REYNOLDS'S REINFORCED CONCRETE DESIGNER'S HANDBOOK

11TH EDITION

CHARLES E. REYNOLDS,
JAMES C. STEEDMAN AND
ANTHONY J. THRELFALL

Reynolds's Reinforced Concrete Designer's Handbook

Reynolds's Reinforced Concrete Designer's Handbook has been completely rewritten and updated for this new edition to take account of the numerous developments in design and practice over the last 20 years. These include significant revisions to British Standards and Codes of Practice, and the introduction of the new Eurocodes. The principal feature of the Handbook is the collection of over 200 full-page tables and charts, covering all aspects of structural analysis and reinforced concrete design. These, together with extensive numerical examples, will enable engineers to produce rapid and efficient designs for a large range of concrete structures conforming to the requirements of BS 5400, BS 8007, BS 8110 and Eurocode 2.

Design criteria, safety factors, loads and material properties are explained in the first part of the book. Details are then given of the analysis of structures ranging from single-span beams and cantilevers to complex multi-bay frames, shear walls, arches and containment structures. Miscellaneous structures such as helical stairs, shell roofs and bow girders are also covered.

A large section of the Handbook presents detailed information concerning the design of various types of reinforced concrete elements according to current design methods, and their use in such structures as buildings, bridges, cylindrical and rectangular tanks, silos, foundations, retaining walls, culverts and subways. All of the design tables and charts in this section of the Handbook are completely new.

This highly regarded work provides in one publication a wealth of information presented in a practical and user-friendly form. It is a unique reference source for structural engineers specialising in reinforced concrete design, and will also be of considerable interest to lecturers and students of structural engineering.

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Reynolds's Reinforced Concrete Designer's Handbook

ELEVENTH EDITION

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Preface to the eleventh edition

Since the last edition of *Reynolds's Handbook*, considerable developments in design and practice have occurred. These include significant revisions to British standard specifications and codes of practice, and the introduction of the Eurocodes. Although current British codes are due to be withdrawn from 2008 onwards, their use is likely to continue beyond that date at least in some English-speaking countries outside the United Kingdom.

One of the most significant changes has been in the system for classifying exposure conditions, and selecting concrete strength and cover requirements for durability. This is now dealt with exclusively in BS 8500, which takes into account the particular cement/combination type. The notation used to define concrete strength gives the cylinder strength as well as the cube strength. For structural design, cube strength is used in the British codes and cylinder strength in the Eurocodes.

The characteristic yield strength of reinforcement has been increased to 500 N/mm^2 (MPa). As a result, new design aids have become necessary, and the *Handbook* includes tables and charts for beams and columns (rectangular and circular) designed to both British and European codes. Throughout the *Handbook*, stress units are given as N/mm^2 for British codes and MPa for European codes. The decimal point is shown by a full stop (rather than a comma) in both cases.

The basic layout of the *Handbook* is similar to the previous edition, but the contents have been arranged in four separate parts for the convenience of the reader. Also, the opportunity has been taken to omit a large amount of material that was no longer relevant, and to revise the entire text to reflect modern design and construction practice. Part 1 is descriptive in form and covers design requirements, loads, materials, structural analysis, member design and forms of construction. Frequent reference is made in Part 1 to the tables that are found in the rest of the *Handbook*. Although specific notes are attached to these tables in Parts 2, 3 and 4, much of the relevant text is embodied in Part 1, and the first part of the *Handbook* should always be consulted.

Part 2 has more detailed information on loads, material properties and analysis in the form of tabulated data and charts for a large range of structural forms. This material is largely independent of any specific code of practice. Parts 3 and 4 cover the design of members according to the requirements of the British and European codes respectively. For each code, the same topics are covered in the same sequence so that the reader can move easily from one code to the other. Each topic is illustrated by extensive numerical examples.

In the Eurocodes, some parameters are given recommended values with the option of a national choice. Choices also exist with regard to certain classes, methods and procedures. The decisions made by each country are given in a national annex. Part 4 of the *Handbook* already incorporates the values given in the UK national annex. Further information concerning the use of Eurocode 2 is given in PD 6687: *Background paper to the UK National Annex to BS EN 1992–1–1.*

The *Handbook* has been an invaluable source of reference for reinforced concrete engineers for over 70 years. I made extensive use of the sixth edition at the start of my professional career 50 years ago. This edition contains old and new information, derived by many people, and obtained from many sources past and present. Although the selection inevitably reflects the personal experience of the authors, the information has been well tried and tested. I owe a considerable debt of gratitude to colleagues and mentors from whom I have learnt much over the years, and to the following organisations for permission to include data for which they hold the copyright:

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Tony Threlfall
Marlow, October 2006

The authors

Charles Edward Reynolds was born in 1900 and received his education at Tiffin Boys School, Kingston-on-Thames, and Battersea Polytechnic. After some years with Sir William Arroll, BRC and Simon Carves, he joined Leslie Turner and Partners, and later C W Glover and Partners. He was for some years Technical Editor of Concrete Publications Ltd and then became its Managing Editor, combining this post with private practice. In addition to the *Reinforced Concrete Designer's Handbook*, of which almost 200,000 copies have been sold since it first appeared in 1932, Charles Reynolds was the author of numerous other books, papers and articles concerning concrete and allied subjects. Among his various professional appointments, he served on the council of the Junior Institution of Engineers, and was the Honorary Editor of its journal at his death on Christmas Day 1971.

James Cyril Steedman was educated at Varndean Grammar School and first was employed by British Rail, whom he joined in 1950 at the age of 16. In 1956 he began working for GKN Reinforcements Ltd and later moved to Malcolm Glover and Partners. His association with Charles Reynolds began when, after the publication of numerous articles in the magazine *Concrete and Constructional Engineering*, he accepted an appointment as Technical Editor of Concrete Publications, a post he held for seven years. He then continued in private practice, combining work for the Publications Division of the Cement and Concrete Association with his own writing and other activities. In 1981 he set up Jacys Computing Services, subsequently devoting much of his time to the development of micro-computer software for reinforced concrete design. He is the joint author, with Charles Reynolds, of *Examples of the Design of Reinforced Concrete Buildings to BS 8110.*

Anthony John Threlfall was educated at Liverpool Institute for Boys, after which he studied civil engineering at Liverpool University. After eight years working for BRC, Pierhead Ltd and IDC Ltd, he took a diploma course in concrete structures and technology at Imperial College. For the next four years he worked for CEGB and Camus Ltd, and then joined the Cement and Concrete Association in 1970, where he was engaged primarily in education and training activities until 1993. After leaving the C&CA, he has continued in private practice to provide training in reinforced and prestressed concrete design and detailing.

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Symbols and abbreviations

The symbols adopted in this book comply, where appropriate, with those in the relevant codes of practice. Although these are based on an internationally agreed system for preparing notations, there are numerous differences between the British and the European codes, especially in the use of subscripts. Where additional symbols are needed to represent properties not used in the codes, these have been selected in accordance with the basic principles wherever possible.

The amount and range of material contained in this book make it inevitable that the same symbols have to be used for different purposes. However, care has been taken to ensure that code symbols are not duplicated, except where this has been found unavoidable. The notational principles adopted for concrete design purposes are not necessarily best suited to other branches of engineering. Consequently, in those tables relating to general structural analysis, the notation employed in previous editions of this book has generally been retained.

Only the principal symbols that are common to all codes are listed here: all other symbols and abbreviations are defined in the text and tables concerned.

A_c	Area of concrete section
A_s	Area of tension reinforcement
A'_s	Area of compression reinforcement
A_{sc}	Area of longitudinal reinforcement in a column
C	Torsional constant
E_c	Static modulus of elasticity of concrete
E_s	Modulus of elasticity of reinforcing steel
F	Action, force or load (with appropriate subscripts)
G	Shear modulus of concrete
G_k	Characteristic permanent action or dead load
I	Second moment of area of cross section
K	A constant (with appropriate subscripts)
L	Length; span
M	Bending moment
N	Axial force
Q_k	Characteristic variable action or imposed load
R	Reaction at support
S	First moment of area of cross section
T	Torsional moment; temperature
V	Shear force
W_k	Characteristic wind load

a	Dimension; deflection
b	Overall width of cross section, or width of flange
d	Effective depth to tension reinforcement
d'	Depth to compression reinforcement
f	Stress (with appropriate subscripts)
f_{ck}	Characteristic (cylinder) strength of concrete
f_{cu}	Characteristic (cube) strength of concrete
f_{yk}	Characteristic yield strength of reinforcement
g_k	Characteristic dead load per unit area
h	Overall depth of cross section

i	Radius of gyration of concrete section
k	A coefficient (with appropriate subscripts)
l	Length; span (with appropriate subscripts)
m	Mass
q_k	Characteristic imposed load per unit area
r	Radius
$1/r$	Curvature
t	Thickness; time
u	Perimeter (with appropriate subscripts)
v	Shear stress (with appropriate subscripts)
x	Neutral axis depth
z	Lever arm of internal forces

α, β	Angle; ratio
α_e	Modular ratio E_s/E_c
γ	Partial safety factor (with appropriate subscripts)
ε_c	Compressive strain in concrete
ε_s	Strain in tension reinforcement
ε'_s	Strain in compression reinforcement
ϕ	Diameter of reinforcing bar
φ	Creep coefficient (with appropriate subscripts)
λ	Slenderness ratio
ν	Poisson's ratio
ρ	Proportion of tension reinforcement A_s/bd
ρ'	Proportion of compression reinforcement A'_s/bd
σ	Stress (with appropriate subscripts)
ψ	Factor defining representative value of action

BS	British Standard
EC	Eurocode
SLS	Serviceability limit state
UDL	Uniformly distributed load
ULS	Ultimate limit state

Part 1

General information

Chapter 1

Introduction

A structure is an assembly of members each of which, under the action of imposed loads and deformations, is subjected to bending or direct force (either tensile or compressive), or to a combination of bending and direct force. These effects may be accompanied by shearing forces and sometimes by torsion. Imposed deformations occur as a result of concrete shrinkage and creep, changes in temperature and differential settlement. Behaviour of the structure in the event of fire or accidental damage, resulting from impact or explosion, may need to be examined. The conditions of exposure to environmental and chemical attack also need to be considered.

Design includes selecting a suitable form of construction, determining the effects of imposed loads and deformations, and providing members of adequate stiffness and resistance. The members should be arranged so as to combine efficient load transmission with ease of construction, consistent with the intended use of the structure and the nature of the site. Experience and sound judgement are often more important than precise calculations in achieving safe and economical structures. Complex mathematics should not be allowed to confuse a sense of good engineering. The level of accuracy employed in the calculations should be consistent throughout the design process, wherever possible.

Structural design is largely controlled by regulations or codes but, even within such bounds, the designer needs to exercise judgement in interpreting the requirements rather than designing to the minimum allowed by the letter of a clause. In the United Kingdom for many years, the design of reinforced concrete structures has been based on the recommendations of British Standards. For buildings, these include 'Structural use of concrete' (BS 8110: Parts 1, 2 and 3) and 'Loading on buildings' (BS 6399: Parts 1, 2 and 3). For other types of structures, 'Design of concrete bridges' (BS 5400: Part 4) and 'Design of concrete structures for retaining aqueous liquids' (BS 8007) have been used. Compliance with the particular requirements of the Building Regulations and the Highways Agency Standards is also necessary in many cases.

Since the last edition of this *Handbook*, a comprehensive set of harmonised Eurocodes (ECs) for the structural and geotechnical design of buildings and civil engineering works has been developed. The Eurocodes were first introduced as Euronorme Voluntaire (ENV) standards, intended for use in conjunction with a national application document (NAD), as an alternative to national codes for a limited number of years. These have now been largely replaced by Euronorme (EN) versions, with each member state adding a National Annex (NA) containing nationally determined parameters in order to implement the Eurocode as a national standard. The relevant documents for concrete structures are EC 0: Basis of structural design, EC 1: Actions on structures, and EC 2: Design of concrete structures. The last document is in four parts, namely – Part 1.1: General rules and rules for buildings, Part 1.2: Structural fire design, Part 2: Reinforced and prestressed concrete bridges, and Part 3: Liquid-retaining and containing structures.

The tables to be found in Parts 2, 3 and 4 of this *Handbook* enable the designer to reduce the amount of arithmetical work involved in the analysis and design of members to the relevant standards. The use of such tables not only increases speed but also eliminates inaccuracies provided the tables are thoroughly understood, and their applications and limitations are realised. In the appropriate chapters of Part 1 and in the supplementary information given on the pages preceding the tables, the basis of the tabulated material is described. Some general information is also provided. The Appendix contains trigonometrical and other mathematical formulae and data.

1.1 ECONOMICAL STRUCTURES

The cost of construction of a reinforced concrete structure is obviously affected by the prices of concrete, reinforcement, formwork and labour. The most economical proportions of materials and labour will depend on the current relationship between the unit prices. Economy in the use of formwork is generally achieved by uniformity of member size and the avoidance of complex shapes and intersections. In particular cases, the use of available formwork of standard sizes may determine the structural arrangement. In the United Kingdom, speed of construction generally has a major impact on the overall cost. Fast-track construction requires the repetitive use of a rapid formwork system and careful attention to both reinforcement details and concreting methods.

There are also wider aspects of economy, such as whether the anticipated life and use of a proposed structure warrant the use of higher or lower factors of safety than usual, or whether the use of a more expensive form of construction is warranted by improvements in the integrity and appearance of the structure. The application of whole-life costing focuses attention on whether the initial cost of a construction of high quality, with little or no subsequent maintenance, is likely to be more economical than a cheaper construction, combined with the expense of maintenance.

The experience and method of working of the contractor, the position of the site and the nature of the available materials, and even the method of measuring the quantities, together with numerous other points, all have their effect, consciously or not, on the designer's attitude towards a contract. So many and varied are the factors involved that only experience and a continuing study of design trends can give reliable guidance. Attempts to determine the most economical proportions for a particular member based only on inclusive prices of concrete, reinforcement and formwork are likely to be misleading. It is nevertheless possible to lay down certain principles.

In broad terms, the price of concrete increases with the cement content as does the durability and strength. Concrete grades are often determined by durability requirements with different grades used for foundations and superstructures. Strength is an important factor in the design of columns and beams but rarely so in the case of slabs. Nevertheless, the same grade is generally used for all parts of a superstructure, except that higher strength concrete may sometimes be used to reduce the size of heavily loaded columns.

In the United Kingdom, mild steel and high yield reinforcements have been used over the years, but grade 500 is now produced as standard, available in three ductility classes A, B and C. It is always uneconomical in material terms to use compression reinforcement in beams and columns, but the advantages gained by being able to reduce member sizes and maintain the same column size over several storeys generally offset the additional material costs. For equal weights of reinforcement, the combined material and fixing costs of small diameter bars are greater than those of large diameter bars. It is generally sensible to use the largest diameter bars consistent with the requirements for crack control. Fabric (welded mesh) is more expensive than bar reinforcement in material terms, but the saving in fixing time will often result in an overall economy, particularly in slabs and walls.

Formwork is obviously cheaper if surfaces are plane and at right angles to each other and if there is repetition of use. The simplest form of floor construction is a solid slab of constant thickness. Beam and slab construction is more efficient structurally but less economical in formwork costs. Two-way beam systems complicate both formwork and reinforcement details with consequent delay in the construction programme. Increased slab efficiency and economy over longer spans may be obtained by using a ribbed form of construction. Standard types of trough and waffle moulds are available in a range of depths. Precasting usually reduces considerably the amount of formwork, labour and erection time. Individual moulds are more expensive but can be used many more times than site formwork. Structural connections are normally more expensive than with monolithic construction. The economical advantage of precasting and the structural advantage of *in situ* casting may be combined in composite forms of construction.

In many cases, the most economical solution can only be determined by comparing the approximate costs of different designs. This may be necessary to decide, say, when a simple cantilever retaining wall ceases to be more economical than one with counterforts or when a beam and slab bridge is more economical than a voided slab. The handbook *Economic Concrete Frame Elements* published by the British Cement Association on behalf of the Reinforced Concrete Council enables designers to rapidly identify least-cost options for the superstructure of multi-storey buildings.

1.2 DRAWINGS

In most drawing offices a practice has been developed to suit the particular type of work done. Computer aided drafting and reinforcement detailing is widely used. The following observations should be taken as general principles that accord with the recommendations in the manual *Standard method of detailing structural concrete* published by the Institution of Structural Engineers (ref. 1).

It is important to ensure that on all drawings for a particular contract, the same conventions are adopted and uniformity of size and appearance are achieved. In the preliminary stages general arrangement drawings of the whole structure are usually prepared to show the layout and sizes of beams, columns, slabs, walls, foundations and other members. A scale of 1:100 is recommended, although a larger scale may be necessary for complex structures. Later, these or similar drawings, are developed into working drawings and should show precisely such particulars as the setting-out of the structure in relation to any adjacent buildings or other permanent works, and the level of, say the ground floor in relation to a fixed datum. All principal dimensions such as distances between columns and walls, and the overall and intermediate heights should be shown. Plans should generally incorporate a gridline system, with columns positioned at the intersections. Gridlines should be numbered 1, 2, 3 and so on in one direction and lettered A, B, C and so on in the other direction, with the sequences starting at the lower left corner of the grid system. The references can be used to identify individual beams, columns and other members on the reinforcement drawings.

Outline drawings of the members are prepared to suitable scales, such as 1:20 for beams and columns and 1:50 for slabs and walls, with larger scales being used for cross sections. Reinforcement is shown and described in a standard way. The only dimensions normally shown are those needed to position the bars. It is generally preferable for the outline of the concrete to be indicated by a thin line, and to show the reinforcement by bold lines. The lines representing the bars should be shown in the correct positions, with due allowance for covers and the arrangement at intersections and laps, so that the details on the drawing represent as nearly as possible the appearance of the reinforcement as fixed on site. It is important to ensure that the reinforcement does not interfere with the formation of any holes or embedment of any other items in the concrete.

A set of identical bars in a slab, shown on plan, might be described as 20H16-03-150B1. This represents 20 number grade 500 bars of 16 mm nominal size, bar mark 03, spaced at 150 mm centres in the bottom outer layer. The bar mark is a number that uniquely identifies the bar on the drawing and the bar bending schedule. Each different bar on a drawing is given a different bar mark. Each set of bars is described only once on the drawing. The same bars on a cross section would be denoted simply by the bar mark. Bar bending schedules are prepared for each drawing on separate forms according to recommendations in BS 8666 *Specification for scheduling, dimensioning, bending and cutting of steel reinforcement for concrete*.

Chapter 2

Design criteria, safety factors and loads

There are two principal stages in the calculations required to design a reinforced concrete structure. In the first stage, calculations are made to determine the effect on the structure of loads and imposed deformations in terms of applied moments and forces. In the second stage, calculations are made to determine the capacity of the structure to withstand such effects in terms of resistance moments and forces.

Factors of safety are introduced in order to allow for the uncertainties associated with the assumptions made and the values used at each stage. For many years, unfactored loads were used in the first stage and total factors of safety were incorporated in the material stresses used in the second stage. The stresses were intended to ensure both adequate safety and satisfactory performance in service. This simple approach was eventually replaced by a more refined method, in which specific design criteria are set and partial factors of safety are incorporated at each stage of the design process.

2.1 DESIGN CRITERIA AND SAFETY FACTORS

A limit-state design concept is used in British and European Codes of Practice. Ultimate (ULS) and serviceability (SLS) limit states need to be considered as well as durability and, in the case of buildings, fire-resistance. Partial safety factors are incorporated into loads (including imposed deformations) and material strengths to ensure that the probability of failure (not satisfying a design requirement) is acceptably low.

In BS 8110 at the ULS, a structure should be stable under all combinations of dead, imposed and wind load. It should also be robust enough to withstand the effects of accidental loads, due to an unforeseen event such as a collision or explosion, without disproportionate collapse. At the SLS, the effects in normal use of deflection, cracking and vibration should not cause the structure to deteriorate or become unserviceable. A deflection limit of span/250 applies for the total sag of a beam or slab relative to the level of the supports. A further limit, the lesser of span/500 or 20 mm, applies for the deflection that occurs after the application of finishes, cladding and partitions so as to avoid damage to these elements. A limit of 0.3 mm generally applies for the width of a crack at any point on the concrete surface.

In BS 5400, an additional partial safety factor is introduced. This is applied to the load effects and takes account of the method of structural analysis that is used. Also there are more load types and combinations to be considered. At the SLS, there are no specified deflection limits but the cracking limits are more critical. Crack width limits of 0.25, 0.15 or 0.1 mm apply according to surface exposure conditions. Compressive stress limits are also included but in many cases these do not need to be checked. Fatigue considerations require limitations on the reinforcement stress range for unwelded bars and more fundamental analysis if welding is involved. Footbridges are to be analysed to ensure that either the fundamental natural frequency of vibration or the maximum vertical acceleration meets specified requirements.

In BS 8007, water-resistance is a primary design concern. Any cracks that pass through the full thickness of a section are likely to allow some seepage initially, resulting in surface staining and damp patches. Satisfactory performance depends upon autogenous healing of such cracks taking place within a few weeks of first filling in the case of a containment vessel. A crack width limit of 0.2 mm normally applies to all cracks, irrespective of whether or not they pass completely through the section. Where the appearance of a structure is considered to be aesthetically critical, a limit of 0.1 mm is recommended.

There are significant differences between the structural and geotechnical codes in British practice. The approach to the design of foundations in BS 8004 is to use unfactored loads and total factors of safety. For the design of earth-retaining structures, CP2 (ref. 2) used the same approach. In 1994, CP2 was replaced by BS 8002, in which mobilisation factors are introduced into the calculation of soil strengths. The resulting values are then used in BS 8002 for both serviceability and ultimate requirements. In BS 8110, the loads obtained from BS 8002 are multiplied by a partial safety factor at the ULS.

Although the design requirements are essentially the same in the British and European codes, there are differences of terminology and in the values of partial safety factors. In the Eurocodes, loads are replaced by actions with dead loads as permanent actions and all live loads as variable actions. Each variable action is given several representative values to be used for particular purposes. The Eurocodes provide a more unified approach to both structural and geotechnical design.

Details of design requirements and partial safety factors, to be applied to loads and material strengths, are given in Chapter 21 for British Codes, and Chapter 29 for Eurocodes.

2.2 LOADS (ACTIONS)

The loads (actions) acting on a structure generally consist of a combination of dead (permanent) and live (variable) loads.

In limit-state design, a design load (action) is calculated by multiplying the characteristic (or representative) value by an appropriate partial factor of safety. The characteristic value is generally a value specified in a relevant standard or code. In particular circumstances, it may be a value given by a client or determined by a designer in consultation with the client.

In BS 8110 characteristic dead, imposed and wind loads are taken as those defined in and calculated in accordance with BS 6399: Parts 1, 2 and 3. In BS 5400 characteristic dead and live loads are given in Part 2, but these have been superseded in practice by the loads in the appropriate Highways Agency standards. These include BD 37/01 and BD 60/94 and, for the assessment of existing bridges, BD 21/01 (refs. 3–5).

When EC 2: Part 1.1 was first introduced as an ENV document, characteristic loads were taken as the values given in BS 6399 but with the specified wind load reduced by 10%. This was intended to compensate for the partial safety factor applied to wind at the ULS being higher in the Eurocodes than in BS 8110. Representative values were then obtained by multiplying the characteristic values by factors given in the NAD. In the EN documents, the characteristic values of all actions are given in EC 1, and the factors to be used to determine representative values are given in EC 0.

2.3 DEAD LOADS (PERMANENT ACTIONS)

Dead loads include the weights of the structure itself and all permanent fixtures, finishes, surfacing and so on. When permanent partitions are indicated, they should be included as dead loads acting at the appropriate locations. Where any doubt exists as to the permanency of the loads, they should be treated as imposed loads. Dead loads can be calculated from the unit weights given in EC 1: Part 1.1, or from actual known weights of the materials used. Data for calculating dead loads are given in *Tables 2.1* and *2.2*.

2.4 LIVE LOADS (VARIABLE ACTIONS)

Live loads comprise any transient external loads imposed on the structure in normal use due to gravitational, dynamic and environmental effects. They include loads due to occupancy (people, furniture, moveable equipment), traffic (road, rail, pedestrian), retained material (earth, liquids, granular), snow, wind, temperature, ground and water movement, wave action and so on. Careful assessment of actual and probable loads is a very important factor in producing economical and efficient structures. Some imposed loads, like those due to contained liquids, can be determined precisely. Other loads, such as those on floors and bridges are very variable. Snow and wind loads are highly dependent on location. Data for calculating loads from stored materials are given in EC 1: Part 1.1.

2.4.1 Floors

For most buildings the loads imposed on floors are specified in loading standards. In BS 6399: Part 1, loads are specified according to the type of activity or occupancy involved. Data for residential buildings, and for offices and particular work areas, is given in *Table 2.3*. Imposed loads are given both as a uniformly distributed load in kN/m^2 and a concentrated load in kN. The floor should be designed for the worst effects of either load. The concentrated load needs to be considered for isolated short span members and for local effects, such as punching in a thin flange. For this purpose, a square contact area with a 50 mm side may be assumed in the absence of any more specific information. Generally, the concentrated load does not need to be considered in slabs that are either solid, or otherwise capable of effective lateral distribution. Where an allowance has to be made for non-permanent partitions, a uniformly distributed load equal to one-third of the load per metre run of the finished partitions may be used. For offices, the load used should not be less than 1.0 kN/m^2.

The floors of garages are considered in two categories, namely those for cars and light vans and those for heavier vehicles. In the lighter category, the floor may be designed for loads specified in the form described earlier. In the heavier category, the most adverse disposition of loads determined for the specific types of vehicle should be considered.

The total imposed loads to be used for the design of beams may be reduced by a percentage that increases with the area of floor supported as given in *Table 2.3*. This does not apply to loads due to storage, vehicles, plant or machinery. For buildings designed to the Eurocodes, imposed loads are given in EC 1: Part 1.1.

In all buildings it is advisable to affix a notice indicating the imposed load for which the floor is designed. Floors of industrial buildings, where plant and machinery are installed, need to be designed not only for the load when the plant is in running order but also for the probable load during erection and testing which, in some cases, may be more severe. Data for loads imposed on the floors of agricultural buildings by livestock and farm vehicles is given in BS 5502: Part 22.

2.4.2 Structures subject to dynamic loads

The loads specified in BS 6399: Part 1 include allowances for small dynamic effects that should be sufficient for most buildings. However, the loading does not necessarily cover conditions resulting from rhythmical and synchronised crowd movements, or the operation of some types of machinery.

Dynamic loads become significant when crowd movements (e.g. dancing, jumping, rhythmic stamping) are synchronised. In practice, this is usually associated with lively pop concerts or aerobics events where there is a strong musical beat. Such activities can generate both horizontal and vertical loads. If the movement excites a natural frequency of the affected part of the structure, resonance occurs which can greatly amplify the response. Where such activities are likely to occur, the structure should be designed to either avoid any significant resonance effects or withstand the anticipated dynamic loads. Some limited guidance on dynamic loads caused by activities such as jumping and dancing is provided in BS 6399: Part 1, Annexe A. To avoid resonance effects, the natural frequency of vibration of the unloaded structure should be greater than 8.4 Hz for the vertical mode, and greater than 4.0 Hz for the horizontal modes.

Different types of machinery can give rise to a wide range of dynamic loads and the potential resonant excitation of the supporting structure should be considered. Where necessary, specialist advice should be sought.

Footbridges are subject to particular requirements that will be examined separately in the general context of bridges.

2.4.3 Parapets, barriers and balustrades

Parapets, barriers, balustrades and other elements intended to retain, stop or guide people should be designed for horizontal loads. Values are given in BS 6399: Part 1 for a uniformly distributed line load and for both uniformly distributed and concentrated loads applied to the infill. These are not taken together but are applied as three separate load cases. The line load should be considered to act at a height of 1.1 m above a datum level, taken as the finished level of the access platform or the pitch line drawn through the nosing of the stair treads.

Vehicle barriers for car parking areas are also included in BS 6399: Part 1. The horizontal force F, as given in the following equation, is considered to act at bumper height, normal to and uniformly distributed over any length of 1.5 m of the barrier. By the fundamental laws of dynamics:

$$F = 0.5mv^2/(\delta_b + \delta_c) \text{ (in kN)}$$

m = gross mass of vehicle (in kg)
v = speed of vehicle normal to barrier, taken as 4.5 m/sec.
δ_b = deflection of barrier (in mm)
δ_c = deformation of vehicle, taken as 100 mm unless better evidence is available

For car parks designed on the basis that the gross mass of the vehicles using it will not exceed 2500 kg (but taking as a representative value of the vehicle population, m = 1500 kg) and provided with rigid barriers ($\delta_b = 0$), F is taken as 150 kN acting at a height of 375 mm above floor level. It should be noted that bumper heights have been standardised at 445 mm.

2.4.4 Roofs

The imposed loads given in *Table 2.4* are additional to all surfacing materials and include for snow and other incidental loads but exclude wind pressure. The snow load on the roof is determined by multiplying the estimated snow load on the ground at the site location and altitude (the site snow load) by an appropriate snow load shape coefficient. The main loading conditions to be considered are:

(a) a uniformly distributed snow load over the entire roof, likely to occur when snow falls with little or no wind;
(b) a redistributed (or unevenly deposited) snow load, likely to occur in windy conditions.

For flat or mono-pitch roofs, it is sufficient to consider the single load case resulting from a uniform layer of snow, as given in *Table 2.4*. For other roof shapes and for the effects of local drifting of snow behind parapets, reference should be made to BS 6399: Part 3 for further information.

Minimum loads are given for roofs with no access (other than that necessary for cleaning and maintenance) and for roofs where access is provided. Roofs, like floors, should be designed for the worst effects of either the distributed load or the concentrated load. For roofs with access, the minimum load will exceed the snow load in most cases.

If a flat roof is used for purposes such as a café, playground or roof garden, the appropriate imposed load for such a floor should be allowed. For buildings designed to the Eurocodes, snow loads are given in EC 1: Part 1.3.

2.4.5 Columns, walls and foundations

Columns, walls and foundations of buildings are designed for the same loads as the slabs or beams that they support. If the imposed loads on the beams are reduced according to the area of floor supported, the supporting members may be designed for the same reduced loads. Alternatively, where two or more floors are involved and the loads are not due to storage, the imposed loads on columns or other supporting members may be reduced by a percentage that increases with the number of floors supported as given in *Table 2.3*.

2.4.6 Structures supporting cranes

Cranes and other hoisting equipment are often supported on columns in factories or similar buildings. It is important that a dimensioned diagram of the actual crane to be installed is obtained from the makers, to ensure that the right clearances are provided and the actual loads are taken into account. For loads due to cranes, reference should be made to BS 2573.

For jib cranes running on rails on supporting gantries, the load to which the structure is subjected depends on the actual disposition of the weights of the crane. The wheel loads are generally specified by the crane maker and should allow for the static and dynamic effects of lifting, discharging, slewing, travelling and braking. The maximum wheel load under practical conditions may occur when the crane is stationary and hoisting the load at the maximum radius with the line of the jib diagonally over one wheel.

2.4.7 Structures supporting lifts

The effect of acceleration must be considered in addition to the static loads when calculating loads due to lifts and similar machinery. If a net static load F is subject to an acceleration a (m/s^2), the resulting load on the supporting structure is approximately $F\ (1 + 0.098a)$. The average acceleration of a passenger lift may be about 0.6 m/s^2 but the maximum acceleration will be considerably greater. BS 2655 requires the supporting structure to be designed for twice the load suspended from the beams, when the lift is at rest, with an overall factor of safety of 7. The deflection under the design load should not exceed span/1500.

2.4.8 Bridges

The analysis and design of bridges is now so complex that it cannot be adequately treated in a book of this nature, and reference should be made to specialist publications. However, for the guidance of designers, the following notes regarding bridge loading are provided since they may also be applicable to ancillary construction and to structures having features in common with bridges.

Road bridges. The loads to be considered in the design of public road bridges in the United Kingdom are specified in the Highways Agency Standard BD 37/01, *Loads for Highway Bridges*. This is a revised version of BS 5400: Part 2, issued by the Department of Transport rather than by BSI. The Standard includes a series of major amendments as agreed by the BSI Technical Committee. BD 37/01 deals with both permanent loads (dead, superimposed dead, differential settlement, earth pressure), and transient loads due to traffic use (vehicular, pedestrian) and environmental effects (wind, temperature). The collision loads in BD 37/01 may be applicable in certain circumstances, where agreed with the appropriate authority, but in most cases the requirements of BD 60/94, *The design of highway bridges for vehicle collision loads* will apply.

Details of live loads due to traffic, to be considered in the design of highway bridges, are given in *Table 2.5*. Two types of standard live loading are given in BD 37/01, to represent normal traffic and abnormal vehicles respectively. Loads are applied to notional lanes of equal width. The number of notional lanes is determined by the width of the carriageway, which includes traffic lanes, hard shoulders and hard strips, and several typical examples are shown diagrammatically in BD 37/01. Notional lanes are used rather than marked lanes in order to allow for changes of use and the introduction of temporary contra-flow schemes.

Type HA loading covers all the vehicles allowable under the Road Vehicles (Construction and Use) and Road Vehicles (Authorised Weight) Regulations. Values are given in terms of a uniformly distributed load (UDL) and a single knife-edge load (KEL), to be applied in combination to each notional lane. The specified intensity of the UDL (kN/m) reduces as the loaded length increases, which allows for two effects. At the shorter end, it allows for loading in the vicinity of axles or bogies being greater than the average loading for the whole vehicle. At the longer end, it takes account of the reducing percentage of heavy goods vehicles contained in the total vehicle population. The KEL of 120 kN is to be applied at any position within the UDL loaded length, and spread over a length equal to the notional lane width. In determining the loads, consideration has been given to the effects of impact, vehicle overloading and unforeseen changes in traffic patterns. The loading derived after application of separate factors for each of these effects was considered to represent an ultimate load, which was then divided by 1.5 to obtain the specified nominal loads.

The loads are multiplied by lane factors, whose values depend on the particular lane and the loaded length. This is defined as the length of the adverse area of the influence line, that is, the length over which the load application increases the magnitude of the effect to be determined. The lane factors take account of the low probability of all lanes being fully loaded at the same time. They also, for the shorter loaded lengths, allow for the effect of lateral bunching of vehicles. As an alternative to the combined loads, a single wheel load of 100 kN applied at any position is also to be considered.

Type HB loading derives from the nature of exceptional industrial loads, such as electrical transformers, generators, pressure vessels and machine presses, likely to use the roads in the neighbouring area. It is represented by a sixteen-wheel vehicle, consisting of two bogies, each one having two axles with four wheels per axle. Each axle represents one unit of loading (equivalent to 10 kN). Bridges on public highways are designed for a specific number of units of HB loading according to traffic use: typically 45 units for trunk roads and motorways, 37.5 units for principal roads and 30 units for all other public roads. Thus, the maximum number of 45 units corresponds to a total vehicle load of 1800 kN, with 450 kN per axle and 112.5 kN per wheel. The length of the vehicle is variable according to the spacing of the bogies, for which five different values are specified. The HB vehicle can occupy any transverse position on the carriageway and is considered to displace HA loading over a specified area surrounding the vehicle. Outside this area, HA loading is applied as specified and shown by diagrams in BD 37/01. The combined load arrangement is normally critical for all but very long bridges.

Road bridges may be subjected to forces other than those due to dead load and traffic load. These include forces due to wind, temperature, differential settlement and earth pressure. The effects of centrifugal action and longitudinal actions due to traction, braking and skidding must also be considered, as well as vehicle collision loads on supports and superstructure. For details of the loads to be considered on highway bridge parapets, reference should be made to BD 52/93 (ref. 6).

In the assessment of existing highway bridges, traffic loads are specified in the Highways Agency document BD 21/01, *The Assessment of Highway Bridges and Structures*. In this case, the type HA loading is multiplied by a reduction factor that varies according to the road surface characteristics, traffic flow conditions and vehicle weight restrictions. Some of the contingency allowances incorporated in the design loading have also been relaxed. Vehicle weight categories of 40, 38, 25, 17, 7.5 and 3 tonnes are considered, as well as two groups of fire engines. For further information on reduction factors and specific details of the axle weight and spacing values in each category, reference should be made BD 21/01.

Footbridges. Details of live loads due to pedestrians, to be considered in the design of foot/cycle track bridges, are given in *Table 2.6*. A uniformly distributed load of 5 kN/m^2 is specified for loaded lengths up to 36 m. Reduced loads may be used for bridges where the loaded length exceeds 36 m, except that special consideration is required in cases where exceptional crowds could occur. For elements of highway bridges supporting footways/cycle tracks, further reductions may be made in the pedestrian live load where the width is greater than 2 m or the element also supports a carriageway. When the footway/cycle track is not protected from vehicular traffic by an effective barrier, there is a separate requirement to consider an accidental wheel loading.

It is very important that consideration is given to vibration that could be induced in foot/cycle track bridges by resonance with the movement of users, or by deliberate excitation. In BD 37/01, the vibration requirement is deemed to be satisfied in cases where the fundamental natural frequency of vibration exceeds 5 Hz for the unloaded bridge in the vertical direction and 1.5 Hz for the loaded bridge in the horizontal direction. When the fundamental natural frequency of vertical vibration f_0 does not exceed 5 Hz, the maximum vertical acceleration should be limited to $0.5\sqrt{f_0}$ m/s^2. Methods for determining the natural frequency of vibration and the maximum vertical acceleration are given in Appendix B of BD 37/01. Where the fundamental natural frequency of horizontal vibration does not exceed 1.5 Hz, special consideration should be given to the possibility of pedestrian excitation of lateral movements of unacceptable magnitude. Bridges possessing low mass and damping, and expected to be used by crowds of people, are particularly susceptible to such vibrations.

Railway bridges. Details of live loads to be considered in the design of railway bridges are given in *Table 2.6*. Two types of standard loading are given in BD 37/01: type RU for main line railways and type RL for passenger rapid transit systems. A further type SW/0 is also included for main line railways.

The type RU loading was derived by a Committee of the International Union of Railways (UIC) to cover present and anticipated future loading on railways in Great Britain and on the Continent of Europe. Nowadays, motive power tends to be diesel and electric rather than steam, and this produces axle loads and arrangements for locomotives that are similar to those for bogie freight vehicles (these often being heavier than the locomotives that draw them). In addition to normal train loading, which can be represented quite well by a uniformly distributed load of 80 kN/m, railway bridges are occasionally subjected to exceptionally heavy abnormal loads. For short loaded lengths it is necessary to introduce heavier concentrated loads to simulate individual axles and to produce high shears at the ends. Type RU loading consists of four concentrated loads of 250 kN, preceded and followed by a uniformly distributed load of 80 kN/m. For a continuous bridge, type SW/0 loading is also to be considered as an additional and separate load case. This loading consists of two uniformly distributed loads of 133 kN/m, each 15 m long, separated by a distance of 5.3 m. Both types of loading, which are applied to each track or as specified by the relevant authority, with half the track load acting on each rail, are to be multiplied by appropriate dynamic factors to allow for impact, lurching, oscillation and other dynamic effects. The factors have been calculated so that, in combination with the specified loading, they cover the effects of slow moving heavy, and fast moving light, vehicles. Exceptional vehicles are assumed to move at speeds not exceeding 80 km/h, heavy wagons at speeds up to 120 km/h and passenger trains at speeds up to 200 km/h.

The type RL loading was derived by the London Transport Executive to cover present and anticipated future loading on lines that carry only rapid transit passenger trains and light engineers' works trains. Passenger trains include a variety of stock of different ages, loadings and gauges used on surface and tube lines. Works trains include locomotives, cranes and wagons used for maintenance purposes. Locomotives are usually of the battery car type but diesel shunt varieties are sometimes used. The rolling stock could include a 30t steam crane, 6t diesel cranes, 20t hopper cranes and bolster wagons. The heaviest train would comprise loaded hopper wagons hauled by battery cars. Type RL loading consists of a single concentrated load of 200 kN coupled with a uniformly distributed load of 50 kN/m for loaded lengths up to 100 m. For loaded lengths in excess of 100 m, the previous loading is preceded and followed by a distributed load of 25 kN/m. The loads are to be multiplied by appropriate dynamic factors. An alternative bogie loading comprising two concentrated loads, one of 300 kN and the other of 150 kN, spaced 2.4 m apart, is also to be considered on deck structures to check the ability of the deck to distribute the loads adequately.

For full details of the locomotives and rolling stock covered by each loading type, and information on other loads to be considered in the design of railway bridges, due to the effects of nosing, centrifugal action, traction and braking, and in the event of derailment, reference should be made to BD 37/01.

2.4.9 Dispersal of wheel loads

A load from a wheel or similar concentrated load bearing on a small but definite area of the supporting surface (called the contact area) may be assumed to be further dispersed over an area that depends on the combined thickness of any surfacing material, filling and underlying constructional material. The width of the contact area of a wheel on a slab is equal to the width of the tyre. The length of the contact area depends on the type of tyre and the nature of the slab surface. It is nearly zero for steel tyres on steel plate or concrete. The maximum contact length is probably obtained with an iron wheel on loose metalling or a pneumatic tyre on an asphalt surface.

The wheel loads, given in BD 37/01 as part of the standard highway loading, are to be taken as uniformly distributed over a circular or square contact area, assuming an effective pressure of 1.1 N/mm^2. Thus, for the HA single wheel load of 100 kN, the contact area becomes a 340 mm diameter circle or a square of 300 mm side. For the HB vehicle where 1 unit of loading corresponds to 2.5 kN per wheel, the side of the square contact area becomes approximately 260 mm for 30 units, 290 mm for 37.5 units and 320 mm for 45 units.

Dispersal of the load beyond the contact area may be taken at a spread-to-depth ratio of 1 horizontally to 2 vertically for asphalt and similar surfacing, so that the dimensions of the contact area are increased by the thickness of the surfacing. The resulting boundary defines the loaded area to be used when checking, for example, the effects of punching shear on the underlying structure.

For a structural concrete slab, 45^o spread down to the level of the neutral axis may be taken. Since, for the purpose of structural analysis, the position of the neutral axis is usually taken at the mid-depth of the section, the dimensions of the contact area are further increased by the total thickness of the slab. The resulting boundary defines the area of the patch load to be used in the analysis.

The concentrated loads specified in BD 37/01 as part of the railway loading will be distributed both longitudinally by the continuous rails to more than one sleeper, and transversely over a certain area of deck by the sleeper and ballast. It may be assumed that two-thirds of a concentrated load applied to one sleeper will be transmitted to the deck by that sleeper and the remainder will be transmitted equally to the adjacent sleeper on either side. Where the depth of ballast is at least 200 mm, the distribution may be assumed to be half to the sleeper lying under the load and half equally to the adjacent sleeper on either side. The load acting on the sleeper from each rail may be distributed uniformly over the ballast at the level of the underside of the sleeper for a distance taken symmetrically about the centreline of the rail of 800 mm, or twice the distance from the centreline of the rail to the nearer end of the sleeper, whichever is the lesser. Dispersal of the loads applied to the ballast may be taken at an angle of 5^o to the vertical down to the supporting structure. The distribution of concentrated loads applied to a track without ballast will depend on the relative stiffness of the rail, the rail support and the bridge deck itself.

2.5 WIND LOADS

All structures built above ground level are affected by the wind to a greater or lesser extent. Wind comprises a random fluctuating velocity component (turbulence or 'gustiness') superimposed on a steady mean component. The turbulence increases with the roughness of the terrain, due to frictional effects between the wind and features on the ground, such as buildings and vegetation. On the other hand, the frictional effects also reduce the mean wind velocity.

Wind loads are dynamic and fluctuate continuously in both magnitude and position. Some relatively flexible structures, such as tall slender masts, towers and chimneys, suspension bridges and other cable-stayed structures may be susceptible to dynamic excitation, in which case lateral deflections will be an important consideration. However, the vast majority of buildings are sufficiently stiff for the deflections to be small, in which case the structure may be designed as if it was static.

2.5.1 Wind speed and pressure

The local wind climate at any site in the United Kingdom can be predicted reliably using statistical methods in conjunction with boundary-layer wind flow models. However, the complexity of flow around structures is not sufficiently well understood to allow wind pressures or distributions to be determined directly. For this reason, the procedure used in most modern wind codes is to treat the calculation of wind speed in a fully probabilistic manner, whilst continuing to use deterministic values of pressure coefficients. This is the approach adopted in BS 6399: Part 2, which offers a choice of two methods for calculating wind loads as follows:

- **standard method** uses a simplified procedure to obtain an effective wind speed, which is used with standard pressure coefficients for orthogonal load cases,
- **directional method** provides a more precise assessment of effective wind speeds for particular wind directions, which is used with directional pressure coefficients for load cases of any orientation.

The starting point for both methods is the basic hourly-mean wind speed at a height of 10 m in standard 'country' terrain, having an annual risk (probability) of being exceeded of 0.02 (i.e. a mean recurrence interval of 50 years). A map of basic wind speeds covering Great Britain and Ireland is provided.

The basic hourly-mean wind speed is corrected according to the site altitude and, if required, the wind direction, season and probability to obtain an effective site wind speed. This is further modified by a site terrain and building height factor to obtain an effective gust wind speed V_e m/s, which is used to calculate an appropriate dynamic pressure $q = 0.613V_e^2$ N/m^2.

Topographic effects are incorporated in the altitude factor for the standard method, and in the terrain and building factor for the directional method. The standard method can be used in hand-based calculations and gives a generally conservative result within its range of applicability. The directional method is less conservative and is not limited to orthogonal design cases. The loading is assessed in more detail, but with the penalty of increased complexity and computational effort. For further details of the directional method, reference should be made to BS 6399: Part 2.

2.5.2 Buildings

The **standard method** of BS 6399: Part 2 is the source of the information in *Tables 2.7–2.9*. The basic wind speed and the correction factors are given in *Table 2.7*. The altitude factor depends on the location of the structure in relation to the local topography. In terrain with upwind slopes exceeding 0.05, the effects of topography are taken to be significant for certain designated zones of the upwind and downwind slopes. In this case, further reference should be made to BS 6399: Part 2. When the orientation of the building is known, the wind speed may be adjusted according to the direction under consideration. Where the building height is greater than the crosswind breadth for the direction being considered, a reduction in the lateral load may be obtained by dividing the building into a number of parts. For buildings in town terrain, the effective height may be reduced as a result of the shelter afforded by structures upwind of the site. For details of the adjustments based on wind direction, division of buildings into parts and the influence of shelter on effective height, reference should be made to BS 6399: Part 2.

When the wind acts on a building, the windward faces are subjected to direct positive pressure, the magnitude of which cannot exceed the available kinetic energy of the wind. As the wind is deflected around the sides and over the roof of the building it is accelerated, lowering the pressure locally on the building surface, especially just downwind of the eaves, ridge and corners. These local areas, where the acceleration of the flow is greatest, can experience very large wind suctions. The surfaces of enclosed buildings are also subjected to internal pressures. Values for both external and internal pressures are obtained by multiplying the dynamic pressure by appropriate pressure coefficients and size effect factors. The overall force on a rectangular building is determined from the normal forces on the windward-facing and leeward-facing surfaces, the frictional drag forces on surfaces parallel to the direction of the wind, and a dynamic augmentation factor that depends on the building height and type.

Details of the dimensions used to define surface pressures and forces, and values for dynamic augmentation factors and frictional drag coefficients are given in *Table 2.8*. Size effect factors, and external and internal pressure coefficients for the walls of rectangular buildings, are given in *Table 2.9*. Further information, including pressure coefficients for various roof forms, free-standing walls and cylindrical structures such as silos, tanks and chimneys, and procedures for more-complex building shapes, are given in BS 6399: Part 2. For buildings designed to the Eurocodes, data for wind loading is given in EC 1: Part 1.2.

2.5.3 Bridges

The approach used for calculating wind loads in BD 37/01 is a hybrid mix of the methods given in BS 6399: Part 2. The directional method is used to calculate the effective wind speed, as this gives a better estimate of wind speeds in towns and for sites affected by topography. In determining the wind speed, the probability factor is taken as 1.05, appropriate to a return period of 120 years. Directional effective wind speeds are derived for orthogonal load cases, and used with standard drag coefficients to obtain wind loads on different elements of the structure, such as decks, parapets and piers. For details of the procedures, reference must be made to BD 37/01.

2.6 MARITIME STRUCTURES

The forces acting upon sea walls, dolphins, wharves, jetties, piers, docks and similar maritime structures include those due to winds and waves, blows and pulls from vessels, the loads from cranes, roads, railways and stored goods imposed on the deck, and the pressures of earth retained behind the structure.

For wharves or jetties of solid construction, the energy of impact due to blows from vessels berthing is absorbed by the mass of the structure, usually without damage to the structure or vessel if fendering is provided. With open construction, consisting of braced piles or piers supporting the deck, in which the mass of the structure is comparatively small, the forces resulting from impact must be considered. The forces depend on the weight and speed of approach of the vessel, on the amount of fendering and on the flexibility of the structure. In general, a large vessel will approach at a low speed and a small vessel at a higher speed. Some typical examples are a 1000 tonne vessel at 0.3 m/s, a 10 000 tonne vessel at 0.2 m/s and a 100 000 tonne vessel at 0.15 m/s. The kinetic energy of a vessel displacing F tonnes approaching at a speed V m/s is equal to $0.514FV^2$ kNm. Hence, the kinetic energy of a 2000 tonne vessel at 0.3 m/s, and a 5000 tonne vessel at 0.2 m/s, is about 100 kNm in each case. If the direction of approach of a vessel is normal to the face of a jetty, the whole of this energy must be absorbed on impact. More commonly, a vessel approaches at an angle with the face of the jetty and touches first at one point, about which the vessel swings. The energy then to be absorbed is $0.514F[(V\sin\theta)^2 - (\rho\omega)^2]$, with θ the angle of approach of the vessel with the face of the jetty, ρ the radius of gyration (m) of the vessel about the point of impact and ω the angular velocity (radians/s) of the vessel about the point of impact. The numerical values of the terms in the expression are difficult to assess accurately, and can vary considerably under different conditions of tide and wind and with different vessels and methods of berthing.

The kinetic energy of approach is absorbed partly by the resistance of the water, but mainly by the fendering, elastic deformation of the structure and the vessel, movement of the ground and also by energy 'lost' upon impact. The relative contributions are difficult to assess but only about half of the total kinetic energy of the vessel may be imparted to the structure and the fendering. The force to which the structure is subjected is calculated by equating the product of the force and half the elastic horizontal displacement of the structure to the kinetic energy imparted. Ordinary timber fenders applied to reinforced concrete jetties cushion the blow, but may not substantially reduce the force on the structure. Spring fenders or suspended fenders can, however, absorb a large proportion of the kinetic energy. Timber fenders independent of the jetty are sometimes provided to protect the structure from impact.

The combined action of wind, waves, currents and tides on a vessel moored to a jetty is usually transmitted by the vessel pressing directly against the side of the structure or by pulls on mooring ropes secured to bollards. The pulls on bollards due to the foregoing causes or during berthing vary with the size of the vessel. For vessels of up to 20 000 tonnes loaded displacement, bollards are required at intervals of 15–30 m with load capacities, according to the vessel displacement, of 100 kN up to 2000 tonnes, 300 kN up to 10 000 tonnes and 600 kN up to 20 000 tonnes.

The effects of wind and waves acting on a marine structure are much reduced if an open construction is adopted and if provision is made for the relief of pressures due to water and air trapped below the deck. The force is not, however, related directly to the proportion of solid vertical face presented to the action of the wind and waves. The pressures imposed are impossible to assess with accuracy, except for sea walls and similar structures where the depth of water at the face of the wall is such that breaking waves do not occur. In this case, the force is due to simple hydrostatic pressure and can be evaluated for the highest anticipated wave level, with appropriate allowance for wind surge. In the Thames estuary, for example, the latter can raise the high-tide level to 1.5 m above normal.

A wave breaking against a sea wall causes a shock pressure additional to the hydrostatic pressure, which reaches its peak value at about mean water level and diminishes rapidly below this level and more slowly above it. The shock pressure can be as much as 10 times the hydrostatic value and pressures up to 650 kN/m^2 are possible with waves 4.5–6 m high. The shape of the face of the wall, the slope of the foreshore, and the depth of water at the wall affect the maximum pressure and the distribution of the pressure. For information on the loads to be considered in the design of all types of maritime structures, reference should be made to BS 6349: Parts 1 to 7.

2.7 RETAINED AND CONTAINED MATERIALS

The pressures imposed by materials on retaining structures or containment vessels are uncertain, except when the retained or contained material is a liquid. In this case, at any depth z below the free surface of the liquid, the intensity of pressure normal to the contact surface is equal to the vertical pressure, given by the simple hydrostatic expression $\sigma_z = \gamma_w z$, where γ_w is unit weight of liquid (e.g. 9.81 kN/m^3 for water). For soils and stored granular materials, the pressures are considerably influenced by the effective shear strength of the material.

2.7.1 Properties of soils

For simplicity of analysis, it is conventional to express the shear strength of a soil by the equation

$$\tau = c' + \sigma'_n \tan\varphi'$$

where $c'$is effective cohesion of soil, φ' is effective angle of shearing resistance of soil, σ'_n is effective normal pressure.

Values of $c'$and φ' are not intrinsic soil properties and can only be assumed constant within the stress range for which they have been evaluated. For recommended fill materials, it is generally sufficient to adopt a soil model with $c' = 0$. Such a model gives a conservative estimate of the shear strength of the soil and is analytically simple to apply in design. Data taken from BS 8002 is given in *Table 2.10* for unit weights of soils and effective angles of shearing resistance.

2.7.2 Lateral soil pressures

The lateral pressure exerted by a soil on a retaining structure depends on the initial state of stress and the subsequent strain within the soil. Where there has been no lateral strain, either because the soil has not been disturbed during construction, or the soil has been prevented from lateral movement during placement, an at-rest state of equilibrium exists. Additional lateral strain is needed to change the initial stress conditions. Depending on the magnitude of the strain involved, the final state of stress in the soil mass can be anywhere between the two failure conditions, known as the active and passive states of plastic equilibrium.

The problem of determining lateral pressures at the limiting equilibrium conditions has been approached in different ways by different investigators. In Coulomb theory, the force acting on a retaining wall is determined by considering the limiting equilibrium of a soil wedge bounded by the rear face of the wall, the ground surface and a planar failure surface. Shearing resistance is assumed to have been mobilised both on the back of the wall and on the failure surface. Rankine theory gives the complete state of stress in a cohesionless soil mass, which is assumed to have expanded or compressed to a state of plastic equilibrium. The stress conditions require that the earth pressure on a vertical plane should act in a direction parallel to the ground surface. Caquot and Kerisel produced tables of earth pressure coefficients derived by a method that directly integrates the equilibrium equations along combined planar and logarithmic spiral failure surfaces.

2.7.3 Fill materials

A wide range of fill materials may be used behind retaining walls. All materials should be properly investigated and classified. Industrial, chemical and domestic waste; shale, mudstone and steel slag; peaty or highly organic soil should not be used as fill. Selected cohesionless granular materials placed in a controlled manner such as well-graded small rock-fills, gravels and sands, are particularly suitable. The use of cohesive soils can result in significant economies by avoiding the need to import granular materials, but may also involve additional problems during design and construction. The cohesive soil should be within a range suitable for adequate compaction. The placement moisture content should be close to the final equilibrium value, to avoid either the swelling of clays placed too dry or the consolidation of clays placed too wet. Such problems will be minimised if the fill is limited to clays with a liquid limit not exceeding 45% and a plasticity index not exceeding 25%. Chalk with a saturation moisture content not exceeding 20% is acceptable as fill, and may be compacted as for a well-graded granular material. Conditioned pulverized fuel ash (PFA) from a single source may also be used: it should be supplied at a moisture-content of 80–100% of the optimum value. For further guidance on the suitability of fill materials, reference should be made to relevant Transport Research Laboratory publications, DoT Standard BD 30/87 (ref. 7) and BS 8002.

2.7.4 Pressures imposed by cohesionless soils

Earth pressure distributions on unyielding walls, and on rigid walls free to translate or rotate about the base, are shown in *Table 2.11*. For a normally consolidated soil, the pressure on the wall increases linearly with depth. Compaction results in higher earth pressures in the upper layers of the soil mass.

Expressions for the pressures imposed in the at-rest, active and passive states, including the effects of uniform surcharge and static ground water, are given in sections 9.1.1–9.1.4. Charts of earth pressure coefficients, based on the work of Caquot and Kerisel (ref. 8), are given in *Tables 2.12–2.14*. These may be used generally for vertical walls with sloping ground or inclined walls with level ground.

2.7.5 Cohesive soils

Clays, in the long term, behave as granular soils exhibiting friction and dilation. If a secant φ' value ($c' = 0$) is used, the procedures for cohesionless soils apply. If tangent parameters (c', φ') are used, the Rankine–Bell equations apply, as given in section 9.1.5. In the short term, if a clay soil is subjected to rapid shearing, a total stress analysis should be undertaken using the undrained shear strength (see BS 8002).

2.7.6 Further considerations

For considerations such as earth pressures on embedded walls (with or without props), the effects of vertical concentrated loads and line loads, and the effects of groundwater seepage, reference should be made to specialist books and BS 8002. For the pressures to be considered in the design of integral bridge abutments, as a result of thermal movements of the deck, reference should be made to the Highways Agency document BA 42/96 (ref. 9).

2.7.7 Silos

Silos, which may also be referred to as bunkers or bins, are deep containers used to store particulate materials. In a deep container, the linear increase of pressure with depth, found in shallow containers, is modified. When a deep container is filled, a slight settlement of the fill activates the frictional resistance between the stored material and the wall. This induces vertical load in the silo wall but reduces the vertical pressure in the material and the lateral pressures on the wall. Janssen developed a theory by which expressions have been derived for the pressures on the walls of a silo containing a granular material having uniform properties. The ratio of horizontal to vertical pressure in the fill is assumed constant, and a Rankine coefficient is generally used. Eccentric filling (or discharge) tends to produce variations in lateral pressure round the silo wall. An allowance is made by considering additional patch loads taken to act on any part of the wall.

Unloading a silo disturbs the equilibrium of the contained mass. If the silo is unloaded from the top, the frictional load on the wall may be reversed as the mass re-expands, but the lateral pressures remain similar to those during filling. With a free-flowing material unloading at the bottom of the silo from the centre of a hopper, two different flow patterns are possible, depending on the characteristics of the hopper and the material. These patterns are termed funnel flow (or core flow) and mass flow respectively. In the former, a channel of flowing material develops within a confined zone above the outlet, the material adjacent to the wall near the outlet remaining stationary. The flow channel can intersect the vertical walled section of the silo or extend to the surface of the stored material. In mass flow, which occurs particularly in steep-sided hoppers, all the stored material is mobilised during discharge. Such flow can develop at varying levels within the mass of material contained in *any* tall silo owing to the formation of a ‘self-hopper’, with high local pressures arising where parallel flow starts to diverge from the walls. Both flow patterns give rise to increases in lateral pressure from the stable, filled condition. Mass flow results also in a substantial local kick load at the intersection of the hopper and the vertical walled section.

When calculating pressures, care should be taken to allow for the inherent variability of the material properties. In general, concrete silo design is not sensitive to vertical wall load, so values of maximum unit weight in conjunction with maximum or minimum consistent coefficients of friction should be used. Data taken from EC 1: Part 4 for the properties of stored materials, and the pressures on the walls and bottoms of silos, are given in *Tables 2.15* and *2.16*.

Fine powders like cement and flour can become fluidised in silos, either owing to rapid filling or through aeration to facilitate discharge. In such cases, the design should allow for both non-fluidised and fluidised conditions.

2.8 EUROCODE LOADING STANDARDS

Eurocode 1: *Actions on Structures* is one of nine international unified codes of practice that have been published by the European Committee for Standardization (CEN). The code, which contains comprehensive information on all the actions (loads) normally necessary for consideration in the design of building and civil engineering structures, consists of ten parts as follows:

1991-1-1 Densities, self-weight and imposed loads
1991-1-2 Actions on structures exposed to fire
1991-1-3 Snow loads
1991-1-4 Wind loads
1991-1-5 Thermal actions
1991-1-6 Actions during execution
1991-1-7 Accidental actions due to impact and explosions
1991-2 Traffic loads on bridges
1991-3 Actions induced by cranes and machinery
1991-4 Actions on silos and tanks

Chapter 3

Material properties

The requirements of concrete and its constituent materials, and of reinforcement, are specified in Regulations, Standards and Codes of Practice. Only those properties that concern the designer directly, because they influence the behaviour and durability of the structure, are dealt with in this chapter.

3.1 CONCRETE

Concrete is a structural material composed of crushed rock, or gravel, and sand, bound together with a hardened paste of cement and water. A large range of cements and aggregates, chemical admixtures and additions, can be used to produce a range of concretes having the required properties in both the fresh and hardened states, for many different structural applications. The following information is taken mainly from ref. 10, where a fuller treatment of the subject will be found.

3.1.1 Cements and combinations

Portland cements are made from limestone and clay, or other chemically similar suitable raw materials, which are burned together in a rotary kiln to form a clinker rich in calcium silicates. This clinker is ground to a fine powder with a small proportion of gypsum (calcium sulphate), which regulates the rate of setting when the cement is mixed with water. Over the years several types of Portland cement have been developed.

As well as cement for general use (which used to be known as ordinary Portland cement), cements for rapid hardening, for protection against attack by freezing and thawing, or by chemicals, and white cement for architectural finishes are also made. The cements contain the same active compounds, but in different proportions. By incorporating other materials during manufacture, an even wider range of cements is made, including air-entraining cement and combinations of Portland cement with mineral additions. Materials, other than those in Portland cements, are used in cements for special purposes: for example, calcium aluminate cement is used for refractory concrete.

The setting and hardening process that occurs when cement is mixed with water, results from a chemical reaction known as hydration. The process produces heat and is irreversible. Setting is the gradual stiffening whereby the cement paste changes from a workable to a hardened state. Subsequently, the strength of the hardened mass increases, rapidly at first but slowing gradually. This gain of strength continues as long as moisture is present to maintain the chemical reaction.

Portland cements can be either inter-ground or blended with mineral materials at the cement factory, or combined with additions in the concrete mixer. The most frequently used of these additional materials in the United Kingdom, and the relevant British Standards, are pulverized-fuel ash (pfa) to BS 3892, fly ash to BS EN 450, ground granulated blastfurnace slag (ggbs) to BS 6699 and limestone fines to BS 7979. Other additions include condensed silica fume and metakaolin. These are intended for specialised uses of concrete beyond the scope of this book.

The inclusion of pfa, fly ash and ggbs has been particularly useful in massive concrete sections, where they have been used primarily to reduce the temperature rise of the concrete, with corresponding reductions in temperature differentials and peak temperatures. The risk of early thermal contraction cracking is thereby also reduced. The use of these additional materials is also one of the options available for minimising the risk of damage due to alkali–silica reaction, which can occur with some aggregates, and for increasing the resistance of concrete to sulfate attack. Most additions react slowly at early stages under normal temperatures, and at low temperature the reaction, particularly in the case of ggbs, can become considerably retarded and make little contribution to the early strength of concrete. However, provided the concrete is not allowed to dry out, the use of such additions can increase the long-term strength and impermeability of the concrete.

When the terms ‘water-cement ratio’ and ‘cement content’ are used in British Standards, these are understood to include combinations. The word ‘binder’, which is sometimes used, is interchangeable with the word ‘cement’ or ‘combination’.

The two methods of incorporating mineral additions make little or no difference to the properties of the concrete, but the recently introduced notation system includes a unique code that identifies both composition and production method. The types of cement and combinations in most common usage are shown with their notation in *Table 2.17*.

Portland cement. The most commonly used cement was known formerly as OPC in British Standards. By grinding the cement clinker more finely, cement with a more rapid early strength development is produced, known formerly as RHPC. Both types are now designated as:

- Portland cement CEM I, conforming to BS EN 197-1

Cements are now classified in terms of both their standard strength, derived from their performance at 28 days, and at an early age, normally two days, using a specific laboratory test based on a standard mortar prism. This is termed the strength class: for example CEM I 42,5N, where 42,5 (N/mm^2) is the standard strength and N indicates a normal early strength.

The most common standard strength classes for cements are 42,5 and 52,5. These can take either N (normal) or R (rapid) identifiers, depending on the early strength characteristics of the product. CEM I in bags is generally a 42,5N cement, whereas CEM I for bulk supply tends to be 42,5R or 52,5 N. Cement corresponding to the former RHPC is now produced in the United Kingdom within the 52,5 strength class. These cements are often used to advantage by precast concrete manufacturers to achieve a more rapid turn round of moulds, and on site when it is required to reduce the time for which the formwork must remain in position. The cements, which generates more early heat than CEM I 42,5N, can also be useful in cold weather conditions.

It is worth noting that the specified setting times of cement pastes relate to the performance of a cement paste of standard consistence in a particular test made under closely controlled conditions of temperature and humidity; the stiffening and setting of concrete on site are not directly related to these standard setting regimes, and are more dependent on factors such as workability, cement content, use of admixtures, the temperature of the concrete and the ambient conditions.

Sulfate-resisting Portland cement SRPC. This is a Portland cement with a low tricalcium aluminate (C_3A) content, for which the British Standard is BS 4027. When concrete made with CEM I cement is exposed to the sulfate solutions that are found in some soils and groundwaters, a reaction can occur between the sulfate and the hydrates from the C_3A in the cement, causing deterioration of the concrete. By limiting the C_3A content in SRPC, cement with a superior resistance to sulfate attack is obtained. SRPC normally has a low-alkali content, but otherwise it is similar to other Portland cements in being non-resistant to strong acids. The strength properties of SRPC are similar to those of CEM I 42,5N but slightly less early heat is normally produced. This can be an advantage in massive concrete and in thick sections. SRPC is not normally used in combination with pfa or ggbs.

Blastfurnace slag cements. These are cements incorporating ggbs, which is a by-product of iron smelting, obtained by quenching selected molten slag to form granules. The slag can be inter-ground or blended with Portland cement clinker at certain cement works to produce:

- Portland-slag cement CEM II/A-S, with a slag content of 6–35% conformimg to BS EN 197-1, or more commonly
- Blastfurnace cement CEM III/A, with a slag content of 36–65% conforming to BS EN 197-1.

Alternatively, the granules may be ground down separately to a white powder with a fineness similar to that of cement, and then combined in the concrete mixer with CEM I cement to produce a blastfurnace cement. Typical mixer combinations of 40–50% ggbs with CEM I cement have a notation CIIIA and, at this level of addition, 28-day strengths are similar to those obtained with CEM I 42,5N.

As ggbs has little hydraulic activity of its own, it is referred to as ‘a latent hydraulic binder’. Cements incorporating ggbs generate less heat and gain strength more slowly, with lower early age strengths than those obtained with CEM I cement. The aforementioned blastfurnace cements can be used instead of CEM I cement but, because the early strength development is slower, particularly in cold weather, it may not be suitable where early removal of formwork is required. They are a moderately, low-heat cement and can, therefore, be used to advantage to reduce early heat of hydration in thick sections. When the proportion of ggbs is 66–80%, CEM III/A and CIIIA become CEM III/B and CIIIB respectively. These were known formerly as high-slag blastfurnace cements, and are specified because of their lower heat characteristics, or to impart resistance to sulfate attack.

Because the reaction between ggbs and lime released by the Portland cement is dependent on the availability of moisture, extra care has to be taken in curing concrete containing these cements or combinations, to prevent premature drying out and to permit the development of strength.

Pulverized-fuel ash and fly ash cements. The ash resulting from the burning of pulverized coal in power station furnaces is known in the concrete sector as pfa or fly ash. The ash, which is fine enough to be carried away in the flue gases, is removed from the gases by electrostatic precipitators to prevent atmospheric pollution. The resulting material is a fine powder of glassy spheres that can have pozzolanic properties: that is, when mixed into concrete, it can react chemically with the lime that is released during the hydration of Portland cement. The products of this reaction are cementitious, and in certain circumstances pfa or fly ash can be used as a replacement for part of the Portland cement provided in the concrete.

The required properties of ash to be used as a cementitious component in concrete are specified in BS EN 450, with additional UK provisions for pfa made in BS 3892: Part 1. Fly ash, in the context of BS EN 450 means ‘coal fly ash’ rather than ash produced from other combustible materials, and fly ash conforming to BS EN 450 can be coarser than that conforming to BS 3892: Part 1.

Substitution of these types of cement for Portland cement is not a straightforward replacement of like for like, and the following points have to be borne in mind when considering the use of pfa concrete:

- Pfa reacts more slowly than Portland cement. At early age and particularly at low temperatures, pfa contributes less strength: in order to achieve the same 28-day compressive strength, the amount of cementitious material may need to be increased, typically by about 10%. The potential strength after three months is likely to be greater than CEM I provided the concrete is kept in a moist environment, for example, in underwater structures or concrete in the ground.
- The water demand of pfa for equal consistence may be less than that of Portland cement.
- The density of pfa is about three-quarters that of Portland cement.
- The reactivity of pfa and its effect on water demand, and hence strength, depend on the particular pfa and Portland cement with which it is used. A change in the source of either material may result in a change in the replacement level required.

- When pfa is to be air-entrained, the admixture dosage rate may have to be increased, or a different formulation that produces a more stable air bubble structure used.

Portland-fly ash cement comprises, in effect, a mixture of CEM I and pfa. When the ash is inter-ground or blended with Portland cement clinker at an addition rate of 20–35%, the manufactured cement is known as Portland-fly ash cement CEM II/B-V conforming to BS EN 197-1. When this combination is produced in a concrete mixer, it has the notation CIIB-V conforming to BS 8500: Part 2.

Typical ash proportions are 25–30%, and these cements can be used in concrete for most purposes. They are likely to have a slower rate of strength development compared with CEM I. When the cement contains 25–40% ash, it may be used to impart resistance to sulfate attack and can also be beneficial in reducing the harmful effects of alkali–silica reaction. Where higher replacement levels of ash are used for improved low-heat characteristics, the resulting product is pozzolanic (pfa) cement with the notation, if manufactured, CEM IV/B-V conforming to BS EN 197-1 or, if combined in the concrete mixer, CIVB-V conforming to BS 8500: Part 2.

Because the pozzolanic reaction between pfa or fly ash and free lime is dependent on the availability of moisture, extra care has to be taken in curing concrete containing mineral additions, to prevent premature drying out and to permit the development of strength.

Portland-limestone cement. Portland cement incorporating 6–35% of carefully selected fine limestone powder is known as Portland-limestone cement conforming to BS EN 197-1. When a 42,5N product is manufactured, the typical limestone proportion is 10–20%, and the notation is CEM II/A-L or CEM II/A-LL. It is most popular in continental Europe but its usage is growing in the United Kingdom. Decorative precast and reconstituted stone concretes benefit from its lighter colouring, and it is also used for general-purpose concrete in non-aggressive and moderately aggressive environments.

3.1.2 Aggregates

The term ‘aggregate’ is used to describe the gravels, crushed rocks and sands that are mixed with cement and water to produce concrete. As aggregates form the bulk of the volume of concrete and can significantly affect its performance, the selection of suitable material is extremely important. Fine aggregates include natural sand, crushed rock or crushed gravel that is fine enough to pass through a sieve with 4 mm apertures (formerly 5 mm, as specified in BS 882). Coarse aggregates comprise larger particles of gravel, crushed gravel or crushed rock. Most concrete is produced from natural aggregates that are specified to conform to the requirements of BS EN 12620, together with the UK Guidance Document PD 6682-1. Manufactured lightweight aggregates are also sometimes used.

Aggregates should be hard and should not contain materials that are likely to decompose, or undergo volumetric changes, when exposed to the weather. Some examples of undesirable materials are lignite, coal, pyrite and lumps of clay. Coal and lignite may swell and decompose, leaving small holes on the surface of the concrete; lumps of clay may soften and form weak pockets; and pyrite may decompose, causing iron oxide stains to appear on the concrete surface. When exposed to oxygen, pyrite has been known to contribute to sulfate attack. High-strength concretes may call for special properties. The mechanical properties of aggregates for heavy-duty concrete floors and for pavement wearing surfaces may have to be specially selected. Most producers of aggregate are able to provide information about these properties, and reference, when necessary, should be made to BS EN 12620.

There are no simple tests for aggregate durability or their resistance to freeze/thaw exposure conditions, and assessment of particular aggregates is best based on experience of the properties of concrete made with the type of aggregate, and knowledge of its source. Some flint gravels with a white porous cortex may be frost-susceptible because of the high water absorption of the cortex, resulting in pop-outs on the surface of the concrete when subjected to freeze/thaw cycles.

Aggregates must be clean and free from organic impurities. The particles should be free from coatings of dust or clay, as these prevent proper bonding of the material. An excessive amount of fine dust or stone ‘flour’ can prevent the particles of stone from being properly coated with cement, and lower the strength of the concrete. Gravels and sands are usually washed by the suppliers to remove excess fines (e.g. clay and silt) and other impurities, which otherwise could result in a poor-quality concrete. However, too much washing can also remove all fine material passing the 0.25 mm sieve. This may result in a concrete mix lacking in cohesion and, in particular, one that is unsuitable for placing by pump. Sands deficient in fines also tend to increase the bleeding characteristics of the concrete, leading to poor vertical finishes due to water scour.

Where the colour of a concrete surface finish is important, supplies of aggregate should be obtained from the one source throughout the job whenever practicable. This is particularly important for the sand – and for the coarse aggregate when an exposed-aggregate finish is required.

Size and grading. The maximum size of coarse aggregate to be used is dependent on the type of work to be done. For reinforced concrete, it should be such that the concrete can be placed without difficulty, surrounding all the reinforcement thoroughly, and filling the corners of the formwork. In the United Kingdom, it is usual for the coarse aggregate to have a maximum size of 20 mm. Smaller aggregate, usually with a maximum size of 10 mm, may be needed for concrete that is to be placed through congested reinforcement, and in thin sections with small covers. In this case the cement content may have to be increased by 10–20% to achieve the same strength and workability as that obtained with a 20 mm maximum-sized aggregate, because both sand and water contents usually have to be increased to produce a cohesive mix. Larger aggregate, with a maximum size of 40 mm, can be used for foundations and mass concrete, where there are no restrictions to the flow of the concrete. It should be noted, however, that this sort of concrete is not always available from ready-mixed concrete producers. The use of a larger aggregate results in a slightly reduced water demand, and hence a slightly reduced cement content for a given strength and workability.

The proportions of the different sizes of particles making up the aggregate, which are found by sieving, are known as the aggregate ‘grading’. The grading is given in terms of the percentage by mass passing the various sieves. Continuously graded aggregates for concrete contain particles ranging in size from the largest to the smallest; in gap-graded aggregates some of the intermediate sizes are absent. Gap grading may be necessary to achieve certain surface finishes. Sieves used for making a sieve analysis should conform to BS EN 933-2. Recommended sieve sizes typically range from 80 to 2 mm for coarse aggregates and from 8 to 0.25 mm for fine aggregates. Tests should be carried out in accordance with the procedure given in BS EN 933-1.

An aggregate containing a high proportion of large particles is referred to as being 'coarsely' graded, and one containing a high proportion of small particles as 'finely' graded. Overall grading limits for coarse, fine and 'all-in' aggregates are contained in BS EN 12620 and PD 6682-1. All-in aggregates, comprising both coarse and fine materials, should not be used for structural reinforced concrete work, because the grading will vary considerably from time to time, and hence from batch to batch, thus resulting in excessive variation in the consistence and the strength of the concrete. To ensure that the proper amount of sand is present, the separate delivery, storage and batching of coarse and fine materials is essential. Graded coarse aggregates that have been produced by layer loading (i.e. filling a lorry with, say, two grabs of material size 10–20 mm and one grab of material size 4–10 mm) are seldom satisfactory because the unmixed materials will not be uniformly graded. The producer should ensure that such aggregates are effectively mixed before loading into lorries.

For a high degree of control over concrete production, and particularly if high-quality surface finishes are required, it is necessary for the coarse aggregate to be delivered, stored and batched using separate single sizes.

The overall grading limits for coarse and fine aggregates, as recommended in BS EN 12620, are given in *Table 2.17*. The limits vary according to the aggregate size indicated as *d/D*, in millimetres, where *d* is the lower limiting sieve size and *D* is the upper limiting sieve size, for example, 4/20. Additionally, the coarseness/fineness of the fine aggregate is assessed against the percentage passing the 0.5 mm sieve to give a *CP*, *MP*, *FP* grading. This compares with the C (coarse), M (medium), F (fine) grading used formerly in BS 882. Good concrete can be made using sand within the overall limits but there may be occasions, such as where a high degree of control is required, or a high-quality surface finish is to be achieved, when it is necessary to specify the grading to even closer limits. On the other hand, sand whose grading falls outside the overall limits may still produce perfectly satisfactory concrete. Maintaining a reasonably uniform grading is generally more important than the grading limits themselves.

Marine-dredged aggregates. Large quantities of aggregates, obtained by dredging marine deposits, have been widely and satisfactorily used for making concrete for many years. If present in sufficient quantities, hollow and/or flat shells can affect the properties of both fresh and hardened concrete, and two categories for shell content are given in BS EN 12620. In order to reduce the corrosion risk of embedded metal, limits for the chloride content of concrete are given in BS EN 206-1 and BS 8500. To conform to these limits, it is necessary for marine-dredged aggregates to be carefully and efficiently washed in fresh water that is frequently changed, in order to reduce the salt content. Chloride contents should be checked frequently throughout aggregate production in accordance with the method given in BS EN 1744-1.

Some sea-dredged sands tend to have a preponderance of one size of particle, and a deficiency in the amount passing the 0.25 mm sieve. This can lead to mixes prone to bleeding, unless mix proportions are adjusted to overcome the problem. Increasing the cement content by 5–10% can often offset the lack of fine particles in the sand. Beach sands are generally unsuitable for good-quality concrete, since they are likely to have high concentrations of chloride due to the accumulation of salt crystals above the high-tide mark. They are also often single-sized, which can make the mix design difficult.

Lightweight aggregates. In addition to natural gravels and crushed rocks, a number of manufactured aggregates are also available for use in concrete. Aggregates such as sintered pfa are required to conform to BS EN 13055-1 and PD 6682-4.

Lightweight aggregate has been used in concrete for many years – the Romans used pumice in some of their construction work. Small quantities of pumice are imported and still used in the United Kingdom, mainly in lightweight concrete blocks, but most lightweight aggregate concrete uses manufactured aggregate.

All lightweight materials are relatively weak because of their higher porosity, which gives them reduced weight. The resulting limitation on aggregate strength is not normally a problem, since the concrete strength that can be obtained still exceeds most structural requirements. Lightweight aggregates are used to reduce the weight of structural elements, and to give improved thermal insulation and fire resistance.

3.1.3 Water

The water used for mixing concrete should be free from impurities that could adversely affect the process of hydration and, consequently, the properties of concrete. For example, some organic matter can cause retardation, whilst chlorides may not only accelerate the stiffening process, but also cause embedded steel such as reinforcement to corrode. Other chemicals, like sulfate solutions and acids, can have harmful long-term effects by dissolving the cement paste in concrete. It is important, therefore, to be sure of the quality of water. If it comes from an unknown source such as a pond or borehole, it needs to be tested. BS EN 1008 specifies requirements for the quality of the water, and gives procedures for checking its suitability for use in concrete.

Drinking water is suitable, of course, and it is usual simply to obtain a supply from the local water utility. Some recycled water is being increasingly used in the interests of reducing the environmental impact of concrete production. Seawater has also been used successfully in mass concrete with no embedded steel. Recycled water systems are usually found at large-scale permanent mixing plants, such as precast concrete factories and ready-mixed concrete depots, where water that has been used for cleaning the plant and washing out mixers can be collected, filtered and stored for re-use. Some systems are able to reclaim up to a half of the mixing water in this way. Large volume settlement tanks are normally required. The tanks do not need to be particularly deep but should have a large surface area and, ideally, the water should be made to pass through a series of such tanks, becoming progressively cleaner at each stage.

3.1.4 Admixtures

An admixture is a material, usually a liquid, which is added to a batch of concrete during mixing to modify the properties of the fresh or the hardened concrete in some way. Most admixtures benefit concrete by reducing the amount of free water needed for a given level of consistence, often in addition to some other specific improvement. Permeability is thereby reduced and durability increased. There are occasions when the use of an admixture is not only desirable, but also essential. Because admixtures are added to concrete mixes in small quantities, they should be used only when a high degree of control can be exercised. Incorrect dosage of an admixture can adversely affect strength and other properties of the concrete. Requirements for the following main types of admixture are specified in BS EN 934-2.

Normal water-reducing admixtures. Commonly known as plasticisers or workability aids, these act by reducing the inter-particle attraction within the cement, to produce a more uniform dispersion of the cement grains. The cement paste is better 'lubricated', and hence the amount of water needed to obtain a given consistency can be reduced. The use of these admixtures can be beneficial in one of three ways:

- When added to a normal concrete at normal dosage, they produce an increase in slump of about 50 mm. This can be useful in high-strength concrete, rich in cement, which would otherwise be too stiff to place.
- The water content can be reduced while maintaining the same cement content and consistence class: the reduction in water/cement ratio (about 10%) results in increased strength and improved durability. This can also be useful for reducing bleeding in concrete prone to this problem; and for increasing the cohesion and thereby reducing segregation in concrete of high consistence, or in harsh mixes that sometimes arise with angular aggregates, or low sand contents, or when the sand is deficient in fines.
- The cement content can be reduced while maintaining the same strength and consistence class. The water/cement ratio is kept constant, and the water and cement contents are reduced accordingly. This approach should never be used if, thereby, the cement content would be reduced below the minimum specified amount.

Too big a dosage may result in retardation and/or a degree of air-entrainment, without necessarily increasing workability, and therefore may be of no benefit in the fresh concrete.

Accelerating water-reducing admixtures. Accelerators act by increasing the initial rate of chemical reaction between the cement and the water so that the concrete stiffens, hardens and develops strength more quickly. They have a negligible effect on consistence, and the 28-day strengths are seldom affected. Accelerating admixtures have been used mainly during cold weather, when the slowing down of the chemical reaction between cement and water at low temperature could be offset by the increased speed of reaction resulting from the accelerator. The most widely used accelerator used to be calcium chloride but, because the presence of chlorides, even in small amounts, increases the risk of corrosion, modern standards prohibit the use of admixtures containing chlorides in all concrete containing embedded metal. Accelerators are sometimes marketed under other names such as hardeners or anti-freezers, but no accelerator is a true anti-freeze, and the use of an accelerator does not avoid the need to protect the concrete in cold weather by keeping it warm (with insulation) after it has been placed.

Retarding water-reducing admixtures. These slow down the initial reaction between cement and water by reducing the rate of water penetration to the cement. By slowing down the growth of the hydration products, the concrete stays workable longer than it otherwise would. The length of time during which concrete remains workable depends on its temperature, consistence class, and water/cement ratio, and on the amount of retarder used. Although the occasions justifying the use of retarders in the United Kingdom are limited, these admixtures can be helpful when one or more of the following conditions apply.

- In warm weather, when the ambient temperature is higher than about 20ºC, to prevent early stiffening ('going-off') and loss of workability, which would otherwise make the placing and finishing of the concrete difficult.
- When a large concrete pour, which will take several hours to complete, must be constructed so that concrete already placed does not harden before the subsequent concrete can be merged with it (i.e. without a cold joint).
- When the complexity of a slip-forming operation requires a slow rate of rise.
- When there is a delay of more than 30 minutes between mixing and placing – for example, when ready-mixed concrete is being used over long-haul distances, or there are risks of traffic delays. This can be seriously aggravated during hot weather, especially if the cement content is high.

The retardation can be varied, by altering the dosage: a delay of 4–6 hours is usual, but longer delays can be obtained for special purposes. While the reduction in early strength of concrete may affect formwork-striking times, the 7-day and 28-day strengths are not likely to be significantly affected. Retarded concrete needs careful proportioning to minimise bleeding due to the longer period during which the concrete remains fresh.

Air-entraining admixtures. These may be organic resins or synthetic surfactants that entrain a controlled amount of air in concrete in the form of small air bubbles. The bubbles need to be about 50 microns in diameter and well dispersed. The main reason for using an air-entraining admixture is that the presence of tiny bubbles in the hardened concrete increases its resistance to the action of freezing and thawing, especially when this is aggravated by the application of de-icing salts and fluids. Saturated concrete – as most external paving will be – can be seriously affected by the freezing of water in the capillary voids, which will expand and try to burst it. If the concrete is air-entrained, the air bubbles, which intersect the capillaries, stay unfilled with water even when the concrete is saturated. Thus, the bubbles act as pressure relief valves and cushion the expansive effect by providing voids into which the water can expand as it freezes, without disrupting the concrete. When the ice melts, surface tension effects draw the water back out of the bubbles.

Air-entrained concrete should be specified and used for all forms of external paving, from major roads and airfield runways down to garage drives and footpaths, which are likely to be subjected to severe freezing and to de-icing salts. The salts may be applied directly, or come from the spray of passing traffic, or by dripping from the underside of vehicles.

Air-entrainment also affects the properties of the fresh concrete. The minute air bubbles act like ball bearings and have a plasticising effect, resulting in a higher consistence. Concrete that is lacking in cohesion, or harsh, or which tends to bleed excessively, is greatly improved by air-entrainment. The risk of plastic settlement and plastic-shrinkage cracking is also reduced. There is also evidence that colour uniformity is improved and surface blemishes reduced. One factor that has to be taken into account when using air-entrainment is that the strength of the concrete is reduced, by about 5% for every 1% of air entrained. However, the plasticising effect of the admixture means that the water content of the concrete can be reduced, which will offset most of the strength loss that would otherwise occur, but even so some increase in the cement content is likely to be required.

High-range water-reducing admixtures. Commonly known as superplasticizers, these have a considerable plasticizing effect on concrete. They are used for one of two reasons:

- To greatly increase the consistence of a concrete mix, so that a ‘flowing’ concrete is produced that is easy both to place and to compact: some such concretes are completely self-compacting and free from segregation.
- To produce high-strength concrete by reducing the water content to a much greater extent than can be achieved by using a normal plasticizer (water-reducing admixture).

A flowing concrete is usually obtained by first producing a concrete whose slump is in the range 50–90 mm, and then adding the superplasticizer, which increases the slump to over 200 mm. This high consistence lasts for only a limited period of time: stiffening and hardening of the concrete then proceed normally. Because of this time limitation, when ready-mixed concrete is being used, it is usual for the superplasticizer to be added to the concrete on site rather than at the batching or mixing plant. Flowing concrete can be more susceptible to segregation and bleeding, so it is essential for the mix design and proportions to allow for the use of a superplasticizer. As a general guide, a conventionally designed mix needs to be modified, by increasing the sand content by about 5%. A high degree of control over the batching of all the constituents is essential, especially the water, because if the consistence of the concrete is not correct at the time of adding the superplasticizer, excessive flow and segregation will occur.

The use of flowing concrete is likely to be limited to work where the advantages, in ease and speed of placing, offset the increased cost of the concrete – considerably more than with other admixtures. Typical examples are where reinforcement is particularly congested, making both placing and vibration difficult; and where large areas, such as slabs, would benefit from a flowing easily placed concrete. The fluidity of flowing concrete increases the pressures on formwork, which should be designed to resist full hydrostatic pressure.

When used to produce high-strength concrete, reductions in water content of as much as 30% can be obtained by using superplasticizers, compared to 10% with normal plasticizers: as a result, 1-day and 28-day strengths can be increased by as much as 50%. Such high-strength water-reduced concrete is used both for high-performance *in situ* concrete construction, and for the manufacture of precast units, where the increased early strength allows earlier demoulding.

3.1.5 Properties of fresh and hardening concrete

Workability. It is vital that the workability of concrete is matched to the requirements of the construction process. The ease or difficulty of placing concrete in sections of various sizes and shapes, the type of compaction equipment needed, the complexity of the reinforcement, the size and skills of the workforce are amongst the items to be considered. In general, the more difficult it is to work the concrete, the higher should be the level of workability. But the concrete must also have sufficient cohesiveness in order to resist segregation and bleeding. Concrete needs to be particularly cohesive if it is to be pumped, or allowed to fall from a considerable height.

The workability of fresh concrete is increasingly referred to in British and European standards as consistence. The slump test is the best-known method for testing consistence, and the slump classes given in BS EN 206-1 are: S1 (10–40 mm), S2 (50–90 mm), S3 (100–150 mm), S4 (160–210 mm). Three other test methods recognised in BS EN 206-1, all with their own unique consistency classes, are namely; Vebe time, degree of compactability and flow diameter.

Plastic cracking. There are two basic types of plastic cracks: plastic settlement cracks, which can develop in deep sections and, often follow the pattern of the reinforcement; and plastic shrinkage cracks, which are most likely to develop in slabs. Both types form while the concrete is still in its plastic state, before it has set or hardened and, depending on the weather conditions, within about one to six hours after the concrete has been placed and compacted. They are often not noticed until the following day. Both types of crack are related to the extent to which the fresh concrete bleeds.

Fresh concrete is a suspension of solids in water and, after it has been compacted, there is a tendency for the solids (both aggregates and cement) to settle. The sedimentation process displaces water, which is pushed upwards and, if excessive, appears as a layer on the surface. This bleed water may not always be seen, since it can evaporate on hot or windy days faster than it rises to the surface. Bleeding can generally be reduced, by increasing the cohesiveness of the concrete. This is usually achieved by one or more of the following means: increasing the cement content, increasing the sand content, using a finer sand, using less water, air-entrainment, using a rounded natural sand rather than an angular crushed one. The rate of bleeding will be influenced by the drying conditions, especially wind, and bleeding will take place for longer on cold days. Similarly, concrete containing a retarder tends to bleed for a longer period of time, due to the slower stiffening rate of the concrete, and the use of retarders will, in general, increase the risk of plastic cracking.

Plastic settlement cracks, caused by differential settlement, are directly related to the amount of bleeding. They tend to occur in deep sections, particularly deep beams, but they may also develop in columns and walls. This is because the deeper the section, the greater the sedimentation or settlement that can occur. However, cracks will form only where something prevents the concrete 'solids' from settling freely. The most common cause of this is the reinforcement fixed at the top of deep sections; the concrete will be seen to 'hang-up' over the bars and the pattern of cracks will directly reflect the layout of the reinforcement below. Plastic settlement cracks can also occur in trough and waffle slabs, or at any section where there is a significant change in the depth of concrete. If alterations to the concrete, for example, the use of an air-entraining or water-reducing admixture, cannot be made due to contractual or economic reasons, the most effective way of eliminating plastic settlement cracking is to re-vibrate the concrete after the cracks have formed. Such re-vibration is acceptable when the concrete is still plastic enough to be capable of being 'fluidized' by a poker, but not so stiff that a hole is left when the poker is withdrawn. The prevailing weather conditions will determine the timing of the operation.

Plastic shrinkage cracks occur in horizontal slabs, such as floors and pavements. They usually take the form of one or more diagonal cracks at 0.5–2 m centres that do not extend to the slab edges, or they form a very large pattern of map cracking. Such cracks are most common in concrete placed on hot or windy days, because they are caused by the rate of evaporation of moisture from the surface exceeding the rate of bleeding. Clearly, plastic shrinkage cracks can be reduced, by preventing the loss of moisture from the concrete surface in the critical first few hours. While sprayed-on resin-based curing compounds are very efficient at curing concrete that has already hardened, they cannot be used on fresh concrete until the free bleed water has evaporated. This is too late to prevent plastic shrinkage cracking, and so the only alternative is to protect the concrete for the first few hours with polythene sheeting. This needs to be supported clear of the concrete by means of blocks or timber, but with all the edges held down to prevent a wind-tunnel effect. It has been found that plastic shrinkage cracking is virtually non-existent when air-entrainment is used.

The main danger from plastic cracking is the possibility of moisture ingress leading to corrosion of any reinforcement. If the affected surface is to be covered subsequently, by either more concrete or a screed, no treatment is usually necessary. In other cases, often the best repair is to brush dry cement (dampened down later) or wet grout into the cracks the day after they form, and while they are still clean; this encourages natural or autogenous healing.

Early thermal cracking. The reaction of cement with water, or hydration, is a chemical reaction that produces heat. If this heat development exceeds the rate of heat loss, the concrete temperature will rise. Subsequently the concrete will cool and contract. Typical temperature histories of different concrete sections are shown in the figure on *Table 2.18*.

If the contraction of the concrete were unrestrained, there would be no cracking at this stage. However, in practice there is nearly always some form of restraint inducing tension, and hence a risk of cracks forming. The restraint can occur due to both external and internal influences. Concrete is externally restrained when, for example, it is cast onto a previously cast base, such as a wall kicker, or between two already hardened sections, such as in infill bay in a wall or slab, without the provision of a contraction joint. Internal restraint occurs, for example, because the surfaces of an element will cool faster than the core, producing a temperature differential. When this differential is large, such as in thick sections, surface cracks may form at an early stage. Subsequently, as the core of the section cools, these surface cracks will tend to close in the absence of any external restraints. Otherwise, the cracks will penetrate into the core, and link up to form continuous cracks through the whole section.

The main factors affecting the temperature rise in concrete are the dimensions of the section, the cement content and type, the initial temperature of the concrete and the ambient temperature, the type of formwork and the use of admixtures. Thicker sections retain more heat, giving rise to higher peak temperatures, and cool down more slowly. Within the core of very thick sections, adiabatic conditions obtain and, above a thickness of about 1.5 m, there is little further increase in the temperature of the concrete. The heat generated is directly related to the cement content. For Portland cement concretes, in sections of thickness 1 m and more, the temperature rise in the core is likely to be about 14ºC for every 100 kg/m^3 of cement. Thinner sections will exhibit lower temperature rises.

Different cement types generate heat at different rates. The peak temperature and the total amount of heat produced by hydration depend upon both the fineness and the chemistry of the cement. As a guide, the cements whose strength develops most rapidly tend to produce the most heat. Sulfate-resisting cement generally gives off less heat than CEM I, and cements that are inter-ground or combined with mineral additions, such as pfa or ggbs, are often chosen for massive construction because of their low heat of hydration.

A higher initial temperature results in a greater temperature rise; for example, concrete in a 500 mm thick section placed at 10ºC could have a temperature rise of 30ºC, but the same concrete placed at 20ºC may have a temperature rise of 40ºC. Steel and GRP formwork will allow the heat generated to be dissipated more quickly than will timber formwork, resulting in lower temperature rises, especially in thinner sections. Timber formwork and/or additional insulation will reduce the temperature differential between the core and the surface of the section, but this differential could increase significantly when the formwork is struck. Retarding water-reducers will delay the onset of hydration, but do not reduce the total heat generated. Accelerating water-reducers will increase the rate of heat evolution and the temperature rise.

The problem of early thermal cracking is usually confined to slabs and walls. Walls are particularly susceptible, because they are often lightly reinforced in the horizontal direction, and the timber formwork tends to act as a thermal insulator, encouraging a larger temperature rise. The problem could be reduced, by lowering the cement content and using cement with a lower heat of hydration, or one containing ggbs or pfa. However, there are practical and economic limits to these measures, often dictated by the specification requirements for the strength and durability of the concrete itself. In practice, cracking due to external restraint is generally dealt with by providing crack control reinforcement and contraction joints. With very thick sections, and very little external restraint, if the temperature differential can be controlled by insulating the concrete surfaces for a few days, cracking can be avoided.

Typical values of the temperature rise in walls and slabs for Portland cement concretes, as well as comparative values for concrete using other cements are given in *Table 2.18*. Further data on predicted temperature rises is given in ref. 11.

3.1.6 Properties of hardened concrete

Compressive strength. The strength of concrete is specified as a strength class or grade, namely the 28-day characteristic compressive strength of specimens made from fresh concrete under standardised conditions. The results of strength tests are used routinely for control of production and contractual conformity purposes. The characteristic strength is defined as that level of strength below which 5% of all valid test results is expected to fall. Test cubes, either 100 mm or 150 mm, are the specimens normally used in the United Kingdom and most other European countries, but cylinders are used elsewhere. Because their basic shapes (ratio of height to cross-sectional dimension) are different, the strength test results are also different, cylinders being weaker than cubes. For normal-weight aggregates, the concrete cylinder strength is about 80% of the corresponding cube strength. For lightweight aggregates, cylinder strengths are about 90% of the corresponding cube strengths.

In British Codes of Practice like BS 8110, strength grades used to be specified in terms of cube strength (e.g. C30), as shown in *Table 3.9*. Nowadays, strength classes are specified in terms of both cylinder strength and equivalent cube strength (e.g. C25/30), as shown in *Tables 3.5* and *4.2*.

In principle, compressive strengths can be determined from cores cut from the hardened concrete. Core tests are normally made only when there is some doubt about the quality of concrete placed (e.g. if the cube results are unsatisfactory), or to assist in determining the strength and quality of an existing structure for which records are not available. Great care is necessary in the interpretation of the results of core tests, and samples drilled from *in situ* concrete are expected to be lower in strength than cubes made, cured and tested under standard laboratory conditions. The standard reference for core testing is BS EN 12504-1.

Tensile strength. The direct tensile strength of concrete, as a proportion of the cube strength, varies from about one-tenth for low-strength concretes to one-twentieth for high-strength concretes. The proportion is affected by the aggregate used, and the compressive strength is therefore only a very general guide to the tensile strength. For specific design purposes, in regard to cracking and shear strength, analytical relationships between the tensile strength and the specified cylinder/cube strength are provided in codes of practice.

The indirect tensile strength (or cylinder splitting strength) is seldom specified nowadays. Flexural testing of specimens may be used on some airfield runway contracts, where the method of design is based on the modulus of rupture, and for some precast concrete products such as flags and kerbs.

Elastic properties. The initial behaviour of concrete under service load is almost elastic, but under sustained loading the strain increases with time. Stress–strain tests cannot be carried out instantaneously, and there is always a degree of non-linearity and a residual strain upon unloading. For practical purpose, the initial deformation is considered to be elastic (recoverable upon unloading), and the subsequent increase in strain under sustained stress is defined as creep. The elastic modulus on loading defined in this way is a secant modulus related to a specific stress level. The value of the modulus of elasticity of concrete is influenced mainly by the aggregate used. With a particular aggregate, the value increases with the strength of the concrete and the age at loading. In special circumstances, For example, where deflection calculations are of great importance, load tests should be carried out on concrete made with the aggregate to be used in the actual structure. For most design purposes, specific values of the mean elastic modulus at 28 days, and of Poisson's ratio, are given in *Table 3.5* for BS 8110 and *Table 4.2* for EC 2.

Creep. The increase in strain beyond the initial elastic value that occurs in concrete under a sustained constant stress, after taking into account other time-dependent deformations not associated with stress, is defined as creep. If the stress is removed after some time, the strain decreases immediately by an amount that is less than the original elastic value because of the increase in the modulus of elasticity with age. This is followed by a further gradual decrease in strain. The creep recovery is always less than the preceding creep, so that there is always a residual deformation.

The creep source in normal-weight concrete is the hardened cement paste. The aggregate restrains the creep in the paste, so that the stiffer the aggregate and the higher its volumetric proportion, the lower is the creep of the concrete. Creep is also affected by the water/cement ratio, as is the porosity and strength of the concrete. For constant cement paste content, creep is reduced by a decrease in the water/cement ratio.

The most important external factor influencing creep is the relative humidity of the air surrounding the concrete. For a specimen that is cured at a relative humidity of 100%, then loaded and exposed to different environments, the lower the relative humidity, the higher is the creep. The values are much reduced in the case of specimens that have been allowed to dry prior to the application of load. The influence of relative humidity on creep is dependent on the size of the member. When drying occurs at constant relative humidity, the larger the specimen, the smaller is the creep. This size effect is expressed in terms of the volume/surface area ratio of the member. If no drying occurs, as in mass concrete, the creep is independent of size.

Creep is inversely proportional to concrete strength at the age of loading over a wide range of concrete mixes. Thus, for a given type of cement, the creep decreases as the age and consequently the strength of the concrete at application of the load increases. The type of cement, temperature and curing conditions all influence the development of strength with age.

The influence of temperature on creep is important in the use of concrete for nuclear pressure vessels, and containers for storing liquefied gases. The time at which the temperature of concrete rises relative to the time at which load is applied affects the creep–temperature relation. If saturated concrete is heated and loaded at the same time, the creep is greater than when the concrete is heated during the curing period prior to the application of load. At low temperatures, creep behaviour is affected by the formation of ice. As the temperature falls, creep decreases until the formation of ice causes an increase in creep, but below the ice point creep again decreases.

Creep is normally assumed to be directly proportional to applied stress within the service range, and the term specific creep is used for creep per unit of stress. At stresses above about one-third of the cube strength (45% cylinder strength), the formation of micro-cracks causes the creep–stress relation to become non-linear, creep increasing at an increasing rate.

The effect of creep is unfavourable in some circumstances (e.g. increased deflection) and favourable in others (e.g. relief of stress due to restraint of imposed deformations, such as differential settlement, seasonal temperature change).

For normal exposure conditions (inside and outside), creep coefficients according to ambient relative humidity, effective section thickness (notional size) and age of loading, are given in *Table 3.5* for BS 8110 and *Table 4.3* for EC 2.

Shrinkage. Withdrawal of water from hardened concrete kept in unsaturated air causes drying shrinkage. If concrete that has been left to dry in air of a given relative humidity is subsequently placed in water (or a higher relative humidity), it will swell due to absorption of water by the cement paste. However, not all of the initial drying shrinkage is recovered even after prolonged storage in water. For the usual range of concretes, the reversible moisture movement represents about 40%–70% of the drying shrinkage. A pattern of alternate wetting and drying will occur in normal outdoor conditions. The magnitude of the cyclic movement clearly depends upon the duration of the wetting and drying periods, but drying is much slower than wetting. The consequence of prolonged dry weather can be reversed by a short period of rain. More stable conditions exist indoors (dry) and in the ground or in contact with water (e.g. reservoirs and tanks).

Shrinkage of hardened concrete under drying conditions is influenced by several factors in a similar manner to creep. The intrinsic shrinkage of the cement paste increases with the water/cement ratio so that, for a given aggregate proportion, concrete shrinkage is also a function of water/cement ratio.

The relative humidity of the air surrounding the member greatly affects the magnitude of concrete shrinkage according to the volume/surface area ratio of the member. The lower shrinkage value of large members is due to the fact that drying is restricted to the outer parts of the concrete, the shrinkage of which is restrained by the non-shrinking core. Clearly, shrinkable aggregates present special problems and can greatly increase concrete shrinkage (ref. 12).

For normal exposure conditions (inside and outside), values of drying shrinkage, according to ambient relative humidity and effective section thickness (notional size), are given in *Table 3.5* for BS 8110 and *Table 4.2* for EC 2.

Thermal properties. The coefficient of thermal expansion of concrete depends on both the composition of the concrete and its moisture condition at the time of the temperature change. The thermal coefficient of the cement paste is higher than that of the aggregate, which exerts a restraining influence on the movement of the cement paste. The coefficient of thermal expansion of a normally cured paste varies from the lowest values, when the paste is either totally dry or saturated, to a maximum at a relative humidity of about 70%. Values for the aggregate are related to their mineralogical composition.

A value for the coefficient of thermal expansion of concrete is needed in the design of structures such as chimneys, tanks containing hot liquids, bridges and other elevated structures exposed to significant solar effects; and for large expanses of concrete where provision must be made to accommodate the effects of temperature change in controlled cracking, or by providing movement joints. For normal design purposes, values of the coefficient of thermal expansion of concrete, according to the type of aggregate, are given in *Table 3.5* for BS 8110 and *Table 4.2* for EC 2.

Short-term stress–strain curves. For normal low to medium strength unconfined concrete, the stress–strain relationship in compression is approximately linear up to about one-third of the cube strength (40% of cylinder strength). With increasing stress, the strain increases at an increasing rate, and a peak stress (cylinder strength) is reached at a strain of about 0.002. With increasing strain, the stress reduces until failure occurs at a strain of about 0.0035. For higher strength concretes, the peak stress occurs at strains > 0.002 and the failure occurs at strains < 0.0035, the failure being progressively more brittle as the concrete strength increases.

For design purposes, the short-term stress–strain curve is generally idealised to a form in which the initial portion is parabolic or linear, and the remainder is at a uniform stress. A further simplification in the form of an equivalent rectangular stress block may be made subsequently. Typical stress–strain curves and those recommended for design purposes are given in *Table 3.6* for BS 8110, and *Table 4.4* for EC 2.

3.1.7 Durability of concrete

Concrete has to be durable in natural environments ranging from mild to extremely aggressive, and resistant to factors such as weathering, freeze/thaw attack, chemical attack and abrasion. In addition, for concrete containing reinforcement, the surface concrete must provide adequate protection against the ingress of moisture and air, which would eventually cause corrosion of the embedded steel.

Strength alone is not necessarily a reliable guide to concrete durability; many other factors have to be taken into account, the most important being the degree of impermeability. This is dependent mainly on the constituents of the concrete, in particular the free water/cement ratio, and in the provision of full compaction to eliminate air voids, and effective curing to ensure continuing hydration.

Concrete has a tendency to be permeable as a result of the capillary voids in the cement paste matrix. In order for the concrete to be sufficiently workable, it is common to use far more water than is actually necessary for the hydration of the cement. When the concrete dries out, the space previously occupied by the excess water forms capillary voids. Provided the concrete has been fully compacted and properly cured, the voids are extremely small, the number and the size of the voids decreasing as the free water/cement ratio is reduced. The more open the structure of the cement paste, the easier it is for air, moisture and harmful chemicals to penetrate.

Carbonation. Steel reinforcement that is embedded in good concrete with an adequate depth of cover is protected against corrosion by the highly alkaline pore water in the hardened cement paste. Loss of alkalinity of the concrete can be caused by the carbon dioxide in the air reacting with and neutralising the free lime. If this reaction, which is called carbonation, reaches the reinforcement, then corrosion will occur in moist environments. Carbonation is a slow process that progresses from the surface, and is dependent on the permeability of the concrete and the humidity of the environment. Provided the depth of cover, and quality of concrete, recommended for the anticipated exposure conditions are achieved, corrosion due to carbonation should not occur during the intended lifetime of the structure.

Freeze/thaw attack. The resistance of concrete to freezing and thawing depends on its impermeability, and the degree of saturation on being exposed to frost; the higher the degree of saturation, the more liable the concrete is to damage. The use of salt for de-icing roads and pavements greatly increases the risk of freeze/thaw damage.

The benefits of air-entrained concrete have been referred to in section 3.1.4, where it was recommended that all exposed horizontal paved areas, from roads and runways to footpaths and garage drives, and marine structures, should be made of air-entrained concrete. Similarly, parts of structures adjacent to highways and in car parks, which could be splashed or come into contact with salt solutions used for de-icing, should also use air-entrained concrete. Alternatively, the cube strength of the concrete should be 50 N/mm^2 or more. Whilst C40/50 concrete is suitable for many situations, it does not have the same freeze/thaw resistance as air-entrained concrete.

Chemical attack. Portland cement concrete is liable to attack by acids and acid fumes, including the organic acids often produced when foodstuffs are being processed. Vinegar, fruit juices, silage effluent, sour milk and sugar solutions can all attack concrete. Concrete made with Portland cement is not recommended for use in acidic conditions where the pH value is 5.5 or less, without careful consideration of the exposure condition and the intended construction. Alkalis have little effect on concrete.

For concrete that is exposed to made-up ground, including contaminated and industrial material, specialist advice should be sought in determining the design chemical class so that a suitable concrete can be specified. The most common form of chemical attack that concretes have to resist is the effect of solutions of sulfates present in some soils and groundwaters.

In all cases of chemical attack, concrete resistance is related to free water/cement ratio, cement content, type of cement and the degree of compaction. Well-compacted concrete will always be more resistant to sulfate attack than one less well compacted, regardless of cement type. Recommendations for concrete exposed to sulfate-containing groundwater, and for chemically contaminated brownfield sites, are incorporated in BS 8500-1.

Alkali–silica reaction. ASR is a reaction that can occur in concrete between certain siliceous constituents present in the aggregate and the alkalis – sodium and potassium hydroxide – that are released during cement hydration. A gelatinous product is formed, which imbibes pore fluid and in so doing expands, inducing an internal stress within the concrete. The reaction will cause damage to the concrete only when the following three conditions occur simultaneously:

- A reactive form of silica is present in the aggregate in critical quantities.
- The pore solution contains ions of sodium, potassium and hydroxyl, and is of a sufficiently high alkalinity.
- A continuing supply of water is available.

If any one of these factors is absent, then damage from ASR will not occur and no precautions are necessary. It is possible for the reaction to take place in the concrete without inducing expansion. Damage may not occur, even when the reaction product is present throughout the concrete, as the gel may fill cracks induced by some other mechanism. Recommendations are available for minimising the risk of damage from ASR in new concrete construction, based on ensuring that at least one of the three aforementioned conditions is absent.

Exposure classes. For design and specification purposes, the environment to which concrete will be exposed during its intended life is classified into various levels of severity. For each category, minimum requirements regarding the quality of the concrete, and the cover to the reinforcement, are given in Codes of Practice. In British Codes, for many years, the exposure conditions were mild, moderate, severe, very severe and most severe (or, in BS 5400, extreme) with abrasive as a further category. Details of the classification system that was used in BS 8110 and BS 5400 are given in *Table 3.9*.

In BS EN 206-1, BS 8500-1 and EC 2, the conditions are classified in terms of exposure to particular actions, with various levels of severity in each category. The following categories are considered:

1. No risk of corrosion or attack
2. Corrosion induced by carbonation
3. Corrosion induced by chlorides other than from seawater
4. Corrosion induced by chlorides from seawater
5. Freeze/thaw attack
6. Chemical attack

If the concrete is exposed to more than one of these actions, the environmental conditions are expressed as a combination of exposure classes. Details of each class in categories 1–5, with descriptions and informative examples applicable in the United Kingdom, are given in *Tables 3.7* and *4.5*. For concrete exposed to chemical attack the exposure classes given in BS EN 206-1 cover only natural ground with static water, which represents a limited proportion of the aggressive ground conditions found in the United Kingdom. In the complementary British Standard BS 8500-1, more comprehensive recommendations are provided, based on the approach used in ref. 13.

On this basis, an ACEC (aggressive chemical environment for concrete) class is determined, according to the chemicals in the ground, the type of soil and the mobility and acidity of the groundwater. The chemicals in the ground are expressed as a design sulfate class (DS), in which the measured sulfate content is increased to take account of materials that may oxidise into sulfate, for example, pyrite, and other aggressive species such as hydrochloric or nitric acid. Magnesium ion content is also included in this classification. Soil is classified as natural or, for sites that may contain chemical residues from previous industrial use or imported wastes, as brownfield. Water in the ground is classified as either static or mobile, and according to its pH value.

Based on the ACEC classification, and according to the size of the section and the selected structural performance level, the required concrete quality expressed as a design chemical class (DC), and any necessary additional protective measures (APMs) can be determined. The structural performance level is classified as low, normal or high, in relation to the intended service life, the vulnerability of the structural details and the security of structures retaining hazardous materials.

Concrete quality and cover to reinforcement. Concrete durability is dependent mainly on its constituents, particularly the free water/cement ratio. The ratio can be reduced, and the durability of the concrete enhanced, by increasing the cement content and/or using admixtures to reduce the amount of free water needed for a particular level of consistence, subject to specified minimum requirements being met for the cement content. By limiting the maximum free water/cement ratio and the minimum cement content, a minimum strength class can be obtained for particular cements and combinations.

Where concrete containing reinforcement is exposed to air and moisture, or is subject to contact with chlorides from any source, the protection of the steel against corrosion depends on the concrete cover. The required thickness is related to the exposure class, the concrete quality and the intended working life of the structure. Recommended values for an intended working life of at least 50 years, are given in *Tables 3.8* and *4.6* (BS 8500), and *3.9* (prior to BS 8500).

Codes of Practice also specify values for the covers needed to ensure the safe transmission of bond forces, and provide an adequate fire-resistance for the reinforced concrete member. In addition, allowance may need to be made for abrasion, or for surface treatments such as bush hammering. In BS 8110, values used to be given for a nominal cover to be provided to all reinforcement, including links, on the basis that the actual cover should not be less than the nominal cover minus 5 mm. In BS 8500, values are given for a minimum cover to which an allowance for tolerance (normally 10 mm) is then added.

Concrete specification. Details of how to specify concrete, and what to specify, are given in BS 8500-1. Three types – designed, prescribed and standardised prescribed concretes – are recognised by BS EN 206-1, but BS 8500 adds two more – designated and proprietary concretes.

Designed concretes are ones where the concrete producer is responsible for selecting the mix proportions, to provide the performance defined by the specifier. Conformity of designed concretes is usually judged by strength testing of 100 mm or 150 mm cubes, which in BS 8500 is the responsibility of the concrete producer. Prescribed concretes are ones where the specification states the mix proportions, in order to satisfy particular performance requirements, in terms of the mass of each constituent. Such concretes are seldom necessary, but might be used where particular properties or special surface finishes are required. Standardised prescribed concretes that are intended for site production, using basic equipment and control, are given in BS 8500-2. Whilst conformity does not depend on strength testing, assumed characteristic strengths are given for the purposes of design. Designated concretes are a wide-ranging group of concretes that provide for most types of concrete construction. The producer must operate a recognized accredited, third party certification system, and is responsible for ensuring that the concrete conforms to the specification given in BS 8500-2. Proprietary concretes are intended to provide for instances when a concrete producer would give assurance of the performance of concrete without being required to declare its composition.

For conditions where corrosion induced by chlorides does not apply, structural concretes should generally be specified as either designated concretes or designed concretes. Where exposure to corrosion due to chlorides is applicable, only the designed concrete method of specifying is appropriate. An exception to this situation is where an exposed aggregate, or tooled finish that removes the concrete surface, is required. In these cases, in order to get an acceptable finish, a special mix design is needed. Initial testing, including trial panels, should be undertaken and from the results of these tests, a prescribed concrete can be specified. For housing applications, both a designated concrete and a standardised prescribed concrete can be specified as acceptable alternatives. This would allow a concrete producer with accredited certification to quote for supplying a designated concrete, and the site contractor, or a concrete producer without accredited certification, to quote for supplying a standardised prescribed concrete.

3.2 REINFORCEMENT

Reinforcement for concrete generally consists of deformed steel bars, or welded steel mesh fabric. Normal reinforcement relies entirely upon the alkaline environment provided by a durable concrete cover for its protection against corrosion. In special circumstances, galvanised, epoxy-coated or stainless steel can be used. Fibre-reinforced polymer materials have also been developed. So far, in the United Kingdom, these materials have been used mainly for external strengthening and damage repair applications.

3.2.1 Bar reinforcement

In the United Kingdom, reinforcing bars are generally specified, ordered and delivered to the requirements of BS 4449. This caters for steel bars with a yield strength of 500 MPa in three ductility classes: grades B500A, B500B and B500C. Bars are round in cross section, having two or more rows of uniformly spaced transverse ribs, with or without longitudinal ribs. The pattern of transverse ribs varies with the grade, and can be used as a means of identification. Information with regard to the basic properties of reinforcing bars to BS 4449, which is in general conformity with BS EN 10080, is given in *Table 2.19*.

All reinforcing bars are produced by a hot-rolling process, in which a cast steel billet is reheated to 1100–1200ºC, and then rolled in a mill to reduce its cross section and impart the rib pattern. There are two common methods for achieving the required mechanical properties in hot-rolled bars: in-line heat treatment and micro-alloying. In the former method, which is sometimes referred to as the quench-and-self-temper (QST) process, high-pressure water sprays quench the bar surface as it exits the rolling mill, producing a bar with a hard tempered outer layer, and a softer more ductile core. Most reinforcing bars in the United Kingdom are of this type, and achieve class B or class C ductility. In the micro-alloying method, strength is achieved by adding small amounts of alloying elements during the steel-making process. Micro-alloy steels normally achieve class C ductility. Another method that can be used to produce high-yield bars involves a cold-twisting process, to form bars that are identified by spiralling longitudinal ribs. This process has been obsolete in the United Kingdom for some time, but round ribbed, twisted bars can be found in some existing structures.

In addition to bars being produced in cut straight lengths, billets are also rolled into coil for diameters up to 16 mm. In this form, the product is ideal for automated processes such as link bending. QST, micro-alloying and cold deformation processes are all used for high-yield coil. Cold deformation is applied by continuous stretching, which is less detrimental to ductility than the cold-twisting process mentioned previously. Coil products have to be de-coiled before use, and automatic link bending machines incorporate straightening rolls. Larger de-coiling machines are also used to produce straight lengths.

3.2.2 Fabric reinforcement

Steel fabric reinforcement is an arrangement of longitudinal bars and cross bars welded together at their intersections in a shear resistant manner. In the United Kingdom, fabric is produced under a closely controlled factory-based manufacturing process to the requirements of BS 4483. In fabric for structural purposes, ribbed bars complying with BS 4449 are used. For wrapping fabric, as described later, wire complying with BS 4482 may be used. Wire can be produced from hot-rolled rod, by either drawing the rod through a die to produce plain wire, or cold rolling the rod to form indented or ribbed wires. In BS 4482, provision is made for plain round wire with a yield strength of 250 MPa, and plain, indented or ribbed wires with a yield strength of 500 MPa.

In BS 4483, provision is made for fabric reinforcement to be either of a standard type, or purpose made to the client's requirements. The standard fabric types have regular mesh arrangements and bar sizes, and are defined by identifiable reference numbers. Type A is a square mesh with identical long bars and cross bars, commonly used in ground slabs. Type B is a rectangular (structural) mesh that is particularly suitable for use in thin one-way spanning slabs. Type C is a rectangular (long) mesh that can be used in pavements, and in two-way spanning slabs by providing separate sheets in each direction. Type D is a rectangular (wrapping) mesh that is used in the concrete encasement of structural steel sections. The stock size of standard fabric sheets is 4.8 m × 2.4 m, and merchant size sheets are also available in a 3.6 m × 2.0 m size. Full details of the preferred range of standard fabric types are given in *Table 2.20*.

Purpose-made fabrics, specified by the customer, can have any combination of wire size and spacing in either direction. In practice, manufacturers may sub-divide purpose-made fabrics into two categories: special (also called scheduled) and bespoke (also called detailed). Special fabrics consist of the standard wire size combinations, but with non-standard overhangs and sheet dimensions up to 12 m × 3.3 m. Sheets with so-called flying ends are used to facilitate the lapping of adjacent sheets.

Bespoke fabrics involve a more complex arrangement in which the wire size, spacing and length can be varied within the sheet. These products are made to order for each contract as a replacement for conventional loose bar assemblies. The use of bespoke fabrics is appropriate on contracts with a large amount of repeatability, and generally manufacturers would require a minimum tonnage order for commercial viability.

3.2.3 Stress–strain curves

For hot-rolled reinforcement, the stress–strain relationship in tension is linear up to yield, when there is a pronounced increase of strain at constant stress (yield strength). Further small increases of stress, resulting in work hardening, are accompanied by considerable elongation. A maximum stress (tensile strength) is reached, beyond which further elongation is accompanied by a stress reduction to failure. Micro-alloy bars are characterised by high ductility (high level of uniform elongation and high ratio of tensile strength/yield strength). For QST bars, the stress–strain curve is of similar shape but with slightly less ductility.

Cold-processed reinforcing steels show continuous yielding behaviour with no defined yield point. The work-hardening capacity is lower than for the hot-rolled reinforcement, with the uniform elongation level being particularly reduced. The characteristic strength is defined as the 0.2% proof stress (i.e. a stress which, on unloading, would result in a residual strain of 0.2%), and the initial part of the stress–strain curve is linear to beyond 80% of this value.

For design purposes, the yield or 0.2% proof condition is normally critical and the stress–strain curves are idealised to a bi-linear, or sometimes tri-linear, form. Typical stress–strain curves and those recommended for design purposes are given in *Table 3.6* for BS 8110, and *Table 4.4* for EC 2.

3.2.4 Bar sizes and bends

The nominal size of a bar is the diameter of a circle with an area equal to the effective cross-sectional area of the bar. The range of nominal sizes (millimetres) is from 6 to 50, with preferred sizes of 8, 10, 12, 16, 20, 25, 32 and 40. Values of the total cross-sectional area provided in a concrete section, according to the number or spacing of the bars, for different bar sizes, are given in *Table 2.20*.

Bends in bars should be formed around standard mandrels on bar-bending machines. In BS 8666, the minimum radius of bend r is standardised as $2d$ for $d \leq 16$, and $3.5d$ for $d \geq 20$, where d is the bar size. Values of r for each different bar size, and values of the minimum end projection P needed to form the bend, are given in *Table 2.19*. In some cases (e.g. where bars are highly stressed), the bars need to be bent to a radius larger than the minimum value in order to satisfy the design requirements, and the required radius R is then specified on the bar-bending schedule.

Reinforcement should not be bent or straightened on site in a way that could damage or fracture the bars. All bars should preferably be bent at ambient temperature, but when the steel temperature is below 5ºC special precautions may be needed, such as reducing the speed of bending or, with the engineer's approval, increasing the radius of bending. Alternatively, the bars may be warmed to a temperature not exceeding 100ºC.

3.2.5 Bar shapes and bending dimensions

Bars are produced in stock lengths of 12 m, and lengths up to 18 m can be supplied to special order. In most structures, bars are required in shorter lengths and often need to be bent. The cutting and bending of reinforcement is generally specified to the requirements of BS 8666. This contains recommended bar shapes, designated by shape code numbers, which are shown in *Tables 2.21* and *2.22*. The information needed to cut and bend the bars to the required dimensions is entered into a bar schedule, an example of which is shown in *Table 2.23*. Each schedule is related to a member on a particular drawing by means of the bar schedule reference number.

In cases where a bar is detailed to fit between two concrete faces, with no more than the nominal cover on each face (e.g. links in beams), an allowance for deviations is required. This is to cater for variation due to the effect of inevitable errors in the dimensions of the formwork, and the cutting, bending and fixing of the bars. Details of the deductions to be made to allow for these deviations, and calculations to determine the bending dimensions in a typical example are given in section 10.3.5, with the completed bar schedule in *Table 2.23*.

3.2.6 Stainless steel reinforcement

The type of reinforcement to be used in a structure is usually selected on the basis of initial costs. This normally results in the use of carbon steel reinforcement, which is around 15% of the cost of stainless steel. For some structures, however, the selective use of stainless steel reinforcement – on exposed surfaces, for example – can be justified. In Highways Agency document BA 84/02, it is recommended that stainless steel reinforcement should be used in splash zones, abutments, parapet edges and soffits, and where the chances of chloride attack are greatest. It is generally considered that, where the concrete is saturated and oxygen movement limited, stainless steel is not required. Adherence to these guidelines can mean that the use of stainless steel reinforcement only marginally increases construction costs, while significantly reducing the whole-life costs of the structure and increasing its usable life.

Stainless steels are produced by adding elements to iron to achieve the required compositional balance. The additional elements, besides chromium, can include nickel, manganese, molybdenum and titanium, with the level of carbon being controlled during processing. These alloying elements affect the steel's microstructure, as well as its mechanical properties and corrosion resistance. Four ranges of stainless steel are produced, two of which are recommended for reinforcement to concrete because of their high resistance to corrosion. Austenitic stainless steels, for which chromium and nickel are the main alloying elements, have good general properties including corrosion resistance and are normally suitable for most applications. Duplex stainless steels, which have high chromium and low nickel contents, provide greater corrosion resistance for the most demanding environments.

In the United Kingdom, austenitic stainless steel reinforcement has been produced to the requirements of BS 6744, which is broadly aligned to conventional reinforcement practice. Thus, plain and ribbed bars are available in the same characteristic strengths and range of preferred sizes as normal carbon steel reinforcement. Traditionally, stainless steel reinforcement has only been stocked in maximum lengths of 6 m, for all sizes. Bars are currently available in lengths up to 12 m for sizes up to 16 mm. For larger sizes, bars can be supplied to order in lengths up to 8 m. Comprehensive data and recommendations on the use of stainless steel reinforcement are given in ref. 14.

3.2.7 Prefabricated reinforcement systems

In order to speed construction by reducing the time needed to fix reinforcement, it is important to be able to pre-assemble much of the reinforcement. This can be achieved on site, given adequate space and a ready supply of skilled personnel. In many cases, with careful planning and collaboration at an early stage, the use of reinforcement assemblies prefabricated by the supplier can provide considerable benefits.

A common application is the use of fabric reinforcement as described in sections 3.2.3 and 10.3.2. The preferred range of designated fabrics can be routinely used in slabs and walls. In cases involving large areas with long spans and considerable repetition, made-to-order fabrics can be specially designed to suit specific projects. Provision for small holes and openings can be made, by cutting the fabric on site after placing the sheets, and adding loose trimming bars as necessary. While sheets of fabric can be readily handled normally, they are awkward to lift over column starter bars. In such cases, it is generally advisable to provide the reinforcement local to the column as loose bars fixed in the conventional manner.

A more recent development is the use of slab reinforcement rolls that can be unrolled directly into place on site. Each made-to-order roll consists of reinforcement of the required size and spacing in one direction, welded to thin metal bands and rolled around hoops that are later discarded. Rolls can be produced up to a maximum bar length of 15 m and a weight of 5 tonnes. The width of the sheet when fully rolled out could be more than 50 m, depending upon the bar size and spacing. The full range of preferred bar sizes can be used, and the bar spacing and length can be varied within the same roll. For each area of slab and for each surface to be reinforced, two rolls are required. These are delivered to site, craned into position and unrolled on continuous bar supports. Each roll provides the bars in one direction, with those in the lower layer resting on conventional spacers or chairs.

The need to provide punching shear reinforcement in solid flat slabs in the vicinity of the columns has resulted in several proprietary reinforcement systems. Vertical reinforcement is required in potential shear failure zones around the columns, until a position is reached at which the slab can withstand the shear stresses without reinforcement. Conventional links are difficult and time-consuming to set out and fix. Single-legged links are provided with a hook at the top and a 90° bend at the bottom. Each link has to be hooked over a top bar in the slab, and the 90° bend pushed under a bottom bar and tied in place.

Shear ladders can be used, in which a row of single-legged links are connected by three straight anchor bars welded to form a robust single unit. The ladders provide the required shear reinforcement and act as chairs to support the top bars. The size, spacing and height of the links can be varied to suit the design requirements. Shear hoops consist of U-shaped links welded to upper and lower hoops to form a three-dimensional unit. By using hoops of increasing size, shear reinforcement can be provided on successive perimeters.

Shear band strips, with a castellated profile, are made from 25 mm wide high-tensile steel strip in a variety of gauges to cater for different shear capacities. The strip has perforated holes along the length to help with anchorage and fixing. The peaks and troughs of the profile are spaced to coincide with the spacing of the main reinforcement. Stud rails consist of a row of steel studs welded to a flat steel strip or a pair of rods. The studs are fabricated from plain or deformed reinforcing bars, with an enlarged head welded to one or both ends. The size, spacing and height of the studs can be varied to suit the shear requirements and the slab depth.

The use of reinforcement continuity strips is a simple and effective means of providing reinforcement continuity across construction joints. A typical application occurs at a junction between a wall and a slab that is to be cast at a later stage. The strips comprise a set of special pre-bent bars housed in a galvanised indented steel casing that is fabricated off-site in a factory-controlled environment. On site, the entire unit is cast into the front face of the wall. After the formwork is struck, the lid of the casing is removed to reveal the legs of the bars contained within the casing. The legs are then straightened outwards by the contractor, ready for lapping with the main reinforcement in the slab. The casing remains embedded in the wall, creating a rebate into which the slab concrete flows and eliminating the need for traditional joint preparation.

3.2.8 Fixing of reinforcement

Reinforcing bars need to be tied together, to prevent their being displaced and provide a rigid system. Bar assemblies and fabric reinforcement need to be supported by spacers and chairs, to ensure that the required cover is achieved and kept during the subsequent placing and compaction of concrete. Spacers should be fixed to the links, bars or fabric wires that are nearest to the concrete surface to which the cover is specified. Recommendations for the specification and use of spacers and chairs, and the tying of reinforcement, are given in BS 7973 Parts 1 and 2. These include details of the number and position of spacers, and the frequency of tying.

3.3 FIRE-RESISTANCE

Building structures need to conform, in the event of fire, to performance requirements stated in the Building Regulations. For stability, the elements of the structure need to provide a specified minimum period of fire-resistance in relation to a standard test. The required fire period depends on the purpose group of the building and the height or, for basements, depth of the building relative to the ground, as given in *Table 3.12*. Building insurers may require longer fire periods for storage facilities, where the value of the contents and the costs of reinstatement of the structure are particularly important.

In BS 8110, design for fire-resistance is considered at two levels. Part 1 contains simple recommendations suitable for most purposes. Part 2 contains a more detailed treatment with a choice of three methods: involving tabulated data, furnace tests or fire engineering calculations. The tabulated data is in the form of minimum specified values of member size and concrete cover. The cover is given to the main reinforcement and, in the case of beams and ribs, can vary in relation to the actual width of the section. The recommendations in Part 1 are based on the same data but the presentation is different in two respects: values are given for the nominal cover to all reinforcement (this includes an allowance for links in the case of beams and columns), and the values do not vary in relation to the width of the section. The required nominal covers to all reinforcement and minimum dimensions for various members are given in *Tables 3.10* and *3.11* respectively.

In the event of a fire in a building, the vulnerable elements are the floor construction above the fire, and any supporting columns or walls. The fire-resistance of the floor members (beams, ribs and slabs) depends upon the protection provided to the bottom reinforcement. The steel begins to lose strength at a temperature of 300ºC, losses of 50% and 75% occurring at temperatures of about 560ºC and 700ºC respectively. The concrete cover needs to be sufficient to delay the time taken to reach a temperature likely to result in structural failure. A distinction is made between simply supported spans, where a 50% loss of strength in the bottom reinforcement could be critical, and continuous spans, where a greater loss is allowed because the top reinforcement will retain its full capacity.

If the cover becomes excessive, there is a risk of premature spalling of the concrete in the event of fire. Concretes made with aggregates containing a high proportion of silica are the most susceptible. In cases where the nominal cover needs to exceed 40 mm, additional measures should be considered and several possible courses of action are described in Part 2 of BS 8110. The preferred approach is to reduce the cover by providing additional protection, in the form of an applied finish or a false ceiling, or by using lightweight aggregates or sacrificial steel. The last measure refers to the provision of more steel than is necessary for normal purposes, so that a greater loss of strength can be allowed in the event of fire. If the nominal cover does exceed 40 mm, then supplementary reinforcement in the form of welded steel fabric should be placed within the thickness of the cover at 20 mm from the concrete surface. There are considerable practical difficulties with this approach and it may conflict with the requirements for durability in some cases.

For concrete made with lightweight aggregate, the nominal cover requirements are all reduced, and the risk of premature spalling only needs to be considered when the cover exceeds 50 mm. The detailed requirements for lightweight aggregate concrete, and guidance on the additional protection provided by selected applied finishes are given in *Table 3.10*.

EC 2 contains a more flexible approach to fire safety design, based on the concept of ‘load ratio’, which is the ratio of the load applied at the fire limit-state to the capacity of the element at ambient temperature.

Chapter 4

Structural analysis

Torsion-less beams are designed as linear elements subjected to bending moments and shear forces. The values for freely supported beams and cantilevers are readily determined by the simple rules of static equilibrium, but the analysis of continuous beams and statically indeterminate frames is more complex. Historically, various analytical techniques have been developed and used as self-contained methods to solve particular problems. In time, it was realised that the methods could be divided into two basic categories: *flexibility* methods (otherwise known as action methods, compatibility methods or force methods) and *displacement* methods (otherwise known as stiffness methods or equilibrium methods). The behaviour of the structure is considered in terms of unknown forces in the first category, and unknown displacements in the second category. For each method, a *particular* solution, obtained by modifying the structure to make it statically determinate, is combined with a *complementary* solution, in which the effect of each modification is determined. Consider the case of a continuous beam. For the flexibility methods, the particular solution involves removing redundant actions (i.e. the continuity between the individual members) to leave a series of disconnected spans. For the displacement methods, the particular solution involves restricting the rotations and/or displacements that would otherwise occur at the joints.

To clarify further the main differences between the methods in the two categories, consider a propped cantilever. With the flexibility approach, the first step is to remove the prop and calculate the deflection at the position of the prop due to the action of the applied loads: this gives the particular solution. The next step is to calculate the concentrated load needed at the position of the prop to restore the deflection to zero: this gives the complementary solution. The calculated load is the reaction in the prop: knowing this enables the moments and forces in the propped cantilever to be simply determined. If the displacement approach is used, the first step is to consider the span as fully fixed at both ends and calculate the moment at the propped end due to the applied loads: this gives the particular solution. The next step is to release the restraint at the propped end and apply an equal and opposite moment to restore the rotation to zero: this gives the complementary solution. By combining the moment diagrams, the resulting moments and forces can be determined.

In general, there are several unknowns and, irrespective of the method of analysis used, the preparation and solution of a set of simultaneous equations is required. The resulting relationship between forces and displacements embodies a series of coefficients that can be set out concisely in matrix form. If flexibility methods are used, the resulting matrix is built up of flexibility coefficients, each of which represents a displacement produced by a unit action. Similarly, if stiffness methods are used, the resulting matrix is formed of stiffness coefficients, each of which represents an action produced by a unit displacement. The solution of matrix equations, either by matrix inversion or by a systematic elimination process, is ideally suited to computer technology. To this end, methods have been devised (the so-called matrix stiffness and matrix flexibility methods) for which the computer both sets up and solves the simultaneous equations (ref. 15).

Here, it is worthwhile to summarise the basic purpose of the analysis. Calculating the bending moments on individual freely supported spans ensures that equilibrium is maintained. The analytical procedure that is undertaken involves linearly transforming these free-moment diagrams in a manner that is compatible with the allowable deformations of the structure. Under ultimate load conditions, deformations at the critical sections must remain within the limits that the sections can withstand and, under service load conditions, deformations must not result in excessive deflection or cracking or both. If the analysis is able to ensure that these requirements are met, it will be entirely satisfactory for its purpose: endeavouring to obtain painstakingly precise results by over-complex methods is unjustified in view of the many uncertainties involved.

To determine at any section the effects of the applied loads and support reactions, the basic relationships are as follows:

Shear force
= Σ(forces on one side of section)
= rate of change of bending moment
Bending moment
= Σ(moments of forces on one side of section)
= $\int$(shear force) = area of shear force diagram
Slope
= (curvature) = area of curvature diagram
Deflection
= $\int$(slope) = area of slope diagram

For elastic behaviour, curvature = M/EI where M is bending moment, E is modulus of elasticity of concrete, I is second moment of area of section. For the purposes of structural analysis to determine bending moments due to applied loads, *I* values may normally be based on the gross concrete section. In determining deflections, however, due allowance needs to be made for the effects of cracking and, in the long term, for the effects of concrete creep and shrinkage.

4.1 SINGLE-SPAN BEAMS AND CANTILEVERS

Formulae to determine the shearing forces, bending moments and deflections produced by various general loads on beams, freely supported at the ends, are given in *Table 2.24*. Similar expressions for some particular load arrangements commonly encountered on beams, either freely supported or fully fixed at both ends, with details of the maximum values, are given in *Table 2.25*. The same information but relating to simple and propped cantilevers is given in *Tables 2.26* and *2.27* respectively. Combinations of loads can be considered by summing the results obtained for each individual load.

In *Tables 2.24–2.27*, expressions are also given for the slopes at the beam supports and the free (or propped) end of a cantilever. Information regarding the slope at other points is seldom required. If needed, it is usually a simple matter to obtain the slope by differentiating the deflection formula with respect to *x*. If the resulting expression is equated to zero and solved to obtain *x*, the point of maximum deflection will have been found. This value of *x* can then be substituted into the original formula to obtain the maximum deflection.

Coefficients to determine the fixed-end moments produced by various symmetrical and unsymmetrical loads on beams, fully fixed at both ends, are given in *Table 2.28*. Loadings not shown can usually be considered by using the tabulated cases in combination. For the general case of a partial uniform or triangular distribution of load placed anywhere on a member, a full range of charts is contained in *Examples of the Design of Reinforced Concrete Buildings*. The charts give deflection and moment coefficients for beams (freely supported or fully fixed at both ends) and cantilevers (simple or propped).

4.2 CONTINUOUS BEAMS

Historically, various methods of structural analysis have been developed for determining the bending moments and shearing forces on beams continuous over two or more spans. Most of these have been stiffness methods, which are generally better suited than flexibility methods to hand computation. Some of these approaches, such as the theorem of three-moments and the methods of fixed points and characteristic points, were included in the previous edition of this *Handbook*. If beams having two, three or four spans are of uniform cross section, and support loads that are symmetrical on each individual span, formulae and coefficients can be derived that enable the support moments to be determined by direct calculation. Such a method is given in *Table 2.37*. More generally, in order to avoid the need to solve large sets of simultaneous equations, methods involving successive approximations have been devised. Despite the general use of computers, hand methods can still be very useful in dealing with routine problems. The ability to use hand methods also develops in the engineer an appreciation of analysis that is invaluable in applying output from the computer.

When bending moments are calculated with the spans taken as the distances between the centres of supports, the critical negative moment in monolithic forms of construction can be considered as that occurring at the edge of the support. When the supports are of considerable width, the span can be taken as the clear distance between the supports plus the effective depth of the beam, or an additional span can be introduced that is equal to the width of the support minus the effective depth of the beam. The load on this additional span should be taken as the support reaction spread uniformly over the width of the support. If a beam is constructed monolithically with a very wide and massive support, the effect of continuity with the span or spans beyond the support may be negligible, in which case the beam should be treated as fixed at the support.

The second moment of area of a reinforced concrete beam of uniform depth may still vary throughout its length, due to variations in the amount of reinforcement and also because, when acting with an adjoining slab, a down-stand beam may be considered as a flanged section at mid-span but a simple rectangular section at the supports. It is common practice, however, to neglect these variations for beams of uniform depth, and use the value of *I* for the plain rectangular section. It is often assumed that a continuous beam is freely supported at the ends, even when beam and support are constructed monolithically. Some provision should still be made for the effects of end restraint.

4.2.1 Analysis by moment distribution

Probably the best-known and simplest system for analysing continuous beams by hand is that of *moment distribution*, as devised by Hardy Cross in 1929. The method, which derives from slope-deflection principles, is described briefly in *Table 2.36*. It employs a system of successive approximations that may be terminated as soon as the required degree of accuracy has been reached. A particular advantage of this and similar methods is that, even after only one distribution cycle, it is often clear whether or not the final values will be acceptable. If not, the analysis can be discontinued and unnecessary work avoided. The method is simple to remember and apply, and the step-by-step procedure gives the engineer a ‘feel’ for the behaviour of the system. It can be applied, albeit less easily, to the analysis of systems containing non-prismatic members and to frames. Hardy Cross moment distribution is described in many textbooks dealing with structural analysis.

Over the years, the Hardy Cross method of analysis begot various offspring. One of these is known as *precise moment distribution* (also called the coefficient of restraint method or direct moment distribution). The procedure is very similar to normal moment distribution, but the distribution and carryover factors are so adjusted that an exact solution is obtained after one distribution in each direction. The method thus has the advantage of removing the necessity to decide when to terminate the analysis. Brief details are given in *Table 2.36* and the method is described in more detail in *Examples of the Design of Reinforced Concrete Buildings* (see also ref. 16).

It should be noted that the load arrangements that produce the greatest negative bending moments at the supports are not necessarily those that produce the greatest positive bending moments in the spans. The design loads to be considered in BS 8110 and EC 2, and the arrangements of live load that give the greatest theoretical bending moments, as well as the less onerous code requirements, are given in *Table 2.29*. Some live load arrangements can result in negative bending moments throughout adjacent unloaded spans.

4.2.2 Redistribution of bending moments

For the ULS, the bending moments obtained by linear elastic analysis may be adjusted on the basis that some redistribution of moments can occur prior to collapse. This enables the effects of both service and ultimate loadings to be assessed, without the need to undertake a separate analysis using plastic-hinge techniques for the ultimate condition. The theoretical justification for moment redistribution is clearly explained in the *Handbook to BS 8110*. Since the reduction of moment at a section assumes the formation of a plastic hinge at that position prior to the ultimate condition being reached, it is necessary to limit the reduction in order to restrict the amount of plastic-hinge rotation and control the cracking that occurs under serviceability conditions. For these reasons, the maximum ratio of neutral axis depth to effective depth, and the maximum distance between tension bars, are each limited according to the required amount of redistribution.

Such adjustments are useful in reducing the inequalities between negative and positive moments, and minimising the amount of reinforcement that must be provided at a particular section, such as the intersection between beam and column, where concreting may otherwise be more difficult due to the congestion of reinforcement. Both BS 8110 and EC 2 allow the use of moment redistribution; the procedure, which may be applied to any system that has been analysed by the so-called exact methods, is described in section 12.3 with an illustrated example provided in *Table 2.33*.

4.2.3 Coefficients for equal loads on equal spans

For beams that are continuous over a number of equal spans, with equal loads on each loaded span, the maximum bending moments and shearing forces can be tabulated. In *Tables 2.30* and *2.31*, maximum bending moment coefficients are given for each span and at each support for two, three, four and five equal spans with identical loads on each span, which is the usual disposition of the dead load on a beam. Coefficients are also given for the most adverse incidence of live loads and, in the case of the support moments, for the arrangements of live load required by BS 8110 (values in square brackets) and by EC 2 (values in curved brackets). It should be noted that the maximum bending moments due to live load do not occur at all the sections simultaneously. The types of load considered are a uniformly distributed load, a central point load, two equal loads applied at the third-points of the span, and trapezoidal loads of various proportions. In *Table 2.32*, coefficients are given for the maximum shearing forces for each type of load, with identical loads on each span and due to the most adverse incidence of live loads.

4.2.4 Bending moment diagrams for equal spans

In *Tables 2.34* and *2.35*, bending moment coefficients for various arrangements of dead and live loads, with sketches of the resulting moment envelopes, are given for beams of two and three spans, and for a theoretically infinite system. This information enables appropriate bending moment diagrams to be plotted quickly and accurately. The load types considered are a uniformly distributed load, a central point load and two equal loads at the third points of the span. Values are given for identical loads on each span (for example, dead load), and for the arrangements of live load required by BS 8110 and EC 2. As the coefficients have been calculated by exact methods, moment redistribution is allowed at the ultimate state in accordance with the requirements of BS 8110 and EC 2. In addition to the coefficients obtained by linear elastic analysis, values are given for conditions in which the maximum support moments are reduced by either 10% or 30%, as described in section 12.3.3. Coefficients are also given for the positive support moments and negative span moments that occur under some arrangements of live load.

4.2.5 Solutions for routine design

A precise determination of theoretical bending moments and shearing forces on continuous beams is not always necessary. It should also be appreciated that the general assumptions of unyielding knife-edge supports, uniform sectional properties and uniform distributions of live load are hardly realistic. The indeterminate nature of these factors often leads in practice to the adoption of values based on approximate coefficients. In *Table 2.29*, values in accordance with the recommendations of BS 8110 and EC 2 are given, for bending moments and shearing forces on uniformly loaded beams of three or more spans. The values are applicable when the characteristic imposed load is not greater than the characteristic dead load and the variations in span do not exceed 15% of the longest span. The same coefficients may be used with service loads or ultimate loads, and the resulting bending moments may be considered to be without redistribution.

4.3 MOVING LOADS ON CONTINUOUS BEAMS

Bending moments caused by moving loads, such as those due to vehicles traversing a series of continuous spans, are most easily calculated with the aid of influence lines. An influence line is a curve with the span of the beam taken as the base, the ordinate of the curve at any point being the value of the bending moment produced at a particular section when a unit load acts at the point. The data given in *Tables 2.38–2.41* enable the influence lines for the critical sections of beams continuous over two, three, four and five or more spans to be drawn. By plotting the position of the load on the beam (to scale), the bending moments at the section being considered can be derived, as explained in the example given in Chapter 12. The curves given for equal spans can be used directly, but the corresponding curves for unequal spans need to be plotted from the data tabulated.

The bending moment due to a load at any point is equal to the ordinate of the influence line at the point multiplied by the product of the load and the span, the length of the shortest span being used when the spans are unequal. The influence lines in the tables are drawn for a symmetrical inequality of spans. The symbols on each curve indicate the section of the beam and the ratio of span lengths to which the curve applies.

4.4 ONE-WAY SLABS

In monolithic building construction, the column layout often forms a rectangular grid. Continuous beams may be provided in one direction or two orthogonal directions, to support slabs that may be solid or ribbed in cross section. Alternatively, the slabs may be supported directly on the columns, as a flat slab. Several different forms of slab construction are shown in *Table 2.42*. These are considered in more detail in the general context of building structures in Chapter 6.

Where beams are provided in one direction only, the slab is a one-way slab. Where beams are provided in two orthogonal directions, the slab is a two-way slab. However, if the longer side of a slab panel exceeds twice the shorter side, the slab is generally designed as a one-way slab. A flat slab is designed as a one-way slab in each direction. Bending moments and shearing forces are usually determined on strips of unit width for solid slabs, and strips of width equal to the spacing of the ribs for ribbed slabs.

The comments in section 4.2.5, and the coefficients for the routine design of beams given in *Table 2.29*, apply equally to one-way spanning slabs. This is particularly true when elastic moments due to service loads are required. However, lightly reinforced slabs are highly ductile members, and allowance is generally made for redistribution of elastic moments at the ULS.

4.4.1 Uniformly distributed load

For slabs carrying uniformly distributed loads and continuous over three or more nearly equal spans, approximate solutions for ultimate bending moments and shearing forces, according to BS 8110 and EC 2, are given in *Table 2.42*. In both cases, the support moments include an allowance for 20% redistribution, but the situation regarding the span moments is somewhat different in the two codes.

In BS 8110, a simplified arrangement of the design loads is permitted, where the characteristic imposed load does not exceed 1.25 × the characteristic dead load or 5 kN/m^2, excluding partitions, and the area of each bay exceeds 30 m^2. Design for a single load case of maximum design load on all spans is considered sufficient, providing the support moments are reduced by 20% and the span moments are increased to maintain equilibrium. Although the resulting moments are compatible with yield-line theory, the span moments are less than those that would occur in the case of alternate spans being loaded with maximum load and minimum load. The implicit redistribution of the span moments, the effect of which on the reinforcement stress under service loads would be detrimental to the deflection of the beam, is ignored in the subsequent design. In EC 2, this simplification is not included and the values given for the span moments are the same as those for beams in *Table 2.29*.

Provision is made in *Table 2.42* for conditions where a slab is continuous with the end support. The restraining element may vary from a substantial wall to a small edge beam, and allowance has been made for both eventualities. The support moment is given as $-0.04Fl$, but the reduced span moment is based on the support moment being no more than $-0.02Fl$.

4.4.2 Concentrated loads

When a slab supported on two opposite sides carries a load concentrated on a limited area of the slab, such as a wheel load on the deck of a bridge, conventional elastic methods of analysis based on isotropic plate theory are often used. These may be in the form of equations, as derived by Westergaard (ref. 17), or influence surfaces, as derived by Pucher (ref. 18). Another approach is to extend to one-way spanning slabs, the theory applied to slabs spanning in two directions. For example, the curves given in *Table 2.47* for a slab infinitely long in the direction l_y can be used to evaluate directly the bending moments in the direction of, and at right angles to, the span of a one-way slab carrying a concentrated load; this method has been used to produce the data for elastic analysis given in *Table 2.45*.

For designs in which the ULS requirement is the main criterion, a much simpler approach is to assume that a certain width of slab carries the entire load. In BS 8110, for example, the effective width for solid slabs is taken as the load width plus $2.4x(1 - x/l)$, x being the distance from the nearer support to the section under consideration and l the span. Thus, the maximum width at mid-span is equal to the load width plus $0.6l$. Where the concentrated load is near an unsupported edge of a slab, the effective width should not exceed $1.2x(1 - x/l)$ plus the distance of the slab edge from the further edge of the load. Expressions for the resulting bending moments are given in *Table 2.45*. For ribbed slabs, the effective width will depend on the ratio of the transverse and longitudinal flexural rigidities of the slab, but need not be taken less than the load width plus $4x/l(1 - x/l)$ metres.

The solutions referred to so far are for single-span slabs that are simply supported at each end. The effects of end-fixity or continuity may be allowed for, approximately, by multiplying the moment for the simply supported case by an appropriate factor. The factors given in *Table 2.45* are derived by elastic beam analysis.

4.5 TWO-WAY SLABS

When a slab is supported other than on two opposite sides only, the precise amount and distribution of the load taken by each support, and consequently the magnitude of the bending moments on the slab, are not easily calculated if assumptions resembling real conditions are made. Therefore, approximate analyses are generally used. The method applicable in any particular case depends on the shape of the slab panel, the conditions of restraint at the supports and the type of load.

Two basic methods are commonly used to analyse slabs that span in two directions. The theory of plates, which is based on elastic analysis, is particularly appropriate to the behaviour under service loads. Yield-line theory considers the behaviour of the slab as a collapse condition approaches. Hillerborg's strip method is a less well-known alternative to the use of yield-line in this case. In some circumstances, it is convenient to use coefficients derived by an elastic analysis with loads that are factored to represent ULS conditions. This approach is used in BS 8110 for the case of a simply supported slab with corners that are not held down or reinforced for torsion. It is also normal practice to use elastic analysis for both service and ULS conditions in the design of bridge decks and liquid-retaining structures. For elastic analyses, a Poisson's ratio of 0.2 is recommended in BS 8110 and BS 5400: Part 4. In EC 2, the values given are 0.2 for uncracked concrete and 0 for cracked concrete.

The analysis must take account of the support conditions, which are often idealised as being free or hinged or fixed, and whether or not the corners of the panels are held down. A free condition refers to an unsupported edge as, for example, the top of a wall of an uncovered rectangular tank. The condition of being freely or simply supported, with the corners not held down, may occur when a slab is not continuous and the edges bear directly on masonry walls or structural steelwork. If the edge of the slab is built into a substantial masonry wall, or is constructed monolithically with a reinforced concrete beam or wall, a condition of partial restraint exists. Such restraint may be allowed for when computing the bending moments on the slab, but the support must be able to resist the torsion and/or bending effects, and the slab must be reinforced to resist the negative bending moment. A slab can be considered as fixed along an edge if there is no change in the slope of the slab at the support irrespective of the incidence of the load. A fixed condition could be assumed if the polar second moment of area of the beam or other support is very large. Continuity over a support generally implies a condition of restraint less rigid than fixity; that is, the slope of the slab at the support depends upon the incidence of load not only on the panel under consideration but also on adjacent panels.

4.5.1 Elastic methods

The so-called exact theory of the elastic bending of plates spanning in two directions derives from work by Lagrange, who produced the governing differential equation for plate bending in 1811, and Navier, who in 1820 described the use of a double trigonometric series to analyse freely supported rectangular plates. Pigeaud and others later developed the analysis of panels freely supported along all four edges.

Many standard elastic solutions have been produced but almost all of these are restricted to square, rectangular and circular slabs (see, for example, refs. 19, 20 and 21). Exact analysis of a slab having an arbitrary shape and support conditions with a general arrangement of loading would be extremely complex. To deal with such problems, numerical techniques such as finite differences and finite elements have been devised. Some notes on finite elements are given in section 4.9.7. Finite-difference methods are considered in ref. 15 (useful introduction) and ref. 22 (detailed treatment). The methods are suited particularly to computer-based analysis, and continuing software developments have led to the techniques being readily available for routine office use.

4.5.2 Collapse methods

Unlike in frame design, where the converse is generally true, it is normally easier to analyse slabs by collapse methods than by elastic methods. The most-widely known methods of plastic analysis of slabs are the yield-line method developed by K W Johansen, and the so-called strip method devised by Arne Hillerborg.

It is generally impossible to calculate the precise ultimate resistance of a slab by collapse theory, since such elements are highly indeterminate. Instead, two separate solutions can be found – one being upper bound and the other lower bound. With solutions of the first type, a collapse mechanism is first postulated. Then, if the slab is deformed, the energy absorbed in inducing ultimate moments along the yield lines is equal to the work done on the slab by the applied load in producing this deformation. Thus, the load determined is the maximum that the slab will support before failure occurs. However, since such methods do not investigate conditions between the postulated yield lines to ensure that the moments in these areas do not exceed the ultimate resistance of the slab, there is no guarantee that the minimum possible collapse load has been found. This is an inevitable shortcoming of upper-bound solutions such as those given by Johansen's theory.

Conversely, lower-bound solutions will generally result in the determination of collapse loads that are less than the maximum that the slab can actually carry. The procedure here is to choose a distribution of ultimate moments that ensures that equilibrium is satisfied throughout, and that nowhere is the resistance of the slab exceeded.

Most of the literature dealing with the methods of Johansen and Hillerborg assumes that any continuous supports at the slab edges are rigid and unyielding. This assumption is also made throughout the material given in Part 2 of this book. However, if the slab is supported on beams of finite strength, it is possible for collapse mechanisms to form in which the yield lines pass through the supporting beams. These beams would then become part of the mechanism considered, and such a possibility should be taken into account when using collapse methods to analyse beam-and-slab construction.

Yield-line analysis. Johansen's method requires the designer to first postulate an appropriate collapse mechanism for the slab being considered according to the rules given in section 13.4.2. Variable dimensions (such as αl_y on diagram (iv)(a) in *Table 2.49*) may then be adjusted to obtain the maximum ultimate resistance for a given load (i.e. the maximum ratio of M/F). This maximum value can be found in various ways: for example by tabulating the work equation as shown in section 13.4.8, using actual numerical values and employing a trial-and-adjustment process. Alternatively, the work equation may be expressed algebraically and, by substituting various values for α, the maximum ratio of M/F may be read from a graph relating α to M/F. Another method is to use calculus to differentiate the equation and then, by setting this equal to zero, determine the critical value of α. This method cannot always be used, however (see ref. 23).

As already explained, although such processes enable the maximum resistance for a given mode of failure to be found, they do not indicate whether the yield-line pattern considered is the critical one. A further disadvantage of such a method is that, unlike Hillerborg's method, it gives no direct indication of the resulting distribution of load on the supports. Although it seems possible to use the yield-line pattern as a basis for apportioning the loaded areas of slab to particular supports, there is no real justification for this assumption (see ref. 23). In spite of these shortcomings, yield-line theory is extremely useful. A considerable advantage is that it can be applied relatively easily to solve problems that are almost intractable by other means.

Yield-line theory is too complex to deal with adequately in this *Handbook*; indeed, several textbooks are completely or almost completely devoted to the subject (refs. 23–28). In section 13.4

and *Tables 2.49* and *2.50*, notes and examples are given on the rules for choosing yield-line patterns for analysis, on theoretical and empirical methods of analysis, on simplifications that can be made by using so-called affinity theorems, and on the effects of corner levers.

Strip method. Hillerborg devised his strip method in order to obtain a lower-bound solution for the collapse load, while achieving a good economical arrangement of reinforcement. As long as the reinforcement provided is sufficient to cater for the calculated moments, the strip method enables such a lower-bound solution to be obtained. (Hillerborg and others sometimes refer to the strip method as the equilibrium theory; this should not, however, be confused with the equilibrium method of yield-line analysis.) In Hillerborg's original theory (now known as the *simple strip method*), it is assumed that, at failure, no load is resisted by torsion and thus, all load is carried by flexure in either of two principal directions. The theory results in simple solutions giving full information regarding the moments over the whole slab to resist a unique collapse load, the reinforcement being placed economically in bands. Brief notes on the use of simple strip theory to design rectangular slabs supporting uniform loads are given in section 13.5 and *Table 2.51*.

However, the simple strip theory is unable to deal with concentrated loads and/or supports and leads to difficulties with free edges. To overcome such problems, Hillerborg later developed his *advanced strip method*, which involves the use of complex moment fields. Although this development extends the scope of the simple strip method, it somewhat spoils the simplicity and directness of the original concept. A full treatment of both the simple and advanced strip theories is given in ref. 29.

A further disadvantage of both Hillerborg's and Johansen's methods is that, being based on conditions at failure only, they permit unwary designers to adopt load distributions that may differ widely from those that would occur under service loads, with the risk of unforeseen cracking. A development that eliminates this problem, as well as overcoming the limitations arising from simple strip theory, is the so-called strip-deflection method due to Fernando and Kemp (ref. 30). With this method the distribution of load in either principal direction is not selected arbitrarily by the designer (as in the Hillerborg method or, by choosing the ratio of reinforcement provided in each direction, as in the yield-line method) but is calculated so as to ensure compatibility of deflection in mutually orthogonal strips. The method results in sets of simultaneous equations (usually eight), the solution of which requires computer assistance.

4.5.3 Rectangular panel with uniformly distributed load

The bending moments in rectangular panels depend on the support conditions and the ratio of the lengths of the sides of the panel. The ultimate bending moment coefficients given in BS 8110 are derived from a yield-line analysis, in which the values of the coefficients have been adjusted to suit the division of the panel into middle and edge strips, as shown in *Table 2.42*. Reinforcement to resist the bending moments calculated from the data given in *Table 2.43* is required only within the middle strips, which are of width equal to three-quarters of the panel width in each direction. The ratio of the negative moment at a continuous edge to the positive moment at mid-span has been chosen as 4/3 to conform approximately to the serviceability requirements. For further details on the derivation of the coefficients, see ref. 31. Nine types of panel are considered in order to cater for all possible combinations of edge conditions. Where two different values are obtained for the negative moment at a continuous edge, because of differences between the contiguous panels, the values may be treated as fixed-end moments and distributed elastically in the direction of span. The procedure is illustrated by means of a worked example in section 13.2.1. Minimum reinforcement as given in BS 8110 is to be provided in the edge strips. Torsion reinforcement is required at corners where either one or both edges of the panel are discontinuous. Values for the shearing forces at the ends of the middle strips are also given in *Table 2.43*.

Elastic bending moment coefficients, for the same types of panel (except that the edge conditions are now defined as fixed or hinged, rather than continuous or discontinuous), are given in *Table 2.44*. The information has been prepared from data given in ref. 21, which was derived by finite element analysis, and includes for a Poisson's ratio of 0.2. For ratios less than 0.2, the positive moments at mid-span are reduced slightly and the torsion moments at the corners are increased. The coefficients may be adjusted to suit a Poisson's ratio of zero, as explained in section 13.2.2.

The simplified analysis due to Grashof and Rankine can be used for a rectangular panel, simply supported on four sides, when no provision is made to resist torsion at the corners or to prevent the corners from lifting. A solution is obtained by considering uniform distributions of load along orthogonal strips in each direction and equating the elastic deflections at the middle of the strips. The proportions of load carried by each strip are then obtained as a function of the ratio of the spans, and the resulting mid-span moments are calculated. Bending moment coefficients for this case are also provided in *Table 2.44*, and basic formulae are given in section 13.2.2.

4.5.4 Rectangular panel with triangularly distributed load

In the design of rectangular tanks, storage bunkers and some retaining structures, cases occur of wall panels spanning in two directions and subjected to triangular distributions of pressure. The intensity of pressure is uniform at any level, but vertically the pressure increases linearly from zero at the top to a maximum at the bottom. Elastic bending moment and shear force coefficients are given for four different types of panel, to cater for the most common combinations of edge conditions, in *Table 2.53*. The information has been prepared from data given in ref. 32, which was derived by finite element analysis and includes for a Poisson's ratio of 0.2. For ratios less than 0.2, the bending moments would be affected in the manner discussed in section 4.5.3.

The bending moments given for individual panels, fixed at the sides, may be applied without modification to continuous walls, provided there is no rotation about the vertical edges. In a square tank, therefore, moment coefficients can be taken directly from *Table 2.53*. For a rectangular tank, distribution of the unequal negative moments at the corners is needed.

An alternative method of designing the panels would be to use yield-line theory. If the resulting structure is to be used to store liquids, however, extreme care must be taken to ensure that the adopted proportions of span to support moment and vertical to horizontal moment conform closely to those given by elastic analyses. Otherwise, the predicted service moments and calculated crack widths will be invalid and the structure may be unsuitable for its intended purpose. In the case of structures with non-fluid contents, such considerations may be less important. This matter is discussed in section 13.6.2.

Johansen has shown (ref. 24), for a panel fixed or freely supported along the top edge, that the total ultimate moment acting on the panel is identical to that on a similar panel with the same total load uniformly distributed. Furthermore, as in the case of the uniformly loaded slab considered in section 13.4.6, a restrained slab may be analysed as if it were freely supported by employing so-called reduced side lengths to represent the effects of continuity or fixity. Of course, unlike the uniformly loaded slab, along the bottom edge of the panel where the loading is greatest, a higher ratio of support to span moment should be adopted than at the top edge of the panel. If the panel is unsupported along the top edge, its behaviour is controlled by different collapse mechanisms. The relevant expressions developed by Johansen (ref. 24) are represented graphically in *Table 2.54*. Triangularly loaded panels can also be designed by means of Hillerborg's strip method (ref. 29), shown also in *Table 2.54*.

4.5.5 Rectangular panels with concentrated loads

Elastic methods can be used to analyse rectangular panels carrying concentrated loads. The curves in *Tables 2.46* and *2.47*, based on Pigeaud's theory, give bending moments on a panel freely supported along all four edges with restrained corners, and carrying a load uniformly distributed over a defined area symmetrically disposed upon the panel. Wheel loads, and similarly highly concentrated loads, are considered to be dispersed through the thickness of any surfacing down to the top of the slab, or farther down to the mid-depth of the slab, as described in section 2.4.9. The dimensions a_x and a_y of the resulting boundary are used to determine a_x/l_x and a_y/l_y, for which the bending moment factors α_{x4} and α_{y4} are read off the curves, according to the ratio of spans $k = l_y/l_x$.

For a total load F acting on the area a_x by a_y, the positive bending moments per unit width of slab are given by the expressions in *Tables 2.46* and *2.47*, in which the value of Poisson's ratio is normally taken as 0.2. The curves are drawn for k values of 1.0, 1.25, $\sqrt{2}$ (= 1.41 approx.), 1.67, 2.0, 2.5 and infinity. For intermediate values of k, the values of α_{x4} and α_{y4} can be interpolated from the values above and below the given value of k. The use of the curves for $k = 1.0$, which apply to a square panel, is explained in section 13.3.2.

The curves for $k = \infty$ apply to panels where l_y is very much greater than l_x, and can be used to determine the transverse and longitudinal bending moments for a long narrow panel supported on the two long edges only. This chart has been used to produce the elastic data for one-way slabs given in *Table 2.45*, as mentioned in section 4.4.2.

For panels that are restrained along all four edges, Pigeaud recommends that the mid-span moments be reduced by 20%. Alternatively, the multipliers given for one-way slabs could be used, if the inter-dependence of the bending moments in the two directions is ignored. Pigeaud's recommendations for the maximum shearing forces are given in section 13.3.2.

To determine the load on the supporting beams, the rules in section 4.6 for a load distributed over the entire panel are sufficiently accurate for a load concentrated at the centre of the panel. This is not always the critical case for live loads, such as a load imposed by a wheel on a bridge deck, since the maximum load on the beam occurs when the wheel is passing over the beam, in which case the beam carries the whole load.

Johansen's yield-line theory and Hillerborg's strip method can also be used to analyse slabs carrying concentrated loads. Appropriate yield-line formulae are given in ref. 24, or the method described in section 13.4.8 may be used. For details of the analysis involved if the advanced strip method is used, see ref. 29.

4.6 BEAMS SUPPORTING RECTANGULAR PANELS

When designing beams supporting a uniformly loaded panel that is freely supported along all four edges or with the same degree of fixity along all four edges, it is generally accepted that each of the beams along the shorter edges of the panel carries load on an area in the shape of a 45° isosceles triangle, whose base is equal to the length of the shorter side, for example, each beam carries a triangularly distributed load. Each beam along the longer edges of the panel carries the load on a trapezoidal area. The amount of load carried by each beam is given by the diagram and expressions in the top left-hand corner of *Table 2.52*. In the case of a square panel, each beam carries a triangularly distributed load equal to one-quarter of the total load on the panel. For beams with triangular and trapezoidal distributions of loading, fixed-end moments and moments for continuous beams are given in *Tables 2.28*, *2.30* and *2.31*.

When a panel is fixed or continuous along one, two or three supports and freely supported on the remaining edges, the sub-division of the total load to the various supporting beams can be determined from the diagrams and expressions on the left-hand side of *Table 2.52*. If the panel is unsupported along one edge or two adjacent edges, the loads on the supporting beams at the remaining edges are as given on the right-hand side of *Table 2.52*. The expressions, which are given in terms of a service load w, may be applied also to an ultimate load n.

For slabs designed in accordance with the BS 8110 method, the loads on the supporting beams may be determined from the shear forces given in *Table 2.43*. The relevant loads are taken as uniformly distributed along the middle three-quarters of the beam length, and the resulting fixed-end moments can be determined from *Table 2.28*.

4.7 NON-RECTANGULAR PANELS

When a panel that is not rectangular is supported along all its edges and is of such proportions that main reinforcement in two directions seems desirable, the bending moments can be determined approximately from the data given in *Table 2.48*. The information, derived from elastic analyses, is applicable to a trapezoidal panel approximately symmetrical about one axis, to a panel that in plan is an isosceles triangle (or nearly so), and to panels that are regular polygons or circular. The case of a triangular panel, continuous or partially restrained along three edges, occurs in pyramidal hopper bottoms. For this case, reinforcement determined for the positive moments should extend over the entire area of the panel, and provision must be made for the negative moments and for the direct tensions that act simultaneously with the bending moments.

If the shape of a panel is approximately square, the bending moments for a square slab of the same area should be used. A slab having the shape of a regular polygon with five or more sides can be treated as a circular slab, with the diameter taken as the mean of the diameters obtained for the inscribed and circumscribed circles: for regular hexagons and octagons, the mean diameters are given in *Table 2.48*.

For a panel, circular in plan, that is freely supported or fully fixed along the circumference and carries a load concentrated symmetrically about the centre on a circular area, the total bending moment to be considered acting across each of two mutually perpendicular diameters is given by the appropriate expressions in *Table 2.48*. These are based on the expressions derived by Timoshenko and Woinowski-Krieger (ref. 20). In general the radial and tangential moments vary according to the position being considered. A circular panel can therefore be designed by one of the following elastic methods:

1. Design for the maximum positive bending moment at the centre of the panel and reduce the amount of reinforcement or the thickness of the slab towards the circumference. If the panel is not truly freely supported at the edge, provide for the appropriate negative bending moment.
2. Design for the average positive bending moment across a diameter and retain the same thickness of slab and amount of reinforcement throughout the entire area of the panel. If the panel is not truly freely supported at the edge, provide for the appropriate negative bending moment.

The reinforcement required for the positive bending moments in each of the preceding methods must be provided in two directions mutually at right angles; the reinforcement for the negative bending moment should be provided by radial bars, normal to and equally spaced around the circumference, or by some equivalent arrangement.

Both circular and other non-rectangular shapes of slab may conveniently be designed for ULS conditions by using yield-line theory: the method of obtaining solutions for slabs of various shapes is described in detail in ref. 24.

4.8 FLAT SLABS

The design of flat slabs, that is, beamless slabs supported directly on columns, has often been based on empirical rules. Modern codes place much greater emphasis on the analysis of such structures as a series of continuous frames. Other methods such as grillage, finite element and yield-line analysis may be employed. The principles described hereafter, and summarised in section 13.8 and *Table 2.55*, are in accordance with the simplified method given in BS 8110. This type of slab can be of uniform thickness throughout or can incorporate thickened drop panels at the column positions. The columns may be of uniform cross section throughout or may be provided with an enlarged head, as indicated in *Table 2.55*.

The simplified method may be used for slabs consisting of rectangular panels, with at least three spans of approximately equal length in each direction, where the ratio of the longer to the shorter side of each panel does not exceed 2. Each panel is divided into column and middle strips, where the width of a column strip is taken as one-half of the shorter dimension of the panel, and bending moments determined for a full panel width are then distributed between column and middle strips as shown in *Table 2.55*. If drops of dimensions not less than one-third of the shorter dimension of the panel are provided, the width of the column strip can be taken as the width of the drop. In this case, the apportionment of the bending moments between column and middle strips is modified accordingly.

The slab thickness must be sufficient to satisfy appropriate deflection criteria, with a minimum thickness of 125 mm, and provide resistance to shearing forces and bending moments. Punching shear around the columns is a critical consideration, for which shear reinforcement can be provided in slabs not less than 200 mm thick. The need for shear reinforcement can be avoided, if drop panels or column heads of sufficient size are provided. Holes of limited dimensions may be formed in certain areas of the slab, according to recommendations given in BS 8110. Larger openings should be appropriately framed with beams designed to carry the slab loads to the columns.

4.8.1 Bending moments

The total bending moments for a full panel width, at principal sections in each direction of span, are given in *Table 2.55*. Panel widths are taken between the centrelines of adjacent bays, and panel lengths between the centrelines of columns. Moments calculated at the centrelines of the supports may be reduced as explained in section 13.8.3. The slab is effectively designed as one-way spanning in each direction, and the comments contained in section 4.4.1 also apply here.

At the edges of a flat slab, the transfer of moments between the slab and an edge or corner column may be limited by the effective breadth of the moment transfer strip, as shown in *Table 2.56*. The structural arrangement should be chosen to ensure that the moment capacity of the transfer strip is at least 50% of the outer support moment given in *Table 2.55*.

4.8.2 Shearing forces

For punching shear calculations, the design force obtained by summing the shear forces on two opposite sides of a column is multiplied by a shear enhancement factor to allow for the effects of moment transfer, as shown in *Table 2.56*. Critical perimeters for punching shear occur at distances of $1.5d$ from the faces of columns, column heads and drops, where d is the effective depth of the slab or drop, as shown in *Table 2.55*.

4.8.3 Reinforcement

At internal columns, two-thirds of the reinforcement needed to resist the negative moments in the column strips should be placed in a width equal to half that of the column strip and central with the column. Otherwise, the reinforcement needed to resist the moment apportioned to a particular strip should be distributed uniformly across the full width of the strip.

4.8.4 Alternative analysis

A more general equivalent frame method for the analysis of flat slabs is described in BS 8110. The bending moments and shearing forces are calculated by considering the structure as a series of continuous frames, transversely and longitudinally. The method is described in detail in *Examples of the design of reinforced concrete buildings*. For further information on both equivalent frame and grillage methods of analysis of flat slab structures, see ref. 33.

4.9 FRAMED STRUCTURES

A structure is statically determinate if the forces and bending moments can be determined by the direct application of the principles of equilibrium. Some examples include cantilevers (whether a simple bracket or a roof of a grandstand), a freely supported beam, a truss with pin-joints, and a three-hinged arch or frame. A statically indeterminate structure is one in which there is a redundancy of members or supports or both, and which can be analysed only by considering the elastic deformations under load. Typical examples of such structures include restrained beams, continuous beams, portal frames and other non-triangulated structures with rigid joints, and two-hinged and fixed-end arches. The general notes relating to the analysis of statically determinate and indeterminate beam systems given in sections 4.1 and 4.2 are equally valid when analysing frames. Providing a frame can be represented sufficiently accurately by an idealised two-dimensional line structure, it can be analysed by any of the methods mentioned earlier (and various others, of course).

The analysis of a two-dimensional frame is somewhat more complex than that of a beam system. If the configuration of the frame or the applied loading (or both) is unsymmetrical, side-sway will almost invariably occur, making the required analysis considerably longer. Many more combinations of load (vertical and horizontal) may need to be considered to obtain the critical moments. Different partial safety factors may apply to different load combinations. The critical design conditions for some columns may not necessarily be those corresponding to the maximum moment: loading producing a reduced moment together with an increased axial thrust may be more critical. However, to combat such complexities, it is often possible to simplify the calculations by introducing a degree of approximation. For instance, when considering wind loads acting on regular multi-bay frames, points of contra-flexure may be assumed to occur at the centres of all the beams and columns (see *Table 2.62*), thus rendering the frame statically determinate. In the case of frames that are not required to provide lateral stability, the beams at each level acting with the columns above and below that level may be considered to form a separate sub-frame for analysis.

Beeby (ref. 34) has shown that, if the many uncertainties involved in frame analysis are considered, there is little to choose as far as accuracy is concerned between analysing a frame as a single complete structure, as a set of sub-frames, or as a series of continuous beams with attached columns. If the effect of the columns is not included in the analysis of the beams, some of the calculated moments in the beams will be greater than those actually likely to occur.

It may not always be possible to represent the true frame as an idealised two-dimensional line structure, and analysis as a fully three-dimensional space frame may be necessary. If the structure consists of large solid areas such as walls, it may not be possible to represent it adequately by a skeletal frame. The finite-element method of analysis is particularly suited to solve such problems and is summarised briefly later.

In the following pages the analysis of primary frames by the methods of slope deflection and various forms of moment distribution is described. Rigorous analysis of complex rigid frames generally requires an amount of calculation out of all proportion to the real accuracy of the results, and some approximate solutions are therefore given for common cases of building frames and similar structures. When a suitable preliminary design has been justified by using approximate methods, an exhaustive exact analysis may be undertaken by employing an established computer program.

4.9.1 Building code requirements

For most framed structures, it is not necessary to carry out a full structural analysis of the complete frame as a single unit, and various simplifications are shown in *Table 2.57.* BS 8110 distinguishes between frames subjected to vertical loads only, because overall lateral stability to the structure is provided by other means, such as shear walls, and frames that are required to support both vertical and lateral loads. Load combinations consisting of (1) dead and imposed, (2) dead and wind, and (3) dead, imposed and wind are also given in *Table 2.57.*

For frames that are not required to provide lateral stability, the construction at each floor may be considered as a separate sub-frame formed from the beams at that level together with the columns above and below. The columns should be taken as fixed in position and direction at their remote ends, unless the assumption of a pinned end would be more reasonable (e.g. if a foundation detail is considered unable to develop moment restraint). The sub-frame should then be analysed for the required arrangements of dead and live loads.

As a further simplification, each individual beam span may be considered separately by analysing a sub-frame consisting of the span in question together with, at each end, the upper and lower columns and the adjacent span. These members are regarded as fixed at their remote ends, with the stiffness of the outer spans taken as only one-half of their true value. This simplified sub-frame should then be analysed for the loading requirements previously mentioned. Formulae giving bending moments due to various loading arrangements acting on the simplified sub-frame, obtained by slope-deflection methods as described in section 14.2.1, are given in *Table 2.61.* Since the method is ‘exact’, the calculated bending moments may be redistributed within the limits permitted by the Codes. The method is dealt with in more detail in *Examples of the design of reinforced concrete buildings*.

BS 8110 also allows analysis of the beams at each floor as a continuous system, neglecting the restraint provided by the columns entirely, so that the continuous beam is assumed to be resting on knife-edge supports. Column moments are then obtained by considering, at each joint, a sub-frame consisting of the upper and lower columns together with the adjacent beams, regarded as fixed at their remote ends and with their stiffness taken as one-half of the true value.

For frames that are required to provide lateral stability to the structure as a whole, load combinations 1 and 3 both need to be considered. For combination 3, the following two-stage method of analysis is allowed for frames of three or more approximately equal bays. First, each floor is considered as a separate sub-frame for the effect of vertical loading as described previously. Next, the complete structural frame is considered for the effect of lateral loading, assuming that a position of contra-flexure (i.e. zero bending moment) occurs at the mid-point of each member. This analysis corresponds to that described for building frames in section 4.11.3, and the method set out in diagram (c) of *Table 2.62* may thus be used. The moments obtained from each of these analyses should then be summed, and compared with those resulting from load combination 1. For tall narrow buildings and other cantilever structures such as masts, pylons and towers, load combination 2 should also be considered.

4.9.2 Moment-distribution method: no sway

In some circumstances, a framed structure may not be subject to side-sway: for example, if the frame is braced by other stiff elements within the structure, or if both the configuration and the loading are symmetrical. Similarly, if a vertically loaded frame is being analysed as a set of sub-frames, as permitted in BS 8110, the effects of any side-sway may be ignored. In such cases, Hardy Cross moment distribution may be used to evaluate the moments in the beam and column system. The procedure, which is outlined in *Table 2.58*, is similar to the one used to analyse systems of continuous beams.

Precise moment distribution may also be used to solve such systems. Here the method, which is also summarised in *Table 2.58*, is slightly more complex to apply than in the equivalent continuous beam case. Each time a moment is carried over, the unbalanced moment in the member must be distributed between the remaining members meeting at the joint in proportion to the relative restraint that each provides. Also, the expression for the continuity factors is more difficult to evaluate. Nevertheless, the method is a valid alternative to the conventional moment-distribution method. It is described in more detail in *Examples of the design of reinforced concrete buildings*.

4.9.3 Moment-distribution method: with sway

If sway occurs, analysis by moment distribution increases in complexity since, in addition to the influence of the original loading with no sway, it is necessary to consider the effect of each degree of sway freedom separately in terms of unknown sway forces. The separate results are then combined to obtain the unknown sway values, and hence the final moments. The procedure is outlined in *Table 2.59*.

The advantages of precise moment distribution are largely nullified if sway occurs, but details of the procedure in such cases are given in ref. 35.

To determine the moments in single-bay frames subjected to side sway, Naylor (ref. 36) devised an ingenious variant of moment distribution, details of which are given in *Table 2.59*. The method can also be used to analyse Vierendeel girders.

4.9.4 Slope-deflection method

The principles of the slope-deflection method of analysing a restrained member are given in *Table 2.60* and section 14.1, together with basic formulae, and formulae for the bending moments in special cases. When there is no deflection of one end of the member relative to the other (e.g. when the supports are not elastic as assumed), when the ends of the member are either hinged or fixed, and when the load on the member is symmetrically disposed, the general expressions are simplified and the resulting formulae for some common cases of restrained members are also given in *Table 2.60*.

The bending moments on a framed structure are determined by applying the formulae to each member successively. The algebraic sum of the bending moments at any joint must equal zero. When it is assumed that there is no deflection (or settlement) a of one support relative to the other, there are as many formulae for the end moments as there are unknowns, and therefore the restraint moments and the slopes at the ends of the members can be evaluated. For symmetrical frames on unyielding foundations, and carrying symmetrical vertical loads, it is common to neglect the change in the position of the joints due to the small elastic contractions of the members, and the assumption of $a = 0$ is reasonably correct. If the foundations or other supports settle unequally under the load, this assumption is not justified and the term a must be assigned a value for the members affected.

If a symmetrical or unsymmetrical frame is subjected to a horizontal force, the resulting sway causes lateral movement of the joints. It is common in this case to assume that there is no elastic shortening of the members. Sufficient formulae to enable the additional unknowns to be evaluated are obtained by equating the reaction normal to the member, that is the shear force on the member, to the rate of change of bending moment. Sway occurs also in unsymmetrical frames subject to vertical loads, and in any frame on which the load is not symmetrically disposed.

Slope-deflection methods have been used to derive bending moment formulae for the simplified sub-frames illustrated on *Table 2.60*. These simplified sub-frames correspond to those referred to in BS 8110, as a basis for determining the bending moments in the individual members of a frame subjected to vertical loads only. The method is described in section 14.2.

An example of applying the slope-deflection formulae to a simple problem of a beam, hinged at one end and framed into a column at the other end, is given in section 14.1.

4.9.5 Shearing forces on members of a frame

The shearing forces on any member forming part of a frame can be simply determined, once the bending moments have been found, by considering the rate of change of the bending moment. The uniform shearing force on a member AB due to end restraint only is $(M_{AB} + M_{BA})/l_{AB}$, account being taken of the signs of the bending moment. Thus if both of the restraint moments are clockwise, the shearing force is the numerical sum of the moments divided by the length of the member. If one restraint moment acts in a direction contrary to the other, the shearing force is the numerical difference in the moments divided by the length of the member. For a member with end B hinged, the shearing force due to the restraint moment at A is M_{AB}/l_{AB}. The variable shearing forces caused by the loads on the member should be algebraically added to the uniform shearing force due to the restraint moments, as indicated for a continuous beam in section 11.1.2.

4.9.6 Portal frames

A common type of frame used in single-storey buildings is the portal frame, with either a horizontal top member, or two inclined top members meeting at the ridge. In *Tables 2.63* and *2.64*, general formulae for the moments at both ends of the columns, and at the ridge where appropriate, are given, together with expressions for the forces at the bases of the columns. The formulae relate to any vertical or horizontal load, and to frames fixed or hinged at the bases. In *Tables 2.65* and *2.66*, corresponding formulae for special conditions of loading on frames of one bay are given.

Frames of the foregoing types are statically indeterminate, but frames with a hinge at the base of each column and one at the ridge, that is, a three-hinged frame, can be readily analysed. Formulae for the forces and bending moments are given in *Table 2.67* for three-hinged frames. Approximate expressions are also given for certain modified forms of these frames, such as when the ends of the columns are embedded in the foundations, and when a tie-rod is provided at eaves level.

4.9.7 Finite elements

In conventional structural analysis, numerous approximations are introduced and the engineer is normally content to accept the resulting simplification. Actual elements are considered as idealised one-dimensional linear members; deformations due to axial force and shear are assumed to be sufficiently small to be neglected; and so on.

In general, such assumptions are valid and the results of the analysis are sufficiently close to the values that would occur in the actual structure to be acceptable. However, when the member sizes become large in relation to the structure they form, the system of skeletal simplification breaks down. This occurs, for example, with the design of such elements as deep beams, shear walls and slabs of various types.

One of the methods developed to deal with such so-called *continuum* structures is that known as finite elements. The structure is subdivided arbitrarily into a set of individual elements (usually triangular or rectangular in shape), which are then considered to be inter-connected only at their corners (nodes). Although the resulting reduction in continuity might seem to indicate that the substitute system would be much more flexible than the original structure, this is not the case if the substitution is undertaken carefully, since the adjoining edges of the elements tend not to separate and thus simulate continuity. A stiffness matrix for the substitute structure can now be prepared, and analysed using a computer in a similar way to that already described.

Theoretically, the pattern of elements chosen might be thought to have a marked effect on the validity of the results. However, although the use of a smaller mesh, consisting of a larger number of elements, can often increase the accuracy of the analysis, it is normal for surprisingly good results to be obtained by experienced analysts when using a rather coarse grid, consisting of only a few large elements.

4.10 COLUMNS IN NON-SWAY FRAMES

In monolithic beam-and-column construction subjected to vertical loads only, provision is still needed for the bending moments produced on the columns due to the rigidity of the joints. The external columns of a building are subjected to greater moments than the internal columns (other conditions being equal). The magnitude of the moment depends on the relative stiffness and the end conditions of the members.

The two principal cases for beam–column connections are at intermediate points on the column (e.g. floor beams) and at the top of the column (e.g. roof beam). Since each member can be hinged, fully fixed or partially restrained at its remote end, there are many possible combinations.

In the first case, the maximum restraint moment at the joint between a beam and an external column occurs when the remote end of the beam is hinged, and the remote ends of the column are fixed, as indicated in *Table 2.60*. The minimum restraint moment at the joint occurs when the remote end of the beam is fixed, and the remote ends of the column are both hinged, as also indicated in *Table 2.60*. Real conditions, in practice, generally lie between these extremes and, with any condition of fixity of the remote ends of the column, the moment at the joint decreases as the degree of fixity at the remote end of the beam increases. With any degree of fixity at the remote end of the beam, the moment at the joint increases very slightly as the degree of fixity at the remote ends of the column increase.

Formulae for maximum and minimum bending moments are given in *Table 2.60* for a number of single-bay frames. The moment on the beam at the joint is divided between the upper and lower columns in the ratio of their stiffness factors *K*, when the conditions at the ends of the two columns are identical. When one column is hinged at the end and the other is fixed, the solution given for two columns with fixed ends can still be used, by taking the effective stiffness factor of the column with the hinged end as 0.75*K*.

For cases where the beam–column connection is at the top of the column, the formulae given in *Table 2.60* may be used, by taking the stiffness factors for the upper columns as zero.

4.10.1 Internal columns

For the frames of ordinary buildings, the bending moments on the upper and lower internal columns can be computed from the expressions given at the bottom of *Table 2.60*; these formulae conform to the method to be used when the beams are analysed as a continuous system on knife-edge supports, as described in clause 3.2.1.2.5 of BS 8110. When the spans are unequal, the greatest bending moments on the column are when the value of M_{es} (see *Table 2.60*) is greatest, which is generally when the longer beam is loaded with (dead + live), load while the shorter beam carries dead load only.

Another method of determining moments in the columns, according to the Code requirements, is to use the simplified sub-frame formulae given on *Table 2.61*. Then considering column SO, for example, the column moment is given by

$$D_{SO}\left(\frac{2D_{TS}F'_T + 4F'_S}{4 - D_{ST}D_{TS}}\right)$$

where D_{SO}, D_{ST} and D_{TS} are distribution factors, F'_S and F'_T are fixed-end moments at S and T respectively (see *Table 2.61*). This moment is additional to any initial fixed-end moment acting on SO.

To determine the maximum moment in the column it may be necessary to examine two separate simplified sub-frames, in which each column is embodied at each floor level (i.e. the column at joint S, say, is part of two sub-frames comprising beams QR to ST, and RS to TU respectively). However, the maximum moments usually occur when the central beam of the sub-frame is the longer of the two beams adjoining the column being investigated, as specified in the Code.

4.10.2 End columns

The bending moments due to continuity between the beams and the columns vary more for end columns than for internal columns. The lack of uniformity in the end conditions affects the moments determined by the simplified method described earlier more significantly than for internal columns. However, even though the values obtained by the simplified methods are more approximate than for internal columns, they are still sufficiently accurate for ordinary buildings. The simplified formulae given on *Table 2.60* conform to clause 3.2.1.2.5 of BS 8110, while the alternative simplified sub-frame method described for internal columns may also be used.

4.10.3 Corner columns

Corner columns are generally subjected to bending moments from beams in two directions at right angles. These moments can be independently calculated by considering two frames (also at right angles), but practical methods of column design depend on both the relative magnitudes of the moments and the direct load, and the relevant limit-state condition. These methods are described in later sections of the *Handbook*.

4.10.4 Use of approximate methods

The methods hitherto described for evaluating the column moments in beam-and-column construction with rigid joints involve significant calculation, including the second moment of area of the members. Often in practice, and especially in the preparation of preliminary schemes, approximate methods are very useful. The final design should be checked by more accurate methods.

The column can be designed provisionally for a direct load increased to allow for the effects of bending. In determining the total column load at any particular level, the load from the floor immediately above that level should be multiplied by the following factors: internal columns 1.25, end columns 1.5 and corner column 2.0.

4.11 COLUMNS IN SWAY FRAMES

In exposed structures such as water towers, bunkers and silos, and in frames that are required to provide lateral stability to a building, the columns must be designed to resist the effects of wind. When conditions do not warrant a close analysis of the bending moments to which a frame is subjected due to wind or other lateral forces, the methods described in the following and shown in *Table 2.62* are sufficiently accurate.

4.11.1 Open braced towers

For columns (of identical cross section) with braced corners forming an open tower, such as that supporting an elevated water tank, the expressions at (a) in *Table 2.62* give bending moments and shearing forces on the columns and braces, due to the effect of a horizontal force at the head of the columns.

In general, the bending moment on the column is the shear force on the column multiplied by half the distance between the braces. If a column is not continuous or is insufficiently braced at one end, as at an isolated foundation, the bending moment at the other end is twice this value.

The bending moment on the brace at an external column is the sum of the bending moments on the column at the points of intersection with the brace. The shearing force on the brace is equal to the change of bending moment, from one end of the brace to the other end, divided by the length of the brace. These shearing forces and bending moments are additional to those caused by the dead weight of the brace and any external loads to which it may be subjected.

The overturning moment on the frame causes an additional direct load on the leeward column and a corresponding relief of load on the windward column. The maximum value of this direct load is equal to the overturning moment at the foot of the columns divided by the distance between the centres of the columns.

The expressions in *Table 2.62* for the bending moments and forces on the columns and braces, apply for columns that are vertical or near vertical. If the columns are inclined, then the shearing force on a brace is $2M_b$ divided by the length of the brace being considered.

4.11.2 Columns supporting massive superstructures

The case illustrated at (b) in *Table 2.62* is common in silos and bunkers where a superstructure of considerable rigidity is carried on comparatively short columns. If the columns are fixed at the base, the bending moment on a single column is $Fh/2J$, where J is the number of columns if they are all of the same size; the significance of the other symbols is indicated in *Table 2.62*.

If the columns are of different sizes, the total shearing force on any one line of columns should be divided between them in proportion to the second moment of area of each column, since they are all deflected by the same amount. If J_1 is the number of columns with second moment of area I_1, J_2 is the number of columns with second moment of area I_2 and so on, the total second moment of area $\Sigma I = J_1 I_1 + J_2 I_2 +$ and so on. Then on any column having a second moment of area Ij, the bending moment is FhIj$/2\Sigma I$ as given in diagram (b) in *Table 2.62*. Alternatively, the total horizontal force can be divided among the columns in proportion to their cross-sectional areas (thus giving uniform shear stress), in which case the formula for the bending moment on any column with cross-sectional area A_j is FhAj$/2\Sigma A$, where ΣA is the sum of the cross-sectional areas of all the columns resisting the total shearing force F.

4.11.3 Building frames

In the frame of a multi-storey, multi-bay building, the effect of the wind may be small compared to that of other loads, and in this case it is sufficiently accurate to divide the horizontal shearing force between the columns on the basis that an end column resists half the amount on an internal column. If in the plane of the lateral force F, J_t is the total number of columns in one frame, the effective number of columns for the purpose of calculating the bending moment on an internal column is $J_t - 1$, the two end columns being equivalent to one internal column; see diagram (c) in *Table 2.62*. In a building frame subjected to wind pressure, the forces on each panel (or storey height) F_1, F_2, F_3 and so on are generally divided into equal shearing forces at the head and base of each storey height of columns. The shearing force at the bottom of any internal column, i storeys from the top, is $(\Sigma F + F_i/2)/(J_t - 1)$, where $\Sigma F = F_1 + F_2 + F_3 + \dots + F_{i-1}$. The bending moment is then the shearing force multiplied by half the storey height.

A bending moment and a corresponding shearing force are caused on the floor beams, in the same way as on the braces of an open braced tower. At an internal column, the sum of the bending moments on the two adjacent beams is equal to the sum of the moments at the base of the upper column and the head of the lower column.

The above method of analysis for determining the effects of lateral loading corresponds to that described in section 4.9.1, and recommended in BS 8110 for a frame of three or more approximately equal bays.

4.12 WALL AND FRAME SYSTEMS

In all forms of construction, the effects of wind force increase in significance as the height of the structure increases. One way of reducing lateral sway, and improving stability, is by increasing the sectional size of the component members of sway frames. However, this will have a direct consequence of increasing storey height and building cost.

Often, a better way is to provide a suitable arrangement of walls linked to flexible frames. The walls can be external or internal, be placed around lift shafts and stairwells to form core structures, or be a combination of types. Sometimes core walls are constructed in advance of the rest of the structure to avoid subsequent delays. The lateral stiffness of systems with a central core can be increased, by providing deep cantilever members at the top of the core structure, to which the exterior columns are connected. Another approach is to increase the load on the central core, by replacing the exterior columns by hangers suspended from the cantilever members at the top of the building. This also avoids the need for exterior columns at ground level, and their attendant foundations. As buildings get taller, the lateral stability requirements are of paramount importance. The structural efficiency can be increased, by replacing the building facade by a rigidly jointed framework, so that the outer shell acts effectively as a closed-box.

Some different structural forms consisting of assemblies of multi-storey frames, shear walls and cores, with an indication of typical heights and proportions, taken from ref. 37, are shown in *Table 2.68*.

4.12.1 Shear wall structures

The lateral stability of low- to medium-rise buildings is often obtained by providing a suitable system of stiff shear walls. The arrangement of the walls should be such that the building is stiff in both flexure and torsion. In rectangular buildings, external shear walls in the short direction can be used to resist lateral loads acting on the wide faces, with rigid frames or infill panels in the long direction. In buildings of square plan form, a strong central service core, surrounded by flexible external frames, can be used. If strong points are placed at both ends of a long building, the restraint provided to the subsequent shrinkage and thermal movements of floors and roof should be carefully considered.

In all cases, the floors and roof are considered to act as stiff plates so that, at each level, the horizontal displacements of all walls and columns are taken to be the same, provided the total lateral load acts through the shear centre of the system. If the total lateral load acts eccentrically, then the additional effect of the resulting torsion moment needs to be considered. The analysis and design of shear wall buildings is covered in ref. 38, from which much of the following treatment is based. Several different plan configurations of shear walls and core units, with notes on their suitability are shown in *Table 2.69*.

4.12.2 Walls without openings

The lateral load transmitted to an individual wall is a function of its position and its relative stiffness. The total deflection of a cantilever wall under lateral load is a combination of bending and shear deformations. However, for a uniformly distributed load, the shear deformation is less than 10% of the total, for $H/D > 3$ in the case of plane walls, and $H/D > 5$ in the case of flanged walls with $B/D = 0.5$ (where B is width of flange, D is depth of web and H is height of wall). Thus, for most shear walls without openings, the dominant mode of deformation is bending, and the stiffness of the wall can be related directly to the second moment of area of the cross section I. Then, for a total lateral load F applied at the shear centre of a system of parallel walls, the shearing force on an individual wall j is $FI_j/\Sigma I_j$.

The position of the shear centre along a given axis y can be readily determined, by calculating the moment of stiffness of each wall about an arbitrary reference point on the axis. The distance from the point to the shear centre, $y_c = \Sigma I_j y_j/\Sigma I_j$.

If the total lateral load acts at distance y_o along the axis, the resulting horizontal moment is $F(y_o–y_c)$. Then, if the torsion stiffness of individual walls is neglected, the total shearing force on wall j is

$$F_j = FI_j/\Sigma I_j + F(y_o - y_c)I_j y_j/\Sigma I_j(y_j - y_c)^2$$

More generalised formulae, in which a wall system is related to two perpendicular axes are given in *Table 2.69*. The above analysis takes no account of rotation at the base of the walls.

4.12.3 Walls containing openings

In the case of walls pierced by openings, the behaviour of the individual wall sections is coupled to a variable degree. The connections between the individual sections are provided either by beams that form part of the wall, or by floor slabs, or by a combination of both. The pierced wall may be analysed by elastic methods in which the flexibility of the coupling elements is represented as a continuous flexible medium. Alternatively, the pierced wall may be idealised as an equivalent plane frame using a ‘wide column’ analogy.

The basis of the continuous connection model is described in section 15.2, and analytical solutions for a wall containing a single line of openings are given in *Table 2.70*.

4.12.4 Interaction of shear walls and frames

The interaction forces between solid walls, pierced walls and frames can vary significantly up the height of a building, as a result of the differences in the free deflected shapes of each structural form. The deformation of solid walls is mainly flexural, whereas pierced walls deform in a shear–flexure mode, and frames deform in an almost pure shear manner. As a result, towards the bottom of a building, solid walls attract load whilst frames and, to a lesser extent, pierced walls shed load. The behaviour is reversed towards the top of a building. Thus, although the distribution of load intensity between the different elements is far from uniform up the building, the total lateral force resisted by each varies by a smaller amount.

As a first approximation, the shearing force at the bottom of each load-resisting element can be determined by considering a single interaction force at the top of the building. Formulae, by which the effective stiffness of pierced walls and frames can be determined, are given in section 15.3.

4.13 ARCHES

Arch construction in reinforced concrete occurs sometimes in roofs, but mainly in bridges. An arch may be three-hinged, two-hinged or fixed-ended (see diagrams in *Table 2.71*), and may be symmetrical or unsymmetrical, right or skew, single or one of a series of arches mutually dependent upon each other. The following consideration is limited to symmetrical and unsymmetrical three-hinged arches, and to symmetrical two-hinged and fixed-end arches; reference should be made to other publications for information on more complex types.

Arch construction may comprise an arch slab (or vault) or a series of parallel arch ribs. The deck of an arch bridge may be supported by columns or transverse walls carried on an arch slab or ribs, when the structure may have open spandrels; or the deck may be below the crown of the arch, either at the level of the springing (as in a bowstring girder) or at some intermediate level. A bowstring girder is generally regarded as a two-hinged arch, with the horizontal component of thrust resisted by a tie, which normally forms part of the deck. If earth or other filling is provided to support the deck, an arch slab and spandrel walls are required and the bridge is a closed or solid-spandrel structure.

4.13.1 Three-hinged arch

An arch with a hinge at each springing and at the crown is statically determinate. The thrusts on the abutments, and the bending moments and shearing forces on the arch itself, are not affected by a small movement of one abutment relative to the other. This type of arch is therefore used when there is a possibility of unequal settlement of the abutments.

For any load in any position, the thrust on the abutments can be determined by the equations of static equilibrium. For the general case of an unsymmetrical arch with a load acting vertically, horizontally or at an angle, the expressions for the horizontal and vertical components of the thrusts are given in the lower part of *Table 2.71*. For symmetrical arches, the formulae given in *Table 2.67* for the thrusts on three-hinged frames apply, or similar formulae can be obtained from the general expressions in *Table 2.71*. The vertical component is the same as the vertical reaction for a freely supported beam. The bending moment at any cross section of the arch is the algebraic sum of the moments of the loads and reactions on one side of the section. There is no bending moment at a hinge. The shearing force is likewise the algebraic sum of the loads and reactions, resolved at right angles to the arch axis at the section, and acting on one side of the section. The thrust at any section is the sum of the loads and reactions, resolved parallel to the axis of the arch at the section, and acting on one side of the section.

The extent of the arch that should be loaded with imposed load to give the maximum bending moment, or shearing force or thrust at a particular cross section can be determined by constructing a series of influence lines. A typical influence line for a three-hinged arch, and the formulae necessary to construct an influence line for unit load in any position, are given in the upper part of *Table 2.71*.

4.13.2 Two-hinged arch

The hinges of a two-hinged arch are placed at the abutments so that, as in a three-hinged arch, only thrusts are transmitted to the abutments, and there is no bending moment on the arch at the springing. The vertical component of the thrust from a symmetrical two-hinged arch is the same as the reaction for a freely supported beam. Formulae for the thrusts and bending moments are given in *Table 2.71*, and notes in section 16.2.

4.13.3 Fixed arch

An arch with fixed ends exerts, in addition to the vertical and horizontal thrusts, a bending moment on the abutments. Like a two-hinged arch and unlike a three-hinged arch, a fixed-end arch is statically indeterminate, and the stresses are affected by changes of temperature and shrinkage of the concrete. As it is assumed in the general theory that the abutments cannot move or rotate, the arch can only be used in such conditions.

A cross section of a fixed-arch rib or slab is subjected to a bending moment and a thrust, the magnitudes of which have to be determined. The design of a fixed arch is a matter of trial and adjustment, since both the dimensions and the shape of the arch affect the calculations, but it is possible to select preliminary sizes that reduce the repetition of arithmetic work to a minimum. A suggested method of determining possible sections at the crown and springing, as given in *Table 2.72* and explained in section 16.3.1, is based on first treating the fixed arch as a hinged arch, and then estimating the size of the cross sections by greatly reducing the maximum stresses.

The general formulae for thrusts and bending moments on a symmetrical fixed arch of any profile are given in *Table 2.72*, and notes on the application and modification of the formulae are given in section 16.3. The calculations necessary to solve the general and modified formulae are tedious, but are eased somewhat by preparing them in tabular form. The form given in *Table 2.72* is particularly suitable for open-spandrel arch bridges, because the appropriate formulae do not assume a constant value of a_I, the ratio of the length of a segment of the arch to the mean second moment of area of the segment.

For large span arches, calculations are made much easier and more accurate by preparing and using influence lines for the bending moment and thrust at the crown, the springing, and the quarter points of the arch. Typical influence lines are given in *Table 2.72*, and such diagrams can be constructed by considering the passage over the arch of a single concentrated unit load, and applying the formulae for this condition. The effect of the dead load, and of the most adverse disposition of imposed load, can be readily calculated from these diagrams. If the specified imposed load includes a moving concentrated load, such as a KEL, the influence lines are almost essential for determining the most adverse position. The case of the positive bending moment at the crown is an exception, when the most adverse position of the load is at the crown. A method of determining the data to establish the ordinates of the influence lines is given in *Table 2.73*.

4.13.4 Fixed parabolic arches

In *Table 2.74* and in section 16.4, consideration is given to symmetrical fixed arches that can have either open or solid spandrels, and be either arch ribs or arch slabs. The method is based on that of Strassner as developed by H Carpenter, and the principal assumption is that the axis of the arch is made to coincide with the line of thrust due to the dead load. This results in an economical structure and a simple calculation method. The shape of the axis of the arch is approximately that of a parabola, and this method can therefore be used only when the designer is free to select the profile of the arch. The parabolic form may not be the most economic for large spans, although it is almost so, and a profile that produces an arch axis coincident with the line of thrust for the dead load plus one-half of the imposed load may be more satisfactory. If the increase in the thickness of the arch from crown to springing is of a parabolic form, only the bending moments and thrusts at the crown and the springing need to be investigated. The necessary formulae are given in section 16.4, where these include a series of coefficients, values of which are given in *Table 2.74*. The application of the method is also illustrated by an example given in section 16.4. The component forces and moments are considered in the following treatment.

The thrusts due to the dead load are relieved somewhat by the effect of the compression causing elastic shortening of the arch. For arches with small ratios of rise to span, and arches that are thick in comparison with the span, the stresses due to arch shortening may be excessive. This can be overcome by introducing temporary hinges at the crown and the springing, which eliminate all bending stresses due to dead load. The hinges are filled with concrete after arch shortening and much of the shrinkage of the concrete have taken place.

An additional horizontal thrust due to a temperature rise or a corresponding counter-thrust due to a temperature fall will affect the stresses in the arch, and careful consideration must be given to the likely temperature range. The shrinkage of the concrete that occurs after completion of the arch produces a counter-thrust, the magnitude of which is modified by creep.

The extent of the imposed load on an arch, necessary to produce the maximum stresses in the critical sections, can be determined from influence lines, and the following values are approximately correct for parabolic arches. The maximum positive moment at the crown occurs when the middle third of the arch is loaded; the maximum negative moment at a springing occurs when four-tenths of the span adjacent to the springing is loaded; the maximum positive moment at the springing occurs when six-tenths of the span furthest away from the springing is loaded. In the expressions given in section 16.4.4, the imposed load is expressed in terms of an equivalent UDL.

When the normal thrusts and bending moments on the main sections have been determined, the areas of reinforcement and stresses at the crown and springing can be calculated. All that now remains is to consider the intermediate sections and determine the profile of the axis of the arch. If the dead load is uniform throughout (or practically so), the axis will be a parabola; but if the dead load is not uniform, the axis must be shaped to coincide with the resulting line of thrust. This can be obtained graphically by plotting force-and-link polygons, the necessary data being the magnitudes of the dead load, the horizontal thrust due to dead load, and the vertical reaction (equal to the dead load on half the span) of the springing. The line of thrust, and therefore the axis of the arch, having been established, and the thickness of the arch at the crown and the springing having been determined, the lines of the extrados and the intrados can be plotted to give a parabolic variation of thickness between the two extremes.

4.14 PROPERTIES OF MEMBERS

4.14.1 End conditions

Since the results given by the more precise methods of elastic analysis vary considerably with the conditions of restraint at the ends of the members, it is important that the assumed conditions are reasonably obtained in the actual construction. Absolute fixity is difficult to attain unless a beam or column is embedded monolithically in a comparatively large mass of concrete. Embedment of a beam in a masonry wall represents more nearly the condition of a hinge, and should normally be considered as such. A continuous beam supported internally on a beam or column is only partly restrained, and where the support at the outer end of an end span is a beam, a hinge should be assumed. With the ordinary type of pad foundation, designed simply for a uniform ground bearing pressure under the direct load on a column, the condition at the foot of the column should also be considered as a hinge. A column built on a pile-cap supported by two, three or four piles is not absolutely fixed, but a bending moment can be developed if the resulting vertical reaction (upwards and downwards) and the horizontal thrust can be resisted by the piles. The foot of a column can be considered as fixed if it is monolithic with a substantial raft foundation.

In two-hinged and three-hinged arches, hinged frames, and some bridge types, where the assumption of a hinged joint must be fully realised, it is necessary to form a definite hinge in the construction. This can be done by inserting a steel hinge (or similar), or by forming a hinge within the frame.

4.14.2 Section properties

For the elastic analysis of continuous structures, the section properties need to be known. Three bases for calculating the second moment of area of a reinforced concrete section are generally recognised in codes of practice, as follows:

1. *The concrete section*: the entire concrete area, but ignoring the reinforcement.

2. *The gross section*: the entire concrete area, together with the reinforcement on the basis of a modular ratio, (i.e. ratio of modulus of elasticity values of steel and concrete).
3. *The transformed section*: the concrete area in compression, together with the reinforcement on the basis of modular ratio.

For methods 2 and 3, the modular ratio should be based on an effective modulus of elasticity of concrete, taking account of the creep effects of long-term loading. In BS 8110, a modular ratio of 15 is recommended unless a more accurate figure can be determined. However, until the reinforcement has been determined, or assumed, calculation of the section properties in this way cannot be made with any precision. Moreover, the section properties vary considerably along the length of the member as the distribution of reinforcement and, for method 3, the depth of concrete in compression change. The extent and effect of cracking on the section properties is particularly difficult to assess for a continuous beam in beam-and-slab construction, in which the beam behaves as a flanged section in the spans where the bending moments are positive, but is designed as a rectangular section towards the supports where the bending moments are negative.

Method 1 is the simplest one to apply and the only practical approach when beginning a new design, but one of the other methods could be used when checking the ability of existing structures to carry revised loadings and, for new structures, when a separate analysis for the SLSs is required. In all cases, it is important that the method used to assess the section properties is the same for all the members involved in the calculation. Where a single stiffness value is to be used to characterise a member, method 1 (or 2) is likely to provide the most accurate overall solution. Method 3 will only be appropriate where the variations in section properties over the length of members are properly taken into account.

4.15 EARTHQUAKE-RESISTANT STRUCTURES

Earthquakes are ground vibrations that are caused mainly by fracture of the earth's crust, or by sudden movement along an already existing fault. During a seismic excitation, structures are caused to oscillate in response to the forced motion of the foundations. The affected structure needs to be able to resist the resulting horizontal load, and also dissipate the imparted kinetic energy over successive deformation cycles. It would be uneconomical to design the structure to withstand a major earthquake elastically, and the normal approach is to provide it with sufficient strength and ductility to withstand such an event by responding inelastically, provided that the critical regions and the connections between members are designed specially to ensure adequate ductility.

Significant advances have been made in the seismic design of structures in recent years, and very sophisticated codes of practice have been introduced (ref. 39). A design horizontal seismic load is recommended that depends on the importance of the structure, the seismic zone, the ground conditions, the natural period of vibration of the structure and the available ductility of the structure. Design load effects in the structure are determined either by linear-elastic structural analysis for the equivalent static loading or by dynamic analysis. When a linear-elastic method is used, the design and detailing of the members needs to ensure that, in the event of a more severe earthquake, the post-elastic deformation of the structure will be adequately ductile. For example, in a multi-storey frame, sufficient flexural and shear strength should be provided in the columns to ensure that plastic hinges form in the beams, in order to avoid a column side-sway mechanism. The proper detailing of the reinforcement is also a very important aspect in ensuring ductile behaviour. At the plastic hinge regions of moment resisting frames, in addition to longitudinal tension reinforcement, it is essential to provide adequate compression reinforcement. Transverse reinforcement is also necessary to act as shear reinforcement, to prevent premature buckling of the longitudinal compression reinforcement and to confine the compressed concrete.

Buildings should be regular in plan and elevation, without re-entrant angles and discontinuities in transferring vertical loads to the ground. Unsymmetrical layouts resulting in large torsion effects, flat slab floor systems without any beams, and large discontinuities in infill systems (such as open ground storeys) should be avoided. Footings should be founded at the same level, and should be interconnected by a mat foundation or by a grid of foundation beams. Only one foundation type should in general be used for the same structure, unless the structure is formed of dynamically independent units.

An alternative to the conventional ductile design approach is to use a seismic isolation scheme. In this case, the structure is supported on flexible bearings, so that the period of vibration of the combined structure and supporting system is long enough for the structure to be isolated from the predominant earthquake ground motion frequencies. In addition, extra damping is introduced into the system by mechanical energy dissipating devices, in order to reduce the response of the structure to the earthquake, and keep the deflections of the flexible system within acceptable limits.

A detailed treatment of the design of earthquake-resisting concrete structures is contained in ref. 40.

Chapter 5

Design of structural members

5.1 PRINCIPLES AND REQUIREMENTS

In modern Codes of Practice, a limit-state design concept is used. Ultimate (ULS) and serviceability (SLS) limit-states are considered, as well as durability and, in the case of buildings, fire-resistance. Partial safety factors are incorporated in both loads and material strengths, to ensure that the probability of failure (i.e. not satisfying a design requirement) is acceptably low. For British Codes (BS 8110, BS 5400, BS 8007), details are given of design requirements and partial safety factors in Chapter 21, material properties in Chapter 22, durability and fire-resistance in Chapter 23. For EC 2, corresponding data are given in Chapters 29, 30 and 31 respectively.

Members are first designed to satisfy the most critical limit-state, and then checked to ensure that the other limit-states are not reached. For most members, the critical condition to be considered is the ULS, on which the required resistances of the member in bending, shear and torsion are based. The requirements of the various SLSs, such as deflection and cracking, are considered later. However, since the selection of an adequate span to effective depth ratio to prevent excessive deflection, and the choice of a suitable bar spacing to avoid excessive cracking, can also be affected by the reinforcement stress, the design process is generally interactive. Nevertheless, it is normal to start with the requirements of the ULS.

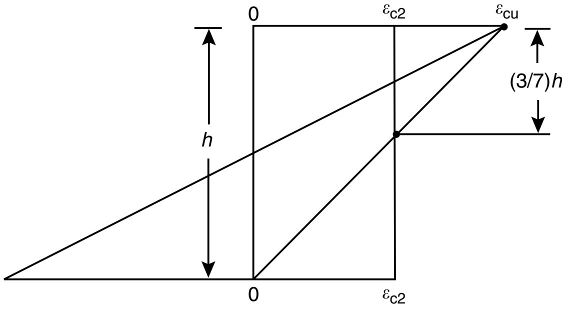
5.2 RESISTANCE TO BENDING AND AXIAL FORCE

Typically, beams and slabs are members subjected to bending while columns are subjected to a combination of bending and axial force. In this context, a beam is defined as a member, in BS 8110, with a clear span not less than twice the effective depth and, in EC 2, as a member with a span not less than three times the overall depth. Otherwise, the member is treated as a deep beam, for which different design methods are applicable. A column is defined as a member, in which the greater overall cross-sectional dimension does not exceed four times the smaller dimension. Otherwise, the member is considered as a wall, for which a different design approach is adopted. Some beams, for example, in portal frames, and slabs, for example, in retaining walls, are subjected to bending and axial force. In such cases, small axial forces that are beneficial in providing resistance to bending are generally ignored in design.

5.2.1 Basic assumptions

For the analysis of sections in bending, or combined bending and axial force, at the ULS, the following basic assumptions are made:

- The resistance of the concrete in tension is ignored.
- The distribution of strain across the section is linear, that is, sections that are plane before bending remain plane after bending, the strain at a point being proportional to its distance from the axis of zero strain (neutral axis). In columns, if the axial force is dominant, the neutral axis can lie outside the section.
- Stress–strain relationships for concrete in compression, and for reinforcement in tension and compression, are those shown in the diagrams on *Table 3.6* for BS 8110 and BS 5400, and *Table 4.4* for EC 2.
- The maximum strain in the concrete in compression is 0.0035, except for EC 2 where the strains shown in the following diagram and described in the following paragraph apply.



Strain distribution at ULS in EC 2

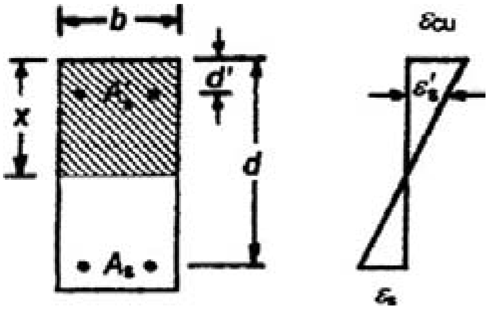
For sections subjected to pure axial compression, the strain is limited to ε_{c2}. For sections partly in tension, the compressive strain is limited to ε_{cu}. For intermediate conditions, the strain diagram is obtained by taking the compressive strain as ε_{c2} at a level equal to 3/7 of the section depth from the more highly compressed face. For concrete strength classes ≤C50/60, the limiting strains are $\varepsilon_{c2} = 0.002$ and $\varepsilon_{cu} = 0.0035$. For higher strength concretes, other values are given in *Table 4.4*.

In all codes, for sections partly in tension, the shape of the basic concrete stress-block is a combination of a parabola and a rectangle. In EC 2, a form consisting of a triangle and a rectangle is also given. In all codes, a simplified rectangular stress distribution may also be used. If the compression zone is rectangular, the compressive force and the distance of the force from the compression face can be readily determined for each stress-block, and the resulting properties are given in section 24.1 for BS 8110, and section 32.1 for EC 2.

The stresses in the reinforcement depend on the strains in the adjacent concrete, which depend in turn on the depth of the neutral axis and the position of the reinforcement in relation to the concrete surfaces. The effect of these factors will be examined separately for beams and columns.

5.2.2 Beams

Depth of neutral axis. This is significant because the value of x/d, where x is the neutral axis depth and d is the effective depth of the tension reinforcement, not only affects the stress in the reinforcement, but also limits the amount of moment redistribution allowed at a given section. In BS 8110 where, because of moment redistribution allowed in the analysis of a member, the design moment is less than the maximum elastic moment, the requirement $x/d \leq (\beta_b - 0.4)$ should be satisfied, where β_b is the ratio of design moment to maximum elastic moment. Thus, for reductions in moment of 10%, 20% and 30%, x/d must not exceed 0.5, 0.4 and 0.3 respectively. In EC 2, as modified by the UK National Annex, similar restrictions apply for concrete strength classes ≤C50/60.



Section Strain diagram

The figure here shows a typical strain diagram for a section containing both tension and compression reinforcement. For the bi-linear stress–strain curve in BS 8110, the maximum design stresses in the reinforcement are $f_y/1.15$ for values of ε_s and $\varepsilon'_s \geq f_y/1.15E_s$. From the strain diagram, this gives:

$$x/d \leq \varepsilon_{cu}/(\varepsilon_{cu} + f_y/1.15E_s) \text{ and } d'/x \leq (\varepsilon_{cu} - f_y/1.15E_s)/\varepsilon_{cu}$$

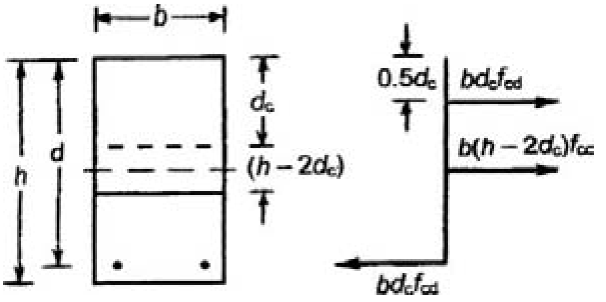
In BS 5400 the reinforcement stress–strain curve is tri-linear, with maximum design stresses of $f_y/1.15$ in tension and $2000f_y/(2300 + f_y)$ in compression. These stresses apply for values of $\varepsilon_s \geq 0.002 + f_y/1.15E_s$ and $\varepsilon'_s \geq 0.002$, giving:

$$x/d \leq \varepsilon_{cu}/(\varepsilon_{cu} + 0.002 + f_y/1.15E_s) \text{ and}$$

$$d'/x \leq (\varepsilon_{cu} - 0.002)/\varepsilon_{cu}$$

With $\varepsilon_{cu} = 0.0035$, $f_y = 500$ N/mm^2 and $E_s = 200$ kN/mm^2, the critical values are $x/d = 0.617$ and $d'/x = 0.38$ for BS 8110, and $x/d = 0.456$ and $d'/x = 0.43$ for BS 5400. For design to EC 2, considerations similar to those in BS 8110 apply.

Effect of axial force. The following figure shows a section that is subjected to a bending moment M and an axial force N, in which a simplified rectangular stress distribution has been assumed for the compression in the concrete. The stress block is shown divided into two parts, of depths d_c and $(h - 2d_c)$, providing resistance to the bending moment M and the axial force N respectively, where $0 < d_c \leq 0.5h$.



Section Forces

The depth d_c (and the force in the tension reinforcement) are determined by the bending moment given by:

$$M = bd_c(d - 0.5d_c)f_{cd}$$

Thus, for analysis of the section, axial forces may be ignored for values satisfying the condition:

$$N \leq b(h - 2d_c)f_{cd}$$

Combining the two requirements gives

$$N \leq bhf_{cd} - 2M/(d - 0.5d_c)$$

In the limit, when $d_c = 0.5h$, this gives

$$N \leq bhf_{cd} - 2M/(d - 0.25h) \cong bhf_{cd} - 3M/h$$

For BS 8110, the condition becomes $N \leq 0.45bhf_{cu} - 3M/h$, which being simplified to $N \leq 0.1bhf_{cu}$ is reasonably valid for $M/bh^2f_{cu} \leq 0.12$. For EC 2, the same condition becomes $N \leq 0.567bhf_{ck} - 3M/h$, which may be reasonably simplified to $N \leq 0.12bhf_{ck}$ for $M/bh^2f_{ck} \leq 0.15$.

Analysis of section. Any given section can be analysed by a trial-and-error process. An initial value is assumed for the neutral axis depth, from which the concrete strains at the reinforcement positions can be calculated. The corresponding stresses in the reinforcement are determined, and the resulting forces in the reinforcement and the concrete are obtained. If the forces are out of balance, the value of the neutral axis depth is changed and the process is repeated until equilibrium is achieved. Once the balanced condition has been found, the resultant moment of all the forces about the neutral axis, or any other suitable point, is calculated.

Singly reinforced rectangular sections. For a section that is reinforced in tension only, and subjected to a moment M, a quadratic equation in x can be obtained by taking moments, for the compressive force in the concrete, about the line of action of the tension reinforcement. The resulting value of x can be used to determine the strain diagram, from which the strain in the reinforcement, and hence the stress, can be calculated. The required area of reinforcement can then be determined from the tensile force, whose magnitude is equal to the compressive force in the concrete. If the calculated value of x exceeds the limit required for any redistribution of moment, then a doubly reinforced section will be necessary.

In designs to BS 8110 and BS 5400, the lever arm between the tensile and compressive forces is to be taken not greater than $0.95d$. Furthermore, it is a requirement in BS 5400 that, if x exceeds the limiting value for using the maximum design stress, then the resistance moment should be at least $1.15M$. Analyses are included in section 24.2.1 for both BS 8110 and BS 5400, and in section 32.2.1 for EC 2. Design charts based on the parabolic-rectangular stress-block for concrete, with $f_y = 500$ N/mm^2, are given in *Tables 3.13*, *3.23* and *4.7* for BS 8110, BS 5400 and EC 2 respectively. Design tables based on the rectangular stress-blocks for concrete are given in *Tables 3.14*, *3.24* and *4.8* for BS 8110, BS 5400 and EC 2 respectively. These tables use non-dimensional parameters and are applicable for values of $f_y \leq 500$ N/mm^2.

Doubly reinforced rectangular sections. A section needing both tension and compression reinforcement, and subjected to a moment M, can be designed by first selecting a suitable value for x, such as the limiting value for using the maximum design stress in the tension reinforcement or satisfying the condition necessary for moment redistribution. The required force to be provided by the compression reinforcement can be derived by taking moments, for the compressive forces in the concrete and the reinforcement, about the line of action of the tensile reinforcement. The force to be provided by the tension reinforcement is equal to the sum of the compressive forces. The reinforcement areas can now be determined, taking due account of the strains appropriate to the value of x selected.

Analyses are included in section 24.2.2 for both BS 8110 and BS 5400, and in section 32.2.2 for EC 2. Design charts based on the rectangular stress-blocks for concrete are given in *Tables 3.15* and *3.16* for BS 8110, *Tables 3.25* and *3.26* for BS 5400 and *Tables 4.9* and *4.10* for EC 2.

Design formulae for rectangular sections. Design formulae based on the rectangular stress-blocks for concrete are given in BS 8110 and BS 5400. In both codes, x is limited to $0.5d$ so that the formulae are automatically valid for redistribution of moment not greater than 10%. The design stress in tension reinforcement is taken $0.87f_y$, although this is only strictly valid for $x/d \leq 0.456$ in BS 5400. The design stresses in any compression reinforcement are taken as $0.87f_y$ in BS 8110 and $0.72f_y$ in BS 5400. Design formulae are given in section 24.2.3 for BS 8110 and BS 5400. Although not included in EC 2, appropriate formulae are given in section 32.2.3.

Flanged sections. In monolithic beam and slab construction, where the web of the beam projects below the slab, the beam is considered as a flanged section for sagging moments. The effective width of the flange, over which uniform conditions of stress can be assumed, is limited to values stipulated in the codes. In most sections, where the flange is in compression, the depth of the neutral axis will be no greater than the flange thickness. In such cases, the section can be considered to be rectangular with b taken as the flange width. If the depth of the neutral axis does exceed the thickness of the flange, the section can be designed by dividing the compression zone into portions comprising the web and the outlying flanges. Details of the flange widths and design procedures are given in sections 24.2.4 for BS 8110 and 32.2.4 for EC 2.

Beam sizes. The dimensions of beams are mainly determined by the need to provide resistance to moment and shear. In the case of beams supporting items such as cladding, partitions or sensitive equipment, service deflections can also be critical. Other factors such as clearances below beams, dimensions of brick and block courses, widths of supporting members and suitable sizes of formwork also need to be taken into account. For initial design purposes, typical span/effective depth ratios for beams in buildings are given in the following table:

Span/effective depth ratios for initial design of beams

Span conditions	Ultimate design load		
	25 kN/m	50 kN/m	100 kN/m
Cantilever	9	7	5
Simply supported	18	14	10
Continuous	22	17	12

The effective span of a continuous beam is generally taken as the distance between centres of supports. At a simple support, or at an encastre′ end, the centre of action may be taken at a distance not greater than half of the effective depth from the face of the support. Beam widths are often taken as half the overall depth of the beam with a minimum of 300 mm. If a much wider band beam is used, the span/effective depth ratio can be increased significantly to the limit necessitated by deflection considerations.

In BS 8110 and BS 5400, to ensure lateral stability, simply supported and continuous beams should be so proportioned that the clear distance between lateral restraints is not greater than $60b_c$ or $250b_c^2/d$, whichever is the lesser. For cantilevers in which lateral restraint is provided only at the support, the clear distance from the end of the cantilever to the face of the support should not exceed $25b_c$ or $100b_c^2/d$, whichever is the lesser one. In the foregoing, b_c is the breadth of the compression face of the beam (measured midway between restraints), or cantilever. In EC 2, second order effects in relation to lateral stability may be ignored if the distance between lateral restraints is not greater than $50b_c(h/b_c)^{1/3}$ and $h \leq 2.5b_c$.

5.2.3 Slabs

Solid slabs are generally designed as rectangular strips of unit width, and singly reinforced sections are normally sufficient. Ribbed slabs are designed as flanged sections, of width equal to the rib spacing, for sagging moments. Continuous ribbed slabs are often made solid in support regions, so as to develop sufficient resistance to hogging moments and shear forces. Alternatively, in BS 8110, ribbed slabs may be designed as a series of simply supported spans, with a minimum amount of reinforcement provided in the hogging regions to control the cracking. The amount of reinforcement recommended is

25% of that in the middle of the adjoining spans extending into the spans for at least 15% of the span length.

The thickness of slabs is normally determined by deflection considerations, which sometimes result in the use of reduced reinforcement stresses to meet code requirements. Typical span/effective depth ratios for slabs designed to BS 8110 are given in the following table:

Span/effective depth ratios for initial design of solid slabs

Span conditions	Characteristic imposed load	
	5 kN/m^2	10 kN/m^2
Cantilever	11	10
Simply supported		
One-way span	27	24
Two-way span	30	27
Continuous		
One-way span	34	30
Two-way span	44	40
Flat slab (no drops)	30	27

In the table here, the characteristic imposed load should include for all finishes, partitions and services. For two-way spans, the ratios given apply to square panels. For rectangular panels where the length is twice the breadth, the ratios given for one-way spans should be used. For other cases, ratios may be obtained by interpolation. The ratios apply to the shorter span for two-way slabs and the longer span for flat slabs. For ribbed slabs, except for cantilevers, the ratios given in the table should be reduced by 20%.

5.2.4 Columns

The second order effects associated with lateral stability are an important consideration in column design. An effective height (or length, in EC 2) and a slenderness ratio are determined in relation to major and minor axes of bending. An effective height, or length, is a function of the clear height and depends upon the conditions of restraint at the ends of the column. A clear distinction exists between a braced column, with effective height ≤ clear height, and an unbraced column, with effective height ≥ clear height. A braced column is one that is fully retrained in position at the ends, as in a structure where resistance to all the lateral forces in a particular plane is provided by stiff walls or bracing. An unbraced column is one that is considered to contribute to the lateral stability of the structure, as in a sway frame.

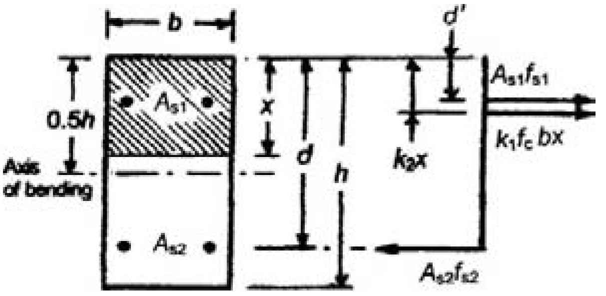
In BS 8110 and BS 5400, a slenderness ratio is defined as the effective height divided by the depth of the cross section in the plane of bending. A column is then considered as either short or slender, according to the slenderness ratios. Braced columns are often short, in which case second order effects may be ignored. In EC 2, the slenderness ratio is defined as the effective length divided by the radius of gyration of the cross section.

Columns are subjected to combinations of bending moment and axial force, and the cross section may need to be checked for more than one combination of values. In slender columns, the initial moments, obtained from an elastic analysis of the structure, are increased by additional moments induced by the deflection of the column. In BS 8110 and EC 2, these additional moments contain a modification factor, the use of which necessitates an iteration process with the factor taken as 1.0 initially. Details of the design procedures are given in *Tables 3.21* and *3.22* for BS 8110, *Tables 3.31* and *3.32* for BS 5400 and *Tables 4.15* and *4.16* for EC 2.

Analysis of section. Any given section can be analysed by a trial-and-error process. For a section bent about one axis, an initial value is assumed for the neutral axis depth, from which the concrete strains at the positions of the reinforcement can be calculated. The resulting stresses in the reinforcement are determined, and the forces in the reinforcement and concrete are evaluated. If the resultant force is not equal to the design axial force N, the value of the neutral axis depth is changed and the process repeated until equality is achieved. The sum of the moments of all the forces about the mid-depth of the section is then the moment of resistance appropriate to N. For a section in biaxial bending, initial values have to be assumed for the depth and the inclination of the neutral axis, and the design process would be extremely tedious without the aid of an interactive computer program.

For design purposes, charts for symmetrically reinforced rectangular and circular sections bent about one axis can be readily derived. For biaxial bending conditions, approximate design methods have been developed that utilise the solutions obtained for uniaxial bending.

Rectangular sections. The figure here shows a rectangular section with reinforcement in the faces parallel to the axis of bending.



Section Forces

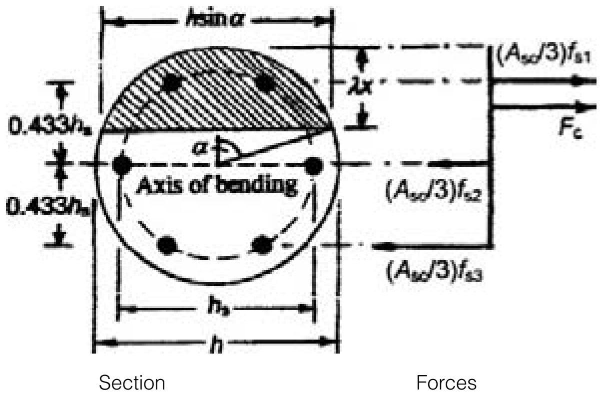
Resolving forces, and taking moments about the mid-depth of the section, gives the following equations for $0 < x \leq h$.

$$N = k_1 bxf_c + A_{s1}f_{s1} - A_{s2}f_{s2}$$
$$M = k_1 bxf_c(0.5h - k_2x) + A_{s1}f_{s1}\ (0.5h - d') + A_{s2}f_{s2}(d - 0.5h)$$

where f_{s1} and f_{s2} are determined by the stress–strain curves for the reinforcement and depend on the value of x. Values of k_1 and k_2 are determined by the concrete stress-block, and f_c is equal to f_{cu} in BS 8110 and BS 5400, and f_{ck} in EC 2.

For symmetrically reinforced sections, $A_{s1} = A_{s2} = A_{sc}/2$ and $d' = h - d$. Design charts based on a rectangular stress-block for the concrete, with values of f_y = 500 N/mm^2, and d/h = 0.8 and 0.85 respectively, are given in *Tables 3.17* and *3.18* for BS 8110, *Tables 3.27* and *3.28* for BS 5400 and *Tables 4.11* and *4.12* for EC 2. Approximate design methods for biaxial bending are given in *Tables 3.21*, *3.31* and *4.16* for design to BS 8110, BS 5400 and EC 2 respectively.

Circular sections. The figure here shows a circular section with six bars spaced equally around the circumference. Six is the minimum number of bars recommended in the codes, and solutions based on six bars will be slightly conservative if more bars are used. The arrangement of bars relative to the axis of bending affects the resistance of the section, and it can be shown that the arrangement in the figure is not the most critical in every case, but the variations are small and may be reasonably ignored.



The following analysis is based on a uniform stress-block for the concrete of depth λx and width $h\sin\alpha$ at the base (as shown in the figure). Resolving forces and taking moments about the mid-depth of the section, where h_s is the diameter of a circle through the centres of the bars, gives the following equations for $0 < x \leq h$.

$$N = [(2\alpha - \sin 2\alpha)/8]h^2 f_{cd} + (A_{sc}/3)(f_{s1} - f_{s2} - f_{s3})$$
$$M = [(3\sin\alpha - \sin 3\alpha)/72]h^3 f_{cd} + 0.433(A_{sc}/3)(f_{s1} + f_{s3})h_s$$

where f_{s1}, f_{s2} and f_{s3} are determined by the stress–strain curves for the reinforcement and depend on the value of x. Values of f_{cd} and λ respectively are taken as $0.45f_{cu}$ and 0.9 in BS 8110, $0.4f_{cu}$ and 1.0 in BS 5400, and $0.51f_{ck}$ and 0.8 in EC 2.

Design charts, derived for values of $f_y = 500$ N/mm², and $h_s/h = 0.6$ and 0.7 respectively, are given in *Tables 3.19* and *3.20* for BS 8110, *Tables 3.29* and *3.30* for BS 5400, and *Tables 4.13* and *4.14* for EC 2. Sections subjected to biaxial moments M_x and M_y can be designed for the resultant moment $M = \sqrt{(M_x^2 + M_y^2)}$.

Design formulae. In BS 8110, two approximate formulae are given for the design of short braced columns under specific conditions. Columns which due to the nature of the structure cannot be subjected to significant moments, for example, columns that provide support to very stiff beams or beams on bearings, may be considered adequate if $N \leq 0.40f_{cu}A_c + 0.67A_{sc}f_y$.

Columns supporting symmetrical arrangements of beams that are designed for uniformly distributed imposed load, and have spans that do not differ by more than 15% of the longer, may be considered adequate if $N \leq 0.35f_{cu}A_c + 0.60A_{sc}f_y$.

BS 5400 contains general formulae for rectangular sections in the form of a trial-and-error procedure, and two simplified formulae for specific applications, details of which are given in *Table 3.32*.

Column sizes. Columns in unbraced structures are likely to be rectangular in cross section, due to the dominant effect of bending moments in the plane of the structure. Columns in braced structures are typically square in cross section, with sizes being determined mainly by the magnitude of the axial loads. In multi-storey buildings, column sizes are often kept constant over several storeys with the reinforcement changing in relation to the axial load. For initial design purposes, typical load capacities for short braced square columns in buildings are given in the following table:

Concrete class	Column size	Reinforcement percentage			
		1%	2%	3%	4%
C25/30	300 × 300	1370	1660	1950	2240
	350 × 350	1860	2260	2650	3050
	400 × 400	2430	2950	3470	3980
	450 × 450	3080	3730	4390	5040
	500 × 500	3800	4610	5420	6230
C32/40	300 × 300	1720	2010	2300	2580
	350 × 350	2350	2740	3130	3520
	400 × 400	3070	3580	4090	4600
	450 × 450	3880	4530	5170	5820
	500 × 500	4790	5590	6390	7190
C40/50	300 × 300	2080	2360	2650	2930
	350 × 350	2830	3220	3600	3990
	400 × 400	3700	4200	4710	5210
	450 × 450	4680	5320	5960	6600
	500 × 500	5780	6570	7360	8150

Ultimate design loads (kN) for short braced columns

In the foregoing table, the loads were derived from the BS 8110 equation for columns that are not subjected to significant moments, with $f_y = 500$ N/mm². In determining the column loads, the ultimate load from the floor directly above the level being considered should be multiplied by the following factors to compensate for the effects of bending: internal column 1.25, edge column 1.5, corner column 2.0. The total imposed loads may be reduced according to the number of floors supported. The reductions, for 2, 3, 4, 5–10 and over 10 floors, are 10%, 20%, 30%, 40% and 50% respectively.

5.3 RESISTANCE TO SHEAR

Much research by many investigators has been undertaken in an effort to develop a better understanding of the behaviour of reinforced concrete subjected to shear. As a result of this research, various theories have been proposed to explain the mechanism of shear transfer in cracked sections, and provide a satisfactory basis for designing shear reinforcement. In the event of overloading, sudden failure can occur at the onset of shear cracking in members without shear reinforcement. As a consequence, a minimum amount of shear reinforcement in the form of links is required in nearly all beams. Resistance to shear can be increased by adding more shear reinforcement but, eventually, the resistance is limited by the capacity of the inclined struts that form within the web of the section.

5.3.1 Members without shear reinforcement

In an uncracked section, shear results in a system of mutually orthogonal diagonal tension and compression stresses. When the diagonal tension stress reaches the tensile strength of the concrete, a diagonal crack occurs. This simple concept rarely applies to reinforced concrete, since members such as beams and slabs are generally cracked in flexure. In current practice, it is more useful to refer to the nominal shear stress $v = V/bd$, where b is the breadth of the section in the tension zone. This stress can then be related to empirical limiting values derived from test data. The limiting value v_c depends on the concrete strength, the effective depth and the reinforcement percentage at the section considered. To be effective, this reinforcement should continue beyond the section for a specified minimum distance as given in Codes of Practice. For values of $v \leq v_c$, no shear reinforcement is required in slabs but, for most beams, a specified minimum amount in the form of links is required.

At sections close to supports, the shear strength is enhanced and, for members carrying generally uniform load, the critical section may be taken at d from the face of the support. Where concentrated loads are applied close to supports, in members such as corbels and pile-caps, some of the load is transmitted by direct strut action. This effect is taken into account in the Codes of Practice by either enhancing the shear strength of the section, or reducing the design load. In members subjected to bending and axial load, the shear strength is increased due to compression and reduced due to tension.

Details of design procedures in Codes of Practice are given in *Table 3.33* for BS 8110, *Table 3.36* for BS 5400 and *Table 4.17* for EC 2.

5.3.2 Members with shear reinforcement

The design of members with shear reinforcement is based on a truss model, in which the tension and compression chords are spaced apart by a system of inclined concrete struts and upright or inclined shear reinforcement. Most reinforcement is in the form of upright links, but bent-up bars may be used for up to 50% of the total shear reinforcement in beams. The truss model results in a force in the tension chord additional to that due to bending. This can be taken into account directly in the design of the tension reinforcement, or indirectly by shifting the bending moment curve each side of any point of maximum bending moment.

In BS 8110, shear reinforcement is required to cater for the difference between the shear force and the shear resistance of the section without shear reinforcement. Equations are given for upright links based on concrete struts inclined at about 45°, and for bent-up bars where the inclination of the concrete struts may be varied between specified limits. In BS 5400, a specified minimum amount of link reinforcement is required in addition to that needed to cater for the difference between the shear force and the shear resistance of the section without shear reinforcement. The forces in the inclined concrete struts are restricted indirectly by limiting the maximum value of the nominal shear stress to specified values.

In EC 2, shear reinforcement is required to cater for the entire shear force and the strength of the inclined concrete struts is checked explicitly. The inclination of the struts may be varied between specified limits for links as well as bent-up bars. In cases where upright links are combined with bent-up bars, the strut inclination needs to be the same for both.

Details of design procedures in Codes of Practice are given in *Table 3.33* for BS 8110, *Table 3.36* for BS 5400 and *Table 4.18* for EC 2.

5.3.3 Shear under concentrated loads

Suspended slabs and foundations are often subjected to large loads or reactions acting on small areas. Shear in solid slabs under concentrated loads can result in punching failures on the inclined faces of truncated cones or pyramids. For design purposes, shear stresses are checked on given perimeters at specified distances from the edges of the loaded area. Where a load or reaction is eccentric with regard to a shear perimeter (e.g. at the edges of a slab, and in cases of moment transfer between a slab and a column), an allowance is made for the effect of the eccentricity. In cases where v exceeds v_c, links, bent-up bars or other proprietary products may be provided in slabs not less than 200 mm deep.

Details of design procedures in Codes of Practice are given in *Table 3.34* for BS 8110, *Tables 3.37* and *3.38* for BS 5400 and *Table 4.19* for EC 2.

5.4 RESISTANCE TO TORSION

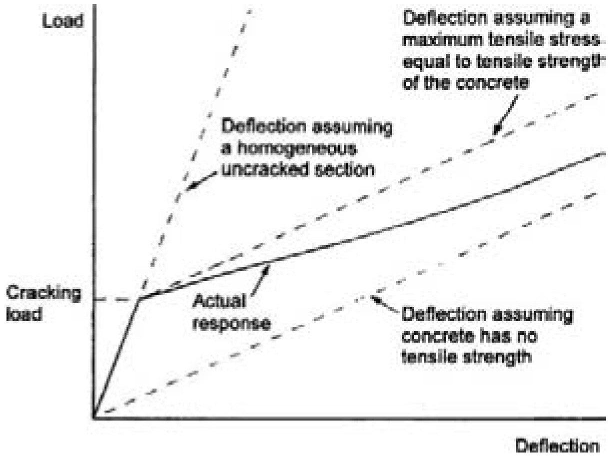
In normal beam-and-slab or framed construction, calculations for torsion are not usually necessary, adequate control of any torsional cracking in beams being provided by the required minimum shear reinforcement. When it is judged necessary to include torsional stiffness in the analysis of a structure, or torsional resistance is vital for static equilibrium, members should be designed for the resulting torsional moment. The torsional resistance of a section may be calculated on the basis of a thin-walled closed section, in which equilibrium is satisfied by a closed plastic shear flow. Solid sections may be modelled as equivalent thin-walled sections. Complex shapes may be divided into a series of sub-sections, each of which is modelled as an equivalent thin-walled section, and the total torsional resistance taken as the sum of the resistances of the individual elements. When torsion reinforcement is required, this should consist of rectangular closed links together with longitudinal reinforcement. Such reinforcement is additional to any requirements for shear and bending.

Details of design procedures in Codes of Practice are given in *Table 3.35* for BS 8110, *Table 3.39* for BS 5400 and *Table 4.20* for EC 2.

5.5 DEFLECTION

The deflections of members under service loading should not impair the appearance or function of a structure. An accurate prediction of deflections at different stages of construction may also be necessary in bridges, for example. For buildings, the final deflection of members below the support level, after allowance for any pre-camber, is limited to span/250. In order to minimise any damage to non-structural elements such as finishes, cladding or partitions, that part of the deflection that occurs after the construction stage is also limited to span/500. In BS 8110, this limit is taken as 20 mm for spans ≥ 10 m.

The behaviour of a reinforced concrete beam under service loading can be divided into two basic phases: before and after cracking. During the uncracked phase, the member behaves elastically as a homogeneous material. This phase is ended by the load at which the first flexural crack forms. The cracks result in a gradual reduction in stiffness with increasing load during the cracked phase. The concrete between the cracks continues to provide some tensile resistance though less, on average, than the tensile strength of the concrete. Thus, the member is stiffer than the value calculated on the assumption that the concrete carries no tension. This additional stiffness, known as 'tension stiffening', is highly significant in lightly reinforced members such as slabs, but has only a relatively minor effect on the deflection of heavily reinforced members. These concepts are illustrated in the following figure.



Schematic load-deflection response

In BS 8110, for the purpose of analysis, 'tension stiffening' is represented by a triangular stress distribution in the concrete, increasing from zero at the neutral axis to a maximum value at the tension face. At the level of the tension reinforcement, the concrete stress is taken as 1 N/mm^2 for short-term loads, and 0.55 N/mm^2 for long-term loads, irrespective of the strain in the tension reinforcement. In EC 2, a more general approach is adopted in which the deformation of a section, which could be a curvature or, in the case of pure tension, an extension, or a combination of these, is calculated first for a homogeneous uncracked section, δ_1, and second for a cracked section ignoring tension in the concrete, δ_2. The deformation of the section under the design loading is then obtained as:

$$\delta = \zeta\delta_2 + (1 - \zeta)\delta_1$$

where ζ is a distribution coefficient that takes account of the degree of cracking according to the nature and duration of the loading, and the stress in the tension reinforcement under the load causing first cracking in relation to the stress under the design service load.

When assessing long-term deflections, allowances need to be made for the effect of concrete creep and shrinkage. Creep can be taken into account by using an effective modulus of elasticity $E_{c,eff} = E_c/(1 + \varphi)$, where E_c is the short-term value and φ is a creep coefficient. Shrinkage deformations can be calculated separately and added to those due to loading.

Generally, explicit calculation of deflections is unnecessary to satisfy code requirements, and simple rules in the form of limiting span/effective depth ratios are provided in BS 8110 and EC 2. These are considered adequate for avoiding deflection problems in normal circumstances and, subject to the particular assumptions made in their derivation, provide a useful basis for estimating long-term deflections of members in buildings, as follows:

$$\text{Deflection} = \frac{\text{actual span/effective depth ratio}}{\text{limiting span/effective depth ratio}} \times \text{span}/250$$

Details of span/effective depth ratios and explicit calculation procedures are given in *Tables 3.40* to *3.42* for BS 8110, and *Tables 4.21* and *4.22* for EC 2.

5.6 CRACKING

Cracks in members under service loading should not impair the appearance, durability or water-tightness of a structure. In BS 8110, for buildings, the design crack width is generally limited to 0.3 mm. In BS 5400, for bridges, the limit varies between 0.25 mm and 0.10 mm depending on the exposure conditions. In BS 8007, for structures to retain liquids, a limit of 0.2 mm usually applies. Under liquid pressure, continuous cracks that extend through the full thickness of a slab or wall are likely to result in some initial seepage, but such cracks are expected to self-heal within a few weeks. If the appearance of a liquid-retaining structure is considered aesthetically critical, a crack width limit of 0.1 mm applies.

In EC 2, for most buildings, the design crack width is generally limited to 0.3 mm, but for internal dry surfaces, a limit of 0.4 mm is considered sufficient. For liquid-retaining structures, a classification system according to the degree of protection required against leakage is introduced. Where a small amount of leakage is acceptable, for cracks that pass through the full thickness of the section, the crack width limit varies according to the hydraulic gradient (i.e. head of liquid divided by thickness of section). The limits are 0.2 mm for hydraulic gradients ≤ 5, reducing uniformly to 0.05 mm for hydraulic gradients ≥ 35.

In order to control cracking in the regions where tension is expected, it is necessary to ensure that the tensile capacity of the reinforcement at yielding is not less than the tensile force in the concrete just before cracking. Thus a minimum amount of reinforcement is required, according to the strength of the reinforcing steel and the tensile strength of the concrete at the time when cracks may first be expected to occur. Cracks due to restrained early thermal effects in continuous walls and some slabs may occur within a few days of the concrete being placed. In other members, it may be several weeks before the applied load reaches a level at which cracking occurs.

Crack widths are influenced by several factors including the cover, bar size, bar spacing and stress in the reinforcement. The stress may need to be reduced in order to meet the crack width limit. Design formulae are given in Codes of Practice in which strain, calculated on the basis of no tension in the concrete, is reduced by a value that decreases with increasing amounts of tension reinforcement. For cracks that are caused by applied loading, the same formulae are used in BS 8110, BS 5400 and BS 8007. For cracks that are caused by restraint to temperature effects and shrinkage, fundamentally different formulae are included in BS 8007. Here, it is assumed that bond slip occurs at each crack, and the crack width increases in direct proportion to the contraction of the concrete.

Generally, for design to BS 8110 and EC 2, there is no need to calculate crack widths explicitly, and simple rules that limit either bar size or bar spacing according to the stress in the reinforcement are provided. Details of both rules and crack width formulae are given in *Table 3.43* for BS 8110 and BS 5400, *Tables 3.44* and *3.45* for BS 8007 and *Tables 4.23–4.25* for EC 2. Additional design aids, derived from the crack width formulae, are provided in *Tables 3.46–3.52* for BS 8007, and *Tables 4.26* and *4.27* for EC 2.

5.7 REINFORCEMENT CONSIDERATIONS

Codes of Practice contain many requirements affecting the reinforcement details such as minimum and maximum areas, anchorage and lap lengths, bends in bars and curtailment. The reinforcement may be curtailed in relation to the bending moment diagram, provided there is always enough anchorage to develop the necessary design force in each bar at every cross section. Particular requirements apply at the positions where bars are curtailed and at simple supports.

Bars may be set out individually, in pairs or in bundles of three or four in contact. For the safe transmission of bond forces, the cover provided to the bars should be not less than the bar size or, for a group of bars in contact, the equivalent diameter of a notional bar with the same cross-sectional area as the group. Gaps between bars (or groups of bars) should be not less than the greater of: (aggregate size plus 5 mm) or the bar size (or equivalent bar diameter for a group). Details of reinforcement limits, and requirements for containing bars in compression, are given in *Table 3.53* for BS 8110, *Table 3.59* for BS 5400 and *Table 4.28* for EC 2.

5.7.1 Anchorage lengths

At both sides of any cross section, the reinforcement should be provided with an appropriate embedment length or other form of end anchorage. In earlier codes, it was also necessary to consider 'local bond' at sections where large changes of tensile force occur over short lengths of reinforcement, and this requirement remains in BS 5400.

Assuming a uniform bond stress between concrete and the surface of a bar, the required anchorage length is given by:

$$\begin{aligned} l_{b,req} &= (\text{design force in bar})/(\text{bond stress} \times \text{perimeter of bar}) \\ &= f_{sd}\,(\pi\phi^2/4)/f_{bd}\,(\pi\phi) = (f_{sd}/f_{bd})(\phi/4) \end{aligned}$$

where f_{sd} is the design stress in the bar at the position from which the anchorage is measured. The design bond stress f_{bd} depends on the strength of the concrete, the type of bar and, in EC 2, the location of the bar within the concrete section during concreting. For example, the bond condition is classified as 'good' in the bottom 250 mm of any section, and in the top 300 mm of a section > 600 mm deep. In other locations, the bond condition is classified as 'poor'. Also in EC 2, the basic anchorage length, in tension, can be multiplied by several coefficients that take account of factors such as the bar shape, the cover and the effect of transverse reinforcement or pressure. For bars of diameter > 40 mm, and bars grouped in pairs or bundles, additional considerations apply. Details of design anchorage lengths, in tension and compression, are given in *Table 3.55* for BS 8110, *Table 3.59* for BS 5400 and *Tables 4.30* and *4.32* for EC 2.

5.7.2 Lap lengths

Forces can be transferred between reinforcement by lapping, welding or joining bars with mechanical devices (couplers). Connections should be placed, whenever possible, away from positions of high stress, and should preferably be staggered. In Codes of Practice, the necessary lap length is obtained by multiplying the required anchorage length by a coefficient.

In BS 8110, for bars in compression, the coefficient is 1.25. For bars in tension, the coefficient is 1.0, 1.4 or 2.0 according to the cover, the gap between adjacent laps in the same layer and the location of the bar in the section. In slabs, where the cover is not less than twice the bar size, and the gap between adjacent laps is not less than six times the bar size or 75 mm, a factor of 1.0 applies. Larger factors are frequently necessary in columns, typically 1.4; and beams, typically 1.4 for bottom bars and 2.0 for top bars. The sum of all the reinforcement sizes in a particular layer should not exceed 40% of the width of the section at that level. When the size of both bars at a lap exceeds 20 mm, and the cover is less than 1.5 times the size of the smaller bar, links at a maximum spacing of 200 mm are required throughout the lap length.

In EC 2, for bars in tension or compression, the lap coefficient varies from 1.0 to 1.5, according to the percentage of lapped bars relative to the total area of bars at the section considered, and transverse reinforcement is required at each end of the lap zone. Details of lap lengths are given in *Table 3.55* for BS 8110, *Table 3.59* for BS 5400 and *Tables 4.31* and *4.32* for EC 2.

5.7.3 Bends in bars

The radius of any bend in a reinforcing bar should conform to the minimum requirements of BS 8666, and should ensure that failure of the concrete inside the bend is prevented. For bars bent to the minimum radius according to BS 8666, it is not necessary to check for concrete failure if the anchorage of the bar does not require a length more than 5 ϕ beyond the end of the bend (see *Table 2.27*). It is also not necessary to check for concrete failure, where the plane of the bend is not close to a concrete face, and there is a transverse bar not less than its own size inside the bend. This applies in particular to a link, which may be considered fully anchored, if it passes round another bar not less than its own size, through an angle of 90°, and continues beyond the end of the bend for a length not less than 8ϕ in BS 8110, and 10ϕ in EC 2.

In cases when a bend occurs at a position where the bar is highly stressed, the bearing stress inside the bend needs to be checked and the radius of bend will need to be more than the minimum given in BS 8666. This situation occurs typically at monolithic connections between members, for example, junction of beam and end column, and in short members such as corbels and pile caps. The design bearing stress is limited according to the concrete strength, and the confinement perpendicular to the plane of the bend. Details of bends in bars are given in *Table 3.55* for BS 8110, *Table 3.59* for BS 5400 and *Table 4.31* for EC 2.

5.7.4 Curtailment of reinforcement

In flexural members, it is generally advisable to stagger the curtailment points of the tension reinforcement as allowed by the bending moment envelope. Bars to be curtailed need to extend beyond the points where in theory they are no longer needed for flexural resistance for a number of reasons, but mainly to ensure that the shear resistance of the section is not reduced locally. Clearly, of course, no reinforcement should be curtailed at a point less than a full anchorage length from a section where it is required to be fully stressed.

In BS 8110 and BS 5400, except at end supports, every bar should extend, beyond the point at which in theory it is no longer required, for a distance not less than the greater of the effective depth of the member or 12 times the bar size. In addition, bars curtailed in a tension zone should satisfy at least one of three alternative conditions: one requires a full anchorage length, one requires the designer to determine the position where the shear resistance is twice the shear force, and the other requires the designer to determine the position where the bending resistance is twice the bending moment. The simplest approach is to comply with the first option, by providing a full anchorage length beyond the point where in theory the bar is no longer required, even if this requires a longer extension than is absolutely necessary in some cases. Details of the requirements are given in *Table 3.56*.

In BS 8110, simplified rules are also given for beams and slabs where the loads are mainly uniformly distributed and, in the case of continuous members, the spans are approximately equal. Details of the rules are given in *Tables 3.57* and *3.58*.

At simple end supports, the tension bars should extend for an effective anchorage length of 12 times the bar size beyond the centre of the support, but no bend should begin before the centre of the support. In cases where the width of the support exceeds the effective depth of the member, the centre of the support may be assumed at half the effective depth from the face of the support. In BS 8110, for slabs, in cases where the design shear force is less than half the shear resistance, anchorage can be obtained by extending the bars beyond the centre of the support for a distance equal to one third of the support width ≥ 30 mm.

In EC 2, the extension a_l of a tension bar beyond the point where in theory it is no longer required for flexural resistance is directly related to the shear force at the section. For members with upright shear links, $a_l = 0.5z\cot\theta$ where z is the lever arm, and θ is the inclination of the concrete struts (see section 35.1.2). Taking $z = 0.9d$, $a_l = 0.45d\cot\theta$, where $\cot\theta$ is selected by the designer in the range $1.0 \leq \cot\theta \leq 2.5$. If the value of $\cot\theta$ used in the shear design calculations is unknown, $a_l = 1.125d$ can be assumed. For members with no shear reinforcement, $a_l = d$ is used. At simple end supports, bottom bars should extend for an anchorage length beyond the face of the support. The tensile force to be anchored is given by $F = 0.5V\cot\theta$, and $F = 1.25V$ can be conservatively taken in all cases. Details of the curtailment requirements are given in *Table 4.32*.

5.8 DEEP BEAMS

In designing normal (shallow) beams of the proportions more commonly used in construction, plane sections are assumed to remain plane after loading. This assumption is not strictly true, but the errors resulting from it only become significant when the depth of the beam becomes equal to, or more than, about half the span. The beam is then classed as a deep beam, and different methods of analysis and design need to be used. These methods take into account, not only the overall applied moments and shears, but also the stress patterns and internal deformations within the beam.

For a single-span deep beam, after the concrete in tension has cracked, the structural behaviour is similar to a tied arch. The centre of the compression force in the arch rises from the support to a height at the crown equal to about half the span of the beam. The tension force in the tie is roughly constant along its length, since the bending moment and the lever arm undergo similar variations along the length of the beam. For a continuous deep beam, the structural behaviour is analogous to a separate tied arch system for each span, combined with a suspension system centred over each internal support.

In BS 8110, for the design of beams of clear span less than twice the effective depth, the designer is referred to specialist literature. In EC 2, a deep beam is classified as a beam whose effective span is less than three times its overall depth. Brief details of suitable methods of design based on the result of extensive experimental work by various investigators are given in ref. 42, and a comprehensive well-produced design guide is contained in ref. 43.

5.9 WALLS

Information concerning the design of load-bearing walls in accordance with BS 8110 is given in section 6.1.8. Retaining walls, and other similar elements that are subjected mainly to transverse bending, where the design vertical load is less than $0.1f_{cu}$ times the area of the cross section, are treated as slabs.

5.10 DETAILS

It has long been realised that the calculated strength of a reinforced concrete member cannot be attained if the details of the required reinforcement are unsatisfactory. Research by the former Cement and Concrete Association and others has shown that this applies particularly at joints and intersections. The details commonly used in wall-to-base and wall-to-wall junctions in retaining structures and containment vessels, where the action of the applied load is to 'open' the corner, are not always effective.

On *Tables 3.62* and *3.63* are shown recommended details that have emerged from the results of reported research. The design information given in BS 8110 and BS 5400 for nibs, corbels and halving joints is included, and supplemented by information given elsewhere. In general, however, details that are primarily intended for precast concrete construction have not been included, as they fall outside the scope of this book.

5.11 ELASTIC ANALYSIS OF CONCRETE SECTIONS

The geometrical properties of various figures, the shapes of which conform to the cross sections of common reinforced concrete members, are given in *Table 2.101*. The data include expressions for the area, section modulus, second moment of area and radius of gyration. The values that are derived from these expressions are applicable in cases when the amount of reinforcement provided need not be taken into account in the analysis of the structure (see section 14.1).

The data given in *Tables 2.102* and *2.103* are applicable to reinforced concrete members, with rectilinear and polygonal cross sections, when the reinforcement provided is taken into account on the basis of the modular ratio. Two conditions are considered: (1) when the entire section is subjected to stress, and (2) when, for members subjected to bending, the concrete in tension is not taken into account. The data given for the former condition are the effective area, the centre and second moment of area, the modulus and radius of gyration. For the condition when a member is subjected to bending and the concrete in tension is assumed to be ineffective, data given include the position of the neutral axis, the lever-arm and the resistance moment.

Design procedures for sections subjected to bending and axial force, with design charts for rectangular and cylindrical columns, are given in *Tables 2.104–2.109*.

Chapter 6

Buildings, bridges and containment structures

The loads and consequent bending moments and forces on the principal types of structural components, and the design resistances of such components, have been dealt with in the preceding chapters. In this chapter some complete structures, comprising assemblies or special cases of such components, and their foundations, are considered.

6.1 BUILDINGS

Buildings may be constructed entirely of reinforced concrete, or one or more elements of the roof, floors, walls, stairs and foundations may be of reinforced concrete in conjunction with a steel frame. Alternatively, the building may consist of interior and exterior walls of cast *in situ* reinforced concrete supporting the floors and roof, with the columns and beams being formed in the thickness of the walls. Again, the entire structure, or parts thereof, may be built of precast concrete elements connected together during construction.

The design of the various parts of a building is the subject of *Examples of the Design of Buildings*. That book includes illustrative calculations and drawings for a typical six-storey multipurpose building. This section provides a brief guide to component design.

6.1.1 Robustness and provision of ties

The progressive collapse of one corner of a London tower block in 1968, as a result of an explosion caused by a gas leak in a domestic appliance on the eighteenth floor, led to recommendations to consider such accidental actions in the design of all buildings. Regulations require a building to be designed and constructed so that, in the event of an accident, the building will not collapse to an extent disproportionate to the cause. Buildings are divided into classes depending on the type and occupancy, including the likelihood of accidents, and the number of occupants that may be affected, with a statement of the design measures to be taken in each of the classes. The BS 8110 normal requirements for 'robustness' automatically satisfy the regulations for all buildings, except those where specific account is to be taken of likely hazards.

The layout and form of the structure should be checked to ensure that it is inherently stable and robust. In some cases, it may be necessary to protect certain elements from vehicular impact, by providing bollards or earth banks. All structures should be able to resist a notional ultimate horizontal force equal to 1.5% of the characteristic dead load of the structure. This force effectively replaces the design wind load in cases where the exposed surface area of the building is small.

Wherever possible, continuous horizontal and vertical ties should be provided throughout the building to resist specified forces. The magnitude of the force increases with the number of storeys for buildings of less than 10 storeys, but remains constant thereafter. The requirements may be met by using reinforcement that is necessary for normal design purposes in beams, slabs, columns and walls. Only the tying forces need to be considered and the full characteristic strength of the reinforcement may be taken into account. Horizontal ties are required in floors and roofs at the periphery, and internally in two perpendicular directions. The internal ties, which may be spread uniformly over the entire building, or concentrated at beam and column positions, are to be properly anchored at the peripheral tie. Vertical ties are required in all columns and load-bearing walls from top to bottom, and all external columns and walls are to be tied into each floor and roof. For regulatory purposes, some buildings are exempt from the vertical tying requirement. Details of the tying requirements are given in *Table 3.54*.

For *in situ* construction, proper attention to reinforcement detailing is all that is normally necessary to meet the tying requirements. Precast forms of construction generally require more care, and recommended details to obtain continuity of horizontal ties are given in the code of practice. If ties cannot be provided, other strategies should be adopted, as described in Part 2 of the code. These strategies are presented in the context of residential buildings of five or more storeys, where each element that cannot be tied is to be considered as notionally removed, one at a time, in each storey in turn. The requirement is that any resulting collapse should be limited in extent, with the remaining structure being able to bridge the gap caused by the removal of the element. If this requirement cannot be satisfied, then the element in question is considered as a key element. In this case, the element and its connections need to be able to resist a design ultimate load of 34 kN/m^2, considered to act from any direction. BS 8110 is vague with regard to the extent of collapse associated with this approach, but a more clearly defined statement is given in the building regulations. Here, a key element is any untied member whose removal would put at risk of collapse, within the storey in question, and the immediately adjacent storeys, more than 15% of the area of the storey (or 70 m^2 if less).

In EC 2, similar principles apply, in that structures not specifically designed to withstand accidental actions, should be provided with a suitable tying system, to prevent progressive collapse by providing alternative load paths after local damage. The UK National Annex specifies compliance with the BS 8110 requirements, as given in *Table 4.29*.

6.1.2 Floors

Suspended concrete floors can be of monolithic construction, in the form of beam-and-slab (solid or ribbed), or flat slab (solid or waffle); or can consist of precast concrete slab units supported on concrete or steel beams; or comprise one of several other hybrid forms. Examples of monolithic forms of construction are shown in the figure on *Table 2.42*.

Two-way beam and solid slab systems can involve a layout of long span secondary beams supported by usually shorter span main beams. The resulting slab panels may be designed as two-way spanning if the longer side is less than twice the shorter side. However, such two-way beam systems tend to complicate both formwork and reinforcement details, with a consequent delay in the construction programme. A one-way beam and solid slab system is best suited to a rectangular grid of columns with long span beams and shorter span slabs. If a ribbed slab is used, a system of long span slabs supported by shorter span beams is preferable. If wide beams are used, the beam can be incorporated within the depth of the ribbed slab.

In BS 8110, ribbed slabs include construction in which ribs are cast *in situ* between rows of blocks that remain part of the completed floor. This type of construction is no longer used in the United Kingdom, although blocks are incorporated in some precast and composite construction. The formers for ribbed slabs can be of steel, glassfibre or polypropylene. Standard moulds are available that provide tapered ribs, with a minimum width of 125 mm, spaced at 600 mm (troughs) and 900 mm (waffles). The ribs are connected by a structural concrete topping with a minimum thickness of 50 mm for trough moulds, and 75 mm for waffle moulds. In most structures, to obtain the necessary fire-resistance, either the thickness of topping has to exceed these minimum values, or a non-structural screed added at a later stage of construction. The spacing of the ribs may be increased to a maximum of 1500 mm, by using purpose-made formers. Comprehensive details of trough and waffle floors are contained in ref. 44.

BS 8110 and EC 2 contain recommendations for both solid and ribbed slabs, spanning between beams or supported directly by columns (flat slabs). Ribs in waffle slabs, and ribs reinforced with a single bar in trough slabs, do not require links unless needed for shear or fire-resistance. Ribs in trough slabs, which are reinforced with more than one bar, should be provided with some links to help maintain the correct cover. The spacing of these links may be in the range 1.0–1.5 m, according to the size of the main bars. Structural toppings are normally reinforced with a welded steel fabric.

Information on the weight of concrete floor slabs is given in *Table 2.1*, and details of imposed loads on floors are given in *Table 2.3*. Detailed guidance on the analysis of slabs is given in Chapters 4 and 13. More general guidance, including initial sizing suggestions, is given in section 5.2.3.

6.1.3 Openings in floors

Large openings (e.g. stairwells) should generally be provided with beams around the opening. Holes for pipes, ducts and other services should generally be formed when the slab is constructed, and the cutting of such holes should not be permitted afterwards, unless done under the supervision of a competent engineer. Small isolated holes may generally be ignored structurally, with the reinforcement needed for a slab without holes simply displaced locally to avoid the hole.

In other cases, the area of slab around an opening, or group of closely spaced holes, needs to be strengthened with extra reinforcement. The cross-sectional area of additional bars to be placed parallel to the principal reinforcement should be at least equal to the area of principal reinforcement interrupted by the opening. Also, for openings of dimensions exceeding 500 mm, additional bars should be placed diagonally across the corners of the opening. Openings with dimensions greater than 1000 mm should be regarded as structurally significant, and the area of slab around the opening designed accordingly.

The effect of an opening in the proximity of a concentrated load, or supporting column, on the shearing resistance of the slab is shown in *Table 3.37*.

6.1.4 Stairs

Structural stairs may be tucked away out of sight within a fire enclosure, or they may form a principal architectural feature. In the former case the stairs can be designed and constructed as simply and cheaply as possible, but in the latter case much more time and trouble is likely to be expended on the design.

Several stair types are illustrated on *Table 2.88*. Various procedures for analysing the more common types of stair have been developed, and some of these are described on *Tables 2.88–2.91*. These theoretical procedures are based on the concept of an idealised line structure and, when detailing the reinforcement for the resulting stairs, additional bars should be included to limit the formation of cracks at the points of high stress concentration that inevitably occur. The ‘three-dimensional’ nature of the actual structure and the stiffening effect of the triangular tread areas, both of which are usually ignored when analysing the structure, will result in actual stress distributions that differ from those calculated, and this must be remembered when detailing. The stair types illustrated on *Table 2.88*, and others, can also be investigated by finite-element methods, and similar procedures suitable for computer analysis. With such methods, it is often possible to take account of the three-dimensional nature of the stair.

Simple straight flights of stairs can span either transversely (i.e. across the flight) or longitudinally (i.e. along the flight). When spanning transversely, supports must be provided on both sides of the flight by either walls or stringer beams. In this case, the waist or thinnest part of the stair construction need be no more than 60 mm thick say, the effective lever arm for resisting the bending moment being about half of the maximum thickness from the nose to the soffit, measured at right angles to the soffit. When the stair spans longitudinally, deflection considerations can determine the waist thickness.

In principle, the design requirements for beams and slabs apply also to staircases, but designers cannot be expected to determine the deflections likely to occur in the more complex stair types. BS 8110 deals only with simple types, and allows a modified span/effective depth ratio to be used. The bending moments should be calculated from the ultimate load due to the total weight of the stairs and imposed load, measured on plan, combined with the horizontal span. Stresses produced by the longitudinal thrust are small and generally neglected in the design of simple systems. Unless circumstances otherwise dictate, suitable step dimensions for a semi-public stairs are 165 mm rise and 275 mm going, which with a 25 mm nosing or undercut gives a tread of 300 mm. Private stairs may be steeper, and those in public buildings should be less steep. In each case, optimum proportions are given by the relationship: $(2 \times \text{rise} + \text{going}) = 600$ mm. Different forms of construction and further details on stair dimensions are given in BS 5395.

Finally, it should be remembered that the prime purpose of a stair is to provide safe pedestrian access between the floors it connects. As such it is of vital importance in the event of a fire, and a principal design consideration must be to provide adequate fire-resistance.

6.1.5 Planar roofs

The design and construction of a flat reinforced concrete roof are essentially the same as for a floor. A water-tight covering, such as asphalt or bituminous felt, is generally necessary and, with a solid slab, some form of thermal insulation is normally required. For ordinary buildings, the slab is generally built level and a drainage slope of the order of 1 in 120 is formed, by adding a mortar topping. The topping is laid directly onto the concrete and below the water-tight covering, and can form the thermal insulation if it is made of a sufficient thickness of lightweight concrete, or other material having low thermal conductivity.

Planar slabs with a continuous steep slope are not common in reinforced concrete, except for mansard roofs. The roof covering is generally of metal or asbestos-cement sheeting, or some lightweight material. Such coverings and roof glazing require purlins for their support and, although these are often of steel, precast concrete purlins are also used, especially if the roof structure is of reinforced concrete.

6.1.6 Non-planar roofs

Roofs that are not planar, other than the simple pitched roofs considered in the foregoing, can be constructed as a series of planar slabs (prismatic or hipped-plate construction), or as single- or double-curved shells. Single-curved roofs, such as segmental or cylindrical shells, are classified as *developable* surfaces. Such surfaces are not as stiff as double-curved roofs or their prismatic counterparts, which cannot be ‘opened up’ into plates without some shrinking or stretching taking place.

If the curvature of a double-curved shell is similar in all directions, the surface is known as *synclastic*. A typical case is a dome, where the curvature is identical in all directions. If the shell curves in opposite directions over certain areas, the surface is termed *anticlastic* (saddle shaped). The hyperbolic-paraboloidal shell is a well-known example, and is the special case where such a double-curved surface is generated by two sets of straight lines. An elementary analysis of some of these structural forms is dealt with in section 19.2 and *Table 2.92*, but reference should be made to specialist publications for more comprehensive analyses and more complex structures. Solutions for many particular shell types have been produced and, in addition, general methods have been developed for analysing shell forms of any shape by means of a computer. Shells, like all statically indeterminate structures, are affected by such secondary effects as shrinkage, temperature change and settlement, and a designer must always bear in mind the fact that the stresses arising from these effects can modify quite considerably those due to normal dead and imposed load. In *Table 2.81*, simple expressions are given for the forces in domed slabs such as are used for the bottoms and roofs of some cylindrical tanks. In a building, a domed roof generally has a much larger rise to span ratio and, where the dome is part of a spherical surface and has an approximately uniform thickness overall, the analysis given in *Table 2.92* applies. Shallow segmental domes and truncated cones are also dealt with in *Table 2.92*.

Cylindrical shells. Segmental or cylindrical roofs are usually designed as shell structures. Thin curved slabs that behave as shells are assumed to offer no resistance to bending, nor to deform under applied distributed loads. Except near edge and end stiffeners, the shell is subjected only to membrane forces, namely a direct force acting longitudinally in the plane of the slab a direct force acting tangentially to the curve of the slab and a shearing force. Formulae for these membrane forces are given in section 19.2.3. In practice, the boundary conditions due to either the presence or absence of edge or valley beams, end diaphragms, continuity and so on affect the displacements and forces that would otherwise occur as a result of membrane action. Thus, as when analysing any indeterminate structure (such as a continuous beam system), the effects due to these boundary restraints need to be combined with the statically determinate stresses arising from the membrane action.

Shell roofs can be arbitrarily subdivided into ‘short’ (where the ratio of length l to radius r is less than about 0.5), ‘long’ (where l/r exceeds 2.5) and ‘intermediate’. For short shells, the influence of the edge forces is slight in comparison with membrane action, and the stresses can be reasonably taken as those due to the latter only. If the shell is long, the membrane action is relatively insignificant, and an approximate solution can be obtained by considering the shell to act as a beam with curved flanges, as described in section 19.2.3.

For the initial analysis of intermediate shells, no equivalent short-cut method has yet been devised. The standard method of solution is described in various textbooks (e.g. refs 45 and 46). Such methods involve the solution of eight simultaneous equations if the shell or the loading is unsymmetrical, or four if symmetry is present, by matrix inversion or other means. By making certain simplifying assumptions and providing tables of coefficients, Tottenham (ref. 47) developed a popular simplified design method, which is rapid and requires the solution of three simultaneous equations only. J D Bennett also developed a method of designing long and intermediate shells, based on an analysis of actual designs of more than 250 roofs. The method, which involves the use of simple formulae incorporating empirical coefficients is summarised on *Tables 2.93* and *2.94*. For further details see ref. 48.

Buckling of shells. A major concern in the design of any shell is the possibility of buckling, since the loads at which buckling occurs, as established by tests, often differ from the values predicted by theory. Ref. 49 indicates that for domes subtending angles of about 90°, the critical external pressure at which buckling occurs, according to both theory and tests, is given by $p = 0.3E(h/r)^2$, where E is the elastic modulus of concrete, and h is the thickness and r the radius of the dome. For a shallow dome with span/rise $\cong$ 10, $p = 0.15E(h/r)^2$. A factor of safety against buckling of 2 to 3 should be adopted. Synclastic shells having a radius ranging from r_1 to r_2 may be considered as an equivalent dome with a radius of $r = \sqrt{}(r_1\ r_2)$.

For a cylindrical shell, buckling is unlikely if the shell is short. In the case of long shells, $p = 0.6E(h/r)^2$.

Anticlastic surfaces are more rigid than single-curved shells and the buckling pressure for a saddle-shaped shell supported on edge stiffeners safely exceeds that of a cylinder having a curvature equal to that of the anticlastic shell at the stiffener. For a hyperbolic-paraboloidal shell with straight boundaries, the buckling load obtained from tests is slightly more than the value given by $n = E(ch)^2/2ab$, where a and b are the lengths of the sides of the shell, c is the rise and h the thickness: this is only half of the value predicted theoretically.

6.1.7 Curved beams

When bow girders, and beams that are not rectilinear in plan, are subjected to vertical loading, torsional moments occur in addition to the normal bending moments and shearing forces. Beams forming a circular arc in plan may comprise part of a complete circular system with equally spaced supports, and equal loads on each span: such systems occur in silos, towers and similar cylindrical structures. Equivalent conditions can also occur in beams where the circle is incomplete, provided the appropriate negative bending and torsional moments can be developed at the end supports. This type of circular beam can occur in structures such as balconies.

On *Tables 2.95–2.97*, charts are given that enable a rapid evaluation of the bending moments, torsional moments and shear forces occurring in curved beams due to uniform and concentrated loads. The formulae on which the charts are based are given in section 19.3 and on the tables concerned. The expressions have been developed from those in ref. 50 for uniform loads, and ref. 51 for concentrated loads. In both cases, the results have been recalculated to take into account values of $G = 0.4E$ and $C = J/2$.

6.1.8 Load-bearing walls

In building codes, for design purposes, a wall is defined as a vertical load-bearing member whose length on plan exceeds four times its thickness. Otherwise, the member is treated as a column, in which case the effects of slenderness in relation to both major and minor axes of bending need to be considered (section 5.2.4). A reinforced wall is one in which not less than the recommended minimum amount of reinforcement is provided, and taken into account in the design. Otherwise, the member is treated as a plain concrete wall, in which case the reinforcement is ignored for design purposes.

A single planar wall, in general, can be subjected to vertical and horizontal in-plane forces, acting together with in-plane and transverse moments. The in-plane forces and moment can be combined to obtain, at any particular level, a longitudinal shear force, and a linear distribution of vertical force. If the in-plane eccentricity of the vertical force exceeds one-sixth of the length of the wall, reinforcement can be provided to resist the tension that develops at one end of the wall. In a plain wall, since the tensile strength of the concrete is ignored, the distribution of vertical load is similar to that for the bearing pressure due to an eccentric load on a footing. Flanged walls and core shapes can be treated in a similar way to obtain the resulting distribution of vertical force. Any unit length of the wall can now be designed as a column subjected to vertical load, combined with bending about the minor axis due to any transverse moment.

In BS 8110, the effective height of a wall in relation to its thickness depends upon the effect of any lateral supports, and whether the wall is braced or unbraced. A braced wall is one that is supported laterally by floors and/or other walls, able to transmit lateral forces from the wall to the principal structural bracing or to the foundations. The principal structural bracing comprise strong points, shear walls or other suitable elements giving lateral stability to a structure as a whole. An unbraced wall provides its own lateral stability, and the overall stability of multi-storey buildings should not, in any direction, depend on such walls alone. The slenderness ratio of a wall is defined as the effective height divided by the thickness, and the wall is considered ‘stocky’ if the slenderness ratio does not exceed 15 for a braced wall, or 10 for an unbraced wall. Otherwise, a wall is considered slender, in which case it must be designed for an additional transverse moment.

The design of plain concrete walls in BS 8110 is similar to that of unreinforced masonry walls in BS 5328. Equations are given for the maximum design ultimate axial load, taking into account the transverse eccentricity of the load, including an additional eccentricity in the case of slender walls. The basic requirements for the design of reinforced and plain concrete walls are summarised in *Table 3.60*.

6.2 BRIDGES

As stated in section 2.4.8, the analysis and design of bridges is now so complex that it cannot be adequately covered in a book of this type, and reference should be made to specialist publications. However, for the guidance of designers who may have to deal with structures having features in common with bridges, brief notes on some aspects of their design and construction are provided. Most of the following information is taken from ref. 52, which also contains other references for further reading.

6.2.1 Types of bridges

For short spans, the simplest and most cost-effective form of deck construction is a cast *in situ* reinforced concrete solid slab. Single span slabs are often connected monolithically to the abutments to form a portal frame. A precast box-shaped reinforced concrete culvert can be used as a simple form of framed bridge, and is particularly economical for short span (up to about 6 m) bridges that have to be built on relatively poor ground, obviating the need for piled foundations.

As the span increases, the high self-weight of a solid slab becomes a major disadvantage. The weight can be reduced, by providing voids within the slab using polystyrene formers. These are usually of circular section enabling the concrete to flow freely under them to the deck soffit. Reinforced concrete voided slabs are economical for spans up to about 25 m. The introduction of prestressing enables such construction to be economical over longer spans, and prestressed voided slabs, with internal bonded tendons, can be used for spans up to about 50 m. If a bridge location does not suit cast *in situ* slab construction, precast concrete beams can be used. Several different types of high quality, factory-made components that can be rapidly erected on site are manufactured. Precast beam construction is particularly useful for bridging over live roads, railways and waterways, where any interruptions to traffic must be minimised. Pre-tensioned inverted T-beams, placed side-by-side and then infilled with concrete, provide a viable alternative to a reinforced concrete solid slab for spans up to about 18 m. Composite forms of construction consisting typically of a 200 mm thick cast *in situ* slab, supported on pre-tensioned beams spaced at about 1.5 m centres, can be used for spans in the range 12–40 m.

For very long spans, prestressed concrete box girders are the usual form for bridge decks – the details of the design being dictated by the method of construction. The span-by-span method is used in multi-span viaducts with individual spans of up to 60 m. A span plus a cantilever of about one quarter the next span is first constructed. This is then prestressed and the falsework moved forward, after which a full span length is formed and stressed back to the previous cantilever. *In situ* construction is used for smaller spans but as spans increase, so also does the cost of the falsework. To minimise the cost, the weight of the concrete to be supported at any one time is reduced, by dividing each span into a series of transverse segments. These segments, which can be cast *in situ* or precast, are normally erected on either side of each pier to form balanced cantilevers and then stressed together. Further segments are then added extending the cantilevers to mid-span, where an *in situ* concrete closure is formed to make the spans continuous. During erection, the leading segments are supported from gantries erected on the piers or completed parts of the deck, and work can advance simultaneously on several fronts. When the segments are precast, each unit is match-cast against the previous one, and then separated for transportation and erection. Finally, an epoxy resin is applied to the matching faces before the units are stressed together.

Straight or curved bridges of single radius, and of constant cross section, can also be built in short lengths from one or both ends. The bridge is then pushed out in stages from the abutments, a system known as incremental launching. Arch bridges, in spans up to 250 m and beyond, can be constructed either *in situ* or using precast segments, which are prestressed together and held on stays until the whole arch is complete.

For spans in excess of 250 m, the decks of suspension and cable-stayed bridges can be of *in situ* concrete – constructed using travelling formwork – or of precast segments stressed together. For a comprehensive treatment of the aesthetics and design of bridges by one of the world's most eminent bridge engineers, see ref. 53. Brief information on typical structural forms and span ranges is given in *Table 2.98*.

6.2.2 Substructures

A bridge is supported at the ends on abutments and may have intermediate piers, where the positions of the supports and the lengths of the spans are determined by the topography of the ground, and the need to ensure unimpeded traffic under the bridge. The overall appearance of the bridge structure is very dependent on the relative proportions of the deck and its supports. The abutments are usually constructed of reinforced concrete but, in some circumstances, mass concrete without reinforcement can provide a simple and durable solution.

Contiguous bored piles or diaphragm walling can be used to form an abutment wall in cases where the wall is to be formed before the main excavation is carried out. Although the cost of this type of construction is high, it can be offset against savings in the amount of land required, the cost of temporary works and construction time. A facing of *in situ* or precast concrete or blockwork will normally be required after excavation. Reinforced earth construction can be used where there is an embankment behind the abutment, in which case a precast facing is often applied. The selection of appropriate ties and fittings is particularly important since replacement of the ties during the life of the structure is very difficult.

Where a bridge is constructed over a cutting, it is usually possible to form a bank-seat abutment on firm undisturbed ground. Alternatively, bank seats can be constructed on piled foundations. However, where bridges over motorways are designed to allow for future widening of the carriageway, the abutment is likely to be taken down to full depth so that it can be exposed at a later date when the widening is carried out.

The design of wing walls is determined by the topography of the site, and can have a major effect on the appearance of the bridge. Wing walls are often taken back at an angle from the face of the abutment for both economy and appearance. Cast *in situ* concrete is normally used, but precast concrete retaining wall units are also available from manufacturers. Concrete crib walling is also used and its appearance makes it particularly suitable for rural situations. Filling material must be carefully selected to ensure that it does not flow out, and the fill must be properly drained. It is important to limit the differential settlement that could occur between an abutment and its wing walls. The problem can be avoided if the wing walls cantilever from the abutment, and the whole structure is supported on one foundation.

The simplest and most economic form of pier is a vertical member, or group of members, of uniform cross section. This might be square, rectangular, circular or elliptical. Shaping of piers can be aesthetically beneficial, but complex shapes will significantly increase the cost unless considerable reuse of the forms is possible. Raking piers and abutments can help to reduce spans for high bridges, but they also require expensive propping and support structures. This in turn complicates the construction process and considerably increases costs.

The choice of foundation to abutments and piers is usually between spread footings and piling. Where ground conditions permit, a spread footing will provide a simple and economic solution. Piling will be needed where the ground conditions are poor and cannot be improved, the bridge is over a river or estuary, the water table is high or site restrictions prevent the construction of a spread footing. It is sometimes possible to improve the ground by consolidating, grouting or applying a surcharge by constructing the embankments well in advance of the bridge structure. Differential settlement of foundations will be affected by the construction sequence, and needs to be controlled. In the early stages of construction, the abutments are likely to settle more than the piers, but the piers will settle later when the deck is constructed.

6.2.3 Integral bridges

For road bridges in the United Kingdom, experience has shown that with all forms of construction, continuous structures are generally more durable than structures with discontinuous spans. This is mainly because joints between spans have often allowed salty water to leak through to piers and abutments. Highways Agency standard BD 57/01 says that, in principle, all bridges should be designed as continuous over intermediate supports unless special circumstances exist. The connections between spans may be made to provide full structural continuity or, in beam and slab construction, continuity of the deck slab only.

Bridges with lengths up to 60 m and skews up to 30° should also be designed as integral bridges, in which the abutments are connected directly to the deck and no movement joints are provided to allow for expansion or contraction. When the designer considers that an integral bridge is inappropriate, the agreement of the overseeing organisation must be obtained. Highways Agency document BA 57/01 has figures indicating a variety of continuity and abutment details.

6.2.4 Design considerations

Whether the bridge is carrying a road, railway, waterway or just pedestrians, it will be subject to various types of load:

- Self-weight, and loads from surfacing, parapets, and so on
- Environmental (e.g. wind, snow, temperature effects)
- Traffic
- Accidental loads (e.g. impact)
- Temporary loads (during construction and maintenance)

Bridges in the United Kingdom are generally designed to the requirements of BS 5400 and several related Highways Agency standards. Details of the traffic loads to be considered for road, railway and footbridges are given in section 2.4.8 and *Tables 2.5* and *2.6*. Details of structural design requirements, including the load combinations to be considered, are given in section 21.2 and *Tables 3.2* and *3.3*.

The application of traffic load to any one area of a bridge deck causes the deck to bend transversely and twist, thereby spreading load to either side. The assessment of how much of the load is shared in this way, and the extent to which it is spread across the deck, depends on the bending, torsion and shear stiffness of the deck in the longitudinal and transverse directions. Computer methods are generally used to analyse the structure for load effects, the most versatile method being grillage analysis, which treats the deck as a two-dimensional series of beam elements in both directions. This method can be used for solid slab, beam and slab and voided slabs where the cross-sectional area of the voids does not exceed 60% of the area of the deck. Box girders are now generally formed as one or two cells without any transverse diaphragms. These are usually quite stiff in torsion, but can distort under load giving rise to warping stresses in the walls and slabs of the box. It is then necessary to use three-dimensional analytical methods such as 3D space frame, folded plate (for decks of uniform cross section), or the generalised 3D finite element method. An excellent treatment of the behaviour and analysis of bridge decks is provided in ref. 54.

It is usual to assume that movement of abutments and wing walls will occur, and to take these into account in the design of the deck and the substructure. Normally the backfill used is a free-draining material, and satisfactory drainage facilities are provided. If these conditions do not apply, then higher design pressures must be considered. Due allowance must be made also for the compaction of the fill during construction, and the subsequent effects of traffic loading. The Highways Agency document BA 42/96 shows several forms of integral abutment, with guidance on their behaviour. Abutments to frame bridges are considered to rock bodily under the effect of deck movements. Embedded abutments, such as piled and diaphragm walls, are considered to flex, and pad foundations to bank seats are considered to slide. Notional earth pressure distributions resulting from deck expansion are also given for frame and embedded abutments.

Creep, shrinkage and temperature movements in bridge decks can all affect the forces applied to the abutments. Piers and to a lesser extent, abutments are vulnerable to impact loads from vehicles or shipping, and must be designed to resist impact or be protected from it. Substructures of bridges over rivers and estuaries are also subjected to scouring and lateral forces due to water flow, unless properly protected.

6.2.5 Waterproofing of bridge decks

Over the years, mastic asphalt has been extensively used for waterproofing bridge decks, but good weather conditions are required if it is to be laid satisfactorily. Preformed bituminous sheeting is less sensitive to laying conditions, but moisture trapped below the sheeting can cause subsequent lifting. The use of hot-bonded heavy-duty reinforced sheet membranes, if properly laid, can provide a completely water-tight layer. The sheets, which are 3–4 mm in thickness, have good puncture resistance, and it is not necessary to protect the membrane from asphalt laid on top. Sprayed acrylic and polyurethane waterproofing membranes are also used. These bond well to the concrete deck surface with little or no risk of blowing or lifting. A tack coat must be applied over the membrane and a protective asphalt layer is placed before the final surfacing is carried out. Some bridges have depended upon the use of a dense, high quality concrete to resist the penetration of water without an applied waterproofing layer. In such cases, it can be advantageous to include silica fume or some similar very fine powdered addition in the concrete.

6.3 CONTAINMENT STRUCTURES

Weights of stored materials are given in EC 1: Part 1.1, and the calculation of horizontal pressures due to liquids and granular materials contained in tanks, reservoirs, bunkers and silos is explained in sections 9.2 and 9.3, in conjunction with *Tables 2.15* and *2.16*. This section deals with the design of containment structures, and the calculation of the forces and bending moments produced by the pressure of the contained materials. Where containers are required to be watertight, the structural design should follow the recommendations given in either BS 8007 or EC 2: Part 3, as indicated in sections 21.3 and 29.4 respectively. In the following notes, containers are conveniently classified as either *tanks* containing liquids, or *bunkers* and s*ilos* containing dry materials.

6.3.1 Underground tanks

Underground storage tanks are subjected to external pressures due to the surrounding earth, in addition to internal water pressure. The empty structure should also be investigated for possible flotation, if the earth can become waterlogged. Earth pressure at-rest conditions should generally be assumed for design purposes, but for reservoirs where the earth is banked up against the walls, it would be more reasonable to assume active conditions. Storage tanks are normally filled to check for watertightness before any backfill material is placed, and there is always a risk that such material could be excavated in the future. Therefore, no reduction to the internal hydrostatic pressure by reason of the external earth pressure should be made, when a tank is full.

The earth covering on the roof of a reservoir, in its final state, acts uniformly over the entire area, but it is usually sensible to treat it as an imposed load. This is to cater for non-uniform conditions that can occur when the earth is being placed in position, and if it becomes necessary to remove the earth for maintenance purposes. Problems can arise in partially buried reservoirs, due to solar radiation causing thermal expansion of the roof. The effect of such movement on a perimeter wall will be minimised, if no connection is made between the roof and the wall until reflective gravel, or some other protective material, has been placed on the roof. Alternatively, restraint to the deflection of the wall can be minimised by providing a durable compressible material between the wall and the soil. This prevents the build-up of large passive earth pressures in the upper portion of the soil, and allows the wall to deflect as a long flexible cantilever.

6.3.2 Cylindrical tanks

The wall of a cylindrical tank is primarily designed to resist ring tensions due to the horizontal pressures of the contained liquid. If the wall is free at the top and free-to-slide at the bottom then, when the tank is full, the ring tension at depth z is given by $n = \gamma z r$, where γ is the unit weight of liquid, and r is the internal radius of the tank. In this condition, when the tank is full, no vertical bending or radial shear exists.

If the wall is connected to the floor in such a way that no radial movement occurs at the base, the ring tension will be zero at the bottom of the wall. The ring tensions are affected throughout the lower part of the wall, and significant vertical bending and radial shear occurs. Elastic analysis can be used to derive equations involving trigonometric and hyperbolic functions, and solutions expressed in the form of tables are included in publications (e.g. refs 55 and 56). Coefficients to determine values of circumferential tensions, vertical bending moments and radial shears, for particular values of the term, height^2/(2 × mean radius × thickness) are given in *Tables 2.75* and *2.76*.

The tables apply to idealised boundary conditions in which the bottom of the wall is either hinged or fixed. It is possible to develop these conditions if an annular footing is provided at the bottom of the wall. The footing should be tied into the floor of the tank to prevent radial movement. If the footing is narrow, there will be little resistance to rotation, and a hinged condition could be reasonably assumed. It is also possible to form a hinge, by providing horizontal grooves at each side of the wall, so that the contact between the wall and the footing is reduced to a narrow throat. The vertical bars are then bent to cross over at the centre of the wall, but this detail is rarely used. At the other extreme, if the wall footing is made wide enough, it is possible to get a uniform distribution of bearing pressure. In this case, there will be no rotation and a fixed condition can be assumed. In many cases, the wall and the floor slab are made continuous, and it is necessary to consider the interaction between the two elements. Appropriate values for the stiffness of the member and the effect of edge loading can be obtained from *Tables 2.76* and *2.77*.

For slabs on an elastic foundation, the values depend on the ratio r/r_k, where r_k is the radius of relative stiffness defined in section 7.2.5. The value of r_k is dependent on the modulus of subgrade reaction, for which data is given in section 7.2.4. Taking $r/r_k = 0$, which corresponds to a 'plastic' soil state, is appropriate for an empty tank liable to flotation.

6.3.3 Octagonal tanks

If the wall of a tank forms, in plan, a series of straight sides instead of being circular, the formwork may be less costly but extra reinforcement, and possibly an increased thickness of concrete, is needed to resist the horizontal bending moments that are produced in addition to the ring tension. If the tank forms a regular octagon, the bending moments in each side are $q\ l^2/12$ at the corners and $q\ l^2/24$ at the centre, where l is the length of the side and q is the 'effective' lateral pressure at depth z. If the wall is free at the top and free-to-slide at the bottom, $q = \gamma z$. In other cases, $q = n/r$ where n is the ring tension at depth z, and r is the 'effective' radius (i.e. half the distance between opposite sides). If the tank does not form a regular octagon, but the length and thickness of the sides are alternately l_1, h_1 and l_2, h_2, the horizontal bending moment at the junction of any two sides is

$$\frac{q}{12}\left[l_1^3 + l_2^3\left(\frac{h_1}{h_2}\right)^3\right]\bigg/\left[l_1 + l_2\left(\frac{h_1}{h_2}\right)^3\right]$$

6.3.4 Rectangular tanks

The walls of large rectangular reservoirs are sometimes built in discontinuous lengths in order to minimise restraints to the effects of early thermal contraction and shrinkage. If the wall base is discontinuous with the main floor slab, each wall unit is designed to be independently stable, and no slip membrane is provided between the wall base and the blinding concrete. Alternatively, the base to each wall unit can be tied into the adjacent panel of floor slab. Roof slabs can be connected to the perimeter walls, or simply supported with a sliding joint between the top of the wall and the underside of the slab. In such forms of construction, except for the effect of any corner junctions, the walls span vertically, either as a cantilever, or with ends that are simply supported or restrained, depending on the particular details.

A cantilever wall is statically determinate and, if supporting a roof, is also isolated from the effect of roof movement. The deflection at the top of the wall is an important consideration, and the base needs to be carefully proportioned in order to minimise the effect of base tilting. The problem of excessive deflection can be overcome, and the wall thickness reduced, if the wall is tied into the roof. If the wall is also provided with a narrow footing tied into the floor, it can be designed as simply supported, although considerable reliance is being put in the ability of the joint to accept continual rotation. If the wall footing is made wide enough, it is possible to obtain a uniform distribution of bearing pressure, in which case there will be no rotation and a fixed condition can be assumed. In cases where the wall and floor slab are made continuous, the interaction between the two elements should be considered.

Smaller rectangular tanks are generally constructed without movement joints, so that structural continuity is obtained in both horizontal and vertical planes. Bending moments and shear forces in individual rectangular panels with idealised edge conditions, when subjected to hydrostatic loading, are given in *Table 2.53*. For a rectangular tank, distribution of the unequal fixity moments obtained at the wall junctions is needed, and moment coefficients for tanks of different span ratios are given in *Tables 2.78* and *2.79*. The shearing forces given in *Table 2.53* for individual panels may still be used.

The tables give values for tanks where the top of the wall is either hinged or free, and the bottom is either hinged or fixed. The edge conditions are generally uncertain, and tend to vary with the loading conditions, as discussed in section 17.2. For the horizontal spans, the shear forces at the vertical edges of one wall result in axial forces in the adjacent walls. Thus, for internal loading, the shear force at the end of a long wall is equal to the tensile force in the short wall, and vice versa. In designing sections, the combined effects of bending moment, axial force and shear force need to be considered.

6.3.5 Elevated tanks

The type of bottom provided to an elevated cylindrical tank depends on the diameter of the tank and the depth of water. For small tanks a flat beamless slab is satisfactory, but beams are necessary for tanks exceeding about 3 m diameter. Some appropriate examples, which include bottoms with beams and domed bottoms, are included in section 17.4 and *Table 2.81*.

It is important that there should be no unequal settlement of the foundations of columns supporting an elevated tank, and a raft should be provided in cases where such problems could occur. In addition to the bending moments and shear forces due to the wind pressure on the tank, as described in sections 2.5 and 8.3, the wind force causes a thrust on the columns on the leeward side and tension in the columns on the windward side. The values of the thrusts and tensions can be calculated from the expressions given for columns supporting elevated tanks in section 17.4.2.

6.3.6 Effects of temperature

If the walls of a tank are subjected to significant temperature effects, due to solar radiation or the storage of warm liquids, the resulting moments and forces need to be determined by an appropriate analysis. The structure can usually be analysed separately for temperature change (expansion or contraction), and temperature differential (gradient through section). For a wall with all of the edges notionally clamped, the temperature differential results in bending moments, causing compression on the warm face and tension on the cold face, given by

$$M = \pm EI\alpha\theta/(1-\nu)h$$

where: E is the modulus of elasticity of concrete, I is second moment of area of the section, h is thickness of wall, α is the coefficient of thermal expansion of concrete, θ is temperature difference between the two surfaces, ν is Poisson's ratio. For cracked sections, ν may be taken as zero, but the value of I should allow for the tension stiffening effect of the concrete. The effect of releasing the notional restraints at edges that are free or hinged modifies the moment field and, in cylindrical tanks, causes additional ring tensions. For further information on thermal effects in cylindrical tanks, reference can be made to either the Australian or the New Zealand standard Code of Practice for liquid-retaining concrete structures.

6.4 SILOS

Silos, which may also be referred to as bunkers or bins, are deep containers used to store particulate materials. In a deep container, the linear increase of pressure with depth, found in shallow containers, is modified. Allowances are made for the effects of filling and unloading, as described in section 2.7.7. The properties of materials commonly stored in silos, and expressions for the pressures set up in silos of different forms and proportions are given in *Tables 2.15* and *2.16*.

6.4.1 Walls

Silo walls are designed to resist the bending moments and tensions caused by the pressure of the contained material. If the wall spans horizontally, it is designed for the combined effects. If the wall spans vertically, horizontal reinforcement is needed to resist the axial tension and vertical reinforcement to resist the bending. In this case, the effect of the horizontal bending moments due to continuity at the corners should also be considered. For walls spanning horizontally, the bending moments and forces depend on the number and arrangement of the compartments. Where there are several compartments, the intermediate walls act as ties between the outer walls. For various arrangements of intermediate walls, expressions for the negative bending moments on the outer walls of the silos are given in *Table 2.80*. Corresponding expressions for the reactions, which are a measure of the axial tensions in the walls, are also given. The positive bending moments can be readily calculated when the negative bending moments at the wall corners are known. An external wall is subjected to the maximum combined effects when the adjacent compartment is full. An internal cross-wall is subjected to the maximum bending moments when the compartment on one side of the wall is full, and to maximum axial tension (but zero bending) when the compartments on both sides are full. In small silos, the proportions of the wall panels may be such that they span both horizontally and vertically, in which case *Table 2.53* can be used to calculate the bending moments.

In the case of an elevated silo, the whole load is generally transferred to the columns by the walls and, when the clear span is greater than twice the depth, the wall can be designed as a shallow beam. Otherwise, the recommendations for deep beams should be followed (see section 5.8 and ref. 43). The effect of wind loads on large structures should be calculated. The effect of both the tensile force in the windward walls of the empty silo and the compressive force in the leeward walls of the full silo are important. In the latter condition, the effect of the eccentric force on the inside face of the wall, due to the proportion of the weight of the contents supported by friction, must be combined with the force due to the wind. At the base and the top of the wall, there are additional bending effects due to continuity of the wall with the bottom and the covers or roof over the compartments.

6.4.2 Hopper bottoms

The design of sloping hopper bottoms in the form of inverted truncated pyramids consists of finding, for each sloping side, the centre of pressure, the intensity of pressure normal to the slope at this point and the mean span. The bending moments at the centre and edge of each sloping side are calculated. The horizontal tensile force is computed, and combined with the bending moment, to determine the horizontal reinforcement required. The tensile force acting along the slope at the centre of pressure is combined with the bending moment at this point, to find the inclined reinforcement needed in the bottom of the slab. At the top of the slope, the bending moment and the inclined component of the hanging-up force are combined to determine the reinforcement needed in the top of the slab.

For each sloping side, the centre of pressure and the mean span can be obtained by inscribing on a normal plan, a circle that touches three of the sides. The diameter of this circle is the mean span, and its centre is the centre of pressure. The total intensity of load normal to the slope at this point is the sum of the normal components of the vertical and horizontal pressures, and the dead weight of the slab. Expressions for determining the pressures on the slab are given in *Table 2.16*. Expressions for determining the bending moments and tensile forces acting along the slope and horizontally are given in *Table 2.81*. When using this method, it should be noted that, although the horizontal span of the slab reduces considerably towards the outlet, the amount of reinforcement should not be reduced below that calculated for the centre of pressure. This is because, in determining the bending moment based on the mean span, adequate transverse support from reinforcement towards the base is assumed.

The hanging-up force along the slope has both vertical and horizontal components, the former being resisted by the walls acting as beams. The horizontal component, acting inwards, tends to produce horizontal bending moments on the beam at the top of the slope, but this is opposed by a corresponding outward force due to the pressure of the contained material. The 'hip-beam' at the top of the slope needs to be designed both to resist the inward pull from the hopper bottom when the hopper is full and the silo above is only partly filled, and also for the case when the arching of the fill concentrates the outward forces due to the peak lateral pressure on the beam during unloading. This is especially important in the case of mass-flow silos (see section 2.7.7).

6.5 BEARINGS, HINGES AND JOINTS

In the construction of frames and arches, hinges are needed at points where it is assumed that there is no bending moment. In bridges, bearings are often required at abutments and piers to transfer loads from the deck to the supports. Various types of bearings and hinges for different purposes are illustrated in *Table 2.99*, with associated notes in section 19.4.1.

Movement joints are often required in concrete structures to allow free expansion and contraction. Fluctuating movements occur due to diurnal solar effects, and seasonal changes of humidity and temperature. Progressive movements occur due to concrete creep, drying shrinkage and ground settlement. Movement joints may also be provided in structures where, because of abrupt changes of loading or ground conditions, pronounced changes occur in the size or type of foundation. Various types of joints for different purposes are illustrated in *Table 2.100*, with associated notes on their construction and application in section 19.4.2.

Chapter 7

Foundations, ground slabs, retaining walls, culverts and subways

7.1 FOUNDATIONS

The design of the foundations for a structure comprises three stages. The first is to determine from an inspection of the site, together with field data on soil profiles and laboratory testing of soil samples, the nature of the ground. The second stage is to select the stratum on which to impose the load, the bearing capacity and the type of foundation. These decisions depend not only on the nature of the ground, but also on the type of structure, and different solutions may need to be considered. Reference should be made to BS 8004: Code of Practice for foundations. The third stage is to design the foundation to transfer and distribute load from the structure to the ground.

7.1.1 Site inspection

The objective of a site inspection is to determine the nature of the top stratum and the underlying strata, in order to detect any weak strata that may impair the bearing capacity of the stratum selected for the foundation. Generally, the depth to which knowledge of the strata is obtained should be not less than one and a half times the width of an isolated foundation, or the width of a structure with closely spaced footings.

The nature of the ground can be determined by digging trial holes, by sinking bores or by driving piles. A trial hole can be taken down to only moderate depths, but the undisturbed soil can be examined, and the difficulties of excavation with the need or otherwise of timbering and groundwater pumping can be determined. Bores can be taken very much deeper than trial holes, and stratum samples at different depths obtained for laboratory testing. A test pile does not indicate the type of soil it has been driven through, but it is useful in showing the thickness of the top crust, and the depth below poorer soil at which a firm stratum is found. A sufficient number of any of these tests should be taken to enable the engineer to ascertain the nature of the ground under all parts of the foundations. Reference should be made to BS 5930: *Code of practice for site investigations*, and BS 1377: *Methods of test for soils for civil engineering purposes*.

7.1.2 Bearing pressures

The pressure that can be safely imposed on a thick stratum of soil commonly encountered is, in some districts, stipulated in local by-laws. The pressures recommended for preliminary design purposes in BS 8004 are given in *Table 2.82*, but these values should be used with caution, since several factors can necessitate the use of lower values. Allowable pressures may generally be exceeded by the weight of soil excavated down to the foundation level but, if this increase is allowed, any fill material applied on top of the foundation must be included in the total load. If the resistance of the soil is uncertain, a study of local records for existing buildings on the same soil can be useful, as may the results of a ground-bearing test.

Failure of a foundation can occur due to consolidation of the ground causing settlement, or rupture of the ground due to shearing. The shape of the surface along which shear failure occurs under a strip footing is an almost circular arc, starting from one edge of the footing, passing under the footing, and then continuing as a tangent to the arc, to intersect the ground surface at an angle depending on the angle of internal friction of the soil. Thus, the average shear resistance depends on the angle of internal resistance of the soil, and on the depth of the footing below the ground surface. In a cohesionless soil, the bearing resistance not only increases as the depth increases, but is proportional to the width of the footing. In a cohesive soil, the bearing resistance also increases with the width of footing, but the increase is less than for a non-cohesive soil.

Except when bearing directly on rock, foundations for all but single-storey buildings, or other light structures, should be taken down at least 1 m below the ground surface, in order to obtain undisturbed soil that is sufficiently consolidated. In clay soils, a depth of at least 1.5 m is needed in the United Kingdom to ensure protection of the bearing stratum from weathering.

7.1.3 Eccentric loads

When a rigid foundation is subjected to concentric loading, that is, when the centre of gravity of the loads coincides with the centre of area of the foundation, the bearing pressure on the ground is uniform and equal to the total applied load divided by the total area. When a load is eccentrically placed on a base, or a concentric load and a bending moment are applied to a base, the bearing pressure is not uniform. For a load that is eccentric about one axis of a rectangular base, the bearing pressure varies from a maximum at the side nearer the centre of gravity of the load to a minimum at the opposite side, or to zero at some intermediate position. The pressure variation is usually assumed to be linear, in which case the maximum and minimum pressures are given by the formulae in *Table 2.82*. For large eccentricities, there may be a part of the foundation where there is no bearing pressure. Although this state may be satisfactory for transient conditions (such as those due to wind), it is preferable for the foundation to be designed so that contact with the ground exists over the whole area under normal service conditions.

7.1.4 Blinding layer

For reinforced concrete footings, or other construction where there is no underlying mass concrete forming an integral part of the foundation, the bottom of the excavation should be covered with a layer of lean concrete, to protect the soil and provide a clean surface on which to place the reinforcement. The thickness of this blinding layer is typically 50–75 mm depending on the surface condition of the excavation.

7.1.5 Foundation types

The most suitable type of foundation depends primarily on the depth at which the bearing stratum lies, and the allowable bearing pressure, which determines the foundation area. Data relating to some common types of separate and combined pad foundations, suitable for sites where the bearing stratum is found close to the surface, are given in *Tables 2.82* and *2.83*. Several types of inter-connected bases and rafts are given in *Table 2.84*. In choosing a foundation suitable for a particular purpose, the nature of the structure should also be considered. Sometimes, it may be decided to accept the risk of settlement in preference to providing a more expensive foundation. For silos and fixed-end arches, the risk of unequal settlement of the foundations must be avoided at all costs, but for gantries and the bases of large steel tanks, a simple foundation can be provided and probable settlement allowed for in the design of the superstructure. In mining districts, where it is reasonable to expect some subsidence, a rigid raft foundation should be provided for small structures to allow the structure to move as a whole. For large structures, a raft may not be economical and the structure should be designed, either to be flexible, or as several separate elements on independent raft foundations.

7.1.6 Separate bases

The simplest form of foundation for an individual column or stanchion is a reinforced concrete pad. Such bases are widely used on ground that is strong and, on weaker grounds, where the structure and the cladding are light and flexible. For bases that are small in area, or founded on rock, a block of plain or nominally reinforced concrete can be used. The thickness of the block is made sufficient for the load to be transferred to the ground under the base at an angle of dispersion through the block of not less than 45° to the horizontal.

To reduce the risk of unequal settlement, the column base sizes for a building founded on a compressible soil should be in proportion to the dead load carried by each column. Bases for the columns of a storage structure should be in proportion to the total load, excluding the effects of wind. In all cases, the pressure on the ground under any base due to combined dead and imposed load, including wind load and any bending at the base of the column, should not exceed the allowable bearing value.

In the design of a separate base, the area of a concentrically loaded base is determined by dividing the maximum service load by the allowable bearing pressure. The subsequent structural design is then governed by the requirements of the ultimate limit state. The base thickness is usually determined by shear considerations, governed by the more severe of two conditions – either shear along a vertical section extending across the full width of the base, or punching shear around the loaded area – where the second condition is normally critical. The critical section for the bending moment at a vertical section extending across the full width of the base is taken at the face of the column for a reinforced concrete column, and at the centre of the base for a steel stanchion. The tension reinforcement is usually spread uniformly over the full width of the base but, in some cases, it may need to be arranged so that there is a concentration of reinforcement beneath the column. Outside this central zone, the remaining reinforcement must still conform to minimum requirements. It is also necessary for tension reinforcement to comply with the bar spacing limitations for crack control.

If the base cannot be placed centrally under the column, the bearing pressure varies linearly. The base is then preferably rectangular, and modified formulae for bearing pressures and bending moments are given in *Table 2.82*. A base supporting, for example, a column of a portal frame may be subjected to an applied moment and horizontal shear force in addition to a vertical load. Such a base can be made equivalent to a base with a concentric load, by placing the base under the column with an eccentricity that offsets the effect of the moment and horizontal force. This procedure is impractical if the direction of the applied moment and horizontal force is reversible, for example, due to wind. In this case, the base should be placed centrally under the column and designed as eccentrically loaded for the two different conditions.

7.1.7 Combined bases

If the size of the bases required for adjacent columns is such that independent bases would overlap, two or more columns can be provided with a common foundation. Suitable types for two columns are shown in *Table 2.83*, for concentrically and eccentrically loaded cases. Reinforcement is required top and bottom, and the critical condition for shear is along a vertical section extending across the full width of the base. For some conditions of loading on the columns, the total load on the base may be concentric, while for other conditions the total load is eccentric, and both cases have to be considered. Some notes on combined bases are given in section 18.1.2.

7.1.8 Balanced and coupled bases

When it is not possible to place an adequate base centrally under a column owing to restrictions of the site, and when for such conditions the eccentricity would result in inadmissible ground pressures, a balanced foundation as shown in *Tables 2.83* and *2.84*, and described in section 18.1.3, is provided. A beam is introduced, and the effect of the cantilever moment caused by the offset column load is counterbalanced by load from an adjacent column. This situation occurs frequently for external columns of buildings on sites in built-up areas.

Sometimes, as in the case of bases under the towers of a trestle or gantry, pairs of bases are subjected to moments and horizontal forces acting in the same direction on each base. In such conditions, the bases can be connected by a stiff beam that converts the effects of the moments and horizontal forces into equal and opposite vertical reactions: then, each base can be designed as concentrically loaded. Such a pair of coupled bases is shown in *Table 2.83*, which also gives formulae for the reactions and the bending moments on the beam.

7.1.9 Strip bases and rafts

When the columns or other supports of a structure are closely spaced in one direction, it is common to provide a continuous base similar to a footing for a wall. Particulars of the design of strip bases are given in *Table 2.83*. Some notes on these bases in relation to the diagrams in *Table 2.84*, together with an example, are given in section 18.1.2.

When the columns or other supports are closely spaced in two directions, or when the column loads are so high and the allowable bearing pressure is so low that a group of separate bases would totally cover the space between the columns, a single raft foundation of one of the types shown at (a)–(d) in *Table 2.84* should be provided. Notes on these designs are given in section 18.1.4.

The analysis of a raft foundation supporting a set of equal loads that are symmetrically arranged is usually based on the assumption of uniformly distributed pressure on the ground. The design is similar to that for an inverted floor, upon which the load is that portion of the ground pressure that is due to the concentrated loads only. Notes on the design of a raft, for which the columns are not symmetrically disposed, are also included in section 18.1.4. An example of the design of a raft foundation is given in *Examples of the Design of Buildings*.

7.1.10 Basements

The floor of a basement, for which a typical cross section is shown at (e) in the lower part of *Table 2.84*, is typically a raft, since the weights of the ground floor over the basement, the walls and other structure above the ground floor, and the basement itself, are carried on the ground under the floor of the basement. For water-tightness, it is common to construct the wall and the floor of the basement monolithically. In most cases, although the average ground pressure is low, the spans are large resulting in high bending moments and a thick floor, if the total load is taken as uniform over the whole area. Since the greater part of the load is transmitted through the walls, and any internal columns, it is more rational and economical to transfer the load on strips and pads placed immediately under the walls and columns. The resulting cantilever action determines the required thickness of these portions, and the remainder of the floor can generally be made thinner.

Where basements are in water-bearing soils, the effect of hydrostatic pressure must be taken into account. The upward water pressure is uniform below the whole area of the floor, which must be capable of resisting the total pressure less the weight of the floor. The walls must be designed to resist the horizontal pressures due to the waterlogged ground, and the basement must be prevented from floating. Two conditions need to be considered. Upon completion, the total weight of the basement and superimposed dead load must exceed the worst credible upward force due to the water by a substantial margin. During construction, there must always be an excess of downward load. If these conditions cannot be satisfied, one of the following steps should be taken:

1. The level of the groundwater near the basement should be controlled by pumping or other means.
2. Temporary vents should be formed in the basement floor, or at the base of the walls, to enable water to freely enter the basement, thereby equalising the external and internal pressures. The vents should be sealed when sufficient dead load from the superstructure has been obtained.
3. The basement should be temporarily flooded to a depth such that the weight of water in the basement, together with the dead load, exceeds the total upward force on the structure.

While the basement is under construction, method 1 normally has to be used, but once the basement is complete, method 3 has the merit of simplicity. Basements are generally designed and constructed in accordance with the recommendations of BS 8102, supplemented by the guidance provided in reports produced by CIRIA (ref. 57). BS 8102 defines four grades of internal environment, each grade requiring a different level of protection against water and moisture ingress. Three types of construction are described to provide either A: tanked, or B: integral or C: drained protection.

Type A refers to concrete or masonry construction where added protection is provided by a continuous barrier system. An external tanking is generally preferred so that any external water pressure will force the membrane against the structure. This is normally only practicable where the construction is by conventional methods in excavation that is open, or supported by temporary sheet piling. The structure should be monolithic throughout, and special care should be taken when a structure is supported on piles to avoid rupture of the membrane, due to settlement of the fill supported by the basement wall.

Type B refers to concrete construction where the structure itself is expected to be sufficient without added protection. A structure designed to the requirements of BS 8007 is expected to inhibit the ingress of water to the level required for a utility grade basement. It is considered that this standard can also be achieved in basements constructed by using diaphragm walls, secant pile walls and permanent sheet piling. If necessary, the performance can be improved by internal ventilation and the addition of a vapour-proof barrier.

Type C refers to concrete or masonry construction where added protection is provided by an internal ventilated drained cavity. This method is applicable to all types of construction and can provide a high level of protection. It is particularly useful for deep basements using diaphragm walls, secant pile walls, contiguous piles or steel sheet piling.

7.1.11 Foundation piers

When a satisfactory bearing stratum is found at a depth of 1.5–5 m below the natural ground level, piers can be formed from the bearing stratum up to ground level. The construction of columns or other supporting members can then begin on the top of the piers at ground level. Such piers are generally square in cross section and most economically constructed in plain concrete. When piers are impractical by reason of the depth at which a firm stratum occurs, or due to the nature of the ground, short bored piles can be used.

7.1.12 Wall footings

When the load on a strip footing is distributed uniformly over the whole length, as in the general case of a wall footing, the principal effects are due to the transverse cantilever action of the projecting portion of the footing. If the wall is of concrete and built monolithically with the footing, the critical bending moment is at the face of the wall. If the wall is of masonry, the maximum bending moment is at the centre of the footing. Expressions for these moments are given in *Table 2.83*. If the projection is less than the thickness of the base, the transverse bending moment may be ignored but the thickness should be such that the shear strength is not exceeded. Whether or not a wall footing is designed for transverse bending, longitudinal reinforcement is generally included, to give some resistance to moments due to unequal settlement and non-uniformity of bearing. In cases where a deep narrow trench is excavated down to a firm stratum, plain concrete fill is normally used.

7.1.13 Foundations for machines

The area of a concrete base supporting a machine or engine must be sufficient to spread the load onto the ground without exceeding the allowable bearing value. It is advantageous, if the centre of area of the base coincides with the centre of gravity of the loads when the machine is working, as this reduces the risk of unequal settlement. If vibration from the machine is transmitted to the ground, the bearing pressure should be considerably lower than normally taken, especially if the ground is clay or contains a large proportion of clay. It is often important that the vibration of a machine should not be transmitted to adjacent structures, either directly or via the ground. In such cases a layer of insulating material should be placed between the concrete base carrying the machine and the ground. Sometimes the base is enclosed in a pit lined with insulating material. In exceptional cases, a machine base may stand on springs, or more elaborate damping devices may be installed. In all cases, the base should be separated from any surrounding area of concrete ground floor.

With light machines the ground bearing pressure may not be the factor that determines the size of the concrete base, as the area occupied by the machine and its frame may require a base of larger area. The position of the holding-down bolts generally determines the length and width of the base, which should extend 150 mm or more beyond the outer edges of the holes left for the bolts. The depth of the base must be such that the bottom is on a satisfactory bearing stratum, and there is enough thickness to accommodate the holding-down bolts. If the machine exerts an uplift force on any part of the base, the dimensions of the base must be such that the part that is subjected to uplift has enough weight to resist the uplift force with a suitable margin of safety. All the supports of any one machine should be carried on a single base, and any sudden changes in the depth and width of the base should be avoided. This reduces the risk of fractures that might result in unequal settlements, which could throw the machine out of alignment. Reinforcement should be provided to resist all tensile forces.

Advice on the design of reinforced concrete foundations to support vibrating machinery is given in ref. 58, which gives practical solutions for the design of raft, piled and massive foundations. Comprehensive information on the dynamics of machine foundations is included in ref. 59.

7.1.14 Piled foundations

Where the upper soil strata is compressible, or too weak to support the loads transmitted by a structure, piles can be used to transmit the load to underlying bedrock, or a stronger soil layer, using end-bearing piles. Where bedrock is not located at a reasonable depth, piles can be used to gradually transmit the structural loads to the soil using friction piles.

Horizontal forces due to wind loading on tall structures, or earth pressure on retaining structures, can be resisted by piles acting in bending or by using raking piles. Foundations for some structures, such as transmission towers and the roofs to sports stadiums, are subjected to upward forces that can be resisted by tension piles. Bridge abutments and piers adjacent to water can be constructed with piled foundations to counter the possible detrimental effects of erosion.

There are two basic categories of piles. Displacement piles are driven into the ground in the form of, either a preformed solid concrete pile or a hollow tube. Alternatively, a void can be formed in the ground, by driving a closed-ended tube, the bottom of which is plugged with concrete or aggregate. This allows the tube to be withdrawn and the void to be filled with concrete. It also allows the base of the pile to be enlarged in order to increase the bearing capacity. Non-displacement or 'cast-in-place' piles are formed by boring or excavating the ground to create a void, into which steel reinforcement and concrete can be placed. In some soils, the excavation needs to be supported to stop the sides from falling in: this is achieved either with casings or by the use of drilling mud (bentonite). For further information on piles, including aspects such as pile driving, load testing and assessment of bearing capacity, reference should be made to specialist textbooks (ref. 60).

7.1.15 Pile-caps

Rarely does a foundation element consist of a single pile. In most cases, piles are arranged in groups or rows with the tops of the piles connected by caps or beams. Generally, concrete is poured directly onto the ground and encases the tops of the piles to a depth of about 75 mm. The thickness of the cap must be sufficient to ensure that the imposed load is spread equally between the piles. For typical arrangements of two to five piles forming a compact group, load can be transmitted by dispersion through the cap. Inclined struts, extending from the load to the top of each pile, are held together by tension reinforcement in the bottom of the cap to form a space frame. The struts are usually taken to intersect at the top of the cap at the centre of the loaded area, but expressions have also been developed that take into account the dimensions of the loaded area. Information regarding the design of such pile-caps, and standardised arrangements and dimensions for groups of two to five piles, are given in *Table 3.61*.

The thickness of a pile-cap designed by dispersion theory is usually determined by shear considerations along a vertical section extending across the full width of the cap. If the pile spacing exceeds three pile diameters, it is also necessary to design for punching shear. In all cases, the shear stress at the perimeter of the loaded area should not exceed the maximum design value related to the compressive strength of the struts. The reinforcement in the bottom of the pile-cap should be provided, at each end, with a full tension anchorage measured from the centre of the pile. Pile-caps can also be designed by bending theory, but this is generally more appropriate where a large number of piles are involved. In such cases, punching shear is likely to be a critical consideration.

7.1.16 Loads on piles in a group

If a group of n piles is connected by a rigid pile-cap, and the centres of gravity of the load F_v and the piles are coincident, each pile will be equally loaded, and will be subjected to a load F_v/n. If the centre of gravity of the load is displaced a distance e from the centre of gravity of the piles, the load on any one pile is

$$F_v\left(\frac{1}{n} \pm \frac{ea_1}{\sum a^2}\right)$$

where $\sum a^2$ is the sum of the squares of the distance of each pile, measured from an axis that passes through the centre of gravity of the group of piles and is at right angles to the line joining this centre of gravity and the centre of gravity of the load, and a_1 is the distance of the pile considered from this axis (positive if on the same side of the axis as the centre of gravity of the load, and negative if on the opposite side). If the structure supported on the group of piles is subjected to a bending moment M, which is transmitted to the foundations, the expression given for the load on any pile can be used by substituting $e = M/F_v$.

The total load that can be carried on a group of piles is not necessarily the safe load calculated for one pile multiplied by the number of piles. Some allowance has to be made for the overlapping of the zones of stress in the soil supporting the piles. The reduction due to this effect is greatest for piles that are supported mainly by friction. For piles supported entirely or almost entirely by end bearing, the maximum safe load on a group cannot greatly exceed the safe bearing load on the area of bearing stratum covered by the group.

7.1.17 Loads on open-piled structures

The loads and forces to which wharves, jetties and similar maritime structures are subjected are dealt with in section 2.6. Such structures can be solid walls made of plain or reinforced concrete, as are most dock walls. A quay or similar waterside wall is more often a sheet pile-wall, as described in section 7.3.3, or it can be an open-piled structure similar to a jetty. The loads on groups of inclined and vertical piles for such structures are considered in *Table 2.85*.

For each probable condition of load, the external forces are resolved into horizontal and vertical components, F_h and F_v, the points of application of which are also determined. If the direction of action and position are opposite to those shown in the diagrams, the signs in the formulae must be changed. It is assumed that the piles are surmounted by a rigid pile-cap or superstructure. The effects on each pile when all the piles are vertical are based on a simple, but approximate, method of analysis. Since a pile offers very little resistance to bending, structures with vertical piles only are not suitable when F_h is dominant. In a group containing inclined piles, F_h can be resisted by a system of axial forces, and the bending moments and shear forces in the piles are negligible. The analysis used in *Table 2.85* is based on the assumption that each pile is hinged at the head and toe. Although this assumption is not accurate, the analysis predicts the behaviour reasonably well. Three designs of the same typical jetty, using different pile arrangements, are given in section 18.2.

7.2 INDUSTRIAL GROUND FLOORS

Most forms of activity in buildings – from manufacturing, storage and distribution to retail and recreation – need a firm platform on which to operate. Concrete ground floors are almost invariably used for such purposes. Although in many parts of the world conventional manufacturing activity has declined in recent years, there has been a steady growth in distribution, warehousing and retail operations, to serve the needs of industry and society. The scale of such facilities, and the speed with which they are constructed, has also increased, with higher and heavier racking and storage equipment being used. These all make greater demands on concrete floors. The following information is taken mainly from ref. 61, where a comprehensive treatment of the subject will be found.

7.2.1 Floor uses

In warehouses, materials handling equipment is used in two distinct areas, according to whether the movement of traffic is free or defined. In free-movement areas, vehicles can travel randomly in any direction. This typically occurs in factories, retail outlets, low-level storage and food distribution centres. In defined-movement areas, vehicles use fixed paths in very narrow aisles. This usually occurs where high-level storage racking is being employed, and distribution and warehouse facilities often combine areas of free movement for low-level activities, such as unloading and packing, alongside areas of defined movement for high-level storage. The two floor uses require different tolerances on surface regularity.

7.2.2 Construction methods

A ground-supported industrial floor slab is made up of layers of materials comprising a sub-base, a slip membrane/methane barrier, and a concrete slab of appropriate thickness providing a suitable wearing surface. Various construction methods can be used to form the concrete slab.

Large areas of floor up to several thousand square metres in extent can be laid in a continuous operation. Fixed forms are used up to 50 m apart at the edges of the area only. Concrete is discharged into the area and spread either manually, or by machine. Surface levels are controlled either manually, using a target staff in conjunction with a laser level transmitter, or by direct control of a laser-guided spreading machine. After the floor has been laid and finished, the area is sub-divided into panels, typically on a 6 m grid in both directions. This is achieved by making saw cuts in the top surface for a depth of at least one-quarter of the depth of the slab, creating a line of weakness in the slab that induces a crack below the saw cut. As a result of concrete shrinkage, each sawn joint will open by a small amount.

With such large-area construction, there are limitations on the accuracy of level and surface regularity that can be achieved, and the construction is most commonly used for free-movement floor areas.

The large-area construction method can also be employed without sub-dividing the area into small panels. In this case, no sawn joints are made, but steel fibres are incorporated in the concrete mix to control the distribution and width of the cracks that occur as a result of shrinkage. The formed joints at the edges of the area will typically open by about 20 mm.

Floors can also be formed as a series of long strips typically 4–6 m wide, with forms along each side. Strips can be laid alternately, with infill strips laid later, or consecutively, or between 'leave-in-place' screed rails. Concrete is poured in a continuous operation in each strip, after which transverse saw cuts are made about 6 m apart to accommodate longitudinal shrinkage. As formwork can be set to tight tolerances, and the distance between the forms is relatively small, the long-strip method lends itself to the construction of very flat floors, and is particularly suitable for defined-movement floor areas.

7.2.3 Reinforcement

Steel fibres, usually manufactured from cold-drawn wire, are commonly used in ground-supported slabs. The fibres vary in length up to about 60 mm, with aspect ratios (length/nominal diameter) from 20 to 100, and a variety of cross sections. In order to increase pull-out resistance the fibres have enlarged, flattened or hooked ends, roughened surface textures or wavy profiles. The composite concrete slab can have considerable ductility dependent on fibre type, dosage, tensile strength and anchorage mechanism. The ductility is commonly measured using the Japanese Standard test method, which uses beams in a third-point loading arrangement. Load-deflection curves are plotted as the load increases until the first crack and then decreases with increasing deflection. The ductility value is expressed as the average load to a deflection of 3 mm divided by the load to first crack. This measure is commonly known as the equivalent flexural strength ratio. In large-area floors with shrinkage joints at the edges only, fibre dosages in the order of 35–45 kg/m^3 are used to control the distribution and width of cracks. In floors with additional sawn joints, fibre dosages in the range 20–30 kg/m^3 are typically used.

In large area floors with additional sawn joints, steel fabric reinforcement (type A) can be placed in the bottom of the slab with typically 50 mm of cover. The proportion of reinforcement used is typically 0.1–0.125% of the effective cross section bd, which is small enough to ensure that the reinforcement will yield at the sawn joints as the concrete shrinks, and also sufficient to provide the slab with adequate rotational capacity after cracking.

7.2.4 Modulus of subgrade reaction

For design purposes, the subgrade is assumed to be an elastic medium characterised by a modulus of subgrade reaction k_s, defined as the load per unit area causing unit deflection. It can be shown that errors of up to 50% in the value of k_s have only a small effect on the slab thickness required for flexural design. However, deflections are more sensitive to k_s values, and long-term settlement due to soil consolidation under load can be much larger than the elastic deflections calculated as part of the slab design.

In principle, the value of k_s used in design should be related to the range of influence of the load, but it is normal practice to base k_s on a loaded area of diameter 750 mm. To this end, it is strongly recommended that the value of k_s is determined from a BS plate-loading test, using a 750 mm diameter plate and a fixed settlement of 1.25 mm. If a smaller plate is used, or a value of k_s appropriate to a particular area is required, the following approximate relationship may be assumed:

$$k_s = 0.5(1 + 0.3/D)^2 k_{0.75}$$

where D is the diameter of the loaded area, and $k_{0.75}$ is a value for D = 0.75 m. This gives values of $k_s/k_{0.75}$ as follows:

D (m)	0.3	0.45	0.75	1.2	∞
$k_s/k_{0.75}$	2.0	1.4	1.0	0.8	0.5

In the absence of more accurate information, typical values of k_s according to the soil type are given in the following table.

Soil type	Values of k_s (MN/m^3)	
	Lower	Upper
Fine or slightly compacted sand	15	30
Well compacted sand	50	100
Very well compacted sand	100	150
Loam or clay (moist)	30	60
Loam or clay (dry)	80	100
Clay with sand	80	100
Crushed stone with sand	100	150
Coarse crushed stone	200	250
Well-compacted crushed stone	200	300

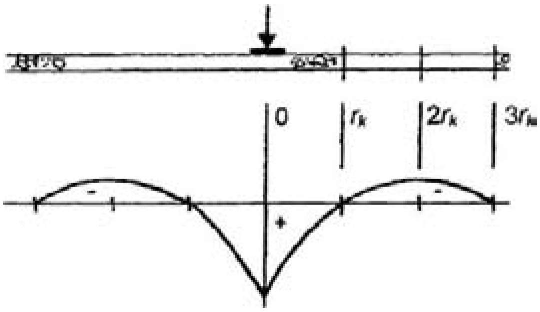
7.2.5 Methods of analysis

Traditionally, ground-supported slabs have been designed by elastic methods using equations developed by Westergaard in the 1920s. Such slabs are relatively thick and an assessment of deflections and other in-service requirements has generally been unnecessary. Using plastic methods of analysis, thinner slabs can be designed, and the need to investigate in-service requirements and load-transfer across joints has become more important. The use of plastic analysis assumes that the slab has adequate ductility after cracking, that is, it contains sufficient fibres or reinforcement, as described in section 7.2.3, to give an equivalent flexural strength ratio in the range 0.3–0.5. Plain concrete slabs, and slabs with less than the minimum recommended amounts of fibres or reinforcement, should still be designed by elastic methods.

Westergaard assumed that a ground-supported concrete slab is a homogeneous, isotropic elastic solid in equilibrium, with the subgrade reactions being vertical only and proportional to the deflections of the slab. He also introduced the concept of the radius of relative stiffness r_k, given by the relationship:

$$r_k = [E_c h^3/12(1-\nu^2)k_s]^{0.25}$$

where E_c is the short-term modulus of elasticity of concrete, h is the slab thickness k_s is the modulus of subgrade reaction and ν is Poisson's ratio. The physical significance of r_k is illustrated in the following figure showing the approximate distribution of elastic bending moments for a single internal concentrated load. The bending moment is positive (tension at the bottom of the slab) with a maximum value at the load position. Along radial lines, it remains positive reducing to zero at r_k from the load. It then becomes negative reaching a maximum at $2r_k$ from the load, with the maximum negative moment (tension at the top of the slab) significantly less than the maximum positive moment. The moment approaches zero at $3r_k$ from the load.



Approximate distribution of elastic bending moments for an internal concentrated load on a ground-supported slab

As the load is increased, the tensile stresses at the bottom of the slab under the load will reach the flexural strength of the concrete. Radial tension cracks will form at the bottom of the slab and, provided there is sufficient ductility, the slab will yield. Redistribution of moments will occur, with a reduction in the positive moment at the load position and a substantial increase in the negative moments some distance away. With further increases in load, the positive moment at the load position will remain constant, and the negative moments will increase until the tensile stresses at the top of the slab reach the flexural strength of the concrete, at which stage failure is assumed. For further information on the analysis and design method with fully worked examples, see ref. 61.

7.3 RETAINING WALLS

Information on soil properties and the pressures exerted by soils on retaining structures is given in section 9.1 and *Tables 2.10–2.14*. This section deals with the design of walls to retain soils and materials with similar engineering properties. In designing to British Codes of Practice, the geotechnical aspects of the design, which govern the size and proportions of the structure, are considered in accordance with BS 8002. Mobilisation factors are introduced into the calculation of the soil strengths, and the resulting pressures are used for both serviceability and ultimate requirements. For the subsequent design of the structure to BS 8110, the earth loads obtained from BS 8002 are taken as characteristic values. In designing to the EC, partial safety factors are applied to the soil properties for the geotechnical aspects of the design, and to the earth loads for the structural design.

7.3.1 Types of retaining wall

Earth retention systems can be categorised into one of two groups, according to whether the earth is stabilised externally or internally, as shown in *Table 2.86*. An externally stabilised system uses an external structural wall to mobilise stabilising forces. An internally stabilised system utilises reinforcements installed within the soil, and extending beyond the potential failure zone.

Traditional retaining walls can be considered as externally stabilised systems, one of the most common forms being the reinforced concrete cantilever wall. Retaining walls on spread foundations, together with gravity structures, support the soil by weight and stiffness to resist forward sliding, overturning and excessive soil movements. The equilibrium of cantilever walls can also be obtained by embedment of the lower part of the wall. Anchored or propped walls obtain their equilibrium partly by embedment of the lower part of the wall, and partly from an anchorage or prop system that provides support to the upper part of the wall.

Internally stabilised walls built above ground are known as reinforced soil structures. By placing reinforcement within the soil, a composite material can be produced that is strong in tension as well as compression. A key aspect of reinforced soil walls is its incremental form of construction, being built up a layer at a time, starting from a small plain concrete strip footing. In this way, construction is always at ground level, the structure is always stable, and progress can be very rapid. The result of the incremental construction is that the soil is partitioned with each layer receiving support from a locally inserted reinforcing element. The process is the opposite of what occurs in a conventional wall, where pressures exerted by the backfill are integrated to produce an overall force to be resisted by the structure. The materials used in a reinforced soil structure comprise a facing (usually reinforced concrete), soil reinforcement (in the form of flat strips, anchors or grids, made from either galvanised steel or synthetic material) and soil (usually a well-graded cohesionless material). Reinforced soil structures are more economic than equivalent structures using externally stabilised methods.

Internal soil stabilisation used in the formation of cuttings or excavations is known as soil nailing. The process is again incremental, with each stage of excavation limited in depth so that the soil is able to support itself. The exposed soil face is protected, usually by a covering of light mesh reinforcement and spray applied concrete. Holes are drilled into the soil, and reinforcement in the form of steel bars installed and grouted. With both reinforced soil and soil nailing, great care is taken to make sure that the reinforcing members do not corrode or deteriorate. Hybrid systems combining elements of internally and externally stabilised soils are also used.

7.3.2 Walls on spread bases

Various walls on spread bases are shown in *Table 2.86*. A cantilever wall is suitable for walls of moderate height. If the soil to be retained can be excavated during construction of the wall, or the wall is required to retain an embankment, the base can project backwards. This is always advantageous, as the earth supported on the base assists in counterbalancing the overturning effect due to the horizontal pressures exerted by the soil. However, a base that projects mainly backwards but partly forwards is usually necessary, in order to limit the bearing pressure at the toe to an allowable value. Sometimes, due to the proximity of adjacent property, it may be impossible to project a base backwards. Under such conditions, where the base projection is entirely forwards, the provision of a key below the base is necessary to prevent sliding, by mobilising the passive resistance of the soil in front of the base.

For wall heights greater than about 8 m, the stem thickness of a cantilever wall becomes excessive. In such cases, a wall with vertical counterforts can be used, in which the slab spans horizontally between the counterforts. For very high walls, in which the soil loading is considerable towards the bottom of the wall, horizontal beams spanning between the counterforts can be used. By graduating the spacing of the beams to suit the loading, the vertical bending moments in each span of the slab can be equalised, and the slab thickness kept the same.

The factors affecting the design of a cantilever slab wall are usually considered per unit length of wall, when the wall is of constant height but, if the height varies, a length of say 3 m could be treated as a single unit. For a wall with counterforts, the length of a unit is taken as the distance between adjacent counterforts. The main factors to be considered in the design of walls on spread bases are stability against overturning, ground bearing pressure, resistance to sliding and internal resistance to bending moments and shearing forces. Suitable dimensions for the base to a cantilever wall can be estimated with the aid of the graph given in *Table 2.86*.

In BS 8002, for design purposes, soil parameters are based on representative shear strengths that have been reduced by applying mobilisation factors. Also, for friction or adhesion at a soil–structure interface, values not greater than 75% of the design shear strength are taken. Allowance is made for a minimum surcharge of 10 kN/m^2 applied to the surface of the retained soil, and for a minimum depth of unplanned earth removal in front of the wall equal to 10% of the wall height, but not less than 0.5 m.

For overall equilibrium, the effects of the disturbing forces acting on the structure should not exceed the effects that can be mobilised by the resisting forces. No additional factors of safety are required with regard to overturning or sliding forwards. For bases founded on clay soils, both the short-term (using undrained shear strength) and long-term (using drained shear strength) conditions should be considered. Checks on ground bearing are required for both the service and ultimate conditions, where the design loading is the same for each, but the bearing pressure distribution is different. For the ultimate condition, a uniform distribution is considered with the centre of pressure coincident with the centre of the applied force at the underside of the base. In general, therefore, the pressure diagram does not extend over the entire base. In cases where resistance to sliding depends on base adhesion, it is unclear as to whether the contact surface length should be based on the service or the ultimate condition.

The foregoing wall movements, due to either overturning or sliding, are independent of the general tendency of a bank or a cutting to slip and carry the retaining wall along with it. The strength and stability of the retaining wall have no bearing on such failures. The precautions that must be taken to prevent such failures are outside the scope of the design of a wall that is constructed to retain the toe of the bank, and are a problem in soil mechanics.

Adequate drainage behind a retaining wall is important to reduce the water pressure on the wall. For granular backfills of high permeability, no special drainage layer is needed, but some means of draining away any water that has percolated through the backfill should be provided, particularly where a wall is founded on an impermeable material. For cohesionless backfills of medium to low permeability, and for cohesive soils, it is usual to provide a drainage layer behind the wall. Various methods can be used, for instance: (a) a blanket of rubble or coarse aggregate, clean gravel or crushed stone; (b) hand-placed pervious blocks as dry walling; (c) graded filter drain, where the back-filling consists of fine-grain material; (d) a geotextile filter used in combination with a permeable granular material. Water entering the drainage layer should drain into a drainage system, which allows free exit of the water either by the provision of weep-holes, or by porous land drains and pipes laid at the bottom of the drainage layer, and led to sumps or sewers via catchpits. Where weep-holes are being used, they should be at least 75 mm in diameter, and at a spacing not more than 1 m horizontally and 1–2 m vertically. Puddled clay or concrete should be placed directly below the weep-holes or pipes, and in contact with the back of the wall, to prevent water from reaching the foundations.

Vertical movement joints should be provided at intervals dependent upon the expected temperature range, the type of the structure and changes in the wall height or the nature of the foundations. Guidance on design options to accommodate movement due to temperature and moisture change are given in BS 8007 and Highways Agency BD 28/87.

7.3.3 Embedded (or sheet) walls

Embedded walls are built of contiguous or interlocking piles, or diaphragm wall panels, to form a continuous structure. The piles may be of timber, or concrete or steel, and have lapped or V-shaped, or tongued and grooved, or interlocking joints between adjacent piles. Diaphragm wall panels are formed of reinforced concrete, using a bentonite or polymer suspension as part of the construction process. Excavation is carried out in the suspension to a width equal to the thickness of the wall required. The suspension is designed to maintain the stability of the slit trench during digging and until the diaphragm wall has been concreted. Wall panels are formed in predetermined lengths with prefabricated reinforcement cages lowered into the trench. Concrete is cast *in situ* and placed by tremie: it is vital that the wet concrete flows freely without segregation so as to surround the reinforcement and displace the bentonite.

Cantilever walls are suitable for only moderate height, and it is preferable not to use cantilever walls when services or foundations are located wholly or partly within the active soil zone, since horizontal and vertical movement in the retained material can cause damage. Anchored or propped walls can have one or more levels of anchor or prop in the upper part of the wall. They can be designed to have fixed or free earth support at the bottom, as stability is derived mainly from the anchorages or props.

Traditional methods of design, although widely used, all have recognised shortcomings. These methods are outlined in annex B of BS 8002, where comments are included on the applicability and limitations of each method. The design of embedded walls is beyond the scope of this *Handbook*, and for further information the reader should refer to BS 8002, Highways Agency BD 42/94 and ref. 62.

7.4 CULVERTS AND SUBWAYS

Concrete culverts, which can be either cast *in situ* or precast, are usually of circular or rectangular cross section. Box type structures can also be used to form subways, cattle creeps or bridges over minor roads.

7.4.1 Pipe culverts

For conducting small streams or drains under embankments, culverts can be built with precast reinforced concrete pipes, which must be strong enough to resist vertical and horizontal pressures from the earth, and other superimposed loads. The pipes should be laid on a bed of concrete and, where passing under a road, should be surrounded with reinforced concrete at least 150 mm thick. The culvert should also be reinforced to resist longitudinal bending resulting from unequal vertical earth pressure and unequal settlement. Due to the uncertainty associated with the magnitude and disposition of the earth pressures, an accurate analysis of the bending moments is impracticable. A basic guide is to take the positive moments at the top and bottom of the pipe, and the negative moments at the ends of a horizontal diameter, as $0.0625qd^2$, where d is the diameter of the circular pipe, and q is the intensity of both the downward pressure on the top and the upward pressure at the bottom, assuming the pressure to be distributed uniformly on a horizontal plane.

7.4.2 Box culverts

The load on the top of a box culvert includes the weights of the earth covering and the top slab, and the imposed load (if any). The weights of the walls and top slab (and any load that is on them) produce an upward reaction from the ground. The weights of the bottom slab and water in the culvert are carried directly on the ground below the slab, and thus have no effect other than their contribution to the total bearing pressure. The horizontal pressure due to the water in the culvert produces an internal triangular load on the walls, or a trapezoidal load if the surface of the water outside the culvert is above the top, when there will also be an upward pressure on the underside of the top slab. The magnitude and distribution of the earth pressure against the sides of the culvert can be calculated in accordance with the information in section 9.1, consideration being given to the risk of the ground becoming waterlogged resulting in increased pressure and the possibility of flotation. Generally, there are only two load conditions to consider:

1. Culvert empty: maximum load on top slab, weight of the walls and maximum earth pressure on walls.
2. Culvert full: minimum load on top slab, weight of the walls, minimum earth pressure and maximum internal hydrostatic pressure on walls (with possible upward pressure on top slab).

In some circumstances, these conditions may not produce the maximum load effects at any particular section, and the effect of every probable combination should be considered. The cross sections should be designed for the combined effects of axial force, bending and shear as appropriate. A simplistic analysis can be used to determine the bending moments produced in a monolithic rectangular box, by considering the four slabs as a continuous beam of four spans with equal moments at the end supports. However, if the bending of the bottom slab tends to produce a downward deflection, the compressibility of the ground and the consequent effect on the bending moments must be taken into account. The loads can be conveniently divided into the following cases:

1. A uniformly distributed load on the top slab, and a uniform reaction from the ground under the bottom slab.
2. A concentrated imposed load on the top slab and a uniform reaction from the ground under the bottom slab.
3. Concentrated loads due to the weight of each wall and a uniform reaction from the ground under the bottom slab.
4. A triangular distributed horizontal pressure on each wall due to the increase in earth pressure in the height of the wall.
5. A uniformly distributed horizontal pressure on each wall due to pressure from the earth and any surcharge above the level of the top slab.
6. Internal horizontal and possibly vertical pressures due to water in the culvert.

Formulae for the bending moments at the corners of the box due to each load case, when the top and bottom slabs are the same thickness, are given in *Table 2.87*. The limiting ground conditions associated with the formulae should be noted.

7.4.3 Subways

The design and construction of buried box type structures, which could be complete boxes, portal frames or structures where the walls are propped by the top slab, are covered by recommendations in Highways Agency standard BD 31/87. These recommendations do not apply to structures that are installed by methods such as thrust boring or pipe jacking.

The nominal superimposed dead load consists of the weight of any road construction materials and the soil cover above the structure. Due to negative arching of the fill material, the structure can be subjected to loads greater than the weight of fill directly above it. An allowance for this effect is made, by considering a minimum load based on the weight of material directly above the structure, and a maximum load equal to the minimum load multiplied by 1.15. The nominal horizontal earth pressures on the walls of the box structure are based on a triangular distribution, with the value of the earth pressure coefficient taken as a maximum of 0.6 and a minimum of 0.2. It is to be assumed that either the maximum or the minimum value can be applied to one wall, irrespective of the value that is applied to the other wall.

Where the depth of cover measured from the finished road surface to the top of the structure is greater than 600 mm, the nominal vertical live loads to be considered are the HA wheel load and the HB vehicle. To determine the nominal vertical live load pressure, dispersion of the wheel loads may be taken to occur from the contact area on the carriageway to the top of the structure at a slope of 2 vertically to 1 horizontally. For structures where the depth of cover is in the range 200–600 mm, full highway loading is to be considered. For HA load, the KEL may be dispersed below the depth of 200 mm from the finished road surface. Details of the nominal vertical live loads are given in sections 2.4.8 and 2.4.9, and *Table 2.5*.

Part 2

Loads, materials and structures

Chapter 8

Loads

In this chapter, unless otherwise stated, all loads are given as characteristic, or nominal (i.e. unfactored) values. For design purposes, each value must be multiplied by the appropriate partial safety factor for the particular load, load combination and limit-state being considered.

Although unit weights of materials should be given strictly in terms of mass per unit volume (e.g. kg/m^3), the designer is usually only concerned with the resulting gravitational forces. To avoid the need for repetitive conversion, unit weights are more conveniently expressed in terms of force (e.g. kN/m^3), where 1 kN may be taken as 102 kilograms.

8.1 DEAD LOAD

The data for the weights of construction materials given in the following tables has been taken mainly from EC 1: Part 1.1, but also from other sources such as BS 648.

8.1.1 Concrete

The primary dead load is usually the weight of the concrete structure. The weight of reinforced concrete varies with the density of the aggregate and the percentage of reinforcement. In UK practice, a value of 24 kN/m^3 has traditionally been used for normal weight concrete with normal percentages of reinforcement, but a value of 25 kN/m^3 is recommended in EC 1. Several typical weights for normal, lightweight and heavyweight (as used for kentledge and nuclear-radiation shielding) concretes are given in *Table 2.1*. Weights are also given for various forms and depths of concrete slabs.

8.1.2 Other construction materials and finishes

Dead loads include such permanent weights as those of the finishes and linings on walls, floors, stairs, ceilings and roofs; asphalt and other applied waterproofing layers; partitions; doors, windows, roof and pavement lights; superstructures of steelwork, masonry or timber; concrete bases for machinery and tanks; fillings of earth, sand, plain concrete or hardcore; cork and other insulating materials; rail tracks and ballasting; refractory linings and road surfacing. In *Table 2*.1, the basic weights of various structural and other materials including metals, stone, timber and rail tracks are given.

The average equivalent weights of various cladding types, as given in *Table 2.2*, are useful in estimating the loads imposed on a concrete substructure. The weights of walls of various constructions are also given in *Table 2.2*. Where a concrete lintel supports a brick wall, it is generally not necessary to consider the lintel as supporting the entire wall above; it is sufficient to allow only for the triangular areas indicated in the diagrams in *Table 2.2*.

8.1.3 Partitions

The weight of a partition is determined by the material of which it is made and the storey height. When the position of the partition is not known, or the use of demountable partitions is envisaged, the equivalent uniformly distributed load given in *Table 2.2* should be considered as an imposed load in the design of the supporting floor slabs.

Weights of permanent partitions, whose position is known, should be included in the dead load. Where the length of the partition is in the direction of span of the slab, an equivalent UDL may be used as given in *Table 2.2*. In the case of brick or similarly bonded partitions continuous over the slab supports, some relief of loading on the slab will occur due to the arching action of the partition, unless this is invalidated by the presence of doorways or other openings. Where the partition is at right angles to the span of the slab, a concentrated line load should be applied at the appropriate position. The slab should then be designed for the combined effect of the distributed floor load and the concentrated load.

8.2 IMPOSED LOADS

Imposed loads on structures include the weights of stored materials and the loads resulting from occupancy and traffic. Comprehensive data regarding the weights of stored materials associated with building, industry and agriculture are given in EC 1: Part 1.1. Data for loads on floors due to livestock and agricultural vehicles are given in BS 5502: Part 22.

8.2.1 Imposed loads on buildings

Data for the vertical loads on floors, and horizontal loads on parapets, barriers and balustrades are given in BS 6399: Part 1. Loads are given in relation to the type of activity/occupancy for which the floor area will be used in service, as follows:

A Domestic and residential activities
B Office and work areas not covered elsewhere

Weights of construction materials and concrete floor slabs

2.1

Type	Material	Weight	Material	Weight
Concrete	Normal weight	kN/m^3	Lightweight (density $\leq$ 2000 kg/m^3)	kN/m^3
	(2000 kg/m^3 < density $\leq$ 2800 kg/m^3)		low strength (insulating)	4–8
	plain or lightly reinforced	24	medium strength (blockwork)	8–16
	reinforced: 2% reinforcement	25	high strength (structural)	16–20
	4% reinforcement	26	Heavyweight (density > 2800 kg/m^3)	30–50
Metals	Aluminium	27	Iron: wrought	76
	Brass, bronze	83–85	Lead	112–114
	Copper	87–89	Steel	77–79
	Iron: cast	71–73	Zinc	71–72
Stone	Basalt	27–31	All-in aggregate	20
	Granite	27–30	Hardcore (consolidated)	19
	Limestone: dense	20–29	Quarry waste	14
	Sandstone	21–27	Stone rubble (packed)	22
	Slate	28	Soils and similar fill materials	*Table 2.10*
Timber	Baltic pine, spruce	5–6	Particleboard:	
	Douglas fir, hemlock	6–7	chipboard	7–8
	Larch, oak (imported), pitch pine, teak	7–8	cement-bonded particle board	12
	Oak (English)	8–9	flake board, strand board, wafer board	7
	Fibre building board:		Plywood:	
	hardboard (standard and tempered)	10	birch plywood	7
	medium density fibreboard	8	blockboard, laminboard, softwood	5
	softboard	4	Wood-wool	6
Miscellaneous	Asphalt: mastic	18–22	per unit area	kN/m^2
	hot-rolled	23	Asphalt, 20 mm thick	0.4
	Asphaltic concrete	24–25	Brickwork and blockwork	*Table 2.2*
	Ballast (normal, e.g. granite)	20	Concrete paving, 50 mm thick	1.2
	Cork (compressed)	4	Granolithic screed, 40 mm thick	1.0
	Glass (in sheets)	25	Lead sheet, 2.5 mm thick	0.3
	Plastics: acrylic sheet	12	Roof cladding and wall sheeting	*Table 2.2*
	Terra cotta (solid)	21	Terrazzo flooring, 25 mm thick	0.6
Rail tracks	Tracks with ballasted bed:		per unit bed length	kN/m
	2 rails UIC 60 with prestressed concrete sleepers and track fastenings			6.0
	2 rails UIC 60 with timber sleepers and track fastenings			3.1
	Tracks without ballasted bed:			
	2 rails UIC 60 with track fastenings			1.7
	2 rails UIC 60 with track fastenings, bridge beam and guard rails			4.9

WEIGHTS OF IN-SITU AND PRECAST CONCRETE FLOORS

Concrete slabs	**Solid slabs**						
	Depth mm	100	125	150	200	250	300
	Weight kN/m^2	2.4	3.0	3.6	4.8	6.0	7.2
	Ribbed slabs (rib spacing 600 mm, rib width 125 mm minimum, rib draw each side 10°, flange thickness 75 mm) Note. For thicker (or thinner) flanges, add (or deduct) 0.6 kN/m^2 per 25 mm concrete						
	Depth mm			250	325	400	475
	Weight kN/m^2: 100% ribbed			3.0	3.6	4.3	5.0
	: 75% ribbed, 25% solid			3.8	4.7	5.6	6.6
	Waffle slabs (rib spacing 900 mm, rib width 125 mm minimum, rib draw each side 1/5, flange thickness 75 mm) Note. For thicker flanges, add 0.6 kN/m^2 per 25 mm concrete						
	Depth mm				300	400	500
	Weight kN/m^2: 100% waffle				3.6	4.8	6.0
	: 75% waffle, 25% solid				4.5	6.0	7.5
	Precast hollow-core units (nominal width 1200 mm) Note. For slabs with structural topping, add 0.6 kN/m^2 per 25 mm concrete (minimum thickness 50 mm)						
	Depth mm	110	150	200	250	300	400
	Weight kN/m^2	2.2	2.4	2.9	3.7	4.1	4.7
	Precast double-tee units (nominal width 2400 mm) Note. For slabs with structural topping, add 0.6 kN/m^2 per 25 mm concrete (minimum thickness 50 mm)						
	Depth mm	325	425	525	625	725	825
	Weight kN/m^2	2.6	2.9	3.3	3.7	4.1	4.5

Weights of roofs and walls

2.2

Roofs

per unit area	kN/m^2
Steel roof trusses in spans up to 25 m	1.0 – 2.0
Corrugated asbestos-cement or steel sheeting, steel purlins etc.	0.4 – 0.5
Patent glazing (with lead-covered astragals), steel purlins etc.	0.4
Slates or tiles, battens, steel purlins etc.	0.7 – 0.9
ditto with boarding, felt etc.	0.8 – 1.1

Walls and sheeting etc.

per unit area	kN/m^2	per unit area	kN/m^2
Blockwork: 200 mm thick		Brickwork: 125 mm thick	
clay: common	3.8	calcium-silicate	2.3
: hollow	2.3	clay: engineering	2.6
concrete (autoclaved aerated)	1.2–1.5	concrete	2.7
ditto (light-weight aggregate): solid	2.6	refractory	1.3
: hollow	2.2	Gypsum panels, 75 mm thick	4.4
ditto (normal-weight aggregate): solid	4.3	Plaster: 2 coats gypsum, 13 mm thick	2.2
: hollow	2.9	Plasterboard: 13 mm thick	1.1

per unit area	kN/m^2
Corrugated asbestos-cement or steel sheeting (including bolts, sheeting rails etc.)	4.3
Steel wall framing (for sheeting or brick panels)	2.4 – 3.4
ditto with brick panels and windows	24
ditto with asbestos-cement or steel sheeting	7.2
Doors (ordinary industrial type: wooden)	3.8
Windows (industrial type: metal or wooden frames)	2.4

Partition loads on slabs

The following symbols are used in the expressions given below:

e effective width of strip supporting partition in m
h distance from face of partition to free edge of slab in m
h_p thickness of partition in m
l effective span of slab in m
w_e equivalent uniformly distributed load in kN/m^2
w_p weight of partition in kN/m

Position of partition unknown – consider as imposed load with $w_e = w_p/3$

Minimum allowance for demountable partitions (offices): $w_e = 1.0$ kN/m^2
Typical allowance for 150 mm solid blockwork with 13 mm gypsum plaster both sides: $w_e = 2.5$ kN/m^2

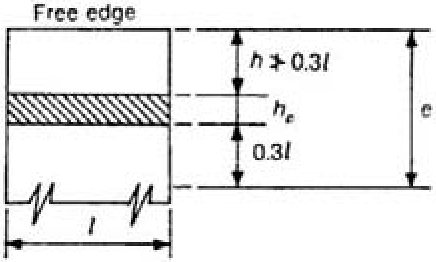
Position of partition known – consider as dead load as follows:

Partition parallel to span: $w_e = w_p/e$

$$e = h_p + h + 0.3l \leq h_p + 0.6l$$

Note. For ribbed slabs, smaller values of e may be appropriate but not less than $h_p + 1.0$ m

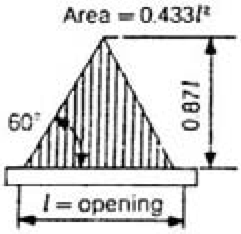
Partition normal to span: treat as concentrated load

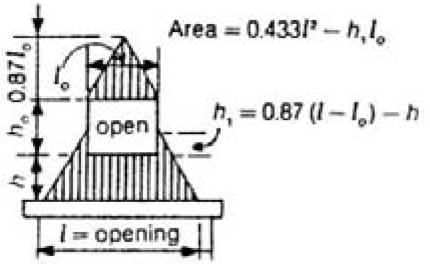


Loads on lintels

Lintels supporting brickwork
(or similarly bonded walls)

Shading denotes area
of wall considered to
be supported by lintel





C	Areas where people may congregate
D	Shopping areas
E	Areas susceptible to the accumulation of goods
F/G	Vehicle and traffic areas

Details of the imposed loads for categories A and B are given in *Table 2.3*. Values are given for uniformly distributed and concentrated loads. These are not to be taken together, but considered as two separate load cases. The concentrated loads normally do not need to be considered for solid or other slabs that are capable of effective lateral distribution. When used for calculating local effects, such as bearing or the punching of thin flanges, a square contact area of 50 mm side should be assumed in the absence of any other specific information.

With certain exceptions, the imposed loads on beams may be reduced according to the area of floor supported. Loads on columns and foundations may be reduced according to either the area of floor or the number of storeys supported. Details of the reductions and the exceptions are given in *Table 2.3*.

Data given in *Table 2.4* for the load on flat or mono-pitch roofs has been taken from BS 6399: Part 3. The loads, which are additional to all surfacing materials, include for snow and other incidental loads but exclude wind pressure. For other roof shapes and the effects of local drifting of snow behind parapets, reference should be made to BS 6399: Part 3.

For building structures designed to meet the requirements of EC 2: Part 1, details of imposed and snow loads are given in EC 1: Parts 1.1 and 1.3 respectively.

8.2.2 Imposed loads on highway bridges

The data given in *Table 2.5* for the imposed load on highway bridges have been taken from the Highways Agency document BD 37/01. Type HA loading consists of two parts: a uniform load whose value varies with the 'loaded length', and a single KEL that is positioned so as to have the most severe effect. The loaded length is the length over which the application of the load increases the effect to be determined. Influence lines may be needed to determine critical loaded lengths for continuous spans and arches. Loading is applied to one or more notional lanes and multiplied by appropriate lane factors. The alternative of a single wheel load also needs to be considered in certain circumstances.

Type HB is a unit loading represented by a 16–wheel vehicle of variable bogie spacing, where one unit of loading is equivalent to 40 kN. The number of units considered for a public highway is normally between 30 and 45, according to the appropriate authority. The vehicle can be placed in any transverse position on the carriageway, displacing HA loading over a specified area surrounding the vehicle.

For further information on the application of combined HA and HB loading, and details of other loads to be considered on highway bridges, reference should be made to BD 37/01 and BD 60/94. For information on loads to be considered for the assessment of existing highway bridges, reference should be made to BD 21/01.

8.2.3 Imposed loads on footbridges

The data given in *Table 2.6* for the imposed load on bridges due to pedestrian traffic have been taken from the Highways Agency document BD 37/01. For further information on the pedestrian loading to be considered on elements of highway or railway bridges that also support footways or cycle tracks, and the serviceability vibration requirements of footbridges, reference should be made to BD 37/01.

8.2.4 Imposed loads on railway bridges

The data given in *Table 2.6* for the imposed load on railway bridges has been taken from the Highways Agency document BD 37/01. Types RU and SW/0 apply to main line railways, type SW/0 being considered as an additional and separate load case for continuous bridges. For bridges with one or two tracks, loads are to be applied to each track. In other cases, loads are to be applied as specified by the relevant authority.

Type RL applies to passenger rapid transit railway systems, where main line locomotives and rolling stock do not operate. The loading consists of a uniform load (or loads, dependent on loaded length), combined with a single concentrated load positioned so as to have the most severe effect. The loading is to be applied to each and every track. An arrangement of two concentrated loads is also to be considered for deck elements, where this would have a more severe effect.

For information on other loads to be considered on railway bridges, reference should be made BD 37/01.

8.3 WIND LOADS

The data given in *Tables 2.7–2.9* for the wind loading on buildings has been taken from the information given for the standard method of design in BS 6399: Part 2. The effective wind speed is determined from *Table 2.7*. Wind pressures and forces on rectangular buildings, as defined in *Table 2.8*, are determined by using standard pressure coefficients given in *Table 2.9*. For data on other building shapes and different roof forms, and details of the directional method of design, reference should be made to BS 6399: Part 2.

Details of the method used to assess wind loads on bridge structures, and the data to be used for effective wind speeds and drag coefficients, are given in BD 37/01. For designs to EC 2, wind loads are given in EC 1: Part 1.2.

Imposed loads on floors of buildings

2.3

Type of use/occupancy for part of building/structure	Examples of specific use		Uniformly distributed load kN/m^2	Concentrated load kN
A Domestic and residential use (also see category C)	All usages within self-contained dwelling units. Communal areas (including kitchens) in blocks of flats with limited use (see note 1); for such areas in other blocks of flats, see category C3		1.5	1.4
	Bedrooms and dormitories except for those in hotels and motels		1.5	1.8
	Bedrooms in hotels and motels. Hospital wards. Toilet areas		2.0	1.8
	Billiard rooms		2.0	2.7
	Communal kitchens (except in blocks of flats as covered by note 1)		3.0	4.5
	Balconies	Single dwelling units and communal areas in blocks of flats with limited use (see note 1)	1.5	1.4
		Guest houses, residential clubs and communal areas in blocks of flats (except as covered by note 1)	Same as rooms to which they give access but not less than 3.0	1.5/m run concentrated at outer edge
		Hotels and motels	Same as rooms to which they give access but not less than 4.0	1.5/m run concentrated at outer edge
B Offices and work areas not covered elsewhere	Operating theatres, X-ray rooms, utility rooms		2.0	4.5
	Work rooms (light industrial) without storage		2.5	1.8
	Offices for general use		2.5	2.7
	Banking halls		3.0	2.7
	Kitchens, laundries, laboratories		3.0	4.5
	Rooms with mainframe computers or similar		3.5	4.5
	Machinery halls, circulation spaces therein		4.0	4.5
	Projection rooms		5.0	Determine for specific use
	Factories, workshops and similar buildings (general industrial)		5.0	4.5
	Foundries		20.0	Determine for specific use
	Catwalks		—	1.0 at 1m ctrs.
	Balconies		Same as rooms to which they give access but not less than 4.0	1.5/m run concentrated at outer edge
	Fly galleries		4.5 kN/m run uniformly distributed over width	—
	Ladders		—	1.5 rung load

Note 1. Communal areas in blocks of flats with limited use refers to blocks of flats not more than three storeys in height, and with not more than four self-contained dwelling units per floor accessible from one staircase. For further details of imposed floor loads applicable to activity/occupancy categories C to G, and details of horizontal imposed loads on parapets, barriers and balustrades, reference should be made to BS 6399: Part 1.

Note 2. For details of imposed floor loads to be used when designing to Eurocode 2, see BS EN 1991-1-1.

Reduction in total distributed imposed floor load according to area of floor or number of floors supported

Beam	Area of floor supported	m^2	0	50	100	150	200	> 250
	Reduction in imposed load on member	%	0	5	10	15	20	25
	Note. Reductions for intermediate areas may be calculated by linear interpolation							
Columns, piers, walls, foundations	Number of floors supported by member		1	2	3	4	5–10	> 10
	Reduction in imposed load on member	%	0	10	20	30	40	50
	Note. Alternatively, the reductions may be based on the floor area supported							

Note. Reductions do not apply to loads due to plant or machinery, or to storage. Otherwise, reductions apply to all imposed loads (including any additional uniformly distributed imposed partition load) for activities described in categories A to D.

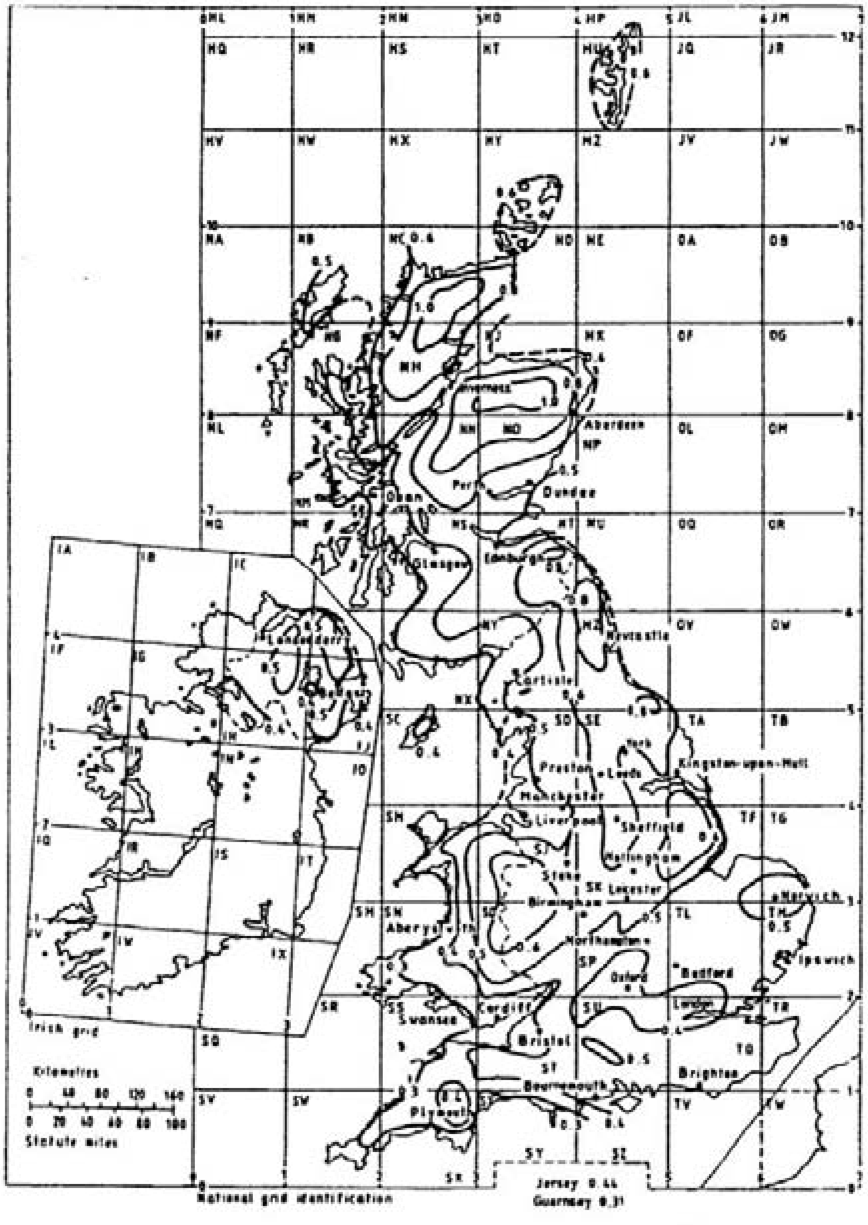
Imposed loads on roofs of buildings

2.4

Type of roof	Type of access	Uniformly distributed load kN/m^2	Concentrated load kN
Flat or monopitch	No access (except for cleaning and maintenance)	$\mu_1 s_o \nless q_1$	0.9
	Note. Above loads assume that spreader boards will be used while any cleaning or maintenance work is in progress on fragile roofs		
	Access	$\mu_1 s_o \nless 1.5$	1.8
	Note. Where access is required for specific usage, above loads should be replaced by the appropriate values for floors, including any appropriate reductions, as given in *Tables 2.5 – 2.7*.		

Site snow load	
Site altitude A	s_o kN/m^2
≤ 100 m	s_b (see isopleths on map)
> 100 m	$s_b + (0.1s_b + 0.09)(A/100 - 1)$
Note. For $A > 500$ m, seek specialist advice.	

Snow load shape coefficient and minimum load				
Angle of pitch of roof (measured from horizontal)	α	$\alpha \leq 30°$	$30° < \alpha < 60°$	$60° \leq \alpha$
Shape coefficient	μ_1	0.8	$0.8(2 - \alpha/30)$	0
Minimum load kN/m^2	q_1	0.6	$0.6(2 - \alpha/30)$	0
Note. Where parapets occur, local snow drifting should be considered.				



Basic snow load on the ground s_b kN/m^2

Imposed loads on bridges – 1

2.5

Highway bridges

HA loading

Type HA loading consists of a uniformly distributed load (UDL) and a knife-edge load (KEL) combined, or of a single wheel load. The carriageway width is divided into notional lanes and the UDL and KEL values given for one notional lane are multiplied by appropriate lane factors. Loadings are interchangeable between lanes and a lane or lanes may be left unloaded if this causes a more severe effect. The UDL varies with the loaded length and the KEL extends over a length equal to the width of the notional lane. The alternative single wheel load is placed at any point on the carriageway and applied over a circular (340 mm diameter) or square (300 mm side) contact area (1.1 N/mm^2 pressure). Dispersal may be taken at spread-to-depth ratios of 1 horizontally to 2 vertically through asphalt or similar, and 1 horizontally to 1 vertically down to the neutral axis of structural concrete slabs.

Number and width b_L of notional lanes according to width of carriageway b (m)	Carriageway width	Notional lanes	Carriageway width	Notional lanes
	$b < 5.00$	1 lane, $b_L = 2.5$m	$10.95 < b \leq 14.60$	4 lanes, $b_L = b/4$
	$5.00 \leq b \leq 7.50$	2 lanes, $b_L = b/2$	$14.60 < b \leq 18.25$	5 lanes, $b_L = b/5$
	$7.50 < b \leq 10.95$	3 lanes, $b_L = b/3$	$18.25 < b \leq 21.90$	6 lanes, $b_L = b/6$

Loaded length L and number of lanes N	Lane factors β_i for first, second, third, and other notional lanes respectively			
	β_1	β_2	β_3	β_n
$L \leq 20$	a_1	a_1	0.6	$0.6a_1$
$20 < L \leq 40$	a_2	a_2	0.6	$0.6a_2$
$40 < L \leq 50$	1.0	1.0	0.6	0.6
$50 < L \leq 112$ and $N < 6$	1.0	$7.1/\surd L$	0.6	0.6
$50 < L \leq 112$ and $N \geq 6$	1.0	1.0	0.6	0.6
$L > 112$ and $N < 6$	1.0	0.67	0.6	0.6
$L > 112$ and $N \geq 6$	1.0	1.0	0.6	0.6

Note 1. N is the total number of notional lanes on the bridge (this includes all the lanes for dual carriageway roads), except for bridges carrying one-way traffic only, where N is taken as twice the number of notional lanes.
Note 2. $a_1 = 0.274\ b_L$ and $a_2 = 0.0137[b_L(40 - L) + 3.65(L - 20)]$.
Note 3. Where there is only one notional lane, the loading on the rest of the carriageway is taken as 5 kN/m^2.

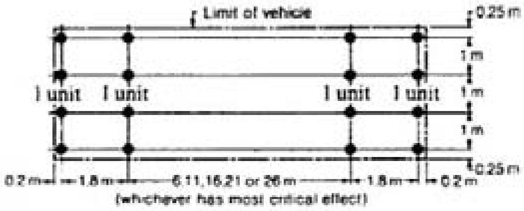
Loaded length L (m)	UDL per notional lane	KEL per notional lane
$L \leq 50$	$W = 336\ (1/L)^{0.67}$ kN/m	120 kN arranged in any direction and at any position within the loaded length
$50 < L \leq 1600$	$W = 36\ (1/L)^{0.10}$ kN/m	
Single wheel load	A 100 kN load applied at any position on the carriageway	

Loaded length m	Load kN/m	Loaded length m	Load kN/m	Loaded length m	Load kN/m
2	211.2	25	38.9	100	22.7
4	132.7	30	34.4	200	21.2
6	101.2	35	31.0	300	20.4
8	83.4	40	28.4	400	19.8
10	71.8	45	26.2	500	19.3
12	63.6	50	24.4	600	19.0
14	57.3	60	23.9	700	18.7
16	52.4	70	23.5	800	18.5
18	48.5	80	23.2	900	18.2
20	45.1	90	23.0	1000	18.1
25	38.9	100	22.7	1600	17.2

HA uniformly distributed load

HB loading

A single HB vehicle is taken to occupy any transverse position on the carriageway, either wholly within one notional lane or straddling two or more notional lanes. No other live loading is taken for a length extending from 25 m in front of the leading axle to 25 m behind the rear axle and a width extending each side of the vehicle to the edge of the notional lane occupied wholly or partially by the vehicle (but not more than 2.5 m either side). Outside this area, HA loading is applied. For further details of the loading arrangements, refer to BD37/01.



HB vehicle – plan and axle arrangement for one unit of loading

Note. One unit of loading is equivalent to 10 kN per axle (i.e. 2.5 kN per wheel). A circular or square contact area, assuming 1.1 N/mm^2 pressure, and load dispersal at spread-to-depth ratios of 1 horizontally to 2 vertically through asphalt or similar, and 1 horizontally to 1 vertically down to the neutral axis of structural concrete slabs may be considered. For public roads in the UK, between 30 and 45 units of loading are normally specified according to use.

Imposed loads on bridges – 2

2.6

Foot/cycle track bridges

Loaded length L (m)	Uniformly distributed load (kN/m^2)	Horizontal load on pedestrian parapets
$L \leq 36$ $36 < L \leq 50$ $50 < L \leq 1600$	5.0 $50W/(L + 270)$ where $W = 336\ (1/L)^{0.67}$ $50W/(L + 270)$ where $W = 36\ (1/L)^{0.10}$	1.4 kN/m length applied at top of parapet

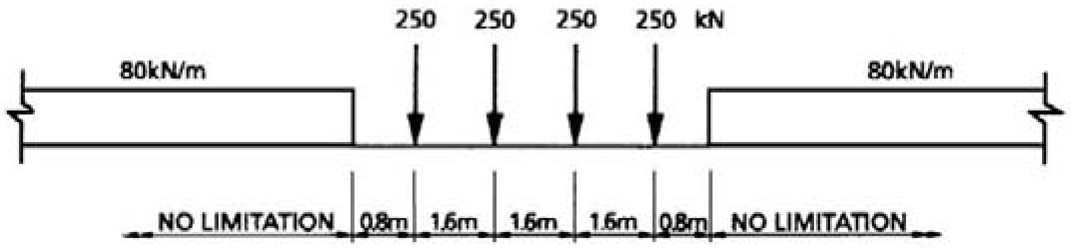
Note 1. Where exceptional crowds are expected and $L > 36$ m, loading is to be agreed with appropriate authority.
Note 2. Consideration to be given to both vertical and horizontal vibration, that could be induced by resonance with the movement of users or by deliberate excitation
Note 3. For elements of highway or railway bridges supporting footways/cycle tracks, the uniformly distributed loads shown above apply for loaded widths not exceeding 2 m. Where the width of the footway/cycle track exceeds 2 m, or the element supports a traffic lane or railway track, the pedestrian load intensity may be reduced. Where a footway/cycle track on a highway bridge is not protected from vehicular traffic by an effective barrier, the effect of an accidental wheel loading should also be considered.

Railway bridges

RU loading

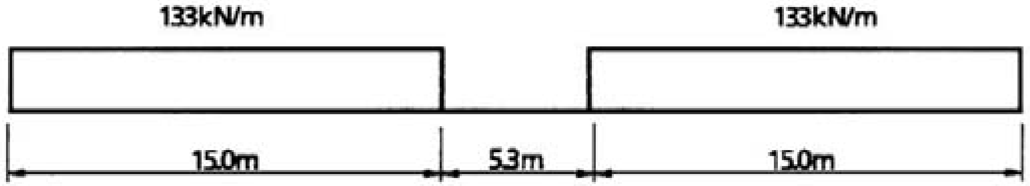
Type RU loading applies to all main line railways of 1.4 m gauge and above. The loading shown below is to be multiplied by the dynamic factors given in the table, where L is the length of the influence line for deflection of the element under consideration. For further information, refer to BD37/01.

Dynamic factors for bending moment and shear			
L (m)	$3.6 \geq L$	$3.6 < L \leq 67$	$L > 67$
Moment Shear	2.00 1.67	$0.73 + 2.16/(\sqrt{L} - 0.2)$ $0.82 + 1.44/(\sqrt{L} - 0.2)$	1.00 1.00



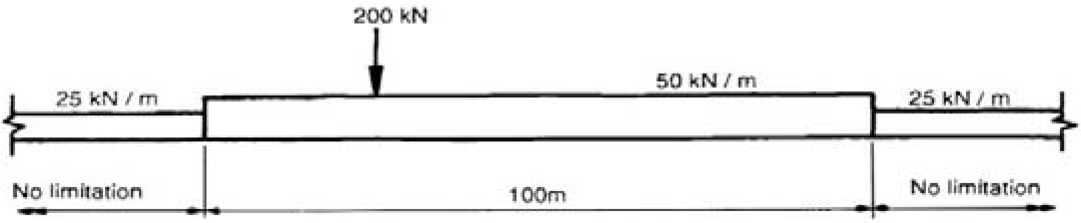
SW/0 loading

Type SW/0 loading applies to continuous bridges on main line railways, as an additional and separate load case to type RU. The loading shown below, which is to be applied without curtailment or repetition along the length of the track, is to be multiplied by the dynamic factors given above for type RU loading.



RL loading

Type RL loading applies only to passenger rapid transit railway systems on lines where main line locomotives and rolling stock do not operate. The loading shown below is to be multiplied by a dynamic factor of 1.2, except for tracks without ballast where, for rail bearers and single-track cross girders, the factor is 1.4. The distributed load may be applied in any number of lengths, but the total length of 50 kN/m intensity should not exceed 100 m on any one track. The concentrated load may be applied at any but only one position. Alternatively for deck elements, two concentrated loads of 300 kN and 150 kN respectively, spaced 2.4 m apart, should be used where this gives a more severe condition.



2.7

Wind speeds (standard method of design)

Symbols:

V_b is basic wind speed in m/s
(see adjoining map)

V_s is site wind speed
$= V_bS_aS_dS_sS_p$ m/s

V_e is effective wind speed
$= V_sS_b$ m/s

where

S_a is altitude factor

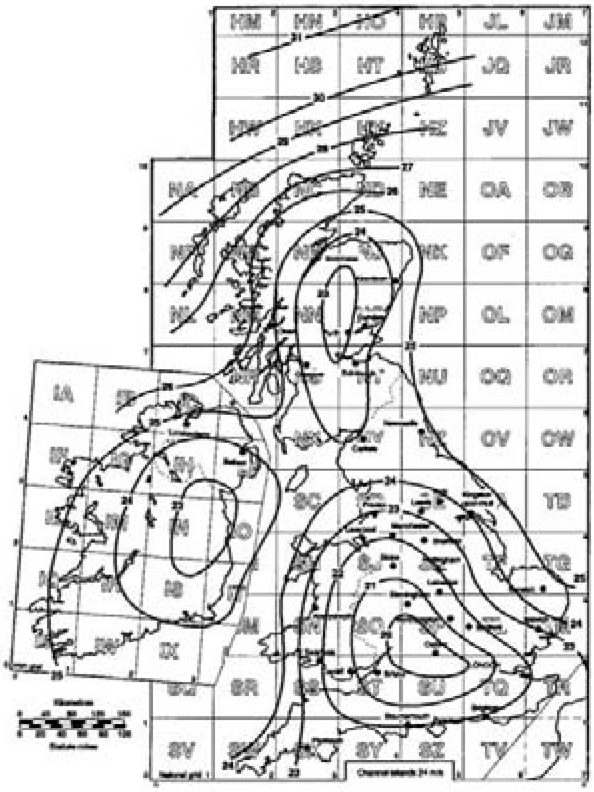
S_d is direction factor

S_s is seasonal factor

S_p is probability factor

Dynamic pressure

$q_s = 0.613V_e^2$ N/m^2



Relationship between effective wind speed and dynamic pressure	
V_e m/s	q_s N/m^2
20	245
22	297
24	353
26	414
28	481
30	552
32	628
34	709
36	794
38	885
40	981
42	1080
44	1190
46	1300
48	1410
50	1530
52	1660
54	1790
56	1920
58	2060
60	2210

Values of wind speed factors	
S_a	In terrain with upwind slopes exceeding 0.05, the effects of topography can be significant in the determination of S_a for sites located within certain zones (see BS 6399: Part 2). For sites where the topography is not considered to be significant, $S_a = 1 + 0.001\ \Delta_s$ where Δ_s is the site altitude in metres above sea level.
S_b	Values of S_b are given in the table below according to the effective height, the site terrain and proximity of the site to the sea. For buildings with height H greater than the crosswind breadth B for the wind direction considered, a reduction in the lateral load can be obtained by dividing the building into a number of parts (see BS 6399: Part 2).
S_d	When the orientation of the building is known, the basic wind speed can be adjusted in accordance with the wind direction (see BS 6399: Part 2). If the orientation is unknown or ignored, S_d should be taken as 1.0 in all directions.
S_s	For specific sub-annual periods, e.g. for temporary works and buildings during construction, the basic wind speed may be reduced (see BS 6399: Part 2, Annex D). For permanent buildings and buildings exposed to the wind for a continuous period of more than 6 months, S_s should be taken as 1.0.
S_p	The risk of the basic wind speed being exceeded from the standard value of $Q = 0.02$ annually can be changed (see BS 6399: Part 2, Annex D). For all normal design situations, S_p should be taken as 1.0.

Values of factor S_b

Site in country					Site in town, extending ≥ 2 km from the site			
Effective height H_e m	Closest distance to sea (km)				Effective height H_e m	Closest distance to sea (km)		
	0	2	10	≥ 100		2	10	≥ 100
≤ 2	1.48	1.40	1.35	1.26	≤ 2	1.18	1.15	1.07
5	1.65	1.62	1.57	1.45	5	1.50	1.45	1.36
10	1.78	1.78	1.73	1.62	10	1.73	1.69	1.58
15	1.85	1.85	1.82	1.71	15	1.85	1.82	1.71
20	1.90	1.90	1.89	1.77	20	1.90	1.89	1.77
30	1.96	1.96	1.96	1.85	30	1.96	1.96	1.85
50	2.04	2.04	2.04	1.95	50	2.04	2.04	1.95
100	2.12	2.12	2.12	2.07	100	2.12	2.12	2.07

Note 1. For buildings in country terrain, and conservatively for buildings in town terrain, the effective height H_e is taken as the maximum height of the building, or particular part of the building, above ground level. Alternatively, for buildings in town terrain, the effective height can be reduced as a result of the shelter afforded by structures located upwind of the site (see BS 6399: Part 2). Interpolation may be used within each table.

Note 2. If $H_e > 100$ m, the directional method of design should be used (see BS 6399: Part 2).

Wind pressures and forces (standard method of design)

2.8

The following symbols are used to define the dimensions of the building and specific surface zones:

Fixed dims. H is height, L is length, W is width

Variable dims. B is crosswind breadth, D is inwind depth
$b = B$, or $b = 2H$, whichever is the smaller

External surface pressure $p_e = q_s C_{pe} C_a$

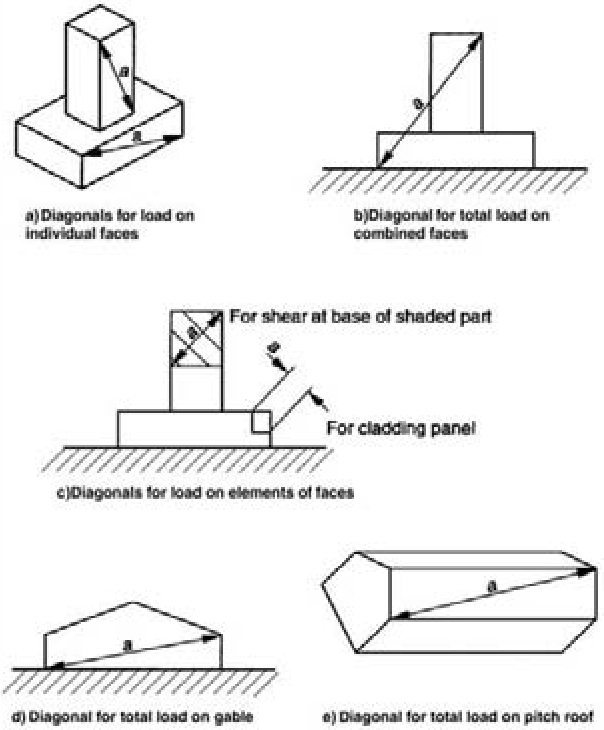
Internal pressure $p_i = q_s C_{pi} C_a$

q_s is dynamic pressure (see *Table 2.7*)
C_{pe} is external pressure coefficient (see *Table 2.9*)
C_{pi} is internal pressure coefficient (see *Table 2.9*)
C_a is a size effect factor (see *Table 2.9*)

Net surface pressure for enclosed building $p = p_e - p_i$

Net surface load (normal to surface) $P = pA$

A is loaded area (see figure below for diagonal dimension)



Definition of diagonal of loaded area

Note. For external pressures, the diagonal dimension a is taken as the largest diagonal of the area over which load sharing takes place. For internal pressures in enclosed buildings, $a = 10 \times \sqrt[3]{\text{internal volume of storey}}$ is taken. For individual structural components, cladding units and their fixings, $a = 5$ m should be taken, unless there is adequate load sharing capacity to justify the use of a diagonal length longer than 5 m.

Overall horizontal force on enclosed building

$$P_{total} = 0.85(\Sigma P_{front} - \Sigma P_{rear})(1 + C_r)$$

ΣP_{front} is horizontal component of surface load summed over windward-facing walls and roofs
ΣP_{rear} is horizontal component of surface load summed over leeward-facing walls and roofs
C_r is dynamic augmentation factor (see below)

As the effect of the internal pressure on the front and rear faces is equal and opposite when they are of equal size, p_i can be ignored in calculating P_{total} for enclosed buildings on level ground. Frictional drag forces on walls parallel to the wind direction where $D > b$, and roofs where $D > b/2$, should be combined with the normal forces in P_{total}.

Frictional drag force on each surface $P_f = q_s C_f A_s C_a$

A_s is area of surface swept by wind as follows:

$A_s = (D - b)H$ for wall $A_s = (D - b/2)B$ for roof

C_f is frictional drag coefficient (see below)

The dynamic augmentation factor depends on the building type factor K_b and the building height H, as follows:

Type	Description	K_b
1	Welded steel unclad frames	8
2	Bolted steel and reinforced concrete unclad frames	4
3	Portal sheds and similar light structures	2
4	Framed buildings with structural walls around lifts and stairs (e.g. partitioned office building)	1
5	Framed buildings with structural walls around lifts and stairs and masonry subdivision walls	0.5

Type	Values of C_r for building height H (m)						
	5	10	20	40	100	200	300
1	0.120	0.172	0.250	—	—	—	—
2	0.060	0.086	0.125	0.184	—	—	—
3	0.030	0.043	0.063	0.092	0.160	0.246	—
4	0.015	0.022	0.032	0.046	0.080	0.123	0.158
5	0.008	0.011	0.016	0.023	0.040	0.062	0.079

Values of C_r are given by the approximate equation:

$$C_r = \frac{K_b(10H)^{0.75}}{320\log(10H)} \quad \text{for } 0 \le C_r \le 0.25 \text{ and } H \le 300 \text{ m}$$

Note. In BS 6399: Part 2 (Annex C), equation (C2) has in the denominator, 800 rather than 320. This seems to be an error, and the equation shown above gives good agreement with the values obtain from Figure 3 of BS 6399: Part2.

The frictional drag coefficient depends on the type of surface, as follows:

Surface type	C_f
Smooth with no variations across wind direction	0.01
Surfaces with corrugations across wind direction	0.02
Surfaces with ribs across wind direction	0.04

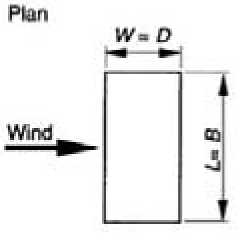
Pressure coefficients and size effect factors for rectangular buildings

2.9

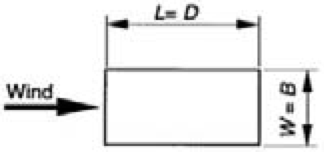
Values of external pressure coefficient C_{pe} for vertical walls

Wind normal to face (front and rear walls)	Building span ratio D/H ≤ 1	Building span ratio D/H ≥ 4	Wind parallel to face (side wall)		Exposure case Isolated	Exposure case Funnelling
Windward face (front)	+ 0.8	+ 0.6	Zone (see key below)	A	− 1.3	− 1.6
Leeward face (rear)	− 0.3	− 0.1		B	− 0.8	− 0.9
Note 1. Interpolation may be used in the range $1 < D/H < 4$.				C	− 0.4	− 0.9

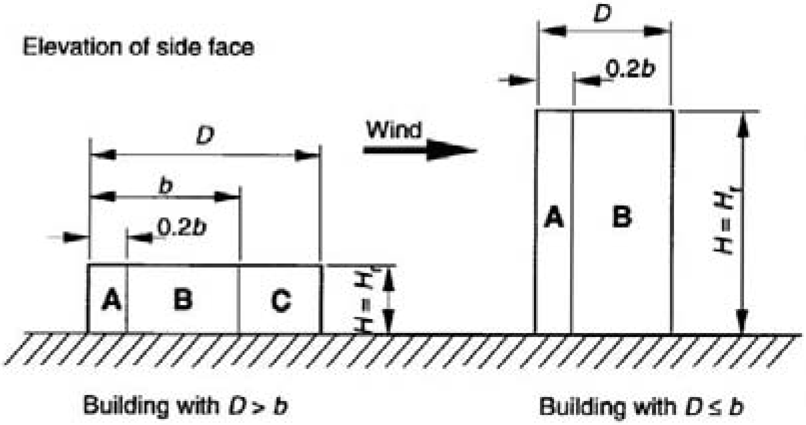
Note 2. The loaded zones on the side faces are divided into vertical strips from the upwind edge of the face in terms of the scaling length $b = B$ or $2H$, whichever is the smaller. Where the walls of two buildings face each other and the gap between them is less than $b/4$ or greater than b, the isolated coefficient should be used. When the gap is $b/2$, the funnelling coefficient should be used. For gaps between $b/4$ and $b/2$, and between $b/2$ and b, linear interpolation may be used.



Wind on long face



Wind on short face



Building with $D > b$

Building with $D \leq b$

Key to pressure coefficient zones on side face

Values of internal pressure coefficient C_{pi} for vertical walls

Enclosed buildings (containing external doors and windows that may be kept closed and where any internal doors are generally open, or are at least three times more permeable than the external doors and windows).	
Two opposite walls equally permeable with other faces impermeable: wind normal to permeable face	+ 0.2
wind normal to impermeable face	− 0.3
Four walls equally permeable with roof impermeable:	− 0.3
Buildings with dominant opening (area of opening ≥ twice sum of openings in other faces).	
Ratio of area of dominant opening to sum of areas of remaining openings = 2	$0.75C_{pe}$
Ratio of area of dominant opening to sum of areas of remaining openings = 3	$0.90C_{pe}$

Values of size effect factor C_a for the standard method of design

Site exposure	Site in country: closest distance to sea (km) < 2	2 to < 10		10 to < 100		≥ 100	Site in town: closest distance to sea (km) 2 to < 10		10 to < 100			≥ 100	
Diagonal a (m)	Effective height H_e (m) > 0	≤ 5	> 5	≤ 20	> 20	> 0	Effective height H_e (m) ≤ 5	> 5	≤ 10	≤ 20	> 20	≤ 10	> 10
≤ 5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
10	0.96	0.95	0.96	0.95	0.96	0.95	0.94	0.96	0.94	0.95	0.96	0.94	0.95
25	0.90	0.88	0.90	0.88	0.90	0.88	0.85	0.90	0.85	0.88	0.90	0.85	0.88
50	0.86	0.83	0.86	0.83	0.86	0.83	0.79	0.86	0.79	0.83	0.86	0.79	0.83
100	0.81	0.78	0.81	0.78	0.81	0.78	0.73	0.81	0.73	0.78	0.81	0.73	0.78
250	0.75	0.70	0.75	0.70	0.75	0.70	0.64	0.75	0.64	0.70	0.75	0.64	0.70
500	0.71	0.65	0.71	0.65	0.71	0.65	0.58	0.71	0.58	0.65	0.71	0.58	0.65
1000	0.67	0.60	0.67	0.60	0.67	0.60	0.52	0.67	0.52	0.60	0.67	0.52	0.60

Note. For external pressures, the diagonal dimension a is the largest diagonal of the area over which load sharing takes place as shown in the figure in *Table 2.8*. For internal pressures in enclosed buildings, $a = 10 \times \sqrt[3]{\text{internal volume of storey}}$; for buildings with dominant openings, $a = 0.2 \times \sqrt[3]{\text{internal volume of storey or room containing dominant opening}}$ or diagonal dimension of dominant opening, whichever is greater; for open-sided buildings, a = diagonal dimension of the open face.

Chapter 9

Pressures due to retained materials

In this chapter, unless otherwise stated, all unit weights and other properties of materials are given as characteristic or representative (i.e. unfactored) values. For design purposes, each value must be modified by appropriate partial safety or mobilisation factors, according to the basis of design and the code of practice employed.

9.1 EARTH PRESSURES

The data given in *Table 2.10* for the properties of soils has been taken from BS 8002. Design values of earth pressure coefficients are based on the design soil strength, which is taken as the lower of the peak soil strength reduced by a mobilisation factor, or the critical state strength.

9.1.1 Pressures imposed by cohesionless soils

For the walls shown in *Table 2.11*, with a uniform normally consolidated soil, a uniformly distributed surcharge and no water pressure, the pressure imposed on the wall increases linearly with depth and is given by:

$$\sigma = K(\gamma z + q)$$

where γ is unit weight of soil, z is depth below surface, q is surcharge pressure (kN/m^2), K is at-rest, active or passive coefficient of earth pressure according to design conditions.

A minimum live load surcharge of 10 kN/m^2 is specified in BS 8002. This may be reasonable for walls 5 m high and above, but appears to be too large for low walls. In this case, values such as 4 kN/m^2 for walls up to 2 m high, 6 kN/m^2 for walls 3 m high and 8 kN/m^2 for walls 4 m high could be used. In BD 37/01, surcharge loads are given of 5 kN/m^2 for footpaths, 10 kN/m^2 for HA loading, 12 kN/m^2 for 30 units of HB loading, 20 kN/m^2 for 45 units of HB loading and, on areas occupied by rail tracks, 30 kN/m^2 for RL loading and 50 kN/m^2 for RU loading.

If static ground water occurs at depth z_w below the surface, the total pressure imposed at $z > z_w$ is given by:

$$\sigma = K\,[\gamma_m z + (\gamma_s - \gamma_w)(z - z_w) + q] + \gamma_w(z - z_w)$$

where γ_m is moist bulk weight of soil, γ_s is saturated bulk weight of soil, γ_w is unit weight of water (9.81 kN/m^3).

9.1.2 At-rest pressures

For a level ground surface and a normally consolidated soil that has not been subjected to removal of overburden, the horizontal earth pressure coefficient is given by:

$$K_o = 1 - \sin\varphi'$$

where φ' is effective angle of shearing resistance of soil.

Compaction of the soil will result in earth pressures in the upper layers of the soil mass that are higher than those given by the above equation. The diagram and equations given in *Table 2.11* can be used to calculate the maximum horizontal pressure induced by the compaction of successive layers of backfill, and determine the resultant earth pressure diagram. The effective line load for dead weight compaction rollers is the weight of the roller divided by its width. For vibratory rollers, the dead weight of the roller plus the centrifugal force caused by the vibrating mechanism should be used. The DOE Specification limits the mass of the roller to be used within 2 m of a wall to 1300 kg/m.

For a vertical wall retaining backfill with a ground surface that slopes upwards, the horizontal earth pressure coefficient may be taken as

$$K_o = (1 - \sin\varphi')(1 + \sin\beta)$$

where β is slope angle. The resultant pressure, which acts in a direction parallel to the ground surface, is given by:

$$\sigma_o = K_o \gamma z / \cos\beta$$

9.1.3 Active pressures

Rankine's theory may be used to calculate the pressure on a vertical plane, referred to as the 'virtual back' of the wall. For a vertical wall and a level ground surface, the Rankine horizontal earth pressure coefficient is given by:

$$K_a = \frac{1 - \sin\varphi'}{1 + \sin\varphi'}$$

The solution applies particularly to the case of a smooth wall or a wall with no relative movement between the soil mass and the back of the wall. The charts given in *Table 2.12*, which are based on the work of Caquot and Kerisel, may be used generally for vertical walls with sloping ground or inclined walls with

Properties of soils

Unit weights of soils (and similar materials)

Granular materials	Moist bulk weight γ_m kN/m^3		Saturated bulk weight γ_s kN/m^3		Cohesive soils	Weight kN/m^3
	Loose	Dense	Loose	Dense		
Gravel	16.0	18.0	20.0	21.0	Peat (very variable)	12.0
Well graded sand and gravel	19.0	21.0	21.5	23.0	Organic clay	15.0
Coarse or medium sand	16.5	18.5	20.0	21.5	Soft clay	17.0
Well graded sand	18.0	21.0	20.5	22.5	Firm clay	18.0
Fine or silty sand	17.0	19.0	20.0	21.5	Stiff clay	19.0
Rock fill	15.0	17.5	19.5	21.0	Hard clay	20.0
Brick hardcore	13.0	17.5	16.5	19.0	Stiff or hard glacial clay	21.0
Slag fill	12.0	15.0	18.0	20.0		
Ash fill	6.5	10.0	13.0	15.0		

Note. Unit weights of fill materials may be determined from standard laboratory compaction tests on representative samples or estimated from records of field compaction tests on similar fills. The values given above are considered to be reasonable, in the absence of reliable test results.

Angle of shearing resistance for siliceous sands and gravels

Angularity	Rounded			Sub-angular			Angular		
Grading	Uniform	Moderate grading	Well graded	Uniform	Moderate grading	Well graded	Uniform	Moderate grading	Well graded
Uniformity coefficient	< 2	2–6	> 6	< 2	2–6	> 6	< 2	2–6	> 6
φ'_{crit}	30°	32°	34°	32°	34°	36°	34°	36°	38°

Overburden pressure kN/m^2	SPT value (number of blows/300 mm)			
≤ 10	≤ 3	7	13	20
40	≤ 5	10	20	30
80	≤ 7	13	27	40
≥ 120	≤ 10	20	40	60
φ'_{max}	φ'_{crit}	$\varphi'_{crit} + 2°$	$\varphi'_{crit} + 6°$	$\varphi'_{crit} + 9°$

Note. The strength and stiffness of cohesionless soils may be determined indirectly by in-situ static or dynamic penetration tests, in accordance with BS 1377: Part 9. Estimated values of the critical state angle of shearing resistance φ'_{crit} are given above, according to the angularity of the particles and grading of the soil. Estimated values of the peak effective angle of shearing resistance φ'_{max} are given, according to the standard penetration test value in relation to the overburden pressure.

Angle of shearing resistance for clay soils

Plasticity index %	φ'_{crit}
15	30°
30	25°
50	20°
80	15°

Note. The shear strengths of clay soils, for both undrained and drained conditions, may be determined from laboratory tests on representative samples, in accordance with BS 1377: Parts 7 and 8. The undrained shear strength may also be determined from in-situ pressuremeter tests. For further information, refer to soil mechanics publications and BS 8002. For the drained condition, the conservative values given above for the critical state angle of shear resistance, according to the plasticity index of the clay, may be used with the effective cohesion $c' = 0$.

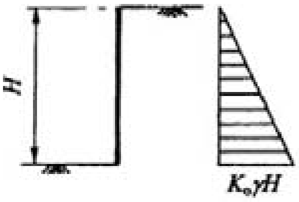
Angle of shearing resistance for rock

Stratum	φ'
Chalk	35°
Clayey marl	28°
Sandy marl	33°
Weak sandstone	42°
Weak siltstone	35°
Weak mudstone	28°

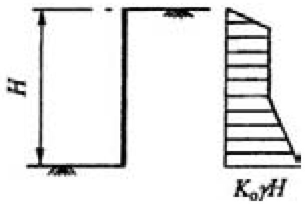
Note. The indicative values given above for the effective angle of friction relate to rocks that can be treated as composed of granular fragments, i.e. they are closely and randomly jointed or otherwise fractured, having a rock quality designation value close to zero. Chalk is defined here as an un-weathered, medium to hard, rubbly to blocky chalk, grade III.

Earth pressure distributions on rigid walls

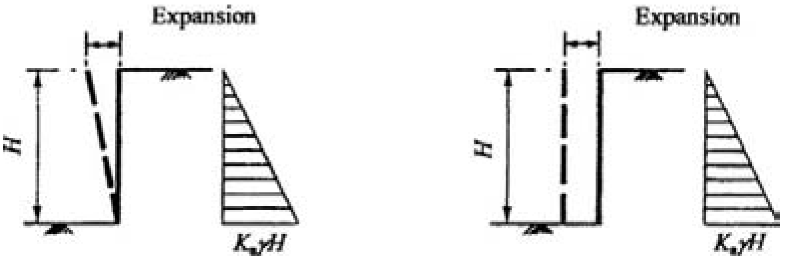
2.11



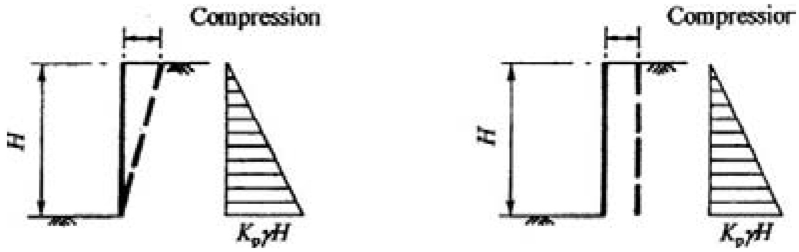
At-rest state for rigid wall



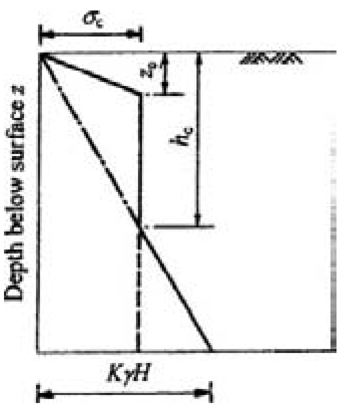
Effects of soil compaction



Active state for rigid wall free to rotate about base or translate



Passive state for rigid wall free to rotate about base or translate



$$\sigma_c = \sqrt{\frac{2Q_1\gamma}{\pi}}$$

$$z_c = K\sqrt{\frac{2Q_1}{\pi\gamma}}$$

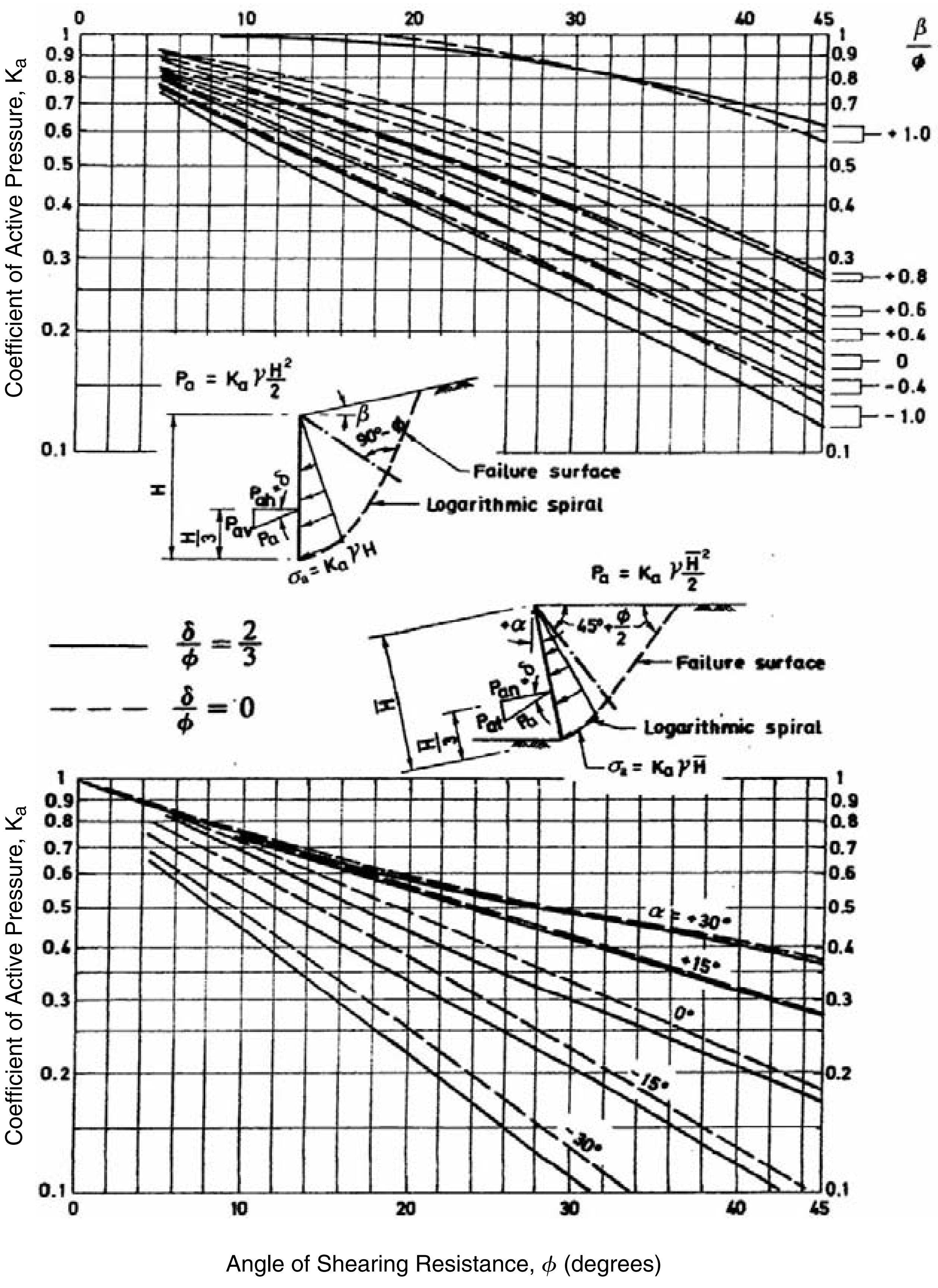
$$h_c = \frac{1}{K}\sqrt{\frac{2Q_1}{\pi\gamma}}$$

K = earth pressure coefficient
$K = K_o$ for unyielding structure
$K = K_a$ for wall free to mobilise fully active state
Q_1 = intensity of effective line load imposed by compaction plant
γ = unit weight of soil
σ = maximum horizontal earth pressure induced by compaction

Horizontal earth pressure distribution resulting from compaction

Active earth pressure coefficients

2.12



level ground. The horizontal and vertical components of resultant pressure are given by:

$$\sigma_{ah} = K_a \gamma z \cos(\alpha + \delta) \quad \text{and} \quad \sigma_{av} = K_a \gamma z \sin(\alpha + \delta)$$

where α is wall inclination to vertical (positive or negative), δ is selected angle of wall friction (taken as positive).

9.1.4 Passive pressures

For a vertical wall and a level ground surface, the Rankine horizontal earth pressure coefficient is given by:

$$K_p = \frac{1 + \sin \varphi'}{1 - \sin \varphi'}$$

The solution applies particularly to the case of a smooth wall or a wall with no relative movement between the soil mass and the back of the wall. The charts given in *Table 2.13*, for vertical walls with sloping ground, and in *Table 2.14*, for inclined walls with level ground, are based on the work of Caquot and Kerisel. The horizontal and vertical components of resultant pressure are given by:

$$\sigma_{ph} = K_p \gamma z \cos(\alpha + \delta) \quad \text{and} \quad \sigma_{pv} = K_p \gamma z \sin(\alpha + \delta)$$

where α is wall inclination to vertical (positive or negative), δ is selected angle of wall friction (taken as negative).

9.1.5 Cohesive soils

If a secant φ' value ($c' = 0$) is selected, the procedures given for cohesionless soils apply. If tangent parameters (c', φ') are to be used, the Rankine–Bell equations may be used as follows:

$$\sigma_a = K_a(\gamma z + q) - 2c'\sqrt{K_a}$$

$$\sigma_p = K_p(\gamma z + q) + 2c'\sqrt{K_p}$$

where c' is effective cohesion. The active earth pressure is theoretically negative to a depth given by:

$$z_o = (2c'/\sqrt{K_a} - q)/\gamma$$

Where cracks, which may form in the tension zone, can become filled with water, full hydrostatic pressure should be considered over the depth z_o. If the surface is protected so that no surface water can accumulate in the tension cracks, the earth pressure should be taken as zero over the depth z_o.

9.1.6 Further considerations

For considerations such as earth pressures on embedded walls (with or without props), the effects of vertical concentrated loads and line loads and the effects of groundwater seepage, reference should be made to BS 8002. For the pressures to be considered in the design of integral bridge abutments, as a result of thermal movements of the deck, reference should be made to Highways Agency document BA 42/96.

9.2 TANKS

The pressure imposed by a contained liquid is given by:

$$\sigma = \gamma_w z$$

where γ_w is unit weight of liquid (see EC 1: Part 1.1), and z is depth below surface. For a fully submerged granular material, the total horizontal pressure on the walls is:

$$\sigma = K(\gamma - \gamma_w)(z - z_o) + \gamma_w z$$

where γ is unit weight of the material (including voids), z_o is depth to top of material, K is material pressure coefficient. If γ_o is unit weight of material (excluding voids), $\gamma = \gamma_o/(1 + e)$ where e is ratio of volume of voids to volume of solids.

The preceding equation applies to materials such as coal or broken stone, with an effective angle of shearing resistance when submerged of approximately 35°. For submerged sand, K should be taken as unity. If the material floats ($\gamma_o < \gamma_w$), the simple hydrostatic pressure applies.

9.3 SILOS

The data given in *Tables 2.15* and *2.16* has been taken from Eurocode 1: Part 4. The pressures apply to silos of the forms shown in *Table 2.15*, subject to the following limitations:

- dimensions $d_c < 50$ m, $h < 100$ m, $h/d_c < 10$.
- eccentricities $e_i < 0.25d_c$, $e_o < 0.25d_c$ with no part of outlet at a distance greater than $0.3d_c$ from centreline of silo.
- filling involves negligible inertia effects and impact loads.
- stored material is free-flowing (cohesion is less than 4 kPa for a sample pre-consolidated to 100 kPa), with a maximum particle size not greater than $0.3d_c$.
- transition between vertical walled section and hopper is on a single horizontal plane.

Dimensions h, h_o, h_1 and z are measured from the equivalent surface, which is a level surface giving the same volume of stored material as the actual surface at the maximum filling.

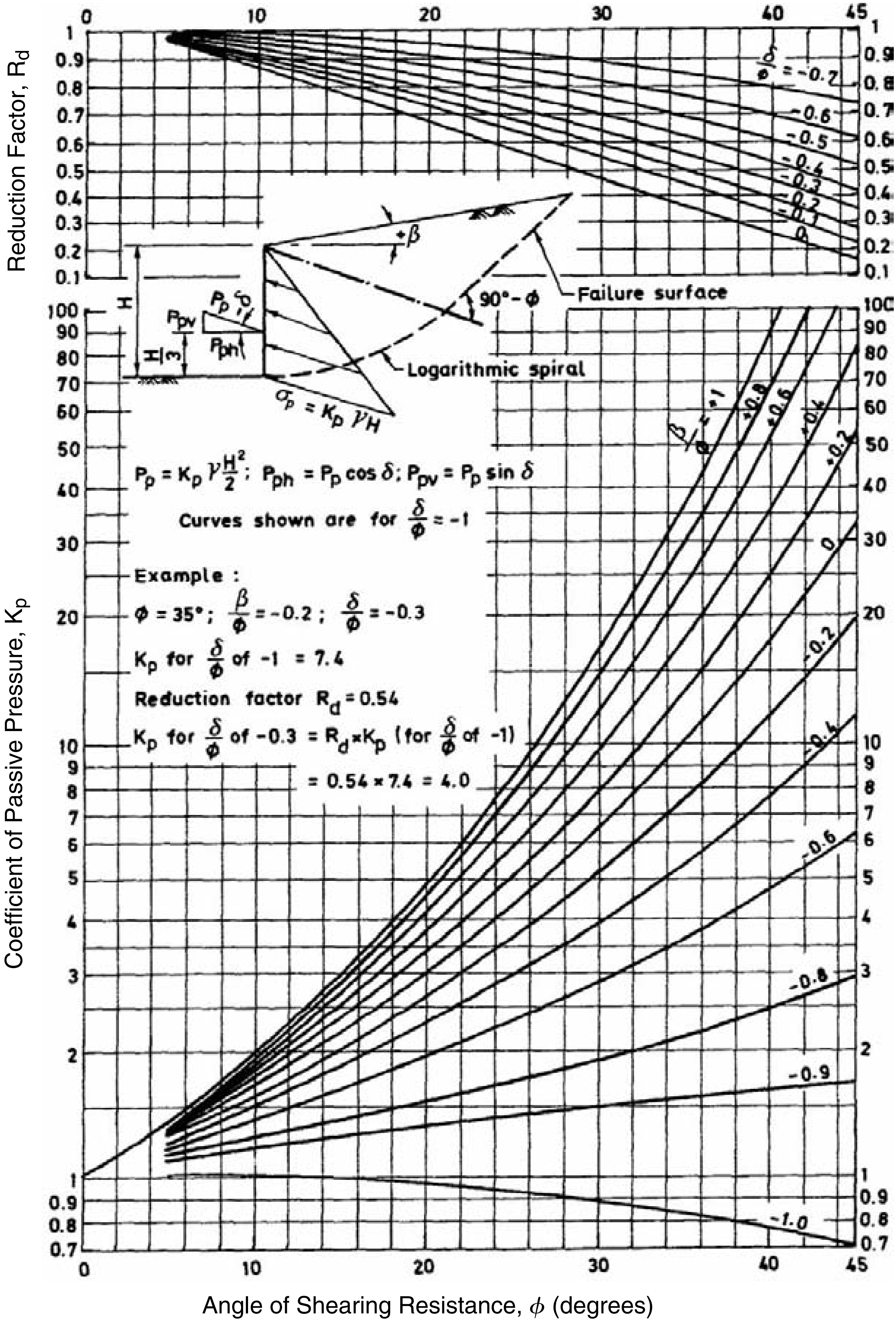
Loads acting on a hopper are shown in *Table 2.15*, where the tensile force at the top of the hopper is required for the design of silo supports or a ring beam at the transition level. The vertical component of the force can be determined from force equilibrium incorporating the vertical surcharge $C_b p_{vo}$ at the transition level and the weight of the hopper contents. The discharge load on the hopper wall is affected by the flow pattern of the stored material, which may be mass flow or funnel flow according to the characteristics of the hopper and the material. The normal load due to p_n is supplemented, for mass flow silos only, by a kick load due to p_s.

Values of material properties and expressions to determine resulting pressures in the vertical walled and bottom sections of a silo are given in *Table 2.16*. For squat silos ($h/d_c < 1.5$), the horizontal pressure p_h may be reduced to zero at the level where the upper surface of the stored material meets the silo wall. Below this level, a linear pressure variation may be assumed taking $K = 1.0$, until this pressure reaches the value appropriate to the depth z below the equivalent surface.

Homogenizing silos, and silos containing powders in which the speed of the rising surface of the material exceeds 10 m/h, should be designed for both the fluidised and non-fluidised conditions. For the fluidised condition, the bulk unit weight of the material may be taken as 0.8γ. For information on test methods to determine the properties of particulate materials, reference should be made to EC 1: Part 4.

Passive earth pressure coefficents – 1

2.13



$P_p = K_p \gamma \frac{H^2}{2}$; $P_{ph} = P_p \cos \delta$; $P_{pv} = P_p \sin \delta$

Curves shown are for $\frac{\delta}{\phi} = -1$

Example :

$\phi = 35°$; $\frac{\beta}{\phi} = -0.2$; $\frac{\delta}{\phi} = -0.3$

K_p for $\frac{\delta}{\phi}$ of $-1 = 7.4$

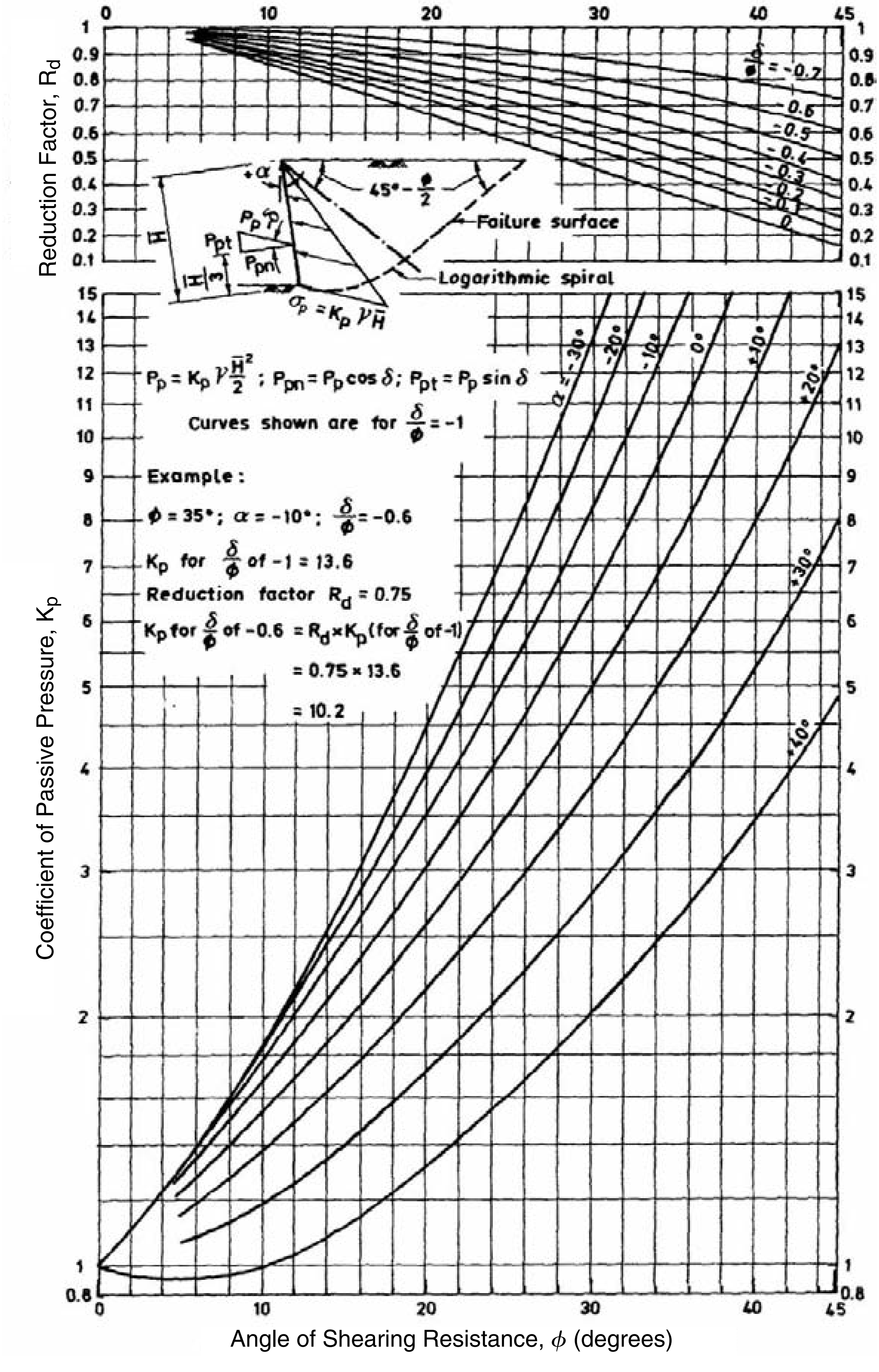
Reduction factor $R_d = 0.54$

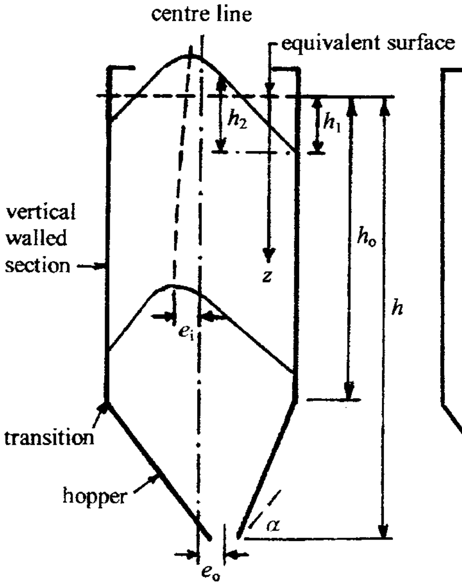
K_p for $\frac{\delta}{\phi}$ of $-0.3 = R_d \times K_p$ (for $\frac{\delta}{\phi}$ of -1)

$= 0.54 \times 7.4 = 4.0$

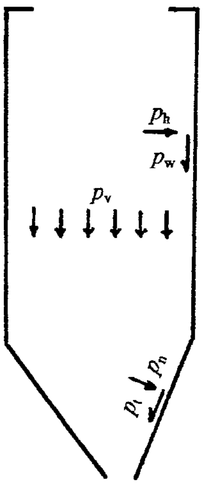
Passive earth pressure coefficents – 2

2.14



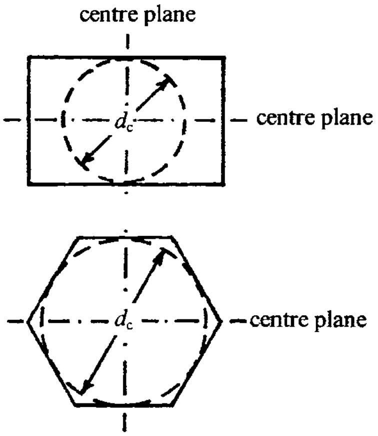


Dimensions

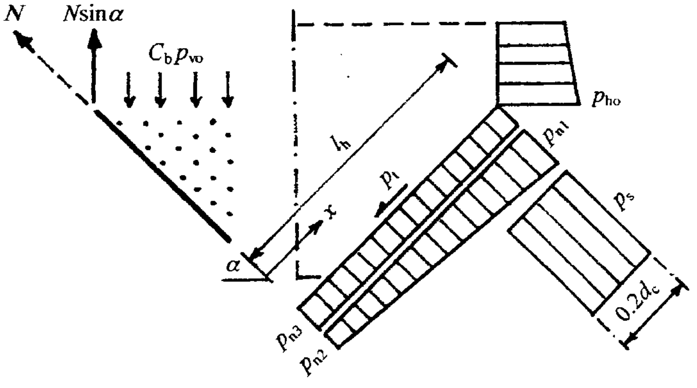


Pressures

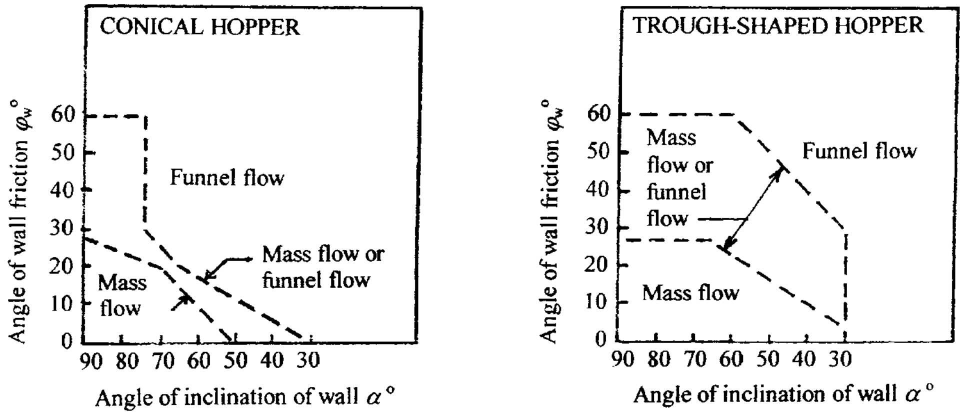
A	cross-sectional area of vertical walled section
C_z	Janssen coefficient
K	ratio of horizontal to vertical pressure
N	tensile force in hopper wall at transition
P_w	vertical load per unit perimeter of vertical wall
U	internal perimeter of vertical walled section
d_c	diameter of inscribed circular cross-section
e_i	eccentricity due to filling
e_o	eccentricity of centre of outlet
h	overall depth measured from equivalent surface at maximum filling to outlet
h_o	depth from equivalent surface to transition
h_l	depth from equivalent surface to lowest point of wall not in contact with stored material
h_2	height of stored material above level h_l
l_h	inclined length of hopper wall
$p_h, p_n, p_p, p_s, p_t, p_v, p_w$	pressures of stored material
x	length along hopper wall measured from outlet
z	depth below equivalent surface
z_o	parameter $(A/UK\mu)$ used to calculate loads
α	angle of inclination of hopper wall to horizontal
γ	bulk unit weight of stored material
μ	coefficient of wall friction (tan φ_w)
φ	effective angle of internal friction of material



Typical cross-sections



Hopper loads and tensile force at top of hopper



Limits of mass flow and funnel flow for conical and trough-shaped hoppers

Silos – 2

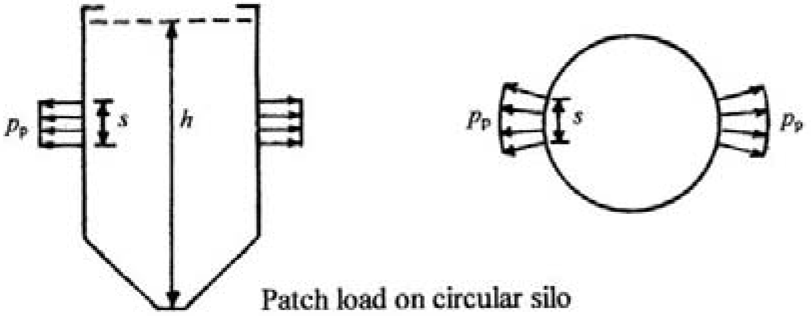
2.16

<table>
<tr><td></td><td colspan="5">Values of loads at depth z below equivalent surface of contained material at maximum filling</td></tr>
<tr><td rowspan="4">Vertical walled sections</td><td colspan="2">Vertical pressure (kN/m^2)</td><td colspan="3">$p_v(z) = \gamma z_o C_z(z)$ where $z_o = (A/UK\mu)$, $C_z(z) = (1 - e^{-z/z_o})$</td></tr>
<tr><td colspan="2" rowspan="2">Horizontal pressure (kN/m^2)
Patch pressure (kN/m^2)
Wall friction pressure (kN/m^2)
Vertical wall load (kN/m)</td><td colspan="2">Filling loads</td><td>Discharge loads</td></tr>
<tr><td colspan="2">$p_h(z) = K\,p_v(z)$
$p_p(z) = 0.2(1 + 4e/d_c)C_p\,p_h(z)$ with $e = e_i$
$p_w(z) = \mu\,p_h(z)$
$P_w(z) = \gamma(A/U)[z - z_0 C_z(z)]$</td><td>$C_h\,p_h(z)$
$C_h\,p_p(z)$ with $e = e_i$ or e_o
$C_w\,p_w(z)$
$C_w\,P_w(z)$</td></tr>
<tr><td colspan="5">Note. Loads are composed of fixed loads, due to $p_h(z)$ and $p_w(z)$, and a free load, due to $p_p(z)$. The patch pressure is taken to act on any part of the wall, on two opposite square areas of side length $s = 0.2d_c$ (see figure below). As a simplification, the most unfavourable load arrangement can be taken by applying the patch at the mid-height of the silo, and using the percentage increase in the wall stresses at that level to increase the wall stresses throughout the silo. For silos, where d_c is less than 5 m, the patch loads may be allowed for by multiplying $p_h(z)$ by $[1.0 + 0.2(1 + 4e/d_c)]$.</td></tr>
<tr><td rowspan="3">Silo bottoms</td><td>Flat
($\alpha \le 20^0$)</td><td>Vertical pressure (kN/m^2)</td><td>$h/d_c \ge 1.5$
$h/d_c < 1.5$</td><td colspan="2">$C_b\,p_{v1}$
$C_b[p_{v1} + (p_{v2} - p_{v3})(1.5d_c - h)/(1.5d_c - h_1)]$ (see *Table 2.15*)
where
$p_{v1} = p_v(z)$ with $z = h$, $p_{v2} = \gamma h_2$, $p_{v3} = p_v(z)$ with $z = h_1$</td></tr>
<tr><td>Hopper
($\alpha > 20^0$)</td><td>Wall normal pressure (kN/m^2)
Wall friction pressure (kN/m^2)</td><td colspan="3">$p_n = p_{n3} + p_{n2} + (p_{n1} - p_{n2})(x/l_h)$ (see *Table 2.15*)
where
$p_{n1} = p_{vo}(C_b\cos^2\alpha + \sin^2\alpha)$, $p_{n2} = C_b\,p_{vo}\cos^2\alpha$, $p_{n3} = 3\gamma(AK/U\sqrt{\mu})\sin^2\alpha$,
$p_{vo} = p_v(z)$ with $z = h_o$, and x is a length between 0 and l_h
$p_t = \mu\,p_n$</td></tr>
<tr><td colspan="5">Note. For mass flow discharge silos only, an additional normal fixed kick load is considered. A pressure $p_s = 2p_h(z)$, with $z = h_o$, is applied over a distance of $0.2d_c$ along the inclined wall and around the perimeter. (see *Table 2.15*)</td></tr>
</table>

Material	Unit weight γ (kN/m^2)	Pressure coefficient K_m	Wall friction coefficient μ_m	Load magnifier C_o
Barley	8.5	0.55	0.45	1.35
Cement	16.0	0.50	0.50	1.40
Clinker	18.0	0.45	0.55	1.40
Dry sand	16.0	0.45	0.50	1.40
Flour	7.0	0.40	0.40	1.45
Fly ash	14.0	0.45	0.55	1.45
Maize	8.5	0.50	0.40	1.40
Sugar	9.5	0.50	0.55	1.40
Wheat	9.0	0.55	0.40	1.30
Coal	10.0	0.50	0.55	1.45
Note. To obtain maximum loads, extreme values of K and μ are taken as follows: p_h ($K = 1.15K_m$, $\mu = 0.9\mu_m$), p_v ($K = 0.9K_m$, $\mu = 0.9\mu_m$), p_w ($K = 1.15K_m$, $\mu = 1.15\mu_m$)				

Load magnifiers	Squat silo		Slender
	$h/d_c \le 1.0$	$1.0 < h/d_c < 1.5$	$h/d_c \ge 1.5$
C_b	1.2	1.2	1.2
C_h	1.0	$1.0 + 2(C_o - 1.0)(h/d_c - 1.0)$	C_o
C_p	0	$2(h/d_c - 1.0)$	1.0
C_w	1.0	$1.0 + 0.2(h/d_c - 1.0)$	1.1
Note. For silos with no discharge flow (i.e. unloaded from top), $C_h = C_w = 1.0$.			

z/z_o	$C_z(z)$
0	0
0.1	0.095
0.2	0.181
0.3	0.259
0.4	0.330
0.5	0.394
0.6	0.451
0.7	0.504
0.8	0.551
0.9	0.594
1.0	0.632
1.1	0.667
1.2	0.699
1.3	0.728
1.4	0.754
1.5	0.777
1.6	0.798
1.7	0.817
1.8	0.835
1.9	0.851
2.0	0.865
2.1	0.878
2.2	0.889
2.3	0.900
2.4	0.909
2.5	0.918
2.6	0.926
2.7	0.933
2.8	0.939
2.9	0.945
3.0	0.950
4.0	0.982
5.0	0.993
∞	1.0



Patch load on circular silo

Chapter 10

Concrete and reinforcement

10.1 CONSTITUENTS OF CONCRETE

10.1.1 Cements and combinations

Manufactured cements are those made in a cement factory. Where a mineral material is included, it is generally added to the cement clinker at the grinding stage. The notation used for these manufactured cements contains the prefix letters CEM. When a concrete producer adds an addition such as pfa or ggbs to CEM I Portland cement in the mixer, the resulting cement is known as a mixer combination, and is denoted by the prefix letter C. Cements and combinations in general use are listed in *Table 2.17*. Further information on the different types and use of cements is given in section 3.1.1.

10.1.2 Aggregates

Overall grading limits for coarse and fine aggregates from natural sources, in accordance with BS EN 12620, are given in *Table 2.17*. Further information is given in section 3.1.2.

10.2 EARLY-AGE TEMPERATURES OF CONCRETE

The calculation of early thermal crack widths in a restrained concrete element requires knowledge of the temperature rise due to the cement hydration. Some typical early temperature histories of various concrete walls, and predicted temperature rises for different cements are given in *Table 2.18*.

The predicted temperature rise values for Portland cement concretes in walls and slabs are taken from BS 8007. These are maximum values, selected from a range of values for Portland cements obtained from different works (ref. 11). The temperature rises given for the other cements, in concrete sections with a minimum dimension of 1 m, should be taken as indicative only, but could be used where other specific information is not available.

10.3 REINFORCEMENT

Reinforcement for concrete generally consists of steel bars, or welded steel mesh fabric, that depend upon the provision of a durable concrete cover for protection against corrosion. The essential properties of bars to BS 4449 and wires to BS 4482, both of which are in general conformity with BS EN 10080, are given in *Table 2.19*. For additional information on the manufacture and properties of steel reinforcement, including stainless steel, refer to section 3.2.

10.3.1 Bars

Bars for normal use produced in the United Kingdom are hot-rolled to a characteristic strength of 500 MPa, and achieve Class B or C ductility. The bars are round in cross section, with sets of parallel transverse ribs separated by longitudinal ribs. The nominal size is the diameter of a circle with an area equal to the effective cross-sectional area of the bar. The maximum overall size is approximately 15% greater than the nominal size. Values of the total cross-sectional area provided in a concrete section, according to the number or spacing of the bars, for different bar sizes, are given in *Table 2.20*.

The type and grade of reinforcement is designated as follows:

Type of steel reinforcement	Notation
Grade B500A, B500B or B500C to BS 4449	H
Grade B500A to BS 4449	A
Grade B500B or B500C to BS 4449	B
Grade B500C to BS 4449	C
A specified grade and type of ribbed stainless steel to BS 6744	S
Reinforcement of a type not included above but with material properties defined in the design or contract specification.	X

Note. In the description B500A and so on, B indicates reinforcing steel.

10.3.2 Fabric

In the United Kingdom, steel fabric reinforcement is generally produced to the requirements of BS 4483, using ribbed bar in accordance with BS 4449. The exception is wrapping fabric, where wire in accordance with BS 4482 may be used. Fabric is produced in a range of standard types, or can be purpose-made to the client's requirements. Full details of the standard fabric types are given in *Table 2.20*.

Type A is a square mesh with identical longitudinal bars and cross bars, commonly used in ground slabs to provide a minimum amount of reinforcement in two directions. Type B is a rectangular (structural) mesh that is particularly suitable for

2.17

Concrete: cements and aggregate grading

Cement types

Cements and combinations in general use

Cement/combination type	Manufactured cement conforming to BS EN 197-1	Mixer combination conforming to BS 8500: Part 2	Notation
Portland cement	CEM I	—	Cement type CEM I is almost entirely of Portland cement clinker and gypsum set regulator. CEM II (and CII) include up to 35% of mineral addition. CEM II (and CIII) include > 35% of blastfurnace slag. CEM IV (and CIV), not in general use, include > 35% of pozzolana (e.g. pfa). In the notation for CEM II and CEM III, the relative proportions of cement clinker are represented by the letters A for high, B for moderate, C for low. In CEM II, the additions are identified by the letters V for siliceous fly ash (e.g. pfa), S for blastfurnace slag, D for silica fume, L or LL for limestone, M for more than one. Strength class and strength development identifiers are added to this notation. The full title, for example, of a manufactured cement with a relatively low proportion of ggbs would be Portland-slag cement: BS EN 197-1 CEM II/A-S 42,5N The strength class is 42.5 (N/mm^2), and the letters L, N, R represent low, normal or rapid early strength development.
Sulfate-resisting Portland cement	SRPC conforms to BS 4027	—	
Portland-fly ash cement (with 21–35% pfa)	CEM II/B-V	CIIB-V	
Portland-slag cement (with 6–20% ggbs)	CEM II/A-S	CIIA-S	
Portland-slag cement (with 21–35% ggbs)	CEM II/B-S	CIIB-S	
Blastfurnace cement (with 36–65% ggbs)	CEM III/A May also conform to BIII/A of BS 146	CIIIA	
Blastfurnace cement (with 66–80% ggbs)	CEM III/B May also conform to BIII/B of BS 146	CIIIB	
Portland-limestone cement (with 6–20% limestone)	CEM II/A-L CEM II/A-LL	CEM IIA-L CEM IIA-LL	

Aggregate grading limits

Overall grading limits for coarse aggregates

Sieve size (mm)	Percentage by mass passing ISO 565 sieve for aggregate size (*d*/*D*)						
	Graded aggregates (category G_C90/15)			Single sized aggregates (category G_C85/20)			
	4/40	4/20	2/14	20/40	10/20	6.3/14	4/10
80	100	—	—	100	—	—	—
40	90–99	100	—	85–99	100	—	—
20	25–70	90–99	98–100	0–20	85–99	98–100	100
14	—	—	90–99	—	—	85–99	98–100
10	—	25–70	—	0–5	0–20	—	85–99
6.3	—	—	25–70	—	—	0–20	—
4	0–15	0–15	—	—	0–5	—	0–20
2	0–5	0–5	0–15	—	—	—	0–5

Overall grading limits for fine aggregates (category G_F85)

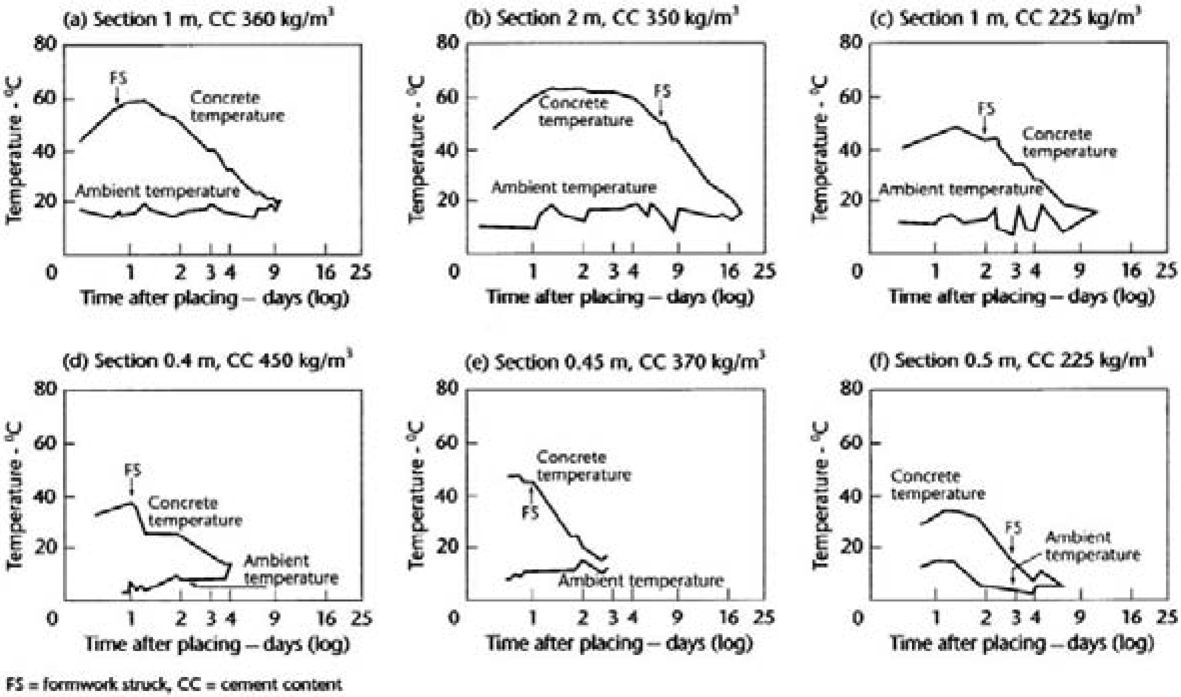
Sieve size (mm)	Percentage by mass passing ISO 565 sieve for aggregate size (*d*/*D*)					
	0/4 (*CP*)	0/4 (*MP*)	0/2 (*MP*)	0/2 (*FP*)	0/1 (*FP*)	Notes
8	100	100	—	—	—	*CP*, *MP*, *FP* classification compares with the former C, M, F classes in BS 882. * Typical grading for sieve sizes 1, 0.25 and 0.063 to be recorded by aggregate producer.
6.3	95–100	95–100	—	—	—	
4	85–99	85–99	100	100	—	
2.8	—	—	95–100	95–100	—	
2	—	—	85–99	85–99	100	
1	*	*	*	*	85–99	
0.5	5–45	30–70	30–70	55–100	55–100	

Note. The aggregate size, e.g. (4/40), shows (lower limiting sieve size *d* / upper limiting sieve size *D*). The grading category, e.g. G_C90/15, shows for coarse aggregate, the minimum % passing *D* / the maximum % passing *d*.

2.18

Concrete: early-age temperatures

Temperature histories



Early temperature history of various concrete walls showing section thickness and cement content

Predicted temperature rises

Temperature rise (°C) for Portland cement concretes in walls and slabs

Section thickness	Walls						Ground slabs and slabs cast on plywood formwork		
	Steel formwork Portland cement content (kg/m³)			18 mm plywood formwork Portland cement content (kg/m³)			Portland cement content (kg/m³)		
mm	325	350	400	325	350	400	325	350	400
300	11	13	15	23	25	31	15	17	21
500	20	22	27	32	35	43	25	28	34
700	28	32	39	38	42	49	32	36	43
1000	38	42	49	42	48	56	39	44	52

Note. The table assumes the following; (a) the formwork is left in position until the peak temperature has passed, (b) the concrete placing temperature is 20°C, (c) the mean daily temperature is 15°C, (d) no allowance has been made for solar heat gain in slabs. For slabs cast on steel formwork, the data for walls and steel formwork may be used.

Temperature rise (°C) for different cements in concrete sections with a minimum dimension of 1m

Cement/combination type	Temperature rise for cement/combination content (kg/m³)			
	350	400	450	500
Portland cement	48	56	—	—
Sulfate-resisting Portland cement	39	46	—	—
Portland-fly ash (35% pfa) cement	32	38	44	50
Portland-slag (50% ggbs) cement	28	32	36	40
Portland-slag (70% ggbs) cement	21	24	27	30

Note. The table assumes the following; (a) the formwork is left in position until the peak temperature has passed, (b) the concrete placing temperature is 20°C, (c) the mean daily temperature is 15°C.

2.19

Reinforcement: general properties

Mechanical and physical properties of bars and wires

Steel reinforcement to BS 4449 (in general conformity with BS EN 10080)

Description	(see Note 1)	Ribbed weldable reinforcing steel in the form of bars, coils or decoiled products.		
Characteristic yield strength (MPa)		500		
Ductility class		A	B	C
Grade designation		B500A	B500B	B500C
Characteristic tensile/yield strength ratio	(see Note 2)	1.05	1.08	1.15
Characteristic total elongation at maximum force (%)		2.5	5.0	7.5
Preferred sizes (mm)	(see Note 3)	8, 10, 12, 16, 20, 25, 32, 40		
Fatigue test stress range (MPa) (5x10^6 stress cycles)	Bar size ≤ 16	200		
	20	185		
	25	170		
	32	160		
	> 32	150		

Note 1. The surface geometry of the bars, as characterised by the dimensions, number, and configuration of transverse and longitudinal ribs, meets the bond requirements of BS EN 1992-1-1, Annex C.

Note 2. For sizes below 8 mm, the values are 1.02 for the strength ratio, and 1% for the total elongation.

Note 3. For bars smaller than 8 mm or larger than 40 mm, the recommended sizes are 6 mm and 50 mm respectively. For the manufacture of welded fabric to BS 4483, the preferred sizes also include 6, 7 and 9 mm.

Note 4. Absolute maximum permissible values are 650 MPa for yield strength, and 1.35 for tensile/yield strength ratio.

Note 5. Where plain round bar of grade 250 MPa is required for dowel bar applications, reference should be made to BS EN 10025-1 or, for use in concrete pavements, BS EN 13877-3.

Steel wire to BS 4482 (in general conformity with BS EN 10080)

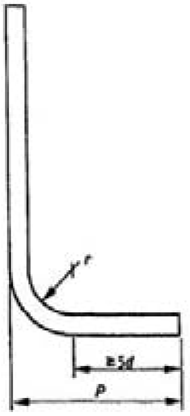
Description of wire		Plain, indented or ribbed		Plain
Grade: characteristic yield strength (MPa)		500		250
	Wire size	< 8	≥ 8	≤ 12
Characteristic tensile/yield strength ratio	(see Note 1)	1.02	1.05	1.15
Characteristic total elongation at maximum force (%)		1.0	2.5	5.0
Preferred sizes (mm)		2.5, 5, 6, 7, 8, 9, 10, 12		

Note 1. Grade 500 ribbed wire in sizes ≥ 8 mm meets the bond, and class A ductility requirements of BS EN 1992-1-1. Even so, to avoid confusion, BS 4482 should not be used to specify wire for applications covered by BS EN 1992-1-1, or for the manufacture of structural welded fabric to BS 4483. In such cases, material should be specified to BS 4449.

Bends in bars

Minimum bend dimensions to BS 8666

Nominal size of bar d mm	Minimum radius for scheduling r mm	Minimum end projection P	
		General and links where bend ≥ 150° (min 5d straight) mm	Normal links where bend < 150° (min 10d straight) mm
6	12	110*	110*
8	16	115*	115*
10	20	120*	130
12	24	125*	160
16	32	130	210
20	70	190	290
25	87	240	365
32	112	305	465
40	140	380	580
50	175	475	725



Note. Due to 'spring back' the actual radius of bend will be slightly greater than shown. Values shown * are governed by the practicalities of bending bars.

Reinforcement: cross-sectional areas of bars and fabric

2.20

	Number of bars	Cross-sectional area of number of bars (mm^2) for size of bars (mm)								
		6	8	10	12	16	20	25	32	40
Bars in specified numbers	1	28	50	78	113	201	314	491	804	1257
	2	57	101	157	226	402	628	982	1608	2513
	3	85	151	236	339	603	942	1473	2413	3770
	4	113	201	314	452	804	1257	1963	3217	5027
	5	141	251	393	565	1005	1571	2454	4021	6283
	6	170	302	471	679	1206	1885	2945	4825	7540
	7	198	352	550	792	1407	2199	3436	5630	8796
	8	226	402	628	905	1608	2513	3927	6434	10050
	9	254	452	707	1018	1810	2827	4418	7238	11310
	10	283	503	785	1131	2011	3142	4909	8042	12570
	11	311	553	864	1244	2212	3456	5400	8847	13820
	12	339	603	942	1357	2413	3770	5890	9651	15080

	Spacing of bars (mm)	Cross-sectional area of bars per unit width (mm^2/m) for size of bars (mm)								
		6	8	10	12	16	20	25	32	40
Bars at specified spacing	75	377	670	1047	1508	2681	4189	6545	10720	—
	100	283	503	785	1131	2011	3142	4909	8042	12570
	125	226	402	628	905	1608	2513	3927	6434	10053
	150	188	335	524	754	1340	2094	3272	5362	8378
	175	162	287	449	646	1149	1795	2805	4596	7181
	200	141	251	393	565	1005	1571	2454	4021	6283
	225	—	223	349	503	894	1396	2182	3574	5585
	250	—	201	314	452	804	1257	1963	3217	5027
	300	—	168	262	377	670	1047	1636	2681	4189
	400	—	—	196	283	503	785	1227	2011	3142
	500	—	—	—	226	402	628	982	1608	2513
	600	—	—	—	—	335	524	818	1340	2094

* 6 mm is a non-preferred size.

Standard fabric types to BS 4483

	Fabric reference	Longitudinal bars			Cross bars			Mass per unit area
		Nominal bar size mm	Pitch of bars mm	Area of bars per unit width mm^2/m	Nominal bar size mm	Pitch of bars mm	Area of bars per unit width mm^2/m	kg/m^2
Standard fabrics	A393	10	200	393	10	200	393	6.16
	A252	8	200	252	8	200	252	3.95
	A193	7	200	193	7	200	193	3.02
	A142	6	200	142	6	200	142	2.22
	B1131	12	100	1131	8	200	252	10.90
	B785	10	100	785	8	200	252	8.14
	B503	8	100	503	8	200	252	5.93
	B385	7	100	383	7	200	193	4.53
	B283	6	100	283	7	200	193	3.73
	C785	10	100	785	6	400	71	6.72
	C636	9	100	636	6	400	71	5.55
	C503	8	100	503	6	400	71	4.51
	C385	7	100	385	6	400	71	3.58
	C283	6	100	283	6	400	71	2.78
	D98	5	200	98	5	200	98	1.54
	D49	2.5	100	49	2.5	100	49	0.77

Notes. Bars used for fabric are in accordance with BS 4449 except for D98 and D49, where wire to BS 4442 may be used. Stock sheet size is 4.8 m (longitudinal bars) x 2.4 m (cross bars). For further information, see section 10.3.2.

thin one-way spanning slabs, where the longitudinal bars provide the main reinforcement, with the cross bars being sufficient to meet the minimum requirements for secondary reinforcement. Type C is a rectangular (long) mesh where the cross bars are minimal, which can be used in any solid slab including column bases, by providing a separate sheet in each direction. Type D is a rectangular (wrapping) mesh that is used in the concrete encasement of structural steel sections.

Standard fabric is normally supplied in stock sheet sizes of 4.8 m (longitudinal bars) × 2.4 m (cross bars), with end overhangs equal to 0.5 × the pitch of the perpendicular bar. To facilitate fixing, and to avoid a build-up of bar layers at the laps, sheets with increased overhangs (flying ends) can be supplied to order. Sheets can also be supplied cut to size, in lengths up to 12 m, and prebent. For guidance on the use of purpose-made fabrics, reference should be made to BS 8666.

10.3.3 Cutting and bending tolerances

Bars are produced in stock lengths of 12 m, and lengths up to 18 m can be supplied to special order. In most structures, bars are required in shorter lengths and often need to be bent. The cutting and bending of reinforcement is generally specified to the requirements of BS 8666. The tolerances on cutting and bending dimensions are as follows:

Cutting and bending processes		Tolerance (mm)
Cutting of straight lengths		± 25
Bending dimension (mm)	≤ 1000	± 5
	> 1000 and ≤ 2000	+ 5, − 10
	> 2000	+ 5, − 25
Length of wires in fabric	$L \leq 5000$	± 25
	$L > 5000$	± $L/200$

10.3.4 Shape codes and bending dimensions

BS 8666 contains details of bar shapes, designated by shape codes as given in *Tables 2.21* and *2.22*. The information needed to cut and bend the bars to the required dimensions is entered into a bar schedule, an example of which is shown in *Table 2.23*. The standard shapes should be used wherever possible, with the relevant dimensions entered into columns *A* to *E* of the bar schedule. All other shapes should be given a shape code 99, with a dimensioned sketch drawn over the columns *A* to *E* using two parallel lines to indicate the bar thickness. One of the bar dimensions should be indicated in parenthesis as a free dimension. Dimensions should be given as a multiple of 5 mm, and the total length, determined in accordance with the equation given in the table, rounded up to a multiple of 25 mm. To facilitate transportation, each bent bar should fit within an imaginary rectangle, the shorter side of which is not longer than 2750 mm.

Most of the shape codes cater for bars bent to the minimum radius taken as $2d$ for $d \leq 16$, and $3.5d$ for $d \geq 20$, where d is the bar size. The minimum straight length needed beyond the end of the curved portion of a bend is $5d$ for a bob and $10d$ for most links. For each bar size, values of the minimum radius r, and the minimum end projection P needed to form a bend, are given in *Table 2.19*.

Bars needing larger radius bends, denoted by R, except for shape codes 12 and 67, should be treated as a shape code 99. For shape code 67, when the radius exceeds the value in the following table, straight bars will be supplied as the required curvature can be obtained during fixing.

Maximum limit for which a preformed radius is required

Bar size (mm)	8	10	12	16
Radius (m)	2.75	3.5	4.25	7.5
Bar size (mm)	20	25	32	40
Radius (m)	14	30	43	58

For shape codes 12, 13, 22 and 33, the largest practical radius for producing a continuous curve is 200 mm and, for a larger radius, a series of short straight sections may be formed.

10.3.5 Deductions for variations

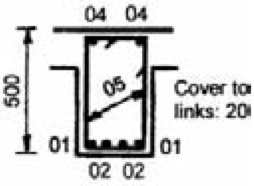
Cover to reinforcement is liable to variation due to the effect of inevitable errors in the dimensions of formwork, and in the cutting, bending and fixing of the bars. In cases where a bar is detailed to fit between two concrete faces, with no more than the nominal cover on each face (e.g. link in a beam), an appropriate allowance for deviations should be applied. The relevant dimension on the schedule should be determined as the nominal dimension of the concrete less the nominal cover on each face less an allowance for deviations as follows:

Total deductions to allow for permissible deviations on member size and in cutting and bending of bars

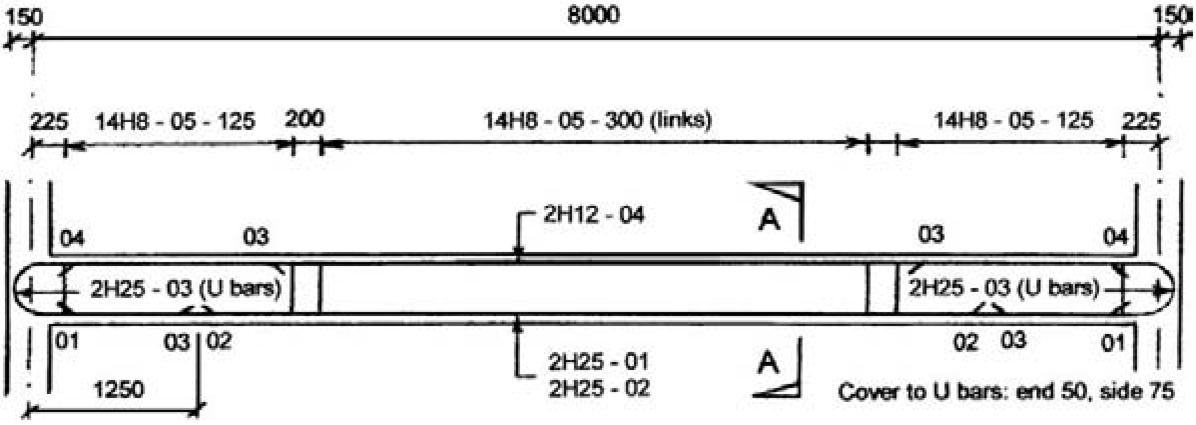
Type of bar	Distance between faces of concrete member	Deduction mm
Links and other bent bars	Not more than 1 m Between 1 m and 2 m Over 2 m	10 15 20
Straight bars	Any length	40

The deductions recommended in the forgoing table are taken from BS 8110, and allow for deviations on the member size of 5 mm for dimensions up to 2 m and 10 mm for dimensions over 2 m. Where the permissible deviations on member size exceed these values, larger deductions should be made or the cover increased.

Example. Determine the relevant bending dimensions for the bars shown in the following beam detail. The completed bar schedule, for 6 beams thus, is given in *Table 2.23*.



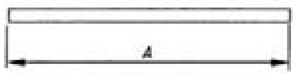
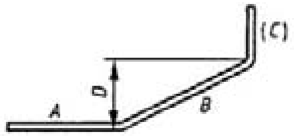
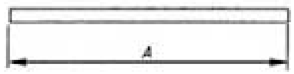
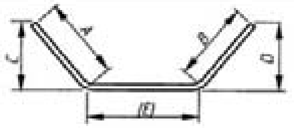
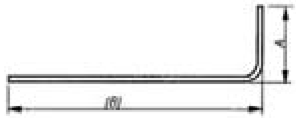
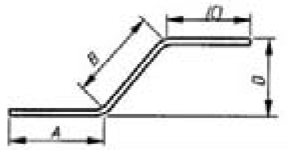
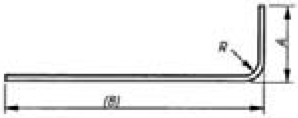
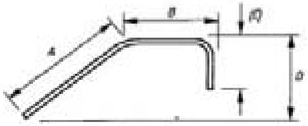
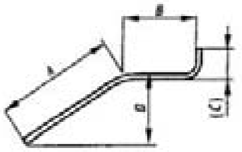
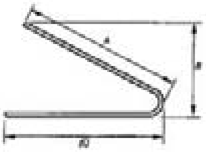
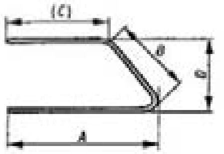
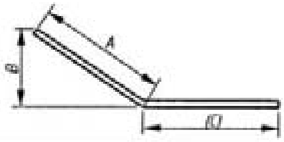
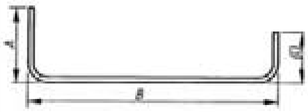
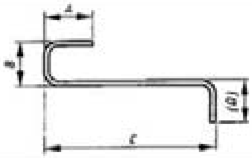
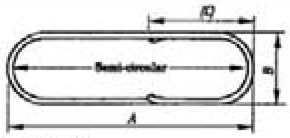
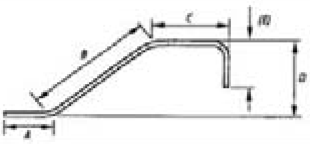
Section A-A



Elevation of beam

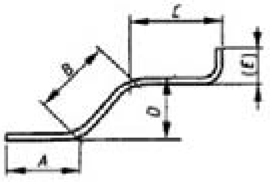
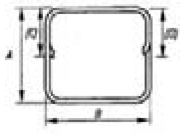
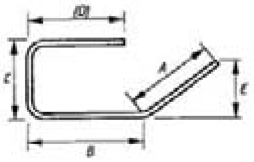
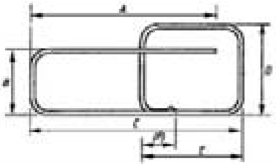
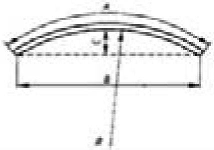
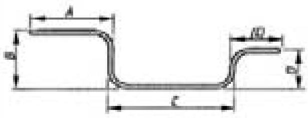
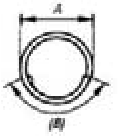
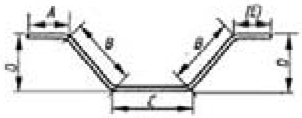
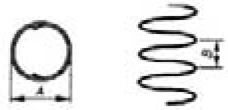
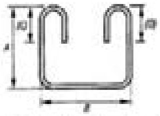
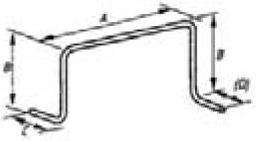
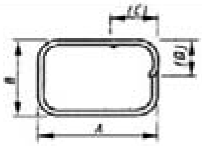
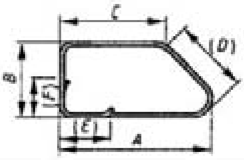
Bar mark	Shape code	Dimensions (mm)	Length (mm) – see *Tables 2.29* and *2.30*
01	00		$L = 8000 - 2 \times 200 = 7600$
02	00		$L = 8000 - 2 \times 1250 = 5500$
03	13	Bar requires radius of bend $R = 6d$ as a design requirement. This necessitates the use of shape code 13, as dimension B would not provide $4d$ length of straight between two bends. $A = 1800$, $C = 1300$ (design requirements) $B = 450 - (2 \times 10) - 10 = 420$ Note. Dimension B is derived from dimension A of bar mark 05, and includes a further deduction of 10 mm for tolerances on cutting and bending.	$L = 1800 + 0.57 \times 420 + 1300 - 1.6 \times 25 = 3300$
04	00		$L = 8000 - 2 \times 200 = 7600$
05	51	$A = 500 - (2 \times 20) - 10 = 450$ $B = 300 - (2 \times 20) - 10 = 250$ $C = D = 115$ (P in *Table 2.19*) $r = 16$ (*Table 2.19*) Note. Dimensions A and B include deductions of 10 mm for permissible deviations.	$L = 2\,(450 + 250 + 115) - (2.5 \times 16) - (5 \times 8) = 1550$

Reinforcement: standard bar shapes and method of measurement – 1

Shape code	Shape	Total length of bar *L* (along centreline)	Shape code	Shape	Total length of bar *L* (along centreline)
00		*A*	24		*A* + *B* + (*C*) *A* and *C* are at 90° to one another
01		*A* (Stock lengths) Dimension should be taken as indicative only. Actual delivery length by agreement with supplier.	25	 If *E* is critical, schedule a 99, with either *A* or *B* as a free dimension.	*A* + *B* + (*E*) *A* and *B* ≥ *P* (*Table 2.19*) See Note 1
11		*A* + (*B*) – 0.5*r* – *d* *A* and (*B*) ≥ *P* (*Table 2.19*)	26		*A* + *B* + (*C*) *A* and *C* ≥ *P* (*Table 2.19*) See Note 1
12	 See Note 3	*A* + (*B*) – 0.43*R* – 1.2*d* *A* and *B* ≥ *R* + 6*d* and ≥ *P* (*Table 2.19*)	27		*A* + *B* + (*C*) – 0.5*r* – *d* *A* and *C* ≥ *P* (*Table 2.19*) See Note 1
13	 See Note 3	*A* + 0.57*B* + (*C*) – .6*d* *A* and *C* ≥ *B*/2 + 5*d* and ≥ *P* (*Table 2.19*) *B* ≥ 2(*r* + *d*)	28		*A* + *B* + (*C*) – 0.5*r* – *d* *A* and *C* ≥ *P* (*Table 2.19*) See Note 1
14		*A* + (*C*) – 4*d* *A* and *C* ≥ *P* (*Table 2.19*) See Note 1	29		*A* + *B* + (*C*) – *r* – 2*d* *A* and *C* ≥*P* (*Table 2.19*) See Note 1
15		*A* + (*C*) *A* and *C* ≥ *P* (*Table 2.19*) See Note 1	31		*A* + *B* + *C* + (*D*) – 1.5*r* – 3*d* *A* and *D* ≥ *P* (*Table 2.19*)
21		*A* + *B* + (*C*) – *r* – 2*d* *A* and *C* ≥ *P* (*Table 2.19*)	32		*A* + *B* + *C* + (*D*) – 1.5*r* – 3*d* *A* and *D* ≥ *P* (*Table 2.19*)
22	 See Note 3	*A* + *B* + *C* + (*D*) – 1.5*r* – 3*d* *A* ≥ *P* (*Table 2.19*) *C* ≥ 2(*r* + *d*) *D* ≥ *C*/2 + 5*d* and ≥ *P* (*Table 2.19*)	33	 See Note 3	2*A* + 1.7*B* + 2© – 4*d* *A* ≥ 12*d* + 30 mm *B* ≥ 2(*r* + *d*) *C* ≥ *B*/2 + 5*d* and *P* (*Table 2.19*)
23		*A* + *B* + © – *r* – 2*d* *A* and *C* ≥ *P* (*Table 2.19*)	34		*A* + *B* + *C* + (*E*) – 0.5*r* – *d* *A* and *E* ≥ *P* (*Table 2.19*) See Note 1

Reinforcement: standard bar shapes and method of measurement – 2

2.22

Shape code	Shape	Total length of bar L (along centreline)	Shape code	Shape	Total length of bar L (along centreline)
35		$A + B + C + (E) - 0.5r - d$ A and E $\geq P$ (*Table 2.19*) See Note 1	63	 If C and D are to be minimised, use $L = 2A + 3B + (14d \geq 150)$	$2A + 3B + 2(C) - 3r - 6d$ C and D are to be equal, $\leq A$, and $\geq P$ (*Table 2.19*)
36		$A + B + C + (D) - r - 2d$ A and D $\geq P$ (*Table 2.19*) See Note 1	64		$A + B + C + 2D + E + (F) - 3r - 6d$ A and F $\geq P$ (*Table 2.19*) See Note 2
41		$A + B + C + D + (E) - 2r - 4d$ A and E $\geq P$ (*Table 2.19*)	67		A If R exceeds value shown in section 10.3.4, straight bars will be supplied.
44		$A + B + C + D + (E) - 2r - 4d$ A and E $\geq P$ (*Table 2.19*)	75		$\pi(A - d) + B$ where B is lap length
46		$A + 2B + C + (E)$ A and E $\geq P$ (*Table 2.19*) See Note 1	77	 C = number of turns	$C\pi(A - d)$ If $B > A/5$, replace $\pi(A - d)$ by $[(\pi(A - d))^2 + B^2]^{0.5}$
47	 If C and D are to be minimised, use $L = 2A + B + (21d \geq 240)$	$2A + B + 2C + 1.5r - 3d$ C and D are to be equal, $\leq A$, and $\geq P$ (*Table 2.19*)	98	 Isometeric sketch	$A + 2B + C + (D) - 2r - 4d$ C and D $\geq P$ (*Table 2.19*)
51	 If C and D are to be minimised, use $L = 2A + 2B + (16d \geq 160)$	$2[A + B + (C)] - 2.5r - 5d$ C and D are to be equal, $\leq A$ or B, $\geq P$ (*Table 2.19*)	99	All other shapes where standard shapes cannot be used. A dimensioned sketch should be drawn over the dimension columns A to E of the bending schedule. Every dimension should be shown, with the dimension that is chosen to allow for permissible deviations shown in parenthesis. Otherwise, the fabricator is free to choose the dimension to allow for tolerance.	To be calculated. See Note 2.
56		$A + B + C + (D) + 2E - 2.5r - 5d$ E and F are to be equal, $\leq A$ or B, $\geq P$ (*Table 2.19*)			

Notes. The values for minimum radius r, and end projection P, as given in *Table 2.19*, apply to all shape codes. Dimensions in parenthesis are the free dimensions to allow for permissible deviations in cutting and bending. If a shape given in this table is required to have a different free dimension, the shape shall be drawn out and given a shape code 99. The length of straight between two bends shall be at least $4d$.

Note 1. The length equations for shape codes 14, 15, 25, 26, 27, 28, 29, 34, 35, 36 and 46 are approximate. When the bend angle exceeds 45°, the length should be calculated allowing for the difference between the specified overall dimensions and the true length measured along the central axis of the bar. When the bend angle approaches 90°, it is preferable to specify shape code 99 with a fully dimensioned sketch.

Note 2. Five bends or more might be impractical within permitted tolerances.

Note 3. For shape codes 12, 13, 22 and 33, the curve may be produced as a series of short straight sections when the radius > 200 mm.

Reinforcement: typical bar schedule

2.23

Reynolds and Associates
Project Physics Block
Client Eastford University

Job No.
Prepared by
Last revised

Drawing No. ☐☐☐☐☐
Date
Sheet No. ☐☐
Status ☐
Revision ☐

Member	Bar mark	Type and size	No. of mbrs	No.of bars in each	Total no. of bars	Length of each bar † mm	Shape code	*A** mm	*B** mm	*C** mm	*D** mm	*E/R** mm	Rev. letter
Beam	01	H25	6	2	12	7600	00						
(see	02	H25		2	12	5500	00						
section	03	H25		4	24	3300	13	1800	420	1300			
10.3.5)	04	H12		2	12	7600	00						
	05	H8		42	252	1550	51	450	250	115			

This schedule conforms to BS8666. * Specified in multiples of 5 mm. † Specified in multiples of 25 mm.
Status: P Preliminary T Tender C Construction

Chapter 11

Cantilevers and single-span beams

The formulae and coefficients in this chapter give values of shearing forces, bending moments, slopes and deflections in terms of the total load on the member. For design purposes, the load F must include the appropriate partial safety factors for the limit-state being considered.

11.1 SIMPLE BEAMS AND CANTILEVERS

The formulae for the reactions, shearing forces and bending moments in freely supported beams (*Tables 2.24* and *2.25*) and simple cantilevers (*Tables 2.26* and *2.27*) are obtained by the rules of static equilibrium. The slope and deflection formulae for freely supported beams and simple cantilevers, and all the formulae for propped cantilevers (*Table 2.27*) are for elastic behaviour and members of constant cross section.

11.2 BEAMS FIXED AT BOTH ENDS

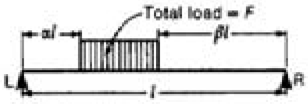
The bending moments on a beam fixed at both ends can be derived from the principle that the area of the free-moment diagram (i.e. the bending moment diagram due to the same load imposed on a freely supported beam of equal span) is equal to the area of the restraint-moment diagram. Also, the centres of area of the two diagrams are in the same vertical line. The shape of the free-moment diagram depends upon the particular characteristics of the imposed load, but the restraint-moment diagram is a trapezium. For loads that are symmetrically disposed on the beam, the centre of area of the free-moment diagram is at the mid-point of the span, and thus the restraint-moment diagram is a rectangle, giving a restraint moment at each support equal to the mean height of the free-moment diagram.

The amount of shearing force in a beam with one or both ends fixed is calculated from the variation of the bending moment along the beam. The shearing force resulting from the restraint moment alone is constant throughout the length of the beam and equal to the difference between the two end-moments divided by the span (i.e. the rate of change of the restraint moment). This shearing force is algebraically added to the shearing force due to the imposed load with the beam taken as freely supported. Thus, the support reaction is the sum (or difference) of the restraint-moment shearing force and the free-moment shearing force. For a beam that is symmetrically loaded with both ends fixed, the restraint moment at each end is the same, and the shearing forces are identical to those for the same beam freely supported. The support reactions are both equal to one-half of the total load on the span. The formulae for the reactions, shearing forces, bending moments, slopes and deflections for fully fixed spans (*Table 2.25*) are for elastic behaviour and members of constant cross section.

11.2.1 Fixed-end moment coefficients

Fixed-end moment coefficients C_{AB} and C_{BA} are given in *Table 2.28* for a variety of unsymmetrical and symmetrical imposed loadings on beams of constant cross section. More complex loading arrangements can generally be formed as a combination of the cases shown, and the resulting fixed-end moments found by superposition. A full range of charts is contained in *Examples of the Design of Reinforced Concrete Buildings* for a member with a partial uniform or triangular distribution of load placed anywhere within the span.

PARTIAL UNIFORM LOAD



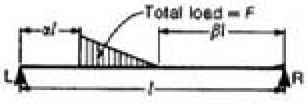
Reactions:
$R_L = \frac{1}{2}F(1-\alpha+\beta)$; $R_R = \frac{1}{2}F(1+\alpha-\beta)$

Shearing forces:
when $x < \alpha$: $V_x = \frac{1}{2}F(1-\alpha+\beta)$
when $\alpha < x < (1-\beta)$: $V_x = F\left[\frac{1}{2}(1-\alpha+\beta) + \frac{(\alpha-x)}{(1-\alpha-\beta)}\right]$
when $x > (1-\beta)$: $V_x = -\frac{1}{2}F(1+\alpha-\beta)$

Bending moments:
when $x < \alpha$: $M_x = \frac{1}{2}Flx(1-\alpha+\beta)$
when $\alpha < x < (1-\beta)$: $M_x = \frac{1}{2}Fl\left[x(1-\alpha+\beta) - \frac{(x-\alpha)^2}{(1-\alpha-\beta)}\right]$
when $x > (1-\beta)$: $M_x = \frac{1}{2}Fl(1-x)(1+\alpha-\beta)$
$M_{x\,max} = \frac{1}{8}(1+\alpha+\beta)(1+\alpha-\beta) \times (1-\alpha+\beta)Fl$ at $\frac{1}{2}(1+\alpha^2-\beta^2)l$

Deflections:
when $x < \alpha$: $a_x = -\frac{Fl^3(1-\alpha+\beta)}{24EI}[(1+2\alpha-\alpha^2-\beta^2)x-2x^3] = a_1$
when $\alpha < x < (1-\beta)$: $a_x = a_1 - \frac{Fl^3(x-\alpha)^4}{24EI(1-\alpha-\beta)}$
when $x > (1-\beta)$: use formula for a_1, transpose α and β and substitute $(1-x)$ for x

PARTIAL TRIANGULAR LOAD



Reactions:
$R_L = \frac{1}{3}F(2-2\alpha+\beta)$; $R_R = \frac{1}{3}F(1+2\alpha-\beta)$

Shearing forces:
when $x < \alpha$: $V_x = \frac{1}{3}F(2-2\alpha+\beta)$
when $\alpha < x < (1-\beta)$: $V_x = -F\left[\frac{1}{3}(1+2\alpha-\beta) - \frac{(1-x-\beta)^2}{(1-\alpha-\beta)^2}\right]$
when $x > (1-\beta)$: $V_x = -\frac{1}{3}F(1+2\alpha-\beta)$

Bending moments:
when $x < \alpha$: $M_x = \frac{1}{3}Flx(2-2\alpha+\beta)$
when $\alpha < x < (1-\beta)$: $M_x = \frac{1}{3}Fl\left[(1+2\alpha-\beta)(1-x) - \frac{(1-x-\beta)^3}{(1-\alpha-\beta)^2}\right]$
when $x > (1-\beta)$: $M_x = \frac{1}{3}Fl(1-x)(1+2\alpha-\beta)$

Deflections:
when $x < \alpha$:

$$a_x = -\frac{Fl^3x(2-2\alpha+\beta)}{162EI}\left[9(1+x)(1-x)-(2-2\alpha+\beta)^2-(1-\alpha-\beta)^2\left\{\frac{3}{2}-\frac{(1-\alpha-\beta)}{5(2-2\alpha+\beta)}\right\}\right] = a_1$$

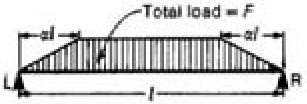
when $\alpha < x < (1-\beta)$:

$$a_x = a_1 - \frac{Fl^3(x-\alpha)^4}{60EI(1-\alpha-\beta)}\left\{5-\frac{(x-\alpha)}{(1-\alpha-\beta)}\right\}$$

when $x > (1-\beta)$:

$$a_x = -\frac{Fl^3(1-x)(1+2\alpha-\beta)}{162EI}\left[9x(2-x)-(1+2\alpha-\beta)^2-(1-\alpha-\beta)^2\left\{\frac{3}{2}+\frac{(1-\alpha-\beta)}{5(1+2\alpha-\beta)}\right\}\right]$$

TRAPEZOIDAL LOAD



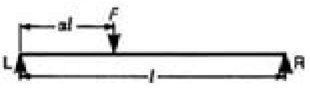
Reactions:
$R_L = R_R = \frac{1}{2}F$

Shearing forces:
when $x < \alpha$: $V_x = \frac{1}{2}F\left[1 - \frac{x^2}{\alpha(1-\alpha)}\right]$
when $\alpha < x \leqslant \frac{1}{2}$: $V_x = \frac{1}{2}F\frac{(1-2x)}{(1-\alpha)}$

Bending moments:
when $x < \alpha$: $M_x = \frac{1}{6}Flx\left[3 - \frac{x^2}{\alpha(1-\alpha)}\right]$
when $\alpha < x \leqslant \frac{1}{2}$: $M_x = \frac{1}{6}Fl\left[\frac{3x(1-x)-\alpha^2}{(1-\alpha)}\right]$
$M_{x\,max} = (3-4\alpha^2)/\{24(1-\alpha)\}Fl$ at midspan

Deflections:
when $x < \alpha$: $a_x = -(Fl^3x/24EI)[(1+\alpha-\alpha^2)-2x^2+\{x^4/5\alpha(1-\alpha)\}]$
when $\alpha < x \leqslant \frac{1}{2}$: $a_x = -[Fl^3/\{24EI(1-\alpha)\}]$
$\times[\{1-2\alpha^2+x(1-x)\}x(1-x)+(\alpha^4/5)]$
$a_{x\,max} = -(4\alpha^2-5)^2Fl^3/\{1920(1-\alpha)EI\}$ at midspan

CONCENTRATED LOAD



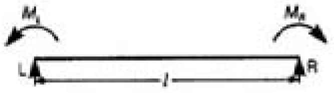
Reactions:
$R_L = F(1-\alpha)$; $R_R = F\alpha$

Shearing forces:
when $x < \alpha$ $V_x = F(1-\alpha)$
when $x > \alpha$ $V_x = -F\alpha$

Bending moments:
when $x < \alpha$ $M_x = Fl(1-\alpha)x$
when $x > \alpha$ $M_x = Fl\alpha(1-x)$
$M_{x\,max} = Fl\alpha(1-\alpha)$ beneath load

Deflections:
when $x < \alpha$ $a_x = (Fl^3(1-\alpha)x/6EI)[\alpha(2-\alpha)-x^2]$
when $x > \alpha$ $a_x = -(Fl^3\alpha(1-x)/6EI)[x(2-x)-\alpha^2]$
when $\alpha \leqslant \frac{1}{2}$: $a_{x\,max} = -Fl^3\alpha(1-\alpha^2)^{3/2}/9\sqrt{(3)}EI$ at $\sqrt{[(1-\alpha^2)/3]}l$ from R.

SUPPORT MOMENTS



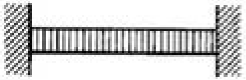
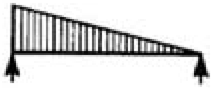
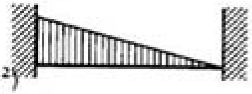
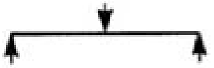
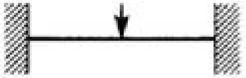
Reactions: $R_L = \frac{M_L - M_R}{l}$; $R_R = \frac{M_R - M_L}{l}$

Shearing force: $V_x = \frac{M_L - M_R}{l}$

Bending moments: $M_x = M_L(1-x) + M_Rx$

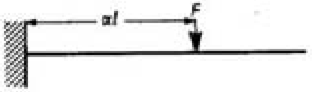
Deflections: $a_x = -[x(1-x)l^2/6EI][(2-x)M_L+(1+x)M_R]$

For notes see *Table 2.26*

	Freely supported span	Fully fixed span
Uniform load	 Reactions: $R_L = R_R = \frac{1}{2}F$ Shearing forces: $V_x = F(\frac{1}{2} - x)$ Bending moments: $M_L = M_R = 0$ $M_x = \frac{1}{2}x(1-x)Fl$ $M_{x\,max} = \frac{1}{8}Fl$ at $\frac{1}{2}l$ Slopes: $\theta_L = \theta_R = \pm Fl^2/24EI$ Deflections: $a_x = -(Fl^3x/24EI)(1-x)(1+x-x^2)$ $a_{x\,max} = -5Fl^3/384EI$ at midspan	 Reactions: $R_L = R_R = \frac{1}{2}F$ Shearing forces: $V_x = F\left(\frac{1}{2} - \frac{x}{l}\right)$ Bending moments: $M_L = M_R = -\frac{1}{12}Fl$ $M_x = \frac{1}{2}Fl[x(1-x) - \frac{1}{6}]$ $M_{x\,max} = \frac{1}{24}Fl$ at midspan Slopes: $\theta_L = \theta_R = 0$ Deflections: $a_x = -Fl^3x^2(1-x)^2/24EI$ $a_{x\,max} = -Fl^3/384EI$ at midspan
Triangular load	*Apex at left-hand end*  Reactions: $R_L = \frac{2}{3}F;\quad R_R = \frac{1}{3}F$ Shearing forces: $V_x = (F/3)(2 - 6x + 3x^2)$ Bending moments: $M_L = M_R = 0$ $M_x = \frac{1}{3}Flx(1-x)(2-x)$ $M_{x\,max} = 2Fl/9\sqrt{3}$ at $(1 - 1/\sqrt{3})l$ from L Slopes: $\theta_L = -2Fl^2/45EI;\quad \theta_R = +7Fl^2/180EI$ Deflections: $a_x = -x(1-x)(2-x)(4+6x-3x^2)Fl^3/180EI$ $a_{x\,max} \simeq -Fl^3/76.7EI$ at $x \simeq 0.4807l$ from L	*Apex at left-hand end*  Reactions: $R_L = \frac{7}{10}F;\quad R_R = \frac{3}{10}F$ Shearing forces: $V_x = (F/10)(7 - 20x + 10x^2)$ Bending moments: $M_L = -Fl/10 \quad M_R = -Fl/15$ $M_x = (Fl/30)(10x^3 - 30x^2 + 21x - 3)$ $M_{x\,max} \simeq Fl/23.32$ at $(1 - \sqrt{0.3})l$ from L Slopes: $\theta_L = \theta_R = 0$ Deflections: $a_x = -(Fl^3/60EI)x^2(1-x)^2(3-x)$ $a_{x\,max} \simeq -Fl^3/382EI$ at $x \simeq 0.4753l$ from L
Central concentrated load	 Reactions: $R_L = R_R = \frac{1}{2}F$ Shearing forces: when $x < \frac{1}{2}$: $V_x = \frac{1}{2}F$ when $x > \frac{1}{2}$: $V_x = -\frac{1}{2}F$ Bending moments: $M_L = M_R = 0$ $M_{x\,max} = \frac{1}{4}Fl$ at midspan Slopes: $\theta_L = -Fl^2/16EI;\quad \theta_R = +Fl^2/16EI$ Deflections: when $x \leqslant \frac{1}{2}$: $a_x = -(Fl^3/48EI)x(3 - 4x^2)$ $a_{x\,max} = -Fl^3/48EI$ at midspan	 Reactions: $R_L = R_R = \frac{1}{2}F$ Shearing forces: when $x < \frac{1}{2}$: $V_x = \frac{1}{2}F$ when $x > \frac{1}{2}$: $V_x = -\frac{1}{2}F$ Bending moments: $M_L = M_R = -\frac{1}{8}Fl$ when $x \leqslant \frac{1}{2}$: $M_x = (Fl/8)(4x - 1)$ when $x \geqslant \frac{1}{2}$: $M_x = (Fl/8)(3 - 4x)$ $M_{x\,max} = \frac{1}{8}Fl$ at midspan Slopes: $\theta_L = \theta_R = 0$ Deflections: when $x \leqslant \frac{1}{2}$: $a_x = -(Fl^3/48EI)x^2(3 - 4x)$ $a_{x\,max} = -Fl^3/192EI$ at midspan

For notes see *Table 2.26*

CONCENTRATED LOAD



Shearing forces: when $x < \alpha$: $V_x = F$

when $x > \alpha$: $V_x = 0$

Bending moments: when $x \leqslant \alpha$: $M_x = -Fl(\alpha - x)$

when $x \geqslant \alpha$: $M_x = 0$

Deflections: when $x \leqslant \alpha$: $a_x = \dfrac{Fl^3}{6EI}x^2(3\alpha - x)$

when $x \geqslant \alpha$: $a_x = -\dfrac{Fl^3}{6EI}\alpha^2(3x - \alpha)$

PARTIAL TRIANGULAR LOAD: *apex at left-hand end*

Shearing forces: when $x \leqslant \alpha$: $V_x = F$

when $\alpha \leqslant x \leqslant (1-\beta)$: $V_x = F\left[1 - \dfrac{(x-\alpha)}{(1-\alpha-\beta)}\left\{2 - \dfrac{(x-\alpha)}{(1-\alpha-\beta)}\right\}\right]$

when $x \geqslant (1-\beta)$: $V_x = 0$

Bending moments: when $x \leqslant \alpha$: $M_x = -\frac{1}{3}Fl(1+2\alpha-\beta-3x)$

when $\alpha \leqslant x \leqslant (1-\beta)$: $M_x = -\frac{1}{3}Fl\left[(1+2\alpha-\beta-3x) + \dfrac{(x-\alpha)^2}{(1-\alpha-\beta)} \times \left\{3 - \dfrac{(x-\alpha)}{(1-\alpha-\beta)}\right\}\right]$

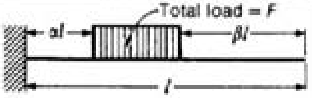
when $x \geqslant (1-\beta)$: $M_x = 0$

Deflections: when $x \leqslant \alpha$: $a_x = (1+2\alpha-\beta-x)Fl^3x^2/6EI(1+2\alpha-\beta-x)$

when $\alpha \leqslant x \leqslant (1-\beta)$: $a_x = -\dfrac{Fl^3}{60EI}\left[10x^2(1+2\alpha-\beta-x) + \dfrac{(x-\alpha)^4}{(1-\alpha-\beta)}\left\{5 - \dfrac{(x-\alpha)}{(1-\alpha-\beta)}\right\}\right]$

when $x \geqslant (1-\beta)$: $a_x = -\dfrac{Fl^3}{60EI}[\alpha\{(1-\beta)^2 + 10x\alpha - 3\alpha^2\} - (1+\alpha-\beta)^2(1+\alpha-\beta-5x)]$

PARTIAL UNIFORM LOAD



Shearing forces: when $x \leqslant \alpha$: $V_x = F$

when $\alpha \leqslant x \leqslant (1-\beta)$: $V_x = F\left[1 - \dfrac{(x-\alpha)}{(1-\alpha-\beta)}\right]$

when $x \geqslant (1-\beta)$: $V_x = 0$

Bending moments when $x \leqslant \alpha$: $M_x = -Fl[\frac{1}{2}(1+\alpha-\beta) - x]$

when $\alpha \leqslant x \leqslant (1-\beta)$: $M_x = -\frac{1}{2}Fl\left[(1+\alpha-\beta-2x) + \dfrac{(x-\alpha)^2}{(1-\alpha-\beta)}\right]$

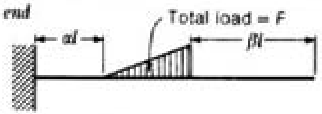
when $x \geqslant (1-\beta)$: $M_x = 0$

Deflections: when $x \leqslant \alpha$: $a_x = -(Fl^3x^2/12EI)[3(1+\alpha-\beta) - 2x]$

when $\alpha \leqslant x \leqslant (1-\beta)$: $a_x = -(Fl^3/24EI)[6x^2(1+\alpha-\beta-\frac{2}{3}x) + \{(x-\alpha)^4/(1-\alpha-\beta)\}]$

when $x \geqslant (1-\beta)$: $a_x = -(Fl^3/24EI)[2\alpha(1-\beta)(1+\alpha-\beta-2x) - (1+\alpha-\beta)^2(1+\alpha-\beta-4x)]$

PARTIAL TRIANGULAR LOAD: *apex at right-hand end*



Shearing forces: when $x \leqslant \alpha$: $V_x = F$

when $\alpha \leqslant x \leqslant (1-\beta)$: $V_x = F\left[1 - \dfrac{(x-\alpha)^2}{(1-\alpha-\beta)^2}\right]$

when $x \geqslant (1-\beta)$: $V_x = 0$

Bending moments: when $x \leqslant \alpha$: $M_x = -\frac{1}{3}Fl(2+\alpha-2\beta-3x)$

when $\alpha \leqslant x \leqslant (1-\beta)$: $M_x = -\frac{1}{3}Fl\left[(2+\alpha-2\beta-3x) + \dfrac{(x-\alpha)^3}{(1-\alpha-\beta)^2}\right]$

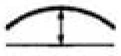
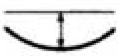
when $x \geqslant (1-\beta)$: $M_x = 0$

Deflections: when $x \leqslant \alpha$: $a_x = -(Fl^3x^2/6EI)(2+\alpha-2\beta-x)$

when $\alpha \leqslant x \leqslant (1-\beta)$: $a_x = -(Fl^3/60EI)[10x^2(2+\alpha-2\beta-x) + \{(x-\alpha)^5/(1-\alpha-\beta)^2\}]$

when $x \geqslant (1-\beta)$: $a_x = -(Fl^3/60EI)[(1-\beta)\{\alpha^2 + 10x(1-\beta) - 3(1-\beta)^2\} - (1+\alpha-\beta)^2(1+\alpha-\beta-5x)]$

KEY TO SIGN CONVENTION FOR TABLES 2.24 TO 2.27

	Reaction	Shearing force	Bending moment	Slope	Deflection
Positive	↑	↑ ↓			
Negative	↓	↓ ↑			

F total load

x distance of point considered from left-hand support in terms of l

Members with fixed ends

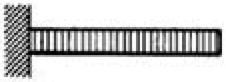
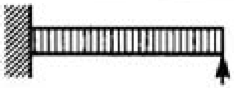
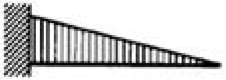
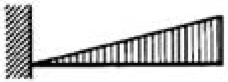
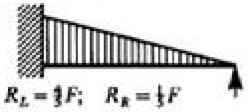
To determine deflection, moment etc. for member with one or both ends fixed or continuous, first calculate deflection, moment etc. for freely supported span. Next, determine deflection, moment etc. throughout span due to action of support moments only. Lastly, obtain final values of deflection, moment etc. by summing foregoing results algebraically.

Slope

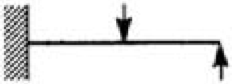
To determine slope at any point, distance xl from left-hand support, differentiate expression for deflection with respect to x.

Moments, shears, deflections: special cases for cantilevers

2.27

<table>
<tr><th></th><th colspan="2">Simple cantilever (fixed at left-hand end)</th><th colspan="2">Propped cantilever (fixed at left-hand support)</th></tr>
<tr><td>Uniform load</td><td colspan="2">
Shearing forces: $M_x = F(1 - x)$
Bending moments: $M_x = -\frac{1}{2}Fl(1-x)^2$
$M_{x\,max} = -\frac{1}{2}Fl$ at $x = 0$
Slopes: $\theta_L = 0;\quad \theta_R = -Fl^2/6EI$
Deflections: $a_x = -(Fl^3x^2/24EI)(6-4x+x^2)$
$a_{x\,max} = -Fl^3/8EI$ at $x = 1$</td><td colspan="2">
Reactions: $R_L = \frac{5}{8}F;\quad R_R = \frac{3}{8}F$
Shearing forces: $V_x = F(\frac{5}{8} - x)$
Bending moments: $M_L = -Fl/8;\quad M_R = 0$
$M_x = (Fl/8)[x(5-4x) - 1]$
$M_{x\,max} = 9Fl/128$ at $x = 5/8$
Slopes $\theta_L = 0;\quad \theta_R = Fl^2/48EI$
Deflections: $a_x = -(Fl^3/48EI)\,x^2(1-x)(3-2x)$
$a_{x\,max} \simeq -Fl^3/185EI$ at $x = 0.5785$ from L</td></tr>
<tr><td>Triangular load</td><td>Apex at l.h. end

Shearing forces: $V_x = F(1-x)^2$
Bending moments:
$M_x = -\frac{1}{3}Fl(1-x)^3$
$M_{x\,max} = -\frac{1}{3}Fl$ at $x = 0$
Slopes: $\theta_L = 0;\quad \theta_R = -Fl^2/12EI$
Deflections:
$a_x = -(Fl^3x^2/60EI)(10-10x+5x^2-x^3)$
$a_{x\,max} = -Fl^3/15EI$ at $x = 1$</td><td>Apex at r.h. end

Shearing forces: $V_x = F(1-x^2)$
Bending moments:
$M_x = -\frac{1}{3}Fl(1-x)^2(2+x)$
$M_{x\,max} = -\frac{2}{3}Fl$ at $x = 0$
Slopes: $\theta_L = 0;\quad \theta_R = -Fl^2/4EI$
Deflections:
$a_x = -(Fl^3x^2/60EI)(20-10x+x^3)$
$a_{x\,max} = -11Fl^3/60EI$ at $x = 1$</td><td>Apex at l.h. end

Reactions: $R_L = \frac{4}{5}F;\quad R_R = \frac{1}{5}F$
Shearing forces: $V_x = F(\frac{4}{5} - 2x + x^2)$
Bending moments:
$M_L = -2Fl/15;\quad M_R = 0$
$M_x = -(5x^2 - 10x + 2)(1-x)Fl/15$
$M_{x\,max} \simeq Fl/16.77$ at $x = 0.5528$ from L
Slopes: $\theta_L = 0;\quad \theta_R = Fl^2/60EI$
Deflections:
$a_x = -x^2(1-x)(2-x)^2Fl^3/60EI$
$a_{x\,max} \simeq -Fl^3/209.6EI$ at $x = 0.5528$ from L</td><td>Apex at r.h. end

Reactions: $R_L = \frac{9}{20}F;\quad R_R = \frac{11}{20}F$
Shearing forces: $V_x = F(\frac{9}{20} - x^2)$
Bending moments:
$M_L = -7Fl/60;\quad M_R = 0$
$M_x = -(20x^3 - 27x + 7)Fl/60$
$M_{x\,max} \simeq Fl/11.82$ at $x = \sqrt{0.45}$ from L
Slopes: $\theta_L = 0;\quad \theta_R = Fl^2/40EI$
Deflections:
$a_x = -x^2(1-x)(7-2x-2x^2)Fl^3/120EI$
$a_{x\,max} \simeq -Fl^3/164EI$ at $x = 0.5975$ from L</td></tr>
<tr><td>Concentrated load</td><td colspan="2">Load at unsupported end

Shearing forces: $V_x = F$
Bending moments: $M_L = -Fl$
$M_x = -F(l-x)$
Slope: $\theta_L = 0;\quad \theta_R = -Fl^2/2EI$
Deflections: $a_x = -\dfrac{Fl^3x^2}{6EI}(3-x)$
$a_{x\,max} = -Fl^3/3EI$ at $x = 1$</td><td colspan="2">Load at centre

Reactions: $R_L = \frac{11}{16}F;\quad R_R = \frac{5}{16}F$
Shearing forces: when $x < \frac{1}{2}$: $V_x = \frac{11}{16}F$
when $x > \frac{1}{2}$: $V_x = -\frac{5}{16}F$
Bending moments: $M_L = -\frac{3}{16}Fl;\quad M_R = 0$
$M_{x\,max} = \frac{5}{32}Fl$ beneath load
Slopes: $\theta_L = 0;\quad \theta_R = Fl^2/32EI$
Deflections: when $x < \frac{1}{2}$: $a_x = -x^2(9-11x)Fl^3/96EI$
when $x > \frac{1}{2}$: $a_x = -(Fl^3/96EI)(x-1)(5x^2-10x+2)$
$a_{x\,max} = -Fl^3/48\sqrt{5}EI$ at $x = 1 - 1/\sqrt{5}$ from L</td></tr>
</table>

For notes see *Table 2.26*

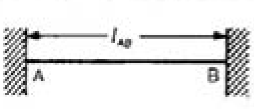
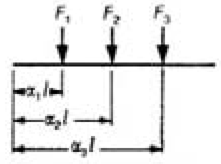
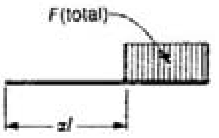
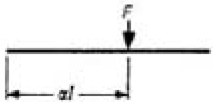
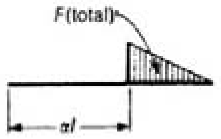
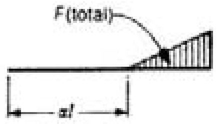
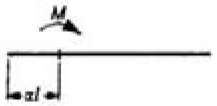
Fixed-end moment coefficients: general data

The fixed-end moment coefficients C_{AB} and C_{BA} can be used as follows:

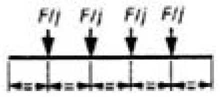
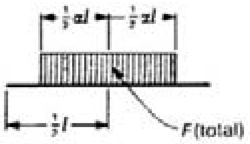
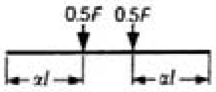
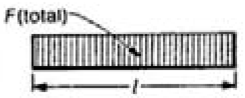
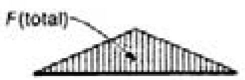
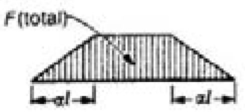
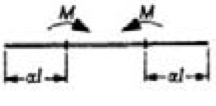
1. **To obtain bending moments at supports of single-span beams fully fixed at both ends** (*Table 2.25*)
   $M_{AB} = -C_{AB}l_{AB}$ and $M_{BA} = -C_{BA}l_{AB}$ (With symmetrical load, $M_{AB} = M_{BA}$)
2. **To obtain fixed-end moments for analysis of continuous beams by moment distribution methods** (*Table 2.36*)
   $FEM_{AB} = C_{AB}l_{AB}$ and $FEM_{BA} = C_{BA}l_{AB}$ (With symmetrical load, $FEM_{AB} = FEM_{BA}$)
3. **To obtain loading factors for analysis of framed structures by slope-deflection methods** (*Table 2.60*)
   $F_{AB} = C_{AB}l_{AB}$ and $F_{BA} = C_{BA}l_{AB}$ (With symmetrical load, $F_{AB} = F_{BA}$)
4. **To obtain loading factors for analysis of portal frames** (*Tables 2.63* and *2.64*)

$$D = \frac{C_{AB} + C_{BA}}{2} l_{AB} \text{ and } z_1 = \frac{C_{AB} + 2C_{BA}}{2(C_{AB} + 2C_{BA})}$$ (With symmetrical loading, $C_{AB} = C_{BA}$ and $z_1 = 0.5$)

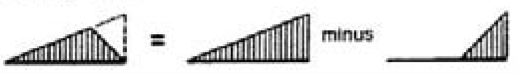
Unsymmetrical loading

	Fixed-end moment coefficients C_{AB}	Fixed-end moment coefficients C_{BA}
 Any number of loads	$\Sigma\alpha(1-\alpha)^2F$	$\Sigma\alpha^2(1-\alpha)F$
	$\frac{(1+\alpha-5\alpha^2+3\alpha^3)}{12}F$	$\frac{(1+\alpha+\alpha^2-3\alpha^3)}{12}F$
	$\alpha(1-\alpha)^2F$	$\alpha^2(1-\alpha)F$
	$\frac{(1+2\alpha-7\alpha^2+4\alpha^3)}{10}F$	$\frac{(1+2\alpha+3\alpha^2-6\alpha^3)}{15}F$
	$\frac{(2-\alpha-4\alpha^2+3\alpha^3)}{30}F$	$\frac{(3+\alpha-\alpha^2-3\alpha^3)}{30}F$
	$(\alpha-1)(3\alpha-1)\frac{M}{l}$	$\alpha(2-3\alpha)\frac{M}{l}$

Symmetrical loading

	Fixed-end moment coefficients $C_{AB} = C_{BA}$
 Any number of loads (j) equally spaced	$\frac{(j+2)}{12(j+1)}F$ j — factor 1 — $0.125F$ 2 — $0.111F$ 3 — $0.104F$ 4 — $0.100F$
	$\frac{(3-\alpha^2)}{24}F$
	$\frac{\alpha(1-\alpha)}{2}F$
	$\frac{1}{12}F$
	$\frac{5}{48}F$
	$\frac{(1+\alpha-\alpha^2)}{12}F$
	$(1-2\alpha)\frac{M}{l}$

Other loadings can generally be considered by combining tabulated cases, thus:



Chapter 12

Continuous beams

The formulae and coefficients in this chapter give values of shearing forces and bending moments in terms of the dead and live loads on the member. For design purposes, these loads must include the appropriate partial safety factors for the limit-state considered and the Code of Practice employed.

For the ULS, the dead load factors are treated differently in BS 8110 and EC 2. For designs to BS 8110, values of either 1.4 or 1.0 are applied separately to each span of the beam. For designs to EC 2, values of either 1.35 or 1.0 are applied to all the spans. If the beam ends with a cantilever, the effect of applying values of either 1.35 or 1.15 separately to the cantilever and the adjacent span should also be considered. Details of the design loads, and of the effects of applying cantilever moments at one or both ends of a continuous beam of two, three, four or five equal spans are given in *Table 2.29*.

12.1 DETERMINATION OF MAXIMUM MOMENTS

12.1.1 Incidence of live load

The values of the bending moments in the spans and at the supports depend upon the incidence of the live load and, for spans that are equal or approximately equal, the dispositions of live load shown in *Table 2.29* give the maximum positive moments in the spans and the maximum negative moments at the supports. Both BS 8110 and EC 2 consider a less severe incidence of live load, when determining the maximum negative moments at the supports. According to BS 8110, the only case that needs to be considered is when all the spans are loaded. According to EC 2, all cases of two adjacent spans loaded should be considered, but the loads shown as optional in *Table 2.29* may be ignored.

Thus, the maximum positive moments due to live load, for the system, are obtained by considering two loading arrangements: one with live load on all the odd-numbered spans and the other with live load on all the even-numbered spans. For designs to BS 8110, the summation of the results for these two cases gives the maximum negative moments.

It should be noted that for designs to EC 2, the UK National Annex allows the BS 8110 loading arrangements to be used as an alternative to those recommended in the base document. In this chapter, the basic arrangements are used.

12.1.2 Shearing forces

The shearing forces in a continuous beam are determined by first considering each span as freely supported, then adding algebraically the rate of change of restraint moment for the particular span. Shearing forces for freely supported spans are readily determined by the rules of static equilibrium. The additional shearing force, which is constant throughout the span, is equal to the difference in the support moments at each end divided by the span.

12.1.3 Maximum positive moments

When the moments at the supports and the shearing forces have been determined, the maximum positive moment in the span can be obtained by first finding the position where the shearing force is zero. The maximum positive moment is then obtained by subtracting the effect of the restraint moments, which varies linearly along the span, from the freely supported moment at this position.

12.2 SOLUTIONS FOR EQUAL SPANS

12.2.1 Coefficients for equal loads on equal spans

Approximate general solutions for the maximum bending moments and shearing forces in uniformly loaded beams of three or more spans are given in *Table 2.29*. Exact solutions for the maximum bending moments in beams of two, three, four or five equal spans are given in *Tables 2.30* and *2.31*, for eight different load distributions. The coefficients given for the support moments due to live load apply to the most onerous loading conditions. For the less severe arrangements described in section 12.1.1, coefficients are shown in the square brackets [ ] for BS 8110, and the curved brackets ( ) for EC 2. The coefficients in *Table 2.32* enable the maximum shearing forces at the supports to be determined.

Example 1. Calculate the maximum ultimate moments in the end and central spans and at the penultimate and interior supports, for a beam continuous over five equal spans of 5 m with characteristic dead and imposed loads of 20 kN/m each, according to the requirements of BS 8110.

Continuous beams: general data

Design loads

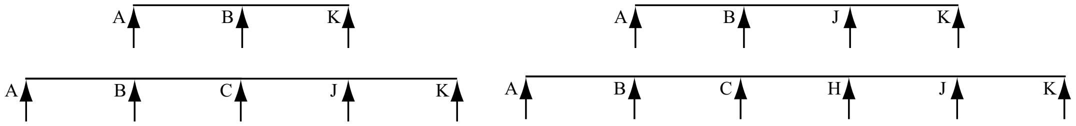
Code	BS 8110		EC 2*	
Load	Dead	Live	Dead	Live
Service	$1.0g_k$	$1.0q_k$	$1.0g_k$	$(1.0 \text{ or } \psi)q_k$
Ultimate	$1.0g_k$	$0.4g_k + 1.6q_k$	$(1.35 \text{ or } 1.0)g_k$	$1.5q_k$

Key: g_k is characteristic dead load, q_k is characteristic imposed load, ψ is a factor whose value depends on the nature and frequency of the load. * If beam ends with a cantilever, consider also: dead = $1.15g_k$ and live = $0.2g_k + 1.5q_k$.

Effect on equal spans of moments applied at end supports

Moment		Applied at A only				Applied at A and K			
Number of spans		2	3	4	5	2	3	4	5
Bending moment	M_A	− 1.000	− 1.000	− 1.000	− 1.000	− 1.000	− 1.000	− 1.000	− 1.000
	M_B	+ 0.250	+ 0.267	+ 0.268	+ 0.268	+ 0.500	+ 0.200	+ 0.286	+ 0.263
	M_C	—	—	− 0.071	− 0.072	—	—	− 0.143	−0.053
	M_D	—	—	—	+ 0.019	—	—	—	− 0.053
	M_E	—	− 0.067	+ 0.218	− 0.005	—	+ 0.200	+ 0.286	+ 0.263
	M_F	0	0	0	0	− 1.000	− 1.000	− 1.000	− 1.000
Shearing force	V_{AR}	+ 1.250	+ 1.267	+ 1.268	+ 1.268	+ 1.500	+ 1.200	+ 1.286	+ 1.263
	V_{BL}	− 1.250	− 1.267	− 1.268	− 1.268	− 1.500	− 1.200	− 1.286	− 1.263
	V_{BR}	− 0.250	− 0.333	− 0.339	− 0.340	− 1.500	0	− 0.429	− 0.316
	V_{CL}	—	—	+ 0.339	+ 0.340	—	—	+ 0.429	+ 0.316
	V_{CR}	—	—	+ 0.089	+ 0.091	—	—	—	0
	V_{HL}	—	—	—	− 0.091	—	—	—	0
	V_{HR}	—	—	—	− 0.024	—	—	+ 0.429	+ 0.316
	V_{JL}	—	+ 0.333	− 0.089	+ 0.024	—	0	− 0.429	− 0.316
	V_{JR}	—	+ 0.067	− 0.018	+ 0.005	—	− 1.200	− 1.286	− 1.263
	V_{KL}	+ 0.250	− 0.067	+ 0.018	− 0.005	+ 1.500	+ 1.200	+ 1.286	+ 1.263

Key:

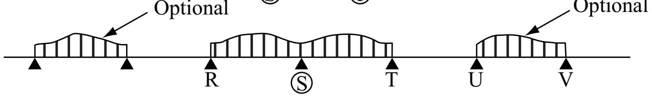


Adjustment to bending moment = M coefficient x applied bending moment
Adjustment to shearing force = (V coefficent x applied bending moment)/span

Live load patterns

To produce maximum positive moment in span ST

To produce maximum negative moment at support S



Simplifications: BS8110: Consider live load on all spans

EC2: Consider live load on spans RS and ST only

Approximate solutions

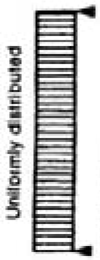
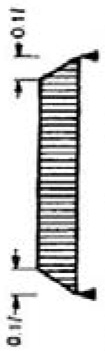
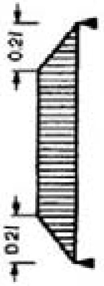
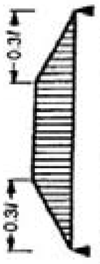
Uniformly loaded continuous beam, freely supported at the ends, with three or more approximately equal spans

Position	Bending moment		Shearing force
	BS 8110	EC 2	BS 8110 and EC 2
At outer support	0	0	$0.45F$
Near middle of end span	$+ 0.09Fl$	$+ 0.09Fl$	—
At first interior support	$- 0.11Fl$	$- 0.11Fl$	$0.60F$
At middle of interior spans	$+ 0.07Fl$	$+ 0.07Fl$	—
At other interior supports	$- 0.08Fl$	$- 0.09Fl$	$0.55F$

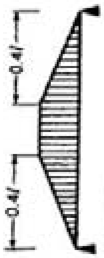
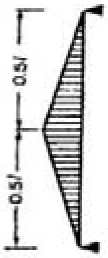
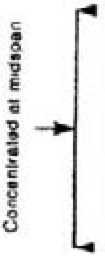
Note: Values apply where characteristic imposed load does not exceed characteristic dead load and variations in span length do not exceed 15% of longest span (F is total design load on span, l is effective span).

Continuous beams: moments from equal loads on equal spans – 1

2.30

Load	All spans loaded (e.g. dead load)	Live load (sequence of loaded spans to give max. bending moment)
	0.125 ▲ 0.070 ▲ 0.070 ▲	0.125 ▲ 0.096 ▲ 0.096 ▲
	0.100 0.100 ▲ 0.080 ▲ 0.025 ▲ 0.080 ▲	[0.100] [0.100] 0.117 0.117 ▲ 0.101 ▲ 0.075 ▲ 0.101 ▲
	0.107 0.071 0.107 ▲ 0.077 ▲ 0.036 ▲ 0.036 ▲ 0.077 ▲	[0.107] [0.071] [0.107] (0.116) (0.107) (0.116) 0.121 0.107 0.121 ▲ 0.099 ▲ 0.081 ▲ 0.081 ▲ 0.099 ▲
	0.105 0.079 0.079 0.105 ▲ 0.078 ▲ 0.033 ▲ 0.046 ▲ 0.033 ▲ 0.078 ▲	[0.105] [0.079] [0.079] [0.105] (0.116) (0.106) (0.106) (0.116) 0.120 0.111 0.111 0.120 ▲ 0.100 ▲ 0.079 ▲ 0.086 ▲ 0.079 ▲ 0.100 ▲
	0.136 ▲ 0.077 ▲ 0.077 ▲	0.136 ▲ 0.105 ▲ 0.105 ▲
	0.109 0.109 ▲ 0.088 ▲ 0.028 ▲ 0.088 ▲	[0.109] [0.109] 0.127 0.127 ▲ 0.111 ▲ 0.083 ▲ 0.111 ▲
	0.117 0.078 0.117 ▲ 0.085 ▲ 0.040 ▲ 0.040 ▲ 0.085 ▲	[0.117] [0.078] [0.117] (0.127) (0.117) (0.127) 0.131 0.117 0.131 ▲ 0.109 ▲ 0.089 ▲ 0.089 ▲ 0.109 ▲
	0.115 0.086 0.086 0.115 ▲ 0.086 ▲ 0.037 ▲ 0.051 ▲ 0.037 ▲ 0.086 ▲	[0.115] [0.086] [0.086] [0.115] (0.126) (0.116) (0.116) (0.126) 0.131 0.121 0.121 0.131 ▲ 0.110 ▲ 0.087 ▲ 0.094 ▲ 0.087 ▲ 0.110 ▲
	0.145 ▲ 0.084 ▲ 0.084 ▲	0.145 ▲ 0.114 ▲ 0.114 ▲
	0.116 0.116 ▲ 0.095 ▲ 0.032 ▲ 0.095 ▲	[0.116] [0.116] 0.135 0.135 ▲ 0.120 ▲ 0.090 ▲ 0.120 ▲
	0.124 0.083 0.124 ▲ 0.092 ▲ 0.045 ▲ 0.045 ▲ 0.092 ▲	[0.124] [0.083] [0.124] (0.135) (0.124) (0.135) 0.140 0.124 0.140 ▲ 0.118 ▲ 0.096 ▲ 0.096 ▲ 0.118 ▲
	0.122 0.092 0.092 0.122 ▲ 0.093 ▲ 0.041 ▲ 0.056 ▲ 0.041 ▲ 0.093 ▲	[0.122] [0.092] [0.092] [0.122] (0.135) (0.123) (0.123) (0.135) 0.139 0.129 0.129 0.139 ▲ 0.119 ▲ 0.095 ▲ 0.102 ▲ 0.095 ▲ 0.119 ▲
	0.151 ▲ 0.090 ▲ 0.090 ▲	0.151 ▲ 0.121 ▲ 0.121 ▲
	0.121 0.121 ▲ 0.102 ▲ 0.036 ▲ 0.102 ▲	[0.121] [0.121] 0.141 0.141 ▲ 0.128 ▲ 0.097 ▲ 0.128 ▲
	0.130 0.086 0.130 ▲ 0.098 ▲ 0.050 ▲ 0.050 ▲ 0.098 ▲	[0.130] [0.086] [0.130] (0.140) (0.130) (0.140) 0.146 0.130 0.146 ▲ 0.126 ▲ 0.103 ▲ 0.103 ▲ 0.126 ▲
	0.127 0.096 0.096 0.127 ▲ 0.099 ▲ 0.046 ▲ 0.062 ▲ 0.046 ▲ 0.099 ▲	[0.127] [0.096] [0.096] [0.127] (0.140) (0.129) (0.129) (0.140) 0.145 0.135 0.135 0.145 ▲ 0.127 ▲ 0.102 ▲ 0.109 ▲ 0.102 ▲ 0.127 ▲

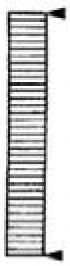
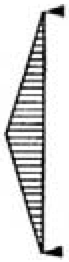
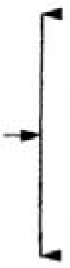
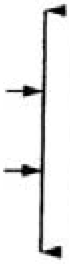
Continuous beams: moments from equal loads on equal spans – 2

Load	All spans loaded (e.g. dead load)	Live load (sequence of loaded spans to give max. bending moment)
	0.155 ▲ 0.094 ▲ 0.094 ▲	0.155 ▲ 0.127 ▲ 0.127 ▲
	0.124 0.124 ▲ 0.107 ▲ 0.040 ▲ 0.107 ▲	[0.124] [0.124] 0.145 0.145 ▲ 0.134 ▲ 0.102 ▲ 0.134 ▲
	0.133 0.089 0.133 ▲ 0.103 ▲ 0.054 ▲ 0.054 ▲ 0.103 ▲	[0.133] [0.089] [0.133] (0.144) (0.133) (0.144) 0.149 0.133 0.149 ▲ 0.132 ▲ 0.109 ▲ 0.109 ▲ 0.132 ▲
	0.131 0.098 0.098 0.131 ▲ 0.104 ▲ 0.050 ▲ 0.066 ▲ 0.050 ▲ 0.104 ▲	[0.131] [0.098] [0.098] [0.131] (0.144) (0.132) (0.132) (0.144) 0.149 0.138 0.138 0.149 ▲ 0.133 ▲ 0.107 ▲ 0.115 ▲ 0.107 ▲ 0.133 ▲
	0.156 ▲ 0.095 ▲ 0.095 ▲	0.156 ▲ 0.129 ▲ 0.129 ▲
	0.125 0.125 ▲ 0.108 ▲ 0.042 ▲ 0.108 ▲	[0.125] [0.125] 0.146 0.146 ▲ 0.136 ▲ 0.104 ▲ 0.136 ▲
	0.134 0.089 0.134 ▲ 0.104 ▲ 0.056 ▲ 0.056 ▲ 0.104 ▲	[0.134] [0.089] [0.134] (0.145) (0.134) (0.145) 0.151 0.134 0.151 ▲ 0.134 ▲ 0.111 ▲ 0.111 ▲ 0.134 ▲
	0.132 0.099 0.099 0.132 ▲ 0.105 ▲ 0.051 ▲ 0.068 ▲ 0.051 ▲ 0.105 ▲	[0.132] [0.099] [0.099] [0.132] (0.145) (0.133) (0.133) (0.145) 0.150 0.139 0.139 0.150 ▲ 0.135 ▲ 0.109 ▲ 0.117 ▲ 0.109 ▲ 0.135 ▲
	0.188 ▲ 0.156 ▲ 0.156 ▲	0.188 ▲ 0.203 ▲ 0.203 ▲
	0.150 0.150 ▲ 0.175 ▲ 0.100 ▲ 0.175 ▲	[0.150] [0.150] 0.175 0.175 ▲ 0.213 ▲ 0.175 ▲ 0.213 ▲
	0.161 0.107 0.161 ▲ 0.170 ▲ 0.116 ▲ 0.116 ▲ 0.170 ▲	[0.161] [0.107] [0.161] (0.174) (0.161) (0.174) 0.181 0.161 0.181 ▲ 0.210 ▲ 0.183 ▲ 0.183 ▲ 0.210 ▲
	0.158 0.118 0.118 0.158 ▲ 0.171 ▲ 0.112 ▲ 0.132 ▲ 0.112 ▲ 0.171 ▲	[0.158] [0.118] [0.118] [0.158] (0.174) (0.160) (0.160) (0.174) 0.179 0.167 0.167 0.179 ▲ 0.211 ▲ 0.181 ▲ 0.191 ▲ 0.181 ▲ 0.211 ▲
	0.167 ▲ 0.111 ▲ 0.111 ▲	0.167 ▲ 0.139 ▲ 0.139 ▲
	0.133 0.133 ▲ 0.122 ▲ 0.033 ▲ 0.122 ▲	[0.133] [0.133] 0.156 0.156 ▲ 0.144 ▲ .0.100 ▲ 0.144 ▲
	0.143 0.095 0.143 ▲ 0.119 ▲ 0.056 ▲ 0.056 ▲ 0.119 ▲	[0.143] [0.095] [0.143] (0.155) (0.143) (0.155) 0.160 0.144 0.160 ▲ 0.143 ▲ 0.111 ▲ 0.111 ▲ 0.143 ▲
	0.140 0.105 0.105 0.140 ▲ 0.120 ▲ 0.050 ▲ 0.061 ▲ 0.050 ▲ 0.120 ▲	[0.140] [0.105] [0.105] [0.140] (0.155) (0.142) (0.142) (0.155) 0.159 0.148 0.148 0.159 ▲ 0.144 ▲ 0.108 ▲ 0.115 ▲ 0.108 ▲ 0.144 ▲

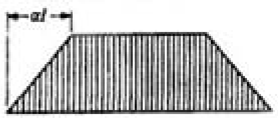
Bending moment = (coefficient) × (total load on one span) × (span)
Bending moment coefficients:
 above line apply to negative bending moment at supports
 below line apply to positive bending moment in span
Coefficients apply when all spans are equal (may be used also when shortest ≥ 85% longest). Loads on each loaded span are same.

Second moment of area is same throughout all spans.
Bending moment coefficients in square brackets (live load) apply if all spans are loaded (i.e. BS 8110 requirements).
Bending moment coefficients in curved brackets (live load) apply if two adjacent spans are loaded (i.e. EC 2 requirements).

Continuous beams: shears from equal loads on equal spans

Load	All spans loaded (e.g. dead load)	Live load (sequence of loaded spans to give max. shearing force)
 Uniformly distributed	0.375 0.625 0.625 0.375	0.438 0.625 0.625 0.438
	0.400 0.500 0.600 0.600 0.500 0.400	0.450 0.583 0.617 0.617 0.583 0.450
	0.393 0.536 0.464 0.607 0.607 0.464 0.536 0.393	0.446 0.603 0.571 0.621 0.621 0.571 0.603 0.446
	0.395 0.526 0.500 0.474 0.605 0.605 0.474 0.500 0.526 0.395	0.447 0.598 0.591 0.576 0.620 0.620 0.576 0.591 0.598 0.447
 Triangularly distributed	0.344 0.656 0.656 0.344	0.422 0.656 0.656 0.422
	0.375 0.500 0.625 0.625 0.500 0.375	0.437 0.605 0.646 0.646 0.605 0.437
	0.366 0.545 0.455 0.634 0.634 0.455 0.545 0.366	0.433 0.628 0.589 0.651 0.651 0.589 0.628 0.433
	0.369 0.532 0.500 0.468 0.631 0.631 0.468 0.500 0.532 0.369	0.434 0.622 0.614 0.595 0.649 0.649 0.595 0.614 0.622 0.434
 Concentrated at midspan	0.313 0.688 0.688 0.313	0.406 0.688 0.688 0.406
	0.350 0.500 0.650 0.650 0.500 0.350	0.425 0.625 0.675 0.675 0.625 0.425
	0.339 0.554 0.446 0.661 0.661 0.446 0.554 0.339	0.420 0.654 0.607 0.681 0.681 0.607 0.654 0.420
	0.342 0.540 0.500 0.460 0.658 0.658 0.460 0.500 0.540 0.342	0.421 0.647 0.636 0.615 0.679 0.679 0.615 0.636 0.647 0.421
 Concentrated at third-points	0.333 0.667 0.667 0.333	0.417 0.667 0.667 0.417
	0.367 0.500 0.633 0.633 0.500 0.367	0.433 0.611 0.656 0.656 0.611 0.433
	0.357 0.548 0.452 0.643 0.643 0.452 0.548 0.357	0.429 0.637 0.595 0.661 0.661 0.595 0.637 0.429
	0.360 0.535 0.500 0.465 0.640 0.640 0.465 0.500 0.535 0.360	0.430 0.631 0.621 0.602 0.659 0.659 0.602 0.621 0.631 0.480

For any trapezoidal load.



SF coefficient = $(k - \frac{1}{2})(1 + \alpha - \alpha^2) + \frac{1}{2}$ where k is SF coefficient for uniform load, read from above table.

Eg. If $\alpha = 0.5$, coefficient at central support of two-span beam is equal to $(0.625 - 0.5)(1 + 0.5 - 0.25) + 0.5 = 0.656$.

The design load consists of a dead load of $1.0g_k = 20$ kN/m and a live load of $(0.4g_k + 1.6q_k) = 40$ kN/m. Then, from *Table 2.30* (using coefficients in square brackets for the live load), the ultimate bending moments are as follows:

Penultimate support:
Dead load: $0.105 \times 20 \times 5^2 = 52.5$ kNm (negative)
Imposed load: $0.105 \times 40 \times 5^2 = \underline{105.0\text{ kNm}}$ (negative)
Total $= \underline{157.5\text{ kNm}}$ (negative)

Interior support:
Dead load: $0.079 \times 20 \times 5^2 = 39.5$ kNm (negative)
Imposed load: $0.079 \times 40 \times 5^2 = \underline{79.0\text{ kNm}}$ (negative)
Total $= \underline{118.5\text{ kNm}}$ (negative)

Near middle of end span:
Dead load: $0.078 \times 20 \times 5^2 = 39.0$ kNm (positive)
Imposed load: $0.100 \times 40 \times 5^2 = \underline{100.0\text{ kNm}}$ (positive)
Total $= \underline{139.0\text{ kNm}}$ (positive)

Middle of central span:
Dead load: $0.046 \times 20 \times 5^2 = 23.0$ kNm (positive)
Imposed load: $0.086 \times 40 \times 5^2 = \underline{86.0\text{ kNm}}$ (positive)
Total $= \underline{109.0\text{ kNm}}$ (positive)

Example 2. Calculate the maximum ultimate moments for example 1, according to the requirements of EC 2.

The design load consists of a dead load of $1.35g_k = 27$ kN/m and a live load of $1.5q_k = 30$ kN/m. Then, from *Table 2.30* (using coefficients in curved brackets for the live load), the ultimate bending moments are as follows:

Penultimate support:
Dead load: $0.105 \times 27 \times 5^2 = 70.9$ kNm (negative)
Imposed load: $0.116 \times 30 \times 5^2 = \underline{87.0\text{ kNm}}$ (negative)
Total $= \underline{157.9\text{ kNm}}$ (negative)

Interior support:
Dead load: $0.079 \times 27 \times 5^2 = 53.3$ kNm (negative)
Imposed load: $0.106 \times 30 \times 5^2 = \underline{79.5\text{ kNm}}$ (negative)
Total $= \underline{132.8\text{ kNm}}$ (negative)

Near middle of end span:
Dead load: $0.078 \times 27 \times 5^2 = 52.7$ kNm (positive)
Imposed load: $0.100 \times 30 \times 5^2 = \underline{75.0\text{ kNm}}$ (positive)
Total $= \underline{127.7\text{ kNm}}$ (positive)

Middle of central span:
Dead load: $0.046 \times 27 \times 5^2 = 31.1$ kNm (positive)
Imposed load: $0.086 \times 30 \times 5^2 = \underline{64.5\text{ kNm}}$ (positive)
Total $= \underline{95.6\text{ kNm}}$ (positive)

Example 3. Calculate the maximum ultimate moments for example 1, according to the requirements of BS 8110, when a 2 m long cantilever is provided at each end of the beam.
Increase in moments due to dead and live loads on cantilevers at critical positions is as follows:

End support
Dead load: $0.5 \times 20 \times 2^2 = 40.0$ kNm (negative)
Imposed load: $0.5 \times 40 \times 2^2 = \underline{80.0\text{ kNm}}$ (negative)
Total $= \underline{120.0\text{ kNm}}$ (negative)

Interior support
From *Table 2.29*, increase due to moments at end supports is $0.053 \times 120 = 6.4$ kNm (negative).

Decrease in moments due to dead load only on cantilevers at critical positions is as follows:

Penultimate support
From *Table 2.29*, decrease due to moments at end supports is $0.263 \times 40 = 10.5$ kNm (positive).

Middle of end span
From *Table 2.29*, decrease due to moments at end supports is $[1.0 - 0.5(1.0 + 0.263)] \times 40 = 14.7$ (negative).

Middle of central span
From *Table 2.29*, decrease due to moments at end supports is $0.053 \times 40 = 2.1$ kNm (negative).

12.3 REDISTRIBUTION OF MOMENTS

As explained in section 4.2.2, for the ULS, both BS 8110 and EC 2 permit the moments determined by a linear elastic analysis to be redistributed, provided that the resulting distribution remains in equilibrium with the loads. Although the conditions affecting the procedure are slightly different in the two codes, the general approach is to reduce the critical moments by a chosen amount, up to the maximum percentage permitted, and determine the revised moments at other positions by equilibrium considerations.

An important point to appreciate is that each particular load combination can be considered separately. Thus, if desired, it is possible to reduce the maximum moments in the spans, and at the supports. For example, the maximum support moments can be reduced to values that are still greater than those that occur with the maximum span moments. The maximum span moments can then be reduced until the corresponding support moments are the same as the (reduced) maximum values.

The principles of static equilibrium require that no changes should be made to the moments in a cantilever or at a freely supported end.

12.3.1 Code requirements

BS 8110 and EC 2 permit the maximum moments to be reduced by up to 30% provided that, in the subsequent design of the relevant sections, the depth of the neutral axis is limited according to the amount of redistribution. (Note that there is no restriction on the maximum percentage increase of moment.) In EC 2, the maximum permitted reduction depends also on the ductility of the reinforcement, being 30% for reinforcement classes B and C but only 20% for class A.

In BS 8110, it is stated that the ultimate resistance moment at a section should be at least 70% of the maximum moment at that section before redistribution. In effect, the process of redistribution alters the positions of points of contra-flexure. The purpose of the code requirement is to ensure that at such points on the diagram of redistributed moments (at which no flexural reinforcement is theoretically required), sufficient reinforcement is provided to cater for the moments that will occur under service loading. The redistribution procedure takes advantage of the ability of continuous beams to develop plastic hinges at critical sections prior to failure, whilst also ensuring that the response remains fully elastic under service loading. The requirements are discussed more fully in books on structural design and in the *Handbook to BS 8110*.

12.3.2 Redistribution procedure

The use of moment redistribution is illustrated in *Table 2.33*, where a beam of three equal spans is examined in accordance with the requirements of BS 8110. The uniformly distributed dead and live loads are each equal to 1200 units per span. The moment diagram for dead load on each span is shown in (a). Moment diagrams for the arrangements of live load that give the maximum moments at the supports and in the spans are shown in (b). The moment envelope obtained by combining the diagrams for dead and live loads is shown in (c). (Note that the vertical scale of diagrams (c)–(f) differs from that of (a) and (b).)

The redistribution procedure is normally used to reduce the maximum support moments. One approach is to reduce these to the values obtained when the span moments are greatest. This is shown in (d), where the support moments have been reduced from −240 to −180, a reduction of 25%. In this case, no other adjustment is needed to the moment envelope. If the maximum support moments are reduced by 30%, from −240 to −168, the span moments must be increased as shown in (e). The 70% requirement, discussed in section 11.3.2, determines the extent of the hogging region in the end span and the minimum value of −21 in the middle span. (For the load cases in EC2, the maximum support moments could be reduced by 30%, from −260 to −182, with no other adjustment needed to the moment envelope.)

If the criterion is to reduce the maximum span moments, in the case of an up-stand beam say, this may be achieved by increasing the corresponding support moments. This is shown in (f), where the maximum moment in the middle span has been reduced by 30%, from 120 to 84, by increasing the support moments from −180 to −216. (Note that the minimum moment in the middle span has increased from −30 to −66.) The moment in the end span has also been reduced by 7%, from 217 to 202. The 70% requirement determines the extent of the sagging regions in both spans. It is clear that any further reduction of moment in the end span would result in a considerable increase in the support moment. For example, a 30% reduction in the end span moment, from 217 to 152, would increase the support moment to −346 and the minimum moment in the middle span to −196.

In view of the many factors involved, it is difficult to give any general rules as to whether to redistribute moments or by how much; such decisions are basically matters of individual engineering judgement. A useful approach is to first calculate the ultimate resistance moments at the support sections, provided by chosen arrangements of reinforcement, and then redistribute the moment diagrams to suit. The span sections can then be designed for the resulting moments, and a check made to ensure that all of the code requirements are satisfied. Moment redistribution, in general, affects the shearing forces at the supports, and it is recommended that beam sections are designed for the greater of the shear forces calculated before and after redistribution.

The use of moment distribution in systems where the beams are analysed in conjunction with adjoining columns requires further consideration. In such cases it is important to ensure that, in any postulated collapse mechanism involving plastic hinges in the columns, these are the last hinges to form. To this end, it is recommended that column sections should be designed for the greater of the moments calculated before and after redistribution.

12.3.3 Bending moment diagrams

The moment diagrams and coefficients given in *Tables 2.34* and *2.35* cater for beams that are continuous over two, three, and four or more equal spans. They apply to cases where the second moment of area of the cross section is constant and the loads on each loaded span are the same. For convenience, coefficients derived by elastic analysis before and after given redistributions, in accordance with the rules of both BS 8110 and EC 2, are tabulated against the location points indicated in the diagrams. For example, M_{12} is the coefficient corresponding to the maximum moment at the central support of a two-span beam, while M_{13} is the coefficient that gives the moment at this support when the moment in the adjoining span is a maximum. Thus, by means of the coefficients given, the appropriate envelope of maximum moments is obtained.

Three load types are considered: UDL throughout each span, a central concentrated load and equal concentrated loads positioned at the third-points of the span. The span moments determined by summing the individual maximum values given separately for dead and live loads in the case of uniform loading, will be approximate but erring on the side of safety, since each maximum value occurs at a slightly different position. The tabulated coefficients may also be used to determine the support moments resulting from combinations of the given load types by summing the results for each type. The corresponding span moments can then be determined as described in section 12.1.3.

Moment coefficients are given for redistribution values of 10% and 30% respectively. For the dead load, all the support moments have been reduced by the full amount, and the span moments increased to suit the adjusted values at the supports. For the live loads, all the support moments and, for 10% redistribution, all the span moments have been reduced by the full amount. For 30% redistribution, each span moment has been reduced to the minimum value required for equilibrium with the new support moments. As a result, the BS 8110 span moment coefficients are the same as those for the dead load. Although there is no particular merit in limiting redistribution to 10%, some BS 8110 design formulae for determining the ultimate resistance moment are related to this condition.

For design purposes, redistribution at a particular section refers to the percentage change in the combined moment due to the dead and live loads. When using the tables, the value for the support moments will be either −10% or −30% but the values for the span moments will need to be calculated for each particular case. Consider, for example, a two-span beam supporting UDLs with $g_k = q_k$ and 30% redistribution, according to the requirements of BS 8110.

The design load consists of a dead load of $1.0g_k$ and a live load of $(0.4g_k + 1.6q_k) = 2.0g_k$. Then, from *Table 2.34*, the ultimate bending moments are as follows:

Before redistribution:

$$M_{11} = (0.070g_k + 0.096 \times 2g_k)l^2 = 0.262g_k l^2$$

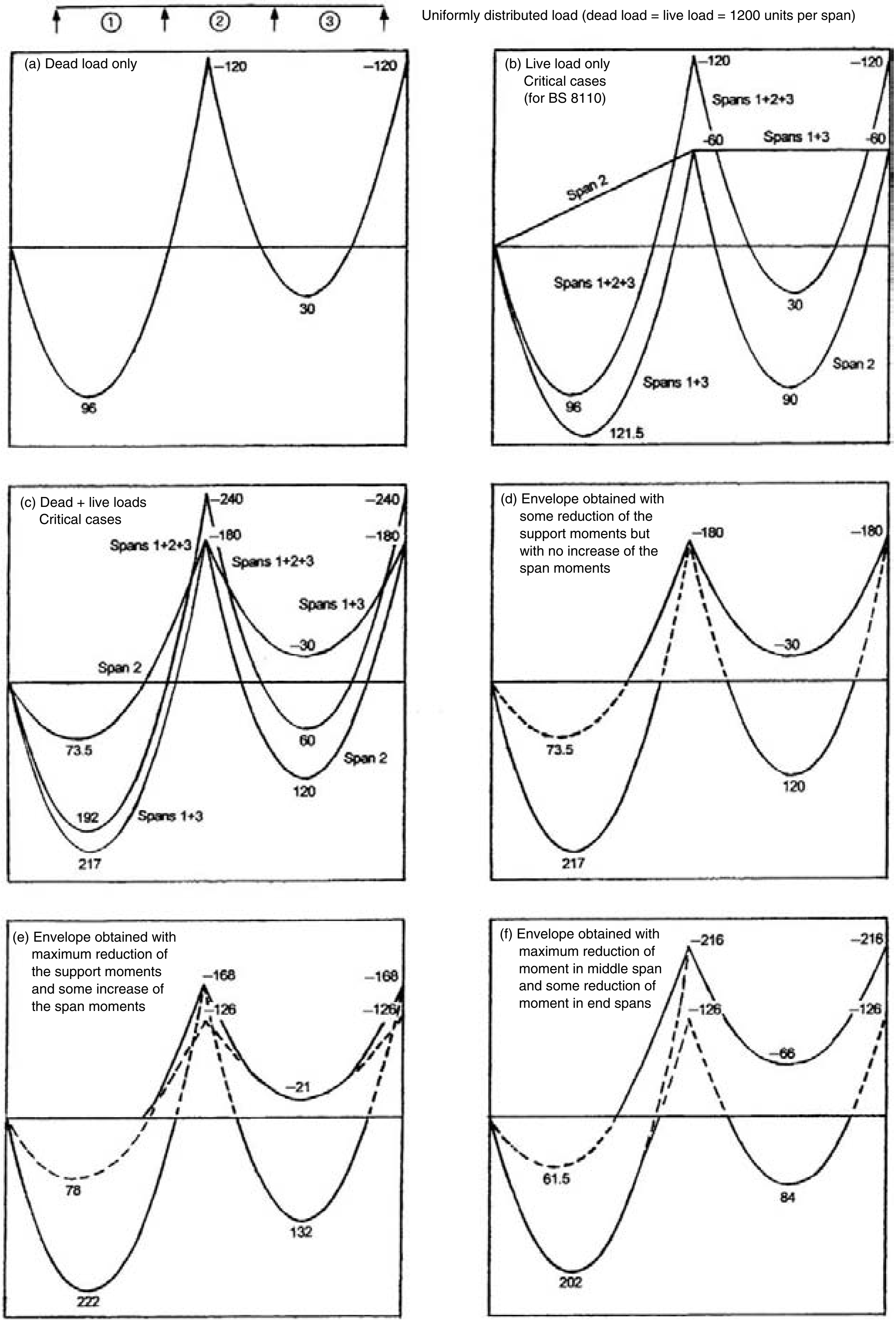
After redistribution:

$$M_{11} = (0.085g_k + 0.085 \times 2g_k)l^2 = 0.255g_k\, l^2$$

$$\%\ \text{Redistribution} = -100 \times (0.262 - 0.255)/0.262 = -3\%.$$

Thus, a full 30% reduction of the maximum support moments is obtained with no increase in the maximum span moment.

Continuous beams: moment redistribution



Uniformly distributed load (dead load = live load = 1200 units per span)

(a) Dead load only

(b) Live load only
Critical cases
(for BS 8110)

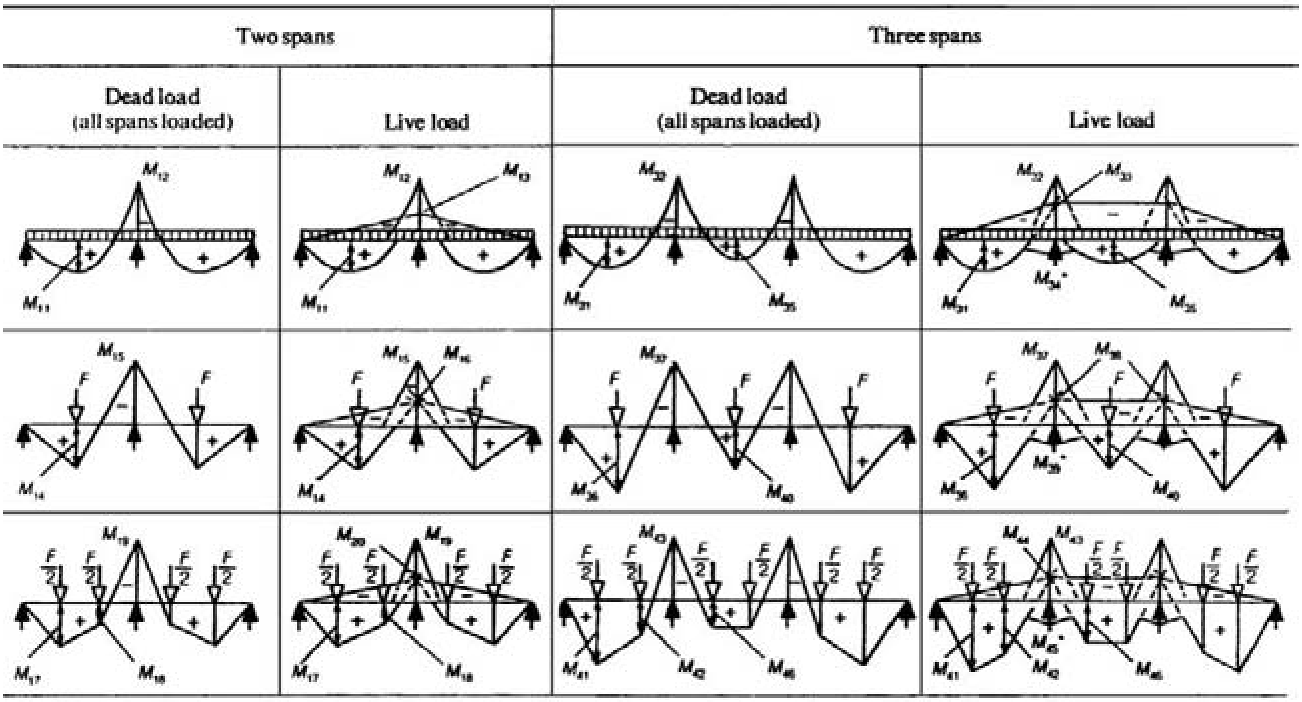
(c) Dead + live loads
Critical cases

(d) Envelope obtained with some reduction of the support moments but with no increase of the span moments

(e) Envelope obtained with maximum reduction of the support moments and some increase of the span moments

(f) Envelope obtained with maximum reduction of moment in middle span and some reduction of moment in end spans

Continuous beams: bending moment diagrams – 1



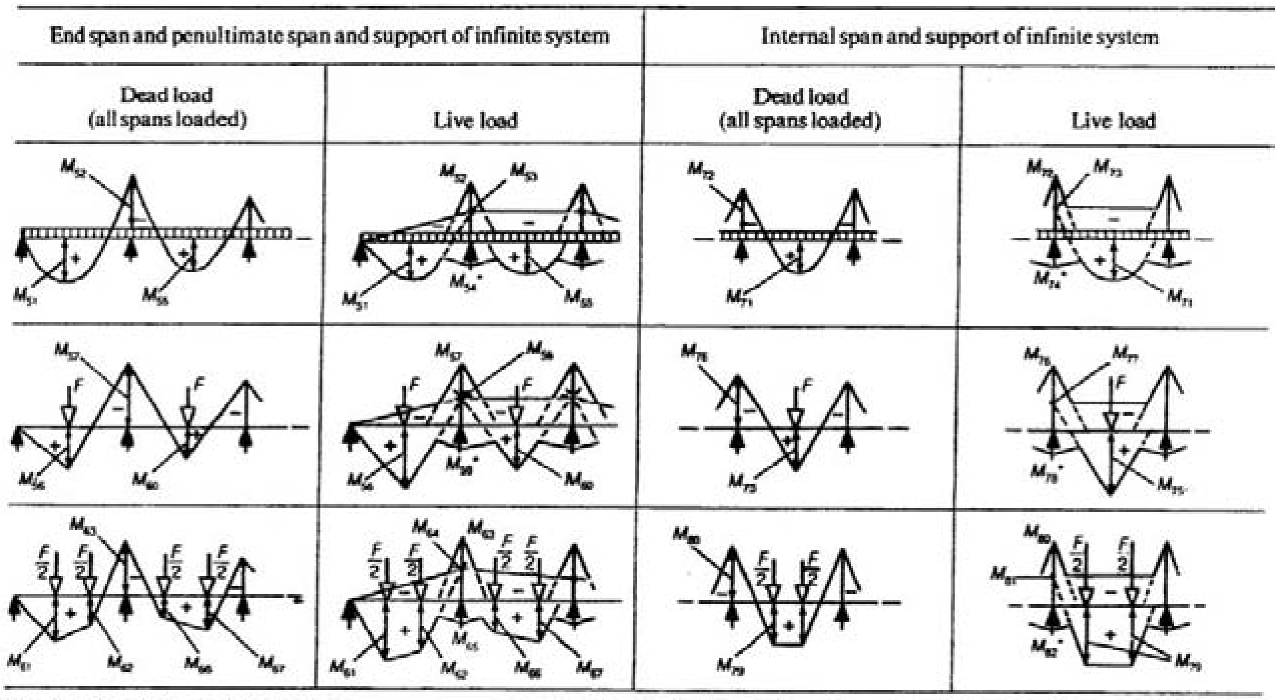
Equal total load *F* on each loaded span
Bending moment = coefficient × *F* × span
Diagrams are symmetrical but are not drawn to scale

Moments indicated thus • do not result from loading arrangement prescribed in Codes, which give zero positive moment at all supports.

Values below indicated thus$^{(11)}$ give maximum percentage reduction of span moment due to live load possible when support moments have been reduced by full 30%.

			Dead load (all spans loaded)			Live load			Live load		
			BS8110 and EC2			BS8110			EC2		
Redistribution			nil	10%	30%	nil	10%	30%	nil	10%	30%
Two spans	Uniform loads	M_{11}	+0.070	+0.075	+0.085	+0.096	+0.086	+0.085$^{(11)}$	+0.096	+0.086	+0.085$^{(11)}$
		M_{12}	−0.125	−0.113	−0.088	−0.125	−0.113	−0.088	−0.125	−0.113	−0.088
		M_{13}	—	—	—	−0.063	−0.085	−0.088	−0.063	−0.085	−0.088
	Central point loads	M_{14}	+0.156	+0.166	+0.184	+0.203	+0.183	+0.184$^{(9)}$	+0.203	+0.183	+0.184$^{(9)}$
		M_{15}	−0.188	−0.169	−0.131	−0.188	−0.169	−0.131	−0.188	−0.169	−0.131
		M_{16}	—	—	—	−0.094	−0.134	−0.131	−0.094	−0.134	−0.131
	Third-point loads	M_{17}	+0.111	+0.117	+0.128	+0.139	+0.125	+0.128$^{(8)}$	+0.139	+0.125	+0.128$^{(8)}$
		M_{18}	+0.056	+0.067	+0.089	+0.111	+0.100	+0.089$^{(20)}$	+0.111	+0.100	+0.089$^{(20)}$
		M_{19}	−0.167	−0.150	−0.117	−0.167	−0.150	−0.117	−0.167	−0.150	−0.117
		M_{20}	—	—	—	−0.083	−0.125	−0.117	−0.083	−0.125	−0.117
Three spans	Uniform loads	M_{31}	+0.080	+0.084	+0.093	+0.101	+0.091	+0.093$^{(13)}$	+0.101	+0.091	+0.087$^{(13)}$
		M_{32}	−0.100	−0.090	−0.070	−0.100	−0.090	−0.070	−0.117	−0.106	−0.082
		M_{33}	—	—	—	−0.050	−0.074	−0.070	−0.050	−0.074	−0.082
		M_{34}*	—	—	—	0.000	0.000	0.000	+0.017	+0.015	+0.012
		M_{35}	+0.025	+0.035	+0.055	+0.075	+0.068	+0.055$^{(27)}$	+0.075	+0.068	+0.053$^{(30)}$
	Central point loads	M_{36}	+0.175	+0.183	+0.198	+0.213	+0.191	+0.198$^{(7)}$	+0.213	+0.191	+0.189$^{(11)}$
		M_{37}	−0.150	−0.135	−0.105	−0.150	−0.135	−0.105	−0.175	−0.158	−0.123
		M_{38}	—	—	—	−0.075	−0.118	−0.105	−0.075	−0.118	−0.123
		M_{39}*	—	—	—	0.000	0.000	0.000	+0.025	+0.023	+0.018
		M_{40}	+0.100	+0.115	+0.145	+0.175	+0.158	+0.145$^{(17)}$	+0.175	+0.158	+0.128$^{(27)}$
	Third-point loads	M_{41}	+0.122	+0.127	+0.136	+0.144	+0.130	+0.136$^{(6)}$	+0.144	+0.130	+0.130$^{(10)}$
		M_{42}	+0.078	+0.087	+0.105	+0.122	+0.110	+0.105$^{(14)}$	+0.122	+0.110	+0.094$^{(23)}$
		M_{43}	−0.133	−0.120	−0.093	−0.133	−0.120	−0.093	−0.156	−0.140	−0.109
		M_{44}	—	—	—	−0.067	−0.110	−0.093	−0.067	−0.110	−0.109
		M_{45}*	—	—	—	+0.000	0.000	0.000	+0.022	+0.020	+0.016
		M_{46}	+0.033	+0.047	+0.074	+0.100	+0.090	+0.074$^{(26)}$	+0.100	+0.090	+0.070$^{(30)}$

Continuous beams: bending moment diagrams – 2



Equal total load F on each loaded span
Bending moment = coefficient × F × span
Diagrams are symmetrical but are not drawn to scale

Moment indicated thus * do not result from loading arrangement prescribed in Codes, which give zero positive moment at all supports.

Values below indicated thus$^{(12)}$ give maximum percentage reduction of span moment due to live load possible when support moments have been reduced by full 30%.

			Dead load (all spans loaded)			Live load			Live load		
			BS8110 and EC2			BS8110			EC2		
Redistribution			nil	10%	30%	nil	10%	30%	nil	10%	30%
End span, and penultimate support and span of infinite system	Uniform loads	M_{51}	+0.078	+0.082	+0.091	+0.100	+0.090	$+0.091^{(9)}$	+0.100	+0.090	$+0.088^{(12)}$
		M_{52}	−0.106	−0.095	−0.074	−0.106	−0.095	−0.074	−0.116	−0.104	−0.081
		M_{53}	—	—	—	−0.053	−0.076	−0.074	−0.054	−0.076	−0.081
		M_{54}*	—	—	—	0.000	0.000	0.000	+0.014	+0.013	+0.010
		M_{55}	+0.034	+0.043	+0.061	+0.079	+0.071	$+0.061^{(23)}$	+0.079	+0.071	$+0.056^{(30)}$
	Central point loads	M_{56}	+0.171	+0.178	+0.195	+0.210	+0.189	+0.195(7)	+0.210	+0.189	$+0.189^{(10)}$
		M_{57}	−0.159	−0.143	−0.111	−0.159	−0.143	−0.111	−0.174	−0.157	−0.122
		M_{58}	—	—	—	−0.079	−0.122	−0.111	−0.079	−0.122	−0.122
		M_{59}*	—	—	—	0.000	0.000	+0.000	+0.021	+0.019	+0.015
		M_{60}	+0.113	+0.127	+0.154	+0.181	+0.163	$+0.154^{(15)}$	+0.181	+0.163	$+0.145^{(30)}$
	Third-point loads	M_{61}	+0.120	+0.124	+0.134	+0.143	+0.129	$+0.134^{(7)}$	+0.143	+0.129	$+0.130^{(9)}$
		M_{62}	+0.072	+0.082	+0.101	+0.119	+0.107	$+0.101^{(6)}$	+0.119	+0.107	$+0.094^{(21)}$
		M_{63}	−0.141	−0.127	−0.099	−0.141	−0.127	−0.099	−0.155	−0.140	−0.109
		M_{64}	—	—	—	−0.071	−0.114	−0.099	−0.072	−0.114	−0.109
		M_{65}*	—	—	—	0.000	0.000	0.000	+0.019	+0.017	+0.013
		M_{66}	+0.038	+0.051	+0.077	+0.103	+0.092	$+0.077^{(25)}$	+0.103	+0.092	$+0.072^{(30)}$
		M_{67}	+0.051	+0.062	+0.086	+0.109	+0.098	$+0.086^{(21)}$	+0.109	+0.098	$+0.076^{(30)}$
Internal span and support of infinite system	Uniform loads	M_{71}	+0.042	+0.050	+0.067	+0.083	+0.075	$+0.067^{(20)}$	+0.083	+0.075	$+0.058^{(30)}$
		M_{72}	−0.083	−0.075	−0.058	−0.083	−0.075	−0.058	−0.106	−0.095	−0.074
		M_{73}	—	—	—	−0.042	−0.050	−0.067	−0.042	−0.050	−0.067
		M_{74}*	—	—	—	0.000	0.000	0.000	+0.028	+0.025	+0.020
	Central point loads	M_{75}	+0.125	+0.138	+0.163	+0.188	+0.169	$+0.163^{(13)}$	+0.188	+0.169	$+0.139^{(26)}$
		M_{76}	−0.125	−0.113	−0.088	−0.125	−0.113	−0.088	−0.159	−0.143	−0.111
		M_{77}	—	—	—	−0.063	−0.081	−0.088	−0.063	−0.081	−0.111
		M_{78}*	—	—	—	0.000	0.000	0.000	+0.043	+0.038	+0.030
	Third-point loads	M_{79}	+0.055	+0.067	+0.089	+0.111	+0.100	$+0.089^{(20)}$	+0.111	+0.100	$+0.078^{(30)}$
		M_{80}	−0.111	−0.100	−0.078	−0.111	−0.100	−0.078	−0.141	−0.127	−0.099
		M_{81}	—	—	—	−0.055	−0.067	−0.078	−0.055	−0.067	−0.089
		M_{82}*	—	—	—	0.000	0.000	0.000	+0.038	+0.034	+0.027

12.4 ANALYSIS BY MOMENT DISTRIBUTION

The Hardy Cross moment distribution method of analysis, in which support moments are derived by a step-by-step process of successive approximations, is described briefly and shown by means of a worked example in *Table 2.36*. The method is able to accommodate span-to-span variations in span length, member size and loading arrangement. The 'precise moment distribution' method avoids the iterative procedure by using more complicated distribution and carry-over factors. Span moments can be determined as described in section 12.1.3.

For continuous beams of two, three or four spans, uniform cross-section and symmetrical loading, the support moments may also be obtained by using the factors in *Table 2.37*.

12.5 INFLUENCE LINES FOR CONTINUOUS BEAMS

The following procedure can be used to determine bending moments at chosen sections in a system of continuous beams, due to a train of loads in any given position.

1. Draw the beam system to a convenient scale.
2. With the ordinates tabulated in the appropriate *Table 2.38*, *2.39*, *2.40* or *2.41*, construct the influence line (for unit load) for the section being considered, selecting a convenient scale for the bending moment.
3. Plot on the influence line diagram the train of loads in what is considered to be the most adverse position.
4. Tabulate the value of (ordinate × load) for each load.
5. Add algebraically the values of (ordinate × load) to obtain the resultant bending moment at the section considered.
6. Repeat for other positions of the load train to ensure that the most adverse position has been considered.

The following example shows the direct use of the tabulated influence lines, for calculating the moments on a beam that is continuous over four spans with concentrated loads applied at specified positions.

Example. Determine the bending moments at the penultimate left-hand support of a system of four spans, having a constant cross section and freely supported at the ends, when loads of 100 kN are applied at the mid-points of the first and third spans from the left-hand end. The end spans are 8 m long and the interior spans are 12 m long.

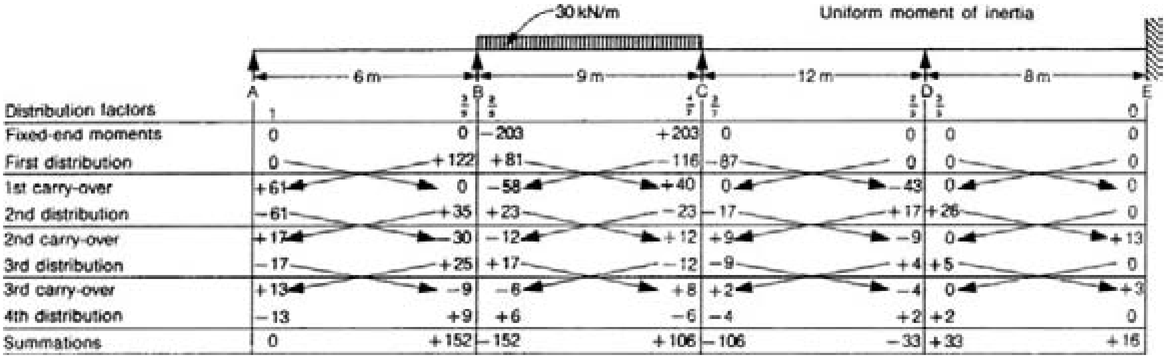
The span ratio is 1:1.5:1.5:1 and the ordinates are obtained from *Table 2.40* for penultimate support C.

With load on first span (ordinate c):
Bending moment $= -(0.082 \times 100 \times 8) = -65.6$ kNm
With load on third span (ordinate m):
Bending moment $= +(0.035 \times 100 \times 8) = \underline{+28.0}$ kNm
Net bending moment at penultimate support $= \underline{-37.6}$ kNm

HARDY CROSS MOMENT DISTRIBUTION

1. Consider each member to be fixed at ends: calculate *fixed-end moments* (*FEMs*) due to external loads on individual members by means of *Table 2.28*.
2. Where members meet, sum of bending moments must equal zero for equilibrium; i.e. at B, $M_{BA}+M_{BC} = 0$. Since ΣFEM (i.e. $FEM_{BA} + FEM_{BC}$) is unlikely to equal zero, a balancing moment of $-\Sigma FEM$ must be introduced at each support to achieve equilibrium.
3. Distribute this balancing moment between members meeting at a joint in proportion to their relative stiffnesses $K = I/l$ *by* multiplying $-\Sigma FEM$ by *distribution factor D* for each member (e.g. at B, $D_{BA} = K_{AB}/(K_{AB} + K_{BC})$ etc. so that $D_{BA} + D_{BC} = 1$. At a free end, $D = 1$; at a fully fixed end, D = 0).
4. Applying a moment at one end of member induces moment of one-half of magnitude and of same sign at opposite end of member (termed *carry-over*). Thus distributed moment $-\Sigma FEM \times D_{BA}$ at B of AB produces a moment of $-(1/2)\Sigma FEM \times D_{BA}$ at A, and so on.
5. These carried-over moments produce further unbalanced moments at supports (e.g. moments carried over from A and C give rise to further moments at B). These must again be redistributed and the carry-over process repeated.
6. Repeat cycle of operations described in steps 2–5 until unbalanced moments are negligible. Then sum values obtained each side of support.

Various simplifications can be employed to shorten analysis. The most useful is that for dealing with a system that is freely supported at the end. If stiffness considered for end span when calculating distribution factors is taken as only three-quarters of actual stiffness, and one-half of fixed-end moment at free support is added to *FEM* at other end of span, the span may then be treated as fixed and no further carrying over from free end back to penultimate support takes place.



	A	B		C		D		E
Distribution factors	1	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{4}{7}$	$\frac{3}{7}$	$\frac{2}{5}$	$\frac{3}{5}$	0
Fixed-end moments	0	0	−203	+203	0	0	0	0
First distribution	0	+122	+81	−116	−87	0	0	0
1st carry-over	+61	0	−58	+40	0	−43	0	0
2nd distribution	−61	+35	+23	−23	−17	+17	+26	0
2nd carry-over	+17	−30	−12	+12	+9	−9	0	+13
3rd distribution	−17	+25	+17	−12	−9	+4	+5	0
3rd carry-over	+13	−9	−6	+8	+2	−4	0	+3
4th distribution	−13	+9	+6	−6	−4	+2	+2	0
Summations	0	+152	−152	+106	−106	−33	+33	+16

PRECISE MOMENT DISTRIBUTION

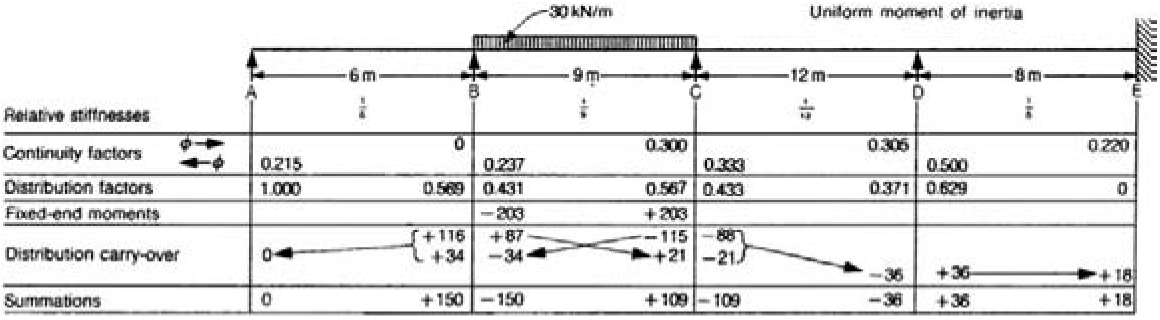
1. Calculate *fixed-end moments* (*FEMs*) as for Hardy Cross moment distribution.
2. Determine *continuity factors* for each span of system from general expression

$$\phi_{n+1} = 1\bigg/\left[2 + \frac{K_{n+1}}{K_n}(2-\phi_n)\right]$$

   where ϕ_n is continuity factor for previous span and K_n and K_{n+1} are stiffnesses of two spans. Work from left to right along system. If left-hand support (A in example below) is free, take $\phi_{AB} = 0$ for first span: if A is fully fixed, $\phi_{AB} = 0.5$. (Intermediate fixity conditions may be assumed if desired, by interpolation.) Repeat the foregoing procedure starting from right-hand end and working to left (to obtain continuity factor ϕ_{AB} for span AB, for example).
3. Calculate *distribution factors* (*DFs*) at junctions between spans from general expression

$$DF_{AB} = \frac{1-2\phi_{AB}}{1-\phi_{AB}\phi_{BA}}$$

   where ϕ_{AB} and ϕ_{BA} are continuity factors obtained in step 2. Note that these distribution factors do not correspond to those used in Hardy Cross moment distribution. Check that, at each support, $\Sigma DF = 1$.
4. Distribute the balancing moments $-\Sigma FEM$ introduced at each support to provide equilibrium for the unbalanced *FEMs* by multiplying by the distribution factors obtained in step 3.
5. Carry over the distributed balancing moments at the supports by multiplying them by the continuity factors obtained in step 2 *by working in opposite direction*. For example, the moment carried over from B to A is obtained by multiplying the distributed moment at B by ϕ_{AB} and so on. This procedure is illustrated in example below. Only a single carry-over operation in each direction is necessary.
6. Sum values obtained to determine final moments.



	A	B		C		D		E
Relative stiffnesses	$\frac{1}{6}$		$\frac{1}{9}$		$\frac{1}{12}$		$\frac{1}{8}$	
Continuity factors $\phi \rightarrow$		0		0.300		0.306		0.220
Continuity factors $\leftarrow \phi$	0.215		0.237		0.333		0.500	
Distribution factors	1.000	0.569	0.431	0.567	0.433	0.371	0.629	0
Fixed-end moments			−203	+203				
Distribution carry-over	0	+116 +34	+87 −34	−115 +21	−88 −21	−36	+36	+18
Summations	0	+150	−150	+109	−109	−36	+36	+18

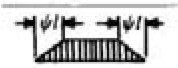
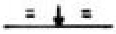
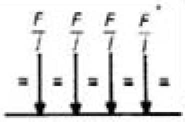
Continuous beams: unequal prismatic spans and loads 2.37

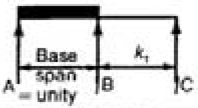
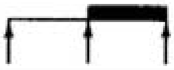
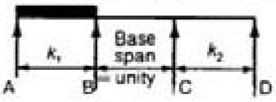
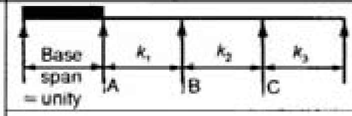
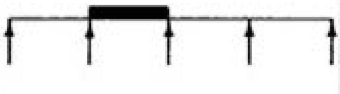
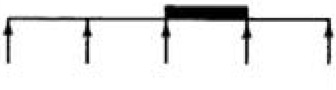
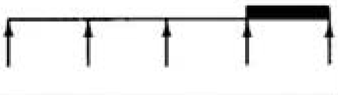
Divide given beam system into a number of similar systems each having one span loaded with a particular type of load. To find the bending moment at any support due to any one of these loads, evaluate the following factors for the particular support and type of load:

- F total load on span being considered
- α load factor (= unity for distributed load)
- Q support moment coefficient
- U moment multiplier (= unity for equal spans)

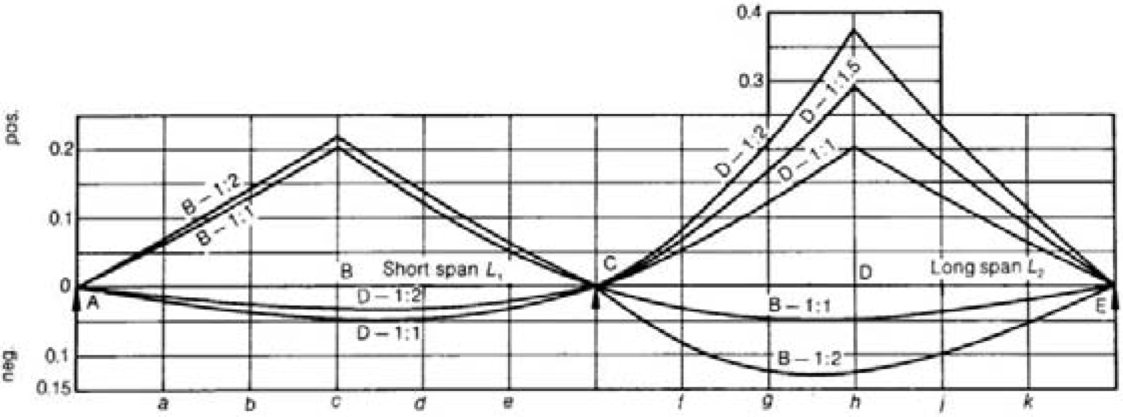
Moment at support $= \alpha QU \times F \times$ base span l

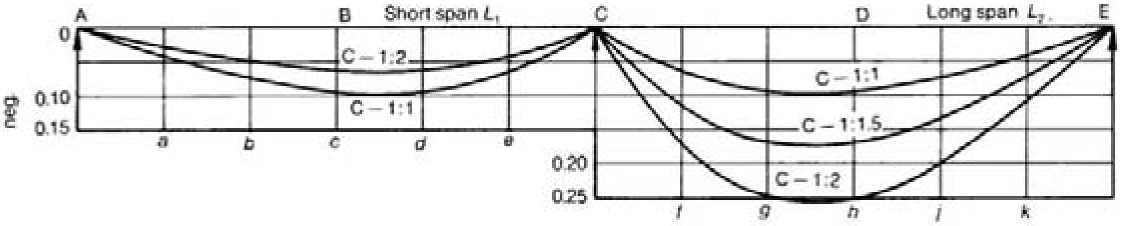
Ratios of remaining spans to base span $= k_1, k_2, k_3$

Type of load	Load factor = α	Max. free bending moment
	1.00	$0.125\,Fl$
	1.25	$0.167\,Fl$
	$1+\psi-\psi^2$	$\dfrac{3-4\psi^2}{24(1-\psi)}Fl$
	1.50	$0.250\,Fl$
	1.33	$0.167\,Fl$
 Any number of loads (j) equally spaced	$\dfrac{2+j}{1+j}$	Even number of loads: $\dfrac{1}{8}\left(\dfrac{2+j}{1+j}\right)Fl$ Odd number of loads: $\dfrac{1}{8}\left(\dfrac{1+j}{j}\right)Fl$

No. of spans	Loaded span	Equal spans Support moment coefficients			Unequal spans Moment multipliers = U	
		Q_A	Q_B	Q_C		
2		—	−0.0625	—	$U_B = \dfrac{2}{1+k_1}$	
		—	−0.0625	—	$U_B = \dfrac{2k_1^2}{1+k_1}$	
	Both spans loaded with identical load	—	−0.1250	—	See note below	
3		—	−0.0667	+0.0167	$U_B = 0.5yU_C$ $U_C = 3k_1^2H$	$x = k_1 + 1$ $y = k_2 + 1$ $H = \dfrac{5}{4xy - 1}$
		—	−0.0500	−0.0500	$U_B = H(y + k_2)$ $U_C = H(x + k_1)$	
		—	+0.0167	−0.0667	$U_B = 3k_2^2H$ $U_C = 0.5xU_B$	
	All spans loaded with identical load	—	−0.1000	−0.1000	For two, three, or four unequal spans loaded simultaneously, determine bending moment for each span loaded separately and add	
4		−0.0670	+0.0179	−0.0045	$U_A = (2/15x)(14 + k_1H_1z)$ $U_B = zH_1$ $U_C = 2k_2H_1$	$x = k_1 + 1$ $y = k_1 + k_2$ $z = k_2 + k_3$ $Y = \dfrac{1}{4xyz - k_1^2z - k_2^2x}$ $H_1 = 14k_1Y$ $H_2 = 14k_1^2Y(x+1)/3$ $H_3 = 14k_2^2Y(k_3+z)/3$ $H_4 = 14k_3^2Y$
		−0.0491	−0.0536	+0.0134	$U_A = (6/11x)k_1 \times [(14k_1/3) - U_B]$ $U_B = zH_2$ $U_C = 2k_2H_2$	
		+0.0134	−0.0536	−0.0491	$U_A = 2k_1H_3$ $U_B = xH_3$ $U_C = (6/11z)k_2 \times [(14k_2/3) - U_B]$	
		−0.0045	+0.0179	−0.0670	$U_A = 2k_1H_4k_2$ $U_B = xH_4k_2$ $U_C = 2H_4(4xy - k_1^2)/15$	
	All spans loaded with identical load	−0.1071	−0.0714	−0.1071	See note above	

Continuous beams: influence lines for two spans





Section	Ratio of spans $L_1:L_2$	Ordinates									
		Shorter span					Longer span				
		a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*j*	*k*
Shorter span midspan B	1:1	0.063	0.130	0.203	0.121	0.052	0.032	0.046	0.047	0.037	0.020
	1:1.5	0.067	0.137	0.213	0.130	0.058	0.058	0.083	0.084	0.067	0.037
	1:2	0.070	0.142	0.219	0.136	0.062	0.085	0.124	0.125	0.099	0.054
Central support C	1:1	0.041	0.074	0.094	0.093	0.064	0.064	0.093	0.094	0.074	0.041
	1:1.5	0.032	0.059	0.075	0.074	0.051	0.115	0.167	0.169	0.133	0.073
	1:2	0.027	0.049	0.063	0.062	0.042	0.170	0.247	0.250	0.198	0.108
Longer span midspan D	1:1	0.020	0.037	0.047	0.046	0.032	0.052	0.121	0.203	0.130	0.063
	1:1.5	0.016	0.030	0.038	0.037	0.025	0.067	0.167	0.291	0.183	0.088
	1:2	0.014	0.025	0.031	0.031	0.021	0.082	0.210	0.375	0.235	0.113

Unequal spans
Data enable influence lines to be drawn for the bending moments produced by a single unit load moving over two unequal spans.

Ordinates for intermediate ratios of spans can be interpolated.

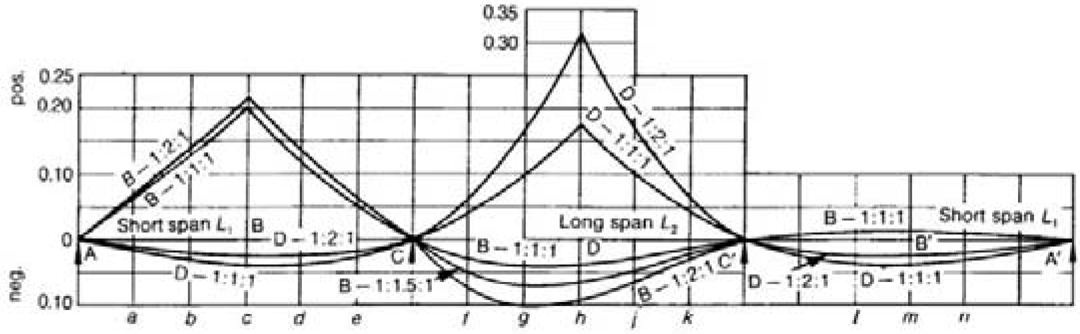
Equal spans
Influence lines marked 1:1 can be used directly (diagram of a succession of loads must be drawn to the same linear scale).

Bending moment due to load F concentrated at any point.
= (ordinate of appropriate influence line)
× (shorter span L_1) × F

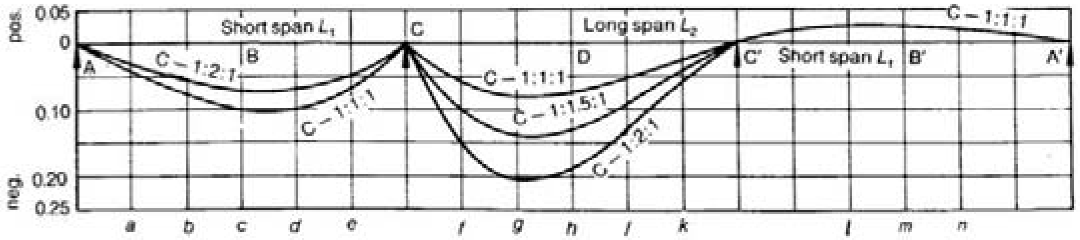
Continuous beams: influence lines for three spans

Three spans (equal or symmetrical inequality)

Influence lines for bending moments
at midspan B of end span AC
and midspan D of central span CC′



Influence lines for bending moments
at interior support C



Section	Ratio of spans $L_1:L_2:L_1$	Ordinates												
		End span AC					Central span CC′					End span C′A′		
		a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*j*	*k*	*l*	*m*	*n*
Midspan B of end span AC	1:1:1	0.062	0.127	0.200	0.117	0.050	0.029	0.040	0.038	0.027	0.013	0.012	0.013	0.010
	1:1.5:1	0.066	0.134	0.209	0.126	0.056	0.051	0.070	0.065	0.046	0.021	0.012	0.012	0.010
	1:2:1	0.068	0.139	0.215	0.132	0.060	0.075	0.102	0.094	0.065	0.029	0.012	0.012	0.009
Interior support C	1:1:1	0.043	0.079	0.100	0.099	0.068	0.057	0.079	0.075	0.054	0.026	0.025	0.025	0.020
	1:1.5:1	0.036	0.065	0.082	0.081	0.056	0.102	0.139	0.130	0.092	0.042	0.024	0.025	0.020
	1:2:1	0.030	0.056	0.070	0.069	0.048	0.151	0.204	0.188	0.129	0.058	0.023	0.023	0.019
Midspan D of central span CC′	1:1:1	0.016	0.030	0.038	0.037	0.025	0.042	0.100	0.175	0.100	0.042	0.037	0.038	0.030
	1:1.5:1	0.013	0.023	0.029	0.028	0.020	0.053	0.135	0.245	0.135	0.053	0.028	0.029	0.023
	1:2:1	0.010	0.019	0.023	0.023	0.016	0.063	0.167	0.313	0.167	0.063	0.023	0.023	0.019

Unequal spans
Data enable influence lines to be drawn for bending moments produced by a single unit load moving over three unequal spans.

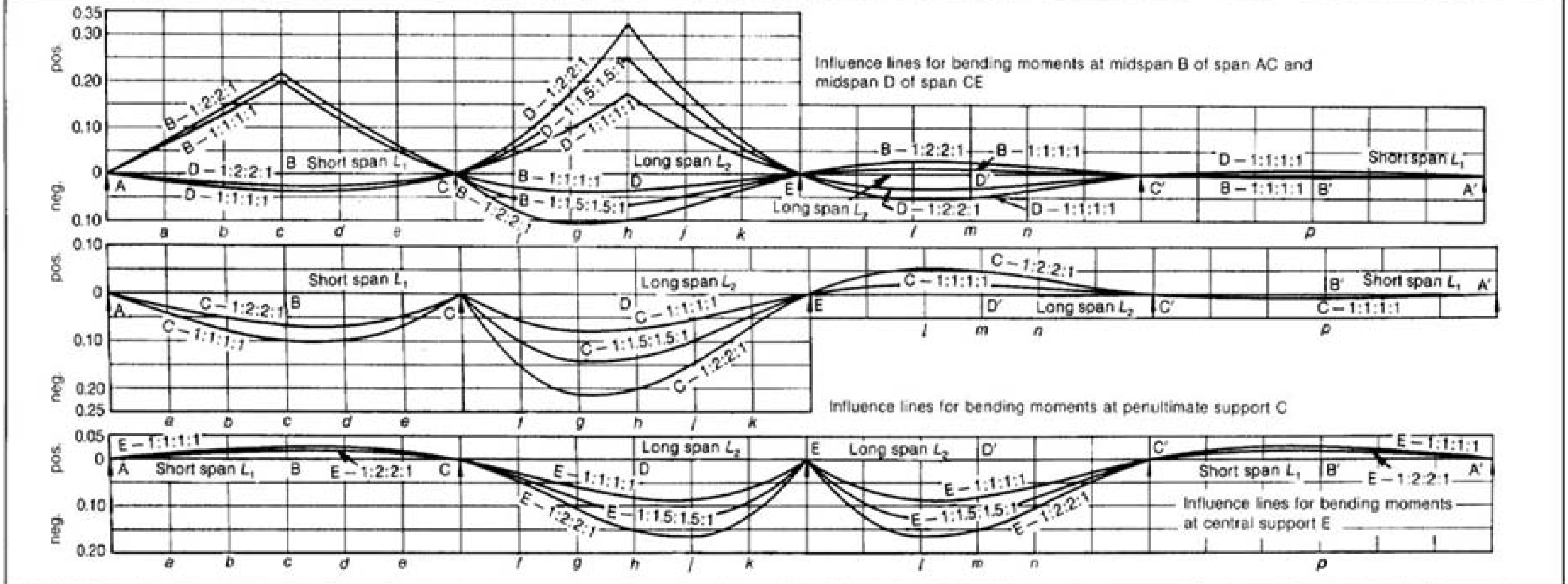
Ordinates for intermediate ratio of spans can be interpolated.

Equal spans
Influence lines marked 1:1:1 can be used directly (diagram of a succession of loads must be drawn to the same linear scale).

Bending moment due to load F concentrated at any poin
= (ordinate of appropriate influence line)
× (end span L_1) × F

Continuous beams: influence lines for four spans

Four spans (equal or symmetrical inequality)



	Section	Ratio of spans	End span AC					Interior span CE					Interior span EC′			End span C′A′
		$L_1:L_2:L_2:L_1$	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*j*	*k*	*l*	*m*	*n*	*p*
Ordinates	Midspan B of end span AC	1:1:1:1	0.062	0.127	0.200	0.117	0.049	0.028	0.039	0.037	0.027	0.013	0.011	0.010	0.007	0.003
		1:1.5:1.5:1	0.066	0.135	0.209	0.126	0.056	0.052	0.071	0.067	0.048	0.023	0.019	0.017	0.012	0.003
		1:2:2:1	0.069	0.140	0.216	0.133	0.060	0.077	0.106	0.100	0.072	0.034	0.027	0.025	0.017	0.003
	Penultimate support C	1:1:1:1	0.043	0.079	0.100	0.099	0.068	0.057	0.078	0.074	0.053	0.025	0.021	0.020	0.015	0.007
		1:1.5:1.5:1	0.035	0.065	0.082	0.081	0.055	0.103	0.142	0.134	0.096	0.046	0.037	0.035	0.025	0.007
		1:2:2:1	0.030	0.054	0.069	0.068	0.047	0.155	0.213	0.200	0.143	0.068	0.055	0.050	0.035	0.006
	Midspan D of interior span CE	1:1:1:1	0.016	0.029	0.037	0.036	0.025	0.041	0.099	0.175	0.098	0.040	0.032	0.030	0.022	0.010
		1:1.5:1.5:1	0.013	0.024	0.030	0.029	0.020	0.054	0.138	0.250	0.140	0.057	0.043	0.041	0.029	0.007
		1:2:2:1	0.011	0.020	0.025	0.025	0.017	0.066	0.175	0.325	0.180	0.073	0.054	0.050	0.035	0.006
	Central support E	1:1:1:1	0.012	0.021	0.027	0.027	0.018	0.028	0.058	0.080	0.085	0.061	0.085	0.080	0.058	0.026
		1:1.5:1.5:1	0.010	0.017	0.022	0.022	0.015	0.038	0.082	0.116	0.124	0.091	0.124	0.116	0.082	0.022
		1:2:2:1	0.008	0.015	0.019	0.019	0.013	0.046	0.104	0.150	0.163	0.121	0.163	0.150	0.104	0.019

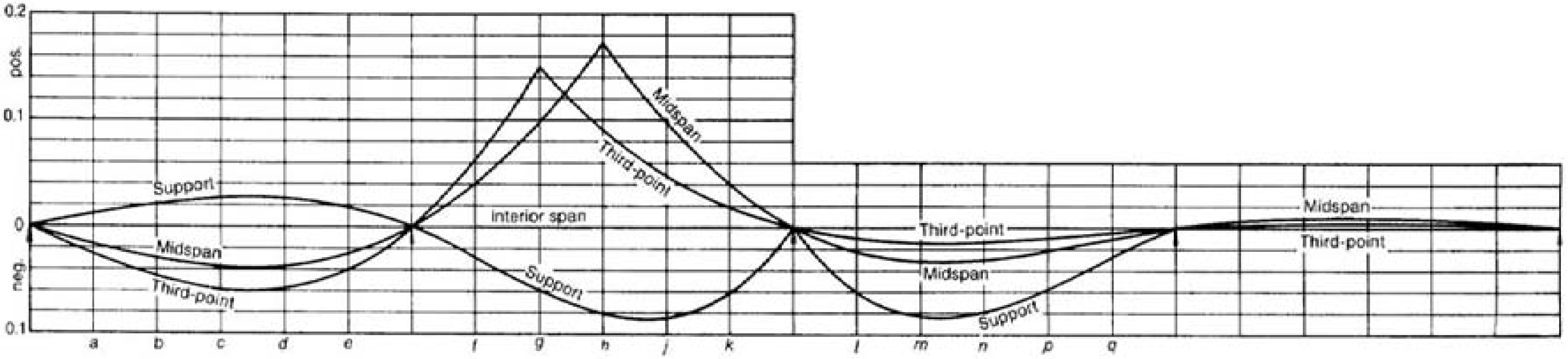
Unequal spans
Data enable influence lines to be drawn for bending moments produced by a single unit load moving over four unequal spans.

Ordinates for intermediate ratios of spans can be interpolated.

Equal spans
Influence lines marked 1:1:1:1 can be used directly (diagram of a succession of loads must be drawn to same linear scale).

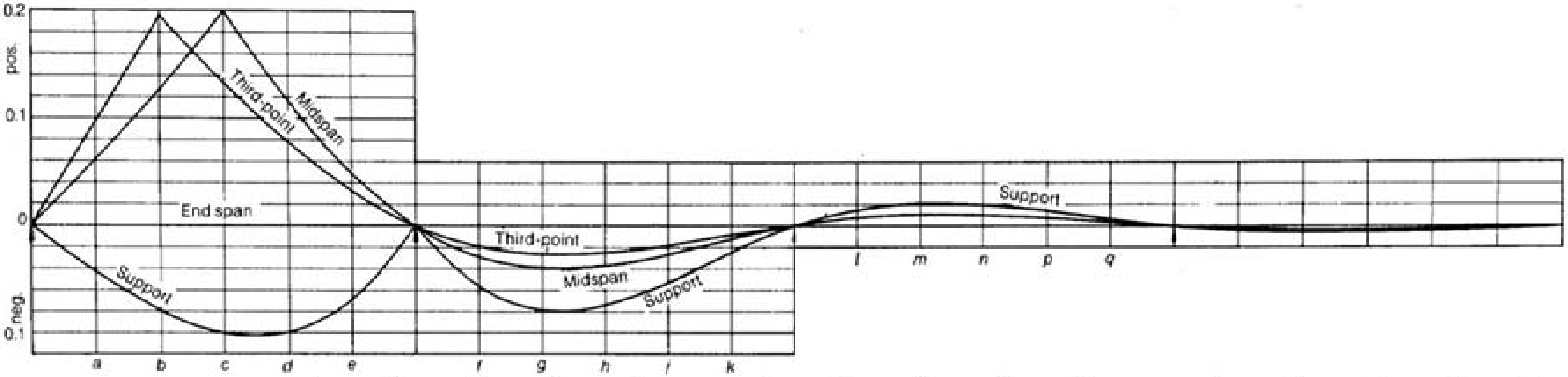
Bending moment due to load F concentrated at any point = (ordinate of appropriate influence line) × (end span L_1) × F

INFLUENCE LINES FOR BENDING MOMENTS AT MIDSPAN AND AT THIRD-POINT OF INTERIOR SPAN AND AT INTERIOR SUPPORT



	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*j*	*k*	*l*	*m*	*n*	*p*	*q*
Midspan	0.016	0.029	0.037	0.036	0.025	0.041	0.098	0.173	0.098	0.040	0.023	0.031	0.030	0.021	0.010
Third-point	0.025	0.046	0.058	0.057	0.039	0.064	0.151	0.091	0.048	0.018	0.010	0.014	0.013	0.009	0.005
Interior support	0.012	0.021	0.027	0.027	0.018	0.028	0.058	0.081	0.085	0.061	0.061	0.084	0.079	0.057	0.027

INFLUENCE LINES FOR BENDING MOMENTS AT MIDSPAN AND AT THIRD-POINT OF END SPAN AND AT PENULTIMATE SUPPORT



	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*j*	*k*	*l*	*m*	*n*	*p*	*q*
Midspan	0.062	0.127	0.200	0.117	0.049	0.028	0.039	0.037	0.026	0.013	0.008	0.010	0.010	0.007	0.003
Third-point	0.097	0.196	0.133	0.078	0.033	0.019	0.026	0.025	0.018	0.008	0.005	0.007	0.007	0.005	0.002
Penultimate support	0.043	0.079	0.100	0.099	0.068	0.057	0.078	0.074	0.053	0.025	0.015	0.021	0.020	0.014	0.007

Influence lines above can be used directly (diagram of a succession of loads must be drawn to the same linear scale).
Bending moment due to load *F* concentrated at any point = (ordinate of appropriate influence line) × (span) × *F*
Ordinates for end span and penultimate support of four equal spans can be used for five or more spans with reasonable accuracy.

Chapter 13

Slabs

In monolithic building construction, concrete floors can take various forms, as shown in *Table 2.42*. Slabs can be solid or ribbed, and can span between beams, in either one or two directions, or be supported directly by columns as a flat slab. Slab elements occur also as decking in bridges and other forms of platform structures, and as walling in rectangular tanks, silos and other forms of retaining structures.

13.1 ONE-WAY SLABS

For slabs carrying uniformly distributed load and continuous over three or more nearly equal spans, approximate solutions for the ultimate bending moments and shearing forces, for both BS 8110 and EC 2, are given in *Table 2.42*. The support moments include an allowance for 20% redistribution in both cases. The differences in the values for the two codes occur as a result of the different load arrangements described in section 4.4.1. However, it should be noted that for designs to EC 2, the UK National Annex allows the use of the BS 8110 simplified load arrangement as an alternative to that recommended in the base document. For two equal spans, the corresponding values for both codes would be:

Position	Moment	Shear
At outer support	0	$0.40F$
Near middle of end span	$+0.08Fl$	—
At interior support	$-0.10Fl$	$0.60F$

For designs where elastic bending moments are required, the coefficients given for beams in *Table 2.29* should be used.

13.2 TWO-WAY SLABS

Various methods, based on elastic or collapse considerations, are used to design slabs spanning in two directions. Elastic methods are appropriate if for example, serviceability checks on crack widths are required, as in the design of bridges and liquid-retaining structures. Collapse methods are appropriate in cases, such as floors in buildings and similar structures, where the main criterion is the ultimate condition, and the serviceability requirements of cracking and deflection are met by compliance with simplified rules.

13.2.1 Uniformly loaded slabs (BS 8110 method)

For rectangular panels carrying uniformly distributed load, where the corners are prevented from lifting and adequate provision is made for torsion, the panel is considered to be divided into middle and edge strips, as shown in *Table 2.42*. The method may be used for continuous slabs, where the characteristic dead and imposed loads on adjacent panels, and the spans perpendicular to the lines of common support, are approximately the same as on the panel being considered.

The bending moments and shearing forces on the middle strips, for nine different panel types, are given in *Table 2.43*. Reinforcement meeting the minimum percentage requirement of the code should be provided in the edge strips. At corners where either one or both edges of the panel are discontinuous, torsion reinforcement is required. This should consist of top and bottom reinforcement, each with layers of bars placed parallel to the sides of the slab and extending from the edges a minimum distance of one-fifth of the shorter span. The area of reinforcement in each of the four layers, as a proportion of that required for the positive moment at mid-span, should be three-quarters where both edges are discontinuous and three-eighths where one edge is discontinuous. At a discontinuous edge, where the slab is monolithic with the support, negative reinforcement equal to a half of that required for the positive moment at mid-span should be provided.

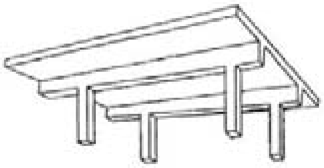
Where, because of differences between contiguous panels, two different values are obtained for the negative moment at a shared continuous edge, these values may be considered as fixed-end moments and moment distribution used to obtain equilibrium in the direction of span. The revised negative moments can then be used to adjust the positive moments at mid-span. For each panel, the sum of the mid-span moment and the average of the support moments should be the same as the original sum for that particular panel.

When the long span exceeds twice the short span, the slab should be designed as spanning in the short direction. In the long direction, the long span coefficient may still be used for the negative moment at a continuous edge.

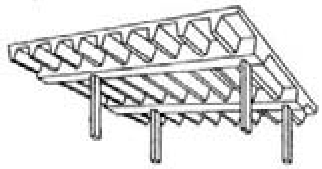
Slabs: general data

Types of slab

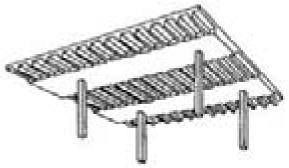
ONE-WAY SLABS



Solid (with beams)

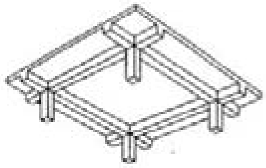


Ribbed (with beams)

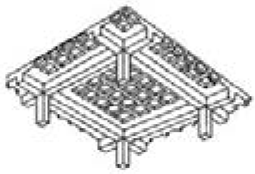


Ribbed (with integral beams)

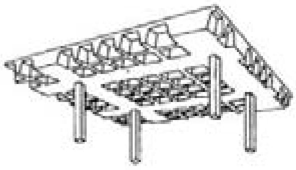
TWO-WAY SLABS



Solid (with beams)

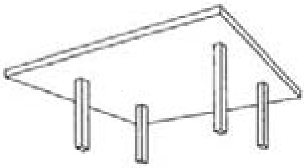


Waffle (with beams)

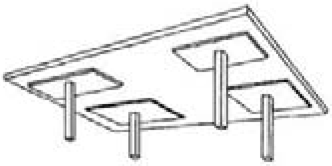


Waffle (with integral beams)

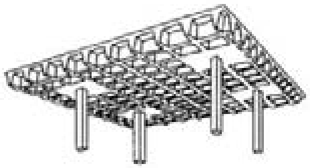
FLAT SLABS



Solid



Solid with drops



Waffle

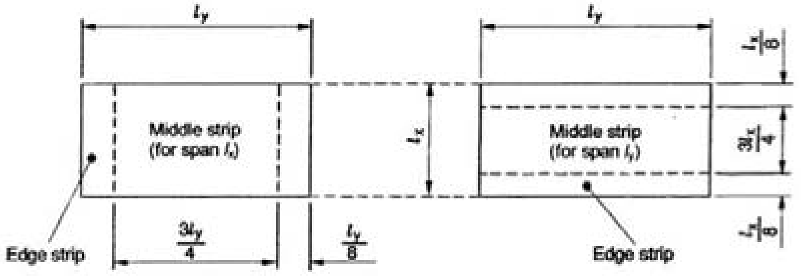
Approximate solutions for one-way slabs

Uniformly loaded one-way slab, freely supported at the ends, with three or more approximately equal spans

Position	Ultimate bending moment		Ultimate shearing force	
	BS 8110	EC 2	BS 8110	EC 2
At outer support	0	0	$0.40F$	$0.45F$
Near middle of end span	$+0.086Fl$	$+0.09Fl$	—	—
At first interior support	$-0.086Fl$	$-0.09Fl$	$0.60F$	$0.60F$
At middle of interior spans	$+0.063Fl$	$+0.07Fl$	—	—
At other interior supports	$-0.063Fl$	$-0.08Fl$	$0.50F$	$0.55F$

Notes: Support moments include allowance for 20% redistribution (F is total design load on span, l is effective span). BS 8110 solutions apply in cases where characteristic imposed load does not exceed 1.25 x characteristic dead load or 5 kN/m^2, and area of each bay exceeds 30 m^2. Where slab is continuous with end support, the following values may be used: moment at outer support $-0.04Fl$, moment near middle of end span $+0.075Fl$, shear at outer support $0.45F$. Eurocode 2 solutions apply in cases where characteristic imposed load does not exceed 1.25 x characteristic dead load.

Two-way slabs (BS 8110)



Division of uniformly loaded rectangular panel into middle and edge strips
(For details of moments and shear forces on middle strips, see *Table 2.43*)

Two-way slabs: uniformly loaded rectangular panels (BS 8110 method)

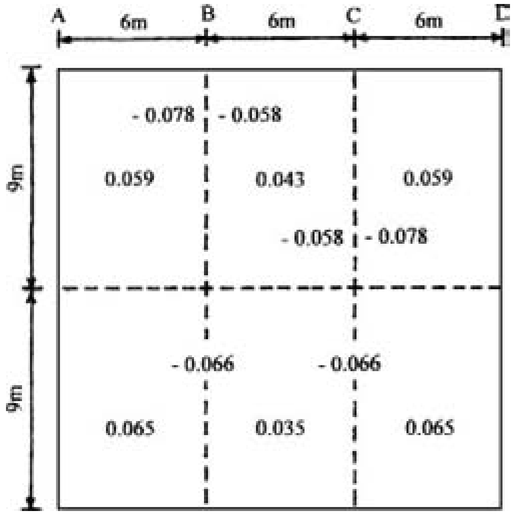
2.43

Rectangular panels supported on four sides with provision for torsion at corners									
Type of panel with moments and shears considered	Short span coefficients β_{mx} and β_{vx} for values of l_y/l_x								Long span coefficients β_{my} and β_{vy}
	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
1. Four edges continuous									
Negative moment at continuous edge	0.032	0.037	0.042	0.046	0.050	0.053	0.059	0.063	0.032
Positive moment at mid-span	0.024	0.028	0.032	0.035	0.037	0.040	0.044	0.048	0.024
Shear force at continuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33
2. One short edge discontinuous									
Negative moment at continuous edge	0.039	0.044	0.048	0.052	0.055	0.058	0.063	0.067	0.037
Positive moment at mid-span	0.029	0.033	0.036	0.039	0.041	0.043	0.047	0.050	0.028
Shear force at continuous edge	0.36	0.39	0.42	0.44	0.45	0.47	0.50	0.52	0.36
Shear force at discontinuous edge	—	—	—	—	—	—	—	—	0.24
3. One long edge discontinuous									
Negative moment at continuous edge	0.037	0.049	0.056	0.062	0.068	0.073	0.082	0.089	0.039
Positive moment at mid-span	0.028	0.036	0.042	0.047	0.051	0.055	0.062	0.067	0.029
Shear force at continuous edge	0.36	0.40	0.44	0.47	0.49	0.51	0.55	0.59	0.36
Shear force at discontinuous edge	0.24	0.27	0.29	0.31	0.32	0.34	0.36	0.38	—
4. Two adjacent edges discontinuous									
Negative moment at continuous edge	0.047	0.056	0.063	0.069	0.074	0.078	0.087	0.093	0.047
Positive moment at mid-span	0.036	0.042	0.047	0.051	0.055	0.059	0.065	0.070	0.036
Shear force at continuous edge	0.40	0.44	0.47	0.50	0.52	0.54	0.57	0.60	0.40
Shear force at discontinuous edge	0.26	0.29	0.31	0.33	0.34	0.35	0.38	0.40	0.26
5. Two long edges continuous									
Negative moment at continuous edge	0.046	0.050	0.054	0.057	0.060	0.062	0.067	0.070	—
Positive moment at mid-span	0.034	0.038	0.040	0.043	0.045	0.047	0.050	0.053	0.034
Shear force at continuous edge	0.40	0.43	0.45	0.47	0.48	0.49	0.52	0.54	—
Shear force at discontinuous edge	—	—	—	—	—	—	—	—	0.26
6. Two short edges continuous									
Negative moment at continuous edge	—	—	—	—	—	—	—	—	0.046
Positive moment at mid-span	0.034	0.046	0.056	0.065	0.072	0.078	0.091	0.100	0.034
Shear force at continuous edge	—	—	—	—	—	—	—	—	0.40
Shear force at discontinuous edge	0.26	0.30	0.33	0.36	0.38	0.40	0.44	0.47	—
7. One long edge continuous									
Negative moment at continuous edge	0.058	0.065	0.071	0.076	0.081	0.084	0.092	0.098	—
Positive moment at mid-span	0.043	0.048	0.053	0.057	0.060	0.063	0.069	0.074	0.043
Shear force at continuous edge	0.45	0.48	0.51	0.53	0.55	0.57	0.60	0.63	—
Shear force at discontinuous edge	0.30	0.32	0.34	0.35	0.36	0.37	0.39	0.41	0.29
8. One short edge continuous									
Negative moment at continuous edge	—	—	—	—	—	—	—	—	0.058
Positive moment at mid-span	0.043	0.054	0.063	0.071	0.078	0.084	0.096	0.105	0.043
Shear force at continuous edge	—	—	—	—	—	—	—	—	0.45
Shear force at discontinuous edge	0.29	0.33	0.36	0.38	0.40	0.42	0.45	0.48	0.30
9. Four edges discontinuous									
Positive moment at mid-span	0.056	0.065	0.074	0.081	0.087	0.092	0.103	0.111	0.056
Shear force at discontinuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33

Note: Maximum values of moment per unit width and shear force per unit width are given by the following relationships, where l_x is short span, l_y is long span and n is design ultimate load per unit area.

Short span: $m_x = \beta_{mx}\, n\, l_x^2$, $v_x = \beta_{vx}\, n\, l_x$ Long span: $m_y = \beta_{my}\, n\, l_x^2$, $v_y = \beta_{vy}\, n\, l_x$

Example. Determine the bending moment coefficients in the short span direction for the slab panel layout shown as follows.



The upper half of the layout shows the coefficients obtained from *Table 2.43*, with $l_y / l_x = 9.0/6.0 = 1.5$, for panel types:

A–B (and C–D): two adjacent edges discontinuous
B–C: one short edge discontinuous

The lower half of the layout shows the coefficients obtained after distribution of the unbalanced support moments at B and C. The effective stiffnesses, allowing for the effects of simple supports at A and D, and carry-over moments at B and C are:

Span A–B (and C–D): $0.75I/l$ Span B–C: $0.5I/l$

Distribution factors at B and C, with no carry-overs, are:

BA (and CD): 0.75/(0.75 + 0.50) = 0.6
BC (and CB): (1.0 − 0.6) = 0.4

Moment coefficients at B and C, after distribution, are

$$-0.078 + 0.6\,(0.078 - 0.058) = -0.066$$

Moment coefficients at mid-span, after redistribution, are:

BA (and CD): 0.059 + 0.5 (0.078 − 0.066) = 0.065
BC: 0.043 − (0.066 − 0.058) = 0.035

13.2.2 Uniformly loaded slabs (elastic analysis)

For rectangular panels carrying uniformly distributed load, where the corners are prevented from lifting and adequate provision is made for torsion, maximum bending and torsion moments are given in *Table 2.44* for nine panel types. Where, in continuous slabs, the edge conditions in contiguous panels result in two different values being obtained for the negative moment at a fixed edge, the moment distribution procedure shown in section 13.2.1 could be used, but this would ignore the inter-dependence of the moments in the two directions. A somewhat complex procedure involving edge stiffness factors is derived, and shown with fully worked examples in ref. 21.

The coefficients include for a Poisson's ratio of 0.2 and have been calculated from data given in ref. 21, which was derived by finite element analysis with Poisson's ratio taken as zero, using the following approximate relationships:

Bending moments $\alpha_{x,\nu} = \alpha_{x0} + \nu\alpha_{y0}$ $\alpha_{y,\nu} = \alpha_{y0} + \nu\alpha_{x0}$
Torsion moments $\alpha_{xy,\nu} = (1 - \nu)\alpha_{xy0}$

where ν is Poisson's ratio, and α_{x0}, α_{y0}, α_{xy0} are coefficients corresponding to $\nu = 0$. Thus, if required, the tabulated values can be readjusted to suit a Poisson's ratio of zero, as follows:

Bending moments $\alpha_{x0} = 1.04\,(\alpha_{x,0.2} - 0.2\,\alpha_{y,0.2})$
$\alpha_{y0} = 1.04\,(\alpha_{y,0.2} - 0.2\,\alpha_{x,0.2})$
Torsion moments $\alpha_{xy0} = 1.25\,\alpha_{xy,0.2}$

For rectangular panels, simply supported on four sides, with no provision to resist torsion at the corners or to prevent the corners from lifting, coefficients taken from BS 8110 are also given in *Table 2.44*. The coefficients, which are derived from the Grashof and Rankine formulae (see section 4.5.3), are given by the following expressions:

$$\alpha_{mx} = \frac{(l_y/l_x)^4}{8\left[1 + (l_y/l_x)^4\right]} \qquad \alpha_{my} = \frac{(l_y/l_x)^2}{8\left[1 + (l_y/l_x)^4\right]}$$

13.2.3 Non-rectangular panels

For a non-rectangular panel supported along all of its edges, bending moments can be determined approximately from the data given in *Table 2.48*. The information, which is based on elastic analysis, is applicable to panels that are trapezoidal, triangular, polygonal or circular. For guidance on using this information, including the arrangement of the reinforcement, reference should be made to section 4.7.

13.3 CONCENTRATED LOADS

13.3.1 One-way slabs

For a slab, simply supported along two opposite edges and carrying a centrally placed load uniformly distributed over a defined area, maximum elastic bending moments are given in *Table 2.45*. The coefficients, which include for a Poisson's ratio of 0.2, have been calculated from the data derived for a rectangular panel infinitely long in one direction. For designs to BS 8110, in which the ULS requirement is the main criterion, a concentrated load placed in any position may be spread over a strip of effective width b_e, as shown in *Table 2.45*. Parallel to the supports, a strip of width $(x + a_y/2)$ equally spaced each side of the load has been considered.

For slabs that are restrained at one or both edges, maximum negative and positive bending moments may be obtained by multiplying the simply supported moment by the appropriate factors given in *Table 2.45*. The factors, which are given for both fixed and continuous conditions, are those appropriate to elastic beam behaviour.

13.3.2 Two-way slabs

For a rectangular panel, freely supported along all four edges and carrying a concentric load uniformly distributed over a defined area, maximum mid-span bending moments based on Pigeaud's theory are given in *Tables 2.46* and *2.47*. Moment coefficients

Two-way slabs: uniformly loaded rectangular panels (elastic analysis)

Rectangular panels supported on four sides with provision for torsion at corners									
Type of panel with moments considered		Coefficients for values of l_y/l_x							
		1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
1. Four edges fixed									
Negative moment at fixed edge	α_{mx}	0.052	0.058	0.064	0.069	0.073	0.077	0.081	0.083
	α_{my}	0.052	0.054	0.056	0.057	0.057	0.057	0.057	0.058
Positive moment at mid-span	α_{mx}	0.021	0.025	0.029	0.032	0.034	0.036	0.040	0.042
	α_{my}	0.021	0.021	0.020	0.020	0.019	0.018	0.018	0.018
2. One short edge hinged (others fixed)									
Negative moment at fixed edge	α_{mx}	0.062	0.068	0.072	0.076	0.079	0.080	0.083	0.084
	α_{my}	0.055	0.057	0.057	0.058	0.058	0.057	0.057	0.056
Positive moment at mid-span	α_{mx}	0.027	0.030	0.033	0.036	0.038	0.039	0.042	0.043
	α_{my}	0.022	0.022	0.021	0.021	0.021	0.021	0.020	0.019
3. One long edge hinged (others fixed)									
Negative moment at fixed edge	α_{mx}	0.055	0.065	0.074	0.082	0.089	0.095	0.107	0.115
	α_{my}	0.062	0.068	0.072	0.076	0.077	0.078	0.080	0.081
Positive moment at mid-span	α_{mx}	0.022	0.026	0.032	0.036	0.040	0.047	0.055	0.061
	α_{my}	0.027	0.028	0.028	0.028	0.027	0.027	0.025	0.022
4. Two adjacent edges hinged (two fixed)									
Negative moment at fixed edge	α_{mx}	0.070	0.079	0.087	0.094	0.100	0.105	0.115	0.120
	α_{my}	0.070	0.074	0.077	0.078	0.078	0.081	0.082	0.082
Positive moment at mid-span	α_{mx}	0.030	0.034	0.038	0.042	0.046	0.050	0.056	0.059
	α_{my}	0.030	0.030	0.030	0.029	0.029	0.029	0.028	0.027
Torsion moment at corner (hinged edges)	α_{mxy}	0.027	0.029	0.031	0.033	0.034	0.035	0.035	0.035
5. Two short edges hinged (others fixed)									
Negative moment at fixed edge	α_{mx}	0.070	0.074	0.077	0.079	0.081	0.083	0.085	0.086
Positive moment at mid-span	α_{mx}	0.032	0.035	0.037	0.039	0.040	0.042	0.044	0.045
	α_{my}	0.022	0.022	0.021	0.021	0.021	0.021	0.022	0.022
6. Two long edges hinged (others fixed)									
Negative moment at fixed edge	α_{my}	0.070	0.079	0.087	0.094	0.100	0.105	0.114	0.120
Positive moment at mid-span	α_{mx}	0.022	0.028	0.035	0.042	0.049	0.056	0.071	0.085
	α_{my}	0.032	0.035	0.037	0.040	0.041	0.042	0.041	0.040
7. One long edge fixed (others hinged)									
Negative moment at fixed edge	α_{mx}	0.084	0.092	0.098	0.104	0.108	0.112	0.118	0.122
Positive moment at mid-span	α_{mx}	0.037	0.041	0.045	0.049	0.051	0.054	0.059	0.062
	α_{my}	0.031	0.030	0.029	0.029	0.028	0.028	0.028	0.029
Torsion moment at corner (hinged edges)	α_{mxy}	0.031	0.033	0.034	0.035	0.035	0.036	0.036	0.036
8. One short edge fixed (others hinged)									
Negative moment at fixed edge	α_{my}	0.084	0.092	0.099	0.104	0.109	0.113	0.118	0.122
Positive moment at mid-span	α_{mx}	0.031	0.039	0.046	0.053	0.060	0.066	0.081	0.093
	α_{my}	0.039	0.042	0.044	0.043	0.043	0.043	0.044	0.044
Torsion moment at corner (hinged edges)	α_{mxy}	0.026	0.035	0.039	0.042	0.044	0.046	0.050	0.053
9. Four edges hinged									
Positive moment at mid-span	α_{mx}	0.044	0.052	0.060	0.066	0.073	0.079	0.091	0.102
	α_{my}	0.044	0.045	0.045	0.045	0.044	0.044	0.044	0.045
Torsion moment at corner	α_{mxy}	0.037	0.040	0.043	0.046	0.048	0.049	0.052	0.053
Rectangular panels simply supported on four sides with corners not held down									
10. Four edges simply-supported									
Positive moment at mid-span	α_{mx}	0.062	0.074	0.084	0.093	0.099	0.104	0.113	0.118
	α_{my}	0.062	0.061	0.059	0.055	0.051	0.046	0.037	0.029

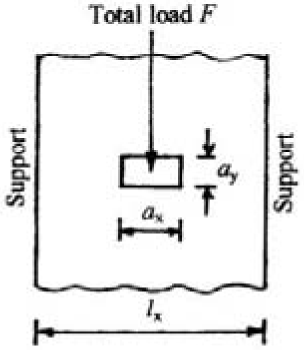
Note: Maximum moments per unit width are given by the following relationships, where l_x is short span, l_y is long span and w is design load per unit area.

Short span: $m_x = \alpha_{mx} w l_x^2$ Long span: $m_y = \alpha_{my} w l_x^2$ Torsion: $m_{xy} = \alpha_{mxy} w l_x^2$

2.45

One-way slabs: concentrated loads

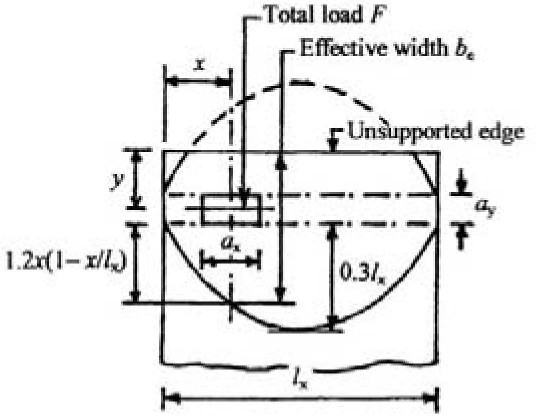
Load at centre of simply supported slab (elastic analysis)

Notation	Values of a_y/l_x		Coefficients for values of a_x/l_x					
			0.1	0.2	0.4	0.6	0.8	1.0
	0.1	α_{mx} α_{my}	0.310 0.246	0.259 0.219	0.199 0.176	0.161 0.146	0.132 0.121	0.109 0.100
	0.2	α_{mx} α_{my}	0.282 0.196	0.243 0.180	0.191 0.152	0.156 0.128	0.129 0.107	0.106 0.089
	0.4	α_{mx} α_{my}	0.240 0.138	0.216 0.131	0.176 0.115	0.146 0.100	0.122 0.085	0.100 0.070
	0.6	α_{mx} α_{my}	0.211 0.105	0.192 0.100	0.161 0.090	0.136 0.079	0.114 0.067	0.094 0.056
	0.8	α_{mx} α_{my}	0.188 0.082	0.173 0.079	0.148 0.071	0.126 0.063	0.106 0.053	0.088 0.045
	1.0	α_{mx} α_{my}	0.169 0.066	0.157 0.063	0.136 0.057	0.116 0.051	0.098 0.044	0.082 0.037

Note. Maximum moments per unit width are given by:

In direction of l_x : $m_x = \alpha_{mx}F$ At right angles to l_x : $m_y = \alpha_{my}F$

Simply supported slab (BS 8110 method)



Effective width of strip:

$$b_e = 1.2x(1-x/l_x) + a_y/2 + y$$
$$\leq 2.4x(1-x/l_x) + a_y$$

Note. Maximum moments per unit width are given by:

In direction of l_x

$$m_x = \frac{Fx}{b_e}\left(1-\frac{x}{l_x}\right)\left(1-\frac{a_x}{2l_x}\right)$$

At right angles to l_x

When $b_e = 2.4\,x\,(1 - x/\,l_x) + a_y$

$$m_y = \frac{Fb_e}{4(2x+a_y)}\left(1-\frac{a_y}{b_e}\right)$$

When $b_e = 1.2\,x\,(1 - x/l_x) + a_y\,/\,2 + y$

$$m_y = \frac{Fb_e}{4(2x+a_y)}\left[4\left(1-\frac{y}{b_e}\right)^2-\frac{a_y}{b_e}\right]$$

$$m_{yx} = \frac{Fb_e}{(2x+a_y)}\left(1-\frac{2y}{b_e}\right)$$ (torsion at unsupported edge)

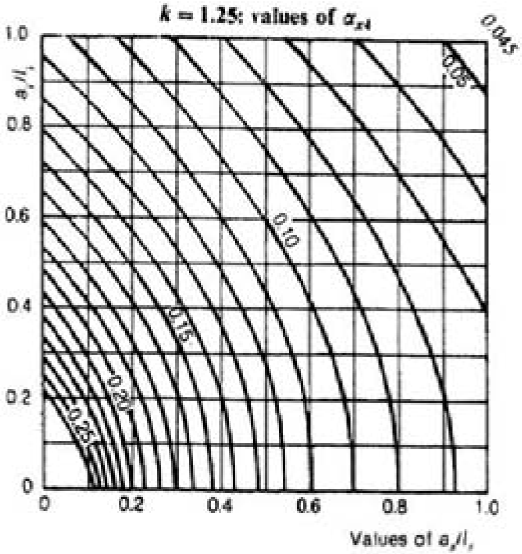
Allowance for end restraint

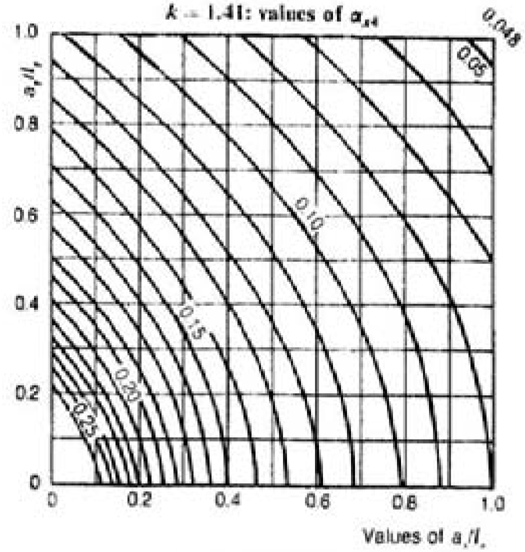
Restraint condition and moment		Multipliers for values of a_x/l_x					
		0	0.2	0.4	0.6	0.8	1.0
Fixed at both ends	negative moment positive moment	0.50 0.50	0.55 0.45	0.59 0.41	0.63 0.37	0.66 0.34	0.67 0.33
Fixed at one end	negative moment positive moment	0.75 0.63	0.82 0.60	0.89 0.59	0.94 0.58	0.98 0.57	1.00 0.56
Multi-span (interior span)	negative moment positive moment	0.32 0.68	0.35 0.65	0.37 0.63	0.40 0.60	0.41 0.59	0.42 0.58
Multi-span (end span)	negative moment positive moment	0.40 0.80	0.44 0.78	0.48 0.77	0.51 0.76	0.53 0.75	0.54 0.75

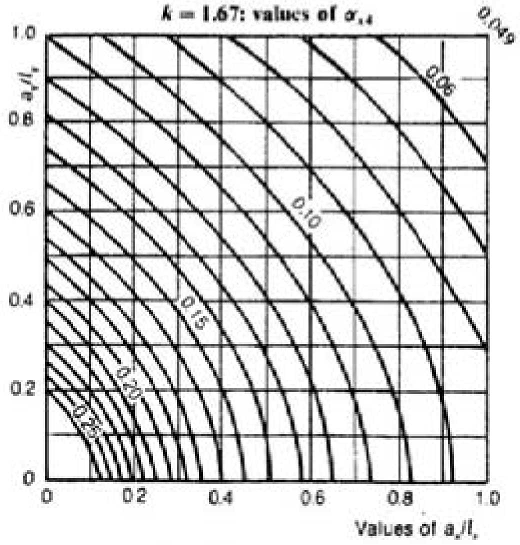
Note. Bending moments are obtained by multiplying the moment for the simply supported case by the appropriate factor. For the general case of a continuous slab, the negative moments for the fixed-end cases should be distributed as necessary to obtain equilibrium. The resulting negative moments can then be used to adjust the positive moments. The values given for the multi-span cases are for load on one span only of a series of five or more equal spans

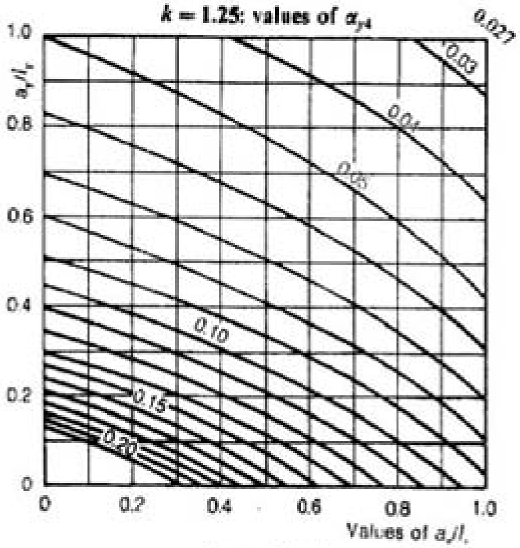
Two-way slabs: rectangular panel with concentric concentrated load – 1

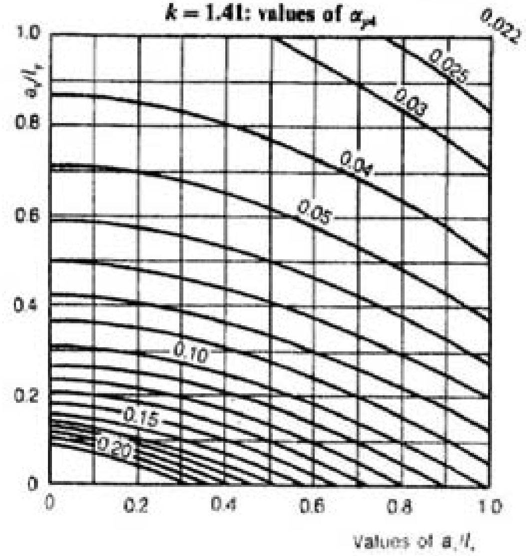
2.46

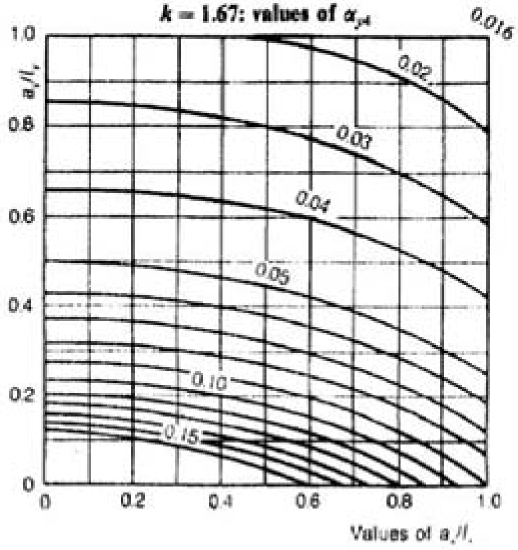

$k = 1.25$: values of α_{x4}


$k = 1.41$: values of α_{x4}

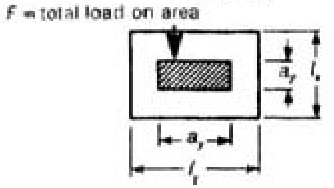

$k = 1.67$: values of α_{x4}


$k = 1.25$: values of α_{y4}


$k = 1.41$: values of α_{y4}

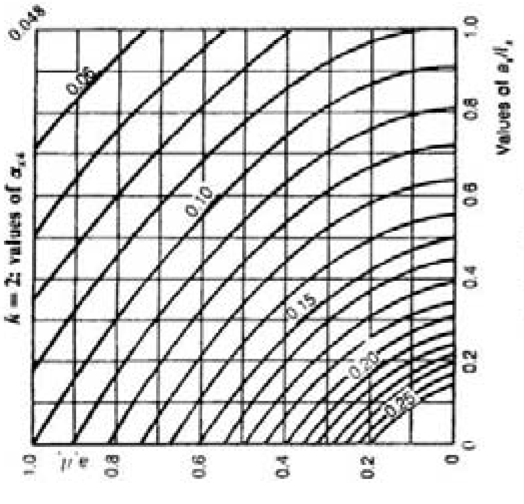

$k = 1.67$: values of α_{y4}

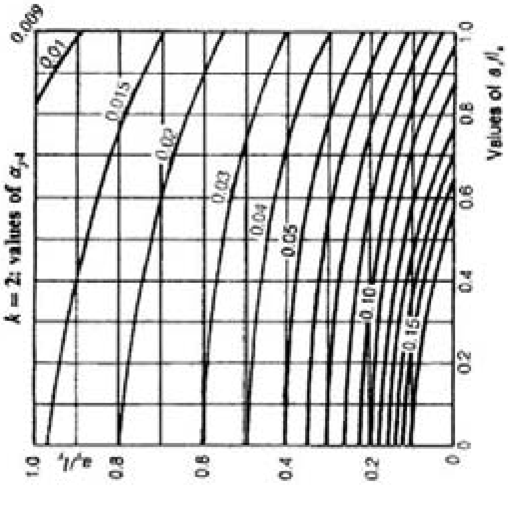
Slabs freely supported along all four edges with corner restraint $k = l_y/l_x$

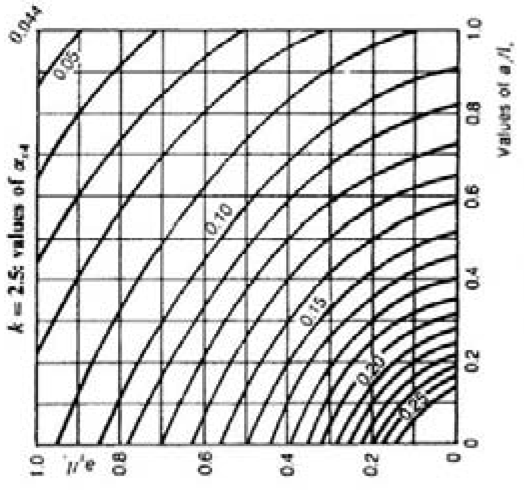


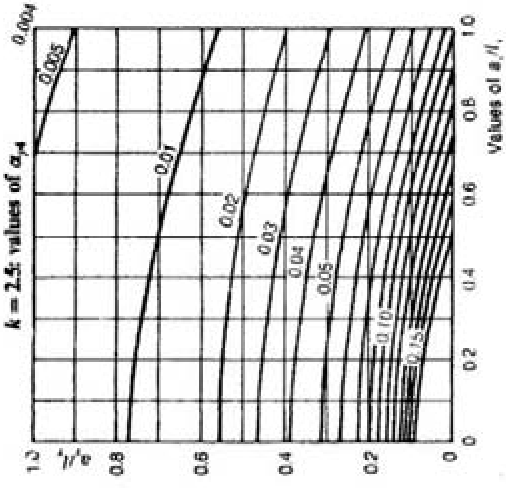
Bending moment across short span = $F(\alpha_{x4} + \nu\alpha_{y4})$
Bending moment across long span = $F(\nu\alpha_{x4} + \alpha_{y4})$
where ν is Poisson's ratio.
BS8110 and BS5400: Part 4 recommend use of $\nu = 0.2$.

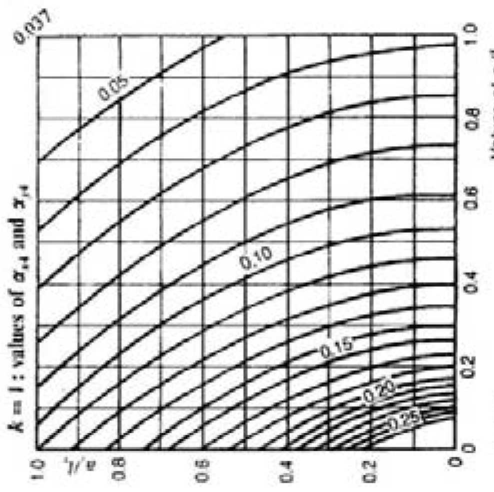
The factors α_{x4} and α_{y4} are non-dimensional and thus the bending moments given by the above expressions are per unit width (i.e. for a load in newtons, the resulting moments are in N m per metre width).

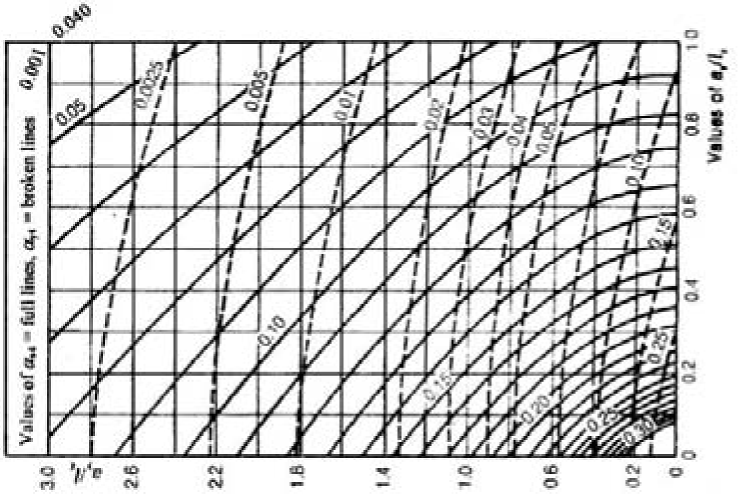

$k = 2$: values of α_{x4}


$k = 2$: values of α_{y4}


$k = 2.5$: values of α_{x4}


$k = 2.5$: values of α_{y4}


$k = 1$: values of α_{x4} and α_{y4}


$k = \infty$

Values of α_{x4} = full lines, α_{y4} = broken lines

Slab supported along two opposite edges only

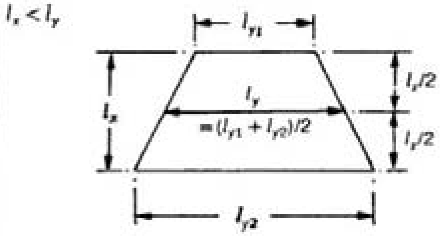
Transverse bending moment = $F(\alpha_{x4} + \nu\alpha_{y4})$

Longitudinal bending moment = $F(\nu\alpha_{x4} + \alpha_{y4})$

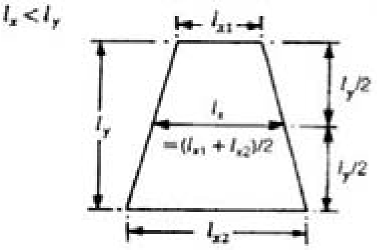
BS8110 and BS5400: Part 4 recommend use of $\nu = 0.2$.

Two-way slabs: non-rectangular panels (elastic analysis)

Trapezium



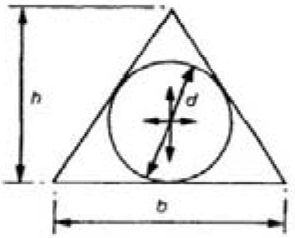
Calculate bending moments as for rectangular panel with $k = l_y/l_x$



If l_{y1} is small compared with l_{y2} or l_{x1} is small compared with l_{x2} } apply rules for triangular panel

---

Isosceles triangle



d = diameter of inscribed circle $= \dfrac{2bh}{b + \sqrt{(b^2 + 4h^2)}}$

Freely supported along all edges (corners restrained):

Bending moment (in two directions at centre of circle) $= +wd^2/16$

Continuous along all sides:

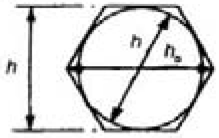
Bending moment (in two directions at centre of circle) $= +wd^2/30$

Bending moment (at sides) $= -wh^2/30$

w is intensity of uniformly distributed load (or intensity of pressure at centre of circle if pressure varies uniformly).
These expressions are valid for values of $v \ngtr 0.2$.

Regular polygon

Five or more sides



h diameter of inscribed circle = distance across flats
h_0 diameter of circumscribed circle = distance across corners
h_1 $(h + h_0)/2 = 1.077h$ for hexagon
$1.041h$ for octagon
Calculate bending moments as for circle of diameter h_1

Circle (diameter = h)

	Freely supported edge	Clamped edge
Concentric concentrated load F uniformly distributed over small area of diameter d	Beneath loaded area $M_r = M_t \ngtr \dfrac{F}{4\pi}\left[1 + (1 + v)\ln\dfrac{h}{d}\right]$ Beneath unloaded area $M_r = -\dfrac{F}{4\pi}(1 + v)\ln\xi$ $M_t = \dfrac{F}{4\pi}[(1 - v) - (1 + v)\ln\xi]$	Beneath loaded area $M_r = M_t \ngtr \dfrac{F}{4\pi}(1 + v)\ln\dfrac{h}{d}$ Beneath unloaded area $M_r = \dfrac{F}{4\pi}\left[\left(\dfrac{d}{2\xi h}\right)^2(1 - v) - (1 + v)\ln\xi - 1\right]$ $M_t = \dfrac{F}{4\pi}\left[\left(\dfrac{d}{2\xi h}\right)^2 v(1 - v) - (1 + v)\ln\xi - v\right]$
Uniformly distributed load w over entire panel	$M_r = \dfrac{wh^2}{64}(3 + v)(1 - \xi^2)$ $M_t = \dfrac{wh^2}{64}[(3 + v) - (1 + 3v)\xi^2]$	$M_r = \dfrac{wh^2}{64}[(1 + v) - (3 + v)\xi^2]$ $M_t = \dfrac{wh^2}{64}[(1 + v) - (1 + 3v)\xi^2]$

Notes

Reinforcement to resist positive bending moments to be provided in two directions mutually at right angles.

M_r moment in radial direction
M_t moment in tangential direction
v Poisson's ratio
ξ $\dfrac{\text{distance of point considered from slab centre}}{\text{radius of slab}}$

For slab continuous at edge, average moments obtained by considering freely supported slab and slab with clamped edge.
If $d <$ half the thickness of slab t, substitute $d' = \sqrt{(1.6d^2 + t^2)} - 0.675t$ for d in above formulae.

given in the charts are used with an appropriate value of Poisson's ratio to calculate the bending moments. The coefficients given at the top right corner of each chart are for the limiting case when the load extends over the entire panel. This case, with Poisson's ratio taken as 0.2, is given also in *Table 2.44*.

When using the chart for square panels ($k = 1.0$), if $a_x = a_y$, $\alpha_{x4} = \alpha_{y4}$ and the resulting bending moment in each direction is given by $F(1 + \nu)\alpha_{x4}$. In other cases, coefficient α_{x4} is based on the direction chosen for a_x and coefficient α_{y4} is obtained by reversing a_x and a_y, as shown in example 1 later.

The maximum shearing forces, V per unit length, on a panel carrying a concentrated load are given by Pigeaud as follows:

$a_x > a_y$ at the centre of length a_x, $V = F/(2a_x + a_y)$
at the centre of length a_y, $V = F/3a_x$

$a_y > a_x$ at the centre of length a_x, $V = F/3a_y$
at the centre of length a_y, $V = F/(2a_y + a_x)$

For panels that are restrained along all four edges, Pigeaud recommends that the mid-span moments be reduced by 20%. Alternatively, the multipliers given for one-way slabs could be used in one or both directions as appropriate, if the interdependence of the bending moments is ignored.

Example 1. Consider a square panel, freely supported along all four edges, carrying a concentric load with $a_x/l_x = 0.8$ and $a_y/l_y = 0.2$. From *Table 2.47* for $k = 1$, the bending moment coefficients are:

For $a_x/l_x = 0.8$ and $a_y/l_y = 0.2$; $\alpha_{x4} = 0.072$
For $a_x/l_x = 0.2$ and $a_y/l_y = 0.8$; $\alpha_{y4} = 0.103$

Maximum bending moments per unit width with $\nu = 0.2$, are:

For span in direction of a_x, $F(0.072 + 0.2 \times 0.103) = 0.093F$
For span in direction of a_y, $F(0.103 + 0.2 \times 0.072) = 0.118F$

Example 2. A bridge deck is formed of a 200 mm thick slab supported by longitudinal beams spaced at 2 m centres. The slab is covered with 100 mm thick surfacing. Determine, for the SLS, the maximum positive bending moments in the slab due to the local effects of live loading.

The critical live load for serviceability is the HA wheel load of 100 kN, to which a partial load factor of 1.2 is applied. For a 100 mm × 100 mm contact area, and allowing for load dispersal through the thickness of the surfacing and down to the mid-depth of the slab (see section 2.4.9), the side of the resulting patch load is (300 + 100 + 200) = 600 mm.

The simply supported bending moment coefficients for a centrally placed load, by interpolation from *Table 2.45*, are:

For $a_x/l_x = a_y/l_x = 600/2000 = 0.3$; $\alpha_{mx} = 0.206$, $\alpha_{my} = 0.145$

Allowing for continuity (interior span) in the direction of l_x and applying a partial load factor of 1.2, the positive bending moments per unit width are:

In direction of l_x

$$m_x = 0.64 \times 0.206 \times 1.2 \times 100 = 15.8 \text{ kNm/m}$$

At right angles to l_x

$$m_y = 0.145 \times 1.2 \times 100 = 17.4 \text{ kNm/m}$$

For design purposes, the bending moments determined earlier will need to be combined with the moments due to the weight of the slab and surfacing, and any transverse effects of the global deck analysis.

Note. Using the method given in BS 8110, with $x = 0.5l_x$, the effective width of the strip is:

$$b_e = 0.6l_x + a_y = 0.6 \times 2.0 + 0.6 = 1.8 \text{ m}$$

Allowing for continuity (interior span) in the direction of l_x and applying a partial load factor of 1.2, the positive bending moments per unit width are:

In direction of l_x

$$m_x = \frac{0.64 \times 1.2 \times 100 \times 1.0}{2 \times 1.8}\left(1 - \frac{0.6}{2 \times 2.0}\right) = 18.1 \text{kNm/m}$$

At right angles to l_x

$$m_y = \frac{1.2 \times 100 \times 1.8}{42 \times 1.0 + 0.6}\left(1 - \frac{0.6}{1.8}\right) = 13.9 \text{kNm/m}$$

13.4 YIELD-LINE ANALYSIS

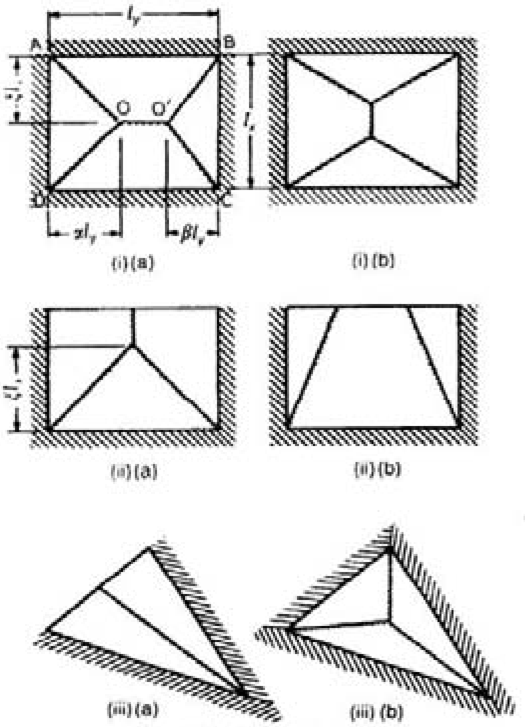
As stated in section 4.5.2, yield-line theory is too complex a subject to deal with adequately in the space available in this *Handbook*. The following notes are therefore intended merely to introduce the designer to the basic concepts, methods and problems involved. For further information, see refs 23 to 28. Application of yield-line theory to the design of rectangular slabs subjected to triangularly distributed loads is dealt with in section 13.6.2.

13.4.1 Basic principles

When a reinforced concrete slab is loaded, cracks form in the regions where the applied moment exceeds the cracking resistance of the concrete. As the load is increased beyond the service value, the concrete continues to crack; eventually, the reinforcement yields and the cracks extend to the corners of the slab, dividing it into several areas separated by so-called yield-lines, as shown in diagrams (i)–(iii) on *Table 2.49*. Any further increase in load will cause the slab to collapse. In the design process, the load corresponding to the formation of the entire system of yield lines is calculated and, by applying suitable partial factors of safety, the resistance moment that must be provided to avoid collapse is determined.

For a slab of given shape, it is usually possible to postulate different modes of failure, the critical mode depending on the support conditions, the panel dimension and the relative proportions of reinforcement provided in each direction. For example, if the slab shown in diagram (i)(a) is reinforced sufficiently strongly in the direction of the shorter span, by comparison with the longer span, this mode of failure will be prevented and that shown in diagram (i)(b) will occur instead. Similarly, if the slab with one edge unsupported shown in diagrams (ii) is loaded uniformly, pattern (b) will occur when the ratio of the longer to shorter side length (or longer to shorter 'reduced side length': see section 13.4.6) exceeds $\sqrt{2}$; otherwise pattern (a) will occur.

All of these patterns may be modified by the formation of corner levers: see section 13.4.9.

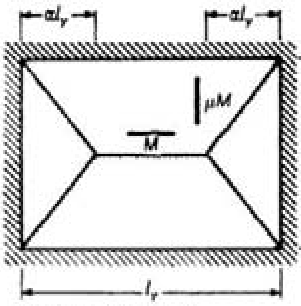


(i)(a) (i)(b)

(ii)(a) (ii)(b)

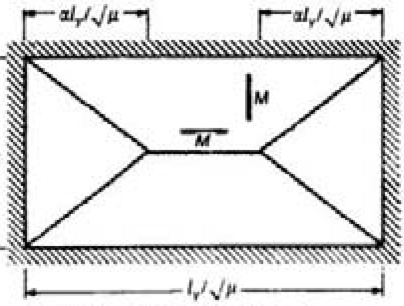
(iii)(a) (iii)(b)

Typical yield-line patterns



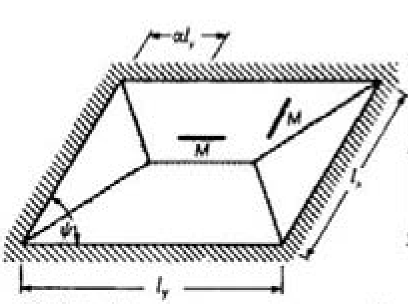
Distributed load = n
Concentrated load = F

(iv)(a) Orthotropically reinforced slab



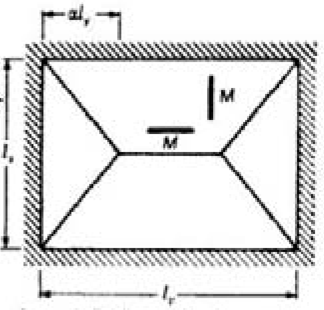
Transformed distributed load = n
Transformed concentrated load = $F/\sqrt{\mu}$

(iv)(b) Affine isotropic slab



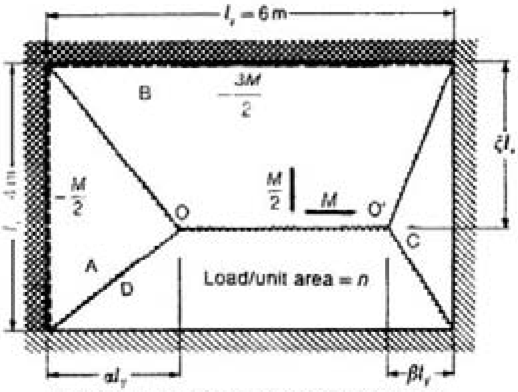
Distributed load = n
Concentrated load = F

(v)(a) Skew slab

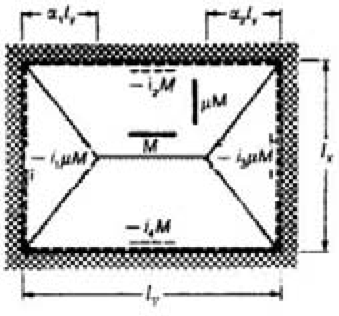


Transformed distributed load = n
Transformed concentrated load = $F/\sin\psi$

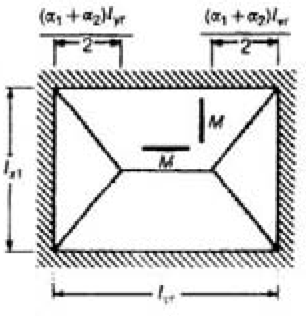
(v)(b) Affine rectangular slab



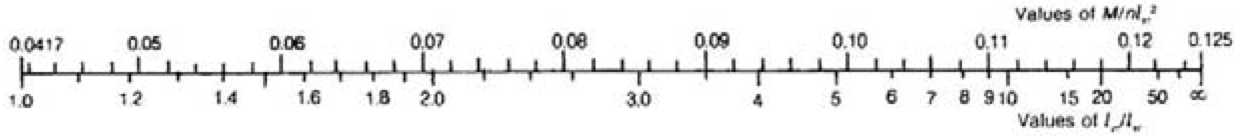
(vii) Slab considered in numerical examples



(vi)(a) Continuous slab



(vi)(b) Affine freely supported slab



Notation commonly adopted

────	positive yield line
-------	negative yield line
────	unsupported slab edge
▨▨▨	freely supported slab edge
▩▩▩	fixed or continuous slab edge
M μM	values and direction of positive resistance moments provided in slab (which act at right angles to direction in which line is drawn). Thus as shown here, resistance (in plane of page) is M vertically and μM horizontally

iM $i\mu M$	value and direction of negative resistance moments
M	principal positive resistance moment
μ	ratio of secondary to principal resistance moment
i_1, i_2, i_3 etc.	ratios of negative resistance moments at supports to positive resistance moment
l_x, l_y	lesser and greater side lengths of slab
l_{xr}, l_{yr}	reduced side lengths
ψ	angle of skew of slab
n	distributed ultimate load
F	concentrated ultimate load

13.4.2 Rules for postulating yield-line patterns

Viable yield-line patterns must comply with the following rules:

1. All yield lines must be straight.
2. A yield line can only change direction at an intersection with another yield-line.
3. A yield line separating two elements of a slab must pass through the intersection of their axes of rotation. (Note: this point may be at infinity.)
4. All reinforcement intercepted by a yield line is assumed to yield at the line.

13.4.3 Methods of analysis

Two basic methods of analysis have been developed. These are commonly referred to as the 'work' or 'virtual work' method and the 'equilibrium' method. The former method involves equating, for the yield-line pattern postulated, the work done by the external loads on the various areas of the slab to obtain a virtual displacement, to the work done by the internal forces in forming the yield lines. When the yield-line pattern is adjusted to its critical dimensions, the ratio of the ultimate resistance to the ultimate load reaches its maximum. When analysing a slab algebraically, this situation can be ascertained by differentiating the expression representing the ratio, and equating the differential to zero in order to establish the critical dimensions. Then by re-substituting these values into the original expression, a formula giving the required ultimate resistance for a slab of given dimensions and loading can be derived.

The so-called equilibrium method is not a true equilibrium method but a variant of the work method, which also gives an upper-bound solution. The method has the great advantage that the resulting equations provide sufficient information of themselves to eliminate the unknown variables, and therefore differentiation is unnecessary. Although there are also other advantages, the method is generally more limited in scope and is not described here: for details see refs 23 and 27.

13.4.4 Virtual-work method

As explained earlier, this method consists of equating the virtual work done by the external loads, in producing a given virtual displacement at some point on the slab, to the work done by the internal forces along the yield lines in rotating the slab elements. To demonstrate the principles involved, an analysis will be given of the freely supported rectangular slab supporting a uniform load and reinforced to resist equal moments M each way, shown in diagram (i)(a) on *Table 2.49*.

Clearly, due to symmetry, yield line OO′ will be midway between AB and CD. Similarly, $\alpha = \beta$ and thus only one dimension is unknown. Consider first the external work done.

The work done by an external load on an individual slab element is equal to the area of the element times the displacement of its centroid times the unit load. Thus for the triangular element ADO with displacement δ at O,

$$\text{work done} = (1/2)(l_x)(\alpha l_y)(\delta/3)n = \alpha\ l_x l_y \delta n/6$$

Similarly, for the trapezoidal area ABOO′ with displacement δ at O and O′,

$$\begin{aligned}\text{work done} &= [(l_x/2)(l_y)(\delta\ /2) - 2(1/2)(l_x/2)(\alpha l_y)(2\delta/3)]\\ &= (3 - 4\alpha)l_x l_y \delta n/12\end{aligned}$$

Thus, since the work done on BCO′ is the same as that done on ADO, and the work done on CDOO′ is the same as that on ABOO′, for the entire slab

$$\begin{aligned}\text{Total external work done} &= 2[\alpha l_x l_y \delta n/6 + (3 - 4\alpha)l_x l_y \delta n/12]\\ &= (3 - 2\alpha)l_x l_y \delta n/6\end{aligned}$$

The internal work done in forming a yield line is equal to the moment along the yield line times the length of the line times the rotation. A useful point to note is that, where a yield line is formed at an angle to the direction of principal moments, instead of considering its true length and rotation, it is usually simpler to consider the components in the direction of the principal moments. For example, for yield line AO, instead of considering the actual length AO and the rotation at right angles to AO, consider length $l_x/2$ and the rotation about AB plus length $\alpha\ l_y$ and the rotation about AD.

Thus, considering the component about AB of the yield line along AOO′ B, the length of the line is l_y, the moment is M and the rotation is $\delta\ /(\ l_x/2)$. Hence,

$$\text{work done} = 2M(l_y/l_x)\delta$$

Similarly, for the yield line along DOO′ C, the work done is again $2M(l_y/l_x)\delta$. (Length OO′ is considered twice, because the rotation between the elements separated by this length is double that occurring over the remaining length.) Now, considering the component about AD of the yield line AOD,

$$\text{work done} = M(l_x/\alpha l_y)\delta$$

Since the work done on yield line BOC is similar, for the entire slab

$$\text{Total internal work done} = 2M(2l_y/l_x + l_x/\alpha l_y)\delta$$

Equating the external work done to the internal work done,

$$(3 - 2\alpha)l_x l_y \delta n/6 = 2M(2l_y/l_x + l_x/\alpha l_y)\delta$$

or $$M = \frac{n}{12}l_x^2\left[\frac{3\alpha - 2\alpha^2}{2\alpha + (l_x/l_y)^2}\right]$$

To determine the critical value of αl_y, the quotient in square brackets must be differentiated and equated to zero. As Jones (ref. 27) has pointed out, to use the well-known relationship

$$\frac{dy}{dx} = \left(v\frac{du}{dx} - u\frac{dv}{dx}\right)\Big/ \mathrm{v}^2 = 0$$

simply as a means of maximising $y = u/v$, it is convenient to rearrange it in the form

$$\frac{u}{v} = \frac{du/dx}{dv/dx}$$

Thus, for the present example,

$$\frac{3\alpha - 2\alpha^2}{2\alpha + (l_x/l_y)^2} = \frac{3 - 4\alpha}{2}$$

This leads to a quadratic in α, the positive root of which is

$$\alpha = \frac{1}{2}\left(\sqrt{\left(\frac{l_x}{l_y}\right)^4 + 3\left(\frac{l_x}{l_y}\right)^2} - \left(\frac{l_x}{l_y}\right)^2\right)$$

When substituted in the original equation, this gives

$$M = \frac{n}{24} l_x^2 \left(\sqrt{3 + \left(\frac{l_x}{l_y}\right)^2} - \frac{l_x}{l_y} \right)^2$$

13.4.5 Concentrated and line loads

Concentrated and line loads are simpler to deal with than uniform loads. When considering the external work done, the contribution of a concentrated load is equal to the load times the relative deflection of the point at which it is applied. In the case of a line load, the external work done over a given slab area is equal to the portion of the load carried on that area times the relative deflection at the centroid of the load.

Yield lines tend to pass beneath heavy concentrated or line loads since this maximises the external work done by such loads. When a concentrated load acts in isolation, a so-called circular fan of yield lines tends to form: this behaviour is complex and reference should be made to specialist textbooks for details (refs 23, 24, 26).

13.4.6 Affinity theorems

Section 13.4.4 illustrates the work involved in analysing a simple freely supported slab with equal reinforcement in each direction (i.e. so-called *isotropic* reinforcement). If different amounts of reinforcement are provided in each direction (i.e. so-called *orthotropic* reinforcement), or if continuity or fixity exists along one or more edges, the formula needs modifying accordingly.

To avoid the need for a vast number of design formulae to cover all conceivable conditions, it is possible to transform most slabs with fixed or continuous edges and orthotropic reinforcement into their simpler freely supported isotropic equivalents by using the following affinity theorems: skew slabs can be transformed similarly into rectangular forms.

1. If an orthotropic slab is reinforced as shown in diagram (iv)(a) on *Table 2.49*, then it can be transformed into the simpler isotropic slab shown in diagram (iv)(b). All loads and dimensions in the direction of the principal co-ordinate axis remain unchanged, but in the affine slab the distances in the direction of the secondary co-ordinate axis are equal to the actual values divided by $\sqrt{\mu}$, and the corresponding total loads are equal to the original values divided by $\sqrt{\mu}$. (The latter requirement means that the intensity of a UDL per unit area remains unchanged by the transformation, since both the area and the total load on that area are divided by $\sqrt{\mu}$.)

   Similarly, a skew slab reinforced as shown in diagram (v)(a) can be transformed into the isotropic slab shown in diagram (v)(b) by dividing the original total load by $\sin\psi$. (As before, this requirement means that the intensity of a UDL per unit area remains unchanged by the transformation.)

   These rules can be combined when considering a skew slab with different reinforcement in each direction; for details, see ref. 28, article 6.

2. By using the reduced side lengths l_{xr} and l_{yr}, an orthotropic slab that is continuous over one or more supports, such as that in diagram (vi)(a) on *Table 2.49*, can be transformed into the simpler freely supported isotropic slab shown in diagram (vi)(b).

For example, for an orthotropic slab with fixed edges that is reinforced for positive moment M and negative moments i_2M and i_4M in span direction l_x, and for positive moment μM and negative moments $\mu i_1 M$ and $\mu i_3 M$ in span direction l_y, it can be shown that

$$M = \frac{n}{24} l_{xr}^2 \left(\sqrt{3 + \left(\frac{l_{xr}}{l_{yr}}\right)^2} - \frac{l_{xr}}{l_{yr}} \right)^2$$

where

$$l_{xr} = \frac{2l_x}{\sqrt{1+i_2} + \sqrt{1+i_4}} \text{ and } l_{yr} = \frac{2l_y}{[\sqrt{1+i_1} + \sqrt{1+i_3}]\sqrt{\mu}}$$

This is identical to the expression derived above for the freely supported isotropic slab but with l_{xr} and l_{yr} substituted for l_x and l_y. Values of M/nl_x^2 corresponding to ratios of l_y/l_x can be read directly from the scale on *Table 2.49*.

The validity of the analysis is based on the assumption that $l_{yr} > l_{xr}$. If this is not the case, the yield line pattern will be as shown in diagram (i)(b) on *Table 2.49*, and l_{xr} and l_{yr} should be transposed as shown in the following example.

Example. Design the slab in diagram (vii) on *Table 2.49* to support an ultimate load n per unit area, assuming that the relative moments of resistance are as shown.

Since $l_x = 4$ m, $i_2 = 3/2$ and $i_4 = 0$,

$$l_{xr} = \frac{2 \times 4}{\sqrt{1 + 3/2} + 1} = 3.10 \text{ m}$$

Since $l_y = 6$ m, $\mu = 1/2$, $i_1 = 1$ and $\mu i_3 = 0$,

$$l_{yr} = \frac{2 \times 6}{(\sqrt{1 + 1} + 1)\sqrt{1/2}} = 7.03 \text{ m}$$

Thus

$$M = \frac{n}{24} l_{xr}^2 \left(\sqrt{3 + \left(\frac{l_{xr}}{l_{yr}}\right)^2} - \frac{l_{xr}}{l_{yr}} \right)^2$$

$$= \frac{n}{24} \times 3.10^2 \left(\sqrt{3 + \left(\frac{3.10}{7.03}\right)^2} - \frac{3.10}{7.03} \right)^2 = 0.726\, n$$

13.4.7 Superposition theorem

A problem that may arise when designing a slab to resist a combination of uniform, concentrated and line loads, some of which may not always occur, is that the critical pattern of yield lines may well vary for different combinations of loads. Also, it is theoretically incorrect to sum the ultimate moments obtained when considering the various loads individually, since these moments may result from different yield line patterns. However, Johansen has established the following superposition theorem:

The sum of the ultimate moments for a series of loads is equal to or greater than those due to the sum of the loads.

In other words, if the ultimate moments corresponding to the yield-line patterns for each load considered separately are added together, the resulting value is equal to greater than that of the system as a whole. This theorem is demonstrated in ref. 28, article 4.

13.4.8 Empirical virtual-work analysis

An important advantage of collapse methods of design is that they can be readily applied to solve problems such as slabs that are irregularly shaped or loaded, or that contain large openings. The analysis of such slabs using elastic methods is by comparison extremely arduous.

To solve such ‘one-off’ problems, it is clearly unrealistic to develop standard algebraic design formulae. The following empirical trial-and-adjustment technique, which involves a direct application of the virtual-work principles, is easy to master and can be used to solve complex problems. It is best illustrated, however, by working through a simple problem such as the one considered in section 13.4.6. There is, of course, no need to employ the procedure in this case. It is used here only to illustrate the method. A more complicated example is given in ref. 28, article 1, on which the description of the method is based.

In addition to the fundamental principles of virtual work discussed in section 13.4.4, the present method depends also on the following principle. If all yield lines (other than those along the supports) are positive and if none of them meets an unsupported edge except at right angles, then no forces due to shear or torsion can occur at the yield lines. Thus, a separate virtual-work balance for each slab area demarcated by the yield lines can be taken.

Example. Consider the slab shown in diagram (vii) on *Table 2.49*, which is continuous over two adjacent edges, freely supported at the others, and subjected to a uniform load n per unit area. The ratios of the moments of resistance provided over the continuous edges, and in the secondary direction to that in the principal direction, are as shown.

The step-by-step trial and adjustment process is as follows:

1. Postulate a likely yield-line pattern.
2. Give a virtual displacement of unity at some point and calculate the relative displacement of any other yield-line intersection points. For the case considered, if O is given a displacement of unity, the displacement at O′ will also be unity, since OO′ is parallel to the axes of rotation of the adjoining slab areas.
3. Choose reasonable arbitrary values for the dimensions that must be determined to define the yield-line pattern. Thus, in the example, initially αl_y is taken as 2 m, βl_y as 1m and ζl_x as 2.5 m.
4. Calculate the actual work done by the load n per unit area and the internal work done by the moments of resistance M, for each separate part of the slab, and thus obtain ratios of M/n for each part.

For example, on area A, the total load is $1/2 \times 4 \times 2 \times n = 4n$. Since the centre of gravity moves through a distance of 1/3, the work done by the load on area A is $4n \times 1/3 = 4n/3$. Now, since O is displaced by unity, the rotation of area A about the support is $1/\alpha l_y = 1/2$. The moments of $M/2$, acting across both the positive and negative yield lines, each exert a total moment of $M/2 \times 4 = 2M$. Thus, the total internal work done in rotating area A is $(2M + 2M) \times 1/2 = 2M$. Equating the internal and external work done on area A gives $2M = 4n/3$, that is, $M/n = 2/3$. Similarly, for area C, $M/n = 1/3$.

For convenience, area B can be divided it into a rectangle (of size 3 m × 2.5 m) and two triangles, and the work done on each part calculated separately. Since the centres of gravity move through a distance of 1/2 for the rectangle and 1/3 for each triangle, the external work done is as follows:

Rectangular area	$3 \times 2.5 \times 1/2 \times n = 3.75n$
Triangular areas	$1/2 \times (2 + 1) \times 2.5 \times 1/3 \times n = \underline{1.25n}$
Total	$= \underline{5.00n}$

Since the rotation is 1/2.5, the work done by the moments is $(1.5M + M) \times 6 \times 1/2.5 = 6M$. Thus the virtual-work ratio is $M/n = 5/6 = 0.833$. Likewise, for area D, $M/n = 3/4$.

5. Sum the separate values of internal and external work done, for the various slab areas, and thus obtain a ratio of $\Sigma M/\Sigma n$ for the entire slab. This ratio will be lower than the critical value, unless the dimensions chosen arbitrarily in step 3 happen to be correct. The calculations are best set out in tabular form as follows:

Area	External work done	Internal work done	M/n
A	$1/2 \times 4 \times 2 \times 1/3 \times n = 1.333n$	$[M/2 + M/2] \times 4 \times 1/2 = 2.000M$	0.667
B	$[3 \times 2.5 \times 1/2 + 1/2 \times 3 \times 2.5 \times 1/3] \times n = 5.000n$	$[3M/2 + M] \times 6 \times 1/2.5 = 6.000M$	0.833
C	$1/2 \times 4 \times 1 \times 1/3 \times n = 0.667n$	$[0 + M/2] \times 4 \times 1/1 = 2.000M$	0.333
D	$[3 \times 1.5 \times 1/2 + 1/2 \times 3 \times 1.5 \times 1/3] \times n = 3.000n$	$[0 + M] \times 6 \times 1/1.5 = 4.000M$	0.750
	Total $= 10.000n$	Total $= 14.000M$	0.714
A	$1/2 \times 4 \times 2.071 \times 1/3 \times n = 1.381n$	$[M/2 + M/2] \times 4 \times 1/2.071 = 1.931M$	0.714
B	$[2.467 \times 2.424 \times 1/2 + 1/2 \times 3.533 \times 2.424 \times 1/3] \times n = 4.418n$	$[3M/2 + M] \times 6 \times 1/2.424 = 6.188M$	0.714
C	$1/2 \times 4 \times 1.462 \times 1/3 \times n = 0.975n$	$[0 + M/2] \times 4 \times 1/1.462 = 1.368M$	0.714
D	$[2.467 \times 1.576 \times 1/2 + 1/2 \times 3.533 \times 1.576 \times 1/3] \times n = 2.872n$	$[0 + M] \times 6 \times 1/1.576 = 3.807M$	0.754
	Total $= 9.646n$	Total $= 13.294M$	0.726

6. By comparing the overall ratio obtained for $\Sigma M/\Sigma n$ with those due to each individual part, it is possible to see how the arbitrary dimensions should be adjusted so that the ratios for the individual parts become approximately equal to each other and to that of the slab as a whole.

The foregoing table shows calculations for initial values of M/n, and also for a set of adjusted values. An examination of the initial values (for which the overall ratio is 0.714) shows that, to obtain the same ratio for each area, M/n needs to be increased for areas A and C, and reduced for areas B and D. For area A, since M/n is proportional to $(\alpha l_y)^2$, αl_y needs to be increased to $\sqrt{(0.714/0.667)} \times 2 = 2.071$. Similarly, for area C, βl_y needs to be increased to $\sqrt{(0.714/0.333)} \times 1 = 1.462$. If, for area B, the external work done is recalculated using the corrected values of αl_y and βl_y, this gives

$$[2.467 \times 2.5 \times 1/2 + 1/2 \times 3.533 \times 2.5 \times 1/3] \times n = 4.556n$$

The internal work done by the moments is unchanged, and so the revised value of M/n is 4.556/6 = 0.759. Thus since, for area B, M/n is proportional to $(\zeta l_x)^2$, ζl_x needs to be reduced to $\sqrt{(0.714/0.759)} \times 2.5 = 2.424$. For area D, the external work done is recalculated using the corrected values of all the variable dimensions, and the revised value of M/n obtained.

7. Repeat steps 4 and 5, using the adjusted values for the arbitrary dimensions, as shown earlier.
8. Repeat this cyclic procedure until reasonable agreement is obtained between the values of M/n. This ratio gives the value of M, for which the required reinforcement must be determined, for a given load n. In the example, the ratios given by the second cycle are quite satisfactory. Note that, although some of the dimensions originally guessed were not particularly accurate, the resulting error in the value of M/n obtained for the whole slab was only about 1.5%, and the required load-carrying capacity is not greatly affected by the accuracy of the arbitrary dimensions.

Concentrated loads and line loads occurring at boundaries between slab areas should be divided equally between the areas that they adjoin, and their contribution to the external work done assessed as described in section 13.4.5.

As in all yield-line theory, the above analysis is only valid if the yield-line pattern considered is the critical one. Where there is a reasonable alternative, both patterns should be investigated to determine which is critical.

13.4.9 Corner levers

Tests and elastic analyses of slabs show that the negative moments along the edges reduce to zero near the corners and increase rapidly away from these points. Thus, in slabs that are fixed or continuous at their edges, negative yield lines tend to form across the corners and, in conjunction with pairs of positive yield lines, result in the formation of additional triangular slab elements known as corner levers, as shown in diagram (i)(a) on *Table 2.50*. If the slab is freely supported, a similar mechanism is induced, causing the corners to lift as shown in diagram (1)(b). If these mechanisms are substituted for the original yield lines running into the corner of the slab, the overall strength of the slab is correspondingly decreased by an amount depending on the factors listed on *Table 2.50*. For a corner lever having an included angle of not less than 90°, the strength reduction is not likely to exceed 8–10%. In such cases, the main reinforcement can be increased slightly, and top reinforcement provided at the corners of the slab to restrict cracking. Recommendations taken from the Swedish Code of Practice are shown in diagram (ii) on *Table 2.50*.

For acute-angled corners, the decrease in strength is more severe. For a triangular slab ABC where no corner angle is less than 30°, Johansen (ref. 25) suggests that the calculated strength without corner-lever action should be divided by a factor k, given by the approximation

$$k = (7.4 - \sin \mathrm{A} - \sin \mathrm{B} - \sin \mathrm{C})/4$$

A mathematical determination of the true critical dimensions of an individual corner lever involves much complex trial and adjustment. However, this is unnecessary, as Jones and Wood (ref. 23) have devised a direct design method that gives corner levers, having dimensions such that the resulting adjustment in strength is similar to that due to the true mechanisms. This design procedure is summarised on *Table 2.50* and illustrated by the following example.

The formulae derived by Jones and Wood, and on which the graphs in *Table 2.50* are based, are as follows:

With fixed edges:

$$k_1 = \left[\frac{1}{K_1 \sin^2(\theta/2)} - 1\right]\sqrt{6(1+i)}\,\sec(\theta/2)$$

$$k_2 = \frac{k_1}{\cos(\theta/2) - \cot\psi \sin(\theta/2)} \text{ where}$$

$$K_1 = \sqrt{4 + 3\cot^2(\theta/2)} - 1 \text{ and } \cot\psi = (K_1 - 1)\tan(\theta/2)$$

With freely supported edges:

$$k_1 = \left[\sqrt{K_2} - 2\sqrt{(1+\mathrm{i})}\right]\sqrt{2/3}\,\sec(\theta/2)$$

$$k_2 = \frac{k_1}{\cos(\theta/2) - \cot\psi \sin(\theta/2)} \text{ where}$$

$$K_2 = (4 + \mathrm{i}) + 3\cot^2(\theta/2) \text{ and}$$

$$\cot\psi = [\sqrt{K_2(1+\mathrm{i})} - (2 + \mathrm{i})]\tan(\theta/2)$$

Example. Calculate the required resistance of the 5 m square slab with fixed edges ($i = 1$) shown on *Table 2.50*.

For the transformed freely supported slab, the reduced side length $l_r = l/\sqrt{(1 + I)} = 5/\sqrt{2} = 3.54$ m. Then, for a square slab, if the formation of corner levers is ignored, the required resistance moment $M = (1/24)nl_r^2 = 0.041 \times 3.54^2 n = 0.521n$.

Now, from the appropriate graph on *Table 2.50*, with $i = 1$ and $\theta = 90°$, read off $k_1 = 1.1$ and $k_2 = 4.2$. Thus, dimensions $a_1 = 1.1\sqrt{0.521} = 0.794$ m and $a_2 = 4.2\sqrt{0.521} = 3.032$ m.

By plotting these values on a diagram of the slab, it is now possible to calculate the revised resistance moment required. If the deflection at the centre is unity, the relative deflection at the apex of the corner levers is 3.032/3.536 = 0.858. Thus, the revised virtual-work equation is

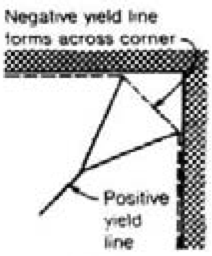
$$4 \times 2M\left[3.412\left(\frac{1}{2.5}\right) + 0.794\sqrt{2}\left(\frac{0.858}{2.471}\right)\right]$$

$$= [(1/3) \times 5^2 - 4 \times (1/2) \times 0.794^2 \times (1/3) \times 0.858]\,n$$

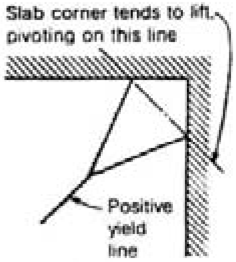
This reduces to $14.036M = 7.973n$ so that $M = 0.568n$.

Two-way slabs: yield-line theory: corner levers

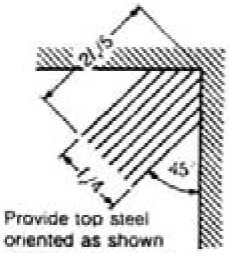
2.50



(i)(a)

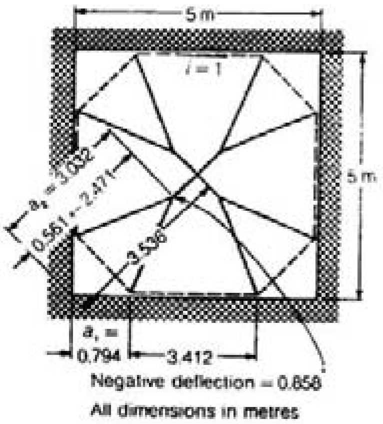


(i)(b)



Provide top steel oriented as shown to resist moment of $nl_xl_y/40$ per unit width

(ii)



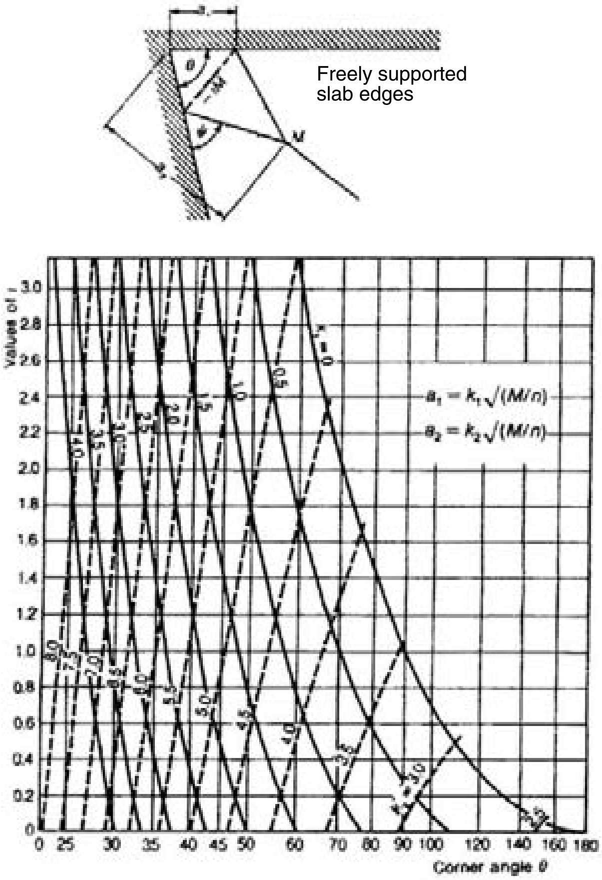
Formation of corner levers (and also circular fans)	
More likely with:	**Less likely with:**
Heavy concentrated loads	Distributed loads only
Acute-angled corners	Obtuse-angled corners
Fixed or continuous edges	Freely supported edges
Unsupported edges (particularly where opposite corners)	No unsupported edges
No top steel in corners	Top steel in corners

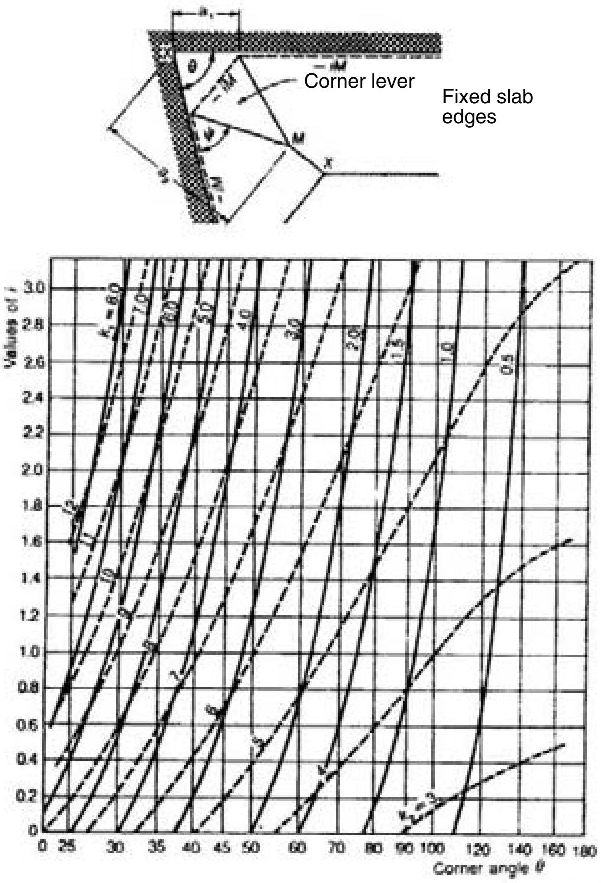
Individual conditions listed are additive. Thus corner levers are most likely to form if several or all factors apply. However, heavy loads is most influential cause.

Design procedure for corner levers

1. Establish the ultimate resistance of the slab without taking corner levers into account.
2. With known values of the corner angle θ and the negative reinforcement factor i, calculate distances a_1 and a_2 by means of the accompanying graphs.
3. Using these dimensions, plot the corner levers on a diagram of the yield lines. If the calculated value of a_2 exceeds the distance XX on the diagram, adopt the length given by the original yield-line pattern.
4. Recalculate the revised ultimate resistance moment for the slab using the new node points established in this way, by means of virtual work.

This procedure is illustrated in the example in section 13.4.9.





Note that this value of M is 9% greater than the uncorrected value; in other words, the load supported by a square slab with a specified moment of resistance is actually 9% less than that calculated, when corner levers are not taken into account.

13.5 HILLERBORG'S SIMPLE STRIP THEORY

13.5.1 Moments in slabs

According to lower-bound (equilibrium) theory, load acting on a slab is resisted by a combination of biaxial bending and torsion. In the simple strip method, the torsion moment is taken as zero and load (or partial load), acting at any position on the slab, is resisted by bending in one of two principal directions. Thus, in diagram (i) on *Table 2.51*, the load acting on the shaded areas is resisted by bending in direction l_y, and the load acting on the remaining area is resisted by bending in direction l_x. In principle, there is an unlimited number of ways of apportioning the load, each of which will lead to a different reinforcement layout while still meeting the collapse criteria. However, the loading arrangement selected should also ensure that the resulting design is simple, economical, and satisfactory with regard to deflection and cracking under service loads.

Some possible ways of apportioning the load on a freely supported rectangular slab are shown in diagrams (i)–(iv) on *Table 2.51*, the notation adopted being given on the table. Perhaps the most immediately obvious arrangement is shown in diagram (i), for which Hillerborg originally suggested that θ could be taken as 45° where both adjacent edges are freely supported. However, in ref. 29 he recommends that θ should be made equal to $\tan^{-1}(l_y/l_x)$, as shown in diagram (i). The disadvantage of the arrangement shown is that the bending moment (and thus the reinforcement theoretically required) varies across strips 2 and 3. Since it is impractical to vary the reinforcement continuously, the usual approach is to calculate the total moment acting on the strip, divide by the width of the strip to obtain the average moment and provide a uniform distribution of reinforcement to resist this moment. To avoid having to integrate across the strip to obtain the total moment, Hillerborg recommends calculating the moment along the centreline of the strip and then multiplying this value by the correction factor

$$1 + \frac{(l_{\max} - l_{\min})^2}{3(l_{\max} + l_{\min})^2}$$

where $l_{\max}$ and $l_{\min}$ are the maximum and minimum loaded lengths of the strip. Strictly speaking, averaging the moments as described violates the principles on which the method is based, and this device should only be used where the factor of safety will not be seriously impaired. If the width of the strip over which the moments are to be averaged is large, it is better to sub-divide it and calculate the average moment for each separate part.

An alternative arrangement that avoids the need to average the moments across the strips is shown in diagram (ii). This has disadvantages in that six different types of strip (and thus six different reinforcement layouts) must be considered, and that, in strip 6, no moment theoretically occurs. Such a strip must, nevertheless, contain distribution reinforcement.

So far, the load on any one separate area has been carried in one direction only. In diagram (iii), however, the loads on the corner areas are so divided that one-half is carried in each direction. Hillerborg (ref. 29) states that this very simple and practical approach never requires more than 10% additional reinforcement, when compared to the theoretically more exact but less practical solution shown in diagram (i), when l_y/l_x is between 1.1 and 4. An additional sophistication that can be introduced is, to apportion the load in each direction in the two-way spanning areas in such a way, that the resulting reinforcement across the shorter span corresponds to the minimum requirement for secondary reinforcement. Details of this and similar stratagems are given in ref. 29.

Diagram (iv) illustrates yet another arrangement that may be considered. By dividing the corner areas into triangles and averaging the moments over these widths as described earlier, Hillerborg shows that the moments in the side strips can be reduced to two-thirds of the values given by the arrangement used in diagram (iii).

13.5.2 Loads on supporting beams

A particular feature of the strip method is that the boundaries between the different loaded areas also define the manner in which the loads are transferred to the supporting beams. For example, in diagram (i) the beams in direction l_x support triangular areas of slab giving maximum loads of $nl_x^2/2l_y$ at their centres.

13.6 BEAMS SUPPORTING RECTANGULAR PANELS

The loads on beams supporting uniformly loaded rectangular slab panels are distributed approximately as a triangular load on the beams along the shorter edges l_x, and a trapezoidal load on the beams along the longer edges l_y, as shown in the diagrams on *Table 2.52*. For a beam supporting a single panel of slab that is either freely supported or subjected to the same degree of restraint along all four edges, where the beam span is equal to the length (or width) of the panel, the equivalent UDL per unit length on the beam for the calculation of bending moments only is as follows:

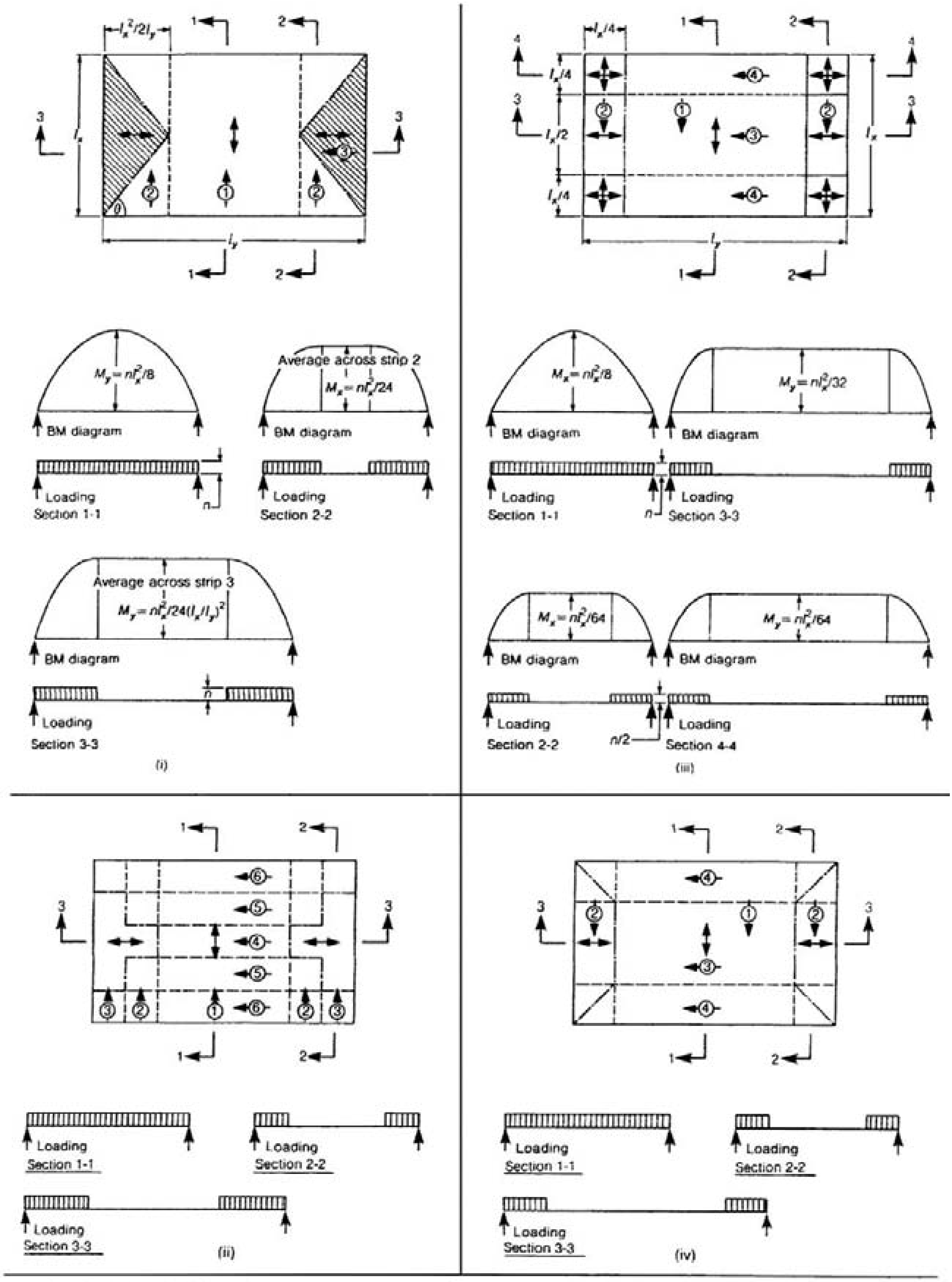
Short-span beam:	$nl_x/3$
Long-span beam:	$(1 - 1/3k^2)\, nl_x/2$

where n is the total UDL per unit area on the slab, appropriate to the limit-state being considered, and $k = l_y/l_x$. For a beam supporting two identical panels, one on either side, the foregoing equivalent loads are doubled. If a beam supports more than one panel in the direction of its length, the distribution of load is in the form of two or more triangles (or trapeziums) and the foregoing formulae are not applicable; in such a case, however, it is sufficiently accurate if the total load on the beam is considered to be uniformly distributed.

For slabs designed in accordance with the BS 8110 method, the loads on the supporting beams may be determined from the slab shear forces given in *Table 2.43*. The loads are to be taken as uniformly distributed along the middle three-quarters of the beam length, where the shear force for the short span of the slab is the load on the long-span beam and vice versa. The resulting beam fixed-end moments can be determined from *Table 2.28*.

2.51

Two-way slabs: Hillerborg's simple strip theory



---- boundary between individual strips

-·-·- boundary between load-carrying areas of slab

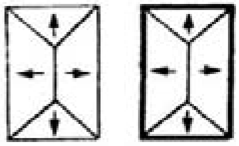
-①→ reference number of strip spanning in direction shown

←→ direction in which load over area indicated is supported

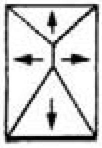
Two-way slabs: rectangular panels: loads on beams (common values)

2.52

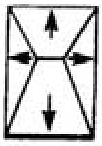
Panels supported along four edges



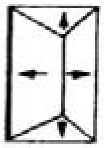
$k > 1$: $R_1 = R_3 = \frac{1}{4}wl_x^2$
$R_2 = R_4 = \frac{1}{2}(k - \frac{1}{2})wl_x^2$
$\alpha = \beta = 1/2k$
$k = 1$: $R_1 = R_2 = R_3 = R_4 = \frac{1}{4}wl_x^2$



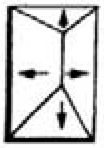
$k \nless 4/3$: $R_1 = \frac{1}{4}wl_x^2$ (min.) $\qquad \alpha = 1/2k$ (min.)
$R_2 = R_4 = \frac{1}{2}(k - \frac{2}{3})wl_x^2$
$R_3 = \frac{5}{12}wl_x^2$ (max.) $\qquad \beta = 5/6k$ (max.)



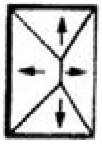
$k \leqslant 4/3$: $R_1 = \frac{3}{5}R_3$ approx. (min.) $\qquad \alpha = 3/8$ (min.)
$\beta = 5/8$ (max.)
$R_2 = R_4 = \frac{3}{16}k^2wl_x^2 \qquad \psi = \xi = 3k/8$
$R_3 = \frac{3}{8}k(1 - \frac{3}{8}k)wl_x^2$ approx. (max.)



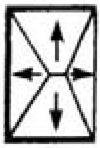
$R_1 = R_3 = \frac{3}{16}wl_x^2$
$R_2 = \frac{3}{5}R_4$ (min.)
$R_4 = \frac{5}{8}(k - \frac{3}{8})wl_x^2$ (max.)
$\alpha = \beta = 3/8k \qquad \psi = \frac{5}{8}$ (max.)



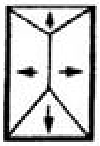
$R_1 = \frac{3}{16}wl_x^2$ (min.) $\qquad \alpha = 3/8k$ (min.)
$R_2 = \frac{3}{5}R_4$ (min.) $\qquad \beta = 5/8k$ (max.)
$R_3 = \frac{5}{16}wl_x^2$ (max.) $\qquad \psi = 5/8$ (max.)
$R_4 = \frac{5}{8}(k - \frac{1}{2})wl_x^2$ (max.)



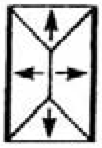
$k \nless 5/4$: $R_1 = R_3 = \frac{5}{16}wl_x^2 \qquad \alpha = \beta = 5/8k$
$R_2 = \frac{3}{5}R_4$ (min.) $\qquad \psi = 5/8$ (max.)
$R_4 = \frac{5}{8}(k - \frac{5}{8})wl_x^2$(max.)



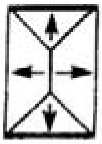
$k \leqslant 5/4$: $R_1 = R_3 = \frac{1}{2}k(1 - \frac{3}{5}k)wl_x^2 \qquad \alpha = \beta = 1/2$
$R_2 = \frac{3}{20}k^2wl_x^2$ (min.)
$R_4 = \frac{1}{4}k^2wl_x^2$ (max.) $\qquad \psi = k/2$
$\xi = 3k/10$



$R_1 = \frac{3}{20}wl_x^2$ (min.) $\qquad \alpha = 3/10k$ (min.)
$R_2 = R_4 = \frac{1}{2}(k - \frac{2}{3})wl_x^2$
$R_3 = \frac{1}{2}wl_x^2$ (max.) $\qquad \beta = 1/2k$ (max.)

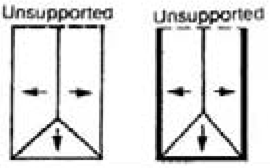


$R_1 = R_3 = \frac{3}{20}wl_x^2$ (min.)
$R_2 = R_4 = \frac{1}{2}(k - \frac{3}{10})wl_x^2$ (max.)
$\alpha = \beta = 3/10k$ (min.)

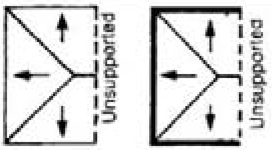


$k \nless 5/3$: $R_1 = R_3 = \frac{5}{12}wl_x^2$ (min.)
$R_2 = R_4 = \frac{1}{2}(k - \frac{5}{6})wl_x^2$ (max.)
$\alpha = \beta = 5/6k$ (min.)

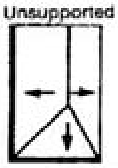
Panels unsupported along one edge



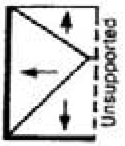
$R_1 = 0$
$R_2 = R_4 = \frac{1}{2}(k - \frac{1}{4})wl_x^2$
$R_3 = \frac{1}{4}wl_x^2$
$\beta = 1/2k$



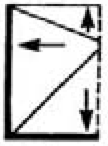
$k \ngtr 2$: $R_1 = R_3 = \frac{1}{2}k(1 - \frac{1}{4}k)wl_x^2$
$R_2 = 0$
$R_4 = \frac{1}{4}k^2wl_x^2$
$\psi = k/2$



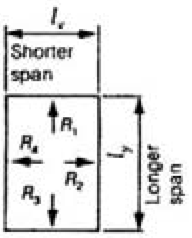
$R_1 = 0 \qquad \beta = 5/8k$
$R_2 = \frac{3}{5}R_4$ (min.) $\qquad \psi = 5/8$
$R_3 = \frac{5}{16}wl_x^2$
$R_4 = \frac{5}{8}(k - \frac{5}{16})wl_x^2$ (max.)



$k \ngtr 8/5$: $R_1 = \frac{3}{5}R_3$ (min.) $\quad R_2 = 0$
$R_3 = \frac{5}{8}k(1 - \frac{5}{16}k)wl_x^2$ (max.)
$R_4 = \frac{5}{16}k^2wl_x^2 \qquad \alpha = 3/8k$ (min.)
$\psi = 5k/8$ (max.)



$k \geqslant 8/5$: $R_1 = \frac{3}{10}wl_x^2$ (min.) $\quad R_2 = 0$
$R_3 = \frac{1}{2}wl_x^2$
$R_4 = (k - \frac{4}{5})wl_x^2$ (max.)
$\alpha = 3/5k \quad \beta = 1/k$



$k = \frac{l_y}{l_x} = \frac{\text{longer span}}{\text{shorter span}}$

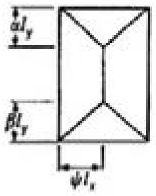
w = intensity of uniformly distributed service load per unit area

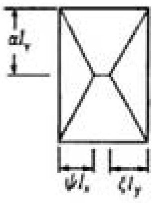
If analysis due to ultimate loads is undertaken, substitute n for w in appropriate formulae

R_1, R_2, R_3, R_4 = total load carried by each support of panel

Condition of supports
- - - - - - - - no support
———— freely supported
▬▬▬▬ continuity or fixity

Loads marked (min.) apply if panel is entirely freely supported along edge indicated: if partially restrained, load will be slightly greater than given and load marked (max.) on opposite edge will be correspondingly reduced.





13.7 TRIANGULARLY DISTRIBUTED LOADS

In the design of rectangular tanks, storage bunkers and some retaining structures, conditions occur of wall panels spanning in two directions, and subjected to distributions of pressure varying linearly from zero at or near the top to a maximum at the bottom. For liquid-retaining structures, with no provision for additional protection in the form of an internal lining or external tanking, an elastic analysis is normally necessary as a basis for checking serviceability cracking. In other cases, an analysis based on collapse methods may be justified.

13.7.1 Elastic analysis

The coefficients given in *Table 2.53* enable the maximum values of bending moments and shearing forces on vertical and horizontal strips of unit width to be determined for panels of different aspect ratios and edge conditions. The latter are taken as fixed at the sides, hinged or fixed at the bottom and hinged or free at the top. The coefficients, which are taken from ref. 32, were derived by a finite element analysis and include for a Poisson's ratio of 0.2. For ratios less than 0.2, the moments could be adjusted in the manner described in section 13.2.2.

The maximum negative bending moment at the bottom edge and the maximum shear forces at the bottom and top edges occur halfway along the panel. The other maximum moments occur at the positions indicated in the following table.

Distance from bottom of panel to position of maximum horizontal moments (negative/positive)

Type of panel	Height/l_z for values of l_x/l_z				
	0.5	1.0	1.5	2.0	4.0
1	0.3	0.5	0.5	0.5	0.5
2	0.3	0.5	0.7/1.0	0.9/1.0	1.0
3	0.3	0.4	0.4	0.4	0.4
4	0.4	0.4	0.5/0.6	0.9/1.0	0.9/1.0

Distance from bottom of panel to position of maximum vertical positive moment

Type of panel	Height/l_z for values of l_x/l_z				
	0.5	1.0	1.5	2.0	4.0
1	0.3	0.5	0.5	0.5	0.5
2	0.3	0.4	0.5	0.6	0.7
3	0.2	0.3	0.3	0.4	0.4
4	0.2	0.3	0.3	0.4	0.4

For a complete map of bending moment values, at intervals of one-tenth of the panel height and length, see ref. 32. The moments obtained for an individual panel apply directly to a square tank with hydrostatic loading. For a rectangular tank, a further distribution of the unequal negative moments at the corners is needed (see *Tables 2.75* and *2.76*).

Example. Determine, due to internal hydrostatic loading, the maximum service moments in the walls of a square tank that can be considered as free along the top edge and hinged along the bottom edge. The tank is 5 m square × 4 m deep, and the water level is to be taken to the top of the walls.

From *Table 2.53*, for panel type 4 with $l_x/l_z = 5/4 = 1.25$, the maximum bending moments are as follows:

Horizontal negative moment at corners

$$m_x = 0.050 \times 9.81 \times 4^3 = 31.4 \text{ kNm/m}$$

Horizontal positive moment (at about $0.5l_z = 2$ m above base)

$$m_x = 0.022 \times 9.81 \times 4^3 = 13.8 \text{ kNm/m}$$

Vertical positive moment (at about $0.3l_z = 1.2$ m above base)

$$m_z = 0.021 \times 9.81 \times 4^3 = 13.2 \text{ kNm/m}$$

13.7.2 Yield-line method

A feature of the collapse methods of designing two-way slabs is that the designer is free to choose the ratios between the total moments in each direction, and between the positive and negative moments. In the case of liquid-retaining structures, where it is important to ensure that the formation of cracks under service load is minimised, the ratios selected should correspond approximately to those given by elastic analysis. The following design procedure is thus suggested:

1. Obtain maximum positive and negative service moment coefficients from *Table 2.53*.
2. Determine μ, i_1 ($= i_3$), and i_4, where $\mu = \alpha_{\text{mh,pos}}/\alpha_{\text{mz,pos}}$, $i_1 = i_3 = \alpha_{\text{mh,neg}}/\alpha_{\text{mh,pos}}$ and $i_4 = \alpha_{\text{mz,neg}}/\alpha_{\text{mz,pos}}$.
3. Calculate l_{xr} and l_{yr}, if the top edge is unsupported, from

$$l_{xr} = \frac{l_x}{\sqrt{1 + i_4}} \quad \text{and} \quad l_{yr} = \frac{2l_y}{[\sqrt{1 + i_1} + \sqrt{1 + i_3}]\sqrt{\mu}}$$

and, if the top edge is freely supported, from

$$l_{xr} = \frac{2l_x}{1 + \sqrt{1 + i_4}} \quad \text{and} \quad l_{yr} = \frac{2l_y}{[\sqrt{1 + i_1} + \sqrt{1 + i_3}]\sqrt{\mu}}$$

4. Obtain M, if the top edge is unsupported, from the chart on *Table 2.54* and, if the top edge is supported, from the scale on *Table 2.49* according to the values of f (or $n = f/2$), l_x (or l_{xr}), l_{yr} and i_4. The basis of this approach is given below.

Top edge of slab unsupported. In ref. 25 Johansen derives the following 'exact' formulae according to the failure mode.

For failure mode 1:

$$M = \frac{fl_x^2}{12k}$$

where k is obtained by solving the quadratic equation

$$4\left[1 + i_4 + \left(\frac{l_x}{l_{yr}}\right)^2\right]k^2 - 4\left[i_4(1 + i_4) + (6 + 4i_4)\left(\frac{l_x}{l_{yr}}\right)^2\right]k + i_4^2\left[1 + i_4 + 4\left(\frac{l_x}{l_{yr}}\right)^2\right] = 0$$

For failure mode 2:

$$M = \frac{fl_{yr}^2}{96}(6 - 8\xi + 3\xi^2)$$

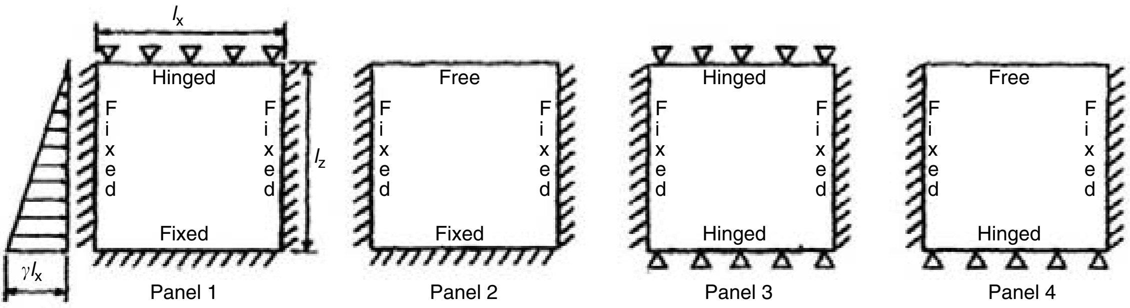
where ξ is obtained by solving the following cubic equation:

$$8\left(\frac{l_{xr}}{l_{yr}}\right)^2\xi^3 + \left[3\text{–}16\left(\frac{l_{xr}}{l_{yr}}\right)^2\right]\xi^2 - 8\xi + 6 = 0$$

Two-way slabs: triangularly distributed load (elastic analysis)

2.53

Rectangular panels with provision for torsion at corners										
Type of panel with moments and shears considered		Coefficients for values of l_x/l_z								
		0.5	0.75	1.0	1.25	1.5	2.0	2.5	3.0	4.0
1. Top hinged, bottom fixed										
Negative moment at side edge	α_{mx}	0.012	0.022	0.029	0.033	0.036	0.037	0.037	0.037	0.037
	α_{mz}	0.002	0.004	0.006	0.007	0.007	0.007	0.007	0.007	0.007
Positive moment for span l_x	α_{mx}	0.006	0.010	0.012	0.013	0.012	0.010	0.009	0.009	0.009
Negative moment at bottom edge	α_{mz}	0.011	0.023	0.035	0.045	0.053	0.062	0.065	0.066	0.067
	α_{mx}	0.002	0.005	0.007	0.009	0.011	0.012	0.013	0.013	0.013
Positive moment for span l_z	α_{mz}	0.003	0.007	0.011	0.016	0.021	0.026	0.028	0.029	0.029
Shear force at side edge	α_{vx}	0.17	0.22	0.24	0.25	0.26	0.26	0.26	0.26	0.26
Shear force at bottom edge	α_{vz}	0.20	0.26	0.32	0.36	0.38	0.40	0.40	0.40	0.40
Shear force at top edge	α_{vz}	0.03	0.05	0.07	0.09	0.11	0.11	0.11	0.11	0.10
2. Top free, bottom fixed										
Negative moment at side edge	α_{mx}	0.012	0.022	0.030	0.037	0.044	0.066	0.082	0.091	0.099
	α_{mz}	0.002	0.004	0.006	0.007	0.009	0.013	0.016	0.017	0.020
Positive moment for span l_x	α_{mx}	0.006	0.010	0.013	0.016	0.021	0.028	0.028	0.024	0.017
Negative moment at bottom edge	α_{mz}	0.011	0.023	0.035	0.048	0.061	0.086	0.109	0.127	0.149
	α_{mx}	0.002	0.005	0.007	0.010	0.012	0.017	0.022	0.025	0.030
Positive moment for span l_z	α_{mz}	0.003	0.007	0.010	0.013	0.015	0.016	0.014	0.011	0.007
Shear force at side edge	α_{vx}	0.17	0.22	0.24	0.25	0.26	0.27	0.33	0.37	0.38
Shear force at bottom edge	α_{vz}	0.19	0.26	0.32	0.36	0.40	0.45	0.48	0.50	0.50
3. Top hinged, bottom hinged										
Negative moment at side edge	α_{mx}	0.014	0.026	0.038	0.047	0.054	0.061	0.063	0.064	0.064
	α_{mz}	0.003	0.005	0.008	0.009	0.011	0.012	0.013	0.013	0.013
Positive moment for span l_x	α_{mx}	0.007	0.012	0.017	0.019	0.021	0.020	0.018	0.017	0.017
Positive moment for span l_z	α_{mz}	0.004	0.009	0.015	0.023	0.031	0.045	0.054	0.059	0.063
Shear force at side edge	α_{vx}	0.20	0.26	0.32	0.35	0.38	0.40	0.41	0.41	0.41
Shear force at bottom edge	α_{vz}	0.11	0.16	0.20	0.23	0.26	0.30	0.32	0.33	0.33
Shear force at top edge	α_{vz}	0.01	0.03	0.05	0.07	0.10	0.13	0.15	0.16	0.17
4. Top free, bottom hinged										
Negative moment at side edge	α_{mx}	0.014	0.026	0.038	0.050	0.063	0.098	0.150	0.205	0.317
	α_{mz}	0.003	0.005	0.008	0.010	0.013	0.020	0.030	0.041	0.063
Positive moment for span l_x	α_{mx}	0.007	0.012	0.017	0.022	0.028	0.046	0.062	0.074	0.089
Positive moment for span l_z	α_{mz}	0.004	0.009	0.014	0.021	0.027	0.037	0.045	0.051	0.058
Shear force at side edge	α_{vx}	0.20	0.26	0.31	0.35	0.37	0.41	0.58	0.76	1.14
Shear force at bottom edge	α_{vz}	0.11	0.15	0.19	0.23	0.26	0.31	0.33	0.36	0.39



Note: Maximum values of moment per unit width and shear force per unit width are given by the following relationships, where l_x is panel length, l_z is panel height and γ is unit weight of liquid. For details of the positions at which the maximum values occur, see section 13.7.1.

Horizontal span: $m_x = \alpha_{mx} \gamma l_z^3$, $v_x = \alpha_{vx} \gamma l_z^2$ Vertical span: $m_z = \alpha_{mz} \gamma l_z^3$, $v_z = \alpha_{vz} \gamma l_z^2$

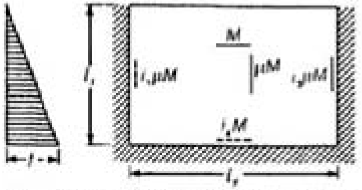
Two-way slabs: triangularly distributed load (collapse method)

2.54

Yield-line theory

Top edge fixed or freely supported

Determine moments from expressions for uniformly loaded slab (see section 13.4.6).



Simple strip theory: top edge fixed or freely supported

If $l_y > l_x$, assuming vertical and horizontal loads are apportioned as shown, if $x/l_x < 3/4$ (note: x is measured in terms of l_x throughout):

Moment on vertical strip 2–5:

$$M_{2-5} = \tfrac{1}{6} f l_x^2 x(1-x)(2-x) + i_2 x + i_4(1-x)$$

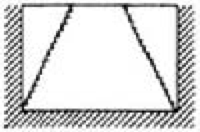
Maximum positive moment occurs at

$$x = 1 - \sqrt{\left[\frac{1}{3} - \frac{2(i_2 - i_4)}{f l_x^2}\right]}$$

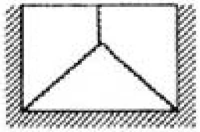
from bottom of wall.

Top edge unsupported

Read M/fl_x^2 corresponding to given values of i_4 and l_{yr}/l_x from accompanying graph.



Failure mode 1



Failure mode 2

Moment on vertical strips 1–4 and 3–6:

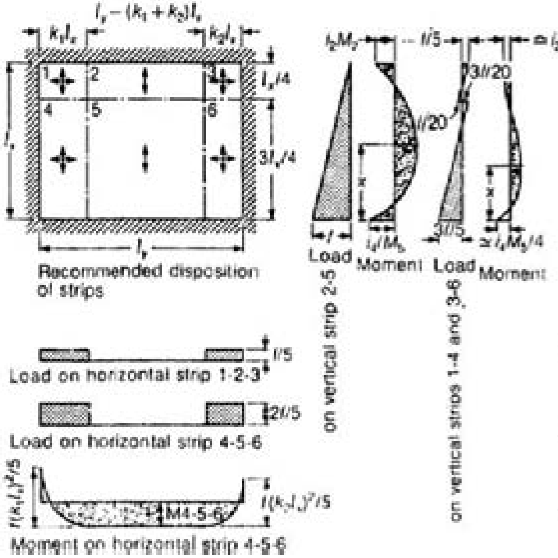
$$M_{3-6} = (1/480) f l_x^2 (80x^2 - 144x + 67)x + (1/4)[i_2 x + i_4(1-x)]$$

Maximum positive moment occurs at

$$x = \frac{3}{5} - \sqrt{\left[\frac{97}{1200} - \frac{1}{2}\frac{(i_2 - i_4)}{f l_x^2}\right]}$$

from bottom of wall.
Design procedure: Assume suitable values for all support moments and for horizontal span moments $M_{1\text{-}2\text{-}3}$ and $M_{4\text{-}5\text{-}6}$. Next calculate k_1 and k_2 and thus evaluate vertical span moments $M_{1\text{-}4}$ $(= M_{3\text{-}6})$ and $M_{2\text{-}5}$ and the positions at which these occur.

Top edge fixed or freely supported



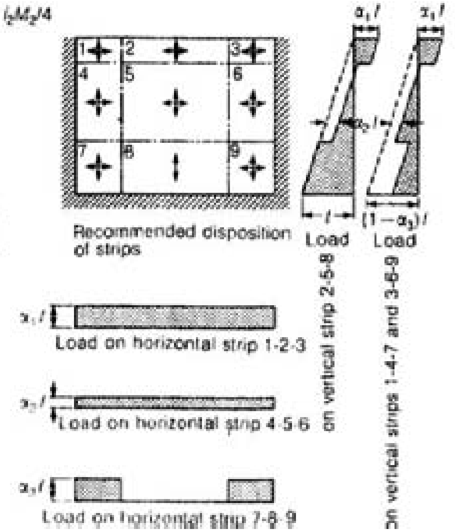
Recommended disposition of strips

Load on horizontal strip 1-2-3

Load on horizontal strip 4-5-6

Moment on horizontal strip 4-5-6

Top edge unsupported

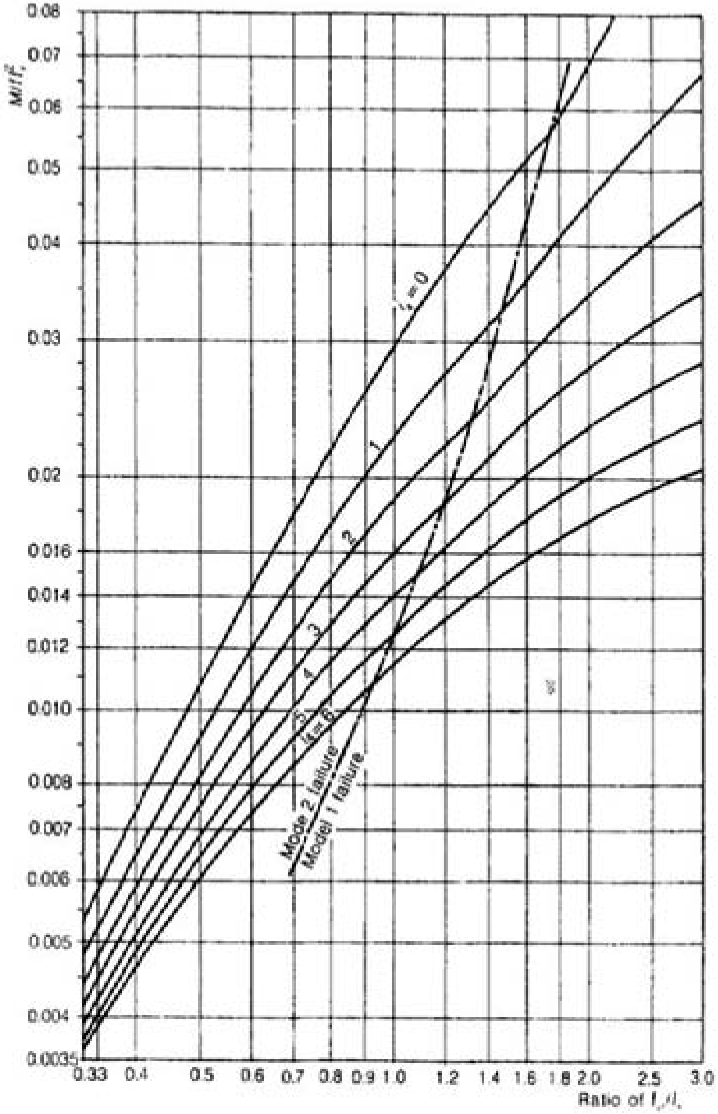


Recommended disposition of strips

Load on horizontal strip 1-2-3

Load on horizontal strip 4-5-6

Load on horizontal strip 7-8-9



The results of these calculations can be plotted graphically, as shown in *Table 2.54*, from which coefficients of M/fl^2_x can be read, corresponding to given values of l_{yr}/l_x and i_4. The chain line on the chart indicates the values at which the failure mode changes.

Top edge of slab supported. In ref. 25 Johansen shows that the total moments, resulting from yield-line analysis for triangularly loaded slabs, correspond to those obtained when the same slab is loaded with a uniform load of one-half the maximum triangular load (i.e. $n = f/2$). Hence, the design expressions in section 13.4.6 and the scale on *Table 2.49* for uniformly loaded slabs can again be used. As before, if the edges of the slab are restrained, the reduced side lengths l_{xr} and l_{yr} should be calculated and substituted in the formula, instead of the actual side lengths l_x and l_y.

Example. Determine moments by yield-line analysis for the walls of the square tank considered in the previous example in section 13.7.1.

Taking the service moment coefficients obtained previously, the following values are the most suitable for:

$$\mu = 0.022/0.021 = 1.0, \quad \text{and} \quad i_1\ (= i_3) = 0.050/0.022 = 2.3$$

Hence,

$$l_{yr} = \frac{2 \times 5}{2\sqrt{1 + 2.3\sqrt{1.0}}} = 2.75 \quad \text{and} \quad l_{yr}/l_x = 2.75/4.0 = 0.70$$

Then, from the chart on *Table 2.54*, with $i_4 = 0$,

$$M/fl_x^2 = 0.018$$

Comparing this value with $\alpha_{mx} = 0.022$, as obtained in the previous example, it can be seen that the elastic moments are 1.2 times those determined by yield-line analysis. Thus, even with a partial load factor of 1.2, the yield-line moments are no greater than the elastic service moments in this case.

Note. It can be seen from the chart on *Table 2.54* that failure mode 2 applies, for which

$$M = \frac{fl_{yr}^2}{96}(6 - 8\xi + 3\xi^2) = 0.018\,fl_x^2$$

Hence

$$3\xi^2 - 8\xi + 6 = 0.018 \times 96 \times (4.0/2.75)^2 = 3.66, \ \xi = 0.335$$

and yield-lines intersect at height $\xi\, l_x = 0.335 \times 4.0 = 1.34$ m above the base.

13.8 FLAT SLABS (SIMPLIFIED METHOD)

The following notes and the data in *Tables 2.55* and *2.56* are based on the recommendations for the simplified method of flat slab design given in BS 8110. Alternatively, and in cases where the following conditions are not met, the structure can be analysed by the equivalent frame method. Other methods of analysis such as grillage, finite-element and yield line may also be employed, in which case the provisions given for the distribution of bending moments are a matter of judgement.

13.8.1 Limitations of method

The system must comprise at least three rows of rectangular panels in each direction. The spans should be approximately equal in the direction being considered, and the ratio of the longer to the shorter sides of the panels should not exceed 2. The conditions allowing the design to be based on the single load case of all spans loaded with the maximum design load, as explained in section 4.4.1 and *Table 2.42*, must be met. All lateral forces must be resisted by shear walls or bracing.

13.8.2 Forms of construction

The slab can be of uniform thickness throughout or thickened drop panels can be introduced at the column positions. Drop panels can extend to positions beyond which the slab can resist punching shear, without needing shear reinforcement. Alternatively, the panels can be further extended to positions where they may be considered to influence the distribution of moments within the slab. In this case, the smaller dimension of the drop panel should be at least one-third of the smaller dimension of the surrounding slab panels.

Columns can be of uniform cross section throughout or can be provided with an enlarged head, the effective dimensions of which are limited according to the depth of the head, as shown in *Table 2.55*. For a flared head, the actual dimension is taken to be the value at a depth 40 mm below the underside of the slab or drop.

The effective diameter of a column, or column head, is the diameter of a circle whose area is equal to the cross-sectional area of the column, or effective column head. In no case, is the effective diameter h_c to be taken greater than one-quarter of the shortest span framing into the column. In cases where the edges of the slab are supported by walls, h_c can be taken as the thickness of the wall.

13.8.3 Bending moments and shearing forces

The total design bending moments and shearing forces given in *Table 2.55* are the same as those given for one-way slabs in *Table 2.42*, where the support moments include for 20% redistribution. The requirements are applied independently in each direction. Any negative moments greater than those at a distance $h_c/2$ from the centreline of the column may be ignored, subject to the following condition being met. In each span, the sum of the maximum positive moment and the average of the negative moments for the whole panel width, where F is the total design load on the panel and l is the panel length between column centrelines, must be not less than

$$(Fl/8)\,(1 - 2h_c/3l)^2$$

This condition is met, in the case of designs that are based on the single load case of all spans loaded with the maximum design load, by taking the design negative moment as the value at a distance $h_c/3$ from the centreline of the column. The moment at this position is obtained approximately by reducing the value at the column centreline by $0.15Fh_c$.

The total design moments on a panel should be apportioned between column and middle strips as shown in *Table 2.55*. In the following table, the moment allocations given in BS 8110 and EC 2 are shown for comparison.

Flat slabs: BS 8110 simplified method – 1

Total design bending moments and shearing forces in direction of span

Member	Slab							Supports	
	Type of connection at outer support				First interior support	Middle interior spans	Other interior supports	Continuous	
Position	Simple		Continuous						
	Outer support	End span	Outer support	End span				Outer	Interior
Moment	0	$0.086Fl$	$-0.04Fl$*	$0.075Fl$	$-0.086Fl$	$0.063Fl$	$-0.063Fl$	$0.04Fl$*	$0.02Fl$
Shear	$0.4F$	—	$0.46F$	—	$0.6F$	—	$0.5F$	—	—

Notes: The support moments include for 20% redistribution (F is total design load on full panel, with width taken between centre-lines of adjacent bays, and l is length between centre-lines of supports). Slab moments at supports may be reduced by $0.15Fh_c$, where h_c is diameter of column or column head, or thickness of wall. Moments at supports are to be divided between upper and lower columns, or walls, in proportion to stiffness. For punching shear considerations, see *Table 2.56*.

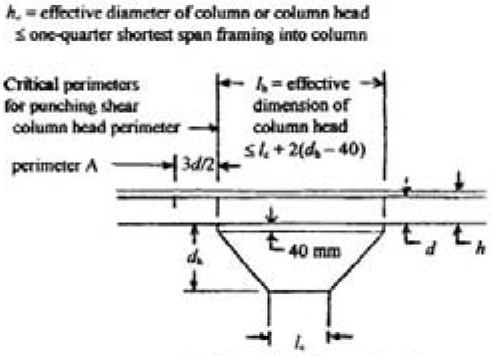
* Moment at outer column is to be taken not greater than $M_{t\,max}$ (see *Table 2.56*). The structural arrangement should be chosen to ensure that $M_{t\,max}$ is not less than $0.02Fl$, in which case there is no need to increase the end span moment.

Distribution of design moments in panels

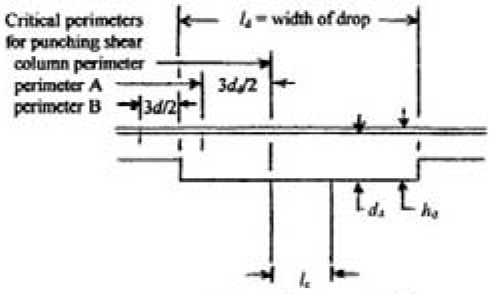
Design moment	Apportionment between column and middle strips expressed as a percentage of total negative or positive design moment	
	Column strip %	Middle strip %
Negative	75	25
Positive	55	45

Notes: For cases where the width of the column strip is taken as equal to that of the drop, and the middle strip is increased in width, the moments apportioned to the middle strip should be increased in proportion to the increased width. The moments apportioned to the column strip may be correspondingly decreased such that the total moments are unchanged. The negative moment apportionment applies at interior columns only. At outer columns, the entire negative moment is to be taken on the effective moment transfer strip, unless an edge beam or strip of slab along the free edge is designed to carry some of the moment into the column by torsion.

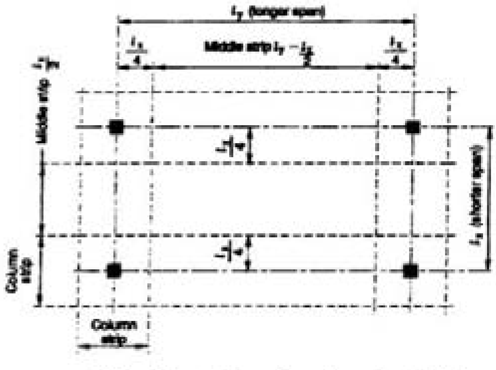
Note: Where an edge beam of depth greater than 1.5 times the slab thickness is provided, the moments for the half-column strip adjacent to the beam should be taken as one-quarter of the values apportioned to a full column strip. The edge beam should be designed to carry a uniformly distributed slab load equal to one-quarter of the total design load for a full panel.



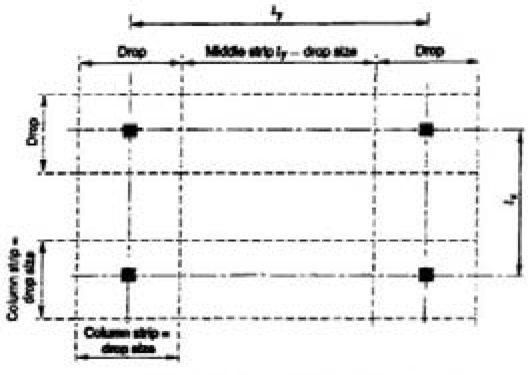
Slab with column head



Slab with drop panel



Slab without drops (or where $l_d < l_x/3$)



Slab with drops (where $l_d \geq l_x/3$)

Flat slabs: BS 8110 simplified method – 2

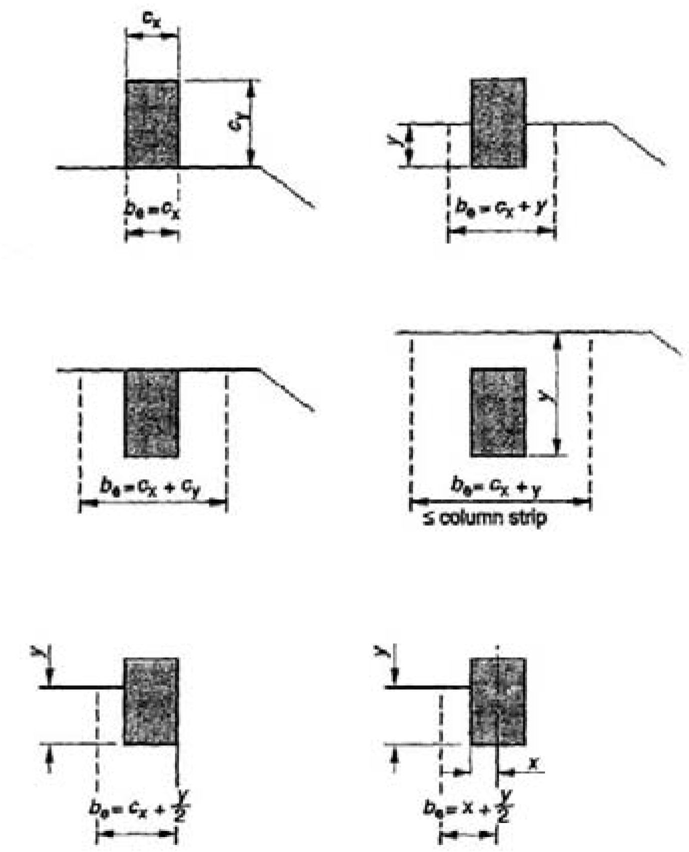
2.56

Effective breadth of moment transfer strip

In general, moments can only be transferred between a slab and an edge or corner column by a narrow column strip. The breadth of the strip b_e, for various typical cases, is shown in the figure opposite. The maximum design moment $M_{t\,max}$ that can be transferred to a column by this strip is given by:

$$M_{t\,max} = 0.15 b_e d^2 f_{cu}$$

where d is the effective depth of the top reinforcement in the column strip, f_{cu} is the concrete grade. Moments in excess of $M_{t\,max}$ may only be transferred to a column, if an edge beam or strip of slab along the free edge is designed to carry the additional moment by torsion.

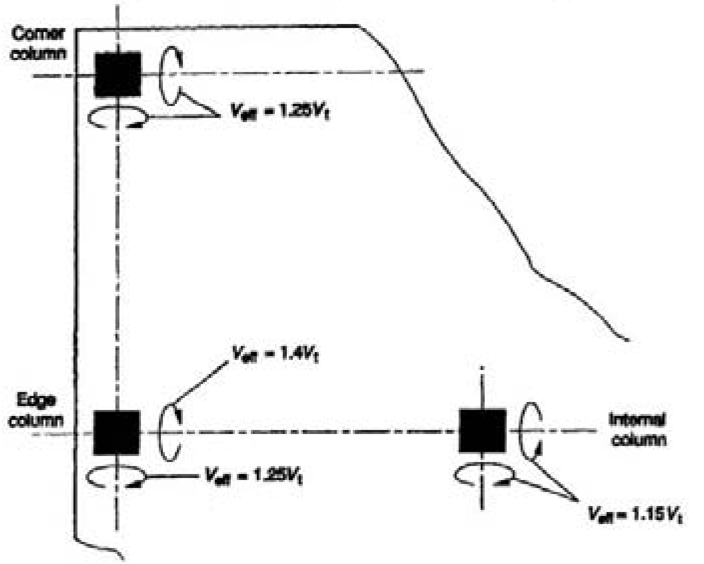


Effective design forces for punching shear around columns

Type of column	Corner	Edge		First internal	Other internal
Axis of bending	Both axes	Parallel to free edge	Perpendicular to free edge	Both axes	Both axes
V_t	$0.46F_e$	$1.0F_e$	$0.46F$	$1.1F$	$1.0F$
V_{eff}/V_t	1.25	1.4	1.25	1.15	1.15

The effective design forces for the calculation of punching shear around the columns, allowing for the effects of moment transfer, are given by V_{eff} as shown in the figure opposite. Values of V_t are given above, where

F is total design load on full panel with width taken between the centre-lines of adjacent bays, F_e is total design load on half-panel with width taken from the edge of the slab to centre-line of first bay (including load due to weight of perimeter walls and cladding).



Design moment		Column strip	Middle strip
BS 8110	Negative Positive	75% 55%	25% 45%
EC 2	Negative Positive	60–80% 50–70%	40–20% 50–30%

At the edges of a slab, the transfer of moments between the slab and an edge or corner column is limited by the effective breadth b_e of the moment transfer strip, given in *Table 2.56*. The maximum design moment that can be transferred by this strip is given by the equation $M_{t\,max} = 0.15 b_e d^2 f_{cu}$ in BS 8110, and $M_{t\,max} = 0.17 b_e d^2 f_{ck}$ in EC 2.

At internal columns, two-thirds of the reinforcement needed to resist the negative moments in the column strips should be placed in a width equal to half that of the column strip, and central with the column. Otherwise, the reinforcement needed to resist the moment apportioned to a particular strip should be distributed uniformly across the width of the strip.

13.8.4 Effective forces for punching shear

The critical consideration for shear in flat slab structures is that of punching shear around the columns. The design force obtained by summing the shear forces on each side of the column is multiplied by an enhancement factor, to allow for the effects of moment transfer, as given in *Table 2.56*. The effective shear force is determined independently in each direction, and the design checked for the worse case.

13.8.5 Reservoir roofs

For reservoir roofs with simply supported ends, where elastic moments are required to check serviceability requirements, the coefficients given for beams in *Table 2.30* could be used. In this case, the negative moments at the centrelines of the columns could be reduced by $0.22Fh_c$, to give approximately the moment at a distance $h_c/2$ from the centreline of the column. This approach will still ensure that the minimum moment requirement mentioned in section 13.8.3 is met.

Chapter 14

Framed structures

When using the formulae and coefficients in this chapter, the loads must include the appropriate partial safety factors for the limit-state being considered. Design loads for the ULS, in accordance with the requirements of BS 8110 and EC 2, are given in *Table 2.57*.

For many framed structures, it is not necessary to carry out a full structural analysis of the complete frame as a single unit. BS 8110 makes a distinction between frames supporting vertical loads only, because lateral stability to the structure as a whole is provided by other means such as shear walls, and frames supporting both vertical and lateral loads. Simplified models for the purpose of analysis, as described in section 4.9.1, are also shown in *Table 2.57*.

The moment-distribution method, used to analyse systems of continuous beams as shown in *Table 2.36*, can be extended to apply to no-sway sub-frames (see section 4.9.2) as shown in *Table 2.58*, and single-bay sway frames (see section 4.9.3) as shown in *Table 2.59*.

14.1 SLOPE-DEFLECTION METHOD OF ANALYSIS

Moment analysis of a restrained member by slope-deflection is based on the following two principles. The difference in slope between any two points in the length of the member is equal to the area of the M/EI diagram between the two points. The distance of any point on the member from a line drawn tangentially to the elastic curve at any other point, the distance being measured normal to the initial position of the member, is equal to the moment (taken about the first point) of the M/EI diagram between these two points. In the foregoing, M represents the bending moment, E the modulus of elasticity of the material, and I the second moment of area of the member. The bending moments that occur at the ends of a member subject to the deformation and restraints shown in the moment diagram at the top of *Table 2.60* are given by the corresponding formulae. The formulae, which have been derived from a combination of the basic principles, are given in a general form and in the special form for members on non-elastic supports.

The stiffness of a member is proportional to EI/l but, as E is assumed to be constant, the term that varies in each member is the stiffness factor $K = I/l$. The terms F_{AB} and F_{BA} relate to the load on the member. When there is no external load, F_{AB} and F_{BA} are zero; when the load is symmetrically disposed, $F_{AB} = F_{BA} = A/l$. Values of F_{AB}, F_{BA} and A/l for different load cases are given in *Table 2.28*.

The conventional signs for slope-deflection analyses are: an external restraint moment acting clockwise is positive; a slope is positive if the rotation of the tangent to the elastic line is clockwise; a deflection in the same direction as a positive slope is positive.

Example. Establish the formulae for the bending moments in a column CAD into which is framed a beam AB. The beam is hinged at B and the column is fixed at C and D (see diagram in *Table 2.60*). The beam only is loaded. Assume there is no displacement of the joint A.

From the general formulae given on *Table 2.60*:

$$M_{AB} = 3EK_{AB}\theta_A - (F_{AB} + F_{BA}/2)$$

$$M_{AC} = 4EK_{AC}\theta_A \quad \text{and} \quad M_{AD} = 4EK_{AD}\theta_A$$

Therefore

$$M_{AB} + M_{AC} + M_{AD} =$$

$$E\theta_A(3K_{AB} + 4K_{AC} + 4K_{AC}) - (F_{AB} + F_{BA}/2) = 0$$

Thus

$$E\theta_A = \frac{F_{AB} + F_{BA}/2}{3K_{AB} + 4K_{AC} + 4K_{AD}}$$

$$M_{AC} = \frac{4K_{AC}(F_{AB} + F_{BA}/2)}{3K_{AB} + 4K_{AC} + 4K_{AD}}$$

$$M_{CA} = 2EK_{AC}\theta_A = M_{AC}/2$$

$$M_{AD} = M_{AC}K_{AD}/K_{AC}$$

$$M_{DA} = M_{AD}/2$$

$$M_{AB} = -\ (M_{AC} + M_{AD})$$

For symmetrical loading:

$$F_{AB} + F_{BA}/2 = 1.5A_{AB}/l_{AB}$$

$$M_{AC} = \frac{6K_{AC}(A_{AB}/l_{AB})}{3K_{AB} + 4K_{AC} + 4D_{AD}}$$

Frame analysis: general data

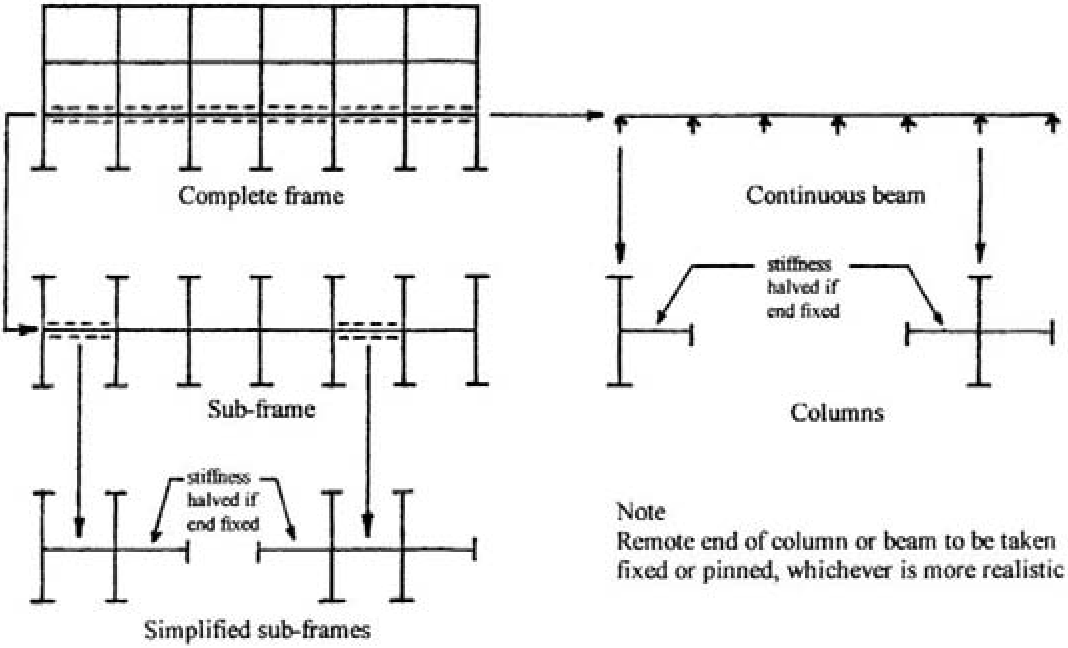
2.57

Design loads

Design loads for ultimate limit state						
Code	BS 8110			Eurocode 2		
Load combination	Dead	Live	Wind	Dead	Live	Wind
1. Dead and imposed	$1.0g_k$	$0.4g_k + 1.6q_k$	—	$(1.35 \text{ or } 1.0)g_k$	$1.5q_k$	—
2. Dead, imposed and wind*	$1.0g_k$	$0.4g_k$	$1.4w_k$	$(1.35 \text{ or } 1.0)g_k$	$1.05^{\blacksquare}q_k$	$1.5w_k$
3. Dead, imposed* and wind	$1.2g_k + 1.2q_k$	—	$1.2w_k$	$(1.35 \text{ or } 1.0)g_k$	$1.5q_k$	$0.9w_k$

Key: g_k is characteristic dead load, q_k is characteristic imposed load, w_k is characteristic wind load. * Indicates leading variable load. $^{\blacksquare}$ Value to be taken as 1.5 for storage areas.

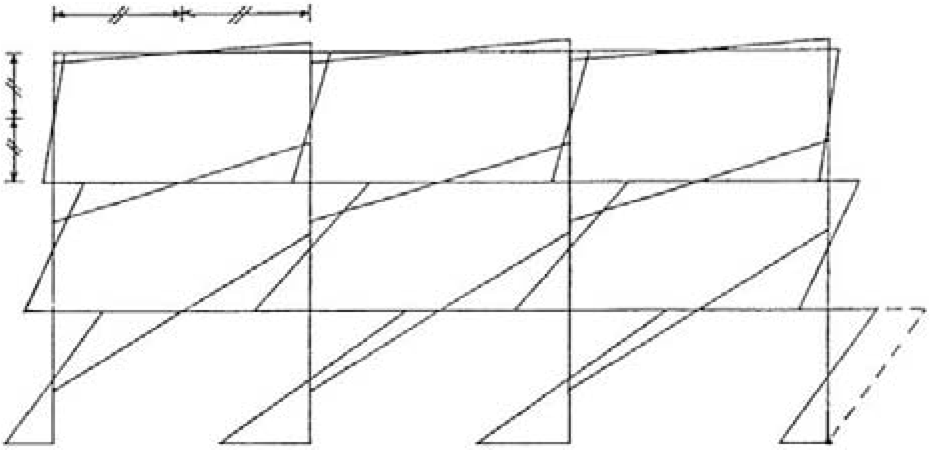
Frames not providing lateral stability



Note
Remote end of column or beam to be taken fixed or pinned, whichever is more realistic

Simplified models for effects of vertical load only

Frames providing lateral stability



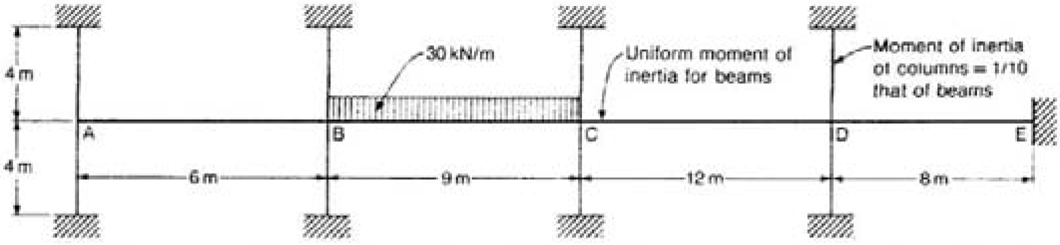
Simplified model for effects of lateral load on frame with three or more approximately equal bays (Note: For effects of vertical load, use simplified model for frame not providing lateral stability)

HARDY CROSS MOMENT DISTRIBUTION

The procedure is basically identical to that when analysing continuous-beam systems (see *Table 2.36*).

1. At each junction of beams and columns, *distribution factors D* are determined for each member (e.g. at B, $D_{BA}=K_{AB}/K_{AB}+K_{BC}+K_{BU}+K_{BL}$ etc.), so that $D_{BA}+D_{BC}+D_{BU}+D_{BL}=1$.
2. The total balancing moment $-\sum FEM$ introduced at each support to obtain joint equilibrium is then distributed between the members meeting at that support by multiplying by these distribution factors, and the resulting moments are carried over to the opposite ends of the members, as shown in *Table 2.36*.
3. Any columns considered pinned at their far ends are best treated as 'equivalent fixed-end members', as shown in *Table 2.36*.

The procedure is illustrated in the following example (UC upper column; LC lower column).



	UC	LC			UC	LC			UC	LC			UC	LC		
Distribution factors	0.115	0.115	0.770	0.508	0.076	0.076	0.340	0.455	0.102	0.102	0.341	0.323	0.097	0.097	0.483	
Fixed-end moments							−203	+203								
1st distribution				+103	+16	+16	+68	−92	−21	−21	−69					
1st carry-over			+52				−46	+34				−35				
2nd distribution	−6	−6	−40	+24	+3	+3	+16	−16	−3	−3	−12	+11	+3	+3	+18	
2nd carry-over			+12	−20			−8	+8			+6	−6				+9
3rd distribution	−1	−1	−10	+14	+2	+2	+10	−6	−1	−1	−6	+2	+1	+1	+2	
3rd carry-over			+7	−5			−3	+5			+1	−3				+1
4th distribution	−1	−1	−5	+4	+1	+1	+2	−2	−1	−1	−2	+1			+2	
Summations	−8	−8	+16	+120	+22	+22	−164	+134	−26	−26	−82	−30	+4	+4	+22	+10

PRECISE MOMENT DISTRIBUTION

The procedure differs only slightly from that for continuous-beam systems described in *Table 2.36*.

1. Calculate fixed-end moments as before.
2. Determine continuity factors for each span from general expression

$$\phi_{n+1}=1\Big/\left\{2+\frac{K_{n+1}}{\Sigma[K_n/(2-\phi_n)]}\right\}$$

   where ϕ_{n+1} and K_{n+1} are continuity factor and stiffness of span being considered, and $\sum[K_n/(2-\phi_n)]$ is sum of values of $K_n/(2-\phi_n)$ of all *remaining* members meeting at joint. If far end of column is fully fixed, $K_n/(2-\phi_n)=2K_n/3$, since $\phi_n=1/2$: if far end is pinned, $K_n/(2-\phi_n)=K_n/2$. As for continuous-beam system, work along system from left to right and then repeat procedure working from right to left.
3. Calculate *distribution factors* (*DFs*) at joints from general expression (given here for member BC at joint B)

$$DF_{BC}=\frac{1-2\phi_{BC}}{1-\phi_{BC}\phi_{CB}}$$

   where ϕ_{BC} and ϕ_{CB} are continuity factors given by step 2. Unlike continuous beam, sum of distribution factors each side of support will not equal unity, due to action of columns. At support B say, sum of distribution factors for columns is $1-DF_{BA}-DF_{BC}$. Then obtain distribution factor for each column by dividing total column distribution factor in proportion to stiffness of columns.
4. Carry over moments at supports as follows. Multiply distributed balancing moment at left-hand end of member by continuity factor obtained by working *from right to left* and carry over this value to right-hand end. At this point, balance carried-over moment by dividing an equal moment of opposite sign between remaining members meeting at that point in proportion to their values of $K_{LR}/(2-\phi_{RL})$. Thus, in example, moment of −30 kN m carried over from C to D is obtained by multiplying −81.1 kN m by 0.368. This moment is balanced at D by moments of 4 kN m (i.e. 30 × 0.0167/0.1167) in each column and 22 kN m (i.e. 30 × 0.0833/0.1167) in beam DE.
5. Undertake one complete carry-over operation working from left to right and then from right to left from each joint at which fixed-end moment occurs and sum results to obtain final moments on system.

For details of structure and loading see diagram above

UC upper column LC lower column

	UC	LC			UC	LC			UC	LC			UC	LC		
Relative stiffnesses	1/40	1/40	1/6		1/40	1/40	1/9		1/40	1/40	1/12		1/40	1/40	1/8	
Continuity $\phi\rightarrow$				0.143				0.344				0.353				0.287
factors $\leftarrow\phi$			0.271				0.301				0.368				0.500	
Distribution factors	0.128	0.128	0.744	0.476	0.089	0.089	0.346	0.443	0.110	0.110	0.337	0.302	0.100	0.100	0.498	
Fixed-end moments							−203	+203								
Distribution				+97	+18	+18	+70	−90	−22	−22	−69					
Carry-over	−8	−8	+16	+23	+4	+4	−31	+21	−4	−4	−13	−30	+4	+4	+22	+11
Summations	−8	−8	+16	+120	+22	+22	−164	+134	−26	−26	−82	−30	+4	+4	+22	+11

NAYLOR'S METHOD FOR SINGLE-BAY FRAMES

1. Consider only one-half of frame as shown. Assuming firstly that loads are applied only horizontally at floor levels, calculate fixed-end moments. Fixed-end moment at each end of each column forming storey = (1/4) × sum of horizontal forces above floor being considered × height of storey (see example).
2. Calculate distribution factors as for normal Hardy Cross moment distribution but assuming stiffness of each horizontal member is six times actual stiffness.
3. Carry out conventional Hardy Cross moment distribution but carrying-over moments equal in value to distributed balancing moments but of *opposite sign*. Procedure is illustrated in example.
4. If interpanel loading occurs, distribute moments in two stages. Firstly, undertake normal Hardy Cross moment distribution for complete frame using distribution factors obtained with true stiffnesses of beams and columns. Secondly, to cater for effects of sway, undertake second distribution using modified distribution factors and carry over as described in steps 1 to 3 above. Sum results obtained from both distributions to obtain final moments.

Effects of side-sway due to unsymmetrical vertical loads on beams may be considered in a similar manner. Firstly, calculate fixed-end moments FEM_{LR} and FEM_{RL} at left-hand (L) and right-hand (R) ends of beam due to given loads. Then analyse structure for fixed-end moments of $(FEM_{LR} + FEM_{RL})/2$ at L and R by normal Hardy Cross method and for fixed-end moments of $(FEM_{LR} - FEM_{RL})$ at L and R respectively as described above. Lastly, sum results obtained by both distributions to obtain final moments.

	A		B			C
Modified stiffness	1		12	1		12
Distribution factors		1/14	6/7	1/14	1/13	12/13
Fixed-end moments	−80.0	−80.0		−40.0	−40.0	
1st distribution		+8.6	+102.8	+8.6	+3.1	+36.9
1st carry-over	−8.6			−3.1	−8.6	
2nd distribution		+0.2	+2.7	+0.2	+0.7	+7.9
2nd carry-over	−0.2			−0.7	−0.2	
3rd distribution		+0.1	+0.5	+0.1		+0.2
Summations	−88.8	−71.1	+106.0	−34.9	−45.0	+45.0

For details of structure and loadings see diagram below

HARDY CROSS MOMENT DISTRIBUTION

1. Assuming firstly that loads are applied only horizontally at floor levels, determine number of degrees of sway freedom of structure (e.g. frame considered in example has two degrees of freedom, as shown in sketches).
2. Assuming that degree of freedom (i.e. sway of upper storey in example given) gives rise to fixed-end moments of unity at ends of upper columns, distribute these moments by conventional Hardy Cross distribution procedure. Consider one-half of frame only, as shown in example, and adopt stiffness for beams of 1.5 times actual stiffness.
3. Repeat procedure to obtain separate distributions of fixed-end moments of unity, corresponding to sway due to each degree of freedom to be considered.
4. Now values of moments obtained from steps 2 and 3 represent effects of applying arbitrary horizontal forces causing sway. To determine actual values of unknown forces $F_1, F_2, \ldots, F_n$ required it is necessary to set up n simultaneous equations relating horizontal force acting above level considered to internal shearing force acting on columns at that level. The internal shearing forces are obtained by multiplying the arbitrary distributed moments by the unknown forces and dividing by storey height. Thus, in example given, simultaneous equation corresponding to shearing forces in upper storey is

$$(1/4) \times 2[-(0.708 + 0.683)F_1 + (0.191 + 0.076)F_2] = -40$$

i.e. $-0.696F_1 + 0.133F_2 = -40$.

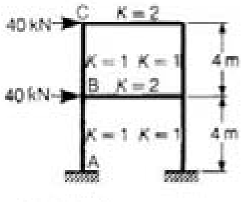
Similarly, for lower storey,

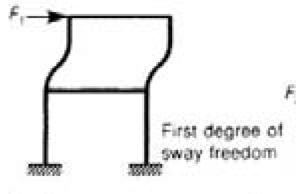
$$(1/4) \times 2[+(0.178 + 0.089)F_1 - (0.899 + 0.797)F_2] = -80$$

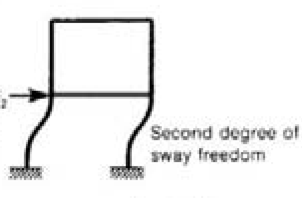
i.e. $+0.133F_1 - 0.848F_2 = -80$. These equations yield $F_1 = +77.83$ and $F_2 = +106.55$.

5. To obtain final moments, multiply moments obtained by distributing each arbitrary fixed-end moment by the appropriate force concerned and sum. For example, final moment in column at base = + 0.089 × 77.83 − 0.899 × 106.55 = − 88.8 kN m.
6. If interpanel loading is applied, distribute fixed-end moments corresponding to this loading throughout frame using normal Hardy Cross moment distribution and assuming no sway occurs, and sum resulting moments obtained and those due to sway analysis to obtain final moments.

Note: precise moment distribution can also be used to solve frames subjected to sway.







A		B		C			A		B		C	
0	1/5	3/5	1/5	1/4	3/4	Distribution factors	0	1/5	3/5	1/5	1/4	3/4
			−1.000	−1.000		Fixed-end moments	−1.000	−1.000				
	−0.200	+0.600	+0.200	+0.250	+0.750	1st distribution		+0.200	+0.600	+0.200		
+0.100			+0.125	+0.100		1st carry-over	+0.100				+0.100	
	+0.025	−0.075	−0.025	−0.025	−0.075	2nd distribution					−0.025	−0.075
−0.012			−0.012	−0.012		2nd carry-over				−0.012		
	+0.003	+0.600	+0.003	+0.003	+0.009	3rd distribution		+0.003	+0.006	+0.003		
+0.001			+0.100	+0.001		3rd carry-over	+0.001				+0.001	
		−0.001			−0.001	4th distribution						−0.001
+0.089	+0.178	+0.530	−0.708	−0.683	+0.683	Summations	−0.899	−0.797	+0.606	+0.191	+0.076	−0.076

Distribution corresponding to 1st deg. of sway freedom

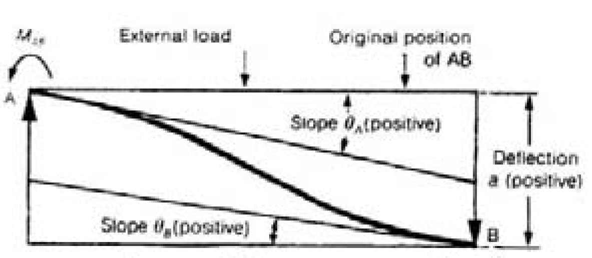
Distribution corresponding to 2nd deg. of sway freedom

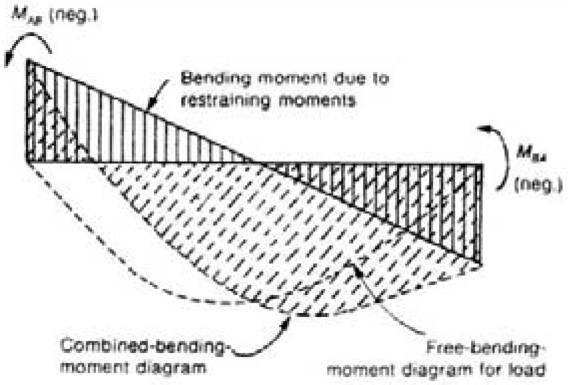
	A		B			C
Final moments	−88.8	−71.2	+106.1	−34.9	−45.1	+45.1

Frame analysis: slope-deflection data

2.60

Basic formulae (slope deflection)





$$M_{AB} = 2EK_{AB}\left(2\theta_A + \theta_B - \frac{3a}{l_{AB}}\right) - F_{AB}$$

$$M_{BA} = 2EK_{AB}\left(2\theta_B + \theta_A - \frac{3a}{l_{AB}}\right) + F_{BA}$$

Notation

The suffixes AB relate to joint or support A of any member AB
The suffixes BA relate to joint or support B of any member AB
} Similarly for other members BC, AC etc

l_{AB} length of member AB
I_{AB} moment of inertia ("concrete units") of AB
K_{AB} stiffness factor of AB $= I_{AB}/l_{AB}$
E modulus of elasticity for concrete
θ_A slope of deformed member AB at A.
With no load on AB, $F_{AB} = F_{BA} = 0$

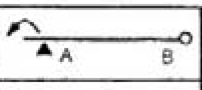
$$F_{AB} = \frac{2A_{AB}}{l_{AB}^2}(2l_{AB} - 3Z_{AB})$$

$$F_{BA} = \frac{2A_{AB}}{l_{AB}^2}(3Z_{AB} - l_{AB})$$

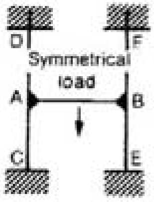
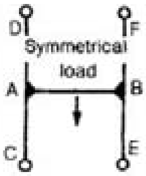
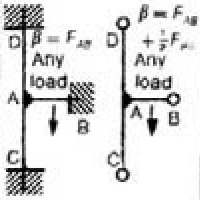
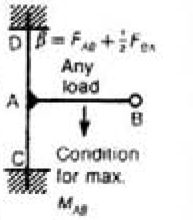
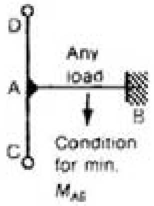
For symmetrical load on AB $F_{AB} = F_{BA} = A_{AB}/l_{AB}$

A_{AB} area of free-bending-moment diagram for load on AB
Z_{AB} distance from A to centroid of free-bending-moment diagram for load on AB

Slope-deflection general formulae non-elastic supports

					
M_{AB}	$2EK_{AB}(2\theta_A + \theta_B) - F_{AB}$	$2EK_{AB}\theta_B - F_{AB}$	$-F_{AB}$	$3EK_{AB}\theta_A$ $-(F_{AB} + \frac{1}{2}F_{BA})$	$-(F_{AB} + \frac{1}{2}F_{BA})$
M_{BA}	$2EK_{AB}(2\theta_B + \theta_A) + F_{BA}$	$4EK_{AB}\theta_B + F_{BA}$	$+F_{BA}$	Zero	Zero

Exterior columns

					
M_{AC}	$\frac{2K_{AC}}{2K_{AC} + 2K_{AD} + K_{AB}} \frac{A_{AB}}{l_{AB}}$	$\frac{3K_{AC}}{3K_{AC} + 3K_{AD} + 2K_{AB}} \frac{A_{AB}}{l_{AB}}$	$\frac{K_{AC}\beta}{K_{AC} + K_{AD} + K_{AB}}$	$\frac{4K_{AC}\beta}{4K_{AC} + 4K_{AD} + 3K_{AB}}$	$\frac{3K_{AC}F_{AB}}{3K_{AC} + 3K_{AD} + 4K_{AB}}$

$M_{AD} = \frac{K_{AD}}{K_{AC}} M_{AC}$ $\quad M_{AB} = -(M_{AC} + M_{AD})$

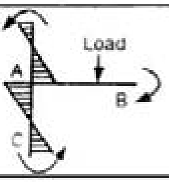
If fixed at C: $M_{CA} = 0.5M_{AC}$ etc.
at hinges: bending moment = 0

Bending moment in external columns of building frames

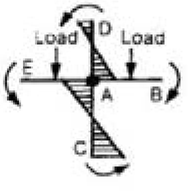
$$M_{AC} = \frac{K_{AC}}{K_{AC} + K_{AD} + \zeta K_{AB}} F_{AB}$$

$$M_{AD} = \frac{K_{AD}}{K_{AC}} M_{AC} \qquad M_{AB} = -(M_{AC} + M_{AD})$$

For frames of one bay width only $\zeta = 1/2$
For frames of more than one bay width $\zeta = 1$
For top storey (e.g. roof beam and top storey column): $K_{AD} = 0$



Interior columns



$$M_{AC} = \frac{K_{AC}}{K_{AC} + K_{AD} + K_{AB} + K_{AE}} M_{es}$$

$$M_{AD} = \frac{K_{AD}}{K_{AC}} M_{AC}$$

M_{es} = either F_{AB} (for dead + live load)
- F_{AE} (for dead load only)
or F_{AE} (for dead + live load)
- F_{AB} (for dead load only)
whichever is the greater.

M_{AB} and M_{AE} are in all cases less than the moments assuming knife-edge support for beam EAB at A.

This method is recommended in BS8110 if beams are analysed as a continuous system on knife-edge supports. When determining the column moments using the formulae shown here, the stiffness of the beams should be taken as one-half of the true value when the remote end is fixed, and three-quarters of the true value when the remote end is pinned.

Note: For values of F_{AB} and F_{BA} see *Table 2.28.*

14.2 CONTINUOUS BEAMS AS FRAME MEMBERS

In many buildings, the interaction of the columns and beams can be considered with sufficient accuracy by applying one of the simplified models shown in *Table 2.57*. The simplified two- or three-span sub-frames in *Table 2.61* are analysed on the assumption that the remote ends of the beams and columns are fixed. Therefore, for any internal span ST, the ends of the beams at R and U, the ends of the lower columns at O and X, and the ends of the upper columns at P and Y are all assumed to be fixed. In addition, the stiffness of the outer beams RS and TU is taken as one-half of the true value. For the fixed-end moments due to normal (i.e. downward-acting) loads, positive numerical values should be substituted into the tabulated expressions. If the resulting sign of the support moment is negative, hogging with tension across the top face of the beam is indicated.

14.2.1 Internal spans

By slope-deflection methods, it can be shown that

$$M_{ST} = -F_{ST} + K_{ST}(\theta_{ST} + \theta_{TS}/2)$$

$$M_{TS} = -F_{TS} + K_{TS}(\theta_{TS} + \theta_{ST}/2)$$

where

$$\theta_{ST} = \frac{(K_{ST}/2)(F_{TS} - F_{TU}) - \sum K_T(F_{SR} - F_{ST})}{\sum K_S \sum K_T - K_{ST}^2/4}$$

$$\theta_{TS} = \frac{(K_{ST}/2)(F_{SR} - F_{ST}) - \sum K_S(F_{TS} - F_{TU})}{\sum K_S \sum K_T - K_{ST}^2/4}$$

$$\sum K_S = \zeta K_{RS} + K_{SO} + K_{SP} + K_{ST}$$

$$\sum K_T = K_{ST} + K_{TX} + K_{TY} + \zeta K_{TU}$$

and ζ is a factor representing the ratio of the assumed to the actual stiffness for the span concerned (i.e. here $\zeta = 1/2$).

By eliminating θ_{ST} and θ_{TS} in the above and rearranging, the following basic formulae are obtained:

$$M_{ST} = -F_{ST} + \frac{D_{ST}}{4 - D_{ST}D_{TS}}\left[2D_{TS}\left(\frac{1}{D_{ST}} - 1\right) \times (F_{TU} - F_{TS}) + (4 - D_{TS})(F_{ST} - F_{SR})\right]$$

$$M_{TS} = -F_{TS} + \frac{D_{TS}}{4 - D_{TS}D_{TS}}\left[2D_{ST}\left(\frac{1}{D_{TS}} - 1\right) \times (F_{ST} - F_{SR}) + (4 - D_{ST})(F_{TU} - F_{TS})\right]$$

These formulae, which are ‘exact’ within the limitations of the fixity conditions of the sub-frame, represent the case of three spans loaded and apply, for example, to the condition of dead load. For design to BS 8110, the maximum moments at supports S and T occur when the live load also is applied to all three spans. For design to EC 2, the maximum moment at support S occurs when the live load is applied to spans RS and ST, and the maximum moment at support T occurs when the live load is applied to spans ST and TU. The maximum positive moment in span ST is obtained when the live load is applied to this span only, for both Codes. The appropriate formulae for these conditions are also given in *Table 2.61*. For the ULS, the BS 8110 loads are dead $1.0g_k$, and live $0.4g_k + 1.6q_k$. The corresponding loads in EC 2 are dead $1.35g_k$, and live $1.5q_k$.

In the foregoing, dead and live loads are applied separately, and the resulting moments are summed. Alternatively, both dead and live loads can be applied in a single operation, by evaluating the basic formulae with fixed-end moment values corresponding to (dead+live) load on the aforesaid spans, and dead load only on the remaining spans. To comply with EC 2, for example, in determining the maximum support moment at S, the fixed-end moments F_{SR}, F_{ST} and F_{TS} should be calculated for a load of $1.35g_k + 1.5q_k$, while F_{TU} should be evaluated for a load of $1.35g_k$ only. This method is used in the following example.

In accordance with both Codes, the moments derived from these calculations may be redistributed if desired. It should be emphasised that, although the diagrams on *Table 2.61* and in the following example are for uniform loads, the method and the formulae are applicable to any type of loading, provided that the appropriate fixed-end moment coefficients, obtained from *Table 2.28* are used.

When the moments M_{ST} and M_{TS} at the supports are known, the positive and negative moments in the spans are obtained by combining the diagram of free moments due to the design loads with the diagram of corresponding support moments.

14.2.2 End spans

The formulae for any interior span ST are rewritten to apply to an end span AB, by substituting A, B, C and so on, for S, T, U etc. (A is the end support and there is no span corresponding to RS). The modified stiffness and distribution factors are given in *Table 2.61*, together with the moment formulae for both spans loaded, and load on span AB only. The dead and live loads should be evaluated, and applied so as to obtain the required support moments, as described in section 14.2.1.

14.2.3 Columns and adjoining spans

The outer members of the sub-frame have been taken as fully fixed at their remote ends. Thus for a member such as RS, the slope-deflection equation is

$$M_{SR} = K_{RS}\,\theta_{SR}$$

Since the rotation of all the members meeting at a joint is the same, $\theta_{SR} = \theta_{ST}$. Thus, by eliminating θ_{SR} and rearranging,

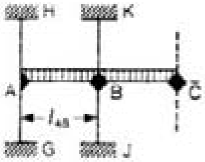
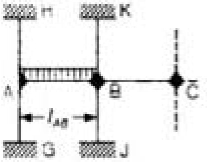
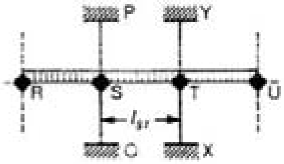
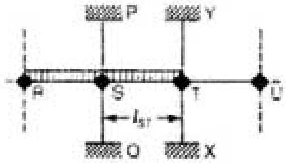
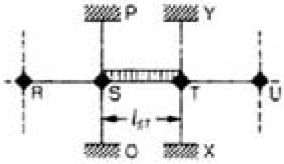
$$M_{SR} = -F_{SR} - \frac{D_{SR}}{4 - D_{ST}D_{TS}} \times \left[2D_{TS}(F_{TU} - F_{TS}) + 4(F_{ST} - F_{SR})\right]$$

Similarly,

$$M_{TU} = -F_{TU} + \frac{D_{TU}}{4 - D_{ST}D_{TS}} \times \left[2D_{ST}(F_{ST} - F_{SR}) + 4(F_{TU} - F_{TS})\right]$$

2.61

Frame analysis: simplified sub-frames

	Arrangement of live load (dead load on all spans)	Bending moments in beam span (at support positions)	Stiffness factors (K) and distribution factors (D)
End span AB	Maximum moment at B  	$M_{AB} = -F_{AB} + \frac{D_{AB}}{4 - D_{AB}D_{BA}}\left[2D_{BA}\left(\frac{1}{D_{AB}} - 1\right) \times (F_{BC} - F_{BA}) + (4 - D_{BA})F_{AB}\right]$ $M_{BA} = -F_{BA} - \frac{D_{BA}}{4 - D_{AB}D_{BA}}\left[2D_{AB}\left(\frac{1}{D_{BA}} - 1\right) \times F_{AB} + (4 - D_{AB})(F_{BC} - F_{BA})\right]$	$K_{AB} = \frac{I_{AB}}{l_{AB}}$ $K_{AG} = \frac{I_{AG}}{l_{AG}}$ $K_{BC} = \frac{I_{BC}}{l_{BC}}$ $K_{BJ} = \frac{I_{BJ}}{l_{BJ}}$ $K_{AH} = \frac{I_{AH}}{l_{AH}}$ $K_{BK} = \frac{I_{BK}}{l_{BK}}$ $D_{AB} = \frac{K_{AB}}{K_{AB} + K_{AG} + K_{AH}}$ $D_{BA} = \frac{K_{AB}}{K_{AB} + \zeta K_{BC} + K_{BJ} + K_{BK}}$
	Maximum moment at A, in span AB, and in columns  	$M_{AB} = -F_{AB} + \frac{D_{AB}}{4 - D_{AB}D_{BA}}\left[-2D_{BA}\left(\frac{1}{D_{AB}} - 1\right) \times F_{BA} + (4 - D_{BA})F_{AB}\right]$ $M_{BA} = -F_{BA} - \frac{D_{BA}}{4 - D_{AB}D_{BA}}\left[2D_{AB}\left(\frac{1}{D_{BA}} - 1\right) \times F_{AB} - (4 - D_{AB})F_{BA}\right]$	
Interior span ST	Maximum moment at S and T (BS8110 requirements)  	$M_{ST} = -F_{ST} + \frac{D_{ST}}{4 - D_{ST}D_{TS}}\left[2D_{TS}\left(\frac{1}{D_{ST}} - 1\right) \times (F_{TU} - F_{TS}) + (4 - D_{TS})(F_{ST} - F_{SR})\right]$ $M_{TS} = -F_{TS} - \frac{D_{TS}}{4 - D_{ST}D_{TS}}\left[2D_{ST}\left(\frac{1}{D_{TS}} - 1\right) \times (F_{ST} - F_{SR}) + (4 - D_{ST})(F_{TU} - F_{TS})\right]$	$K_{RS} = \frac{I_{RS}}{l_{RS}}$ $K_{ST} = \frac{I_{ST}}{l_{ST}}$ $K_{TU} = \frac{I_{TU}}{l_{TU}}$ $K_{SO} = \frac{I_{SO}}{l_{SO}}$ $K_{SP} = \frac{I_{SP}}{l_{SP}}$ $K_{TX} = \frac{I_{TX}}{l_{TX}}$ $K_{TY} = \frac{I_{TY}}{l_{TY}}$ $D_{ST} = \frac{K_{ST}}{\zeta K_{RS} - K_{ST} + K_{SO} + K_{SP}}$ $D_{TS} = \frac{K_{ST}}{K_{ST} + \zeta K_{TU} + K_{TX} + K_{TY}}$
	Maximum moment at S* (EC2 requirements)  	$M_{ST} = -F_{ST} + \frac{D_{ST}}{4 - D_{ST}D_{TS}}\left[-2D_{TS}\left(\frac{1}{D_{ST}} - 1\right) \times F_{TS} + (4 - D_{TS})(F_{ST} - F_{SR})\right]$ $M_{TS} = -F_{TS} - \frac{D_{TS}}{4 - D_{ST}D_{TS}}\left[2D_{ST}\left(\frac{1}{D_{TS}} - 1\right) \times (F_{ST} - F_{SR}) - (4 - D_{ST})F_{TS}\right]$	
	Maximum moment in span ST and in columns  	$M_{ST} = -F_{ST} + \frac{D_{ST}}{4 - D_{ST}D_{TS}}\left[-2D_{TS}\left(\frac{1}{D_{ST}} - 1\right) \times F_{TS} + (4 - D_{TS})F_{ST}\right]$ $M_{TS} = -F_{TS} - \frac{D_{TS}}{4 - D_{ST}D_{TS}}\left[2D_{ST}\left(\frac{1}{D_{TS}} - 1\right) \times F_{ST} - (4 - D_{ST})F_{TS}\right]$	* For maximum moment at T, put live load on spans ST and TU. In all subscripts, replace R by U and interchange S and T.

Notation: F_{AB} etc. numerical value of fixed-end moment (negative) at A etc. due to load on AB etc.
l_{AB} etc. length of member AB etc. I_{AB} etc. second moment of area of member AB etc.

Note: To comply with BS8110 requirements for analysing simplified sub-frames, take $\zeta = 0.5$.

The expressions for the moments in the columns are similar to the foregoing, but F_{SR} and F_{TU} should be replaced by the initial fixed-end moment in the column concerned (normally zero), and the appropriate distribution factor for the column should be substituted for D_{SR} or D_{TU}.

Example. Determine the critical ultimate bending moments in beam ST of the system shown in the following figure below, which represents part of a multi-storey frame, in accordance with the requirements of BS 8110 and EC 2 respectively.

Stiffness values (mm³)

$$K_{RS} = \frac{20.16 \times 10^9}{6 \times 10^3} = 3.36 \times 10^6$$

$$K_{ST} = K_{TU} = \frac{21.85 \times 10^9}{8 \times 10^3} = 2.73 \times 10^6$$

$$\text{For upper columns: } K = \frac{2.13 \times 10^9}{4 \times 10^3} = 0.53 \times 10^6$$

$$\text{For lower columns: } K = \frac{3.42 \times 10^9}{4 \times 10^3} = 0.86 \times 10^6$$

Distribution factors (if stiffness values are divided by 10^6)

$$D_{ST} = \frac{2.73}{0.5 \times 3.36 + 2.73 + 0.53 + 0.86} = \frac{2.73}{5.80} = 0.471$$

$$D_{TS} = \frac{2.73}{2.73 + 0.5 \times 2.73 + 0.53 + 0.86} = \frac{2.73}{5.49} = 0.497$$

Support-moment equations

$$\begin{aligned} M_{ST} = & -F_{ST} + [0.471/(4 - 0.471 \times 0.497)] \\ & \times [2 \times 0.497(1/0.471 - 1)(F_{TU} - F_{TS}) \\ & + (4 - 0.497)(F_{ST} - F_{SR})] \\ = & -F_{ST} + 0.125[1.116(F_{TU} - F_{TS}) + 3.503(F_{ST} - F_{SR})] \end{aligned}$$

$$\begin{aligned} M_{TS} = & -F_{TS} - [0.497/(4 - 0.471 \times 0.497)] \\ & \times [2 \times 0.471(1/0.497 - 1)(F_{ST} - F_{SR}) \\ & + (4 - 0.471)(F_{TU} - F_{TS})] \\ = & -F_{TS} - 0.132[0.953(F_{ST} - F_{SR}) + 3.529(F_{TU} - F_{TS})] \end{aligned}$$

BS 8110 requirements

Fixed-end moments

For dead load only: $1.0g_k = 8$ kN/m

$$F_{RS} = F_{SR} = 8 \times 6^2/12 = 24 \text{ kNm}$$
$$F_{ST} = F_{TS} = F_{TU} = F_{UT} = 8 \times 8^2/12 = 42.7 \text{ kNm}$$

For dead +live load:

$$1.4g_k + 1.6q_k = 1.4 \times 8 + 1.6 \times 10 = 27.2 \text{ kN/m}$$
$$F_{RS} = F_{SR} = 27.2 \times 6^2/12 = 81.6 \text{ kNm}$$
$$F_{ST} = F_{TS} = F_{TU} = F_{UT} = 27.2 \times 8^2/12 = 145.1 \text{ kNm}$$

Maximum moments on beam ST

At S (dead +live load on all spans)

$$\begin{aligned} M_{ST} &= -145.1 + 0.125 \\ &\quad \times [1.116(145.1 - 145.1) + 3.503(145.1 - 81.6)] \\ &= -145.1 + 27.8 = -117.3 \text{ kNm} \end{aligned}$$

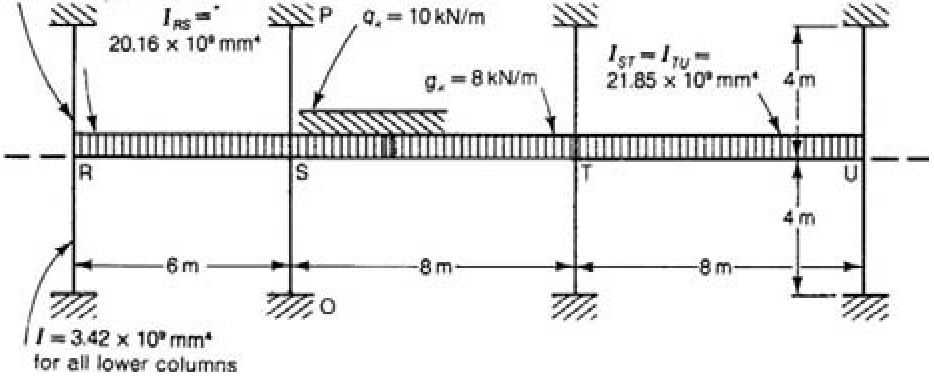
At T (dead +live load on all spans)

$$\begin{aligned} M_{TS} &= -145.1 - 0.132 \\ &\quad \times [0.953(145.1 - 81.6) + 3.529(145.1 - 145.1)] \\ &= -145.1 - 8.0 = -153.1 \text{ kNm} \end{aligned}$$

Mid-span (dead +live load on ST, dead load on RS and TU)

$$\begin{aligned} M_{ST} &= -145.1 + 0.125 \\ &\quad \times [1.116(42.7 - 145.1) + 3.503(145.1 - 24.0)] \\ &= -145.1 + 38.7 = -106.4 \text{ kNm} \end{aligned}$$

$$\begin{aligned} M_{TS} &= -145.1 - 0.132 \\ &\quad \times [0.953(145.1 - 24.0) + 3.529(42.7 - 145.1)] \\ &= -145.1 + 32.5 = -112.6 \text{ kNm} \end{aligned}$$



Maximum positive span moment is then approximately

$$1.5F_{ST} - 0.5(M_{ST} + M_{TS}) = 1.5 \times 145.1 - 0.5(106.4 + 112.6)$$
$$= 217.6 - 109.5 = 108.1 \text{ kNm}$$

Maximum moments on column at S (dead +live load on ST, dead load on RS and TU). Distribution factors for lower and upper columns respectively are:

$$D_{SO} = \frac{0.86}{5.80} = 0.148 \qquad D_{SP} = \frac{0.53}{5.80} = 0.091$$

Bending moments for lower and upper columns respectively are:

$$M_{SO} = \left[0.148/(4-0.471 \times 0.497)\right]$$
$$\times \left[2 \times 0.497(145.1-42.7) + 4(145.1-24.0)\right]$$
$$= 23.1 \text{ kNm}$$
$$M_{SP} = \left[0.091/(4-0.471 \times 0.497)\right]$$
$$\times \left[2 \times 0.497(145.1-42.7) + 4(145.1-24.0)\right]$$
$$= 14.2 \text{ kNm}$$

Alternatively, using the method shown on *Table 2.60* for an interior column, when the adjoining beams are analysed as a continuous system on knife-edge supports,

$$\Sigma K_S = 0.5K_{RS} + 0.5\, K_{ST} + K_{SO} + K_{SP}$$
$$= (1.68 + 1.37 + 0.53 + 0.86) \times 10^6 = 4.44 \times 10^6 \text{ mm}^3$$

$$D_{SO} = \frac{0.86}{4.44} = 0.194 \qquad D_{SP} = \frac{0.53}{4.44} = 0.119$$

Maximum unbalanced fixed-end moment at S

$$(F_{ST} - F_{RS}) = 145.1 - 24.0 = 121.1 \text{ kNm}$$
$$M_{SO} = 0.194 \times 121.1 = 23.5 \text{ kNm}$$
$$M_{SP} = 0.119 \times 121.1 = 14.4 \text{ kNm}$$

It will be seen that, in this example, the results obtained by the two methods are almost identical.

EC 2 requirements

Fixed-end moments. For dead load only:

$$1.35g_k = 1.35 \times 8 = 10.8 \text{ kN/m}$$
$$F_{RS} = F_{SR} = 10.8 \times 6^2/12 = 32.4 \text{ kNm}$$
$$F_{ST} = F_{TS} = F_{TU} = F_{UT} = 10.8 \times 8^2/12 = 57.6 \text{ kNm}$$

For dead +live load:

$$1.35g_k + 1.5q_k = 1.35 \times 8 + 1.5 \times 10 = 25.8 \text{ kN/m}$$
$$F_{RS} = F_{SR} = 25.8 \times 6^2/12 = 77.4 \text{ kNm}$$
$$F_{ST} = F_{TS} = F_{TU} = F_{UT} = 25.8 \times 8^2/12 = 137.6 \text{ kNm}$$

Maximum moments on beam ST. At S (dead +live load on RS and ST, dead load on TU)

$$M_{ST} = -137.6 + 0.125$$
$$\times[1.116(57.6 - 137.6) + 3.503(137.6 - 77.4)]$$
$$= -137.6 + 15.2 = -122.4 \text{ kNm}$$

At T (dead +live load on ST and TU, dead load on RS)

$$M_{TS} = -137.6 - 0.132$$
$$\times[0.953(137.6 - 32.4) + 3.529(137.6 - 137.6)]$$
$$= -137.6 - 13.2 = -150.8 \text{ kNm}$$

Mid-span (dead +live load on ST, dead load on RS and TU)

$$M_{ST} = -137.6 + 0.125$$
$$\times[1.116(57.6 - 137.6) + 3.503(137.6 - 32.4)]$$
$$= -137.6 + 34.9 = -102.7 \text{ kNm}$$
$$M_{TS} = -137.6 - 0.132$$
$$\times[0.953(137.6 - 32.4) + 3.529(57.6 - 137.6)]$$
$$= -137.6 + 24.0 = -113.6 \text{ kNm}$$

Maximum positive span moment is then approximately

$$1.5F_{ST} - 0.5(M_{ST} + M_{TS}) = 1.5 \times 137.6 - 0.5(102.7 + 113.6)$$
$$= 206.4 - 108.1 = 98.3 \text{ kNm}$$

Maximum moments on column at S (dead +live load on ST, dead load on RS and TU)

$$M_{SO} = \left[0.148/(4 - 0.471 \times 0.497)\right]$$
$$\times \left[2 \times 0.497(137.6 - 57.6) + 4(137.6 - 32.4)\right]$$
$$= 19.7 \text{ kNm}$$
$$M_{SP} = \left[0.091/(4 - 0.471 \times 0.497)\right]$$
$$\times \left[2 \times 0.497(137.6 - 57.6) + 4(137.6 - 32.4)\right]$$
$$= 12.1 \text{ kNm}$$

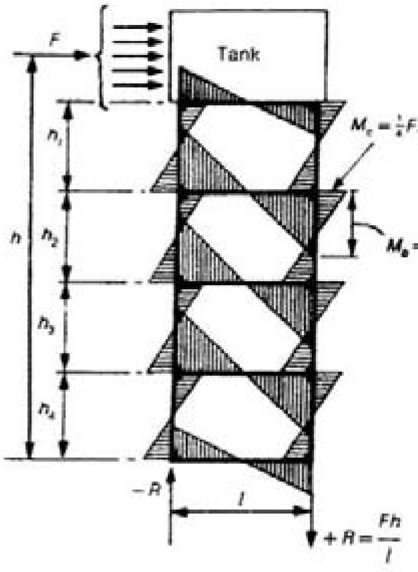
14.3 EFFECTS OF LATERAL LOADS

For many structures, a close analysis of the bending moments to which a frame is subjected due to wind or other horizontal loads is unwarranted. In such cases, the methods illustrated in *Table 2.62* are sufficiently accurate. Further information on the use of these methods is given in section 4.11.

14.4 PORTAL FRAMES

General formulae for the bending moments in single-storey, single-bay rigid frames are given in *Table 2.63* (rectangular frames) and *Table 2.64* (gable frames). The formulae, which relate to any vertical or horizontal load, cater for frames with the columns fixed or hinged at the base. Formulae for specific load cases are given in *Tables 2.65* and *2.66*. Formulae for the forces and bending moments in frames with a hinge at the base of each column and one at the ridge (i.e. three-hinged frames) are given in *Table 2.67*.

(a) Open braced tower



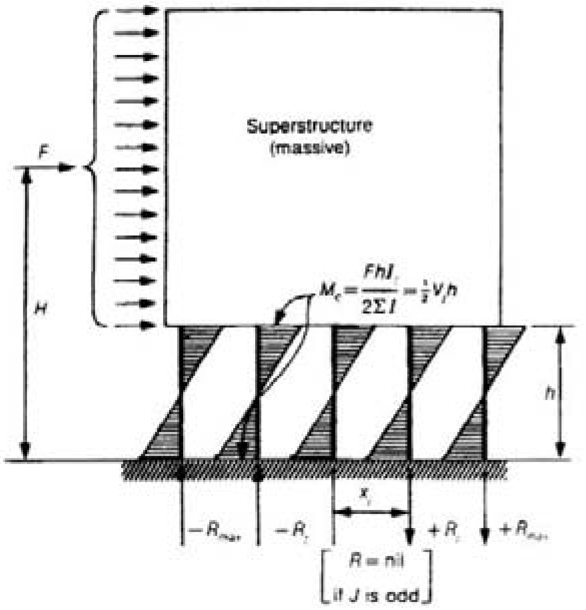
Shearing forces:

on column: $V_c = \frac{1}{2}F$

on brace: $V_b = 2M_b/l$

Assumption: both columns of same size and vertical or nearly vertical.

(b) Substructure of silo (or similar structure)



$$R_j = \frac{FHx_j}{\Sigma x^2}$$

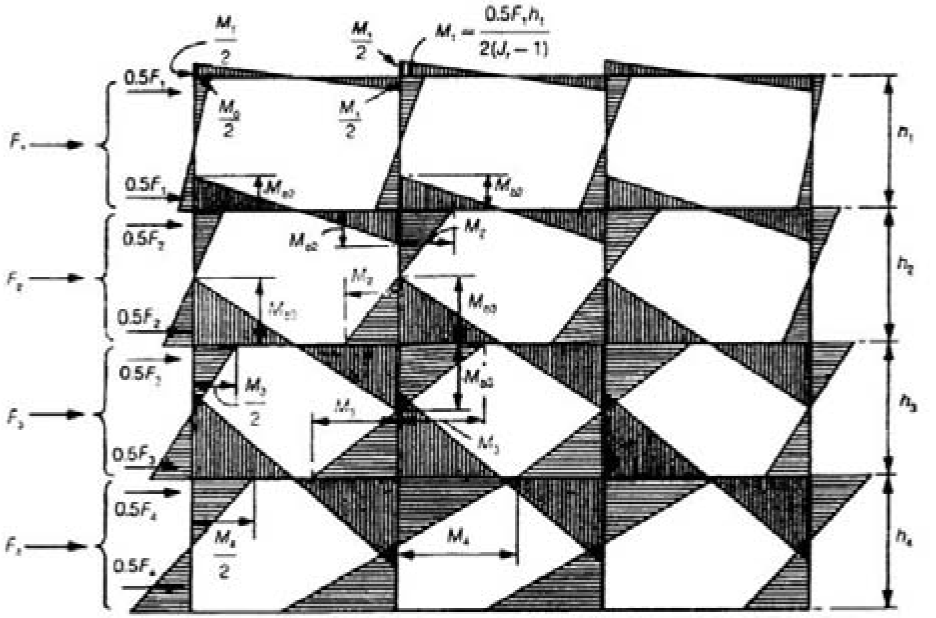
I_j moment of inertia of any column j (columns of different sizes.)

J total number of columns supporting superstructure

Shearing force on column j: $V_j = FI_j/\Sigma I$

If all columns are same size: $M_c = Fh/2J$

(c) Frame of multistorey building



$$M_j = \frac{\Sigma_1^{j-1}F + F_j/2}{J_t - 1}\left(\frac{h_j}{2}\right)$$

$$M_{bj} = \frac{1}{2}(M_j + M_{j+1})$$

Example with $J_t = 4$:

$$M_1 = \frac{F_1h_1}{12}$$

$$M_2 = \frac{h_2}{6}(F_1 + \frac{1}{2}F_2)$$

$$M_3 = \frac{h_3}{6}(F_1 + F_2 + \frac{1}{2}F_3)\text{etc.}$$

$$M_{b1} = \frac{1}{2}M_1$$

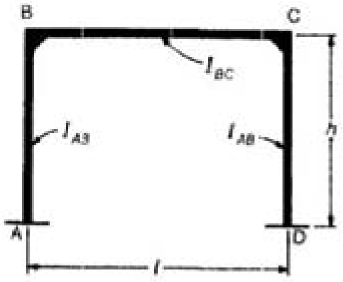
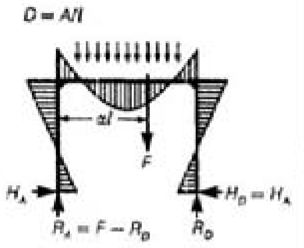
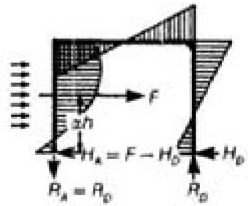
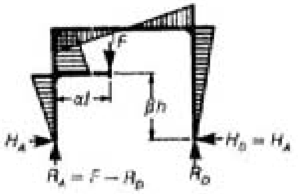
$$M_{b2} = \frac{1}{2}(M_1 + M_2)$$

$$M_{b3} = \frac{1}{2}(M_2 + M_3)\text{etc.}$$

Assumption: exterior column subjected to half shearing force on interior column.

J_t total number of columns in frame

Note: Explanations of the effects of wind forces (or other lateral loads) on the structures illustrated in *Table 2.62* are given in section 4.11.

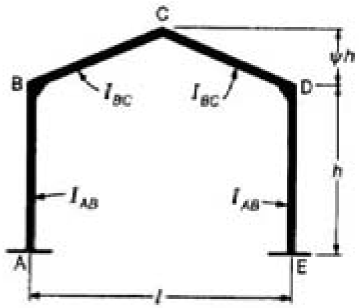
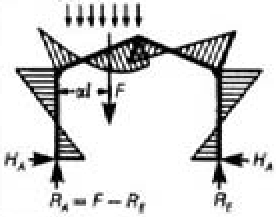
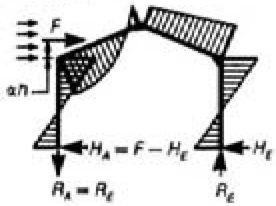
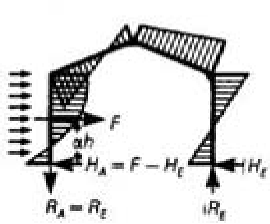
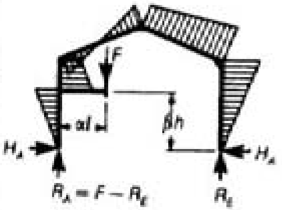
Frame form	Loading	Reactions and bending moments		
		Fixed feet		Hinged feet
	$D = A/l$ 	$H_A = 3D/k_1h$ $R_D = \alpha F - 2k_2'D$ $M_A = \left(\frac{1}{k_1} - k_2'\right)D$ $M_B = H_Ah - M_A$	$M_C = H_Ah - M_D$ $M_D = \left(\frac{1}{k_1} + k_2'\right)D$ $k_2' = 3(1 - 2z_1)/k_2$	$H_A = 3D/k_3h$ $R_D = \alpha F$ $M_A = M_D = 0$ $M_B = M_C = H_Ah$
	$D = A/l$ 	$H_D = \frac{1}{2}\alpha F + \frac{3D}{k_1h}(k_5z_1 - k_6)$ $C_1 = \frac{D}{k_1}[k_3(1 - z_1) - Kz_1]$ $C_2 = \frac{1}{2k_2}[F\alpha hk_4 + 6KD]$ $R_D = (M_B + M_C)/l$	$M_A = C_1 + C_2$ $M_B = h(H_D - \alpha F) + M_A$ $M_C = H_Dh - M_D$ $M_D = C_1 - C_2$	$H_D = \frac{1}{2}\alpha F + \frac{3Kz_1D}{k_3h}$ $R_D = F\alpha h/l$ $M_A = M_D = 0$ $M_B = h[H_A - F(1 - \alpha)]$ $M_C = H_Dh$
$K = \frac{I_{BC}h}{I_{AB}l}$ $k_1 = K + 2$ $k_2 = 6K + 1$ $k_3 = 2K + 3$ $k_4 = 3K + 1$ $k_5 = 2K + 1$ $k_6 = K + 1$	⊗ 	$H_A = \frac{3F\alpha l}{2k_1h}[2K\beta(1 - \beta) + \beta(2 - \beta)]$ $R_D = \frac{6F\alpha K\beta}{k_2}$ $M_A = \frac{F\alpha l}{2k_1k_2}\{k_2[3k_6(1 - \beta)\beta + 2k_1(\beta - 1)] + \beta(k_2 - k_1)\}$ $M_B = \frac{F\alpha l}{2k_1k_2}[3(\beta k_2 - k_5) + 13]K\beta$ $M_C = \frac{F\alpha l}{2k_1k_2}[3(\beta k_2 - 3k_5) - 5]K\beta$ $M_D = \frac{F\alpha l}{2k_1k_2}\{k_2[3k_6(1 - \beta) + 1] + k_1\}\beta$		$H_A = \frac{3F\alpha l}{2hk_3}[(1 - \beta^2)K + 1]$ $R_D = \alpha F$ $M_A = M_D = 0$ $M_B = F\alpha l - H_Ah$ $M_C = H_Ah$

Notes for *Tables 2.63* and *2.64*.

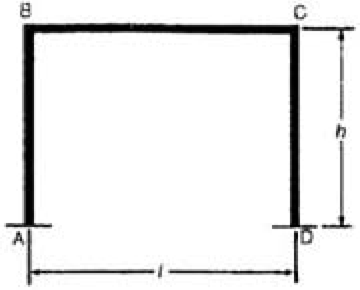
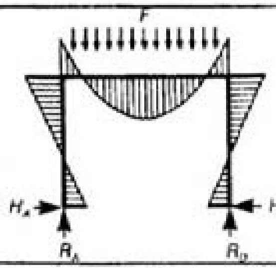
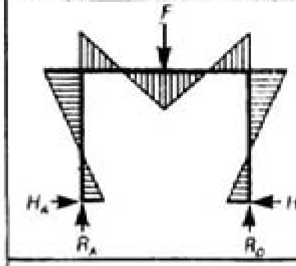
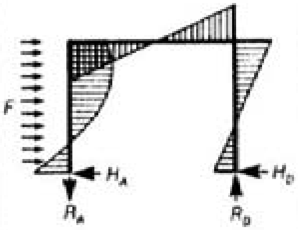
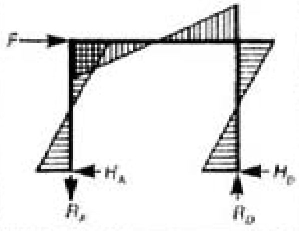
Formulae (except X and Y) apply to any type of loading and give numerical values of reactions and moments; see loading diagrams for direction of action.

Centre of gravity of total load F is at αl or αh. Centroid of free-bending-moment diagram is at z_1l, z_1h, $z_1\psi h$ or $\frac{1}{2}z_1l$ from left-hand support (vertical loads) or lower support (horizontal loads)

$$D = \frac{\text{area of free-bending-moment diagram}}{\text{loaded length}} = \frac{A}{l}, \frac{A}{l/2}, \frac{A}{h} \text{ or } \frac{A}{\psi h}; \text{ see } \textit{Table 2.28}.$$

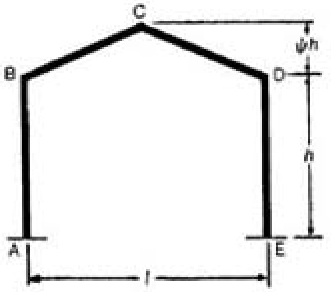
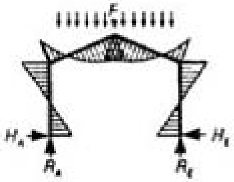
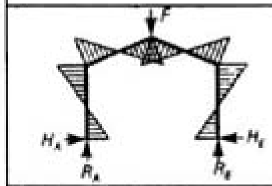
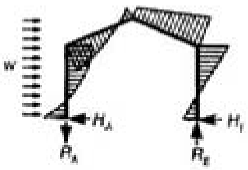
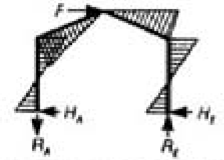
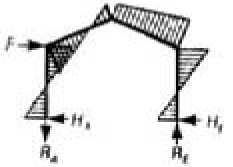
Frame form	Loading	Reactions and bending moments Fixed feet	Hinged feet
 $K=\frac{I_{BC}h}{I_{AB}\sqrt{[(l/2)^2+(\psi h)^2]}}$ $k_4=3K+1$ $k_7=K+3+\psi(3+\psi)$ $k_8=2K+6+3\psi+\alpha\psi(3+2\psi)$ $k_9=2K+6+3\psi$ $k_{10}=3K+\psi(4K+1)$ $k_{11}=K-\psi+2z_1\psi(1+K)$ $k_{12}=K(K+4+6\psi)+\psi^2(4K+1)$ $k_{13}=K+\psi(2K+1)$ $k_{14}=K-\psi(3+2\psi)+3z_1\psi(2+K+\psi)$ $k_{15}=(K+4+3\psi)K$ $k_{16}=\frac{3(1-z_1)D}{3K+1}$ $k_{17}=K+2+\psi-2z_1(K+1)$ $k_{18}=2(K+3)+2\psi(\psi+3)-3z_1(K+2)-3z_1\psi$	$D=A/(l/2)$ 	$H_A=\frac{\frac{1}{2}F\alpha lk_{10}+3k_{11}D}{hk_{12}}$ $\quad R_E=F\alpha-\frac{k_{16}}{l}$ $M_A=\frac{\frac{1}{2}F\alpha lk_{13}+Dk_{14}}{k_{12}}-\frac{1}{2}k_{16}$ $\quad M_B=H_Ah-M_A$ $M_C=\frac{1}{2}(F\alpha l+M_A+M_E)-H_Ah(1+\psi)$ $\quad M_D=H_Ah-M_E$ $M_E=\frac{\frac{1}{2}F\alpha lk_{13}+Dk_{14}}{k_{12}}+\frac{1}{2}k_{16}$	$H_A=\frac{F\alpha l(3+2\psi)+6(1+z_1\psi)D}{4hk_7}$ $R_E=\alpha F$ $M_A=M_E=0$ $M_B=M_D=H_Ah$ $M_C=\frac{1}{2}F\alpha l-H_Ah(1+\psi)$
	$D=A/\psi h$ 	$H_E=\frac{Fh(k_{15}+\alpha k_{10})+6k_{11}D}{2hk_{12}}$ $\quad R_E=\frac{1}{l}[Fh(1+\alpha)-M_E-M_A]$ $M_A=\frac{1}{k_{12}}\left[\frac{Fh\psi}{2}(1-\alpha)k_{13}-Dk_{14}\right]+\frac{Fh(3\psi+2)+6(1-z_1)D}{4k_4}$ $M_B=H_Ah-M_A$ $M_C=\frac{1}{2}[Fh(1+\alpha)-M_A+M_E]-H_Eh(1+\psi)$ $\quad M_D=H_Eh-M_E$ $M_E=\frac{1}{k_{12}}\left[\frac{Fh\psi}{2}(1-\alpha)k_{13}-Dk_{14}\right]-\frac{Fh(3\psi+2)+6(1-z_1)D}{4k_4}$	$H_E=\frac{Fhk_8+6(1+z_1\psi)D}{4hk_7}$ $R_E=Fh(1+\alpha)/l$ $M_A=M_E=0$ $M_B=H_Ah$ $M_C=\frac{1}{2}Fh(1+\alpha)-H_Eh(1+\psi)$ $M_D=H_Eh$
	$D=A/h$ 	$H_E=\frac{1}{2hk_{12}}(F\alpha hk_{15}-6Kk_{17}D)$ $\quad R_E=(F\alpha h-M_E-M_A)/l$ $M_A=\frac{1}{k_{12}}(\frac{1}{2}F\alpha h\psi k_{13}+Kk_{18}D)+\frac{1}{4k_4}[F\alpha h(3K+2)+6KD]$ $M_B=h(F\alpha-H_E)-M_A$ $\quad M_C=\frac{1}{2}(F\alpha h-M_A+M_E)-H_Eh(1+\psi)$ $M_D=H_Eh-M_E$ $M_E=\frac{1}{k_{12}}(\frac{1}{2}F\alpha h\psi k_{13}+Kk_{18}D)-\frac{1}{4k_4}[F\alpha h(3K+2)+6KD]$	$H_E=\frac{F\alpha hk_9+6Kz_1D}{4hk_7}$ $R_E=F\alpha h/l$ $M_A=M_E=0$ $M_B=h(F\alpha-H_E)$ $M_C=\frac{1}{2}F\alpha h-H_Eh(1+\psi)$ $M_D=H_Eh$
	Ⓨ 		$H_A=\frac{3F\alpha l[K(1-\beta^2)+2+\psi]}{4hk_7}$ $R_E=\alpha F$ $\quad M_A=M_E=0$ $M_B=F\alpha l-H_Ah$ $M_C=\frac{1}{2}F\alpha l-H_Ah(1+\psi)$ $M_D=H_Ah$ when also hinged at C: $H_A=\frac{F\alpha l}{2h(1+\psi)}$ and $M_C=0$

Rectangular frames: special cases

Frame form	Loading	Reactions and bending moments		
		Fixed feet		Hinged feet
		$H_A = H_D = \frac{Fl}{4hk_1}$ $R_A = R_D = \frac{1}{2}F$	$M_A = M_D = \frac{Fl}{12k_1}$ $M_B = M_C = \frac{Fl}{6k_1}$	$H_A = H_D = \frac{Fl}{4hk_3}$ $R_A = R_D = \frac{1}{2}F$ $M_A = M_D = 0$ $M_B = M_C = H_A h = \frac{Fl}{4k_3}$
		$H_A = H_D = \frac{3Fl}{8hk_1}$ $R_A = R_D = \frac{1}{2}F$	$M_A = M_D = \frac{Fl}{8k_1}$ $M_B = M_C = \frac{Fl}{4k_1}$	$H_A = H_D = \frac{3Fl}{8hk_3}$ $R_A = R_D = \frac{1}{2}F$ $M_A = M_D = 0$ $M_B = M_C = H_A h = \frac{3Fl}{8k_3}$
$F =$ total load $I_{AB} = I_{CD}$ $K = \frac{I_{BC}h}{I_{AB}l}$ $k_1 = K + 2$ $k_2 = 6K + 1$ $k_3 = 2K + 3$ $k_4 = 3K + 1$		$H_A = F - H_D$ $H_D = \frac{Fk_3}{8k_1}$ $R_A = -\frac{FhK}{lk_2} = -R_D$ $M_A = \frac{Fh}{4}\left[\frac{K+3}{6k_1} + \frac{4K+1}{k_2}\right]$	$M_B = h(H_A - \frac{1}{2}F) - M_A$ $M_C = H_D h - M_D$ $M_D = \frac{Fh}{4}\left[\frac{K+3}{6k_1} - \frac{4K+1}{k_2}\right]$	$H_A = \frac{F}{8}\left(\frac{6k_3 - K}{k_3}\right)$ $H_D = F - H_A$ $R_D = -R_A = \frac{Fh}{2l}$ $M_A = M_D = 0$ $M_B = h(\frac{1}{2}F - H_D) = \frac{3Fhk_1}{8k_3}$ $M_C = H_D h = \frac{Fh}{8}\left(\frac{2k_3 + K}{k_3}\right)$
		$H_A = H_D = \frac{1}{2}F$ $R_A = -R_D = -\frac{3FhK}{lk_2}$	$M_A = M_D = \frac{Fhk_4}{2k_2}$ $M_B = M_C = \frac{3FhK}{2k_2}$	$H_A = H_D = \frac{1}{2}F$ $R_D = -R_A = \frac{Fh}{l}$ $M_A = M_D = 0$ $M_B = M_C = \frac{1}{2}Fh$

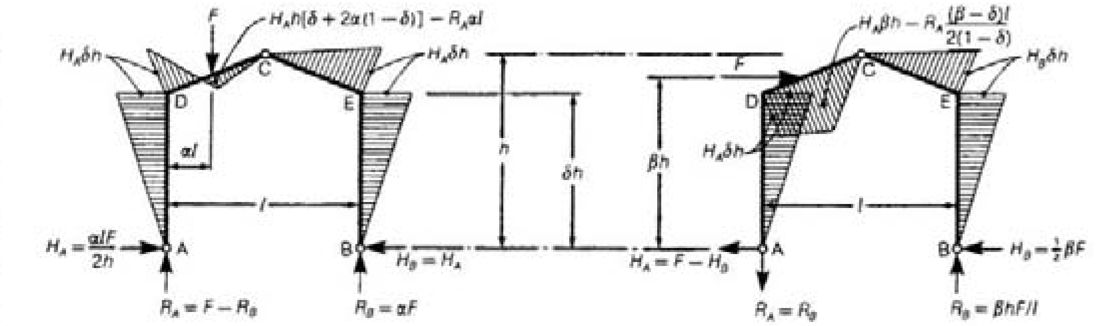
Note: formulae give numerical values of reactions and bending moments: see diagrams for direction of action.

Gable frames: special cases

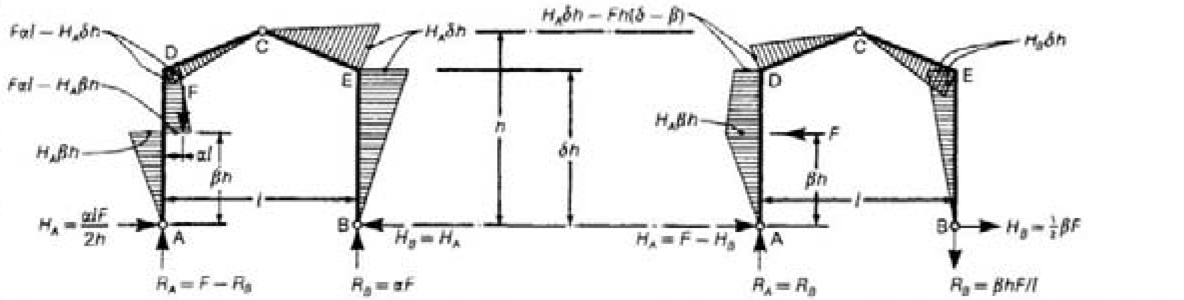
Frame form	Loading	Reactions and bending moments	
		Fixed feet	Hinged feet
 F = total load $I_{AB} = I_{DE}$ $I_{BC} = I_{CD}$ $K = \frac{I_{BC}h}{I_{AB}\sqrt{(l/2)^2 + (\psi h)^2}}$ $k_1 = K + 2$ $k_4 = 3K + 1$ $k_7 = K + 3 + \psi(3 + \psi)$ $k_9 = 2K + 3(2 + \psi)$ $k_{10} = 3K + \psi(4K + 1)$ $k_{12} = K(K + 4 + 6\psi) + \psi^2(4K + 1)$ $k_{13} = K + \psi(2K + 1)$ $k_{15} = K(K + 4 + 3\psi)$ $k_{19} = 8(K + 3) + 5\psi(4 + \psi)$ $k_{20} = K + 3 + 2\psi$ $k_{21} = 2K(K + 4 + 5\psi) + \psi^2(5K + 1)$ $k_{22} = 2(10K + 3) + \psi(9K + 6) + \psi^2$		$H_A = H_E = \frac{Fl}{8h}\left[\frac{4K + \psi(5K + 1)}{k_{12}}\right]$ $R_A = R_E = \frac{1}{2}F$ $M_A = M_E = \frac{Fl}{48}\left[\frac{K(8 + 15\psi) + \psi(6 - \psi)}{k_{12}}\right]$ $M_B = M_D = H_A h - M_A$ $M_C = H_A h(1 + \psi) - \frac{1}{8}Fl - M_A$	$H_A = H_E = \frac{Fl}{32h}\left(\frac{8 + 5\psi}{k_7}\right)$ $R_A = R_E = \frac{1}{2}F \qquad M_A = M_E = 0$ $M_B = M_D = H_A h$ $M_C = H_A h(\psi + 1) - \frac{Fl}{8}$
		$H_A = H_E = \frac{Fl}{4h}\left(\frac{k_{10}}{k_{12}}\right)$ $R_A = R_E = \frac{1}{2}F$ $M_A = M_E = \frac{Fl}{4}\left(\frac{k_{13}}{k_{12}}\right)$ $M_B = M_D = H_A h - M_A$ $M_C = H_A h(1 + \psi) - \frac{1}{4}Fl - M_A$	$H_A = H_E = \frac{Fl}{8h}\left(\frac{3 + 2\psi}{k_7}\right)$ $R_A = R_E = \frac{1}{2}F \qquad M_A = M_E = 0$ $M_B = M_D = H_A h$ $M_C = H_A h(1 + \psi) - \frac{1}{4}Fl$
	F(total) = $wh(1 + \psi)$ 	$H_E = \frac{wh}{4k_{12}}(Kk_{20} + \psi k_{21}) \qquad H_A = F - H_E$ $R_E = -R_A = \frac{wh^2}{8lk_4}[4K(1 + 3\psi) + \psi^2(12K + 5)]$ $M_A = \frac{wh^2}{24}\left\{\left[\frac{K(K + 6 + 15\psi) + \psi^2 k_{22}}{k_{12}}\right] + \frac{12k_5 + \psi[12(3K + 2) + 3\psi]}{2k_4}\right\}$ $M_B = H_A h - M_A - \frac{1}{2}wh^2$ $M_C = H_E h(1 + \psi) - \frac{1}{2}lR_E - M_E \qquad M_D = H_E h - M_E$ $M_E = \frac{wh^2}{24}\left\{\left[\frac{K(K + 6 + 15\psi) + \psi^2 k_{22}}{k_{12}}\right] - \frac{12k_5 + \psi[12(3K + 2) + 3\psi]}{2k_4}\right\}$	$H_E = \frac{wh}{16k_7}(2k_9 + K + \psi k_{19})$ $H_A = wh(1 + \psi) - H_E$ $R_E = -R_A = \frac{wh^2}{2l}[1 + \psi(2 + \psi)]$ $M_A = M_E = 0$ $M_B = H_A h - \frac{1}{2}wh^2$ $M_C = H_E h(1 + \psi) - \frac{1}{2}R_E l$ $M_D = H_E h$
		$H_A = H_E = \frac{1}{2}F$ $R_E = -R_A = \frac{1}{l}[Fh(1 + \psi) - 2M_E]$ $M_A = M_E = \frac{Fh}{4}\left(\frac{3K + 2}{2K + 1}\right)$ $M_B = M_D = H_A h - M_A$ $M_C = 0$	$H_A = H_E = \frac{1}{2}F$ $R_E = -R_A = \frac{1}{l}[Fh(1 + \psi)]$ $M_A = M_E = 0$ $M_B = M_D = \frac{1}{2}Fh \qquad M_C = 0$
		$H_E = \frac{FK}{2}\left(\frac{k_{15}}{k_{12}}\right) \qquad H_A = F - H_E \qquad M_B = H_A h - M_A$ $R_E = -R_A = \frac{3Fh}{2l}\left(\frac{K}{3K + 1}\right)$ $M_C = H_E h(1 + \psi) - \frac{1}{2}lR_E - M_E$ $M_D = H_E h - M_E$ $M_A = \frac{Fh}{2}\left(\frac{\psi k_{13}}{k_{12}} + \frac{k_4 + 1}{2k_4}\right)$ $M_E = \frac{Fh}{2}\left(\frac{\psi k_{13}}{k_{12}} - \frac{k_4 + 1}{2k_4}\right)$	$H_E = \frac{Fk_9}{4k_7} \qquad R_E = -R_A = \frac{Fh}{l}$ $H_A = F - H_E \qquad M_A = M_E = 0$ $M_B = H_A h \qquad M_D = H_E h$ $M_C = H_E h(1 + \psi) - \frac{1}{2}Fh$

Note: formulae give numerical values of reactions and bending moments: see diagrams for direction of action.

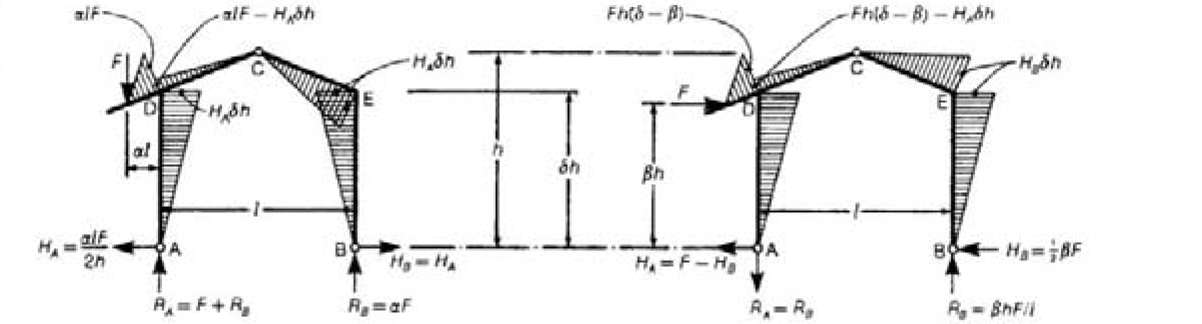
Three-hinged frame with normal loading



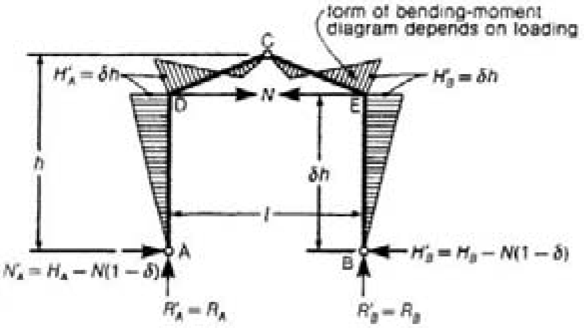
Three-hinged frame carrying crane



Three-hinged frame with overhang



Three-hinged frame with tie-rod

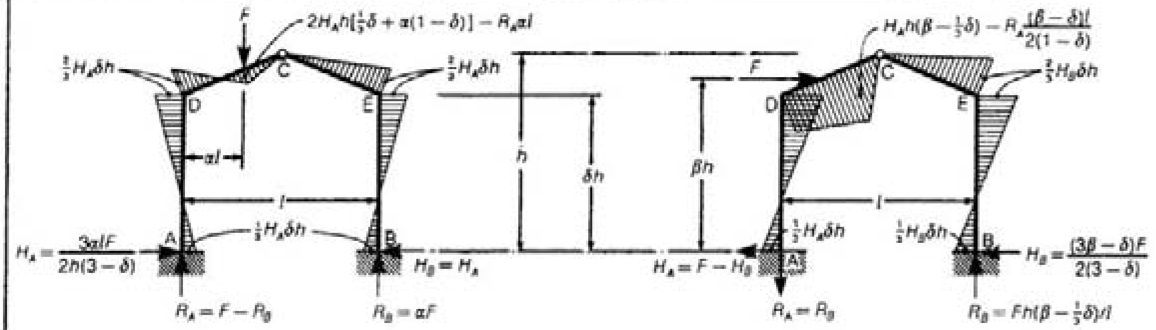


Force in tie-rod $= N = \dfrac{H_A + H_B}{2(1-\delta) + 6lE_cI/A_{tie}E_s(\delta h)^3}$

Stress in tie-rod $= N/A'_{tie}$

H_A, H_B, R_A and R_B	Reactions for three-hinged frame without tie-rod: H_A and H_B assumed acting inwards; negative if acting outwards
E_c	elastic modulus of concrete; E_s = ditto for tie-rod
A_{tie}	normal cross-sectional area of tie-rod
A'_{tie}	minimum cross-sectional area of tie-rod
I	mean moment of inertia (concrete units) of frame

Frame with embedded legs and hinge at ridge



Chapter 15

Shear wall structures

In modern multi-storey buildings, lateral stability is provided by a system of frames or walls, or a combination of both. Notes on wall and frame systems are given in section 4.12 and different structural forms, with information on typical building heights and proportions given in ref. 37, are shown in *Table 2.68*.

15.1 LAYOUT OF SHEAR WALLS AND ALLOCATION OF LATERAL LOADS

Arrangements of shear walls and core units should be such that the building is stiff in both flexure and torsion. Different plan configurations, with remarks as to their suitability, are shown in *Table 2.69*. The lateral load transmitted to each wall is a function of its stiffness, and its position in relation to the shear centre of the system. The location of the shear centre can be readily determined by calculating the moment of stiffness of the walls about an arbitrary reference point, as shown in *Table 2.69*. The lateral load on each wall can then be determined from the generalised formulae given also in *Table 2.69*. For most walls without openings, the dominant mode of deformation is bending (see section 4.12.2). In this case, K may be replaced with I in the generalised formulae, where I is the second moment of area of the cross section.

15.2 PIERCED SHEAR WALLS

In the case of walls pieced by openings, the behaviour of the individual wall sections is coupled to a variable degree. The deflected shape of the pierced wall is a combination of frame and wall action. The wall may be idealised as a plane frame, or analysed by elastic methods in which the flexibility of the beams is represented as a continuous flexible medium.

Solutions using the continuous connection model for a wall containing a single line of openings are given in *Table 2.70*. The total lateral load F is considered in three different forms: a concentrated load applied at the top of the wall, a uniform load distribution and a triangular load distribution with the maximum value at the top of the wall. Formulae are given for the maximum axial force at the base of each wall section, the maximum shear force on a connecting beam (and the height of the beam above the base of the wall) and the maximum deflection at the top of the wall. Formulae, whereby values at any level can be determined, are given in ref. 38.

The main two parameters that define the performance of the wall are α and β, which depend on the geometrical properties of the wall and beam elements, and on the number of lines of openings. The formulae in *Table 2.70* cater for a wall with dissimilar cross sections on either side of a single line of openings. For identical cross sections, the formulae become

$$\alpha^2 = \frac{\beta^2}{l}\left[l^2 + \frac{4I_1}{A_1}\right] \qquad \beta^2 = \frac{6lI_e}{I_1 h b_e^3}$$

where A_1 and I_1 refer to each portion of the wall. For a wall with two symmetrical lines of openings, the formulae for the parameters become

$$\alpha^2 = \frac{\beta^2}{l}\left[2l^2 + \frac{(2I_1 + I_2)}{A_1}\right] \qquad \beta^2 = \frac{12lI_e}{(2I_1 + I_2)hb_e^3}$$

where A_1 and I_1 refer to each outer portion, and I_2 refers to the central portion of the wall. Similarly, the moments become

$$M_1 = \frac{(M_0 - Nl)I_1}{2I_1 + I_2} \qquad M_2 = \frac{(M_0 - Nl)I_2}{2I_1 + I_2}$$

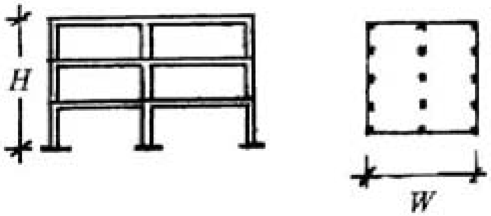
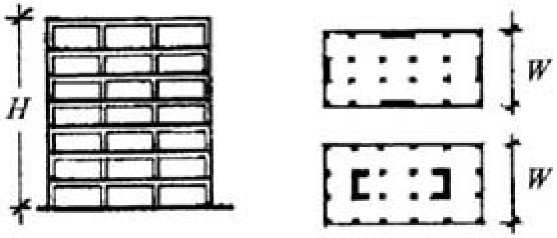
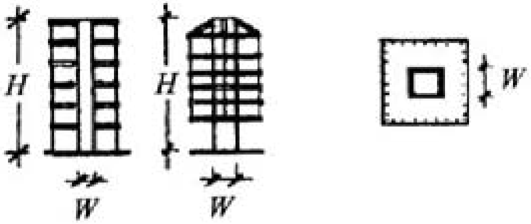
There is no axial force in the central portion of the wall.

15.3 INTERACTION OF SHEAR WALLS AND FRAMES

Although the interaction forces between solid walls, pierced walls and frames can vary considerably up the height of a building, the effect on the total lateral force resisted by each element is less significant. As a first approximation, in order to determine the forces at the bottom of each load-resisting element, it is normally sufficient to consider the effect of a single interaction force at the top of the building. This is equivalent to load sharing in terms of relative stiffness. The location of the shear centre and the allocation of the lateral load can be determined as indicated in *Table 2.69*, using the following formulae for the stiffness of each element.

$$\text{Solid wall} \quad K = \frac{3EI_w}{H^3}$$

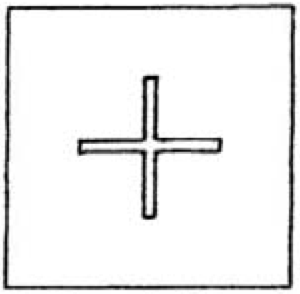
$$\text{Pierced wall } K = \frac{3E\sum I_w}{H^3\left[1 - \frac{\beta^2 l}{\alpha^2}\left(1 + \frac{3}{(\alpha H)^2} + \frac{3}{(\alpha H)^3}\right)\right]}$$

Structure	Section and plan	Typical heights (storeys)	Typical values of H/W	Remarks
Multi-storey rigid frames		5–15	1–5	Rigid frames with no extra vertical bracing can be economic up to 15 storeys
Shear walls and/or cores with flexible or rigid frames		10–55	4–5	Shear walls or cores interact with rigid frames to provide stiff vertical bracing system for heights over 30 storeys
Framed tube and core		40–65	6–7	Also known as tube in tube system
Core structure with flexible frames, or suspended floors, connected to deep cantilever trusses		10–30	8–12	With suspended floor system plan area is limited and core provides all lateral stability

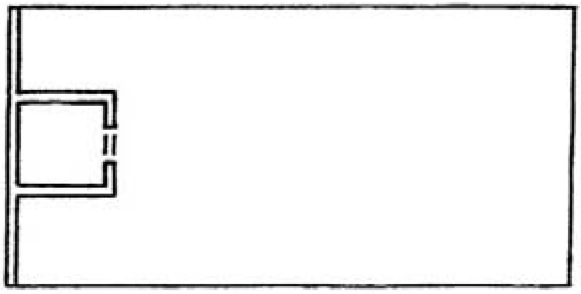
2.69

Shear wall layout and lateral load allocation

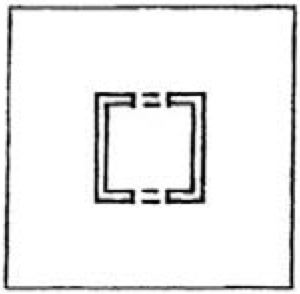
Plan configuration of shear walls



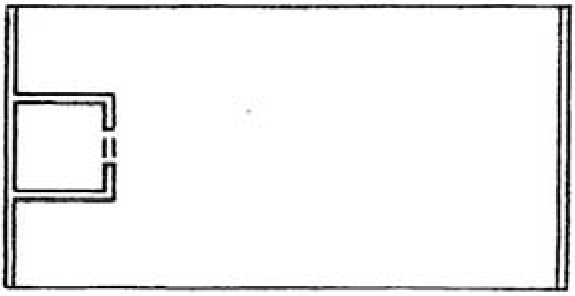
Cruciform walls able to resist biaxial bending but not torsion



Walls able to resist biaxial bending, but poor layout resulting in large torsion on core of limited capacity

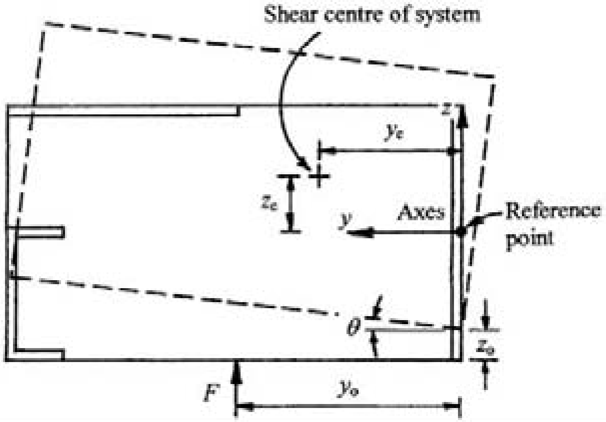


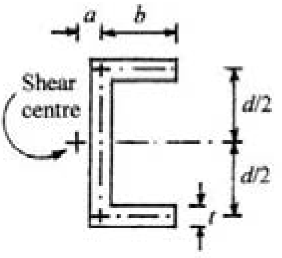
Core unit able to resist both biaxial bending and torsion



Walls able to resist biaxial bending, with good layout resulting in little torsion and considerable resistance

Location of shear centre and allocation of lateral load





$$a \approx \frac{(bd)^2 t}{4I_z}$$

Distances of shear centre of wall system from perpendicular co-ordinate axes *y* and *z* through any reference point are:

$$y_c = \frac{\sum K_z y}{\sum K_z} \qquad z_c = \frac{\sum K_y z}{\sum K_y}$$

K_y and K_z are stiffness values of individual wall units (in relation to a unit force applied at top of wall unit) in the directions *y* and *z* respectively.

y and *z* are distances from reference point to the shear centres of individual wall units. For core units with asymmetric openings, see ref. 38.

Shearing forces on individual wall units, in directions *y* and *z* respectively, from total force *F* in direction *z* are:

$$F_y = \frac{F(y_o - y_c)K_y(z - z_c)}{\sum K_z(y - y_c)^2 + \sum K_y(z - z_c)^2 + \sum K_\theta} \qquad F_z = \frac{FK_z}{\sum K_z} + \frac{F(y_o - y_c)K_z(y - y_c)}{\sum K_z(y - y_c)^2 + \sum K_y(z - z_c)^2 + \sum K_\theta}$$

For planar walls, K_y may be neglected for walls in direction *z*, and K_z may be neglected for walls in direction *y*. The torsion stiffness, K_θ, of individual walls may generally be neglected, but see ref. 33 for guidance on core units. For most walls without openings, K_y and K_z may be replaced by second moment of area values, I_y and I_z, in the directions *y* and *z* respectively. For structural interaction of planar walls, pierced walls and frames, see section 15.3.

Analysis of pierced shear walls

2.70

Distribution of total lateral load F

Concentrated load F at top of wall

Maximum axial force in wall (bottom of wall)

$$N = \frac{FH\beta^2}{\alpha^2}\left[1 - \frac{\sinh \alpha H}{\alpha H \cosh \alpha H}\right] \approx \frac{FH\beta^2}{\alpha^2}\left(1 - \frac{1}{\alpha H}\right)$$

Maximum shear force in beam (top of wall)

$$V_b = \frac{Fh\beta^2}{\alpha^2}\left[1 - \frac{1}{\cosh \alpha H}\right] \approx \frac{Fh\beta^2}{\alpha^2}$$

Maximum lateral deflection (top of wall)

$$a = \frac{FH^3}{3E(I_1 + I_2)}\left[1 - \frac{\beta^2 l}{\alpha^2}\left(1 - \frac{3}{(\alpha H)^2} + \frac{3}{(\alpha H)^3}\right)\right]$$

Uniform load of intensity F/H

Maximum axial force in wall (bottom of wall)

$$N = \frac{FH\beta^2}{2\alpha^2}\left[1 - \frac{2(\alpha H \sinh \alpha H - \cosh \alpha H + 1)}{(\alpha H)^2 \cosh \alpha H}\right]$$

$$\approx \frac{FH\beta^2}{2\alpha^2}\left[1 - \frac{2}{\alpha H} + \frac{2}{(\alpha H)^2}\right]$$

Maximum shear force in beam (see below for x/H)

$$V_b = \frac{Fh\beta^2 K_v}{\alpha^2} \quad \text{(see below for } K_v\text{)}$$

αH	2	4	6	8	10	16
x/H	0.50	0.38	0.30	0.26	0.23	0.18
K_v	0.25	0.42	0.54	0.62	0.67	0.76

Maximum lateral deflection (top of wall)

$$a = \frac{FH^3}{8E(I_1 + I_2)}\left[1 - \frac{\beta^2 l}{\alpha^2}\left(1 - \frac{4}{(\alpha H)^2} + \frac{8}{(\alpha H)^3} - \frac{8}{(\alpha H)^4}\right)\right]$$

Triangular load of intensity $2F/H$ at top of wall

Maximum axial force in wall (bottom of wall)

$$N = \frac{FH\beta^2}{1.5\alpha^2}\left[1 - \frac{1.5(\alpha H)^2 \sinh \alpha H + 3(\alpha H - \sinh \alpha H)}{(\alpha H)^3 \cosh \alpha H}\right]$$

$$\approx \frac{FH\beta^2}{1.5\alpha^2}\left[1 - \frac{1.5}{\alpha H} + \frac{3}{(\alpha H)^3}\right]$$

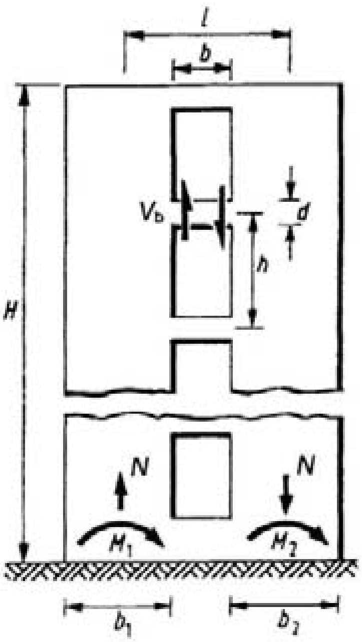
Maximum shear force in beam (see below for x/H)

$$V_b = \frac{Fh\beta^2 K_v}{\alpha^2} \quad \text{(see below for } K_v\text{)}$$

αH	2	4	6	8	10	16
x/H	0.80	0.45	0.37	0.33	0.29	0.22
K_v	0.34	0.57	0.71	0.79	0.85	0.92

Maximum lateral deflection (top of wall)

$$a = \frac{11FH^3}{60E(I_1 + I_2)}\left[1 - \frac{\beta^2 l}{11\alpha^2}\left(11 - \frac{40}{(\alpha H)^2} + \frac{60}{(\alpha H)^3} - \frac{120}{(\alpha H)^5}\right)\right]$$



$$\alpha^2 = \frac{\beta^2}{l}\left[l^2 + \frac{A_1 + A_2}{A_1 A_2}(I_1 + I_2)\right]$$

$$\beta^2 = \frac{12lI_e}{(I_1 + I_2)hb_e^3} \qquad I_e = \frac{I_b}{1 + 2.4(d/b)^2}$$

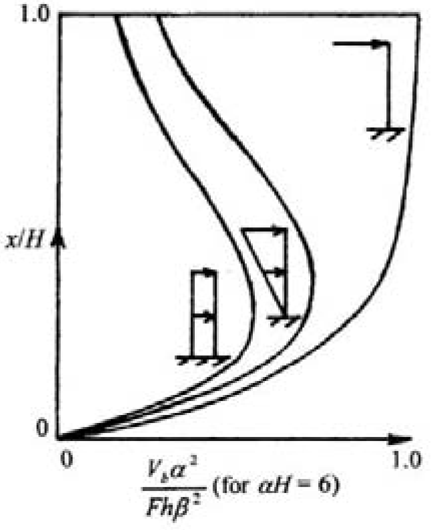
$b \leq b_e = (2b + 5d)/3 \leq b + d$ $\qquad$ $M_b = V_b b/2$

I_b, I_1 and I_2 are the second moment of areas of the beam and wall sections respectively

l is the distance between the centres of area of the wall sections

$$M_1 = \frac{(M_0 - Nl)I_1}{I_1 + I_2} \qquad M_2 = \frac{(M_o - Nl)I_2}{I_1 + I_2}$$

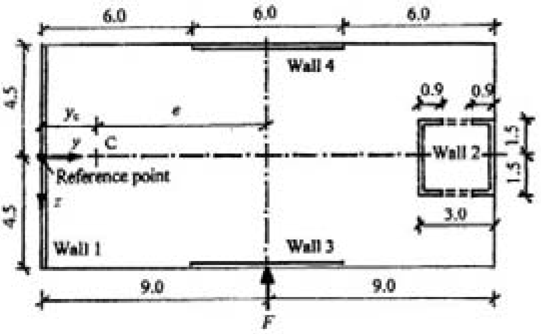
M_o = FH, $FH/2$ and $FH/1.5$ for concentrated, uniform and triangular loads respectively



$$\text{Frame } K = \frac{12E\sum I_c}{h^2H\left(k_c + \dfrac{l_b\sum I_c}{h\sum I_b}\right)}$$

For the frame, $\sum I_c$ and $\sum I_b$ are the sums of the second moments of area of the columns and beams respectively, for the lowest bay of height h and beam span l_b. The factor k_c allows for a reduction of I_c over the height of the building and is given by $k_c = \log_e c/(c - 1)$, where $c = I_{c\,\text{top}}/I_{c\,\text{bottom}}$.

Example 1. For the shear wall layout in the following figure, determine the location of the shear centre, and the lateral load applied to each wall, using the formulae in *Table 2.69*. All dimensions are in metres and the walls are 200 mm thick.



Layout of shear walls

From symmetry, the shear centre of the wall system must be on the y-axis. Similarly, the shear centre of the two channel sections (taken together) is at the mid-point of the core unit. The second moment of area values (mm^4) of walls 1 and 2 in direction z (discounting the stiffness of walls 3 and 4) are:

$$I_{1z} = \frac{0.2 \times 9.0^3}{12} = 12.15 \quad I_{2z} = \frac{1.8\times 3.0^3 - 1.4\times 2.6^3}{12} = 2.0$$

The distance of the shear centre C from the reference point is

$$y_c = \frac{12.15 \times 0.1 + 2.00 \times 16.5}{12.15 + 2.00} = 2.42\text{m}$$

Eccentricity of total lateral force is $e = 9.0 - 2.42 = 6.58$ m
The second moment of area values (mm^4) of walls 3 and 4 in direction y are:

$$I_{3y} = I_{4y} = \frac{0.2\times 6.0^3}{12} = 3.6$$

Lateral forces on each wall in terms of total force F are:

$$F_{1z} = \frac{12.15F}{12.15 + 2.0}$$

$$+ \frac{12.15(0.1 - 2.42)(F \times 6.58)}{12.15 \times 2.32^2 + 2.0 \times 14.08^2 + 2 \times 3.6 \times 4.4^2}$$

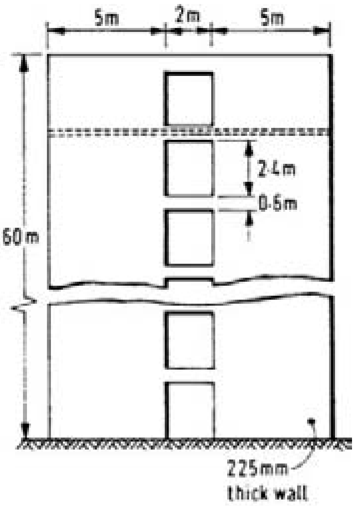
$$= \frac{12.15F}{14.15} - \frac{185.5F}{601.28} = (0.859 - 0.309)\,F = 0.55F$$

$$F_{2z} = \frac{2.0F}{14.15} + \frac{2.0(16.5 - 2.42)(F \times 6.58)}{601.28}$$

$$= (0.141 + 0.309)\,F = 0.45F$$

$$F_{3y} = -F_{4y} = \frac{3.6(4.4 - 0)(F \times 6.58)}{601.28} = 0.17F$$

Example 2. The elevation of a pierced shear wall is shown in the following figure. Identical walls are provided at each end of a 24 m long rectangular building. The building is subjected to a characteristic wind pressure of 1.25kN/m^2 acting on the broad face, and the resistance provided by any other frames parallel to the walls may be ignored. The resulting forces and bending moments acting on the walls are to be determined.



Elevation of shear wall

If the total wind force acting on the building is shared equally between the two walls, the horizontal force to be resisted by each wall is $F = 1.25 \times 12 \times 60 = 900$ kN.

With reference to *Table 2.70*, the following values apply:

$A_1 = A_2 = 0.225 \times 5 = 1.125$ m^2
$I_1 = I_2 = 0.225 \times 5^3/12 = 2.344$ m^4
$I_b = 0.225 \times 0.6^3/12 = 0.00405$ m^4

$$I_e = \frac{I_b}{1 + 2.4(d/b)^2} = \frac{0.00405}{1 + 2.4 \times (0.6/2)^2} = 0.00333$$

$$b_e = (2b + 5d)/3 = (2 \times 2 + 5 \times 0.6)/3 = 2.33$$

$$\beta^2 = \frac{12lI_e}{(I_1 + I_2)hb_e^3} = \frac{12 \times 7 \times 0.00333}{2 \times 2.344 \times 3 \times 2.33^3} = 0.00157$$

$$\alpha^2 = \frac{\beta^2}{l}\left[l^2 + \frac{A_1 + A_2}{A_1A_2}(I_1 + I_2)\right]$$

$$= \frac{0.00157}{7}\left[7^2 + \frac{2 \times 2 \times 2.344}{1.125}\right] = 0.01286$$

Hence, $\alpha = 0.113$, $\beta = 0.0396$, $\alpha H = 0.113 \times 60 = 6.8$. Maximum axial force at bottom of each wall element:

$$N \approx \frac{FH\beta^2}{2\alpha^2}\left[1 - \frac{2}{\alpha H} + \frac{2}{(\alpha H)^2}\right]$$

$$= \frac{900 \times 60 \times 0.00157}{2 \times 0.01286}\left[1 - \frac{2}{6.8} + \frac{2}{6.8^2}\right] = 2470 \text{ kN}$$

Moment at bottom of each wall element:

$$\begin{aligned} M_1 = M_2 &= (FH/2 - Nl)/2 \\ &= (900 \times 60/2 - 2470 \times 7)/2 = 4855 \text{ kNm} \end{aligned}$$

Maximum shear force in beams, for $\alpha H = 6.8$, occurs where $x/H = 0.28$ (5th or 6th floor level) and $K_v = 0.57$. Then,

$$V_b = \frac{Fh\beta^2 K_v}{\alpha^2} = \frac{900 \times 3 \times 0.00157 \times 0.57}{0.01286} = 188 \text{ kN}$$

Maximum bending moment in beam

$$M_b = V_b\, b/2 = 188 \times 2/2 = 188 \text{ kNm}$$

For subsequent design purposes, the forces and moments due to the characteristic wind load must be multiplied by a partial safety factor appropriate to the load combination.

Chapter 16

Arches

In this chapter elastic analyses in terms of characteristic loads and service stresses are indicated. Where limit-state design methods are employed, care must be taken to include the appropriate partial safety factors for the load combination and limit-state being considered. Arch structures may be either three-hinged, or two-hinged or fixed-ended, as shown in the diagrams at the top of *Table 2.71*.

16.1 THREE-HINGED ARCH

For the general case of an unsymmetrical three-hinged arch with a load acting vertically, horizontally or at an angle, the expressions for the horizontal and vertical components of the thrusts on the abutments are given in the lower part of *Table 2.71*. For symmetrical arches, the formulae for three-hinged portal frames given in *Table 2.67* are applicable. The bending moment at each hinge is zero, and at any particular section, the bending moment, shearing force and axial thrust may be determined by considering the loads and abutment reactions on one side of the section. Further information regarding the extent of the arch that should be loaded with imposed load, in order to produce the maximum effect at a particular section, is given in section 4.13.1.

16.2 TWO-HINGED ARCH

For a symmetrical two-hinged arch, the vertical component of thrust on the abutments is the same as for a freely supported beam. The horizontal component of thrust H is given by the formula in *Table 2.71*, where M_x is the bending moment on a section at distance x from the crown: with the arch considered as a freely supported beam, M_x is given by the corresponding expression in *Table 2.71*.

The summations $\Sigma M_x y a_I$ and $\Sigma y^2 a_I$ are taken over the whole length of the arch. In the formula for H, which allows for the elastic shortening of the arch, A is the average equivalent area of the arch rib or slab, and a is the length of a short segment of the axis of the arch, where the coordinates of a are x and y as shown in *Table 2.71*. If I is the second moment of area of the arch at x, then $a_I = a/I$. The bending moment at any section is given by $M_d = M_x - Hy$.

The procedure involves dividing the axis of the arch into an even number of segments. The calculations can be facilitated, by tabulating the successive steps. The total bending moment is required, generally, only at the crown ($x = 0$, $y = y_c$) and the first quarter-point ($x = 0.25l$). The moment M_c at the crown is the bending moment for a freely supported beam minus Hy_c. For the maximum positive moment at the crown, the sum of the values of M_c for all elements of dead load is added to the values of M_c for only those values of imposed load that give positive values of M_c. For the maximum negative moment at the crown, the sum of the values of M_c for all elements of dead load is added to the values of M_c for only those values of imposed load that give negative values of M_c. The moment at the first quarter-point is the bending moment for a freely supported beam minus Hy_q, where y_q is the vertical ordinate of the first quarter-point. The bending moment is combined with the normal component of H. For an arch of large span, it is worthwhile preparing the influence lines ($F = 1$) for the bending moments at the crown and at the first quarter-point.

16.3 FIXED ARCH

16.3.1 Determination of thickness

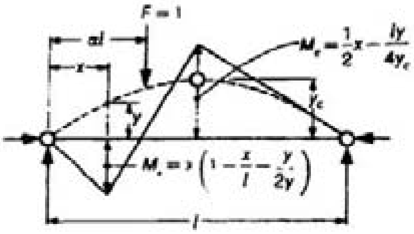
The diagram at the top of *Table 2.72* shows an approximate method of determining the section thickness at the crown and the springing, for a symmetrical arch with fixed ends. Draw a horizontal line through the crown C and find G, the point of intersection of the line with the vertical through the centre of gravity of the total load on half the span of the arch. Set out length GT equal to the dead load on the half span, drawn to a convenient scale; draw a horizontal line through T to intersect GS extended at R. Draw lines RK perpendicular to GR, and GK parallel to the tangent to the arch axis at S. On the same scale that was used to draw GT, measure TR, which equals H_c, and GK, which equals H_s. If f_{cc} is the maximum allowable compressive stress in the concrete, b the assumed breadth of the arch (1 m for a slab), h_c the thickness at the crown, and h_s the thickness at the springing, then approximately

$$h_c = 1.7H_c/bf_{cc} \quad h_s = 2H_s/bf_{cc}$$

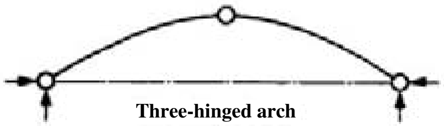
The method applies to spans from 12 to 60 m, and span/rise ratios from 4 to 8. The method does not depend on knowing the profile of the arch (except for solid-spandrel earth-filled arches, where the dead load is largely dependent on the shape of the arch), but the span and rise must be known. With h_c and h_s thus determined, the thrusts and bending moments at the crown,

Arches: three-hinged and two-hinged arches

Three-hinged arch **Influence line for section at *x***

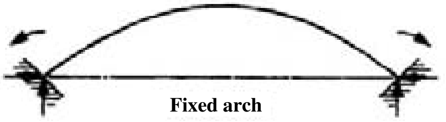


Types of arches



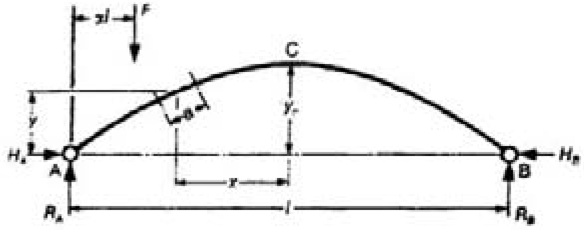
Three-hinged arch

Two-hinged arch



Fixed arch

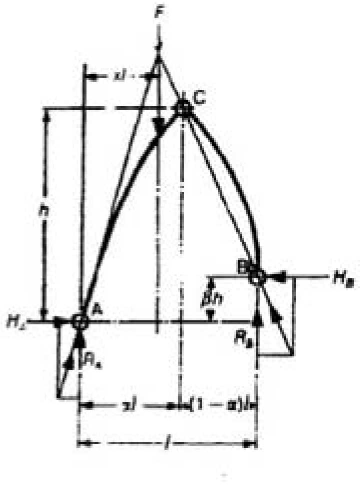
Two-hinged arch

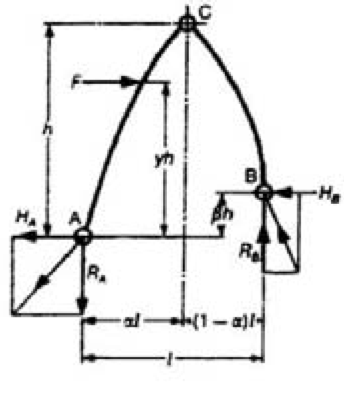


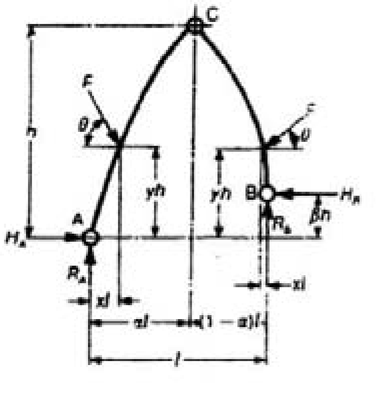
Bending moment at *x*: $M_x = F\alpha l\left(\frac{x}{l} + \frac{1}{2}\right)$

Horizontal thrust: $H_A = H_B = \frac{\Sigma M_x y a_l}{(\Sigma y^2 a_l) + l/A}$

Unsymmetrical three-hinged arches **General case**







Reaction formulae for general case:

$$R_A = \sum_A^C\left[1 - \frac{(1-\beta)x}{(1-\alpha\beta)}\right]F\sin\theta + \frac{1}{1-\alpha\beta}\sum_B^C xF\sin\theta + \frac{h}{(1-\alpha\beta)l}\left[\sum_B^C(y-\beta)F\cos\theta - (1-\beta)\sum_A^C yF\cos\theta\right]$$

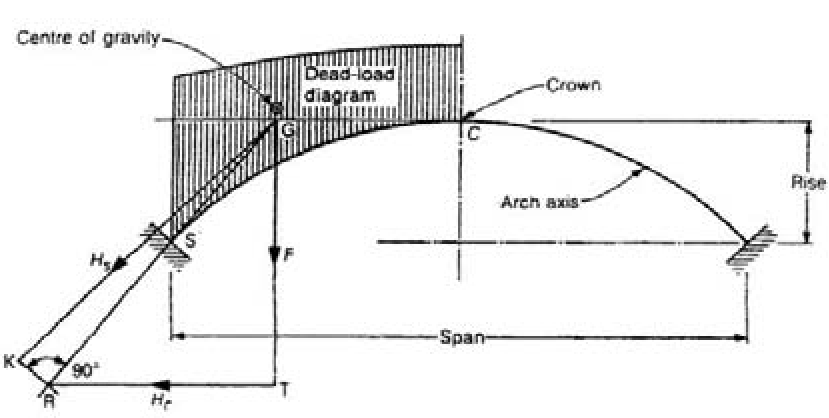
$$R_B = \frac{1-\beta}{1-\alpha\beta}\sum_A^C xF\sin\theta + \sum_B^C\left[1 - \frac{x}{1-\alpha\beta}\right]F\sin\theta + \frac{h}{(1-\alpha\beta)l}\left[(1-\beta)\sum_A^C yF\cos\theta - \sum_B^C(y-\beta)F\cos\theta\right]$$

$$H_A = \frac{l}{(1-\alpha\beta)h}\left[(1-\alpha)\sum_A^C xF\sin\theta + \alpha\sum_B^C xF\sin\theta\right] + \frac{\alpha}{(1-\alpha\beta)}\sum_B^C(y-\beta)F\cos\theta - \sum_A^C\left[1 - \frac{(1-\alpha)y}{(1-\alpha\beta)}\right]F\cos\theta$$

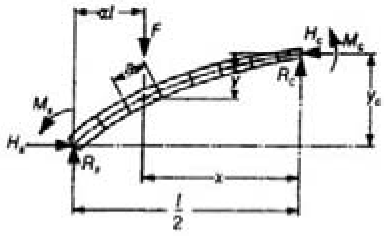
$$H_B = \frac{l}{(1-\alpha\beta)h}\left[(1-\alpha)\sum_A^C xF\sin\theta + \alpha\sum_B^C xF\sin\theta\right] + \frac{1-\alpha}{(1-\alpha\beta)}\sum_A^C yF\cos\theta - \sum_B^C\left[1 - \frac{(y-\beta)\alpha}{(1-\alpha\beta)}\right]F\cos\theta$$

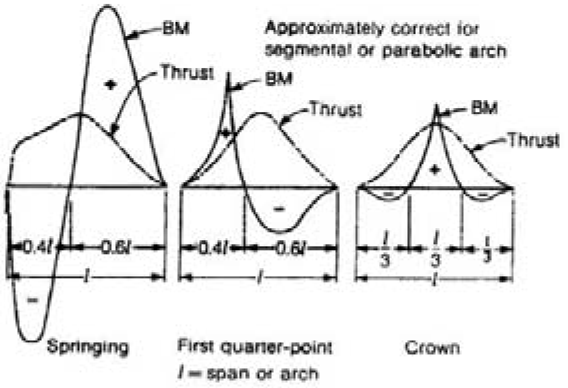
Note: see section 16.2 for explanation of symbols in and notes on the formulae for two-hinged arches.

Determination of trial sizes – Approximate method



Notation and typical influence lines

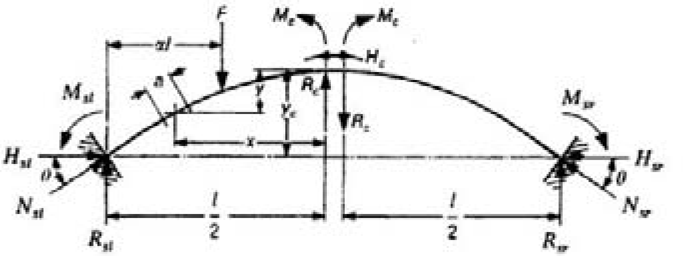




T rise or fall of temperature

λ coefficient of linear expansion per degree C

General formulae



$M_f = [x - l(\frac{1}{2} - \alpha)]F$

$a_I = a/I$

$a_A = a/A$

Load F on left-hand side: $H_c = \dfrac{\Sigma a_I \Sigma M_f y a_I - \Sigma y a_I \Sigma M_f a_I}{2[\Sigma a_I \Sigma y^2 a_I - (\Sigma y a_I)^2]}$ $\quad M_c = \dfrac{\Sigma M_f a_I - 2H_c \Sigma y a_I}{2\Sigma a_I}$ $\quad R_c = \dfrac{\Sigma M_f x a_I}{2\Sigma x^2 a_I}$

Temperature and shrinkage: $H_{c2} = \dfrac{(\pm T)\lambda l E_c \Sigma a_I}{2[\Sigma a_I \Sigma y^2 a_I - (\Sigma y a_I)^2]}$ $\quad M_{c2} = -\dfrac{H_{c2} \Sigma y a_I}{\Sigma a_I}$

Arch shortening: $H_{c1} = -\dfrac{\Sigma a_A \Sigma a_I (H_c + H_{c2})}{\Sigma a_I \Sigma y^2 a_I - (\Sigma y a_I)^2}$ $\quad M_{c1} = -\dfrac{H_{c1} \Sigma y a_I}{\Sigma a_I}$

Summations are taken over left-hand half of arch: note that summations involving M_f include only segments within αl.

At crown: net thrust $H_c^* = H_c + H_{c1} + H_{c2}$ (note signs of H and M)

net bending moment $M_c^* = M_c + M_{c1} + M_{c2}$

At springings: $M_{sl} = M_c^* + H_c^* y_c + \frac{1}{2} l R_c - F\alpha l$ $\quad R_{sl} = F - R_c$

$M_{sr} = M_c^* + H_c^* y_c - \frac{1}{2} l R_c$ $\quad R_{sr} = R_c$; $\quad H_{sl} = H_{sr} = H_c^*$

$N_{sl} = H_{sl} \cos \theta + R_{sl} \sin \theta$ $\quad N_{sr} = H_{sr} \cos \theta + R_{sr} \sin \theta$

At any section x_q from crown: in left-hand half: $M_q = M_c^* + H_c^* y_q + R_c x_q - F[x_q - l(\frac{1}{2} - \alpha)]$

(include term for F only if within x_q)

$H_q = H_c^*$ $\quad R_q = R_c - F$

in right-hand half: $M_{qr} = M_c^* + H_c^* y_q - R_c x_q$ $\quad H_{qr} = H_c^*$ $\quad R_{qr} = R_c$

Note: for an explanation of the approximate method of determining the size of a fixed arch, see section 16.3.1

springing and quarter-points can be obtained and the stresses on the assumed sections calculated. If the stresses are shown to be unsuitable, other dimensions must be tried and the calculations reworked.

16.3.2 Determination of load effects

The following method is suitable for determining load effects in any symmetrical fixed arch, if the dimensions and shape of the arch are known, or assumed, and if the shape of the arch must conform to a particular profile. Reference should be made to section 4.13.3 for general comments on this method.

On half of the arch drawn to scale, as in *Table 2.72*, plot the arch axis. Divide the half-arch into k segments, such that each segment has the same ratio $a_I = a/I$, where a is the length and I is the mean second moment of area of the segment, based on the thickness of the arch measured normal to the axis, with allowance being made for the reinforcement.

Coordinates x and y relative to the axis of the arch at the crown are determined by measurement to the centre of length of each segment. Calculate, separately, the dead and imposed loads on each segment. Assume each load acts at the centre of length of the segment. In an open-spandrel arch, the dead and imposed loads are concentrated on the arch at the column positions: these should be taken as the centre of the segments, but it may not then be possible to maintain a constant value of a_I, and the value of a_I for each segment must be calculated; the general formulae in *Table 2.72* are then applicable.

For constant values of a_I, the forces and bending moment at the crown are as follows:

$$H_c = \frac{k\sum M_1 y - \sum y \sum M_1}{2\left[k\sum y^2 - \left(\sum y\right)^2\right]}$$

$$R_c = \sum M_1 x/2\sum x^2$$

$$M_c = \frac{\sum M_1 - 2H_c\sum y}{2k}$$

The summations are taken over one-half of the arch. The term M_1 is the moment at the centre of the segment of all the loads from the centre of the segment to the crown. Summations are also made for all the loads on the other half of the arch, for which R_c is negative.

Due to the elastic shortening of the arch resulting from H_c,

$$H_{c1} = \frac{H_c k\sum(a/A)}{a_I\left[k\sum y^2 - \left(\sum y\right)^2\right]}$$

$$M_{c1} = -H_{c1}a\sum y/k$$

where A is the cross-sectional area of the segment calculated on the same basis as I.

Due to a rise ($+T$) or a fall ($-T$) in concrete temperature,

$$H_{c2} = \frac{(\pm T)k\lambda E_c l_1}{2a_I\left[k\sum y^2 - \left(\sum y\right)^2\right]}$$

$$M_{c2} = -H_{c2}\sum y/k$$

where λ is the coefficient of linear expansion of the concrete, E_c is the short-term modulus of elasticity of the concrete, and l_1 is the length of the arch axis. Arch shortening due to H_{c2} is neglected. The effect of concrete shrinkage can be treated as a temperature fall in which T is replaced by $(\varepsilon_{cs}\phi)/\lambda$, where ε_{cs} is the shrinkage strain (allowing for reinforcement restraint), and ϕ is a reduction coefficient (typically taken as 0.43) to allow for the effect of creep.

The foregoing formulae are used to determine the effects of the various design loads in the following procedure. Calculate $(H_c - H_{c1})$, R_c and $(M_c - M_{c1})$ for the dead load. Calculate, for each load separately, values of H_{c2} and M_{c2}, for temperature rise, temperature fall and concrete shrinkage. Calculate, for the imposed load (represented as an equivalent uniform load), $(H_c - H_{c1})$ and $(M_c - M_{c1})$. To obtain the maximum positive bending moment at the crown (and the horizontal thrust), the imposed load should be applied to the middle third of the arch, more or less. (By considering the effect of the imposed load on one segment more, and one segment less, than those in the middle third of the arch, the number of segments that should be loaded to give the maximum positive bending moment at the crown can be determined.) With the imposed load applied only to those segments that are unloaded when calculating the maximum positive moment at the crown, the maximum negative moment at the crown due to the imposed load is obtained. The maximum bending moments due to the imposed load are each combined with the bending moments due to dead load and arch shortening, and with the bending moments due to temperature change and concrete shrinkage, in such a way that the most adverse total values are obtained. The corresponding thrusts are also calculated and combined with the appropriate bending moments to check the design conditions at the crown.

The bending moment at the springing, due to load at a point between the springing and the crown of the arch, distant αl from the springing (where l is the span of the arch), is

$$M_s = (M_c - M_{c1}) + (H_c - H_{c1})y_c + R_c l/2 - F\alpha l$$

where y_c is the rise of the arch. For the dead load, the values determined for the crown are substituted in this expression, with the term $F\alpha l$ replaced by $\Sigma F\ [(l/2) - x]$. To obtain the maximum negative bending moment at the springing, the imposed load is applied to those parts of the arch extending 0.4 of the span from the springing, more or less. (As before, the effect of applying the imposed load to one segment more, and one segment less, than this distance should be determined to ensure that the most adverse loading disposition has been considered.) By applying load to only those segments that are unloaded when calculating the maximum negative bending moment, the maximum positive bending moment is obtained. These maximum bending moments are each combined with the bending moments due to dead load, temperature change and concrete shrinkage, to obtain the most adverse total values. The conditions at the springing are then checked for the combined effects of the most adverse bending moments and the corresponding normal thrusts.

The normal thrust at the springing is given by

$$N_s = (H_c - H_{c1})\cos\theta + R_s\sin\theta$$

In this expression, the vertical component of thrust, given by R_s = (total load on half-arch) $- R_c$, is calculated for the loads used to determine $(H_c - H_{c1})$.

The shearing force at the springing is given by

$$V_s = (H_c - H_{c1})\sin\theta + R_s\cos\theta$$

In this expression, the maximum value is generally obtained when the imposed load extends over the whole arch. At the crown, the maximum shearing force is the maximum value of R_c due to any combination of dead and imposed load.

The bending moment at a section with coordinates x_q and y_q, due to load at a point between the springing and the crown of the arch, distant αl from the springing, is

$$M_q = M_c + H_c y_q + R_c x_q - F[x_q - (0.5 - \alpha)l]$$

At the quarter-point, $x_q = l/4$, and the procedures to determine the maximum bending moment, normal thrust and shearing force are similar to those described above. The method given in *Table 2.73* can be used to obtain data for influence lines.

16.4 FIXED PARABOLIC ARCH

Formulae and guidance on using the data in *Table 2.74* are given below. See section 4.13.4 for further comments.

16.4.1 Dead load and arch shortening

The horizontal thrust due to dead load alone is $H = k_1 g l^2/y_c$, where g is the dead load per unit length at the crown, l is the span, and y_c is the rise of the arch axis. The coefficient k_1 depends on the dead load at the springing, which varies with the rise/span ratio and type of structure; that is, whether the arch is open spandrel, or solid spandrel, or whether the dead load is uniform throughout the span.

An elastic shortening results from the thrust along the arch axis (assuming rigid abutments). The counter-thrust H_1, while slightly reducing the thrust due to the dead load, renders this thrust eccentric, and produces a positive bending moment at the crown and a negative bending moment at each springing. If h is the thickness of the arch at the crown,

$$H_1 = -k_2\left(\frac{h}{y_c}\right)^2 H$$

in which the coefficient k_2 depends on the relative thickness at the crown and the springing.

Due to dead load and elastic shortening, the net thrusts H_c and H_s at the crown and the springing respectively, acting parallel to the arch axis at these points, are given by

$$H_c = H - H_1 \quad H_s = \frac{H}{\cos\theta} - H_1 \cos\theta$$

where θ is the angle between the horizontal and the tangent to the arch axis at the springing, with $\cos\theta$ given in *Table 2.74*.

The bending moments due to the eccentricities of H_c and H_s are given by $M_c = k_3 y_c H_1$ and $M_s = (k_3 - 1)\, y_c H_1$ respectively.

16.4.2 Temperature change

The additional horizontal thrust due to a rise in temperature, or corresponding counter-thrust due to a fall in temperature is given by

$$H_2 = \pm k_4\left(\frac{h}{y_c}\right)^2 hT$$

If T is the temperature change in °C, with h and y_c in metres, then H_2 is in kN per metre width of arch. The values of k_4 in *Table 2.74* are based on an elastic modulus $E_c = 20$ kN/mm^2, and a coefficient of linear expansion $\lambda = 12$ micro-strain/°C. For any other values of E_c and λ, k_4 should be multiplied by a value of $(E_c/20)(\lambda/12)$. At the crown, the increase or decrease in normal thrust due to a change in temperature is H_2, and the bending moment is $-k_3 y_c H_2$, account being taken of the sign of H_2. The normal thrust at the springing due to a change of temperature is $H_2\cos\theta$, and whether this thrust increases or decreases the thrust due to dead load, depends on the sign of H_2. At the springing, the bending moment is $(1 - k_3)y_c H_2$, the sign being the same as that of H_2.

16.4.3 Shrinkage of concrete

The effect of concrete shrinkage can be treated as a fall in temperature in which T is replaced by $(\varepsilon_{cs}\phi)/(\lambda = 12)$, where ε_{cs} (micro-strain) is the shrinkage (allowing for reinforcement restraint), and ϕ is a reduction coefficient (taken as 0.43) to allow for the effect of creep.

16.4.4 Imposed load

The maximum bending moments, and corresponding thrusts and reactions, are given by the following expressions, where q per unit length is the intensity of uniform load equivalent to the specified imposed load.

At the crown	positive bending moment = $k_5\, ql^2$ horizontal thrust = $k_6\, ql^2/y_c$
At the springing	negative bending moment = $k_7\, ql^2$ horizontal thrust = $k_8\, ql^2/y_c$ vertical reaction = $k_9\, ql$
	positive bending moment = $k_{10}\, ql^2$ horizontal thrust = $k_{11}\, ql^2/y_c$ vertical reaction = $k_{12}\, ql$

The normal thrust at the springing, where H is the horizontal thrust and R is the vertical reaction, is given by

$$N = H\cos\theta + R\sqrt{(1 - \cos^2\theta)}.$$

16.4.5 Dimensions of arch

The line of thrust, and therefore the arch axis, can be plotted as described in section 4.13.4. The thickness of the arch at the crown and the springing having been determined, the lines of the extrados and intrados can be plotted to give a parabolic variation of thickness between the two extremes. Thus, the thickness normal to the axis of the arch at any point is given by $[(h_s - h)\alpha^2 + h]$, where α is the ratio of the distance of the point from the crown, measured along the axis of the arch, to half the length of the axis of the arch.

Example. Determine the bending moments and thrusts on a fixed parabolic arch slab for an open-spandrel bridge.

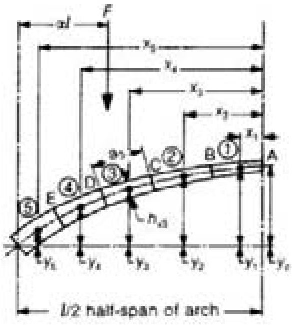
Specified values

Span: 50 m measured horizontally between the intersection of the arch axis and the abutment. Rise: 7.5 m in the arch axis.
Thickness: 900 mm at springing, 600 mm at crown.
Dead load: 12 kN/m^2, imposed load: 15 kN/m^2.
Temperature range: ±24°C, $\lambda = 12$ micro-strain per °C.

Arches: computation chart for symmetrical fixed-ended arch

2.73

Dimensional properties													Unit load at A (crown)				Unit load at B				Unit load at C etc.		
Seg. no.	x	y	h_s	a	A	I_s	$a_I = a/I$	$a_I x$	$a_I x^2$	$a_I y$	$a_I y^2$	$a_A = a/A$	M_f ($= x$)	$M_f a_I = a_I x$	$M_f a_I x = a_I x^2$	$M_f a_I y = a_I xy$	M_f	$M_f a_I$	$M_f a_I x$	$M_f a_I y$	C Sub-headings as B	D	E
1	√	√	√	√	√	√	√	√	√	√	√	√	√	—	—	√	—	—	—	—	—	—	—
2	√	√	√	√	√	√	√	√	√	√	√	√	√	—	—	√	√	√	√	√	—	—	—
3	√	√	√	√	√	√	√	√	√	√	√	√	√	—	—	√	√	√	√	√	√		—
4	√	√	√	√	√	√	√	√	√	√	√	√	√	—	—	√	√	√	√	√	√	√	—
5 etc.	√	√	√	√	√	√	√	√	√	√	√	√	√	—	—	√	√	√	√	√	√	√	√
Summations on left-hand half of arch						$\sum_0^{0.5l} =$	S_s	S_{1A}	S_{2A}	S_{sy}	S_{sy2}	S_{sA}		S_{1A}	S_{2A}	S_{3A}	$\sum_0^{\alpha l} =$	S_{1B}	S_{2B}	S_{3B}	Totals similar to B		



Denominator (den.)
$$= (S_s \times S_{sy2}) - (S_{sy})^2$$

Horizontal thrust at crown
$$H_c = \frac{(S_s \times S_3) - (S_{sy} \times S_1)}{2 \times \text{denominator}}$$

Bending moment at crown
$$M_c = \frac{S_1 - (2H_c \times S_{sy})}{2S_s}$$

Shearing force at crown and springing
$R_c = S_2/2S_{2A}$; $R_s = 1 - R_c$

Temperature and shrinkage
$$H_{c2} = \pm \frac{T\lambda E_c S_s}{2 \times \text{denominator}}$$
$$M_{c2} = -H_{c2} S_{sy}/S_s$$

Arch shortening:
Horizontal thrust at crown
$$H_{c1} = -\frac{S_{sA} \times S_s \times (H_c \text{ or } H_{c2})}{\text{denominator}}$$
BM at crown
$$M_{c1} = -H_{c1} S_{sy}/S_s$$

Bending moments at springings:
Left-hand support: $M_{sl} = M_c^* + H_c^* y_s + R_c \tfrac{1}{2}l - F\alpha l$
Right-hand support: $M_{sr} = M_c^* + H_c^* y_s - R_c \tfrac{1}{2}l - M_f$
(M_f is zero when load is on left-hand half of arch)
$H_c^* = H_c - (H_{c1}$ due to $H_c)$
$M_c^* = M_c - (M_{c1}$ for H_{c1} due to $H_c)$

Ordinates of influence lines

Unit load at		A (crown) $x = 0.5$	B ($x =$ √)	C ($x =$ √)	D etc. ($x =$ √)
1	$S_s \times S_3$	√	√	√	√
2	$S_{sy} \times S_1$	√	√	√	√
3	(1)–(2)	√	√	√	√
4	$H_c = (3)/(2 \times \text{den.})$	√	√	√	√
5	$2(4) \times S_{sy}$	√	√	√	√
6	$S_1 - (5)$	√	√	√	√
7	$M_c = (6)/2S_s$	√	√	√	√
8	$R_c = S_2/(2S_{2A})$	√	√	√	√
9	$R_s = 1 - (8)$	√	√	√	√
10	(4) or H_{c2}	√	√	√	√
11	$(10) \times S_{sA} \times S_s$	√	√	√	√
12	$H_{c1} = -(11)/\text{den.}$	√	√	√	√
13	$M_{c1} = -(12) \times S_{sy}/S_s$	√	√	√	√
14	$[(4) - (12)] \times y_s$	√	√	√	√
15	(7) − (13)	√	√	√	√
16	(14) + (15)	√	√	√	√
17	$\tfrac{1}{2}l \times (8)$	√	√	√	√
18	$M_{sl} = (16) + (17) - F\alpha l$	√	√	√	√
19	$M_{sr} = (16) - (17)$	√	√	√	√

Bending moment at quarter-point: repeat similar to foregoing with corresponding formulae

Dead loads (effects calculated from influence lines)

Segment	Loads			At crown								M_{sl} at springing				Quarter point	Imposed loads
				H_c		H_{c1}		M_c		M_{c1}		LH segments		RH segments			
	Arch: $24 \times 10^3 \times ah_s$	Fill and other dead load	Total dead load (½ arch)	Ordinate	Product	Ordinate	Product	Ordinate	Product	Ordinate	Product	Ordinate	Product	Ordinate	Product		
1	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	Repeat for M_q	Maximum thrusts, shearing forces and bending moments due to imposed loads are calculated from the influence lines for the most adverse position of loads.
2	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√		
3	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√		
4	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√		
5 etc.	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√		
		$\Sigma =$	F_g	$\Sigma H_c =$	√	$\Sigma H_{c1} =$	√	$\Sigma M_c =$	√	$\Sigma M_{c1} =$	√	$\Sigma M_{sl} =$	√	$\Sigma M_{sr} =$	√		
$R_s = F_g$	$R_c = 0$			Total $H_c^* = 2(\Sigma H_c - \Sigma H_{c1}) =$ √				Total $M_c^* = 2(\Sigma M_c - \Sigma M_{c1}) =$ √				Net $M_s = \Sigma M_{sl} \pm \Sigma M_{sr} =$ √					

Resultant thrusts, bending moments and shearing forces due to dead and imposed loads are calculated by substituting in formulae.

Note: Factors in this chart are non-dimensional except for weight of segments.

Arches: fixed-ended parabolic arches

Type		Uniform dead load				Open spandrel				Solid spandrel			
Rise/span		0.10	0.15	0.20	0.25	0.10	0.15	0.20	0.25	0.10	0.15	0.20	0.25
Inclination of axis of arch at springing: $\cos\theta$		0.930	0.848	0.781	0.709	0.918	0.820	0.740	0.650	0.893	0.764	0.665	0.565
Horizontal thrust due to dead load: values of k_1	—	0.125	0.125	0.125	0.125	0.135	0.140	0.144	0.148	0.160	0.176	0.190	0.204
Horizontal thrust due to arch shortening: values of k_2	h_s/h												
	1.25	1.10	1.07	1.03	0.99	1.13	1.08	1.03	1.00	1.19	1.13	1.08	1.00
	1.50	1.42	1.37	1.32	1.25	1.44	1.39	1.33	1.27	1.53	1.48	1.42	1.33
	1.75	1.68	1.63	1.58	1.53	1.73	1.68	1.63	1.58	1.86	1.82	1.76	1.69
Moments due to arch shortening, temperature change and eccentricity of thrust: values of k_3	1.25	0.284	0.293	0.300	0.307	0.279	0.280	0.281	0.282	0.255	0.261	0.265	0.270
	1.50	0.248	0.253	0.258	0.263	0.240	0.243	0.247	0.251	0.224	0.226	0.228	0.230
	1.75	0.223	0.227	0.231	0.235	0.218	0.220	0.222	0.224	0.200	0.200	0.200	0.200
Horizontal force due to temperature change: values of k_4	1.25	321	305	293	280	326	311	294	276	343	319	300	274
	1.50	432	413	396	380	441	422	401	381	472	448	422	394
	1.75	538	518	497	477	552	532	510	484	592	566	542	520
Horizontal thrusts due to imposed load	k_6	0.059	0.059	0.059	0.059	0.062	0.064	0.065	0.066	0.070	0.074	0.077	0.080
	k_8	0.039	0.039	0.039	0.039	0.038	0.038	0.037	0.037	0.037	0.035	0.033	0.032
	k_{11}	0.086	0.086	0.086	0.086	0.088	0.089	0.090	0.092				
	h_s/h												
	1.25									0.093	0.097	0.098	0.100
	1.50									0.095	0.098	0.101	0.103
	1.75									0.097	0.100	0.104	0.106
Vertical reactions due to imposed load	k_v	0.358	0.358	0.358	0.358	0.354	0.352	0.350	0.349	0.342	0.337	0.330	0.321
	k_{12}	0.149	0.149	0.149	0.149	0.150	0.151	0.153	0.155	0.160	0.164	0.170	0.177
Bending moments due to imposed load: Values of $k_5 \times 10^4$	h_s/h												
	1.25	48	49	51	52	52	54	57	60	60	69	77	84
	1.50	45	46	46	47	48	50	52	54	56	62	68	75
	1.75	42	43	43	44	44	46	48	50	52	58	63	68
Bending moments due to imposed load: Values of k_7	1.25	0.019	0.019	0.018	0.018	0.018	0.018	0.017	0.017	0.017	0.015	0.014	0.014
	1.50	0.021	0.021	0.020	0.020	0.020	0.020	0.019	0.018	0.018	0.017	0.016	0.015
	1.75	0.022	0.022	0.022	0.022	0.022	0.021	0.020	0.020	0.020	0.018	0.017	0.016
Bending moments due to imposed load: Values of k_{10}	1.25	0.019	0.019	0.018	0.018	0.020	0.021	0.021	0.021	0.024	0.025	0.025	0.026
	1.50	0.021	0.020	0.020	0.020	0.022	0.023	0.023	0.023	0.026	0.027	0.028	0.028
	1.75	0.022	0.022	0.022	0.022	0.024	0.025	0.025	0.025	0.029	0.030	0.031	0.031

Note: see section 16.4 for formulae in which coefficients k_1, k_2 etc. should be substituted, as shown in the example in that section.

$E_c = 28$ kN/mm^2, $\varepsilon_{cs} = 200$ micro-strain, $\phi = 0.43$.

Geometrical properties

$$\frac{h_s}{h} = \frac{900}{600} = 1.5 \qquad \frac{y_c}{l} = \frac{7.5}{50} = 0.15$$

Inclination of arch axis at springing (*Table 2.73*): $\cos\theta = 0.82$
A strip of slab 1 m wide is considered. The coefficients, taken from *Table 2.74*, are substituted in the expressions given in section 16.4.4. Allowing for self-weight of arch slab,

$$\text{Total dead load at crown} = 12 + 0.6 \times 24 = 26.4 \text{ kN/m}^2$$

Loads and resulting bending moments and thrusts	*Moment* kNm/m	*Thrust* kN/m
Horizontal thrusts due to dead load and other actions.		
Dead load ($k_1 = 0.140$)		
$H = 0.140 \times 26.4 \times (50^2/7.5)$		1232
Arch shortening ($k_2 = 1.39$)		
$H_1 = -1.39 \times (0.6/7.5)^2 \times 1232$		−11
Temperature change ($k_4 = 422$)		
$H_2 = \pm\, 422 \times (28/20) \times (0.6/7.5)^2 \times 0.6 \times 24$		±55
Concrete shrinkage ($k_4 = 422$)		
$H_3 = -422 \times (28/20) \times (0.6/7.5)^2 \times 0.6 \times (200/12) \times 0.43$		−16
Crown: maximum positive bending moment and corresponding thrust		
Dead load and arch shortening ($k_3 = 0.243$, thrusts H and H_1 as above)	20	
$M_c = 0.243 \times 7.5 \times 11$		
$H_c = 1232 - 11$		1221
Temperature fall (thrust H_2 as above)		−55
$M_c = 0.243 \times 7.5 \times 55$	100	
Shrinkage (thrust H_3 as above)		−16
$M_c = 0.243 \times 7.5 \times 16$	29	
Imposed load ($k_5 = 0.005$, $k_6 = 0.064$)		
$M_c = 0.005 \times 15 \times 50^2$	188	
$H_c = 0.064 \times 15 \times (50^2/7.5)$		320
Totals	337	1470
Springing: maximum negative bending moment and corresponding thrust		
Dead load and arch shortening		
$M_s = (1 - 0.243) \times 7.5 \times 11$	−63	
$H_s = (1232/0.82) - 11 \times 0.82$		1494
Temperature fall		
$M_s = (1 - 0.243) \times 7.5 \times 55$	−312	
$H_s = -55 \times 0.82$		−45
Shrinkage		
$M_s = (1 - 0.243) \times 7.5 \times 16$	−91	
$H_s = -16 \times 0.82$		−13
Imposed load ($k_7 = 0.020$, $k_8 = 0.038$, $k_9 = 0.352$)		
$M_s = -0.020 \times 15 \times 50^2$	−750	
$H = 0.038 \times 15 \times (50^2/7.5) = 190$ kN/m, $R = 0.352 \times 15 \times 50 = 264$ kN/m		
$N = 190 \times 0.82 + 264\sqrt{(1 - 0.82^2)}$		307
Totals	−1216	1743
Springing: maximum positive bending moment and corresponding thrust		
Dead load and arch shortening (values as before)	−63	1494
Temperature rise (−values for temperature fall)	312	45
Shrinkage (neglect as partial in short term and beneficial in long term)		
Imposed load ($k_{10} = 0.023$, $k_{11} = 0.089$, $k_{12} = 0.151$)		
$M_s = 0.023 \times 15 \times 50^2$	863	
$H = 0.089 \times 15 \times (50^2/7.5) = 445$ kN/m, $R = 0.151 \times 15 \times 50 = 113$ kN/m		
$N = 445 \times 0.82 + 113\sqrt{(1 - 0.82^2)}$		430
Totals	1112	1969

Chapter 17
Containment structures

In the following, containers are conveniently categorised as *tanks* containing liquids, and *bunkers and silos* containing dry materials, each category being subdivided into cylindrical and rectangular structures. The intensity of pressure on the walls of the structure is considered to be uniform at any level, but vertically the pressure increases linearly from zero at the top to a maximum at the bottom.

17.1 CYLINDRICAL TANKS

If the wall of a cylindrical tank has a sliding joint at the base and is free at the top, then when the tank is full, no radial shear or vertical bending occurs. The circumferential tension at depth z below the top is given by $n = \gamma\, r_i\, z$ per unit height, where r_i is the internal radius of the tank and γ is unit weight of the liquid. If the wall is supported at the base in such a way that no radial movement can occur, radial shear and vertical bending result, and the circumferential tension is always zero at the bottom of the wall. Values of circumferential tensions, vertical moments and radial shears, according to values of the term, height^2/(2 × mean radius × thickness), can be obtained from *Tables 2.75* and *2.76*.

The tables apply to walls with a free top and a bottom that is either fixed or hinged. The coefficients have been derived by elastic analysis and allow for a Poisson's ratio of 0.2. For further information on the tables, reference should be made to publications on cylindrical tanks, such as refs 55 and 56. If an annular footing is provided at the base of the wall, a hinged detail can be formed although this is rarely done. The footing normally needs to be tied into the floor of the tank to prevent radial movement. Reliance solely on the frictional resistance of the ground to the radial force on the footing is generally inadequate and always uncertain. If the joint between the wall and the footing is continuous, it is possible to develop a fixed condition by widening the footing until a uniform distribution of bearing pressure is obtained. In many cases, the wall and the floor are made continuous, and it then becomes necessary to consider the structural interaction of a cylindrical wall and a ground supported circular slab. Appropriate values for the stiffness of the member and the effect of edge loading can be obtained from *Table 2.76* for walls, and *Table 2.77* for slabs.

For slabs on an elastic foundation, the values depend on the ratio r/r_k, where r_k is the radius of relative stiffness defined in section 7.2.5. The value of r_k is dependent on the modulus of subgrade reaction, for which data is given in section 7.2.4. Taking $r/r_k = 0$, which corresponds to a 'plastic' soil state, is appropriate for an empty tank liable to flotation.

Example 1. Determine, due to internal hydrostatic loading, maximum service values for circumferential tension, vertical moment and radial shear in the wall of a cylindrical tank that is free at the top edge and hinged at the bottom. The wall is 300 mm thick, the tank is 6 m deep, the mean radius is 10 m, and the water level is taken to the top of the wall.

From *Table 2.75*, for $l_z^2/2rh = 6^2/(2 \times 10 \times 0.3) = 6$,

$$\alpha_n = 0.643 \text{ at } z/l_z = 0.7,\ \alpha_m = 0.008 \text{ at } z/l_z = 0.8,$$
$$\alpha_v = 0.110 \text{ at } z/l_z = 1.0$$
$$n = \alpha_n \gamma l_z r = 0.643 \times 9.81 \times 6 \times 10 = 378.5 \text{ kN/m}$$
$$m = \alpha_m \gamma l_z^3 = 0.008 \times 9.81 \times 6^3 = 17.0 \text{ kNm/m}$$
$$v = \alpha_v \gamma l_z^2 = 0.110 \times 9.81 \times 6^2 = 38.9 \text{ kN/m}$$

Example 2. Determine, for the tank considered in example 1, the corresponding values, if the wall is fixed at the bottom.

From *Table 2.75*, for $l_z^2/2rh = 6^2/(2 \times 10 \times 0.3) = 6$,

$$\alpha_n = 0.514 \text{ at } z/l_z = 0.6,\ \alpha_m = 0.005 \text{ at } z/l_z = 0.7,$$
$$\alpha_{mb} = -0.019 \text{ and } \alpha_v = 0.197 \text{ at } z/l_z = 1.0$$
$$n = \alpha_n \gamma l_z r = 0.514 \times 9.81 \times 6 \times 10 = 302.5 \text{ kN/m}$$
$$m = \alpha_m \gamma l_z^3 = 0.005 \times 9.81 \times 6^3 = 10.6 \text{ kNm/m}$$
$$m = \alpha_m \gamma l_z^3 = -0.019 \times 9.81 \times 6^3 = -40.3 \text{ kNm/m}$$
$$v = \alpha_v \gamma l_z^2 = 0.197 \times 9.81 \times 6^2 = 69.6 \text{ kN/m}$$

Consider a straight wall that is centrally placed on a footing of width $b = (2a + h)$, where a represents the distance from edge of footing to face of wall. The weight of liquid per unit length of wall, on the inside of the footing, is given by $\gamma l_z a$. Assuming a uniform distribution of bearing pressure due to the liquid load, the bending moment about the centre of the wall due to the pressure on the toe of the footing is:

$$m = \gamma l_z a\,(a/b)\,(a + h)/2 \quad \text{giving} \quad a^2(a + h) = 2(m/\gamma l_z)b$$

Substituting for b, h and $m/\gamma\, l_z = 40.3/(9.81 \times 6) = 0.685$, the following cubic equation in a is obtained:

$$a^2(a + 0.3) = 1.37 \times (2a + 0.3)$$

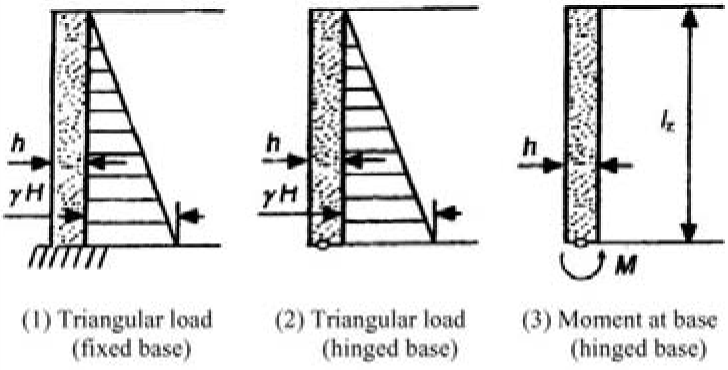
Cylindrical tanks: elastic analysis – 1

2.75

Coefficients for circumferential tensions, vertical moments and radial shears in wall of constant thickness

Load case	α	z/l_z	Values of coefficient α for values of $l_z^2/2rh$								
			2	3	4	5	6	8	10	12	16
(1) Triangular load (fixed base)	α_{n1}	0	0.234	0.134	0.067	0.025	0.018	- 0.011	- 0.011	- 0.005	0
		0.5	**0.274**	**0.362**	**0.429**	**0.477**	0.504	0.534	0.542	0.543	0.531
		0.6	0.232	0.330	0.409	0.469	**0.514**	**0.575**	**0.608**	0.628	0.641
		0.7	0.172	0.262	0.334	0.398	0.447	0.530	0.589	**0.633**	**0.687**
		0.8	0.104	0.157	0.210	0.259	0.301	0.381	0.440	0.494	0.582
		0.9	0.031	0.052	0.073	0.092	0.112	0.151	0.179	0.211	0.265
	α_{m1}	0.6	**0.0115**	**0.0097**	**0.0077**	**0.0059**	0.0046	0.0028	0.0019	0.0013	0.0004
		0.7	0.0075	0.0077	0.0069	0.0059	**0.0051**	**0.0038**	**0.0029**	0.0023	0.0013
		0.8	– 0.0021	0.0012	0.0023	0.0028	0.0029	0.0029	0.0028	**0.0026**	**0.0019**
		0.9	– 0.0185	– 0.0119	– 0.0080	– 0.0058	– 0.0041	– 0.0022	– 0.0012	– 0.0005	0.0001
		1.0	**– 0.0436**	**– 0.0333**	**– 0.0268**	**– 0.0222**	**– 0.0187**	**– 0.0146**	**– 0.0122**	**– 0.0104**	**– 0.0079**
	α_{v1}	1.0	**0.299**	**0.262**	**0.236**	**0.213**	**0.197**	**0.174**	**0.158**	**0.145**	**0.127**
(2) Triangular load (hinged base)	α_{n2}	0	0.205	0.074	0.017	– 0.008	– 0.011	– 0.015	– 0.008	– 0.002	0.002
		0.5	**0.434**	0.506	0.545	0.562	0.566	0.564	0.552	0.541	0.521
		0.6	0.419	**0.519**	**0.579**	**0.617**	0.639	0.661	0.666	0.664	0.650
		0.7	0.369	0.479	0.553	0.606	**0.643**	**0.697**	**0.730**	**0.750**	0.764
		0.8	0.280	0.375	0.447	0.503	0.547	0.621	0.678	0.720	**0.776**
		0.9	0.151	0.210	0.256	0.294	0.327	0.386	0.433	0.477	0.536
	α_{m2}	0.6	0.0199	0.0127	0.0083	0.0057	0.0039	0.0020	0.0011	0.0005	– 0.0004
		0.7	**0.0219**	0.0152	0.0109	0.0080	0.0062	0.0038	0.0025	0.0017	0.0008
		0.8	0.0205	**0.0153**	**0.0118**	**0.0094**	**0.0078**	**0.0057**	0.0043	0.0032	0.0022
		0.9	0.0145	0.0111	0.0092	0.0078	0.0068	0.0054	**0.0045**	**0.0039**	**0.0029**
	α_{v2}	1.0	**0.189**	**0.158**	**0.137**	**0.121**	**0.110**	**0.096**	**0.087**	**0.079**	**0.068**
(3) Moment at hinged base	α_{n3}	0	– 0.68	– 1.78	– 1.87	– 1.54	– 1.04	– 0.24	0.21	0.32	0.22
		0.5	3.69	4.29	4.31	3.93	3.34	2.05	0.82	– 0.18	– 1.30
		0.6	4.30	5.66	6.34	6.60	6.54	5.87	4.79	3.52	1.12
		0.7	**4.54**	**6.58**	8.19	9.41	10.3	11.3	11.6	11.3	9.67
		0.8	4.08	6.55	**8.82**	**11.0**	**13.1**	**16.5**	19.5	21.8	24.5
		0.9	2.75	4.73	6.81	9.02	11.4	16.1	**20.9**	**25.7**	**34.7**
	α_{m3}	0.6	0.193	0.087	0.023	– 0.015	– 0.037	– 0.062	– 0.067	– 0.064	– 0.051
		0.7	0.340	0.227	0.150	0.095	0.057	0.002	– 0.031	– 0.049	– 0.066
		0.8	0.519	0.426	0.354	0.296	0.252	0.178	0.123	0.081	0.025
		0.9	0.748	0.692	0.645	0.606	0.572	0.515	0.467	0.424	0.354
		1.0	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**
	α_{v3}	1.0	**– 2.57**	**– 3.18**	**– 3.68**	**– 4.10**	**– 4.49**	**– 5.18**	**– 5.81**	**– 6.38**	**– 7.36**

Load Cases (top of wall free)



(1) Triangular load (fixed base)

(2) Triangular load (hinged base)

(3) Moment at base (hinged base)

For α values shown above, positive signs indicate for:

(α_n) tension, (α_m) tension in outside face, (α_v) force acting inward

Circumferential tensions, vertical moments and radial shears, at depths denoted by z/l_z, are given by the following equations, where l_z is height of wall, r is radius to centre of wall, z is depth from top of wall and γ is unit weight of liquid.

Load cases (1) and (2):

Hoop tension: $n = \alpha_n \gamma l_z r$ (per unit height)

Vertical moment: $m = \alpha_m \gamma l_z^3$ (per unit length)

Radial shear: $v = \alpha_v \gamma l_z^2$ (per unit length)

Load case (3): M = edge moment per unit length

Hoop tension: $n = \alpha_n M r / l_z^2$ (per unit height)

Vertical moment: $m = \alpha_m M$ (per unit length)

Radial shear: $v = \alpha_v M / l_z$ (per unit length)

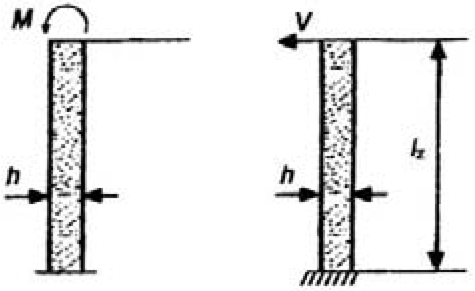
Cylindrical tanks: elastic analysis – 2

2.76

Coefficients for circumferential tensions and vertical moments in wall of constant thickness

Load case	α	z/l_z	Values of coefficient α for values of $l_z^2/2rh$								
			2	3	4	5	6	8	10	12	16
(4) Moment at top	α_{n4}	0	**−13.63**	**−20.45**	**−27.26**	**−34.08**	**−40.89**	**−54.52**	**−68.15**	**−81.78**	**−109.0**
		0.1	−7.43	−9.43	−10.77	−11.61	−12.03	−11.90	−10.77	−8.87	−3.46
		0.2	−2.98	−2.22	−0.87	0.85	2.78	6.95	11.18	**15.85**	**22.45**
		0.3	−0.02	1.92	4.03	6.11	8.05	**11.33**	**13.76**	15.38	16.66
		0.4	1.74	3.81	**5.62**	**7.04**	**8.09**	9.21	9.33	8.78	6.65
		0.5	2.60	**4.25**	5.31	5.86	6.00	5.48	4.40	3.15	0.90
	α_{m4}	0	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**
		0.1	0.943	0.918	0.894	0.872	0.850	0.810	0.773	0.738	0.675
		0.2	0.810	0.738	0.675	0.619	0.568	0.481	0.408	0.347	0.249
		0.3	0.646	0.534	0.443	0.369	0.306	0.210	0.140	0.088	0.022
		0.4	0.481	0.347	0.249	0.176	0.121	0.046	0.003	−0.022	−0.041
		0.5	0.333	0.196	0.109	0.052	0.014	−0.026	−0.041	−0.043	−0.034
(5) Shear at top (fixed base)	α_{n5}	0	**5.12**	**6.32**	**7.34**	**8.22**	**9.02**	**10.42**	**11.67**	**12.76**	**14.74**
		0.1	3.83	4.37	4.73	4.99	5.17	5.36	5.43	5.41	5.22
		0.2	2.68	2.70	2.60	2.45	2.27	1.85	1.43	1.03	0.33
		0.3	1.74	1.43	1.10	0.79	0.50	0.02	−0.36	−0.63	−0.96
		0.4	1.02	0.58	0.19	−0.11	−0.34	−0.63	−0.78	−0.83	−0.76
		0.5	0.52	0.02	−0.26	−0.47	−0.59	−0.66	−0.62	−0.52	−0.32
	α_{m5}	0.1	−0.077	−0.072	−0.068	−0.064	−0.062	−0.057	**−0.053**	**−0.049**	**−0.044**
		0.2	−0.115	−0.100	**−0.088**	**−0.078**	**−0.070**	**−0.058**	−0.049	−0.042	−0.031
		0.3	**−0.126**	**−0.100**	−0.081	−0.067	−0.056	−0.041	−0.029	−0.022	−0.012
		0.4	−0.119	−0.086	−0.063	−0.047	−0.036	−0.021	−0.012	−0.007	−0.001
		0.5	−0.103	−0.066	−0.043	−0.028	−0.018	−0.007	−0.002	0	0.002
		1.0	0.019	0.024	0.019	0.011	0.006	0.001	0	0	0

Load Cases (top of wall free)



(4) Moment at top (any base)

(5) Shear at top (fixed base)

For α values shown above, positive signs indicate for: (α_n) tension, (α_m) tension in outside face

Circumferential tensions and vertical moments, at depths denoted by z/l_z, are given by the following equations, where l_z is height of wall, r is radius to centre of wall, z is depth from top of wall and γ is unit weight of liquid.

Load case (4): M is edge moment per unit length

Hoop tension: $n = \alpha_n M r / l_z^2$ (per unit height)

Vertical moment: $m = \alpha_m M$ (per unit length)

Load case (5): V is edge shear per unit length

Hoop tension: $n = \alpha_n V r / l_z$ (per unit height)

Vertical moment: $m = \alpha_m V l_z$ (per unit length)

Note. Coefficients for load case (4) apply to a semi-infinite cylinder. Since the effect of the moment dies out rapidly as z/l_z increases, the same values may be used for all base conditions with errors that are reasonably small for $l_z^2/2rh > 2$, and negligible for $l_z^2/2rh > 8$.

Coefficients for rotational stiffness of wall and fixed edge moment (FEM) for load cases (1) and (5)

	$l_z^2/2rh$	2	3	4	5	6	8	10	12	16	20
Stiffness	α_w	0.445	0.548	0.635	0.713	0.783	0.903	1.010	1.108	1.281	1.430
FEM	α_{w1}	−0.0436	−0.0333	−0.0268	−0.0222	−0.0187	−0.0146	−0.0122	−0.0104	−0.0079	−0.0063
	α_{w5}	−0.019	−0.024	−0.019	−0.011	−0.006	−0.001	0	0	0	0

Rotational stiffness and fixed edge moments are given by the following equations:

Rotational stiffness of wall (hinged base and free top): $K_w = \alpha_w E_c h^3 / l_z$ where E_c is modulus of elasticity of concrete

Fixed edge moment for load case (1): $M_w = \alpha_{w1} \gamma l_z^3$ Fixed edge moment for load case (5): $M_w = \alpha_{w5} V l_z$

Cylindrical tanks: elastic analysis – 3

Coefficients for bending moments in a uniform circular slab on an elastic foundation

Load case	r/r_k	Radial coefficient α_r for values of r_x/r						Tangential coefficient α_t for values of r_x/r					
		1.0	0.8	0.6	0.4	0.2	0	1.0	0.8	0.6	0.4	0.2	0
(1) Edge moment (no shear restraint)	0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	1	1.0	0.998	0.993	0.988	0.984	0.982	0.993	0.991	0.988	0.984	0.983	0.982
	2	1.0	0.977	0.906	0.828	0.770	0.749	0.908	0.876	0.832	0.789	0.759	0.749
	3	1.0	0.923	0.707	0.481	0.323	0.266	0.723	0.627	0.495	0.375	0.295	0.266
	4	1.0	0.855	0.513	0.207	0.017	–0.046	0.576	0.426	0.236	0.081	–0.015	–0.046
	5	1.0	0.773	0.353	0.059	–0.081	–0.118	0.492	0.308	0.102	–0.034	–0.100	–0.118
	6	1.0	0.680	0.215	–0.018	–0.078	–0.083	0.441	0.232	0.035	–0.057	–0.081	–0.083
	7	1.0	0.583	0.105	–0.051	–0.049	–0.034	0.405	0.175	0.001	–0.049	–0.042	–0.034
	8	1.0	0.488	0.028	–0.055	–0.023	–0.005	0.378	0.132	–0.020	–0.034	–0.014	–0.005
	9	1.0	0.400	–0.020	–0.045	–0.007	0.006	0.358	0.098	–0.025	–0.020	0	0.006
	10	1.0	0.319	–0.044	–0.029	0.001	0.006	0.341	0.072	–0.025	–0.010	0.004	0.006
(2) Edge shear (no edge rotation)	0	–0.250	–0.106	0.006	0.086	0.134	0.150	–0.050	0.022	0.078	0.118	0.142	0.150
	1	–0.249	–0.105	0.006	0.086	0.133	0.149	–0.050	0.022	0.079	0.117	0.141	0.149
	2	–0.240	–0.098	0.009	0.081	0.123	0.137	–0.048	0.021	0.073	0.109	0.130	0.137
	3	–0.211	–0.072	0.017	0.066	0.090	0.097	–0.042	0.019	0.059	0.082	0.094	0.097
	4	–0.172	–0.039	0.025	0.046	0.048	0.047	–0.034	0.017	0.040	0.047	0.047	0.047
	5	–0.139	–0.013	0.029	0.029	0.019	0.014	–0.028	0.014	0.025	0.022	0.016	0.014
	6	–0.116	0.002	0.027	0.017	0.005	0	–0.023	0.012	0.016	0.009	0.003	0
	7	–0.100	0.010	0.021	0.009	0	–0.003	–0.020	0.010	0.010	0.003	0.002	–0.003
	8	–0.088	0.013	0.016	0.003	–0.001	–0.002	–0.018	0.009	0.006	0	–0.002	–0.002
	9	–0.078	0.015	0.011	0.001	–0.001	–0.001	–0.016	0.007	0.003	0	–0.001	–0.001
	10	–0.071	0.015	0.007	0	–0.001	0	–0.014	0.006	0.002	0	0	0

Note. Radial and tangential moments per unit width, at positions denoted by r_x/r, are given by the following equations, where r is radius of slab and r_x is distance from centre of slab. For α values shown above, positive signs indicate tension at top, compression at bottom.

	Radial moment	Tangential moment
Load case (1), where M is edge moment per unit length (rotation inward)	$m_r = \alpha_r M$	$m_t = \alpha_t M$
Load case (2), where Q is edge load per unit length (deflection downward)	$m_r = \alpha_r Q r$	$m_t = \alpha_t Q r$

The radius of relative stiffness r_k is given by the following equation, where E_c is modulus of elasticity of concrete, h is slab thickness, and k_s is modulus of subgrade reaction (see section 7.2.4 for further information):

$r_k = [E_c h^3/12(1-\nu^2)k_s]^{0.25}$ where ν is Poisson's ratio. For $\nu = 0.2$, $r_k = [E_c h^3/11.52\,k_s]^{0.25}$

Coefficients for rotational stiffness of slab and fixed edge moment (FEM) for load case (2)

	r/r_k	0	1	2	3	4	5	6	7	8	9	10
Stiffness	α_s	0.104	0.105	0.118	0.159	0.222	0.285	0.346	0.407	0.468	0.529	0.590
FEM	α_{s2}	-0.250	-0.249	-0.240	-0.211	-0.172	-0.139	-0.116	-0.100	-0.088	-0.078	-0.071

Rotational stiffness and fixed edge moments are given by the following equations:

Rotational stiffness of slab: $K_s = \alpha_s E_c h^3/r$ where E_c is modulus of elasticity of concrete

Fixed edge moment for load case (2): $M_s = \alpha_{s2} Q r$ where Q is edge load per unit length acting downward

Solution of the equation by trial and error gives $a = 1.6$ m and width $b = 2 \times 1.6 + 0.3 = 3.5$ m. This width is correct for the annular footing provided the footing is positioned so that its centre of area coincides with the centre of the wall.

If c represents the distance from the outer edge of the footing to the centreline of the wall then, for a unit length of wall, the lengths of the trapezoidal area are $(r + c)/r$ for the outer edge and $(r + c - b)/r$ for the inner edge. The usual formula for a trapezoidal area gives:

$$[(r + c) + 2(r + c - b)]\, b/r = 3[(r + c) + (r + c - b)]\, c/r$$

Substituting for b and r, and rearranging the terms gives:

$$6c^2 + 39c - 80.5 = 0 \quad \text{from which} \quad c = 1.65 \text{ m}$$

Thus, a fixed edge condition can be obtained by providing a 3.5 m wide footing, with the outer edge of the footing 1.5 m from the outer face of the wall.

Example 3. Determine, for the tank considered in example 1, the corresponding values if the wall is continuous with the floor slab. The slab is 400 mm thick and the soil on which the tank is to be built is described as well-compacted sand.

Properties of cylindrical wall. From *Table 2.76*, with $l_z^2/2rh = 6$, $\alpha_w = 0.783$ and the wall stiffness is given by:

$$K_w = \alpha_w E_c h^3/l_z = (0.783 \times 0.3^3/6)\, E_c = 0.0035\, E_c$$

Also, $\alpha_{w1} = 0.0187$ and the fixed edge moment when the tank is full is given by:

$$M_w = \alpha_{w1} \gamma l_z^3 = -\,0.0187 \times 9.81 \times 6^3 = -\,39.6 \text{ kNm/m}$$

Properties of circular slab on elastic subgrade. From section 7.2.4, for well-compacted sand, a mean value of 75 MN/m^3 can be taken for the modulus of subgrade reaction.

From *Table 2.77*, for a slab on an elastic subgrade, the radius of relative stiffness, with $\nu = 0.2$, is given by:

$$r_k = [E_c h^3/11.52\, k_s]^{0.25}$$

Therefore, taking $E_c = 33$ kN/mm^2 and $k_s = 75$ MN/m^3,

$$r_k = [33 \times 10^9 \times 0.4^3/(11.52 \times 75 \times 10^6)]^{0.25} = 1.25 \text{ m}$$

With $r/r_k = 8$, $\alpha_s = 0.468$ and the slab stiffness is given by:

$$K_s = \alpha_s E_c h^3/r = (0.468 \times 0.4^3/10)\, E_c = 0.0030\, E_c$$

The unit edge load on the slab due to the 'effective' weight of the wall is

$$Q = 0.3 \times 6 \times (24 - 9.81) = 25.5 \text{ kN/m}$$

With $r/r_k = 8$, $\alpha_{s2} = 0.088$ and the corresponding fixed edge moment is given by:

$$M_s = \alpha_{s2} \quad Qr = -0.088 \times 25.5 \times 10 = -22.5 \text{ kNm/m}$$

Moment distribution at joint. Since the calculated fixed edge moments for the wall and the slab both act in the same direction, the joint will rotate when the notional restraint is removed. This will induce additional moments and change the circumferential tensions in the wall. At the joint, the induced moments will be proportional to the relative stiffness values of the two elements, according to the following distribution factors:

$$\text{wall: } \frac{0.0035}{0.0035 + 0.0030} = 0.54, \quad \text{slab: } \frac{0.0030}{0.0035 + 0.0030} = 0.46$$

Element	Wall	Slab
Distribution factor	0.54	0.46
Fixed end moment	−39.6	−22.5
Induced moment	33.5	28.6
Final moment	−6.1	6.1

It can be seen from the moments calculated for the wall that the joint rotation is close to that for a hinged base condition. The use of a thinner slab or a lower value for the modulus of subgrade reaction will increase the rotation, until a 'closing' corner moment develops, and the circumferential tensions in the wall exceed those obtained for a hinged base.

Final forces and moments. The final circumferential tensions and vertical moments, at various levels in the wall, can be obtained by combining the results for load cases (1) and (3) in *Table 2.75*, where M is the induced moment. The following equations apply:

$$\begin{aligned} n &= \alpha_{n1} \gamma\, l_z\, r + \alpha_{n3} M r/l_z^2 \\ &= (9.81 \times 6 \times 10)\, \alpha_{n1} + (33.5 \times 10/6^2)\, \alpha_{n3} \\ &= 588.6\, \alpha_{n1} + 9.3\, \alpha_{n3} \text{ kN/m} \\ m &= \alpha_{m1}\, \gamma\, l_z^3 + \alpha_{m3} M = (9.81 \times 6^3)\, \alpha_{m1} + 33.5\, \alpha_{m3} \\ &= 2119\, \alpha_{m1} + 33.5\, \alpha_{m3} \end{aligned}$$

The resulting values, for different values of z/l_z are shown in the following tables:

Circumferential tensions in wall (kN/m)

z/l_z	Load case (1)		Load case (3)		Final force
	α_{n1}	588.6 α_{n1}	α_{n3}	9.3 α_{n3}	
0.5	0.504	296.7	3.34	31.1	327.8
0.6	0.514	302.5	6.54	60.8	363.3
0.7	0.447	263.1	10.3	95.8	358.9
0.8	0.301	177.2	13.1	121.8	299.0
0.9	0.112	65.9	11.4	106.0	171.9

Vertical moments in wall (kNm/m)

z/l_z	Load case (1)		Load case (3)		Final moment
	α_{m1}	2119 α_{m1}	α_{m3}	33.5 α_{m3}	
0.6	0.0046	9.8	−0.037	−1.2	8.6
0.7	0.0051	10.8	0.057	1.9	12.7
0.8	0.0029	6.2	0.252	8.4	14.6
0.9	−0.0041	−8.7	0.572	19.2	10.5
1.0	−0.0187	−39.6	1.0	33.5	−6.1

The final radial shear at the base of the wall is given by:

$$\begin{aligned} v &= \alpha_{v1} \gamma l_z^2 + \alpha_{v3} M/l_z \\ &= 0.197 \times 9.81 \times 6^2 - 4.49 \times 33.5/6 = 44.5 \text{ kN/m} \end{aligned}$$

The final radial and tangential moments in the floor slab can be obtained by combining the results for load cases (1) and (2) in *Table 2.77*. The following equations apply:

$$m_r = \alpha_{r1}M + \alpha_{r2}\,Qr \quad \text{and} \quad m_t = \alpha_{t1}M + \alpha_{t2}\,Qr$$

Radial moments in slab (kNm/m)

r_x/r	Load case (1)		Load case (2)		Final moment
	α_{r1}	$28.6\alpha_{r1}$	α_{r2}	$255\alpha_{r2}$	
1.0	1.0	28.6	−0.088	−22.5	6.1
0.8	0.488	14.0	0.013	3.3	17.3
0.6	0.028	0.8	0.016	4.1	4.9
0.4	−0.055	−1.6	0.003	0.8	−0.8
0.2	−0.023	−0.7	−0.001	−0.3	−1.0
0	−0.005	−0.2	−0.002	−0.5	−0.7

Tangential moments in slab (kNm/m)

r_x/r	Load case (1)		Load case (2)		Final moment
	α_{t1}	$28.6\alpha_{t1}$	α_{t2}	$255\alpha_{t2}$	
1.0	0.378	10.8	−0.018	−4.6	6.2
0.8	0.132	3.8	0.009	2.3	6.1
0.6	−0.020	−0.6	0.006	1.5	0.9
0.4	−0.034	−1.0	0	0	−1.0
0.2	−0.014	−0.4	−0.002	−0.5	−0.9
0	−0.005	−0.2	−0.002	−0.5	−0.7

Example 4. Determine, for the tank considered in example 3, the factor of safety against flotation, and the moments in the slab, if the worst credible groundwater level is 1.5 m above the underside of the slab.

The radius of the slab is 10.3 m and radius to the centre of the wall is 10.15 m. If the weight of the wall is spread uniformly over the full area of the slab, the total downward pressure due to the weight of the slab and the wall is:

$$[0.4 + 0.3 \times 6 \times (2 \times 10.15/10.3^2)] \times 24 = 17.8\ \text{kN/m}^2$$

The upward pressure due to a 1.5 m depth of groundwater is 14.7 kN/m², giving a safety factor of 17.8/14.7 = 1.21, which satisfies the BS 8007 minimum requirement of 1.1. The unit edge load on the slab due to the weight of the wall is

$$Q = 0.3 \times 6 \times 24 = 43.2\ \text{kN/m}$$

Taking $r/r_k = 0$ (for a ‘plastic’ soil condition), $\alpha_s = 0.104$ and $\alpha_{s2} = -\ 0.250$, giving the following values:

$$K_s = \alpha_s E_c h^3/r = (0.104 \times 0.4^3/10)E_c = 0.0007\ E_c$$
$$M_s = \alpha_{s2}Qr = -\,0.250 \times 43.2 \times 10 = -108\ \text{kNm/m}$$

The wall stiffness is the same as before and the effect of the groundwater (1.1 m from the top of the slab) on the wall may be taken as a simple cantilever, giving values:

$$K_w = \alpha_w E_c h^3/l_z = (0.783 \times 0.3^3/6)\ E_c = 0.0035\ E_c$$
$$M_w = \alpha_{w1}\gamma l_z^{\,3} = -9.81 \times 1.1^3/6 = -2.2\ \text{kNm/m}$$

The new moment distribution factors are as follows:

$$\text{wall: } \frac{0.0035}{0.0035 + 0.0007} = 0.84, \quad \text{slab: } \frac{0.0030}{0.0035 + 0.0007} = 0.16$$

Element	Wall	Slab
Distribution factor	0.84	0.16
Fixed end moment	−2.2	−108.0
Induced moment	92.6	17.6
Final moment	90.4	−90.4

It will be seen that, in this example, the final edge moment on the slab is close to a fixed edge condition, since the stiffness of the wall is much greater than that of the slab. Final values for the moments and forces in the slab and the wall can now be calculated as shown in example 3.

17.2 RECTANGULAR TANKS

The bending moments obtained by *Table 2.53*, for individual rectangular panels with triangularly distributed loads, may be applied without modification to continuous walls providing there is no rotation about the vertical edges. In a square tank, therefore, moment coefficients can be taken directly from *Table 2.53*. For a rectangular tank, distribution of the unequal fixity moments at the wall junctions is needed and moment coefficients for tanks of different span ratios are given in *Tables 2.78* and *2.79*. The shearing forces given in *Table 2.53* for the individual panels may still be used.

The tables give values for idealised edge conditions at the top and bottom of the wall. The top of a wall should be taken as free for an open tank, and when a sliding joint is provided between a roof and the top of the wall. If a wall is continuous with the roof, the joint will rotate when the tank is full and the condition will tend towards hinged. When there is earth loading on the wall, a closing-corner moment will develop when the tank is empty. At the bottom of the wall, a hinged condition may be created by providing a narrow wall footing tied into the floor slab, or by adopting a reinforced hinge detail. Where the tank wall is continuous with the floor, the deformation of the floor resulting from the wall loading is often complex, and highly dependent on the assumed ground conditions. The edge condition at the bottom of the wall is generally uncertain, but will tend towards hinged when the tank is full. When there is earth loading on the wall, the edge condition will tend towards fixed when the tank is empty. If the floor is extended outwards to form a toe, the condition at the base will be affected in the way discussed in section 17.1.

In considering the horizontal spans, the shear forces at the vertical edges of one wall result in axial forces in the adjacent walls. Thus, for internal loading on a rectangular tank, the shear force at the end of the long wall is equal to the tensile force in the short wall, and vice versa. In designing sections, the combined effects of the bending moment, the axial force and the shear force need to be considered.

Example 5. Determine, due to internal hydrostatic loading, the maximum service moments and shear forces in the walls of a rectangular tank that can be considered as free along the top edge and hinged along the bottom edge. The tank is 6m long,

Rectangular tanks: triangularly distributed load (elastic analysis) – 1

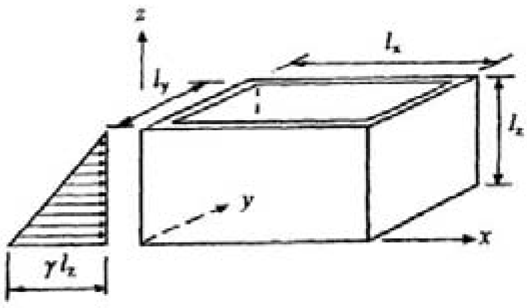
2.78

Span ratios and moments considered		(1) Top hinged, bottom fixed					(2) Top free, bottom fixed				
		Coefficients for short span ratio l_y/l_z					Coefficients for short span ratio l_y/l_z				
		0.5	1.0	1.5	2.0	3.0	0.5	1.0	1.5	2.0	3.0
Long span ratio l_x/l_z = 4.0											
Negative moment at corners	α_{mx}	0.022	0.032	0.036	0.037	0.037	0.057	0.056	0.069	0.081	0.095
($\alpha_{my} = \alpha_{mx}$ and $\alpha_{mz,y} = \alpha_{mz,x}$)	$\alpha_{mz,x}$	0.004	0.006	0.007	0.007	0.007	0.011	0.011	0.014	0.016	0.019
Positive moment for span l_x	α_{mx}	0.009	0.009	0.009	0.009	0.009	0.016	0.017	0.017	0.017	0.017
Positive moment for span l_y	α_{my}	0.003	0.012	0.012	0.010	0.009	0.001	0.007	0.017	0.027	0.024
Negative moment at bottom	$\alpha_{mz,x}$	0.067	0.067	0.067	0.067	0.067	0.152	0.152	0.151	0.150	0.149
(1) $\alpha_{mx} = 0.013$, (2) $\alpha_{mx} = 0.030$	$\alpha_{mz,y}$	0.005	0.033	0.053	0.062	0.066	0	0.019	0.050	0.081	0.126
	α_{my}	0.001	0.007	0.011	0.012	0.013	0	0.004	0.010	0.016	0.025
Positive moment for span l_z	$\alpha_{mz,x}$	0.029	0.029	0.029	0.029	0.029	0.007	0.007	0.006	0.006	0.007
	$\alpha_{mz,y}$	0.003	0.011	0.021	0.026	0.029	0.007	0.012	0.016	0.016	0.011
Long span ratio l_x/l_z = 3.0											
Negative moment at corners	α_{mx}	0.022	0.032	0.036	0.037	0.037	0.054	0.053	0.066	0.078	0.091
($\alpha_{my} = \alpha_{mx}$ and $\alpha_{mz,y} = \alpha_{mz,x}$)	$\alpha_{mz,x}$	0.004	0.006	0.007	0.007	0.007	0.011	0.011	0.013	0.016	0.017
Positive moment for span l_x	α_{mx}	0.009	0.009	0.009	0.009	0.009	0.022	0.022	0.023	0.024	0.024
Positive moment for span l_y	α_{my}	0.003	0.012	0.012	0.010	0.009	0.001	0.007	0.017	0.027	0.024
Negative moment at bottom	$\alpha_{mz,x}$	0.067	0.066	0.066	0.066	0.066	0.134	0.133	0.131	0.129	0.127
(1) $\alpha_{mx} = 0.013$, (2) $\alpha_{mx} = 0.025$	$\alpha_{mz,y}$	0.005	0.033	0.053	0.062	0.066	0	0.020	0.051	0.082	0.127
	α_{my}	0.001	0.007	0.011	0.012	0.013	0	0.004	0.010	0.016	0.025
Positive moment for span l_z	$\alpha_{mz,x}$	0.029	0.029	0.029	0.029	0.029	0.009	0.009	0.010	0.010	0.011
	$\alpha_{mz,y}$	0.003	0.011	0.021	0.026	0.029	0.006	0.012	0.016	0.016	0.011
Long span ratio l_x/l_z = 2.0											
Negative moment at corners	α_{mx}	0.022	0.032	0.036	0.037		0.041	0.042	0.054	0.066	
($\alpha_{my} = \alpha_{mx}$ and $\alpha_{mz,y} = \alpha_{mz,x}$)	$\alpha_{mz,x}$	0.004	0.006	0.007	0.007		0.008	0.008	0.011	0.013	
Positive moment for span l_x	α_{mx}	0.010	0.010	0.010	0.010		0.029	0.029	0.028	0.028	
Positive moment for span l_y	α_{my}	0.003	0.012	0.012	0.010		0.001	0.009	0.019	0.028	
Negative moment at bottom	$\alpha_{mz,x}$	0.063	0.063	0.062	0.062		0.097	0.095	0.090	0.086	
(1) $\alpha_{mx} = 0.012$, (2) $\alpha_{mx} = 0.017$	$\alpha_{mz,y}$	0.005	0.033	0.053	0.062		0	0.023	0.056	0.086	
	α_{my}	0.001	0.007	0.011	0.012		0	0.005	0.011	0.017	
Positive moment for span l_z	$\alpha_{mz,x}$	0.027	0.026	0.026	0.026		0.015	0.015	0.016	0.016	
	$\alpha_{mz,y}$	0.003	0.011	0.021	0.026		0.006	0.011	0.016	0.016	
Long span ratio l_x/l_z = 1.5											
Negative moment at corners	α_{mx}	0.022	0.032	0.036			0.028	0.036	0.044		
($\alpha_{my} = \alpha_{mx}$ and $\alpha_{mz,y} = \alpha_{mz,x}$)	$\alpha_{mz,x}$	0.004	0.006	0.007			0.006	0.007	0.009		
Positive moment for span l_x	α_{mx}	0.012	0.012	0.012			0.025	0.024	0.021		
Positive moment for span l_y	α_{my}	0.003	0.012	0.012			0.001	0.010	0.021		
Negative moment at bottom	$\alpha_{mz,x}$	0.056	0.054	0.053			0.071	0.067	0.061		
(1) $\alpha_{mx} = 0.011$, (2) $\alpha_{mx} = 0.012$	$\alpha_{mz,y}$	0.005	0.033	0.053			0.001	0.028	0.061		
	α_{my}	0.001	0.007	0.011			0	0.006	0.012		
Positive moment for span l_z	$\alpha_{mz,x}$	0.022	0.021	0.021			0.016	0.015	0.015		
	$\alpha_{mz,y}$	0.003	0.011	0.021			0.005	0.010	0.015		
Long span ratio l_x/l_z = 1.0											
Negative moment at corners	α_{mx}	0.020	0.029				0.021	0.030			
($\alpha_{my} = \alpha_{mx}$ and $\alpha_{mz,y} = \alpha_{mz,x}$)	$\alpha_{mz,x}$	0.004	0.006				0.004	0.006			
Positive moment for span l_x	α_{mx}	0.013	0.012				0.016	0.013			
Positive moment for span l_y	α_{my}	0.003	0.012				0.003	0.013			
Negative moment at bottom	$\alpha_{mz,x}$	0.039	0.035				0.041	0.035			
(1) $\alpha_{mx} = 0.007$, (2) $\alpha_{mx} = 0.007$	$\alpha_{mz,y}$	0.006	0.035				0.005	0.035			
	α_{my}	0.001	0.007				0.001	0.007			
Positive moment for span l_z	$\alpha_{mz,x}$	0.013	0.011				0.011	0.010			
	$\alpha_{mz,y}$	0.003	0.011				0.003	0.010			
For details of tank dimensions and notes, see *Table 2.79*.											

Rectangular tanks: triangularly distributed load (elastic analysis) – 2

2.79

Span ratios and moments considered		(3) Top hinged, bottom hinged					(4) Top free, bottom hinged				
		Coefficients for short span ratio l_y/l_z					Coefficients for short span ratio l_y/l_z				
		0.5	1.0	1.5	2.0	3.0	0.5	1.0	1.5	2.0	3.0
Long span ratio $l_x/l_z = 4.0$											
Negative moment at corners	α_{mx}	0.037	0.050	0.059	0.062	0.064	0.216	0.187	0.191	0.209	0.261
($\alpha_{my} = \alpha_{mx}$ and $\alpha_{mz,y} = \alpha_{mz,x}$)	$\alpha_{mz,x}$	0.007	0.010	0.012	0.012	0.013	0.043	0.037	0.038	0.042	0.052
Positive moment for span l_x	α_{mx}	0.017	0.017	0.017	0.017	0.017	0.091	0.092	0.092	0.091	0.090
Positive moment for span l_y	α_{my}	0	0.014	0.021	0.020	0.017	0	0	0.004	0.024	0.070
Positive moment for span l_z	$\alpha_{mz,x}$	0.063	0.063	0.063	0.063	0.063	0.059	0.059	0.059	0.059	0.058
	$\alpha_{mz,y}$	0.001	0.013	0.030	0.044	0.059	0	0.006	0.018	0.031	0.049
Long span ratio $l_x/l_z = 3.0$											
Negative moment at corners	α_{mx}	0.037	0.050	0.059	0.062	0.064	0.142	0.124	0.132	0.152	0.205
($\alpha_{my} = \alpha_{mx}$ and $\alpha_{mz,y} = \alpha_{mz,x}$)	$\alpha_{mz,x}$	0.007	0.010	0.012	0.012	0.013	0.028	0.025	0.026	0.030	0.041
Positive moment for span l_x	α_{mx}	0.017	0.017	0.017	0.017	0.017	0.080	0.081	0.080	0.078	0.074
Positive moment for span l_y	α_{my}	0	0.015	0.021	0.020	0.017	0	0	0.012	0.034	0.074
Positive moment for span l_z	$\alpha_{mz,x}$	0.060	0.059	0.059	0.059	0.059	0.053	0.053	0.053	0.052	0.051
	$\alpha_{mz,y}$	0.001	0.013	0.030	0.044	0.059	0	0.008	0.021	0.034	0.051
Long span ratio $l_x/l_z = 2.0$											
Negative moment at corners	α_{mx}	0.036	0.049	0.058	0.061		0.071	0.065	0.078	0.098	
($\alpha_{my} = \alpha_{mx}$ and $\alpha_{mz,y} = \alpha_{mz,x}$)	$\alpha_{mz,x}$	0.007	0.010	0.012	0.012		0.014	0.013	0.016	0.020	
Positive moment for span l_x	α_{mx}	0.019	0.019	0.020	0.020		0.055	0.055	0.051	0.046	
Positive moment for span l_y	α_{my}	0	0.015	0.021	0.020		0	0.005	0.022	0.046	
Positive moment for span l_z	$\alpha_{mz,x}$	0.048	0.046	0.045	0.045		0.040	0.040	0.038	0.037	
	$\alpha_{mz,y}$	0.001	0.013	0.030	0.045		0.001	0.011	0.025	0.037	
Long span ratio $l_x/l_z = 1.5$											
Negative moment at corners	α_{mx}	0.033	0.046	0.054			0.042	0.050	0.063		
($\alpha_{my} = \alpha_{mx}$ and $\alpha_{mz,y} = \alpha_{mz,x}$)	$\alpha_{mz,x}$	0.007	0.009	0.011			0.008	0.010	0.013		
Positive moment for span l_x	α_{mx}	0.021	0.021	0.021			0.037	0.035	0.028		
Positive moment for span l_y	α_{my}	0	0.015	0.021			0	0.011	0.028		
Positive moment for span l_z	$\alpha_{mz,x}$	0.035	0.032	0.031			0.030	0.029	0.027		
	$\alpha_{mz,y}$	0.002	0.013	0.031			0.001	0.013	0.027		
Long span ratio $l_x/l_z = 1.0$											
Negative moment at corners	α_{mx}	0.025	0.038				0.027	0.038			
($\alpha_{my} = \alpha_{mx}$ and $\alpha_{mz,y} = \alpha_{mz,x}$)	$\alpha_{mz,x}$	0.005	0.008				0.005	0.008			
Positive moment for span l_x	α_{mx}	0.018	0.017				0.021	0.017			
Positive moment for span l_y	α_{my}	0.002	0.017				0.001	0.017			
Positive moment for span l_z	$\alpha_{mz,x}$	0.018	0.015				0.017	0.014			
	$\alpha_{mz,y}$	0.002	0.015				0.002	0.014			



Dimensions of tank

Note: Maximum values of moment per unit width are given by the following relationships, where l_x, l_y and l_z are length, breadth and height respectively of the tank, and γ is unit weight of liquid.

Horizontal (long span): $m_x = \alpha_{mx}\gamma l_z^3$

Horizontal (short span): $m_y = \alpha_{my}\gamma l_z^3$

Vertical (long wall): $m_{z,x} = \alpha_{mz,x}\gamma l_z^3$

Vertical (short wall): $m_{z,y} = \alpha_{mz,y}\gamma l_z^3$

Maximum values of shear per unit width may be determined for each wall, according to value of l_x/l_z or l_y/l_z, from *Table 2.53*.

4m wide and 4m deep, and the water level is to be taken to the top of the walls.

From *Table 2.79*, for $l_x/l_z = 6/4 = 1.5$, $l_y/l_z = 4/4 = 1.0$ and top edge free, maximum bending moments are as follows:

Horizontal negative moment at corners

$$m_x = m_y = 0.050 \times 9.81 \times 4^3 = 31.4 \text{ kNm/m}$$

Horizontal positive moments (at about $0.5l_z$ = 2m above base)

$$m_x = 0.035 \times 9.81 \times 4^3 = 22.0 \text{ kNm/m (long wall)}$$

$$m_y = 0.011 \times 9.81 \times 4^3 = 6.9 \text{ kNm/m (short wall)}$$

Vertical positive moments (at about $0.3l_z$ = 1.2m above base)

$$m_{z,x} = 0.029 \times 9.81 \times 4^3 = 18.2 \text{ kNm/m (long wall)}$$

$$m_{z,y} = 0.013 \times 9.81 \times 4^3 = 8.2 \text{ kNm/m (short wall)}$$

From *Table 2.53*, for panel type 4 with $l_h/l_z = 1.5$ (long wall) and 1.0 (short wall), maximum shear forces are as follows:

Horizontal shear forces at side edges

$$v_{h,x} = 0.37 \times 9.81 \times 4^2 = 58.1 \text{ kN/m (long wall)}$$

$$v_{h,y} = 0.31 \times 9.81 \times 4^2 = 48.7 \text{ kN/m (short wall)}$$

Vertical shear forces at bottom edge

$$v_{z,x} = 0.26 \times 9.81 \times 4^2 = 40.8 \text{ kN/m (long wall)}$$

$$v_{z,y} = 0.19 \times 9.81 \times 4^2 = 29.8 \text{ kN/m (short wall)}$$

From the above, the horizontal tensile forces in the walls are

$$n_{h,x} = 48.7 \text{ kN/m (long wall)}, \; n_{h,y} = 58.1 \text{ kN/m (short wall)}$$

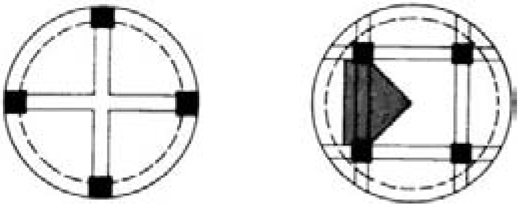
17.3 SILOS

Notes on the properties of stored materials and the pressures set up in silos of different forms and proportions are given in sections 2.7.7 and 9.3, and *Tables 2.15* and *2.16*. Notes on the design of silo walls are given in section 6.4.1. For rectangular silos that are divided into several compartments, where the walls span horizontally, expressions for the bending moments and reactions are given in *Table 2.80*.

17.4 BOTTOMS OF ELEVATED TANKS AND SILOS

17.4.1 Tanks

The figure here shows the floor of an elevated cylindrical tank supported by beams in two different arrangements.



(a) (b)

In (a) each beam spans between opposite columns and carries one-quarter of the load on the floor. The remaining half of the floor load, the weight of the wall and the load from the roof are transferred to the columns through the wall. In (b) each length of beam between the columns carries the load on the shaded area, with the remainder of the floor load, the weight of the wall and the load from the roof being equally divided between the eight cantilever lengths. An alternative to (b) is to place the columns almost under the wall, in which case the cantilevers are unnecessary, but secondary beams might be required.

For a tank of large diameter, a domed bottom of one of the types shown in *Table 2.81* is more economical and, although the formwork is much more costly, the saving in concrete and reinforcement compared to beam-and-slab construction can be considerable. Ring beams A and C in the case of a simple domed bottom resist the horizontal component of thrust from the dome, and the thickness of the dome is determined by the magnitude of the thrust. Expressions for the thrust and the vertical shearing force around the edge of the dome, and the resultant circumferential tension in the ring beam are given in *Table 2.81*. Domes, used to form the bottom or the roof, can also be analysed by the method given in *Table 2.92*.

A bottom consisting of a central dome and an outer conical part, as illustrated in *Table 2.81*, is economical for the largest tanks. This form of construction is traditionally known as an Intze tank. The outward thrust from the top of the conical part is resisted by the ring beam B, and the difference between the inward thrust from the bottom of the conical part and the outward thrust from the domed part is resisted by the ring beam A. Expressions for the forces are given in *Table 2.81*, and the proportions of the conical and domed parts can be chosen so that the resultant thrust on beam A is zero. Suitable proportions for bottoms of this type are given in *Table 2.81*, and the volume of a tank with these proportions is $0.604\, d_o^3$, where d_o is the diameter of the tank. The tank wall should be designed as described previously, account being taken of the vertical bending at the base of the wall, and the effect of this bending on the conical part. The floor must be designed to resist, in addition to the forces and bending moments already described, any radial tension due to the vertical bending of the wall.

Example 6. Determine for service loads the principal forces in the bottom of a cylindrical tank of the Intze type, where:

$$d_o = 10 \text{ m}, \; d = 8 \text{ m}, \; \phi = 48^\circ \text{ (giving } \cot\phi = 0.900\text{)},$$

$$\theta = 40^\circ \text{ (giving } \cot\theta = 1.192 \text{ and } \mathrm{cosec}\,\theta = 1.55\text{)},$$

$$F_1 = 2500 \text{ kN}, \; F_2 = 2800 \text{ kN, and } F_3 = 1300 \text{ kN}.$$

Then, from *Table 2.81*, the following values are obtained:

Unit vertical shearing force at periphery of dome

$$V_1 = F_1/(\pi d) = 2500/(8\pi) = 100 \text{ kN/m}$$

Unit thrust at periphery of dome

$$N_1 = V_1 \mathrm{cosec}\,\theta = 100 \times 1.55 = 155 \text{ kN/m}$$

(Note. The required thickness of the dome at the springing is determined by the values of N_1 and V_1)

Unit outward horizontal thrust from dome on ring beam A

$$H_1 = V_1 \cot\theta = 100 \times 1.192 = 119.2 \text{ kN/m}$$

Unit vertical shearing force at inner edge of cone

$$V_2 = (F_2 + F_3)/(\pi d) = (2800 + 1300)/(8\pi) = 163 \text{ kN/m}$$

Unit inward horizontal thrust from cone on ring beam A

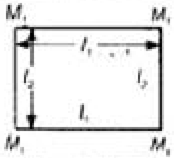
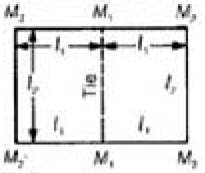
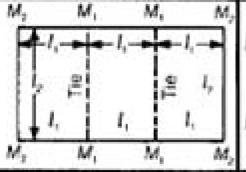
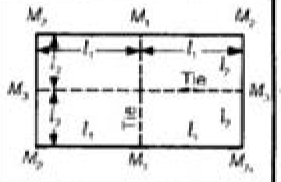
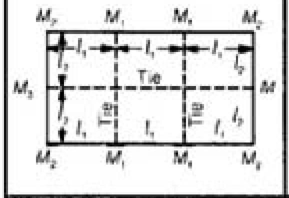
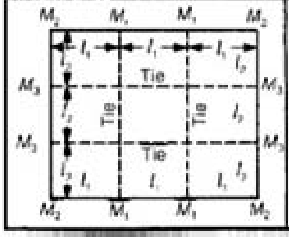
$$H_2 = V_2 \cot\phi = 163 \times 0.900 = 146.8 \text{ kN/m}$$

Circumferential compression in ring beam A

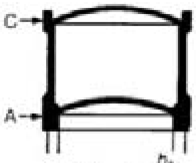
$$N_A = 0.5d(H_2 - H_1) = 0.5 \times 8 \times (146.8 - 119.2) = 110.4 \text{ kN}$$

Rectangular containers spanning horizontally: moments in walls

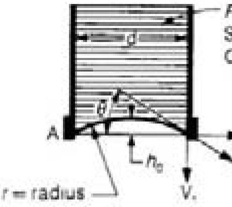
2.80

Form of container	Support-moment formulae	Bending moments $M = ql_1^2/k$.			Bending-moment coefficients k				Reaction formulae (i.e. shearing force)
		Ratio of l_2/l_1							
		1/2	3/4	1	5/4	3/2	7/4	2	
	$M_1 = \frac{q(l_1^3 + l_2^3)}{12(l_1 + l_2)}$	16.00	14.77	12	9.14	6.86	5.19	4.00	$R_{t1} = \frac{1}{2}ql_1$ $R_{t2} = \frac{1}{2}ql_2$
	$M_1 = \frac{q(l_1^3 + 3l_1^2 l_2 - l_2^3)}{12(l_1 + 2l_2)}$	10.11	10.61	12	15.02	22.59	60.63	—	$R_{t1} = \frac{1}{2}ql_1 + \frac{M_2 - M_1}{l_1}$
	$M_2 = \frac{q(l_1^3 + 2l_2^3)}{12(l_1 + 2l_2)}$	19.20	16.27	12	8.56	6.19	4.61	3.53	$R_{t2} = \frac{1}{2}ql_2$
	$M_1 = \frac{q(3l_1^3 + 6l_1^2 l_2 - l_2^3)}{12(3l_1 + 5l_2)}$	11.23	11.44	12	12.99	14.61	17.32	22.29	$R_{t1}(\text{end span}) = \frac{1}{2}ql_1 + \frac{M_2 - M_1}{l_1}$
	$M_2 = \frac{q(3l_1^3 - 5l_2^3)}{12(3l_1 + 5l_2)}$	18.21	15.85	12	8.70	6.34	4.73	3.63	$R_{t1}(\text{central span}) = \frac{1}{2}ql_1$ $R_{t2} = \frac{1}{2}ql_2$
	$M_1 = \frac{q(l_1 + l_2)(2l_1 - l_2)}{24}$	10.67	10.97	12	14.22	19.20	34.91	—	$R_{t1} = \frac{1}{2}ql_1 + \frac{M_2 - M_1}{l_1}$
	$M_2 = \frac{q(l_1^3 + l_2^3)}{12(l_1 + l_2)}$	16.00	14.77	12	9.14	6.86	5.19	4.00	$R_{t2} = \frac{1}{2}ql_2 + \frac{M_2 - M_3}{l_2}$
	$M_3 = \frac{q(l_1 + l_2)(2l_2 - l_1)}{24}$	—	27.43	12	7.11	4.80	3.49	2.67	
	$M_1 = \frac{q(6l_1^3 + 6l_1^2 l_2 - l_2^3)}{12(6l_1 + 5l_2)}$	11.49	11.61	12	12.73	13.94	15.89	19.20	$R_{t1}(\text{end span}) = \frac{1}{2}ql_1 + \frac{M_2 - M_1}{l_1}$
	$M_2 = \frac{q(6l_1^3 + 5l_2^3)}{12(6l_1 + 5l_2)}$	15.40	14.43	12	9.32	7.08	5.40	4.17	$R_{t1}(\text{central span}) = \frac{1}{2}ql_1$
	$M_3 = \frac{q(5l_2^3 + 9l_1 l_2^2 - 3l_1^3)}{12(6l_1 + 5l_2)}$	—	28.04	12	7.06	4.75	3.45	2.63	$R_{t2} = \frac{1}{2}ql_2 + \frac{M_2 - M_3}{l_2}$
	$M_1 = \frac{q(5l_1^3 + 6l_1^2 l_2 - l_2^3)}{60(l_1 + l_2)}$	11.43	11.57	12	12.80	14.12	16.27	20.00	$R_{t1}(\text{end span}) = \frac{1}{2}ql_1 + \frac{M_2 - M_1}{l_1}$
	$M_2 = \frac{q(l_1^3 + l_2^3)}{12(l_1 + l_2)}$	16.00	14.77	12	9.14	6.86	5.19	4.00	$R_{t1}(\text{central span}) = \frac{1}{2}ql_1$
	$M_3 = \frac{q(5l_2^3 + 6l_1 l_2^2 - l_1^3)}{60(l_1 + l_2)}$	80.00	23.41	12	7.44	5.11	3.74	2.86	$R_{t2}(\text{end span}) = \frac{1}{2}ql_2 + \frac{M_2 - M_3}{l_2}$ $R_{t2}(\text{central span}) = \frac{1}{2}ql_2$

Domed bottoms



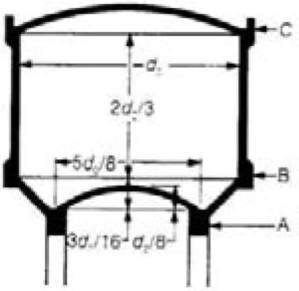
$\cos\theta = 1 - \frac{h_s}{r}$

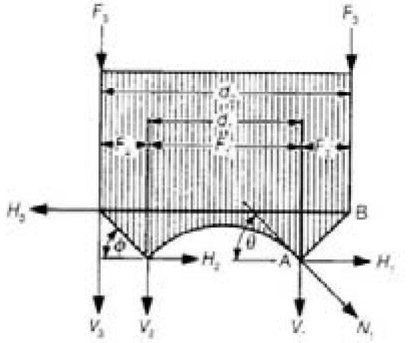


F_1: weight of contents above dome including weight of dome
Shearing force $V_1 = F_1/\pi d$ (force per unit length of perimeter)
Circumferential tension in beam at $A = H_1 d/2 = 0.16F_1 \cot\theta$

$H_1 = V_1 \cot\theta$

N_1 = thrust at perimeter of dome = V_1 cosec θ per unit length of perimeter





Reactions from cone Reactions from dome

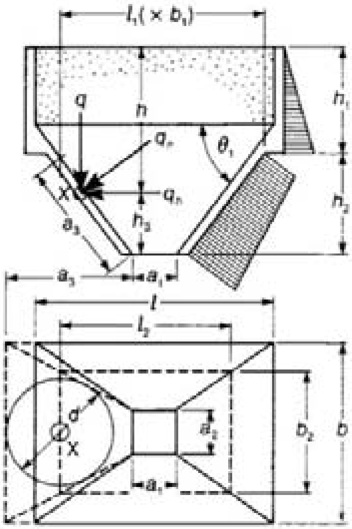
Values of V_1, N_1 and H_1 as for simple dome above

$$\left.\begin{aligned} V_2 &= \frac{F_2 + F_3}{\pi d} \\ H_2 &= V_2 \cot\phi \\ V_3 &= \frac{F_3}{\pi d_0} \\ H_3 &= V_3 \cot\phi \end{aligned}\right\} \text{per unit length of appropriate periphery}$$

(Ideal case occurs when $H_1 = H_2$)

Circumferential tension in beam at $A = \frac{1}{2}d(H_1 - H_2)$
F_1 weight of contents above dome including weight of dome
F_2 weight of contents above cone including weight of cone
F_3 weight of wall etc. and all loads on roof including weight of roof etc.

Hopper bottoms



D weight of contents per unit volume
g_s weight of sloping slab per unit area
G_x total weight of hopper bottom below level of X
G total weight of hopper bottom of depth h_2
$F_x = D\{\frac{1}{3}h_3[a_1a_2 + b_2l_2 + \sqrt{(a_1a_2b_2l_2)}] + b_2l_2h\} + G_x$
$F = D\{\frac{1}{3}h_2[a_1a_2 + b_1l_1 + \sqrt{(a_1a_2b_1l_1)}] + b_1l_1h_1\} + G$
$q = Dh$; $q_h = k_2Dh$; $q_n = q_h \sin^2\theta_1 + q\cos^2\theta_1 + g_s\cos\theta_1$

Horizontal effects at midspan and corners of hopper bottom:
Bending moment = $\pm(1/32)q_n d^2$ per unit length of sloping slab
Direct tension = $(1/2)q_n l_2 \sin\theta_1$ per unit length of sloping slab

Effects along slope:
Bending moment at centre and top of sloping slab = $\pm(1/32)q_n d^2$

$$\left.\begin{aligned} &\text{Direct tension (in direction of slope)} = \frac{F_x}{2\sin\theta_1(l_2 + b_2)} \text{ at centre} \\ &\qquad = \frac{F}{2\sin\theta_1(l_1 + b_1)} \text{ at top} \\ &\text{Vertical hanging-up force} = \frac{F}{2(l_1 + b_1)} \text{ at top} \end{aligned}\right\} \text{per unit width of sloping slab}$$

(Note. If H_1 exceeds H_2, the circumferential force is tensile. The ideal case occurs when $H_1 = H_2$ and $N_A = 0$: see as follows.)

Unit vertical shearing force at outer edge of cone

$$V_3 = F_3/(\pi d_o) = 1300/(10\pi) = 41.4 \text{ kN/m}$$

Unit outward horizontal thrust from cone on ring beam B

$$H_3 = V_3 \cot \phi = 41.4 \times 0.900 = 37.2 \text{ kN/m}$$

Circumferential tension in ring beam B

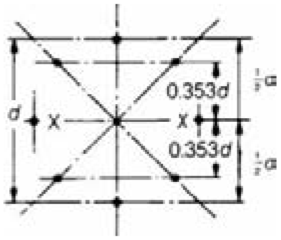
$$N_B = 0.5 d_o H_3 = 0.5 \times 10 \times 37.2 = 186.2 \text{ kN}$$

The vertical wall must be reinforced for the circumferential tension due to the horizontal pressure of the contained liquid, given by $n = 0.5\gamma d_o z$ per unit depth. The conical part of the tank must be reinforced to resist circumferential tension, and the reinforcement may be either distributed over the height of the conical portion, or concentrated in the ring beams at the top and bottom. In large-diameter Intze tanks, the width of ring beam A can be considerable and, if this is so, the weight of water immediately above the beam should not be taken to contribute to the forces on the dome and conical part. With a wide beam, F_1 is taken as the weight of the contents over the net area of the dome and d as the internal diameter of the ring beam; F_2 is taken as the weight of the contents over the net area of the conical part and d, for use with F_2, as the external diameter of the ring beam. If this were to be done for a ring beam of reasonable width in the forgoing example, H_1 and H_2 would be more nearly balanced.

17.4.2 Columns supporting elevated tanks

For a group of four columns on a square grid, the thrusts and tensions in the columns, due to wind loading on the tank, can be calculated as follows. When the wind blows normal to the side of the group, the thrust on each column on the leeward side, and the tension in each column on the windward side, is $M_w/2d$, where d is the column spacing and M_w is the total moment due to the wind. When the wind blows normal to a diagonal of the group, the thrust on a leeward corner column, and tension in a windward corner column is $M_w/d\sqrt{2}$. For any other arrangement, the force on any column can be calculated from the equivalent second moment of area of the group.

Consider the case when the wind blows normal to the X-X axis of the group of eight columns shown in the following figure. The second moment of area of the group of columns about the axis is $2 \times (d/2)^2 + 4 \times (0.353\ d)^2 = 1.0\ d^2$. The thrust on the extreme leeward column is $(0.5d/1.0\ d^2)\ M_w = M_w/2d$. The forces on each of the other columns in the group can be determined similarly, by substituting the appropriate value for the distance of the column from the axis.



17.4.3 Silos

Notes on the pressures set up on hopper bottoms are given in sections 2.7.7 and 9.3, and *Tables 2.15* and *2.16*. Notes on the design of hopper bottoms in the form of inverted truncated pyramids are given in section 6.4.2. Expressions for bending moments and tensile forces are given in *Table 2.81*.

Chapter 18

Foundations and retaining walls

18.1 PAD FOUNDATIONS

Some general notes on the design of foundations are given in section 7.1. The size of a pad or spread foundation is usually determined using service loads and allowable bearing values. The subsequent structural design is then determined by the requirements of the ULS. Presumed allowable bearing values recommended for preliminary design purposes in BS 8004 are given in *Table 2.82*.

18.1.1 Separate bases

An introduction to separate bases is given in section 7.1.6. Diagrams of bearing pressure distributions, and expressions for pressures and maximum bending moments in rectangular bases subjected to concentric and eccentric loading are given in *Table 2.82*.

Example 1. The distribution of bearing pressure is required under a base 3 m long, 2.5 m wide and 600 mm thick, when it supports a concentrated load of 1000 kN at an eccentricity of 300 mm in relation to its length.

Weight of base: $F_b = 3.0 \times 2.5 \times 0.6 \times 24 = 108$ kN
Total load: $F_{tot} = F_b + F_v = 108 + 1000 = 1108$ kN

Eccentricity of total load,

$$e_{tot} = F_v e / F_{tot} = (1000 \times 0.3)/1108 = 0.27 \text{ m}$$

Since $e_{tot} < l/6 = 3.0/6 = 0.5$ m, the bearing pressure diagram is trapezoidal, and the maximum and minimum pressures are:

$$f = (1 \pm 6e_{tot}/l)F_{tot}/bl = (1 \pm 6 \times 0.27/3.0)[1108/(2.5 \times 3.0)]$$
$$= (1.54 \text{ and } 0.46) \times 148 = 228 \text{ kN/m}^2 \text{ and } 68 \text{ kN/m}^2$$

Note. For the structural design of the base, F_b and F_v should include safety factors appropriate to the ULS. Bearing pressures corresponding to these values of F_b and F_v, reduced by F_b/bl, should then be used to determine bending moments and shear forces for the subsequent design.

18.1.2 Combined bases

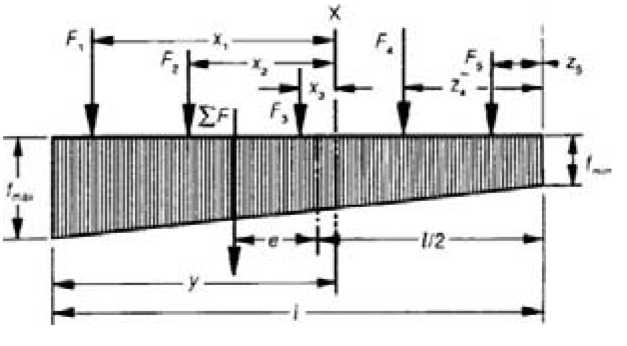
When more than one column or load is carried on a single base, the centre of gravity of the total load should coincide, if possible, with the centre of area of the base. Then, assuming a rigid base, the resulting bearing pressure will be uniformly distributed. The base should be symmetrically disposed about the line of the loads, and can be rectangular or trapezoidal on plan as shown in *Table 2.83*. Alternatively, it could be made up as a series of rectangles as shown at (b) in *Table 2.84*. In this last case, each rectangle should be proportioned so that the load upon it acts at the centre of the area, with the area of the rectangle being equal to the load divided by an allowable bearing pressure, and the value of the pressure being the same for each rectangle.

If it is not practical to proportion the combined base in this way, then the total load will be eccentric. If the base is thick enough to be considered to act as a rigid member, the ground bearing pressure will vary according to the diagram shown at (c) in *Table 2.84*. For a more flexible base, the pressure will be greater immediately under the loads, giving a distribution of pressure as shown at (d) in *Table 2.84*.

In the case of a uniform distribution or linear variation of pressure, the longitudinal bending moment on the base at any section is the sum of the anti-clockwise moments of the loads to the left of the section, minus the clockwise moment due to the ground pressure between the section and the left-hand end of the base. This method of analysis gives larger values for longitudinal bending moments than if a non-linear variation is assumed. Formulae for combined bases carrying two loads are given in *Table 2.83*.

Example 2. A strip base, 15 m long and 1.5 m wide, carries a line of five unequal concentrated loads arranged eccentrically as shown in the following figure. The bending moment is to be determined at the position of load F_2, where the values of the loads and the distances from RH end are as follows:

$$F_1 = 500 \text{ kN}, F_2 = 450 \text{ kN}, F_3 = 400 \text{ kN}, F_5 = 300 \text{ kN}$$
$$z_1 = 14 \text{ m}, z_2 = 11 \text{ m}, z_3 = 8 \text{ m}, z_4 = 5 \text{ m}, z_5 = 1.5 \text{ m}$$



Foundations: presumed allowable bearing values and separate bases

2.82

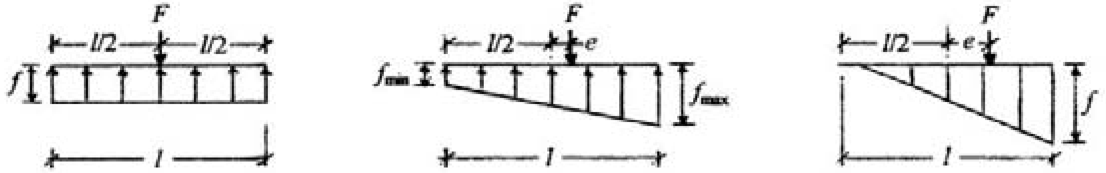
Presumed allowable bearing value

Category	Type of rock or soil	Bearing value kN/m^2	Remarks
Rocks	Strong igneous and gneissic rocks in sound condition	10 000	Values are based on the assumption that the foundation is taken down to unweathered rock.
	Strong limestones and strong sandstones	4 000	
	Schists and slates	3 000	
	Strong shales, strong mudstones and strong siltstones	2 000	
Non-cohesive soils	Dense gravel, or dense sand and gravel	> 600	Values apply where width of foundation is not less than 1 m. Groundwater level is assumed to be not above the bottom of the foundation.
	Medium dense gravel, or medium dense sand and gravel	200–600	
	Loose gravel, or loose sand and gravel	< 200	
	Compact sand	> 300	
	Medium dense sand	100–300	
	Loose sand	< 100	
Cohesive soils	Very stiff boulder clays and hard clays	300–600	Clays and silts are susceptible to long-term consolidation settlement.
	Stiff clays	150–300	
	Firm clays	75–150	
	Soft clays and silts	< 75	

Notes. These values, which apply under static loading, are for preliminary design purposes only, and may need to be altered upwards or downwards. No addition has been made for the depth of embedment of the foundation. Values for non-cohesive soils depend on degree of looseness. No values are given for very soft clays and silts, peat and organic soils, or made ground and fill.

Total bearing pressure

Distribution of bearing pressure on rectangular base for concentric load, and load eccentric about one axis only



Concentric load ($e = 0$)

$k = 1$

Eccentric load ($e \leq l/6$)

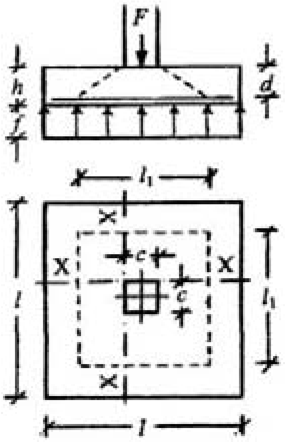
$k = 1 \pm \frac{6e}{l}$

Eccentric load ($e > l/6$)

$k = \frac{4l}{3(l-2e)}$

$f = kF/bl$ where F is total load on foundation (including weight of base), b and l are width and length of foundation.

Bearing pressures for structural design of base



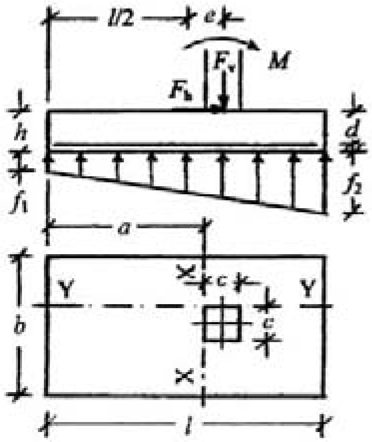
For structural design (to BS 8110):

$f = F/l^2$ where $F = 1.4G_k + 1.6Q_k$

Bending moment at X-X, $M = fl(l-c)^2/2$

Punching shear stress at critical perimeter

$v = f(l^2 - l_1^2)/4l_1d$ where $l_1 = c + 3d$



For structural design, where all loads are design ultimate loads:

$f_1 = [F_v - (F_b + F_v)(6e_{tot}/l)]/bl$, $f_2 = [F_v + (F_b + F_v)(6e_{tot}/l)]/bl$

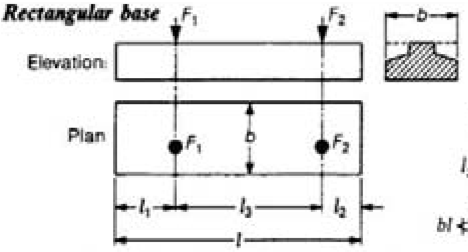
F_b is load due to weight of base $e_{tot} = (F_ve + F_bh + M)/(F_b + F_v)$

Bending moments at X-X and Y-Y:

$M_X = (a^2b/2)[f_1 + (a/3l)(f_2 - f_1)]$ $M_Y = F_v(b-c)^2/8b$

Concentric load

Rectangular base



$$f_{net} = \frac{F_1 + F_2}{bl}$$

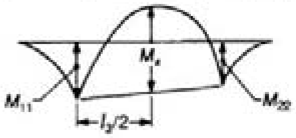
$$l_1 = l - (l_2 + l_3)$$

$$l_2 = \tfrac{1}{2}l - \frac{F_1 l_3}{F_1 + F_2}$$

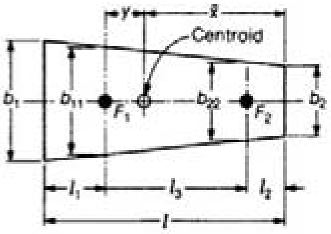
$$bl \nless \frac{F_1 + F_2 + F_{base}}{f_{lim}}$$

$M_{11} = -\frac{1}{2} f_{net} b l_1^2$ $\quad M_x = +\frac{1}{8} f_{net} b l_3^2$ $\quad M_{22} = -\frac{1}{2} f_{net} b l_2^2$

Transverse bending moment (at each load) $= -\frac{1}{8} bF_1$ (or F_2)



Trapezoidal base



$$\bar{x} = \frac{l}{3}\left[\frac{2b_1 + b_2}{b_1 + b_2}\right]$$

$$y = \frac{F_2 l_3}{F_1 + F_2}$$

$$l_1 = l - (y + \bar{x})$$

$$l_2 = l - (l_3 + l_1)$$

$$f_{net} = \frac{2(F_1 + F_2)}{l(b_1 + b_2)}$$

$$b_{11} = b_2 + (b_1 - b_2)\frac{l - l_1}{l}$$

$$\tfrac{1}{2}l(b_1 + b_2) \nless \frac{F_1 + F_2 + F_{base}}{f_{lim}} \qquad b_{22} = b_2 + (b_1 - b_2)\frac{l_2}{l}$$

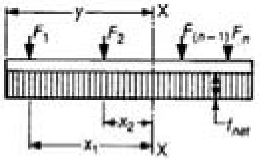
Longitudinal bending moments (total)
(diagram similar to rectangular base concentrically loaded)

$M_{11} = -\frac{1}{6} l_1^2 (2b_1 + b_{11}) f_{net}$ $\quad M_{22} = -\frac{1}{6} l_2^2 (2b_2 + b_{22}) f_{net}$

$M_x = +\frac{1}{16} l_3^2 (b_{11} + b_{22}) f_{net}$ (approx.)

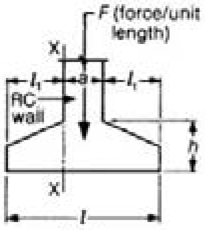
Transverse bending moments $= -\frac{1}{8} F_1 b_{11}$ at F_1: $-\frac{1}{8} F_2 b_{22}$ at F_2

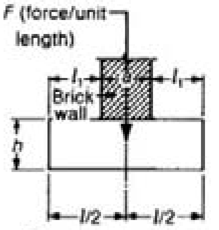
General case for any number of loads



Bending moment
at X-X:
$= \Sigma Fx - \frac{1}{2} b y^2 f_{net}$

Wall footings



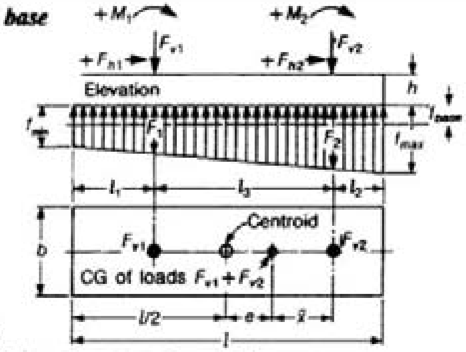


$$l \nless \frac{F + F_{base}}{F_{lim}} \qquad f_{net} = \frac{F}{l}$$

$M_x = \frac{1}{2} f_{net} l_1^2$ per unit length $\quad M_x = \frac{1}{4} f_{net} l_1 l$ per unit length

Eccentric load (note sign and direction of F_v, F_h, M, e etc)

Rectangular base



$$\bar{x} = \frac{F_{v1} l_3}{F_{v1} + F_{v2}}$$

$$e = \tfrac{1}{2}l - (l_2 + \bar{x})$$

$$M_0 = (F_{v1} + F_{v2})e + M_1 + M_2 + (F_{h1} + F_{h2})h$$

$$\left.\begin{matrix} f_{max} \\ f_{max} \end{matrix}\right\} \frac{1}{bl}\left[(F_{v1} + F_{v2} + F_{base}) \pm \frac{6M_0}{l}\right] \begin{matrix} \nless f_{lim} \\ \nless 0 \end{matrix}$$

$$f_1 = f_{min} + \frac{l_1}{l}(f_{max} - f_{min}); \qquad f_2 = f_{min} + \frac{l_1 + l_3}{l}(f_{max} - f_{min})$$

Longitudinal bending moments (total)
(diagram similar to rectangular base loaded concentrically)

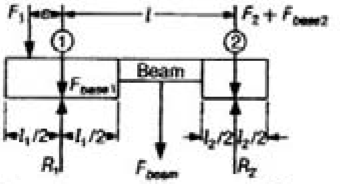
$M_{11} = -\frac{1}{6} b l_1^2 (2f_{min} + f_1 - 3f_{base})$

$M_x = +\frac{1}{16} b l_3^2 (f_1 + f_2)$ (approx.)

$M_{22} = -\frac{1}{6} b l_2^2 (2f_{max} + f_2 - 3f_{base})$

Transverse bending moments (at each load) $= -\frac{1}{8} bF_{v1}$ (or F_{v2})

Balanced bases



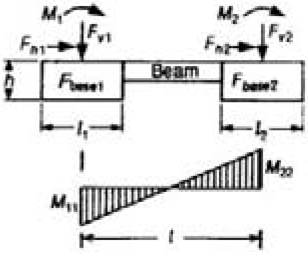
$$R = \frac{F_1 e}{l}$$

$R_1 = F_1 + R + F_{base1} + \frac{1}{2} F_{beam}$ $\quad b_1 l_1 \nless \frac{R_1}{f_{lim}}$ $\quad b_2 l_2 \nless \frac{R_2}{f_{lim}}$

$R_2 = F_2 - R + F_{base2} + \frac{1}{2} F_{beam}$

Longitudinal bending moment varies from $F_1 e$ at centre-line of ① to zero at centre-line of ②

Coupled bases



$R_1 = F_{v1} - R + F_{base1}$

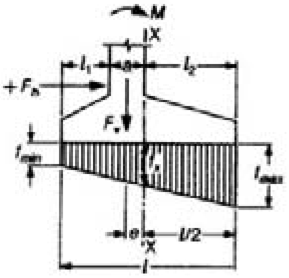
$R_2 = F_{v2} + R + F_{base2}$

$R = (M_{11} + M_{22})/l$

$M_{11} = M_1 + F_{h1} h$

$M_{22} = M_2 + F_{h2} h$

Wall footing



M = moment per unit length
F_h, F_v = load per unit length
$M_0 = M + F_h h \quad F_v e$

$$\left.\begin{matrix} f_{max} \\ f_{min} \end{matrix}\right\} \frac{1}{l}\left(F_v \pm \frac{6M_0}{l}\right) \begin{matrix} \nless f_{lim} \\ \nless 0 \end{matrix}$$

$$f_x = f_{min} + \frac{l_1 + a}{l}(f_{max} - f_{min})$$

$M_x = \frac{1}{6} l_2^2 (f_x + 2f_{max} - 3f_{base})$
per unit length

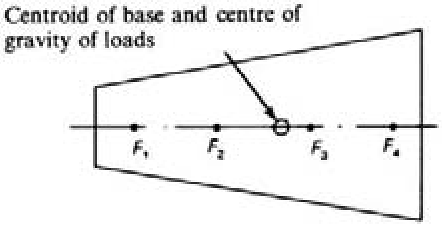
Shearing force at X $= \frac{1}{2} l_2 (f_x + f_{max} - 2f_{base})$ per unit length

The notation used in the diagrams above is as follows: F, F_v = vertical imposed load, F_h = horizontal imposed load (kN), f_{lim} = allowable bearing pressure, f_{base} = pressure due to weight of base, f_{net} = pressure due to imposed loads only (kN/m^2)

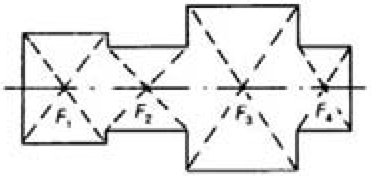
Foundations: inter-connected bases and rafts

2.84

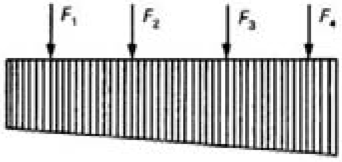
Combined foundations for line of loads



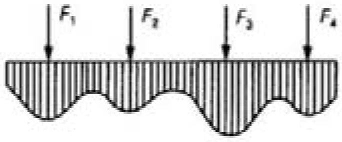
(a) *Trapezoidal*



(b) *Rectangular*

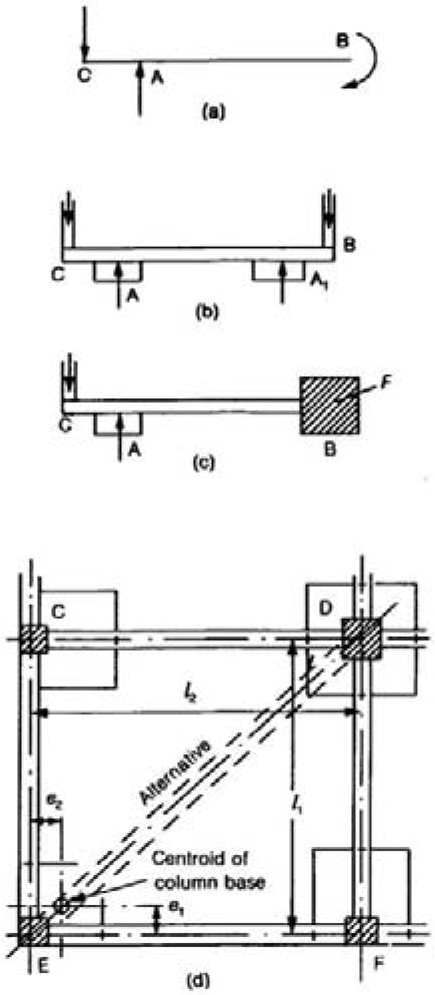


(c) *Uniform variation of pressure* (when CG and centroid not concident)

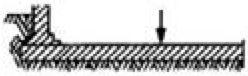


(d) *Non-uniform variation of pressure*

Balanced foundations

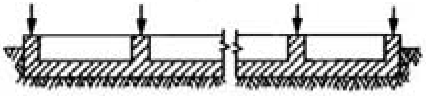


Rafts

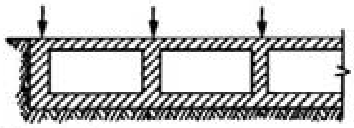


(a) *Solid raft*

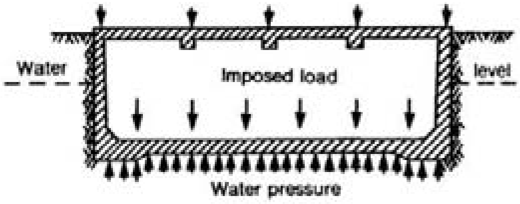
(b) *Solid raft with thickening at edge*



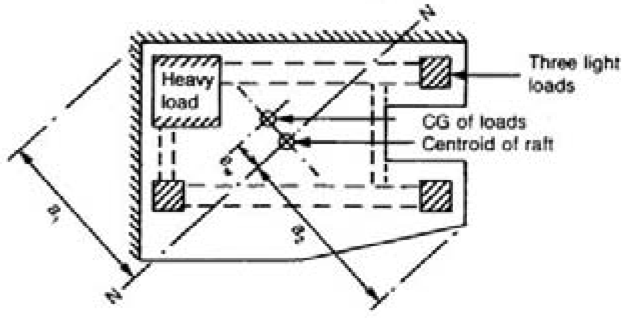
(c) *Beam-and-slab raft*



(d) *Cellular raft*



(e) *Basement in wet ground*



(f) *Plan of eccentrically loaded raft*

The first step is to determine the eccentricity of the total load, and check that $e = (\Sigma Fz/\Sigma F - l/2) \leq l/6$. Hence,

$$\Sigma Fz = 500 \times 14 + 450 \times 11 + 400 \times 8 + 350 \times 5 + 300 \times 1.5 = 17350 \text{ kNm}$$

$$\Sigma F = 500 + 450 + 400 + 350 + 300 = 2000 \text{ kN}$$

$$e = 17350/2000 - 15/2 = 1.175 \text{ m} < l/6 = 2.5 \text{ m}$$

The maximum and minimum bearing pressures can now be calculated from the formula in *Table 2.82*, where

$$k = (1 \pm 6e/l) = (1 \pm 6 \times 1.175/15) = 1.47 \text{ or } 0.53$$

Hence, with $\Sigma F/bl = 2000/(1.5 \times 15) = 88.9 \text{ kN/m}^2$

$$f_{\text{max}} = 1.47 \times 88.9 = 130.7 \text{ kN/m}^2$$

$$f_{\text{min}} = 0.53 \times 88.9 = 47.1 \text{ kN/m}^2$$

Then, for any section X-X, at distance y from the left-hand end of the base, the bearing pressure is

$$f_{\text{x}} = f_{\text{max}} - (f_{\text{max}} - f_{\text{min}})\, y/l$$

Considering the loads to the left of section X-X, where x is the distance of a load from the section, the resultant bending moment on the base at section X-X is

$$M = \Sigma Fx - (2f_{\text{max}} + f_{\text{x}})\, by^2/6$$

Thus, at the position of load F_2, where $y = 4$ m and $x_1 = 3$ m,

$$f_{\text{x}} = 130.7 - (130.7 - 47.1) \times 4/15 = 108.4 \text{ kN/m}^2$$

$$M = 500 \times 3 - (2 \times 130.7 + 108.4) \times 1.5 \times 4^2/6 = 20.8 \text{ kNm}$$

18.1.3 Balanced bases

With reference to the diagrams on the upper right-hand side of *Table 2.84*, (a) shows a system in which beam BC rests on a base at A, supports a column on the overhanging end C, and is counterbalanced at B. The reaction at A, which depends on the relative values of BC and BA, can be provided by a base designed for a concentric load. The counterbalance at B could be provided by load from another column as at (b), in which case the dead load on this column needs to be sufficient to counterbalance the dead and imposed loads on the column at C, and vice versa. It is often possible to arrange for base A_1 to be positioned directly under column B. Formulae giving the values of the reactions at A and A_1 are given in *Table 2.83*, where various combinations of values for F_1 and F_2 usually need to be considered. Thus, if F_1 varies from $F_{1,\text{max}}$ to $F_{1,\text{min}}$, and F_2 varies from $F_{2,\text{max}}$ to $F_{2,\text{min}}$, reaction R (see diagram in *Table 2.83*) will vary from

$$R_{\text{max}} = e\, F_{1,\text{max}}/l \quad \text{to} \quad R_{\text{min}} = e\, F_{1,\text{min}}/l$$

Hence, R_1 and R_2 can have the following values:

$$R_{1,\text{max}} = F_{1,\text{max}} + F_{\text{base 1}} + R_{\text{max}} + F_{\text{beam}}/2$$

$$R_{1,\text{min}} = F_{1,\text{min}} + F_{\text{base 1}} + R_{\text{min}} + F_{\text{beam}}/2$$

$$R_{2,\text{max}} = F_{2,\text{max}} + F_{\text{base 2}} - R_{\text{min}} + F_{\text{beam}}/2$$

$$R_{2,\text{min}} = F_{2,\text{min}} + F_{\text{base 2}} - R_{\text{max}} + F_{\text{beam}}/2$$

Therefore, base 1 must be designed for a maximum load of $R_{1,\text{max}}$ and base 2 for $R_{2,\text{max}}$, but $R_{2,\text{min}}$ must always be positive. From the reactions, the shearing forces and bending moments on the beam can be calculated. In the absence of a convenient column load being available at B, a suitable anchorage must be provided by other means, such as a counterweight in mass concrete as at (c), or the provision of tension piles. If the column to be supported is a corner column loading the foundation eccentrically in two directions, one parallel to each building line, as at (d), it is sometimes possible to use a diagonal balancing beam anchored by an internal column D. In other cases, however, the two wall beams meeting at the column can be designed as balancing beams to overcome the double eccentricity. For beam EC, the cantilever moment is $F_{\text{E}}\, e_1$, where F_{E} is the column load, and the upward force on column C is $F_{\text{E}}\, e_1/(l_1 - e_1)$. For beam EF, the corresponding values are $F_{\text{E}}\, e_2$ and $F_{\text{E}}\, e_2/(l_2 - e_2)$ respectively.

18.1.4 Rafts

The required thickness of a raft foundation is determined by the shearing forces and bending moments, which depend on the magnitude and spacing of the loads. If the thickness does not exceed 300 mm, a solid slab as at (a) in the lower part of *Table 2.84* is generally the most convenient form. If a slab at ground level is required, it is usually necessary to thicken the slab at the edge, as at (b), to ensure that the edge of the raft is deep enough to avoid weathering of the ground under the raft. If a greater thickness is required, beam-and-slab construction designed as an inverted floor, as at (c), is more efficient. In cases where the total depth required exceeds 1 m, or where a level top surface is required, a cellular construction consisting of a top and bottom slab with intermediate ribs, as at (d), can be adopted.

When the columns on a raft are not equally loaded, or are not symmetrically arranged, the raft should be designed so that the centre of area coincides with the centre of gravity of the loads. In this case, the pressure on the ground is uniform, and the required area is equal to the total load (including the weight of the raft) divided by the allowable bearing value. If the coincidence of the centre of area of the raft and the centre of gravity of the loads is impractical, due to the extent of the raft being limited on one or more sides, the shape of the raft on plan should be so that the eccentricity e_{w} of the total load F_{tot} is kept to a minimum, as in the example shown at (f).

The maximum bearing pressure (which occurs at the corner shown at distance a_1 from axis N-N on the plan, and should not exceed the allowable bearing value) is given by:

$$f_{\text{max}} = \frac{F_{\text{tot}}}{A_{\text{raft}}} + \frac{F_{\text{tot}}\, e_{\text{w}}\, a_1}{I_{\text{raft}}}$$

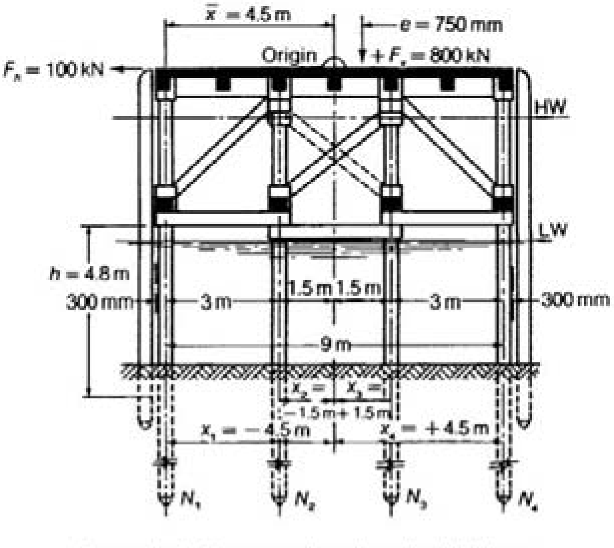
where A_{raft} is the total area of the raft, and I_{raft} is the second moment of area about the axis N-N, which passes through the centre of area and is normal to the line joining the centre of area and the centre of gravity of the loads. The pressure along axis N-N is $F_{\text{tot}}/A_{\text{raft}}$, and the minimum pressure (at the corner shown at distance a_2 from axis N-N) is given by:

$$f_{\text{min}} = \frac{F_{\text{tot}}}{A_{\text{raft}}} - \frac{F_{\text{tot}}\, e_{\text{w}}\, a_2}{I_{\text{raft}}}$$

When the three pressures have been determined, the pressure at any other point, or the mean pressure over any area, can be assessed. Having arranged for a rational system of beams or ribs to divide the slab into suitable panels, as suggested by the broken lines at (f), the panels of slab and the beams can be designed for the bending moments and shear forces due to the net upward pressures to which they are subjected.

18.2 OPEN-PILED STRUCTURES

Expressions from which the loads on groups of inclined and vertical piles, supporting jetties and similar structures, can be obtained are given in *Table 2.85*. For each probable condition of load, the forces acting on the superstructure are resolved into horizontal and vertical components, F_h and F_v, the points of application of which are also determined. If the direction of action and position of the forces are opposite to those in the diagrams, the signs in the formulae must be changed.



Example 1. Cross section through piled jetty

Example 1: vertical piles only. The adjacent figure shows a cross section through a jetty, where the loads apply to one row of piles (i.e. $n = 1$ for each line and $\Sigma n = 4$). Since the group is symmetrical, $\bar{x} = 0.5 \times 9.0 = 4.5$ m.

$$M = F_v\, e - F_h\, h = 800 \times 0.75 - 100 \times 4.8 = 120 \text{ kNm}$$

The calculations for the loads on the piles are shown in the following table, from which the maximum load on any pile is 212 kN. For each pile, the shear force is 100/4 = 25 kN, and the bending moment is 25 × 4.8/2 = 60 kNm.

Pile no.	x (m)	x^2 (m^2)	k_w $\left(\frac{1}{\sum n}\right)$	k_m $\left(\frac{x}{nI}\right)$	Axial load ($k_w F_v + k_m M$)
N_1	−4.5	20.25	+0.25	$-\frac{4.5}{45} = -0.100$	(0.25 X 800) − (0.100 X 120) = 188kN
N_2	−1.5	2.25	+0.25	$-\frac{1.5}{45} = -0.033$	200 − (0.033 X 120) = 196kN
N_3	+1.5	2.25	+0.25	+0.033	200 + 4 = 204kN
N_4	+4.5	20.25	+0.25	+0.100	200 + 12 = 212kN
$I = \Sigma nx^2 = 45.00$ m^4					

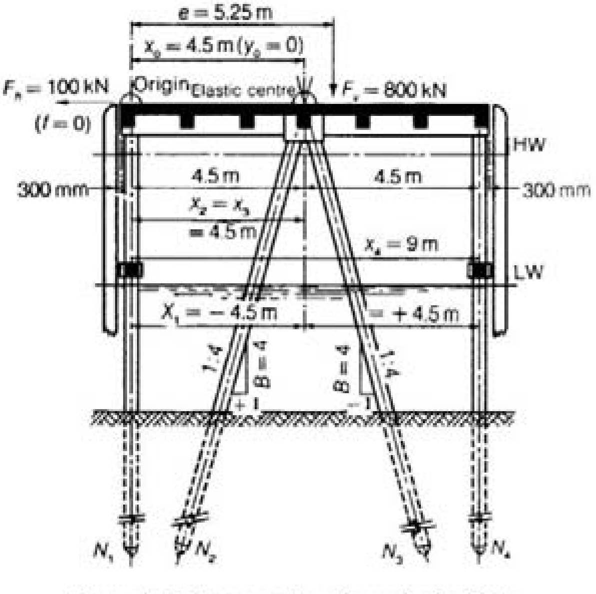
Example 2: vertical and inclined piles. The adjacent figure shows a cross section through a jetty, where all the piles are driven to the same depth. Since A is the same for all the piles, if $J = A/l$ is taken as unity for piles N_1 and N_4 then, for piles N_2 and N_3, $J = 4/\sqrt{(1 + 4^2)} = 0.97$. Since the group is symmetrical: $\Sigma_2 = 0$, Σ_3 not needed, $\Sigma_4 = 0$, $k = \Sigma_1 \Sigma_5$, $x_o = 9/2 = 4.5$ m, and $y_o = 0$.

$$M = F_v\, e - F_h\, h = 800(5.25 - 4.5) + 0 = 600 \text{ kNm}$$

The calculation is then as shown in the following table, from which $k = 3.826 \times 0.114 = 0.436$. The axial loads on the piles are given by $N_x = k_p\,(k_w\, F_v + k_h\, F_h + k_m\, M)$, hence

$$N_1 = 1.0 \times (0.261 \times 800 + 0 - 0.111 \times 600) = 142.1 \text{ kN}$$
$$N_2 = 0.941 \times (0.261 \times 800 + 2.19 \times 100 + 0) = 402.6 \text{ kN}$$
$$N_3 = 0.941 \times (0.261 \times 800 - 2.19 \times 100 + 0) = -9.6 \text{ kN}$$
$$N_4 = 1.0 \times (0.261 \times 800 + 0 + 0.111 \times 600) = 275.5 \text{ kN}$$

Thus the maximum load on any pile is 402.6 kN. Note that the weight of the pile has to be added to the above values.

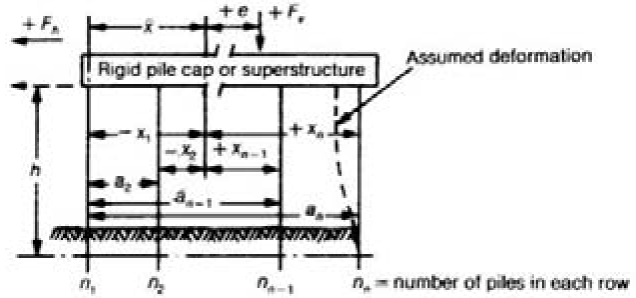


Example 2. Cross section through piled jetty

Foundations: loads on open-piled structures

2.85

Vertical piles only



$\bar{x} = \Sigma an/\Sigma n$ (for symmetrical group $\bar{x} = a_n/2$)

$I = \Sigma nx^2$

Axial load on any pile:

$N_x = k_w F_v + k_m M$ plus weight of pile

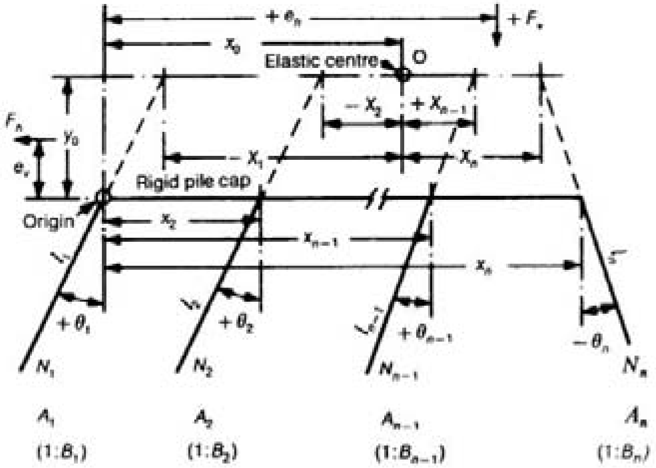
$M = F_v e - F_h h$

$k_w = 1/\Sigma n; \quad k_m = x/I$

Shearing force on any pile $= V = F_h/\Sigma n$

Bending moment on any pile $= Vh/2$

Groups containing inclined piles with or without vertical piles



N_n pile reference and load

A_n cross-sectional area

(1:B_n) slope of pile = tan θ

E is assumed to be constant

Axial load on any pile:

$N_x = k_p(k_w F_v + k_h F_h + k_m M)$

$M = F_v(e_h - x_0) + F_h(y_0 - e_v)$

Co-ordinates of elastic centre:

$x_0 = (\Sigma_3\Sigma_5 - \Sigma_2\Sigma_4)/k$

$y_0 = (\Sigma_2\Sigma_3 - \Sigma_1\Sigma_4)/k$

$k = \Sigma_1\Sigma_5 - \Sigma_2^2$

$J = A/l$

Summations	Piles inclined towards right	Vertical piles	Piles inclined towards left
$\Sigma_1 = \Sigma J \cos^2\theta$	$+\Sigma_R \frac{A}{l}\frac{B^2}{1+B^2}$	$+\Sigma_V \frac{A}{l}$	$+\Sigma_L \frac{A}{l}\frac{B^2}{1+B^2}$
$\Sigma_2 = \Sigma J \cos\theta \sin\theta$	$+\Sigma_R \frac{A}{l}\frac{B}{1+B^2}$	Nil	$-\Sigma_L \frac{A}{l}\frac{B}{1+B^2}$
$\Sigma_3 = \Sigma xJ \cos^2\theta$	$+\Sigma_R \frac{A}{l}\frac{B^2}{1+B^2}x$	$+\Sigma_V \frac{A}{l}x$	$+\Sigma_L \frac{A}{l}\frac{B^2}{1+B^2}x$
$\Sigma_4 = \Sigma xJ \cos\theta \sin\theta$	$+\Sigma_R \frac{A}{l}\frac{B}{1+B^2}x$	Nil	$-\Sigma_L \frac{A}{l}\frac{B}{1+B^2}x$
$\Sigma_5 = \Sigma J \sin^2\theta$	$+\Sigma_R \frac{A}{l}\frac{1}{1+B^2}$	Nil	$+\Sigma_L \frac{A}{l}\frac{1}{1+B^2}$
$\Sigma_6 = \Sigma X^2 J \cos^2\theta$	$+\Sigma_R \frac{A}{l}\frac{B^2}{1+B^2}X^2$	$+\Sigma_V \frac{A}{l}X^2$	$+\Sigma_L \frac{A}{l}\frac{B^2}{1+B^2}X^2$
$X = x - x_0 + y_0 \tan\theta$	$x - x_0 + \frac{y_0}{B}$	$x - x_0$	$x - x_0 - \frac{y_0}{B}$
Coefficients in formula for N_x			
$k_p = J_x \cos\theta_x = \frac{A_x}{l_x}\frac{B}{\sqrt{(1+B^2)}}$	$+\frac{A}{l}\frac{B}{\sqrt{(1+B^2)}}$	$+\frac{A}{l}$	$+\frac{A}{l}\frac{B}{\sqrt{(1+B^2)}}$
$k_w = (\Sigma_5 - \tan\theta\,\Sigma_2)/k$	$+\left(\Sigma_5 - \frac{\Sigma_2}{B}\right)\Big/k$	$+\frac{\Sigma_5}{k}$	$+\left(\Sigma_5 + \frac{\Sigma_2}{B}\right)\Big/k$
$k_h = (\tan\theta\,\Sigma_1 - \Sigma_2)/k$	$+\left(\frac{\Sigma_1}{B} - \Sigma_2\right)\Big/k$	$-\frac{\Sigma_2}{k}$	$-\left(\frac{\Sigma_1}{B} + \Sigma_2\right)\Big/k$
$k_m = X/I = X/\Sigma_6$	$+\frac{X}{\Sigma_6}$	$+\frac{X}{\Sigma_6}$	$+\frac{X}{\Sigma_6}$

Note on symmetrical groups $\Sigma_2 = 0$; $k = \Sigma_1\Sigma_5$; $x_0 = \frac{1}{2}x_n$; Σ_3 is not required

Pile no.	$\sum_1$	$\sum_5$	x (m)	X (m)	$\sum_6 = \mathrm{I}$	k_p	k_w	k_h	k_m
N_1	$+1$	0	0	-4.5	$+20.25$	$+1$	$+\frac{0.114}{0.436}$ $= +0.261$	0	$-\frac{4.5}{40.5}$ $= -0.111$
N_2	$+\frac{0.97 \times 4^2}{1+4^2}$ $= +0.913$	$+\frac{0.97}{1+4^2}$ $= +0.057$	$+4.5$	0	0	$+\frac{0.97 \times 4}{\sqrt{(1+4^2)}}$ $= +0.941$	$+0.261$	$+\frac{3.826}{4 \times 0.436}$ $= +2.19$	0
N_3	$+0.913$	$= +0.057$	$+4.5$	0	0	$= +0.941$	$+0.261$	$= -2.19$	0
N_4	$+1$	0	$+9$	$+4.5$	$+20.25$	$+1$	$+0.261$	0	$+0.111$
Totals	$+3.826$	$+0.114$	—	—	$+40.5$	—	—	—	—

Example 3: inclined piles only. The adjacent figure shows a cross section through a jetty, where $\cot \theta = 5$. Since the values of A and l are the same for each pile, unity may be substituted for $J = A/l$. For piles N_1 and N_3, $B = +5$ and, for piles N_2 and N_4, $B = -5$. Since the group is symmetrical: $\Sigma_2 = 0$, Σ_3 not needed, $\Sigma_4 = 0$, $k = \Sigma_1 \Sigma_5$, $x_o = 3$ m, and $y_o = 0$.

$$M = F_v e - F_h h = 800(3.75 - 3.0) + 0 = 600 \text{ kNm}$$

The calculation is then as shown in the following table, from which $k = 3.826 \times 0.1538 = 0.5917$. The axial loads on the piles are given by $N_x = k_p (k_w F_v + k_h F_h + k_m M)$, hence

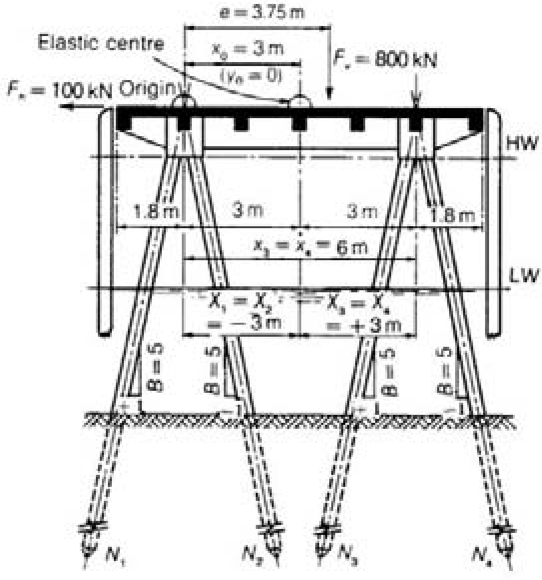
$$N_1 = 0.9806 \times (0.26 \times 800 + 1.3 \times 100 - 0.0867 \times 600)$$
$$= 0.9806 \times (208 + 130 - 52) = 280.4 \text{ kN}$$

$$N_2 = 0.9806 \times (208 - 130 - 52) = 25.5 \text{ kN}$$

$$N_3 = 0.9806 \times (208 + 130 + 52) = 382.4 \text{ kN}$$

$$N_4 = 0.9806 \times (208 - 130 + 52) = 127.5 \text{ kN}$$

Thus the maximum load on any pile is 382.4 kN, to which the weight of the pile has to be added.



Example 3. Cross section through piled jetty

Pile no.	$\sum_1$	$\sum_5$	x (m)	X (m)	$\sum_6 = \mathrm{I}$	k_p	k_w	k_h	X/I
N_1	$\frac{5^2}{1+5^2}$ $= +0.9615$	$\frac{1}{1+5^2}$ $= +0.0385$	0	-3.0	$0.9615(-3)^2$ $= +8.65$	$\frac{5}{\sqrt{(1+5^2)}}$ $= +0.9806$	$\frac{0.1538}{0.5917}$ $= +0.26$	$+\frac{3.846}{5 \times 0.5917}$ $= +1.3$	$-\frac{3}{34.62}$ $= -0.0867$
N_2	$+0.9615$	$+0.0385$	0	-3.0	$+8.65$	$+0.9806$	$+0.26$	$-\frac{3.846}{5 \times 0.5917}$ $= -1.3$	-0.0867
N_3	$+0.9615$	$+0.0385$	$+6.0$	$+3.0$	$+8.65$	$+0.9806$	$+0.26$	$+1.3$	$+\frac{3}{34.62}$ $= +0.0867$
N_4	$+0.9615$	$+0.0385$	$+6.0$	$+3.0$	$+8.65$	$+0.9806$	$+0.26$	-1.3	$+0.0867$
Total	$+3.846$	$+0.1538$	—	—	$+34.62$	—	—	—	—

Notes on design examples (1)–(3). In each case, the pile group is symmetrical and is subjected to the same imposed loads. Examples (2) and (3) are special cases of symmetrical groups for which $\Sigma_4 = 0$ and therefore $y_o = 0$; this condition applies only when the inclined piles are in symmetrical pairs, with both pairs meeting at the same pile-cap.

Example (3) requires the smallest pile, but the difference in the maximum load is small between (2) and (3). Although the maximum load on a pile is least in (1), the bending moment requires a pile of greater cross-sectional area to provide the necessary resistance to combined bending and thrust. If the horizontal force is increased, the superiority of (3) is greater. If $F_h = 200$ kN (instead of 100 kN), the maximum pile loads are: 236 kN (and a large bending moment of 120 kNm) in (1), 609 kN in (2), and 510 kN in (3). Arrangement (2) is the most suitable when F_h is small. If $F_h = 10$ kN, the maximum pile loads are: 255 kN (and a bending moment of 6 kNm) in (1), 217 kN in (2), and 268 kN in (3). Arrangement (1) is the most suitable when there is no horizontal load.

18.3 RETAINING WALLS

Various types of earth retention systems, for which notes are given in section 7.3.1, are shown in *Table 2.86*. Information on the pressures exerted by soils on retaining structures is given in section 9.1 and *Tables 2.10* to *2.14*.

18.3.1 Walls on spread bases

Several walls on spread bases, for which notes are given in section 7.3.2, are shown in *Table 2.86*. Suitable dimensions for a base to a cantilever wall can be estimated with the aid of the chart given in *Table 2.86*, which is a modified version of a chart published in ref. 63. The modified chart is applicable to design methods in which the soil parameters incorporate either mobilisation factors as in BS 8002, or partial factors of safety as in the Eurocodes.

Stability against overturning is assured over the entire range of the chart, and the maximum bearing pressure under service conditions can be investigated for all types of soil. A uniform surcharge that is small compared to the total forces acting on the wall can be simply represented by an equivalent height of soil: l can be replaced by $l_e = l + q/\gamma$, where q is the surcharge pressure. In more general cases, $l_e = 3M_h/F_h$ and $\gamma = 2F_h/K_A l_e^2$ can be used, where F_h is the total horizontal force and M_h is the bending moment about underside of base due to F_h.

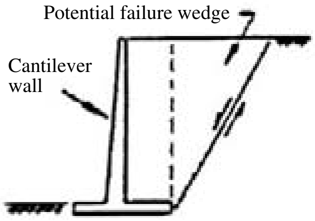
The chart contains two curves showing where the bearing pressure is uniform, and triangular, respectively. A uniform bearing condition is important when it is necessary to avoid tilting in order to minimise deflection at the top of the wall. It is generally advisable to maintain ground contact over the full area of the base, especially for clays where the occurrence of ground water beneath the heel could soften the formation.

Further information on the use of the chart, for walls that are designed in accordance with BS 8002 and BS 8110, is given in section 28.4.

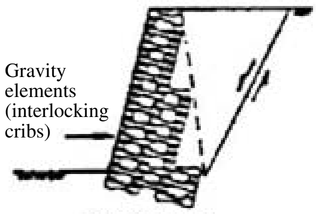
18.4 BOX CULVERTS

Formulae for the bending moments in the corners of a box culvert due to symmetrical load cases are given in *Table 2.87*. Some notes on the different load cases and assumed ground conditions are given in section 7.4.2. Design requirements of Highways Agency BD 31/87 are summarised in section 7.4.3.

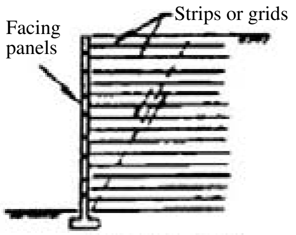
Earth retention systems



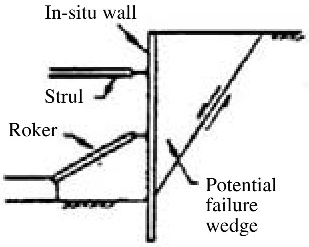
Cantilever wall on spread base



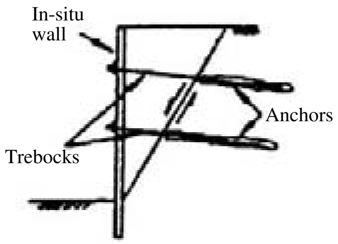
Gravity wall



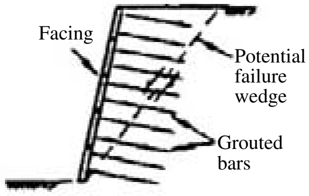
Reinforced soil



Embedded wall (propped)



Embedded wall (anchored)



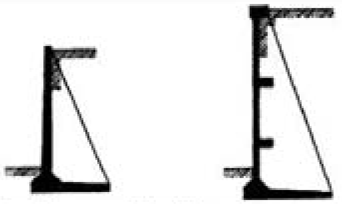
Soil nailing

EXTERNALLY STABILISED

INTERNALLY STABILISED

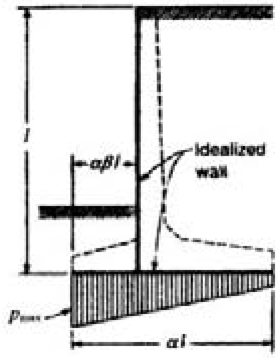
Cantilever wall on spread base

Simple cantilever walls



Cantilever walls with counterforts

Values of the controlling proportions α and β, for the idealised wall of zero thickness shown below and for given values of $\xi = p_{max}/\gamma l$ and $\psi = \tan \delta_b / \sqrt{K_A}$, where γ is unit weight of soil, δ_b is angle of base friction, and K_A is fully active earth pressure coefficient, can be obtained from the adjoining chart (see also 18.3.1).



(a) $p_{max} = \dfrac{4F_v}{3(\alpha l - 2e)}$ for $e \geq \dfrac{\alpha l}{6}$

(b) $p_{max} = \dfrac{F_v}{\alpha l}(1 + \dfrac{6e}{\alpha l})$ for $e \leq \dfrac{\alpha l}{6}$

$e = M_b/F_v + \alpha l/2 - x \leq \alpha l/6$

F_v is resultant of all vertical loads including weight of wall, and weight of earth and surcharge on base
M_b is bending moment about underside of base due to all horizontal forces acting on full height of wall
x is distance from toe of base to line of action of F_v

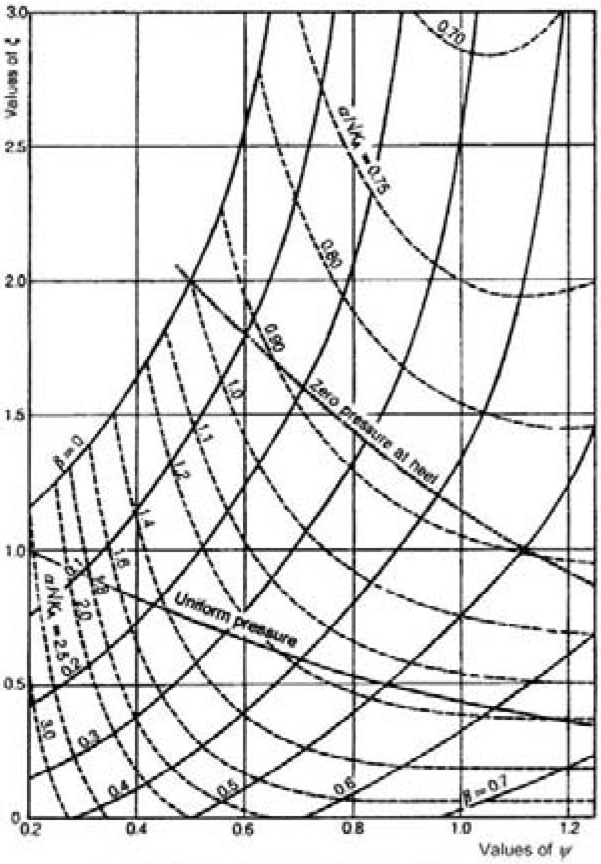


Chart for cantilever wall on spread base

Rectangular culverts

2.87

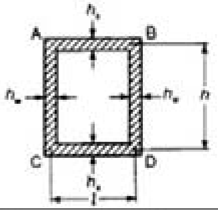
Bending moments (per unit length of culvert)

$M_A = M_B$ $\quad$ $M_C = M_D$

Pressures and uniform loads are per unit area of walls or slab.

Loads F and G are total loads per unit length of culvert. h and l are measured between centres of walls or slabs.

q_1 = pressure transferred to soil.



$k = \frac{h}{l}\left(\frac{h_s}{h_w}\right)^3$

$K_1 = k + 1$

$K_2 = k + 2$

$K_3 = k + 3$

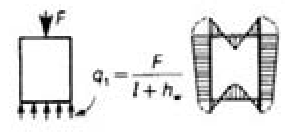
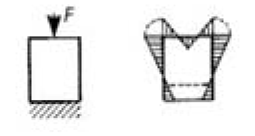
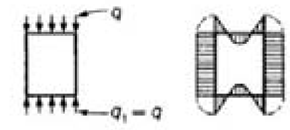
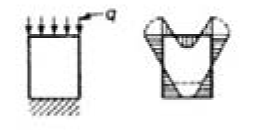
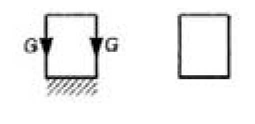
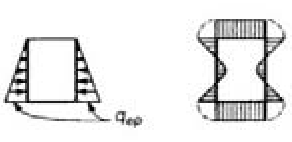
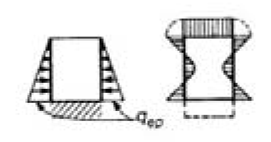
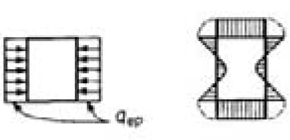
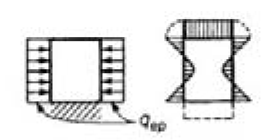
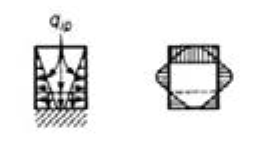
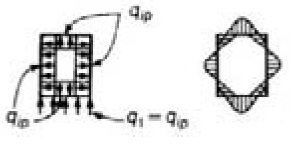
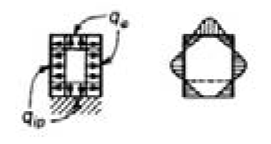
$K_4 = 4k + 9$

$K_5 = 2k + 3$

$K_6 = k + 6$

$K_7 = 2k + 7$

$K_8 = 3k + 8$

Loading	Condition of supporting ground (limiting cases)			
	Highly compressible		Non-compressible	
Concentrated load on roof		$M_A = -\frac{FlK_4}{24K_1K_3}$ $M_C = \frac{K_6}{K_4}M_A$		$M_A = -\frac{Fl}{4K_2}$ $M_C = -\frac{M_A}{2}$
Uniform load on roof		$\left.\begin{matrix}M_A\\M_C\end{matrix}\right\} = -\frac{ql^2}{12K_1}$		$M_A = -\frac{ql^2}{6K_2}$ $M_C = -\frac{M_A}{2}$
Weight of walls		$M_A = +\frac{q_1l^2k}{12K_1K_3}$ $M_C = -\frac{K_5}{k}M_A$		$M_A = M_C = 0$
Earth pressure on walls		$M_A = -\frac{q_{ep}h^2kK_7}{60K_1K_3}$ $M_C = \frac{K_8}{K_7}M_A$		$M_A = -\frac{q_{ep}h^2k}{30K_2}$ $M_C = \frac{K_8}{2k}M_A$
Earth (surcharge) pressure on walls		$\left.\begin{matrix}M_A\\M_C\end{matrix}\right\} = -\frac{q_{ep}h^2k}{12K_1}$		$M_A = -\frac{q_{ep}h^2k}{12K_2}$ $M_C = \frac{K_3}{k}M_A$
Hydrostatic (internal) pressure		$M_A = +\frac{q_{ip}h^2kK_7}{60K_1K_3}$ $M_c = \frac{K_8}{K_7}M_A$		$M_A = +\frac{q_{ip}h^2k}{30K_2}$ $M_C = \frac{K_8}{2k}M_A$
Excess hydrostatic (internal) pressure		$M_A = +\frac{q_{ip}(h^2kK_3 + l^2K_5)}{12K_1K_3}$ $M_C = +\frac{q_{ip}k(h^2K_3 - l^2)}{12K_1K_3}$		$M_A = +\frac{q_{ip}(h^2k + 2l^2)}{12K_2}$ $M_C = +\frac{q_{ip}(h^2K_3 - l^2)}{12K_2}$

Chapter 19

Miscellaneous structures and details

19.1 STAIRS

Some general notes on stairs are given in section 6.1.4. For details of characteristic imposed loads on stairs and landings, and horizontal loads on balustrades, refer to BS 6399: Part 1. Stairs forming monolithic structures are generally designed for the uniformly distributed imposed load only. Stairs that are formed of separate treads (usually precast), cantilevering from a wall or central spine beam, must be designed also for the alternative concentrated load. Some general information on stair types and dimensions is given in *Table 2.88*, and comprehensive data is given in BS 5395.

19.1.1 Simple stairs

The term 'simple' is used here for a staircase that spans in the direction of the stair flight between beams, walls or landings located at the top and bottom of the flight. The staircase may include a section of landing spanning in the same direction and continuous with the stair flight. For such staircases, the following statements are contained in BS 8110.

When staircases surrounding open wells include two spans that intersect at right angles, the load on the area common to both spans may be divided equally between the spans. When the staircase is built at least 110 mm into a wall along part of all of its length, a 150 mm wide strip adjacent to the wall may be deducted from the loaded area. When the staircase is built monolithically at its ends into structural members spanning at right angles to its span, the effective span of the stairs should be taken as the horizontal distance between the centrelines of the supporting members, where the width of each member is taken not more than 1.8 m. For a simply supported staircase, the effective span should be taken as the horizontal distance between the centrelines of the supports, or the clear distance between the faces of supports plus the effective depth of the stair waist, whichever is the lesser. If a stair flight occupies at least 60% of the effective span, the permissible span/effective depth ratio calculated for a slab may be increased by 15%.

19.1.2 Free-standing stairs

In ref. 64, Cusens and Kuang employ strain–energy principles to determine expressions relating the horizontal restraint force H and moment M_o at the mid-point of a free-standing stair. By solving the two resulting equations simultaneously, the values of H and M_o obtained can then be substituted into general expressions, to determine the forces and moments at any point along the structure.

It is possible, by neglecting subsidiary terms, to simplify the basic equations produced by Cusens and Kuang. If this is done, the expressions given in *Table 2.88* are obtained, and these yield H and M_o directly. Comparisons of the solutions obtained by these simplified expressions, with those obtained using the expressions presented by Cusens and Kuang, show that the resulting variations are negligible for values in the range encountered in concrete design.

The expressions given in *Table 2.88* are based on a value of $G/E = 0.4$, with C taken as half of the St. Venant value for plain concrete. As assumed by Cusens and Kuang, only half of the actual width of the landing is considered to determine its second moment of area.

Example 1. Design a free-standing staircase, assumed to be fully fixed at the ends, to support total ultimate loads per unit length $n_f = 16.9$ kN/m and $n_l = 15.0$ kN/m. The dimensions are as follows: $a = 2.7$ m, $b = 1.4$ m, $b_1 = 1.8$ m, $h_f = 100$ mm, $h_l = 175$ mm and $\phi = 30^o$.

From the expressions given in *Table 2.88*, $H = 81.86$ kN/m and $M_o = 35.87$ kNm/m. If Cusens and Kuang's more exact expressions are used to analyse the structure, $H = 81.89$ kN/m and $M_o = 35.67$ kNm/m. Thus, the errors involved by using the approximate expressions are negligible for H, and about 0.5% for M_o. If these values of H and M_o are substituted into the other expressions in *Table 2.88*, corresponding values of M_v, M_h and T at any point in the structure can be found, for various load combinations, as shown in the table in page no. 208.

19.1.3 Sawtooth stairs

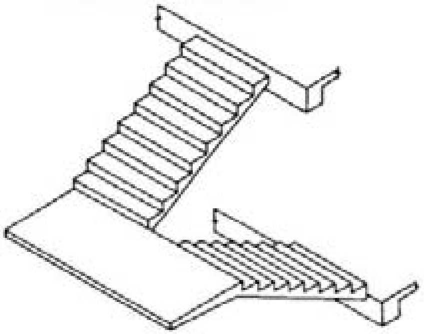
In ref. 65, Cusens shows that, if axial shortening is neglected and the strain–energy due to bending only is considered, the mid-span moments for a so-called sawtooth stairs are given by the general expression:

$$M_s = \frac{nl^2(k_{11} + kk_{12})}{j^2(k_{13} + kk_{14})}$$

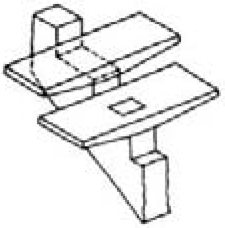
where k = (stiffness of tread)/(stiffness of riser) and j is the number of treads.

Stairs: general information

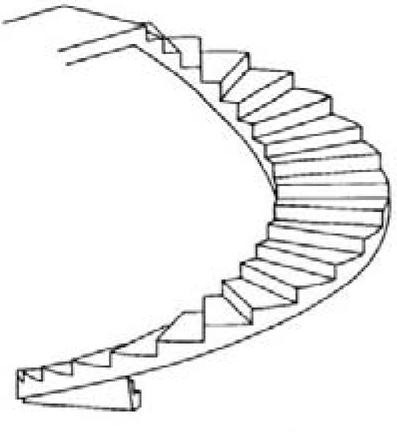
Common stair types



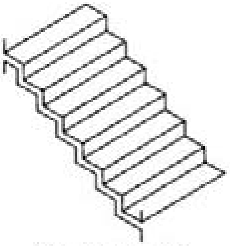
Free-standing (or scissor) stairs (unsupported landing)



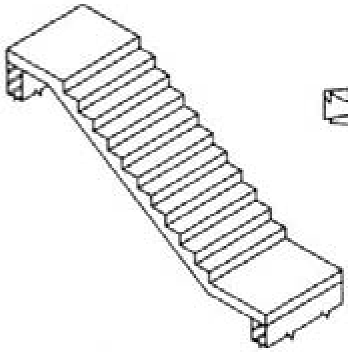
Precast cantilever treads supported by spine beam



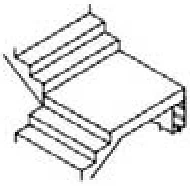
Helical stairs



Sawtooth stairs



Simple stairs



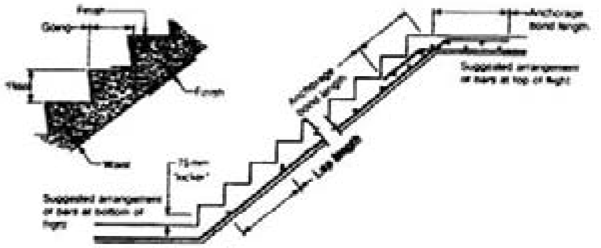
Landing for simple stairs

Dimensions

Optimum dimensions for stairs (BS 5395)

Usage	Rise (mm)	Going (mm)	Minimum width (mm)
Public	150	300	1000
Semi-public	165	275	1000
Private	175	250	800

Note. Optimum combination of rise and going are given by 2 x rise + going = 600 mm



Typical reinforcement arrangement for simple stair

Free-standing stairs

If flights are freely supported at A and A':

$$H=\left[n_l(b_1+b)\left(1+\frac{1}{2}\frac{b}{a}\sec\phi\right)+n_f a\cos\phi\right]\Big/2\tan\phi$$

If flights are fully fixed at A and A':

$$H=\left[n_l(b_1+b)\left(4+3\frac{b}{a}\sec\phi\right)+3n_f a\cos\phi\right]\Big/$$

$$\left\{2\tan\phi\,4+3(b_1/a)^2\Big/\left[\frac{0.72}{1+(h_f/b)^2}+\frac{1}{K}\right]\right\}$$

$$M_0=[Hb_1\tan\phi-\tfrac{1}{4}n_l(b_1^2-b^2)]\Big/\left[\frac{1.44K}{1+(h_f/b)^2}+2\right]$$

where $K=\left(\frac{h_f}{h_l}\right)^3\left(\frac{b_1}{a}\right)\sec^2\phi$

Then for OB, at any point distance y from O:

$$M_v=-M_0-\tfrac{1}{2}n_l y^2 \qquad M_h=-Hy \qquad T=-\tfrac{1}{2}n_l by$$

For BC, at any point distance y from O:

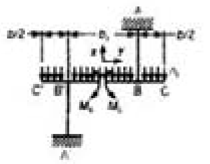
$$M_v=-\tfrac{1}{2}n_l[\tfrac{1}{2}(b_1+b)-y]^2 \qquad M_h=0$$
$$T=-\tfrac{1}{2}n_l b[\tfrac{1}{2}(b_1+b)-y]$$

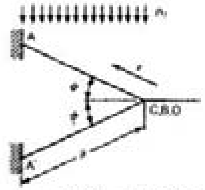
For AB, at any point distance x from B:

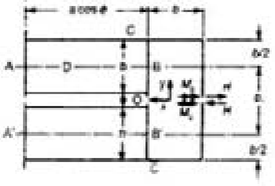
$$M_v=Hx\sin\phi-\tfrac{1}{2}n_l(b_1+b)(x\cos\phi+\tfrac{1}{4}b)-\tfrac{1}{2}n_f x^2\cos^2\phi$$
$$M_h=-\tfrac{1}{2}Hb_1\cos\phi-[M_0+\tfrac{1}{8}n_l(b_1^2-b^2)]\sin\phi$$
$$T=-\tfrac{1}{2}Hb_1\sin\phi+[M_0+\tfrac{1}{8}n_l(b_1^2-b^2)]\cos\phi$$

Additional notation

a	length of flight
b	width of flight and landing
b_1	distance between centre-lines of flights
H, M_0	horizontal restraint force and restraint moment at cut, respectively
h_f, h_l	slab depth of flight and of landing, respectively
M_h, M_v, T	horizontal and vertical bending moments and torsional moment at any point, respectively
n_f, n_l	ultimate load per unit length on flight and on landing, respectively
x, y	distances measured along flight and along Y-axis respectively
ϕ	slope of flight measured from horizontal







Application of imposed load	Values of M_v (kNm/m)				Values of M_h (kNm/m)		Values of T (kNm/m)	
	At O	At B	At D	At A	At B in OB	Throughout AB	At B in BC	Throughout AB
Throughout	−35.87	−16.80	−1.16	−8.61	−73.67	−82.94	−7.35	−3.69
Flights only	−24.81	−9.22	+1.60	−10.67	−50.36	−56.68	−4.03	−2.55
Landing only	−31.32	−16.80	−3.14	−3.33	−64.86	−73.03	−7.35	−3.22

Note. At point B, the expressions give theoretical values for M_v (with imposed load applied throughout) that reduce abruptly from – 41.95 kNm/m in OB to –3.68 kNm/m in BC, due to the intersection with flight AB. Since the members in the actual structure are of finite width, Cusens and Kuang recommend redistributing these moments across the intersection between the flight and the landing to give a value of (–41.95 – 3.68)/2 = − 22.82 kNm/m.

If j is odd:

$$k_{11} = j^2/16 + j(j-1)(j-2)/48$$
$$k_{12} = (j-1)^2/16 + (j-1)(j-2)(j-3)/48$$
$$k_{13} = j/2,\ k_{14} = (j-1)/2$$

If j is even:

$$k_{11} = j(j-1)(j-2)/48$$
$$k_{12} = (j-1)(j-2)(j-3)/48$$
$$k_{13} = (j-1)/2,\ k_{14} = (j-2)/2$$

The chart on *Table 2.89* gives coefficients to determine the support moments for various values of j and k. Having found the support moment, the maximum mid-span bending moment can be determined by using the appropriate expression on the table and deducting the support moment.

Typical bending moment and shearing force diagrams for a stair are also shown on *Table 2.89*, together with suggested arrangements of reinforcement. Because of the stair profile, stress concentrations occur in the re-entrant corners, and the real stresses to be resisted will be larger than those obtained from the moments. To resist these increased stresses, Cusens recommends providing twice the reinforcement theoretically required, unless suitable fillets or haunches are incorporated at the junctions, in which case the reinforcement need only be about 10% more than is theoretically necessary. The method of reinforcing shown in diagram (a) is very suitable, but is generally only practical if haunches are provided. Otherwise, the arrangement shown in diagram (b) should be adopted. A further possibility is to arrange the bars as shown in diagram (a) on *Table 3.63* for wall-to-wall corners.

Example 2. A sawtooth stairs has seven treads, each 300 mm wide with risers 180 mm high, the thickness of both being 100 mm. The stairs, which are 1.0 m wide, are to be designed to support a characteristic imposed load of 3.0 kN/m^2 to the requirements of BS 8110.

The self-weight of the treads and risers (assuming no finishes are required) is:

$$g_k = 0.1 \times 24 \times (0.3 + 0.18)/0.3 = 3.84\ \text{kN/m}^2$$

For design to BS 8110, total design ultimate load for a 1.0 m wide stair is given by:

$$n = 1.4\, g_k + 1.6 q_k = 1.4 \times 3.84 + 1.6 \times 3.0 = 10.18\ \text{kN/m}$$

Since $l_t = 300$ mm, $l_r = 180$ mm and $h_t = h_r = 100$ mm,

$$k = h_t^3 l_r / h_r^3 l_t = 180/300 = 0.6$$

From the chart on *Table 2.89*, for 7 treads and k = 0.6, the support moment coefficient is – 0.088. Thus,

$$M_s = -0.088 n l^2 = -0.088 \times 10.18 \times (0.3 \times 7)^2 = -3.95\ \text{kNm}$$

Since $j = 7$ is odd, the free bending moment is given by:

$$M = (nl^2/8)(j^2 + 1)/j^2 = 10.18 \times 2.1^2/8 \times 50/49 = 5.73\ \text{kNm}$$

Hence, the maximum moment at mid-span is

$$M_0 = M - M_s = 5.73 - 3.95 = 1.78\ \text{kNm}$$

19.1.4 Helical stairs

By using strain–energy principles it is possible to formulate, for symmetrically loaded helical stairs fully fixed at the ends, the following two simultaneous equations in M_0 and H:

$$\begin{aligned} &M_0[K_1(k_5 + \tfrac{1}{2}\sin 2\theta) + k_5 k_7] \\ &\quad + HR_2[k_4(k_7 - K_1)\tan\phi + k_5 \sin\phi \cos\phi\ (1 - K_2)] \\ &\quad + nR_1^2[K_1(k_5 + \tfrac{1}{2}\sin 2\theta - \sin\theta) + k_5 k_7 + k_6 k_7 R_2/R_1] = 0 \end{aligned}$$

$$\begin{aligned} &M_0[k_4(k_7 - K_1) + k_5 (k_7 - K_2)] \\ &\quad + HR_2[1/2K_1 \tan\phi\ (\tfrac{1}{3}\theta^3 - \tfrac{1}{2}\theta^2 \sin 2\theta - 2k_4) \\ &\quad + 1/2k_7 \tan\phi\ (\tfrac{1}{3}\theta^3 + \tfrac{1}{2}\theta^2 \sin 2\theta + 2k_4) + 2k_4 \tan\phi\ (k_7 - K_2) \\ &\quad + k_5 \cos^2\phi\ (\tan\phi + K_2 \cot\phi)] + nR_1^2[K_1(k_6 - k_4) + k_4 k_7 \\ &\quad + k_7\ (\theta^2 \sin\theta + 2k_6)R_2/R_1 + (k_7 - K_2)(k_5 + k_6 R_2/R_1)] = 0 \end{aligned}$$

where

$$k_4 = \tfrac{1}{4}\theta\cos 2\theta - \tfrac{1}{8}\sin 2\theta, \quad k_5 = \tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta$$
$$k_6 = \theta\cos\theta - \sin\theta, \quad k_7 = \cos^2\phi + K_2 \sin^2\phi$$
$$K_1 = GC/EI_1, \quad K_2 = GC/EI_2 \quad \text{and} \quad \theta = \beta/2$$

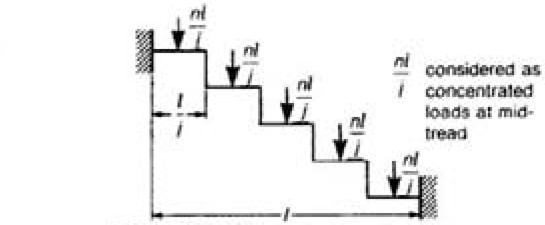
The equations can be solved on a programmable calculator or larger machine to obtain coefficients k_1 and k_2, representing M_0 and H respectively. If the resulting values of M_0 and H are then substituted into the equations given on *Table 2.89*, the bending and torsional moments, shearing forces and thrusts at any point along the stair can be easily calculated. The critical quantity controlling helical stair design is usually the vertical moment M_{vs} at the supports, and a further coefficient can be derived to give this moment directly.

In ref. 66, Santathadaporn and Cusens give 36 design charts for helical stairs, covering ranges of β of 60° to 720°, ϕ of 20° to 50°, b/h of 0.5 to 16 and R_1/R_2 of 1.0 to 1.1, for $G/E = 3/7$. The four design charts provided on *Tables 2.90* and *2.91* have been recalculated for $G/E = 0.4$, with C taken as half of the St. Venant value for plain concrete. These charts cover ranges of β of 60° to 360° and ϕ of 20° to 40°, with values for b/h of 5 and 10 and R_1/R_2 of 1.0 and 1.1, being the ranges met most

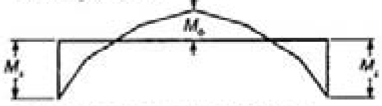
2.89

Stairs: sawtooth and helical stairs

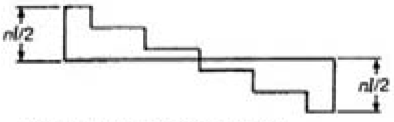
Sawtooth stairs



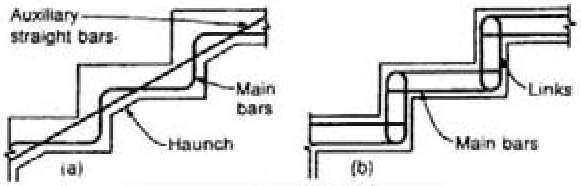
Loading diagram



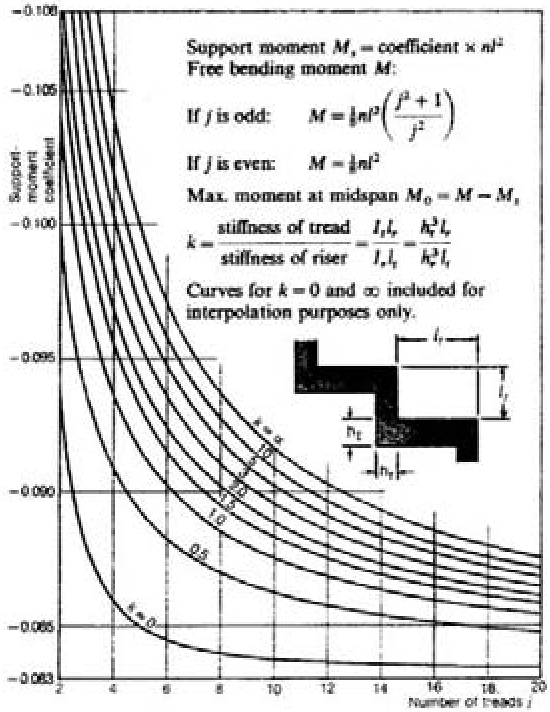
Typical bending-moment diagram



Typical shearing-force diagram



Possible arrangements of reinforcement



Support moment M_s = coefficient × nl^2

Free bending moment M:

If j is odd: $M = \frac{1}{8}nl^2\left(\frac{j^2+1}{j^2}\right)$

If j is even: $M = \frac{1}{8}nl^2$

Max. moment at midspan $M_0 = M - M_s$

$$k = \frac{\text{stiffness of tread}}{\text{stiffness of riser}} = \frac{I_t l_r}{I_r l_t} = \frac{h_t^3 l_r}{h_r^3 l_t}$$

Curves for $k = 0$ and ∞ included for interpolation purposes only.

Helical stairs

At any point along stair

Lateral moment: $M_n = M_0 \sin\theta \sin\phi - HR_2\theta \tan\phi \cos\theta \sin\phi - HR_2 \sin\theta \cos\phi + nR_1 \sin\phi(R_1 \sin\theta - R_2\theta)$

Torsional moment: $T = (M_n \sin\theta - HR_2\theta \cos\theta \tan\phi + nR_1^2 \sin\theta - nR_1R_2\theta)\cos\phi + HR_2 \sin\theta \sin\phi$

Vertical moment: $M_y = M_0 \cos\theta + HR_2\theta \tan\phi \sin\theta - nR_1^2(1 - \cos\theta)$

Thrust: $N = -H \sin\theta \cos\phi - nR_1\theta \sin\phi$

Lateral shearing force across stair: $V_n = nR_1\theta \cos\phi - H \sin\theta \sin\phi$

Radial horizontal shearing force: $V_h = H \cos\theta$

where

Redundant moment acting tangentially at midspan: $M_0 = k_1 nR_2^2$

Horizontal redundant force at midspan: $H = k_2 nR_2$

Vertical moment at supports: $M_v = k_3 nR_2^2$

Additional notation

I_1, I_2 second moment of area of stair section about horizontal axis and axis normal to slope, respectively

n total loading per unit length projected along centre-line of load

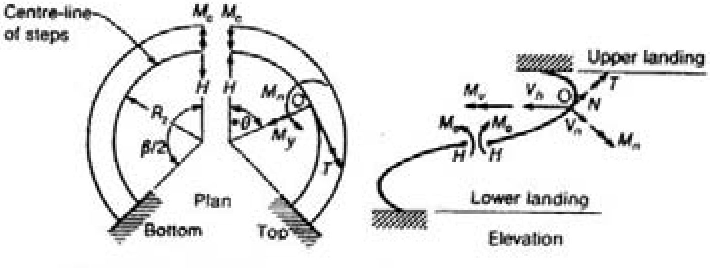
R_1 radius of centre-line of loading = $(2/3)(R_0^3 - R_i^3)/(R_0^2 - R_i^2)$

R_2 radius of centre-line of steps = $(1/2)(R_i + R_0)$, where R_i and R_0 are the internal and external radii of the stair, respectively

θ angle subtended in plan between point considered and midpoint of stair

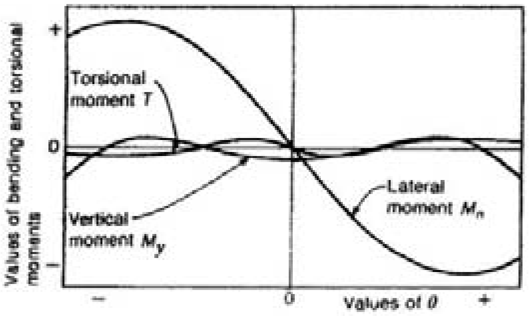
β total angle subtended by helix in plan

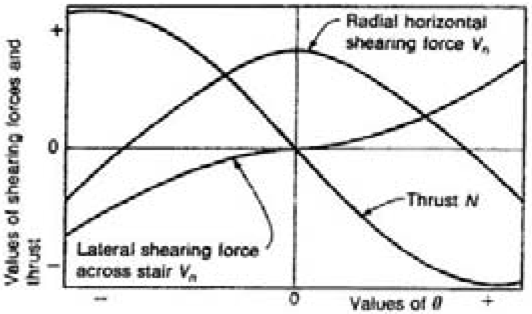
ϕ slope of tangent to helix centre-line measured from horizontal



Plan

Elevation

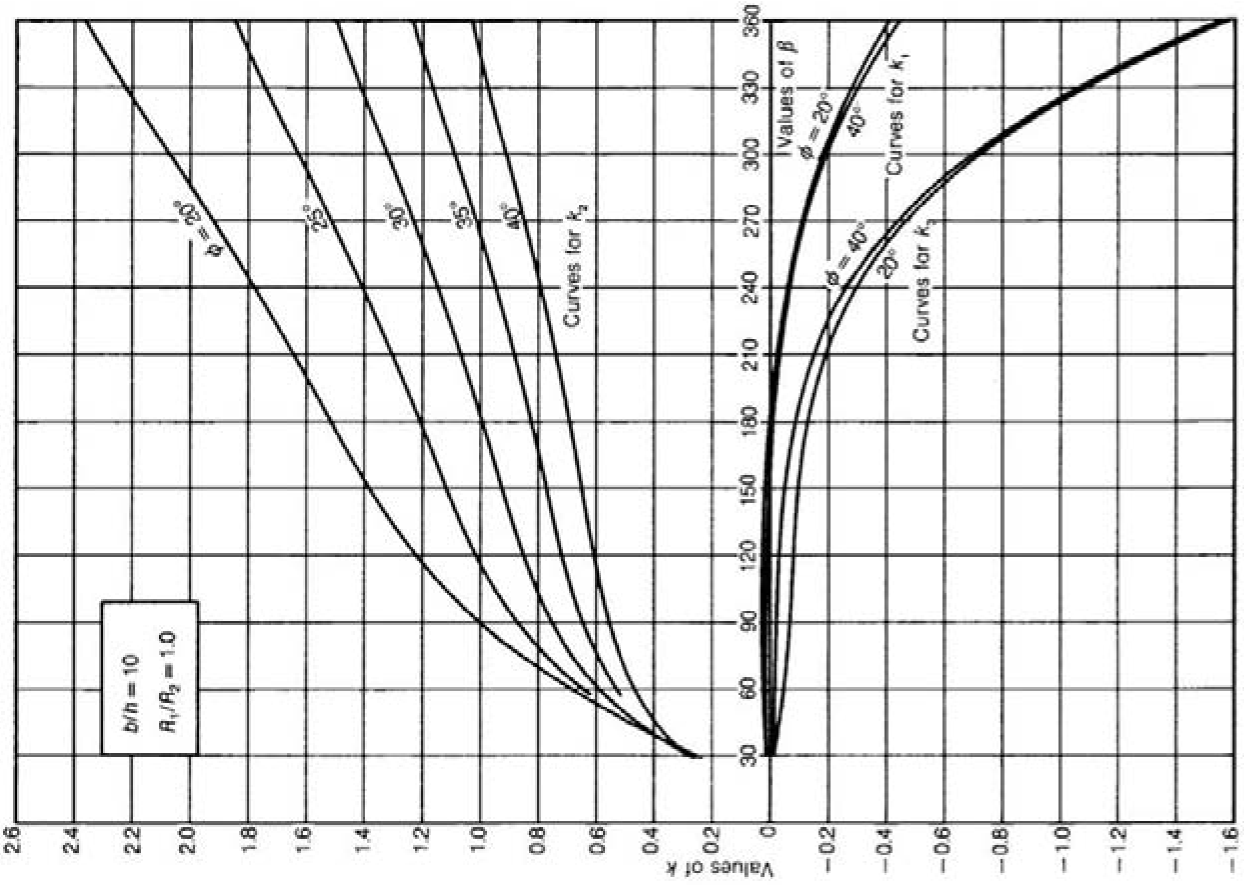


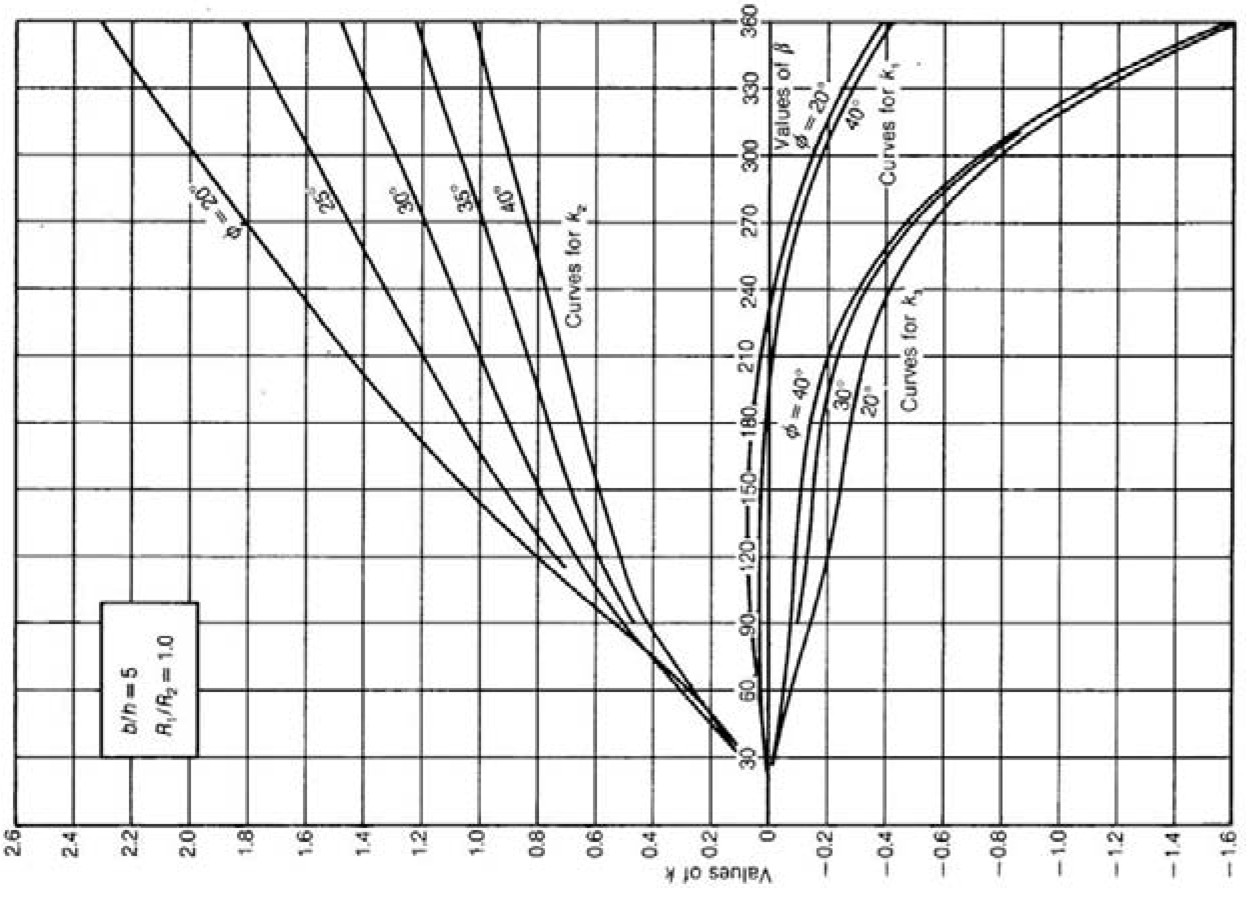


Note. For values of design coefficients k_1, k_2 and k_3, see *Tables 2.90* and *2.91*.

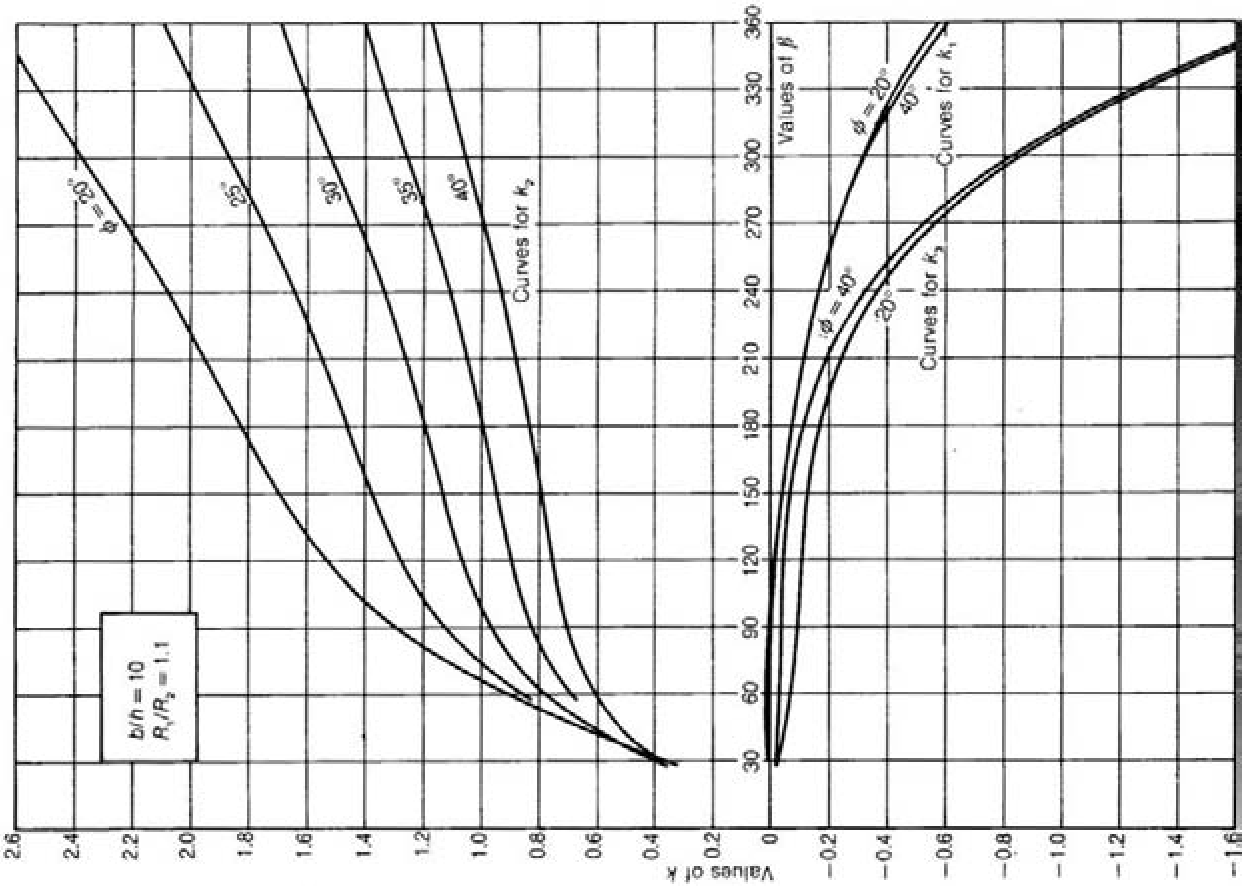
2.90

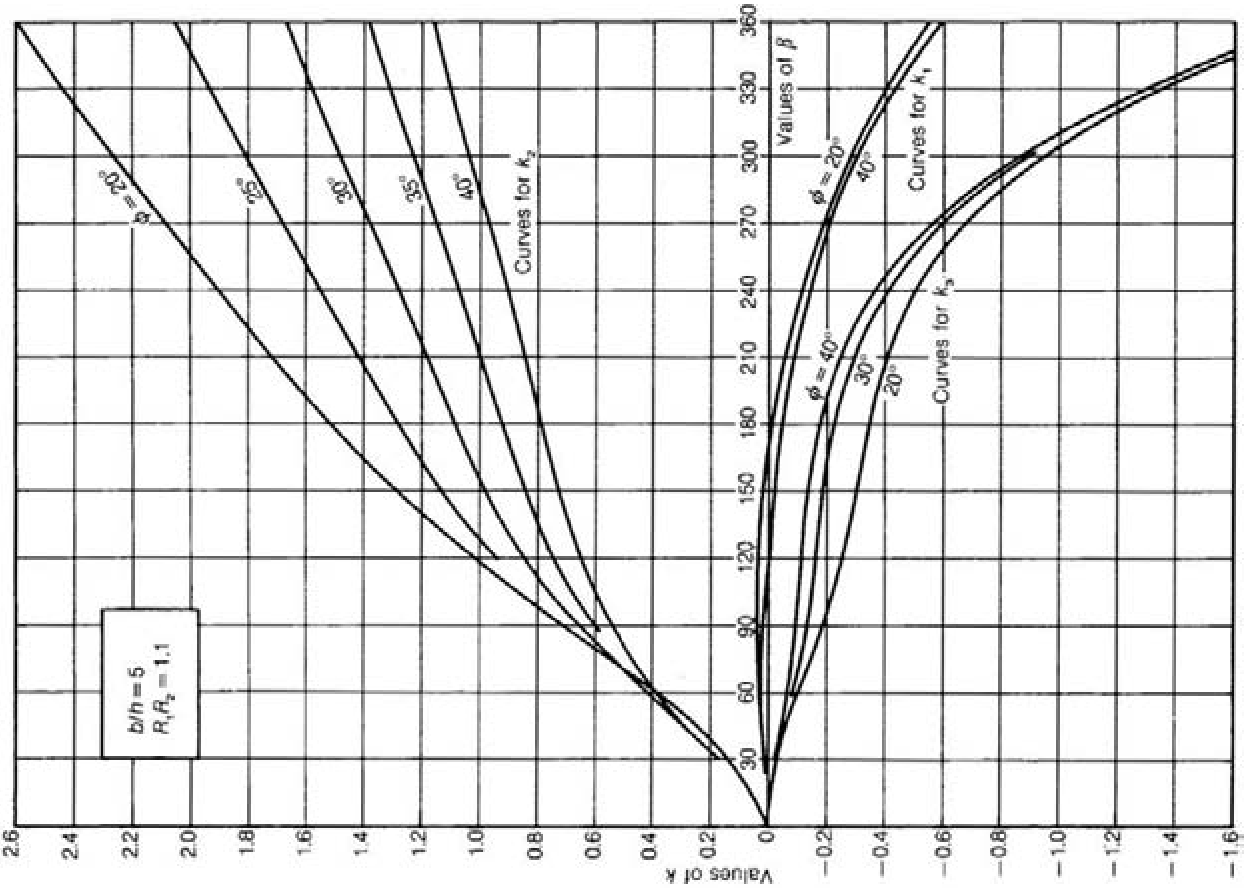
Design coefficients for helical stairs – 1





Design coefficients for helical stairs – 2





frequently in helical stair design. Interpolation between the various curves and charts will be sufficiently accurate for preliminary design purposes.

Example 3. A helical stairs, having an angle of inclination to the horizontal plane of $\phi = 25^\circ$, is to be designed to support a characteristic imposed load of 3.0 kN/m^2 to the requirements of BS 8110. The stairs are to be 1.2 m wide, with a minimum slab thickness of 120 mm. The radius to the inside of the stair is $R_i = 900$ mm, and the angle turned through is $\beta = 240^\circ$.

Assuming the mean thickness on plan of the stairs (including treads and finishes) is 220 mm, the self-weight of the stairs is $0.22 \times 24 = 5.3$ kN/m^2, and the design ultimate load intensity:

$$n = 1.4 \times 5.3 + 1.6 \times 3.0 = 12.22 \text{ kN/m}^2$$

The radius of the centreline of the load is given by:

$$R_1 = \frac{2(R_o^3 - R_i^3)}{3(R_o^2 - R_i^2)} = \frac{2(2.1^3 - 0.9^3)}{3(2.1^2 - 0.9^2)} = 1.58 \text{ m}$$

The radius of the centreline of the stairs

$$R_2 = 0.9 + 0.5 \times 1.2 = 1.5 \text{ m}$$

Hence, $R_1/R_2 = 1.58/1.5 = 1.05$, $b/h = 1200/120 = 10$, and from the charts on *Tables 2.90* and *2.91*, interpolating as necessary, $k_1 = -0.12$, $k_2 = +1.52$ and $k_3 = -0.32$. Thus, for a 1.2 m wide stairs, the total design values are:

$$M_0 = 1.2k_1nR_2{}^2 = -0.12 \times 12.22 \times 1.5^2 \times 1.2 = -3.96 \text{ kNm}$$

$$H = 1.2k_2nR_2 = 1.52 \times 12.22 \times 1.5 \times 1.2 = 33.4 \text{ kN}$$

$$M_{vs} = 1.2k_3nR_2{}^2 = -0.32 \times 12.22 \times 1.5^2 \times 1.2 = -10.56 \text{ kNm}$$

The slab should now be checked to ensure that the thickness provided is sufficient to resist M_{vs}. Then, assuming this is so, the foregoing values of M_0 and H can be substituted into the equations for M_v, M_n, T, N, V_n and V_h given on *Table 2.89* to obtain moments and forces at any point along the stairs. For example, where $\theta = 60^\circ$, $M_v = 1.11$ kNm, $M_n = -48.17$ kNm, $T = 0.05$ kNm, $N = -36.5$ kN, $V_n = 9.68$ kN and $V_h = 16.7$ kN. Typical distributions of moments and forces along the stair are shown on *Table 2.89.*

19.2 NON-PLANAR ROOFS

19.2.1 Prismatic structures

To design a simple prismatic roof or any structure comprising a number of planar slabs for service load, the resultant loads Q acting at right angles to each slab and the unbalanced thrusts N acting in the plane of each slab are determined first, taking into account the thrust of one slab on another. The slabs are then designed to resist the transverse bending moments due to the loads Q, assuming continuity, combined with the thrusts N. The longitudinal forces F due to the slabs bending in their own plane under the loads N are, for any two adjacent slabs AB and BC, calculated from formula (2) in *Table 2.92*, where M_{AB} and M_{BC} are found from formula (1) if the structure is freely supported at the end of length L. For each pair of slabs AB-BC, BC-CD and so on, there is an equation like formula (2) containing three unknown forces F. If there are n pairs, there are $(n - 1)$ equations and $(n + 1)$ unknowns. The conditions at the outer edges, a and z, of the end slabs determine the forces F at these edges; for example, if the outer edges are unsupported, $F_a = F_z = 0$. The simultaneous equations are solved for the remaining unknown forces F_A, F_B, F_C and so on. The longitudinal stress at any junction B is calculated from the formula (in *Table 2.92*) for f_b. Variation of the longitudinal stress from one function to the next is rectilinear. If f_b is negative, the stress is tensile and should be resisted by reinforcement. The shearing stresses are generally small.

19.2.2 Domes

A dome is designed for the total vertical load only, that is, for the weights of the slab, any covering on the slab, any ceiling or other distributed load suspended from the slab, and the imposed load. The service load intensity w is the equivalent load per unit area of surface of the dome. Horizontal service loads due to wind and the effects of shrinkage and changes in temperature can be allowed for by assuming an ample normal load, or by inserting more reinforcement than that required for the normal load alone, or by designing for stresses well below permissible values, or by combining any or all of these methods.

Segmental domes. Referring to the diagram and formulae in *Table 2.92*, for a unit strip at S, the circumferential force acting in a horizontal plane is T, and the corresponding force acting tangentially to the surface of the dome (the meridional thrust) is N. At the plane where θ is $51^\circ 48'$, that is, the plane of rupture, $T = 0$. Above this plane, T is compressive reaching a maximum value of $0.5wr$ at the crown of the dome ($\theta = 0$). Below this plane, T is tensile equalling $0.167wr$ when $\theta = 60^\circ$ and $wr = 90^\circ$. The meridional thrust N is $0.5wr$ at the crown, $0.618wr$ at the plane of rupture, $0.667wr$ when $\theta = 60^\circ$, and wr when $\theta = 90^\circ$, that is, N increases from the crown towards the support and has its greatest value at the support.

For a concentrated load F applied at the crown of the dome, T and N are given by the appropriate formulae in *Table 2.92*, where T is tensile and N is compressive. The load is assumed to be concentrated on so small an area at the crown that it can be taken as a point load. The theoretical stress at the crown is therefore infinite, but the practical impossibility of obtaining a point load invalidates the use of the formulae when $\theta = 0$ or very nearly so. For domes of varying thickness, see ref. 67.

For a shallow dome, approximate analysis only is sufficient and appropriate formulae are given in *Table 2.92*.

Conical domes. In a conical dome, the circumferential forces are compressive throughout and, for any horizontal plane at distance x from the apex, are given by the expression for T in *Table 2.92*, the corresponding force in the direction of the slope being N. The horizontal outward force per unit length of the circumference at the bottom of the slope T_r, needs to be resisted by the supports or a bottom ring beam.

19.2.3 Segmental shells

General notes on the design of cylindrical shell roofs and the use of *Tables 2.93* and *2.94* are given in section 6 1.6.

Membrane action. Consideration of membrane action only gives the following membrane forces per unit width of slab due to the uniform load shown on *Table 2.92*: stresses are obtained by dividing by the thickness of the shell h. Negative values of V indicate tension in the direction corresponding to an increase in

Non-planar roofs: general data

Prismatic structure (hipped plate)

N_{AB} etc. thrust in plane of slab AB etc. due to component of load and weight and of thrust in slab BC etc.

$$M_{AB} = \frac{1}{8} N_{AB} L^2; \qquad M_{BC} = \frac{1}{8} N_{BC} L^2, \text{ etc.} \qquad (1)$$

Longitudinal stresses (tensile if negative) at junction B:

$$f_B = \frac{2}{h_{AB} l_{AB}} \left(\frac{3M_{AB}}{l_{AB}} - F_A - 2F_B \right)$$

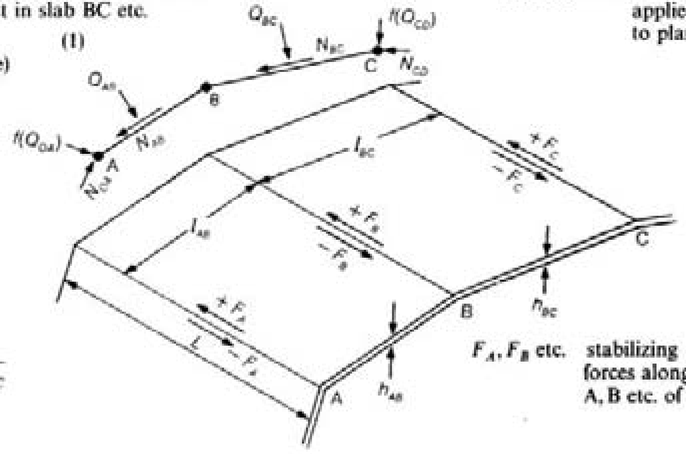
$$= \frac{2}{h_{BC} l_{BC}} \left(2F_B + F_C - \frac{3M_{BC}}{l_{BC}} \right)$$

(similarly for junction C etc.)

Formulae for stabilizing forces, slabs AB and BC:

$$\frac{F_A}{h_{AB} l_{AB}} + 2F_B \left(\frac{1}{h_{AB} l_{AB}} + \frac{1}{h_{BC} l_{BC}} \right) + \frac{F_C}{h_{BC} l_{BC}}$$

$$= 3 \left(\frac{M_{AB}}{h_{AB} l_{AB}^2} + \frac{M_{BC}}{h_{BC} l_{BC}^2} \right) \qquad (2)$$



Q_{AB}, Q_{BC} etc. loads applied normal to plane of slabs

F_A, F_B etc. stabilizing forces along junctions A, B etc. of slabs

Dome

Segmental dome

T circumferential force (in horizontal plane) in unit strip at S.

N meridional thrust (acting tangentially) in unit strip at S.

Uniform load w per unit area of surface of dome:

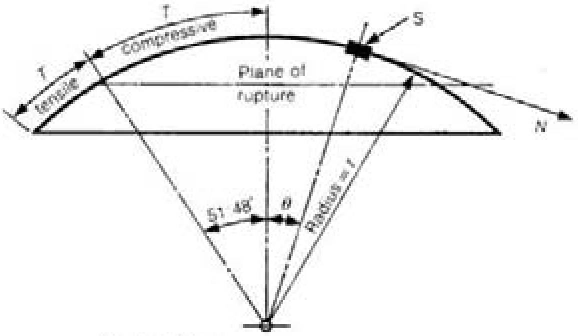
$$T = wr \left[\frac{1 - \cos\theta - \cos^2\theta}{1 + \cos\theta} \right]; \qquad N = -\frac{wr}{1 + \cos\theta}$$

At crown: $T = N = \frac{1}{2} wr$ (compression)

Load F concentrated at crown:

$$T = \frac{F}{2\pi r} \operatorname{cosec}^2\theta; \qquad N = -\frac{F}{2\pi r \sin^2\theta}$$

(Formulae not applicable to small values of θ.)



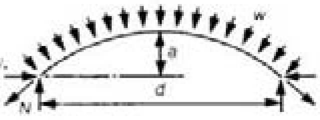
Shallow segmental dome (approx. method)

$$r = \frac{d^2}{8a} + \frac{1}{2}a$$

Total load on supports $= R = 2\pi raw$.

At springing: $N = -\dfrac{2Rr}{\pi d^2}$

Tensile force in ring beam at springing: $T_r = \dfrac{R(r - a)}{\pi d}$



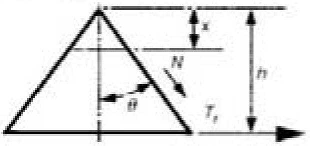
Conical dome

In horizontal plane at x:

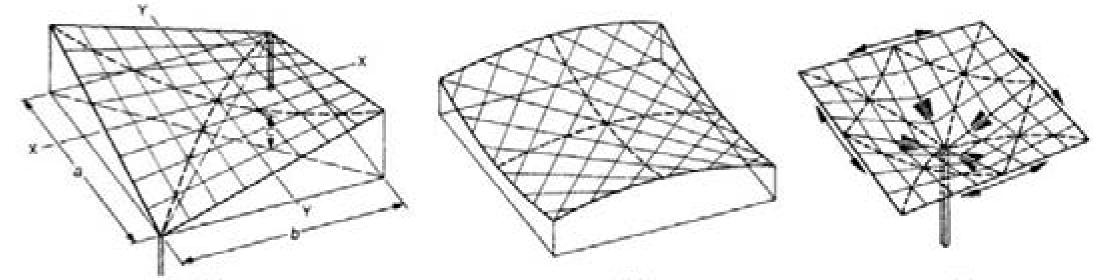
$T = -wx\tan^2\theta$ (i.e. compression) and $N = -\dfrac{wx}{2\cos^2\theta}$

At bottom:

$T_r = -wh\dfrac{\tan\theta}{2\cos\theta}$ per unit length of circumference

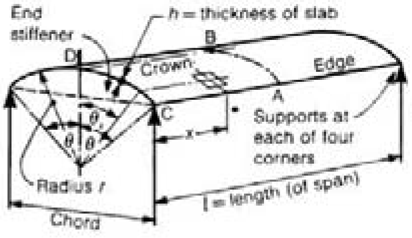


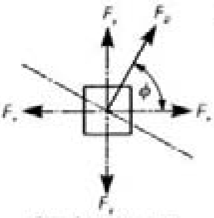
Hyperbolic paraboloid



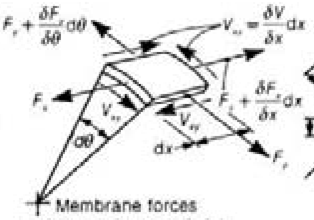
(a) (b) (c)

Cylindrical shell

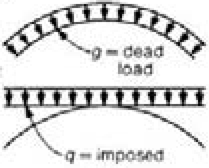




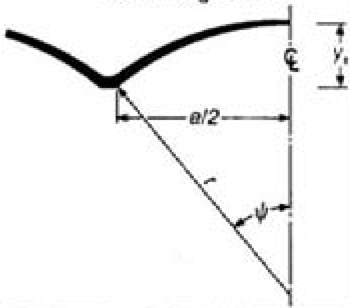
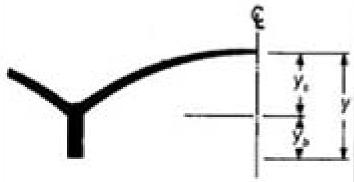
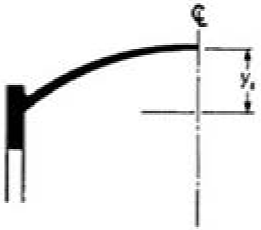
Principal stresses



Membrane forces acting on element of slab



Distribution of loads

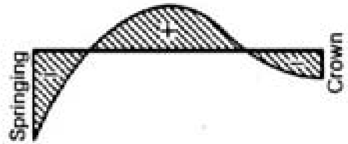
		Feather-edged shell	Shell with valley beams	Shell with edge beams
				
	Assumptions made	No valley beam No horizontal displacement or rotation at shell edge	Half-shell acts with half valley beam No horizontal displacement or rotation of valley beam	Edge beam does not deflect vertically but has no horizontal restraint except its own stiffness
	Minimum value⁽¹⁾ of y	$\frac{l}{30}\left(1+\frac{l}{60}\right)$	$\frac{l}{30}\left(1+\frac{l}{60}\right)$	$\frac{l}{45}\left(1+\frac{l}{60}\right)$
Principal forces and moments required for design	Load per unit length supported by half-shell	$F_s = wr\psi + \frac{1}{2}W_b$	$F_s = 1.19\,W y_s/y$	Read from graph on opposite page
	Load per unit length supported by half-beam	—	$F_b = W - F_s$	$F_b = wr\psi - F_s$
	Total tension in shell valley at midspan	$N_{ts} = K_1 F_s l^2/y_s$ where $K_1 = 1.7$ when $K\psi \ngtr 1.75$ $K_1 = \frac{9.35}{9-2K\psi}$ when $K\psi > 1.75^{(2)}$	$N_{ts} = \frac{K_2 W l^2}{y}\left(\frac{y_s}{y} - 0.48\right)$ where $K_2 = 1.78$ when $K\psi \ngtr 1.75$ $K_2 = \frac{9.8}{9-2K\psi}$ when $K\psi > 1.75^{(2)}$	$N_{ts} = \frac{K_3 F_s l^2}{y_s}$ where $K_3 = 0.17$ when $K\psi \ngtr 1.5$ $K_3 = \frac{1.02}{9-2K\psi}$ when $K\psi > 1.5^{(2)}$
	Total tension in beam at midspan	—	$N_{tb} = \frac{K_2 W l^2}{y}\left(1.33 - \frac{y_s}{y}\right)$	$N_{tb} = \frac{22 K_3 F_s l^2}{3 y_s}$
	Total shear in half-shell	$\sum V = 9.25\,Fl/\psi$	$\sum V = 8.45\,Fl/\psi$	$\sum V = 8.60\,Fl/\psi$
	Maximum positive transverse bending moment⁽³⁾	$M_{dt} = \frac{6 \times 10^{-5} F^2 l}{y_s}$	$M_{dt} = \frac{4.4 \times 10^{-5} F^2 l}{y}$	$M_{dt} = \frac{5.2 \times 10^{-5} F^2 l}{y_s}$
	Transverse tension in shell	$N_t = 11.8\,wr\sqrt[4]{(l/a)}$	$N_t = 10.2\,wr\sqrt[4]{(l/a)}$	$N_t = 10.8\,wr$

Notes

1. If value of y adopted is less than that given by appropriate expression, max. longitudinal compressive stress and/or deflection at crown may be excessive and empirical method is inapplicable.

2. In such cases (i.e. 'short' shells) transverse bending moments M_{dt} given by formulae are less reliable and longitudinal moments may be significant, so caution should be adopted.

3. The distribution of transverse moments is as shown in sketch below, maximum value occurring at springing, where shell is thickened to at least 150 mm. Negative moment at crown is about 2/3 to 3/4 of max. positive moment, and steel provided top and bottom throughout shell to resist max. positive moment will suffice.

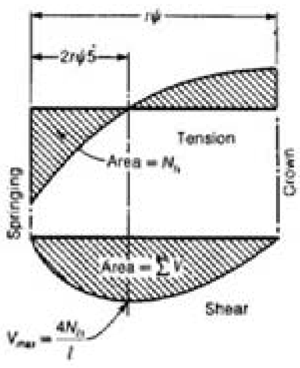
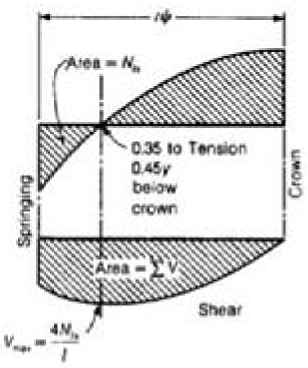
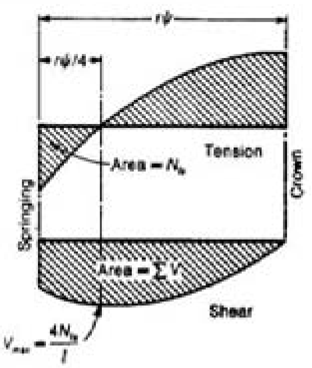


Typical distribution of M_{dt}

4. Beams should be designed as follows:

Valley beams: to resist load F_b plus direct tension (N_{tb} at midspan) plus horizontal shear between shell and valley beam varying from $4/lN_{tb}$ at support to zero at midspan

Edge beams: as continuous beam over internal columns to resist loads as valley beam.

Distributions of principal tension and shearing force in half-shell			
Reinforcement required[4]	Total tensile reinforcement required in half-shell = N_{ls}/f_{st}. Total shearing reinforcement required at 45° across corner = $\sum V/1.41 f_{stv}$. (The point at which this steel may be curtailed can be estimated by means of the shearing-force diagrams above.) Fabric reinforcement required to resist transverse moments = $M_{dt}/z f_{st}$ (top and bottom)[6].		

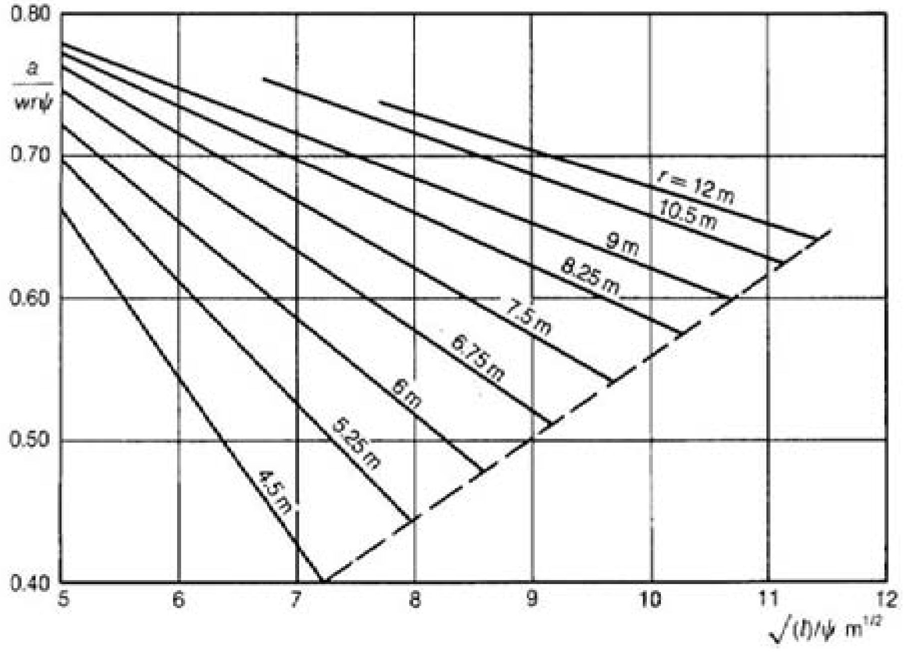
5. If a exceeds 15 m, buckling is possible and empirical method is inapplicable unless shell has vertically supported edge beams.

6. Formulae give values of M_{dt} for shell thickness of 65 mm. Thicker shells induce higher moments but if fabric is designed for thickness of 65 mm it will suffice for thicker shell, owing to increase in effective depth.

7. If $y_b < 300$ mm, shell should be designed as feather-edged.

Notation

a	chord width of shell[5] (m)
F_b	load per unit length carried by beam (kg/m)
F_s	load per unit length carried by half-shell (kg/m)
f_{st}	permissible stress in tension reinforcement
f_{stv}	permissible stress in shearing reinforcement
K	shell parameter: $K = 0.925 \times \sqrt[4]{(r^3/l^2 t)}$
l	span of shell (m)
M_{dt}	transverse bending moment in shell (positive for tension on underside) (N m per metre)
N_{lb}	longitudinal tension in beam at midspan (N)
N_{ls}	longitudinal tension in shell valley at midspan (N)
N_t	transverse tension in shell (N)
r	radius of shell (m)
t	thickness of shell (m)
V	shear force per unit length (N/m)
$\sum V$	sum of all shear forces in half-shell at stiffening beam (N)
W	total load per unit length of half-shell and valley beam, including all imposed loads and thickenings (kg/m)
W_b	load per unit length along valley additional to shell and valley beam load (kg/m)
w	load per unit area of shell, including self-weight, finishes and imposed load (kg/m²)
y	total depth from soffit of valley beam to crown of shell (m)
y_s	depth of shell from springing to crown (m)
y_b	depth of downstand of valley beam[7] (m)
z	lever-arm (within shell thickness)
ψ	angle subtended by half-shell (i.e. angle between radius and vertical at springing)



x and a decrease in θ_x. In the case of F, positive values indicate tension. Reinforcement should be provided approximately in line with, and to resist, the principal tensile forces. If the shell is supported along any edges, the forces will be modified accordingly. The values of the forces at any point are as follows:

Tangential force

$$F_y = -(g + q\cos\theta_x)\, r\cos\theta_x$$

Longitudinal force

$$F_x = -(1 - x)(x/r)[g\cos\theta_x + 1.5q(\cos^2\theta_x - \sin^2\theta_x)]$$

Shearing force

$$V_{xy} = (2x - l)(g + 1.5q\cos\theta_x)\sin\theta_x$$

Principal forces (due to membrane forces only)

$$F_p = 0.5\{(F_x + F_y) \pm [(F_x - F_y)^2 + 4V^2{}_{xy}]^{0.5}\}$$

$$\tan 2\phi = \frac{2V_{xy}}{F_x - F_y}$$

At A (mid-point at edge: $\theta_x = \theta$, $x = l/2$)

$$F_{yA} = -(g + q\cos\theta)\, r\cos\theta$$
$$F_{xA} = -(l^2/4r)[g\cos\theta + 1.5q(\cos^2\theta - \sin^2\theta)]$$
$$V_{xyA} = 0$$

At B (mid-point at crown: $\theta_x = 0$, $x = l/2$)

$$F_{yB} = -(g + q)r$$
$$F_{xB} = -(l^2/4r)(g + 1.5q)$$
$$V_{xyB} = 0$$

At C (support at edge: $\theta_x = \theta$, $x = 0$)

$$F_{yC} = -(g + q\cos\theta)\, r\cos\theta$$
$$F_{xC} = 0$$
$$V_{xyC} = -(g + 1.5q\cos\theta)\, l\sin\theta$$

At D (support at crown: $\theta_x = 0$, $x = 0$)

$$F_{yD} = -(g + q)\, r$$
$$F_{xD} = 0,\ V_{xyD} = 0$$

Beam action. If the ratio of the length of a cylindrical shell to its radius, l/r, is not less than 2.5, the longitudinal forces can be approximated with reasonable accuracy by calculating the second moment of area I_{xx} and the vertical distance from the neutral axis to the crown $\bar{y}$ from the approximate expressions:

$$I_{xx} \cong R^2h\,(R - 3h/2)(\alpha^2 + \alpha\sin\alpha\cos\alpha - 2\sin^2\alpha)/\alpha$$
$$\bar{y} \cong R - [(R - 2h/3)\sin\alpha]/\alpha$$

Then, if n is the total uniform load per unit area acting on the shell (i.e. including self-weight etc.), the maximum bending moment in a freely supported shell is given by:

$$M = (2\theta rn)l^2/8 = \theta rnl^2/4$$

Hence, from the relationship $M/I_{xx} = f/y$ the horizontal forces at mid-span at the crown and springing of a shell of thickness h are given by the expressions:

$$F_x(\text{top}) = \frac{M\bar{y}h}{I_{xx}},\quad F_x(\text{bottom}) = \frac{M[r(1 - \cos\theta) - \bar{y}]h}{I_{xx}}$$

At the supports the total shearing force in the shell is

$$V = (2\theta rn)l/2 = \theta rnl$$

The shearing stress at any point is then given by $v = (VA\bar{x})/(2hI_{xx})$

where $A\bar{x}$ is the first moment of area about the neutral axis of the area of cross section of shell, above the point at which the shearing stress is being determined.

The principal shortcoming of this approximate analysis is that it does not indicate in any simple manner the magnitude of the transverse moments that occur in the shell. However, a tabular method has been devised by which these moments can be evaluated indirectly from the lateral components of the shearing stresses: for details see ref. 68.

19.2.4 Hyperbolic-paraboloidal shells

The simplest type of double-curved shell is generated by the intersection of two separate sets of inclined straight lines (parallel to axes XX and YY respectively, as shown in the diagrams on *Table 2.92*). Vertical sections through the shell at angles XX or YY are parabolic in shape, while horizontal sections through the surface form hyperbolas: hence, the name hyperbolic paraboloid.

Individual units such as those shown in diagrams (a) and (b) in *Table 2.92* can be used separately, being supported on columns or buttresses located at either the higher or the lower corners. Alternatively, groups of units can be combined to achieve roofs having attractive and unusual shapes, such as is shown in diagram (c). Some idea of the more unlikely forms that can be achieved may be obtained from ref. 69.

If the shell is shallow and the loading is uniform, the shell behaves as a membrane transferring uniform compressive and tensile forces of $F/2c$ (where F is the total load on the unit, and c is the rise), acting parallel to the directions of principal curvature, to the edges of the shell. The edge forces are then transmitted back to the supports along beams at the shell edges. The main problems that arise when designing these shells are the interaction between the shell and the supporting edge members, the design of the buttresses or ties needed to resist the horizontal component of the forces at the supports, and the fact that excessive deflections at unsupported edges lead to stresses that differ considerably from those predicted by simplified theories.

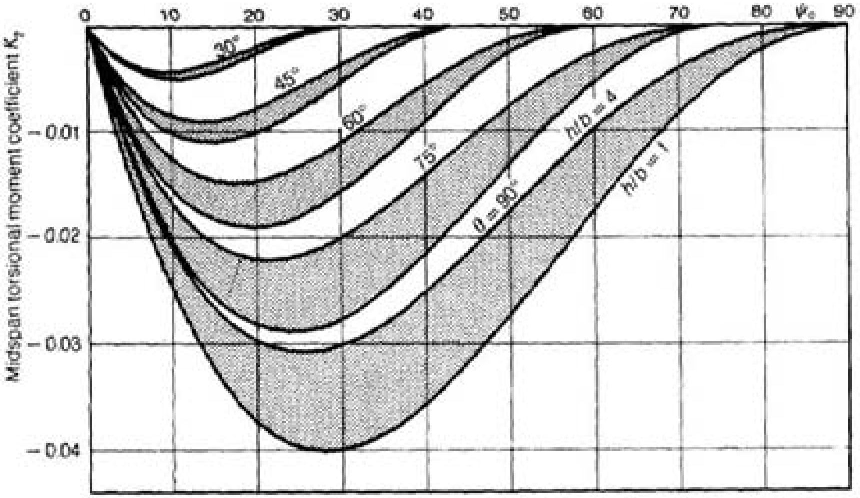
Increasing the sharpness of curvature of the shell increases its stability, and reduces the forces and reactions within the shell, but to avoid the need for top forms the maximum slope should not exceed 45°: this corresponds to a value of $1/\sqrt{2}$ for the ratio c/a or c/b in the diagrams on *Table 2.92*. To ensure stability if a single unit is used, the ratio should be not less than 1/5. A useful introduction to the theory and design of hyperbolic-paraboloidal shells is given in ref. 70.

19.3 BEAMS CURVED ON PLAN

Some general notes on beams forming a circular arc on plan are given in section 6.1.7. The following analyses apply only if the appropriate negative bending and torsional moments can be developed at the end supports.

19.3.1 Concentrated loads

If a beam LR (see *Table 2.95*) curved on plan is subjected to a concentrated load F, such that the angle between the radius at



Between L and F (i.e. $\psi_0 \leqslant \psi \leqslant \theta$):

$M = Fr[K_1 \cos\psi - K_2 \sin\psi + K_3 \sin\psi - \sin(\psi - \psi_0)]$

$T = Fr\{K_1 \sin\psi + K_2 \cos\psi + K_3(1 - \cos\psi) - [1 - \cos(\psi - \psi_0)]\}$

$V = F(1 - K_3)$

Between F and O (i.e. $0 \leqslant \psi \leqslant \psi_0$):

$M = Fr(K_1 \cos\psi - K_2 \sin\psi + K_3 \sin\psi)$

$T = Fr[K_1 \sin\psi + K_2 \cos\psi + K_3(1 - \cos\psi)]$

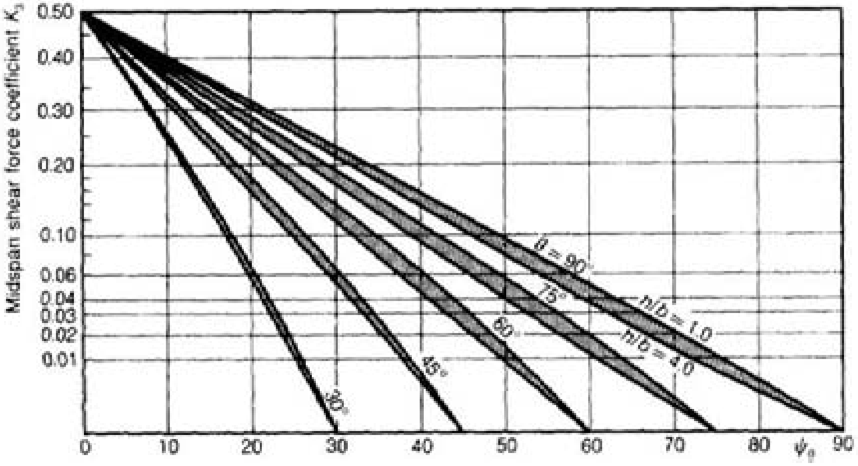
$V = -FK_3$

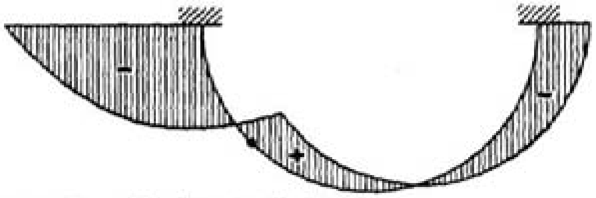
Between O and R (i.e. $0 \leqslant \psi \leqslant \theta$)

$M = Fr(K_1 \cos\psi + K_2 \sin\psi - K_3 \sin\psi)$

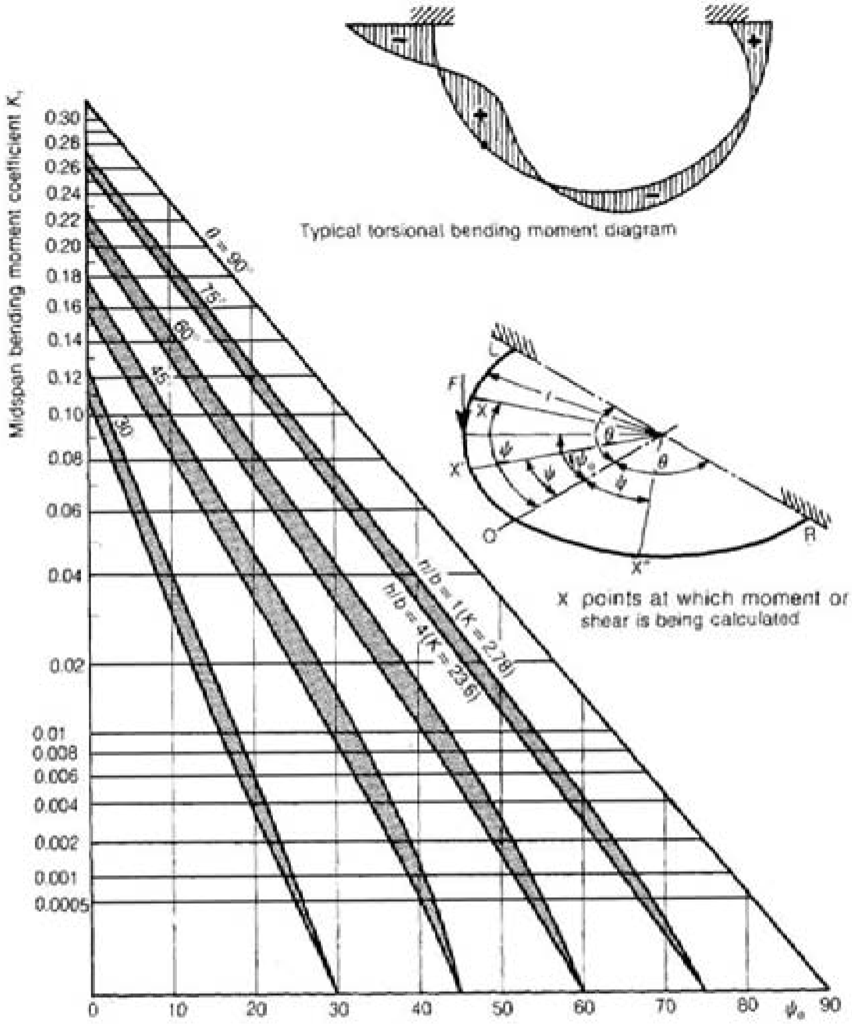
$T = Fr[-K_1 \sin\psi + K_2 \cos\psi + K_3(1 - \cos\psi)]$

$V = -FK_3$





Typical flexural bending moment diagram



Typical torsional bending moment diagram the point of application F of load F, and the radius at the mid-point O of the beam, is ψ_0, the following expressions are applicable at any point X between F and L (i.e. $\psi \geq \psi_0$):

$$M = M_0\cos\psi - T_0\sin\psi + V_0 r\sin\psi - Fr\sin(\psi - \psi_0)$$

$$T = M_0\sin\psi + T_0\cos\psi + V_0 r(1 - \cos\psi) - Fr[1 - \cos(\psi - \psi_0)]$$

$$V = -V_0 + F$$

where M_0, T_0 and V_0 are respectively the bending moment, the torsional moment and the shearing force at mid-span, r is the radius of curvature in plan and ψ is the angle defining the position of X, as shown in the diagram on *Table 2.95*.

If X is between F and O (i.e. X′, $\psi \leq \psi_0$), terms containing F are equal to zero. If X is between O and R (i.e. X″), signs of the terms containing $\sin\psi$ should also be reversed. Now by writing $M_0 = K_1 Fr$, $T_0 = K_2 Fr$ and $V_0 = K_3 F$,

$$K_1 = \frac{k_1}{k_2}, \quad K_2 = \frac{k_3 k_8 - k_5 k_6}{k_4 k_8 - k_5 k_7} \quad \text{and} \quad K_3 = \frac{k_4 k_6 - k_3 k_7}{k_4 k_8 - k_5 k_7}$$

where k_1, k_2, k_3 and so on are given by the following expressions, in which $K = EI/GC$ (i.e. flexural rigidity/torsional rigidity).

$$\begin{aligned}
k_1 &= \tfrac{1}{4}(K - 1)\sin\psi_0(\sin 2\theta - \sin 2\psi_0) \\
&\quad - \tfrac{1}{2}(K - 1)\cos\psi_0(\sin^2\theta - \sin^2\psi_0) \\
&\quad - \tfrac{1}{2}(K + 1)(\theta - \psi_0)\sin\psi_0 - K(\cos\theta - \cos\psi_0) \\
k_2 &= (K + 1)\theta - 1/2(K - 1)\sin 2\theta \\
k_3 &= K(\sin\theta - \sin\psi_0) - 1/4\,(K - 1)\cos\psi_0(\sin 2\theta - \sin 2\psi_0) \\
&\quad - \tfrac{1}{2}(K - 1)\sin\psi_0(\sin^2\theta - \sin^2\psi_0) \\
&\quad - \tfrac{1}{2}(K + 1)(\theta - \psi_0)\cos\psi_0 \\
k_4 &= (K + 1)\theta + \tfrac{1}{2}(K - 1)\sin 2\theta \\
k_5 &= 2K\sin\theta - (K + 1)\theta - \tfrac{1}{2}(K - 1)\sin 2\theta \\
k_6 &= \tfrac{1}{4}(K - 1)\cos\psi_0(\sin 2\theta - \sin 2\psi_0) + K(\theta - \psi_0) \\
&\quad + \tfrac{1}{2}(K + 1)(\theta - \psi_0)\cos\psi_0 \\
&\quad + \tfrac{1}{2}(K - 1)\sin\psi_0(\sin^2\theta - \sin^2\psi_0) \\
&\quad - K(1 + \cos\psi_0)(\sin\theta - \sin\psi_0) \\
&\quad + K\sin\psi_0(\cos\theta - \cos\psi_0) \\
k_7 &= k_5 \\
k_8 &= \tfrac{1}{2}(K - 1)\sin 2\theta - 4K\sin\theta + (3K + 1)\theta
\end{aligned}$$

For rectangular beams, the graphs provided on *Table 2.95* enable values of K_1, K_2 and K_3 to be read directly, for given values of θ, ψ_0 and h/b (i.e. depth/width). Values of $G = 0.4E$ and $C = J/2$, as recommended in BS 8110, have been used.

19.3.2 Uniform load

For a curved beam with a UDL over the entire length, owing to symmetry, the torsional moment (and also the shearing force) at mid-span is zero. By integrating the foregoing formulae, the bending and torsional moments at any point X along the beam are given by the expressions:

$$M = M_0\cos\psi - nr^2(1 - \cos\psi)$$

$$T = M_0\sin\psi - nr^2(\psi - \sin\psi)$$

If $M_0 = K_4 nr^2$, where

$$K_4 = \frac{4[(1 + K)\sin\theta - K\theta\cos\theta]}{2\theta(1 + K) + \sin 2\theta(1 - K)} - 1$$

these expressions can be rearranged to give the bending and torsional moments at the supports, and the maximum positive moments in the span in terms of non-dimensional factors K_4, K_5, K_6 and K_7 as shown on *Tables 2.96* and *2.97*. The factors can be read from the charts given on the tables.

Example 1. A bow girder, 450 mm deep and 450 mm wide, has a radius of 4 m and subtends an angle of 90°. The ends are rigidly fixed, and the total UDL is 200 kN. The maximum moments are to be determined.

The distributed load per unit length,

$$n = 200/(\pi r/2) = 200/(\pi \times 4/2) = 31.8\ \text{kN/m}$$

From the charts on *Tables 2.96* and *2.97* (with $h/b = 1.0$ and $\theta = 45°$): $K_4 = 0.086$, $K_5 = -0.23$, $K_6 = -0.0175$, $K_7 = 0.023$, with $\psi_1 = 23°$ and $\psi_2 = 40°$.

Maximum positive bending moment (at midspan)

$$K_4 nr^2 = 0.086 \times 31.8 \times 4^2 = 43.8\ \text{kNm}$$

Maximum negative bending moment (at supports)

$$K_5 nr^2 = -0.23 \times 31.8 \times 4^2 = -117\ \text{kNm}$$

Zero bending moment occurs at $\psi_1 = 23°$.

Maximum negative torsional moment (at supports)

$$K_6 nr^2 = -0.0175 \times 31.8 \times 4^2 = -8.9\ \text{kNm}$$

Maximum positive torsional moment (at $\psi_1 = 23°$)

$$K_7 nr^2 = 0.023 \times 31.8 \times 4^2 = 11.7\ \text{kNm}$$

Zero torsional moment occurs at $\psi_2 = 40°$.

Example 2. The beam in example 1 supports a concentrated load of 200 kN at a point where the radius subtends an angle of 15° from the left-hand end (i.e. $\psi_0 = 30°$). Moments and shearing forces at midspan and the supports are required.

From the charts on *Table 2.95* (with $h/b = 1.0$, $\theta = 45°$ and $\psi_0 = 30°$): $K_1 = 0.015$, $K_2 = -0.0053$, $K_3 = 0.067$.

At midspan ($\psi = 0$),

$$M_0 = 0.015 \times 200 \times 4 = 12\ \text{kNm}$$

$$T_0 = -0.0053 \times 200 \times 4 = -4.3\ \text{kNm}$$

$$V_0 = 0.067 \times 200 = 13.4\ \text{kN}$$

At left-hand support ($\psi = 45°$),

$$\begin{aligned}
M &= M_0\cos 45° - T_0\sin 45° + V_0\, r\sin 45° - Fr\sin(45° - 30°) \\
&= [12 - (-4.3) + 13.4 \times 4) \times 0.707 - 200 \times 4 \times 0.259 \\
&= -158\ \text{kNm}
\end{aligned}$$

$$\begin{aligned}
T &= M_0\sin 45° + T_0\cos 45° + V_0\, r\,(1 - \cos 45°) \\
&\quad - Fr\,(1 - \cos 15°) \\
&= (12 - 4.3) \times 0.707 + 13.4 \times 4 \times 0.293 - 200 \times 4 \times 0.034 \\
&= -6.1\ \text{kNm}
\end{aligned}$$

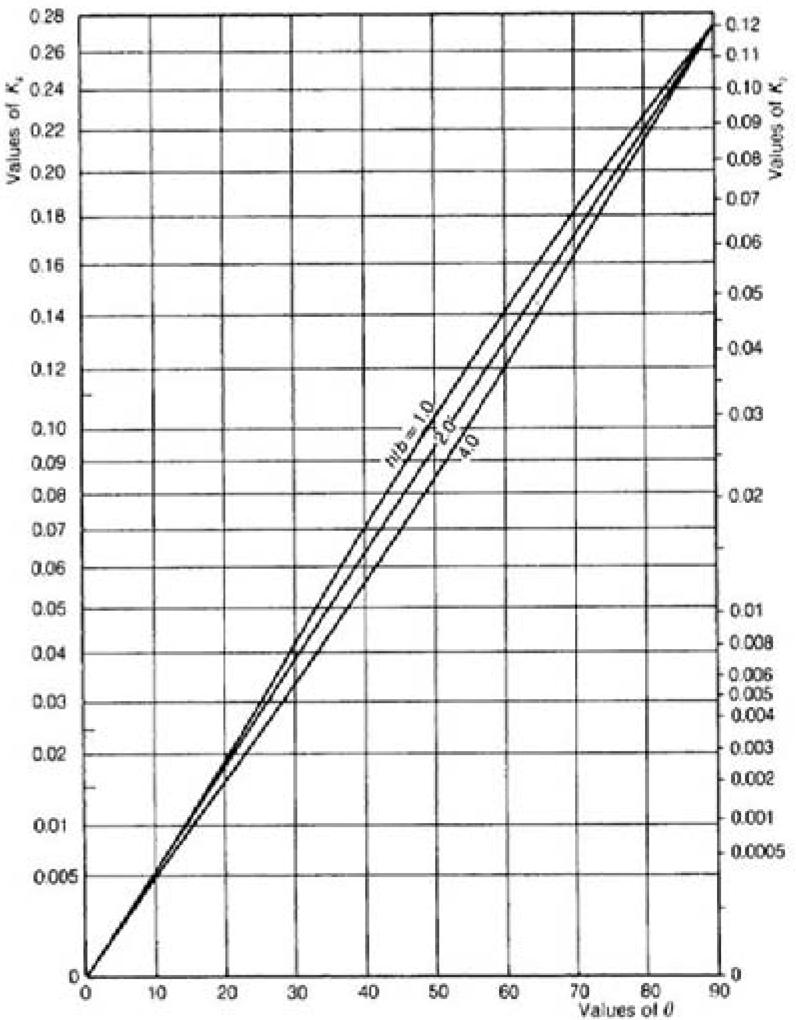
$$V = V_0 - F = 13.4 - 200 = 186.6\ \text{kN}$$

At right-hand support ($\psi = 45°$),

$$\begin{aligned}
M &= M_0\cos 45° + T_0\sin 45° - V_0\, r\sin 45° \\
&= [12 - 4.3 - 13.4 \times 4) \times 0.707 = -32.4\ \text{kNm}
\end{aligned}$$

$$\begin{aligned}
T &= -M_0\sin 45° + T_0\cos 45° + V_0\, r\,(1 - \cos 45°) \\
&= (-12 - 4.3) \times 0.707 + 13.4 \times 4 \times 0.293 \\
&= 4.2\ \text{kNm}
\end{aligned}$$

$$V = V_0 = 13.4\ \text{kN}$$



At angle ψ from centre-line of beam: ultimate bending moment

$M = nr^2[\cos\psi(K_4 + 1) - 1]$ where

$K_4 = 4[\sin\psi(1 + K) - \psi\cos\psi K]/[2\psi(1 + K) + \sin 2\psi(1 - K)] - 1$

At midspan, where $\psi = 0$, $M = K_4 nr^2$

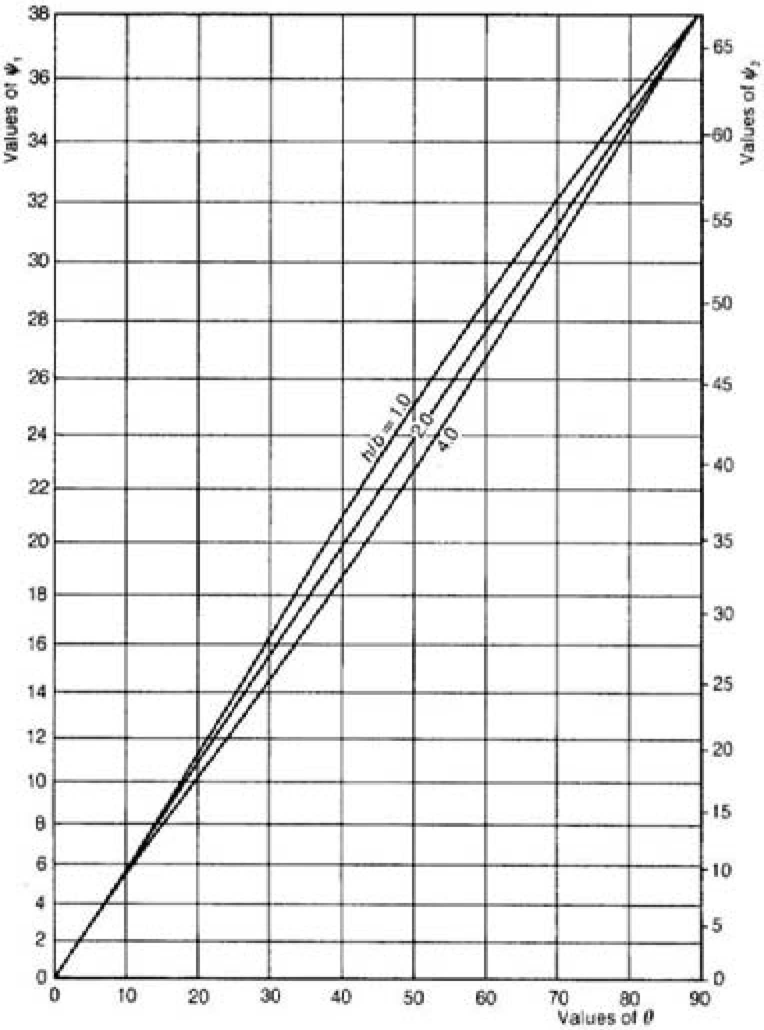
At support, where $\psi = \theta$, $M = K_5 nr^2$, where $K_5 = \cos\theta(K_4 + 1) - 1$

$M = 0$ occurs at ψ_1 where

$\psi_1 = \cos^{-1}(1/K_4 + 1)$

$T = 0$ occurs at ψ_2 where

$\psi_2 = \sin\psi_2(K_4 + 1)$



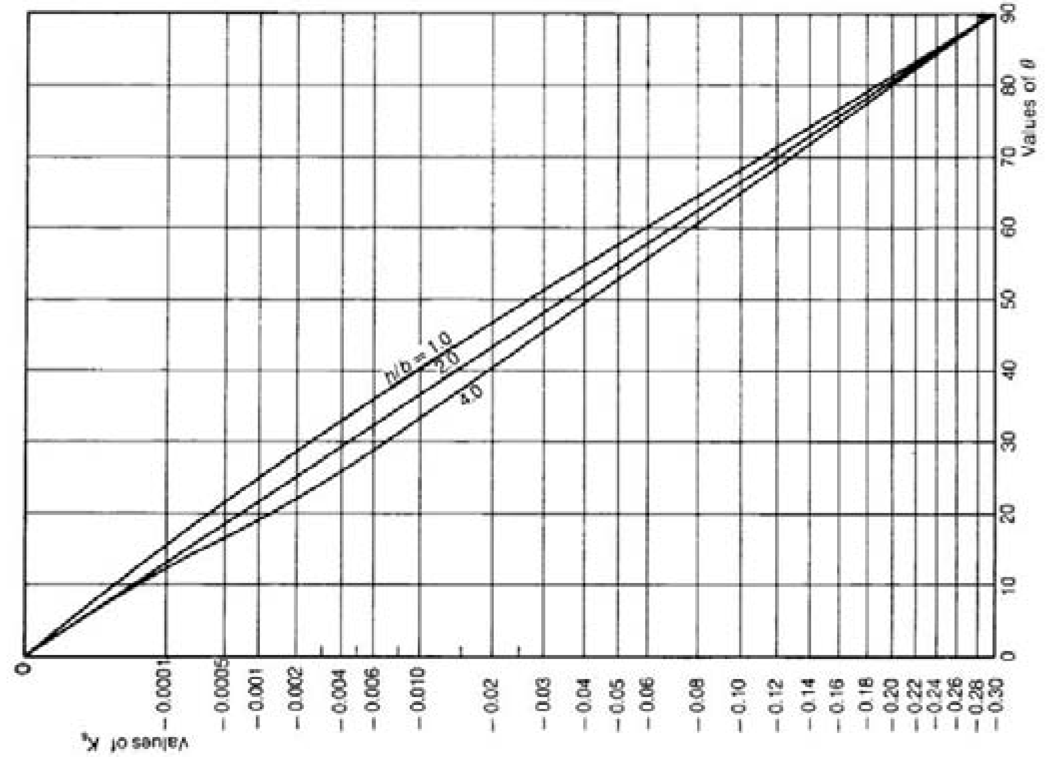
At angle ψ from centre-line of beam: ultimate torsional moment

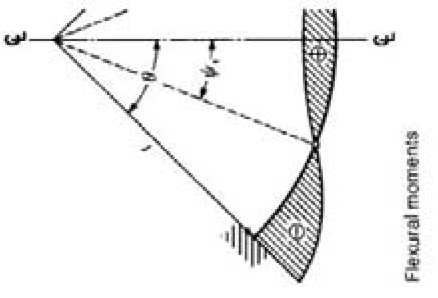
$T = nr^2[\sin\psi(K_4 + 1) - \psi]$

At midspan, where $\psi = 0$, $T = 0$

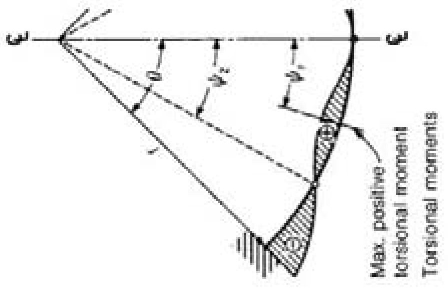
At support, where $\psi = \theta$, $T = K_6 nr^2$, where $K_6 = \sin\theta(K_4 + 1) - \theta$

At $\psi = \psi_1$, $T = K_7 nr^2$ (maximum positive value)

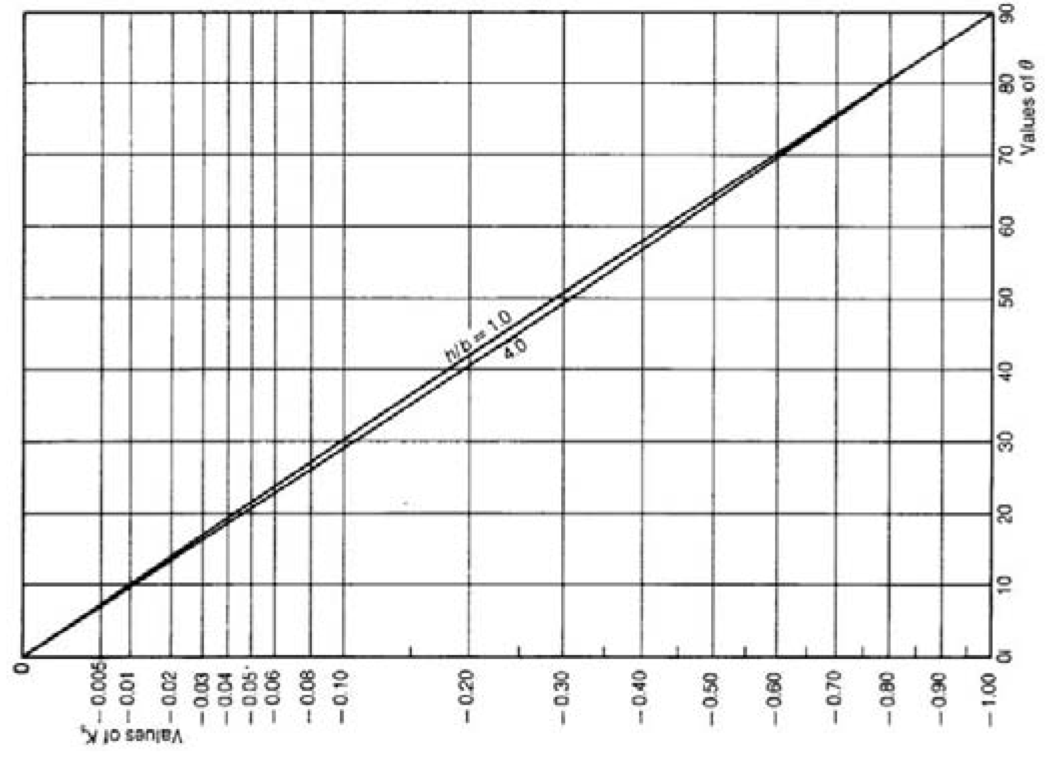




Flexural moments



Torsional moments



19.4 BEARINGS, HINGES AND JOINTS

A comprehensive guide on bridge bearings and expansion joints, including a treatment of the design techniques used to accommodate movements in bridges, is contained in ref. 71. Various types of bridges, for which notes are given in section 6.2, and typical span ranges, are shown in *Table 2.98*.

19.4.1 Hinges and bearings

A hinge is an element that can transmit thrust and transverse force, but permits rotation without restraint. If it is vital for such action to be fully realised, a steel hinge can be provided. Alternatively, hinges that are monolithic with the member can be formed, as indicated at (a) and (b) in *Table 2.99*. The 'Mesnager' hinge shown at (a) has been used, for example, in the frames of large bunkers to isolate the container from the substructure, or to provide a hinge at the base of the columns of a hinged frame bridge. The hinge-bars '*a*' resist the entire horizontal shear force, and the so-called throat of concrete at D must be sufficient to transfer the full compressive force from the upper to the lower part of the member. The hinge-bars should be bound together by links '*d*', and the main vertical bars '*e*' should terminate on each side of the slots B and C. It may be advantageous during construction to provide bars extending across the slots, and then cut these bars on completion of the frame. The slots should be filled with a bituminous material, or lead, or a similar separating layer.

The Freyssinet hinge shown at (b) has largely superseded the Mesnager hinge. In this case, the large compressive stress across the throat results in a high shearing resistance, and the inclusion of bars crossing the throat can adversely affect the hinge. Tests have shown that, as a result of biaxial or triaxial restraint, such hinges can withstand compressive stresses of several times the cube strength without failure occurring. The bursting tension on each side of the throat normally governs the design of this type of hinge.

A number of other types of hinges and bearings that have been used on various occasions are shown in *Table 2.99*:

(c) a hinge formed by the convex end of a concrete member bearing in a concave recess in the foundations
(d) a hinge suitable for the bearing of a girder where rotation, but not sliding, is required
(e) a bearing for a girder where sliding is required
(f) a mechanical hinge suitable for the base of a large portal frame, or the abutment of a large hinged arch bridge
(g) a hinge suitable for the crown of a three-hinged arch when the provision of a mechanical hinge is not justified
(h) a bearing suitable for the support of a freely suspended span on a cantilever in an articulated bridge.

Bridge bearings have to be able to accommodate the rotations resulting from deflection of the deck under load. They also have to be able to accommodate horizontal movements of the deck caused by prestress, creep, shrinkage and temperature change. Some bearings allow horizontal movement in one direction only and are restrained in the other direction, whilst other types allow movement in any direction. Elastomeric bearings that are formed of layers of steel plate embedded in rubber can accommodate small horizontal shear movements.

PTFE (polytetrafluoroethylene) bearings can give unlimited free sliding between low friction PTFE surfaces and steel plates. Pot bearings that incorporate rubber discs allow for small rotations, while spherical bearings that move on a PTFE surface permit larger rotations. Mechanical bearings, such as rockers and rollers, can be used to provide either longitudinal fixity or resistance to lateral force. Pot bearings, special guide bearings or pin bearings are also used for this purpose. Bearings need to be inspected regularly, and may require maintenance or replacement during the lifetime of the bridge. As this can be both difficult and expensive, it is very important that the structure is designed to make inspection, maintenance and replacement possible.

19.4.2 Movement joints

Movement joints are often required to allow free expansion and contraction due to temperature changes and shrinkage in such structures as retaining walls, reservoirs, roads and long buildings. In order to allow unrestrained deformation of the walls of cylindrical containers, sliding joints can be provided at the bottom and top of the wall. Several types of joints for various purposes are shown in *Table 2.100*. Figures (a)–(f) show some of the joint details recommended in BS 8007.

Expansion joints at wide spacing may be desirable in large areas of walls and roofs that are not protected from solar heat gain, or where a contained liquid is subjected to a substantial temperature range. Except for structures designed to be fully continuous, contraction joints of the type described in section 26.2.2, and at the maximum spacing specified in *Table 3.45* should be provided. The reinforcement should be curtailed to form a complete movement joint, or made 50% continuous to form a partial movement joint. Waterstops are positioned at the centre of wall sections, and at the underside of floor slabs that are supported on a smooth layer of blinding concrete. In basement walls, waterstops are best positioned at the external face where they are supported by the earth.

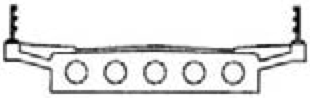
The recommendations of BS 8007 with regard to the spacing of vertical joints may be applied also to earth-retaining walls. For low walls with thin stems, simple butt joints are generally used. However, the effect of unequal deflection or tilting of one part of a wall relative to the next will show at the joints. For retaining walls higher than about 1 m, a keyed joint can be used. Alternatively, dowels passing through the joint, with the ends on one side greased and sheathed, can be used.

Figure (g) shows alternative details at the joint between the wall and floor of a cylindrical tank, to minimise or eliminate restraint at this position. In the first case, rubber or neoprene pads with known shear deformation characteristics are used. In the second case, action depends on a sliding membrane of PTFE or similar material. These details are most commonly used for prestressed cylindrical tanks.

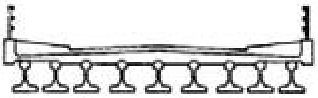
Movement joints in buildings should divide the structure into individual sections, passing through the whole structure above ground level in one plane. Joints at least 25 mm wide should be provided at about 50 m centres both longitudinally and transversely. In the top storey, and for open buildings and exposed slabs, additional joints should be provided to give a spacing of about 25 m. Joints also need to be incorporated in finishes and cladding at movement joint locations. Joints in walls should be made at column positions, in which case a double

Deck types and span range

Reinforced concrete solid slab

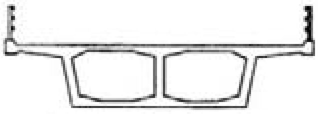


Voided slab

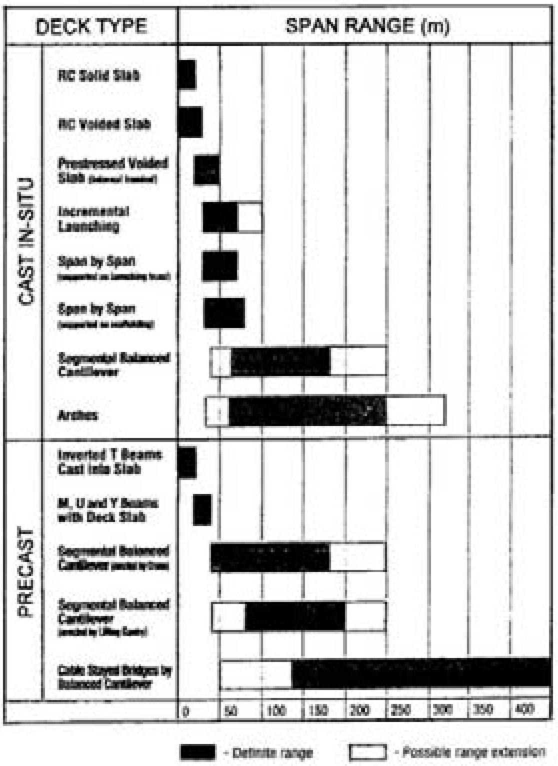


Precast beam and in-situ slab

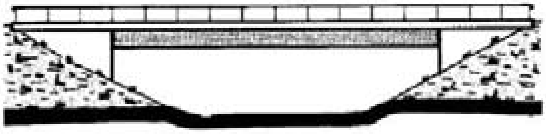
Single cell box section



Multi cell box section



Structural forms



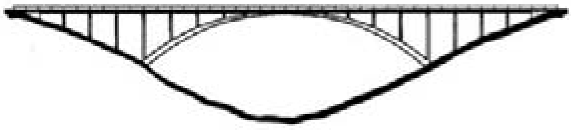
Single span beam or slab bridge

Frame bridge

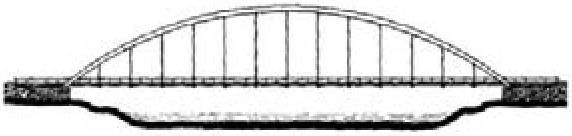
Strut frame bridge

Three span beam bridge with curved haunches

Continuous beam viaduct of uniform depth with varying spans



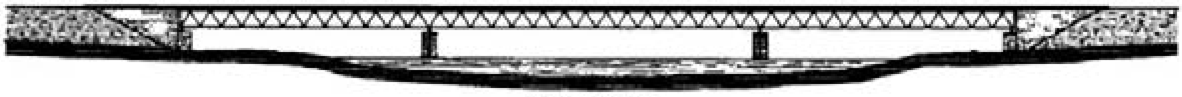
Arch beam bridge



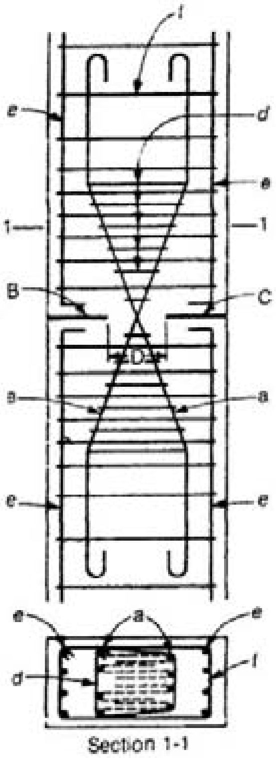
Tied arch or bowstring bridge

Cable stayed bridge

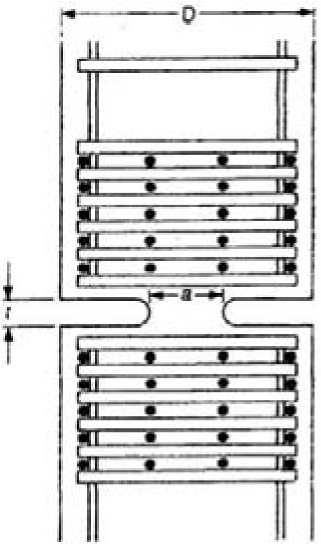
Suspension bridge



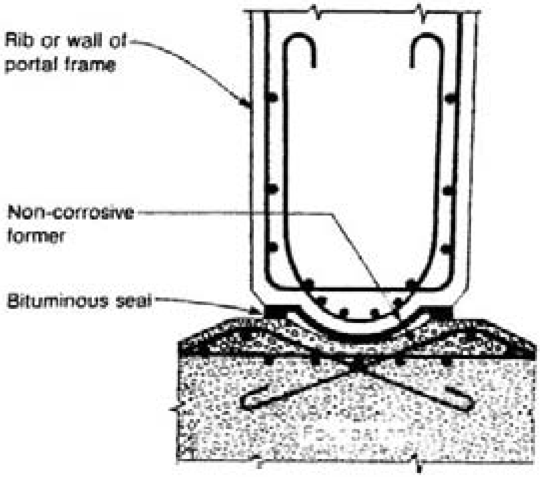
Truss bridge



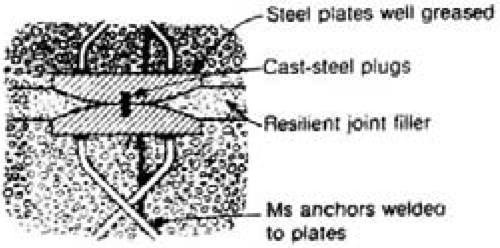
(a) Mesnager hinge



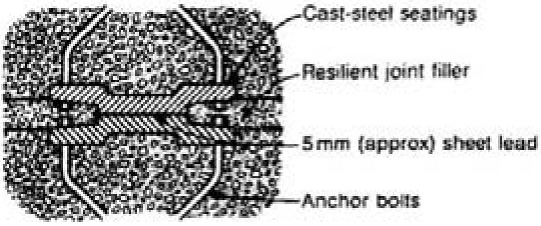
(b) Freyssinet hinge



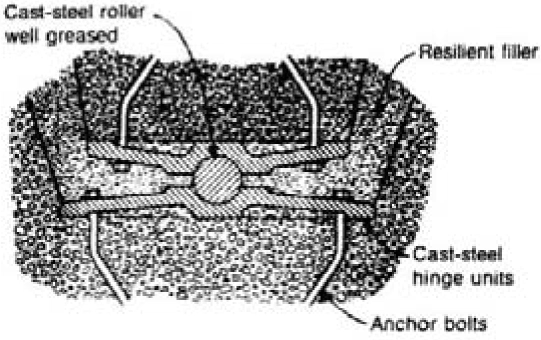
(c) Hinge at foundation



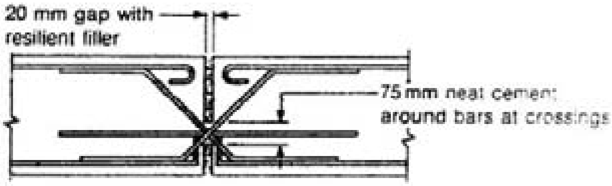
(d) Fixed-end bearing (hinge)



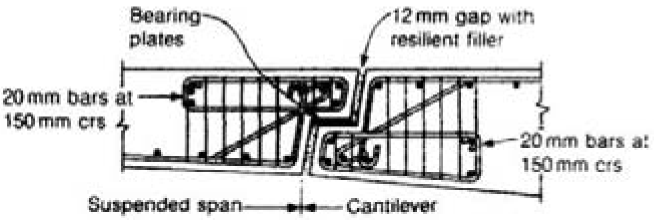
(e) Free-end bearing (hinge)



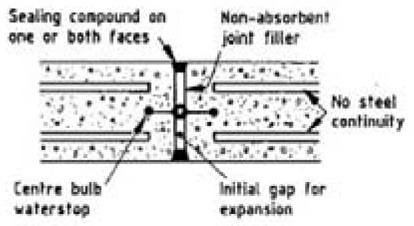
(f) Mechanical hinge



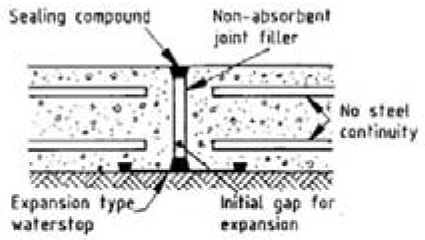
(g) Hinge at crown of arch



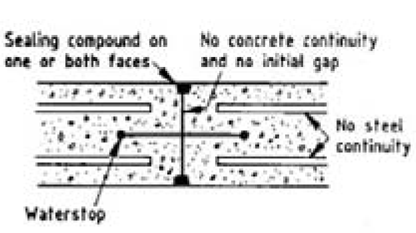
(h) Suspension bearing



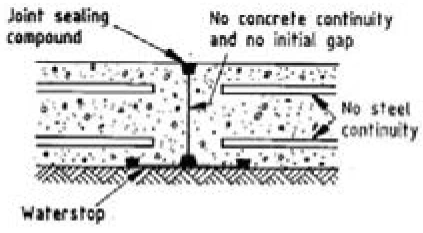
(a) Expansion joint in reservoir wall



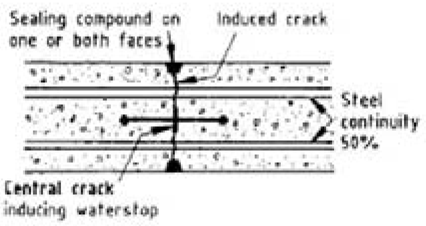
(b) Expansion joint in reservoir floor



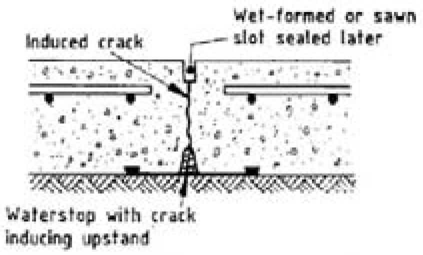
(c) Complete contraction joint in wall



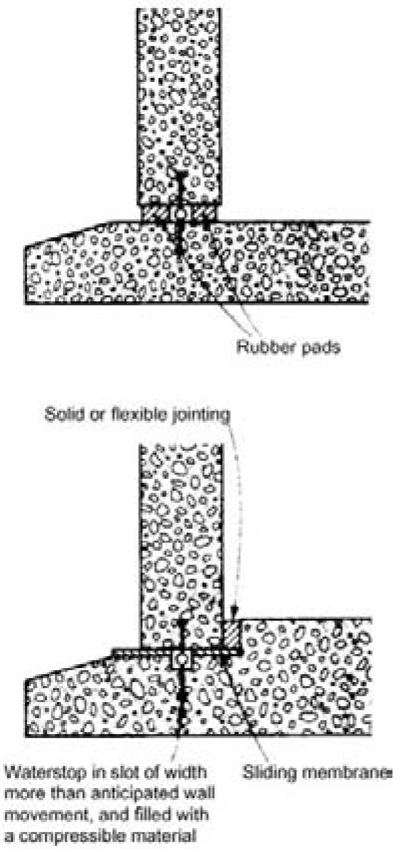
(d) Complete contraction joint in floor



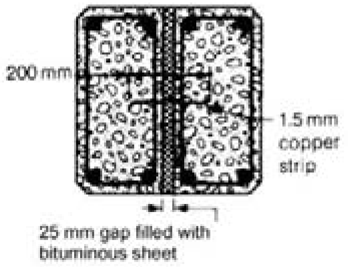
(e) Induced partial contraction joint in wall



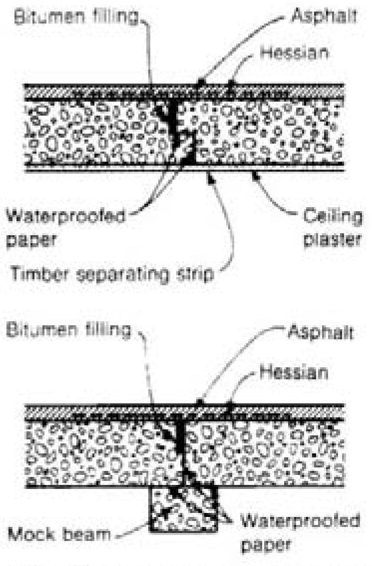
(f) Induced complete contraction joint in floor



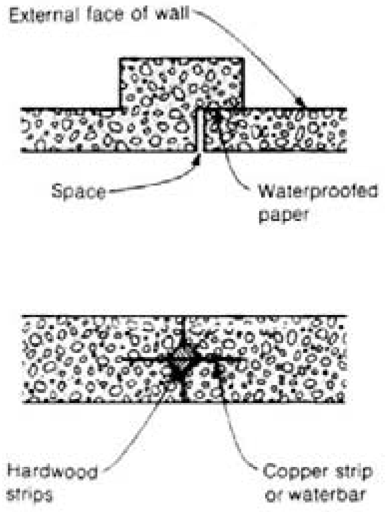
(g) Alternative designs for joints at base of wall to cylindrical tank



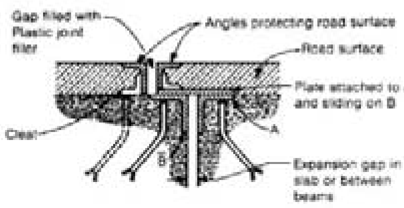
(h) Expansion joint at column



(j) Alternative designs for joints in roof slabs



(k) Alternative designs for joints in external walls of buildings



(i) Expansion joint in bridge deck

column as shown at (h) can be provided. The copper strip or other similar type of waterbar must be notched where the links occur, the ends of the notched pieces being bent horizontally or cut off. At joints in suspended floors and flat roofs, a double beam can be provided. Joints in floors should be sealed to prevent the accumulation of rubbish. Roof joints should also be provided with waterstops. The provision of joints in large-area industrial ground floor slabs is considered in section 7.2.2 and recommended details are given in ref. 61.

Expansion joints in bridges need to be either waterproof or designed to allow for drainage, and should not badly disrupt the riding quality of the deck. Joints should also be designed to require minimal maintenance during their lifetime, and be able to be replaced if necessary. Compressible materials such as neoprene or rubber can accommodate small movements. In this case, joints can be buried and covered by the surfacing. This type of joint, which consists of a small gap covered by a galvanised steel plate, and a band of rubberised bitumen flexible binder to replace part of the surfacing, is known as an asphaltic plug. To accommodate larger movements, a flexible sealing element supported by steel edge beams is required. Mechanical joints based on interlocking sets of steel toothed plates can be used for very large movements.

Chapter 20

Elastic analysis of concrete sections

The elastic analysis of a reinforced concrete section, by the modular ratio method, is applicable to the behaviour of the section under service loads only. The strength of the concrete in tension is neglected, and a linear stress–strain relationship is assumed for both concrete and reinforcement. The strain distribution across the section is also assumed to be linear. Thus, the strain at any point on the section is proportional to the distance of the point from the neutral axis and, since the stress–strain relationship is linear, the stress in the concrete is also proportional to the distance from the neutral axis. This gives a triangular distribution of stress, ranging from zero at the neutral axis to a maximum at the outermost point on the compression face. Assuming no slipping occurs between the reinforcement and the surrounding concrete, the strain in both materials is the same, and the ratio of the stresses in the two materials depends on the ratio of the modulus of elasticity of steel and concrete, known as the modular ratio α_e. The value of E_s is taken as 200 kN/mm^2, but the value of E for concrete depends on several factors, including the aggregate type, the concrete strength, and the load duration. Commonly adopted values for sustained loads, are 15 for normal-weight concrete and 30 for lightweight concrete. The geometrical properties of reinforced concrete sections can be expressed in equivalent concrete units, by multiplying the reinforcement area by α_e.

20.1 PURE BENDING

Expressions for the properties of common reinforced concrete sections are given in *Tables 2.102* and *2.103*. For sections that are entirely in compression, where the presence of the reinforcement is ignored, simplified expressions are given in *Table 2.101*. The maximum stress in the concrete is given by $f_c = M\bar{x}/I$ for sections entirely in compression. In other cases, $f_c = M/K_2\,z$ unless expressed otherwise, and the stress in the outermost tension reinforcement is given by $f_s = \alpha_e f_c\,(d/x - 1)$.

Expressions for the properties of rectangular and flanged sections are also given in *Table 3.42*, in connection with the serviceability calculation procedures contained in BS 8110, BS 5400 and BS 8007 (see Chapter 26).

20.2 COMBINED BENDING AND AXIAL FORCE

The general analysis of any section, subjected to direct thrust and uniaxial bending, is considered in *Table 2.104*. In the case of symmetrically reinforced rectangular columns, the design charts on *Tables 2.105* and *2.106* apply. The design charts on *Table 2.107* apply to annular sections, such as hollow masts. Information on uniaxial bending combined with direct tension, and biaxial bending and direct force, is given in *Tables 2.108* and *2.109* respectively.

20.2.1 Symmetrically reinforced rectangular section

For a symmetrically reinforced rectangular section subjected to axial force N and bending moment M, by equating forces and taking moments about the mid-depth of the section, the following expressions are obtained:

(1) For values of $x > h$ (i.e. entire section in compression),

$$\frac{N}{bhf_c} = \frac{A}{bh} - \left(\frac{Ah^2}{21}\right)\left(\frac{M}{bh^2 f_c}\right) \qquad \text{where}$$

$$A = [1 + (\alpha_e - 1)\,\rho]bh$$

$$I = \left\{\frac{1}{12} + \left[\rho(\alpha_e - 1)\left(\frac{d}{h} - 1\right)\right]^2 bh^3\right\}$$

(2) For values of $x < h$ (i.e. one face in tension),

$$\frac{N}{bhf_c} = 0.5\frac{x}{h} + 0.5\rho(\alpha_e - 1)\left(1 - \frac{h-d}{x}\right) - 0.5\rho\alpha_e\left(\frac{d}{h} - 1\right)$$

$$\frac{M}{bh^2 f_c} = 0.5\frac{x}{h} + \left(0.5 - \frac{x}{3h}\right) + 0.5\rho(\alpha_e - 1)\left(1 - \frac{h-d}{x}\right)$$

$$\left(\frac{d}{h} - 0.5\right) + 0.5\rho\alpha_e\left(\frac{d}{x} - 1\right)\left(\frac{d}{h} - 0.5\right)$$

where f_c is maximum stress in concrete at compression face, and $\rho = A_{sc}/bh$ is total reinforcement ratio.

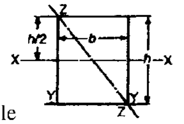
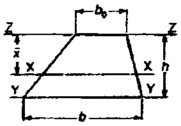
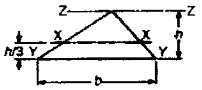
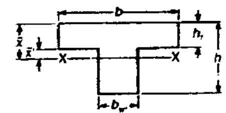
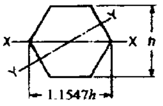
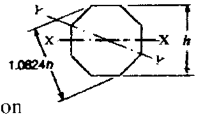
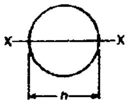
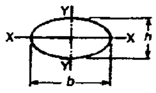
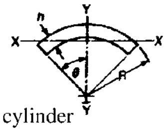
The stress in the tension reinforcement is then given by:

$$f_s = \alpha_e f_c(d/x - 1)$$

For $\alpha_e = 15$, the charts given on *Tables 2.105* and *2.106* can be used directly. For other values of α_e, the curves for ρ may be considered to represent values of $[(\alpha_e - 1)/14]\,\rho$ in region (1) and $(\alpha_e/15)\,\rho$ in region (2). For given values of M, N and f_c, the required value of ρ can be readily determined. For given values

2.101

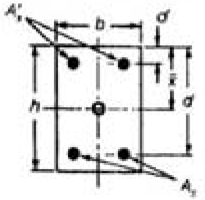
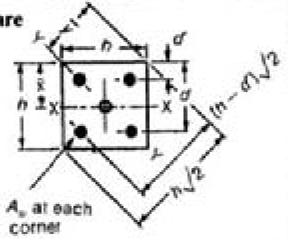
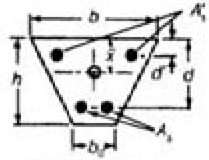
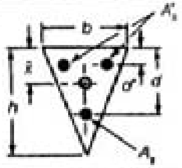
Geometric properties of uniform sections

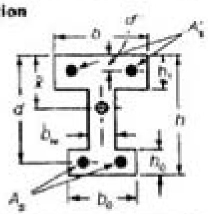
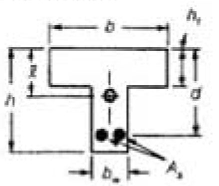
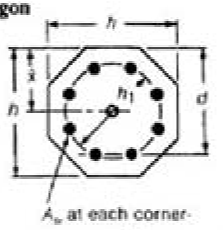
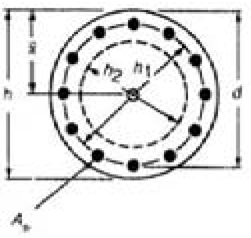
Section	Area A	Section modulus J_c	Second moment of area I	Radius of gyration i
 Rectangle	bh	About XX: $\frac{1}{6}bh^2$ About ZZ: $\frac{1}{6}\frac{b^2h^2}{\sqrt{(b^2+h^2)}}$	About XX: $\frac{1}{12}bh^3$ About YY: $\frac{1}{3}bh^3$ About ZZ: $\frac{1}{6}\frac{b^3h^3}{(b^2+h^2)}$	About XX: $h/\sqrt{12}=0.2887h$
 Trapezium	$\frac{1}{2}h(b_0+b)$	About XX to b_0: $\frac{(b^2+4bb_0+b_0^2)h^2}{12(b+2b_0)}$ To b: $I_{xx}/(h-\bar{x})$ where $\bar{x}=\frac{1}{3}h\left(\frac{2b+b_0}{b+b_0}\right)$	About XX: $\frac{(b^2+4bb_0+b_0^2)h^3}{36(b+b_0)}$ About YY: $I_{xx}+A(h-\bar{x})^2$ About ZZ: $I_{xx}+A(\bar{x})^2$	About XX: $\sqrt{\frac{I_{xx}}{A}}$
 Triangle	$\frac{1}{2}bh$	About XX to apex: $\frac{1}{24}bh^2$ About XX to base: $\frac{1}{12}bh^2$	About XX: $\frac{1}{36}bh^3$ About YY: $\frac{1}{12}bh^3$ About ZZ: $\frac{1}{4}bh^3$	About XX: $h/\sqrt{18}=0.2357h$
 Flanged section	$bh_f+b_w(h-h_f)$	About XX: $\frac{I_{xx}}{\bar{x}}$ or $\frac{I_{xx}}{h-\bar{x}}$ $\bar{x}=\frac{h_f^2(b-b_w)+h^2b_w}{2[h_f(b-b_w)+hb_w]}$	About XX: $\frac{1}{3}\left[b\bar{x}^3+b_w(h-\bar{x})^3-(\bar{x}')^3(b-b_w)\right]$ where $\bar{x}'=\bar{x}-h_f$	About XX: $\sqrt{\frac{I_{xx}}{A}}$
 Hexagon	$0.8660h^2$ (side $=0.5774h$)	About XX: $0.1203h^3$ About YY: $0.1042h^3$	About XX or YY: $0.0601h^4$	About XX or YY: $0.2635h$
 Octagon	$0.8284h^2$ (side $=0.4142h$)	About XX: $0.1095h^3$ About YY: $0.1011h^3$	About XX or YY: $0.0547h^4$	About XX or YY: $0.2570h$
 Circle	$\frac{1}{4}\pi h^2=0.7854h^2$	About XX: $\frac{1}{32}\pi h^3=0.0982h^3$	About XX: $\frac{1}{64}\pi h^4=0.0491h^4$	About XX: $0.2500h$
 Ellipse	$\frac{1}{4}\pi bh=0.7854bh$	About XX: $\frac{1}{32}\pi bh^2=0.0982bh^2$	About XX: $\frac{1}{64}\pi bh^3=0.0491bh^3$	About XX: $0.2500h$
 Sector of cylinder	$\theta h(2R-h)$	About XX: to top I_{xx}/y_t^* About XX: to bottom I_{xx}/y_b^*	About XX: I_{xx}^* About YY: I_{yy}^*	About XX: $\sqrt{\frac{I_{xx}}{A}}$

$$y_t^*=R\left\{1-\frac{2\sin\theta}{3\theta}\left(1-\frac{h}{R}+\frac{1}{2-(h/R)}\right)\right. \qquad y_b^*=R\left[\frac{2\sin\theta}{3\theta(2-(h/R))}+\left(1-\frac{h}{R}\right)\frac{2\sin\theta-3\theta\cos\theta}{3\theta}\right]$$

About XX: $I_{xx}^*=R^3h\left\{\left[1-\frac{3h}{2R}+\left(\frac{h}{R}\right)^2-\frac{1}{4}\left(\frac{h}{R}\right)^3\right]\left[\theta+\sin\theta\cos\theta-\frac{2\sin^2\theta}{\theta}\right]+\frac{h^2\sin^2\theta}{3R^2\theta(2-(h/R))}\left[1-\frac{h}{R}+\frac{1}{6}\left(\frac{h}{R}\right)^2\right]\right\}$

About YY: $I_{yy}^*=R^3h\left\{\left[1-\frac{3h}{2R}+\left(\frac{h}{R}\right)^2-\frac{1}{4}\left(\frac{h}{R}\right)^3\right]\left[\theta-\sin\theta\cos\theta\right]\right\}$

	Entire cross-section subjected to stress	Bending only with concrete ineffective in tension compression zone at top (as drawn)
Notation (additional to data on diagrams) Geometrical properties are expressed in equivalent concrete units	A_{tr} effective area $\bar{x}$ position of centroid from top edge I_{XX} moment of inertia about centroidal axis Modulus of section: For top edge: $J_0 = \frac{I_{XX}}{\bar{x}}$ For bottom edge: $J_h = \frac{I_{XX}}{h-\bar{x}}$ } unless expressed otherwise Radius of gyration: $i = \sqrt{\frac{I_{XX}}{A_{tr}}}$	Distance of neutral axis below top edge $= x$ Related to maximum stresses: $x = d \Big/ \left(1 + \frac{f_{st}}{\alpha_e f_{cr}}\right)$ Lever-arm $= z$ Compression-reinforcement factor $K_1 = A'_s(\alpha_e - 1)\left(\frac{x-d'}{x}\right)$ Moment of resistance: M_d (compression) $= zK_2 f_{cr}$ M_d (tension) $= zA_s f_{st}$ } unless expressed otherwise
Rectangle 	$A_{tr} = bh + (\alpha_e - 1)(A_s + A'_s)$ $\bar{x} = \frac{1}{A_{tr}}[\frac{1}{2}bh^2 + (\alpha_e - 1)(A_s d + A'_s d')]$ $= \frac{1}{2}h$ if $A_s = A'_s$ $I_{XX} = \frac{1}{3}b[\bar{x}^3 + (h-\bar{x})^3] + (\alpha_e - 1)[A_s(d-\bar{x})^2 + A'_s(\bar{x}-d')^2]$ $= \frac{1}{12}bh^3 + 2A_s(\alpha_e - 1)(\frac{1}{2}h - d')^2$ if $A_s = A'_s$	$x = \sqrt{\left\{\left[\alpha_e \frac{A_s}{b} + (\alpha_e - 1)\frac{A'_s}{b}\right]^2 + 2\left[\alpha_e \frac{A_s}{b} + (\alpha_e - 1)\frac{A'_s}{b}\frac{d'}{d}\right]\right\}}$ $- \left[\alpha_e \frac{A_s}{b} + (\alpha_e - 1)\frac{A'_s}{b}\right]$ $z = \frac{1}{K_2}[\frac{1}{2}bx(d - \frac{1}{3}x) + K_1(d-d')]$ $K_2 = \frac{1}{2}bx + K_1$ If $A'_s = 0$, $z = d - \frac{1}{3}x$ and M_d(compression) $= \frac{1}{2}bxzf_{cr} = K_{conc}bd^2$
Square 	$A_{tr} = h^2 + 4(\alpha_e - 1)A_{sl}$ About axis XX: $\bar{x} = \frac{1}{2}h$ $I_{XX} = \frac{1}{12}h^4 + 4A_{sl}(\alpha_e - 1)(\frac{1}{2}h - d')^2$ $J_0 = J_h = \frac{2I_{XX}}{h}$ About diagonal axis YY: $\bar{y} = 0.707h$ $I_{YY} = \frac{1}{12}h^4 + 2A_{sl}(\alpha_e - 1)(\frac{1}{2}h - d')^2$ $J_0 = J_h = \frac{2\sqrt{(2)}I_{YY}}{h}$	About axis XX: formulae as for rectangular section with $A_s = A'_s = 2A_{sl}$ and $b = h$. About diagonal axis YY: $x \ngtr 0.707h$ $z = \frac{1}{K_2}\left[\frac{x^2}{3\sqrt{2}}\left(d\sqrt{2} - \frac{4x}{9}\right) + A_{sl}(\alpha_e - 1)\left(\frac{x - d'\sqrt{2}}{x}\right)(h - 2d')\sqrt{2}\right]$ $K_2 = \frac{x^2}{3\sqrt{2}} + A_{sl}(\alpha_e - 1)\left(\frac{x - d'\sqrt{2}}{x}\right)$ $A_s = A_{sl}$
Trapezium 	$A_{tr} = \frac{1}{2}(b + b_0)h + (\alpha_e - 1)(A_s + A'_s)$ $\bar{x} = \frac{1}{A_{tr}}\left[\frac{h^2}{6}(b + 2b_0) + (\alpha_e - 1)(A_s d + A'_s d')\right] = \frac{h}{3}\left(\frac{b + 2b_0}{b + b_0}\right)$ approx. $I_{XX} = \left[\frac{(b + b_0)^2 + 2bb_0}{b + b_0}\right]\frac{h^3}{36} + (\alpha_e - 1)[A_s(d-\bar{x})^2 + A'_s(\bar{x}-d')^2]$	$z = \frac{1}{K_2}\left\{\frac{1}{2}x\left[b(d - \frac{1}{3}x) - \frac{x}{3h}(b - b_0)\left(d - \frac{4x}{9}\right)\right] + K_1(d-d')\right\}$ $K_2 = \frac{1}{2}x\left[b - \frac{x}{3h}(b - b_0)\right] + K_1$
Triangle 	$A_{tr} = \frac{1}{2}bh + (\alpha_e - 1)(A_s + A'_s)$ $\bar{x} = \frac{1}{A_{tr}}[\frac{1}{6}bh^2 + (\alpha_e - 1)(A_s d + A'_s d')] = \frac{1}{3}h$ approx. $I_{XX} = \frac{bh^3}{36} + (\alpha_e - 1)[A_s(d-\bar{x})^2 + A'_s(\bar{x}-d')^2]$	$z = \frac{1}{K_2}\left\{\frac{bx}{2}\left[(d - \frac{1}{3}x) - \frac{x}{3h}\left(d - \frac{4x}{9}\right)\right] + K_1(d-d')\right\}$ $K_2 = \frac{1}{2}bx\left(1 - \frac{x}{3h}\right) + K_1$

I-section  	$A_{tr} = bh_f + b_0h_0 + b_w(h - h_f - h_0) + (\alpha_e - 1)(A_s + A'_s)$ $\bar{x} = \frac{1}{2A_{tr}}[(b - b_w)h_f^2 + (b_0 - b_w)(2h - h_0)h_0 + b_wh^2 + 2(\alpha_e - 1)(A_sd + A'_sd')]$ $I_{xx} = \frac{1}{3}[b\bar{x}^3 - (b - b_w)(\bar{x} - h_f)^3 + b_0(h - \bar{x})^3 - (b_0 - b_w)(h - \bar{x} - h_0)^3] + (\alpha_e - 1)[A'_s(\bar{x} - d')^2 + A_s(d - \bar{x})^2]$	If $x \ngtr h_f$: use formulae for rectangle If $x > h_f$: $z = \frac{1}{K_2}\left[\frac{1}{2}bx(d - \frac{1}{3}x) - \frac{1}{6x}(b - b_w)(x - h_f)(3d - 2h - x) - K_1(d - d')\right]$ $K_2 = \frac{1}{2}\left[bx - \frac{1}{x}(b - b_w)(x - h_f)\right] + K_1$
T- and L-sections  	$A_{tr} = bh_f + b_w(h - h_f) + (\alpha_e - 1)A_s$ $\bar{x} = \frac{1}{2A_{tr}}[bh_f^2 + b_w(h - h_f)(h + h_f) + 2A_sd(\alpha_e - 1)]$ $I_{xx} = \frac{1}{3}[b\bar{x}^3 - (b - b_w)(\bar{x} - h_f)^3 + b_w(h - \bar{x})^3] + (\alpha_e - 1)A_s(d - \bar{x})^2$	If $x \ngtr h_f$: use formulae for rectangle If $x > h_f$: $x = \frac{(1/2)bh_f^2 + A_s\alpha_ed}{bh_f + A_s\alpha_e}$ $z = d - \left(\frac{3x - 2h_f}{2x - h_f}\right)\frac{h_f}{3} = d - \frac{1}{2}h_f$ approx. M_d (compression) $= (2x - h_f)\frac{zf_{cr}bh_f}{2x}$
Octagon  $d = h - d' \quad d' = \frac{1}{2}h - 0.462h_1$	$A_{tr} = 0.828h^2 + 8A_{sl}(\alpha_e - 1)$ $\bar{x} = \frac{1}{2}h$ $I_{xx} = 0.055h^4 + 4A_{sl}(\frac{1}{2}h_1)^2(\alpha_e - 1)$ $J_0 = J_h = 0.109h^3 + 4A_{sl}\frac{(h_1/2)^2}{h}(\alpha_e - 1)$	$z = \frac{1}{K_2}\left[0.207hx\left(h - \frac{x}{8}\right) + K_3 + 2A_{sl}(\alpha_e - 1)\left(\frac{x - d'}{x}\right)(d - d')\right]$ $K_2 = 0.207hx + K_4 + 2A_{sl}(\alpha_e - 1)\left(\frac{x - d'}{x}\right)$ If $x \ngtr 0.29h$: $K_3 = K_4\left(d - \frac{4x}{9}\right) \qquad K_4 = \frac{1}{3}x^2$ If $x > 0.29h$: $K_3 = K_4\left[d - 0.195h\left(\frac{x - 0.22h}{x - 0.195h}\right)\right] \qquad K_4 = \frac{0.086h^2}{x}(x - 0.195h)$
Circle and annulus  a = depth of individual bar from top of section	For annulus: $A_{tr} = 0.7854(h^2 - h_2^2) + (\alpha_e - 1)\Sigma A_{sl}$ $\bar{x} = 0.5h$ $I_{xx} = \frac{1}{64}\pi(h^4 - h_2^4) + (\alpha_e - 1)\frac{1}{2}(\frac{1}{2}h_1)^2\Sigma A_{sl}$ $J_0 = J_h = \frac{\pi}{32h}(h^4 - h_2^4) + (\alpha_e - 1)\frac{(h_1/2)^2}{h}\Sigma A_{sl}$ For circle: use above formulae with $h_2 = 0$	For circle: M_d(compression) $= M_d$(tension) $= [K_5(a_t - a_c) + K_6(a_t - a'_c)]f_{cr}$ $K_5 = \left(\frac{h^2}{4x} - \frac{h - x}{3}\right)\sqrt{[(h - x)x]} + \frac{h^2}{8x}(h - 2x)\sin^{-1}\left(\frac{h - 2x}{h}\right)$ $K_6 = \frac{\alpha_e - 1}{x}\Sigma_0^x(x - a)A_{sl}$ $a_t = \frac{\Sigma_x^d(a - x)aA_{sl}}{\Sigma_x^d(a - x)A_{sl}} \qquad a'_c = \frac{\Sigma_0^x(x - a)aA_{sl}}{\Sigma_0^x(x - a)A_{sl}}$ $a_c = \frac{1}{K_5}\left\{\left[\frac{2}{3}x^2 - \frac{7}{12}h\left(x + \frac{h}{4}\right) + \frac{5}{32x}h^3\right]\sqrt{[(h - x)x]} + \frac{1}{2}\sqrt{[(h - x)^3x^3]} + \frac{h^3}{8x}(\frac{5}{8}h - x)\sin^{-1}\left(\frac{h - 2x}{h}\right)\right\}$ For annulus: if $x \ngtr (h - h_2)/2$ use above formulae; otherwise use graphical method

Uniaxial bending and compression (modular ratio)

Compressive stresses only

Equivalent area of strip: $\delta A_{tr} = b_s h_s + (\alpha_e - 1)\delta A'_s$

Equivalent area of transformed section: $A_{tr} = \sum \delta A_{tr}$

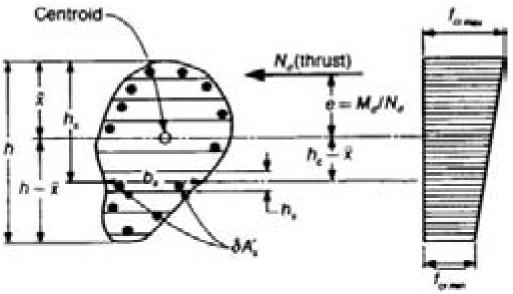
Depth of centroid of transformed section: $\bar{x} = \sum \delta A_{tr} h_c / A_{tr}$

Moment of inertia of transformed section about centroid:

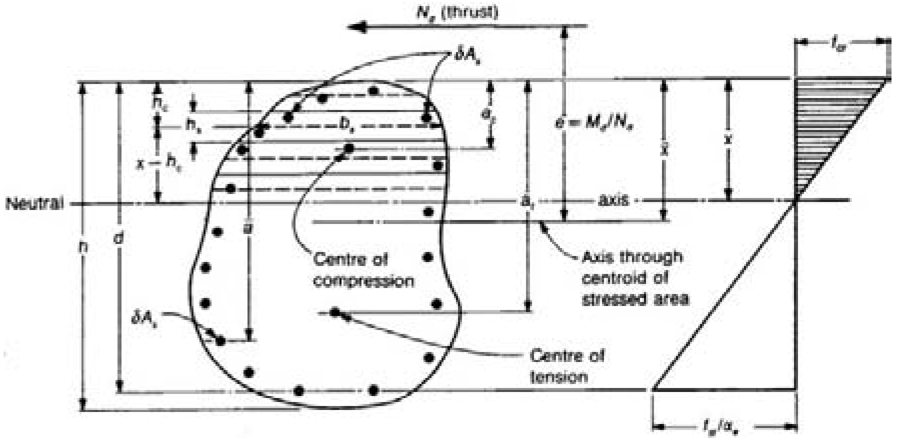
$$I_{tr} = \sum \delta A_{tr}\left[\frac{(h_s)^2}{12} + (h_c - \bar{x})^2\right]$$

Compressive stresses:

$$f_{cr\,max} = \frac{N_d}{A_{tr}} + \frac{M_d \bar{x}}{I_{tr}} \qquad f_{cr\,min} = \frac{N_d}{A_{tr}} - \frac{M_d(h - \bar{x})}{I_{tr}}$$



Combined compressive and tensile stresses



Assume a value of x.

Depth to centre of tension: $a_t = \sum Sa / \sum S$, where $S = (a - x)\delta A_s$

If all bars are of the same size: $a_t = \sum a(a - x)/\sum(a - x)$

Equivalent area of strip: $\delta A_{tr} = b_s h_s + (\alpha_e - 1)\delta A'_s$

Depth to centre of compression: $a_c = \sum(x - h_c)h_c \delta A_{tr} / \sum(x - h_c)\delta A_{tr}$

Position of centroid of stressed area:

$$\bar{x} = \frac{\alpha_e \sum \delta A_s a + \sum \delta A_{tr} h_c}{\alpha_e \sum \delta A_s + \sum \delta A_{tr}}$$

Maximum stresses:

$$f_{cr} = \frac{N_d x(e + a_t - \bar{x})}{(a_t - a_c)\sum(x - h_c)\delta A_{tr}}$$

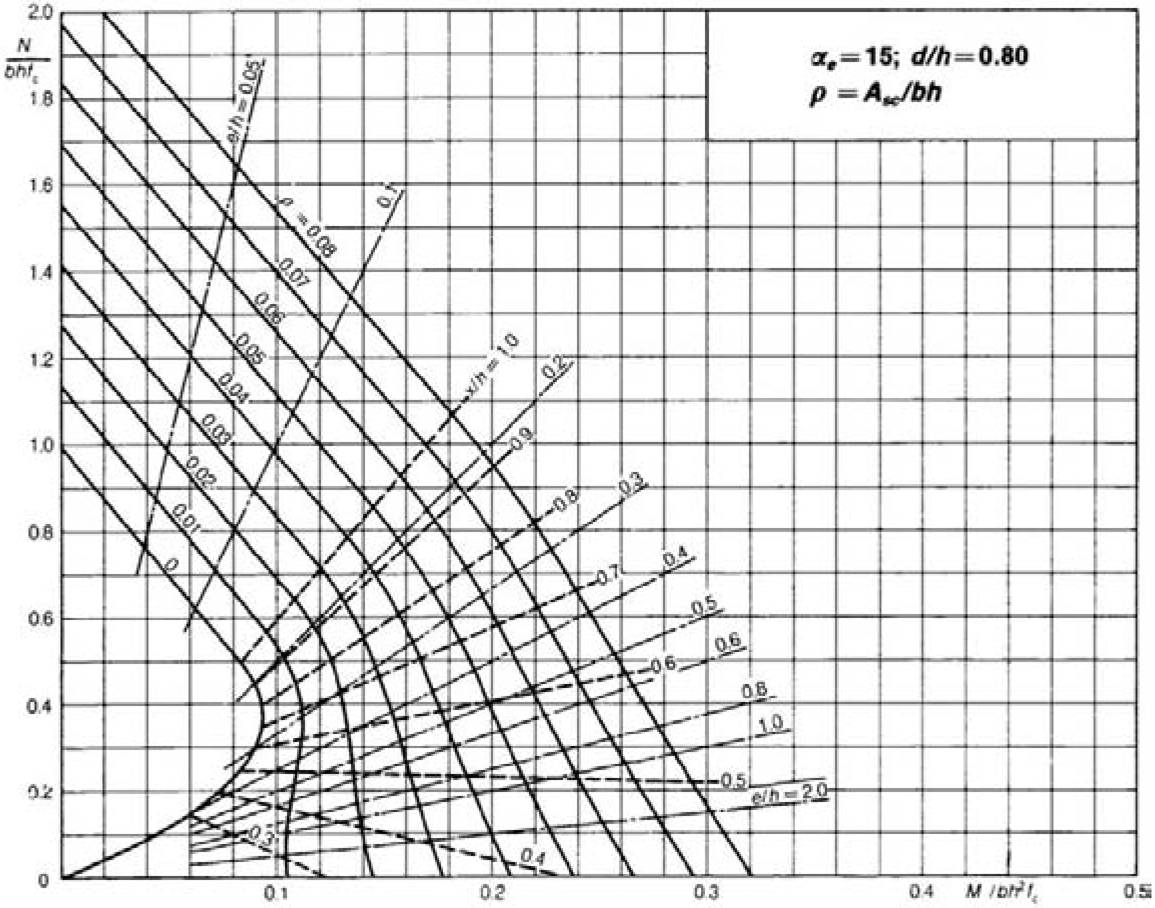
$$f_{st} = \frac{(d - x)}{\sum S}\left[\frac{f_{cr}}{x}\sum(x - h_c)\delta A_{tr} - N_d\right]$$

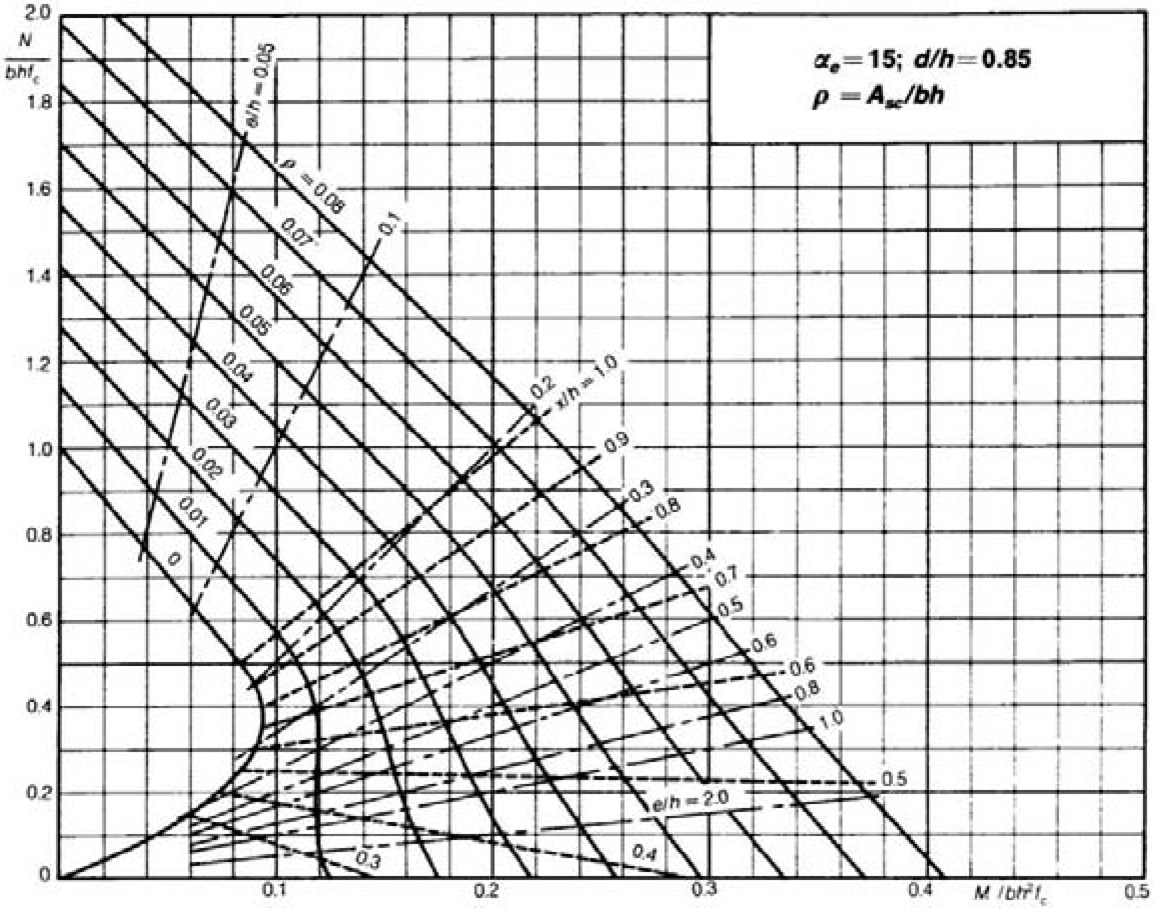
Finally, check the assumed value of x by substituting these stresses in

$$\frac{x}{d} = \frac{1}{1 + f_{st}/(\alpha_e f_{cr})}$$

Symmetrically reinforced rectangular columns (modular ratio) – 1

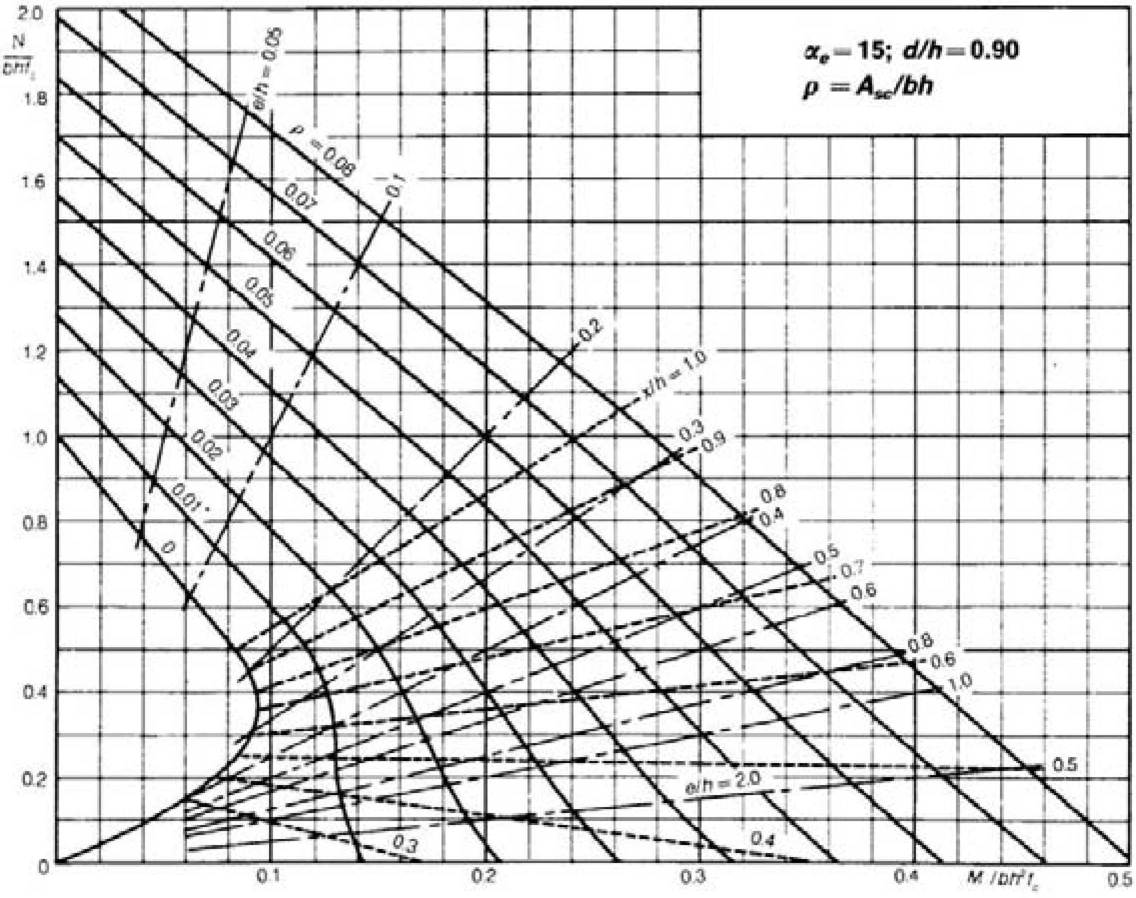
2.105

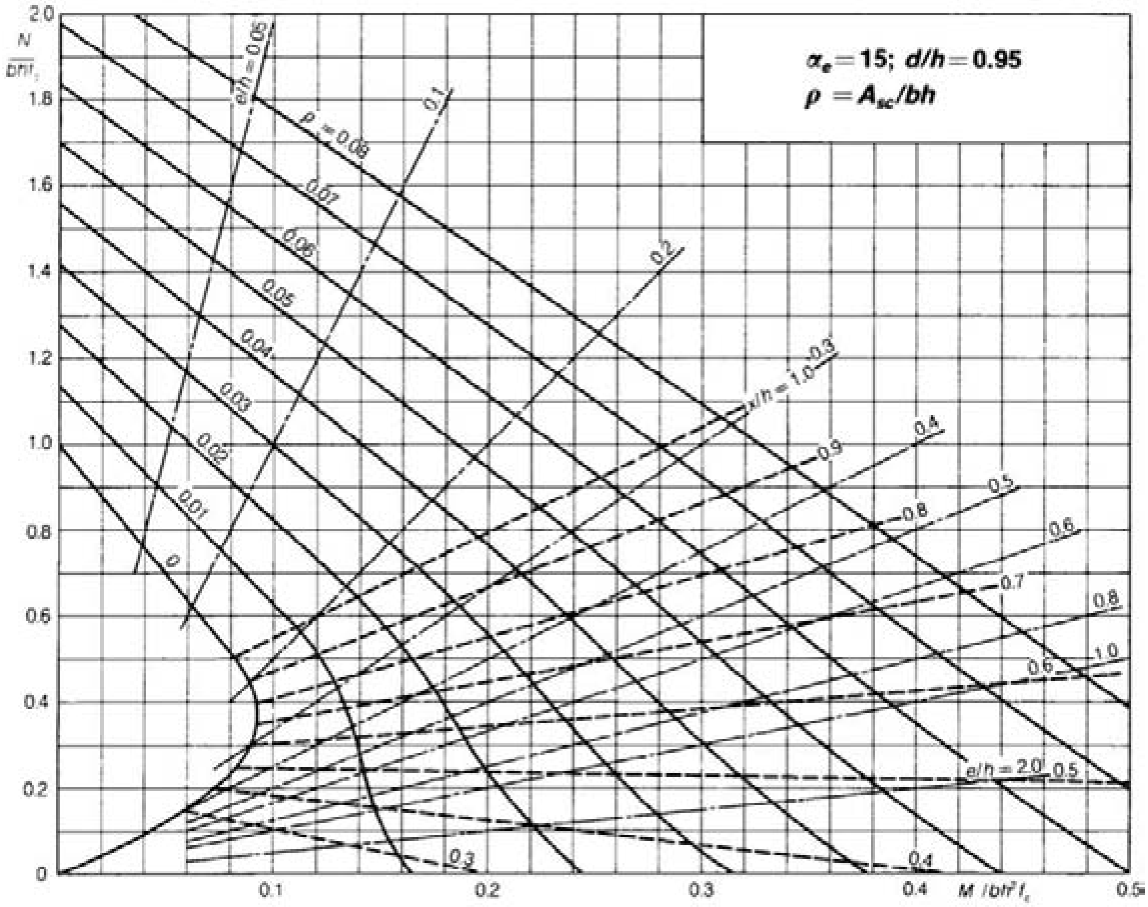


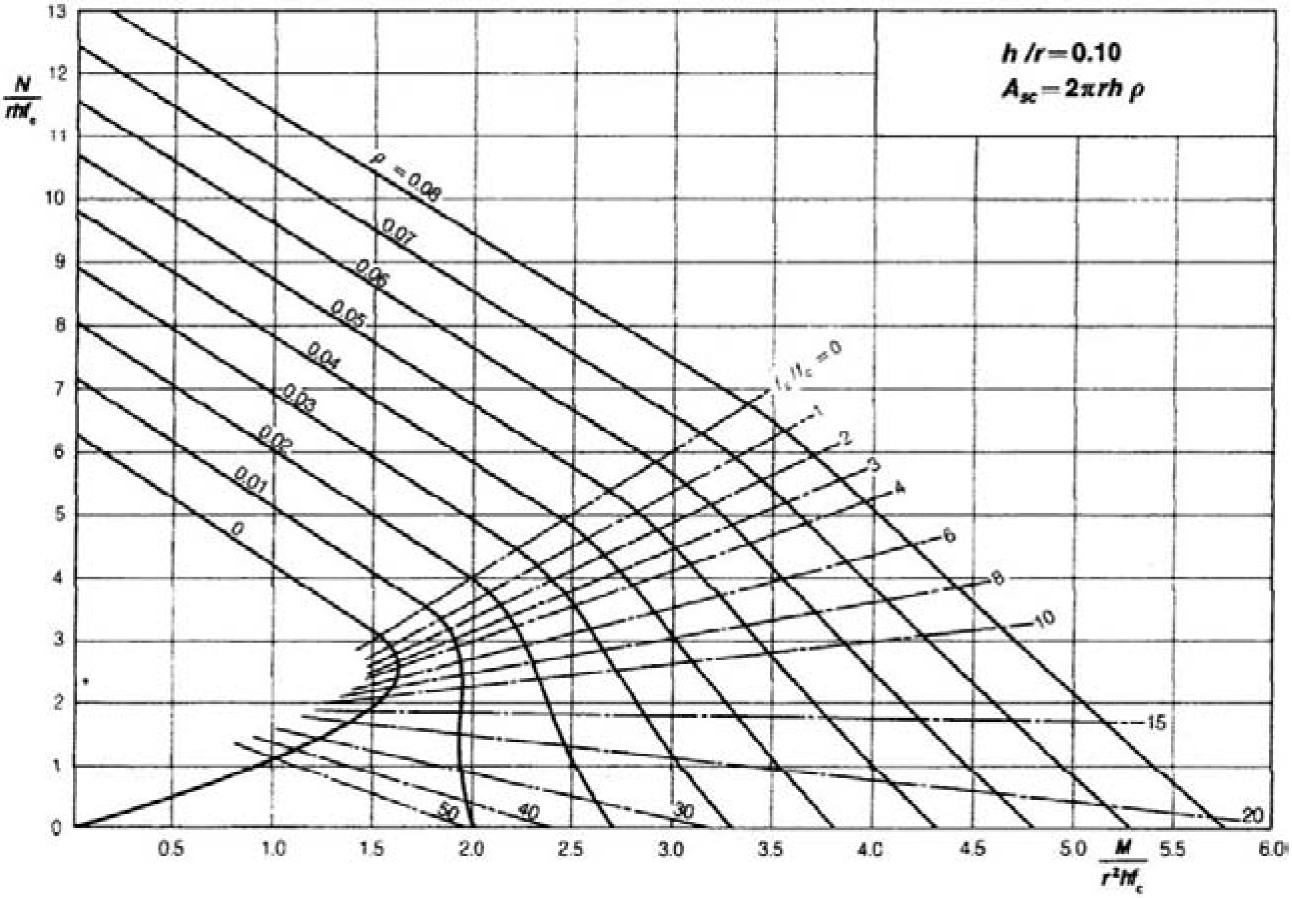


Symmetrically reinforced rectangular columns (modular ratio) – 2

2.106






$h/r = 0.10$
$A_{sc} = 2\pi rh\rho$
$\frac{N}{rhf_c}$
$\frac{M}{r^2hf_c}$
$\rho = 0.08$
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
$f_s/f_c = 0$
1
2
3
4
6
8
10
15
20
30
40
50
0
1
2
3
4
5
6
7
8
9
10
11
12
13
0.5
1.0
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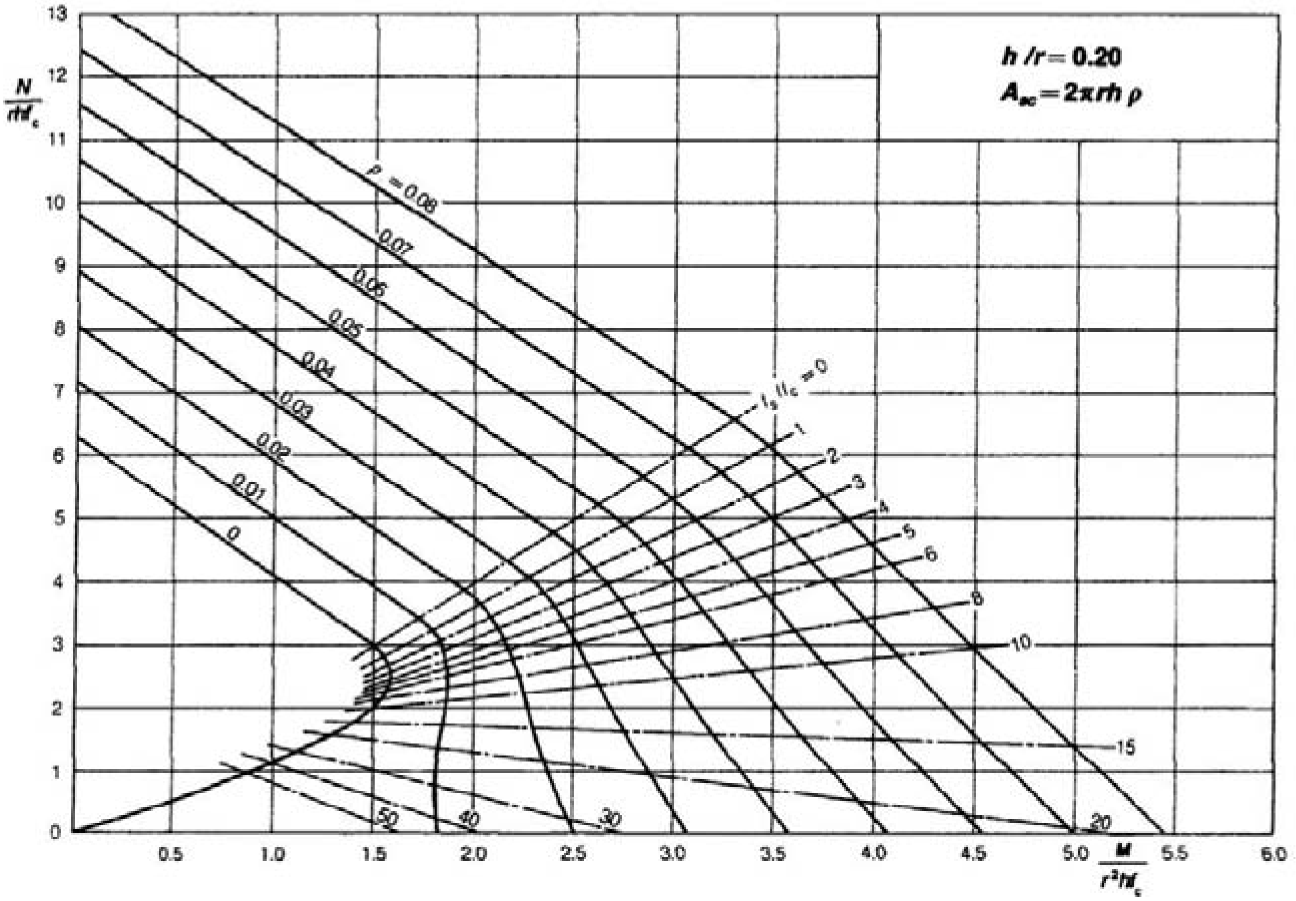
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0


$h/r = 0.20$
$A_{sc} = 2\pi rh\rho$
$\frac{N}{rhf_c}$
$\frac{M}{r^2hf_c}$
$\rho = 0.08$
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
$f_s/f_c = 0$
1
2
3
4
5
6
8
10
15
20
30
40
50
0
1
2
3
4
5
6
7
8
9
10
11
12
13
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0

Uniaxial bending and tension (modular ratio)

Tensile stresses only: $e \ngtr z_s - \bar{x}_s$ (concrete ineffective in tension)

Irregular groups of bars

Group A_{s1} = bars in tension due to action of moment only

$$\bar{x}_s = \frac{A_{s1}z_s}{A_{s1} + A_{s2}}$$

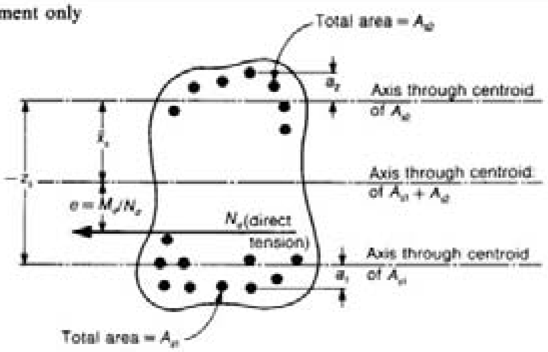
Average tensile stresses in reinforcement:

In group A_{s1}: $f_{st1} = \dfrac{N_d(e + \bar{x}_s)}{z_s A_{s1}}$

In group A_{s2}: $f_{st2} = \dfrac{N_d(z_s - e - \bar{x}_s)}{z_s A_{s2}}$

Maximum tensile stress (in outer bar or bars in group A_{s1}):

$$f_{st1\,max} = f_{st2} + \left(\frac{z_s + a_1}{z_s}\right)(f_{st1} - f_{st2})$$



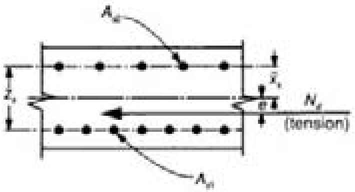
Regular groups of bars

If $A_{s1} \neq A_{s2}$, $\bar{x}_s = \dfrac{A_{s1}z_s}{A_{s1} + A_{s2}}$

Tensile stresses in reinforcement:

In group A_{s1}: $f_{st1} = \dfrac{N_d(e + \bar{x}_s)}{z_s A_{s1}}$

In group A_{s2}: $f_{st2} = \dfrac{N_d(z_s - e - \bar{x}_s)}{z_s A_{s2}}$



If $A_{s1} = A_{s2} = A_s$ at each face,

Tensile stresses:

In group A_{s1}: $f_{st1} = \dfrac{N_d[e + z_s/2]}{z_s A_s}$

In group A_{s2}: $f_{st2} = \dfrac{N_d[(z_s/2) - e]}{z_s A_s}$

Compressive and tensile stresses (concrete ineffective in tension)

Non-rectangular section

Assume depth to neutral axis x
Depth to centre of tension:

$$a_t = \frac{\sum Sa}{\sum S}$$

where $S = (a - x)\delta A_s$
If all bars are of same size:

$$a_t = \frac{\sum a(a - x)}{\sum (a - x)}$$

Equivalent area of strip:

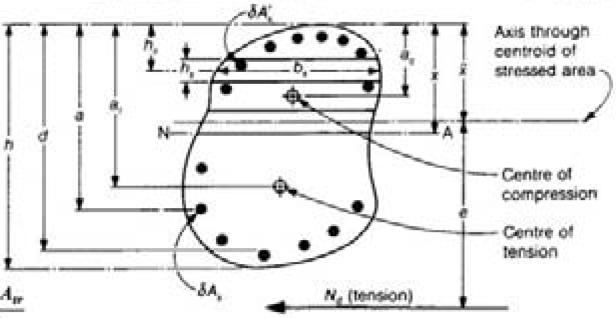
$\delta A_{tr} = b_s h_s + (\alpha_e - 1)\delta A_s$

Depth to centre of compression $a_c = \dfrac{\sum (x - h_c)h_c\delta A_{tr}}{\sum (x - h_c)\delta A_{tr}}$

Position of centroid of stressed area: $\bar{x} = \dfrac{\alpha_e \sum \delta A_s a + \sum h_c \delta A_{tr}}{\alpha_e \sum \delta A_s + \sum \delta A_{tr}}$

Maximum stresses: $f_{cr} = \dfrac{N_d x(e - a_t + \bar{x})}{(a_t - a_c)\sum (x - h_c)\delta A_{tr}}$; $\quad f_{st} = \dfrac{(d - x)}{\sum S}\left[\dfrac{f_{cr}}{x}\sum (x - h_c)\delta A_{tr} + N_d\right]$

Finally check assumed value of x.



Rectangular section

$e > z_s - \bar{x}$

Assume value of x; evaluate x/d and $K_1 = bx$.

$$\bar{x} = \frac{(K_1 x/2) + \alpha_e A_s d + (\alpha_e - 1)A_s'd'}{K_1 + \alpha_e A_s + (\alpha_e - 1)A_s'} (= \tfrac{1}{2}h \text{ approx.})$$

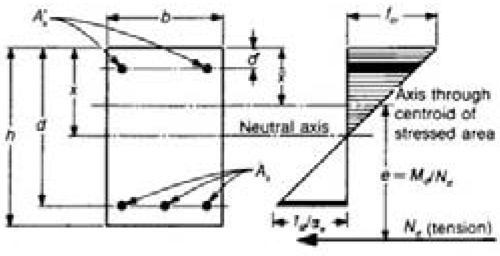
Calculate $\beta_1 = \dfrac{e + \bar{x}}{d} - 1$; $\quad \beta_2 = \dfrac{1}{2}\dfrac{x}{d}\left(1 - \dfrac{1}{3}\dfrac{x}{d}\right)$

$$\beta_3 = (\alpha_e - 1)\left(1 - \frac{d'}{x}\right)$$

Determine stresses by substituting in

$$f_{cr} = \frac{N_d\beta_1}{\beta_2 bd + \beta_3 A_s'(1 - d'/d)}$$

$f_{st} = [f_{cr}(\tfrac{1}{2}K_1 + \beta_3 A_s') + N_d]/A_s$ Finally check assumed value of x.



$e > \tfrac{2}{3}h$

Calculate f_{cr} and f_{st} due to moment only and determine x for these stresses.

Evaluate $f_c = \dfrac{N_d}{bx + \alpha_e A_s + (\alpha_e - 1)A_s'}$. Then $f_{cr\,max} = f_{cr} - f_c$ and $f_{st\,max} = f_{st} + \alpha_e f_c$

Approx. method for slabs

Evaluate $e = M_d/N_d$ and $e_1 = e + \tfrac{1}{2}h - d$; then $A_{s\,req} = \dfrac{N_d}{f_{st}}\left(1 + \dfrac{e_1}{z}\right)$ or $f_{st} = \dfrac{N_d}{A_s}\left(1 + \dfrac{e_1}{z}\right)$

Rectangular member subjected to bending about two axes

Compressive stresses only developed

Section moduli:

J_{xo} about X-X, at edge A-B $= I_x/\bar{x}$

J_{xh} about X-X, at edge C-D $= I_x/(h-\bar{x})$

J_{yo} about Y-Y, at edge B-C $= I_y/\bar{y}$

J_{yb} about Y-Y, at edge A-D $= I_y/(b-\bar{y})$

M_{dy} bending moment in plane Y-Y

M_{dx} bending moment in plane X-X

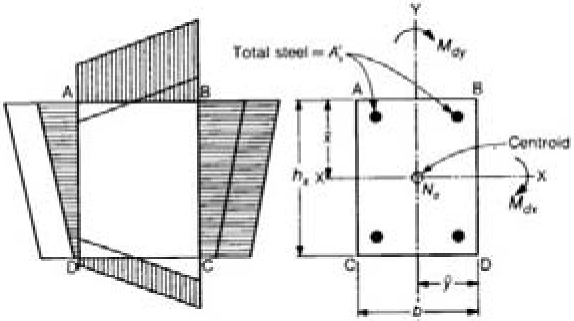
A_{tr} transformed area $= bh + (\alpha_e - 1)A'_s$

Stresses: $f_{cc} = N_d/A_{tr}$ where $N_d =$ concentric thrust; if no thrust, $f_{cc} = 0$.

At A: $f_{cr} = \left(f_{cc} + \dfrac{M_{dy}}{J_{xo}}\right) - \dfrac{M_{dx}}{J_{yb}}$

At B: $f_{cr} = \left(f_{cc} + \dfrac{M_{dy}}{J_{xo}}\right) + \dfrac{M_{dx}}{J_{yo}}$ (= maximum value)

At C: $f_{cr} = \left(f_{cc} - \dfrac{M_{dy}}{J_{xh}}\right) + \dfrac{M_{dx}}{J_{yo}}$



At D: $f_{cr} = \left(f_{cc} - \dfrac{M_{dy}}{J_{xh}}\right) - \dfrac{M_{dx}}{J_{yb}}$ (= minimum value)

Tensile and compressive stresses

Notation as above, where applicable.

Also $J_x = \frac{1}{6}xb^2 + \dfrac{(\alpha_e - 1)(A_s + A'_s)z_b^2}{2b}$

Approximate stresses when $M_{dy} > M_{dx}$:

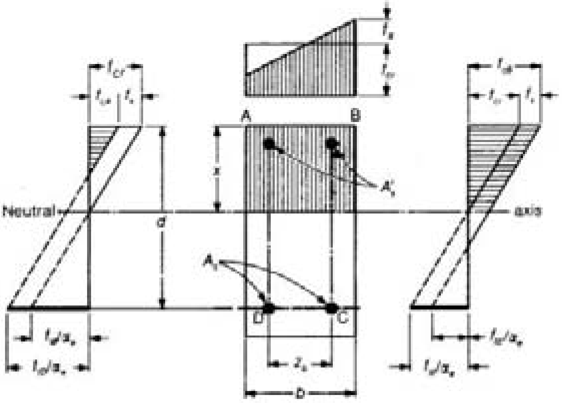
1. Calculate f_{st}, f_{cr} and x for M_{dy} combined with N_d or for M_{dy} alone if no thrust.
2. Calculate $M_{dx}/J_x = f_x$.
3. Resultant stresses are as follows:

Compression in concrete at A: $f_{cA} = f_{cr} - f_x$

Compression in concrete at B: $f_{cB} = f_{cr} + f_x$ (= maximum value)

Tension in reinforcement at C: $f_{tC} = f_{st} - \alpha_e f_x$

Tension in reinforcement at D: $f_{tD} = f_{st} + \alpha_e f_x$ (= maximum value)



Principal stresses

f_{pt} principal tensile stress

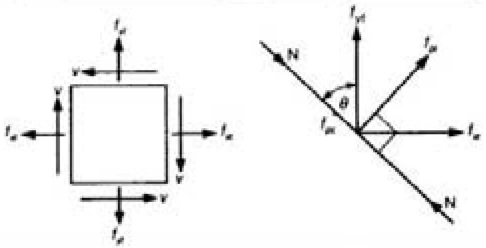
f_{pc} principal compressive stress

θ inclination of principal plane N-N

f_{xt}, f_{yt} applied tensile (positive) stresses

f_{xc}, f_{yc} applied compressive (negative) stresses

v applied shearing stress



f_{xt} and f_{yt} tensile	$f_{pt} = -\frac{1}{2}(f_{xt} + f_{yt}) - \sqrt{\{[\frac{1}{2}(f_{xt} - f_{yt})]^2 + v^2\}}$ $f_{pc} = -\frac{1}{2}(f_{xt} + f_{yt}) + \sqrt{\{[\frac{1}{2}(f_{xt} - f_{yt})]^2 + v^2\}}$	$\tan 2\theta = 2v/(f_{xt} - f_{yt})$
f_{xt} tensile $f_{yt} = 0$	$f_{pt} = -\frac{1}{2}[\sqrt{(f_{xt}^2 + 4v^2)} + f_{xt}]$ $f_{pc} = \frac{1}{2}[\sqrt{(f_{xt}^2 + 4v^2)} - f_{xt}]$	$\tan 2\theta = 2v/f_{xt}$
f_{xc} compressive $f_{yt} = 0$	$f_{pt} = -\frac{1}{2}[\sqrt{(f_{xc}^2 + 4v^2)} - f_{xc}]$ $f_{pc} = \frac{1}{2}[\sqrt{(f_{xc}^2 + 4v^2)} + f_{xc}]$	$\tan 2\theta = -2v/f_{xc}$
v only	$f_{pt} = -v$ $f_{pc} = +v$ $\theta = 45°$	

of M, N and ρ, the value of f_c is obtained by finding the point of intersection of the ρ curve and the e/h line. The e/h line can be obtained by interpolation, or can be drawn as follows: on the gridline for $N/bhf_c = 1.0$, find the point where M/bh^2f_c is equal to the value of e/h; through this point draw a line from the origin. The value of x/h can be obtained by interpolation, or by solving the equation:

$$(x/h)^2 - 2[N/bhf_c - (\alpha_e - 0.5)\rho](x/h) - (\alpha_e + d/h - 1)\rho = 0$$

Example 1. A 400 × 400 column, reinforced with 4H32 bars, is subjected to values of $M = 120$ kNm and $N = 500$ kN due to service loads. The maximum stresses in the concrete and the reinforcement are to be determined, assuming $\alpha_e = 15$.

$$\rho = A_{sc}/bh = 3217/400^2 = 0.02$$
$$e/h = M/Nh = 120/(500 \times 0.4) = 0.60$$

Allowing for 35 mm nominal cover to H8 links,

$$d = 400 - (35 + 8 + 32/2) = 340 \text{ mm}, \; d/h = 340/400 = 0.85$$

From the chart for $d/h = 340/400 = 0.85$ on *Table 2.105*, at the intersection of $\rho = 0.02$ and $e/h = 0.6$,

$$N/bhf_c = 0.25, \; x/h = 0.51$$
$$f_c = 500 \times 10^3/(0.25 \times 400^2) = 12.5 \text{ N/mm}^2$$
$$x = 0.51 \times 400 = 204 \text{ mm}$$
$$f_s = \alpha_e f_c\,(d/x - 1)$$
$$= 15 \times 12.5 \times (340/204 - 1) = 125 \text{ N/mm}^2$$

20.2.2 Uniformly reinforced annular section

The charts given on *Table 2.107* are based on the assumption that the bars may be represented, with little loss of accuracy, by a notional ring of reinforcement having the same total cross-sectional area and located at the centre of the section. If $\alpha_e = 15$, the charts can be used directly. For other values of α_e, the curves for ρ and f_s/f_c may be considered to represent values of $(\alpha_e/15)\rho$ and $(15/\alpha_e)(f_s/f_c)$ respectively.

Example 2. A cylindrical shaft with a mean radius of 1 m and a thickness of 100 mm is reinforced with 42H16 vertical bars located at the centre of the section. The shaft is subjected to values of $M = 2700$ kNm and $N = 3600$ kN due to service loads, and the stresses in the concrete and the reinforcement are to be determined, assuming $\alpha_e = 15$.

$$\rho = A_{sc}/2\pi rh = 8446/(2 \times \pi \times 1000 \times 100) = 0.0135$$
$$e/h = M/Nh = 2400/(3200 \times 0.1) = 7.5$$

A line corresponding to $e/h = 7.5$ can be drawn on the chart for $h/r = 0.10$ on *Table 2.107*, from the origin to a point such as $N/rhf_c = 6$, $M/r^2hf_c = 4.5$. At the intersection of this line with an interpolated curve for $\rho = 0.0135$,

$$N/rhf_c = 2.6, \; f_s/f_c = 6$$
$$f_c = 3200 \times 10^3/(2.6 \times 1000 \times 100) = 12.3 \text{ N/mm}^2$$
$$f_s = 6 \times 12.3 = 74 \text{ N/mm}^2$$

Part 3

Design to British Codes

Chapter 21

Design requirements and safety factors

In most British Codes, the design requirements are set out in relation to specified limit-state conditions. Calculations to determine the ability of a member (or assembly of members) to satisfy a particular limit state are undertaken using design loads and design strengths (or stresses). These design values are determined from characteristic loads and characteristic strengths of materials (or stress limits), by the application of partial safety factors specified in the Code concerned.

21.1 BUILDINGS

For buildings and other structures designed to BS 8110, the characteristic values of dead load G_k (*Tables 2.1* and *2.2*), imposed load Q_k (*Tables 2.3* and *2.4*), and wind load W_k (*Tables 2.7–2.9*) are specified in BS 6399: Parts 1, 2 and 3.

Design loads are given by:

$$\text{design load} = F_k \times \gamma_f$$

where F_k is equal to G_k, Q_k or W_k as appropriate, and γ_f is the partial safety factor appropriate to the load, load combination, and limit-state being considered.

The characteristic strength of a material f_k means that value of the cube strength of concrete f_{cu}, or the yield strength of reinforcement f_y, below which 5% of all possible test results would be expected to fall. In practice, for concrete specified in accordance with BS 8500, a dual classification is used, for example C25/30, in which the characteristic strength of cylinder test specimens is followed by the characteristic strength of cube test specimens. The characteristic strength of reinforcement is taken as the value specified in BS 4449 or BS 4482.

Design strengths are given by:

$$\text{design strength} = f_k/\gamma_m$$

where f_k is either f_{cu} or f_y as appropriate, and γ_m is the partial safety factor appropriate to the material, and limit-state being considered. The appropriate factors are already incorporated in the design equations provided in the Code.

Details of the design requirements, and partial safety factors for the ULS, are given in *Table 3.1*. For the serviceability limit states, when calculations are required, the partial safety factors are taken as unity. However, for most designs, explicit calculations are unnecessary, and the design requirement is met by complying with simple rules.

For the ULS, adverse and beneficial values are given for dead and imposed loads, and these are to be applied separately to the loads on different parts of the structure to cause the most severe effects. Thus, for load combination 1, design loads may vary from place to place with a maximum of $(1.4G_k + 1.6Q_k)$ and a minimum of $1.0G_k$ (see also *Table 2.30*). For load combination 2, the maximum wind load of $1.4W_k$ with the minimum dead load of $1.0G_k$ can result in a critical equilibrium condition for tower structures. For load combinations 2 and 3, the design wind loads can sometimes be less than the minimum notional horizontal load of $0.015G_k$ associated with the requirement for robustness.

The overall dimensions and stability of earth-retaining and foundation structures are to be determined in accordance with the appropriate codes for earth-retaining structures (BS 8002) and foundations (BS 8004). Design loads given in BS 8110 are then used in to establish section sizes and reinforcement areas. The factor γ_f should be applied to all earth and water pressures, except those that are derived from equilibrium with other design loads, such as bearing pressures below foundations.

21.2 BRIDGES

For bridges and other structures designed to BS 5400: Part 4, nominal and design values of dead load, superimposed dead load, wind load, temperature, live loads for highway, footway and railway bridges (*Tables 2.5* and *2.6*) and other loads are given in the Highways Agency Standard BD 37/01.

Design loads are given by:

$$\text{design load} = F_k \times \gamma_{fl}$$

where F_k is the specified nominal load, and γ_{fl} is the partial safety factor appropriate to the load, load combination and limit-state being considered. A separate partial safety factor is then used to allow for inaccuracies in the method of analysis.

Design load effects are given by:

$$\text{design load effect} = \text{effect of design load} \times \gamma_{f3}$$

where γ_{f3} is the partial safety factor appropriate to the method of analysis and limit-state being considered. If the method of analysis is linear elastic, it is possible to replace γ_{fl} by $\gamma_{fl} \times \gamma_{f3}$ in the calculation of the design loads.

3.1

Design requirements and partial safety factors (BS 8110)

	Limit state		Design requirement	Means of compliance
Design requirements	Ultimate	Structural stability	Structure, whose resistance is based on the design strengths of materials, should be able to support without collapse, the design effects of specified combinations of design loads.	By calculation (normally), or based on model or prototype tests where necessary.
		Robustness	Structure should not be unreasonably affected by accidents, resulting in local damage or the failure of single elements.	By calculation and attention to layout and detailing.
	Serviceability	Cracking	Design surface crack width due to applied loads, or thermal and shrinkage effects, not greater than 0.3 mm.	Limiting bar spacing rules, or by calculation.
		Deflection (due to vertical loads)	Final deflection below level of supports not greater than $l/250$, where l is span of member, or length of cantilever. Deflection after installation of elements, such as cladding and partitions, not greater than lesser of $l/500$ or 20 mm.	Limiting span/effective depth ratios, or by calculation.
		Vibration	Avoidance of discomfort or alarm to occupants, structural damage, or interference with proper function.	Consult specialist literature if consideration is needed.
	Other considerations	Durability	Structure should perform satisfactorily in the anticipated environment for its intended lifetime, with all embedded metal adequately protected from corrosion.	Minimum concrete strength class and cover according to exposure classification.
		Fire resistance	Structural stability should be adequate for the appropriate period of time required by regulations.	Minimum concrete size and cover, or by test or analysis.
		Fatigue	Consideration to be given to the effects of imposed loading that is predominately cyclical.	Consult specialist literature if consideration is needed.

Partial safety factors for the ultimate limit state

Load combinations and values of γ_f for the ultimate limit state

Load combination (see Notes 1 and 2)	Load type					
	Dead		Imposed		Wind	Earth^a and water^b pressure
	Adverse	Beneficial	Adverse	Beneficial		
1. Dead and imposed (with earth and water pressure)	1.4	1.0	1.6	0	—	$1.2^{a,b}$ 1.0^a
2. Dead andwind (with earth and water pressure)	1.4	1.0	—	—	1.4	$1.2^{a,b}$ 1.0^a
3. Dead and wind and imposed (with earth and water pressure)	1.2	1.2	1.2	1.2	1.2	$1.2^{a,b}$ 1.0^a

Note 1. When considering (i) the effect of exceptional loads due to misuse or accident, or (ii) the continued stability of a structure after sustaining local damage; the value of γ_f should be taken as 1.05 on (i) defined loads and loads likely to be acting simultaneously, or (ii) loads likely to occur before measures are taken to repair or offset the effect of damage. The loads to be taken into account are (a) dead load, (b) one-third of wind load, (c) one-third of imposed load, except for buildings used primarily for storage or industrial purposes where 100% of imposed load should be considered.

Note 2. For fire engineering calculations in accordance with BS 8110: Part 2, the value of γ_f should be taken as 1.05 on dead loads and 1.0 on imposed loads.

[a] The earth pressure is that obtained from BS 8002, including an appropriate mobilisation factor. The more onerous of the two factored conditions should be taken. The value of γ_f should be taken as 1.2, unless an allowance for unplanned excavation in accordance with BS 8002, 3.2.2.2 is included in the calculations, when it may be taken as 1.0.

[b] Where the maximum credible water level can be clearly defined, γ_f may be taken as 1.2. Otherwise use $\gamma_f = 1.4$.

Values of γ_m for the ultimate limit state

Concrete				Reinforcement
Compression	Shear	Bond	Bearing etc.	
1.5*	1.25	1.4	≥ 1.5	1.15*

* When considering the effects of exceptional loads or local damage, and when used in fire engineering calculations, the values of γ_m may be taken as 1.3 for concrete and 1.0 for reinforcement.

The characteristic strength of a material f_k means that value of the cube strength of concrete f_{cu}, or the yield strength of reinforcement f_y, below which 5% of all possible test results would be expected to fall. In practice, characteristic values are specified in the manner described for design according to BS 8110 in section 21.1. The characteristic stress is the stress value at the assumed limit of linearity on the stress–strain curve for the material.

Design strengths are given by:

$$\text{design strength} = f_k/\gamma_m$$

where f_k is either f_{cu} or f_y as appropriate, and γ_m is the partial safety factor appropriate to the material, and limit-state being considered. The appropriate factors are already incorporated in the design equations provided in the Code.

Design stress limits (serviceability) are given by:

$$\text{design stress limit} = \text{characteristic stress}/\gamma_m$$

Details of the design requirements, characteristic stresses and partial safety factors are given in *Tables 3.2* and *3.3*.

21.3 LIQUID-RETAINING STRUCTURES

For liquid-containing or liquid-excluding structures designed to BS 8007, the design basis is similar to that in BS 8110 but modified for the limit-state of cracking. Separate calculations of crack width are required for the effect of applied loads and the effect of temperature and moisture change.

Details of the design requirements and partial safety factors (updated according to BS 8110) are given in *Table 3.4*.

Design requirements and partial safety factors (BS 5400) – 1

3.2

		Limit state		Design requirement	Means of compliance
Design requirements	Ultimate	Rupture or instability		Structure, whose resistance is based on the design strengths of materials, should be able to support without collapse, the design effects of specified combinations of design loads.	By calculation (as given in BS 5400: Part 4).
	Serviceability	Cracking		Design crack widths under load combination 1, with live loading generally as HA only, to be limited according to exposure classification (*Table 3.9*): 0.25 mm (moderate or severe), 0.15 mm (very severe), and 0.10 mm (extreme).	By calculation (as given for load in BS 5400: Part 4 and BD 24/92, and for thermal movements in BD 28/87).
		Vibration		For foot/cycle track bridges, fundamental natural frequency of vibration to exceed 5 Hz vertically for unloaded bridge, and 1.5 Hz horizontally for loaded bridge. Otherwise, the maximum accelerations are to comply with specified limits. For standard highway and railway loading, dynamic effects are covered by impact allowance in the nominal live load.	By calculation (as given in Appendix B of BD 37/01). Deemed to satisfy.
		Stresses		Stresses limited to specified design values. Characteristic stress limits are $0.5f_{cu}$ for concrete in compression, and $0.75f_y$ for reinforcement in tension and compression.	By calculation, if necessary.
	Other considerations	Durability		Structure should perform satisfactorily in the anticipated environment for its intended lifetime, with all embedded metal adequately protected from corrosion.	Minimum concrete strength class and cover accordingto exposure classification.
		Fatigue		For non-welded reinforcing bars, under load combination 1, with live loading as HA only, stress range to be limited. For spans in range 5–200 m, limits are 155 N/mm^2 for bar sizes ≤ 16 mm, and 120 N/mm^2 for bar sizes > 16 mm. For welded bars, compliance with criteria in BS 5400: Part 10.	By calculation.
		Deflection (due to vertical loads)		Avoidance of drainage difficulties or violation of minimum specified clearances, control of profile during construction.	By calculation, if necessary.

Partial safety factors for loads in each load combination (Highways Agency BD 37/01) – continued on *Table 3.3*

Load		Limit state	γ_{fl} to be taken in combination				
			1	2	3	4	5
Dead[c] (concrete)	([a] γ_{fl} should be taken as 1.2 when dead load is not accurately assessed, i.e. actual weights of concrete and reinforcement)	ULS[a]	1.15	1.15	1.15	1.15	1.15
		SLS	1.00	1.00	1.00	1.00	1.00
Superimposed dead[c] (surfacing)	([b] γ_{fl} may be reduced to 1.2 for ULS and 1.0 for SLS subject to the approval of the appropriate authority)	ULS[b]	1.75	1.75	1.75	1.75	1.75
		SLS[b]	1.20	1.20	1.20	1.20	1.20
Superimposed dead[c] (other)	([c] For all dead and superimposed dead, γ_{fl} should be taken as 1.0 where this has a more severe total effect)	ULS	1.20	1.20	1.20	1.20	1.20
		SLS	1.00	1.00	1.00	1.00	1.00
Wind	During erection ([d] γ_{fl} should be taken as 1.0 for relieving effect of wind)	ULS[d]		1.10			
		SLS		1.00			
	With dead plus superimposed dead load only ([d] γ_{fl} should be taken as 1.0 for relieving effect of wind)	ULS[d]		1.40			
		SLS		1.00			
	With dead, superimposed dead, other combination 2 loads ([d] γ_{fl} should be taken as 1.0 for relieving effect of wind)	ULS[d]		1.10			
		SLS		1.00			
Temperature	Restraint to movement, except frictional	ULS			1.30		
		SLS			1.00		
	Frictional bearing restraint	ULS					1.30
		SLS					1.00
	Effect of temperature difference	ULS			1.00		
		SLS			0.80		
Differential settlement		ULS	1.20	1.20	1.20	1.20	1.20
		SLS	1.00	1.00	1.00	1.00	1.00
Erection: temporary loads		ULS		1.15	1.15		
		SLS		1.00	1.00		
Exceptional loads		To be assessed and agreed between the engineer and the appropriate authority					

Design requirements and partial safety factors (BS 5400) – 2

3.3

Partial safety factors for loads in each load combination (Highways Agency BD 37/01) – continued from *Table 3.2*

Load			Limit state	γ_{fL} to be taken in combination				
				1	2	3	4	5
Earth pressure (retained fill and/or live load)	Vertical loads	([d] γ_{fL} should be taken as 1.0 where this has a more severe total effect)	ULS[d]	1.20	1.20	1.20	1.20	1.20
			SLS	1.00	1.00	1.00	1.00	1.00
	Non-vertical loads	([d] γ_{fL} should be taken as 1.0 where this has a more severe total effect)	ULS[d]	1.50	1.50	1.50	1.50	1.50
			SLS	1.00	1.00	1.00	1.00	1.00
Highway bridges live loading	HA alone		ULS	1.50	1.25	1.25		
			SLS	1.20	1.00	1.00		
	HA with HB, or HB alone		ULS	1.30	1.10	1.10		
			SLS	1.10	1.00	1.00		
	Footway and cycle track loading		ULS	1.50	1.25	1.25		
			SLS	1.00	1.00	1.00		
	Accidental wheel loading (to be considered as acting alone and not with any other primary live load)		ULS	1.50				
			SLS	1.20				
Loads due to vehicle collision with parapets (assessment of local effects)	Parapet load	Low and normal containment	ULS				1.50	Each secondary live load is considered separately with other combination 4 loads as appropriate
			SLS				1.20	
		High containment	ULS				1.40	
			SLS				1.15	
	Associated primary live load	Low, normal and high containment	ULS				1.30	
			SLS				1.10	
Loads due to vehicle collision with parapets (assessment of global effects)	Parapet load (massive structures):							
	bridge superstructures and non-elastomeric bearings		ULS				1.25	
	bridge substructures, and wing and retaining walls		ULS				1.00	
	elastomeric bearings		SLS				1.00	
	Parapet load (light structures):							
	bridge superstructures and non-elastomeric bearings		ULS				1.40	
	bridge substructures, and wing and retaining walls		ULS				1.40	
	elastomeric bearings		SLS				1.00	
	Associated primary live load (all structures):							
	bridge superstructures and non-elastomeric bearings		ULS				1.25	
	bridge substructures, and wing and retaining walls		ULS				1.25	
	elastomeric bearings		SLS				1.00	
Vehicle collision loads on bridge supports and superstructures	Assessment of local and global effects on:							
	all elements except elastomeric bearings		ULS				1.50	
	elastomeric bearings		SLS				1.00	
Longitudinal load	HA and associated primary live load		ULS				1.25	
			SLS				1.00	
	HB and associated primary live load		ULS				1.10	
			SLS				1.00	
Centrifugal load and associated primary live load			ULS				1.50	
			SLS				1.00	
Accidental skidding load and associated primary live load			ULS				1.25	
			SLS				1.00	
Foot/cycle track bridges	Live load and effects due to parapet load		ULS	1.50	1.25	1.25		
			SLS	1.00	1.00	1.00		
	Vehicle collision loads on supports and superstructures (this is the only secondary live load to be considered)		ULS				1.50	
Railway bridges	Type RU and RL, and SW/0 primary and secondary live loading		ULS	1.40	1.20	1.20		
			SLS	1.10	1.00	1.00		

Partial safety factors for loads due to concrete shortening, for load effects and for material properties (BS 5400: Part4)

Limit-state	Values of γ_{fL} for loads due to concrete creep and shrinkage	Values of γ_{f3} for assessment of design load effects	Values of γ_m for analysis of sections		
			Concrete in compression		Reinforcement
			Bending	Axial load	
ULS	1.20	1.10[a]	1.50[b]	1.50[b]	1.15
SLS	1.00	1.00	1.00[c]	1.33[c]	1.00

[a] Increased to 1.15 for plastic method of analysis. [b] Applied to characteristic strength. [c] Applied to characteristic stress limit.

3.4

Design requirements and partial safety factors (BS 8007)

	Limit state		Design requirement	Means of compliance
Design requirements	Ultimate	Structural stability	Structure, whose resistance is based on the design strengths of materials, should be able to support without collapse, the design effects of specified combinations of design loads.	By calculation in accordance with BS 8110.
	Serviceability	Cracking	Design surface crack width due to applied loads, or thermal and shrinkage effects, not greater than: Severe or very severe exposure 0.2 mm Critical aesthetic appearance 0.1 mm	Allowable steel stresses or by calculation using equations in BS 8007.
		Deflection	Final deflection not greater than *l*/250, where *l* is span of member, or length of cantilever (including walls).	Limiting span/effective depth ratios or by calculation.
		Flotation	For structures subject to groundwater pressure, where the groundwater level can be reliably assessed, the deadweight of the empty structure with any anchoring devices should give a minimum factor of safety of 1.1 against uplift forces.	By calculation.
	Other considerations	Durability	Structure should perform satisfactorily in the anticipated environment for its intended lifetime (40–60 years), with all embedded metal adequately protected from corrosion. Exposure classification not less than severe (*Table 3.9*). For extended design life, consideration should be given to increasing the cement content or the cover, or using special reinforcement (galvanized, epoxy-coated, stainless steel).	Minimum concrete strength class and cover according to exposure classification. For severe exposure: concrete grade C35A nominal cover 40 mm. Otherwise, as BS 8110.

Partial safety factors

Load combinations and values of γ_f for the ultimate limit state (see Note 1)

Load combination	Load type					
	Dead (see Note 2)		Imposed		Wind	Earth^a and water^b pressure
	Adverse	Beneficial	Adverse	Beneficial		
1. Dead and imposed (with earth and water pressure)	1.4	1.0	1.6	0	—	$1.2^{a,b}$ 1.0^{a}
2. Dead and wind (with earth and water pressure)	1.4	1.0	—	—	1.4	$1.2^{a,b}$ 1.0^{a}
3. Dead and wind and imposed (with earth and water pressure)	1.2	1.2	1.2	1.2	1.2	$1.2^{a,b}$ 1.0^{a}

Note 1. Values of γ_f are those given in BS 8110: Part 1: 1997. In BS 8007, which refers specifically to BS 8110: Part 1: 1985, the value for retained liquid loads is given as 1.4 for load combinations 1 and 2, and 1.2 for load combination 3. For containment structures, liquid levels should be taken to the tops of walls, assuming that liquid outlets are blocked.
Note 2. Earth covering on reservoir roofs may be taken as dead load, but due account should be taken of construction loads from plant and heaped earth, which may exceed the intended design load.

[a] The earth pressure is that obtained from BS 8002, including an appropriate mobilisation factor. The more onerous of the two factored conditions should be taken. The value of γ_f should be taken as 1.2, unless an allowance for unplanned excavation in accordance with BS 8002, 3.2.2.2 is included in the calculations, when it may be taken as 1.0.
[b] Where the maximum credible water level can be clearly defined, γ_f may be taken as 1.2. Otherwise, use $\gamma_f = 1.4$.

Values of γ_m for the ultimate limit state

Concrete				Reinforcement
Compression	Shear	Bond	Bearing etc.	
1.5	1.25	1.4	≥ 1.5	1.15

Note. For the serviceability limit states, the values of γ_f and γ_m should be taken as 1.0. For containment structures, the liquid level should be taken to the working top or overflow level, as appropriate to working conditions.

Chapter 22
Properties of materials

22.1 CONCRETE

22.1.1 Strength and elastic properties

The characteristic strength of concrete means that value of the 28-day cube strength below which 5% of all valid test results is expected to fall. In BS 8500, compressive strength classes are expressed in terms of both characteristic cylinder strength and characteristic cube strength. The recommended compressive strength classes, and mean values of the static modulus of elasticity at 28 days are given in *Table 3.5*.

In BS 8110, for normal-weight concrete, the mean value of the static modulus of elasticity of concrete at 28 days is given by the expression:

$$E_{c,28} = 20 + 0.2f_{cu,28} \text{ (kN/mm}^2\text{)}$$

where $f_{cu,28}$ is the cube strength at 28 days. The mean value of the modulus of elasticity at an age $t \geq 3$ days can be estimated from the expression:

$$E_{c,t} = E_{c,28} (0.4 + 0.6f_{cu,t}/f_{cu,28})$$

where $f_{cu,t}$ is the cube strength at age t. Where deflections are of great importance, and test data for concrete made with the aggregate to be used in the structure is not available, a range of values for $E_{c,28}$ based on (mean value ± 6 kN/mm^2) should be considered.

In BS 5400, slightly higher mean values of the modulus of elasticity are used, given by the expression:

$$E_c = 19 + 0.3f_{cu} \text{ (kN/mm}^2\text{)}$$

where f_{cu} is the cube strength at the age considered.

22.1.2 Creep and shrinkage

The creep strain in concrete may be assumed to be directly proportional to the applied stress, for stresses not exceeding about one-third of the cube strength at the age of loading. For design to BS 8110, values of the creep coefficient (creep per unit of stress), according to the ambient relative humidity, the effective section thickness and the age of loading, can be obtained from the figure in *Table 3.5*. Creep strain is partly recoverable if the stress is reduced. The final recovery (after 1 year) is approximately 0.3 × (stress reduction)/E_u, where E_u is the modulus of elasticity at the age of unloading.

For design to BS 8110, an estimate of the drying shrinkage of plain concrete, according to the ambient relative humidity, the effective section thickness and the original water content, can be obtained from the figure in *Table 3.5*. Aggregates with a high moisture movement, such as some Scottish dolerites and whinstones, and gravels containing these rocks, produce concrete with a high initial drying shrinkage (ref. 12). Also, aggregates with a low elastic modulus may result in higher than normal concrete shrinkage.

In BS 5400, values are obtained for the creep coefficient as a product of five partial coefficients, and for the shrinkage strain as a product of four partial coefficients, where the coefficients are obtained from a series of figures.

22.1.3 Thermal properties

For design to BS 8110, values of the coefficient of thermal expansion of concrete, according to aggregate type, are given in *Table 3.5*. In BS 5400, a value of 12×10^{-6} per °C is generally taken, except when limestone aggregates are used, when a value of 9×10^{-6} per °C is recommended.

22.1.4 Stress–strain curves

Typical short-term stress–strain curves for normal strength concretes in compression, as described in section 3.1.6, and the idealised curve given for design purposes in BS 8110 and BS 5400, are shown in *Table 3.6*. In the expression given for the maximum design stress, 0.67 takes account of the ratio between the cube strength and the strength in bending.

22.2 REINFORCEMENT

22.2.1 Strength and elastic properties

The characteristic strength of steel reinforcement made to the requirements of BS 4449 is 500 N/mm^2. BS 4449 caters for round ribbed bars in three ductility classes: grades B500A, B500B and B500C. Fabric reinforcement is manufactured using bars to BS 4449, except for wrapping fabric, where wire to BS 4482 with a characteristic strength of 250 N/mm^2 may be used. For more information on types, properties and sizes of

Concrete (BS 8110): strength and deformation characteristics

3.5

Strength and elastic properties

Characteristic cube strength at 28 days f_{cu} (N/mm^2)	Concrete strength class (normal-weight aggregates)	Static modulus of elasticity at 28 days $E_{c,28}$ (kN/mm^2)	Concrete strength class (lightweight aggregates)	Notes
20	NA	24	LC18/20	See section 22.1.1 for further information on modulus of elasticity. For concrete that is made with lightweight aggregates, the given values of $E_{c,28}$ should be multiplied by the term $(w/2000)^2$, where w is the density of the concrete (kg/m^3).
25	C20/25	25	LC22/25	
30	C25/30	26	LC27/30	
35	C28/35	27	LC32/35	
40	C32/40	28	LC37/40	
45	C35/45	29	LC41/45	
50	C40/50	30	LC45/50	
55	C45/55	31	LC50/55	
60	C50/60	32	LC55/60	
Poisson's ratio	Where linear elastic analysis is appropriate, Poisson's ratio may be taken as 0.2.			

Creep and shrinkage

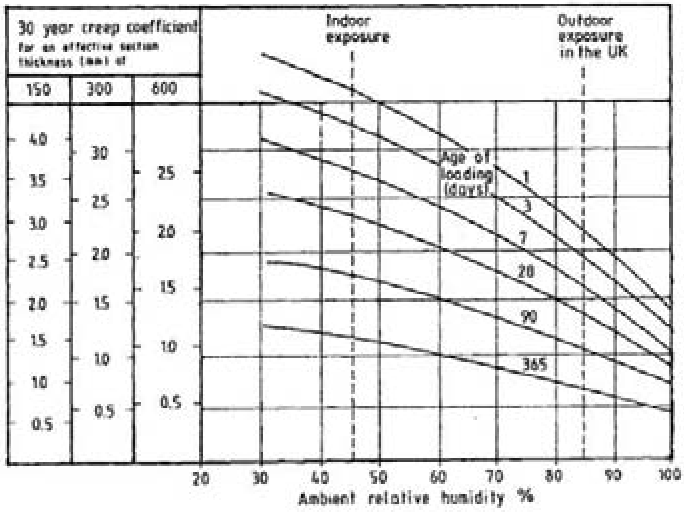
The final (30 year) creep strain in concrete can be predicted from:

$$\varepsilon_{cc} = \frac{stress}{E_t}\phi$$

where

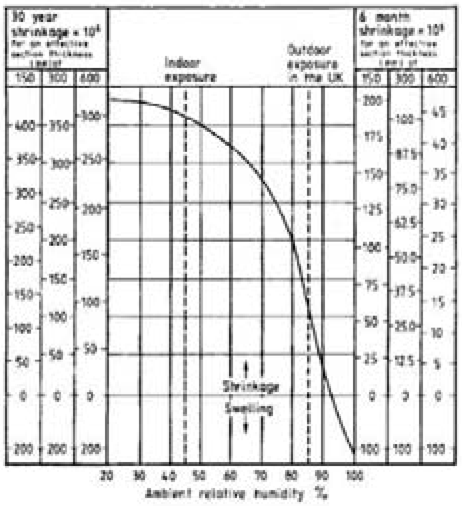
E_t is the modulus of elasticity of the concrete at age of loading t,
ϕ is the creep coefficient (estimated from the figure below).

In the figure, the effective section thickness is defined, for uniform sections, as twice the cross-sectional area divided by the length of the exposed perimeter. If drying is prevented by immersion in water or by sealing, the effective section thickness may be taken as 600 mm. For general design purposes, suitable values of relative humidity for indoor and outdoor exposure in the UK are 45% and 85%. It can be assumed that about 40%, 60% and 80% of the final creep develops during the first 1month, 6 months and 30 months under load, when concrete is exposed to conditions of constant relative humidity.



Creep coefficient for normal-weight concrete

The drying shrinkage of plain concrete may be estimated from the figure below. See notes on creep coefficient for recommendations on the effective section thickness and ambient relative humidity. The given shrinkage values relate to concrete of normal workability made without water-reducing admixtures; i.e. concretes with an original water content of about 190 L/m^3. In other cases, shrinkage may be taken as directly proportional to water content, within the range 150 L/m^3 to 230 L/m^3. For outdoor exposure in the UK, concrete will undergo seasonal cyclic strains of ± 0.4 times the 30-year shrinkage in addition to the average shrinkage strain; the maximum shrinkage will occur at the end of each summer. For guidance on the shrinkage of reinforced concrete, see section 19.



Drying shrinkage of normal-weight concrete

Thermal

Thermal expansion of rock group and related concrete

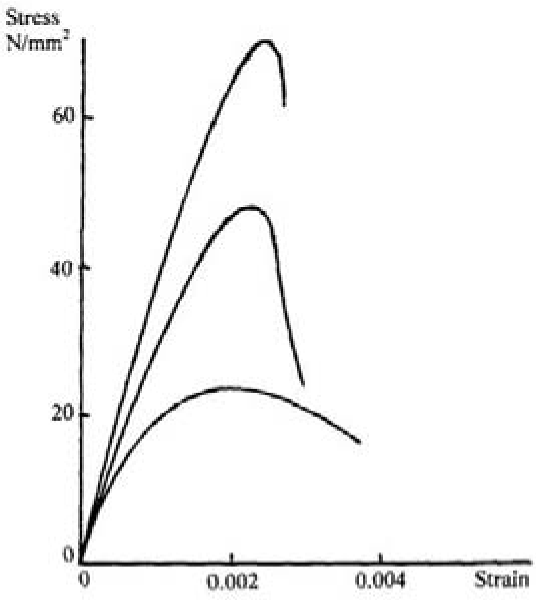
Aggregate type	Typical coefficient of expansion (x 10^{-6}/°C)	
	Aggregate	Concrete
Flint, quartzite	11	12
Granite, basalt	7	10
Limestone	6	8

Note. The coefficient of thermal expansion of concrete is dependent mainly on the expansion coefficients of the aggregate and cement paste, and the degree of saturation of the concrete. The given values are for concrete in a partially dry state. In the saturated state, the coefficient will be approximately 2x10^{-6}/°C less than the tabulated value.

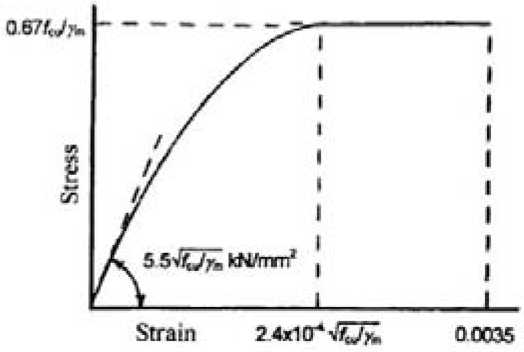
Stress-strain curves (BS 8110 and BS 5400): concrete and reinforcement

3.6

Concrete



TYPICAL STRESS–STRAIN CURVES



DESIGN STRESS–STRAIN CURVE

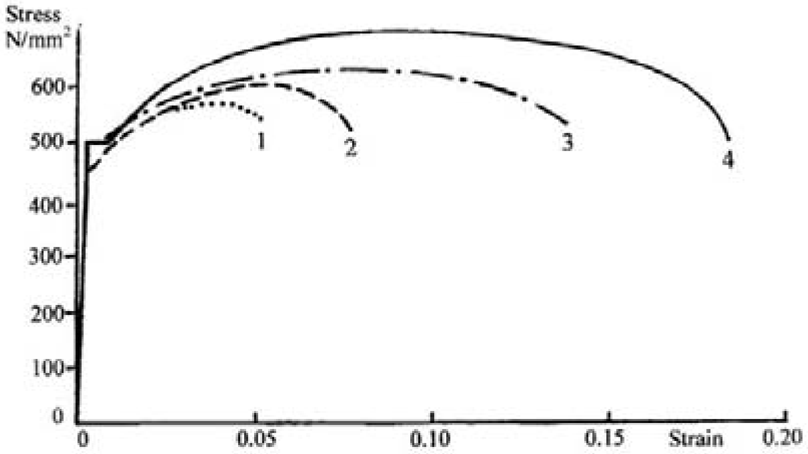
Note 1. In the expressions given, f_{cu} is in N/mm².

Note 2. A simplified rectangular stress distribution may also be assumed with the design stress taken as follows

BS 8110: $0.67f_{cu}/\gamma_m$ for a depth from the compression face of 0.9 times the depth of the compression zone.

BS 5400: $0.6f_{cu}/\gamma_m$ for the entire compression zone.

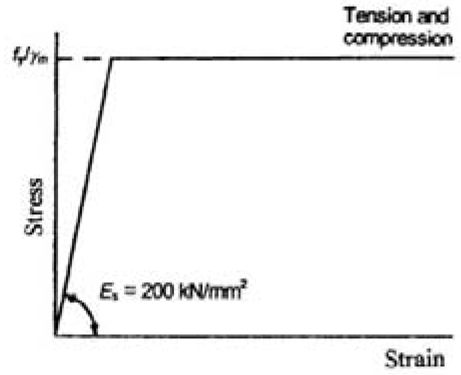
Reinforcement



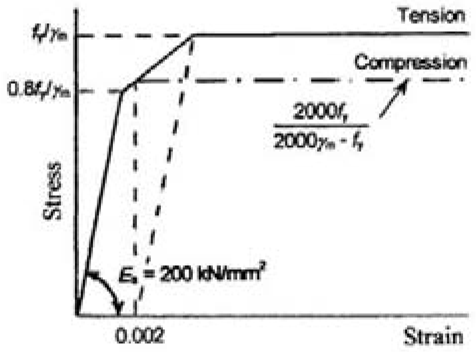
TYPICAL STRESS–STRAIN CURVES

Types of reinforcing steel

1. Cold-rolled wire (used in fabric reinforcement).
2. Cold-stretched coil (used in automatic bending machines for link reinforcement).
3. Hot-rolled (in-line heat treated, QST process) bars.
4. Hot-rolled (micro-alloy) bars.



DESIGN STRESS–STRAIN CURVE (BS 8110)



DESIGN STRESS–STRAIN CURVE (BS 5400)

bar and fabric reinforcement, reference should be made to section 10.3 and *Tables 2.19* and *2.20*.

22.2.2 Stress–strain curves

Typical stress–strain curves for reinforcing steels in tension as described in section 3.2.3, and the idealised curves given for design purposes in BS 8110 and BS 5400 respectively, for reinforcement in tension or compression, are shown in *Table 3.6*. For design purposes, the modulus of elasticity of all reinforcing steels is taken as 200 kN/mm^2. The BS 5400 stress–strain curve is the one that was used in CP110, prior to that code being superseded by BS 8110.

Chapter 23

Durability and fire-resistance

In the following, the term nominal cover is used to describe the design cover shown on the drawings. It is the required cover to the first layer of bars, including links. The nominal cover should not be less than the values needed for durability and fire-resistance, nor less than the nominal maximum size of aggregate. Also, the nominal cover should be sufficient to ensure that the cover to the main bars is not less than the bar size or, for a group of bars in contact, the equivalent bar size. For a group of bars, the equivalent bar size is the diameter of a circle whose cross-sectional area is equal to the sum of the areas of the bars in the group.

23.1 DURABILITY

23.1.1 Exposure classes

Details of the classification system used in BS EN 206-1 and BS 8500-1, with informative examples applicable in the UK, are given in *Table 3.7*. Often, the concrete can be exposed to more than one of the actions described in the table, in which case a combination of the exposure classes will apply. At the time of drafting this *Handbook*, the amendments necessitated by the introduction of BS 8500 have not been incorporated in BS 5400 or BS 8007 (which is based on BS 8110:1985). The system of exposure classes, concrete grades and covers, used prior to BS 8500, is given in *Table 3.9*.

23.1.2 Concrete strength classes and covers

Concrete durability is dependent mainly on its constituents, and limitations on the maximum free water/cement ratio and the minimum cement content are specified for each exposure class. These limitations result in minimum concrete strength classes for particular cements. For reinforced concrete, the protection of the steel against corrosion depends on the cover. The required thickness of cover is related to the exposure class, the concrete quality and the intended working life of the structure. Details of the recommendations in BS 8500 are given in *Table 3.8*.

The values given for the minimum cover apply for ordinary carbon steel in concrete without special protection, and for structures with an intended working life of at least 50 years. The values given for the nominal cover include an allowance for tolerance of 10 mm, which is recommended for buildings and is normally also sufficient for other types of structures.

For uneven concrete surfaces (e.g. ribbed finish or exposed aggregate), the cover should be increased by at least 5 mm. If concrete is cast against an adequate blinding, the nominal cover should generally be at least 40 mm. For concrete cast directly against the earth, the nominal cover should generally be not less than 75 mm

23.2 FIRE-RESISTANCE

23.2.1 Building regulations

The minimum period of fire-resistance required for elements of the structure, according to the purpose group of a building and its height or, for basements, depth relative to the ground are given in *Table 3.12*. Building insurers may require longer fire periods for storage facilities.

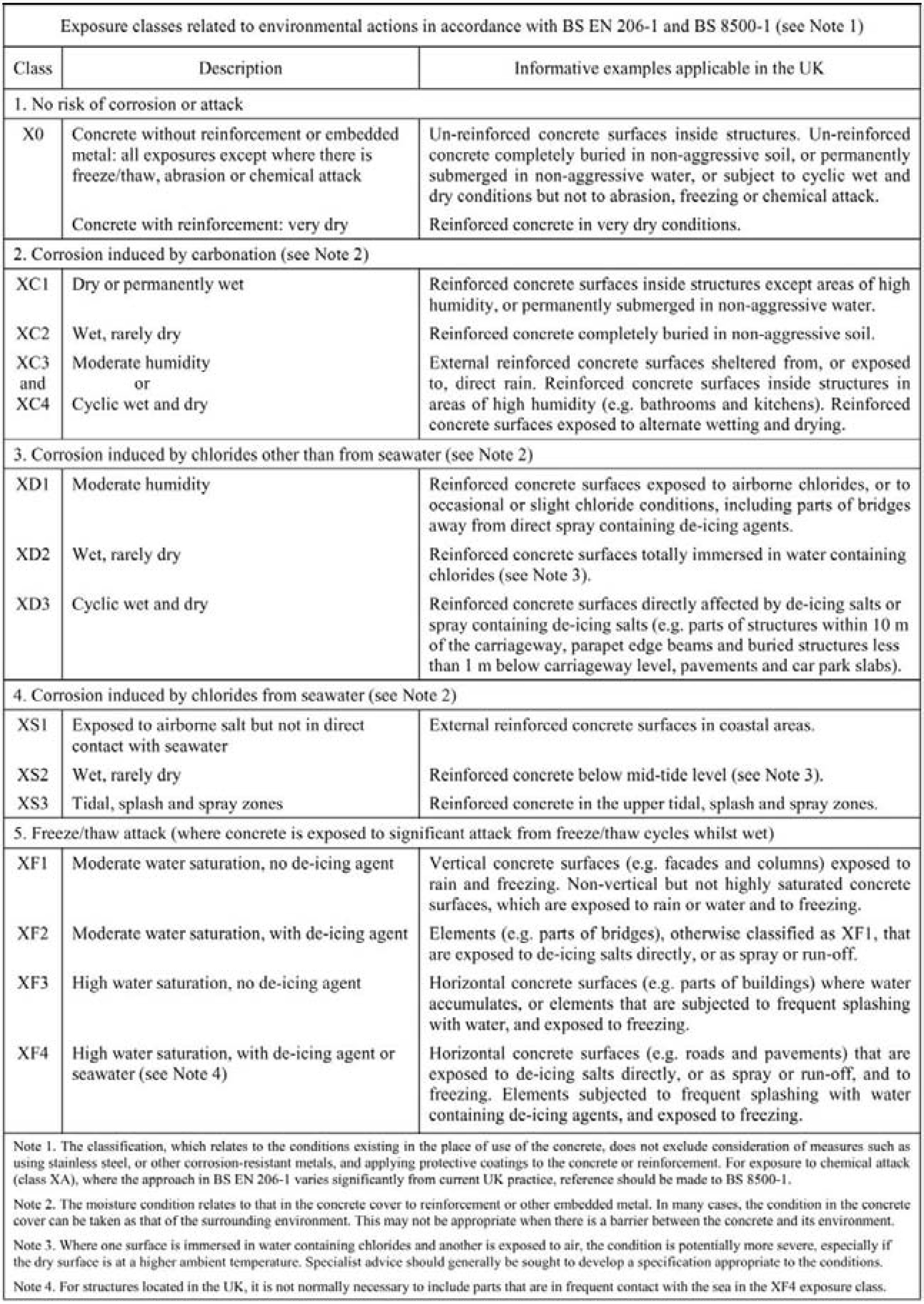
23.2.2 Nominal covers and minimum dimensions

The recommendations in BS 8110 regarding nominal cover for different periods of fire-resistance are given in *Table 3.10*. In the table, the cover applies to links for beams and columns, but to main bars for floor slabs and ribs (even if links are provided). For two-way spanning solid slabs, the cover may be taken as the average for each direction. For beams, floors and ribs, the requirements apply to the reinforcement in the bottom and side faces only. The minimum thickness of floors includes any concrete screed on the top surface. This is particularly important for ribbed slabs where the structural flange could be no more than 75 mm thick. The values given in the table apply to members whose dimensions comply with the minimum values given in *Table 3.11*.

For cases where it is considered that special measures need to be taken to prevent the spalling of concrete, a summary of the recommendations in BS 8110-2 is given in *Table 3.10*.

3.7

Exposure classification (BS 8500)



Exposure classes related to environmental actions in accordance with BS EN 206-1 and BS 8500-1 (see Note 1)		
Class	**Description**	**Informative examples applicable in the UK**
1. No risk of corrosion or attack		
X0	Concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack	Un-reinforced concrete surfaces inside structures. Un-reinforced concrete completely buried in non-aggressive soil, or permanently submerged in non-aggressive water, or subject to cyclic wet and dry conditions but not to abrasion, freezing or chemical attack.
	Concrete with reinforcement: very dry	Reinforced concrete in very dry conditions.
2. Corrosion induced by carbonation (see Note 2)		
XC1	Dry or permanently wet	Reinforced concrete surfaces inside structures except areas of high humidity, or permanently submerged in non-aggressive water.
XC2	Wet, rarely dry	Reinforced concrete completely buried in non-aggressive soil.
XC3 and XC4	Moderate humidity or Cyclic wet and dry	External reinforced concrete surfaces sheltered from, or exposed to, direct rain. Reinforced concrete surfaces inside structures in areas of high humidity (e.g. bathrooms and kitchens). Reinforced concrete surfaces exposed to alternate wetting and drying.
3. Corrosion induced by chlorides other than from seawater (see Note 2)		
XD1	Moderate humidity	Reinforced concrete surfaces exposed to airborne chlorides, or to occasional or slight chloride conditions, including parts of bridges away from direct spray containing de-icing agents.
XD2	Wet, rarely dry	Reinforced concrete surfaces totally immersed in water containing chlorides (see Note 3).
XD3	Cyclic wet and dry	Reinforced concrete surfaces directly affected by de-icing salts or spray containing de-icing salts (e.g. parts of structures within 10 m of the carriageway, parapet edge beams and buried structures less than 1 m below carriageway level, pavements and car park slabs).
4. Corrosion induced by chlorides from seawater (see Note 2)		
XS1	Exposed to airborne salt but not in direct contact with seawater	External reinforced concrete surfaces in coastal areas.
XS2	Wet, rarely dry	Reinforced concrete below mid-tide level (see Note 3).
XS3	Tidal, splash and spray zones	Reinforced concrete in the upper tidal, splash and spray zones.
5. Freeze/thaw attack (where concrete is exposed to significant attack from freeze/thaw cycles whilst wet)		
XF1	Moderate water saturation, no de-icing agent	Vertical concrete surfaces (e.g. facades and columns) exposed to rain and freezing. Non-vertical but not highly saturated concrete surfaces, which are exposed to rain or water and to freezing.
XF2	Moderate water saturation, with de-icing agent	Elements (e.g. parts of bridges), otherwise classified as XF1, that are exposed to de-icing salts directly, or as spray or run-off.
XF3	High water saturation, no de-icing agent	Horizontal concrete surfaces (e.g. parts of buildings) where water accumulates, or elements that are subjected to frequent splashing with water, and exposed to freezing.
XF4	High water saturation, with de-icing agent or seawater (see Note 4)	Horizontal concrete surfaces (e.g. roads and pavements) that are exposed to de-icing salts directly, or as spray or run-off, and to freezing. Elements subjected to frequent splashing with water containing de-icing agents, and exposed to freezing.

Note 1. The classification, which relates to the conditions existing in the place of use of the concrete, does not exclude consideration of measures such as using stainless steel, or other corrosion-resistant metals, and applying protective coatings to the concrete or reinforcement. For exposure to chemical attack (class XA), where the approach in BS EN 206-1 varies significantly from current UK practice, reference should be made to BS 8500-1.

Note 2. The moisture condition relates to that in the concrete cover to reinforcement or other embedded metal. In many cases, the condition in the concrete cover can be taken as that of the surrounding environment. This may not be appropriate when there is a barrier between the concrete and its environment.

Note 3. Where one surface is immersed in water containing chlorides and another is exposed to air, the condition is potentially more severe, especially if the dry surface is at a higher ambient temperature. Specialist advice should generally be sought to develop a specification appropriate to the conditions.

Note 4. For structures located in the UK, it is not normally necessary to include parts that are in frequent contact with the sea in the XF4 exposure class.

Concrete quality and cover requirements for durability (BS 8500)

3.8

Recommended limiting values for concrete quality (see Note 1) and cover to all reinforcement (including links)

Exposure class (*Table 3.7*)	Maximum water/ cement ratio	Minimum cement content kg/m^3	Minimum strength class (*Table 3.5*) for cement type (see Note 2)			Minimum cover (see Note 3) mm	Nominal cover (see Note 4) mm
			Group 4	Group 5	Group 6		
XC1	0.70	240	C20/25	C20/25	C20/25	15	25
XC2	0.65	260	C25/30	C25/30	C25/30	25	35
XC3/XC4 (cement type IVB invalid)	0.65	260	C25/30	C25/30	C25/30	35	45
	0.60	280	C28/35	C28/35	C28/35	30	40
	0.55	300	C32/40	C32/40	C32/40	25	35
	0.45	340	C40/50	C40/50	C40/50	20	30
XD1	0.60	300	C28/35	C28/35	C28/35	35	45
	0.55	320	C32/40	C32/40	C32/40	30	40
	0.45	360	C40/50	C40/50	C40/50	25	35
XD2	0.55	320	C28/35	C25/30	C20/25	40	50
	0.50	340	C32/40	C28/35	C25/30	35	45
	0.40	380	C40/50	C35/45	C32/40	30	40
XD3	0.45	360	C35/45	—	—	50	60
	0.40	380	C40/50	—	—	45	55
	0.35	380	C45/55	—	—	40	50
	0.50	340	—	C28/35	C25/30	50	60
	0.45	360	—	C32/40	C28/35	45	55
	0.40	380	—	C35/45	C32/40	40	50
XS1	0.50	340	C35/45	C32/40	—	40	50
	0.45	360	C40/50	C35/45	—	35	45
	0.35	380	C50/60	C45/55	—	30	40
	0.55	320	—	—	C20/25	40	50
	0.50	340	—	—	C25/30	35	45
	0.40	380	—	—	C32/40	30	40
XS2	0.55	320	C28/35	C25/30	C20/25	40	50
	0.50	340	C32/40	C28/35	C25/30	35	45
	0.40	380	C40/50	C35/45	C32/40	30	40
XS3	0.40	380	C40/50	—	—	50	60
	0.35	380	C45/55	—	—	45	55
	0.50	340	—	C28/35	C25/30	50	60
	0.45	360	—	C32/40	C28/35	45	55
	0.40	380	—	C35/45	C32/40	40	50

Note 1. The concrete quality recommendations apply for a maximum aggregate size of 20 mm, and an intended working life for the structure of at least 50 years. For concrete subject to freeze/thaw action and concrete in aggressive ground conditions reference should be made to Annex A of BS 8500-1.

Note 2. The cement (or combination) types are those listed in Table A.17 of BS 8500-1 (see also *Table 2.17*), as follows:

Group 4	Portland cement, Portland-slag cement, Sulfate-resisting Portland cement
Group 5	Portland-fly ash cement, Blastfurnace cement (36–65% ggbs)
Group 6	Blastfurnace cement (66–80% ggbs), Pozzolanic cement

Note 3. The minimum cover values apply for ordinary carbon steel in concrete without special protection, and an intended working life for the structure of at least 50 years.

Note 4. The allowance for tolerance of 10 mm is generally acceptable for buildings, and is normally also sufficient for other types of structures. In certain situations, this allowance may be reduced (e.g. where fabrication is subjected to a quality assurance system, in which the monitoring includes measurements of the concrete cover; or where, for precast elements, it can be assured that a very sensitive measurement device is used for monitoring and non-conforming members are rejected.

Exposure conditions, concrete and cover requirements (prior to BS 8500)

3.9

BS 8110 requirements

Environment	Exposure conditions
Mild	Concrete surfaces protected against weather or aggressive conditions.
Moderate	Concrete surfaces in outdoor conditions, but sheltered from severe rain or freezing whilst wet. Concrete surfaces continuously under non-aggressive water. Concrete in contact with non-aggressive soil. Concrete subject to condensation.
Severe	Concrete surfaces exposed to severe rain, alternate wetting and drying, occasional freezing or severe condensation.
Very severe	Concrete surfaces occasionally exposed to seawater spray or de-icing salts (directly or indirectly). Concrete surfaces exposed to corrosive fumes or severe freezing conditions whilst wet.
Most severe	Concrete surfaces frequently exposed to seawater spray or de-icing salts (directly or indirectly). Concrete in seawater tidal zone down to 1 m below lowest water level.

Nominal cover to all reinforcement (including links) to meet durability requirements

Conditions of exposure	Nominal cover (mm)				
Mild	25	20	20[a]	20[a]	20[a]
Moderate	—	35	30	25	20
Severe	—	—	40	30	25
Very severe	—	—	50[b]	40[b]	30
Most severe	—	—	—	—	50
Lowest grade of concrete	C30	C35	C40	C45	C50

[a] These covers may be reduced to 15 mm providing the nominal maximum size of aggregate does not exceed 15 mm.

[b] Where concrete is subject to freezing whilst wet, air entrainment should be used, in which case strength grade may be reduced by 5.

BS 5400 requirements

Environment	Exposure conditions
Moderate	Concrete surfaces above ground level and fully sheltered against rain, de-icing salts and spray from sea water: e.g. surfaces protected by bridge deck waterproofing or by permanent formwork, interior surfaces of pedestrian subways, voided superstructures or cellular abutments. Concrete surfaces permanently saturated by water with a pH > 4.5.
Severe	Concrete surfaces exposed to driving rain, or alternate wetting and drying: e.g. walls and structure supports remote from the carriageway, bridge deck soffits, and buried parts of structures.
Very severe	Concrete surfaces directly affected by de-icing salts, or spray from sea water: e.g. walls and structure supports adjacent to the carriageway, parapet edge beams, concrete adjacent to the sea.
Extreme	Concrete surfaces exposed to abrasive action by sea water, or water with a pH ≤ 4.5: e.g. marine structures, parts of structure in contact with moorland water.

Nominal cover to all reinforcement (including links) to meet durability requirements (see Note)

Conditions of exposure	Nominal cover (mm)			
Moderate	45	35	30	25
Severe	—	45[b]	35	30
Very severe	—	[a]	50[b]	40
Extreme	—	—	65[b]	55
Lowest grade of concrete	C25	C30	C40	C50 and over

Note. Highways Agency BD57/01 requires these covers to be increased by at least 10 mm for cast in-situ constuction.

[a] For parapet beams only, grade 30 concrete is permitted provided it is air entrained and the nominal cover is 60 mm.

[b] Where surface is liable to freezing whilst wet, air entrained concrete should be used.

3.10

Fire resistance requirements (BS 8110) – 1

Normal-weight aggregate concrete

Nominal cover to all reinforcement (including links) to meet specified periods of fire resistance

Fire period (*Table 3.12*)	Nominal cover (mm)						
	Beams[a]		Floors		Ribs		Columns[a]
hours	Simply supported	Continuous	Simply supported	Continuous	Simply supported	Continuous	
0.5	20[b]	20[b]	20[b]	20[b]	20[b]	20[b]	20[b]
1.0	20[b]	20[b]	20	20	20	20[b]	20[b]
1.5	20	20[b]	25	20	35	20	20
2.0	40	30	35	25	(45)	35	25
3.0	(60)	40	(45)	35	(55)	40	25
4.0	(70)	(50)	(55)	(45)	(65)	(50)	25

Note. The table relates to members of the minimum dimensions given in *Table 3.11*. Where values are shown in parenthesis, additional measures should be taken to reduce the risk of spalling (see separate table below).

[a] For the purposes of assessing a nominal cover for beams and columns, an allowance for links of 10 mm has been made to cover the range from 8 mm to 12 mm.

[b] These covers may be reduced to 15 mm providing the nominal maximum size of aggregate does not exceed 15 mm.

Lightweight aggregate concrete

Nominal cover to all reinforcement (including links) to meet specified periods of fire resistance

Fire period (*Table 3.12*)	Nominal cover (mm)						
	Beams[a]		Floors		Ribs		Columns[a]
hours	Simply supported	Continuous	Simply supported	Continuous	Simply supported	Continuous	
0.5	15	15	15	15	15	15	15
1.0	15	15	15	15	15	15	15
1.5	15	15	20	20	20	20	15
2.0	30	15	25	20	30	25	15
3.0	40	25	35	25	40	25	15
4.0	50	35	45	35	50	35	15

Note. The table relates to concrete with lightweight aggregate of 15 mm maximum nominal size, and to members of the minimum dimensions given in *Table 3.11*.

[a] For the purposes of assessing a nominal cover for beams and columns, an allowance for links of 10 mm has been made to cover the range from 8 mm to 12 mm.

Measures to prevent spalling

In cases where the nominal cover to the outermost layer of reinforcement exceeds 40 mm for normal-weight concrete, or 50 mm for lightweight aggregate concrete, there is a risk of spalling in the event of fire. Where this could endanger the member, measures should be taken to avoid its occurrence. The cover can be reduced to an acceptable value by:

(a) The application of a finish by hand or spray of plaster or vermiculite.
(b) The provision of a false ceiling as a fire barrier.
(c) The use of lightweight aggregate concrete.
(d) The use of sacrificial tensile reinforcement.

The following guidance can be given regarding the additional protection provided by selected applied finishes:

15 mm mortar, or gypsum plaster ≡ 25 mm concrete

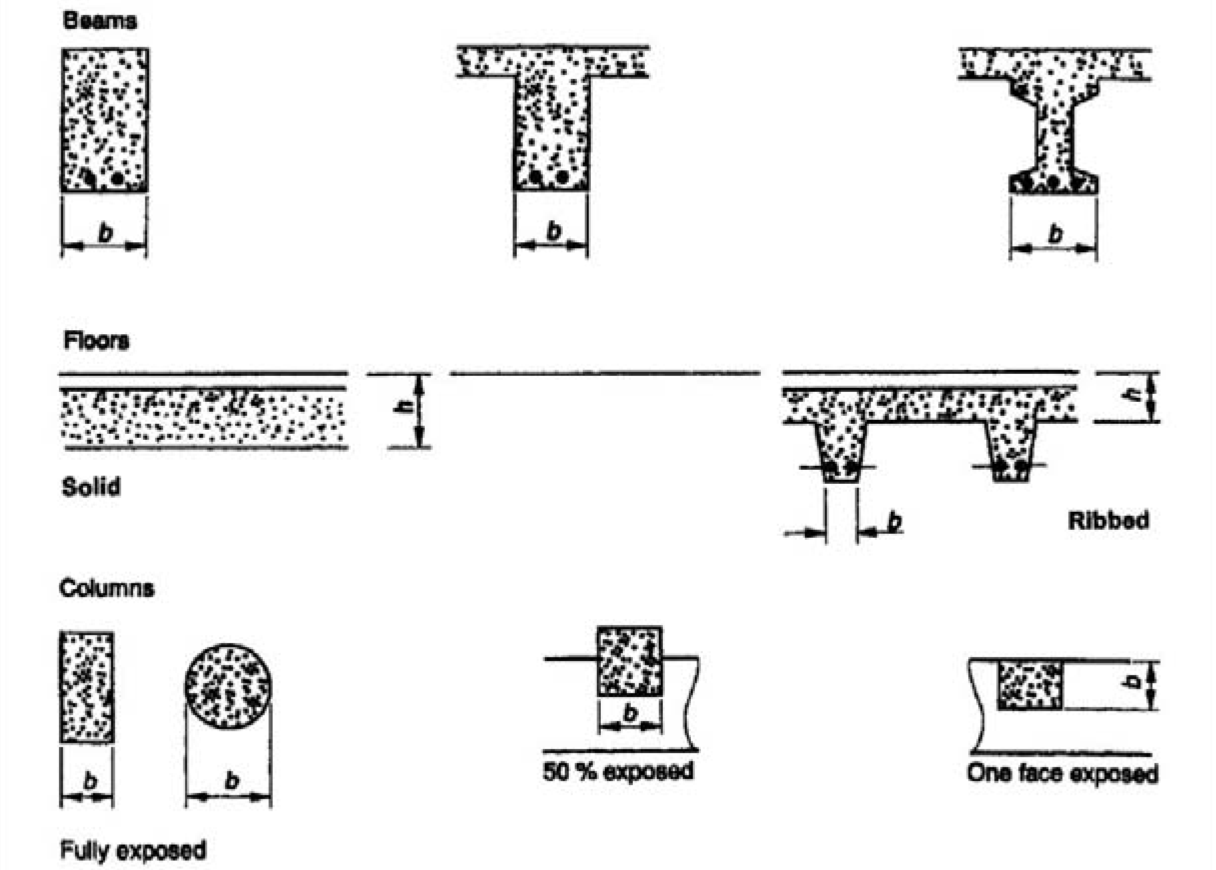
25 mm lightweight plaster, sprayed lightweight insulation, or vermiculite slabs
≡ 25 mm concrete (≤ 2 hours), 12.5 mm concrete (> 2 hours)

Alternatively, supplementary reinforcement (e.g. welded steel fabric) placed within the cover, at 20 mm from the face of the concrete, has been used to prevent spalling. There are practical difficulties in keeping the fabric in place and in compacting the concrete. In certain circumstances, there would also be a conflict with the durability requirements.

3.11

Fire resistance requirements (BS 8110) – 2

Minimum dimensions of members



Fire resistance period	Minimum beam width (b)	Minimum rib width (b)	Minimum thickness of floor (h)	Minimum column width (b)			Minimum wall thickness for reinforcement percentage p		
				Fully exposed mm	50% exposed mm	One face exposed mm	$p <$ 0.4	$0.4 < p <$ 1.0	$p >$ 1.0
hours	mm	mm	mm				mm	mm	mm
0.5	200	125	75	150	125	100	150	100	75
1.0	200	125	95	200	160	120	150	120	75
1.5	200	125	110	250	200	140	175	140	100
2.0	200	125	125	300	200	160	—	160	100
3.0	240	150	150	400	300	200	—	200	150
4.0	280	175	170	450	350	240	—	240	180

Note. The minimum dimensions given above relate specifically to the nominal covers given in *Table 3.10*. If the actual dimensions of the member exceed the minimum values, it may be possible to reduce the nominal covers. For further information, reference should be made to Section 4 of BS 8110-2.

3.12

Building regulations: minimum fire periods

Purpose group of building	Minimum fire periods (hours) for elements of structure					
	Basement storey (note 1)		Ground or upper storey			
	Depth (m) of lowest basement		Height (m) of top floor above ground in building or separated part of a building			
	≤ 10	> 10	≤ 5	≤ 18	≤ 30	> 30
1. Residential (domestic):						
a. Flats and maisonettes	1.0	1.5	0.5[1]	1.0[2]	1.5[2]	2.0[2]
b. and c. Dwelling houses	0.5[1]	—	0.5[1]	1.0[3]	—	—
2. Residential						
a. Institutional (note 2)	1.0	1.5	0.5[1]	1.0	1.5	2.0[4]
b. Other residential	1.0	1.5	0.5[1]	1.0	1.5	2.0[4]
3. Office:						
– not sprinklered	1.0	1.5	0.5[1]	1.0	1.5	—
– sprinklered (note 3)	1.0	1.0	0.5[1]	0.5[1]	1.0	2.0[4]
4. Shop and commercial:						
– not sprinklered	1.0	1.5	1.0	1.0	1.5	—
– sprinklered (note 3)	1.0	1.0	0.5[1]	1.0	1.0	2.0[4]
5. Assembly and recreation:						
– not sprinklered	1.0	1.5	1.0	1.0	1.5	—
– sprinklered (note 3)	1.0	1.0	0.5[1]	1.0	1.0	2.0[4]
6. Industrial:						
– not sprinklered	1.5	2.0	1.0	1.5	2.0	—
– sprinklered (note 3)	1.0	1.5	0.5[1]	1.0	1.5	2.0[4]
7. Storage and other non-residential:						
a. Any building or part not described elsewhere						
– not sprinklered	1.5	2.0	1.0	1.5	2.0	—
– sprinklered (note 3)	1.0	1.5	0.5[1]	1.0	1.5	2.0[4]
b. Car park for light vehicles						
– open-sided car park (note 4)	—	—	0.25[5]	0.25[5]	0.25[5]	1.0
– any other car park	1.0	1.5	0.5[1]	1.0	1.5	2.0[4]

Note 1. The floor over a basement (or topmost basement, if there is more than one basement) should meet the provisions for either the basement, or the ground and upper storeys, whichever period is higher.
Note 2. Multi-storey hospitals designed in accordance with the NHS Firecode should have a minimum of 1 hour standard.
Note 3. Sprinklered means that the building is fitted throughout with an automatic sprinkler system meeting the relevant recommendations of BS 5306 Fire extinguishing installations and equipment on premises: Part 2 specification for sprinkler systems; i.e. the relevant occupancy rating together with additional requirements for life safety.
Note 4. The car park should comply with the relevant provisions in the guidance on B3, Section 12 of the Regulations.

[1] Increased to a minimum of 1.0 hour for compartment walls separating buildings.
[2] Reduced to 0.5 hour for any floor within a maisonette, but not if the floor contributes to the support of the building.
[3] Reduced to 0.5 hour for 3-storey dwelling houses, but minimum of 1.0 hour for compartment walls separating buildings.
[4] Reduced to 1.5 hours for elements not forming part of the structural frame.
[5] Increased to 0.5 hour for elements protecting means of escape and 1.0 hour for compartment walls separating buildings.

Chapter 24

Bending and axial force

24.1 DESIGN ASSUMPTIONS

Basic assumptions regarding the design of cracked concrete sections at the ULS are outlined in section 5.2. The tensile strength of the concrete is neglected and strains are evaluated on the assumption that plane sections before bending remain plane after bending. Reinforcement stresses are then derived from these strains on the basis of the design stress–strain curves shown on *Table 3.6*. The BS 5400 curve is the one that was used in CP 110 prior to that code being superseded by BS 8110. For the concrete stresses, alternative assumptions are permitted. The design stress–strain curve for concrete shown on *Table 3.6* gives a stress distribution that is a combination of a parabola and rectangle. The form of the data governing the shape of the curve causes the relative proportions of the two parts to vary as the concrete strength changes. Thus the total compressive force provided by the concrete is not linearly related to f_{cu}, and the position of the centroid of the stress-block changes with f_{cu}.

Alternatively, an equivalent rectangular stress distribution in the concrete may be assumed, as noted on *Table 3.6*. The BS 5400 rectangular stress-block is the one that was used in CP 110 prior to that code being superseded by BS 8110. In BS 5400, the use of the rectangular stress-block is prohibited in flanged, ribbed and voided sections where the neutral axis lies outside the flange, although there appears to be no logical reason for this restriction.

For a rectangular area of width b and depth x, the total compressive force is given by $k_1 f_{cu} bx$ and the distance of the force from the compression face is given by $k_2 x$, where values of k_1 (allowing for the term $0.67/\gamma_m$)and k_2 are given in the following table, according to the shape of the stress-block.

Properties of concrete stress-blocks for rectangular area

Shape	f_{cu} N/mm^2	k_1	k_2
Parabolic- rectangular (*Table 3.6*)	25	0.405	0.456
	30	0.401	0.452
	35	0.397	0.448
	40	0.394	0.445
	50	0.388	0.439
BS 8110 rectangular		0.402	0.450
BS 5400 rectangular		0.400	0.500

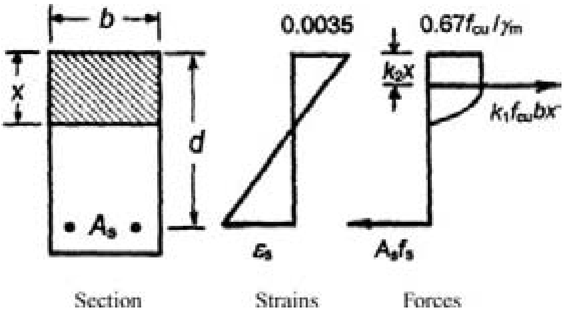
Using the rectangular stress-block is conservative in BS 5400 but gives practically the same result as that obtained with the parabolic-rectangular form in BS 8110.

24.2 BEAMS AND SLABS

Beams and slabs are generally subjected to bending only but, sometimes, are also required to resist an axial force, for example, in a portal frame, or in a floor acting as a prop between basement walls. Axial thrusts not exceeding $0.1 f_{cu}$ times the area of the cross section may be ignored in the analysis of the section, since the effect of the axial force is to increase the moment of resistance. Where a section is designed to resist bending only, the value of the lever arm should not be taken greater than 0.95 times the effective depth of the reinforcement.

In cases where, as a result of moment redistribution allowed in the analysis of the member, the design moment is less than the maximum elastic moment at the section, the neutral axis depth should satisfy the requirement $x/d \leq (\beta_b - 0.4)$, where β_b is the ratio of design moment to maximum elastic moment.

24.2.1 Singly reinforced rectangular sections



The lever arm between the forces shown in the figure here is given by $z = (d - k_2x)$, from which $x = (d - z)/k_2$.

Taking moments for the compressive force about the line of action of the tensile force gives

$$M = k_1 f_{cu} bxz = k_1 f_{cu} bz(d - z)/k_2$$

The solution of the resulting quadratic equation in z gives

$$z/d = 0.5 + \sqrt{0.25 - (k_2/k_1)K} \leq 0.95 \text{ where } K = M/bd^2 f_{cu}$$

Taking moments for the tensile force about the line of action of the compressive force gives

$$M = A_s f_s z, \text{ from which } A_s = M/f_s z$$

The strain in the reinforcement $\varepsilon_s = 0.0035(1 - x/d)/(x/d)$ and from the BS 8110 design stress–strain curve, the stress in the reinforcement is given by:

$$f_s = \varepsilon_s E_s = 700(1 - x/d)/(x/d) \leq f_y/1.15$$

Thus for design to BS 8110, $f_s = f_y/1.15$ for values of

$$x/d \leq 805/(805 + f_y) = 0.617 \text{ for } f_y = 500 \text{ N/mm}^2$$

From the BS 5400 design stress–strain curve, the stress in the reinforcement is given by:

$$f_s = \varepsilon_s E_s = 700(1 - x/d)/(x/d) \leq 0.8f_y/1.15 \text{ or}$$

$$0.8f_y/1.15 < f_s = \left(\frac{200f_y}{2300 + f_y}\right)\left(\frac{3.5 + 4.5x/d}{x/d}\right) \leq f_y/1.15$$

Thus for design to BS 5400, $f_s < 0.8f_y/1.15$ for values of

$$x/d > 805/(805 + 0.8f_y) = 0.668 \text{ for } f_y = 500 \text{ N/mm}^2$$

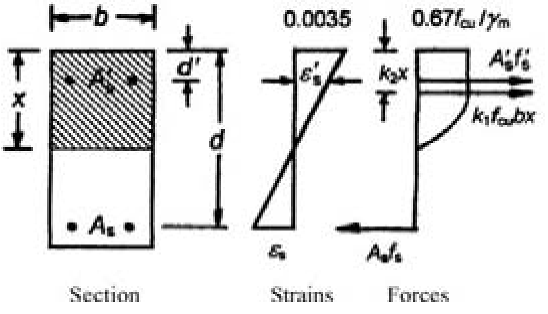
and $f_s = f_y/1.15$ for values of

$$x/d \leq 805/(1265 + f_y) = 0.456 \text{ for } f_y = 500 \text{ N/mm}^2$$

Design charts for $f_y = 500$ N/mm^2, derived on the basis of the parabolic-rectangular stress-block for the concrete, are given in *Table 3.13* for BS 8110 and *Table 3.23* for BS 5400. In the case of BS 5400, unless the section is proportioned such that $f_s = f_y/1.15$, the moment of resistance should provide at least 1.15 times the applied design moment.

Design tables, based on the rectangular stress-blocks for the concrete, are given in *Table 3.14* for BS 8110 and *Table 3.24* for BS 5400. The tables use non-dimensional parameters and are valid for $f_y \leq 500$ N/mm^2. The formulae used to derive the tables, and the limitations when redistribution of moment has been allowed in the analysis of the member, are also given.

24.2.2 Doubly reinforced rectangular sections



The forces provided by the concrete and the reinforcement are shown in the figure here. Taking moments for the two compressive forces about the line of action of the tensile force gives

$$M = k_1 f_{cu} bx(d - k_2 x) + A'_s f'_s(d - d')$$

The strain in the reinforcement $\varepsilon'_s = 0.0035(1 - d'/x)$ and from the BS 8110 design stress–strain curve, the stress is given by:

$$f'_s = \varepsilon'_s E_s = 700(1 - d'/x) \leq f_y/1.15$$

Thus for design to BS 8110, $f'_s = f_y/1.15$ for values of

$$x/d \geq [805/(805 - f_y)](d'/d) = 2.64(d'/d) \text{ for } f_y = 500\text{N/mm}^2$$

From the BS 5400 design stress-strain curve, the stress in the reinforcement is given by:

$$f'_s = \varepsilon'_s E_s = 700(1 - d'/x) \leq 0.8f_y/1.15 \qquad \text{or}$$

$$0.8f_y/1.15 < f'_s = \left(\frac{200f_y}{2300 + f_y}\right)\left(11.5 - 3.5\frac{d'}{x}\right) \leq \left(\frac{2000f_y}{2300 + f_y}\right)$$

Thus for design to BS 5400, $f'_s < 0.8f_y/1.15$ for values of

$$x/d < [805/(805 - 0.8f_y)](d'/d) = 2(d'/d) \text{ for } f_y = 500\text{N/mm}^2$$

and $f'_s = 2000f_y(2300 + f_y)$ for values of $x/d \geq (7/3)(d'/d)$

Equating the tensile and compressive forces gives

$$A_s f_s = k_1 f_{cu} bx + A'_s f'_s$$

where the stress in the tension reinforcement is given by the expressions used in section 24.2.1.

Design charts based on the rectangular stress-blocks for concrete, and for $d'/d = 0.1$ and 0.15 respectively, are given in *Tables 3.15* and *3.16* for BS 8110, and *Tables 3.25* and *3.26* for BS 5400. The charts use non-dimensional parameters and were determined for $f_y = 500$ N/mm^2, but may be safely used for $f_y \leq 500$ N/mm^2. In determining the forces in the concrete, no reduction has been made for the area of concrete displaced by the compression reinforcement.

24.2.3 Design formulae for rectangular sections

Design formulae based on the rectangular stress-blocks for concrete are given in BS 8110 and BS 5400. In both codes, x is limited to $0.5d$ so that the formulae are automatically valid for redistribution of moment not exceeding 10%.

The stress in the tension reinforcement is taken as $0.87f_y$ in both codes, although this is only strictly valid for $x/d \leq 0.456$ in BS 5400. The stress in the compression reinforcement is taken as $0.87f_y$ in BS 8110, and $0.72f_y$ in BS 5400. The code equations, which follow from the analyses in sections 24.2.1 and 24.2.2, take different forms in BS 8110 and BS 5400.

In BS 8110, the requirement for compression reinforcement depends on the value of $K = M/bd^2 f_{cu}$ compared to K' where:

$$K' = 0.156 \quad \text{for } \beta_b \geq 0.9$$

$$K' = 0.402(\beta_b - 0.4) - 0.18(\beta_b - 0.4)^2 \text{ for } 0.9 > \beta_b \geq 0.7$$

β_b is the ratio, design moment to maximum elastic moment

For $K \leq K'$, compression reinforcement is not required and

$$A_s = M/0.87f_y z$$

where

$$z = d\{0.5 + \sqrt{0.25 - K/0.9}\} \leq 0.95d \text{ and } x = (d - z)/0.45$$

For $K > K'$, compression reinforcement is required and

$$A'_s = (K - K')\, bd^2 f_{cu}/0.87f_y\,(d - d')$$
$$A_s = A'_s + K'\, bd^2 f_{cu}/0.87f_y z$$

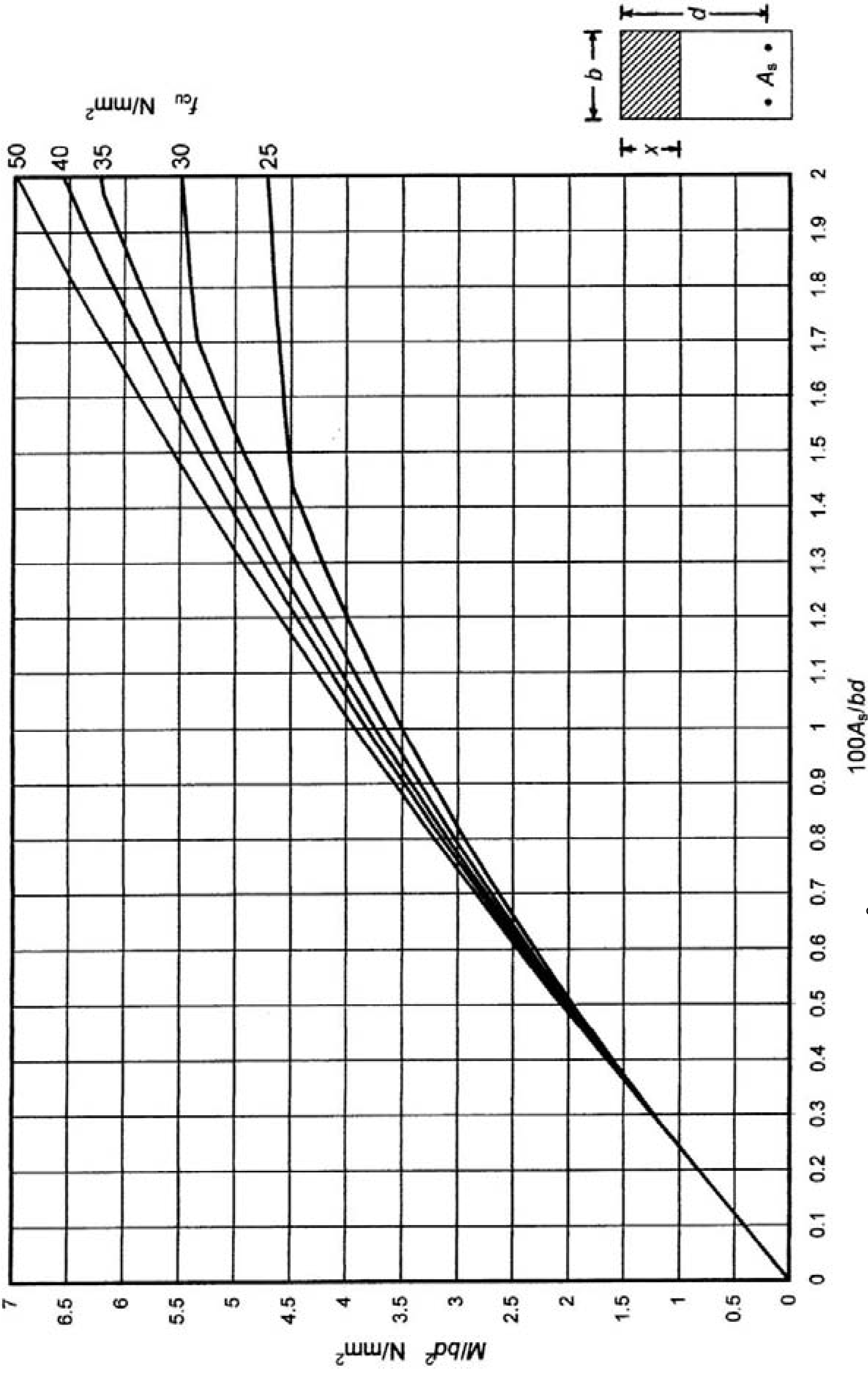
where

$$z = d\{0.5 + \sqrt{0.25 - K'/0.9}\} \text{ and } x = (d - z)/0.45$$

For $d'/x > 0.375$ (for $f_y = 500$ N/mm^2), A'_s should be replaced by $1.6(1 - d'/x)\,A'_s$ in the equations for A'_s and A_s.

BS 8110 Design chart for singly reinforced rectangular beams

3.13



Singly reinforced beams ($f_y = 500$ N/mm^2)

BS 8110 Design table for singly reinforced rectangular beams

3.14

$\frac{M}{bd^2 f_{cu}}$	$\frac{A_s f_y}{bdf_{cu}}$	$\frac{x}{d}$	$\frac{z}{d}$
≤0.043	1.21K	0.111	0.950
0.044	0.054	0.115	0.948
0.046	0.056	0.120	0.946
0.048	0.059	0.126	0.943
0.050	0.061	0.131	0.941
0.052	0.064	0.137	0.938
0.054	0.066	0.142	0.936
0.056	0.069	0.148	0.933
0.058	0.072	0.154	0.931
0.060	0.074	0.160	0.928
0.062	0.077	0.165	0.926
0.064	0.080	0.171	0.923
0.066	0.082	0.177	0.920
0.068	0.085	0.183	0.918
0.070	0.088	0.189	0.915
0.072	0.091	0.195	0.912
0.074	0.094	0.201	0.910
0.076	0.096	0.207	0.907
0.078	0.099	0.213	0.904
0.080	0.102	0.219	0.901
0.082	0.105	0.225	0.899
0.084	0.108	0.232	0.896
0.086	0.111	0.238	0.893
0.088	0.114	0.244	0.890
0.090	0.117	0.250	0.887
0.092	0.120	0.257	0.884
0.094	0.123	0.263	0.882
0.096	0.126	0.270	0.879
0.098	0.129	0.276	0.876
0.100	0.132	0.283	0.873
0.102	0.135	0.290	0.870
0.104	0.138	0.296	0.867
0.106	0.141	0.303	0.864
0.108	0.144	0.310	0.861
0.110	0.148	0.317	0.857
0.112	0.151	0.324	0.854
0.114	0.154	0.331	0.851
0.116	0.157	0.338	0.848
0.118	0.161	0.345	0.845
0.120	0.164	0.352	0.842
0.122	0.167	0.359	0.838
0.124	0.171	0.367	0.835
0.126	0.174	0.374	0.832
0.128	0.178	0.382	0.828
0.130	0.181	0.389	0.825
0.132	0.185	0.397	0.821
0.134	0.188	0.405	0.818
0.136	0.192	0.412	0.814
0.138	0.196	0.420	0.811
0.140	0.199	0.428	0.807
0.142	0.203	0.436	0.804
0.144	0.207	0.444	0.800
0.146	0.211	0.453	0.796
0.148	0.215	0.461	0.793
0.150	0.219	0.470	0.789
0.152	0.223	0.478	0.785
0.154	0.227	0.487	0.781
0.156	0.231	0.496	0.777
0.158	0.235	0.505	0.773
0.160	0.239	0.514	0.769
0.162	0.244	0.523	0.765
0.164	0.248	0.533	0.760
0.166	0.252	0.542	0.756
0.168	0.257	0.552	0.752
0.170	0.262	0.562	0.747
0.172	0.266	0.572	0.743
0.174	0.271	0.582	0.738
0.176	0.276	0.593	0.733
0.178	0.280	0.603	0.728

Redistribution %	Moment ratio β_b	Limiting values		
		$\frac{x}{d}$	$\frac{M}{bd^2 f_{cu}}$	$\frac{A_s f_y}{bdf_{cu}}$
0	1.00	0.60	0.176	0.276
5	0.95	0.55	0.166	0.252
10	0.90	0.50	0.156	0.231
15	0.85	0.45	0.144	0.207
20	0.80	0.40	0.132	0.185
25	0.75	0.35	0.118	0.161
30	0.70	0.30	0.104	0.138

Formulae used to determine limiting values:

$x/d = (\beta_b - 0.4) \qquad K = 0.402(\beta_b - 0.4) - 0.18(\beta_b - 0.4)^2$

Formulae used in table above

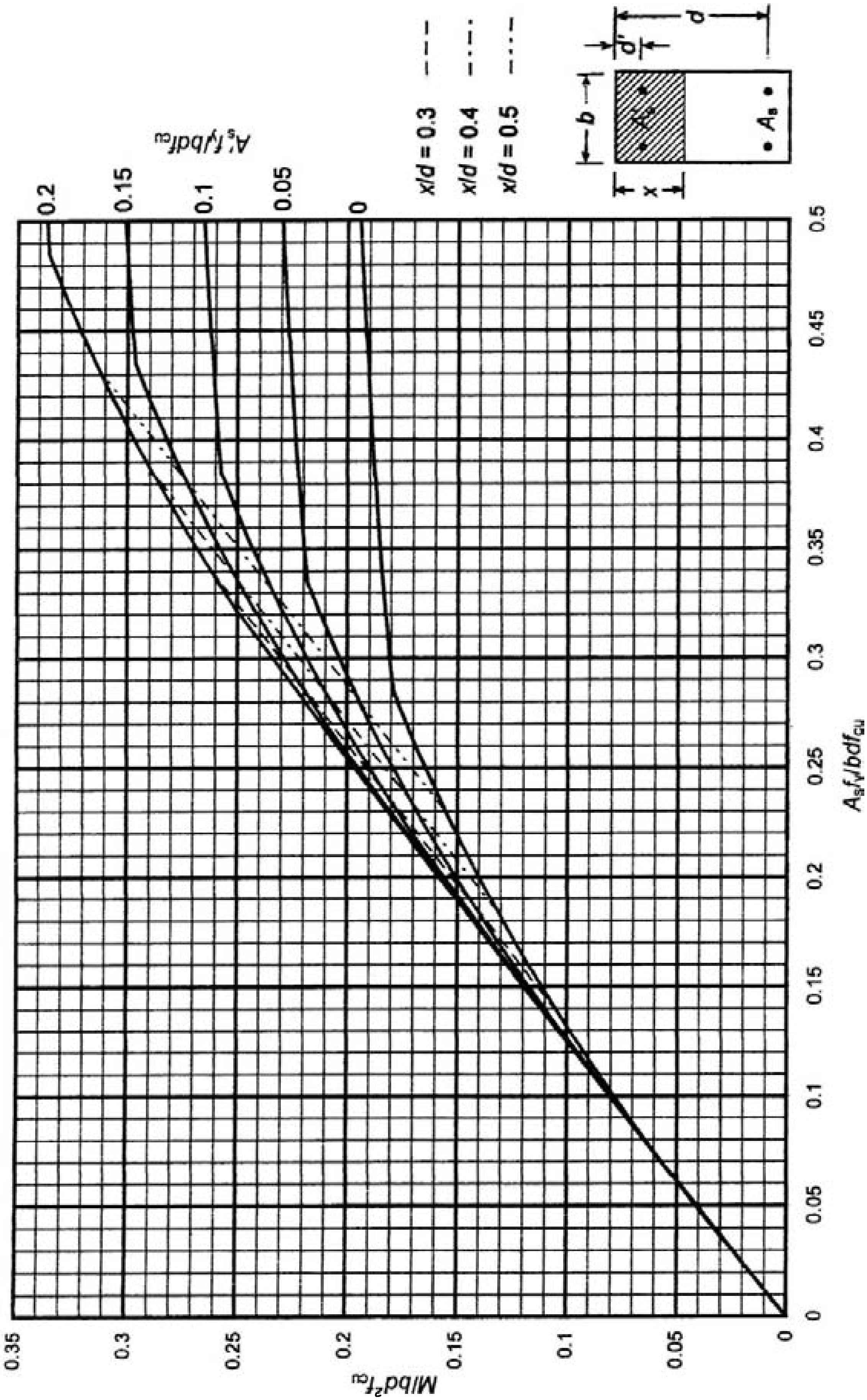
$$\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{K}{0.9}} \leq 0.95$$

$$\frac{x}{d} = \frac{1 - z/d}{0.45} \qquad \frac{A_s f_y}{bdf_{cu}} = \frac{K}{0.87\,z/d}$$

where $K = \dfrac{M}{bd^2 f_{cu}}$

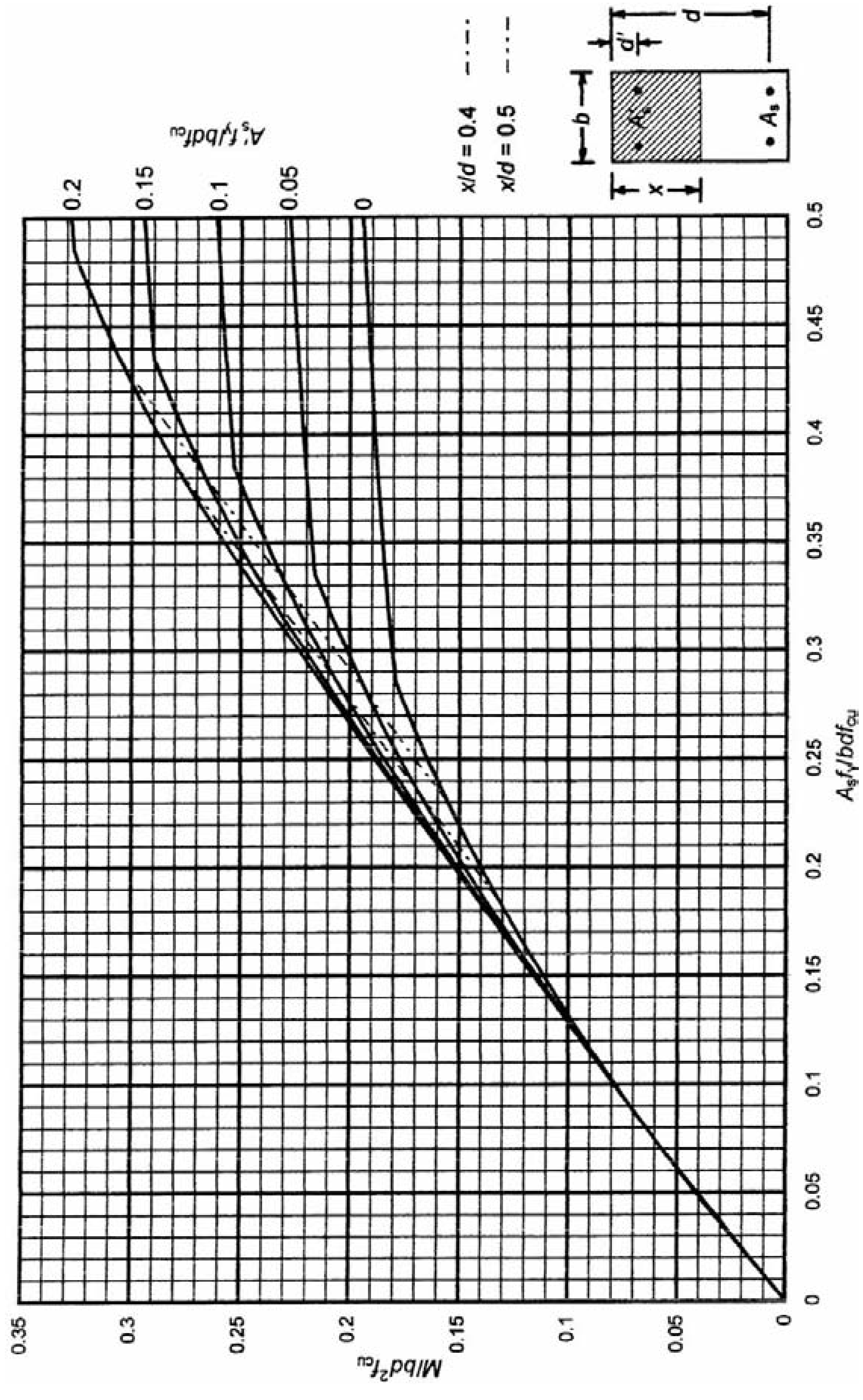
BS 8110 Design chart for doubly reinforced rectangular beams – 1

3.15



Doubly reinforced beams ($f_y = 500$ N/mm^2, $d'/d = 0.1$)

BS 8110 Design chart for doubly reinforced rectangular beams – 2



Doubly reinforced beams ($f_y = 500$ N/mm^2, $d'/d = 0.15$)

In BS 5400, the moment of resistance of a section without compression reinforcement is given by the equations:

$$M_u = A_s(0.87f_y)z \leq 0.15bd^2f_{cu}$$

where

$$z = d\,(1 - 1.1A_sf_y/bdf_{cu}) \leq 0.95d$$

For a section with compression reinforcement, the moment of resistance is given by the equations:

$$M_u = 0.15bd^2f_{cu} + A'_s(0.72f_y)(d - d')$$

$$A_s(0.87f_y) = 0.2bdf_{cu} + A'_s(0.72f_y)$$

The equations, which are based on $x = 0.5d$, are considered to be valid for values of $d'/d \leq 0.2$. A variant on the equations, with x taken as a variable, is included in Highways Agency BD44/95 for assessment purposes. These equations should not be used for values of x greater than $0.5d$.

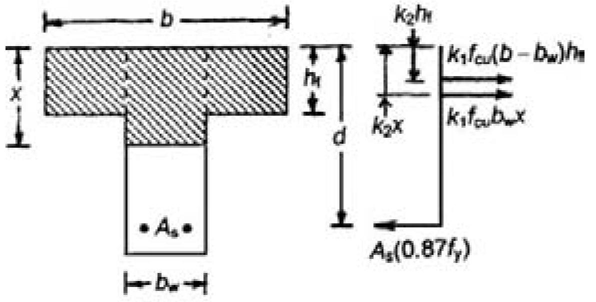
24.2.4 Flanged sections

In monolithic beam and slab construction, where the web of the beam projects below the slab, the beam is considered as a flanged section for sagging moments. The effective width of the flange may be taken as follows:

T beam: $b = b_w + 0.2l_z \leq$ actual flange width
L beam: $b = b_w + 0.1l_z \leq$ actual flange width

l_z is the distance between points of zero moment which, for a continuous beam, may be taken as 0.7 times effective span.

In most sections where the flange is in compression, the depth of the neutral axis will be no greater than the thickness of the flange. In this case, the section can be considered to be rectangular with b taken as the flange width. The condition regarding the neutral axis depth can be confirmed initially by showing that $M \leq k_1f_{cu}bh_f\,(d - k_2h_f)$, where h_f is the thickness of the flange. Alternatively, the section can be considered to be rectangular initially, and the neutral axis depth can be checked subsequently.



Section Forces

The figure here shows a flanged section where the neutral axis depth is greater than the flange thickness. The concrete force can be divided into two components and the required area of tension reinforcement is then given by:

$$A_s = A_{s1} + k_1f_{cu}\,(b - b_w)h_f\,/0.87f_y$$

where

A_{s1} = area of reinforcement required to resist a moment M_1 applied to a rectangular section, of width b_w, and

$$M_1 = M - k_1f_{cu}\,(b - b_w)h_f\,(d - k_2h_f) \leq 0.15bd^2f_{cu}$$

Using the rectangular concrete stress-blocks in the foregoing equations, gives $k_1 = 0.4$, with $k_2 = 0.45$ for BS 8110 and 0.5 for BS 5400. This approach gives solutions that are 'correct' when $x = h_f$, but become slightly more conservative as $(x - h_f)$ increases. A different approach is used in BS 8110 resulting in solutions that are 'correct' when $x = 0.5d$, but increasingly conservative as $(x - h_f)$ decreases. As a result, the solution when $x = h_f$ does not agree with that obtained by considering the section as rectangular with b taken as the flange width.

24.2.5 General analysis of sections

The analysis of a section of any shape, with any arrangement of reinforcement, involves a trial-and-error process. An initial value is assumed for the neutral axis depth, from which the concrete strains at the positions of the reinforcement can be calculated. The corresponding stresses in the reinforcement are determined, and the resulting forces in the reinforcement and the concrete are obtained. If the forces are out of balance, the value of the neutral axis depth is changed and the process is repeated until equilibrium is achieved. Once the balanced condition has been found, the resultant moment of the forces about the neutral axis, or any other point, is calculated.

Example 1. The beam shown in the following figure is to be designed to the requirements of BS 8110. The design loads on each span are as follows, where $G_k = 160$ kN and $Q_k = 120$ kN:

$$F_{max} = 1.4G_k + 1.6Q_k = 416\text{ kN},\ F_{min} = 1.0G_k = 160\text{ kN}$$

The section design is to be based on the following values:

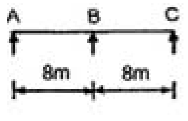
$$f_{cu} = 40\text{ N/mm}^2,\ f_y = 500\text{ N/mm}^2,\ \text{cover to links} = 25\text{ mm.}$$

For sagging moments, effective width of flange

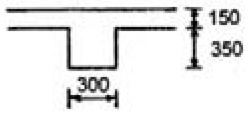
$$b = b_w + 0.2l_z = 300 + 0.2(0.7 \times 8000) = 1420\text{ mm}$$

Allowing for 8 mm links and 32 mm main bars,

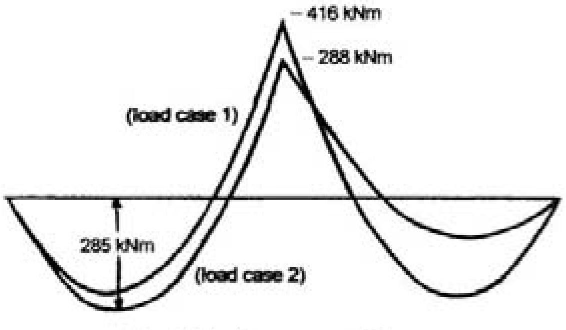
$$d = 500 - (25 + 8 + 16) = 450\text{ mm say.}$$



Spans



Cross section



Elastic bending moment diagram

In the calculations that follow, solutions are obtained using charts and equations, to demonstrate the use of each method.

Maximum sagging moment. For section to be designed as rectangular with b taken as the flange width, bending moment should satisfy the condition:

$$M \le k_1 f_{cu} b h_f \,(d - k_2 h_f)$$

$$= 0.4 \times 40 \times 1420 \times 150 \times (450 - 0.45 \times 150) \times 10^{-6}$$

$$= 1303 \text{ kNm } (>285 \text{ kNm})$$

$$M/bd^2 = 285 \times 10^6/(1420 \times 450^2) = 0.99 \text{ N/mm}^2$$

From chart in *Table 3.13*, $100A_s/bd = 0.24$,

$$A_s = 0.0024 \times 1420 \times 450 = 1534 \text{ mm}^2$$

Alternatively, $K = M/bd^2 f_{cu} = 0.99/40 = 0.0248$
From *Table 3.14*, $A_s f_y/bdf_{cu} = 1.21K = 0.0300$

$$A_s = 0.03 \times 1420 \times 450 \times 40/500 = 1534 \text{ mm}^2$$

Alternatively, by calculation or from *Table 3.14*,

$$z/d = 0.5 + \sqrt{0.25 - 0.0248/0.9} = 0.972 \le 0.95$$

Hence $A_s = M/0.87 f_y z$ gives

$$A_s = 285 \times 10^6/(0.87 \times 500 \times 0.95 \times 450) = 1533 \text{ mm}^2$$

Using 2H32 gives 1608 mm^2

Maximum hogging moment

$$K = M/bd^2 f_{cu} = 416 \times 10^6/(300 \times 450^2 \times 40) = 0.171$$

From *Table 3.14*, $A_s f_y/bdf_{cu} = 0.264$

$$A_s = 0.264 \times 300 \times 450 \times 40/500 = 2851 \text{ mm}^2$$

Using 4H32 gives 3217 mm^2

Although this is a valid solution, it may be possible to reduce the area of tension reinforcement to a more suitable value, by allowing for some compression reinforcement. Consider the use of 2H25 with $d' = 45$ mm ($d'/d = 0.1$).

$$A'_s f_y/bdf_{cu} = 982 \times 500/(300 \times 450 \times 40) = 0.09$$

From the chart in *Table 3.15*, $A_s f_y/bdf_{cu} = 0.225$

$$A_s = 0.225 \times 300 \times 450 \times 40/500 = 2430 \text{ mm}^2$$

A solution can also be obtained using the design equations, as follows:

With 2H25 for A'_s and assuming $f'_s = 0.87 f_y$,

$$K' = K - A'_s\,(0.87 f_y)(d - d')/bd^2 f_{cu}$$

$$= 0.171 - 982 \times 0.87 \times 500 \times 405/(300 \times 450^2 \times 40) = 0.100$$

$$z/d = 0.5 + \sqrt{0.25 - 0.100/0.9} = 0.873$$

$$x/d = (1 - z/d)/0.45 = (1 - 0.873)/0.45 = 0.282$$

$$d'/x = (d'/d)(x/d) = 0.1/0.282 = 0.355$$

Since $d'/x < 0.375$, $f'_s = 0.87 f_y$ is valid.

$$A_s = A'_s + K'\,bd^2 f_{cu}/0.87 f_y z$$

$$= 982 + 0.100 \times 300 \times 450^2 \times 40/(0.87 \times 500 \times 0.873 \times 450)$$

$$= 2404 \text{ mm}^2 \text{ (compared to 2430 mm}^2 \text{ obtained from chart)}$$

Using 3H32 gives 2413 mm^2

Example 2. Suppose that in the previous example the maximum hogging moment at B is reduced by 30% to 291 kNm.

$$K = M/bd^2 f_{cu} = 291 \times 10^6/(300 \times 450^2 \times 40) = 0.120$$

$$\beta_b = 291/416 = 0.70,\ x/d \le (\beta_b - 0.4) = 0.30$$

From chart in *Table 3.15*, keeping to left of line for $x/d = 0.3$

$$A'_s f_y/bdf_{cu} = 0.025,\ A_s f_y/bdf_{cu} = 0.158$$

$$A'_s = 0.025 \times 300 \times 450 \times 40/500 = 270 \text{ mm}^2$$

$$A_s = 0.158 \times 300 \times 450 \times 40/500 = 1706 \text{ mm}^2$$

A solution can also be obtained by using the design equations with $x/d = 0.3$ as follows:

$$K' = 0.402(\beta_b - 0.4) - 0.18(\beta_b - 0.4)^2$$

$$= 0.402 \times 0.3 - 0.18 \times 0.3^2 = 0.104 \ (<K = 0.120)$$

$$z/d = 0.5 + \sqrt{0.25 - 0.104/0.9} = 0.867$$

$$A'_s = (K - K')\,bd^2 f_{cu}/0.87 f_y\,(d - d')$$
$$= 0.016 \times 300 \times 450^2 \times 40\,/\,(0.87 \times 500 \times 405) = 221 \text{ mm}^2$$

$$A_s = A'_s + K' bd^2 f_{cu}/0.87 f_y z$$
$$= 221 + 0.104 \times 300 \times 450^2 \times 40/(0.87 \times 500 \times 0.867 \times 450)$$
$$= 1710 \text{ mm}^2$$

Using 2H25 and 1H32 gives 1786 mm^2

Since the reduced hogging moment for load case 1 is still greater than the elastic hogging moment for load case 2, the design sagging moment remains the same as in example 1.

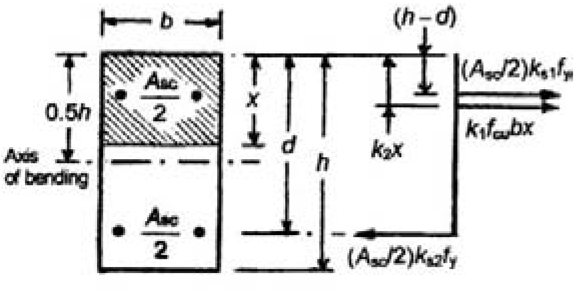
In the foregoing examples, at the bottom of the beam, 2H32 bars would run the full length of each span with 2H25 splice bars at support B. Other bars would be curtailed according to the bending moment requirements and detailing rules.

24.3 COLUMNS

In the Codes of Practice, a column is a compression member whose greater overall cross-sectional dimension does not exceed four times its smaller dimension. An effective height and a slenderness ratio are determined in relation to major and minor axes of bending. An effective height is a function of the clear height and depends upon the restraint conditions at the ends of the column. A slenderness ratio is defined as the effective height divided by the depth of the cross section in the plane of bending. The column is then considered to be either short or slender, according to the slenderness ratios.

Columns are subjected to combinations of bending moment and axial force, and the cross section may need to be checked for more than one combination of values. In slender columns, from an elastic analysis of the structure, the initial moment, is increased by an additional moment induced by the deflection of the column. In BS 8110, this additional moment contains a modification factor K, the use of which results in an iteration process with K taken as 1.0 initially. The design charts in this chapter contain sets of K lines as an aid to the design process. Details of the design procedures are given in *Tables 3.21* and *3.22* for BS 8110, and *Tables 3.31* and *3.32* for BS 5400.

24.3.1 Rectangular columns



Section Forces

The figure here shows a rectangular section in which the reinforcement is disposed equally on two opposite sides of a horizontal axis through the mid-depth. By resolving forces and taking moments about the mid-depth of the section, the following equations are given for $0 < x/h \leq 1.0$.

$$N/bhf_{cu} = k_1(x/h) + 0.5(A_{sc}f_y/bhf_{cu})(k_{s1} - k_{s2})$$

$$M/bh^2f_{cu} = k_1(x/h)\{0.5 - k_2(x/h)\} + 0.5\,(A_{sc}f_y/bhf_{cu})(k_{s1} + k_{s2})\,(d/h - 0.5)$$

For BS 8110, the stress factors, k_{s1} and k_{s2}, are given by:

$$k_{s1} = 1.4(x/h + d/h - 1)/(x/h) \leq 0.87$$

$$k_{s2} = 1.4(d/h - x/h)/(x/h) \leq 0.87$$

The maximum axial force N_{uz} is given by the equation

$$N_{uz}/bhf_{cu} = 0.45 + 0.87(A_{sc}f_y/bhf_{cu})$$

Design charts, based on the rectangular stress-block for the concrete, and for values of $d/h = 0.8$ and 0.85, are given in *Tables 3.17* and *3.18* respectively. On each curve, a straight line has been taken between the point where $x/h = 1.0$ and the point where $N = N_{uz}$. The charts, which were determined for $f_y = 500$ N/mm², may be safely used for $f_y \leq 500$ N/mm². In determining the forces in the concrete, no reduction has been made for the area of concrete displaced by the compression reinforcement. In the design of slender columns, the K factor is used to modify the deflection corresponding to a load N_{bal}, which for a symmetrically reinforced rectangular section is given as $0.25bdf_{cu}$. In the charts, N_{bal} is taken as the value at which M is a maximum. A line corresponding to N_{bal} passes through a cusp on each curve. For $N \leq N_{bal}$, K is taken as 1.0. For $N > N_{bal}$, K can be determined from the lines on the chart.

For BS 5400, the stress factors, k_{s1} and k_{s2}, are given by:

$$k_{s1} = 1.4(x/h + d/h - 1)/(x/h) \leq 0.7$$

$$0.7 \leq k_{s1} = 0.25(3.3x/h + d/h - 1)/(x/h) \leq 0.714$$

$$k_{s2} = 1.4(d/h - x/h)/(x/h) \leq 0.7$$

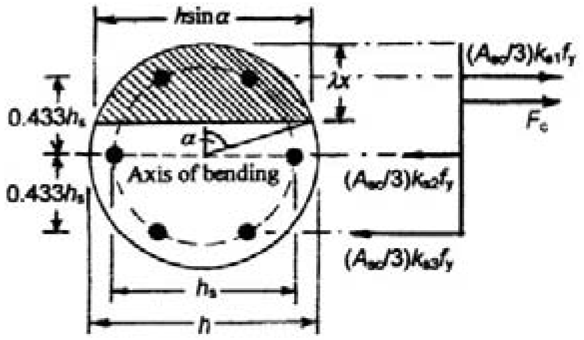
$$0.7 \leq k_{s2} = 0.25(d/h + 1.3x/h)/(x/h) \leq 0.87$$

Design charts, based on the rectangular stress-block for the concrete, and for values of $d/h = 0.8$ and 0.85, are given in *Tables 3.27* and *3.28* respectively. On each curve, a straight line has been taken between the point where $x/h = 0.8$ and the point where $N = N_{uz}$. The use of the rectangular stress-block results in N_{uz} being given by the equation

$$N_{uz}/bhf_{cu} = 0.4 + 0.714(A_{sc}f_y/bhf_{cu})$$

There are no K lines on the charts, as no modification factor is used in the design of slender columns to BS 5400.

24.3.2 Circular columns



Section Forces

The figure here shows a circular section containing six bars spaced equally around the circumference. Solutions based on six bars will be slightly conservative if more bars are used. The arrangement of the bars relative to the axis of bending affects the resistance of the section, and the arrangement shown in the figure is not the most critical in every case. For some combinations of bending moment and axial force, if the arrangement shown is rotated through 30°, a slightly more critical condition results, but the differences are small and may be reasonably ignored.

The following analysis is based on a uniform stress-block for the concrete, of depth λx and width $h\sin\alpha$ at the base (as shown in figure). Negative axial forces are included in order to cater for members such as tension piles. By resolving forces and taking moments about the mid-depth of the section, the following equations are obtained, where $\alpha = \cos^{-1}(1 - 2\lambda x/h)$ for $0 < x \leq 1.0$, and h_s is the diameter of a circle through the centres of the bars:

$$N/h^2f_{cu} = k_c(2\alpha - \sin 2\alpha)/8 + (\pi/12)(A_{sc}f_y/A_cf_{cu})(k_{s1} - k_{s2} - k_{s3})$$

$$M/h^3f_{cu} = k_c(3\sin\alpha - \sin 3\alpha)/72 + (\pi/27.7)(A_{sc}f_y/A_cf_{cu})(h_s/h)(k_{s1} + k_{s3})$$

The minimum axial force N_{min} is given by the equation

$$N_{min}/h^2f_{cu} = -\,0.87(\pi/4)(A_{sc}f_y/A_cf_{cu})$$

For BS 8110, $k_c = 0.45$, $\lambda = 0.9$ and the stress factors, k_{s1}, k_{s2} and k_{s3}, are given by:

$$-\,0.87 \leq k_{s1} = 1.4(0.433h_s/h - 0.5 + x/h)/(x/h) \leq 0.87$$

$$-\,0.87 \leq k_{s2} = 1.4(0.5 - x/h)/(x/h) \leq 0.87$$

$$-\,0.87 \leq k_{s3} = 1.4(0.5 + 0.433h_s/h - x/h)/(x/h) \leq 0.87$$

The maximum axial force N_{uz} is given by the equation

$$N_{uz}/h^2 f_{cu} = (\pi/4)\{0.45 + 0.87(A_{sc} f_y / A_c f_{cu})\}$$

Design charts for values of $h_s/h = 0.6$ and 0.7, are given in *Tables 3.19* and *3.20* respectively. The statements in section 24.3.1 on the derivation and use of the charts for rectangular sections apply also to those for circular sections.

For BS 5400, $k_c = 0.4$, $\lambda = 1.0$ and the stress factors, k_{s1}, k_{s2} and k_{s3}, are given by:

$$-\,0.87 \le k_{s1} = 0.25(0.433 h_s/h - 0.5 - 1.3x/h)/(x/h) \le -\,0.7$$
$$-\,0.7 \le k_{s1} = 1.4(0.433 h_s/h - 0.5 + x/h)/(x/h) \le 0.7$$
$$0.7 \le k_{s1} = 0.25(0.433 h_s/h - 0.5 + 3.3x/h)/(x/h) \le 0.714$$

$$-\,0.714 \le k_{s2} = 0.25(0.5 - 3.3x/h)/(x/h) \le -\,0.7$$
$$-\,0.7 \le k_{s2} = 1.4(0.5 - x/h)/(x/h) \le 0.7$$
$$0.7 \le k_{s2} = 0.25(0.5 + 1.3\,x/h)/(x/h) \le 0.87$$

$$-\,0.714 \le k_{s3} = 0.25(0.433 h_s/h + 0.5 - 3.3x/h)/(x/h) \le 0.7$$
$$-\,0.7 \le k_{s3} = 1.4(0.433 h_s/h + 0.5 - x/h)/(x/h) \le 0.7$$
$$0.7 \le k_{s3} = 0.25(0.433 h_s/h + 0.5 + 1.3x/h)/(x/h) \le 0.87$$

The maximum axial force N_{uz} is given by the equation

$$N_{uz}/h^2 f_{cu} = (\pi/4)\{0.4 + 0.72(A_{sc} f_y / A_c f_{cu})\}$$

Design charts for values of $h_s/h = 0.6$ and 0.7, are given in *Tables 3.29* and *3.30* respectively. The statements in section 24.3.1 on the derivation and use of the charts for rectangular sections apply also to those for circular sections.

24.3.3 Design formulae for short braced columns

Approximate formulae are given in BS 8110 for the design of short braced columns under specific conditions. Where, due to the nature of the structure, a column cannot be subjected to significant moments, it may be considered adequate if the design ultimate axial load, $N \le 0.4 f_{cu} A_c + 0.75 A_{sc} f_y$.

Columns supporting symmetrical arrangements of beams that are designed for uniformly distributed imposed load, and have spans that do not differ by more than 15% of the longer, may be considered adequate if $N \le 0.35 f_{cu} A_c + 0.67 A_{sc} f_y$.

24.3.4 General analysis of column sections

Any given cross section can be analysed by a trial-and-error process. For a section bent about one axis, an initial value is assumed for the neutral axis depth, from which the concrete strains at the positions of the reinforcement can be calculated. The resulting stresses in the reinforcement are determined, and the forces in the reinforcement and concrete evaluated. If the resultant force is not equal to the design axial force N, the value of the neutral axis depth is changed and the process repeated until equality is achieved. The resultant moment of all the forces about the mid-depth of the section is then the moment of resistance appropriate to N. This approach is used to analyse a rectangular section in example 6.

Example 3. A 300 mm square braced column designed to BS 8110, for the following requirements:

$$l_o = 3.75 \text{ m and } \beta = 0.9 \text{ in both directions}$$
$$M_x = 54 \text{ kNm},\ M_y = 0,\ N = 1800 \text{ kN}$$
$$f_{cu} = 40 \text{ N/mm}^2,\ f_y = 500 \text{ N/mm}^2,\ \text{cover to links} = 35 \text{ mm}$$

Since $l_e/h = 0.9 \times 3750/300 = 11.25 < 15$, the column is short.

$$M_{min} = N(0.05h) = 1800 \times 0.05 \times 0.3 = 27 \text{ kNm } (< M_x)$$

Allowing for 8 mm links and 32 mm main bars,

$$d = 300 - (35 + 8 + 16) = 240 \text{ mm say}$$
$$M/bh^2 f_{cu} = 54 \times 10^6/(300 \times 300^2 \times 40) = 0.05$$
$$N/bhf_{cu} = 1800 \times 10^3/(300 \times 300 \times 40) = 0.5$$

From the design chart for $d/h = 240/300 = 0.8$,

$$A_{sc} f_y / bhf_{cu} = 0.22 \ (\textit{Table 3.17})$$
$$A_{sc} = 0.22 \times 300 \times 300 \times 40/500 = 1584 \text{ mm}^2$$

Using 4H25 gives 1963 mm²

Example 4. A 300 mm circular braced column designed to BS 8110, for the same requirements as example 3:

Allowing for 8 mm links and 32 mm main bars,

$$h_s = 300 - 2 \times (35 + 8 + 16) = 180 \text{ mm say}$$
$$M/h^3 f_{cu} = 54 \times 10^6/(300^3 \times 40) = 0.05$$
$$N/h^2 f_{cu} = 1800 \times 10^3/(300^2 \times 40) = 0.5$$

From the design chart for $h_s/h = 180/300 = 0.6$,

$$A_{sc} f_y / A_c f_{cu} = 0.52 \ (\textit{Table 3.19})$$
$$A_{sc} = 0.52 \times (\pi/4) \times 300^2 \times 40/500 = 2940 \text{ mm}^2$$

Using 6H25 gives 2945 mm²

Example 5. The column in example 3, but designed for biaxial bending with $M_y = 25$ kNm, and all other requirements as before:

Since $h' = b'$ and $M_x > M_y$, the section may be designed for an increased moment about the x-x axis (see *Table 3.21*):

$$\beta = 1 - (7/6)(N/bhf_{cu}) = 1 - (7/6) \times 0.5 = 0.42$$
$$M'_x = M_x + \beta\, M_y = 54 + 0.42 \times 25 = 64.5 \text{ kNm}$$
$$M/bh^2 f_{cu} = 64.5 \times 10^6/(300 \times 300^2 \times 40) = 0.06$$

From the design chart for $d/h = 240/300 = 0.8$,

$$A_{sc} f_y / bhf_{cu} = 0.26 \ (\textit{Table 3.17})$$
$$A_{sc} = 0.26 \times 300 \times 300 \times 40/500 = 1872 \text{ mm}^2$$

Using 4H25 gives 1963 mm²

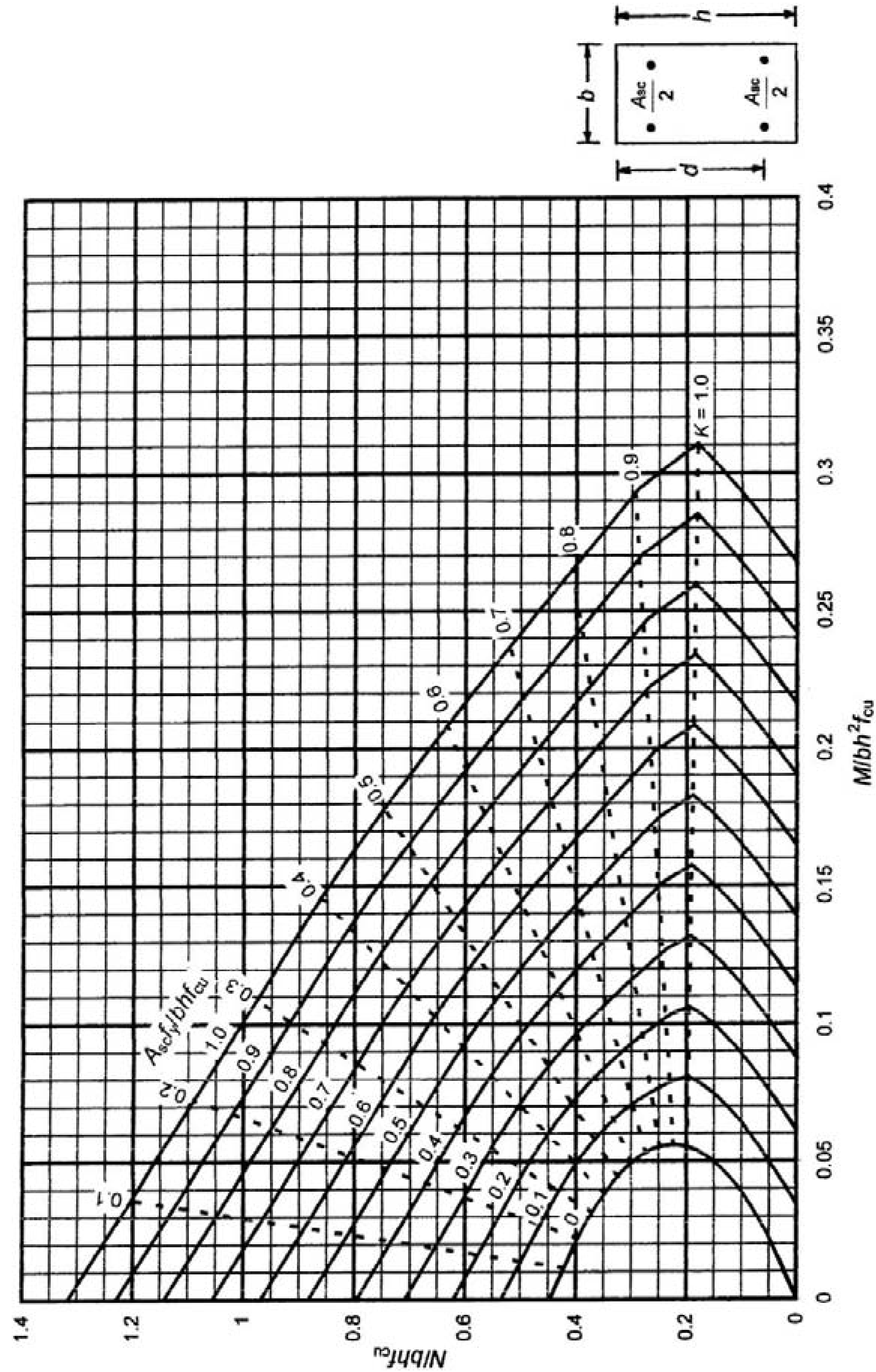
Example 6. A 300 mm square short column designed to BS 5400 for the following requirements:

$$M_x = 60 \text{ kNm},\ M_y = 40 \text{ kNm},\ N = 1800 \text{ kN}$$
$$f_{cu} = 40 \text{ N/mm}^2,\ f_y = 500 \text{ N/mm}^2,\ d = 240 \text{ mm}$$

The section may be designed by assuming the reinforcement (4H32 say) and checking the condition (see *Table 3.31*):

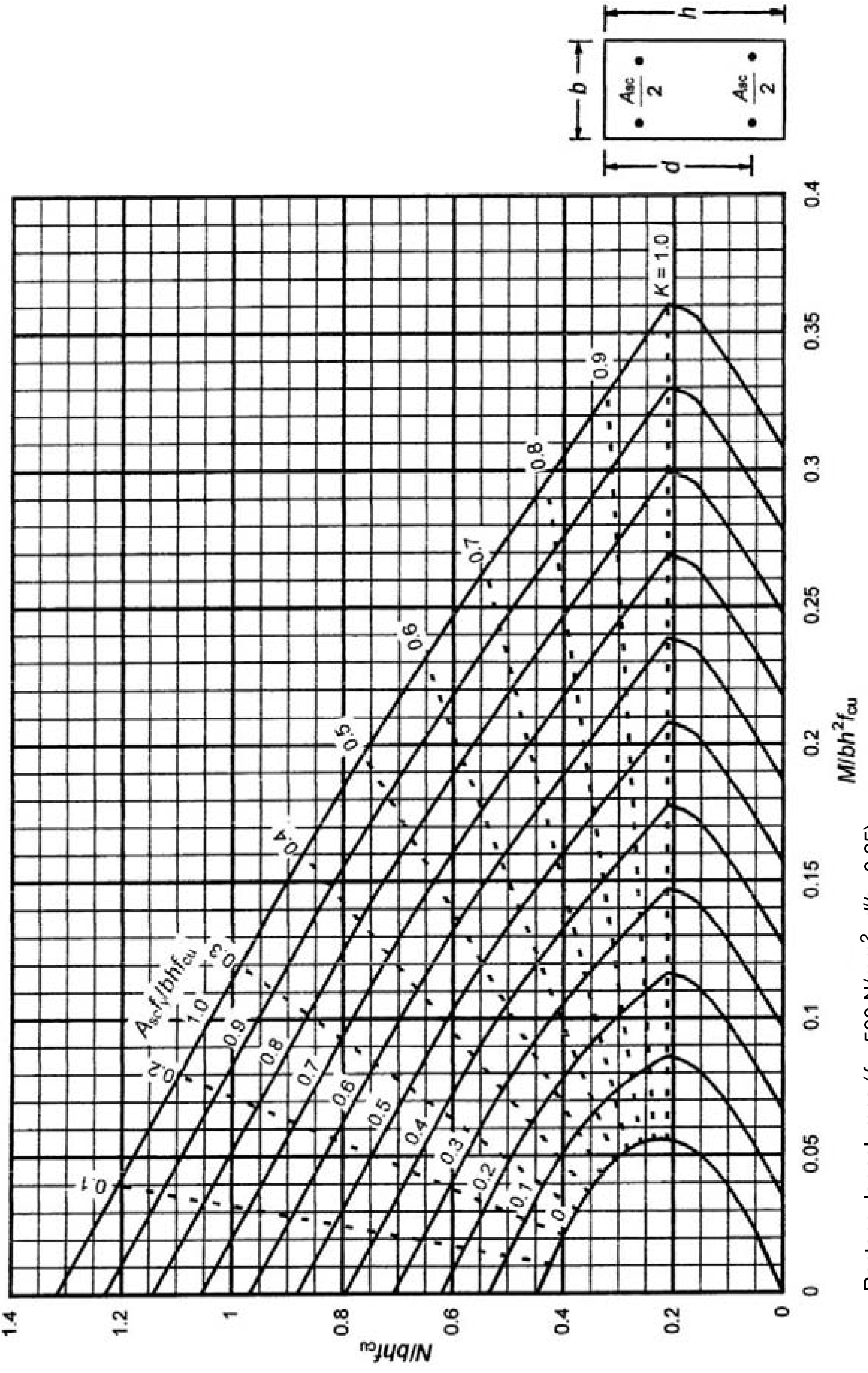
$$A_{sc} f_y / bhf_{cu} = 3217 \times 500/(300 \times 300 \times 40) = 0.45$$
$$N/bhf_{cu} = 1800 \times 10^3/(300 \times 300 \times 40) = 0.5$$

BS 8110 Design chart for rectangular columns – 1



Rectangular columns ($f_y = 500$ N/mm^2, $d/h = 0.8$)

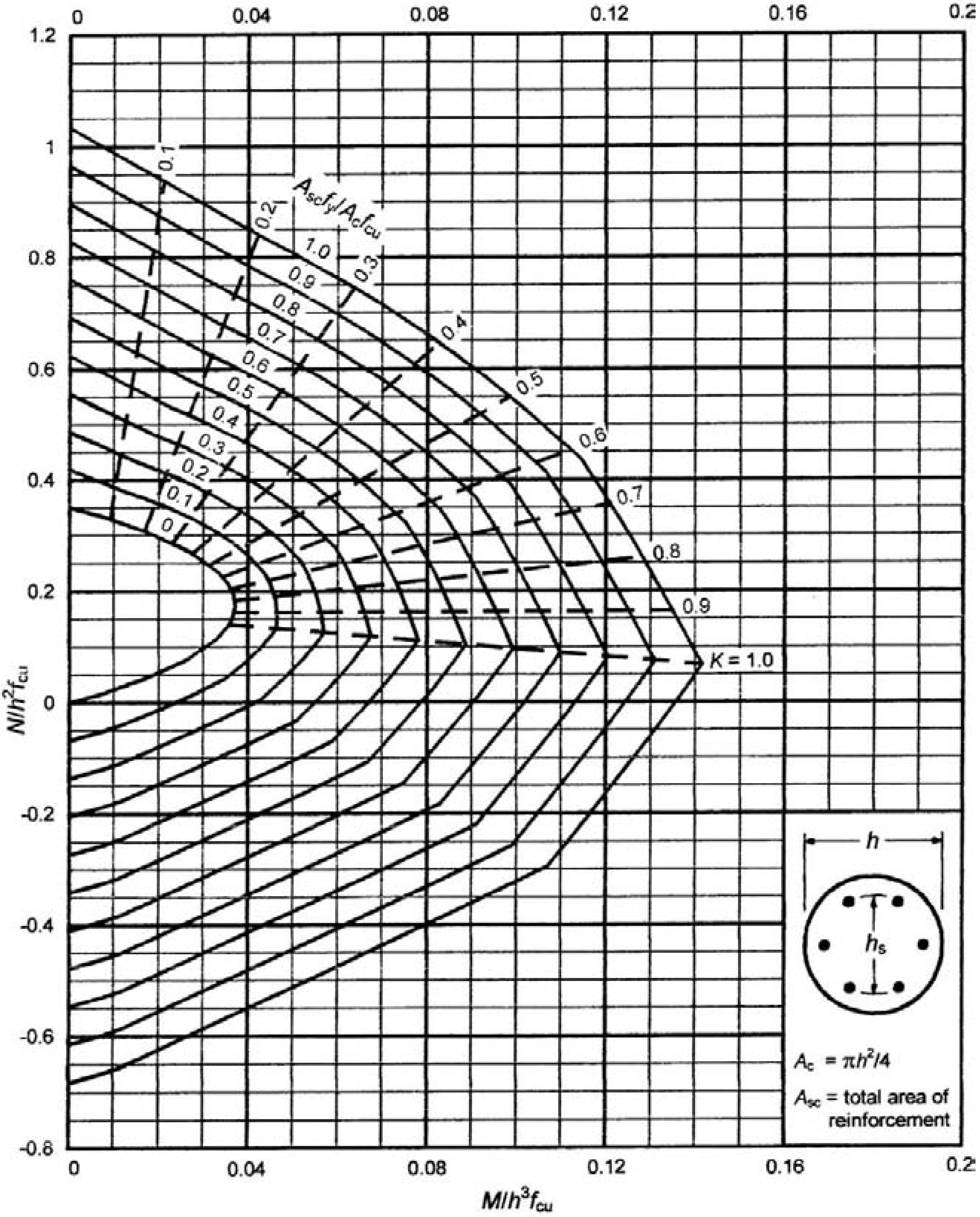
BS 8110 Design chart for rectangular columns – 2



Rectangular columns ($f_y = 500$ N/mm^2, $d/h = 0.85$)

BS 8110 Design chart for circular columns – 1

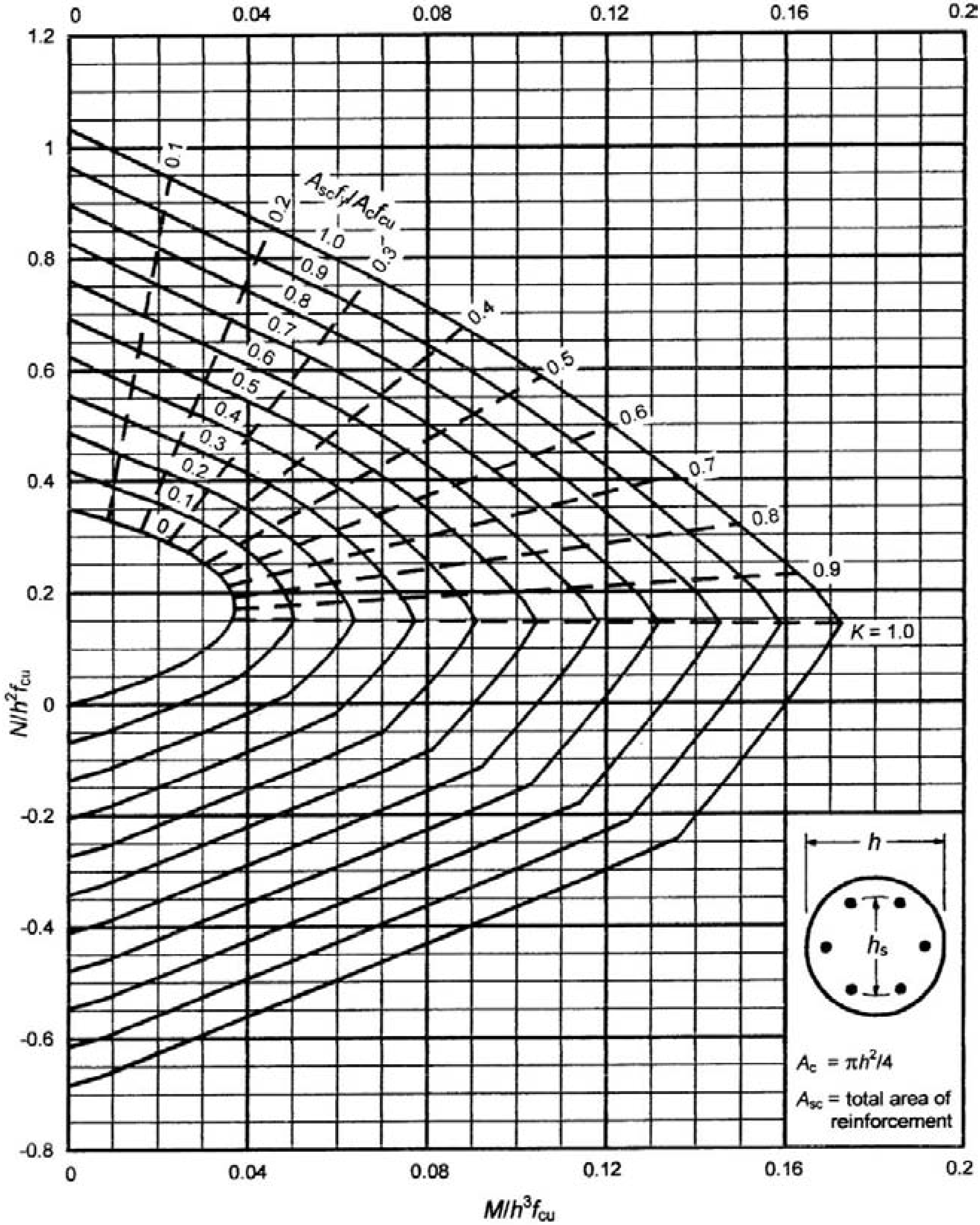
3.19



Circular columns ($f_y = 500$ N/mm^2, $h_S/h = 0.6$)

3.20

BS 8110 Design chart for circular columns – 2



Circular columns ($f_y = 500$ N/mm^2, $h_S/h = 0.7$)

BS 8110 Design procedure for columns – 1

3.21

Effective height

Values of β (see note 1)						
Type of column (see note 2)	Braced			Unbraced		
Restraint condition at end 1	Restraint condition at end 2			Restraint condition at end 2		
	1	2	3	1	2	3
1. Connected monolithically to members on either side, of depth not less than overall dimension of column in plane considered. At bases designed to support load and resist column moment.	0.75	0.80	0.90	1.2	1.3	1.6
2. Connected monolithically to members on either side, of depth less than the overall dimension of column in plane considered.	0.80	0.85	0.95	1.3	1.5	1.8
3. Connected to members not specifically designed to restrain rotation, but providing some nominal restraint (e.g. bases designed to support load only).	0.90	0.95	1.00	1.6	1.8	—
4. Unrestrained in position and direction (e.g. free end of a cantilever column).	—	—	—	2.2	—	—

Note 1. Effective height in plane considered is given by $l_e = \beta l_o$ where l_o is clear height between end restraints.

Note 2. A column may be considered braced in a given plane if lateral stability to the structure as a whole is provided by walls, or bracing, designed to resist all lateral forces in that plane. Otherwise, it should be considered as unbraced.

Note 3. For framed structures, β may also be calculated from the following equations:

Braced columns: lesser of $\beta = 0.7 + 0.05(\alpha_{c,1} + \alpha_{c,2}) \leq 1.0$ and $\beta = 0.85 + 0.05\alpha_{c,min} \leq 1.0$
Unbraced columns: lesser of $\beta = 1.0 + 0.15(\alpha_{c,1} + \alpha_{c,2})$ and $\beta = 2.0 + 0.3\alpha_{c,min}$

where $\alpha_{c,1}$ and $\alpha_{c,2}$ are the ratios, sum of column stiffnesses to sum of beam stiffnesses, at ends 1 and 2 respectively, and $\alpha_{c,min}$ is the lesser of $\alpha_{c,1}$ and $\alpha_{c,2}$. The stiffness is taken as I/l for members continuous at the remote end, and $0.75I/l$ for members pinned at the remote end, where I is the second moment of area of the cross-section and l is the length of the member. Only members properly framed into the ends of the column in the appropriate plane of bending should be considered. In flat slab construction, the beam stiffness should be based on the dimensions of the column strip. At bases designed to resist column moments, α_c may be taken as 1.0. At bases not designed to resist column moments, and in cases where beams are nominally simply supported, α_c should be taken as 10.

Short and slender columns

A column is considered as short if, in each plane of buckling, the ratio l_e/h is less than 15 for a braced column, or less than 10 for an unbraced column, where h is the depth of the section in the particular plane. Otherwise it is considered as slender. The ratio l_o/b should not exceed 60 and, if one end of an unbraced column is unrestrained in a given plane (e.g. a cantilever column), the ratio $l_o h/b^2$ should not exceed 100, where h and b are the larger and smaller dimensions of the section respectively. No check is normally necessary on the deflection of unbraced columns if, in the direction and at the level considered, the average value of l_e/h for all the columns does not exceed 30.

At no section in a column should the design moment be taken less than Ne_{min}, where $e_{min} = 0.05h \leq 20$ mm. In cases of biaxial bending, it is only necessary to satisfy the minimum requirement about one axis at a time. In the design of slender columns, account needs to be taken of the additional moments induced in the column by its deflection. Where l_e/h exceeds 20 and either one or both ends of the column are connected monolithically to other members (e.g. a base, beams or slabs), then these members should also be designed to withstand the additional moments applied by the ends of the column. Where there are columns both above and below a joint, the members should be designed to withstand the sum of the additional moments at the ends of the two columns. For slender columns of constant cross-section having a symmetrical arrangement of reinforcement, equations for the design moments are given in *Table 3.22*.

Biaxial bending

A symmetrically-reinforced square or rectangular section (see figure) may be designed to support an increased moment about one axis:

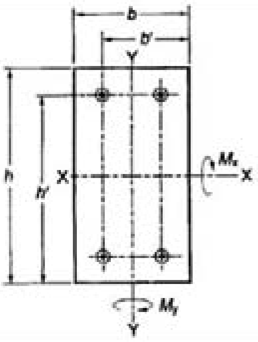
For $M_x / h' \geq M_y / b'$, design for $M'_x = M_x + \beta(h'/b')M_y$

For $M_x / h' < M_y / b'$, design for $M'_y = M_y + \beta(b'/h')M_x$

where $\beta = 1 - (7/6)(N/bhf_{cu}) \geq 0.3$

Circular sections should be designed for the resultant uniaxial moment:

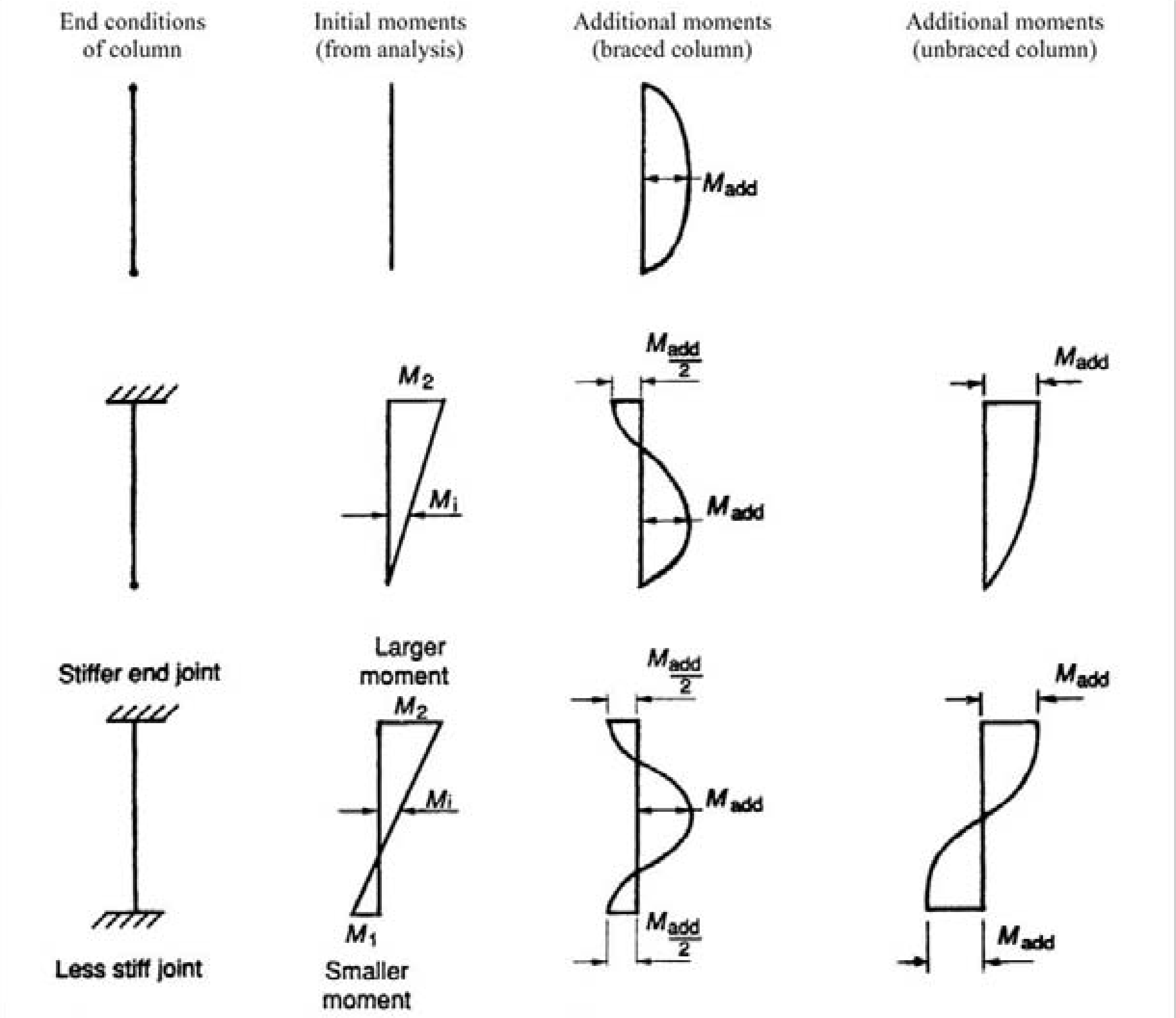
$$M = \sqrt{(M_x^2 + M_y^2)}$$



3.22

BS 8110 Design procedure for columns – 2

Moment diagrams for slender columns



Design moments in slender columns

Case	Initial conditions	Total design moments $M_t = M_i + M_{add} = M_i + Na_u$
1	Column bent about a major axis (but see 3)	$M_{tx} = M_{ix} + N(Kh/2000)(l_e/b)^2$
2	Column bent about a minor axis	$M_{ty} = M_{iy} + N(Kb/2000)(l_e/b)^2$
3	Column bent about both axes, or column (with $l_{ex}/h > 20$ or $h \geq 3b$) bent about a major axis	$M_{tx} = M_{ix} + N(Kh/2000)(l_{ex}/h)^2$ $M_{ty} = M_{iy} + N(Kb/2000)(l_{ey}/b)^2$

Notes. In the above, M_{ix} and M_{iy} are the initial moments (from analysis) about the major and minor axes respectively. Dimensions h and b ($\leq h$) are the overall depths of the cross-section for bending about the major and minor axes, and for cases 1 and 2, l_e is the greater of l_{ex} and l_{ey}. For circular sections, $M_t = M_i + N(Kh/2000)(l_e/h)^2$, where h is the diameter of the section. In the expressions for M_{add}, K is a modification factor derived from the following equation:

$$K = (N_{uz} - N)/(N_{uz} - N_{bal}) \leq 1.0 \qquad \text{where} \qquad N_{uz} = 0.45f_{cu}A_c + 0.87f_yA_{sc}$$

Values of N_{bal} may be taken as $0.2bhf_{cu}$ for symmetrically-reinforced rectangular sections, and $0.15h^2f_{cu}$ for circular sections. Appropriate values of K may be found iteratively. Alternatively, it will always be conservative to use $K = 1$.

For braced columns bent about a single axis, the assumed initial moment at the point of maximum additional moment is given by $M_i = 0.4M_1 + 0.6M_2 \geq 0.4M_2$, where M_1 is the smaller initial end moment and M_2 is the larger initial end moment. Assuming that the column is bent in double curvature, M_1 should be taken as negative and M_2 as positive. The maximum design moment is then obtained as the greatest of the following: (a) M_2, (b) $M_i + M_{add}$, (c) $M_1 + M_{add}/2$, (d) Ne_{min}, where the positions of moments (a)–(c) are shown in the figure above.

For unbraced columns, the maximum design moment at the stiffer end joint is equal to $M_2 + M_{add}$. The moment at the less stiff joint may be reduced to $M_1 + (\alpha_{c,2}/\alpha_{c,1})M_{add}$, where $\alpha_{c,1}$ and $\alpha_{c,2}$ are as defined in *Table 3.21*. At any given level, the ends of all the columns are usually constrained to deflect sideways by the same amount. In such cases, when calculating $M_{add} = Na_u$ in the above equations, the deflection a_u may be taken as the average for all the columns.

From the design chart for $d/h = 240/300 = 0.8$,

$$M_u/bh^2f_{cu} = 0.075,\ N_{uz}/bhf_{cu} = 0.72\ (\textit{Table 3.27})$$

$$\alpha^n = 0.67 + 1.67(N/N_{uz}) = 0.67 + 1.67(0.5/0.72) = 1.8$$

Since the column is square,

$$M_{ux} = M_{uy} = 0.075 \times 300 \times 300^2 \times 40 \times 10^{-6} = 81 \text{ kNm}$$

$$\left[\frac{M_x}{M_{ux}}\right]^{\alpha_n} + \left[\frac{M_y}{M_{uy}}\right]^{\alpha_n} = \left[\frac{60}{81}\right]^{1.8} + \left[\frac{40}{81}\right]^{1.8} = 0.86$$

Since this value is less than 1.0, 4H32 are sufficient.

Example 7. The column in example 3, but taken as unbraced ($\beta = 1.6$) in the direction of M_x, with all other requirements as before:

Since $l_{ex}/h = 1.6 \times 3750/300 = 20 > 10$, the column is slender.

The additional bending moment about the x-x axis is given by:

$$M_{add} = N(Kh/2000)(l_{ex}/b)^2$$

With $K = 1.0$ initially, and since $b = h$

$$M_{add} = 1800 \times 0.3/2000 \times 20^2 = 108 \text{ kNm}$$
$$M = M_i + M_{add} = 54 + 108 = 162 \text{ kNm}$$

$$M/bh^2f_{cu} = 162 \times 10^6/(300 \times 300^2 \times 40) = 0.15$$

$N/bhf_{cu} = 0.5$ as before, and from the design chart

$$A_{sc}f_y/bhf_{cu} = 0.63\ (\textit{Table 3.17})$$
$$A_{sc} = 0.63 \times 300 \times 300 \times 40/500 = 4536 \text{ mm}^2$$

This requires 4T40 but it can be seen from the chart that, with $A_{sc}f_y/bhf_{cu} = 0.63$, K is about 0.6. If we use 4H32,

$$A_{sc}f_y/bhf_{cu} = 3217 \times 500/(300 \times 300 \times 40) = 0.45$$

$N/bhf_{cu} = 0.5$ as before, and from the design chart

$M/bh^2f_{cu} = 0.107$ and $K = 0.53$

With $K = 0.53$, corresponding to $A_{sc}f_y/bhf_{cu} = 0.45$

$$M_{add} = 0.53 \times 108 = 57 \text{ kNm},\ M = 54 + 57 = 111 \text{ kNm}$$
$$M/bh^2f_{cu} = 111 \times 10^6/(300 \times 300^2 \times 40) = 0.103 < 0.107$$

Thus 4H32, which gives 3217 mm^2, is sufficient.

Note that K can also be calculated from the equations given in BS 8110, as follows:

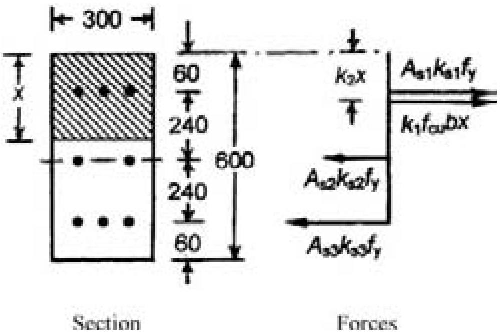
$$N_{uz}/bhf_{cu} = 0.45 + 0.87(A_{sc}f_y/bhf_{cu}) = 0.45 + 0.87 \times 0.45 = 0.84$$

$$N_{bal}/bhf_{cu} = 0.25(d/h) = 0.25 \times 0.8 = 0.2$$

$$K = (N_{uz} - N)/(N_{uz} - N_{bal}) = (0.84 - 0.5)/(0.84 - 0.2) = 0.53$$

Example 8. The following figure shows a rectangular section reinforced with 8H32. The ultimate moment of resistance of the section about the major axis is to be determined in accordance with the following requirements:

$$N = 2500 \text{ kN},\ f_{cu} = 40 \text{ N/mm}^2,\ f_y = 500 \text{ N/mm}^2$$



Consider the bars in each half of the section to be replaced by an equivalent pair of bars. Depth to the centre of area of the bars in one half of the section = 60 + 240/4 = 120 mm. The section can now be considered to be reinforced with four bars of area $A_{sc}/4$, where $d = 600 - 120 = 480$ mm.

$$A_{sc}f_y/bhf_{cu} = 6434 \times 500/(300 \times 600 \times 40) = 0.45$$
$$N/bhf_{cu} = 2500 \times 10^3/(300 \times 600 \times 40) = 0.35$$

From the design chart for $d/h = 480/600 = 0.8$,

$$M/bh^2f_{cu} = 0.14\ (\textit{Table 3.17})$$
$$M = 0.14 \times 300 \times 600^2 \times 40 \times 10^{-6} = 605 \text{ kNm}$$

The solution can be checked, using a trial-and-error process to analyse the original section, as follows:

$$N = k_1f_{cu}bx + (A_{s1}k_{s1} - A_{s2}k_{s2} - A_{s3}k_{s3})f_y$$

where $d/h = 540/600 = 0.9$, and k_{s1}, k_{s2} and k_{s3}, are given by:

$$k_{s1} = 1.4(x/h + d/h - 1)/(x/h) \le 0.87$$
$$k_{s2} = 1.4(0.5 - x/h)/(x/h) \le 0.87$$
$$k_{s3} = 1.4(d/h - x/h)/(x/h) \le 0.87$$

With $x = 300$ mm, $x/h = 0.5$, $k_{s1} = 0.87$, $k_{s2} = 0$ and $k_{s3} = 0.87$

$$N = 0.4 \times 40 \times 300 \times 300 \times 10^{-3} = 1440 \text{ kN } (<2500)$$

With $x = 360$ mm, $x/h = 0.6$, $k_{s2} = -0.233$, $k_{s3} = 0.7$

$$N = 0.4 \times 40 \times 300 \times 360 \times 10^{-3} + (2413 \times 0.87 + 1608 \times 0.233 - 2413 \times 0.7) \times 500 \times 10^{-3} = 1728 + 392 = 2120 \text{ kN } (<2500)$$

With $x = 390$ mm, $x/h = 0.65$, $k_{s2} = -0.323$, $k_{s3} = 0.538$

$$N = 0.4 \times 40 \times 300 \times 390 \times 10^{-3} + (2413 \times 0.87 + 1608 \times 0.323 - 2413 \times 0.538) \times 500 \times 10^{-3} = 1872 + 660 = 2532 \text{ kN } (>2500)$$

With $x = 387$ mm, $x/h = 0.645$, $k_{s2} = -0.315$, $k_{s3} = 0.553$

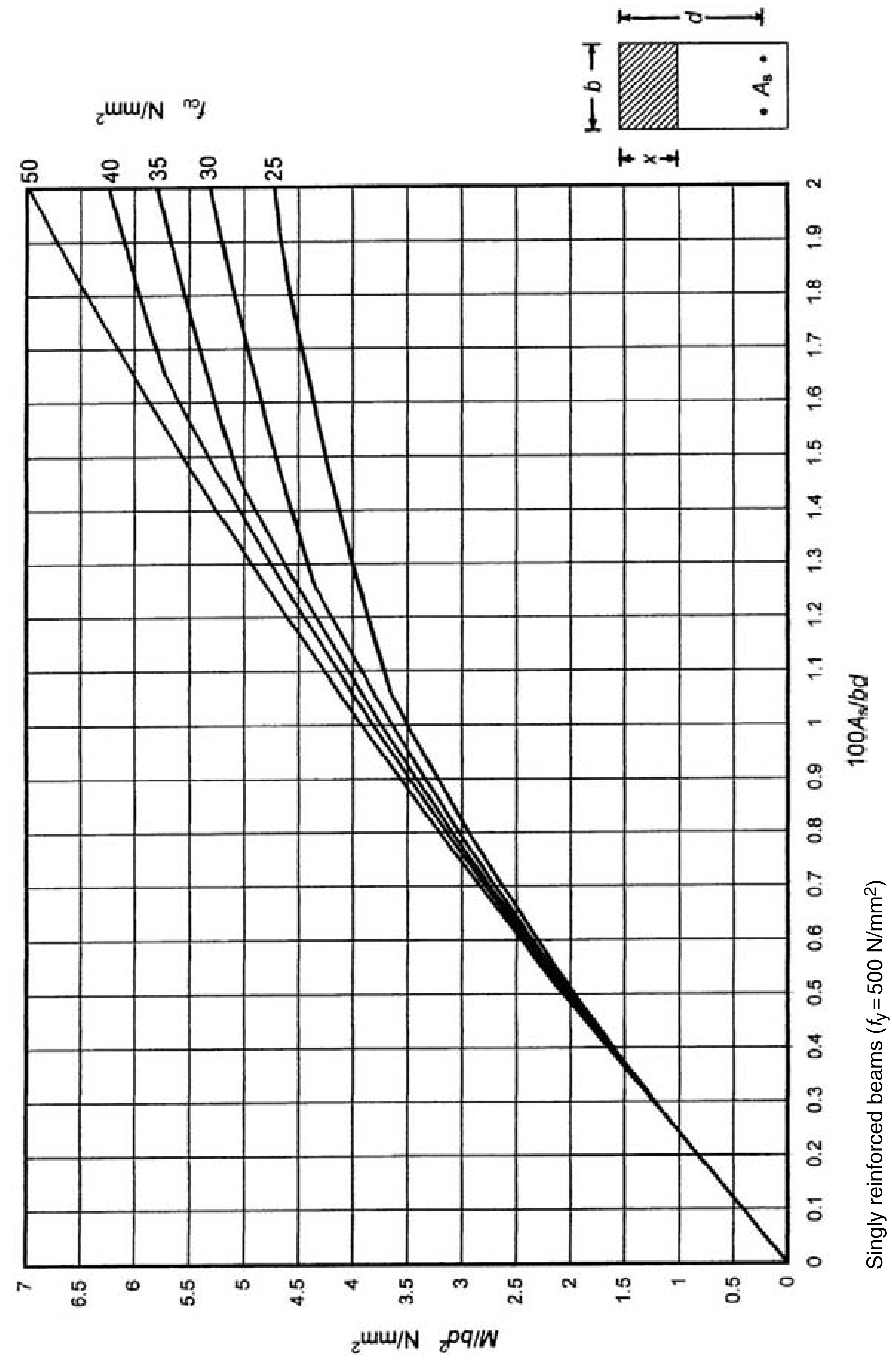
$$N = 0.4 \times 40 \times 300 \times 387 \times 10^{-3} + (2413 \times 0.87 + 1608 \times 0.315 - 2413 \times 0.553) \times 500 \times 10^{-3} = 1858 + 636 = 2494 \text{ kN } (\approx 2500)$$

Taking moments about the mid-depth of the section gives:

$$\begin{aligned} M &= k_1f_{cu}bx(0.5h - k_2x) + (A_{s1}k_{s1} + A_{s3}k_{s3})(d - 0.5h)f_y \\ &= 0.4 \times 40 \times 300 \times 387 \times (300 - 0.45 \times 387) \times 10^{-6} \\ &\quad + (2413 \times 0.87 + 2413 \times 0.553)(540 - 300) \times 500 \times 10^{-6} \\ &= 233 + 412 = 645 \text{ kNm } (>605 \text{ obtained before}) \end{aligned}$$

BS 5400 Design chart for singly reinforced rectangular beams

3.23



Singly reinforced beams (f_y=500 N/mm²)

BS 5400 Design table for singly reinforced rectangular beams

3.24

$\frac{M}{bd^2 f_{cu}}$	$\frac{A_s f_y}{bdf_{cu}}$	$\frac{x}{d}$	$\frac{z}{d}$
≤ 0.038	1.21K	0.100	0.950
0.040	0.049	0.106	0.947
0.042	0.051	0.111	0.944
0.044	0.054	0.117	0.942
0.046	0.056	0.123	0.939
0.048	0.059	0.128	0.936
0.050	0.062	0.134	0.933
0.052	0.064	0.140	0.930
0.054	0.067	0.146	0.927
0.056	0.070	0.151	0.924
0.058	0.072	0.157	0.921
0.060	0.075	0.163	0.918
0.062	0.078	0.169	0.915
0.064	0.081	0.175	0.912
0.066	0.083	0.181	0.909
0.068	0.086	0.188	0.906
0.070	0.089	0.194	0.903
0.072	0.092	0.200	0.900
0.074	0.095	0.206	0.897
0.076	0.098	0.213	0.894
0.078	0.101	0.219	0.891
0.080	0.104	0.225	0.887
0.082	0.107	0.232	0.884
0.084	0.110	0.238	0.881
0.086	0.113	0.245	0.877
0.088	0.116	0.252	0.874
0.090	0.119	0.258	0.871
0.092	0.122	0.265	0.867
0.094	0.125	0.272	0.864
0.096	0.128	0.279	0.861
0.098	0.131	0.286	0.857
0.100	0.135	0.293	0.854
0.102	0.138	0.300	0.850
0.104	0.141	0.307	0.846
0.106	0.145	0.314	0.843
0.108	0.148	0.322	0.839
0.110	0.151	0.329	0.835
0.112	0.155	0.337	0.832
0.114	0.158	0.344	0.828
0.116	0.162	0.352	0.824
0.118	0.165	0.360	0.820
0.120	0.169	0.368	0.816
0.122	0.173	0.376	0.812
0.124	0.176	0.384	0.808
0.126	0.180	0.392	0.804
0.128	0.184	0.400	0.800
0.130	0.188	0.408	0.796
0.132	0.192	0.417	0.792
0.134	0.196	0.426	0.787
0.136	0.200	0.434	0.783
0.138	0.204	0.443	0.778
0.140	0.208	0.452	0.774
0.142	0.212	0.462*	0.769
0.144	0.216	0.471*	0.765
0.146	0.221	0.480*	0.760
0.148	0.225	0.490*	0.755
0.150	0.230	0.500*	0.750

The following formulae are used in the table above:

$$\frac{z}{d} = 0.5 + \sqrt{0.25 - 1.25K} \leq 0.95 \qquad \frac{x}{d} = \frac{1 - z/d}{0.5} \qquad \frac{A_s f_y}{bdf_{cu}} = \frac{K}{0.87z/d}$$

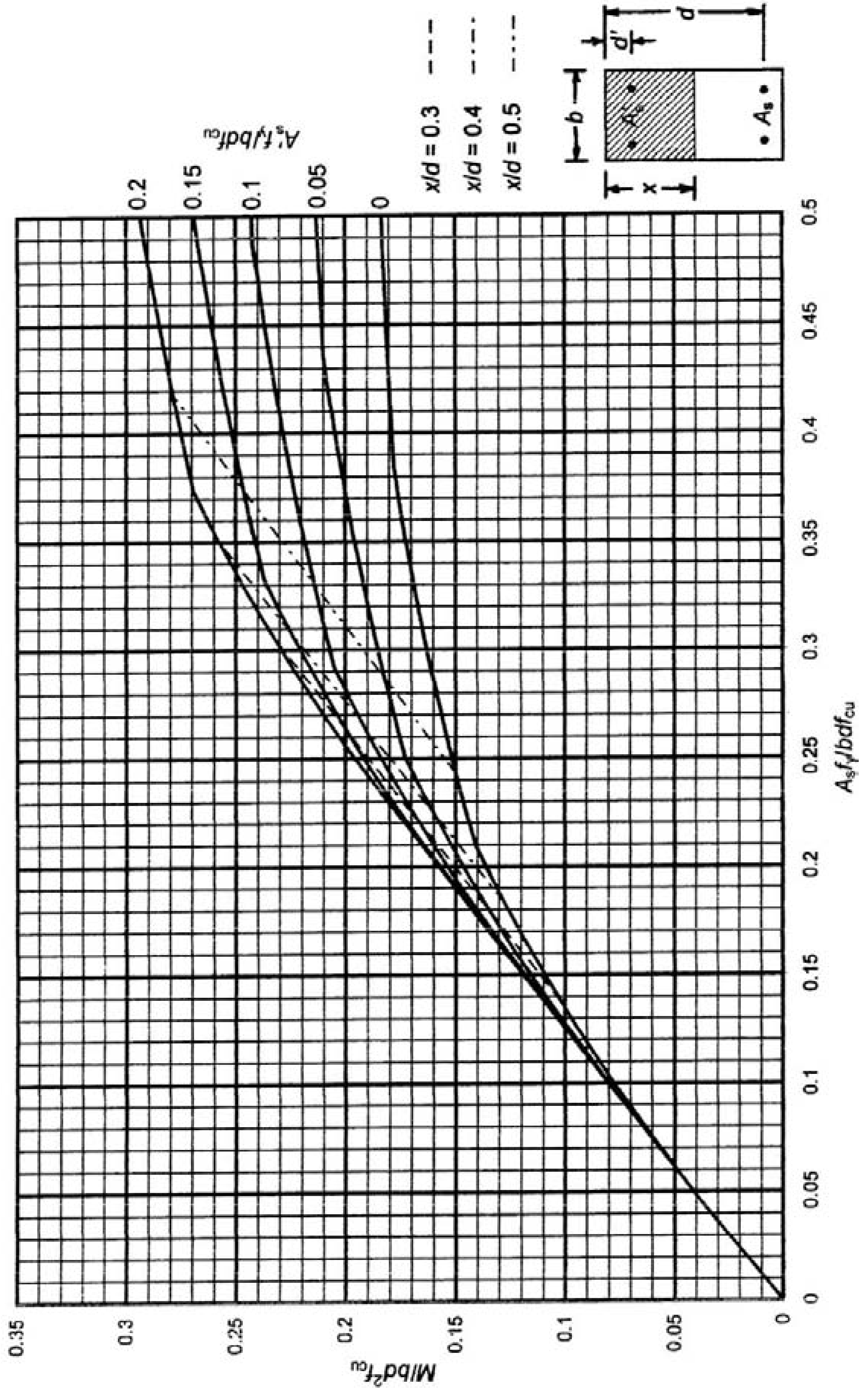
where $K = \frac{M}{bd^2 f_{cu}}$

The tabulated data include for γ_m values of 1.5 for concrete and 1.15 for steel. If the table is used for assessment purposes in accordance with Highways Agency document BD44/95, in designs where characteristic strengths are replaced by worst credible strengths, f_{cu} should be multiplied by (1.5/1.2), and f_y should be multiplied by (1.15/1.1). If measured values of the effective depth d are used, in addition to the worst credible steel strength, f_y may be multiplied by (1.15/1.05).

* For $f_y = 500$ and $\gamma_m = 1.15$, values of $x/d > 0.456$ are cases where the reinforcement stress should be less than $0.87f_y$, but the formulae are considered valid for all values of $K \leq 0.15$.

BS 5400 Design chart for doubly reinforced rectangular beams – 1

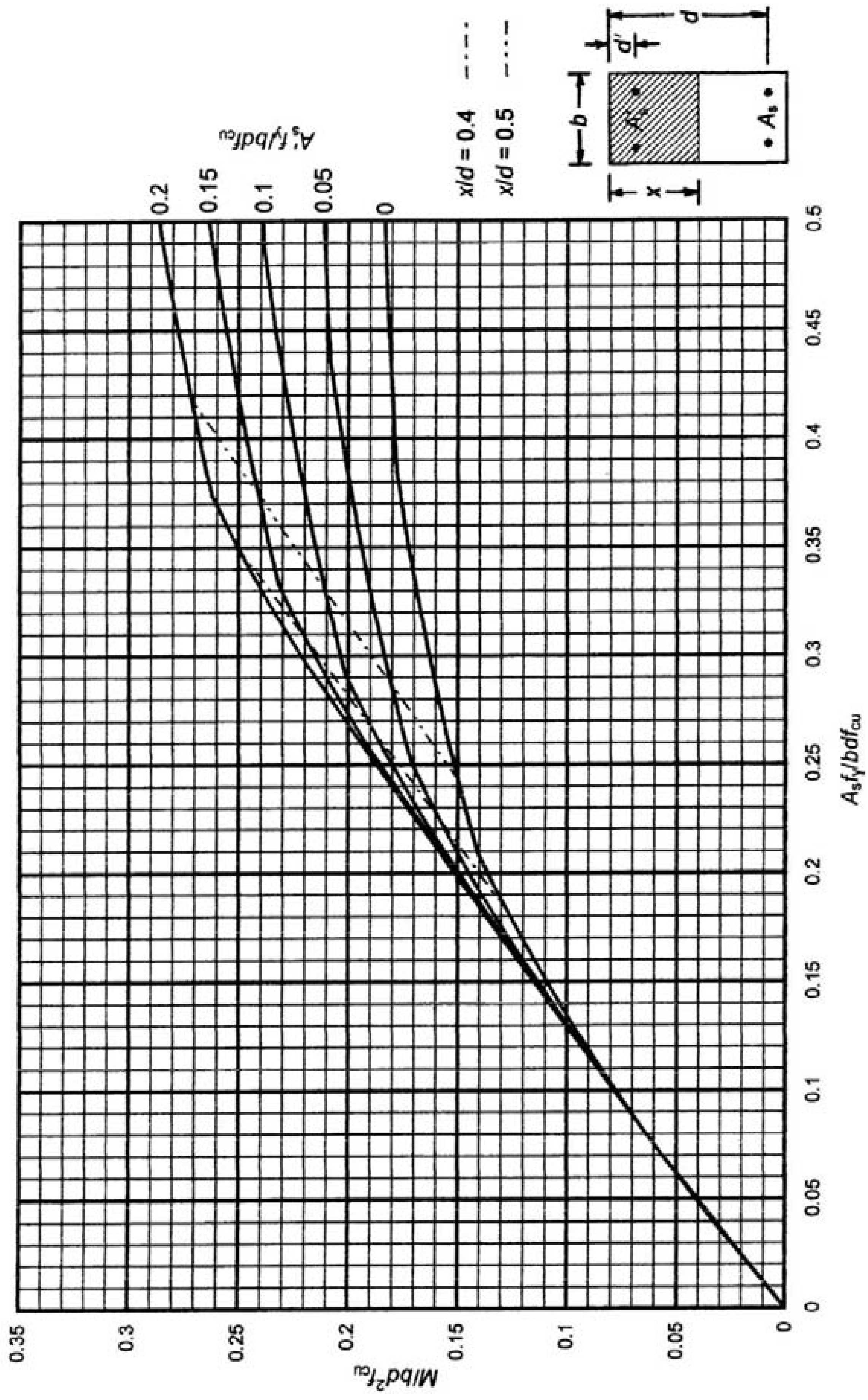
3.25



Doubly reinforced beams ($f_y = 500$ N/mm^2, $d'/d = 0.1$)

BS 5400 Design chart for doubly reinforced rectangular beams – 2

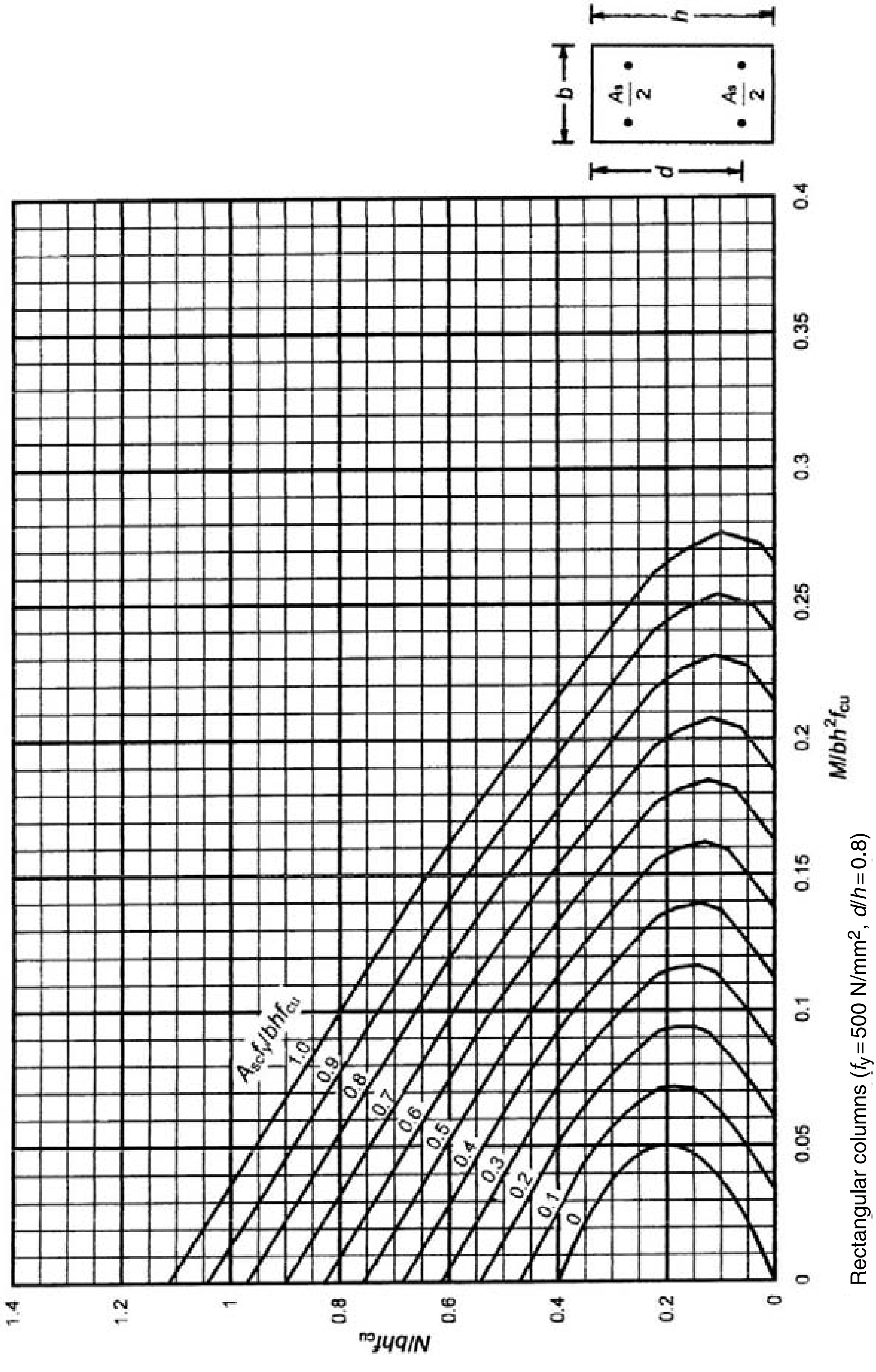
3.26



Doubly reinforced beams ($f_y = 500$ N/mm^2, $d'/d = 0.15$)

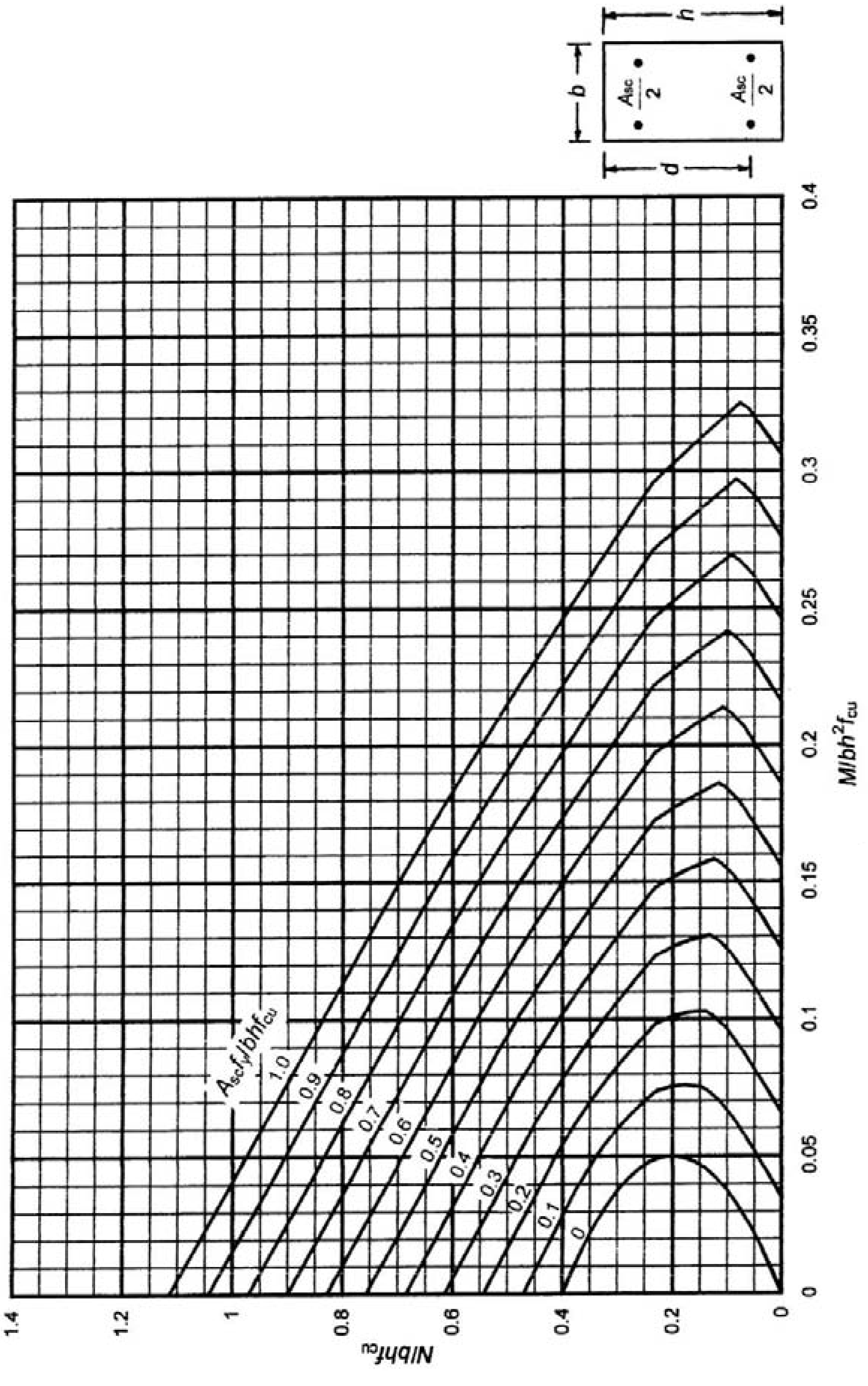
BS 5400 Design chart for rectangular columns – 1

3.27



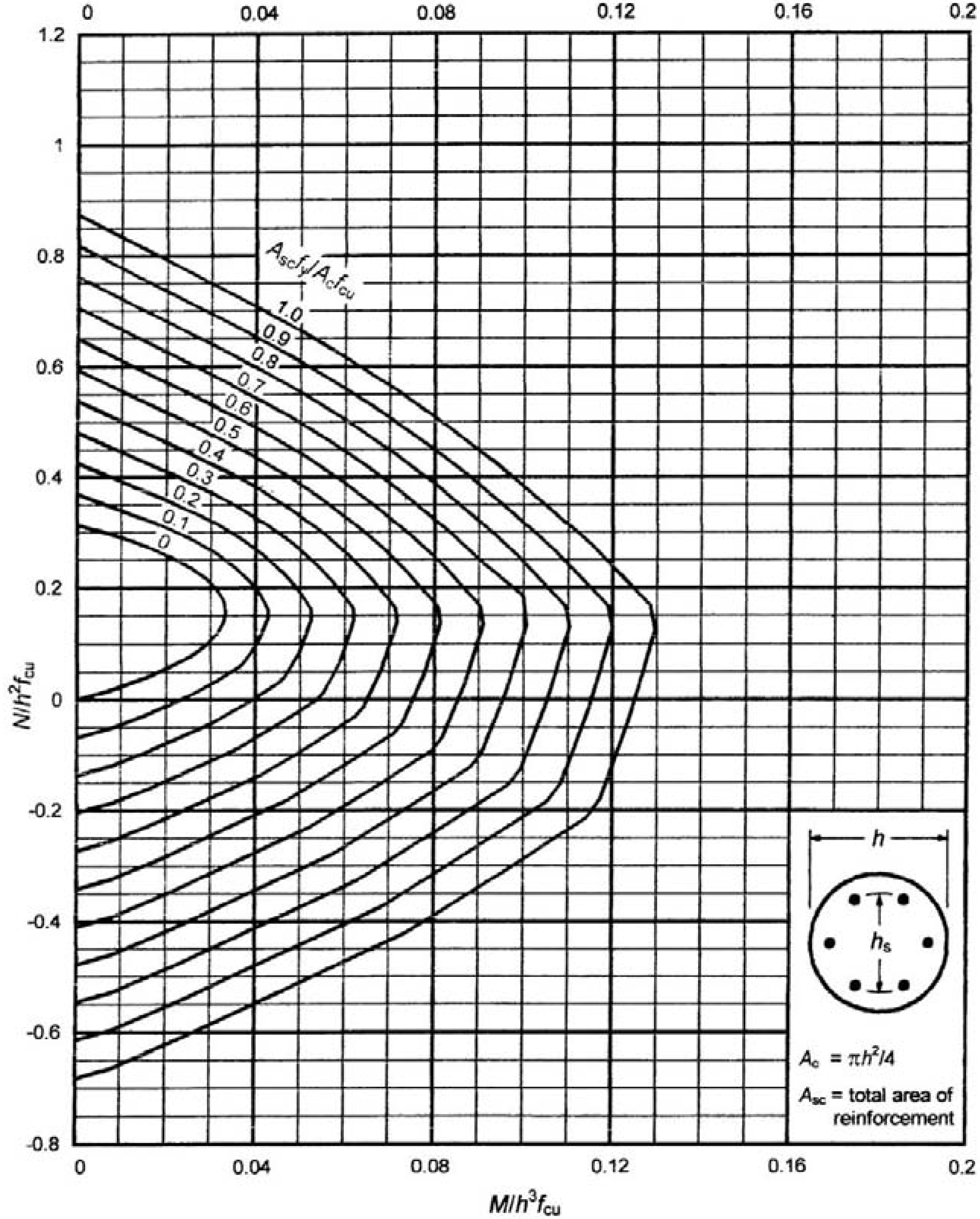
Rectangular columns ($f_y = 500$ N/mm^2, $d/h = 0.8$)

BS 5400 Design chart for rectangular columns – 2



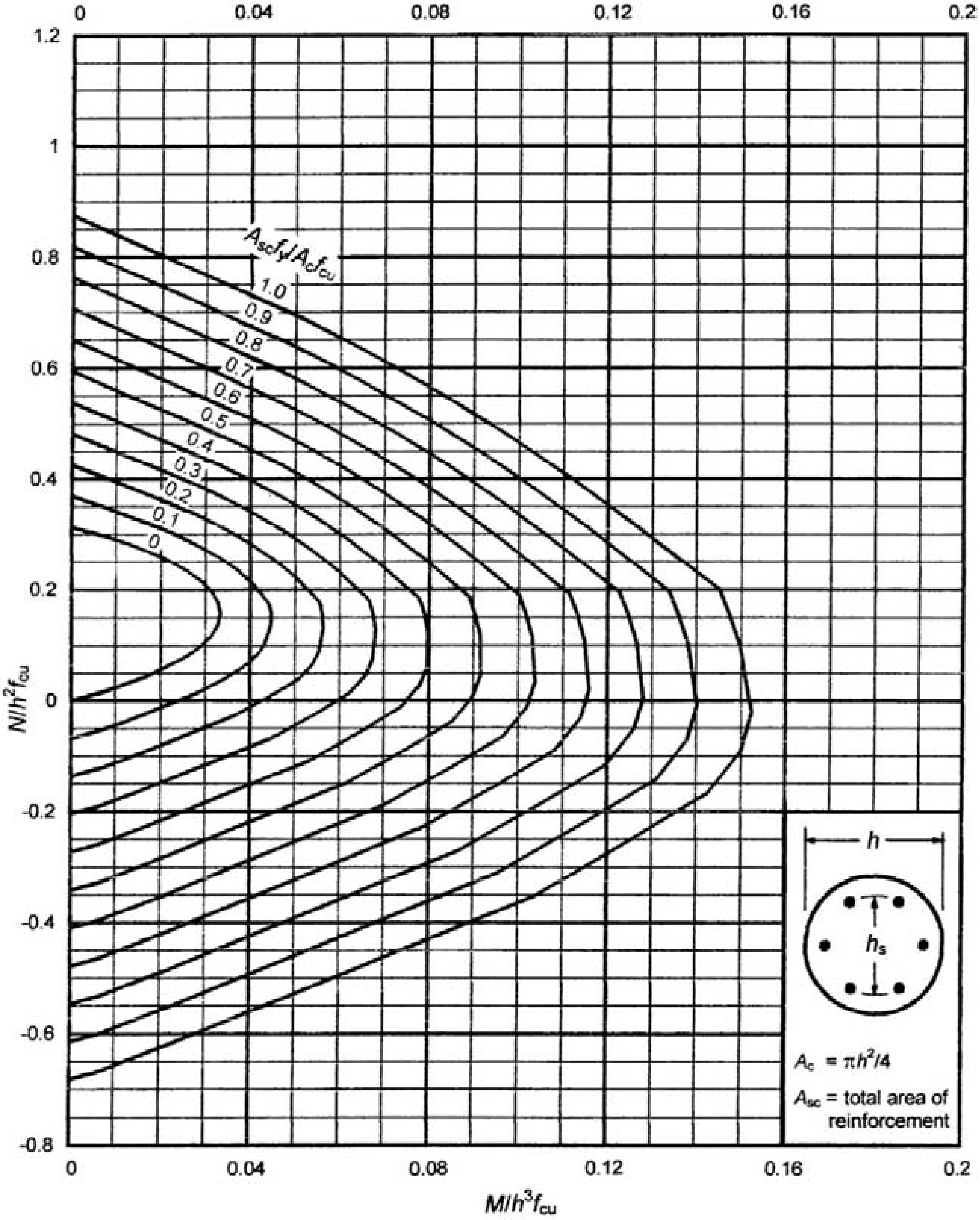
Rectangular columns (f_y=500 N/mm^2, d/h=0.85)

BS 5400 Design chart for circular columns – 1



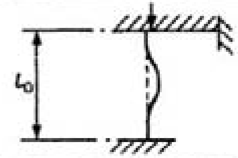
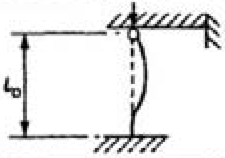
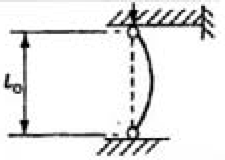
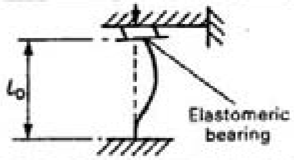
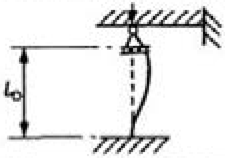
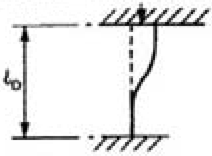
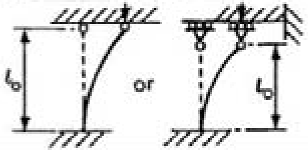
Rectangular columns ($f_y = 500$ N/mm^2, $h_S/h = 0.6$)

BS 5400 Design chart for circular columns – 2



Circular columns ($f_y = 500$ N/mm^2, $h_S/h = 0.7$)

BS 5400 Design procedure for columns – 1

	Case	Idealized column and buckling mode	Restraints			Effective Height, l_e
			Location	Position	Rotation	
Effective height	1		Top	Full	Full	$0{\cdot}70\, l_o$
			Bottom	Full	Full	
	2		Top	Full	None	$0{\cdot}85\, l_o$
			Bottom	Full	Full	
	3		Top	Full	None	$1{\cdot}0\, l_o$
			Bottom	Full	None	
	4		Top	None	None	$1{\cdot}3\, l_o$
			Bottom	Full	Full	
	5		Top	None	None	$1{\cdot}4\, l_o$
			Bottom	Full	Full	
	6		Top	None	Full	$1{\cdot}5\, l_o$
			Bottom	Full	Full	
	7		Top	None	None	$2{\cdot}3\, l_o$
			Bottom	Full	Full	

Note 1. In the above, l_o is the clear height between end restraints in the plane of buckling considered. A column is considered as short if, in each plane of buckling, the ratio l_e/h is less than 12, where h is the depth of the section in the particular plane. Otherwise it is considered as slender. In each plane of buckling, the ratio l_o/h should not exceed 60.

Note 2. For a short column bent about the minor axis, the moment obtained by analysis should be increased by Ne_a, where $e_a = 0.05h \leq 20$ mm. For a short column bent about the major axis, or biaxially bent, the moment about each axis should be increased by $e_a = 0.03h \leq 20$ mm. In the design of slender columns, account needs to be taken of the additional moments induced in the column by its deflection. For slender columns of constant cross-section having a symmetrical arrangement of reinforcement, equations for the total design moments are given in *Table 3.32*.

Biaxial bending

A symmetrically-reinforced square or rectangular section may be designed as being bent separately about each axis in turn, providing the following criterion is satisfied:

$$\left[\frac{M_x}{M_{ux}}\right]^{\alpha_n} + \left[\frac{M_y}{M_{uy}}\right]^{\alpha_n} \leq 1.0 \qquad \text{with} \qquad 1.0 \leq \alpha_n = 0.67 + 1.67\,(N/N_u) \leq 2.0 \qquad \text{where}$$

M_x, M_y are applied moments about the x-x (major) and y-y (minor) axes respectively, including the moments Ne_a.
M_{ux}, M_{uy} are resistance moments about the x-x and y-y axes respectively, corresponding to the axial load capacity of the section ignoring all bending, given by $N_u = 0.45 f_{cu} A_c + f_{yc} A_{sc}$, and N is the applied axial load.

Circular sections should be designed for the resultant uniaxial moment: $M = \sqrt{(M_x^2 + M_y^2)}$

BS 5400 Design procedure for columns – 2

Design moments in slender columns

Case	Initial conditions	Total design moments in symmetrically reinforced columns of rectangular or circular cross-section
1	Column (with $h < 3b$) bent about a major axis	$M_{tx} = M_{ix} + N(h/1750)(1 - 0.0035l_e/b)(l_e/b)^2$
2	Column bent about a minor axis	$M_{ty} = M_{iy} + N(b/1750)(1 - 0.0035l_e/b)(l_e/b)^2$
3	Column bent about both axes, or column (with $h \geq 3b$) bent about a major axis	$M_{tx} = M_{ix} + N(h/1750)(1 - 0.0035l_{ex}/h)(l_{ex}/h)^2$ $M_{ty} = M_{iy} + N(b/1750)(1 - 0.0035l_{ey}/b)(l_{ey}/b)^2$

Notes. In the above, M_{ix} and M_{iy} are the initial moments (from analysis) about the x-x (major) and y-y (minor) axes respectively. Values of the initial moments should be taken not less than Ne_a, where $e_a = 0.05h \leq 20$ mm for case 1, and $e_a = 0.05b \leq 20$ mm for case 2. For case 3, M_{ix} and M_{iy} should be increased by Ne_a, where $e_a = 0.03h \leq 20$ mm for M_{ix}, and $e_a = 0.03b \leq 20$ mm for M_{iy}. Dimensions h and b ($\leq h$) are the overall depths of the cross-section for bending about the major and minor axes respectively and, for cases 1 and 2, l_e is the greater of l_{ex} and l_{ey}. For circular sections, $M_t = M_i + N(h/1750)(1 - 0.0035l_e/h)(l_e/h)^2$, where h is the diameter of the section.

For a column fixed in position at both ends where no transverse loads occur in its height, the initial moment may be reduced to $M_i = 0.4M_1 + 0.6M_2 \geq 0.4\ M_1$, where M_1 is the smaller initial end moment (assumed negative if the column is bent in double curvature) and M_2 is the larger initial end moment (assumed positive), provided that $M_t \geq M_2$.

Design formulae for rectangular sections

The following formulae may be used to determine the resistance of a rectangular cross-section with reinforcement in the two faces parallel to the axis of bending, whether the reinforcement is symmetrical or not:

$$N_u = 0.4f_{cu}bd_c + A'_{s1}f_{yc} + A_{s2}f_{s2} \qquad M_u = 0.2f_{cu}bd_c(h - d_c) + A'_{s1}f_{yc}(0.5h - d') - A_{s2}f_{s2}(0.5h - d_2)$$

where

N_u and M_u are the combined axial load and bending resistance of the section for the particular value of d_c assumed
b is the breadth of the section
d_c is the assumed depth of concrete in compression, where $2.33\,d' \leq d_c \leq h$
d' is the depth from the surface to the reinforcement in the more highly compressed face
d_2 is the depth from the surface to the reinforcement in the other face
h is the overall depth of the section in the plane of bending
f_{cu} is the characteristic cube strength of the concrete
f_{yc} is the design compressive strength of the reinforcement, taken as $2000f_y/(2300 + f_y)$
  $f_{yc} = 0.714\,f_y$ when $f_y = 500$ N/mm^2
f_{s2} is the stress in the reinforcement A_{s2}, given by the following expressions (when $f_y = 500$ N/mm^2):
  $f_{s2} = -\,0.87f_y$ for $d_c \leq 0.46(h - d_2)$
  $f_{s2} = -\,0.25[(h - d_2)/d_c + 1.3]\,f_y$ for $0.46(h - d_2) < d_c < 0.67(h - d_2)$
  $f_{s2} = -\,1.4[(h - d_2)/d_c - 1]\,f_y$ for $0.67(h - d_2) \leq d_c \leq h$
A'_{s1} is the area of compression reinforcement in the more highly compressed face
A_{s2} is the area of reinforcement in the other face, which may be in compression or tension (negative) as the resultant eccentricity of the load increases and the value of d_c decreases from h to $2.33\,d'$

The following formulae apply when the full depth of the section is in compression:

$$N_u = 0.4f_{cu}bh + A'_{s1}f_{yc} + A_{s2}f_{s2} \qquad M_u = A'_{s1}f_{yc}(0.5h - d') - A_{s2}f_{s2}(0.5h - d_2) \qquad 1.4(d_2/h)f_y \leq f_{s2} \leq f_{yc} = 0.714\,f_y$$

The following simplified formulae may also be used as appropriate:

(a) Where $e = M/N \leq (0.5h - d')$ and $N \leq 0.45b(h - 2e)f_{cu}$, only nominal reinforcement is required.
(b) Where $e = M/N > (0.5h - d_2)$, the axial load may be ignored and the column section designed as a beam to resist an increased moment $M_a = M + N(0.5h - d_2)$. The required area of tension reinforcement can then be calculated as $A_s = (M_a/z - N)/(0.87f_y)$, where z is the lever arm appropriate to the increased moment.

Chapter 25

Shear and torsion

25.1 SHEAR RESISTANCE

25.1.1 Shear stress

In BS 8110, the design shear stress at any cross section in a member of uniform depth is calculated from:

$$v = V/b_v d$$

where

V is the shear force due to ultimate loads
b_v is the breadth of the section, which for a flanged section is taken as the average width of the web below the flange
d is the effective depth to the tension reinforcement

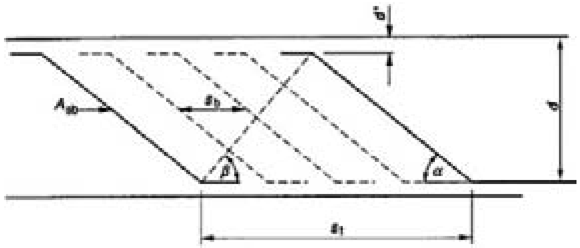
For a member of varying depth, the shear force is calculated as $V \pm (M \tan\theta_s)/d$, where θ_s is the angle between the tension reinforcement and the compression face of the member. The negative sign applies when moment and effective depth both increase in the same direction. In no case should v exceed the lesser of $0.8\sqrt{f_{cu}}$ or 5 N/mm^2, whatever the reinforcement. In BS 5400, b is used in place of b_v, and the maximum value of v is taken as the lesser of $0.75\sqrt{f_{cu}}$ or 4.75 N/mm^2.

25.1.2 Concrete shear stress

In BS 8110, the design concrete shear stress v_c is a function of the concrete grade, the effective depth and the percentage of effective tension reinforcement at the section considered. It is often convenient to determine v_c at the section where the reinforcement is least, and use the same value throughout the member. For sections at distance $a_v \leq 2d$ from the face of a support, or concentrated load, v_c may be multiplied by $2d/a_v$, providing the tension reinforcement is adequately anchored. Alternatively, as a simplification for beams carrying uniform load, the section at distance d from the face of a support may be designed without using this enhancement, and the same reinforcement provided at sections closer to the support. In BS 5400, the design concrete shear stress is obtained as $\xi_s v_c$, where v_c is the ultimate shear stress and ξ_s is a depth factor.

25.1.3 Shear reinforcement

Requirements for shear reinforcement depend on the value of v in relation to v_c (or $\xi_s v_c$). In slabs, no shear reinforcement is required provided v does not exceed the concrete shear stress. In all beams of structural importance, a minimum amount of shear reinforcement in the form of links, equivalent to a shear resistance $V_{sv} = 0.4\, b_v d$, is required. The shear resistance can be increased, by introducing more links, or bent-up bars can be used to provide up to 50% of the total shear reinforcement. The contribution of the shear reinforcement is determined on the basis of a truss analogy, in which the bars act as tension members and inclined struts form within the concrete, as shown in the figure here.



System of bent-up bars used as shear reinforcement

In the figure, the truss should be chosen so that both α and β are $\geq 45°$, and $s_t \leq 1.5d$. The design shear resistance provided by a system of bent-up bars is then given by:

$$V_{sb} = (A_{sb}/s_b)(0.87 f_y)(s_t \sin\alpha)$$

where $s_t = (d - d')(\cot\alpha + \cot\beta) \leq 1.5d$

For bars bent-up at 45°, with $f_y = 500$ N/mm^2,

$$V_{sb} = 0.461 A_{sb}\,(d/s_b) \geq 0.307 A_{sb} \text{ kN}$$

For bars bent-up at 60°, with $f_y = 500$ N/mm^2,

$$V_{sb} = 0.594\, A_{sb}\,(d - d')/s_b \geq 0.376\, A_{sb} \text{ kN}$$

In BS 8110, the shear resistance of members containing shear reinforcement is taken as the sum of the resistances provided separately by the shear reinforcement and the concrete. The strut analogy results in an additional longitudinal tensile force that is effectively taken into account in the curtailment rules for the longitudinal reinforcement. In BS 5400, the minimum amount of shear reinforcement is required in addition to that needed to cater for the difference between the design shear force and the concrete shear resistance. It is also necessary to design the longitudinal reinforcement for the additional force.

Details of the design procedures for determining the shear resistances of members are given in *Table 3.33* for BS 8110, and *Table 3.36* for BS 5400.

BS 8110 Shear resistance

3.33

Members without shear reinforcement

The design shear resistance of a flexural member without shear reinforcement is given by $V_c = v_c b_v d$, where:

$$v_c = \left(\frac{0.27}{\gamma_m}\right)\left(\frac{400}{d}\right)^{1/4}\left(\frac{100A_s f_{cu}}{b_v d}\right)^{1/3} \quad \text{with} \quad \left(\frac{400}{d}\right)^{1/4} \geq 0.67, \;\; 0.15 \leq \left(\frac{100A_s}{b_v d}\right) \leq 3.0, \;\; f_{cu} \leq 40 \;\; \text{and} \;\; \gamma_m = 1.25$$

A_s is the area of longitudinal tension reinforcement that extends for a distance $\geq d$ beyond the section considered. (At supports, the full area of tension reinforcement may be used, providing the requirements for anchorage are met)
b_v is the section breadth (for a flanged section, b_v should be taken as the average width of the rib below the flange)
d is the effective depth of the tension reinforcement

For sections at distance $a_v \leq 2d$ from the face of a support, or concentrated load, v_c may be replaced by $(2d/a_v)v_c$, but $V/b_v d$ should not exceed the lesser of $0.8\sqrt{f_{cu}}$ or 5 N/mm^2. The tension reinforcement should extend on each side of the section for a distance ≥ d or have an equivalent anchorage. See also the alternative method described in section 25.1.2.

For a flexural member subjected also to axial load N (positive in compression, negative in tension), v_c may be replaced by $v_c + 0.6(N/A_c)(Vh/M) \leq v_c + 0.6(N/A_c)$, where h is the overall depth and A_c is the gross area of the concrete section.

$\frac{100A_s}{b_v d}$	Design concrete shear stress v_c (N/mm^2) for values of d (mm) and f_{cu} = 25 N/mm^2 (see Notes)								
	100	125	150	200	300	400	600	1200	≥ 2000
≤ 0.15	0.47	0.45	0.43	0.40	0.36	0.33	0.30	0.25	0.22
0.25	0.56	0.53	0.51	0.47	0.43	0.40	0.36	0.30	0.27
0.50	0.71	0.67	0.64	0.60	0.54	0.50	0.45	0.38	0.33
0.75	0.81	0.77	0.73	0.68	0.62	0.57	0.52	0.43	0.38
1.0	0.89	0.84	0.81	0.75	0.68	0.63	0.57	0.48	0.42
1.5	1.02	0.97	0.92	0.86	0.78	0.72	0.65	0.55	0.48
2.0	1.12	1.06	1.02	0.95	0.85	0.79	0.72	0.60	0.53
≥ 3.0	1.29	1.22	1.16	1.08	0.98	0.91	0.82	0.69	0.61

Notes. For f_{cu} = 30, 35 and ≥ 40 N/mm^2, values of v_c in table should be multiplied by 1.06, 1.12 and 1.17 respectively. For members with shear reinforcement, where d > 400 mm, values of v_c corresponding to d = 400 mm may be used.

Members with shear reinforcement

The design shear resistance of a member provided with upright links and a system of bent-up bars is given by:

$$V_u = v_c b_v d + (A_{sv}/s_v)(0.87f_{yv})d + (A_{sb}/s_b)(0.87f_y)(s_t \sin\alpha) \quad \text{where} \quad s_t = (d - d')(\cot\alpha + \cot\beta) \leq 1.5d$$

A_{sb} is cross-sectional area of bent up bars	α is angle (≥ 45°) between bent-up bar and axis of member
A_{sv} is cross-sectional area of vertical legs of links	β is angle (≥ 45°) between 'compression strut' of a system of bent-up bars and axis of member
d' is depth from concrete face to top of bent-up bar	s_b is spacing of bent-up bars along the member
f_y is characteristic strength of bent-up bars	s_v is spacing of links along the member
f_{yv} is characteristic strength of link reinforcement	

At no section should $V/b_v d$ exceed the lesser of $0.8\sqrt{f_{cu}}$ or 5 N/mm^2, whatever shear reinforcement is provided.

Form and area of shear reinforcement to be provided in beams (where $f_{yv} \leq 500$ N/mm^2)

Value of $v = V/b_v d$ (N/mm^2)	Form of shear reinforcement	Area of shear reinforcement
$v < 0.5v_c$ throughout whole beam	Minimum links desirable but may be omitted in members such as lintels.	None specified
$0.5v_c < v \leq (v_c + 0.4)$	Links at spacing ≤ 0.75d for full length of beam (transverse spacing of legs ≤ d)	$A_{sv}/s_v = 0.4b_v/0.87f_{yv}$ (see *Table 3.35* for areas of links)
$(v_c + 0.4) < v \leq 0.8\sqrt{f_{cu}}$ or 5 N/mm^2 whichever is the lesser	Links or links combined with bent-up bars, but not more than 50% of shear resistance provided by reinforcement to be in the form of bent-up bars	Where links only are provided: $A_{sv}/s_v = b_v(v - v_c)/0.87f_{yv}$ Where bent-up bars are provided, see equation and section 25.1.3

Form and area of shear reinforcement to be provided in solid slabs

Value of $v = V/b_v d$ (N/mm^2)	Form of shear reinforcement	Area of shear reinforcement
$v < v_c$	None required	Not applicable
$v_c < v \leq (v_c + 0.4)$	In slabs ≥ 200 mm deep, links at spacing ≤ d in regions where $v > v_c$	As beams
$(v_c + 0.4) < v \leq 0.8\sqrt{f_{cu}}$ or 5 N/mm^2 whichever is the lesser	In slabs ≥ 200 mm deep, any combination of links and bent-up bars at spacing ≤ d	As beams

Note. If shear reinforcement is required in sections at distance $a_v \leq 2d$ from the face of a support, the total area to be provided within the middle three-quarters of a_v is given by $\Sigma A_{sv} = a_v b_v(v - 2dv_c/a_v)/0.87f_{yv} \geq 0.4a_v b_v/0.87f_{yv}$.

25.1.4 Shear under concentrated loads

The maximum shear stress at the edge of a concentrated load should not exceed the lesser of $0.8\sqrt{f_{cu}}$ or 5 N/mm^2. Shear in solid slabs under concentrated loads can result in punching failures on the inclined faces of truncated cones or pyramids. For calculation purposes, the shear perimeter is taken as the boundary of the smallest rectangle that nowhere comes closer to the edges of a loaded area than a specified distance. The shear capacity is checked first on a perimeter at distance $1.5d$ from the edge of the loaded area. If the calculated shear stress is no greater than v_c, no shear reinforcement is needed. If the shear stress exceeds v_c, shear reinforcement is required within the failure zone, and further checks are needed on successive perimeters at intervals of $0.75d$, until a perimeter is reached where shear reinforcement is no longer required.

Details of design procedures for shear under concentrated loads are given in *Table 3.34* for BS 8110, and *Tables 3.37* and *3.38* for BS 5400.

25.1.5 Shear in bases

The shear strength of pad footings near concentrated loads is governed by the more severe of the following two conditions:

(a) Shear along a vertical section extending for the full width of the base. In BS 8110, the concrete shear stress v_c may be multiplied by $2d/a_v$, for all values of $a_v \leq 2d$. In BS 5400, the critical section is taken at distance d from the face of the load with no enhancement of the concrete shear stress.

(b) Punching shear around the loaded area, as described in section 25.1.4. The reaction resulting from the soil bearing pressure within the shear perimeter may be deducted from the design load on the column, when calculating the design shear force acting on the section.

The shear strength of pile caps is normally governed by the shear along a vertical section extending for the full width of the cap. The critical section for shear is assumed to be located at 20% of the pile diameter from the near face of the pile. The design shear force acting on this section is taken as the whole of the reaction from the piles with centres lying outside the section. In BS 8110, the design concrete shear stress may be multiplied by $2d/a_v$, where a_v is the distance from the column face to the critical section, for strips of width up to three times the pile diameter centred on each pile. In BS 5400, this enhancement may be applied to strips of width equal to one pile diameter centred on each pile. For pile caps designed by truss analogy, 80% of the tension reinforcement should be concentrated in these strips.

25.1.6 Bottom loaded beams

Where load is applied near the bottom of a section, sufficient vertical reinforcement to transmit the load to the top of the section should be provided in addition to any reinforcement required to resist shear.

25.2 DESIGN FOR TORSION

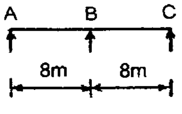
In normal beam-and-slab or framed construction, calculations for torsion are not usually necessary, adequate control of any torsional cracking in beams being provided by the required minimum shear reinforcement. When it is judged necessary to include torsional stiffness in the analysis of a structure, or torsional resistance is vital for static equilibrium, members should be designed for the resulting torsional moment. The torsional resistance of a section may be calculated on the basis of a thin-walled closed section, in which equilibrium is satisfied by a closed plastic shear flow. Solid sections may be modelled as equivalent thin-walled sections. Complex shapes may be divided into a series of sub-sections, each of which is modelled as an equivalent thin-walled section, and the total torsional resistance taken as the sum of the resistances of the individual elements. When torsion reinforcement is required, this should consist of rectangular closed links together with longitudinal reinforcement. Such reinforcement is additional to any requirements for shear and bending.

Details of design procedures for torsion are given in *Table 3.35* for BS 8110, and *Table 3.39* for BS 5400.

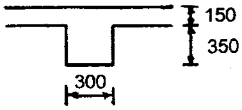
Example 1. The beam shown in the following figure is to be designed for shear to the requirements of BS 8110. Details of the design loads and the bending requirements, for which the tension reinforcement comprises 2H32 (bottom) and 3H32 (top, at support B), are contained in example 1 of Chapter 24. The design of the section is to be based on the following values:

$$f_{cu} = 40 \text{ N/mm}^2, f_y = 500 \text{ N/mm}^2, d = 450 \text{ mm}$$

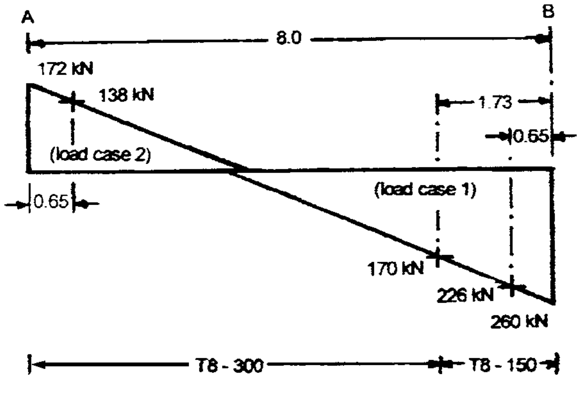
In the following calculations, a simplified approach is used in which the critical section for shear is taken at distance d from the face of the support with no enhancement of the concrete shear stress. In addition, the value of v_c is determined for the section where the tension reinforcement is least, and the same value of v_c used throughout. Since the beam will be provided with shear reinforcement, the value of v_c may be taken as that obtained for a section with $d = 400$ mm.



Spans



Cross section



Shear force diagram

Based on 2H32 as effective tension reinforcement,

$100A_s/b_v d = 100 \times 1608/(300 \times 450) = 1.19$

$v_c = 0.78$ N/mm^2 (*Table 3.33*, for $d = 400$ and $f_{cu} = 40$)

BS 8110 Shear under concentrated loads

3.34

Design procedure

For design purposes, a shear perimeter is taken as the boundary of the smallest rectangle that nowhere comes closer to the edges of a loaded area than a specified distance. The shear capacity is checked first on a perimeter distant $1.5d$ from the edge of the loaded area, where d is the mean effective depth for all effective tension reinforcement crossing the perimeter. If the calculated shear stress v does not exceed v_c, no shear reinforcement is needed.

If v exceeds v_c, shear reinforcement is required within the failure zone that exists between the shear perimeter and the edge of the loaded area. In zones where $v_c < v \leq 2v_c$, shear reinforcement in the form of links may be provided in solid slabs ≥ 200 mm deep. No provision is made for cases where $v > 2v_c$. Further checks should be made on successive perimeters at intervals of $0.75d$, until a perimeter is reached where shear reinforcement is no longer needed. Each check is associated with a $1.5d$ wide failure zone that overlaps the previous failure zone by $0.75d$.

The maximum design shear stress at the edge of a concentrated load, or the face of a column or column head, should not exceed the lesser of $0.8\sqrt{f_{cu}}$ or 5 N/mm^2. The value of the maximum stress is given by $v_{max} = V/u_0d$, where V is the design value of the concentrated load, and u_0 is the length of the perimeter that touches the loaded area reduced where necessary at the edges of the slab, or for the effect of openings (see *Tables 3.37* and *3.38*).

For pile caps, no check for punching is needed in cases where the spacing of the piles is not greater than three times the pile diameter (taken as the diameter of the inscribed circle for non-circular piles). In other cases, the shear stress should be checked on a rectangular perimeter whose sides are at distances a_v from the column face, where a_v is equal to the distance from the column face to the inner edge of the piles plus one-fifth of the pile diameter. For sides where $a_v < 1.5d$, v_c may be replaced by $(1.5d/a_v)v_c$, provided this value does not exceed the lesser of $0.8\sqrt{f_{cu}}$ or 5 N/mm^2.

Effective shear forces

In flat slab structures, the punching shear forces around the columns should be increased to allow for the effects of moment transfer. The design effective shear force V_{eff} at the perimeter considered should be taken as follows:

Column	Internal	Edge		Corner
Axis of bending	Both	Perpendicular to edge	Parallel to edge	Both
V_{eff}	$(1 + 1.5M_t/V_tx)\,V_t$	$(1.25 + 1.5M_t/V_tx)\,V_t$	$1.25V_t$	$1.25V_t$

Values of V_{eff} should be determined independently for the moments and shears about both axes of the column and the design checked for the worse case, where:

M_t is the design moment transmitted to the column at the connection. For an internal column, M_t is the difference in the slab moments on either side of the axis of bending. In cases where the equivalent frame method of analysis is used and pattern loads are considered, M_t may be reduced by 30%.

V_t is the design shear force transmitted to the column at the connection. For an internal column, V_t is the sum of the shear forces on either side of the axis of bending.

x is the length of the side of the shear perimeter parallel to the axis of bending.

Alternatively, for braced structures with approximately equal spans, values of $V_{eff} = 1.15V_t$ for internal columns, and $V_{eff} = 1.4V_t$ for edge columns bent about the axis perpendicular to the edge, may be taken (see, also *Table 2.56*).

For pad footings, the load resulting from the soil bearing pressure within the shear perimeter may be deducted from the design load on the column, when calculating the design shear force transmitted to the footing.

Shear reinforcement

In slabs ≥ 200 mm deep, for $v_c < v \leq 2v_c$, shear links may be provided in accordance with the following equations:

Shear stress $v = V_{eff}/ud$	Total area of shear reinforcement
$v_c < v \leq 1.6v_c$	$\Sigma A_{sv}\sin\alpha = (v - v_c)ud/0.87f_{yv} \geq 0.4\ ud/0.87f_{yv}$
$1.6v_c < v \leq 2v_c$	$\Sigma A_{sv}\sin\alpha = (3.5v - 5v_c)ud/0.87f_{yv} \geq 0.4\ ud/0.87f_{yv}$

Notes. ΣA_{sv} is the total area of shear reinforcement required in the failure zone, u is the effective length of the outer perimeter of the failure zone, reduced where necessary for the effect of openings or unsupported edges (see *Tables 3.37* and *3.38*), and α is the angle between the shear reinforcement and the plane of the slab.

The shear reinforcement required within each failure zone should be distributed on at least two perimeters. The first perimeter of reinforcement should be located at approximately $0.5d$ from the inner edge of the failure zone and should contain not less then 40% of the total amount of reinforcement required within the zone. The spacing of perimeters of shear reinforcement should not exceed $0.75d$ and the spacing of the shear reinforcement around any perimeter should not exceed $1.5d$. Shear reinforcement should be anchored round at least one layer of tension reinforcement. The shear reinforcement within the overlapping portion of adjacent failure zones may be taken into account as contributing to the requirements for both zones. No provision is made in the code for the use of shear reinforcement other than links, or for cases where $v > 2v_c$.

3.35

BS 8110 Design for torsion

Analysis of structure

If required in structural analysis or design, the torsional rigidity of a section ($G \times C$) may be calculated by assuming the shear modulus $G = 0.42E$ and taking the torsional constant C as half the St. Venant value calculated for the plain concrete section.

The St. Venant value for a rectangular section is:

$\beta h^3_{\text{min}} h_{\text{max}}$

where

h_{max} and h_{min} are the dimensions of the section

Values of coefficient β						
$h_{\text{max}}/h_{\text{min}}$	1	1.5	2	3	5	> 5
β	0.14	0.20	0.23	0.26	0.29	0.33

The St. Venant value for a non-rectangular section can be obtained by dividing the section into a series of rectangles, and summing the values obtained for each rectangle. The sub-division of the section should be arranged so as to maximise the total value obtained. This will generally be achieved if the widest rectangle is made as long as possible.

Torsional shear stress

Rectangular sections — Calculate as $v_t = \dfrac{2T}{h^2_{\text{min}}(h_{\text{max}} - h_{\text{min}}/3)}$ where T = design ultimate torsional moment

T-, L- or I-sections — Divide section into component rectangles so as to maximise the term $\Sigma h^3_{\text{min}} h_{\text{max}}$, and treat each component as a rectangular section subjected to a torsional moment equal to:

$T\,[\,h^3_{\text{min}} h_{\text{max}} / \Sigma h^3_{\text{min}} h_{\text{max}}\,]$

Hollow sections — Box sections in which the wall thickness exceeds one-quarter of the overall dimension of the section in the direction of measurement may be treated as solid sections.

Torsion reinforcement requirements

Reinforcement to be provided for combined shear and torsion (see Notes)

Shear stress	Torsional shear stress	
	$v_t \le v_{t,\text{min}}$	$v_t > v_{t,\text{min}}$
$v \le v_c + 0.4$	Minimum shear reinforcement $A_{sv}/s_v = 0.4b_v/0.87f_{yv}$	Designed torsion reinforcement but not less than minimum shear reinforcement
$v > v_c + 0.4$	Designed shear reinforcement $A_{sv}/s_v = b_v(v - v_c)/0.87f_{yv}$	Designed shear reinforcement and designed torsion reinforcement

Notes. At no section should ($v + v_t$) exceed v_{tu} nor, in small sections where the larger centre-to-centre dimension of a rectangular link, $y_1 < 550$ mm, should v_t exceed $v_{tu}(y_1/550)$, where $v_{tu} = 0.8\sqrt{f_{cu}}$ or 5 N/mm², whichever is the lesser. No torsion reinforcement is required in sections where $v_t \le v_{t,\text{min}} = 0.067\sqrt{f_{cu}}$ or 0.4 N/mm², whichever is the lesser.

Torsion reinforcement should consist of rectangular closed links together with longitudinal reinforcement. The areas of these reinforcements, which is additional to any requirements for shear and bending, should be such that:

$$\frac{A_{sv}}{s_v} \ge \frac{T}{0.8x_1y_1(0.87f_{yv})} \quad \text{and} \quad A_s \ge \frac{T(x_1 + y_1)}{0.8x_1y_1(0.87f_y)}$$

A_s is total area of the longitudinal bars

A_{sv} is area of 2 legs of a rectangular closed link

x_1 and y_1 are centre-to-centre dimensions of links

The longitudinal torsion reinforcement should be distributed evenly round the inside perimeter of the links. The clear distance between the bars should not exceed 300 mm and at least four bars, one in each corner of the links, should be used. In a region where reinforcement is needed for bending and torsion, the sum of both requirements can be provided by using larger bars than those required for bending alone. The torsion reinforcement should extend for a distance at least equal to the largest dimension of the section beyond the section where theoretically it is no longer required. In T-, L- or I-sections, the reinforcement should be detailed so that the link cages interlock and tie the component rectangles together. The area properties of link cages, according to size and spacing, are shown below.

Areas of 2-leg links

Values of A_{sv}/s_v for 2 legs of link reinforcement (mm²/mm)

Spacing s_v (mm)	Size of bar (mm)				Spacing s_v (mm)	Size of bar (mm)			
	8	10	12	16		8	10	12	16
75	1.34	2.09	3.01	5.36	200	0.50	0.78	1.13	2.01
100	1.00	1.57	2.26	4.02	225	0.44	0.70	1.00	1.79
125	0.80	1.25	1.81	3.21	250	0.40	0.63	0.90	1.61
150	0.67	1.04	1.51	2.68	275	0.36	0.57	0.82	1.46
175	0.57	0.90	1.29	2.30	300	0.33	0.52	0.75	1.34

Note. For torsion, s_v should not exceed the least of x_1, $y_1/2$ or 200 mm. For shear, s_v should not exceed $0.75d$ for beams and d for solid slabs.

BS 5400 Shear resistance

3.36

Members without shear reinforcement

The design shear resistance of a flexural member without shear reinforcement is given by $V_c = \xi_s v_c bd$, where:

$$v_c = \left(\frac{0.27}{\gamma_m}\right)\left(\frac{100A_s f_{cu}}{bd}\right)^{1/3} \quad \text{with} \quad \xi_s = \left(\frac{500}{d}\right)^{1/4} \geq 0.7, \quad 0.15 \leq \left(\frac{100A_s}{bd}\right) \leq 3.0, \quad f_{cu} \leq 40 \quad \text{and} \quad \gamma_m = 1.25$$

A_s is the area of longitudinal tension reinforcement that continues for a distance $\geq d$ beyond the section considered. (At supports, the full area of tension reinforcement may be used, provided the requirements for anchorage are met)

b is the breadth of the section which, for a flanged section, should be taken as the width of the rib

d is the effective depth of the tension reinforcement

For beam sections at distance $a_v \leq 2d$ from the face of a support, the front edge of a rigid bearing or centre line of a flexible bearing, $\xi_s v_c$ may be replaced by $\xi_s(2d/a_v)v_c$, but V/bd should not exceed the lesser of $0.75\sqrt{f_{cu}}$ or 4.75 N/mm². The tension reinforcement should continue to the support and have an anchorage equivalent to 20 times the bar size.

f_{cu} N/mm²	Ultimate concrete shear stress v_c (N/mm²) for values of $100A_s/bd$								
	≤ 0.15	0.25	0.50	0.75	1.0	1.5	2.0	2.5	≥ 3.0
25	0.34	0.40	0.50	0.57	0.63	0.72	0.80	0.85	0.91
30	0.36	0.42	0.53	0.61	0.67	0.76	0.85	0.91	0.97
35	0.37	0.44	0.56	0.64	0.70	0.80	0.89	0.96	1.02
≥ 40	0.39	0.47	0.59	0.67	0.74	0.84	0.93	1.00	1.06

Depth factor ξ_s for values of effective depth d (mm)

d	≤ 100	150	200	300	400	500	1000	1500	≥ 2000
ξ_s	1.50	1.35	1.25	1.15	1.05	1.00	0.85	0.75	0.70

Members with shear reinforcement

The design shear resistance of a member provided with upright links and a system of bent-up bars is given by:

$$V_u = (\xi_s v_c - 0.4)bd + (A_{sv}/s_v)(0.87f_{yv})d + (A_{sb}/s_b)(0.87f_y)(s_t \sin\alpha) \quad \text{where} \quad s_t = (d - d')(\cot\alpha + \cot\beta) \leq 1.5d$$

A_{sb} is cross-sectional area of bent up bars	α is angle (≥ 45°) between bent-up bar and axis of member
A_{sv} is cross-sectional area of vertical legs of links	β is angle (≥ 45°) between 'compression strut' of a system of bent-up bars and axis of member
d' is depth from concrete face to top of bent-up bar	
f_y is characteristic strength of bent-up bars	s_b is spacing of bent-up bars along the member
f_y is characteristic strength of link reinforcement	s_v is spacing of links along the member

At any cross-section, an additional area of longitudinal reinforcement is required in the tension zone (in excess of that required for bending) given by $A_{sa} = 0.5V/0.87f_y$. In no case should V/bd exceed the lesser of $0.75\sqrt{f_{cu}}$ or 4.75 N/mm².

Form and area of shear reinforcement to be provided in beams

Value of $v = V/bd$ (N/mm²)	Form of shear reinforcement	Area of shear reinforcement
$v \leq \xi_s v_c$	Links at spacing ≤ 0.75d in direction of span and at right angles to span	$A_{sv}/s_v = 0.4b/0.87f_{yv}$ (see *Table 3.39* for areas of links)
$\xi_s v_c < v \leq 0.75\sqrt{f_{cu}}$ or 4.75 N/mm² whichever is the lesser	Links or links combined with bent-up bars, but not more than 50% of shear resistance provided by reinforcement to be in the form of bent-up bars	Where links only are provided: $A_{sv}/s_v = b(v + 0.4 - \xi_s v_c)/0.87f_{yv}$ Where links and bent-up bars are provided, see general equation

Form and area of shear reinforcement to be provided in solid slabs

Value of $v = V/bd$ (N/mm²)	Form of shear reinforcement	Area of shear reinforcement
$v \leq \xi_s v_c$	None required	Not applicable
$\xi_s v_c < v \leq 0.75\sqrt{f_{cu}}$ or 4.75 N/mm² whichever is the lesser	In slabs ≥ 200 mm deep, as beams except link spacing ≤ d	As beams

Columns

For a column or load-bearing wall subjected to uniaxial shear, the value of $\xi_s v_c$ obtained above may be multiplied by $(1 + 0.05N/A_c)$, where A_c is the gross area of the concrete section and N is the ultimate axial load. Members subjected to biaxial shear should be designed such that:

$$\frac{V_x}{V_{ux}} + \frac{V_y}{V_{uy}} \leq 1.0$$

V_x and V_y are the applied shears for the x-x and y-y axes respectively

V_{ux} and V_{uy} are the corresponding shear resistances provided by the concrete and link reinforcement, derived allowing for the axial load enhancement factor

For a circular column, the area of longitudinal reinforcement to be used to determine v_c should be taken as that in the half of the column opposite the extreme compression fibre. In the shear calculations, d is taken as the distance from the centroid of this reinforcement to the extreme compression fibre, and b is taken as the diameter of the section.

3.37

BS 5400 Shear under concentrated loads – 1

Design procedure

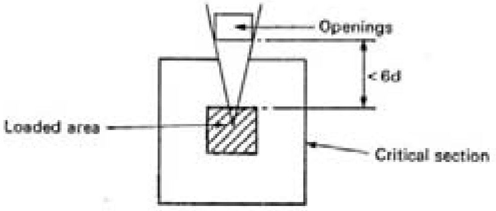
For design purposes, a shear perimeter is taken as the boundary of the smallest rectangle that nowhere comes closer to the edges of a loaded area than a specified distance. Details of loaded areas for wheel loads, allowing for dispersal down to the top surface of the concrete slab, are given in section 2.4.9. The shear capacity is checked first at a critical section distant $1.5d$ from the edge of the loaded area, as shown in figure (a) on *Table 3.38*, where d is the effective depth to the tension reinforcement. If a part of the perimeter of the critical section cannot, physically, extend $1.5d$ from the boundary of the loaded area, that part should be taken as far from the loaded area as is physically possible and the value of v_c for that part may be increased to $(1.5d/a_v)v_c$, where a_v is the distance from the boundary of the loaded area to the perimeter actually considered. Near unsupported edges and for cantilever slabs, the critical section should be taken as the worst case obtained from figures (a), (b) or (c) on *Table 3.38*. For a group of concentrated loads, adjacent loaded areas should be considered singly and in combination.

The overall shear resistance should be taken as the sum of the shear resistances for each portion of the critical section. The value of $100A_s/b_sd$, which is used to determine v_c for each portion, should be based on the tension reinforcement associated with that portion as shown in the figures. If the ultimate shear force V, due to concentrated loads, does not exceed the value of V_c determined from the formulae in *Table 3.38*, no further check is needed. If V does exceed V_c, shear reinforcement in the form of links may be provided in slabs ≥ 200 mm deep. Further checks should be made on successive perimeters at intervals of $0.75d$ out from the critical section until a perimeter is reached where shear reinforcement is no longer needed. The maximum design shear stress at the edge of a concentrated load, or the face of a column, should not exceed the lesser of $0.75\sqrt{f_{cu}}$ or 4.75 N/mm^2.

For pad footings, the load resulting from the soil bearing pressure within the shear perimeter may be deducted from the column load, when calculating the design shear force transmitted to the footing.

Openings

When openings in slabs or footings are located at a distance less than $6d$ from the edge of a concentrated load or the face of a column, the part of the periphery of the critical section that is enclosed by radial projections taken from the centre of the loaded area to the corners of the opening should be considered ineffective. Where a hole is adjacent to a column face, and its greatest width is less than one-quarter of the column side or one half of the slab depth, whichever is the lesser, its presence may be ignored.



Openings in slabs

Shear reinforcement

In solid slabs ≥ 200 mm deep, for $V_c < V$, shear links may be provided in accordance with the following equation:

$$\Sigma A_{sv} = (V - V_c)/0.87f_{yv} \geq 0.4\Sigma bd/0.87f_{yv}$$

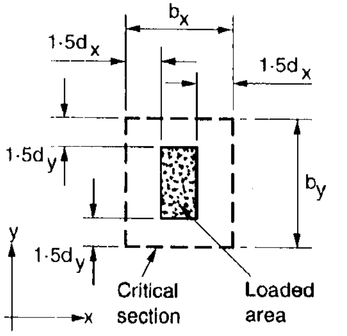
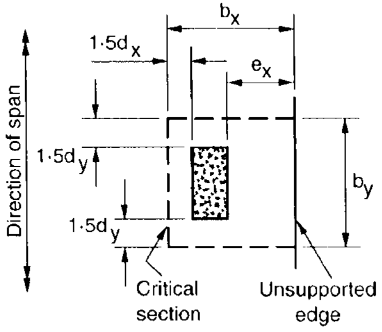
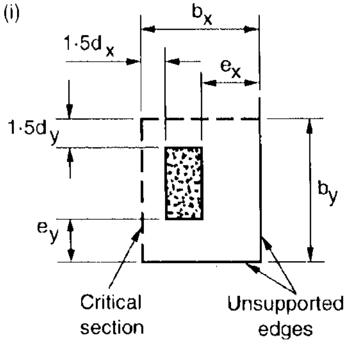
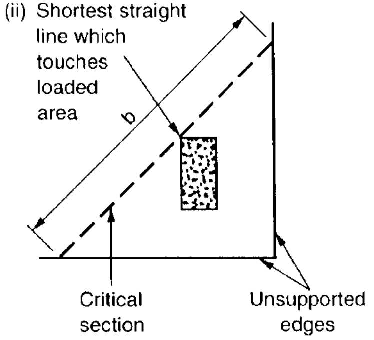
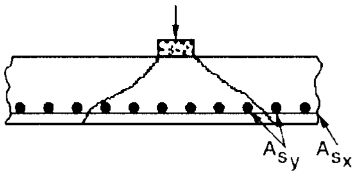
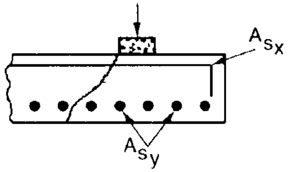
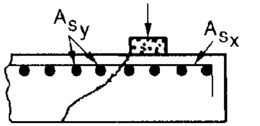
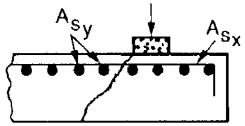
ΣA_{sv} is the area of shear reinforcement to be provided as described below
Σbd is the area of the section under consideration
f_{yv} is the characteristic strength of the shear reinforcement

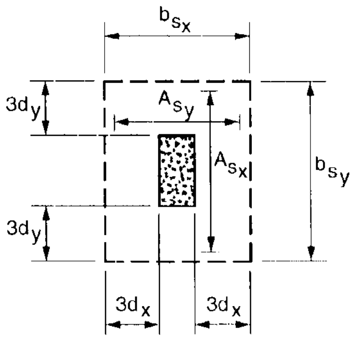
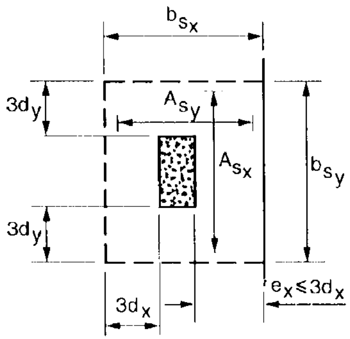
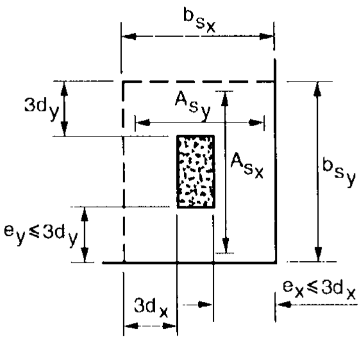
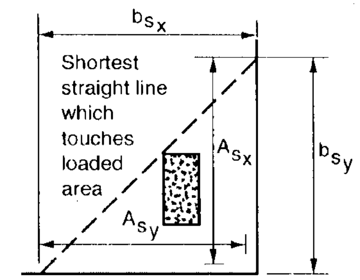
The prescribed area of shear reinforcement should be provided at the critical section, and a similar amount on a parallel perimeter at a distance $0.75d$ inside the critical section. If shear reinforcement is needed at perimeters further out than the critical section, the prescribed area should be provided at the section under consideration. The spacing of the shear reinforcement should not exceed $0.75d$, and each link should be anchored at both ends by passing round the main reinforcement. If inclined links are used A_{sv} should be replaced by $A_{sv}\sin\alpha$, where α is the angle between the links and the plane of the slab.

3.38

BS 5400 Shear under concentrated loads – 2

<table>
<tr><th colspan="5">Parameters for shear in solid slabs under concentrated loads</th></tr>
<tr><td>Load position</td><td>(a) Load at middle of slab</td><td>(b) Load at edge of slab</td><td colspan="2">(c) Load at corner of cantilever slab</td></tr>
<tr><td>Critical section for calculating shear resistance V_c.

Critical sections (a), (b) and (c) (i) are assumed to have squared corners for rectangular and circular loaded areas.</td>
<td> </td>
<td> </td>
<td>(i)
 </td>
<td> </td></tr>
<tr><td>Idealised mode of failure (only tension reinforcement shown)</td>
<td> </td>
<td> </td>
<td> </td>
<td> </td></tr>
<tr><td>Parameters used to derive v_c from <i>Table 3.36</i> for each portion of critical section.

Note. A_s should include only tensile reinforcement that is effectively anchored.</td>
<td> </td>
<td> </td>
<td> </td>
<td> The ratio of reinforcement should be taken as the average of the two ratios of reinforcement in the two directions $\frac{100A_s}{b_s d}$</td></tr>
<tr><td>Shear resistance V_c</td><td>$\Sigma\xi_{sx}v_c bd$ for 4 critical portions</td><td>$0.8\Sigma\xi_{sx}v_c bd$ for 3 critical portions</td><td>$0.8\Sigma\xi_{sx}v_c bd$ for 2 critical portions</td><td>$0.25(\xi_{sx} + \xi_{sy})\ v_c b\ (d_x + d_y)$</td></tr>
</table>

BS 5400 Design for torsion

3.39

Torsional shear stress

Rectangular sections $v_t = \dfrac{2T}{h_{min}^2 (h_{max} - h_{min}/3)}$

T is the ultimate torsional moment
h_{max} and h_{min} are the dimensions of the section

T-, L- or I-sections Divide the section into component rectangles so as to maximise the term $\Sigma h_{min}^3 h_{max}$, and treat each of the components as a rectangular section subjected to a torsional moment equal to:

$$T\,[\,h_{min}^3 h_{max} / \Sigma h_{min}^3 h_{max}\,]$$

Hollow sections $v_t = \dfrac{T}{2h_{wo} A_o}$

A_o is the area enclosed by the median wall line
h_{wo} is the relevant wall thickness

Torsion reinforcement requirements

Reinforcement to be provided for combined shear and torsion (see Notes)

Shear stress	Torsional shear stress	
	$v_t \le v_{t,min}$	$v_t > v_{t,min}$
$v \le \xi_s v_c$	Minimum shear reinforcement $A_{sv}/s_v = 0.4b/0.87f_{yv}$	Designed torsion reinforcement but not less than minimum shear reinforcement
$v > \xi_s v_c$	Designed shear reinforcement $A_{sv}/s_v = b(v + 0.4 \ - \xi_s v_c)/0.87f_{yv}$	Designed shear reinforcement and designed torsion reinforcement

Notes. At no section should (v + v_t) exceed v_{tu} nor, in small sections where the larger centre-to-centre dimension of a rectangular link, $y_1 < 550$ mm, should v_t exceed $v_{tu}(y_1/550)$, where v_{tu} is the lesser of $0.75\sqrt{f_{cu}}$ or 4.75 N/mm^2. Torsion reinforcement is not required in sections where $v_t \le v_{t,min} = 0.067\sqrt{f_{cu}}$ or 0.42 N/mm^2, whichever is the lesser.

Torsion reinforcement should consist of rectangular closed links together with longitudinal reinforcement. The area of the reinforcement, which is additional to any requirements for shear and bending, should be such that:

Rectangular sections $\dfrac{A_{st}}{s_v} \ge \dfrac{T}{1.6x_1 y_1 (0.87 f_{yv})}$ and $\dfrac{A_{sL}}{s_L} \ge \dfrac{A_{st}}{s_v}\left(\dfrac{f_{yv}}{f_{yL}}\right)$

Hollow sections $\dfrac{A_{st}}{s_v} \ge \dfrac{T}{2A_o (0.87 f_{yv})}$ and $\dfrac{A_{sL}}{s_L} \ge \dfrac{A_{st}}{s_v}\left(\dfrac{f_{yv}}{f_{yL}}\right)$

A_{sL} is the area of one longitudinal torsion bar
A_{st} is the area of one leg of a torsion link
s_L is the spacing of the longitudinal bars
s_v is the spacing of the links along the member
x_1 is the smaller centreline dimension of the link
y_1 is the larger centreline dimension of the link

The area of either the links or the longitudinal reinforcement may be reduced by up to 20% provided that the product (A_{st}/s_v) x (A_{sL}/s_L) remains unchanged. The total area of longitudinal torsion reinforcement is $2(A_{sL}/s_L)(x_1 + y_1)$.

The longitudinal torsion reinforcement should be distributed evenly round the inside perimeter of the links, and such that there is a bar in each corner of the links. In a region where tension reinforcement is needed for bending and torsion, the sum of both requirements can be provided by using larger bars than those required for bending alone. In a region that is subjected to simultaneous compression due to bending, the area of longitudinal torsion reinforcement in the compression zone may be reduced by an amount equal to the magnitude of the compressive force/$0.87f_{yL}$. In the case of beams, the depth of compression zone used to calculate the compressive force may be taken as twice the cover to the torsion links. In T-, L- or I-sections, the reinforcement should be detailed so that the link cages interlock and tie the component rectangles together. The area properties of link cages, according to size and spacing, are shown below.

Areas of 2-leg links

Values of A_{sv}/s_v (= 2 A_{st}/s_v) for 2 legs of link reinforcement (mm^2/mm)

Spacing s_v (mm)	Size of bar (mm)				Spacing s_v (mm)	Size of bar (mm)			
	8	10	12	16		8	10	12	16
75	1.34	2.09	3.01	5.36	200	0.50	0.78	1.13	2.01
100	1.00	1.57	2.26	4.02	225	0.44	0.70	1.00	1.79
125	0.80	1.25	1.81	3.21	250	0.40	0.63	0.90	1.61
150	0.67	1.04	1.51	2.68	275	0.36	0.57	0.82	1.46
175	0.57	0.90	1.29	2.30	300	0.33	0.52	0.75	1.34

Note. For torsion, s_v should not exceed the least of $(x_1 + y_1)/4$, 16 times diameter of longitudinal corner bar or 300 mm. For shear, s_v should not exceed $0.75d$ for beams and d for solid slabs.

Minimum link requirements are given by:

$$A_{sv}/s_v = 0.4b_v/0.87f_{yv} = 0.4 \times 300/(0.87 \times 500)$$
$$= 0.28\ \text{mm}^2/\text{mm}$$
$$s_v \le 0.75d = 0.75 \times 450 = 337.5\ \text{mm}$$

From *Table 3.35*, H8-300 provides 0.33 mm^2/mm

Shear resistance of section containing H8-300 is given by:

$$V_u = v_c b_v d + (A_{sv}/s_v)(0.87f_{yv})d$$
$$= (0.78 \times 300 + 0.33 \times 0.87 \times 500) \times 450 \times 10^{-3} = 170\ \text{kN}$$

Based on a support of width 400 mm, distance from centre of support to critical section = 200 + 450 = 650 mm. The design load is 416/8 = 52 kN/m, and the shear forces at the critical sections are:

$$\text{End A, } V = 172 - 0.65 \times 52 = 138\ \text{kN} < 170\ \text{kN (H8-300)}$$
$$\text{End B, } V = 260 - 0.65 \times 52 = 226\ \text{kN} > 170\ \text{kN}$$
$$v = V/b_v d = 226 \times 10^3/(300 \times 450) = 1.68\ \text{N/mm}^2$$

Area of links required at end B is given by:

$$A_{sv}/s_v = (v - v_c)b_v/0.87f_{yv} = (1.68 - 0.78) \times 300/(0.87 \times 500)$$
$$= 0.62\ \text{mm}^2/\text{mm}$$

From *Table 3.35*, H8-150 provides 0.67 mm^2/mm

Note that if the concrete shear strength is taken as $(2d/a_v)v_c$ for $a_v \le 2d$, critical section is at $a_v = 2d$ and distance from centre of support to critical section = 200 + 900 = 1100 mm. Here, $V = 203$ kN, $v = 1.50\ \text{N/mm}^2$, $A_{sv}/s_v = 0.50\ \text{mm}^2/\text{mm}$ and, from *Table 3.35*, H8-200 would be sufficient.

Example 2. A 700 mm thick solid slab bridge deck is supported at the end abutment on bearings spaced at 1.5 m centres. The maximum bearing reaction resulting from the worst arrangement of the design loads is 625 kN. The tension reinforcement at the end of the span is H25-200. The slab is to be designed for shear to the requirements of BS 5400, using the following values:

$$f_{cu} = 40\ \text{N/mm}^2, f_y = 500\ \text{N/mm}^2, d = 620\ \text{mm}$$

If the critical section for punching is taken at $1.5d = 930$ mm from the edge of each bearing, clearly the critical perimeters from adjacent bearings will overlap. Therefore, the slab will be designed for shear along a vertical section extending for the full width of the slab. Assuming that the bearing reaction can be spread over a slab strip equal in width to the spacing of the bearings,

$$v = V/bd = 625 \times 10^3/(1500 \times 620) = 0.67\ \text{N/mm}^2$$

Based on H25-200 as effective tension reinforcement,

$$100A_s/bd = 100 \times 2454/(1000 \times 620) = 0.40$$
$$v_c = 0.54\ \text{N/mm}^2, \xi_s = 0.95\ (\textit{Table 3.36}, \text{ for } f_{cu} = 40)$$

Area of links required at end of span is given by:

$$A_{sv}/s_v = (v + 0.4 - \xi_s v_c)b/0.87f_{yv}$$
$$= (0.67 + 0.4 - 0.95 \times 0.54) \times 1000/(0.87 \times 500)$$
$$= 1.28\ \text{mm}^2/\text{mm}$$

Consider H10 links and transverse spacing of legs at 200 mm to suit spacing of tension reinforcement, that is, 5 legs per metre. Then, required longitudinal spacing of links is given by,

$$s_v \le 5 \times 78/1.28 = 304\ \text{mm}$$

Provide H10-300 with legs at 200 mm centres transversely.

Example 3. A 280 mm thick flat slab is supported by 400 mm square columns arranged on a 7.2 m square grid. The slab, which has been designed using the simplified method for determining moments, contains as tension reinforcement in the top of the slab at an interior support, within a 1.8 m wide strip central with the column, H16-180 in each direction. The slab is to be designed to the requirements of BS 8110 (see *Table 3.34*), for a shear force resulting from the maximum design load applied to all panels adjacent to the column of $V_t = 954$ kN.

$$f_{cu} = 40\ \text{N/mm}^2, f_y = 500\ \text{N/mm}^2, d = 240\ \text{mm (average)}$$

For design using the simplified method, the design effective shear force at an interior column $V_{eff} = 1.15V = 1098$ kN.

The maximum design shear stress at the column face,

$$V_{eff}/u_o d = 1098 \times 10^3/(4 \times 400 \times 240) = 2.86\ \text{N/mm}^2\ (\le 5.0)$$

Based on H16-180 as effective tension reinforcement,

$$100A_s/b_v d = 100 \times 201/(180 \times 240) = 0.46$$
$$v_c = 0.65\ \text{N/mm}^2\ (\textit{Table 3.33}, \text{ for } d = 240 \text{ and } f_{cu} = 40)$$

The length of the first critical perimeter at $1.5d$ from the face of the column is $4 \times (3d + 400) = 4480$ mm. Thus, the design shear stress at the first critical perimeter,

$$v = 1098 \times 10^3/(4480 \times 240) = 1.02\ \text{N/mm}^2\ (= 1.57v_c)$$

Since $v_c < v \le 1.6v_c$ and $(v - v_c) < 0.4\ \text{N/mm}^2$, the total area of vertical links required within the failure zone is given by:

$$\Sigma A_{sv} = 0.4ud/0.87f_{yv} = 0.4 \times 4480 \times 240/(0.87 \times 500)$$
$$= 989\ \text{mm}^2$$

20H8 will provide 1006 mm^2, which should be arranged on two perimeters at $0.5d = 120$ mm, and $1.25d = 300$ mm, from the column face. The inner perimeter should contain at least 40% of the total, that is, 8H8, with 12H8 on the outer perimeter.

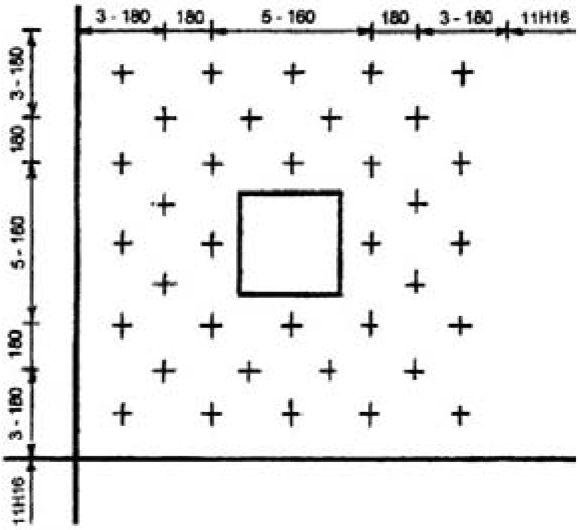
The length of the second critical perimeter at $2.25d$ from the face of the column is $4 \times (4.5d + 400) = 5920$ mm. Thus, the design shear stress at the second critical perimeter,

$$v = 1098 \times 10^3/(5920 \times 240) = 0.77\ \text{N/mm}^2\ (> v_c)$$
$$\Sigma A_{sv} = 0.4 \times 5920 \times 240/(0.87 \times 500) = 1307\ \text{mm}^2$$

12H8 are already provided by the outer perimeter of bars in the first failure zone. A further perimeter containing 16H8, to give a total of 28H8, will provide 1407 mm^2 in the second failure zone. The length of the third critical perimeter at $3d$ from the face of the column is $4 \times (6d + 400) = 7360$ mm. Thus, the design shear stress at the third critical perimeter,

$$v = 1098 \times 10^3/(7360 \times 240) = 0.62\ \text{N/mm}^2\ (< v_c)$$

The reinforcement layout is shown in the figure following, where + indicates the link positions, and the spacing of the tension reinforcement has been adjusted so that the links can be anchored round the tension bars.



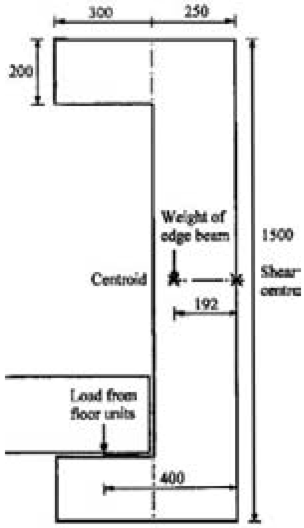
Example 4. The following figure shows a channel section edge beam, on the bottom flange of which bear 8 m long simply supported contiguous floor units. The edge beam, which is continuous over several 14 m spans, is prevented from lateral rotation at its supports. The positions of the centroid and the shear centre of the section are shown in the figure, and the beam is to be designed to the requirements of BS 8110.

Characteristic loads:
floor units: dead 3.5 kN/m^2, imposed 2.5 kN/m^2
edge beam: dead 12 kN/m

Design ultimate loads:

$$\begin{array}{ll} \text{floor units } (1.4 \times 3.5 + 1.6 \times 2.5) \times 8/2 & = 35.6 \\ \text{edge beam } 1.4 \times 12 & = \underline{16.8} \\ & \quad \underline{52.4} \text{ kN/m} \end{array}$$

f_{cu} = 40 N/mm^2, f_y = 500 N/mm^2, d = 1440 mm



Bending moment, shear force and torsional moment (about shear centre of section) at interior support (other than first):

M = – 0.08 × 52.4 × 14^2 = – 822 kNm (*Table 2.29*)
V = 52.4 × 14/2 = 367 kN
T = (35.6 × 0.400 + 16.8 × 0.192) × 14/2 = 122 kNm

(Note: In calculating V and T, a coefficient of 0.5 rather than 0.55 has been used since the dead load is dominant, and the critical section may be taken at the face of the support.)

Considering beam as one large rectangle of size 250 × 1500 and two small rectangles of size 200 × 300,

$$\Sigma h_{\min}{}^3 h_{\max} = 250^3 \times 1500 + 2 \times 200^3 \times 300$$
$$= (23.4 + 2 \times 2.4) \times 10^9 = 28.2 \times 10^9$$

Torsional moment to be considered on large rectangle:

$$T_1 = 122 \times 23.4/28.2 = 101.2 \text{ kNm}$$

Torsional moment to be considered on each small rectangle:

$$T_2 = 122 \times 2.4/28.2 = 10.4 \text{ kNm}$$

Reinforcement required in large rectangle

Bending (see *Table 3.14*)

$K = M/bd^2f_{cu} = 822 \times 10^6/(550 \times 1440^2 \times 40) = 0.018$
Since $K \leq 0.043$, $A_s = M/0.87f_yz$ where $z = 0.95d$

$A_s = 822 \times 10^6/(0.87 \times 500 \times 0.95 \times 1440) = 1382$ mm^2

Shear (see *Table 3.33*)

$v = V/b_vd = 367 \times 10^3/(250 \times 1440) = 1.02$ N/mm^2
$100A_s/b_vd = 100 \times 1382/(250 \times 1440) = 0.38$
$v_c = 0.53$ N/mm^2 (for $d = 400$ and $f_{cu} = 40$)

$A_{sv}/s_v = b_v(v - v_c)/0.87f_{yv} = 250 \times (1.02 - 0.53)/(0.87 \times 500)$
= 0.28 mm^2/mm (total for all vertical legs)

Additional requirement for bottom loaded beam

$A_{sv}/s_v = 35.6/(0.87 \times 500) = 0.08$ mm^2/mm (for inner leg)

Torsion (see *Table 3.35*)

$v_t = 2T_1/[h_{\min}{}^2(h_{\max} - h_{\min}/3)]$
$= 2 \times 101.2 \times 10^6/[250^2 \times (1500 - 250/3)] = 2.29$ N/mm^2
$(v + v_t) = 1.02 + 2.29 = 3.31$ N/mm^2 ($< v_{tu} = 5.0$)

Assuming 30 mm cover to links, dimensions of links:

$$x_1 = 250 - 2 \times 35 = 180 \text{ mm}, \; y_1 = 1500 - 2 \times 35 = 1430 \text{ mm}$$

$A_{sv}/s_v = T_1/[0.8x_1y_1(0.87f_y)]$
$= 101.2 \times 10^6/(0.8 \times 180 \times 1430 \times 0.87 \times 500)$
= 1.13 mm^2/mm (total for 2 outer legs)

Total link requirement for shear, torsion and bottom load, assuming single links with 2 legs:

$$A_{sv}/s_v = 0.28 + 1.13 + 2 \times 0.08 = 1.57 \text{ mm}^2\text{/mm}$$
$$s_v \leq \text{least of } x_1 = 180 \text{ mm}, \; y_1/2 = 715 \text{ mm or } 200 \text{ mm}$$

From *Table 3.35*, H10-100 provides 1.57 mm^2/mm

Total area of longitudinal reinforcement for torsion:

$$A_s = (A_{sv}/s_v)(f_{yv}/f_y)(x_1 + y_1)$$
$$= 1.13 \times 1.0 \times (180 + 1430) = 1819 \text{ mm}^2$$

Area of longitudinal reinforcement required in part of the section between centrelines of flanges (1300 mm apart)

$$= 1819 \times 1300/(180 + 1430) = 1469 \text{ mm}^2$$

From *Table 2.28*, 14H12 provides 1583 mm^2

Total area of tension reinforcement required at top of beam for bending and torsion

$$= 1382 + 0.5 \times (1819 - 1469) = 1557 \text{ mm}^2$$

From *Table 2.28*, 2H32 provides 1608 mm^2

Reinforcement required in small rectangles

With link dimensions taken as $x_1 = 200 - 2 \times 35 = 130$ mm, and $y_1 = 300 - 35 = 265$ mm, since $y_1 < 550$ mm, v_t should not exceed $v_{tu}y_1/550 = 5.0 \times 265/550 = 2.4$ N/mm^2.

$$v_t = 2T_2/[h_{min}{}^2(h_{max} - h_{min}/3)]$$
$$= 2 \times 10.4 \times 10^6/[200^2 \times (300 - 200/3)]$$
$$= 2.23 \text{ N/mm}^2 \; (<2.4)$$

Area of link reinforcement required for torsion:

$$A_{sv}/s_v = T_2/[0.8x_1y_1(0.87f_y)]$$
$$= 10.4 \times 10^6/(0.8 \times 130 \times 265 \times 0.87 \times 500)$$
$$= 0.87 \text{ mm}^2/\text{mm (total for 2 outer legs)}$$

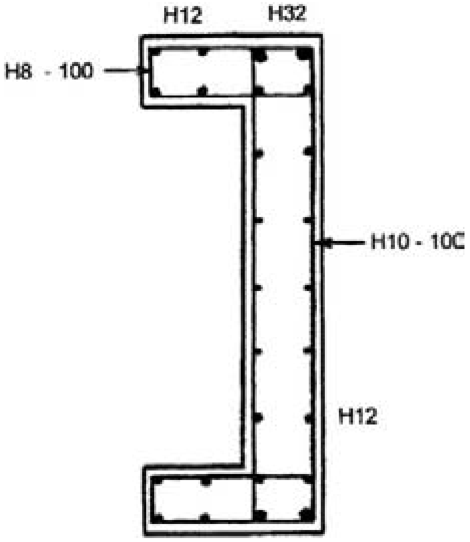
$s_v \leq$ least of $x_1 = 130$ mm, $y_1/2 = 132$ mm or 200 mm

From *Table 3.35*, H8-100 provides 1.00 mm^2/mm

Area of longitudinal reinforcement for torsion:

$$A_s = (A_{sv}/s_v)(f_{yv}/f_y)(x_1 + y_1)$$
$$= 0.87 \times 1.0 \times (130 + 265) = 344 \text{ mm}^2$$

From *Table 2.28*, 4H12 provides 452 mm^2



The lower rectangle should also be designed for the bending and shear resulting from the load applied by the floor units.

Chapter 26

Deflection and cracking

26.1 DEFLECTION

Deflections of members under service load should not impair the appearance or function of a structure. For bridges, there are no specific limits in BS 5400, but allowances are needed to ensure that minimum clearances and satisfactory drainage are provided. Accurate predictions of deflections at different stages of construction are also required.

In BS 8110, the final deflection of members below the level of the supports, after allowance for any pre-camber, is limited to span/250. A further limit, to be taken as the lesser of span/500 or 20 mm, applies to the increase in deflection that occurs after the application of finishes, cladding or partitions in order to minimise any damage to such elements. These requirements may be met by limiting the span/effective depth ratio of the member to the values given in *Table 3.40*. In this table, the design service stress in the tension reinforcement is shown as $f_s = (5/8)(f_y/\beta_b)(A_{s\ req}/A_{s\ prov})$. In BS 8110 at the time of writing, the term (5/8), which is applicable to $\gamma_m = 1.15$, is given incorrectly as (2/3), which is applicable to $\gamma_m = 1.05$.

The span/effective depth ratio limits take account of normal creep and shrinkage but, if these are likely to be particularly high (e.g. free shrinkage strain >0.00075 or creep coefficient >3), the permissible span/effective depth ratio derived from the table should be reduced by up to 15%. The limiting ratios may be used also for designs where lightweight aggregate concrete is used, except that for all beams and slabs where the characteristic imposed load exceeds 4 kN/mm^2, the values derived from *Table 3.40* should be multiplied by 0.85.

In special circumstances, when the calculation of deflection is considered necessary, the methods described in *Tables 3.41* and *3.42* can be used. Careful consideration is needed in the case of cantilevers, where the usual formulae assume that the cantilever is rigidly fixed and remains horizontal at the root. Where the cantilever forms the end of a continuous beam, the deflection at the end of the cantilever is likely to be either increased or decreased by an amount $l\theta$, where l is the length of the cantilever measured to the centre of the support, and θ is the rotation at the support. Where a cantilever is connected to a substantially rigid structure, some root rotation will still occur, and the effective length should be taken as the length to the face of the support plus half the effective depth.

26.2 CRACKING

26.2.1 Buildings and bridges

Cracking of members under service load should not impair the appearance or durability of the structure. In BS 8110, for buildings, the design surface crack width is generally limited to 0.3 mm. For members such as beams and slabs in which the nominal cover does not exceed 50 mm, the crack width requirement may be met by limiting the gaps between tension bars to specified values. In circumstances where calculation is considered necessary, crack width formulae are provided. Details of the bar spacing rules (see section 26.1 for comment on f_s) and the crack width formulae are given in *Table 3.43*.

In BS 5400, the design crack width limits apply only for load combination 1 where, for highway bridges, the live load is generally taken as HA. Crack widths are calculated for a surface taken at a distance from the outermost reinforcement equal to the nominal cover required for durability. The design crack width limits vary according to the exposure conditions as follows: 0.25 mm (moderate or severe exposure), 0.15 mm (very severe exposure), and 0.10 mm (extreme exposure). In many cases, these requirements are critical and details of the crack width formulae are given in *Table 3.43*.

In BS 8110, for cracking due to the effects of applied loads, the modulus of elasticity of concrete is taken as $E_c/2$, where values of E_c are given in *Table 3.5*. In BS 5400, a value in the range E_c to $E_c/2$ is taken according to the proportion of live to permanent load.

Cracking due to restrained early thermal effects is considered in BS 8110: Part 2, and Highways Agency BD28/87. In these documents, the restrained early thermal contraction is given by $\varepsilon_t = 0.8R\alpha T$, where α is a coefficient of expansion for the mature concrete, R is a restraint factor, and T is a temperature differential or fall. The following values of R are given:

Type of pour and restraint	R
Base cast onto blinding	0.1–0.2
Slab cast onto formwork	0.2–0.4
Wall cast onto base slab	0.6–0.8
Infill bays	0.8–1.0

BS 8110 Deflection – 1

3.40

Span/effective depth ratios

In the absence of specific deflection calculations, the span/effective depth ratios of beams and slabs should satisfy the following requirement:

$l/d \leq$ basic ratio $\times \alpha_s \times \alpha'_s$ where values of the basic ratio, and modification factors α_s and α'_s are given below.

Basic span/effective depth ratios for rectangular sections according to support conditions		
Simply supported	Continuous	Cantilever
20	26	7

Note 1. For flanged sections with $b_w/b \leq 0.3$, the basic ratios for rectangular sections should be multiplied by 0.8. For values of $b_w/b > 0.3$, the basic ratios for rectangular sections should be multiplied by $(5 + 2b_w/b)/7$.

Note 2. For two-way spanning slabs, the ratio should be based on the shorter span. For flat slabs, the check should be carried out for the more critical direction, and the basic ratio should be multiplied by 0.9, except for slabs with drops of gross width in both directions not less than one-third of the respective spans.

Note 3. For spans > 10 m, where it is necessary to limit the increase in deflection that occurs after the application of finishes, cladding or partitions, the basic ratios should be multiplied by 10/span. This multiplier effectively changes the limit on total deflection from span/250 to 40 mm. For a cantilever > 10 m, the deflection should be calculated.

Modification factor α_s, which depends on the amount of tension reinforcement and its service stress, is given by:

$$\alpha_s = 0.55 + (477 - f_s)/[120 \text{ x } (0.9 + M/bd^2)] \leq 2.0$$

M is the design ultimate moment (maximum moment in span or, for a cantilever, at the support)
b is the width of a rectangular section or, for a flanged section, the effective width of the compression flange
f_s is the design service stress in the tension reinforcement, which may be estimated from $f_s = (5/8)(f_y/\beta_b)(A_{s\,req}/A_{s\,prov})$
$A_{s\,req}/A_{s\,prov}$ is the ratio, area of reinforcement required to area of reinforcement provided, for design moment M
β_b is ratio, M/(maximum elastic moment at section), in continuous spans where redistribution of moment is involved

For designs where $f_y = 500$ N/mm^2, and $f_s = (5/8)f_y = 312$ N/mm^2 may be assumed ($\beta_b \geq 1.0$ and $A_{s\,req} \geq A_{s\,prov}$), values of α_s for different values of M/bd^2 are as follows:

M/bd^2	0.50	0.75	1.0	1.5	2.0	3.0	4.0	5.0	6.0
α_s	1.53	1.38	1.27	1.12	1.02	0.90	0.83	0.78	0.75

Modification factor α'_s, which depends on the amount of compression reinforcement, is given by:

$$\alpha'_s = 1 + (100\,A'_{s\,prov}/bd)/(3 + 100\,A'_{s\,prov}/bd) \leq 1.5$$

$A'_{s\,prov}$ is the area of reinforcement provided in the compression zone, whether or not effectively tied with links.

The foregoing recommendations were derived for uniformly loaded members, and include for some rotational restraint at end supports. Basic ratios of 18 (both ends pinned), 24 (one end pinned and one end fixed) and 30 (both ends fixed) can be obtained for a rectangular section. For a cantilever, no allowance has been made for rotation of the support.

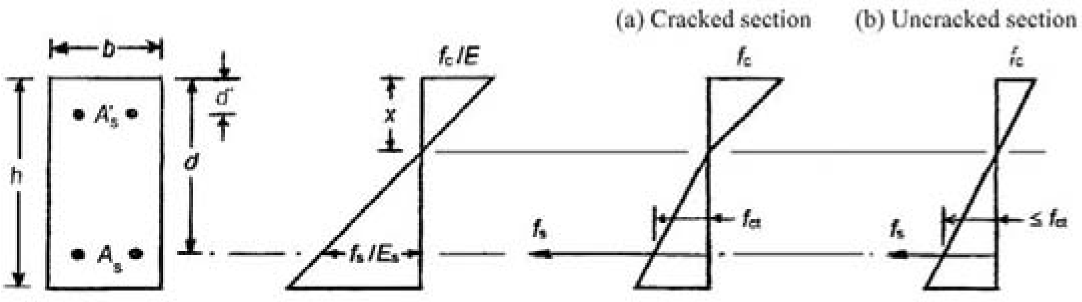
For a cantilever with no rotation of the support, where the overall depth reduces linearly from h at the support to h_o at the free end, the basic ratio should be multiplied by the following factors:

Load distribution on cantilever	Modification factor for values of h_o/h							
	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3
Uniform load	1.00	0.94	0.88	0.81	0.75	0.68	0.61	0.54
Triangular load (zero at free end)	1.25	1.19	1.12	1.06	0.99	0.92	0.85	0.78
Concentrated load at free end	0.75	0.69	0.63	0.58	0.52	0.46	0.40	0.33

Calculations

In special circumstances, when the calculation of deflection is considered necessary, the method given in *Table 3.41* can be used. For dead load, the characteristic value should generally be used. For imposed load, the characteristic value should be used in limit state calculations, and the expected value in best estimate calculations. The proportion of imposed load considered as permanent should be taken as 25% for normal domestic or office occupancy, and at least 75% for structures used for storage. For structural analysis where a single value of stiffness is used for each member, the stiffness of the uncracked concrete section should be used. If a more sophisticated method of analysis is used, in which variations in section properties over the length of the member are considered, it may be more appropriate to use the stiffness of the cracked transformed section at highly stressed sections. In deflection calculations, there are several factors that are often difficult to assess, but which can have a considerable effect on the reliability of the result. These include the assumptions made regarding the restraints provided by supports, the age of members when load is first applied, the stages at which subsequent load is applied, and the effects of finishes and rigid partitions. A reasonable approach may be to assess maximum and minimum values for the influence of these effects and take the average.

Curvatures



The curvature $1/r_b$ of a section subjected to simple bending can be determined from the following relationships:

For (a) and (b) $\dfrac{1}{r_b} = \dfrac{f_c}{xE} = \dfrac{f_s}{(d-x)E_s}$ Alternatively, for (b) $\dfrac{1}{r_b} = \dfrac{M}{EI_o}$ where

E is modulus of elasticity of concrete. Use E_c for short-term loading, and $E_{eff} = E_c/(1 + \phi)$ for long-term loading where ϕ is a creep coefficient. For values of E_c and ϕ, see section 22.1 and *Table 3.5*.
E_s is modulus of elasticity of reinforcement, taken as 200 kN/mm^2
I_o is second moment of area of gross concrete section (see *Table 3.42*)
M is moment due to design service loading at section considered
f_c is calculated stress in concrete at compression face, f_s is calculated stress in tension reinforcement

The solution of the above relationship for (a) requires a trial-and-error approach. Allowing for a small approximation, whereby x is determined for the transformed section, the following simpified relationship can be derived:

$\dfrac{1}{r_b} = \dfrac{M - M_{ct}}{EI_c} \ge \dfrac{M}{EI_o}$ $M_{ct} = b_t f_{ct}(h-x)^3/3(d-x)$ where

I_c is second moment of area of cracked transformed section (see *Table 3.42*)
b_t is width of tension zone (i.e. width of rectangular section, or width of web for a flanged section)
x is neutral axis depth of cracked transformed section (see *Table 3.42*)
f_{ct} is tensile stress in concrete at centroid of tension reinforcement, taken as 1 N/mm^2 (instantaneously) reducing to 0.55 N/mm^2 (long-term)

The shrinkage curvature $1/r_{cs}$ of a section can be determined from the following relationship:

$\dfrac{1}{r_{cs}} = \dfrac{\varepsilon_{cs} E_s S_s}{E_{eff} I}$ where

I is second moment of area of section, taken as either I_c or I_o depending on the value used to derive $1/r_b$
S_s is first moment of area of reinforcement about centroid of transformed or gross section, whichever is appropriate, given by $S_s = A_s(d-x) - A'_s(x-d')$
ε_{cs} is free shrinkage strain (see section 22.1 and *Table 3.5*)

Deflections

Deflections can be determined from the relationship $1/r_x = d^2a/dx^2$, where $1/r_x$ is the curvature and a is the deflection at x, by calculating curvatures at successive sections along a member and using a numerical integration technique. Alternatively, the deflection at the mid-span of a beam, or the end of a cantilever, is given approximately by:

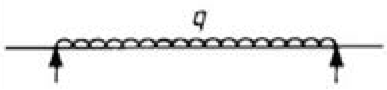
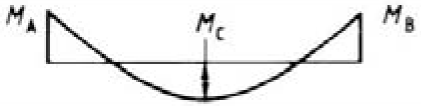
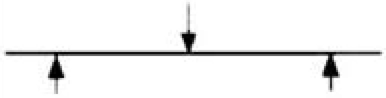
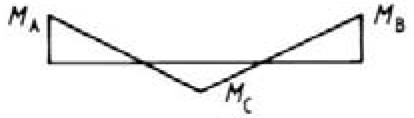
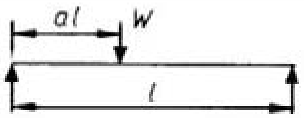
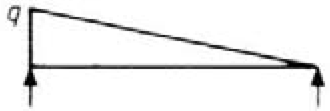
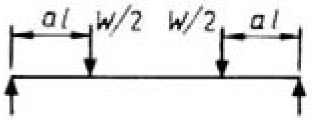
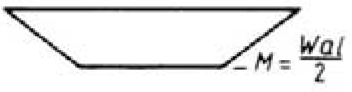
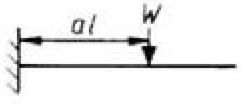
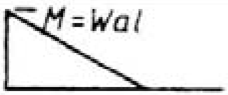
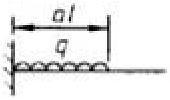
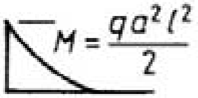
$a = \Sigma Kl^2(1/r_b) + Kl^2(1/r_{cs})$ where

K is a factor that, for a member of constant cross section, is related to the shape of the bending moment diagram (see *Table 3.42*). For concrete shrinkage, K is equal to the bending moment coefficient for uniform dead load applicable to the maximum sagging moment (e.g. $K = 0.125$ for pinned ends) or, for a cantilever, $K = 0.5$.
l is the effective span of the member.
$1/r_b$ is the curvature due to loading (at the position of maximum sagging moment or, for a cantilever, the support).
$1/r_{cs}$ is the curvature due to concrete shrinkage.

The maximum deflection should be taken as the sum of the long-term deflections due to permanent load and concrete shrinkage, and the short-term deflection due to transient load. For complex load arrangements, the value of $1/r_b$ due to the total load, and a K value appropriate to the bending moment diagram due to the total load should be used. When summing the deflections due to permanent loads and transient loads for the cracked section, the stiffening effect of the concrete in the tension zone (represented by M_{ct} in the simplified relationship) should be allowed only once.

BS 8110 Deflection – 3

	Rectangular section (or flanged section where $x \le h_f$)	Flanged section (where $x > h_f$)
Properties of sections	Cracked transformed section (where $\alpha_e = E_s/E_c$ or E_s/E_{eff}, $\rho = A_s/bd$, $\rho' = A'_s/bd$)	
	$x/d = \sqrt{\beta^2 + 2\beta'} - \beta$ where $\beta = \alpha_e\rho + (\alpha_e - 1)\rho'$ $\quad \beta' = \alpha_e\rho + (\alpha_e - 1)\rho' d'/d$ $\frac{I_c}{bd^3} = \frac{1}{3}\left(\frac{x}{d}\right)^3 + \alpha_e\rho\left(1 - \frac{x}{d}\right)^2 + (\alpha_e - 1)\rho'\left(\frac{x}{d} - \frac{d'}{d}\right)^2$	$\beta = [\alpha_e\rho + (\alpha_e - 1)\rho' + (1 - b_w/b)(h_f/d)](b/b_w)$ $\beta' = [\alpha_e\rho + (\alpha_e - 1)\rho' d'/d + 0.5(1 - b_w/b)(h_f/d)^2](b/b_w)$ $\frac{I_c}{bd^3} = \frac{1}{3}\left[\left(\frac{x}{d}\right)^3 - \left(1 - \frac{b_w}{b}\right)\left(\frac{x}{d} - \frac{h_f}{d}\right)^3\right]$ $+ \alpha_e\rho\left(1 - \frac{x}{d}\right)^2 + (\alpha_e - 1)\rho'\left(\frac{x}{d} - \frac{d'}{d}\right)^2$
	Uncracked gross section (where $\alpha_e = E_s/E_c$ or E_s/E_{eff}, $\rho_o = A_s/bh$, $\rho'_o = A'_s/bh$)	
	$\frac{x}{h} = \frac{1 + 2(\alpha_e - 1)(\rho_o d/h + \rho'_o d'/h)}{2[1 + (\alpha_e - 1)(\rho_o + \rho'_o)]}$ $\frac{I_o}{bh^3} = \frac{1}{3}\left[\left(\frac{x}{h}\right)^3 + \left(1 - \frac{x}{h}\right)^3\right]$ $+ (\alpha_e - 1)\left[\rho_o\left(\frac{d}{h} - \frac{x}{h}\right)^2 + \rho'_o\left(\frac{x}{h} - \frac{d'}{h}\right)^2\right]$	$\frac{x}{h} = \frac{(b_w/b) + (1 - b_w/b)(h_f/h)^2 + 2(\alpha_e - 1)(\rho_o d/h + \rho'_o d'/h)}{2[(b_w/b) + (1 - b_w/b)(h_f/h) + (\alpha_e - 1)(\rho_o + \rho'_o)]}$ $\frac{I_o}{bh^3} = \frac{1}{3}\left[\left(\frac{x}{h}\right)^3 + \left(\frac{b_w}{b}\right)\left(1 - \frac{x}{h}\right)^3 - \left(1 - \frac{b_w}{b}\right)\left(\frac{x}{h} - \frac{h_f}{h}\right)^3\right]$ $+ (\alpha_e - 1)\left[\rho_o\left(\frac{d}{h} - \frac{x}{h}\right)^2 + \rho'_o\left(\frac{x}{h} - \frac{d'}{h}\right)^2\right]$

	Loading	Bending moment diagram	Values of K
Deflection coefficients			$K = 0.104\,(1 - \beta/10)$ where $\beta = (M_A + M_B)/M_C$ For pinned ends, $K = 0.104$
			$K = 0.083\,(1 - \beta/4)$ where $\beta = (M_A + M_B)/M_C$
			$K = \frac{3 - 4a^2}{48(1 - a)}$ For $a = 0.5$, $K = 0.083$
			$K = 0.102$
			$K = 0.125 - \frac{a^2}{6}$
			$K = \frac{a(3 - a)}{6}$ for end deflection For $a = 1.0$, $K = 0.333$
			$K = \frac{a(4 - a)}{12}$ for end deflection For $a = 1.0$, $K = 0.25$

BS 8110 (and BS 5400) Cracking

3.43

BS 8110 bar spacing rules

For beams, in the absence of specific crack width calculations, the spacing of bars in tension (ignoring bars of size less than 0.45 × size of largest bar) where the cover does not exceed 50 mm should satisfy the following requirements:

Clear distance between bars in tension $a_b \leq (47\,000)/f_s \leq 300$ mm where $f_s = (5/8)(f_y/\beta_b)(A_{s\,req}/A_{s\,prov})$

Distance between side face of beam and nearest longitudinal bar in tension $a_c \leq (23\,500)/f_s \leq 150$ mm

For beams of overall depth $h > 750$ mm, longitudinal bars should be provided at a spacing $s_b \leq 250$ mm near the side faces of the beam. These bars should be distributed over a distance $\geq (2/3)h$ measured from the tension face. The bar size should be $\geq \sqrt{(s_b b/f_y)}$, where b (taken ≤ 500 mm) is the breadth of the beam at the point considered.

For $f_s = (5/8)f_y$ and $f_y = 500$ N/mm^2, the following values apply:

$a_b \leq 150$ mm $a_c \leq 75$ mm For $h > 750$ mm, longitudinal bars near side faces to be T16 – 250 for $b \geq 500$ mm

For slabs, in the absence of specific crack width calculations, the spacing of bars in tension where the cover is not greater than 50 mm should satisfy the following requirements:

For $h \leq 200$ mm or $100A_s/\text{bd} < 0.3$,	$a_b \leq$ lesser of $3d$ or 750 mm
For $h > 200$ mm and $0.3 \leq 100A_s/\text{bd} < 1.0$	$a_b \leq$ least of $(47\,000/f_s)/(100A_s/bd)$ or $3d$ or 750 mm
For $h > 200$ mm and $100A_s/\text{bd} \geq 1.0$	$a_b \leq$ least of $(47\,000/f_s)$ or $3d$ or 750 mm

BS 8110 crack width calculations

The design width of a crack at a particular point on the surface of a member, for values of $f_s \leq 0.8f_y$, may be calculated from the following relationships:

$$w_{cr} = \frac{3a_{cr}\varepsilon_m}{1+2(a_{cr}-c_{min})/(h-x)} \qquad \text{where} \quad \varepsilon_m = \varepsilon_1 - \frac{b_t(h-x)(a'-x)}{3E_s A_s(d-x)} = \left(\frac{f_s}{E_s} - \frac{b_t(h-x)}{3E_s A_s}\right)\left(\frac{a'-x}{d-x}\right)$$

A_s	is area of tension reinforcement
E_s	is modulus of elasticity of reinforcement
a'	is distance from compression face to level at which crack width is being determined
a_{cr}	is distance from point considered to surface of nearest longitudinal bar
b_t	is width of section at centroid of tension steel
c_{min}	is minimum cover to tension steel
f_s	is calculated stress in tension reinforcement, ignoring stiffening effect of concrete in tension zone
x	is neutral axis depth of cracked transformed section, taking modulus of elasticity of concrete as $E_c/2$
w_{cr}	is design surface crack width with an acceptable probability of being exceeded
ε_m	is average strain at level where cracking is being considered
ε_1	is calculated strain at level considered, ignoring stiffening effect of concrete in tension zone

Concrete shrinkage may generally be ignored, but if an abnormally high value ($\varepsilon_{cs} > 0.0006$) is expected, the value of ε_m should be increased by $0.5\varepsilon_{cs}$. A negative value of ε_m indicates that the section is uncracked.

BS 5400 crack width equations

The design width of a crack at a particular point on the surface of a member, taken at distance c_{min} from the outermost bar, for solid rectangular sections, the stems of T beams, and other solid sections shaped without re-entrant angles, may be calculated from the following relationships (notation as in BS 8110 unless defined below):

$$w_{cr} = \frac{3a_{cr}\varepsilon_m}{1+2(a_{cr}-c_{min})/(h-d_c)} \qquad \text{where} \quad \varepsilon_m = \varepsilon_1 - \left(\frac{3.8b_t h(a'-d_c)}{\varepsilon_s A_s(h-d_c)}\right)\left(1-\frac{M_q}{M_g}\right) \text{x}\, 10^{-9} \leq \varepsilon_1$$

M_g	is bending moment at section considered due to permanent loads
M_q	is bending moment at section considered due to live loads
c_{min}	is minimum cover to outermost reinforcement
d_c	is depth of concrete in compression (if $d_c = 0$, design crack width should be calculated as $w_{cr} = 3a_{cr}\varepsilon_m$)
ε_s	is calculated strain at centroid of tension reinforcement, ignoring stiffening effect of concrete in tension zone

When the axis of the bending moment and the direction of the tensile reinforcement resisting the bending moment are not orthogonal (e.g. in a skew slab), then A_s should be taken as $\Sigma\,(A_t \cos^4\alpha_1)$ where A_t is the area of reinforcement in a particular direction, and α_1 is the angle between the axis of the design moment and the direction of the reinforcement.

For flanges in overall tension, including tensile zones of box beams and voided slabs, the design crack width should be calculated as $w_{cr} = 3a_{cr}\varepsilon_m$, where ε_m is determined from the equation given above.

The spacing of all bars in tension should not exceed 300 mm, and the spacing of transverse bars in slabs with circular voids should not exceed twice the minimum flange thickness.

26.2.2 Liquid-retaining structures

In BS 8007, for structures where the retention or exclusion of liquid is a prime consideration, a design surface crack width limit of 0.2 mm generally applies. In cases where the surface appearance is considered to be aesthetically critical, the limit is taken as 0.1 mm. Under liquid pressure, cracks that extend through the entire thickness of a slab or a wall are expected to result in some initial seepage, but it is assumed that such cracks will heal autogeneously within 21 days for a 0.2 mm design crack width, and 7 days for a 0.1 mm design crack width. Separate calculations using basically different crack width formulae are used for the effects of applied loads and the effects of temperature and moisture change. Details of the formulae for each type of cracking are given in *Table 3.44*.

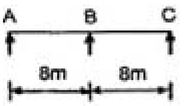
In order to control any potential cracking due to the effects of restrained thermal contraction and shrinkage, three design options are given in which the reinforcement requirements are related to the incidence of any movement joints. Details of the design options are given in *Table 3.45*, where the joint spacing requirements refer to joints as being either complete or partial. Contraction joints can be formed by casting against stop ends or by incorporating crack-inducing waterstops. The joints are complete, if all of the reinforcement is discontinued at the joints, and partial, if only 50% of the reinforcement is discontinued. The joints need to incorporate waterstops and surface sealants to ensure a liquid-tight structure.

The reinforcement needed to control cracking in continuous or semi-continuous construction depends on the magnitude of the restrained contraction. The restraint factor R (i.e. ratio of restrained to free contraction) may be taken as 0.5 generally, but more specific values for some common situations are also given in *Table 3.45*. For particular sections and arrangements of reinforcement, limiting values for restrained contraction strain are given in *Table 3.46*.

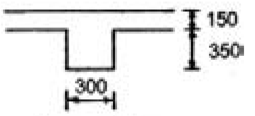
For cracking due to applied loading, and concrete classes that are typically either C28/35 or C32/40, the effective modular ratio $\alpha_e = 2E_s/E_c$ may be taken as 15. In this case, for singly reinforced rectangular sections, the elastic properties of the transformed section in flexure are given in *Table 3.47*. For particular sections and arrangements of reinforcement, limiting values are given for service moments in *Tables 3.48–3.50*, and direct tensile forces in *Tables 3.51* and *3.52*.

Example 1. The beam shown in the following figure is to be checked for deflection and cracking to the requirements of BS 8110. The designs for bending and shear are contained in example 1 of Chapters 24 and 25 respectively. The tension reinforcement, comprising 3H32 (top at B) and 2H32 (bottom), is based on the following values:

$$f_{cu} = 40 \text{ N/mm}^2, f_y = 500 \text{ N/mm}^2, \text{ cover to links} = 25 \text{ mm}$$



Spans



Cross section

For the bottom reinforcement, with d = 450 mm:

$$b = 1420 \text{ mm}, M/bd^2 = 0.99 \text{ N/mm}^2$$
$$A_{s\,req} = 1534 \text{ mm}^2, A_{s\,prov} = 1608 \text{ mm}^2$$

For continuous support conditions and a flanged section with $b_w/b = 300/1420 = 0.21 \leq 0.3$, *Table 3.40* gives

$$\text{Basic span/effective depth ratio} = 0.8 \times 26 = 20.8$$

The estimated service stress in the reinforcement, for a design with no redistribution of the ultimate moment, is given by:

$$f_s = (5/8)f_y\,(A_{s\,req}/A_{s\,prov})$$
$$= (5/8) \times 500 \times 1534/1608 = 298 \text{ N/mm}^2$$

Modification factor for tension reinforcement

$$\alpha_s = 0.55 + (477 - f_s)/120(0.9 + M/bd^2) \leq 2.0$$
$$= 0.55 + (477 - 298)/[120 \times (0.9 + 0.99)] = 1.34$$

Ignoring modification factor for compression reinforcement,

$$\text{Limiting span/effective depth ratio} = 20.8 \times 1.34 = 27.9$$

$$\text{Actual span/effective depth ratio} = 8000/450 = 17.8$$

Allowing for H8 links with 25 mm cover, the clear distance between the H32 bars in the bottom of the beam is given by:

$$a_b = 300 - 2 \times (25 + 8 + 32) = 170 \text{ mm}$$

From *Table 3.43*, the limiting distance is given by:

$$a_b \leq 47\,000/f_s = 47\,000/298 = 158 \text{ mm } (<170 \text{ mm})$$

The clear distance between the H32 bars could be reduced to 150 mm by increasing the side cover to the links to 35 mm. Alternatively the bars could be changed to 2H25 and 2H20.

Example 2. A 280 mm thick flat slab, supported by columns arranged on a 7.2 m square grid, is to be designed to the requirements of BS 8110. Bending moments are to be determined using the simplified method, where the total design ultimate load on a panel is 954 kN, and the slab is to be checked for deflection.

$$f_{cu} = 40 \text{ N/mm}^2, f_y = 500 \text{ N/mm}^2, \text{ cover to bars} = 25 \text{ mm}$$

Allowing for the use of H12 bars in each direction, and based on the bars in the second layer of reinforcement:

$$d = 280 - (25 + 12 + 6) = 235 \text{ mm}, \quad l/d = 7200/235 = 30.6$$

In the following calculations, α_s is based on the average value of M/bd^2 determined for a full panel width. From *Table 2.62* the design ultimate bending moment, for an end span with a continuous connection at the outer support, is given by:

$$M = 0.075Fl = 0.075 \times 954 \times 7.2 = 515 \text{ kNm}$$
$$M/bd^2 = 515 \times 10^6/(7200 \times 235^2) = 1.30 \text{ N/mm}^2$$

From *Table 3.40*, for a continuous flat slab without drops, the basic span/effective depth ratio = 26 × 0.9 = 23.4

Modification factor for tension reinforcement, if $f_s = (5/8)f_y$:

$$\alpha_s = 0.55 + (477 - f_s)/120(0.9 + M/bd^2) \leq 2.0$$
$$= 0.55 + (477 - 312)/[120 \times (0.9 + 1.3)] = 1.17$$

Assuming that no compression reinforcement is provided,

$$\text{Limiting span/effective depth ratio} = 23.4 \times 1.17 = 27.4$$

The limiting value of l/d can be raised to 30.6 by increasing α_s to 1.31, which reduces f_s to 276 N/mm², so that the area of tension reinforcement determined for the ULS should be multiplied by the factor 312/276 = 1.13.

For an interior span, where the bending moment coefficient is 0.063, compared to 0.075 for the end span:

$$M/bd^2 = 1.30 \times 0.063/0.075 = 1.09 \text{ N/mm}^2$$

$$\alpha_s = 0.55 + (477 - 312)/[120 \times (0.9 + 1.09)] = 1.24 \ (<1.31)$$

3.44

BS 8007 Cracking

Cracking due to loading

The design width of a crack at a particular point on the surface of a member, for values of $f_c \le 0.45 f_{cu}$ and $f_s \le 0.8 f_y$, may be calculated from the following relationships:

(Members in flexure, or combined flexure and axial load where $d < x < 0$)

$$w = \frac{3a_{cr}\varepsilon_m}{1 + 2(a_{cr} - c_{min})/(h - x)}$$

For $w_{lim} = 0.2$ mm, $\varepsilon_m = \left(\frac{f_s}{E_s} - \frac{b_t(h-x)}{3E_s A_s}\right)\left(\frac{a' - x}{d - x}\right)$

For $w_{lim} = 0.1$ mm, $\varepsilon_m = \left(\frac{f_s}{E_s} - \frac{1.5b_t(h-x)}{3E_s A_s}\right)\left(\frac{a' - x}{d - x}\right)$

(Members in direct tension, or combined flexure and axial load where the whole section is in tension) $w = 3a_{cr}\varepsilon_m$

For $w_{lim} = 0.2$ mm, $\varepsilon_m = \frac{f_s}{E_s} - \frac{2b_t h}{3E_s A_s}$

For $w_{lim} = 0.1$ mm, $\varepsilon_m = \frac{f_s}{E_s} - \frac{b_t h}{E_s A_s}$

A_s	is area of tension reinforcement
E_s	is modulus of elasticity of reinforcement
a'	is distance from compression face to level at which crack width is being determined
a_{cr}	is distance from point considered to surface of nearest longitudinal bar
b_t	is width of section at centroid of tension steel
c_{min}	is minimum cover to tension steel
f_s	is calculated stress in tension reinforcement, ignoring stiffening effect of concrete in tension zone
x	is neutral axis depth of cracked transformed section, taking modulus of elasticity of concrete as $E_c/2$
w	is design crack width with an acceptable probability of being exceeded
ε_m	is average strain at level where cracking is being considered

For members subjected to combined flexure and axial tension where the whole section is in tension, in the expression for ε_m, the second term should be multiplied by a factor between 0.5 when $x = 0$, and 1.0 for direct tension.

Cracking due to temperature and moisture effects

The design width of a crack in continuous (option 1) and semi-continuous (option 2) construction of thin cross section may be calculated from the following relationships (see *Table 3.45* for design options):

$w_{max} = s_{max}\,\varepsilon$ $\quad s_{max} = (f_{ct}/f_b)(\phi/2\rho)$ $\quad \varepsilon = \varepsilon_{cs} + R\varepsilon_{te} - (100 \times 10^{-6})$ $\quad \varepsilon_{te} = \alpha(T_1 + T_2)$

T_1	is estimated temperature fall between hydration peak and ambient at time of construction (see *Table 2.18*). For design purposes, values of T_1 should be taken ≥ 20°C for walls and ≥ 15 °C for slabs.
T_2	is estimated further fall in temperature because of seasonal variations. For semi-continuous construction, where movement joints are provided as for option 2 in *Table 3.45*, T_2 may be omitted.
R	is a restraint factor, which may be taken as 0.5 for immature concrete with rigid end restraints. Values of R for some common situations are given in *Table 3.45*. Otherwise, $R = 0.5$ may be used generally.
f_b	is average bond strength between immature concrete and reinforcement, where f_{ct}/f_b is taken as 1.0 for plain bars and 0.67 for deformed type 2 bars. For square-mesh fabric reinforcement, in which the cross-wires are not smaller than the main wires, f_{ct}/f_b may be taken as 0.8 for plain wires and 0.5 for deformed type 2 wires.
f_{ct}	is tensile strength of immature concrete, taken at age of 3 days, given by $f_{ct} = 0.12 f_{cu}^{0.7} \ge 1.6$ N/mm^2
s_{max}	is likely maximum spacing of cracks when $\rho \ge \rho_{crit}$
w_{max}	is estimated maximum crack width arising from restrained drying shrinkage and thermal contraction.
α	is coefficient of expansion of mature concrete (see *Table 3.5*).
ε	is effective strain which may usually be taken as ε_{te}
ε_{cs}	is estimated shrinkage strain. For concrete exposed to normal UK climatic conditions, the shrinkage strain less its associated creep strain is generally less than 100×10^{-6}, unless high shrinkage aggregates are used.
ε_{te}	is estimated total thermal contraction after peak temperature arising from hydration effects
ϕ	is size of each reinforcing bar
ρ	is ratio, area of reinforcement to gross area of concrete, provided in a surface zone (see below).
ρ_{crit}	is critical reinforcement ratio, given by $\rho_{crit} = f_{ct}/f_y$. For $f_{ct} = 1.6$ N/mm^2 and $f_y = 500$ N/mm^2, $\rho_{crit} = 0.0032$.

The reinforcement ratios needed to limit the value of w_{max} to values of either 0.2 mm or 0.1 mm (see *Table 3.4*) should be provided in surface zones of the following thickness, where h is the overall thickness of the section:

Walls and suspended slabs: surface zone thickness = $0.5h \le 250$ mm (each side)

Ground slabs: surface zone thickness = $0.5h \le 250$ mm (top), 100 mm (bottom, for slabs with $h \ge 300$ mm)

From the above relationships, the reinforcement ratio required to limit the calculated crack width to w_{lim} is given by:

$\rho = (f_{ct}/f_b)(\varepsilon\phi/2w_{lim}) \ge \rho_{crit} = f_{ct}/f_y$ where, in most cases, $\varepsilon = 0.5\varepsilon_{te}$ may be used.

Maximum values of $\varepsilon\,(\times 10^{-6})$ for given bar arrangements in specified surface zones are given in *Table 3.46*.

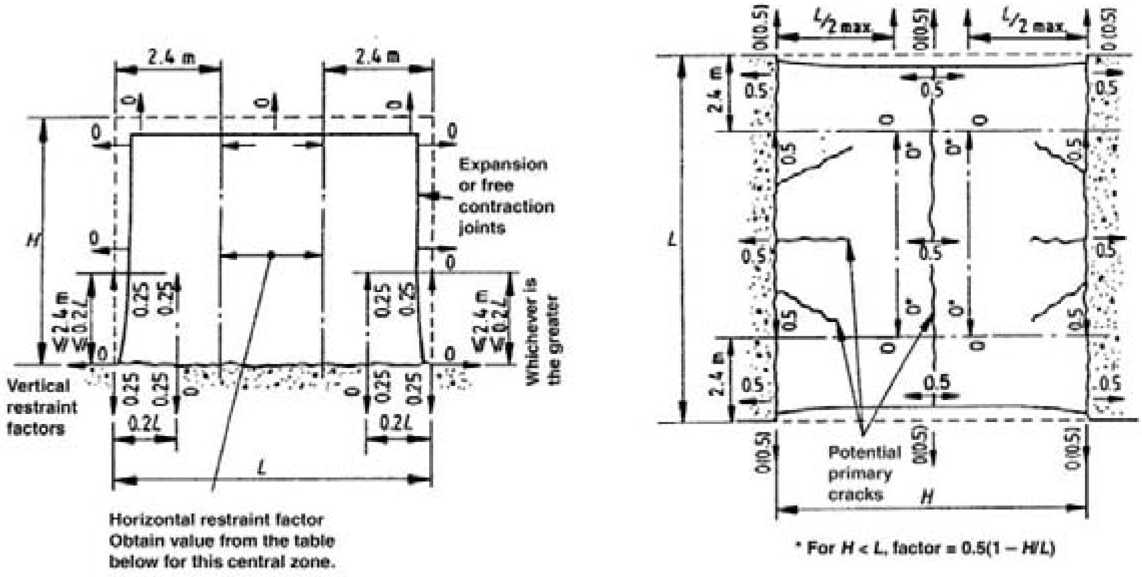
3.45

BS 8007 Design options and restraint factors

Design options

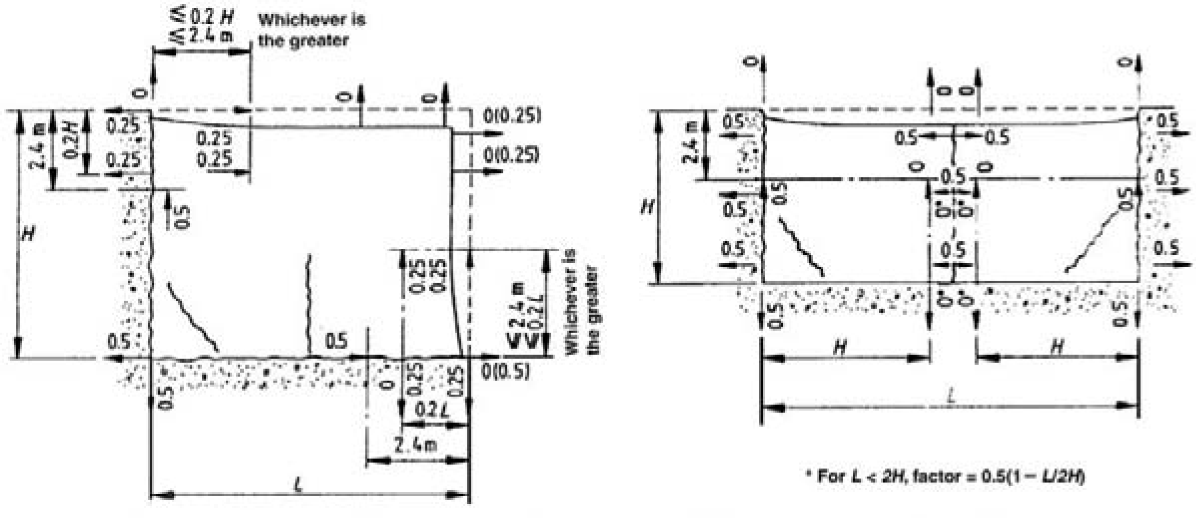
Option	Type of construction	Maximum spacing (m) of movement joints		Reinforcement ratio ρ
1	Continuous	No joints other than widely spaced expansion joints		$(f_{ct}/f_b)(\phi/w_{max})\varepsilon \geq \rho_{crit}$
2	Semi-continuous	(a) Complete joints: (b) Alternate partial and complete joints: (c) Partial joints:	15 m 11.25 m 7.5 m	
3	Free to move	(a) Complete joints: (b) Alternate partial and complete joints: (c) Partial joints:	$4.8 + w_{max}/\varepsilon$ $0.5s_{max} + 2.4 + w_{max}/\varepsilon$ $s_{max} + w_{max}/\varepsilon$	$2/3\rho_{crit}$

Values of restraint factor R for various concrete placing sequences



(a) Wall panel on base

(b) Horizontal slab between rigid restraints



(c) Continuous wall – sequential bay construction

(d) Continuous wall – alternate bay construction

Panel type (a)	Value of horizontal restraint factor R in central zone according to ratio L/H			
Ratio L/H	≤ 2	3	4	≥ 8
Top of panel	0	0.05	0.3	0.5
Base of panel	0.5	0.5	0.5	0.5

Note. H is height or width to a free edge, and L is distance between complete movement joints.

Note. For external restraint to seasonal temperature variation T_2 in ground slabs cast on blinding, $R = 0.5$ at mid-length reducing uniformly to zero at the ends may be taken, for 30 m lengths and over.

BS 8007 Design table for cracking due to temperature effects

3.46

Limiting crack width = 0.2 mm

Thickness of zone mm	Bar type and size	Maximum value of ε (x 10^{-6}) for bar spacing (mm)								
		300	250	225	200	175	150	125	100	75
100	H10			209*	235	269	314	377	471	628
	H12	188*	226*	251*	282	323	377	452	565	754
125	H12		180	201	226	258	301	362	452	603
	H16	201	241	268	301	344	402	482	603	804
150	H12			167	188	215	251	301	377	502
	H16	167	201	223	251	287	335	402	502	670
175	H12				161	184	215	258	323	430
	H16	143	172	191	215	246	287	344	430	574
200	H12					161	188	226	282	377
	H16		150	167	188	215	251	301	377	502
	H20	157	188	209	235	269	314	377	471	628
225	H12						167	201	251	335
	H16		134	149	167	191	223	268	335	446
	H20	139	167	186	209	239	279	335	418	558
250	H12							181	226	301
	H16			134	150	172	201	241	301	402
	H20	125	150	167	188	215	251	301	377	502

* These values apply only to the bottom surface zones of ground slabs and wall bases of overall thickness ≥ 300 mm.

Limiting crack width = 0.1 mm

Thickness of zone mm	Bar type and size	Maximum value of ε (x 10^{-6}) for bar spacing (mm)								
		300	250	225	200	175	150	125	100	75
100	H10			104	117	134	157	188	235	314
	H12	94	113	125	141	161	188	226	282	377
	H16	125	150	167	188	215	251	301	377	502
125	H12		90	100	113	129	150	181	226	301
	H16	100	120	134	150	172	201	241	301	402
	H20	125	150	167	188	215	251	301	377	502
150	H12			83	94	107	125	150	188	251
	H16	83	100	111	125	143	167	201	251	335
	H20	104	125	139	157	179	209	251	314	419
175	H12				80	92	107	129	161	215
	H16	71	86	95	107	123	143	172	215	287
	H20	89	107	119	134	153	179	215	269	359
200	H16		75	83	94	107	125	150	188	251
	H20	78	94	104	117	134	157	188	235	314
	H25	98	117	130	147	168	196	235	294	392
225	H16		67	74	83	95	111	134	167	223
	H20	69	83	93	104	119	139	167	209	279
	H25	87	104	116	130	149	174	209	261	349
250	H16			67	75	86	100	120	150	201
	H20	62	75	83	94	107	125	150	188	251
	H25	78	94	104	117	134	157	188	235	314

The values of ε in the above tables, where the bar spacing is taken ≤ 2 x surface zone thickness or 300 mm, whichever is the lesser, are derived from the relationship: $\varepsilon = 3(\rho/\phi)w_{\lim}$ where $\rho \geq 0.0032$ is the reinforcement ratio in the surface zone.

To determine the value of ε due to temperature and moisture effects to be considered for design purposes, see *Table 3.44*.

BS 8007 Elastic properties of cracked rectangular sections in flexure

3.47

Elastic properties of singly reinforced rectangular sections subjected to bending (or combined bending and tension)

$\frac{100A_s}{bd}$	$\frac{100M}{f_s bd^2}$	$\frac{x}{d}$	$\frac{100A_s}{bd}$	$\frac{100M}{f_s bd^2}$	$\frac{x}{d}$	$\frac{100A_s}{bd}$	$\frac{100M}{f_s bd^2}$	$\frac{x}{d}$
0.10	0.095	0.159	0.80	0.697	0.384	1.50	1.259	0.483
0.12	0.113	0.173	0.82	0.714	0.388	1.52	1.274	0.485
0.14	0.131	0.185	0.84	0.730	0.392	1.54	1.290	0.487
0.16	0.150	0.196	0.86	0.747	0.395	1.56	1.306	0.489
0.18	0.168	0.207	0.88	0.763	0.398	1.58	1.321	0.491
0.20	0.186	0.217	0.90	0.779	0.402	1.60	1.337	0.493
0.22	0.203	0.226	0.92	0.796	0.405	1.62	1.353	0.495
0.24	0.221	0.235	0.94	0.812	0.408	1.64	1.368	0.497
0.26	0.239	0.243	0.96	0.828	0.412	1.66	1.384	0.499
0.28	0.257	0.251	0.98	0.845	0.415	1.68	1.399	0.501
0.30	0.274	0.258	1.00	0.861	0.418	1.70	1.415	0.503
0.32	0.292	0.266	1.02	0.877	0.421	1.72	1.430	0.505
0.34	0.309	0.272	1.04	0.893	0.424	1.74	1.446	0.507
0.36	0.327	0.279	1.06	0.909	0.427	1.76	1.461	0.509
0.38	0.344	0.286	1.08	0.925	0.430	1.78	1.477	0.511
0.40	0.361	0.292	1.10	0.941	0.433	1.80	1.492	0.513
0.42	0.378	0.298	1.12	0.957	0.436	1.82	1.508	0.515
0.44	0.396	0.303	1.14	0.973	0.438	1.84	1.523	0.517
0.46	0.413	0.309	1.16	0.989	0.441	1.86	1.539	0.518
0.48	0.430	0.314	1.18	1.005	0.444	1.88	1.554	0.520
0.50	0.447	0.319	1.20	1.021	0.446	1.90	1.569	0.522
0.52	0.464	0.325	1.22	1.037	0.449	1.92	1.585	0.524
0.54	0.481	0.330	1.24	1.053	0.452	1.94	1.600	0.526
0.56	0.498	0.334	1.26	1.069	0.454	1.96	1.616	0.527
0.58	0.514	0.339	1.28	1.085	0.457	1.98	1.631	0.529
0.60	0.531	0.344	1.30	1.101	0.459	2.00	1.646	0.531
0.62	0.548	0.348	1.32	1.117	0.462			
0.64	0.565	0.353	1.34	1.133	0.464			
0.66	0.581	0.357	1.36	1.149	0.467			
0.68	0.598	0.361	1.38	1.164	0.469			
0.70	0.615	0.365	1.40	1.180	0.471			
0.72	0.631	0.369	1.42	1.196	0.474			
0.74	0.648	0.373	1.44	1.212	0.476			
0.76	0.665	0.377	1.46	1.227	0.478			
0.78	0.681	0.381	1.48	1.243	0.480			

The values in the table above are derived from the following equations:

$$\frac{x}{d} = \sqrt{\left(\frac{\alpha_e A_s}{bd}\right)^2 + \frac{2\alpha_e A_s}{bd}} - \frac{\alpha_e A_s}{bd} \qquad \frac{100M}{f_s bd^2} = \frac{100A_s}{bd}(1 - \frac{x}{3d}) \qquad \alpha_e = \frac{2E_s}{E_c} = 15$$

For a section subjected to combined bending and tension, where $M/N > (d - 0.5h)$, the value of M in these equations should be replaced by $M_1 = M - N(d - 0.5h)$. The total area of reinforcement required is then given by $A_s + N/f_s$. The analysis of such a section, containing a given area of reinforcement, involves an iterative process in order to determine the values of x and f_s (see example 5).

A_s is area of tension reinforcement to resist M or M_1
M is bending moment due to design service loading
N is direct tension due to design service loading
E_c is modulus of elasticity of concrete
E_s is modulus of elasticity of reinforcement

b is breadth of section
d is effective depth of tension reinforcement
f_s is stress in tension reinforcement
h is overall depth of section
x is neutral axis depth

For a section where $M/N < (d - 0.5h)$, tension reinforcement is required on both faces, given by the equations:

$$A_{s1} = \frac{0.5N}{f_s}\left(1 + \frac{M/N}{d - 0.5h}\right) \text{ on face in tension due to } M \qquad A_{s2} = \frac{0.5N}{f_s}\left(1 - \frac{M/N}{d - 0.5h}\right) \text{ on other face}$$

For a comprehensive treatment of crack width calculations for combined bending and direct tension, see ref. 41.

BS 8007 Design table for cracking due to flexure in slabs – 1

	Thickness of slab mm	Bar type and size	Maximum service moment and stress in tension bars	Bar spacing (mm)							
				300	250	225	200	175	150	125	100
w_{lim} = 0.2 mm (cover to bars = 40 mm)	200	H10	M kNm/m				17.0	18.5	20.5	23.5	28.1
			f_s N/mm^2				*304*	*290*	*278*	*268*	*258*
		H12	M kNm/m				20.3	22.4	25.2	29.3	35.5
			f_s N/mm^2				*257*	*249*	*243*	*237*	*233*
		H16	M kNm/m				28.3	31.7	36.4	42.9	52.6
			f_s N/mm^2				*210*	*208*	*206*	*205*	*204*
	250	H12	M kNm/m		25.7	27.3	29.5	32.5	36.6	42.7	52.0
			f_s N/mm^2		*301*	*290*	*279*	*270*	*263*	*257*	*253*
		H16	M kNm/m		33.7	36.6	40.4	45.5	52.4	62.4	77.5
			f_s N/mm^2		*230*	*226*	*223*	*221*	*220*	*220*	*222*
		H20	M kNm/m		43.9	48.3	53.9	61.3	71.4	85.7	107.1
			f_s N/mm^2		*198*	*197*	*197*	*197*	*199*	*201*	*204*
	300	H12	M kNm/m			37.2	40.0	43.9	49.3	57.2	69.7
			f_s N/mm^2			*314*	*302*	*290*	*281*	*274*	*270*
		H16	M kNm/m	39.6	44.8	48.6	53.5	60.1	69.4	82.9	103.7
			f_s N/mm^2	*255*	*243*	*238*	*234*	*232*	*231*	*232*	*235*
		H20	M kNm/m	49.3	57.4	63.1	70.6	80.6	94.3	114.1	144.1
			f_s N/mm^2	*209*	*205*	*204*	*204*	*205*	*207*	*211*	*216*
	350	H16	M kNm/m	50.8	57.2	61.7	67.8	76.0	87.6	104.6	131.1
			f_s N/mm^2	*272*	*256*	*250*	*245*	*242*	*241*	*242*	*245*
		H20	M kNm/m	62.0	71.9	79.0	88.2	100.7	118.1	143.5	182.5
			f_s N/mm^2	*218*	*212*	*210*	*210*	*211*	*214*	*219*	*226*
		H25	M kNm/m	79.8	94.9	105.5	119.5	138.1	163.8	200.8	256.5
			f_s N/mm^2	*184*	*184*	*186*	*188*	*192*	*197*	*203*	*211*
	400	H16	M kNm/m	63.4	70.8	76.1	83.3	93.1	106.9	127.5	159.8
			f_s N/mm^2	*289*	*271*	*263*	*257*	*253*	*251*	*251*	*254*
		H20	M kNm/m	76.0	87.6	95.9	107.0	122.0	143.1	174.1	222.0
			f_s N/mm^2	*227*	*220*	*217*	*217*	*218*	*220*	*226*	*233*
		H25	M kNm/m	96.1	114.0	126.8	143.6	166.2	197.7	243.5	313.3
			f_s N/mm^2	*189*	*188*	*189*	*192*	*196*	*201*	*209*	*218*
	450	H16	M kNm/m	77.3	85.7	91.9	100.1	111.4	127.6	151.7	189.8
			f_s N/mm^2	*307*	*286*	*277*	*269*	*264*	*260*	*260*	*263*
		H20	M kNm/m	91.2	104.4	114.1	126.9	144.4	169.2	205.8	262.9
			f_s N/mm^2	*237*	*228*	*225*	*224*	*224*	*227*	*232*	*240*
		H25	M kNm/m	113.5	134.2	149.0	168.7	195.3	232.7	287.3	371.3
			f_s N/mm^2	*194*	*192*	*193*	*196*	*200*	*206*	*214*	*224*
	500	H16	M kNm/m		102.1	109.0	118.3	131.2	149.7	177.3	221.2
			f_s N/mm^2		*302*	*291*	*282*	*275*	*270*	*269*	*271*
		H20	M kNm/m	107.7	122.6	133.5	148.1	168.1	196.5	238.8	305.0
			f_s N/mm^2	*248*	*237*	*233*	*231*	*230*	*233*	*238*	*246*
		H25	M kNm/m	132.1	155.5	172.5	194.9	225.5	268.7	332.2	430.6
			f_s N/mm^2	*199*	*197*	*198*	*200*	*203*	*209*	*218*	*229*
	600	H20	M kNm/m	144.9	163.1	176.5	194.5	219.4	255.2	308.7	393.3
			f_s N/mm^2	*271*	*256*	*250*	*246*	*244*	*245*	*249*	*256*
		H25	M kNm/m	173.2	202.0	223.0	251.0	289.6	344.4	425.7	552.9
			f_s N/mm^2	*212*	*207*	*207*	*208*	*211*	*217*	*226*	*238*
		H32	M kNm/m	225.2	272.1	306.1	351.2	412.7	499.3	626.3	821.9
			f_s N/mm^2	*173*	*176*	*179*	*183*	*190*	*199*	*210*	*224*

BS 8007 Design table for cracking due to flexure in slabs – 2

3.49

	Thickness of slab mm	Bar type and size	Maximum service moment and stress in tension bars		Bar spacing (mm)							
					300	250	225	200	175	150	125	100
$w_{\lim} = 0.2$ mm (cover to bars = 52 mm)	200	H10	M	kNm/m				14.1	15.1	16.5	18.4	21.2
			f_s	N/mm^2				*275*	*258*	*243*	*227*	*212*
		H12	M	kNm/m				16.4	17.8	19.7	22.2	26.0
			f_s	N/mm^2				*227*	*216*	*206*	*196*	*186*
		H16	M	kNm/m				22.0	24.2	27.1	31.1	36.8
			f_s	N/mm^2				*178*	*173*	*167*	*162*	*155*
	250	H12	M	kNm/m		22.5	23.7	25.3	27.4	30.2	34.3	40.4
			f_s	N/mm^2		*280*	*267*	*255*	*243*	*231*	*221*	*210*
		H16	M	kNm/m		28.6	30.7	33.4	36.9	41.7	48.3	58.0
			f_s	N/mm^2		*208*	*202*	*196*	*191*	*186*	*182*	*177*
		H20	M	kNm/m		36.2	39.3	43.3	48.4	55.1	64.4	77.7
			f_s	N/mm^2		*174*	*171*	*169*	*166*	*164*	*161*	*158*
	300	H12	M	kNm/m			33.3	35.4	38.2	42.1	47.8	56.3
			f_s	N/mm^2			*295*	*280*	*266*	253	*241*	*230*
		H16	M	kNm/m	35.3	39.4	42.2	45.9	50.8	57.4	66.8	80.9
			f_s	N/mm^2	*240*	*224*	*217*	*211*	*206*	*201*	*197*	*193*
		H20	M	kNm/m	43.1	49.3	53.6	59.1	66.3	76.0	89.6	109.5
			f_s	N/mm^2	*192*	*185*	*182*	*179*	*177*	*176*	*175*	*173*
	350	H16	M	kNm/m	46.3	51.4	55.0	59.6	65.9	74.4	86.7	105.2
			f_s	N/mm^2	*258*	*240*	*232*	*225*	*219*	*214*	*209*	*205*
		H20	M	kNm/m	55.6	63.4	68.9	76.0	85.3	98.1	116.2	143.0
			f_s	N/mm^2	*203*	*195*	*191*	*189*	*187*	*186*	*185*	*185*
		H25	M	kNm/m	70.1	81.9	90.1	100.6	114.3	132.8	158.7	196.3
			f_s	N/mm^2	*169*	*166*	*165*	*165*	*166*	*167*	*168*	*169*
	400	H16	M	kNm/m	58.7	64.7	69.0	74.6	82.2	92.7	107.9	130.9
			f_s	N/mm^2	*278*	*257*	*247*	*239*	*231*	*225*	*220*	*216*
		H20	M	kNm/m	69.3	78.7	85.3	94.0	105.5	121.4	144.0	177.8
			f_s	N/mm^2	*214*	*205*	*201*	*197*	*195*	*194*	*194*	*194*
		H25	M	kNm/m	86.1	100.5	110.5	123.5	140.7	164.1	197.0	245.5
			f_s	N/mm^2	*175*	*172*	*171*	*171*	*172*	*173*	*175*	*177*
	450	H16	M	kNm/m	72.4	79.4	84.4	91.0	99.9	112.4	130.5	158.1
			f_s	N/mm^2	*297*	*273*	*262*	*252*	*244*	*237*	*231*	*226*
		H20	M	kNm/m	84.2	95.2	103.0	113.2	126.9	145.9	173.1	214.1
			f_s	N/mm^2	*226*	*214*	*210*	*206*	*203*	*202*	*201*	*202*
		H25	M	kNm/m	103.2	120.1	132.1	147.6	168.2	196.5	236.6	296.2
			f_s	N/mm^2	*182*	*178*	*177*	*177*	*178*	*179*	*182*	*185*
	500	H16	M	kNm/m		95.4	101.2	108.7	119.0	133.4	154.4	186.6
			f_s	N/mm^2		*290*	*277*	*266*	*256*	*248*	*241*	*235*
		H20	M	kNm/m	100.5	113.0	121.9	133.7	149.6	171.6	203.5	251.8
			f_s	N/mm^2	*238*	*224*	*219*	*214*	*211*	*209*	*208*	*209*
		H25	M	kNm/m	121.5	141.0	154.8	172.8	196.9	230.1	277.4	348.3
			f_s	N/mm^2	*188*	*184*	*182*	*182*	*183*	*185*	*187*	*191*
	600	H20	M	kNm/m	137.2	152.7	163.9	178.8	199.0	227.2	268.3	331.2
			f_s	N/mm^2	*262*	*245*	*237*	*231*	*226*	*223*	*221*	*221*
		H25	M	kNm/m	162.0	186.4	203.9	226.9	257.9	301.0	363.0	456.7
			f_s	N/mm^2	*203*	*196*	*194*	*192*	*193*	*194*	*197*	*201*
		H32	M	kNm/m	207.5	246.9	275.0	311.6	360.5	427.6	523.1	665.3
			f_s	N/mm^2	*163*	*163*	*164*	*167*	*170*	*174*	*180*	*185*

BS 8007 Design table for cracking due to flexure in slabs – 3

	Thickness of slab mm	Bar type and size	Maximum service moment and stress in tension bars	Bar spacing (mm)							
				300	250	225	200	175	150	125	100
w_{lim} = 0.2 mm (cover to bars = 56 mm)	200	H10	M kNm/m				13.3	14.2	15.3	16.9	19.3
			f_s N/mm^2				*266*	*249*	*232*	*216*	*199*
		H12	M kNm/m				15.3	16.5	18.1	20.3	23.5
			f_s N/mm^2				*218*	*207*	*196*	*184*	*173*
		H16	M kNm/m				20.2	22.1	24.6	27.9	32.7
			f_s N/mm^2				*169*	*163*	*157*	*150*	*142*
	250	H12	M kNm/m		21.5	22.6	24.0	25.9	28.4	32.0	37.3
			f_s N/mm^2		*274*	*260*	*247*	*234*	*222*	*210*	*198*
		H16	M kNm/m		27.1	28.9	31.3	34.5	38.6	44.4	52.8
			f_s N/mm^2		*201*	*195*	*188*	*182*	*177*	*171*	*165*
		H20	M kNm/m		34.0	36.7	40.2	44.7	50.6	58.6	70.1
			f_s N/mm^2		*167*	*164*	*160*	*157*	*154*	*150*	*146*
	300	H12	M kNm/m			32.1	34.0	36.5	40.1	45.1	52.6
			f_s N/mm^2			*289*	*274*	*259*	245	*231*	*218*
		H16	M kNm/m	34.0	37.7	40.3	43.6	48.0	54.0	62.3	74.7
			f_s N/mm^2	*235*	*219*	*211*	*204*	*198*	*192*	*187*	*181*
		H20	M kNm/m	41.2	46.8	50.7	55.7	62.2	70.8	82.8	100.2
			f_s N/mm^2	*187*	*179*	*175*	*172*	*169*	*167*	*164*	*161*
	350	H16	M kNm/m	44.9	49.6	52.9	57.2	62.8	70.6	81.6	98.1
			f_s N/mm^2	*254*	*235*	*227*	*219*	*212*	*205*	*200*	*194*
		H20	M kNm/m	53.6	60.8	65.8	72.3	80.8	92.3	108.4	132.2
			f_s N/mm^2	*199*	*190*	*186*	*182*	*179*	*177*	*175*	*173*
		H25	M kNm/m	67.1	77.9	85.4	95.0	107.4	124.0	147.0	180.1
			f_s N/mm^2	*164*	*161*	*159*	*158*	*158*	*158*	*158*	*157*
	400	H16	M kNm/m	57.2	62.8	66.8	72.0	78.9	88.5	102.3	122.9
			f_s N/mm^2	*274*	*252*	*242*	*233*	*225*	*218*	*211*	*205*
		H20	M kNm/m	67.2	75.9	82.0	90.0	100.6	115	135.4	165.6
			f_s N/mm^2	*211*	*200*	*195*	*191*	*188*	*186*	*184*	*183*
		H25	M kNm/m	83.0	96.3	105.6	117.5	133.2	154.3	183.9	227.1
			f_s N/mm^2	*171*	*167*	*166*	*165*	*165*	*165*	*166*	*166*
	450	H16	M kNm/m	70.9	77.4	82.0	88.2	96.4	107.8	124.3	149.2
			f_s N/mm^2	*294*	*269*	*258*	*247*	*238*	*229*	*222*	*216*
		H20	M kNm/m	82.0	92.3	99.5	109.0	121.6	138.9	163.6	200.6
			f_s N/mm^2	*222*	*210*	*205*	*200*	*197*	*194*	*192*	*191*
		H25	M kNm/m	100.0	115.8	126.9	141.2	160.1	185.9	222.1	275.6
			f_s N/mm^2	*178*	*173*	*172*	*171*	*171*	*171*	*173*	*174*
	500	H16	M kNm/m		93.3	98.7	105.7	115.2	128.5	147.7	176.8
			f_s N/mm^2		*286*	*273*	*261*	*250*	*241*	*232*	*225*
		H20	M kNm/m	98.2	109.9	118.3	129.3	143.9	164.2	193.2	236.9
			f_s N/mm^2	*235*	*220*	*214*	*209*	*205*	*202*	*200*	*198*
		H25	M kNm/m	118.2	136.4	149.3	166.0	188.2	218.6	261.7	325.6
			f_s N/mm^2	*185*	*179*	*178*	*177*	*176*	*177*	*178*	*180*
	600	H20	M kNm/m	134.7	149.4	159.9	173.8	192.6	218.8	256.5	313.7
			f_s N/mm^2	*260*	*241*	*233*	*227*	*221*	*217*	*213*	*211*
		H25	M kNm/m	158.5	181.5	197.9	219.4	248.2	288.0	344.7	429.8
			f_s N/mm^2	*200*	*192*	*189*	*188*	*187*	*187*	*189*	*191*
		H32	M kNm/m	201.8	239.0	265.3	299.4	344.7	406.5	493.6	622.1
			f_s N/mm^2	*160*	*159*	*160*	*161*	*164*	*167*	*171*	*175*

BS 8007 Design table for cracking due to direct tension in walls – 1

3.51

	Thickness of wall mm	Bar type and size	Maximum service tension force and stress in bars		Bar spacing (EF) mm							
					300	250	225	200	175	150	125	100
$w_{\lim}$ = 0.2 mm (cover to bars = 40 mm)	200	H12	N	kN/m				278	319	378	470	620
			f_s	N/mm^2				*246*	*247*	*251*	*260*	*274*
		H16	N	kN/m				394	467	574	739	1008
			f_s	N/mm^2				*196*	*203*	*214*	*230*	*251*
		H20	N	kN/m				545	661	830	1090	1513
			f_s	N/mm^2				*174*	*184*	*198*	*217*	*241*
	250	H16	N	kN/m		337	375	427	500	608	772	1041
			f_s	N/mm^2		*210*	*210*	*213*	*218*	*227*	*240*	*259*
		H20	N	kN/m		435	496	578	894	863	1124	1547
			f_s	N/mm^2		*173*	*178*	*184*	*193*	*206*	*224*	*246*
		H25	N	kN/m		592	688	818	1002	1271	1682	2348
			f_s	N/mm^2		*151*	*158*	*167*	*179*	*194*	*214*	*239*
	300	H16	N	kN/m	319	370	408	460	534	641	806	1075
			f_s	N/mm^2	*238*	*230*	*229*	*229*	*232*	*239*	*251*	*267*
		H20	N	kN/m	388	469	529	611	727	897	1157	1580
			f_s	N/mm^2	186	*187*	*190*	*195*	*203*	*214*	*230*	*252*
		H25	N	kN/m	498	625	721	851	1035	1304	1715	2382
			f_s	N/mm^2	*152*	*159*	*165*	*174*	*185*	*199*	*218*	*243*
	350	H16	N	kN/m		404	442	494	567	674	839	1108
			f_s	N/mm^2		*251*	*247*	*246*	*247*	*252*	*261*	*276*
		H20	N	kN/m	422	502	562	645	761	930	1190	1613
			f_s	N/mm^2	*202*	*200*	*202*	*205*	*212*	*222*	*237*	*257*
		H25	N	kN/m	531	659	754	885	1069	1337	1748	2415
			f_s	N/mm^2	*162*	*168*	*173*	*180*	*191*	*204*	*223*	*246*
$w_{\lim}$ = 0.1 mm (cover to bars = 40 mm)	200	H12	N	kN/m				272	293	322	368	443
			f_s	N/mm^2				*241*	*227*	*214*	*204*	*196*
		H16	N	kN/m				330	367	420	503	637
			f_s	N/mm^2				*164*	*160*	*157*	*156*	*159*
		H20	N	kN/m				406	464	548	678	890
			f_s	N/mm^2				*129*	*129*	*131*	*135*	*142*
	250	H16	N	kN/m		335	354	380	417	470	553	687
			f_s	N/mm^2		*208*	*198*	*189*	*182*	*176*	*172*	*171*
		H20	N	kN/m		384	414	456	514	598	728	940
			f_s	N/mm^2		*153*	*149*	*145*	*143*	*143*	*145*	*150*
		H25	N	kN/m		463	510	576	668	802	1008	1341
			f_s	N/mm^2		*118*	*117*	*117*	*119*	*123*	*128*	*137*
	300	H16	N	kN/m	**360**	**385**	404	430	467	520	603	737
			f_s	N/mm^2	***268***	***240***	*226*	*214*	*203*	*194*	*188*	*183*
		H20	N	kN/m	**394**	434	464	506	564	648	778	990
			f_s	N/mm^2	**188**	*173*	*166*	*161*	*157*	*155*	*155*	*158*
		H25	N	kN/m	449	513	560	626	718	852	1058	1391
			f_s	N/mm^2	*137*	*131*	*129*	*128*	*128*	*130*	*135*	*142*
	350	H16	N	kN/m		**435**	**454**	480	517	570	653	787
			f_s	N/mm^2		***271***	***254***	*239*	*225*	*213*	*203*	*196*
		H20	N	kN/m	**444**	484	514	556	614	698	828	1040
			f_s	N/mm^2	**212**	*193*	*184*	*177*	*171*	*167*	*165*	*166*
		H25	N	kN/m	499	563	610	676	768	902	1108	1441
			f_s	N/mm^2	*153*	*143*	*140*	*138*	*137*	*138*	*141*	*147*

Note. Values shown in bold type for $w_{\lim}$ = 0.1 mm exceed those obtained for $w_{\lim}$ = 0.2 mm as a result of formulae used for ε_m

3.52

BS 8007 Design table for cracking due to direct tension in walls – 2

	Thickness of wall mm	Bar type and size	Maximum service tension force and stress in bars		Bar spacing (EF) mm							
					300	250	225	200	175	150	125	100
$w_{\lim} = 0.2$ mm (cover to bars = 52 mm)	200	H12	N	kN/m				271	307	360	438	561
			f_s	N/mm^2				*240*	*238*	*239*	*242*	*248*
		H16	N	kN/m				380	446	539	679	898
			f_s	N/mm^2				*189*	*194*	*201*	*211*	*224*
		H20	N	kN/m				522	626	773	992	1336
			f_s	N/mm^2				*166*	*174*	*185*	197	*213*
	250	H16	N	kN/m		331	366	413	479	573	712	931
			f_s	N/mm^2		*206*	*205*	*206*	*209*	*214*	*222*	*232*
		H20	N	kN/m		425	481	556	659	806	1026	1369
			f_s	N/mm^2		*169*	*172*	*177*	*184*	*193*	*204*	*218*
		H25	N	kN/m		575	663	781	944	1176	1521	2061
			f_s	N/mm^2		*147*	*152*	*159*	*168*	*180*	*194*	*210*
	300	H16	N	kN/m	316	364	399	447	512	606	745	965
			f_s	N/mm^2	*236*	*226*	*224*	*222*	*223*	*226*	*232*	*240*
		H20	N	kN/m	383	459	514	589	692	840	1059	1403
			f_s	N/mm^2	*183*	*183*	*184*	*188*	*193*	*201*	*211*	*223*
		H25	N	kN/m	489	608	696	815	977	1210	1555	2094
			f_s	N/mm^2	*150*	*155*	*160*	*166*	*174*	*185*	*198*	*213*
	350	H16	N	kN/m		397	433	480	546	639	779	998
			f_s	N/mm^2		*247*	*242*	*239*	*238*	*239*	*242*	*248*
		H20	N	kN/m	417	492	548	622	726	873	1092	1436
			f_s	N/mm^2	*199*	*196*	*196*	*198*	*202*	*208*	*217*	*229*
		H25	N	kN/m	523	642	730	848	1011	1243	1588	2127
			f_s	N/mm^2	*160*	*163*	*167*	*173*	*180*	*190*	*202*	*217*
$w_{\lim} = 0.1$ mm (cover to bars = 52 mm)	200	H12	N	kN/m				269	287	313	352	414
			f_s	N/mm^2				*238*	*222*	*208*	*195*	*183*
		H16	N	kN/m				323	356	403	473	582
			f_s	N/mm^2				*161*	*155*	*150*	*147*	*145*
		H20	N	kN/m				394	446	520	630	801
			f_s	N/mm^2				*126*	*124*	*124*	*125*	*128*
	250	H16	N	kN/m		332	350	373	406	453	523	632
			f_s	N/mm^2		*207*	*196*	*186*	*177*	*169*	*163*	*157*
		H20	N	kN/m		379	407	444	496	570	679	851
			f_s	N/mm^2		*151*	*146*	*142*	*138*	*136*	*135*	*136*
		H25	N	kN/m		454	498	557	639	755	927	1197
			f_s	N/mm^2		*116*	*114*	*114*	*114*	*115*	*118*	*122*
	300	H16	N	kN/m	**357**	**382**	**400**	423	456	503	573	682
			f_s	N/mm^2	***267***	***238***	***224***	*211*	*199*	*188*	*178*	*170*
		H20	N	kN/m	**392**	429	457	494	546	620	729	901
			f_s	N/mm^2	***187***	*171*	*164*	*158*	*152*	*148*	*145*	*144*
		H25	N	kN/m	445	504	548	607	689	805	977	1247
			f_s	N/mm^2	*136*	*128*	*126*	*124*	*123*	*123*	*125*	*127*
	350	H16	N	kN/m		**432**	**450**	473	506	553	623	732
			f_s	N/mm^2		***269***	***252***	*236*	*220*	*206*	*194*	*182*
		H20	N	kN/m	**442**	479	507	544	596	670	779	951
			f_s	N/mm^2	***211***	*191*	*182*	*173*	*166*	*160*	*155*	*152*
		H25	N	kN/m	495	554	598	657	739	855	1027	1297
			f_s	N/mm^2	*151*	*141*	*137*	*134*	*132*	*131*	*131*	*132*

Note. Values shown in bold type for $w_{\lim} = 0.1$ mm exceed those obtained for $w_{\lim} = 0.2$ mm as a result of formulae used for ε_m

In this case, increasing α_s to 1.31 reduces f_s to 295 N/mm^2, so that the area of tension reinforcement should be multiplied by the factor 312/295 = 1.06.

Example 3. A simply supported 700 mm thick solid slab bridge deck is to be designed for bending and checked for cracking and fatigue to the requirements of BS 5400.

$$f_{cu} = 40 \text{ N/mm}^2, f_y = 500 \text{ N/mm}^2, d = 620 \text{ mm}$$

The maximum longitudinal ultimate moment at mid-span for load combination 1 (37.5 units HB live load) is 950 kNm/m

$$M/bd^2 f_{cu} = 950 \times 10^6/(1000 \times 620^2 \times 40) = 0.062$$
$$A_s f_y/bdf_{cu} = 0.078 \text{ (\textit{Table 3.24})}$$
$$A_s = 0.078 \times 1000 \times 620 \times 40/500 = 3869 \text{ mm}^2\text{/m}$$

Although H25-125 gives 3927 mm^2/m, this is unlikely to be sufficient with regard to cracking and fatigue, which will be checked on the basis of H25-100 (4909 mm^2/m).

The maximum longitudinal service moment at mid-span for load combination 1 (HA live load) is $M_s = 600$ kNm/m, with

$$M_g = 325 \text{ kNm/m}, M_q = 275 \text{ kNm/m}, M_q/M_g = 0.85$$

In the following calculations, the modulus of elasticity of the concrete is taken as a value reflecting the relative proportions of live load and permanent load. Taking E_c for live load and $E_c/2$ for permanent load, the effective value is given by:

$$E_{eff} = [1 - 0.5/(1 + M_q/M_g)]E_c$$

From section 22.1.1, $E_c = 19 + 0.3f_{cu} = 31$ kN/mm^2

$$E_{eff} = [1 - 0.5/(1 + 0.85)] \times 31 = 22.6 \text{ kN/mm}^2$$

From *Table 3.42*, the neutral axis depth x (or d_c) is given by:

$$x/d = \sqrt{\beta^2 + 2\beta} - \beta \text{ where } \beta = (E_s/E_{eff})(A_s/bd)$$
$$\beta = (200/22.6) \times 4909/(1000 \times 620) = 0.07$$
$$x/d = \sqrt{0.07^2 + 2 \times 0.07} - 0.07 = 0.31, x = 192 \text{ mm}$$

Stress in tension reinforcement is given by:

$$f_s = M_s/A_s(d - x/3)$$
$$= 600 \times 10^6/[4909 \times (620 - 192/3)] = 220 \text{ N/mm}^2$$

Strain in tension reinforcement

$$\varepsilon_s = f_s/E_s = 220/(200 \times 10^3) = 0.0011$$

Strain at surface taken at distance c_{min} from outermost bars, ignoring stiffening effect of concrete,

$$\varepsilon_1 = \varepsilon_s (h - x - 10)/(d - x)$$
$$= 0.0011 \times (700 - 192 - 10)/(620 - 192) = 0.00128$$

Stiffening effect of concrete (with $a' = h$)

$$\varepsilon_2 = \left(\frac{3.8b_t h}{\varepsilon_s A_s}\right)\left(1 - \frac{M_q}{M_g}\right) \times 10^{-9}$$
$$= 3.8 \times 1000 \times 700 \times 0.15 \times 10^{-9}/(0.0011 \times 4909)$$
$$= 0.00007$$

Strain at surface, allowing for stiffening effect of concrete,

$$\varepsilon_m = \varepsilon_1 - \varepsilon_2 = 0.00128 - 0.00007 = 0.00121$$

Distance from surface of bar, where s_b is bar spacing, to point midway between bars, on surface taken at distance c_{min} from outermost bars, is given by:

$$a_{cr} = \sqrt{(s_b/2)^2 + (h - d - 10)^2} - \phi/2$$
$$= \sqrt{(50)^2 + (70)^2} - 12.5 = 73.5 \text{ mm}$$

The maximum design crack width is given by:

$$w_{cr} = \frac{3a_{cr}\varepsilon_m}{1 + 2(a_{cr} - c_{min})/(h - d_c)}$$
$$= 3 \times 73.5 \times 0.00121/[1 + 2 \times (73.5 - 45)/(700 - 192)]$$
$$= 0.24 \text{ mm}$$

From *Table 3.2*, the limiting design crack width for a bridge deck soffit (severe exposure) is 0.25 mm.

The service stress range in the reinforcement resulting from the live load moment M_q is given by:

$$(M_q/M_s)f_s = (275/600) \times 220 = 101 \text{ N/mm}^2$$

From *Table 3.2*, the limiting value, for spans in the range 5–200 m and bar sizes >16 mm, is 120 N/mm^2.

Example 4. The wall of a cylindrical tank, 7.5 m deep and 15 m diameter, is 300 mm thick. The wall, which is continuous with the base slab, is to be designed for temperature effects, and those due to internal hydrostatic pressure when the tank is full of liquid.

Design crack width 0.2 mm $f_{cu} = 40$ N/mm^2
Cover to horizontal bars 52 mm $f_y = 500$ N/mm^2

Effects of temperature change. Allowing for concrete grade C32/40, with 350 kg/m^3 Portland cement, at a placing temperature of 20°C and a mean ambient temperature during construction of 15°C, the temperature rise for concrete placed within 18 mm plywood formwork:

$$T_1 = 25^\circ\text{C (\textit{Table 2.18})}$$

As the wall is to be designed to resist hoop tension, there will be no vertical movement joints and allowance must be made for a fall in temperature due to seasonal variations. Allowing for $T_2 = 15$°C, restraint factor R taken as 0.5 and coefficient of thermal expansion α taken as 12×10^{-6} per °C (*Table 3.5*), restrained total thermal contraction after the peak temperature arising from hydration effects is given by:

$$R\alpha\,(T_1 + T_2) = 0.5 \times 12 \times 10^{-6} \times (25 + 15) = 240 \times 10^{-6}$$

From *Table 3.46*, for 0.2 mm crack width and a surface zone thickness of 300/2 = 150 mm, H16-200 (EF) will suffice.

Effects of hydrostatic load. Suppose that an elastic analysis of the tank, assuming a floor 300 mm thick, indicates a service maximum circumferential tension of 400 kN/m. This value occurs at a depth of 6 m, and above this level the hoop tensions can be assumed to reduce approximately linearly to near zero at the top of the wall.

In BS 8110, for direct tension and $f_y = 500$ N/mm^2, minimum reinforcement equal to 0.45% of the concrete cross section is required. Hence, for a wall 300 mm thick, the minimum area of reinforcement required on each face:

$$A_{s\,min} = 0.0045 \times 1000 \times 150 = 675 \text{ mm}^2\text{/m (H16-300)}$$

From *Table 3.51*, for a 0.2 mm crack width and 40 mm cover, the following values are obtained for a 300 mm thick wall:

H16-225 (EF) provides for $N = 408$ kN/m
H16-300 (EF) provides for $N = 319$ kN/m

In order to cater for the effects of both temperature change and hydrostatic load, H16-225 (EF) can be used for a height of

3.0 m say, and minimum reinforcement of H16-300 (EF) for the remaining height of 4.5 m.

Example 5. A 200 mm thick roof slab to a reservoir is to be designed for serviceability cracking to the requirements of BS 8007.

Design crack width 0.2 mm Cover to bars 40 mm

(a) Sliding layer is provided between slab and perimeter wall. Maximum service moment: $M = 25$ kNm/m.

From *Table 3.48*, H12-150 caters for $M = 25.2$ kNm/m with $f_s = 243$ N/mm^2.

(b) Slab is tied to perimeter wall. Maximum service moment and direct tension: $M = 25$ kNm/m and $N = 40$ kN/m.

$$h = 200 \text{ mm}, d = 200 - (40 + 12/2) = 154 \text{ mm}$$

Since $M/N = (25/40) \times 10^3 = 625 \text{ mm} > (d - 0.5h) = 54$ mm, one face will remain in compression, and the section can be designed for a reduced moment $M_1 = M - N(d - 0.5h)$ with the tensile force acting at the level of the reinforcement.

$$M_1 = 25 - 40 \times 0.054 = 22.8 \text{ kNm/m}$$

From solution (a) above, take $f_s = 240$ N/mm^2 as a trial value.

$$100M_1/f_s bd^2 = 100 \times 22.8 \times 10^6/(240 \times 1000 \times 154^2) = 0.400$$

From *Table 3.47*, by interpolation: $100A_{s1}/bd = 0.445$

$$A_{s1} = 0.00445 \times 1000 \times 154 = 685 \text{ mm}^2\text{/m}$$

Total area of reinforcement required is given by:

$$A_s = A_{s1} + N/f_s = 685 + 40 \times 10^3/240 = 852 \text{ mm}^2\text{/m}$$

Using H12-125 gives 905 mm^2/m. This may not necessarily be sufficient, because the stiffening effect of the concrete inherent in solution (a) is reduced by the additional tension. A crack width calculation is needed to confirm the solution.

The stress in the reinforcement is given approximately by:

$$f_s = 240 \times 852/905 = 226 \text{ N/mm}^2$$
$$100M_1/f_s bd^2 = 0.400 \times 240/226 = 0.425$$

From *Table 3.47*, by interpolation: $100A_{s1}/bd = 0.474$

$$A_{s1} = 0.00474 \times 1000 \times 154 = 730 \text{ mm}^2\text{/m}$$
$$A_s = A_{s1} + N/f_s = 730 + 40 \times 10^3/226 = 907 \text{ mm}^2\text{/m}$$

This is near enough to the area given by H-125, no further iteration is needed, and $100M_1/f_s bd^2 = 0.425$ may be assumed.

From *Table 3.47*, by interpolation: $x/d = 0.312$, $x = 48$ mm

Strain in the reinforcement,

$$\varepsilon_s = f_s/E_s = 226/(200 \times 10^3) = 0.00113$$

Strain at surface, ignoring stiffening effect of concrete,

$$\varepsilon_1 = \varepsilon_s(h - x)/(d - x)$$
$$= 0.00113 \times (200 - 48)/(154 - 48) = 0.00162$$

Stiffening effect of concrete at surface (with $a' = h$)

$$\varepsilon_2 = b_t(h - x)^2/3E_s A_s(d - x)$$
$$= 1000 \times (200 - 48)^2/[3 \times 200 \times 10^3 \times 905 \times (154 - 48)]$$
$$= 0.00040$$

Strain at surface, allowing for stiffening effect of concrete,

$$\varepsilon_m = \varepsilon_1 - \varepsilon_2 = 0.00162 - 0.00040 = 0.00122$$

Distance from surface of bar, where s_b is bar spacing, to point on surface midway between bars, is given by:

$$a_{cr} = \sqrt{(s_b/2)^2 + (h - d)^2} - \phi/2$$
$$= \sqrt{(62.5)^2 + (46)^2} - 6 = 71.6 \text{ mm}$$

The maximum design crack width is given by:

$$w_{cr} = \frac{3a_{cr}\varepsilon_m}{1 + 2(a_{cr} - c_{min})/(h - x)}$$
$$= 3 \times 71.6 \times 0.00122/[1 + 2 \times (71.6 - 40)/(200 - 48)]$$
$$= 0.19 \text{ mm}$$

Chapter 27

Considerations affecting design details

Codes of Practice contain numerous requirements that affect the reinforcing details such as minimum and maximum areas, tying provisions, anchorage and curtailment.

Bars may be arranged individually, in pairs or in bundles of three or four in contact. In BS 8110, for the safe transmission of bond forces, the cover provided to the bars should be not less than the bar size or, for a group of bars in contact, the equivalent diameter of a notional bar with the same total cross-sectional area as the group. In BS 5400, the forgoing cover requirement is increased by 5 mm. Requirements for cover with regard to durability and fire-resistance are given in Chapter 23. Gaps between bars (or groups of bars) generally should be not less than the greater of (h_{agg} + 5 mm), where h_{agg} is the maximum size of the coarse aggregate, or the bar size (or the equivalent bar size for bars in groups). Details of reinforcement limits are given in *Table 3.53* for BS 8110, and *Table 3.59* for BS 5400.

27.1 TIES IN STRUCTURES

For robustness, the necessary interaction between elements is obtained by tying the structure together. Where the structure is divided into structurally independent sections, each section should have an appropriate tying system. In the design of ties, the reinforcement may be assumed to act at its characteristic strength, and only the specified tying forces need to be taken into account. Reinforcement provided for other purposes may be considered to form part of, or the whole of the ties. Details of the tying requirements in BS 8110 are given in *Table 3.54*.

27.2 ANCHORAGE AND LAP LENGTHS

At both sides of any cross section, bars should be provided with an appropriate embedment length or other form of end anchorage. In BS 5400, it is also necessary to consider 'local bond' where large changes of tensile force occur over short lengths of reinforcement. Critical sections for local bond are at simply supported ends, at points where tension bars stop and at points of contra-flexure. However, the last two points need not be considered if the anchorage bond stresses in the continuing bars do not exceed 0.8 times the ultimate values.

The radius of any bend in a reinforcing bar should conform to the minimum requirements of BS 8666, and should ensure that the bearing stress at the mid-point of the curve does not exceed the maximum value given in BS 8110 or BS 5400 as appropriate. A link may be considered fully anchored, if it passes round another bar not less than its own size, through an angle of 90°, and continues beyond the end of the bend for a minimum length of eight diameters. Details of anchorage lengths, local bond stresses, and bends in bars are given in *Tables 3.55* and *3.59*, for BS 8110 and BS 5400 respectively.

In BS 8007, for horizontal bars in direct tension, the design ultimate anchorage bond stress is taken as 70% of the value given in BS 8110. Also, for sections where bars are needed to control cracking due to temperature and moisture effects, the required anchorage bond length $l_{ab} = \phi/6\,\rho$,where $\rho \geq \rho_{crit}$ is the reinforcement ratio in the surface zone (*Table 3.44*). This value can exceed the anchorage bond length in BS 8110.

Laps should be located, if possible, away from positions of maximum moment and should preferably be staggered. Laps in fabric should be layered or nested to keep the lapped wires or bars in one plane. BS 8110 requires that, at laps, the sum of all the reinforcement sizes in a particular layer should not exceed 40% of the breadth of the section at that level. When the size of both bars at a lap exceeds 20 mm, and the cover is less than 1.5 times the size of the smaller bar, links of size not less than one-quarter the size of the smaller bar, and spacing not greater then 200 mm, should be provided throughout the lap length. Details of lap lengths are given in *Tables 3.55* and *3.59* for BS 8110 and BS 5400 respectively.

27.3 CURTAILMENT OF REINFORCEMENT

In flexural members, it is generally advisable to stagger the curtailment points of the tension reinforcement as allowed by the bending moment envelope. Curtailed bars should extend beyond the points where in theory they are no longer needed, in accordance with certain conditions. Details of the general curtailment requirements in BS 8110 are given in *Table 3.56*. Simplified rules for beams and slabs are also shown in *Tables 3.57* and *3.58*. In BS 5400, the general curtailment procedure is the same as that in BS 8110, except for the requirement at a simply supported end where condition (3) does not apply.

Example 1. The design of the beam shown in the following figure is given in example 1 of Chapters 24 (bending) and 25 (shear). The design ultimate loads on each span are F_{max} = 416 kN and F_{min} = 160 kN. The main reinforcement is as follows: in the

3.53

BS 8110 Reinforcement limits

Minimum areas

Minimum areas of grade 500 reinforcement according to condition	$A_{s,min}$ (mm^2)
Tension reinforcement in sections subjected mainly to pure tension	$0.0045A_c$
Tension reinforcement in sections subjected to flexure:	
T-beam with flange in tension	$0.0026b_wh$
L-beam with flange in tension	$0.0020b_wh$
Flanged beam with web in tension ($b_w/b < 0.4$)	$0.0018b_wh$
Flanged beam with web in tension ($b_w/b \geq 0.4$)	$0.0013b_wh$
Rectangular section (in solid slabs, this requirement applies in both directions)	$0.0013bh$
Compression reinforcement (where such reinforcement is required for the ultimate limit state)	
General rule	$0.004A_{cc}$
Simplified rules for particular cases:	
Rectangular column or wall	$0.004A_c$
Flanged beam with flange in compression	$0.004bh_f$
Flanged beam with web in compression	$0.002b_wh$
Rectangular beam	$0.002A_c$
Transverse reinforcement in beam flanges (near top surface over full effective flange width)	$0.0015h_fl$
'Anticrack' reinforcement in plain walls (in each direction)	$0.0025A_c$

Note 1. A_c is total area of concrete, A_{cc} is area of concrete in compression, b is width of section or width of flange as appropriate, b_w is average width of concrete below flange, h_f is thickness of flange, and l is length of flange.
Note 2. For reinforced columns, minimum number of bars is 4 in rectangular columns and 6 in circular columns, with bar size not less than 12 mm.
Note 3. Columns or walls with less than the minimum areas of reinforcement should be designed as plain concrete.

Minimum requirements of grade 500 reinforcement in solid slabs

Thickness of slab	Maximum spacing of main or secondary reinforcement according to bar size (mm)			Thickness of slab	Maximum spacing of main or secondary reinforcement according to bar size (mm)		
mm	8	10	12	mm	8	10	12
100	225			200	200	300	450
125	300			225	175	275	400
150	250	375		250	150	250	350
175	225	350	450	300	125	200	300

Note. Values of maximum bar spacing are limited to $3(h - 25)$, where h is slab thickness.

Maximum areas

Maximum areas of main reinforcement according to member	$A_{s,max}$ (mm^2)
Beams (tension or compression reinforcement)	$0.04A_c$
Columns (longitudinal reinforcement)	
Vertically-cast columns	$0.06A_c$
Horizontally-cast columns	$0.08A_c$
At laps in all columns	$0.10A_c$
Walls (vertical reinforcement)	$0.04A_c$

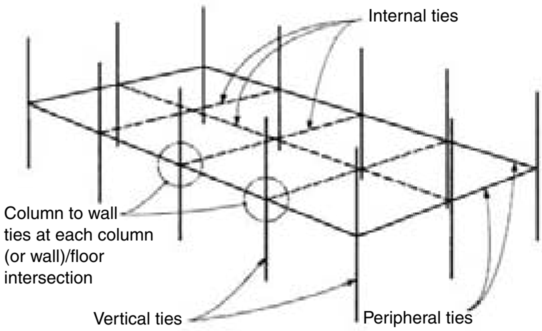
Containment of bars in compression

Minimum requirements for containment of compression reinforcement according to member	
Beam or column	Links or ties, at least one-quarter the size of the largest compression bar or 6 mm, whichever is the greater, at a maximum spacing of 12 times the size of the smallest compression bar. Every corner bar, and each alternate bar (or pair or bundle) in an outer layer of reinforcement, should be supported by a link passing round the bar and having an included angle not greater than 135°. For circular columns, circular ties passing round the bars or groups provide sufficient support.
Reinforced wall	Horizontal bars, at least one-quarter the size of the largest compression bar or 6 mm, whichever is the greater, providing not less than 0.25% of the concrete area. If the vertical compression reinforcement exceeds 2% of the concrete area, links at least one-quarter the size of the largest compression bar or 6 mm, whichever is the greater, should be provided through the thickness of the wall. The link spacing should not exceed twice the wall thickness, in either the horizontal or the vertical direction, nor 16 times the bar size in the vertical direction. All vertical compression should be enclosed by a link, and no bar should be further than 200 mm from a restrained bar, at which a link passes round the bar with an included angle of not more than 90°.

BS 8110 Provision of ties

3.54

System



Wherever possible, a system of effectively fully continuous horizontal and vertical ties should be provided to satisfy the requirements given below. If appropriate ties cannot be provided, some other design strategy should be examined, as described in section 6.1.1, and Part 2 of BS 8110.

Requirements

Type of tie	Requirement
Peripheral	At each floor and roof level, an effectively continuous tie, located within 1.2 m of the outside edges of the building, or within a perimeter wall. The tensile force to be resisted is given by: $F_t = (20 + 4n_o) \le 60$ kN where: n_o is the number of storeys in the structure
Internal	At each floor and roof level, in two directions approximately at right angles, effectively continuous ties anchored to the peripheral tie at each end (unless continuing as ties to columns or walls). They may, in whole or in part, be spread evenly in slabs, or may be grouped at or in beams, walls or other appropriate positions. If grouped, they should be spaced generally at not more than $1.5l_r$ where l_r is the greater distance between the centres of the columns, frames or walls supporting any two adjacent spans in the direction of the tie under consideration. In walls they should be within 0.5 m of the top or bottom of the floor or roof slab. The tensile force to be resisted is given by: $F_{t,int} = \left(\frac{g_k + q_k}{7.5}\right)\left(\frac{l_r}{5}\right)F_t \ge F_t$ (kN/m width) where: $(g_k + q_k)$ is the sum of the characteristic dead and imposed loading, l_r is as defined above. When walls occur in plan in one direction only (e.g. cross-wall or spine-wall construction), the value of l_r used to assess the tie force in the direction parallel to the wall should be taken as the lesser of the actual length of the wall, or the length that may be considered lost in the event of an accident. This length should be taken as the length between adjacent lateral supports, or the length between a lateral support and a free edge, as appropriate.
Horizontal to columns and walls	Each external column and, if the peripheral tie is not located within the wall, every metre length of external wall carrying vertical load should be anchored or tied horizontally into the structure at each floor and roof level. The tensile force to be resisted is given by the greater of: (a) $2F_t$ [or $(l_s/2.5)F_t$ if less, where l_s is the floor to ceiling height in metres]; or (b) 3% of the total design ultimate vertical load carried by the column or wall at that level For corner columns, ties able to resist the tensile force should be provided in each of two directions, approximately at right angles. When the peripheral tie is located within a wall, only such horizontal tying as is required to anchor the internal ties to the peripheral tie needs to be considered.
Vertical	Each column and each wall carrying vertical load should be tied continuously from the highest to the lowest level. The tensile force to be resisted is the maximum design accidental load received by the column or wall from any one storey. For this purpose, γ_f should be taken as 1.05, and the following loads should be taken into account: dead load; one-third of imposed load, except for buildings used predominantly for storage or industrial purposes, or where the imposed loads are permanent, when full imposed load should be taken; one-third of wind load.

Horizontal ties, with a minimum capacity of $2F_t$ in each spanning direction, should be positioned so as to interact with the vertical structure. For columns, this can generally be achieved by ensuring that two bottom bars in each direction pass directly through the column. Where top bars are used as ties, they should be restrained by links in the beam/slab.

In the design of ties, reinforcement may be assumed to act at its characteristic strength, and only the specified tying forces need be taken into account. Reinforcement provided for other purposes may be considered to form part of, or the whole of the ties. At re-entrant corners, or at substantial changes in construction, care should be taken to ensure that ties are adequately anchored or otherwise made effective. A tie may be considered anchored to another tie at right angles if the bars of the former tie extend: either 12ϕ, or an equivalent anchorage, beyond all the bars of the other tie; or, an effective anchorage length (based on the force in the bars) beyond the centre-line of the bars of the other tie. The anchorage and lap lengths given in *Table 3.55*, including the multiplier $A_{s,req}/A_{s,prov}$ may be used for this purpose.

BS 8110 Anchorage requirements

3.55

Anchorage and lap lengths

The required anchorage bond length for a bar, acting at the maximum design stress, is given by: $l_{ba} = (0.87f_y/4f_{bu})\phi$

f_{bu}	is the ultimate anchorage bond stress given by $f_{bu} = \beta\sqrt{f_{cu}}$
β	is a coefficient dependent on bar type. For bars in tension, in slabs, or in beams where minimum links have been provided, $\beta = 0.28$, 0.4 and 0.5 for plain, type 1 deformed, and type 2 deformed bars respectively. If minimum links are not provided in beams, $\beta = 0.28$ should be used. For fabric, where the number of welded intersections within the anchorage length is at least four, $\beta = 0.65$ may be used. Otherwise, a value of β appropriate to the type of bar should be used. For bars in compression, values of β may be increased by 25%.
ϕ	is the bar size. For a group of bars curtailed at the same position, ϕ should be taken as the equivalent bar size. If bars in a group are curtailed at different positions, with an anchorage bond length or more between each position, ϕ may be taken as the value for a single bar.

The required lap length for a bar, acting at the maximum design stress, is given by: $l_{bl} = \alpha_l l_{ba}$

α_l	is a coefficient dependent on the conditions. For bars in tension, $\alpha_l = 1.0$ except: (a) where a lap occurs at the top of a section as cast and the cover is less than 2ϕ, $\alpha_l = 1.4$; (b) where a lap occurs at the corner of a section and the cover to either face is less than 2ϕ, or the gaps between adjacent laps in the same layer are less than $6\phi \geq 75$ mm, $\alpha_l = 1.4$; (c) where conditions (a) and (b) apply, $\alpha_l = 2.0$. For bars in compression, $\alpha_l = 1.25$.

Ultimate anchorage bond lengths and lap lengths as a multiple of bar size (for grade 500, type 2 deformed bars)

Condition	Concrete cube strength f_{cu} (N/mm^2)				
	25	30	35	40	50
Tension anchorage length	44	40	38	35	31
Compression anchorage length	35	32	30	28	25
Tension lap length ($\alpha_l = 1.0$)	44	40	38	35	31
Tension lap length ($\alpha_l = 1.4$)	62	56	52	49	44
Tension lap length ($\alpha_l = 2.0$)	88	80	75	70	62
Compression lap length ($\alpha_l = 1.25$)	44	40	38	35	31

Note. When the stress in the reinforcement is less than $0.87f_y$, anchorage lengths and lap lengths may be multiplied by $A_{s,req}/A_{s,prov}$. Minimum lap lengths: for bars, greater of 15 times bar size or 300 mm; for fabric, 250 mm.

Bends in bars

For a 90° bend, the effective anchorage length for that portion of the bar between the start of the bend and a point four times the bar size beyond the end of the bend may be taken as the greater of: either, four times the internal radius of the bend with a maximum of 12 times the bar size, or the actual length of bar. For bars bent to the minimum radius according to BS 8666, the effective anchorage length of a 90° bend is 8ϕ for $\phi \leq 16$, and 12ϕ for $\phi > 16$. Any length of bar in excess of four times the bar size beyond the end of the bend, and which is within the anchorage region, may also be taken into account. In this case, the internal radius of the bend should satisfy the following relationship:

$$r \geq \frac{F_{bt}(1 + 2\phi/a_b)}{2f_{cu}\phi} \geq r_{min} \text{ (according to BS 8666)}$$

where

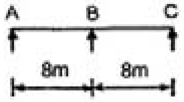
F_{bt}	is the tensile force due to the design ultimate loads in a bar, or group of bars in contact, at the start of a bend
a_b	is the centre-to-centre distance between bent bars (or groups of bars) perpendicular to the plane of bend. If, at the position of the bend, the bar (or group of bars) is adjacent to the face of the member, a_b should be taken as the cover to the bar plus the bar size.
r	is the internal radius of the bend
ϕ	is the bar size or, for bars in a group, the equivalent bar size

f_{cu} N/mm^2	Minimum value of r as a multiple of bar size (with $f_s = 0.87 \times 500 = 435$ N/mm^2) for values of a_b/ϕ							
	2	3	4	5	6	8	10	12
25	13.7	11.4	10.3	9.6	9.1	8.5	8.2	8.0
30	11.4	9.5	8.5	8.0	7.6	7.1	6.8	6.6
35	9.8	8.1	7.3	6.8	6.5	6.1	5.9	5.7
40	8.5	7.1	6.4	6.0	5.7	5.3	5.1	5.0
50	6.8	5.7	5.1	4.8	4.6	4.3	4.1	4.0

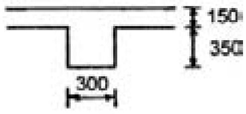
Note. Tabulated values of r may be multiplied by $f_s/435$, where f_s is the stress in the reinforcement at the start of the bend. Minimum values of r for bending according to BS 8666 are 2ϕ for $\phi \leq 16$, and 3.5ϕ for $\phi > 16$.

spans 2H32 (bottom); at support B, 3H32 (top) and 2H25 (bottom). The width of each support is 400 mm, and bars are to be curtailed according to the requirements of BS 8110. In the following calculations, the use of the general curtailment procedure (*Table 3.56*) and the simplified curtailment rules (*Table 3.57*) are both examined.

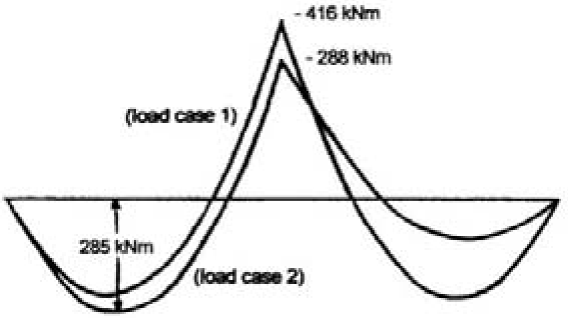
$$f_{cu} = 40 \text{ N/mm}^2, f_y = 500 \text{ N/mm}^2, d = 450 \text{ mm}$$



Spans



Cross section



Elastic bending moment diagram

End anchorage. At the bottom of each span, the 2H32 will be continued to the supports. At the end support (simple), an effective anchorage of 12ϕ is required beyond the centre of bearing. This can be obtained by providing a 90° bend with an internal radius of 3.5ϕ, provided the bend does not start before the centreline of the support. Allowing for 50 mm end cover to the bars, the distance from the outside face of the support to the start of the bend is $50 + 4.5\phi = 194$ mm. This would be satisfactory since the width of the support is 400 mm. If the support width were any less, the 2H32 could be stopped at the face of the support and lapped with 2H25, in which case it would be necessary to reassess the shear design.

Curtailment points for top bars. The resistance moment provided by 2H32 can be determined as follows:

$$A_s f_y / bdf_{cu} = 1608 \times 500/(300 \times 450 \times 40) = 0.149$$
$$M/bd^2 f_{cu} = 0.111 \text{ (Table 3.14)}$$
$$M = 0.111 \times 300 \times 450^2 \times 40 \times 10^{-6} = 270 \text{ kNm}$$

For load case 1, reaction at A (or C) is given by:

$$R_A = 0.5F_{max} - M_B/L = 0.5 \times 416 - 416/8 = 156 \text{ kN}$$

Distance x from A to point where $M = 270$ kNm is given by:

$$0.5(F_{max}/L)\, x^2 - R_A x = 0.5 \times (416/8)x^2 - 156x = 270$$

Hence $0.5x^2 - 3x - 5.2 = 0$, giving $x = 7.4$ m. Thus, of the 3H32 required at B, one bar is no longer needed for flexure at a distance of $(8.0 - 7.4) = 0.6$ m from B.

The bar to be curtailed needs to extend beyond this point for a distance not less than $d \geq 12\,\phi = 450$ mm, to a position where one of the following conditions is satisfied:

1. A full tension anchorage length beyond the theoretical point of curtailment, that is, $35\phi = 35 \times 32 = 1.1$ m
2. A point where the shear force is no more than half the design shear resistance at the section. From the shear design calculations in Chapter 25, with H8-300 links, $V_u = 170$ kN. For load case 1, distance x from B to the point where shear force is 170/2 = 85 kN, is given by:

   $$x = [1 - (85 + 156)/416]L = 0.42 \times 8 = 3.36 \text{ m}$$

3. A point where the bending moment is no more than half the design resistance moment at the section. As in the calculations above, but with $M = 270/2 = 135$ kNm, $0.5x^2 - 3x - 2.6 = 0$, giving $x = 6.8$ m, and a distance from B of $(8.0 - 6.8) = 1.2$ m.

From the foregoing, it can be seen that if the bar were curtailed at a distance of 1.2 m from B, this would satisfy condition (3) and be more than 450 mm beyond the theoretical curtailment point at 0.6 m from B. However, the bar should extend 1.3 m from B in order to provide a full tension anchorage of 1.1 m beyond the face of the support. It can be seen from the foregoing calculations that checking conditions (2) and (3) is a tedious process, and complying with condition (1) is a more practical approach, even though it would mean curtailing the bar at $(0.6 + 1.1) = 1.7$ m from B in this example.

Suppose that the remaining 2H32 are continued to the point of contra-flexure in span BC for load case 2.

The reaction at support C is given by

$$R_C = 0.5F_{min} - M_B/L = 0.5 \times 160 - 288/8 = 44 \text{ kN}$$

Distance from B to point of contra-flexure is given by:

$$x = L(1 - 2R_C/F_{min}) = 8 \times (1 - 2 \times 44/160) = 3.6 \text{ m}$$

The bars need to extend beyond this point for a distance not less than $d \geq 12\phi = 450$ mm. Link support bars, say 2H12 with a lap of 300 mm, could be used for the rest of the span.

If the simplified curtailment rules are applied, one bar out of three may be curtailed at $0.15L \geq 45\phi = 1.45$ m from the face of the support, that is, at 1.65 m from B. The other two bars may be curtailed at $0.25L = 2.0$ m from the face of the support, that is, at 2.2 m from B. Beyond this point, bars giving an area not less than 20% of the area required at B should be provided, that is, $0.2 \times 2413 = 483$ mm^2 (2H20 gives 628 mm^2). Since the bars are in the top of a section as cast, where the cover is less than 2ϕ and the gap between adjacent laps is not less than 6ϕ, $\alpha_1 = 1.4$ (see *Table 3.55*) and the required lap length is:

$$l_{bl} = 49\phi\,(A_{s,req}/A_{s,prov}) = 49 \times 20 \times 483/628 = 750 \text{ mm}$$

Example 2. A typical floor to an 8-storey building consists of a 280 mm thick flat slab, supported by columns arranged on a 7.2 m square grid. The slab, for which the characteristic loading is 8.0 kN/m^2 dead and 4.5 kN/m^2 imposed, is to be provided with ties to the requirements of BS 8110. The design ultimate load on a panel is 954 kN, and bending moments are to be determined by the simplified method (see section 13.8).

$$f_{cu} = 40 \text{ N/mm}^2, f_y = 500 \text{ N/mm}^2, \text{cover to bars} = 25 \text{ mm}$$

Allowing for the use of H12 bars in each direction, and based on the bars in the second layer of reinforcement:

$$d = 280 - (25 + 12 + 6) = 235 \text{ mm say}$$

From *Table 2.55*, the design ultimate sagging moment for an interior panel is given by:

$$M = 0.063Fl = 0.063 \times 954 \times 7.2 = 433 \text{ kNm}$$

The total panel moment is to be apportioned between column and middle strips, where the width of each strip is 3.6 m. For the column strip with 55% of the panel moment,

$$M = 0.55 \times 433 = 238 \text{ kNm}$$
$$M/bd^2f_{\mathrm{cu}} = 238 \times 10^6/(3600 \times 235^2 \times 40) = 0.030$$

From *Table 3.14*, since $M/bd^2f_{\mathrm{cu}} < 0.043$, $z/d = 0.95$. Hence,

$$A_{\mathrm{s}} = 238 \times 10^6/(0.87 \times 500 \times 0.95 \times 235)$$
$$= 2504 \text{ mm}^2 \text{ (24H12-150 gives 2714 mm}^2\text{)}$$

For the middle strip with 45% of the panel moment,

$$M = 0.45 \times 433 = 195 \text{ kNm}$$
$$M/bd^2f_{\mathrm{cu}} = 195 \times 10^6/(3600 \times 235^2 \times 40) = 0.025 \ (< 0.043)$$

$$A_{\mathrm{s}} = 195 \times 10^6/(0.87 \times 500 \times 0.95 \times 230)$$
$$= 2008 \text{ mm}^2 \text{ (18H12-200 gives 2036 mm}^2\text{)}$$

For the peripheral tie, the tensile force is given by:

$$F_{\mathrm{t}} = (20 + 4n_{\mathrm{o}}) \leq 60 \text{ kN} = (20 + 4 \times 8) = 52 \text{ kN}$$

The required area of reinforcement, acting at its characteristic strength, is given by:

$$A_{\mathrm{s}} = F_{\mathrm{t}}/f_{\mathrm{y}} = 52 \times 10^3/500 = 104 \text{ mm}^2 \text{ (1H12)}$$

For the internal ties, the tensile force is given by:

$$F_{t,\mathrm{int}} = \left(\frac{g_{\mathrm{t}} + q_{\mathrm{k}}}{7.5}\right)\left(\frac{l_{\mathrm{r}}}{5}\right) \mathrm{F_t} \geq \mathrm{F_t} \text{ kN/m}$$

$$= \left(\frac{8.0 + 4.5}{7.5}\right)\left(\frac{7.2}{5}\right) \times 52 = 125 \text{ kN/m}$$

If the internal ties are spread evenly in the slab, the required area of reinforcement acting at its characteristic strength,

$$A_{\mathrm{s}} = 125 \times 10^3/500 = 250 \text{ mm}^2\text{/m (H12-400)}$$

In this case, alternate bars in both column and middle strips need to be made effectively continuous.

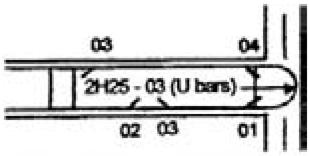
If the internal ties are concentrated at the column lines, the total area of reinforcement required in each group,

$$A_{\mathrm{s}} = 250 \times 7.2 = 1800 \text{ mm}^2 \text{ (16H12 gives 1810 mm}^2\text{)}$$

In this case, the bars in the middle two-thirds of each column strip need to be made effectively continuous. Since the bars are located at the bottom of the slab, and the gap between each set of lapped bars exceeds 6ϕ, $\alpha_1 = 1.0$ and a lap length of 35ϕ is sufficient (*Table 3.55*, for $f_{\mathrm{cu}} = 40$ N/mm^2).

Example 3. The following figure shows details of the reinforcement at the junction between a 300 mm wide beam and a 300 mm square column. Bars 03 need to develop the maximum design stress at the column face, and the required radius of bend is to be checked in accordance with the requirements of BS 8110.

$$f_{\mathrm{cu}} = 40 \text{ N/mm}^2, f_{\mathrm{y}} = 500 \text{ N/mm}^2$$



Cover to U bars: end 50, side 75

The minimum radius of bend of the bars depends on the value of a_{b}/ϕ , where a_{b} is taken as either centre-to-centre distance between bars, or (side cover plus bar size), whichever is less. Hence $a_{\mathrm{b}} = (300 - 75 \times 2 - 25) = 125 \leq (75 + 25) = 100$ mm.

From *Table 3.55*, for $a_{\mathrm{b}}/\phi = 100/25 = 4$, $r_{\mathrm{min}} = 6.4\phi$. This value can be reduced slightly by considering the stress in the bar at the start of the bend. If $r = 6\phi$, distance from face of column to start of bend $= 300 - 50 - 7 \times 25 = 75$ mm (i.e. 3ϕ). From *Table 3.55*, the required anchorage length is 35ϕ, and $r_{\mathrm{min}} = (1 - 3\phi/35\phi\ ) \times 6.4\phi = 5.9\phi$. Thus $r = 6\phi$ is sufficient.

Example 4. A 700 mm thick solid slab bridge deck is simply supported at the end abutments. The design of the slab to the requirements of BS 5400 is given in example 2 of Chapter 25 (shear) and example 3 of Chapter 26 (bending, cracking and fatigue). The tension reinforcement at the end of the span is H25-200, and the maximum design shear force is 625 kN acting on a 1.5 m wide strip of slab. A local bond check is required.

$$f_{\mathrm{cu}} = 40 \text{ N/mm}^2, f_{\mathrm{y}} = 500 \text{ N/mm}^2, d = 620 \text{ mm}$$

Bar perimeter provided by H25-200 in a 1.5 m strip of slab,

$$\Sigma u_{\mathrm{s}} = (1500/200) \times 25\pi = 589 \text{ mm}$$

The local bond stress is given by the relationship

$$f_{\mathrm{bs}} = V/\Sigma u_{\mathrm{s}}d = 625 \times 10^3/(589 \times 620) = 1.71 \text{ N/mm}^2$$

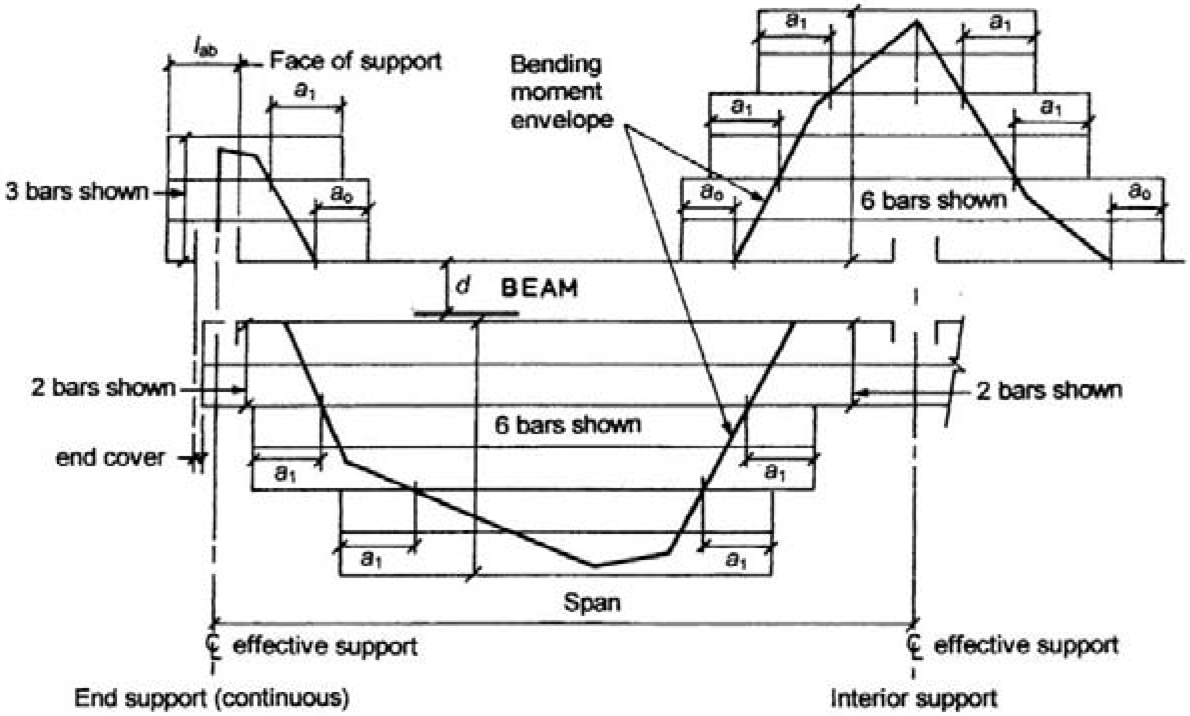
From *Table 3.59*, ultimate local bond stress $f_{\mathrm{lbu}} = 4.0$ N/mm^2.

BS 8110 Curtailment requirements

In every member, except at end supports, every bar should extend beyond the point at which in theory it is no longer needed for flexural resistance, for a distance not less than the greater of the effective depth of the member, or twelve times the bar size. In addition, bars that are curtailed in a tension zone should satisfy one of the following conditions:

(1) The bar extends for an anchorage bond length, appropriate to the maximum design stress $0.87f_y$, beyond the point at which in theory it is no longer needed (i.e. the point where the design resistance moment, considering only the continuing bars, is equal to the design moment).
(2) The bar ends at a point where the design shear resistance is at least twice the design shear force at the section.
(3) The bar ends at a point where the design resistance moment is at least twice the design bending moment at the section (i.e. the design bending moment is no more than half the value at the theoretical curtailment point).

The simplest approach is to satisfy condition (1), as shown in the following example in which $a_o = d \geq 12\phi$, where d is the effective depth and ϕ is the bar size, and $a_1 = l_{ba} \geq d$, where l_{ba} is the anchorage bond length (see *Table 3.55*).



Example of general curtailment procedure using option (1) for bars curtailed in a tension zone

At a simply-supported end of a member, each tension bar should satisfy one of the following conditions:

(1) The bar extends for an effective anchorage length equivalent to 12ϕ beyond the centre-line of the support, but no bend should begin before the centre-line of the support.
(2) The bar extends for an effective anchorage length equivalent to 12ϕ beyond a point $d/2$ from the face of the support, but no bend should begin before a point $d/2$ from the face of the support.
(3) For slabs, where the design ultimate shear stress is less than $0.5v_c$ (see *Table 3.33*), the bar extends for a distance beyond the centre-line of the support, equal to $w/3 \geq 30$ mm where w is the support width.

For bars bent to the minimum radius according to BS 8666, the minimum support width needed to satisfy condition (1) is given by $w = (2c + 9\phi)$ for $\phi > 16$ mm, where c is the end cover to the bar. For $\phi \leq 16$ mm, the minimum width of support is given by $w = (2c + 6\phi)$ provided the bar extends eight times the bar size beyond the end of the bend.

If the end of a beam is monolithically connected to a column but the beam is designed on the assumption of a simple support, with some nominal top reinforcement to control cracking, the anchorage of the bottom reinforcement should satisfy either (1) or (2). If the design resistance of the beam in bending and shear is based on the top reinforcement, the top bars should extend into the span for a distance of at least $3d$ from the face of the support.

Where a beam or slab extends beyond the end support to form a cantilever, care should be taken to ensure that the top reinforcement in the cantilever extends beyond the point of contra-flexure in the adjacent span.

3.57

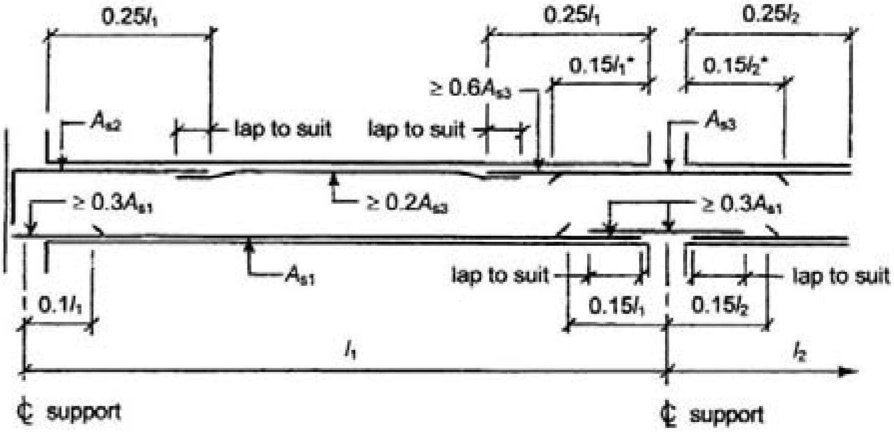
BS 8110 Simplified curtailment rules for beams

The curtailment of reinforcement shown in the figures below may be used for beams in the following circumstances:

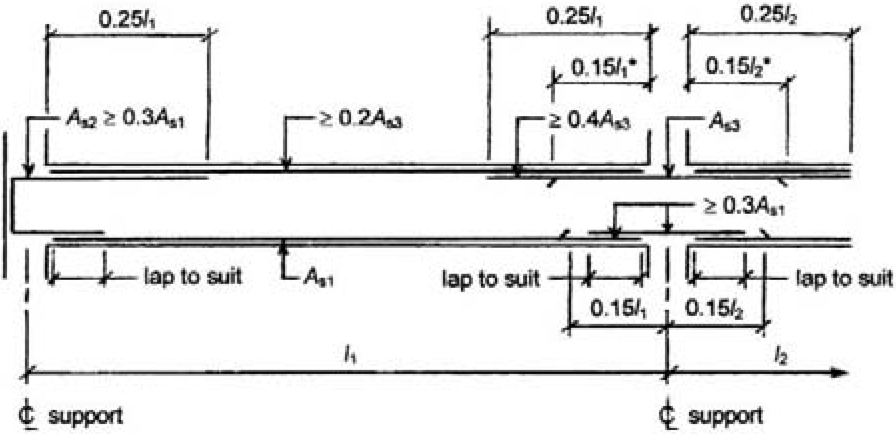
(1) The beams are designed for predominantly uniformly distributed loads.
(2) In the case of continuous beams, the spans are approximately equal.

The details shown in the figures do not necessarily provide sufficient areas of reinforcement, where bars are required to act in compression, or to satisfy tying provisions (see *Table 3.54*). Dimensions shown thus, $0.15l_1$*, should be not less than a tension anchorage length (see *Table 3.55*), which may be conservatively taken as 45 times the bar size.

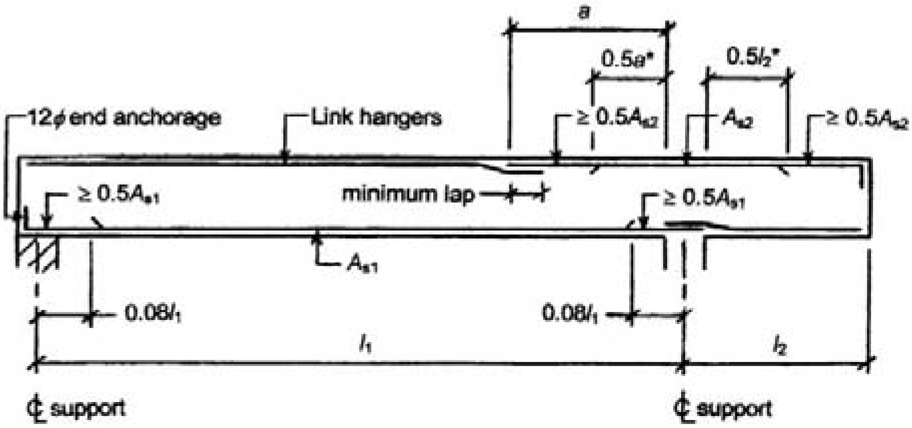
Typical examples



Example showing curtailment rules for continuous beam with restraint at end support



Alternative use of curtailment rules for continuous beam with restraint at end support



Example showing curtailment rules for simply supported beam with cantilever

Note. In the figure above, $a = (2M_2/G_k + d)$, where G_k is the total characteristic dead load on span l_1, M_2 is the design ultimate moment at the centre-line of the support for cantilever l_2, and d is the effective depth of the reinforcement.

BS 8110 Simplified curtailment rules for slabs

3.58

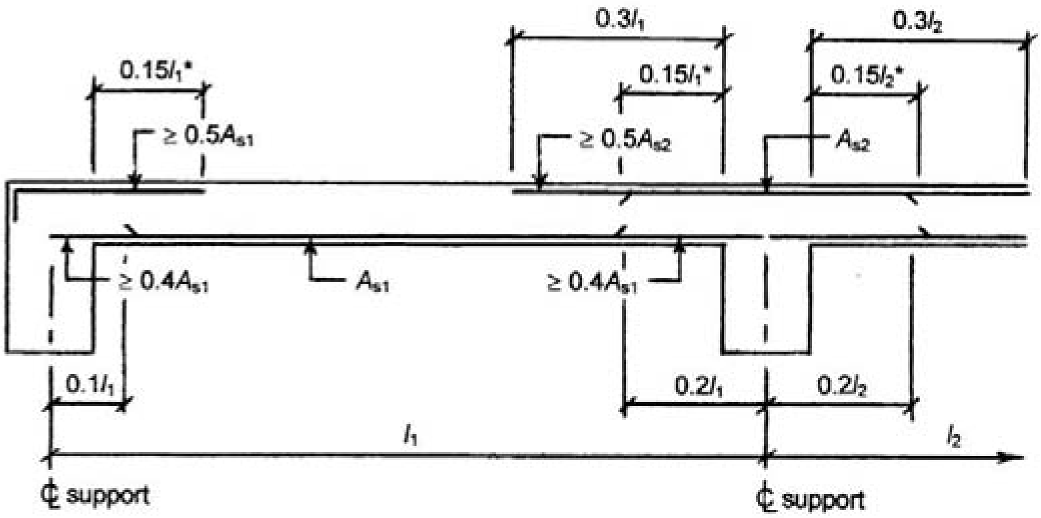
Typical examples

The curtailment of reinforcement shown in the figures below may be used for slabs in the following circumstances:

(1) The slabs are designed for predominantly uniformly distributed loads.

(2) In the case of continuous slabs, design has been carried out for the single load case of maximum design load on all spans, and the spans are approximately equal (see *Tables 2.49,2.50* and *2.62*).

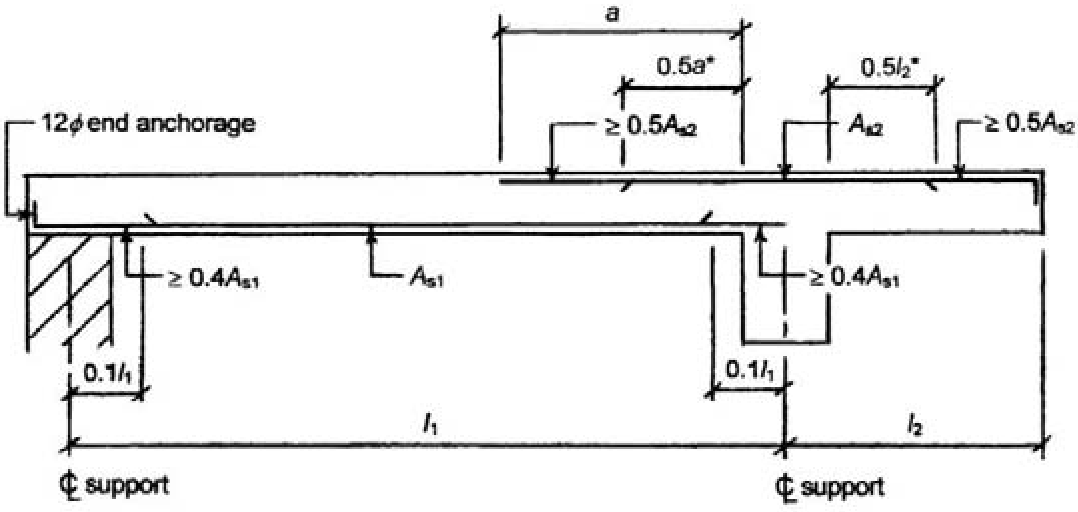
The details shown in the figures do not necessarily provide sufficient areas of reinforcement to comply with minimum requirements, or to satisfy tying provisions (see *Table 3.54*). Dimensions shown thus, 0.15l_1*, should be not less than a tension anchorage length (see *Table 3.55*), which may be conservatively taken as 45 times the bar size.



Example showing curtailment rules for continuous slab with nominal restraint at end support

Note. In the figure above, the shear resistance at the LH end may be based on the bottom reinforcement, if the bars are anchored in accordance with the requirements for a simple support. Otherwise, the bars may be stopped at the centre of the support, in which case the shear resistance should be based on the top reinforcement.

For two-way spanning rectangular panels, details of the reinforcement requirements in the middle and edge strips, and to provide for torsion at the corners, are given in section 13.2.1. For flat slabs, details of the reinforcement distribution in the column and edge strips are given in section 13.8.3. At a free edge, reinforcement for negative design moments in the direction perpendicular to the edge (except for the effective moment transfer strips at the outer columns) is needed, only if moments arise from loading on any extension of the slab beyond the column centrelines. However, minimum top reinforcement ($A_s \geq 0.0012bh$ for $f_y = 500$ N/mm^2) should be provided extending 0.15l_1* into the span.



Example showing curtailment rules for simply supported slab with cantilever

Note. In the figure above, $a = [l_2 (nl_2/g_k l_1) + d]$, where n is the design ultimate load per m^2 on the cantilever, g_k is the characteristic dead load per m^2 on the span l_1, and d is the effective depth of the reinforcement. If the slab adjacent to the cantilever spans in a direction parallel to the beam, $a = [l_2 (n/g_k)^{0.5} + d]$.

Minimum reinforcement

Minimum areas of grade 500 reinforcement according to condition	$A_{s,min}$ (mm^2)
Main tension reinforcement in a beam or slab (where b is breadth of a rectangular section, or average breadth excluding the compression flange for a non-rectangular section)	$0.0015bd$
Secondary reinforcement in a solid slab	$0.0012bd$
Main reinforcement in a column (where N kN is the ultimate axial load)	$0.3N \leq 0.01A_c$
Transverse reinforcement in flange of voided slab (where l is length of flange)	
Bottom or predominantly tensile flange (where h_f is minimum flange thickness)	$(0.01h_f \leq 1.5)l$
Top or predominantly compressive flange (ditto)	$(0.007h_f \leq 1)l$

Anchorage bond, local bond and lap requirements

The required anchorage bond length of a bar, acting at the maximum design stress, is given by: $l_{ba} = (0.87f_y/4\alpha_n f_{bu})\phi$

f_{bu} is the ultimate anchorage bond stress given in the table below.
α_n is a reduction factor for the effective perimeter of bars in a group. For a single bar, $\alpha_n = 1.0$. For 2, 3 or 4 bars in a group, $\alpha_n = 0.8$. 0.6 and 0.4 respectively.
ϕ is the bar size for a single bar, or each bar in a group.

The local bond stress at any section is given by: $f_{bs} = [V \pm (M/d)\tan\theta_s]/\Sigma u_s d \leq f_{lbu}$

M is the design ultimate moment at the section considered, where the negative sign applies when the moment is increasing numerically in the same direction as the effective depth d of the section.
V is the design ultimate shear force at the section considered.
f_{lbu} is the ultimate local bond stress given in the table below.
Σu_s is the sum of the effective perimeters of the tension reinforcement. For n bars in a group, $u_s = \alpha_n n(\pi\phi)$.
θ_s is the angle between the compression face of the section and the tension reinforcement.

Bar type	Ultimate anchorage bond stress f_{bu} for values of f_{cu} (N/mm^2)								Ultimate local bond stress f_{lbu} for values of f_{cu} (N/mm^2)			
	Tension				Compression							
	20	25	30	≥ 40	20	25	30	≥ 40	20	25	30	≥ 40
Plain	1.2	1.4	1.5	1.9	1.5	1.7	1.9	2.3	1.7	2.0	2.2	2.7
Deformed: type 1	1.7	1.9	2.2	2.6	2.1	2.4	2.7	3.2	2.1	2.5	2.8	3.4
Deformed: type 2	2.2	2.5	2.8	3.3	2.7	3.1	3.5	4.1	2.6	2.9	3.3	4.0

The required lap length for a bar, acting at the maximum design stress, is given by: $l_{bl} = \alpha_l l_{ba}$

α_l is a coefficient. If, for lapped bars in the corner of a section, the cover to both faces is at least 2ϕ and, for sets of lapped bars in the same layer, the gaps between the sets are at least 150 mm, $\alpha_l = 1.0$; if either or both of the previous conditions are not satisfied, $\alpha_l = 2.0$ for bars at the top of a section as cast; otherwise, $\alpha_l = 1.4$.

Ultimate anchorage bond lengths and lap lengths as a multiple of bar size (for grade 500, type 2 deformed bars)

Condition	Tension for values of f_{cu} (N/mm^2)				Compression for values of f_{cu} (N/mm^2)			
	20	25	30	≥ 40	20	25	30	≥ 40
Anchorage length	50	44	39	33	41	35	31	27
Lap length ($\alpha_l = 1.0$)	50	44	39	33	41	35	31	27
Lap length ($\alpha_l = 1.4$)	70	62	55	47	57	49	44	37
Lap length ($\alpha_l = 2.0$)	100	88	78	66	81	70	62	53

Bends in bars

For a 90° bend, the effective anchorage length for that portion of the bar between the start of the bend and a point four times the bar size beyond the end of the bend may be taken as four times the internal radius of the bend $\leq 24\phi$. For bars bent to the minimum radius according to BS 8666, the effective anchorage length of a 90° bend is 8ϕ for $\phi \leq 16$, and 14ϕ for $\phi > 16$. Any length of bar in excess of four times the bar size beyond the end of the bend, and which is within the anchorage region, may also be taken into account. In this case, the design bearing stress inside the bend should satisfy the following relationship:

$$\text{Bearing stress} = \frac{F_{bt}}{r\phi} \leq \frac{1.5f_{cu}}{1+2(\phi/a_b)} \qquad \text{where}$$

F_{bt} is the tensile force due to the design ultimate loads in a bar, or group of bars in contact, at the start of a bend
a_b is the centre-to-centre distance between bent bars (or groups of bars) perpendicular to the plane of bend. If, at the position of the bend, the bar (or group of bars) is adjacent to the face of the member, a_b should be taken as the cover to the bar plus the bar size.
r is the internal radius of the bend
ϕ is the bar size or, for bars in a group, the equivalent bar size

Chapter 28

Miscellaneous members and details

28.1 LOAD-BEARING WALLS

In BS 8110, for the purpose of design, a wall is defined as a vertical load-bearing member whose length on plan exceeds four times its thickness. Otherwise, the member is treated as a column. A reinforced wall is one in which not less than the recommended minimum amount of vertical reinforcement is provided, and taken into account in the design. Otherwise, the member is treated as a plain concrete wall, in which case the reinforcement is ignored for the purpose of design. Limiting reinforcement requirements are given in *Table 3.53*. Bearing stresses under concentrated loads should not exceed $0.6f_{cu}$ for concrete strength classes $\geq$ C20/25. Design requirements for reinforced and plain concrete walls are given in *Table 3.60*.

28.2 PAD BASES

Notes on the distribution of pressure under pad foundations are given in section 18.1, and values for the structural design of separate bases are given in *Table 2.82*. Critical sections for bending are taken at the face a concrete column, or the centre of a steel stanchion. The design moment is taken as that due to all external loads and reactions to one side of the section.

Generally, tension reinforcement may be spread uniformly across the width of the base, but the following requirement should be satisfied where c is the column width, and l_c is the distance from the centre of a column to the edge of the pad. If $l_c > 0.75(c + 3d)$, two-thirds of the reinforcement should be concentrated within a zone that extends on either side for a distance no more than $1.5d$ from the face of the column. For bases with more than one column in the direction considered, l_c should be taken as either half the column spacing, or the distance to the edge of the pad, whichever is the greater.

The pad should be examined for normal shear and punching shear. Normal shear is checked on vertical sections extending across the full width of the base. Within any distance $a_v < 2d$ from the face of the column, the shear strength may be taken as $(2d/a_v)v_c$. For a concentric load, the critical position occurs at $a_v = a/2 \leq 2d$, where a is the distance from the column face to the edge of the base. For an eccentric load, checks can be made at $a_v = 0.5d$, d, and so on to find the critical position.

Punching shear is checked on a perimeter at a distance $1.5d$ from the face of the column. The shear force at this position is that due to the effective ground pressure acting on the area outside the perimeter. For a concentric load with $a > 1.5d$, the check for punching shear is the critical shear condition. If the main reinforcement is taken into account in the determination of v_c, the bars should extend a distance d beyond the shear perimeter. In this case, the bars need to extend $2.5d$ beyond the face of the column, and will need to be bobbed at the end unless $a > 2.5d$, in which case straight bars would suffice.

The maximum clear spacing between the bars, a_b, should satisfy the following requirements, for $f_y = 500$ N/mm^2:

$100A_s/bd$	<0.3	0.3	0.4	0.5	0.6	0.75	≥1.0
a_b (mm)	750	500	375	300	250	200	150

Example 1. A base is required to support a 600 mm square column subjected to vertical load only, for which the values are 4600 kN (service) and 6800 kN (ultimate). The allowable ground bearing value is 300 kN/m^2.

$f_{cu} = 35$ N/mm^2, $f_y = 500$ N/mm^2, nominal cover = 50 mm

Allowing 10 kN/m^2 for ground floor loading and extra over soil displaced by concrete, the net allowable bearing pressure can be taken as 290 kN/m^2. Area of base required

$A_{base} = 4600/290 = 15.86$ m^2. Provide base 4.0 m square.

Distance from face of column to edge of base, $a = 1700$ mm.

Take depth of base $\geq 0.5a$, say $h = 850$ mm

Allowing for 25 mm main bars, average effective depth,

$$d = 850 - (50 + 25) = 775 \text{ mm}$$

Bearing pressure under base due to ultimate load on column,

$$p_u = 6800/4^2 = 425 \text{ kN/m}^2$$

Bending moment on base at face of column,

$$M = p_u la^2/2 = 425 \times 4 \times 1.7^2/2 = 2456 \text{ kNm}$$

$$K = M/bd^2 f_{cu} = 2456 \times 10^6/(4000 \times 775^2 \times 35) = 0.0292$$

From *Table 3.14*, since $K \leq 0.043$, $A_s f_y/bd f_{cu} = 1.21K$ and

$$A_s = 1.21 \times 0.0292 \times 4000 \times 775 \times 35/500 = 7670 \text{ mm}^2$$

3.60

BS 8110 Load-bearing walls

	Reinforced concrete walls ($A_{sc} \ge 4h$ mm^2/m)	Plain concrete walls
Braced walls	**Effective height:** l_e to be determined as for columns (see *Table 3.21*) **Design procedure:** **Stocky wall** ($l_e/h \le 15$, where h is wall thickness) Design unit length of wall as a short column bent about the minor axis, with $e_{min} = 0.05h \le 20$ mm (see section 24.3.1 and *Tables 3.17* and *3.18*) Alternatively, for a wall supporting an approximately symmetrical arrangement of slabs (uniform load and spans differing by no more than 15%): Design ultimate axial load per unit length is given by: $n_w \le 0.35 f_{cu} h + 0.67 A_{sc} f_y$ where A_{sc} is area of compression reinforcement per unit length **Slender wall** ($15 < l_e/h \le 40$ for $A_{sc} < 10h$, 45 otherwise) Design unit length of wall as a slender column bent about the minor axis (see *Table 3.22*). If only one layer of centrally placed reinforcement is provided, double the additional moment due to slenderness. **Note.** For walls with significant in-plane bending, see the procedure described under general below.	**Effective height:** If lateral supports resist lateral deflection and rotation: l_e = 0.75 x clear distance between lateral supports or 2 x distance of lateral support from free edge If lateral supports resist lateral deflection only: l_e = 1.0 x distance between centres of lateral supports or 2.5 x distance of lateral support from free edge **Design equations:** **Stocky wall** ($l_e/h \le 15$) Design ultimate axial load per unit length is given by: $n_w \le 0.3(1 - 2e_x/h) f_{cu} h$ where e_x is resultant eccentricity of load at right-angles to plane of wall, with $e_{min} = 0.05h \le 20$ mm. **Slender wall** ($15 < l_e/h \le 30$) Design ultimate axial load per unit length is given by the lesser of the following: $n_w \le 0.3(1 - 2e_x/h) f_{cu} h$ or $n_w \le 0.3[1 - 1.2e_x/h - 0.0008(l_e/h)^2] f_{cu} h$ where e_x is as defined above for a stocky wall
Unbraced walls	**Effective height:** l_e to be determined as for columns (see *Table 3.21*) **Design procedure:** **Stocky wall** ($l_e/h \le 10$, where h is wall thickness) Consider as described above for braced column, except that alternative equation does not apply. **Slender wall** ($10 < l_e/h \le 30$) Consider as described above for braced column.	**Effective height:** If wall supports at its top a roof or floor slab spanning at right angles to the wall, $l_e = 1.5 l_o$ where l_o is height of wall above a lateral support. Otherwise, $l_e = 2 l_o$. Slenderness limit: $l_e/h \le 30$ **Design equations:** Design ultimate axial load per unit length is given by the lesser of the following: $n_w \le 0.3(1 - 2e_{x,1}/h) f_{cu} h$ or $n_w \le 0.3[1 - 2e_{x,2}/h - 0.0008(l_e/h)^2] f_{cu} h$ where $e_{x,1}$ is resultant eccentricity of load at right-angles to plane of wall at top of wall $e_{x,2}$ is resultant eccentricity of load at right-angles to plane of wall at bottom of wall
General	**Braced and unbraced walls** A braced wall is one where the reactions to lateral forces acting on the wall are provided by lateral supports, which are able to transmit the lateral forces from the braced wall to the principal structural bracing or to the foundations. Lateral supports can be horizontal or vertical elements, such as floors or crosswalls. Principal structural bracing will be strong points, shear walls, or similar stiff elements providing lateral stability to the structure as a whole. An unbraced wall is one providing its own lateral stability, and the overall stability of multi-storey buildings, in any direction, should not depend on unbraced walls alone. **Design procedure for walls with significant in-plane bending (e.g. shear walls)** Determine distribution of vertical load along wall due to axial load and in-plane bending by elastic analysis, assuming no tension in concrete. Design unit length of wall at critical positions for appropriate combination of vertical load and transverse moment or eccentricity. For the analysis of shear wall structures, see section 4.12 and chapter 15. **Eccentricity of loads on plain concrete walls** Design loads from a concrete floor or roof may be assumed to act at one-third of the bearing area from the loaded face. It should be noted that loads may be applied to walls at greater than half the thickness of the wall through fittings such as joist hangers. For a braced wall at any level, the transverse eccentricity of the resultant load with respect to the axial plane of the wall may be calculated on the assumption that, immediately above a lateral support, the eccentricity of all the loads above that level is zero. **Conditions under which resistance to rotation of a lateral support may be assumed** (a) where the lateral support and the braced wall are concrete walls detailed to provide bending restraint; or (b) where a concrete floor has a bearing on at least two-thirds of the wall thickness, or the connection provides bending restraint.	

From *Table 2.20*,

$$16\text{H}25\text{-}250 \text{ gives } 7854 \text{ mm}^2, \text{ and}$$

$$100A_s/bd = 100 \times 7854/(4000 \times 775) = 0.25$$

Critical perimeter for punching shear occurs at 1.5d from face of column, where the length of side of the perimeter

$$l_1 = c + 3d = 600 + 3 \times 775 = 2925 \text{ mm}$$

Hence,

$$V = f(l^2 - l_1^2) = 425 \times (4^2 - 2.925^2) = 3164 \text{ kN}$$

$$v = V/4l_1d = 3164 \times 10^3/(4 \times 2925 \times 775) = 0.35 \text{ N/mm}^2$$

$$v_c = 0.216\left(\frac{400}{d}\right)^{1/4}\left(\frac{100A_sf_{cu}}{bd}\right)^{1/3}$$

$$= 0.216 \times (400/775)^{1/4}(0.25 \times 35)^{1/3} = 0.38 \text{ N/mm}^2 (> v)$$

Critical position for shear on vertical section across full width of base occurs at distance $a_v = a/2 = 850 \text{ mm} \leq 2d$ from face of column, where

$$V = 425 \times 4 \times 1.7/2 = 1445 \text{ kN}$$

$$v = V/bd = 1445 \times 10^3/(4000 \times 775) = 0.47 \text{ N/mm}^2$$

$$v_c(2d/a_v) = 0.38 \times (2 \times 775/850) = 0.69 \text{ N/mm}^2 (> v)$$

28.3 PILE-CAPS

In BS 8110 (and BS 5400), a pile-cap may be designed by either bending theory or truss analogy. In the latter case, the truss is of a triangulated form with nodes at the centre of the loaded area, and at the intersections of the centrelines of the piles with the tension reinforcement, as shown for compact groups of two to five piles in *Table 3.61*. Expressions for the tensile forces are given, taking into account the dimensions of the column, and also simplified expressions when the column dimensions are ignored. Bars to resist the tensile forces are to be located within zones extending not more than 1.5 times the pile diameter either side of the centre of the pile. The bars are to be provided with a tension anchorage beyond the centres of the piles, and the bearing stress on the concrete inside the bend in the bars should be checked (see *Table 3.55*).

The design shear stress calculated at the perimeter of the column should not exceed the lesser of $0.8\sqrt{f_{cu}}$ or 5 N/mm². Critical perimeters for checking shear resistance are shown in *Table 3.61*, where the whole of the shear force from piles with centres lying outside the perimeter should be taken into account. The shear resistance is normally governed by shear along a vertical section extending across the full width of the cap, where the design concrete shear stress may be enhanced as shown in *Table 3.61*. If the pile spacing exceeds 3 times the pile diameter, punching shear should also be considered.

In BS 5400, shear enhancement applies to zones of width equal to the pile diameter only, and the concrete shear stress is taken as the average for the whole section. Also, the check for punching shear is made at a distance of 1.5d from the face of the column, with no enhancement of the shear strength.

The following values are recommended for the thickness of pile-caps, where h_p is the pile diameter:

$$\text{For } h_p \leq 550 \text{ mm}, h = (2h_p + 100) \text{ mm}$$

$$\text{For } h_p > 550 \text{ mm}, h = 8(h_p - 100)/3 \text{ mm}$$

Example 2. A pile-cap is required for a group of 4 × 450 mm diameter piles, arranged at 1350 mm centres on a square grid. The pile-cap supports a 450 mm square column subjected to an ultimate design load of 4000 kN.

$$f_{cu} = 35 \text{ N/mm}^2, f_y = 500 \text{ N/mm}^2$$

Allowing for an overhang of 150 mm beyond the face of the pile, size of pile-cap = 1350 + 450 + 300 = 2100 mm square.

Take depth of pile-cap as $(2h_p + 100) = 1000$ mm.

Assuming tension reinforcement to be 100 mm up from base of pile-cap, $d = 1000 - 100 = 900$ mm.

Using truss analogy with the apex of the truss at the centre of the column, the tensile force between adjacent piles is

$$F_t = \frac{Nl}{8d} = \left(\frac{4000 \times 1350}{8 \times 900}\right) = 750 \text{ kN in each zone}$$

$$A_s = F_t/0.87f_y = 750 \times 10^3/(0.87 \times 500) = 1724 \text{ mm}^2$$

Providing 4H25 gives 1963 mm², and since the pile spacing is not more than 3 times the pile diameter, bars may be spread uniformly across the pile-cap giving a total of 8H25-275 in each direction, so that

$$100A_s/bh = 100 \times 3926/(2100 \times 1000) = 0.19 (> 0.13 \text{ min})$$

Critical position for shear on vertical section across full width of pile-cap occurs at distance from face of column given by:

$$a_v = 0.5(l - c) - 0.3h_p$$

$$= 0.5 \times (1350 - 450) - 0.3 \times 450 = 315 \text{ mm}$$

Portion of column load carried by two piles is 2000 kN, thus

$$v = V/bd = 2000 \times 10^3/(2100 \times 900) = 1.06 \text{ N/mm}^2$$

$$v_c = 0.216\left(\frac{400}{d}\right)^{1/4}\left(\frac{100A_sf_{cu}}{bd}\right)^{1/3}$$

$$= 0.216 \times (400/900)^{1/4}(0.19 \times 35)^{1/3} = 0.33 \text{ N/mm}^2$$

$$v_c (2d/a_v) = 0.33 \times (2 \times 900/315) = 1.88 \text{ N/mm}^2 (> v)$$

Shear stress calculated at perimeter of column is

$$v = V/ud = 4000 \times 10^3/(4 \times 450 \times 900) = 2.47 \text{ N/mm}^2$$

Maximum shear strength = $0.8\sqrt{f_{cu}} = 4.73 \text{ N/mm}^2 (> v)$

Taking a_b as either centre-to-centre distance between bars, or (side cover plus bar size), whichever is less,

$$a_b = 275 \leq (75 + 25) = 100 \text{ mm}, a_b/\phi = 100/25 = 4$$

From *Table 3.55*, minimum radius of bend $r_{min} = 7\phi$ say.

28.4 RETAINING WALLS ON SPREAD BASES

General notes on the design of walls to BS 8002 are given in section 7.3.2. Design values of earth pressure coefficients are based on a design soil strength, which is taken as the lower of either the peak soil strength reduced by a mobilisation factor, or the critical state strength. Design values of the soil strength using effective stress parameters are given by:

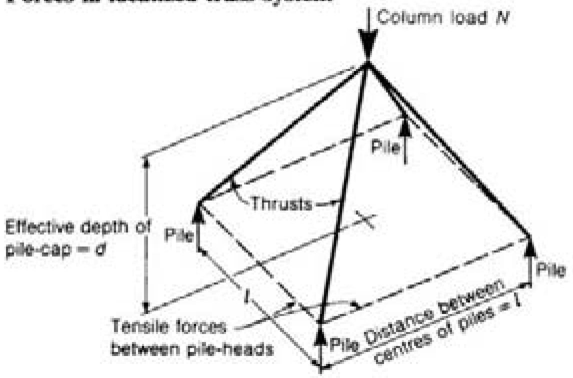
$$\text{design } \tan\varphi' = (\tan\varphi'_{max})/1.2 \leq \tan\varphi'_{crit}$$

$$\text{design } c' = c'/1.2 \leq \tan\varphi'_{crit}$$

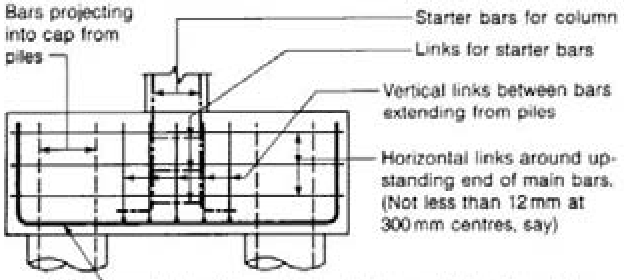
BS 8110 Pile-caps

3.61

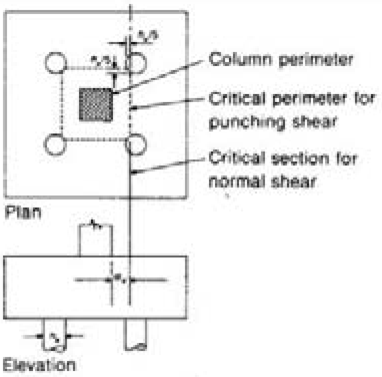
Forces in idealized truss system



Design of reinforcement



Main bars designed to resist tensile force obtained from the table below ($A_s = F_t / 0.87f_y$). Bars to be located within zone that extends no more than 1.5 x pile diameter either side of centre of pile, with tension anchorage beyond centre of pile.



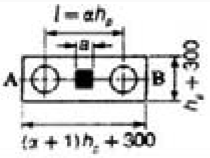
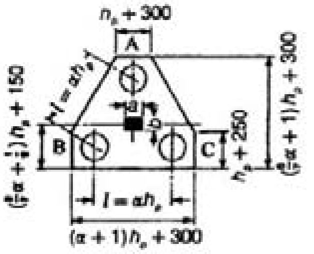
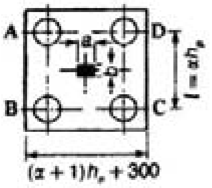
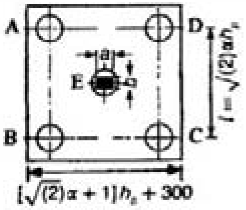
Shear resistance on a vertical plane:

For $l \le 3h_p$, $V_c = v_c(2d/a_v)bd$
For $l > 3h_p$, $V_c = \Sigma[v_{c1}(2d/a_v)b_1 d] + v_{c2}(b - \Sigma b_1)d$

where
b is overall width of pile-cap at critical section
b_v is width of zone containing main bars extending not more than 1.5 x pile diameter each side of centre of pile
v_{c1} is design concrete shear stress appropriate to width b_1
v_{c2} is design concrete shear stress appropriate to $(b - \Sigma b_1)$

Punching shear resistance ($V \le v_{c,max}$ x column perimeter)

For $l \le 3h_p$, no check required
For $l > 3h_p$, $V_c = v_c(1.5d/a_v)(4l - 2.4h_c)d$

Number of piles	Dimensions of pile-cap	Tensile force F_t between piles	
		Dimensions of column considered	Simplified
2		$\frac{N}{12ld}(3l^2 - a^2)$	$\frac{Nl}{4d}$
3		$F_{t(AB)} = F_{t(AC)} = \frac{N}{18ld}(2l^2 - b^2)$ $F_{t(BC)} = \frac{N}{36ld}(4l^2 + b^2 - 3a^2)$	$\frac{Nl}{9d}$
4		$F_{t(AD)} = F_{t(BC)} = \frac{N}{24ld}(3l^2 - a^2)$ $F_{t(AB)} = F_{t(CD)} = \frac{N}{24ld}(3l^2 - b^2)$	$\frac{Nl}{8d}$
5		$F_{t(AD)} = F_{t(BC)} = \frac{N}{30ld}(3l^2 - a^2)$ $F_{t(AB)} = F_{t(CD)} = \frac{N}{30ld}(3l^2 - b^2)$	$\frac{Nl}{10d}$

Notation: a, b are column dimensions, h_p is pile diameter, α is pile spacing factor (usually between 2 and 3 depending on conditions) where c', $\varphi'_{\max}$ and φ'_{crit} are representative (i.e. conservative) values of effective cohesion, peak effective angle of shearing resistance and critical state angle of shearing resistance for the soil. In the absence of reliable site investigation and soil test data, values may be derived from *Table 2.10*.

Design values of friction and adhesion at the soil-structure interface (wall or base) are given by:

design $\tan\delta$ (or δ_b) $= 0.75 \times$ design $\tan\varphi'$

design c_w (or c_b) $= 0.75 \times c_{ud} = 0.5\ c_u$ in which

$c_{ud} = c_u/1.5$, where c_u is undrained shear strength

A minimum surcharge of 10 kN/m^2 applied to the surface of the retained soil, and a minimum depth of unplanned earth removal in front of the wall, equal to 10% of the wall height but not less than 0.5 m, should be considered. Wall friction should be ignored in the determination of K_A.

Suitable dimensions for the base to a cantilever wall can be estimated with the aid of the chart given in *Table 2.86*. For sliding, the chart is valid for non-cohesive soils only. Thus, for bases founded on clay soils, the long-term condition can be investigated by using φ'_{crit}, with $c' = 0$. For the short-term condition, the ratio β does not enter into the calculations for sliding and, taking the contact surface length as the full width of the base, α is given by $\alpha = K_A\gamma l/2c_b$. When α has been determined from this equation, the curve for $\alpha/\sqrt{K_A}$ on the chart can be used to check the values of β and ξ that were obtained for the long-term condition.

Example 3. A cantilever retaining wall on a spread base is required to support level ground and a footpath adjacent to a road. The existing ground may be excavated as necessary to construct the wall, and the excavated ground behind the wall is to be reinstated by backfilling with a granular material. A graded drainage material will be provided behind the wall, with an adequate drainage system at the bottom.

Height of fill to be retained: 4.0 m above top of base
Surcharge: 100 mm surfacing plus 5 kN/m^2 live load
(design for minimum value of 10 kN/m^2)

Properties of retained soil (well graded sand and gravel):
unit weight of soil $\gamma = 20$ kN/m^3

$\varphi'_{\max} = \varphi'_{\text{crit}} = 35^{\circ}$, design $\varphi' = \tan^{-1}[(\tan 35^{\circ})/1.2] = 30^{\circ}$
$K_A = (1 - \sin\varphi')/(1 + \sin\varphi') = 0.33$

Properties of sub-base soil (medium sand):
allowable bearing value $p_{\max} = 200$ kN/m^2

$\varphi'_{\max} = 35^{\circ}$, $\varphi'_{\text{crit}} = 32^{\circ}$, design $\varphi' = 30^{\circ}$ (as fill)
design $\tan\delta_b = 0.75$ design $\tan\varphi' = 0.43$

Take thickness of both wall (at bottom of stem) and base to be equal to (height of fill)/10 = 4000/10 = 400 mm

Height of wall to underside of base, $l = 4.0 + 0.4 = 4.4$ m.

Allowing for surcharge, equivalent height of wall

$$l_e = l + q/\gamma = 4.4 + 10/20 = 4.9 \text{ m}$$
$$\xi = p_{\max}/\gamma l_e = 200/(20 \times 4.9) \approx 2.0$$
$$\psi = \tan\delta_b/\sqrt{K_A} = 0.43/\sqrt{0.33} = 0.75$$

From *Table 2.86*, $\alpha/\sqrt{K_A} = 0.8$, $\beta = 0.18$. Hence,

$$\text{Width of base} = \alpha l_e = (0.8\sqrt{0.33}) \times 4.9 = 2.25 \text{ m}$$
$$\text{Toe projection} = \beta(\alpha l_e) = 0.18 \times 2.25 = 0.4 \text{ m}$$

Example 4. The sub-base for the wall described in example 3 is a clay soil with properties as given here. All other values are as specified in example 3.

Properties of sub-base soil (firm clay):
allowable bearing value $p_{\max} = 100$ kN/m^2, $c_u = 50$ kN/m^2

$c_{ud} = 50/1.5 = 33.3$ kN/m^2, design $c_b = 50/2 = 25$ kN/m^2
$\varphi'_{\text{crit}} = 25^{\circ}$ (assumed plasticity index 30%)
design $\tan\delta_b = 0.75$ design $\tan\varphi'_{\text{crit}} = 0.33$

For the long-term condition:

$$\xi = p_{\max}/\gamma l_e = 100/(20 \times 4.9) \approx 1.0$$
$$\psi = \tan\delta_b/\sqrt{K_A} = 0.33/\sqrt{0.33} = 0.57$$

From *Table 2.86*, $\alpha/\sqrt{K_A} = 1.15$, $\beta = 0.24$. Hence,

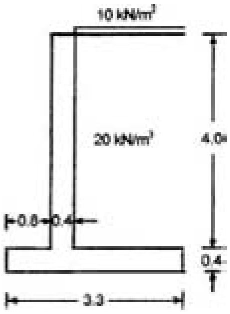
$$\text{Width of base} = \alpha l_e = (1.15\sqrt{0.33}) \times 4.9 = 3.3 \text{ m say}$$
$$\text{Toe projection} = \beta(\alpha l_e) = 0.24 \times 3.3 = 0.8 \text{ m}$$

For the short-term condition:

$$\alpha = K_A\ \gamma l/2c_b = 0.33 \times 20 \times 4.9/(2 \times 25) = 0.65$$
$$\alpha/\sqrt{K_A} = 0.65/\sqrt{0.33} = 1.13\ (<1.15)$$

Since this value is less than that calculated for the long-term condition, the base dimensions are satisfactory.

Example 5. The wall obtained in example 4, a cross section through which is shown here, is to be designed to BS 8002.



The vertical loads and bending moments about the front edge of the base are:

		Load (kN)	Moment (kNm)
Surcharge	10×2.1	$= 21.0$	$\times 2.25 = 47.3$
Backfill	$20 \times 2.1 \times 4.0$	$= 168.0$	$\times 2.25 = 378.0$
Wall stem	$24 \times 0.4 \times 4.0$	$= 38.4$	$\times 1.0 = 38.4$
Wall base	$24 \times 0.4 \times 3.3$	$= 31.7$	$\times 1.65 = 52.3$
	Totals	$F_v = 259.1$	516.0

The horizontal loads and bending moments about the bottom of the base are:

		Load (kN)	Moment (kNm)
Surcharge	$0.33 \times 10 \times 4.4$	$= 14.5$	$\times 4.4/2 = 31.9$
Backfill	$0.33 \times 20 \times 4.4^2/2$	$= 63.9$	$\times 4.4/3 = 93.7$
	Totals	$F_h = 78.4$	125.6

Resultant moment $M_{\text{net}} = 516 - 125.6 = 390.4$ kNm

Distance from front edge of base to resultant vertical force

$$a = M_{net}/F_v = 390.4/259.1 = 1.50 \text{ m}$$

Eccentricity of vertical force relative to centreline of base

$$e = 3.3/2 - 1.5 = 0.15 \text{ m } (< 3.3/6 = 0.55 \text{ m})$$

Maximum pressure at front of base

$$p_{max} = (259.1/3.3)(1 + 6 \times 0.15/3.3) = 100 \text{ kN/m}^2$$

Minimum pressure at back of base

$$p_{min} = (259.1/3.3)(1 - 6 \times 0.15/3.3) = 57 \text{ kN/m}^2$$

For the ultimate bearing condition, a uniform distribution is considered of length $l_b = 2a = 2 \times 1.5 = 3.0$ m. Pressure

$$p_u = F_v/l_b = 259.1/3.0 = 86.4 \text{ kN/m}^2$$

The ultimate bearing resistance is given by the equation:

$$q_u = (2 + \pi)\, c_{ud}\, i_c \quad \text{where } i_c = 0.5[1 + \sqrt{1 - F_h/(c_{ud}\, l_b)}]$$
$$i_c = 0.5[1 + \sqrt{1 - 78.4/(33.3 \times 3.0)}] = 0.73$$
$$q_u = (2 + \pi) \times 33.3 \times 0.73 = 125 \text{ kN/m}^2 \ (> p_u = 86.4)$$

Resistance to sliding (long-term)

$$= F_v \tan \delta_b = 259.1 \times 0.33 = 85.5 \text{ kN } (> F_h = 78.4)$$

Resistance to sliding (short-term)

$$= (\alpha l_e)\, c_b = 3.3 \times 25 = 82.5 \text{ kN } (> F_h = 78.4)$$

For resistance to sliding (short-term), the contact surface has been taken as the full width of the base. This is considered reasonable, since base adhesion is taken as only $0.75c_{ud}$. If the contact surface is based on the pressure diagram assumed for the ultimate bearing condition, the resistance to sliding is reduced to: $l_b\, c_b = 3.0 \times 25 = 75$ kN.

Example 6. The structural design of the wall in example 5 is to be in accordance with the requirements of BS 8110.

$$f_{cu} = 35 \text{ N/mm}^2, f_y = 500 \text{ N/mm}^2, \text{ nominal cover} = 40 \text{ mm}$$

Allowing for H16 bars with 40 mm cover,

$$d = 400 - (40 + 8) = 352 \text{ mm}$$

For the ULS, values of γ_f are taken as 1.2 for the horizontal loads, and 1.4 for all the vertical loads.

The ultimate bending moment at the bottom of the wall stem:

$$M = 1.2 \times 0.33 \times (10 \times 4^2/2 + 20 \times 4^3/6) = 116.2 \text{ kNm/m}$$
$$M/bd^2 f_{cu} = 116.2 \times 10^6/(1000 \times 352^2 \times 35) = 0.0268$$

From *Table 3.14*, since $K \leq 0.043$, $z = 0.95d$

$$A_s = 116.2 \times 10^6/[(0.87 \times 500)(0.95 \times 352)] = 799 \text{ mm}^2/\text{m}$$

From *Table 2.20*, H16-250 gives 804 mm^2/m

The ultimate shear force at the bottom of the wall stem:

$$V = 1.2 \times 0.33 \times (10 \times 4 + 20 \times 4^2/2) = 79.2 \text{ kN/m}$$
$$V/bd = 79.2 \times 10^3/(1000 \times 352) = 0.23 \text{ N/mm}^2 \ (< v_c)$$

From *Table 3.43*, the clear distance between bars should not exceed $(47\,000/f_s)/(100A_s/bd) \leq 750$ mm. Thus, with

$$f_s = 0.87 f_y/\gamma_f = 0.87 \times 500/1.2 = 362 \text{ N/mm}^2$$
$$a_b \leq (47\,000/362)/[100 \times 804/(1000 \times 352)] = 568 \text{ mm}$$

Since the γ_f values used for the horizontal and vertical loads are not the same, the bearing pressures must be recalculated.

$$F_v = 1.4 \times 259.1 = 362.7 \text{ kN},\ M_v = 1.4 \times 516 = 722.4 \text{ kNm}$$
$$M_{net} = 722.4 - 1.2 \times 125.6 = 571.7 \text{ kNm}$$
$$a = 571.7/362.7 = 1.575 \text{ m},\ e = 1.65 - 1.575 = 0.075 \text{ m}$$
$$p_{max} = (362.7/3.3)(1 + 6 \times 0.075/3.3) = 125 \text{ kN/m}^2$$
$$p_{min} = (362.7/3.3)(1 - 6 \times 0.075/3.3) = 95 \text{ kN/m}^2$$

Bearing pressure under base at inside face of wall

$$p_{wall} = 95 + (125 - 95)(2.1/3.3) = 95 + 19 = 114 \text{ kN/m}^2$$

Bending moment on base at inside face of wall

$$M = 1.4 \times (10 + 20 \times 4 + 24 \times 0.4) \times 2.1^2/2$$
$$-(95 \times 2.1^2/2 + 19 \times 2.1^2/6) = 84 \text{ kNm}$$
$$A_s = 84 \times 10^6/[(0.87 \times 500)(0.95 \times 352)] = 578 \text{ mm}^2/\text{m}$$

Use H16-250 to fit in with vertical bars in wall.

Shear force on base at inside face of wall

$$V = 1.4 \times (10 + 20 \times 4 + 24 \times 0.4) \times 2.1$$
$$-(95 \times 2.1 + 19 \times 2.1/2) = 73.4 \text{ kN}$$
$$V/bd = 73.4 \times 10^3/(1000 \times 352) = 0.21 \text{ N/mm}^2 \ (< v_c)$$

For the base, the bending moment and shear force have been calculated for a bearing pressure diagram that varies linearly as indicated in BS 8110. If the pressure diagram assumed for the ultimate bearing condition in example 5 is taken,

$$p_u = F_v/2a = 362.7/(2 \times 1.575) = 115.1 \text{ kN/m}^2$$
$$M = 1.4 \times (10 + 20 \times 4 + 24 \times 0.4) \times 2.1^2/2$$
$$-115.1 \times (3.15 - 1.2)^2/2 = 88.6 \text{ kNm}$$
$$V = 1.4 \times (10 + 20 \times 4 + 24 \times 0.4) \times 2.1$$
$$-115.1 \times (3.15 - 1.2) = 68.4 \text{ kN}$$

28.5 RECOMMENDED DETAILS

The information given in *Tables 3.62* and *3.63* has been taken from several sources, including BS 8110, research undertaken by the Cement and Concrete Association (C&CA), and reports published by the Concrete Society.

28.5.1 Continuous nibs

The BS 8110 recommendations are shown in *Table 3.62*. As a result of investigations by the C&CA, various methods of reinforcing continuous nibs were put forward. Method (a) is efficient but it is difficult to incorporate the bars in shallow nibs, if the bends in the bars are to meet the minimum code requirements. Method (b) is reasonably efficient and it is a simple matter to anchor the bars at the outer face of the nib. The C&CA recommendations were based on an assumption of truss action. BS 8110 suggests that such nibs should be designed as short cantilever slabs, but both methods lead to similar amounts of reinforcement.

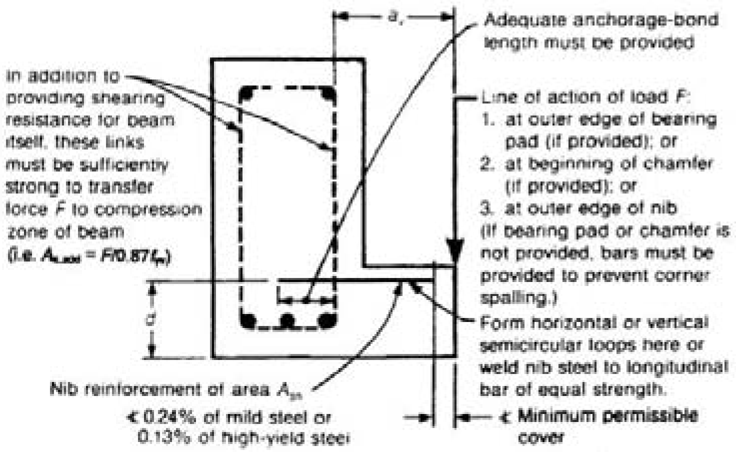
28.5.2 Corbels

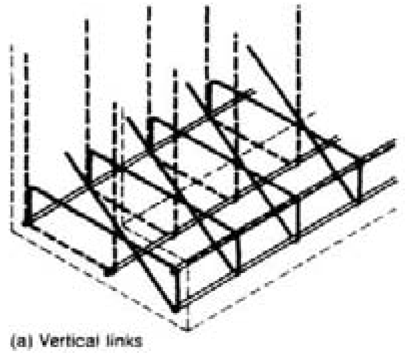
The information in *Table 3.62* is based on the requirements of BS 8110 and BS 5400, supplemented by recommendations

Recommended details: nibs, corbels and halving joints

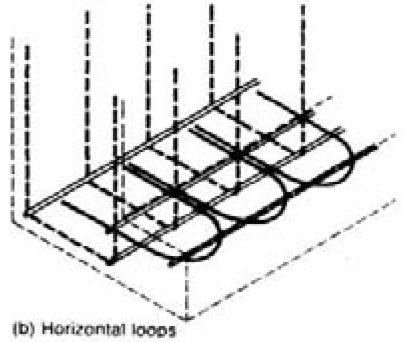
3.62

Continuous nibs





(a) Vertical links

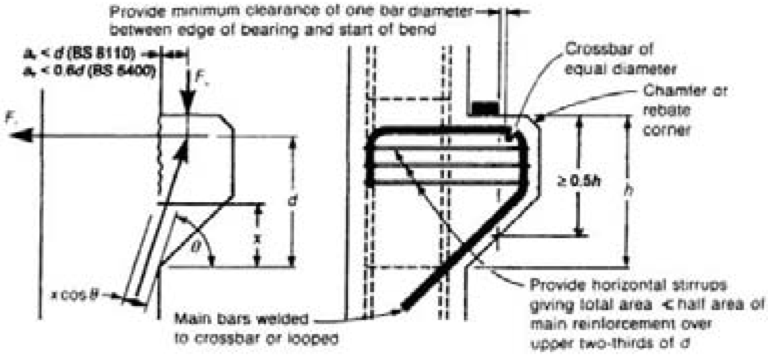


(b) Horizontal loops

Design procedure in accordance with BS 8110

1. Determine $A_{sn} = M/0.87f_yz$, where $M = Fa_v$
2. Check that V_c exceeds F, where
   $V_c = v_c(2d/a_v)bd \leq V_{c,\max}$
   and v_c is obtained from *Table 3.33.*
3. Determine $A_{s,add}$ to be provided by inner legs of links in beam, in addition to link requirements for shear and torsion.

Corbels



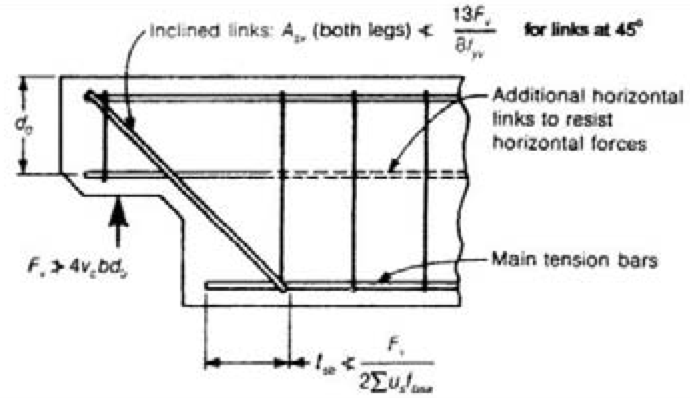
$$\left[k_2^2 + \frac{k_1k_2}{k_v}\left(\frac{a_v}{d}\right)\right]\left(\frac{x}{d}\right)^2 - \left[2k_2 + \frac{k_1}{k_v}\left(\frac{a_v}{d}\right)\right]\left(\frac{x}{d}\right) + \left[1 + \left(\frac{a_v}{d}\right)^2\right] = 0$$

where values of k_1 and k_2 are given in section 24.1, and $k_v = F_v/bdf_{cu}$

Design procedure

1. Determine depth d based on shear considerations, where
   $V_c = v_c(2d/a_v)bd \leq V_{c,\max}$
   and v_c is obtained from *Table 3.33* or *Table 3.36.*
2. Provide area of main reinforcement
   $A_s = F_va_v/(d - k_2x)f_{yd}$
   where f_{yd} is the design stress in the reinforcement, appropriate to the value of x/d, where x/d is obtained from the quadratic equation given below the figure opposite.
3. In BS 8110, $A_s \geq 0.5F_v/f_{yd}$ should be provided. In BS 5400, $A_s \geq 0.004bh$ should be provided.

Halving joints

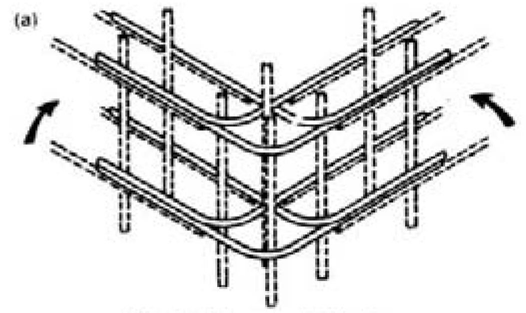


Note. If the main reinforcement is bent or hooked vertically, the inclined links should be anchored by bending them parallel to the main reinforcement. In this case, and if bent-up bars are used instead of inclined links, the bearing stress inside the bends should satisfy the relationship given in *Table 3.59.*

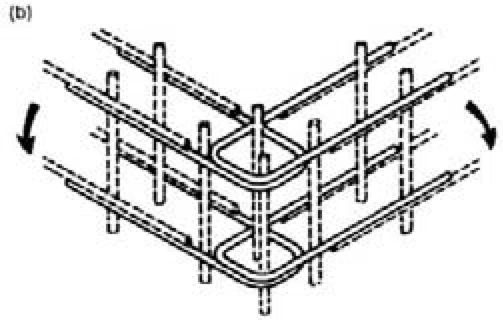
Recommended details: intersections of members

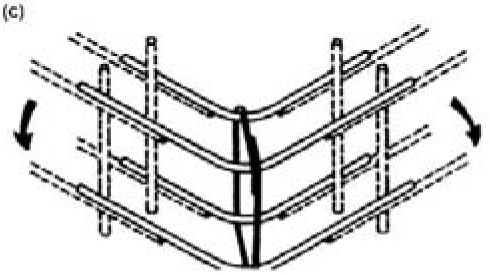
3.63

Wall-to-wall intersections

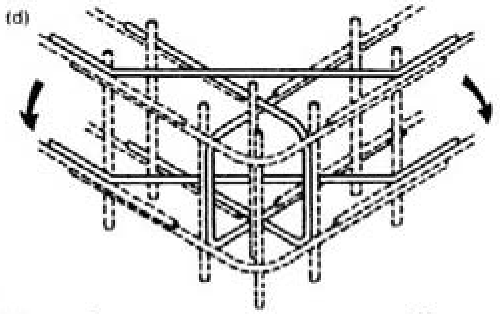


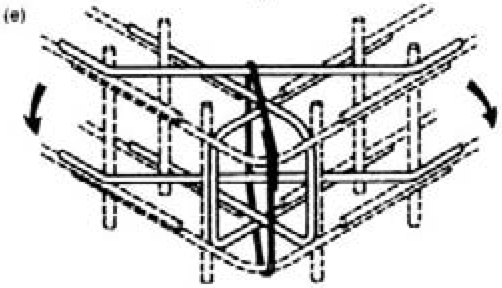
(This detail is only suitable where moments tend to close corner)

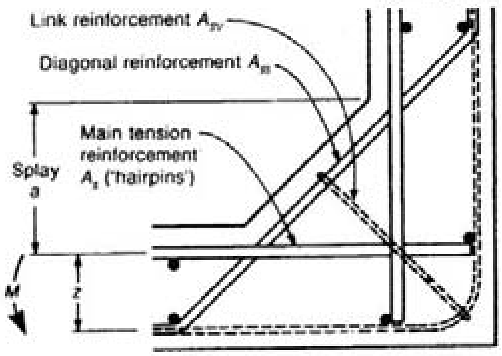




$$A_s = \frac{M}{0.87 f_y z} \qquad A_{sv} = \sqrt{(2)} f_y A_s / f_{yv}$$



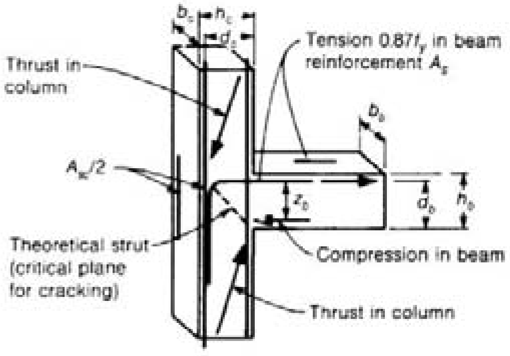




$$A_s = \frac{M}{0.87 f_y z} \qquad A_{ss} = \frac{\sqrt{(2)}(M - 0.6 \times 0.87 A_s f_y z)}{0.87 f_{ys}(a + z)}$$

If A_s (high-yield steel) $> bd/100$, $A_{sv} = 0.6 f_y A_s / f_{yv}$

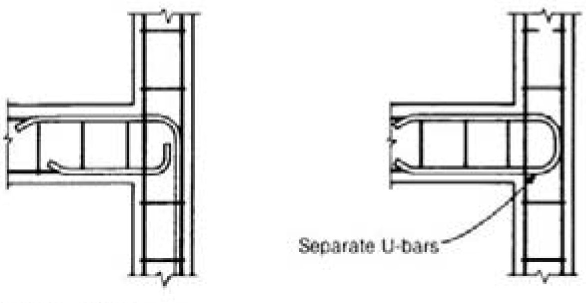
Beam-column intersections



Limit-state of serviceability

$$\frac{5N_d}{b_c h_c}\left\{\sqrt{\left[1 + \left(\frac{3M_d}{z_b N_d}\right)^2\right]} - 1\right\} \not> f_{cu}$$

N_d thrust in column *above* beam and
M_d moment in beam, both *due to service loads*



Ultimate limit-state

$$A_s \not> \left(3 + \frac{2d_c}{z_b}\right)\frac{\beta v_c b_c d_c}{0.87 f_y}$$

β ratio of ultimate resistance moment in beam at column face after redistribution to that before redistribution
v_c unit shearing resistance for column section corresponding to f_{cu} and $\rho = A_{sc}/2b_c d_c$

taken from C&CA research reports. For the system of forces shown in the figure, the inclined force in the concrete

$$F_c = F_v/\sin\theta = k_1 f_{cu} b(x\cos\theta) \quad \text{where } \tan\theta = (d - k_2 x)/a_v$$

In these expressions, k_1 and k_2 are properties of the concrete stress block as given in section 24.1. From these expressions, a quadratic equation in x/d (given in the table) can be derived. In BS 8110, for values of $x/d \le (1 - 2a_v/d)/k_2$, a minimum value of $F_t = 0.5F_v$ is taken for the tensile force.

In the detail shown in the table, the main bars are bent back to form a loop. If the bars are welded to a crossbar, they can be curtailed at the outer edge of the corbel. In this case, two additional small diameter bars are needed to support the horizontal stirrups. If the stirrups are required to pass outside the main bars in the column, they should be detailed as two lapping U-bars for ease of assembly.

28.5.3 Halving joints

The recommendations given in BS 5400, as a result of work carried out by the C&CA, are summarised in *Table 3.62*. The inclined links must intersect the line of action of F_v. If this cannot be ensured (e.g. the inclined links could be displaced), or if horizontal forces can occur at the joint, horizontal links must also be provided as shown.

28.5.4 Reinforcement details at frame corners

Research has shown that when frame corners are subjected to bending moments tending to close the corner, the most likely cause of premature failure is due to bearing under the bend of the tension bars at the outside of the corner. Provided that the radius of the bend is gradual and that sufficient anchorage is given for the lapping bars, the use of simple details as shown in (a) or (b) on *Table 3.63* is recommended.

With 'opening corners', the problems are somewhat greater and tests have shown that some details can fail at well below their calculated strength. In this case, the detail shown in (d) is recommended. If at all possible, a concrete splay should be formed within the corner, and the diagonal reinforcement A_{ss} provided with appropriate cover. If a splay is impracticable, the diagonal bars should be included within the corner itself.

Detail (d) is suitable for reinforcement amounts up to about 1%. If more than this is required, transverse links should be included as shown in (e). The arrangement shown in (c) could be used, but special attention needs to be paid to bending and fixing the diagonal links, which must be designed to resist all the force in the main tension bars. Care must also be taken to provide adequate cover to the bars at the inside of the corner.

28.5.5 Beam-column intersections

Research has shown that the forces in a joint between a beam and an end column are as shown in the sketch on *Table 3.63*. Diagonal tensile forces occur at right angles to the theoretical strut that is shown. To ensure that, as a result, diagonal cracks do not form across the corner, a design limit that is related to the service condition is shown in the table. To ensure that the joint has sufficient ultimate strength, an expression has also been developed for a minimum amount of reinforcement that is needed to extend from the top of the beam into the column at the junction. However, for floor beams, it has been shown that the U-bar detail shown in the table is satisfactory.

While research indicates that, unless it is carefully detailed as described, the actual strength of the joint between a beam and an end column could be as little as half of the calculated moment capacity, it seems that internal beam-column joints have considerable reserves of strength. Joints having a beam on one side of a column and a short cantilever on the other are more prone to loss of strength, and it is desirable in such circumstances to detail the joint as for an end column, with the beam reinforcement turned down into the column and the cantilever reinforcement extending into the beam.

Example 7. A corbel is required to support a design ultimate vertical load of 500 kN at a distance of 200 mm from the face of a column. The load is applied through a bearing pad, and the 300 mm wide corbel is to be designed to BS 8110.

$$f_{cu} = 30 \text{ N/mm}^2, f_y = 500 \text{ N/mm}^2, \text{cover} = 40 \text{ mm}$$

Minimum reinforcement, based on $F_t = 0.5F_v$ and $f_{yd} = 0.87f_y$,

$$A_s = 0.5 \times 500 \times 10^3/(0.87 \times 500) = 575 \text{ mm}^2$$

Calculate shear resistance based on 2H20 ($A_s = 628$ mm²) for values of $d \ge 400$ mm, where $a_v = 200$ mm, $b = 300$ mm, v_c is obtained from *Table 3.33*, and $V_c = v_c\,(2d/a_v)bd$.

d mm	$100A_s/bd$	v_c N/mm²	$(2d/a_v)v_c$ N/mm²	V_c ($\times 10^{-3}$) kN
500	0.42	0.50	2.50	375
550	0.38	0.49	2.70	445
600	0.35	0.47	2.82	508

Assuming that A_s will need to be increased as a result of the corbel analysis, with a corresponding increase in V_c, consider $h = 600$ mm with $d = 550$ mm. Hence

$$k_v = F_v/bdf_{cu} = 500 \times 10^3/(300 \times 550 \times 30) = 0.10$$
$$a_v/d = 200/550 = 0.36,\ k_1 = 0.40,\ k_2 = 0.45 \text{ (section 24.1)}$$

From *Table 3.62*, in the quadratic equation for x/d,

$$k_2^2 + \frac{k_1 k_2}{k_v}\left(\frac{a_v}{d}\right) = 0.45^2 + 0.40 \times 0.45 \times 0.36/0.1 = 0.85,$$

$$2k_2 + \frac{k_1}{k_v}\left(\frac{a_v}{d}\right) = 2 \times 0.45 + 0.40 \times 0.36/0.1 = 2.34,$$

$$1 + \left(\frac{a_v}{d}\right)^2 = 1 + 0.36^2 = 1.13 \quad \text{giving the equation}$$

$$0.85(x/d)^2 - 2.34(x/d) + 1.13 = 0 \quad \text{from which } x/d = 0.625$$

Hence, $x = 0.625 \times 550 = 344$ mm, and the strain in the bars

$$\varepsilon_s = 0.0035(d - x)/x = 0.0035 \times 206/344 = 0.0021$$

$$f_{yd} = \varepsilon_s E_s = 0.0021 \times 200 \times 10^3 = 420 \text{ N/mm}^2 \ (\leq 0.87 f_y)$$

Since $(1 - 2a_v/d)/k_2 = (1 - 2 \times 200/550)/0.45 = 0.622 < x/d$

$$F_t = F_v a_v/(d - k_2 x) > F_v/2 \text{ and } A_s = F_v a_v/(d - k_2 x) f_{yd}$$

$$A_s = 500 \times 10^3 \times 200/[(550 - 0.45 \times 344) \times 420] = 603 \text{ mm}^2$$

In this case, 2H20 giving 628 mm^2 is sufficient for the tensile force but insufficient for the shear resistance. Changing the reinforcement to 3H20 gives 942 mm^2 and increases the shear resistance as follows:

$$100A_s/bd = 100 \times 942/(300 \times 550) = 0.57$$

$$v_c = 0.56 \text{ N/mm}^2, \ V_c = v_c\,(2d/a_v)bd = 508 \text{ kN}$$

Minimum area of horizontal links = 0.5 × 942 = 471 mm^2, to be provided by 3H10 links (2 legs per link) which should be located in the tension zone (i.e. extending over a depth of about 200 mm below the main bars).

Part 4

Design to European Codes

Chapter 29

Design requirements and safety factors

In the Eurocodes (ECs), design requirements are set out in relation to specified limit-state conditions. Calculations to determine the ability of a member (or assembly of members) to satisfy a particular limit-state are undertaken using design actions (loads or deformations) and design strengths. These design values are determined from either characteristic actions or representative actions, and from characteristic strengths of materials, by the application of partial safety factors.

29.1 ACTIONS

Characteristic values of the actions to be used in the design of buildings and civil engineering structures are given in several parts of EC 1: *Actions on structures*, as follows:

1991-1-1	Densities, self-weight and imposed loads
1991-1-2	Actions on structures exposed to fire
1991-1-3	Snow loads
1991-1-4	Wind loads
1991-1-5	Thermal actions
1991-1-6	Actions during execution
1991-1-7	Accidental actions due to impact and explosions
1991-2	Traffic loads on bridges
1991-3	Actions induced by cranes and machinery
1991-4	Actions on silos and tanks

A variable action (e.g. imposed load, snow load, wind load, thermal action) has the following representative values:

characteristic value	Q_k
combination value	$\psi_0 Q_k$
frequent value	$\psi_1 Q_k$
quasi-permanent value	$\psi_2 Q_k$

The characteristic and combination values are used for the verification of the ultimate and irreversible serviceability limit-states. The frequent and quasi-permanent values are used for the verification of ULSs involving accidental actions, and reversible SLSs. The quasi-permanent values are also used for the calculation of long-term effects.

Design actions (loads) are given by:

$$\text{design action (load)} = \gamma_F \times \psi\, F_k$$

where F_k is the specified characteristic value of the action, γ_F is the value of the partial safety factor for the action (γ_A for accidental actions, γ_G for permanent actions, γ_Q for variable actions) and the limit state being considered, and ψ is either 1.0, ψ_0, ψ_1 or ψ_2. Recommended values of γ_F and ψ are given in EC 0: *Basis of structural design*.

29.2 MATERIAL PROPERTIES

The characteristic strength of a material f_k means that value of either the cylinder strength f_{ck} or the cube strength $f_{ck,cube}$ of concrete, or the yield strength f_{yk} of reinforcement, below which not more than 5% of all possible test results are expected to fall. In practice the concrete strength is selected from a set of strength classes, which in EC 2 are based on the characteristic cylinder strength. The application rules in EC 2 are valid for reinforcement in accordance with BS EN 10080, whose specified yield strength is in the range 400–600 MPa.

Design strengths are given by:

$$\text{design strength} = f_k/\gamma_M$$

where f_k is either f_{ck} or f_{yk} as appropriate, and γ_M is the value of the partial safety factor for the material (γ_C for concrete, γ_S for steel reinforcement) and the limit-state being considered, as specified in EC 2.

29.3 BUILDINGS

Details of the design requirements and partial safety factors are given in *Table 4.1*. Appropriate combinations of design actions and values of ψ are given on page 336.

29.4 CONTAINMENT STRUCTURES

For structures containing liquids or granular solids, the main representative value of the variable action resulting from the retained material should be taken as the characteristic value for all design situations. Appropriate characteristic values are given in EC 1: Part 4: *Actions in silos and tanks*.

For the ULS, where the maximum level of a retained liquid can be clearly defined and the effective density of the liquid (allowing for any suspended solids) will not vary significantly, the value of γ_Q applied to the resulting characteristic action may be taken as 1.2. Otherwise, and for retained granular materials in silos, $\gamma_Q = 1.5$ should be used.

Limit-state and design consideration*	Combination of design actions
Ultimate (persistent and transient actions)	$\Sigma\gamma_{G,j}\, G_{k,j} + \gamma_{Q,1}\, Q_{k,1} + \Sigma\gamma_{Q,i}\, \psi_{0,i}\, Q_{k,i}$ (j ≥ 1, i > 1)
Ultimate (accidental action)	$(A_d + \Sigma G_{k,j} + (\psi_{1,1} \text{ or } \psi_{2,1})\, Q_{k,1} + \Sigma\psi_{2,i}\, Q_{k,i}$ (j ≥ 1, i > 1)
Serviceability (function, including damage to structural and non-structural elements, e.g. partition walls etc.)	$\Sigma G_{k,j} + Q_{k,1} + \Sigma\psi_{0,i}\, Q_{k,i}$ (j ≥ 1, i > 1)
Serviceability (comfort to user, use of machinery, avoiding ponding of water, etc.)	$\Sigma G_{k,j} + \psi_{1,1}\, Q_{k,1} + \Sigma\psi_{2,i}\, Q_{k,i}$ (j ≥ 1, i > 1)
Serviceability (appearance)	$\Sigma G_{k,j} + \Sigma\psi_{2,i}\, Q_{k,i}$ (j ≥ 1, i ≥ 1)

Note: In the combination of design actions shown above, $Q_{k,1}$ is the leading variable action and $Q_{k,i}$ are any accompanying variable actions. Where necessary, each action in turn should be considered as the leading variable action.
* Serviceability design consideration and associated combination of design actions as specified in the UK National Annex.

Combinations of design actions on buildings

Values of ψ for variable actions (* as specified in the UK National Annex)	ψ_0	ψ_1	ψ_2
Imposed loads (Category and type, see EN 1991-1-1)			
A: domestic, residential area, B: office area	0.7	0.5	0.3
C: congregation area, D: shopping area	0.7	0.7	0.6
E: storage area	1.0	0.9	0.8
F: traffic area (vehicle weight ≤ 30 kN)	0.7	0.7	0.6
G: traffic area (30 kN < vehicle weight ≤ 160 kN)	0.7	0.5	0.3
H: roof	0.7	0	0
Snow loads (see EN 1991-1-3)			
Sites located at altitude > 1000 m above sea level	0.7	0.5	0.2
Sites located at altitude ≤ 1000 m above sea level	0.5	0.2	0
Wind loads (see EN 1991-1-4)	0.5*	0.2	0
Thermal actions (see EN 1991-1-5)	0.6	0.5	0

Values of ψ for variable actions on buildings

For the SLS of cracking, a classification of liquid-retaining structures in relation to the required degree of protection against leakage, and the corresponding design requirements given in EC 2: Part 3, are detailed here. Silos containing dry materials may generally be designed as Class 0, but where the stored material is particularly sensitive to moisture, class 1, 2 or 3 may be appropriate.

Class	Leakage requirements	Design provisions
0	Leakage acceptable or irrelevant.	The provisions in EN 1992-1-1 may be adopted (see *Table 4.1*)
1	Leakage limited to small amount. Some surface staining or damp patches acceptable.	The width of any cracks that can be expected to pass through the full thickness of the section should be limited to w_{k1} given by: $0.05 \leq w_{k1} = 0.225(1 - h_w/45h) \leq 0.2$ mm where h_w/h is the hydraulic gradient (i.e. head of liquid divided by thickness of section) at the depth under consideration. Where the full thickness of the section is not cracked, the provisions in EN 1992-1-1 apply (see *Table 4.1*).
2	Leakage minimal. Appearance not to be impaired by staining.	Cracks that might be expected to pass through the full thickness of the section should be avoided, unless measures such as liners or water bars are included.
3	No leakage permitted	Special measures (e.g. liners or prestress) are required to ensure watertightness.

Note: In classes 1 and 2, to provide adequate assurance that cracks do not pass through the full width of a section, the depth of the compression zone should be at least $0.2h \leq 50$ mm under quasi-permanent loading for all design conditions. The depth should be calculated by linear elastic analysis assuming that concrete in tension is neglected.

Classification of liquid-retaining structures

Design requirements and partial safety factors (EC 2: Part 1)

4.1

	Limit state		Design requirement	Means of compliance
Design requirements for buildings	Ultimate	Structural resistance	Structure, whose resistance is based on the design strengths of materials, should be able to support without collapse, the design effects of specified combinations of design actions.	By calculation (normally), or based on model or prototype tests where necessary.
		Fatigue	Consideration to be given to the effects of imposed actions that are predominately cyclical.	By calculation, if necessary.
	Serviceability	Cracking	Design surface crack width under quasi-permanent load, or due to restrained deformations, ≤ 0.3 mm in general. This limit may be relaxed for exposure classes X0 and XC1, if there is no specific requirement with regard to appearance. For exposure class XD3, special measures may be needed.	Minimum reinforcement with maximum bar size/spacing, or by calculation.
		Deflection (due to vertical loads)	Final sag relative to supports, under characteristic loads and after allowance for any pre-camber, $\leq l/250$, where l is span of member, or length of cantilever. Deflection that occurs after construction, under characteristic loads, $\leq l/500$.	Limiting span/effective depth ratios, or by calculation.
		Stress limitation	For concrete surfaces exposed to chlorides or freeze/thaw, compressive stress under characteristic loads to be $\leq 0.6f_{ck}$. Non-linear creep to be considered, if compressive stress under quasi-permanent loads exceeds $0.45f_{ck}$. Tensile stress in reinforcement under characteristic loads to be $\leq 0.8f_{yk}$, and due to imposed deformation to be $\leq 1.0f_{yk}$.	By calculation, if necessary.
		Vibration	Avoidance of discomfort or alarm to occupants, structural damage, or interference with proper function.	Consult specialist literature if consideration is needed.
	Other	Durability	Structure should perform satisfactorily in the anticipated environment for its design working life, with all embedded metal adequately protected from corrosion.	Minimum concrete strength class, and minimum cover to reinforcement.
		Fire resistance	Structural resistance should be adequate for the appropriate period of time required by regulations.	Minimum concrete size and cover, or by test or analysis.

	Limit state	Values of γ_f for actions (see Note 4)				Values of γ_M for materials	
		Permanent γ_G (see Note 1)		Variable γ_Q (see Note 2)		Concrete	Reinforcing steel
		Unfavourable	Favourable	Unfavourable	Favourable	γ_C	γ_S
Partial safety factors for structural design (see Note 5)	Ultimate (see Note 3)	1.35	1.0	1.5	0	1.5	1.15
	Serviceability	1.0	1.0	1.0	0	1.0	1.0

Note 1. Permanent actions include self-weight of structural and non-structural components, and direct actions resulting from soil (using un-factored soil properties) and water within soil. For the ultimate limit state, a factor of either 1.35 or 1.0 should be applied to both the unfavourable part and the favourable part of the action, whichever gives the more unfavourable effect at the section considered. In cases where the verification of static equilibrium is required, the simultaneous application of 1.35 to the unfavourable part and 1.15 to the favourable part of the action should also be considered. Indirect actions caused by concrete shrinkage and uneven settlement should generally be considered for serviceability limit states, but need not be considered for ultimate limit states provided the elements have sufficient ductility and rotation capacity. If concrete shrinkage is considered for ultimate limit states, $\gamma_{SH} = 1.0$ should be used.

Note 2. Variable actions include imposed loads, snow loads, wind loads and thermal actions. Representative values of the variable actions, incorporating ψ factors appropriate to the design situation, are obtained as given in section 31.3.

Note 3. For accidental design situations, γ_A (unfavourable) = 1.0, $\gamma_C = 1.2$ and $\gamma_S = 1.0$.

Note 4. Other sets of γ_f values are possible, as specified in the National Annexe for the particular member state.

Note 5. In the geotechnical design of foundations and earth-retaining structures, where partial safety factors are applied to the soil properties: for ultimate limit states, γ_G (unfavourable) = 1.0 and γ_Q (unfavourable) = 1.3.

Chapter 30
Properties of materials

30.1 CONCRETE

30.1.1 Strength and elastic properties

The characteristic strength of concrete is defined as that level of compressive strength below which 5% of all valid test results is expected to fall. Strength classes are specified in terms of both cylinder strength and equivalent cube strength. Recommended strength classes with indicative values for the secant modulus of elasticity at 28 days are given in *Table 4.2*. The values given for normal-weight concrete are appropriate for concretes made with quartzite aggregates. For limestone and sandstone aggregates, these values should be reduced by 10% and 30% respectively. For basalt aggregates the values should be increased by 20%.

Variation of the secant modulus of elasticity with time can be estimated by the expression:

$$E_{cm}(t) = [f_{cm}(t)/f_{cm}]^{0.3}\ E_{cm}$$

where $E_{cm}(t)$ and $f_{cm}(t)$ are values at age t days, and E_{cm} and f_{cm} are values determined at age 28 days.

30.1.2 Creep and shrinkage

The creep strain in concrete may be assumed to be directly proportional to the applied stress for stresses not exceeding $0.45f_{ck}(t_0)$, where t_0 is age of concrete at the time of loading. Values of the final creep coefficient, $\varphi(\infty, t_o)$, and the creep development coefficient, $\beta_c(t, t_0)$, according to the time under load, can be obtained from *Table 4.3*. The procedure used to determine the final creep coefficient is as follows:

1. Determine point corresponding to age of loading t_o on the appropriate curve (N for normally hardening cement, R for rapidly hardening cement, S for slowly hardening cement).
2. Construct secant line from origin of curve to the point corresponding to t_o.
3. Determine point corresponding to the notional size of the member h_o on the appropriate curve for the concrete strength class.
4. Cross horizontally from the point determined in 3, to intersect the secant line determined in 2.
5. Drop vertically from the intersection point determined in 4, to obtain the required creep coefficient $\varphi(\infty, t_o)$.

When the applied stress exceeds $0.45f_{ck}(t_0)$ at time of loading, creep non-linearity should be considered and $\varphi(\infty, t_o)$ should be increased as indicated in *Table 4.2*.

The total shrinkage strain is composed of two components: autogenous shrinkage strain and drying shrinkage strain. The autogenous shrinkage strain develops during hardening of the concrete: a major part therefore develops in the early days after casting. It should be considered specifically when new concrete is cast against hardened concrete. Drying shrinkage strain develops slowly, as it is a function of the migration of the water through the hardened concrete. Final values of each component of shrinkage strain, and development coefficients with time, are given in *Table 4.2*.

30.1.3 Thermal properties

Values of the coefficient of thermal expansion of concrete for normal design purposes are given in *Table 4.2*.

30.1.4 Stress–strain curves

Idealised stress–strain curves for concrete in compression are given in *Table 4.4*. Curve A is part parabolic and part linear, and curve B is bi-linear. A simplified rectangular diagram is also given as a further option.

30.2 REINFORCEMENT

30.2.1 Strength and elastic properties

The characteristic yield strength of reinforcement according to EN 10080 is required to be in the range 400–600 MPa. The characteristic yield strength of reinforcement complying with BS 4449 is 500 MPa. For further information on types, properties and preferred sizes of reinforcement, reference should be made to section 10.3 and *Tables 2.19* and *2.20*.

30.2.2 Stress–strain curves

Idealised bi-linear stress–strain curves for reinforcement in tension or compression are shown in *Table 4.4*. Curve A has an inclined top branch up to a specified strain limit, and curve B has a horizontal top branch with no need to check the strain limit. For design purposes, the modulus of elasticity of steel reinforcement is taken as 200 GPa.

4.2

Concrete (EC 2): strength and deformation characteristics – 1

Strength and elastic properties

Normal-weight concrete				Lightweight aggregate concrete		
Concrete strength class	Characteristic cylinder strength at 28 days f_{ck} (MPa)	Characteristic cube strength at 28 days $f_{ck,cube}$ (MPa)	Modulus of elasticity at 28 days E_{cm} (GPa)	Concrete strength class	Characteristic cylinder strength at 28 days f_{lck} (MPa)	Characteristic cube strength at 28 days $f_{lck,cube}$ (MPa)
C20/25	20	25	30	LC20/22	20	22
C25/30	25	30	31	LC25/28	25	28
C30/37	30	37	33	LC30/33	30	33
C35/45	35	45	34	LC35/38	35	38
C40/50	40	50	35	LC40/44	40	44
C45/55	45	55	36	LC45/50	45	50
C50/60	50	60	37	LC50/55	50	55
C55/67	55	67	38	LC55/60	55	60
C60/75	60	75	39	LC60/66	60	66
C70/85	70	85	41	LC70/77	70	77
C80/95	80	95	42	LC80/88	80	88
C90/105	90	105	44	—	—	—
Secant modulus of elasticity (σ_c = 0 to 0.4 f_{cm}) at 28 days		$E_{cm} = 22(f_{cm}/10)^{0.3}$ where $f_{cm} = f_{ck} + 8$ (MPa)		$E_{lcm} = E_{cm}(\rho/2200)^2$ where ρ is oven-dry density of concrete (kg/m^3)		
Poisson's ratio		May normally be taken as 0.2 for un-cracked concrete and 0 for cracked concrete.				

Creep

Creep strain in concrete at time t, for a constant stress $\sigma_c \leq 0.45 f_{ck}(t_0)$ applied at time t_0, can be predicted from:

$$\varepsilon_{cc}(t, t_0) = \frac{\sigma_c}{E_c}\ \varphi(\infty, t_0)\ \beta_c(t, t_0)$$

where E_c is the tangent modulus which may be taken as $1.05E_{cm}$. $\varphi(\infty, t_0)$ is the final creep coefficient and $\beta_c(t, t_0)$ is a creep development coefficient, values of which for inside (RH = 50%) and outside (RH = 80%) conditions, can be estimated from *Table 4.3*. For cases where $\sigma_c > 0.45 f_{ck}(t_0)$, creep non-linearity should be considered and $\varphi(\infty, t_0)$ should be multiplied by $\exp\{1.5[\sigma_c/f_{cm}(t_0) - 0.45]\}$, where $f_{cm}(t_0)$ is the mean compressive strength at time t_0. For lightweight aggregate concrete, the values given for $\varphi(\infty, t_0)$ should be multiplied by $(\rho/2200)^2$.

Shrinkage

Concrete strength class	Basic shrinkage strains			Time after start of shrinkage	h_0 mm	100	200	300	≥ 500
	$\varepsilon_{ca}(\infty)$ ($\times 10^{-6}$)	$\varepsilon_{cd,0}$ ($\times 10^{-6}$)			k_h	1.0	0.85	0.75	0.70
		RH = 50%	RH = 80%		$\beta_{as}(t)$	$\beta_{ds}(t)$ for values of h_0			
C20/25	25	560	310	1 month	0.67	0.43	0.21	0.13	0.06
C30/37	50	500	280	3 months	0.85	0.69	0.44	0.30	0.17
C40/50	75	450	250	6 months	0.93	0.82	0.62	0.46	0.29
C50/60	100	400	230	1 year	0.98	0.90	0.76	0.63	0.45
C90/105	200	260	150	∞	1.0	1.0	1.0	1.0	1.0

Total shrinkage strain in concrete, at time t after shrinkage starts can be predicted from:

$$\varepsilon_{cs}(t) = \varepsilon_{ca}(\infty)\beta_{as}(t) + k_h\ \varepsilon_{cd,0}\ \beta_{ds}(t)$$

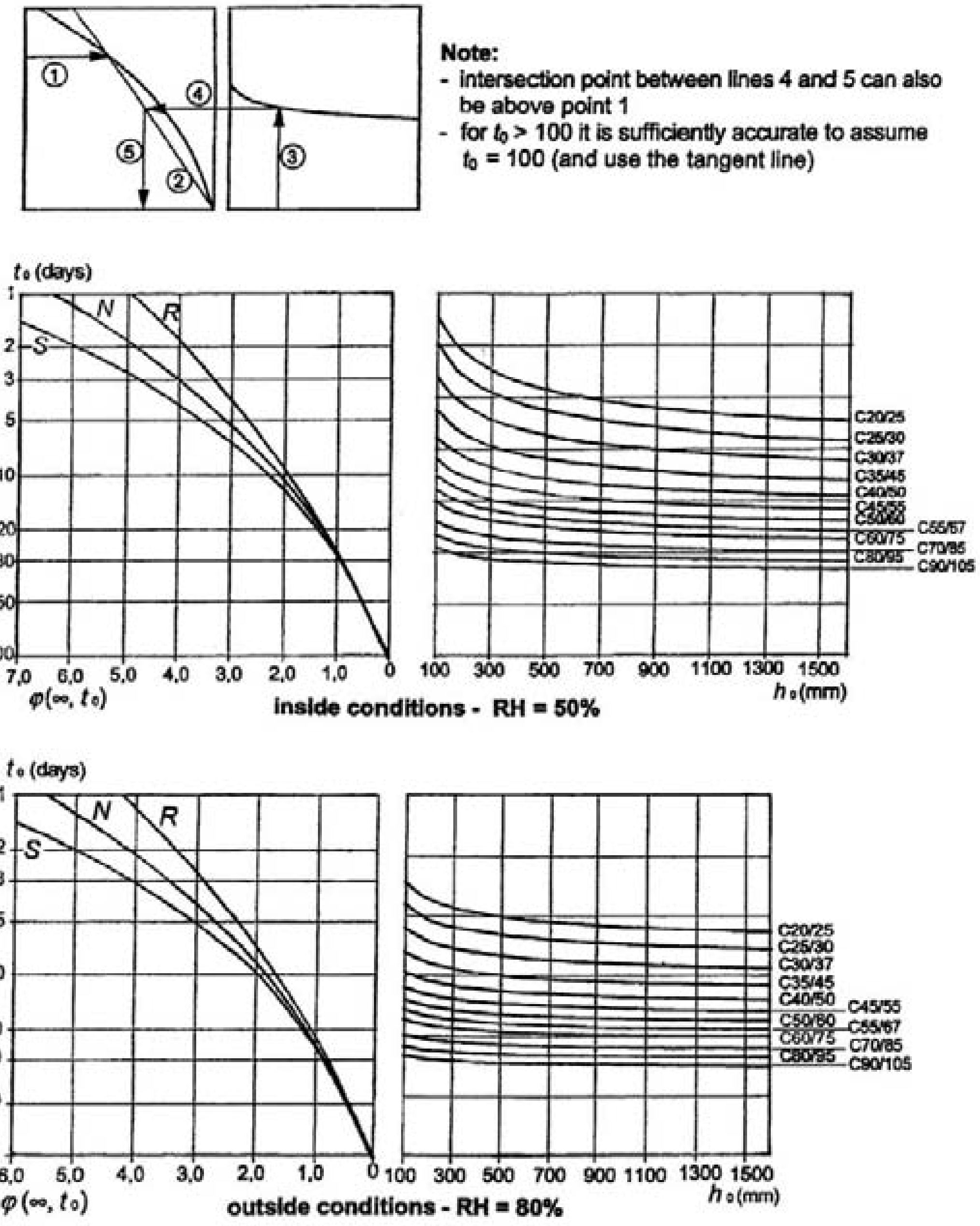
where $\varepsilon_{ca}(\infty)$ is autogenous shrinkage, $k_h\ \varepsilon_{cd,0}$ is drying shrinkage, $\beta_{as}(t)$ and $\beta_{ds}(t)$ are development coefficients. The notional size of the member $h_0 = 2A_c/u$, where A_c is cross-sectional area and u is length of perimeter exposed to drying.

The values given for $\varepsilon_{cd,0}$ apply to normal-weight concrete and cement class N. These should be increased by 30% for cement class R, and may be reduced by 20% for cement class S. For lightweight aggregate concrete, the values given for $\varepsilon_{cd,0}$ should be multiplied by 1.2. The values given for $\varepsilon_{ca}(\infty)$ apply where no supply of water from the aggregate to the drying microstructure is possible. In other conditions, the values of $\varepsilon_{ca}(\infty)$ will be considerably reduced.

Thermal

For normal-weight concrete, the linear coefficient of thermal expansion may be taken as $10\text{x}10^{-6}/^{\circ}\text{K}$, in the absence of more accurate information. For lightweight aggregate concrete, depending on the type of aggregate, the coefficient can vary over a wide range between about $4\text{x}10^{-6}$ and $14\text{x}10^{-6}/^{\circ}\text{K}$. For design purposes, in cases where the calculation is of minor significance, a value of $8\text{x}10^{-6}/^{\circ}\text{K}$ may be taken.

Creep coefficients (see also *Table 4.2*)



Note:
- intersection point between lines 4 and 5 can also be above point 1
- for $t_0 > 100$ it is sufficiently accurate to assume $t_0 = 100$ (and use the tangent line)

Final creep coefficient $\varphi(\infty, t_0)$ for normal-weight concrete

Creep development coefficient $\beta_c(t, t_0)$ for relative humidity (RH) and notional size of member* (mm)

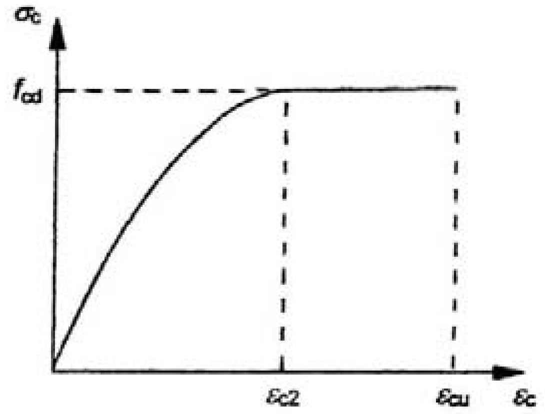
Time under load $(t - t_0)$	Inside conditions (RH = 50%)				Outside conditions (RH = 80%)			
	100	200	300	≥ 500	100	200	300	≥ 500
1 month	0.46	0.42	0.39	0.35	0.44	0.39	0.36	0.32
3 months	0.62	0.57	0.53	0.48	0.59	0.53	0.49	0.44
6 months	0.72	0.67	0.63	0.58	0.69	0.63	0.59	0.54
1 year	0.82	0.77	0.73	0.68	0.79	0.73	0.69	0.64
∞	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

* Notional size of member $h_0 = 2A_c/u$, where A_c is cross-sectional area and u is length of perimeter exposed to drying.

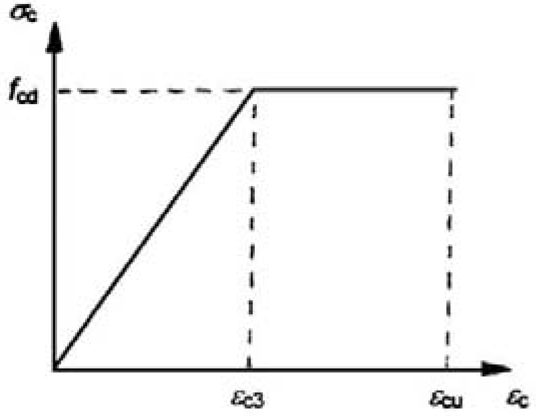
4.4

Stress–strain curves (EC 2): concrete and reinforcement

Concrete



DESIGN STRESS-STRAIN CURVE A



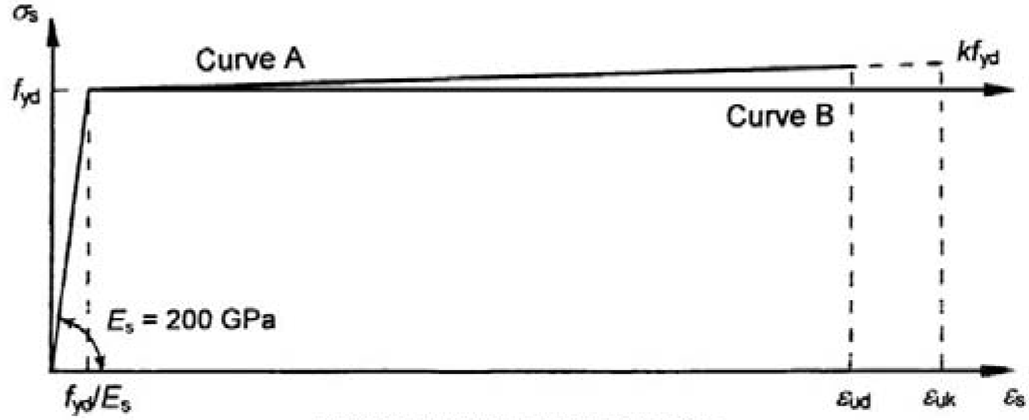
DESIGN STRESS-STRAIN CURVE B

Note 1. In the alternative stress-strain curves above, the maximum design stress $f_{cd} = \alpha_{cc} f_{ck}/\gamma_c$, where values of the strains ε_{c2}, ε_{c3} and ε_{cu} are given in the table below. In the UK National Annex, α_{cc}= 0.85 for compression in flexure and axial load, but may be taken as 1.0 for other phenomena (e.g. struts in truss model assumed for shear resistance).

Note 2. A simplified rectangular stress distribution may also be assumed with the stress taken as ηf_{cd} for a depth from the extreme compression fibre equal to λ times the depth of the compression zone, where values of η and λ are given in the table below according to the value of f_{ck}. If the width of the compression zone decreases in the direction of the extreme compression fibre, the value of ηf_{cd} should be reduced by 10%.

f_{ck} (MPa)	≤ 50	55	60	70	80	90
ε_{c2}	0.0020	0.0022	0.0023	0.0024	0.0025	0.0026
ε_{c3}	0.00175	0.0018	0.0019	0.0020	0.0022	0.0023
ε_{cu}	0.0035	0.0031	0.0029	0.0027	0.0026	0.0026
η	1.0	0.975	0.95	0.90	0.85	0.80
λ	0.8	0.7875	0.775	0.75	0.725	0.70

Reinforcement



DESIGN STRESS-STRAIN CURVES

Note 1. In the alternative stress-strain curves shown above, the top branch of the curve is taken as follows:

Curve A: an inclined top branch with a strain limit of ε_{ud} and a maximum stress of $k f_{yd}$ at ε_{uk}.

Curve B: a horizontal top branch with no need to check the strain limit.

Note 2. The design stress $f_{yd} = f_{yk}/\gamma_s$, the limiting strain $\varepsilon_{ud} = 0.9\,\varepsilon_{uk}$, and values of k, ε_{uk} and ε_{ud} are given in the table below according to the ductility class of the reinforcement (see *Table 2.27*).

Ductility class	k	ε_{uk}	ε_{ud}
A	1.05	0.025	0.0225
B	1.08	0.050	0.0450
C	1.15	0.075	0.0675

Chapter 31

Durability and fire-resistance

In the following, the concrete cover to the first layer of bars, as shown on the drawings, is described as the nominal cover. It is defined as a minimum cover plus an allowance in design for deviation. A minimum cover is required to ensure the safe transmission of bond forces, the protection of steel against corrosion, and an adequate fire-resistance. In order to transmit bond forces safely and ensure adequate concrete compaction, the minimum cover should be not less than the bar diameter or, for bundled bars, the equivalent diameter of a notional bar having the same cross-sectional area as the bundle.

31.1 DURABILITY

31.1.1 Exposure classes

Details of the classification system used in BS EN 206–1 and BS 8500-1, with informative examples applicable in the United Kingdom are given in *Table 4.5*. Often, the concrete can be exposed to more than one of the actions described in the table, in which case a combination of the exposure classes will apply.

31.1.2 Concrete strength classes and covers

Concrete durability is dependent mainly on its constituents, and limitations on the maximum free water/cement ratio and the minimum cement content are specified for each exposure class. These limitations result in minimum concrete strength classes for particular cements. For reinforced concrete, the protection of the steel against corrosion depends on the cover. The required thickness of cover is related to the exposure class, the concrete quality and the intended working life of the structure. Details of the recommendations in BS 8500 are given in *Table 4.6*.

The values given for the minimum cover apply for ordinary carbon steel in concrete without special protection, and for structures with an intended working life of at least 50 years. The values given for the nominal cover include an allowance for tolerance of 10 mm, which is recommended for buildings and is normally also sufficient for other types of structures.

For uneven concrete surfaces (e.g. ribbed finish or exposed aggregate), the cover should be increased by at least 5 mm. If *in-situ* concrete is placed against another concrete element (precast or *in-situ*), the minimum cover to the reinforcement at the interface need be no more than that recommended for adequate bond, provided the following conditions are met: the concrete strength class is at least C25/30, the exposure time of the concrete surface to an outdoor environment is no more than 28 days, and the interface has been roughened.

Where concrete is cast against prepared ground (including blinding), the nominal cover should be at least 50 mm. For concrete cast directly against the earth, the nominal cover should be at least 75 mm.

31.2 FIRE-RESISTANCE

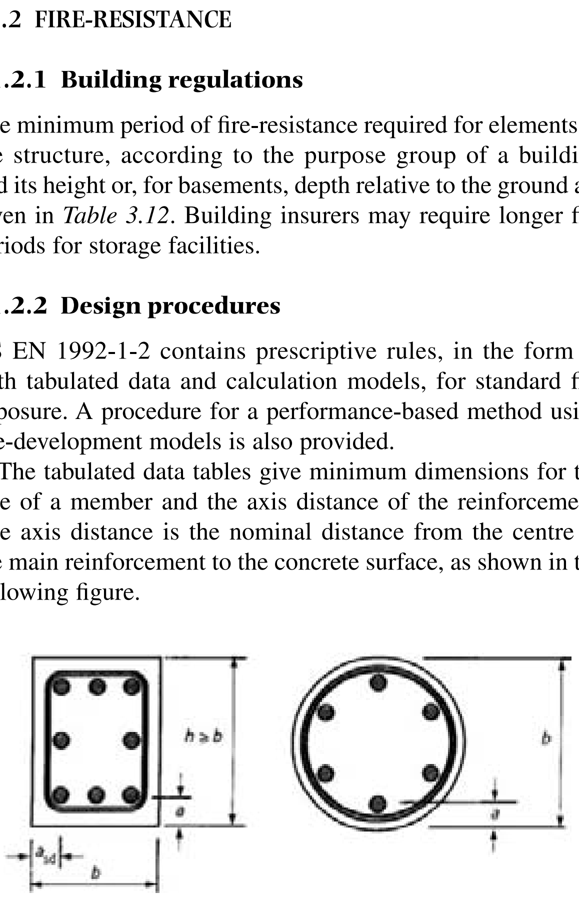
31.2.1 Building regulations

The minimum period of fire-resistance required for elements of the structure, according to the purpose group of a building and its height or, for basements, depth relative to the ground are given in *Table 3.12*. Building insurers may require longer fire periods for storage facilities.

31.2.2 Design procedures

BS EN 1992-1-2 contains prescriptive rules, in the form of both tabulated data and calculation models, for standard fire exposure. A procedure for a performance-based method using fire-development models is also provided.

The tabulated data tables give minimum dimensions for the size of a member and the axis distance of the reinforcement. The axis distance is the nominal distance from the centre of the main reinforcement to the concrete surface, as shown in the following figure.

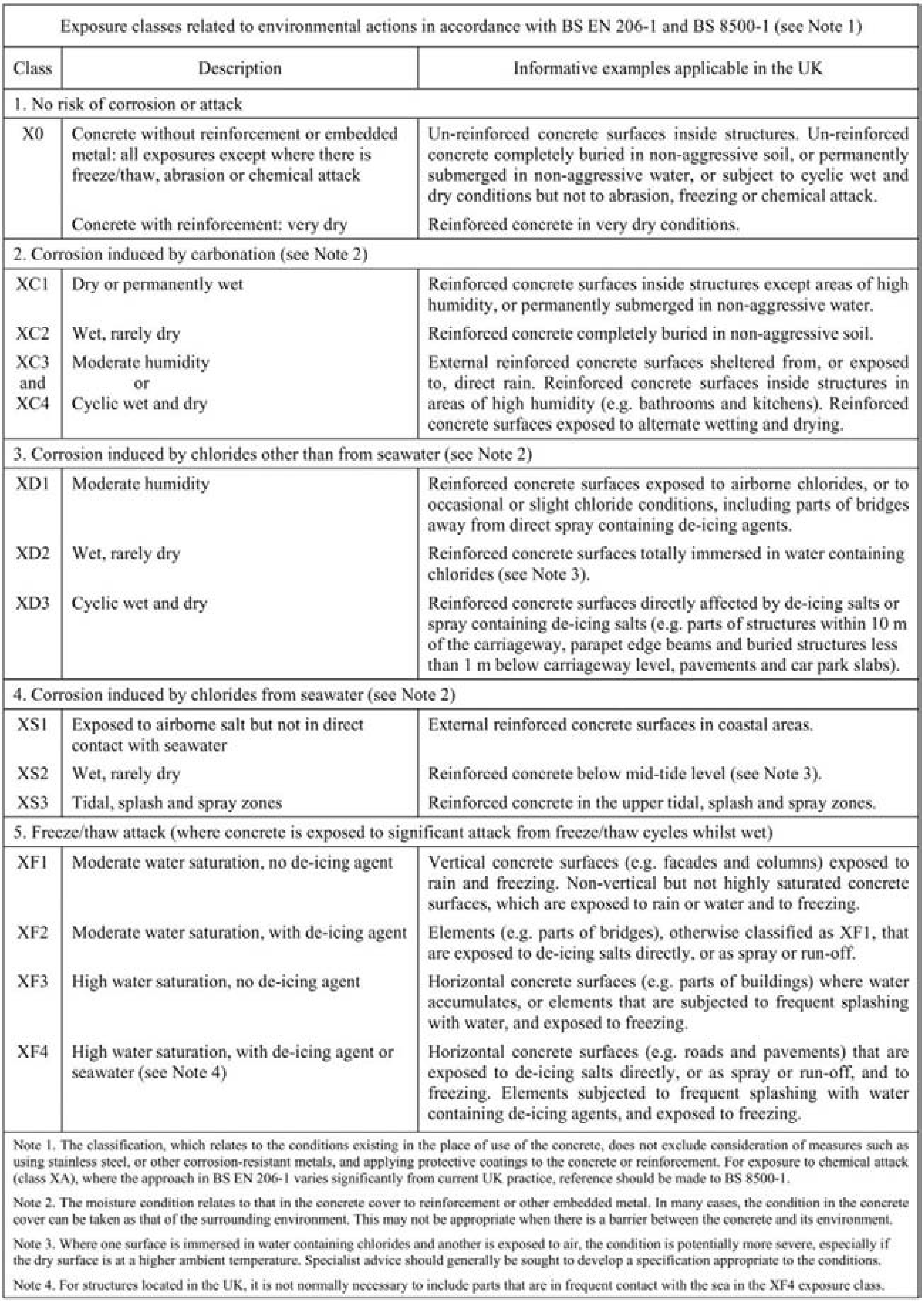


Section through member showing nominal axis distance

Tabulated data is given for beams, slabs and braced columns, for which provision is made for the load level to be taken into account. In many cases, for fire periods up to about two hours, the cover required for other purposes will control. For further information on all the design procedures, reference should be made to BS EN 1992-1-2.

4.5

Exposure classification (BS 8500)



Exposure classes related to environmental actions in accordance with BS EN 206-1 and BS 8500-1 (see Note 1)		
Class	Description	Informative examples applicable in the UK
1. No risk of corrosion or attack		
X0	Concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack	Un-reinforced concrete surfaces inside structures. Un-reinforced concrete completely buried in non-aggressive soil, or permanently submerged in non-aggressive water, or subject to cyclic wet and dry conditions but not to abrasion, freezing or chemical attack.
	Concrete with reinforcement: very dry	Reinforced concrete in very dry conditions.
2. Corrosion induced by carbonation (see Note 2)		
XC1	Dry or permanently wet	Reinforced concrete surfaces inside structures except areas of high humidity, or permanently submerged in non-aggressive water.
XC2	Wet, rarely dry	Reinforced concrete completely buried in non-aggressive soil.
XC3 and XC4	Moderate humidity or Cyclic wet and dry	External reinforced concrete surfaces sheltered from, or exposed to, direct rain. Reinforced concrete surfaces inside structures in areas of high humidity (e.g. bathrooms and kitchens). Reinforced concrete surfaces exposed to alternate wetting and drying.
3. Corrosion induced by chlorides other than from seawater (see Note 2)		
XD1	Moderate humidity	Reinforced concrete surfaces exposed to airborne chlorides, or to occasional or slight chloride conditions, including parts of bridges away from direct spray containing de-icing agents.
XD2	Wet, rarely dry	Reinforced concrete surfaces totally immersed in water containing chlorides (see Note 3).
XD3	Cyclic wet and dry	Reinforced concrete surfaces directly affected by de-icing salts or spray containing de-icing salts (e.g. parts of structures within 10 m of the carriageway, parapet edge beams and buried structures less than 1 m below carriageway level, pavements and car park slabs).
4. Corrosion induced by chlorides from seawater (see Note 2)		
XS1	Exposed to airborne salt but not in direct contact with seawater	External reinforced concrete surfaces in coastal areas.
XS2	Wet, rarely dry	Reinforced concrete below mid-tide level (see Note 3).
XS3	Tidal, splash and spray zones	Reinforced concrete in the upper tidal, splash and spray zones.
5. Freeze/thaw attack (where concrete is exposed to significant attack from freeze/thaw cycles whilst wet)		
XF1	Moderate water saturation, no de-icing agent	Vertical concrete surfaces (e.g. facades and columns) exposed to rain and freezing. Non-vertical but not highly saturated concrete surfaces, which are exposed to rain or water and to freezing.
XF2	Moderate water saturation, with de-icing agent	Elements (e.g. parts of bridges), otherwise classified as XF1, that are exposed to de-icing salts directly, or as spray or run-off.
XF3	High water saturation, no de-icing agent	Horizontal concrete surfaces (e.g. parts of buildings) where water accumulates, or elements that are subjected to frequent splashing with water, and exposed to freezing.
XF4	High water saturation, with de-icing agent or seawater (see Note 4)	Horizontal concrete surfaces (e.g. roads and pavements) that are exposed to de-icing salts directly, or as spray or run-off, and to freezing. Elements subjected to frequent splashing with water containing de-icing agents, and exposed to freezing.

Note 1. The classification, which relates to the conditions existing in the place of use of the concrete, does not exclude consideration of measures such as using stainless steel, or other corrosion-resistant metals, and applying protective coatings to the concrete or reinforcement. For exposure to chemical attack (class XA), where the approach in BS EN 206-1 varies significantly from current UK practice, reference should be made to BS 8500-1.

Note 2. The moisture condition relates to that in the concrete cover to reinforcement or other embedded metal. In many cases, the condition in the concrete cover can be taken as that of the surrounding environment. This may not be appropriate when there is a barrier between the concrete and its environment.

Note 3. Where one surface is immersed in water containing chlorides and another is exposed to air, the condition is potentially more severe, especially if the dry surface is at a higher ambient temperature. Specialist advice should generally be sought to develop a specification appropriate to the conditions.

Note 4. For structures located in the UK, it is not normally necessary to include parts that are in frequent contact with the sea in the XF4 exposure class.

Concrete quality and cover requirements for durability (BS 8500)

4.6

Recommended limiting values for concrete quality (see Note 1) and cover to all reinforcement (including links)

Exposure class (*Table 4.5*)	Maximum water/ cement ratio	Minimum cement content kg/m^3	Minimum strength class (*Table 4.2*) for cement type (see Note 2)			Minimum cover (see Note 3) mm	Nominal cover (see Note 4) mm
			Group 4	Group 5	Group 6		
XC1	0.70	240	C20/25	C20/25	C20/25	15	25
XC2	0.65	260	C25/30	C25/30	C25/30	25	35
XC3/XC4 (cement type IVB invalid)	0.65	260	C25/30	C25/30	C25/30	35	45
	0.60	280	C28/35	C28/35	C28/35	30	40
	0.55	300	C32/40	C32/40	C32/40	25	35
	0.45	340	C40/50	C40/50	C40/50	20	30
XD1	0.60	300	C28/35	C28/35	C28/35	35	45
	0.55	320	C32/40	C32/40	C32/40	30	40
	0.45	360	C40/50	C40/50	C40/50	25	35
XD2	0.55	320	C28/35	C25/30	C20/25	40	50
	0.50	340	C32/40	C28/35	C25/30	35	45
	0.40	380	C40/50	C35/45	C32/40	30	40
XD3	0.45	360	C35/45	—	—	50	60
	0.40	380	C40/50	—	—	45	55
	0.35	380	C45/55	—	—	40	50
	0.50	340	—	C28/35	C25/30	50	60
	0.45	360	—	C32/40	C28/35	45	55
	0.40	380	—	C35/45	C32/40	40	50
XS1	0.50	340	C35/45	C32/40	—	40	50
	0.45	360	C40/50	C35/45	—	35	45
	0.35	380	C50/60	C45/55	—	30	40
	0.55	320	—	—	C20/25	40	50
	0.50	340	—	—	C25/30	35	45
	0.40	380	—	—	C32/40	30	40
XS2	0.55	320	C28/35	C25/30	C20/25	40	50
	0.50	340	C32/40	C28/35	C25/30	35	45
	0.40	380	C40/50	C35/45	C32/40	30	40
XS3	0.40	380	C40/50	—	—	50	60
	0.35	380	C45/55	—	—	45	55
	0.50	340	—	C28/35	C25/30	50	60
	0.45	360	—	C32/40	C28/35	45	55
	0.40	380	—	C35/45	C32/40	40	50

Note 1. The concrete quality recommendations apply for a maximum aggregate size of 20 mm, and an intended working life for the structure of at least 50 years. For concrete subject to freeze/thaw action and concrete in aggressive ground conditions reference should be made to Annex A of BS 8500-1.

Note 2. The cement (or combination) types are those listed in Table A.17 of BS 8500-1 (see also *Table 2.17*), as follows:

Group 4	Portland cement, Portland-slag cement, Sulfate-resisting Portland cement
Group 5	Portland-fly ash cement, Blastfurnace cement (36–65% ggbs)
Group 6	Blastfurnace cement (66–80% ggbs), Pozzolanic cement

Note 3. The minimum cover values apply for ordinary carbon steel in concrete without special protection, and an intended working life for the structure of at least 50 years.

Note 4. The allowance for tolerance of 10 mm is generally acceptable for buildings, and is normally also sufficient for other types of structures. In certain situations, this allowance may be reduced (e.g. where fabrication is subjected to a quality assurance system, in which the monitoring includes measurements of the concrete cover; or where, for precast elements, it can be assured that a very sensitive measurement device is used for monitoring and non-conforming members are rejected.

Chapter 32

Bending and axial force

32.1 DESIGN ASSUMPTIONS

Basic assumptions regarding the design of cracked concrete sections at the ULS are outlined in section 5.2. The tensile strength of the concrete is neglected and strains are evaluated on the assumption that plane sections before bending remain plane after bending. Reinforcement stresses are then derived from these strains on the basis of the design stress-strain curves shown on *Table 4.4*. Two alternatives are given in which the top branch of the curve is taken as either horizontal with no need to check the strain limit, or inclined up to a specified strain limit. For the concrete stresses, three alternative assumptions are permitted as shown on *Table 4.4*. The design stress–strain curves give stress distributions that are a combination of either a parabola and a rectangle, or a triangle and a rectangle. Alternatively, a rectangular concrete stress distribution may be assumed. Whichever alternative is used, the proportions of the stress-block and the maximum strain are constant for $f_{ck} \leq 50$ MPa, but vary as the concrete strength changes for $f_{ck} > 50$ MPa.

For a rectangular area of width b and depth x, the total compressive force is given by $k_1 f_{ck} bx$ and the distance of the force from the compression face is given by $k_2 x$, where values of k_1 (including for the term α_{cc}/γ_c) and k_2 according to the shape of the stress block are given in the table opposite.

32.2 BEAMS AND SLABS

Beams and slabs are generally subjected to bending only but, sometimes, are also required to resist an axial force, for example in a portal frame, or in a floor acting as a prop between basement walls. Axial thrusts not greater than $0.12 f_{ck}$ times the area of the cross section may be ignored in the analysis of the section, since the effect of the axial force is to increase the moment of resistance.

In cases where, as a result of moment redistribution allowed in the analysis of the member, the design moment is less than the maximum elastic moment at the section, the necessary ductility may be assumed without explicit verification if the neutral axis depth satisfies the limits given in the table opposite.

f_{ck} MPa	Shape of stress-block	k_1	k_2	ε_{cu}
≤50	Parabola-rectangle	0.459	0.416	0.0035
	Triangle-rectangle	0.425	0.389	
	Rectangular	0.453	0.400	
55	Parabola-rectangle	0.433	0.400	0.0031
	Triangle-rectangle	0.402	0.375	
	Rectangular	0.435	0.394	
60	Parabola-rectangle	0.417	0.392	0.0029
	Triangle-rectangle	0.381	0.363	
	Rectangular	0.417	0.388	
70	Parabola-rectangle	0.399	0.383	0.0027
	Triangle-rectangle	0.357	0.351	
	Rectangular	0.382	0.375	
80	Parabola-rectangle	0.385	0.378	0.0026
	Triangle-rectangle	0.327	0.340	
	Rectangular	0.349	0.363	
90	Parabola-rectangle	0.378	0.375	0.0026
	Triangle-rectangle	0.316	0.337	
	Rectangular	0.317	0.350	

Properties of concrete stress-blocks for rectangular area

f_{ck} MPa	Maximum value of x/d* at section where moment redistribution is required
≤50	$(\delta - 0.4)$
55	$0.95\ (\delta - 0.4)$
60	$0.92\ (\delta - 0.4)$
70	$0.90\ (\delta - 0.4)$
≥80	$0.88\ (\delta - 0.4)$

In the above expressions, δ is the ratio of the design moment to the maximum elastic moment for values of:

$1.0 > \delta \geq 0.7$ where class B and C reinforcement is used.

$1.0 > \delta \geq 0.8$ where class A reinforcement is used.

*Based on values given in the UK National Annex.

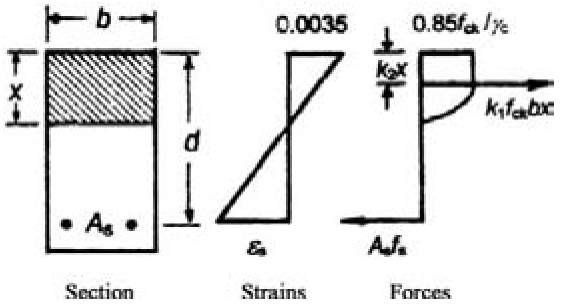
Limits of x/d for moment redistribution

Where plastic analysis is used, the necessary ductility may be assumed without explicit verification if the neutral axis depth at any section satisfies the following requirement:

$$x/d \leq 0.25 \text{ for concrete strength classes} \leq \text{C50/60}$$
$$x/d \leq 0.15 \text{ for concrete strength classes} > \text{C50/60}$$

The following analyses and resulting design charts and tables are applicable to concrete strength classes ≤C50/60.

32.2.1 Singly reinforced rectangular sections



The lever arm between the forces shown in the figure here is given by $z = (d - k_2x)$, from which $x = (d - z)/k_2$.

Taking moments for the compressive force about the line of action of the tensile force gives

$$M = k_1 f_{ck} bxz = k_1 f_{ck} bz(d - z)/k_2$$

The solution of the resulting quadratic equation in z gives

$$z/d = 0.5 + \sqrt{0.25 - (k_2 - k_1)\mu} \quad \text{where } \mu = M/bd^2f_{ck}$$

Taking moments for the tensile force about the line of action of the compressive force gives

$$M = \mathrm{A}_s f_s\, z, \text{ from which } A_s = M/f_s\, z$$

The strain in the reinforcement $\varepsilon_s = 0.0035(1 - x/d)/(x/d)$ and from the design stress–strain curves, the stress is given by:

$$f_s = \varepsilon_s E_s = 700(1 - x/d)/(x/d) \leq k_s f_{yk}/1.15$$

If the top branch of the design stress–strain curve is taken as horizontal (curve B), $k_s = 1.0$ and $f_s = f_{yk}/1.15$ for values of

$$x/d \leq 805/(805 + f_{yk}) = 0.617 \text{ for } f_{yk} = 500 \text{ MPa}$$

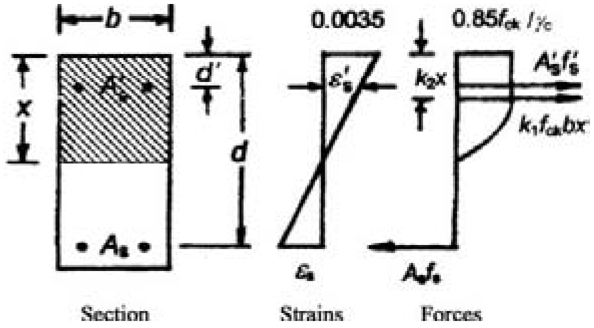
A design chart for $f_{yk} = 500$ MPa, derived on the basis of the parabolic-rectangular stress-block for the concrete and curve B for the reinforcement, is given in *Table 4.7.*

If the top branch of the design stress–strain curve is taken as inclined (curve A), the value of k_s depends on the strain and the ductility class of the reinforcement. The use of curve A is advantageous in reducing the reinforcement requirement. The maximum reduction, for a lightly reinforced section, is close to 5%, 8% and 15% for reinforcement ductility classes A, B and C respectively. For more heavily reinforced sections, the reductions are less and the particular ductility class makes very little difference to the values obtained.

A design table, based on the rectangular stress-block for the concrete and design curve A for the reinforcement, is given in *Table 4.8*. The table uses non-dimensional parameters, and is valid for values of $f_{ck} \leq 50$ MPa. Values of k_s, determined for $f_{yk} = 500$ MPa, were based on the most critical ductility class in each case, namely class A for $M/bd^2f_{ck} \leq 0.046$ and class B for $M/bd^2f_{ck} > 0.046$. This can be seen from the following table, which gives values of A_sf_{yk}/bdf_{ck} for each ductility class in the range up to $M/bd^2f_{ck} = 0.048$.

M/bd^2f_{ck}	A_sf_{yk}/bdf_{ck} for ductility class		
	A	B	C
0.010	0.0111	0.0108	0.0102
0.012	0.0134	0.0130	0.0123
0.014	0.0156	0.0152	0.0144
0.016	0.0179	0.0174	0.0165
0.018	0.0202	0.0197	0.0186
0.020	0.0224	0.0219	0.0207
0.022	0.0247	0.0241	0.0228
0.024	0.0270	0.0264	0.0252
0.026	0.0293	0.0286	0.0276
0.028	0.0317	0.0309	0.0300
0.030	0.0340	0.0331	0.0324
0.032	0.0363	0.0354	0.0349
0.034	0.0387	0.0379	0.0373
0.036	0.0410	0.0403	0.0398
0.038	0.0434	0.0428	0.0423
0.040	0.0457	0.0453	0.0448
0.042	0.0481	0.0478	0.0473
0.044	0.0505	0.0503	0.0498
0.046	0.0529	0.0528	0.0523
0.048	0.0553	0.0554	0.0549

32.2.2 Doubly reinforced rectangular sections



The forces provided by the concrete and the reinforcement, are shown in the figure here. Taking moments for the two compressive forces about the line of action of the tensile force gives

$$M = k_1 f_{ck} bx(d - k_2x) + A_s' f_s'(d - d')$$

The strain in the reinforcement $\varepsilon_s' = 0.0035(1 - d'/x)$ and from design stress–strain curve B, the stress is given by:

$$f_s' = \varepsilon_s' E_s = 700(1 - d'/x) \leq f_{yk}/1.15$$

Thus, $f_s' = f_{yk}/1.15$ for values of

$$x/d \geq [805/(805 - f_{yk})](d'/d) = 2.64(d'/d) \text{ for } f_{yk} = 500 \text{ MPa}$$

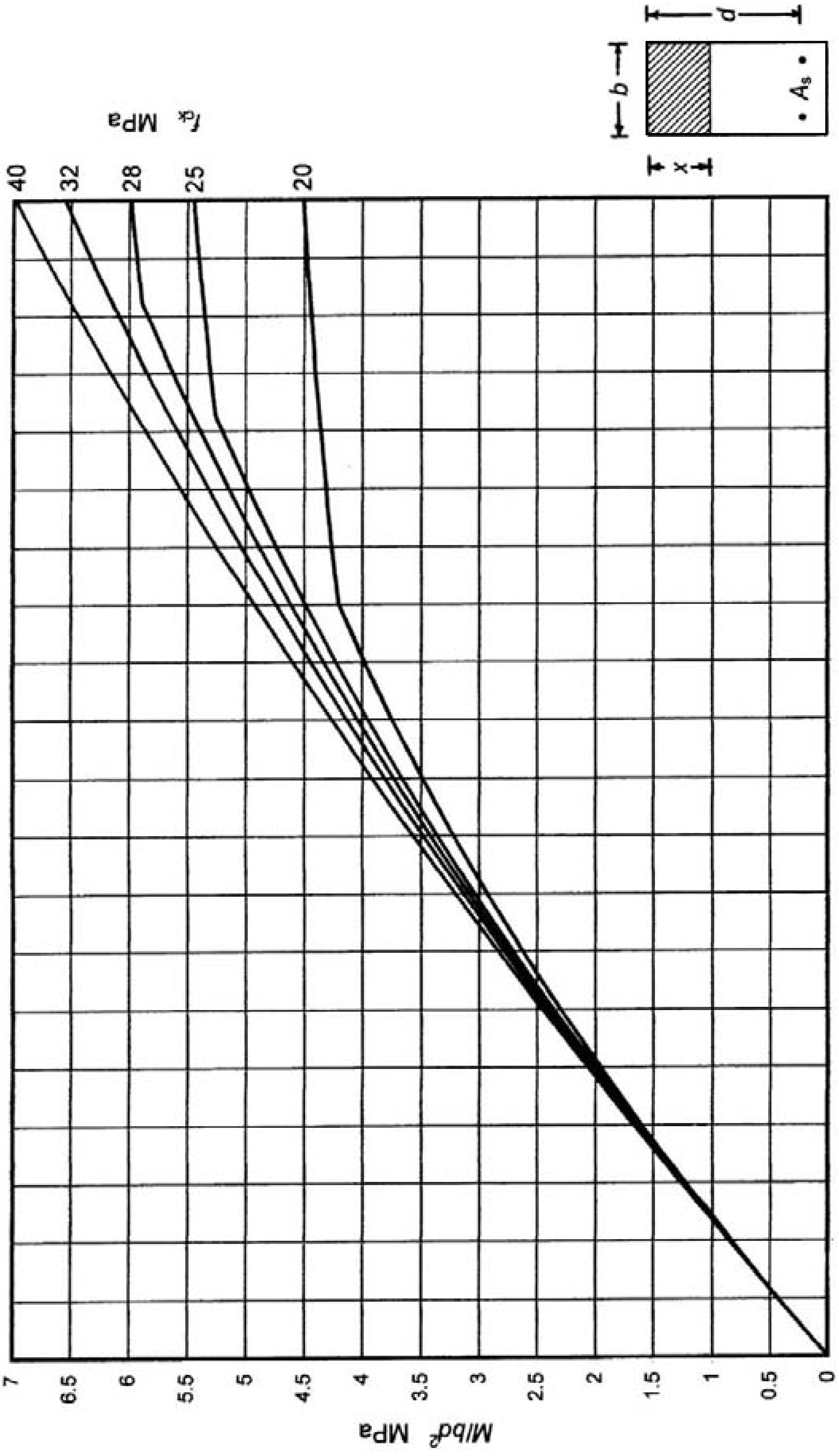
Equating the tensile and compressive forces gives

$$A_s f_s = k_1 f_{ck} bx + A_s' f_s'$$

where the stress in the tension reinforcement is given by the expression derived in section 32.2.1.

EC 2 Design chart for singly reinforced rectangular beams

4.7



Singly reinforced beams (f_{yK}=500 MPa)

EC 2 Design table for singly reinforced rectangular beams

4.8

$\frac{M}{bd^2f_{ck}}$	$\frac{A_sf_{yk}}{bdf_{ck}}$	$\frac{x}{d}$	$\frac{z}{d}$
0.010	0.011	0.022	991
0.012	0.014	0.027	0.989
0.014	0.016	0.031	0.987
0.016	0.018	0.036	0.986
0.018	0.020	0.040	0.984
0.020	0.023	0.045	0.982
0.022	0.025	0.050	0.980
0.024	0.027	0.054	0.978
0.026	0.030	0.059	0.977
0.028	0.032	0.063	0.975
0.030	0.034	0.068	0.973
0.032	0.037	0.073	0.971
0.034	0.039	0.077	0.969
0.036	0.041	0.082	0.967
0.038	0.044	0.087	0.965
0.040	0.046	0.092	0.963
0.042	0.048	0.096	0.961
0.044	0.051	0.101	0.960
0.046	0.053	0.106	0.958
0.048	0.056	0.111	0.956
0.050	0.058	0.116	0.954
0.052	0.061	0.121	0.952
0.054	0.063	0.125	0.950
0.056	0.066	0.130	0.948
0.058	0.069	0.135	0.946
0.060	0.071	0.140	0.944
0.062	0.074	0.145	0.942
0.064	0.076	0.150	0.940
0.066	0.079	0.155	0.938
0.068	0.082	0.160	0.936
0.070	0.084	0.165	0.934
0.072	0.087	0.170	0.932
0.074	0.090	0.176	0.930
0.076	0.092	0.181	0.928
0.078	0.095	0.186	0.926
0.080	0.098	0.191	0.924
0.082	0.100	0.196	0.921
0.084	0.103	0.202	0.919
0.086	0.106	0.207	0.917
0.088	0.109	0.212	0.915
0.090	0.112	0.217	0.913
0.092	0.115	0.223	0.911
0.094	0.117	0.228	0.909
0.096	0.120	0.234	0.907
0.098	0.123	0.239	0.904
0.100	0.126	0.245	0.902
0.102	0.129	0.250	0.900
0.104	0.132	0.256	0.898
0.106	0.135	0.261	0.896
0.108	0.138	0.267	0.893
0.110	0.141	0.272	0.891
0.112	0.144	0.278	0.889
0.114	0.147	0.284	0.887
0.116	0.150	0.289	0.884
0.118	0.153	0.295	0.882
0.120	0.156	0.301	0.880
0.122	0.159	0.307	0.877
0.124	0.162	0.313	0.875
0.126	0.165	0.319	0.873
0.128	0.168	0.324	0.870
0.130	0.171	0.330	0.868
0.132	0.174	0.336	0.865
0.134	0.178	0.343	0.863
0.136	0.181	0.349	0.861
0.138	0.184	0.355	0.858
0.140	0.187	0.361	0.856
0.142	0.190	0.367	0.853
0.144	0.194	0.373	0.851
0.146	0.197	0.380	0.848
0.148	0.200	0.386	0.846
0.150	0.204	0.393	0.843
0.152	0.207	0.399	0.840
0.154	0.211	0.405	0.838
0.156	0.214	0.412	0.835
0.158	0.218	0.419	0.833
0.160	0.221	0.425	0.830
0.162	0.225	0.432	0.827
0.164	0.228	0.439	0.824
0.166	0.232	0.446	0.822
0.168	0.235	0.452	0.819
0.170	0.239	0.459	0.816
0.172	0.243	0.466	0.814
0.174	0.247	0.473	0.811
0.176	0.250	0.480	0.808
0.178	0.254	0.488	0.805
0.180	0.258	0.495	0.802
0.182	0.262	0.502	0.799
0.184	0.266	0.510	0.796
0.186	0.270	0.517	0.793
0.188	0.274	0.525	0.790
0.190	0.278	0.532	0.787
0.192	0.282	0.540	0.784
0.194	0.286	0.548	0.781
0.196	0.290	0.556	0.778
0.198	0.294	0.564	0.775
0.200	0.298	0.572	0.771
0.202	0.303	0.580	0.768
0.204	0.307	0.588	0.765
0.206	0.311	0.597	0.761
0.208	0.316	0.605	0.758
0.210	0.320	0.614	0.754

Redistribution %	Moment ratio δ	Limiting values		
		$\frac{x}{d}$	$\frac{M}{bd^2f_{ck}}$	$\frac{A_sf_{yk}}{bdf_{ck}}$
0*	1.00	0.60	0.207	0.313
5	0.95	0.55	0.194	0.286
10	0.90	0.50	0.181	0.260
15	0.85	0.45	0.167	0.234
20	0.80	0.40	0.152	0.207
25	0.75	0.35	0.136	0.181
30	0.70	0.30	0.120	0.156

Formulae used to determine limiting values:

$x/d = (\delta - 0.4)$ $\mu = 0.453\ (x/d)\ (1 - 0.4\ x/d)$

* Value included for purposes of interpolation. Limiting values do not apply in cases where $\delta \geq 1.0$.

Formulae used in table above

$$\frac{z}{d} = 0.5 + \sqrt{0.25 - 0.882\mu} \qquad \frac{x}{d} = \frac{1 - z/d}{0.4}$$

$$\frac{A_sf_{yk}}{bdf_{ck}} = \frac{1.15\mu}{k_s z/d} \qquad \text{where } \mu = \frac{M}{bd^2f_{ck}}$$

Formulae are valid for values of $f_{ck} \leq 50$ MPa. Values of k_s were determined using stress-strain curve A in *Table 4.4*, for $f_{yk} = 500$ MPa, in each case taking the lowest value for ductility class A, B or C. For further information regarding values for a particular ductility class, and cases where $f_{ck} > 50$ MPa, reference should be made to section 32.2.1.

Design charts, based on the rectangular stress-block for the concrete, design curve B for the reinforcement, and for values of $d'/d = 0.1$ and 0.15 respectively, are given in *Tables 4.9* and *4.10*. The charts, which use non-dimensional parameters, were determined for $f_{yk} = 500$ MPa, but may be safely used for $f_{yk} \leq 500$ MPa. In determining the forces in the concrete, no reduction has been made for the area of concrete displaced by the compression reinforcement.

32.2.3 Design formulae for rectangular sections

No design formulae are given in the code but the following are valid for values of $f_{ck} \leq 50$ MPa and $f_{yk} \leq 500$ MPa. The formulae are based on the rectangular stress-block for the concrete, and stresses of $0.87f_{yk}$ in tension and compression reinforcement. The compression reinforcement requirement depends on the value of $\mu = M/bd^2f_{ck}$ compared to μ' where:

$$\mu' = 0.210 \qquad \text{for } \delta \geq 1.0$$

$$\mu' = 0.453(\delta - 0.4) - 0.181(\delta - 0.4)^2 \qquad \text{for } \delta < 1.0$$

δ is the ratio, design moment to maximum elastic moment, where $\delta \geq 0.7$ for class B and C reinforcement, and $\delta \geq 0.8$ for class A reinforcement

For $\mu > \mu'$, compression reinforcement is not required and

$$A_s = M/0.87f_{yk}z \qquad \text{where}$$

$$z = d\{0.5 + \sqrt{0.25 - 0.882\mu}\} \qquad \text{and } x = (d - z)/0.4$$

For $\mu > \mu'$, compression reinforcement is required and

$$A'_s = (\mu - \mu')bd^2f_{ck}/0.87f_{yk}(d - d')$$

$$A_s = A'_s + \mu' bd^2f_{ck}/0.87f_{yk}z \qquad \text{where}$$

$$z = d\{0.5 + \sqrt{0.25 - 0.882\mu'}\} \qquad \text{and } x = (d - z)/0.4$$

For $d'/x > 0.375$ (for $f_y = 500$ MPa), A'_s should be replaced by $1.6(1 - d'/x)A'_s$ in the equations for A'_s and A_s.

32.2.4 Flanged sections

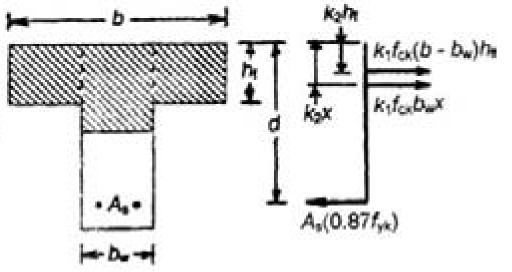
In monolithic beam and slab construction, where the web of the beam projects below the slab, the beam is considered as a flanged section for sagging moments. The effective width of the flange, over which uniform conditions of stress can be assumed, may be taken as $b_{eff} = b_w + b'$, where

$$b' = 0.1(a_w + l_0) \leq 0.2l_0 \leq 0.5a_w \quad \text{for L beams}$$

$$b' = 0.2(a_w + l_0) \leq 0.4l_0 \leq 1.0a_w \quad \text{for T beams}$$

In the aforesaid expressions, b_w is the web width, a_w is the clear distance between the webs of adjacent beams, and l_0 is the distance between points of zero bending moment for the beam. If l_{eff} is the effective span, l_0 may be taken as $0.85l_{eff}$ when there is continuity at one end of the span only, and $0.7l_{eff}$ when there is continuity at both ends of the span. For up-stand beams, when designing for hogging moments, l_0 may be taken as $0.3l_{eff}$ at internal supports and $0.15l_{eff}$ at end supports.

In most sections, where the flange is in compression, the depth of the neutral axis will be no greater than the thickness of the flange. In this case, the section can be considered to be rectangular with b taken as the flange width. The condition regarding the neutral axis depth can be confirmed initially by showing that $M \leq k_1f_{ck}bh_f(d - k_2h_f)$, where h_f is the thickness of the flange. Alternatively, the section can be considered to be rectangular initially, and the neutral axis depth can be checked subsequently.



Section Forces

The figure here shows a flanged section where the neutral axis depth is greater than the flange thickness. The concrete force can be divided into two components and the required area of tension reinforcement is then given by:

$A_s = A_{s1} + k_1f_{ck}(b - b_w)h_f/0.87f_{yk}$ where

A_{s1} = area of reinforcement required to resist a moment M_1 applied to a rectangular section, of width b_w, and

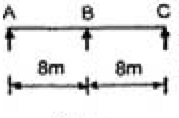
$$M_1 = M - k_1f_{ck}(b - b_w)h_f(d - k_2h_f) \leq \mu' bd^2f_{ck}$$

Using the rectangular concrete stress-block in the forgoing equations, gives $k_1 = 0.45$ and $k_2 = 0.4$. This approach gives solutions that are 'correct' when $x = h_f$, but become slightly more conservative as $(x - h_f)$ increases.

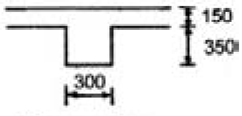
32.2.5 General analysis of sections

The analysis of a section of any shape, with any arrangement of reinforcement, involves a trial-and-error process. An initial value is assumed for the neutral axis depth, from which the concrete strains at the positions of the reinforcement can be calculated. The corresponding stresses in the reinforcement are determined, and the resulting forces in the reinforcement and the concrete are obtained. If the forces are out of balance, the value of the neutral axis depth is changed and the process is repeated until equilibrium is achieved. Once the balanced condition has been found, the resultant moment of the forces about the neutral axis, or any convenient point, is calculated.

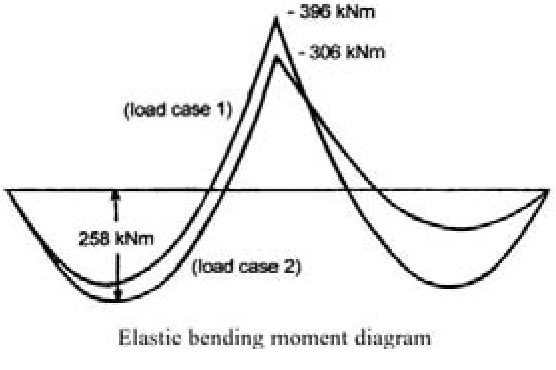
Example 1



Spans



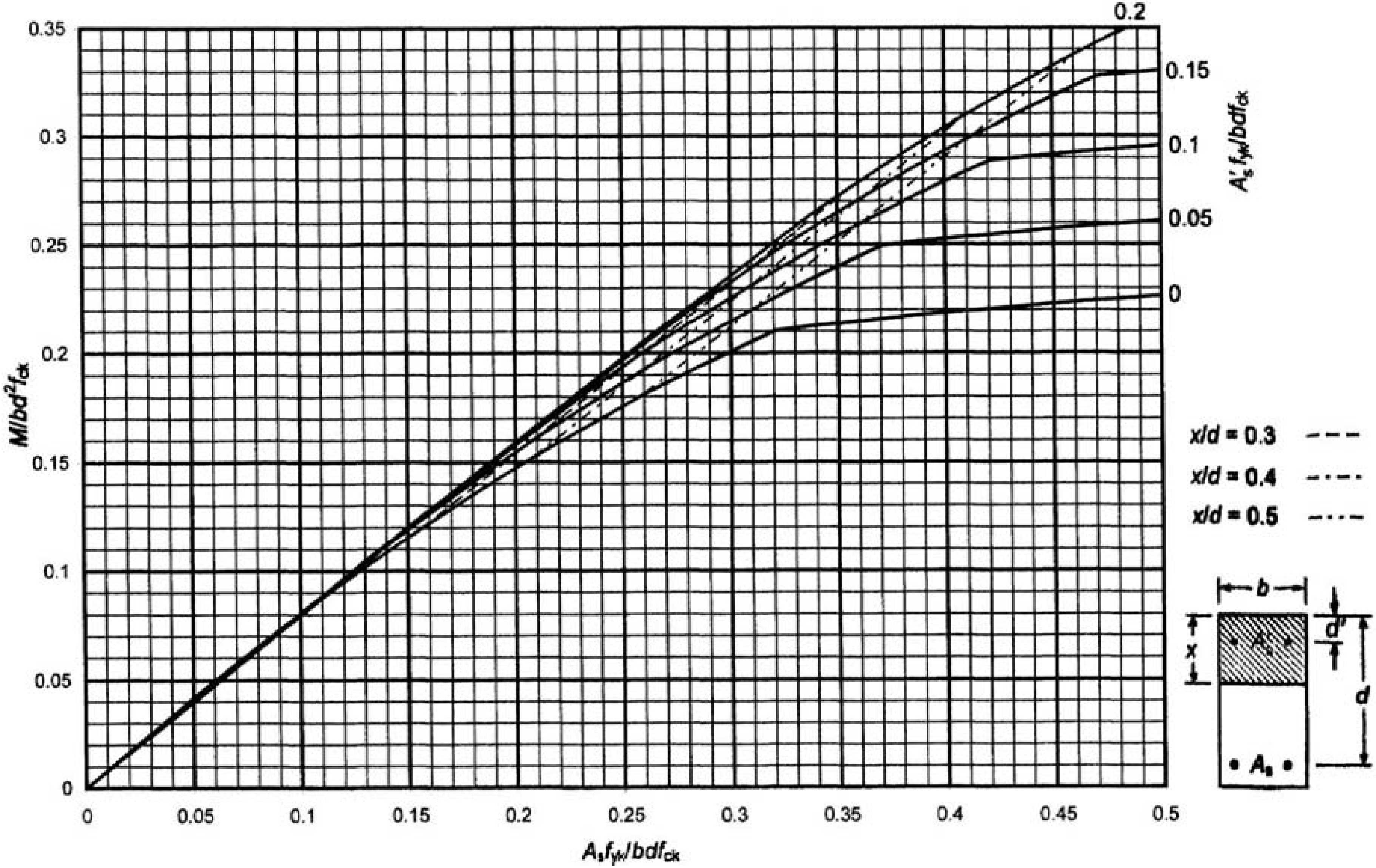
Cross section



Elastic bending moment diagram

4.9

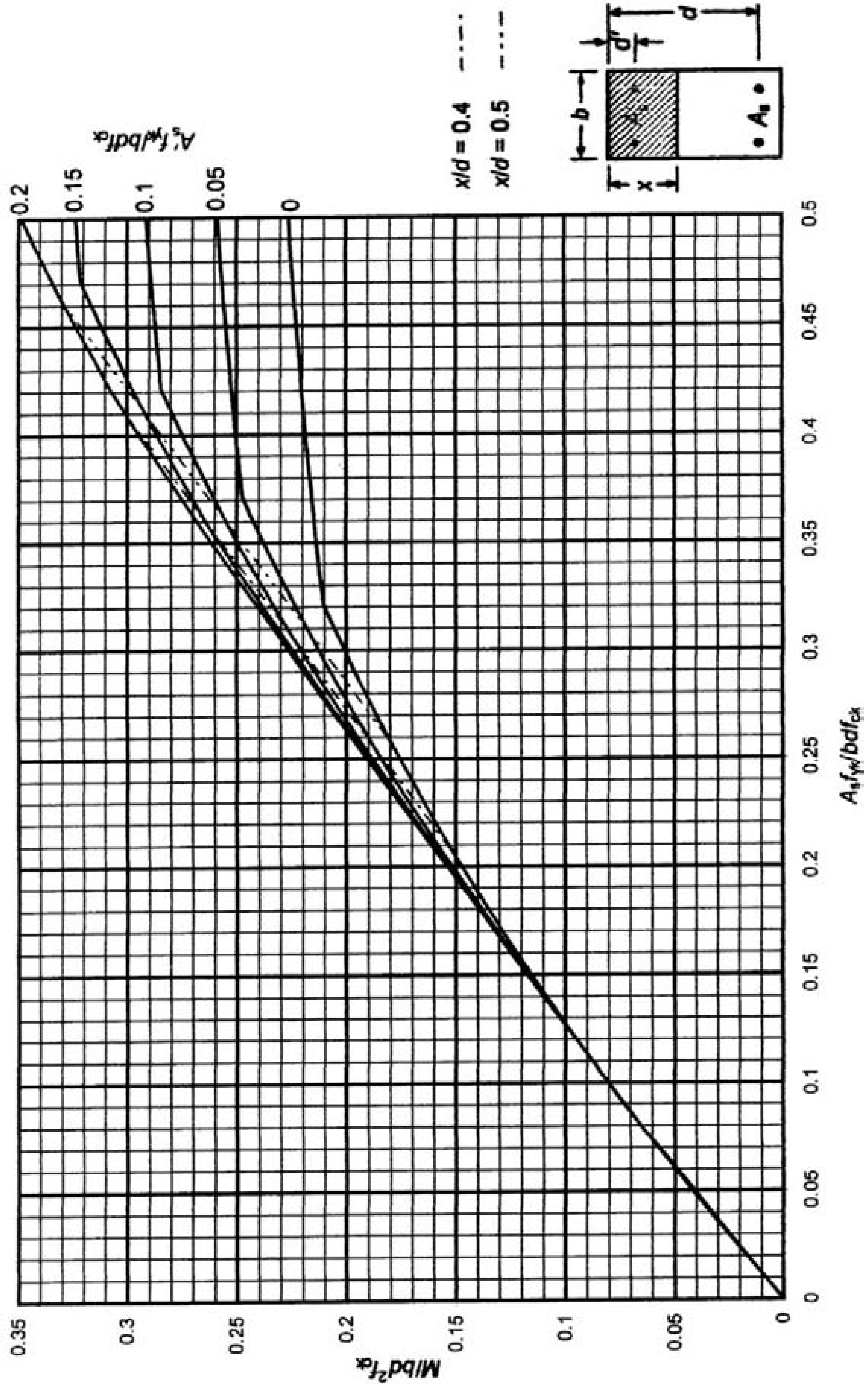
EC 2 Design chart for doubly reinforced rectangular beams – 1



Doubly reinforced beams ($f_{yk}=500$ MPa, $d'/d=0.1$)

4.10

EC 2 Design chart for doubly reinforced rectangular beams – 2



Doubly reinforced beams (f_{yk} = 500 MPa, d'/d = 0.15)

The beam shown in the figure here is part of a monolithic beam and slab floor, in which the beams are spaced at 5 m centres. The maximum and minimum design loads on each span are as follows, where $G_k = 160$ kN and $Q_k = 120$ kN:

$$F_{max} = 1.35G_k + 1.5Q_k = 396 \text{ kN}, F_{min} = 1.35\ G_k = 216 \text{ kN}$$

The section design is to be based on the following values:

$$f_{ck} = 32 \text{ MPa}, f_{yk} = 500 \text{ MPa, cover to links} = 35\text{mm.}$$

For sagging moments, b' and b_{eff} are given by:

$$\begin{aligned} b' &= 0.2(a_w + l_0) \leq 0.4\ l_0 \leq a_w \\ &= 0.2(4700 + 0.85 \times 8000) = 2300 \text{ mm} \\ b_{eff} &= b_w + b' = 300 + 2300 = 2600 \text{ mm} \end{aligned}$$

Allowing for 8 mm links and 32 mm main bars,

$$d = 500 - (35 + 8 + 16) = 440 \text{ mm say.}$$

In the calculations that follow, solutions are obtained using charts and equations, to demonstrate the use of each method.

Maximum sagging moment. For section to be designed as rectangular with b taken as the flange width, bending moment should satisfy the condition:

$$\begin{aligned} M &\leq k_1 f_{ck} b h_f\ (d - k_2 h_f) \\ &= 0.45 \times 32 \times 2600 \times 150 \times (440 - 0.4 \times 150) \times 10^{-6} \\ &= 2134 \text{ kNm } (>258 \text{ kNm}) \end{aligned}$$

$$M/bd^2 = 258 \times 10^6/\ (2600 \times 440^2) = 0.51 \text{ MPa}$$

From chart in *Table 4.7*, $100A_s/bd = 0.12$,

$$A_s = 0.0012 \times 2600 \times 440 = 1373 \text{ mm}^2$$

Alternatively, by calculation or from *Table 4.8*,

$$\mu = M/bd^2 f_{ck} = 0.51/32 = 0.016$$

$$z/d = 0.5 + \sqrt{0.25 - 0.882 \times 0.016} = 0.985$$

Hence $A_s = M/0.87 f_{yk} z$ gives

$$A_s = 258 \times 10^6/\ (0.87 \times 500 \times 0.985 \times 440) = 1369 \text{ mm}^2$$

Using 3H25 gives 1473 mm^2

The above solutions are based on design stress-strain curve B for the reinforcement. Solutions based on curve A also can be obtained from the table in section 32.2.1, as follows:

$M/bd^2 f_{ck}$	$A_s f_{yk}/bdf_{ck}$ for ductility class		
	A	B	C
0.016	0.0179	0.0174	0.0165
A_s (mm^2)	1310	1274	1208

Maximum hogging moment

$$\mu = M/bd^2 f_{ck} = 396 \times 10^6/\ (300 \times 440^2 \times 32) = 0.213$$

From *Table 4.8*, this appears to be just beyond the range for a singly reinforced section. From the chart in *Table 4.9*, a value of $A_s f_{yk}/bdf_{ck} = 0.34$ can be obtained. Although this is a valid solution, it should be possible to reduce the area of tension reinforcement to a more suitable value, by allowing for some compression reinforcement. Consider the use of 2H25, which gives $d' = 55$ mm ($d'/d = 55/440 = 0.125$).

$$A'_s f_{yk}/bdf_{ck} = 982 \times 500/(300 \times 440 \times 32) = 0.116$$

Interpolating from charts in *Tables 4.9* and *4.10*, with

$$\begin{aligned} &A'_s f_{yk}/bdf_{ck} = 0.1, A_s f_{yk}/bdf_{ck} = 0.285 \text{ (for } d'/d = 0.125) \\ &A_s = 0.285 \times 300 \times 440 \times 32/500 = 2408 \text{ mm}^2 \end{aligned}$$

Using 3H32 gives 2413 mm^2

A solution can also be obtained using the design equations, as follows: $f'_s = 0.87 f_y$ is valid for $x/d \geq (d'/d)/0.375 = 0.333$.

With $x/d = 0.333$ and $z/d = (1 - 0.4x/d) = 0.867$,

$$\mu' = 0.453(x/d)(z/d) = 0.453 \times 0.333 \times 0.867 = 0.131$$

$$\begin{aligned} A'_s &= (\mu - \mu')\ bd^2 f_{ck}\ /\ 0.87 f_{yk}\ (d - d') \\ &= (0.213 - 0.131) \times 300 \times 440^2 \times 32/ \\ &\quad (0.87 \times 500 \times 385) \\ &= 910 \text{ mm}^2 \text{ (2H25 gives 982 mm}^2) \end{aligned}$$

$$\begin{aligned} A_s &= A'_s + \mu' bd^2 f_{ck}\ /0.87 f_{yk}\ z \\ &= 910 + 0.131 \times 300 \times 440^2 \times 32/ \\ &\quad (0.87 \times 500 \times 0.867 \times 440) \\ &= 2377 \text{ mm}^2 \text{ (3H32 gives 2413 mm}^2) \end{aligned}$$

Example 2. Suppose that in the previous example the maximum hogging moment at B is reduced by 20% to 317 kNm. This is valid for reinforcement of all ductility classes.

$$\begin{aligned} &\mu = M/bd^2 f_{ck} = 317 \times 10^6/\ (300 \times 440^2 \times 32) = 0.171 \\ &\delta = 317/396 = 0.80, x/d \leq (\ \delta - 0.4) = 0.4 \end{aligned}$$

From chart in *Table 4.10*, keeping to left of line for $x/d = 0.4$,

$$\begin{aligned} &A'_s f_{yk}/bdf_{ck} = 0.05, A_s f_{yk}/bdf_{ck} = 0.23 \\ &A'_s = 0.05 \times 300 \times 440 \times 32/500 = 423 \text{ mm}^2 \end{aligned}$$

Using 2H20 gives 628 mm^2 (instead of 2H25 in example 1)

$$A_s = 0.23 \times 300 \times 440 \times 32/500 = 1943 \text{ mm}^2$$

Using 4H25 gives 1963 mm^2 (instead of 3H32 in example 1)

A solution can also be obtained by using the design equations as in example 1, with $x/d = 0.333$, $z/d = 0.867$, $\mu' = 0.131$:

$$\begin{aligned} A'_s &= (\ \mu - \mu')bd^2 f_{ck}\ /\ 0.87 f_{yk}\ (d - d') \\ &= 0.04 \times 300 \times 440^2 \times 32\ /\ (0.87 \times 500 \times 385) \\ &= 444 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} A_s &= A'_s + \mu' bd^2 f_{ck}\ /0.87 f_{yk} z \\ &= 444 + 0.131 \times 300 \times 440^2 \times 32/ \\ &\quad (0.87 \times 500 \times 0.867 \times 440) \\ &= 1912 \text{ mm}^2 \end{aligned}$$

Since the reduced hogging moment for load case 1 is still greater than the elastic hogging moment for load case 2, the design sagging moment remains the same as in example 1.

In the foregoing examples, at the bottom of the beam, 2H25 bars would run the full length of each span, with 2H25 or 2H20 splice bars at support B. Other bars would be curtailed to suit the bending moment requirements and detailing rules.

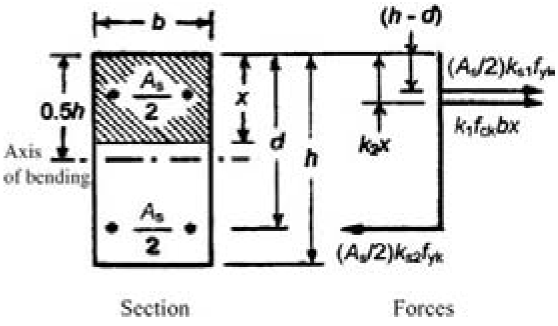
32.3 COLUMNS

In the Code of Practice, a column is taken to be a compression member whose greater overall cross sectional dimension is not more than four times its smaller dimension. An effective length and a slenderness ratio are determined in relation to its major and minor axes of bending. The effective length is a function of the clear height and depends upon the restraint conditions at the ends of the column. A slenderness ratio is defined as the effective length divided by the radius of gyration of the uncracked concrete section. Columns should generally be designed for both first order and second order effects, but second order effects may be ignored provided the slenderness ratio does not exceed a particular limiting value. This can vary considerably and has to be determined from an equation involving several factors. These generally need to be calculated but default values are also given.

Columns are subjected to combinations of bending moment and axial force, and the cross section may need to be checked for more than one combination of values. Several methods of analysis, of varying complexity, are available for determining second order effects. Many columns can be treated as isolated members, and a simplified method of design using equations based on an estimation of curvature is commonly used. The equations contain a modification factor K_r, the use of which results in an iteration process with K_r taken as 1.0 initially. The design procedures are shown in *Tables 4.15* and *4.16*.

In the code, for sections subjected to pure axial load, the concrete strain is limited to 0.002 for values of $f_{ck} \leq 50$ MPa. In this case, the design stress in the reinforcement should be limited to 400 MPa. However, in other parts of the code, the design stress in this condition is shown as $f_{yd} = f_{yk}/\gamma_s = 0.87f_{yk}$. In the derivation of the charts in this chapter, which apply for all values of $f_{ck} \leq 50$ MPa and $f_{yk} \leq 500$ MPa, the maximum compressive stress in the reinforcement was taken as $0.87f_{yk}$. The charts contain sets of K_r lines to aid the design process.

32.3.1 Rectangular columns



The foregoing figure shows a rectangular section in which the reinforcement is disposed equally on two opposite sides of a horizontal axis through the mid-depth. By resolving forces and taking moments about the mid-depth of the section, the following equations are obtained for $0 < x/h \leq 1.0$.

$$N/bhf_{ck} = k_1(x/h) + 0.5(A_sf_{yk}/bhf_{ck})(k_{s1} - k_{s2})$$

$$M/bh^2f_{ck} = k_1(x/h)\{0.5 - k_2(x/h)\} + 0.5(A_sf_{yk}/bhf_{ck})(k_{s1} + k_{s2})(d/h - 0.5)$$

The stress factors, k_{s1} and k_{s2}, are given by:

$$k_{s1} = 1.4(x/h + d/h - 1)/(x/h) \leq 0.87$$
$$k_{s2} = 1.4(d/h - x/h)/(x/h) \leq 0.87$$

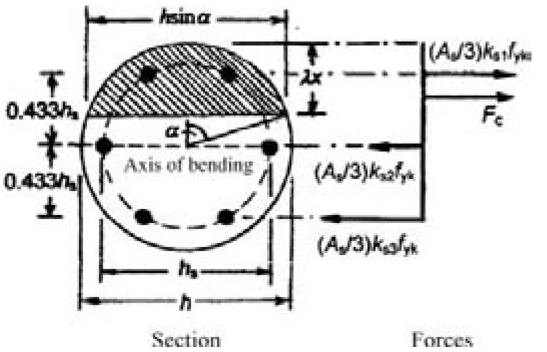
The maximum axial force N_u is given by the equation

$$N_u/bhf_{ck} = 0.567 + 0.87(A_sf_{yk}/bhf_{ck})$$

Design charts, based on the rectangular stress-block for the concrete, and for values of $d/h = 0.8$ and 0.85, are given in *Tables 4.11* and *4.12* respectively. On each curve, a straight line has been taken between the point where $x/h = 1.0$ and the point where $N = N_u$. The charts, which were determined for $f_{yk} = 500$ MPa, may be safely used for $f_y \leq 500$ MPa. In determining the forces in the concrete, no reduction has been made for the area of concrete displaced by the compression reinforcement. In the design of slender columns, the K_r factor is used to modify the deflection corresponding to a load N_{bal} at which the moment is a maximum. A line corresponding to N_{bal} passes through a cusp on each curve. For $N \leq N_{bal}$, the K_r value is taken as 1.0. For $N > N_{bal}$, K_r can be determined from the lines on the chart.

32.3.2 Circular columns

The following figure shows a circular section with six bars spaced equally around the circumference. Solutions based on six bars will be slightly conservative if more bars are used. The arrangement of the bars relative to the axis of bending affects the resistance of the section, and the arrangement shown in the figure is not the most critical in every case. For some combinations of bending moment and axial force, if the arrangement shown is rotated through 30°, a slightly more critical condition results, but the differences are small and may be reasonably ignored.



The following analysis is based on a uniform stress-block for the concrete, of depth λx and width $h\sin\alpha$ at the base (as shown in figure). Negative axial forces are included in order to cater for members such as tension piles. By resolving forces and taking moments about the mid-depth of the section, the following equations are obtained, where $\alpha = \cos^{-1}(1 - 2\lambda x/h)$ for $0 < x \leq 1.0$, and h_s is the diameter of a circle through the centres of the bars:

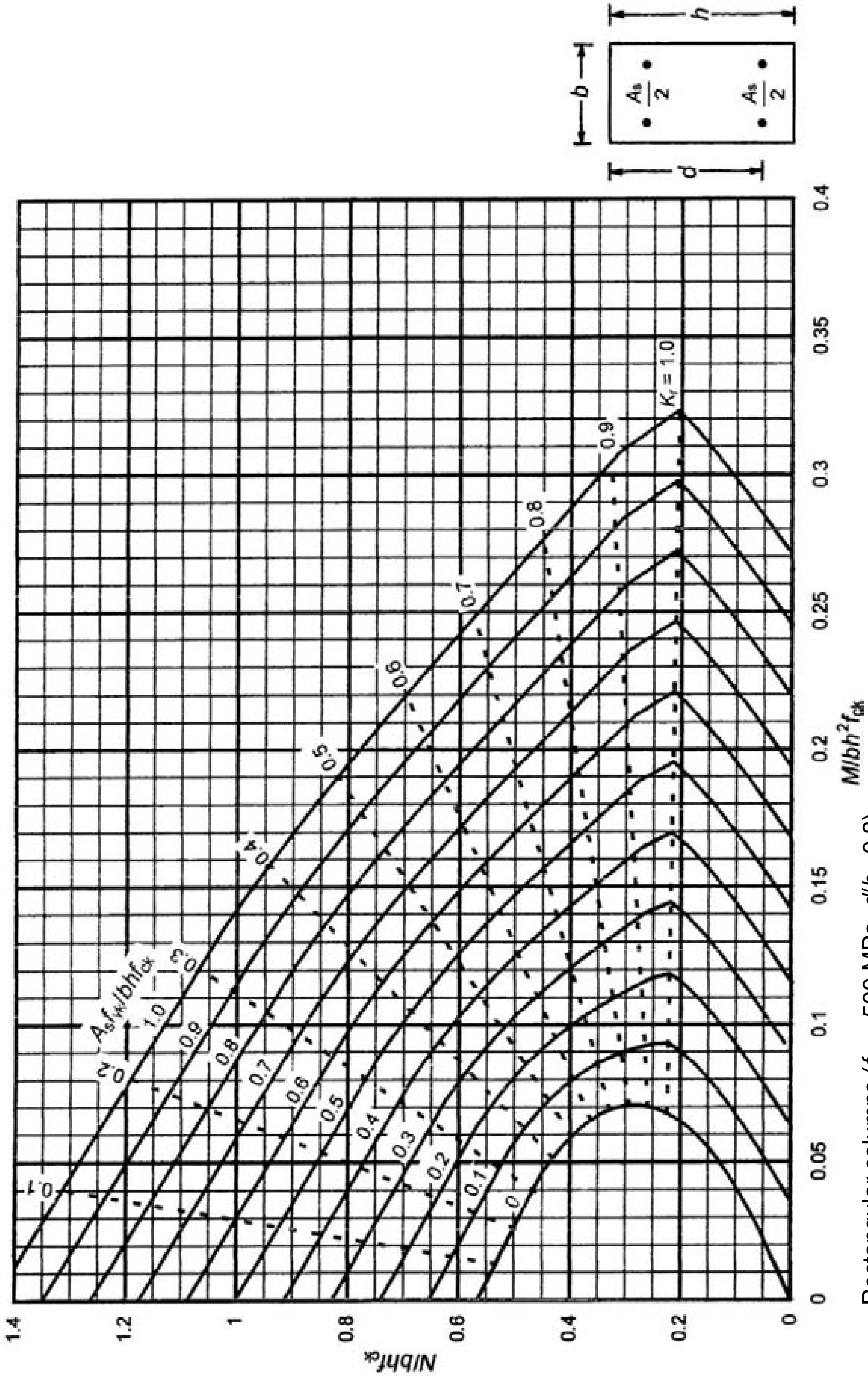
$$N/h^2f_{ck} = k_c(2\alpha - \sin 2\alpha)/8 + (\pi/12)(A_sf_{yk}/A_cf_{ck})(k_{s1} - k_{s2} - k_{s3})$$
$$M/h^3f_{ck} = k_c(3\sin\alpha - \sin 3\alpha)/72 + (\pi/27.7)(A_sf_{yk}/A_cf_{ck})(h_s/h)(k_{s1} + k_{s3})$$

Because the width of the compression zone decreases in the direction of the extreme compression fibre, the design stress in the concrete has to be reduced by 10%. Thus, in the above equations: $k_c = 0.9 \times 0.567 = 0.51$ and $\lambda = 0.8$.

The stress factors, k_{s1}, k_{s2} and k_{s3}, are given by:

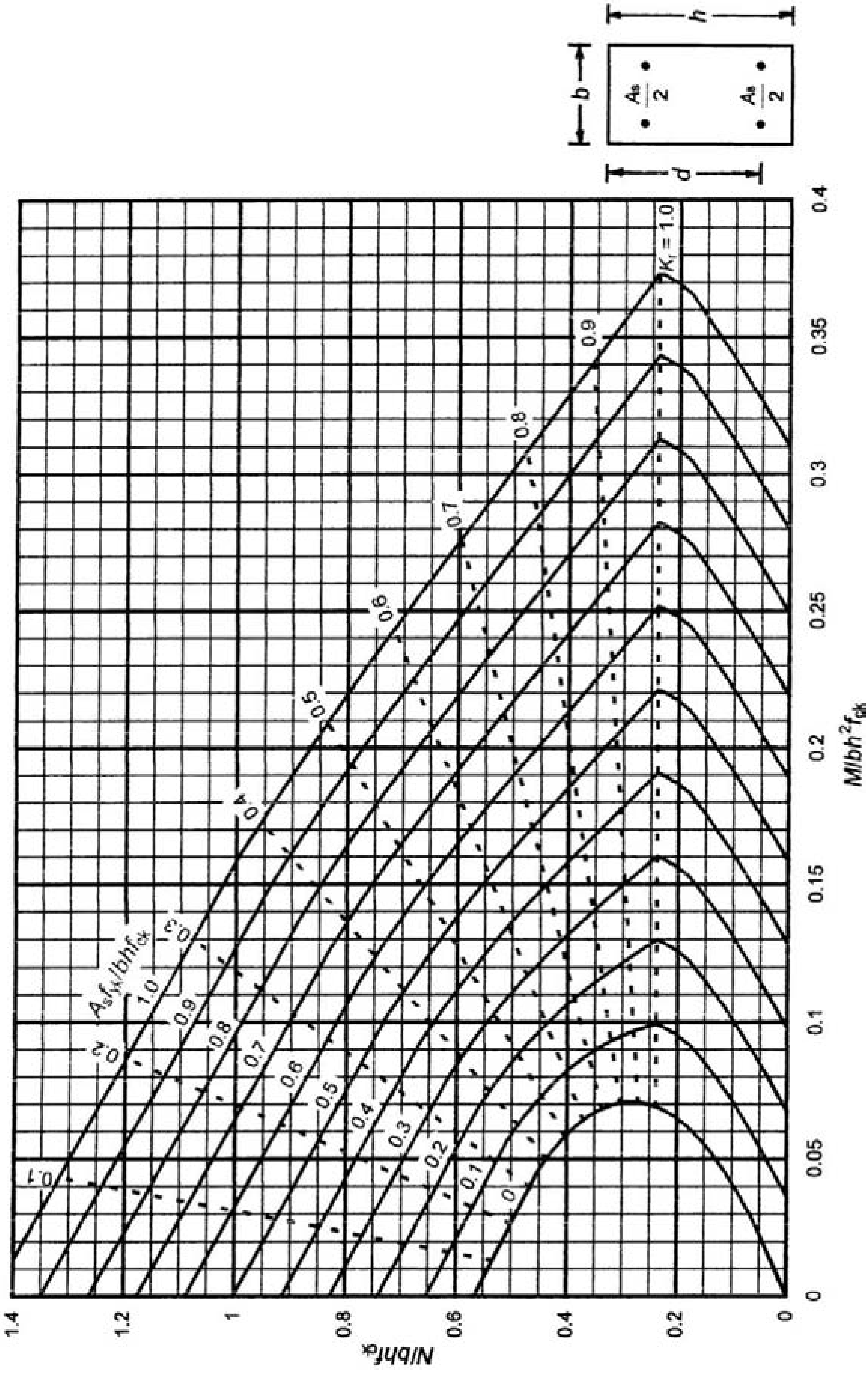
$$-0.87 \leq k_{s1} = 1.4(0.433h_s/h - 0.5 + x/h)/(x/h) \leq 0.87$$
$$-0.87 \leq k_{s2} = 1.4(0.5 - x/h)/(x/h) \leq 0.87$$
$$-0.87 \leq k_{s3} = 1.4(0.5 + 0.433h_s/h - x/h)/(x/h) \leq 0.87$$

EC 2 Design chart for rectangular columns – 1



Rectangular columns ($f_{yk} = 500$ MPa, $d/h = 0.8$)

4.12



Rectangular columns (f_{yk} = 500 MPa, d/h = 0.85)

To avoid irregularities in the charts, the reduced design stress in the concrete is used to determine the maximum axial force N_u, which is given by the equation:

$$N_u/h^2f_{ck} = (\pi/4)\{0.51 + 0.87(A_sf_{yk}/A_cf_{ck})\}$$

The minimum axial force N_{min} is given by the equation:

$$N_{min}/h^2f_{ck} = -0.87(\pi/4)(A_sf_{yk}/A_cf_{ck})$$

Design charts for values of $h_s/h = 0.6$ and 0.7, are given in *Tables 4.13* and *4.14* respectively. The statements in section 32.3.1 on the derivation and use of the charts for rectangular sections apply also to those for circular sections.

32.3.3 General analysis of column sections

Any given cross section can be analysed by a trial-and-error process. For a section bent about one axis, an initial value is assumed for the neutral axis depth, from which the concrete strains at the positions of the reinforcement can be calculated. The resulting stresses in the reinforcement are determined, and the forces in the reinforcement and concrete evaluated. If the resultant force is not equal to the design axial force N, the value of the neutral axis depth is changed and the process repeated until equality is achieved. The resultant moment of all the forces about the mid-depth of the section is then the moment of resistance appropriate to N. This approach is used to analyse a rectangular section in example 6.

Example 3. A 300 mm square braced column designed for the following requirements:

$l = 5.0$ m, $k = 0.675$ at both joints in both directions
$M_{02} = 40$ kNm, $M_{01} = -20$ kNm about x-x axis
M_0 = negligible about y-y axis, $N = 1720$ kN
$f_{ck} = 32$ MPa, $f_{yk} = 500$ MPa, cover to links = 35 mm

Allowing for 8 mm links and 32 mm main bars,

$$d = 300 - (35 + 8 + 16) = 240 \text{ mm say}$$

From *Table 4.15*, effective length for a braced column, where the joint stiffness is the same at both ends, is given by

$$l_0 = [0.5(0.45 + 2k)/(0.45 + k)] \times l = 0.8 \times 5.0 = 4.0 \text{ m}$$

Slenderness ratio $\lambda = l_0/i = 4000/(300/\sqrt{12}) = 46.2$

From Table 4.15, with $M_{01} = -0.5M_{02}$, $C = 2.2$ and

$$\lambda_{lim} = 34/\sqrt{N/A_cf_{ck}} = 34/\sqrt{1720 \times 10^3/(300^2 \times 32)} = 44$$

Since $\lambda > \lambda_{lim}$, second order moments need to be considered.

Minimum design moment, with $e_0 = h/30 = 300/30 \geq 20$ mm,

$$M_{min} = Ne_0 = 1720 \times 0.02 = 34 \text{ kNm}$$

Additional first order moment resulting from imperfections, with $0.67 \leq \alpha_h = 2/\sqrt{l} = 2/\sqrt{(5.0)} = 0.9 \leq 1.0$:

$$M_i = N(\alpha_h l_0/400) = 1720 \times (0.9 \times 4.0/400) = 16 \text{ kNm}$$

Total first order moment for section at end 2 of the column,

$$M_x = M_{02} + M_i = 40 + 16 = 56 \text{ kNm } (>M_{min})$$
$$M/bh^2f_{ck} = 56 \times 10^6/(300 \times 300^2 \times 32) = 0.065$$
$$N/bhf_{ck} = 1720 \times 10^3/(300 \times 300 \times 32) = 0.600$$

From the design chart for $d/h = 240/300 = 0.8$,

$$A_sf_{yk}/bhf_{ck} = 0.25 \text{ (\textit{Table 4.11})}$$
$$A_s = 0.25 \times 300 \times 300 \times 32/500 = 1440 \text{ mm}^2$$

Using 4H25 gives 1963 mm^2

The section where the second order moment is greatest may be designed by first assuming the reinforcement (4H25 say):

$A_sf_{yk}/bhf_{ck} = 1963 \times 500/(300 \times 300 \times 32) = 0.34$
$N_u/bhf_{ck} = 0.86$ (*Table 4.11*) and, for $N/bhf_{ck} = 0.6$,
$M_u/bh^2f_{ck} = 0.09$, $K_r = 0.4$ (*Table 4.11*)
$M_{ux} = M_{uy} = 0.09 \times 300 \times 300^2 \times 32 \times 10^{-6} = 78$ kNm

Second order moment resulting from deflection, with $K_r = 0.4$ and, for $f_{ck} = 32$ MPa and $\lambda = 46$, $K_\varphi = 1.3$ (*Table 4.16*):

$$M_2 = N(K_rK_\varphi l_0^2/d)/2000$$
$$= 1720 \times (0.4 \times 1.3 \times 4.0^2/0.24)/2000 = 30 \text{ kNm}$$

Equivalent first order moment (near mid-height of column):

$$M_{0e} = 0.6M_{02} + 0.4M_{01} \geq 0.4M_{02} = 0.4 \times 40 = 16 \text{ kNm}$$

Total design moments (near mid-height of column)

$$M_x = M_{0e} + M_i + M_2 = 16 + 16 + 30 = 62 \text{ kNm } (>M_{min})$$
$$M_y = M_2 = 30 \text{ kNm}$$

Design for biaxial bending may be ignored if the following two conditions are satisfied: (a) $0.5\lambda_y \leq \lambda_x \leq 2\lambda_y$ and (b) for a square column, M_x is either $\leq 0.2M_y$ or $\geq 5M_y$. In this case, since $M_x \cong 2M_y$, condition (b) is not satisfied and a further check is necessary, as follows:

For $N/N_u = 0.60/0.86 < 0.7$

$$\alpha_n = 0.92 + 0.83(N/N_u) = 1.50$$

Hence

$$\left[\frac{M_x}{M_{ux}}\right]^{\alpha_n} + \left[\frac{M_y}{M_{uy}}\right]^{\alpha_n} = \left[\frac{62}{78}\right]^{1.5} + \left[\frac{30}{78}\right]^{1.5} = 0.95$$

Since this value is less than 1.0, 4H25 is sufficient.

Example 4. A 350 mm circular braced column designed for the same requirements as example 3. Thus $l_0 = 4.0$ m as before.

Slenderness ratio $\lambda = l_0/i = 4000/(350/4) = 45.7$

Cross-sectional area $A_c = (\pi/4) \times 350^2 = 96.2 \times 10^3$ mm^2

$$\lambda_{lim} = 34/\sqrt{N/A_cf_{ck}} = 34/\sqrt{1720/(96.2 \times 32)} = 45.5$$

Since $\lambda \cong \lambda_{lim}$, second order moments need not be considered.
Allowing for 8 mm links and 20 mm main bars,

$$h_s = 350 - 2 \times (35 + 8 + 10) = 244 \text{ mm}$$

For the section at end 2 of the column,

$$M/h^3f_{ck} = 56 \times 10^6/(350^3 \times 32) = 0.041$$
$$N/h^2f_{ck} = 1720 \times 10^3/(350^2 \times 32) = 0.44$$

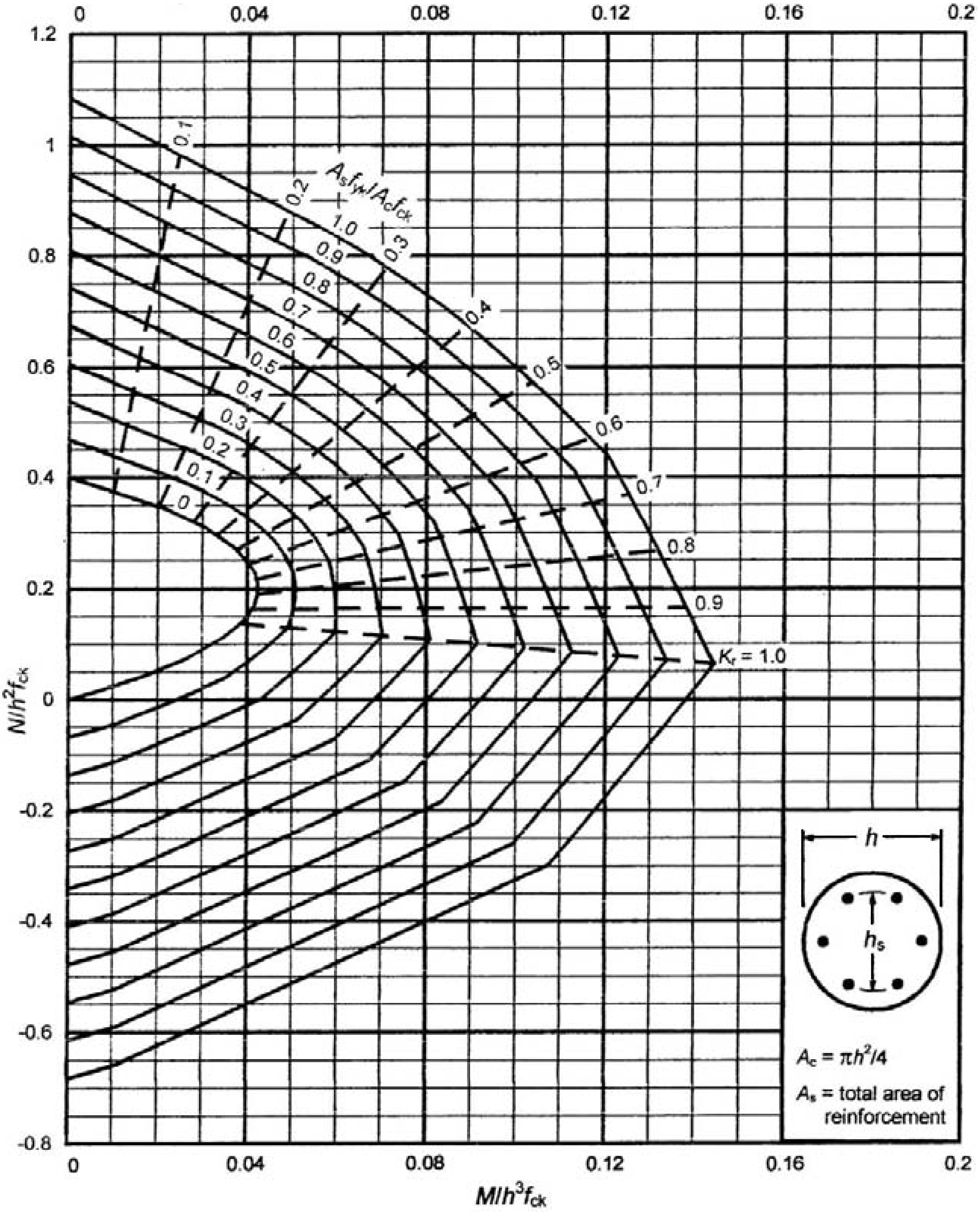
From the design chart for $h_s/h = 244/350 = 0.7$

$$A_sf_{yk}/A_cf_{ck} = 0.26 \text{ (\textit{Table 4.14})}$$
$$A_s = 0.26 \times 96.2 \times 10^3 \times 32/500 = 1600 \text{ mm}^2$$

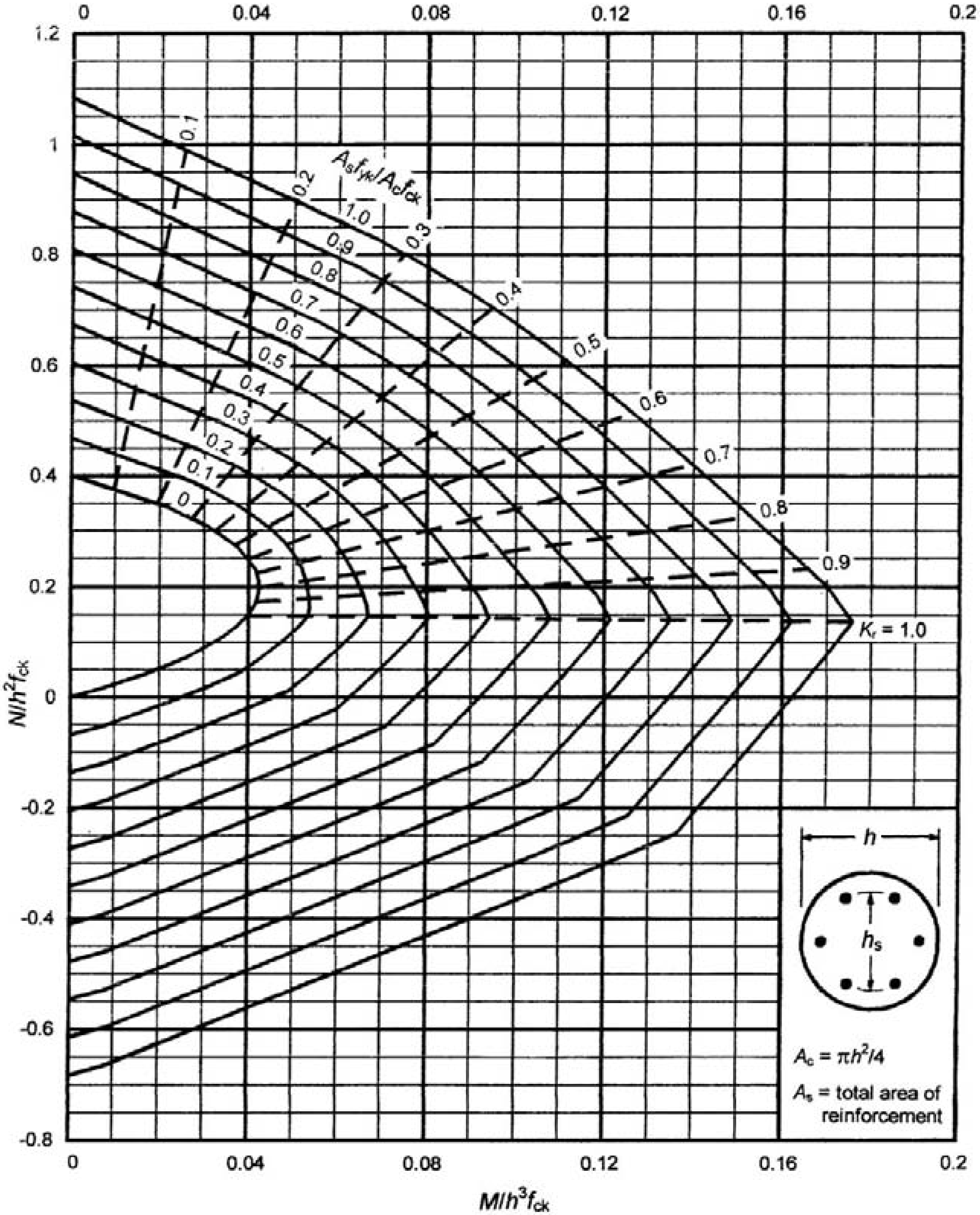
Using 6H20 gives 1885 mm^2

EC 2 Design chart for circular columns – 1

4.13



Circular columns ($f_{yk} = 500$ MPa, $h_s/h = 0.6$)

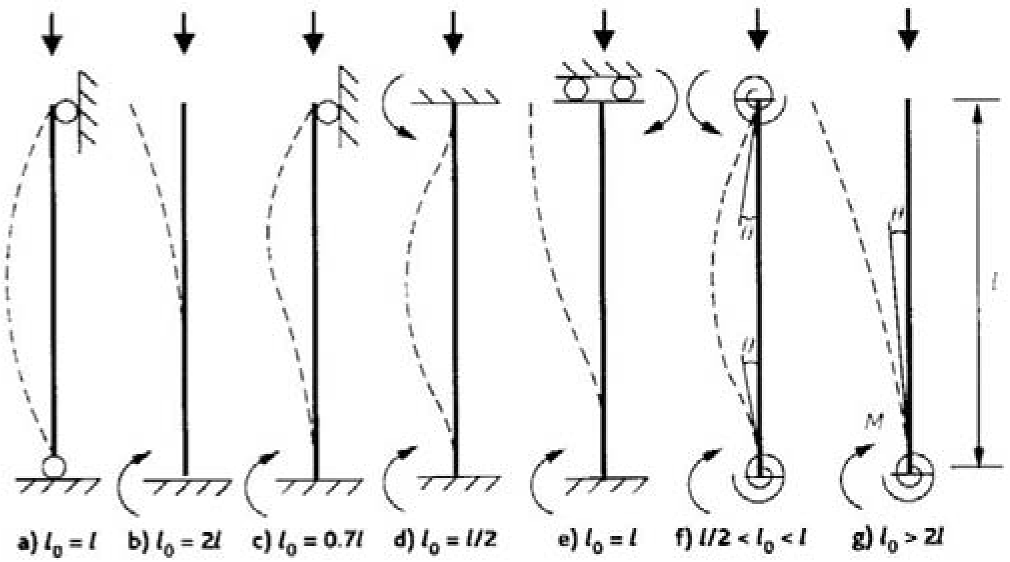


Circular columns ($f_{yk} = 500$ MPa, $h_s/h = 0.7$)

4.15

EC 2 Design procedure for columns – 1

Effective length



Examples of buckling modes and corresponding effective lengths for isolated members

Note 1. In the above, l is the clear height between end restraints in the plane of buckling considered.

Note 2. Columns, which in analysis and design are assumed not to contribute to the overall horizontal stability of a structure in a given plane, are defined as braced in that plane (examples a, c, d, f). Otherwise, they are considered as unbraced (examples b, e, g).

Note 3. For columns in regular frames, the effective length may be calculated from the following equations:

Braced: $l_0 = 0.5l\sqrt{(1+\alpha_1)(1+\alpha_2)}$ where $\alpha_1 = k_1/(0.45 + k_1)$, $\alpha_2 = k_2/(0.45 + k_2)$

Unbraced: $l_0 = l\sqrt{(1+\beta_1)(1+\beta_2)}$ where $\beta_1 = k_1/(1 + k_1)$, $\beta_2 = k_2/(1 + k_2)$ or

$l_0 = l\sqrt{(1+10\beta_3)}$ where $\beta_3 = k_1\, k_2\, /(k_1 + k_2)$, whichever is greater

In the above equations, k_1 and k_2 are the relative flexibilities of rotational restraint at nodes 1 and 2 respectively. If the stiffness of adjacent columns does not vary significantly (say, difference not exceeding 15% of the higher value), the relative flexibility may be taken as the stiffness of the column under consideration divided by the sum of the stiffness of the beams (or, for an end column, the stiffness of the beam) attached to the column in the appropriate plane of bending. Otherwise, the effective column stiffness should be taken as the sum of the stiffness of the columns above and below the node. The stiffness of a member is $4EI/l$ for members fixed at the remote end, and $3EI/l$ for members pinned at the remote end, where I is the second moment of area of the cross-section allowing for the effect of cracking (for beams, 50% of the value for the uncracked section could be used), and l is the length of the member. In flat slabs, the beam stiffness should be based on the dimensions of the column strip. At nodes where the beams are considered as nominally simply-supported, and at bases not designed to resist column moments, k should be taken as 10. At bases designed to resist column moments, k may be taken as 1.0.

Slenderness ratio

The slenderness ratio $\lambda = l_0/i$, where i is the radius of gyration of the uncracked concrete section ($i = h/4$ for circular sections, and $h/\sqrt{12}$ for rectangular sections, h being the depth of the section in the plane of bending). Second order effects may be ignored if $\lambda \leq \lambda_{\text{lim}}$, where $\lambda_{\text{lim}} = 20\ (A \times B \times C)/\sqrt{N/A_c f_{ck}}$ and $C = (1.7 - M_{01}/M_{02})$, where M_{01} and M_{02} are the first order end moments with $|M_{02}| \geq |M_{01}|$. Values of $A = 0.7$ and $B = 1.1$ may be used in the absence of specific data (see Eurocode 2), from which the following values of λ_{lim} are obtained for particular cases.

Case	Design condition	C	$\sqrt{N/A_c f_{ck}}\ \lambda_{\text{lim}}$
1	Braced column with $M_{01} = -M_{02}$ (i.e. equal and opposite moments)	2.7	41.6
2	Braced column with $M_{01} = 0$ (i.e. pinned at one end)	1.7	26.2
3	Braced column with $M_{01} = M_{02}$ (eg. moments predominately due to imperfections or transverse loading) and unbraced columns in general	0.7	10.8

In cases of biaxial bending, the slenderness criterion should be checked separately for each direction. Depending on the outcome of the check, second order effects (a) may be ignored in both directions, (b) should be taken into account in one direction, or (c) should be taken into account in both directions.

4.16

EC 2 Design procedure for columns – 2

Design moments

Column sections should be designed for the design axial load N and, in each direction, design moments as follows:

$M = (M_0 + M_i) + M_2 \geq Ne_0$

where

$e_0 = h/30 \geq 20$ mm

M_0 is the first order moment obtained by elastic analysis of the structure

M_i is an additional first order moment resulting from imperfections

M_2 is a nominal second order moment resulting from deflection

Note. In the Eurocode, M_0 is used to represent the total first order moment including the effect of imperfections. The approach adopted here is more sensible when using the following equation for M_{0e}.

In braced columns, differing first order end moments M_{01} and M_{02} may be replaced by an equivalent moment:

$M_{0e} = 0.6M_{02} + 0.4M_{01} \geq 0.4M_{02}$ But M should be taken not less than $(M_{02} + M_i)$ or $(M_{01} + M_i) + 0.5M_2$

In the above equation, M_{02} is the larger first order end moment, and M_{01} and M_{02} should have opposite numerical signs if the column is bent in double curvature. In unbraced columns, $M_0 = M_{02}$.

The additional first order moment resulting from imperfections is given by:

$M_i = N(\alpha_h l_0/400)$ $0.67 \leq \alpha_h = 2/\sqrt{l} \leq 1.0$ where l is the length of the column

The nominal second order moment resulting from deflection is given approximately by:

$M_2 = N(0.2f_{yk}/E_s)(K_r K_\varphi l_0^2/d)$ Hence $M_2 = N(K_r K_\varphi l_0^2/d)/2000$ for $f_{yk} = 500$ MPa and $E_s = 200$ Gpa

In the above equation, l_0 is the effective length of the column (see *Table 4.15*) and d is the distance from the more highly compressed face to the reinforcement at the opposite face. If the reinforcement is not all concentrated on opposite faces, but part is distributed parallel to the plane of bending, $d = h/2 + i_s$ where i_s is the radius of gyration of the total reinforcement area. If the first order moment is constant, consideration should be given to increasing the value of M_2 given by the above equation. Increasing the value by 25% (i.e. in the denominator, replacing 2000 by 1600) would be appropriate for constant curvature (i.e. constant total moment).

K_r is a correction factor derived from $K_r = (N_u - N)/(N_u - N_{bal}) \leq 1.0$, where $N_u = 0.567f_{ck}A_c + 0.87f_{yk}A_s$, and N_{bal} is the axial load at the maximum resistance moment, which may be taken as $0.225f_{ck}A_c$. The appropriate value of K_r may be found iteratively. Alternatively, it will always be conservative to use $K_r = 1.0$.

K_φ is a creep effect factor derived from $K_\varphi = (1 + \beta\varphi_{ef}) \geq 1.0$, in which $\beta = (0.35 + f_{ck}/200 - \lambda/150)$, and φ_{ef} is the effective creep ratio given by $\varphi_{ef} = \varphi(\infty, t_0) \times (M_{0qp}/M_0)$ where M_{0qp} is the first order moment in quasi-permanent load combination (SLS), and values of $\varphi(\infty, t_0)$ can be determined from *Table 4.3*.

For inside conditions (RH = 50%), age of concrete at time of loading $t_0 \geq 30$ days, and notional size $h_0 = 150$ mm: values of $\varphi(\infty, t_0)$ vary between 3.0 for $f_{ck} = 20$ MPa and 1.5 for $f_{ck} = 50$ MPa. Taking $\varphi_{ef} = (2/3)\varphi(\infty, t_0)$ gives values for the creep effect factor K_φ as shown in the following table:

f_{ck} MPa	φ_{ef}	K_φ for values of λ					
		15	30	45	60	75	90
20	2.0	1.7	1.5	1.3	1.1	1.0	1.0
50	1.0	1.5	1.4	1.3	1.2	1.1	1.0

Biaxial bending

As a first step, a separate design in each principal direction may be made, ignoring biaxial bending. Imperfections need be considered only in the direction where they have the more unfavourable effect. No further check is needed if the following two conditions are satisfied: (a) $0.5\lambda_y \leq \lambda_x \leq 2\lambda_y$, (b) M_x/h is either $\leq 0.2M_y/b$ or $\geq 5M_y/b$. In the foregoing, λ_x and λ_y are slenderness ratios, M_x and M_y are design moments including necessary second order moments, h and b are section depths, in respect of the x-x and y-y axes respectively.

If the foregoing conditions are not satisfied, the section should be designed for biaxial bending. A symmetrically reinforced rectangular section may be designed as being bent separately about each axis in turn, providing the following criterion is satisfied.

$$\left[\frac{M_x}{M_{ux}}\right]^{\alpha_n} + \left[\frac{M_y}{M_{uy}}\right]^{\alpha_n} \leq 1.0$$

For $N/N_u < 0.7$: $\alpha_n = 0.92 + 0.83\,(N/N_u) \geq 1.0$

For $N/N_u \geq 0.7$: $\alpha_n = 0.33 + 1.67\,(N/N_u) \leq 2.0$

M_x, M_y are applied moments about the x-x and y-y axes respectively, including necessary second order moments

M_{ux}, M_{uy} are resistance moments about the x-x and y-y axes respectively, corresponding to the axial load capacity of the section ignoring all bending, given by $N_u = 0.567f_{ck}A_c + 0.87f_{yk}A_s$

For circular sections, $\alpha_n = 2$, or the section can be designed for the resultant uniaxial moment: $M = \sqrt{\left(M_x^2 + M_y^2\right)}$.

Example 5. A 400 mm circular unbraced column designed for the same requirements as example 3. Thus, $k = 0.675$ as before.

From *Table 4.15*, effective length for an unbraced column, where the joint stiffness is the same at both ends, is given by

$$l_0 = [(1 + 2k)/(1 + k)] \times l = 1.4 \times 5.0 = 7.0 \text{ m}$$

Slenderness ratio $\lambda = l_0/i = 7000/(400/4) = 70$

Cross-sectional area $A_c = (\pi/4) \times 400^2 = 125.7 \times 10^3 \text{ mm}^2$

$$\lambda_{\text{lim}} = 10.8/\sqrt{N/A_c f_{ck}} = 10.8/\sqrt{1720/(125.7 \times 32)} = 16.5$$

Since $\lambda > \lambda_{\text{lim}}$, second order moments need to be considered. Allowing for 8 mm links and 32 mm main bars,

$$h_s = 400 - 2 \times (35 + 8 + 16) = 280 \text{ mm say}$$

Additional first order moment resulting from imperfections,

$$M_i = N(\alpha_h l_0/400) = 1720 \times (0.9 \times 7.0/400) = 27 \text{ kNm}$$

Second order moment resulting from deflection, with $K_r = 1.0$ (max), $K_\varphi = 1.1$ for $f_{ck} = 32$ MPa and $\lambda = 70$ (*Table 4.16*), and $d = h/2 + 0.35h_s = 298$ mm:

$$M_2 = N(K_r K_\varphi l_0^2/d)/2000$$
$$= 1720 \times (1.0 \times 1.1 \times 7.0^2/0.298)/2000 = 155 \text{ kNm}$$

Total design moments at end 2 of column

$$M_x = M_{02} + M_i + M_2 = 40 + 27 + 155 = 222 \text{ kNm } (>M_{\text{min}})$$
$$M_y = M_2 = 155 \text{ kNm}$$

Resultant uniaxial moment at end 2 of column

$$M = \sqrt{(M_x^2 + M_y^2)} = \sqrt{(222^2 + 155^2)} = 270 \text{ kNm}$$

$$M/h^3 f_{ck} = 270 \times 10^6/(400^3 \times 32) = 0.13$$
$$N/h^2 f_{ck} = 1720 \times 10^3/(400^2 \times 32) = 0.34$$

From the design chart for $h_s/h = 280/400 = 0.7$

$$A_s f_{yk}/A_c f_{ck} = 0.84 \text{ (Table 4.14)}$$
$$A_s = 0.84 \times 125.7 \times 10^3 \times 32/500 = 6758 \text{ mm}^2$$

It can be seen from the chart that $K_r < 1.0$. Using 8H32, gives

$$A_s f_{yk}/A_c f_{ck} = 6434 \times 500/(125.7 \times 10^3 \times 32) = 0.80$$
For $N/h^2 f_{ck} = 0.34$, $M/h^3 f_{ck} = 0.125$ and $K_r = 0.75$

Hence, $M_u = 0.125 \times 400^3 \times 32 \times 10^{-6} = 256$ kNm

With $K_r = 0.75$, modified $M_2 = 0.75 \times 155 = 116$ kNm

Total design moments at end 2 of column

$$M_x = M_{02} + M_i + M_2 = 40 + 27 + 116 = 183 \text{ kNm}$$
$$M_y = M_2 = 116 \text{ kNm}$$

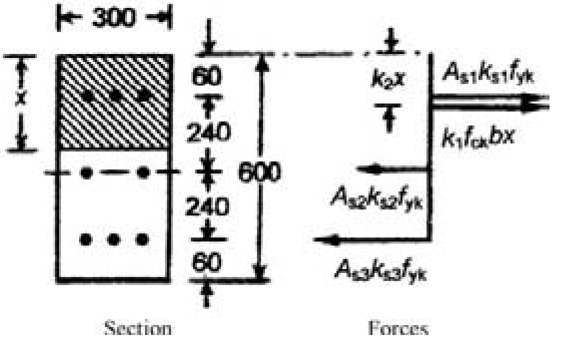
Resultant uniaxial moment at end 2 of column

$$M = \sqrt{(183^2 + 116^2)} = 217 \text{ kNm}$$

Since $M < M_u$, 8H32 is sufficient.

Example 6. The column section in the following figure is reinforced with 8H32 arranged as shown. The moment of resistance about the major axis is to be obtained for the following requirements:

$$N = 2300 \text{ kN}, f_{ck} = 32 \text{ MPa}, f_{yk} = 500 \text{ MPa}$$



Consider the bars in each half of the section to be replaced by an equivalent pair of bars. Depth to the centroid of the bars in one half of the section = 60 + 240/4 = 120 mm. The section is now considered to be reinforced with four equivalent bars, where $d = 600 - 120 = 480$ mm.

$$A_s f_{yk}/bhf_{ck} = 6434 \times 500/(300 \times 600 \times 32) = 0.56$$
$$N/bhf_{cu} = 2300 \times 10^3/(300 \times 600 \times 32) = 0.40$$

From the design chart for $d/h = 480/600 = 0.8$,

$$M_u/bh^2 f_{ck} = 0.18 \text{ (Table 4.11)}$$
$$M_u = 0.18 \times 300 \times 600^2 \times 32 \times 10^{-6} = 622 \text{ kNm}$$

The solution can be checked, using a trial-and-error process to analyse the original section, as follows:

The axial load on the section is given by:

$$N = k_1 f_{ck} bx + (A_{s1}k_{s1} - A_{s2}k_{s2} - A_{s3}k_{s3})f_{yk}$$

where $d/h = 540/600 = 0.9$, and k_{s1}, k_{s2} and k_{s3}, are given by:

$$k_{s1} = 1.4(x/h + d/h - 1)/(x/h) \leq 0.87$$
$$k_{s2} = 1.4(0.5 - x/h)/(x/h) \leq 0.87$$
$$k_{s3} = 1.4(d/h - x/h)/(x/h) \leq 0.87$$

With $x = 300$ mm, $x/h = 0.5$, $k_{s1} = 0.87$, $k_{s2} = 0$ and $k_{s3} = 0.87$

$$N = 0.45 \times 32 \times 300 \times 300 \times 10^{-3} = 1296 \text{ kN } (<2300)$$

With $x = 360$ mm, $x/h = 0.6$, $k_{s2} = -0.233$, $k_{s3} = 0.7$

$$N = 0.45 \times 32 \times 300 \times 360 \times 10^{-3} + (2413 \times 0.87$$
$$+ 1608 \times 0.233 - 2413 \times 0.7) \times 500 \times 10^{-3}$$
$$= 1555 + 392 = 1947 \text{ kN } (<2300)$$

With $x = 390$ mm, $x/h = 0.65$, $k_{s2} = -0.323$, $k_{s3} = 0.538$

$$N = 0.45 \times 32 \times 300 \times 390 \times 10^{-3} + (2413 \times 0.87$$
$$+ 1608 \times 0.323 - 2413 \times 0.538) \times 500 \times 10^{-3}$$
$$= 1685 + 660 = 2345 \text{ kN } (>2300)$$

With $x = 387$ mm, $x/h = 0.645$, $k_{s2} = -0.315$, $k_{s3} = 0.553$

$$N = 0.45 \times 32 \times 300 \times 387 \times 10^{-3} + (2413 \times 0.87$$
$$+ 1608 \times 0.315 - 2413 \times 0.553) \times 500 \times 10^{-3}$$
$$= 1672 + 636 = 2308 \text{ kN}$$

Since the internal and external forces are now sensibly equal, taking moments about the mid-depth of the section gives:

$$M_u = k_1 f_{ck} bx(0.5h - k_2 x) + (A_{s1}k_{s1} + A_{s3}k_{s3})(d - 0.5h)f_y$$
$$= 0.45 \times 32 \times 300 \times 387 \times (300 - 0.4 \times 387) \times 10^{-6}$$
$$+(2413 \times 0.87 + 2413 \times 0.553)(540 - 300)$$
$$\times 500 \times 10^{-6}$$
$$= 243 + 412 = 655 \text{ kNm } (>622 \text{ obtained before})$$

Chapter 33

Shear and torsion

33.1 SHEAR RESISTANCE

33.1.1 Members without shear reinforcement

The design shear resistance at any cross section of a member not requiring shear reinforcement can be calculated as:

$$V_{\mathrm{Rd,c}} = v_{\mathrm{Rd,c}} b_w d$$

where

b_w is the minimum width of section in the tension zone

d is the effective depth to the tension reinforcement

$v_{\mathrm{Rd,c}}$ is the design concrete shear stress

The design concrete shear stress is a function of the concrete strength, the effective depth and the reinforcement percentage at the section considered. To be effective, this reinforcement should extend a distance $\geq (l_{\mathrm{bd}} + d)$ beyond the section, where l_{bd} is the design anchorage length. At a simple support, for a member carrying predominantly uniform load, the length l_{bd} may be taken from the face of the support. The design shear resistance of members with and without axial load can be determined from the data given in *Table 4.17*.

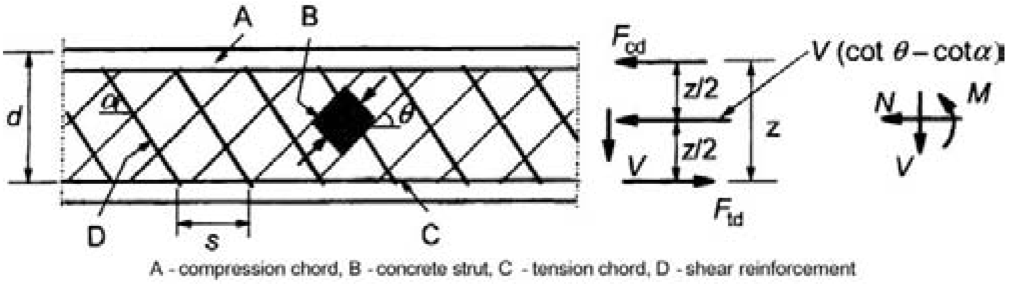
In the UK National Annex, it is recommended that the shear strength of concrete strength classes higher than C50/60 is determined by tests, unless there is evidence of satisfactory past performance of the particular concrete mix including the aggregates used. Alternatively, the shear strength should be limited to that given for concrete strength class C50/60.

33.1.2 Members with shear reinforcement

The design of members with shear reinforcement is based on a truss model, in which the compression and tension chords are spaced apart by a system of inclined concrete struts and vertical or inclined shear reinforcement. Angle α between the reinforcement and the axis of the member should be $\geq 45^{\circ}$.

Angle θ between the struts and the axis of the member may be selected by the designer within the limits $1.0 \leq \cot\theta \leq 2.5$ generally. However, for elements in which shear co-exists with externally applied tension, $\cot\theta$ should be taken as 1.0. The web forces are $V\sec\theta$ in the struts and $V\sec\alpha$ in the shear reinforcement over a panel length $l = z\ (\cot\alpha + \cot\theta)$, where z may normally be taken as $0.9d$. The width of each strut is equal to $z\ (\cot\alpha + \cot\theta)\ \sin\theta$ and the design value of the maximum shear force $V_{\mathrm{Rd,max}}$ is limited to the compressive resistance provided by the struts, which includes a strength reduction factor for concrete cracked in shear. The least shear reinforcement is required when $\cot\theta$ is such that $V = V_{\mathrm{Rd,max}}$. The truss model results in a force ΔF_{td} in the tension chord that is additional to the force M/z due to bending, but the sum $\Delta F_{\mathrm{td}} + M/z$ need not be taken greater than $M_{\max}/z$, where $M_{\max}$ is the maximum moment in the relevant hogging or sagging region. The additional force ΔF_{td} can be taken into account by shifting the bending moment curve each side of any point of maximum moment by an amount $a_l = 0.5z(\cot\theta - \cot\alpha)$. For members without shear reinforcement, $a_l = d$ should be used. The curtailment of the longitudinal reinforcement can then be based on the modified bending moment diagram. A design procedure to determine the required area of shear reinforcement, and details of the particular requirements for beams and slabs, are given in *Table 4.18*.

For most beams, a minimum amount of shear reinforcement in the form of links is required, irrespective of the magnitude of the shear force. Thus, there is no need to determine $V_{\mathrm{Rd,c}}$.



A - compression chord, B - concrete strut, C - tension chord, D - shear reinforcement

Truss model and notation for members with shear reinforcement

4.17

EC 2 Shear resistance – 1

Members without shear reinforcement

The design shear resistance of a flexural member without shear reinforcement is given by $V_{\mathrm{Rd,c}} = v_{\mathrm{Rd,c}} b_w d$, where:

$$v_{\mathrm{Rd,c}} = \left(\frac{0.18k}{\gamma_c}\right)\left(\frac{100A_{sl}f_{ck}}{b_w d}\right)^{1/3} \geq v_{\min} = 0.035k^{3/2}f_{ck}^{\ 1/2} \quad \text{with} \quad k = 1 + \sqrt{\frac{200}{d}} \leq 2.0, \quad \left(\frac{100A_{sl}}{b_w d}\right) \leq 2.0 \quad \text{and} \quad \gamma_c = 1.5$$

A_{sl} is area of longitudinal tension reinforcement that extends a distance $\geq (l_{bd} + d)$ beyond section considered
b_w is minimum width of section within tension zone (i.e. between neutral axis and level of tension reinforcement)
d is effective depth of tension reinforcement
l_{bd} is design anchorage length of longitudinal tension reinforcement

For members carrying predominantly uniform load, $V_{\mathrm{Rd,c}}$ need not be checked at sections less than d from the face of a support, provided $V \leq V_{\mathrm{Rd,max}} = 0.2\,(1 - f_{ck}/250) f_{ck} b_w d$ at the support. For members with load applied on the upper face at distance $a_v \leq 2d$ from the edge of a support (or centre of bearing, where flexible bearings are used), the contribution to the resulting shear force may be reduced by multiplying this load by $\beta = a_v/2d \geq 0.25$. The tension reinforcement should be fully anchored at the support, and the shear force calculated without using β should not exceed $V_{\mathrm{Rd,max}}$.

For a flexural member subjected also to axial load N (positive compression, negative tension), $v_{\mathrm{Rd,c}}$ may be replaced by $v_{\mathrm{Rd,c}} + 0.15N/A_c \leq v_{\mathrm{Rd,c}} + 0.02f_{ck}$, where A_c is the gross area of the concrete section.

f_{ck} MPa	$\dfrac{100A_{sl}}{b_w d}$	Design concrete shear stress $v_{\mathrm{Rd,c}}$ (MPa) for values of d (mm)								
		≤ 200	250	300	400	500	700	1000	1500	2000
20	≤ 0.15	*0.44*	*0.40*	*0.38*	*0.35*	*0.32*	*0.30*	*0.27*	*0.25*	*0.23*
	0.40	0.48	0.45	0.43	0.41	0.39	0.37	0.35	0.33	0.31
	0.60	0.55	0.52	0.50	0.47	0.45	0.42	0.40	0.37	0.36
	0.80	0.60	0.57	0.55	0.51	0.49	0.46	0.44	0.41	0.40
	1.0	0.65	0.61	0.59	0.55	0.53	0.50	0.47	0.44	0.43
	1.5	0.74	0.70	0.67	0.63	0.61	0.57	0.54	0.51	0.49
	≥ 2.0	0.82	0.77	0.74	0.70	0.67	0.63	0.59	0.56	0.54
25	≤ 0.15	*0.49*	*0.45*	*0.42*	*0.39*	*0.36*	*0.33*	*0.30*	*0.28*	*0.26*
	0.40	0.52	0.49	0.47	0.44	0.42	0.39	0.37	0.35	0.34
	0.60	0.59	0.56	0.54	0.50	0.48	0.45	0.43	0.40	0.39
	0.80	0.65	0.62	0.59	0.55	0.53	0.50	0.47	0.44	0.43
	1.0	0.70	0.66	0.63	0.60	0.57	0.54	0.51	0.48	0.46
	1.5	0.80	0.76	0.73	0.68	0.65	0.61	0.58	0.55	0.53
	≥ 2.0	0.88	0.84	0.80	0.75	0.72	0.68	0.64	0.60	0.58
28	≤ 0.20	*0.52*	*0.48*	*0.45*	*0.41*	*0.38*	*0.35*	*0.32*	*0.29*	*0.28*
	0.40	0.54	0.51	0.49	0.46	0.44	0.41	0.39	0.36	0.35
	0.60	0.61	0.58	0.56	0.52	0.50	0.47	0.44	0.42	0.40
	0.80	0.67	0.64	0.61	0.58	0.55	0.52	0.49	0.46	0.44
	1.0	0.73	0.69	0.66	0.62	0.59	0.56	0.53	0.50	0.48
	1.5	0.83	0.79	0.76	0.71	0.68	0.64	0.60	0.57	0.55
	≥ 2.0	0.92	0.87	0.83	0.78	0.75	0.70	0.66	0.62	0.60
32	≤ 0.20	*0.56*	*0.51*	*0.48*	*0.44*	*0.41*	*0.37*	*0.34*	*0.31*	*0.30*
	0.40	0.56	0.53	0.51	0.48	0.46	0.43	0.40	0.38	0.37
	0.60	0.64	0.61	0.58	0.55	0.52	0.49	0.46	0.44	0.42
	0.80	0.71	0.67	0.64	0.60	0.58	0.54	0.51	0.48	0.46
	1.0	0.76	0.72	0.69	0.65	0.62	0.58	0.55	0.52	0.50
	1.5	0.87	0.82	0.79	0.74	0.71	0.67	0.63	0.59	0.57
	≥ 2.0	0.96	0.91	0.87	0.82	0.78	0.73	0.69	0.65	0.63
40	≤ 0.20	*0.62*	*0.57*	*0.54*	*0.49*	*0.46*	*0.42*	*0.38*	*0.35*	*0.33*
	0.40	*0.62*	0.57	0.55	0.51	0.49	0.46	0.44	0.41	0.40
	0.60	0.69	0.65	0.63	0.59	0.56	0.53	0.50	0.47	0.45
	0.80	0.76	0.72	0.69	0.65	0.62	0.58	0.55	0.52	0.50
	1.0	0.82	0.78	0.74	0.70	0.67	0.63	0.59	0.56	0.54
	1.5	0.94	0.89	0.85	0.80	0.76	0.72	0.68	0.64	0.62
	≥ 2.0	1.03	0.98	0.94	0.88	0.84	0.79	0.75	0.70	0.68
50	≤ 0.25	*0.70*	*0.64*	*0.60*	*0.55*	*0.51*	*0.47*	*0.43*	*0.39*	*0.37*
	0.50	0.70	0.66	0.64	0.60	0.57	0.54	0.51	0.48	0.46
	0.75	0.80	0.76	0.73	0.68	0.65	0.61	0.58	0.55	0.53
	1.0	0.88	0.84	0.80	0.75	0.72	0.68	0.64	0.60	0.58
	1.5	1.01	0.96	0.92	0.86	0.82	0.77	0.73	0.69	0.66
	≥ 2.0	1.11	1.05	1.01	0.95	0.91	0.85	0.80	0.76	0.73

Note. Values of $v_{\mathrm{Rd,c}}$ shown in italic are determined by $v_{\min}$. For concrete strength classes > C50/60, see section 33.1.1.

EC 2 Shear resistance – 2

4.18

Members with shear reinforcement

The design shear resistance of a flexural member with inclined shear reinforcement is given by:

$V_{Rd,s} = (A_{sw}/s) f_{ywd}\, l \sin\alpha \le V_{Rd,max} = 0.4\,(1 - f_{ck}/250)\, f_{ck}\, b_w\, l \sin^2\theta$ with $\Delta F_{td} = 0.5V(\cot\theta - \cot\alpha)$

A_{sw}	is total cross-sectional area of shear reinforcement at section considered
ΔF_{td}	is additional force in longitudinal tension reinforcement, but $\Delta F_{td} + M/z$ need not be taken greater than M_{max}/z where M and V are co-existent values of bending moment and shear force, and M_{max} is the maximum moment within the hogging or sagging region that contains the section considered
b_w	is minimum width of section between tension and compression chords
f_{ywd}	is design yield strength of shear reinforcement ($f_{ywd} \le 0.87 f_{yk}$)
l	is length increment given by $l = z\,(\cot\alpha + \cot\theta)$. In regions where there is no discontinuity of V, the required shear reinforcement for any length l may be determined using the smallest value of V in the increment.
s	is spacing of shear reinforcement along longitudinal axis of member
z	is inner lever arm, corresponding to M_{max} (for a member of constant depth). For a member without axial force the approximate value $z = 0.9d$ may normally be used.
α	is angle ($45° \le \alpha \le 90°$) between shear reinforcement and longitudinal axis of member
θ	is angle ($1.0 \le \cot\theta \le 2.5$) between concrete compression strut and longitudinal axis of member

For members with upright ($\alpha = 90°$) shear reinforcement, $l = z\cot\theta$ and the design shear resistance is given by:

$V_{Rd,s} = (A_{sw}/s) f_{ywd}\, z \cot\theta \le V_{Rd,max} = 0.4\,(1 - f_{ck}/250)\, f_{ck}\, b_w\, z \sin^2\theta \cot\theta$ with $\Delta F_{td} = 0.5V\cot\theta$

The smallest value of A_{sw}/s (and largest value of ΔF_{td}) is obtained, when $V = V_{Rd,max}$ is solved to find $\cot\theta \le 2.5$. Values of $\cot\theta$ can be determined from the table below, according to the value of $v_w = V/(1 - f_{ck}/250) f_{ck}\, b_w\, z \le v_{w,max}$. If desired, the value of ΔF_{td} may be reduced (and A_{sw}/s increased) by using smaller values of $\cot\theta \ge 1.0$.

α	$\cot\theta$	2.5	2.2	2.0	1.8	1.6	1.4	1.2	1.0
90°	$v_{w,max} = 0.4\sin^2\theta\cot\theta$	≤ 0.138	0.150	0.160	0.170	0.180	0.189	0.197	0.200
45°	$v_{w,max} = 0.4\sin^2\theta(1 + \cot\theta)$	≤ 0.193	0.219	0.240	0.264	0.292	0.324	0.360	0.400
Note. For members with upright links combined with bent-up bars, values of $v_{w,max}$ should be interpolated pro rata.									

Example. Suppose $V/(1 - f_{ck}/250) f_{ck}\, b_w\, z = 0.16$. Then, for upright links ($\alpha = 90°$), the smallest value of A_{sw}/s is obtained with $\cot\theta = 2.0$ and $\Delta F_{td} = 0.5V\cot\theta = 1.0V$. Any value of $\cot\theta$ between 2.0 and 1.0 would also be valid. For 60% upright links and 40% bars bent-up at 45°, the smallest values of A_{sw}/s are obtained with $\cot\theta = 2.5$, for which $v_{w,max} = 0.6 \times 0.138 + 0.4 \times 0.193 = 0.16$, and $\Delta F_{td} = 0.5V(\cot\theta - 0.4\cot\alpha) = 0.5(2.5 - 0.4 \times 1.0)V = 1.05V$.

Form of shear reinforcement		
Value of V	Solid slabs	Beams
$V \le V_{Rd,c}$	None required	Minimum links with $s \le 0.75d(1 + \cot\alpha)$, and transverse spacing of legs $s_t \le 0.75d \le 600$ mm. However, links may be omitted in members such as lintels with span ≤ 2 m, which contribute little to the overall stability of a structure.
$V_{Rd,c} < V \le V_{Rd,max}/3$	In slabs ≥ 200 mm deep, links with $s \le 0.75d(1 + \cot\alpha)$ and $s_t \le 1.5d$, or bent-up bars with $s_b \le d$.	Links, or links combined with bent-up bars at a longitudinal spacing $s_b \le 0.6d(1 + \cot\alpha)$, but not more than 50% of shear reinforcement in form of bent-up bars.
$V_{Rd,max}/3 < V \le V_{Rd,max}$	As above but not more than 50% of shear reinforcement as bent-up bars.	

Required area of shear reinforcement: $\dfrac{A_{sw}}{s} = \dfrac{V/\sin\alpha\,(\cot\alpha + \cot\theta)}{f_{ywd}\, z} \ge \rho_{w,min}\, b_w \sin\alpha$ $\rho_{w,min} = (0.08\sqrt{f_{ck}})/f_{yk}$

$(\alpha = 90°)\ \dfrac{A_{sw}}{s} = \dfrac{V/\cot\theta}{f_{ywd}\, z} \ge \rho_{w,min}\, b_w$ $(\alpha = 45°)\ \dfrac{A_{sw}}{s} = \dfrac{V\sqrt{2}/(1 + \cot\theta)}{f_{ywd}\, z} \ge (\rho_{w,min}/\sqrt{2})\, b_w$

For members with load applied on the upper face at a distance $a_v \le 2d$ from the edge of a support (or centre of bearing, where flexible bearings are used), the contribution to the resulting shear force may be reduced by multiplying this load by $\beta = a_v/2d \ge 0.25$. The shear force calculated in this way should satisfy the condition $V \le A_{sw} f_{ywd} \sin\alpha$, where $A_{sw} f_{ywd}$ is the resistance of the shear reinforcement provided within the middle three-quarters of a_v. The reduction is valid only if the longitudinal tension reinforcement is fully anchored at the support.

In members with inclined chords, the shear components of the design forces in the chords may be added to the design shear resistance provided by the reinforcement. In checking that the design shear force does not exceed $V_{Rd,max}$, the same shear components may be deducted from the shear force resulting from the design loads.

33.1.3 Shear under concentrated loads

In slabs and column bases, the maximum shear stress at the perimeter of a concentrated load should not exceed $v_{Rd,max}$. Shear in solid slabs under concentrated loads can result in punching failures on the inclined faces of truncated cones or pyramids. For design purposes, a control perimeter forming the shortest boundary that nowhere comes closer to the perimeter of the loaded area than a specified distance should be considered. The basic control perimeter may generally be taken at a distance $2d$ from the perimeter of the loaded area.

If the maximum shear stress here is no greater than $v_{Rd,c}$, no shear reinforcement is required. Otherwise, the position of the control perimeter at which the maximum shear stress is equal to $v_{Rd,c}$ should be determined, and shear reinforcement provided in the zone between this control perimeter and the perimeter of the loaded area.

For flat slabs with enlarged column heads (or drop panels), where d_H is the effective depth at the face of the column and the column head (or drop) extends a distance $l_H > 2d_H$ beyond the face of the column, a basic control perimeter at a distance $2d_H$ from the column face should be considered. In addition, a basic control perimeter at a distance $2d$ from the column head (or drop) should be considered.

Control perimeters (in part or as a whole) at distances less than $2d$ should also be considered where a concentrated load is applied close to a supported edge, or is opposed by a high pressure (e.g. soil pressure on bases). In such cases, values of $v_{Rd,c}$ may be multiplied by $2d/a$, where a is the distance from the edge of the load to the control perimeter. For column bases, the favourable action of the soil pressure may be taken into account when determining the shear force acting at the control perimeter.

Details of design procedures for shear under concentrated loads are given in *Table 4.19*.

33.1.4 Bottom loaded beams

Where load is applied near the bottom of a section, sufficient vertical reinforcement to transmit the load to the top of the section should be provided in addition to any reinforcement required to resist shear.

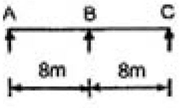
33.2 DESIGN FOR TORSION

In normal beam-and-slab or framed construction, calculations for torsion are not usually necessary, adequate control of any torsional cracking in beams being provided by the required minimum shear reinforcement. When it is judged necessary to include torsional stiffness in the analysis of a structure, or torsional resistance is vital for static equilibrium, members should be designed for the resulting torsional moment.

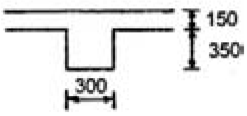
The torsional resistance may be calculated on the basis of a thin-walled closed section, in which equilibrium is satisfied by a plastic shear flow. A solid section may be modelled as an equivalent thin-walled section. Complex shapes may be divided into a series of sub-sections, each of which is modelled as an equivalent thin-walled section, and the total torsional resistance taken as the sum of the resistances of the individual elements. When torsion reinforcement is required, this should consist of rectangular closed links together with longitudinal reinforcement. Such reinforcement is additional to the requirements for shear and bending. Details of a suitable design procedure for torsion are given in *Table 4.20*.

Example 1. The beam shown in the following figure, which was designed for bending in example 1 of Chapter 32, is to be designed for shear. The maximum design load is 49.5 kN/m and the design is based on the following values:

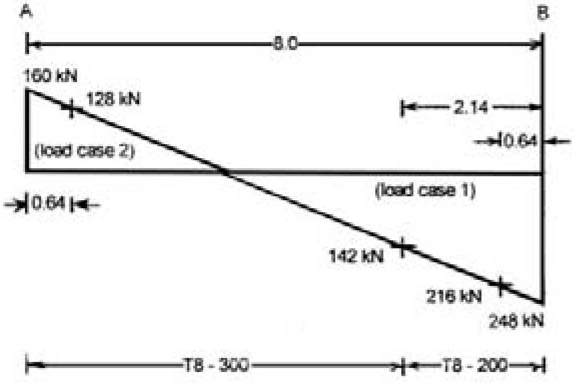
$$f_{ck} = 32 \text{ MPa}, f_{ywk} = 500 \text{ MPa}, d = 440 \text{ mm}$$



Spans



Cross section



Shear force diagram

Since the load is uniformly distributed, the critical section for shear may be taken at distance d from the face of the support. Based on a support width of 400 mm, distance from centre of support to critical section = 200 + 440 = 640 mm. At end B,

$$\begin{aligned} V &= 248 - 0.64 \times 49.5 = 216 \text{ kN} \\ \nu_w &= V/[b_w\, z\, (1 - f_{ck}/250)f_{ck}] \\ &= 216 \times 10^3/[300 \times 0.9 \times 440 \times (1 - 32/250) \times 32] \\ &= 0.065 \end{aligned}$$

From *Table 4.18*, since $\nu_w < 0.138$, $\cot\theta = 2.5$ may be used. Hence, area of links required is given by:

$$\begin{aligned} A_{sw}/s &= V/f_{ywd}\, z \cot\theta \\ &= 216 \times 10^3/(0.87 \times 500 \times 0.9 \times 440 \times 2.5) \\ &= 0.50 \text{ mm}^2/\text{mm} \end{aligned}$$

From *Table 4.20*, H8-200 provides 0.50 mm²/mm

4.19

EC 2 Shear under concentrated loads

Design procedure

For design purposes, a control perimeter is taken as the shortest boundary that nowhere comes closer to the perimeter of a loaded area than a specified distance. The basic control perimeter may generally be taken at a distance $2d$ from the perimeter of the loaded area, where d is taken as the mean of the effective depths of the reinforcement in two orthogonal directions (for flat slabs with enlarged column heads, or drop panels, see section 33.1.3). If the maximum shear stress at the basic control perimeter is no greater than $v_{Rd,c}$ (see *Table 4.17*), no shear reinforcement is required. In evaluating $v_{Rd,c}$, the reinforcement ratio A_{sl}/bd should be taken as $\sqrt{(\rho_x\ .\ \rho_y)} \leq 0.02$, where ρ_x and ρ_y are the reinforcement ratios (averaged over a width equal to the column width plus $3d$ each side) in two orthogonal directions.

In column bases, checks should also be made on control perimeters at distances $< 2d$, in which case the value of $v_{Rd,c}$ may be multiplied by $2d/a$, where a is the distance from the edge of the loaded area to the control perimeter. In determining the shear force, allowance may be made for the favourable action of the soil pressure within the control perimeter. If the depth of the base reduces towards the edge, d may be taken as the value at the loaded area.

If v exceeds $v_{Rd,c}$ at a control perimeter, shear reinforcement in the form of links, bent-down bars or other products may be provided in slabs ≥ 200 mm thick. The position of the control perimeter at which the maximum shear stress is equal to $v_{Rd,c}$ should be determined, and shear reinforcement provided in the zone between this control perimeter and the loaded area. Adjacent to the load, the maximum shear stress should not exceed $v_{Rd,max} = 0.2(1 - f_{ck}/250)f_{ck}$.

Maximum shear stress

Where a load or reaction is eccentric with regard to a control perimeter (e.g. at the edge of a slab, and in cases of moment transfer between a slab and a column), the maximum shear stress is given by $v = \beta V/u_i\, d$, where V is the design value of the load or reaction, β is a coefficient (see below), and u_i is the length of the control perimeter being considered, reduced where necessary at the edge of a slab or for the effect of openings (see *Table 3.37*).

For flat slab structures, values of β for internal rectangular columns can be determined precisely from equations given in Eurocode 2, but the following values are adequate for most purposes, where M is the moment transmitted to the column, and c_1 and c_2 are column dimensions respectively parallel to and perpendicular to the plane of the moment:

(for $c_2 \leq c_1$), $\beta = 1 + \dfrac{1.8(M/V)}{c_2 + 4d}$ (for $c_2 \geq 2c_1$), $\beta = 1 + \dfrac{1.5(M/V)}{c_2 + 4d}$ (for $c_1 < c_2 < 2c_1$), β by interpolation

For internal circular columns $\beta = 1 + \dfrac{0.6\pi(M/V)}{D + 4d}$ where D is the diameter of the column

For column bases, the above β expressions apply, except that $4d$ should be replaced by $2a$ where $a \leq 2d$ is the distance from the edge of the loaded area to the control perimeter considered. For concentric loading, $\beta = 1.0$.

For edge and corner rectangular columns in flat slab structures, where the eccentricity is towards the interior of the slab, $\beta = u_1/u_{1,ef}$ where u_1 is the basic control perimeter, and $u_{1,ef}$ is a reduced basic control perimeter. Thus, the maximum shear stress is given by $v = \beta V/u_1\, d = V/u_{1,eff}\, d$, where u_1 and $u_{1,eff}$ are given by the following expressions:

(edge) $u_1 = 2a_1 + c_2 + 2\pi d$ $u_{1,ef} = c_1 + c_2 + 2\pi d \leq c_2 + (2\pi + 3)d$
(corner) $u_1 = a_1 + a_2 + \pi d$ $u_{1,ef} = 0.5(c_1 + c_2) + \pi d \leq 0.5c_1 + (\pi + 1.5)d \leq 0.5c_2 + (\pi + 1.5)d$

(In the above expressions, a_1 and a_2 are dimensions from the edges of the slab to the interior edges of the column)

For columns in flat slab structures whose lateral stability is not dependent on frame action, and where the adjacent spans do not differ in length by more than 25%, the following approximate values of β may be used:

(internal) $\beta = 1.15$ (edge) $\beta = 1.4$ (corner) $\beta = 1.5$

Adjacent to a loaded area, the maximum shear stress is given by $v = \beta V/u_0\, d$, where u_0 is the perimeter of the load reduced where necessary for the effect of openings. For columns in flat slab structures, the following values apply:

(internal) $u_0 = 2c_1 + 2c_2$ (edge) $u_0 = 2c_1 + c_2 \leq c_2 + 3d$ (corner) $u_0 = c_1 + c_2 \leq c_1 + 1.5d \leq c_2 + 1.5d$

Shear reinforcement

Where shear reinforcement is needed in slabs ≥ 200 mm deep, the following requirements apply:

$A_{sw} = (v - 0.75v_{Rd,c})\, u_1\, s_r/(1.5 f_{ywd,ef} \sin\alpha) \geq \rho_{w,min}\, u_1\, s_r/(1.5\sin\alpha + \cos\alpha)$ where $\rho_{w,min} = (0.08\sqrt{f_{ck}})/f_{yk}$

A_{sw} is total cross-sectional area of one perimeter of shear reinforcement around loaded area
$f_{ywd,ef}$ is effective design strength of shear reinforcement given by $f_{ywd,ef} = 250 + 0.25d \leq f_{ywd}$ (MPa with d in mm)
s_r is radial spacing of perimeters of shear reinforcement
α is angle between shear reinforcement and plane of slab

Link reinforcement should be provided on at least two perimeters, with the first perimeter at a distance from the loaded area between $0.3d$ and $0.5d$. The last perimeter should be at a distance $\geq 1.5d$ from the loaded area, and $\leq 1.5d$ inside the control perimeter where reinforcement is no longer required. Links should be arranged such that $s_r \leq 0.75d$, with the spacing of the legs of the links around a perimeter $s_t \leq 1.5d$ within the first control perimeter, and $\leq 2d$ for parts of perimeters outside the first control perimeter that are assumed to contribute to the shear resistance.

4.20

EC 2 Design for torsion

Design procedure

The torsional resistance of a member may be calculated on the basis of a thin-walled closed section. Solid sections may be modelled as equivalent thin-walled sections. Complex shapes may be considered as a series of sub-sections, with each one modelled as an equivalent thin-walled section, and the total torsional resistance taken as the sum of the values obtained for the individual elements. The distribution of the acting torsional moment over the sub-sections should be in proportion to their un-cracked torsional stiffness values. Thus, T-, L- or I-sections may be divided into component rectangles, and the individual elements designed for torsional moments equal to $T\,[\,h_{\min}^3 h_{\max} / \Sigma\, h_{\min}^3 h_{\max}\,]$, where $h_{\max}$ and $h_{\min}$ are the maximum and minimum dimensions of the particular rectangles.

For approximately rectangular solid sections, minimum shear reinforcement as required for beams (see *Table 4.18*) is sufficient if the condition $T/T_{\mathrm{Rd,c}} + V/V_{\mathrm{Rd,c}} \leq 1.0$ is satisfied, where $T_{\mathrm{Rd,c}}$ is the design torsional resistance (see below) and $V_{\mathrm{Rd,c}}$ is the design shear resistance (see *Table 4.17*) for the uncracked section. The maximum resistance of a solid cross-section subjected to torsion and shear is given by:

$$T/2A_k\,t_{\mathrm{ef,i}} + V/b_w z = 0.34\,(1 - f_{ck}/250)\,f_{ck}\sin^2\theta\cot\theta \qquad \text{where } 1.0 \leq \cot\theta \leq 2.5$$

Torsion cracking moment

The design torsional resistance of an uncracked equivalent thin-walled closed section is given by:

$$T_{\mathrm{Rd,c}} = 2A_k\,t_{\mathrm{ef,i}} f_{\mathrm{ctd}} \qquad \text{where}$$

A is overall area of cross-section within outer circumference, including inner hollow areas
A_k is area enclosed by centre-lines of connecting walls, including inner hollow areas
f_{ctd} is design tensile strength of concrete, taken as $0.14 f_{ck}^{2/3}$ for $f_{ck} \leq 50$ MPa
$t_{\mathrm{ef,i}}$ is effective wall thickness, taken as $A/u \geq$ twice distance from outer circumference to centre of longitudinal bars, but not greater than actual wall thickness for a non-solid section
u is outer circumference of cross-section

Reinforcement requirements

Torsion reinforcement should consist of rectangular closed links at 90° to the longitudinal axis of the member together with longitudinal reinforcement. The areas of reinforcement required for torsion and shear, should be such that:

$$\frac{A_{sl}}{s_1} \geq \frac{T\cot\theta}{2A_k f_{yd}} \qquad \frac{A_{st}}{s} \geq \frac{T/\cot\theta}{2A_k f_{ywd}} \qquad \frac{A_{sw}}{s} \geq \frac{V/\cot\theta}{f_{ywd}\,z}$$

A_{sl} is area of one longitudinal torsion bar
A_{st} is area of one leg of a torsion link
A_{sw} is area of vertical legs of shear links

f_{yd} and f_{ywd} are design yield strengths of reinforcement ($\leq 0.87 f_{yk}$)
s is spacing of link reinforcement along longitudinal axis of member
s_l is spacing of longitudinal bars around inner periphery of links

For closed links with 2 vertical legs, total area required for torsion and shear is given by:

$$\frac{A_s}{s} \geq \left(\frac{T}{A_k} + \frac{V}{z}\right)\left(\frac{1/\cot\theta}{f_{ywd}}\right) \qquad \text{where } v_w = \frac{T/2A_k t_{\mathrm{ef,i}} + V/b_w z}{(1 - f_{ck}/250) f_{ck}} \leq v_{w,\max} = 0.4\sin^2\theta\cot\theta$$

Values of $\cot\theta$, depending on the value of v_w, may be selected from the data given in *Table 4.18*.

The longitudinal spacing of the links in a section or a component rectangle should not exceed the least of $u/8$, $0.75d$ or $h_{\min}$, where $h_{\min}$ is the lesser dimension of the cross-section. Longitudinal reinforcement should be distributed evenly round the inner periphery of the links. The spacing of the bars should not exceed 350 mm and at least four bars, one in each corner of the links, should be used. In regions where reinforcement is needed for bending and torsion, the sum of both requirements can be provided by using larger bars than those required for bending alone. In T-, L- or I-sections, the reinforcement should be detailed so that the link cages interlock and tie the component rectangles together. Area properties of link cages, according to size and spacing, are shown below.

Areas of 2-leg links

Values of A_s/s for 2 legs of link reinforcement (mm^2/mm)

Spacing s (mm)	Size of bar (mm)				Spacing s (mm)	Size of bar (mm)			
	8	10	12	16		8	10	12	16
75	1.34	2.09	3.01	5.36	200	0.50	0.78	1.13	2.01
100	1.00	1.57	2.26	4.02	225	0.44	0.70	1.00	1.79
125	0.80	1.25	1.81	3.21	250	0.40	0.63	0.90	1.61
150	0.67	1.04	1.51	2.68	275	0.36	0.57	0.82	1.46
175	0.57	0.90	1.29	2.30	300	0.33	0.52	0.75	1.34

Minimum requirements for vertical links are given by:

$$\begin{aligned} A_{\mathrm{sw}}/s &= (0.08\sqrt{f_{\mathrm{ck}}})\, b_{\mathrm{w}} / f_{\mathrm{yk}} = (0.08\sqrt{32}) \times 300/500 \\ &= 0.27 \text{ mm}^2/\text{mm} \\ s &\leq 0.75d = 0.75 \times 440 = 330 \text{ mm} \end{aligned}$$

From *Table 4.20*, H8-300 provides 0.33 mm^2/mm

$$\begin{aligned} V_{\mathrm{Rd,s}} &= (A_{\mathrm{sw}}/s)\, f_{\mathrm{ywd}}\, z \cot\theta \\ &= 0.33 \times 0.87 \times 500 \times 0.9 \times 440 \times 2.5 \times 10^{-3} \\ &= 142 \text{ kN} \end{aligned}$$

At end A, $V = 160 - 0.64 \times 49.5 = 128$ kN ($< V_{\mathrm{Rd,s}} = 142$ kN)

Example 2. A 250 mm thick flat slab is supported by 400 mm square columns arranged on a 7.2 m square grid. The slab contains as tension reinforcement in the top of the slab at an interior support, within a 1.8 m wide strip central with the column, H16-150 in each direction. Lateral stability of the structure does not depend on frame action, and the design shear force resulting from the maximum design load applied to all panels adjacent to the column is $V = 854$ kN.

$$f_{\mathrm{ck}} = 40 \text{ MPa}, f_{\mathrm{ywk}} = 500 \text{ MPa}, d = 210 \text{ mm (average)}$$

Since the lateral stability of the structure does not depend on frame action, β may be taken as 1.15 (*Table 4.19*).

Maximum shear stress adjacent to the column face,

$$\begin{aligned} \beta\, V/u_{\mathrm{o}}d &= 1.15 \times 854 \times 10^3/(4 \times 400 \times 210) = 2.93 \text{ MPa} \\ v_{\mathrm{Rd,max}} &= 0.2\,(1 - f_{\mathrm{ck}}/250) f_{\mathrm{ck}} \\ &= 0.2 \times (1 - 40/250) \times 40 = 6.72 \text{ MPa } (>2.93) \end{aligned}$$

Based on H16-150 as effective tension reinforcement,

$$\begin{aligned} 100A_{\mathrm{sl}}/b_{\mathrm{w}}d &= 100 \times 201/(150 \times 210) = 0.64 \\ v_{\mathrm{Rd,c}} &= 0.70 \text{ MPa } (\textit{Table 4.17}, \text{ for } d = 210 \text{ and } f_{\mathrm{ck}} = 40) \end{aligned}$$

The length of the first control perimeter at $2d$ from the face of the column is $4 \times 400 + 4\,\pi\, d = 4239$ mm. Thus, the maximum shear stress at the first control perimeter,

$$\beta V/u_1 d = 1.15 \times 854 \times 10^3/(4239 \times 210) = 1.10 \text{ MPa}$$

Since $v > v_{\mathrm{Rd,c}}$, shear reinforcement is needed, where effective design strength $f_{\mathrm{ywd,ef}} = 250 + 0.25d = 300$ MPa. The area needed in one perimeter of vertical shear reinforcement at maximum radial spacing $s_{\mathrm{r}} = 0.75d = 150$ mm say, is given by:

$$\begin{aligned} A_{\mathrm{sw}} &= (v - 0.75 v_{\mathrm{Rd,c}})\, u_1\, s_{\mathrm{r}} / 1.5\, f_{\mathrm{ywd,ef}} \\ &= (1.10 - 0.75 \times 0.70) \times 4239 \times 150/(1.5 \times 300) \\ &= 813 \text{ mm}^2 \\ &\geq \rho_{\mathrm{w,min}}\, u_1\, s_{\mathrm{r}} / 1.5 = (0.08\sqrt{f_{\mathrm{ck}}})\, u_1\, s_{\mathrm{r}} / 1.5 f_{\mathrm{yk}} \\ &= (0.08\sqrt{40}) \times 4239 \times 150/(1.5 \times 500) = 429 \text{ mm}^2 \end{aligned}$$

Using 12H10 gives 942 mm^2

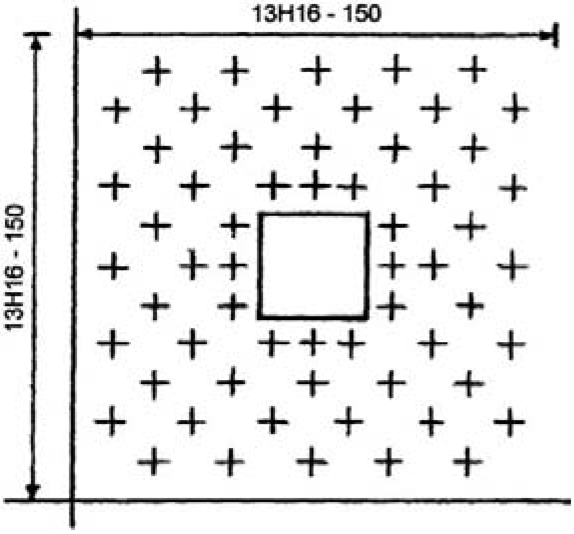
Length of control perimeter at which $v = v_{\mathrm{Rd,c}}$ is given by:

$$u = \beta\, V/d\, v_{\mathrm{Rd,c}} = 1.15 \times 854 \times 10^3/(210 \times 0.70) = 6681 \text{ mm}$$

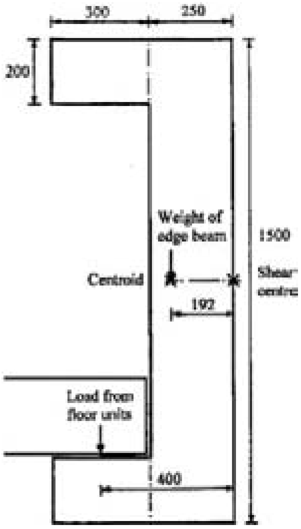
Distance of this control perimeter from face of column is:

$$a = (6681 - 4 \times 400)/2\,\pi = 809 \text{ mm}$$

The distance of the final perimeter of reinforcement from the control perimeter where $v = v_{\mathrm{Rd,c}}$ should be $\leq 1.5d = 315$mm. Thus, 4 perimeters of reinforcement with $s_{\mathrm{r}} = 150$ mm, and the first perimeter at 100 mm from the face of the column, would be suitable. The reinforcement layout is shown in the following figure, where + indicates the link positions, and the links can be anchored round the tension bars.



Example 3. The following figure shows a channel section edge beam, on the bottom flange of which bear 8 m long simply supported contiguous floor units. The beam is continuous in 14 m spans and is prevented from lateral rotation at the supports. The centroid and the shear centre of the section are shown.



Characteristic loads:

floor units: dead 3.5 kN/m^2, imposed 2.5 kN/m^2
edge beam: dead 12 kN/m

Design ultimate loads:

floor units $(1.35 \times 3.5 + 1.5 \times 2.5) \times 8/2 = 33.8$
edge beam $1.35 \times 12 = \underline{16.2}$
$\underline{50.0}$ kN/m

$$f_{ck} = 32\text{ MPa}, f_{yk} = 500\text{ MPa}, d = 1440\text{ mm}$$

Bending moment, shear force and torsional moment (about shear centre of section) at interior support (other than first):

$M = -0.09 \times 50 \times 14^2 = -882$ kNm (*Table 2.29*)
$V = 0.5 \times 50 \times 14 = 350$ kN
$T = 0.5 \times (33.8 \times 0.400 + 16.2 \times 0.192) \times 14 = 117$ kNm

(Note: In calculating V and T, a coefficient of 0.5 rather than 0.55 has been used since the dead load is dominant and the critical section may be taken at the face of the support.)

Considering beam as one large rectangle of size 250 × 1500 and two small rectangles of size 200 × 300,

$$\begin{aligned}\Sigma h_{min}{}^3 h_{max} &= 250^3 \times 1500 + 2 \times 200^3 \times 300 \\ &= (23.4 + 2 \times 2.4) \times 10^9 = 28.2 \times 10^9\end{aligned}$$

Torsional moment to be considered on large rectangle:

$$T_1 = 117 \times 23.4/28.2 = 97\text{ kNm}$$

Torsional moment to be considered on each small rectangle:

$$T_2 = 117 \times 2.4/28.2 = 10\text{ kNm}$$

Reinforcement required in large rectangle

Shear and torsion (see *Table 4.20*). Assuming 30 mm cover to H10 links, distance from surface of concrete to centre of H12 longitudinal bars = 46 mm.

$$\begin{aligned}t_{ef,i} &= A/u = 250 \times 1500/[2 \times (250 + 1500)] \\ &= 107\text{ mm } (\geq 2 \times 46 = 92\text{ mm}) \\ A_k &= (250 - 107) \times (1500 - 107) = 199.2 \times 10^3\text{ mm}^2\end{aligned}$$

For values of $(1 - f_{ck}/250)f_{ck} = (1 - 32/250) \times 32 = 27.9$ MPa and $z = 1440 - 100 = 1340$ mm (to centre of flange)

$$\begin{aligned}\nu_w &= [T_1/2A_k\, t_{ef,i} + V/b_w\, z]/(1 - f_{ck}/250)f_{ck} \\ &= [97/(2 \times 199.2 \times 107) + 350/(250 \times 1340)] \times 10^3/27.9 \\ &= 0.119\end{aligned}$$

Since $\nu_w < 0.138$, $\cot\theta = 2.5$ may be used (*Table 4.18*). For a system of closed links, total area required in two legs for torsion and shear is given by:

$$\begin{aligned}A_s/s &= (T_1/A_k + V/\, z)/f_{ywd} \cot\theta \\ &= (97/199.2 + 350/1340) \times 10^3/(0.87 \times 500 \times 2.5) \\ &= 0.69\text{ mm}^2/\text{mm}\end{aligned}$$

The inner legs of the links are also subjected to a vertical tensile force resulting from the load of 33.8 kN/m applied by the floor units. Additional area required in inner legs:

$$A_s/s = 33.8/(0.87 \times 500) = 0.08\text{ mm}^2/\text{mm}$$

Total area required in two legs for torsion, shear and the additional vertical tensile force:

$$A_s/s = 0.69 + 2 \times 0.08 = 0.85\text{ mm}^2/\text{mm}$$

The area of longitudinal reinforcement required for torsion is given by:

$$\begin{aligned}A_{sl}/s_l &= T\cot\theta\ /2A_k f_{yd} \\ &= 97 \times 10^3 \times 2.5\ /(2 \times 199.2 \times 0.87 \times 500) \\ &= 1.40\text{ mm}^2/\text{mm}\end{aligned}$$

Different combinations of links and longitudinal bars can be obtained by changing the value of cot θ as follows:

Bars	Links		Longitudinal	
cot θ	A_s/s mm^2/mm	Size and spacing	A_{sl}/s_l mm^2/mm	Size and spacing
2.5	0.85	H10-175	1.40	H12-150
2.0	1.02	H10-150	1.12	H12-200
1.6	1.24	H10-125	0.90	H12-250

$s \leq$ least of $u/8 = 3500/8 = 437.5$ mm, $0.75d = 1080$ mm or $h_{min} = 250$ mm, $s_l \leq 350$ mm

Bending (see *Table 4.8*)

$\mu = M/bd^2 f_{ck} = 882 \times 10^6/(550 \times 1440^2 \times 32) = 0.024$
$A_s f_{yk}/bdf_{ck} = 0.027$ and $x/d = 0.054$ (i.e. $x = 78 < 200$ mm)

$$A_s = 0.027 \times 550 \times 1440 \times 32/500 = 1369\text{ mm}^2$$

Total area of longitudinal bars required at top of beam for bending and torsion (equivalent to 2H12 say)

$$= 1369 + 226 = 1595\text{ mm}^2$$

From *Table 2.28*, 2H32 provides 1608 mm^2

Reinforcement required in small rectangles

Torsion. Assuming 30 mm cover to H8 links, distance from surface of concrete to centre of H12 longitudinal bars = 44 mm.

$$\begin{aligned}t_{ef,i} &= A/u = 200 \times 300/[2 \times (200 + 300)] \\ &= 60\ (\geq 2 \times 44 = 88\text{ mm}) \\ A_k &= (200 - 88) \times (300 - 88) = 23.7 \times 10^3\text{ mm}^2\end{aligned}$$

$$\begin{aligned}\nu_w &= (T_2/2A_k\, t_{ef,i})/(1 - f_{ck}/250)f_{ck} \\ &= 10 \times 10^3/(2 \times 23.7 \times 88 \times 27.9) = 0.086\end{aligned}$$

Since $\nu_w \leq 0.138$, cot $\theta = 2.5$ may be used. For a system of closed links, area required in two legs is given by:

$$\begin{aligned}A_{st}/s &= T_2/A_k f_{ywd} \cot\theta \\ &= 10 \times 10^3/(23.7 \times 0.87 \times 500 \times 2.5) \\ &= 0.39\text{ mm}^2/\text{mm}\end{aligned}$$

The lower rectangle is also subjected to bending resulting from the load of 33.8 kN/m applied by the floor units. The distance of the load from the centre of the inner leg of the links in the large rectangle is 150 + 35 = 185 mm.

$$M = 33.8 \times 0.185 = 6.25\text{ kNm}$$

Taking the lever arm for the small rectangle as the distance between the centres of the top and bottom arms of the links, $z = 132$ mm. Additional area required in top arms of links:

$$\begin{aligned}A_s/s &= M/f_{yd}\, z = 6.25 \times 10^3/(0.87 \times 500 \times 132) \\ &= 0.11\text{ mm}^2/\text{mm}\end{aligned}$$

Total area required in two arms for torsion and bending:

$$A_s/s = 0.39 + 2 \times 0.11 = 0.61\text{ mm}^2/\text{mm}$$

The area of longitudinal reinforcement required for torsion is given by:

$$\begin{aligned}A_{sl}/s_l &= T\cot\theta/2A_k f_{yd} \\ &= 10 \times 10^3 \times 2.5/(2 \times 23.7 \times 0.87 \times 500) \\ &= 1.21\text{ mm}^2/\text{mm}\end{aligned}$$

Different combinations of links and longitudinal bars can be obtained by changing the value of cotθ as follows:

Bars	Links		Longitudinal	
cot θ	A_s/s mm^2/mm	Size and spacing	A_{sl}/s_l mm^2/mm	Size and spacing
2.5	0.61	H10-150	1.21	H12-175
1.8	0.76	H10-125	0.88	H12-250

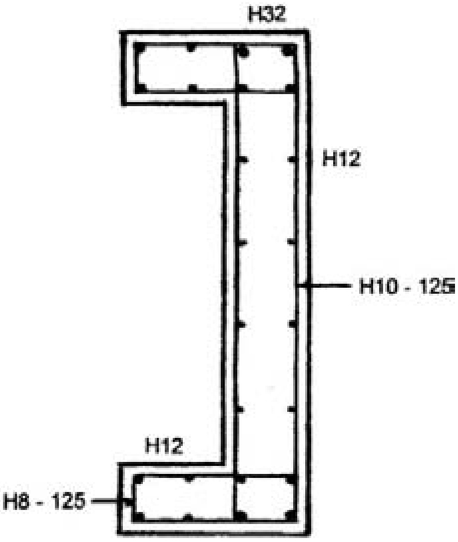
$$s \leq \text{lesser of } u/8 = 1000/8 = 125 \text{ mm or } h_{\min} = 200 \text{ mm}$$

The lower rectangle is also subjected to shear in the vertical longitudinal plane, for which

$$V/b_w\, d = 33.8 \times 10^3/(1000 \times 166) = 0.21 \text{ MPa}$$

From *Table 4.17*, $v_{\min} = 0.56$ MPa ($f_{ck} = 32$, $d \leq 200$)

From the foregoing calculations, the reinforcement shown in the figure opposite provides a practical arrangement, in which the links comprise H10-125 for the large rectangle and H8-125 for the small rectangles. The longitudinal bars are all H12-250, apart from the 2H32 bars at the top of the large rectangle.



Chapter 34

Deflection and cracking

34.1 DEFLECTION

Deflections of members under service load should not impair the function or appearance of a structure. For buildings, the design requirements, and associated combinations of design actions, to be considered are given in section 29.3. The final deflection of members below the level of the supports under characteristic loading, after allowance for any pre-camber, is generally limited to span/250. A further limit of span/500 applies to the increase in deflection that occurs after the construction stage, in order to minimise any damage to both structural and non-structural elements. The requirements may be met by complying with the limits on span/effective depth ratio given in *Table 4.21*.

In special circumstances, when the calculation of deflection is considered necessary, an adequate prediction can be made using the methods given in *Table 4.22*. Careful consideration is needed in the case of cantilevers, where the usual formulae assume that the cantilever is rigidly fixed and remains horizontal at the root. Where the cantilever forms the end of a continuous beam, the deflection at the end of the cantilever is likely to be either increased or decreased by an amount $l\theta$, where l is the length of the cantilever measured to the centre of the support, and θ is the rotation at the support. Where a cantilever is connected to a substantially rigid structure, some root rotation will still occur, and the effective length should be taken as the length to the face of the support plus half the effective depth.

34.2 CRACKING

34.2.1 Building structures

Cracks in members under service load should not impair the appearance or durability of the structure. For buildings, the design requirements are given in Table 4.1. The calculated crack width under quasi-permanent loading, or as a result of restrained deformations, is generally limited to 0.3 mm. For dry surfaces inside buildings, where crack width has no effect on durability, a limit of 0.4 mm is recommended where there is a need to ensure an acceptable appearance. However, in the UK National Annex, a limit of 0.3 mm is required in this situation. In the regions of concrete members where tension is expected, a calculated minimum amount of reinforcement is needed in order to control cracking, as given in *Table 4.23*. Where minimum reinforcement is provided, the crack width requirements may be met by direct calculation, or by limiting either the bar size or the bar spacing, as given in *Table 4.24*. For the calculation of crack widths due to restrained imposed deformation, no guidance is given in Part 1 of the code but the following equation is given in PD 6687 (see preface).

$$\varepsilon_{\text{sm}} - \varepsilon_{\text{cm}} = 0.8R\varepsilon_{\text{imp}}$$

where R is a restraint factor (see section 26.2.1) and ε_{imp} is the free strain due to temperature fall or drying shrinkage.

34.2.2 Liquid-retaining structures

For structures containing liquids, design requirements related to leakage considerations are given in section 29.4. Where a small amount of leakage with related surface staining or damp patches is acceptable, for cracks that can be expected to pass through the full thickness of the section, the calculated crack width is limited to a value that varies according to the hydraulic gradient (i.e. head of liquid divided by thickness of section). The limits are 0.2 mm for hydraulic gradients ≤ 5, reducing uniformly to 0.05 mm for hydraulic gradients ≥ 35. Thus, the limits for a 300 mm thick wall to a 7.5 m deep tank would be: 0.2 mm at 1.5 m below the top, 0.15 mm at 4.5 m below the top, and 0.1 mm at 7.5 m below the top. The limits apply under the quasi-permanent loading combination, where the full characteristic value is taken for hydrostatic loading. For members in axial tension, where at least the minimum reinforcement is provided, the crack width requirements may be met by direct calculation, or by limiting either the bar size or the bar spacing, as given in *Table 4.25*.

In cases of bending, with or without axial force, where the full thickness of the section is not cracked, and not less than 0.2 times the section thickness ≤ 50 mm is in compression, the crack width limit is 0.3 mm and *Table 4.24* applies.

For cracking due to restraint of imposed deformations such as shrinkage and early thermal movements, two distinct types of restraint are considered. For a concrete element restrained at its ends (e.g. an infill bay with construction joints between the new section of concrete and the pre-existing sections), the crack formation is similar to that caused by external loading. An appropriate expression for the tensile strain contributing to the crack width is given in *Table 4.25* and, for specified values of

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Span-effective depth ratios

In the absence of specific deflection calculations, the span/effective depth ratios of beams and slabs should satisfy the following requirement:

$l/d \leq$ basic ratio x α_s x β_s where the basic ratio = $20K$, and values of K, α_s and β_s are given below.

Structural system	K	Basic ratio
Simply-supported member (beam, one-way spanning slab, or two-way spanning slab)	1.0	20
End span of continuous member (beam, one-way spanning slab, or two-way spanning slab continuous over one long side)	1.3	26
Interior span of continuous member (beam, one-way spanning slab, or two-way spanning slab)	1.5	30
Slab supported on columns without beams (flat slab)	1.2	24
Cantilever	0.4	8

Note 1. For flanged sections with $b/b_w \geq 3$, the basic ratios for rectangular sections should be multiplied by 0.8. For values of $b/b_w < 3$, the basic ratios for rectangular sections should be multiplied by $(11 - b/b_w)/10$.
Note 2. The ratio should be based on the shorter span for two-way spanning slabs, and the longer span for flat slabs.
Note 3. For beams and slabs, other than flat slabs, with spans exceeding 7 m, which support partitions liable to be damaged by excessive deflections, the basic ratio should be multiplied by 7/span. For flat slabs, where one or both spans exceeds 8.5 m, which support partitions liable to be damaged by excessive deflections, the basic ratio should be multiplied by 8.5/span.

Modification factor α_s, which depends on the concrete strength f_{ck} and reinforcement percentages, is given by:

For $100A_s/bd < 0.1f_{ck}^{0.5}$ $\alpha_s = 0.55 + 0.0075f_{ck}/(100A_s/bd) + 0.005f_{ck}^{0.5}[f_{ck}^{0.5}/(100A_s/bd) - 10]^{1.5}$

For $100A_s/bd \geq 0.1f_{ck}^{0.5}$ $\alpha_s = 0.55 + 0.0075f_{ck}/[100(A_s - A_s')/bd] + 0.013f_{ck}^{0.25}(100\,A_s'/bd)^{0.5}$

$\alpha_s = 0.55 + 0.0075f_{ck}/(100A_s/bd)$ when singly reinforced

f_{ck} MPa	α_s (singly reinforced section) for values of $100A_s/bd$									
	0.25	0.3	0.4	0.5	0.6	0.75	1.0	1.25	1.5	2.0
20	1.65	1.29	0.95	0.85	0.80	0.75	0.70	0.67	0.65	0.63
25	2.09	1.61	1.12	0.93	0.86	0.80	0.74	0.70	0.67	0.64
28	2.38	1.81	1.23	0.98	0.90	0.83	0.76	0.72	0.69	0.65
32	2.78	2.10	1.39	1.07	0.95	0.87	0.79	0.74	0.71	0.67
40	3.64	2.72	1.74	1.29	1.05	0.95	0.85	0.79	0.75	0.70
50	4.81	3.57	2.24	1.60	1.18	1.05	0.93	0.85	0.80	0.74

Modification factor $\beta_s = 310/\sigma_s$, where σ_s is the stress in the tension reinforcement under the characteristic loading. It will normally be conservative to assume $\beta_s = (500/f_{yk})(A_{s,prov}/A_{s,req})$ for values of $A_{s,prov}/A_{s,req} \leq 1.5$.

A_s is area of tension reinforcement required to resist the design ultimate moment (maximum moment in span or, for a cantilever, at the support). $A_{s,prov}/A_{s,req}$ is the ratio, area reinforcement provided to area reinforcement required.
A_s' is area of compression reinforcement required to resist the design ultimate moment (as for A_s above)

The recommendations were derived for uniformly loaded members of constant depth. For a cantilever of overall depth h at the support reducing linearly to h_o at the free end, the basic ratio should be multiplied by the following factors:

Load distribution on cantilever	Modification factor for values of h_o/h							
	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3
Uniform load	1.00	0.94	0.88	0.81	0.75	0.68	0.61	0.54
Triangular load (zero at free end)	1.25	1.19	1.12	1.06	0.99	0.92	0.85	0.78
Concentrated load at free end	0.75	0.69	0.63	0.58	0.52	0.46	0.40	0.33

Calculations

In special circumstances, when the calculation of deflection is considered necessary, the method given in *Table 4.22* can be used. For permanent loads, characteristic values should generally be used. For variable load, the characteristic value should be used in limit state calculations, and the expected value in best estimate calculations. The proportion of variable load considered as quasi-permanent should be taken as $\psi_2 Q_k$, where values of ψ_2 are given in section 29.3. For structural analysis where a single value of stiffness is used for a member, the stiffness of the uncracked concrete section should be used. If a more sophisticated method of analysis is used, in which variations in section properties over the length of the member are considered, it may be more appropriate to use the stiffness of the cracked transformed section at highly stressed sections. In deflection calculations, several factors that are often difficult to assess can have a considerable effect on the reliability of the result. These include the assumptions made regarding the restraints provided by supports, the age of members when load is first applied, the stages at which subsequent load is applied, and the effects of finishes and rigid partitions. A reasonable approach may be to assess maximum and minimum values for the influence of these effects and take the average.

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4.22

Curvatures

Members, which are not expected to be loaded above a level that would cause the tensile strength of the concrete to be exceeded anywhere within the member, should be considered as uncracked. Members, which are expected to crack but may not be fully cracked, will behave in a manner intermediate between the uncracked and fully cracked conditions.

The curvature $1/r_b$ of a section subjected to pure bending can be determined from the following relationships:

For an uncracked section: $\frac{1}{r_b} = \frac{M}{EI_o}$ For a cracked section: $\frac{1}{r_b} = \frac{M}{EI_c}\left[\zeta + (1-\zeta)\frac{I_c}{I_o}\right]$ where

E is the modulus of elasticity of the concrete. Use E_{cm} for short-term loading, and $E_{c,eff} = E_{cm}/[1 + \varphi(\infty,t_0)]$ for long-term loading where $\varphi(\infty,t_0)$ is a creep coefficient. For values of E_{cm} and $\varphi(\infty,t_0)$, see *Tables 4.2* and *4.3*.

I_c is the second moment of area of the cracked transformed section. See *Table 3.42* for section properties.

I_o is the second moment of area of the gross concrete section. See *Table 3.42* for section properties.

M is the moment due to the characteristic loading at the section considered

M_{cr} is the cracking moment given by $M_{cr} = f_{ctm}I_o/(h-x)$. See *Table 3.42* for section properties.

f_{ctm} is the mean value of the axial tensile strength of concrete given by $f_{ctm} = 0.3f_{ck}^{(2/3)}$ for values of $f_{ck} \leq 50$ MPa.

ζ is a distribution coefficient allowing for tension stiffening, given by:

$$\zeta = 1 - \beta\left(\frac{M_{cr}}{M}\right)^2$$

β is a coefficient that takes account of the influence of the duration of the loading, or of repeated loading on the average strain. $\beta = 1.0$ for single short-term loading. $\beta = 0.5$ for sustained loads or many cycles of repeated loading.

The shrinkage curvature $1/r_{cs}$ of a section can be determined from the following relationships:

For an uncracked section: $\frac{1}{r_{cs}} = \frac{\varepsilon_{cs}E_sS_o}{E_{c,eff}I_o}$ For a cracked section: $\frac{1}{r_{cs}} = \frac{\varepsilon_{cs}E_sS_c}{E_{c,eff}I_c}\left[\zeta + (1-\zeta)\frac{I_cS_o}{I_oS_c}\right]$ where

E_s is the modulus of elasticity of the reinforcement taken as 200 GPa

S_c is the first moment of area of the reinforcement about the centroid of the cracked section given by: $S_c = A_s(d-x_c) - A'_s(x_c - d')$ where x_c is the neutral axis depth

S_o is the first moment of area of the reinforcement about the centroid of the uncracked section given by: $S_o = A_s(d-x_o) - A'_s(x_o - d')$ where x_o is the neutral axis depth

ε_{cs} is the free shrinkage strain (see *Table 4.2*)

Deflections

Deflections can be determined from the relationship $1/r_x = d^2a/dx^2$, where $1/r_x$ is the curvature and a is the deflection at x, by calculating curvatures at successive sections along a member and using a numerical integration technique. Alternatively, the deflection at the mid-span of a beam, or the end of a cantilever, is given approximately by:

$a = \Sigma Kl^2(1/r_b) + Kl^2(1/r_{cs})$ where

K is a factor that, for a member of constant cross-section, is related to the shape of the bending moment diagram (see *Table 3.42*). For concrete shrinkage, K is equal to the bending moment coefficient for uniform dead load applicable to the maximum sagging moment (e.g. $K = 0.125$ for pinned ends) or, for a cantilever, $K = 0.5$.

l is the effective span of the member.

$1/r_b$ is the curvature due to loading (at the position of maximum sagging moment or, for a cantilever, the support).

$1/r_{cs}$ is the curvature due to concrete shrinkage

The maximum long-term deflection should be taken as the sum of the long-term deflections due to permanent load, quasi-permanent load and concrete shrinkage, and the short-term deflection due to transient load. For complex load arrangements, the value of $1/r_b$ due to the total load, and a K value appropriate to the bending moment diagram due to the total load should be used.

4.23

EC 2 Cracking – 1

Minimum areas of reinforcement

A minimum amount of bonded reinforcement is required to control cracking in regions where tension is expected. The amount may be estimated from equilibrium between the tensile force in concrete just before cracking and the tensile force in reinforcement at yielding, or at a lower stress if necessary to limit the crack width. A procedure for calculating crack widths, and a simplified alternative approach by limiting either bar size or bar spacing, are given in *Table 4.24*.

The minimum area of reinforcement required in the tension zone of a section subjected to pure bending or tension, is given by the following relationship:

$A_{s,min} = k_c\, k f_{ct,eff} A_{ct} / \sigma_s$ where

A_{ct} is the area of concrete within the tensile zone, taken as that part of the section calculated to be in tension just before formation of the first crack. For boxes and flanged sections, the individual parts of the section should be treated separately. If the effect of the reinforcement on the properties of the uncracked section is ignored, the depth of the tensile zone, for a rectangular section, is h for tension, and $0.5h$ for pure bending.

$f_{ct,eff}$ is the mean value of the tensile strength of the concrete effective at a time when the cracks are first expected to appear. For general design purposes, $f_{ct,eff} = f_{ctm}$, where $f_{ctm} = 0.3f_{ck}^{(2/3)}$ for values of $f_{ck} \leq 50$ MPa. For conditions where cracking is expected to occur at a time t earlier than 28 days (e.g. early thermal cracking), $f_{ct,eff} = [f_{cm}(t)/(f_{ck} + 8)]\, f_{ctm}$ where $f_{cm}(t)$ is the mean concrete compressive strength at an age of t days.

k is a coefficient that allows for the effect of non-uniform self-equilibrating stresses, leading to corresponding reductions of restraint forces. For webs with $h \leq 300$ mm, or flanges with $b \leq 300$ mm, $k = 1.0$. For webs with $h \geq 800$ mm, or flanges with $b \geq 800$ mm, $k = 0.65$. Intermediate values may be interpolated.

k_c is a coefficient that takes account of the stress distribution within the section immediately prior to cracking, and of the change of lever arm. For pure tension, $k_c = 1.0$. For pure bending, in rectangular sections and for the webs of boxes and flanged sections, $k_c = 0.4$. For bending combined with axial force, and the flanges of boxes and flanged sections, see the equations given below.

σ_s is the maximum stress permitted in the reinforcement immediately after formation of the first crack, which may be taken as f_{yk}. However, a lower stress may be necessary to satisfy the design crack width limit under the relevant combination of loads (see section 34.2 and *Table 4.24*).

For bending combined with axial force, in rectangular sections and for the webs of boxes and flanged sections, values of k_c are given by the following relationships:

Bending and compression: For $h \leq 1$ m, $k_c = 0.4(1 - \sigma_c / 1.5 f_{ct,eff})$ For $h > 1$ m, $k_c = 0.4(1 - \sigma_c / 1.5 h f_{ct,eff})$

Bending and tension: $k_c = 0.4(1 + 1.5\sigma_c / f_{ct,eff}) \leq 1.0$

For the flanges of box sections and flanged sections in all conditions: $k_c = 0.9F_{cr}/A_{ct}f_{ct,eff} \geq 0.5$

F_{cr} is the tensile force in the flange immediately prior to cracking for the cracking moment calculated with $f_{ct,eff}$.

σ_c is the mean stress in the concrete (taken positive for compression and tension) due to the axial force, under the relevant combination of loads.

Minimum reinforcement percentages for $f_{ct,eff} = f_{ctm}$ and $\sigma_s = f_{yk} = 500$ MPa						
Condition	Depth of section or width of flange	Minimum reinforcement percentage $100A_{s,min}/A_{ct}$ for values of f_{ck}				
Pure tension	f_{ck} MPa	25	28	32	40	50
Rectangular section or flanged section (web and flange treated separately)	h or $b_f \leq 300$ mm	0.52	0.55	0.60	0.70	0.82
	h or $b_f \geq 800$ mm	0.34	0.36	0.39	0.45	0.53
Pure bending	f_{ck} MPa	25	28	32	40	50
Rectangular section, or flanged section (web in tension)	$h \leq 300$ mm	0.21	0.22	0.24	0.28	0.33
	$h \geq 800$ mm	0.14	0.15	0.16	0.18	0.21

Note 1. Intermediate values may be interpolated for values of h or b_f between 300 mm and 800 mm.

Note 2. For flanged sections where the flange is in tension, and all sections subjected to bending combined with axial force, the minimum reinforcement percentage varies according to the value of k_c determined as above.

Note 3. In addition to the above requirements, the area of main tension reinforcement in beams and slabs should be not less than $A_{s,min} = 0.26(f_{ctm}/f_{yk})\,b_t d \geq 0.0013\,b_t d$, where b_t is the mean width of the tension zone.

Beams with a total depth ≥ 1000 mm, where the main reinforcement is concentrated in only a small proportion of the depth, should be provided with additional skin reinforcement to control cracking on the side faces of the beam. This reinforcement should be located within the links, and evenly distributed between the level of the main reinforcement and the neutral axis. The area of the skin reinforcement should be not less than the value of $A_{s,min}$ obtained with $k = 0.5$ and $\sigma_s = f_{yk}$. The maximum bar size or spacing may be obtained from *Table 4.24*, by assuming pure tension and a stress of half the value assessed for the main reinforcement.

EC 2 Cracking – 2

4.24

Control of cracking without direct calculation

In buildings, for slabs subjected to bending without significant axial tension, no specific measures are necessary to control cracking, provided the overall depth does not exceed 200 mm, and the specified detailing rules are observed.

In general, for members where the minimum reinforcement described in *Table 4.23* is provided, crack widths are not likely to be excessive if the following limitations on either bar size or bar spacing are observed:

Stress in reinforcement σ_s MPa	Maximum bar size ϕ_s* (mm) for crack width		Maximum bar spacing (mm) for crack width		Stress in reinforcement σ_s MPa	Maximum bar size ϕ_s* (mm) for crack width		Maximum bar spacing (mm) for crack width	
	0.4 mm	0.3 mm	0.4 mm	0.3 mm		0.4 mm	0.3 mm	0.4 mm	0.3 mm
160	40	32	300	300	320	12	10	150	100
200	32	25	300	250	360	10	8	100	50
240	20	16	250	200	400	8	6		
280	16	12	200	150	450	6	5		

For cracking caused dominantly by restraint, compliance with maximum bar size applies where $\sigma_s = k_c\, k\, f_{ct,eff}\, A_{ct}/A_s$. For cracking caused mainly by loading, compliance with either maximum bar size or maximum bar spacing may be used, where σ_s is calculated on the basis of a cracked section under the relevant combination of loads.

The maximum bar size ϕ_s* obtained from the table should be adjusted to ϕ_s, as follows:

If at least part of section is in compression: $\phi_s = \phi_s^* (f_{ct,eff}/2.9)[k_c h_{cr}/2(h-d)]$

If all of section is in tension: $\phi_s = \phi_s^* (f_{ct,eff}/2.9)[h_{cr}/4(h-d)]$

In the above, h_{cr} is the depth of the tensile zone immediately prior to cracking, $(h-d)$ is the distance from a tensile face to the centroid of the nearest layer of reinforcement, k_c and $f_{ct,eff}$ are defined in *Table 4.23*.

Calculation of crack width

The maximum crack spacing may be calculated from the following relationships:

$s_{r,max} = 3.4c + 0.425 k_1 k_2 (A_{c,eff}/A_s)\phi$ for $s_b \le 5(c + \phi/2)$ $\qquad s_{r,max} = 1.3(h-x)$ for $s_b > 5(c + \phi/2)$

The design crack width at a concrete surface may be calculated from the following relationships:

$$w_k = (\varepsilon_{sm} - \varepsilon_{cm})\, s_{r,max} = \left(\frac{\sigma_s}{E_s} - \varepsilon_{cm}\right) s_{r,max} \ge 0.6\frac{\sigma_s}{E_s}\, s_{r,max} \qquad \varepsilon_{cm} = k_t\left(1 + \frac{\alpha_e A_s}{A_{c,eff}}\right)\left(\frac{A_{c,eff}\, f_{ct,eff}}{A_s E_s}\right)$$

$A_{c,eff}$	is the effective area of concrete in tension surrounding the reinforcement. The depth of the area should be taken as the least of $2.5(h-d)$, $(h-x)/3$ or $h/2$.
A_s	is the area of reinforcement within the tensile zone, where $A_s \ge A_{s,min}$ (see *Table 4.23*)
E_s	is the modulus of elasticity of the reinforcement taken as 200 GPa
c	is the cover to the tension reinforcement
$f_{ct,eff}$	is the mean value of the tensile strength of the concrete effective at a time when the cracks are first expected to appear. For further information, see *Table 4.23*.
k_1	is a coefficient that takes account of the bond properties of the reinforcement. $k_1 = 0.8$ for high bond bars.
k_2	is a coefficient that takes account of the distribution of strain. $k_2 = 0.5$ for bending, 1.0 for pure tension. For cases of eccentric tension or for local areas, $k_2 = (\varepsilon_1 + \varepsilon_2)/2\varepsilon_1$, where ε_1 and ε_2 are respectively the greater and lesser strains at the boundaries of the section considered, assessed on the basis of a cracked section.
k_t	is a factor dependent on the duration of the load. $k_t = 0.6$ for short term loading, 0.4 for long term loading.
s_b	is the bar spacing (centre-to-centre)
$s_{r,max}$	is the maximum crack spacing. For members reinforced in two orthogonal directions, where the angle between the axes of principal stress and the direction of the reinforcement exceeds 15°, the crack spacing should be calculated from: $s_{r,max} = (s_{r,max,x}\, s_{r,max,y})/(s_{r,max,x} \sin\theta + s_{r,max,y} \cos\theta)$, where $s_{r,max,x}$ and $s_{r,max,y}$ are the crack spacings in the x and y directions respectively, and θ is the angle between the reinforcement in the x direction and the direction of the principal tensile stress.
w_k	is the design surface crack width
α_e	is the modular ratio E_s/E_{cm}, where $E_{cm} = 22(f_{cm}/10)^{0.3}$ Gpa and $f_{cm} = (f_{ck} + 8)$ MPa
ϕ	is the bar size. Where a mixture of bar sizes is used in a section, an equivalent size ϕ_{eq} should be used. For a section with n_1 bars of size ϕ_1 and n_2 bars of size ϕ_2, the equivalent size $\phi_{eq} = (n_1\phi_1^2 + n_2\phi_2^2)/(n_1\phi_1 + n_2\phi_2)$.
σ_s	is the stress in the tension reinforcement under the relevant combination of loads (section 34.2), including the effect of imposed deformations, assuming a cracked section.

For walls subjected to early thermal contraction, where the bottom of the wall is restrained by a previously cast base and the horizontal reinforcement area $A_s < A_{s,min}$, it may be assumed that $s_{r,max}$ = 1.3 times the height of the wall.

4.25

EC 2 Cracking – 3

Design for imposed deformation

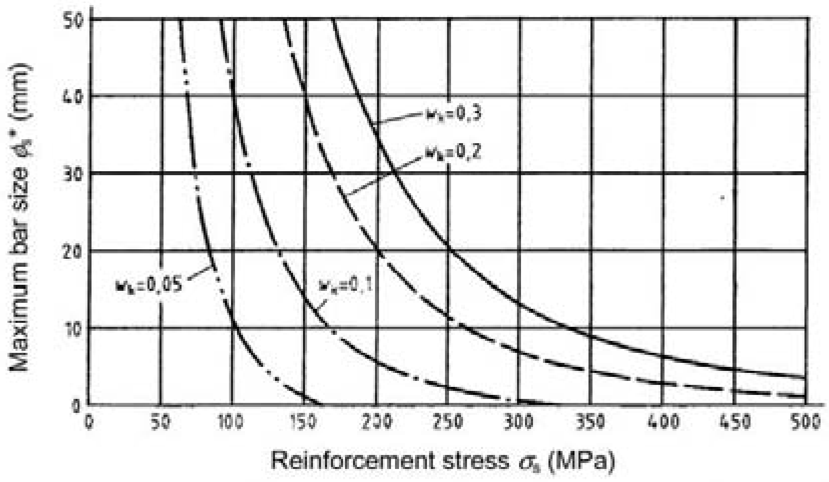
The recommendations in Part 3 of the Eurocode allow for two main options as follows: (a) design for full restraint, in which no movement joints are provided except for some widely spaced joints that may be needed where substantial imposed deformation is expected, or (b) design for free movement, in which any potential cracking is controlled by the proximity of the joints. For (b), complete movement joints spaced at no more than 1.5 times wall height ≥ 5m, and minimum reinforcement as given in *Table 4.28*, should be provided. For (a), the following information should be used in conjunction with the crack spacing and crack width expressions given in *Table 4.24*.

For a wall or slab panel restrained at its ends, the mean tensile strain contributing to cracking may be calculated from the expression: $(\varepsilon_{sm} - \varepsilon_{cm}) = 0.5\, k_c\, k\, f_{ct,eff}(1 + \alpha_e A_s/A_{ct})/(A_s/A_{ct})E_s$

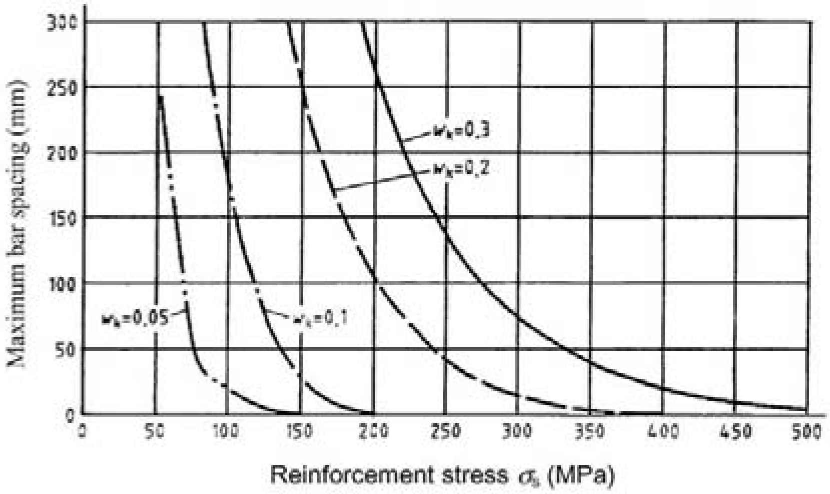
For long panels restrained along one or more edges, a value of $(\varepsilon_{sm} - \varepsilon_{cm}) = R_{ax}\, \varepsilon_{free}$ can be taken, where R_{ax} is a restraint factor, and ε_{free} is the strain that would occur if the panel was completely unrestrained. Values of R_{ax} for some common situations are given in *Table 3.45*, or may be taken as 0.5 generally. Values of ε_{free} due to early thermal movements are given by $\varepsilon_{free} = \alpha T_1$, where α is the coefficient of expansion of mature concrete (see *Table 3.5*) and T_1 is the estimated fall of temperature between hydration peak and ambient at the time of construction (see *Table 2.19*).

Control of cracking without direct calculation for members in axial tension

The recommendations in Part 3 of the Eurocode indicate that, for members subjected to axial tension, the limits on bar size and bar spacing given in *Table 4.24* may be replaced by values obtained from the following charts. For cracking caused dominantly by restraint, compliance with the maximum bar size applies with $\sigma_s = k_c\, k\, f_{ct,eff}\, A_{ct}/A_s$. For cracking caused mainly by loading, compliance with either the maximum bar size or the maximum bar spacing may be used, where σ_s is calculated on the basis of a cracked section under the relevant combination of loads.



The maximum bar size ϕ_s^* obtained from the chart should be adjusted to $\phi_s = \phi_s^*(f_{ct,eff}/2.9)[h/10(h - d)]$



Alternatively, for specified values of cover, section thickness and reinforcement content, maximum values of $f_{ct,eff}$ are given in *Table 4.26*, and maximum values of $R_{ax}\, \varepsilon_{free}$ are given in *Table 4.27*.

cover, section thickness and reinforcement content, maximum values of $f_{\text{ct,eff}}$ are given in *Table 4.26*.

For a concrete panel restrained along an edge (e.g. a wall cast onto a pre-existing stiff base), the formation of the crack only influences the distribution of stresses locally, and the crack width becomes a function of the restrained strain rather than the tensile strain capacity of the concrete. In this case, the tensile strain contributing to the crack width is taken as $R_{\text{ax}}\,\varepsilon_{\text{free}}$ where typical values of $\varepsilon_{\text{free}}$ can be estimated from the information given in *Table 4.25*. The restraint factor R_{ax} may be taken as 0.5 generally, or reference can be made to *Table 3.45*, where values are indicated for panels restrained along one, two or three edges respectively. For specified values of cover, section thickness and reinforcement content, maximum values of $R_{\text{ax}}\,\varepsilon_{\text{free}}$ are given in *Table 4.27*.

It will be found that the calculated strain contributing to the crack width for a panel restrained at its ends is normally more than $R_{\text{ax}}\,\varepsilon_{\text{free}}$. Thus, the reinforcement required to limit a crack width to the required value is greater for a panel restrained at its ends than for a panel restrained along one or more edges. Also, for a specific crack width, the reinforcement needed for a panel restrained along an edge is less than that in BS 8007, since the design crack spacing is less than that in BS 8007.

Example 1. The beam shown in the following figure is to be checked for deflection and cracking. The design for bending and shear is shown in example 1 of Chapters 32 and 33 respectively. The reinforcement in the bottom of each span, 3H25 (1473 mm^2), is based on the following values:

$$f_{\text{ck}} = 32\text{ MPa}, f_{\text{yk}} = 500\text{ MPa}, b_{\text{eff}} = 2600\text{ mm}, d = 440\text{ mm}$$

Spans Cross section

The actual span/effective depth ratio = 8000/440 = 18.2

From *Table 4.21*, for the end span of a continuous beam, and a flanged section with $b/b_{\text{w}} = 2600/300 = 8.67 \geq 3$,

Basic span/effective depth ratio = 0.8 × 26 = 20.8

For members supporting partitions liable to be damaged by excessive deflections, the basic ratio should be multiplied by 7/span. In this case, the basic ratio = 20.8 × 7/8 = 18.2.

Since $100A_{\text{s req}}/bd = 100 \times 1373/(2600 \times 440) = 0.12$ is small, the modification factor α_{s} is large (> 3), and the limiting ratio is more than three times the actual value.

The neutral axis depth for the uncracked section, ignoring the effect of the reinforcement, is given by:

$$x = \frac{b_{\text{w}}h^2 + (b_{\text{f}} - b_{\text{w}})h_{\text{f}}^2}{2[b_{\text{w}}h + (b_{\text{f}} - b_{\text{w}})h_{\text{f}}]} = \frac{300 \times 500^2 + 2300 \times 150^2}{2[300 \times 500 + 2300 \times 150]}$$
$$= 128\text{ mm } (< h_{\text{f}} = 150\text{ mm})$$

Area of tension zone is given by:

$$A_{\text{ct}} = b_{\text{w}}\,(h - h_{\text{f}}) + b_{\text{f}}\,(h_{\text{f}} - x) = 300 \times 350 + 2600 \times 22$$
$$= 162 \times 10^3\text{ mm}^2$$

From *Table 4.23*, for $f_{\text{ck}} = 32$ MPa and bending of a section with $h = 500$ mm, by interpolation, $100A_{\text{s,min}}/A_{\text{ct}} = 0.21$.

$$A_{\text{s,min}} = 0.0021 \times 162 \times 10^3 = 340\text{ mm}^2\ (< 1473\text{ mm}^2)$$
$$\geq 0.26\,(f_{\text{ctm}}/f_{\text{yk}})\,b_{\text{t}}d \geq 0.0013\,b_{\text{t}}d$$
$$= 0.26 \times (3.0/500) \times 300 \times 440 = 206\text{ mm}^2$$

The design ultimate load is 396 kN and the quasi-permanent load, where the value of ψ_2 is obtained from section 29.3, is

$$G_{\text{k}} + \psi_2\,Q_{\text{k}} = 160 + 0.3 \times 120 = 196\text{ kN}$$

Hence, the stress in the reinforcement under quasi-permanent loading, is given approximately by:

$$\sigma_{\text{s}} = (196/396)(0.87f_{\text{yk}})(A_{\text{s req}}\,/A_{\text{s prov}})$$

Thus, for the bars in the bottom of the beam

$$\sigma_{\text{s}} = (196/396)(0.87 \times 500)(1373/1473) = 200\text{ MPa}$$

From *Table 4.24*, for $w_{\text{k}} = 0.3$ mm, the crack width criterion is met if $\phi^*_{\text{s}} \leq 25$ mm, or the bar spacing ≤ 250 mm.

The adjusted maximum bar size is given by:

Depth of tension zone, $h_{\text{cr}} = h - x = 500 - 128 = 372$ mm

$$\phi_{\text{s}} = \phi^*_{\text{s}}\,(f_{\text{ct,eff}}\,/2.9)[k_{\text{c}}\,h_{\text{cr}}\,/2(h - d)]$$
$$= 25 \times (3.0/2.9) \times [0.4 \times 372/(2 \times 60)] = 32\text{ mm}$$

Note. It can be seen from the foregoing, that all of the criteria are comfortably satisfied, and the checks for deflection and cracking are hardly necessary in this example.

Example 2. A 250 mm thick flat slab is supported by columns, which are arranged on a 7.2 m square grid. The characteristic loads are 7.2 kN/m^2 dead and 4.5 kN/m^2 imposed, and the slab is to be checked for deflection and cracking.

$$f_{\text{ck}} = 32\text{ MPa}, f_{\text{yk}} = 500\text{ MPa, cover to bars} = 25\text{ mm}$$

Allowing for the use of H12 bars in each direction, and based on the bars in the second layer of reinforcement:

$$d = 250 - (25 + 12 + 6) = 205\text{ mm}, l/d = 7200/205 = 35$$

From *Table 4.21*, for a flat slab, with spans ≤ 8.5 m,

Basic span/effective depth ratio = 24

Total design ultimate load for a square panel is given by:

$$F = (1.35 \times 7.2 + 1.5 \times 4.5) \times 7.2^2 = 854\text{ kN}$$

From *Table 2.62* the design ultimate bending moment, for an end span with a continuous connection at the outer support, is

$$M = 0.075Fl = 0.075 \times 854 \times 7.2 = 461\text{ kNm}$$
$$M/bd^2f_{\text{ck}} = 461 \times 10^6/(7200 \times 205^2 \times 32) = 0.048$$
$$A_{\text{s}}f_{\text{yk}}/bdf_{\text{ck}} = 0.056\ (\textit{Table 4.8})$$
$$100A_{\text{s}}/bd = 100 \times 0.056 \times 32/500 = 0.36$$

From *Table 4.21*, for $100A_{\text{s}}/bd < 0.1f_{\text{ck}}^{0.5} = 0.1 \times 32^{0.5} = 0.57$

$$\alpha_{\text{s}} = 0.55 + 0.0075f_{\text{ck}}/(100A_{\text{s}}/bd)$$
$$+ 0.005f_{\text{ck}}^{0.5}[f_{\text{ck}}^{0.5}/(100A_{\text{s}}/bd) - 10]^{1.5}$$
$$= 0.55 + 0.0075 \times 32/0.36$$
$$+ 0.005 \times 32^{0.5} \times (32^{0.5}/0.36 - 10)^{1.5} = 1.60$$

EC 2 Early thermal cracking in end restrained panels

4.26

For a concrete element restrained at its ends, the mean tensile strain contributing to cracking, with $k_c = 1.0$ (pure tension), may be calculated from the expression: $(\varepsilon_{sm} - \varepsilon_{cm}) = 0.5\, k f_{ct,eff}(1 + \alpha_e A_s/A_{ct})/(A_s/A_{ct})E_s$

With $k_1 = 0.8$ (high bond bars) and $k_2 = 1.0$ (pure tension), the maximum crack spacing: $s_{r,max} = 3.4[c + 0.1(A_{c,eff}/A_s)\phi]$

With $\alpha_e = 6$, the design crack width: $w_k = 1.7k(1 + 6A_s/A_{ct})[\ c + 0.1(A_{c,eff}/A_s)\phi]\, f_{ct,eff}\ /(A_s/A_{ct})E_s$

Maximum values of $f_{ct,eff}$ for $w_k = 0.2$ mm are given below. For other values of w_k, multiply values of $f_{ct,eff}$ by $5w_k$.

	Thickness of section (mm)	Bar size (EF)	Maximum values of $f_{ct,eff}$ (MPa) according to bar spacing (mm) for $w_k = 0.2$ mm								
			300	250	225	200	175	150	125	100	75
Cover = 40 mm	200	H12								1.70	2.72
		H16					1.41	1.83	2.47	3.53	5.45
		H20		1.38	1.65	2.02	2.52	3.23	4.30	6.00	8.98
	250	H16							1.76	2.55	4.05
		H20				1.38	1.73	2.25	3.03	4.30	6.59
		H25		1.69	2.03	2.47	3.07	3.93	5.20	7.21	10.7
	300	H16							1.49	2.15	3.40
		H20					1.46	1.90	2.57	3.66	5.65
		H25		1.38	1.65	2.02	2.52	3.23	4.31	6.02	9.03
	350	H16								1.93	3.06
		H20						1.71	2.31	3.30	5.12
		H25			1.48	1.81	2.27	2.92	3.90	5.48	8.26
	400	H16								1.77	2.81
		H20						1.56	2.12	3.04	4.72
		H25			1.36	1.66	2.08	2.69	3.60	5.07	7.68
Cover = 50 mm	200	H12								1.60	2.51
		H16					1.34	1.72	2.31	3.26	4.95
		H20		1.31	1.57	1.90	2.36	3.01	3.96	5.47	8.06
	250	H16							1.61	2.31	3.59
		H20				1.31	1.64	2.11	2.82	3.97	5.98
		H25		1.61	1.92	2.33	2.88	3.66	4.80	6.58	9.59
	300	H16							1.22	1.76	2.78
		H20						1.57	2.11	3.01	4.63
		H25			1.42	1.73	2.15	2.76	3.66	5.09	7.58
	350	H16								1.58	2.50
		H20						1.41	1.90	2.72	4.20
		H25				1.51	1.88	2.42	3.22	4.52	6.79
	400	H20							1.74	2.50	3.87
		H25				1.38	1.73	2.22	2.97	4.18	6.31
		H32		1.73	2.06	2.50	3.10	3.93	5.16	7.09	10.3
Cover = 60 mm	200	H12								1.50	2.33
		H16						1.63	2.16	3.02	4.54
		H20			1.49	1.80	2.22	2.81	3.68	5.03	7.31
	250	H16							1.52	2.16	3.32
		H20					1.56	2.00	2.65	3.68	5.48
		H25		1.54	1.83	2.20	2.71	3.42	4.45	6.05	8.70
	300	H16								1.63	2.54
		H20						1.49	2.00	2.81	4.27
		H25			1.36	1.65	2.04	2.60	3.42	4.72	6.94
	350	H20							1.62	2.30	3.55
		H25					1.65	2.11	2.81	3.92	5.87
		H32		1.72	2.05	2.47	3.04	3.84	4.99	6.77	9.73
	400	H20							1.48	2.12	3.28
		H25					1.48	1.90	2.53	3.55	5.35
		H32		1.49	1.77	2.15	2.66	3.37	4.41	6.04	8.79

4.27

EC 2 Early thermal cracking in edge restrained panels

For a long concrete panel restrained along an edge, the mean tensile strain contributing to cracking may be taken as $R_{ax}\,\varepsilon_{free}$.

With $k_1 = 0.8$ (high bond bars) and $k_2 = 1.0$ (pure tension), the maximum crack spacing: $s_{r,max} = 3.4[c + 0.1(A_{c,eff}/A_s)\phi]$

The design crack width: $w_k = 3.4[\,c + 0.1(A_{c,eff}/A_s)\phi]\,R_{ax}\,\varepsilon_{free}$

Maximum values of $R_{ax}\,\varepsilon_{free}$ for $w_k = 0.2$ mm are given below. For other values of w_k, multiply values of $R_{ax}\,\varepsilon_{free}$ by $5w_k$.

	Thickness of section (mm)	Bar size (EF)	Maximum values of $R_{ax}\,\varepsilon_{free}$ (x 10^{-6}) according to bar spacing (mm) for $w_k = 0.2$ mm								
			300	250	225	200	175	150	125	100	75
Cover = 40 mm	200	H10	139	164	180	200	224	255	295	351	434
		H12	164	192	211	233	260	295	341	402	492
		H16	211	246	268	295	328	369	422	492	590
		H20	254	295	321	351	388	434	492	567	670
	250	H12	145	170	187	207	232	264	305	363	447
		H16	180	211	231	254	284	321	369	434	527
		H20	211	246	268	295	328	369	422	492	590
	300	H25	244	284	309	338	375	419	476	550	652
		H32	284	328	356	388	428	476	536	614	719
	Note. Values of $R_{ax}\,\varepsilon_{free}$ given for section thickness $h = 250$ mm apply for $h \geq 230$ mm (H12), $h \geq 240$ mm (H16) and $h \geq 250$ mm (H20). Values of $R_{ax}\,\varepsilon_{free}$ given for $h = 300$ mm apply for $h \geq 262.5$ mm (H25) and $h \geq 280$ mm (H32).										
Cover = 50 mm	200	H10	136	159	175	193	216	244	281	331	404
		H12	160	186	204	224	249	281	322	377	454
		H16	203	236	257	281	311	347	393	454	536
		H20	244	281	304	332	364	404	454	517	602
	250	H12	131	154	169	186	208	236	272	322	393
		H16	169	197	215	236	262	295	337	393	472
		H20	204	236	257	281	311	347	393	454	536
	300	H12	119	139	153	169	190	215	249	296	364
		H16	148	174	190	209	233	263	303	356	431
		H20	175	204	222	244	271	304	347	404	484
	350	H25	204	236	257	281	311	347	393	454	536
		H32	238	275	297	324	357	396	445	508	592
	Note. Values of $R_{ax}\,\varepsilon_{free}$ given for section thickness $h = 300$ mm apply for $h \geq 280$ mm (H12), $h \geq 290$ mm (H16) and $h \geq 300$ mm (H20). Values of $R_{ax}\,\varepsilon_{free}$ given for $h = 350$ mm apply for $h \geq 312.5$ mm (H25) and $h \geq 330$ mm (H32).										
Cover = 60 mm	200	H10	133	155	170	187	208	234	268	314	378
		H12	155	181	197	216	239	268	305	354	421
		H16	197	227	246	268	295	328	369	421	491
		H20	234	268	289	314	343	378	421	476	546
	250	H12	128	150	164	181	201	227	260	305	369
		H16	164	190	207	227	251	281	319	369	437
		H20	197	227	246	268	295	328	369	421	491
	300	H12	109	128	141	155	174	197	227	268	328
		H16	141	164	179	197	219	246	281	328	393
		H20	170	197	214	234	259	289	328	378	447
	350	H12	100	118	130	143	160	182	211	250	307
		H16	126	148	161	178	198	224	257	301	364
		H20	149	174	189	208	231	259	295	343	410
		H25	180	208	226	247	272	304	343	394	464
		H32	219	251	271	295	323	358	400	454	524
	400	H25	174	202	220	240	265	296	335	386	455
		H32	205	236	255	278	306	339	381	434	504
	Note. Values of $R_{ax}\,\varepsilon_{free}$ given for section thickness $h = 350$ mm apply for $h \geq 330$ mm (H12), $h \geq 340$ mm (H16) and $h \geq 350$ mm (H20). Values of $R_{ax}\,\varepsilon_{free}$ given for $h = 400$ mm apply for $h \geq 362.5$ mm (H25) and $h \geq 380$ mm (H32).										

Limiting span/effective depth ratio = $1.6 \times 24 = 38.4$ (> 35)

From Table 4.23, for $f_{ck} = 32$ MPa and bending of a section with $h \leq 300$ mm, $100A_{s,min}/A_{ct} = 0.24$. With $A_{ct} = 0.5bh$,

$$\begin{aligned} 100A_{s,min}/bd &= 0.24 \times 0.5 \times 250/205 = 0.15 \\ &\geq 0.26\ (f_{ctm}/f_{yk}) \geq 0.0013 \\ &= 0.26 \times 3.0/500 = 0.16\ (< 0.36) \end{aligned}$$

The design ultimate load is 854 kN and the quasi-permanent load, where the value of ψ_2 is obtained from section 29.3, is

$$G_k + \psi_2\, Q_k = (7.2 + 0.3 \times 4.5) \times 7.2^2 = 443 \text{ kN}$$

Hence, the stress in the reinforcement under quasi-permanent loading is given approximately by:

$$\sigma_s = (443/854)(0.87f_{yk}) = (443/854)(0.87 \times 500) = 226 \text{ MPa}$$

From *Table 4.24*, for $w_k = 0.3$ mm, the crack width criterion is met if $\phi^*_s \leq 16$ mm, or the bar spacing ≤ 200 mm.

The adjusted maximum bar size, with $h_{cr} = 0.5h$, is given by:

$$\begin{aligned} \phi_s &= \phi^*_s\ (f_{ct,eff}/2.9)[k_c\, h_{cr}/2(h-d)] \\ &= 16 \times (3.0/2.9) \times [0.4 \times 125/(2 \times 45)] = 9 \text{ mm} \end{aligned}$$

Area of reinforcement required to give $100A_s/bd = 0.36$ is

$$A_s = 0.0036 \times 1000 \times 205 = 738 \text{ mm}^2\text{/m (H12-150)}$$

Example 3. The wall of a cylindrical tank, 7.5 m deep and 15 m diameter, is 300 mm thick. The wall, which is continuous with the base slab, is to be designed for temperature effects, and those due to internal hydrostatic pressure when the tank is full of liquid.

Design class 1 (see section 29.4)	$f_{yk} = 500$ MPa
Cover to horizontal bars 40 mm	$f_{ck} = 32$ MPa

Effects of temperature change. With $f_{ctm} = 0.3 f_{ck}^{(2/3)} = 3.0$ MPa, and assuming that early thermal cracks will occur at a time when $f_{cm}(t) = 24$ MPa,

$$f_{ct,eff} = [f_{cm}(t)/(f_{ck} + 8)]\, f_{ctm} = [24/(32 + 8)] \times 3.0 = 1.8 \text{ MPa}$$

The limiting crack width varies according to the hydraulic gradient (depth of liquid / thickness of section). If the wall is designed to the recommendations for a panel restrained at its ends, then suitable reinforcement details for 40 mm cover and $f_{ct,eff} = 1.8$ MPa, selected from *Table 4.26*, are given here.

Depth (m)	Hydraulic gradient	Design crack width (mm)	Reinforcement required (EF)
1.5	5	0.2	H20-150
4.5	15	0.15	H20-125
7.5	25	0.1	H20-100

Note: The table for $w_k = 0.2$ mm was used throughout, by taking effective values of $f_{ct,eff} = 1.8/(5w_k)$ MPa.

If the wall is designed to the recommendations for a panel restrained along the edge, the restrained tensile strain needs to be estimated, as follows:

Allowing for concrete grade C32/40, with 350 kg/m³ Portland cement, at a placing temperature of 20°C and a mean ambient temperature during construction of 15°C, the temperature rise for concrete placed within 18 mm plywood formwork:

$$T_1 = 25^\circ\text{C } (\textit{Table 2.19})$$

As the wall is to be designed to resist hoop tension, there will be no vertical movement joints and allowance must be made for a fall in temperature due to seasonal variations. Allowing for $T_2 = 15$°C, restraint factor R_{ax} taken as 0.5 and coefficient of thermal expansion α taken as 12×10^{-6} per °C (*Table 3.5*), restrained total thermal contraction after the peak temperature arising from hydration effects is given by:

$$R_{ax}\alpha\,(T_1 + T_2) = 0.5 \times 12 \times 10^{-6} \times (25 + 15) = 240 \times 10^{-6}$$

Hence, suitable reinforcement details for 40 mm cover and $R_{ax}\varepsilon = 240 \times 10^{-6}$, selected from *Table 4.27*, are given here.

Depth (m)	Hydraulic gradient	Design crack width (mm)	Reinforcement required (EF)
1.5	5	0.2	H20-250
4.5	15	0.15	H20-150
7.5	25	0.1	H20-100

Note: The table for $w_k = 0.2$ mm was used throughout, for effective values of $R_{ax}\varepsilon = [240/(5w_k)] \times 10^{-6}$. For H20 bars, values for a 250 mm thick section apply for $h \geq 250$ mm.

From *Table 4.23*, the minimum reinforcement percentage for $f_{ct,eff} = f_{ctm}$, in the case of a rectangular section in pure tension with $h = 300$ mm and $f_{ck} = 32$ MPa, is 0.60. For the control of early thermal cracking, the value is $(1.8/3.0) \times 0.6 = 0.36$, and the minimum area of reinforcement required on each face:

$$A_{s,min} = 0.0036 \times 150 \times 1000 = 540 \text{ mm}^2\text{/m (H16-300)}$$

Clearly, the reinforcement needed for thermal crack control greatly exceeds this minimum requirement. In the lower part of the wall, the reinforcement provided is H20-100 (EF), and the corresponding stress at a cracked section is:

$$\sigma_s = f_{ct,eff}\,(A_{ct}/A_s) = 1.8 \times 150 \times 1000/3142 = 86 \text{ MPa}$$

This solution can be checked approximately by reference to the chart for maximum bar size on *Table 4.25*, as follows:

For $\sigma_s = 86$ MPa and $w_k = 0.1$ mm, $\phi_s^* = 55$ mm say

$$\begin{aligned} \phi_s &= \phi_s{}^* \frac{f_{ct,eff}\, 0.1h}{2.9\,(h-d)} \\ &= 55 \times (1.8/2.9) \times (0.1 \times 300/50) = 20 \text{ mm} \end{aligned}$$

Effects of hydrostatic load. Suppose that an elastic analysis of the tank, assuming a floor 300 mm thick, indicates a service maximum circumferential tension of 400 kN/m. This value occurs at a depth of 6 m, where the design crack width is 0.125 mm. Above this level, the tensions can be assumed to reduce approximately linearly to near zero at the top of the wall.

For a section reinforced with H20-100 (EF), the stress in the reinforcement $\sigma_s = 400 \times 10^3/6284 = 64$ MPa. Since this is less than the stress due to $f_{ct,eff}$, the reinforcement needed for thermal crack control is also sufficient for the circumferential tension. This solution can be checked also by reference to the charts on *Table 4.25*, which show that with $\sigma_s = 64$ MPa, the bar size and the bar spacing are of no consequence.

Chapter 35

Considerations affecting design details

The code contains many requirements that affect the details of the reinforcement such as minimum and maximum areas, tying provisions, anchorage and curtailment.

Bars may be set out individually, or grouped in bundles of two or three in contact. Bundles of four bars may also be used for vertical bars in compression, and for bars in a lapped joint. For the safe transmission of bond forces, the cover provided to the bars should be not less than the bar diameter or, for a bundle, the equivalent diameter ($\leq$ 55 mm) of a notional bar with the same sectional area as the bundle. Requirements for cover with regard to durability are given in Chapter 31. Gaps between bars (or bundles) generally should be not less than the greatest of: (d_g + 5 mm) where d_g is the maximum aggregate size, the bar diameter (or equivalent bar diameter for a bundle), or 20 mm. Details of reinforcement limits are given in *Table 4.28*.

At intermediate supports of continuous flanged beams, the total area of tension reinforcement should be spread over the effective width of the flange, but a part of the reinforcement may be concentrated over the web width.

35.1 TIES IN STRUCTURES

Structures not specifically designed to withstand accidental actions should be provided with a suitable tying system, to prevent progressive collapse by providing alternative load paths after local damage. Where the structure is divided into structurally independent sections, each section should have an appropriate tying system. The reinforcement providing the ties may be assumed to act at its characteristic strength, and only the specified tying forces need to be taken into account. Reinforcement required for other purposes may be considered to form part of, or the whole of the ties. Details of the tying requirements, as specified in the UK National Annex, are given in *Table 4.29*.

35.2 ANCHORAGE AND LAP LENGTHS

At both sides of any cross section, bars should be provided with an appropriate embedment length or other form of end anchorage. For bent bars, the basic tension anchorage length is measured along the centreline of the bar from the section in question to the end of the bar, where:

$$l_{bd} = \alpha_1 \, \alpha_2 \, \alpha_3 \, \alpha_4 \, \alpha_5 \, l_{b,rqd} \geq l_{b,min}$$

As a simplified alternative, a tension anchorage for a standard bend, hook or loop may be provided as an equivalent length $l_{b,eq} = \alpha_1 \, l_{b,rqd}$ (see figure here), where α_1 is taken as 0.7 for covers perpendicular to the bend $\geq 3\phi$. Otherwise $\alpha_1 = 1.0$.

Bends or hooks do not contribute to compression anchorages. Details of anchorage lengths are given in *Table 4.30*.

Laps should be located, if possible, away from positions of maximum moment and should generally be staggered. Details of lap lengths are given in *Table 4.31*.

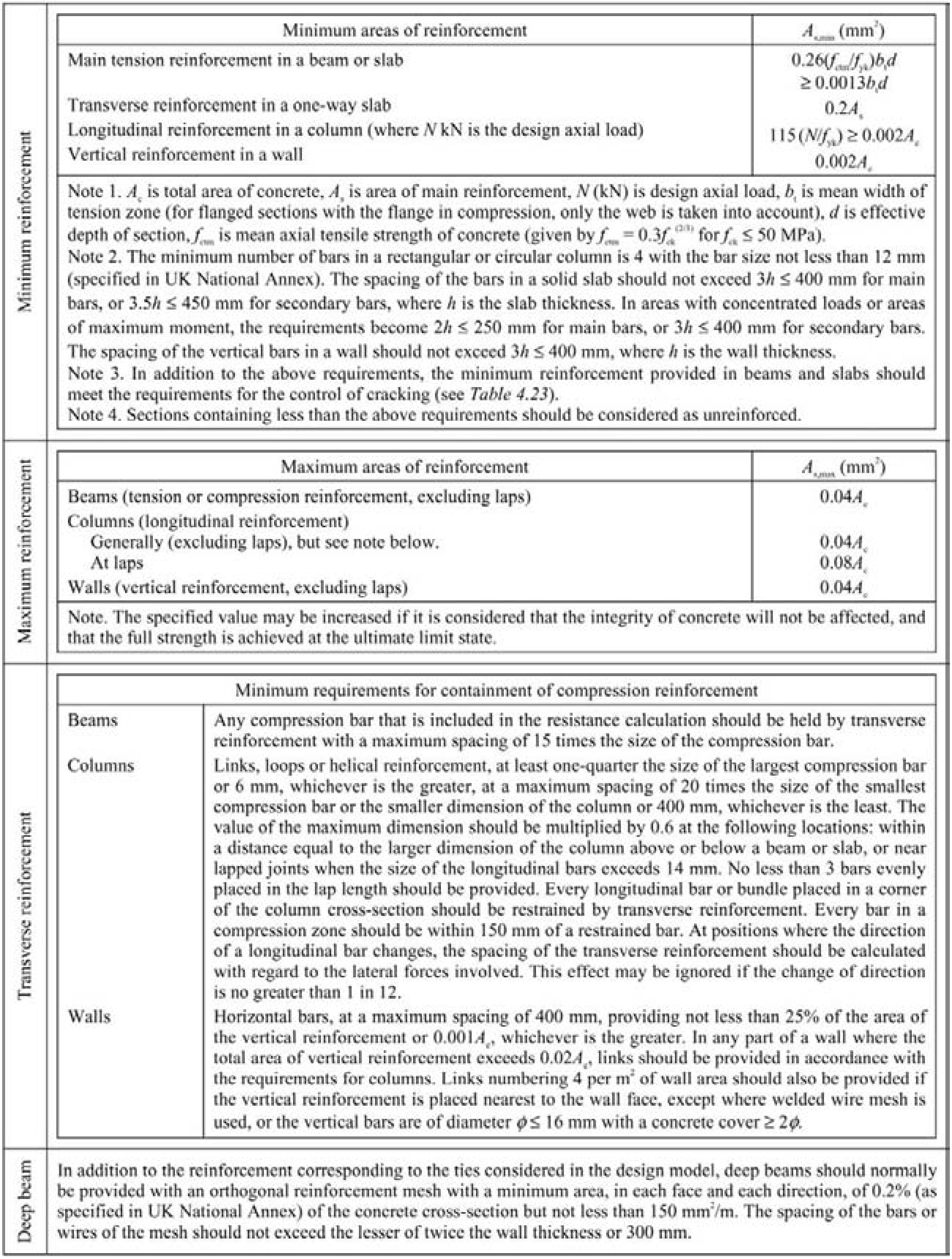
The radius of any bend in a reinforcing bar should conform to the minimum requirements of BS 8666, and should ensure that failure of the concrete inside the bend is prevented. A link may be considered fully anchored, if it passes round another bar of not less than its own diameter, through an angle of 90°, and continues beyond the end of the bend for a minimum length of 10 diameters $\geq$ 70 mm. Details of bends in bars are given in *Table 4.31*. Additional rules for large diameter bars ($>$ 40 mm according to the UK National Annex) and bundles are given in *Table 4.32*.

35.3 CURTAILMENT OF REINFORCEMENT

In flexural members, it is generally advisable to stagger the curtailment points of the tension reinforcement as allowed by the bending moment envelope. Bars should be curtailed in accordance with the rules set out in *Table 4.32* and illustrated in the figure on page 387. Except at end supports, every tension bar should extend beyond the point at which in theory it is no longer needed for flexural resistance for a distance not less than a_l. The bar should also extend beyond the point at which it is fully required to provide flexural resistance for a distance not less than $a_l + l_{bd}$. At a simple end support, the bars should extend for the anchorage length l_{bd} necessary to develop the force ΔF_{td}.

EC 2 Reinforcement limits

4.28



Minimum reinforcement

Minimum areas of reinforcement	$A_{s,min}$ (mm^2)
Main tension reinforcement in a beam or slab	$0.26(f_{ctm}/f_{yk})b_t d$ $\geq 0.0013 b_t d$
Transverse reinforcement in a one-way slab	$0.2A_s$
Longitudinal reinforcement in a column (where N kN is the design axial load)	$115(N/f_{yk}) \geq 0.002A_c$
Vertical reinforcement in a wall	$0.002A_c$

Note 1. A_c is total area of concrete, A_s is area of main reinforcement, N (kN) is design axial load, b_t is mean width of tension zone (for flanged sections with the flange in compression, only the web is taken into account), d is effective depth of section, f_{ctm} is mean axial tensile strength of concrete (given by $f_{ctm} = 0.3f_{ck}^{(2/3)}$ for $f_{ck} \leq 50$ MPa).

Note 2. The minimum number of bars in a rectangular or circular column is 4 with the bar size not less than 12 mm (specified in UK National Annex). The spacing of the bars in a solid slab should not exceed $3h \leq 400$ mm for main bars, or $3.5h \leq 450$ mm for secondary bars, where h is the slab thickness. In areas with concentrated loads or areas of maximum moment, the requirements become $2h \leq 250$ mm for main bars, or $3h \leq 400$ mm for secondary bars. The spacing of the vertical bars in a wall should not exceed $3h \leq 400$ mm, where h is the wall thickness.

Note 3. In addition to the above requirements, the minimum reinforcement provided in beams and slabs should meet the requirements for the control of cracking (see *Table 4.23*).

Note 4. Sections containing less than the above requirements should be considered as unreinforced.

Maximum reinforcement

Maximum areas of reinforcement	$A_{s,max}$ (mm^2)
Beams (tension or compression reinforcement, excluding laps)	$0.04A_c$
Columns (longitudinal reinforcement)	
Generally (excluding laps), but see note below.	$0.04A_c$
At laps	$0.08A_c$
Walls (vertical reinforcement, excluding laps)	$0.04A_c$

Note. The specified value may be increased if it is considered that the integrity of concrete will not be affected, and that the full strength is achieved at the ultimate limit state.

Transverse reinforcement

Minimum requirements for containment of compression reinforcement	
Beams	Any compression bar that is included in the resistance calculation should be held by transverse reinforcement with a maximum spacing of 15 times the size of the compression bar.
Columns	Links, loops or helical reinforcement, at least one-quarter the size of the largest compression bar or 6 mm, whichever is the greater, at a maximum spacing of 20 times the size of the smallest compression bar or the smaller dimension of the column or 400 mm, whichever is the least. The value of the maximum dimension should be multiplied by 0.6 at the following locations: within a distance equal to the larger dimension of the column above or below a beam or slab, or near lapped joints when the size of the longitudinal bars exceeds 14 mm. No less than 3 bars evenly placed in the lap length should be provided. Every longitudinal bar or bundle placed in a corner of the column cross-section should be restrained by transverse reinforcement. Every bar in a compression zone should be within 150 mm of a restrained bar. At positions where the direction of a longitudinal bar changes, the spacing of the transverse reinforcement should be calculated with regard to the lateral forces involved. This effect may be ignored if the change of direction is no greater than 1 in 12.
Walls	Horizontal bars, at a maximum spacing of 400 mm, providing not less than 25% of the area of the vertical reinforcement or $0.001A_c$, whichever is the greater. In any part of a wall where the total area of vertical reinforcement exceeds $0.02A_c$, links should be provided in accordance with the requirements for columns. Links numbering 4 per m^2 of wall area should also be provided if the vertical reinforcement is placed nearest to the wall face, except where welded wire mesh is used, or the vertical bars are of diameter $\phi \leq 16$ mm with a concrete cover $\geq 2\phi$.

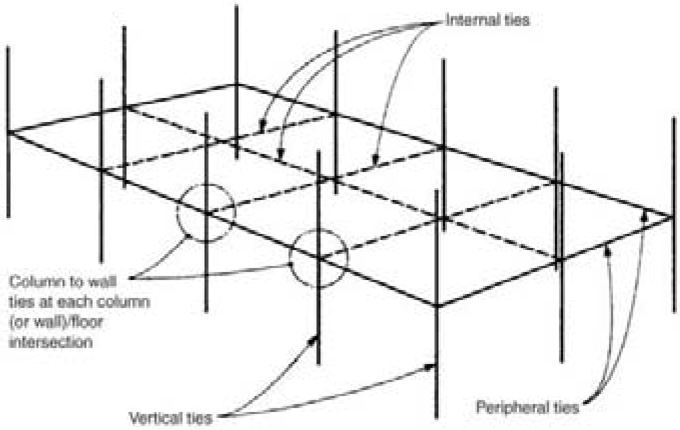
Deep beam

In addition to the reinforcement corresponding to the ties considered in the design model, deep beams should normally be provided with an orthogonal reinforcement mesh with a minimum area, in each face and each direction, of 0.2% (as specified in UK National Annex) of the concrete cross-section but not less than 150 mm^2/m. The spacing of the bars or wires of the mesh should not exceed the lesser of twice the wall thickness or 300 mm.

EC 2 Provision of ties

4.29

System



The recommendations in the UK National Annex are the same as those in BS 8110. In structures that are not specifically designed to withstand accidental actions, a system of effectively continuous horizontal and vertical ties should be provided in accordance with the requirements below.

Requirements

Type of tie	Requirement
Peripheral	At each floor and roof level, an effectively continuous tie, located within 1.2 m of the outside edges of the building, or within a perimeter wall. The tensile force to be resisted is given by: $F_{tie,per} = (20 + 4n_o) \le 60$ kN where: n_o is the number of storeys in the structure
Internal	At each floor and roof level, in two directions approximately at right angles, effectively continuous ties anchored to the peripheral tie at each end (unless continuing as ties to columns or walls). They may, in whole or in part, be spread evenly in slabs, or may be grouped at or in beams, walls or other appropriate positions. If grouped, they should be spaced generally at not more than $1.5l_r$ where l_r is the greater distance between the centres of the columns, frames or walls supporting any two adjacent spans in the direction of the tie under consideration. In walls they should be within 0.5 m of the top or bottom of the floor or roof slab. The tensile force to be resisted is given by: $F_{tie,int} = \left(\dfrac{g_k + q_k}{7{\cdot}5}\right)\left(\dfrac{l_r}{5}\right)F_t \ge F_t$ (kN/m width) where: $F_t = (20 + 4n_o) \le 60$ where: $(g_k + q_k)$ is the sum of the characteristic dead and imposed loading, l_r is as defined above. When walls occur in plan in one direction only (e.g. cross-wall or spine-wall construction), the value of l_r used to assess the tie force in the direction parallel to the wall should be taken as the lesser of the actual length of the wall, or the length that may be considered lost in the event of an accident. This length should be taken as the length between adjacent lateral supports, or the length between a lateral support and a free edge, as appropriate.
Horizontal to columns and walls	Each external column and, if the peripheral tie is not located within the wall, every metre length of facade wall carrying vertical load should be anchored or tied horizontally into the structure at each floor and roof level. The tensile force to be resisted, $F_{tie,col} = F_{tie,fac}$ is given by the greater of: (a) $2F_t$ [or $(l_s/2.5)F_t$ if less, where l_s is the floor to ceiling height in metres]; or (b) 3% of the total design ultimate vertical load carried by the column or wall at that level For corner columns, ties able to resist the tensile force should be provided in each of two directions, approximately at right angles. When the peripheral tie is located within a wall, only such horizontal tying as is required to anchor the internal ties to the peripheral tie needs to be considered.
Vertical	Each column and each wall carrying vertical load should be tied continuously from the highest to the lowest level. The tensile force to be resisted is the maximum design accidental load received by the column or wall from any one storey. For this purpose, γ_f should be taken as 1.0, and the following loads should be taken into account: dead load; one-third of imposed load, except for buildings used predominantly for storage or industrial purposes, or where the imposed loads are permanent, when full imposed load should be taken; one-third of wind load.

In the design of the ties, the reinforcement may be assumed to act at its characteristic strength, and only the specified tying forces need be taken into account. Reinforcement provided for other purposes may be considered to form part of, or the whole of the ties. At re-entrant corners, or at substantial changes in construction, care should be taken to ensure that ties are adequately anchored or otherwise made effective. A tie may be considered anchored to another tie at right angles if the bars of the former tie extend: either 12ϕ, or an equivalent anchorage, beyond all the bars of the other tie; or, an effective anchorage length (based on the force in the bars) beyond the centre-line of the bars of the other tie. The anchorage and lap lengths given in *Table 4.30*, including the multiplier $A_{s,req}/A_{s,prov}$ may be used for this purpose.

4.30

EC 2 Anchorage requirements

Anchorage lengths

The design anchorage length of a bar in tension is given by:

$l_{bd} = \alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 l_{b,rqd} \geq l_{b,min}$ where $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and $\alpha_5 \leq 1.0$, and product $(\alpha_2 \alpha_3 \alpha_5) \geq 0.7$.

The design anchorage length of a bar in compression is given by: $l_{bd} = \alpha_4 l_{b,rqd} \geq l_{b,min}$

c_d is either the cover to the bar or half the gap between adjacent bars in the same tensile plane, whichever is the lesser. For straight bars, the least cover to any face applies: for bars bent perpendicular to the tensile plane, or looped within the tensile plane, the cover perpendicular to the plane of the bend applies.

f_{bd} is the design ultimate bond stress given, for ribbed bars, by $f_{bd} = 2.25 \eta_1 \eta_2 f_{ctd}$.

f_{ctd} is the design concrete tensile strength given by $f_{ctd} = 0.21 f_{ck}^{(2/3)}$ for $f_{ck} \leq 50$ MPa.

$l_{b,min}$ is the minimum anchorage length taken as follows: in tension, the greatest of $0.3 l_{b,rqd}$ or 10ϕ or 100 mm; in compression, the greatest of $0.6 l_{b,rqd}$ or 10ϕ or 100 mm.

$l_{b,rqd}$ is the basic required anchorage length given by $l_{b,rqd} = \sigma_{sd}/4 f_{bd}$, where $\sigma_{sd} \leq 0.87 f_{yk}$ is the design stress in the bar at the section in question. See tabulated values below for grade 500 ribbed bars, and $\sigma_{sd} = 0.87 f_{yk}$.

α_1 is for the effect of the shape of the bars: for bent bars where $c_d \geq 3\phi$, $\alpha_1 = 0.7$. Otherwise, $\alpha_1 = 1.0$.

α_2 is for the effect of cover: for straight bars where $c_d \geq 3\phi$, or bent bars where $c_d \geq 5\phi$, $\alpha_2 = 0.7$; for straight bars where $\phi < c_d < 3\phi$, or bent bars where $3\phi < c_d < 5\phi$, values of α_2 in the range $1.0 > \alpha_2 > 0.7$ may be interpolated.

α_3 is for the effect of confinement by transverse reinforcement (not welded to the main reinforcement): for bars in the corner of a link where $\lambda \geq 3$, or not in the corner of a link where $\lambda \geq 6$, $\alpha_3 = 0.7$; for bars in the corner of a link where $0 < \lambda < 3$, or not in the corner of a link where $0 < \lambda < 6$, α_3 values in the range $1.0 > \alpha_3 > 0.7$ may be interpolated.

α_4 is for the influence of one or more welded transverse bars or wires $(\phi_t \geq 0.6\phi)$ along the design anchorage length l_{bd}: where the distance of the transverse bar or wire from the section in question is $\geq 5\phi$, $\alpha_4 = 0.7$. For direct supports, l_{bd} may be taken less than $l_{b,min}$ provided there is at least one welded transverse bar or wire over the support, at a distance ≥ 15 mm from the face of the support.

α_5 is for the effect of pressure p transverse to the plane of splitting along the design anchorage length l_{bd}: where $p \geq 7.5$ MPa, $\alpha_5 = 0.7$; where $0 < p < 7.5$ MPa, α_5 values in the range $1.0 > \alpha_5 > 0.7$ may be interpolated.

η_1 is a coefficient related to the bond condition, and the bar position during concreting. The bond condition is classified as 'good' in sections ≤ 250 mm deep, in the bottom 250 mm of sections ≤ 600 mm deep, and in all but the top 300 mm of sections > 600 mm deep. 'Good' applies also, in sections of any depth, to bent-up bars inclined at α to the horizontal, where $45° \leq \alpha \leq 90°$. In all other cases, and where slip-forms are used, the conditions are classified as 'poor'. For 'good' conditions, $\eta_1 = 1.0$; for 'poor' conditions, $\eta_1 = 0.7$.

η_2 is a coefficient related to the bar diameter. For $\phi \leq 32$ mm, $\eta_2 = 1.0$; for $\phi > 32$ mm, $\eta_2 = 1.32 - \phi/100$.

λ is the effective transverse reinforcement ratio, given by $\lambda = (\Sigma A_{st}/A_s - 0.25)$ for a beam and $\lambda = \Sigma A_{st}/A_s$ for a slab, where ΣA_{st} is area of transverse reinforcement along length l_{bd}, A_s is area of largest anchored bar.

Basic required anchorage length $l_{b,rqd}$ as a multiple of bar size ≤ 32 mm (with $\sigma_{sd} = 0.87 \times 500 = 435$ MPa)

Condition	Bond	Concrete cylinder strength f_{ck} (MPa)					
		20	25	28	32	40	50
Anchorage length (tension or compression)	Good Poor	47 67	41 59	38 54	35 50	30 43	26 37

Note 1. The above values should be divided by 0.92 for $\phi = 40$ mm, and 0.82 for $\phi = 50$ mm.

Note 2. For values of $\sigma_s < 435$ MPa, the above values may be multiplied by $(\sigma_s/435)$.

Note 3. The above values of $l_{b,rqd}$ may be used also for the design anchorage length l_{bd}, but lower values of l_{bd} can be obtained in some cases. For bars in tension: lower values of l_{bd} apply for straight bars with $c_d > \phi$ (α_2), other than straight bars with $c_d > 3\phi$ (α_1 and α_2), bars confined by links (α_3), the influence of welded transverse bars (α_4), and the effect of lateral pressure (α_5). For bars in compression: lower values of l_{bd} apply for the influence of welded transverse bars (α_4).

In special circumstances, additional anchorage may be obtained by purpose-designed transverse welded bars bearing on the concrete, for further details of which reference should be made to clause 8.6 of Eurocode 2. For details of bends in bars, see *Table 4.31*. For large diameter bars (> 40 mm) and bundled bars, see *Table 4.32*.

4.31

EC 2 Laps and bends in bars

Lap lengths

The design lap length for a bar is given by: $l_0 = \alpha_6 l_{bd} \geq l_{0,min}$

where: $\lambda = (\Sigma A_{st}/A_s - 1)$ is used to determine α_3, and α_6 is taken as 1.0, in the evaluation of l_{bd}.

$l_{0,min}$ is the minimum lap length taken as the greatest of $0.3\alpha_6 l_{b,rqd}$ or 15ϕ or 200 mm.

α_6 is a coefficient related to the percentage of lapped bars, given by $1.0 \leq \alpha_6 = (\rho_1/25)^{0.5} \leq 1.5$. For bars of the same diameter, typical values of α_6 according to the proportion of bars lapped are: one in four, $\alpha_6 = 1.0$; one in three, $\alpha_6 = 1.15$; one in two $\alpha_6 = 1.4$; two in three, $\alpha_6 = 1.5$.

ρ_1 is the percentage of lapped bars, relative to the total area of parallel bars, at the section considered. In this calculation, bars containing laps that are staggered by a distance $\geq 0.65l_0$, relative to the lapped bars being considered, may be taken as not lapped.

If the gap between two lapped bars exceeds the lesser of 4ϕ or 50 mm, the lap length should be increased by the value of the gap. The stagger between laps in adjacent pairs of lapped bars should be not less than $1.3l_0$. The clear distance between adjacent pairs of lapped bars should be not less than $2\phi \geq 20$ mm. For bars in tension, where the forgoing provisions are met and the bars are all in one layer, all of the bars may be lapped. Where the bars are in several layers, no more than half of the bars should be lapped. All bars in compression and all secondary (distribution reinforcement) may be lapped at the same section. For large diameter bars (> 40 mm) and bundled bars, see *Table 4.32*.

For bars in tension, transverse reinforcement should be provided in the lap zone as follows:

(1) Where the diameter of the lapped bars is < 20 mm, or the percentage of lapped bars in any section is < 25%, any transverse reinforcement or links necessary for other reasons may be assumed sufficient for the transverse forces associated with the lap.

(2) Where the diameter of the lapped bars is ≥ 20 mm, the transverse reinforcement should have a total area not less than the area of one lapped bar. The transverse bars should be placed outside and perpendicular to the direction of the lapped reinforcement. Half of the required area of transverse bars should be provided at each end of the lap, over a length of $l_0/3$ with bars at a spacing ≤ 150 mm. If more than 50% of the main reinforcement is lapped at one section, and the distance between adjacent laps is $\leq 10\phi$, the transverse reinforcement should be in the form of links or U bars anchored into the body of the section.

For bars in compression, in addition to the rules for bars in tension, one bar of the transverse reinforcement should be placed outside each end of the lap length, and within 4ϕ of the ends of the lap.

For welded mesh fabric made of ribbed wires, the main wires may be lapped by intermeshing (3 layers of wires) or by layering (4 layers of wires) of the fabric. For intermeshed fabric, the lapping arrangements given above for bars apply, taking $\alpha_1 = 1.0$. For layered fabric, the main wires should be lapped in zones where $\sigma_s \leq 0.7f_{yk}$ and the percentage that may be lapped at any section is 100% if $(A_s/s)_{prov} \leq 1200$ mm²/m, and 60% if $(A_s/s)_{prov} > 1200$ mm²/m, where s is the spacing of the wires. For secondary wires, the required lap lengths are: for $\phi \leq 6$ mm, $l_0 \geq 150$ mm with at least 1 wire pitch; for $6 < \phi \leq 8.5$ mm, $l_0 \geq 250$ mm with 2 wire pitches; $8.5 < \phi \leq 12$ mm, $l_0 \geq 350$ mm with 2 wire pitches.

Bends in bars

For bars bent to the minimum radius according to BS 8666, there is no need for a check to avoid concrete failure if one of the following conditions exists: either the anchorage of the bar does not require a length more than 5ϕ beyond the end of the bend, or the plane of the bend is not close to a concrete face and there is a transverse bar of diameter $\geq \phi$ inside the bend. Otherwise the internal radius of the bend should satisfy the following relationship:

$$r \geq \frac{F_{bt}(1 + 2\phi/a_b)}{4f_{cd}\phi} \geq r_{min} \text{ (according to BS 8666)}$$

where

F_{bt} is the tensile force due to the design ultimate loads in a bar, or group of bars in contact, at the start of a bend.

a_b is half the centre-to-centre distance between bent bars (or groups of bars) perpendicular to the plane of the bend. If, at the position of the bend, the bar (or group of bars) is adjacent to the face of the member, a_b should be taken as the cover to the bar plus half the bar size.

f_{cd} is the design compressive strength of the concrete, given by $f_{cd} = 0.567f_{ck}$ where $f_{ck} \leq 55$ MPa.

f_{ck} MPa	Minimum value of r as a multiple of bar size (with $\sigma_s = 0.87 \times 500 = 435$ MPa) for values of a_b/ϕ						
	1.5	2	2.5	3	4	5	6
20	17.6	15.1	13.6	12.6	11.3	10.5	10.0
25	14.1	12.1	10.8	10.0	9.0	8.4	8.0
28	12.6	10.8	9.7	9.0	8.1	7.5	7.2
32	11.0	9.4	8.5	7.8	7.1	6.6	6.3
40	8.8	7.5	6.8	6.3	5.6	5.3	5.0
50	7.0	6.0	5.4	5.0	4.5	4.2	4.0

Note. Tabulated values of r may be multiplied by $\sigma_s/435$, where σ_s is the stress in the reinforcement at the start of the bend. Minimum values of r for bending according to BS 8666 are 2ϕ for $\phi \leq 16$, and 3.5ϕ for $\phi > 16$.

EC 2 Rules for curtailment, large diameter bars and bundles 4.32

Curtailment of reinforcement	In every member, except at end supports, every tension bar should extend beyond the point at which in theory it is no longer needed for flexural resistance for a distance a_l. The bar should also extend beyond the point at which it is fully needed for flexural resistance for a distance $a_l + l_{bd}$. For members without shear reinforcement, $a_l = d$. For members with shear reinforcement, $a_l = 0.5z(\cot\theta - \cot\alpha)$, as described in section 33.1.2. When the shear reinforcement consists of upright links ($\alpha = 90°$), $a_l = 0.5z\cot\theta$ where $1.0 \le \cot\theta \le 2.5$. Taking $z = 0.9d$, $a_l = 0.45d\cot\theta$. Thus, $a_l = 1.125d$ can be safely assumed in the absence of a more precise value based on the shear design calculations. Similarly, this value could also be conservatively taken when the shear resistance is provided by links combined with bent-up bars. The anchorage length of a bent-up bar should be not less than $1.3l_{bd}$ in the tension zone, and $0.7l_{bd}$ in the compression zone, measured from the point of intersection of the axes of the bent-up bar and the longitudinal reinforcement. At all supports, the area of bottom reinforcement to be provided should be at least 25% generally, and 50% in simply supported slabs, of the area provided in the span. At end supports, with little or no fixity assumed in design, the tensile force to be anchored is given by $F = (a_l/z)V + N$, where N (tension positive, compression negative) is the axial force, if any, applied to the member. Thus, when $N = 0$, $F = 0.5V\cot\theta$, and a value of $F = 1.25V$ could be taken in all cases. The required anchorage length l_{bd} should be taken from the front edge of the bearing area (direct support), or face of the supporting member (indirect support). In evaluating l_{bd}, the effect of transverse pressure may be taken into account for direct supports. At intermediate supports, the anchorage length for the bottom bars should be not less than 10ϕ for straight bars. Where continuity of the bottom reinforcement is necessary for design purposes, this may be achieved by using lapped bars with the laps located outside the support if required. In monolithic construction, even when simple supports have been assumed in design, the section at supports should be designed for a bending moment arising from partial fixity of at least 25% (as specified in UK National Annex) of the maximum moment in the span. The reinforcement should extend from the face of the support at least 0.2 times the length of the adjacent span. In slabs, at a free unsupported edge, the section should be reinforced with a longitudinal bar top and bottom contained by transverse U bars, with legs of length not less than twice the slab thickness.
Large diameter bars and bundles	Large diameter bars Bars of diameter > 40 mm (according to UK National Annex) should generally be anchored using mechanical devices. Alternatively, they may be anchored as straight bars, but links should be provided as confining reinforcement. In cases where there is no transverse compression, the area of reinforcement to be provided by the links, additional to that required for shear, is as follows: $A_{sh} = 0.25A_s n_1$ (parallel to the tension face), $A_{sv} = 0.25A_s n_2$ (perpendicular to the tension face), where A_s is the area of an anchored bar, n_1 is the number of layers with bars anchored at the same point in the member, n_2 is the number of bars anchored in each layer. Large diameter bars may only be lapped in concrete sections of minimum dimension 1 m, or where $\sigma_s \le 0.7f_{yk}$. Otherwise, mechanical devices (couplers) should be used. Bundled bars Bars may be grouped in bundles of two, three or four in contact. If two touching bars are located one above the other, in a section where the bond conditions are 'good' (see η_1 in *Table 4.30*), they may be treated as separate bars. Bundles of four bars may only be used for vertical bars in compression, and for bars in a lapped joint. The equivalent diameter of a bundle is the diameter of a notional bar having the same total cross-sectional area, given by $\phi_n = \phi\sqrt{n_b} \le 55$ mm, where n_b is the number of bars in the bundle. Anchoring bundles of bars Bundles of bars in tension may be curtailed over end and intermediate supports. Bundles with $\phi_n < 32$ mm may be curtailed near a support without staggering the bars. Bundles with $\phi_n \ge 32$ mm should be curtailed as follows: for a bundle of three bars, a stagger of at least $1.3l_{bd}$ between first and second bars, and l_{bd} between second and last bars should be provided. Where the stagger exceeds $1.3l_{b,rqd}$ (based on the bar diameter), the bar diameter may be used to assess l_{bd}. Otherwise, the equivalent diameter of the bundle should be used. Bundles of bars in compression may be anchored without staggering the bars. For bundles with $\phi_n \ge 32$ mm, at least four links having a diameter ≥ 12 mm at the end of the bundle, and a further link just beyond the end of the bundle, should be provided. Lapping bundles of bars Bundles of two bars with $\phi_n < 32$ mm may be lapped without staggering the bars, where l_0 is based on the equivalent diameter of the bundle. For bundles of two bars with $\phi_n \ge 32$ mm, or three bars, laps in individual bars should be staggered by at least $1.3l_0$, where l_0 is based on a single bar. There should be no more than four bars at any position. Surface reinforcement Where, for the main tension bars in a beam, ϕ or $\phi_n > 32$ mm, surface reinforcement is needed within the tension zone. This should consist of wire mesh or small diameter bars at a spacing ≤ 150 mm, placed outside the links. The minimum area of surface reinforcement to be provided is 2%, parallel to the main bars, and 1%, perpendicular to the main bars, of the area of tensile concrete external to the links, where the depth of the tension zone is taken ≤ 600 mm.

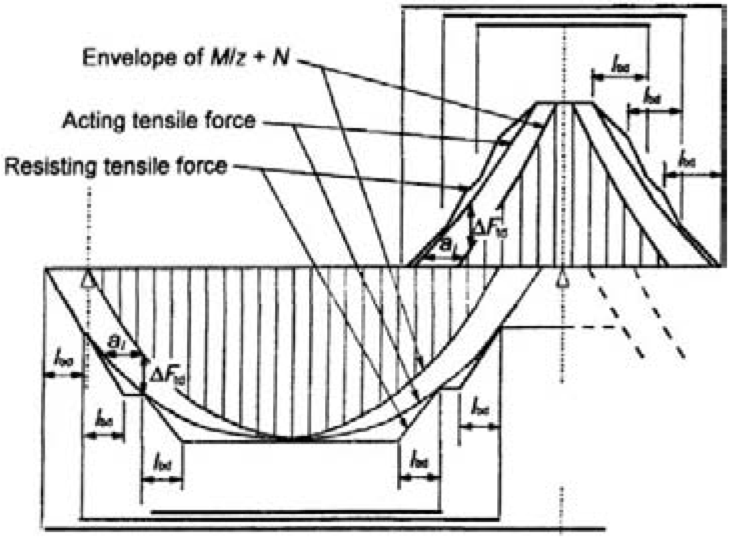
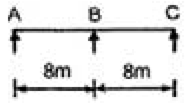


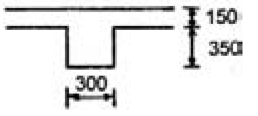
Illustration of the curtailment of longitudinal reinforcement taking account of resistance within anchorage lengths

Example 1. The beam shown in the following figure, which was designed in example 1 of Chapters 32 (bending) and 33 (shear), is to be checked for the reinforcement details. The design ultimate loads are $F_{max} = 396$ kN, and $F_{min} = 216$ kN, on each span. The width of each support is 400 mm, and the main reinforcement is as follows: spans 3H25 (bottom); support B, 3H32 (top) and 2H25 (bottom).

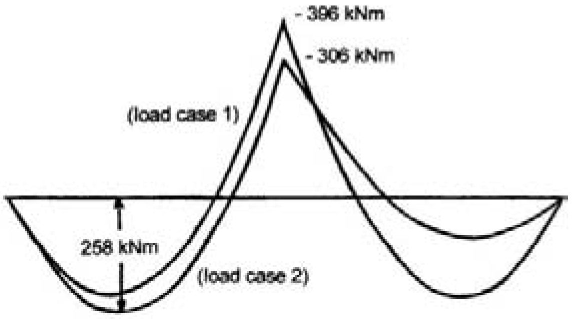
$f_{ck} = 32$ MPa, $f_{yk} = 500$ MPa, cover to links = 35 mm.



Spans



Cross section



Elastic bending moment diagram

End anchorage. At the bottom of each span, 2H25 ($\geq 25\%$ of area provided in the span) will be continued to the support. At the end support, the tensile force to be anchored is $F = 0.5V\cot\theta$, in which θ is the inclination of the concrete strut required for shear design. In the shear design calculations in chapter 33, $V = 128$ kN at the critical section and $V_{Rd,s} = 142$ kN when $\cot\theta = 2.5$. Thus, $\cot\theta = (V/V_{Rd,s}) \times 2.5 = 2.25$ could be used. At the face of the support, $V = 160 - 0.2 \times 49.5 = 150$ kN. With $\cot\theta = 2.25$,

$$F = 0.5 \times 150 \times 2.25 = 169 \text{ kN}$$
$$\sigma_s = F/A_s = 169 \times 10^3/982 = 172 \text{ MPa}$$

For bars in the bottom of the section, the bond condition is classified as 'good'. Thus, from *Table 4.30*

$$l_{b,rqd} = 35\phi \times (\sigma_s/435) = 35 \times 25 \times (172/435) = 346 \text{ mm}$$

The design anchorage length is given by:

$$l_{bd} = \alpha_1\, \alpha_2\, \alpha_3\, \alpha_4\, \alpha_5\, l_{b,rqd} \geq l_{b,min}$$

Coefficients α_1 and α_2 depend on c_d, which is taken as either the cover to the main bar or half the gap between the main bars, whichever is the lesser. With 35 mm cover to H8 links, cover to main bars is 45 mm and the gap between the bars is $300 - 2 \times (45 + 25) = 160$ mm. Hence, $c_d = 45$ mm (or 1.8ϕ).

Since $c_d < 3\phi$, $\alpha_1 = 1.0$ for both bent bars and straight bars. Hence, using the simplified approach (see section 35.2), for both straight bars or standard bends, $l_{b,eq} = \alpha_1\, l_{b,rqd} = 346$ mm.

For a 400 mm wide support, allowing for 50 mm end cover to the bars, the anchorage length provided from the near face of the support is 350 mm ($> l_{b,eq}$).

For the basic approach, the following values can be obtained:

For straight bars, $\alpha_2 = 1 - 0.15(c_d/\phi - 1) = 0.88$.

With no transverse reinforcement provided within the bearing length, $\alpha_3 = 1.0$ and $\alpha_4 = 1.0$.

Transverse pressure due to reaction on support is given by:

$$p = V/(\text{bearing area}) = 150 \times 10^3/(400 \times 300) = 1.25 \text{ MPa}$$

Hence, with $p = 1.25$ MPa, $\alpha_5 = 1 - 0.04p = 0.95$, and

$$l_{bd} = \alpha_2 \alpha_5 l_{b,rqd} = 0.88 \times 0.95 \times 346 = 290 \text{ mm}$$
$$\geq l_{b,min} = 10\phi = 250 \text{ mm } (\geq 0.3 l_{b,rqd} \text{ or } 100 \text{ mm})$$

Curtailment points for bottom bars. The resistance moment provided by 2H25 at the bottom of the beam may be determined as follows:

$$A_s f_{yk}/bdf_{ck} = 982 \times 500/(2600 \times 440 \times 32) = 0.0134$$
$$M/bd^2 f_{cu} = 0.012 \text{ (\textit{Table 4.8} or section 32.2.1)}$$
$$M = 0.012 \times 2600 \times 440^2 \times 32 \times 10^{-6} = 193 \text{ kNm}$$

Reaction at A, for load case 2, is $R_A = 160$ kN

Distance x from A to point where $M = 193$ kNm is given by:

$$R_A x - 0.5(F_{max}/L)x^2 = 160x^2 - 0.5 \times (396/8)x^2 = 193$$

Hence $0.5x^2 - 3.23x + 3.9 = 0$, giving $x = 1.6$ m and 4.85 m.

Thus, of the 3H25 required in the span, one bar is no longer needed for flexure at 1.6 m and 4.85 m from the end support. At these points, $V = 80$ kN and $\cot\theta = (80/142) \times 2.5 = 1.4$ is sufficient. Thus, the bar needs to extend beyond these points for a distance $a_l = 0.45d \cot\theta = 0.45 \times 440 \times 1.4 = 280$ mm.

Curtailment points for top bars. The resistance moment provided by 2H32 can be determined as follows:

$A_s f_{yk}/bdf_{ck} = 1608 \times 500/(300 \times 440 \times 32) = 0.190$
$M/bd^2 f_{cu} = 0.142$ (*Table 4.8*)
$M = 0.142 \times 300 \times 440^2 \times 32 \times 10^{-6} = 264$ kNm

For load case 1, reaction at A (or C) is given by:

$$R_A = 0.5F_{max} - M_B/L = 0.5 \times 396 - 396/8 = 148 \text{ kN}$$

Distance x from A to point where $M = 264$ kNm is given by:

$$0.5(F_{max}/L)x^2 - R_A x = 0.5 \times (396/8)\, x^2 - 148\, x = 264$$

Hence $0.5x^2 - 3x - 5.3 = 0$, giving $x = 7.4$ m. Thus, of the 3H32 required at B, one bar is no longer required for flexure at distance $(8.0 - 7.4) = 0.6$ m from B. If this distance is less than $l_{b,\,rqd}$, the point of curtailment will be determined by the need to develop the full force in the bar at B. Here, $\cot\theta = 2.5$ giving $a_l = 0.45d\cot\theta = 1.125d$. As the bars are effectively in a slab of thickness ≤ 250 mm, it seems reasonable to assume 'good' bond conditions giving $l_{b,\,rqd} = 35\phi$ (*Table 4.30*). Thus, distance from B (edge of support, say) at which one bar may be curtailed is $a_l + l_{b,\,rqd} = 1.125 \times 440 + 35 \times 32 = 1615$ mm.

Suppose that the remaining bars are continued to the point of contra-flexure in span BC for load case 2.

The reaction at support C is given by

$$R_C = 0.5F_{min} - M_B/L = 0.5 \times 216 - 306/8 = 70 \text{ kN}$$

Distance from B to point of contra-flexure is given by:

$$x = L(1 - 2R_C/F_{min}) = 8 \times (1 - 2 \times 70/216) = 2.8 \text{ m}$$

Here $V = 70$ kN and $\cot\theta = (70/142) \times 2.5 = 1.25$ is sufficient. Thus, distance from B at which the remaining bars may be curtailed is $2800 + 0.45 \times 440 \times 1.25 = 3050$ mm.

Link support bars, say 2H12, could be used for the remainder of the span.

Example 2. A typical floor to an 8-storey building consists of a 250 mm thick flat slab, supported by columns arranged on a 7.2 m square grid. The slab, for which the characteristic loading is 7.2 kN/m² dead and 4.5 kN/m² imposed, is to be provided with ties to the requirements of the UK National Annex. The design panel load is 854 kN, and bending moments are to be determined by the simplified method (see section 13.8).

$$f_{ck} = 32 \text{ MPa}, f_{yk} = 500 \text{ MPa}, \quad \text{cover to bars} = 25 \text{ mm}$$

Allowing for the use of H12 bars in each direction, and based on the bars in the second layer of reinforcement:

$$d = 250 - (25 + 12 + 6) = 205 \text{ mm say}$$

From *Table 2.55*, the design ultimate sagging moment for an interior panel is given by:

$$M = 0.063Fl = 0.063 \times 854 \times 7.2 = 388 \text{ kNm}$$

The total panel moment is to be apportioned between column and middle strips, where the width of each strip is 3.6 m. For the column strip with 60% of the panel moment,

$M = 0.6 \times 388 = 233$ kNm
$M/bd^2 f_{ck} = 233 \times 10^6/(3600 \times 205^2 \times 32) = 0.048$
$A_s f_{yk}/bdf_{ck} = 0.056$ (*Table 4.8*)
$A_s = 0.056 \times 3600 \times 205 \times 32/500$
$= 2645$ mm² (24H12-150 gives 2714 mm²)

For the middle strip with 40% of the panel moment,

$M = 0.4 \times 388 = 155$ kNm
$M/bd^2 f_{ck} = 155 \times 10^6/(3600 \times 205^2 \times 32) = 0.032$
$A_s f_{yk}/bdf_{ck} = 0.037$ (*Table 4.8*)
$A_s = 0.037 \times 3600 \times 205 \times 32/500$
$= 1748$ mm² (16H12-225 gives 1810 mm²)

For the peripheral tie, the tensile force is given by:

$$F_{tie,per} = (20 + 4n_o) \leq 60 \text{ kN} = (20 + 4 \times 8) = 52 \text{ kN}$$

The required area of reinforcement, acting at its characteristic strength, is given by:

$$A_s = F_{tie,per}/f_{yk} = 52 \times 10^3/500 = 104 \text{ mm}^2 \text{ (1H12)}$$

For the internal ties, the tensile force is given by:

$$F_{tie,int} = \left(\frac{g_k + q_k}{7.5}\right)\left(\frac{l_r}{5}\right)F_t \geq F_t \text{ kN/m}$$

With $F_t = (20 + 4n_o) \leq 60$ kN $= (20 + 4 \times 8) = 52$ kN,

$$F_{tie,int} = \left(\frac{7.2 + 4.5}{7.5}\right)\left(\frac{7.2}{5}\right) \times 52 = 117 \text{ kN/m}$$

If the internal ties are spread evenly in the slab, the required area of reinforcement acting at its characteristic strength,

$$A_s = 117 \times 10^3/500 = 234 \text{ mm}^2\text{/m (H12-450)}$$

In this case, at least every third bar in the column strips and every other bar in the middle strips need to be continuous.

If the internal ties are concentrated at the column lines, the total area of reinforcement required in each group,

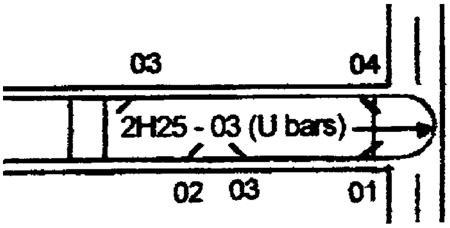
$$A_s = 234 \times 7.2 = 1685 \text{ mm}^2 \text{ (16H12 gives 1810 mm}^2\text{)}$$

In this case, the bars in the middle two-thirds of each column strip need to be continuous. For sections ≤ 250 mm deep, the bond condition is 'good' and $l_{b,rqd} = 35\,\phi$ (*Table 4.30*). Laps in adjacent pairs of lapped bars should be staggered by $1.3l_0$, where l_0 is the design lap length (*Table 4.31*). With $l_{bd} = l_{b,rqd}$ and $\alpha_6 = 1.4$, for one in two bars lapped at the same section,

$$l_0 = \alpha_6\, l_{bd} = 1.4 \times 35 \times 12 = 600 \text{ mm say.}$$

Example 3. The following figure shows details of the reinforcement at the junction between a 300 mm wide beam and a 300 mm square column. Bars 03 need to develop the maximum design stress at the column face, and the radius of bend necessary to avoid failure of the concrete inside the bend is to be determined.

$$f_{ck} = 32 \text{ MPa}, f_{yk} = 500 \text{ MPa}$$



Cover to U bars: end 50, side 75

The minimum radius of bend of the bars depends on the value of a_b/ϕ, where a_b is taken as half the centre-to-centre distance between adjacent bars or, for bars adjacent to the face of a member, the side cover plus half the bar size. Thus, in this case, $a_b = 75 + 25/2 = 87.5$ mm.

From *Table 4.31*, for $a_b/\phi = 87.5/25 = 3.5$, $r_{min} = 7.4\phi$. This value can be reduced slightly by taking into account the stress reduction in the bar between the edge of the support and the start of the bend. If $r = 7\phi$, distance from face of column to start of bend = $300 - 50 - 8 \times 25 = 50$ mm (i.e. 2ϕ).

From *Table 4.30*, for 'good' bond conditions, the required anchorage length is 35ϕ and $r_{min} = (1 - 2\phi/35\phi) \times 7.4\phi = 7\phi$. Thus, $r = 7\phi$ is sufficient.

Chapter 36

Foundations and earth-retaining walls

36.1 GEOTECHNICAL DESIGN

Eurocode 7 provides in outline all the requirements for the design of geotechnical structures. It classifies structures into three categories according to their complexity and associated risk, but concentrates on the design of conventional structures with no exceptional risk. These include spread, raft and pile foundations, retaining structures, bridge piers and abutments, embankments and tunnels. Limit-states of stability, strength and serviceability need to be considered. The requirements of the ULS and SLS may be satisfied by the following methods, alone or in combination: calculations, prescriptive measures, testing, observational procedures. The calculation method adopted in the United Kingdom for the ULS requires the consideration of two combinations of partial factors for actions and soil parameters, as shown here.

Partial safety factors for the ULS

Combination	Safety factor on actions γ_F		Safety factor for soil parameters# γ_M		
	γ_G	γ_Q	$\gamma_{\varphi'}$	$\gamma_{c'}$	γ_{cu}
1	1.35*	1.5*	1.0	1.0	1.0
2	1.0	1.3*	1.25	1.25	1.4

* If the action is favourable to the effect being considered, values of $\gamma_G = 1.0$ and $\gamma_Q = 0$ should be used.

\# Subscripts refer to the following soil parameters:

φ' is angle of shearing resistance (in terms of effective stress), and factor $\gamma_{\varphi'}$ is applied to $\tan\varphi'$
c' is cohesion intercept (in terms of effective stress)
c_u is undrained shear strength

Generally, combination 2 determines the size of the structure with regard to overall stability, bearing capacity, sliding and settlement, and combination 1 governs the structural design of the members. The required size of spread foundations may be determined by ULS calculations, using soil parameter values derived from the geotechnical design report for the project. Alternatively, the size may be determined by limiting the bearing pressure under the characteristic loads to a prescribed value, or a calculated allowable bearing pressure may be used. For the SLS, the settlement of spread foundations should be checked by calculation or may, in the case of firm to stiff clays, be taken as satisfactory if the ratio of design ultimate bearing capacity to service load is ≥ 3. This approach is not valid for soft clays and settlement calculations should always be carried out in such cases.

36.2 PAD BASES

Critical sections for bending are taken at the face of a column or the centre of a steel stanchion. The design moment is taken as that due to all external loads and reactions to one side of the section. Punching resistance should be verified at control perimeters within $2d$ from the column periphery. For bases without shear reinforcement, the design shear resistance is:

$$v_{Rd} = v_{Rd,c}\,(2d/a_v) \geq v_{min}\,(2d/a_v)$$

where a_v ($\leq 2d$) is the distance from the column periphery to the control perimeter being considered. The net applied force $V_{red} = V - \Delta V$, where V is the applied column load and ΔV is the resulting upward force within the control perimeter. For concentric loading, the punching shear stress is $v = V_{red}/ud$, where u is the length of the control perimeter. For eccentric loading, the maximum shear stress is $\beta V_{red}/ud$, where β is a magnification factor determined from equations given in EC 2. At the column perimeter, the punching shear stress should not exceed $v_{Rd,max} = 0.2(1 - f_{ck}/250)f_{ck}$.

Normal shear resistance should also be verified on vertical sections at distance d from the column face extending across the full width of the base, where the design shear resistance is $v_{Rd,c} \geq v_{min}$. Alternatively, it would be reasonable to check at sections within $2d$ from the column face, using the enhanced design shear resistance given for punching shear. In this case, for concentric loading, the critical position for normal shear and punching shear occurs at $a_v = a/2 \leq 2d$, where a is the distance from the column face to the edge of the base. For eccentric loading, checks can be made at $a_v = 0.5d$, d, and so on to find the critical position.

If the tension reinforcement is included in the determination of $v_{Rd,c}$, the bars should extend for at least $(d + l_{bd})$ beyond the section considered (see also EC 2, section 9.8.2.2). If the tension reinforcement is ignored in the shear calculations, straight bars will usually suffice.

Example 1. A base is required to support a 600 mm square column subjected to vertical load only, for which the values are 4250 kN (service) and 6000 kN (ultimate). The allowable ground bearing value is 300 kN/m^2 (kPa).

$f_{ck} = 32$ MPa, $f_{yk} = 500$ MPa, nominal cover = 50 mm

Allowing 10 kN/m^2 for ground floor loading, and extra over soil displaced by concrete, the net allowable bearing pressure can be taken as 290 kN/m^2. Area of base required

$A_{base} = 4250/290 = 14.7$ m^2. Provide base 4.0 m square.

Distance from face of column to edge of base, $a = 1700$ mm.

Taking depth of base $\geq 0.5a$, say $h = 900$ mm.

Allowing for 25 mm main bars, average effective depth,

$$d = 900 - (50 + 25) = 825 \text{ mm}$$

Bearing pressure under base due to ultimate load on column,

$$p_u = 6000/4^2 = 375 \text{ kN/m}^2$$

Bending moment on base at face of column,

$$M = p_u l a^2/2 = 375 \times 4 \times 1.7^2/2 = 2168 \text{ kNm}$$

$$M/bd^2 f_{ck} = 2168 \times 10^6/(4000 \times 825^2 \times 32) = 0.025$$

From *Table 4.8*, $A_s f_{yk}/bdf_{ck} = 0.0285$, and

$$A_s = 0.0285 \times 4000 \times 825 \times 32/500 = 6020 \text{ mm}^2$$

From *Table 2.20*,

13H25-300 gives 6380 mm^2, and

$100A_s/bh = 100 \times 6380/(4000 \times 900) = 0.18$ ($>$ 0.13 min)

For values of $G_k = 2500$ kN, $Q_k = 1750$ kN and $\psi_2 = 0.3$, the quasi-permanent load is

$$G_k + \psi_2 Q_k = 2500 + 0.3 \times 1750 = 3025 \text{ kN}$$

Hence, the stress in the reinforcement under quasi-permanent loading is given approximately by:

$$\sigma_s = (3025/6000)(0.87 f_{yk})(A_{s\,req}/A_{s\,prov})$$
$$= (3025/6000)(0.87 \times 500)(6020/6380) = 207 \text{ MPa}$$

From *Table 4.24*, for $w_k = 0.3$ mm, the crack width criterion is met if $\phi^*_s \leq 23$ mm, or the bar spacing ≤ 240 mm.

The adjusted maximum bar size, with $h_{cr} = 0.5h$, is given by:

$$\phi_s = \phi^*_s (f_{ct,eff}/2.9)[k_c h_{cr}/2(h - d)]$$
$$= 23 \times (3.0/2.9) \times [0.4 \times 450/(2 \times 75)] = 28 \text{ mm}$$

In this case, H25-300 satisfies the bar size criterion.

For members without shear reinforcement, distance from face of column to critical position for punching shear (or normal shear) is given by $a_v = 0.5a = 850$ mm, where perimeter

$$u = 4c + 2\pi a_v = 4 \times 600 + 2\pi \times 850 = 7740 \text{ mm}$$

Area of base inside critical perimeter is

$$A_u = 4a_v c + \pi a_v^2 = 4 \times 0.85 \times 0.6 + \pi \times 0.85^2 = 4.31 \text{ m}^2$$

Hence, the net applied force and resulting shear stress are as follows:

$$V_{red} = V - \Delta V = 375 \times (4^2 - 4.31) = 4384 \text{ kN}$$
$$v = V/ud = 4384 \times 10^3/(7740 \times 825) = 0.69 \text{ MPa}$$

From *Table 4.17*, for $f_{ck} = 32$ MPa and $100A_{sl}/bd \leq 0.20$, the design concrete shear stress $v_{Rd,c}$ is determined by v_{min}.

$$v_{min} = 0.035k^{3/2}f_{ck}^{1/2} \quad \text{where } k = 1 + (200/d)^{1/2} \leq 2.0$$
$$k = 1 + (200/825)^{1/2} = 1.49$$
$$v_{min} = 0.035 \times 1.49^{3/2} \times 32^{1/2} = 0.36 \text{ MPa}$$

Hence, design shear resistance,

$$v_{Rd} = v_{min}(2d/a_v) = 0.36 \times (2 \times 825/850) = 0.70 \text{ MPa } (>v)$$

At the column perimeter, ignoring the small reduction ΔV

$$v = V/ud = 6000 \times 10^3/(4 \times 600 \times 825) = 3.03 \text{ MPa}$$

$$v_{Rd,\,max} = 0.2\,(1 - f_{ck}/250)f_{ck}$$
$$= 0.2 \times (1 - 32/250) \times 32 = 5.58 \text{ MPa } (>v)$$

36.3 PILE-CAPS

A pile-cap may be designed by either bending theory or truss analogy (i.e. strut-and-tie). In the latter case, the truss is of a triangulated form with nodes at the centre of the loaded area, and at the intersections of the centrelines of the piles with the tension reinforcement, as shown for compact groups of two to five piles in *Table 3.61*. Expressions for the tensile forces are given, taking into account the dimensions of the column, and also simplified expressions when the column dimensions are ignored. Bars to resist the tensile forces are to be located within zones extending not more than 1.5 times the pile diameter either side of the centre of the pile. The bars are to be provided with a tension anchorage beyond the centres of the piles. The compression caused by the pile reaction may be assumed to spread at 45° angles from the edge of the pile, and taken into account when calculating the anchorage length. The bearing stress on the concrete inside the bend in the bars should be checked (see *Table 4.31*).

Example 2. A pile-cap is required for a group of 4 × 450 mm diameter piles, arranged at 1350 mm centres on a square grid. The pile-cap supports a 450 mm square column subjected to an ultimate design load of 4000 kN.

$f_{ck} = 32$ MPa, $f_{yk} = 500$ MPa

Allowing for an overhang of 150 mm beyond the face of the pile, size of pile-cap = 1350 + 450 + 300 = 2100 mm square.

Take depth of pile-cap as $(2h_p + 100) = 1000$ mm.

Assuming tension reinforcement to be 100 mm up from base of pile-cap, $d = 1000 - 100 = 900$ mm.

Using truss analogy with the apex of the truss at the centre of the column, the tensile force between adjacent piles is

$$F_t = \frac{Nl}{8d} = \frac{4000 \times 1350}{8 \times 900} = 750 \text{ kN in each zone}$$
$$A_s = F_t/0.87f_{yk} = 750 \times 10^3/(0.87 \times 500) = 1724 \text{ mm}^2$$

For a pile spacing $\leq$ three times pile diameter, the bars may be spread uniformly across the cap, and a total for two ties of 8H25-250 (giving 3926 mm^2) in each direction can be used.

$100A_s/bd = 100 \times 3926/(2100 \times 900) = 0.20$ ($>$ 0.13 min)

The bars should be provided with a tension anchorage beyond the centre of the piles (see *Table 4.30*). For 25 mm diameter bars spaced at 250 mm centres, half the gap between bars gives $c_b = 112.5$ mm (i.e. 4.5ϕ). So, for bent bars and 'good' bond conditions, $l_{bd} = \alpha_1 l_{b,rqd} = 0.7 \times 35\phi = 25\phi$.

Taking a_b as either half the centre-to-centre distance between bars, or (side cover plus half bar size), whichever is less,

$$a_b = 250/2 \leq 175 = 125 \text{ mm}, \ a_b/\phi = 125/25 = 5$$

From *Table 4.31*, minimum radius of bend $r_{min} = 7\phi$ say.

Consider the critical section for shear to be located at 20% of the pile diameter inside the pile-cap. Distance of this section from the column face,

$$\begin{aligned} a_v &= 0.5(l - c) - 0.3h_p \\ &= 0.5 \times (1350 - 450) - 0.3 \times 450 = 315 \text{ mm} \end{aligned}$$

Length of corresponding square perimeter for punching shear

$$u = 4\ (l - 0.6h_p) = 4 \times (1350 - 0.6 \times 450) = 4320 \text{ mm}$$

Since length of perimeter of pile-cap = 4 × 2100 = 8400 mm is less than $2u$, normal shear extending across the full width of the pile-cap is more critical than punching shear.

The contribution of the column load to the shear force may be reduced by applying a factor $\beta = a_v/2d$, where $0.5d \leq a_v \leq 2d$. Since $a_v/d = 315/900 = 0.35$ (<0.5), $\beta = 0.25$.

$$v = V/bd = 0.25 \times 2000 \times 10^3/(2100 \times 900) = 0.27 \text{ MPa}$$

From *Table 4.17*, for $f_{ck} = 32$ MPa and $100A_{sl}/bd \leq 0.20$, the design concrete shear stress $v_{Rd,c}$ is determined by v_{min}.

$$v_{min} = 0.035k^{3/2}f_{ck}^{1/2} \quad \text{where } k = 1 + (200/d)^{1/2} \leq 2.0$$

$$k = 1 + (200/900)^{1/2} = 1.47$$

$$v_{min} = 0.035 \times 1.47^{3/2} \times 32^{1/2} = 0.35 \text{ MPa } (>v)$$

Shear stress calculated at perimeter of column,

$$v = V/ud = 4000 \times 10^3/(4 \times 450 \times 900) = 2.47 \text{ MPa}$$

$$\begin{aligned} v_{Rd,\max} &= 0.2\ (1 - f_{ck}/250)\, f_{ck} \\ &= 0.2 \times (1 - 32/250) \times 32 = 5.58 \text{ MPa } (>v) \end{aligned}$$

36.4 RETAINING WALLS ON SPREAD BASES

General notes on walls on spread bases are given in section 7.3.2. For design purposes, the characteristic soil parameter, which is defined as a cautious estimate of the value affecting the occurrence of the limit-state, is divided by a partial safety factor (see section 36.1). Design values of the soil strength at the ULS (combination 2) are given by:

$$\tan \varphi'_d = (\tan \varphi')/1.25 \qquad \text{and} \qquad c'_d = c'/1.25$$

where c' and φ' are characteristic values of cohesion intercept and angle of shearing resistance (in terms of effective stress).

Design values for shear resistance at the interface of the base and sub-soil, of friction (for drained conditions) and adhesion (for undrained conditions) are given by:

$\tan \delta_d = \tan \varphi'_d$ for cast *in-situ* concrete

$\tan \delta_d = \tan (2/3)\ \varphi'_d$ for precast concrete

$c_{ud} = c_u/1.4$ where c_u is undrained shear strength

Walls should be checked for ULS of overall stability, bearing resistance and sliding. The resistance of the ground should be determined for both long-term (i.e. drained) and short-term (i.e. undrained) conditions where appropriate.

For eccentric loading, the ground bearing is assumed to be uniformly distributed and coincident with the line of action of the resultant applied load. The traditional practice of using characteristic actions and allowable bearing pressures to limit ground deformation, and check bearing resistance, may also be adopted by mutual agreement. This approach assumes a linear variation of bearing pressure for eccentric loading, and it is still necessary to consider the ULS for the structural design and to check sliding.

The partial safety factors for the SLS are given as unity, but it is often prudent to use the ULS for the active force (as in BS 8002). In this case, suitable dimensions for the wall base can be estimated with the aid of the chart given in *Table 2.86*. Here, the value p_{max} applies for a linear pressure variation, and if the ground pressure is uniform and centred on the centre of gravity of the applied load, the contact length is $\delta(\alpha l)$, where δ depends on whether the solution is (a) above, or (b) below, the curve for 'zero pressure at heel' shown on the chart, as follows:

(a) $\delta = 4(1 - \beta)/3\xi \leq 2/3$ and $p = 0.75\, p_{max}$

(b) $\delta = 4/3 - \xi/3(1 - \beta) > 2/3$ and $p = (1 - \beta)\gamma l/\delta$

For sliding, the chart applies directly to non-cohesive soils. Thus, for bases founded on clay, the long-term condition can be investigated by using φ', with $c' = 0$. For the short-term condition, the ratio β does not enter into the calculations for sliding and α is given by $\alpha = K_A\ \gamma l/2\delta c_d$. When α has been determined from this equation, the curve for $\alpha/\sqrt{K_A}$ on the chart can be used to check the values of β and ξ that were obtained for the long-term condition.

Example 3. A cantilever retaining wall on a spread base is required to support level ground and a footpath adjacent to a road. The existing ground may be excavated as necessary to construct the wall, and the excavated ground behind the wall is to be reinstated by backfilling with a granular material. A graded drainage material will be provided behind the wall, with an adequate drainage system at the bottom.

Height of fill to be retained: 4.0 m above top of base

Surcharge: 5 kN/m^2

Properties of retained soil (well graded sand and gravel):
unit weight of soil $\gamma = 20$ kN/m^3
$\varphi' = 35^o$, $\varphi'_d = \tan^{-1}$ [(tan 35^o)/1.25] = 29^o
$K_A = (1 - \sin \varphi'_d)/(1 + \sin \varphi'_d) = 0.35$

Properties of sub-base soil (medium sand):
allowable bearing value $f_{max} = 200$ kN/m^2 (kPa)
$\varphi' = 35^o$, $\varphi'_d = 29^o$ (as fill)
$\tan \delta_d = \tan\varphi'_d = 0.55$

Take thickness of both wall (at bottom of stem) and base to be equal to (height of fill)/10 = 4000/10 = 400 mm

Height of wall to underside of base, $l = 4.0 + 0.4 = 4.4$ m.

Allowing for surcharge, equivalent height of wall

$$l_e = l + q/\gamma = 4.4 + 5/20 = 4.65 \text{ m}$$

$$\xi = p_{\max}/\gamma l_e = 200/(20 \times 4.65) = 2.15$$

$$\psi = \tan\delta_d/\sqrt{}K_A = 0.55/\sqrt{}0.35 = 0.93$$

From *Table 2.86*, $\alpha /\sqrt{}K_A = 0.745, \beta = 0.38$. Hence,

Width of base $= \alpha\, l_e = (0.745\sqrt{}0.35) \times 4.65 = 2.1$ m say

Toe projection $= \beta(\alpha l_e) = 0.38 \times 2.1 = 0.8$ m

Since the solution lies above the curve for 'zero pressure at heel' on the chart, for uniform bearing centred on the centre of gravity of the applied load,

$\delta = 4(1 - \beta)/3\xi = 4 \times (1 - 0.38)/(3 \times 2.15) = 0.385$ and
$p = 0.75\, p_{\max} = 0.75 \times 200 = 150$ kN/m^2 (kPa)

Example 4. The sub-base for the wall described in example 3 is a clay soil with properties as given below. All other values are as specified in example 3.

Properties of sub-base soil (firm clay):

allowable bearing value $p_{\max} = 100$ kN/m^2 (kPa)
$c_u = 50$ kN/m^2, $c_{ud} = c_u/1.4 = 50/1.4 = 35$ kN/m^2
$\varphi' = 25^o$, $\varphi'_d = \tan^{-1}$ [(tan 25^o)/1.25] = 20.5^o
$\tan\delta_d = \tan\varphi'_d = 0.37$

For the long-term condition:

$$\xi = p_{\max}/\gamma l_e = 100/(20 \times 4.65) = 1.08$$

$$\psi = \tan\delta_d/\sqrt{}K_A = 0.37/\sqrt{}0.35 = 0.625$$

From *Table 2.86*, $\alpha /\sqrt{}K_A = 1.07, \beta = 0.25$. Hence,

Width of base $= \alpha\, l_e = (1.07\sqrt{}0.35) \times 4.65 = 3.0$ m say

Toe projection $= \beta\, (\alpha l_e) = 0.25 \times 3.0 = 0.8$ m say

Since the solution lies below the curve for 'zero pressure at heel' on the chart, for uniform bearing centred on the centre of gravity of the applied load,

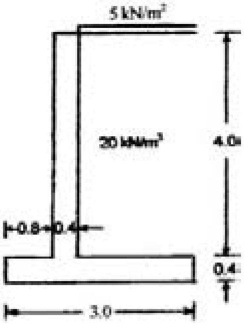
$\delta = 4/3 - \xi/3(1 - \beta) = 4/3 - 1.08/3(1 - 0.25) = 0.85$
$p = (1 - \beta)\gamma l/\delta = (1 - 0.25) \times 20 \times 4.65/0.85 = 82$ kN/m^2

For the short-term condition:

$\alpha = K_A\, \gamma l/2\delta c_{ud} = 0.35 \times 20 \times 4.65/(2 \times 0.85 \times 35) = 0.55$
$\alpha/\sqrt{}K_A = 0.55/\sqrt{}0.35 = 0.93$

Since this value is less than that obtained for the long-term condition, the base dimensions are satisfactory.

Example 5. The wall obtained in example 4, a cross section through which is shown below, is to be designed according to EC 7.



The vertical loads and bending moments about the front edge of the base are:

		Load (kN)	Moment (kNm)	
Surcharge	5 × 1.8	= 9.0	× 2.1	= 18.9
Backfill	20 × 1.8 × 4.0	= 144.0	× 2.1	= 302.4
Wall stem	24 × 0.4 × 4.0	= 38.4	× 1.0	= 38.4
Wall base	24 × 0.4 × 3.0	= 28.8	× 1.5	= 43.2
	Totals	F_v = 220.2		M_v = 402.9

The horizontal loads and bending moments about the bottom of the base are:

		Load (kN)	Moment (kNm)	
Surcharge	0.35 × 5 × 4.4	= 7.7	× 4.4/2	= 17.0
Backfill	0.35 × 20 × 4.4^2/2	= 67.8	× 4.4/3	= 99.4
	Totals	F_h = 75.5		M_h = 116.4

Resultant moment $M_{net} = 402.9 - 116.4 = 286.5$ kNm

Distance from front edge of base to resultant vertical force

$$a = M_{net}/F_v = 286.5/220.2 = 1.30 \text{ m}$$

Eccentricity of vertical force relative to centreline of base

$$e = 3.0/2 - 1.30 = 0.20 \text{ m } (< 3.0/6 = 0.5 \text{ m})$$

Maximum pressure at front of base

$$p_{\max} = (220.2/3.0)(1 + 6 \times 0.20/3.0) = 103 \text{ kN/m}^2$$

Minimum pressure at back of base

$$p_{\min} = (220.2/3.0)(1 - 6 \times 0.20/3.0) = 44 \text{ kN/m}^2$$

For the ultimate bearing condition, a uniform distribution is considered of length $l_b = 2a = 2 \times 1.3 = 2.6$ m giving

$$p_u = F_v/l_b = 220.2/2.6 = 85 \text{ kN/m}^2$$

The ultimate bearing resistance is given by the equation:

$q_u = (2 + \pi)\, c_{ud}\, i_c$ where $i_c = 0.5[1 + \sqrt{1 - F_h/(c_{ud} l_b)}]$
$i_c = 0.5[1 + \sqrt{1 - 75.5/(35 \times 2.6)}] = 0.70$
$q_u = (2 + \pi) - 35 - 0.70 = 126$ kN/m^2 ($> p_u = 85$)

Resistance to sliding (long-term)

$$= F_v \tan\delta_b = 220.2 \times 0.37 = 81.5 \text{ kN } (> F_h = 75.5)$$

Resistance to sliding (short-term)

$$= c_{ud}\, l_b = 35 \times 2.6 = 91 \text{ kN } (> F_h = 75.5)$$

Example 6. The structural design of the wall in example 5 is to be in accordance with the requirements of EC 2.

$$f_{ck} = 32 \text{ MPa}, f_{yk} = 500 \text{ MPa, nominal cover} = 40 \text{ mm}$$

Allowing for H16 bars with 40 mm cover,

$$d = 400 - (40 + 8) = 352 \text{ mm}$$

For the ULS (combination 1), $\gamma_F = 1.35$ for all permanent actions, and $\gamma_M = 1.0$. Thus,

$$\varphi'_d = 35^o, \text{ and } K_A = (1 - \sin\varphi'_d)/(1 + \sin\varphi'_d) = 0.27$$

The ultimate bending moment at the bottom of the wall stem:

$$M = 1.35 \times 0.27 \times (5 \times 4^2/2 + 20 \times 4^3/6) = 92.4 \text{ kNm/m}$$

(Note that for combination 2, $\gamma_F K_A = 1.0 \times 0.35$ which is less than $\gamma_F K_A = 1.35 \times 0.27 = 0.365$ for combination 1)

$$M/bd^2 f_{ck} = 92.4 \times 10^6/(1000 \times 352^2 \times 32) = 0.023$$

From *Table 4.8*, $A_s f_{yk}/bd\,f_{ck} = 0.026$, and

$$A_s = 0.026 \times 1000 \times 352 \times 32/500 = 586 \text{ mm}^2/\text{m}$$

From *Table 2.20*, H12-150 gives 754 mm^2/m

The ultimate shear force at the bottom of the wall stem:

$$V = 1.35 \times 0.27 \times (5 \times 4 + 20 \times 4^2/2) = 65.6 \text{ kN/m}$$

From *Table 4.17*, $v_{min} = 0.035k^{3/2} f_{ck}{}^{1/2}$ where

$$k = 1 + (200/d)^{1/2} = 1 + (200/352)^{1/2} = 1.75\ (\leq 2.0)$$
$$v_{min} = 0.035 \times 1.75^{3/2} \times 32^{1/2} = 0.46 \text{ MPa}$$
$$V/bd = 65.6 \times 10^3/(1000 \times 352) = 0.19 \text{ MPa}\ (< v_{min})$$

Since the loads are permanent, the stress in the reinforcement under service loading is given approximately by:

$$\begin{aligned}\sigma_s &= (0.87 f_{yk}/\gamma_F)(A_{s\,req}/A_{s\,prov})\\ &= (0.87 \times 500/1.35)(586/754) = 250 \text{ MPa}\end{aligned}$$

From *Table 4.24*, for $w_k = 0.3$ mm, the crack width criterion is met if $\phi^*_s \leq 15$ mm, or the bar spacing ≤ 185 mm.

The adjusted maximum bar size, with $h_{cr} = 0.5h$, is given by:

$$\begin{aligned}\phi_s &= \phi^*_s\,(f_{ct,eff}/2.9)[k_c\,h_{cr}/2(h-d)]\\ &= 15 \times (3.0/2.9) \times [0.4 \times 200/(2 \times 48)] = 13 \text{ mm}\end{aligned}$$

In this case, H12-150 meets both requirements.

For the wall base, the loads and bending moments calculated for combination 2 (see example 4) can be modified to suit the revised parameters for combination 1, as follows:

$$F_v = 1.35 \times 220.2 = 297.3 \text{ kN}$$
$$M_v = 1.35 \times 402.9 = 543.9 \text{ kNm}$$
$$M_h = (0.365/0.35) \times 116.4 = 121.4 \text{ kNm}$$

Resultant moment $M_{net} = 543.9 - 121.4 = 422.5$ kNm

Distance from front edge of base to resultant vertical force

$$a = M_{net}/F_v = 422.5/297.3 = 1.42 \text{ m}$$

Bearing contact length $l_b = 2a = 2 \times 1.42 = 2.84$ m

$$p_u = F_v/l_b = 297.3/2.84 = 104.7 \text{ kN/m}^2$$

Note that values of both p_u and q_u are greater for combination 1 than for combination 2, but 2 is still critical for bearing.

Bending moment on base at inside face of wall

$$\begin{aligned}M &= 1.35 \times (5 + 20 \times 4 + 24 \times 0.4) \times 1.8^2/2\\ &\quad - 104.7 \times (2.84 - 1.2)^2/2 = 66.1 \text{ kNm}\end{aligned}$$

Shear force on base at inside face of wall

$$\begin{aligned}V &= 1.35 \times (5 + 20 \times 4 + 24 \times 0.4) \times 1.8\\ &\quad - 104.7 \times (2.84 - 1.2) = 58.2 \text{ kNm}\end{aligned}$$

The bending moment and shear force are both less than the values at the bottom of the wall stem. Thus, H12-150 can be used to fit in with the vertical bars in the wall.

Appendix

Mathematical formulae and data

MATHEMATICAL AND TRIGONOMETRICAL FUNCTIONS

Trigonometrical formulae

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta - \tan^2\theta = 1$$
$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

$$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$
$$\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$$
$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$
$$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$

$$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$$

$$\tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}$$

$$\sin\theta + \sin\phi = 2\sin\tfrac{1}{2}(\theta + \phi)\cos\tfrac{1}{2}(\theta - \phi)$$
$$\sin\theta - \sin\phi = 2\cos\tfrac{1}{2}(\theta + \phi)\sin\tfrac{1}{2}(\theta - \phi)$$
$$\cos\theta + \cos\phi = 2\cos\tfrac{1}{2}(\theta + \phi)\cos\tfrac{1}{2}(\theta - \phi)$$
$$\cos\theta - \cos\phi = -2\sin\tfrac{1}{2}(\theta + \phi)\sin\tfrac{1}{2}(\theta - \phi)$$
$$\sin\theta\sin\phi = \tfrac{1}{2}[\cos(\theta - \phi) - \cos(\theta + \phi)]$$
$$\sin\theta\cos\phi = \tfrac{1}{2}[\sin(\theta + \phi) + \sin(\theta - \phi)]$$
$$\cos\theta\cos\phi = \tfrac{1}{2}[\cos(\theta + \phi) + \cos(\theta - \phi)]$$

OTHER DATA

Factors

$\pi = 355/113$ (approx.) $= 22/7$ (approx.) $= 3.141\,592\,654$ (approx.)

One radian $= 180^\circ/\pi = 57.3^\circ$ (approx.) $= 57.295\,779\,5$ (approx.)

Length of arc subtended by an angle of one radian = radius of arc

One degree Fahrenheit = 5/9 degree centigrade or Celsius

Temperature of $t\,^\circ$F $= (t - 32)/1.8\,^\circ$C

Temperature of $t\,^\circ$C $= (1.8t + 32)\,^\circ$F

Base of Napierian logarithms, $e = 193/71$ (approx.) $= 2721/1001$ (approx.) $= 2.718\,281\,828$ (approx.)

To convert common into Napierian logarithms, multiply by 76/33 (approx.) = 3919/1702 (approx.) = 2.302 585 093 (approx.).

Nominal value of $g = 9.806\,65\ \text{kg/s}^2 = 32.174\ \text{ft/s}^2$

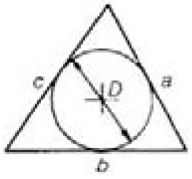
Inscribed circle

Diameter of inscribed circle of a triangle:

$$D = 2b\sqrt{\left[a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2\right]}\bigg/(a + b + c)$$

For isosceles triangle, $a = c$:

$$D = \frac{b\sqrt{(4a^2 - b^2)}}{2a + b}$$



Solution of triangles

Applicable to any triangle ABC in which $AB = c$, $BC = a$, $AC = b$:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\text{area} = \frac{bc\sin A}{2} = \frac{ac\sin B}{2} = \frac{ab\sin C}{2}$$

$$= \sqrt{[s(s - a)(s - b)(s - c)]} \quad \text{where} \quad s = (a + b + c)/2.$$

$$\sin\frac{A}{2} = \sqrt{\left[\frac{(s - b)(s - c)}{bc}\right]}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

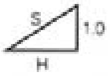
Roots of quadratics

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} = -\frac{b}{2a}\left[1 \mp \sqrt{\left(1 - \frac{4ac}{b^2}\right)}\right]$$

Applications

1. Roof slopes, $s = \sqrt{(1 + H^2)}$:



   (a) Limiting slope for inclined roof loading = 20°:

$$H = \cot 20° = 2.7475$$

   Therefore limiting slope = 1:2.75.

   (b) Limiting slope for inclined roofs = 10°:

$$H = \cot 10° = 5.6713$$

   Therefore limiting slope = 1:5.67.

2. Earth pressures:

$$k_2 = \frac{1 - \sin\theta}{1 + \sin\theta} = \tan^2\left(45° - \frac{\theta}{2}\right)$$

$$\frac{1}{k_2} = \frac{1 + \sin\theta}{1 - \sin\theta} = \tan^2\left(45° + \frac{\theta}{2}\right)$$

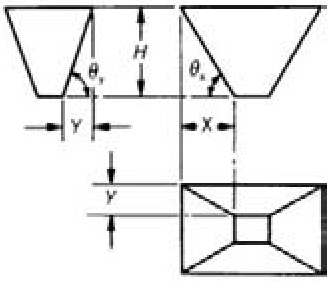
3. Hopper bottom slopes:
   Specified minimum slope in valley = ϕ.

$$Y/X = R$$

$$X = H \cot\phi/\sqrt{(1 + R^2)}$$

$$\tan\theta_x = \tan\phi\sqrt{(1 + R^2)}$$

$$\tan\theta_y = \tan\phi\sqrt{(1 + R^2)}/R$$



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