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Exploring the Health State of a Population by Dynamic Modeling Methods

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Exploring the Health State of a Population by Dynamic Modeling Methods

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Preface

We are happy to have this book ready after several years of work, presentations, special publications, computer work and program development, and application to numerous data sets and further exploration of many old and new demographic parameters. Modeling, measuring, and quantifying health state was an important task of our studies.

The relatively new branch of stochastic theory and more specifically the first exit time or hitting time section provided an essential tool in our studies along with stochastic simulations, important to reproduce the provided data sets and validate the models proposed and used.

The data fitting techniques and related programs are presented. Many new and old terms are explored and quantitatively estimated, especially the health state or vitality of a population and the deterioration and related functions.

Having introduced and developed a strong tool to quantitatively analyze the health state of a population, new interesting features emerged including the deterioration function, a function expressing the curvature of the health state function. Even more the estimation of the healthy life years lost was possible by applying several methods and techniques and hence the healthy life expectancy; aging and longevity were studied as well.

As for every new tool proposed, the health state function gave rise to a quantitative estimation of the particular stages of human development and construction of related tables for specific countries.

The book provides the appropriate comparative applications and statistics as connecting tools accompanied by the existing literature, and as such it will be a valuable source to demographers, health scientists, statisticians, economists, and sociologists.

Athens, Greece — Christos H. Skiadas
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June 2017

Contents

Chapter 1
Life Expectancy, Deterioration Function and Application to Halley Breslau Data

1.1 Introduction

The methods and techniques related to life expectancy started to grow from the proposal of Life Tables system by Edmond Halley (1693) in his study on Breslau birth and death data. The related methods include mathematical models starting from the famous Gompertz (1825) model. The Gompertzian influence on using the mortality data to construct life tables was more strong than expected. He had proposed a model and a method to cope with the data. The model was relatively simple but quite effective because he could, by applying his model, to account for the main part of the data set. The method he proposed and applied was based on a data transformation by adding, dividing and finally taking logarithms of the raw data in an approach leading to a linearization of the original distribution. This was very important during Gompertz days when the calculations where very laborious. Furthermore, this method of data linearization made possible the wide dissemination, approval and use by the actuarial people of the Gompertz model and the later proposed Gompertz-Makeham (Makeham 1860) variation. However, the use of the logarithmic transformation of the data points until nowadays turned to be a strong drawback in the improvement of the related fields. By taking logarithms of a transformed form of the data it turns to have very high absolute values for the data points at the beginning and at the end of the time interval, the later resulting in serious errors due to the appearance of very high values at high ages. To overcome these problems resulting from the use of logarithmic transformations several methods and techniques have being proposed and applied to the data thus making more complicated the use of the transformed data. The approaches to find the best model to fit to the transformed data leaded to more and more complicated models with the Heligman and Pollard (1980) 8-Component Model and similar models to be in use today. The task was mainly to find models to fit to data well,

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instead to search for models with a good explanatory ability. Technically the used methodology is directed to the actuarial science and practice and to applications in finance and insurance (see the method proposed by Lee and Carter 1992).

Another quite annoying thing by applying logarithms is that the region around the maximum point of the raw data death distribution including this maximum point along with the left and right inflection points are all on the almost straight line part of the logarithmic curve. From the modelling point of view it is quite difficult to find from the transformed data the characteristics of the original data distribution. Even more when constructing the life tables the techniques developed require the calculation of the probabilities and then the construction of a model population usually of 100,000 people. The reconstruction of the original data distribution from the model population provides a distribution which is not so-close to the original one. Here we propose a method to use models, methods and techniques on analysing data without transforming the data by taking logarithms. While this methodology has obvious advantages it faces the problem of introducing it in a huge worldwide system based on the traditional use of methods, models and techniques arising from the Gompertzian legacy. To cope with the well established life table data analyses we present our work as to test the existing results, to give tools for simpler applications and more important to make reliable predictions and forecasts. We give much attention to the estimation and analysis of the life expectancy and the life expectancy at birth as are the most important indicators for policy makers, practitioners and of course researchers from various scientific fields.

1.2 The Deterioration Function: Further Analysis

Consider the death probability density function $g(t)$

$$g(t) = \frac{|H(t) - tH'(t)|}{\sigma\sqrt{2\pi t^3}} \exp^{-\frac{(H(t))^2}{2\sigma^2 t}}, \tag{1.1}$$

where $H(t)$ is the Health State function.

When the health state function takes the simple form $H(t) = l - (bt)^c$ the following formula for the death probability density function arises.

$$g(t) = \frac{|l + (c-1)(bt)^c|}{\sigma\sqrt{2\pi t^3}} \exp^{-\frac{(l-(bt)^c)^2}{2\sigma^2 t}}, \tag{1.2}$$

A simpler form is provided for $\sigma = 1$ and by setting $k = \frac{1}{\sqrt{2\pi}}$

$$g(t) = \frac{|k(l + (c-1)(bt)^c)|}{\sqrt{t^3}} \exp^{-\frac{(l-(bt)^c)^2}{2t}}, \tag{1.3}$$

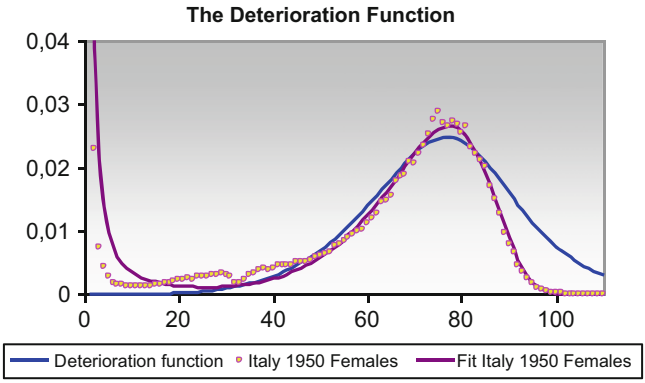

Fig. 1.1 Deterioration function, raw and estimated data for Italy

The Deterioration Function for the model $H(t) = l - (bt)^c$ or for the model $H(t) = (bt)^c$ is expressed by the following formula (Skiadas and Skiadas 2011). This formula provides the value of the curvature at every point $(H(t), t)$.

$$K(t) = \frac{|c(c-1)b^c t^{(c-2)}|}{\sqrt{(1 + c^2 b^{2c} t^{(2c-2)})^3}}, \tag{1.4}$$

This is a bell-shaped distribution presented in Fig. 1.1. The first exit time IM-model (Skiadas and Skiadas 2007, 2010a,b, 2011; Janssen and Skiadas 1995) is applied to the female mortality data of Italy for the year 1950.

$$g(t) = \frac{|k(l + (c-1)(bt)^c)|}{\sqrt{t^3}} \exp^{-\frac{(l-(bt)^c)^2}{2t}}, \tag{1.5}$$

A simpler 3 parameter version of this model arises when the infant mortality is limited thus turning the parameter l to be: $l = 0$ and the last formula takes the simpler form:

$$g(t) = \frac{|k((c-1)(bt)^c|}{\sqrt{t^3}} \exp^{-\frac{((bt)^c)^2}{2t}}, \tag{1.6}$$

The data, the fitting and the deterioration curves are illustrated in Fig. 1.1. The deterioration function starts from very low values at the first stages of the lifetime and is growing until a high level and then gradually decreases. As the main human characteristics remain relatively unchanged during last centuries it is expected that the deterioration function and especially the maximum point should remain relatively stable in previous time periods except of course of the last decades when the changes of the way of living and the progress of biology and medicine tend to shift the maximum deterioration point to the older age periods.

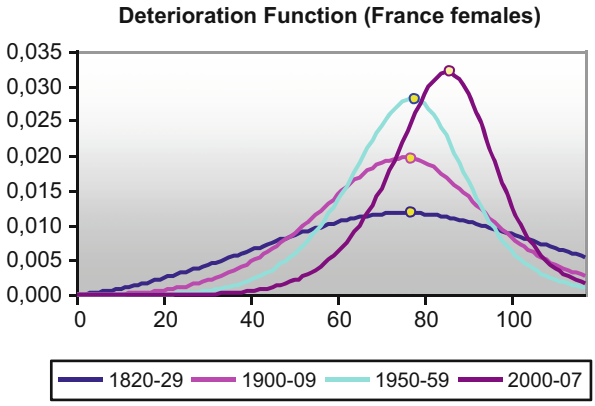

Fig. 1.2 Deterioration function for females in France

We can also explore the Greenwood and Irwin (1939) argument for a late-life mortality deceleration or the appearance of mortality plateaus at higher ages by observing the shape of the deterioration function. As we can see in Figs. 1.1 and 1.2 the deterioration tends to decrease in higher ages and especially after reaching the maximum value, leading to asymptotically low levels thus explaining what we call as Mortality Leveling-off. The deterioration of the organism tends to zero at higher ages. Economos (1979, 1980) observed a mortality leveling-off in animals and manufactured items. The characteristic non-symmetric bell-shaped form of the deterioration function is illustrated in Fig. 1.2 for females in France and for various periods from 1820 to 2007. The IM-model is applied to the data provided by the human mortality database for France in groups of 10 years. The form of the deterioration function is getting sharper as we approach resent years. The improvement of the way of living is reflected in the left part of the deterioration function. The graph for 1820–1829 presents the deterioration starting from very early ages. Instead the graph for the period 1900–1909 shows an improvement in the early deterioration period. The improvement continues for 1950–1959 and 2000–2007 periods.

However, the maximum point of these curves is achieved at almost the same year of age from 1820–1829 until the 1950–1959 period. This is an unexpected result as it was supposed that the general improvement of the way of living would result in a delay in the deterioration mechanisms. To clarify this observation we will further analyze the deterioration function. A main characteristic of the deterioration function is its maximum point achieved at:

$$T_{max} = \left[\frac{c-2}{(2c-1)c^2b^{2c}}\right]^{\frac{1}{2c-2}}, \tag{1.7}$$

We will apply this formula to mortality data of several countries and for various time periods. The task is to explore our argument for the stability of the maximum deterioration point around certain age limits and of a shift of this point to higher ages. Figure 1.3 for females in France from 1816 to 2007 is quite interesting. Four graphs are illustrated. The data are summarized in Table 1.1.

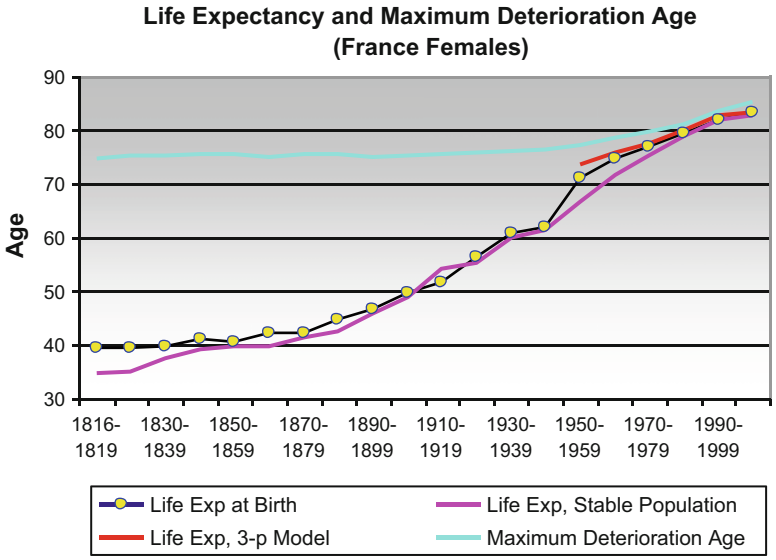

Fig. 1.3 Life expectancy at birth and maximum deterioration age for France

Table 1.1 Life expectancy at birth and maximum deterioration age for France female

Years	HM-database	IM-model	3p-model	Max Det
1816–1819	39.54	34.69		74.79
1820–1829	39.58	34.98		75.2
1830–1839	39.66	37.39		75.36
1840–1849	41.19	39.15		75.67
1850–1859	40.43	39.67		75.43
1860–1869	42.27	39.74		75.11
1870–1879	42.33	41.51		75.63
1880–1889	44.81	42.54		75.45
1890–1899	46.72	45.77		74.98
1900–1909	49.83	48.87		75.23
1910–1919	51.53	54.06		75.53
1920–1929	56.41	55.41		75.79
1930–1939	60.89	60.13		76.06
1940–1949	62.00	61.50		76.37
1950–1959	71.16	66.68	73.72	77.15
1960–1969	74.39	71.6	75.78	78.61
1970–1979	76.89	75.25	77.54	79.86
1980–1989	79.45	78.75	80.04	81.34
1990–1999	81.81	81.96	82.7	83.51
2000–2007	83.47	82.87	83.27	85.26

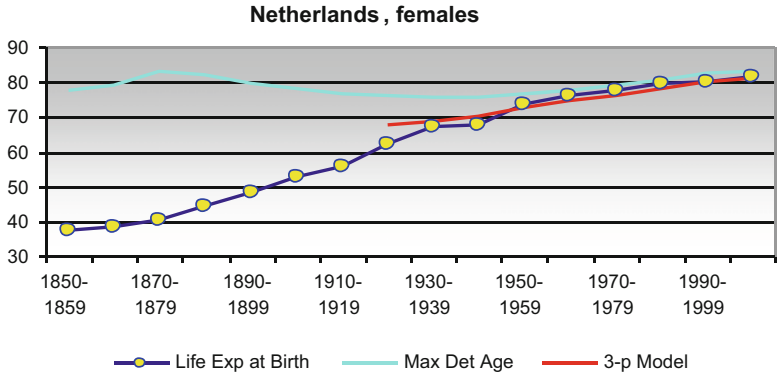

Fig. 1.4 Life expectancy at birth and maximum deterioration age for Netherlands

The data for the life expectancy at birth (blue line) are collected from the human mortality database for France (females in 10 year groups). The life expectancy at birth estimates based only on the fitting estimations of the IM model (based on the death data only) is presented with a magenta line. As the infant mortality becomes negligible during last decades we can estimate the life expectancy at birth by based on a 3-parameters alternative of the IM-model (red line). Note that according to the Keyfitz and Caswell (2005) theory for a stable population the life expectancy estimates tend to coincide when using different methods. Here the three different estimates for the life expectancy at birth are similar for the last 20 years. That is interesting is that the life expectancy at birth is approaching the maximum deterioration age (light blue line) which may be seeing as a plateau for the life expectancy at birth.

Figure 1.3 illustrates the life expectancy at birth (blue line) with data collected from the human mortality database (females in 10 year groups). As the infant mortality becomes negligible during last decades we can estimate the life expectancy at birth by based on a 3-parameters alternative of the IM-model (red line). The maximum deterioration age is expressed by a cyan line. The results for the four countries studied, (A) Netherlands, Fig. 1.4, (B) Denmark, Fig. 1.5, (C) Italy, Fig. 1.6 and (D) Norway, Fig. 1.7 are similar to the previous application for France.

The age year where the maximum value for the deterioration function is achieved for females in various countries and for several time periods is illustrated in Fig. 1.8. The data for females from various countries for 10 year periods from the human mortality data base are used and the Infant Mortality First Exit Density Model (IM-model) is applied. The maximum deterioration age for females was between 72 and 84 years from 1830 to 1950 for the countries studied (Table 1.2). For all the cases a continuous growth appears for the maximum deterioration age after 1950 until now. That is more important is that the mean value of the

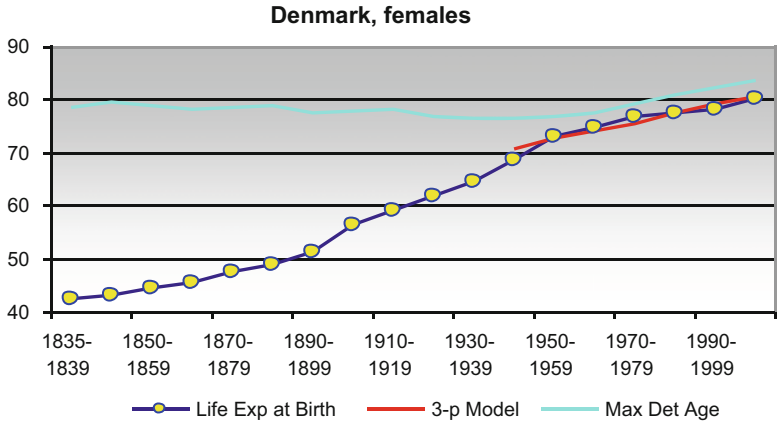

Fig. 1.5 Life expectancy at birth and maximum deterioration age for Denmark

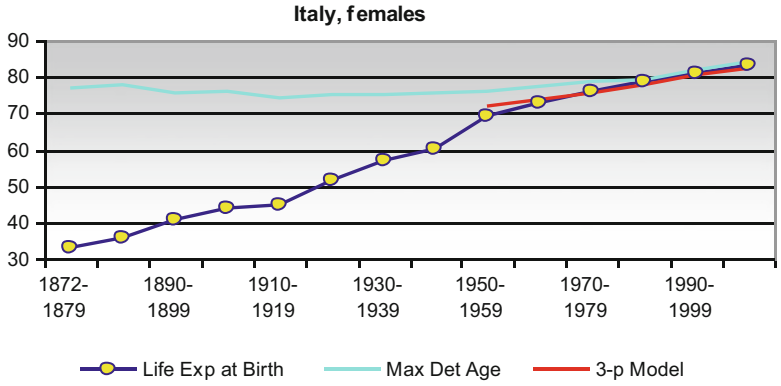

Fig. 1.6 Life expectancy at birth and maximum deterioration age for Italy

maximum deterioration age was between 76 and 78 years for 140 years (1830–1970) irrespective of the fluctuations in the life expectancy supporting the argument for an aging mechanism in the human genes. However, the scientific and medical developments after 1950 gave rise in a gradual increase of the level of the maximum deterioration age from 77 to 84 years in the last 60 years (1950–2010).

1.3 Stability of the Deterioration Function Characteristics

The derivation of the deterioration function of a population allows us to make early estimates for the life expectancy. The main assumption is based on accepting a theory for an internal deterioration mechanism driven by a code which governs the

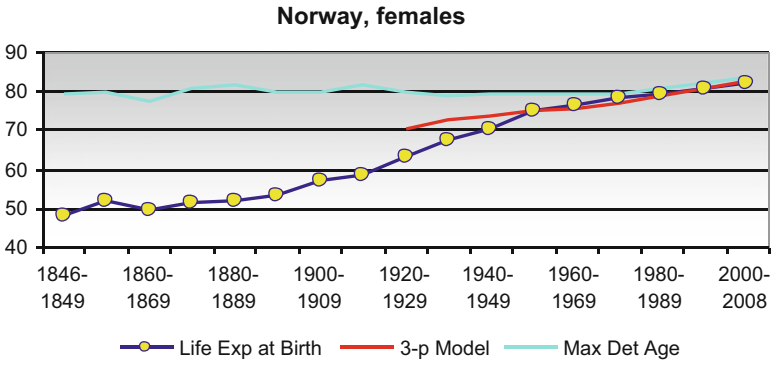

Fig. 1.7 Life expectancy at birth and maximum deterioration age for Norway

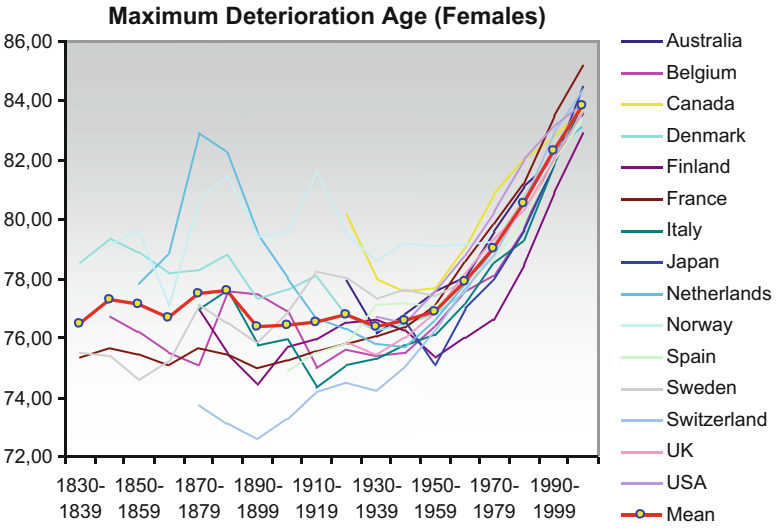

Fig. 1.8 Maximum deterioration age for 15 countries and mean value

life expectancy. If this assumption holds the deterioration function should include information for the future life expectancy even when using data from periods when the mean life duration was relatively small. In the previous chapter we have found that the maximum value of the deterioration function in many countries was set at high levels even when dealing with mortality data coming from various countries and from the last two centuries. The next very important point is to estimate the total effect of the deterioration to a population in the course of the life time termed as DTR. This is expressed by the following summation formula: $DTR = \int_0^t tK(t)dt \approx \Sigma_0^t tK(t)$.

Where t is the age and $K(t)$ is the deterioration function. The last formula expresses the expectation that an individual will survive from the deterioration

Table 1.2 Maximum deterioration age for 15 countries and mean value

The maximum deterioration age in 15 countries (Females)																
Year	Australia	Belgium	Canada	Denmark	Finland	France	Italy	Japan	Netherlands	Norway	Spain	Sweden	Switzerland	UK	USA	Mean
1830–1839				78.54		75.36						75.50				76.47
1840–1849		76.70		79.35		75.67				79.15		75.42				77.26
1850–1859		76.20		78.91		75.43			77.85	79.61		74.61				77.10
1860–1869		75.50		78.18		75.11			78.86	77.07		75.21				76.65
1870–1879		75.10		78.29	76.98	75.63	76.99		82.88	80.70		77.12	73.70			77.49
1880–1889		77.60		78.79	75.46	75.45	77.65		82.25	81.48		76.52	73.14			77.59
1890–1899		77.50		77.32	74.44	74.98	75.76		79.51	79.42		75.87	72.61			76.38
1900–1909		76.90		77.64	75.69	75.23	75.95		77.96	79.55	74.88	76.86	73.26			76.39
1910–1919		74.99		78.11	75.98	75.53	74.34		76.68	81.63	75.48	78.25	74.17			76.52
1920–1929	77.94	75.60	80.15	76.76	76.51	75.79	75.11		76.34	79.52	75.78	78.01	74.49	75.87		76.76
1930–1939	76.14	75.40	78.00	76.25	76.64	76.06	75.29		75.81	78.61	77.11	77.34	74.21	75.47	76.71	76.36
1940–1949	76.82	75.50	77.60	76.47	76.26	76.37	75.76	76.57	75.69	79.22	77.16	77.61	75.03	75.99	76.52	76.57
1950–1959	77.56	76.40	77.68	76.81	75.36	77.15	76.10	75.10	76.60	79.09	77.05	77.43	76.24	76.89	77.59	76.87
1960–1969	78.04	77.60	79.03	77.57	76.02	78.61	77.15	77.00	77.84	79.14	77.92	77.87	77.71	78.20	78.82	77.90
1970–1979	79.59	78.10	80.88	79.12	76.60	79.86	78.54	77.97	78.99	79.27	78.69	79.19	78.78	79.38	80.29	79.02
1980–1989	81.14	79.70	82.04	80.79	78.51	81.24	79.30	79.67	80.80	80.55	79.99	80.78	80.64	80.29	82.06	80.50
1990–1999	82.07	82.20	82.79	82.22	80.95	83.51	81.88	81.93	82.38	82.16	82.00	82.47	82.96	82.10	83.13	82.32
2000–2007	83.59	83.50	83.70	83.64	82.96	85.26	83.97	84.54	83.18	83.47	83.37	83.96	84.41	83.66	83.92	83.81

caused in his organism by the deterioration mechanism. The result is given in years of age in a Table like the classical life tables. The deterioration function is estimated from 0 to 117 years a limit set according to the existing death data sets. The estimated life expectancy levels are not the specific levels at the dates of the calculation but refer to future dates when the external influences, illnesses and societal causes will be reduced to a minimum following the advancement of our population status. The life expectancy levels seem to be reached in the recent years in some countries and in the forthcoming decades for others. DTR will be a strong indicator for the level of life expectancy of a specific population mainly caused by the DNA and genes. Due to its characteristics DTR can also be estimated from only mortality data (number of deaths per age or death distribution) thus making simpler the handling of this indicator even when population data are missing or are not well estimated. Another important point of the last formula is that we can find an estimator of the life expectancy in various age periods and to construct a life table. As it is expected the existence of a deterioration law will result in a population distribution over time thus making possible the construction of life tables by using the population distribution resulting from the deterioration law.

As the introduction of the deterioration function and the DTR indicator are quite new terms introduced when using the stochastic modeling techniques and the first exit time or hitting time theory we have applied the DTR and other forms resulting from the deterioration function to the mortality data for countries included in the Human Mortality Database (HMD). A main advantage by using these data sets is that are systematically collected and developed as to be able to make applications and comparisons between countries. The death data for 10-year periods are preferred as to avoid local fluctuations. However, the results are also strong when using and the other data sets from HMD for 5-year of 1-year periods.

The DTR is estimated for 15 countries for large time periods. As it is presented in Fig. 1.9 the DTR for 150 years from 1830 to 1980 was between 78 and 84 years of age with the mean value to be from 79.80 to 81.98 years (see Table 1.3). The lowest value 79.80 years was achieved the period 1950–1959. The main conclusion is that the DTR can be used as a measure of the future life expectancy levels. An example is based on the estimates for Sweden from 1751. The same method can be applied for other countries.

The next Fig. 1.10 illustrates the graphs from the DTR application to Sweden (females) for the period 1751–1759 (red line) and 1950–1959 (brown line). The blue line expresses the life expectancy estimates for 2000–2008 downloaded from the human mortality database. Our application includes the periods 1800–1809, 1850–1859 and 2000–2008. The results of the DTR method refer to the future limits of the life expectancy in connection to the limits of the human organism and the developments of science. Our results for 2000–2008 suggest a future level for the life expectancy at birth in Sweden at 84.89 years, 2.34 years higher than the period 2000–2008.

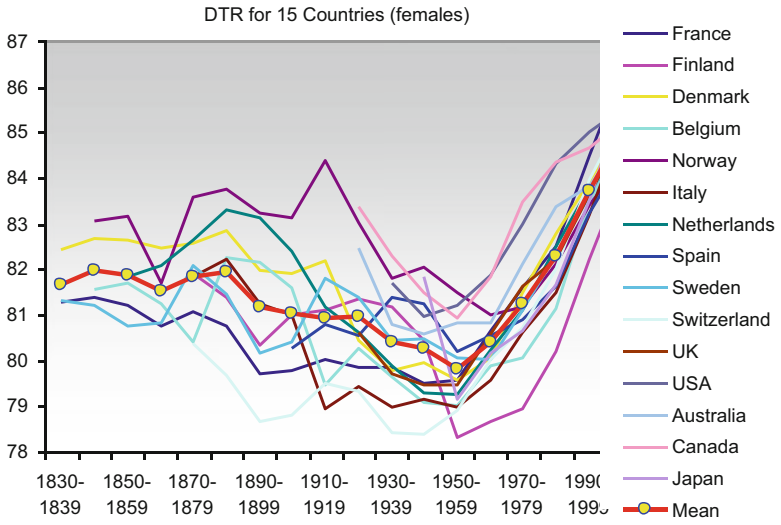

Fig. 1.9 DTR for 15 countries and mean value

1.4 Estimation of the Life Expectancy by the DTR System

An indicator of the health state is the estimated maximum deterioration age (Max Det) which is stable and independent of the age level selected. The maximum deterioration age during recent years tends to coincide with the life expectancy at birth estimated with the classical techniques of constructing life tables. Both the DTR and the Max Det are quite good measures of the life expectancy now and in the future as is presented in Fig. 1.11 where the mean values for the 15 countries studied are given.

The influence of the life level T for the estimation of life expectancy by the DTR system in Sweden (females) is illustrated in Fig. 1.12. Two scenarios are selected for the estimation of the future life expectancy at birth. In the first a $T = 117$ year level is accepted (dark brawn line) and in the second $T = 110$. As it was expected the higher level for T suggests a higher level for the life expectancy at birth via the DTR method. However, both scenarios tend to coincide in recent years something that it is quite useful in estimating the future trends for the life expectancy development (see Table 1.4). That it is important with the DTR system is that we can construct life tables for future dates thus doing forecasts. Instead with the Max Det we can have only an estimate for the future levels of life expectancy at birth but not for the life expectancy in other ages. The standard life expectancy at birth is presented (blue line) and the Max Deterioration points are also presented (light blue line).

Table 1.3 The DTR effect for 15 countries and mean value

The DTR effect in 15 countries (Females)																
Year	Australia	Belgium	Canada	Denmark	Finland	France	Italy	Japan	Netherlands	Norway	Spain	Sweden	Switzerland	UK	USA	Mean
1830–1839				82.43		81.29						81.30				81.67
1840–1849		81.55		82.68		81.39				83.07		81.20				81.98
1850–1859		81.70		82.65		81.20			81.84	83.16		80.76				81.89
1860–1869		81.23		82.47		80.75			82.07	81.71		80.81				81.51
1870–1879		80.41		82.57	81.92	81.07	81.85		82.62	83.59		82.07	80.37			81.83
1880–1889		82.25		82.86	81.37	80.76	82.22		83.30	83.77		81.44	79.66			81.96
1890–1899		82.16		81.98	80.34	79.70	81.23		83.12	83.22		80.15	78.67			81.17
1900–1909		81.61		81.89	81.01	79.77	80.98		82.38	83.13	80.26	80.42	78.82			81.03
1910–1919		79.45		82.20	81.10	80.01	78.93		81.18	84.39	80.78	81.81	79.49			80.93
1920–1929	82.47	80.28	83.38	80.43	81.35	79.86	79.42		80.63	83.02	80.54	81.37	79.32	80.61		80.98
1930–1939	80.80	79.64	82.30	79.77	81.16	79.86	78.98		79.88	81.80	81.40	80.45	78.42	79.72	81.70	80.42
1940–1949	80.58	79.09	81.48	79.94	80.46	79.50	79.16	81.85	79.28	82.05	81.25	80.48	78.39	79.46	80.97	80.26
1950–1959	80.82	79.02	80.92	79.57	78.30	79.59	78.98	79.16	79.26	81.48	80.20	80.06	78.91	79.48	81.21	79.80
1960–1969	80.81	79.90	81.82	80.05	78.66	80.66	79.58	80.15	80.22	81.01	80.55	80.02	80.02	80.58	81.89	80.39
1970–1979	82.13	80.07	83.49	81.47	78.95	81.59	80.60	80.64	81.10	81.18	80.89	81.12	80.72	81.63	82.98	81.24
1980–1989	83.37	81.15	84.33	82.77	80.21	82.49	81.49	81.67	82.51	82.12	81.64	82.31	82.00	82.21	84.32	82.31
1990–1999	83.82	83.47	84.68	83.91	82.21	84.49	83.18	83.52	83.80	83.41	83.25	83.61	83.97	83.66	85.01	83.73
2000–2007	84.87	84.79	85.24	85.14	84.07	86.36	85.15	85.78	84.46	84.39	84.45	84.89	85.36	84.99	85.53	85.03

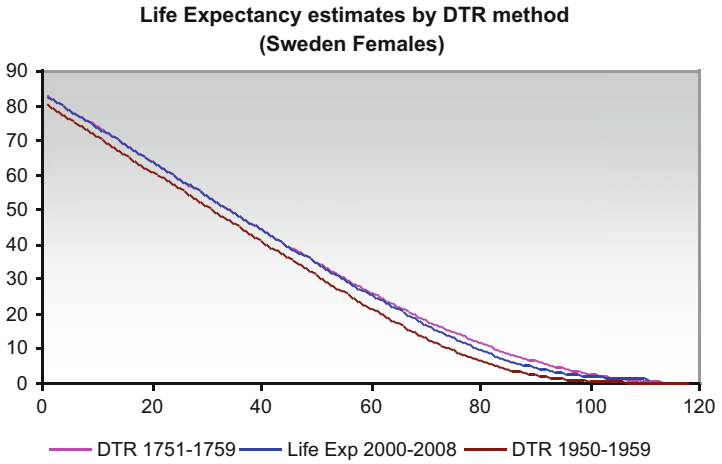

Fig. 1.10 DTR application to Sweden (females) for the period 1751–1759 (*red line*) and 1950–1959 (*brown line*). Life expectancy estimates for 2000–2008 are provided from the human mortality database (*blue line*)

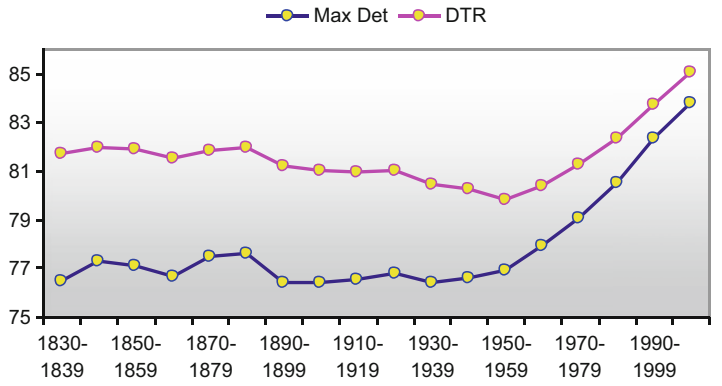

Fig. 1.11 Maximum deterioration age and DTR for 15 countries

1.5 The Halley Life Table

Edmund Halley published his famous paper in 1693. It was a pioneering study indicating of how a scientist of a high caliber could cope to a precisely selected data sets. Halley realized that to construct a life table from only mortality data was fusible only on the basis of a stationary population (Keyfitz and Caswell 2005) by means of a population where births and deaths are almost equal and the incoming and outgoing people are limited. This was the case of the Breslau city in Silesia (now Wroclaw). The birth and death data sent to Halley gave him the opportunity to construct a life table and present his results in the paper on An Estimate of the Degrees of the Mortality of Mankind, drawn from curious Tables of the Births and Funerals at the City of Breslau; with an Attempt to ascertain the Price of Annuities

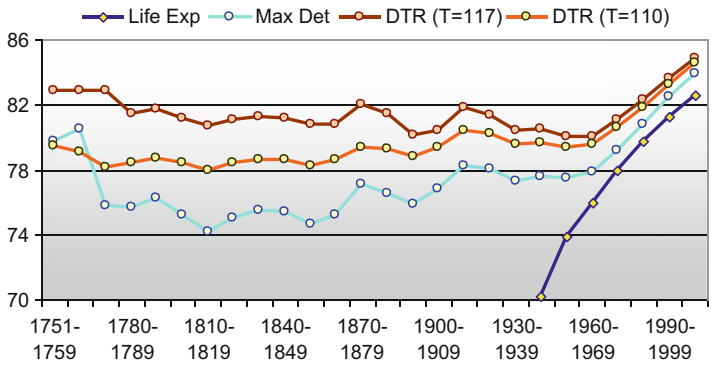

Fig. 1.12 Life expectancy, maximum deterioration age and DTR for Sweden, females

Table 1.4 Maximum deterioration age and DTR for Sweden female

Year	Max Det	DTR
1830–1839	76.47	81.67
1840–1849	77.26	81.98
1850–1859	77.10	81.89
1860–1869	76.65	81.51
1870–1879	77.49	81.83
1880–1889	77.59	81.96
1890–1899	76.38	81.17
1900–1909	76.39	81.03
1910–1919	76.52	80.93
1920–1929	76.76	80.98
1930–1939	76.36	80.42
1940–1949	76.57	80.26
1950–1959	76.87	79.80
1960–1969	77.90	80.39
1970–1979	79.02	81.24
1980–1989	80.50	82.31
1990–1999	82.32	83.73
2000–2007	83.81	85.03

upon Lives. The same time period was also proposed a method for handling life tables by Graunt (1676). For more information on the history and the development of actuarial science see the related history by Haberman and Sibbett (1995). The purpose of this part is first to use the Halleys life table data in order to construct a mortality curve by applying a stochastic model resulting from the first exit time theory. After applying the model and constructing the mortality curve we find the deterioration function for the specific population of Breslau at the years studied by Halley and thus making possible to find the maximum deterioration age and constructing a graph for the vitality of the population, a term proposed by Halley and used many years later by Strehler and Mildvan (1960) who also suggest the term vitality of a person in a stochastic modeling of the human life. In Table 1.5 the first

Table 1.5 Halley Breslau data

Age	Persons (Total)	Persons	Age	Persons (Total)	Persons
1	1000	145	51	335	11
2	855	57	52	324	11
3	798	38	53	313	11
4	760	28	54	302	10
5	732	22	55	292	10
6	710	18	56	282	10
7	692	12	57	272	10
8	680	10	58	262	10
9	670	9	59	252	10
10	661	8	60	242	10
11	653	7	61	232	10
12	646	6	62	222	10
13	640	6	63	212	10
14	634	6	64	202	10
15	628	6	65	192	10
16	622	6	66	182	10
17	616	6	67	172	10
18	610	6	68	162	10
19	604	6	69	152	10
20	598	6	70	142	11
21	592	6	71	131	11
22	586	7	72	120	11
23	579	6	73	109	11
24	573	6	74	98	10
25	567	7	75	88	10
26	560	7	76	78	10
27	553	7	77	68	10
28	546	7	78	58	9
29	539	8	79	49	8
30	531	8	80	41	7
31	523	8	81	34	6
32	515	8	82	28	5
33	507	8	83	23	3
34	499	9	84	20	
35	490	9	85		
36	481	9	86		
37	472	9	87		
38	463	9	88		
39	454	9	89		
40	445	9	90		

(continued)

Table 1.5 (continued)

Age	Persons (Total)	Persons	Age	Persons (Total)	Persons
41	436	9	91		
42	427	10	92		
43	417	10	93		
44	407	10	94		
45	397	10	95		
46	387	10	96		
47	377	10	97		
48	367	10	98		
49	357	11	99		
50	346	11	100		

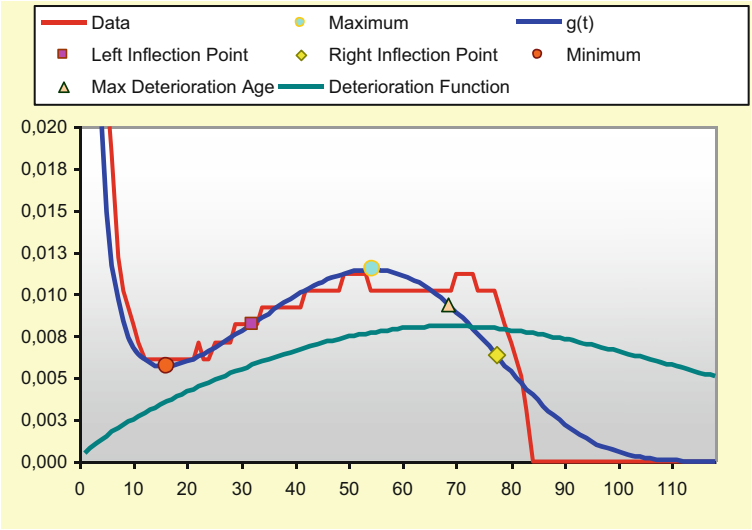

Fig. 1.13 Fit curve, data plot and deterioration curve for Halley data

two columns include the Breslau life table data from the Halley paper whereas in the third column we have constructed the deaths per age as the difference between two consecutive rows of the second column. The data from the third column are inserted into our Excel program of the fist exit time distribution function and the results are presented in Fig. 1.13.

The estimated best fit is presented with a blue line. The parameter estimates and the values for the characteristic points are given in the Table 1.6.

From the next Figure and Table the estimated maximum death rate is at the age of 53.3 years, the right inflection point is at 76.2 years, the left inflection point is at 30.9 years and the minimum at 14.6 years. A very important characteristic of the health state of the population is given by estimating the age where the maximum

Table 1.6 Parameter estimates and characteristic points for Halley Breslau data fit

Characteristic points of graph	Year	g(t)	g'(t)	Parameter
Maximum	53.3	0.011457	0	c = 2.72
Left inflection point	30.9	0.008182	0.0001081	b = 0.03425
Right inflection point	76.2	0.006412	-0.000470	l = 0.25907
Minimum	14.6	0.005679	0	k = 0.61215
Maximum deterioration age	67.4			

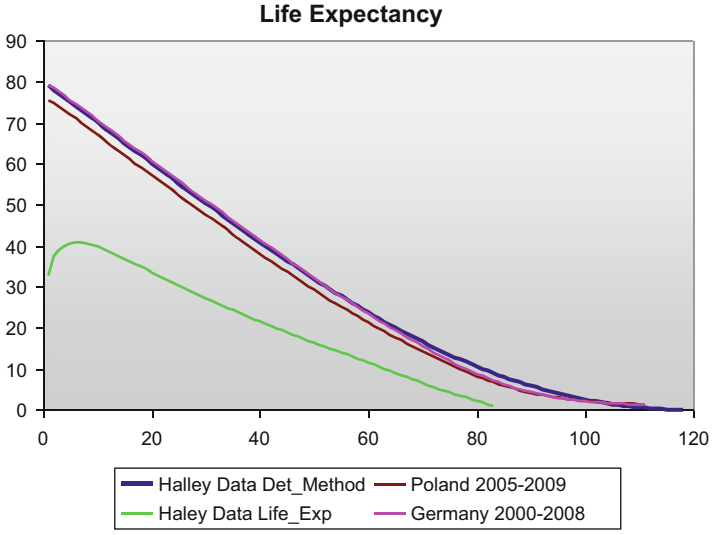

Fig. 1.14 Life expectancy curves for Breslau data

deterioration takes place. This is estimated at the age of 67.39 years and it is the maximum of the deterioration function presented by a green curve in the graph.

The DTR system as presented earlier provides a method to find a future life expectancy based on the deterioration function. The surprising result is that the estimated life expectancy is quite close to the values for Germany from 2000 to 2008 and higher than the related values for Poland (2005–2009) as provided by the Human Mortality Database. The estimates are presented in Fig. 1.14 along with the estimates for the life expectancy estimated by the Breslau data (see the green line in the graph).

Illustration of the development of the maximum deterioration age from the Halley days until today is given in Fig. 1.15. The maximum deterioration age for the Breslau data (1687–1691) was 67.39 years and continued to increase by 4.75 years per century.

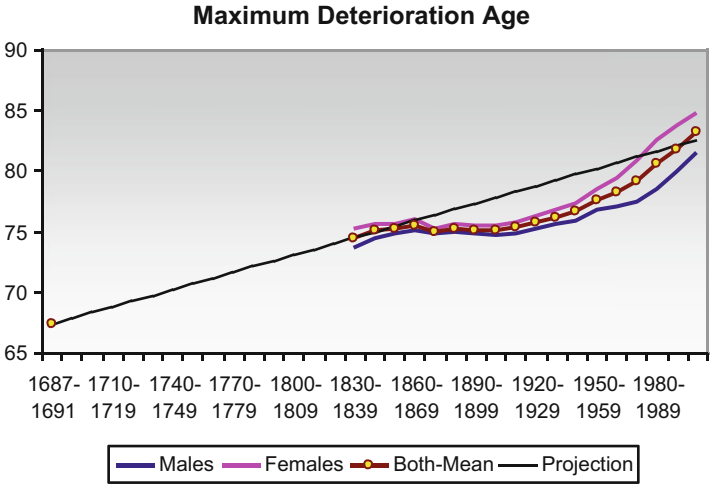

Fig. 1.15 Maximum deterioration age for various time periods

1.6 The Program

A computer program (IM-model-DTR-Life-Tables) was developed to be able to make the necessary computations related to this paper. Furthermore the program estimates the life expectancy tables by based on the mortality and population data. The life expectancy is also estimated by based on the fitting curve thus making more accurate the related estimations. The program and the related theory can be found in the website: http://www.cmsim.net. The program is developed in Excel 2003 and it is very easy to use without any special tool.

1.7 Summary and Conclusions

We have developed and applied a new theoretical framework for analyzing mortality data. The work starting several years ago was based on the stochastic theory and the derivation of a first exit time distribution function suitable for expressing the human mortality. Furthermore we have explored on how we could model the so-called *vitality* of a person or the opposite term the deterioration of an organism and to provide a function, the deterioration function, which could be useful for sociologists, police makers and the insurance people in making their estimates and plan the future.

Acknowledgements The data used are downloaded from the Human Mortality Database at: http://www.mortality.org or from the statistical year-books of the countries studied.

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Chapter 2
A Quantitative Method for Estimating the Human Development Stages Based on the Health State Function Theory and the Resulting Deterioration Process

2.1 Introduction: The Health State Function Theory

The advances in stochastic theory and the resulting first exit time or hitting time theories gave rise to the development a new theoretical and applied approach for estimating the health state function (HSF) of a population and finding various very important parameters useful in many applications including the life expectancy and the healthy life expectancy. In this study we propose quantitative methods of estimating the human development age groups based on the findings of the health state function theory. We briefly present the first exit time theory used to find the health state function of a population (HSF) and then we give the details of the new theoretical approach with the appropriate applications to support and validate the theoretical assumptions. In developing the health state function theory the original problem was to give a definition of the health state. It is not an easy task to find an exact and precise definition of health of an individual. Even more it is relatively difficult to define and measure the health state of a person. However, as for very many language terms introduced centuries or thousand of years ago the approval and use from the public is straightforward. According to the World Health Organization (WHO) definition in 1946 health is: "a state of complete physical, mental, and social well-being and not merely the absence of disease or infirmity." When we started working on the paper of 1995 (Janssen and Skiadas 1995, Dynamic Modelling of Life-Table Data, Applied Stochastic Models and Data Analysis, vol. 11, No 1, 35–49, 1995) it was clear that a definition of health was very important for our study. As we where ready to use the relatively new tools of stochastic theory, that it was essential it was to define the health state of an individual in the course of time. We called S_t the health state at a time t. Accordingly, when the time is replaced by the age x, the health state is denoted by S_x. We soon get to the conclusion that an estimation of the health state was very difficult following the luck of a precise definition and measure of health of the individual. Health and health state

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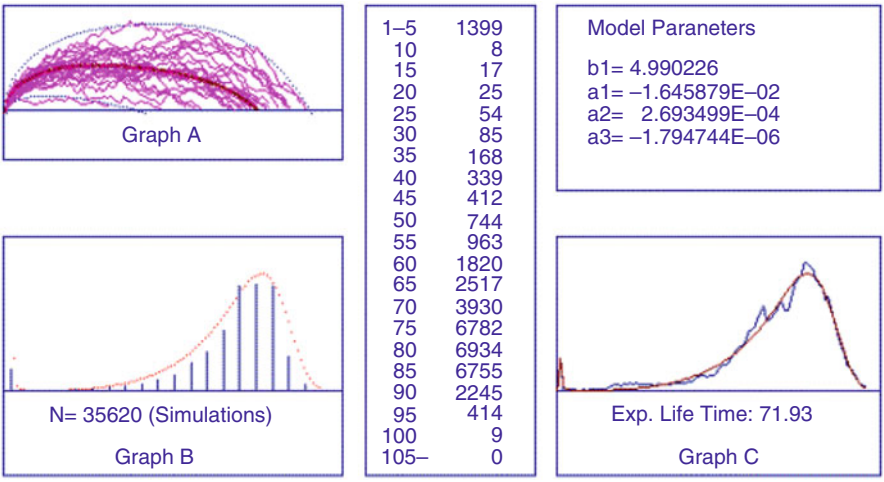

Fig. 2.1 Health state function, model fitting, stochastic paths and stochastic simulation for Greece, males (1992)

are stochastic variables that is quantities which present slow, sharp or even dramatic changes over time (see Fig. 2.1 Graph A for few stochastic paths of individuals; the light red lines). However, for the modern stochastic theory there is no need to find the course of the health state for every individual. The task is to find the summation of many health states of the individuals that is to define and find the Health State of a Population. In this case the health state of a population could be a smooth function over time or better over age group; a function which can be approximated by a relatively simple mathematical form expressed by the dark red curve in Fig. 2.1. However, several corrections and approximations were needed for almost two decades to find a convenient form for the Health State Function as it is illustrated in Fig. 2.2. Both presentations provide similar results for the characteristic points (maximum or minimum) with the exemption of the order of these points that is the highest local maximum is in the adolescence age period and the lower local maximum is found at ages from 30 to 40 years of age with the recent estimates while the opposite was estimated with the old method. The local minimum remain the same between 18 and 30 years of age in both methods of health state estimation. However, the estimation of age variations and classifications remain similar thus giving to the old method an advantage because of the simplicity of applications. Note that in both cases the health state curve follows the same path after reaching the local maximum between 30 and 40 years.

The next problem we had was the selection of data. The selection of data related to the health or the health state was a difficult or impossible task due to the luck of systematically collected data sets for long periods of time along with the previously mentioned problems with the definition of health and the health state. Especially when trying to collect survey data from various countries and population groups

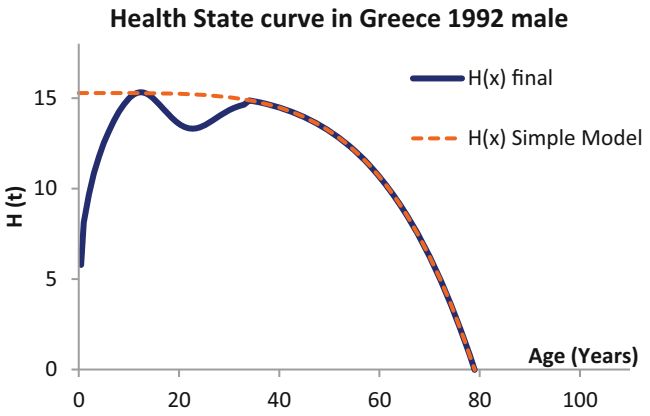

Fig. 2.2 The final form of the health state function for Greece, males (1992)

the main problem is on how individuals respond on questions related to their health and health state. Cultural, economic or societal issues take particular importance in answering the related questions.

The only reliable data sets are the population and death data selected for centuries by the bureau of the census of every country. These data sets almost precisely reply to the question Alive or Dead. Alive represents the number of the members of a population for an age group and Dead represents the number of dead of the members of the same group of population at the same time period. We assume that death is the cause of reaching a very low health level for the health state while alive is a higher level health status. The resulting death time series include vital information for the health state of a population as definitely include the picture of the population members reaching a low health level at specific years of age as a distribution over years of age. This is called the hitting time or first exit time distribution in terms of the stochastic theory. There are many stochastic theory technicalities related on how to find the health state function (the cause) from the result that is the distribution function (the death distribution) but finally we arrived to a final solution in the paper of 1995. The application was done for data in France and Belgium thus giving the form for the health state of the population in these countries. An application for Greece (males 1992) is illustrated in Fig. 2.1. The first exit time model proposed was applied to the data by using a nonlinear regression analysis algorithm. Graph C of Fig. 2.1 presents the data (blue line) and fit curve (red line) and the estimated expected life time. In top right the estimated parameter values are given. The resulting Health State Function is estimated by using these parameters and is the basis of Graph A of Fig. 2.1(dark red line). Several health state random realizations for individuals are presented (light red lines). End of life results when these paths approach the bar of the minimum health. The next step is to verify the validity of our theory by means of solving the inverse problem that is: to find the death distribution function when the health state function is known or estimated, by performing

stochastic simulations. 35620 stochastic path realizations are done and the results are summarized in a Table in the middle of Fig. 2.1 in 5 year age groups. The related graph is Graph B of Fig. 2.1 where the simulation bars approach the death probability distribution (dotted line) relatively well. By using the results of the theory which we have applied we arrive to the very important point that is to find the form of the health state function of a population by only death results. Of course we normalize the death distribution by using the population distribution. In other words we take into account the number of the population per age group. In the case of a stable population in terms of Keyfitz and Caswell (2005) we do not need such adaptation and only the number of deaths per age is needed. We have also proposed a method of finding the death distribution $g(x)$ by using the mortality data usually referred as $\mu(x)$. For more information and theoretical and applied details see our references (Skiadas 2011a,b, 2012 and Skiadas and Skiadas 2010, 2011, 2012a,b, 2013).

A next approach came only recently (Skiadas 2012) by solving the inverse problem on how to find the health state function of a population over age from the death distribution expressed by the death probability density function presented in Fig. 2.3. This is found after dividing the death data of a population per year of age by the sum of deaths for all age years thus providing a data set with sum equal to unity. In this way it is easy to compare death data from various time periods or from different population groups. The case in our example is for males in USA the year 2000 and both the original data points and the fit curve by using our model are present. Figure 2.4 illustrates the health state function resulting from the same data and from the fit curve and Fig. 2.5 presents the heath state change (first derivative) of the USA males in 2000 from data and curve fit. The continuous line in Fig. 2.5 refers to the first difference (the derivative) of the health state function.

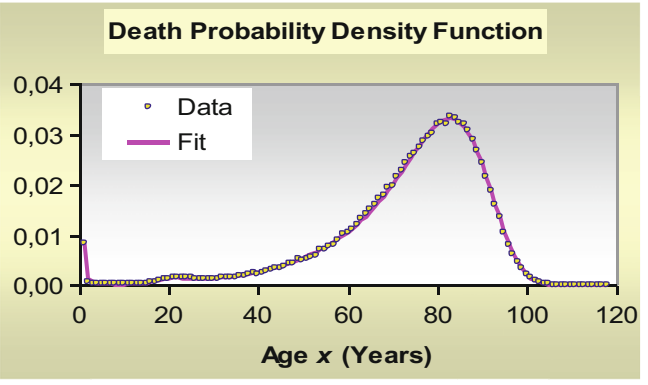

Fig. 2.3 The death density function

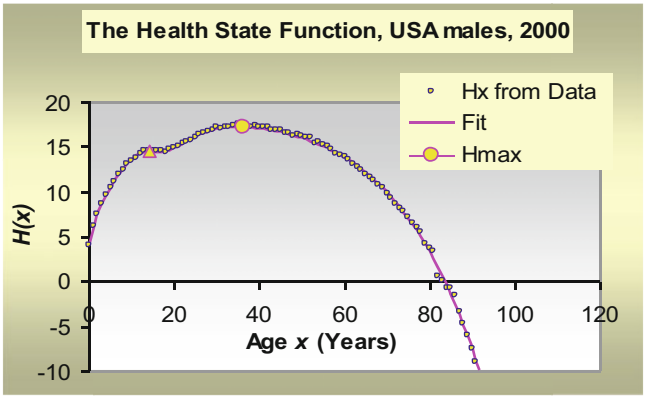

Fig. 2.4 The health state function of a population

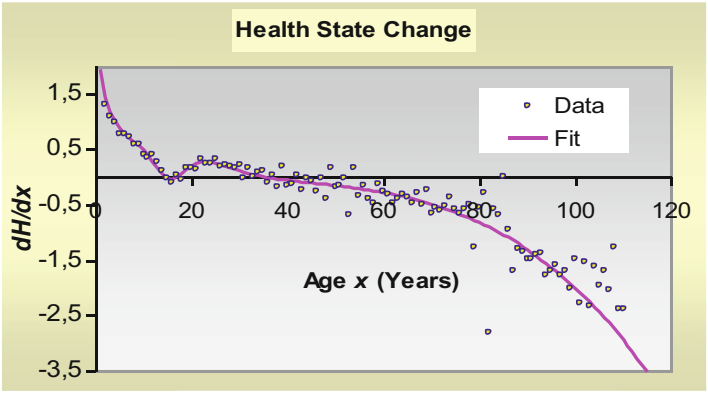

Fig. 2.5 The health state change of a population

2.2 Human Development Stages Based on the Health State Function

The Health State Theory and the resulting models and applications provide the opportunity to explore the development of the early and middle stages of the human life span. This is feasible by estimating the first and second variation of the health state changes or in mathematical notation to estimate the first and second derivative of the health state function (see the second variation in Fig. 2.6).

There are three main graphs to analyze the human development based on the health state function of a population: the health state graph, the first difference graph and the second difference graph. Each graph provides characteristic milestones of the human development (see Figs. 2.7 and 2.8 for males and females in USA the year 2000). For males in USA in 2000 the health state graph (Magenta curve in

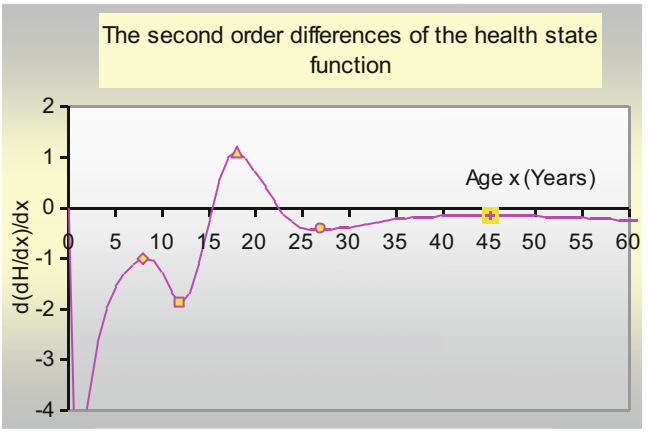

Fig. 2.6 The second order differences of the health state function

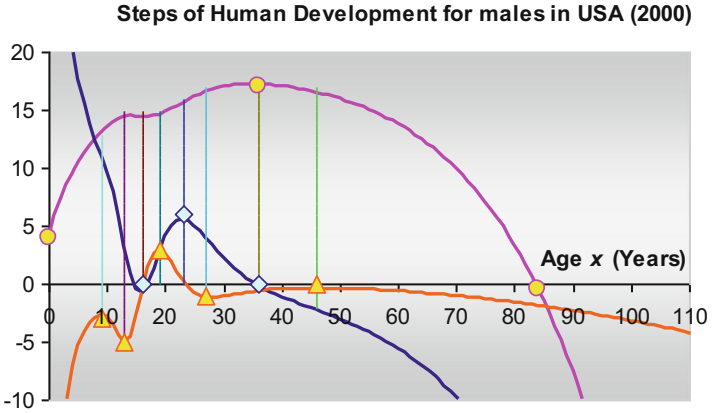

Fig. 2.7 Health state function, first and second order differences for males

Fig. 2.7) has a starting point at zero age (health level: 4.00) a maximum health age at 36 years of age (17.22) and a zero health state at 84 years of age. After this year of age the health is in a critical and supercritical condition. The death probability is declining exponentially. The graph of the first differences expresses the speed of health state change (Blue curve in Fig. 2.7). The first part of this graph starts from very high positive values at birth and fast decreases at a value close to zero (−0.75) at 16 years of age. It follows a growing part to a maximum (5.6) at 23 years of age. Then the decline is continuous passing from positive to negative values at 36'years of age corresponding to the health state maximum. The acceleration of the health state is illustrated in the graph of the second differences (Orange curve in Fig. 2.7) and provides characteristic values for the years of age 9, 13, 19, 27 and 46. All these time steps correspond to changes of the slop of the speed of health change (the first

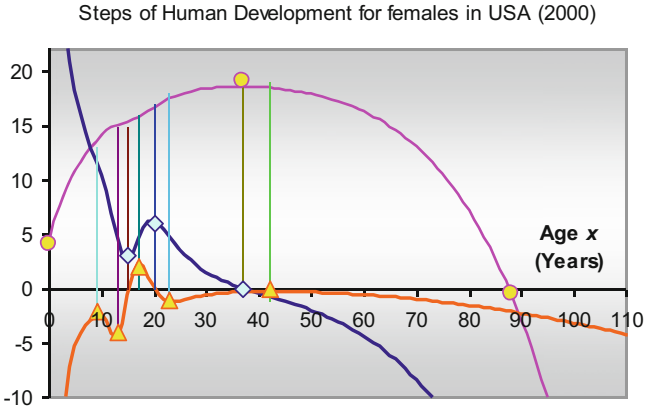

Fig. 2.8 Health state function, first and second order differences for females

differences graph) thus indicating characteristic changes in human development. The last one, that is 46 years of age, corresponds to the starting point of faster and faster declining of the speed of health changes. This means that is the starting point of the deterioration of the human organism via the aging process. Then the decline of the health state is a merely continuous process in terms of the health state function. Instead the first part of the life span from zero age until the maximum health state (36 years for males in USA the year 2000) is extremely interesting indicating on how the human complex organism is developed.

The results of the application for USA males, the year 2000 are summarized in Tables 2.1 and 2.2 and illustrated in the previous Figs. 2.7 and 2.8. In the same Tables related results are given for UK, Australia, Canada, Germany, France, Italy and Japan for males and females in 2000. As it was expected there are very many similarities especially for the countries USA, UK, Australia and Canada. Our estimates provide useful information for the Pre-Adolescence, Early-Adolescence, Late-Adolescence age years ranging from 9 (minimum) to 21 (maximum) years of age for males and 8 (minimum) to 19 (maximum) years of age for females and followed by the First, Second and Third Stage of Adult Development starting from 19 (minimum) to 37 (maximum) years of age for males and from 16 (minimum) to 41 (maximum) years of age for females. The next age groups refer to Early Middle ages from 33 (minimum) to 48 (maximum) years of age for males and 37 (minimum) to 46 (maximum) years of age for females, Middle+Old ages from 43 (minimum) to 85 (maximum) years of age for males and 41 (minimum) to 91 (maximum) years of age for females and Very Old ages from 83+ (minimum) years of age for males and 87+ (minimum) years of age for females.

Table 2.1 Human development age groups based on the health state function (male, 2000)

Country	Pre adolescence	Early adolescence	Late adolescence	First stage of adult development	Second stage of adult development	Third stage of adult development	Early middle ages	Middle and old ages	Very old ages
USA	9–13	13–16	16–19	19–23	23–27	27–36	36–46	46–84	84–
UK	9–12	12–17	17–20	20–25	25–32	32–37	37–48	48–83	83–
Australia	9–12	12–17	17–21	21–27	27–34	34–41	41–48	48–85	85–
Canada	8–12	12–16	16–19	19–24	24–29	29–37	37–47	47–85	85–
Germany	12–15	15–17	17–19	19–23	23–26	26–34	34–43	43–83	83–
France	11–15	15–17	17–19	19–22	22–26	26–33	33–43	43–85	85–
Italy	9–12	12–17	17–20	20–24	24–30	30–37	37–47	47–84	84–
Japan	11–14	14–16	16–19	19–23	23–26	26–36	36–43	43–86	86–

Table 2.2 Human development age groups based on the health state function (female, 2000)

Country	Pre adolescence	Early adolescence	Late adolescence	First stage of adult development	Second stage of adult development	Third stage of adult development	Early middle ages	Middle and old ages	Very old ages
USA	9–13	13–15	15–17	17–20	20–23	23–37	37–42	42–88	88–
UK	10–13	13–16	16–18	18–21	21–25	25–38	38–43	43–87	87–
Australia	10–13	13–16	16–18	18–21	21–25	25–41	41–43	43–89	89–
Canada	8–10	10–14	14–16	16–20	20–24	24–40	40–43	43–89	89–
Germany	10–13	13–16	16–18	18–21	21–24	24–40	40–41	41–88	88–
France	9–11	11–15	15–16	16–19	19–23	23–41	41–45	45–90	90–
Italy	8–12	12–16	16–19	19–24	24–30	30–41	41–42	42–89	89–
Japan	12–16	15–17	17–19	19–21	21–24	24–40	40–46	46–91	91–

2.3 Human Development Stages Based on the Death Probabilities

Another point is to explore the steps of human development by using the traditional method of computing the death probabilities as a function of age $\mu(x)$ of a population. The estimation of the death probabilities is the basis of the classical method of constructing life tables in actuarial science. The death probability (the probability of dying within a specific age year) is estimated by dividing the number of deaths in a specific year of age by the living population in the same year of age. In general, the probability of death increases over time thus growing in the opposite direction of the health state of the population. However, a mortality method of defining the age groups based on the death probabilities may give interesting findings compared with those provided by the health state method. The resulting graphs of an application in USA males in 2000 are presented in Fig. 2.9. The graphs

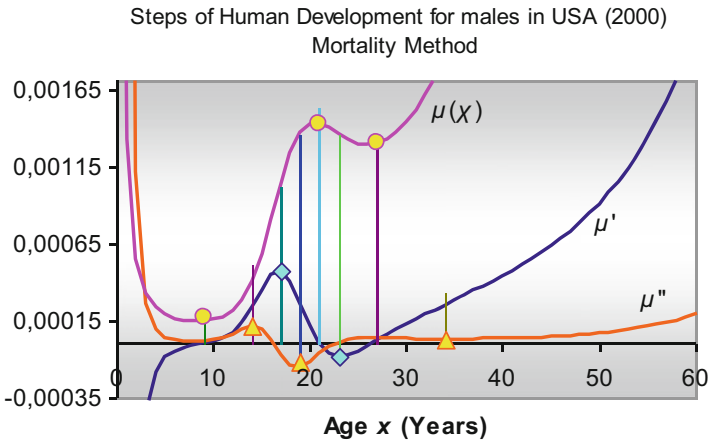

Fig. 2.9 Mortality function and first and second order differences

represent the death probabilities $\mu(x)$, the fist differences $\mu(x)'$ and the second differences $\mu(x)''$. The characteristic points estimated are at 9, 14, 17, 19, 21, 23, 27 and 34 years of age. The values are very close or identical to the estimates based on the health state function.

2.4 An Application to Disease Data in USA

There are two shortcomings by using the death probabilities method. While the year with the maximum health is estimated with a small decline of two years from the health state method estimates (34 years instead of 36), the other estimate related to the early middle ages period at 46 years of age is totally missing. This is a very important age step connected with the acceleration of the decline of the health state. That is why it is the starting point of diseases like prostate cancer for males and uterine cancer with females. Cancer and other data from USA and UK support this argument. Table 2.3 includes related data for USA in 2007.

The death rates per year of age for USA the year 2007 based on the previous Table 2.3 are presented in Fig. 2.10. The cases selected from the USA data base include diseases which start from the age of the maximum health state. Significant values of the order of 0.4% of the total causes appear for the age group 35–44 years. Instead the previous years cases (from 0 to 34 years) account only for the 0.072% of the total cases. After the age group 35–44 the death rates follow an exponential trend. The estimates based on the USA 2007 data for males and females provide the following age groups (see Table 2.4). The decline starts after the maximum health state level at 36 years of age but the main causes start from 45 years of age a key point characterizing the onset of middle and old ages.

Table 2.3 Death rates for USA 2007 for various causes

USA 2007												
Cause of death	Under 1 year	1–4 years	5–14 years	15–24 years	25–34 years	35–44 years	45–54 years	55–64 years	65–74 years	75–84 years	85 years and over	Sum
Malignant neoplasm of larynx	0	0	0	2	2	55	450	932	989	871	333	3.634
Malignant neoplasms of corpus uteri and uterus, part unspecified	0	0	0	2	31	168	584	1.583	2.021	1.960	1.107	7.456
Malignant neoplasm of prostate	0	1	0	1	1	21	428	2.271	5.716	11.257	9.397	29.093
Malignant neoplasm of bladder	1	0	0	0	7	93	570	1.564	2.817	5.009	3.782	13.843
Multiple myeloma and immunoproliferative neoplasms	0	0	0	2	7	155	705	1.788	2.917	3.853	1.879	11.306
Parkinson's disease	0	0	0	2	2	12	60	396	2.310	9.363	7.911	20.056
Alzheimer's disease	0	0	0	0	1	8	95	728	3.984	23.009	46.804	74.629
Other acute ischemic heart diseases	2	0	0	3	17	109	376	679	740	1.021	1.145	4.092
Atherosclerosis	1	0	0	2	1	27	134	350	829	2.298	4.590	8.232
Emphysema	3	0	1	1	10	60	486	1.590	3.294	4.835	2.509	12.789
Infections of kidney	5	1	1	8	6	30	59	64	91	170	193	628
Hyperplasia of prostate	0	0	0	0	0	1	1	12	47	147	283	491
Inflammatory diseases of female pelvic organs	4	0	0	1	5	6	14	19	22	21	24	116
Total	16	2	2	24	90	745	3.962	11.976	25.777	63.814	79.957	186.365

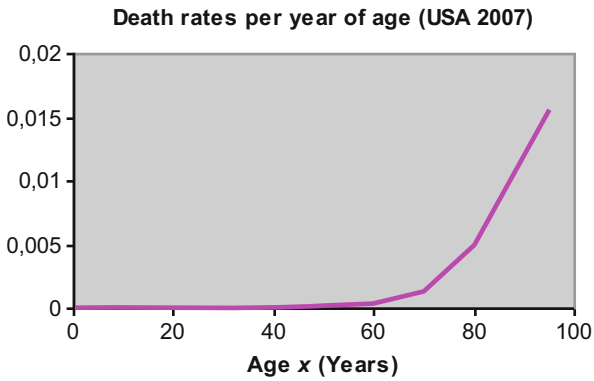

Fig. 2.10 Death rates for USA 2007 based on the Table 2.3 data (total)

2.5 Human Development Age Groups Based on the Deterioration Function

The age groups after 40–45 years of age until 85–90 years where the mean zero health state level appears is a quite large period of the life time and it is worth noting to make separations in specific age groups or subgroups. For this case we can use the deterioration function Det and the first Det', second Det'' and third Det''' differences (derivatives) of this function in order to define and characterize these groups. The deterioration function is a measure of the curvature of the health state function $H(x)$ and thus it can be used to find how and how fast the HSF is changing. The formula measuring the curvature requires the calculation of the first and second derivatives of the health state function and has the form $Det = |\ddot{H}|/(1+\dot{H}^2)^{3/2}$. The deterioration function is a very good measure of the decline of the human health state in the time course. Furthermore we can estimate the total influence of the deterioration process ($TDET$) by the following formula:

$$TDET = \int_{x(minDet)}^{x} Det(s)ds \approx \sum_{x(minDet)}^{x} Det(s) \tag{2.1}$$

Where the starting year denoted by x_{minDet} is at the minimum deterioration age and the final year denoted by $x = x_{110}$ is at the 110 year of age.

Figure 2.11 illustrates the deterioration function and the first, second and third differences (derivatives) for USA in 2007 (males). The deterioration function (light blue line) starts from very high positive values at birth and declines until age 41 where the minimum deterioration age appears, it grows to a maximum at 76 years and then continuously declines. It is worth noting that the first part of this function from birth until the minimum corresponds to the adult development stage. The part from the minimum until the late age years corresponds to the human deterioration life period. The first difference (blue line) has maximum and minimum corresponding to 61 and 90 years of age respectively. These age year steps represent

Table 2.4 Human development age groups based on the health state function (total population, 2007)

Country	Pre adolescence	Early adolescence	Late adolescence	First stage of adult development	Second stage of adult development	Third stage of adult development	Early middle ages	Middle and old ages	Very old ages
USA	10–14	14–17	17–19	19–23	23–28	28–36	36–45	45–88	88–

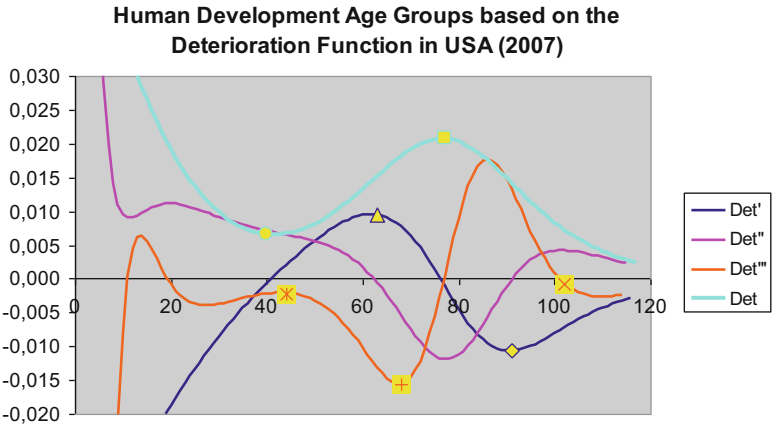

Fig. 2.11 The Deterioration function and the three differences of this function

the points where the exponential growth or decline of the deterioration function slows down. The first difference expresses the speed of the deterioration. The speed is positive for the first period of the deterioration (from 41 to 76 years of age) and then is negative for all the period of over 76 years of age. The second difference (magenta line) of the deterioration function provides an estimate of changes at 101 years of age. The second difference expresses the acceleration of the deterioration function. The acceleration is positive until 61 years of age then is negative until 90 years of age and then is positive for the rest of the life span. The third difference (orange line) of the deterioration function provides three characteristic points at 67, 85 and 109 years of age. The estimated values for USA males and females for 2000 are summarized in Table 2.5. In the same Table the Total Deterioration (TDET) is estimated in the last year of each age group along with the percentage of the total deterioration in the last year of each age group. The estimates for both males and females are very close each other especially as regards the % total deterioration. According to these findings the total deterioration is 24.0% and 23.8% at 60 years of age for males and females respectively. The related values are 33.2 and 32.5 at 65 and 66 years of age for males and females. Total deterioration close to 50% is estimated at the maximum deterioration age at 74 (TDET = 52.6%) and 75 (TDET = 53.1%) years of age for males and females respectively. The total deterioration is close to 3/4 of the final, that is 74.4 for males and 74.8 for females in age years 84 (males) and 85 (females). In 100 years of age the total deterioration is 95.0 for males and 94.3 for females whereas this is 99.2% for males and 99.1% for females at 108 years of age while we have selected TDET = 100% at 110 years of age. This selection was done because the data provided by the Human Mortality Database cover the range from 0 to 110 years of age.

An application to Sweden (males) gave interesting results regarding the development of the Deterioration and the Total Deterioration in the time course. Sweden provides reliable death and population data for the last two and a half centuries.

Table 2.5 Human development age groups based on the deterioration function, total deterioration and % deterioration in USA (2000)

Age groups, Total Det[a], %Tot Det[b]	First deterioration stage	Second deterioration stage	Light disabilities	Moderate and severe disabilities	Old ages (many disabilities)	Very old ages (critical health)	Critical ages (very critical health)	Highly critical ages
Age groups	41–60	60–65	65–74	74–84	84–89	89–100	100–108	108–
Total Det	0.21	0.29	0.46	0.65	0.72	0.83	0.867	0.874
Male %Tot Det	24	33.2	52.6	74.4	82.4	95	99.2	100
Age groups	40–61	61–66	66–75	75–85	85–89	89–100	100–108	108–
Total Det	0.22	0.3	0.49	0.69	0.75	0.87	0.915	0.923
Female %Tot Det	23.8	32.5	53.1	74.8	81.3	94.3	99.1	100

[a]The total deterioration is estimated for the final year of the age group
[b]The % total deterioration is based on a 100% total deterioration in the year 110

In our study we explore data from 1800, 1900, 2000 and 2010 for males and the results are presented in Table 2.6. The main finding is that the Total Deterioration Percentage in every group is relatively stable for all this 200 years period. Instead the years of age for the various groups are moved to higher ages supporting the argument of an improvement of the health state during a large period of time mainly due to the advances in medicine and social and economic welfare of the country. Figure 2.12 illustrates these findings for Sweden. The gap between age groups is smaller as we move to higher age groups. Following the results presented in Tables 2.5 and 2.6 we can form the following age groups already presented in these Tables. The First Deterioration age group is at 20%–25% TDET, the Second Deterioration age group is at 30%–33% TDET. It follows a Light Disability Stage group at 50%–55% TDET, a Moderate and Severe Disabilities age group at 70%–75% TDET and an Old Ages group with many disabilities at 80%–90% TDET. Then the next two periods refer to Critical Health state with TDET at 90%–95% and Very Critical Health with TDET at 97%–99% years of age. The final group refers to Highly Critical ages with TDET close to 100%. Accordingly the Table 2.7 for males in year 2000 for USA, Australia, UK, Canada, Germany, France, Italy and Japan is constructed. The findings indicate a similar behavior for the age groups proposed supporting our theory and method for constructing these age groups based on death and population data.

2.6 Further Analysis and Quantification

It should be noted that many studies in last decades emphasize on the estimation of the healthy life expectancy and the loss of healthy life years by light, moderate or severe disability causes. These studies provide tables presenting the healthy life expectancy and the loss of healthy life years in various countries. They give important information in countries and government officials, social and economic agencies, insurance companies and actuaries in order to adapt their plans according to the new findings and to forecast the future. However, the healthy life expectancy as the life expectancy are mainly statistical estimates and provide a part of the hidden information in life table data. Instead with the method proposed we extract vital information for the health condition of various age groups by using death and population data. The simplest is to obtain the related data from an international data base as of the World Health Organization or the Human Mortality Database (HMD) used in this study. We use the Deaths 1×1 and the Population 1×1 data for USA, Sweden and the other countries from HMD and we use the program Excel File of the SKI-6-Parameters-22-06-2012 Program (zip format 7 Mb) which we have developed in Excel, from the website http://www.cmsim.net/id24.html. By inserting the death and population data for a country for a specific year in columns P and Q in the Excel program, the Health State Function and the Deterioration Function are calculated. Another method is to insert in column R of the Excel program the values for $\mu(x)$ included in files as the life table data mltper 1×1 for males in the HMD or

Table 2.6 Human development age groups based on the deterioration function, total deterioration and % deterioration for males in Sweden (1800–2010)

Age groups, Total Det[a], %Tot Det[b]	First deterioration stage	Second deterioration stage	Light disabilities	Moderate and severe disabilities	Old ages (many disabilities)	Very old ages (critical health)	Critical ages (very critical health)	Highly critical ages
Age groups	31–49	49–54	54–64	64–75	75–80	80–93	93–102	102–
Total Det	0.23	0.31	0.5	0.69	0.77	0.89	0.93	0.96
1800 % Tot Det	24	32.3	52.1	71.9	80.2	92.7	96.9	100
Age groups	34–55	55–60	60–68	68–76	76–80	80–89	89–96	96–
Total Det	0.22	0.32	0.52	0.72	0.8	0.91	0.96	1.01
1900 % Tot Det	21.8	31.7	51.5	71.3	79.2	90.1	95	100
Age groups	40–58	58–63	63–72	72–81	81–86	86–97	97–105	105–
Total Det	0.21	0.3	0.48	0.66	0.74	0.85	0.89	0.907
2000 % Tot Det	23.2	33.1	52.9	72.8	81.6	93.7	98.1	100
Age groups	40–59	59–65	65–73	73–81	81–87	87–95	95–102	102–
Total Det	0.19	0.3	0.49	0.67	0.77	0.87	0.91	0.937
2010 % Tot Det	20.3	32	52.3	71.5	82.2	92.8	97.1	100

[a]The Total Deterioration is estimated for the final year of the age group
[b]The % Total Deterioration is based on a 100% total deterioration in the year 110

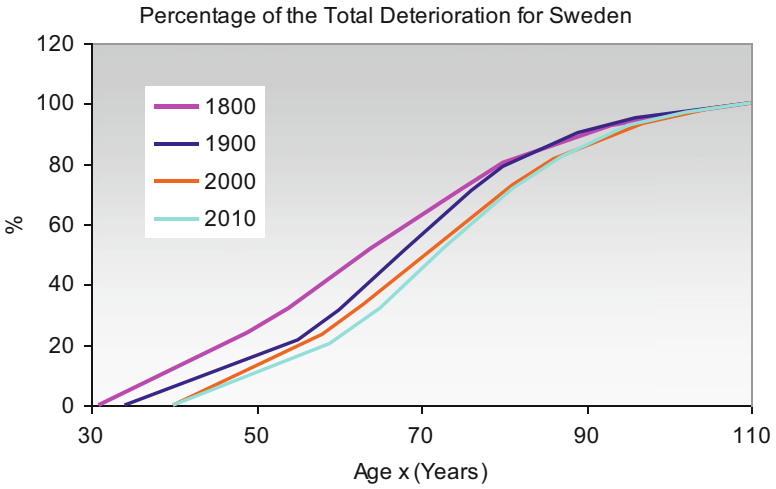

Fig. 2.12 The effect of the total deterioration in Sweden (males)

to obtain the death distribution from a Life Table, insert the data directly in column Q and set the indicator equal to 0 in cell AB24 of the Excel program. When we use the life table mltper 1×1 for males from the HMD Database the death distribution is given in column *dx* for the age years from 0 to 110.

Our results should also be useful to scientists working in the fields of human development and the related theories as are the Erikson's theory of personality and the proposed Erikson's stages of psychosocial development, the Piaget, Sullivan and others in providing a quantitative measure of the proposed age groups of these theories. A first approach is given in Table 2.8 from results estimated from males and females data in USA (2000). The pre-adolescence, early-adolescence and late-adolescence periods are defined thus quantifying the age groups suggested by the related theories by using the Health State Function Theory. The results from the deterioration function approach provide a better classification for adulthood period (see Tables 2.5, 2.6, and 2.7).

2.7 Final Classification Groups

There are two main classification approaches proposed here: the human development age group classification based on the Health State Function which gives very good and detailed classification for the first part of the life course from birth until the age of the maximum health state and the early middle ages, and the classification based on the Deterioration Function which gives a very detailed classification for the rest of the life time period after the year of the maximum health state.

Table 2.7 Human development age groups based on the deterioration function for males (2000) in several countries

Country	First deterioration stage	Second deterioration stage	Light disabilities stage	Moderate and severe disabilities stage	Old ages (many disabilities)	Very old ages (critical health)	Critical ages (very critical health)
USA	41–60	60–65	65–74	74–84	84–89	89–100	100–
UK	41–56	56–60	60–70	70–83	83–88	88–101	101–
Australia	39–57	57–62	62–73	73–82	82–87	87–99	99–
Canada	40–57	57–63	63–72	72–84	84–88	88–100	100–
Germany	40–57	57–62	62–72	72–83	83–88	88–100	100–
France	40–60	60–65	65–74	74–84	84–88	88–98	98–
Italy	40–57	57–62	62–71	71–83	83–88	88–101	101–
Japan	40–58	58–64	64–73	73–84	84–88	88–100	100–

Table 2.8 Human development age groups based on the health state function

Methods	Pre adolescence	Early adolescence	Late adolescence	First stage of adult development	Second stage of adult development	Third stage of adult development	Early middle ages	Middle and old ages	Very old ages
Erikson	Competence	Adolescence		Early adulthood		[a]Adulthood (27–64), old ages (64–)			
Sullivan	Pre adolescence	Early adolescence	Late adolescence						
Piaget	Concrete operations	Formal operational stage							
Males USA 2000	9–13	13–16	16–19	19–23	23–27	27–36	36–46	46–84	84–
Females USA 2000	9–13	13–15	15–17	17–20	20–23	23–37	37–42	42–89	89–

[a]The correct group formation according to Erikson is found by using the deterioration function group classification. In this case there appear age groups close to 65 years (see the previous Tables 2.5, 2.6, and 2.7 for validation)

The final classification form for USA males and females in 2000 is presented in Tables 2.5, 2.6, and 2.9. The main groups proposed include the Pre-adolescence period from 9–13 years of age for both males and females, the Adolescence period (13–16 years for males and 13–15 years for females), the Adult Development (19–36 years for males and 17–37 years for females), the Middle ages (36–65 years for males and 37–66 years for females), the Old ages (65–89 years for males and 66–89 years for females) and the Very Old and Critical Ages (over 89 years of age for both males and females). Except of the Pre-adolescence period the other main groups are divided in subgroups. The Adolescence period is divided in two subgroups the Early Adolescence (13–16 years for males and 13–15 for females) and the Late Adolescence (16–19 years for males and 15–17 years for females). The Adult Development period is divided in three subgroups the First Stage of Adult Development (19–23 years for males and 17–20 years for females), the Second Stage of Adult Development (23–27 years for males and 20–23 for females) and the Third Stage of Adult Development (27–36 years for males and 23–37 years for females). The Middle Ages period is divided in three subgroups the Early Middle Ages period (36–46 years for males and 37–42 years for females), the First Deterioration Stage (46–60 years for males and 41–61 years for females) and the Second Deterioration Stage (60–65 years for males and 61–66 years for females). The Old Ages period is divided in three subgroups the Light Disabilities Stage (65–74 years for males and 66–75 years for females), the Moderate and Severe disabilities Stage (74–84 years for males and 75–85 years for females) and the Many Disabilities Stage (84–89 years of age for both males and females). The Very Old and Critical Ages period is divided in three subgroups the Very Old Ages subgroup (89–100 years of age for males and females) characterized by critical health condition, the Critical Ages subgroup (100–108 years for males and females) characterized by very critical health and the Highly Critical Health subgroup (over 108 years of age for both males and females) characterized by high uncertainty regarding survival. The deterioration of the human organism is very high in this age group.

2.8 Conclusions

The method of estimating the health state function of a population, the deterioration function and the first and second and third differences (derivatives) provided a method to estimate the Human Development Age Groups. The resulting applications in various countries supported the argument of characteristic age groups. The proposed human development age group selection and classification will be useful in scientific fields like medicine, sociology, biology, anthropology, psychology, gerontology, probability and statistics and many others. We provide a method to quantify the human development age groups thus giving reliable tools for further studies.

Table 2.9 Human development age groups based on the health state function and the deterioration function (USA 2000)

	Adolescence			Adult development			Middle ages			Old ages			Very old and critical ages		
Sex	Pre adolescence	Early adolescence	Late adolescence	First stage of adult development	Second stage of adult development	Third stage of adult development	Early middle ages	First deterioration stage	Second deterioration stage	Light disabilities stage	Moderate and severe disabilities stage	Many disabilities stage	Very old ages (critical health)	Critical ages (critical health)	Highly critical ages
Male	9–13	13–16	16–19	19–23	23–27	27–36	36–46	46–60	60–65	65–74	74–84	84–89	89–100	100–108	108–
Female	9–13	13–15	15–17	17–20	20–23	23–37	37–42	42–61	61–66	66–75	75–85	85–89	89–100	100–108	108–

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Chapter 3
Estimating the Healthy Life Expectancy from the Health State Function of a Population in Connection to the Life Expectancy at Birth

3.1 Introduction

In previous studies (Skiadas 2011a,b, 2012; Skiadas and Skiadas 2007, 2008, 2010a,b, 2011a,b) we have introduced the Health State Function (HSF) of a Population and we have applied to population and mortality data in various countries. In these studies we have further analyzed and expanded the theory proposed by Janssen and Skiadas (1995). Following the traditional methods (Graunt 1676; Halley 1693; Gompertz 1825) developed in Demography and Actuarial Science we have proposed methods and techniques to estimate the HSF directly from the data sets, without fitting any intermediate function as is also the case for finding the mortality μ_x from the same data sets. We have also suggested a method for presenting both HSF and μ_x in the same graph (Skiadas 2012). Having introduced a function expressing the development of the health state of a population according to the age of this population we can estimate several interesting and important characteristics related to the health state. The graph for the HSF versus age (see Fig. 3.1) is of a non-symmetric parabola form showing a plateau like shape during 30–45 years and then slowly decreasing until a fast decrease stage at old ages. (In our notation we use the term t for the age instead of x preferred by actuaries). This is because the same theory and practice can apply to other disciplines and especially when estimating the life time of machines and complex systems (Skiadas 1986).

The health state function applied in this study is a good approximation of the final form presented in Fig. 3.2 where the final health state form along with the simple model health state approximation. The latter is quite important for defining a health state trajectory where the final form asymptotically approaches. The comparative applications done between countries are not affected by the use of the approximation of the health state.

By observing the above graph in Fig. 3.1 we can immediately see that the area between the health state curve and the horizontal axis (OMCO) represents the total

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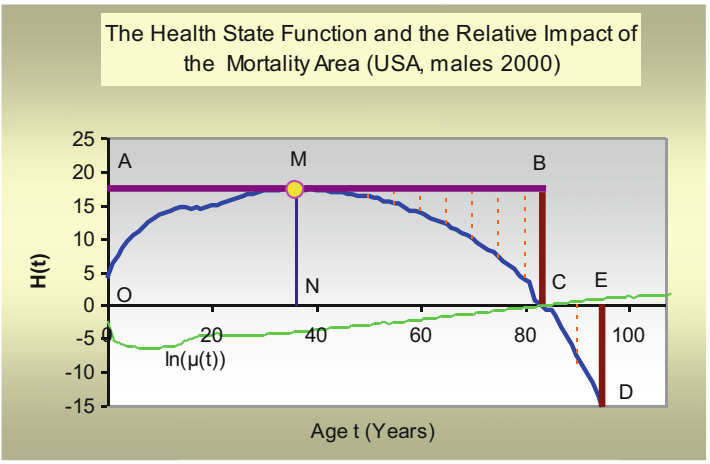

Fig. 3.1 The impact of the mortality area to health state

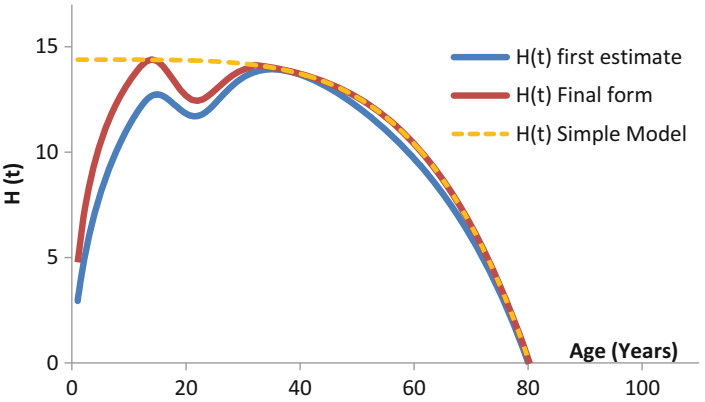

Fig. 3.2 Health state first estimate, simple model approach and final form

health dynamics (THD) of the population. Of particular importance is also the area of the health rectangle (OABC) which includes the health state curve. This rectangle is divided in two rectangular parts the smaller (OAMN) indicating the first part of the human life until reaching the point M at the highest level of health state (usually the maximum is between 30 and 45 years) and the second part (NMBC) characterized by the gradual deterioration of the human organism until the zero level of the health state. This zero point health age C is associated with the maximum death rate. After this point the health state level appears as negative in the graph and characterizes a part of the human life totally unstable with high mortality; this is also indicated by a positively increasing form of the logarithm of the force of mortality $\ln(\mu_x)$. We call the second rectangle NMBC as the deterioration rectangle. Instead the first rectangle OAMN is here called as the development rectangle. For both cases we can

find the relative impact of the area inside each rectangle but outside the health state area to the overall health state. In this chapter we analyze the relative impact of the deterioration area MBCM indicated by dashed lines in the deterioration rectangle. It should be noted that if no-deterioration mechanism was present or the repairing mechanism was perfect the health state should continue following the straight line AMB parallel to the X-axis at the level of the maximum health state. The smaller the deterioration area related to the health state area, the higher the healthy life of the population. This comparison can be done by estimating the related areas and making a simple division. However, when trying to expand the human life further than the limits set by the deterioration mechanisms the percentage of the non-healthy life years becomes higher. This means that we need to divide the total rectangle area by that of the deterioration area to find an estimate for the lost healthy life years. It is clear that if we do not correct the deterioration mechanisms the loss of healthy years will become higher as the expectation of life becomes larger. This is already observed in the estimates of the World Health Organization (WHO) in the World Health Report for 2000 (WHO 2000) where the healthy years lost for females are higher than the corresponding values for males. The females show higher life expectancy than males but also higher values for the healthy life years lost to disability. The proposed loss of healthy life yearsİ indicator is given by:

$$LHLY_1 = \lambda \frac{OABC}{THD_{ideal}} . \frac{THD_{ideal}}{MBCM} = \lambda \frac{OABC}{MBCM} \tag{3.1}$$

Where THD_{ideal} is the ideal total health dynamics of the population and the parameter λ expresses years and should be estimated according to the specific case. For comparing the related results in various countries we can set $\lambda = 1$. When OABC approaches the THD_{ideal} as is the case of several countries in nowadays the loss of healthy life years indicator LHLY can be expressed by other forms. Another point is the use of the (ECD) area in improving forecasts especially when using the 5-year abridged life tables as is the case of the data for all the WHO Countries. In this case the expanded loss of healthy life years indicator LHLY will take the following two forms:

$$LHLY_2 = \lambda \frac{OMCO + ECD}{MBCM} \tag{3.2}$$

$$LHLY_3 = \lambda \frac{OABC + ECD}{MBCM} \tag{3.3}$$

It is clear that the last form will give higher values than the previous one. The following scheme applies: $LHLY_1 < LHLY_2 < LHLY_3$. It remains to explore the forecasting ability of the three forms of the loss of healthy life years indicator by applying LHLY to life tables provided by WHO or by the Human Mortality Database or by other sources.

3.2 Related Theory

Life expectancy and life expectancy at birth are indicators that should be related to the health state of a population. However, a crucial aspect in defining the connection between the health state and life expectancy of a population was the luck of a quantitative definition and measure of the health state of a population. Recently we have succeeded in estimating the health state of a population by using the data sets for the yearly deaths per year of age and the age distribution of the population for the same year. The used data are provided by the human mortality database for 35 countries. The indicator for the health state of a population is based (Skiadas 2012) on the following probability density function $g(t)$

$$g(t) = \frac{k}{\sqrt{t^3}} \exp^{-\frac{(H_t)^2}{2t}} \tag{3.4}$$

Where H_t is the Health State Function and k is a parameter and $g(t)$ is given by using the data sets provided by the data bases. A simple estimate is achieved by an Excel program (SKI-6-Parameters) provided in the website http://www.cmsim.net. The Health State is found by solving the above relation for H_t. The resulting form is:

$$H_t = \left(-2t \ln \frac{g(t)\sqrt{t^3}}{k} \right)^{1/2} \tag{3.5}$$

The parameter k is defined by the relation

$$k = max(g(t)\sqrt{t^3}) \tag{3.6}$$

The total estimate of the health state of the population is given by the relation

$$H_{total} = \int_0^{t(H=0)} H_s ds \approx \sum_0^{t(H=0)} H_s \Delta_s \tag{3.7}$$

Where $t(H = 0)$ is the age of the maximum death rate corresponding to zero health state $H_t = 0$ and $\Delta_t = (t + j - t)$ where j is the interval between age groups. The total health state H_{total} accounts for the surface covered by the horizontal axis X and the positive part of the health state function (see Fig. 3.1). It must be noted that the negative part of the health state function do not produce stochastic paths for the health state as is already known from the theory of the first exit time of a stochastic process. This is clearly illustrated in the next Fig. 3.3 for males in USA (2000).

Other indicators are the age of the maximum health state and the maximum health state. Both estimates are not very precise due to the almost flat shape of the health state curve around the maximum (see Fig. 3.1). Instead the total health state H_{total} accounts for the main time period of the healthy life time expressed by the positive

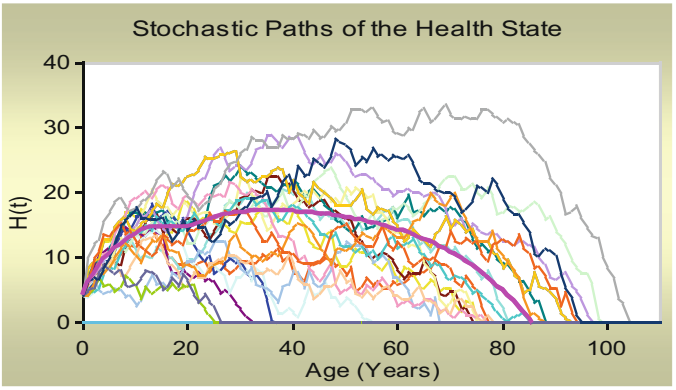

Fig. 3.3 Health state function and stochastic paths

part of the health state function. The ranking of 36 Countries (males and females) selected from the Human Mortality Database according to their scores for the total health state H_{total} (THS) the year 2000 are given in Table 3.1. The related values for the Life Expectancy at Birth ($LEB = ex$) are also included in the same Table along with the estimated LEB after finding the relationship between THS and LEB which follows a linear function for males (see Fig. 3.4) and females (see Fig. 3.5) of the form $LEB = 34.88 + 0.0352 * THS$ and $LEB = 39.67 + 0.0307 * THS$ respectively. As was expected the total health state for females is higher than males in the same country and at the same year.

The next step is to find the Life Expectancy at Birth versus the Total Health State during time for the same country. The cases for males and females in USA from 1933 to 2007 are explored. That it is found is a linear relationship for the first period from 1933 to 1960 for males (slope 0.0785) and another linear part from 1960 to 2007 with lower slope (0.0364). For females the slope is 0.0517 for the first period and 0.0394 for the second period. The findings are in favor of a slower development of the life expectancy at birth related to the growth of the total health state of the population. The argument is further explored by using the data for Sweden (females) from 1751 to 2007. A sigmoid like function is applied indicating that the life expectancy at birth merely tends to a level even if the total health state tends to increase considerably during time (see Fig. 3.6).

3.3 Application to WHO Data

A complete data set for almost all the countries of the World can be found and download from the WHO database in Excel format. For our application here we use the LT1990200020009whs2011 package downloaded from the WHO database and including the Life Tables for all WHO Countries for 1990, 2000 and 2009.

Table 3.1 Total health state and life expectancy at birth for various countries

Males	THS	LEB	LEB estimated	Females	THS	LEB	LEB estimated
Sweden	1220	77.40	77.85	Japan	1451	84.58	84.18
Italy	1207	76.55	77.39	Switzerland	1427	82.62	83.45
Japan	1206	77.70	77.36	Spain	1406	82.72	82.80
Australia	1202	77.12	77.22	France	1405	82.80	82.77
Luxemburg	1201	74.45	74.29	Italy	1397	82.50	82.53
Switzerland	1178	76.95	76.37	Luxemburg	1397	80.74	80.64
Canada	1174	76.64	76.23	Sweden	1391	82.04	82.34
Norway	1172	75.95	76.16	Australia	1385	82.41	82.16
Netherlands	1170	75.53	76.09	Norway	1365	81.37	81.54
Belgium	1165	74.58	75.91	Austria	1363	81.10	81.48
Israel	1163	76.69	75.84	Canada	1361	81.83	81.42
UK	1161	75.43	75.77	Germany	1354	81.01	81.21
New Zealand	1159	76.09	75.70	New Zealand	1350	81.24	81.08
France	1158	75.24	75.67	Finland	1347	81.02	80.99
Germany	1158	74.96	75.67	Belgium	1345	80.91	80.93
Austria	1146	75.10	75.25	Netherlands	1340	80.57	80.78
Finland	1129	74.15	74.65	Israel	1333	81.00	80.56
Spain	1120	75.77	74.33	Portugal	1318	80.30	80.10
Denmark	1119	74.44	74.29	Chile	1311	79.96	79.89
Portugal	1098	73.30	73.55	UK	1298	80.23	79.49
USA	1093	74.19	73.38	Slovenia	1294	79.76	79.37
Taiwan	1083	73.32	73.03	Slovakia	1289	77.33	79.21
Ireland	1081	73.98	72.96	Ireland	1287	79.23	79.15
Czech Republic	1078	71.56	72.85	Denmark	1279	79.12	78.91
Chile	1067	73.89	72.46	USA	1268	79.52	78.57
Slovenia	1039	72.18	71.48	Taiwan	1263	79.22	78.42
Slovakia	1013	69.02	70.56	Czech Republic	1258	78.34	78.26
Bulgaria	998	68.30	70.03	Poland	1236	77.95	77.59
Poland	985	69.57	69.57	Bulgaria	1208	75.01	76.73
Hungary	934	67.53	67.78	Lithuania	1201	77.37	76.51
Lithuania	925	66.74	67.46	Latvia	1200	76.08	76.48
Latvia	860	64.87	65.17	Estonia	1197	76.20	76.06
Estonia	860	65.20	65.08	Hungary	1193	76.08	76.27
Belarus	795	63.31	61.61	Ukraine	1149	73.52	74.92
Ukraine	791	62.08	62.74	Belarus	1138	74.67	74.58
Russia	714	58.99	60.03	Russia	1080	72.25	72.80

We expand the Table (see Tables 3.2, 3.3, and 3.4) to the right by estimating the Loss of Healthy Life Years (LHLY1, LHLY2 and LHLY3) in the columns next to the Life Expectancy column (see Tables 3.2, 3.3, and 3.4). After estimating the LHLY we estimate the Healthy Life Years (HALE1, HALE2 and HALE3) from

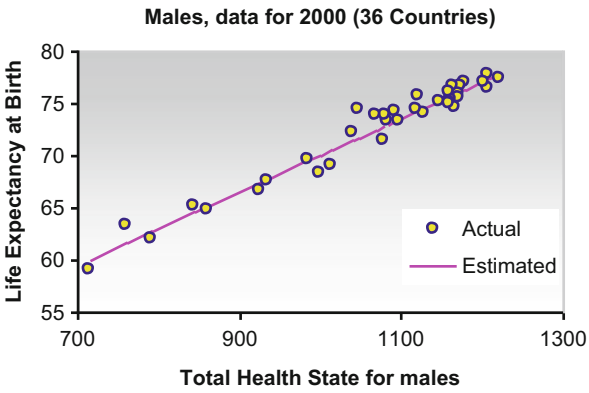

Fig. 3.4 Life expectancy at birth versus total health state

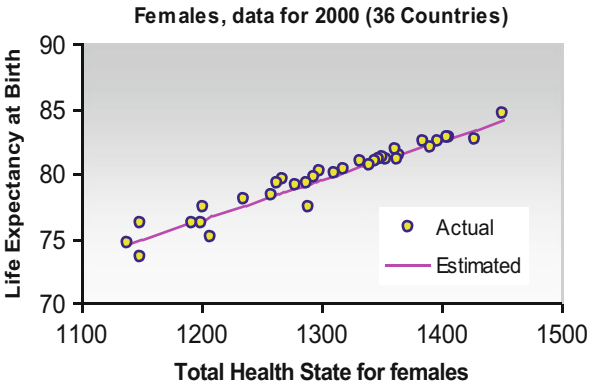

Fig. 3.5 Life expectancy at birth versus total health state

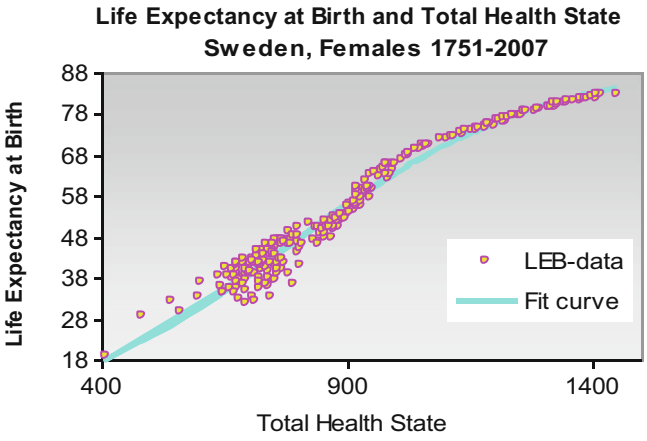

Fig. 3.6 Life expectancy at birth versus total health state in Sweden

Table 3.2 Estimation of the total health state and life expectancy at birth and other parameters

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Iso	Country	Year	Sex	Age x	${}_nM_x$	${}_nq_x$	l_x	${}_nd_x$	${}_nL_x$	T_x	e_x	*LHLY3*	*HALE3*	*LHLY1*	*HALE2*
AFG	Afghanistan	2009	Males	0	0.15973	0.14366	100,000	14,366	89,944	4,656,098	46.6	11.3	35.3	9.7	36.9
AFG	Afghanistan	2009	Males	1	0.01980	0.07561	85,634	6,475	326,995	4,566,154	53.3		40.4		42.2
AFG	Afghanistan	2009	Males	5	0.00438	0.02166	79,159	1,714	391,509	4,239,159	53.6		40.6		42.4
AFG	Afghanistan	2009	Males	10	0.00227	0.01127	77,445	873	385,042	3,847,650	49.7		37.6		39.3
AFG	Afghanistan	2009	Males	15	0.00183	0.00909	76,572	696	381,122	3,462,607	45.2		34.2		35.8
AFG	Afghanistan	2009	Males	20	0.00562	0.02769	75,876	2,101	374,130	3,081,486	40.6		30.8		32.1
AFG	Afghanistan	2009	Males	25	0.00758	0.03720	73,775	2,744	362,017	2,707,356	36.7		27.8		29.0
AFG	Afghanistan	2009	Males	30	0.00924	0.04515	71,031	3,207	347,138	2,345,340	33.0		25.0		26.1
AFG	Afghanistan	2009	Males	35	0.01123	0.05462	67,824	3,704	329,860	1,998,201	29.5		22.3		23.3
AFG	Afghanistan	2009	Males	40	0.01334	0.06457	64,120	4,140	310,249	1,668,342	26.0		19.7		20.6
AFG	Afghanistan	2009	Males	45	0.01635	0.07853	59,980	4,710	288,123	1,358,093	22.6		17.1		17.9
AFG	Afghanistan	2009	Males	50	0.02098	0.09969	55,269	5,510	262,573	1,069,970	19.4		14.7		15.4
AFG	Afghanistan	2009	Males	55	0.02976	0.13850	49,760	6,892	231,569	807,397	16.2		12.3		12.8
AFG	Afghanistan	2009	Males	60	0.03929	0.17886	42,868	7,668	195,171	575,828	13.4		10.1		10.6
AFG	Afghanistan	2009	Males	65	0.05483	0.24110	35,200	8,487	154,785	380,657	10.8		8.2		8.5
AFG	Afghanistan	2009	Males	70	0.08127	0.33774	26,714	9,022	111,012	225,872	8.5		6.4		6.7
AFG	Afghanistan	2009	Males	75	0.11943	0.45985	17,691	8,135	68,118	114,860	6.5		4.9		5.1
AFG	Afghanistan	2009	Males	80	0.17730	0.61423	9,556	5,870	33,106	46,742	4.9		3.7		3.9
AFG	Afghanistan	2009	Males	85	0.25197	0.77295	3,686	2,849	11,308	13,636	3.7		2.8		2.9
AFG	Afghanistan	2009	Males	90	0.34413	0.84661	837	709	2,059	2,328	2.8		2.1		2.2
AFG	Afghanistan	2009	Males	95	0.46466	0.88463	128	114	244	269	2.1		1.6		1.7
AFG	Afghanistan	2009	Males	100	0.60981	1	15	15	24	24	1.6		1.2		1.3

AFG	Afghanistan	2009	Females	0	0.13477	0.12316	100,000	12,316	91,379	5,024,534	50.2	12.3	37.9	10.5	39.7
AFG	Afghanistan	2009	Females	1	0.01955	0.07470	87,684	6,550	335,017	4,933,155	56.3		42.5		44.6
AFG	Afghanistan	2009	Females	5	0.00447	0.02210	81,134	1,793	401,188	4,598,138	56.7		42.8		44.9
AFG	Afghanistan	2009	Females	10	0.00267	0.01327	79,341	1,053	394,072	4,196,950	52.9		39.9		41.9
AFG	Afghanistan	2009	Females	15	0.00385	0.01907	78,288	1,493	387,706	3,802,878	48.6		36.7		38.5
AFG	Afghanistan	2009	Females	20	0.00560	0.02761	76,795	2,120	378,673	3,415,171	44.5		33.6		35.2
AFG	Afghanistan	2009	Females	25	0.00628	0.03093	74,675	2,310	367,598	3,036,498	40.7		30.7		32.2

Table 3.3 Estimation of the total health state and life expectancy at birth and other parameters (expanded to the right)

Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF
LHLY	HALE1		dx/Sum(dx)		dx-normalized		kx (max)				Health state	Max health			
7.2	39.4	0.1437	0.1437	1	0.14366	0.1437	10.79	0.1437	2.939		2.93917799	12.39	2.9392	0	0
	45.1	0.0647	0.0647	4	0.01619	0.0458		0.1831	4.675		4.67455791		4.6746	0	0
	45.4	0.0171	0.0171	5	0.00343	0.0504		0.2519	8.025		8.02537239		8.0254	0	0
	42.1	0.0087	0.0087	5	0.00175	0.0637		0.3185	10.626		10.6263154		10.6263	0	0
	38.3	0.0070	0.0070	5	0.00139	0.0891		0.4454	12.390		12.3899422		12.3899	15	15
	34.4	0.0210	0.0210	5	0.00420	0.4044		2.0219	11.745		11.7451492		11.7451	0	15
	31.1	0.0274	0.0274	5	0.00549	0.7276		3.6378	11.843		11.8427389		11.8427	0	15
	27.9	0.0321	0.0321	5	0.00641	1.1070		5.5352	11.883		11.8826296		11.8826	0	15
	25.0	0.0370	0.0370	5	0.00741	1.6001		8.0006	11.724		11.7238039		11.7238	0	15
	22.0	0.0414	0.0414	5	0.00828	2.1737		10.8686	11.464		11.463674		11.4637	0	15
	19.1	0.0471	0.0471	5	0.00942	2.9389		14.6944	10.941		10.9405346		10.9405	0	15
	16.4	0.0551	0.0551	5	0.01102	4.0136		20.0679	10.046		10.0457475		10.0457	0	15
	13.7	0.0689	0.0689	5	0.01378	5.7763		28.8817	8.369		8.36858793		8.3686	0	15
	11.3	0.0767	0.0767	5	0.01534	7.3064		36.5319	6.901		6.90064826		6.9006	0	15
	9.1	0.0849	0.0849	5	0.01697	9.1011		45.5057	4.746		4.7463878		4.7464	0	15
	7.2	0.0902	0.0902	5	0.01804	10.7948		53.9742	0.00	70	0		0.0000		15
	5.5	0.0813	0.0813	5	0.01627	10.7796		53.8981	0.463		−0.46296406		0.0000		15
	4.1	0.0587	0.0587	5	0.01174	8.5584		42.7919	6.133		−6.13265955		0.0000		15
	3.1	0.0285	0.0285	5	0.00570	4.5443		22.7214	12.199		−12.1989286		0.0000		15
	2.4	0.0071	0.0071	5	0.00142	1.2309		6.1547	19.879		−19.8790326		0.0000		15
	1.8	0.0011	0.0011	5	0.00023	0.2145		1.0723	27.430		−27.4298071		0.0000		15
	1.4	0.0001	0.0001	5	0.00003	0.0305		0.1523	34.437		−34.4366585		0.0000		15

8.6	41.6	0.1232	0.1232	1	0.12316	0.1232	13.57	0.1232	3.067		3.06661523	13.30	3.0666	0	0
	46.7	0.0655	0.0655	4	0.01638	0.0463		0.1853	4.767		4.76658043		4.7666	0	0
	47.0	0.0179	0.0179	5	0.00359	0.0527		0.2635	8.162		8.16152356		8.1615	0	0
	43.9	0.0105	0.0105	5	0.00211	0.0768		0.3842	10.669		10.6689334		10.6689	0	0
	40.3	0.0149	0.0149	5	0.00299	0.1911		0.9555	11.679		11.6793576		11.6794	0	0
	36.9	0.0212	0.0212	5	0.00424	0.4080		2.0402	12.132		12.1316313		12.1316	0	0
	33.7	0.0231	0.0231	5	0.00462	0.6125		3.0625	12.692		12.692379		12.6924	0	0

Table 3.4 Estimation of the total health state and life expectancy at birth and other parameters (expanded to the right)

AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	AQ	AR
							Exp health	Max*Exp		Health state	*Total*
0	0.00	0.00	2.939	2.939	0.00	0.0212	0.0212	32.86	407	2.93918	675.1
0	0.00	0.00	18.698	18.698	0.00	0.0338	0.1014			18.69823	
0	0.00	0.00	40.127	40.127	0.00	0.0580	0.4062			40.12686	
0	0.00	0.00	53.132	53.132	0.00	0.0768	0.9219			53.13158	
15	0.000	61.950	61.950	61.950	0.00	0.0896	1.5228			61.94971	
15	3.224	58.726	58.726	58.726	0.00	0.0849	1.8681			58.72575	
15	2.736	59.214	59.214	59.214	0.00	0.0856	2.3118			59.21369	
15	2.537	59.413	59.413	59.413	0.00	0.0859	2.7491			59.41315	
15	3.331	58.619	58.619	58.619	0.00	0.0848	3.1362			58.61902	
15	4.631	57.318	57.318	57.318	0.00	0.0829	3.4810			57.31837	
15	7.247	54.703	54.703	54.703	0.00	0.0791	3.7176			54.70267	
15	11.721	50.229	50.229	50.229	0.00	0.0726	3.7767			50.22874	
15	20.107	41.843	41.843	41.843	0.00	0.0605	3.4487			41.84294	
15	27.446	34.503	34.503	34.503	0.00	0.0499	3.0932			34.50324	
15	38.218	23.732	23.732	23.732	0.00	0.0343	2.2992			23.73194	
0	0.00	0.00	0.00	0.00	0.00	0.0000	0.0000			0.0000	
0	0.00	0.00	0.00	2.315	2.315	0.0000	0.0000			0.0000	
0	0.00	0.00	0.00	30.663	30.663	0.0000	0.0000			0.0000	
0	0.00	0.00	0.00	60.995	60.995	0.0000	0.0000			0.0000	
0	0.00	0.00	0.00	99.395	99.395	0.0000	0.0000			0.0000	
0	0.00	0.00	0.00	137.149	137.149	0.0000	0.0000			0.0000	
0	0.00	0.00	0.00	172.183	172.183	0.0000	0.0000			0.0000	
0	0.00	0.00	3.067	3.067	0.00	0.0192	0.0192	35.67	474	3.06662	779.8
0	0.00	0.00	19.066	19.066	0.00	0.0299	0.0897			19.06632	
0	0.00	0.00	40.808	40.808	0.00	0.0512	0.3585			40.80762	
0	0.00	0.00	53.345	53.345	0.00	0.0669	0.8033			53.34467	
0	0.00	0.00	58.397	58.397	0.00	0.0733	1.2458			58.39679	
0	0.00	0.00	60.658	60.658	0.00	0.0761	1.6747			60.65816	
0	0.00	0.00	63.462	63.462	0.00	0.0796	2.1503			63.46189	

$HALE = ex - LHLY$, where ex is the Life Expectancy. In the same columns we estimate the HALE for all the age period of the human life as is the case for ex. We use the data ndx form the column I as starting values of the calculations beginning from column S. Other very important indicators are also estimated in the related columns. The values for $k(t)$ are provided in column W and the maximum k_{max} is given in column X. The Health State Function values are given in column AB whereas the maximum Health State is given in column AC. The Expected Healthy Age is given in column AO. The expected healthy age is estimated by

$$Texp = \int_0^{t(H=0)} H_t^* sds \approx \sum_0^{t(H=0)} H_s \Delta_s \tag{3.8}$$

Where H_t^* is the normalized value of the Health State calculated in column AN and $t(H = 0)$ is the age where the Health State is zero. Another characteristic indicator is given in column AP by multiplying the maximum health state with the expected healthy age. The resulting indicator characterizes the health status of the population. Finally a very important indicator is given in column AR. This indicator is measuring the Total Health State of the population and it is expressed by the area (OMCO) in Fig. 3.1. It is estimated by the following formula:

$$H_{total} = \int_0^{t(H=0)} H_s ds \approx \sum_0^{t(H=0)} H_t \Delta_t \tag{3.9}$$

The Total Health State indicator provides another tool to classify the various countries. The rankings estimated are analogues to the life expectancy at birth. However, this indicator can give more precise results. Even more, as is the case when introducing new methods and tools, new frontiers open for analysis and research.

The HALE3 results of the application to the WHO data for 2000 for males are summarized in Table 3.5. In the same Table we have included the HALE (WHO) data provided from the WHO report. The HALE3 where preferred for comparison as are more close to the HALE (WHO) estimates. Figure 3.7 illustrates the country ranking according to HALE3 and Fig. 3.8 the country ranking according to HALE (WHO). In both cases the estimates are close enough and we have checked a 3 year confidence interval quite reasonable if we take into consideration the uncertainty in the estimates presented in the HALE (WHO) report. Grenada and Quatar estimates are far from the confidence intervals while the other countries behavior supports our methodology. Figure 3.9 illustrates the HALE3 and HALE (WHO) when the country ranking is according to the life expectancy at birth ex.

Another very interesting point is to check our HALE estimates to the estimates published in The Lancet by Colin D Mathers, Ritu Sadana, Joshua A Salomon, Christopher JL Murray, Alan D Lopez, 2001 in a paper titled Healthy life expectancy in 191 countries, 1999 (Mathers et al. 2001). Mathers et al. study is based on the 1999 WHO data. For our comparison we have the 2000 data from WHO. However, the one year interval is quite small and we proceed in comparing both results presented in Table 3.6. We keep the notation of the authors for the healthy life expectancy as DALE in the Table 3.6 and in the following Fig. 3.10 where the WHO member countries are ranked according to the life expectancy at birth. Our estimates from HALE3 estimator are in relative accordance to DALE estimated by Mathers et al.

3.4 Conclusions

We have proposed and applied a method to estimate the Health State Function of a Population and various related characteristics and estimators. We have used the related theoretical framework to estimate the Healthy Life Expectancy of the

Table 3.5 Comparing the HALE estimates

Rank	Country males 2000	HALE3	HALE (WHO)	Rank	Country males 2000	HALE3	HALE (WHO)	Rank	Country males 2000	HALE3	HALE (WHO)
1	Japan	69.9	71.2	65	Romania	61.3	59.5	129	Sao Tome and Principe	54.4	50.3
2	Switzerland	69.6	70.4	66	Suriname	58.0	59.5	130	Russian Federation	49.2	50.3
3	Sweden	67.4	70.1	67	Qatar	72.4	59.3	131	Mongolia	46.2	50.3
4	Iceland	69.9	69.8	68	Poland	61.4	59.3	132	Pakistan	50.8	50.2
5	Andorra	68.7	69.8	69	Oman	61.8	59.2	133	Bhutan	48.5	50.1
6	Greece	65.5	69.7	70	Iran (Islamic Republic of)	54.4	59.0	134	Kyrgyzstan	56.3	49.6
7	Australia	69.6	69.6	71	Fiji	56.7	58.7	135	Tajikistan	53.5	49.6
8	Italy	66.5	69.5	72	Mauritius	60.0	58.6	136	Yemen	49.5	48.9
9	New Zealand	66.1	69.5	73	Colombia	59.6	58.6	137	Myanmar	48.4	47.7
10	Monaco	68.0	69.4	74	Sri Lanka	55.7	58.6	138	Nepal	50.4	47.5
11	Israel	69.5	69.3	75	Libyan Arab Jam	61.4	58.4	139	Gambia	45.4	47.3
12	Denmark	66.6	68.9	76	Algeria	59.4	58.4	140	Gabon	45.8	46.8
13	Norway	64.9	68.8	77	Ecuador	58.2	58.4	141	Papua New Guinea	50.2	46.6
14	Malta	68.3	68.7	78	Saudi Arabia	61.5	58.3	142	Ghana	45.4	46.5
15	Spain	65.7	68.7	79	Viet Nam	57.7	58.2	143	Comoros	46.2	46.2
16	France	68.5	68.5	80	Jordan	57.6	58.2	144	Sudan	47.2	45.7
17	Canada	69.8	68.3	a1	Samoa	55.0	58.2	145	Cambodia	45.4	45.6
18	United Kingdom	66.5	68.3	82	Solomon Islands	57.1	58.0	146	Senegal	47.3	45.2
19	Netherlands	66.6	68.2	83	Belize	56.9	58.0	147	Equatorial Guinea	41.9	44.9
20	Austria	66.5	68.1	84	Peru	58.9	57.8	148	Lao People's Democratic Republic	48.4	43.7
21	Ireland	65.1	67.8	85	Thailand	53.5	57.7	149	Madagascar	47.6	43.2
22	Belgium	65.1	67.7	86	Saint Kitts and Nevis	59.7	57.6	150	Benin	42.7	43.1
23	Luxembourg	66.0	67.6	87	Bahamas	54.6	57.2	151	South Africa	44.1	43.0

24	Germany	66.5	67.4	88	Egypt	58.0	57.1	152	Togo	42.5	42.7
25	Singapore	66.6	66.8	89	Philippines	59.4	57.0	153	Congo	42.6	42.5
26	Cyprus	62.6	66.4	90	Seychelles	58.7	57.0	154	Mauritania	46.3	42.1
27	Finland	64.5	66.1	91	Armenia	60.2	56.9	155	Nigeria	39.4	42.1
28	United States America	64.9	65.7	92	Cape Verde	58.0	56.9	156	Eritrea	46.2	41.4
29	Cuba	67.4	65.1	93	Turkey	60.0	56.8	157	Haiti	43.4	41.3
30	Kuwait	64.6	64.6	94	Indonesia	55.4	56.5	158	Kenya	43.0	41.2
31	Slovenia	64.7	64.5	95	Albania	55.0	56.5	159	Cameroon	41.0	40.9
32	Costa Rica	68.4	64.2	96	Palau	52.7	56.5	160	Guinea	39.5	40.4
33	Portugal	62.1	63.9	97	Tuvalu	54.7	56.4	161	Zimbabwe	38.2	39.6
34	The Former Yugoslavia	57.3	63.9	98	Estonia	59.7	56.2	162	Côte d'Ivoire	40.2	39.1
35	Brunei Darussalam	68.6	63.8	99	Georgia	59.8	56.1	163	Swaziland	39.1	38.8
36	Chile	63.7	63.5	100	Vanuatu	57.8	56.0	164	United Republic	41.5	38.6
37	Republic of Korea	64.3	63.2	101	Nicaragua	61.2	55.8	165	Chad	39.7	38.6
38	Dominica	60.8	63.2	102	Micronesia (Federated States of)	55.6	55.8	166	Liberia	40.5	38.2
39	Mexico	63.3	63.1	103	Honduras	52.4	55.8	167	Botswana	41.3	38.1
40	Bahrain	59.4	63.0	104	Republic of Moldova	55.8	55.4	16a	Guinea-Bissau	34.1	36.7
41	Czech Republic	64.7	62.9	105	Belarus	55.7	55.4	169	Namibia	41.5	36.5
42	Jamaica	61.4	62.9	106	Hungary	60.4	55.3	170	Uganda	36.7	36.2
43	Panama	65.4	62.6	107	Morocco	59.7	55.3	171	Angola	34.3	36.2
44	United Arab Emirates	66.1	62.3	108	El Salvador	55.3	55.3	172	Lesotho	38.4	36.1
45	Barbados	59.4	62.3	109	Brazil	59.5	54.9	173	Ethiopia	36.3	35.7
46	Bosnia and Herzegovina	62.7	62.1	110	Dem. People's Republic of Korea	54.1	54.9	174	Djibouti	44.9	35.6
47	Grenada	38.3	62.1	111	Marshall Islands	49.9	54.8	175	Somalia	40.0	35.5
48	Argentina	63.4	61.8	112	Dominican Republic	62.5	54.7	176	Burkina Faso	40.3	35.4

(continued)

Table 3.5 (continued)

Rank	Country males 2000	HALE3	HALE (WHO)	Rank	Country males 2000	HALE3	HALE (WHO)	Rank	Country males 2000	HALE3	HALE (WHO)
49	Uruguay	63.7	61.7	113	Maldives	59.1	54.2	177	Afghanistan	33.8	35.1
50	Antigua and Barbuda	59.7	61.7	114	Lithuania	60.1	53.6	178	Mali	40.8	34.8
51	Tunisia	61.2	61.0	115	Guatemala	49.3	53.5	179	Central African Republic	39.0	34.7
52	Bulgaria	59.4	61.0	116	Azerbaijan	51.6	53.3	180	Dem. Rep. of the Congo	34.9	34.4
53	China	61.9	60.9	117	Kiribati	48.5	52.8	181	Niger	41.9	33.9
54	Croatia	56.3	60.8	11a	Uzbekistan	57.2	52.7	182	Burundi	38.5	33.9
55	Saint Lucia	64.3	60.7	119	Iraq	54.5	52.6	183	Zambia	34.7	33.7
56	Venezuela	62.8	60.4	120	Ukraine	52.3	52.3	184	Rwanda	38.2	32.0
57	Cook Islands	58.9	60.4	121	India	50.a	52.2	185	Mozambique	38.7	31.5
58	Lebanon	60.5	60.3	122	Latvia	57.4	51.4	186	Malawi	35.0	31.4
59	Trinidad and Tobago	54.5	60.3	123	Guyana	55.1	51.4	187	Sierra Leone	29.7	29.7
60	Paraguay	62.0	59.9	124	Bolivia	51.2	51.4				
61	Saint Vincent and the Grenadines	58.2	59.7	125	Turkmenistan	48.1	51.2				
62	Malaysia	58.1	59.7	126	Bangladesh	51.2	50.6				
63	Syrian Arab Republic	61.4	59.6	127	Kazakhstan	48.1	50.5				
64	Slovakia	61.2	59.6	128	Nauru	43.9	50.4				

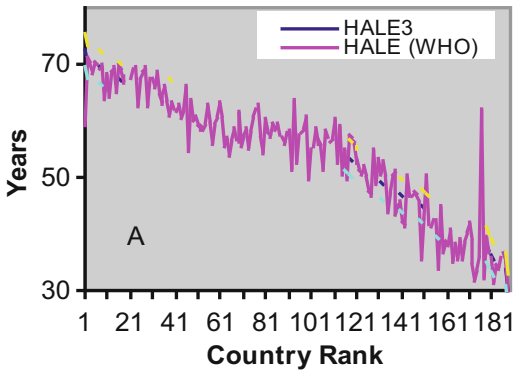

Fig. 3.7 Application for WHO data, males 2000. Ranking according to HALE3

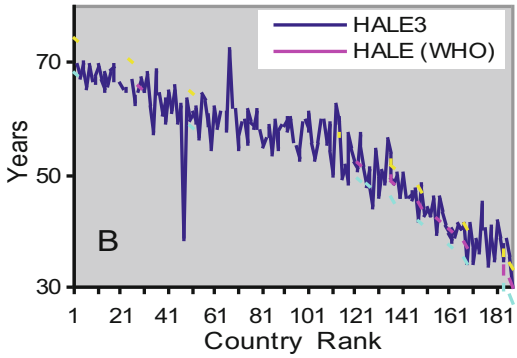

Fig. 3.8 Application for WHO data, males 2000. Ranking according to HALE (WHO)

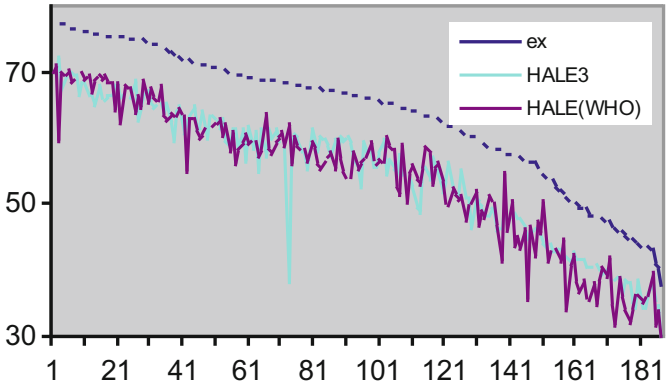

Fig. 3.9 WHO data, males 2000, ranking according to ex

population and we have done applications by using the life table data for the member countries of the World Health Organization by comparing our results with the provided from WHO. The findings indicate that the proposed methodology can be a useful tool for estimating the loss of healthy life yearsİ and then to find the Healthy

Table 3.6 Comparing our HALE 2000 estimates with DALE 1999

Rank	Country	HALE3 2000	DALE 999	Rank	Country	HALE3 2000	DALE 1999	Rank	Country	HALE3 2000	DALE 1999
1	Japan	71.9	74.5	65	Cook Islands	60.8	63.4	129	Sao Tome and Principe	58.2	53.5
2	Australia	70.8	73.2	66	Kuwait	68.4	63.2	130	Bolivia	52.2	53.3
3	France	70.4	73.1	67	Estonia	62.5	63.1	131	India	50.9	53.2
4	Sweden	71.4	73.0	68	Paraguay	66.0	63.0	132	Vanuatu	60.9	52.8
5	Spain	69.7	72.8	69	Oman	62.5	63.0	133	Nauru	51.3	52.5
6	Italy	70.6	72.7	70	Ukraine	59.5	63.0	134	Bhutan	49.0	51.8
7	Switzerland	70.5	72.5	71	Colombia	63.8	62.9	135	Myanmar	50.1	51.6
8	Greece	69.6	72.5	72	Turkey	61.0	62.9	136	Bangladesh	50.4	49.9
9	Monaco	69.9	72.4	73	Tonga	59.2	62.9	137	Yemen	49.6	49.7
10	San Marino	73.1	72.3	74	Sri Lanka	59.1	62.8	138	Nepal	50.6	49.5
11	Andorra	69.6	72.3	75	Mauritius	64.5	62.7	139	Gambia	45.4	48.3
12	Canada	71.3	72.0	76	Suriname	59.5	62.7	140	Gabon	46.5	47.8
13	Netherlands	70.7	72.0	77	Dominican Republic	63.7	62.5	141	Papua New Guinea	50.4	47.0
14	United Kingdom	70.8	71.7	78	Romania	63.0	62.3	142	Comoros	47.2	46.8
15	Norway	70.2	71.7	79	China	62.2	62.3	143	Lao People's Democratic Republic	48.5	46.1
16	Austria	70.4	71.6	80	Latvia	61.6	62.2	144	Cambodia	46.2	45.7
17	Belgium	69.7	71.6	81	Belarus	60.7	61.7	145	Ghana	45.6	45.5
18	Luxembourg	70.6	71.1	82	Niue	64.4	61.6	146	Congo	41.6	45.1
19	Iceland	71.3	70.8	83	Algeria	60.3	61.6	147	Senega.	48.0	44.6

20	Malta	71.7	70.5	84	Saint Kitts and Nevis	60.0	61.6	148	Equatorial Guinea	40.9	44.1
21	Finland	69.1	70.5	85	El Salvador	62.6	61.5	149	Haiti	42.1	43.8
22	Germany	70.6	70.4	86	Republic of Moldova	58.1	61.5	150	Sudan	46.4	43.0
23	Israel	70.5	70.4	87	Tunisia	62.3	61.4	151	Côte d'Ivoire	38.9	42.8
24	United States of America	69.3	70.0	88	Malaysia	59.9	61.4	152	Benin	42.6	42.2
25	Dominica	65.3	69.8	89	Russian Federation	57.2	61.3	153	Cameroon	37.9	42.2
26	Cyprus	63.6	69.8	90	Honduras	57.8	61.1	154	Mauritania	46.3	41.4
27	Ireland	66.6	69.6	91	Ecuador	63.8	61.0	155	Togo	43.1	40.7
28	Denmark	70.8	69.4	92	Belize	62.7	60.9	156	South Africa	45.7	39.8
29	Singapore	71.4	69.3	93	Lebanon	62.0	60.6	157	Kenya	42.4	39.3
30	Portugal	68.1	69.3	94	Iran (Islamic Republic of)	59.3	60.5	158	Nigeria	35.9	38.3
31	New Zealand	70.7	69.2	95	Uzbekistan	59.0	60.2	159	Swaziland	40.1	38.1
32	Chile	69.0	68.6	96	Guyana	58.6	60.2	160	Angola	34.7	38.0
33	Slovenia	69.2	68.4	97	Thailand	57.0	60.2	161	Djibouti	45.2	37.9
34	Czech Republic	66.7	68.0	98	Jordan	61.5	60.0	162	Guinea	37.6	37.8
35	Jamaica	64.3	67.3	99	Albania	56.4	60.0	163	Eritrea	47.8	37.7
36	Uruguay	68.2	67.0	100	Indonesia	59.4	59.7	164	Afghanistan	34.7	37.7
37	Croatia	59.8	67.0	101	Micronesia (Federated States of)	59.0	59.6	165	Guinea-Bissau	35.0	37.2
38	Costa Rica	69.5	66.7	102	Fiji	60.8	59.4	166	Lesotho	37.6	36.9
39	Argentina	67.7	66.7	103	Peru	60.4	59.4	167	Madagascar	47.9	36.6

(continued)

Table 3.6 (continued)

Rank	Country	HALE3 2000	DALE 999	Rank	Country	HALE3 2000	DALE 1999	Rank	Country	HALE3 2000	DALE 1999
40	Armenia	61.6	66.7	104	Libyan Arab Jam	62.4	59.3	168	Somalia	38.8	36.4
41	Slovakia	65.4	66.6	105	Seychelles	61.5	59.3	169	United Republic of Tanzania	40.0	36.0
42	Saint Vincent and the Grenadines	59.8	66.4	106	Bahamas	61.9	59.1	170	Central African Republic	35.9	36.0
43	Georgia	61.7	66.3	107	Brazil	61.3	59.1	171	Namibia	43.6	35.6
44	Poland	65.5	66.2	108	Morocco	60.8	59.1	172	Burkina Faso	38.3	35.5
45	Panama	66.5	66.0	109	Palau	63.2	59.0	173	Burundi	36.8	34.6
46	Antigua and Barbuda	63.1	65.8	110	Philippines	59.7	58.9	174	Mozambique	35.7	34.4
47	Grenada	47.7	65.5	111	Syrian Arab Rep	62.5	58.8	175	Liberia	37.5	34.0
48	United Arab Em	66.7	65.4	112	Egypt	61.3	58.5	176	Ethiopia	38.3	33.5
49	Mexico	67.5	65.0	113	VietNam	61.5	58.2	177	Mali	37.9	33.1
50	Saint Lucia	67.2	65.0	114	Nicaragua	64.1	58.1	178	Zimbabwe	39.3	32.9
51	Republic of Korea	65.8	65.0	115	Cape Verde	59.4	57.6	179	Rwanda	37.2	32.8
52	Venezuela	64.7	65.0	116	Tuvalu	53.8	57.4	180	Uganda	38.7	32.7
53	Barbados	62.1	65.0	117	Tajikistan	56.1	57.3	181	Botswana	42.1	32.3
54	Bosnia and He	67.1	64.9	118	Marshall Islands	47.7	56.8	182	Zambia	36.0	30.3
55	Trinidad and T	61.3	64.6	119	Kazakhstan	54.8	56.4	183	Malawi	36.0	29.4

56	Saudi Arabia	62.5	64.5	120	Kyrgyzstan	57.4	56.3	184	Niger	39.8	29.1
57	Brunei Darussalam	69.7	64.4	121	Pakistan	50.3	55.9	185	Sierra Leone	31.8	25.9
58	Bulgaria	63.0	64.4	122	Iraq	59.2	55.3				
59	Bahrain	60.3	64.4	123	Kiribati	50.1	55.3				
60	Lithuania	65.0	64.1	124	Solomon Islands	60.5	54.9				
61	Hungary	64.8	64.1	125	Turkmenistan	53.7	54.3				
62	The Former Yugoslavia	62.7	63.7	126	Guatemala	52.8	54.3				
63	Azerbaijan	55.8	63.7	127	Maldives	58.9	53.9				
64	Qatar	72.2	63.5	128	Mongolia	50.3	53.8				

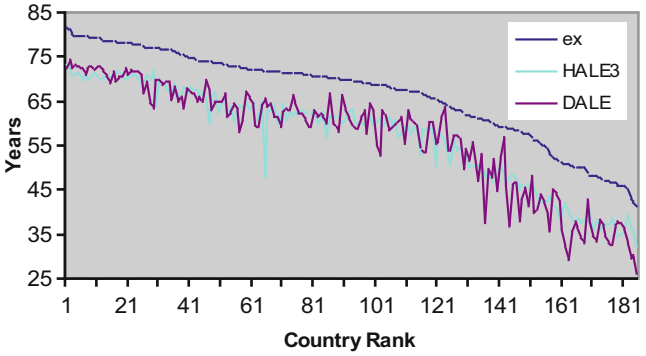

Fig. 3.10 DALE 1999, HALE3 2000 and ex 2000

Life Expectancy of a population. Even more, by these applications, we straighten the proposed theory for the health state of a population. We do believe that interesting applications is various fields may arise.

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Chapter 4
The Health-Mortality Approach in Estimating the Healthy Life Years Lost Compared to the Global Burden of Disease Studies and Applications in World, USA and Japan

4.1 Introduction

Starting from the late 1980s a Global Burden of Disease (GBD) study was applied in many countries reflecting the optimistic views of many researchers and policy makers worldwide to quantify the health state of a population or a group of persons. In the time course they succeeded in establishing an international network collecting and providing adequate information to calculate health measures under terms as Loss of Healthy Life Years (LHLY) or Healthy Life Expectancy (HALE). The latter tends to be a serious measure important for the policy makers and national and international health programs. So far the process followed was towards statistical measures including surveys and data collection using questionnaires and disability and epidemiological data as well (McDowell 2006). They faced many views referring to the definition of health and to the inability to count the various health states and of course the different cultural and societal aspects of the estimation of health by various persons worldwide. Further to any objections posed when trying to quantify health, the scientific community had simply to express with strong and reliable measures that millions of people for centuries and thousands of years expressed and continue to repeat every day: That their health is good, fair, bad or very bad. As for many decades the public opinion is seriously quantified by using well established statistical and poll techniques it is not surprising that a part of these achievements helped to improve, establish and disseminate the health state measures. However, a serious scientific part is missing or it is not very much explored that is to find the model underlying the health state measures. Observing the health state measures by country from 1990 until nowadays it is clear that the observed and estimated health parameters follow a rather systematic way. If so why not to find the process underlying these measures? It will support the provided health measures with enough documentation while new horizons will open towards better estimates and data validation. From the early 1990s we have introduced and applied

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methods, models and techniques to estimate the health state of a population. The related results appear in several publications and we have already observed that our estimates are related or closely related to the provided by the World Health Organization (WHO) and other agencies as Eurostat or experts as the REVES group. However, our method based on a difficult stochastic analysis technique, is not easy to use especially by practitioners. The last four centuries demography and demographers are based on the classical Life Tables. Thus here we propose a very simple model based on the mortality μ_x of a population provided in a classical life table. To compare our results with those provided by WHO we use the μ_x included in the WHO abridged life tables. Our estimates are compared with the HALE estimates for all the WHO countries. Even more we provide the related simple program in Excel which provides immediately the Life Expectancy, the Loss of Healthy Life Years and the Healthy Life Expectancy estimate. The comparisons suggest an improved WHO estimate for the majority of the countries. There are countries results differing from the model and need further study.

4.1.1 Further Details

The Global Burden of Disease Study explored the health status of the population of all the countries members of the World Health Organization (WHO). It is a large team work started more than 25 years ago (see Murray and Lopez 1997, 2000; Mathers et al. 2000; Salomon et al. 2012; Murray et al. 2015; Hausman 2012; Vos et al. 2010; Robine et al. 1999; WHO 2001, 2002, 2004, 2013, 2014 and many other publications). The last years, with the financial support of the Bill and Melinda Gates foundation, the work was expanded via a large international group of researchers. The accuracy of the data collection methods was improved along with the data development and application techniques. So far the health status indicators were developed and gradually were established under terms as healthy life expectancy and loss of healthy life years. Methods and techniques developed during the seventies and eighties as the Sullivan method (Sullivan 1971) were used quite successfully. Several publications are done with the most important included in The Lancet under the terms DALE and HALE whereas a considerable number can be found in the WHO and World Bank publications. The same half part of a century several works appear in the European Union exploring the same phenomenon and providing more insight to the estimation of the health state of a population and providing tools for the estimation of severe, moderate and light disability. The use of these estimates from the health systems and the governments is obvious. To a surprise the development of the theoretical tools was not so large. The main direction was towards to surveys and collection of mass health state data instead of developing and using theoretical tools. The lessons learned during the last centuries were towards the introduction of models in the analysis of health and mortality. The classical examples are Edmund Halley for Life Tables and Benjamin Gompertz for the law of mortality and many others. Today our ability to use mass storage tools as

the computers and the extensive application of surveys and polls to many political, social and economic activities directed the main health state studies. In other words we give much attention to opinions of the people for their health status followed by extensive health data collection. However, it remains a serious question: can we validate the health status results? As it is the standard procedure in science a systematic study as the Global Burden of Disease should be validated by one or more models. Especially as these studies are today the main tool for the health programs of many countries the need of verification is more important. People reply according to their experience. Two main approaches arise: The mortality focus approach and the health status approach. Although both look similar responds may have significant differences. The main reason is that health is a rather optimistic word opposed to the pessimistic mortality term. Twenty years ago we provided a model to express the health state of a population. We developed and expanded this model leading to a system providing health status indexes. Here we propose several methodologies to estimate the health indexes and to compare with the provided by WHO.

4.2 The Mortality Approach

4.2.1 The Simplest Model

We need a simple model to express the health status. The best achievement should be to propose a model in which the health measure should be presented by only one main parameter. We thus propose a two parameter model with one crucial health parameter:

$$\mu_x = (\frac{x}{T})^b, \tag{4.1}$$

The parameter T represents the age at which $\mu_x = 1$ and b is a crucial health state parameter expressing the curvature of μ_x. As the health state is improved b gets higher values. This simple model is illustrated in Fig. 4.1 where μ_x is plotted against the age x. The straight line (OA) expresses the simplest case with $b = 1$. As b is taking higher values the form of the graph for μ_x follows an exponential path of the curve (OCA). For high values of b the curve approaches asymptotically the path (OBA)

The main task is to find the area E_x under the curve OCABO in the mortality diagram (see Fig. 4.1) which is a measure of the mortality effect. This is done by estimating the integral

$$E_x = \int_0^T (\frac{x}{T})^b dx = \frac{T}{b+1}(\frac{x}{T})^b, \tag{4.2}$$

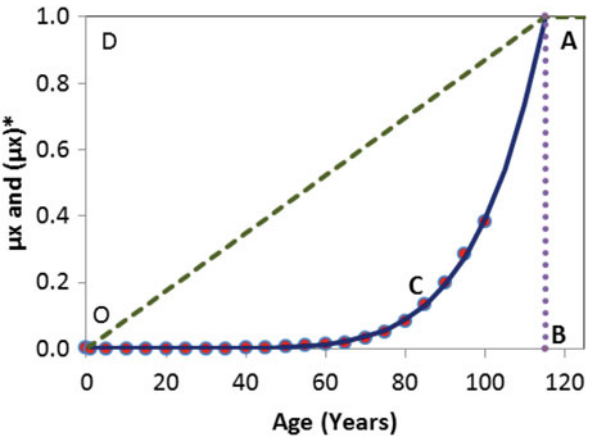

Fig. 4.1 The mortality diagram

The resulting value for E_x in the interval $[0, T]$ is given by the simple form:

$$E_{mortality} = \frac{T}{b+1} \tag{4.3}$$

It is clear that the total information for the mortality is the area provided under the curve μ_x and the horizontal axis. The total area E_{total} of the healthy and mortality part of the life span is nothing else but the area included into the rectangle of length T and height 1 that is $E_{total} = T$. The health area is given by

$$E_{health} = T - E_{mortality} = T - \frac{T}{b+1} = \frac{bT}{b+1}, \tag{4.4}$$

Then a very simple relation arises for the fraction $E_{health}/E_{mortality}$ that is

$$\frac{E_{health}}{E_{mortality}} = b \tag{4.5}$$

This is the simplest indicator for the loss of health status of a population. As we have estimated by another method it is more close to the severe disability causes indicator. The relation $E_{total}/E_{mortality}$ provides another interesting indicator of the form:

$$\frac{E_{total}}{E_{mortality}} = b + 1, \tag{4.6}$$

This indicator is more appropriate for the severe and moderate disability causes indicator (It is compatible with our estimates using the health state approach). It provides larger values for the disability measures as the E_{total} is larger or the $E_{mortality}$ area is smaller by means that as we live longer the disability period becomes larger. This method suggests a simple but yet interesting tool for classification of various

countries and populations, for the loss of healthy life years. A correction multiplier λ should be added for specific situations so that the estimator of the loss of healthy life years should be of the form:

$$LHLY = \lambda \frac{E_{total}}{E_{mortality}} = \lambda(b+1)$$

$$LHLY = \lambda \frac{E_{total}}{E_{mortality}} = b+1$$

However, for comparisons between countries it is sufficient to select $\lambda = 1$. Even more the selection of $\lambda = 1$ is appropriate when we would like to develop a quantitative measure for the LHLY without introducing the public opinion for the health status and the estimates for the cause of diseases and other disability measures. From another point of view the influence of the health status of the society to the public opinions related to health may cause differences in the values for LHLY estimated with the HALE method thus a value for λ larger or smaller than unity is needed. By means that we will have to measure not exactly the health status but the public opinion related to the health status, the latter leading in a variety of health estimates in connection to socioeconomic and political situation along with crucial health information from the mass media. Both measures, the standard measure with $\lambda = 1$ and the flexible one with λ different from 1 could be useful for decision makes and health policy administrators and governmental planners. To our great surprise our model by selecting $\lambda = 1$ provided results very close to those provided by WHO as it is presented in the following Tables and in other applications. It is clear that we have found an interesting estimator for the loss of healthy life years. Our idea to find the loss of healthy life years as a fraction of surfaces in a mortality diagram was proven to be quite important for expressing the health state measures. A more detailed method based on the health state stochastic theory is presented in the book on The Health State Function of a Population and related publications (see Skiadas and Skiadas 2010b, 2012a, 2015) where more health estimators are found.

4.2.1.1 Application Details

As our method needs life table data we prefer to use full life tables when available. The Human Mortality Database is preferred for a number of countries providing full life tables. However, only a small part of the world countries are included and thus we also use the abridged life tables provided by the World Health Organization. The new abridged life tables from WHO including data from 0 to 100 years provide good results when applying our method. Instead the previous life tables (0 to 85 years) are not easily applied. It could be possible to use these life tables by expanding from 85 to 100 years. For both the abridged and the full life table data we have developed the appropriate models and estimation programs in Excel thus make it easy to use.

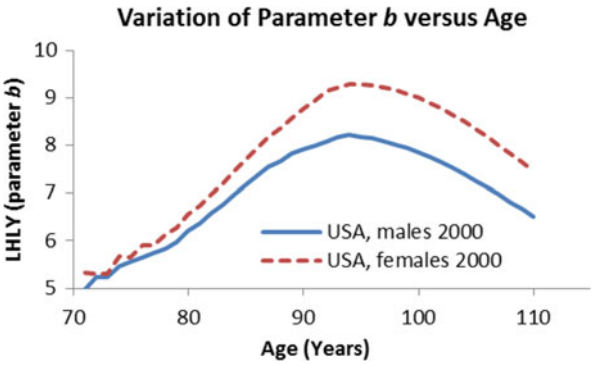

Fig. 4.2 Development of the health parameter

4.2.1.2 Stability of the Coefficients of the Simple Model

Here we discuss some important issues regarding the application of the simple model proposed by Eq. (4.1). To apply this model to data we use a non-linear regression analysis technique by using a Levenberg-Marquardt algorithm. The data are obtained from the WHO database providing abridged life tables of the 0–100 years form. The important part of the model is the parameter b expressing the loss of healthy life years. Even more b can express the curvature of mortality function μ_x. Applying the model to data we need a procedure for the selection of the most appropriate value for b.

4.2.1.3 When *b* Should Be Accepted

The simpler is to find if b follows a systematic change versus age. We start by selecting all the n data points $(m_0, m_1, \ldots, m_n)$ for μ_x to find b and then we select $n-1, n-2, \ldots, n-m$ for a sufficient number of $m < n$. As is presented in Fig. 4.2 the parameter b follows a systematic change. The example is for USA males and females the year 2000 and the data are from the full life tables of the Human Mortality Database. As it is expected b is larger for females than for males. In both cases a distinct maximum value in a specific year of age appears. Accordingly a specific minimum appears for the other not so important parameter T (see Fig. 4.3). It is clear that only the specific maximum value for b should be selected. Even more the estimates for the maximum b account for a local minimum for the first difference dx' of dx provided from the life table. Next Fig. 4.4 illustrates this case for USA males the year 2000 along with a fit curve from our model SK-6. The maximum b is at 94 years for males and females the same as for the minimum of the first difference corresponding to the right inflection point of the death curve dx. Table 4.1 includes the parameter estimates for b and T the year 2000 for USA males and females.

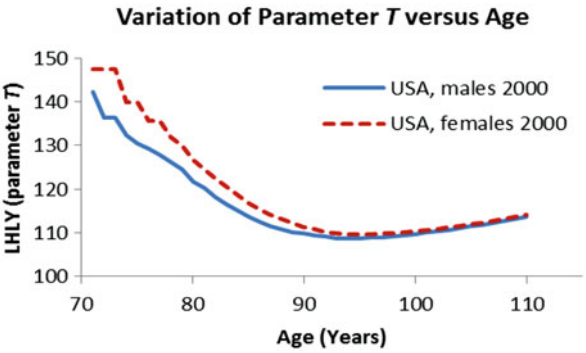

Fig. 4.3 Development of T parameter

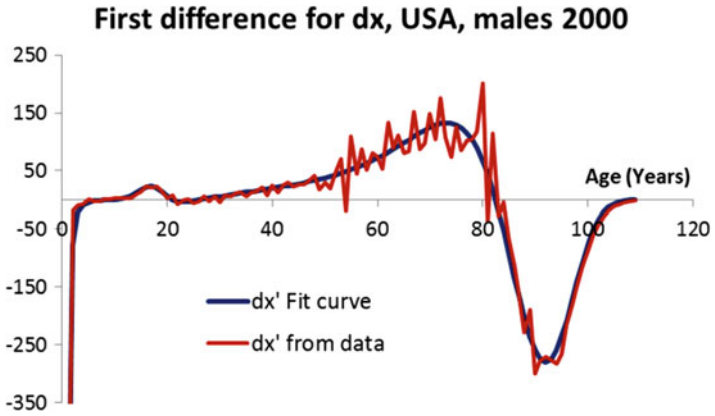

Fig. 4.4 First difference (derivative) of *dx* versus age

4.2.2 Estimation Without a Model (Direct Estimation)

As the needed data sets in the form of m_x or q_x data are provided from the life tables we have developed a method of direct estimation of the loss of healthy life year estimators directly from the life table by expanding the life table to the right.

$$b + 1 = \frac{E_{total}}{E_{mortality}} = \frac{xm_x}{\sum_0^x m_x}, \tag{4.7}$$

$$b = \frac{E_{health}}{E_{mortality}} = \frac{xm_x - \sum_0^x m_x}{\sum_0^x m_x} = \frac{xm_x}{\sum_0^x m_x} - 1, \tag{4.8}$$

The only needed is to estimate the above fractions from the life table data. A similar indicator results by selecting the q_x data from the life table and using the:

Table 4.1 Parameter estimates for the model (USA, 2000)

Age	Females		Males		Age	Females		Males	
Years	b	T	b	T	Years	b	T	b	T
71	5.318	147.5	4.975	142.3	91	8.942	110.7	7.992	109.4
72	5.308	147.5	5.244	136.4	92	9.143	110.0	8.081	109.1
73	5.296	147.5	5.231	136.4	93	9.224	109.8	8.173	108.8
74	5.663	140.0	5.459	132.3	94	9.291	109.6	8.218	108.6
75	5.649	140.0	5.559	130.5	95	9.286	109.6	8.189	108.7
76	5.905	135.6	5.642	129.2	96	9.263	109.6	8.148	108.8
77	5.896	135.6	5.736	127.8	97	9.224	109.7	8.094	109.0
78	6.146	131.9	5.844	126.3	98	9.167	109.9	8.027	109.2
79	6.280	130.1	5.981	124.5	99	9.093	110.1	7.947	109.4
80	6.551	126.8	6.214	121.8	100	9.002	110.3	7.856	109.7
81	6.748	124.6	6.368	120.2	101	8.896	110.6	7.754	110.0
82	6.972	122.5	6.587	118.2	102	8.775	110.8	7.642	110.3
83	7.209	120.4	6.774	116.6	103	8.641	111.2	7.521	110.7
84	7.453	118.5	6.981	115.0	104	8.495	111.5	7.391	111.0
85	7.710	116.8	7.186	113.6	105	8.339	111.9	7.255	111.4
86	7.947	115.3	7.378	112.5	106	8.173	112.3	7.114	111.8
87	8.185	114.0	7.546	111.5	107	8.000	112.7	6.967	112.3
88	8.369	113.1	7.665	110.9	108	7.822	113.1	6.818	112.7
89	8.579	112.2	7.826	110.1	109	7.638	113.5	6.666	113.2
90	8.778	111.3	7.916	109.8	110	7.452	114.0	6.512	113.6

$$b + 1 = \frac{E_{total}}{E_{mortality}} = \frac{xq_x}{\sum_0^x q_x}, \tag{4.9}$$

$$b = \frac{E_{health}}{E_{mortality}} = \frac{xq_x - \sum_0^x q_x}{\sum_0^x q_x} = \frac{xq_x}{\sum_0^x q_x} - 1, \tag{4.10}$$

In both cases the results are similar as it is presented in the following Figs. 4.5 and 4.6. The estimates from m_x are slightly larger than from q_x. In both cases the b estimators growth to a maximum at old ages and then decline. The selected b or $b + 1$ indicator for the life years lost from birth are those of the maximum value. A smoothing technique averaging over 5 years estimators is used to avoid sharp fluctuations in the maximum range area for the direct method. For the Model method a simple 3 point averaging gives good results. The maximum HLYL for the direct estimation is 9.84 for m_x and 9.26 for q_x. For the Model estimation with m_x data the related HLYL is 10.0. As we have estimated for other cases both the estimation of the b indicator by this direct method and the method by using a model give similar results.

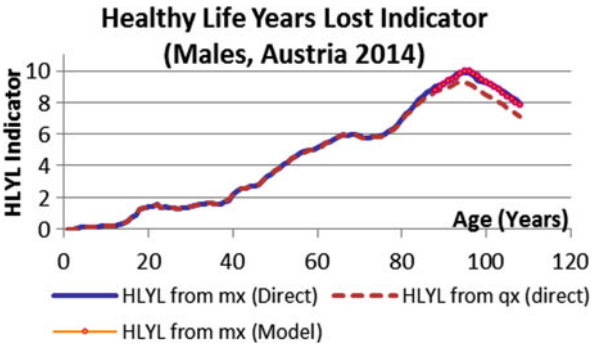

Fig. 4.5 Estimation of the HLYL indicator (b) by the direct method and by the simple model (Full results)

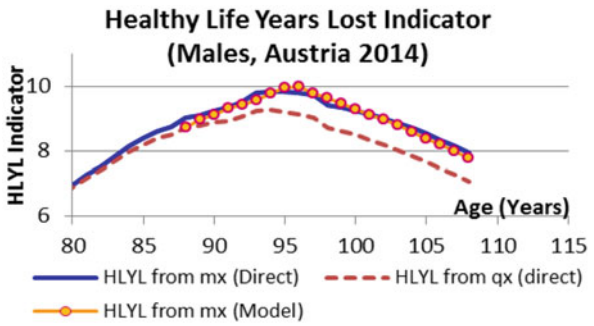

Fig. 4.6 Estimation of the HLYL indicator (b) by the direct method and by the simple model (expanded around the maximum)

4.2.3 More Details: The Gompertz and the Weibull Distributions

It should be noted that a more convenient (Gompertz 1825) model form is provided by Carriere (1992) in the form $\mu_x = Bc^x$, where B and c are parameters. This is close to our simple model selected. However, we have also selected and applied the following form for the probability density function of the Gompertz model:

$$f_x = e^{-k+bx-e^{-l+bx}}, \tag{4.11}$$

The characteristic parameter expressing the loss of healthy life years is the parameter l. this is also demonstrated by observing the cumulative distribution function of the form:

$$F_x = e^{-e^{-l+bx}}, \tag{4.12}$$

The related survival function is

$$S_x = 1 - e^{-e^{-l+bx}}, \tag{4.13}$$

The probability density function is:

$$f_x = be^{-l+bx}e^{-e^{-l+bx}}, \tag{4.14}$$

And the hazard function is

$$h(x) = \frac{f_x}{F_x} = be^{-l+bx} = e^{\ln b - l + bx} = e^{-k+bx}, \tag{4.15}$$

Thus explaining the above Gompertz form selected ($k = l - \ln(b)$). The selected value for the estimation of the healthy life years lost is provided by the parameter l. In the same paper Carriere suggests the use the Weibull model. This model has density function (b and T are parameters):

$$f_x = \frac{b}{T}(\frac{x}{T})^{b-1}e^{-(\frac{x}{T})^b}, \tag{4.16}$$

The Weibull model provides an important form for the hazard function:

$$h(x) = \frac{b}{T}(\frac{x}{T})^{b-1}, \tag{4.17}$$

Even more the cumulative hazard is given by:

$$H(x) = (\frac{x}{T})^b, \tag{4.18}$$

Another important point is that the Cumulative Hazard provided by the Weibull model is precisely the form for the simple model presented earlier and the parameter b expresses the healthy life years lost.

4.3 The Health State Models

4.3.1 The Health State Distribution

Although the health state models are introduced from 1995 (see Janssen and Skiadas 1995 and more publications from Skiadas and Skiadas 2007, 2010b, 2012a, 2014) few applications appear. The main reason is due to the very laborious first exit time stochastic theory needed and that it is assumed that the use of the Gompertz and the Weibull models along with the related extensions give enough tools for the practical applications. This is not correct as the first exit time stochastic models are produced by using one of the most elegant and accurate methodology to model the health-death process as it is demonstrated in the following. The probability distribution of the general health state model is of the form (see Jennen and Lerche 1981):

$$f(x) = \frac{|H - xH'|}{\sigma\sqrt{2\pi x^3}} e^{-\frac{[H(x)]^2}{2\sigma^2 x}}, \tag{4.19}$$

For the main applications in Demography we can set $\sigma = 1$ reducing to the simpler from:

$$f(x) = \frac{|H - xH'|}{\sqrt{2\pi x^3}} e^{-\frac{[H(x)]^2}{2x}}, \tag{4.20}$$

While the simpler form arises for the following health state function

$$H(x) = l - (bx)^c, \tag{4.21}$$

That is

$$f(x) = \frac{|l + (c-1)(bx)^c|}{\sqrt{2\pi x^3}} e^{-\frac{[l-(bx)^c]^2}{2x}}, \tag{4.22}$$

The simpler model of this form arises when $c = 1$ and it is the so-called Inverse Gaussian expressing the probability density function for the first exit time of a linearly decaying process:

$$f(x) = \frac{|l|}{\sqrt{2\pi x^3}} e^{-\frac{[l-(bx)]^2}{2x}}, \tag{4.23}$$

Applications of this or similar type forms can found in Ting and Whitmore (2006) and in Weitz and Fraser (2001). The last model as right skewed cannot express the human death process expressed by a highly left skewed probability density function. Instead the previous 4-parameter model is applied very successfully. Even more this form is very flexible providing very good fitting in the case of high levels of infant mortality, as it was the case for time periods some decades ago and also for nowadays when infant mortality is relatively low. Two different options arise for the model. That corresponding to the health state estimation with the parameter l expressing the high level of the health state and represented with Figs. 4.7 and 4.9 and another form with low levels for the parameter l expressing the Infant Mortality (see Figs 4.8 and 4.10). In the latter case the form of the density function is:

$$f(x) = \frac{2|l + (c-1)(bx)^c|}{\sqrt{2\pi x^3}} e^{-\frac{[l-(bx)^c]^2}{2x}}, \tag{4.24}$$

When the parameter l is very small a 2-parameter model termed here as the Half-Inverse Gaussian distribution results:

$$f(x) = \frac{2|(c-1)(bx)^c|}{\sqrt{2\pi x^3}} e^{-\frac{[-(bx)^c]^2}{2x}}, \tag{4.25}$$

The name arises from the similarity of this form with the Half-Normal distribution.

Fig. 4.7 First exit time model applied in USA death probability density for females the year 1950

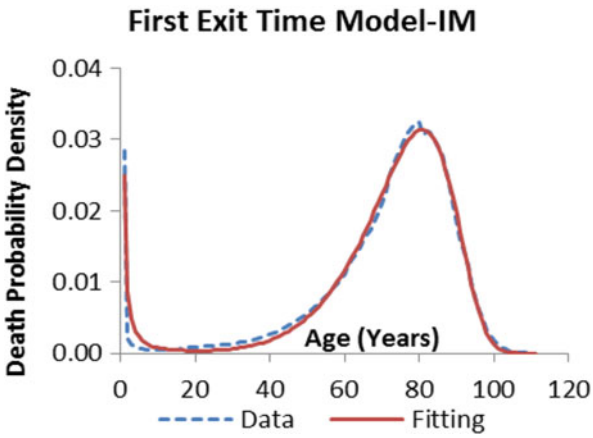

Fig. 4.8 First exit time model-IM including the infant mortality applied in USA death probability density for females the years 1950

The advantage of the proposed half-inverse Gaussian or IM-Model for the infant mortality modeling is obvious in the case of the application in USA females in 1950. The IM-Model provides a fairly well $R^2 = 0.990$ instead of $R^2 = 0.920$ for the Health State Model which provides similar results with the 2-parameter model (see the Table 4.2). The resulting R^2 for the year 2010 in USA females are similar as the infant mortality is relatively small (see Figs. 4.9 and 4.10 and Table 4.2).

4.3.2 An Important Extension: The Simplest IM-Model

Jennen (1985) suggested a second order approximation to improve the previous model with the first order approximation form:

$$f(x) = \frac{|H - xH'|}{\sqrt{2\pi x^3}} e^{-\frac{[H(x)]^2}{2x}}, \tag{4.26}$$

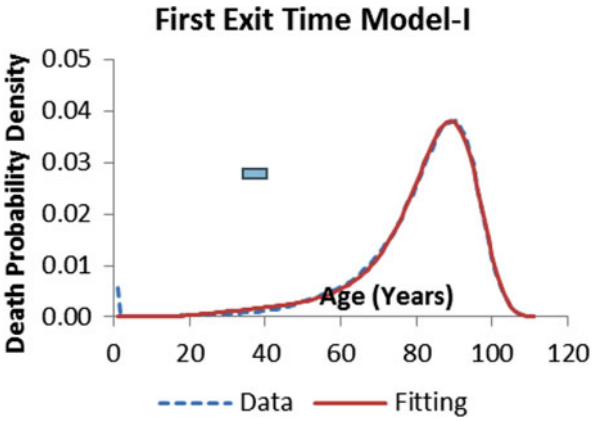

Fig. 4.9 First exit time model applied in USA death probability density for females the year 2010

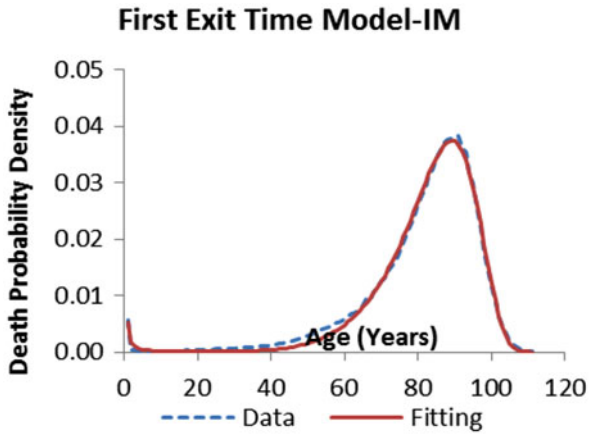

Fig. 4.10 First exit time model-IM including the infant mortality applied in USA death probability density for females the years 2010

Table 4.2 Parameters of the models

Year	2010	2010	2010	1950	1950	1950
Parameter/R^2	Health state	IM model	2-parameter	Health state	IM model	2-parameter
c	5.28	7.91	7.91	4.18	6.26	6.27
b	0.0192	0.0148	0.0148	0.0239	0.0173	0.0173
l	13.84	0.0066	–	13.05	0.0314	–
R^2	0.993	0.995	0.993	0.920	0.990	0.927

However, we propose and apply here a simpler form adequate for the applications in demography data:

$$f(x) = \frac{2}{\sqrt{2\pi}} \left[\frac{|H - xH'|}{\sqrt{x^3}} + \frac{k\sqrt{x^3}H''}{2|H - xH'|} \right] e^{-\frac{[H(x)]^2}{2x}}, \tag{4.27}$$

The parameter k expresses the level of the influence of the second order correction term. When $k = 0$ the last equation form reduces to the first order approximation. The next step is to use the expression presented earlier for $H(x)$ to find the advanced form of IM-model:

$$f(x) = \frac{2}{\sqrt{2\pi}}\left[\frac{|l + (c-1)(bx)^c|}{\sqrt{x^3}} - \frac{k\sqrt{x^3}c(c-1)b^c x^{(c-2)}}{2|l + (c-1)(bx)^c|}\right] e^{-\frac{[l-(bx)^c]^2}{2x}}, \qquad (4.28)$$

This is the simpler 4-parameter model providing quite well fitting for the logarithm of the force of mortality, providing not only good estimates for the infant mortality but also very good estimates for all the period of the life time for males and females as is illustrated in Figs. 4.11, 4.12, 4.13, 4.14, 4.15, and 4.16. We have thus demonstrated that the model proposed in 1995 and the new versions and advanced forms provided in several publications and in this paper, approach fairly well the mortality data sets provided by the bureau of the census and statistical agencies. This is important in order to straighten the findings when applying the first exit time theory to life table data.

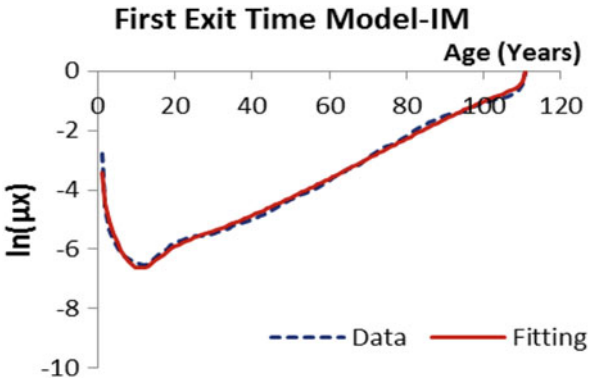

Fig. 4.11 The first exit time model-IM including the infant mortality applied in the USA force of mortality data in logarithmic form for males the year 1933

Fig. 4.12 The first exit time model-IM including the infant mortality applied in the USA force of mortality data in logarithmic form for females the year 1933

Fig. 4.13 The first exit time model-IM including the infant mortality applied in the USA force of mortality data in logarithmic form for males the year 1950

Fig. 4.14 The first exit time model-IM including the infant mortality applied in the USA force of mortality data in logarithmic form for females the year 1950

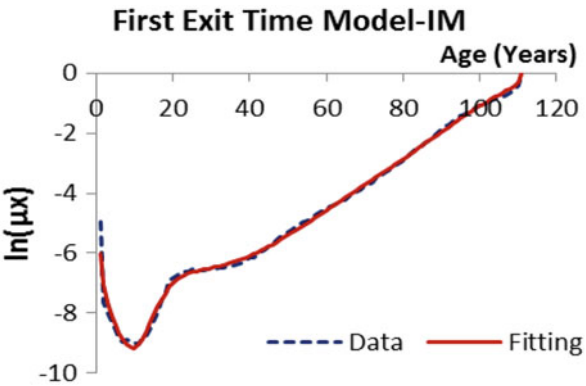

Fig. 4.15 The first exit time model-IM including the infant mortality applied in the USA force of mortality data in logarithmic form for males the year 2010

Fig. 4.16 The first exit time model-IM including the infant mortality applied in the USA force of mortality data in logarithmic form for females the year 2010

4.3.3 The Health State Function and the Relative Impact on Mortality

Considering the high importance of the proposed model and the related indicator for the verification of the GBD results we proceed in the introduction of a second method based on the health state of the population instead of the previous one which was based on mortality. This model was proposed earlier (see Skiadas and Skiadas 2010b, 2012a, 2013, 2014). These works were based on an earlier publication modeling the health state of a population via a first exit time stochastic methodology. Here we develop a special application adapted to WHO data provided as abridged life tables (0 to 100 with 5 year periods). First we expand the abridged life table to full and then we estimate the health indicators and finally the loss of healthy life years indicators.

By observing the above graph (Fig. 4.17) we can immediately see that the area between the health state curve and the horizontal axis (OMCO) represents the total health dynamics (THD) of the population. Of particular importance is also the area of the health rectangle (OABC) which includes the health state curve. This rectangle is divided in two rectangular parts the smaller (OAMN) indicating the first part of the human life until reaching the point M at the highest level of health state (usually the maximum is between 30 and 45 years) and the second part (NMBC) characterized by the gradual deterioration of the human organism until the zero level of the health state. This zero point health age C is associated with the maximum death rate. After this point the health state level appears as negative in the graph and characterizes a part of the human life totally unstable with high mortality; this is also indicated by a positively increasing form of the logarithm of the force of mortality $ln(\mu x)$. We call the second rectangle NMBC as the deterioration rectangle. Instead the first rectangle OAMN is here called as the development rectangle. For both cases we can find the relative impact of the area inside each rectangle but outside the health state area to the overall health state. In this study we analyze the relative impact

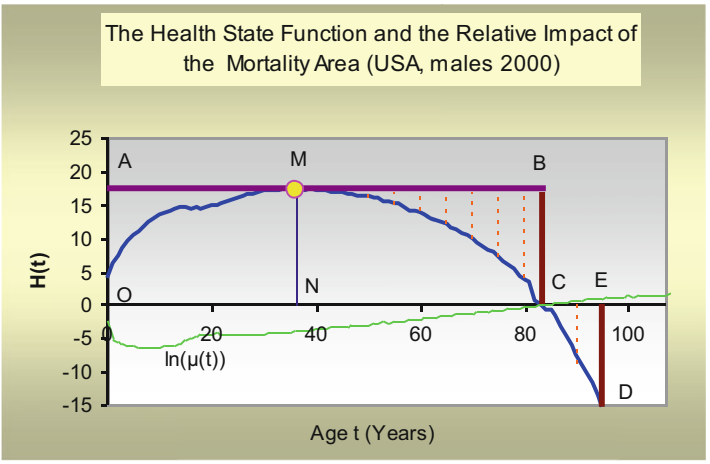

Fig. 4.17 The impact of the mortality area to health state

of the deterioration area MBCM indicated by dashed lines in the deterioration rectangle. It should be noted that if no-deterioration mechanism was present or the repairing mechanism was perfect the health state should continue following the straight line AMB parallel to the X-axis at the level of the maximum health state. The smaller the deterioration area related to the health state area, the higher the healthy life of the population. This comparison can be done by estimating the related areas and making a simple division. However, when trying to expand the human life further than the limits set by the deterioration mechanisms the percentage of the non-healthy life years becomes higher. This means that we need to divide the total rectangle area by that of the deterioration area to find an estimate for the lost healthy life years. It is clear that if we dont correct the deterioration mechanisms the loss of healthy years will become higher as the expectation of life becomes larger. This is already observed in the estimates of the World Health Organization (WHO) in the World Health Report for 2000 where the lost healthy years for females are higher than the corresponding values for males. The females show higher life expectancy than males but also higher values for the lost healthy years. The proposed loss of healthy life years indicator is given by:

$$LHLY_1 = \lambda \frac{OABC}{THD_{ideal}} \cdot \frac{THD_{ideal}}{MBCM} = \lambda \frac{OABC}{MBCM}$$

Where THD_{ideal} is ideal total health dynamics of the population and the parameter l expresses years and should be estimated according to the specific case. For comparing the related results in various countries we can set $\lambda = 1$. When OABC approaches the THD_{ideal} as is the case of several countries in nowadays the loss of healthy life years indicator LHLY can be expressed by other forms. Another point is the use of the (ECD) area in improving forecasts especially when using the

5-year life tables as is the case of the data for all the WHO Countries. In this case the expanded loss of healthy life years indicator LHLY will take the following two forms:

$$LHLY_2 = \lambda \frac{OMCO + ECD}{MBCM}$$

$$LHLY_3 = \lambda \frac{OABC + ECD}{MBCM}$$

It is clear that the last form will give higher values than the previous one. The following scheme applies: $LHLY1 < LHLY2 < LHLY3$. It remains to explore the forecasting ability of the three forms of the loss of healthy life years indicator by applying LHLY to life tables provided by WHO or by the Human Mortality Database or by other sources.

As for the previous case here important is the loss of health state area MBCM whereas the total area including the healthy and non-healthy part is included in OABC+ECD.

$$LHLY_2 = \lambda \frac{OABC + ECD}{MBCM}$$

Details and applications are included in the book on The Health State Function of a Population, the supplement of this book and other publications (see Skiadas 2011a,b,c, 2012 and Skiadas and Skiadas 2008, 2010a,b, 2011a,b, 2012a,b, 2013). It is important that we can explore the health state of a population by using the mortality approach with the Simple Model proposed herewith and the health state function approach as well. The latter method provides many important health measures than the simple model.

4.4 Applications

4.4.1 Comparative Application for the World and World Regions

The Table 4.3 includes our estimates for the healthy life expectancy at birth for the years 2000 and 2012 by applying the proposed mortality model and the health state model (HSM), and the estimates of WHO referred as HALE and included in the WHO websites (August 2015). Our estimates for the mortality model are based on $LHLY = b + 1 = E_{total}/E_{mortality}$. The main finding is that our models verify the WHO (HALE) estimates based on the Global Burden of Disease Study. Our results are quite close (with less to one year difference) to the estimates for the World, the High Income Countries, the African region, the European region and Western Pacific and differ by 1–2 years for the Eastern Mediterranean region and

Table 4.3 Comparison of model and HSM estimates with WHO (HALE) results

	Healthy Life Exp						Life Exp (LE)			
	2000			2012			2000		2012	
Sex/Region	HALE	Model	HSM	HALE	Model	HSM	HALE	Model	HALE	Model
Both sexes combined										
World	58.0	58.4	58.2	61.7	62.5	61.9	66.2	66.2	70.3	70.3
High income countries	67.3	67.1	67	69.8	69.6	69.2	76.0	76.0	78.9	78.9
African Region	43.1	42.8	42.8	49.6	49.9	49.6	50.2	50.2	57.7	57.7
Region of the Americas	64.9	65.7	65.4	67.1	67.7	67.2	73.9	73.9	76.4	76.3
Eastern Mediterranean	55.4	56.9	56.6	58.3	59.7	59.4	64.9	64.9	67.8	67.8
European Region	63.9	63.9	63.9	66.9	67.2	67.0	72.4	72.4	76.1	76.0
South East Asian Region	54.2	56.3	55.6	58.5	60.6	60.0	62.9	63.0	67.5	67.5
Western Pacific Region	64.8	63.9	64.2	68.1	67.3	67.5	72.3	72.3	75.9	75.9
Males										
World	56.4	56.6	56.2	60.1	60.4	60.0	63.9	63.9	68.1	68.0
High income countries	64.7	64.1	64.2	67.5	67	67	72.4	72.3	75.8	75.7
African Region	42.4	41.6	42.3	48.8	48.6	48.6	49.0	49.0	56.3	56.3
Region of the Americas	62.7	63.1	62.5	64.9	65.1	64.6	70.8	70.8	73.5	73.5
Eastern Mediterranean	54.8	55.7	55.6	57.4	58.2	57.9	63.6	63.6	66.1	66.1
European Region	60.7	60.4	61.1	64.2	64.3	64.5	68.2	68.2	72.4	72.4
South East Asian Region	53.5	55.4	54.6	57.4	59.2	58.6	61.6	61.7	65.7	65.7
Western Pacific Region	63.0	61.8	62.0	66.6	65.2	65.7	70.0	70.0	73.9	73.9
Females										
World	59.7	60.3	59.9	63.4	64.3	64.1	68.5	68.5	72.7	72.6
High income countries	70.0	69.7	69.6	72.0	71.8	72.1	79.6	79.5	82.0	81.9
African Region	43.8	43.8	43.5	50.4	51.2	50.5	51.4	51.4	59	59.1
Region of the Americas	67.2	68.0	67.8	69.1	69.9	69.8	77.0	76.9	79.3	79.2
Eastern Mediterranean	56.1	58.2	57.8	59.2	61.3	61.0	66.4	66.4	69.7	69.6
European Region	67.1	67.6	67.3	69.6	70.0	69.7	76.7	76.6	79.6	79.6
South East Asian Region	55.0	57.2	56.4	59.7	62.0	61.7	64.3	64.4	69.4	69.4
Western Pacific Region	66.7	65.7	66.1	69.8	68.9	69.1	74.8	74.8	78.1	78.0

the South East Asian region. In the last two cases the collection of data and the accuracy of the information sources may lead to high uncertainty of the related health state estimates. This is demonstrated in the provided confidence intervals for the estimates in countries of these regions in the studies by Salomon et al. (2012) and the Report of WHO (2001) for the HLE of the member states (2000). From the Salomon et al. study we have calculated a mean confidence interval of 5.5 years for males and 6.8 for females for the year 2000. We thus propose to base the future works on the system we propose and to use it to calibrate the estimates especially for the countries providing of low accuracy data. To support future studies we have formulated an easy to use framework in Excel. The only needed is to insert data for μx in the related column of the program. The program estimates the life expectancy, the loss of healthy life years and the healthy life expectancy.

4.4.2 Application to USA 2008

Both the Gompertz and the Weibull health estimators are calculated by the appropriate computer program. The results are compared with those of the methods proposed earlier thus providing enough evidence for a successful application. The estimates of the WHO are also included in the related table. The task to find an alternative of the WHO and other estimates for the Healthy Life Years Lost is highly supported by using a series of methods leading to similar and easily reproducible results. Only few detailed publications appear in order to use for comparative applications. It is highly appreciated that one paper by Chang et al. for USA data for 2008 and of Yong and Saito (2009) on healthy life expectancy in Japan: 1986–2004, published in Demographic Research are of particular importance for our comparative applications. A very important paper by Chang et al. (2015) appeared in the Journal of Public Health. It includes calculations of the Life Expectancy (LE) and the Healthy Life Expectancy (HLE) for the United States population the year 2008 by sex and race/ethnicity. Our task was to find good estimates compared to the authors results by a different methodology than the survey data collection and the Sullivan method followed. Following the above provided models and estimation techniques, three different methods are selected to estimate the HLE from only the life table data sets (mortality data). The benefits are: Simple estimation, estimates for all the period where mortality data exist, comparison with existing estimates from other methods, fix the weights needed for other measurements, provide a simple methodology for health decision makers to organize future plans. As an example the estimates for USA (1950) are LE (68.0 years) and HLE (61.3 years). We have calculated similar results for the first part of the Table 1 of the Chang et al. paper related to all races and for both sexes, male and female (see our Table 4.4). A power model, a Weibull model and a Gompertz model are used to first estimate the Healthy Life Years Lost (HLYL) and then find the Healthy Life Expectancy (HLE) from

Table 4.4 Comparing Chang et al. estimates

Life Expectancy (LE) Healthy Life Expectancy (HLE) and Healthy Life Years Lost (HLYL) by sex for the United States population, 2008																														
	Both sexes										Male										Female									
Age	LE	HLE				Plausibility	HLYL				LE	HLE				Plausibility	HLYL				LE	HLE				Plausibility	HLYL			
(Years)	Pub	Pub	SK	W	G	range	Pub	SK	W	G	Pub	Pub	SK	W	G	range	Pub	SK	W	G	Pub	Pub	SK	W	G	range	Pub	SK	W	G
<1	78.1	69.3	68.9	69.7	69.8	(68.4–70.3)	8.8	9.2	8.4	8.3	75.6	67.6	66.8	67.7	67.9	(66.8–69.2)	8.0	8.8	7.9	7.7	80.6	70.9	70.7	71.2	71.4	(69.8–72.6)	9.7	9.9	9.4	9.2
1–4	77.6	68.7	68.4	69.2	69.3	(67.9–69.8)	8.9	9.2	8.4	8.3	75.1	67.1	66.3	67.2	67.4	(66.1–68.5)	8.0	8.8	7.9	7.7	80.1	70.3	70.2	70.7	70.9	(69.1–71.8)	9.8	9.9	9.4	9.2
5–9	73.7	64.9	64.6	65.3	65.5	(64.1–66.0)	8.8	9.1	8.4	8.2	71.2	63.2	62.5	63.3	63.5	(62.2–64.6)	8.0	8.7	7.9	7.7	76.1	66.5	66.2	66.8	66.9	(65.2–67.9)	9.6	9.9	9.3	9.2
10–14	68.8	60.0	59.7	60.5	60.6	(59.2–61.1)	8.8	9.1	8.3	8.2	66.3	58.4	57.6	58.5	58.7	(57.4–59.7)	7.9	8.7	7.8	7.6	71.2	61.6	61.4	61.9	62.1	(60.4–63.0)	9.6	9.8	9.3	9.1
15–19	63.8	55.1	54.8	55.6	55.7	(54.4–56.2)	8.7	9.0	8.2	8.1	61.3	53.5	52.7	53.6	53.8	(52.6–54.8)	7.8	8.6	7.7	7.5	66.2	56.7	56.5	57.1	57.2	(55.5–58.1)	9.5	9.7	9.1	9.0
20–24	59.0	50.4	50.2	50.9	51.1	(49.7–51.5)	8.6	8.8	8.1	7.9	56.6	48.8	48.2	49.0	49.2	(47.9–50.1)	7.8	8.4	7.6	7.4	61.3	52.0	51.8	52.3	52.5	(50.8–53.3)	9.3	9.5	9.0	8.8
25–29	54.3	45.9	45.7	46.4	46.6	(45.2–46.9)	8.4	8.6	7.9	7.7	52.0	44.4	43.8	44.6	44.8	(43.5–45.6)	7.6	8.2	7.4	7.2	56.5	47.3	47.2	47.8	47.9	(46.2-48.6)	9.2	9.3	8.7	8.6
30–34	49.5	41.4	41.2	41.9	42.0	(40.7–42.3)	8.1	8.3	7.6	7.5	47.3	39.9	39.4	40.2	40.3	(39.1–41.1)	7.4	7.9	7.1	7.0	51.6	42.7	42.6	43.1	43.3	(41.7–44.0)	8.9	9.0	8.5	8.3
35–39	44.8	36.8	36.8	37.5	37.6	(36.2–37.8)	8.0	8.0	7.3	7.2	42.6	35.4	35.0	35.7	35.9	(34.7–36.6)	7.2	7.6	6.9	6.7	46.8	38.2	38.2	38.7	38.8	(37.2–39.4)	8.6	8.6	8.1	8.0
40–14	40.1	32.5	32.5	33.1	33.3	(31.9–33.4)	7.6	7.6	7.0	6.8	38.0	31.0	30.7	31.4	31.6	(30.3–32.2)	7.0	7.3	6.6	6.4	42.0	33.8	33.8	34.2	34.4	(32.9–34.9)	8.2	8.2	7.8	7.6
45–49	35.5	28.3	28.3	28.9	29.0	(27.8–29.2)	7.2	7.2	6.6	6.5	33.5	26.9	26.6	27.3	27.5	(26.3–28.0)	6.6	6.9	6.2	6.0	37.3	29.6	29.5	30.0	30.1	(28.7–30.7)	7.7	7.8	7.3	7.2
50–54	31.0	24.3	24.3	24.8	24.9	(23.8–25.1)	6.7	6.7	6.2	6.1	29.1	22.9	22.7	23.3	23.4	(22.4–24.0)	6.2	6.4	5.8	5.7	32.8	25.5	25.5	25.9	26.1	(24.8–26.6)	7.3	7.3	6.9	6.7
55–59	26.8	20.6	20.6	21.1	21.2	(20.2–21.4)	6.2	6.2	5.7	5.6	25.0	19.3	19.1	19.7	19.8	(18.8–20.3)	5.7	5.9	5.3	5.2	28.4	21.8	21.7	22.1	22.2	(21.1–22.7)	6.6	6.7	6.3	6.2
60–64	22.7	17.2	17.0	17.5	17.6	(16.8–17.9)	5.5	5.7	5.2	5.1	21.0	16.0	15.6	16.1	16.3	(15.5–16.9)	5.0	5.4	4.9	4.7	24.1	18.2	18.0	18.3	18.4	(17.6–19.1)	5.9	6.1	5.8	5.7
65–69	18.8	14.0	13.8	14.2	14.3	(13.7–14.7)	4.8	5.0	4.6	4.5	17.3	13.0	12.5	13.0	13.1	(12.6–13.8)	4.3	4.8	4.3	4.2	20.0	14.9	14.6	14.9	15.0	(14.3–15.7)	5.1	5.4	5.1	5.0
70–74	15.2	11.1	10.8	11.2	11.3	(10.8–11.7)	4.1	4.4	4.0	3.9	13.9	10.2	9.8	10.1	10.2	(9.9–11.0)	3.7	4.1	3.8	3.7	16.2	11.7	11.5	11.7	11.8	(11.3–12.5)	4.5	4.7	4.5	4.4
75–79	11.8	8.4	8.1	8.4	8.5	(8.2–9.0)	3.4	3.7	3.4	3.3	10.7	7.7	7.2	7.6	7.6	(7.5–8.4)	3.0	3.5	3.1	3.1	12.6	8.9	8.7	8.9	8.9	(8.6–9.6)	3.7	3.9	3.7	3.7
80–84	8.9	6.1	6.0	6.2	6.3	(6.0–6.7)	2.8	2.9	2.7	2.6	8.0	5.5	5.3	5.5	5.6	(5.3–6.1)	2.5	2.7	2.5	2.4	9.5	6.6	6.4	6.5	6.6	(6.4–7.2)	2.9	3.1	3.0	2.9
85+	6.4	4.3	4.3	4.5	4.5	(4.3–4.8)	2.1	2.1	1.9	1.9	5.7	3.7	3.7	3.9	3.9	(3.6–4.3)	2.0	2.0	1.8	1.8	6.8	4.6	4.6	4.7	4.7	(4.5–5.1)	2.2	2.2	2.1	2.1

Pub Chang et al. estimates, *SK* Skiadas model estimates, *W* Weibull estimates, *HLE-G* Gompertz estimates

the simple relation HLE=LE-HLYL. We have verified that our results are within the provided plausibility range suggested by Chang et al. and very close to their estimates for all the life period. Instead for the second part of Table 1 of the authors our estimates differ considerably from the related figures provided. Specifically for Hispanic we have estimated 8.9 HLYL corresponding to 72.1 HLE instead of 13.4 (67.6 HLE) of the authors and for Non-Hispanic black we estimated 7.0 HLYL (66.7 HLE) instead of 12.3 (61.4 HLE) of the authors. The other estimates for Non-Hispanic white 7.9 HLYL (70.5 HLE) of the authors is in agreement with our estimates of 8.3 HLYL (70.1 HLE). It is important to clarify the cause of the differences as the perfect agreement between both methods will straighten the HLE estimators. We have provided the related estimation programs in the webpage http://www.smtda.net/demographics2016.html to support comparative applications.

4.4.3 Application in Japan 1986–2004

The results from Yong and Saito (2009) on healthy life expectancy in Japan: 1986–2004, published in Demographic Research are used for our comparative applications. The authors applied the Sullivan method to data collected from a large national survey in Japan. The number of responders and the methodology applied assures relatively good results adequate for a comparative study. Especially the part of the study related to not so poor and poor state of health was selected for our comparisons. This is because our theory presented above suggests the estimation of the health state as a fraction of areas related to mortality and health as it is presented in Fig. 4.1 and the related theory. The impact on the public opinion regarding the health state is due to the E_{health}, the $E_{mortality}$ and the total Health-Mortality area E_{total}. The impact is expressed by the exponent b or b+1 depending on the form of the social status of the society and the male-female differentiation regarding the adoption and spread of the information for health, disability and mortality. We apply the Direct Estimation as presented above in Japan from 1947 to 2012 for m_x and q_x data included in the full life tables provided by the Human mortality Database (HMD) thus estimating the parameters b and $b + 1$ as is illustrated in Fig. 4.18 for Japan (males). In the same Figures we also have included the Healthy Life Years Lost (HLYL) from the HALE estimates of the World Health Organization. Although the results for the years 1990, 2000a, 2010, 2012 and 2013 are within the region defined by the four curves, there are significant differences in the estimates in 2000b, 2001, 2002 and 2007 underestimating the HLYL due to improvements in the methodology and the use of new epidemiological data. In the Annex Table of the World Health Report 2001 (WHO 2001) and the related of 2002 (WHO 2002) write:

> Healthy life expectancy estimates published here are not directly comparable to those published in the World Health Report 2000, due to improvements in survey methodology and the use of new epidemiological data for some diseases. See Statistical Annex notes (pp.130–135). The figures reported in this Table along with the data collection and estimation methods have been largely developed by WHO and do not necessarily reflect

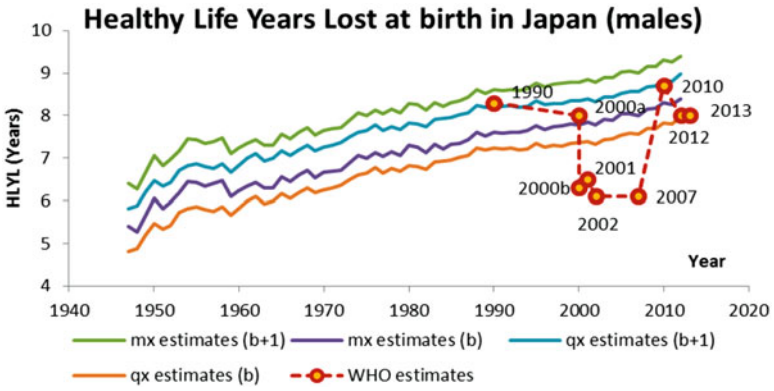

Fig. 4.18 Comparing the HLYL with a direct method to the WHO estimates (Japan, males)

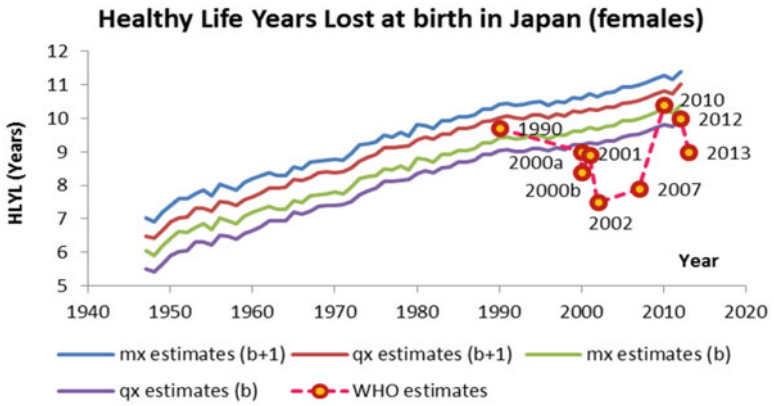

Fig. 4.19 Comparing the HLYL with a direct method to the WHO estimates (Japan, females)

official statistics of Member States. Further development in collaboration with Member States is underway for improved data collection and estimation methods (WHO 2001).

Healthy life expectancy estimates published here are not directly comparable to those published in The World Health Report 2001, because of improvements in survey methodology and the use of new epidemiological data for some diseases and revisions of life tables for 2000 for many Member States to take new data into account (see Statistical Annex explanatory notes). The figures reported in this Table along with the data collection and estimation methods have been largely developed by WHO and do not necessarily reflect official statistics of Member States. Further development in collaboration with Member States is under way for improved data collection and estimation methods (WHO 2002).

Figure 4.19 illustrates the healthy life years lost for females in Japan following the same procedure as for males. As before significant differences appear especially

for the years 2002 and 2007. Even more it is clear that the differences not following a clear trend are due to the ongoing process of the estimation team of WHO to arrive in a best estimate method. To this end the recently provided estimates for 2000a, 2012 and 2013 (presented without decimal points) for males and females are closer to the results from our methodology.

Figure 4.20a–h illustrate the expected number of years in poor health for 25, 45, 65 and 85 years men and women following the Yong-Saito findings and the direct application results based on m_x and q_x estimates included in Tables 4.5 and 4.6 (see end of the paper for these Tables and Fig. 4.20a–h). For both cases (men and women) the majority of the Yong-Saito estimates are within the interval suggested with our calculations. As for both men and women the Yong-Saito findings suggest a declining pattern for the healthy life years lost until 1995, followed by an increasing trend not explained by significant variations of the mortality trends or of the life expectancy, we have to explore socioeconomic factors influencing the responses to questionnaires and in a second stage the changes in the health state of a population. So, huge changes in LHLY could be expected to arise in very special morbidity cases as from the spread of epidemics. Instead the growing unemployment rate in Japan leading to a maximum in 2004 along with the slowdown of the economy and the related economic indicators can explain the relative changes in the public opinion regarding the health state. After all as the surveys cover 280,000 households and data on over 750,000 individuals were collected, the uncertainty degree should be very low. Specific sociological surveys are needed to explore the influence of socioeconomic and political factors not only to the health state but to the way the responders reply to a specific questionnaire.

4.4.4 Application in the World

Another application is presented in Table 4.7 where the mortality model and the WHO (HALE) results from 1990 to 2013 are compared for the WHO member countries.

Figure 4.21 illustrates the estimates of the arithmetic mean for the loss of healthy life years (LHLY) for various time periods with the proposed mortality model (circles) versus the HALE method (rhombus) for the countries of WHO (males). The model estimates and the HALE differ by 1.0 years in 1990, 0.4 years in 2000, 0.6 years in 2012 and 0.6 years in 2013. Several options of the related figures for HALE are included starting from the first estimates in 1990 until the estimates of 2013. The estimates for 1990, 2000, 2012 and 2013 are provided in the last WHO websites and can be accepted as the official estimates whereas the HALE estimates for 2001, 2002 and 2010 are also included.

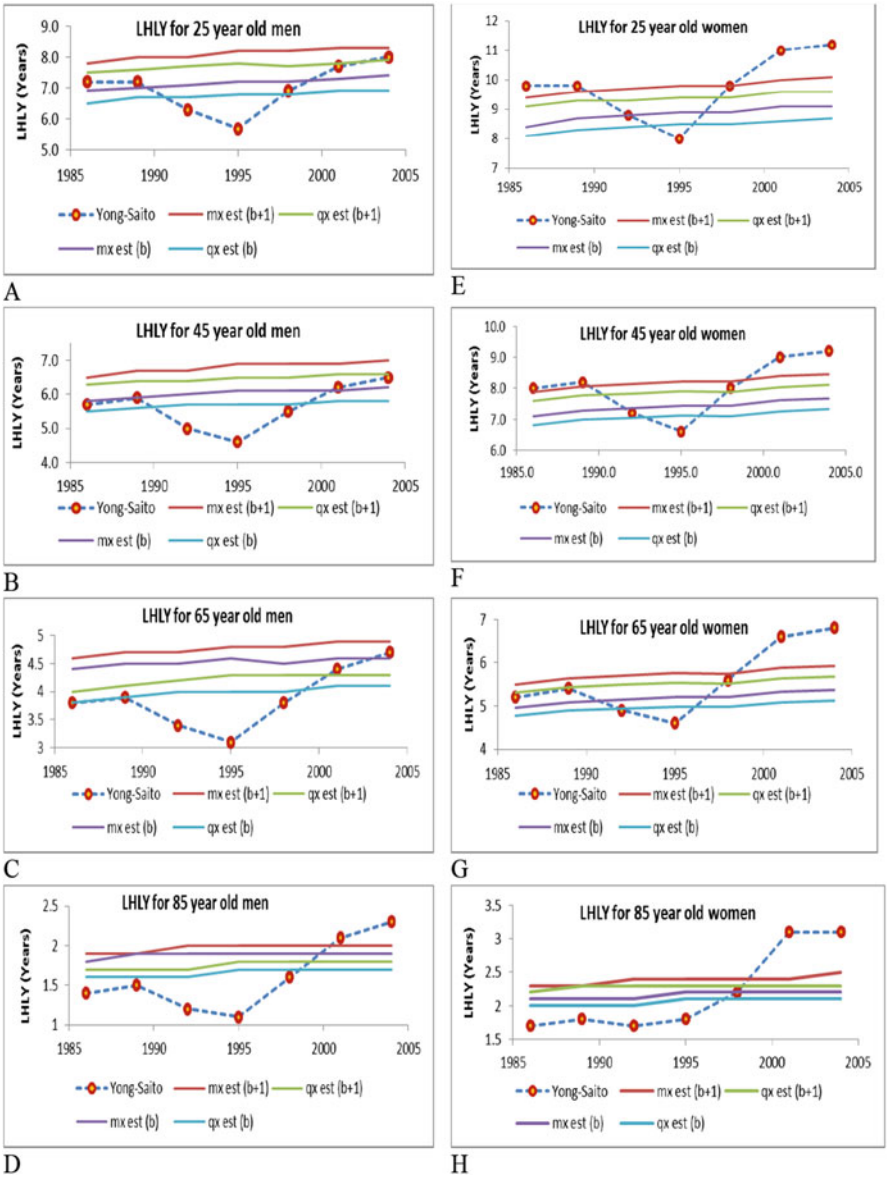

Fig. 4.20 Expected number of years in poor health for 25, 45, 65 and 85 years men and women (Yong-Saito findings and direct application results based on m_x and q_x)

Table 4.5 Life expectancy and healthy life expectancy of Japanese men, 1986–2004

Life expectancy and healthy life expectancy for Japanese men, 1986–2004						
		Expected number of years in poor health				
Year	Life expectancy	Yong-Saito	$m_x(b+1)$	$q_x(b+1)$	$m_x(b)$	$q_x(b)$
At birth men						
1986	**75.3**		8.3	8.0	7.3	7.0
1989	**76.0**		8.5	8.2	7.5	7.2
1992	**76.1**		8.6	8.2	7.6	7.2
1995	**76.4**		8.8	8.3	7.8	7.3
1998	**77.2**		8.8	8.3	7.8	7.3
2001	**78.0**		8.8	8.4	7.8	7.4
2004	**78.6**		8.9	8.4	7.9	7.4
25 year old men						
1986	**51.4**	**7.2**	7.8	7.5	6.9	6.5
1989	**52.0**	**7.2**	8.0	7.6	7.0	6.7
1992	**52.1**	**6.3**	8.0	7.7	7.1	6.7
1995	**52.3**	**5.7**	8.2	7.8	7.2	6.8
1998	**53.0**	**6.9**	8.2	7.7	7.2	6.8
2001	**53.8**	**7.7**	8.3	7.8	7.3	6.9
2004	**54.3**	**8.0**	8.3	7.9	7.4	6.9
45 year old men						
1986	**32.4**	**5.7**	6.5	6.3	5.8	5.5
1989	**32.9**	**5.9**	6.7	6.4	5.9	5.6
1992	**33.0**	**5.0**	6.7	6.4	6.0	5.7
1995	**33.3**	**4.6**	6.9	6.5	6.1	5.7
1998	**34.0**	**5.5**	6.9	6.5	6.1	5.7
2001	**34.8**	**6.2**	6.9	6.6	6.1	5.8
2004	**35.3**	**6.5**	7.0	6.6	6.2	5.8
65 year old men						
1986	**15.9**	**3.8**	4.6	4.0	4.4	3.8
1989	**16.2**	**3.9**	4.7	4.1	4.5	3.9
1992	**16.3**	**3.4**	4.7	4.2	4.5	4.0
1995	**16.5**	**3.1**	4.8	4.3	4.6	4.0
1998	**17.1**	**3.8**	4.8	4.3	4.5	4.0
2001	**17.8**	**4.4**	4.9	4.3	4.6	4.1
2004	**18.2**	**4.7**	4.9	4.3	4.6	4.1
85 year old men						
1986	**4.8**	**1.4**	1.9	1.7	1.8	1.6
1989	**4.9**	**1.5**	1.9	1.7	1.9	1.6
1992	**4.9**	**1.2**	2.0	1.7	1.9	1.6
1995	**5.1**	**1.1**	2.0	1.8	1.9	1.7
1998	**5.5**	**1.6**	2.0	1.8	1.9	1.7
2001	**5.9**	**2.1**	2.0	1.8	1.9	1.7
2004	**6.1**	**2.3**	2.0	1.8	1.9	1.7

Table 4.6 Life expectancy and healthy life expectancy of Japanese women, 1986–2004

Life expectancy and healthy life expectancy for Japanese men, 1986–2004						
		Expected number of years in poor health				
Year	Life expectancy	Yong-Saito	$m_x(b+1)$	$q_x(b+1)$	$m_x(b)$	$q_x(b)$
At birth men						
1986	**81.0**		10.0	9.7	9.0	8.7
1989	**81.8**		10.3	9.9	9.3	8.9
1992	**82.3**		10.4	10.0	9.4	9.0
1995	**82.8**		10.5	10.1	9.5	9.1
1998	**83.9**		10.5	10.1	9.5	9.1
2001	**84.9**		10.7	10.3	9.7	9.3
2004	**85.5**		10.8	10.3	9.8	9.3
25 year old women						
1986	**56.7**	**9.8**	9.4	9.1	8.4	8.1
1989	**57.5**	**9.8**	9.6	9.3	8.7	8.3
1992	**57.9**	**8.8**	9.7	9.3	8.8	8.4
1995	**58.6**	**8.0**	9.8	9.4	8.9	8.5
1998	**59.6**	**9.8**	9.8	9.4	8.9	8.5
2001	**60.5**	**11.0**	10.0	9.6	9.1	8.6
2004	**61.1**	**11.2**	10.1	9.6	9.1	8.7
45 year old women						
1986.0	**37.4**	**8.0**	7.9	7.6	7.1	6.8
1989.0	**38.1**	**8.2**	8.1	7.8	7.3	7.0
1992.0	**38.5**	**7.2**	8.1	7.8	7.4	7.1
1995.0	**39.1**	**6.6**	8.2	7.9	7.4	7.1
1998.0	**40.2**	**8.0**	8.2	7.9	7.4	7.1
2001.0	**41.0**	**9.0**	8.4	8.0	7.6	7.3
2004.0	**41.6**	**9.2**	8.5	8.1	7.7	7.3
65 year old women						
1986	**19.3**	**5.2**	5.5	5.3	5.0	4.8
1989	**20.0**	**5.4**	5.6	5.4	5.1	4.9
1992	**20.3**	**4.9**	5.7	5.5	5.1	4.9
1995	**20.9**	**4.6**	5.7	5.5	5.2	5.0
1998	**22.0**	**5.6**	5.7	5.5	5.2	5.0
2001	**22.7**	**6.6**	5.9	5.6	5.3	5.1
2004	**23.3**	**6.8**	5.9	5.7	5.4	5.1
85 year old women						
1986	**5.7**	**1.7**	2.3	2.2	2.1	2.0
1989	**6.0**	**1.8**	2.3	2.3	2.1	2.0
1992	**6.1**	**1.7**	2.4	2.3	2.1	2.0
1995	**6.7**	**1.8**	2.4	2.3	2.2	2.1
1998	**7.4**	**2.2**	2.4	2.3	2.2	2.1
2001	**7.8**	**3.1**	2.4	2.3	2.2	2.1
2004	**8.1**	**3.1**	2.5	2.3	2.2	2.1

Table 4.7 Mortality model and the WHO (HALE) results compared

Mean values of the mortality model for the Loss of Healthy Life Years (LHLY) and the related results from the HALE method for the WHO countries												
Type	HALE	MODEL	HALEb	HALE	MODEL	HALE	HALE	HALE	HALE	MODEL	HALE	MODEL
Year	1990	1990	2000	2000	2000	2001	2002	2010	2012	2012	2013	2013
Males	8.7	7.7	8.4	8.2	7.8	9.7	7.2	9.3	8.8	8.2	8.9	8.3
Females	10.3	8.6	11.0	9.5	8.8	10.0	8.9	10.8	10.2	9.3	10.1	9.3

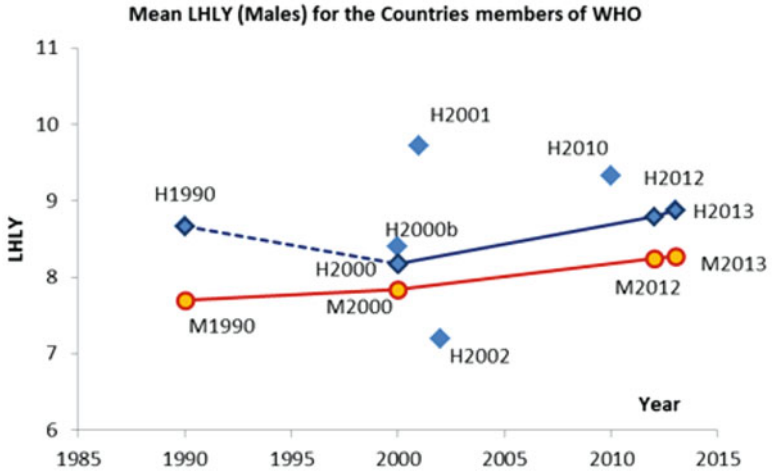

Fig. 4.21 Mean estimates for the healthy life years lost for the Model and HALE (males)

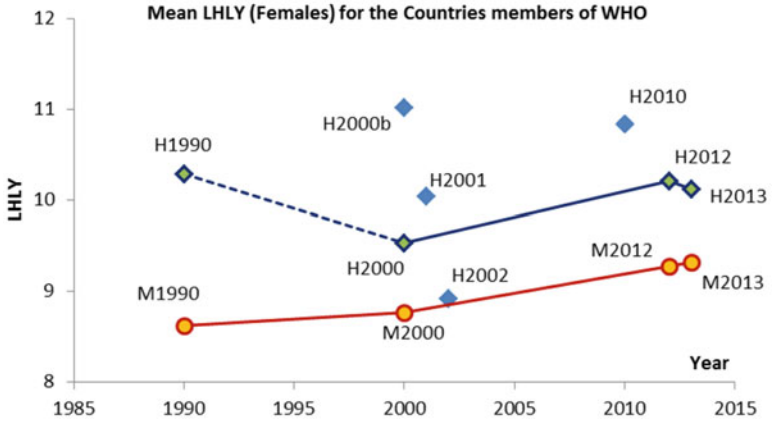

Fig. 4.22 Mean estimates for the healthy life years lost for the Model and HALE (females)

Figure 4.22 summarizes the estimates of the mean of the loss of healthy life years (LHLY) for various time periods with the proposed model (circles) versus the HALE method (rhombus) for the countries of WHO (females). The model estimates (females) and the HALE differ by 1.7 years in 1990, 0.7 years in 2000, 0.9 years in 2012 and 0.8 years in 2013. For females as for males the estimated differences between model and HALE are higher in 1990 than for the following years due to the higher values of the HALE estimates.

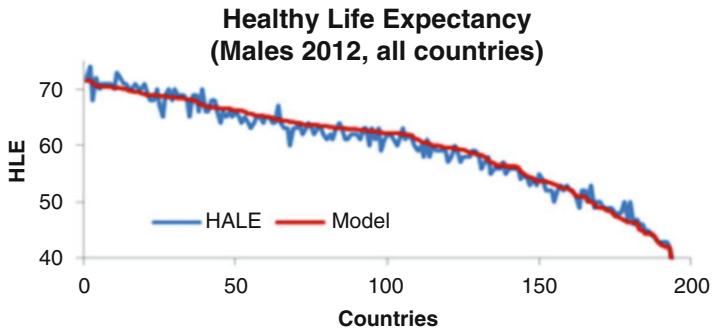

Fig. 4.23 Healthy life expectancy (HLE) for males (2012) for all the WHO countries. Model and HALE

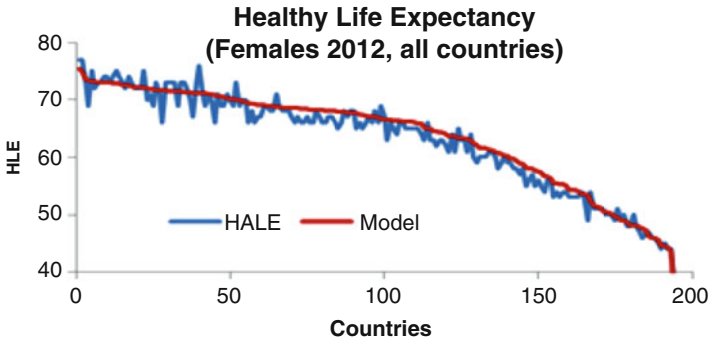

Fig. 4.24 Healthy life expectancy (HLE) for females (2012) for all the WHO countries. Model and HALE

Figure 4.23 illustrates the Healthy life expectancy (HLE) for males (2012) for all the WHO countries, estimated by the Model (red line) and HALE estimates (Blue line). The mean value is 59.6 for HALE and 60.2 years for the model.

Figure 4.24 illustrates the Healthy life expectancy (HLE) for females (2012) for all the WHO countries, estimated by the Model (red line) and HALE estimates (Blue line). The mean value is 63.1 for HALE and 63.8 years for the model. The full estimated figures are included in Tables 4.8, 4.9 and 4.10 in the end of the paper.

Table 4.8 Mortality model and the WHO (HALE) results compared

Country – 2012 – Females	HALE	MODEL	Country – 2012 – Males	HALE	MODEL
Afghanistan	49	53.4	Afghanistan	49	51.0
Albania	66	65.1	Albania	64	63.9
Algeria	63	64.4	Algeria	62	61.9
Andorra	74	73.0	Andorra	70	68.6
Angola	45	44.5	Angola	43	42.2
Antigua and Barbuda	66	67.4	Antigua and Barbuda	63	64.0
Argentina	69	69.9	Argentina	64	65.2
Armenia	66	66.2	Armenia	60	59.9
Australia	74	72.8	Australia	71	70.4
Austria	73	71.5	Austria	69	68.3
Azerbaijan	65	66.4	Azerbaijan	61	62.2
Bahamas	67	67.1	Bahamas	62	62.9
Bahrain	66	68.5	Bahrain	66	66.9
Bangladesh	61	63.7	Bangladesh	60	62.5
Barbados	69	70.5	Barbados	64	66.2
Belarus	68	68.6	Belarus	59	59.9
Belgium	73	71.4	Belgium	69	67.9
Belize	66	68.1	Belize	61	62.2
Benin	51	51.4	Benin	50	49.1
Bhutan	59	62.1	Bhutan	58	60.3
Bolivia (Plurinational State of)	61	61.2	Bolivia (Plurinational State of)	58	57.5
Bosnia and Herzegovina	70	70.2	Bosnia and Herzegovina	66	66.7
Botswana	53	55.3	Botswana	52	53.4
Brazil	67	69.2	Brazil	62	63.3
Brunei Darussalam	69	69.0	Brunei Darussalam	68	66.8

(continued)

Table 4.8 (continued)

Country – 2012 – Females	HALE	MODEL	Country – 2012 – Males	HALE	MODEL
Bulgaria	68	67.7	Bulgaria	63	63.0
Burkina Faso	51	50.5	Burkina Faso	50	49.0
Burundi	49	49.7	Burundi	46	46.4
Cabo Verde	66	68.2	Cabo Verde	61	62.4
Cambodia	63	66.5	Cambodia	59	62.3
Cameroon	49	49.1	Cameroon	48	47.3
Canada	73	73.0	Canada	71	70.0
Central African Republic	44	43.7	Central African Republic	43	42.1
Chad	44	44.4	Chad	43	42.9
Chile	72	73.2	Chile	68	68.9
China	69	66.7	China	67	64.6
Colombia	70	72.0	Colombia	66	66.6
Comoros	54	55.2	Comoros	53	52.7
Congo	51	51.5	Congo	49	49.0
Cook Islands	66	67.9	Cook Islands	63	65.6
Costa Rica	71	71.9	Costa Rica	68	68.9
Côte d'Ivoire	46	45.9	Côte d'Ivoire	45	44.4
Croatia	70	71.0	Croatia	65	66.5
Cuba	69	71.9	Cuba	65	68.7
Cyprus	76	71.1	Cyprus	73	70.2
Czech Republic	71	70.4	Czech Republic	66	66.2
Democratic People's Republic of Korea	65	63.4	Democratic People's Republic of Korea	59	58.4
Democratic Republic of the Congo	45	45.6	Democratic Republic of the Congo	43	42.4
Denmark	72	72.2	Denmark	69	68.9
Djibouti	53	54.3	Djibouti	52	51.8

Dominica	65	68.0	Dominica	61	63.2
Dominican Republic	67	69.2	Dominican Republic	65	68.1
Ecuador	68	67.7	Ecuador	64	63.1
Egypt	63	65.2	Egypt	60	61.2
El Salvador	66	69.3	El Salvador	59	61.0
Equatorial Guinea	48	48.7	Equatorial Guinea	47	46.2
Eritrea	55	58.1	Eritrea	53	54.4
Estonia	71	71.0	Estonia	63	63.5
Ethiopia	56	56.2	Ethiopia	54	53.7
Fiji	62	64.4	Fiji	58	59.6
Finland	73	71.6	Finland	69	67.7
France	74	72.5	France	69	68.3
Gabon	54	55.1	Gabon	53	53.8
Gambia	53	53.7	Gambia	52	50.9
Georgia	68	68.6	Georgia	62	62.8
Germany	73	71.5	Germany	70	68.5
Ghana	54	52.5	Ghana	53	50.1
Greece	73	71.1	Greece	69	68.7
Grenada	66	67.3	Grenada	60	62.1
Guatemala	65	65.9	Guatemala	60	59.5
Guinea	50	50.3	Guinea	49	48.6
Guinea-Bissau	47	47.4	Guinea-Bissau	46	45.1
Guyana	57	58.9	Guyana	52	52.5
Haiti	53	55.5	Haiti	50	53.1
Honduras	65	67.4	Honduras	62	63.6
Hungary	69	69.1	Hungary	63	64.2
Iceland	73	71.5	Iceland	72	70.2

(continued)

Table 4.8 (continued)

Country – 2012 – Females	HALE	MODEL	Country – 2012 – Males	HALE	MODEL
India	58	60.8	India	56	58.2
Indonesia	64	63.7	Indonesia	61	60.4
Iran (Islamic Republic of)	65	66.1	Iran (Islamic Republic of)	63	63.7
Iraq	63	64.4	Iraq	58	58.4
Ireland	73	73.0	Ireland	70	69.7
Israel	73	72.5	Israel	71	70.3
Italy	74	73.0	Italy	71	70.3
Jamaica	66	66.9	Jamaica	62	62.2
Japan	77	75.0	Japan	72	70.5
Jordan	65	66.2	Jordan	64	63.6
Kazakhstan	64	63.0	Kazakhstan	56	56.5
Kenya	54	53.7	Kenya	52	51.0
Kiribati	60	60.3	Kiribati	56	56.4
Kuwait	67	68.5	Kuwait	68	68.5
Kyrgyzstan	63	64.6	Kyrgyzstan	58	58.7
Lao People's Democratic Republic	58	58.9	Lao People's Democratic Republic	56	56.5
Latvia	68	68.9	Latvia	61	62.0
Lebanon	71	71.7	Lebanon	68	68.3
Lesotho	44	44.5	Lesotho	42	42.0
Liberia	53	54.1	Liberia	52	52.2
Libya	65	67.7	Libya	64	64.5
Lithuania	70	69.6	Lithuania	61	61.8
Luxembourg	73	73.5	Luxembourg	70	70.5
Madagascar	56	58.0	Madagascar	54	55.5
Malawi	51	49.6	Malawi	50	46.8

Malaysia	66	67.1	Malaysia	63	64.3
Maldives	67	68.1	Maldives	66	67.0
Mali	48	48.1	Mali	49	48.4
Malta	72	71.4	Malta	70	69.3
Marshall Islands	61	63.2	Marshall Islands	57	59.6
Mauritania	54	55.5	Mauritania	52	52.9
Mauritius	68	68.3	Mauritius	62	62.2
Mexico	69	70.2	Mexico	65	65.1
Micronesia (Federated States of)	60	61.6	Micronesia (Federated States of)	59	59.7
Monaco	75	72.1	Monaco	70	67.2
Mongolia	63	63.3	Mongolia	56	56.2
Montenegro	67	68.6	Montenegro	65	65.3
Morocco	61	63.6	Morocco	60	60.7
Mozambique	46	45.9	Mozambique	45	44.4
Myanmar	58	59.3	Myanmar	56	56.1
Namibia	59	60.5	Namibia	55	56.4
Nauru	69	73.3	Nauru	64	66.6
Nepal	60	60.9	Nepal	58	59.0
Netherlands	72	72.2	Netherlands	70	70.3
New Zealand	73	71.4	New Zealand	71	69.1
Nicaragua	66	68.6	Nicaragua	61	63.0
Niger	50	50.4	Niger	50	50.2
Nigeria	47	46.3	Nigeria	46	44.9
Niue	66	68.3	Niue	62	63.4
Norway	72	72.2	Norway	70	69.4
Oman	67	71.2	Oman	65	66.4
Pakistan	57	57.9	Pakistan	56	56.4

(continued)

Table 4.8 (continued)

Country – 2012 – Females	HALE	MODEL	Country – 2012 – Males	HALE	MODEL
Palau	64	65.8	Palau	61	62.7
Panama	69	71.1	Panama	65	66.0
Papua New Guinea	55	57.7	Papua New Guinea	52	53.6
Paraguay	67	68.0	Paraguay	63	63.1
Peru	68	69.4	Peru	66	66.5
Philippines	63	63.4	Philippines	57	57.9
Poland	71	71.1	Poland	64	65.5
Portugal	73	73.0	Portugal	69	68.4
Qatar	66	71.6	Qatar	68	71.1
Republic of Korea	75	72.8	Republic of Korea	70	68.8
Republic of Moldova	66	66.5	Republic of Moldova	59	59.9
Romania	69	68.8	Romania	63	62.6
Russian Federation	66	66.3	Russian Federation	57	56.4
Rwanda	56	57.6	Rwanda	55	55.1
Saint Kitts and Nevis	66	68.0	Saint Kitts and Nevis	61	62.9
Saint Lucia	66	70.8	Saint Lucia	60	64.2
Saint Vincent and the Grenadines	65	66.1	Saint Vincent and the Grenadines	61	63.4
Samoa	66	68.4	Samoa	62	62.9
San Marino	73	69.9	San Marino	72	71.5
Sao Tome and Principe	59	60.2	Sao Tome and Principe	56	56.8
Saudi Arabia	66	68.3	Saudi Arabia	64	64.8
Senegal	56	56.6	Senegal	54	54.3
Serbia	67	68.2	Serbia	63	64.3

Seychelles	71	68.8	Seychelles	63	62.0
Sierra Leone	39	38.2	Sierra Leone	39	38.1
Singapore	77	75.3	Singapore	74	71.4
Slovakia	70	69.8	Slovakia	64	64.9
Slovenia	73	72.3	Slovenia	67	68.7
Solomon Islands	60	61.6	Solomon Islands	58	59.2
Somalia	46	47.6	Somalia	44	43.8
South Africa	53	54.3	South Africa	49	49.6
South Sudan	48	48.2	South Sudan	47	46.4
Spain	75	73.3	Spain	71	70.0
Sri Lanka	68	68.6	Sri Lanka	63	63.5
Sudan	54	56.7	Sudan	52	53.6
Suriname	68	67.1	Suriname	63	63.6
Swaziland	47	47.8	Swaziland	44	44.3
Sweden	73	71.7	Sweden	71	69.5
Switzerland	74	72.8	Switzerland	71	70.3
Syrian Arab Republic	65	66.1	Syrian Arab Republic	55	53.7
Tajikistan	60	61.7	Tajikistan	59	59.6
Thailand	68	69.0	Thailand	63	62.5
The FYROM	68	67.9	The FYROM	65	65.3
Timor-Leste	58	59.7	Timor-Leste	55	57.0
Togo	50	48.1	Togo	50	46.4
Tonga	61	61.3	Tonga	64	65.7
Trinidad and Tobago	64	66.3	Trinidad and Tobago	58	60.4
Tunisia	67	68.4	Tunisia	65	65.1

(continued)

Table 4.8 (continued)

Country – 2012 – Females	HALE	MODEL	Country – 2012 – Males	HALE	MODEL
Turkey	67	67.8	Turkey	63	62.4
Turkmenistan	59	59.8	Turkmenistan	53	52.4
Tuvalu	60	62.4	Tuvalu	57	59.3
Uganda	50	50.4	Uganda	49	48.2
Ukraine	67	66.6	Ukraine	59	58.6
United Arab Emirates	66	69.4	United Arab Emirates	66	67.4
United Kingdom	72	72.5	United Kingdom	70	69.6
United Republic of Tanzania	53	54.3	United Republic of Tanzania	51	51.1
United States of America	71	70.7	United States of America	68	66.8
Uruguay	70	71.4	Uruguay	65	66.2
Uzbekistan	62	64.0	Uzbekistan	59	59.4
Vanuatu	63	65.0	Vanuatu	61	62.1
Venezuela (Bolivarian Republic of)	69	70.6	Venezuela (Bolivarian Republic of)	63	62.7
Vietnam	69	71.5	Vietnam	62	63.8
Yemen	55	57.4	Yemen	54	54.7
Zambia	50	49.0	Zambia	48	47.5
Zimbabwe	51	51.3	Zimbabwe	48	47.2

Table 4.9 Healthy life years lost to disability: mortality model and HALE results for males

Estimates of the mortality model for the Loss of Healthy Life Years (LHLY) for males for the WHO member countries and the related results from the HALE method of the World Health Organization												
	HALE	Model	HALE	HALE	Model	HALE				Model	HALE	Model
Countries/Year	1990	1990	2000b	2000	2000	2001	2002	2010	2012	2012	2013	2013
Afghanistan	8.9	6.9	9.1	9	7.2	10.0	6.6	9.7	9	7.5	11	7.5
Albania	9.3	8.6	7.9	8	8.6	10.4	7.8	9.5	9	8.6	9	8.6
Algeria	9.8	8.3	9.7	9	8.3	11.9	7.9	10.5	8	8.3	8	8.3
Andorra	10.6	8.8	7.3	9	8.7	7.4	7.0	11.5	10	10.6	9	10.6
Angola	6.4	7.2	8.1	6	7.4	8.4	6.3	8.2	7	7.7	7	7.7
Antigua and Barbuda	9.7	7.6	10.1	9	8.5	11.8	8.9	12.9	10	9.3	10	9.2
Argentina	8.5	7.1	8.4	9	7	9.5	8.3	9.0	9	7.5	9	7.5
Armenia	8.7	6.6	7.5	8	7.4	10.8	7.6	9.0	8	7.2	8	7.3
Australia	9.7	8.1	6.9	9	9.3	7.3	7.0	10.8	10	10.0	9	9.9
Austria	9.1	8.4	6.8	9	9.1	7.0	7.1	10.7	10	9.9	11	10.0
Azerbaijan	8.1	6.9	8.4	8	6.9	10.4	7.2	9.0	8	7.3	9	7.2
Bahamas	10.8	8.6	10.8	8	9.5	14.1	8.4	12.1	10	9.4	11	9.4
Bahrain	9.4	7.9	9.7	10	8	9.9	7.9	10.1	10	9.3	10	9.3
Bangladesh	9.4	7.4	9.8	10	7.2	10.2	7.3	12.4	9	6.9	10	6.9
Barbados	8.1	8.7	9.3	9	8.7	9.5	7.6	7.7	11	8.8	11	8.8
Belarus	9.2	6.9	6.6	8	6.5	9.0	6.1	10.2	9	6.7	9	6.6
Belgium	9.9	7.9	6.9	9	8.8	7.1	6.3	11.6	10	9.8	9	9.8
Belize	8.2	8.7	11.1	10	8.3	11.4	9.0	8.5	11	9.5	11	9.5
Benin	8.6	8.0	8.5	8	8	10.9	6.6	9.4	7	8.3	7	8.3
Bhutan	8.7	6.5	10.3	8	6.8	10.5	7.3	9.6	9	7.3	9	7.3
Bolivia	9.1	7.9	9.5	8	8.1	13.1	8.2	9.7	8	7.9	8	8.0
Bosnia and Herzegovina	9.3	7.1	6.6	9	6.2	9.3	7.0	11.0	10	7.8	9	7.8

(continued)

Table 4.9 (continued)

Estimates of the mortality model for the Loss of Healthy Life Years (LHLY) for males for the WHO member countries and the related results from the HALE method of the World Health Organization												
	HALE	Model	HALE	HALE	Model	HALE				Model	HALE	Model
Countries/Year	1990	1990	2000b	2000	2000	2001	2002	2010	2012	2012	2013	2013
Botswana	8.8	9.7	6.5	6	6.7	6.4	4.2	9.4	9	7.7	10	8.0
Brazil	9.1	6.6	9.5	8	6.2	13.3	8.5	9.3	8	6.9	9	6.3
Brunei Darussalam	8.1	8.6	9.6	7	8.4	12.8	9.7	8.6	8	8.8	8	8.9
Bulgaria	7.6	7.5	6.3	8	7.8	7.5	6.2	7.4	9	7.7	9	7.6
Burkina Faso	7.3	7.9	7.2	7	8.1	8.3	5.6	7.5	7	8.2	8	8.2
Burundi	6.6	7.3	6.7	6	7.2	6.8	5.3	7.5	8	7.5	7	7.6
Cambodia	8.5	6.2	7.8	9	6.4	10.3	6.3	8.7	10	7.1	10	7.1
Cameroon	8.4	8.0	8.1	7	7.8	10.1	6.0	8.1	11	8.0	10	8.0
Canada	9.3	7.9	7.7	9	8.8	8.4	7.1	10.2	7	9.5	8	9.6
Cape Verde	9.2	7.7	9.6	9	7.9	13.6	7.9	10.1	10	8.3	9	8.4
Central African Republic	6.6	7.5	6.9	6	7.4	9.7	5.1	5.9	7	7.8	7	7.8
Chad	7.9	7.2	8.7	6	7.3	11.1	6.4	8.2	7	7.5	7	7.5
Chile	8.7	7.2	9	9	7.2	8.7	8.5	9.3	9	7.8	9	7.9
China	7.2	8.0	8	8	8.8	7.7	6.5	7.4	7	9.3	7	9.2
Colombia	8.9	8.4	8.6	9	7.1	11.4	9.7	9.3	11	7.8	10	9.8
Comoros	7.4	7.2	9.1	7	7.3	12.8	7.8	8.2	7	7.5	7	7.5
Congo	7.6	7.9	7.7	7	7.9	10.9	6.3	7.9	8	8.2	8	8.2
Cook Islands		7.8	8.3	10	9.3	11.6	8.6		10	7.7	11	7.7
Costa Rica	9.9	7.1	9.2	10	7.4	11.1	9.5	9.8	9	8.0	9	8.1
Cote d'lvoire	7.9	7.7	7.2	6	7.5	8.7	5.4	7.4	7	7.9	7	7.9
Croatia	8.8	7.0	9	9	7.3	9.2	7.2	9.8	9	7.5	10	7.5
Cuba	9.5	8.6	8.6	10	7.7	10.0	7.9	12.6	11	7.6	12	8.1
Cyprus	9.9	8.9	8.4	7	10.5	9.4	8.8	10.5	8	9.4	7	9.4

Czech Republic	8.1	7.3	8.6	9	8	8.1	6.6	9.5	9	8.7	9	8.7
Democratic People's Republic of Korea	7.7	7.7	9.6	5	7.1	10.5	6.4	7.7	7	7.5	6	7.5
Democratic Republic of the Congo	8.1	7.7	7.2	7	7.7	9.8	6.0	8.1	7	7.8	8	7.8
Denmark	9.1	7.5	5.3	9	7.9	5.5	6.3	10.5	10	8.7	9	8.7
Djibouti	8.3	7.8	7.8	7	7.8	10.0	6.1	9.3	8	8.0	8	8.1
Dominica	9.4	8.3	9.4	10	8.7	12.2	9.1	11.8	11	8.8	11	8.8
Dominican Republic	9.1	8.8	10.8	9	6.7	11.1	7.7	11.2	12	7.2	11	6.4
Ecuador	9.5	6.7	9.9	9	6.7	11.1	8.1	10.0	9	9.7	9	9.7
Egypt	10.1	7.7	8.3	9	7.6	8.9	7.4	10.5	9	7.6	8	7.6
El Salvador	9.0	6.9	11	8	6	12.7	9.3	9.4	9	6.6	8	6.7
Equatorial Guinea	6.6	7.6	8.7	7	7.6	10.6	7.2	8.0	8	7.8	8	7.9
Eritrea	7.6	5.6	7.7	5	5.3	9.9	6.5	8.7	9	6.4	8	6.4
Estonia	7.6	7.1	9.3	8	6.7	7.7	6.0	8.9	9	7.8	9	7.3
Ethiopia	6.0	7.6	7.1	7	8	10.0	6.1	8.1	8	8.6	9	8.7
Fiji	8.8	7.6	8.3	9	7	11.0	7.7	8.5	9	7.0	9	7.0
Finland	10.0	7.6	7.6	8	8.5	6.8	6.1	11.8	10	9.9	10	9.8
France	9.4	8.6	6.7	8	9.2	6.6	6.7	10.5	10	10.2	10	10.2
Gabon	8.0	8.3	7.8	9	8.2	9.8	7.1	7.6	9	8.6	9	8.5
Gambia	8.1	8.3	8.6	7	8.4	11.1	6.9	8.5	7	8.5	8	8.5
Georgia	8.0	7.4	9.6	8	7.5	7.9	6.2	8.1	8	7.6	9	7.6
Germany	9.1	8.0	6.9	8	8.9	6.8	5.9	10.4	9	9.8	10	9.8
Ghana	8.9	9.9	8.5	8	10.5	10.0	7.2	8.7	8	11.2	9	10.0
Greece	9.5	8.4	5.7	9	8.7	6.5	6.7	10.1	9	9.1	10	9.5
Grenada	9.3	7.3	8.8	9	7	9.7	7.5	11.2	9	7.3	10	7.3
Guatemala	8.6	6.8	10.1	8	8.5	12.2	8.2	8.8	9	8.7	8	8.7
Guinea	7.8	8.2	8.6	7	8.2	10.1	7.0	8.6	8	8.4	8	8.4
Guinea-Bissau	7.4	7.8	7.7	8	8	9.8	6.1	8.1	7	8.0	8	8.0

(continued)

Table 4.9 (continued)

Estimates of the mortality model for the Loss of Healthy Life Years (LHLY) for males for the WHO member countries and the related results from the HALE method of the World Health Organization

Countries/Year	HALE 1990	Model 1990	HALE 2000b	HALE 2000	Model 2000	HALE 2001	2002	2010	2012	Model 2012	HALE 2013	Model 2013
Guyana	8.5	6.0	10.1	8	6.6	9.7	8.4	10.6	8	7.5	8	7.5
Haiti	8.2	7.7	8.4	8	7.8	7.1	5.6	4.7	11	7.9	11	7.9
Honduras	9.2	8.0	10.6	10	8	12.3	7.9	9.5	10	8.2	10	8.2
Hungary	8.1	7.3	11	10	7.3	9.3	6.8	9.3	10	7.1	10	7.1
Iceland	11.5	8.5	7.3	9	9.3	7.6	6.3	13.1	9	11.0	10	10.3
India	8.0	5.9	7.6	9	6	8.4	6.8	8.3	8	6.2	9	6.2
Indonesia	8.4	7.5	6.9	8	7.8	8.3	7.5	8.4	8	8.1	8	8.1
Iran, Islamic Republic of	9.3	7.4	9.1	9	8.3	10.9	10.4	10.1	9	8.5	9	8.5
Iraq	9.6	8.1	9.2	8	8.2	11.0	10.3	9.8	8	7.7	8	7.7
Ireland	8.8	7.6	6.3	8	8.3	6.1	6.3	10.4	10	9.2	10	9.2
Israel	9.6	8.5	7.3	10	9.1	8.1	6.9	10.9	10	9.8	10	9.7
Italy	9.2	8.2	6.4	9	8.9	7.0	6.0	10.6	9	9.8	9	9.9
Jamaica	9.7	8.9	10	8	9	9.9	6.9	12.3	11	9.4	11	9.3
Japan	8.3	8.9	6.3	8	9	6.5	6.1	8.7	8	9.4	8	9.5
Jordan	9.8	8.1	10.3	8	8.3	11.4	9.0	10.9	8	8.5	8	8.6
Kazakhstan	7.5	6.0	7.5	7	6	9.8	6.1	7.4	7	6.7	7	6.6
Kenya	8.6	8.3	7	6	8	8.7	5.7	8.5	7	8.3	8	8.4
Kiribati	8.4	7.4	7.6	8	7.6	10.6	9.5	8.2	8	7.9	8	7.9
Kuwait	11.0	8.5	9.6	9	9.2	10.8	8.2	10.8	10	9.6	10	9.6
Kyrgyzstan	8.2	5.7	10.4	7	7.4	12.5	8.2	8.1	8	6.8	8	6.8
Lao People's Democratic Republic	7.3	7.1	8.6	8	7.5	11.1	7.0	8.3	8	7.9	9	7.9
Latvia	8.1	6.8	12.8	8	6.6	10.1	6.6	8.9	8	7.0	8	7.0

Lebanon	9.1	8.0	8.9	9	8.6	11.1	8.4	10.3	10	9.3	9	9.4
Lesotho	7.6	7.1	5.9	6	6.7	6.9	3.3	6.4	7	6.9	7	6.9
Liberia	7.4	7.5	8.4	8	7.7	9.3	6.5	8.9	9	8.3	9	8.3
Libyan Arab Jamahiriya	10.5	8.0	9.2	9	8.3	11.4	8.1	10.7	9	8.7	9	8.8
Lithuania	8.4	6.6	13.3	9	6.8	10.8	7.2	8.7	8	6.6	9	6.7
Luxembourg	9.0	7.7	6.3	9	8.3	6.4	6.4	11.1	10	9.1	10	9.2
Madagascar	7.9	7.1	8.5	8	6.8	11.1	7.2	9.2	8	7.0	9	7.0
Malawi	7.2	9.3	5.8	6	9.5	6.7	4.8	7.2	9	10.8	8	11.1
Malaysia	8.9	7.1	8.6	8	7.2	11.7	8.0	8.7	9	7.2	9	7.2
Maldives	9.2	8.3	10.4	8	6.7	14.3	7.5	10.2	10	8.7	10	8.8
Mali	6.8	8.0	7.9	7	8	10.5	6.4	8.1	8	8.5	7	8.5
Malta	9.7	8.2	6.7	9	8.5	8.2	6.2	10.4	10	9.5	9	9.2
Marshall Islands	9.4	7.5	7.9	8	7.9	10.3	7.2	8.8	10	8.3	10	8.3
Mauritania	9.3	8.3	9.6	9	8.4	11.4	6.9	9.8	9	8.6	9	8.6
Mauritius	7.8	7.4	9.1	8	7.6	11.0	8.1	8.5	8	8.1	8	8.1
Mexico	7.8	8.8	7.9	8	8.9	9.0	8.3	7.8	8	7.6	8	8.8
Micronesia, Federated States of	8.4	7.9	8	9	7.9	10.6	7.9	8.2	9	8.1	9	8.1
Monaco		8.7	7.4	9	8.9	7.5	7.1		9	11.4	9	11.4
Mongolia	7.2	7.5	10.9	7	7.4	11.4	6.8	7.3	8	7.3	7	7.3
Morocco	10.3	8.1	10.8	9	8.2	12.6	9.4	10.6	9	8.2	9	8.2
Mozambique	7.2	7.3	6.4	6	7.5	9.3	4.9	7.1	7	7.9	7	7.9
Myanmar		7.3	8.5	8	7.5	8.2	6.3		8	7.7	8	7.7
Namibia	8.3	7.4	6.3	8	7.3	8.6	5.2	8.4	9	7.9	10	8.1
Nauru		7.4	8.3	11	7.8	9.9	6.9		11	8.1	11	8.1
Nepal	8.7	7.2	11	8	7.6	9.9	7.4	10.1	9	7.9	9	8.0
Netherlands	9.3	7.7	7.3	9	9.1	7.1	6.3	10.6	9	9.8	9	9.9
New Zealand	9.7	7.9	6.4	9	8.6	6.9	7.2	10.9	9	9.9	9	9.9

(continued)

Table 4.9 (continued)

Estimates of the mortality model for the Loss of Healthy Life Years (LHLY) for males for the WHO member countries and the related results from the HALE method of the World Health Organization

	HALE	Model	HALE	HALE	Model	HALE				Model	HALE	Model
Countries/Year	1990	1990	2000b	2000	2000	2001	2002	2010	2012	2012	2013	2013
Nicaragua	9.4	8.4	10.6	9	7.6	12.7	8.2	9.6	8	7.4	9	7.4
Niger	6.6	8.0	8.8	7	8.2	10.2	6.8	8.4	9	8.4	8	8.4
Nigeria	8.1	7.9	7.7	7	7.9	10.6	6.8	8.8	7	8.2	7	8.2
Niue		7.8	8.7	10	8	11.3	8.6		10	8.2	10	8.2
Norway	10.7	8.2	6.9	10	9.2	6.8	5.9	12.2	11	10.1	11	9.9
Oman	9.7	7.8	10.3	9	7.7	10.4	8.3	10.2	9	7.8	9	7.8
Pakistan	8.6	8.0	10	9	8	10.7	6.9	8.7	8	8.1	9	8.1
Palau		7.5	8.2	9	7.9	11.4	7.7		10	8.2	10	8.3
Panama	9.7	8.0	8.9	10	8	10.8	8.5	9.3	9	8.1	9	8.2
Papua New Guinea	7.9	5.7	8.5	7	5.9	10.5	7.0	7.9	8	6.2	8	6.2
Paraguay	10.3	8.9	10.3	9	7.5	12.9	9.1	9.7	9	8.6	9	8.6
Peru	9.8	9.0	8.9	9	8.4	11.5	7.9	10.4	10	8.9	10	8.9
Philippines	9.2	7.6	7.7	9	7.8	13.1	8.0	9.2	8	7.3	8	7.3
Poland	8.3	7.1	10	9	7.4	7.8	7.5	9.3	9	7.2	10	7.3
Portugal	8.9	7.9	7.8	8	8.4	8.5	6.9	9.9	9	9.0	10	9.0
Qatar	11.8	9.8	11.1	11	8	11.5	8.2	12.7	11	7.9	11	8.0
Republic of Korea	7.5	7.4	7.3	8	7.7	6.7	6.9	8.6	8	8.9	8	9.2
Republic of Moldova	7.8	6.8	7.7	7	6.6	10.0	6.8	8.0	7	6.6	7	6.7
Romania	8.2	7.5	6.8	8	7.6	9.2	7.0	8.7	8	7.9	8	7.8
Russian Federation	7.7	6.6	9.1	8	6.5	7.4	5.5	7.7	8	6.9	8	6.9
Rwanda	6.8	7.3	6.5	7	8.1	7.3	5.6	8.8	9	8.2	9	8.4
Saint Kitts and Nevis		8.2	8.4	8	9	10.2	8.7		10	8.0	11	8.0
Saint Lucia	9.5	7.8	8.5	9	7	10.7	8.6	11.9	11	7.2	12	7.2

Saint Vincent and the Grenadines	9.3	7.8	8	8	7.8	10.3	7.9	11.6	11	8.2	11	8.2
Samoa	8.7	6.3	8.5	8	6.8	11.0	7.6	8.6	8	7.2	8	7.2
San Marino		9.7	6.5	9	10.5	7.2	6.3		10	10.3	10	10.3
Sao Tome and Principe	9.0	7.9	10	8	8	14.8	7.5	9.7	9	8.2	9	8.2
Saudi Arabia	10.8	8.1	9.7	10	8.2	10.9	8.6	11.1	10	9.1	9	9.1
Senegal	7.9	8.3	8.8	8	8.3	11.3	7.3	8.7	9	8.5	8	8.5
Seychelles	7.4	6.8	9.5	6	8.7	11.3	9.6	7.1	6	7.4	7	7.5
Sierra Leone	7.2	7.1	7.3	5	7	8.6	5.1	8.9	6	7.4	7	7.5
Singapore	8.0	8.1	8.6	6	8.4	8.6	8.6	9.2	6	8.7	6	9.7
Slovakia	8.5	7.3	9.7	9	7.4	7.7	6.7	9.2	9	7.3	9	7.4
Slovenia	8.6	7.9	7.4	9	8.2	7.0	6.1	10.2	11	8.2	11	8.3
Solomon Islands	7.9	7.3	8.6	9	7.6	12.0	8.3	7.5	9	7.9	8	7.9
Somalia	7.1	6.9	8.3	7	7	8.5	6.9	7.8	7	7.1	8	7.2
South Africa	8.1	6.4	6.6	7	6.5	7.7	5.5	8.3	7	6.7	8	6.7
Spain	8.5	8.1	6.6	8	8.5	6.6	6.2	9.6	8	9.1	9	9.0
Sri Lanka	9.4	6.3	9	8	6.1	11.5	8.0	9.3	8	8.0	9	7.9
Sudan	10.1	7.2	9.8	9	7.2	11.2	7.8	11.0	9	7.4	9	7.4
Suriname	9.1	8.9	8.5	10	9.9	10.0	7.6	11.6	12	10.9	11	11.1
Swaziland	8.4	7.6	6	7	7.3	6.4	3.7	7.0	8	7.9	8	7.8
Sweden	9.9	8.6	7.2	10	9.5	7.2	6.2	11.2	10	10.4	10	10.4
Switzerland	9.1	8.8	6.2	9	9.8	6.2	6.6	10.6	11	10.3	10	10.4
Syrian Arab Republic	10.1	8.3	9.7	10	8.5	10.7	8.5	10.5	7	8.5	8	8.7
Tajikistan	8.2	7.7	10.8	7	7.7	12.8	7.9	8.7	8	7.8	8	7.8
Thailand	8.5	7.2	8.4	8	8	9.3	8.4	8.2	8	8.6	8	8.7
The FYROM	8.9	7.7	6.3	9	7.4	8.5	7.2	9.6	9	8.1	10	8.2
Togo	8.4	9.3	7.9	8	10.3	9.7	6.5	8.3	8	10.7	8	10.7
Tonga	9.7	6.9	8.1	8	7.2	11.0	8.2	8.4	10	7.9	10	8.0

(continued)

Table 4.9 (continued)

Estimates of the mortality model for the Loss of Healthy Life Years (LHLY) for males for the WHO member countries and the related results from the HALE method of the World Health Organization

Countries/Year	HALE 1990	Model 1990	HALE 2000b	HALE 2000	Model 2000	HALE 2001	2002	2010	2012	Model 2012	HALE 2013	Model 2013
Trinidad and Tobago	8.6	6.5	8.2	8	6.5	8.4	7.3	10.5	9	6.6	9	6.6
Tunisia	9.0	8.5	8.2	8	8.5	10.1	8.2	9.5	9	8.5	9	8.6
Turkey	8.4	8.4	10	9	8.9	8.5	6.7	9.4	9	9.3	9	9.3
Turkmenistan	7.5	6.9	8.8	6	6.9	12.1	7.1	8.3	7	7.2	7	7.2
Tuvalu		6.1	7.2	9	6.3	9.9	7.0		9	6.7	9	6.7
Uganda	7.6	7.4	7.2	6	7.3	9.0	6.2	8.2	8	7.9	8	8.0
Ukraine	8.0	6.9	10.3	7	6.8	9.3	6.8	7.9	7	7.1	7	7.1
United Arab Emirates	10.1	7.6	10	9	7.9	9.0	7.8	10.6	10	8.3	9	8.4
United Kingdom	9.5	7.6	6.5	10	8.6	6.6	6.7	10.7	10	9.2	10	9.2
United Republic of Tanzania	8.3	7.8	7.2	7	7.9	9.5	5.5	9.1	8	8.3	9	8.3
United States of America	8.7	7.6	8.2	8	8.5	8.0	7.4	9.7	9	9.4	8	9.4
Uruguay	8.3	8.3	8.4	8	6.7	9.7	8.0	8.6	8	7.2	9	7.2
Uzbekistan	8.4	7.5	9.4	7	7.3	11.7	7.6	8.5	8	7.1	8	7.1
Vanuatu	8.1	7.6	8.2	9	7.9	11.0	8.0	7.9	9	8.2	10	8.2
Venezuela, Bolivarian Republic of	8.9	7.6	10.1	9	7.9	13.7	9.3	8.6	9	9.3	9	9.3
Vietnam	8.8	6.8	8.5	9	6.7	11.0	7.4	9.0	9	6.8	9	6.8
Yemen	9.5	7.4	10.4	8	7.5	12.9	10.8	10.2	8	7.6	9	7.6
Zambia	7.3	6.7	5.5	5	6.8	6.2	4.3	7.5	7	7.8	8	7.9
Zimbabwe	9.6	8.7	5.8	5	6.7	5.6	3.9	7.8	9	8.7	8	8.9
Method	HALE	Model	HALEb	HALE	Mode	HALE	HALE	HALE	HALE	Mode	HALE	Mode
Year	1990	1990	2000	2000	2000	2001	2002	2010	2012	2012	2013	2013
Mean	8.67	**7.69**	8.41	8.18	**7.83**	9.73	7.19	9.33	8.79	**8.25**	8.88	8.28

Table 4.10 Healthy life years lost to disability: mortality model and HALE results for females

Estimates of the mortality model for the Loss of Healthy Life Years (LHLY) for females for the WHO member countries and the related results from the HALE method of the World Health Organization												
	HALE	Model	HALE	HALE	Model	HALE				Model	HALE	Model
Countries/Year	1990	1990	2000b	2000	2000	2001	2002	2010	2012	2012	2013	2013
Afghanistan	10.1	7.1	12.5	11	7.5	8.1	7.7	11.1	12	7.9	12	8.0
Albania	11.0	11.3	10.6	9	10.4	11.7	10.8	11.1	9	10.1	10	10.0
Algeria	11.2	8.9	12.9	10	8.9	11.2	9.6	11.9	10	8.9	11	9.0
Andorra	12.5	11.1	10.1	11	11.3	10.0	9.1	13.0	12	12.5	12	13.0
Angola	8.3	7.4	10.8	7	7.6	6.5	6.9	9.9	7	7.9	7	8.0
Antigua and Barbuda	11.2	9.5	14.5	9	9.2	10.9	10.3	13.5	11	9.7	11	10.0
Argentina	10.2	9.0	11.9	10	8.8	12.0	10.0	10.6	10	9.5	11	10.0
Armenia	10.7	7.9	10.1	10	8.3	11.9	10.4	11.3	9	8.9	9	9.0
Australia	11.2	9.7	8.8	11	10.4	9.5	8.7	12.0	11	11.6	11	12.0
Austria	10.7	10.4	8.9	10	11.1	8.8	8.6	12.1	11	11.8	11	12.0
Azerbaijan	10.5	7.8	11.4	9	7.8	11.2	10.0	11.1	10	8.6	10	9.0
Bahamas	12.2	9.7	15.7	11	11.3	12.5	9.7	13.9	12	10.8	11	11.0
Bahrain	9.9	7.9	12.4	12	7.6	12.2	10.1	11.2	12	9.6	12	10.0
Bangladesh	11.5	7.9	12.9	10	7.8	9.2	9.3	12.3	10	7.7	10	8.0
Barbados	10.4	8.4	13.4	12	8.7	10.6	9.8	10.4	13	10.2	13	10.0
Belarus	11.2	8.4	9.2	10	8.1	11.4	9.4	11.7	10	9.1	10	9.0
Belgium	11.7	10.2	9.9	10	10.6	9.4	8.2	12.1	11	11.4	11	11.0
Belize	10.3	9.8	14.3	11	9.5	11.3	10.2	10.8	12	9.8	12	10.0
Benin	9.2	8.5	11.9	9	8.3	9.2	7.9	10.2	9	8.6	9	9.0

(continued)

Table 4.10 (continued)

Estimates of the mortality model for the Loss of Healthy Life Years (LHLY) for females for the WHO member countries and the related results from the HALE method of the World Health Organization

Countries/Year	HALE 1990	Model 1990	HALE 2000b	HALE 2000	Model 2000	HALE 2001	2002	2010	2012	Model 2012	HALE 2013	Model 2013
Bhutan	9.4	6.0	14.3	8	6.1	9.9	9.5	10.2	10	6.4	9	6.0
Bolivia	11.1	8.5	12.1	9	8.8	10.7	9.4	10.7	9	8.6	9	9.0
Bosnia and Herzegovina	10.8	8.8	9.4	10	7.1	11.5	10.0	12.7	11	9.3	10	9.0
Botswana	10.7	7.9	7.9	6	7.4	5.9	5.2	11.1	10	7.5	10	8.0
Brazil	10.0	8.2	12.7	10	7.7	11.0	9.8	10.5	10	8.1	11	7.0
Canada	11.1	9.5	9.8	10	10.3	10.4	8.3	11.8	8	10.7	9	11.0
Cape Verde	12.0	8.4	12.3	12	9.0	11.3	10.0	12.7	11	9.7	11	10.0
Central African Republic	8.6	7.6	8.9	7	7.2	7.7	6.1	7.6	8	7.9	8	8.0
Chad	9.2	7.4	11.2	7	7.4	8.7	7.6	9.2	8	7.6	8	8.0
Chile	10.2	8.7	12.1	10	8.7	11.7	10.3	10.5	11	9.2	11	9.0
China	8.0	8.7	9.7	7	9.7	8.4	7.6	8.6	8	10.1	8	10.0
Colombia	11.0	8.7	11.8	12	7.5	12.7	10.0	11.2	14	8.4	12	11.0
Comoros	8.6	7.6	12.3	8	7.7	11.0	9.6	9.3	9	7.9	8	8.0
Congo	9.8	8.1	10.1	8	7.9	8.7	7.2	10.0	9	8.4	9	8.0
Cook Islands		7.4	11.0	11	8.1	11.4	11.5		12	10.1	12	10.0
Costa Rica	11.4	9.2	12.4	10	8.8	11.6	10.3	11.4	10	9.2	10	9.0
Cote d'lvoire	10.1	8.0	9.5	7	7.4	7.7	6.7	9.6	8	7.8	8	8.0
Croatia	11.3	9.3	10.6	10	9.6	10.2	9.3	11.6	11	9.9	11	10.0
Cuba	11.0	10.1	10.9	11	8.2	10.8	9.8	12.9	13	8.7	12	9.0
Cyprus	11.9	11.6	12.7	8	12.6	12.0	10.6	12.3	8	12.7	8	13.0
Czech Republic	10.3	9.2	9.9	9	9.8	9.3	8.1	11.1	10	10.6	10	11.0
Democratic People's Rep. of Korea	8.7	9.8	11.2	8	9.2	10.3	7.4	8.9	8	9.6	8	10.0

Democratic Republic of the Congo	9.2	7.7	9.6	8	7.7	8.2	7.0	9.6	8	7.9	8	8.0
Denmark	10.6	9.1	8.4	9	8.9	8.7	8.4	11.5	11	9.6	11	10.0
Djibouti	9.6	8.2	10.1	9	8.2	8.1	7.4	10.3	10	8.5	9	9.0
Dominica	10.7	9.3	12.2	10	9.7	11.2	10.2	12.9	12	9.1	12	9.0
Brunei Darussalam	9.8	8.5	12.7	10	8.0	12.2	11.9	10.2	9	9.3	10	9.0
Bulgaria	8.9	9.3	9.2	9	9.5	9.6	8.5	8.8	10	10.0	10	10.0
Burkina Faso	8.5	8.2	9.5	7	8.3	7.2	6.3	9.3	8	8.4	8	8.0
Burundi	7.7	7.2	8.5	7	7.3	6.6	6.2	8.4	8	7.6	9	8.0
Cambodia	9.6	6.6	9.8	10	7.0	9.1	7.6	10.1	12	8.0	12	8.0
Cameroon	9.9	8.0	10.5	8	7.8	8.4	7.3	9.7	12	8.1	11	8.0
Dominican Republic	10.7	9.8	14.0	10	7.6	10.7	9.6	11.8	12	8.5	10	7.0
Ecuador	11.1	7.4	12.0	11	7.4	10.8	9.4	11.3	10	10.5	11	11.0
Egypt	12.0	8.2	12.0	11	8.2	10.8	8.8	12.6	11	8.3	11	8.0
El Salvador	11.2	7.9	13.9	10	7.1	11.5	10.4	11.2	11	7.1	11	7.0
Equatorial Guinea	8.7	7.6	11.4	8	7.6	9.2	8.5	10.7	10	7.9	9	8.0
Eritrea	9.1	6.2	10.4	8	6.7	9.1	8.6	10.0	11	7.5	10	8.0
Estonia	10.0	8.9	11.0	9	9.2	10.4	8.1	11.3	10	10.3	11	10.0
Ethiopia	6.9	7.6	9.6	8	8.0	8.5	7.7	8.8	9	9.1	9	9.0
Fiji	9.9	8.4	10.7	10	8.0	11.0	9.7	9.8	11	8.1	10	8.0
Finland	12.5	9.8	9.5	10	10.9	8.8	8.0	13.7	11	11.8	11	12.0
France	11.7	10.9	10.2	11	11.4	9.5	8.8	12.4	11	12.3	11	12.0
Gabon	10.6	8.6	10.4	9	8.5	9.0	8.8	10.5	10	8.8	10	9.0
Gambia	9.7	8.4	12.1	9	8.6	10.1	8.4	9.8	10	8.9	9	9.0
Georgia	10.5	8.8	11.6	10	8.9	10.2	8.4	11.0	10	9.2	10	9.0
Germany	10.9	10.0	9.2	10	10.5	8.9	7.6	11.9	10	11.6	10	12.0
Ghana	10.5	11.2	11.0	9	10.5	9.2	8.5	10.6	10	11.2	9	11.0

(continued)

Table 4.10 (continued)

Estimates of the mortality model for the Loss of Healthy Life Years (LHLY) for females for the WHO member countries and the related results from the HALE method of the World Health Organization

Countries/Year	HALE 1990	Model 1990	HALE 2000b	HALE 2000	Model 2000	HALE 2001	2002	2010	2012	Model 2012	HALE 2013	Model 2013
Greece	11.4	10.1	8.5	10	11.0	8.9	8.1	11.7	10	12.2	11	12.0
Grenada	10.9	8.8	11.5	11	9.3	9.7	8.9	11.8	12	9.9	11	10.0
Guatemala	9.8	7.2	12.6	10	8.9	11.9	9.1	10.2	10	9.1	10	9.0
Guinea	8.9	8.3	11.9	9	8.1	9.1	8.2	9.9	9	8.5	9	9.0
Guinea-Bissau	8.7	8.2	10.5	8	8.1	8.2	7.2	9.1	9	8.2	8	8.0
Guyana	10.5	6.6	14.2	9	7.8	10.0	9.7	11.5	10	7.8	10	8.0
Haiti	8.7	8.0	11.2	8	8.3	7.4	6.9	6.5	11	8.2	11	8.0
Honduras	11.0	8.8	13.2	11	8.9	10.7	9.9	11.0	12	9.1	12	9.0
Hungary	10.4	8.8	10.7	10	9.2	10.5	8.6	11.1	11	9.5	11	10.0
Iceland	12.8	9.8	9.3	10	10.7	9.4	8.2	14.5	11	12.1	11	12.0
India	9.2	6.4	11.0	10	6.7	10.4	8.4	9.8	10	7.1	9	7.0
Indonesia	9.2	8.1	9.1	9	8.4	10.1	9.1	9.3	9	8.8	9	9.0
Iran, Islamic Republic of	11.5	7.6	11.4	10	8.9	13.2	12.5	12.5	11	9.8	11	10.0
Iraq	10.5	8.9	12.1	11	9.1	9.6	11.6	10.5	11	9.1	10	9.0
Ireland	10.4	9.1	8.8	9	9.5	8.9	8.2	11.7	10	10.1	10	10.0
Israel	10.8	9.4	10.0	10	10.2	10.0	9.0	12.0	11	11.2	10	11.0
Italy	11.2	10.2	9.6	11	10.8	9.3	7.8	12.0	11	11.9	11	12.0
Jamaica	11.0	9.3	11.5	11	9.7	10.0	8.6	12.7	11	10.1	11	10.0
Japan	9.7	10.9	8.4	9	11.1	8.9	7.5	10.4	10	11.7	9	12.0
Jordan	11.3	8.7	13.6	10	8.9	13.6	10.9	11.9	10	9.2	11	9.0
Kazakhstan	10.0	7.3	10.3	8	7.5	11.3	9.6	9.8	8	9.3	9	9.0
Kenya	9.7	8.5	9.4	7	8.4	7.5	7.1	10.1	8	8.7	9	9.0

Kiribati	9.3	8.0	10.1	10	8.3	10.5	11.0	10.3	9	8.7	9	9.0
Kuwait	12.6	9.7	12.0	11	9.8	10.2	10.6	12.6	12	10.0	12	10.0
Kyrgyzstan	10.3	7.1	13.2	9	8.9	12.8	10.6	10.5	9	8.3	9	8.0
Lao People's Democratic Republic	8.4	7.4	10.4	9	7.9	9.6	9.2	9.3	9	8.5	10	9.0
Latvia	10.5	8.7	11.6	10	9.2	11.1	8.3	11.3	11	9.6	10	10.0
Lebanon	10.6	8.8	12.2	10	9.3	9.8	10.4	11.4	11	10.1	11	10.0
Lesotho	10.1	7.4	7.7	6	7.2	6.3	5.0	8.1	8	7.2	8	7.0
Liberia	8.9	7.9	11.7	8	8.2	8.3	6.7	10.0	10	8.7	10	9.0
Libyan Arab Jamahiriya	12.5	8.8	12.4	12	9.1	10.8	10.5	12.9	12	9.5	12	9.0
Lithuania	10.6	9.0	14.0	10	9.5	12.6	9.9	10.9	11	9.9	9	10.0
Luxembourg	11.2	9.4	8.7	10	9.9	9.0	8.0	12.3	11	10.4	11	10.0
Madagascar	9.0	7.3	12.0	9	7.1	9.7	8.4	10.4	9	7.5	10	8.0
Malawi	8.2	8.9	7.4	6	8.6	6.3	5.8	8.5	9	10.0	9	10.0
Malaysia	10.1	8.2	10.7	10	8.9	11.2	10.0	10.1	10	9.1	10	9.0
Maldives	9.7	8.1	13.8	9	8.3	10.1	9.0	11.5	10	9.8	11	10.0
Mali	8.2	8.0	10.5	8	8.0	8.5	7.4	9.3	9	8.5	9	9.0
Malta	11.6	10.9	8.6	10	10.6	9.5	8.3	12.4	11	11.3	10	12.0
Marshall Islands	10.7	8.0	10.4	9	8.5	9.6	8.9	10.2	11	9.1	12	9.0
Mauritania	10.5	8.7	12.7	11	8.8	9.5	8.2	10.7	11	9.0	10	9.0
Mauritius	9.5	8.2	12.2	10	8.8	17.2	10.9	10.1	10	9.5	10	9.0
Mexico	9.4	9.8	10.9	9	9.9	11.8	9.3	9.3	10	8.3	9	10.0
Micronesia, Federated States of	9.3	8.3	10.3	10	8.4	10.3	9.6	9.7	10	8.6	9	9.0
Monaco		11.2	10.5	11	10.9	10.5	9.3		11	13.3	11	13.0
Mongolia	8.3	8.6	12.4	8	8.1	10.3	8.0	9.0	9	8.4	8	8.0
Morocco	12.0	8.7	16.0	11	8.8	15.5	11.9	12.5	12	8.9	11	9.0
Mozambique	9.1	7.5	8.4	8	7.6	8.3	6.4	8.8	8	7.9	8	8.0
Myanmar		7.8	10.7	9	8.0	8.5	8.4		10	8.3	9	8.0

(continued)

Table 4.10 (continued)

Estimates of the mortality model for the Loss of Healthy Life Years (LHLY) for females for the WHO member countries and the related results from the HALE method of the World Health Organization												
	HALE	Model	HALE	HALE	Model	HALE				Model	HALE	Model
Countries/Year	1990	1990	2000b	2000	2000	2001	2002	2010	2012	2012	2013	2013
Namibia	10.2	8.0	7.9	8	6.7	8.0	6.7	9.8	10	8.1	10	8.0
Nauru		8.1	11.1	13	9.2	9.6	9.0		14	9.6	14	10.0
Nepal	9.1	7.4	13.8	9	7.9	8.8	9.1	10.7	9	8.5	10	9.0
Netherlands	11.6	9.1	9.7	11	10.2	9.6	8.5	12.4	11	11.3	11	11.0
New Zealand	11.0	10.3	8.9	10	10.7	9.4	9.0	12.0	11	11.4	11	11.0
Nicaragua	11.0	8.2	13.0	11	7.2	10.7	9.3	11.2	10	7.5	11	8.0
Niger	7.8	8.2	11.5	8	8.3	8.5	7.5	9.3	9	8.5	8	9.0
Nigeria	9.2	7.9	10.3	7	7.8	8.9	7.8	9.6	8	8.2	8	8.0
Niue		8.9	11.4	11	9.2	11.6	11.3		12	9.4	12	9.0
Norway	12.6	9.7	9.1	12	10.7	9.3	8.1	13.4	12	11.2	12	11.0
Oman	11.8	7.6	13.2	12	7.3	12.9	11.1	12.5	11	7.1	12	7.0
Pakistan	9.3	8.4	14.7	10	8.5	10.0	9.3	9.8	9	8.5	10	8.0
Palau		8.0	10.4	10	8.5	10.7	10.4		11	9.1	11	9.0
Panama	11.6	11.3	11.0	11	8.9	11.1	10.2	11.2	11	9.0	10	9.0
Papua New Guinea	8.3	6.5	10.4	9	6.7	9.5	9.1	8.8	10	6.9	10	7.0
Paraguay	11.5	9.6	12.3	12	8.8	11.0	10.5	11.2	11	9.7	11	10.0
Peru	10.7	9.1	11.8	10	8.6	10.8	9.6	11.0	11	9.4	11	9.0
Philippines	11.0	8.4	10.2	11	9.1	11.7	10.2	10.6	9	8.8	9	9.0
Poland	10.5	9.7	13.4	10	9.6	11.5	10.2	11.2	11	9.8	10	10.0
Portugal	10.9	9.9	10.7	10	10.4	10.7	8.8	11.6	11	10.9	11	11.0
Qatar	13.3	9.2	13.2	12	7.6	11.2	10.0	14.7	14	7.9	13	8.0

Republic of Korea	9.1	8.9	9.5	9	10.1	8.4	8.6	10.1	10	11.6	10	12.0
Republic of Moldova	9.8	7.7	8.9	9	7.7	10.9	9.2	10.0	9	8.0	9	8.0
Romania	9.8	8.7	9.5	9	8.4	11.2	9.7	10.3	9	9.3	9	9.0
Russian Federation	10.3	8.4	11.4	9	8.3	10.4	7.7	10.2	9	8.8	9	9.0
Rwanda	8.1	7.4	8.7	7	8.3	6.8	6.6	10.7	10	8.7	10	9.0
Saint Kitts and Nevis		9.5	10.5	10	10.4	10.0	9.1		12	9.5	12	10.0
Saint Lucia	11.1	7.8	10.9	12	7.2	10.6	10.2	12.4	13	8.0	13	8.0
Saint Vincent and the Grenadines	11.1	8.8	11.3	10	8.8	10.2	9.8	12.0	11	10.0	11	10.0
Samoa	9.9	7.3	11.3	10	7.8	10.4	9.4	10.2	11	8.2	10	8.0
San Marino		12.0	9.5	11	12.2	9.8	8.1		11	13.6	11	14.0
Sao Tome and Principe	10.3	8.1	12.2	10	8.2	10.3	9.0	11.5	11	8.4	10	8.0
Saudi Arabia	12.9	9.6	12.8	12	10.0	11.0	11.0	13.3	12	9.2	12	9.0
Senegal	9.9	8.7	11.6	10	8.7	9.5	8.4	10.6	10	9.0	10	9.0
Seychelles	9.6	9.0	13.8	8	10.1	13.6	12.3	9.1	7	9.6	7	10.0
Sierra Leone	8.9	7.4	9.6	6	7.4	6.9	5.8	10.2	7	7.6	6	8.0
Singapore	9.7	9.1	11.3	8	9.8	11.6	10.4	10.7	8	9.4	7	11.0
Slovakia	11.1	9.3	12.3	9	9.4	10.7	8.9	10.8	10	10.0	10	10.0
Slovenia	11.0	10.0	10.2	11	10.3	9.2	8.2	11.8	11	11.1	12	11.0
Solomon Islands	8.6	7.5	11.3	10	8.0	11.5	10.3	8.7	10	8.5	9	8.0
Somalia	8.3	7.1	11.2	8	7.3	7.9	8.1	9.0	9	7.5	9	7.0
South Africa	10.2	7.7	8.6	9	7.8	7.6	7.3	9.6	9	8.0	10	8.0
Spain	10.4	10.4	9.8	10	11.0	9.6	7.7	11.2	10	11.7	11	12.0
Sri Lanka	11.1	7.7	11.7	10	8.5	11.4	10.3	11.2	10	9.7	10	10.0
Sudan	11.4	7.5	13.4	10	7.7	9.8	9.4	12.6	11	7.9	11	8.0

(continued)

Table 4.10 (continued)

Estimates of the mortality model for the Loss of Healthy Life Years (LHLY) for females for the WHO member countries and the related results from the HALE method of the World Health Organization

Countries/Year	HALE 1990	Model 1990	HALE 2000b	HALE 2000	Model 2000	HALE 2001	2002	2010	2012	Model 2012	HALE 2013	Model 2013
Suriname	10.7	11.5	11.9	12	13.7	10.0	10.0	12.2	12	13.1	12	13.0
Swaziland	10.2	7.1	8.0	7	7.1	6.1	5.2	8.1	8	7.6	8	8.0
Sweden	11.7	10.4	9.2	10	11.0	9.1	7.9	12.3	11	11.8	11	12.0
Switzerland	11.0	10.9	8.8	11	11.9	8.4	8.1	12.1	11	12.2	11	12.0
Syrian Arab Republic	12.1	8.7	12.9	11	9.1	12.7	10.5	12.7	11	9.5	11	10.0
Tajikistan	9.8	8.0	12.7	8	7.4	13.7	10.1	10.5	9	7.6	9	8.0
Thailand	10.0	8.4	10.5	10	8.7	11.5	10.2	9.7	11	9.5	10	10.0
The FYROM	10.5	8.4	8.9	10	8.6	11.0	10.2	10.8	10	9.7	10	10.0
Togo	10.1	9.5	10.3	9	10.5	8.2	7.7	10.1	9	10.9	9	11.0
Tonga	10.3	7.7	10.8	10	7.9	10.5	9.6	10.6	9	8.0	9	8.0
Trinidad and Tobago	10.8	7.2	10.7	10	7.6	10.6	8.6	12.0	11	7.7	11	8.0
Tunisia	10.7	10.7	11.7	10	9.8	9.8	10.3	11.4	11	9.5	10	10.0
Turkey	10.8	9.1	12.0	11	9.9	10.1	9.3	11.7	11	10.6	12	11.0
Turkmenistan	9.3	7.7	11.9	8	7.4	12.7	9.7	10.4	8	7.6	9	8.0
Tuvalu		6.9	10.0	9	1 7.0	1 9.7	8.3		10	7.3	10	7.0
Uganda	8.8	7.7	9.4	6	7.3	7.9	7.2	9.7	8	8.0		80
Ukraine	10.2	8.9	12.0	8	8.8	11.5	9.4	10.0	9	9.0	9	9.0
United Arab Emirates	11.6	7.7	12.5	11	7.9	11.5	10.9	12.4	11	8.3	11	8.0
United Kingdom	10.9	9.1	8.5	10	9.8	9.0	8.4	11.8	11	10.2	11	10.0
United Republic of Tanzania	9.3	8.0	9.6	7	8.1	7.9	6.8	10.0	10	8.7	10	9.0
United States of America	10.5	9.1	10.7	10	9.6	10.7	8.5	11.0	10	10.3	10	10.0

Uruguay	10.2	10.0	11.4		8.8	109	9.9	10.4	11	9.4	11	9.0
Uzbekistan	10.5	7.1	12.2	9	6.9	12.4	10.0	10.6	10	7.5	10	8.0
Vanuatu	9.1	8.1	10.8	11	8.5	10.8	9.8	9.5	11	9.0	10	9.0
Venezuela, Bolivarian Republic of	10.5	8.5	12.3	10	8.8	11.5	10.1	10.7	11	9.1	11	9.0
Vietnam	10.1	8.2	11.3	11	8.5	10.4	9.3	10.5	11	8.6	10	9.0
Yemen	10.3	7.8	12.7	10	7.9	10.2	11.5	11.0	10	8.0	11	8.0
Zambia	8.6	7.4	7.2	6	6.8	5.6	5.3	8.6	8	8.9	9	9.0
Zimbabwe	11.0	8.9	7.9	6	7.3	5.5	4.7	9.0	9	8.8	9	9.0
Method	HALE	MODEL	HALEb	HALE	MODE	HALE	HALE	HALE	HALE	MODE	HALE	MODE
Year	1990	1990	2000	2000	2000	2001	2002	2010	2012	2012	2013	2013
Mean	10.3	**8.6**	11	9.5	**8.8**	10	8.9	10.8	10.2	**9.3**	10.1	9.3

4.5 Discussion and Conclusions

The GBD study criticized by Williams (see Murray and Lopez 2000) whereas many comments from people from social sciences and philosophy refer to the impossibility to define health and, as a consequence, to measure it. The main problem is that we cannot have flexibility in finding an estimate of health the way we do with other measures of the human organism and related activities. So far if we measure health by collecting surveys it is clear that the uncertainty is relatively high. Even more if we decide for an accepted health state estimate (see Sanders 1964 and related studies during 1960s and 1970s) it remains the problem of accepting a unit of measure. The quantitative methods we propose overcome many of the objections posed. That we have achieved is to propose and apply several quantitative methods and techniques leading to estimates of the healthy life years lost, that more than to be close to the WHO results, provide enough evidence for estimating and quantifying the health state of a population.

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Chapter 5
The Health State Status of the US States for the Period 1989–1991 (Decennial Life Tables)

5.1 The Health State Theory: Mortality Versus Health State

While mortality is well defined if we know the number of deaths at specific age fraction of a population in a period of time, the health state is a crucial parameter for the condition of the members of a population connected to mortality but not explicitly derived with any of the specific estimation methods in use in demography and the related sciences as physiology, health sciences, insurance, probability and statistics. However, health is well accepted in our vocabulary and used in the every day life as the characteristic mean that differentiates life and death. As we can estimate the death probability density function we could use a simple measure of health by estimating the remaining population size by deleting the number of deaths from an original population at specific time periods of one year in the life tables or larger periods as in the abridged life tables where the main part of the selected periods of time is five years of age. The life tables accepted and used for few centuries in demography and actuarial science refer to an hypothetical population of 100,000 people which gradually deteriorates as the total number of deaths increases with age. Accordingly the total population becomes zero exactly at the age when the total number of deaths becomes equal to the number of the original population. However, it is hard to accept the curve of the declining population as the measure of the health state of the population. By observing Fig. 5.1 it is clear that the death density curve has a maximum at high ages and at the same age the population declining curve has an inflection point thus changing slope from negative to positive although it continues to decline. Without smoothing the data we can easily see from the first difference (derivative) of the death distribution that a slowdown is presented after reaching the age of the maximum deaths that is after 80 years of age in our example. The death process is accelerated until the maximum and then it declines following a negative process for the first differences with a minimum after 90 years of age. Then the continues decline slows down until the very high ages. Figure 5.2

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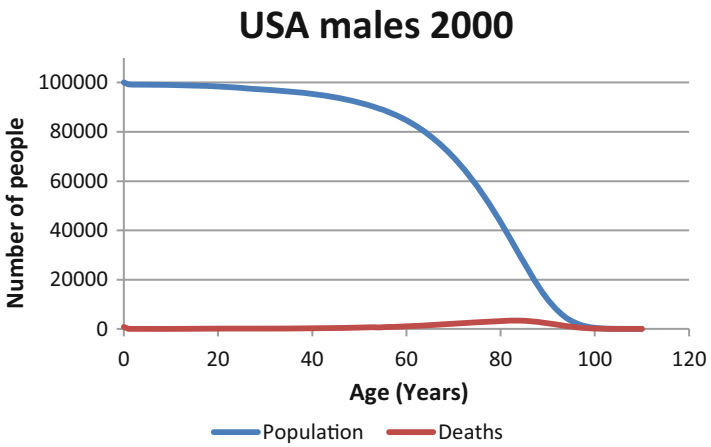

Fig. 5.1 Population and deaths

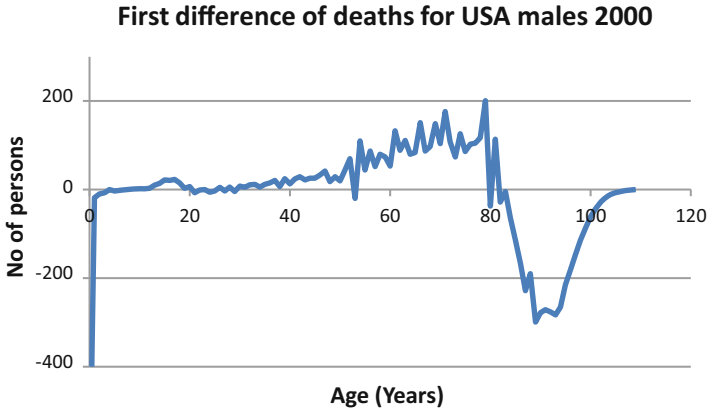

Fig. 5.2 First difference of deaths

illustrates the situation. Of course better results could be achieved when smoothing data. Similar results can be found be exploring the first second and higher order derivatives of μ_x or m_x after the appropriate smoothing.

After the above analysis the need of further analysis of the case of health versus mortality immediately appear. The point at the age year corresponding to the maximum number of deaths is of particular importance. Instead this point do not appear in the classical logarithmic approach for μ_x or m_x as it is illustrated in Fig. 5.4. The first task is to propose a model and a method to reproduce the death density function. Until now the problem was solved by introducing an exponential function of the Gompertz (1825) type, similar or equivalent with appropriate transformations. A second but also very important point is to explore the health state function. This should be a function that has a minimum point or even zero point

close to the age corresponding to the maximum number of deaths. Accordingly this is not the population curve presented in Fig. 5.1 as this curve has only an inflection point at the age of the maximum deaths and continues to steadily decrease. The remaining simple and smooth curves are of the form

$$H_x = l - (bx)^c \tag{5.1}$$

where b, c, l are parameters and H_x is the Health State Function

Then we use the following simple 3-Parameter model to estimate the parameters and then the health state function $H(x)$.

$$g(x) = \frac{|l + (c-1)(bx)^c|}{\sqrt{2\pi x^3}} e^{-\frac{(l-(bx)^c)^2}{2x}}, \tag{5.2}$$

We turn now to have an estimate of the form of the unknown health state function $H(x)$ when the death probability density is given $g(x)$. The resulting function is of the form

$$H(x) = \left| -2x \ln \frac{g(x)\sqrt{x^3}}{k} \right|^{1/2}, \tag{5.3}$$

To assure a positive sign in the between brackets term in the right hand side of the last formula the following relation holds

$$k \geq g(x)\sqrt{x^3}, \tag{5.4}$$

Thus we can immediately have an illustration of the form of the function $H(x)$ by introducing the values for the deaths $g(x)$ per age x from any annual data sets from the human mortality databases in the above formula (5.3) after estimating the parameter k.

Application to the USA 1989–1991 data sets provide the curves illustrated in Fig. 5.3. The continuously declining curve is provided by the above model (5.1) whereas the first approximation of the Health State H_x is given by (5.3). The final form is found with our method proposed to approach the first approximation of H_x the simple smooth curve thus closing the gap between applications and achieving the final form for H_x to provide the local maximum in the adolescence age period to be the full maximum while the next maximum point at the ages from 30 to 40 years to stay in lower levels as it is expected. The local minimum remains in the age interval from 18 to 30 years of age although for some time periods and in various countries it could disappear especial for females.

In many applications the simpler form of the quadratic like curve of Fig. 5.4 gives adequate results especially when comparing different countries or states as in the case with the United States states presented in this chapter. That it is different is the maximum health state age which is found at the local maximum in ages from 30

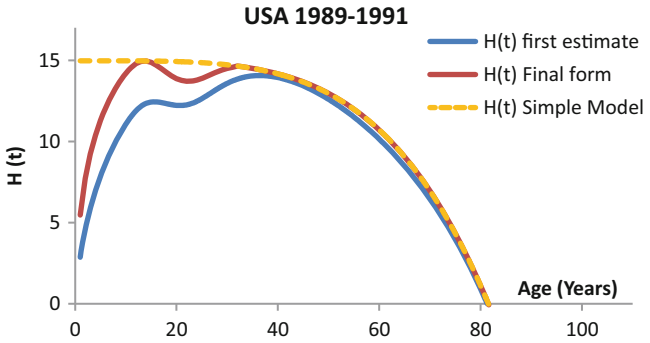

Fig. 5.3 Health state function

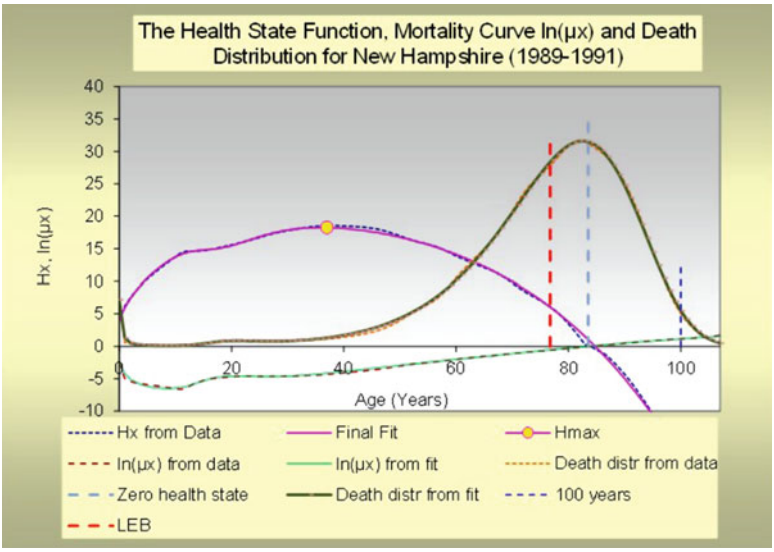

Fig. 5.4 Main health state and mortality characteristics

to 40 years. The quadratic like health state form is very important for the estimation of the total health state that is the total area under this curve.

The very important finding from the above analysis is that the curve expressing the health state is a smooth declining curve approaching zero near the age of the maximum number of deaths. This could also be verified by applying stochastic simulations via a technique presented in the appropriate chapter of this book. From the theoretical point of view of defining the health state of a population interesting publications are due to McDowell (2006), Sanders (1964), Strehler and Mildvan (1960), Torrance (1976) and many others while the quantitative part was partially covered by the paper by Strehler (1960) in a publication in Science (1960). However, the approach to use the first exit time theory came only after the development of

the related theory during 1970s. Weitz and Fraser (2001) in 2001 proposed the Inverse Gaussian, the simplest first exit time density model, to model the "health status" of Mediterranean Flies using one of the best data tables for this species. However, these data provide a death density function with maximum at very early ages. It looks like the opposite to the human death probability density providing a maximum at high ages. Thus an extension of the Inverse Gaussian should be done to cover the human health status cases. However, Weitz and Fraser by accepting and applying this model suggested a linear declining form for the health state process. Six years earlier (1995) in a publication in the Applied Stochastic Models and Data Analysis (ASMDA) Journal we have solved the problem by finding the appropriate "Health State" forms and applied to the Life Table data in Belgium and France (see Janssen and Skiadas 1995). Many publications followed during last 20 years exploring special parts of the health state process (see Skiadas 2011; Skiadas and Skiadas 2010a,b, 2011, 2014, 2015). The use of the health state process in various applications provide a strong tool for many applications as here with the US States.

5.2 Application to United States states

We apply the health state function theory to explore the health state status of the USA States for the period 1989–1991. The data are from the official decennial life tables. We first use the New Hampshire data for the first application presented in the next figure and then we apply the same theory to 50 USA States and give comparative results.

Figure 5.4 illustrates the main futures of the human health state and mortality theory. Three main graphs are present the Death Distribution, the Health State Function and the Mortality Curve. The example used is for New Hampshire U.S. decennial life tables for 1989–1991 provided by the US Department of Health and Human Services, National Center for Health Statistics, Centers for Disease Control and Prevention, Division of Vital Statistics. For the three graphs presented two main futures are given: the estimates from the data provided and the estimates after the fitting by using the SKI-1995 model and the related program in Excel provided in the http://www.cmsim.net website. The fitting is almost perfect. From the data sets we form and present the death distribution (The scale of the graph is adapted). Few important futures are illustrated in the above graph.

1. The maximum number of deaths appear at 83.5 years of age
2. The death distribution around this year of age is of a non-symmetric bell-shaped form. The main task of future studies is to explore the mechanisms related to the form of this distribution and on how we can expand the region around of the peak of the deaths to the right.
3. A 33.3% of the total number of deaths appear in the age interval ± 5 years from the maximum (78–88 years of age). This is 33.2% for USA 1990 data.

4. A 58.1% of the total number of deaths appear in the age interval ±10 years from the maximum (73–93 years of age). The related value for USA 1990 is 57.0%. This part of the death data, almost the 2/3 of the total deserves special attention. Any improvement by shifting the death distribution to the right will provide valuable help in millions of people.
5. The number of deaths from 100+ is only a 1.9% of the total number of deaths. (For USA 1990 is 1.1%). This is a very small amount distributed at the right hand part of the tail of the death distribution so that it is very difficult to collect any reliable information. That is why the studies on centenarians and super-centenarians face problems.
6. The number of deaths from 0 to 25 is only a 1.9% of the total number of deaths. It is similar to the number of deaths for 100+ years of age. This is 2.6% for USA 1990 data.

The Life Expectancy at Birth (LEB) is estimated at 76.2 years of age (red line in the graph). LEB is the most popular indicator as it is used by actuaries and insurance companies to calculate the pension funds. However, LEB is a statistic indicator and the large public confuses this indicator with the year of the maximum number of deaths. LEB is always several years lower than the age year of the maximum death rate as is illustrated in the graph. For earlier time periods when infant mortality was extremely high LEB differs significantly from the age year of the maximum death rate. The use of the force of mortality μ_x and its logarithmic form $\ln(\mu_x)$ do not help much as it provides a linear form for the age years higher than 30. The theory of the health state of a population instead includes the empirical observations related to the health state starting from lower values at birth, increasing until maturity and then decreasing at higher ages. The theory includes many theoretical and technical details developed last decades and based on the modern theory of the first exit time of a stochastic process from a barrier. Although the full knowledge of the theory requires high level mathematics and statistics the applications are feasible by using the Excel software provided in the http://www.cmsim.net website. The health state function H_x for New Hampshire is presented in the above Fig. 5.4. The main futures of the health state function are the following:

1. The health state function is zero close to the age year of the maximum death rate.
2. The health state function provides a maximum at a specific age ranging from 30 to 45 years. The level of this maximum can be used to rank countries and regions. For New Hampshire it is 37 years of age at a level of 18.54. It is 37.52 years of age for USA in 1990 at a level of 17.56.
3. A more accurate estimate related to this maximum is the expected healthy age. For New Hampshire is 38.41 years of age. It is 38.97 years for USA in 1990.
4. Calculating the area under the health state function from zero age until the age of zero health state we have a clear estimate of the health condition of a population. The related number of the Total Health State is 1130 for New Hampshire (1989–1991) and 1110 for USA in 1990. Estimates for Sweden for a period of the last 250 years follow (Table 5.1). The Total Health State improved and the Life Expectancy at Birth as well; the later increasing by 40.9 years of age in 250 years. Instead the age of the maximum death rate increased by only 12 years

Table 5.1 Estimates for Sweden

Year	Max death rate	Life expectancy at birth	Total health state
1751	74	38.7	737
1800	71	32.9	646
1850	73	44.5	724
1900	79	51.9	866
1950	80	71.0	1076
2000	86	79.6	1291

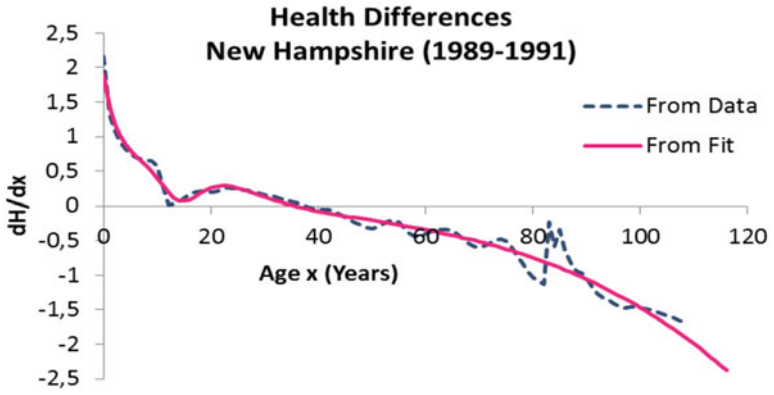

Fig. 5.5 The health state differences as the first derivative of the health state function

from 74 years in 1751 to 86 years in 2000. Contrary to the general opinion the maximum death rate of the population of Sweden was at the relatively high age of 74 years in 1751 almost 2 times more than the LEB years of age. This is an indication that the governing mechanisms for the human life duration are relatively stable and special attention is needed in organizing future studies.

5. A local maximum of the health state appears in 12 years of age from data sets (at 14.59 health level) and at 15 years of age from the fit curve (at 14.78 health level) as illustrated in Fig. 5.4. As the case is very sensitive we estimate the Health State Differences presented in Fig. 5.5. The level of health state achieved in this young age accounts for the 78.7% of the maximum health state. Furthermore Fig. 5.5 provides a clear view of the course of health state changes in a population as a function of age. The changes expressed as the first derivative of the health state function (dH/dx) are positive but declining from birth until the end of the first decade or of the beginning of the second decade of the life span when it is close to zero thus providing a local maximum for H_x, then increases until a maximum (for New Hampshire is estimated at 23 years of age from the data sets and 22 from the fit curve) and then continuously decline passing from positive to negative values. The zero point is achieved close to the year of the maximum health state.

   The Fig. 5.6 left and right illustrate the estimates for the Total Health State (THS) and the Life Expectancy at Birth (LEB) for the US States for the period

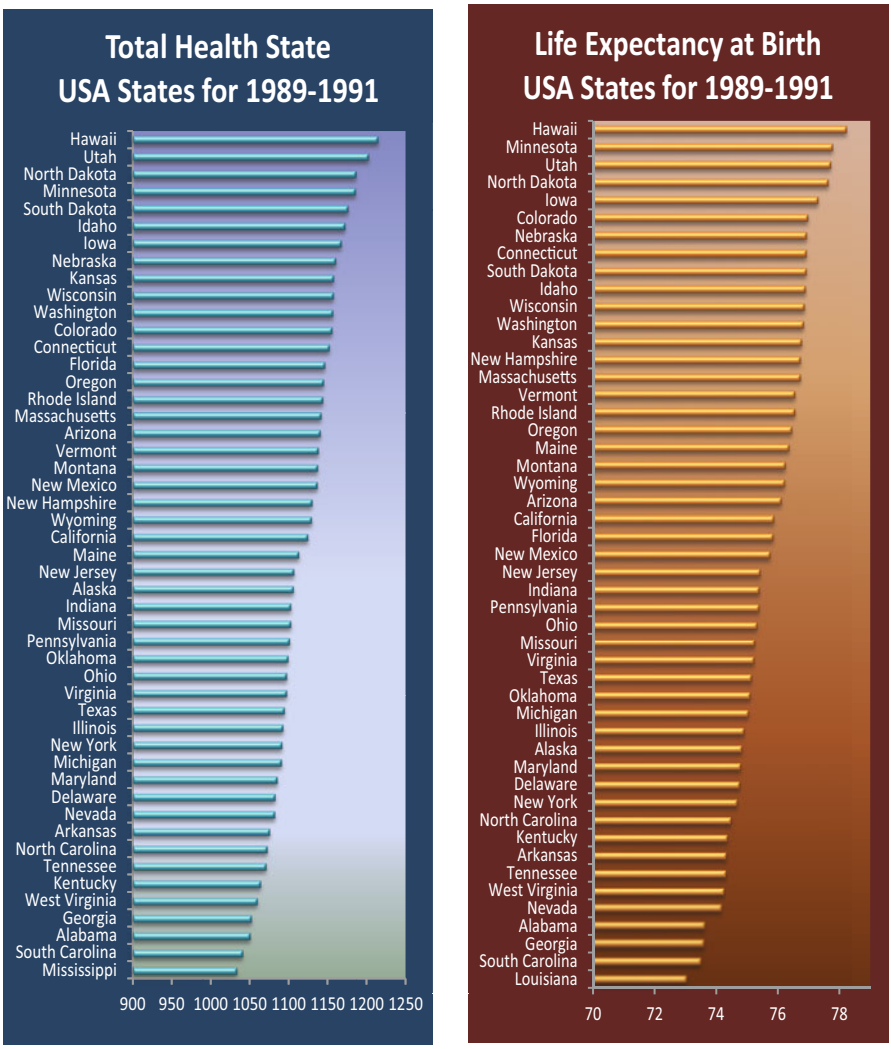

Fig. 5.6 *Left*: Total health state estimates. *Right*: Life expectancy at birth estimates

1989–1991 (Decennial Life Tables). Observing the rank of the particular States we found clear connections between THS and LEB. The States presenting highest Total Health State show high Life Expectancy at Birth as well (Fig. 5.7).

The Life Expectancy at Birth versus the Total Health State for the US States (1989–1991) is presented in Fig. 5.6 along with the linear trend line with equation: $y = 0.0283x + 44.061$. The relationship is evident. It is further demonstrated in the next comparative Table 5.2. The US States are classified according to Life Expectancy at Birth and in the next column the Total Health State ranking appears. The last column indicates how many places moved up

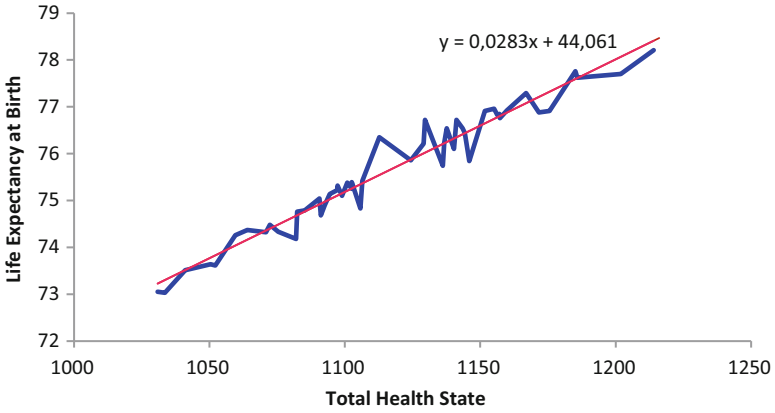

Fig. 5.7 The LEB versus THS for 50 USA States

Table 5.2 Life expectancy and total health state rankings of US States

LEB and THS rankings for US States					
Rank	Life expectancy at birth		Total health state		Places +up/−down
1	78.22	Hawaii	Hawaii	1214	0
2	78.10	Minnesota	Utah	1202	1
3	77.95	Utah	North Dakota	1186	1
4	77.76	North Dakota	Minnesota	1185	−2
5	77.63	Iowa	South Dakota	1176	4
6	77.47	Colorado	Idaho	1172	4
7	77.34	Nebraska	Iowa	1167	−2
8	77.34	Connecticut	Nebraska	1160	−1
9	77.28	South Dakota	Kansas	1157	4
10	77.27	Idaho	Wisconsin	1157	1
11	77.19	Wisconsin	Washington	1156	1
12	77.19	Washington	Colorado	1155	−6
13	77.17	Kansas	Connecticut	1152	−5
14	77.15	New Hampshire	Florida	1146	10
15	77.06	Massachusetts	Oregon	1144	3
16	77.03	Vermont	Rhode Island	1143	1
17	76.96	Rhode Island	Massachusetts	1141	−2
18	76.79	Oregon	Arizona	1140	4
19	76.77	Maine	Vermont	1138	−3
20	76.58	Montana	Montana	1137	0
21	76.58	Wyoming	New Mexico	1136	4
22	76.29	Arizona	New Hampshire	1130	−8

(continued)

Table 5.2 (continued)

LEB and THS rankings for US States					
Rank	Life expectancy at birth		Total health state		Places +up/−down
23	76.00	California	Wyoming	1129	−2
24	75.91	Florida	California	1124	−1
25	75.86	New Mexico	Maine	1113	−6
26	75.82	New Jersey	New Jersey	1106	0
27	75.81	Indiana	Alaska	1106	9
28	75.78	Pennsylvania	Indiana	1103	−1
29	75.76	Ohio	Missouri	1103	1
30	75.70	Missouri	Pennsylvania	1101	−2
31	75.67	Virginia	Oklahoma	1099	2
32	75.57	Texas	Ohio	1097	−3
33	75.55	Oklahoma	Virginia	1097	−2
34	75.44	Michigan	Texas	1094	−2
35	75.24	Illinois	Illinois	1093	0
36	75.23	Alaska	New York	1091	3
37	75.19	Maryland	Michigan	1091	−3
38	75.02	Delawae	Maryland	1085	−1
39	74.90	New York	Delaware	1082	−1
40	74.90	North Carolina	Nevada	1082	5
41	74.89	Kentucky	Arkansas	1075	1
42	74.82	Arkansas	North Carolina	1072	−2
43	74.72	Tennessee	Tennessee	1071	0
44	74.64	West Virginia	Kentucky	1064	−3
45	74.22	Nevada	West Virginia	1060	−1
46	74.02	Alabama	Georgia	1052	1
47	73.99	Georgia	Alabama	1050	−1
48	73.93	South Carolina	South Carolina	1041	0
49	73.50	Louisiana	Mississippi	1034	1
50	73.45	Mississippi	Louisiana	1031	−1

or down every State. Six States are exactly classified. Seventeen States change only one position up or down. Nine States moved to 2 places up or down, 6 States moved to 3 places, 5 states moved to 4 places up or down, whereas 2 States moved to 5 places and 2 to 6 places. The remaining three States are New Hampshire (8 places down), Alaska (9 places up) and Florida (10 places up). USA with 1110 for the THS will be ranked between Maine (1113) and New Jersey (1106) in a place between 25 and 26 in the middle of the US States. Instead according to LEB (75.24 years) USA should be ranked in place 35 of 50 States with Illinois.

6. The most important futures of the Health State Function of a Population is the estimation of the Loss of Health Life Years (LHLY) and then the calculation of the Healthy Life Expectancy (HLE) as the difference between the Life

Expectancy at Birth (LEB) and LHLY that is HLE = LEB − LHLY. There are three special cases. In the most important we estimate the loss of healthy life years under severe causes and we calculate the healthy life expectancy at birth (HLEB) under severe causes. The method used is applied to the World Health Organization (WHO) member states for the years 1990, 2000 and 2009. The application for USA States (1989–1991) is presented in Fig. 5.8 left. Minnesota is ranked first with 71.93 years and Louisiana with 67.44 healthy life years is in the last place. The gap is 4.49 healthy life years. Minnesota, Hawaii, Utah, Connecticut, Iowa, North Dakota, Wisconsin, New Hampshire, Nebraska and Massachusetts form the first decade whereas West Virginia, Kentucky, Nevada, Georgia, New York, Arkansas, Alabama, South Carolina, Mississippi and Louisiana are the last ten states in the rank.

7. The estimates of the Healthy Life Expectancy at Birth under all causes are also presented. This is an indicator including severe, moderate and light causes for loss of healthy life years (Fig. 5.8 right). As it is expected the related indicator for the Healthy Life Expectancy at Birth (HLEB) under all causes provides lower values for the expected healthy years of age than the previous one. However, it is an important estimator for the health policy planers especially when estimate the expenses for the health care system. New Hampshire (63.00 years), Maine, Vermont, Iowa, Nebraska, Minnesota, Massachusetts, Wisconsin, Colorado and Washington are the first ten with the highest healthy life years. The lower ten positions are covered by Arkansas, Louisiana, South Carolina, Arizona, Georgia, Alabama, New Mexico, Mississippi, New York and Florida (57.12 years). The gap from the first to the last one is 5.88 life years. For USA 1990 the HLEB (severe causes) is 68.69 years higher than Missouri (69.54 years) and lower than Florida (69.71 years) in a place between 28 and 29.
8. A comparative study is presented in Table 5.3 including the estimates for the healthy life expectancy at birth under severe and under all causes of disabilities for the USA States from 1989–1991. The rankings differ significantly in the two estimates. The main reason is that by estimating all causes of disabilities (severe, moderate and light) the light causes responsible for the loss of several life years of age are higher or lower in places with special characteristics for the way of living. The main positive changes (+up) were for West Virginia (+25), Delaware (+24), Kentucky (+23), Alaska (+22), Maine (+14), Vermont (+14), Indiana (+13), Ohio (+11) and Virginia (+10). The main negative changes (−down) were for Hawaii (−35), Connecticut (−23), Florida (−22), Arizona (−21), Utah (−21), Kansas (−16), New Mexico (−16), North Dakota (−15), California (−14), South Dakota (−11) and Rhode Island (−10). For USA 1990 the HLEB (all causes) is 59.37 years higher than South Carolina (59.23 years) and lower than Louisiana (59.41 years) in a place between 42 and 43.

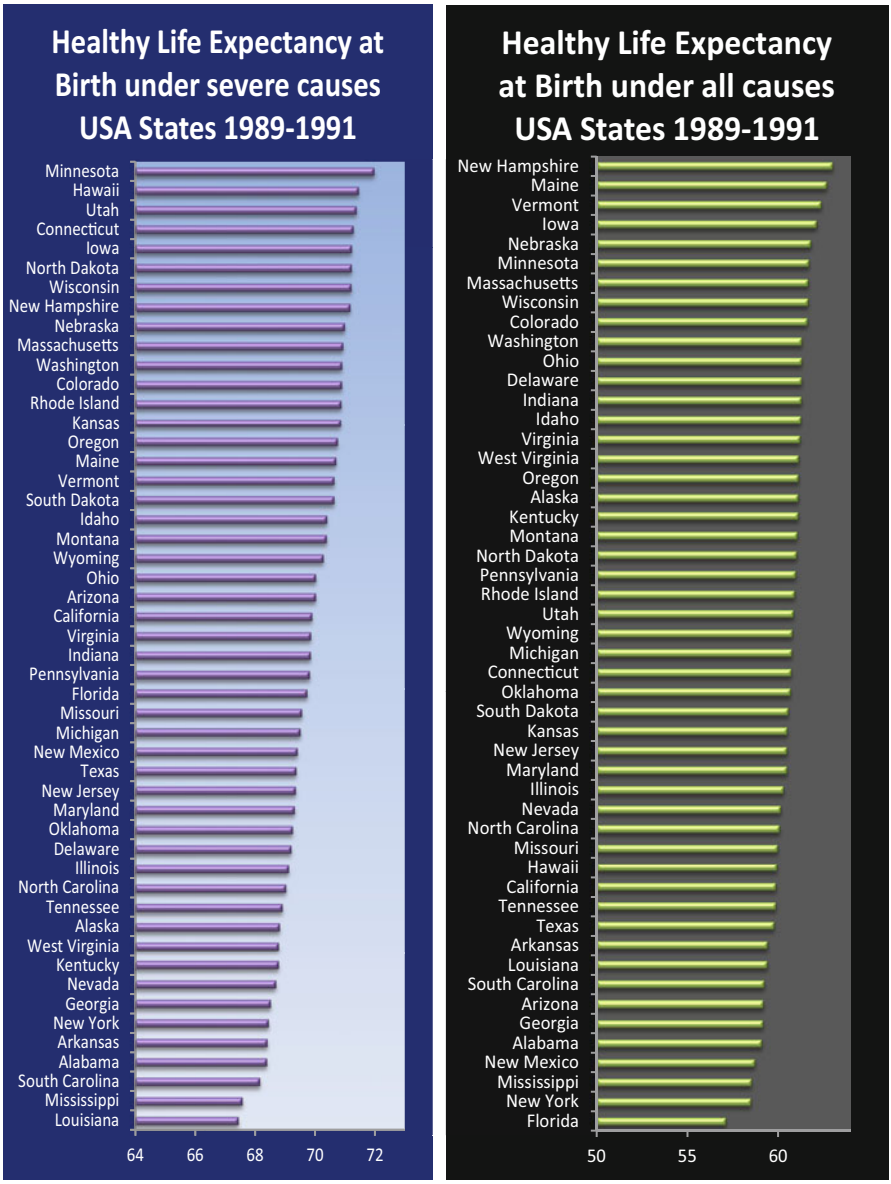

Fig. 5.8 *Left*, HLEB (severe causes). *Right*, HLEB (all causes)

Table 5.3 Healthy life expectancy rankings for US States (severe and all disability causes)

Healthy life expectancy rankings for US States					
Rank	Healthy life expectancy at birth (severe causes)		Healthy life expectancy at birth (all causes)		Places +up/−down
1	71.93	Minnesota	New Hampshire	63.00	7
2	71.42	Hawaii	Maine	62.67	14
3	71.35	Utah	Vermont	62.36	14
4	71.24	Connecticut	Iowa	62.12	1
5	71.18	Iowa	Nebraska	61.80	4
6	71.18	North Dakota	Minnesota	61.69	−5
7	71.17	Wisconsin	Massachusetts	61.65	3
8	71.14	New Hampshire	Wisconsin	61.64	−1
9	70.95	Nebraska	Colorado	61.60	3
10	70.90	Massachusetts	Washington	61.29	1
11	70.86	Washington	Ohio	61.29	11
12	70.86	Colorado	Delaware	61.29	24
13	70.84	Rhode Island	Indiana	61.28	13
14	70.82	Kansas	Idaho	61.24	5
15	70.72	Oregon	Virginia	61.20	10
16	70.67	Maine	West Virginia	61.12	25
17	70.61	Vermont	Oregon	61.11	−2
18	70.60	South Dakota	Alaska	61.10	22
19	70.37	Idaho	Kentucky	61.10	23
20	70.35	Montana	Montana	61.04	0
21	70.25	Wyoming	North Dakota	61.01	−15
22	70.00	Ohio	Pennsylvania	60.96	5
23	70.00	Arizona	Rhode Island	60.90	−10
24	69.88	California	Utah	60.84	−21
25	69.83	Virginia	Wyoming	60.78	−4
26	69.83	Indiana	Michigan	60.75	4
27	69.80	Pennsylvania	Connecticut	60.71	−23
28	69.71	Florida	Oklahoma	60.67	7
29	69.54	Missouri	South Dakota	60.55	−11
30	69.49	Michigan	Kansas	60.49	−16
31	69.39	New Mexico	New Jersey	60.48	2
32	69.35	Texas	Maryland	60.48	2
33	69.33	New Jersey	Illinois	60.29	4
34	69.30	Maryland	Nevada	60.13	9
35	69.23	Oklahoma	North Carolina	60.07	3

(continued)

Table 5.3 (continued)

Healthy life expectancy rankings for US States					
Rank	Healthy life expectancy at birth (severe causes)		Healthy life expectancy at birth (all causes)		Places +up/−down
36	69.18	Delaware	Missouri	59.97	−7
37	69.10	Illinois	Hawaii	59.93	−35
38	69.01	North Carolina	California	59.88	−14
39	68.90	Tennessee	Tennessee	59.87	0
40	68.80	Alaska	Texas	59.77	−8
41	68.77	West Virginia	Arkansa s	59.42	5
42	68.77	Kentucky	Louisia na	59.41	8
43	68.68	Nevada	South Carolina	59.23	5
44	68.50	Georgia	Arizona	59.18	−21
45	68.44	New York	Georgia	59.16	−1
46	68.40	Arkansas	Alabama	59.10	1
47	68.38	Alabama	New Mexico	58.72	−16
48	68.15	South Carolina	Mississippi	58.53	1
49	67.57	Mississippi	New York	58.50	−4
50	67.44	Louisiana	Florida	57.12	−22

5.3 Summary and Conclusions

The Health Status of the US States for the period 1989–1991 is explored by using the stochastic theory of the first exit time or hitting time of a stochastic process from a barrier. The systematic study included the parameter estimation of models used and then the ranking of the States according to specific parameters related to the health state as the life expectancy at birth and the healthy life expectancy. Special attention was given to estimate the Light, Severe and Total disability causes and then to rank the US States accordingly. The US States are also classified according to the total health state estimated by the stochastic theory proposed. The total health state is a crucial parameter for the health level of a specific population.

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Chapter 6
Life Expectancy at Birth, Estimates and Forecasts in the Netherlands (Females)

6.1 Introduction

Long term predictions of the Life Expectancy and the Life Expectancy at Birth (LEB) is a very critical issue for long range planning for countries and national and international organizations as the World Health Organization (WHO). Whereas for short term forecasts several tools are in use (Heligman and Pollard 1980; Makeham 1860; Lee and Carter 1992) leading to reliable predictions, the long term estimates are a difficult task. High uncertainty is inherent in exploring the future trends especially in estimating the LEB for various countries. As the LEB is growing for many decades the issue is to explore systematic growth changes leading to better medium and long term forecasts. An important development is coming from the recent introduction and estimation from demographic data of a deterioration function and the associated maximum deterioration age (Det) (see the recent papers Skiadas and Skiadas 2011a,b; Skiadas 2011a,b and download from http://www.cmsim.org). The value of Det is higher than the LEB. However, in nowadays the LEB is growing shortening the gap from Det and in some cases as in Japan (females) the LEB is very close to Det. It is expected that the Life Expectancy at Birth will increase in all the countries thus approaching asymptotically to Det. Another point is the estimate of a future level of the Life Expectancy at Birth based on the deterioration function which we call the DTR system. The future life expectancy at birth estimated by the DTR system is higher than the Det and provides another level for LEB, whereas Det and DTR approach each other in the long term. Instead to use only the data for the annual values of LEB for making predictions we have two more data sets for Det and DTR thus improving the reliability of forecasts.

C.H. Skiadas, C. Skiadas, *Exploring the Health State of a Population by Dynamic Modeling Methods*, The Springer Series on Demographic Methods and Population Analysis 45, DOI 10.1007/978-3-319-65142-2_6

6.2 General Theory on Stochastic Modeling of Health State

The first paper related to this theory on stochastic modeling of the health state of an individual was published in 1995 (Janssen and Skiadas 1995). An application on the Belgium and France data was presented by the authors in the Royal Association of Belgian Actuaries (ARAB/KVBA, founded in 1895) in a meeting in 1995 celebrating the 100 years of the Association. The modeling approach was focused on finding the distribution of the first exit time of a diffusion process expressing the health state of a person from a barrier. The related theory can be found in Janssen and Skiadas (1995), Skiadas and Skiadas (2007, 2010a,b, 2011a) and recently in the International Encyclopedia of Statistical Science, Springer (2011) (Skiadas and Skiadas 2011c; Skiadas 2011a). The publications (Skiadas and Skiadas 2007, 2010a,b, 2011a,c) are focused on the development and application of a first exit time model for mortality including the infant mortality. The model termed as First Exit Time-IM is expressed by the following probability density function

$$g(t) = \frac{k(l + (c+1)(bt)^c)}{\sqrt{t^3}} e^{-\frac{(l-(bt)^c)^2}{2t}},$$

where k, l, c and b are parameters and $k = \frac{1}{\sqrt{2\pi}}$. This model arises from the more general one of the form:

$$g(t) = \frac{(l + (c+1)(bt)^c)}{\sigma\sqrt{2\pi t^3}} e^{-\frac{(l-(bt)^c)^2}{2\sigma^2 t}},$$

by setting $\sigma = 1$.

The parameter l accounts for the infant mortality. A simpler 3-parameter version of this model arises when the infant mortality is limited thus turning the parameter l to be: $l = 0$ and the last formula takes the simpler form:

$$g(t) = \frac{k((c+1)(bt)^c)}{\sqrt{t^3}} e^{-\frac{((bt)^c)^2}{2t}},$$

In the first model the health state $H(t)$ is expressed by the simple relation

$$H(t) = l - (bt)^c,$$

whereas this formula becomes $H(t) = (bt)^c$ for the second and simpler model. In both cases a characteristic relation is given by the next function expressing the curvature of the health state function during time (time here is the age of the individual)

$$K(t) = \frac{|c(c-1)b^c t^{c-2}|}{(1 + c^2 b^{2c} t^{2c-2})^{3/2}}$$

The curvature $K(t)$ gives a measure of the deterioration of the human organism or of loss of vitality in terms of Halley (1693) and Strehler and Mildvan (1960). We call $K(t)$ the Deterioration Function which provides a bell-shaped curve. The maximum of the deterioration function (*Det*) is achieved at the age T_{Deter} which is given by the formula

$$T_{Deter} = \left[\frac{c-2}{(2c-1)c^2b^{2c}}\right]^{1/(2c-2)}$$

A search in several countries shows that *Det* is slowly changing during the last centuries and practically remaining almost stable until 1950 then showing a steady increase from 1950 until nowadays. A consequence of the existence of the deterioration function $K(t)$ and its maximum at *Det* is the postulate that life expectancy at birth will tend to approach *Det* during time. The next very important point is to estimate the total effect of the deterioration of a population in the course of the life time termed as *DTR*. This is expressed by the following summation formula:

$$DTR = \int_0^t sK(s)ds \approx \sum_0^t sK(s)$$

Where t is the age and $K(t)$ is the deterioration function. The last formula expresses the expectation that an individual will survive from the deterioration caused in his organism by the deterioration mechanism. The result is given in years of age and we can construct a Table like the classical life tables. The *DTR* provides the future trends for the life expectancy and it is very important in doing forecasts along with *Det*. The Life Expectancy at Birth (*LEB*) can also be estimated by a similar summation formula

$$LEB = \int_0^t sg(s)ds \approx \sum_0^t sg(s)$$

The integral gives as an immediate summation whereas the sum in the right hand side of the last formula leads to the estimation of *LEB* by the classical Life Table method (Graunt 1676; Halley 1693; Haberman and Sibbett 1995; Keyfitz and Caswell 2005). All the needed estimates are fusible by using a program in Excel which can be downloaded from: http://www.cmsim.net/id13.html. The program estimates the life expectancy from data, from the fitting curve, by calculating the integral and by estimating the future values of the life expectancy by using the *DTR* method.

The next two Figs. 6.1 and 6.2 illustrate special cases for Netherlands (1930 and 2006). The data points, the fitting curve $g(t)$ and the deterioration function appear.

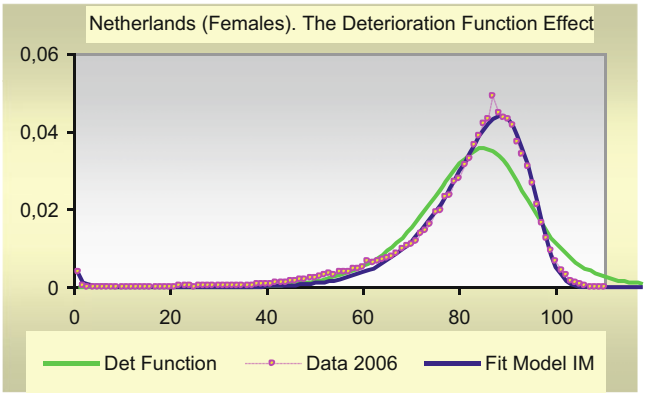

Fig. 6.1 Application in Netherlands for 2006

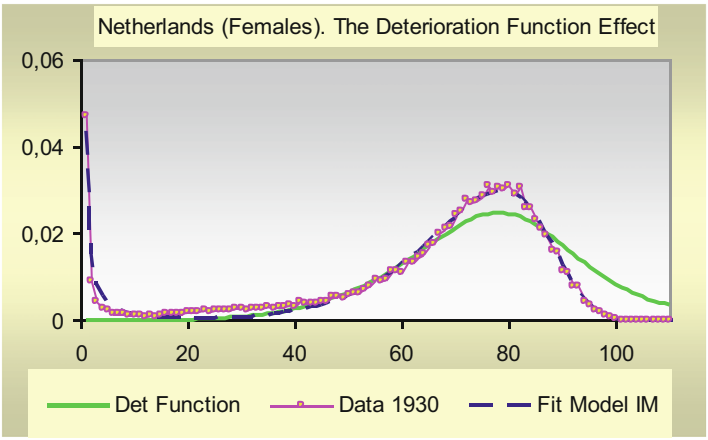

Fig. 6.2 Application in Netherlands for 1930

6.3 Modeling and Applications in the Netherlands (Females): First Method of Forecasts

In the following we do forecasts for the Life Expectancy at Birth by using the data provided by the Human Mortality Database (HMD). The population and death data (per age) for females are introduced into the IM-First Exit Time (Skiadas and Skiadas 2007, 2010a,b) nonlinear regression analysis program (http://www.cmsim.net/id20.html) estimating all the necessary parameters including LEB (from data, from fitting and the mean value), Det and DTR.

We use the population and death data from the Human Mortality Database for Netherlands (females) from 1900 to 2006. We estimate the Life Expectancy at Birth (LEB), the Maximum deterioration age (Det) and the future life expectancy based

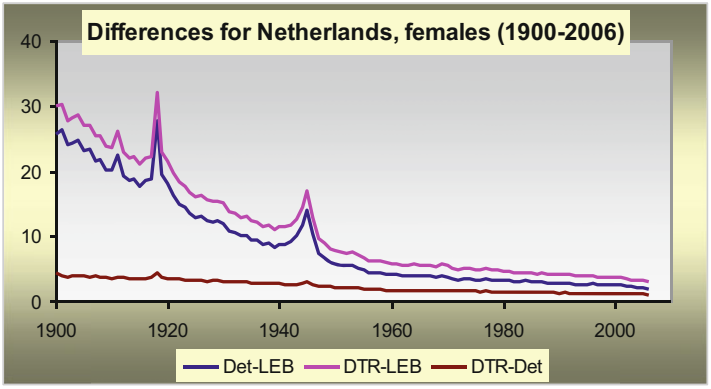

Fig. 6.3 Differences between LEB, Det and DTR

on the DTR Method (DTR). As it can be easily verified from applications in several countries the LEB tends to increase during time approaching the Det which tends to coincide with the DTR as it is illustrated in Fig. 6.3 where the differences (Det-LEB), (DTR-LEB) and (DTR-Det) appear. By observing Fig. 6.3 it is clear that a systematic relation between the three estimates is present.

In the following we will use the Det and DTR estimates along with the LEB to improve life expectancy predictions. The key point is the deterioration function and the maximum point of this function corresponding to the age with the maximum deterioration of the human organism (see the main ideas in Economos (1980), Gompertz (1825), Greenwood and Irwin (1939), Halley (1693), Janssen and Skiadas (1995), Skiadas and Skiadas (2010b, 2011b), Skiadas (2011a), and Strehler and Mildvan (1960)). As far as the other causes of death will be diminished due to the improvements in science and the social and community efforts to improve our way of living, the LEB age will by close to the Det age, whereas the future life expectancy estimated by the DTR system will approach the Det values. In other words the future values of the life expectancy at birth can be found by simply shifting the Det and DTR trends to the right as it is illustrated in Fig. 6.4. We can immediately have an estimate for the future trend of LEB by starting from the value of LEB for 1900 ($f_0 = 48.98$ years) and assuming an upper limit $F(1900) = DTR(1900) = 78.86$. The next step is to find the parameter b of the next equation as to fit the first data sets for LEB after 1900. The first 10 data points (1900–1909) are sufficient for a good prediction of LEB for almost 40 years (see Fig. 6.4, blue line). The next step is to find the time lag between LEB and the right shift of the Det. After shifting the Det to the right (see Fig. 6.4) we shift the DTR to the right as to fit on the "right shifted Det". We thus form a series (by averaging) composed from the values of LEB and the shifted Det and DTR. Then we use this series as an entry in a nonlinear regression analysis program for the estimate of the best fit. As the system tends to stabilize to an upper limit we use the following equation:

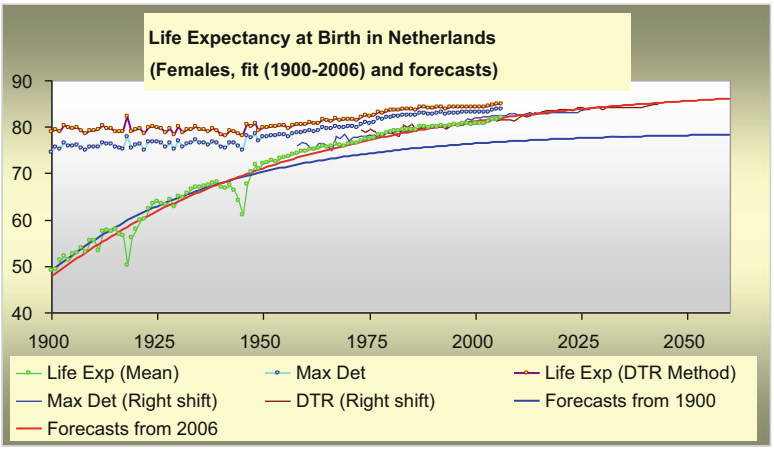

Fig. 6.4 Fit and forecasts

Table 6.1 Regression analysis parameters for right shifted data

F	f_0	a
88.89	47.77	0.01676

$$F = (F - f_0)e^{-at}$$

Where F is the upper limit, f_0 is the starting point and a is a parameter expressing the speed of growth. The regression analysis gave the following values (Table 6.1)

The LEB is estimated from 1900–2006 data for females in the Netherlands. The estimates based on the 1950–2006 data for females in Netherlands are also estimated. For the latter case the estimates are also illustrated in Fig. 6.5. The LEB, Det and DTR values are estimated from 1950–2006. Det and DTR are shifted to the right as in the previous case and estimates are done from 2006. The fitting by using the previous equation and the nonlinear regression gave the following values (Table 6.2). The upper limit is lower (87.70 years) than in the previous case (88.89 years).

The predicted values by using the data from 1900 to 2006 and 1950 to 2006 are calculated. Characteristic values are included in the next Table 6.3. In the mean time World Health Organization has estimated and published the final estimates for the next 10 years after our study. The provided figures for the years 2010 and 2015 are very close to our forecasting values (see the Table 6.3).

We turn out to explore two special cases of very low life expectancy at birth in Netherlands that of the year 1918 due to the influenza pandemic and of 1945 due to the after the Second World War effects. The resulting values are compared to the Det and DTR values for the same time period. As it is presented in Fig. 6.6E both Det and DTR show almost stable behavior as they represent the effects of deterioration of the human organism. The year 1945 a mortality excess appear (see Fig. 6.6B) distributed to all the ages. It is demonstrated by a local minimum in the LEB

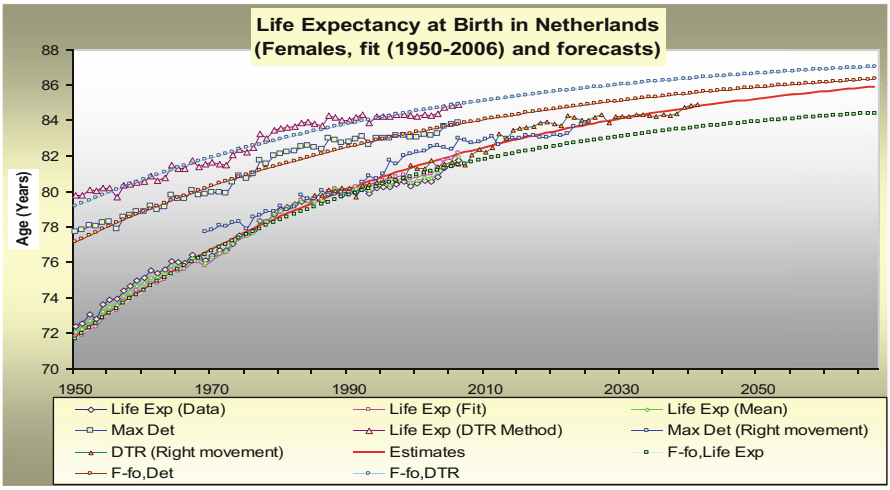

Fig. 6.5 Fit and forecasts from 2006 in Netherlands

Table 6.2 Regression analysis parameters for 1950–2006 estimates

F	f_0	a
87.70	71.75	0.01862

Table 6.3 Predicted values for life expectancy at birth for two time periods

Year	Life expectancy at birth, forecasts (data from 1900–2006)	Life expectancy at birth, forecasts (data from 1950–2006)	WHO recent data
2010	82.37	82.48	82.70
2015	82.90	82.95	83.60
2020	83.38	83.37	
2025	83.82	83.75	
2030	84.23	84.10	
2035	84.60	84.42	
2040	84.95	84.71	
2045	85.27	84.98	
2050	85.56	85.22	
2055	85.82	85.44	
2060	86.07	85.64	

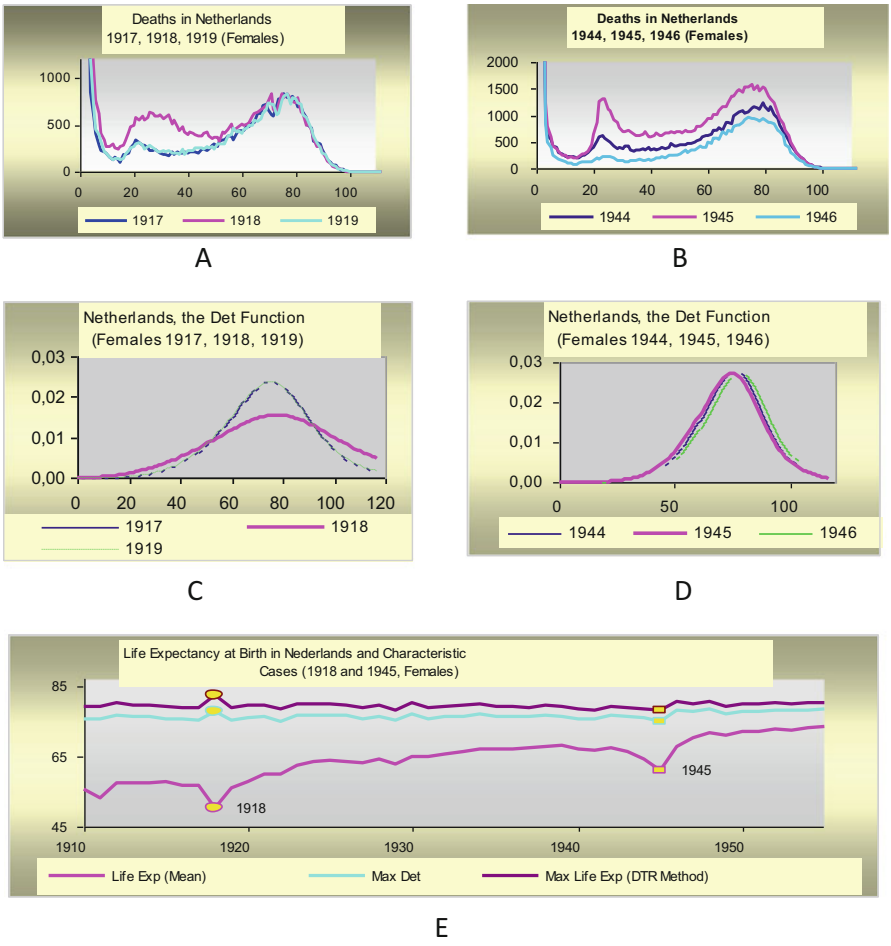

Fig. 6.6 (**A**, **B**, **C**, **D**, **E**). Special cases

diagram. The corresponding influence to Det and DTR is limited as it was expected by the related theory. More interesting is the case of the influenza pandemic in 1918. As it is illustrated in Fig. 6.6A the mortality excess is distributed in the age group from 15 to 50 years approximately. The resulting value for LEB is a local minimum as it was expected, whereas, Det and DTR show a local maximum. This can be explained by observing Fig. 6.6C. The graphs expressing the deterioration function for the years 1917 and 1919 almost coincide whereas the related graph for the year 1918 is completely different expressing the influence of pandemic influenza. Instead for the years 1944, 1945 and 1946 the three curves for the deterioration function are very close each other (Fig. 6.6D).

6.4 Parameter Analysis of the IM-Model: Second Method of Forecasts (Classical)

By using the $g(t)$ formula for the death distribution we apply the nonlinear regression analysis program to the Netherlands data for females from 1850 to 2006 and we estimate the parameters of the model. The parameters show systematic changes over time so that future predictions are possible. The parameter b follows an exponential decay process presented in Fig. 6.7. The fitting parameters are summarized in Table 6.4. The parameter b approaches the lower limit at $b = 0.01175$. The last period (1960–2006) can also be modeled by a simple line of the form $b = a + ct$ where $a = 0.01594$ and $c = -0.00003116$. The linear form can apply for short and medium term predictions.

The parameter l accounts for the infant mortality. As it is illustrated in Fig. 6.8 this parameter follows a negative exponential process tending to very low values close to zero. The nonlinear regression analysis fitting gave the next values for the negative exponential function applied (Table 6.5). The limit of the parameter l is found to be $l = 0.001329$ indicating the successful application of the social health services in the infant section.

The parameter c follows a growing process as it is illustrated in Fig. 6.9. The total period explored is between 1850 and 2006. This period is divided in two periods (1850–1963) and (1964–2006) by observing the related data (see Fig. 6.9). The following exponential function is applied for the two periods

$$Y = F + (f_0 - F)e^{at}$$

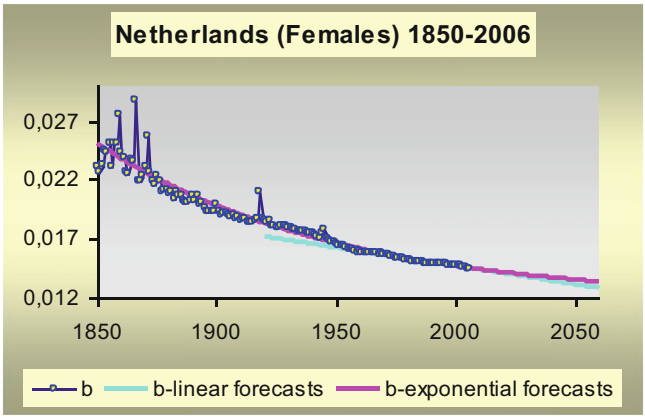

Fig. 6.7 Parameter b: fit and forecasts

Table 6.4 Regression analysis parameters for parameter b

F	f_0	a
0.01175	0.02509	0.010074

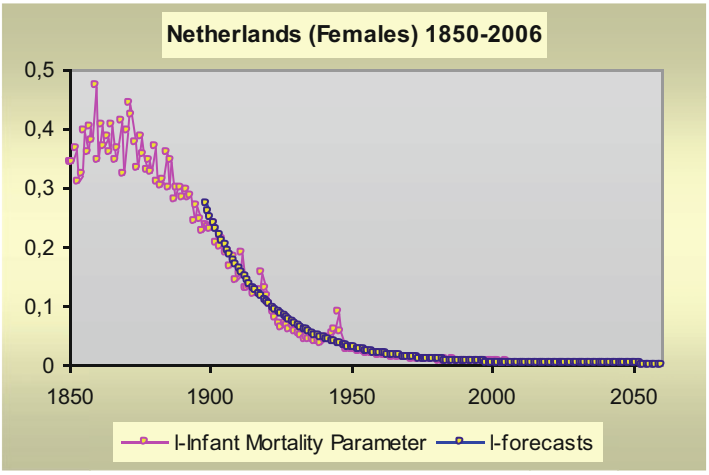

Fig. 6.8 Infant mortality parameter l

Table 6.5 Regression analysis parameters for parameter l

F	f_0	a
0.001329	0.2505	0.04329

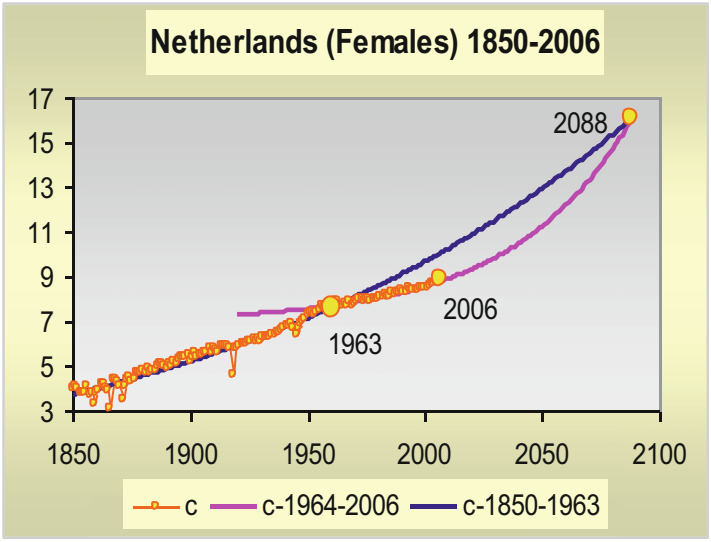

Fig. 6.9 Exponent c: fit and forecasts

The parameters from the nonlinear regression analysis fitting are summarized in Table 6.6. The forecast based on the period (1850–1963) suggests higher values for c than the forecast based on the (1964–2006) data. However, both curves cross each other at 2088 (see Fig. 6.9).

Table 6.6 Regression analysis parameters for parameter c

Time period	F	f_0	a
1850–1963	-1.2738	3.7108	0.005235
1964–2006	6.9492	7.7386	0.01969

Table 6.7 Netherlands (females) life expectancy at birth for various periods

Parameter	2006	2020	2040	2060	2080
b	0.01443	0.01407	0.01345	0.01282	0.0122
l	0.004531	0.002711	0.00191	0.001574	0.001432
c	8.922	9.327	10.475	12.176	14.698

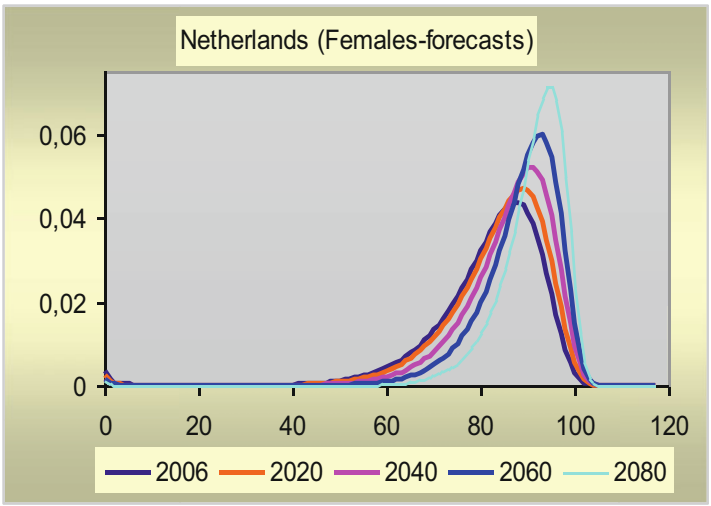

Fig. 6.10 Forecasts of death rates in Netherlands (2020, 2040, 2060 and 2080)

By estimating the future trends of the parameters b, l and c we can calculate the death distribution for several time periods. We have selected the parameters forecasts for the years 2020, 2040, 2060 and 2080. The values for these parameters are summarized in Table 6.7. The related graphs are illustrated in Fig. 6.10.

We also explore the behavior of the death distribution at the right inflection point. As it was already tested for the case of Sweden (Skiadas and Skiadas 2011b) the tangent at the right inflection point of the death distribution tends to shift to a position perpendicular to the X axis, whereas the right displacement of this line is slower (see Fig. 6.11). In other words the characteristic tangent which we call as *the longevity tangent* (Skiadas and Skiadas 2011b) looks like a barrier set by the human organism to our efforts for longevity. The results for Netherlands (Females) support the previous findings for Sweden (Females).

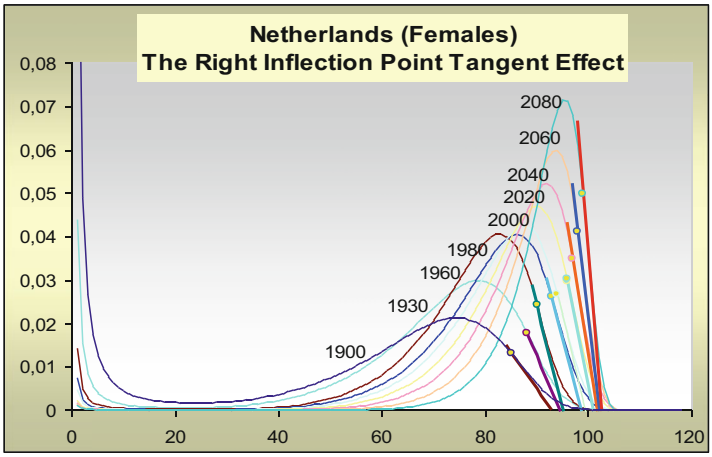

Fig. 6.11 The vertical shift effect of the tangent at the right inflection point

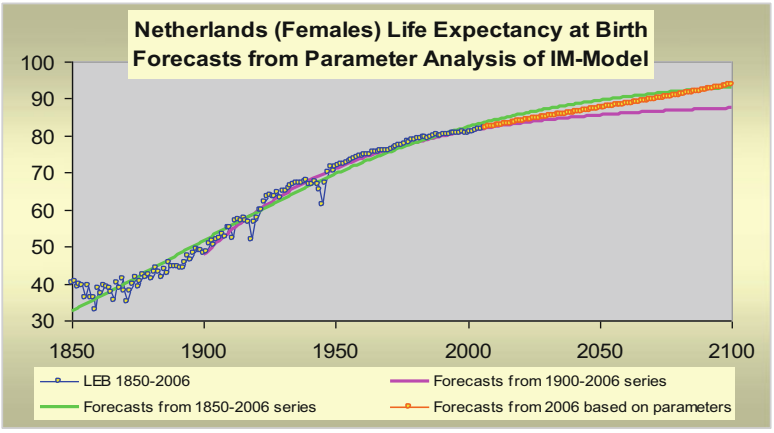

Fig. 6.12 Estimating the life expectancy at birth by using the parameter forecasts of the IM-Model (Netherlands, females)

The estimates and forecasts for the parameters of the $g(t)$ provide another method for estimating the future trends for the life expectancy at birth (LEB). In Fig. 6.12 we provide the graphs for the estimates of LEB based on the series (1850–2006) and (1900–2006). For both cases we fit a Logistic model to the data. This model has the form

$$LEB(t) = \frac{F}{1 + ((F - f_0)/f_0)e^{-at}}$$

Table 6.8 Regression analysis parameters for LEB

Time period	F	f_0	a
1850–2006	96.18	32.53	0.01636
1900–2006	88.89	47.17	0.01676

Table 6.9 Netherlands (females) life expectancy at birth for various periods

Year	1900–2006	1950–2006	1850–2006	2006+
2010	82.37	82.48	84.16	82.63
2015	82.90	82.95	85.00	83.28
2020	83.38	83.37	85.78	83.91
2025	83.82	83.75	86.51	84.52
2030	84.23	84.10	87.20	85.13
2035	84.60	84.42	87.85	85.72
2040	84.95	84.71	88.45	86.30
2045	85.27	84.98	89.01	86.88
2050	85.56	85.22	89.53	87.46
2055	85.82	85.44	90.02	88.04
2060	86.07	85.64	90.48	88.62
2065	86.30	85.83	90.90	89.21
2070	86.51	85.99	91.30	89.82
2075	86.70	86.14	91.66	90.43
2080	86.87	86.28	92.00	91.06
2085	87.04	86.41	92.32	91.72
2090	87.19	86.52	92.61	92.40
2095	87.32	86.63	92.88	93.10
2100	87.45	86.72	93.13	93.84

The estimated parameters of the Logistic model are presented in Table 6.8. For both time periods studied the growth parameter a obtains similar values, whereas the use of all the data points (1850–2006) gives an estimated for the life expectancy upper limit $F = 96.18$ higher than the upper limit estimated from the series (1900–2006) that is $F = 88.89$. The LEB, estimated by using the parameters b, l and c, the function $g(t)$ and the related program, gives us a graph (Fig. 6.12) between the two other estimates. The Table 6.9 includes the predictions based on the two methods proposed. The second method based on the analysis of the parameters gives higher future values for the life expectancy at birth. However, by combining both methods we can expect to improve forecasts and especially by making long range estimates.

6.5 Conclusions

We have applied a new theoretical framework to forecast the future life expectancy and life expectancy at birth in the Netherlands (Females). We also do predictions based on a classical forecasting methodology. The resulting figures are close to

those suggested by the Dutch Actuarial Association in the Projection Table 2010–2060 (Dutch Actuarial Association 2010). However, the two methods are based on a different underlying theory. The method used here can give reliable fitting even by estimating values in the past. By using the 1950–2006 data for Netherlands (Females) we have good predictions for the past period (1900–1950) and for the future (2006–2060).

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Chapter 7
Remarks and Findings on "Evidence for a Limit to Human Life Span"

7.1 The Mortality Probability Density Function and Related Modeling

Following the open questions addressed after the publication in Nature by Dong et al. (2016) titled "Evidence for a limit to human life span" we search for the advancement of the existing models, methods and techniques to improve fit and forecasts. The main tool is the classical Life Table and the probabilistic methods in hand. The mortality probability density function is derived from the life tables in the form of dx provided that every element of dx is divided by 100,000. Note that life tables provide results for a hypothetical population of 100,000. By dividing by this number the resulting death probability density function occurs (see the solid curves in Fig. 7.1a, b). Note that this death probability density (pdf) function is not that provided by the direct number of deaths during a specific period of time as it is presented in Fig. 7.1a, b (dashed curves). The clear advantage of selecting the death probability density from the dx estimates of the life tables is that the death probability resulting from dx includes the variations in the population over time and age as the life table construction is based on $m_x = D_x/P_x$, where D_x is the number of deaths and P_x is the population at age x and m_x is the mortality. The Health State Function should be based both on deaths and the population and thus on the mortality probability density function and not on the probability density function based only on deaths. Having introduced the mortality probability density function (pdf), the classical model used is Gompertz (1825) with a good fit for the middle and high age groups. Over the years the Gompertz model, in the exponential or simpler in a logarithmic form is applied to the simpler data form provided as m_x or q_x in the life tables or as the logarithms of m_x or q_x.

C.H. Skiadas, C. Skiadas, *Exploring the Health State of a Population by Dynamic Modeling Methods*, The Springer Series on Demographic Methods and Population Analysis 45, DOI 10.1007/978-3-319-65142-2_7

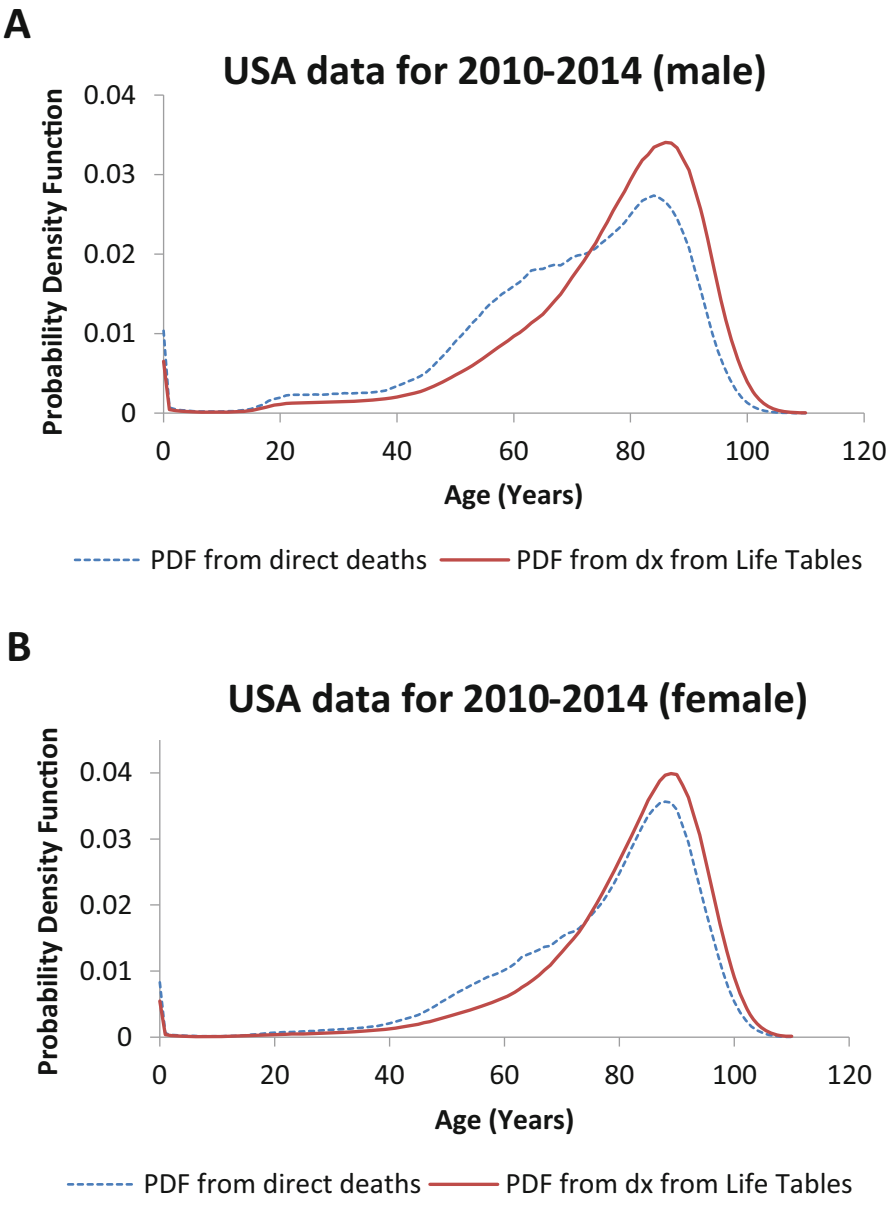

Fig. 7.1 Death probability density forms

The illustrations in Fig. 7.1 provide a clear view of the death phenomenon in a population in the form of a probability density function. As a density function should have a variance or a standard deviation σ expressing the dispersion of deaths

around a mean value. The smaller the σ the sharper the form of the death pdf. A limit for the human life span is expected if:

(A) σ tends to zero or
(B) the right hand side of the pdf remains unchanged or
(C) the pdf is stable.

In both B and C cases it remains the problem of very few death cases appearing at the very right of the pdf tails. These data points appear with very low probabilities, sometimes once per millions of deaths. Many years are needed for countries like France or with larger population to find an exceptional longevity case. It is not clear if these exceptional and rare cases could be effectively determined.

The pdf includes important information for longevity studies. However, the Gompertz or related models are not very satisfactory in expressing important issues, as the standard deviation σ is given with a very complicated formula. Even more the fitting is poor in the past historical data. That is important is the generating function of the mortality pdf that is the health state $H(x)$ or the health status of the individuals. This was achieved by Weitz and Fraser (2001) for the case of the Carey Medfly data (Carey et al. 1992). The authors Weitz and Fraser (2001) supposed that the health state of an individual is a stochastic process following higher and lower values in the time course and death occurs when this process hits for the first time a barrier set at zero for the health state. Then, the summation of all the death instances at every age, form the death density function which is turn to be a pdf after normalization. Weitz and Fraser (2001) have used the theory for the first exit or hitting time first published by Schrödinger (1915) and Smoluchowsky (1915). However, although the methodology could be applied to the human population, the form of the distribution provided for medflies was not appropriate. Fortunately a generalization of the first exit time theory was done by Janssen and Skiadas (1995) whereas the exact formula was published by Skiadas and Skiadas (2010b) and applied in Skiadas and Skiadas (2014, 2015).

$$g(x) = \frac{|H_x - xH'_x|}{\sigma\sqrt{2\pi x^3}} e^{-\frac{[H_x]^2}{2\sigma^2 x}} \tag{7.1}$$

This general form gave good results for a simple Health State Function selection that is $H_x = l - (bx)^c$, with l, b, c as parameters.

The pdf $g(x)$ is given by

$$g(x) = \frac{|l + (c-1)(bx)^c|}{\sigma\sqrt{2\pi x^3}} e^{-\frac{[l-(bx)^c]^2}{2\sigma^2 x}} \tag{7.2}$$

for the Skiadas and Skiadas (2010b) applications and by

$$g(x) = \frac{|l|}{\sigma\sqrt{2\pi x^3}} e^{-\frac{[l-(bx)]^2}{2\sigma^2 x}} \tag{7.3}$$

for the Weitz and Fraser (2001) applications. Clearly Weitz and Fraser formula is a special case of the Skiadas and Skiadas (2010b) arising when the exponent $c = 1$. for both cases the parameter $l = 1$ to obtain the results of Figs. 7.2 and 7.3 where the parameter σ is estimated along with the other parameters. An alternative case is to set $\sigma = 1$ and estimate l and the other parameters.

Estimating the data points at the very end to the right of the mortality probability density problem is a very delicate task as the data points selected are very few and the related models are hard to achieve a good fitting. The classical approach was to follow statistics of extremes and the related theory. Statistics of extremes is an alternative of coping with the high age life spans. Fisher, Tippett, Gnedenko and Gumbell are between the founders of this theory. However, it is not clear on how an application could be successful for the human life span data. The simpler approach is first to transform data in a convenient way and then to find the proper way to cope with data with a particular method or model. The Extreme value theoretical approaches suggest mainly a fast declining negative exponential like equation forms generated by one, two or more negative exponents depending on how fast is the declining process. For a very fast declining form the double exponential Gumbell law is of particular importance.

7.2 Maximum Death Age

The first step is to transform the data by taking logarithms, a convenient step before any further study. We thus have immediately an option to see visually the decline of the process in the last stages of the life span. We select a 5 years time period from 1980 to 2014. The estimated maximum death age covers 7 time periods from 1980–1984 to 2010–2014 for USA females. As the death population is in logarithmic scale one corresponds to 10 persons and zero to one person dead. Accordingly 2, corresponds to 100 persons, 3–1000 and so on. For the period studied the deaths at 110 years of age range from 32 in 1980–1984 to 108 in 1995–1999 where the larger number appear (see Figs. 7.4, 7.5, and 7.6).

As the probability of death in very high ages is very low the number of people surviving in high ages is very critical (see in Fig. 7.7 for details in four countries and at the Halley (1693) Breslau data). Theoretically for a huge population surviving at high ages the existence of super-centenarians is larger. That is why United States have a relatively large number compared to other countries with smaller population.

The case we also study here is to estimate for which time period the appearance of long living people was higher. Note that in the cases presented above the population over 100 was not the same but it was precisely the number provided by the Human Mortality Database (HMD) death tables. This number was larger and larger moving from the older time periods to the most recent. In Fig. 7.8 provided from the cohort estimates we use the number of deaths at 100 years of age (123587) from 1980 to 2004 for USA females as the starting point for all estimations. The red line expresses the mean value of the five periods estimated. All estimates provide good

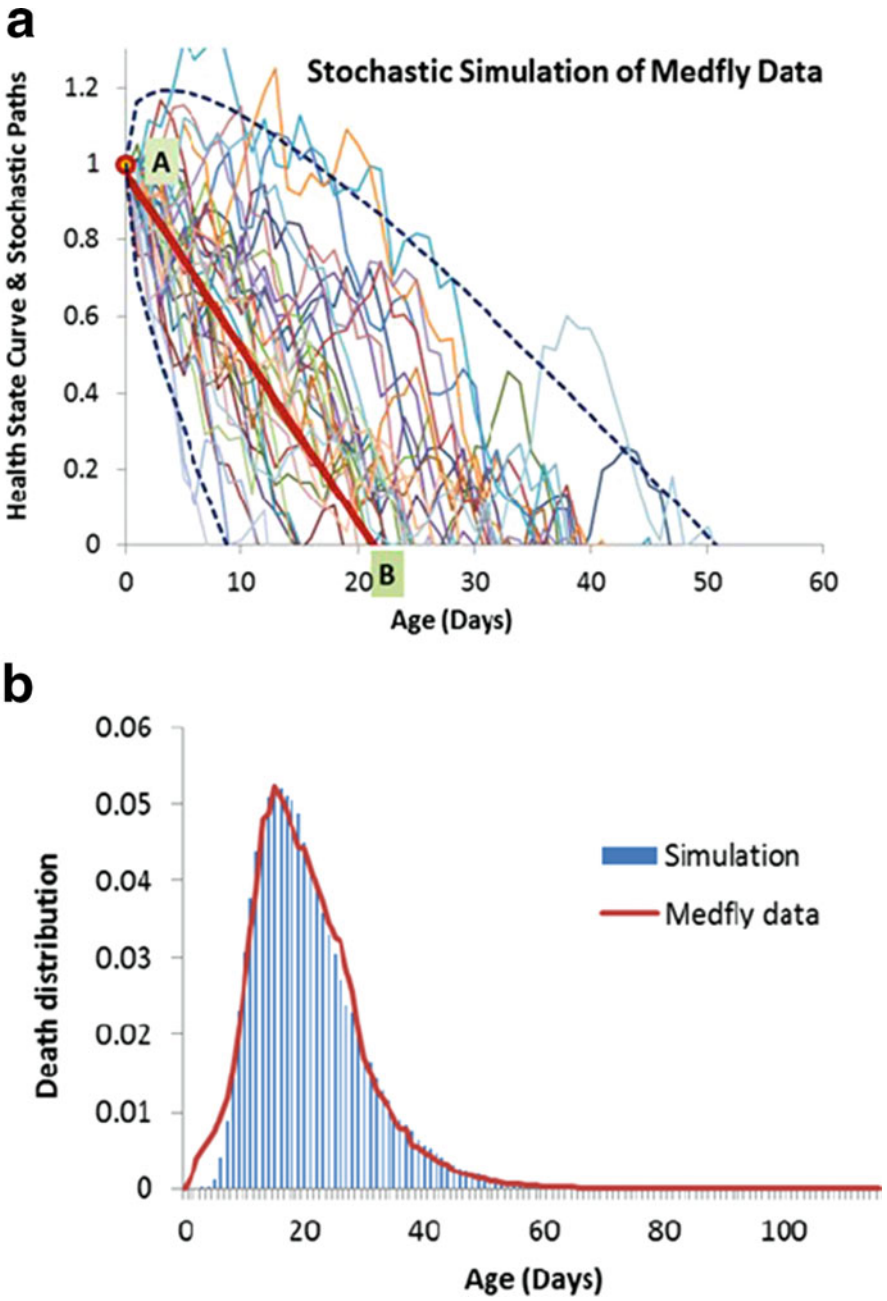

Fig. 7.2 The Weitz and Fraser Medfly data case

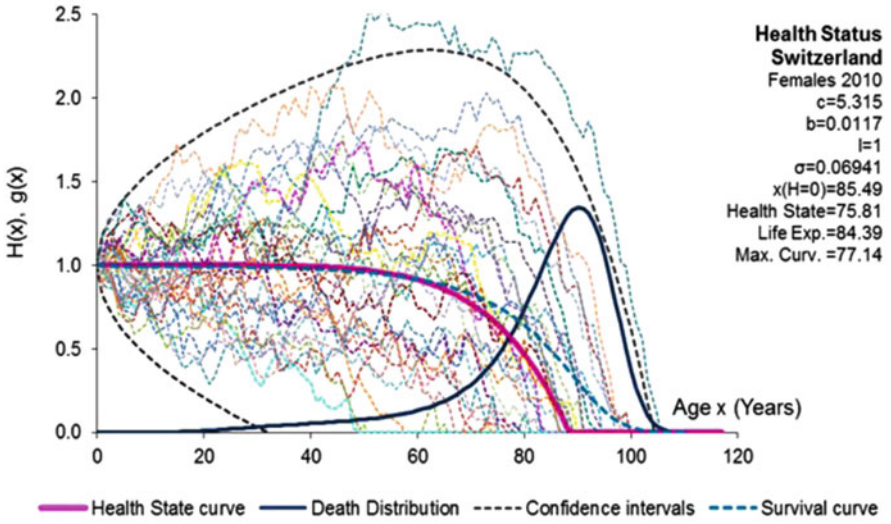

Fig. 7.3 The human data case

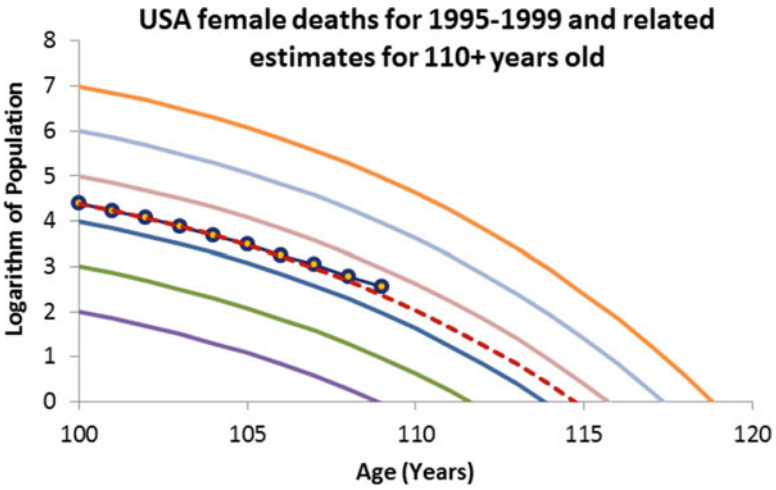

Fig. 7.4 Fit on death data at very high ages for USA female (1995–1999). Five year periods

probabilities of survival for at least one person at 116 years of age. The highest probability belongs to the 1995–1999 group followed by the 2000–2004 group whereas the probabilities for the three remaining time periods (1980–1984, 1985–1989 and 1990–1994) are lower but yet in the vicinity of year 116.

The survivals at very high ages are presented in this Fig. 7.9 where the curve expresses the mean value of the estimates presented earlier. For 129,660 deaths at 100 years of age we have only 3 survivals at 115, 1 survival in 116 where we have

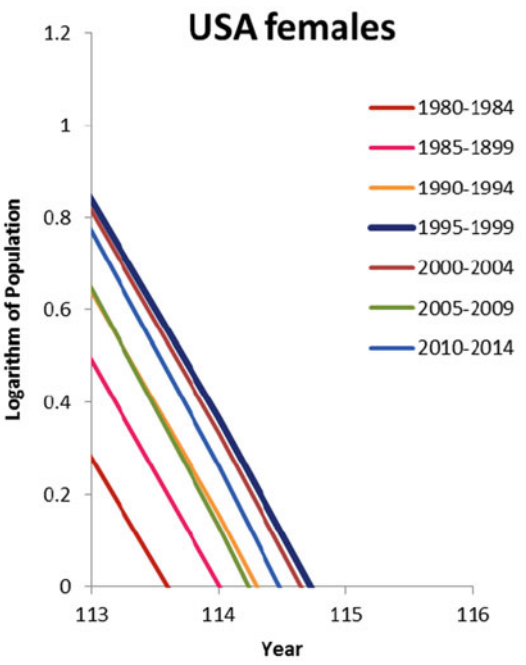

Fig. 7.5 Fit on death data at very high ages (113+) for USA female. Five year periods

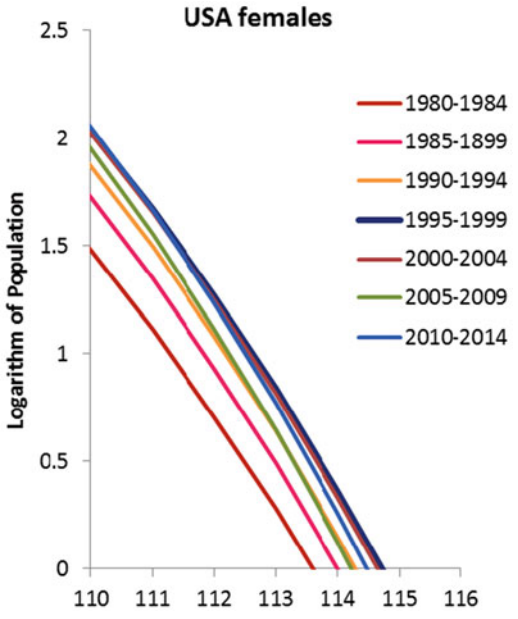

Fig. 7.6 Fit on death data at very high ages for USA female. Five year periods

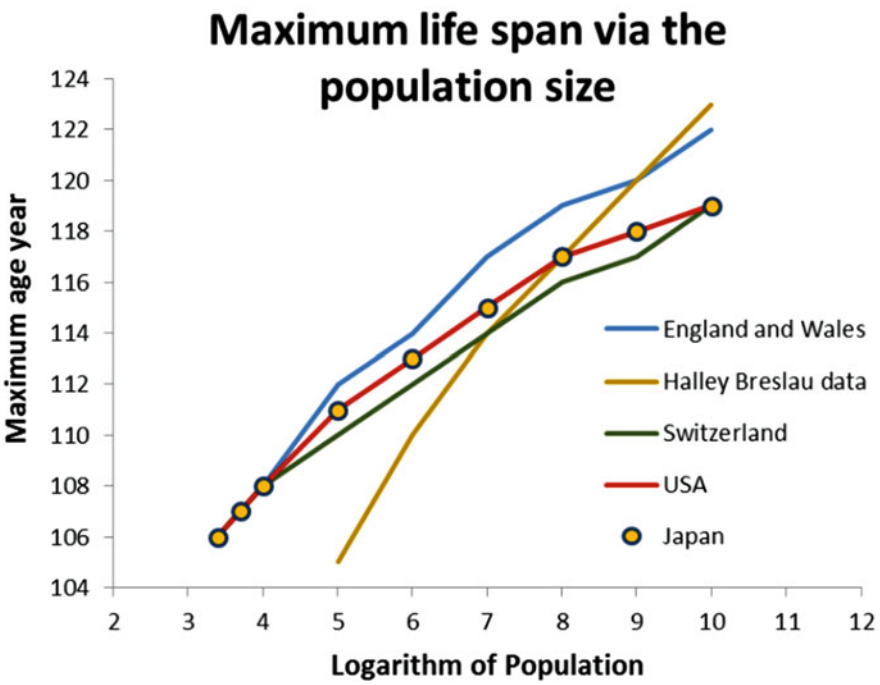

Fig. 7.7 Maximum life span via the population size

Fig. 7.8 Fit on death data at very high ages for USA female. Five year periods

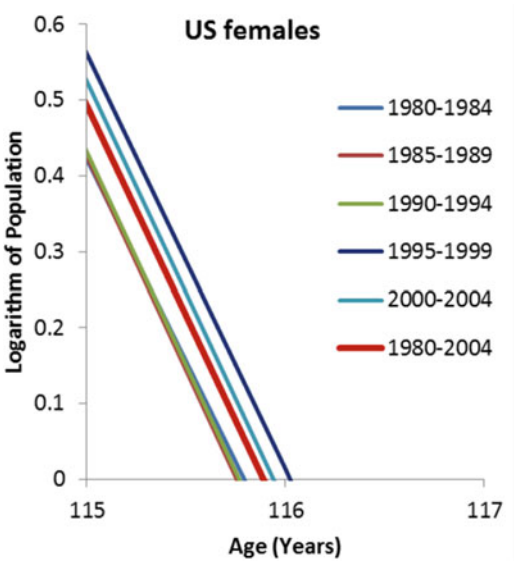

Fig. 7.9 Survivals at very high ages for USA female

a probability $0.87 \approx 1$, whereas, for age 117 we have 0.22 and only 0.049 in 118, dropping very fast to 0.010 for 119 and to 0.0016 probability for 120 years (Fig. 7.9). According to these figures the survival at 120 is not zero but it is very low by means that the population surviving after 100 should be some hundreds time larger than today. Note that for the 129,660 deaths at 100 years of age the number of deaths for 100+ is 277,841 whereas for 110+ is 1926. Assuming that we need a figure larger than 0.5 and smaller than 1.5 to expect at least one survival we observe that we have to triple the population to reach 117 years, to multiply by 11 to reach 118, by 50 to reach 119 and by 312 to reach 120. It is unfortunate that a huge population surviving over 100 years of age is needed to expect to reach very high ages. There is not a clear barrier but a very low survival probability (Fig. 7.10).

7.3 Data Description

Data Description: according to the 2014 US death base from the Human Mortality Database (HMD) 72 super-centenarians appear (64 females and 8 males). Considering the related number of deaths for females and males (1,298,177 and 1,328,240 respectively) the probability of appearance in the death base of 2014 for United States of one super-centenarian is only 5 per 100,000 for females and 6 per 1,000,000 for males. It looks like to be impossible to model such a small quantity by a convenient model or method. Clearly larger data sets are needed. Fortunately US data are quite large. Selecting data from 34 years from 1980 to 2014 the super centenarians are 2576 females and 653 males for 39,617,539 and 40,794,889 females and males deaths respectively for all 34 years of age.

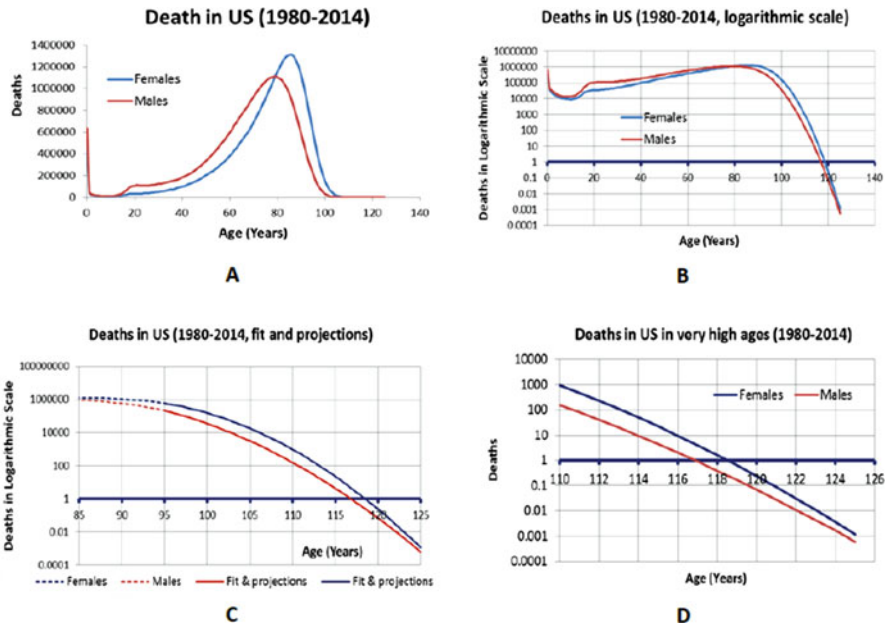

Fig. 7.10 Logarithmic transformation of USA male and female death data. Fit and projections

7.4 Data Transformation, Fit and Projections

The data handle in demography has benefited by a relatively simple transformation coming from the Gompertz days, by means of the logarithmic transformation. The only we have to do is to plot the data in a logarithmic chart where in the upper part the population figures are presented whereas in the lower part the numbers expressed in decimal places express the probability of finding a person alive in the related age. This method of data reduction provides a visual illustration as presented in the Fig. 7.11 where data for females in 2014 and for 1980–2014 for females as well are presented. In both cases the data visualization is similar with the second data curve to be placed in a higher position. The data structure in the ages over 95 or 100 years are quite smooth providing a unique opportunity to fit a relatively simple quadratic model of the form

$$y = ax^2 - bx + c$$

to the data from 100 to 110 of from 95 to 110. Both applications give a very good fit and the relative projections presented in Fig. 7.11. The logarithmic form provides very interesting facilities the most important being the direct estimation of the person died at the higher age from the point where the projection curve cuts the horizontal line. This point is higher in the case of the larger population as is the case

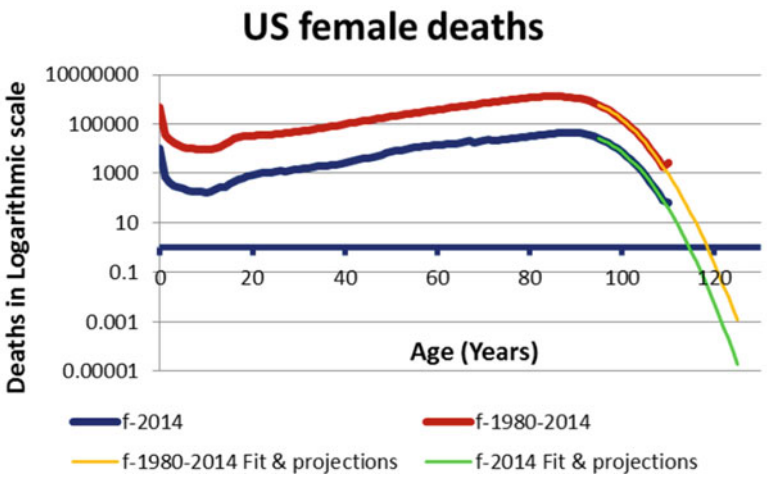

Fig. 7.11 Logarithmic transformation of USA female death data. Fit and projections

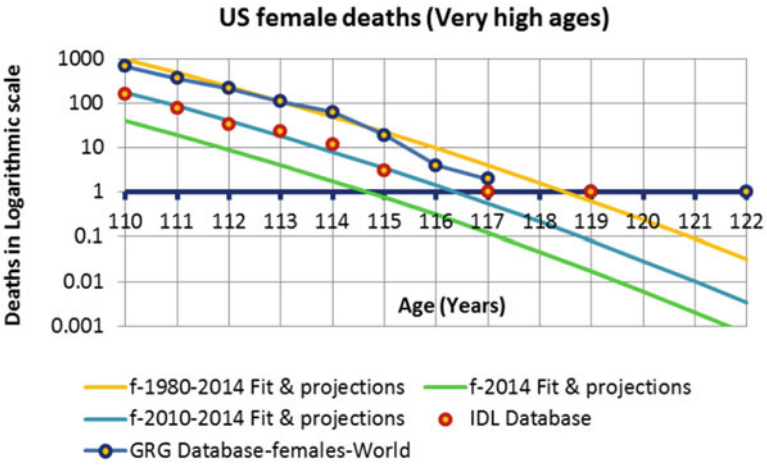

Fig. 7.12 USA female deaths at high ages for various time periods. Fit and projections

of females in US from 1980 to 2014 where the US female deaths for 34 years are summarized. For 2014 females the higher age for a super-centenarian is 114.5 years whereas for 1980–2014 is at 118.5 years. For the latter case the age year of 121 could be achieved for a population ten times larger (see the next Fig. 7.12). The projections line for females in US from 2010 to 2014 perfectly fits to the data provided from the IDL Database (http://www.supercentenarians.org/). The super centenarian data provided by the Gerontology Research Group for females in the World fit to the projected line for 1980–2014 data for US females.

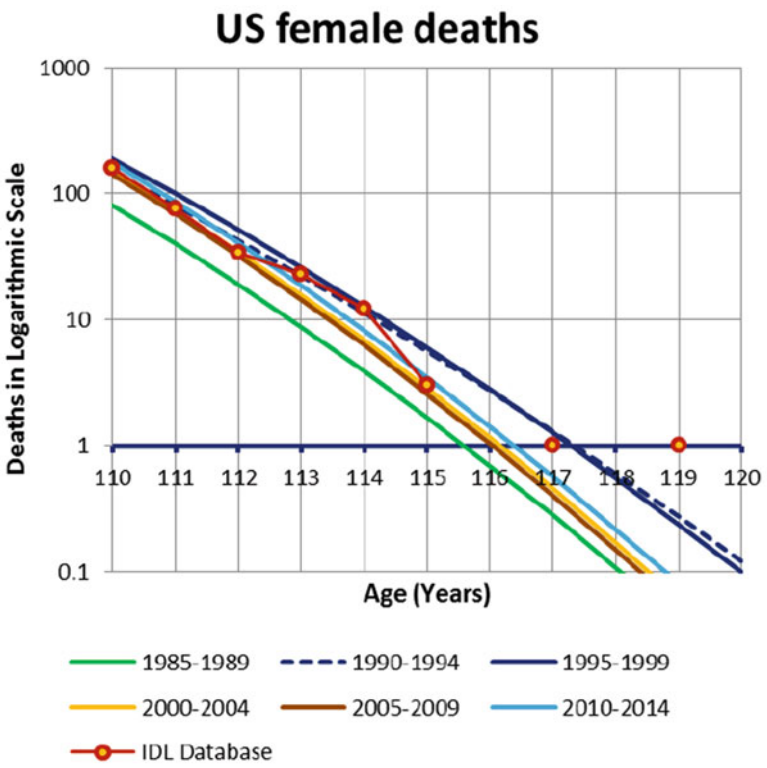

Fig. 7.13 Five year period USA female deaths at high ages. Fit and projections

In Fig. 7.13 the projections of six periods from 1985 to 2014 are illustrated along with the data for females in US provided by the IDL database. The periods 1990–1994 and 1995–1999 are characterized by higher levels of the maximum age at death with one death at 117 year of age whereas all the periods assume higher values from 115 years of age for at least one survival. In all the cases studied a survival from age higher than 118 years could appear if the population size will be 10 times higher. Clearly as we deal with 5 year groups for females in US it is likely to have super-centenarians at ages 118 and higher in a time horizon of the next 50 years. More optimistic forecasts are coming from the lines for 1990–1994 and 1995–1999 leading to estimates of 120 years for the next 50 years. For both time periods the appearance of 117 year old super-centenarians is more probable than for the other periods and the same hold for the 119 years. As far as we are dealing with a large population or we have enough time to wait to obtain a quite large population the probability to find a super-centenarian in a higher age is larger. However, the logarithmic low suggests an exponential increase for the number of the population needed to pass from one age barrier to the next. However, there is no age limit like a barrier. Simply the probability of finding super-centenarians at very high ages is so small that a huge population is needed exceeding the World population.

7.5 Use of Life Table Data

The above methodology could apply to the death distribution provided as *dx* from the life table data. This is a good method to compare different populations size and various time periods. The shortcomings come from the structure of the life tables referring to a hypothetical population of 100,000 people. However, if we know the population size for the specific year and country, we can have a direct estimate of the super-centenarians. Example from the 2000–2009 life tables from US and France is illustrated in the next two Figs. 7.14 and 7.15. The life tables are dealing with a hypothetical population of 100,000 people. Both results for US and France are quite close each other in the centenarians and super-centenarians age periods. As the number of deaths is only 100,000 the expected number of super-centenarians is small (8 for US and 8 for France as well). The projections provide all super-centenarians below 115 years of age. However, this number is expected to rise depending on the number of deaths or of the hypothetical population size. As we already know the number of female deaths for USA from 1933 to 2014 (73,336,683) and for France from 1816 to 2014 (68,038,639) we have a quite large population number to find a significant number of super-centenarians over 115 years of age. In both cases a super-centenarian is expected to appear at 119 years of age as in US, Sarah Knauss (1880–1999). In France Jeanne Calment has the age record at 122 years. Following figure the probability to live 121 is ten times smaller than to reach 119 for both US and France and one hundred times smaller to reach 125 years of age. However, as it was in France for Jeanne Calment, to reach such a high age has a small probability but not zero. High age super-centenarians could appear in quite large populations or in small or large populations but in a quite large time horizon.

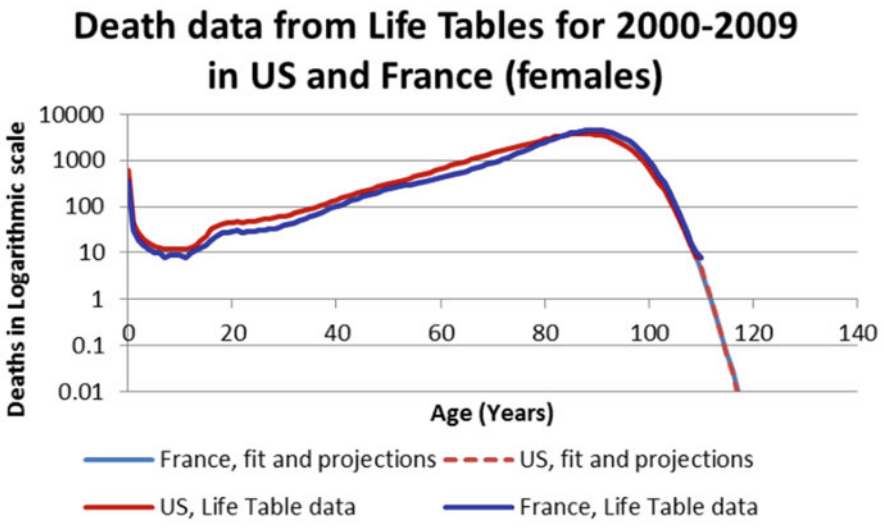

Fig. 7.14 Logarithmic life table in USA and France

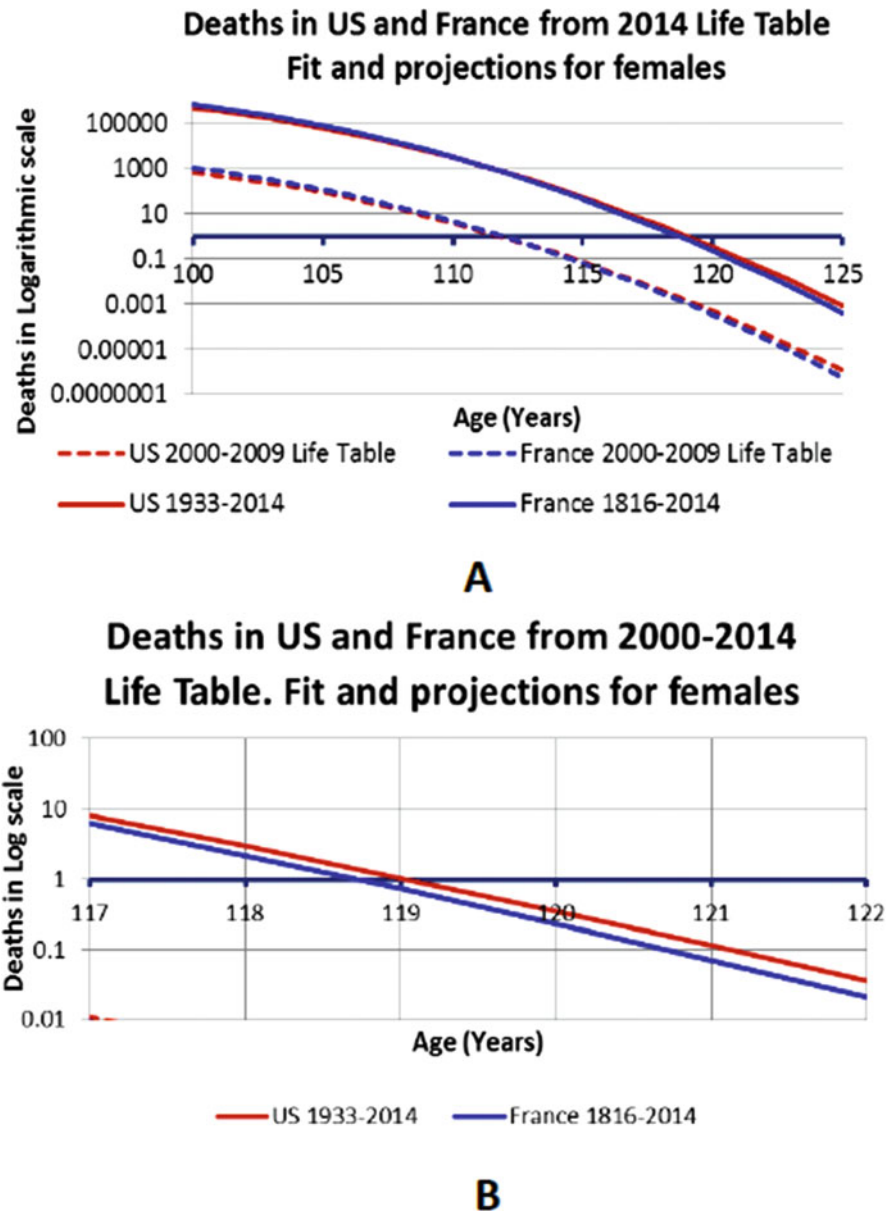

Fig. 7.15 Deaths in USA and France. Fit and projections

7.6 Model Application Method

The provided data transformation followed by fitting and the related projections are important tools to find the time course of the super-centenarians development. However, a model application method is also important to cross-validate the previous estimates in the extreme right of the death curve of the population. A convenient model was developed by using the first exit time theory and applied to MedFly data by Weitz and Fraser (2001) and by (2010a,b; 2014) to the human population. The latter model provides good fit to the data from 10 years until the end of the life span. Only the first part including the infant mortality is excluded. This part has not real effect in the super-centenarians estimation.

7.7 Fit and Simulation

A good fit model is important but it should be verified by an effective projection study. The fit and projection study is illustrated in the next Fig. 7.16. So far we have used a simple model of the form

$$\log(g(x)) = c - b(x - 100) - a(x - 100)^2$$

to fit and calculate projections up to 125 years of age. The logarithmic form was quite appropriate to model the fast declining mortality data for the years 100+ and more importantly for 110+. However, a second model based on the first exit time theory could be appropriate in order to have a second exploration of the super centenarian case. The simpler form of the first exit time stochastic model transformed to account for the logarithmic data sets is given by:

$$\log(g(x)) = k - (3/2)log(x) - (bx)^c.$$

For both models the application to the female data in US from 1980 to 2014 provides close results illustrated in the following Fig. 7.16. For both cases deaths are predicted at 115 years of age and one in the year 118 for the stochastic model and at 118.5 year for the simple model. The probability of living at ages higher than 120 is very small and a hundred times smaller from 120 to 125.

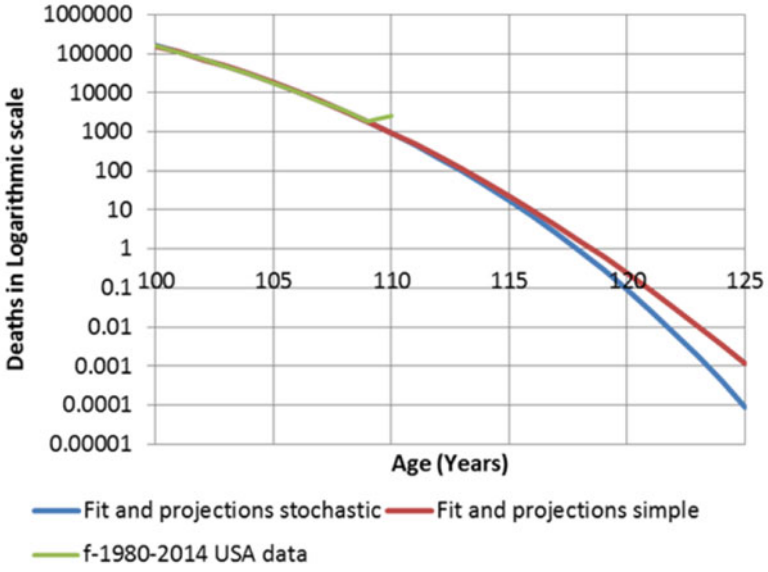

Fig. 7.16 Female supercentenarian deaths, two methods of fit and forecasts in USA

7.8 The Methodology

We select the 100–110 years data from 1980 to 2012 from the death data for Italy (females) provided by the HMD. We fit to the logarithm of the 100–109 data sets the simple model and we do projections up to 120 years of age. Now we deal with different super centenarian data provided by several sources. The most reliable data provided by the IDL data base (see the blue circles in the Fig. 7.17) refer to 31 super centenaries whereas the data selected for HMD refer to 108 super centenarians. We have used the HMD to find the trend for the projections done after 110 years of age. We also have estimated 96 deaths for 110+ years of age instead of 108 provided by the HMD database, that is a 89% success. The next step is to assume an analogous trend followed by the data provided by the IDL database and the only needed is to shift the projections curve to a lower level so that the total estimated super centenarian deaths from 110+ to be exactly 31 females as provided by IDL database. This is achieved by multiplying by 0.32 the HMD projections. The resulting curve (green color) perfectly fits to the IDL data. Also by multiplying the HMD projections by 1.59 the estimated deaths are 152 as are downloaded in January 8, 2017 by a gerontology database at http://gerontology.wikia.com/wiki/List_of_Italian_supercentenarians.

We have succeeded to find a convenient method to cope with the super centenarian data trend and then to have a better view of the future development of the long lived people. The same approach is accepted for the US death data for females from 1980 to 2014 as are provided by the HMD. The fit and the projections are illustrated in the next Fig. 7.18. The female deaths provided from HMD for 110+ for 1980–

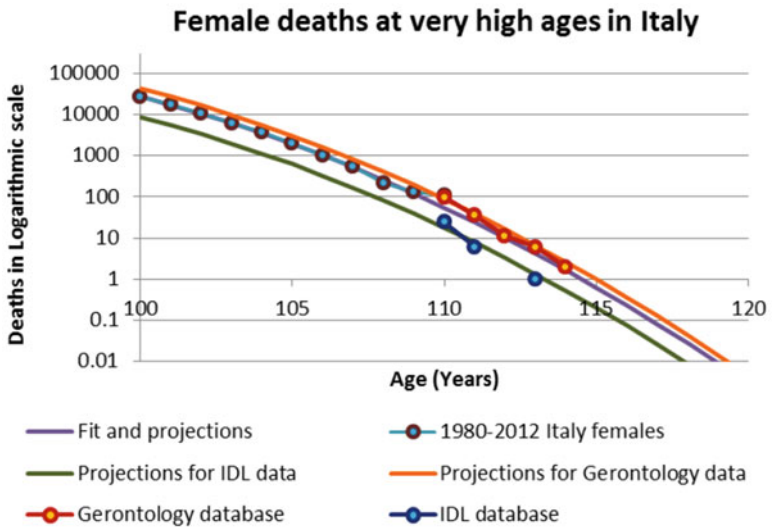

Fig. 7.17 Female Supercentenarian deaths, fit and projections in Italy

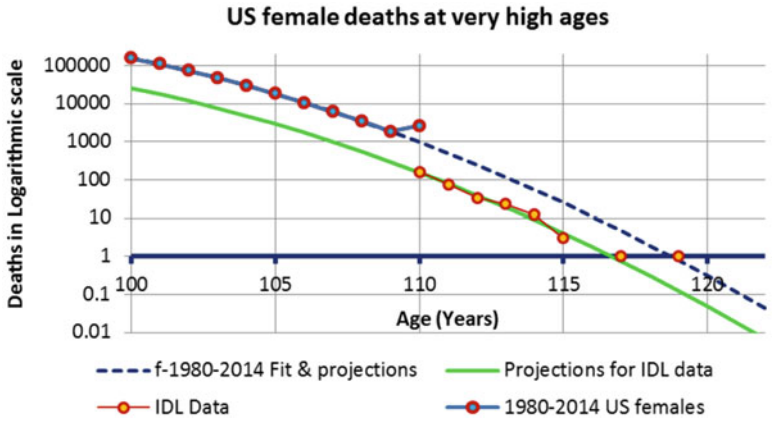

Fig. 7.18 Female supercentenarian deaths, fit and forecasts in USA

2014 are 2576 whereas the projected are 1930 covering the 75% of the total sum. The provided 309 deaths from IDL data is the basis for finding the trajectory for this case presented by the green line in the same Fig. 7.18. We multiply by 0.16 the HMD projections. The used methodology provides a good method to locate the trajectory for the IDL data whereas it can estimate some extra points placed outside this trajectory by means the case surviving at 119 years of age. The latter case is within the space located by the upper curve which is produced from the 1980–2014 population size that is 39,617,539 and 2,576 deaths for 110+ years of age. The super-centenarians deaths from IDL data base refer to a population size of 6,338,806 that is close to a 5 year sum of death females in US.

7.9 Summary and Conclusions

Following the open questions addressed after the publication in Nature by Dong et al. titled "Evidence for a limit to human life span" we have proposed new analytic tools methods and techniques to estimate the number of centenarians and super-centenarians in a society. We propose and apply related equation forms and we use the data from Gerontology and related data bases to validate the models and do projections.

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Chapter 8
Stages of Human Development: The Life-Span Approach and Related Applications and Comparisons

8.1 Introduction

Following the Paul B. Baltes (1987) theoretical propositions of Life-Span Developmental Psychology presented in his seminal review paper in Developmental Psychology in 1987 we have tested his arguments based on the findings of other scientific fields and especially from the Human Population and Mortality studies. Human mortality studies started few centuries ago (Halley 1693), long before the development of Psychology, Sociology and the related fields. Human population and mortality studies benefited from the existing data sets in providing well established quantitative measures for the human life-span and the related studies. From the other part Psychology and Sociology have already an immense literature of well-developed qualitative and quantitative studies. As the main aim in all these fields is the exploration of the very many aspects of humans, the human development and the human life activities, it is worth noting to explore how the findings in one or more fields could apply to others. It is not surprising that Baltes suggest as a pioneer in life-span development studies Johannes Nikolaus Tetens (1777). Tetens expertise includes philosophy, actuarial science and mathematical statistics, enough material to understand the various aspects of the life-span process. Quetelet (1835) was also involved in related studies while he has also published some of the works of P. F. Verhulst (1838) on demography.

Baltes sees an interconnection of psychology and sociology as he writes: *Especially within sociology, the study of the life course and of the interage and intergenerational fabric of society is enjoying a level of attention comparable with that of the life-span approach in psychology. He continuous: These observations on literature, art history, and social images of the life course suggest that the field of life-span development is by no means an invention of developmental psychologists. Rather, its recent emergence in psychology reflects the perhaps belated effort on the part of psychologists to attend to an aspect of the human condition that is part and*

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parcel of our everyday cultural knowledge systems about living organisms. Such social images suggest that the life course is something akin to a natural, social category of knowledge about ontogenesis and the human condition.

Baltes accepts the following definition: *Life-span developmental psychology involves the study of constancy and change in behavior throughout the life course (ontogenesis), from conception to death. If one would try to find a similar definition from the part of human health and mortality by means of human biology or human sociology it should be: Life-span biology (or sociology) involves the study of constancy and change in human development throughout the life course, from conception (or birth) to death.*

Baltes in 1987 sees the following research scheme: *The current research scene suggests that in the immediate future life-span developmental psychology will not be identified with a single theory. It is above all a subject matter divided into varying scholarly specializations. The most general orientation toward this subject matter is simply to view behavioral development as a lifelong process.*

These views continue until today thus making the multidisciplinary studies very important.

8.2 The Health State Approach and Applications

Last 20 years, interesting studies related to healthy life span can have applications in psychology, sociology and the related fields. The human biology could be also important influencing changes in the psychological life-span development. Of course it is quite difficult and perhaps impossible to propose the theory of life span as Baltes had already pointed out. Clearly changes in our health state may affect our psychology. The task is to find a classification related to the life-span by using existing data and applications. First of all changes in the health state of an individual may affect many human characteristics including those studied by developmental psychology. The form of the health state is unpredictable on an individual basis due to the high uncertainty inherent thus forming a stochastic process in a mathematical or statistical notation. Several realizations of the health state of a person during the life span are given in Fig. 8.1. The mean value of many health states of a group of persons form a relatively smooth curve presented by the magenta curve (smooth line) in the same figure. This curve expressing the health state of the population is estimated from death and population data provided from the statistical yearbooks. The related statistical methods are presented in several publications (Skiadas and Skiadas 2010, 2014, 2015; Janssen and Skiadas 1995). The main finding is that a peak of the health state of a population is in a range from 30 to 40 years of age for many of the developed countries. A detailed search for USA males in the year 2000 is presented below.

According to the previous Fig. 8.2 the health state of the male population in USA the year 2000 is at the level 4.0 at birth and then it is growing until a maximum level of 17.22 in age 35.0, then the health state continuously decreases. On age 83.6 years

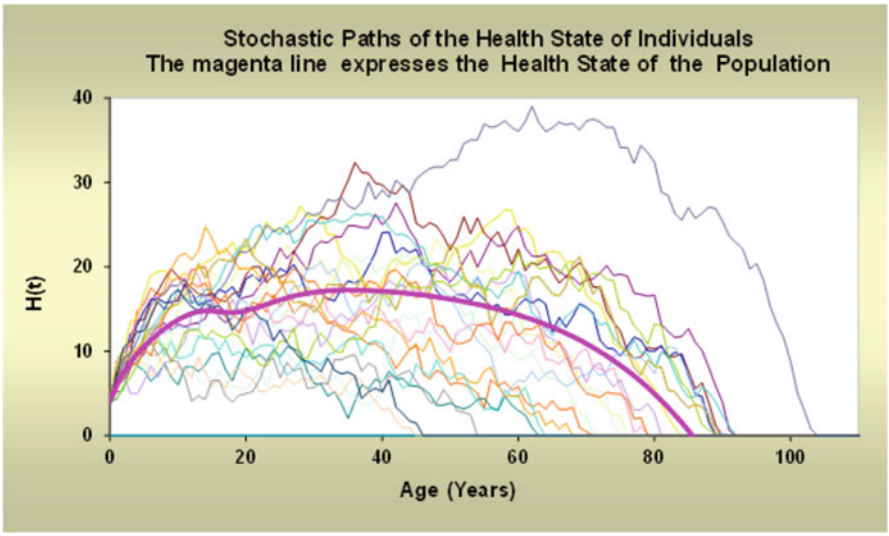

Fig. 8.1 The health state for USA in 2007

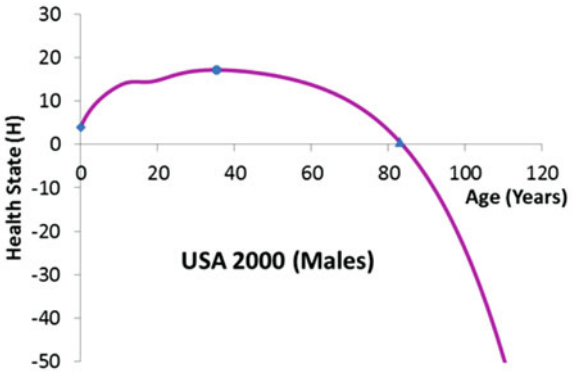

Fig. 8.2 The health state status of the USA male population

the health state is zero, where the maximum death rate appears, and it is negative for higher ages characterizing the critical health state period of life. The health state as presented in the last Figure is a relatively smooth curve. However, the details should be observed in the first, second and higher order differences (derivatives). The first difference expresses the speed of health state changes whereas the second difference expresses the acceleration of the health state changes.

The first difference (see Fig. 8.3) has two characteristic points by means of local maximum and minimum speed rates for the health state. The local minimum is at 14.8 years of age, whereas the local maximum is at 23.5 years of age for males in USA (2000). The age at the local maximum is very crucial for adult development. It characterizes the onset of the second (middle) stage of adult development.

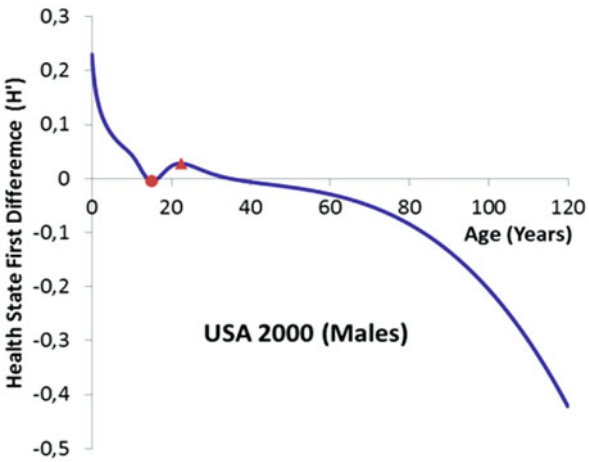

Fig. 8.3 The first difference of the health state of the USA male population

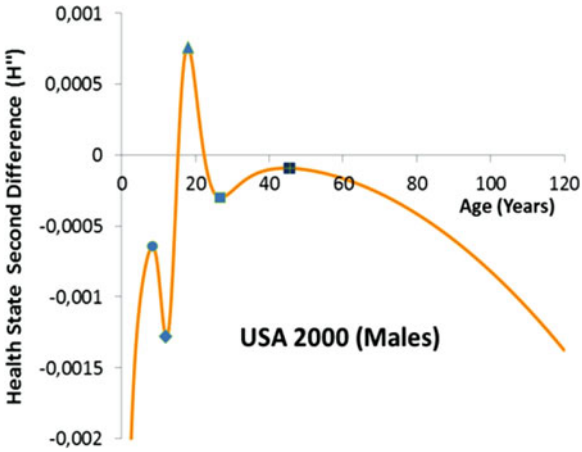

Fig. 8.4 The second difference of the health state of the USA male population

The second difference (Fig. 8.4) expressing the acceleration of the health state has 5 local maxima and minima indicating characteristic age years for the life-span. The first maximum at 8.4 years for USA (2000) males is the starting point for the pre-adolescence period ending at the next minimum at the 12.4 years of age. The following maximum at 17.9 years characterizes the end of the middle adolescence period of life and the onset to the late adolescence period whereas the next minimum at 27.4 years of age is the starting point for the third stage of adult development. The next local maximum at 44.4 years of age indicates the end of the early middle age period which starts from the maximum health state at age 35.0 and the beginning of middle and old age periods. Another very important issue is related to the age period with positive values of the second differences for the health state. This period is precisely the same with the age period from minimum to maximum first stage differences and it is from 14.8 to 23.5 years of age for USA males characterizing the late adolescence and the first stage of adult

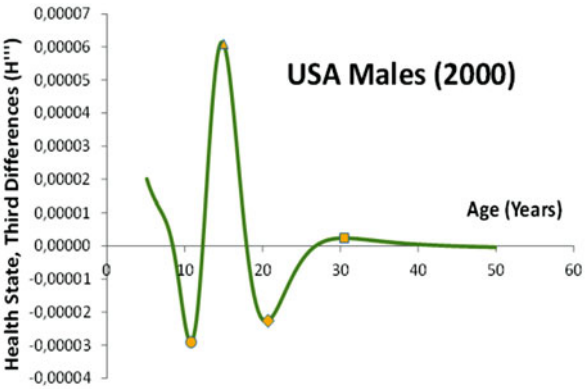

Fig. 8.5 The third difference of the health state of the USA male population

development. This age period is the most important for the development of many human characteristics including biological, psychological and social parameters. However, the human development continuous as long as the first difference of the health state is positive. The end of this period coincides with a peak of the health state during 30 years of age.

More details are obtained from the third differences of the health state (see Fig. 8.5). There are four local maximum and minimum characteristic points the first at 10.8 years of age, the second at 14.8 years of age that is equivalent to the local minimum of the first difference of the health state, the third at 20.5 years of age and the fourth at 30.4 years of age. The local minimum at 20.5 years of age is very important as it characterizes the transition from the late adolescence period of life to the early stage of adult development. The value for the local minimum at 10.8 years of age can be used to further subdivide the pre-adolescence period (8.4–12.4 years of age). The value for the local maximum at 30.4 years is useful to subdivide the late stage of adult development (27.4–35 years of age) in two subgroups. The most important in our analysis is the local minimum at 20.5 years of age. This is also more stable during our analysis than the estimates of the other characteristic points of the third order differences.

Summarizing the findings from the analysis of the health state and the first, second and higher order differences for US States (1999–2001) for males and females we form the following Tables 8.1 and 8.2. Clearly the characteristic life-span periods are presented including the three adolescence periods (pre, early and late adolescence) the early (first), middle (second) and late (third) stage of adult development, the middle and the old ages and the very old ages. The Tables 8.3 and 8.5 and the complete Table 8.4 include all the information for the human development age groups for males and females in USA for 2000 by averaging the 1998–2002 data.

That is demonstrated in Tables 8.1 and 8.2 is a very good formulation of particular stages of life-span known in psychology and related fields. The totally new is the ability to define the particular stages quantitatively and to compare between various time periods or countries. We also explain why skills and abilities

Table 8.1 Human development age groups in US States (males)

U.S. States (Males 1999–2001)																					U.S. States (Males 1999–2001)											
Human development stages[a]																					Human deterioration stages[a]											
U.S. States	Age groups for US States estimated with the health state methodology (Years of age)																				Human development age groups based on the deterioration function (Years of age)											
Alabama	0		6.9		11.2		14.2		18.1		21.3		25.2		30.0		35.0		81.4		46.0		62.4		66.4		77.0		79.6		96.9	
Alaska	0		8.2		10.7		14.3		18.9		22.8		27.3		33.0		38.8		83.9		43.7		60.5		65.0		75.1		77.9		95.3	
Arizona	0		8.8		11.0		14.2		18.3		21.8		25.9		31.0		36.2		85.6		48.4		66.3		70.8		82.0		84.5		102.8	
Arkansas	0		7.0		11.2		14.4		18.4		21.8		25.8		30.8		35.6		82.0		45.6		61.9		65.9		76.4		79.4		97.0	
California	0		8.9		11.1		14.2		18.1		21.5		25.5		30.4		36.3		84.7		46.7		63.2		67.2		77.9		80.9		98.7	
Colorado	0		7.7		10.0		13.5		17.8		21.6		25.9		31.4		37.2		85.3		45.4		63.0		67.6		77.9		80.1		96.9	
Connecticut	0		8.0		10.4		14.0		18.6		22.4		27.0		32.7		37.5		84.8		46.7		63.3		67.4		77.9		80.5		97.8	
Delaware	0		8.3		10.6		14.0		18.3		22.4		26.3		31.7		37.5		83.8		46.7		63.5		67.8		78.4	M	81.0		98.5	
District of Columbia	0		6.7		11.0		13.8		17.3		20.4		23.9		28.3		32.9		80.4		47.1		65.2		69.8		80.6	O	82.1		98.8	
Florida	0		9.0		11.2		14.5		18.6		22.1	F	26.3	S	31.5	T	36.2		85.1		48.1		65.8		70.2		81.3	D	83.8		101.9	
Georgia	0		6.9		11.0		14.0		17.8		21.1	I	25.0	E	29.8	H	35.0		81.6		45.8		61.6		65.4		75.8	E	78.7		96.1	V
Hawaii	0		6.5		9.2		13.1		18.0		22.2	R	27.2	C	33.3	I	37.2		86.0		48.0		65.1		65.7		76.1	R	79.1	O	96.1	E
Idaho	0		6.6		10.7		13.7		17.6		20.9	S	24.8	O	29.6	R	37.0		84.9		44.6		61.4	S	65.7		76.1	A	79.1	L	96.8	R
Illinois	0	I	9.0		11.1		14.3		18.3		21.6	T	25.6	N	30.6	D	36.3		82.9		46.0	F	62.1	E	66.0	L	76.4	T	79.1	D	96.4	Y
Indiana	0	N	6.9		11.1		14.2		18.1		21.4		25.3	D	30.2		36.0		82.5		45.5	I	61.2	C	65.0	I	75.4	E	78.4		95.9	
Iowa	0	F	7.1		11.3		14.1		17.8		20.9	S	24.6		29.2	S	36.5		84.6		44.8	R	61.4	O	65.7	G	76.0		78.7	A	96.0	O
Kansas	0	A	6.6		10.8		14.0	M	18.0		21.5	T	25.5	S	30.6	T	37.0	M	83.6		45.4	S	61.3	N	65.2	H	75.5	&	78.5	G	96.0	L
Kentucky	0	N	6.8		10.8	E	14.0	I	18.1	L	21.5	G	25.6	T	30.6	A	35.5	I	81.2		45.0	T	60.6	D	64.4	T	74.6		77.6	E	94.7	D
Louisiana	0	C	9.1	P	11.4	A	14.7	D	18.9	A	22.4	E	26.6	A	31.9	G	36.0	D	81.4		45.3		61.9		66.1		76.6	S	79.0	S	96.1	
Maine	0	Y	6.8	R	11.1	R	14.3	D	18.4	T	21.8		25.9	G	30.9	E	37.2	D	83.3	V	44.5	D	60.1	D	64.0	D	74.1	E	77.3		94.6	A
Maryland	0		8.8	E	11.1	L	14.4	L	18.5	E	22.1		26.2	E	31.4		36.5	L	83.2	E	46.6	E	63.3	E	67.4	I	78.0	V	80.4	–	97.6	G

Massachusetts	0	&	6.9	–	10.7	Y	14.1	E	18.3		22.0	O	26.2		31.5	O	36.5	E	83.8	R	45.7	T	61.5	T	65.4	S	75.7	E	78.7		96.1	E
Michigan	0		9.2	A	11.4		14.6		18.7	A	22.2	F	26.3	O	31.4	F	36.2		83.2	Y	46.3	E	62.5	E	66.5	A	77.0	R	79.9	M	97.4	S
Minnesota	0	E	6.8	D	10.8	A	13.9	A	18.0	D	21.4		25.4	F	30.4		37.1	A	85.1		44.7	R	61.5	R	65.9	B	76.1	E	78.9	A	96.1	
Mississippi	0	A	6.9	O	11.0	D	14.3	D	18.6	O	22.2	A	26.5		31.8	A	35.5	N	80.7	O	45.0	I	61.5	I	65.6	I	76.0		78.5	N	95.5	–
Missouri	0	R	7.0	L	11.2	O	14.2	O	17.9	L	21.0	D	24.7	A	29.3	D	35.5	D	83.2	L	46.2	O	63.0	O	67.2	L	77.7	D	80.3	Y	97.8	
Montana	0	L	7.2	E	10.9	L	14.5	L	19.0	E	22.9	U	27.4	D	33.1	U	38.1		84.5	D	43.5	R	60.7	R	65.3	I	75.6	I	78.5		96.2	C
Nebraska	0	Y	7.5	S	11.8	E	14.3	E	17.5	S	20.3	L	23.5	U	27.5	L	36.8	O	84.6		44.7	A	61.0	A	65.2	T	75.6	S	78.8	D	96.7	R
Nevada	0		7.2	C	11.5	S	14.4	S	18.2	C	21.5	T	25.3	L	30.0	T	35.2	L	82.6	A	46.4	T	62.6	T	66.5	I	77.2	A	80.0	I	97.7	I
Ness Hampshire	0	C	6.6	E	10.6	C	13.8	C	17.9	E	21.3		25.3	T	30.4		36.8	D	84.1	G	45.1	I	60.9	I	64.8	E	75.0	B	78.2	S	95.7	T
New Jersey	0	H	9.2	N	11.6	E	15.2	E	19.8	N	23.6	D	28.2		33.8	D	37.4		83.3	E	45.4	O	61.5	O	65.6	S	75.8	I	78.6	A	95.7	I
New Mexico	0	I	7.7	C	10.3	N	14.1	N	18.9	C	23.0	E	27.9	D	33.9	E	38.1	A	85.1	S	46.9	N	65.1	N	69.9		80.5	L	82.4	B	99.6	C
New York	0	L	8.9	E	11.2	C	14.5	C	18.8	E	22.4	V	26.7	E	32.0	V	36.4	G	83.9		46.6		62.9		67.0	S	77.5	I	80.4	I	98.0	A
North Carolina	0	D	8.4		10.6	E	13.8	E	17.9		21.4	E	25.4	V	30.5	E	35.2	E	82.6		46.8	S	63.3	S	67.4	T	78.1	T	80.6	L	98.1	L
North Dakota	0	H	7.5		9.9		13.4		17.9		21.8	L	26.3	E	31.9	L	37.5	S	84.3		45.3	T	61.3	T	65.3	A	75.5	I	78.6	I	96.1	
Ohio	0	O	7.0		11.1		14.2		18.1		21.4	O	26.3	L	30.2	O	35.5		82.8		46.2	A	62.1	A	65.9	G	76.4	E	79.3	T	98.8	H
Oklahoma	0	O	7.1		9.6		13.1		17.6		21.5	P	26.0	O	31.7	P	35.4		824		46.4	G	62.9	G	66.9	E	77.6	S	80.2	I	97.6	E
Oregon	0	D	6.7		10.4		13.7		18.0		21.5	M	25.8	P	31.0	M	36.5		84.6		46.1	E	62.8	E	67.1		77.6		80.3	E	97.8	A
Pennsylvania	0		9.5		11.7		14.8		18.8		22.2	E	26.2	M	31.2	E	36.5		83.1		45.9		62.0		66.0		76.4	S	79.3	S	96.7	L
Rhode Island	0		7.3		11.3		14.4		18.3		21.7	N	25.6	E	30.5	N	35.9		84.5		46.0		62.8		67.0		77.5	T	80.1		97.5	T
South Carolina	0		6.9		11.1		14.3		18.3		21.7	T	25.8	N	30.8	T	35.1		81.6		46.3		62.9		66.9		77.5	A	80.0		97.2	H
South Dakota	0		6.4		10.3		13.4		17.4		20.7		24.7	T	29.6		36.4		84.9		46.0		63.2		67.6		78.3	G	80.9		96.7	
Tennessee	0		6.8		11.0		14.1		18.1		21.5		25.6		30.6		35.4		81.7		46.0		62.3		66.3		76.8	E	79.4		96.6	
Texas	0		7.1		11.3		14.3		18.2		21.0		24.5		28.8		36.2		83.9		46.1		62.2		66.1		76.6		79.7		97.5	
Utah	0		7.2		9.8		13.5		18.2		22.2		26.9		32.7		37.6		85.9		45.8		63.3		67.9		78.4		80.8		96.1	
Vermont	0		7.2		11.5		14.7		18.8		22.3		26.3		31.4		37.5		84.4		44.1		60.9		65.4		75.5		78.1		95.1	

(continued)

Table 8.1 (continued)

U.S. States (Males 1999–2001)												U.S. States (Males 1999–2001)					
Human development stages[a]												Human deterioration stages[a]					
U.S. States	Age groups for US States estimated with the health state methodology (Years of age)											Human development age groups based on the deterioration function (Years of age)					
Virginia	0	6.8	10.9	14.1	18.1	21.5	25.5	30.5	36.1	83.1		46.1	62.0	65.9	76.3	79.1	96.4
Washington	0	7.1	11.3	14.2	14.3	21.1	24.8	29.4	36.3	84.8		45.5	62.3	665	77.0	79.8	97.3
West Virgin	0	7.1	11.3	14.3	18.2	21.5	25.3	30.1	35.0	82.1		46.3	62.8	66.8	77.4	80.1	97.7
Wisconsin	0	7.4	11.7	14.5	18.0	21.0	24.5	28.8	36.2	84.3		45.2	61.6	65.7	76.0	79.0	96.6
Wyoming	0	7.9	11.8	14.9	18.8	22.1	26.0	30.9	38.9	83.9		41.8	59.7	647	74.3	87.2	92.5
Average	0	7.5	10.9	14.1	18.2	21.7	25.8	30.9	36.3	83.5		45.8	62.4	66.5	77.0	79.9	97.1

[a]Theory developed by: C. H. Skiadas and C. Skiadas, *The Health State Function of a Population*, (2nd Ed., January 2013) www.cmsim.net, http://www.amazon.com/Health-State-Function-Population/dp/6l88046505/ref=sr_1_1?s=books&ie=UTF8&qid=1364343495&sr=1-1

Table 8.2 Human development age groups in US States (females)

U.S. States (Females 1999–2001)																					U.S. States (Females 1999–2001)											
Human development stages[a]																					Human deterioration stages[a]											
U.S. States	Age groups for US States estimated with the health state methodology (Years of age)																				Human development age groups based on the deterioration function (Years of age)											
Alabama[b]	0		8.3		10.8		14.5		19.3		23.3		28.0		33.9		37.6		87.4		47.2		65.3		70.1		80.7		83.0		100.5	
Alaska	0		7.3		11.6		14.7		18.7		22.0		26.0		30.9		38.6		87.5		44.9		61.9		66.5		76.8		79.9		97.8	
Arizona	0		7.4		9.6		12.9		17.1		20.6		24.8		30.0		37.4		89.6		46.7		64.6		69.4		80.3		83.4		102.1	
Arkansas	0		7.2		9.9		13.3		17.6		21.2		25.5		3.09		36.9		87.8		47.0		64.3		68.7		79.7		83.0		101.7	
California	0		7.3		10.3		13.2		16.9		20.1		23.8		28.5		36.8		89.3		46.7		63.9		68.4		79.2		82.7		101.5	
Colorado	0		6.7		10.0		12.9		16.5		19.5		23.1		27.7		37.8		69.0		44.3		62.3		67.3		77.6		80.5		98.4	
Connecticut[b]	0		8.9		10.4		13.4		17.1		19.9		23.8		29.4		38.0		89.2		47.2		64.6		69.2		79.8		82.5		100.3	
Delaware	0		7.7		9.0		12.3		16.4		20.0		24.1		29.3		36.5		88.1		47.4		65.1		69.7		80.5		83.1		101.1	
District of Columbia[b]	0		8.7		10.5		13.5		18.0		23.5		28.3		32.6		33.7		88.0		50.4		70.2		75.3		86.9	M	0.0		107.1	
Florida	0		6.4		9.2		13.2		18.2		22.5		27.6	S	33.9		37.9		90.4		48.8		67.3		72.1		83.3	O	86.0		104.6	
Georgia[b]	0		8.3		10.6		14.0		18.3		21.9	F	26.2	E	31.5	T	37.4		86.4		45.3		62.5		67.1		77.3	D	79.8		96.9	V
Hawaii	0		6.0		8.9		13.2		18.6		23.1	I	28.6	C	35.3	H	39.0		92.1		47.4		67.1		72.5		83.1	E	85.0		102.4	E
Idaho[b]	0		7.8		11.0		13.7		17.2		20.2	R	23.7	O	28.1	I	39.2		88.1		44.3		61.9		66.8		76.7	R	79.2	O	96.0	R
Illinois	0	I	7.1		10.3		13.3		17.0		20.2	S	23.9	N	28.6	R	36.3		88.4		47.3	F	64.4	S	68.7		79.6	A	82.9	L	101.6	Y
Indiana	0	N	7.4		11.1		13.9		17.5		20.6	T	24.2	D	28.7	D	36.3		87.6		47.0	I	63.6	E	67.8	L	78.6	T	82.0	D	100.6	
Iowa	0	F	6.9		10.1		12.9		16.6		19.7		23.4		27.9		38.4		88.7		43.2	R	60.8	C	65.7	I	75.9	E	79.0		96.9	O
Kansas	0	A	6.7		10.0		12.7		16.3		19.3	S	22.8	S	27.2	S	36.6	M	89.0		46.6	S	64.0	O	68.6	G	79.4		82.7	A	101.3	L
Kentucky[b]	0	N	8.8		10.4	E	14.2	M	19.0		23.1	T	27.9	T	33.9	T	37.9	I	86.2		44.9	T	62.2	N	66.9	H	77.0	&	79.3	G	96.2	D
Louisiana	0	C	6.3	P	9.0	A	12.8	I	17.7	L	21.8	A	26.7	A	33.8	A	37.9	D	86.5		44.1		62.7	D	67.9	T	77.7		90.6	E	96.0	
Maine	0	Y	6.8	R	8.7	R	11.9	D	15.9	A	19.4	G	23.4	G	28.5	G	38.0	D	86.5	V	40.2	D	58.7		68.5		73.1	S	85.4	S	90.5	A
Maryland[b]	0		9.1	E	10.5	L	14.4	D	19.4	T	23.7	E	28.7	E	34.9	E	38.4	L	87.2	E	44.2	E	62.5	D	67.5	D	77.3	E	90.0		95.3	G
Massachusetts	0	&	7.6	–	9.8	Y	1.32	L	17.4	E	21.0		25.2		30.5		37.2	E	89.1	R	45.9	T	63.3	E	67.9	I	78.5	V	81.6	–	99.9	E

(continued)

Table 8.2 (continued)

U.S. States (Females 1999–2001)																				U.S. States (Females 1999–2001)																	
Human development stages[a]																				Human deterioration stages[a]																	
U.S. States	Age groups for US States estimated with the health state methodology (Years of age)																			Human development age groups based on the deterioration function (Years of age)																	
Michigan	0		7.1	A	10.1		13.1	E	17.0		20.2	O	24.1	O	28.9	O	36.2		88.0	Y	46.8	E	64.0	T	68.5	S	79.3	E	82.4		100.9	S
Minnesota	0	E	7.0	D	10.2	A	13.3		17.3	A	20.8	F	24.8	F	29.8	F	39.0	A	88.8		43.6	R	60.9	E	65.7	A	75.7	R	78.8	M	96.5	
Mississippi	0	A	7.0	O	9.5	D	13.2	A	17.8	D	21.7		26.3		32.1		36.1	N	87.3	O	48.3	I	66.2	R	70.8	B	82.0	E	84.6	A	103.0	–
Missouri	0	R	7.1	L	10.7	O	13.4	D	16.7	O	19.6	A	22.9	A	27.1	A	36.1	D	87 9	L	46.4	O	63.8	I	68.3	I	79.1		82.1	N	100.4	
Montana	0	L	7.7	E	11.5	L	13.7	O	16.7	L	19.2	D	22.0	D	25.7	D	37.1	O	88.7	D	44.6	R	62.5	O	67.4	T	77.8	D	80.7	Y	98.7	C
Nebraska[b]	0	Y	7.8	S	10.7	E	13.3	L	16.7	E	19.7	U	23.0	U	27.3	U	39.2	L	88.2		43.4	A	61.0	R	66.0	I	75.9	I	78.6		95.7	R
Nevada	0		7.9	C	11.7	S	14.3	E	17.8	S	20.7	L	24.1	L	28.3	L	35.7	D	87.6	A	47.6	T	64.5	A	68.7	E	79.7	S	83.0	D	101.7	I
Ness Hampshire	0		8.2	E	11.9	C	13.9	S	16.5	C	18.8	T	21.4	T	24.6	T	36.8		88.2	G	44.5	I	61.1	T	65.5	S	75.7	A	79.2	I	97.3	T
New Jersey[b]	0	C	9.0	N	10.7	E	14.4	C	17.3	E	19.1		24.2		27.7		38.2		88.2	E	46.0	O	64.1	I	69.0		79.1	B	81.1	S	97.8	I
New Mexico	0	H	6.2	C	9.0	N	13.1	E	18.3	N	22.7	D	27.9	D	34.4	D	37.1		89.1	S	40.6	N	61.2	O	66.9	S	75.5	I	86.1	A	90.9	C
New York[b]	0	I	7.5	E	9.1	C	12.4	N	17.4	C	21.6	E	26.7	E	32.9	E	37.0	A	89.1		47.6		65.1	N	69.6	T	80.5	L	83.6	B	102.2	A
North Carolina	0	L	7.0		10.3	E	13.3	C	17.1	E	20.4	V	24.3	V	29.1	V	36.0	G	88.3		48.2	S	65.5		69.9	A	81.0	I	84.1	I	102.8	L
North Dakota	0	D	6.1		8.5		11.9	E	16.4		20.1	E	24.5	E	30.0	E	37.8	E	90.6		47.9	T	66.3	S	71.1	G	81.9	T	84.3	L	102.0	
Ohio[b]	0	H	8.8		10.4		13.9		18.3		22.1	L	26.6	L	32.1	L	37.8	S	86.6		44.1	A	61.6	T	66.3	E	76.2	I	89.9	I	95.3	H
Oklahoma	0	O	7.3		10.9		13.6		17.0		20.0	O	23.4	O	27.7	O	35.5		87.4		47.1	G	64.4	A	68.8		79.7	E	82.8	T	101.3	E
Oregon[b]	0	O	7.8		11.1		13.6		16.9		19.6	P	22.8	P	26.9	P	37.2		87.7		45.4	E	62.3	G	66.8		77.0	S	79.9	I	97.4	A
Pennsylvania	0	D	7.5		9.9		13.5		18.1		22.0	M	26.5	M	32.2	M	37.6		87.8		45.1		62.6	E	67.3		77.6		80.4	E	97.9	L
Rhode Island	0		7.8		10.2		12.5		18.2		19.6	E	26.5	E	32.1	E	37.1		89.4		47.2		64.7		89.3		80.1	S	83.2	S	101.7	T
South Carolina[b]	0		8.4		10.8		14.2		18.6		22.3	N	26.7	N	32.2	N	37.0		87.5		46.8		65.0		69.8		80.4	T	82.5		99.8	H

South Dakota[b]	0		5.1		8.3		13.0		18.9		24.0	T	29.9	T	37.3	T	40.1		90.0		46.0		64.3	69.3		79.7	A	82.3		100.0
Tennessee	0		6.3		8.8		12.5		17.2		21.1		25.8		31.7		36.6		87.4		47.1		64.5	69.0		79.8	G	82.7		101.0
Texas	0		7.3		10.9		14.8		16.9		19.8		23.2		27.5		36.4		86.8		46.1		63.1	67.5		78.2	E	81.7		100.3
Utah[b]	0		8.6		10.3		12.5		18.4		23.3		29.2		36.5		40.4		88.1		41.9		61.7	67.3		76.2		87.4		92.3
Vermont	0		8.8		12.6		14.8		17.7		20.2		23.0		26.6		38.2		87.3		41.1		58.5	63.4		73.2		88.2		93.7
Virginia[b]	0		8.2		11.6		14.1		17.3		20.0		23.2		27.2		37.7		86.8		43.8		61.5	66.4		76.1		89.1		94.4
Washington	0		8.0		11.6		14.2		17.2		19.8		22.9		26.7		36.9		88.6		45.1		62.3	67.0		77.4		80.6		98.8
West Virgin[b]	0		7.8		10.1		13.5		17.8		21.5		25.8		31.2		38.1		85.7		45.1		62.1	66.6		76.5		89.9		96.2
Wisconsin	0		7.2		10.6		13.3		16.7		19.6		23.0		27.2		37.3		89.3		45.6		62.9	67.5		78.1		81.5		100.1
Wyoming	0		9.2		12.9		15.1		18.0		20.4		23.2		26.8		37.1		86.6		42.2		59.2	63.9		74.2		77.8		96.1
Average	0		7.5		10.3		13.5		17.5		21.0		25.1		30.1		37.5		88.2		45.6		63.3	68.1		78.5		81.4		96.9
Vermont	0		8.8		12.6		14.8		17.7		20.2		23.0		26.6		38.2		87.3		41.1		58.5	63.4		73.2		88.2		93.7
Virginia[b]	0		8.2		11.6		14.1		17.3		20.0		23.2		27.2		37.7		86.8		43.8		61.5	66.4		76.1		89.1		94.4
Washington	0		8.0		11.6		14.2		17.2		19.8		22.9		26.7		36.9		88.6		45.1		62.3	67.0		77.4		80.6		98.8
West Virgin[b]	0		7.8		10.1		13.5		17.8		21.5		25.8		31.2		38.1		85.7		45.1		62.1	66.6		76.5		89.9		96.2
Wisconsin	0		7.2		10.6		13.3		16.7		19.6		23.0		27.2		37.3		89.3		45.6		62.9	67.5		78.1		81.5		100.1
Wyoming	0		9.2		12.9		15.1		18.0		20.4		23.2		26.8		37.1		86.6		42.2		59.2	63.9		74.2		77.8		96.1
Average	0		7.5		10.3		13.5		17.5		21.0		25.1		30.1		37.5		88.2		45.6		63.3	68.1		78.5		81.4		96.9

[a]Theory developed by: C. H. Skiadas and C. Skiadas, *The Health State Function of a Population*, (2nd Ed., January 2013) www.cmsim.net

[b]Second method of estimation http://www.amazon.com/Health-State-Function-Population/dp/6188046505/ref=sr_1_1?s=books&ie=UTF8&qid=1364343495&sr=1-1

Table 8.3 Human development age groups based on the health state function for USA for the period 2000 (average of 1998–2002)

Sex	Pre adolescence	Early adolescence	Middle adolescence	Late adolescence	Early stage of adult development	Middle stage of adult development	Late stage of adult development	Middle and old ages	Very old ages
Males	8.4–12.4	12.4–14.8	14.8–17.9	17.9–20.5	20.5–23.5	23.5–27.4	27.4–35.0	35.0–83.6	83.6–
Females	9.3–12.4	12.4–14.0	14.0–16.2	16.2–18.0	18.0–20.1	20.1–22.7	22.7–36.6	36.6–87.6	87.6–
Average	8.8–12.4	12.4–14.4	14.4–17.1	17.1–19.3	19.3–21.8	21.8–25.1	25.1–35.8	35.8–85.6	85.6–

Table 8.4 Complete human development age groups based on the health state function for USA for the period 2000 (average of 1998–2002)

Sex	Pre adolescence	Pre adolescence	Early adolescence	Middle adolescence	Late adolescence	Early stage of adult development	Middle stage of adult development	Late stage of adult development	Late stage of adult development	Middle and old ages	Middle and old ages	Very old ages
Males	8.4–10.7	10.7–12.4	12.4–14.8	14.8–17.9	17.9–20.5	20.5–23.5	23.5–27.4	27.4–30.4	30.4–35.0	35.0–44.4	44.4–83.6	83.6–
Females	9.3–11.3	11.3–12.4	12.4–14.0	14.0–16.2	16.2–18.0	18.0–20.1	20.1–22.7	22.7–24.6	24.6–36.6	36.6–41.1	41.1–87.6	87.6–
Average	8.8–11.0	11.0–12.4	12.4–14.4	14.4–17.1	17.1–19.3	19.3–21.8	21.8–25.1	25.1–35.8	27.5–35.8	35.8–42.8	42.8–85.6	85.6–

Table 8.5 Human development age groups based on the health state function for 35 countries for the period 2000 (average of 1998–2002)

Sex	Pre adolescence	Early adolescence	Middle adolescence	Late adolescence	Early stage of adult development	Middle stage of adult development	Late stage of adult development	Middle and old ages	Very old ages
Males	8.8–12.7	12.7–15.2	15.2–18.5	18.5–21.3	21.3–24.5	24.5–28.6	28.6–34.8	34.8–81.3	81.3–
Females	9.1–12.0	12.0–13.9	13.9–16.3	16.3–18.4	18.4–20.9	20.9–24.0	24.0–39.7	39.7–86.3	86.3–
Average	9.0–12.3	12.3–14.8	14.8–17.4	17.4–19.8	19.8–22.7	22.7–26.3	26.3–37.2	37.2–83.8	83.8–

approach maximum levels in specific age periods. We can also find the average values for males and females for the period 1998–2002 for USA. In Table 8.3 the related values for males and females are given along with the average estimation for both males and females. A peak in the health state is at 35.0 years of age for males, 36.6 years for females and 35.8 for the average. The pre-adolescence period starts at 8.4 years for males and 9.3 for females, the early adolescence periods starts at 12.4 years for males and females, the middle adolescence period starts at 14.8 years for males and 14.0 years for females, while the late adolescence starts at 17.9 years for males and 16.2 years for females. Accordingly the onset to the early stage of adult development is 20.5 years for males and 18.0 years for females, the middle stage of adult development begins at 23.5 years for males and 20.1 years for females and the late stage of adult development starts at 27.4 years of age for males and 22.7 years for females.

The results for USA (2000) for males and females (Table 8.3) are compared to (see Table 8.5) the average for 35 countries (Australia, Austria, Belarus, Belgium, Bulgaria, Canada, Chile, Czech Republic, Denmark, Estonia Finland, France, Germany, Greece, Hungary, Ireland, Israel, Italy, Japan, Latvia, Lithuania, Luxembourg, Netherlands, New Zealand, Norway, Poland, Portugal, Russia, Slovakia, Slovenia, Spain, Sweden, Switzerland, UK, USA) for the same time period (1998–2002). The similarities are obvious supporting our ability to define and estimate the human development age groups in various countries.

8.3 Human Development Age Groups Based on the Deterioration Function

The age groups around 40 years of age until 85–90 years where the zero health state level appears is a quite large period of the life time and it is worth noting to make separations in specific age groups or subgroups. For this case we can use the deterioration function Det and the first Det', second Det'' and third Det''' differences (derivatives) of this function in order to define and characterize these groups. The deterioration function is a measure of the curvature of the health state function $H(x)$ and thus it can be used to find how and how fast the HSF is changing. The formula measuring the curvature requires the calculation of the first and second derivatives of the health state function (HSF) and has the form:

$$Det = \frac{|H''|}{(1 + H'^2)^{3/2}} \tag{8.1}$$

The deterioration function is a very good measure of the decline of the human health state in the time course. Furthermore we can estimate the total influence of the deterioration process (TDET) by the following formula:

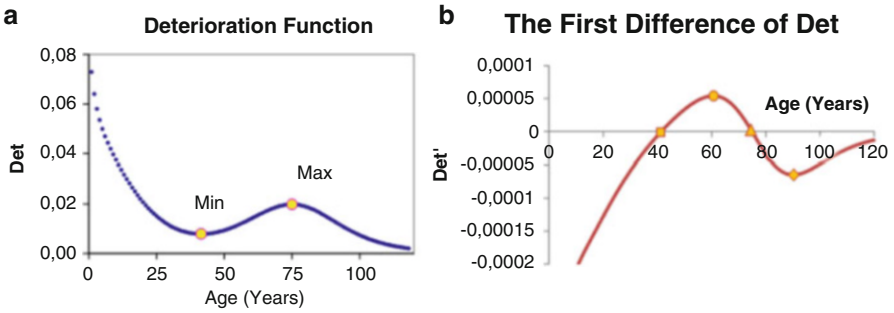

Fig. 8.6 (**a**). The deterioration function, (**b**). The first difference of Det

$$TDET = \int_{x(minDet)}^{x} Det(s)ds \approx \sum_{x(minDet)}^{x} Det(s) \tag{8.2}$$

Where the starting year denoted by x_{minDet} is at the minimum deterioration age and the final year denoted by $x = x_{110}$ is at the 110 year of age.

Figure 8.6a, b illustrate the deterioration function and the first differences (derivatives) for USA in 2000 (males, 1998–2002). The deterioration function (Fig. 8.6a) starts from very high positive values at birth and declines until age 41.5 years where the minimum deterioration age appears, it grows to a maximum at 75.0 years and then continuously declines. It is worth noting that the first part of this function from birth until the minimum corresponds to the adult development stage. The part from the minimum until the late age years corresponds to the human deterioration life period. The first difference (Fig. 8.6b) has maximum and minimum corresponding to 60.8 and 90.2 years of age respectively. These age year steps represent the points where the exponential growth or decline of the deterioration function slows down. The first difference expresses the speed of the deterioration. The speed is positive for the first period of the deterioration (from 41.5 to 75.0 years of age) and then is negative for all the period of over 75.0 years of age. The second difference expresses the acceleration of the deterioration function. The acceleration is positive until 60.8 years of age then is negative until 90.2 years of age and then is positive for the rest of the life span. The estimated values for the deterioration stages for US States males and females for 1999–2001 are presented in Tables 8.1 and 8.2. The average values are included in Table 8.6.

Table 8.7 summarizes the results from the 35 countries study. The first deterioration stage of the organism starts close to 40 years (41.4 years for males and 39.2 years for females) and ends almost 20 years later at 59.4 years for both males and females. The second deterioration stage covers 4.9 years for males (59.4–64.3) and 5.5 years for females (59.4–64.9). The light disabilities stage covers 9 years for males (64.3–73.3) and 8.1 years for females (64.9–73.0). The moderate and severe disabilities stage covers 11.5 years for males (73.3–84.8) and 9.9 years for females (73.0–82.9). The next period includes the old ages with many disabilities and covers

Table 8.6 Human development age groups based on the deterioration function for US States for the period 1999–2001

Human development age groups based on the deterioration function for US States for the period 1999–2001						
Average values	First deterioration stage	Second deterioration stage	Light disabilities stage	Moderate & severe disabilities Stage	Old ages-many disabilities	Very old ages-critical health
Males	45.8–62.4	62.4–66.5	66.5–77.0	77.0–79.9	79.9–97.1	97.1–
Females	45.6–63.3	63.3–68.1	68.1–78.5	78.5–81.4	81.4–98.8	98.9–
Average	45.7–62.8	62.8–67.3	67.3–77.7	77.7–80.6	80.6–98.0	98.0–

Table 8.7 Human development age groups based on the deterioration function for 35 countries for the period 2000 (average of 1998–2002)

Human development age groups based on the deterioration function for 35 countries for the period 2000 (average of 1998–2002)						
	First deterioration stage	Second deterioration stage	Light disabilities stage	Moderate & severe disabilities stage	Old ages-many disabilities	Very old ages-critical health
Males	41.4–59.4	59.4–64.3	64.3–73.3	73.3–84.8	84.8–89.7	89.7–
Females	39.2–59.4	59.4–64.9	64.9–73.0	73.0–82.9	82.9–87.4	87.4–
Average	40.3–59.4	59.4–64.6	64.6–73.2	73.2–83.9	83.8–88.6	88.6–

4.9 years for males (84.8–89.7) and 4.5 years for females (82.9–87.4). The final period covers the very old ages with many disabilities and critical health starting from 89.7 years for males and 87.4 years for females.

8.4 Applications

A relatively recent research from Laura T. Germine, Bradley Duchaine, Ken Nakayama (2011) suggests a classification of cognitive development which is in accordance to our studies related to a health state function developed from population and death data sets. According to their study cognitive development and aging meet. They found that Face learning ability peaks after age 30. A peak was found at 31.4 years for both males and females during the late stage of adult development following our estimates in Tables 8.3, 8.4 and 8.5.

Interesting are the results from the same study of an experiment with adult and children faces recognition. The upright adult faces recognition performance peaked at 31.6 years of age and for children faces at 30.1. Also the results for inverted children faces peaked at 26.9 years for males and 21.8 for females. For these cases the estimated years of age are within the middle stage of adult development as

presented in Table 8.3 (23.5–27.4 years for males) and (20.1–22.7 years for females) and in Table 8.5 (24.5–28.6 years for males) and (20.9–24.0 years for females). According to Li et al. (2004) several critical cognitive characteristics peak after 20 years of age although crystallized intelligence peaks near 40 years of age at the end of the late stage of adult development and near the maximum health state age. This stage is in agreement with our findings for the health state of a population. The main cognitive characteristics studied from Li et al. (2004) peak at the local maximum of the first difference of the health state which is from 20 to 25 years (see Fig. 8.3 and Tables 8.3, 8.4 and 8.5). For males, in the 35 countries studied, the peak is at 24.5 years of age, whereas for females the peak is at 20.9 years of age. For USA (2000) males the peak is at 23.5 years and 20.1 years for females. Another study refers to the development of chess ratings during the life span. Philip E. Ross (2013) in IEEE Spectrum presents the development of chess ratings for grandmasters based on the 1965 ELO (a standard measure of the chess performance). According to this study the peak is in the mid-30 years of age. Robert W. Howard (2005) explored the case for the top 10, 50, 100 and 500 chess players finding the median for 2005 to be close to 30 years of age for the top 10 and top 50 group and between 30 and 31 for the top 100 group, whereas for top 500 group the median age is between 33 and 34 years.

We calculated 30.6 years as the mean age for the top 100 chess players from data from the FIDE official website (August 2013 http://ratings.fide.com/toplist.phtml). The related age year for women was 29.8 years. These findings support the argument that the best performance of chess players appear during the third stage of adult development when the health state is close to the peak and the speed of the health state is positive.

The top 50 golf professional players as of 20 of August 2013 show an average age of 32.44 years whereas the average age is 34.6 years for the first 10 in the rank. The average age for the 27 golf champions from 1986 to 2012 is 32.4 years. As for the chess players the golf professional players performance peaks at the late stage of adult development when the health state is close to the maximum level (see Figs. 8.1 and 8.2).

In athletics the average age of athletes when achieving the world records as of August 2013 is 26.5 years for men and 27.2 for women. According to Tables 8.3, 8.4 and 8.5, men world champions in athletics are mainly in the middle stage of adult development while women in the late stage.

The graph of the age changes in the similarities test as presented in Rozencwajg and Bertoux (2008) is in an almost perfect agreement with the health state graph illustrated in Fig. 8.2.

8.5 Discussion

The aim of this study was on finding the effect of the health state on various human characteristics during the life span. That it is demonstrated from the applications presented refers to peaks achieved in the years of age from the first to the late period

of the adult development by means from the local maximum of the first difference of the health state until the maximum health state level corresponding to zero value for the first differences. Accordingly it is expected to have the main number of peaks for several human characteristic performances from 24.5 to 34.8 years of age for males and from 20.9 to 39.7 years of age for females for the 35 countries studied.

Another important stage of human development is characterized by positive values for the second differences of the health state. This period covers (see Table 8.3) the late adolescence and the early stage of adult development from 18.5 to 24.5 years of age for males with a peak at 21.3 and from 16.3 to 20.9 for females with a peak at 18.4 years of age. According to these findings, for the 35 countries studied (see Table 8.3), the late adolescence period starts 2.2 years earlier for females (16.3 years) than for males (18.5 years). The end of the late adolescence period is 2.9 years earlier for females (18.4 years) than for males (21.3 years). The end of the early stage of adult development is 3.6 years of age earlier for females (20.9 years) than for males (24.5 years).

According to the findings presented in Table 8.3 the middle stage of adult development ends 4.6 years earlier for females at 24.0 years than for males ending at 28.6 years of age. Female maturity is faster in the three very important stages of the life span (late adolescence, early and middle stage of adult development). Instead the late stage of adult development is larger for females ($39.7-24.0 = 15.7$ years) than for males ($34.8-28.6 = 6.2$ years). The next period covers the middle and old ages from 39.7 to 86.3 years for females and from 34.8 to 81.3 years of age for males. The following group includes the very old ages. The last two groups need further analysis which is possible by applying special techniques presented by Skiadas and Skiadas in several publications (Skiadas and Skiadas 2010, 2014, 2015).

The following Table 8.8 summarizes the human development stages for males and females in USA (2000) including the related stages proposed by Erikson Erikson (1959), Sullivan and Piaget schools. The stages proposed by these scientists and many co-workers have explored many aspects of the human development both qualitatively and quantitatively based on longitudinal studies. Our method defines precisely the related stages thus giving a new tool to scientists especially from the developmental psychology field. The information provided is based on death and population data sets used to define the health state of a population during the life span. It remains to researchers of particular fields to use the provided information in connection to their results from longitudinal and other studies.

Concluding we have proposed and applied a method to define and calculate the main stages of human development during the life span by using the health state of a population and the related first and second differences. Following the calculation of the health state from death and population data sets provided by the bureau of the census of a country or from specialized databases as is the human mortality database (HMD) we have introduced biological parameters in the life span approach of human development. We have thus succeeded to provide a methodology to support Paul B. Baltes arguments for a multidisciplinary study of the life span Developmental Psychology. More studies on various fields of psychology, sociology, neurology, neurophysiology and other fields will support and expand our results (Table 8.8).

Table 8.8 Human development age groups based on the health state function

Human development age groups based on the health state function									
Methods	Pre adolescence	Early adolescence	Late adolescence	First stage of adult development	Second stage of adult development	Third stage of adult development	Early middle ages	Middle + old ages	Very old Ages
Erikson's stages	Competence	Adolescence		Early adulthood		Adulthood (27–64), Old Ages (64-)			
Sullivan's	Pre-adolescence	Early adolescence	Late adolescence						
Piaget's	Concrete operations	Formal operational stage							
Males USA 2000	8.4–12.4	12.4–14.8	14.8–17.9	17.9–20.5	20.5–23.5	23.5–27.4	27.4–35.0	35.0–83.6	83.6–
Females USA 2000	9.3–12.4	12.4–14.0	14.0–16.2	16.2–18.0	18.0–20.1	20.1–22.7	22.7–36.6	36.6–87.6	87.6–

Appendix

The related theory and applications along with the estimation programs are given in several publications and in the web at http://www.cmsim.net.

A snapshot of the Human Development Program in Excel is given in Fig. 8.7. The program provides the human development stages from death and population data for males, females and total (both). If mortality data μ_x are available it should be inserted in column R starting from R26 Cell. There are four Excel worksheets: the main called Ski-6-Parameters, the supporting Gompertz model, the USA-Deaths+Population-1933–2007 including the death and population data for USA from 1933 to 2007 and the 7-Countries-2000 including the death and population data for seven countries for the year 2000 (Australia, Canada, France, Germany, Italy, Japan and UK). The data for a specific year for males or females including 111 data points from age 0 to 110 are copied and then pasted in the main Excel worksheet Ski-6-Parameters in P and Q columns starting from P26 and Q26 cells. The program is semi-automatic and the main starting parameters are arranged according to the data for the main developed countries in the last decade. The only needed is to change the value of the parameter in Cell C2 to achieve a minimum for the sum of squared errors in Cell D3. The program provides very many characteristic parameters useful in several scientific fields as are the life expectancy and the healthy life expectancy, the health state and an estimate for the total health state and several Tables and Figures.

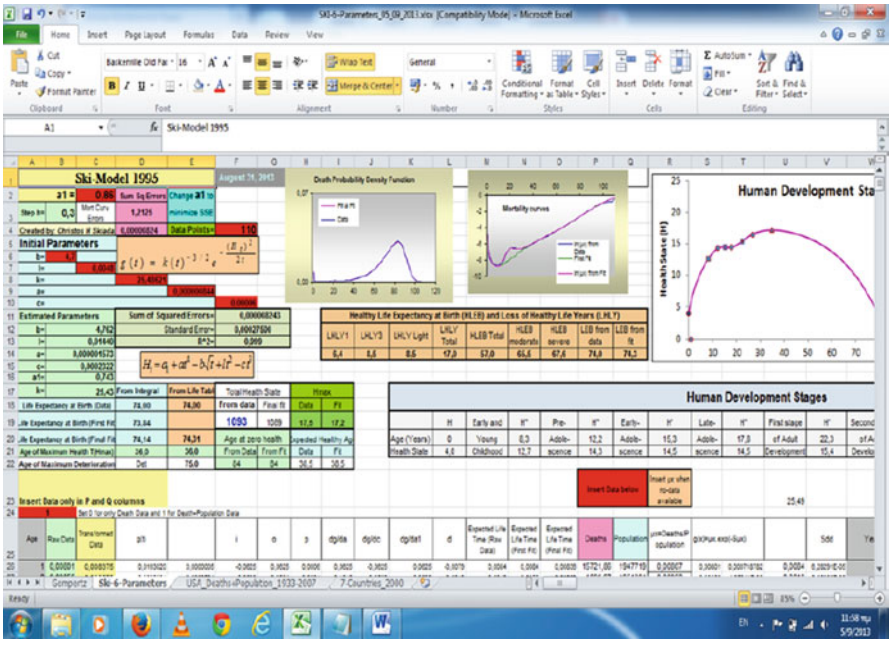

Fig. 8.7 Snapshot of the human development program in excel

The Human Development Stages based on the health state of the population are presented in the table included in Cells K17 to AC21. To the right of the spreadsheet in Cells AF17 to AT20 the Human Development Age Groups based on the Deterioration Function are presented in the related Table (see Table 8.9). In this Table further analysis of the human development ages in the middle and old ages is given. The estimates included in this table along with the related from the healthy life expectancy table give enough information for the human development age groups after the health state peak when the human deterioration is growing. The following Tables 8.10 and 8.11 summarize the estimates for US States (1999–2001) for Life expectancy, Healthy life expectancy and Loss of healthy life years along with the Expected healthy age, the Maximum health state and the Total Health State.

Table 8.9 Human development age groups based on the deterioration function

Human development age groups based on the deterioration function														
	Det	First deterioration stage	**Det′**	Second deterioration stage	**Det″**	Light disabilities Stage	**Det**	Moderate & severe disabilities Stage	**Det″**	Old ages (many disabilities)	**Det′**	Very old ages (critical health)	**Det″**	Critical ages (very critical heath)
Age (Years)	**41,4**		**60,8**		**66,1**		**74,9**		**85,2**		**90**		**100,5**	

Table 8.10 Estimates for US States (1999–2001, male)

U.S. States (Males 1999–2001)										U.S. States (Males 1999–2001)					
Healthy Life Expectancy at Birth (HELB) and Loss of Healthy Life Years (LHLY)[a]										Expected healthy age (Yean)[a]		Maximum health state (Hmax)[a]		Total health state[a]	
U.S. States	LHLY1	LHLY2	LHLY light	LHLY total	HELB total	HELB moderate & severe	HELB severe	LEB data	LEB from fit	From data	From fit	From data	From fit	Front data	From fit
Alabama	5.6	7.6	4.9	12.5	59.1	64.0	66.0	71.6	71.8	36.7	36.8	16.6	16.5	987	986
Alaska	6.0	7.6	6.1	13.7	60.7	66.7	68.4	74.4	74.6	38.7	38.7	17.4	17.2	1073	1070
Arizona	5.5	6.7	6.1	12.9	62.6	68.7	69.9	75.4	75.7	38.5	38.6	16.9	16.9	1061	1061
Arkansas	5.4	7.1	5.3	12.4	59.9	65.1	66.9	72.3	72.5	37.1	37.2	16.7	16.6	1004	1003
California	5.7	7.1	6.3	13.4	62.9	69.2	70.5	76.3	76.6	38.5	38.5	17.8	17.8	1105	1105
Colorado	5.6	7.0	7.5	14.5	62.1	69.5	70.9	76.5	76.8	39.0	39.0	17.8	17.8	1122	1122
Connecticut	6.0	7.4	6.1	13.5	62.9	69.0	70.4	76.4	76.7	38.6	38.6	17.9	17.8	1117	1115
Delaware	5.9	7.5	5.5	13.0	61.5	67.0	68.6	74.5	74.7	37.9	38.0	17.2	17.2	1061	1060
District of Columbia	5.4	7.2	4.2	11.4	57.4	61.6	63.5	68.8	69.0	35.5	35.7	16.0	15.5	908	907
Florida	5.3	6.5	6.6	13.2	62.0	68.6	69.9	75.2	75.5	38.3	38.4	17.0	16.9	1059	1059
Georgia	5.8	7.8	4.9	12.7	59.9	64.8	66.8	72.5	72.8	36.9	37.0	17.0	17.0	1015	1016
Hawaii	5.3	6.5	6.9	13.4	64.0	70.9	72.1	77.4	77.7	39.0	39.0	17.9	17.9	1138	1138
Idaho	6.2	7.7	6.1	13.8	62.6	68.7	70.2	76.4	76.8	38.9	38.9	17.9	17.8	1115	1114
Illinois	6.0	7.7	5.2	12.9	61.2	66.4	68.2	74.2	74.4	37.7	37.7	17.5	17.4	1063	1062

Indiana	5.7	7.6	5.6	13.1	60.7	66.2	68.1	73.8	74.1		37.5	37.6	17.4	17.4	1051	1051
Iowa	5.9	7.5	6.6	14.1	62.3	68.9	70.5	76.4	76.7		38.6	38.8	18.1	18.1	1122	1124
Kansas	5.8	7.4	6.1	13.5	61.6	67.7	69.3	75.1	75.4		38.2	38.3	17.8	17.8	1090	1089
Kentucky	5.7	7.8	5.2	13.1	59.4	64.7	66.8	72.5	72.8		36.8	36.9	17.1	17.0	1017	1017
Louisiana	5.8	7.9	4.8	12.7	58.6	63.4	65.5	71.4	71.5		36.8	36.9	16.4	16.3	982	981
Maine	5.7	7.4	6.5	13.9	61.6	68.1	69.8	75.5	75.8		38.1	38.2	18.1	18.0	1102	1102
Maryland	5.6	7.3	5.9	13.2	60.6	66.5	68.2	73.8	74.0		37.7	37.7	17.1	17.1	1044	1043
Massachusetts	5.8	7.3	6.2	13.5	62.5	68.7	70.3	76.1	76.4		38.2	38.2	18.0	18.0	1114	1113
Michigan	5.6	7.3	6.0	13.3	60.9	66.9	68.6	74.2	74.5		37.7	37.8	17.3	17.3	1058	1057
Minnesota	5.5	6.9	7.7	14.6	62.3	70.1	71.4	77.0	77.3		39.0	39.0	18.2	18.2	1140	1140
Mississippi	5.8	8.0	4.4	12.4	58.1	62.5	64.8	70.5	70.7		36.4	36.5	16.2	16.1	964	964
Missouri	5.5	7.2	6.1	13.3	60.6	66.7	68.3	73.8	74.1		37.6	37.7	17.1	17.1	1042	1041
Montana	6.1	7.8	6.4	14.1	61.3	67.7	69.3	75.4	75.6		38.7	38.8	17.3	17.2	1082	1081
Nebraska	5.9	7.3	6.6	13.9	62.3	68.9	70.3	76.2	76.6		38.7	38.7	18.1	18.0	1113	1112
Nevada	5.5	7.2	5.6	12.8	60.8	66.4	68.1	73.6	73.9		37.2	37.3	17.0	17.0	1032	1032
Ness Hampshire	5.6	7.2	7.0	14.1	62.4	69.3	70.9	76.5	76.9		38.4	38.5	18.3	18.2	1125	1124
New Jersey	6.0	7.8	6.0	13.7	61.3	67.2	69.1	75.0	75.3		37.9	38.0	17.6	17.5	1086	1085
New Mexico	5.7	7.1	6.6	13.7	61.0	67.6	69.0	74.7	74.9		38.6	38.7	16.8	16.7	1058	1056
New York	5.7	7.1	5.9	13.1	62.3	68.2	69.7	75.4	75.7		38.0	38.1	17.5	17.5	1084	1083
North Carolina	5.5	7.2	5.4	12.6	60.7	66.1	67.8	73.3	73.6		37.2	37.3	16.9	16.9	1026	1026
North Dakota	5.9	7.4	6.6	14.0	62.2	68.8	70.3	76.2	76.6		38.6	38.7	18.1	18.0	1119	1118
Ohio	5.8	7.5	5.4	12.9	61.3	66.7	68.4	74.2	74.5		37.5	37.6	17.4	17.4	1059	1059
Oklahoma	5.5	7.2	5.6	12.8	60.2	65.8	67.5	73.0	73.3		37.2	37.3	16.8	16.7	1017	1017
Oregon	5.7	7.2	6.4	13.6	62.4	68.8	70.3	76.1	76.4		38.4	38.5	17.7	17.7	1101	1102
Pennsylvania	5.7	7.4	6.1	13.5	60.9	67.0	68.7	74.3	74.6		37.7	37.8	17.4	17.4	1062	1061
Rhode Island	5.6	7.0	6.8	13.8	62.3	69.1	70.5	76.1	76.4		38.3	38.4	17.7	17.8	1106	1106

(continued)

Table 8.10 (continued)

U.S. States (Males 1999–2001)										U.S. States (Males 1999–2001)					
Healthy Life Expectancy at Birth (HELB) and Loss of Healthy Life Years (LHLY)[a]										Expected healthy age (Yean)[a]		Maximum health state (Hmax)[a]		Total health state[a]	
U.S. States	LHLY1	LHLY2	LHLY light	LHLY total	HELB total	HELB mod-erate & severe	HELB severe	LEB data	LEB from fit	From data	From fit	From data	From fit	Front data	From fit
South Carolina	5.7	7.7	4.7	12.4	59.5	64.3	66.3	71.9	72.1	36.7	36.8	16.5	16.5	992	992
South Dakota	5.9	7.3	6.1	13.4	62.0	68.1	69.5	75.4	75.8	38.6	38.7	17.4	17.4	1081	1081
Tennessee	5.8	7.8	4.7	12.5	59.7	64.4	66.4	72.2	72.5	36.8	37.0	16.7	16.7	1002	1002
Texas	5.6	7.2	6.1	13.3	61.1	67.2	68.8	74.4	74.7	37.7	37.8	17.3	17.3	1055	1055
Utah	5.9	7.2	6.9	14.0	63.0	69.9	71.2	77.1	77.4	39.2	39.2	17.7	17.7	1128	1128
Vermont	6.0	7.6	6.8	14.4	62.0	68.8	70.4	76.4	76.7	38.6	38.7	18.0	17.9	1121	1120
Virgin	5.6	7.3	6.2	13.5	61.2	67.4	69.1	74.7	75.1	37.7	37.8	17.6	17.7	1078	1078
Washington	5.9	7.4	6.4	13.7	62.7	69.1	70.5	76.4	76.8	38.6	38.7	17.9	17.9	1116	1115
West Virgin	5.2	6.9	5.9	12.9	60.1	66.1	67.8	73.0	73.3	37.0	37.0	16.8	16.8	1012	1012
Wisconsin	5.6	7.1	6.8	14.0	61.9	68.7	70.2	75.9	76.2	38.5	38.5	18.0	17.9	1107	1107
Wyoming	6.3	8.3	7.0	15.3	59.7	66.7	68.8	75.0	75.3	38.9	39.0	17.3	17.4	1094	1092
Average	5.7	7.4	6.0	13.4	61.2	67.2	68.9	74.6	74.9	37.9	38.0	17.4	17.3	1067	1066

[a]Theory developed by: C. H. Skiadas and C. Skiadas, *The Health State Function of a Population*, (2nd Ed., January 2013). www.cmsim.net, http://www.amazon.com/Health-State-Function-Population/dp/6l88046505/ref=sr_1_1?s=books&ie=UTF8&qid=1364343495&sr=1-1

Table 8.11 Estimates for US States (1999–2001, female)

U.S. States (Females 1999–2001)										U.S. States (Females 1999–2001)					
Healthy Life Expectancy at Birth (HLEB) and Loss of Healthy Life Years (LHLY)[a]										Expected healthy age (Years)[a]		Maximum health state (Hmax)[a]		Total health state[a]	
U.S. States	LHLY1	LHLY2	LHLY light	LHLY total	HELB total	HELB moderate & severe	HELB severe	LEB data	LEB from fit	From data	From fit	From data	From fit	Front data	From fit
Alabama	5.9	7.1	7.4	14.4	64.1	71.5	72.6	78.5	78.8	39.8	39.9	17.8	17.9	1166	1166
Alaska	6.0	7.0	8.0	15.0	64.6	72.6	73.7	79.6	80.1	40.3	40.4	18.9	18.8	1217	1216
Arizona	5.9	6.7	8.3	15.0	66.3	74.6	75.4	81.3	81.9	40.9	40.9	18.6	18.8	1239	1242
Arkansas	6.0	7.0	7.1	14.1	65.1	72.2	73.2	79.2	79.7	39.9	40.0	18.2	18.2	1176	1176
California	5.7	6.6	8.6	15.2	66.7	75.2	76.1	81.8	82.4	40.7	40.8	19.1	19.1	1261	1263
Colorado	5.9	6.9	8.5	15.4	66.0	74.5	75.5	81.4	81.9	40.9	41.0	19.1	19.1	1260	1262
Connecticut	5.9	6.8	8.6	15.4	66.4	75.0	75.9	81.8	82.2	40.8	40.9	19.2	19.2	1278	1277
Delaware	5.4	6.4	8.4	14.8	65.2	73.6	74.6	80.0	80.4	40.1	40.2	18.4	18.5	1207	1208
District of Columbia	5.2	6.1	6.6	12.7	64.9	71.6	72.4	77.7	78.1	38.8	39.0	16.8	16.9	1106	1106
Florida	5.4	6.2	8.7	14.9	66.6	75.3	76.0	81.4	81.9	41.0	41.1	18.4	18.5	1234	1236
Georgia	5.9	7.3	7.6	14.9	63.6	71.2	72.5	78.5	78.8	39.6	39.6	18.3	18.4	1184	1185
Hawaii	5.6	6.3	10.4	16.7	67.0	77.4	78.0	83.7	84.2	42.1	42.2	19.1	19.2	1323	1325
Idaho	6.2	7.4	8.7	16.1	64.7	73.4	74.6	80.7	81.0	40.9	40.9	19.1	19.2	1263	1262
Illinois	5.4	6.4	8.3	14.6	65.8	74.1	75.0	80.5	81.0	40.2	40.2	18.7	18.8	1220	1222
Indiana	5.9	6.9	7.5	14.3	65.3	72.8	73.8	79.7	80.1	39.9	40.0	18.6	18.7	1205	1206

(continued)

Table 8.11 (continued)

U.S. States (Females 1999–2001)										U.S. States (Females 1999–2001)					
Healthy Life Expectancy at Birth (HLEB) and Loss of Healthy Life Years (LHLY)[a]										Expected healthy age (Years)[a]		Maximum health state (Hmax)[a]		Total health state[a]	
U.S. States	LHLY1	LHLY2	LHLY light	LHLY total	HELB total	HELB moderate & severe	HELB severe	LEB data	LEB from fit	From data	From fit	From data	From fit	Front data	From fit
Iowa	5.9	6.9	8.9	15.8	65.9	74.7	75.7	81.6	82.1	41.0	41.0	19.3	19.5	1280	1282
Kansas	5.6	6.4	8.2	14.6	66.5	74.6	75.5	81.1	81.6	40.6	40.7	18.9	18.9	1239	1240
Kentucky	5.9	7.3	7.8	15.2	63.3	71.1	72.5	78.4	78.8	39.6	39.6	18.3	18.4	1186	1185
Louisiana	6.0	7.4	8.1	15.5	62.2	70.2	71.7	77.7	78.0	39.6	39.7	17.9	18.0	1165	1167
Maine	6.4	8.1	8.9	16.9	63.0	71.8	73.4	79.9	80.3	40.4	40.5	19.5	19.5	1263	1266
Maryland	5.9	7.2	8.9	16.1	63.2	72.1	73.4	79.3	79.6	40.1	40.2	18.5	18.6	1224	1224
Massachusetts	5.7	6.6	9.0	15.6	66.3	75.3	76.2	81.9	82.4	40.7	40.7	19.2	19.3	1274	1275
Michigan	5.3	6.2	8.1	14.3	65.7	73.8	74.7	80.0	80.6	40.0	40.1	18.5	18.7	1209	1211
Minnesota	6.1	7.0	8.7	15.8	66.3	75.0	76.0	82.0	82.5	41.1	41.2	19.8	19.8	1303	1304
Mississippi	5.4	6.4	7.4	13.9	63.9	71.4	72.4	77.8	78.2	39.4	39.5	17.6	17.6	1134	1135
Missouri	5.7	6.6	7.8	14.4	65.2	73.0	74.0	79.7	80.1	40.1	40.1	18.4	18.6	1201	1203
Montana	6.1	7.1	8.3	15.4	65.4	73.7	74.6	80.8	81.2	40.8	40.9	19.0	19.0	1245	1248
Nebraska	6.3	7.5	8.7	16.1	64.9	73.5	74.7	81.0	81.4	40.9	41.0	19.3	19.4	1273	1274
Nevada	5.7	6.6	7.5	14.1	65.3	72.8	73.8	79.4	79.9	39.7	39.8	18.3	18.5	1189	1190
New Hampshire	5.8	6.8	8.8	15.6	66.1	74.9	75.9	81.7	82.2	40.5	40.6	19.8	19.8	1286	1287
New Jersey	5.8	6.9	8.8	15.8	64.8	73.6	74.7	80.5	80.9	40.4	40.5	18.8	18.9	1249	1249
New Mexico	6.8	8.3	10.5	18.8	61.5	71.9	73.5	80.2	80.6	41.3	41.4	18.9	18.8	1274	1274
New York	5.5	6.4	8.6	15.0	66.3	75.0	75.8	81.3	81.9	40.5	40.5	18.8	18.9	1243	1245

North Carolina	5.3	6.2	8.2	14.4	65.4	73.6	74.5	79.7	80.2		40.0	40.1	18.3	18.5	1196	1198
North Dakota	5.8	6.5	8.9	15.4	67.3	76.3	77.0	82.7	83.3		41.4	41.5	19.3	19.5	1297	1300
Ohio	6.3	7.7	7.7	15.4	63.8	71.5	72.9	79.2	79.6		39.9	39.9	18.7	18.8	1221	1221
Oklahoma	5.5	6.6	7.7	14.3	64.5	72.2	73.3	78.8	79.3		39.6	39.7	18.1	18.3	1170	1173
Oregon	5.9	7.0	8.1	15.1	65.5	73.6	74.7	80.6	81.0		40.3	40.3	19.2	19.2	1253	1252
Pennsylvania	5.9	7.0	8.1	15.1	65.0	73.1	74.2	80.1	80.6		40.2	40.3	18.8	18.9	1233	1234
Rhode Island	5.8	6.6	8.5	15.1	66.5	75.0	75.8	81.6	82.1		40.7	40.8	19.1	19.1	1260	1262
South Carolina	5.9	7.1	7.6	14.7	64.0	71.6	72.8	78.7	79.0		39.8	39.9	17.9	18.1	1177	1178
South Dakota	6.0	6.8	9.2	16.0	65.9	75.1	76.0	82.0	82.1		41.6	41.7	19.0	19.0	1276	1275
Tennessee	5.7	6.7	7.6	14.3	64.5	72.1	73.2	78.9	79.3		39.7	39.7	18.1	18.2	1170	1172
Texas	5.4	6.4	8.0	14.4	65.9	73.9	74.9	80.3	80.8		40.2	40.2	18.7	18.8	1216	1218
Utah	6.6	8.0	10.0	18.0	63.2	73.2	74.6	81.2	81.6		41.2	41.3	19.1	19.2	1298	1299
Vermont	6.0	7.3	8.8	16.1	64.4	73.2	74.5	80.5	81.1		40.5	40.5	19.4	19.6	1268	1270
Virginia	6.1	7.5	8.1	15.6	64.0	72.1	73.5	79.6	80.0		40.1	40.2	18.9	19.1	1239	1240
Washington	5.9	6.9	8.5	15.3	66.0	74.5	75.4	81.4	81.9		40.6	40.6	19.3	19.3	1263	1264
West Virginia	6.2	7.7	7.0	14.7	63.4	70.4	71.9	78.1	78.3		39.5	39.6	18.5	18.5	1184	1182
Wisconsin	5.9	6.8	8.7	15.4	66.4	75.0	75.9	81.8	82.4		40.9	41.0	19.2	19.3	1270	1271
Wyoming	6.2	7.5	7.5	15.0	63.8	71.3	72.6	78.8	79.4		39.9	39.9	18.6	18.6	1194	1194
Average	5.9	6.9	8.3	15.2	65.0	73.4	74.4	80.3	80.7		40.4	40.4	18.7	18.8	1231	1232

[a]Theory developed by: C. H. Skiadas and C. Skiadas, *The Health State Function of a Population*, (2nd Ed., January 2013). www.cmsim.net, http://www.amazon.com/Health-State-Function-Population/dp/6l88046505/ref=sr_1_1?s=books&ie=UTF8&qid=1364343495&sr=1-1

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Chapter 9
Derivation and Validation of the Health State Function Form of a Population

9.1 Derivation of the Final Health State Form

Following several publications (Skiadas and Skiadas 2007, 2010a,b, 2011, 2014, 2015; Skiadas 2011) we have proposed the following Simple Model related to the Health State process $H(x)$ of a population and the death probability density $g(x)$:

$$g(x) = \frac{|H_x - xH'_x|}{\sigma\sqrt{2\pi x^3}} e^{-\frac{[H_x]^2}{2\sigma^2 x}}$$

By including the diffusion coefficient σ into the parameters of the health state function $H(x)$ at age x we arrive to the simpler form:

$$g(x) = \frac{|H_x - xH'_x|}{\sqrt{2\pi x^3}} e^{-\frac{[H_x]^2}{2x}} \tag{9.1}$$

The same form arises by setting $\sigma = 1$ in the first form above.

There are two methods to estimate $H(x)$ when the death distribution $g(x)$ is provided or another convenient form as is the dx estimates from the life tables:

1. To invert the last formula and find immediately $H(x)$
2. To find an analytic form of $H(x)$, introduce it in Eq. (9.1) and then estimate the parameters of this formula by applying regression analysis techniques.

Clearly inversion of formula (9.1) is the best technique as no any assumption for the analytic form of $H(x)$ is needed. However, the term $|H_x - xH'_x|$ makes the inversion not possible. Instead the inversion is possible for the simple form:

$$g(x) = \frac{k}{\sqrt{2\pi x^3}} e^{-\frac{[H_x]^2}{2x}} \tag{9.2}$$

C.H. Skiadas, C. Skiadas, *Exploring the Health State of a Population by Dynamic Modeling Methods*, The Springer Series on Demographic Methods and Population Analysis 45, DOI 10.1007/978-3-319-65142-2_9

Providing the following form for the health state function:

$$H(x) = \left| -2x \ln \frac{\sqrt{2\pi x^3} g(x)}{k} \right|^{1/2} \tag{9.3}$$

In the last form the parameter k should to be estimated. However, as it was already published in a previous study to preserve continuity and to keep the logarithm values negative k should be equal to:

$$k = max(\sqrt{2\pi x^3} g(t)) \tag{9.4}$$

After estimating k from Eq. (9.4), $H(x)$ could be estimated from (9.3). Now another critical question arises: how close is the $H(x)$ estimate from (9.3) related to the correct estimate from (9.1). Or better is any method leading to the correct estimate from (9.1) when a first estimate is done from (9.3)? We are going to find such a method by using a relatively simple but yet quite good model for the estimation of $H(x)$ that is given by the formula (b, c, l are parameters):

$$H(x) = l - (bx)^c \tag{9.5}$$

Inserting $H(x)$ from (9.5) in (9.1) the following form results:

$$g(x) = \frac{|l + (c-1)(bx)^c|}{\sqrt{2\pi x^3}} e^{-\frac{[l-(bx)^c]^2}{2x}} \tag{9.6}$$

For $c = 1$ the relation (9.6) reduces to the Inverse Gaussian a model applied by Weitz and Fraser (2001) to data for Mediterranean flies:

$$g(x) = \frac{|l|}{\sqrt{2\pi x^3}} e^{-\frac{[l-(bx)]^2}{2x}} \tag{9.7}$$

By introducing $H(x)$ from (9.5) to (9.2) the following approximation of (9.6) results:

$$g(x) = \frac{k}{\sqrt{2\pi x^3}} e^{-\frac{[l-(bx)^c]^2}{2x}} \tag{9.8}$$

Our aim is to estimate $H(x)$ from both (9.6) and (9.8) by applying non-linear regression analysis techniques and find a method to correct the findings from (9.8) in order to be as close as possible to the related estimates of $H(x)$ from (9.6). First of all we observe that an inversion of (9.1) will give the following form for $H(x)$:

$$H(x) = \left| -2x \ln \frac{\sqrt{2\pi x^3} g(x)}{|H_x - xH'_x|} \right|^{1/2} \tag{9.9}$$

Clearly the term $|H_x - xH'_x|$ in (9.9) cannot be estimated. However, we can use the estimates for $H(x)$ from (9.3) to use in the term $|H_x - xH'_x|$ in (9.9) along with a correction parameter k_1 as it appears in (9.10):

$$H(x) = \left| -2x \ln \frac{\sqrt{2\pi x^3} g(x)}{k_1 |H_x - xH'_x|} \right|^{1/2} \tag{9.10}$$

In the last form the parameter k_1 should to be estimated. However, as it was already the case for k in (9.4) to preserve continuity and to keep the logarithm values negative k_1 should be equal to:

$$k_1 = max \left(\frac{\sqrt{2\pi x^3} g(t)}{|H_x - xH'_x|} \right) \tag{9.11}$$

9.2 First Application

We fit the models (9.6) and (9.8) to the 2000 mortality data set for males in USA by using a non-linear regression analysis algorithm we have programmed in Excel solver. Both models provide similar R^2, the Approximation model giving relatively better estimates due to the extra parameter k. The fitting to the data is similar to both models as is illustrated in Fig. 9.1a, b. Even more the estimates of the year of the maximum death rate are very close each other (81.5 for the Complete Model and 81.9 for the Approximation). A difference appears for the year of the zero health state $H(x = 0)$ that is 79.2 for the Complete Model and 84.5 for the Approximation. The Approximation tends to overestimate the health state $H(x)$ as is illustrated in Fig. 9.2. Instead the Corrected curve for $H(x)$ tends to coincide with that of the Complete Model especially at the age of zero health state $H(x = 0)$ where this age is

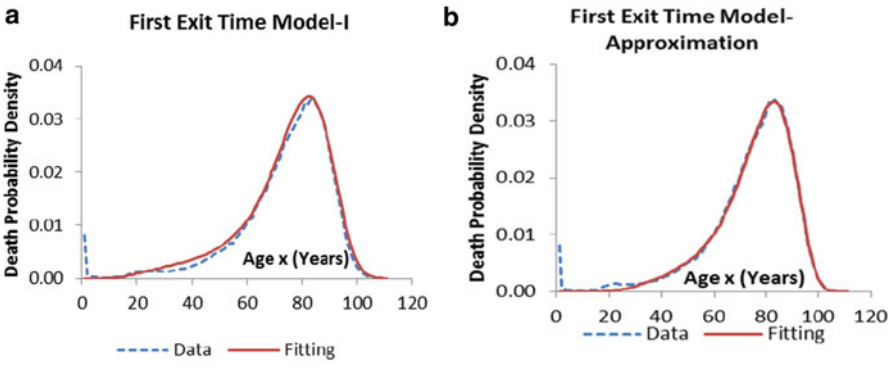


Fig. 9.1 The death probability density using two models

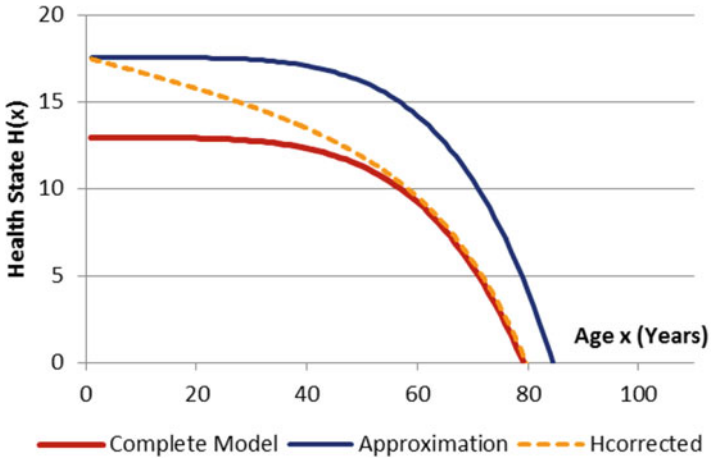
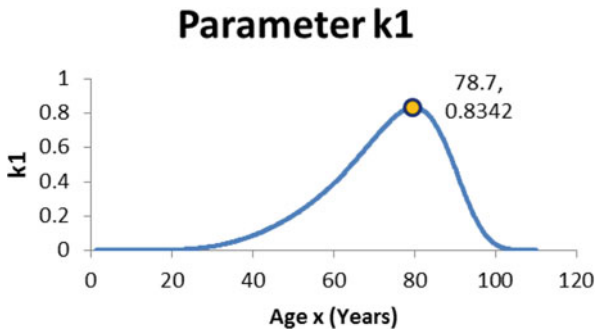


Fig. 9.2 First health state approximations

Fig. 9.3 Estimation of k_1. First method



79.2 for the Complete Model and 79.7 for the Corrected form of the Approximation presented with dashed line in Fig. 9.2. The parameter of the Corrected form is $k_1 = 0.8342$.

The estimation of parameter k_1 from $k_1 = max\left(\frac{\sqrt{2\pi x^3}g(t)}{|H_x - xH'_x|}\right)$ is quite stable as is illustrated in Fig. 9.3 where the function $k_1(x)$ is presented. The $g(x)$ figures are from the provided data and $H(x)$ comes from the estimates of the Approximation Model. The approximation for the first derivative of $H(x)$ is $H'_x \approx H_{x+0.1} - H_x$. We preferred to choose a step $x = 0.1$ to obtain better estimation.

9.3 Second Application

The same data for USA males (2000) are used for this second application. In this case we estimate $H(x)$ applying a simpler form of (9.12) resulting from (9.2) by inserting $\sqrt{2\pi}$ in the parameter k so that $k = k^*/\sqrt{2\pi}$

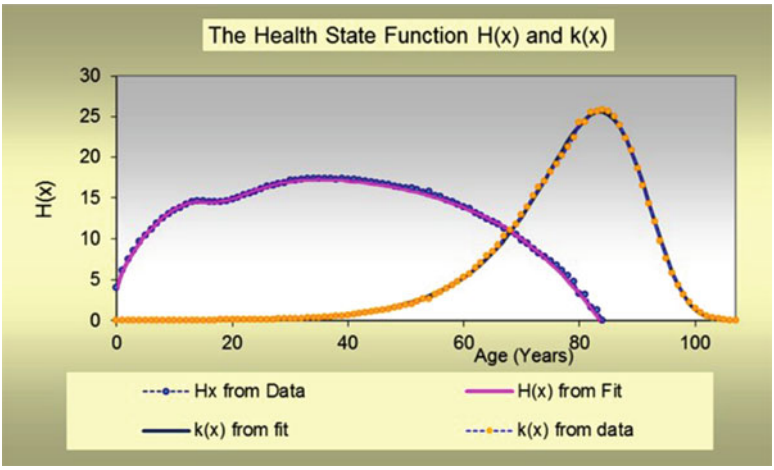


Fig. 9.4 The health state function from the 1995 application

$$g(x) = \frac{k}{\sqrt{x^3}} e^{-\frac{[H_x]^2}{2x}} \tag{9.12}$$

Accordingly from (9.12) the following form for the health state function results:

$$H(x) = \left| -2x \ln \frac{\sqrt{x^3} g(x)}{k} \right|^{1/2} \tag{9.13}$$

By using (9.13) directly to the data set we have immediately the first approximation for $H(x)$ presented in Fig. 9.4. The parameter k is estimated from $k(x) = \sqrt{x^3} g(x)$. The estimated value is $k = max(\sqrt{x^3} g(x))$. The graph for $k(x)$ is illustrated in Fig. 9.4 along with the $H(x)$ graph. However, the direct estimation of $H(x)$ from data series is vulnerable due to fluctuations especially when estimating the first derivative of $H(x)$ from $H'_x \approx H_{x+0.1} - H_x$. The simplest solution is first to smooth the data series and after to proceed in the estimation of $H(x)$. An efficient smoothing method is by fitting a good model to data series. For this application we use the SKI-6 model an extended form of a six parameter model introduced by Janssen and Skiadas (1995). The fit curve for $H(x)$ and the resulting $k(x)$ are illustrated in Fig. 9.4. Both the direct estimation and the fit estimation provide a first approximation for $H(x)$. A better one will follow by introducing the technique already applied in the previous example.

The estimation of k_1 follows the lines from the previous case. The maximum value appears in Fig. 9.5.

The steps followed for the final estimation of H(x) (see the thick blue line in Fig. 9.6) start from:

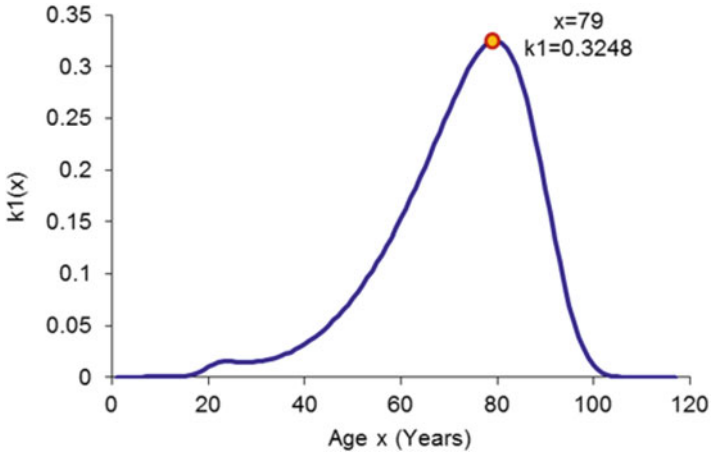


Fig. 9.5 Estimation of parameter k_1

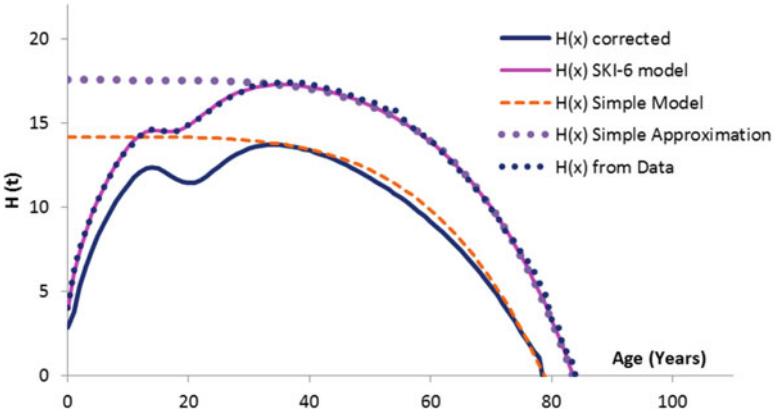


Fig. 9.6 Estimations leading to the final form for $H(x)$

1. Estimation of H(x) directly from the data series (curve with small blue dots)
2. Fit of the Simple Approximation model to data (curve with violet dots)
3. Fit of the Simple Model to data (dashed curve)
4. Fit of the SKI-6 Model to data (Magenta curve)
5. Final form of the H(x) corrected (Blue curve)

As there is not an alternative method to compare the final $H(x)$ corrected we have used the Simple Model which is easily applied to data sets and is compared to the $H(x)$ corrected. Clearly both curves are very close each other for the ages higher than thirty five years. The curves resulting from the SKI-6 model and the direct application to data, approach very well each other. In this case the Simple approximation model approach the other two curves after forty years of age.

9.4 Final Health State Form

So far the Health State found for $H(x)$ includes two local maximum points one in the adolescence period of age and one in the period from 30 to 40 years of age and a local minimum for the young ages from 18 to 30 years of age. However, another correction should be done as the first maximum in the adolescence period of age should be higher than the second one as it is expected from the everyday life experience and the findings of the smaller number of deaths at this period of age. The correction is needed because the first approximation of the first exit time densities that we have used with relation (9.1) covers quite well the main part of the diffusion process except of the first part covering the young and very young ages. Fortunately there is a second order approximation which can be used to produce the Final Form of the Health State Function of a Population. We use the simple model form to locate the main directions of the process like a trajectory and we use a convenient form of the second approximation to provide the requested form of health state $H(x)$. From (9.10) the form of the Health State takes the form

$$H(x) = \left| -2a_1 x^{a_2} \ln \frac{\sqrt{2\pi x^3}\, g(x)}{k_1 |H_x - x H_x'|} \right|^{1/2} \tag{9.14}$$

Where a_1 and a_2 are parameters estimated by a nonlinear regression (Levenberg 1944; Marquardt 1963) as to best fit the health state $H(x)$ to the trajectory set by the simple model. As it is illustrated in the next figure the Final Health State model (continuous line) approaches the simple model in the two maximum points where the first maximum is higher than the second and both the Health State curve and the simple model curve coincide at the same point on the X axis. The parameter values are $a_1 = 2.4$ and $a_2 = 0.75$ for USA 2000 male data and $a_1 = 4.5$ and $a_2 = 0.55$ for USA 2000 female data. The related values for USA 2014 male are $a_1 = 1.7$ and $a_2 = 0.85$ and $a_1 = 3.15$ and $a_2 = 0.65$ for female (Figs. 9.7 and 9.8).

9.5 Validation of the Final Health State Form by Stochastic Simulations

So far we have derived a simple model to express the Health State Function and cover the main part of the life span except of the infant mortality period. First we have to explore if the inverse holds by means to reconstruct the death probability density function when we know

1. The parameters of the health state function in the simple form and
2. The health state function in the final form

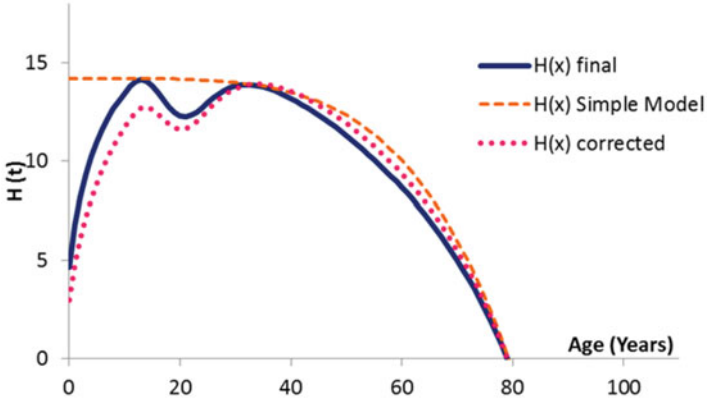


Fig. 9.7 Last approximation form

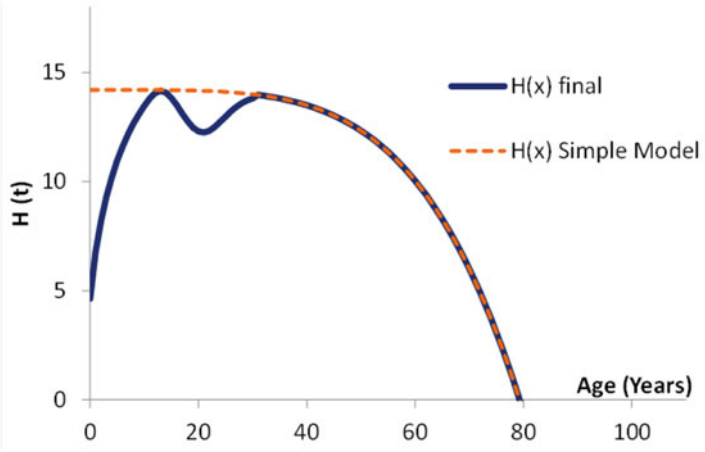


Fig. 9.8 The final health state form

The first case is illustrated in Fig. 9.9 for female in USA in 2010. The Health State function is expressed by the simple model form $H_x = 1-(bx)^c$. The parameters b, c and σ are estimated by a non linear regression on the following simple model form:

$$g(x) = \frac{|1 + (c-1)(bx)^c|}{\sigma\sqrt{2\pi x^3}} e^{-\frac{[1-(bx)^c]^2}{2\sigma^2 x}} \tag{9.15}$$

The stochastic simulation is presented in Fig. 9.9a where stochastic paths appear along with the mean value that is the health state of the population H_x. After 200,000 realizations the resulting simulation graph is illustrated in Fig. 9.9b along with the continuous line expressing the death probability density $g(t)$. The adaptation is quite good.

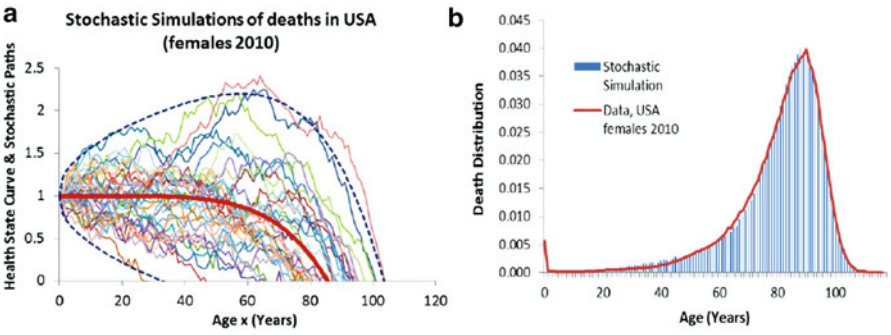


Fig. 9.9 The simple model health state form

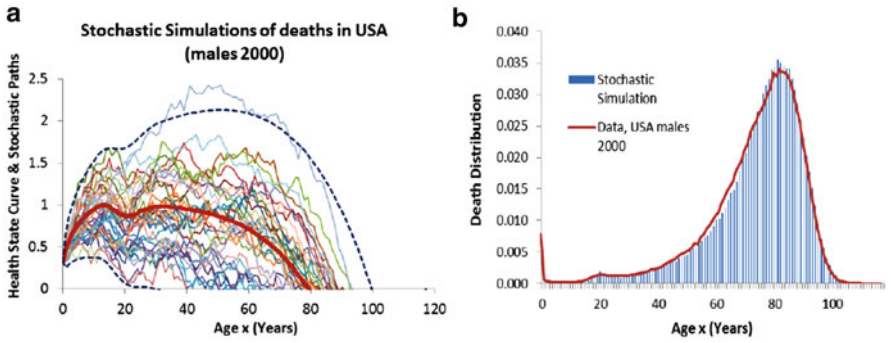


Fig. 9.10 The final health state form

Instead for the case of males the simple model is not so successful because of the extra death instances in the age years from 18 to 30. In this case the health state function in the final form is required. The estimation of the simple model parameters is also an important part and then the final health state form is derived as it was presented earlier in this note. The health state and the stochastic paths are illustrated in Fig. 9.10a whereas the simulation histogram appears in Fig. 9.10b after 200,000 stochastic realizations. In this case the extra deaths in the early years are also presented in the simulations graph.

9.6 Summary and Conclusions

We have proposed and analyzed a Simple and a Final model form for the Health State Function of a Population. the forms are presented analytically along with particular details for application when the death data are available. Stochastic simulations have done verifying the good applicability of the models along with

the establishment of the Health State curve as a good measure of the health status of a population. These quantitative health estimates can have now many interesting applications in finding important health parameters including the healthy life years and the life years lost to disabilities.

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Chapter 10
The Health Status of a Population: Health State and Survival Curves and HALE Estimates

10.1 Introduction

The form of the survival curve resulting from a classical life table is a measure of the health state or health status of a population. Much attention has given to the exploration of the part of the influence of disability on the form of the survival curve thus providing methods for estimating the loss of healthy life years in a population. However, the form of the survival curve depends on the dispersion of deaths around a mean value, measured by the standard deviation σ thus making the survival curve not very appropriate when doing comparisons between different populations. An approach based on a simple Inverse Gaussian model and applied to Carey et al. (1992) data is due to Weitz and Fraser (2001). The Inverse Gaussian form cannot apply to the human population data. The needed general form is given in Skiadas and Skiadas (2010) whereas the theoretical and technical issues are presented in Janssen and Skiadas (1995).

The main points in estimating the Health State Model (HSM) are: to introduce a "system of measure" a "metric" that is independent of the standard deviation, to effectively estimate the health state, to reproduce the population behavior, to compare the health state in various countries and populations for the same or various time periods, to be able to calculate to cost of the health improvement of the population, to estimate the insurance cost and the insurance policies.

10.1.1 The Related Theory

Following the first exit time theory we assume that the health state of an individual is expressed by a stochastic function denoted by S_x and the associated stochastic paths over time t or age x are estimated after integrating the stochastic differential

C.H. Skiadas, C. Skiadas, *Exploring the Health State of a Population by Dynamic Modeling Methods*, The Springer Series on Demographic Methods and Population Analysis 45, DOI 10.1007/978-3-319-65142-2_10

$$dS_x = h_x dx + \sigma dW_x \tag{10.1}$$

With drift h_x and finding the formula for the stochastic paths S_x

$$S_x = \int_0^x h_s ds + \sigma \int_0^x dW_s = H_x + \sigma W_x \tag{10.2}$$

where we assume $W_0 = 0$

W_x is the Wiener process and the Health State is provided as the integral of the instantaneous change h_x

$$h_x = \frac{dH_x}{dx} \tag{10.3}$$

The death occurs when $S_x = 0$ and from (10.4) follows that

$$H_x + \sigma W_x = 0 \tag{10.4}$$

The simpler form for the Health State H_x should be a decreasing process of the form (see related bibliography in Janssen and Skiadas 1995 and Skiadas and Skiadas 2010, 2014, 2015):

$$H_x = l - (bx)^c \tag{10.5}$$

Where l, b and c are parameters. The form of (10.2) becomes

$$l - (bx)^c + \sigma W_x = 0 \tag{10.6}$$

This is a very important relation providing different ways of simulation of the stochastic process. This is demonstrated by observing the new form of (10.2) that is

$$S_x = l - (bx)^c + \sigma W_x \tag{10.7}$$

This form is important for constructing stochastic paths for the health state S_x. The stochastic paths ($l = 1$) are presented in the next Fig. 10.1a where the Health State curve is expressed by the heavy red curve, the confidence intervals by the dashed blue curves and the stochastic paths are illustrated by the light lines. Every stochastic path expresses the health state of an individual during the life course. The end comes when the stochastic path reaches the level zero in a particular age. Then a point is added to the previous points produced in this age and in this particular place. The same is done for all the 117 age years studied producing the histogram for the death probability density of Fig. 10.1b after 260,000 stochastic realizations. The data are from the life table of the Human Mortality Database (HMD) for USA females in 2010. The produced simulations are very close to the real data as is expressed by observing the histograms (note that as the data in the life tables are constructed

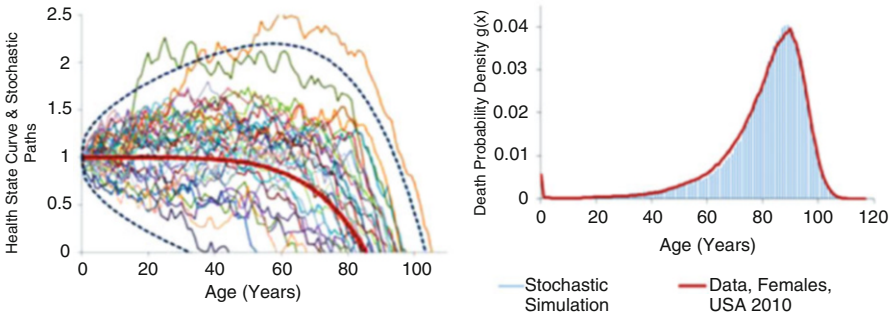

Fig. 10.1 Health state curve and stochastic paths (*left*). Death probability density (*right*)

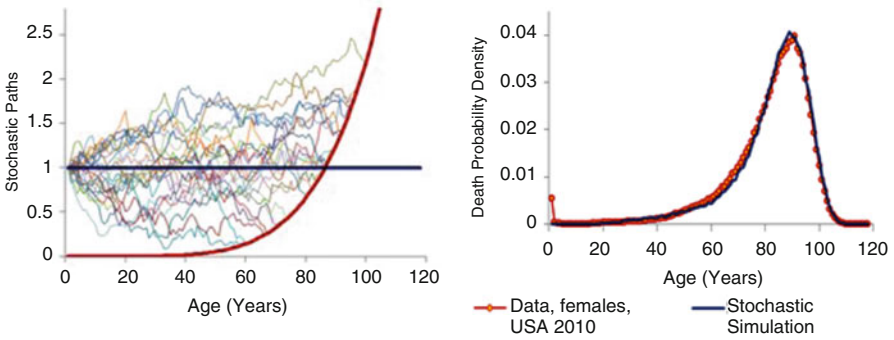

Fig. 10.2 Health state curve and stochastic paths (*left*). Death probability density (*right*)

for 100,000 population and the total area of the death probability density should be equal to one we have to divide the numbers for $D(x)$ provided by the HMD by 100,000). As we have done successive simulations we can easily find the Life Expectancy from the simulations to be 81.41 years of age very close to the 81.22 years provided by the HMD. Note that in our simulations we have not taken into consideration the infant mortality.

The next form arises from (10.7) by a simple transformation

$$S_x + (bx)^c = l + \sigma W_x \tag{10.8}$$

In this case we obtain the same simulations results for the death probability density (see Fig. 10.2b) as from the previous case though the simulation process presented in Fig. 10.2a is different. In this case the stochastic paths generated from the standard Wiener process start from $l = 1$ and are developed horizontally until to reach for the first time the curve expressed by the $z(x) = (bx)^c$. This case is more appropriate from an explanatory point of view as the curve expressed by $f(x) = (bx)^c$ follows a continuously increasing path analogous to the force of mortality curve. The application is for the USA females at in 2010.

A third form is given by

$$S_x - l + (bx)^c = \sigma W_x \tag{10.9}$$

The three forms (10.7), (10.8) and (10.9) are mathematically equivalent. However, they provide three distinct simulation opportunities very important to explain the development of the health status and the development of the death probability density function. According to the theory developed (Janssen and Skiadas 1995) first it was solved the associated Fokker-Planck equation for the appropriate boundary conditions in order to find the transition probability density function.

10.1.2 The General Case

In the general case we have worked with the form

$$dS_x = h_x dx + \sigma_x dW_x \tag{10.10}$$

obtaining the following transition probability density form

$$f(x) = \frac{1}{\int_0^x \sigma(s)ds\sqrt{2\pi x}} e^{-\frac{H_x^2}{2x(\int_0^x \sigma(s)ds)^2}} \tag{10.11}$$

Where

$$H_x = \int_0^x h_s ds$$

For a stable σ the simple form arises

$$f(x) = \frac{1}{\sigma\sqrt{2\pi x}} e^{-\frac{H_x^2}{2\sigma^2 x}} \tag{10.12}$$

Then we could find the formula for the first exit time probability density function $g(x)$ for the health state stochastic process crossing or hitting for the first time a barrier set to zero.

$$g(x) = \frac{|H_x - tH'_x|}{\sigma\sqrt{2\pi x^3}} e^{-\frac{H_x^2}{2\sigma^2 x}} \tag{10.13}$$

10.2 The Weitz and Fraser Paper Revisited

Fifteen years after the Weitz and Fraser (2001) paper only few applications appear though the high importance of this paper for the establishment of a system of measuring the health state or health status of a population. Perhaps the few applications are due to the paper title "Explaining mortality rate plateaus" instead of the use of the first exit time or hitting time theory in relation to the health status. Weitz and Fraser introduced the simpler form for the health state, presented here by Eq. (10.5), but with the parameter $c = 1$. This is a form not convenient for the human population health status but it could be used to model the health status of a large group of Medflies systematically studied by Carey et al. (1992).

$$H_x = l - (bx) \tag{10.14}$$

This form for the health status provides the following probability density function for the first exit time from a barrier set at zero level, that is:

$$g(x) = \frac{l}{\sigma\sqrt{2\pi x^3}} e^{-\frac{(l-bx)^2}{2\sigma^2 x}} \tag{10.15}$$

This is the classical form of the so-called Inverse Gaussian known at least from 1915. When trying to estimate the parameters of (10.15) from the mortality data $d(x)$ or $g(x)$ by a non-linear regression analysis it is obvious that we cannot estimate simultaneously σ with the two parameters b and l. Instead by using the transformation $b^* = b/\sigma$ and $l^* = l/\sigma$ we arrive in the following (10.16) form providing the new parameters b^* and l^* as functions of σ. Instead Weitz and Fraser selected to set $l = 1$ so that they estimated σ and b. Both methods can be useful for the estimates that follow. Note that setting a health state level equal to unity was proposed by Torrance (1976).

$$g(x) = \frac{(l/\sigma)}{\sqrt{2\pi x^3}} e^{-\frac{((l/\sigma)-(b/\sigma)x)^2}{2x}} = \frac{l^*}{\sqrt{2\pi x^3}} e^{-\frac{(l^*-b^*x)^2}{2x}} \tag{10.16}$$

10.2.1 Deterministic and Stochastic Case

An interesting question is related to the deterministic and stochastic case of the problem by means of estimating the mean value of the process. This is illustrated in Fig. 10.3a where the mean value of the deterministic process is the line AB and the mean value of the stochastic process is precisely the same line AB. That is different is that the deterministic line is the result of the mean value of N members of the population identically distributed with $\sigma = 0$ whereas, in the stochastic case, the mean value provided in line AB is the average of the stochastic paths of N members of the population. Although no-stochastic path is identical to another the mean value

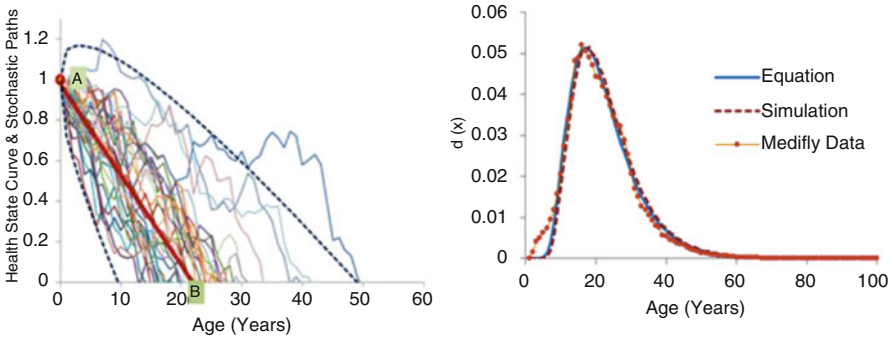

Fig. 10.3 Stochastic paths (*left*). Stochastic simulation for Carey Medfly (*right*)

is identical to the deterministic case. That it is different is that $\sigma > 0$. From (10.2) and (10.14) the stochastic paths are of the form:

$$S_x = H_x + \sigma W_x = l - bx + \sigma W_x \tag{10.17}$$

The deterministic case ($\sigma = 0$) is given by:

$$S_x = H_x = l - bx \tag{10.18}$$

Following the deterministic case death occurs at $H_x = 0$ at age $x = l/b$ (point B of Fig. 10.3a). For the stochastic case the estimation procedure based on (10.16) will find $l^* = l/\sigma$ and $b^* = b/\sigma$. For this case death occurs at $H_x = 0$ at age $x = l^*/b^* = (l/\sigma)/(b/\sigma) = l/b$ that is precisely the same as in the deterministic case as it is independent of σ. Note that the stochastic case provides the death probability density form of Fig. 10.3b whereas the deterministic case will distribute all deaths exactly at point B of Fig. 10.3a thus producing a sharp distribution form at Fig. 10.3b. The higher the standard deviation σ results in larger dispersion in the death distribution. Instead the starting point of the process (the health state l at birth) is at level l for the deterministic process (point A, Fig. 10.3a) whereas it is at level $l^* = l/\sigma$ for the stochastic process by means that the starting point for the stochastic process resulting from the estimation from (10.16) can be in a large variety of places in the Y axis depending on the selection of σ by means that the location of the point A is not definite and the same holds for the estimation of the level of the health state at birth. Instead it is perfectly located the zero health state at point B (Fig. 10.3a).

For the deterministic case we can estimate the total health state estimated as the area of the triangle (OAB) that is $H_{total} = (OA) * (OB)/2 = l(l/b)/2 = (l^2/b)/2$. The mean value is $H_{mean} = l/2$. For the stochastic case the total health state is $H_{total} = (OA) * (OB)/2 = l^*(l/b)/2 = (l/\sigma)(l/b)/2$. The mean value is $H_{mean} = l^*/2 = (l/\sigma)/2$. This value is equal to the deterministic case only when $\sigma = 1$. Comparing different cases it is advisable to set $l^* = 1$. The resulting mean health

state level for the stochastic process is: $H_{mean} = l^*/2 = (l/\sigma)/2 = 1/2$. The total health state is estimated as: $H_{total} = (OA) * (OB)/2 = l^*(l^*/b^*)/2 = (l^*/b^*)/2$. The estimates of Weitz and Fraser for Carey Medfly data by the maximum likelihood estimation method give: $\sigma^* = 0.0975$ corresponding to $l^* = 1/\sigma^* = 10.256$ and $b^* = 0.448$. Our estimates by a Levenberg-Marquardt nonlinear regression analysis algorithm provide: $l^* = 10.532$ corresponding to $\sigma^* = 1/l^* = 0.0949$ and $b^* = 0.480$. The total health state is $H_{total} = (1/b^*)/2 = 11.446$ for Weitz and Frazer and 10.967 for our estimates. The advanced method applying (10.5) for the same data sets provides the following parameter estimates: $c^* = 1.321, b^* = 0.2370$ and $l^* = 8.572$. The total health state is $H_{total} = 12.211$.

10.2.2 The Main Points in Estimating the Health State of a Population

The Main Points in Estimating the Health State of a Population are the following:

1. To introduce a "system of measure" a "metric" that is independent of the related situation
2. To effectively estimate the health state
3. To reproduce the population behavior by means of finding the death probability density, the life expectancy and other measures
4. To compare the health state in various countries and populations for the same or various time periods
5. To find a measure of the health state for all the age period by estimating a measure of the total health state as a summation or integration for all the period of the life span
6. To be able to calculate to cost of the health improvement of the population
7. To estimate the insurance cost and the insurance policies

10.3 The More General Case

Recall the health state form $H_x = l - (bx)^c$ and using (10.13) the following form arises for the probability density function for the first exit time from a barrier set at zero level, that is:

$$g(x) = \frac{l + (c-1)(bx)^c}{\sigma\sqrt{2\pi x^3}} e^{-\frac{(l-(bx)^c)^2}{2\sigma^2 x}} \tag{10.19}$$

By using the transformation $b^* = b/\sigma^{1/c}$ and $l^* = l/\sigma$.

We arrive in the following (10.20) form providing the new parameters b^* and l^* as functions of σ.

$$g(x) = \frac{l^* + (c-1)(b^*x)^c}{\sqrt{2\pi x^3}} e^{-\frac{(l^* - (b^*x)^c)^2}{2x}} \tag{10.20}$$

The following alternative of $H_x = l - (bx)^c$ accepts the simpler transformation $b^* = b/\sigma$ and $l^* = l/\sigma$

$$H_x = l - bx^c \tag{10.21}$$

However, during the applications, the parameter b is very small causing problems in the fitting process. Both Health State forms provide the same measures for the total health state. This is achieved by calculating the integral from age zero ($x = 0$) to age $x = T$ of the zero health state.

$$H_{total} = \int_0^T (l^* - (b^*x)^c)dx = \left[xl^* - \frac{(b^*)^c x^{c+1}}{c+1}\right]_0^T = Tl^* \frac{c}{c+1} \tag{10.22}$$

Note that from $H_x = l - (bx)^c$, $H = 0$ for $T = l^{*(1/c)}/b^*$. The very important achievement is that by setting $l^* = 1$ in (10.22) the total health state is given by:

$$H_{total} = T\frac{c}{c+1} = \frac{l^{*(1/c)}}{b^*} \cdot \frac{c}{c+1} \tag{10.23}$$

Note that the total health state is the more convenient estimator for the comparisons between different populations and countries. It is also in accordance with the main part of the life expectancy at birth ranking though a refined ranking emerges for some countries. Accordingly the mean health state H_{mean} is given by:

$$H_{mean} = \frac{c}{c+1} \tag{10.24}$$

We can easily find that for the simple case ($c = 1$) the mean health state is $H_{mean} = 1/2$ and the total health state is $H_{total} = T/2 = (l^*/b^*)/2$. Note that the total health state is the more convenient estimator for the comparisons between different populations and countries. The main idea is coming from establishing a health state level equal to unity (see Fig. 10.4a, b) at birth by setting $l^* = 1$. This is an independent measure for every population and country. The next step is to find the total health state by evaluating the integral in (10.22) providing the simple measure in (10.23). By using the transformation $b^* = b/\sigma$ and $l^* = l/\sigma$ the right hand side of (10.23) is given by

$$H_{total} = T\frac{c}{c+1} = \frac{l^{*(1/c)}}{b^*} \cdot \frac{c}{c+1} = \frac{l^{1/c}/\sigma^{1/c}}{b/\sigma^{1/c}} \cdot \frac{c}{c+1} = \frac{l^{1/c}}{b} \cdot \frac{c}{c+1} \tag{10.25}$$

The very important achievement from (10.25) is that the total health state is independent of σ. When the exponent c is large the total health state is of the order of $l^{1/c}/b$. This is a simple method expressing the health state in terms of age as it is done for the life expectancy.

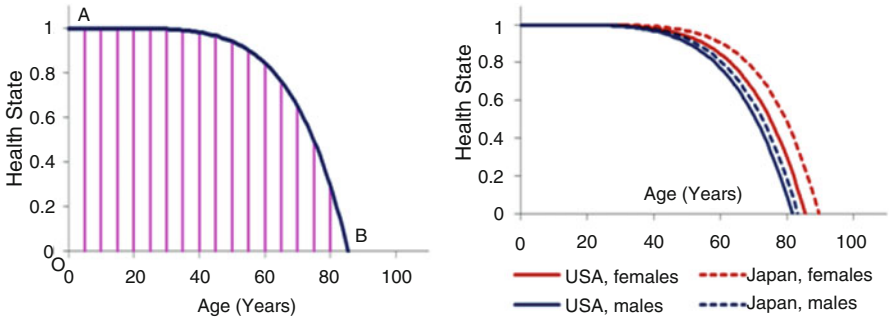

Fig. 10.4 Health state versus age and total health state area (*left*). Comparing health state (*right*)

10.4 Connection of the Health State Curve to the Survival Curve

As it was presented earlier the health state curve is independent of σ. Instead the survival curve $l(x)$ is connected to σ as can be verified by stochastic simulations. Even more the survival curve is connected with the death probability which includes σ as is expressed in (10.26).

$$Survival = 1 - \int_0^x g(s)ds = 1 - \int_0^x \frac{|l + (c-1)(bs)^c|}{\sigma\sqrt{2\pi s^3}} e^{-\frac{(l-(bs)^c)^2}{2\sigma^2 s}} \tag{10.26}$$

Where the integral is normalized so that

$$\int_0^\infty g(s)ds = 1 \tag{10.27}$$

Recalling (10.13) a very important relation for Life Expectancy is given by

$$LE = \int_0^x sg(s)ds = \int_0^x \frac{|H_s - tH_s'|}{\sigma\sqrt{2\pi s^3}} e^{-\frac{H_s^2}{2\sigma^2 s}} \tag{10.28}$$

From the last formula the relation between the life expectancy and the health state is clarified. For the simple case for the health state $H_x = l - bx$ the resulting relation is

$$LE = \int_0^x sg(s)ds = \int_0^x \frac{l}{\sigma\sqrt{2\pi s^3}} e^{-\frac{(l-bx)^2}{2\sigma^2 s}} \tag{10.29}$$

Following the above theory we have estimated the health state for the females in USA for 2010 and we have reproduced the results by applying stochastic simulations (see Fig. 10.5a). The health state curve for USA females in 2010 is

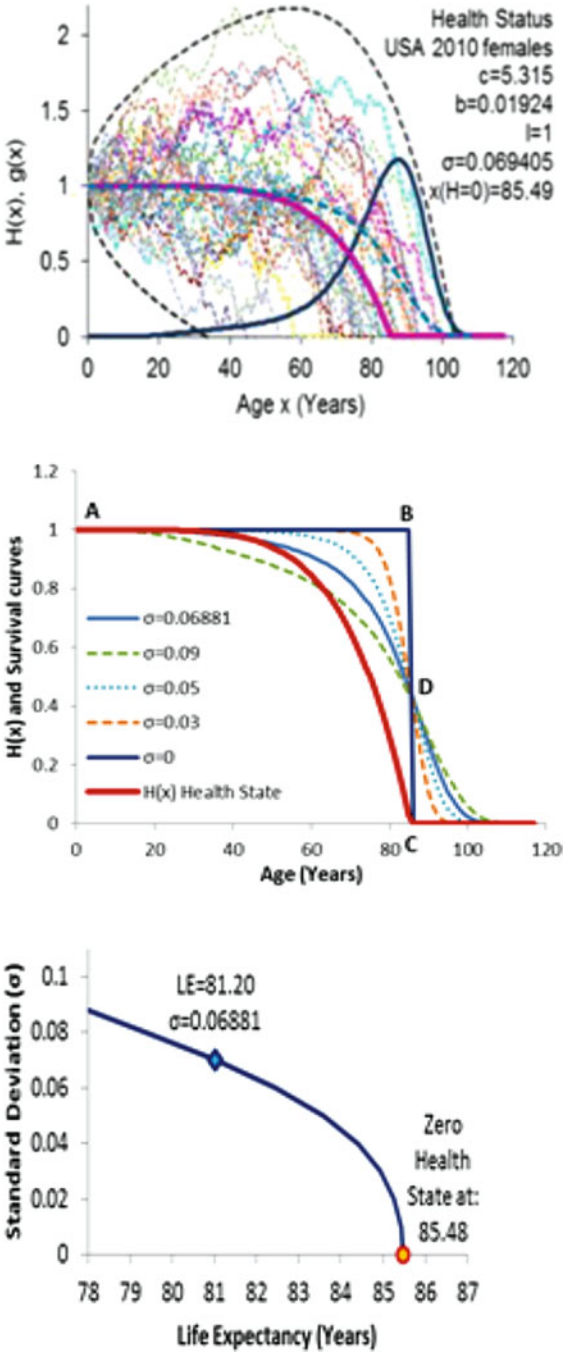

Fig. 10.5 Stochastic simulations (*up*). Health state and survival (*middle*). LE and σ (*down*)

illustrated by the heavy curve (see Fig. 10.5b). The corresponding survival curve (continuous line) for this related case is presented with $\sigma = 0.06881$. The curve with higher value for $\sigma = 0.09$ is presented by a dashed line whereas the lower values for σ ($\sigma = 0.05$, dotted light curve and $\sigma = 0.03$ dashed upper curve) appear as well. The very important case with $\sigma = 0$ corresponding to the deterministic case is illustrated with a heavy line (ABDC). The latter ends at the point C at the zero health state age. This is the case with the total rectangularization process. In this case the Life Expectancy asymptotically approaches the zero health state age (point C of Fig. 10.5a). All the survival curves cross around the point D at the zero health state age.

10.5 Comparing the Health State or Health Status and Life Expectancy

From equation (10.26) for the survivorship we have estimated the Life Expectancy for various values of the standard deviation σ. The results are illustrated in Fig. 10.5c. The Life Expectancy at Birth is systematically related to the standard deviation σ. At $\sigma = 0$ the Life Expectancy is equal to the age at zero Health Status (application for USA, females, 2010). The life expectancy as it is given in the life table from Human Mortality Database is 81.20 years and we estimated that this is achieved for $\sigma = 0.06881$.

10.5.1 The Time Development of the Standard Deviation σ

As far as the standard deviation σ is estimated by the proposed method we have calculated the related figures for USA and Switzerland males and females and presented in Fig. 10.6. The standard deviation and thus the dispersion are larger for males.

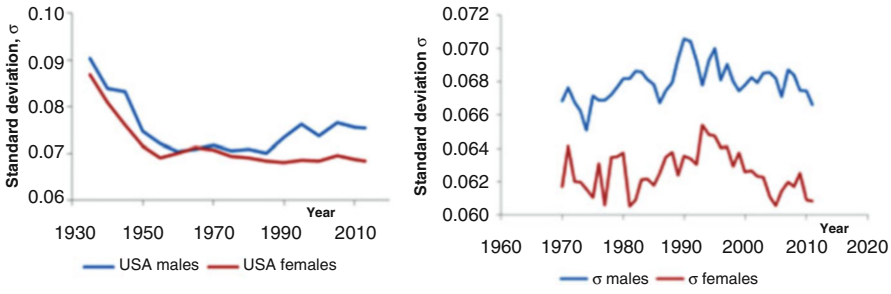

Fig. 10.6 The time development of the standard deviation σ for USA (*left*), and for Switzerland (*right*)

10.6 Health State, Life Expectancy and HALE Estimates

The proposed methodology for estimating the Health State or Health Status of a population provides a tool for comparing the HS with the healthy life expectancy as it is presented by the HALE measures of the World Health Organization (WHO). Related information and publications are included in very many references (see Murray et al. (2015), Robine et al. (1999), Salomon et al. (2012) and WHO annexes from 2000–2015 WHO (2000, 2002, 2004, 2013, 2014) and many publications in Lancet). The very important property of the HS measure is that it is independent of the dispersion expressed by the standard deviation σ. It will be the same if the dispersion is large or small. Instead the life expectancy is influenced by the dispersion as it was provided in formulas (10.28) and (10.29) and presented in Fig. 10.5b. So far we can estimate the LE and the HS for every time period as far as life table data are provided. Table 10.1 includes our estimates for the HS and LE for 36 countries. These countries are included in the Human Mortality Database and are extensively studied. The countries are ranked according to the HS.

There is relatively good agreement with the LE rankings although differences appear as it was expected. The measure based on HS is also a strong methodological tool avoiding dispersion. We can call it an absolute measure of the Health State (HE) of a population. Following our introduction of the HS as an absolute measure of health state we search if this estimate could be a measure or metric of the Healthy Life Expectancy estimates as are expressed by the so-called HALE system. However, HALE estimates are strongly influenced by the dispersion σ and could not be compared with the HS. The various healthy life expectancy (HLE) estimates by using only the life table data are presented in Skiadas (ArXiv.org, March 2016) with good results compared to the HALE estimates. Here we use one of these estimates called Direct Estimation from the Life Tables (see Skiadas, ArXiv.org, March 2016, pp. 7–8). This is illustrated in several figures for few of the countries presented in Table 10.1. Fortunately the HALE data provided include the estimated confidence intervals thus improving comparisons as is presented in Figs. 10.7a–h and 10.8i–n where the confidence bars appear. The HS is expressed by the heavy blue curve and the HLE by light red curve whereas the HALE estimates are presented by rhombus with confidence bars.

It is demonstrated that the Health State is growing for the years studied (more than 40 years starting 1970). Only for males in the Czech Republic was a decline few years before 1990 and then a continuous increase until nowadays. The healthy life expectancy (HLE) was not increasing as fast as the Health State (HS) in several cases.

Table 10.1 Health state and life expectancy for 36 countries

Males			Females		
Country	HS	LE	Country	HS	LE
Switzerland	70.58	80.05	Japan	76.49	86.30
Australia	70.57	79.87	France	75.87	84.69
New Zealand	69.93	79.16	Switzerland	75.46	84.39
France	69.67	78.04	Spain	74.78	85.01
Canada	69.56	79.17	Australia	74.74	84.25
Sweden	69.51	79.51	Luxembourg	74.30	83.18
Japan	69.51	79.56	Finland	74.23	83.24
Italy	69.30	79.49	Italy	74.21	84.49
Spain	68.97	79.01	Canada	73.95	83.51
Norway	68.96	78.84	Sweden	73.95	83.47
Austria	68.42	77.67	Austria	73.89	83.13
Luxembourg	68.40	77.94	New Zealand	73.85	82.96
UK	68.31	78.37	Belgium	73.78	82.65
Ireland	68.24	78.26	Norway	73.69	83.15
Greece	68.16	77.99	Netherlands	73.42	82.72
Portugal	67.96	76.74	Germany	73.30	82.71
Belgium	67.94	77.38	Slovenia	72.61	82.62
Israel	67.92	79.70	Portugal	72.31	83.04
Netherlands	67.91	78.78	Ireland	72.29	82.77
Finland	67.60	76.72	UK	72.17	82.35
USA	67.52	76.38	Israel	72.06	83.50
Germany	67.42	77.67	USA	71.86	81.22
Denmark	66.60	77.12	Greece	71.71	83.17
Scotland	66.11	76.24	Taiwan	71.26	82.37
Taiwan	65.72	76.24	Poland	70.99	80.47
Slovenia	65.70	76.28	Estonia	70.93	80.55
Czech Republic	63.24	74.47	Czech Republic	70.90	80.64
Poland	60.35	72.16	Lithuania	70.54	78.75
Slovakia	59.69	71.73	Denmark	70.13	81.33
Bulgaria	59.47	70.31	Scotland	70.09	80.63
Estonia	59.44	70.83	Slovakia	68.93	79.15
Lithuania	55.15	67.55	Bulgaria	68.82	77.25
Ukraine	54.61	65.20	Belarus	67.16	76.49
Latvia	53.55	67.43	Latvia	66.95	77.42
Russia	52.68	63.06	Russia	66.81	74.87
Belarus	52.06	64.60	Ukraine	66.27	75.19

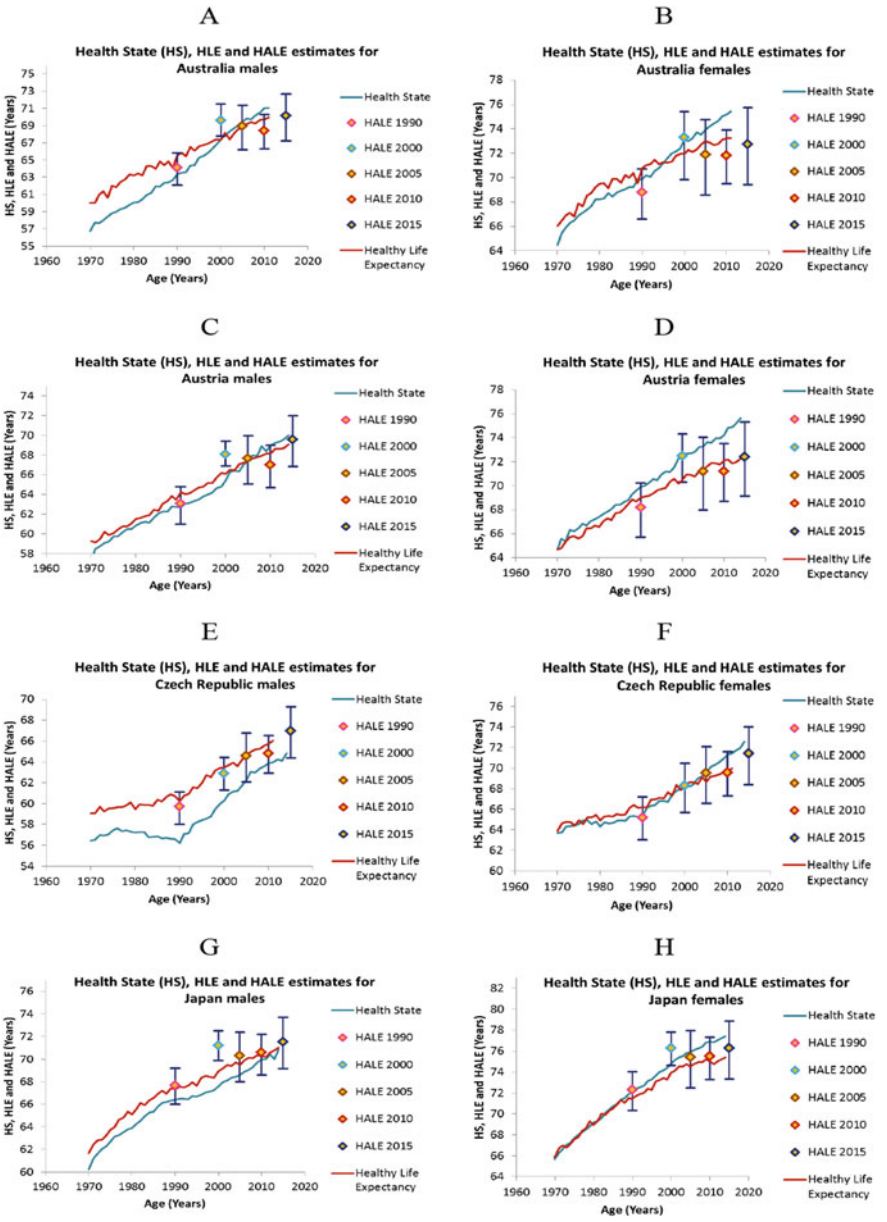

Fig. 10.7 Health state, healthy life expectancy and HALE for various countries (first part)

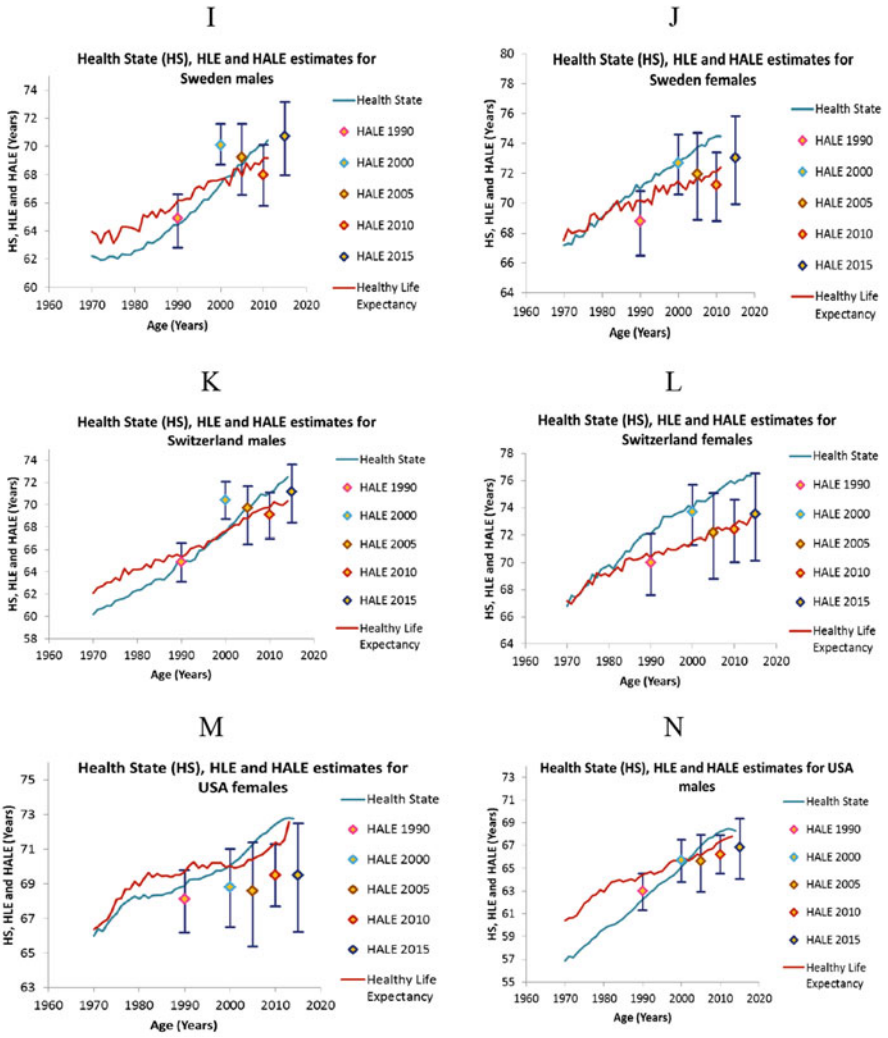

Fig. 10.8 Health state, healthy life expectancy and HALE for various countries (second part)

10.7 Further Discussion

We have introduced and estimated the Health State of a population along with the associated Health State Curve and related parameters as the Age of zero Health State and the Age of the Maximum Curvature of the health state curve. This is also an important age year where the maximum decline of the health state appears. Furthermore we have connected the health state with the survival curve and the standard deviation σ as it is estimated from the Death Probability Density Function.

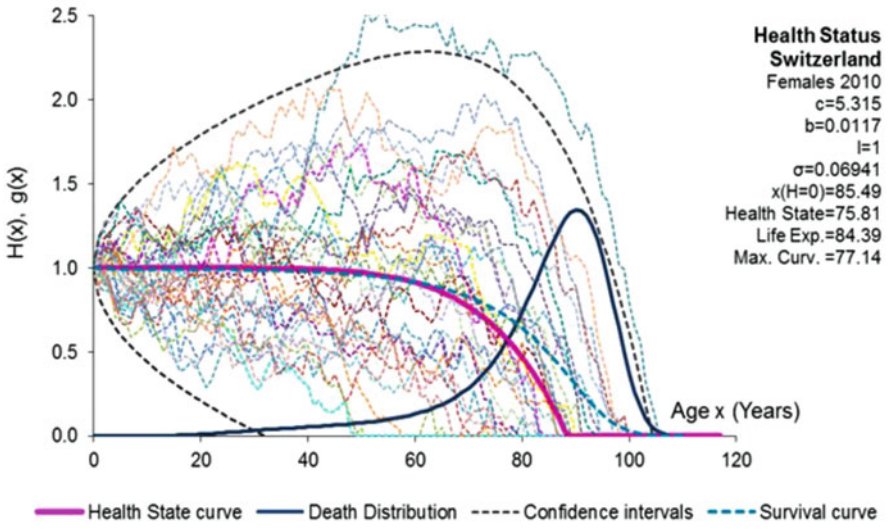

Fig. 10.9 Health state curve, survival curve, death distribution, stochastic paths and confidence intervals for females in Switzerland 2010

Stochastic simulations are also important to verify the theory and to reproduce the provided data after a quite large number of stochastic paths generated. The main part of the theoretical and applied findings is illustrated in Fig. 10.9 for females in Switzerland 2010. The model parameters estimated are $b = 0.0117, c = 5.315, \sigma = 0.06941$ for $l = 1$. The Health State estimated (HS = 78.85) is precisely the area under the Health State Curve whereas the Life Expectancy estimated (LE = 84.39) is the area under the Survival Curve. The Maximum Curvature is at 77.14 years defining the maximum deterioration stage.

10.8 Conclusion

We have presented and applied the general theory of the health state of a population in connection to the survival curves and life expectancy. We have provided a measure for the Health State or Health Status which is independent of the dispersion parameter expressed by the standard deviation σ. We also have provided enough evidence of the relation of life expectancy and the health state or health status estimator. We complete our work with stochastic simulations thus effectively reproducing the death distribution. We also provide illustrations with our estimated Health Status and a comparative approach of Healthy Life Expectancy and the HALE measures in several countries.

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Chapter 11
Theoretical Approach to Health State Modeling

11.1 The Stochastic Model

Following the Janssen and Skiadas (1995) lines of stochastic analysis we assume that the health state $S = S(t)$ of an individual at age t follows a stochastic process of the form:

$$dS(t) = h(t)dt + \sigma(t)dW(t), \tag{11.1}$$

where $h(t)$ is the drift coefficient or the infinitesimal mean and $\sigma(t)$ the variance parameter or the infinitesimal variance or the diffusion coefficient and $W(t)$ the standard Wiener process. The latter is the best alternative to reproduce a stochastic process of Brownian motion type that is a random process to account for the random changes of our health state. Accordingly an equation of the last type can model the time course of a complex system as are several complicated machines or automata. The last equation is immediately integrable provided we have selected appropriate initial conditions as $S(t = 0) = S(0)$.

$$S(t) = S(0) + \int_0^t h(s)ds + \int_0^s \sigma(s)dW(s), \tag{11.2}$$

This equation form gives a large number of stochastic paths for the health state $S(t)$ of an individual. However, it should be noted that these paths following a random process with drift are artificial realizations that can not calculated in the real life for a specific individual else we have a perfect inspection system estimating the health state in real time; that is impossible so far. However, we can find methods to estimate special characteristics of this system of stochastic paths provided by the last equation as is the summation of infinitesimal mean that is the mean value $H(t)$ of the health state over time given by

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$$H(t) = S(0) + \int_0^t h(s)ds, \tag{11.3}$$

Even so it is not feasible to calculate $H(t)$ immediately. Fortunately there are several theoretical approaches to find the time development of $H(t)$ from the advances in physics, mathematics and applied mathematics, and probability and statistics. The first approach is by observing that as $H(t)$ expresses the Health State of a large ensemble of persons, a population, it should be a declining function of time or better age with the exception of the first stages of the human development and especially the infancy.

11.2 General Solution

Up to now an application to living organisms, Medflies, make it possible to estimate $H(t)$ based on a theory developed in 1915 by Schrödinger (1915) and Smoluchowsky (1915) in two independent publications in the same scientific journal. In this theory a simple linear declining function is accepted for $H(t)$ and the parameter estimates are done from a non linear fitting on the data of a death distribution. This is the so-called Inverse Gaussian distribution used by Weitz and Fraser (2001). Six years earlier we have solved the problem of finding a method of estimating not only a simple linear $H(t)$ but a non linear one which is important to model the health state of a population. The death data used are coming from the life tables and we had interesting and very important results and applications.

However, to find the appropriate form of $H(t)$ for human populations, a series of delicate mathematical calculations are needed. We preferred estimates leading to closed form solutions thus providing important and easy applied tools for scientists from several fields and those non familiar with stochastic theory methodology and practice. The important steps leading to the final forms for estimating $H(t)$ are given in the following.

The first step is the formulation of the transition probability density function $p(S,t)$, that is a function expressing the probability for the health state S of an individual to move from one point at time t to the next. This is achieved by calculating a Chapman-Kolmogorov equation for the discrete case. However, we use here the continuous alternative of this equation that is the following Fokker-Planck equation:

$$\frac{\partial p(S,t)}{\partial t} = -h(t)\frac{\partial [p(S,t)]}{\partial S} + \frac{1}{2}[\sigma(t)]^2\frac{\partial^2 [p(S,t)]}{\partial S^2}, \tag{11.4}$$

This partial differential equation for S and t is solved for the following appropriate boundary conditions

$$p(S(t), t_0; S_0, t_0) = \delta(S(t) - S_0), \tag{11.5}$$

$$\frac{\partial p[S(t), t_0; S_0, t]}{\partial S(t)} \to 0 \quad \text{as} \quad S(t) \to \pm\infty \tag{11.6}$$

For the solution we use the method of characteristic functions. The characteristic function $\phi(S, t)$ is introduced by the following equation

$$\phi(S, t) = \int_{-\infty}^{+\infty} p(S, t; S_0, t_0) \exp(isS) ds, \tag{11.7}$$

Integrating by parts and using the Fokker-Plank equation we arrive at

$$\frac{\partial \phi}{\partial t} = ish(t)\phi - \frac{1}{2}[\sigma(t)]^2 s^2 \phi, \tag{11.8}$$

which with the initial conditions proposed

$$\phi(s, t_0) = \exp(isS_0), \tag{11.9}$$

is solved providing the following expression for ϕ

$$\phi(s, t_0) = \exp\left[is\left[S_0 + \int_{t_0}^{t} h(t')dt'\right] - \frac{1}{2}s^2 \int_{t_0}^{t} \left[\sigma(t')\right]^2 dt'\right], \tag{11.10}$$

This is the characteristic function of a Gaussian with mean

$$\left[S_0 + \int_{t_0}^{t} h(t')dt'\right], \tag{11.11}$$

and variance

$$\left[\sigma(t')\right]^2 dt', \tag{11.12}$$

Considering Eq. 11.3 and $t_0 = 0$ the solution is

$$p(t) = \frac{1}{[2\pi \int_0^t [\sigma(s)]^2 ds]^{1/2}} \exp\left[-\frac{[H(t)]^2}{2\int_0^t [\sigma(s)]^2 ds}\right], \tag{11.13}$$

11.3 Specific Solution

When $\sigma(t) = \sigma$ a simple presentation of the transition probability density function during time is given by:

$$p(t) = \frac{1}{\sigma\sqrt{2\pi t}} \exp\left[-\frac{[H(t)]^2}{2\sigma^2 t}\right], \tag{11.14}$$

Having estimated the transition probability density function for the continuous process we can find the first exit time probability density function for the process reaching a barrier.

11.4 A First Approximation Form

A convenient form is provided in several references as an approximation of the form:

$$g(t) = \frac{|H - tH'|}{t} p(t), \tag{11.15}$$

By using the estimated $p(t)$ we arrive at the following form for the first exit time probability density function

$$g(t) = \frac{|H - tH'|}{\sigma\sqrt{2\pi t^3}} \exp\left[-\frac{[H(t)]^2}{2\sigma^2 t}\right], \tag{11.16}$$

The last formula is coming from a first approximation of the first exit time densities with good results in relatively simpler cases (see Skiadas and Skiadas 2010, 2014, 2015).

11.5 A Second Approximation Form

For more complicated cases a second approximation was proposed of the form:

$$g(t) = \frac{1}{\sigma\sqrt{2\pi}} \left[\frac{|H - tH'|}{\sqrt{t^3}} + \frac{\sqrt{t^3}H''}{2|H - tH'|^2}\right] \exp\left[-\frac{[H(t)]^2}{2\sigma^2 t}\right], \tag{11.17}$$

The last formula is also an approximation and higher order terms are omitted.

However, for applications in demography the following approximation formula proved to be quite important providing good fitting to the death data sets of a population. This model became very flexible by introducing an extra parameter k and simplifying the second term.

$$g(t) = \frac{2}{\sigma\sqrt{2\pi}} \left[\frac{|H - tH'|}{\sqrt{t^3}} + \frac{k\sqrt{t^3}H''}{2|H - tH'|}\right] \exp\left[-\frac{[H(t)]^2}{2\sigma^2 t}\right], \tag{11.18}$$

Furthermore we can arrive in a very interesting formula by selecting the following form for $H(t)$:

$$H(t) = l - (bt)^c, \tag{11.19}$$

where l, b, c are parameters

The resulting first and second order approximations are:

$$g(t) = \frac{|l + (c-1)(bt)^c|}{\sigma\sqrt{2\pi t^3}} \exp\left[-\frac{[l-(bt)^c]^2}{2\sigma^2 t}\right], \tag{11.20}$$

$$g(t) = \frac{2}{\sigma\sqrt{2\pi}}\left[\frac{|l + (c-1)(bt)^c|}{\sqrt{t^3}} - \frac{k\sqrt{t^3}c(c-1)b^c t^{(c-2)}}{2|l + (c-1)(bt)^c|}\right] \exp\left[-\frac{[l-(bt)^c]^2}{2\sigma^2 t}\right], \tag{11.21}$$

As for fitting this formula and the previous simpler forms to data sets it is not possible to estimate the parameters of the model along with σ, two options are selected; that is to set $\sigma = 1$ and estimate l, b, c or to set $l = 1$ and estimate b, c, σ. The latter is very important when stochastic simulations are needed. It is also useful for applications on health state estimates. It was selected from Weitz and Fraser (2001) for the application in Medflies and from Skiadas and Skiadas (2010, 2014, 2015) for many applications and in this book as well.

11.6 Summary and Conclusions

The theoretical approach to Health State modeling presented here was based on the first exit time or hitting time theory for a stochastic process crossing a barrier leaded to Health State equation forms. The probability density function form was derived and the first and second order approximation of the first exit time densities was proposed. Several applications follow in the chapters of this book and in other publications and presentations as well.

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