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Sarmila Sahoo

Design Aids for Stiffened Composite Shells with Cutouts

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Sarmila Sahoo
Department of Civil Engineering
Heritage Institute of Technology Kolkata
Kolkata, West Bengal
India

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Preface

Laminated composites are increasingly being used nowadays in aerospace, civil, marine and other related weight-sensitive engineering applications that require high strength-to-weight and stiffness-to weight ratios. Some of the structural components of aircrafts and missile and ship structures can be idealized as composite shell panels. Quite often, to save weight and also to provide a facility for inspection, cutouts are provided in these panels. In practice, the margin of the cutouts must be stiffened to take account of stress concentration effects. Also, there can be some instruments directly fixed on these panels, and the safety of these instruments can be dependent on the vibration characteristics of the panels. Hence, free vibration studies on laminated composite shell panels with cutouts are of interest to both structural engineers and materials scientists.

This book focuses on the free vibrations of graphite-epoxy laminated composite stiffened shells with cutouts both in terms of the natural frequencies and mode shapes. The dynamic analysis of shell structures, which may have complex geometry and arbitrary loading and boundary conditions, is solved efficiently by the finite element method, including cutouts in shells. The results may be readily used by practising engineers dealing with stiffened composite shells with cutouts. Several shell forms, viz. cylindrical shell, hypar shell, conoidal shell, spherical shell, saddle shell, hyperbolic paraboloidal shell and elliptic paraboloidal shell, are considered in the book. The dynamic characteristics of stiffened composite shells with cutouts are described in terms of the natural frequency and mode shapes. The size of the cutouts and their positions with respect to the shell centre are varied for different edge constraints of cross-ply and angle-ply laminated composite shells. The effects of these parametric variations on the fundamental frequencies and mode shapes are discussed in detail. The information regarding the behaviour of stiffened shells with cutouts for a wide spectrum of eccentricity and boundary conditions for cross-ply and angle-ply shells may be used as design aids for structural engineers. It is believed that this book will have a significant contribution to the existing literature from the point of view of both industrial importance and academic interest.

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About the Author

Dr. Sarmila Sahoo received her Bachelor of Engineering (Civil Engineering) from Bengal Engineering College (D.U.), Shibpur (presently Indian Institute of Engineering Science and Technology, Shibpur), Master of Civil Engineering (Structural Engineering) from Jadavpur University and Doctor of Philosophy (Engineering) from Jadavpur University. A national scholarship holder throughout her career, she was the recipient of GATE fellowship from Ministry of Human Resource Development and Senior Research Fellowship from the Council of Scientific and Industrial Research, Govt. of India. She has worked in the Design and Development Unit at Conveyor and Ropeway Services Pvt. Ltd. She has earlier taught Civil Engineering at Meghnad Saha Institute of Technology, Kolkata. Presently, she is working as Associate Professor of Civil Engineering at Heritage Institute of Technology, Kolkata. Her research interests are the finite element method, vibration of plates and shells and composite materials. She is actively engaged in research and currently supervising a number of research students for their doctoral thesis. She has published over 40 research articles in international journals and conference proceedings. She is associated with the editorial and review board of a number of international journals. She is a member of the International Association of Engineers.

Chapter 1
Fundamental Consideration

Abstract The importance and usefulness of shell structures made of laminated composites are presented in this chapter. Different shell forms along with the necessity of stiffeners and cutouts are also discussed. Extensive literature survey is carried out and it is found that studies on free vibration aspects of laminated composite shell panels with cutout are scanty. The mathematical formulation for finite element analysis of stiffened shell panels with cutout is presented and the same is validated through solution of benchmark problems.

Keywords Shell structure • Laminated composite • Finite element • Stiffeners • Cutout

Notations

a, b	length and width of shell in plan
a', b'	length and width of cutout in plan
b_{st}	width of stiffener in general
b_{sx}, b_{sy}	width of X and Y stiffeners respectively
B_{sx}, B_{sy}	strain displacement matrix of stiffener element
d_{st}	depth of stiffener in general
d_{sx}, d_{sy}	depth of X and Y stiffeners respectively
e	eccentricity of stiffeners with respect to mid surface of shell
e_{sx}, e_{sy}	eccentricities of X and Y stiffeners with respect to mid surface of shell
E_{11}, E_{22}	elastic moduli
G_{12}, G_{13}, G_{23}	shear moduli of a lamina with respect to 1, 2 and 3 axes of fibre
h	shell thickness
M_{sxx}, M_{syy}	moment resultants of stiffeners
np	number of plies in a laminate
N_{sxx}, N_{syy}	axial force resultants of stiffeners
Q_{sxxz}, Q_{syyz}	transverse shear resultants of stiffeners

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R_{xx}, R_{yy}, R_{xy}	radii of curvature of shell, radius of cross curvature of shell
T_{sxx}, T_{syy}	torsion resultants of stiffeners
u_{sx}, w_{sx}	axial and transverse translational degrees of freedom at each node of X-stiffener element
v_{sy}, w_{sy}	axial and transverse translational degrees of freedom at each node of Y-stiffener element
x, y, z	local co-ordinate axes
X, Y, Z	global co-ordinate axes
z_k	distance of bottom of the kth ply from mid-surface of a laminate
α_{sx}, β_{sx}	rotational degrees of freedom at each node of X-stiffener element
α_{sy}, β_{sy}	rotational degrees of freedom at each node of Y-stiffener element
δ_{sxi}, δ_{syi}	nodal displacement of stiffener element
ν_{12}, ν_{21}	Poisson's ratios
ρ	density of material
ω	natural frequency
$\overline{\omega}$	non-dimensional natural frequency $= \omega a^2 \left(\rho / E_{22} h^2\right)^{1/2}$

1.1 Introduction

A shell is defined as an arbitrarily curved structural surface, which resists externally superimposed loads by combined inplane thrusts as well as bending of the surface. This coupling of inplane forces and bending moments renders the shell forms with high strength. Thus a shell form gains its strength through its form rather than through its mass. The doubly curved, non-developable surfaces are specialization of such structural elements and have added efficiency both in terms of strength and aesthetics.

Many examples of shells are encountered in nature like the eggs, nuts, human and animal skulls and so on. From the very dawn of civilization, man was attracted towards shells realizing their potentialities and the primitive Chou vases, Greek urns and curved roofs made by him bear the signature of his liking. The first prototype submarine in 1620, the earliest pressure cooker in 1688, the use of balloon as a thin-walled pressure vessel in the eighteenth century and the pneumatic tyres in 1845 are some of the earliest applications of shell structures. Among the shells of civil engineering interests, the Pantheon of Rome built in AD 125 and the Mosque of Santa Sophia, Istanbul, built in AD 538 are examples of earlier shell roofs with large spans. Subsequently, in countless places, shell forms were used in civil engineering practices as roofs, radomes, foundations, grain-storage structures, chimneys, water tanks, cooling towers, etc. From the early years of human civilization up to the present time, the course of shell construction, application and research has been following a meandering path with the emphasis being shifted from one aspect to another from time to time.

Man took a leaf out of the nature to conceive of constructing stiffened structures. Stiffened plate/shell structures have been widely used in a variety of engineering structures such as chimneys, pipes and conduits, bridges, aircrafts, ships and offshore structures. The idea that structures can be made of thin plates/shells and may be further strengthened by integrating them with a row of ribs appeared as a revolutionary concept as more economy in weight is achieved in the process without compromising with the strength or stability.

The extensive use of stiffened structural elements in engineering started in the nineteenth century with the application of steel plates for hulls of ships, steel bridges and aircraft structures. With the development of improved structural materials such as steel and concrete and that of better fastening devices such as welding, stiffened steel structures have become most cost-effective and efficient. These developments paved the way for wide applications of stiffened structural elements in various branches of engineering, a few of which are cited below as examples:

1. Aircraft structure
2. Ship hauls
3. Offshore structures
4. Dock gates/lock gates
5. Road bridges
6. Rocket launching pedestals
7. Floors and roofs of buildings
8. Retaining walls
9. Storage tanks
10. Railway cars
11. Large transportation carrier panels

As the use of stiffened structures gained popularity, the engineers started thinking of using different materials in different applications depending on the case specific requirements. For example, boats and small ships were constructed using wood, large ships with steel, aircrafts using aluminium alloy and buildings using reinforced concrete and/or steel. The search in the field of advanced materials resulted in the development of lightweight laminated composites with high stiffness-to-weight as well as strength-to-weight characteristics. The potentially high resistance of these materials to environmental degradation results in lower life-cycle costs and they may be tailored to any desired shape. Due to numerous advantages the laminated composites can offer, these materials started being applied in different branches of engineering. Bandyopadhyay (2000), Karbhari and Zhao (2000) highlighted such applications in their papers with special emphasis on civil engineering applications.

The use of composites in engineering structures evolved as a result of a necessity during the Second World War when there was concern for the loss of supply of bauxite and hence aluminium alloys. A new material named ‘Gordon Aerolite’ was developed in Britain by Aero Research Limited (at present a part of the Ciba Geigy Group) as an alternative to aluminium alloy for aircraft construction. This material was a fibre-reinforced plastic with untwisted flax fibres embedded in phenolic resin.

The fuselage with its frames, stringers and covering skin was constructed with this material. This was the world's first composite used in modern structures.

Broadly composites are combinations of two or more materials to achieve specific properties. From an engineering point of view, a composite may be defined as a combination of materials, which differ in composition or form on macroscopic level and remain chemically inactive with respect to each other to form a useful material. Commonly used composites are of two types, i.e. particulate and fibrous.

A particulate composite consists of particle reinforcements embedded in a continuous phase (matrix) of other material. Both particle and matrix may be metallic or non-metallic. Fibrous composites consist of a large number of strong, stiff, continuous or chopped fibres embedded in a matrix having a large length-to-diameter ratio (100 or more) to ensure a reinforcing action. They are made of either inorganic (glass, carbon, boron) or organic (aramid or kevlar) materials. The matrices can also be metallic (aluminium) or non-metallic (polyester, epoxy, phenolic resins and ceramics).

Lamina is formed when an array of fibres is given a resin bath and hardened. The matrix transfers the loads to fibres. A number of laminae or plies are bonded together in different orientations to form a laminate. Laminas are homogeneous and anisotropic in macroscopic scale, and laminate is orthotropic. These are used in different structural forms such as beams, plates, shells, stiffened plates, stiffened shells, etc. The low cost of assembly, ease of repair, high specific stiffness, high specific strength, excellent damage tolerance and superior fatigue response characteristics propelled the use of laminated composites in several important engineering fields.

The exact analysis of stiffened plates/shells based on the theory of elasticity is rarely performed due to its tedious and difficult computational procedures for evaluation of deflection and stresses. Analysis of stiffened shells has been approached from three different angles. In the first approach, the elastic properties of stiffeners are distributed uniformly along the orthogonal directions and the stiffened plate/shell is replaced by an equivalent orthotropic plate/shell of constant thickness. The orthotropic plate/shell theory demands equal and close spacing of the stiffeners. Also the stiffeners are converted into an 'equivalent plate/shell'. In this approach the evaluation of stresses in the plate and the stiffeners separately becomes difficult. Further, the evaluation of properties of the equivalent plate/shell becomes a difficult proposition. These limitations restrict the usefulness of the technique. In the second type of modelling, the stiffened plate/shell is idealized as grillage, which is a plane structure of intersecting beams and carries lateral load through the action of beam bending. There are basically two drawbacks in this approach. Firstly, the centroidal planes of beams in different directions are assumed to be coincident, which affect the accuracy of stresses calculated. Secondly, that the beam properties are derived based on effective width of plate/shell is an issue which remains inconclusive. Further, common parts of the same plate (effective width) are considered to be effective for both the intersecting beams in two different directions. Therefore, this model has restrictive usage. The third category of modelling is most realistic and accurate. In this method the plate/shell sheet and the array of beams are treated as separate entities initially. The stiffness of the stiffened structures is derived by adding the individual stiffness by maintaining the compatibility of the deformation at the interfaces. This methodology makes the analysis sufficiently involved and complex, but it gives a better picture of structural behaviour. However, the advent of high-speed digital computers and parallel development of numerical techniques of structural analysis have made the analysis of structures simpler and more accurate. There are a number of numerical methods that consider the plate/shell and the stiffeners as separate entities. They are:

1. Finite difference method
2. Energy methods, such as Rayleigh's method, Rayleigh-Ritz method, Galerkin method, etc.
3. Dynamic relaxation method
4. Finite element method

Out of these, the finite element *method is the most* versatile due to its ability to consider any arbitrary geometry, loading and boundary conditions. The review of the literature shows that the researchers from different corners of the world are engaged in the analysis of composite stiffened plates and shells by finite element approach and the present approach also uses this technique to analyse the free vibrations of composite stiffened shells.

Stiffened shell structures find extensive use in aerospace, civil, marine and other engineering applications as mentioned earlier. In civil engineering construction, cylindrical, elliptic paraboloid, hyperbolic paraboloid, conoidal and hypar shells are commonly used as roofing units. Among these the doubly curved shells are architecturally appealing and frequently favoured for roofing large column-free areas. The synclastic elliptic paraboloid shells are both structurally stiff and architecturally acceptable due to their elegant appearance and surface geometry. The anticlastic hyperbolic paraboloid shells are preferred in many places due to their high aesthetics. The aesthetically appealing conoidal shells are singly ruled surfaces, easy to cast and can allow entry of north light. The anticlastic doubly curved hypar shells having only the twist of curvature are preferred in many situations due to their architectural beauty. The surfaces are doubly ruled and hence easy to cast and fabricate. The above types of shells are often stiffened to achieve greater strength with relatively less amount of material which results in considerable weight reduction.

As major structural components of ships and aircrafts, the stiffened plates/shells are subjected to both static and dynamic loading of varying magnitudes. The static loads are caused by the internal and/or external pressure in fuel cells of aircrafts, long pipes and conduit structures, dead and live loads on the ship decks and similar other loads. Dynamic effects are caused by the aerodynamic loadings on aircrafts, hydrodynamic loadings on ship hulls, engine vibrations and such other forces. Shell forms used as roofing units have to withstand various distributions of dead and live loads apart from the dynamic loads from wind, earthquake, snowfall, etc. Hence, the knowledge of their static as well as dynamic behaviour is important from the standpoint of analysis and design.

It is evident from the review of literature (presented in the next section) that extensive research about bending, free and forced vibrations, stability, etc., is going on both about isotropic and composite shells all over the world. The advent of high-speed computers is helping the applications of the numerical techniques like finite difference, finite element, boundary element and others to analyse arbitrary shell geometry subjected to complicated loadings and boundary conditions. Of the different shell forms cylindrical, spherical and saddle ones have received some attention from researchers although much is left to be done. The other forms like the skewed hypars and the conoids which are ruled surfaces and industrially important have appeared at a very fewer places in the literature. It is found from a study of the technical reports and journals that studies on static and dynamic behaviour of doubly curved isotropic and composite shells like hyperbolic paraboloid, hypar and conoidal shells are far from complete and hence define a rich area of research.

Composite shell structures find extensive use in aerospace, civil and marine engineering applications. In practical civil engineering applications, the necessity of covering large column-free open areas often raises an issue. To cover large column-free open spaces as in airports, parking lots, hangars and the like, thin shells are preferred to flat plates. Such areas in medical plants and automobile industries require entry of north light through the roofing units. Quite often, cutouts are provided in shell panels to save weight and at times also to provide a facility for inspection. The margins of the cutouts are provided with stiffeners to take account of stress concentration effects. In civil engineering construction, cylindrical, conoidal, hypar, spherical, saddle, hyperbolic paraboloid (among the anticlastic) and elliptic paraboloid (among the synclastic) shells are used as roofing units to cover large column-free areas. Nowadays, civil engineers prefer use of laminated composites to fabricate these shell forms as the high specific stiffness and strength properties of these materials result in less gravity forces and mass-induced forces (seismic force) compared to their isotropic counterparts. All these taken together reduce the foundation costs to a great extent. Realizing the importance of laminated composite shells in the industry, several aspects of shell behaviour such as bending, buckling, vibration, impact, etc., are being reported by different researchers. The present discussion is, however, restricted to the free vibration behaviour of composite stiffened shell panels with cutout.

1.2 Literature Review

The uses of shell structures and related research have a long history and the literature that has accumulated in this area is vast. In this section, a ‘historical review’ is presented to give a picture of the course of development of shell research which started as early as the first half of the nineteenth century (1833). Introduction of laminated composites as a construction material was a significant development. These materials started being applied in different places. In this chapter attention is mainly focused on laminated composite shells from 1953 onwards although some relevant literatures on isotropic shells are also reviewed. The review is presented in a chronological manner as far as possible.

The analysis of thin shells attracted attention of researchers from the first half of the nineteenth century when Lame and Clapeyron (1833) analysed shells of revolution under symmetric loading. Aron (1874) was the first to analyse the shell problem using the general theory of elasticity and obtained the components of bending and twist in a form that were independent of the tangential displacements. It was based on the Kirchhoff's hypothesis (1876) for plates where normal to the middle surface before deformation was assumed to remain normal to the middle surface after deformation. Love (1888) observed that this was not strictly correct and derived accurate formulae for bending of shells. Love was the first investigator to present a good approximation of thin shell bending behaviour, based on the classical theory of elasticity. Sechler (1974) in his book presented the historical course of shell research and design.

The membrane theory proposed by Lame and Clapeyron (1833) neglected all moment expressions, and the flexural strains were assumed to be zero or negligible compared to the axial strains. Beltrami (1881) established the general form of equations of this theory. The limitations of the membrane theory resulted in the investigations carried out by Marguerre (1938), Vlasov (1947) and Reissner (1955) who developed the general theory of shell bending. Then onwards, both the membrane and bending theories were found to be applicable to analyse different problems depending upon the nature of the shell forms. Besides static analysis, vibration characteristics of shallow spherical shells were considered by Reissner (1946, 1955) and other authors (Hoppmann 1961; Johnson and Reissner 1958; Kalnins and Naghdi 1960) using analytical methods.

While the theory of shell structures was being simplified and improved from time to time by many researchers, another group of investigators started developing exotic materials with high strength and stiffness properties. This resulted in the use of laminated composite materials to fabricate shell forms. As a result, bending analysis of laminated composite shells emerged as a new field. The first analysis that combined bending-stretching coupling was due to Ambartsumyan (1953). He looked into the behaviour of laminated orthotropic shells. Subsequently, Dong et al. (1962) introduced a theory of thin shells laminated with anisotropic material by extending the theory of Stavsky (1963), proposed for plates, to Donnell's shallow shell theory (1933). Cheng and Ho (1963) studied buckling characteristics of anisotropic cylindrical shells using Flügge's shell theory.

Vlasov (1964) continued to work on the membrane theory, while Nazarov (1956) and Wang (1953) concentrated on the bending theories of shells. Flügge and Conrad (1956) proposed more suitable expressions for the set of differential equations and improved the bending theory of Marguerre (1938). The nonlinear vibrations of shells received attention from Grigolyuk (1955). The membrane theory of hyperbolic and elliptic paraboloid was extended by Parme (1956) to obtain the stresses expressing the shell surface in Cartesian coordinates. With the progress of shell research, different theories were proposed by Sanders (1959), Koiter (1960), Naghdi (1963) and Goldenveizer (1968). The theories due to Koiter

(1960) and Budiansky and Sanders (1963) appeared to be more popular because they were consistent with the basic Love-Kirchhoff hypotheses. Among the other theories proposed, membrane theories of Fischer (1960), Flugge and Geyling (1957), Soare (1966) and Ramaswamy and Rao (1961) and bending theories of Bongard (1959) and Chetty and Tottenham (1964) are worth mentioning.

A general heterogeneous equation for the linear analysis of thin shallow second-order shells of positive, zero or negative Gaussian curvature was derived by Munro (1961). Apeland and Popov (1961) discussed the bending analysis of translational shells of rectangular planforms and hypars. Russel and Gerstle (1967, 1968) further extended the theoretical and experimental investigations of Brebbia (1966) related to umbrella-like hyperbolic paraboloid shells. Forced vibrations of axisymmetric shallow spherical shells were reported by Reisman and Culowski (1968). Different methods that were being adopted to solve shell problems were the variational techniques like Ritz, collocation, subdomain (Biezenokoch), least squares, Galerkin, Kantorovich and Trefftz methods. The area of shell research received a new dimension when in 1961 the finite element method was suggested to be used by Greene et al. (1961) utilizing flat displacement elements. These elements were found to be good for a limited class of problems, but when the membrane-flexural stiffnesses were strongly coupled, the element failed completely. Grafton and Strome (1963) first developed the singly curved element. Since then, finite element method acquired a prominent position as an efficient analytical tool in shell research.

Apart from the numerical approaches, investigations into the bending theory of shells were carried out by Iyengar and Srinivasan (1968) employing equations of Bongard (1959) by expressing the displacement functions satisfying the boundary conditions and by Ramaswamy (1971) expressing the problem in terms of three coupled differential equations.

Doubly curved finite element was used by Greene et al. (1968) for the dynamic analysis of shells. Connor and Brebbia (1967) used a four-noded rectangular shallow shell element having three radii of curvature and five degrees of freedom per node. The concept of isoparametric curved elements, where the displacement and coordinates are interpolated from the nodal values by the same set of shape functions, was first introduced by Ergatoudis et al. (1968), and applications of finite elements to the analysis of plates and shells were reported by Gallagher (1969). Ahmad et al. (1970) introduced the concept of treating shell element as a special case of three-dimensional analysis. Dhatt (1970) solved thin shell problems with curved triangular element using discrete Kirchhoff's hypothesis, and translational shells were analysed by Choi and Schnobrich (1970) using finite element method.

Bandyopadhyay and Ray (1971–72) studied the performance of hypars in elastic and ultimate ranges experimentally. The buckling criterion of different shell forms was studied by Abel and Billington (1972), while Gergely (1972) used the energy principle to estimate the buckling load. Vibration and stability of laminated composite cylindrical and other shallow shell forms were reported by Dong (1968) and Dulaska (1969). Olson and Lindberg (1968) studied dynamic characteristics of doubly curved shells with triangular finite elements, while Brebbia and Hadid (1971) solved conoidal shell problems using rectangular finite element.

Wu (1971), who presented equations of large deformations for dynamic analysis, investigated the nonlinear problem of shells. Vibrations of doubly curved shells on rectangular planform with shear diaphragm boundaries were studied by Leissa and Kadi (1971) using the exact Navier method. They studied the effects of curvature and inplane inertia on natural frequencies. Ghosh (1973) worked on behaviour of right parabolic conoidal shells both analytically and experimentally. The study by Bhattacharya (1972), in relation to the analysis of hyperbolic paraboloid shells using simultaneous partial differential equations, was for better understanding of their nonlinear behaviour. The bending analysis of this class of shells was carried out by Schnobrich (1972). Shoeb and Schnobrich (1972) implemented the incremental approach to study the characteristics of shells made of ductile materials. Shallow elliptical domes were analysed for bending, buckling and vibration by Jones and Mazumdar (1974).

Hadid and Brebbia (1975) reported a review of different methods of shell analysis and also solved problems of hyperbolic paraboloid, pinched cylinder and conoid. Bandyopadhyay (1978) reported a general formulation for doubly curved shells in terms of three displacement components and furnished the differential equations of equilibrium of shells considering the effect of radial shear.

The laminated composite shells occupied a prominent position in the minds of researchers due to the high specific strength and stiffness properties from the sixth decade of the last century. Widera and his colleagues (1970, 1980) derived a first-approximation theory for the unsymmetric deformation of non-homogeneous, anisotropic, elastic cylindrical shells by integrating the elastic equations asymptotically. For a homogeneous, isotropic material, the theory reduced to that due to Donnell. The transverse shear deformation effects in composites were incorporated in the works of Gulati and Essenberg (1967) and Zukas and Vinson (1971). The shear deformation theory put forward by Whitney and Sun (1973, 1974) included both transverse shear deformation and transverse normal strain together with the expansional strains.

The finite element method, due to its versatility, started being used for the analysis of laminated composite shells. Earlier studies in this aspect were due to Lakshiminarayana and Viswanath (1976) and Rao (1978) who developed a high-precision triangular cylindrical and a rectangular shallow finite element, respectively. Since then the finite element method began to be used widely and improved elements started being developed and used from time to time. It is interesting to note that researchers concentrated on parallel developments of refined theories as well as better finite elements.

Bert et al. [Bert and Reddy (1982), Bert and Kumar (1982)] presented exact solutions for bending and vibrations of bimodulus composite simply supported cylindrical and spherical shells subjected to sinusoidal distribution of transverse load. Natural frequencies and mode shapes of graphite-epoxy cantilevered plates and shells were studied by Crawley (1979). The investigation was experimental and results were compared with those obtained by Ritz and finite element methods.

Donnell's shallow shell theory was used by Soldatos (1983) to study the vibration of laminated cylindrical shells that were not necessarily shallow. Exact solution of moderately thick laminated composite shells was reported by Reddy (1984) by extending Sanders' shell theory, and it was reported that antisymmetric angle-ply laminations with simply supported boundaries do not admit exact solutions. Soldatos (1984) compared the different shell theories of Donnell, Flugge and Vlasov for studying vibration characteristics of composite cylindrical shells. Research in the field of isotropic shells also continued simultaneously, and Aditya and Bandyopadhyay (1989) presented a generalized formulation for the doubly curved shells using the finite element method.

With the use of different finite elements, the concept of degenerated shell elements was used by Chao and Reddy (1984) to study bending and vibration of cylindrical and spherical shells. Reddy and Chandrashekhara (1985) used a nine-noded finite element and reported geometrically nonlinear transient response of laminated composite cylindrical and spherical shells. Panda and Natarajan (1981) developed an element extending the work of Ahmad et al. (1970). Although vibrations of composite plates with cutouts received some attention from Bert and Reddy (1982), perforated composite shells were not analysed. Reddy (1981) presented a review paper dealing with the finite element modelling of composite plates and shells which gave an elaborate picture of the evolution of different elements used in shell analysis.

Vibrations of shallow isotropic shells with different boundary conditions were investigated by Leissa and his colleagues [Leissa et al. (1981, 1983), Narita and Leissa (1984)] using the Ritz method with algebraic polynomials. Natural frequencies and mode shapes were presented for saddle, cylindrical and spherical shells of rectangular planform with different parametric variations. Coleby and Majumdar (1982) reported vibration of simply supported shells of elliptic planform. Xiang-Sheng (1985) studied forced vibrations of elastic shallow shells under moving loads.

Higher-order shear deformation theories of laminated composites were mostly based on assuming a displacement field and were cumbersome and computationally more demanding. Also, these theories assumed a constant thickness-wise shear stress and required a shear correction factor. To overcome these shortcomings, Reddy and Liu (1985) presented a simple shear deformation theory assuming a displacement field where the inplane displacements were assumed as cubic functions of the thickness coordinate and the transverse displacement was assumed to be constant through the thickness. This theory, which led to a parabolic distribution of transverse shear stress, neglected the stretching of the normal and was used to obtain exact results for deflection and natural frequencies of simply supported cross-ply shells. Bhimaraddi (1984) presented a shear deformation theory where a parabolic distribution of transverse shear stress was assumed.

Librescu (1987) developed a refined geometrically nonlinear theory of anisotropic symmetrically laminated composite shallow shells incorporating transverse shear deformation and transverse normal stress effects. Librescu and Stein (1988) studied the post-buckling behaviour of symmetrically laminated panels by applying that theory. This theory, however, assumed material linearity. Weichert (1988) developed a rate theory for shells considering anisotropic elastic-plastic behaviour using first-order shear approximation taking into account the geometrically nonlinear effects. Exact solution of shear-flexible doubly curved antisymmetric angle-ply shells using Sanders' approximations to develop a uniform shear deformation theory was reported by Chaudhuri and Abu-Arja (1988). Kapania (1989), highlighting the developments in the analysis of composite shells from 1982 to 1989, presented an excellent review work.

Ghosh and Bandyopadhyay (1989) worked on bending analysis of isotropic conoidal shells, while Qatu (1989) investigated bending and free vibrations of laminated shallow shells, and Chao and Tung (1989) studied response of clamped orthotropic hemispherical shells subjected to step pressure and blast loads. Chandrashekhara (1989) used a nine-noded isoparametric finite element for free vibration analysis of composite cylindrical and spherical shells. Hence it is evident that different investigators analysed isotropic and composite shells simultaneously for both bending and vibration characteristics.

The shell theories developed and used are transverse shear deformable analogues of either quasi-shallow Donnell-type or the first-order approximation Love-type and Sanders' type classical ones. A comparison of these theories done by Soldatos (1984) revealed that the Sanders' type approximations are mathematically superior. The parabolic transverse shear deformation theories led to the appearance of some higher-order moments and transverse shear resultants that have no physical meaning. To remove this difficulty, Soldatos (1991) proposed a refined theory satisfying zero shear traction boundaries and applied it to the free vibration analysis of antisymmetric angle-ply laminated composite circular cylindrical shells.

Jaishen and Lei (1991) worked out closed-form solutions of nonlinear dynamic response of shallow shells. Qatu and Leissa (1991a) obtained natural frequencies of cantilevered doubly curved laminated composite shells using Ritz method. They also studied the vibrations of twisted composite cantilever plates having the radius of cross curvature. Nonlinear free and forced vibrations of isotropic and composite shells were reported by Tsai and Palazotto (1991) using a curved quadrilateral finite element. Chia and Chia (1992) solved problems of nonlinear vibration of moderately thick composite spherical shells by the method of harmonic balance. Bending analysis of laminated composite elliptic paraboloid and conoid was reported by Dey et al. (1994).

Sheinman and Reichman (1992) studied the buckling and vibration characteristics of shallow composite shell panels by employing Ritz method. A refined theory of composite shallow shells was developed by Touratier (1992) in which a simple shear deformation theory considered cosine distribution of transverse shear strains and tangential stress-free boundaries. Bending and vibration of some isotropic and composite shells were studied. An excellent review work was done by Qatu (1992) discussing the developments of shallow shell vibration research. Using a doubly curved laminated composite shear-flexible shell element, the transient response was studied by Ganapathi and Varadan (1992), considering geometric nonlinearity using von Karman's strain-displacement relations. Hwang and Foster (1992) presented free vibration results of axisymmetric shallow spherical shell with a circular hole. Both analytical and finite element solutions were obtained and mutually compared.

A refined higher-order shear deformation theory was reported by Mallikarjuna and Kant (1992) taking up problems of laminated composite and sandwich shells under static load and comparing the results with those obtained by first-order shear deformation theory developed therein and also with results obtained from closed-form solutions. A finite strip analysis of diaphragm-supported composite shells was conducted by Mohd and Dawe (1993). Vibrations of isotropic shallow shells with two adjacent edges clamped and others free were studied by Qatu and Leissa (1993) by employing Ritz method. Bhaskar and Varadan (1993) used Navier approach together with Laplace transform technique to assess interlaminar stresses in composite shells under transient loading, while Ritz minimization procedure was used by Liew and Lim (1994) to analyse vibrations of perforated isotropic shallow shells. Kant et al. (1994) investigated shell dynamics with three-dimensional degenerated finite elements. Cylindrical and spherical cap problems were solved. Vibrations of moderately thick spherical shells with large amplitudes were reported by Sathyamoorthy (1994) using the Galerkin method. Ganapathi et al. (1994) investigated behaviour of spherical shells subjected to periodic load using a shear-flexible element. Different aspects of laminated composite shell behaviour were taken up by several investigators. Jing Hung-Sying and Teng Kuan-Goang (1995) studied bending behaviour, while Ye and Soldatos (1995) investigated buckling characteristics. Mizusawa and Kito (1995), Sathyamoorthy (1995), Piskunov et al. (1994) and Tessler et al. (1995) put their emphasis on vibrations of laminated composite shells. Ye and Han (1995) also investigated a nonlinear bending and buckling behaviour of polar orthotropic shells of revolution by means of a semi-analytical method. Noor and Burton (1989, 1990) and Burton and Noor (1995) reported excellent review work where they focused on the assessment of computational model used by different authors to analyse multilayered composite plates and shells and sandwich panels. The aspect of shear deformation was specially highlighted in one of their papers.

A simple higher-order shear deformation theory of laminated composite shells was developed by Xiao-Ping (1996). Using Love's first-order geometric approximation and Donnell's simplification, the governing equations of shallow shells were established by him. Khatri and Asnani (1996) presented a vibration and damping analysis of fibre-reinforced composite conical shells. In their study the shell forms which were discussed were mostly of non-civil engineering applications. Qatu (1996) obtained the natural frequencies of cantilevered shallow shells having triangular and trapezoidal planforms used the Ritz method with algebraic polynomials. Suzuki et al. (1996) presented an exact solution for analysing free vibrations of laminated composite, noncircular thick cylindrical shells. Schokker et al. (1996) and Nemeth (1996) concentrated on buckling behaviour.

Free vibration analysis of laminated anisotropic shallow shells including transverse shear deformation was done by Wang and Schweizerhof (1997). They used boundary-domain element method. The effects of both inplane and rotary inertia on the natural frequencies were included. Gendy et al. (1997) carried out free vibration and stability analyses of laminated composite plates and shells with hybrid/mixed formulation. At the same time, Aksu (1997) introduced a finite element formulation for the free, undamped vibration analysis of a shell of general shape with a curved isoparametric trapezoidal element, including thickness shear deformations and without neglecting z/R in comparison with unity. He considered the rotary inertia effects in consistent mass matrix. Large amplitude free vibration behaviour of doubly curved shallow open shells with simply supported edges was presented by Shin (1997). Gautham and Ganesan (1997) examined the free vibration characteristics of isotropic and laminated orthotropic spherical caps. Sivasubramonian et al. (1997) studied free vibration of curved panels with cutouts and Crossland and Dickinson (1997) studied the free vibration of thin shallow shells with slits on rectangular planform.

Besides these, Chun and Lam (1998), Tan (1998), Korjakin et al. (1998), Lim et al. (1998), Laura et al (1999), Sivakumar et al. (1999), Sivasubramonian et al. (1999), Reddy and Palaninathan (1999), Hu and Tsai (1999), Yadav and Verma (2001), Anlas and Goker (2001), Korjakin et al. (2001) and Lee and Han (2001) investigated dynamic aspects of isotropic and composite plates and shells. Kistler and Waas (1999) used static response characteristics as a tool for understanding transverse impact response of thin fibre-reinforced composite cylindrical panels. The effect of impact velocity, panel curvature, thickness, both inplane and out-of-plane boundary conditions and the validity of linear and nonlinear plate theory on the resulting impact force and panel displacement were investigated. Qatu (1999) derived accurate equations of elastic deformation for laminated composite deep, thick shells. The equations included shells with a pre-twist and accurate force and moment resultants were derived, which were considerably different than those used for plates. Toorani and Lakis (2000) developed general equations of anisotropic plates and shells including transverse shear deformations, rotary inertia and initial curvature effects. These equations were applied to different geometries, such as cylindrical, spherical and conoidal shells and as well as to rectangular and circular plates. Abe et al. (2000) examined nonlinear free vibration characteristics of first and second vibration modes of laminated shallow shells with rigidly clamped edges. Nonlinear equations of motion for the shells based on the first-order shear deformation and classical shell theories were derived by means of Hamilton's principle. Lok and Cheng (2001) developed a closed-form solution for the forced response of orthotropic thick plate and sandwich panel. Nath and Shukla (2001) presented the nonlinear transient analysis of the shear deformable laminated composite plates, subjected to step, ramp and sinusoidal loading. The clamped, simply supported, free and their combinations (non-Levy type) of boundary conditions were considered.

Turkmen (2002) in his paper presented the results from a theoretical and experimental investigation of the dynamic response of cylindrically curved laminated composite shells subjected to normal blast loading. Ganapathi et al. (2002) did the dynamic analysis of laminated cross-ply noncircular thick cylindrical shells subjected to thermal/mechanical load based on higher-order theory. Zhang (2001,

2002) presented the vibration analysis of rotating laminated composite cylindrical shells using the wave propagation approach.

While many researchers simplified and improved the theory of shell structures with isotropic and laminated composite materials from time to time, a group of investigators started working on stiffened shells due to their superior performance under different loading conditions and cost-effectiveness. There were two main types of analyses considered in the literature on this subject depending upon whether the stiffeners were treated by averaging their properties over the surface of the shell or by considering them as discrete elements. The first method was particularly applicable to surfaces having a large number of closely and equally spaced stiffeners. The second method could be more general as it could accommodate any distribution of the stiffeners and present a clear picture of the behaviour of the structures. Earlier, the usual method of analysing the free vibration of cylindrical shells was to treat the structure as an orthotropic continuum with effective extensional and flexural stiffness by considering the stiffeners to be closely spaced and by averaging the stiffening effects over the surface of the shell. This approach was adopted by Hoppmann II (1958) and Miller (1957). Weingarten (1965) also used linear shell theory to develop an analysis for predicting the natural frequencies of ring-stiffened simply supported conical shells by finding an equivalent orthotropic shell in which the Galerkin method was adopted for the solution. An experimental investigation was also performed in conjunction with the analysis. The free flexural vibration of cylindrical shells stiffened by equidistant ring frame was investigated by Wah (1966) using finite difference calculus.

Egle and Sewall (1968) studied the vibration of orthogonally stiffened cylindrical shells treating the stiffeners as discrete elements using energy methods for various end conditions. Egle and Soder (1969) also reported a similar and more comprehensive analysis on the vibration of stiffened cylindrical shells according to Flügge's shell theory considering more exact transverse stiffening effects. McDonald (1970) reported the vibration of rib-stiffened freely supported cylindrical shells by solving the coupled equations of motion of the shell and stiffeners directly, neglecting the inplane and rotary inertia. The effects of inplane and rotary inertia on the frequencies of eccentrically stiffened simply supported shells using energy method was studied by Parthan and Johns (1969). A theoretical analysis for the free vibration of clamped-free and clamped ring-stiffened circular cylindrical shells was developed by Sharma and Johns (1971) using the Rayleigh-Ritz technique. Flügge's theory was used and the results were compared with those obtained from Timoshenko-Love theory. A more exact analysis of the vibration of simply supported cylindrical shells with discrete longitudinal stiffeners was reported by Rinehart and Wang (1972) using the energy method, in which Vlasov's thin-walled beam theory (Vlasov, 1961) was used for the stringers, and Donnell's approximate shell theory (1933) and Flügge's more exact shell theory (Flügge 1960) were used for the skin. Sinusoidal waveform was considered in longitudinal direction, and mode shapes in circumferential direction were represented by Fourier series. Wang and Rinehart (1974) also performed an exact analysis of longitudinally stiffened cylindrical shells with any edge supporting conditions. The structural system was treated as an isotropic cylinder interacting with a set of discrete thin-walled stringers having no restriction to the number, spacing and cross-sectional shapes of stringers. Stress and stability analyses of stiffened cylindrical shells were also reported by Wang (1970) and Wang and Lin (1973), respectively.

The finite element method became a popular tool of analysis of stiffened shells also. As a result, the research on stiffened shells got a new dimension. Schmit (1968), Carr and Clough (1969) analysed eccentrically stiffened shell structures by approximating stiffeners by the finite element mesh consisting of the same elements as used for the shell. But, difficulty was experienced in maintaining the compatibility, while introducing a substantial number of additional nodes. In another approach, the finite element method was presented by Kohnke and Schnobrich (1972) using a 48 degrees of freedom shell element (Bogner et al. 1967) and a 16 degrees of freedom beam element for the static analysis of circular cylinders with eccentric axial and hoop stiffeners. The relatively complicated shell element was chosen because it had the capacity to retain compatibility even for elements intersecting at sharp angles. However, one inherent deficiency of the basic deformation functions used herein was that the rigid body modes were not explicitly included for the curved surface. In this formulation, the stiffness matrix of a beam was added to that of the remainder of the structures, thus requiring no new generalized coordinates. In performing this addition, the eccentricity was accommodated through a coordinate transformation.

Mukhopadhyay and Satsangi (1983) introduced an isoparametric stiffened plate bending element for the analysis of ship structures. A new approach was presented to cater for stiffeners in which the stiffener positions and properties remain undisturbed in the formulation. This is a distinct improvement over the existing lumped stiffener and orthotropic plate models. Deb and Booton (1988) presented a linear finite element model based on Mindlin's shear distortion theory for bending of eccentrically stiffened plates subjected to transverse loading. Omurtag and Aköz (1992, 1993) presented a mixed finite element formulation for the bending analysis of eccentrically stiffened cylindrical shells. In this method, the geometric and dynamic boundary conditions were obtained for stiffened cylindrical shells using Gâteaux differential. An isoparametric finite element formulation was employed to derive element matrices using a rectangular four-noded shell and two-noded straight-circular space bar elements with 36 and 24 degrees of freedom, respectively. The eccentricity of space bars was included in the formulation of finite element matrices. Variable cross-section property was also introduced. The bending analysis of plates/shells was also reported by Talaslidis and Sous (1992) using simple flat triangular elements of plates/shells and appropriate beam elements. The basic theory included the effects of transverse shear strains and the variational principle of Hu-Washizu which served as the starting point of discretization. Appropriate transformations and properly chosen shear constraints yielded simple plate elements with only six degrees of freedom: three vertical displacements at the corner nodes and three tangential components of rotations at three mid-side nodes. These could be combined to four-triangle macroelements. C^0 beam elements were used for stiffeners, which were compatible with these macroelements. This formulation was extended for the prediction of the bending behaviour of stiffened plates and shells and elastoplastic behaviour of square plates to study their performances. A new higher-order arbitrarily stiffened triangular shallow shell finite element was introduced by Sinha et al. (1992) to investigate the behaviour of stiffened plates and shells. Sinha and Mukhopadhyay (1997) modified the same formulation to study stiffened plates and shells having varying thickness in the skin as well as in the stiffening members under various static and dynamic loading conditions.

In the dynamic analysis of isotropic stiffened shells, Al-Najafi and Warburton (1970) used the finite element method employing axisymmetric elements to investigate the natural frequencies and mode shapes of thin circular cylindrical shells with stiffening rings. Each stiffening ring was treated as a discrete element. Mead and Bardell (1986, 1987) analysed the free vibration aspects of a circular thin cylindrical shell with discrete axial and periodic circumferential stiffeners by wave propagation method using periodic elements. Mustafa and Ali (1987a) studied the free vibration characteristics of stiffened cylindrical shells and orthogonally stiffened cylindrically curved panels employing an eight-noded orthogonally stiffened super shell finite element. Experimental measurements of natural frequencies and mode shapes of an orthogonally stiffened panel were carried out to substantiate the theoretical predictions. Mustafa and Ali (1987b) also applied the structural symmetry techniques with appropriate constraints on the boundaries of parts of a shell structure for the free vibration analysis of closed cylindrical and conical shells to predict natural frequencies and mode shapes. Half and quarter models of shell were analysed using semi-loof and facet shell finite elements. Bardell and Mead (1989a) established the stiffness and mass matrices of a cylindrically curved rectangular panel employing the hierarchical finite element method the stiffness and mass matrices were combined with periodic structure theory to analyse an orthogonally stiffened cylindrical shell with discrete line simple supports. A thin cylindrical shell stiffened by equi-pitched, identical stringers axially and equi-pitched identical frames circumferentially was analysed by them (Bardell and Mead 1989b) as a two-dimensional periodic structure by using wave propagation technique in conjunction with the hierarchical finite element method. The dynamic analysis of stiffened plates and shells using spline Gauss collocation method was carried out by Cheng and Dade (1990). Srinivasan and Krishnan (1990) investigated the dynamic response of orthogonally stiffened cylindrical shell panels clamped on all edges and subjected to a uniformly distributed step load using a series solution combined with Galerkin's method and normal mode method for free vibration and dynamic response, respectively. The wave propagation technique to the finite element model of a single periodic unit to determine elastic wave propagation characteristics in orthogonally stiffened cylindrical shells was applied by Accorsi and Bennett (1991). Mecitouglu and Dockmeci (1991) investigated the free vibration analysis of thin, stiffened, shallow cylindrical shells by collocation method within the frame of the theory of classical thin orthotropic shallow shells.

Jiang and Olson (1991) presented a numerical model for large deflection, elastoplastic analysis of cylindrical shell structures under air blast loading conditions, which was based on a transversely curved finite strip formulation and was capable of simulating nonlinear transient behaviour of isotropic and stringer-stiffened shells. Langley (1992) developed the dynamic stiffness technique based on a singly curved orthogonally stiffened shell element, which had a constant radius of curvature and was simply supported along the curved edges. The same method was applied to a range of stiffened circular cylinders, a cylinder with an internal floor and a five-panel/six-stringer array. Combined shells, cylinder-cone-plate and stiffened shells were analysed by Sivadas and Ganesan (1993) for their free vibration characteristics using a higher-order semi-analytical finite element. Liu and Chen (1992) investigated the free vibrations of a skew cantilever stiffened plate by finite element method. The influences of stiffener length, depth, number and location were examined. Sheikh and Mukhopadhyay (1993) reported similar analysis for plates on arbitrary planform by the general spline finite strip method. Chen et al. (1994a, b) also worked in this area. Rao et al. (1993) investigated the natural frequencies of submerged stiffened plates/shells using a triangular shallow shell finite element. By applying the same method Sinha and Mukhopadhyay (1994, 1995a) studied the free vibration and transient response of eccentric stiffened plates and shallow shells. In a review paper by Sinha and Mukhopadhyay (1995b), it was clearly pointed out that, until 1995, only the cylindrical configuration for stiffened shell panels received considerable attention. Chen et al. (1994a, b) did the buckling analysis of ring-stiffened cylindrical shells with cutouts by mixed method of finite strip and finite element. Chen et al. (1996) in their paper presented the construction, verification and utilization of a finite element model for longitudinally stiffened large diameter fabricated steel cylinders subjected to bending. The model was based on a large deformation elastoplastic incremental formulation. Three-dimensional degenerated plate-shell elements were used for both shells and stiffeners. The model incorporates both initial imperfections and residual stresses. Jiang and Olson (1994) developed a super finite element with C^1 shell element and curved beam element for the free vibration analysis of stiffened cylindrical shells. Stanley and Ganesan (1997) investigated natural frequencies of stiffened circular cylindrical shells for short and long shells by semi-analytical finite element method. The free vibration characteristics of longitudinally stiffened square cylindrical panels with symmetrical square cutouts using the finite element method were investigated by Sivasubramonian et al. (1999).

With the progress of research on isotropic stiffened shells, the laminated composite stiffened shells also were receiving attention from researchers. Venkatesh and Rao (1983) introduced a finite element analysis of laminated shells reinforced with laminated stiffeners using a rectangular laminated anisotropic shallow thin finite element with 48 degrees of freedom. They combined a laminated anisotropic curved beam having 16 degrees of freedom with the shell element. Compatibility between the shell and the stiffener was maintained all along their junction line. Symmetrically stiffened isotropic plate and shell problems were solved and compared with those available in the literature to establish the accuracy of the method. The same method was also extended to analyse the bending behaviour of an eccentrically stiffened laminated cantilever shell to show its ability. Venkatesh and Rao (1985) worked further on stiffened doubly curved laminated anisotropic shells of revolution by combining a 40 d. o. f. shell element with a 16 d. o. f. stiffener element. The model was validated by solving problems of eccentrically stiffened composite cylindrical and hyperboloid shells. Bhimaraddi et al. (1989) presented a finite element static analysis of laminated shells of revolution with orthogonally stiffened laminated stiffeners using shell of revolution element and curved beam element with higher-order theory. These two elements were isoparametric in which the effects of shear deformation and rotary inertia were taken into account. The main drawback of the proposed element was that it could only handle the shells of revolution.

A continuum based, two-dimensional degenerated shell element with a compatible degenerated beam element for stiffeners was developed by Liao and Reddy (1990) for the geometric nonlinear analysis of laminated, anisotropic, stiffened shells. An iterative solution procedure, either Newton-Raphson method or modified Riks method, was employed to trace the nonlinear equilibrium path. Goswami and Mukhopadhyay (1994) reported the static analysis of composite stiffened shells with the help of two different elements—the nine-noded Lagrangian isoparametric element and the heterosis element. In the formulation, the stiffeners need not be placed along the nodal lines as it could take care of the placement of the stiffener anywhere within the shell element. They also extended the same formulation for the geometrically nonlinear static and transient dynamic response analysis of laminated composite stiffened shells (Goswami and Mukhopadhyay, 1995a, b, c). The random response analysis of composite stiffened shells was also reported by Goswami (1997) by extending the above formulation.

It is worth mentioning that the extensive research work was carried out on the free vibration behaviour of isotropic/laminated stiffened plates by several investigators. Some of the notable works were due to Olson and Hazell (1977), Mukherjee and Mukhopadhyay (1988), Bhimaraddi et al. (1989), Harik and Guo (1993), Chen et al. (1994), Chandrashekhara and Kolli (1997), Sivakumar et al. (1999) and Zeng and Bert (2001).

The free vibration of stiffened composite cylindrical shells with a rectangular cutout was investigated by Lee and Kim (1997, 1998b) experimentally as well as analytically. In the analytical solutions, Love's shell theory was used to derive the governing equations of composite cylindrical shell and the discrete stiffener theory was used to consider the ring-stiffened effects. They (Lee and Kim 1998a) also extended the same analytical solutions for the free vibration of rotating composite cylindrical shells with axial stiffeners (stringers) and circumferential stiffeners (rings). Ryu et al. (2000) recently reported a study on stress analysis of composite stiffened cylindrical shells with circular and elliptic cutouts.

Barut et al. (2000) developed a new stiffened shell element combining shallow beam and shallow shell elements for geometrically nonlinear analysis of stiffened composite laminates under mechanical loading. The formulation of this element was based on the principle of virtual displacement in conjunction with the co-rotational form of total Lagrangian description of motion. Ruotolo (2001) presented a comparison of some thin shell theories used for the dynamic analysis of isotropic and laminated composite stiffened cylinders. Moreover, Prusty and Satsangi (2001a, b) studied the transient dynamic response of stiffened composite plates and shells using an eight-noded isoparametric shell element for the shell and a three-noded curved isoparametric stiffener element for the stiffener.

Recent literature shows that Nayak and Bandyopadhyay (2002a, b), Rikards et al. (2001), Prusty et al. (2001) and Prusty and Satsangi (2001a, 2001b) took up different aspects of laminated composite stiffened shell behaviour. Qatu (2002a, b) reviewed in details the research papers on shell dynamics for both isotropic and composite materials and observed that among stiffened panels, only cylindrical and spherical forms received some attention, while information about other forms like skewed hypars and conoids are scanty. In a recent paper by Kang and Leissa (2005), a three-dimensional analysis was presented for determining the free vibration frequencies and mode shapes of thick, complete (nontruncated) conical shells of revolution, while vibration of stiffened composite hypar shells received limited attention only in a recent paper published by Nayak and Bandyopadhyay (2005) considering biaxially stiffened, antisymmetric laminated shells only. The cases of uniaxially stiffened shells and symmetric laminations were excluded.

The volume of the literatures that exist on the shell research when examined carefully reflects the fact that investigators started research on shells as early as second half of the nineteenth century. The initial studies were about bending analysis of shells of simple geometry and boundary conditions like cylindrical configuration. The earlier work of Lame and Clapeyron (1833), Beltrami (1881) and others considered the membrane theory and later Aron (1874), Love (1888) and others like Nazarov (1956) and Wang (1953) worked on bending theory. Different approaches considering the shear deformation and other relevant aspects started being proposed and refined from time to time. The historical background of shell analysis was summarized by Hadid and Brebbia (1975). Subsequently, a number of informative review papers were published by different researchers like Noor and Burton (1989, 1990), Kapania (1989), Sinha and Mukhopadhyay (1995b), Liew et al. (1997) and Qatu (2002a, b). These review papers give a comprehensive picture about the chronological development of shell research and one finds that from the second half of the twentieth century, the numerical approaches like the Galerkin, finite difference, finite strip and finite element methods started gaining popularity. The researchers had realized that closed-form solution of shell problems could not be reached except for very simple surface geometries, loading and boundary conditions. They also realized that the configuration like folded plates, conoidal, saddle, spherical, elliptic and hyperbolic paraboloid and hypar shells can offer a number of parallel advantages that suit to the requirements of the industry. In fact in industrial applications a shell may have complicated boundary condition and may be subjected to complex loading. The advent of very high-speed computers in the second half of the twentieth century was a major development which paved the way for researchers to get involved in the analysis and design of shells of arbitrary geometry and loading conditions using numerical techniques. The shell surfaces often need to be stiffened for enhanced strength and stability to resist point loads and for avoiding stress concentration around cutouts. As the numerical approaches like finite element become popular to investigators, they started venturing to analyse stiffened shells with cutouts. Another major development which occurred in the second half of the twentieth century was the introduction of laminated composite in the weight-sensitive branches of engineering including civil engineering. As a result, as the research on isotropic shell went on, a parallel course of research on laminated composite shells started. The earlier studies on composite shells were due to Widera and his colleagues (1970, 1980), Lakshminarayana and Viswanath (1976), Rao (1978) and others.

The literature reflects the fact that among the shell configurations, the cylindrical and spherical ones received most attention and the industrially important conoids [Chakravorty and Bandyopadhyay (1995), Chakravorty et al. (1996, 1998), Ghosh and Bandyopadhyay (1989) and Nayak and Bandyopadhyay (2002b)] also received considerable focus. Hypar shell has a structural resemblance with the twisted plate which has wide applications in mechanical engineering and was studied by a number of investigators like Kielb et al. (1985), Seshu and Ramamurti (1989), Qatu and Leissa (1991b) and others. Skewed hypar shells of civil engineering application received limited attention from Schwarte (1994), Chakravorty et al. (1998) and Nayak and Bandyopadhyay (2002a, 2005). Application of doubly curved shells in structures often necessitates provision of cutouts for the passage of light, service lines and also sometimes for alteration of the resonant frequency. The free vibration of composite as well as isotropic plate with cutout was studied by different researchers from time to time. Reddy (1982) investigated large amplitude flexural vibration of composite plate with cutout. Malhotra et al. (1989) studied free vibration of composite plate with cutout for different boundary conditions. One of the early reports on free vibration of curved panels with cutout was due to Sivasubramonian et al. (1997, 1999). They analysed the effect of cutouts on the natural frequencies of plates with some classical boundary conditions. The plate had a curvature in one direction and was straight in the other. The effect of fibre orientation and size of cutout on natural frequency on orthotropic square plates with square cutout was studied using Rayleigh-Ritz method. Later Sivakumar et al. (1999), Rossi (1999), Huang and Sakiyama (1999) and Hota and Padhi (2007) studied free vibration of plate with various cutout geometries. Chakravorty et al. (1998) analysed the effect of concentric cutout on different shell options. Sivasubramonian et al. (1997, 1999) studied the effect of curvature and cutouts on square panels with different boundary conditions. The size of the cutout (symmetrically located) as well as curvature of the panels is varied. Hota and Chakravorty (2007) published useful information about free vibration of stiffened conoidal shell roofs with cutout. Later Nanda and Bandyopadhyay (2007) and Kumar et al. (2013) studied the effect of different parametric variations on free vibration of cylindrical shell with cutout using first-order shear deformation theory (FSDT) and higher-order shear deformation theory (HOSDT), respectively.

It is noted from the literature review that free vibration study of laminated composite shell panels with cutouts are scanty in literature. Thus the present study intends to consider the free vibration behaviour of stiffened shells panels with cutouts by varying the size and position of the cutouts.

1.3 Mathematical Formulation

A laminated composite shell of uniform thickness h (Fig. 1.1) and radius of curvature R_{xx} and R_{yy} is considered. Keeping the total thickness the same, the thickness may consist of any number of thin laminae each of which may be arbitrarily oriented at an angle θ with reference to the x-axis of the coordinate system. The constitutive equations for the shell are given by (a list of notations is given):

$$\{F\} = [E]\{\varepsilon\} \tag{1.1}$$

$$\text{where,} \quad \{F\} = \left\{ N_x, \quad N_y, \quad N_{xy}, \quad M_x, \quad M_y, \quad M_{xy}, \quad Q_x, \quad Q_y \right\}^T,$$

$$[E] = \begin{bmatrix} [A] & [B] & [0] \\ [B] & [D] & [0] \\ [0] & [0] & [S] \end{bmatrix}, \quad \{\varepsilon\} = \left\{ \varepsilon_x^0, \quad \varepsilon_y^0, \quad \gamma_{xy}^0, \quad k_x, \quad k_y, \quad k_{xy}, \quad \gamma_{xz}^0, \quad \gamma_{yz}^0 \right\}^T.$$

The force and moment resultants are expressed as

$$\begin{aligned} &\left\{ N_x, \quad N_y, \quad N_{xy}, \quad M_x, \quad M_y, \quad M_{xy}, \quad Q_x, \quad Q_y \right\}^T \\ &= \int\limits_{-h/2}^{h/2} \left\{ \sigma_x, \quad \sigma_y, \quad \tau_{xy}, \quad \sigma_z.z, \quad \sigma_y.z, \quad \tau_{xy}.z, \quad \tau_{xz}, \quad \tau_{yz} \right\}^T dz \end{aligned} \tag{1.2}$$

The submatrices $[A]$, $[B]$, $[D]$ and $[S]$ of the elasticity matrix $[E]$ are functions of Young's moduli, shear moduli and the Poisson's ratio of the laminates. They also depend on the angle which the individual lamina of a laminate makes with the global x-axis. The detailed expressions of the elements of the elasticity matrix are available in several references including Vasiliev et al. (1993) and Qatu (2004).

The strain-displacement relations on the basis of improved first-order approximation theory for thin shell (Dey et al. 1992) are established as

$$\begin{aligned} \left\{ \varepsilon_x, \quad \varepsilon_y, \quad \gamma_{xy}, \quad \gamma_{xz}, \quad \gamma_{yz} \right\}^T &= \left\{ \varepsilon_x^0, \quad \varepsilon_y^0, \quad \gamma_{xy}^0, \quad \gamma_{xz}^0, \quad \gamma_{yz}^0 \right\}^T \\ &+ z\left\{ k_x, \quad k_y, \quad k_{xy}, \quad k_{xz}, \quad k_{yz} \right\}^T \end{aligned} \tag{1.3}$$

where the first vector is the mid-surface strain for a shell and the second vector is the curvature.

1.3.1 Formulation for Shell

An eight-noded curved quadratic isoparametric finite element (Fig. 1.2a) is used for shell analysis. The five degrees of freedom taken into consideration at each node are

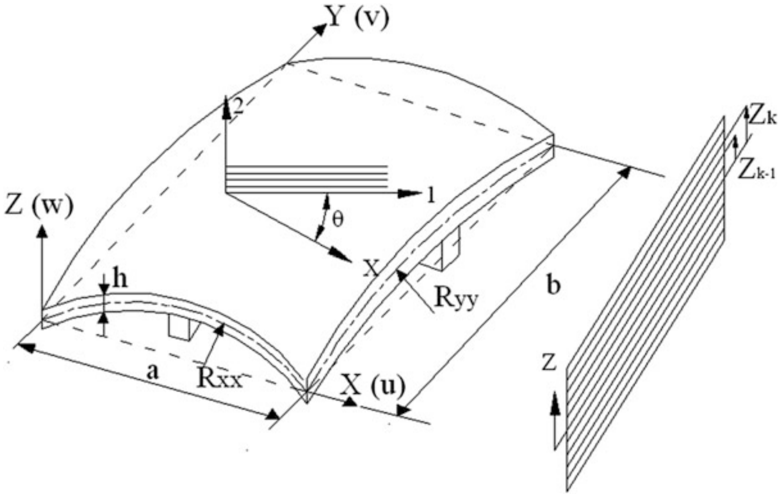

Fig. 1.1 Laminations in a stiffened shell

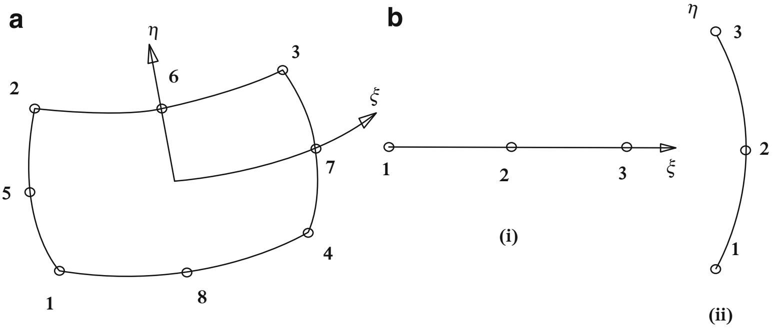

Fig. 1.2 (**a**) Eight-noded shell element with isoparametric coordinates. (**b**) Three-noded stiffener elements, (i) x-stiffener and (ii) y-stiffener (Adapted from Sahoo (2014), with permission from Elsevier)

u, v, w, α and β. The following expressions establish the relations between the displacement at any point with respect to the coordinates ξ and η and the nodal degrees of freedom:

$$u = \sum_{i=1}^{8} N_i u_i \quad v = \sum_{i=1}^{8} N_i v_i \;\; w = \sum_{i=1}^{8} N_i w_i \;\; \alpha = \sum_{i=1}^{8} N_i \alpha_i \;\; \beta = \sum_{i=1}^{8} N_i \beta_i \tag{1.4}$$

where the shape functions derived from a cubic interpolation polynomial are

$$\begin{array}{ll} N_i = (1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i+\eta\eta_i-1)/4, & \text{for } i=1,2,3,4 \\ N_i = (1+\xi\xi_i)(1-\eta^2)/2, & \text{for } i=5,7 \\ Ni = (1+\eta\eta_i)\left(1-\xi^2\right)/2, & \text{for } i=6,8 \end{array} \tag{1.5}$$

The generalized displacement vector of an element is expressed in terms of the shape functions and nodal degrees of freedom as

$$[u] = [N]\{d_e\} \tag{1.6}$$

$$\text{i.e.,}\quad \{u\} = \begin{Bmatrix} u \\ v \\ w \\ \alpha \\ \beta \end{Bmatrix} = \sum_{i=1}^{8} \begin{bmatrix} N_i & & & & \\ & N_i & & & \\ & & N_i & & \\ & & & N_i & \\ & & & & N_i \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \alpha_i \\ \beta_i \end{Bmatrix}$$

1.3.1.1 Element Stiffness Matrix

The strain-displacement relation is given by

$$\{\varepsilon\} = [B]\{d_e\}, \tag{1.7}$$

$$\text{where}\quad [B] = \sum_{i=1}^{8} \begin{bmatrix} N_{i,x} & 0 & -\dfrac{N_i}{R_{xx}} & 0 & 0 \\ 0 & N_{i,y} & -\dfrac{N_i}{R_{yy}} & 0 & 0 \\ N_{i,y} & N_{i,x} & -2N_i/R_{xy} & 0 & 0 \\ 0 & 0 & 0 & N_{i,x} & 0 \\ 0 & 0 & 0 & 0 & N_{i,y} \\ 0 & 0 & 0 & N_{i,y} & N_{i,x} \\ 0 & 0 & N_{i,x} & N_i & 0 \\ 0 & 0 & N_{i,y} & 0 & N_i \end{bmatrix} \tag{1.8}$$

The element stiffness matrix is

$$[K_e] = \iint [B]^T [E][B]dxdy \tag{1.9}$$

1.3.1.2 Element Mass Matrix

The element mass matrix is obtained from the integral

$$[M_e] = \iint [N]^T [P][N]dxdy, \tag{1.10}$$

where, $[N] = \sum_{i=1}^{8} \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix}$, $[P] = \sum_{i=1}^{8} \begin{bmatrix} P & 0 & 0 & 0 & 0 \\ 0 & P & 0 & 0 & 0 \\ 0 & 0 & P & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$,

$$\text{in which} \quad P = \sum_{k=1}^{np} \int_{z_{k-1}}^{z_k} \rho dz \quad \text{and} \quad I = \sum_{k=1}^{np} \int_{z_{k-1}}^{z_k} z^2 \rho dz \tag{1.11}$$

1.3.2 Formulation for Stiffener

Three-noded curved isoparametric beam elements (Fig. 1.2b) are used to model the stiffeners, which are taken to run only along the boundaries of the shell elements. In the stiffener element, each node has four degrees of freedom, i.e. u_{sx}, w_{sx}, α_{sx} and β_{sx} for x-stiffener and v_{sy}, w_{sy}, α_{sy} and β_{sy} for y-stiffener. The generalized force-displacement relation of stiffeners can be expressed as

$$\begin{aligned} &x\text{-stiffener}: \quad \{F_{sx}\} = [D_{sx}]\{\varepsilon_{sx}\} = [D_{sx}][B_{sx}]\{\delta_{sxi}\}; \\ &y\text{-stiffener}: \quad \{F_{sy}\} = [D_{sy}]\{\varepsilon_{sy}\} = [D_{sy}][B_{sy}]\{\delta_{syi}\} \end{aligned} \tag{1.12}$$

$$\begin{aligned} &\text{where, } \{F_{sx}\} = [N_{sxx} \quad M_{sxx} \quad T_{sxx} \quad Q_{sxxz}]^T; \\ &\{\varepsilon_{sx}\} = [u_{sx.x} \quad \alpha_{sx.x} \quad \beta_{sx.x} \quad (\alpha_{sx} + w_{sx.x})]^T \text{and} \{F_{sy}\} \\ &\qquad = [N_{syy} \quad M_{syy} \quad T_{syy} \quad Q_{syyz}]^T; \\ &\{\varepsilon_{sy}\} = [v_{sy.y} \quad \beta_{sy.y} \quad \alpha_{sy.y} \quad (\beta_{sy} + w_{sy.y})]^T \end{aligned}$$

The generalized displacements of the x-stiffener and the shell are related by the transformation matrix $\{\delta_{sxi}\} = [T_x]\{\delta\}$ where

$$[T_x] = \begin{bmatrix} 1 + \frac{e_{sx}}{R_{xx}} & \text{symmetric} & & \\ 0 & 1 & & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.13}$$

The generalized displacements of the y-stiffener and the shell are related by the transformation matrix $\left\{\delta_{syi}\right\} = \left[T_y\right]\{\delta\}$ where

$$\left[T_y\right] = \begin{bmatrix} 1+\dfrac{e_{sy}}{R_{yy}} & \text{symmetric} & & \\ 0 & 1 & & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.14}$$

These transformations are required due to curvature of x-stiffener and y-stiffener. In the above equations, e_{sx} and e_{sy} are the eccentricities of the x-stiffener and y-stiffener. $\{\delta\}$ is the appropriate portion of the displacement vector of the shell.

Elasticity matrices are as follows:

$$[D_{sx}] = \begin{bmatrix} A_{11}b_{sx} & B'_{11}b_{sx} & B'_{12}b_{sx} & 0 \\ B'_{11}b_{sx} & D'_{11}b_{sx} & D'_{12}b_{sx} & 0 \\ B'_{12}b_{sx} & D'_{12}b_{sx} & \frac{1}{6}(Q_{44}+Q_{66})d_{sx}b_{sx}^3 & 0 \\ 0 & 0 & 0 & b_{sx}S_{11} \end{bmatrix}$$

$$\left[D_{sy}\right] = \begin{bmatrix} A_{22}b_{sy} & B'_{22}b_{sy} & B'_{12}b_{sy} & 0 \\ B'_{22}b_{sy} & \frac{1}{6}(Q_{44}+Q_{66})b_{sy} & D'_{12}b_{sy} & 0 \\ B'_{12}b_{sy} & D'_{12}b_{sy} & D'_{11}d_{sy}b_{sy}^3 & 0 \\ 0 & 0 & 0 & b_{sy}S_{22} \end{bmatrix}.$$

$$\text{where,} \quad D'_{ij} = D_{ij} + 2eB_{ij} + e^2A_{ij}; \quad B'_{ij} = B_{ij} + eA_{ij}, \tag{1.15}$$

and A_{ij}, B_{ij}, D_{ij} and S_{ij} are as explained in (Sahoo and Chakravorty 2004).

Here the shear correction factor is taken as 5/6 for the stiffeners. The sectional parameters are calculated with respect to the mid-surface of the shell by which the effect of eccentricities of stiffeners is automatically included. The element stiffness matrices are of the following forms.

$$\begin{aligned} &\text{for } x\text{-stiffener}: \quad [K_{xe}] = \int [B_{sx}]^T[D_{sx}][B_{sx}]dx; \\ &\text{for } y\text{-stiffener}: \quad \left[K_{ye}\right] = \int \left[B_{sy}\right]^T\left[D_{sy}\right]\left[B_{sy}\right]dy \end{aligned} \tag{1.16}$$

The integrals are converted to isoparametric coordinates and are carried out by 2-point Gauss quadrature. Finally, the element stiffness matrix of the stiffened shell is obtained by appropriate matching of the nodes of the stiffener and shell elements through the connectivity matrix and is given as

$$[K_e] = [K_{she}] + [K_{xe}] + [K_{ye}]. \tag{1.17}$$

The element stiffness matrices are assembled to get the global matrices.

Element mass matrix for stiffener element is the following:

$$\begin{aligned} [M_{sx}] &= \int [N]^T [P][N]dx \quad \text{for } x\text{-stiffener} \\ \text{and} \quad [M_{sy}] &= \int [N]^T [P][N]dy \quad \text{for } y\text{-stiffener} \end{aligned} \tag{1.18}$$

Here, $[N]$ is a 3×3 diagonal matrix:

$$[P] = \sum_{i=1}^{3} \begin{bmatrix} \rho.b_{sx}d_{sx} & 0 & 0 & 0 \\ 0 & \rho.b_{sx}d_{sx} & 0 & 0 \\ 0 & 0 & \rho.b_{sx}d_{sx}^2/12 & 0 \\ 0 & 0 & 0 & \rho\left(b_{sx}.d_{sx}^3 + b_{sx}^3.d_{sx}\right)/12 \end{bmatrix} \text{for } x\text{-stiffener}$$

$$[P] = \sum_{i=1}^{3} \begin{bmatrix} \rho.b_{sy}d_{sy} & 0 & 0 & 0 \\ 0 & \rho.b_{sy}d_{sy} & 0 & 0 \\ 0 & 0 & \rho.b_{sy}d_{sy}^2/12 & 0 \\ 0 & 0 & 0 & \rho\left(b_{sy}.d_{sy}^3 + b_{sy}^3.d_{sy}\right)/12 \end{bmatrix} \text{for } y\text{-stiffener}$$

The mass matrix of the stiffened shell element is the sum of the matrices of the shell and the stiffeners matched at the appropriate nodes:

$$[M_e] = [M_{she}] + [M_{xe}] + [M_{ye}]. \tag{1.19}$$

The element mass matrices are assembled to get the global matrices.

1.3.3 Cutout Consideration

The code developed can take the position and size of cutout as input. The programme is capable of generating non-uniform finite element mesh all over the shell surface. So the element size is gradually decreased near the cutout margins (Fig. 1.3). Such finite element mesh is redefined in steps and a particular grid is chosen to obtain the fundamental frequency when the result does not improve by more than one percent on further refining. Convergence of results is ensured in all the problems taken up here.

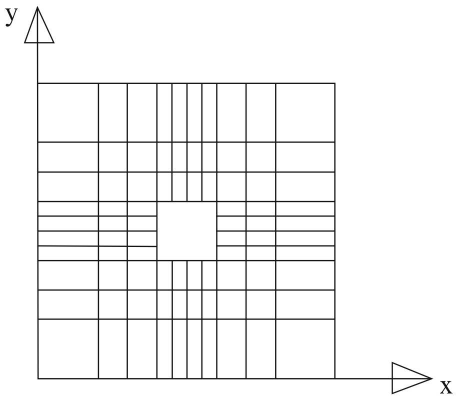

Fig. 1.3 Typical 10 × 10 non-uniform mesh arrangements drawn to scale (Adapted from Sahoo (2014), with permission from Elsevier)

1.3.4 Solution Procedure

The free vibration analysis involves determination of natural frequencies from the condition

$$\left|[K] - \omega^2[M]\right| = 0 \tag{1.20}$$

This is a generalized eigenvalue problem and is solved by the subspace iteration algorithm.

1.4 Validation Study

First the validation study of the proposed finite element shell model in the presence of cutout is carried out. The results of Table 1.1 show that the agreement of present results with the earlier ones is excellent and the correctness of the stiffener formulation is established. Here monotonic convergence is noted as the mesh is made progressively finer. Free vibration of corner point supported, simply supported and clamped spherical shells of $(0/90)_4$ lamination with cutouts is also considered. The fundamental frequencies of spherical shell with cutout obtained by the present method agree well with those reported by Chakravorty et al. (1998) as evident from Table 1.2, establishing the correctness of the cutout formulation in doubly curved shells. Thus it is evident that the finite element model proposed here can successfully analyse vibration problems of stiffened composite shell panels with cutouts which is reflected by close agreement of present results with benchmark ones.

Table 1.1 Natural frequencies (Hz) of centrally stiffened clamped square plate

Mode no.	Mukherjee and Mukhopadhyay (1988)	Nayak and Bandyopadhyay (2002a) N8 (FEM)	Nayak and Bandyopadhyay (2002a) N9 (FEM)	Present method
1	711.8	725.2	725.1	733

a = $b = 0.2032$ m, shell thickness = 0.0013716 m, stiffener depth 0.0127 m, stiffener width = 0.00635 m, stiffener eccentric at bottom
Material property: $E = 6.87 \times 10^{10}$ N/m^2, $\nu = 0.29$, $\rho = 2823$ kg/m^3

Table 1.2 Nondimensional fundamental frequencies ($\overline{\omega}$) for laminated composite spherical shell with cutout

a'/a	CS Chakravorty et al.(1998)	CS Present model	SS Chakravorty et al.(1998)	SS Present model	CL Chakravorty et al.(1998)	CL Present model
0.0	34.948	34.601	47.109	47.100	118.197	117.621
0.1	35.175	35.926	47.524	47.114	104.274	104.251
0.2	36.528	36.758	48.823	48.801	98.299	97.488
0.3	37.659	37.206	50.925	50.920	113.766	113.226
0.4	39.114	39.412	53.789	53.788	110.601	110.094

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = h/R_{yy} = 1/300$, CS = corner point supported, SS = simply supported, CL = clamped

1.5 Present Scope

Free vibration behaviour of stiffened shells with cutouts is studied considering different boundary conditions. The variation of fundamental frequency and mode shapes due to change in eccentricity of cutout along x and y direction is also considered. Numerical results are then obtained for several shells with cross-ply and angle-ply laminations. The shell thickness h is taken to be constant in all cases and lamina properties taken are $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$.

The shells considered are of square plan form ($a = b$) and the cutouts are also taken to be square in plan ($a' = b'$). The cutout sizes (i.e. a'/a) are varied from 0 to 0.4 and boundary conditions are varied along the four edges. The stiffeners are placed along the cutout periphery and extended up to the edge of the shell. The boundary conditions are designated by describing the support clamped or simply supported as C or S taken in an anticlockwise order from the edge $x = 0$. This means a shell with CSCS boundary is clamped along $x = 0$, simply supported along $y = 0$ and clamped along $x = a$ and simply supported along $y = b$ (Fig. 1.4). The material and geometric properties of shells and cutouts are mentioned along with the figures.

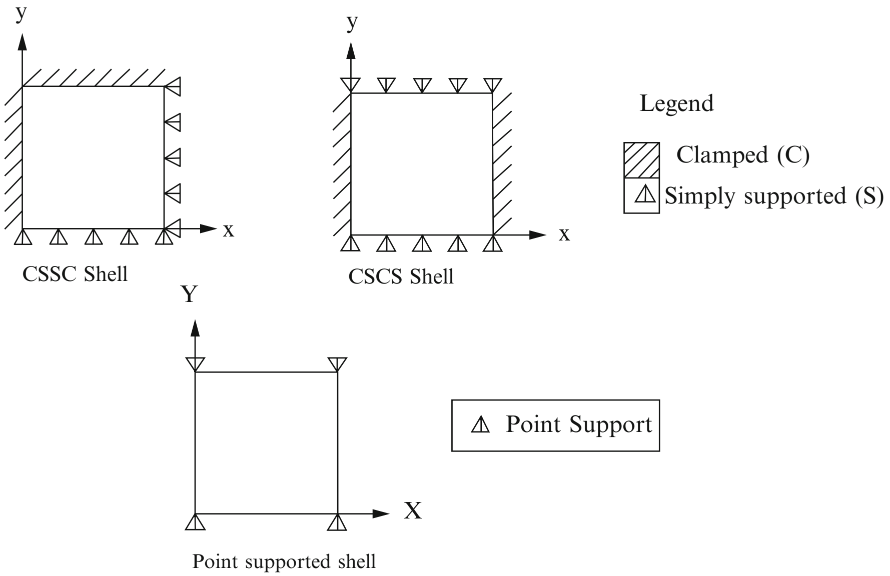

Fig. 1.4 Arrangement of boundary constraints (Adapted from Sahoo and Chakravorty (2005), with permission from SAGE)

1.6 Closure

This chapter contains the general introduction. In this chapter the review of the existing literature is done meticulously. The available literature is thoroughly analysed and critically discussed to identify the lacunae present therein. Based on the elaborate review exercise, the actual scope of the present work is outlined. Having defined the scope, it contains the mathematical formulation employed in the present analysis. Results are obtained for some specific benchmark problems solved by earlier investigators to establish the validity of the present formulation.

References

Abe A, Kobayashi Y, Yamada G (2000) Non-linear vibration characteristics of clamped laminated shallow shells. J Sound Vib 234(3):405–426

Abel JF, Billington DP (1972) Stability analysis of cooling towers – a review of current methods. In: Shell structures and climatic influences, proceedings of International Association for Shell Structures Symposium, July, University of Calgary, Alberta, Canada

Accorsi ML, Bennett MS (1991) A finite element based method for the analysis of free wave propagation in stiffened cylinders. J Sound Vib 148(2):279–292

Aditya AK, Bandyopadhyay JN (1989) Study of the shell characteristics of paraboloid of revolution shell structure using finite element method. Comput Struct 32(2):423–432

Ahmad S, Irons BM, Zienkiewicz OC (1970) Analysis of thick and thin shell structures of curved finite elements. Int J Numer Methods Eng 2:419–450

Aksu T (1997) A finite element formulation for free vibration analysis of shells of general shape. Comput Struct 65(5):687–694

Al-Najafi AMJ, Warburton GB (1970) Free vibration of ring-stiffened cylindrical shells. J Sound Vib 13(1):9–25

Ambartsumyan SA (1953) Calculation of laminated anisotropic shells. Izvestiia Akademiia Nauk Armenskoi SSR. Ser Fiz Mat Est Tekh Nauk 6(3):15

Anlas G, Goker G (2001) Vibration analysis of skew fibre-reinforced composite laminated plates. J Sound Vib 242(2):265–276

Apeland K, Popov EP (1961) Analysis of bending stresses in translational shells. In: Proceedings of the colloquium on simplified calculation methods, Brussels, pp 9–43

Aron H (1874) Das Gleichgewitcht und die Bewegund einer underlich dunner, beliebig, gekrummten, elastischen Schale. J fur reine und ang Math 78

Bandyopadhyay JN (1978) Analytical study of the shell characteristics of hyperbolic paraboloid and paraboloid of revolution shell structures. Ph.D. dissertation, Department of Civil Engineering, I.I.T., Kharagpur

Bandyopadhyay JN (2000) Applications of fibre reinforced composites in civil and other engineering structures. In: Proceedings of structural engineering convention – an international meet, January, I.I.T., Bombay, India

Bandyopadhyay JN, Ray DP (1971–72) Reinforced concrete hyperbolic paraboloid and cylindrical shells, Part-I, Behavior of hyperbolic paraboloid shells in elastic and ultimate ranges. In: Proceedings of Indian Society of Theoretical and Applied Mechanics, Allahabad, pp 117–127

Bardell NS, Mead DJ (1989a) Free vibration of an orthogonally stiffened cylindrical shell, Part I: discrete line simple supports. J Sound Vib 134(1):29–54

Bardell NS, Mead DJ (1989b) Free vibration of an orthogonally stiffened cylindrical shell, Part II: discrete general stiffeners. J Sound Vib 134(1):55–72

Barut A, Madenci E, Tessler A, Starnes JH Jr (2000) A new stiffened shell element for geometrically nonlinear analysis of composite laminates. Comput Struct 77:11–40

Beltrami E (1881) Sullequilibrio delle super ficite flessibilied in esterdibill., Mem R. Acad. Sci. di Bologna

Bert CW, Kumar M (1982) Vibration of cylindrical shells of bimodulus composite materials. J Sound Vib 81(1):107–121

Bert CW, Reddy VS (1982) Cylindrical shells of bimodulus material. J Eng Mech Div ASCE 8:675–688

Bhaskar K, Varadan TK (1993) Interlaminar stresses in composite cylindrical shells under transient loads. J Sound Vib 168(3):469–477

Bhattacharya AP (1972) Bending analysis of hyperbolic paraboloid shells. Meddelelse nr. 23, Institutt for Mckanikk, Norges Tekniske Hogskde

Bhimaraddi A (1984) A higher-order theory for free vibration analysis of circular cylindrical shells. Int J Solids Struct 20(7):623–630

Bhimaraddi A, Carr AJ, Moss PJ (1989) Finite element analysis of laminated shells of revolution with laminated stiffeners. Comput Struct 33(1):295–305

Bogner FK, Fox RL, Schmit LA (1967) A cylindrical shell discrete element. AIAA J 5(4):745–750

Bongard W (1959) Zurtheorite and berechnung von scalentragwerken in form gleichseltiger hyparbolischer paraboloide. Bautechnik-Archiv 15

Brebbia CA (1966) An experimental and theoretical investigation into hyperbolic paraboloid shells. University of Southampton, Civil Engineering, Report no. CE/2/66

Brebbia CA, Hadid HA (1971) Analysis of plates and shells using rectangular curved elements. University of Southampton Civil Engineering, Report no. CE/5/71

Budiansky B, Sanders JL (1963) On the best first-order linear shell theory. Progress in Applied Mechanics, The Prager Anniversary Volume, Macmillan, pp 129–140

Burton WS, Noor AK (1995) Assessment of computational models for sandwich panels and shells. Comput Methods Appl Mech Eng 124:125–151

Carr AJ, Clough RW (1969) Dynamic earthquake behaviour of shell roofs. In: Proceeding of the 4th world conference on earthquake engineering, Santiago, Chile

Chakravorty D, Bandyopadhyay JN (1995) On the free vibration of shallow shells. J Sound Vib 185(4):673–684

Chakravorty D, Bandyopadhyay JN, Sinha PK (1996) Finite element free vibration analysis of doubly curved laminated composite shells. J Sound Vib 191(4):491–504

Chakravorty D, Sinha PK, Bandyopadhyay JN (1998) Application of FEM on free and forced vibration of laminated shells. ASCE J Eng Mech 124(1):1–8

Chandrashekhara K (1989) Free vibrations of anisotropic laminated doubly curved shells. Comput Struct 33(1–3):435–440

Chandrashekhara K, Kolli M (1997) Free vibration of eccentrically stiffened laminated plates. J Reinf Plast Compos 16(10):884–902

Chao WC, Reddy JN (1984) Analysis of laminated composite shells using a degenerated 3-d element. Int J Numer Methods Eng 20:1991–2007

Chao CC, Tung TP (1989) Step presence and blast response of clamped orthotropic hemispherical shells. Int J Impact Eng 8(3):191–207

Chaudhuri RA, Abu-Arja KR (1988) Exact solution of shear-flexible doubly curved antisymmetric angle-ply shells. Int J Eng Sci 26:587–604

Chen CJ, Liu W, Chern SM (1994a) Vibration analysis of stiffened plates. Comput Struct 50(4):471–480

Chen W, Ren W, Zhang W (1994b) Buckling analysis of ring-stiffened cylindrical shells with cutouts by mixed method of finite strip and finite element. Comput Struct 53(4):811–816

Chen Q, Elwi AE, Kulak GL (1996) Finite element analysis of longitudinally stiffened cylinders in bending. J Eng Mech 122(11):1060–1068

Cheng SP, Dade C (1990) Dynamic analysis of stiffened plates and shells using spline Gauss collocation method. Comput Struct 36(4):623–629

Cheng S, Ho BP (1963) Stability of heterogeneous aeolotropic cylindrical shells under combined loading. AIAA J 1(4):892–898

Chetty SKM, Tottenham H (1964) An investigation into the bending analysis of hyperbolic paraboloid shells. Indian Concr J 38:248–158

Chia CY, Chia DS (1992) Nonlinear vibration of moderately thick antisymmetric angle-ply shallow spherical shell. Comput Struct 44(4):797–805

Choi CK, Schnobrich WC (1970) Finite element analysis of translation shells. University of Illinois, Structural Research no. 368

Chun L, Lam KY (1998) Dynamic response of fully-clamped laminated composite plates subjected to low-velocity impact of a mass. Int J Solids Struct 35(11):963–979

Coleby JR, Majumdar J (1982) Vibrations of simply supported shallow shells on elliptical bases. J Appl Mech Trans ASME 49:227–229

Connor JJ, Brebbia CA (1967) Stiffness matrix for shallow rectangular shell element. J Eng Mech Div Proc ASCE 93(EM5):43–55

Crawley EF (1979) The natural modes of graphite/epoxy cantilever plates and shells. J Compos Mater 13:195–205

Crossland JA, Dickinson SM (1997) The free vibration of thin rectangular planform shallow shells with slits. J Sound Vib 199(3):513–521

Deb A, Booton M (1988) Finite element models for stiffened plates under transverse loading. Comput Struct 28(3):361–372

Dey A, Bandyopadhyay JN, Sinha PK (1992) Finite element analysis of laminated composite paraboloid of revolution shells. Comput Struct 44(3):675–682

Dey A, Bandyopadhyay JN, Sinha PK (1994) Behaviour of paraboloid of revolution shell using cross-ply and anti-symmetric angle-ply laminates. Comput Struct 52(6):1301–1308

Dhatt GS (1970) An efficient triangular finite element. AIAA J 8(11):2100–2102

Dong SB (1968) Free vibrations of laminated orthotropic cylindrical shells. J Acoust Soc Am 44(6):1628–1635

Dong SB, Pister KS, Taylor RL (1962) On the theory of laminated anisotropic shells and plates. J Aerosp Sci 29:969–975

Donnell LH (1933) Stability of thin walled tubes in torsion. NACA Report 479

Dulaska E (1969) Vibration and stability of anisotropic shallow shells. Acta Tech 65(3/4):225–260

Egle DM, Sewall JL (1968) An analysis of free vibration of orthogonally stiffened cylindrical shells with stiffeners treated as discrete elements. AIAA J 6:518–526

Egle DM, Soder KE Jr (1969) A theoretical analysis of the free vibration of discretely stiffened cylindrical shells with arbitrary end conditions. NASA CR 1316

Ergatoudis I, Irons BM, Zienkiewicz OC (1968) Curved isoparametric quadrilateral elements for finite element analysis. Int J Solids Struct 4:31–42

Fischer L (1960) Determination of membrane stresses on elliptic paraboloid using polynomials. J Am Concr Inst 32(4):433–441

Flügge W (1960) Stresses in shells. Julins Springer, Berlin

Flügge W, Conrad DA (1956) Singular solution in the theory of shallow shells. Tech. Report No. 101, Division of Engineering Mechanics, Stanford University

Flügge W, Geyling FT (1957) A general theory of deformation of membrane shells. Int Assoc Bridg Struct Eng 17:23

Gallagher RH (1969) Analysis of plate and shell structures – applications of finite element method in engineering. Vanderbilt University, ASCE Publication

Ganapathi M, Varadan TK (1992) Application of a field consistent shear flexible element for nonlinear dynamic analysis of laminated shells. Finite Elem Analy Des 12(2):105–116

Ganapathi M, Vardan TK, Balamurugan V (1994) Dynamic instability of laminated composite curved panels using finite element method. Comput Struct 53(2):335–342

Ganapathi M, Patel BP, Pawargi DS (2002) Dynamic analysis of laminated cross-ply composite non-circular thick cylindrical shells using higher-order theory. Int J Solids Struct 39:5945–5962

Gautham BP, Ganesan N (1997) Free vibration characteristics of isotropic and laminated orthotropic spherical caps. J Sound Vib 204(1):17–40

Gendy AS, Saleeb AF, Mikhail SN (1997) Free vibrations and stability analysis of laminated composite plates and shells with hybrid/mixed formulation. Comput Struct 63(6):1149–1163

Gergely P (1972) Buckling of orthotropic hyperbolic paraboloid shells. J Struct Div Proc ASCE 98 (ST1):613–699

Ghosh B (1973) Analytical and experimental study of right shallow parabolic conoidal shells. Ph.D. thesis, Jadavpur University, India

Ghosh B, Bandyopadhyay JN (1989) Bending analysis of conoidal shells using curved quadratic isoparametric element. Comput Struct 33(3):717–728

Goldenveizer AL (1968) Method for justifying and refining the theory of shells. App Math Mech (Translation of Prikl Mat Mekh) 32:704–718

Goswami S (1997) Response of composite stiffened shells under stochastic excitation. J Reinf Plast Compos 16(16):1492–1522

Goswami S, Mukhopadhyay M (1994) Finite element analysis of laminated composite stiffened shell. Reinf Plast Compos 13(7):574–616

Goswami S, Mukhopadhyay M (1995a) Geometrically nonlinear transient dynamic response of laminated composite stiffened shells. J Reinf Plast Compos 14(6):618–640

Goswami S, Mukhopadhyay M (1995b) Geometrically nonlinear analysis of laminated composite stiffened shells. J Reinf Plast Compos 14(12):1317–1336

Goswami S, Mukhopadhyay M (1995c) Finite element free vibration analysis of laminated composite stiffened shells. J Compos Mater 29(18):2388–2422

Grafton PE, Strome DR (1963) Analysis of axi-symmetric shells by the direct stiffness method. AIAA J 1:2342–2347

Greene BE, Strome DR, Weikel RC (1961) Application of the stiffness method of analysis of shell structures. In: Proceedings Aviation conference of ASME, Los Angeles, CA

Greene BE, Jones RE, Strome DR (1968) Dynamic analysis of shells using doubly curved finite elements. In: Proceedings of 2nd conference on matrix methods structural mechanics, AFFDL-TR-68-150, pp 185–212

Grigolyuk EI (1955) Nonlinear vibrations and stability of shallow rods and shells. Izv Akad Nauk SSSR, OTN3, pp 33 (in Russian)

Gulati ST, Essenberg F (1967) Effects of anisotropy in axi-symmetric cylindrical shells. J Appl Mech 34:650–666

Hadid HA, Brebbia CA (1975) A review of different methods of shell analysis. J Struct Eng 3(1):1–16

Harik IE, Guo M (1993) Finite element analysis of eccentrically stiffened plates in free vibration. Comput Struct 49(6):1007–1015

Hoppmann WH II (1958) Some characteristics of flexural vibrations of orthogonally stiffened cylindrical shells. J Acoust Soc Am 30(1):77–82

Hoppmann WH II (1961) Frequencies of vibration of shallow spherical shells. J Appl Mech Trans ASME 28:306–307

Hota SS, Chakravorty D (2007) Free vibration of stiffened conoidal shell roofs with cutouts. J Vib Control 13(3):221–240

Hota SS, Padhi P (2007) Vibration of plates with arbitrary shapes of cutouts. J Sound Vib 302(4–5):1030–1036

Hu HT, Tsai JY (1999) Maximization of the fundamental frequencies of laminated cylindrical shells with respect to fibre orientations. J Sound Vib 225(4):723–740

Huang M, Sakiyama T (1999) Free vibration analysis of rectangular plates with variously-shaped holes. J Sound Vib 226(4):769–786

Hwang DY, Foster WA Jr (1992) Analysis of axi-symmetric free vibration of isotropic shallow spherical shells with a circular hole. J Sound Vib 157(2):331–343

Iyengar KTS, Srinivasan RS (1968) Bending analysis of hyperbolic paraboloid shells. Struct Eng 46(12):397–401

Jaishen F, Lei F (1991) Closed form solutions of nonlinear dynamic response in shallow shells. Appl Math Model 15(8):416–424

Jiang J, Olson MD (1991) Non-linear dynamic analysis of blast loaded cylindrical structures. Comput Struct 41(1):41–52

Jiang J, Olson MD (1994) Vibration analysis of orthogonally stiffened cylindrical shells using super finite elements. J Sound Vib 173(1):73–83

Jing Hung-Sying, Teng Kuan-Goang (1995) Analysis of thick laminated anisotropic cylindrical shells using a refined shell theory. Int J Solids Struct 32(10):1459–1476

Johnson MW, Reissner E (1958) On transverse vibrations of shallow spherical shells. Q Appl Math 15(4):365–380

Jones RE, Mazumdar J (1974) A method of static analysis of shallow shells. AIAA J 12(8): 1134–1136

Kalnins A, Naghdi PM (1960) Axi-symmetric vibrations of shallow elastic spherical shells. J Acoust Soc Am 32:342–347

Kang J, Leissa AW (2005) Free vibrations of thick, complete conical shells of revolution from a three-dimensional theory. J Appl Mech 72(5):797–800

Kant T, Kumar S, Singh UP (1994) Shell dynamics with three-dimensional degenerate finite elements. Comput Struct 50(1):135–146

Kapania RK (1989) A review on the analysis of laminated shells. J Press Vessel Technol 111:88–96

Karbhari VM, Zhao L (2000) Use of composites for 21st century civil infrastructure. Comput Methods Appl Mech Eng 185:433–454

Khatri KN, Asnani NT (1996) Vibration and damping analysis of fibre reinforced composite material conical shells. J Sound Vib 193(3):581–595

Kielb RE, Leissa AW, Macbain JC (1985) Vibration of twisted cantilever plates – a comparison of theoretical results. Int J Numer Methods Eng 21:1365–1380

Kirchhoff G (1876) Vor elesungen Uber Mathematische Physik. Mechanik 1

Kistler LS, Waas AM (1999) On the response of curved laminated panels subjected to transverse impact loads. Int J Solids Struct 36:1311–1327

Kohnke PC, Schnobrich WC (1972) Analysis of eccentrically stiffened cylindrical shells. J Struct Div ASCE 98(ST7):1493–1510

Koiter WT (1960) A consistent first-approximation in the general theory of thin elastic shells. In: Proceeding of the symposium on theory of thin elastic shells, IUTAM Delft, pp 24–28, Aug 1953, North Holland Publishing Co., Amsterdam, pp 12–33

Korjakin A, Rikards R, Altenbach H, Chate A (1998) Analysis of free damped vibrations of laminated composite conical shells. Compos Struct 41:39–47

Korjakin A, Rikards R, Altenbach H, Chate A (2001) Free damped vibrations of sandwich shells of revolution. J Sandw Struct Mater 3:171–196

Kumar A, Chakrabarti A, Bhargava P (2013) Vibration of composite cylindrical shells with cutouts using higher order theory. Int J Sci Ind Res 5(4):199–202

Lakshiminarayana HV, Viswanath S (1976) A high precision triangular laminated anisotropic cylindrical shell finite element. Comput Struct 8:633–640

Lame G, Clapeyron BPE (1833) Memoire Sur I'equilibre interieur des corps solids. Mem Pres Par Div Savants 4

Langley RS (1992) A dynamic stiffness technique for the vibration analysis of stiffened shell structures. J Sound Vib 156(3):521–540

Laura PAA, Avalos DR, Larrondo HA (1999) Forced vibrations of simply supported anisotropic rectangular plates. J Sound Vib 220(1):178–185

Lee SJ, Han SE (2001) Free-vibration analysis of plates and shells with a nine-node assumed natural degenerated shell element. J Sound Vib 241(4):605–633

Lee YS, Kim YW (1997) Vibration analysis for composite cylindrical shells with a rectangular cutout. In: Proceeding of the ICCM-II, Gold Coast, 14–18 July, II, pp 84–93

Lee YS, Kim YW (1998a) Vibration analysis of rotating composite cylindrical shells with orthogonal stiffeners. Comput Struct 69:271–281

Lee YS, Kim YW (1998b) Vibration analysis of stiffened composite cylindrical shells with a rectangular cutout. In: Proceeding of the first Asian-Australian Conference on Composite Materials(ACCM-I), Osaka, Japan, 7–9 Oct, 318/1-318/4

Leissa AW, Kadi AS (1971) Curvature effects on shallow shell vibrations. J Sound Vib 16(2):173–187

Leissa AW, Lee JK, Wang AJ (1981) Vibrations of cantilevered shallow cylindrical shells of rectangular planform. J Sound Vib 78(3):311–328

Leissa AW, Lee JK, Wang AJ (1983) Vibrations of cantilevered doubly-curved shallow shells. Int J Solids Struct 19(5):411–424

Liao CL, Reddy JN (1990) Analysis of anisotropic, stiffened composite laminates using a continuum-based shell element. Comput Struct 34(6):805–815

Librescu L (1987) Refined Geometrically nonlinear theories of anisotropic laminated shells. Q Appl Math 45:1–27

Librescu L, Stein M (1988) Nonlinear theory and postbuckling behaviour of shear deformable symmetrically laminated composite panels. In: Proceedings of 16th ICAS congress of aeronautical sciences, Jerusalem, Israel

Liew KM, Lim CW (1994) Vibration of perforated doubly curved shallow shells with rounded corners. Int J Solids Struct 31(11):1519–1536

Liew KM, Lim CW, Kitipornchai S (1997) Vibration of shallow shells: a review with bibliography. Appl Mech Rev 50(8):431–444

Lim CW, Liew KM, Kitipornchai S (1998) Vibration of cantilevered laminated composite shallow conical shells. Int J Solids Struct 35(15):1695–1707

Liu WH, Chen WC (1992) Vibration analysis of skew cantilever plates with stiffeners. J Sound Vib 159(1):1–11

Lok TS, Cheng QH (2001) Bending and forced vibration response of a clamped orthotropic thick plate and sandwich panel. J Sound Vib 245(1):63–78

Love AEH (1888) On the small free vibrations and deformations of thin elastic shells. Philos Trans R Soc 179(A)

Malhotra SK, Ganesan N, Veluswami MA (1989) Vibration of composite plate with cutouts. J Aeronaut Soc India 41:61–64

Mallikarjuna, Kant T (1992) A general fibre-reinforced composite shell element based on a refined shear deformation theory. Comput Struct 42(3):381–388

Marguerre K (1938) Zurtheorie der gekrummten platte grossen formanderung., Proceedings of Fifth International Congress of Applied Mechanics

McDonald D (1970) A problem in the free vibration of stiffened cylindrical shells. AIAA J 8:252–258

Mead DJ, Bardell NS (1986) Free vibration of a thin cylindrical shell with discrete axial stiffeners. J Sound Vib 111(2):229–250

Mead DJ, Bardell NS (1987) Free vibration of a thin cylindrical shell with periodic circumferential stiffeners. J Sound Vib 115(3):499–520

Mecitouglu Z, Dockmeci MC (1991) Free vibrations of a thin, stiffened cylindrical shallow shell. AIAA J 30(3):848–850

Miller PR (1957) Free vibrations of a stiffened cylindrical shell. ARC Report and Memoranda no. 3154, pp 1–38

Mizusawa T, Kito H (1995) Vibration of antisymmetric angle-ply laminated cylindrical panels by the spline finite strip method. Comput Struct 56(4):589–604

Mohd S, Dawe DJ (1993) Finite strip vibration analysis of composite prismatic shell structures with diaphragm engs. Comput Struct 49(5):753–765

Mukherjee A, Mukhopadhyay M (1988) Finite element free vibration of eccentrically stiffened plates. Comput Struct 30(6):1303–1317

Mukhopadhyay M, Satsangi SK (1983) Isoparametric stiffened plate bending element for the analysis of Ships' structures., The Royal Institute of Naval Architects

Munro J (1961) The linear analysis of thin shallow shells. Inst Civ Eng Proc 19:291–306

Mustafa BAJ, Ali R (1987a) Prediction of natural frequency of vibration of stiffened cylindrical shells and orthogonally stiffened curved panels. J Sound Vib 113(2):317–327

Mustafa BAJ, Ali R (1987b) Free vibration analysis of multisymmetric stiffened shells. Comput Struct 27(6):803–810

Naghdi PM (1963) Foundations of elastic shell theory, vol IV, Progress in solid mechanics. North Holland Publishing Co., Amsterdam

Nanda N, Bandyopadhyay JN (2007) Nonlinear free vibration analysis of laminated composite cylindrical shells with cutouts. J Reinf Plast Compos 26(14):143–1427

Narita Y, Leissa AW (1984) Vibrations of corner supported shallow shells of rectangular planform. Earthq Eng Struct Dyn 12:651–661

Nath Y, Shukla KK (2001) Non-linear transient analysis of moderately thick laminated composite plates. J Sound Vib 247(3):509–526

Nayak AN, Bandyopadhyay JN (2002a) On the free vibration of stiffened shallow shells. J Sound Vib 255(2):357–382

Nayak AN, Bandyopadhyay JN (2002b) Free vibration analysis and design aids of stiffened conoidal shells. J Eng Mech 128(4):419–427

Nayak AN, Bandyopadhyay JN (2005) Free vibration analysis of laminated stiffened shells. ASCE J Eng Mech 131(1):100–105

Nazarov AA (1949) (English Translation in NASA TN 1426, 1956), On the theory of thin shells. Prikl Mat Mek 13:547–550

Nemeth MP (1996) Buckling and post-buckling behavior of laminated composite plates with a cutout. NASA Technical paper 3587

Noor AK, Burton WS (1989) Assessment of shear deformation theories for multilayered composite plates. Appl Mech Rev 42(1):1–13

Noor AK, Burton WS (1990) Assessment of computational models for multilayered composite shells. Appl Mech Rev 43(4):67–97

Olson MD, Hazell CR (1977) Vibration studies on some integral rib-stiffened plates. J Sound Vib 50(1):43–61

Olson MD, Lindberg GM (1968) Vibration analysis of cantilevered curved-plates using a new cylindrical shell finite element. In: Proceedings 2nd conference on matrix methods structural mechanics, AFFDL-TR-68-150, pp 247–270

Omurtag MH, Aköz AY (1992) Mixed finite element formulation of eccentrically stiffened cylindrical shells. Comput Struct 42(5):751–768

Omurtag MH, Aköz AY (1993) A compatible cylindrical shell element for stiffened cylindrical shells in a mixed finite element formulation. Comput Struct 49(2):363–370

Panda SC, Natarajan R (1981) Analysis of laminated composite shell structures by finite element method. Comput Struct 14:225–230

Parme AL (1956) Hyperbolic paraboloid and other shells of double curvatures. J Struct Div Proc ASCE 82(ST5):1057-1m to 32

Parthan S, Johns DJ (1969) Effect of inplane and rotary inertia on the frequencies of eccentrically stiffened cylindrical shells. In: Proceedings of AIAA structural dynamics of aeroelasticity specialist conference, New Orleans, pp 253–262

Piskunov VG, Verijenko VE, Adali S, Tabakov PY (1994) Transverse shear and normal deformation higher-order theory for the solution of dynamic problems of laminated plates and shells. Comput Struct 31(24):3345–3374

Prusty BG, Satsangi SK (2001a) Analysis of stiffened shell for ships and ocean structures by finite element method. Ocean Eng 28:621–638

Prusty BG, Satsangi SK (2001b) Finite element transient dynamic analysis of laminated stiffened shells. J Sound Vib 248(2):215–233

Prusty BG, Ray C, Satsangi SK (2001) First ply failure analysis of stiffened panels – a finite element approach. Compos Struct 51:73–81

Qatu MS (1989) Free vibration and static analysis of laminated composite shallow shells. Ph.D. dissertation, Ohio State University

Qatu MS (1992) Review of shallow shell vibration research. Shock Vib Dig 24(9):3–15

Qatu MS (1996) Vibration analysis of cantilevered shallow shells with triangular and trapezoidal planforms. J Sound Vib 191(2):219–231

Qatu MS (1999) Accurate equations for laminated composite deep thick shells. Int J Solids Struct 36:2917–2941

Qatu MS (2002a) Recent research advances in the dynamic behaviour of shells: 1989–2000-Part 1: laminated composite shells. Appl Mech Rev 50(4):325–349

Qatu MS (2002b) Recent research advances in the dynamic behaviour of shells: 1989–2000-Part 2: laminated composite shells. Appl Mech Rev 50(5):415–434

Qatu MS (2004) Vibration of laminated shells and plates. Elsevier, Amsterdam

Qatu MS, Leissa AW (1991a) Natural frequencies for cantilevered doubly-curved laminated composite shallow shells. Comput Struct 17(3):227–256

Qatu MS, Leissa AW (1991b) Vibration studies for laminated composite twisted cantilever plates. Int J Mech Sci 33(11):927–940

Qatu MS, Leissa AW (1993) Vibrations of shallow shells with two adjacent edges clamped and others free. J Mech Struct Mach 21(3):285–301

Ramaswamy GS (1971) Design and construction of concrete shell roofs. Tata McGraw-Hill, New Delhi

Ramaswamy GS, Rao K (1961) The membrane theory applied to hyperbolic paraboloid shells. Indian Concr J 35:156–171

Rao KP (1978) A rectangular laminated anisotropic shallow thin shell finite element. Comput Methods Appl Mech Eng 15:13–33

Rao PS, Sinha G, Mukhopadhyay M (1993) Vibration of submerged stiffened plates by the finite element method. Int Shipbuild Prog 40(423):261–292

Reddy JN (1981) Finite element modeling of layered anisotropic composite plates and shells. Shock Vib Dig 13(12):3–12

Reddy JN (1982) Large amplitude flexural vibration of layered composite plates with cutouts. J Sound Vib 83(1):1–10

Reddy JN (1984) Exact solutions of moderately thick laminated shells. J Eng Mech Div ASCE 110 (EM5):794–809

Reddy JN, Chandrashekhara K (1985) Geometrically non-linear transient analysis of laminated doubly curved shells. Int J Non-linear Mech 20(2):79–80

Reddy JN, Liu CF (1985) A higher-order shear deformation theory of laminated elastic shells. Int J Eng Sci 23(3):319–330

Reddy ARK, Palaninathan R (1999) Free vibration of skew laminates. Comput Struct 70:415–423

Reisman H, Culowski PM (1968) Forced axi-symmetric motion of shallow spherical shells. J Eng Mech Div 94:653–670

Reissner E (1946) On vibrations of shallow spherical shells. J Appl Phys 17:1038–1042

Reissner E (1955) On transverse vibrations of thin shallow elastic shells. Q Appl Math 13:169–176

Rikards R, Chate A, Ozolinsh O (2001) Analysis for buckling and vibrations of composite stiffened shells and plates. Compos Struct 51:361–370

Rinehart SA, Wang JTS (1972) Vibration of simply supported cylindrical shells with longitudinal stiffeners. J Sound Vib 24:151–161

Rossi RE (1999) Transverse vibrations of thin, orthotropic rectangular plates with rectangular cutouts with fixed boundaries. J Sound Vib 221(4):733–736

Ruotolo R (2001) A comparison of some thin shell theories used for the dynamic analysis of stiffened cylinders. J Sound Vib 243(5):847–860

Russel RR, Gerstle KH (1967) Bending of hyperbolic paraboloid structures. J Struct Div Proc ASCE 93(ST3):181–199

Russel RR, Gerstle KH (1968) Hyperbolic paraboloid structures on four supports. J Struct Div Proc ASCE 94(ST4):995–1010

Ryu CH, Lee YS, Choi MH, Kim YW (2000) A study on stress analysis of composite cylindrical shells with a circular or elliptic cutout. In: Proceeding of the second Asian-Australian Conference on Composite Materials (ACCM-2000), 18–20 Aug, pp 915–920

Sahoo S (2014) Laminated composite stiffened shallow spherical panels with cutouts under free vibration - A finite element approach. Eng Sci Technol Int J 17:247–259

Sahoo S, Chakravorty D (2004) Finite element bending behaviour of composite hyperbolic paraboloid shells with various edge conditions. J Strain Anal Eng Des 39(5):499–513

Sahoo S, Chakravorty D (2005) Finite element vibration characteristics of composite hypar shallow shells with various edge supports. J Vib Control 11(10):1291–1309

Sahoo S, Chakravorty D (2008) Bending of composite stiffened hypar shell roofs under point load. J Eng Mech 134(6):441–454

Sanders JL Jr (1959) An improved first approximation theory for thin shells. NASA TR-R24

Sathyamoorthy M (1994) Vibrations of moderately thick shallow spherical shells at large amplitudes. J Sound Vib 172(1):63–70

Sathyamoorthy M (1995) Nonlinear vibrations of moderately thick orthotropic shallow spherical shells. Comput Struct 57(1):59–65

Schmit LA (1968) Developments in discrete element finite deflection structural analysis by functional minimization. Tech. Report, AFFDL-TR-68-126, Wright-Patterson Air Force Base, Dayton, OH

Schnobrich WC (1972) Analysis of hipped roof hyperbolic paraboloid structures. J Struct Div Proc ASCE 98(ST7):1575–1583

Schokker A, Sridharan S, Kasagi A (1996) Dynamic buckling of composite shells. Comput Struct 59(1):43–53

Schwarte J (1994) Vibrations of corner point supported rhombic hypar shells. J Sound Vib 175(1):105–114

Sechler EE (1974) In: Fung YC, Sechler EE (eds) The historical developments of shell research and design, thin shell structures, theory, experiments and design. Prentice-Hall Inc, Englewood Cliffs, pp 3—25

Seshu P, Ramamurti V (1989) Vibration of twisted composite plates. J Aeronaut Soc India 41:65–70

Sharma CB, Johns DJ (1971) Vibration characteristics of a clamped-free and clamped ring stiffened circular cylindrical shells. J Sound Vib 14(4):459–474

Sheikh AH, Mukhopadhyay M (1993) Free vibration analysis of stiffened plates with arbitrary planform by the general spline finite strip method. J Sound Vib 162(1):147–164

Sheinman I, Reichman Y (1992) A study of buckling and vibration of laminated shallow curved panels. Int J Solids Struct 29(11):1329–1338

Shin DK (1997) Large amplitude free vibration behavior of doubly curved shallow open shells with simply-supported edges. Comput Struct 62(1):35–49

Shoeb NA, Schnobrich WC (1972) Elastic plastic analysis of cylindrical shells. J Eng Mech Div Proc ASCE 98(EM1):47–60

Sinha G, Mukhopadhyay M (1994) Finite element free vibration analysis of stiffened shells. J Sound Vib 171(4):529–548

Sinha G, Mukhopadhyay M (1995a) Transient dynamic response of arbitrary stiffened shells by finite element method. J Vib Acoust Trans ASME 117:11–16

Sinha G, Mukhopadhyay M (1995b) Static and dynamic analysis of stiffened shells – a review. Proc Indian Natl Sci Acad 61A(3–4):195–219

Sinha G, Mukhopadhyay M (1997) Static, free and forced vibration analysis of arbitrary non-uniform shells with tapered stiffeners. Comput Struct 62(5):919–933

Sinha G, Sheikh AH, Mukhopadhyay M (1992) A new finite element model for the analysis of arbitrary stiffened shells. Finite Elem Anal Des 12:241–271

Sivadas KR, Ganesan N (1993) Free vibration analysis of combined and stiffened shells. Comput Struct 46(3):537–546

Sivakumar K, Iyengar NGR, Deb K (1999) Free vibration of laminated composite plates with cutout. J Sound Vib 221(3):443–470

Sivasubramonian B, Kulkarni AM, Rao GV, Krishnan A (1997) Free vibration of curved panels with cutouts. J Sound Vib 200(2):227–234

Sivasubramonian B, Rao GV, Krishnan A (1999) Free vibration of longitudinally stiffened curved panels with cutout. J Sound Vib 226(1):41–55

Soare M (1966) A numerical approach to the bending theory of hyperbolic shells – 1 and 2. Indian Concr J February: 63–69 and 113–119

Soldatos KP (1983) Free vibrations of antisymmetric angle-ply laminated circular cylindrical shell panels. Q J Mech Appl Math 36(2):207–221

Soldatos KP (1984) A comparison of some shell theories used for the dynamic analysis of cross-ply laminated circular cylindrical panels. J Sound Vib 97(2):305–319

Soldatos KP (1991) A refined laminated plate and shell theory with applications. J Sound Vib 144:109–129

Srinivasan RS, Krishnan PA (1990) Response of orthogonally stiffened cylindrical shell panels. AIAA J 28(6):1144–1145

Stanley AJ, Ganesan N (1997) Free vibration characteristics of stiffened cylindrical shells. Comput Struct 65(1):33–45

Stavsky Y (1963) Thermoelasticity of heterogeneous aelotropic plates. J Eng Mech Div ASCE 89(EM2):89–105

Suzuki K, Shikanai G, Leissa AW (1996) Free vibrations of laminated composite non-circular thick cylindrical shells. Int J Solids Struct 33(27):4079–4100

Talaslidis D, Sous I (1992) A discrete kirchhoff triangular element for the analysis of thin stiffened shells. Comput Struct 43(4):663–674

Tan DY (1998) Free vibration analysis of shells of revolution. J Sound Vib 213(1):15–33

Tessler A, Tsui T, Saether E (1995) A(1,2) – order theory for elasto dynamic analysis of thick orthotropic shells. Comput Struct 32(22):3237–3260

Toorani MH, Lakis AA (2000) General equations of anisotropic plates and shells including transverse shear deformations, rotary inertia and initial curvature effects. J Sound Vib 237(4):561–615

Touratier M (1992) A refined theory of laminated shallow shells. Int J Solids Struct 29(11): 1401–1415

Tsai CT, Palazotto AN (1991) On the finite element analysis of non-linear vibration for cylindrical shells with high order shear deformation theory. Int J Non-linear Mech 26(3/4):379–388

Turkmen HS (2002) Structural response of laminated composite shells subjected to blast loading: comparison of experimental and theoretical methods. J Sound Vib 249(4):663–678

Vasiliev VV, Jones RM, Man LL (1993) Mechanics of composite structures. Taylor and Francis, Washington, DC

Venkatesh A, Rao PK (1983) Analysis of laminated shells with laminated stiffeners using rectangular shell finite elements. Comput Methods Appl Mech Eng 38:255–272

Venkatesh A, Rao PK (1985) Analysis of laminated shells of revolution with laminated stiffeners using a double curved quadrilateral finite element. Comput Struct 20(4):669–682

Vlasov VZ (1947) Membrane theory of thin shells formed by second order surfaces. Prikl Mat Mekh, Akademiya Nauk, S.S.S.R., XI:4

Vlasov VZ (1961) Thin walled elastic beams. Israel program for science translations (Translation from Russian), Jerusalem

Vlasov VZ (1964) General theory of shells and its application in engineering. English translation of 1949 Russian Edition, NASA, USA, No. N64-19883

Wah T (1966) Flexural vibration of ring-stiffened cylindrical shells. J Sound Vib 3:242–251

Wang CT (1953) Applied elasticity. McGraw-Hill Book Company, New York

Wang JTS (1970) Orthogonally stiffened cylindrical shells subjected to internal pressure. AIAA J 8(3):455–461

Wang JTS, Lin YJ (1973) Stability of discretely stringer-stiffened cylindrical shells. AIAA J 11(6):810–814

Wang JTS, Rinehart SA (1974) Free vibration of longitudinally stiffened cylindrical shells. J Appl Mech 41:1087–1093

Wang J, Schweizerhof K (1997) Free vibration of laminated anisotropic shallow shells including transverse shear deformation by the boundary-domain element method. Comput Struct 62(1):151–156

Weichert D (1988) Anisotropic elastic–plastic shells at moderate rotations. In: Proceeding of the 7th ASME engineering mechanics specialty conference, Blacksburg, VA, p 282

Weingarten VI (1965) Free vibrations of ring-stiffened conical shells. AIAA J 3(8):1475–1481

Whitney JM, Sun CT (1973) A higher order theory for extensional motion of laminated isotropic shells and plates. J Sound Vib 30:85

Whitney JM, Sun CT (1974) A refined theory for laminated anisotropic cylindrical shells. J Appl Mech 41:471–476

Widera GEO, Chung SW (1970) A theory for non-homogeneous isotropic cylindrical shells. J Appl Math Phys 21:378–399

Widera GEO, Logan DL (1980) Refined theories for non-homogeneous anisotropic cylindrical shells: Part I – Derivation. J Eng Mech Div ASCE 106(EM6):1053–1074

Wu CI (1971) On vibrations of laminated anisotropic plates and shells. Ph.D. dissertation, University of Delaware

Xiang-Sheng C (1985) Forced vibrations of elastic shallow shells due to moving loads. Appl Math Mech 6(3):233–240

Xiao-ping S (1996) An improved simple higher-order theory for laminated composite shells. Comput Struct 60(3):343–350

Yadav D, Verma N (2001) Free vibration of composite circular cylindrical shells with random material properties. Part II: Applications. Compos Struct 51:371–380

Ye Z, Han RPS (1995) On the nonlinear analysis of orthotropic shallow shells of revolution. Comput Struct 55(2):325–331

Ye J, Soldatos KP (1995) Three dimensional buckling analysis of laminated composite hollow cylinders and cylindrical panels. Int J Solids Struct 32(13):1949–1962

Zeng JA, Bert CW (2001) A differential quadrature analysis of vibration for rectangular stiffened plates. J Sound Vib 241(2):247–252

Zhang XM (2001) Vibration analysis of cross-ply laminated composite cylindrical shells using the wave propagation approach. Appl Acoust 62:1221–1228

Zhang XM (2002) Parametric analysis of frequency of rotating laminated composite cylindrical shells with the wave propagation approach. Comput Method Appl Mech Eng 191:2029–2043

Zukas JA, Vinson JR (1971) Laminated transversely isotropic cylindrical shells. J Appl Mech 38:400–407

Chapter 2
Stiffened Cylindrical Shell with Cutout

Abstract Cylindrical shell panels made of laminated composites with cutout are investigated for free vibration behaviour. Finite element model is used for studying the dynamic behaviour of stiffened shells using eight-noded curved quadratic isoparametric element for shell and a three-noded beam element for stiffener. Size of the cutouts and their positions with respect to the shell centre are varied for different laminations and edge constraints. The results presented in the form of figures and tables are analysed to suggest guidelines for selection of optimum size and position of the cutout with respect to shell centre considering different laminations and practical boundary conditions.

Keywords Cylindrical shell • Cutout • Eccentricity • Fundamental frequency • Mode shapes

2.1 Introduction

Vibration of shells is a topic of extensive research in mechanical and structural dynamics. Among different shell geometries, circular cylindrical shells have wide applications in all types of industries. They are extensively used in many engineering fields such as civil, aerospace, chemical, mechanical, naval and nuclear. Vibration of cylindrical shells has received extensive attention from many researchers. Arnold and Warburton (1949, 1953) started the study of vibration of cylindrical shells. Again, Warburton (1965) contributed information about the vibration of thin cylindrical shell. Later, Leissa et al. (1981) and Chung (1981) studied the free vibration characteristics of cylindrical shells. Free vibration of antisymmetric angle-ply laminated circular cylindrical panels was studied by Soldatos (1983). A review on vibration of isotropic and multilayered laminated cylindrical shell is available in an excellent article by Noor and Burton (1990). Leissa (1973) compiled a monograph containing detailed discussion of shell vibration problems and different aspects of vibration of cylindrical shells with different shell theories and boundary conditions. Vibration of cylindrical shells continues to find wide acceptance among researchers (Singh and Gupta 1994; Lam and Loy 1995a, b; Lim and Liew 1995; Loy et al. 1997; Naeem and Sharma 2000; Zhang

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et al. 2001; Xuebin 2008). Though Dennis and Palazotto (1990) and Nanda and Bandyopadhyay (2007) concentrated on static and dynamic response, respectively, of a cylindrical composite shell panel with cutout based on a geometrically nonlinear theory, free vibration of composite thin cylindrical shells in the presence of cutout is scanty in the existing body of literature.

Thus it is found that results of free vibration characteristics of isotropic as well as composite stiffened shell panels with cutout are limited in the existing body of literature. Chakravorty et al. (1998) provide some results of free vibration of cylindrical shell with concentric cutout, while free vibration behaviour of composite stiffened cylindrical shell panels in the presence of both concentric and eccentric cutouts has recently been considered (Sahoo 2015).

2.2 Problem

In the present chapter, laminated stiffened cylindrical shells for different laminations and three types of boundary conditions such as clamped, simply supported and point supported boundary conditions with cutouts (Fig. 2.1) are studied for free vibration behaviour. The laminations considered are two-layered, three-layered and four-layered, symmetric and antisymmetric cross-ply and angle-ply. The variation of fundamental frequencies and mode shapes due to change in size of cutout is studied. Among them, the laminations which are performing best from different cross-ply and angle-ply shells, both symmetric and antisymmetric are chosen. Those shells are further studied varying the number of boundary constraints, size of the cutout and eccentricity along x and y direction. Thus numerical results are obtained for several cylindrical shells with four-layered symmetric and antisymmetric cross and angle-ply shells.

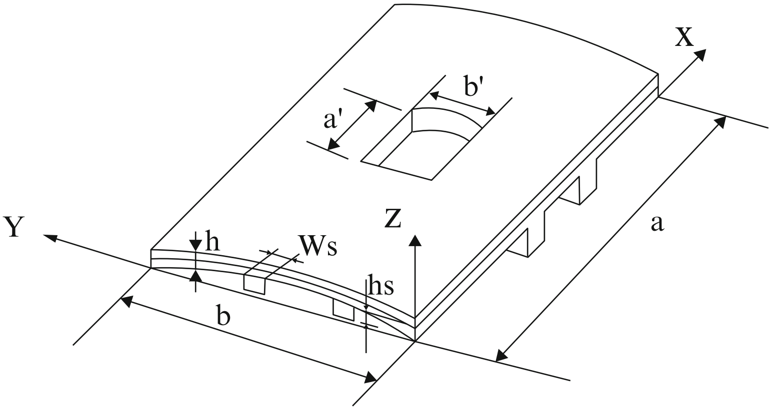


Fig. 2.1 Cylindrical shell with concentric cutout stiffened along the margins

2.3 Results and Discussion

2.3.1 *Free Vibration Behaviour of Shells with Concentric Cutouts*

Table 2.1 shows the fundamental frequencies of two-, three- and four-layered symmetric and antisymmetric cross-ply and angle-ply shells for simply supported, clamped and point supported boundary conditions with cutout. Tables 2.2 and 2.3 furnish the results of nondimensional frequency ($\overline{\omega}$) of 0/90/0/90, 0/90/90/0, +45/−45/+45/−45 and +45/−45/−45/+45 stiffened cylindrical shells for different numbers of boundary constraints with cutout. In both the cases, size of the square cutout is varied from 0.1 to 0.4.

2.3.1.1 Effect of Cutout Size on Fundamental Frequency

From Tables 2.1, 2.2 and 2.3, it is seen that the fundamental frequency increases if a cutout is provided in a stiffened shell. This increasing trend is observed for both symmetric and antisymmetric cross-ply and angle-ply shells. This initial increase in frequency is due to the increased numbers of stiffeners. As stiffeners are provided along the periphery of the cutout, so with the introduction of cutout numbers of stiffeners are increased from two to four. Thus, due to introduction of cutout, loss of mass gets significance over loss of stiffness and hence the frequency increases. It is seen from Table 2.1 that for increase in size of cutout with $a'/a = 0.2$, the frequencies increase. But with further increase in cutout size, fundamental frequency may increase or decrease. It is also observed that for clamped shells with the increase in cutout size up to $a'/a = 0.3$, fundamental frequencies increase for all the laminations considered here. But in case of simply supported shells, reverse trend is observed for angle-ply laminations. With the increase in cutout size beyond $a'/a = 0.1$, fundamental frequencies decrease in case of angle-ply shells with simply supported boundary conditions. Again behaviours of point supported shells are the same as clamped shells. When the cutout size is further increased without changing the number and dimensions of the stiffeners, the shell surface incurs loss of both mass and stiffness. So lamination and boundary conditions interact jointly in vibration behaviour of stiffened shell with cutout. From Table 2.1, it is also observed that four-layered cross-ply and angle-ply shells exhibit better performances than two- and three-layered shells. So four-layered cross-ply and angle-ply shells have been chosen for further study.

Table 2.2 shows results of four-layered antisymmetric cross-ply (0/90/0/90) and symmetric cross-ply (0/90/90/0) shells for different numbers of boundary constraints. Similarly Table 2.3 shows results of four-layered antisymmetric angle-ply (+45/−45/+45/−45) and symmetric angle-ply (+45/−45/−45/+45) shells for different numbers of boundary constraints. Here also cutout size is increased from $a'/a = 0.1$ to 0.4. It is evident from Tables 2.2 and 2.3 that when the cutout size is

Table 2.1 Nondimensional fundamental frequencies ($\overline{\varpi}$) for laminated composite stiffened cylindrical shell for different sizes of central square cutout, different laminations and boundary conditions

Boundary conditions	Laminations	Cutout size (a'/a)				
		0	0.1	0.2	0.3	0.4
Clamped	0/90	68.96	81.09	95.30	100.42	98.95
	90/0	69.15	81.16	96.06	101.21	99.87
	0/90/0	97.66	107.23	119.53	117.01	117.60
	90/0/90	82.02	101.86	104.85	105.36	100.29
	0/90/0/90	89.34	103.45	111.30	110.68	110.34
	90/0/90/0	89.66	103.55	112.02	111.18	110.85
	0/90/90/0	98.14	108.25	114.16	112.18	113.30
	90/0/0/90	85.11	104.19	107.52	109.35	105.37
	+45/−45	82.19	92.43	98.82	98.01	96.36
	+45/−45/+45	91.03	100.92	100.61	101.01	101.96
	+45/−45/+45/−45	91.97	102.69	102.62	102.44	103.10
	+45/−45/−45/+45	91.31	101.90	102.07	102.32	103.28
Simply supported	0/90	36.42	42.61	45.96	47.98	49.83
	90/0	37.08	43.54	47.09	49.22	51.10
	0/90/0	38.47	45.88	47.75	49.66	51.74
	90/0/90	38.99	46.38	48.39	50.37	52.47
	0/90/0/90	38.28	45.61	47.63	49.63	51.90
	90/0/90/0	38.64	46.14	48.22	50.27	52.58
	0/90/90/0	38.67	46.02	47.97	49.97	52.19
	90/0/0/90	39.01	46.40	48.37	50.31	52.48
	+45/−45	53.74	64.05	62.22	60.67	58.90
	+45/−45/+45	66.02	69.57	65.16	61.97	60.08
	+45/−45/+45/−45	66.46	69.98	66.24	62.95	61.00
	+45/−45/−45/+45	65.81	70.47	66.29	62.86	60.79
Point supported	0/90	18.95	23.15	24.06	24.71	23.87
	90/0	19.33	23.28	24.27	25.05	24.19
	0/90/0	19.09	23.32	24.67	26.98	24.94
	90/0/90	18.79	22.98	23.68	24.44	24.56
	0/90/0/90	21.17	26.40	26.45	26.37	25.27
	90/0/90/0	21.47	26.60	26.64	26.60	25.48
	0/90/90/0	20.34	25.31	25.96	26.31	25.04
	90/0/0/90	20.28	24.80	25.15	25.48	24.97
	+45/−45	19.66	23.39	24.94	25.43	24.69
	+45/−45/+45	20.70	24.20	25.38	26.15	26.02
	+45/−45/+45/−45	23.94	28.20	29.14	28.87	28.05
	+45/−45/−45/+45	22.60	26.42	27.33	27.82	27.33

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 0$, $h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 2.2 Nondimensional fundamental frequencies ($\overline{\omega}$) for laminated composite cross-ply stiffened cylindrical shell for different sizes of central square cutout and different boundary conditions

Laminations	Boundary conditions	Cutout size (a'/a)				
		0	0.1	0.2	0.3	0.4
0/90/90/0	CCCC	98.14	108.25	114.16	112.18	113.30
	CSCC	57.35	68.46	70.99	74.37	77.08
	CCSC	93.32	100.20	109.39	107.87	108.90
	CSSC	52.76	61.65	64.11	67.06	69.64
	CSCS	48.08	59.15	61.36	64.68	68.11
	SCSC	93.51	99.91	106.41	105.10	106.13
	CSSS	42.71	51.79	53.56	55.99	58.63
	SSSC	49.38	56.96	59.32	61.95	64.43
	SSSS	38.67	46.02	47.97	49.97	52.19
	Point supported	20.34	25.30	25.96	26.31	25.04
0/90/0/90	CCCC	89.34	103.45	111.30	110.68	110.34
	CSCC	58.20	69.33	71.62	74.68	77.20
	CCSC	79.56	88.92	100.14	105.27	105.56
	CSSC	51.96	61.01	63.51	66.34	68.85
	CSCS	50.14	60.80	62.97	66.29	69.93
	SCSC	80.08	87.50	98.66	101.94	102.35
	CSSS	43.31	52.21	53.98	56.44	59.21
	SSSC	47.84	55.87	58.06	60.59	62.95
	SSSS	38.28	45.61	47.63	49.63	51.90
	Point supported	21.17	26.39	26.45	26.37	25.27

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 0$, $h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

increased, fundamental frequency is increased in most cases of cross-ply shells up to $a'/a = 0.3$. But for angle-ply shells, a reverse trend is observed after $a'/a = 0.2$. So for angle-ply shells, with the increase of cutout size, fundamental frequencies decrease in some cases. In this case, loss of stiffness is significant than loss of mass. Hence fundamental frequency decreases except for CCSC and SCSC angle-ply shells. From Tables 2.1, 2.2 and 2.3, it is found that with the introduction of a cutout up to $a'/a = 0.2$, on shell surface, the fundamental frequency increases almost all the combination of lamination and boundary conditions considered here. Hence for functional requirements, concentric cutouts with stiffened margins may be incorporated safely on shell surfaces up to $a'/a = 0.2$.

2.3.1.2 Effect of Boundary Conditions

According to the number of boundary constraints, the six groups of boundary conditions considered are:

Table 2.3 Nondimensional fundamental frequencies ($\overline{\varpi}$) for laminated composite angle-ply stiffened cylindrical shell for different sizes of central square cutout and different boundary conditions

		Cutout size (a'/a)				
Laminations	Boundary conditions	0	0.1	0.2	0.3	0.4
+45/−45/−45/+45	CCCC	91.31	101.90	102.07	102.32	103.28
	CSCC	76.98	86.63	86.39	86.16	85.56
	CCSC	82.29	90.56	91.24	91.86	93.21
	CSSC	72.00	79.84	79.24	78.13	77.10
	CSCS	75.80	82.45	81.38	80.61	80.83
	SCSC	79.50	86.63	86.77	87.10	88.38
	CSSS	69.91	76.03	74.41	72.81	72.27
	SSSC	67.62	74.52	73.84	72.14	70.75
	SSSS	65.81	70.47	66.29	62.86	60.79
	Point supported	22.60	26.42	27.33	27.82	27.33
+45/−45/+45/−45	CCCC	91.97	102.69	102.62	102.45	103.10
	CSCC	76.40	85.72	86.20	85.75	85.46
	CCSC	82.59	90.90	91.59	91.96	93.21
	CSSC	71.04	78.24	78.37	77.73	77.50
	CSCS	75.63	82.45	81.52	80.60	80.81
	SCSC	79.87	87.01	87.10	87.22	88.42
	CSSS	70.90	75.77	74.41	72.81	72.33
	SSSC	67.62	74.37	73.98	72.27	71.04
	SSSS	66.46	69.98	66.24	62.95	61.00
	Point supported	23.94	28.20	29.14	28.87	28.05

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 0$, $h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Group I: CCCC shells
Group II: CSCC and CCSC shells
Group III: CSSC, CSCS and SCSC shells
Group IV: CSSS and SSSC shells
Group V: SSSS shells
Group VI: corner point supported

The combinations in a particular group have equal number of boundary reactions.

It is seen from Tables 2.2 and 2.3 that shells belonging to the same groups of boundary combinations may not have close values for fundamental frequencies. Thus different boundary conditions may be regrouped according to performance. Based on the values of $\overline{\varpi}$, the following groups may be identified:

For cross-ply shells:
Group I: Contains CCCC, CCSC and SCSC boundaries which exhibit relatively high frequencies for both symmetric and antisymmetric

Group II: Contains CSCC, CSSC, CSCS, SSSC and CSSS which exhibit intermediate values of frequencies for both symmetric and antisymmetric

Group III: Contains SSSS which exhibit relatively lower values of frequencies for both symmetric and antisymmetric

Group IV: Contains corner point supported boundaries which exhibit the lowest values of frequencies for both symmetric and antisymmetric

Similarly for angle-ply shells:

Group I: Contains CCCC, CCSC, CSCC, CSSC, SCSC and CSCS boundaries which exhibit relatively high frequencies for both symmetric and antisymmetric

Group II: Contains CSSS, SSSC and SSSS boundaries which exhibit intermediate values of frequencies for both symmetric and antisymmetric

Group III: Contains corner point supported shells for both symmetric and antisymmetric

From Tables 2.2 and 2.3, it is seen that the free vibration characteristics largely depend on the arrangement rather than the actual number of boundary constraints. From Tables 2.2 and 2.3, it is also seen that when the circular edge along $x = a$ is released from clamped to simply supported, the change of frequency for both cross-ply and angle-ply shells is not so significant. In case of symmetric cross-ply shell, this change is hardly 4–7 %. But for cross-ply shells, both symmetric and antisymmetric, if the edge along any one of the straight edges is released, fundamental frequency decreases significantly than that of a clamped shell. For cross-ply shell this decrease in frequency is 30–42 %, whereas for angle-ply shell this decrease in frequency is 15–17 % than that of a clamped shell. For cross-ply shells if both the straight edges, i.e. along $y = 0$ and $y = b$ are released, frequency values undergo marked decrease. The results indicate that the edge along $y = 0$ or $y = b$ should preferably be clamped in order to achieve higher frequency values and if one of the straight edges has to be released for functional reason, the edge along $x = 0$ and $x = a$ of a cylindrical shell must be clamped to make up for the excessive loss of frequency. When two adjacent edges are released, the effect is the same as when two straight edges are released. For angle-ply shells also, if any two adjacent edges are released, the decrease in fundamental frequency is more than when two alternate edges are released. But for angle-ply shells, with the introduction of simply supported edges, the decrease in frequency is not so significant.

Tables 2.4 and 2.5 are prepared from Tables 2.2 and 2.3, respectively, for cutouts with $a'/a = 0.2$. The frequency of a corner point supported shell is assigned a value of 0 and a fully clamped shell is assigned a value 100. Marks are assigned to each boundary condition according to this scale. Hence Tables 2.4 and 2.5 show the efficiency of a particular boundary condition in improving the frequency of a shell, considering that of clamped shell as the upper limit. A practising engineer can realize at a glance from these tables the efficiency of a particular clamping option and make suitable arrangement in improving the fundamental frequency of a shell with minimum number of boundary constraints relative to that of a clamped shell.

Table 2.4 Clamping options for cross-ply stiffened cylindrical shells with central cutouts having a'/a ratio 0.2

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of no. of restraints
0/90/90/0	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Good improvement	25
	1	Edge along $x = 0$ (CSSS)	Marked improvement	31
		One straight edge along $y = b$ (SSSC)	Marked improvement	38
	2	(a) Two alternate edges along $x = 0$ and $x = a$ (CSCS)	Marked improvement	40
		(b) Two straight edges along $y = 0$ and $y = b$(SCSC)	Remarkable improvement	91
		(c) Any two edges except the above option (CSSC)	Marked improvement	43
	3	Three edges (CSCC)	Marked improvement	51
		Three edges (CCSC)	Remarkable improvement	95
	4	All sides (CCCC)	Frequency attains highest value	100
0/90/0/90	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Good improvement	25
	1	Edge along $x = 0$ (CSSS)	Marked improvement	32
		One straight edge along $y = b$ (SSSC)	Marked improvement	37
	2	(a) Two alternate edges along $x = 0$ and $x = a$ (CSCS)	Marked improvement	43
		(b) Two straight edges along $y = 0$ and $y = b$ (SCSC)	Remarkable improvement	85
		(c) Any two edges except the above option (CSSC)	Marked improvement	44

(continued)

Table 2.4 (continued)

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of no. of restraints
	3	Three edges (CSCC)	Marked improvement	53
		Three edges (CCSC)	Remarkable improvement	87
	4	All sides (CCCC)	Frequency attains highest value	100

2.3.1.3 Mode Shape

The typical mode shapes corresponding to the fundamental modes of vibration for different sizes of cutouts are plotted in Figs. 2.2 and 2.3 for antisymmetric cross-ply and angle-ply shells, respectively. The boundary condition considered here is CCSC. Similarly, Figs. 2.4 and 2.5 show fundamental mode of vibration for corner point supported shells. The normalized displacements are drawn with the shell midsurface as the reference for all the support condition and for all the lamination used here. Except corner point supported shells, the fundamental modes are clearly a bending mode for cross-ply and angle-ply shells. The fundamental modes for corner point supported shells are complicated. With the introduction of cutout, mode shapes remain almost similar. When the size of the cutout is increased from 0.2 to 0.4, the fundamental modes of vibration do not change to an appreciable amount.

2.3.2 *Effect of Eccentricity of Cutout Position*

2.3.2.1 Fundamental Frequency

The effect of eccentricity of cutout positions on fundamental frequencies is considered for symmetric and antisymmetric cross and angle-ply laminations, from the results obtained for different locations of a cutout with $a'/a = 0.2$. The nondimensional coordinates of the cutout centre $\left(\bar{x}=\frac{x}{a}, \bar{y}=\frac{y}{a}\right)$ was varied from 0.2 to 0.8 along x and y directions. But the distance of a cutout margin from the shell boundary was less than 0.1. The margins of cutouts were stiffened with four stiffeners. The study is carried out for all the ten boundary conditions. Tables 2.6 and 2.7 express the fundamental frequency of a shell with an eccentric cutout as a percentage (r) of fundamental frequency of a shell with a concentric cutout.

It can be seen that eccentricity of the cutout along the length of the shell towards the circular edges makes it more flexible. It is true for both symmetric and antisymmetric shells. It is also seen that, in almost all the cases, r value is maximum

Table 2.5 Clamping options and their relative performance for angle-ply stiffened cylindrical shells with central cutouts having a'/a ratio 0.2

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of no. of restraints
+45/−45/−45/+45	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Marked improvement	52
	1	Edge along $x = 0$ (CSSS)	Marked improvement	63
		One straight edge along $y = b$ (SSSC)	Marked improvement	62
	2	(a) Two alternate edges along $x = 0$ and $x = a$ (CSCS)	Marked improvement	72
		(b) Two straight edges along $y = 0$ and $y = b$ (SCSC)	Remarkable improvement	80
		(c) Any two edges except the above option (CSSC)	Marked improvement	69
	3	Three edges (CSCC)	Remarkable improvement	79
		Three edges (CCSC)	Remarkable improvement	86
	4	All sides (CCCC)	Frequency attains highest value.	100
+45/−45/+45/−45	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Marked improvement	50
	1	Edge along $x = 0$ (CSSS)	Marked improvement	62
		One straight edge along $y = b$ (SSSC)	Marked improvement	61
	2	(a) Two alternate edges along $x = 0$ and $x = a$ (CSCS)	Marked improvement	71
		(b) Two straight edges along $y = 0$ and y = b(SCSC)	Remarkable improvement	79
		(c) Any two edges except the above option (CSSC)	Marked improvement	67

(continued)

Table 2.5 (continued)

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of no. of restraints
	3	Three edges (CSCC)	Remarkable improvement	78
		Three edges (CCSC)	Remarkable improvement	85
	4	All sides (CCCC)	Frequency attains highest value	100

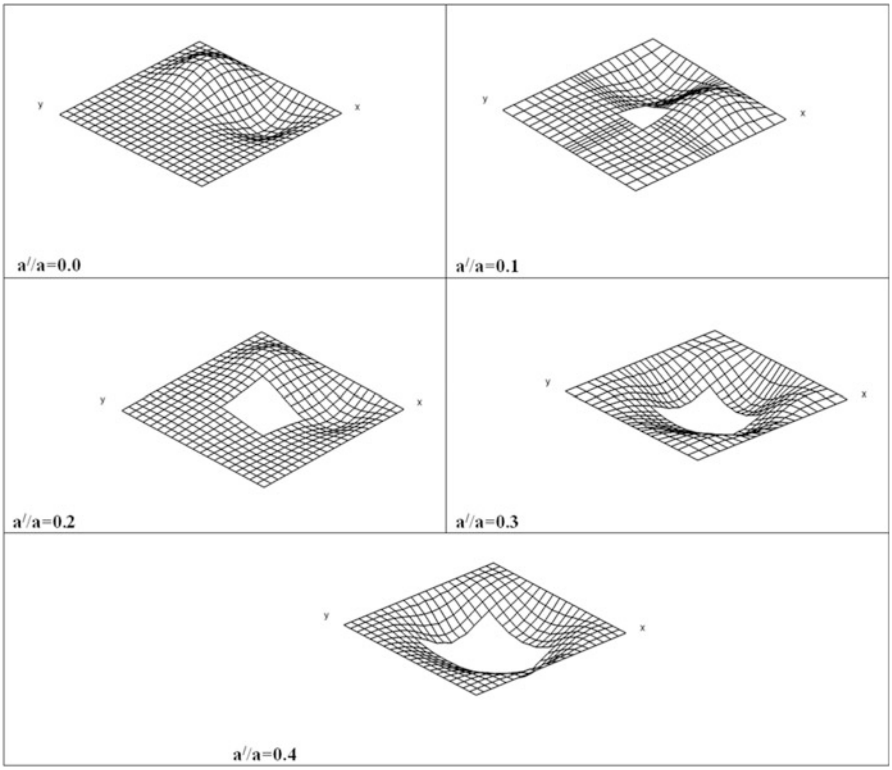


Fig. 2.2 First mode shapes of laminated composite (0/90/0/90)/CCSC stiffened cylindrical shell for different sizes of central square cutout

along $\bar{x} = 0.5$. It is noticed that towards the simply supported circular edge, r value is greater than that of the clamped circular edge for both cross and single ply shells. This means that a designer should preferably provide an eccentric cutout towards the simply supported circular edge if he has to place it along the length.

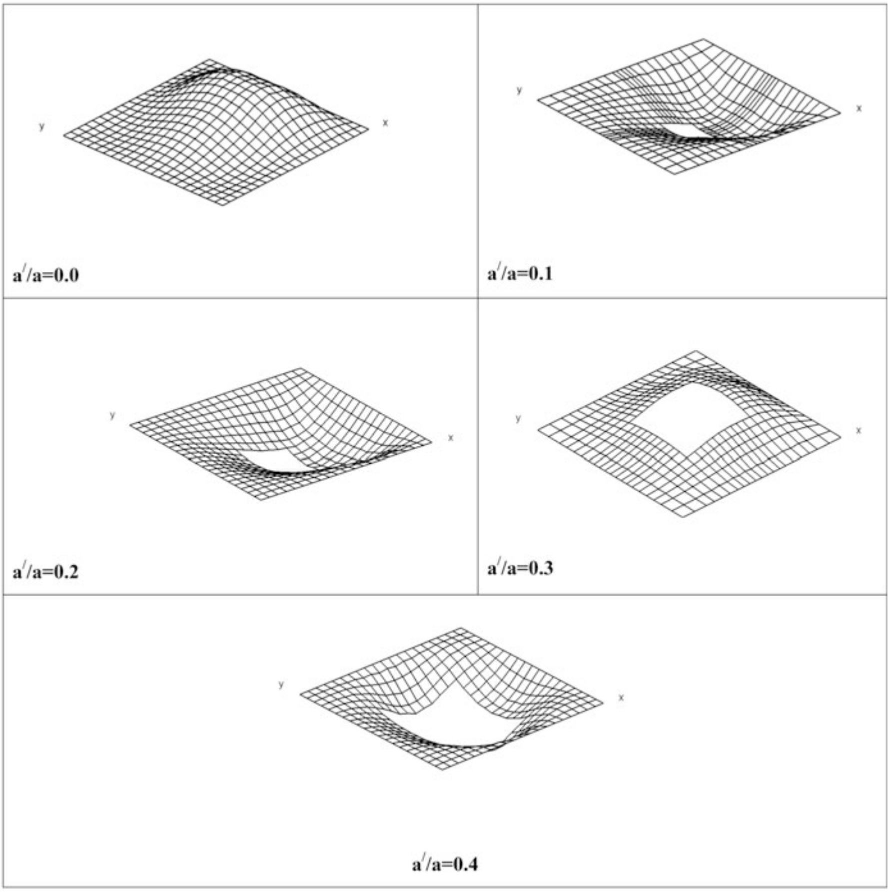


Fig. 2.3 First mode shapes of laminated composite (+45/−45/+45/−45)/CCSC stiffened cylindrical shell for different sizes of central square cutout

It is observed from Tables 2.6 and 2.7 that if the eccentricity of a cutout is varied along the width, the shell becomes stiffer when the cutout shifts towards simply supported edges. So, for functional purposes, if a shift of central cutout is required, eccentricity of a cutout along the width should preferably be towards the simply supported straight edge. But when two opposite straight edges are simply supported or clamped, then a shift of cutout towards the boundary is preferable in the second case. For shells with two straight edges of identical boundary condition, the maximum fundamental frequency occurs along $\overline{y} = 0.5$. For corner point supported shells, the maximum fundamental frequency always occurs when cutout centre varies within central 20 % zone along the width of the shell.

Tables 2.8 and 2.9 provide the maximum values of *r* together with the position of the cutout for cross-ply and angle-ply shells, respectively. These tables also show the rectangular zones within which *r* is always greater than or equal to 90 for both

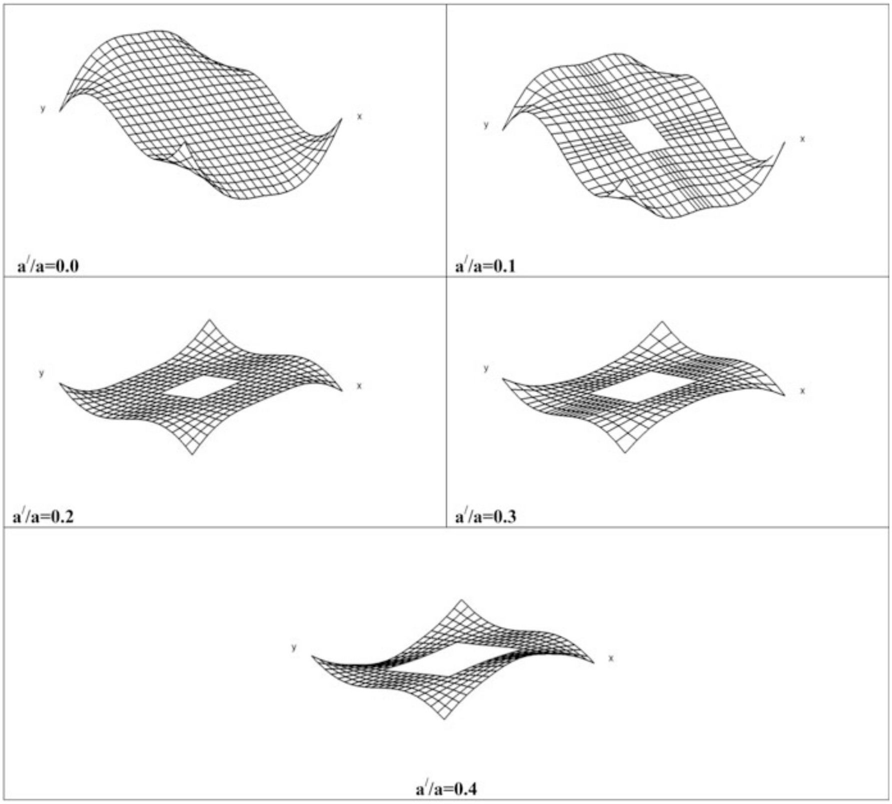


Fig. 2.4 First mode shapes of laminated composite (0/90/0/90)/point supported stiffened cylindrical shell for different sizes of central square cutout

symmetric and antisymmetric laminations. It is to be further noted that, for symmetric and antisymmetric laminations, the rectangular zones are almost similar both for cross-ply and angle-ply shells. Again, a point to be noted is that, although the zone rectangular in plan has been identified, at some other points *r* may have values greater than or equal to 90. These tables indicate the maximum eccentricity of a cutout which can be permitted. With these eccentricities the fundamental frequency of a concentrically punctured shell will not reduce to a significant amount. Thus these tables will help practising engineers in making proper choice of cutout size and position.

2.3.2.2 Mode Shape

The mode shapes corresponding to the fundamental modes of vibration are plotted in Figs. 2.6, 2.7, 2.8, 2.9, 2.10 and 2.11 for antisymmetric cross-ply and angle-ply shell of CCCC, CCSC and SCSC shells for some eccentric position of the cutout.

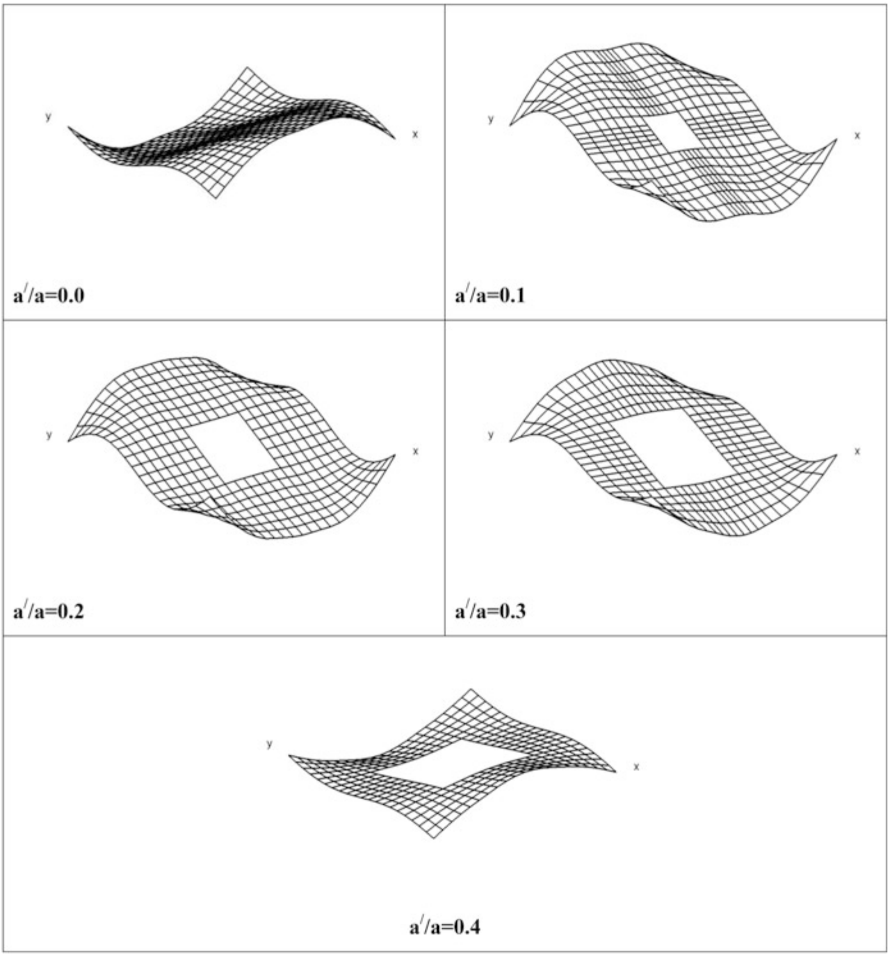


Fig. 2.5 First mode shapes of laminated composite (+45/−45/+45/−45)/point supported stiffened cylindrical shell for different sizes of central square cutout

All the mode shapes are bending mode. For different positions of cutout, mode shapes are almost similar to one another and only the crest and trough position changes.

2.4 Closure

Free vibration characteristics of stiffened cylindrical roof panels with cutouts show that the arrangement of boundary constraints along the four edges rather than their actual number is more important. If cross-ply fully clamped shell is released for any

Table 2.6 Values of ‘r’ for cross-ply cylindrical shells

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
CCCC	0/90/90/0	0.2	82.10	85.97	89.18	89.38	89.19	85.97	82.10
		0.3	81.81	85.75	90.50	91.84	90.50	85.75	81.81
		0.4	82.29	86.92	93.22	96.10	93.22	86.92	82.29
		0.5	82.75	87.89	95.56	100.00	95.56	87.89	82.75
		0.6	82.29	86.92	93.22	96.10	93.22	86.92	82.29
		0.7	81.81	85.75	90.50	91.84	90.50	85.75	81.81
		0.8	81.96	85.80	89.09	89.37	89.16	85.83	82.01
	0/90/0/90	0.2	71.34	78.68	88.30	90.61	88.23	78.68	71.34
		0.3	71.05	78.22	87.96	92.36	87.96	78.22	71.05
		0.4	70.93	78.20	88.54	95.88	88.54	78.20	70.93
		0.5	71.11	78.49	89.10	100.00	89.10	78.49	71.11
		0.6	70.94	78.20	88.54	95.88	88.54	78.20	70.93
		0.7	71.05	78.22	87.96	92.36	87.96	78.22	71.05
		0.8	71.14	78.45	88.04	90.59	88.24	78.51	71.20
CSCC	0/90/90/0	0.2	74.40	80.43	87.77	92.14	87.77	80.43	74.33
		0.3	80.69	86.98	94.35	98.39	94.35	87.00	80.63
		0.4	84.41	91.08	98.59	102.45	98.61	91.11	84.35
		0.5	81.79	88.34	95.90	100.00	96.00	88.45	81.81
		0.6	75.38	81.41	88.77	93.31	88.96	81.56	75.46
		0.7	69.97	75.60	82.67	87.20	82.86	75.77	70.05
		0.8	66.81	72.24	78.94	83.07	79.08	72.36	66.88
	0/90/0/90	0.2	74.21	80.68	88.44	92.75	88.44	80.68	74.16
		0.3	80.33	86.93	94.58	98.65	94.58	86.93	80.27
		0.4	84.31	91.23	98.80	102.60	98.81	91.26	84.26
		0.5	79.74	87.34	95.73	100.00	95.81	87.45	79.80
		0.6	72.87	80.38	88.97	93.48	89.08	80.51	72.95
		0.7	68.47	75.65	83.69	87.81	83.78	75.75	68.53
		0.8	66.39	73.16	80.49	84.11	80.56	73.23	66.42
CCSC	0/90/90/0	0.2	81.92	85.44	90.26	92.45	92.37	89.64	85.38
		0.3	81.78	85.20	90.07	94.12	93.88	89.43	85.14
		0.4	81.84	85.53	91.15	97.14	96.72	90.64	85.65
		0.5	81.96	85.86	92.00	100.00	99.65	91.61	86.11
		0.6	81.84	85.53	91.15	97.18	96.72	90.64	85.65
		0.7	81.78	85.20	90.07	94.12	93.88	89.43	85.14
		0.8	81.79	85.29	90.06	92.43	92.35	89.51	85.28
	0/90/0/90	0.2	71.94	77.98	87.13	98.18	97.79	87.21	78.80
		0.3	71.78	77.70	86.66	98.33	97.53	86.75	78.51
		0.4	71.62	77.54	86.55	99.32	98.20	86.75	78.43
		0.5	71.68	77.68	86.78	100.00	98.78	87.07	78.65
		0.6	71.62	77.54	86.55	99.32	98.73	86.75	78.43
		0.7	71.78	77.70	86.66	98.33	97.53	86.75	78.51
		0.8	71.76	77.78	86.89	98.12	97.72	87.04	78.66

(continued)

Table 2.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
CSSC	0/90/90/0	0.2	74.40	80.32	87.99	94.67	93.48	86.18	78.44
		0.3	78.82	84.96	92.84	99.39	98.67	91.70	83.75
		0.4	80.94	87.30	95.34	101.98	101.67	94.91	86.79
		0.5	79.38	85.49	93.29	100.00	99.69	92.93	85.20
		0.6	75.53	81.14	88.39	95.04	94.24	87.33	80.35
		0.7	72.02	77.26	84.07	90.45	89.10	82.17	75.76
		0.8	69.91	75.00	81.53	87.43	85.76	79.04	72.95
	0/90/0/90	0.2	73.58	79.81	87.94	95.09	94.60	87.14	78.70
		0.3	78.21	84.71	92.91	99.78	99.48	92.41	84.00
		0.4	79.99	86.88	95.39	102.31	102.31	95.58	87.07
		0.5	76.76	83.67	92.43	100.00	100.25	92.93	84.35
		0.6	71.97	78.55	87.17	95.04	94.90	87.29	78.96
		0.7	68.75	75.09	83.36	90.79	90.41	83.01	75.07
		0.8	67.31	73.48	81.33	88.08	87.58	80.68	73.15
CSCS	0/90/90/0	0.2	65.27	71.56	79.32	84.08	79.30	71.54	65.27
		0.3	71.38	77.75	85.50	90.14	85.50	77.75	71.38
		0.4	78.46	84.99	92.50	96.61	92.50	84.97	78.46
		0.5	82.63	89.23	96.40	100.00	96.41	89.23	82.63
		0.6	78.46	84.97	92.50	96.61	92.50	84.97	78.46
		0.7	71.38	77.75	85.50	90.14	85.50	77.75	71.38
		0.8	65.16	71.41	79.14	83.91	79.14	71.38	65.12
	0/90/0/90	0.2	66.38	73.40	81.25	85.28	81.25	73.40	66.36
		0.3	71.03	78.42	86.58	90.74	86.58	78.42	71.03
		0.4	77.35	84.75	92.77	96.76	92.77	84.74	77.34
		0.5	82.63	89.33	96.49	100.00	96.49	89.33	82.63
		0.6	77.35	84.75	92.77	96.76	92.77	84.75	77.35
		0.7	71.03	78.42	86.60	90.74	86.60	78.42	71.03
		0.8	66.33	73.32	81.10	85.14	81.10	73.27	66.30
SCSC	0/90/90/0	0.2	83.94	87.70	92.75	94.44	92.75	87.70	83.94
		0.3	83.79	87.46	92.57	95.72	92.57	87.46	83.79
		0.4	83.92	87.84	93.68	98.12	93.68	87.84	83.92
		0.5	84.09	88.23	94.54	100.00	94.57	88.21	84.09
		0.6	83.98	87.85	93.68	98.14	93.68	87.84	83.92
		0.7	83.79	87.46	92.57	95.72	92.57	87.46	83.79
		0.8	83.82	87.56	92.56	94.42	92.68	87.60	83.87
	0/90/0/90	0.2	72.68	79.00	88.34	98.67	88.34	79.00	72.68
		0.3	72.49	78.69	87.87	98.71	87.87	78.69	72.49
		0.4	72.40	78.58	87.78	99.45	87.78	78.58	72.40
		0.5	72.51	78.76	88.02	100.00	88.02	78.76	72.51
		0.6	72.40	78.58	87.78	100.20	87.78	78.58	72.40
		0.7	72.49	78.69	87.87	98.71	87.87	78.69	72.49
		0.8	72.50	78.80	88.10	98.60	88.27	78.88	72.58

(continued)

Table 2.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
CSSS	0/90/90/0	0.2	66.65	72.78	80.71	88.07	86.99	79.22	71.43
		0.3	71.60	77.80	85.74	92.87	92.18	84.80	77.00
		0.4	76.53	82.95	90.94	97.70	97.50	90.76	82.84
		0.5	78.92	85.49	93.50	100.00	100.06	93.75	85.75
		0.6	76.53	82.95	90.94	97.70	97.50	90.76	82.84
		0.7	71.98	77.80	85.74	92.87	92.18	84.80	77.02
		0.8	66.37	72.50	80.43	87.83	86.78	79.00	71.19
	0/90/0/90	0.2	66.30	72.86	81.25	88.70	88.74	81.46	72.88
		0.3	70.49	77.20	85.72	93.16	93.28	86.05	77.49
		0.4	75.53	82.29	90.66	97.74	97.98	91.26	82.88
		0.5	78.73	85.40	93.44	100.00	100.31	94.20	86.14
		0.6	75.53	82.29	90.66	97.74	97.98	91.26	82.88
		0.7	70.49	77.20	85.72	93.16	93.28	86.05	77.49
		0.8	66.15	72.69	81.05	88.48	88.53	81.27	72.73
SSSC	0/90/90/0	0.2	76.26	83.95	92.14	96.73	92.14	83.95	76.26
		0.3	79.79	87.64	95.73	99.88	95.73	87.64	79.77
		0.4	81.66	89.48	97.45	101.43	97.45	89.48	81.64
		0.5	81.02	88.35	96.02	100.00	96.02	88.35	81.02
		0.6	78.61	85.15	92.41	96.56	92.40	85.15	78.61
		0.7	76.15	82.06	88.98	93.29	88.98	82.06	76.15
		0.8	74.63	80.23	86.88	91.12	86.87	80.21	74.61
	0/90/0/90	0.2	75.18	83.65	92.34	96.99	92.34	83.65	75.18
		0.3	78.94	87.50	95.97	100.17	95.97	87.50	78.94
		0.4	80.61	89.17	97.61	101.65	97.61	89.17	80.59
		0.5	79.00	87.13	95.66	100.00	95.66	87.13	79.00
		0.6	75.82	83.33	91.82	96.62	91.82	83.31	75.82
		0.7	73.44	132.17	88.79	93.61	88.77	80.50	73.44
		0.8	72.39	79.23	87.12	91.66	87.13	79.21	72.39
SSSS	0/90/90/0	0.2	70.86	78.63	87.18	92.20	87.18	78.63	70.86
		0.3	74.84	82.70	90.99	95.48	90.99	82.70	74.84
		0.4	78.47	86.53	94.62	98.60	94.62	86.53	78.47
		0.5	80.03	88.26	96.25	100.00	96.25	88.26	80.03
		0.6	78.47	86.53	94.64	98.60	94.62	86.53	78.47
		0.7	74.84	82.70	90.99	95.48	90.99	82.70	74.84
		0.8	70.52	78.34	86.91	91.99	86.99	78.40	70.56
	0/90/0/90	0.2	70.61	79.05	87.91	92.65	87.91	79.05	70.61
		0.3	74.26	82.62	91.29	95.74	91.29	82.62	74.05
		0.4	77.96	86.35	94.67	98.68	94.67	86.35	77.96
		0.5	79.82	88.24	96.28	100.00	96.28	88.24	79.82
		0.6	77.96	86.35	94.67	98.68	94.67	86.35	77.96
		0.7	74.26	82.62	91.29	95.74	91.29	82.62	74.24
		0.8	70.38	78.82	87.70	92.44	87.72	78.84	70.38

(continued)

Table 2.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
CS	0/90/90/0	0.2	69.92	81.01	94.49	104.89	94.45	80.97	69.88
		0.3	66.29	77.39	90.72	101.39	90.72	77.39	66.26
		0.4	63.17	74.38	88.06	100.19	88.06	74.31	63.14
		0.5	61.94	73.23	87.17	100.00	87.17	73.19	61.90
		0.6	63.17	74.38	88.10	100.19	88.06	74.35	63.14
		0.7	66.29	77.43	90.76	101.43	90.72	77.39	66.26
		0.8	66.22	78.51	92.95	104.20	94.07	79.97	67.76
	0/90/0/90	0.2	72.89	84.05	95.43	102.04	95.43	84.01	72.85
		0.3	69.60	80.68	92.40	99.89	92.40	80.68	69.57
		0.4	67.75	79.24	91.64	99.85	91.64	79.24	67.71
		0.5	67.22	78.94	91.61	100.00	91.61	78.94	67.22
		0.6	67.75	79.24	91.64	99.85	91.64	79.24	67.71
		0.7	69.60	80.68	92.44	99.89	92.40	80.68	69.57
		0.8	69.94	81.85	94.25	101.66	95.05	83.06	71.08

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 0$, $h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 2.7 Values of 'r' for angle-ply cylindrical shells

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
CCCC	+45/−45/−45/+45	0.2	73.32	76.88	82.80	90.17	85.22	79.27	75.40
		0.3	75.63	79.66	85.88	92.80	88.88	82.53	77.93
		0.4	79.75	84.44	90.96	97.01	93.80	87.31	81.93
		0.5	85.48	90.35	96.26	100.00	96.26	90.35	85.48
		0.6	81.94	87.32	93.80	97.01	90.96	84.44	79.75
		0.7	77.93	82.53	88.88	92.80	85.88	79.66	75.63
		0.8	75.40	79.27	85.21	90.15	82.78	76.87	72.33
	+45/−45/+45/−45	0.2	74.09	77.65	83.40	89.83	83.42	77.69	74.15
		0.3	76.29	80.57	86.83	92.59	86.84	80.62	76.36
		0.4	80.18	85.32	92.00	96.93	92.00	85.35	80.23
		0.5	85.27	90.33	96.25	100.00	96.25	90.33	85.27
		0.6	80.23	85.36	92.00	96.93	92.00	85.32	80.18
		0.7	76.36	80.62	86.84	92.59	86.83	80.57	76.29
		0.8	74.15	77.69	83.41	89.82	83.40	77.64	74.09
CSCC	+45/−45/−45/+45	0.2	80.72	85.77	93.27	100.22	95.64	88.25	82.73
		0.3	87.10	91.34	97.92	103.41	100.28	94.02	89.14
		0.4	92.09	96.69	101.53	104.47	101.70	96.21	90.17
		0.5	85.63	91.91	97.89	100.00	95.69	89.13	83.23
		0.6	80.22	85.88	92.27	95.02	89.98	83.23	77.82
		0.7	77.53	82.69	88.99	92.33	87.20	80.54	75.38
		0.8	76.98	82.85	87.72	91.62	86.56	80.10	75.14

(continued)

Table 2.7 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
	+45/−45/ +45/−45	0.2	81.64	86.97	94.47	100.48	94.58	87.20	81.98
		0.3	88.27	92.88	99.39	103.89	99.25	92.82	88.28
		0.4	90.84	96.87	102.11	104.90	102.05	96.97	91.01
		0.5	84.20	90.24	96.55	100.00	96.84	90.32	83.86
		0.6	78.89	84.16	90.57	94.72	90.96	84.28	78.50
		0.7	76.36	81.22	87.48	91.89	87.83	81.42	76.19
		0.8	75.97	80.55	86.59	91.16	86.84	80.75	76.02
CCSC	+45/−45/ −45/+45	0.2	76.60	78.35	82.31	89.77	94.40	87.68	82.35
		0.3	78.74	80.94	85.23	92.63	97.23	91.12	85.46
		0.4	82.07	84.89	89.62	96.70	100.72	95.74	89.76
		0.5	85.21	88.58	93.56	100.00	102.43	97.93	92.34
		0.6	83.42	86.95	92.26	98.90	99.02	92.88	87.44
		0.7	80.58	83.51	88.43	95.69	94.71	87.98	82.89
		0.8	78.53	80.78	85.14	92.74	91.91	85.03	80.01
	+45/−45/ +45/−45	0.2	77.28	79.18	83.18	90.60	92.68	86.00	81.83
		0.3	79.17	81.73	86.30	93.72	95.64	89.10	83.78
		0.4	82.18	85.42	90.56	97.63	99.76	93.92	88.11
		0.5	84.83	88.14	93.36	100.00	102.55	97.99	92.30
		0.6	82.39	85.61	90.76	97.88	99.98	94.04	88.15
		0.7	79.43	82.05	86.78	94.29	95.79	89.16	83.76
		0.8	77.54	79.58	83.83	91.41	92.72	85.99	80.91
CSSC	+45/−45/ −45/+45	0.2	81.50	85.65	91.53	99.44	101.72	94.23	86.67
		0.3	87.99	90.80	95.44	102.16	104.59	99.14	91.17
		0.4	88.20	93.97	98.38	103.26	103.91	98.38	90.96
		0.5	84.43	90.31	95.94	100.00	98.45	92.36	85.98
		0.6	80.83	85.80	91.44	95.92	93.69	87.42	81.81
		0.7	78.76	83.13	88.54	93.75	91.65	85.26	79.71
		0.8	78.14	82.19	87.44	93.49	91.77	85.20	79.08
	+45/−45/ +45/−45	0.2	81.66	86.63	93.03	101.14	101.72	93.91	86.81
		0.3	87.72	92.09	97.33	103.92	104.80	99.00	91.95
		0.4	87.70	93.90	99.13	104.16	104.93	101.31	93.24
		0.5	84.04	89.58	95.16	100.00	100.05	94.69	88.15
		0.6	80.43	85.01	90.38	95.62	95.24	89.51	83.60
		0.7	78.46	82.56	87.84	93.57	92.88	86.93	81.18
		0.8	78.13	82.12	87.37	94.19	92.61	86.42	80.39
CSCS	+45/−45/ −45/+45	0.2	77.10	82.24	89.19	94.49	90.77	84.27	79.23
		0.3	78.40	83.63	90.40	95.63	92.76	86.48	81.03
		0.4	80.97	86.72	93.32	97.97	96.14	90.24	83.77
		0.5	85.11	93.00	98.02	100.00	98.02	93.00	85.12
		0.6	83.77	90.24	96.14	97.97	93.32	86.72	80.97
		0.7	81.03	86.48	92.76	95.63	90.40	83.63	78.39
		0.8	79.23	84.26	90.73	94.43	89.13	82.22	77.10

(continued)

Table 2.7 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
	+45/−45/ +45/−45	0.2	77.80	82.83	89.45	93.84	89.24	82.65	77.74
		0.3	79.44	84.81	91.29	95.12	90.86	84.51	79.45
		0.4	81.98	88.26	94.68	97.73	94.19	88.05	82.19
		0.5	85.06	94.34	98.27	100.00	98.27	94.34	85.06
		0.6	82.19	88.06	94.19	97.73	94.68	88.26	81.98
		0.7	79.45	84.51	90.86	95.12	91.29	84.81	79.44
		0.8	77.74	82.64	89.21	93.77	89.41	82.79	77.80
SCSC	+45/−45/ −45/+45	0.2	79.19	81.79	86.25	94.12	89.17	84.25	81.06
		0.3	81.55	84.38	89.07	96.02	92.32	87.02	83.40
		0.4	84.82	88.23	93.14	98.51	95.60	90.27	86.17
		0.5	87.42	91.47	96.51	100.00	96.51	91.47	87.42
		0.6	86.17	90.27	95.60	98.51	93.14	88.23	84.82
		0.7	83.40	87.02	92.32	96.02	89.07	84.38	81.55
		0.8	81.06	84.25	89.17	94.12	86.24	81.79	79.19
	+45/−45/ +45/−45	0.2	79.72	82.58	87.13	94.85	87.81	83.00	80.00
		0.3	81.96	85.21	90.21	96.36	90.61	85.46	82.16
		0.4	84.99	88.83	94.10	98.63	94.17	88.90	85.10
		0.5	87.01	91.16	96.36	100.00	96.36	91.16	87.01
		0.6	85.10	88.90	94.18	98.63	94.10	88.83	84.98
		0.7	82.16	85.46	90.61	96.36	90.21	85.21	81.96
		0.8	80.00	83.00	87.80	94.85	87.13	82.58	79.72
CSSS	+45/−45/ −45/+45	0.2	78.91	83.94	90.10	94.10	96.13	89.60	83.60
		0.3	79.68	85.19	90.93	96.73	96.83	91.53	85.43
		0.4	81.09	87.62	93.37	98.15	98.59	94.33	88.29
		0.5	82.66	91.76	97.58	100.00	99.02	95.58	89.06
		0.6	82.42	89.93	95.81	98.90	96.44	90.74	85.28
		0.7	81.36	87.19	92.96	97.42	94.69	88.59	82.87
		0.8	80.43	85.35	91.36	97.15	94.54	87.60	81.68
	+45/−45/ +45/−45	0.2	79.91	84.28	90.34	96.59	94.58	87.89	82.27
		0.3	81.12	86.66	92.22	97.02	94.93	89.11	83.63
		0.4	81.92	89.26	95.04	98.62	96.90	91.61	86.12
		0.5	82.48	91.94	97.59	100.00	99.15	96.10	89.24
		0.6	81.47	88.15	93.86	98.21	97.78	92.89	86.94
		0.7	80.45	85.71	91.32	96.63	95.71	90.19	84.16
		0.8	79.83	84.56	90.49	96.47	95.00	88.27	82.38
SSSC	+45/−45/ −45/+45	0.2	83.97	90.37	97.20	104.39	100.41	92.31	83.40
		0.3	87.42	94.75	100.49	105.53	102.75	95.57	85.87
		0.4	86.29	95.40	101.33	104.23	101.15	94.11	85.62
		0.5	85.54	92.33	98.00	100.00	96.14	89.91	83.22
		0.6	83.26	88.91	94.28	96.28	91.96	86.25	80.85
		0.7	81.47	86.63	92.05	94.88	90.22	84.51	79.29
		0.8	80.31	85.71	91.43	95.59	90.66	84.55	78.37

(continued)

Table 2.7 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
	+45/−45/ +45/−45	0.2	82.87	90.27	97.63	104.79	99.82	92.67	85.23
		0.3	85.62	94.54	101.12	105.80	102.24	96.12	87.74
		0.4	85.37	93.86	100.81	104.35	101.78	95.46	87.16
		0.5	83.21	90.08	96.32	100.00	101.72	91.71	84.69
		0.6	81.20	86.55	92.20	96.17	93.74	88.21	82.37
		0.7	80.04	84.73	90.23	94.73	91.77	86.35	80.94
		0.8	79.56	84.56	90.24	95.42	91.70	86.05	80.24
SSSS	+45/−45/ −45/+45	0.2	85.07	90.83	97.04	101.89	98.36	92.41	86.66
		0.3	86.09	91.51	96.33	100.17	98.39	93.95	87.92
		0.4	87.65	92.91	96.98	99.80	98.91	95.60	89.15
		0.5	89.11	96.36	98.54	100.00	98.54	96.36	89.11
		0.6	89.17	95.61	98.91	99.80	96.98	92.91	87.65
		0.7	87.92	93.95	98.39	100.17	96.33	91.51	86.09
		0.8	86.53	92.40	98.33	101.84	97.00	90.80	85.02
	+45/−45/ +45/−45	0.2	85.99	91.27	97.30	101.43	97.24	91.27	86.01
		0.3	87.77	93.39	97.74	99.95	96.47	91.67	86.58
		0.4	89.05	94.97	98.49	99.74	97.13	93.07	87.67
		0.5	88.90	96.23	98.49	100.00	98.49	96.23	88.90
		0.6	87.67	93.07	97.13	99.74	98.49	94.97	89.05
		0.7	86.58	91.67	96.47	99.95	97.74	93.39	87.77
		0.8	85.91	91.26	97.19	101.37	97.28	91.26	85.99
CS	+45/−45/ −45/+45	0.2	81.81	93.38	101.24	102.45	97.55	87.56	76.80
		0.3	79.51	90.30	98.72	100.80	96.01	86.79	77.57
		0.4	78.41	88.62	97.11	100.18	95.57	87.19	78.05
		0.5	78.01	87.82	95.98	100.00	95.98	87.78	78.01
		0.6	78.08	87.19	95.61	100.18	97.07	88.62	78.38
		0.7	77.57	86.79	96.01	100.80	98.72	90.30	79.47
		0.8	76.03	86.83	97.07	101.98	100.51	91.55	79.29
	+45/−45/ +45/−45	0.2	78.17	88.92	98.32	102.02	97.46	87.23	76.46
		0.3	76.90	86.89	96.19	100.21	95.54	85.69	75.88
		0.4	76.01	85.96	95.16	99.69	94.72	85.35	75.67
		0.5	75.57	85.52	94.61	100.00	94.65	85.52	75.57
		0.6	75.67	85.35	94.72	99.69	95.13	85.96	76.01
		0.7	75.88	85.69	95.50	100.21	96.19	86.89	76.87
		0.8	74.91	85.72	96.29	100.96	97.29	87.51	76.36

Table 2.8 Maximum values of r with corresponding coordinates of cutout centres and zones where $r \geq 90$ for cross-ply stiffened cylindrical shells

Laminations	0/90/90/0			0/90/0/90		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value of $r \geq 90$	Maximum values of r	Coordinate of cutout centre	Area in which the value of $r \geq 90$
CCCC	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,	100.00	$\bar{x} = 0.5$	$\bar{x} = 0.5$,
		$\bar{y} = 0.5$	$0.3 \leq \bar{y} \leq 0.7$		$\bar{y} = 0.5$	$0.2 \leq \bar{y} \leq 0.8$
CSCC	102.45	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,	102.60	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,
		$\bar{y} = 0.4$	$0.3 \leq \bar{y} \leq 0.5$		$\bar{y} = 0.4$	$0.3 \leq \bar{y} \leq 0.5$
CCSC	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,	100.00	$\bar{x} = 0.5$	$0.5 \leq \bar{x} \leq 0.6$,
		$\bar{y} = 0.5$	$0.2 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.5$	$0.2 \leq \bar{y} \leq 0.8$
CSSC	101.98	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.7$,	102.31	$\bar{x} = 0.5$	$0.5 \leq \bar{x} \leq 0.6$,
		$\bar{y} = 0.4$	$0.3 \leq \bar{y} \leq 0.5$		$\bar{y} = 0.4$	$0.2 \leq \bar{y} \leq 0.7$
CSCS	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,
		$\bar{y} = 0.5$	$0.4 \leq \bar{y} \leq 0.6$		$\bar{y} = 0.5$	$0.4 \leq \bar{y} \leq 0.6$
SCSC	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,	100.20	$\bar{x} = 0.5$	$\bar{x} = 0.5$,
		$\bar{y} = 0.5$	$0.2 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.6$	$0.2 \leq \bar{y} \leq 0.8$
CSSS	100.06	$\bar{x} = 0.6$	$0.4 \leq \bar{x} \leq 0.7$,	100.31	$\bar{x} = 0.6$	$0.4 \leq \bar{x} \leq 0.7$,
		$\bar{y} = 0.5$	$0.4 \leq \bar{y} \leq 0.6$		$\bar{y} = 0.5$	$0.4 \leq \bar{y} \leq 0.6$
SSSC	101.43	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,	101.65	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,
		$\bar{y} = 0.4$	$0.2 \leq \bar{y} \leq 0.6$		$\bar{y} = 0.4$	$0.2 \leq \bar{y} \leq 0.6$
SSSS	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,
		$\bar{y} = 0.5$	$0.3 \leq \bar{y} \leq 0.7$		$\bar{y} = 0.5$	$0.3 \leq \bar{y} \leq 0.7$
CS	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,
		$\bar{y} = 0.2$	$0.2 \leq \bar{y} \leq 0.3$ and $0.7 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.5$	$0.2 \leq \bar{y} \leq 0.8$

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 0$, $h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 2.9 Maximum values of r with corresponding coordinates of cutout centres and zones where $r \geq 90$ for angle-ply stiffened cylindrical shells

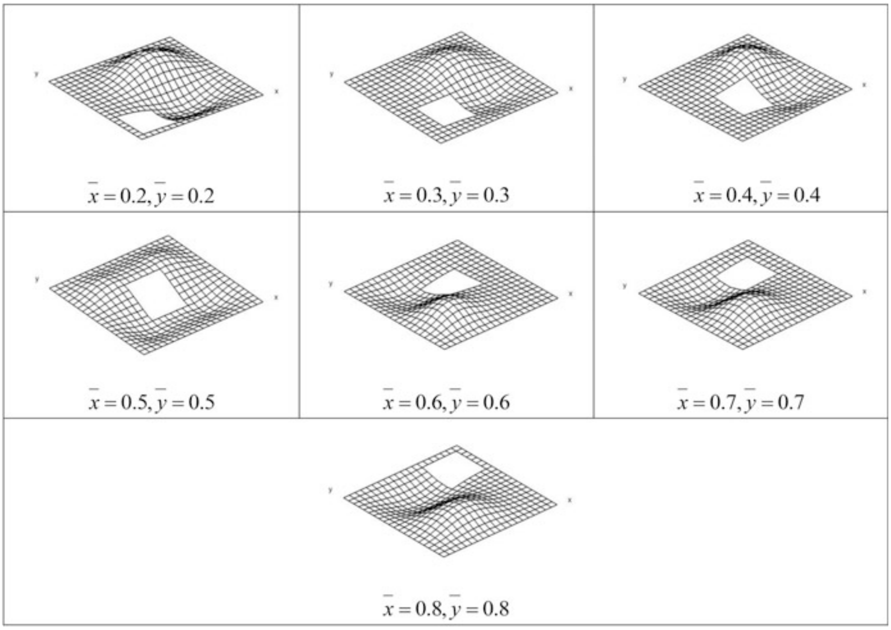
Laminations	+45/−45/−45/+45			+45/−45/+45/−45		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value of $r \geq 90$	Maximum values of r	Coordinate of cutout centre	Area in which the value of $r \geq 90$
CCCC	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,
		$\bar{y} = 0.5$	$0.4 \leq \bar{y} \leq 0.6$		$\bar{y} = 0.5$	$0.4 \leq \bar{y} \leq 0.6$
CSCC	104.47	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,	104.90	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,
		$\bar{y} = 0.4$	$0.2 \leq \bar{y} \leq 0.5$		$\bar{y} = 0.4$	$0.2 \leq \bar{y} \leq 0.6$
CCSC	102.43	$\bar{x} = 0.6$	$0.5 \leq \bar{x} \leq 0.7$,	102.55	$\bar{x} = 0.6$	$0.5 \leq \bar{x} \leq 0.6$,
		$\bar{y} = 0.5$	$0.3 \leq \bar{y} \leq 0.6$		$\bar{y} = 0.5$	$0.2 \leq \bar{y} \leq 0.8$
CSSC	104.59	$\bar{x} = 0.6$	$0.4 \leq \bar{x} \leq 0.7$,	104.93	$\bar{x} = 0.6$	$0.4 \leq \bar{x} \leq 0.7$,
		$\bar{y} = 0.3$	$0.2 \leq \bar{y} \leq 0.5$		$\bar{y} = 0.4$	$0.2 \leq \bar{y} \leq 0.6$
CSCS	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,
		$\bar{y} = 0.5$	$0.3 \leq \bar{y} \leq 0.7$		$\bar{y} = 0.5$	$0.3 \leq \bar{y} \leq 0.7$
SCSC	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.7$,	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,
		$\bar{y} = 0.5$	$0.4 \leq \bar{y} \leq 0.5$		$\bar{y} = 0.5$	$0.3 \leq \bar{y} \leq 0.7$
CSSS	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,
		$\bar{y} = 0.5$	$0.2 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.5$	$0.2 \leq \bar{y} \leq 0.8$
SSSC	105.53	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7$,	105.80	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7$,
		$\bar{y} = 0.3$	$0.2 \leq \bar{y} \leq 0.5$; $0.4 \leq \bar{x} \leq 0.6$		$\bar{y} = 0.3$	$0.2 \leq \bar{y} \leq 0.5$; $0.4 \leq \bar{x} \leq 0.6$
			$0.6 \leq \bar{y} \leq 0.8$			$0.6 \leq \bar{y} \leq 0.8$
SSSS	101.89	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7$,	101.43	$\bar{x} = 0.5$;	$0.3 \leq \bar{x} \leq 0.7$,
		$\bar{y} = 0.2$	$0.2 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.2$	$0.2 \leq \bar{y} \leq 0.8$

(continued)

Table 2.9 (continued)

Laminations	+45/−45/−45/+45			+45/−45/+45/−45		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value of $r \geq 90$	Maximum values of r	Coordinate of cutout centre	Area in which the value of $r \geq 90$
CS	102.45	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,	102.02	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$,
		$\bar{y} = 0.2$	$0.2 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.2$	$0.2 \leq \bar{y} \leq 0.8$

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 0$, $h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$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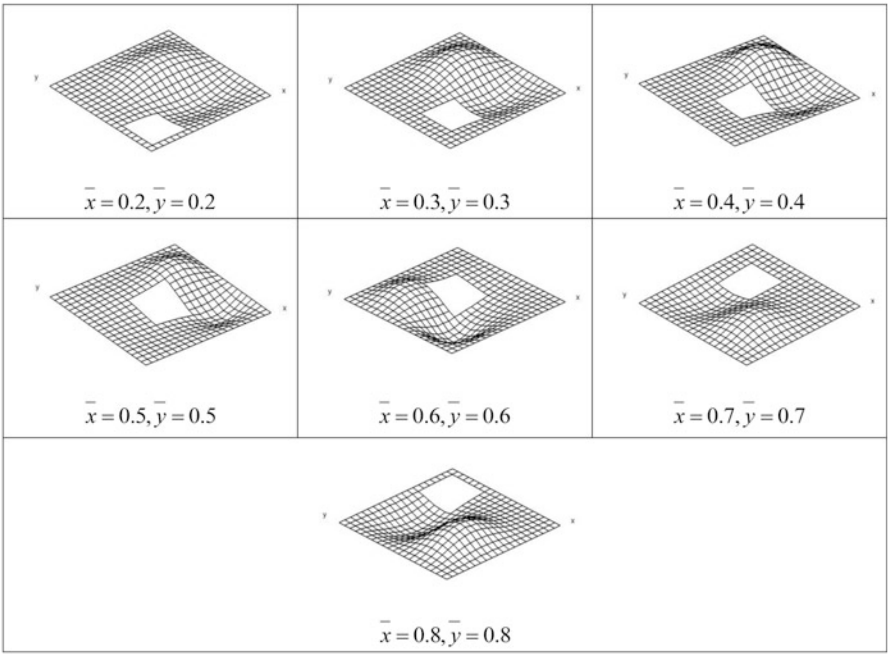
$\bar{x} = 0.6, \bar{y} = 0.6$
$\bar{x} = 0.7, \bar{y} = 0.7$
$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 2.6 First mode shapes of laminated composite (0/90/0/90) stiffened cylindrical shell for different positions of square cutout with CCCC boundary condition

functional reason, then opposite edges must be released to avoid excessive loss in frequency. But for angle-ply shells if any two edges are released, the change in fundamental frequency is not so significant. The relative free vibration performances of shells for different combinations of edge conditions along the four sides will be useful in decision-making for practising engineers. The information regarding the behaviour of stiffened cylindrical shells with eccentric cutouts for varying eccentricity and boundary conditions for cross-ply and angle-ply shells may act as useful design aids for structural engineers. The specific zones are



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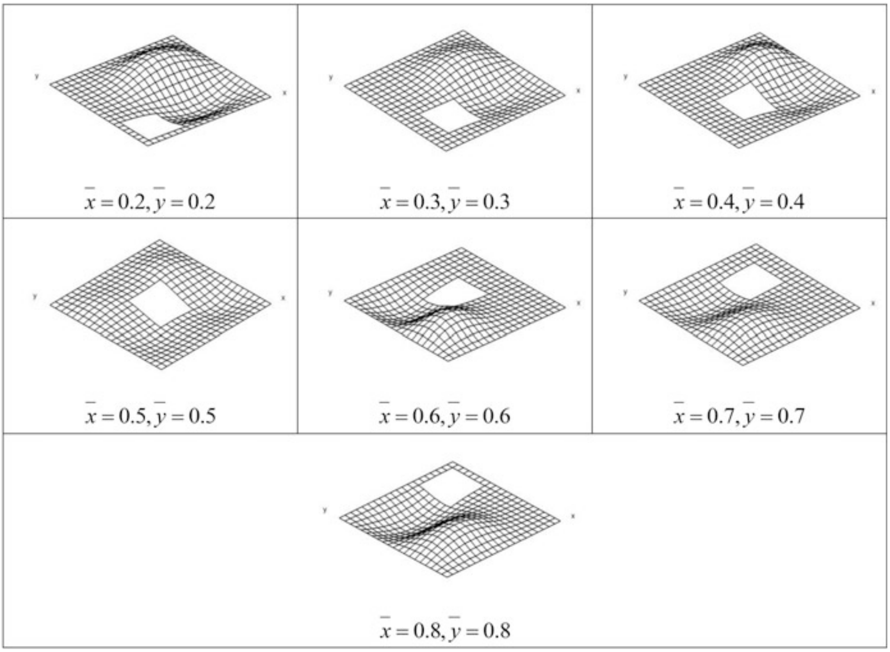
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 2.7 First mode shapes of laminated composite (0/90/0/90) stiffened cylindrical shell for different positions of square cutout with CCSC boundary condition



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$\bar{x} = 0.4, \bar{y} = 0.4$

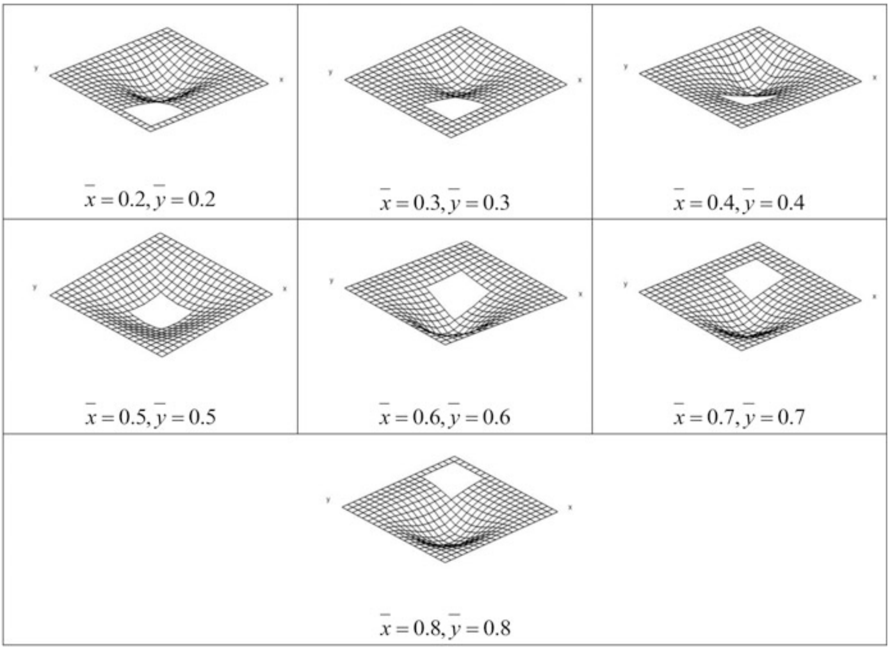
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 2.8 First mode shapes of laminated composite (0/90/0/90) stiffened cylindrical shell for different positions of square cutout with SCSC boundary condition



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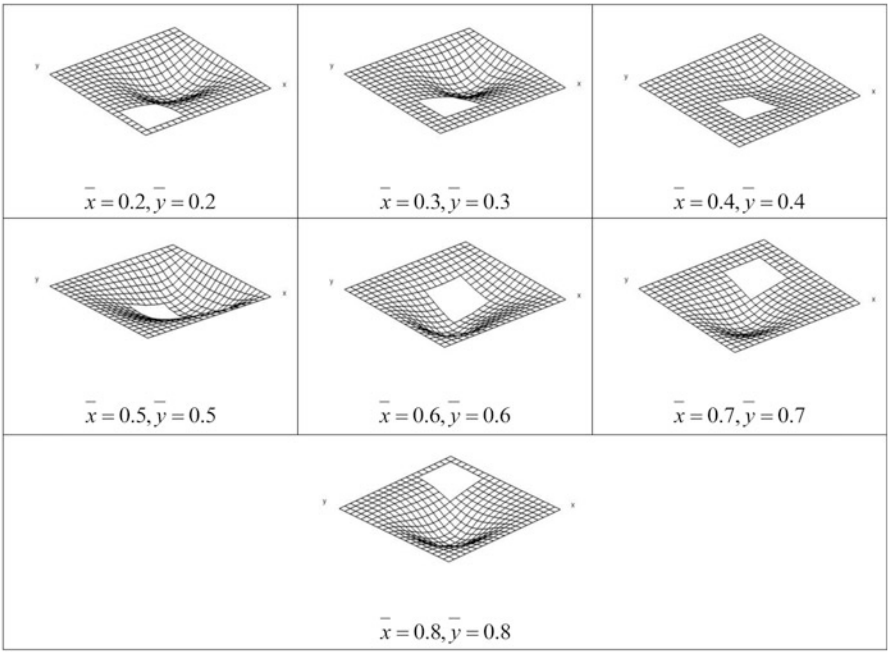
$\bar{x} = 0.5, \bar{y} = 0.5$ $\bar{x} = 0.6, \bar{y} = 0.6$ $\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 2.9 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened cylindrical shell for different positions of square cutout with CCCC boundary condition



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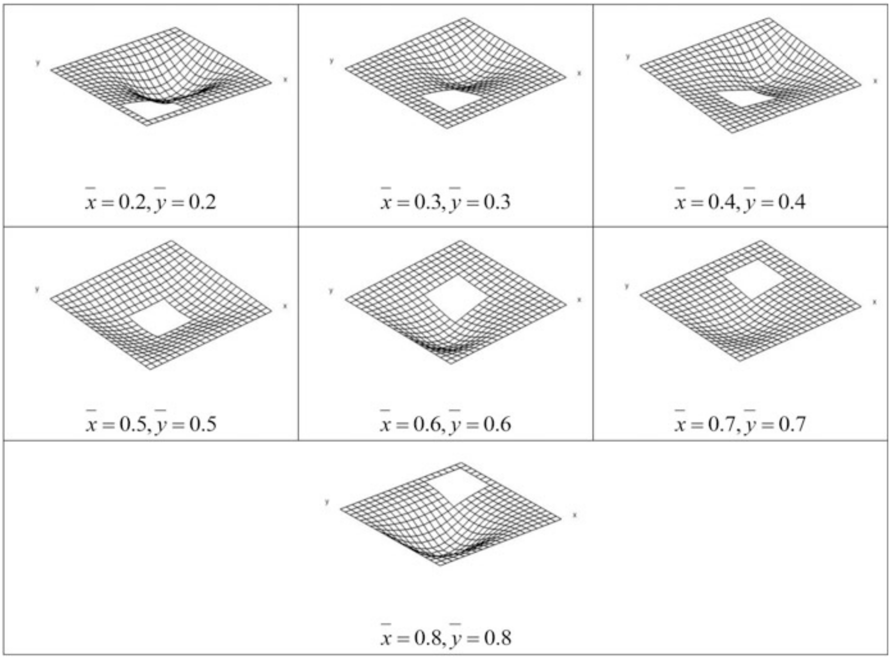
$\bar{x} = 0.5, \bar{y} = 0.5$ $\bar{x} = 0.6, \bar{y} = 0.6$ $\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 2.10 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened cylindrical shell for different positions of square cutout with CCSC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 2.11 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened cylindrical shell for different positions of square cutout with SCSC boundary condition

identified within which the cutout centre may be moved so that the loss of frequency is less than 10 % with respect to a shell with a central cutout. This will enable an engineer to make a decision regarding the allowable eccentricity of the cutout centre.

References

Arnold RN, Warburton GB (1949) Flexural vibrations of the walls of thin cylindrical shell having freely supported ends. Proc R Soc Lond A 197:238–256

Arnold RN, Warburton GB (1953) The flexural vibration of thin cylinders. Proc Inst Mech Eng A 167:62–80

Chakravorty D, Sinha PK, Bandyopadhyay JN (1998) Applications of FEM on free and forced vibration of laminated shells. ASCE J Eng Mech 124(1):1–8

Chung H (1981) Free vibration analysis of cylindrical shells. J Sound Vib 74(3):331–350

Dennis ST, Palazotto AN (1990) Static response of a cylindrical composite panel with cutouts using geometrically nonlinear theory. AIAA J 28(6):1082–1088

Lam KY, Loy CT (1995a) Effect of boundary conditions on frequencies of a multilayered cylindrical shell. J Sound Vib 188(3):363–384

Lam KY, Loy CT (1995b) Free vibration of a rotating multi-layered cylindrical shell. Int J Solids Struct 32(5):647–663

Leissa AW (1973) Vibration of shells. NASA SP-288, Reprinted by Acoustical Society of America, America Institute of Physics, 1993

Leissa AW, Lee JK, Wang AJ (1981) Vibrations of cantilevered shallow cylindrical shells of rectangular planform. J Sound Vib 78(3):311–328

Lim CW, Liew KM (1995) A higher order theory for vibration of shear deformable cylindrical shallow shells. Int J Mech Sci 37(3):277–295

Loy CT, Lam KM, Shu C (1997) Analysis of cylindrical shells using generalized differential quadrature. J Shock Vib 4(3):193–198

Naeem MN, Sharma CB (2000) Prediction of natural frequencies for thin circular cylindrical shells. Proc Inst Mech Eng 214C:1313–1327

Nanda N, Bandyopadhyay JN (2007) Nonlinear free vibration analysis of laminated composite cylindrical shells with cutouts. J Rreinf Plast Compos 26(14):1413–1427

Noor AK, Burton WS (1990) Assessment of computational models for multi-layered composite shells. Appl Mech Rev 43:67–97

Sahoo S (2015) Laminated composite stiffened cylindrical shell panels with cutouts under free vibration. Int J Manuf Mater Mech Eng 5(3):37–63

Singh SP, Gupta K (1994) Damped free vibration of layered composite cylindrical shells. J Sound Vib 172(2):191–209

Soldatos KP (1983) Free vibrations of antisymmetric angle-ply laminated circular cylindrical panels. Q J Mech Appl Math 36(2):207–221

Warburton GB (1965) Vibration of thin cylindrical shell. J Mech Eng Sci 7:399–407

Xuebin L (2008) Study on free vibration analysis of circular cylindrical shells using wave propagation. J Sound Vib 311:667–682

Zhang XM, Liu GR, Lam KY (2001) Vibration analysis of thin cylindrical shells using wave propagation approach. J Sound Vib 239(3):397–403

Chapter 3
Stiffened Hypar Shell with Cutout

Abstract Free vibration characteristics of stiffened composite hypar shell (hyperbolic paraboloidal shells bounded by straight edges) in the presence of cutout are considered following a generalized finite element formulation using eight-noded curved quadratic isoparametric element for shell with a three-noded beam element for stiffener. The size of the cutouts and their positions with respect to the shell centre for different edge constraints are varied to obtain the results in the form of figures and tables. The results are analysed to achieve guidelines for the selection of optimum size and position of the cutout with respect to shell centre considering different practical constraints.

Keywords Hypar shell • Cutout • Eccentricity • Fundamental frequency • Mode shapes

3.1 Introduction

The use of laminated composites in civil engineering applications has motivated the researchers to analyse the different aspects of composite structural elements including different forms of shells. A skewed hypar shell is very appealing from an aesthetic point of view and attractive from a casting point of view being doubly ruled and easy to cast. Moreover, entry of north light is possible with this shell geometry, and due to this advantage, roofing units are made with this shell. In general, thin-walled shells exhibit improved performances with stiffeners, particularly when the shell surface needs provision for cutouts. Cutout is sometimes necessary in roof structure for entry of light, to provide accessibility to other parts of the structure, for venting and at times for alteration of resonant frequency. Thus knowledge of the free vibration characteristics of stiffened composite skewed hypar shell with cutout is essential for using these forms effectively.

Reddy (1982) considered the finite element analysis of composite plate with cutout and presented the effects of parametric variations on linear and nonlinear frequencies. Later, Malhotra et al. (1989) considered orthotropic square plates with square cutouts to observe the effect of fibre orientation and size of cutout on natural frequency for different boundary conditions using Rayleigh-Ritz method.

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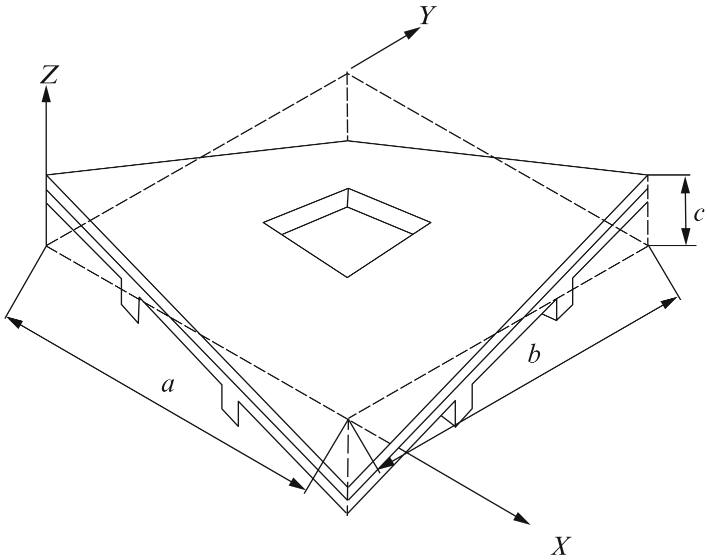
Sivasubramonian et al. (1997) reported free vibration of curved panels with cutout to analyse the effect of cutouts on the natural frequencies for classical boundary conditions. Later Sivakumar et al. (1999), Rossi (1999), Huang and Sakiyama (1999) and Hota and Padhi (2007) considered free vibration of plate with various cutout geometries. Chakravorty et al. (1998) reported some results of the effect of concentric cutout on natural frequency of different shell options. Sivasubramonian et al. (1999) studied the free vibration behaviour of longitudinally stiffened square panels with symmetrical square cutouts using finite element method and varying the size of the cutout (symmetrically located) as well as curvature of the panels. Hota and Chakravorty (2007) provided useful information about free vibration of stiffened conoidal shell roofs with cutout. Later Nanda and Bandyopadhyay (2007) investigated the effect of different parametric variations on nonlinear free vibration characteristics of cylindrical shell with cutout using an eight-noded C^0 continuity, isoparametric quadrilateral element. Detailed free vibration analysis of composite hypar shells with cutout both unstiffened (Sahoo 2011) and stiffened (Sahoo 2012) has been considered with different practical boundary conditions.

Thus it is found that results of free vibration characteristics of isotropic as well as composite stiffened composite shell panels with cutout are scanty in the existing body of literature. Chakravorty et al. (1998) deal with some results of free vibration of hypar shell with concentric cutout, but detailed information regarding the free vibration behaviour of composite stiffened hypar shell with cutout is needed for the effective use of these shell forms. In the present chapter, the free vibration of stiffened hypar shell with cutouts (Fig. 3.1) is considered for different boundary conditions. The variation of fundamental frequency with change in eccentricity of cutout along x and y direction is also considered.

3.2 Problem

In order to consider extensively the effect of cutout size and position on the free vibration response, problems for hypar shells with combinations of different laminations and clamped, simply supported and corner point supported shells have been solved. Among them four-layered cross-ply and angle-ply laminations, both symmetric and antisymmetric, have been chosen for further study. The selection of the four-layered laminations is based on a study by Sahoo and Chakravorty (2005, 2006) which reports that repeating 0/90 unit and +45/−45 unit more than once keeping the total shell thickness constant does not improve the fundamental frequency to an appreciable extent. In this part, numbers of boundary constraints along the four edges have been changed and its effects on fundamental frequencies have been studied from the view point of practical situations.



Surface equation: $z = \frac{4c}{ab}(x-a/2)(y-b/2)$

Fig. 3.1 Surface of a skewed hypar shell with cutout

3.3 Results and Discussion

3.3.1 Free Vibration Behaviour of Shells with Concentric Cutouts

Table 3.1 furnishes the results of nondimensional frequency ($\overline{\omega}$) of stiffened hypar shells with cutout for two-, three- and four-layered cross- and angle-ply laminations both symmetric and antisymmetric, for clamped, simply supported and point supported boundary conditions. Tables 3.2 and 3.3 furnish the fundamental frequencies only for four-layered cross- and angle-ply shells, respectively, for both symmetric and antisymmetric laminations. Here numbers of boundary constraints have been varied from maximum to minimum.

3.3.1.1 Effect of Cutout Size on Fundamental Frequency

From Tables 3.1, 3.2 and 3.3, it is observed that when a cutout is provided to a stiffened shell, the fundamental frequency increases in general. This trend is observed for both symmetric and antisymmetric cross-ply and angle-ply shells. This initial increase in frequency is due to the increased stiffness. As with the introduction of cutout, numbers of stiffeners are increased from two to four in the

Table 3.1 Nondimensional fundamental frequencies ($\overline{\omega}$) for laminated composite stiffened hypar shell for different sizes of central square cutout, different laminations and boundary conditions

Boundary conditions	Laminations	Cutout size (a'/a)				
		0	0.1	0.2	0.3	0.4
Clamped	0/90	112.18	114.10	111.83	109.80	107.20
	90/0	112.18	114.10	111.83	109.80	107.20
	0/90/0	114.61	116.47	114.25	112.61	111.31
	90/0/90	114.45	116.32	114.15	112.55	111.26
	0/90/0/90	114.31	116.21	114.07	112.30	110.75
	90/0/90/0	114.31	116.21	114.07	112.30	110.75
	0/90/90/0	114.92	116.74	114.56	112.91	111.63
	90/0/0/90	114.92	116.75	114.59	112.93	111.62
	+45/−45	162.57	187.44	183.90	148.87	138.52
	+45/−45/+45	179.93	196.96	200.70	163.49	149.94
	+45/−45/+45/−45	186.81	201.93	205.67	169.43	155.18
	+45/−45/−45/+45	182.33	202.42	205.53	170.13	156.27
Simply supported	0/90	47.97	56.07	61.73	68.06	71.63
	90/0	47.97	56.07	61.74	68.06	71.62
	0/90/0	55.10	64.78	68.03	71.72	73.24
	90/0/90	55.81	65.48	68.80	72.48	73.93
	0/90/0/90	58.48	66.44	71.07	75.44	77.37
	90/0/90/0	58.47	66.44	71.07	75.44	77.35
	0/90/90/0	56.93	66.25	69.67	73.42	75.06
	90/0/0/90	57.55	66.88	70.35	74.12	75.68
	+45/−45	48.20	57.64	63.11	70.83	74.37
	+45/−45/+45	64.37	73.48	76.42	81.21	81.35
	+45/−45/+45/−45	64.06	73.09	77.42	82.12	83.21
	+45/−45/−45/+45	66.41	75.12	78.50	83.05	83.65
Point supported	0/90	13.62	13.98	13.79	14.26	14.21
	90/0	13.62	13.97	13.79	14.26	14.21
	0/90/0	13.78	14.16	14.15	13.90	14.18
	90/0/90	12.93	13.39	13.26	13.04	13.23
	0/90/0/90	16.02	16.21	16.02	16.48	16.42
	90/0/90/0	15.99	16.24	16.03	16.48	16.44
	0/90/90/0	14.85	15.14	15.09	15.56	15.73
	90/0/0/90	14.29	14.65	14.50	15.05	15.10
	+45/−45	13.39	13.66	13.76	14.21	14.49
	+45/−45/+45	11.94	12.07	12.03	14.62	14.59
	+45/−45/+45/−45	18.21	18.53	18.63	19.16	19.48
	+45/−45/−45/+45	14.55	14.72	14.74	15.12	15.39

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $c/a = 0.2$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 3.2 Nondimensional fundamental frequencies ($\overline{\omega}$) for laminated composite cross-ply stiffened hypar shell for different sizes of central square cutout and different boundary conditions

Laminations	Boundary conditions	Cutout size (a'/a)				
		0	0.1	0.2	0.3	0.4
0/90/90/0	CCCC	114.92	116.74	114.56	112.91	111.63
	CSCC	107.48	108.67	106.92	104.79	102.30
	CCSC	109.78	110.60	108.90	107.13	105.66
	CCCS	107.48	108.66	106.92	104.69	102.28
	CSSC	104.11	104.06	101.92	98.36	94.30
	CCSS	104.11	104.06	101.91	98.34	94.29
	CSCS	70.08	81.68	86.94	94.21	95.84
	SCSC	87.76	95.63	99.78	100.59	100.93
	CSSS	63.11	73.41	77.15	80.87	81.04
	SSSC	73.86	80.58	85.28	86.79	84.22
	SSCS	63.11	73.40	77.15	80.87	81.04
	SSSS	56.93	66.25	69.67	73.42	75.06
	Point supported	14.85	15.17	15.09	15.56	15.73
0/90/0/90	CCCC	114.31	116.21	114.07	112.3	110.75
	CSCC	107.82	108.94	107.31	105.38	103.29
	CCSC	108.46	109.49	107.81	105.71	103.5
	CCCS	107.82	108.92	107.31	105.32	103.27
	CSSC	103.72	103.71	101.68	98.09	93.95
	CCSS	103.74	103.75	101.73	98.13	93.98
	CSCS	76.63	86.19	92.88	97.89	97.71
	SCSC	76.36	85.92	92.59	98.75	98.13
	CSSS	66.84	75.33	80.12	83.35	82.55
	SSSC	66.78	75.24	80.08	83.47	82.64
	SSCS	66.85	75.39	80.13	83.35	82.55
	SSSS	58.48	66.44	71.07	75.44	77.37
	Point supported	16.02	16.25	16.02	16.48	16.42

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $c/a = 0.2$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

present analysis. For studying the effect of cutout size, in more details the ratio of the fundamental frequency of a punctured shell with increased size of cutout is compared to that of a full shell. It is evident from Tables 3.1, 3.2 and 3.3 that in all the cases with the introduction of cutout with $a'/a = 0.1$, the frequencies increase. But with further increase in cutout size, i.e. when $a'/a = 0.2$, fundamental frequency may increase or decrease. It is to be noted that for shells with minimum number of boundary constraints, i.e. corner point supported shell with increase in cutout size from 0.1 to 0.2, fundamental frequencies decrease, but with further increase in size of the cutout, fundamental frequencies increase. In the case of shells with increased number of boundary constraints, fundamental frequencies increase with increase in cutout size. But the reverse is the case for shells with maximum number of boundary constraints. Here fundamental frequencies decrease with increase in

Table 3.3 Nondimensional fundamental frequencies ($\overline{\omega}$) for laminated composite angle-ply stiffened hypar shell for different sizes of central square cutout and different boundary conditions

		Cutout size (a'/a)				
Laminations	Boundary conditions	0	0.1	0.2	0.3	0.4
+45/−45/−45/+45	CCCC	189.33	202.42	205.53	170.13	156.27
	CSCC	145.97	150.58	139.99	132.63	129.34
	CCSC	145.25	148.88	138.97	130.41	128.00
	CCCS	145.88	149.62	139.45	130.80	128.44
	CSSC	111.08	117.56	115.91	116.31	111.62
	CCSS	115.64	122.05	121.30	119.56	113.49
	CSCS	86.23	97.20	102.36	110.76	114.25
	SCSC	86.12	96.99	102.16	110.50	114.13
	CSSS	76.47	86.76	90.99	96.98	97.77
	SSSC	76.43	86.67	90.88	96.82	97.78
	SSCS	76.47	86.80	90.99	96.98	97.77
	SSSS	66.41	75.12	78.50	84.05	83.65
	Point supported	14.55	14.72	14.74	15.12	15.39
+45/−45/+45/−45	CCCC	186.81	201.93	205.67	169.43	155.18
	CSCC	144.1	148.32	138.81	131.46	129.59
	CCSC	143.78	147.81	138.72	130.65	129.36
	CCCS	144.11	148.19	138.77	130.59	129.3
	CSSC	112.15	118.44	117.47	117.18	112.04
	CCSS	112.01	117.6	117.24	116.39	111.87
	CSCS	82.24	93.66	99.79	108.57	113.37
	SCSC	82.39	93.66	99.86	108.63	113.65
	CSSS	73.07	83.38	88.58	94.96	96.92
	SSSC	73.21	83.43	88.66	95.02	97.14
	SSCS	73.07	83.41	88.58	94.96	96.92
	SSSS	64.06	73.09	77.42	82.12	83.21
	Point supported	18.21	18.53	18.63	19.16	19.48

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $c/a = 0.2$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

cutout size beyond $a'/a = 0.1$. With the increase in cutout size, number and dimensions of the stiffeners do not change but the shell surface incurs loss of both mass and stiffness. In the case of shells with increased number of boundary constrains (shells with more clamped edges), loss of stiffness is more significant than loss of mass. Hence fundamental frequencies decrease except in the case of clamped angle-ply shells. In the case of shells with less number of boundary constraints, with increase in cutout sizes, loss of mass is more significant than loss of stiffness. So, fundamental frequencies increase with the increase in cutout sizes in the case of shells with less number of boundary constraints. As fundamental frequencies increase in most of the cases considered here up to $a'/a = 0.2$, this leads to the practical consideration that concentric cutouts with stiffened margins may be incorporated safely on shell surfaces for functional requirements up to $a'/a = 0.2$.

3.3.1.2 Effect of Boundary Conditions on Fundamental Frequency

The boundary conditions have been divided into five groups, so that the combinations in a particular group include equal number of boundary reactions. The groups are of the following forms:

Group I: contains CCCC shells both cross-ply and angle-ply
Group II: contains CSCC, CCSC and CCCS shells both cross-ply and angle-ply
Group III: contains CSSC, CCSS, CSCS and SCSC shells both cross-ply and angle-ply
Group IV: contains SSSS shells both cross-ply and angle-ply
Group V: corner point supported shells both cross-ply and angle-ply

As evident from Tables 3.2 and 3.3, fundamental frequencies of members belonging to different boundary combinations may be regrouped according to performance.

According to the values of ($\overline{\omega}$), Group III may be subdivided into Group IIIa and Group IIIb for both cross-ply and angle-ply shells.

Group I: contains CCCC shells which exhibit relatively high frequencies both for symmetric and antisymmetric
Group II: contains CSCC, CCSC and CCCS shells which exhibit intermediate values of frequencies both for symmetric and antisymmetric
Group IIIa: contains CSSC and CCSS shells which exhibit relatively lower values of frequencies both for symmetric and antisymmetric
Group IIIb: contains CSCS and SCSC shells which exhibit lower values of frequencies than previous one both for symmetric and antisymmetric
Group IV: contains SSSS shells which exhibit lower values of frequencies than previous one both for symmetric and antisymmetric
Group V: contains corner point supported shells which exhibit lowest values of frequencies both for symmetric and antisymmetric

This observation reveals that the impact of number of boundary constraints is more important than their arrangement. But the impact of arrangement of boundary constraints has a great impact in case of the shells which have two clamped edges. It is found that when two adjacent edges are clamped, the frequency attains higher value than when two alternate edges are clamped.

The frequencies are further studied and marks are given to the options of clamping the edges of a shell in order of gradually improving performances. Tables 3.4 and 3.5 furnish such clamping options for cross-ply and angle-ply shells, respectively. The scale is chosen like this: 0 is assigned to a corner point supported shell and 100 to a clamped shell. These marks are furnished for cutouts with $a'/a = 0.2$. These tables will be helpful to a practising engineer dealing with such shell structures. If one takes the frequency of a clamped shell as upper limit and that of the corner point supported as lower limit, one can easily realize the efficiency of a particular boundary condition.

Table 3.4 Clamping options for cross-ply hypar shells with central cutouts having a'/a ratio 0.2

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of clamping
0/90/90/0	0	Corner point supported, with least number of boundary constraints	–	0
	0	Simply supported, no edges clamped (SSSS)	Good improvement	55
	1	(a) Along $x = 0$ (CSSS)	Good improvement	62
		(b) Along $x = a$ (SSCS)	Good improvement	62
		(c) Along $y = b$ (SSSC)	Good improvement	71
	2	(a) Two alternate edges (CSCS)	Good improvement	72
		(b) Two alternate edges (SCSC)	Marked improvement	85
		(c) Two adjacent edges (CSSC, CCSS)	Marked improvement	87
	3	(a) Three edges excluding $y = 0$ CSCC	Remarkable improvement and frequency becomes almost equal to that of fully clamped shells	92
		(b) Three edges excluding $x = a$ CCSC		94
		(c) Three edges excluding $y = b$ CCCS		92
	4	All sides (CCCC)	Frequency attains a maximum value	100
0/90/0/90	0	Corner point supported, with least number of boundary constraints	–	0
	0	Simply supported, no edges clamped (SSSS)		56
	1	(a) Along $x = 0$ (CSSS)	Good improvement	65
		(b) Along $x = a$ (SSCS)		
		(c) Along $y = b$ (SSSC)		

(continued)

Table 3.4 (continued)

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of clamping
	2	(a) Two alternate edges (CSCS)	Very good improvement	78
		(b) Two alternate edges (SCSC)		
		(c) Two adjacent edges (CSSC, CCSS)	Marked improvement	87
	3	(a) Three edges excluding $y = 0$ CSCC	Remarkable improvement and frequency becomes almost equal to that of fully clamped shells	94
		(b) Three edges excluding $x = a$ CCSC		
		(c) Three edges excluding $y = b$ CCCS		
	4	All sides (CCCC)	Frequency attains a maximum value	100

3.3.1.3 Mode Shapes

The mode shapes corresponding to the fundamental modes of vibration for CCCS and SSSS shells with different sizes of the cutouts are plotted in Figs. 3.2, 3.3, 3.4 and 3.5 for cross-ply and angle-ply shells. The fundamental mode is clearly a bending mode. With the introduction of cutout, mode shapes remain almost similar. When the size of the cutout is increased from 0.2 to 0.4, the fundamental modes of vibration do not change to an appreciable amount.

3.3.2 *Effect of Eccentricity of Cutout Position*

3.3.2.1 Fundamental Frequency

The influence of cutout positions on fundamental frequencies is studied from the results obtained for different locations of a cutout with $a'/a = 0.2$. As earlier the nondimensional coordinates of the cutout centre ($\bar{x} = x/a, \bar{y} = y/a$) are varied from 0.2 to 0.8 along both the plan directions. The study is carried out for all the 13 boundary conditions for cross-ply and angle-ply hypar shells. Both symmetric

Table 3.5 Clamping options for angle-ply hypar shells with central cutouts having a'/a ratio 0.2

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of clamping
+45/−45/ −45/+45	0	Corner point supported, with least number of boundary constraints	–	0
	0	Simply supported, no edges clamped (SSSS)	Slight improvement	33
	1	(a) Along $x = 0$ (CSSS)	Slight improvement	40
		(b) Along $x = a$ (SSCS)		
		(c) Along $y = b$ (SSSC)		
	2	(a) Two alternate edges (CSCS, SCSC)	Slight improvement	46
		(b) Two adjacent edges (CSSC)	Good improvement	53
		(c) Two adjacent edges (CCSS)	Good improvement	56
	3	Three edges excluding $y = 0$ CSCC	Good improvement	65
		Three edges excluding $x = a$ CCSC		
		Three edges excluding $y = b$ CCCS		
	4	All sides (CCCC)	Frequency attains a maximum value	100
+45/−45/ +45/−45	0	Corner point supported, with least number of boundary constraints		0
	0	Simply supported, no edges clamped (SSSS)	Slight improvement	31
	1	(a) Along $x = 0$ (CSSS)	Slight improvement	37
		(b) Along $x = a$ (SSCS)		
		(c) Along $y = b$ (SSSC)		
	2	(a) Two alternate edges (CSCS, SCSC)	Slight improvement	44
		(b) Two adjacent edges (CSSC,CCSS)	Good improvement	53

(continued)

Table 3.5 (continued)

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of clamping
	3	Three edges excluding $y=0$ CSCC	Good improvement	64
		Three edges excluding $x=a$ CCSC		
		Three edges excluding $y=b$ CCCS		
	4	All sides (CCCC)	Frequency attains a maximum value	100

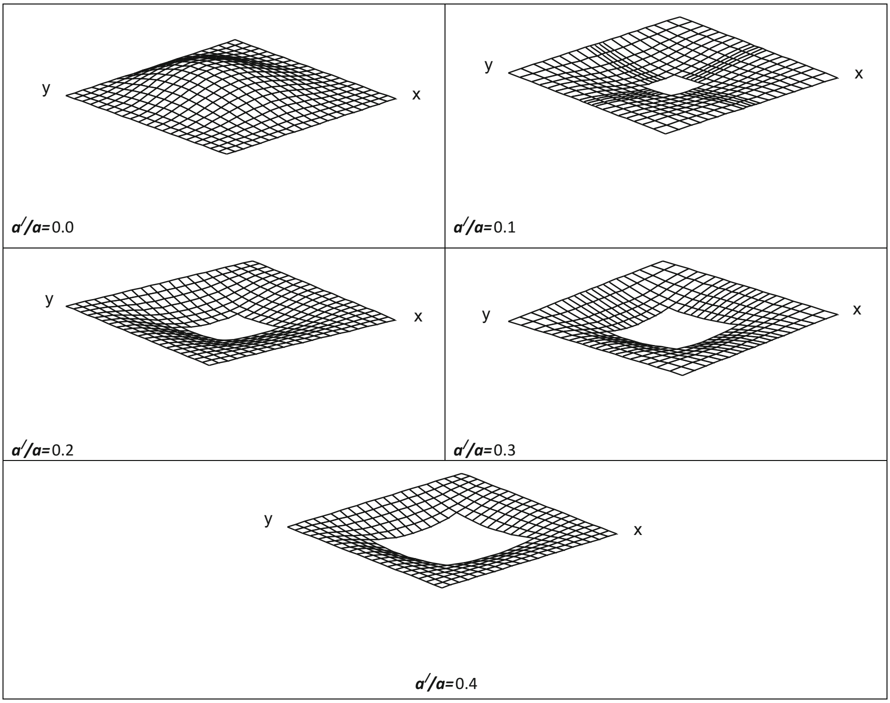
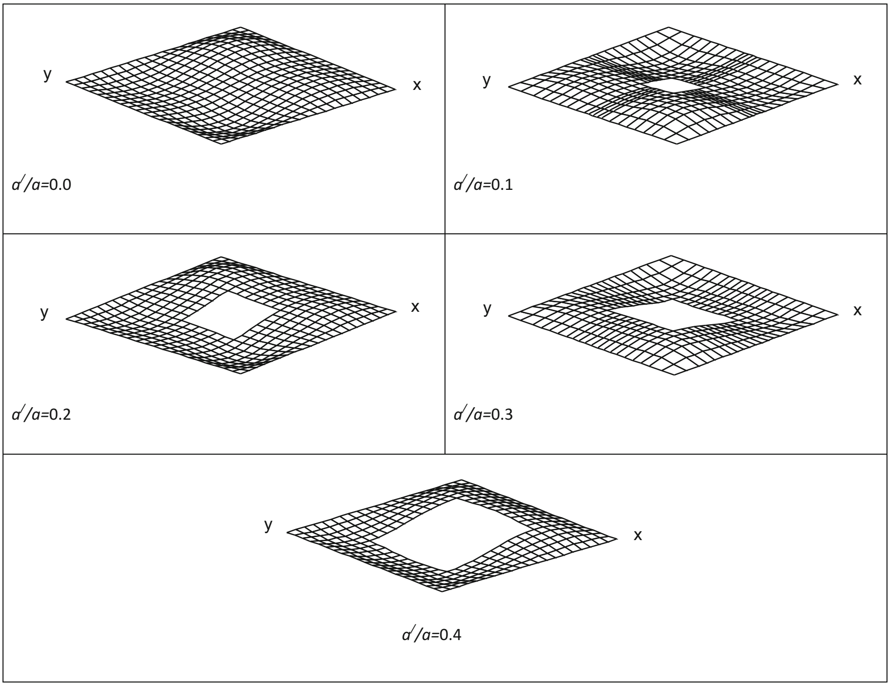


Fig. 3.2 First mode shapes of laminated composite (0/90/0/90)/CCCS stiffened hypar shell for different sizes of central square cutout

and antisymmetric laminations have been considered. The r values (ratio of the fundamental frequency of a shell with an eccentric puncture to that of a shell with concentric puncture in percentage) are given in Tables 3.6 and 3.7 for cross-ply and angle-ply hypar shells, respectively.



$a'/a=0.0$

$a'/a=0.1$

$a'/a=0.2$

$a'/a=0.3$

$a'/a=0.4$

Fig. 3.3 First mode shapes of laminated composite (0/90/0/90)/SSSS stiffened hypar shell for different sizes of central square cutout

It may be seen that fundamental frequency is maximum when cutout is along the centre line of the shell in the case of a shell with four edges clamped for symmetric and antisymmetric cross- and angle-ply shells. But, in the case of shells with four edges simply supported, only antisymmetric cross-ply shell shows a similar trend. Other simply supported shells, like symmetric cross-ply and symmetric and antisymmetric angle-ply, show maximum fundamental frequency when the cutout centre is along the diagonal. For shells with one edge simply supported and others clamped, fundamental frequencies increase with the increase in eccentricity towards the simply supported edges. This observation is true for cross-ply as well as angle-ply shells both for symmetric and antisymmetric laminations. For cross-ply shells, both symmetric and antisymmetric, having two opposite edges clamped and other opposite edges simply supported, fundamental frequencies increase when the cutout centres are placed along the centre line of the shell equidistant from two simply supported edges. But angle-ply shells with the same boundary condition become stiffer when the cutout centre is shifted towards the clamped edges. It is further noticed that the shells with three edges simply supported and other clamped fundamental frequency is maximum when cutout centre is along the line which is equidistant from two simply supported edges. This observation is true for cross-ply shells. For angle-ply shells maximum frequency occurs for variation of cutout centre within central 40 % zone. Along the other direction, shifting of cutout centres towards the simply supported edge which is opposite to clamped edge makes the

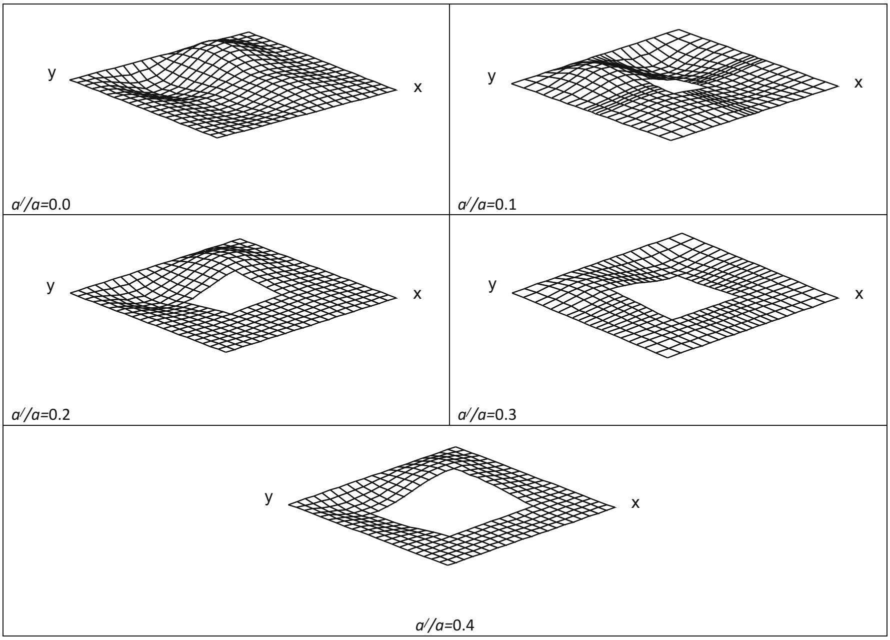


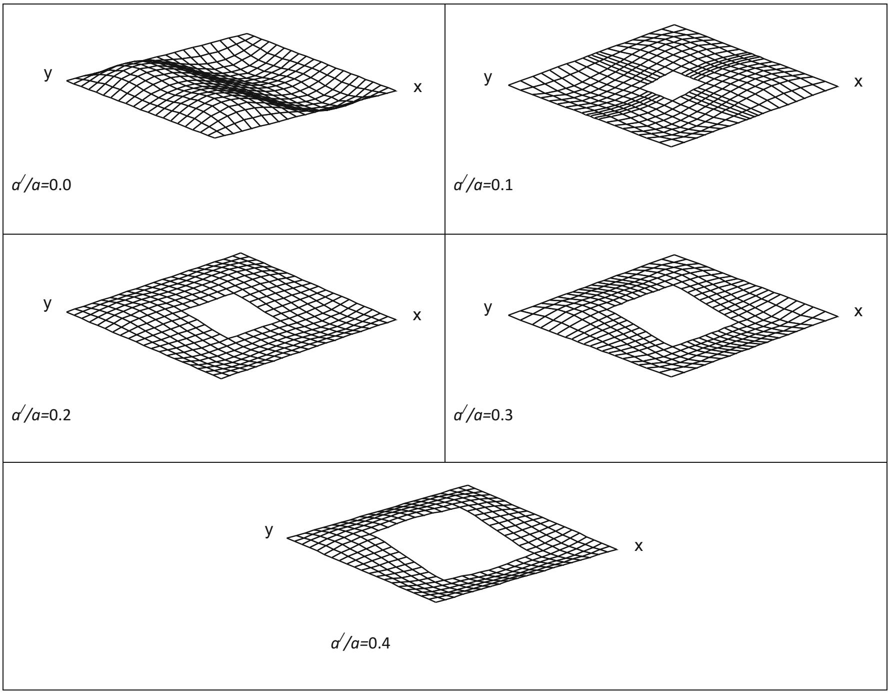
Fig. 3.4 First mode shapes of laminated composite (+45/−45/+45/−45)/CCCS stiffened hypar shell for different sizes of central square cutout

shell stiffer. This is also true for cross-ply shells. But reverse is the case for angle-ply shells. In the case of an angle-ply shell, the shifting of cutout centre towards the simply supported edges makes the shell more flexible. For corner point supported shells, coordinates of cutout centre towards the boundary of the shells make it stiffer and hence the fundamental frequency increases.

In Tables 3.8 and 3.9, maximum values of r along with $\bar{x}$ and $\bar{y}$ is provided. Here, $\bar{x}$ and $\bar{y}$ denote the nondimensional coordinates of the cutout centre. Tables 3.8 and 3.9 also present specification of the rectangular zones within which r values are always greater than or equal to 95. So the centre of the cutout may be varied within that zone. It is to be further noted that, besides the rectangular zones identified in the tables, r may have similar values. So the cutout centre may be placed at some points beyond the zones indicated in Table 3.8 and 3.9. These tables will be helpful to practising engineers to get an idea of the reduction in value of the fundamental frequency of a concentrically punctured shell due to change in eccentricity of a cutout. Thus, these tables will also help to decide the maximum eccentricity which can be permitted.

3.3.2.2 Mode Shape

The mode shapes corresponding to the fundamental modes of vibration are plotted in Figs. 3.6, 3.7, 3.8, 3.9, 3.10, 3.11, 3.12 and 3.13 for cross-ply and angle-ply shell of CCCC, CCSC, CSSC and corner point supported shells for different eccentric



$a'/a = 0.0$

$a'/a = 0.1$

$a'/a = 0.2$

$a'/a = 0.3$

$a'/a = 0.4$

Fig. 3.5 First mode shapes of laminated composite (+45/−45/+45/−45)/SSSS stiffened hypar shell for different sizes of central square cutout

positions of the cutout. In Figs 3.6, 3.7, 3.8, 3.9, 3.10 and 3.13, $\bar{x}$ and $\bar{y}$ denote the nondimensional coordinates of the cutout centre. All the mode shapes are bending mode, except corner point supported shells. It is found that for different positions of cutout, mode shapes are somewhat similar to one another and only the crest and trough position changes.

3.4 Closure

Free vibration problems of stiffened hypar shell roofs with cutout show that concentric cutouts may be incorporated safely on stiffened hypar shell surfaces for functional requirements up to $a'/a = 0.2$. The arrangement of boundary constraint along the four edges rather than their actual number is more important. Fundamental frequency undergoes marked improvement when the edge is changed to a clamped one from a simply supported condition. The relative free vibration performances of stiffened hypar shells for different combinations of edge conditions along the four sides will be very useful in decision-making for practising engineers. The information regarding behaviour of stiffened hypar shell with eccentric cutouts for a wide range of eccentricity and boundary conditions may be used as design aids by structural engineers.

Table 3.6 Values of ‘r’ for cross-ply hypar shells

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
CCCC	0/90/90/0	0.2	97.68	98.19	99.00	99.61	99.00	98.19	97.68
		0.3	98.49	98.95	99.51	99.83	99.51	98.95	98.49
		0.4	99.50	99.74	99.90	99.95	99.90	99.74	99.49
		0.5	100.03	100.11	100.06	100.00	100.06	100.11	100.03
		0.6	99.49	99.74	99.90	99.95	99.90	99.74	99.49
		0.7	98.49	98.95	99.51	99.83	99.51	98.95	98.49
		0.8	97.68	98.18	98.96	99.58	98.94	98.17	97.68
	0/90/0/90	0.2	97.52	98.19	99.14	99.69	99.12	98.17	97.49
		0.3	98.23	98.83	99.51	99.87	99.51	98.81	98.22
		0.4	99.25	99.61	99.86	99.96	99.86	99.6	99.23
		0.5	99.86	100.03	100.03	100	100.03	100.03	99.86
		0.6	99.23	99.6	99.86	99.97	99.86	99.61	99.25
		0.7	98.22	98.81	99.51	99.86	99.51	98.83	98.23
		0.8	97.49	98.16	99.09	99.68	99.1	98.16	97.51
CSCC	0/90/90/0	0.2	99.46	99.58	99.97	100.22	99.97	99.58	99.42
		0.3	100.34	100.18	100.23	100.30	100.23	100.18	100.31
		0.4	100.33	99.97	100.03	100.14	100.03	99.97	100.32
		0.5	99.06	98.76	99.35	100.00	99.35	98.75	99.06
		0.6	97.22	96.76	97.72	99.86	97.69	96.75	97.19
		0.7	95.48	94.40	93.95	93.70	93.85	94.35	95.41
		0.8	94.11	92.12	89.78	88.63	89.67	92.02	94.03
	0/90/0/90	0.2	99.53	99.69	100.03	100.24	100.03	99.68	99.47
		0.3	100.24	100.15	100.25	100.35	100.26	100.16	100.21
		0.4	100.09	99.84	100	100.19	100.02	99.86	100.1

(continued)

Table 3.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
		0.5	97.94	98.62	99.33	100	99.34	98.63	98.79
		0.6	97.34	97.15	98.3	99.78	98.29	97.16	97.35
		0.7	96.37	95.99	96.98	99.42	96.98	96	96.38
		0.8	95.87	95.29	95.69	96.68	95.7	95.3	95.89
CCSC	0/90/90/0	0.2	97.05	97.39	98.02	98.94	99.95	100.18	99.71
		0.3	96.86	97.31	97.99	98.89	99.80	100.20	99.96
		0.4	98.08	98.53	99.02	99.50	99.98	100.33	100.32
		0.5	99.51	99.82	99.95	100.00	100.16	100.43	100.51
		0.6	98.08	98.53	99.02	99.50	99.98	100.33	100.32
		0.7	96.86	97.31	98.00	98.89	99.80	100.20	99.96
		0.8	97.05	97.38	97.97	98.88	99.93	100.15	99.71
	0/90/0/90	0.2	96.21	96.77	97.75	99.07	100.12	100.13	99.48
		0.3	95.57	96.35	97.51	98.83	99.83	100.09	99.72
		0.4	95.66	97.17	98.52	99.42	99.96	100.24	100.13
		0.5	95.95	99.59	99.9	100	100.13	100.35	100.38
		0.6	95.58	97.08	98.46	99.39	99.96	100.24	100.13
		0.7	95.52	96.3	97.46	98.8	99.83	100.1	99.72
		0.8	96.21	96.75	97.68	98.99	100.1	100.12	99.49
CCCS	0/90/90/0	0.2	94.01	92.02	89.69	88.62	89.69	92.02	94.01
		0.3	95.39	94.34	93.86	93.68	93.85	94.34	95.39
		0.4	97.18	96.75	97.69	99.86	97.69	96.75	97.18
		0.5	99.06	98.75	99.35	100.00	99.35	98.76	99.06
		0.6	100.32	99.97	100.03	100.14	100.03	99.97	100.32
		0.7	100.31	100.18	100.23	100.30	100.23	100.18	100.31
		0.8	99.25	99.36	99.75	100.03	99.78	99.35	99.23

	0/90/0/90	0.2	95.89	95.31	95.71	96.64	95.67	95.29	95.87
		0.3	96.38	96	96.97	99.41	96.96	95.99	96.37
		0.4	97.35	97.16	98.29	99.78	98.29	97.15	97.34
		0.5	98.79	98.63	99.34	100	99.33	98.62	98.78
		0.6	100.1	99.86	100.02	100.19	100	99.84	100.08
		0.7	100.21	100.16	100.26	100.35	100.25	100.15	100.21
		0.8	99.32	99.49	99.83	100.05	99.83	99.47	99.31
CSSC	0/90/90/0	0.2	99.52	99.36	99.49	99.94	100.55	100.90	100.84
		0.3	100.10	99.90	99.90	100.10	100.45	100.80	101.02
		0.4	100.86	100.45	100.28	100.30	100.38	100.48	100.67
		0.5	99.27	98.96	99.25	100.00	100.30	99.96	99.83
		0.6	97.65	97.04	97.38	98.94	100.23	99.09	98.75
		0.7	96.91	95.72	95.32	95.63	96.61	97.16	97.60
		0.8	96.68	94.70	92.89	91.74	91.89	93.94	96.43
	0/90/0/90	0.2	98.79	98.82	99.2	99.86	100.55	100.89	100.79
		0.3	98.95	99.27	99.66	100.08	100.48	100.8	100.95
		0.4	98.27	100.33	100.3	100.38	100.43	100.46	100.57
		0.5	96.25	97.59	98.9	100	100.32	99.93	99.75
		0.6	96.09	96.3	97.29	99.15	100.22	99.4	99.01
		0.7	96.62	96.19	96.67	98.31	100.17	98.95	98.66
		0.8	97.57	96.83	96.72	97.23	98.56	98.61	98.7
CCSS	0/90/90/0	0.2	96.61	94.64	92.84	91.76	91.90	93.94	96.43
		0.3	96.87	95.69	95.28	95.62	96.61	97.16	97.59
		0.4	97.64	97.04	97.37	98.95	100.24	99.10	98.76
		0.5	99.28	98.97	99.26	100.00	100.31	99.97	99.84
		0.6	100.87	100.46	100.29	100.31	100.39	100.49	100.68
		0.7	100.09	99.91	99.91	100.11	100.46	100.81	101.03
		0.8	99.39	99.23	99.28	99.72	100.35	100.73	100.69

(continued)

Table 3.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
	0/90/0/90	0.2	97.54	96.8	96.73	97.24	98.51	98.56	98.64
		0.3	96.61	96.18	96.66	98.3	100.13	98.91	98.62
		0.4	96.14	96.34	97.32	99.15	100.2	99.37	98.99
		0.5	96.32	97.69	98.95	100	100.29	99.9	99.73
		0.6	98.28	100.3	100.27	100.34	100.38	100.41	100.53
		0.7	98.84	99.19	99.6	100.03	100.44	100.76	100.91
		0.8	98.65	98.67	98.92	99.56	100.32	100.69	100.63
CSCS	0/90/90/0	0.2	81.53	81.54	81.70	81.72	81.70	81.54	81.53
		0.3	90.59	90.69	90.00	89.43	90.00	90.69	90.59
		0.4	98.15	98.09	97.24	96.53	97.24	98.09	98.15
		0.5	101.46	101.25	100.63	100.00	100.63	101.24	101.46
		0.6	98.15	98.10	97.24	96.53	97.24	98.09	98.15
		0.7	90.59	90.69	90.00	89.43	90.00	90.69	90.59
		0.8	81.14	81.19	81.32	81.40	81.34	81.31	81.19
	0/90/0/90	0.2	86.53	86.51	86.95	86.16	86.97	86.49	86.48
		0.3	92.9	92.77	92.84	92.87	92.84	92.75	92.84
		0.4	96.52	96.4	97.17	97.7	97.17	96.38	96.47
		0.5	97.17	97.41	98.85	100	98.84	97.41	97.17
		0.6	96.47	96.39	97.17	97.7	97.17	96.4	96.5
		0.7	92.84	92.75	92.84	92.88	92.84	92.77	92.9
		0.8	86.17	86.18	86.66	87.02	86.73	86.33	86.33
SCSC	0/90/90/0	0.2	92.32	94.30	94.08	93.77	94.08	94.30	92.32
		0.3	92.64	94.91	95.04	94.92	95.04	94.91	92.64
		0.4	93.93	96.65	97.46	97.65	97.46	96.65	93.93

		0.5	94.70	98.50	99.89	100.00	99.89	98.49	94.70
		0.6	93.93	96.65	97.46	97.65	97.46	96.65	93.93
		0.7	92.64	94.91	95.04	94.92	95.04	94.91	92.64
		0.8	92.28	94.28	93.98	93.71	94.02	94.28	92.29
	0/90/0/90	0.2	86.38	92.89	97.14	98.25	97.12	92.87	86.36
		0.3	86.43	92.92	96.97	98.23	96.93	92.87	85.96
		0.4	86.9	92.92	97.45	99.25	97.39	92.87	86.84
		0.5	87.13	92.85	97.71	100	97.71	92.85	87.13
		0.6	86.84	92.87	97.39	99.25	97.45	92.92	86.9
		0.7	86.39	92.87	96.93	98.22	96.95	92.92	86.43
		0.8	86.27	92.76	97.09	98.14	97.05	92.8	86.31
CSSS	0/90/90/0	0.2	82.70	82.64	82.57	82.66	83.37	84.10	83.90
		0.3	91.81	92.08	91.26	90.55	91.68	93.66	93.74
		0.4	98.85	99.20	98.04	97.14	98.65	101.34	101.24
		0.5	101.88	102.23	100.97	100.00	101.69	104.54	104.17
		0.6	98.85	99.21	98.06	97.14	98.65	101.34	101.24
		0.7	91.81	92.08	91.26	90.56	91.68	93.66	93.74
		0.8	82.27	82.31	82.24	82.24	82.90	83.80	83.54
	0/90/0/90	0.2	87.55	87.4	87.57	88.03	88.73	89.14	88.85
		0.3	94.37	94.44	94.06	94.01	95.18	96.65	96.62
		0.4	98.85	98.84	98.33	98.33	99.99	101.94	101.87
		0.5	100.37	100.28	99.82	100	101.89	103.9	103.68
		0.6	98.76	98.77	98.29	98.31	99.99	101.94	101.87
		0.7	94.28	94.37	94.02	93.98	95.16	96.63	96.59
		0.8	87.17	87.1	87.29	87.74	88.4	88.85	88.6

(continued)

Table 3.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
SSSC	0/90/90/0	0.2	94.28	99.70	102.27	102.97	102.27	99.70	94.27
		0.3	94.56	99.94	102.65	103.41	102.65	99.94	94.56
		0.4	94.39	98.84	101.38	102.27	101.38	98.84	94.39
		0.5	93.60	97.40	99.38	100.00	99.38	97.40	93.60
		0.6	92.55	96.54	97.85	97.97	97.85	96.54	92.55
		0.7	91.92	96.21	97.21	97.08	97.21	96.21	91.92
		0.8	92.00	96.17	97.16	97.12	97.19	96.15	92.01
	0/90/0/90	0.2	89.01	96.86	102.03	103.78	102.03	96.86	88.99
		0.3	89.2	96.84	102.06	103.95	102.06	96.82	89.16
		0.4	88.7	95.26	100.02	101.88	100.03	95.26	88.7
		0.5	87.94	94.02	98.32	100	98.39	94.1	88
		0.6	87.47	94.07	98.36	99.88	98.44	94.18	87.55
		0.7	87.29	94.47	98.9	100.4	98.98	94.56	87.38
		0.8	87.34	94.26	98.78	100.33	98.84	94.35	87.42
SSCS	0/90/90/0	0.2	83.90	84.11	83.37	82.66	82.57	82.64	82.70
		0.3	93.74	93.66	91.68	90.56	91.26	92.09	91.81
		0.4	101.24	101.32	98.65	97.14	98.06	99.20	98.85
		0.5	104.17	104.55	101.69	100.00	100.97	102.23	101.88
		0.6	101.24	101.32	98.65	97.14	98.06	99.20	98.85
		0.7	93.75	93.66	91.68	90.56	91.26	92.09	91.81
		0.8	83.42	83.63	82.86	82.32	82.13	82.41	82.26
	0/90/0/90	0.2	88.83	89.09	88.68	88	87.53	87.34	87.45
		0.3	96.58	96.62	95.15	93.97	94.01	94.37	94.27
		0.4	101.85	101.94	99.98	98.3	98.27	98.77	98.78

		0.5	103.67	103.9	101.89	100	99.82	100.28	100.36
		0.6	101.87	101.94	99.98	98.33	98.32	98.83	98.83
		0.7	96.61	96.65	95.17	94	94.05	94.44	94.37
		0.8	88.43	88.7	88.33	87.77	87.26	87.21	87.22
SSSS	0/90/90/0	0.2	82.78	83.61	83.49	83.25	83.49	83.59	82.78
		0.3	92.32	93.38	92.39	91.53	92.39	93.41	92.34
		0.4	98.16	100.32	98.91	97.67	98.91	100.33	98.16
		0.5	100.10	104.15	101.55	100.00	101.55	104.15	100.11
		0.6	98.21	100.37	98.91	97.69	98.92	100.33	98.16
		0.7	92.34	93.40	92.39	91.53	92.39	93.40	92.34
		0.8	82.42	83.29	83.11	82.83	82.89	83.39	82.32
	0/90/0/90	0.2	83.79	86.96	87.85	88.06	87.93	87.06	83.88
		0.3	87.24	93.04	94.61	94.75	94.7	93.12	87.28
		0.4	88.2	94.89	98.09	98.78	98.2	94.94	88.2
		0.5	88.38	95.08	99.09	100	99.09	95.08	88.38
		0.6	88.21	94.95	98.2	98.78	98.09	94.89	88.2
		0.7	87.28	93.12	94.68	94.75	94.61	93.04	87.24
		0.8	83.72	86.9	87.65	87.76	87.44	86.79	83.43
CS	0/90/90/0	0.2	114.25	112.72	113.59	114.25	113.65	112.72	114.18
		0.3	104.37	102.78	103.25	103.51	103.18	102.72	104.44
		0.4	101.79	99.80	100.20	100.46	100.27	99.93	101.86
		0.5	101.13	99.40	99.67	100.00	99.73	99.34	101.19
		0.6	101.92	99.87	100.33	100.46	100.33	99.80	101.72
		0.7	104.44	102.72	103.11	103.38	103.18	102.58	104.37
		0.8	113.06	111.60	112.39	112.59	112.06	112.06	113.32

(continued)

Table 3.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
	0/90/0/90	0.2	109.05	105.06	104.87	104.56	104.37	104.37	108.11
		0.3	104.87	101.00	100.75	100.44	100.25	100.31	103.81
		0.4	104.06	100.12	100.06	100.00	99.94	100.00	103.50
		0.5	103.68	100.06	99.94	100.00	100.00	100.12	103.62
		0.6	103.43	99.81	99.81	100.06	100.06	100.31	104.12
		0.7	103.81	100.19	100.19	100.44	100.69	100.94	104.81
		0.8	107.62	103.56	103.50	103.50	103.75	104.99	108.61

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $c/a = 0.2$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 3.7 Values of ‘*r*’ for angle-ply hypar shells

			$\bar{x}$						
Edge condition	Laminations	$\bar{y}$	0.2	0.3	0.4	0.5	0.6	0.7	0.8
CCCC	+45/−45/−45/+45	0.2	76.40	77.64	79.63	82.22	79.95	79.07	76.59
		0.3	77.71	82.23	87.03	90.42	88.00	85.01	79.20
		0.4	79.74	86.49	93.32	96.76	95.19	87.99	80.05
		0.5	82.03	89.55	95.88	100.00	95.89	89.52	82.07
		0.6	80.05	87.92	95.20	96.78	93.29	86.36	79.74
		0.7	79.20	84.98	88.17	90.32	86.31	82.23	77.71
		0.8	76.40	78.79	79.77	82.19	79.60	77.42	76.04
	+45/−45/+45/−45	0.2	75.5	77.52	79.62	81.91	79.62	77.53	75.48
		0.3	77.73	83.09	88.13	90.22	87.48	83.1	77.73
		0.4	79.88	88.13	94.67	97.1	94.66	87.7	79.88
		0.5	81.64	90.22	96.18	100	96.18	89.43	81.65
		0.6	79.88	87.48	94.67	97.1	94.67	87.56	79.88
		0.7	77.73	83.1	87.78	90.23	87.45	83.09	77.73
		0.8	75.23	77.24	79.46	81.91	79.59	77.26	75.22
CSCC	+45/−45/−45/+45	0.2	106.88	110.29	112.60	113.67	113.12	111.85	107.32
		0.3	108.56	115.65	119.35	119.54	119.95	116.04	108.81
		0.4	110.89	112.23	112.27	110.81	111.93	112.37	108.37
		0.5	105.99	106.52	102.75	100.00	101.72	106.69	104.81
		0.6	106.09	102.06	96.79	93.74	95.30	100.48	102.53
		0.7	100.81	97.41	92.59	89.70	90.95	95.27	99.19
		0.8	95.11	92.89	88.96	86.54	87.35	90.64	93.82
	+45/−45/+45/−45	0.2	107.37	111.09	113.43	114.35	113.44	111.12	107.33
		0.3	109.36	116.53	120.39	120.22	120.33	116.71	109.43
		0.4	110.05	113.46	112.76	111.31	112.73	113.62	110.23

(continued)

Table 3.7 (continued)

Edge condition	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
		0.5	106.26	107.25	102.28	100	102.23	107.25	106.1
		0.6	103.13	100.64	95.53	93.26	95.47	100.66	102.59
		0.7	98.44	94.11	90.94	89	90.86	94.96	97.47
		0.8	92.85	89.98	87.17	85.77	87.11	90.07	92.03
CCSC	+45/−45/−45/+45	0.2	94.37	99.43	102.42	104.61	108.17	108.98	108.08
		0.3	91.33	95.97	101.22	106.96	112.11	116.04	112.82
		0.4	87.90	91.34	95.66	102.15	112.37	120.77	114.21
		0.5	86.72	89.65	93.59	100.00	111.05	120.25	114.61
		0.6	88.99	92.42	96.55	102.57	112.41	119.99	113.53
		0.7	93.34	97.72	102.32	106.89	112.83	116.85	111.13
		0.8	94.80	100.14	104.88	106.51	110.23	109.33	107.84
	+45/−45/+45/−45	0.2	92.25	97.51	102.65	106.18	110.36	109.26	107.52
		0.3	90.31	95.06	100.71	107.15	113.38	116.61	111.43
		0.4	87.39	90.93	95.43	102.23	112.88	120.74	113.94
		0.5	86.01	88.94	93.11	100	111.51	120.59	114.85
		0.6	87.39	90.93	95.43	102.23	112.88	120.71	113.96
		0.7	90.31	95.07	100.71	107.15	113.39	116.77	111.39
		0.8	92.22	97.4	102.31	106.04	110.01	109.17	107.57
CCCS	+45/−45/−45/+45	0.2	94.02	91.02	87.62	86.55	88.88	93.13	94.51
		0.3	99.05	95.63	91.22	89.74	92.53	97.60	99.94
		0.4	102.40	100.87	95.56	93.78	96.81	102.37	104.57
		0.5	104.75	106.93	101.92	100.00	102.80	106.92	106.16
		0.6	108.30	112.20	111.96	110.71	112.31	112.65	109.85
		0.7	108.85	115.86	120.17	119.71	119.53	116.07	108.95
		0.8	106.57	111.09	112.66	113.37	112.36	109.76	106.50

	+45/−45/+45/ −45	0.2	91.86	90.02	87.14	85.78	87.14	90.02	91.86
		0.3	97.09	94.85	90.89	89	90.89	94.85	97.09
		0.4	102.26	100.62	95.5	93.27	95.5	100.62	102.25
		0.5	105.99	107.28	102.26	100	102.26	107.28	105.93
		0.6	110.5	113.49	112.73	111.27	112.72	113.49	110.06
		0.7	109.37	116.56	120.35	120.21	120.35	116.58	109.37
		0.8	106.23	109.95	112.53	113.65	112.87	110.26	106.66
CSSC	+45/−45/−45/+45	0.2	98.29	102.68	104.53	103.36	102.12	101.54	99.39
		0.3	94.68	98.79	102.21	103.51	103.97	103.93	101.79
		0.4	91.55	94.83	98.46	101.28	103.01	104.04	102.46
		0.5	92.11	94.96	98.19	100.00	101.04	103.32	103.52
		0.6	96.67	99.62	100.71	98.40	98.24	101.73	104.31
		0.7	101.01	102.95	99.97	95.31	94.67	98.27	102.18
		0.8	99.60	100.45	96.59	91.94	91.04	93.95	97.64
	+45/−45/+45/−45	0.2	95.93	100.5	103.22	102.74	101.52	100.22	97.64
		0.3	93.28	97.34	101.01	102.67	103.21	102.67	100.37
		0.4	90.61	94.01	97.93	100.97	102.56	103.17	101.65
		0.5	91.13	94.4	98.11	100	100.55	102.33	102.63
		0.6	95.01	98.53	100.05	97.71	97.28	100.34	102.78
		0.7	99.07	101.19	98.19	93.94	93.32	96.59	99.91
		0.8	96.67	98.42	94.33	90.33	89.66	92.33	95.22
CCSS	+45/−45/−45/+45	0.2	92.65	95.68	92.90	88.91	88.25	91.22	93.04
		0.3	95.75	100.22	97.27	92.76	91.78	94.93	97.27
		0.4	93.49	97.85	99.79	97.19	96.05	98.47	100.31
		0.5	89.49	93.32	97.59	100.00	99.74	100.40	100.45

(continued)

Table 3.7 (continued)

Edge condition	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
		0.6	88.83	92.32	96.50	100.04	101.42	101.07	99.32
		0.7	91.80	95.52	98.92	100.61	101.07	100.02	97.63
		0.8	93.03	97.36	100.54	100.53	99.27	97.20	94.43
	+45/−45/+45/−45	0.2	97.17	98.62	94.38	90.41	89.83	92.61	95.51
		0.3	99.04	101.42	98.17	93.99	93.49	96.79	100.11
		0.4	95	98.7	100.12	97.75	97.43	100.53	102.97
		0.5	91.08	94.55	98.15	100	100.66	102.49	102.78
		0.6	90.55	94.13	97.97	101	102.65	103.29	101.75
		0.7	93.3	97.5	101.13	102.8	103.35	102.81	100.5
		0.8	95.63	100.33	103.41	102.77	101.64	100	97.48
CSCS	+45/−45/−45/+45	0.2	96.05	96.73	95.28	94.61	95.95	97.29	96.63
		0.3	103.41	104.47	101.13	99.07	101.28	104.24	103.42
		0.4	102.48	104.02	101.73	100.06	101.74	103.62	102.38
		0.5	101.20	102.80	101.34	100.00	101.34	102.80	101.20
		0.6	102.38	103.62	101.74	100.06	101.73	104.02	102.48
		0.7	103.42	104.24	101.28	99.07	101.13	104.47	103.41
		0.8	95.81	96.59	95.40	94.26	94.72	96.11	95.30
	+45/−45/+45/−45	0.2	95.87	96.13	94.92	94.15	94.92	96.13	95.83
		0.3	103.42	104.06	100.92	98.89	100.92	104.06	103.42
		0.4	103.08	104.22	101.8	99.99	101.8	104.22	103.08
		0.5	102.11	103.48	101.55	100	101.55	103.48	102.11
		0.6	103.08	104.23	101.8	100	101.8	104.22	103.08
		0.7	103.42	104.06	100.92	98.89	100.92	104.06	103.42
		0.8	94.95	95.31	94.23	93.78	94.29	95.6	95.17

SCSC	+45/−45/−45/+45	0.2	96.57	103.71	102.58	101.25	102.41	103.57	96.92
		0.3	97.23	104.69	104.19	102.98	103.73	104.35	97.61
		0.4	95.72	101.28	101.85	101.46	101.74	101.35	96.31
		0.5	95.01	99.17	100.01	100.00	100.01	99.17	95.01
		0.6	96.31	101.35	101.74	101.46	101.85	101.28	95.72
		0.7	97.61	104.35	103.73	102.97	104.19	104.69	97.23
		0.8	96.92	103.54	102.28	101.16	102.54	103.71	96.56
	+45/−45/+45/−45	0.2	96.22	103.6	103.18	102.26	103.18	103.6	96.22
		0.3	96.53	104.15	104.36	103.67	104.36	104.15	96.53
		0.4	95.31	100.98	101.85	101.68	101.85	100.98	95.31
		0.5	94.52	98.92	99.94	100	99.94	98.93	94.52
		0.6	95.31	100.98	101.85	101.68	101.85	100.98	95.31
		0.7	96.53	104.15	104.36	103.67	104.36	104.15	96.53
		0.8	96.23	103.59	103.13	102.19	103.13	105.58	96.22
CSSS	+45/−45/−45/+45	0.2	94.93	96.37	94.38	91.64	90.91	91.93	91.41
		0.3	103.06	105.66	102.48	97.70	96.30	97.85	97.47
		0.4	103.33	105.80	103.66	99.81	98.33	99.25	98.99
		0.5	102.37	104.48	103.02	100.00	98.75	99.49	99.22
		0.6	103.46	105.37	103.34	99.77	98.60	99.71	99.49
		0.7	103.75	105.68	102.40	97.78	96.66	98.23	98.15
		0.8	94.59	96.20	94.50	91.41	90.23	90.70	90.64
	+45/−45/+45/−45	0.2	94.97	96.22	94.35	91.61	90.5	91.12	90.97
		0.3	103.71	105.89	102.58	97.88	96.51	98.02	98.03
		0.4	104.04	106.17	103.76	99.83	98.56	94.14	99.79
		0.5	103.07	105.12	103.26	100	98.84	99.89	99.89
		0.6	104.04	106.17	103.76	99.83	98.56	99.78	99.79
		0.7	103.71	105.89	102.58	97.89	96.51	98.03	98.03
		0.8	94.35	95.69	93.98	91.17	90.05	90.5	90.42

(continued)

Table 3.7 (continued)

Edge condition	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
SSSC	+45/−45/−45/+45	0.2	91.20	98.01	99.34	99.09	98.82	97.24	91.22
		0.3	91.43	98.20	99.67	99.45	99.19	97.79	91.97
		0.4	90.86	96.74	98.61	98.73	98.32	96.42	91.16
		0.5	92.12	97.92	99.76	100.00	99.82	97.90	92.02
		0.6	95.15	102.61	103.53	103.15	103.79	102.71	94.86
		0.7	97.05	106.02	105.68	104.62	105.93	105.96	96.91
		0.8	95.60	104.15	103.60	102.32	103.38	103.41	95.37
	+45/−45/+45/−45	0.2	90.77	97.71	99.57	99.72	99.58	97.7	90.77
		0.3	91.11	97.83	99.65	99.8	99.65	97.83	91.11
		0.4	90.67	96.5	98.5	98.79	98.5	96.51	90.67
		0.5	91.95	98	99.83	100	99.83	98	91.95
		0.6	94.82	102.81	103.94	103.44	103.94	102.81	94.81
		0.7	96.72	106.2	106.42	105.32	106.42	106.2	96.72
		0.8	95.38	104.11	104.24	103.18	104.22	104.09	95.38
SSCS	+45/−45/−45/+45	0.2	91.24	91.37	90.65	91.77	94.74	96.64	95.23
		0.3	98.15	98.23	96.66	97.78	102.40	105.68	103.75
		0.4	99.49	99.71	98.60	99.76	103.34	105.37	103.46
		0.5	99.23	99.49	98.75	100.00	103.02	104.48	102.37
		0.6	98.99	99.25	98.33	99.81	103.66	105.80	103.33
		0.7	97.47	97.85	96.30	97.70	102.48	105.66	103.06
		0.8	90.42	90.98	90.17	91.34	93.69	95.81	93.98
	+45/−45/+45/−45	0.2	90.97	91.14	90.5	91.61	94.35	96.23	94.99
		0.3	98.03	98.03	96.51	97.88	102.58	105.89	103.71
		0.4	95.29	99.78	98.56	99.83	103.76	106.17	104.04

		0.5	99.89	99.89	98.84	100	103.26	105.12	103.07
		0.6	99.81	99.79	98.56	99.83	103.76	106.17	104.04
		0.7	98.03	98.03	96.51	97.89	102.59	105.89	103.71
		0.8	89.91	90.07	89.62	91.29	93.56	95.74	94.07
SSSS	+45/−45/−45/+45	0.2	92.57	93.41	91.31	90.13	91.35	93.55	91.45
		0.3	92.98	100.65	99.17	97.29	98.57	100.41	93.17
		0.4	91.04	98.87	100.94	100.14	100.57	98.36	91.16
		0.5	89.97	97.07	100.00	100.00	99.99	97.07	89.97
		0.6	91.16	98.36	100.57	100.14	100.94	98.87	91.04
		0.7	93.17	100.41	98.57	97.29	99.18	100.65	92.98
		0.8	91.18	92.97	90.90	89.81	90.71	92.87	92.38
	+45/−45/+45/−45	0.2	90.91	91.7	89.71	88.65	89.71	91.7	90.91
		0.3	91.48	99.37	97.5	95.99	97.5	99.37	91.48
		0.4	89.64	97.39	100.37	99.5	100.37	97.39	89.64
		0.5	88.62	95.93	99.42	100	99.42	95.93	88.62
		0.6	89.64	97.39	100.37	99.49	100.37	97.39	89.64
		0.7	91.48	99.37	97.51	95.99	97.51	99.37	91.48
		0.8	90.69	91.02	89.15	88.3	89.06	91.21	90.3
CS	+45/−45/−45/+45	0.2	109.16	105.77	105.36	106.58	108.89	112.82	122.52
		0.3	106.11	102.65	101.49	101.63	102.44	105.50	114.59
		0.4	106.24	101.90	100.41	100.20	100.88	103.19	110.52
		0.5	107.80	102.24	100.34	100.00	100.27	102.17	107.73
		0.6	110.52	103.19	100.81	100.20	100.47	101.83	106.17
		0.7	114.72	105.50	102.44	101.63	101.56	102.71	106.17
		0.8	121.44	112.35	108.62	106.11	105.16	105.77	109.23

(continued)

Table 3.7 (continued)

Edge condition	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
	+45/−45/+45/−45	0.2	103.86	102.36	102.25	102.47	102.31	102.36	103.86
		0.3	102.63	101.72	101.13	100.91	101.13	101.66	102.58
		0.4	103.01	101.34	100.48	100.27	100.48	101.34	103.01
		0.5	103.86	101.72	100.32	100.00	100.32	101.66	103.81
		0.6	103.01	101.34	100.48	100.21	100.48	101.40	103.01
		0.7	102.58	101.66	101.18	100.91	101.18	101.72	102.58
		0.8	103.81	102.31	101.93	101.77	102.09	102.36	103.65

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $c/a = 0.2$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 3.8 Maximum values of r with corresponding coordinates of cutout centres and zones where $r \geq 95$ for cross-ply hypar shells

Laminations	0/90/90/0			0/90/0/90		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$
CCCC	100.11	(0.3, 0.5), (0.7, 0.5)	$0.2 \leq \bar{x} \leq 0.8$	100.03	(0.3, 0.5), (0.4, 0.5), (0.6, 0.5), (0.7, 0.5)	$0.2 \leq \bar{x} \leq 0.8$
			$0.2 \leq \bar{y} \leq 0.8$			$0.2 \leq \bar{y} \leq 0.8$
CSCC	100.34	(0.2, 0.3)	$0.2 \leq \bar{x} \leq 0.8$	100.35	(0.5, 0.3)	$0.2 \leq \bar{x} \leq 0.8$
			$0.2 \leq \bar{y} \leq 0.6$			$0.2 \leq \bar{y} \leq 0.8$
CCSC	100.51	(0.8, 0.5)	$0.2 \leq \bar{x} \leq 0.8$	100.38	(0.8, 0.5)	$0.2 \leq \bar{x} \leq 0.8$
			$0.2 \leq \bar{y} \leq 0.8$			$0.2 \leq \bar{y} \leq 0.8$
CCCS	100.32	(0.2, 0.6), (0.8, 0.6)	$0.2 \leq \bar{x} \leq 0.8$	100.35	(0.5, 0.7)	$0.2 \leq \bar{x} \leq 0.8$
			$0.4 \leq \bar{y} \leq 0.8$			$0.2 \leq \bar{y} \leq 0.8$
CSSC	101.02	(0.8, 0.3)	$0.2 \leq \bar{x} \leq 0.8$	100.95	(0.8, 0.3)	$0.2 \leq \bar{x} \leq 0.8$
			$0.2 \leq \bar{y} \leq 0.7$			$0.2 \leq \bar{y} \leq 0.8$
CCSS	100.87	(0.2, 0.6)	$0.2 \leq \bar{x} \leq 0.8$	100.91	(0.8, 0.6)	$0.2 \leq \bar{x} \leq 0.8$
			$0.3 \leq \bar{y} \leq 0.8$			$0.2 \leq \bar{y} \leq 0.8$
CSCS	101.46	(0.2, 0.5), (0.8, 0.5)	$0.2 \leq \bar{x} \leq 0.8$	100.00	(0.5, 0.5)	$0.2 \leq \bar{x} \leq 0.8$
			$0.4 \leq \bar{y} \leq 0.6$			$0.4 \leq \bar{y} \leq 0.6$
SCSC	100.00	(0.5, 0.5)	$0.3 \leq \bar{x} \leq 0.7$	100.00	(0.5, 0.5)	$0.4 \leq \bar{x} \leq 0.6$
			$0.4 \leq \bar{y} \leq 0.6$			$0.2 \leq \bar{y} \leq 0.8$
CSSS	104.54	(0.7, 0.5)	$0.2 \leq \bar{x} \leq 0.8$	103.90	(0.7, 0.5)	$0.2 \leq \bar{x} \leq 0.8$
			$0.4 \leq \bar{y} \leq 0.6$			$0.4 \leq \bar{y} \leq 0.6$
SSSC	103.41	(0.5, 0.3)	$0.3 \leq \bar{x} \leq 0.7$	103.95	(0.5, 0.3)	$0.4 \leq \bar{x} \leq 0.6$
			$0.2 \leq \bar{y} \leq 0.8$			$0.2 \leq \bar{y} \leq 0.8$

(continued)

Table 3.8 (continued)

Laminations	0/90/90/0			0/90/0/90		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$
SSCS	104.55	(0.3, 0.5), (0.3, 0.5),	$0.2 \leq \bar{x} \leq 0.8$	103.90	(0.3, 0.5)	$0.2 \leq \bar{x} \leq 0.8$
			$0.4 \leq \bar{y} \leq 0.6$			$0.4 \leq \bar{y} \leq 0.6$
SSSS	104.15		$0.2 \leq \bar{x} \leq 0.8$	100.00	(0.5, 0.5)	$0.4 \leq \bar{x} \leq 0.6$
		(0.7, 0.5)	$0.4 \leq \bar{y} \leq 0.6$			$0.4 \leq \bar{y} \leq 0.6$
cs	114.25	(0.2, 0.2), (0.5, 0.2)	$0.2 \leq \bar{x} \leq 0.8$	109.05	(0.2, 0.2)	$0.2 \leq \bar{x} \leq 0.8$
			$0.2 \leq \bar{y} \leq 0.8$			$0.2 \leq \bar{y} \leq 0.8$

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $c/a = 0.2$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 3.9 Maximum values of r with corresponding coordinates of cutout centres and zones where $r \geq 95$ for angle-ply hypar shells

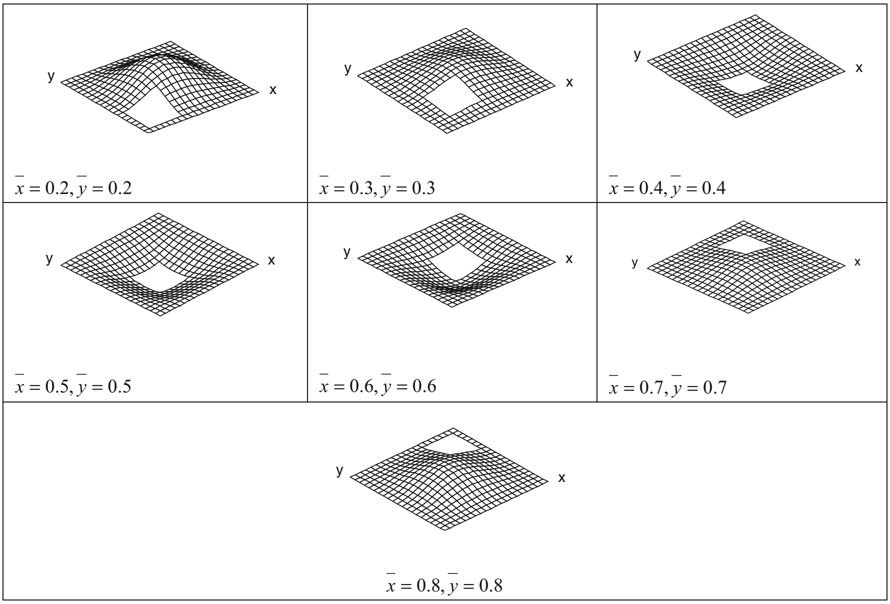
Laminations	+45/−45/−45/+45			+45/−45/+45/−45		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$
CCCC	100.00	(0.5, 0.5)	$0.4 \leq \bar{x} \leq 0.6$	100.00	(0.5, 0.5)	$0.4 \leq \bar{x} \leq 0.6$
			$0.4 \leq \bar{x} \leq 0.6$			$\bar{y} = 0.5$
CSCC	119.95	(0.6, 0.3)	$0.2 \leq \bar{x} \leq 0.8$	120.39	(0.4, 0.3)	$0.2 \leq \bar{x} \leq 0.8$
			$0.2 \leq \bar{y} \leq 0.6$			$0.2 \leq \bar{y} \leq 0.6$
CCSC	119.99	(0.7, 0.6)	$0.4 \leq \bar{x} \leq 0.8$	120.74	(0.7, 0.4)	$0.5 \leq \bar{x} \leq 0.8$
			$0.2 \leq \bar{y} \leq 0.8$			$0.2 \leq \bar{y} \leq 0.8$
CCCS	120.17	(0.4, 0.7)	$0.2 \leq \bar{x} \leq 0.8$	120.35	(0.4, 0.7), (0.6, 0.7)	$0.2 \leq \bar{x} \leq 0.8$
			$0.4 \leq \bar{y} \leq 0.8$			$0.4 \leq \bar{y} \leq 0.8$
CSSC	104.53	(0.4, 0.2)	$0.3 \leq \bar{x} \leq 0.8$	103.22	(0.4, 0.2)	$0.4 \leq \bar{x} \leq 0.8$
			$0.2 \leq \bar{y} \leq 0.8$			$0.2 \leq \bar{y} \leq 0.6$

(continued)

Table 3.9 (continued)

Laminations	+45/−45/−45/+45			+45/−45/+45/−45		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$
CCSS	101.42	(0.6, 0.6)	$0.4 \leq \bar{x} \leq 0.8$	103.35	(0.6, 0.7)	$0.2 \leq \bar{x} \leq 0.8$
			$0.4 \leq \bar{y} \leq 0.8$			$0.4 \leq \bar{y} \leq 0.8$
CSCS	104.47	(0.3, 0.3), (0.7, 0.7)	$0.2 \leq \bar{x} \leq 0.8$	104.23	(0.3, 0.6)	$0.2 \leq \bar{x} \leq 0.8$
			$0.2 \leq \bar{y} \leq 0.8$			$0.3 \leq \bar{y} \leq 0.7$
SCSC	104.69	(0.3, 0.3), (0.7, 0.7)	$0.2 \leq \bar{x} \leq 0.8$	105.58	(0.7, 0.8)	$0.3 \leq \bar{x} \leq 0.7$
			$0.2 \leq \bar{y} \leq 0.8$			$0.2 \leq \bar{y} \leq 0.8$
CSSS	105.80	(0.3, 0.4)	$0.2 \leq \bar{x} \leq 0.8$	106.17	(0.3, 0.4), (0.3, 0.6)	$0.2 \leq \bar{x} \leq 0.8$
			$0.3 \leq \bar{y} \leq 0.7$			$0.3 \leq \bar{y} \leq 0.7$
SSSC	106.02	(0.3, 0.7)	$0.3 \leq \bar{x} \leq 0.7$	106.42	(0.4, 0.7), (0.6, 0.7)	$0.3 \leq \bar{x} \leq 0.7$
			$0.2 \leq \bar{y} \leq 0.8$			$0.2 \leq \bar{y} \leq 0.8$
SSCS	105.80	(0.7, 0.6)	$0.2 \leq \bar{x} \leq 0.8$	106.17	(0.7, 0.4), (0.7, 0.6)	$0.2 \leq \bar{x} \leq 0.8$
			$0.3 \leq \bar{y} \leq 0.7$			$0.3 \leq \bar{y} \leq 0.7$
SSSS	100.94	(0.4, 0.4), (0.6, 0.6)	$0.3 \leq \bar{x} \leq 0.7$	100.37	(0.4, 0.4), (0.6, 0.4)	$0.3 \leq \bar{x} \leq 0.7$
			$0.3 \leq \bar{y} \leq 0.7$		(0.4, 0.6), (0.6, 0.6)	$0.3 \leq \bar{y} \leq 0.7$
CS	122.52	(0.8, 0.2)	$0.2 \leq \bar{x} \leq 0.8$	103.86	(0.2, 0.2), (0.8, 0.2)	$0.2 \leq \bar{x} \leq 0.8$
			$0.2 \leq \bar{y} \leq 0.8$			$0.2 \leq \bar{y} \leq 0.8$

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $c/a = 0.2$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$



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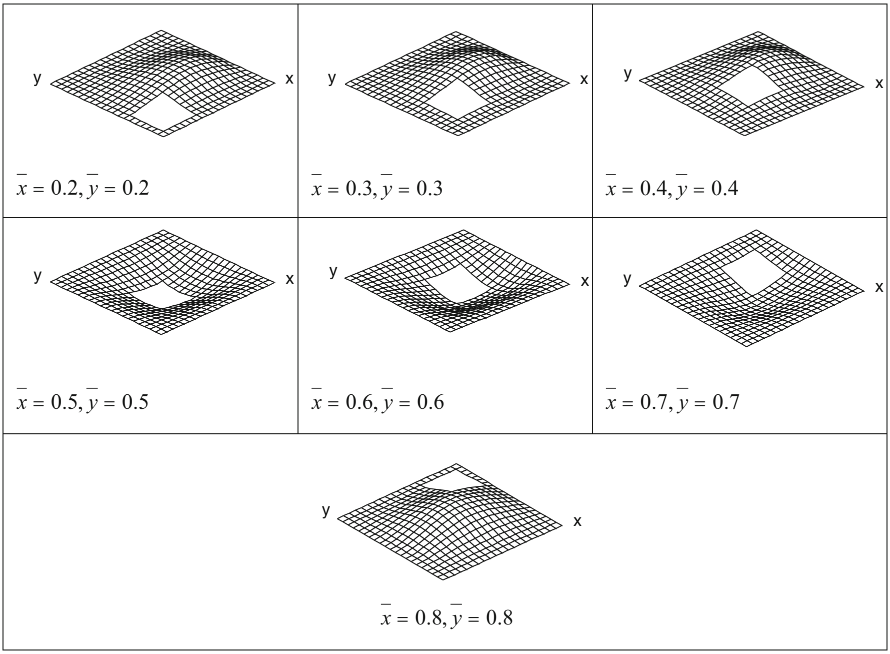
$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 3.6 First mode shapes of laminated composite (0/90/0/90) stiffened hypar shell for different positions of square cutout with CCCC boundary condition



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$\bar{x} = 0.4, \bar{y} = 0.4$

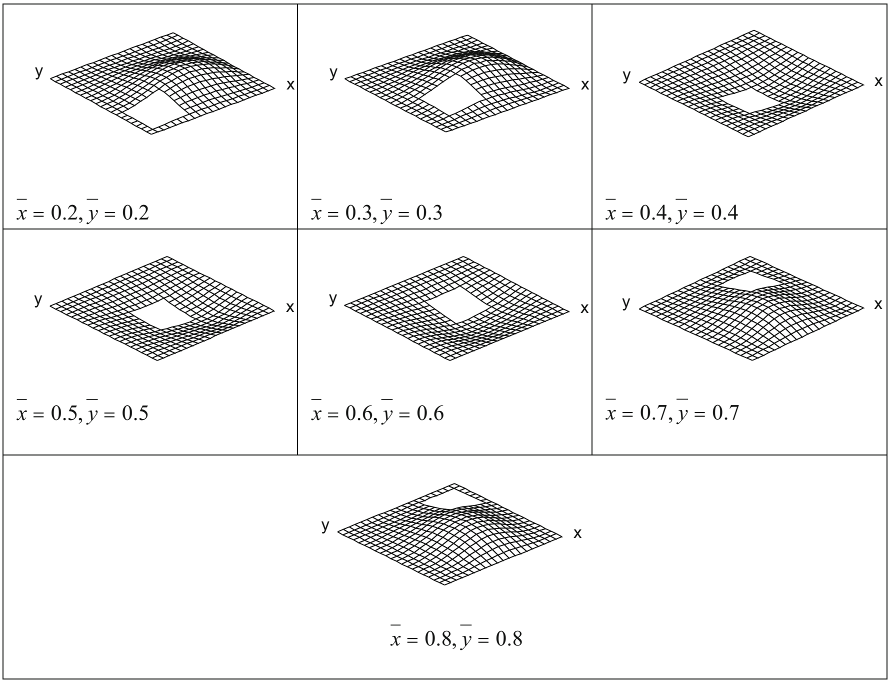
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 3.7 First mode shapes of laminated composite (0/90/0/90) stiffened hypar shell for different positions of square cutout with CCSC boundary condition



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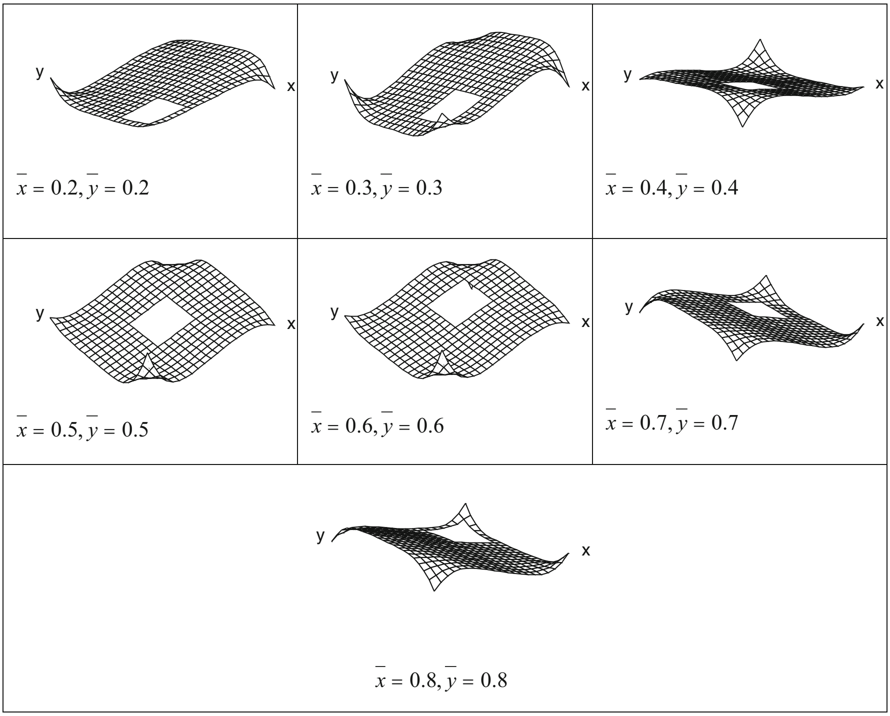
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 3.8 First mode shapes of laminated composite (0/90/0/90) stiffened hypar shell for different positions of square cutout with CSSC boundary condition



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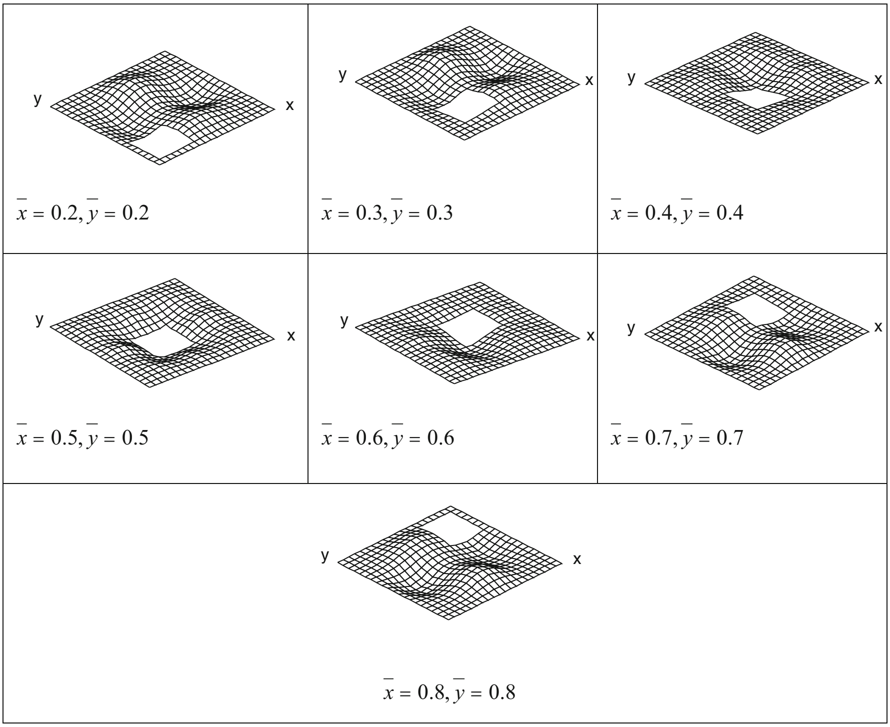
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 3.9 First mode shapes of laminated composite (0/90/0/90) stiffened hypar shell for different positions of square cutout with point supported boundary condition



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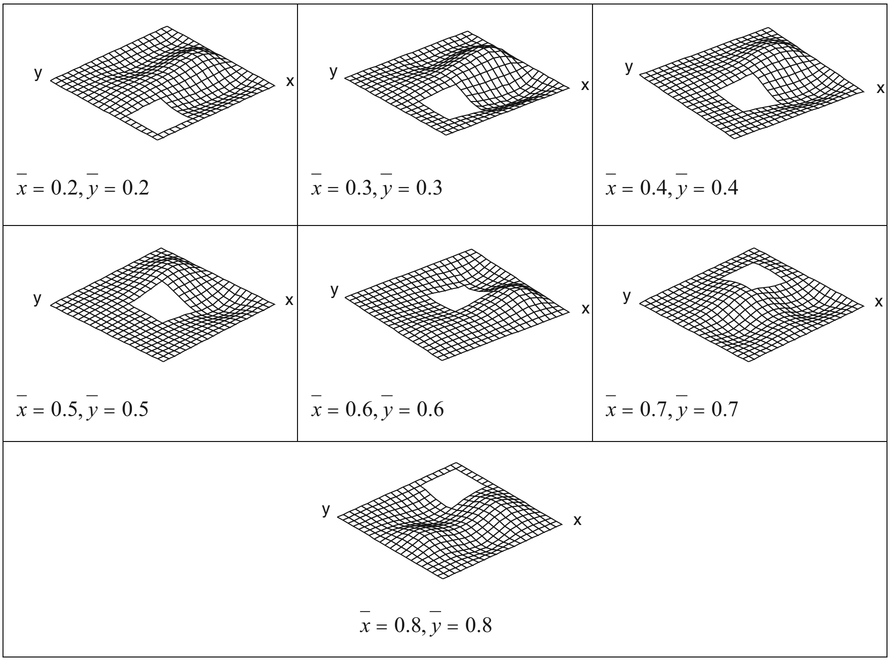
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 3.10 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened hypar shell for different positions of square cutout with CCCC boundary condition



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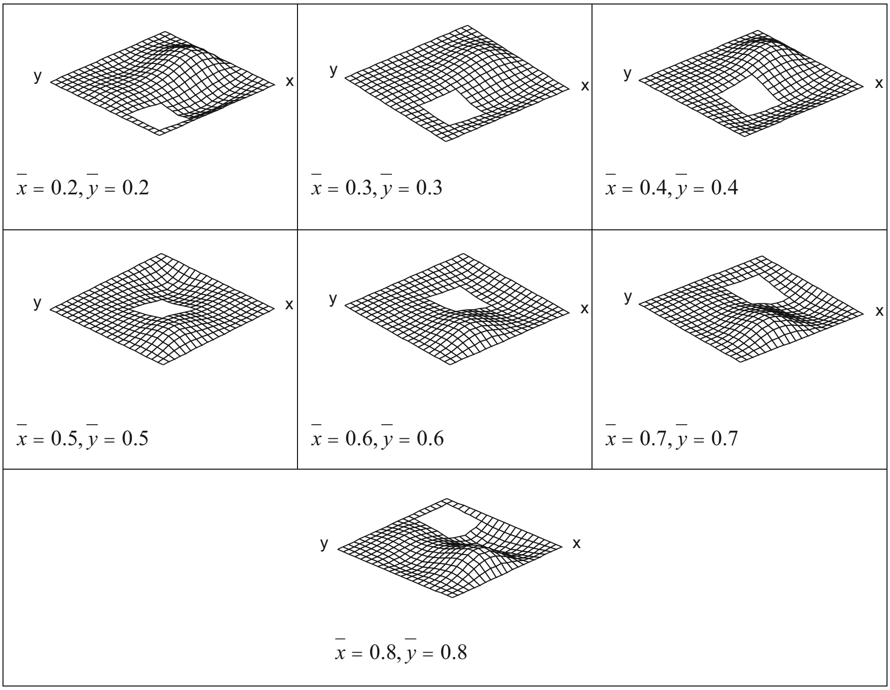
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 3.11 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened hypar shell for different positions of square cutout with CCSC boundary condition



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$\bar{x} = 0.4, \bar{y} = 0.4$

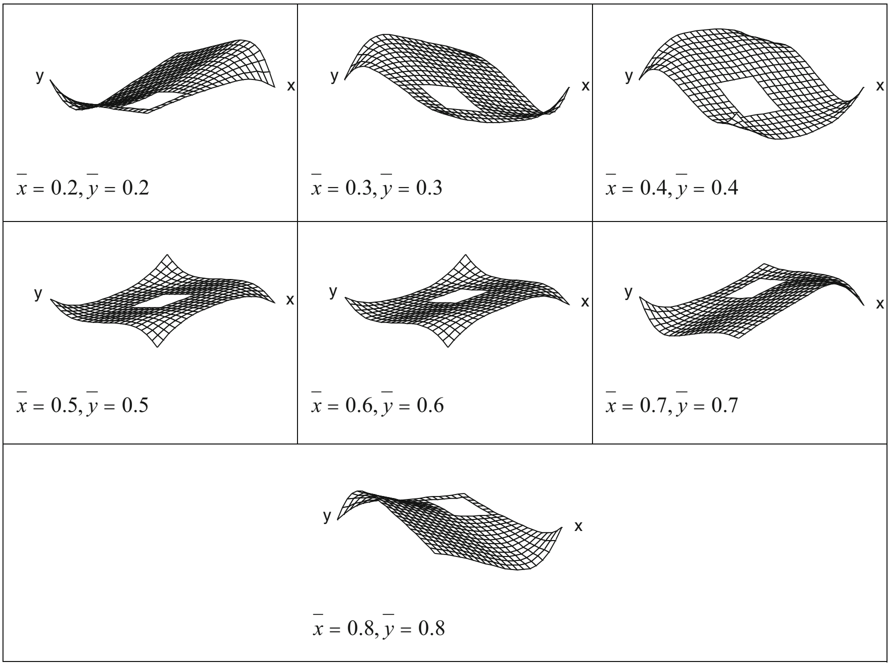
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 3.12 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened hypar shell for different positions of square cutout with CSSC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 3.13 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened hypar shell for different positions of square cutout with point supported boundary condition

References

Chakravorty D, Sinha PK, Bandyopadhyay JN (1998) Applications of FEM on free and forced vibration of laminated shells. ASCE J Eng Mech 124(1):1–8

Hota SS, Chakravorty D (2007) Free vibration of stiffened conoidal shell roofs with cutouts. J Vib Control 13(3):221–240

Hota SS, Padhi P (2007) Vibration of plates with arbitrary shapes of cutouts. J Sound Vib 302:1030–1036

Huang M, Sakiyama T (1999) Free vibration analysis of rectangular plates with variously shaped holes. J Sound Vib 226(4):769–786

Malhotra SK, Ganesan N, Veluswami MA (1989) Vibration of composite plates with cut-outs. J Aeronaut Soc India 41:61–64

Nanda N, Bandyopadhyay JN (2007) Nonlinear free vibration analysis of laminated composite cylindrical shells with cutouts. J Reinf Plast Compos 26(14):1413–1427

Reddy JN (1982) Large amplitude flexural vibration of layered composite plates with cutouts. J Sound Vib 83(1):1–10

Rossi RE (1999) Transverse vibrations of thin orthotropic rectangular plates with rectangular cutouts with fixed boundaries. J Sound Vib 221(4):733–736

Sahoo S (2011) Free vibration of laminated composite hypar shell roofs with cutouts. Adv Acoust Vib 2011:13p, Article ID 403684

Sahoo S (2012) Behaviour and optimization aids of composite stiffened hypar shell roofs with cutout under free vibration. ISRN Civil Eng 2012: 14p, Article ID 989785

Sahoo S, Chakravorty D (2005) Finite element vibration characteristics of composite hypar shallow shells with various edge supports. J Vib Control 11(10):1291–1309

Sahoo S, Chakravorty D (2006) Stiffened composite hypar shell roofs under free vibration: behaviour and optimization aids. J Sound Vib 295:362–377

Sivakumar K, Iyengar NGR, Deb K (1999) Free vibration of laminated composite plates with cutout. J Sound Vib 221(3):443–470

Sivasubramonian B, Kulkarni AM, Rao GV, Krishnan A (1997) Free vibration of curved panels with cutouts. J Sound Vib 200(2):227–234

Sivasubramonian B, Rao GV, Krishnan A (1999) Free vibration of longitudinally stiffened curved panels with cutout. J Sound Vib 226(1):41–55

Chapter 4
Stiffened Conoidal Shell with Cutout

Abstract Analysis of natural frequency and mode shape of a stiffened composite conoidal shell with cutout is done with the help of finite element method. Dynamic characteristics of stiffened composite conoidal shells with cutout are analysed for varying size of the cutouts and their positions with respect to the shell centre for different boundary conditions of cross-ply and angle-ply laminated composite conoids. The detailed effects of these parametric variations on the fundamental frequencies and mode shapes are considered. The results may be readily used by practising engineers for appropriate selection of stiffened composite conoids with the provision of cutouts either central or eccentric.

Keywords Conoidal shell • Cutout • Eccentricity • Fundamental frequency • Mode shapes

4.1 Introduction

Among the different shell panels commonly used as civil engineering roofing units, the conoidal shell provides a number of advantages. Conoidal shells can cover large column-free areas, allow entry of north light and provide ease of casting due to ruled surfaces. Thus, this shell is preferable particularly in medical, chemical and food processing industries where entry of north light is desirable. Moreover, these industries often require cutouts in the shell panel for the passage of light, service lines and also sometimes for alteration of resonant frequency. The margins of the cutouts are generally stiffened to take account of stress concentration effects. An in-depth study of vibration is required for proper use of these curved forms. A generalized finite element formulation for the doubly curved laminated composite shell (discussed in Chap. 1) has been adopted using eight-noded curved quadratic isoparametric finite elements including three radii of curvature. Some of the important contributions on the study of conoidal shells by different researchers are briefly reviewed here.

The research on conoidal shell was started by Hadid (1964) who analysed static characteristics of conoidal shells using the variational method. Then it was followed by researchers like Brebbia and Hadid (1971), Choi (1984), Ghosh and

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Springer Transactions in Civil and Environmental Engineering,
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Bandyopadhyay (1989, 1990), Dey et al. (1992) and Das and Bandyopadhyay (1993). Dey et al. (1992) contributed significantly on static analysis of conoidal shell. Chakravorty et al. (1995a) used the finite element analysis to consider the free vibration characteristics of shallow isotropic conoids and observed the effects of excluding some of the inertia terms from the mass matrix on the first four natural frequencies. Chakravorty et al. (1995b, 1996, 1998) also considered free and forced vibration characteristics of graphite-epoxy composite conoidal shells with regular boundary conditions. Later, Nayak and Bandyopadhyay (2002a, b, 2005, 2006) reported free vibration of stiffened isotropic and composite conoidal shells. Das and Chakravorty (2007, 2008) considered static and vibration characteristics of un-punctured and unstiffened composite conoids. Hota and Chakravorty (2007) studied isotropic punctured conoidal shells with complicated boundary conditions along the four edges. It is also seen from the recent reviews (Qatu et al. 2010, 2012) that dynamic characteristics of stiffened conoidal shells with cutout are limited in the literature. A recent study (Sahoo 2013) has focused on the free vibrations of graphite-epoxy laminated composite stiffened conoids with cutout in terms of the natural frequencies and mode shapes.

4.2 Problem

Problems of conoidal shells with cutout are considered for varying size and position of cutout along both of the plan directions of the shell for different practical edge conditions. The laminations considered are both symmetric and antisymmetric cross-ply and angle-ply. In order to compare the effect of cutout size on free vibration response, results of un-punctured conoidal shells are also included in the study. Figure 4.1 shows a conoidal shell having a concentric cutout stiffened along the margins.

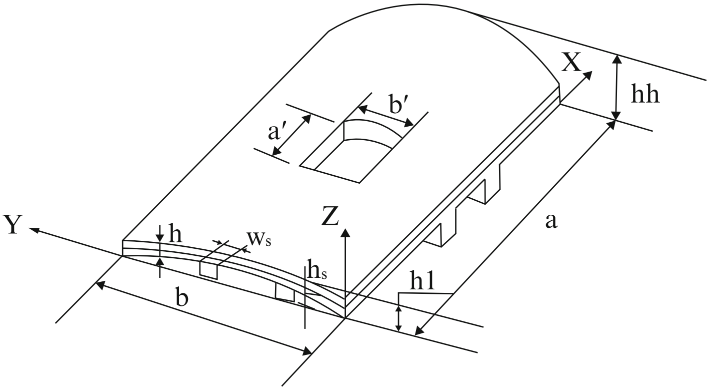


Fig. 4.1 Conoidal shell with a concentric cutout stiffened along the margins

4.3 Results and Discussion

4.3.1 Free Vibration Behaviour of Shells with Concentric Cutouts

Table 4.1 shows the nondimensional fundamental frequencies for clamped, simply supported and point supported conoidal shell panels for varying laminations. Here the effect of cutout size on fundamental frequencies of shells with different laminations and boundary conditions has been studied. Among them, only the four-layered symmetric and antisymmetric cross- and angle-ply laminations are chosen for further study. Tables 4.2 and 4.3 provide the nondimensional fundamental frequency $\overline{\omega}$ of cross-ply and angle-ply stiffened conoidal shell roofs for varying cutout sizes and different combinations of edge conditions.

4.3.1.1 Effect of Cutout Size on Fundamental Frequency

From Tables 4.1, 4.2 and 4.3, it is observed that if a cutout is provided in a stiffened shell, the fundamental frequencies increase. This increasing trend continues up to $a'/a = 0.3$ for both cross- and angle-ply shells. But further increase in cutout size (i.e. $a'/a = 0.3$–0.4) fundamental frequency may decrease in some cases. The initial increase in frequency may be due to the fact that when a cutout is made in an un-punctured surface the number of stiffeners increases from two to four. With further increase in cutout size, the number and dimensions of the stiffeners do not change, but the shell surface incurs loss of both mass and stiffness. As the cutout grows in size, the loss of mass becomes more significant than that of stiffness, and hence the frequency increases. But, in some cases, shells with further increase in the size of the cutout, the loss of stiffness gradually becomes more important than that of mass, resulting in decrease in fundamental frequency. This leads to the engineering conclusion that cutouts with stiffened margins may always safely be provided on shell surfaces for functional requirements.

4.3.1.2 Effect of Boundary Conditions on Fundamental Frequency

The boundary conditions are divided into six groups, considering number of boundary constraints. The combinations in a particular group have equal number of boundary reactions. These groups are:

Group I: CCCC shells
Group II: CSCC, CCSC and SCCC shells
Group III: CSSC, SSCC, CSCS and SCSC shells
Group IV: CSSS, SSSC and SSCS shells
Group V: SSSS shells
Group VI: Corner point supported shell

Table 4.1 Nondimensional fundamental frequencies ($\overline{\varpi}$) for laminated composite stiffened conoidal shell for different sizes of central square cutout, different laminations and boundary conditions

		Cutout size (a'/a)				
Boundary conditions	Laminations	0	0.1	0.2	0.3	0.4
Clamped	0/90	90.70	104.25	113.47	119.47	115.84
	90/0	90.79	104.29	113.84	120.47	116.95
	0/90/0	113.72	123.88	131.37	134.68	131.92
	90/0/90	99.97	116.16	119.77	122.84	118.64
	0/90/0/90	105.78	118.96	124.69	128.33	125.71
	90/0/90/0	105.91	119.10	125.01	128.85	126.27
	0/90/90/0	113.30	123.99	130.89	133.96	131.29
	90/0/0/90	102.84	118.45	122.12	125.46	121.88
	+45/−45	121.75	148.40	153.11	142.00	135.56
	+45/−45/+45	129.40	152.59	161.27	164.45	156.01
	+45/−45/+45/−45	137.54	163.04	168.73	168.14	103.10
	+45/−45/−45/+45	135.26	160.70	167.27	166.95	160.14
Simply supported	0/90	49.84	52.45	55.84	61.19	66.34
	90/0	49.82	52.22	56.12	61.78	67.19
	0/90/0	55.42	57.59	60.52	64.66	68.91
	90/0/90	53.51	56.43	60.71	65.82	70.44
	0/90/0/90	54.73	56.89	60.44	65.11	69.76
	90/0/90/0	54.77	56.99	60.67	65.48	70.27
	0/90/90/0	55.86	57.97	61.12	65.48	69.98
	90/0/0/90	54.31	56.89	60.93	65.84	70.43
	+45/−45	76.05	79.81	80.71	84.85	85.91
	+45/−45/+45	85.10	90.03	90.67	93.33	90.27
	+45/−45/+45/−45	89.90	92.69	94.39	95.48	92.78
	+45/−45/−45/+45	89.53	93.32	94.55	96.19	92.88
Point supported	0/90	16.87	17.41	18.48	20.13	21.61
	90/0	18.33	18.94	19.82	21.43	22.68
	0/90/0	19.70	20.42	21.56	23.17	24.36
	90/0/90	17.77	18.38	19.36	21.34	22.81
	0/90/0/90	21.18	21.69	22.62	24.23	25.54
	90/0/90/0	21.96	22.50	23.33	24.93	26.13
	0/90/90/0	20.99	21.64	22.68	24.53	25.93
	90/0/0/90	19.91	20.40	21.24	22.89	24.48
	+45/−45	19.53	19.94	20.60	21.86	23.48
	+45/−45/+45	20.87	22.09	22.75	23.78	25.11
	+45/−45/+45/−45	26.40	26.83	27.37	28.54	30.18
	+45/−45/−45/+45	24.06	25.05	25.58	26.73	28.23

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $a/hh = 5$, $hl/hh = 0.25$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 4.2 Nondimensional fundamental frequencies ($\overline{\omega}$) for laminated composite cross-ply stiffened conoidal shell for different sizes of central square cutout and different boundary conditions

Laminations	Boundary conditions	Cutout size (a'/a)				
		0	0.1	0.2	0.3	0.4
0/90/90/0	CCCC	113.30	124.00	130.89	133.96	131.29
	CSCC	79.90	88.03	91.44	97.34	98.22
	CCSC	112.39	123.22	128.26	129.14	125.69
	CCCS	79.25	85.62	90.84	96.19	98.09
	CSSC	78.12	85.34	88.21	92.81	92.21
	CCSS	77.51	83.15	87.68	91.82	92.09
	CSCS	69.51	74.79	78.46	84.71	90.17
	SCSC	108.09	115.82	121.46	124.17	121.70
	CSSS	65.67	69.40	72.16	76.74	80.40
	SSSC	69.08	73.13	77.32	82.05	84.32
	SSCS	62.59	65.53	69.27	74.27	79.65
	SSSS	55.86	57.97	61.12	65.48	69.98
	Point supported	20.99	21.61	22.68	24.53	25.93
0/90/0/90	CCCC	105.77	118.91	124.39	127.48	124.49
	CSCC	79.35	87.90	91.47	96.48	97.05
	CCSC	104.03	117.32	120.85	122.59	119.59
	CCCS	79.00	86.46	91.15	95.86	97.01
	CSSC	76.79	84.42	87.45	91.15	90.36
	CCSS	76.46	83.13	87.17	90.64	90.32
	CSCS	70.87	76.03	80.22	86.60	91.99
	SCSC	96.46	106.19	112.26	116.54	114.87
	CSSS	65.76	69.53	72.87	77.60	81.14
	SSSC	65.74	70.72	75.08	79.98	82.22
	SSCS	62.33	65.25	69.38	74.72	80.43
	SSSS	54.77	56.68	59.28	62.37	65.21
	Point supported	21.18	21.69	22.62	24.23	25.54

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $a/hh = 5$, $hl/hh = 0.25$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

It is observed from Tables 4.2 and 4.3 that fundamental frequencies of members belonging to same groups of boundary combinations may not have close values. So the different boundary conditions may be regrouped according to performance. According to the values of $\overline{\omega}$, the following groups may be identified.

For cross-ply shells:

Group I: Contains CCCC, CCSC and SCSC boundaries which exhibit relatively high frequencies

Group II: Contains CSCC, CCCS, CSSC, CCSS, CSCS, SSSC, CSSS and SSCS which exhibit intermediate values of frequencies

Table 4.3 Nondimensional fundamental frequencies ($\overline{\omega}$) for laminated composite angle-ply stiffened conoidal shell for different sizes of central square cutout and different boundary conditions

Laminations	Boundary conditions	Cutout size (a'/a)				
		0	0.1	0.2	0.3	0.4
+45/−45/−45/+45	CCCC	135.26	160.70	167.27	166.95	160.14
	CSCC	122.63	135.89	138.67	141.08	132.84
	CCSC	131.42	156.03	164.65	156.53	148.92
	CCCS	127.91	142.08	144.81	143.71	135.93
	CSSC	113.70	124.84	126.29	119.31	106.24
	CCSS	119.83	131.32	131.82	120.69	108.47
	CSCS	118.25	129.05	132.49	136.54	128.18
	SCSC	115.82	128.46	134.56	141.46	143.48
	CSSS	102.43	107.53	109.69	110.90	102.52
	SSSC	100.95	107.94	110.47	110.16	101.15
	SSCS	102.07	107.80	111.57	119.47	123.29
	SSSS	89.53	93.32	94.55	96.19	92.88
	Point supported	24.06	25.05	25.58	26.73	28.23
+45/−45/+45/−45	CCCC	137.54	149.75	156.93	154.61	147.03
	CSCC	127.64	139.98	142.62	142.21	133.74
	CCSC	133.18	157.49	165.99	157.62	149.89
	CCCS	123.91	136.05	139.27	140.96	134.92
	CSSC	119.31	129.28	129.53	119.81	107.44
	CCSS	114.18	124.24	126.96	119.41	107.34
	CSCS	119.19	129.26	133.12	136.96	128.42
	SCSC	117.63	129.25	135.32	141.11	143.74
	CSSS	102.75	106.96	109.58	110.37	103.19
	SSSC	105.21	111.03	113.48	110.79	101.83
	SSCS	103.07	107.68	111.86	118.83	123.54
	SSSS	89.82	91.66	94.39	89.28	83.30
	Point supported	26.40	26.83	27.37	28.54	30.18

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $a/hh = 5$, $hl/hh = 0.25$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Group III: Contains SSSS and corner point supported boundaries which exhibit relatively low values of frequencies

Similarly for angle-ply shells:

Group I: Contains CCCC, CCSC, CSCC, CCCS, CSSC, CCSS, SCSC and CSCS boundaries which exhibit relatively high frequencies

Group II: Contains SSSC, CSSS, SSCS and SSSS boundaries which exhibit intermediate values of frequencies

Group III: Contains corner point supported shells which exhibit relatively low values of frequencies

It is clearly seen from the present results that the free vibration characteristics largely depend on the arrangement of boundary constraints rather than their actual number. It can be seen from the present study that if the higher parabolic edge along $x = a$ is changed from clamped to simply supported, there is hardly any change of frequency for cross-ply shell. But for antisymmetric angle-ply shells, if the edge along the higher parabolic edge is released, fundamental frequency even increases than that of a clamped shell. For cross-ply shells, if the edge along $y = 0$ or $y = b$ is released, i.e. along the straight edges, frequency values undergo marked decrease. The results indicate that the straight edges should preferably be clamped in order to achieve higher frequency. But if these edges have to be released for functional reason, the parabolic edges of a conoid must be clamped to get better performance. But for angle-ply shells, if any two edges are released, the change in fundamental frequency is not so significant. This is true for both symmetric and antisymmetric laminations.

Tables 4.4 and 4.5 show the efficiency of a particular clamping option in improving the fundamental frequency of a shell with minimum number of boundary constraints up to a value that of a clamped shell. Marks are assigned to each boundary combination in a scale where the frequency of a corner point supported shell is assigned 0 and that of a fully clamped shell is assigned 100. These marks are furnished for cutouts with $a'/a = 0.2$. These tables will help a practising engineer to realize at a glance the efficiency of a particular boundary condition in improving the frequency of a shell, taking that of clamped shell as the upper limit.

4.3.1.3 Mode Shapes

The mode shapes corresponding to the fundamental modes of vibration are plotted in Figs. 4.2, 4.3, 4.4 and 4.5 for SSCS and SSSS boundary conditions for cross-ply and angle-ply shells, respectively. The normalized displacements are drawn with the shell mid-surface as the reference. The fundamental mode is clearly seen to bending mode for all the boundary conditions for cross-ply and angle-ply shells, except corner point supported shell. The fundamental mode shapes are complicated for corner point supported shells. When cutout is introduced, mode shapes remain almost similar. When the size of the cutout is increased from 0.2 to 0.4, the fundamental modes of vibration do not change to an appreciable amount.

4.3.2 Effect of Eccentricity of Cutout Position

4.3.2.1 Fundamental Frequency

The effect of eccentricity of cutout positions is studied for different locations of a cutout with $a'/a = 0.2$. The nondimensional coordinates of the cutout centre $\left(\bar{x} = \frac{x}{a}, \bar{y} = \frac{y}{a}\right)$ are varied from 0.2 to 0.8 along each direction in such a way that

Table 4.4 Clamping options for cross-ply conoidal shells with central cutouts having a'/a ratio 0.2

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of clamping
0/90/90/0	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Good improvement	36
	1	(a) Higher parabolic edge along $x = a$ (SSCS)	Marked improvement	43
		(b) Lower parabolic edge along $x = 0$ (CSSS)	Marked improvement	46
		(c) One straight edge along $y = b$ (SSSC)	Marked improvement	50
	2	(a) Two alternate edges including the higher and lower parabolic edge $x = 0$ and $x = a$ (CSCS)	Marked improvement	52
		(b) Two straight edges along $y = 0$ and $y = b$ (SCSC)	Remarkable improvement	91
		(c) Any two edges except the above option (CCSS, CSSC)	Marked improvement	60–61
	3	Three edges including the two parabolic edges (CCCS, CSCC)	Marked improvement	63–64
		Three edges excluding the higher parabolic edge along $x = a$ (CCSC)	Remarkable improvement	98
	4	All sides (CCCC)	Frequency attains highest value	100
0/90/0/90	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Good improvement	35
	1	(a) Higher parabolic edge along $x = a$ (SSCS)	Marked improvement	46
		(b) Lower parabolic edge along $x = 0$ (CSSS)	Marked improvement	49
		(c) One straight edge along $y = b$ (SSSC)	Marked improvement	52
	2	(a) Two alternate edges including the higher and lower parabolic edge $x = 0$ and $x = a$ (CSCS)	Marked improvement	57
		(b) Two straight edges along $y = 0$ and $y = b$ (SCSC)	Remarkable improvement	88
		(c) Any two edges except the above option (CSSC, CCSS)	Marked improvement	63

(continued)

Table 4.4 (continued)

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of clamping
	3	Three edges including the two parabolic edges (CSCC, CCCS)	Marked improvement	45
		Three edges excluding the higher parabolic edge along $x = a$ (CCSC)	Remarkable improvement	96
	4	All sides (CCCC)	Frequency attains highest value	100

the distance of a cutout margin from the shell boundary is not less than one tenth of the plan dimension of the shell. The margins of cutouts are stiffened with four stiffeners. The study is carried out for all the 13 boundary conditions for both cross-ply and angle-ply shells. The fundamental frequency of a shell with an eccentric cutout is expressed as a percentage of fundamental frequency of a shell with a concentric cutout. This percentage is denoted by r. In Tables 4.6 and 4.7, such results are furnished.

It is noted that eccentricity of the cutout along the length of the shell towards the parabolic edges makes it more flexible. It is also observed that eccentricity towards the lower parabolic edge r value is greater than that of the higher parabolic edge. This means that if a designer needs to provide an eccentric cutout along the length, it should preferably be placed towards the lower height boundary. This is true for cross-ply shells. But reverse is the case for angle-ply shells. For angle-ply shells, when cutout shifts towards the lower parabolic edge, the shell becomes more flexible. Again for corner point supported shells, r values increase towards the parabolic edges and become maximum along the lower parabolic edge. Most of the cross-ply shells yield the maximum value of r along $\overline{x} = 0.5$, some yields maximum values of r along $\overline{x} = 0.4$, and remaining few cases show maximum values within $\overline{x} = 0.4$–0.5.

It may be noted from Table 4.6 that if the eccentricity of a cutout is varied along the width, the shell becomes stiffer when the cutout shifts towards simply supported edges. So, for functional purposes, if a shift of central cutout is desired, eccentricity of a cutout along the width should preferably be placed towards the simply supported straight edge. For shells having two straight edges of identical boundary condition, the maximum fundamental frequency occurs along $\overline{y} = 0.5$. For corner point supported shells, the maximum fundamental frequency always occurs along the boundary of the shell. All these are true for cross-ply shells only. For an angle-ply shell, the trend is not uniform; the boundary conditions and the fundamental frequency behave in a complex manner as clearly observed from Table 4.7. But for

Table 4.5 Clamping options for angle-ply conoidal shells with central cutouts having a'/a ratio 0.2

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of clamping
+45/−45/−45/+45	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Marked improvement	49
	1	(a) Higher parabolic edge along $x = a$ (SSCS)	Marked improvement	60
		(b) Lower parabolic edge along $x = 0$ (CSSS)	Marked improvement	59
		(c) One straight edge along $y = b$ (SSSC)	Marked improvement	59
	2	(a) Two alternate edges including the higher and lower parabolic edge $x = 0$ and $x = a$ (CSCS)	Remarkable improvement	75
		(b) Two straight edges along $y = 0$ and $y = b$(SCSC)	Remarkable improvement	77
		(c) Any two edges except the above option (CSSC, CCSS)	Remarkable improvement	71–75
	3	Three edges including the two parabolic edges (CSCC, CCCS)	Remarkable improvement	80–84
		Three edges excluding the higher parabolic edge along $x = a$ (CCSC)	Frequency attains more than a fully clamped shell	98
	4	All sides (CCCC)	Remarkable improvement	100
+45/−45/+45/−45	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Marked improvement	52
	1	(a) Higher parabolic edge along $x = a$ (SSCS)	Marked improvement	65
		(b) Lower parabolic edge along $x = 0$ (CSSS)	Marked improvement	64
		(c) One straight edge along $y = b$ (SSSC)	Marked improvement	66
	2	(a) Two alternate edges including the higher and lower parabolic edge $x = 0$ and $x = a$ (CSCS)	Remarkable improvement	81
		(b) Two straight edges along $y = 0$ and $y = b$(SCSC)	Remarkable improvement	83
		(c) Any two edges except the above option (CSSC, CCSS)	Remarkable improvement	77–79

(continued)

Table 4.5 (continued)

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of clamping
	3	Three edges including the two parabolic edges (CSCC, CCCS)	Remarkable improvement	86–89
		Three edges excluding the higher parabolic edge along $x = a$ (CCSC)	Frequency attains more than a fully clamped shell	107
	4	All sides (CCCC)	Remarkable improvement	100

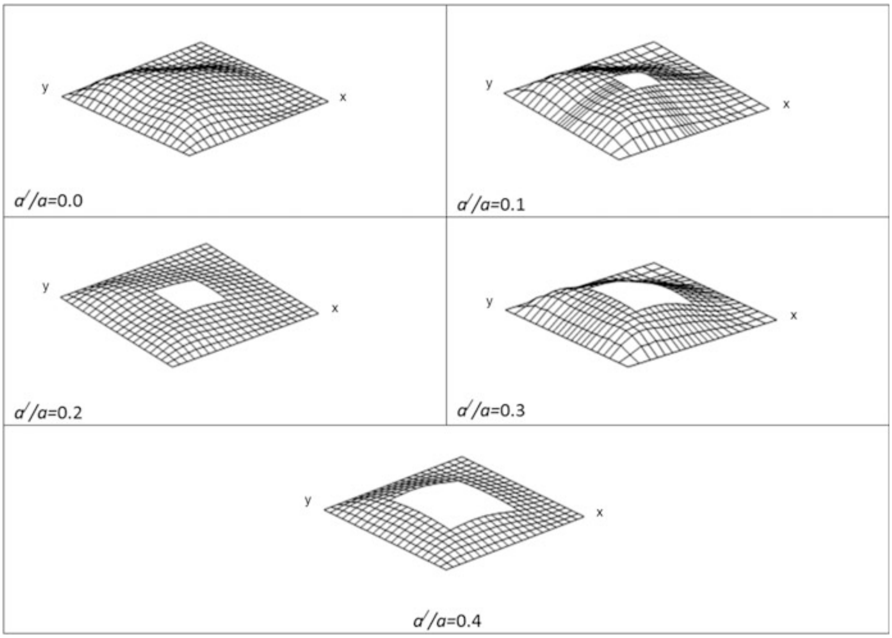


Fig. 4.2 First mode shapes of laminated composite (0/90/0/90) stiffened conoidal shell for different sizes of central square cutout and SSCS boundary condition

corner point supported angle-ply shells, the maximum values of r are along the boundary.

Table 4.8 and 4.9 show the maximum values of r together with the position of the cutout. These tables also identify the rectangular zones within which r is always greater than or equal to 90. It is to be noted that at some other points r values may have similar values, but only the zone rectangular in plan has been identified. These tables indicate the maximum eccentricity of a cutout which can be permitted if the

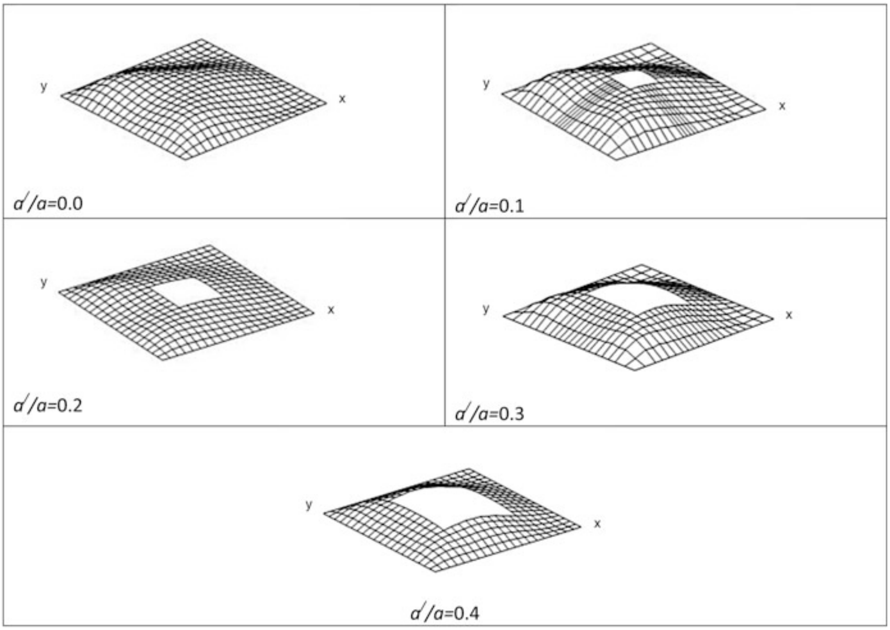


Fig. 4.3 First mode shapes of laminated composite (0/90/0/90) stiffened conoidal shell for different sizes of central square cutout and SSSS boundary condition

fundamental frequency of a concentrically punctured shell does not reduce by a drastic amount. So these tables will be helpful to practising engineers dealing with such shell forms.

4.3.2.2 Mode Shape

The mode shapes corresponding to the fundamental modes of vibration are plotted in Figs. 4.6, 4.7, 4.8, 4.9, 4.10, 4.11, 4.12 and 4.13 for cross-ply and angle-ply shells of CCCC, CCSC, SCSC and SSSC boundary conditions for different eccentric positions of the cutout. All the mode shapes are bending modes. It is found that for different positions of cutout, mode shapes are somewhat similar and only the crest and trough positions change.

4.4 Closure

The present analysis on conoidal shell shows that cutouts with stiffened margins may always safely be introduced on shell surfaces for functional purposes. For dynamic characteristics, the arrangement of boundary constraints along the four

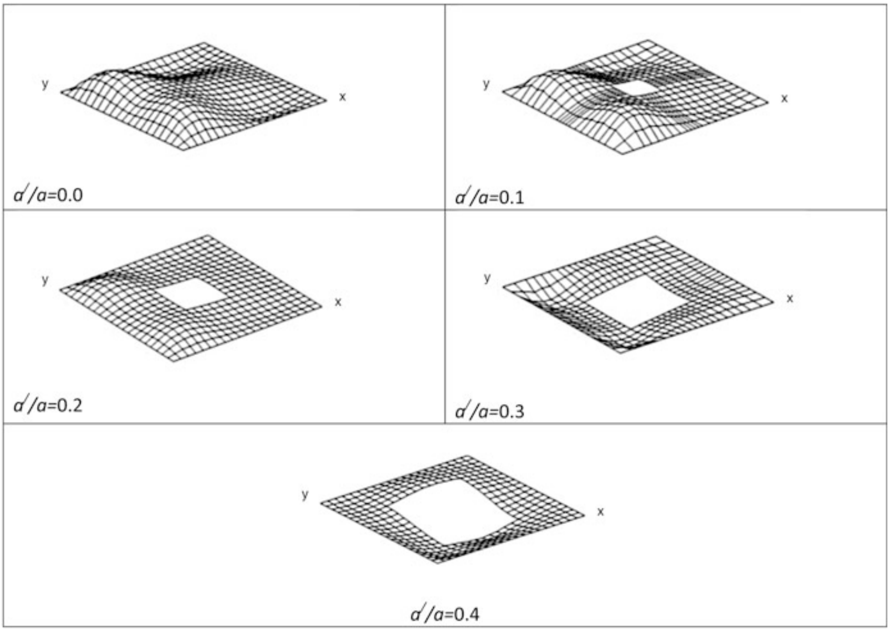


Fig. 4.4 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened conoidal shell for different sizes of central square cutout and SSCS boundary condition

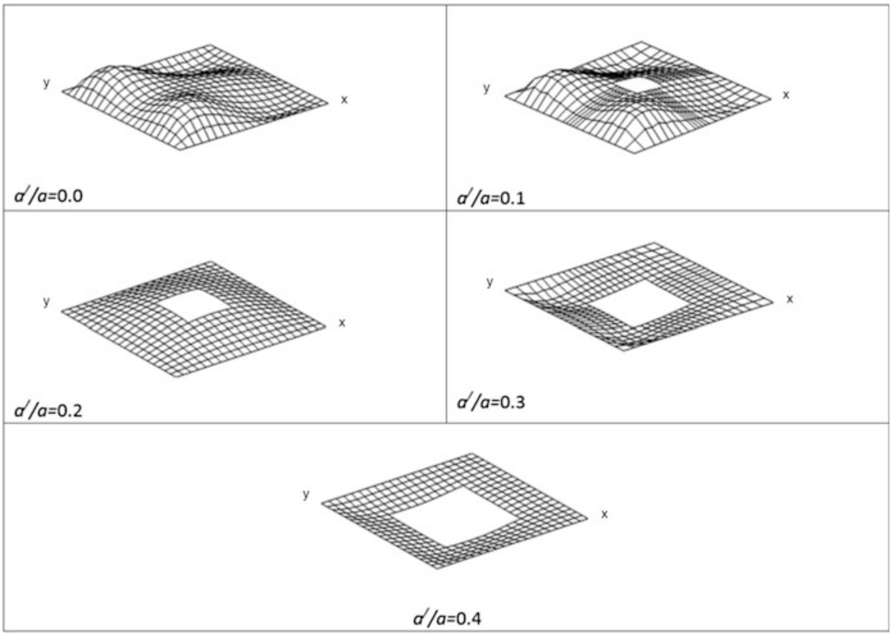


Fig. 4.5 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened conoidal shell for different sizes of central square cutout and SSSS boundary condition

Table 4.6 Values of ‘*r*’ for cross-ply conoidal shells

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
CCCC	0/90/90/0	0.2	87.09	93.28	100.52	101.49	94.45	88.51	84.35
		0.3	86.87	92.97	100.53	102.37	94.77	88.52	84.20
		0.4	86.26	92.02	99.14	100.98	93.81	87.81	83.64
		0.5	86.05	91.66	98.45	100.00	93.28	87.50	83.47
		0.6	86.26	92.02	99.14	100.98	93.81	87.81	83.64
		0.7	86.87	92.97	100.53	102.37	94.77	88.52	84.20
		0.8	87.03	93.17	100.41	101.44	94.42	88.47	84.32
	0/90/0/90	0.2	82.95	90.62	99.50	101.71	93.09	85.11	79.44
		0.3	82.56	90.02	99.16	103.03	93.74	85.18	79.19
		0.4	81.95	89.12	97.77	101.28	92.55	84.27	78.41
		0.5	81.96	88.98	97.22	100.02	91.85	83.96	78.25
		0.6	81.95	89.12	97.77	101.28	92.56	84.27	78.41
		0.7	82.56	90.02	99.16	103.04	93.74	85.18	79.19
		0.8	82.89	90.51	99.41	101.69	93.07	85.09	79.42
CSCC	0/90/90/0	0.2	97.15	103.41	107.02	102.84	95.75	90.32	87.28
		0.3	100.58	107.28	111.14	108.08	101.70	96.07	92.30
		0.4	99.04	105.63	109.68	107.13	101.31	96.04	92.22
		0.5	95.81	101.32	103.60	100.00	94.96	91.12	88.45
		0.6	93.08	97.12	97.74	93.65	89.38	86.60	84.80
		0.7	91.40	94.25	94.18	90.42	86.81	84.53	83.05
		0.8	90.78	93.06	92.81	89.45	86.24	84.15	82.76
	0/90/0/90	0.2	93.86	100.89	106.42	103.74	95.96	89.33	85.17
		0.3	96.37	104.67	110.51	108.66	101.82	94.92	89.73
		0.4	93.86	102.09	108.28	106.92	100.35	93.65	88.51
		0.5	90.59	97.87	102.33	100	93.88	88.42	84.45

		0.6	88.15	94.26	97.22	94.59	89.29	84.75	81.46
		0.7	87.11	92.08	94.30	92.06	87.58	83.56	80.48
		0.8	87.07	91.38	93.32	91.46	87.38	83.53	80.52
CCSC	0/90/90/0	0.2	85.69	91.37	98.42	101.65	95.63	89.61	85.16
		0.3	85.33	90.82	97.92	102.26	96.05	89.77	85.19
		0.4	84.63	89.75	96.42	100.90	95.23	89.21	84.81
		0.5	84.41	89.34	95.77	100.00	94.72	88.94	84.66
		0.6	84.63	89.75	96.42	100.90	95.23	89.21	84.81
		0.7	85.33	90.82	97.92	102.26	96.05	89.77	85.19
		0.8	85.62	91.24	98.28	101.58	95.59	89.58	85.15
	0/90/0/90	0.2	80.92	87.87	96.51	101.95	95.02	87.04	81.09
		0.3	80.43	87.12	95.77	102.73	95.73	87.27	81.11
		0.4	79.62	85.98	94.27	101.07	94.67	86.60	80.68
		0.5	79.45	85.71	93.79	100	93.99	86.37	80.63
		0.6	79.61	85.98	94.27	101.07	94.67	86.60	80.68
		0.7	80.43	87.12	95.78	102.73	95.73	87.27	81.11
		0.8	80.84	87.74	96.39	101.91	95.01	87.02	81.08
CCCS	0/90/90/0	0.2	90.79	93.53	93.01	89.37	86.12	84.08	82.72
		0.3	91.47	94.73	94.37	90.33	86.67	84.42	82.98
		0.4	93.32	97.66	97.96	93.56	89.26	86.53	84.76
		0.5	96.21	101.93	103.92	100.00	94.94	91.15	88.53
		0.6	99.52	106.29	110.18	107.42	101.55	96.28	92.49
		0.7	101.06	107.95	111.76	108.60	102.14	96.49	92.70
		0.8	97.21	103.79	107.54	103.37	96.22	90.61	87.49
	0/90/0/90	0.2	87.18	91.75	93.55	91.68	87.29	83.46	80.48
		0.3	87.18	92.36	94.48	92.06	87.49	83.46	80.42
		0.4	88.31	94.56	97.43	94.59	89.19	84.66	81.41

(continued)

Table 4.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
		0.5	90.82	98.20	102.57	100	93.77	88.34	84.43
		0.6	94.11	102.44	108.56	106.99	100.35	93.67	88.57
		0.7	96.61	105.03	110.82	108.87	101.97	95.06	89.88
		0.8	93.76	101.04	106.55	103.96	96.17	89.38	85.12
CSSC	0/90/90/0	0.2	96.12	102.60	107.60	104.78	97.85	91.94	87.78
		0.3	97.27	103.87	109.43	108.42	102.78	97.09	92.64
		0.4	95.68	101.84	106.90	106.25	101.61	96.72	92.63
		0.5	94.23	99.64	102.56	100.00	95.67	92.08	89.20
		0.6	93.21	97.46	98.39	94.68	90.67	87.97	86.01
		0.7	92.45	95.62	95.74	92.10	88.54	86.27	84.71
		0.8	92.11	94.76	94.68	91.37	88.20	86.10	84.65
	0/90/0/90	0.2	91.56	99.05	106.09	105.72	98.53	91.43	86.00
		0.3	91.47	99.49	107.46	108.70	103.15	96.32	90.41
		0.4	89.31	96.96	104.29	105.67	101.03	94.90	89.41
		0.5	88.00	95.09	100.52	100	95.16	90.04	85.74
		0.6	87.24	93.63	97.47	95.88	91.22	86.82	83.29
		0.7	87.03	92.62	95.57	94.03	89.94	85.98	82.70
		0.8	87.17	92.30	94.86	93.62	89.89	86.07	82.82
CCSS	0/90/90/0	0.2	92.14	95.21	94.84	91.26	88.02	85.98	84.55
		0.3	92.53	96.07	95.89	91.96	88.36	86.11	84.59
		0.4	93.45	97.96	98.59	94.56	90.49	87.83	85.93
		0.5	94.62	100.19	102.87	100.00	95.62	92.06	89.22
		0.6	96.12	102.42	107.38	106.54	101.82	96.93	92.86
		0.7	97.72	104.46	110.01	108.90	103.17	97.46	93.00
		0.8	95.96	102.69	108.05	105.25	98.31	92.19	87.94

	0/90/0/90	0.2	87.34	92.67	95.10	93.61	89.77	85.97	82.75
		0.3	87.15	92.88	95.73	94.02	89.81	85.85	82.61
		0.4	87.41	93.90	97.66	95.86	91.08	86.69	83.20
		0.5	88.22	95.39	100.74	100	95.03	89.93	85.68
		0.6	89.54	97.26	104.56	105.76	101.01	94.90	89.44
		0.7	91.70	99.80	107.75	108.90	103.29	96.44	90.53
		0.8	91.03	98.86	106.13	105.86	98.73	91.45	85.90
CSCS	0/90/90/0	0.2	88.90	91.80	92.00	88.73	85.87	84.64	84.41
		0.3	92.90	95.72	95.62	92.24	89.43	88.29	87.89
		0.4	96.78	99.62	99.76	96.90	94.29	92.96	92.12
		0.5	99.13	101.71	102.08	100.00	97.96	96.74	95.73
		0.6	96.78	99.62	99.76	96.90	94.29	92.96	92.12
		0.7	92.90	95.70	95.62	92.24	89.43	88.29	87.89
		0.8	88.85	91.73	91.87	88.68	85.81	84.62	84.41
	0/90/0/90	0.2	87.18	90.17	91.94	90.28	87.53	86.03	85.51
		0.3	90.24	93.67	95.13	93.17	90.30	88.54	87.30
		0.4	93.27	97.39	98.93	97.17	94.30	92.11	89.89
		0.5	95.71	99.84	101.26	100	97.77	96.42	93.01
		0.6	93.27	97.39	98.93	97.15	94.29	92.10	89.88
		0.7	90.23	93.67	95.13	96.91	90.29	88.54	87.30
		0.8	87.05	90.06	91.80	90.24	87.50	86.02	85.51
SCSC	0/90/90/0	0.2	89.76	95.79	102.31	100.92	94.06	88.79	85.07
		0.3	89.35	95.25	102.13	101.64	94.40	88.93	85.09
		0.4	88.27	93.88	100.53	100.72	93.82	88.56	84.90
		0.5	87.68	93.19	99.63	100.00	93.45	88.36	84.80
		0.6	88.27	93.88	100.53	100.72	93.82	88.56	84.90
		0.7	89.35	95.25	102.13	101.64	94.40	78.54	85.09
		0.8	89.69	95.67	102.18	100.87	94.02	88.76	85.06

(continued)

Table 4.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
	0/90/0/90	0.2	86.11	93.71	101.98	100.98	91.91	84.86	79.92
		0.3	85.57	92.99	101.67	102.01	92.32	84.98	79.91
		0.4	84.34	91.53	100.01	100.90	91.66	84.57	79.68
		0.5	83.78	90.95	99.22	100	91.32	84.47	79.66
		0.6	84.34	91.53	100.01	100.90	91.66	84.57	79.68
		0.7	85.57	92.99	101.67	102.01	92.33	84.98	79.91
		0.8	86.04	93.59	101.88	100.97	91.89	84.85	79.91
CSSS	0/90/90/0	0.2	92.90	95.93	96.41	93.46	90.63	89.26	88.54
		0.3	96.12	98.84	98.91	95.97	93.50	92.57	92.17
		0.4	98.16	100.60	100.75	98.57	96.87	96.37	95.95
		0.5	98.79	101.09	101.44	100.00	99.03	98.95	98.52
		0.6	98.16	100.60	100.75	98.57	96.87	96.37	95.95
		0.7	96.12	98.84	98.92	95.97	93.49	92.57	92.17
		0.8	92.84	95.84	96.27	93.36	90.58	89.22	88.53
	0/90/0/90	0.2	90.96	94.02	96.24	95.32	93.01	91.54	90.57
		0.3	93.33	96.65	98.46	97.32	95.16	93.91	92.93
		0.4	94.99	98.15	99.74	99.09	97.66	96.79	95.58
		0.5	96.16	98.43	100.04	100	99.38	99.25	98.90
		0.6	94.98	98.16	99.74	99.09	97.66	96.78	95.57
		0.7	93.33	96.65	98.46	97.32	95.16	93.91	92.93
		0.8	90.83	93.90	96.03	95.23	92.97	91.51	90.58
SSSC	0/90/90/0	0.2	99.19	108.21	111.59	105.56	97.85	92.43	89.02
		0.3	102.92	110.57	113.37	108.61	101.58	96.04	92.18
		0.4	103.49	109.66	110.88	106.03	99.86	94.99	91.41

		0.5	102.20	107.11	105.73	100.00	94.93	91.39	88.74
		0.6	100.81	104.51	101.49	95.47	91.18	88.61	86.78
		0.7	99.90	102.72	99.38	93.70	89.86	87.73	86.34
		0.8	99.52	101.98	98.65	93.31	89.72	87.74	86.51
	0/90/0/90	0.2	97.80	107.46	112.24	106.67	97.84	91.03	86.34
		0.3	100.32	109.21	113.74	109.52	101.48	94.42	89.13
		0.4	99.54	107.57	110.64	105.97	98.63	92.34	87.47
		0.5	97.80	105.13	105.98	100	93.28	88.23	84.43
		0.6	96.81	103.14	102.65	96.37	90.21	85.95	82.90
		0.7	96.66	101.97	101.06	95.20	89.52	85.60	82.86
		0.8	96.78	101.61	100.55	94.96	89.46	85.65	82.98
SSCS	0/90/90/0	0.2	94.79	97.63	95.24	90.63	87.77	86.89	86.96
		0.3	98.31	100.72	98.25	93.95	91.45	90.70	90.52
		0.4	101.15	103.48	101.52	97.79	95.39	94.28	93.47
		0.5	102.37	104.68	103.19	100.00	97.63	96.16	94.82
		0.6	101.15	103.48	101.52	97.79	95.39	94.28	93.47
		0.7	98.33	100.72	98.25	93.95	91.47	90.70	90.52
		0.8	94.72	97.55	95.12	90.57	87.71	86.86	86.95
	0/90/0/90	0.2	93.16	96.47	95.57	91.66	88.79	87.81	87.73
		0.3	96.52	99.49	98.33	94.51	91.80	90.73	90.03
		0.4	99.29	102.15	101.31	97.90	95.24	93.80	92.36
		0.5	100.56	103.32	102.91	100	97.63	96.28	94.95
		0.6	99.29	102.16	101.31	97.90	95.24	93.80	92.36
		0.7	96.51	99.48	98.33	94.51	91.80	90.73	90.02
		0.8	93.06	96.36	95.44	91.63	88.75	87. 80	87.73
SSSS	0/90/90/0	0.2	95.57	100.72	99.95	95.99	93.23	92.21	91.74
		0.3	99.69	103.45	101.80	97.79	95.35	94.65	94.29
		0.4	102.55	105.12	103.04	99.28	97.12	96.48	95.91

(continued)

Table 4.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
		0.5	103.68	105.68	103.52	100.00	98.00	97.33	96.60
		0.6	102.55	105.12	103.04	99.28	97.12	96.47	95.91
		0.7	99.69	103.45	101.80	97.79	95.35	94.65	94.29
		0.8	95.44	100.61	99.85	95.91	93.19	92.18	91.70
	0/90/0/90	0.2	91.51	96.36	98.30	97.49	95.59	93.69	91.98
		0.3	95.25	98.79	99.93	98.90	97.11	95.56	94.41
		0.4	97.75	100.11	100.76	99.69	98.06	96.68	95.61
		0.5	98.75	100.50	101.01	100	98.51	97.23	96.15
		0.6	97.74	100.11	100.76	99.69	98.06	96.68	95.61
		0.7	95.25	98.79	99.92	98.90	97.11	95.56	94.41
		0.8	91.45	96.29	98.15	97.41	95.57	93.67	91.95
CS	0/90/90/0	0.2	136.33	130.07	118.12	108.07	103.88	104.28	111.46
		0.3	132.23	126.01	113.32	102.82	98.24	98.68	106.97
		0.4	128.48	124.16	111.64	100.62	95.94	96.69	105.16
		0.5	126.76	123.41	111.20	100.00	95.33	96.25	104.81
		0.6	128.44	124.16	111.60	100.57	95.94	96.74	105.16
		0.7	132.23	125.97	113.32	102.78	98.24	98.63	106.92
		0.8	132.89	126.76	115.52	105.38	101.32	101.81	109.52
	0/90/0/90	0.2	137.89	128.05	114.30	104.65	100.79	101.36	109.08
		0.3	135.21	125.57	111.23	101.09	97.01	97.83	106.31
		0.4	132.32	124.34	110.52	100.19	95.99	97.05	105.59
		0.5	131.08	123.72	110.37	100	95.81	96.91	105.50
		0.6	132.29	124.36	110.55	100.21	96.02	97.09	105.60
		0.7	135.23	125.58	111.22	101.09	96.98	97.82	106.30
		0.8	135.48	125.75	112.44	102.53	98.80	99.58	107.92

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $a/hh = 5$, $hl/hh = 0.25$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 4.7 Values of ‘r’ for angle-ply conoidal shells

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
CCCC	+45/−45/−45/+45	0.2	73.92	82.11	89.22	85.66	81.28	76.76	71.67
		0.3	75.85	83.67	91.98	90.65	86.03	81.07	75.61
		0.4	78.04	86.07	95.35	96.81	90.72	84.14	77.77
		0.5	76.16	85.44	96.19	100.00	92.10	83.98	76.98
		0.6	74.48	83.54	94.07	92.91	85.68	78.61	72.71
		0.7	73.77	82.45	90.23	85.39	78.96	73.20	68.37
		0.8	72.74	81.19	87.27	81.62	76.12	71.12	66.51
	+45/−45/+45/−45	0.2	72.82	78.24	82.76	83.83	81.57	76.80	71.92
		0.3	74.88	80.77	86.55	88.89	86.82	81.25	75.55
		0.4	76.70	83.52	91.23	95.70	93.79	87.11	80.04
		0.5	77.11	84.77	94.01	100	97.40	89.41	81.52
		0.6	76.81	83.65	91.33	95.68	93.76	87.16	80.11
		0.7	75.01	80.91	86.63	88.86	86.80	81.30	75.62
		0.8	72.65	77.91	82.27	83.76	81.58	76.63	71.76
CSCC	+45/−45/−45/+45	0.2	83.47	93.38	100.58	95.30	89.67	86.32	82.22
		0.3	86.77	96.05	104.95	103.58	99.62	95.10	87.79
		0.4	85.62	93.21	101.85	105.04	103.07	97.54	89.15
		0.5	80.90	88.66	97.16	100.00	98.90	94.21	85.77
		0.6	79.27	86.60	94.19	96.14	94.09	87.60	80.42
		0.7	78.47	85.45	92.73	94.06	89.72	82.75	76.57
		0.8	78.08	85.15	92.47	93.11	87.65	81.15	75.24
	+45/−45/+45/−45	0.2	81.08	91.06	97.80	91.45	86.03	83.24	78.66
		0.3	83.53	93.13	102.17	99.59	94.92	89.19	82.21
		0.4	84.07	93.82	101.70	103.81	101.50	94.60	86.41

(continued)

Table 4.7 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
		0.5	83.00	90.43	97.73	100	99.47	96.30	87.55
		0.6	81.28	87.97	94.61	96.33	95.82	92.08	84.90
		0.7	79.65	86.29	92.87	94.10	92.14	86.70	80.22
		0.8	78.63	85.62	92.37	92.59	89.17	83.32	77.14
CCSC	+45/−45/−45/+45	0.2	71.50	78.96	87.42	86.48	82.25	77.69	72.51
		0.3	73.21	80.40	89.00	91.11	86.95	81.96	76.40
		0.4	74.27	82.16	91.08	96.73	91.58	84.95	78.35
		0.5	73.16	81.44	91.44	100.00	93.14	84.93	77.72
		0.6	71.25	79.22	89.41	93.48	86.71	79.55	73.43
		0.7	70.78	78.67	87.74	86.15	79.91	74.10	69.15
		0.8	70.45	78.23	86.07	82.45	77.04	72.03	67.37
	+45/−45/+45/−45	0.2	70.08	77.73	85.94	83.80	79.23	74.36	69.33
		0.3	71.07	78.70	87.77	88.37	83.34	77.77	72.30
		0.4	71.98	80.02	89.90	95.29	89.76	82.91	76.26
		0.5	72.47	81.21	91.40	100	93.47	85.28	77.81
		0.6	71.94	79.89	89.67	95.31	89.87	83.04	76.36
		0.7	71.09	78.67	87.71	88.51	83.48	77.89	72.37
		0.8	70.01	77.57	85.79	83.97	79.34	74.23	69.12
CCCS	+45/−45/−45/+45	0.2	78.61	85.65	92.04	92.28	89.95	85.46	79.49
		0.3	79.93	86.28	92.45	94.00	93.12	89.43	83.43
		0.4	82.22	88.39	94.44	96.33	96.24	93.27	86.69
		0.5	83.91	91.46	98.02	100.00	99.34	94.57	86.62
		0.6	82.17	91.79	101.41	101.60	96.76	89.48	82.43
		0.7	81.85	91.39	99.41	95.13	89.15	83.28	77.49
		0.8	80.04	89.63	95.35	88.09	82.14	78.83	74.75

	+45/−45/+45/ −45	0.2	78.06	85.09	91.57	92.62	90.17	84.85	78.65
		0.3	78.79	85.58	91.98	93.76	92.49	87.57	81.17
		0.4	80.50	87.43	93.91	95.95	95.85	92.34	85.56
		0.5	82.84	90.48	97.66	100	99.88	96.86	88.99
		0.6	85.40	94.49	102.33	104.31	102.18	96.77	88.82
		0.7	85.31	94.80	103.14	100.95	96.52	91.37	84.62
		0.8	81.94	91.75	98.55	93.01	87.37	84.65	80.32
CSSC	+45/−45/−45/+45	0.2	82.55	92.56	103.77	101.50	95.78	92.26	88.01
		0.3	84.77	93.81	106.03	108.03	104.66	100.86	92.87
		0.4	84.27	93.29	100.94	104.77	103.36	99.13	92.22
		0.5	81.27	88.63	96.23	100.00	99.50	96.90	91.79
		0.6	79.71	86.84	94.14	97.55	97.70	95.07	87.43
		0.7	79.50	86.29	93.34	96.96	97.29	90.04	82.92
		0.8	79.27	85.94	93.06	97.13	95.33	87.63	81.13
	+45/−45/+45/−45	0.2	80.25	90.35	102.00	98.15	92.56	90.10	85.81
		0.3	81.69	90.91	103.71	105.16	101.55	97.77	88.93
		0.4	81.34	89.74	99.85	103.87	102.50	98.78	91.05
		0.5	81.53	89.78	96.78	100	99.64	97.69	92.23
		0.6	81.91	89.29	95.25	97.88	98.40	97.87	92.32
		0.7	81.21	88.07	94.48	97.33	98.43	94.98	87.25
		0.8	80.28	87.19	94.25	97.48	97.83	90.75	83.64
CCSS	+45/−45/−45/+45	0.2	80.11	86.66	93.51	96.87	97.78	93.50	86.56
		0.3	81.72	87.92	93.81	96.97	98.59	97.51	90.86
		0.4	82.47	89.46	94.87	97.78	98.99	99.11	93.98
		0.5	80.28	87.52	96.12	100.00	100.33	99.48	93.28
		0.6	79.41	87.15	97.93	102.87	101.67	97.83	89.11
		0.7	79.94	88.72	100.76	100.96	96.14	91.04	84.49
		0.8	79.01	88.46	101.15	94.48	88.58	85.73	81.85

(continued)

Table 4.7 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
	+45/−45/+45/ −45	0.2	79.27	85.73	92.22	96.18	97.75	92.44	85.31
		0.3	80.02	86.47	92.66	96.15	97.78	95.35	88.24
		0.4	81.07	87.90	94.08	97.11	98.18	97.53	92.58
		0.5	82.09	90.36	97.02	100	100.12	98.61	94.25
		0.6	82.52	90.83	101.80	104.55	103.37	100.35	93.76
		0.7	82.94	91.82	103.72	105.73	102.49	99.51	91.40
		0.8	80.35	90.09	100.96	99.20	93.71	91.35	87.51
CSCS	+45/−45/−45/+45	0.2	81.27	89.52	96.57	95.21	91.31	88.75	83.98
		0.3	88.30	90.94	97.74	99.31	98.71	95.69	88.50
		0.4	85.99	92.69	99.41	101.17	101.91	99.06	91.05
		0.5	81.96	89.98	98.11	100.00	100.45	97.12	88.48
		0.6	80.09	87.82	95.34	97.18	96.29	90.75	83.19
		0.7	79.36	86.88	94.00	95.15	91.85	85.55	79.03
		0.8	78.17	86.14	93.40	92.28	86.62	82.19	77.05
	+45/−45/+45/−45	0.2	78.16	86.02	92.99	92.45	88.85	85.81	80.40
		0.3	79.87	86.98	93.77	95.53	94.67	90.30	83.26
		0.4	81.38	88.48	95.48	97.76	98.51	95.34	87.72
		0.5	83.42	91.21	98.29	100	101.46	99.52	90.64
		0.6	83.29	90.59	97.46	99.34	99.88	96.91	89.04
		0.7	81.81	89.18	96.02	97.21	95.83	91.36	84.35
		0.8	79.31	87.55	94.71	93.33	88.91	85.86	80.62
SCSC	+45/−45/−45/+45	0.2	86.37	95.34	98.05	91.30	88.83	86.41	82.05
		0.3	88.00	96.97	102.45	96.18	92.74	89.81	84.77
		0.4	89.12	99.21	107.01	100.14	95.07	91.30	85.60

		0.5	88.05	99.04	108.52	100.00	93.91	89.52	84.01
		0.6	85.99	96.14	103.82	95.35	89.74	85.17	80.17
		0.7	85.62	95.33	97.75	89.66	85.11	81.12	76.97
		0.8	85.43	94.70	93.38	85.59	82.15	78.94	75.24
	+45/−45/+45/−45	0.2	85.13	94.33	95.14	88.09	85.18	82.43	78.47
		0.3	86.02	95.54	99.99	92.93	89.05	85.59	80.89
		0.4	86.85	97.20	105.50	97.94	92.88	88.93	83.29
		0.5	87.28	98.40	107.89	100	94.35	90.35	84.26
		0.6	86.65	96.78	104.68	97.46	92.68	88.89	83.21
		0.7	85.82	95.21	99.43	92.52	89.00	85.63	80.78
		0.8	84.80	93.90	94.87	87.93	85.32	82.39	78.17
CSSS	+45/−45/−45/+45	0.2	87.72	95.78	103.52	105.37	104.34	102.76	97.67
		0.3	90.68	98.31	104.50	105.38	106.22	107.00	101.57
		0.4	90.18	99.36	104.49	102.62	102.10	102.39	99.47
		0.5	87.07	95.61	102.29	100.00	99.12	99.80	98.42
		0.6	85.61	93.62	100.42	99.94	99.64	100.77	98.47
		0.7	85.66	93.39	100.00	101.32	102.07	102.32	93.57
		0.8	84.40	91.99	99.12	102.01	101.62	97.80	90.89
	+45/−45/+45/−45	0.2	84.94	92.47	99.49	102.08	102.50	102.21	95.70
		0.3	87.00	94.42	100.61	101.92	103.36	105.52	99.08
		0.4	87.48	95.49	101.49	100.52	100.61	102.25	100.82
		0.5	87.93	97.25	102.63	100	99.42	100.62	99.68
		0.6	88.55	96.98	102.73	101.72	101.67	102.89	100.81
		0.7	88.61	96.44	102.88	103.97	105.12	106.83	99.48
		0.8	85.79	93.99	101.84	104.21	103.55	102.66	95.38

(continued)

Table 4.7 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
SSSC	+45/−45/−45/+45	0.2	91.98	102.59	106.89	100.09	94.51	90.59	87.50
		0.3	92.95	103.87	110.74	106.18	101.27	96.60	90.78
		0.4	90.74	100.64	106.83	104.02	99.26	94.70	89.26
		0.5	87.15	96.26	102.19	100.00	96.23	92.79	88.40
		0.6	85.72	94.33	99.78	98.01	95.46	92.98	88.90
		0.7	85.62	93.65	98.84	97.37	95.75	93.17	86.80
		0.8	85.29	93.10	98.22	96.66	95.48	91.38	85.28
	+45/−45/+45/−45	0.2	90.30	101.51	103.78	96.22	91.65	89.27	87.14
		0.3	91.10	102.46	108.0	102.69	98.67	95.77	90.18
		0.4	90.30	101.40	106.03	102.76	98.70	95.25	90.04
		0.5	89.52	99.11	102.72	100	96.67	94.02	90.03
		0.6	88.79	96.86	100.66	98.47	96.35	94.74	91.56
		0.7	88.17	95.67	99.70	97.82	96.80	96.04	90.72
		0.8	87.40	94.93	98.93	96.57	95.99	95.43	88.37
SSCS	+45/−45/−45/+45	0.2	92.30	101.49	103.52	97.97	94.17	91.01	87.48
		0.3	94.13	102.82	105.58	101.72	99.61	97.60	91.62
		0.4	94.80	103.67	105.98	101.53	98.66	96.41	92.15
		0.5	91.48	101.25	104.70	100.00	96.88	95.03	91.57
		0.6	89.68	99.37	103.24	98.93	96.14	94.05	88.21
		0.7	89.17	98.56	102.10	97.73	95.01	91.48	85.09
		0.8	88.13	97.51	100.43	94.16	90.59	88.21	83.47
	+45/−45/+45/−45	0.2	88.80	97.76	99.94	94.98	92.17	90.69	87.60
		0.3	90.15	98.97	101.93	98.67	97.22	96.20	89.40
		0.4	91.31	100.31	103.41	99.75	97.84	97.30	92.29

		0.5	92.29	101.66	104.36	100	97.49	96.68	93.64
		0.6	92.52	101.54	104.51	100.36	98.03	97.31	92.92
		0.7	91.86	100.92	103.93	99.90	98.07	97.44	90.29
		0.8	89.93	99.40	101.68	95.91	92.707	91.70	87.79
SSSS	+45/−45/−45/+45	0.2	96.09	104.72	108.96	106.02	103.34	100.52	96.41
		0.3	97.69	106.18	109.15	106.06	104.22	102.25	97.78
		0.4	96.71	105.24	106.86	102.50	100.07	98.58	95.67
		0.5	93.95	102.75	104.67	100.00	97.66	96.89	95.21
		0.6	92.51	101.49	104.90	101.02	99.18	98.82	96.93
		0.7	92.68	101.75	105.90	103.29	102.21	101.81	96.63
		0.8	91.59	100.33	105.61	103.45	102.12	102.13	95.10
	+45/−45/+45/−45	0.2	88.81	93.99	97.54	99.44	99.34	97.46	93.66
		0.3	90.12	95.85	98.24	99.30	99.56	98.87	96.46
		0.4	89.80	95.33	98.006	97.77	96.85	96.01	94.85
		0.5	89.67	94.93	97.58	100	95.46	94.36	93.38
		0.6	90.25	95.51	98.51	98.72	97.79	96.67	95.01
		0.7	90.94	96.73	99.73	101.29	101.51	100.52	97.14
		0.8	88.91	94.65	99.28	101.66	101.06	99.26	95.53
CS	+45/−45/−45/+45	0.2	129.98	125.45	112.90	104.30	101.56	101.13	104.18
		0.3	126.15	122.44	110.56	104.03	100.20	100.16	103.67
		0.4	121.42	119.70	109.07	100.94	99.10	99.49	102.46
		0.5	117.79	117.24	107.90	100.00	98.40	99.02	101.02
		0.6	117.63	116.42	107.35	99.96	98.36	98.75	100.31
		0.7	120.13	117.24	108.41	101.60	99.69	99.80	102.03
		0.8	119.94	117.04	110.13	104.34	103.21	103.64	106.49

(continued)

Table 4.7 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
	+45/−45/+45/−45	0.2	124.61	120.40	110.99	104.15	102.64	103.58	108.89
		0.3	120.49	117.59	108.49	101.85	100.60	101.66	107.32
		0.4	117.05	116.18	107.58	100.43	99.26	100.69	105.79
		0.5	115.79	115.79	107.43	100	98.79	100.43	105.33
		0.6	118.00	116.84	107.79	100.62	99.24	100.34	105.58
		0.7	122.49	118.78	109.26	102.38	100.47	100.98	106.53
		0.8	122.08	118.09	109.56	102.71	101.20	101.87	107.39

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $a/hh = 5$, $hl/hh = 0.25$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 4.8 Maximum values of r with corresponding coordinates of cutout centres and zones where $r \geq 90$ for cross-ply conoidal shells

Laminations	0/90/90/0			0/90/0/90		
Boundary conditions	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 90$	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 90$
CCCC	102.37	$\bar{x}=0.5, \bar{y}=0.3, 0.7$	$0.3 \leq \bar{x} \leq 0.6, 0.2 \leq \bar{y} \leq 0.8$	103.04	$\bar{x}=0.5, \bar{y}=0.7$	$\bar{x}=0.6, 0.2 \leq \bar{y} \leq 0.8$
CSCC	111.14	$\bar{x}=0.4, \bar{y}=0.3$	$0.2 \leq \bar{x} \leq 0.7, 0.6 \leq \bar{y} \leq 0.5; 0.2 \leq \bar{x} \leq 0.4, 0.6 \leq \bar{y} \leq 0.8$	110.51	$\bar{x}=0.4, \bar{y}=0.3$	$0.3 \leq \bar{x}$ 0.5, $0.6 \leq \bar{y} \leq 0.8$
CCSC	102.26	$\bar{x}=0.5, \bar{y}=0.3$; $\bar{x}=0.5, \bar{y}=0.7$	$0.3 \leq \bar{x} \leq 0.6, 0.2 \leq \bar{y} \leq 0.8$	102.73	$\bar{x}=0.5, \bar{y}=0.3$ $\bar{x}=0.5, \bar{y}=0.7$	$\bar{x}=0.4, 0.6, 0.2 \leq \bar{y} \leq 0.8$
CCCS	110.18	$\bar{x}=0.4, \bar{y}=0.6$	$0.2 \leq \bar{x} \leq 0.5, 0.2 \leq \bar{y} \leq 0.8; 0.6 \leq \bar{x} \leq 0.7, 0.5 \leq \bar{y} \leq 0.8$	110.82	$\bar{x}=0.4, \bar{y}=0.7$	$0.3 \leq \bar{x}$ 0.5, $0.2 \leq \bar{y} \leq 0.4$
CSSC	109.43	$\bar{x}=0.4, \bar{y}=0.3$	$0.2 \leq \bar{x} \leq 0.7, 0.2 \leq \bar{y} \leq 0.5; 0.2 \leq \bar{x} \leq 0.5, 0.6 \leq \bar{y} \leq 0.8$	108.70	$\bar{x}=0.5, \bar{y}=0.3$	$0.3 \leq \bar{x}$ 0.5, $0.6 \leq \bar{y} \leq 0.8$
CCSS	108.90	$\bar{x}=0.5, \bar{y}=0.7$	$0.2 \leq \bar{x} \leq 0.5, 0.2 \leq \bar{y} \leq 0.4; 0.2 \leq \bar{x} \leq 0.7, 0.5 \leq \bar{y} \leq 0.8$	108.90	$\bar{x}=0.5, \bar{y}=0.7$	$0.3 \leq \bar{x}$ 0.5, $0.2 \leq \bar{y} \leq 0.4$
CSCS	102.08	$\bar{x}=0.4, \bar{y}=0.5$	$0.2 \leq \bar{x} \leq 0.5, 0.3 \leq \bar{y} \leq 0.7$;	101.26	$\bar{x}=0.4, \bar{y}=0.5$	$0.3 \leq \bar{x}$ 0.5, $0.2 \leq \bar{y} \leq 0.3$

(continued)

Table 4.8 (continued)

Laminations	0/90/90/0			0/90/0/90		
Boundary conditions	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 90$	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 90$
			$0.6 \leq \bar{x} \leq 0.8$, $0.4 \leq \bar{y} \leq 0.6$			$0.7 \leq \bar{y} \leq 0.8$
SCSC	102.31	$\bar{x} = 0.4$, $\bar{y} = 0.2$	$0.3 \leq \bar{x} \leq 0.6$, $0.2 \leq \bar{y} \leq 0.8$	102.01	$\bar{x} = 0.5$, $\bar{y} = 0.7$	$\bar{x} = 0.3$, 0.5, $0.2 \leq \bar{y} \leq 0.8$
CSSS	101.44	$\bar{x} = 0.4$, $\bar{y} = 0.5$	$0.2 \leq \bar{x} \leq 0.6$, $0.2 \leq \bar{y} \leq 0.8$; $0.7 \leq \bar{x} \leq 0.8$, $0.3 \leq \bar{y} \leq 0.7$	100.04	$\bar{x} = 0.4$, $\bar{y} = 0.5$	$0.2 \leq \bar{x}$ 0.8, $0.2 \leq \bar{y} \leq 0.8$
SSSC	113.37	$\bar{x} = 0.4$, $\bar{y} = 0.3$	$0.2 \leq \bar{x} \leq 0.5$, $0.2 \leq \bar{y} \leq 0.8$; $0.6 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.4$	113.74	$\bar{x} = 0.4$, $\bar{y} = 0.3$	$0.6 \leq \bar{x}$ 0.7, $0.2 \leq \bar{y} \leq 0.4$
SSCS	104.68	$\bar{x} = 0.3$, $\bar{y} = 0.5$	$0.2 \leq \bar{x} \leq 0.5$, $0.2 \leq \bar{y} \leq 0.8$; $0.6 \leq \bar{x} \leq 0.8$, $0.3 \leq \bar{y} \leq 0.7$	103.32	$\bar{x} = 0.3$, $\bar{y} = 0.5$	$0.5 \leq \bar{x}$ 0.8, $0.3 \leq \bar{y} \leq 0.7$
SSSS	105.68	$\bar{x} = 0.3$, $\bar{y} = 0.5$	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.8$	101.01	$\bar{x} = 0.4$, $\bar{y} = 0.5$	$\bar{x} = 0.8$, $0.2 \leq \bar{y} \leq 0.8$
CS	136.33	$\bar{x} = 0.2$, $\bar{y} = 0.2$	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.8$	137.89	$\bar{x} = 0.2$, $\bar{y} = 0.2$	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.8$

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $a/hh = 5$, $hl/hh = 0.25$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$,$\nu_{12} = \nu_{21} = 0.25$

Table 4.9 Maximum values of r with corresponding coordinates of cutout centres and zones where $r \geq 90$ for angle-ply conoidal shells

Laminations	+45/−45/−45/+45			+45/−45/+45/−45		
Boundary conditions	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 90$	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 90$
CCCC	100.00	$\bar{x}=0.5, \bar{y}=0.5$	$\bar{x}=0.4,$ 0.5, $0.3 \leq \bar{y} \leq 0.6$	100.00	$\bar{x}=0.5, \bar{y}=0.5$	$\bar{x}=0.4,$ 0.6, $0.4 \leq \bar{y} \leq 0.6$
CSCC	105.04	$\bar{x}=0.5, \bar{y}=0.4$	$0.3 \leq \bar{x}$ 0.5, $0.2 \leq \bar{y} \leq 0.4$; $0.4 \leq \bar{x}$ 0.5, $0.5 \leq \bar{y} \leq 0.8$	103.81	$\bar{x}=0.5, \bar{y}=0.4$	$\bar{x}=0.3,$ $0.2 \leq \bar{y} \leq 0.5$; $0.6 \leq \bar{x}$ 0.7, $0.3 \leq \bar{y} \leq 0.6$
CCSC	100.00	$\bar{x}=0.5, \bar{y}=0.5$	$0.4 \leq \bar{x} \leq 0.6,$ $0.4 \leq \bar{x} \leq 0.5$	100.00	$\bar{x}=0.5, \bar{y}=0.5$	$0.4 \leq \bar{x} \leq 0.6, \bar{y}=0.5$
CCCS	101.60	$\bar{x}=0.5, \bar{y}=0.6$	$0.4 \leq \bar{x} \leq 0.6,$ $0.2 \leq \bar{y} \leq 0.6$	104.31	$\bar{x}=0.5, \bar{y}=0.6$	$0.4 \leq \bar{x} \leq 0.6,$ $0.2 \leq \bar{y} \leq 0.4$
CSSC	108.03	$\bar{x}=0.5, \bar{y}=0.3$	$0.3 \leq \bar{x} \leq 0.7,$ $0.2 \leq \bar{y} \leq 0.4$; $0.4 \leq \bar{x} \leq 0.7,$ $0.5 \leq \bar{y} \leq 0.6$	105.16	$\bar{x}=0.5, \bar{y}=0.3$	$0.4 \leq \bar{x} \leq 0.7,$ $0.7 \leq \bar{y} \leq 0.8$; $\bar{x}=0.8,$ $0.4 \leq \bar{y} \leq 0.6$
CCSS	102.87	$\bar{x}=0.5, \bar{y}=0.6$	$0.4 \leq \bar{x} \leq 0.7,$ $0.2 \leq \bar{y} \leq 0.7$	105.73	$\bar{x}=0.5, \bar{y}=0.7$	$\bar{x}=0.5,$ 0.7, $0.2 \leq \bar{y} \leq 0.4$; $\bar{x}=0.3,$ $0.5 \leq \bar{y} \leq 0.7$
CSCS	101.91	$\bar{x}=0.6, \bar{y}=0.5$	$0.4 \leq \bar{x} \leq 0.6,$ $0.2 \leq \bar{y} \leq 0.7$	101.46	$\bar{x}=0.6, \bar{y}=0.5$	$0.4 \leq \bar{x} \leq 0.7, \bar{y}=0.3$
SCSC	108.52	$\bar{x}=0.4, \bar{y}=0.5$	$0.3 \leq \bar{x} \leq 0.5,$ $0.2 \leq \bar{y} \leq 0.6$	107.89	$\bar{x}=0.4, \bar{y}=0.5$	$0.5 \leq \bar{x} \leq 0.6,$ $0.4 \leq \bar{y} \leq 0.6$

(continued)

Table 4.9 (continued)

Laminations	+45/−45/−45/+45			+45/−45/+45/−45		
Boundary conditions	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 90$	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 90$
CSSS	107.00	$\bar{x}=0.7, \bar{y}=0.3$	$0.3 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.8$	106.83	$\bar{x}=0.7, \bar{y}=0.7$	$\bar{x}=0.3$, $0.2 \leq \bar{y} \leq 0.8$
SSSC	106.89	$\bar{x}=0.4, \bar{y}=0.2$	$0.3 \leq \bar{x} \leq 0.7$, $0.2 \leq \bar{y} \leq 0.8$	108.10	$\bar{x}=0.4, \bar{y}=0.3$	$0.7 \leq \bar{x} \leq 0.8$, $0.3 \leq \bar{y} \leq 0.7$
SSCS	105.98	$\bar{x}=0.4, \bar{y}=0.4$	$0.3 \leq \bar{x} \leq 0.7$, $0.2 \leq \bar{y} \leq 0.7$	104.51	$\bar{x}=0.4, \bar{y}=0.6$	$\bar{x}=0.2$, 0.8, $0.3 \leq \bar{y} \leq 0.7$
SSSS	109.15	$\bar{x}=0.4, \bar{y}=0.3$	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.8$	101.66	$\bar{x}=0.5, \bar{y}=0.8$	$\bar{x}=0.3$, 0.8, $0\ .2 \leq \bar{y} \leq 0.8$
CS	129.98	$\bar{x}=0.2, \bar{y}=0.2$	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.8$	124.61	$\bar{x}=0.2, \bar{y}=0.2$	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.8$

$a/b=1$, $a/h=100$, $a'/b'=1$, $a/hh=5$, $hl/hh=0.25$; $E_{11}/E_{22}=25$, $G_{23}=0.2E_{22}$, $G_{13}=G_{12}=0.5E_{22}$, $\nu_{12}=\nu_{21}=0.25$

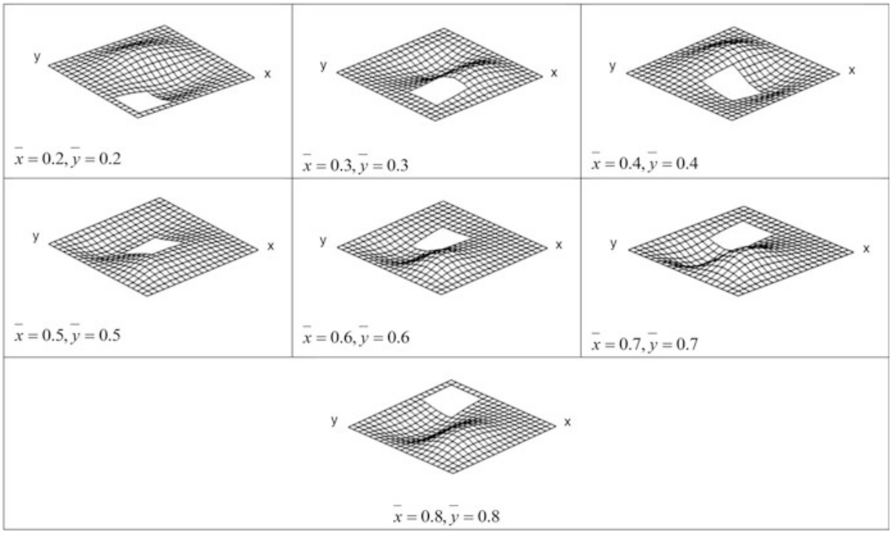
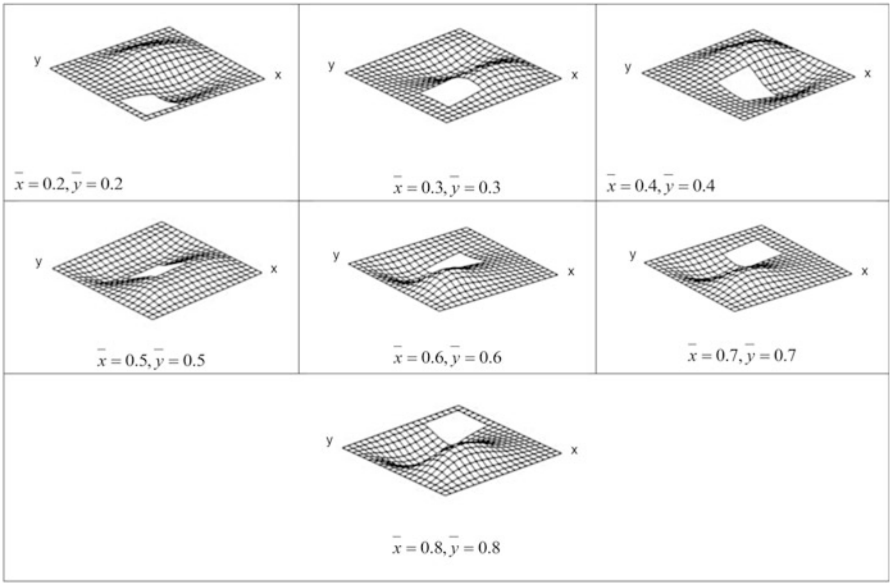


Fig. 4.6 First mode shapes of laminated composite (0/90/0/90) stiffened conoidal shell for different position of central square cutout and CCCC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

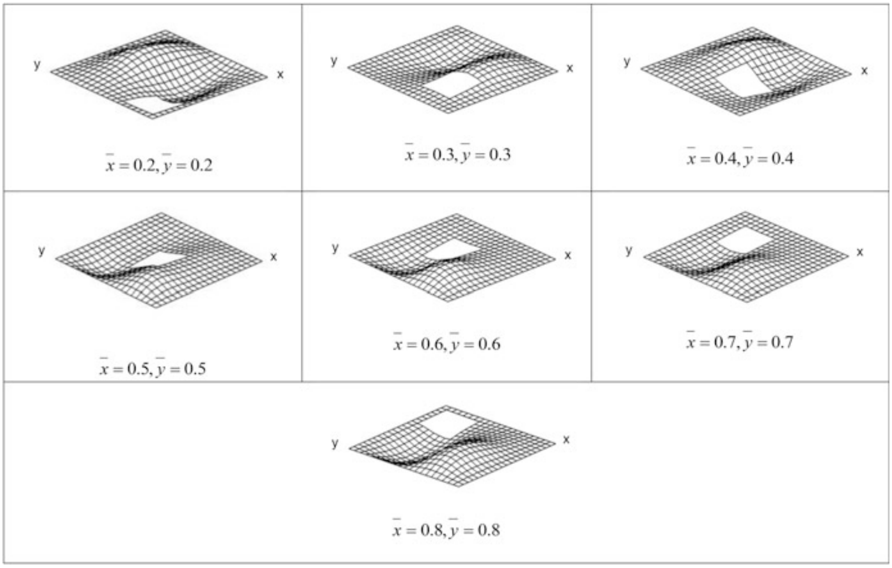
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 4.7 First mode shapes of laminated composite (0/90/0/90) stiffened conoidal shell for different position of central square cutout and CCSC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

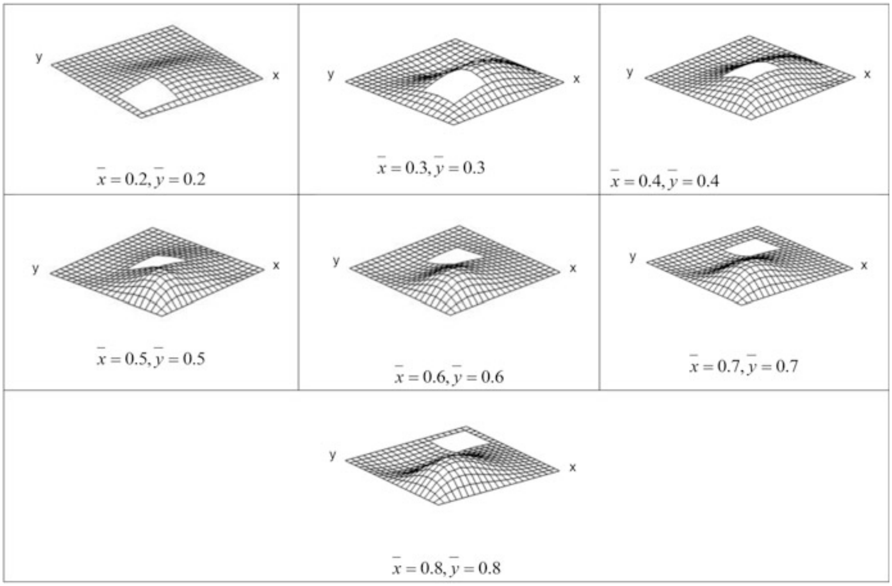
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 4.8 First mode shapes of laminated composite (0/90/0/90) stiffened conoidal shell for different position of central square cutout and SCSC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

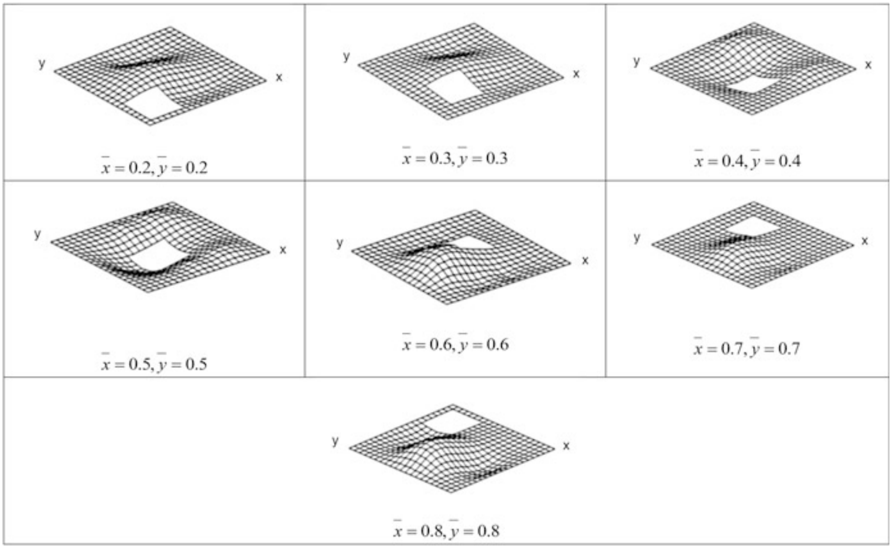
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 4.9 First mode shapes of laminated composite (0/90/0/90) stiffened conoidal shell for different position of central square cutout and SSSC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

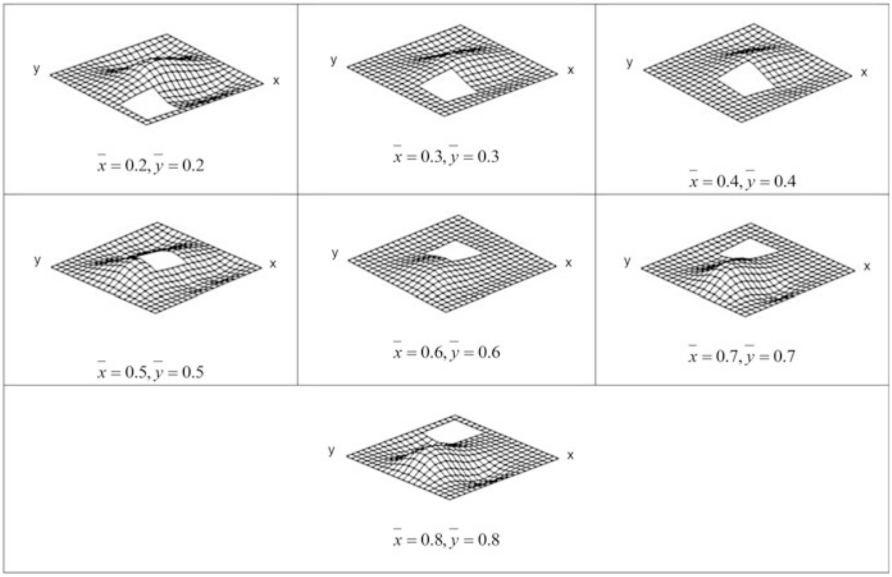
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 4.10 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened conoidal shell for different position of central square cutout and CCCC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

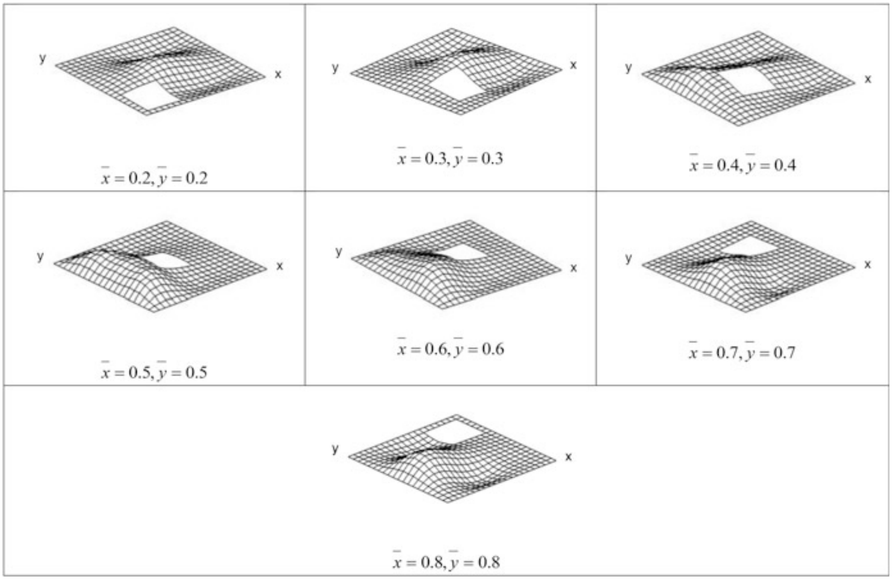
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 4.11 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened conoidal shell for different position of central square cutout and CCSC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

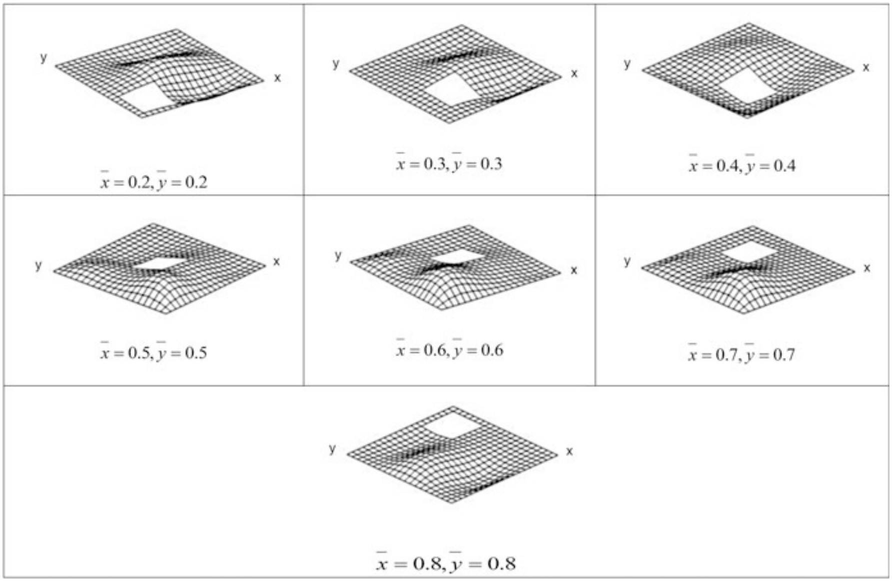
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 4.12 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened conoidal shell for different position of central square cutout and SCSC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 4.13 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened conoidal shell for different position of central square cutout and SSSC boundary condition

edges rather than their actual number is more important. The relative free vibration performances of shells for different combinations of boundary conditions along the four edges will be useful in decision-making for the selection of boundary constraints by practising engineers. The data on the characteristics of stiffened conoids with eccentric cutouts for a wide range of eccentricity and boundary conditions for both cross-ply and angle-ply shells may be used as design tools for structural engineers.

References

Brebbia C, Hadid H (1971) Analysis of plates and shells using rectangular curved elements. CE/5/71 Civil engineering department, University of Southampton

Chakravorty D, Bandyopadhyay JN, Sinha PK (1995a) Free vibration analysis of point supported laminated composite doubly curved shells – a finite element approach. Comput Struct 54(2):191–198

Chakravorty D, Bandyopadhyay JN, Sinha PK (1995b) Finite element free vibration analysis of point supported laminated composite cylindrical shells. J Sound Vib 181(1):43–52

Chakravorty D, Bandyopadhyay JN, Sinha PK (1996) Finite element free vibration analysis of doubly curved laminated composite shells. J Sound Vib 191(4):491–504

Chakravorty D, Bandyopadhyay JN, Sinha PK (1998) Application of FEM on free and forced vibration of laminated shells. ASCE J Eng Mech 124(1):1–8

Choi CK (1984) A conoidal shell analysis by modified isoparametric element. Comput Struct 18(5):921–924

Das AK, Bandyopadhyay JN (1993) Theoretical and experimental studies on conoidal shells. Comput Struct 49(3):531–536

Das HS, Chakravorty D (2007) Design aids and selection guidelines for composite conoidal shell roofs – a finite element application. J Reinf Plast Compos 26(17):1793–1819

Das HS, Chakravorty D (2008) Natural frequencies and mode shapes of composite conoids with complicated boundary conditions. J Reinf Plast Compos 27(13):1397–1415

Dey A, Bandyopadhyay JN, Sinha PK (1992) Finite element analysis of laminated composite conoidal shell structures. Comput Struct 43(30):469–476

Ghosh B, Bandyopadhyay JN (1989) Bending analysis of conoidal shells using curved quadratic isoparametric element. Comput Struct 33(3):717–728

Ghosh B, Bandyopadhyay JN (1990) Approximate bending analysis of conoidal shells using the Galerkin methods. Comput Struct 36(5):801–805

Hadid HA (1964) An analytical and experimental investigation into the bending theory of elastic conoidal shells. PhD dissertation, University of Southampton

Hota S, Chakravorty D (2007) Free vibration of stiffened conoidal shell roofs with cutouts. J Vib Control 13(3):221–240

Nayak AN, Bandyopadhyay JN (2002a) On the free vibration of stiffened shallow shells. J Sound Vib 255(2):357–382

Nayak AN, Bandyopadhyay JN (2002b) Free vibration and design aids of stiffened conoidal shells. ASCE J Eng Mech 128(4):419–427

Nayak AN, Bandyopadhyay JN (2005) Free vibration analysis of laminated stiffened shells. ASCE J Eng Mech 131(1):100–105

Nayak AN, Bandyopadhyay JN (2006) Dynamic response analysis of stiffened conoidal shells. J Sound Vib 291(3–5):1288–1297

Qatu MS, Sullivan RW, Wang W (2010) Recent research advances on the dynamic analysis of composite shells: 2000–2009. Compos Struct 93:14–31

Qatu MS, Asadi E, Wang W (2012) Review of recent literature on static analyses of composite shells: 2000–2010. Open J Compos Mater 2:61–86

Sahoo S (2013) Dynamic characters of stiffened composite conoidal shell roofs with cutout: design aids and selection guidelines. J Eng 2013:18p, Article ID 230120

Chapter 5
Stiffened Spherical Shell with Cutout

Abstract In this chapter, free vibration characteristics of laminated composite stiffened spherical shells with cutouts are studied using finite element method employing eight-noded curved quadratic isoparametric element for shell with three-noded curved beam element for stiffener formulation. Dynamic characters of stiffened spherical shells with different sizes and positions of the cutouts with respect to the shell centre for different edge constraints are analysed in the form of figures and tables. The results are analysed to arrive at guidelines for the selection of optimum size and position of the cutout with respect to shell centre considering different practical constraints.

Keywords Spherical shell • Cutout • Eccentricity • Fundamental frequency • Mode shapes

5.1 Introduction

Many structural components of aircrafts, missile and ship structures can be idealized as composite shell panels. Among various shell forms of different geometries, spherical shell, shell of revolution with curved meridian, is the most commonly used in various types of industries. Spherical pressure vessel is common in the chemical and process industries. Cutouts are provided in these shell panels to save weight and at times to provide a facility for inspection. In usual practice, the margin of the cutouts is stiffened to take account of stress concentration effects. In some situations, there can be some instruments directly fixed on these panels, and the safety of these instruments depends on the vibration characteristics of the panels. Hence free vibration studies on spherical shell panels with cutouts are of interest to structural engineers.

Kapania (1989), Noor and Burton (1990) and Reddy (1981) have proposed different computational models for the analysis of laminated composites. Chao and Reddy (1984) studied the dynamic response of simply supported cylindrical and spherical shells. The transient response of spherical and cylindrical shells with various boundary conditions and loading was reported by Reddy and Chandrashekhara (1985). Chao and Tung (1989) investigated the dynamic response

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of axisymmetric polar orthotropic hemispherical shells. Later free vibration study of doubly curved shells was performed by Qatu (1991), Liew and Lim (1994, 1995), Chakravorty et al. (1995, 1996, 1998), Shin (1997) and Tan (1998). Kant et al. (1994) analysed problems of a clamped spherical and simply supported cylindrical cap under external pressure. Sathyamoorthy (1995) reported the nonlinear vibration of orthotropic spherical shells. Later in 1997, Gautham and Ganesan reported free vibration characteristics of isotropic and laminated orthotropic spherical caps, while Chia and Chia (1997) reported nonlinear vibration of antisymmetric angle-ply shallow spherical shell having moderate thickness. Free vibration of curved panels with cutouts was reported by Sivasubramonian et al. (1997). Qatu et al. (2010) reviewed the literature on the vibration aspects of composite shells available during 2000–2009 and observed that closed cylindrical shells have received greater attention. Other shell geometries have also been investigated. Among those conical shells and shallow shells on rectangular, triangular, trapezoidal, circular, elliptical, rhombic or other planforms are receiving considerable attention. Shallow spherical shells also received some attention. Wang et al. (2002) considered wave propagation of stresses in orthotropic thick-walled spherical shells. Lellep and Hein (2002) studied optimization of shallow spherical shells under impact loading. Dai and Wang (2005) analysed stress wave propagation in laminated piezoelectric spherical shells subjected to thermal shock and electric excitation. Dynamic stability of spherical shells was evaluated by Ganapathi (2007) and Park and Lee (2009). Shallow spherical shells on rectangular or circular planform (spherical cap) with cutout (stiffened along the margin) have recently been reported in the literature (Sahoo 2014). The present chapter focuses on the free vibration behaviour of composite shallow spherical shell with cutout (stiffened along the margin), with concentric and eccentric cutouts, and considers the shells to have various combinations of edge conditions.

5.2 Problem

The effect of cutout size on free vibration response has been studied considering problems for different cross-ply and angle-ply spherical shells with symmetric and antisymmetric laminations and different boundary conditions like clamped, simply supported and point supported boundary conditions. Among them, only four-layered cross-ply and angle-ply shells having both symmetric and antisymmetric laminations have been chosen for further study. These multilayered shells with different numbers of boundary constraints have been further solved to analyse the free vibration responses due to change in position of cutout. Figure 5.1 shows a spherical shell panel having a concentric cutout and stiffened along the cutout margins.

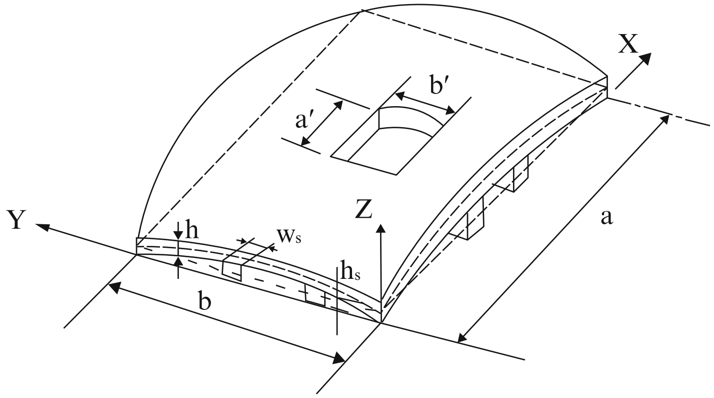


Fig. 5.1 Spherical shell with a concentric cutout stiffened along the margins (Adapted from Sahoo (2014) with permission from Elsevier)

5.3 Results and Discussion

5.3.1 Free Vibration Behaviour of Shells with Concentric Cutouts

5.3.1.1 Effect of Cutout Size on Fundamental Frequency

Table 5.1 furnishes the results of nondimensional frequency ($\overline{\omega}$) of stiffened spherical shells with cutout for different two-, three- and four-layered symmetric and antisymmetric cross- and angle-ply shells. It is seen from Table 5.1 that, with the introduction of cutout in a stiffened shell with simply supported and corner supported boundary conditions, fundamental frequencies increase. But in the case of a shell with clamped boundary, this unified trend is not observed. Although angle-ply shells and two-layered cross-ply shells exhibit increased performances with the introduction of cutout, two- and three-layered cross-ply shells exhibit reverse trend. With further increase in cutout sizes, fundamental frequencies increase. The stiffnesses of multilayered clamped shells are much higher than shells with simply supported and corner point supported shells. So loss of stiffness through introduction of cutout has significant effect on the total stiffness of the shell. Moreover the effect of addition of mass is more significant than effect of addition of stiffness due to increased number of stiffeners (as the numbers of stiffeners are increased from two to four due to introduction of cutout). As a result, fundamental frequencies decrease in clamped shells.

From Tables 5.2 and 5.3, it is observed that when a cutout is provided in a stiffened shell, the fundamental frequency increases in almost all the cases. This trend is observed for both cross-ply and angle-ply shells except CCCC cross-ply shells. This initial increase in frequency is due to the fact that with the introduction of cutout, the number of stiffeners increases from two to four in the present study. It

Table 5.1 Nondimensional fundamental frequencies ($\overline{\varpi}$) for laminated composite stiffened spherical shell for different sizes of central square cutout, different laminations and boundary conditions

		Cutout size (a'/a)				
Boundary conditions	Laminations	0	0.1	0.2	0.3	0.4
Clamped	0/90	81.93	92.44	108.17	118.17	121.81
	90/0	81.95	92.44	108.17	118.21	121.88
	0/90/0	131.32	115.87	131.36	139.32	138.18
	90/0/90	131.14	115.84	131.33	139.24	138.11
	0/90/0/90	122.96	112.03	128.97	137.61	138.63
	90/0/90/0	115.08	112.03	128.97	137.66	138.69
	0/90/90/0	133.71	116.64	132.62	139.86	138.42
	90/0/0/90	134.07	116.61	132.64	138.80	138.38
	+45/−45	114.11	125.22	136.48	137.28	127.88
	+45/−45/+45	117.30	129.20	138.06	137.65	133.34
	+45/−45/+45/−45	129.15	141.07	147.00	140.95	135.56
	+45/−45/−45/+45	129.48	139.67	145.95	140.67	135.64
Simply supported	0/90	56.07	61.46	66.84	69.83	72.42
	90/0	56.04	61.42	66.79	69.82	72.44
	0/90/0	57.55	64.85	68.27	71.44	75.11
	90/0/90	57.57	64.87	68.29	71.45	75.09
	0/90/0/90	57.48	64.72	68.25	71.49	75.35
	90/0/90/0	57.47	64.71	68.24	71.48	75.35
	0/90/90/0	57.71	64.97	68.71	71.63	75.43
	90/0/0/90	57.72	64.99	68.43	71.64	75.42
	+45/−45	82.82	92.02	97.21	100.73	97.03
	+45/−45/+45	89.42	99.80	103.92	103.88	99.10
	+45/−45/+45/−45	91.48	101.30	105.41	105.20	100.13
	+45/−45/−45/+45	91.06	101.18	105.09	105.48	100.18
Point supported	0/90	18.94	23.15	24.06	24.71	23.87
	90/0	19.33	23.28	24.27	25.05	24.19
	0/90/0	19.09	23.32	24.67	26.98	24.94
	90/0/90	26.69	29.44	33.04	37.88	33.74
	0/90/0/90	36.98	38.21	38.78	40.43	42.18
	90/0/90/0	37.10	38.17	38.74	40.40	41.16
	0/90/90/0	30.35	32.93	35.84	38.29	39.87
	90/0/0/90	30.32	32.92	35.77	38.19	39.74
	+45/−45	26.93	29.51	32.49	36.37	37.33
	+45/−45/+45	25.85	28.01	30.88	34.45	36.75
	+45/−45/+45/−45	33.89	36.75	38.57	41.43	41.75
	+45/−45/−45/+45	29.48	31.81	34.42	37.99	39.95

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 5.2 Nondimensional fundamental frequencies ($\overline{\omega}$) for laminated composite cross-ply stiffened spherical shell for different sizes of central square cutout and different boundary conditions

Laminations	Boundary conditions	Cutout size (a'/a)				
		0	0.1	0.2	0.3	0.4
0/90/90/0	CCCC	133.72	116.64	132.62	139.86	138.42
	CSCC	87.40	97.11	108.64	114.76	113.95
	CSSC	71.63	79.78	83.43	87.12	91.02
	CSCS	88.72	91.63	106.06	108.99	107.52
	CSSS	64.38	72.84	75.61	78.94	82.58
	SSSS	57.71	64.97	68.41	71.63	75.43
	Point supported	30.35	32.92	35.84	38.29	39.87
0/90/0/90	CCCC	122.17	112.03	128.97	137.61	138.63
	CSCC	90.87	99.49	112.31	115.09	116.30
	CSSC	70.90	78.91	83.00	86.71	90.79
	CSCS	95.10	95.40	109.58	109.97	110.73
	CSSS	64.66	72.92	75.91	79.34	83.30
	SSSS	57.48	64.72	68.25	71.49	75.35
	Point supported	36.98	37.92	38.78	40.43	41.18

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

is also evident from Table 5.2 and 5.3 that when the cutout size is increased, fundamental frequency is increased in all cases except CSSC angle-ply shells. Here, when a'/a ratio changes from 0.1 to 0.2, fundamental frequency decreases slightly. It is further noted that for angle-ply shells with the increase in cutout size, fundamental frequency increases up to $a'/a = 0.2$, but with further increase in cutout size, reverse trend is observed. So for angle-ply shells with the increase of cutout size, loss of stiffness is more significant than loss of mass. Hence fundamental frequency decreases except in the case of CSCS and corner point supported angle-ply shells ($a'/a = 0.2$–0.3). As with the introduction of a cutout of $a'/a = 0.2$, on shell surface, the frequency increases in most of the cases; this leads to the engineering conclusion that concentric cutouts with stiffened margins may be provided safely on spherical shell surfaces for functional requirements up to $a'/a = 0.2$.

5.3.1.2 Effect of Boundary Conditions on Fundamental Frequency

The boundary conditions considered here are CCCC, CSCC, CSSC, CSCS, SSSS and corner point supported.

It is seen from Table 5.2 and 5.3 that fundamental frequencies of members belonging to same number of boundary constraints like CSSC and CSCS have not close values. So the boundary constraint is not the sole criteria for its performance but their arrangement has a greater impact. It can be seen from the present study that

Table 5.3 Nondimensional fundamental frequencies ($\overline{\varpi}$) for laminated composite angle-ply stiffened spherical shell for different sizes of central square cutout and different boundary conditions

		Cutout size (a'/a)				
Laminations	Boundary conditions	0	0.1	0.2	0.3	0.4
+45/−45/−45/+45	CCCC	129.48	139.67	145.95	140.67	135.64
	CSCC	109.08	118.15	119.55	118.66	115.62
	CSSC	103.08	112.93	112.57	109.50	106.12
	CSCS	100.86	110.23	115.69	116.23	112.44
	CSSS	95.86	105.27	109.50	106.67	103.04
	SSSS	91.06	101.18	105.09	105.48	100.18
	Point supported	29.48	31.81	34.42	37.99	39.95
+45/−45/+45/−45	CCCC	129.15	141.07	147.00	140.95	135.56
	CSCC	108.21	116.74	118.97	117.96	115.44
	CSSC	102.59	111.21	111.08	108.88	106.62
	CSCS	100.28	109.61	115.53	116.13	112.36
	CSSS	95.56	104.82	109.31	106.47	103.04
	SSSS	91.47	101.30	105.41	105.20	100.13
	Point supported	33.89	36.75	38.57	41.43	41.75

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

if one edge is released from clamped to simply supported, the change of frequency is lowest in case of antisymmetric cross-ply shells. For other laminations considered here, the change in frequency is greater than the previous one. Again, if the two adjacent edges are released, the decrease in fundamental frequency is very much significant for cross-ply shells. This is true for both symmetric and antisymmetric cross-ply shells. But in the case of angle-ply shells, when two adjacent edges are released, the decrease in frequency is not so significant. Further, if two alternate edges are released from clamped to simply supported, fundamental frequency does not change to a great extent with respect to a clamped shell. For cross-ply shells with the introduction of three or four simply supported edges, frequency values undergo marked decrease, but for angle-ply shells with the introduction of more number of simply supported edges, the frequency value does not change so drastically. The results indicate that two alternate edges should preferably be clamped in order to achieve higher frequency values.

Tables 5.4 and 5.5 are prepared from Tables 5.2 and 5.3 to reveal the efficiency of a particular clamping option to improve the fundamental frequency of a shell with minimum number of boundary constraints relative to that of a clamped shell. Marks are assigned to each boundary combination in a scale ranging from 0 to 100. The frequency of a corner point supported shell is assigned a value of 0 and that of a fully clamped shell is assigned a value of 100. These marks are furnished for cutouts with $a'/a = 0.2$. These tables will enable a practising engineer to decide at a glance the efficiency of a particular boundary condition while selecting such shell forms.

Table 5.4 Clamping options for cross-ply spherical shells with central cutouts having a'/a ratio 0.2

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of no. of restraints
0/90/90/0	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Good improvement	34
	1	One edge (CSSS)	Good improvement	41
	2	(a) Two alternate edges (CSCS)	Marked improvement	73
		(b) Two adjacent edges (CSSC)	Good improvement	49
	3	Three edges (CSCC)	Marked improvement	75
	4	All sides (CCCC)	Frequency attains highest value	100
0/90/0/90	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Good improvement	33
	1	One edge (CSSS)	Good improvement	41
	2	(a) Two alternate edges (CSCS)	Marked improvement	79
		(b) Two adjacent edges (CSSC)	Good improvement	49
	3	Three edges (CSCC)	Marked improvement	82
	4	All sides (CCCC)	Frequency attains highest value	100

5.3.1.3 Mode Shape

Figures 5.2, 5.3, 5.4 and 5.5 show the mode shapes corresponding to the fundamental modes of vibration for CCCC and CSCS cross-ply and angle-ply shells, respectively. The mode shapes are plotted considering the shell mid-surface as the reference. The normalized displacements of the shells are such that the maximum

Table 5.5 Clamping options for angle-ply spherical shells with central cutouts having a'/a ratio 0.2

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of no. of restraints
+45/−45/−45/+45	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Very good improvement	63
	1	One edge (CSSS)	Very good improvement	67
	2	(a) Two alternate edges (CSCS)	Marked improvement	73
		(b) Two edges (CSSC)	Very good improvement	70
	3	Three edges (CSCC)	Marked improvement	76
	4	All sides (CCCC)	Frequency attains highest value	100
+45/−45/+45/−45	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Very good improvement	62
	1	One edge (CSSS)	Very good improvement	65
	2	(a) Two alternate edges (CSCS)	Marked improvement	71
		(b) Two edges (CSSC)	Very good improvement	67
	3	Three edges (CSCC)	Marked improvement	74
	4	All sides (CCCC)	Frequency attains highest value	100

displacement is unity. The fundamental mode is clearly a bending mode for all edge conditions for cross-ply and angle-ply shells, except corner point supported shell. For corner point supported shells, the fundamental mode shapes are complicated.

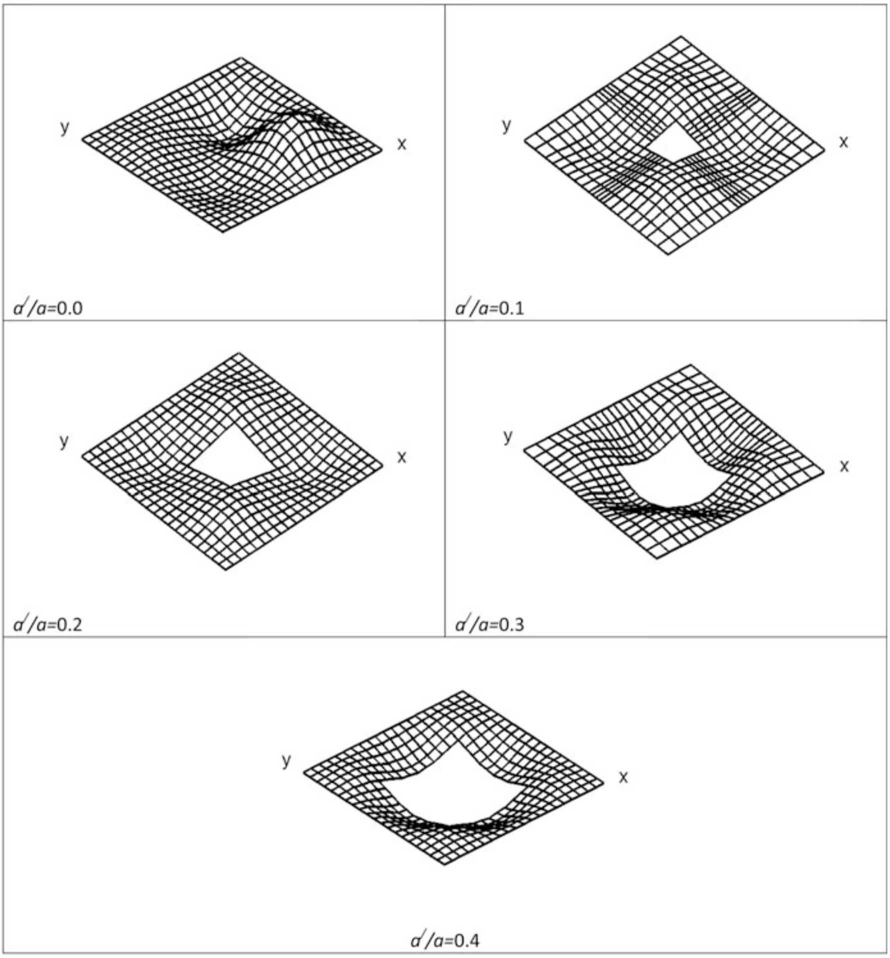


Fig. 5.2 First mode shapes of laminated composite (0/90/0/90)/CCCC stiffened spherical shell for different sizes of central square cutout and boundary conditions

5.3.2 *Effect of Eccentricity of Cutout Position*

5.3.2.1 Fundamental Frequency

The effects of eccentricities of cutout position on fundamental frequencies are shown in Tables 5.6 and 5.7. Tables 5.6 and 5.7 are prepared from the results obtained for different locations of a cutout with $a'/a = 0.2$. The position of the cutout centre $\left(\bar{x} = \frac{x}{a}, \bar{y} = \frac{y}{a}\right)$ is varied from 0.2 to 0.8 along each directions. Here also the distance of a cutout margin from the shell boundaries is maintained as one tenth of the plan dimension of the shell. The boundaries of cutouts are stiffened with four stiffeners. The study is carried out for all the seven boundary conditions for both

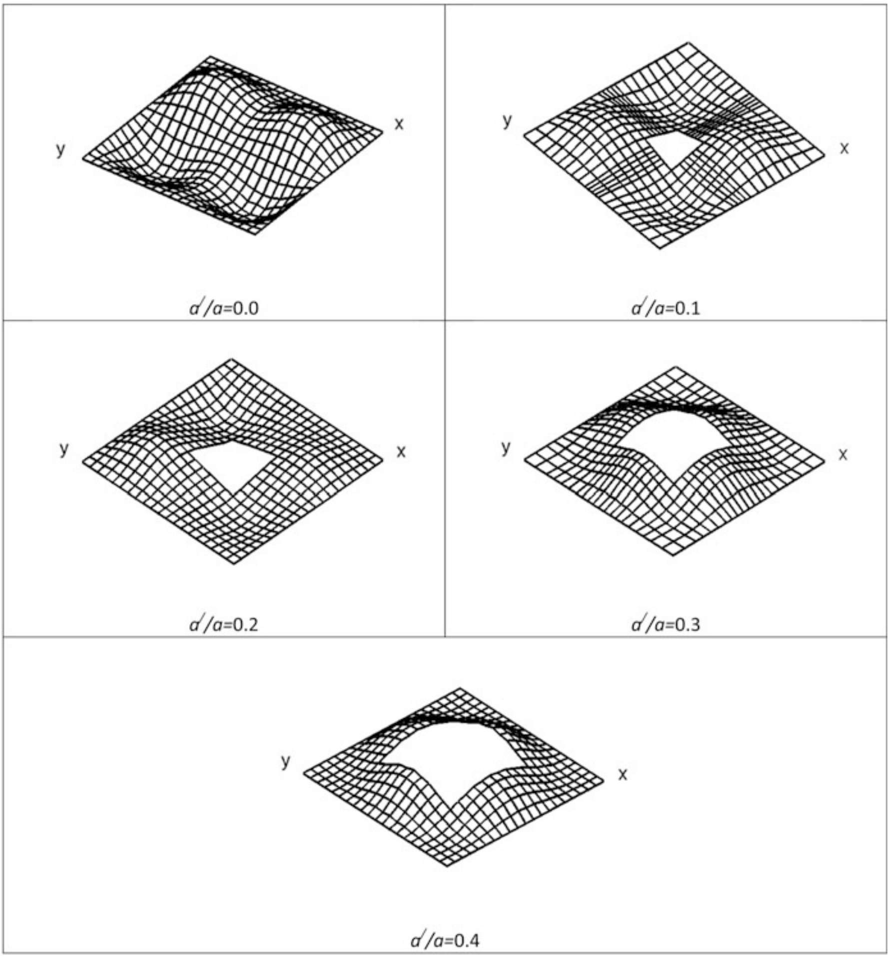


Fig. 5.3 First mode shapes of laminated composite (0/90/0/90)/CSCS stiffened spherical shell for different sizes of central square cutout and boundary conditions

cross-ply and angle-ply shells. The fundamental frequency of a shell with an eccentric cutout is expressed as a percentage (r) of fundamental frequency of a shell with a concentric cutout obtainable from Tables 5.2 and 5.3. In Tables 5.6 and 5.7, such r values are furnished.

It can be seen that eccentricity of the cutout along the length and width of the shell towards the edges makes it more flexible. This observation is true for both symmetric and antisymmetric cross-ply and angle-ply shells. It is also seen that in almost all the cases r value is maximum in and around $\bar{x} = 0.5$ and $\bar{y} = 0.5$. It is noticed that for CSCC shells towards the simply supported edges r value is greater than that of the opposite clamped edges. Similarly for CSSC shells, eccentricity towards the two adjacent simply supported edges shows greater stiffness than the

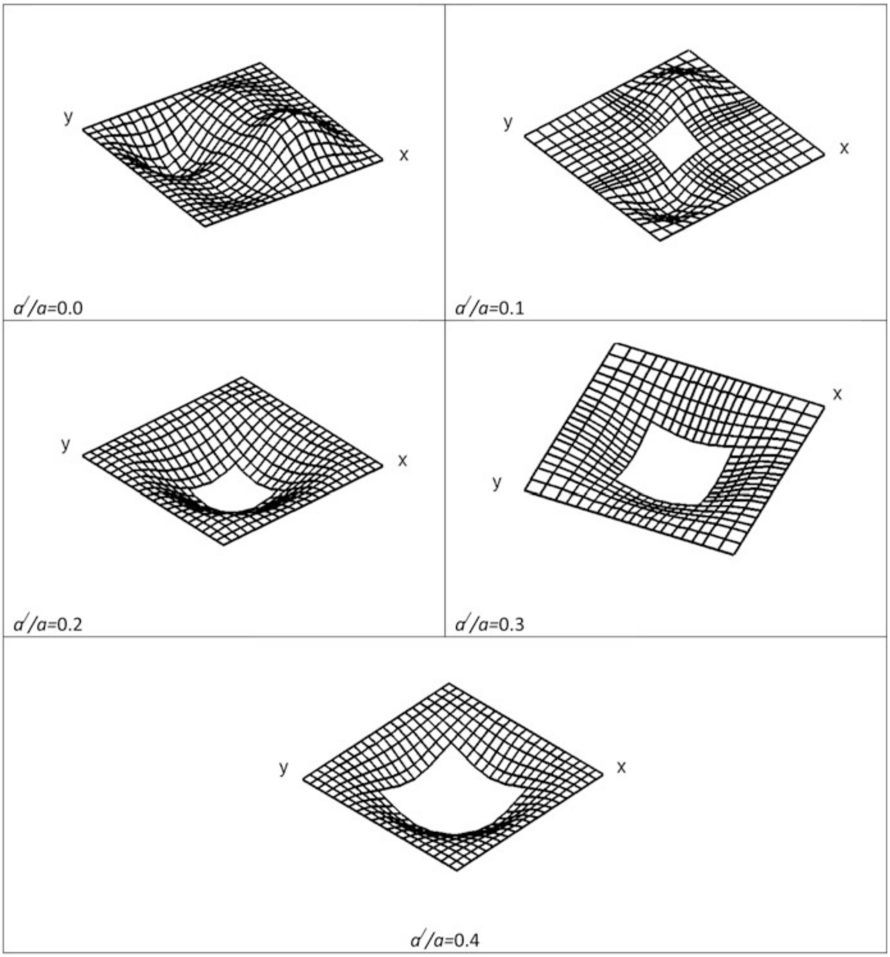
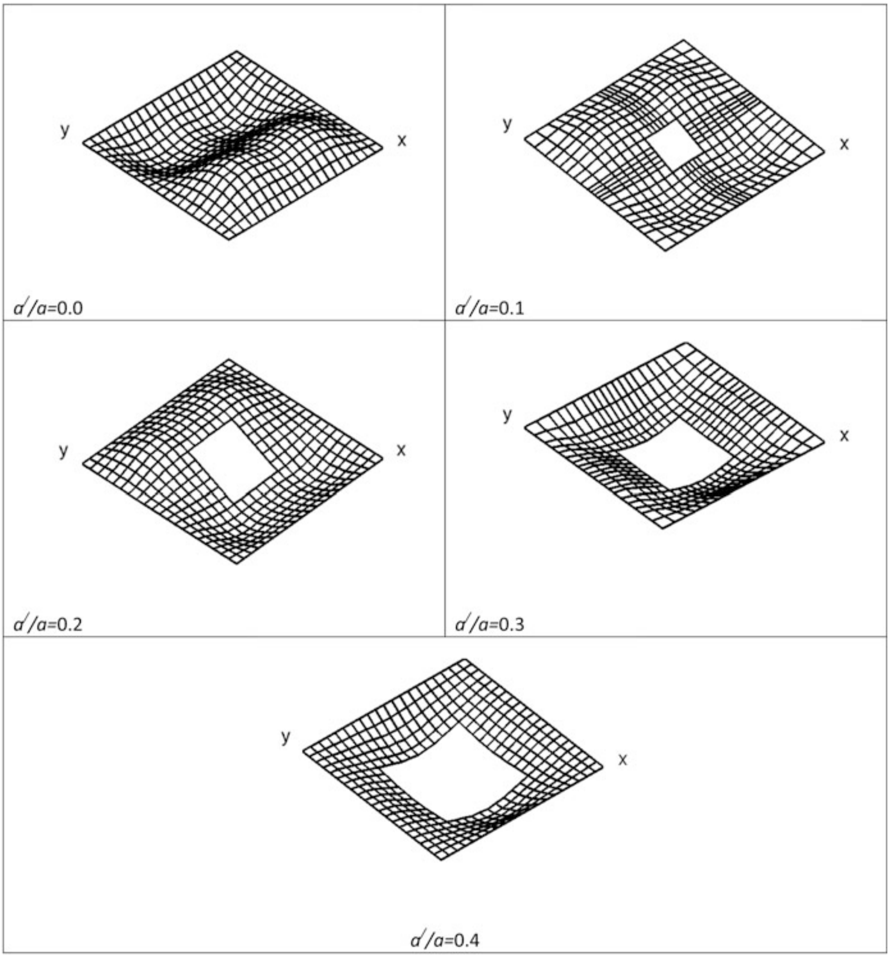


Fig. 5.4 First mode shapes of laminated composite (+45/−45/+45/−45)/CCCC stiffened spherical shell for different sizes of central square cutout and boundary conditions

eccentricity towards the opposite clamped edges. But in the case of shells with two opposite edges simply supported and others clamped, eccentricity towards clamped edges shows greater stiffness. Moreover, when three edges are simply supported, the eccentricity towards the simply supported edge opposite to the clamped one shows higher frequency value. For corner point supported shells, the maximum fundamental frequency always occurs along the diagonal of the shell.

Tables 5.8 and 5.9 are prepared from Tables 5.6 and 5.7 and provide the maximum values of r together with the position of the cutout. These tables also provide the rectangular zones within which r is always greater than or equal to 90. These tables further provide information regarding the maximum eccentricity of a cutout which can be permitted. So these tables will help practising engineers.



$a'/a=0.0$ $a'/a=0.1$ $a'/a=0.2$ $a'/a=0.3$ $a'/a=0.4$

Fig. 5.5 First mode shapes of laminated composite (+45/−45/+45/−45)/CSCS stiffened spherical shell for different sizes of central square cutout and boundary conditions

5.3.2.2 Mode Shape

To study the behaviour of the shells for different eccentric positions of the cutout, the mode shapes are plotted in Figs. 5.6, 5.7, 5.8 and 5.9 for cross-ply and angle-ply shells of CCCC and CSCC boundary conditions. Only the antisymmetric laminations have been considered. All the mode shapes are bending mode. It is found that for different positions of cutout, mode shapes do not change. The only changes that occur in mode shapes are the position of the crest and trough.

Table 5.6 Values of ‘r’ for cross-ply spherical shells

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
CCCC	0/90/90/0	0.2	85.56	84.31	85.30	87.51	85.30	84.31	85.54
		0.3	89.12	87.72	88.38	89.91	88.37	87.66	88.89
		0.4	94.26	93.55	94.14	95.00	94.14	93.55	98.16
		0.5	102.38	98.76	99.24	100.00	99.23	98.70	102.28
		0.6	94.25	93.56	94.14	95.00	94.15	93.56	94.68
		0.7	89.15	87.84	88.37	89.93	88.37	87.66	88.89
		0.8	85.72	84.27	85.27	87.49	85.30	84.29	85.52
	0/90/0/90	0.2	89.69	89.25	91.19	94.17	91.19	89.25	89.79
		0.3	89.22	89.68	92.15	94.67	92.15	89.67	89.22
		0.4	91.15	92.14	94.92	97.51	94.92	92.14	91.15
		0.5	94.08	94.68	97.53	100.00	97.53	94.68	94.43
		0.6	91.15	92.14	94.92	97.51	94.94	92.14	91.15
		0.7	89.22	89.68	92.15	94.67	92.15	89.67	89.22
		0.8	89.42	89.11	91.08	94.81	91.17	89.18	89.56
CSCC	0/90/90/0	0.2	90.59	88.71	86.64	85.43	86.64	88.70	90.57
		0.3	94.82	94.62	94.30	93.86	94.30	94.62	94.82
		0.4	96.93	99.06	102.83	105.27	102.82	99.06	96.92
		0.5	95.91	97.00	98.99	100.00	98.97	96.99	95.66
		0.6	88.95	87.54	87.48	87.62	87.55	87.58	88.40
		0.7	81.78	80.52	80.49	80.74	80.55	80.57	81.37
		0.8	78.02	76.89	77.05	77.46	77.11	76.92	77.63
	0/90/0/90	0.2	92.33	90.94	90.03	89.61	90.02	90.94	92.36
		0.3	93.92	94.57	95.37	95.92	95.37	94.57	93.92
		0.4	94.60	97.17	101.36	104.31	101.36	97.17	94.60

(continued)

Table 5.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
		0.5	93.94	95.20	97.87	100.00	97.99	95.25	93.80
		0.6	89.61	88.50	89.47	90.54	89.57	88.58	89.20
		0.7	85.30	84.08	84.73	85.54	84.79	84.13	84.91
		0.8	83.12	82.02	82.62	83.36	82.66	82.07	82.77
CSSC	0/90/90/0	0.2	81.19	83.72	87.95	93.17	91.33	84.69	78.29
		0.3	85.40	88.41	93.08	98.23	97.55	91.78	85.83
		0.4	88.16	91.57	96.58	101.71	102.24	97.72	92.39
		0.5	87.04	90.12	94.74	100.00	101.56	97.83	93.14
		0.6	82.79	85.07	88.84	94.14	96.43	92.53	88.07
		0.7	78.63	80.46	83.78	88.83	91.19	87.35	83.22
		0.8	75.97	77.71	80.87	85.50	87.43	83.93	80.13
	0/90/0/90	0.2	80.71	83.47	88.00	93.16	91.86	85.07	78.07
		0.3	84.93	88.07	92.92	98.14	97.66	91.64	85.14
		0.4	88.11	91.69	96.80	101.86	102.36	97.71	92.02
		0.5	86.47	89.59	94.40	100.00	101.69	98.01	93.13
		0.6	81.61	84.02	88.34	94.23	96.40	92.57	87.73
		0.7	77.78	79.96	83.98	89.34	91.19	87.63	83.14
		0.8	75.60	77.77	81.53	86.18	87.58	84.43	80.36
CSCS	0/90/90/0	0.2	75.94	75.46	73.95	73.02	73.95	75.46	75.94
		0.3	80.99	80.28	79.07	78.37	79.07	80.28	80.99
		0.4	89.35	88.11	87.27	86.75	87.27	88.11	89.10
		0.5	94.90	96.26	98.85	100.00	98.91	96.26	94.90
		0.6	89.60	88.11	87.27	86.75	87.27	88.11	89.10
		0.7	80.99	80.28	79.07	78.37	79.07	80.28	80.99
		0.8	75.94	75.46	73.94	72.98	73.93	75.46	75.93

	0/90/0/90	0.2	82.20	81.61	80.79	80.39	80.79	81.61	82.20
		0.3	85.44	84.60	84.24	84.24	84.24	84.59	85.43
		0.4	90.52	89.68	90.01	90.51	90.01	89.68	90.51
		0.5	93.88	95.61	98.86	100.00	98.49	95.61	93.88
		0.6	90.52	89.68	90.01	90.51	90.01	89.68	90.51
		0.7	85.44	84.59	84.24	84.24	84.24	84.60	85.43
		0.8	90.40	81.61	80.78	80.36	80.77	81.61	82.20
CSSS	0/90/90/0	0.2	76.68	78.32	81.58	86.72	87.96	82.56	77.04
		0.3	80.70	82.62	86.15	91.46	93.27	88.44	83.23
		0.4	85.29	87.78	91.81	96.98	98.69	94.66	89.74
		0.5	87.78	90.76	95.20	100.00	101.42	98.12	93.60
		0.6	85.29	87.78	91.81	96.98	98.69	94.66	89.74
		0.7	80.70	82.62	86.14	91.46	93.27	88.44	83.23
		0.8	76.55	78.20	81.44	86.58	87.77	82.34	76.84
	0/90/0/90	0.2	76.78	78.86	82.65	87.54	88.24	83.26	77.38
		0.3	80.28	82.49	86.54	91.89	93.24	88.70	83.14
		0.4	84.72	87.30	91.61	96.98	98.59	94.65	89.45
		0.5	87.67	90.73	95.22	100.00	101.42	98.21	93.64
		0.6	84.72	87.30	91.61	96.98	98.59	94.65	89.45
		0.7	80.28	82.49	86.54	91.89	93.24	88.70	83.14
		0.8	76.70	78.78	82.53	87.41	88.05	83.09	77.22
SSSS	0/90/90/0	0.2	75.91	79.97	84.80	88.58	84.80	79.96	75.91
		0.3	80.94	84.87	89.59	92.98	89.59	84.87	80.94
		0.4	86.23	90.26	94.87	97.65	94.87	90.26	86.23
		0.5	89.14	93.23	97.68	100.00	97.68	93.23	89.14
		0.6	86.23	90.26	94.87	97.65	94.87	90.26	86.23
		0.7	80.94	84.87	89.59	92.98	89.59	84.87	80.94
		0.8	75.72	79.78	84.61	88.44	84.64	79.78	75.73

(continued)

Table 5.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
	0/90/0/90	0.2	75.63	80.13	85.27	88.67	85.27	80.13	75.63
		0.3	80.19	84.50	89.63	92.97	89.63	84.50	80.19
		0.4	85.48	89.74	94.68	97.60	94.68	89.74	85.48
		0.5	89.04	93.23	97.70	100.00	97.70	93.23	89.04
		0.6	85.48	89.74	94.68	97.60	94.68	89.74	85.48
		0.7	80.19	99.15	89.63	92.97	89.63	84.50	80.19
		0.8	75.52	80.00	85.11	88.53	85.11	79.97	75.49
CS	0/90/90/0	0.2	100.06	99.39	97.38	96.04	97.38	99.39	100.06
		0.3	101.42	100.28	97.85	96.34	97.82	100.28	101.42
		0.4	103.49	101.98	99.14	97.49	99.14	101.98	103.49
		0.5	107.00	105.52	101.65	100.00	101.76	104.77	106.92
		0.6	103.49	101.95	99.11	97.49	99.11	101.95	103.49
		0.7	101.42	100.28	97.82	96.32	97.82	100.25	101.42
		0.8	99.97	99.33	97.35	95.98	97.32	99.30	99.94
	0/90/0/90	0.2	97.50	99.25	101.08	102.24	101.08	99.25	97.50
		0.3	98.76	99.72	100.80	101.86	100.77	99.72	98.76
		0.4	99.61	99.85	100.21	100.52	100.23	99.85	99.61
		0.5	99.95	99.79	99.85	100.00	99.92	99.79	99.92
		0.6	99.61	99.85	100.23	100.67	100.23	99.85	99.61
		0.7	98.74	99.72	100.77	101.47	100.77	99.72	98.74
		0.8	97.16	98.99	100.93	102.19	100.85	98.99	97.16

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 5.7 Values of ‘r’ for angle-ply spherical shells

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
CCCC	+45/−45/−45/+45	0.2	77.83	79.13	80.45	81.97	82.25	81.13	79.51
		0.3	79.07	81.41	84.02	86.78	86.26	83.71	81.08
		0.4	80.34	83.95	88.36	93.51	91.04	86.19	82.14
		0.5	81.86	86.65	93.42	100.00	93.39	86.65	81.84
		0.6	82.14	86.19	91.04	93.49	88.36	83.95	80.34
		0.7	81.08	83.71	86.26	86.78	84.02	81.41	79.07
		0.8	79.50	81.12	82.23	81.99	80.45	79.12	77.81
	+45/−45/+45/−45	0.2	78.58	79.77	80.67	81.18	80.71	79.82	78.62
		0.3	79.71	82.12	84.52	86.13	84.61	82.21	79.77
		0.4	80.56	84.46	89.07	93.23	89.17	84.54	80.61
		0.5	81.06	86.02	93.14	100.00	93.12	86.01	81.06
		0.6	80.62	84.54	89.16	93.20	89.07	84.46	80.57
		0.7	79.77	82.21	84.61	86.12	84.52	82.12	80.57
		0.8	78.59	79.82	80.71	81.17	80.66	79.76	78.56
CSCC	+45/−45/−45/+45	0.2	91.32	93.12	96.10	99.15	98.32	95.60	93.50
		0.3	95.32	98.54	101.87	105.82	104.63	101.18	97.23
		0.4	92.99	98.13	104.26	105.80	102.51	97.35	92.51
		0.5	88.36	92.63	97.78	100.00	95.79	90.59	86.37
		0.6	86.78	90.41	94.91	97.37	93.27	88.47	84.61
		0.7	87.94	91.07	94.75	97.41	93.50	89.44	85.96
		0.8	89.27	92.46	95.17	98.42	94.09	91.15	87.86
	+45/−45/+45/−45	0.2	93.10	94.97	97.58	99.39	97.52	94.91	93.07
		0.3	97.00	100.52	103.77	106.35	103.73	100.55	96.78
		0.4	93.25	98.20	103.56	106.09	103.61	98.04	92.49

(continued)

Table 5.7 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
		0.5	87.73	91.89	96.84	100.00	96.76	91.40	86.85
		0.6	85.98	89.65	94.14	97.27	93.96	89.31	85.40
		0.7	87.24	90.54	94.35	97.34	93.98	90.28	86.98
		0.8	89.03	92.31	95.07	98.42	94.42	91.97	88.96
CSSC	+45/−45/−45/+45	0.2	88.38	87.50	87.31	89.34	94.80	99.92	95.76
		0.3	93.00	92.03	91.55	93.68	99.87	105.63	100.07
		0.4	93.56	96.05	96.20	98.53	104.13	100.57	95.39
		0.5	90.02	94.24	97.41	100.00	99.36	94.47	90.02
		0.6	88.70	91.62	94.04	97.43	97.00	92.29	87.97
		0.7	89.41	90.86	91.55	94.15	96.80	92.75	88.16
		0.8	88.72	89.14	88.39	89.74	93.65	93.67	89.02
	+45/−45/+45/−45	0.2	90.45	89.36	89.05	91.18	96.30	99.86	96.66
		0.3	94.43	93.77	93.58	96.07	102.18	107.04	99.83
		0.4	94.52	97.58	98.07	100.83	106.34	102.31	96.40
		0.5	90.12	94.36	97.22	100.00	101.46	96.22	91.35
		0.6	88.52	91.83	93.98	97.19	98.21	93.77	89.28
		0.7	89.72	91.65	91.76	94.27	97.79	94.00	89.62
		0.8	90.28	89.40	88.22	89.87	94.49	94.64	90.66
CSCS	+45/−45/−45/+45	0.2	89.91	93.51	96.27	99.84	97.59	94.81	91.27
		0.3	87.88	91.52	95.77	99.65	97.16	93.26	89.77
		0.4	85.55	89.64	94.88	99.37	96.33	91.21	87.12
		0.5	85.09	89.45	95.42	100.00	95.42	89.45	85.09
		0.6	87.12	91.21	96.33	99.37	94.88	89.64	85.55
		0.7	89.77	93.26	97.16	99.65	95.77	91.52	87.88
		0.8	91.11	94.75	97.58	100.16	96.20	93.49	89.81

	+45/−45/+45/ −45	0.2	91.02	94.01	96.45	99.50	97.09	94.47	90.94
		0.3	88.82	92.32	96.24	99.60	96.62	92.50	88.86
		0.4	86.21	90.38	95.54	99.31	95.68	90.47	86.27
		0.5	85.01	89.55	95.59	100.00	95.59	89.55	85.01
		0.6	86.27	90.47	95.68	99.31	95.54	90.38	86.21
		0.7	88.86	92.50	96.62	99.60	96.24	92.32	88.82
		0.8	90.80	94.45	97.06	100.07	96.42	94.00	90.90
CSSS	+45/−45/−45/+45	0.2	84.30	85.37	85.79	87.69	92.13	94.58	91.71
		0.3	87.64	91.46	90.90	92.68	97.44	94.86	90.53
		0.4	85.72	90.41	93.89	96.82	98.61	93.64	88.94
		0.5	85.06	89.82	95.48	100.00	97.95	92.41	87.59
		0.6	87.11	91.08	94.65	98.16	97.62	92.39	87.70
		0.7	89.12	91.14	92.29	94.60	98.30	93.53	88.79
		0.8	87.52	88.32	88.39	89.87	92.96	93.90	88.92
	+45/−45/+45/−45	0.2	88.51	87.70	87.16	88.52	92.22	95.28	91.11
		0.3	89.64	91.83	91.96	93.79	98.01	94.67	90.12
		0.4	86.75	91.19	94.79	98.08	98.61	93.27	88.45
		0.5	85.10	89.92	95.57	100.00	98.13	92.51	87.51
		0.6	86.31	90.42	93.71	97.00	97.83	92.88	88.13
		0.7	88.97	91.11	91.52	93.61	98.09	94.16	89.73
		0.8	88.27	87.75	87.42	88.94	92.82	94.91	90.64
SSSS	+45/−45/−45/+45	0.2	84.42	86.74	86.40	86.28	87.76	89.19	87.60
		0.3	86.94	92.06	91.78	91.50	92.55	92.48	89.30
		0.4	86.58	91.93	96.16	96.95	96.25	92.62	87.90
		0.5	86.40	91.58	97.00	100.00	97.00	91.58	86.40
		0.6	87.89	92.63	96.25	96.95	96.16	91.94	86.58
		0.7	89.30	92.48	92.55	91.50	91.78	92.06	86.94
		0.8	87.36	89.12	87.72	86.21	86.23	86.58	84.31

(continued)

Table 5.7 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
	+45/−45/+45/ −45	0.2	89.43	88.84	86.94	85.92	86.78	88.54	88.65
		0.3	89.03	93.12	92.17	91.19	91.70	92.48	88.75
		0.4	87.10	92.29	96.15	96.64	95.38	91.81	86.96
		0.5	86.05	91.27	96.70	100.00	96.70	91.27	86.05
		0.6	86.97	91.81	95.38	96.64	96.15	92.26	87.09
		0.7	88.76	92.48	91.70	91.19	92.15	93.12	89.03
		0.8	88.49	88.45	86.70	85.84	86.78	88.74	89.44
CS	+45/−45/−45/+45	0.2	106.01	108.77	105.52	103.66	105.06	107.21	106.36
		0.3	110.46	113.60	107.06	104.53	105.58	107.93	107.93
		0.4	106.01	107.21	103.43	101.54	103.02	105.84	105.58
		0.5	104.18	104.82	101.63	100.00	101.63	104.82	104.18
		0.6	105.58	105.84	103.02	101.54	103.40	107.23	106.01
		0.7	107.93	107.93	105.58	104.56	107.06	113.07	109.65
		0.8	106.22	107.12	104.91	103.57	105.29	108.69	105.78
	+45/−45/+45/−45	0.2	94.84	97.72	96.21	94.66	95.90	97.85	96.06
		0.3	98.60	102.64	99.71	97.72	99.01	101.06	98.70
		0.4	96.81	100.03	101.56	99.77	100.13	99.30	96.47
		0.5	95.18	98.06	99.90	100.00	99.90	98.06	95.18
		0.6	96.47	99.33	100.13	99.79	101.56	100.05	96.81
		0.7	98.73	101.09	98.99	97.72	99.71	102.64	98.60
		0.8	95.85	97.80	95.88	94.61	96.16	97.64	94.66

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 5.8 Maximum values of r with corresponding coordinates of cutout centres and zones where $r \geq 90$ for cross-ply spherical shells

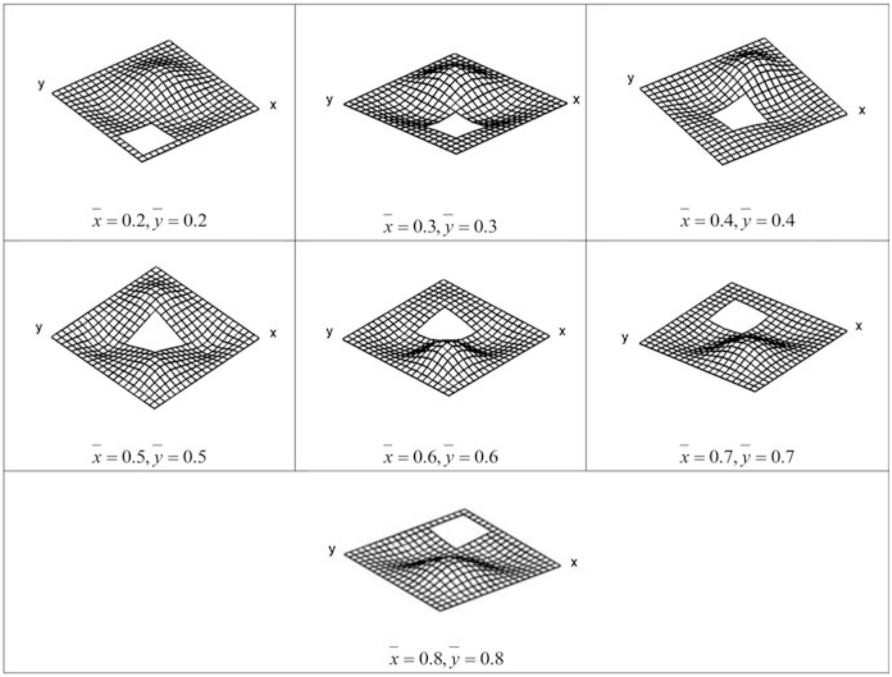
Laminations	0/90/90/0			0/90/0/90		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value of $r \geq 90$	Maximum values of r	Coordinate of cutout centre	Area in which the value of $r \geq 90$
CCCC	102.38	$\bar{x} = 0.2$ $\bar{y} = 0.5$	$0.2 \leq \bar{x}$ 0.8, $0.4 \leq \bar{y} \leq 0.6$	100.00	$\bar{x} = 0.5$ $\bar{y} = 0.5$	$0.4 \leq \bar{x}$ 0.6, $0.2 \leq \bar{y} \leq 0.8$; $0.2 \leq \bar{x}$ 0.3, $0.4 \leq \bar{y} \leq 0.6$; $0.7 \leq \bar{x}$ 0.8, $0.4 \leq \bar{y} \leq 0.6$
CSCC	105.27	$\bar{x} = 0.5$ $\bar{y} = 0.4$	$0.2 \leq \bar{x} \leq 0.8$, $0.3 \leq \bar{y} \leq 0.5$	104.31	$\bar{x} = 0.5$ $\bar{y} = 0.4$	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.5$
CSSC	102.24	$\bar{x} = 0.6$ $\bar{y} = 0.4$	$0.3 \leq \bar{x} \leq 0.8$, $0.4 \leq \bar{y} \leq 0.5$	102.36	$\bar{x} = 0.6$ $\bar{y} = 0.4$	$0.5 \leq \bar{x} \leq 0.7$, $0.3 \leq \bar{y} \leq 0.6$
CSCS	100.00	$\bar{x} = 0.5$ $\bar{y} = 0.5$	$0.2 \leq \bar{x} \leq 0.8$, $\bar{y} = 0.5$	100.00	$\bar{x} = 0.5$ $\bar{y} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$, $0.4 \leq \bar{y} \leq 0.6$
CSSS	101.42	$\bar{x} = 0.6$ $\bar{y} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$, $0.4 \leq \bar{y} \leq 0.6$	101.42	$\bar{x} = 0.6$ $\bar{y} = 0.5$	$0.4 \leq \bar{x} \leq 0.7$, $0.4 \leq \bar{y} \leq 0.6$
SSSS	100.00	$\bar{x} = 0.5$ $\bar{y} = 0.5$	$0.3 \leq \bar{x} \leq 0.7$, $0.4 \leq \bar{y} \leq 0.6$	100.00	$\bar{x} = 0.5$ $\bar{y} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$, $0.4 \leq \bar{y} \leq 0.6$
CS	107.00	$\bar{x} = 0.2$ $\bar{y} = 0.5$	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.8$	102.24	$\bar{x} = 0.5$, $\bar{y} = 0.2$ $\bar{x} = 0.5$, $\bar{y} = 0.8$	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.8$

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 5.9 Maximum values of r with corresponding coordinates of cutout centres and zones where $r \geq 90$ for angle-ply spherical shells

Laminations	+45/−45/−45/+45			+45/−45/+45/−45		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value of $r \geq 90$	Maximum values of r	Coordinate of cutout centre	Area in which the value of $r \geq 90$
CCCC	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.5$	100.00	$\bar{x} = 0.5$	$\bar{x} = 0.5$, $0.4 \leq \bar{y} \leq 0.6$; $0.4 \leq \bar{x} \leq 0.6$, $\bar{y} = 0.5$
		$\bar{y} = 0.5$	$0.5 \leq \bar{y} \leq 0.6$		$\bar{y} = 0.5$	
CSCC	105.82	$\bar{x} = 0.5$	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.4$; $0.3 \leq \bar{x} \leq 0.6$, $0.5 \leq \bar{y} \leq 0.8$	106.35	$\bar{x} = 0.5$	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.4$; $0.3 \leq \bar{x} \leq 0.6$, $0.7 \leq \bar{y} \leq 0.8$
		$\bar{y} = 0.3$			$\bar{y} = 0.3$	
CSSC	104.13	$\bar{x} = 0.7$	$0.2 \leq \bar{x} \leq 0.8$, $0.3 \leq \bar{y} \leq 0.5$; $0.3 \leq \bar{x} \leq 0.7$, $0.6 \leq \bar{y} \leq 0.7$	107.04	$\bar{x} = 0.7$	$0.5 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.5$; $0.2 \leq \bar{x} \leq 0.4$, $0.3 \leq \bar{y} \leq 0.5$
		$\bar{y} = 0.3$			$\bar{y} = 0.3$	
CSCS	100.16	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7$, $0.2 \leq \bar{y} \leq 0.3$; $0.4 \leq \bar{x} \leq 0.6$, $0.4 \leq \bar{y} \leq 0.8$	100.00	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7$, $0.2 \leq \bar{y} \leq 0.8$
		$\bar{y} = 0.8$			$\bar{y} = 0.5$	
CSSS	100.00	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7$, $0.3 \leq \bar{y} \leq 0.7$	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.7$, $0.3 \leq \bar{y} \leq 0.7$
		$\bar{y} = 0.5$			$\bar{y} = 0.5$	
SSSS	100.00	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7$, $0.3 \leq \bar{y} \leq 0.7$	100.00	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7$, $0.3 \leq \bar{y} \leq 0.7$
		$\bar{y} = 0.5$			$\bar{y} = 0.5$	
CS	113.60	$\bar{x} = 0.3$	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.8$	102.64	$\bar{x} = 0.7$, $\bar{y} = 0.7$	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.8$
		$\bar{y} = 0.3$			$\bar{x} = 0.3$, $\bar{y} = 0.3$	

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

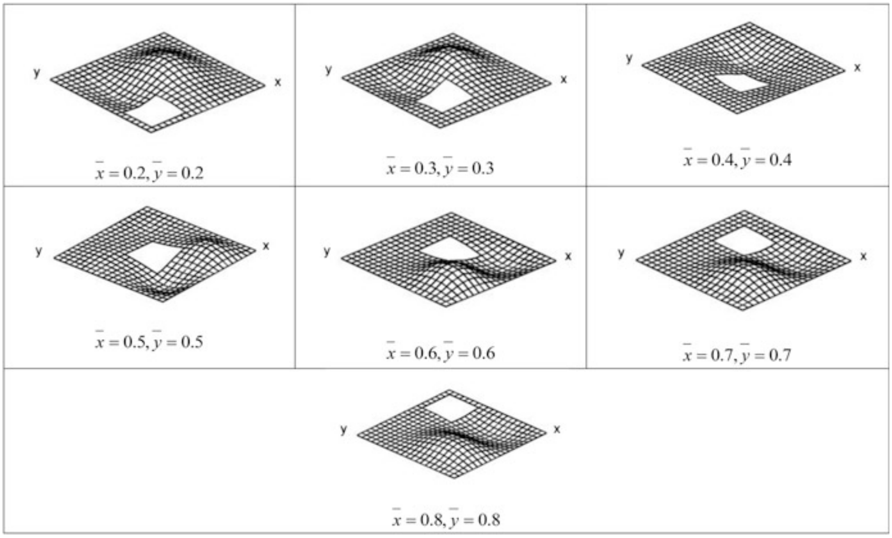


$\bar{x} = 0.2, \bar{y} = 0.2$ $\bar{x} = 0.3, \bar{y} = 0.3$ $\bar{x} = 0.4, \bar{y} = 0.4$

$\bar{x} = 0.5, \bar{y} = 0.5$ $\bar{x} = 0.6, \bar{y} = 0.6$ $\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 5.6 First mode shapes of laminated composite (0/90/0/90) stiffened spherical shell for different positions of square cutout with CCCC boundary condition

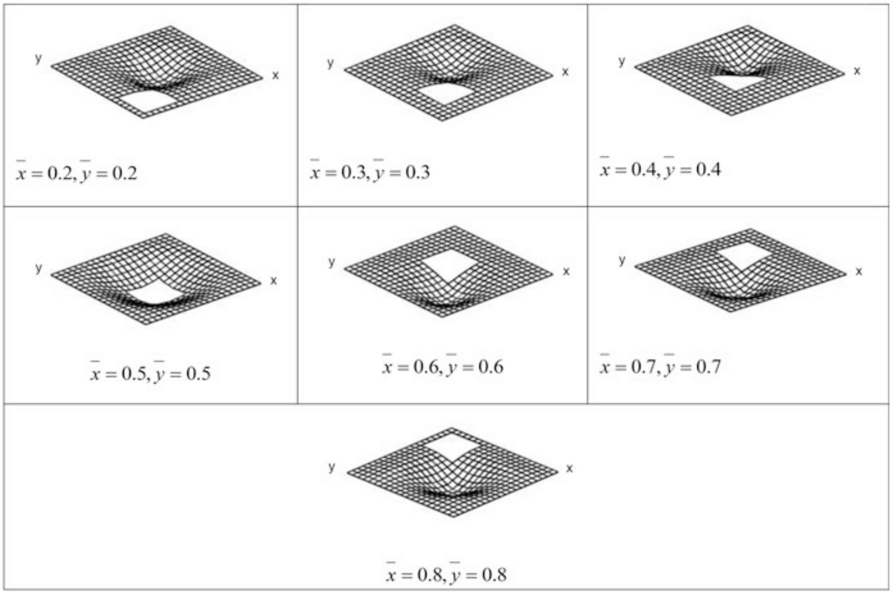


$\bar{x} = 0.2, \bar{y} = 0.2$ $\bar{x} = 0.3, \bar{y} = 0.3$ $\bar{x} = 0.4, \bar{y} = 0.4$

$\bar{x} = 0.5, \bar{y} = 0.5$ $\bar{x} = 0.6, \bar{y} = 0.6$ $\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 5.7 First mode shapes of laminated composite (0/90/0/90) stiffened spherical shell for different positions of square cutout with CSCC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

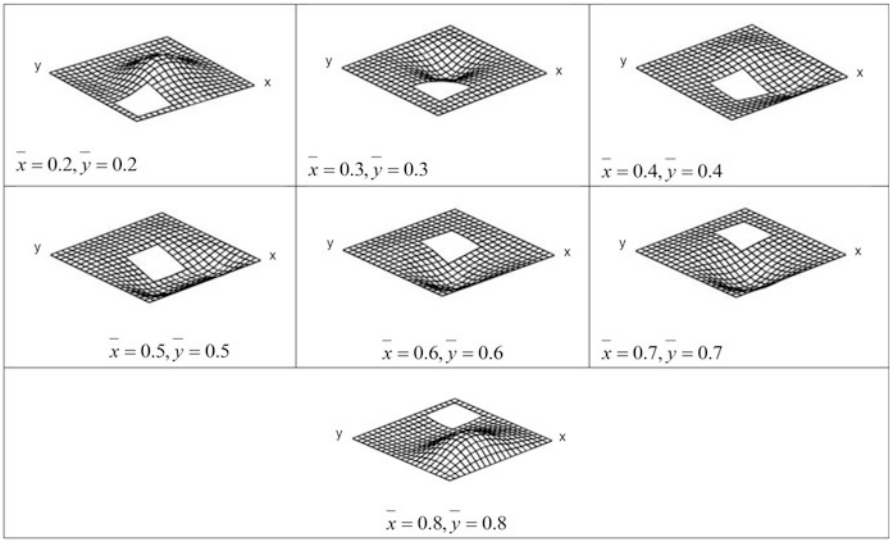
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 5.8 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened spherical shell for different positions of square cutout with CCCC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 5.9 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened spherical shell for different positions of square cutout with CSCC boundary condition

5.4 Closure

Free vibration characteristics of stiffened spherical panels in presence of cutouts show that the arrangement of boundary constraints along the edges is more important than their actual number. If fully clamped shell is released for any functional reason, then opposite edges must be released to avoid excessive loss in frequency. The relative free vibration performances of shells for different combinations of edge constraints along four sides will be useful in decision-making for practising engineers. The data on the behaviour of stiffened spherical shell with eccentric cutouts for a wide range of eccentricity and edge conditions for both cross-ply and angle-ply shells may be useful as design tools for structural engineers dealing with such shell forms.

References

Chakravorty D, Sinha PK, Bandyopadhyay JN (1995) Free vibration analysis of point supported laminated composite doubly curved shells- a finite element approach. Comput Struct 54(2):191–207

Chakravorty D, Sinha PK, Bandyopadhyay JN (1996) Finite element free vibration analysis of doubly curved laminated composite shells. J Sound Vib 191(4):491–504

Chakravorty D, Sinha PK, Bandyopadhyay JN (1998) Applications of FEM on free and forced vibration of laminated shells. ASCE J Eng Mech 124(1):1–8

Chao WC, Reddy JN (1984) Analysis of laminated composite shells using a degenerated 3-d element. Int J Numer Methods Eng 20:1991–2007

Chao CC, Tung TP (1989) Step pressure and blast response of clamped orthotropic hemispherical shells. Int J Impact Eng 8(3):191–207

Chia CY, Chia DS (1997) Nonlinear vibration of moderately thick anti-symmetric angle-ply shallow spherical shell. Comput Struct 44(4):797–805

Dai HL, Wang X (2005) Stress wave propagation in laminated piezoelectric spherical shells under thermal shock and electric excitation. Eur J Mech A Solids 24:263–276

Ganapathi M (2007) Dynamic stability characteristics of functionally graded materials shallow spherical shells. Compos Struct 79:338–343

Gautham BP, Ganesan N (1997) Free vibration characteristics of isotropic and laminated orthotropic spherical caps. J Sound Vib 204(1):17–40

Kant T, Kumar S, Singh UP (1994) Shell dynamics with three dimensional finite elements. Comput Struct 50(1):135–146

Kapania RK (1989) A review on the analysis of laminated shells. J Press Vessel Technol 111(2):88–96

Lellep J, Hein H (2002) Optimization of clamped plastic shallow shells subjected to initial impulsive loading. Eng Optim 34:545–556

Liew KM, Lim CW (1994) Vibration of perforated doubly-curved shallow shells with rounded corners. Int J Solids Struct 31(11):1519–1536

Liew KM, Lim CW (1995) Vibration behaviour of doubly-curved shallow shells of curvilinear planform. J Eng Mech 121(12):1277–1283

Noor AK, Burton WS (1990) Assessment of computational models for multi-layered composite shells. Appl Mech Rev 43:67–97

Park T, Lee SY (2009) Parametric instability of delaminated composite spherical shells subjected to in-plane pulsating forces. Compos Struct 91:196–204

Qatu MS (1991) Natural frequencies for cantilevered doubly-curved laminated composite shallow shells. Compos Struct 17:227–255

Qatu MS, Sullivan RW, Wang W (2010) Recent research advances on the dynamic analysis of composite shells: 2000–2009. Compos Struct 93:14–31

Reddy JN (1981) Finite element modeling of layered, anisotropic composite plates and shells: review of recent research. Shock Vib Dig 13(12):3–12

Reddy JN, Chandrashekhara K (1985) Geometrically nonlinear transient analysis of laminated doubly curved shells. Int J Nonlin Mech 20(2):79–90

Sahoo S (2014) Laminated composite stiffened shallow spherical panels with cutouts under free vibration – a finite element approach. Eng Sci Technol Int J 17(4):247–259

Sathyamoorthy M (1995) Nonlinear vibrations of moderately thick orthotropic shallow spherical shells. Comput Struct 57(1):59–65

Shin DK (1997) Large amplitude free vibration behavior of doubly curved shallow open shells with simply supported edges. Comput Struct 62(1):35–49

Sivasubramonian B, Kulkarni AM, Venkateswara Rao G, Krishnan A (1997) Free vibration of curved panels with cutouts. J Sound Vib 200(2):227–234

Tan DY (1998) Free vibration analysis of shells of revolution. J Sound Vib 213(1):15–33

Wang X, Lu G, Guillow SR (2002) Stress wave propagation in orthotropic laminated thick-walled spherical shells. Int J Solids Struct 39:3027–4037

Chapter 6
Stiffened Saddle Shell with Cutout

Abstract In this chapter, free vibration characteristics of laminated composite stiffened saddle shells with cutouts are studied using finite element method employing eight-noded curved quadratic isoparametric element for shell with a three-noded curved beam element for stiffener. Free vibration behaviour of stiffened saddle shells with varying size and position of the cutouts with respect to the shell centre for different edge constraints is studied to arrive at useful conclusions for the shell designers. The results are analysed to provide guidelines for the selection of optimum size and position of the cutout with respect to shell centre considering various practical boundary conditions.

Keywords Saddle shell • Cutout • Eccentricity • Fundamental frequency • Mode shapes

6.1 Introduction

Saddle shell is one of the different shell forms used as roofing units. Examples of such saddle roofs are Warszawa Ochota railway station, Church Army Chapel, Blackheath, the Calgary Saddledome and London VeloPark. Kapania (1989), Noor and Burton (1990) and Reddy (1981) proposed various computational models for laminated composites. Chao and Reddy (1984) considered the dynamic response of simply supported cylindrical and spherical shells. Reddy and Chandrashekhara (1985) considered the transient response of spherical and cylindrical shells with various boundary conditions and loading. Chao and Tung (1989) investigated the dynamic response of axisymmetric polar orthotropic hemispherical shells. Later on, a number of researchers like Qatu (1991), Liew and Lim (1994, 1995), Chakravorty et al. (1995, 1996, 1998), Shin (1997) and Tan (1998) studied free vibration of doubly curved shells. Kant et al. (1994) studied clamped spherical and simply supported cylindrical cap under external pressure. Sathyamoorthy (1995) reported the nonlinear vibration of moderately thick orthotropic spherical shells. Gautham and Ganesan (1997) studied isotropic and laminated orthotropic spherical caps under free vibration, while Chia and Chia (1997) considered antisymmetric angle-ply shallow spherical shell having moderate thickness and subjected to

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nonlinear vibration. Free vibration of curved panels having cutouts was studied by Sivasubramonian et al. (1997). Qatu et al. (2010) reviewed the vibration studies of composite shells between 2000 and 2009 and observed that most of the researchers concentrated on closed cylindrical shells. Other shell forms like conical shells and shallow shells on rectangular, triangular, trapezoidal, circular, elliptical, rhombic or other planforms also received considerable attention. Shallow spherical shells also received some attention. Wang et al. (2002) investigated wave propagation of stresses in orthotropic thick-walled spherical shells. Lellep and Hein (2002) considered shallow spherical shells under impact loading for optimization. Dai and Wang (2005) analysed stress wave propagation in laminated piezoelectric spherical shells subjected to thermal shock and electric excitation. Dynamic stability of spherical shells was evaluated by Ganapathi (2007) and Park and Lee (2009). But, saddle shells on rectangular planform with cutout (stiffened along the margin) are rare in the existing literature (Sahoo 2014). The present chapter focuses on the free vibration behaviour of composite saddle shell with cutout (stiffened along the margin), with concentric and eccentric cutouts, and considers the shells to have various combinations of edge conditions.

6.2 Problem

Figure 6.1 shows a saddle shell having a concentric cutout stiffened along the margin. In a view to study the effect of cutout size and position on the free vibration response, saddle shells with different cross- and angle-ply laminations (both symmetric and antisymmetric) and different boundary conditions are considered. Among the laminations considered in the first phase of the study, the best two both from cross-ply and angle-ply are chosen for further study. The positions of the cutouts are varied along both of the plan directions of these shells for different practical edge conditions to study the effect of eccentricity of cutout on dynamic response.

6.3 Results and Discussion

6.3.1 *Free Vibration Behaviour of Shells with Concentric Cutouts*

Tables 6.1, 6.2 and 6.3 furnish the results of nondimensional frequency ($\overline{\omega}$) of four-layered symmetric and antisymmetric cross-ply and angle-ply stiffened saddle shells with cutout.

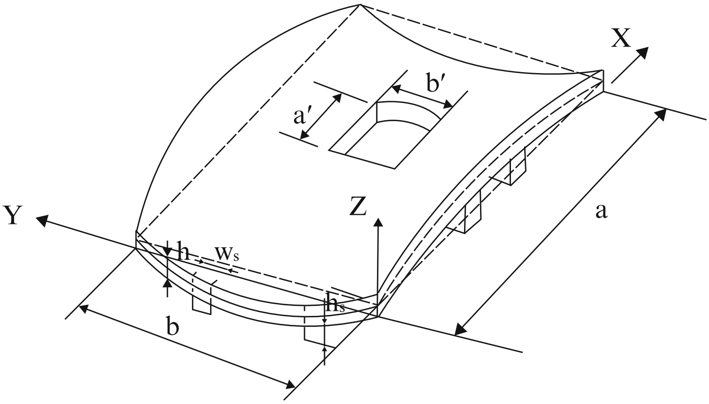


Fig. 6.1 Saddle shell with a concentric cutout stiffened along the margins

6.3.1.1 Effect of Cutout Size on Fundamental Frequency

From Tables 6.1, 6.2 and 6.3, it is observed that when a cutout is provided in a stiffened shell, the fundamental frequency improves in most of the cases, except some cases of clamped shells. This improving trend is observed for both cross-ply and angle-ply shells. It is evident from Tables 6.1, 6.2 and 6.3 that in all the cases with the introduction of cutout with $a'/a = 0.1$, the frequencies improve. With further increase in cutout size, fundamental frequencies also increase. As with the introduction of a cutout of $a'/a = 0.4$, in shell surface, the frequency increases in most of the cases; this leads to the practical consideration that concentric cutouts with stiffened margins may be introduced safely on shell surfaces for functional purposes up to $a'/a = 0.4$.

6.3.1.2 Effect of Boundary Conditions on Fundamental Frequency

The boundary conditions may be arranged in the following order, considering number of boundary constraints: CCCC, CSCC, CSSC, CSCS, CSSS, SSSS and corner point supported.

Tables 6.4 and 6.5 furnish the efficiencies of particular shells with different numbers of edge constraints relative to that of a clamped shell. Marks are assigned to each boundary combination in a scale assigning a value of 0 to the frequency of a corner point supported shell and 100 to that of a fully clamped shell. These marks are furnished for cutouts with $a'/a = 0.2$. It is seen from Tables 6.4 and 6.5 that fundamental frequencies of members belonging to the same number of boundary constraints may not have close values. So the boundary constraint is not the sole criteria for its performance. The free vibration characteristics mostly depend on the arrangement of boundary constrains rather than their actual number is evident from the present study. It can be seen from the present study that if one edge is

Table 6.1 Nondimensional fundamental frequencies ($\overline{\varpi}$) for laminated composite stiffened saddle shell for different sizes of central square cutout, different laminations and boundary conditions

		Cutout size (a'/a)				
Boundary conditions	Laminations	0	0.1	0.2	0.3	0.4
Clamped	0/90	69.78	85.51	105.90	112.27	118.89
	90/0	69.99	85.83	106.36	112.14	118.90
	0/90/0	128.90	110.47	130.60	137.48	137.12
	90/0/90	128.08	111.26	132.37	137.39	136.69
	0/90/0/90	93.16	107.18	129.70	135.35	138.56
	90/0/90/0	98.06	107.28	129.87	135.27	138.51
	0/90/90/0	115.67	111.59	132.59	137.90	137.91
	90/0/0/90	134.80	112.14	133.90	137.88	137.41
	+45/−45	74.22	87.58	96.68	100.54	102.48
	+45/−45/+45	79.53	96.27	101.31	106.02	111.11
	+45/−45/+45/−45	81.34	98.71	103.20	107.23	112.06
	+45/−45/−45/+45	80.92	98.08	102.70	107.19	112.36
Simply supported	0/90	45.68	49.83	64.16	68.77	69.05
	90/0	45.92	49.91	64.51	69.21	69.34
	0/90/0	48.10	63.13	67.68	70.85	73.32
	90/0/90	49.99	65.24	68.94	71.67	73.51
	0/90/0/90	49.10	63.99	68.27	71.36	73.77
	90/0/90/0	49.22	64.13	68.47	71.59	73.95
	0/90/90/0	48.72	63.74	68.13	71.27	73.80
	90/0/0/90	50.07	65.31	69.09	71.89	73.97
	+45/−45	48.37	60.68	68.26	71.66	73.62
	+45/−45/+45	49.93	65.49	69.92	72.42	76.43
	+45/−45/+45/−45	51.61	67.00	71.16	74.09	77.28
	+45/−45/−45/+45	50.90	66.43	70.65	73.87	77.05
Point supported	0/90	19.84	24.43	29.05	35.56	42.41
	90/0	19.71	23.27	28.87	35.44	42.41
	0/90/0	21.64	25.67	30.82	39.98	39.10
	90/0/90	21.86	24.87	30.13	35.13	39.98
	0/90/0/90	27.81	30.65	35.72	41.93	47.84
	90/0/90/0	27.71	30.54	35.61	41.85	47.79
	0/90/90/0	24.91	28.66	34.21	40.73	48.36
	90/0/0/90	25.11	27.83	32.79	39.02	45.27
	+45/−45	23.80	27.85	33.59	40.19	43.36
	+45/−45/+45	26.54	29.89	34.14	39.33	42.25
	+45/−45/+45/−45	34.28	36.70	42.48	45.71	47.15
	+45/−45/−45/+45	30.11	33.76	38.34	44.39	47.04

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = -h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 6.2 Nondimensional fundamental frequencies ($\overline{\varpi}$) for laminated composite cross-ply stiffened saddle shell for different sizes of central square cutout and different boundary conditions

		Cutout size (a'/a)				
Laminations	Boundary conditions	0	0.1	0.2	0.3	0.4
0/90/90/0	CCCC	115.67	111.59	132.59	137.90	137.91
	CSCC	75.82	89.04	103.47	118.77	120.59
	CSSC	62.82	73.75	82.13	87.56	91.39
	CSCS	77.79	83.20	101.79	113.39	116.05
	CSSS	56.45	70.71	75.43	79.43	82.74
	SSSS	48.72	63.74	68.13	71.27	73.80
	Point supported	24.91	28.65	34.21	40.73	48.36
0/90/0/90	CCCC	93.08	107.18	129.7	135.35	138.56
	CSCC	79.68	92.13	108.7	120.34	121.57
	CSSC	61.39	72.13	81.88	87.4	91.17
	CSCS	84.89	87.82	107.4	116.61	118.1
	CSSS	57.39	69.92	76.06	80.13	83.51
	SSSS	49.1	63.99	68.27	71.36	73.77
	Point supported	27.81	30.62	35.72	41.93	47.84

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = -h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 6.3 Nondimensional fundamental frequencies ($\overline{\varpi}$) for laminated composite angle-ply stiffened saddle shell for different sizes of central square cutout and different boundary conditions

		Cutout size (a'/a)				
Laminations	Boundary conditions	0	0.1	0.2	0.3	0.4
+45/−45/−45/+45	CCCC	80.92	98.08	102.70	107.19	112.36
	CSCC	75.67	91.40	94.89	98.34	101.35
	CSSC	68.59	82.22	86.11	89.02	91.64
	CSCS	72.50	87.03	90.79	94.06	97.52
	CSSS	63.84	77.19	81.01	84.20	87.16
	SSSS	50.90	66.43	70.65	73.87	77.05
	Point supported	30.11	33.76	38.34	44.39	47.04
+45/−45/+45/−45	CCCC	81.34	98.71	103.2	107.23	112.06
	CSCC	75.81	91.13	95.02	98.14	101.27
	CSSC	68.36	81.17	85.72	89	92.23
	CSCS	73	87.56	91.25	94.6	97.53
	CSSS	64.37	77.65	81.45	84.33	87.34
	SSSS	51.61	67	71.16	74.09	77.28
	Point supported	34.28	36.7	42.48	45.71	47.15

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = -h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

released from clamped to simply supported, the change of frequency is not very significant for angle-ply shells. But in the case of cross-ply shells, this decrease in frequency is large. For cross-ply shells, if the two alternate edges are released from

Table 6.4 Clamping options for cross-ply saddle shells with central cutouts having a'/a ratio 0.2

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of no. of restraints
0/90/90/0	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Good improvement	34
	1	One edge (CSSS)	Good improvement	42
	2	(a) Two alternate edges (CSCS)	Marked improvement	69
		(b) Two adjacent edges (CSSC)	Good improvement	49
	3	Three edges (CSCC)	Marked improvement	70
	4	All sides (CCCC)	Frequency attains highest value	100
0/90/0/90	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Good improvement	35
	1	One edge (CSSS)	Good improvement	43
	2	(a) Two alternate edges (CSCS)	Marked improvement	76
		(b) Two adjacent edges (CSSC)	Good improvement	49
	3	Three edges (CSCC)	Marked improvement	78
	4	All sides (CCCC)	Frequency attains highest value	100

clamped to simply supported, fundamental frequency does not change to a great extent, whereas if the two adjacent edges are released, fundamental frequency decreases significantly than that of a clamped shell. For angle-ply shells, these changes in frequencies are less. For both types of shells, if all the edges are simply supported, frequency values undergo marked decrease. The results indicate that the two alternate edges should preferably be clamped in order to achieve higher frequency values.

Table 6.5 Clamping options for angle-ply saddle shells with central cutouts having a'/a ratio 0.2

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of no. of restraints
+45/−45/−45/+45	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Good improvement	50
	1	One edge (CSSS)	Marked improvement	66
	2	(a) Two alternate edges (CSCS)	Remarkable improvement	82
		(b) Two adjacent edges (CSSC)	Marked improvement	74
	3	Three edges (CSCC)	Remarkable improvement	88
	4	All sides (CCCC)	Frequency attains highest value	100
+45/−45/+45/−45	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Good improvement	47
	1	One edge (CSSS)	Marked improvement	64
	2	(a) Two alternate edges (CSCS)	Remarkable improvement	80
		(b) Two adjacent edges (CSSC)	Marked improvement	71
	3	Three edges (CSCC)	Remarkable improvement	87
	4	All sides (CCCC)	Frequency attains highest value	100

These tables will help a shell designer to observe at a glance the efficiency of a particular boundary combination in improving the frequency of a shell, taking that of clamped shell as the upper limit.

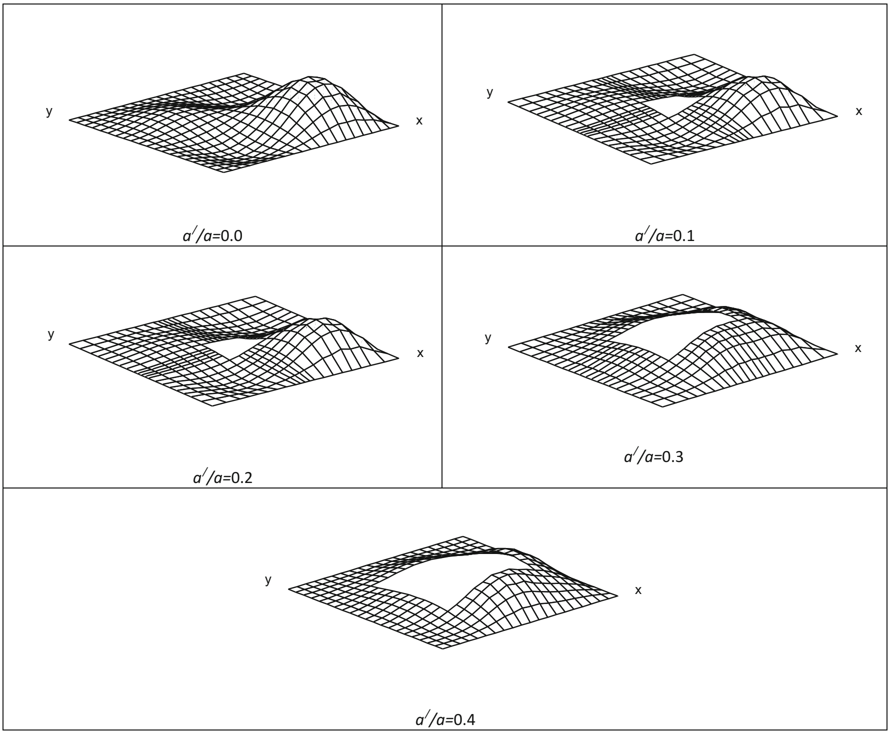
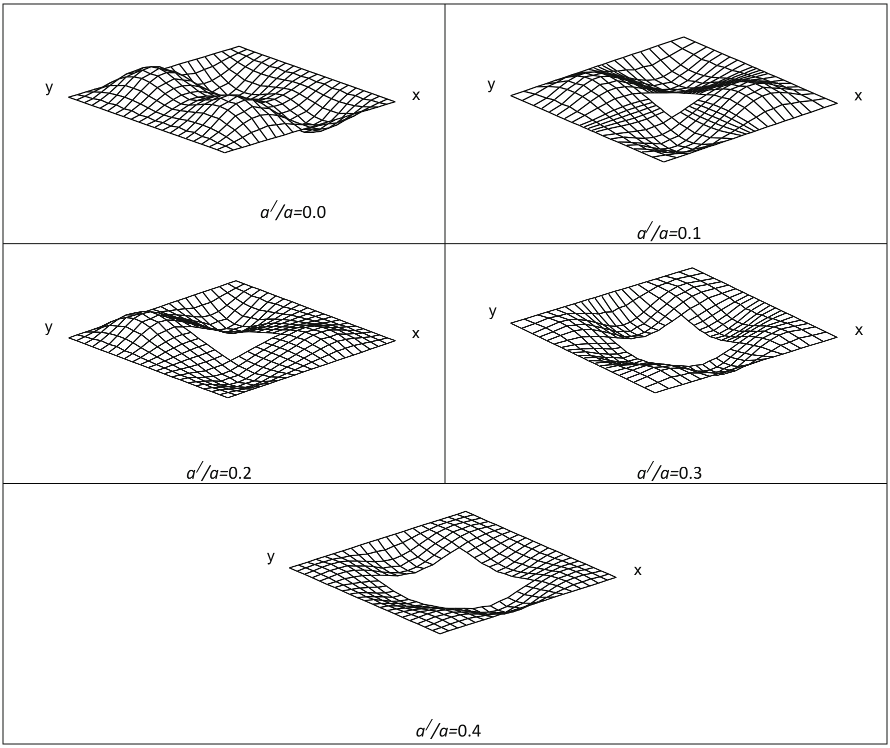


Fig. 6.2 First mode shapes of laminated composite (0/90/0/90) stiffened saddle shell for different sizes of central square cutout and CSSC boundary condition

6.3.1.3 Mode Shape

The mode shapes corresponding to the fundamental modes of vibration are plotted in Figs. 6.2, 6.3, 6.4 and 6.5 for cross-ply and angle-ply shells with CSSC and CSCS boundary conditions for different sizes of the cutout. The fundamental mode is a bending mode for all the edge conditions for cross-ply and angle-ply shell, except corner point supported shell. For corner point supported shells, the fundamental mode shapes are complicated. With the introduction of cutout, mode shapes do not change much. When the size of the cutout is increased from 0.2 to 0.4, the fundamental modes of vibration do not change to an appreciable amount.



$a'/a=0.0$

$a'/a=0.1$

$a'/a=0.2$

$a'/a=0.3$

$a'/a=0.4$

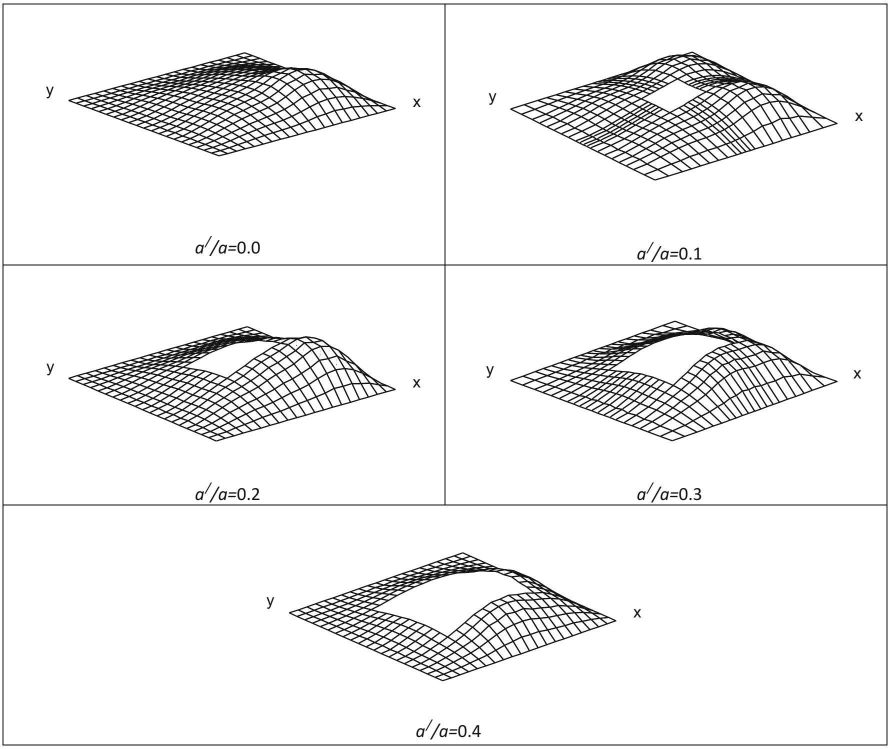
Fig. 6.3 First mode shapes of laminated composite (0/90/0/90) stiffened saddle shell for different sizes of central square cutout and CSCS boundary condition

6.3.2 *Effect of Eccentricity of Cutout Position*

6.3.2.1 Fundamental Frequency

The effect of eccentricity of cutout positions on fundamental frequencies is considered from the results obtained for different locations of a cutout with $a'/a = 0.2$. The nondimensional coordinates of the cutout centre $\left(\bar{x} = \frac{x}{a}, \bar{y} = \frac{y}{a}\right)$ were varied from 0.2 to 0.8 along each direction, so that the distance of a cutout margin from the shell boundary did not fall below one tenth of the plan dimension of the shell. The margins of cutouts were stiffened with four stiffeners. The study was carried out for all the seven boundary conditions for both cross-ply and angle-ply shells. The fundamental frequency of a shell with an eccentric cutout is expressed as a percentage of fundamental frequency of a shell with a concentric cutout. This percentage is denoted by r. In Tables 6.6 and 6.7, such results are furnished.

It can be observed that eccentricity of the cutout along the length of the shell towards the edges makes it more flexible. It is also seen that almost in all the cases, r value is maximum in and around $\bar{x} = 0.5$ and $\bar{y} = 0.5$. It is noticed that towards the



$a'/a=0.0$ $a'/a=0.1$ $a'/a=0.2$ $a'/a=0.3$ $a'/a=0.4$

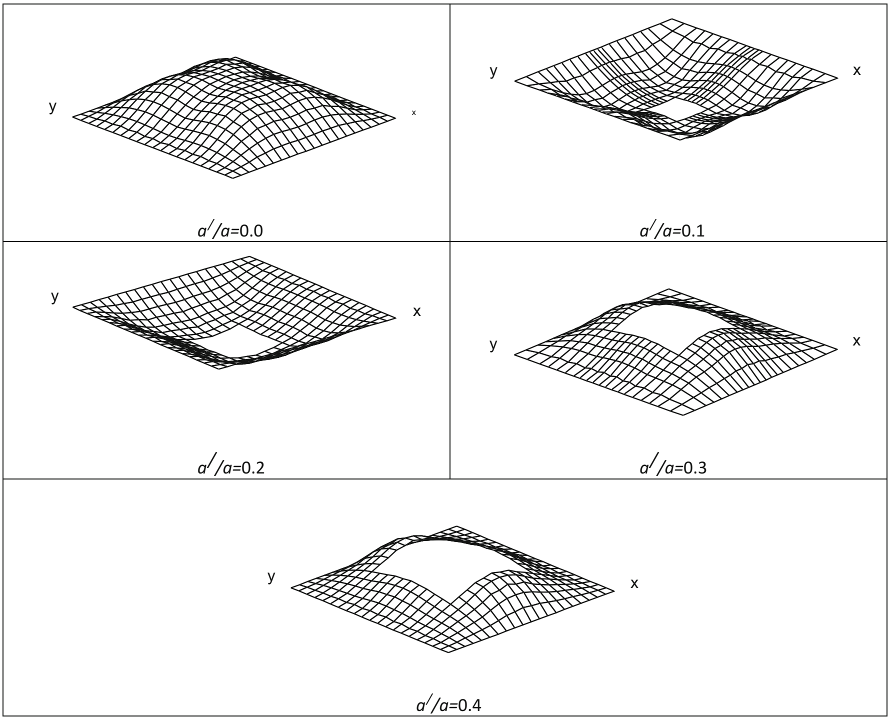
Fig. 6.4 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened saddle shell for different sizes of central square cutout and CSSC boundary condition

simply supported edge *r* value is greater than that of the clamped edge when only one edge is simply supported. When two adjacent edges are simply supported, the same trend is observed. But for cross-ply shells, reverse trend is observed when alternate edges are simply supported. Moreover, when three edges are simply supported, the simply supported edge opposite to the clamped one shows more frequency value than that of the clamped edge.

Tables 6.8 and 6.9 provide the maximum values of *r* together with the position of the cutout. These tables also identify the rectangular zones within which *r* is always greater than or equal to 90. These tables indicate the maximum eccentricity of a cutout which can be permitted if the fundamental frequency of a concentrically punctured shell needs not to reduce a drastic amount.

6.3.2.2 Mode Shape

The mode shapes corresponding to the fundamental modes of vibration are plotted in Figs. 6.6, 6.7, 6.8 and 6.9 for cross-ply and angle-ply shells of CCCC and CSCC boundary conditions for different eccentric positions of the cutouts. All the mode shapes



$a'/a=0.0$

$a'/a=0.1$

$a'/a=0.2$

$a'/a=0.3$

$a'/a=0.4$

Fig. 6.5 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened saddle shell for different sizes of central square cutout and CSCS boundary condition

are bending mode. It is observed that for different positions of cutouts, mode shapes are somewhat similar to one another and only the crest and trough position changes.

6.4 Closure

The arrangement of boundary constraints along the four edges rather than their actual number is more important. If cross-ply fully clamped shell is released for any functional reason, then opposite edges must be released to avoid excessive loss in frequency. The practising engineers will be benefitted from the relative free vibration performances of shells for different combinations of edge conditions along the four sides. The data on the behaviour of stiffened saddle shell with eccentric cutouts for a wide range of eccentricity and edge conditions for cross-ply and angle-ply shells may also be used as design aids for structural designers. The present study identifies the specific zones within which the cutout centre may be moved so that the loss of frequency is less than 10 % with respect to a shell with a central cutout. That will help an engineer to make a decision regarding the maximum allowance of eccentricity of the cutout centre.

Table 6.6 Values of ‘*r*’ for cross-ply saddle shells

			$\bar{x}$						
Edge condition	Laminations	$\bar{y}$	0.2	0.3	0.4	0.5	0.6	0.7	0.8
CCCC	0/90/90/0	0.2	83.66	82.46	83.53	85.99	83.53	82.46	83.63
		0.3	85.79	84.96	86.23	88.66	86.23	84.96	85.78
		0.4	90.82	90.75	92.32	94.40	92.32	90.75	90.90
		0.5	97.22	96.11	98.22	100.00	98.21	96.09	96.84
		0.6	90.88	90.75	92.32	94.40	92.32	90.75	91.97
		0.7	85.81	84.96	86.23	88.66	86.23	84.96	85.78
		0.8	83.54	82.39	83.46	85.99	83.51	82.43	83.57
	0/90/0/90	0.2	91.20	85.18	88.20	91.75	88.17	85.18	85.03
		0.3	84.39	85.22	89.00	93.85	89.00	85.22	84.39
		0.4	86.94	88.31	92.41	96.84	92.41	88.31	86.94
		0.5	90.16	91.96	96.15	100.00	96.15	91.96	90.16
		0.6	86.94	88.31	92.41	96.84	92.41	88.31	86.94
		0.7	84.39	85.22	89.00	93.64	89.00	85.22	84.39
		0.8	84.76	84.97	88.01	91.77	88.13	85.07	87.34
CSCC	0/90/90/0	0.2	90.60	89.95	89.82	89.68	89.82	89.94	90.57
		0.3	96.52	96.87	97.92	98.50	97.92	96.87	96.51
		0.4	99.68	102.72	107.84	110.72	107.82	102.71	99.61
		0.5	96.39	96.96	98.87	100.00	98.90	96.97	95.95
		0.6	87.67	86.06	86.18	86.51	86.29	86.10	95.76
		0.7	80.49	78.85	78.74	79.05	78.83	78.89	80.07
		0.8	76.84	75.31	75.24	75.58	75.32	75.36	76.48
	0/90/0/90	0.2	91.12	90.60	91.90	93.11	91.90	90.60	91.10
		0.3	94.86	95.19	97.57	99.66	97.57	95.18	94.86
		0.4	95.93	99.74	105.62	109.00	106.03	99.76	95.88
		0.5	93.08	93.79	97.03	100.00	97.33	93.89	92.79

		0.6	86.93	85.83	87.77	89.93	87.92	85.92	86.50
		0.7	82.66	81.28	82.85	84.66	82.94	81.34	82.26
		0.8	80.75	79.37	80.75	82.34	80.82	79.43	80.40
CSSC	0/90/90/0	0.2	68.16	73.37	81.21	91.82	94.67	85.74	77.84
		0.3	73.80	79.72	88.31	98.84	100.77	92.01	83.68
		0.4	77.18	83.71	93.15	103.94	105.50	96.63	87.54
		0.5	74.69	80.19	88.51	100.00	103.75	93.85	84.61
		0.6	67.93	71.87	78.53	89.52	96.01	85.79	77.40
		0.7	61.82	65.16	71.35	82.22	88.70	78.91	71.28
		0.8	58.13	61.54	67.89	78.94	84.38	74.78	67.54
	0/90/0/90	0.2	67.33	73.00	81.56	92.87	96.24	87.04	78.21
		0.3	72.96	79.10	88.04	99.06	101.48	92.50	83.52
		0.4	76.94	83.79	93.47	104.13	105.86	97.08	87.64
		0.5	73.28	78.60	87.24	100.00	104.47	94.43	83.29
		0.6	65.80	70.00	77.99	91.48	98.34	87.23	77.64
		0.7	60.39	64.56	72.67	86.24	92.93	81.74	72.47
		0.8	57.62	62.10	70.46	83.79	89.52	78.58	69.52
CSCS	0/90/90/0	0.2	73.62	73.16	72.13	71.46	72.13	73.16	73.62
		0.3	78.67	77.98	77.30	76.91	77.30	77.98	78.67
		0.4	86.83	86.06	85.90	85.79	85.90	86.06	86.83
		0.5	94.54	95.95	98.55	100.00	98.55	95.95	94.54
		0.6	86.83	86.06	85.90	85.79	85.90	86.06	86.83
		0.7	78.67	77.98	77.30	76.91	77.30	77.98	78.67
		0.8	73.61	73.16	72.12	71.43	72.11	73.16	73.61
	0/90/0/90	0.2	78.77	78.00	78.58	79.36	78.58	78.00	78.77
		0.3	81.64	80.85	81.94	83.18	81.94	80.85	81.64
		0.4	86.58	86.04	87.77	89.54	87.77	86.04	86.58

(continued)

Table 6.6 (continued)

Edge condition	Laminations	$\overline{y}$	$\overline{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
		0.5	92.51	94.34	97.12	100.00	97.09	94.34	92.51
		0.6	86.58	86.04	87.77	89.54	87.77	86.04	86.58
		0.7	81.64	80.85	81.94	83.18	81.94	80.85	81.64
		0.8	78.76	78.00	78.58	79.35	78.58	78.00	78.77
CSSS	0/90/90/0	0.2	52.63	56.91	64.60	77.46	85.22	74.35	65.85
		0.3	58.93	63.37	70.97	83.55	91.37	80.58	71.91
		0.4	72.94	71.72	80.10	92.66	99.03	88.35	79.00
		0.5	70.57	77.32	87.38	100.00	103.35	93.25	83.27
		0.6	66.31	71.72	80.10	92.66	99.03	88.35	79.01
		0.7	58.93	63.37	70.98	83.55	91.38	80.58	71.91
		0.8	52.49	56.81	64.54	77.38	85.17	74.25	65.73
	0/90/0/90	0.2	53.39	58.77	68.30	82.91	90.38	78.57	68.38
		0.3	58.51	63.66	72.90	87.51	94.99	83.39	73.30
		0.4	65.33	70.81	80.02	94.19	100.46	89.57	79.40
		0.5	70.35	77.20	87.33	100.00	103.67	93.82	83.51
		0.6	65.33	70.81	80.02	94.20	100.46	89.57	79.40
		0.7	58.51	63.66	72.89	87.51	94.99	83.39	73.30
		0.8	53.31	58.73	68.25	82.84	90.28	78.49	68.28
SSSS	0/90/90/0	0.2	50.45	58.45	69.15	83.31	69.15	58.45	50.45
		0.3	57.32	65.24	75.71	88.58	75.71	65.24	57.32
		0.4	65.08	73.80	84.78	95.64	84.78	73.80	65.08
		0.5	69.47	79.26	91.19	100.00	91.19	79.25	69.47
		0.6	65.08	73.80	84.78	95.64	84.78	73.80	65.08
		0.7	57.32	65.24	75.71	88.58	75.71	65.24	57.32
		0.8	50.26	58.33	69.06	83.22	69.09	58.36	50.30

	0/90/0/90	0.2	50.81	60.23	73.00	87.40	73.00	60.25	50.81
		0.3	56.41	65.40	77.74	91.59	77.74	65.40	56.41
		0.4	63.78	72.87	84.90	96.78	84.90	72.87	63.78
		0.5	69.18	79.20	91.26	100.00	91.26	79.20	69.18
		0.6	63.78	72.87	84.90	96.78	84.90	72.87	63.78
		0.7	56.41	65.40	77.75	91.59	77.74	65.40	56.41
		0.8	50.71	60.19	72.95	87.33	72.96	60.19	50.73
CS	0/90/90/0	0.2	82.99	92.93	103.86	110.03	103.86	92.93	82.99
		0.3	89.83	96.84	102.95	105.20	102.95	96.84	89.83
		0.4	97.81	101.32	102.08	101.34	102.05	101.32	97.81
		0.5	105.99	104.24	101.67	100.00	101.67	104.24	105.99
		0.6	97.81	101.32	102.05	101.34	102.05	101.32	97.81
		0.7	89.83	96.81	102.95	105.17	102.95	96.81	89.83
		0.8	82.93	92.87	103.77	110.00	103.80	92.87	82.93
	0/90/0/90	0.2	81.33	91.69	101.88	106.75	101.88	91.69	81.33
		0.3	87.40	94.82	101.06	103.50	101.04	94.82	87.40
		0.4	94.65	98.49	100.45	101.04	100.45	98.49	94.65
		0.5	101.51	100.62	100.11	100.00	100.11	100.62	101.51
		0.6	94.65	98.46	100.45	101.04	100.45	98.49	94.65
		0.7	87.40	94.82	101.04	103.50	101.04	94.82	93.00
		0.8	81.30	91.66	101.82	106.69	101.79	91.63	81.27

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = -\ h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 6.7 Values of ‘*r*’ for angle-ply saddle shells

Edge condition	Laminations	$\overline{y}$	$\overline{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
CCCC	+45/−45/−45/+45	0.2	62.02	66.76	73.99	82.14	75.98	68.32	63.15
		0.3	67.55	71.62	79.20	87.58	81.83	73.75	68.17
		0.4	73.52	78.93	86.85	94.85	89.80	81.54	75.53
		0.5	81.19	86.87	94.51	100.00	94.51	86.87	81.19
		0.6	75.53	81.54	89.80	94.85	86.85	78.92	73.52
		0.7	68.17	73.75	81.83	87.57	79.20	71.62	66.57
		0.8	63.14	68.31	75.97	82.13	73.99	66.75	62.01
	+45/−45/+45/−45	0.2	62.14	67.01	74.35	81.73	74.25	66.95	62.10
		0.3	66.84	72.12	79.94	87.32	79.80	72.03	66.79
		0.4	73.89	79.66	87.87	94.77	87.69	79.53	73.81
		0.5	80.79	86.62	94.42	100.00	94.42	86.62	80.79
		0.6	73.81	79.53	87.69	94.77	87.87	79.66	73.89
		0.7	66.79	72.03	79.80	87.32	79.94	72.12	66.84
		0.8	62.10	66.94	74.23	81.72	74.34	67.00	62.13
CSCC	+45/−45/−45/+45	0.2	66.53	71.59	79.13	87.06	81.46	73.47	67.92
		0.3	71.69	77.08	85.04	93.27	87.89	79.47	73.47
		0.4	79.14	84.82	92.89	100.07	95.57	87.44	81.20
		0.5	81.19	87.68	95.54	100.00	94.01	85.78	79.46
		0.6	72.86	79.21	87.55	92.61	85.58	77.39	71.43
		0.7	67.08	72.85	80.77	86.23	79.31	71.61	66.15
		0.8	63.40	68.70	76.16	81.87	75.22	67.93	62.84
	+45/−45/+45/−45	0.2	66.99	72.23	79.93	86.96	79.79	72.14	66.93
		0.3	72.28	77.96	86.19	93.34	86.02	77.86	72.22
		0.4	79.85	85.92	94.21	100.31	94.02	8.92	79.73
		0.5	80.52	86.75	94.69	100.00	94.61	86.31	79.75

		0.6	72.23	78.15	86.18	92.38	86.09	77.79	71.77
		0.7	66.63	72.04	79.59	85.89	79.49	71.76	66.29
		0.8	63.09	68.09	75.23	81.47	75.12	67.87	62.86
CSSC	+45/−45/−45/+45	0.2	76.47	69.90	76.19	85.47	88.56	80.27	73.88
		0.3	71.56	75.86	82.56	91.95	95.04	86.69	79.87
		0.4	78.09	82.89	90.04	99.01	101.74	94.54	87.41
		0.5	76.86	82.78	91.17	100.00	100.27	92.42	84.69
		0.6	69.93	75.47	83.66	93.31	92.46	83.88	76.38
		0.7	64.79	69.60	76.95	86.59	86.10	77.71	70.71
		0.8	61.41	65.57	72.22	81.65	81.87	73.77	67.24
	+45/−45/+45/−45	0.2	67.62	71.93	78.69	88.09	87.51	79.16	73.02
		0.3	73.11	77.87	85.09	94.56	93.95	85.32	78.76
		0.4	79.35	84.54	92.11	100.94	101.35	93.36	86.30
		0.5	76.87	82.51	90.76	100.00	101.20	93.33	85.17
		0.6	69.88	75.13	82.93	92.58	93.30	84.62	76.75
		0.7	64.93	69.54	76.54	85.90	86.63	78.14	70.92
		0.8	61.76	65.78	72.13	81.23	82.09	73.92	67.34
CSCS	+45/−45/−45/+45	0.2	64.81	69.95	77.29	84.03	79.02	71.47	65.95
		0.3	68.33	73.85	81.64	88.85	84.00	75.93	69.92
		0.4	73.76	79.73	87.96	95.21	90.89	82.55	75.99
		0.5	81.62	87.63	95.20	100.00	95.20	87.63	81.62
		0.6	75.99	82.55	90.89	95.21	87.96	79.73	73.75
		0.7	69.92	75.93	84.00	88.85	81.64	73.85	68.33
		0.8	65.94	71.45	78.97	83.95	77.23	69.92	64.79
	+45/−45/+45/−45	0.2	65.00	70.15	77.46	83.55	77.68	70.47	65.30
		0.3	68.71	74.33	82.18	88.50	82.47	74.77	69.17
		0.4	74.35	80.57	88.92	95.05	89.24	81.16	75.05

(continued)

Table 6.7 (continued)

Edge condition	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
		0.5	81.86	87.95	95.39	100.00	95.39	87.95	81.90
		0.6	75.05	81.16	89.24	95.05	88.92	80.57	74.35
		0.7	69.17	74.77	82.47	88.50	82.18	74.33	68.71
		0.8	65.29	70.44	77.63	83.47	77.42	70.13	64.99
CSSS	+45/−45/−45/+45	0.2	60.65	65.20	72.53	82.66	86.32	78.35	71.30
		0.3	64.84	70.08	77.77	87.89	91.47	83.30	75.79
		0.4	70.30	76.16	84.47	94.59	98.01	90.31	82.38
		0.5	75.68	82.30	91.01	100.00	101.49	94.56	87.00
		0.6	71.84	78.30	87.16	96.65	95.68	87.11	79.57
		0.7	66.19	71.84	80.09	90.06	89.31	80.73	73.55
		0.8	61.81	66.65	74.32	84.40	84.58	76.37	69.55
	+45/−45/+45/−45	0.2	62.07	66.70	74.06	83.99	85.39	77.42	70.77
		0.3	66.19	71.52	79.36	89.37	90.44	82.20	75.17
		0.4	71.50	77.58	86.11	95.97	97.07	89.02	81.61
		0.5	75.65	82.34	91.09	100.00	101.52	94.65	86.92
		0.6	70.51	76.70	85.24	95.03	96.11	87.64	79.66
		0.7	64.81	70.30	78.26	88.19	89.43	80.91	73.46
		0.8	60.37	65.02	72.49	82.53	84.37	76.26	69.33
SSSS	+45/−45/−45/+45	0.2	55.17	65.44	77.00	86.79	79.18	67.62	57.18
		0.3	60.04	70.13	81.77	91.49	84.49	72.70	62.24
		0.4	66.19	76.53	88.14	96.94	90.77	78.92	67.98
		0.5	70.64	81.76	93.40	100.00	93.40	81.76	70.63
		0.6	67.98	78.92	90.79	96.94	88.14	76.53	66.17
		0.7	62.24	72.70	84.49	91.49	81.77	70.13	60.04
		0.8	57.04	67.53	79.14	86.75	76.97	65.38	55.06

	+45/−45/+45/−45	0.2	56.00	66.46	78.09	86.57	77.50	66.16	56.07
		0.3	60.91	71.33	83.09	91.39	82.70	71.14	61.06
		0.4	66.81	77.63	89.45	96.92	89.15	77.50	67.02
		0.5	70.52	81.69	93.41	100.00	93.41	81.69	70.52
		0.6	67.03	77.50	89.15	96.92	89.45	77.63	66.81
		0.7	61.06	71.14	82.70	91.39	83.09	71.33	60.91
		0.8	55.96	66.09	77.46	86.54	78.08	66.41	55.89
CS	+45/−45/−45/+45	0.2	67.45	76.42	85.65	87.98	78.09	69.59	62.42
		0.3	73.03	81.79	91.05	91.42	81.09	72.80	66.35
		0.4	80.02	88.32	97.00	96.09	85.55	77.46	71.67
		0.5	79.26	84.38	92.12	100.00	92.12	84.38	79.24
		0.6	71.67	77.49	85.58	96.09	97.00	88.32	79.99
		0.7	66.35	72.80	81.09	91.44	91.05	81.77	73.03
		0.8	62.42	69.59	78.09	87.92	85.55	76.40	67.37
	+45/−45/+45/−45	0.2	59.06	66.85	75.42	81.90	75.35	66.74	59.09
		0.3	63.54	70.88	79.61	86.39	79.54	70.90	63.63
		0.4	69.44	76.32	85.12	92.28	85.15	76.44	69.37
		0.5	76.67	83.22	91.74	100.00	91.74	83.22	76.67
		0.6	69.40	76.44	85.17	92.28	86.16	76.32	69.44
		0.7	63.63	70.93	79.54	86.39	79.61	70.88	63.54
		0.8	59.09	66.76	75.38	81.87	75.35	66.83	59.06

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = -h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 6.8 Maximum values of r with corresponding coordinates of cutout centres and zones where $r \geq 90$ for cross-ply saddle shells

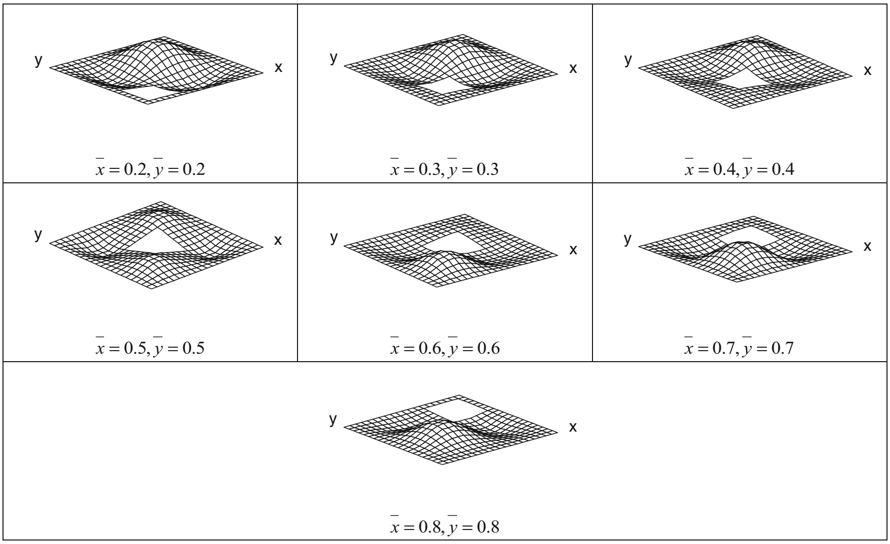
Laminations	0/90/90/0			0/90/0/90		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value of $r \geq 90$	Maximum values of r	Coordinate of cutout centre	Area in which the value of $r \geq 90$
CCCC	100.00	(0.5, 0.5)	$0.2 \leq \bar{x} \leq 0.8$, $0.4 \leq \bar{y} \leq 0.6$.	100.00	(0.5, 0.5)	$0.4 \leq \bar{x} \leq 0.6$, $0.4 \leq \bar{y} \leq 0.6$.
CSCC	110.72	(0.5, 0.4)	$0.2 \leq \bar{x} \leq 0.8, 0.3 \leq \bar{y} \leq 0.5$.	109.00	(0.5, 0.4)	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.5$.
CSSC	105.50	(0.6, 0.4)	$0.5 \leq \bar{x} \leq 0.7$, $0.2 \leq \bar{y} \leq 0.5$.	105.86	(0.6, 0.4)	$0.5 \leq \bar{x} \leq 0.6$, $0.2 \leq \bar{y} \leq 0.6$.
CSCS	100.00	(0.5, 0.5)	$0.2 \leq \bar{x} \leq 0.8$, $\bar{y} = 0.5$.	100.00	(0.5, 0.5)	$0.2 \leq \bar{x} \leq 0.8$, $\bar{y} = 0.5$.
CSSS	103.35	(0.6, 0.5)	$0.5 \leq \bar{x} \leq 0.6$, $0.4 \leq \bar{y} \leq 0.6$.	103.67	(0.6, 0.5)	$0.5 \leq \bar{x} \leq 0.6$, $0.4 \leq \bar{y} \leq 0.6$.
SSSS	100.00	(0.5, 0.5)	$\bar{x} = 0.5$, $0.4 \leq \bar{y} \leq 0.6$.	100.00	(0.5, 0.5)	$\bar{x} = 0.5$, $0.3 \leq \bar{y} \leq 0.7$.
CS	110.03	(0.5, 0.2)	$0.3 \leq \bar{x} \leq 0.7, 0.2 \leq \bar{y} \leq 0.8$.	106.75	(0.5, 0.2)	$0.3 \leq \bar{x} \leq 0.7$, $0.2 \leq \bar{y} \leq 0.8$.

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = -h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 6.9 Maximum values of r with corresponding coordinates of cutout centres and zones where $r \geq 90$ for angle-ply saddle shells

Laminations	+45/−45/−45/+45			+45/−45/+45/−45		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value of $r \geq 90$	Maximum values of r	Coordinate of cutout centre	Area in which the value of $r \geq 90$
CCCC	100.00	(0.5, 0.5)	$\bar{x} = 0.5, 0.4 \leq \bar{y} \leq 0.6$.	100.00	(0.5, 0.5)	$\bar{x} = 0.5, 0.4 \leq \bar{y} \leq 0.6$.
CSCC	100.07	(0.5, 0.4)	$0.4 \leq \bar{x} \leq 0.6, 0.4 \leq \bar{y} \leq 0.5$.	100.31	(0.5, 0.4)	$0.4 \leq \bar{x} \leq 0.6, 0.4 \leq \bar{y} \leq 0.5$.
CSSC	101.74	(0.6, 0.4)	$0.5 \leq \bar{x} \leq 0.6, 0.3 \leq \bar{y} \leq 0.6$.	101.35	(0.6, 0.4)	$0.5 \leq \bar{x} \leq 0.6, 0.3 \leq \bar{y} \leq 0.6$.
CSCS	100.00	(0.5, 0.5)	$0.4 \leq \bar{x} \leq 0.6, 0.5 \leq \bar{y} \leq 0.6$.	100.00	(0.5, 0.5)	$\bar{x} = 0.5, 0.4 \leq \bar{y} \leq 0.6$.
CSSS	100.00	(0.5, 0.5)	$0.5 \leq \bar{x} \leq 0.7, 0.4 \leq \bar{y} \leq 0.5$.	101.52	(0.6, 0.5)	$0.4 \leq \bar{x} \leq 0.6, 0.5 \leq \bar{y} \leq 0.6$.
SSSS	100.00	(0.5, 0.5)	$0.5 \leq \bar{x} \leq 0.6, 0.4 \leq \bar{y} \leq 0.5$.	100.00	(0.5, 0.5)	$\bar{x} = 0.5, 0.3 \leq \bar{y} \leq 0.7$.
CS	100.00	(0.5, 0.5)	$0.4 \leq \bar{x} \leq 0.5, 0.3 \leq \bar{y} \leq 0.5$.	100.00	(0.5, 0.5)	$\bar{x} = 0.5, 0.4 \leq \bar{y} \leq 0.6$ $0.4 \leq \bar{x} \leq 0.6, \bar{y} = 0.5$.

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = -h/R_{yy} = 1/300$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

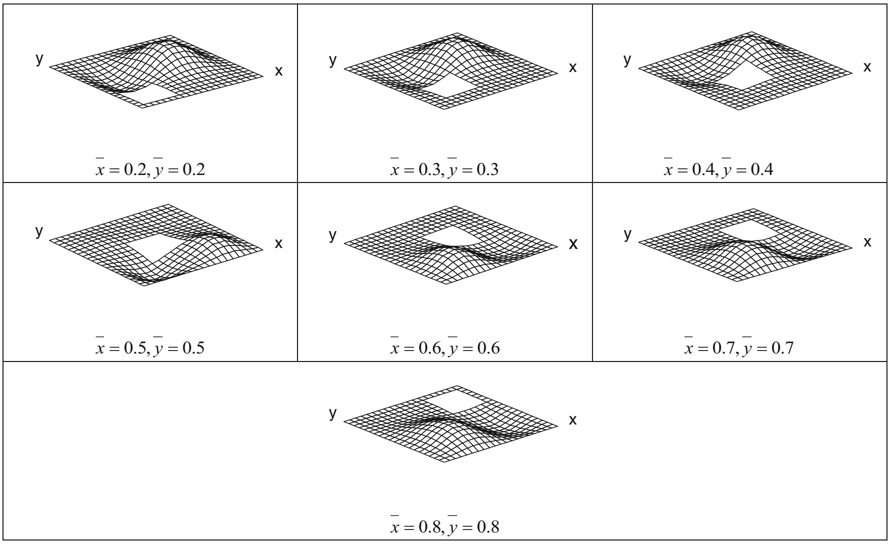
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 6.6 First mode shapes of laminated composite (0/90/0/90) stiffened saddle shell for different positions of square cutout with CCCC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

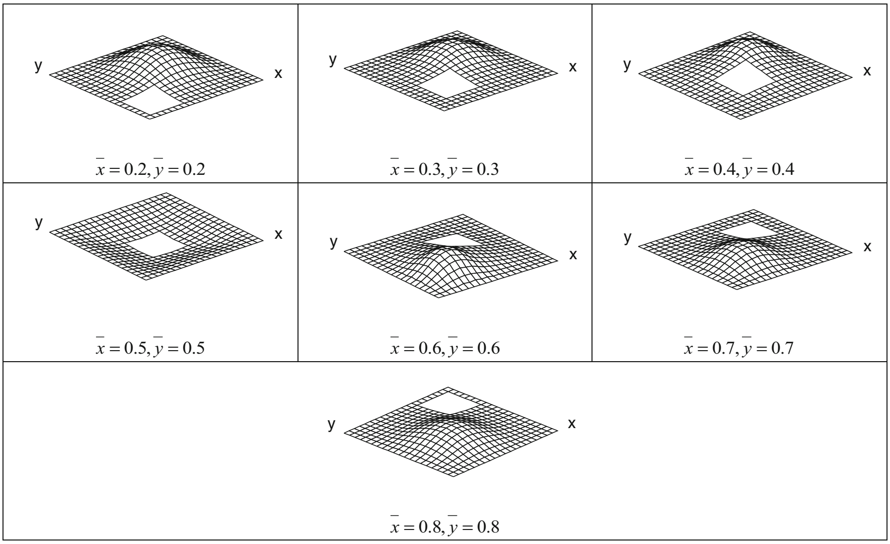
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 6.7 First mode shapes of laminated composite (0/90/0/90) stiffened saddle shell for different positions of square cutout with CSCC boundary condition

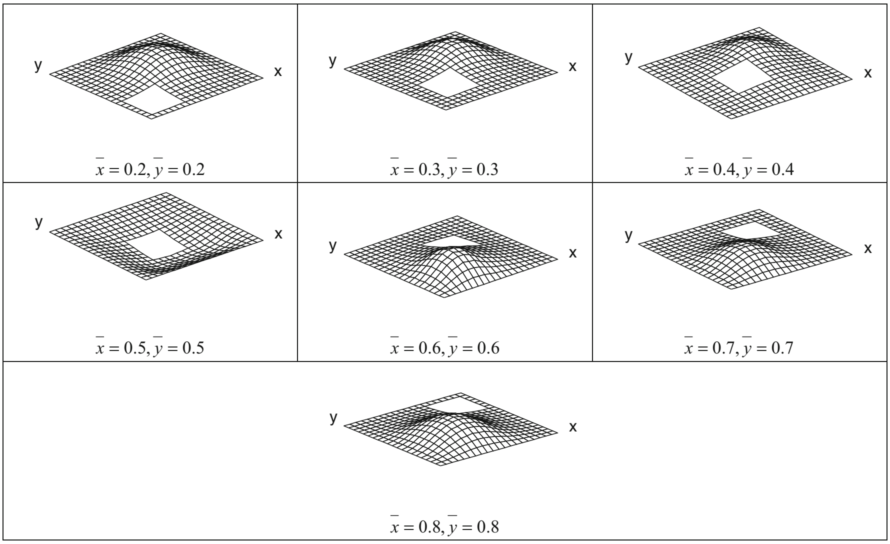


$\bar{x} = 0.2, \bar{y} = 0.2$ $\bar{x} = 0.3, \bar{y} = 0.3$ $\bar{x} = 0.4, \bar{y} = 0.4$

$\bar{x} = 0.5, \bar{y} = 0.5$ $\bar{x} = 0.6, \bar{y} = 0.6$ $\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 6.8 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened saddle shell for different positions of square cutout with CCCC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$ $\bar{x} = 0.3, \bar{y} = 0.3$ $\bar{x} = 0.4, \bar{y} = 0.4$

$\bar{x} = 0.5, \bar{y} = 0.5$ $\bar{x} = 0.6, \bar{y} = 0.6$ $\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 6.9 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened saddle shell for different positions of square cutout with CSCC boundary condition

References

Chakravorty D, Sinha PK, Bandyopadhyay JN (1995) Free vibration analysis of point supported laminated composite doubly curved shells – a finite element approach. Comput Struct 54(2):191–207

Chakravorty D, Sinha PK, Bandyopadhyay JN (1996) Finite element free vibration analysis of doubly curved laminated composite shells. J Sound Vib 191(4):491–504

Chakravorty D, Sinha PK, Bandyopadhyay JN (1998) Applications of FEM on free and forced vibration of laminated shells. ASCE J Eng Mech 124(1):1–8

Chao WC, Reddy JN (1984) Analysis of laminated composite shells using a degenerated 3-d element. Int J Numer Methods Eng 20:1991–2007

Chao CC, Tung TP (1989) Step pressure and blast response of clamped orthotropic hemispherical shells. Int J Impact Eng 8(3):191–207

Chia CY, Chia DS (1997) Nonlinear vibration of moderately thick anti-symmetric angle-ply shallow spherical shell. Comput Struct 44(4):797–805

Dai HL, Wang X (2005) Stress wave propagation in laminated piezoelectric spherical shells under thermal shock and electric excitation. Eur J Mech A Solids 24:263–276

Ganapathi M (2007) Dynamic stability characteristics of functionally graded materials shallow spherical shells. Compos Struct 79:338–343

Gautham BP, Ganesan N (1997) Free vibration characteristics of isotropic and laminated orthotropic spherical caps. J Sound Vib 204(1):17–40

Kant T, Kumar S, Singh UP (1994) Shell dynamics with three dimensional finite elements. Comput Struct 50(1):135–146

Kapania RK (1989) A review on the analysis of laminated shells. J Press Vessel Technol 111(2):88–96

Lellep J, Hein H (2002) Optimization of clamped plastic shallow shells subjected to initial impulsive loading. Eng Optim 34:545–556

Liew KM, Lim CW (1994) Vibration of perforated doubly-curved shallow shells with rounded corners. Int J Solids Struct 31(11):1519–1536

Liew KM, Lim CW (1995) Vibration behaviour of doubly-curved shallow shells of curvilinear planform. J Eng Mech 121(12):1277–1283

Noor AK, Burton WS (1990) Assessment of computational models for multi-layered composite shells. Appl Mech Rev 43:67–97

Park T, Lee SY (2009) Parametric instability of delaminated composite spherical shells subjected to in-plane pulsating forces. Compos Struct 91:196–204

Qatu MS (1991) Natural frequencies for cantilevered doubly-curved laminated composite shallow shells. Compos Struct 17:227–255

Qatu MS, Sullivan RW, Wang W (2010) Recent research advances on the dynamic analysis of composite shells: 2000–2009. Compos Struct 93:14–31

Reddy JN (1981) Finite element modeling of layered, anisotropic composite plates and shells: review of recent research. Shock Vib Dig 13(12):3–12

Reddy JN, Chandrashekhara K (1985) Geometrically nonlinear transient analysis of laminated doubly curved shells. Int J Nonlinear Mech 20(2):79–90

Sahoo S (2014) Free vibration of laminated composite stiffened saddle shell roofs with cutout. IOSR J Mech Civ Eng ICAET-2014:30–34

Sathyamoorthy M (1995) Nonlinear vibrations of moderately thick orthotropic shallow spherical shells. Comput Struct 57(1):59–65

Shin DK (1997) Large amplitude free vibration behavior of doubly curved shallow open shells with simply supported edges. Comput Struct 62(1):35–49

Sivasubramonian B, Kulkarni AM, Venkateswara Rao G, Krishnan A (1997) Free vibration of curved panels with cutouts. J Sound Vib 200(2):227–234

Tan DY (1998) Free vibration analysis of shells of revolution. J Sound Vib 213(1):15–33

Wang X, Lu G, Guillow SR (2002) Stress wave propagation in orthotropic laminated thick-walled spherical shells. Int J Solids Struct 39:3027–4037

Chapter 7
Stiffened Hyperbolic Paraboloid Shell with Cutout

Abstract This chapter considers dynamic characteristics of stiffened composite hyperbolic paraboloid shell panel with cutout under free vibration in terms of natural frequency and mode shapes. Finite element method is used for the analysis by combining an eight-noded curved shell element with a three-noded curved beam element. The variations in cutout size as well as positions of the cutouts with respect to the shell centre are considered for different boundary conditions of cross-ply and angle-ply laminated shells. The effects of these parametric variations on the fundamental frequencies and mode shapes are studied in details to arrive at a set of conclusions of practical engineering relevance.

Keywords Hyperbolic paraboloid shell • Cutout • Eccentricity • Fundamental frequency • Mode shapes

7.1 Introduction

In civil engineering practice, it is often a necessity to cover large open areas without any column. In such applications as in airports, parking lots, hangars and the like, thin shells are preferred over flat plates. Such areas in medical plants and automobile industries require entry of north light through the roofing units. In civil engineering construction, conoidal, hyperbolic paraboloid (among the anticlastic) and elliptic paraboloid (among the synclastic) shells are used as roofing units to cover large column-free areas. The hyperbolic paraboloid shells are aesthetically suitable although they offer less stiffness than other doubly curved shells. Realizing the importance of laminated composite doubly curved shells in the industry, several aspects of shell behaviour including static, dynamic, buckling, impact, etc., are being reported by different researchers. The present chapter highlights the free vibration behaviour of composite stiffened hyperbolic paraboloid shell panels with cutout.

No wonder a number of researchers are working to explore different behavioural aspects of laminated doubly curved shells. Researchers like Ghosh and Bandyopadhyay (1990), Dey et al. (1992, 1994) and Chakravorty et al. (1995, 1996) reported static and dynamic behaviour of doubly curved shells made of

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laminated composites. Later Nayak and Bandyopadhyay (2002, 2005, 2006), Das and Chakravorty (2007, 2008, 2010, 2011) and Pradyumna and Bandyopadhyay (2008, 2011) reported static, dynamic and instability behaviour of laminated doubly curved shells. The free vibration of both isotropic and composite plates with cutout has received attention of different researchers from time to time. Reddy (1982) investigated composite plates with cutout for flexural vibration of large amplitude. Malhotra et al. (1989) studied the effect of different boundary conditions on composite plate with cutout under free vibration. Sivasubramonian et al. (1997) reported free vibration of curved panels with cutout with classical boundary conditions. Rayleigh-Ritz method was used to study the effect of fibre orientation and size of cutout on natural frequency on orthotropic square plates with square cutout. Later Sivakumar et al. (1999), Rossi (1999), Huang and Sakiyama (1999) and Hota and Padhi (2007) considered free vibration of plate with various cutout geometries. Chakravorty et al. (1998) analysed the effect of concentric cutout on different shell forms. Sivasubramonian et al. (1999) studied the effect of curvature and cutouts on square panels with different boundary conditions by varying the size of the cutout (symmetrically located) as well as curvature of the panels. Hota and Chakravorty (2007) provided useful information on stiffened conoidal shell roofs with cutout subjected to free vibration. Later Nanda and Bandyopadhyay (2007) and Kumar et al. (2013) considered the effect of different parametric variations on free vibration of cylindrical shell with cutout using first-order shear deformation theory (FSDT) and higher-order shear deformation theory (HYSD), respectively.

It is noted from the literature review that free vibration study of laminated composite hyperbolic paraboloid shell panels with cutout has received little attention (Sahoo 2016) so far although the importance of this shell form has been noted. The present chapter is devoted to the study of the free vibration behaviour of stiffened hyperbolic paraboloid shell panels with cutout by varying the size and position of the cutouts.

7.2 Problem

Problems of hyperbolic paraboloid shell panels (Fig. 7.1) with different laminations both cross- and angle-ply and different boundary conditions have been solved using finite element method in order to study the dynamic response and the effect of cutout size and position on it. The positions of the cutouts are also shifted along both of the plan directions of the shell for some laminations for different practical edge conditions to evaluate the effect of eccentricity of cutout on the dynamic response.

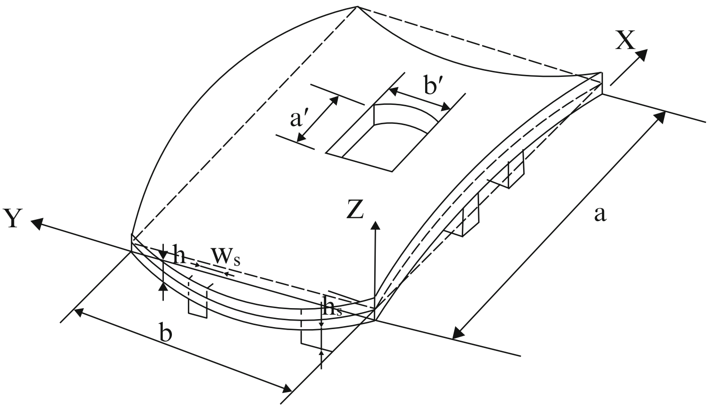


Fig. 7.1 Hyperbolic paraboloidal shell with a concentric cutout stiffened along the margins

7.3 Results and Discussion

7.3.1 Free Vibration Behaviour of Shells with Concentric Cutouts

Tables 7.1, 7.2 and 7.3 furnish the results of nondimensional frequency $(\overline{\omega})$ of stiffened hyperbolic paraboloid shells with cutout. In the first phase, two-, three- and four-layered symmetric and antisymmetric, cross- and angle-ply laminations have been chosen. These shells are analysed for clamped, simply supported and point supported boundary conditions. Among them, the laminations which are exhibiting better performances are chosen for further study. Thus four laminations are considered in the second phase of analysis. The effect of size of cutout and its position on the free vibration response of these shells is studied for various combinations of edge conditions along the four edges.

7.3.1.1 Effect of Cutout Size on Fundamental Frequency

From Tables 7.1, 7.2, and 7.3, it is observed that when a cutout is provided in a stiffened shell, the fundamental frequency improves in all the cases. This improving trend is observed for both cross-ply and angle-ply shells. This initial increase in frequency is due to the fact that with the introduction of cutout, numbers of stiffeners increase from two to four in the present study. It is evident from Tables 7.1, 7.2 and 7.3 that in all the cases when cutout is provided with $a'/a = 0.3$, the frequencies increase. But with further increase in cutout size, fundamental frequencies decrease in few cases. As with the introduction of a cutout of $a'/a = 0.3$, on shell surface, the frequency increases in all the cases; this leads to the

Table 7.1 Nondimensional fundamental frequencies ($\overline{\omega}$) for laminated composite stiffened hyperbolic paraboloid shell for different sizes of central square cutout, different laminations and boundary conditions

		Cutout size (a'/a)				
Boundary conditions	Laminations	0	0.1	0.2	0.3	0.4
Clamped	0/90	76.55	93.44	114.76	118.96	125.79
	90/0	76.73	93.98	115.57	118.92	125.61
	0/90/0	111.32	115.89	137.49	155.18	157.13
	90/0/90	105.06	118.43	142.49	153.91	145.40
	0/90/0/90	98.07	113.96	138.47	150.92	153.79
	90/0/90/0	98.07	114.21	138.94	150.90	153.55
	0/90/90/0	104.24	117.21	139.79	155.00	157.43
	90/0/0/90	111.97	119.46	144.13	156.19	148.30
	+45/−45	86.43	99.14	113.00	117.25	115.80
	+45/−45/+45	98.36	114.35	120.51	123.73	126.66
	+45/−45/+45/−45	100.24	117.38	122.58	125.03	127.81
	+45/−45/−45/+45	99.67	116.51	122.05	124.99	128.13
Simply supported	0/90	60.65	52.97	70.60	87.40	84.06
	90/0	39.95	53.37	71.10	86.96	84.19
	0/90/0	63.47	70.04	86.95	95.67	96.77
	90/0/90	68.44	72.53	94.11	98.66	95.92
	0/90/0/90	66.21	68.82	88.25	97.80	96.83
	90/0/90/0	66.48	69.02	88.62	98.42	97.24
	0/90/90/0	64.63	71.16	91.13	96.79	97.42
	90/0/0/90	68.54	72.96	94.55	99.41	97.30
	+45/−45	59.64	70.85	88.88	95.55	92.36
	+45/−45/+45	71.35	87.41	94.68	98.51	98.32
	+45/−45/+45/−45	73.24	90.05	96.76	99.28	99.61
	+45/−45/−45/+45	72.61	89.47	96.05	99.11	99.37
Point supported	0/90	20.91	26.64	33.48	41.55	49.81
	90/0	20.59	26.34	33.19	41.39	47.66
	0/90/0	23.06	28.42	34.05	42.03	42.85
	90/0/90	24.46	29.23	36.42	39.95	48.43
	0/90/0/90	28.56	33.66	40.85	49.17	57.67
	90/0/90/0	28.38	33.49	40.73	49.18	57.82
	0/90/90/0	25.89	31.64	38.35	46.61	53.07
	90/0/0/90	27.06	31.64	38.70	46.95	54.95
	+45/−45	26.39	32.67	40.02	48.96	52.75
	+45/−45/+45	30.74	35.66	41.23	44.56	51.57
	+45/−45/+45/−45	36.17	42.04	49.86	59.38	61.29
	+45/−45/−45/+45	34.44	39.97	46.40	55.09	61.30

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 1/300$, $R_{xx}/R_{yy} = -1.5$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 7.2 Nondimensional fundamental frequencies ($\overline{\omega}$) for laminated composite cross-ply stiffened hyperbolic paraboloidal shell for different sizes of central square cutout and different boundary conditions

		Cutout size (a'/a)				
Laminations	Boundary conditions	0	0.1	0.2	0.3	0.4
0/90/90/0	CCCC	104.21	117.21	139.79	155.00	157.43
	CSCC	79.32	94.17	111.86	129.18	131.77
	CCSC	96.15	106.07	123.62	142.36	143.51
	CCCS	78.99	92.31	111.13	128.37	131.51
	CSSC	69.38	80.04	94.69	109.11	111.21
	CCSS	69.32	79.82	94.65	109.07	111.19
	CSCS	87.92	86.46	107.33	126.02	128.52
	SCSC	100.44	104.16	122.09	139.55	140.44
	CSSS	63.30	73.69	90.73	102.68	104.34
	SSSC	67.72	77.92	93.63	103.87	104.39
	SSCS	63.33	73.78	90.73	102.68	104.34
	SSSS	64.63	71.16	91.13	96.79	97.42
	Point supported	25.89	31.63	38.35	46.61	53.07
0/90/0/90	CCCC	103.94	118.21	142.85	155.42	157.23
	CSCC	82.21	97.37	117.73	133.23	129.29
	CCSC	84.52	96.86	115.87	133.89	137.92
	CCCS	81.95	96.1	117.23	132.71	129.06
	CSSC	66.1	77.91	94.04	109.39	109.2
	CCSS	66.04	77.73	94.02	109.38	109.17
	CSCS	92.07	91.93	114.27	130.15	127.7
	SCSC	90.59	93.61	114.08	131.02	132.55
	CSSS	61.19	73.11	90.99	104.18	104.03
	SSSC	62.26	74.04	94.4	103.78	102.22
	SSCS	61.19	73.18	90.99	104.18	104.03
	SSSS	66.21	68.82	88.25	97.8	96.83
	Point supported	28.56	33.64	40.85	49.17	57.67

$a/b=1$, $a/h=100$, $a'/b'=1$, $h/R_{xx}=1/300$, $R_{xx}/R_{yy}=-1.5$; $E_{11}/E_{22}=25$, $G_{23}=0.2E_{22}$, $G_{13}=G_{12}=0.5E_{22}$, $\nu_{12}=\nu_{21}=0.25$

observation that concentric cutouts with stiffened margins may be incorporated safely on shell surfaces for functional purposes up to $a'/a=0.3$.

7.3.1.2 Effect of Boundary Conditions on Fundamental Frequency

The boundary conditions may be arranged in the following order, considering number of boundary constraints: CCCC, CSCC, CCSC, CCCS, CSSC, CCSS,

Table 7.3 Nondimensional fundamental frequencies ($\overline{\varpi}$) for laminated composite angle-ply stiffened hyperbolic paraboloidal shell for different sizes of central square cutout and different boundary conditions

Laminations	Boundary conditions	Cutout size (a'/a)				
		0	0.1	0.2	0.3	0.4
+45/−45/−45/+45	CCCC	99.67	116.51	122.05	124.99	128.13
	CSCC	92.06	109.01	114.16	115.50	113.34
	CCSC	93.85	107.92	113.37	116.78	119.98
	CCCS	91.37	107.10	113.46	115.10	113.26
	CSSC	84.58	97.81	106.10	108.69	107.94
	CCSS	82.86	95.60	103.59	108.56	109.04
	CSCS	89.64	105.25	110.68	112.01	110.84
	SCSC	91.82	105.93	110.55	113.61	116.84
	CSSS	81.33	94.01	102.20	105.19	105.18
	SSSC	80.59	94.88	101.63	105.04	105.15
	SSCS	81.33	94.36	102.20	105.19	105.18
	SSSS	72.61	89.47	96.05	99.11	99.37
	Point supported	34.43	39.97	46.40	55.09	61.30
+45/−45/+45/−45	CCCC	101.36	119.09	123.64	126.15	129.41
	CSCC	91.93	108.2	114.13	115.26	113.36
	CCSC	94.41	108.5	113.84	116.84	119.99
	CCCS	91.7	107.51	113.95	115.13	113.35
	CSSC	84.44	96.76	105.31	108.66	108.55
	CCSS	83.77	96.27	105.06	108.64	108.7
	CSCS	90.06	105.92	111.18	112.05	110.86
	SCSC	92.44	106.55	111.01	113.69	116.84
	CSSS	82.22	93.92	102.95	105.28	105.35
	SSSC	81.12	95.43	102.26	105.14	105.35
	SSCS	82.35	94.2	102.95	105.28	105.35
	SSSS	73.24	90.05	96.76	99.26	99.61
	Point supported	36.17	42.04	49.86	59.38	61.29

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 1/300$, $R_{xx}/R_{yy} = -1.5$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

CSCS, SCSC, CSSS, SSSC, SSCS, SSSS and corner point supported. Tables 7.4 and 7.5 are prepared with the consideration that a value of 0 is assigned to the frequency of a corner point supported shell and a value of 100 is assigned to that of a fully clamped shell. Marks are assigned to each boundary combination in that scale. These marks are considered for cutouts with $a'/a = 0.2$. These tables will enable a practising engineer to judge the relative performance of these boundary constraints in the shell forms in the presence of cutouts.

Table 7.4 Clamping options for cross-ply hyperbolic paraboloidal shells with central cutouts having a'/a ratio 0.2

	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of no. of restraints
0/90/90/0	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Good improvement	52
	1	(a) Hyperbolic edge along $x = a$ (SSCS)	Good improvement	52
		(b) Hyperbolic edge along $x = 0$ (CSSS)	Good improvement	52
		(c) One parabolic edge along $y = b$ (SSSC)	Good improvement	54
	2	(a) Two hyperbolic edges $x = 0$ and $x = a$ (CSCS)	Marked improvement	68
		(b) Two parabolic edges along $y = 0$ and $y = b$ (SCSC)	Remarkable improvement	83
		(c) Any two edges except the above option (CSSC, CCSS)	Good improvement	56
	3	Three edges including the two hyperbolic edges (CSCC, CCCS)	Marked improvement	72
		Three edges excluding the hyperbolic edge along $x = a$ (CCSC)	Remarkable improvement	84
	4	All sides (CCCC)	Frequency attains highest value	100
0/90/0/90	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Good improvement	46
	1	(a) Hyperbolic edge along $x = a$ (SSCS)	Good improvement	49
		(b) Hyperbolic edge along $x = 0$ (CSSS)	Good improvement	49
		(c) One parabolic edge along $y = b$ (SSSC)	Good improvement	53
	2	(a) Two hyperbolic edges $x = 0$ and $x = a$ (CSCS)	Marked improvement	72
		(b) Two parabolic edges along $y = 0$ and $y = b$ (SCSC)	Marked improvement	72
		(c) Any two edges except the above option (CSSC, CCSS)	Good improvement	52

(continued)

Table 7.4 (continued)

	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of no. of restraints
	3	Three edges including the two hyperbolic edges (CSCC, CCCS)	Marked improvement	75
		Three edges excluding the hyperbolic edge along $x = a$ (CCSC)	Marked improvement	74
	4	All sides (CCCC)	Frequency attains highest value	100

It is seen from Table 7.4 and 7.5 that fundamental frequencies of members belonging to the same number of boundary constraints may not have close values for all the cases considered here. So the boundary constraint is not the sole criteria for its performance. The free vibration characteristics mostly depend on the arrangement of boundary constrains rather than their actual number. It is seen that if the hyperbolic edge along $x = a$ is released from clamped to simply supported, the change of frequency is more in the case of a cross-ply shells than that for angle-ply shells. Again, if the two adjacent edges are released, fundamental frequency decreases more significantly than that of a shell in which two alternate edges are released. This is true for both cross- and angle-ply shells. For cross-ply shells, if three or four edges are simply supported, frequency values undergo marked decrease, but for angle-ply shells, with the introduction of more number of simply supported edges, the frequency value does not change so drastically. The results indicate that two alternate edges need to be preferably clamped in order to attain higher frequency values.

7.3.1.3 Mode Shapes

The mode shapes corresponding to the fundamental modes of vibration are plotted in Figs. 7.2, 7.3, 7.4 and 7.5 for cross-ply and angle-ply shells. The normalized displacements are drawn with the consideration that the shell mid-surface is the reference for all the support conditions and all the laminations. The fundamental mode is either a bending mode or torsion mode for all the boundary conditions for cross-ply and angle-ply shells. When a cutout is introduced, the mode shapes remain almost similar. With the increase in the size of the cutout from 0.2 to 0.4, the fundamental modes of vibration do not change to an appreciable amount.

Table 7.5 Clamping options for angle-ply hyperbolic paraboloidal shells with central cutouts having a'/a ratio 0.2

	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of no. of restraints
+45/−45/−45/+45	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Marked improvement	66
	1	(a) Hyperbolic edge along $x = a$ (SSCS)	Marked improvement	74
		(b) Hyperbolic edge along $x = 0$ (CSSS)	Marked improvement	74
		(c) One parabolic edge along $y = b$ (SSSC)	Marked improvement	73
	2	(a) Two hyperbolic edges $x = 0$ and $x = \mathrm{a}$ (CSCS)	Remarkable improvement	85
		(b) Two parabolic edges along $y = 0$ and $y = b$ (SCSC)	Remarkable improvement	85
		(c) Any two adjacent edges (CSSC)	Marked improvement	79
		(d) Any two adjacent edges (CCSS)	Marked improvement	76
	3	Three edges including the two hyperbolic edges (CSCC, CCCS)	Remarkable improvement	90
		Three edges excluding the hyperbolic edge along $x = a$ (CCSC)	Remarkable improvement	89
	4	All sides (CCCC)	Frequency attains highest value	100
+45/−45/+45/−45	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Marked improvement	64
	1	(a) Hyperbolic edge along $x = a$ (SSCS)	Marked improvement	72
		(b) Hyperbolic edge along $x = 0$ (CSSS)	Marked improvement	72
		(c) One parabolic edge along $y = b$ (SSSC)	Marked improvement	71
	2	(a) Two hyperbolic edges $x = 0$ and $x = a$ (CSCS)	Remarkable improvement	83
		(b) Two parabolic edges along $y = 0$ and $y = b$ (SCSC)	Remarkable improvement	83

(continued)

Table 7.5 (continued)

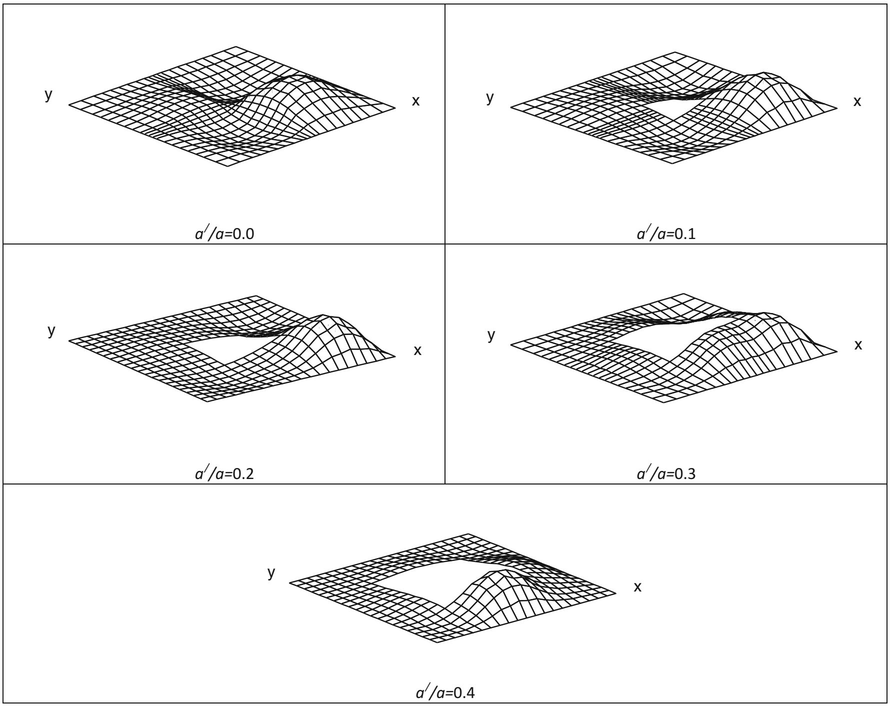
	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of no. of restraints
		(c) Any two edges except the above option (CSSC, CCSS)	Marked improvement	75
	3	Three edges including the two hyperbolic edges (CSCC, CCCS)	Remarkable improvement	87
		Three edges excluding the hyperbolic edge along $x = a$ (CCSC)	Remarkable improvement	87
	4	All sides (CCCC)	Frequency attains highest value	100

7.3.2 *Effect of Eccentricity of Cutout Position*

7.3.2.1 Fundamental Frequency

The results obtained for different locations of a cutout with $a'/a = 0.2$ are further analysed to study the effect of eccentricity of cutout positions on fundamental frequencies. The nondimensional coordinates of the cutout centre $\left(\bar{x} = \frac{x}{a}, \bar{y} = \frac{y}{a}\right)$ are varied from 0.2 to 0.8 along each direction in such a way that the distance of a cutout margin from the shell boundary does not fall below one tenth of the plan dimension of the shell. The margins of cutouts are stiffened with four stiffeners. The study is carried out for all the 13 boundary conditions for both cross-ply and angle-ply shells. The fundamental frequency of a shell having an eccentric cutout is expressed as a percentage of fundamental frequency of a shell having a concentric cutout. This percentage is denoted by r and the results are provided in Tables 7.6 and 7.7.

It is observed that eccentricity of the cutout along the length of the shell towards the clamped edges makes it more flexible. It is also observed that almost in all the cases r value is maximum in and around $\bar{x} = 0.5$ and $\bar{y} = 0.5$. When the edge, opposite to a clamped edge, is simply supported, r value first increases towards the simply supported edge and then decreases. But, when two opposite edges are simply supported, r value decreases towards the simply supported edges. Again in the case of an angle-ply shell, if the simply supported edge is the hyperbolic one, then r value decreases towards the edge. So, for functional purposes, if a shift of central cutout is required, eccentricity of a cutout along the length or width should preferably be towards the simply supported edge which is opposite to a clamped



a'/a=0.0

a'/a=0.1

a'/a=0.2

a'/a=0.3

a'/a=0.4

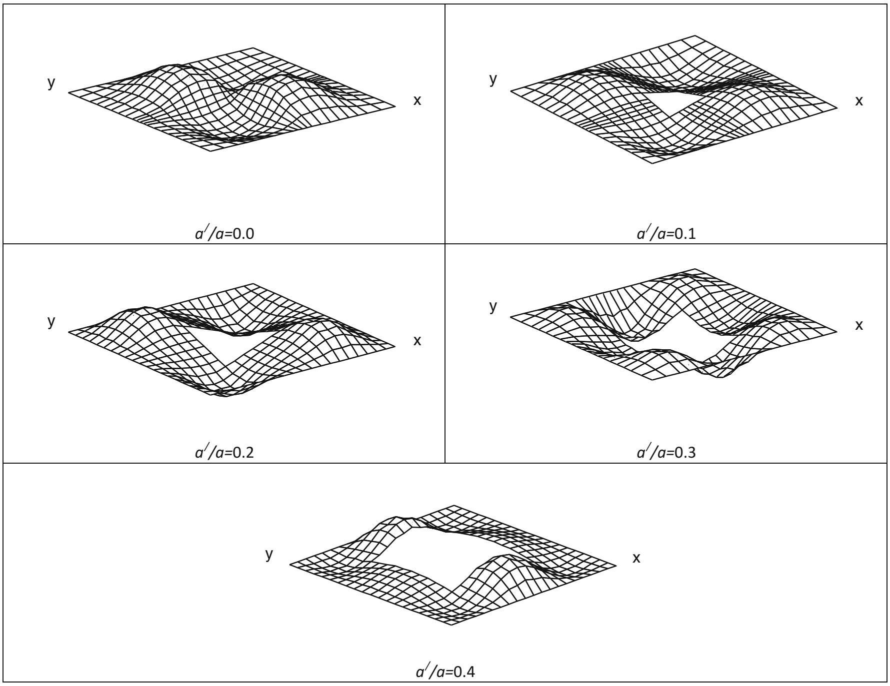
Fig. 7.2 First mode shapes of laminated composite (0/90/0/90) stiffened hyperbolic paraboloidal shell for different sizes of central square cutout and CSSC boundary conditions

edge for cross-ply shells. For angle-ply shells, eccentricity towards simply supported hyperbolic edge should be avoided.

Tables 7.8 and 7.9 provide the maximum values of *r* together with the position of the cutout. These tables also identify the rectangular zones within which *r* is always greater than or equal to 95. These tables will enable practising engineers in deciding the proper placement of the cutout.

7.3.2.2 Mode Shapes

The mode shapes corresponding to the fundamental modes of vibration are plotted in Figs.7.6, 7.7, 7.8, 7.9, 7.10, 7.11, 7.12 and 7.13 for cross-ply and angle-ply shell of CCCC CCSC, SCSC and SSSC boundary conditions for different eccentric positions of the cutout. All the mode shapes are either bending or torsion mode. It is observed that for different positions of cutout, mode shapes are somewhat similar to one another and only the crest and trough position changes.



a'/a=0.0

a'/a=0.1

a'/a=0.2

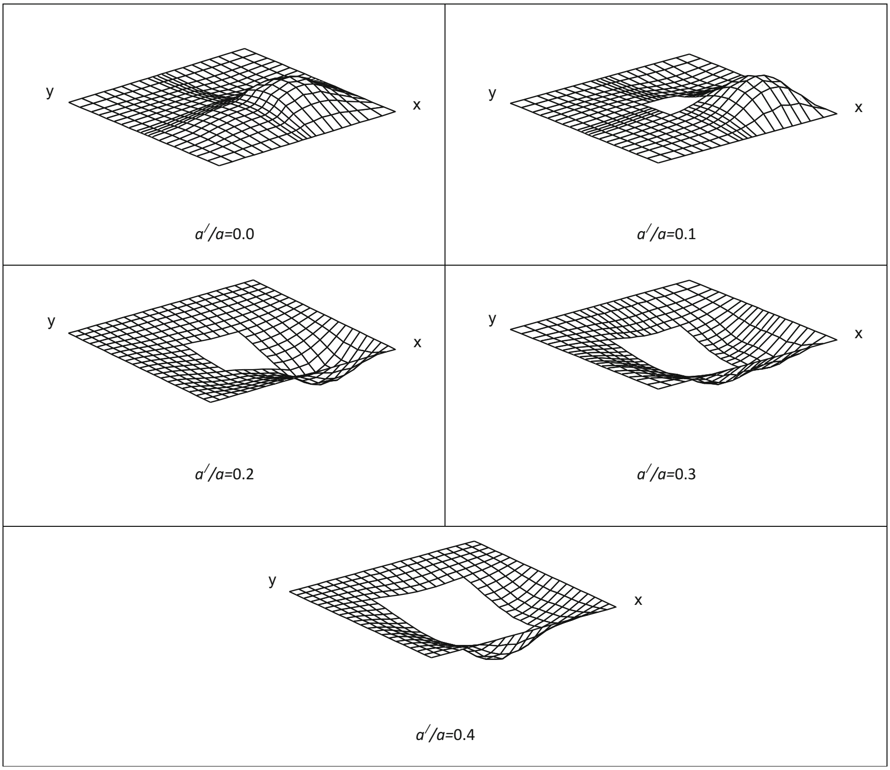
a'/a=0.3

a'/a=0.4

Fig. 7.3 First mode shapes of laminated composite (0/90/0/90) stiffened hyperbolic paraboloidal shell for different sizes of central square cutout and CSCS boundary conditions

7.4 Closure

Free vibration characteristics mostly depend on how the boundary constraints along the four edges are arranged rather than their actual number. If two edges are released for any functional reason, then two alternate edges must release instead of two adjacent edges. The relative performances of shells in regard to free vibration characteristics for different combinations of edge conditions along the four sides are believed to be very helpful in decision-making for practising engineers. For functional purposes, if a shift of central cutout is required, eccentricity of a cutout along the length or width should preferably be towards the simply



a'/a=0.0

a'/a=0.1

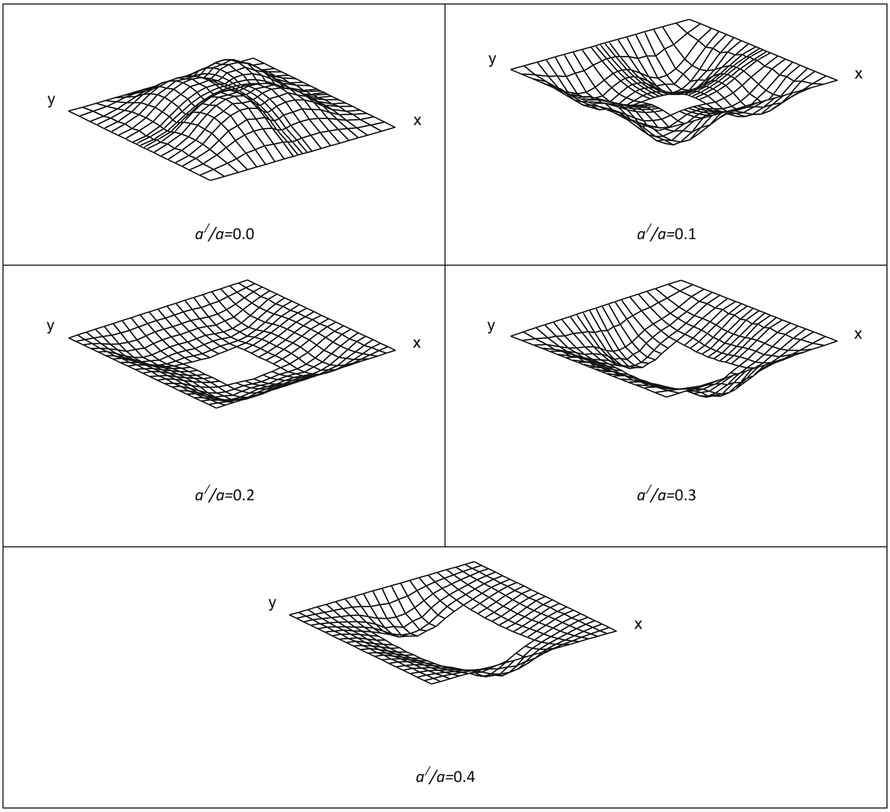
a'/a=0.2

a'/a=0.3

a'/a=0.4

Fig. 7.4 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened hyperbolic paraboloidal shell for different sizes of central square cutout and CSSC boundary conditions

supported edge which is opposite to a clamped edge for cross-ply shells. For angle-ply shells eccentricity towards simply supported hyperbolic edge should be avoided. The data regarding the behaviour of stiffened hyperbolic paraboloid shell with eccentric cutouts for a wide range of eccentricity and boundary conditions for cross-ply and angle-ply shells will be helpful design aids for structural engineers dealing with such shell structures.



a'/a=0.0

a'/a=0.1

a'/a=0.2

a'/a=0.3

a'/a=0.4

Fig. 7.5 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened hyperbolic paraboloidal shell for different sizes of central square cutout and CSCS boundary conditions

Table 7.6 Values of ‘r’ for cross-ply hyperbolic paraboloidal shells

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
CCCC	0/90/90/0	0.2	89.09	90.82	94.20	97.90	94.20	90.81	89.08
		0.3	87.70	89.45	93.11	97.48	93.11	89.45	87.70
		0.4	89.58	91.24	94.77	98.64	94.77	91.24	89.58
		0.5	91.95	93.63	96.96	100.00	96.96	93.63	91.95
		0.6	89.58	91.24	94.77	98.64	94.77	91.24	89.58
		0.7	87.70	89.45	93.11	97.48	93.11	89.45	87.70
		0.8	88.85	90.56	93.88	97.90	94.11	90.61	88.91
	0/90/0/90	0.2	82.88	86.09	92.60	99.80	92.60	86.10	82.88
		0.3	81.62	84.83	91.57	99.64	91.57	84.83	81.62
		0.4	83.23	86.19	92.48	99.75	92.48	86.19	83.23
		0.5	85.60	88.38	94.14	100.00	94.14	88.38	85.60
		0.6	83.23	86.19	92.48	99.85	92.48	86.19	83.23
		0.7	81.62	84.83	91.57	99.64	91.57	84.83	81.69
		0.8	82.62	85.82	92.31	99.78	92.52	85.88	82.72
CSCC	0/90/90/0	0.2	95.76	98.63	101.84	103.62	101.83	98.63	95.75
		0.3	97.34	102.57	107.80	110.44	107.81	102.58	97.35
		0.4	96.74	101.33	108.80	118.09	108.96	101.36	96.65
		0.5	92.79	94.34	97.49	100.00	97.66	94.41	92.45
		0.6	87.32	87.05	88.41	98.82	88.60	87.12	86.84
		0.7	84.18	83.46	84.49	85.77	84.65	83.53	83.73
		0.8	83.37	82.72	83.75	84.96	83.90	82.81	83.00
	0/90/0/90	0.2	91.49	96.28	100.95	105.01	100.95	96.31	91.51
		0.3	91.46	96.97	104.34	110.17	104.36	97.00	91.46
		0.4	91.09	95.56	103.48	112.01	103.62	95.62	91.01
		0.5	88.02	89.69	94.66	100.00	94.97	89.83	87.79
		0.6	84.01	84.46	88.60	93.32	88.84	84.59	83.72
		0.7	82.15	82.44	86.47	90.93	86.64	82.54	81.88
		0.8	81.98	82.37	86.32	90.59	86.51	82.48	81.78
CCSC	0/90/90/0	0.2	80.87	83.53	89.02	99.01	103.19	96.08	91.14
		0.3	80.35	83.00	88.42	98.24	102.22	95.45	90.92
		0.4	80.64	83.24	88.71	98.88	103.07	95.51	90.31
		0.5	81.17	83.80	89.42	100.00	104.29	95.94	90.09
		0.6	80.64	83.24	88.71	98.88	103.07	95.51	90.31
		0.7	80.35	83.00	88.42	98.24	102.22	95.45	90.92
		0.8	80.70	83.34	88.73	98.94	103.01	95.83	90.97
	0/90/0/90	0.2	73.72	77.82	85.97	100.54	106.08	94.91	87.48
		0.3	73.16	77.31	85.52	100.21	105.48	94.52	87.44
		0.4	73.36	77.32	85.32	99.77	105.38	94.06	86.47
		0.5	73.88	77.79	85.71	100.00	105.75	94.03	85.90
		0.6	73.36	77.32	85.32	99.77	105.38	94.06	86.47
		0.7	73.16	77.31	85.52	100.21	105.48	94.52	87.44
		0.8	73.49	77.59	85.71	100.50	105.99	94.68	87.29

(continued)

Table 7.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
CCCS	0/90/90/0	0.2	83.55	83.26	84.19	85.14	84.19	83.26	83.55
		0.3	84.26	83.96	84.89	85.90	84.88	83.96	84.26
		0.4	87.41	87.57	88.80	89.91	88.80	87.57	87.41
		0.5	93.06	94.92	97.89	100.00	97.89	94.92	93.06
		0.6	97.27	101.96	109.54	123.05	109.35	101.96	97.27
		0.7	97.97	103.23	108.52	111.16	108.51	103.23	97.97
		0.8	95.72	99.06	102.48	104.26	102.47	99.07	95.91
	0/90/0/90	0.2	82.11	82.75	86.69	90.77	86.69	82.75	82.11
		0.3	82.17	82.77	86.77	91.09	86.77	82.77	82.17
		0.4	84.01	84.80	88.90	93.41	88.90	84.80	84.01
		0.5	88.12	90.05	94.98	100.00	94.98	90.05	88.11
		0.6	91.38	95.94	103.84	113.05	103.84	95.94	91.38
		0.7	91.82	97.37	104.78	110.54	104.78	97.37	91.82
		0.8	90.00	95.75	101.33	105.46	101.35	96.32	90.68
CSSC	0/90/90/0	0.2	71.91	79.82	90.94	105.34	103.51	91.42	81.58
		0.3	76.06	84.77	97.05	111.76	109.58	97.04	86.44
		0.4	76.78	84.91	96.55	112.82	113.81	99.77	88.26
		0.5	71.96	77.67	86.27	100.00	104.79	92.31	82.45
		0.6	65.73	70.08	77.28	89.88	94.36	83.59	75.28
		0.7	61.75	65.95	73.15	85.93	89.06	78.68	70.97
		0.8	60.26	64.82	72.38	85.51	87.40	76.91	69.26
	0/90/0/90	0.2	71.77	79.96	91.50	107.07	107.29	93.76	82.91
		0.3	76.23	85.35	98.03	113.08	112.28	98.86	87.59
		0.4	75.40	82.61	93.92	112.04	116.08	101.44	89.19
		0.5	68.44	73.67	83.16	100.00	107.26	92.77	81.94
		0.6	62.08	66.80	76.05	93.07	99.39	85.43	75.41
		0.7	58.92	63.95	73.64	91.03	95.87	81.94	72.16
		0.8	58.21	63.64	73.59	91.04	94.92	80.97	71.21
CCSS	0/90/90/0	0.2	60.29	64.88	72.46	85.55	87.13	76.82	69.24
		0.3	61.74	65.98	73.17	85.96	88.70	78.55	70.94
		0.4	65.73	70.11	77.31	89.89	93.92	83.45	75.26
		0.5	71.97	77.71	86.30	100.00	104.38	92.20	82.44
		0.6	76.81	84.94	96.58	112.86	113.77	99.75	88.26
		0.7	76.10	84.81	97.09	111.81	109.62	97.07	86.47
		0.8	71.48	79.44	90.68	105.35	103.53	91.32	81.41
	0/90/0/90	0.2	58.25	63.72	73.68	91.06	94.76	80.91	71.20
		0.3	58.90	63.97	73.65	91.02	95.64	81.83	72.12
		0.4	62.06	66.82	76.06	93.08	99.06	85.29	75.37
		0.5	68.42	73.69	83.16	100.00	106.87	92.61	81.89
		0.6	75.41	82.63	93.93	112.05	116.01	101.38	89.16
		0.7	76.25	85.36	98.05	113.10	112.30	98.88	87.60
		0.8	71.31	79.61	91.38	107.05	107.30	93.70	82.78

(continued)

Table 7.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
CSCS	0/90/90/0	0.2	82.94	82.40	82.54	82.67	82.53	82.40	82.94
		0.3	84.82	84.47	85.00	85.48	85.00	84.47	84.82
		0.4	88.75	89.08	90.31	91.26	90.31	89.08	88.75
		0.5	93.70	96.63	98.42	100.00	98.42	96.63	93.70
		0.6	88.75	89.08	90.31	91.26	90.31	89.08	88.75
		0.7	84.83	84.47	85.00	85.48	85.00	84.47	84.82
		0.8	82.93	82.38	82.51	82.66	82.52	82.39	82.93
	0/90/0/90	0.2	82.39	82.75	86.02	89.24	86.02	82.75	82.39
		0.3	83.15	83.62	87.24	91.00	87.24	83.62	83.15
		0.4	85.43	86.25	90.30	94.62	90.30	86.25	85.43
		0.5	89.61	90.53	95.16	100.00	95.16	90.53	89.46
		0.6	85.43	86.25	90.29	94.62	90.30	86.25	85.43
		0.7	83.15	83.62	87.24	91.00	87.24	83.62	83.15
		0.8	82.33	82.71	85.99	89.24	86.01	82.73	82.38
SCSC	0/90/90/0	0.2	79.29	83.00	89.32	99.59	89.32	83.00	79.29
		0.3	79.31	82.86	88.93	99.17	88.93	82.86	79.31
		0.4	78.97	82.72	89.07	99.16	89.07	82.72	78.97
		0.5	78.85	82.88	89.58	100.00	89.58	82.88	78.85
		0.6	78.97	82.72	89.07	99.87	89.07	82.72	78.97
		0.7	79.31	82.86	88.93	98.75	88.93	82.86	79.31
		0.8	79.12	82.81	89.03	99.17	89.24	82.84	79.20
	0/90/0/90	0.2	71.41	76.93	86.16	101.86	86.16	76.93	71.41
		0.3	71.56	76.94	86.04	100.83	86.04	76.94	71.56
		0.4	70.99	76.53	85.67	100.01	85.67	76.53	70.99
		0.5	70.69	76.48	85.81	100.00	85.81	76.48	70.69
		0.6	70.99	76.53	85.67	100.14	85.67	76.53	70.99
		0.7	71.56	76.94	86.04	100.56	86.04	76.94	71.56
		0.8	71.19	76.70	85.90	100.67	86.09	76.77	71.29
CSSS	0/90/90/0	0.2	51.33	57.82	67.63	82.96	84.15	72.06	62.83
		0.3	55.42	61.59	70.76	85.37	88.03	76.33	67.22
		0.4	61.04	67.79	76.99	91.12	95.07	83.04	73.27
		0.5	65.13	74.29	87.10	100.00	104.47	89.74	78.06
		0.6	61.04	67.79	76.99	91.12	95.06	83.04	73.27
		0.7	55.43	61.59	70.76	85.37	88.03	76.33	67.22
		0.8	51.27	57.80	67.60	82.94	84.13	72.03	62.79
	0/90/0/90	0.2	51.72	59.07	70.82	89.60	92.64	77.23	65.92
		0.3	54.74	61.40	72.46	90.93	95.30	80.27	69.26
		0.4	59.74	66.11	76.46	94.37	100.12	85.32	74.24
		0.5	65.28	73.99	82.80	100.00	107.08	91.45	79.13
		0.6	59.74	66.11	76.46	94.37	100.12	85.32	74.24
		0.7	54.74	61.40	72.46	90.93	95.30	80.26	69.26
		0.8	51.71	59.06	70.79	89.58	92.61	77.22	65.90

(continued)

Table 7.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
SSSC	0/90/90/0	0.2	69.80	79.08	91.01	102.97	91.01	79.08	69.80
		0.3	73.61	83.66	96.66	108.19	96.66	83.66	73.61
		0.4	73.41	83.03	95.76	110.20	95.75	83.03	73.40
		0.5	67.68	75.36	85.52	100.00	85.52	75.36	67.68
		0.6	61.54	67.96	76.68	90.04	76.79	67.96	61.54
		0.7	58.03	64.14	72.68	86.39	72.68	64.14	58.03
		0.8	56.92	63.18	71.96	85.77	72.00	63.20	56.92
	0/90/0/90	0.2	68.33	77.85	90.13	102.99	90.13	77.85	68.33
		0.3	72.36	82.73	96.11	107.63	96.11	82.73	72.36
		0.4	69.89	78.93	91.63	109.01	91.63	78.92	69.89
		0.5	62.18	69.81	81.00	100.00	81.00	69.81	62.18
		0.6	56.27	63.36	74.18	90.79	74.18	63.36	56.27
		0.7	53.77	60.90	71.95	88.98	71.95	60.90	53.77
		0.8	53.46	60.75	71.94	89.13	71.98	60.77	53.50
SSCS	0/90/90/0	0.2	62.83	72.05	84.15	82.96	67.62	57.82	51.33
		0.3	67.22	76.33	88.03	85.37	70.75	61.59	55.42
		0.4	73.27	83.04	95.06	91.12	76.99	67.79	61.04
		0.5	78.04	89.74	104.47	100.00	87.10	74.29	65.13
		0.6	73.27	83.04	95.06	91.12	76.99	67.79	61.04
		0.7	67.22	76.33	88.03	85.37	70.75	61.59	55.42
		0.8	62.78	72.03	84.13	82.94	67.60	57.80	51.28
	0/90/0/90	0.2	65.92	77.23	92.64	89.60	70.82	59.07	51.72
		0.3	69.26	80.26	95.30	90.93	72.46	61.40	54.74
		0.4	74.24	85.32	100.12	94.37	76.46	66.11	59.74
		0.5	79.13	91.45	107.08	100.00	82.80	73.99	65.28
		0.6	74.24	85.32	100.12	94.37	76.46	66.11	59.74
		0.7	69.25	80.26	95.30	90.93	72.46	61.40	54.73
		0.8	65.90	77.22	92.63	89.58	70.79	59.05	51.70
SSSS	0/90/90/0	0.2	46.61	55.16	66.25	81.99	66.24	55.16	46.61
		0.3	50.60	58.74	69.22	84.36	69.22	58.74	50.60
		0.4	56.16	64.73	75.23	89.84	75.23	64.73	56.16
		0.5	60.47	71.24	85.11	100.00	85.11	71.24	60.47
		0.6	56.16	64.72	75.22	89.84	75.22	64.72	56.16
		0.7	50.60	58.74	69.22	84.36	69.22	58.74	50.59
		0.8	46.56	55.14	66.21	81.97	66.21	55.14	46.57
	0/90/0/90	0.2	48.35	58.19	71.73	90.91	71.73	58.19	48.35
		0.3	51.21	60.39	73.30	92.10	73.30	60.39	51.21
		0.4	56.14	64.92	77.21	95.34	77.21	64.91	56.14
		0.5	62.58	72.14	83.37	100.00	83.37	72.14	62.57
		0.6	56.14	64.91	77.21	95.33	77.21	64.91	56.14
		0.7	51.21	60.39	73.30	92.10	73.30	60.39	51.21
		0.8	48.34	58.18	71.69	92.03	71.71	58.16	48.34

(continued)

Table 7.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
CS	0/90/90/0	0.2	91.37	102.29	109.41	112.10	109.41	102.29	91.37
		0.3	93.48	100.29	104.82	106.65	104.82	100.29	93.48
		0.4	95.96	98.46	100.57	101.67	100.55	98.46	95.96
		0.5	97.29	97.68	98.93	100.00	98.93	97.68	97.29
		0.6	95.93	98.46	100.55	101.67	100.55	98.46	95.93
		0.7	93.46	100.29	104.82	106.65	104.82	100.29	93.46
		0.8	91.21	102.01	109.05	112.02	109.20	102.11	91.29
	0/90/0/90	0.2	87.88	97.94	104.11	107.44	104.11	97.94	87.88
		0.3	88.96	95.74	101.44	104.94	101.42	95.74	88.94
		0.4	90.09	93.54	98.07	101.64	98.07	93.51	90.09
		0.5	90.55	92.51	96.52	100.00	96.52	92.51	90.53
		0.6	90.09	93.54	98.09	101.64	98.07	93.54	90.09
		0.7	88.94	95.74	101.42	104.94	101.42	95.74	88.94
		0.8	87.81	97.72	103.79	107.37	103.84	97.72	87.83

$a/b=1$, $a/h=100$, $a'/b'=1$, $h/R_{xx}=1/300$, $R_{xx}/R_{yy}=-1.5$; $E_{11}/E_{22}=25$, $G_{23}=0.2E_{22}$, $G_{13}=G_{12}=0.5E_{22}$, $\nu_{12}=\nu_{21}=0.25$

Table 7.7 Values of '*r*' for angle-ply hyperbolic paraboloidal shells

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
CCCC	+45/−45/−45/+45	0.2	64.48	68.81	74.98	86.01	77.10	69.71	65.10
		0.3	67.37	72.04	79.38	89.61	81.24	73.36	68.28
		0.4	72.36	77.53	85.30	95.14	87.85	79.43	73.72
		0.5	79.86	85.80	93.45	100.00	93.45	85.81	79.85
		0.6	73.72	79.43	87.85	95.14	85.30	77.53	72.36
		0.7	68.28	73.36	81.24	89.61	79.37	72.04	67.37
		0.8	65.10	69.71	77.09	86.01	75.79	68.80	64.47
	+45/−45/+45/−45	0.2	65.33	69.87	77.01	86.18	76.94	69.82	65.30
		0.3	68.42	73.41	81.02	89.95	80.91	73.34	68.38
		0.4	73.58	79.17	87.27	95.50	87.15	79.08	73.54
		0.5	80.82	86.30	93.83	99.99	93.83	86.30	80.82
		0.6	73.54	79.08	87.15	95.50	87.27	79.17	73.58
		0.7	68.38	73.34	80.91	89.95	81.02	73.41	68.42
		0.8	65.30	69.82	76.93	86.18	77.01	69.87	65.33
CSCC	+45/−45/−45/+45	0.2	67.69	72.71	80.45	91.00	81.93	73.80	68.54
		0.3	71.40	76.57	84.50	95.05	86.55	78.02	72.45
		0.4	76.83	82.57	90.85	100.39	93.51	84.63	78.23
		0.5	74.22	82.19	92.70	100.00	90.93	80.80	73.20
		0.6	68.03	74.99	84.88	93.89	83.64	74.21	67.51
		0.7	64.65	70.96	80.13	89.59	79.34	70.54	64.44
		0.8	63.06	68.90	77.45	87.05	76.98	68.64	62.94
	+45/−45/+45/−45	0.2	68.16	73.25	81.05	90.98	80.86	73.14	68.11
		0.3	71.86	77.24	85.42	95.23	85.21	77.12	71.80
		0.4	77.25	83.43	92.15	100.74	91.89	83.28	77.16
		0.5	73.80	81.40	91.44	100.00	91.51	81.00	73.14

		0.6	67.84	74.51	83.86	93.67	83.01	74.27	67.50
		0.7	64.65	70.70	79.37	89.27	79.35	70.50	64.44
		0.8	63.14	68.73	76.84	86.65	76.79	68.54	62.96
CCSC	+45/−45/−45/+45	0.2	66.40	69.56	74.73	83.45	82.98	75.02	70.03
		0.3	69.18	72.75	78.37	87.45	87.40	78.94	73.45
		0.4	73.84	77.98	84.23	93.60	94.34	85.44	79.24
		0.5	80.07	85.09	91.58	100.00	99.69	91.91	85.44
		0.6	74.61	79.36	86.40	96.24	91.65	83.35	77.66
		0.7	69.75	73.77	80.03	89.87	85.38	77.50	72.42
		0.8	66.83	70.30	75.90	85.26	81.57	74.04	69.35
	+45/−45/+45/−45	0.2	66.83	70.25	75.70	84.69	81.61	74.12	69.42
		0.3	69.50	73.43	79.50	89.00	85.70	77.71	72.51
		0.4	74.02	78.64	85.43	95.16	92.27	83.71	77.79
		0.5	79.35	85.01	91.52	100.00	99.77	92.15	85.30
		0.6	73.49	77.95	84.60	94.33	92.55	83.90	77.91
		0.7	69.02	72.80	78.68	88.04	85.93	77.85	72.60
		0.8	66.42	69.69	74.95	83.78	81.77	74.24	69.50
CCCS	+45/−45/−45/+45	0.2	62.81	68.45	76.77	87.25	77.93	69.33	63.45
		0.3	64.19	70.17	78.91	89.67	80.63	71.40	65.05
		0.4	67.11	73.61	82.91	93.83	85.40	75.45	68.44
		0.5	72.65	79.98	89.97	100.00	93.26	82.70	74.67
		0.6	78.57	85.13	94.07	100.94	91.41	83.08	77.30
		0.7	72.89	78.50	87.08	95.63	85.02	77.04	71.84
		0.8	68.97	74.26	82.43	91.52	80.93	73.15	68.11
	+45/−45/+45/−45	0.2	62.89	68.42	76.65	86.68	76.96	68.84	63.25
		0.3	64.33	70.32	79.12	94.53	79.49	70.81	64.75
		0.4	67.33	74.01	83.55	93.64	83.99	74.63	67.95

(continued)

Table 7.7 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
		0.5	72.89	80.62	91.07	100.00	91.58	81.53	73.91
		0.6	77.26	83.41	92.03	100.89	92.29	83.56	77.38
		0.7	71.92	77.24	85.34	95.38	85.56	77.36	71.97
		0.8	68.21	73.25	80.98	91.09	81.16	73.36	68.26
CSSC	+45/−45/−45/+45	0.2	69.10	73.06	79.10	88.58	88.11	79.36	73.63
		0.3	73.03	77.08	83.32	92.98	92.98	83.85	77.72
		0.4	74.51	81.79	89.56	99.20	100.07	90.86	84.06
		0.5	69.09	76.57	87.49	100.00	97.03	86.25	77.31
		0.6	64.63	70.95	80.28	92.99	89.55	79.24	71.26
		0.7	62.05	67.49	75.79	87.95	85.03	75.33	68.16
		0.8	60.74	65.66	73.43	85.01	82.53	73.35	66.76
	+45/−45/+45/−45	0.2	70.74	74.99	81.37	91.13	87.59	79.20	73.63
		0.3	74.11	78.70	85.59	95.80	92.22	83.45	77.51
		0.4	74.46	82.22	91.92	101.92	99.20	90.05	83.45
		0.5	69.20	76.62	87.32	100.00	98.33	86.96	77.50
		0.6	65.00	71.37	80.57	92.91	90.40	79.69	71.36
		0.7	62.67	68.23	76.47	88.23	85.58	75.63	68.27
		0.8	61.49	66.58	74.37	85.63	82.84	73.55	66.89
CCSS	+45/−45/−45/+45	0.2	61.33	66.51	74.63	86.21	85.04	75.38	68.48
		0.3	62.72	68.20	76.52	88.52	87.97	77.67	70.14
		0.4	65.10	71.32	80.40	92.90	93.11	82.11	73.76
		0.5	69.36	76.53	86.99	100.00	101.24	90.02	80.60
		0.6	75.10	82.93	94.13	103.89	99.69	90.68	84.31
		0.7	75.33	80.00	87.18	97.90	92.96	84.20	78.26
		0.8	71.60	75.88	82.47	92.84	88.57	80.04	74.35

	+45/−45/+45/−45	0.2	61.59	66.54	74.25	85.53	83.24	74.26	67.84
		0.3	62.68	68.11	76.35	88.25	85.96	76.40	69.40
		0.4	64.90	71.18	80.40	92.98	90.77	80.55	72.79
		0.5	68.99	76.33	87.09	100.00	98.65	87.98	79.22
		0.6	74.54	82.21	91.17	101.11	99.67	90.40	83.69
		0.7	73.79	78.07	84.68	94.79	92.65	83.76	77.73
		0.8	69.78	73.91	80.17	89.99	87.97	79.50	73.84
CSCS	+45/−45/−45/+45	0.2	63.66	69.67	78.33	88.83	79.54	70.57	64.37
		0.3	65.42	71.66	80.69	91.53	82.45	72.92	66.32
		0.4	68.49	75.25	84.84	95.74	87.39	77.13	69.82
		0.5	73.28	81.51	92.00	100.00	92.00	81.51	73.27
		0.6	69.82	77.13	87.39	95.74	84.84	75.25	68.49
		0.7	66.32	72.93	82.45	91.53	80.69	71.66	65.42
		0.8	64.36	70.57	79.54	88.81	78.32	69.66	63.65
	+45/−45/+45/−45	0.2	63.68	69.60	78.21	88.23	78.50	69.99	64.04
		0.3	65.52	71.78	80.89	91.10	81.26	72.28	65.96
		0.4	68.67	75.63	85.47	95.54	85.92	76.26	69.28
		0.5	73.12	81.73	92.91	100.00	92.91	81.73	73.41
		0.6	69.28	76.26	85.93	95.54	85.47	75.63	68.67
		0.7	65.96	72.28	81.26	91.10	80.89	71.78	65.52
		0.8	64.03	69.99	78.49	88.21	78.19	69.59	63.67
SCSC	+45/−45/−45/+45	0.2	68.05	71.32	76.63	85.53	77.83	72.07	68.48
		0.3	70.86	74.57	80.35	89.45	82.05	75.62	71.48
		0.4	75.52	79.89	86.31	95.28	88.54	81.33	76.43
		0.5	81.78	86.99	93.62	100.00	93.62	86.99	81.78
		0.6	76.43	81.34	88.54	95.28	86.31	79.89	75.52
		0.7	71.48	75.62	82.05	89.45	80.35	74.57	70.86
		0.8	68.48	72.07	77.83	85.53	76.62	71.31	68.04

(continued)

Table 7.7 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
	+45/−45/+45/−45	0.2	68.49	72.01	77.63	85.85	76.85	71.44	68.07
		0.3	71.21	75.27	81.51	89.85	80.67	74.62	70.71
		0.4	75.78	80.57	87.54	95.55	86.70	79.88	75.23
		0.5	81.15	86.92	93.58	100.00	93.58	86.92	81.15
		0.6	75.23	79.88	86.70	95.55	87.54	80.57	75.78
		0.7	70.71	74.62	80.67	89.85	81.51	75.27	71.21
		0.8	68.07	71.44	76.85	85.85	77.62	72.01	68.49
CSSS	+45/−45/−45/+45	0.2	59.77	65.68	74.73	86.80	85.53	75.49	68.13
		0.3	62.14	67.94	76.89	89.32	88.58	77.94	70.16
		0.4	64.43	70.87	80.70	93.71	93.79	82.56	74.15
		0.5	66.57	74.06	86.39	100.00	98.20	87.63	78.53
		0.6	65.27	71.97	82.47	96.00	91.08	80.43	72.50
		0.7	62.91	68.77	77.96	90.90	86.69	76.50	69.04
		0.8	60.71	66.36	75.28	87.76	84.23	74.49	67.35
	+45/−45/+45/−45	0.2	60.83	66.45	75.08	86.92	84.29	74.84	67.87
		0.3	62.69	68.44	77.35	89.77	87.22	77.24	69.89
		0.4	64.63	71.18	81.32	94.53	92.17	81.64	73.73
		0.5	66.22	73.74	86.32	100.00	98.45	87.71	77.97
		0.6	64.61	71.24	81.38	94.37	91.27	80.25	71.93
		0.7	62.24	68.23	77.26	89.64	86.45	76.05	68.40
		0.8	59.80	65.71	74.67	86.70	83.67	68.04	66.65
SSSC	+45/−45/−45/+45	0.2	71.68	76.13	82.52	92.25	84.03	77.28	72.73
		0.3	75.73	80.22	86.84	96.57	88.78	81.44	76.57
		0.4	76.72	85.30	93.26	101.99	95.77	84.03	75.15
		0.5	70.07	79.04	90.79	100.00	88.08	77.00	68.43

		0.6	65.14	73.05	83.30	93.47	81.39	71.56	63.79
		0.7	62.66	69.44	78.62	89.12	77.47	68.42	61.63
		0.8	61.62	67.55	76.12	86.66	75.53	66.76	60.63
	+45/−45/+45/−45	0.2	72.56	77.15	83.76	92.20	82.30	75.80	71.28
		0.3	76.08	80.93	88.07	96.76	86.86	79.98	75.31
		0.4	75.75	84.47	94.51	101.97	93.41	84.10	75.00
		0.5	69.58	78.21	89.52	100.00	88.70	77.13	68.11
		0.6	65.07	72.73	82.60	93.54	81.81	71.60	63.43
		0.7	62.89	69.52	78.40	89.06	77.65	68.44	61.36
		0.8	61.94	67.85	76.23	86.47	75.46	66.86	60.70
SSCS	+45/−45/−45/+45	0.2	67.35	74.49	84.24	87.77	75.30	66.39	60.74
		0.3	69.04	76.50	86.69	90.89	77.96	68.77	62.91
		0.4	72.50	80.43	91.08	96.00	82.46	71.96	65.27
		0.5	78.53	87.63	98.20	100.00	86.39	74.06	66.57
		0.6	74.15	82.57	93.80	93.71	80.70	70.87	64.43
		0.7	70.16	77.94	88.59	89.32	76.89	67.94	62.14
		0.8	68.12	75.49	85.53	86.78	74.67	65.64	59.74
	+45/−45/+45/−45	0.2	66.65	73.88	83.68	86.72	74.69	65.74	59.83
		0.3	68.40	76.05	86.45	89.64	77.26	68.23	62.24
		0.4	71.93	80.26	91.27	94.37	81.37	71.24	64.61
		0.5	77.97	87.71	98.45	100.00	86.32	73.74	66.22
		0.6	73.73	81.65	92.17	94.53	81.31	71.18	64.63
		0.7	69.89	77.24	87.23	89.77	77.35	68.44	62.69
		0.8	67.86	74.84	84.28	86.90	75.03	66.40	60.80
SSSS	+45/−45/−45/+45	0.2	57.04	66.24	77.39	89.39	78.13	67.30	58.44
		0.3	59.16	68.00	79.32	92.06	80.67	69.35	60.61
		0.4	62.88	71.80	83.36	96.28	85.52	73.52	64.40
		0.5	68.34	78.20	90.48	100.00	90.48	78.20	68.34

(continued)

Table 7.7 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
		0.6	64.40	73.53	85.52	96.28	83.36	71.80	62.87
		0.7	60.62	69.35	80.67	92.06	79.32	68.00	59.16
		0.8	58.43	67.30	78.12	89.38	77.32	66.22	57.03
	+45/−45/+45/−45	0.2	57.38	66.48	77.43	88.99	77.61	66.75	57.63
		0.3	59.29	68.18	79.58	91.81	80.01	68.85	59.97
		0.4	62.77	71.98	83.90	96.20	84.43	72.80	63.84
		0.5	68.08	78.21	91.21	100.00	91.21	78.21	68.08
		0.6	63.84	72.81	84.43	96.20	83.90	71.98	62.77
		0.7	59.97	68.85	80.01	91.81	79.58	68.18	59.29
		0.8	57.63	66.74	77.60	88.98	77.38	66.46	57.37
CS	+45/−45/−45/+45	0.2	72.72	82.11	93.00	96.83	83.45	73.17	66.16
		0.3	74.59	83.13	93.60	97.50	83.75	74.22	67.93
		0.4	73.73	81.34	91.59	98.86	85.28	75.86	69.38
		0.5	71.08	78.04	88.02	100.00	88.00	78.04	71.08
		0.6	69.38	75.88	85.28	98.86	91.59	59.44	73.73
		0.7	67.93	74.25	83.77	97.52	93.58	83.13	74.59
		0.8	66.16	73.13	83.36	96.79	92.44	81.88	72.61
	+45/−45/+45/−45	0.2	64.80	72.96	83.43	92.90	85.34	74.61	66.23
		0.3	66.53	74.25	84.50	95.15	86.34	76.11	68.49
		0.4	67.75	75.73	86.40	98.11	88.03	77.54	69.61
		0.5	68.95	77.16	88.29	100.00	88.29	77.16	68.95
		0.6	69.63	77.56	88.03	98.09	86.40	63.44	67.75
		0.7	68.51	76.13	86.34	95.17	84.52	74.25	66.53
		0.8	66.21	74.55	85.22	92.90	83.23	72.88	64.76

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 1/300$, $R_{xx}/R_{yy} = -1.5$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 7.8 Maximum values of r with corresponding coordinates of cutout centres and zones where $r \geq 95$ for cross-ply hyperbolic paraboloidal shells

Laminations	0/90/90/0			0/90/0/90		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$
CCCC	100.00	(0.5,0.5)	$\bar{x} = 0.5$	100.00	(0.5,0.5)	$\bar{x} = 0.5$
			$0.2 \leq \bar{y} \leq 0.8$			$0.2 \leq \bar{y} \leq 0.8$
CSCC	118.09	(0.5,0.4)	$0.2 \leq \bar{x} \leq 0.8$	112.01	(0.5,0.4)	$0.3 \leq \bar{x} \leq 0.7$
			$0.2 \leq \bar{y} \leq 0.4$			$0.2 \leq \bar{y} \leq 0.4$
CCSC	104.29	(0.6,0.5)	$0.5 \leq \bar{x} \leq 0.7$	106.08	(0.6,0.2)	$0.5 \leq \bar{x} \leq 0.6$
			$0.2 \leq \bar{y} \leq 0.8$			$0.2 \leq \bar{y} \leq 0.8$
CCCS	123.05	(0.5,0.6)	$0.3 \leq \bar{x} \leq 0.7$	113.05	(0.5,0.6)	$0.3 \leq \bar{x} \leq 0.7$
			$0.5 \leq \bar{y} \leq 0.8$			$0.6 \leq \bar{y} \leq 0.8$
CSSC	113.81	(0.6,0.4)	$0.5 \leq \bar{x} \leq 0.6$	116.08	(0.6,0.4)	$0.5 \leq \bar{x} \leq 0.6$
			$0.2 \leq \bar{y} \leq 0.5$			$0.2 \leq \bar{y} \leq 0.5$
CCSS	113.77	(0.6,0.6)	$0.5 \leq \bar{x} \leq 0.6$	116.01	(0.6,0.6)	$0.5 \leq \bar{x} \leq 0.6$
			$0.5 \leq \bar{y} \leq 0.8$			$0.5 \leq \bar{y} \leq 0.8$
CSCS	100.00	(0.5,0.5)	$0.3 \leq \bar{x} \leq 0.7$	100.00	(0.5,0.5)	$0.4 \leq \bar{x} \leq 0.6$
			$\bar{y} = 0.5$			$\bar{y} = 0.5$
SCSC	100.00	(0.5,0.5)	$\bar{x} = 0.5$	101.86	(0.5,0.2)	$\bar{x} = 0.5$
			$0.2 \leq \bar{y} \leq 0.8$			$0.2 \leq \bar{y} \leq 0.8$
CSSS	104.47	(0.6,0.5)	$\bar{x} = 0.6$	107.08	(0.6,0.5)	$\bar{x} = 0.6$
			$0.4 \leq \bar{y} \leq 0.6$			$0.3 \leq \bar{y} \leq 0.7$
SSSC	110.20	(0.5,0.4)	$\bar{x} = 0.5$	109.01	(0.5,0.4)	$\bar{x} = 0.5$
			$0.2 \leq \bar{y} \leq 0.5$			$0.2 \leq \bar{y} \leq 0.5$
SSCS	104.47	(0.4,0.5)	$\bar{x} = 0.4$	107.08	(0.4,0.5)	$\bar{x} = 0.4$
			$0.4 \leq \bar{y} \leq 0.6$			$0.3 \leq \bar{y} \leq 0.7$
SSSS	100.00	(0.5,0.5)	$\bar{x} = 0.5$	100.00	(0.5,0.5)	$\bar{x} = 0.5$
			$\bar{y} = 0.5$			$\bar{y} = 0.5$

(continued)

Table 7.8 (continued)

Laminations	0/90/90/0			0/90/0/90		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$
CS	112.10	(0.5,0.2)	$0.3 \leq \bar{x} \leq 0.7$ $0.2 \leq \bar{y} \leq 0.8$	107.37	(0.5,0.8)	$0.4 \leq \bar{x} \leq 0.6$ $0.2 \leq \bar{y} \leq 0.8$

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 1/300$, $R_{xx}/R_{yy} = -1.5$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 7.9 Maximum values of r with corresponding coordinates of cutout centres and zones where $r \geq 95$ for angle-ply hyperbolic paraboloidal shells

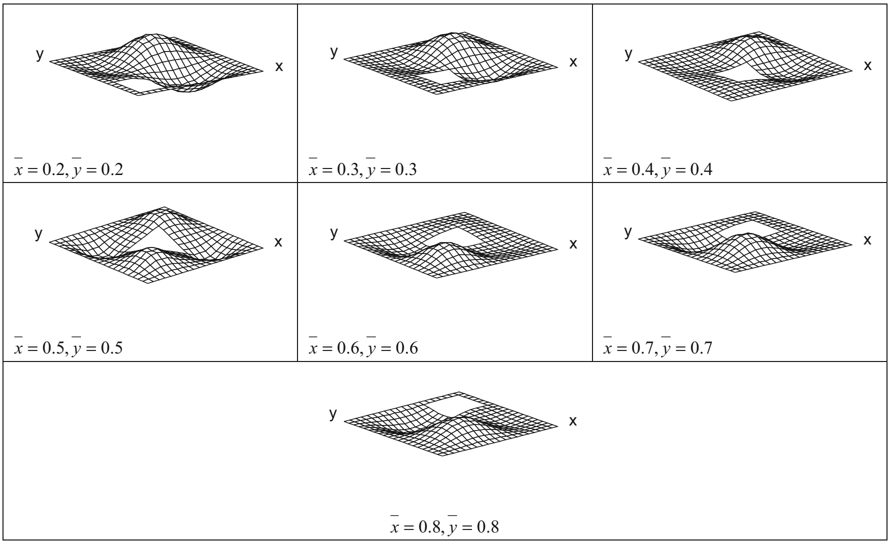
Laminations	+45/−45/−45/+45			+45/−45/+45/−45		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$
CCCC	100.00	(0.5,0.5)	$\bar{x} = 0.5$ $0.4 \leq \bar{y} \leq 0.6$	100.00	(0.5,0.5)	$\bar{x} = 0.5$ $0.4 \leq \bar{y} \leq 0.6$
CSCC	100.39	(0.5,0.4)	$\bar{x} = 0.5$ $0.3 \leq \bar{y} \leq 0.5$	100.74	(0.5,0.4)	$\bar{x} = 0.5$ $0.3 \leq \bar{y} \leq 0.5$
CCSC	100.00	(0.5,0.5)	$0.5 \leq \bar{x} \leq 0.6$ $\bar{y} = 0.5$	100.00	(0.5,0.5)	$\bar{x} = 0.5, \bar{y} = 0.4, 0.5$ $\bar{x} = 0.6, \bar{y} = 0.5$
CCCS	100.94	(0.5,0.6)	$\bar{x} = 0.5$ $0.5 \leq \bar{y} \leq 0.7$	100.89	(0.5,0.6)	$\bar{x} = 0.5$ $0.5 \leq \bar{y} \leq 0.7$
CSSC	100.07	(0.6,0.4)	$0.5 \leq \bar{x} \leq 0.6$ $0.4 \leq \bar{y} \leq 0.5$	101.91	(0.5,0.4)	$\bar{x} = 0.5$ $0.3 \leq \bar{y} \leq 0.5$ $\bar{x} = 0.6$ $0.4 \leq \bar{y} \leq 0.5$
CCSS	103.89	(0.5,0.6)	$0.5 \leq \bar{x} \leq 0.6$ $0.5 \leq \bar{y} \leq 0.6$	101.11	(0.5,0.6)	$0.5 \leq \bar{x} \leq 0.6$ $0.5 \leq \bar{y} \leq 0.6$
CSCS	100.00	(0.5,0.5)	$\bar{x} = 0.5$ $0.4 \leq \bar{y} \leq 0.6$	100.00	(0.5,0.5)	$\bar{x} = 0.5$ $0.4 \leq \bar{y} \leq 0.6$

(continued)

Table 7.9 (continued)

Laminations	+45/−45/−45/+45			+45/−45/+45/−45		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$
SCSC	100.00	(0.5,0.5)	$\bar{x} = 0.5$ $0.4 \leq \bar{y} \leq 0.6$	100.00	(0.5,0.5)	$\bar{x} = 0.5$ $0.4 \leq \bar{y} \leq 0.6$
CSSS	100.00	(0.5,0.5)	$\bar{x} = 0.5,0.6$ $\bar{y} = 0.5$	100.00	(0.5,0.5)	$\bar{x} = 0.5,0.6$ $\bar{y} = 0.5$
SSSC	101.99	(0.5,0.4)	$\bar{x} = 0.5$ $0.3 \leq \bar{y} \leq 0.5$	101.97	(0.5,0.4)	$\bar{x} = 0.5$ $0.3 \leq \bar{y} \leq 0.5$
SSCS	100.00	(0.5,0.5)	$\bar{x} = 0.4,0.5$ $\bar{y} = 0.5$	100.00	(0.5,0.5)	$\bar{x} = 0.4,0.5$ $\bar{y} = 0.5$
SSSS	100.00	(0.5,0.5)	$\bar{x} = 0.5$ $0.4 \leq \bar{y} \leq 0.6$	100.00	(0.5,0.5)	$\bar{x} = 0.5$ $0.4 \leq \bar{y} \leq 0.6$
CS	100.00	(0.5,0.5)	$\bar{x} = 0.5$ $0.2 \leq \bar{y} \leq 0.8$	100.00	(0.5,0.5)	$\bar{x} = 0.5$ $0.3 \leq \bar{y} \leq 0.7$

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 1/300$, $R_{xx}/R_{yy} = -1.5$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

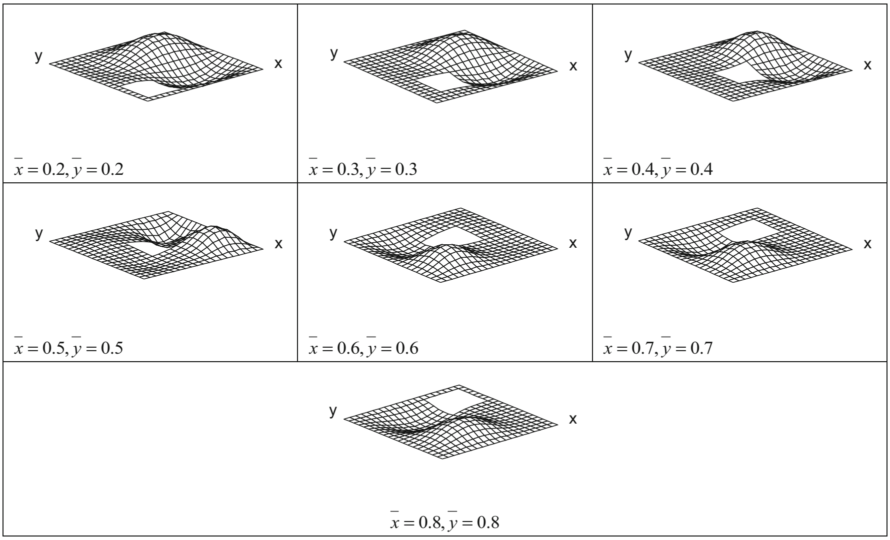
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 7.6 First mode shapes of laminated composite (0/90/0/90) stiffened hyperbolic paraboloidal shell for different position of central square cutout and CCCC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$
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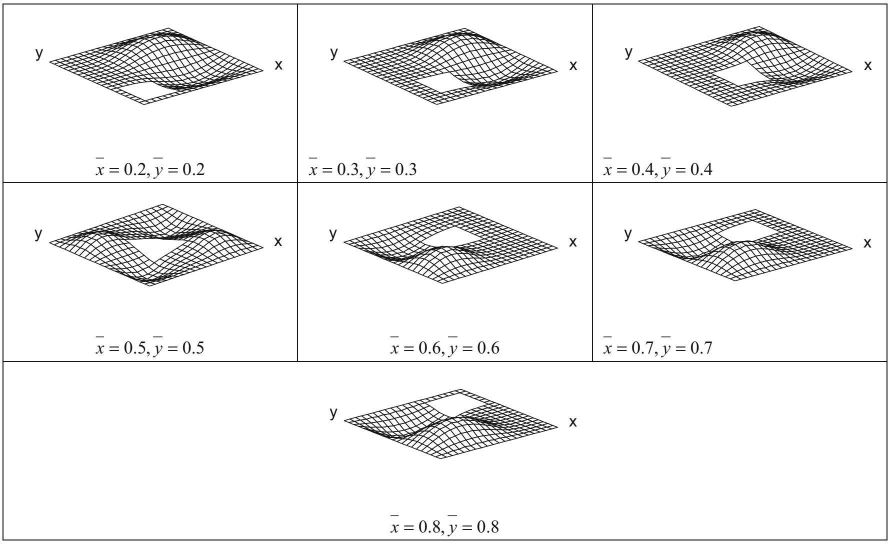
$\bar{x} = 0.4, \bar{y} = 0.4$
$\bar{x} = 0.5, \bar{y} = 0.5$
$\bar{x} = 0.6, \bar{y} = 0.6$
$\bar{x} = 0.7, \bar{y} = 0.7$
$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 7.7 First mode shapes of laminated composite (0/90/0/90) stiffened hyperbolic paraboloidal shell for different position of central square cutout and CCSC boundary condition



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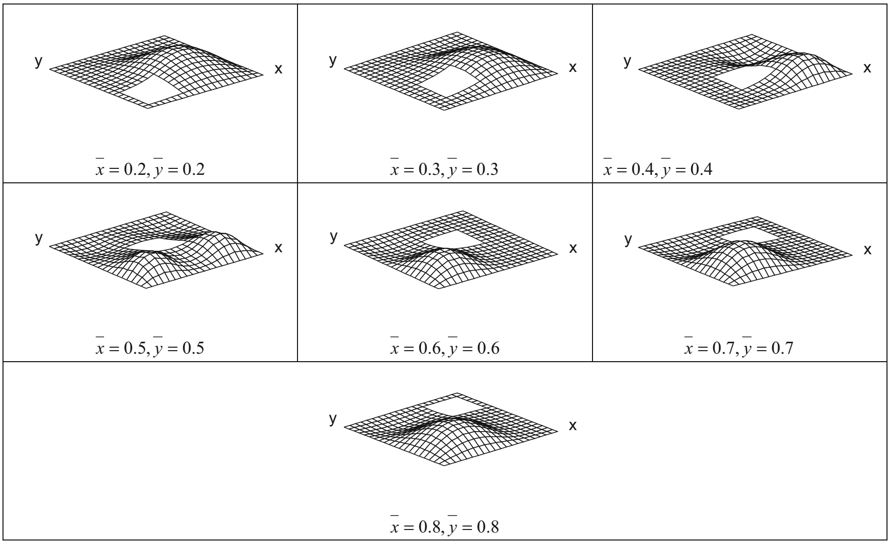
$\bar{x} = 0.3, \bar{y} = 0.3$
$\bar{x} = 0.4, \bar{y} = 0.4$
$\bar{x} = 0.5, \bar{y} = 0.5$
$\bar{x} = 0.6, \bar{y} = 0.6$
$\bar{x} = 0.7, \bar{y} = 0.7$
$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 7.8 First mode shapes of laminated composite (0/90/0/90) stiffened rectangular hyperbolic paraboloidal shell for different position of central square cutout and SCSC boundary condition



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$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

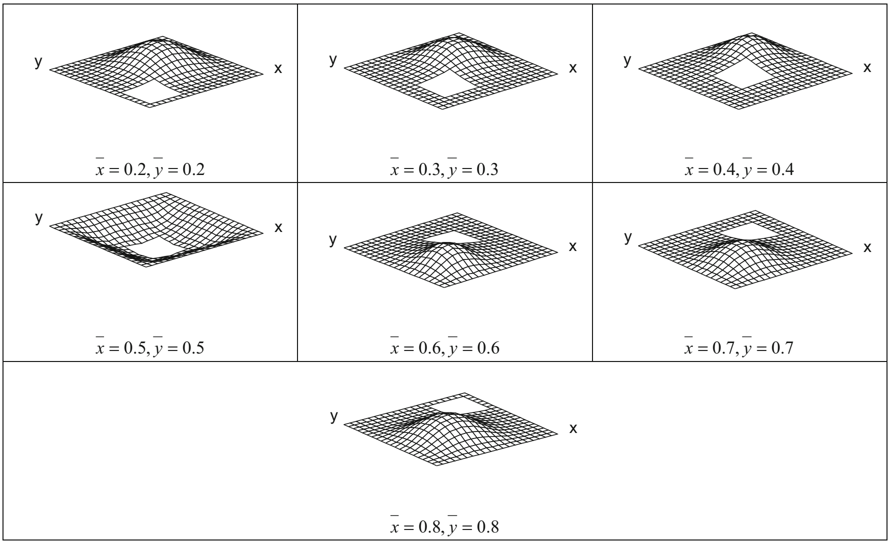
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 7.9 First mode shapes of laminated composite (0/90/0/90) stiffened hyperbolic paraboloidal shell for different position of central square cutout and SSSC boundary condition



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$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

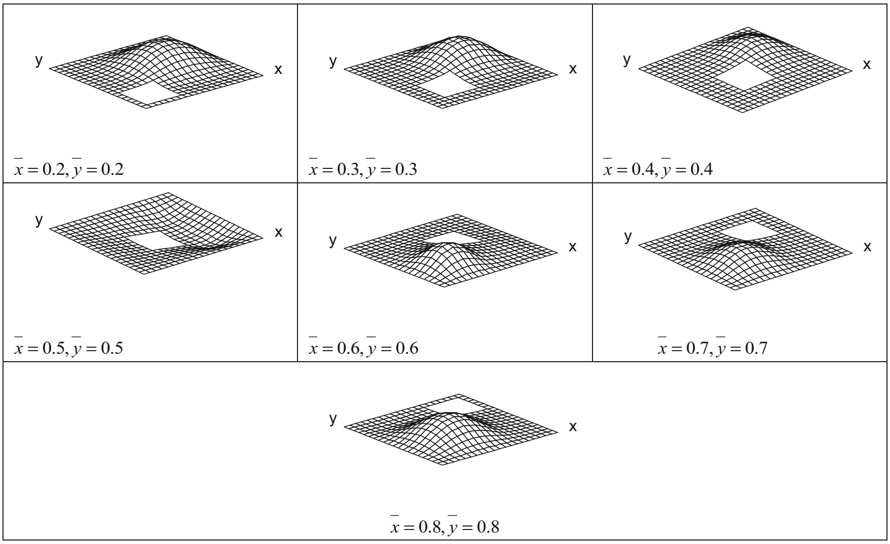
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 7.10 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened hyperbolic paraboloidal shell for different position of central square cutout and CCCC boundary condition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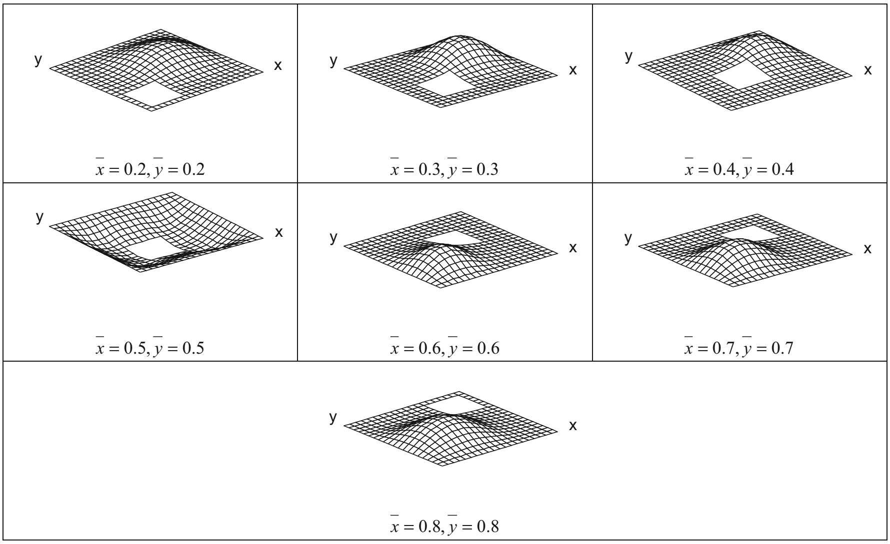
$\bar{x} = 0.5, \bar{y} = 0.5$ $\bar{x} = 0.6, \bar{y} = 0.6$ $\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 7.11 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened hyperbolic paraboloidal shell for different position of central square cutout and CCSC boundary condition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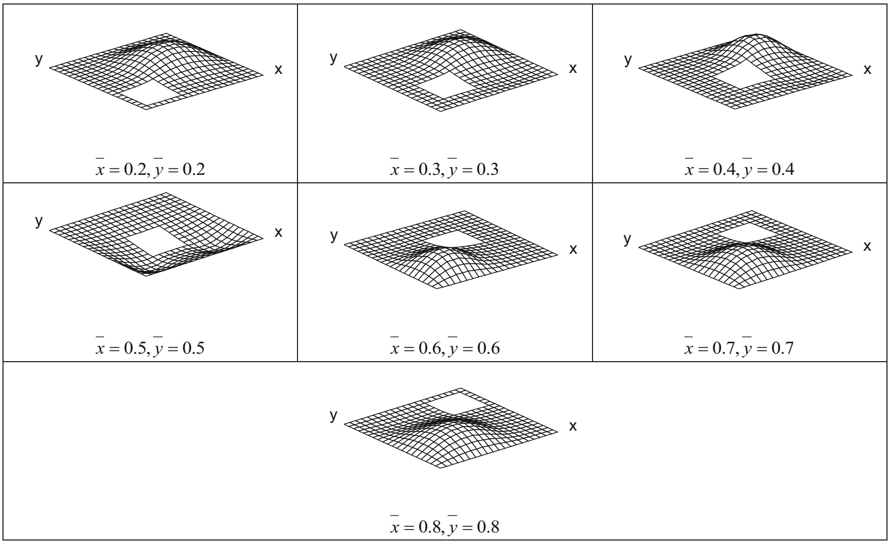
$\bar{x} = 0.5, \bar{y} = 0.5$ $\bar{x} = 0.6, \bar{y} = 0.6$ $\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 7.12 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened hyperbolic paraboloidal shell for different position of central square cutout and SCSC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 7.13 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened hyperbolic paraboloidal shell for different position of central square cutout and SSSC boundary condition

References

Chakravorty D, Sinha PK, Bandyopadhyay JN (1995) Free vibration analysis of point supported laminated composite doubly curved shells – a finite element approach. Comput Struct 54(2):191–207

Chakravorty D, Sinha PK, Bandyopadhyay JN (1996) Finite element free vibration analysis of doubly curved laminated composite shells. J Sound Vib 191(4):491–504

Chakravorty D, Sinha PK, Bandyopadhyay JN (1998) Applications of FEM on free and forced vibration of laminated shells. J Eng Mech 124(1):1–8

Das HS, Chakravorty D (2007) Design aids and selection guidelines for composite conoidal shell roofs – a finite element application. J Reinf Plast Compos 26:1793–1819

Das HS, Chakravorty D (2008) Natural frequencies and mode shapes of composite conoids with complicated boundary conditions. J Reinf Plast Compos 27:1397–1415

Das HS, Chakravorty D (2010) Finite element application in analysis and design of point supported composite conoidal shell roofs suggesting selection guidelines. J Strain Anal Eng Des 45(3):165–177

Das HS, Chakravorty D (2011) Bending analysis of stiffened composite conoidal shell roofs through finite element application. J Compos Mater 45:525–542

Dey A, Bandyopadhyay JN, Sinha PK (1992) Finite element analysis of laminated composite paraboloid of revolution shells. Comput Struct 44(3):675–682

Dey A, Bandyopadhyay JN, Sinha PK (1994) Technical note: behaviour of paraboloid of revolution shell using cross-ply and anti-symmetric angle-ply laminates. Comput Struct 52(6):1301–1308

Ghosh B, Bandyopadhyay JN (1990) Analysis of paraboloid of revolution type shell structures using isoparametric doubly curved shell elements. Comput Struct 36(5):791–800

Hota SS, Chakravorty D (2007) Free vibration of stiffened conoidal shell roofs with cutouts. J Vib Control 13(3):221–240

Hota SS, Padhi P (2007) Vibration of plates with arbitrary shapes of cutouts. J Sound Vib 302(4-5):1030–1036

Huang M, Sakiyama T (1999) Free vibration analysis of rectangular plates with variously-shaped holes. J Sound Vib 226(4):769–786

Kumar A, Chakrabarti A, Bhargava P (2013) Vibration of composite cylindrical shells with cutouts using higher order theory. Int J Sci Ind Res 5(4):199–202

Malhotra SK, Ganesan N, Veluswami MA (1989) Vibration of composite plate with cutouts. J Aeronaut Soc India 41:61–64

Nanda N, Bandyopadhyay JN (2007) Nonlinear free vibration analysis of laminated composite cylindrical shells with cutouts. J Reinf Plast Compos 26(14):143–1427

Nayak AN, Bandyopadhyay JN (2002) Free vibration analysis and design aids of stiffened conoidal shells. J Eng Mech 128:419–427

Nayak AN, Bandyopadhyay JN (2005) Free vibration analysis of laminated stiffened shells. J Eng Mech 131:100–105

Nayak AN, Bandyopadhyay JN (2006) Dynamic response analysis of stiffened conoidal shells. J Sound Vib 291:1288–1297

Pradyumna S, Bandyopadhyay JN (2008) Static and free vibration analyses of laminated shells using a higher order theory. J Reinf Plast Compos 27:167–186

Pradyumna S, Bandyopadhyay JN (2011) Dynamic instability behaviour of laminated hypar and conoid shells using a higher-order shear deformation theory. Thin Walled Struct 49:77–84

Reddy JN (1982) Large amplitude flexural vibration of layered composite plates with cutouts. J Sound Vib 83(1):1–10

Rossi RE (1999) Transverse vibrations of thin, orthotropic rectangular plates with rectangular cutouts with fixed boundaries. J Sound Vib 221(4):733–736

Sahoo S (2016) Performance evaluation of free vibration of laminated composite stiffened hyperbolic paraboloid shell panel with cutout. Int J Eng Technol 7:1–24

Sivakumar K, Iyengar NGR, Deb K (1999) Free vibration of laminated composite plates with cutout. J Sound Vib 221(3):443–465

Sivasubramonian B, Kulkarni AM, Rao GV, Krishnan A (1997) Free vibration of curved panels with cutouts. J Sound Vib 200(2):227–234

Sivasubramonian B, Rao GV, Krishnan A (1999) Free vibration of longitudinally stiffened curved panels with cutout. J Sound Vib 226(1):41–55

Chapter 8
Stiffened Elliptic Paraboloid Shell with Cutout

Abstract In this chapter free vibration study of laminated composite stiffened elliptic paraboloid shell having cutout has been presented in terms of natural frequency and mode shape. Cross- and angle-ply shells with different edge conditions have been studied varying the size and position of the cutouts. The present free vibration study includes the effects of these parametric variations on natural frequencies and mode shapes and set of inferences of practical engineering significances.

Keywords Elliptic paraboloid shell • Cutout • Eccentricity • Fundamental frequency • Mode shapes

8.1 Introduction

The analysis of thin shells attracted attention of researchers from the first half of the nineteenth century. The researchers had realized that the configuration like folded plates, conoidal, saddle, spherical, elliptic and hyperbolic paraboloid and hypar shells can offer a number of parallel advantages that suit to the requirements of the industry. Though researchers like Chakravorty et al. (1998), Sivasubramonian et al. (1999), Hota and Chakravorty (2007), Nanda and Bandyopadhyay (2007), and Kumar et al. (2013) published useful information about free vibration of shells with cutout, scrutiny of the existing literature clearly shows that the information available for free vibration behaviour of stiffened elliptic paraboloid shells with cutouts is limited (Sahoo 2015). Shells of double curvature, particularly elliptic paraboloids, are capable to span over relatively large areas without the help of intermediate supports in comparison with flat plates and cylindrical panels of similar proportions. This aspect in particular attracts the designers to use such shell forms in places of large column free areas. Moreover, elliptic parabolic shells are acceptable from architectural aesthetics point of view and stiff enough from structural point due to their surface geometry. Thus an in-depth study of this form of doubly curved composite shell with cutout is needed in order to utilize the fullest potential of these shell forms.

S. Sahoo, *Design Aids for Stiffened Composite Shells with Cutouts*,
Springer Transactions in Civil and Environmental Engineering,
DOI 10.1007/978-981-10-2062-9_8

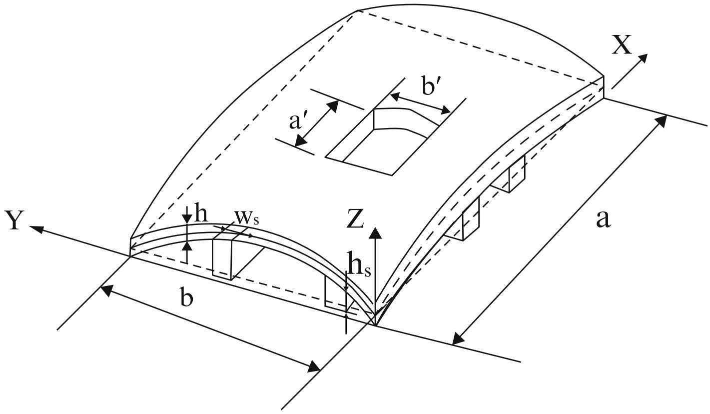


Fig. 8.1 Elliptic paraboloidal shell with a concentric cutout stiffened along the margins

8.2 Problem

Figure 8.1 shows the elliptic paraboloid shell with a concentric cutout having stiffeners along the margin of the cutout. Numerical results are obtained using finite element approach for several elliptic paraboloid shells with different laminations and boundary conditions varying the sizes of the central cutout. Among the laminations considered in the first phase of the study, four-layered symmetric and antisymmetric cross- and angle-ply laminations are chosen for further study. In the second phase of the study, shells are analysed by varying boundary constraints, cutout size and cutout position.

8.3 Results and Discussion

8.3.1 *Free Vibration Behaviour of Shells with Concentric Cutouts*

Table 8.1 contains the variation of nondimensional fundamental frequency ($\overline{\omega}$) of different symmetric and antisymmetric cross- and angle-ply laminated composite stiffened elliptic paraboloid shells with clamped, simply supported and corner point supported boundary conditions for different sizes of the cutout. Tables 8.2 and 8.3 show the same for four-layered cross-ply (0/90/90/0 and 0/90/0/90) and four-layered angle-ply (+45/−45/−45/+45 and +45/−45/+45/−45) stiffened elliptic paraboloid shells for variation in number of boundary constraints along the four edges.

Table 8.1 Nondimensional fundamental frequencies ($\overline{\omega}$) for laminated composite stiffened elliptic paraboloid shell for different sizes of the central square cutout, different lamination and boundary conditions

Boundary conditions	Laminations	Cutout size (a'/a)				
		0	0.1	0.2	0.3	0.4
Clamped	0/90	96.99	101.46	115.12	128.02	130.50
	90/0	94.41	101.48	115.20	128.30	131.00
	0/90/0	135.04	122.62	136.18	153.86	155.35
	90/0/90	144.78	122.87	136.41	154.43	146.25
	0/90/0/90	139.73	119.13	133.82	152.26	152.99
	90/0/90/0	140.62	119.16	133.89	152.43	153.67
	0/90/90/0	146.63	123.24	137.13	154.85	157.81
	90/0/0/90	136.38	123.75	137.72	153.20	145.39
	+45/−45	129.22	142.64	152.56	142.65	130.22
	+45/−45/+45	132.43	144.18	153.17	160.69	153.98
	+45/−45/+45/−45	143.55	156.08	165.22	166.77	157.25
	+45/−45/−45/+45	143.47	155.61	163.76	166.29	157.41
Simply supported	0/90	65.02	69.43	75.33	78.47	81.09
	90/0	65.21	69.71	75.67	78.91	81.55
	0/90/0	66.28	72.88	76.67	80.22	84.45
	90/0/90	66.49	73.04	76.99	80.69	84.79
	0/90/0/90	66.29	72.77	76.75	80.43	84.75
	90/0/90/0	66.41	72.95	76.95	80.67	85.01
	0/90/90/0	66.51	73.04	76.93	80.60	84.97
	90/0/0/90	66.60	73.14	77.07	88.77	84.99
	+45/−45	89.60	99.83	102.70	106.01	106.86
	+45/−45/+45	95.43	105.46	108.23	111.58	112.88
	+45/−45/+45/−45	96.82	106.74	109.73	112.61	114.30
	+45/−45/−45/+45	97.16	107.00	109.73	112.73	114.16
Point supported	0/90	26.23	28.94	32.69	37.61	38.60
	90/0	27.62	30.42	34.99	38.48	38.69
	0/90/0	30.25	32.60	36.39	40.86	35.06
	90/0/90	25.34	28.39	32.36	36.46	35.42
	0/90/0/90	35.65	38.04	40.62	43.15	42.99
	90/0/90/0	36.95	39.30	42.19	43.20	43.02
	0/90/90/0	33.76	36.07	38.98	40.81	41.52
	90/0/0/90	28.96	31.79	35.55	40.62	41.98
	+45/−45	25.68	28.44	31.77	35.61	37.32
	+45/−45/+45	26.48	28.68	31.56	34.89	37.68
	+45/−45/+45/−45	32.82	35.56	38.02	41.03	42.20
	+45/−45/−45/+45	29.97	32.28	35.07	38.76	41.05

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 1/300$, $R_{xx}/R_{yy} = 1.5$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 8.2 Nondimensional fundamental frequencies ($\overline{\omega}$) for laminated composite cross-ply stiffened elliptic paraboloidal shell for different sizes of the central square cutout and different boundary conditions

Laminations	Boundary conditions	Cutout size (a'/a)				
		0	0.1	0.2	0.3	0.4
0/90/90/0	CCCC	146.63	123.24	137.13	154.85	157.81
	CSCC	95.57	103.75	115.13	120.33	120.48
	CCSC	109.78	115.60	126.61	137.79	141.75
	CCCS	95.30	102.40	114.62	120.29	120.47
	CSSC	81.78	88.81	93.26	97.36	101.69
	CCSS	81.79	88.81	93.26	97.35	101.69
	CSCS	102.07	96.96	109.71	112.82	111.62
	SCSC	111.33	113.93	124.44	133.87	134.54
	CSSS	72.73	80.55	83.67	87.39	91.66
	SSSC	76.49	83.54	87.19	91.04	95.26
	SSCS	72.73	79.62	83.67	87.39	91.66
	SSSS	66.51	73.04	76.93	80.60	84.97
	Point supported	33.76	36.06	38.98	40.81	41.52
0/90/0/90	CCCC	139.71	119.13	133.82	152.26	152.99
	CSCC	98	105.54	117.64	120.51	121.84
	CCSC	99.19	106.35	118.08	129.78	133.29
	CCCS	97.8	104.59	117.34	120.5	121.82
	CSSC	80.43	87.38	92.46	96.68	100.71
	CCSS	80.43	87.38	92.46	96.68	100.71
	CSCS	106.82	100.62	113.02	113.71	114.7
	SCSC	100.3	103.75	115.68	124.68	125.6
	CSSS	72.96	80.66	83.91	87.7	92.14
	SSSC	75.27	82.63	86.14	89.89	93.65
	SSCS	72.96	79.7	83.91	87.7	92.14
	SSSS	66.29	72.77	76.75	80.43	84.75
	Point supported	35.65	38.04	40.62	43.15	42.99

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 1/300$, $R_{xx}/R_{yy} = 1.5$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

8.3.1.1 Effect of Cutout Size on Fundamental Frequency

The effect of cutout size on fundamental frequency of composite shells with different boundary conditions can be studied using the results furnished in Tables 8.1, 8.2 and 8.3. All the stiffened shells exhibit greater stiffness with introduction of cutout. This initial increase in frequency is due to the fact that with the introduction of cutout, numbers of stiffeners are increased from two to four in the present study. Only one exception is there. In case of clamped shell, with the introduction of cutout, fundamental frequency decreases. In this case, though the number of stiffeners increases, loss of stiffness is relatively more pronounced. Tables 8.1, 8.2 and 8.3 provide further information about the effect of cutout size

Table 8.3 Nondimensional fundamental frequencies ($\overline{\varpi}$) for laminated composite angle-ply stiffened elliptic paraboloidal shell for different sizes of the central square cutout and different boundary conditions

		Cutout size (a'/a)				
Laminations	Boundary conditions	0	0.1	0.2	0.3	0.4
+45/−45/−45/+45	CCCC	143.47	155.61	163.76	166.29	157.41
	CSCC	119.69	129.01	130.73	131.18	126.74
	CCSC	127.58	135.70	139.32	140.56	138.51
	CCCS	119.15	127.90	130.11	130.34	126.55
	CSSC	113.44	123.66	125.30	125.05	121.32
	CCSS	112.46	122.76	124.34	124.23	121.79
	CSCS	109.69	118.31	122.67	126.69	125.66
	SCSC	119.49	127.66	133.98	139.96	136.58
	CSSS	103.89	112.08	115.57	119.01	119.74
	SSSC	107.57	117.78	119.24	119.05	116.84
	SSCS	102.80	112.27	115.57	119.01	119.74
	SSSS	97.16	107.00	109.73	112.73	114.16
	Point supported	29.97	32.28	35.07	38.76	41.05
+45/−45/+45/−45	CCCC	143.55	156.08	165.22	166.77	157.25
	CSCC	118.68	127.5	129.9	130.18	126.45
	CCSC	127.22	135.27	139.29	140.28	138.42
	CCCS	118.65	127.28	129.87	129.98	126.43
	CSSC	112.52	122.38	124.23	124.1	121.69
	CCSS	112.58	122.42	124.66	124.11	121.12
	CSCS	108.89	117.52	122.31	126.27	125.59
	SCSC	118.85	127.01	133.74	139.51	136.51
	CSSS	102.38	111.57	115.4	118.74	119.75
	SSSC	107.64	117.59	119.26	118.89	116.87
	SSCS	102.37	111.77	115.4	118.74	119.75
	SSSS	96.82	106.74	109.73	112.61	114.3
	Point supported	32.82	35.56	38.02	41.03	42.2

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 1/300$, $R_{xx}/R_{yy} = 1.5$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

on fundamental frequency. With the introduction of a cutout up to $a'/a = 0.3$, on the shell surface, the frequency increases in almost all the cases. Thus it can be concluded that concentric cutouts with stiffened margins may be introduced safely on shell surfaces for functional purposes up to $a'/a = 0.3$.

8.3.1.2 Effect of Boundary Conditions on Fundamental Frequency

The boundary conditions arranged according to the increased number of boundary constraints are as follows: corner point supported, SSSS, SSCS, SSSC, CSSS, SSSC, CSCS, SCSC, CSSC, CCSS, CSCC, CCCS, CCSC and CCCC.

Table 8.4 Clamping options for cross-ply elliptic paraboloidal shells with central cutouts having a'/a ratio 0.2

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of no of restraints
0/90/90/0	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Good improvement	39
	1	(a) Elliptic edge along $x = a$ (SSCS)	Good improvement	46
		(b) Elliptic edge along $x = 0$ (CSSS)	Good improvement	46
		(c) One parabolic edge along $y = b$ (SSSC)	Good improvement	49
	2	(a) Two elliptic edges $x = 0$ and $x = a$ (CSCS)	Marked improvement	72
		(b) Two parabolic edges along $y = 0$ and $y = b$ (SCSC)	Marked improvement	87
		(c) Any two adjacent edges (CSSC, CCSS)	Good improvement	55
	3	3 edges including the two elliptic edges (CSCC, CCCS)	Marked improvement	78
		3 edges excluding the elliptic edge along $x = a$ (CCSC)	Marked improvement	89
	4	All sides (CCCC)	Frequency attains highest value	100
0/90/0/90	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Good improvement	39
	1	(a) Elliptic edge along $x = a$ (SSCS)	Good improvement	46
		(b) Elliptic edge along $x = 0$ (CSSS)	Good improvement	46
		(c) One parabolic edge along $y = b$ (SSSC)	Good improvement	49

(continued)

Table 8.4 (continued)

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of no of restraints
	2	(a) Two elliptic edges $x = 0$ and $x = a$ (CSCS)	Marked improvement	78
		(b) Two parabolic edges along $y = 0$ and $y = b$ (SCSC)	Marked improvement	81
		(c) Any two adjacent edges (CSSC, CCSS)	Good improvement	56
	3	3 edges including the two elliptic edges (CSCC, CCCS)	Marked improvement	83
		3 edges excluding the elliptic edge along $x = a$ (CCSC)	Marked improvement	83
	4	All sides (CCCC)	Frequency attains highest value	100

As evident from Tables 8.4 and 8.5, the fundamental frequencies of members belonging to the same number of boundary constraints may not have the same value. Though stiffness depends on the number of boundary constraints, arrangement of boundary constraints is more significant than number of boundary constraints to increase the stiffness of the stiffened shells with cutout. Tables 8.4 and 8.5 show the efficiency of a particular boundary combination in increasing the fundamental frequency. Marks are assigned to each boundary combination in a scale where a value of 0 is assigned to the minimum frequency (corner point supported shell) and a value of 100 is assigned to the maximum frequency (clamped shell). These marks are furnished for cutouts with $a'/a = 0.2$ These tables will help a structural engineer to overview at a glance the efficiency of a particular boundary combination in increasing the frequency of a shell, taking that of clamped shell as the upper limit.

It can be seen from the present study that if the one edge is released from clamped to simply supported, the change of frequency is more in case of an angle-ply shells than that for a cross-ply shells. Again, if the two adjacent edges are released, fundamental frequency decreases remarkably, whereas if the two alternate edges are released, the decrease in frequency is far less. This change in frequency is very much significant in case of cross-ply shells. For cross-ply shells if three or four edges are simply supported, frequency values undergo marked decrease, but for angle-ply shells with the introduction of more number of simply supported edges, the frequency value does not change so drastically. The results

Table 8.5 Clamping options for angle-ply elliptic paraboloidal shells with central cutouts having a'/a ratio 0.2

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of no of restraints
+45/−45/−45/+45	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Very good improvement	58
	1	(a) Elliptic edge along $x = a$ (SSCS)	Very good improvement	63
		(b) Elliptic edge along $x = 0$ (CSSS)	Very good improvement	63
		(c) One parabolic edge along $y = b$ (SSSC)	Very good improvement	65
	2	(a) Two elliptic edges $x = 0$ and $x = a$ (CSCS)	Very good improvement	68
		(b) Two parabolic edges along $y = 0$ and $y = b$ (SCSC)	Marked improvement	77
		(c) Any two adjacent edges (CSSC, CCSS)	Marked improvement	70
	3	3 edges including the two elliptic edges (CSCC, CCCS)	Marked improvement	74
		3 edges excluding the elliptic edge along $x = a$ (CCSC)	Remarkable improvement	81
	4	All sides (CCCC)	Frequency attains highest value	100
+45/−45/+45/−45	0	Corner point supported	–	0
	0	Simply supported no edges clamped (SSSS)	Good improvement	56
	1	(a) Elliptic edge along $x = a$ (SSCS)	Very good improvement	61
		(b) Elliptic edge along $x = 0$ (CSSS)	Very good improvement	61
		(c) One parabolic edge along $y = b$ (SSSC)	Very good improvement	64

(continued)

Table 8.5 (continued)

Laminations	Number of sides to be clamped	Clamped edges	Improvement of frequencies with respect to point supported shells	Marks indicating the efficiencies of no of restraints
	2	(a) Two elliptic edges $x = 0$ and $x = a$ (CSCS)	Very good improvement	66
		(b) Two parabolic edges along $y = 0$ and $y = b$ (SCSC)	Marked improvement	75
		(c) Any two adjacent edges (CSSC, CCSS)	Very good improvement	68
	3	3 edges including the two elliptic edges (CSCC, CCCS)	Marked improvement	72
		3 edges excluding the elliptic edge along $x = a$ (CCSC)	Remarkable improvement	80
	4	All sides (CCCC)	Frequency attains highest value	100

indicate that particularly for cross-ply shells, two alternate edges should preferably be clamped in order to achieve higher frequency values.

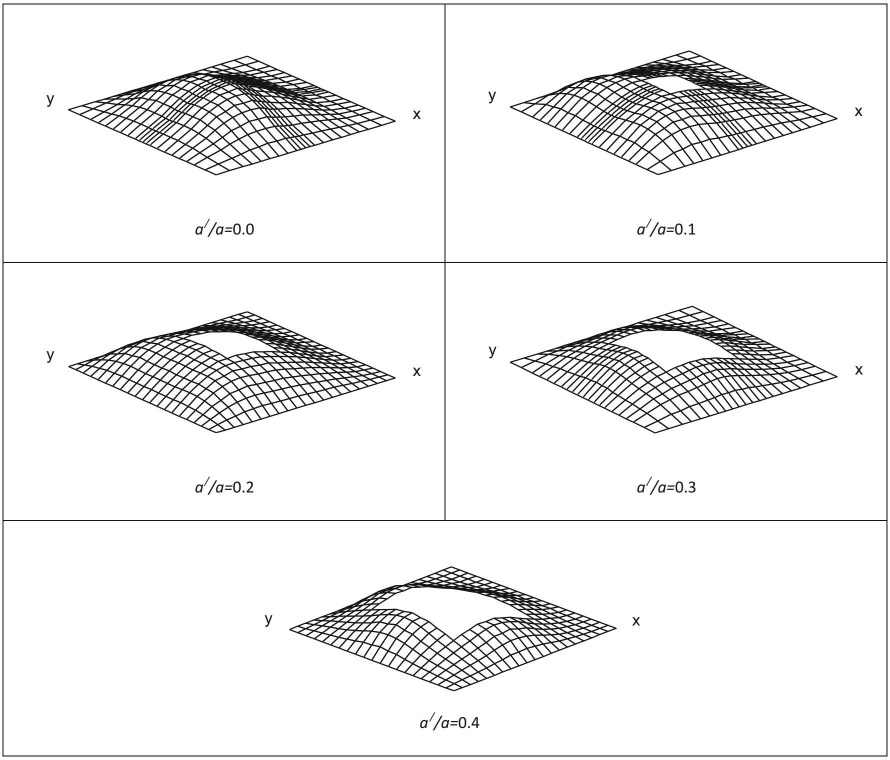
8.3.1.3 Mode Shapes

The mode shapes of the fundamental modes of vibration are shown in Figs. 8.2, 8.3, 8.4, 8.5, 8.6 and 8.7 for SSCS, SSSS and point supported boundary conditions for different sizes of the cutout. Both cross-ply and angle-ply stiffened elliptic paraboloid shells are considered. For corner point supported shells, the fundamental mode shapes appear complicated. When cutout is provided, mode shapes remain almost similar. When the size of the cutout is changed from 0.2 to 0.4, the fundamental modes of vibration do not change to a great extent.

8.3.2 Effect of Eccentricity of Cutout Position

8.3.2.1 Fundamental Frequency

The effect of eccentricity of cutout positions on fundamental frequencies is studied for different locations of a cutout with $a'/a = 0.2$. The nondimensional coordinates



a'/a=0.0

a'/a=0.1

a'/a=0.2

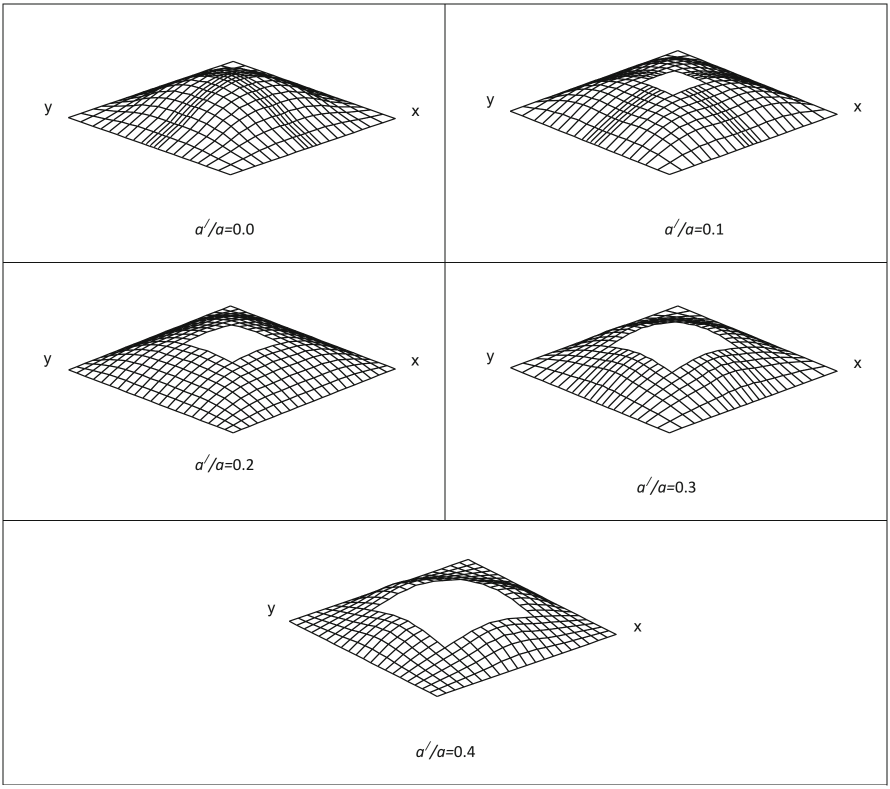
a'/a=0.3

a'/a=0.4

Fig. 8.2 First mode shapes of laminated composite (0/90/0/90) stiffened elliptic paraboloidal shell for different sizes of central square cutout and SSCS boundary condition

of the cutout centres $\left(\bar{x} = \frac{x}{a}, \bar{y} = \frac{y}{a}\right)$ are varied from 0.2 to 0.8 along each direction, so that the distance of a cutout margin from the shell boundary is not less than one-tenth of the plan dimension of the shell. The margins of cutouts are stiffened with four stiffeners. The study is carried out for all the boundary conditions for both cross-ply and angle-ply shells. The fundamental frequency of a shell with an eccentric cutout is expressed as a percentage of fundamental frequency of a shell with a concentric cutout. This percentage is denoted by r and furnished in Tables 8.6 and 8.7.

It is observed that eccentricity of the cutout along the length of the shell towards the clamped edges makes it more flexible. When edge opposite to a clamped edge is simply supported, r value first increases towards the simply supported edge and then decreases. This is true for both cross-ply and angle-ply shells. For cross-ply shells, when two opposite edges are simply supported, r value decreases towards the simply supported edges. But in case of an angle-ply shell, if the two opposite edges are simply supported, then r value first increases and then decreases towards the boundary. So, for functional purposes, if a shift of central cutout is required, eccentricity of a cutout along the length or width should preferably be towards the simply supported edge but not towards very near to the boundary for angle-ply



a'/a=0.0

a'/a=0.1

a'/a=0.2

a'/a=0.3

a'/a=0.4

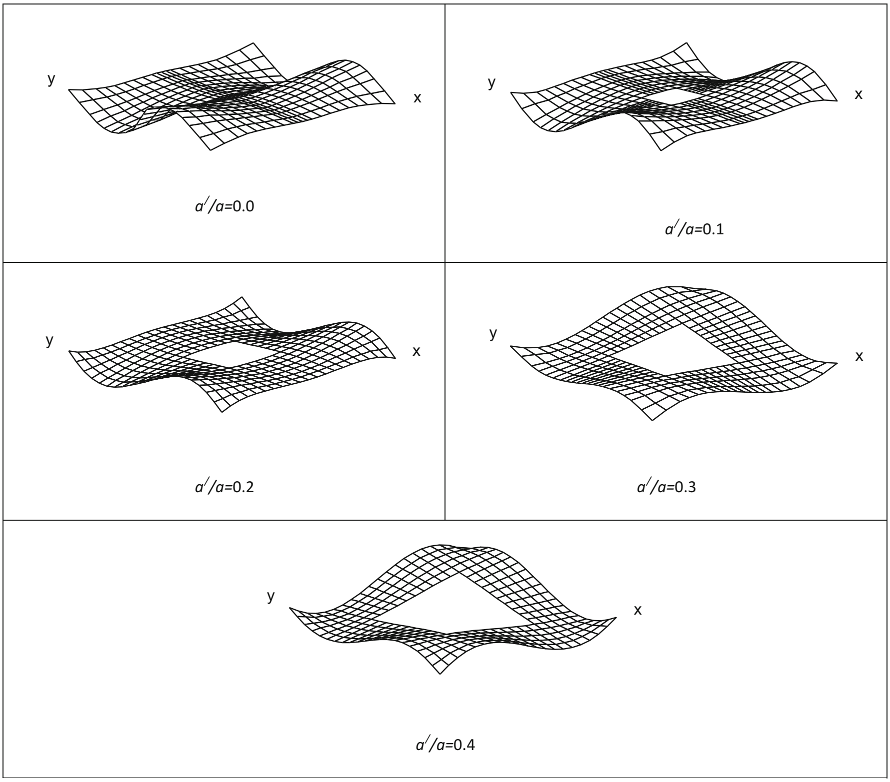
Fig. 8.3 First mode shapes of laminated composite (0/90/0/90) stiffened elliptic paraboloidal shell for different sizes of central square cutout and SSSS boundary condition

shells. But in case of cross-ply shells, eccentricity of a cutout should be towards a simply supported edge, which is opposite to a clamped edge.

Tables 8.8 and 8.9 provide the maximum values of r together with the position of the cutout. These tables also identify the rectangular zones within which r is always greater than or equal to 95. These tables will help practising engineers in deciding the placement of cutout in the shell structures.

8.3.2.2 Mode Shape

The mode shapes corresponding to the fundamental modes of vibration are shown in Figs. 8.8, 8.9, 8.10, 8.11, 8.12, 8.13, 8.14 and 8.15 for cross-ply and angle-ply shells of different boundary conditions for different eccentric positions of the cutout. All the mode shapes are either bending or torsional mode. It is observed that for different positions of cutout mode shapes are somewhat similar to one another, only the crest and trough positions change.



$a'/a = 0.0$

$a'/a = 0.1$

$a'/a = 0.2$

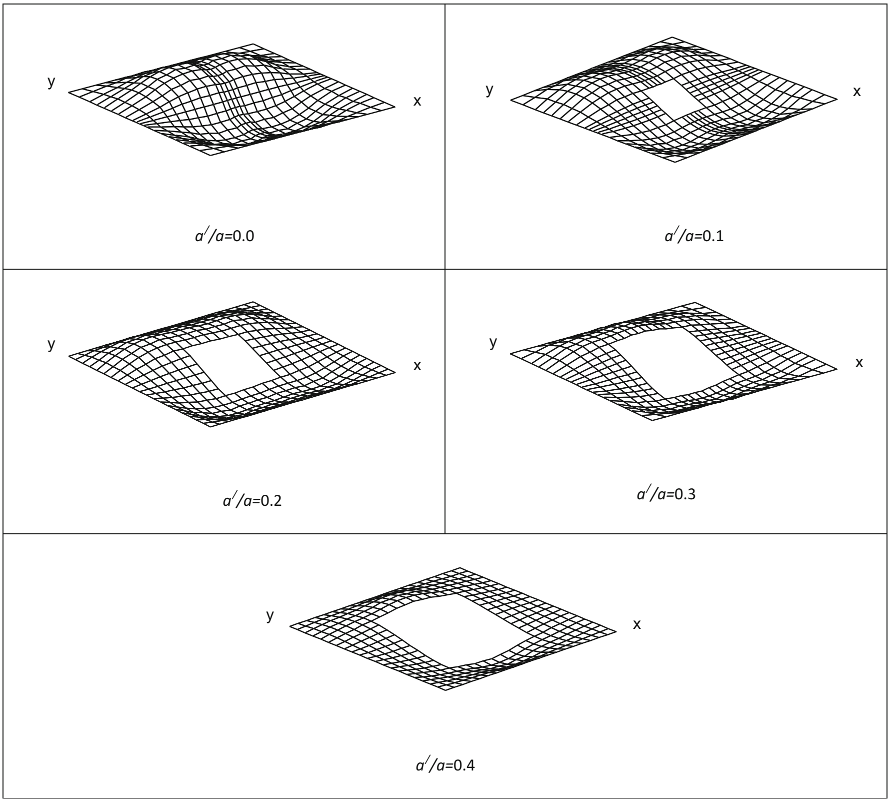
$a'/a = 0.3$

$a'/a = 0.4$

Fig. 8.4 First mode shapes of laminated composite (0/90/0/90) stiffened elliptic paraboloidal shell for different sizes of central square cutout and point supported boundary condition

8.4 Closure

Analysis of free vibration problems of stiffened elliptic parabolic shell panels with cutouts reveals that the arrangement of boundary constraints along the four edges is more important than their actual number. The relative free vibration performances of stiffened shells with cutout for different boundary combination are expected to be very helpful for practising engineers. For cross-ply shells eccentricity towards the simply supported edge which is opposite to a clamped edge is preferable. For angle-ply shells eccentricity towards simply supported edge is preferable. These results may be helpful as design aids for structural engineers as it provides information regarding the behaviour of stiffened elliptic parabolic shell with eccentric cutouts for a wide range of eccentricity and edge conditions for cross-ply and angle-ply shells.



$a'/a=0.0$

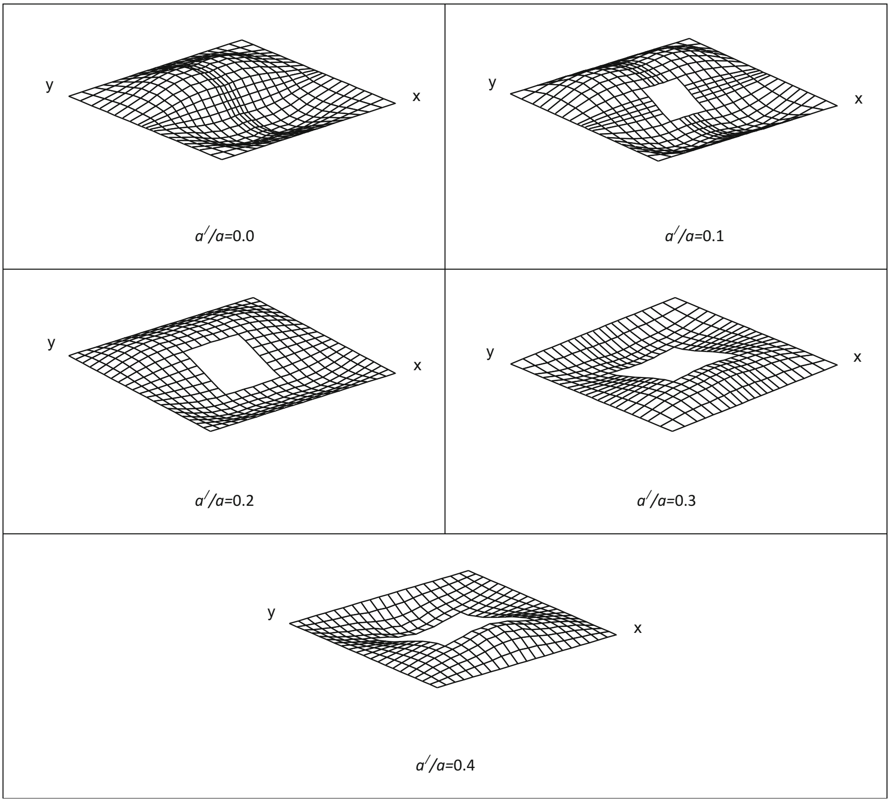
$a'/a=0.1$

$a'/a=0.2$

$a'/a=0.3$

$a'/a=0.4$

Fig. 8.5 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened elliptic paraboloidal shell for different sizes of central square cutout and SSCS boundary condition



$a'/a=0.0$

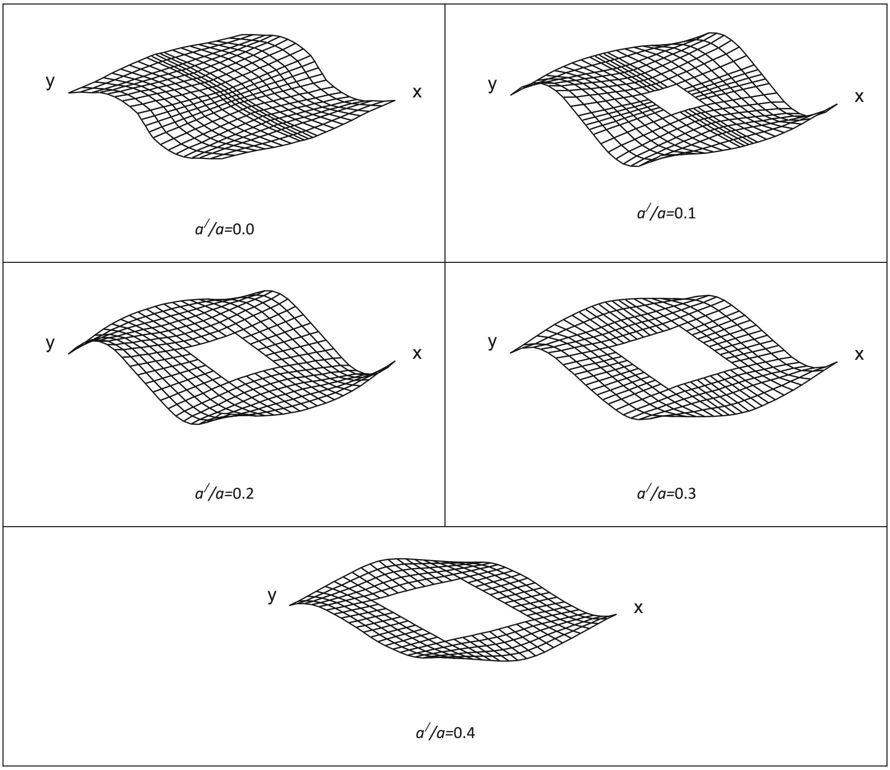
$a'/a=0.1$

$a'/a=0.2$

$a'/a=0.3$

$a'/a=0.4$

Fig. 8.6 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened elliptic paraboloidal shell for different sizes of central square cutout and SSSS boundary condition



$a'/a=0.0$

$a'/a=0.1$

$a'/a=0.2$

$a'/a=0.3$

$a'/a=0.4$

Fig. 8.7 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened elliptic paraboloidal shell for different sizes of central square cutout and point supported boundary condition

Table 8.6 Values of ‘r’ for cross-ply elliptic paraboloidal shells

			$\bar{x}$						
Edge conditions	Laminations	$\bar{y}$	0.2	0.3	0.4	0.5	0.6	0.7	0.8
CCCC	0/90/90/0	0.2	96.08	97.55	100.18	103.06	100.50	97.55	96.48
		0.3	94.65	96.02	98.61	101.25	98.61	96.02	94.65
		0.4	95.72	96.67	98.59	100.20	98.59	96.67	95.71
		0.5	97.24	97.86	99.07	100.00	99.07	97.86	97.24
		0.6	95.71	96.67	98.59	100.19	98.59	96.67	95.71
		0.7	94.65	96.02	98.61	101.13	98.61	96.02	94.65
		0.8	95.82	97.32	100.15	102.98	100.12	97.37	95.89
	0/90/0/90	0.2	91.56	94.52	99.99	106.19	99.98	94.51	91.56
		0.3	90.17	92.97	98.13	104.72	98.14	92.97	90.17
		0.4	91.13	93.27	97.36	101.19	97.36	93.27	91.13
		0.5	92.77	94.26	97.32	100	97.32	94.25	92.76
		0.6	91.13	93.27	97.36	101.20	97.36	93.27	91.13
		0.7	90.17	92.97	98.13	103.92	98.14	92.97	90.17
		0.8	91.29	94.22	99.64	106.12	99.86	94.28	91.37
CSCC	0/90/90/0	0.2	94.60	95.28	96.19	96.46	96.19	95.29	94.60
		0.3	95.70	97.98	101.62	103.99	101.62	97.99	95.70
		0.4	96.30	99.14	104.00	108.10	104.00	99.15	96.30
		0.5	95.41	96.90	98.94	100.00	98.97	96.91	95.21
		0.6	91.60	90.65	90.93	91.24	91.03	90.71	91.04
		0.7	87.90	86.75	87.08	87.48	87.15	86.81	87.37
		0.8	86.57	85.63	86.13	86.67	86.22	85.71	86.13
	0/90/0/90	0.2	92.98	94.81	97.51	99.58	97.51	94.81	92.99
		0.3	93.52	96.56	101.13	105.32	101.13	96.57	93.53
		0.4	94.02	97.09	102.07	105.84	102.07	97.11	94.02
		0.5	93.46	94.87	97.65	100	97.78	94.93	93.35

		0.6	90.92	90.64	92.24	93.71	92.36	90.73	90.64
		0.7	89.24	88.69	90.10	91.35	90.18	88.76	88.93
		0.8	89.00	88.52	89.93	91.16	90.04	88.60	88.72
CCSC	0/90/90/0	0.2	88.47	90.50	94.52	101.63	105.83	99.68	95.74
		0.3	87.95	89.92	93.82	100.71	103.40	98.29	94.68
		0.4	87.96	89.71	93.39	100.13	102.08	96.52	92.39
		0.5	88.23	89.85	93.40	100.00	101.53	95.71	91.15
		0.6	87.96	89.71	93.39	100.11	102.04	96.52	92.39
		0.7	87.95	89.92	93.82	100.71	103.45	98.29	94.68
		0.8	88.33	90.34	94.28	101.51	105.31	99.51	95.55
	0/90/0/90	0.2	83.73	86.74	92.51	102.51	107.23	98.91	93.17
		0.3	83.10	86.09	91.82	101.71	105.11	97.36	91.99
		0.4	83.06	85.73	91.07	100.59	103.21	95.04	89.13
		0.5	83.33	85.79	90.85	100	102.12	93.80	87.52
		0.6	83.06	85.73	91.07	100.59	103.20	95.04	89.13
		0.7	83.10	86.09	91.82	101.71	105.11	97.36	91.99
		0.8	83.54	86.53	92.27	102.40	107.14	98.67	92.98
CCCS	0/90/90/0	0.2	86.42	85.98	86.42	86.84	86.42	85.98	86.42
		0.3	87.65	87.06	87.32	87.63	87.32	87.06	87.64
		0.4	91.37	90.97	91.17	91.34	91.17	90.98	91.37
		0.5	95.64	97.30	99.18	100.00	99.19	97.30	95.64
		0.6	96.73	99.58	104.44	108.58	104.44	99.58	96.73
		0.7	96.13	98.42	102.07	104.45	102.07	98.42	96.13
		0.8	94.48	95.27	96.36	96.75	96.40	95.28	94.54
	0/90/0/90	0.2	88.91	88.75	90.14	91.28	90.14	88.75	88.90
		0.3	89.07	88.87	90.26	91.45	90.26	88.87	89.07
		0.4	90.80	90.83	92.40	93.76	92.40	90.83	90.80

(continued)

Table 8.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
		0.5	93.57	95.08	97.82	100	97.82	95.08	93.57
		0.6	94.25	97.34	102.31	106.1	102.31	97.34	94.25
		0.7	93.76	96.80	101.39	105.58	101.39	96.80	93.76
		0.8	92.19	94.34	97.46	99.75	97.51	94.51	92.39
CSSC	0/90/90/0	0.2	84.42	87.41	92.04	96.33	93.15	86.33	79.66
		0.3	87.90	91.25	96.10	100.40	98.68	92.90	86.96
		0.4	89.59	93.08	97.98	102.56	102.49	98.08	93.04
		0.5	87.96	90.86	95.12	100.00	101.00	97.29	93.06
		0.6	84.57	86.81	90.37	95.07	96.05	92.33	88.48
		0.7	81.95	83.97	87.21	91.26	91.81	88.48	85.06
		0.8	80.61	82.59	85.59	87.96	89.17	86.36	83.30
	0/90/0/90	0.2	84.56	87.63	92.36	96.84	94.20	87.16	80.03
		0.3	88.23	91.68	96.61	101.01	99.45	93.40	87.01
		0.4	89.69	93.31	98.43	103.22	103.29	98.83	93.48
		0.5	86.68	89.50	94.15	100	101.47	97.52	92.85
		0.6	82.76	85.11	89.33	94.97	96.30	92.28	87.84
		0.7	80.45	82.72	86.63	91.35	92.16	88.72	84.83
		0.8	79.53	81.75	85.33	89.18	89.69	86.86	83.47
CCSS	0/90/90/0	0.2	80.62	82.60	85.61	88.92	89.14	86.34	83.28
		0.3	81.95	83.97	87.21	91.26	91.76	88.45	85.04
		0.4	84.57	86.82	90.38	95.07	96.01	92.29	88.46
		0.5	87.96	90.86	95.12	100.00	100.97	97.27	93.05
		0.6	89.58	93.08	97.98	102.56	102.49	98.08	93.04
		0.7	87.88	91.25	96.10	100.40	98.68	92.90	86.95
		0.8	84.07	87.06	91.70	96.19	92.93	86.02	79.37

	0/90/0/90	0.2	79.55	81.78	85.37	89.18	89.68	86.85	83.46
		0.3	80.45	82.72	86.63	91.35	92.14	88.70	84.80
		0.4	82.76	85.12	89.33	94.97	96.28	92.25	87.82
		0.5	86.68	89.50	94.15	100	101.45	97.49	92.83
		0.6	89.68	93.31	98.43	103.22	103.29	98.82	93.48
		0.7	88.21	91.68	96.61	101.00	99.45	93.40	87.01
		0.8	84.21	87.30	92.09	96.69	93.98	86.89	79.80
CSCS	0/90/90/0	0.2	86.01	85.32	83.57	82.53	83.57	85.32	86.01
		0.3	88.92	87.91	86.72	85.99	86.72	87.91	88.86
		0.4	93.50	92.91	92.35	91.92	92.35	92.91	93.49
		0.5	95.93	98.05	100.10	100.00	100.05	98.05	95.93
		0.6	93.59	92.91	92.35	91.92	92.35	92.91	93.48
		0.7	88.88	87.91	86.72	85.99	86.72	87.91	88.85
		0.8	86.01	85.32	83.56	82.50	83.56	85.31	86.01
	0/90/0/90	0.2	89.93	89.24	88.60	88.27	88.60	89.24	89.96
		0.3	91.09	90.34	90.39	90.56	90.39	90.34	91.08
		0.4	93.18	93.05	93.91	94.69	93.91	93.05	93.19
		0.5	94.49	96.72	98.67	100	98.63	96.72	94.49
		0.6	93.18	93.05	93.91	94.69	93.91	93.05	93.19
		0.7	91.08	90.34	90.39	90.56	90.39	90.34	91.07
		0.8	89.89	89.23	88.59	88.25	88.59	89.23	89.95
SCSC	0/90/90/0	0.2	87.87	90.68	95.32	102.36	95.32	90.68	87.87
		0.3	87.54	90.20	94.65	101.76	94.65	90.20	87.54
		0.4	86.35	89.15	93.64	100.93	93.64	89.15	86.35
		0.5	85.66	88.63	93.22	100.00	93.22	88.62	85.66
		0.6	86.35	89.15	93.64	101.36	93.64	89.15	86.35
		0.7	87.54	90.20	94.65	101.77	94.65	90.20	87.54
		0.8	87.74	88.47	95.07	102.23	95.23	90.55	87.79

(continued)

Table 8.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
	0/90/0/90	0.2	82.72	86.72	93.25	102.59	93.25	86.72	82.72
		0.3	82.30	86.18	92.57	102.07	92.57	86.18	82.30
		0.4	80.76	84.77	91.12	101.41	91.12	84.77	80.76
		0.5	79.84	84.01	90.38	100	90.37	84.01	79.83
		0.6	80.76	84.77	91.12	101.52	91.12	84.77	80.76
		0.7	82.30	86.18	92.57	102.11	92.57	86.18	82.30
		0.8	82.54	86.52	93.02	102.52	93.18	86.58	82.62
CSSS	0/90/90/0	0.2	80.28	82.44	85.99	90.08	88.87	83.66	78.36
		0.3	83.26	85.55	89.24	93.64	93.35	88.65	83.78
		0.4	86.60	89.27	93.26	97.71	98.09	94.02	89.47
		0.5	88.48	91.55	95.84	100.00	100.62	97.29	93.12
		0.6	86.59	89.27	93.26	97.71	98.09	94.02	89.47
		0.7	83.26	85.55	89.23	93.64	93.35	88.65	83.78
		0.8	80.18	82.34	85.87	89.98	88.73	83.48	78.18
	0/90/0/90	0.2	80.04	82.38	86.12	90.10	89.36	84.28	78.66
		0.3	82.65	85.07	89.04	93.53	93.52	88.86	83.63
		0.4	85.97	88.64	92.83	97.54	98.11	93.98	89.07
		0.5	88.38	91.49	95.81	100	100.72	97.45	93.21
		0.6	85.97	88.64	92.83	97.54	98.11	93.98	89.07
		0.7	82.65	85.07	89.04	93.53	93.52	88.86	83.63
		0.8	79.98	82.30	86.04	90.01	89.21	84.13	78.52
SSSC	0/90/90/0	0.2	80.66	86.54	92.82	96.81	92.82	86.54	80.66
		0.3	86.65	91.87	97.43	100.56	97.43	91.87	86.65
		0.4	90.54	94.99	99.82	102.39	99.82	94.99	90.54

		0.5	89.40	93.05	97.36	100.00	97.36	93.05	89.40
		0.6	85.76	88.89	92.81	95.53	92.81	88.89	85.76
		0.7	83.14	86.01	89.57	91.96	89.57	86.01	83.14
		0.8	81.91	84.63	87.85	89.80	87.85	84.63	81.91
	0/90/0/90	0.2	80.54	86.67	93.10	97.09	93.12	86.67	80.54
		0.3	86.64	92.12	97.83	100.98	97.83	92.12	86.64
		0.4	90.77	95.33	100.28	102.88	100.28	95.33	90.77
		0.5	87.67	91.57	96.62	100	96.62	91.57	87.67
		0.6	83.52	87.06	91.84	95.36	91.84	87.06	83.52
		0.7	81.37	84.66	88.97	91.86	88.97	84.66	81.37
		0.8	80.62	83.74	87.56	89.85	87.57	83.74	80.64
SSCS	0/90/90/0	0.2	78.36	83.66	88.87	90.08	85.99	82.44	80.28
		0.3	83.78	88.65	93.35	93.64	89.23	85.55	83.26
		0.4	89.47	94.02	98.09	97.71	93.26	89.27	86.59
		0.5	93.12	97.29	100.62	100.00	95.84	91.55	88.48
		0.6	89.47	94.02	98.09	97.71	93.26	89.27	86.60
		0.7	83.78	88.65	93.35	93.64	89.23	85.55	83.26
		0.8	78.22	83.52	88.74	90.01	85.90	82.34	80.18
	0/90/0/90	0.2	78.66	84.28	89.36	90.10	86.12	82.37	80.04
		0.3	83.63	88.86	93.52	93.53	89.04	85.07	82.65
		0.4	89.07	93.98	98.11	97.54	92.83	88.64	85.97
		0.5	93.20	97.44	100.72	100	95.81	91.49	88.38
		0.6	89.07	93.98	98.11	97.54	92.83	88.64	85.97
		0.7	83.63	88.86	93.52	93.53	89.04	85.08	82.65
		0.8	78.56	84.17	89.24	90.04	86.04	82.30	79.97
SSSS	0/90/90/0	0.2	78.81	83.18	87.92	90.95	87.92	83.18	78.81
		0.3	83.04	87.04	91.55	94.36	91.55	87.04	83.04

(continued)

Table 8.6 (continued)

Edge conditions	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
		0.4	87.39	91.28	95.59	98.05	95.59	91.28	87.39
		0.5	89.96	93.83	97.92	100.00	97.92	93.83	89.96
		0.6	87.39	91.28	95.59	98.05	95.59	91.28	87.39
		0.7	83.04	87.04	91.55	94.36	91.55	87.04	83.04
		0.8	78.67	83.04	87.78	90.85	87.81	83.01	78.66
	0/90/0/90	0.2	78.59	83.13	87.99	90.85	87.99	83.13	78.59
		0.3	82.32	86.55	91.32	94.19	91.32	86.55	82.32
		0.4	86.57	90.67	95.28	97.93	95.28	90.66	86.57
		0.5	89.92	93.81	97.93	100	97.93	93.81	89.92
		0.6	86.57	90.66	95.28	97.93	95.28	90.66	86.57
		0.7	82.32	86.55	91.32	94.19	91.32	86.55	82.32
		0.8	78.49	83.04	87.87	90.76	87.86	83.00	78.46
CS	0/90/90/0	0.2	103.77	103.36	101.67	100.41	101.64	103.36	103.77
		0.3	104.18	103.36	101.39	100.03	101.39	103.36	104.18
		0.4	104.59	103.34	101.15	99.79	101.15	103.34	104.59
		0.5	105.64	103.49	101.23	100.00	101.26	103.49	105.18
		0.6	104.59	103.31	101.15	99.79	101.15	103.31	104.57
		0.7	104.16	103.36	101.39	100.05	101.39	103.36	104.16
		0.8	103.64	103.31	101.64	100.38	101.59	103.28	103.62
	0/90/0/90	0.2	102.07	103.47	105.37	108.62	105.34	103.47	102.07
		0.3	101.38	102.39	103.96	106.47	103.96	102.39	101.38
		0.4	99.29	99.63	83.95	102.26	100.64	99.63	99.29
		0.5	98.03	98.06	98.77	100	98.74	98.03	98.03
		0.6	99.29	99.63	100.64	102.26	100.64	99.63	99.29
		0.7	101.40	102.39	103.99	106.62	103.96	102.36	101.38
		0.8	101.65	103.03	104.80	107.16	104.63	103.00	101.67

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 1/300$, $R_{xx}/R_{yy} = 1.5$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 8.7 Values of '*r*' for angle-ply elliptic paraboloidal shells

Edge condition	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
CCCC	+45/−45/−45/+45	0.2	79.63	82.52	84.69	86.55	86.44	84.09	80.83
		0.3	80.31	83.88	87.29	90.27	89.28	85.74	81.80
		0.4	80.14	84.52	89.80	95.65	91.74	86.14	81.57
		0.5	80.40	85.27	91.82	100.00	91.79	85.23	80.42
		0.6	81.59	86.15	91.79	95.43	89.82	84.52	80.16
		0.7	81.81	85.74	89.27	90.33	87.29	83.89	80.31
		0.8	80.75	84.06	86.41	86.35	84.71	82.49	79.51
	+45/−45/+45/−45	0.2	79.66	82.85	84.84	85.74	84.86	82.87	79.69
		0.3	80.31	84.19	87.56	89.71	87.66	84.28	80.35
		0.4	80.05	84.59	90.03	95.29	90.17	84.67	80.08
		0.5	79.67	84.63	91.24	100	91.22	84.61	79.67
		0.6	80.09	84.67	90.19	94.95	90.06	84.59	80.08
		0.7	80.36	84.28	87.66	89.54	87.56	84.20	80.32
		0.8	79.57	82.84	84.84	85.66	84.87	82.81	79.57
CSCC	+45/−45/−45/+45	0.2	95.68	98.16	101.71	106.80	103.77	100.19	97.29
		0.3	96.65	102.54	108.05	109.11	107.65	103.17	97.35
		0.4	92.27	97.19	102.40	104.43	101.12	95.98	91.00
		0.5	88.05	92.43	97.54	100.00	96.10	90.67	85.94
		0.6	87.87	91.90	96.45	98.78	95.52	90.51	85.93
		0.7	89.93	93.81	97.46	99.30	97.11	93.04	88.61
		0.8	89.90	94.41	97.88	99.11	97.72	93.98	89.15
	+45/−45/+45/−45	0.2	97.21	99.98	103.64	107.41	103.36	99.85	97.14
		0.3	97.50	103.57	108.60	109.94	108.48	103.37	97.07
		0.4	92.15	97.10	102.06	104.71	102.12	96.68	91.33

(continued)

Table 8.7 (continued)

Edge condition	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
		0.5	87.35	91.82	96.90	100	96.91	91.46	86.53
		0.6	87.12	91.38	96.09	98.75	95.98	91.19	86.61
		0.7	89.41	93.62	97.54	99.38	97.34	93.61	89.28
		0.8	89.58	94.39	98.14	99.38	98.18	94.80	89.75
CCSC	+45/−45/−45/+45	0.2	90.54	89.76	87.75	88.24	94.12	98.16	93.53
		0.3	91.90	92.13	90.70	91.71	98.00	100.05	94.09
		0.4	91.62	94.16	94.19	96.04	102.50	100.72	94.29
		0.5	91.58	96.53	97.47	100.00	106.50	100.03	93.76
		0.6	92.69	96.16	95.98	98.19	103.50	98.94	92.86
		0.7	92.08	94.09	92.71	93.93	99.12	98.03	92.46
		0.8	89.64	91.52	89.75	90.33	95.21	96.45	92.08
	+45/−45/+45/−45	0.2	90.79	90.78	88.46	88.76	94.18	97.45	92.96
		0.3	92.55	92.96	91.44	92.48	98.49	99.29	93.22
		0.4	92.00	94.63	94.78	96.94	103.47	99.94	93.45
		0.5	91.54	96.78	97.42	100	106.45	100.15	93.66
		0.6	92.14	95.54	95.18	97.14	103.07	99.81	93.50
		0.7	92.21	93.32	91.74	92.90	98.85	99.17	93.24
		0.8	90.44	90.67	88.63	89.30	94.91	97.64	92.86
CCCS	+45/−45/−45/+45	0.2	88.98	93.87	97.69	99.20	98.16	94.78	89.95
		0.3	88.54	93.01	97.11	99.39	97.76	94.19	90.01
		0.4	85.93	90.48	95.47	98.85	96.76	92.28	87.99
		0.5	85.90	90.58	95.93	100.00	97.87	92.83	88.22
		0.6	90.92	95.87	100.93	104.40	102.74	97.62	92.53
		0.7	97.46	103.33	107.88	109.33	108.46	103.01	97.03
		0.8	96.89	100.21	103.75	107.19	101.49	98.32	96.08

	+45/−45/+45/ −45	0.2	89.73	94.81	98.21	99.41	98.18	94.41	89.34
		0.3	89.28	93.62	97.36	99.39	97.57	93.64	89.20
		0.4	86.61	91.19	95.99	98.76	96.10	91.41	86.94
		0.5	86.51	91.44	96.88	100	96.91	91.85	87.19
		0.6	91.29	96.65	102.07	104.69	102.08	97.11	92.04
		0.7	97.05	103.36	108.51	109.93	108.62	103.60	97.45
		0.8	97.11	99.63	103.03	107.13	102.98	99.55	96.72
CSSC	+45/−45/−45/+45	0.2	96.15	95.95	95.32	96.78	101.60	101.71	97.91
		0.3	94.78	99.07	99.21	100.76	105.54	102.30	97.23
		0.4	90.69	95.76	100.74	103.70	101.27	96.47	91.65
		0.5	87.01	91.82	97.10	100.00	97.24	92.31	87.57
		0.6	86.97	91.48	96.22	99.00	96.77	92.15	87.20
		0.7	88.79	93.16	97.00	99.37	97.89	93.79	88.50
		0.8	88.45	93.17	96.69	97.50	97.95	94.17	88.82
	+45/−45/+45/−45	0.2	96.07	97.95	96.72	98.01	101.80	100.96	97.75
		0.3	95.28	100.08	100.59	102.37	106.09	102.77	97.35
		0.4	90.75	95.97	101.16	104.11	102.58	97.75	92.51
		0.5	86.40	91.38	96.69	100	98.25	93.44	88.45
		0.6	86.00	90.90	95.98	98.98	97.24	92.86	87.83
		0.7	87.80	92.80	97.46	99.69	98.05	94.24	89.08
		0.8	87.80	92.74	95.59	96.69	98.21	94.82	89.48
CCSS	+45/−45/−45/+45	0.2	85.11	89.65	92.74	94.65	98.01	94.86	89.79
		0.3	85.31	90.84	96.69	99.37	98.37	94.71	89.82
		0.4	83.82	89.33	95.42	99.24	97.95	93.80	89.02
		0.5	84.36	89.67	95.71	100.00	99.03	94.56	89.83
		0.6	89.17	94.48	100.15	103.76	102.89	98.31	93.32
		0.7	88.93	99.91	101.63	103.44	105.69	102.30	97.10
		0.8	93.42	99.95	98.04	99.24	100.83	99.16	96.21

(continued)

Table 8.7 (continued)

Edge condition	Laminations	$\bar{y}$	$\bar{x}$						
			0.2	0.3	0.4	0.5	0.6	0.7	0.8
	+45/−45/+45/ −45	0.2	87.45	92.91	95.60	95.95	98.42	94.43	88.94
		0.3	87.26	92.71	97.53	99.91	98.29	94.11	88.91
		0.4	84.96	90.37	95.92	99.20	97.19	92.68	87.80
		0.5	84.85	90.37	96.37	100	97.75	93.00	88.27
		0.6	89.04	94.69	100.51	103.81	101.83	97.06	92.14
		0.7	94.10	99.54	100.63	102.25	106.26	102.45	97.05
		0.8	94.40	97.96	96.84	98.09	101.74	100.69	96.82
CSCS	+45/−45/−45/+45	0.2	92.43	96.67	100.06	101.52	100.89	98.07	93.92
		0.3	92.81	97.09	100.60	102.07	101.00	97.99	94.07
		0.4	88.77	93.18	98.06	100.83	98.32	93.85	89.78
		0.5	86.92	91.26	96.60	100.00	96.60	91.26	86.92
		0.6	89.78	93.85	98.32	100.83	98.06	93.18	88.77
		0.7	94.07	97.99	101.00	102.07	100.60	97.09	92.81
		0.8	93.85	97.86	100.59	101.20	99.55	96.45	92.38
	+45/−45/+45/−45	0.2	93.56	98.10	101.23	101.93	100.57	97.29	92.94
		0.3	93.52	97.73	101.01	102.31	100.96	97.51	93.28
		0.4	89.31	93.65	98.28	100.90	98.29	93.49	89.13
		0.5	86.91	91.36	96.69	100	96.69	91.36	86.91
		0.6	89.13	93.49	98.29	100.90	98.28	93.65	82.77
		0.7	93.28	97.51	100.96	102.31	101.01	97.73	93.52
		0.8	92.92	97.13	100.29	101.59	100.83	97.83	93.46
SCSC	+45/−45/−45/+45	0.2	93.01	93.04	90.04	88.69	91.31	94.66	92.59
		0.3	93.91	95.45	93.10	92.07	94.29	97.11	94.18
		0.4	93.96	97.57	96.82	96.62	97.88	99.22	94.78

		0.5	94.40	99.89	99.94	100.00	99.94	99.90	94.44
		0.6	94.85	99.25	97.88	96.62	96.82	97.58	93.98
		0.7	94.19	97.11	94.29	92.07	93.10	95.45	93.71
		0.8	92.50	94.60	91.27	88.68	90.01	93.00	92.60
	+45/−45/+45/−45	0.2	93.51	94.06	90.70	88.69	90.56	93.94	92.74
		0.3	94.58	96.25	93.72	92.19	93.76	96.61	93.70
		0.4	94.49	98.00	97.26	96.75	97.57	98.98	94.12
		0.5	94.44	100.09	99.96	100	99.96	100.04	94.44
		0.6	94.30	99.03	97.57	96.75	97.26	98.03	94.55
		0.7	93.72	96.62	93.76	92.19	93.72	96.25	94.58
		0.8	92.63	93.90	90.50	88.68	90.65	94.01	93.43
CSSS	+45/−45/−45/+45	0.2	89.89	93.79	96.71	99.90	101.19	97.91	93.50
		0.3	90.53	95.72	100.30	102.42	101.34	97.86	93.59
		0.4	87.36	92.46	97.88	101.00	99.44	95.34	90.93
		0.5	85.89	90.72	96.24	100.00	98.30	93.73	89.18
		0.6	88.85	93.43	98.17	100.98	99.39	94.96	90.27
		0.7	93.08	97.65	101.07	102.28	100.83	96.87	92.31
		0.8	92.80	97.26	100.38	101.03	99.49	95.98	91.68
	+45/−45/+45/−45	0.2	92.34	97.05	99.89	102.08	101.06	97.11	92.43
		0.3	92.26	97.42	101.57	103.12	101.63	97.54	92.86
		0.4	88.26	93.24	98.33	101.33	99.72	95.24	90.48
		0.5	85.90	90.81	96.30	100	98.29	93.65	88.99
		0.6	87.98	92.83	97.85	100.79	99.22	94.97	90.35
		0.7	91.92	96.81	100.79	102.33	101.07	97.37	92.87
		0.8	91.86	96.25	99.20	101.30	100.71	97.26	92.76
SSSC	+45/−45/−45/+45	0.2	95.26	98.21	97.95	97.85	99.85	100.16	94.72
		0.3	95.07	100.27	102.07	102.37	103.43	100.22	94.47

(continued)

Table 8.7 (continued)

Edge condition	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
		0.4	91.40	96.58	101.31	102.87	99.97	94.78	89.59
		0.5	88.31	93.52	98.30	100.00	96.60	91.00	85.61
		0.6	87.74	93.11	97.69	99.56	96.52	90.70	84.82
		0.7	88.59	94.06	98.22	99.38	97.48	91.59	85.39
		0.8	88.43	93.94	97.71	95.49	94.94	90.78	85.16
	+45/−45/+45/−45	0.2	95.24	100.24	98.64	97.67	98.86	100.02	95.33
		0.3	94.79	100.29	102.55	102.03	102.87	99.87	94.26
		0.4	90.56	95.77	100.92	102.99	100.82	95.44	89.91
		0.5	86.98	92.26	97.33	100	97.48	92.02	86.40
		0.6	86.37	91.96	97.05	99.55	97.12	91.82	85.92
		0.7	87.23	93.18	98.30	99.97	98.31	93.23	87.05
		0.8	87.15	93.08	97.28	95.80	98.16	93.43	87.21
SSCS	+45/−45/−45/+45	0.2	91.88	96.31	100.01	101.43	100.81	97.58	92.98
		0.3	92.31	96.88	100.83	102.28	101.07	97.63	93.08
		0.4	90.27	94.96	99.39	100.98	98.16	93.43	88.85
		0.5	89.18	93.73	98.30	100.00	96.24	90.72	85.89
		0.6	90.93	95.34	99.44	101.00	97.87	92.46	87.36
		0.7	93.59	97.86	101.34	102.42	100.30	95.79	90.53
		0.8	93.25	97.59	100.87	99.84	96.27	93.70	89.85
	+45/−45/+45/−45	0.2	92.91	97.63	101.14	101.82	99.56	96.53	91.98
		0.3	92.87	97.37	101.07	102.33	100.79	96.82	91.91
		0.4	90.35	94.97	99.22	100.79	97.85	92.83	87.98
		0.5	88.99	93.65	98.30	100	96.30	90.81	85.90
		0.6	90.48	95.24	99.72	101.33	98.32	93.24	88.26

		0.7	92.86	97.54	101.63	103.12	101.57	97.44	92.25
		0.8	92.25	96.84	100.73	102.03	99.38	96.74	92.22
SSSS	+45/−45/−45/+45	0.2	89.12	93.78	97.87	100.50	100.68	96.91	91.93
		0.3	89.73	94.89	99.75	102.03	100.92	96.97	92.05
		0.4	88.20	93.50	98.58	100.87	97.52	94.31	89.35
		0.5	87.42	92.50	97.52	100.00	98.83	92.50	87.42
		0.6	89.35	94.31	98.83	100.87	98.58	93.50	88.20
		0.7	92.06	96.97	100.92	102.06	99.75	94.90	89.73
		0.8	91.51	96.45	100.22	100.27	97.33	93.47	88.91
	+45/−45/+45/−45	0.2	91.17	96.36	100.55	102.18	100.03	95.81	90.74
		0.3	91.11	96.35	100.87	102.54	100.75	96.14	90.94
		0.4	88.70	93.93	98.75	100.97	98.79	93.90	88.66
		0.5	87.30	92.47	97.51	100	97.51	92.47	87.30
		0.6	88.66	93.90	98.79	100.97	98.75	93.93	88.70
		0.7	90.94	96.14	100.75	102.54	100.87	96.35	91.11
		0.8	90.45	95.42	99.57	101.70	99.99	95.97	90.86
CS	+45/−45/−45/+45	0.2	115.48	121.67	117.59	115.26	115.97	115.51	111.61
		0.3	112.40	114.09	111.41	110.09	111.63	113.06	111.12
		0.4	105.76	106.33	103.79	102.77	104.88	107.56	106.93
		0.5	104.48	104.68	101.63	100.00	101.63	104.68	104.45
		0.6	106.93	107.56	104.88	102.77	103.82	106.33	105.76
		0.7	111.12	113.06	111.63	110.09	111.41	114.06	112.40
		0.8	111.46	115.23	115.31	114.34	116.54	121.39	115.37

(continued)

Table 8.7 (continued)

Edge condition	Laminations	$\bar{y}$	$\bar{x}$ 0.2	0.3	0.4	0.5	0.6	0.7	0.8
	+45/−45/+45/ −45	0.2	106.23	111.34	112.81	111.57	112.47	110.47	105.50
		0.3	105.47	108.78	108.65	107.73	108.76	108.42	104.76
		0.4	100.53	102.66	102.71	102.13	102.87	102.76	100.53
		0.5	98.68	100.50	100.47	100	100.47	100.50	98.66
		0.6	100.53	102.76	102.87	102.13	102.71	102.66	100.53
		0.7	104.76	108.42	108.76	107.73	108.65	108.84	105.47
		0.8	105.39	110.49	111.89	110.60	112.07	111.13	106.15

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 1/300$, $R_{xx}/R_{yy} = 1.5$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 8.8 Maximum values of r with corresponding coordinates of cutout centres and zones where $r \geq 95$ for cross-ply elliptic paraboloidal shells

Laminations	0/90/90/0			0/90/0/90		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$
CCCC	103.06	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7$	106.18	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.5$
		$\bar{y} = 0.2$	$0.2 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.2$	$0.2 \leq \bar{y} \leq 0.8$
CSCC	108.10	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7$	105.84	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$
		$\bar{y} = 0.4$	$0.2 \leq \bar{y} \leq 0.5$		$\bar{y} = 0.4$	$0.2 \leq \bar{y} \leq 0.5$
CCSC	105.83	$\bar{x} = 0.6$	$0.5 \leq \bar{x} \leq 0.7$	107.23	$\bar{x} = 0.6$	$0.5 \leq \bar{x} \leq 0.7$
		$\bar{y} = 0.2$	$0.2 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.2$	$0.2 \leq \bar{y} \leq 0.8$
CCCS	108.58	$\bar{x} = 0.5$	$0.2 \leq \bar{x} \leq 0.8$	106.12	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$
		$\bar{y} = 0.6$	$0.5 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.6$	$0.5 \leq \bar{y} \leq 0.8$
CSSC	102.56	$\bar{x} = 0.5$	$0.5 \leq \bar{x} \leq 0.6$	103.29	$\bar{x} = 0.6$	$0.5 \leq \bar{x} \leq 0.6$
		$\bar{y} = 0.4$	$0.3 \leq \bar{y} \leq 0.6$		$\bar{y} = 0.4$	$0.3 \leq \bar{y} \leq 0.5$
CCSS	102.56	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.7$	103.29	$\bar{x} = 0.6$	$0.5 \leq \bar{x} \leq 0.6$
		$\bar{y} = 0.6$	$0.5 \leq \bar{y} \leq 0.6$		$\bar{y} = 0.6$	$0.5 \leq \bar{y} \leq 0.7$
CSCS	100.10	$\bar{x} = 0.4$	$0.2 \leq \bar{x} \leq 0.8$	100.00	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7$
		$\bar{y} = 0.5$	$\bar{y} = 0.5$		$\bar{y} = 0.5$	$\bar{y} = 0.5$
SCSC	102.36	$\bar{x} = 0.5$	$\bar{x} = 0.5$	102.59	$\bar{x} = 0.5$	$\bar{x} = 0.5$
		$\bar{y} = 0.2$	$0.2 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.2$	$0.2 \leq \bar{y} \leq 0.8$
CSSS	100.62	$\bar{x} = 0.6$	$0.5 \leq \bar{x} \leq 0.6$	100.72	$\bar{x} = 0.6$	$0.5 \leq \bar{x} \leq 0.6$
		$\bar{y} = 0.5$	$0.4 \leq \bar{y} \leq 0.6$		$\bar{y} = 0.5$	$0.4 \leq \bar{y} \leq 0.6$
SSSC	102.39	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$	102.88	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$
		$\bar{y} = 0.4$	$0.3 \leq \bar{y} \leq 0.5$		$\bar{y} = 0.4$	$0.3 \leq \bar{y} \leq 0.5$
SSCS	100.62	$\bar{x} = 0.4$	$0.4 \leq \bar{x} \leq 0.5$	100.72	$\bar{x} = 0.4$	$0.4 \leq \bar{x} \leq 0.5$
		$\bar{y} = 0.5$	$0.4 \leq \bar{y} \leq 0.6$		$\bar{y} = 0.5$	$0.4 \leq \bar{y} \leq 0.6$

(continued)

Table 8.8 (continued)

Laminations	0/90/90/0			0/90/0/90		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value r $\geq$ 95	Maximum values of r	Coordinate of cutout centre	Area in which the value r $\geq$ 95
SSSS	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$	100.00	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6$
		$\bar{y} = 0.5$	$0.4 \leq \bar{y} \leq 0.6$		$\bar{y} = 0.5$	$0.4 \leq \bar{y} \leq 0.6$
CS	105.64	$\bar{x} = 0.2$	$0.2 \leq \bar{x} \leq 0.8$	108.62	$\bar{x} = 0.5$	$0.2 \leq \bar{x} \leq 0.8$
		$\bar{y} = 0.5$	$0.2 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.2$	$0.2 \leq \bar{y} \leq 0.8$

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 1/300$, $R_{xx}/R_{yy} = 1.5$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$

Table 8.9 Maximum values of r with corresponding coordinates of cutout centres and zones where $r \geq 95$ for angle-ply elliptic paraboloidal shells

Laminations	+45/−45/−45/+45			+45/−45/+45/−45		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$
CCCC	100.00	$\bar{x} = 0.5$	$\bar{x} = 0.5$	100.00	$\bar{x} = 0.5$	$\bar{x} = 0.5$
		$\bar{y} = 0.5$	$0.4 \leq \bar{y} \leq 0.6$		$\bar{y} = 0.5$	$0.4 \leq \bar{y} \leq 0.5$
CSCC	109.11	$\bar{x} = 0.5$	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.3$;	109.94	$\bar{x} = 0.5$	$0.2 \leq \bar{x} \leq 0.8$, $0.2 \leq \bar{y} \leq 0.3$;
		$\bar{y} = 0.3$	$0.4 \leq \bar{x} \leq 0.6$, $0.4 \leq \bar{y} \leq 0.8$;		$\bar{y} = 0.3$	$0.4 \leq \bar{x} \leq 0.6$, $0.4 \leq \bar{y} \leq 0.8$;
CCSC	106.50	$\bar{x} = 0.6$	$0.6 \leq \bar{x} \leq 0.7$, $0.3 \leq \bar{y} \leq 0.4$;	106.45	$\bar{x} = 0.6$	$0.3 \leq \bar{x} \leq 0.4$, $0.5 \leq \bar{y} \leq 0.6$;
		$\bar{y} = 0.5$	$0.3 \leq \bar{x} \leq 0.7$, $0.5 \leq \bar{y} \leq 0.6$		$\bar{y} = 0.5$	$\bar{x} =$ 0.5,$0.4 \leq \bar{y} \leq 0.6$;
						$\bar{x} = 0.6$, $0.3 \leq \bar{y} \leq 0.7$;
						$\bar{x} = 0.7$, $0.2 \leq \bar{y} \leq 0.8$.

(continued)

Table 8.9 (continued)

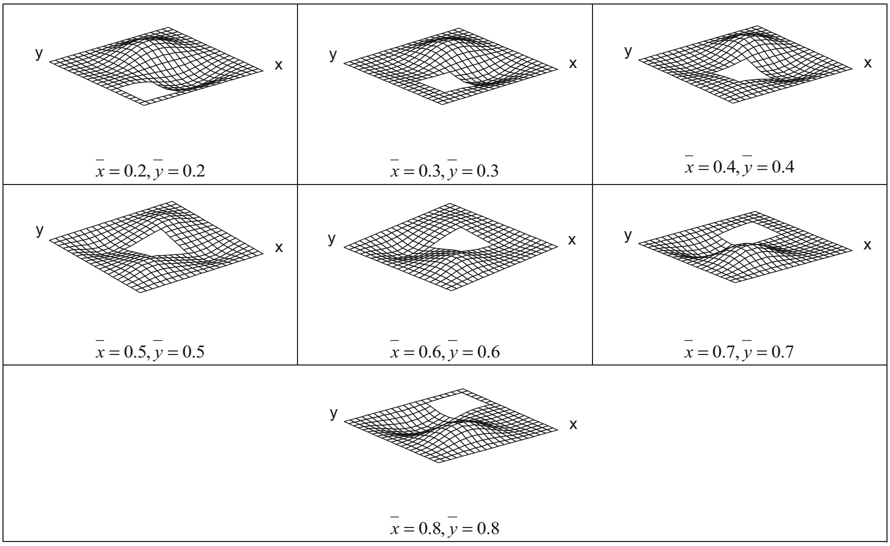
Laminations	+45/−45/−45/+45			+45/−45/+45/−45		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$
CCCS	109.33	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6, 0.2 \leq \bar{y} \leq 0.5$;	109.93	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6, 0.2 \leq \bar{y} \leq 0.6$;
		$\bar{y} = 0.7$	$0.3 \leq \bar{x} \leq 0.7, 0.6 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.7$	$0.2 \leq \bar{x} \leq 0.8, 0.7 \leq \bar{y} \leq 0.8$
CSSC	105.54	$\bar{x} = 0.6$	$0.3 \leq \bar{x} \leq 0.7, 0.2 \leq \bar{y} \leq 0.4$;	106.09	$\bar{x} = 0.6$	$0.2 \leq \bar{x} \leq 0.8, 0.2 \leq \bar{y} \leq 0.3$;
		$\bar{y} = 0.3$	$0.4 \leq \bar{x} \leq 0.6, 0.5 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.3$	$0.4 \leq \bar{x} \leq 0.6, 0.4 \leq \bar{y} \leq 0.8$
CCSS	105.69	$\bar{x} = 0.6$	$0.4 \leq \bar{x} \leq 0.6, 0.3 \leq \bar{y} \leq 0.6$;	103.81	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.6, 0.2 \leq \bar{y} \leq 0.6$;
		$\bar{y} = 0.7$	$0.3 \leq \bar{x} \leq 0.8, 0.7 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.6$	$0.3 \leq \bar{x} \leq 0.8, 0.7 \leq \bar{y} \leq 0.8$
CSCS	102.07	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7, 0.2 \leq \bar{y} \leq 0.3$ & $0.7 \leq \bar{y} \leq 0.8$	102.31	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7, 0.2 \leq \bar{y} \leq 0.3$ & $0.7 \leq \bar{y} \leq 0.8$;
		$\bar{y} = 0.3{,}0.7$	$0.4 \leq \bar{x} \leq 0.6, 0.4 \leq \bar{y} \leq 0.6$		$\bar{y} = 0.7$	$0.4 \leq \bar{x} \leq 0.6, 0.4 \leq \bar{y} \leq 0.6$
SCSC	100.00	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7, 0.4 \leq \bar{y} \leq 0.6$	100.04	$\bar{x} = 0.7$	$\bar{x} = 0.3, 0.7$ $0.3 \leq \bar{y} \leq 0.7$;
		$\bar{y} = 0.5$			$\bar{y} = 0.5$	$0.4 \leq \bar{x} \leq 0.6, 0.4 \leq \bar{y} \leq 0.6$
CSSS	102.42	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.7, 0.2 \leq \bar{y} \leq 0.6$;	103.12	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7, 0.2 \leq \bar{y} \leq 0.3$;
		$\bar{y} = 0.3$	$0.3 \leq \bar{x} \leq 0.7{,}0.7 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.3$	$0.4 \leq \bar{x} \leq 0.6{,}0.4 \leq \bar{y} \leq 0.8$
SSSC	103.43	$\bar{x} = 0.6$	$0.3 \leq \bar{x} \leq 0.7, 0.2 \leq \bar{y} \leq 0.3$;	102.99	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7, 0.2 \leq \bar{y} \leq 0.4$;
		$\bar{y} = 0.3$	$0.4 \leq \bar{x} \leq 0.6, 0.4 \leq \bar{y} \leq 0.7$		$\bar{y} = 0.4$	$0.4 \leq \bar{x} \leq 0.6, 0.5 \leq \bar{y} \leq 0.8$

(continued)

Table 8.9 (continued)

Laminations	+45/−45/−45/+45			+45/−45/+45/−45		
Boundary condition	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$	Maximum values of r	Coordinate of cutout centre	Area in which the value $r \geq 95$
SSCS	102.42	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7, 0.2 \leq \bar{y} \leq 0.3$;	103.12	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7, 0.2 \leq \bar{y} \leq 0.3$ & $0.7 \leq \bar{y} \leq 0.8$;
		$\bar{y} = 0.7$	$0.4 \leq \bar{x} \leq 0.6, 0.4 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.7$	$0.4 \leq \bar{x} \leq 0.6, 0.4 \leq \bar{y} \leq 0.6$
SSSS	102.06	$\bar{x} = 0.5$	$0.4 \leq \bar{x} \leq 0.7, 0.2 \leq \bar{y} \leq 0.3$;	102.54	$\bar{x} = 0.5$	$0.3 \leq \bar{x} \leq 0.7, 0.2 \leq \bar{y} \leq 0.3$ & $0.7 \leq \bar{y} \leq 0.8$;
		$\bar{y} = 0.7$	$0.4 \leq \bar{x} \leq 0.6, 0.4 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.7$	$0.4 \leq \bar{x} \leq 0.6, 0.4 \leq \bar{y} \leq 0.6$
CS	121.67	$\bar{x} = 0.3$	$0.2 \leq \bar{x} \leq 0.8$	112.47	$\bar{x} = 0.6$	$0.2 \leq \bar{x} \leq 0.8$
		$\bar{y} = 0.2$	$0.2 \leq \bar{y} \leq 0.8$		$\bar{y} = 0.2$	$0.2 \leq \bar{y} \leq 0.8$

$a/b = 1$, $a/h = 100$, $a'/b' = 1$, $h/R_{xx} = 1/300$, $R_{xx}/R_{yy} = 1.5$; $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{13} = G_{12} = 0.5E_{22}$, $\nu_{12} = \nu_{21} = 0.25$



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

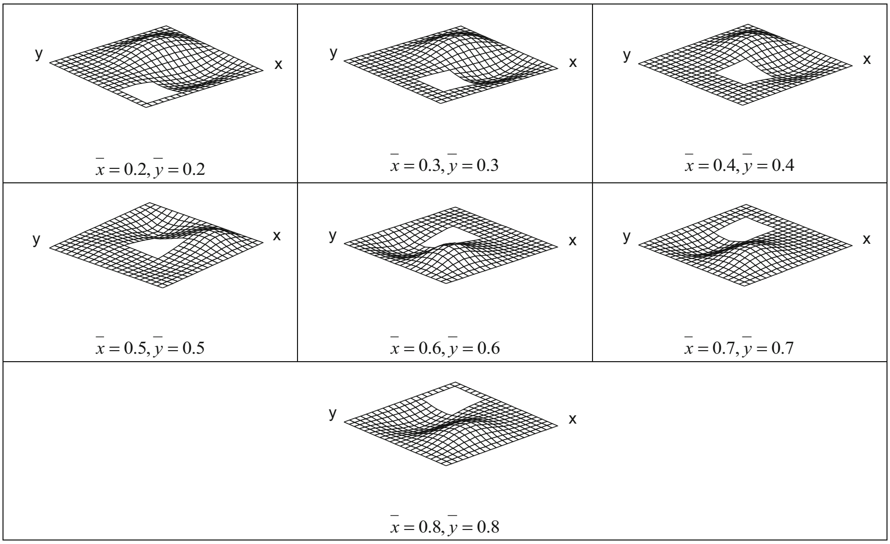
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 8.8 First mode shapes of laminated composite (0/90/0/90) stiffened elliptic paraboloidal shell for different position of central square cutout and CCCC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

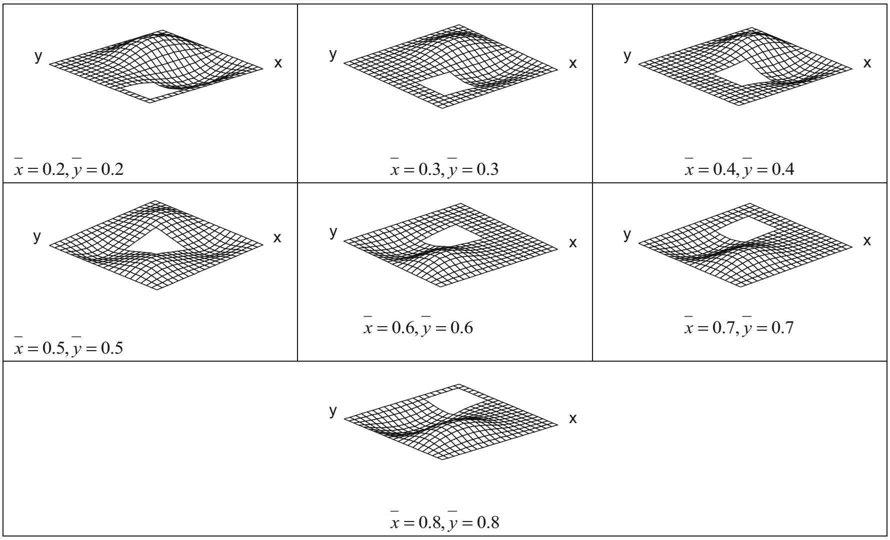
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 8.9 First mode shapes of laminated composite (0/90/0/90) stiffened elliptic paraboloidal shell for different position of central square cutout and CCSC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

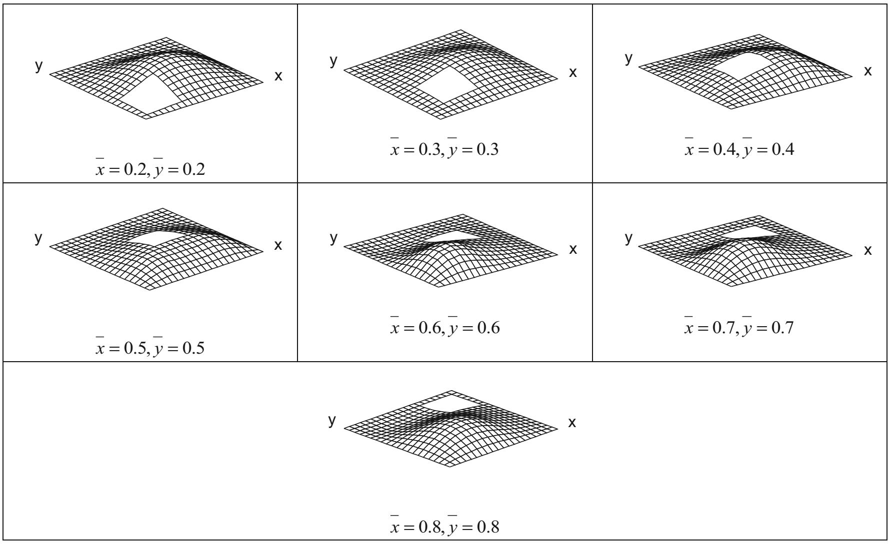
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 8.10 First mode shapes of laminated composite (0/90/0/90) stiffened elliptic paraboloidal shell for different position of central square cutout and SCSC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

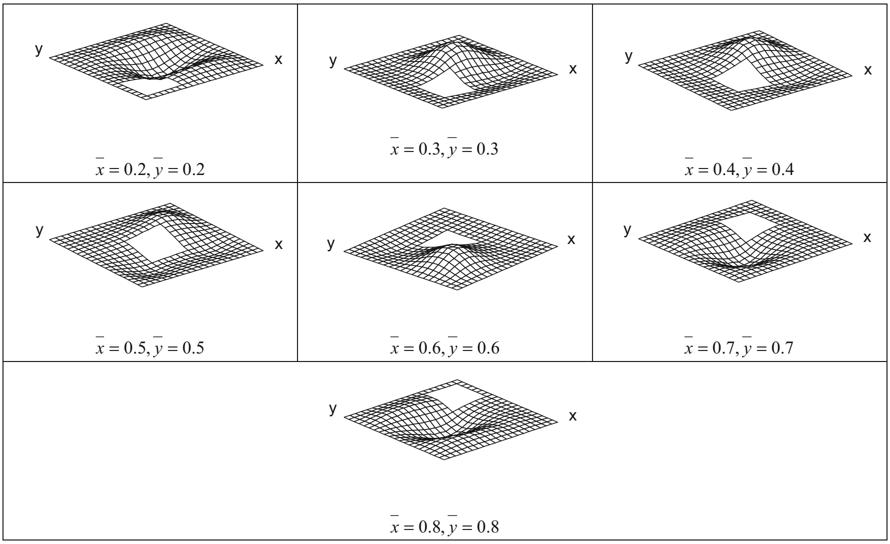
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 8.11 First mode shapes of laminated composite (0/90/0/90) stiffened elliptic paraboloidal shell for different position of central square cutout and SSSC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

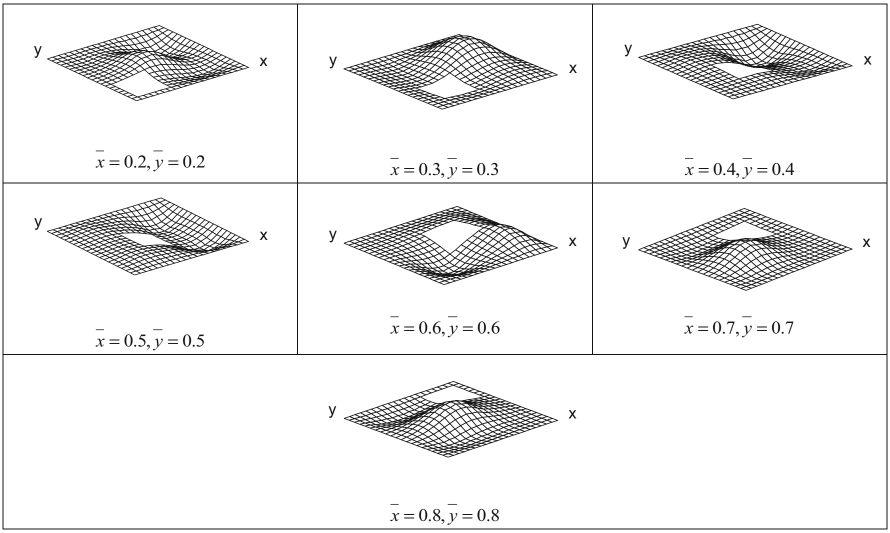
$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 8.12 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened elliptic paraboloidal shell for different position of central square cutout and CCCC boundary condition

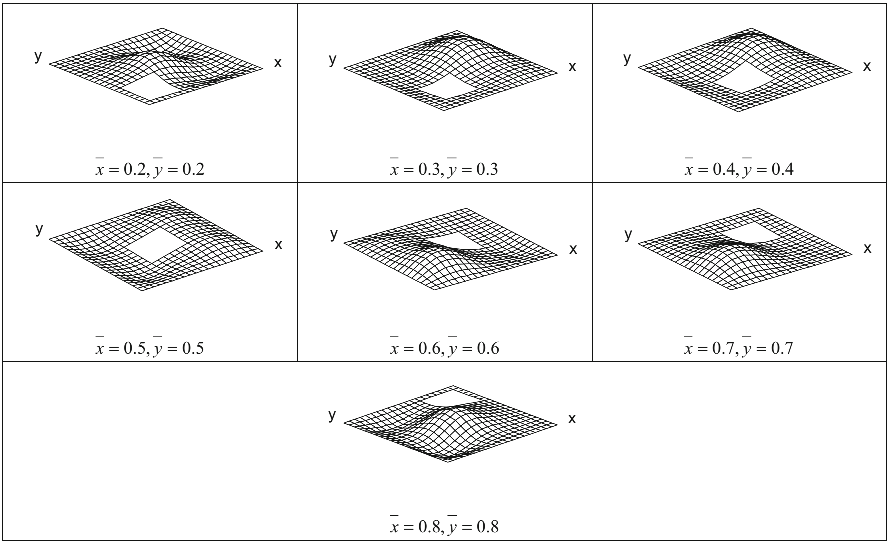


$\bar{x} = 0.2, \bar{y} = 0.2$ $\bar{x} = 0.3, \bar{y} = 0.3$ $\bar{x} = 0.4, \bar{y} = 0.4$

$\bar{x} = 0.5, \bar{y} = 0.5$ $\bar{x} = 0.6, \bar{y} = 0.6$ $\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 8.13 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened elliptic paraboloidal shell for different position of central square cutout and CCSC boundary condition

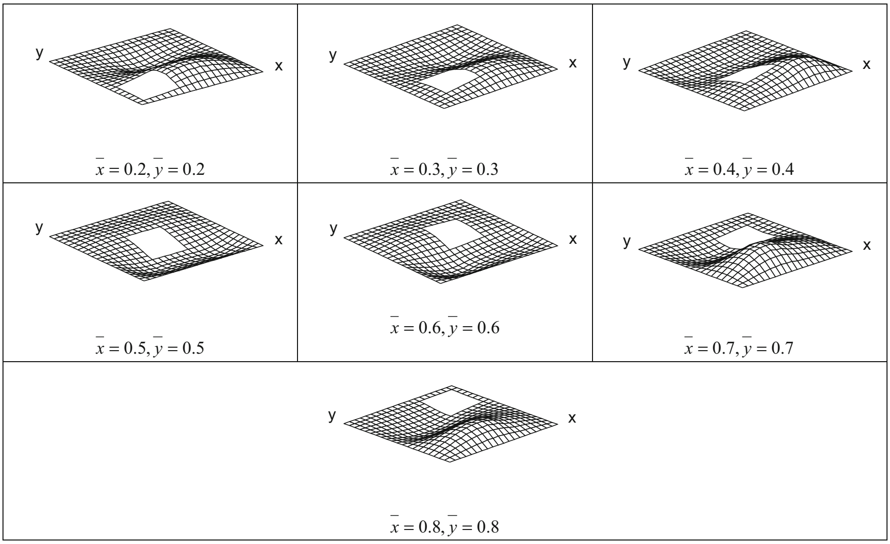


$\bar{x} = 0.2, \bar{y} = 0.2$ $\bar{x} = 0.3, \bar{y} = 0.3$ $\bar{x} = 0.4, \bar{y} = 0.4$

$\bar{x} = 0.5, \bar{y} = 0.5$ $\bar{x} = 0.6, \bar{y} = 0.6$ $\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 8.14 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened elliptic paraboloidal shell for different position of central square cutout and SCSC boundary condition



$\bar{x} = 0.2, \bar{y} = 0.2$

$\bar{x} = 0.3, \bar{y} = 0.3$

$\bar{x} = 0.4, \bar{y} = 0.4$

$\bar{x} = 0.5, \bar{y} = 0.5$

$\bar{x} = 0.6, \bar{y} = 0.6$

$\bar{x} = 0.7, \bar{y} = 0.7$

$\bar{x} = 0.8, \bar{y} = 0.8$

Fig. 8.15 First mode shapes of laminated composite (+45/−45/+45/−45) stiffened elliptic paraboloidal shell for different position of central square cutout and SSSC boundary condition

References

Chakravorty D, Sinha PK, Bandyopadhyay JN (1998) Applications of FEM on free and forced vibration of laminated shells. J Eng Mech 124(1):1–8

Hota SS, Chakravorty D (2007) Free vibration of stiffened conoidal shell roofs with cutouts. J Vib Control 13(3):221–240

Kumar A, Chakrabarti A, Bhargava P (2013) Vibration of composite cylindrical shells with cutouts using higher order theory. Int J Sci Ind Res 5(4):199–202

Nanda N, Bandyopadhyay JN (2007) Nonlinear free vibration analysis of laminated composite cylindrical shells with cutouts. J Reinf Plast Compos 26(14):143–1427

Sahoo S (2015) Free vibration behavior of laminated composite stiffened elliptic parabolic shell panel with cutout. Curved Layer Struct 2(1):162–182

Sivasubramonian B, Rao GV, Krishnan A (1999) Free vibration of longitudinally stiffened curved panels with cutout. J Sound Vib 226(1):41–55