$ and specific speed is 16.5, then find (a) diameter of the turbine and (b) diameter of the jet.
[Ans: (a) 1.78 m and (b) 0.14 m]
9.13. Draw and Explain the main parts of a radial flow reaction turbine.
9.14. Draw the inlet and outlet velocity triangles for an inward flow reacton turbine.
9.15. Define the following terms for a radial flow reaction turbine.
(a) Hydraulic efficiency
(b) overall efficiency.
9.16. An inward flow reaction turbine has outer and inner diameters of the wheel as 1 m and 0.5 m respectively. The vanes are radial at the inlet and the discharge is radial at the outlet. The water enters the vanes at an angle of 10°. Assuming the velocity of flow to be constant and is equal to 3 m/s, find (a) the speed of wheel and (b) the vane angle at outlet. [Ans: (a) 325 rpm, (b) 19.43°]
9.17. An inward flow reaction turbine has external and internal diameters as 1.2 m and 0.6 m respectively. The velocity of flow through the runner is constant and

is equal to 1.8 m/s. Determine (a) discharge through the runner and (b) width at outlet if the width at inlet is 20 cm. *(MKU–April '97)*

[Ans: (a) 1.36 m^3/s, (b) 40 cm]

9.18. A reaction turbine works at 500 rpm under a head of 100 m. The diameter of the turbine at inlet is 100 cm and the flow area is 0.35 m^2. The angles made by absolute and rela tive velocities at inlet are 15° and 60° respectively with the tangential velocity. Determine (a) the volume flow rate, (b) the power developed and (c) efficiency. Assume whirl at outlet to be zero.

[Ans: (a) 2.9 m^3/s, (b) 2356 kW and (c) 82.65%]

9.19. A Francis turbine with an overall efficiency of 70% is required to produce 147 kW. It is working under a head of 8 m. The peripheral velocity = $0.3\sqrt{2gH}$ and the radial velocity of flow at inlet is $0.96\sqrt{2gH}$. The wheel runs at 200 rpm and the hydraulic losses in the turbine are 20% of the available energy. Assuming radial discharge, determine (a) the guide blade angle, (b) the wheel vane angle at inlet, (c) diameter of the wheel at inlet and (d) width of the wheel at inlet.

[Ans: (a) 35.45°, (b) 42.54°, (c) 36 cm and (d) 20 cm]

9.20. An outward flow reaction turbine has internal and external diameters of the runner as 0.5 m and 1.0 m respectively. The guide blade angle is 15° and velocity of flow through the runner is constant and equal to 3 m/s. If the speed of the tur bine is 250 rpm; head on the turbine is 10 m and discharge at outlet is radial, determine (a) the runer vane angles at inlet and outlet; (b) work done by the water on the runner per second per kg of water and (c) hydraulic efficiency.

[Ans: (a) 32.5°, 12.5°, (b) 73.3 W/(kg/s) and (c) 74.7%]

9.21. An inward flow reaction turbine has external and internal diameters as 1.0 m and 0.6 m respectively. The hydraulic efficiency of the turbine is 90% when the head on the turbine is 36 m. The velocity of flow at outlet is 2.5 m/s and discharge at outlet is radial. If the vane angle at outlet is 15° and width of the wheel is 10 cm at inlet and outlet, determine: (a) the guide blade angle, (b) speed of the turbine, (c) vane angle at inlet, (d) volume flow rate and (e) power developed.

[Ans: (a) 4.2°, (b) 297 rpm, (C) 17.1°, (d) 0.47 m^3/s and (e) 150 kW]

9.22. An inward flow reaction turbine has an exit diameter of 1 metre and its breadth at inlet is 25 cm. If the velocity of flow at inlet is 2 m/s, find (a) the mass of water passing through the turbine per second. Assume 10% of the area of flow is blocked by blade thickness. If the speed of the runner is 210 rpm and guide blades make an angle of 10° to the wheel tangent, draw the inlet velocity triangle and find (b) the runner vane angle at inlet, (c) velocity of wheel at inlet, (d) the velocity of water leaving the guide vanes and (e) the relative velocity of water entering the runner blade.

[Ans: (a) 1413.6 kg/s, (b) 80.1°, (c) 11 m/s, (d) 11.52 m/s and (e) 2.03 m/s]

9.23. Draw a sketch of a Kaplan turbine and describe the working principle of the main parts of the turbine.

9.24. Draw the velocity triangles of a Kaplan axial flow turbine and derive the expression for work done.

9.25. A Kaplan turbine working under a head of 20 m develops 12,000 kW. The outer diameter of the runner is 3.5 m and hub diameter 1.75 m. The guide blade angle at the entrance edge of the runner is 35°. The hydraulic and overall efficiencies of the turbine are 88% and 84% respectively. If the velocity of whirl is zero at outlet, determine

(a) Runner inlet and outlet vane angles at the extreme edge of the runner and *(MU–Oct.'98, BU–Nov. '96)*

(b) Speed of the turbine. *(MKU–Nov. '97)*

[Ans: (a) 103.64° and 40.16° and (b) 65.32 rpm]

9.26. A Kaplan turbine runner is to be designed to develop 9100 kW. The net available head is 5.6 m. If the speed ratio = 2.09, flow ratio = 0.68, overall eficiency 86% and the diameter of the boss is 1/3 of the diameter of the runner. Find (a) the diameter of the runner, (b) speed and (c) the specific speed of the turbine.

[Ans: (a) 6.21 m (b) 67.5 rpm and (c) 746]

9.27. The hub diameter of a Kaplan turbine, working under a head of 12 m, is 0.35 times the runner diameter. The turbine is running at 100 rpm. If the vane angle of the extreme edge of the runner at outlet is 15° and flow ratio 0.6, find (a) diameter of the runner, (b) diameter of the hub and (c) the discharge through the runner. The velocity of whirl at the outlet is assumed as zero.

[Ans: (a) 6.55 m, (b) 2.3 m and (c) 271.8 m^3/s]

9.28. A Kaplan turbine has the hub diameter of 2.0 m and runner diameter of 5.0 m. If it develops 25,000 kW when running at 150 rpm under a head of 25 m, with η_H and η_o of 90% and 85% respectively, determine the discharge through the turbine and guide blade angle at inlet.

[Ans: (a) 119.93 m^3/s and (b) 52.29°]

9.29. A Kaplan turbine working under a head of 4 m produces 11000 kW. Speed ratio 1.2 and flow ratio 0.65, hub diameter is 0.36 times the outer diameter of runner. Taking overall efficiency as 85%, find the diameter and speed of runner.

[Ans: (a) 9.15 m and (b) 22.19 rpm]

9.30. A Kaplan turbine has a specific speed of 450. If it develops 10,100 kW under a head of 20 m, find the diameter of the runner. Take speed ratio as 2.

[Ans: 4 m]

9.31. What are the bases of selection of a turbine at a particular place?

9.32. Describe the theory of draft tube and draw the different types of draft tubes.

9.33. Explain the phenomenon of cavitation in turbines.

9.34. Explain with a neat sketch the double regulation governing method in an impulse turbine.

9.35. With a neat diagram, explain the method of governing a reaction turbine.

9.36. Describe the governing method used for an axial flow turbine.

10

POWER TRANSMITTING TURBO MACHINES

INTRODUCTION

Turbomachines are categorised as

(a) Power absorbing turbomachines
Example Pumps, fans, blowers, and compressors.

(b) Power generating turbomachines
Example Steam/gas turbines, and hydraulic turbines.

(c) Power transmitting turbomachines
Example Fluid couplings and torque convertors.

The power generating and power absorbing turbomachines are used respectively to produce and absorb power, the fluids flowing through them experience a change in enthalpy between the inlet and the outlet. In a power transmitting turbomachine, the fluid is totally contained in a rigid casing and does not cross the system boundary.

The power transmitting turbomachines essentially serve the same purpose as mechanical transmission devices such as a gear train and clutch assembly, in transmitting power between two different shafts. In a gear train, the drive is positive i.e. if the driver shaft rotates, the driven shaft will necessarily rotate. In a power transmitting turbomachine, there is no positive drive, since the driver and driven shafts are not mechanically coupled.

THE HYDRAULIC COUPLING

The fluid or hydraulic coupling is a device used for transmitting power from driving shaft to driven shaft with the help of fluid (generally oil). Oil is used as the working fluid because of its stability, non-corrosive nature and lubricating properties. There is no mechanical connection between the two shafts.

The hydraulic coupling consists of a radial pump impeller mounted on a driving shaft and a radial flow reaction turbine mounted on the driven shaft. Both the impeller and runner are identical in shape and together form a casing which is completely enclosed and filled with oil.

(a) Working Principle

In the beginning, both the shafts are at rest. When the driving shaft is rotated, the oil starts moving from the inner radius to the outer radius of the pump impeller as shown in the Fig. 10.1. The pressure energy and kinetic energy of the oil increase at the outer radius of the pump impeller.

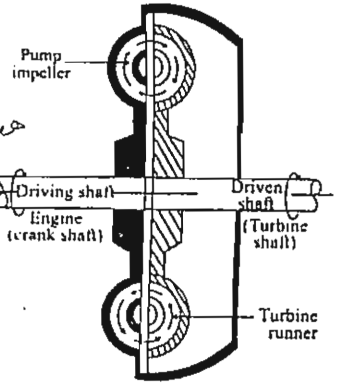


Figure 10.1 Fluid (or) Hydraulic coupling

The energy-increased oil then enters the runner of the reaction turbine at the outer radius and flows from the outer radius to the inner radius of the turbine runner. The oil while flowing through the runner, transfers its energy to the blades of the runner and makes the runner to rotate. Thus, finally, the driven shaft rotates. The oil then flows back into the pump impeller. Thus, a continuous circulation of oil occurs and hence continuous rotation of both shafts are maintained.

The power is transmitted hydraulically from the driving shaft to the driven shaft and the driven shaft is free from engine vibrations.

The speed of the driven shaft is always less than the speed of the driving shaft by about 2 per cent. The efficiency of the power transmission by hydraulic coupling is about 98%.

(b) Efficiency

The efficiency of hydraulic coupling is derived in the following equation.

$$\eta_{\text{fluid coupling}} = \frac{\text{Power output}}{\text{Power input}} = \frac{\text{Power transmitted to the driven shaft}}{\text{Power available at the driving shaft}}$$

But power at any shaft is given by

$$\text{Power} = \frac{2\pi NT}{60}$$

(or)

$$\boxed{\text{Power} \propto (\text{Speed} \times \text{Torque})}$$

Let,

N_A – Speed of driving shaft
N_B – Speed of driven shaft
T_A – Torque at driving shaft
T_B – Torque at driven shaft

Then, power available at shaft A is

$$P_A \propto N_A T_A$$

and power transmitted to shaft B is

$$P_B \propto N_B T_B$$

The efficiency equation then becomes

$$\eta_{FC} = \frac{N_B \times T_B}{N_A \times T_A}$$

But $T_A = T_B$ i.e the torque transmitted is the same.

$$\boxed{\eta_{FC} = \frac{N_B}{N_A} = \frac{\text{Driven shaft speed}}{\text{Driving shaft speed}}}$$

(c) Slip

Slip of fluid coupling is defined as the ratio of the difference of the speeds of the driving and driven shafts to the speed of the driving shaft. Mathematically,

$$\text{Slip } \sigma = \frac{N_A - N_B}{N_A} = 1 - \frac{N_B}{N_A}$$

or

$$\boxed{\sigma = 1 - \eta_{FC}}$$

The fluid coupling effects a smooth transfer of power from the engine to the transmission. All jerking and roughness is eliminated by the use of the fluid coupling. This provides smooth take off and reduces the wear and strain on the drive train.

At high engine speeds, the coupling is very efficient. It gives one to one ratio between driven and driving members. At medium speeds, the coupling is not quite as effective. At low engine speeds there is little power transfer. When the engine speed is low, there is no power transfer. This is same as having a conventional clutch in the disengaged position. This allows the coupling to act as a clutch. As the engine speed is increased, power transfer becomes more effective.

The fluid coupling cannot increase the torque above that produced by the crank shaft.

TORQUE CONVERTER

The hydraulic torque converter is a device used to transmit increased or decreased power from one shaft to another. The torque transmitted at the driven shaft may be more or less than the torque available at the driving shaft. The torque at the driven shaft may be increased by about five times the torque available at the driving shaft with an efficiency of about 90%.

A torque converter (Fig. 10.2) comprises of (i) pump impeller coupled to the driving shaft (ii) turbine runner coupled to the driven shaft and (iii) stator or fixed guide vane arranged between the pump impeller and the turbine runner.

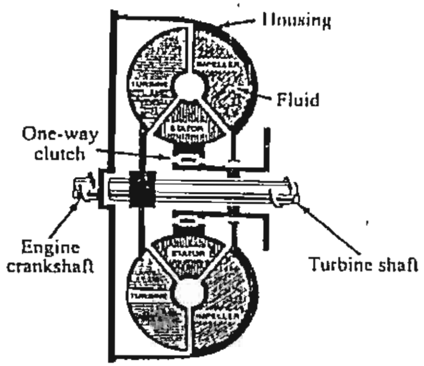


Figure 10.2 Torque converter

Oil flowing from the pump impeller to the turbine runner exerts a torque on the stationary vanes. These vanes change the direction of flow of oil, thereby making a possible torque and speed transformation.

The torque relationship is described as

$$T_B = T_S + T_A$$

where

T_A – Torque at driving shaft

T_B – Torque at driven shaft

T_S – Torque at stationary vanes

The power at any shaft is proportional to the product of torque and shaft speed.

Then, power available at shaft A is

$$P_A = \omega_A T_A$$

and power available at shaft B is

$$P_B = \omega_B T_B \quad \text{or}$$
$$= \omega_B (T_A + T_S)$$

Efficiency of torque converter

$$\eta_{tc} = \frac{\text{Power output}}{\text{Power input}}$$
$$= \frac{\omega_B (T_A + T_S)}{\omega_A T_A}$$
$$= \frac{\omega_B}{\omega_A}\left(1 + \frac{T_S}{T_A}\right)$$

Obviously, when there are no guide vanes, torque converter reduces to flange coupling, because, when

$$T_S = 0$$

Then,

$$\eta_{tc} = \frac{\omega_B}{\omega_A} = \eta_{FC}$$

Further depending upon the design and orientation of guide vanes, the torque converter may function as a torque mulitiplier or torque divider.

Torque converter acts as a torque multiplier i.e., when T_S is positive. To achieve this, the guide vanes are designed to receive a torque from the oil in a direction opposite to that exerted on the driven shaft.

If the guide vanes are designed such that they receive a torque from the oil in the same direction as that of the driven shaft, then the torque converter will act as a torque divider i.e. when T_S is negative.

Usually the torque converters are employed for torque augmentation i.e. for increasing the torque. Large magnification are obtained by having two or more sets of turbine runner and fixed guide vanes.

Torque converters find application in diesel locomotives, earth moving machinery and automobile power transmitting units.

At low speed ratios, torque converters are more economical than fluid coupling. Conversely when the speed ratio approaches unity, the fluid coupling is economical.

For optimum advantages in a system, the transmission system is so designed that the unit acts as a converter at low speed ratios and as a coupling at high speed ratios i.e. at speed ratios above 0.5.

CHARACTERISTICS OF FLUID COUPLING & CONVERTER

The characteristics of fluid coupling and Torque convertor are shown in Figures 10.3 and 10.4. The efficiency and Torque ratio (T_B/T_A) are functions of speed ratio (N_B/N_A). The torque ratio of a torque convertor falls with increasing speed ratios, while at the same time the efficiency increases. At a speed ratio between 0.65 and 0.7 (design value), the transmission efficiency reaches its maximum value of about 85%. If the speed ratio exceeds the design value, the efficiency decreases quite rapidly.

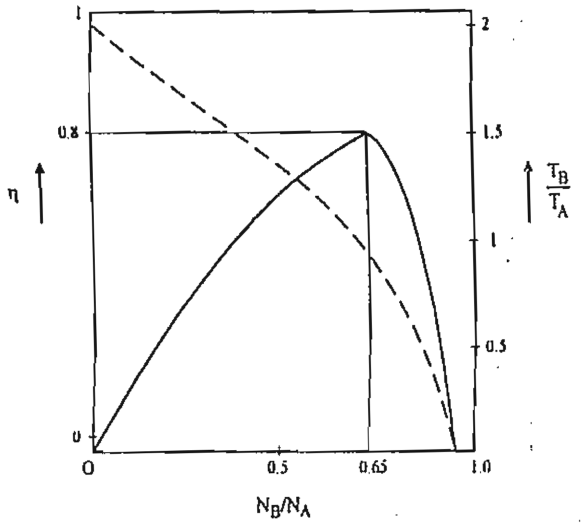


Figure 10.3 Torque converter characteristics

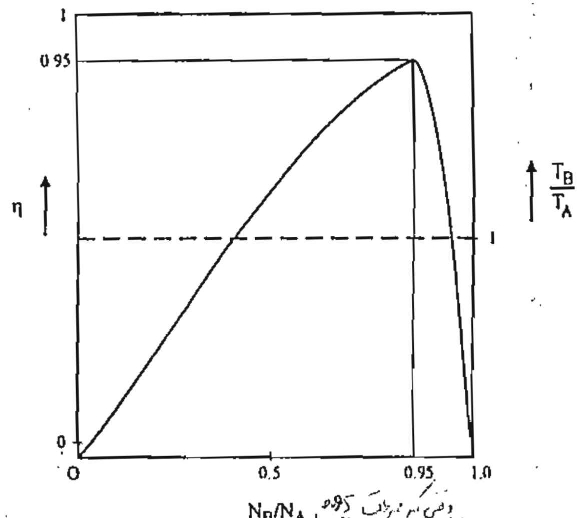


Figure 10.4 Fluid coupling characteristics

But the efficiency of the fluid coupling increases continuously with speed ratio, reaching a maximum value of about 95% when the speed ratio is 0.95. However, the rise in efficiency of the fluid coupling is not as fast as that of the torque convertor during initial stages. Infact, the torque convertor, running at its design ratio (0.65 – 0.7), has a higher efficiency than that of the fluid coupling at the same speed

ratio. Thus the torque convertor is a more efficient power transmitting device at low speed ratios, while the fluid coupling is more efficient device at speed ratio nearer to 0.95 or 0.98.

For automobiles, the speed ratio varies between 0 and 0.98. Therefore, it is usual to have a combination of fluid coupling and hydraulic convertor, so as to avoid the inefficient range of operation of each device. At low speed ratios, the unit act as a torque convertor while at high speed ratios, the same unit works as a fluid coupling with torque ratio of unity and its efficiency increases to about 0.98 with an increasing speed ratio.

EXERCISES

10.1. What are power transmitting turbomachines?
10.2. What is hydraulic coupling?
10.3. Explain with a neat sketch, the working of fluid coupling.
10.4. Derive the efficiency of a fluid coupling.
10.5. Define: Slip of a fluid coupling. Show that Slip $\sigma = 1 - \eta$ where η is the fluid coupling efficiency.
10.6. In a fluid coupling, the speeds of the driving and driven shafts are 800 rpm and 780 rpm respectively, find
(a) the efficiency of the hydraulic coupling and (b) the slip of the coupling.
[**Ans:** (a) 97.5% and (b) 2.5%]
10.7. What is a Torque coverter?
10.8. How does a torque converter differ from a fluid coupling?
10.9. Draw a neat sketch and explain the working, principle of a torque converter?
10.10. Derive the expression for the efficiency of a torque converter?
10.11. What is the role of guide vanes in a torque converter?
10.12. Compare torque converter and fluid coupling on the basis of speed ratio.
10.13. Write a note on the characteristics of a fluid coupling.
10.14. Discuss the characteristics of torque convertor.

APPENDIX

AXIAL FLOW COMPRESSOR BLADE DESIGN (FOR COMPRESSOR PROBLEMS)

```
# include <stdio.h>
# include <math.h>
# include <conio.h>
main ( )
{
double a1, a2, b1, b2, i, j, k, l, A1, A2, B1, B2, q, r, s, t;
float td, WF, U, Ca, Cp = 1005.0, R, Um, Ut, Ur;
int x;
char Q;
printf ("enter temperature difference =");
scanf ("%f", &td);
printf ("enter workdone factor =");
scanf ("%f", &WF);
printf ("enter mean blade speed m/sec) =");
scanf ("%f", &Um);
printf ("enter axial velocity (m/sec) =");
scanf ("%f", &Ca);
printf ("enter the blade tip speed (m/sec) =");
scanf ("%f", &Ut);
printf ("enter the blade root speed (m/sec) =");
scanf ("%f", &Ur);
printf ("constant reaction blade ? =");
scanf ("%c", &Q);
clrscr ( );
if (Q = = 'y')     /*CONSTANT REACTION BLADE DESIGN */
{
for (x=1; x <=3; x++)
{
if (x == 1)
{
 U = Ut;
  printf ("blade tip angles (deg.) : \n\n");
 }
else if (x == 2)
{
 U = Um;
  printf ("mean blade angles (deg.) :\n\n");
 }
```

```
else
{
 U = Ur;
  printf ("blade root angles (deg.) :\n\n");
 }
i = (td * Cp)/(WF * U * Ca);
j = (0.5 * 2 * U) / Ca;
k = atan ((i + j) / 2);
b1 = (180 * k) / 3.14;
printf ("inlet blade angle = %f \n", b1);
l = atan (j - tan (k));
b2 = (180 * l)/3.14;
printf ("outlet blade angle = %f \n", b2);
printf ("inlet air angle = %f \n", a1 = b2);
printf ("outlet air angle = %f \n\n", a2 = b1);
}
}
else
{
clrscr ( );     /*FREE VORTEX BLADE DESIGN * /
i = (td * Cp) / (WF * Um * Ca);
j = (0.5 * 2 * Um) / Ca;
k = atan ( (i + j) / 2);
b1 = (180 * k) / 3.14;
printf ("mean blade angles (deg.) :\n \n");
printf ("inlet blade angle = %f \n", b1);
l = atan (j - tan (k) );
b2 = (180 * l) / 3.14;
printf ("outlet blade angle = %f \n", b2);
printf ("inlet air angle = %f \n", a1 = b2);
printf ("outlet air angle = %f \n \n", a2 = b1);
for (x = 1; x < = 2; x + +)
{
if (x==1)
{
 U = Ut;
  printf ("blade tip angles (deg.) : \n \n");
 }
else
 {
 U = Ur;
  printf ("blade root angles (deg.) : \n\n");
 }
A1 = atan ( (Um / U)* tan (1));
a1 = (180*A1) / 3.14;
printf ("inlet air angle = %f \n", a1);
B1 = atan ((U / Ca) - tan (A1));
b1 = (180*B1) / 3.14;
printf ("in let blade angle = %f \n", b1);
A2 = atan ((Um/U)* tan (k));
a2 = (180*A2) / 3.14;
printf ("outlet air angle = %f \n", a2);
B2 = atan ((U / Ca) - tan (A2));
b2 = (180*B2) / 3.14;
printf ("outlet blade angle = %f \n", b2);
R = Ca / (2*U) * (tan (B1) + tan (B2));
printf ("degree of reaction = %f \n\n", R);
}
}
}
```


INPUT

enter temperature difference = 20
enter workdone factor = 0.92
enter mean blade speed (m/sec) = 210
enter axial velocity (m/sec) = 157.5
enter blade tip speed (m/sec) = 262.5
enter blade root speed (m/sec) = 157.5
constant reaction blade ? = y

OUTPUT

Blade tip angles (deg.):
inlet blade angle = 47.687020
outlet blade angle = 29.659747
inlet air angle = 29.659747
outlet air angle = 47.687020
Mean blade angles (deg.):
inlet blade angle = 44.935090
outlet blade angle = 18.601836
inlet air angle = 18.601836
outlet air angle = 44.935090
Blade root angles (deg.):
inlet blade angle = 43.261696
outlet blade angle = 3.414273
inlet air angle = 3.414273
outlet air angle = 43.261696