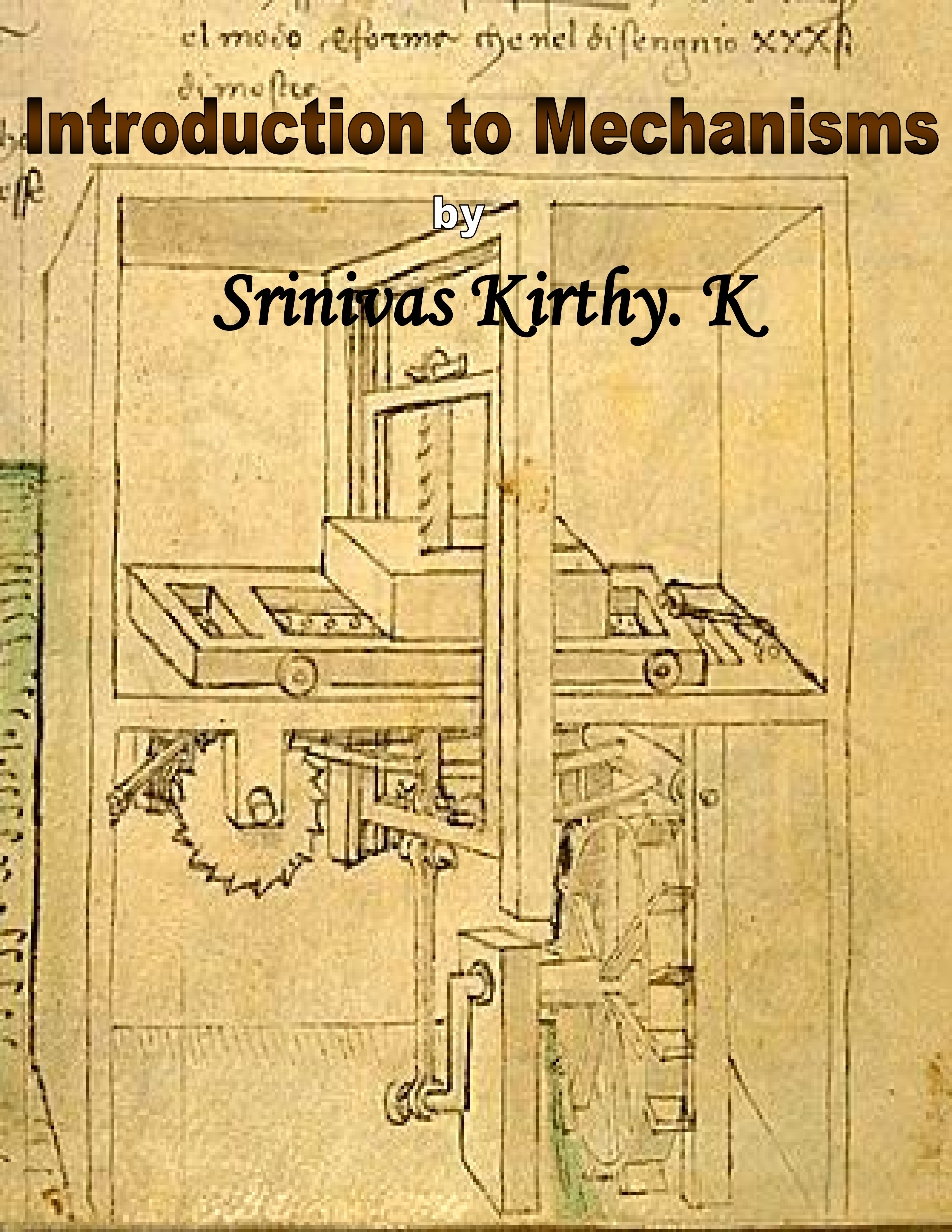
Introduction to Mechanisms

by

Srinivas Kirthy. K



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1 Physical Principles

This chapter introduces the basic physical principles behind mechanisms as well as basic concepts and principles required for this course.

1.1 Force and Torque

1.1.1 Force

Force: *an agent or influence that, if applied to a free body results chiefly in an acceleration of the body and sometimes in elastic deformation and other effects.*

Every day we deal with ***forces*** of one kind or another. A pressure is a force. The earth exerts a force of attraction for all bodies or objects on its surface. To study the forces acting on objects, we must know how the forces are applied, the direction of the forces and their value. Graphically, forces are often represented by a vector whose end represents the point of action.

A *mechanism* is what is responsible for any action or reaction. *Machines* are based on the idea of transmitting forces through a series of predetermined motions. These related concepts are the basis of dynamic movement.

1.1.2 Torque

Torque: *Something that produces or tends to produce rotation and whose effectiveness is measured by the product of the force and the perpendicular distance from the line of action of the force to the axis of rotation.*

Consider the lever shown in Figure 1-1. The lever is a bar that is free to turn about the fixed point, A, called the *fulcrum*; a weight acts on the one side of the lever, and a balancing force acts on the other side of the lever.

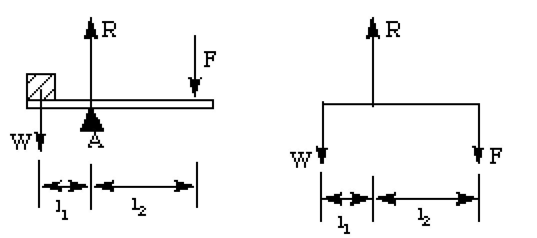

Figure 1-1 A lever with balanced forces

To analyze levers, we need to find the ***torques*** of the forces acting on the lever. To get the *torque* of force W about point A, multiply W by l_1, its distance from A. Similarly F x l_2 is the *torque* of F about fulcrum A.

1.2 Motion

***Motion**: a change of position or orientation.*

1.2.1 Motion Along a Straight Path

We begin our study of motion with the simplest case, motion in a straight line.

1. **Position and displacement along a line**
   The first step in the study of motion is to describe the position of a moving object. Consider a car on an east-west stretch of straight highway. We can describe the ***displacement*** of the car by saying "the car is 5 kilometers west of the center town". In this description, we specified two factors, the original point of measure and the direction of the ***displacement***.
2. **Velocity**
   We can define the ***velocity*** of an object moving steadily as its displacement per unit time:

$$v = \frac{x_2 - x_1}{t_2 - t_1} = \frac{d}{t}$$

   (1-1)

   where t = t2 - t1 is the time interval during which the displacement occurred. When velocity varies, we can let the time interval become infinitesimally small, thus

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

   (1-2)

3. **Acceleration**
   Acceleration is the variation of the velocity in a unit time period. If the velocity changes in a constant rate, then we can describe the acceleration by

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

   (1-3)

More generally, acceleration is

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

(1-4)

1.2.2 Linear Motion in Space

The picture becomes more complicated when the motion is not merely along a straight line, but rather extends into a plane. Here we can describe the motion with a vector which includes the magnitude and the direction of movement.

1. ***Position vector*** and ***displacement vector***
   The directed segment which describes the position of an object relative to an origin is the *position vector*, as $\mathbf{d}_1$ and $\mathbf{d}_2$ in Figure 1-2

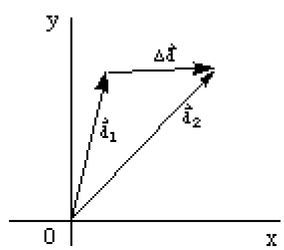

Figure 1-2 Position vector and displacement vector

If we wish to describe a motion from position $\mathbf{d}_1$ to position $\mathbf{d}_2$, for example, we can use vector $\mathbf{d}_1$, the vector starts at the point described by $\mathbf{d}_1$ and goes to the point described by $\mathbf{d}_2$, which is called the *displacement vector*.

$$\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$$

(1-5)

2. ***Velocity vector***
   For a displacement $\Delta\mathbf{d}$ occurring in a time interval Δt, the average velocity during the interval is

$$\vec{v}_{ave} = \frac{\Delta\vec{d}}{\Delta t}$$

(1-6)

Clearly $\mathbf{V}_{ave}$ has the direction of $\Delta\mathbf{d}$.

In the limit as delta t approaches zero, the instantaneous velocity is

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t}$$

(1-7)

The direction of **V** is the direction of $\Delta\mathbf{d}$ for a very small displacement; it is therefore along, or tangent to, the path.

3. ***Acceleration vector***
   The instantaneous acceleration is the limit of the ratio $\Delta\mathbf{V}/\Delta t$ as Δt becomes very small:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

(1-8)

1.2.3 Motion of a Rigid Body in a Plane

The previous sections discuss the motion of particles. For a rigid body in a plane, its motion is often more complex than a particle because it is comprised of a linear motion and a rotary motion. Generally, this kind of motion can be decomposed into two motions (Figure 1-3), they are:

1. The linear motion of the center of the mass of the rigid body. In this part of the motion, the motion is the same as the motion of a particle on a plane.
2. The rotary motion of the rigid body relative to its *center of mass*.

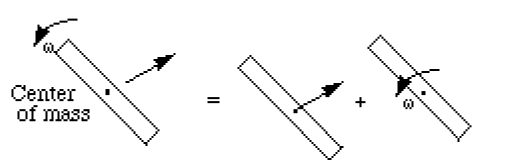

Figure 1-3 Motion of a rigid body in a plane

1.3 Newton's Law of Motion

1.3.1 Newton's First Law

When no force is exerted on a body, it stays at rest or moves in a straight line with constant speed. This principle of inertia is also known as ***Newton's first law.*** It is from this law that Newton was able to build up our present understanding of dynamics.

1.3.2 Newton's Second Law

From our daily life, we can can observe that:

1. When a force **F** is applied on an object, Δ**V**, the change of the velocity of the object, increases with the length of time delta t increases;
2. The greater the force **F**, the greater Δ**V**; and
3. The larger the body (object) is, the less easily accelerated by forces.

It is convenient to write the proportionality between **F**Δt and Δ**V** in the form:

$$F\Delta t = m\Delta V$$

(1-9)

The proportionality constant *m* varies with the object. This constant *m* is refered to as the ***inertial mass*** of the body. The relationship above embodies ***Newton's law of motion*** (***Newton's second law***). As

$$\frac{\Delta V}{\Delta t} = a$$

(1-10)

in which *a* is the acceleration of the object. We have

$$F = ma$$

(1-11)

If m = 1 kg and a = 1m/sec^2, than F = 1 *newton*.

Forces and accelerations are vectors, and Newton's law can be written in vector form.

$$\vec{F} = m\vec{a}$$

(1-12)

1.4 Momentum and Conservation of Momentum

1.4.1 Impulse

Try to make a baseball and a cannon ball roll at the same speed. As you can guess, it is harder to get the cannon ball going. If you apply a constant force **F** for a time Δt, the change in velocity is given by Equation 1-9. So, to get the same Δ**v**, the product **F**Δt must be greater the greater the mass m you are trying to accelerate.

To throw a cannon ball from rest and give it the same final velocity as a baseball (also starting from rest), we must push either harder or longer. What counts is the product **F**Δt. This product **F**Δt is the natural measure of how hard and how long we push to change a motion. It is called the ***impulse*** of the force.

1.4.2 Momentum

Suppose we apply the same *impulse* to a baseball and a cannon ball, both initially at rest. Since the initial value of the quantity ***mv*** is zero in each case, and since equal impulses are applied, the final values m**v** will be equal for the baseball and the cannon ball. Yet, because the mass of the cannon ball is much greater than the mass of the baseball, the velocity of the cannon ball will be much less than the velocity of the baseball. The product ***mv***, then, is quite a different measure of the motion than simply **v** alone. We call it the ***momentum* p** of the body, and measure it in kilogram-meters per second.

$$\vec{p} = m\vec{v}$$

(1-13)

Velocity and *momentum* are quite different concepts: velocity is a *kinematical* quantity, whereas momentum is a *dynamic* one, connected with the causes of changes in the motion of masses.

Because of its connection with the *impulse* which occurs naturally in Newton's law (Equation 1-9), we expect *momentum* to fit naturally into Newtonian dynamics. Newton did express his law of motion in terms of the *momentum*, which he called the *quantity of motion*. We can express Newton's law in terms of the change in *momentum* instead of change in *velocity*:

$$\vec{F}\Delta t = m\Delta\vec{v} = m(\vec{v}' - \vec{v})$$

(1-14)

where **v** and **v**' are the velocities before and after the impulse. The right-hand side of the last equation can be written as

$$m(\vec{v}' - \vec{v}) = m\vec{v}' - m\vec{v} = \vec{p}' - \vec{p} = \Delta\vec{p}$$

(1-15)

the change in the *momentum*. Therefore

$$\vec{F}\Delta t = \Delta\vec{p}$$

(1-16)

or, in other words, the *impulse* equals the change in *the momentum*.

1.4.3 Conservation of Momentum

In Figure 1-4 a moving billiard ball collides with a billiard ball at rest. The incident ball stops and the ball it hits goes off with the same velocity with which the incident ball came in. The two billiard balls have the same mass. Therefore, the *momentum* of the second ball after the collision is the same as that of the incident ball before collision. The incident ball has lost all its *momentum*, and the ball it struck has gained exactly the *momentum* which the incident ball lost.

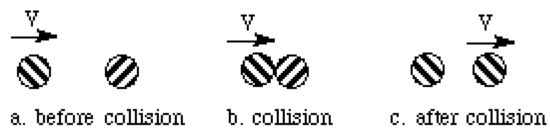

Figure 1-4 Collision of billiard balls

This phenomenon is consistent with *the law of conservation of momentum* which says that the total *momentum* is constant when two bodies interact.

1.5 Work, Power and Energy

1.5.1 Work

Work is a force applied over a distance. If you drag an object along the floor you do *work* in overcoming the friction between the object and the floor. In lifting an object you do *work* against gravity which tends to pull the object toward the earth. Steam in a locomotive cylinder does *work* when it expands and moves the piston against the resisting forces. ***Work*** is the product of the resistance overcome and the distance through which it is overcome.

1.5.2 Power

Power *is the rate at which* *work* *is done.*

In the British system, power is expressed in foot-pounds per second. For larger measurements, the horsepower is used.

1horsepower = 550ft *lb/s = 33,000ft*lb/min

In SI units, power is measured in joules per second, also called the **watt** (W).

1hp = 746 W = 0.746kW

1.5.3 Energy

All object possess energy.   This can come from having work done on it at some point in time. Generally, there are two kinds of energy in mechanical systems, **potential** and **kinetic**. Potential energy is due to the position of the object and kinetic energy is due to its movement.

For example, an object set in motion can overcome a certain amount of resistance before being brought to rest, and the energy which the object has on account of its motion is used up in overcoming the resistance, bring the object to rest. *Fly wheels* on engines both receive and give up *energy* and thus cause the *energy* to return more smoothly throughout the *stroke*.

Elevated weights have power to do *work* on account of their elevated position, as in various types of hammers, *etc.*

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2 Mechanisms and Simple Machines

***Mechanism**: the fundamental physical or chemical processes involved in or responsible for an action, reaction or other natural phenomenon.*

***Machine**: an assemblage of parts that transmit forces, motion and energy in a predetermined manner.*

***Simple Machine**: any of various elementary mechanisms having the elements of which all machines are composed. Included in this category are the lever, wheel and axle, pulley, inclined plane, wedge and the screw.*

The word ***mechanism*** has many meanings. In *kinematics*, a mechanism is a means of transmitting, controlling, or constraining relative movement (Hunt 78). Movements which are electrically, magnetically, pneumatically operated are excluded from the concept of mechanism. The central theme for mechanisms is rigid bodies connected together by joints.

A ***machine*** is a combination of rigid or resistant bodies, formed and connected do that they move with definite relative motions and transmit force from the source of power to the resistance to be overcome. A machine has two functions: transmitting definite relative motion and transmitting force. These functions require strength and rigidity to transmit the forces.

The term ***mechanism*** is applied to the combination of geometrical bodies which constitute a machine or part of a machine. A ***mechanism*** may therefore be defined as a combination of rigid or resistant bodies, formed and connected so that they move with definite relative motions with respect to one another (Ham *et al*. 58).

Although a truly *rigid body* does not exist, many engineering components are rigid because their deformations and distortions are negligible in comparison with their relative movements.

The *similarity* between *machines* and *mechanisms* is that

- they are both combinations of rigid bodies
- the relative motion among the rigid bodies are definite.

The *difference* between *machine* and *mechanism* is that machines transform energy to do work, while mechanisms so not necessarily perform this function. The term ***machinery*** generally means machines and mechanisms. Figure 2-1 shows a picture of the main part of a diesel engine. The mechanism of its cylinder-link-crank parts is a *slider-crank mechanism*, as shown in Figure 2-2.

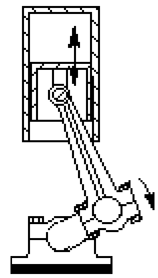

Figure 2-1 Cross section of a power cylinder in a diesel engine

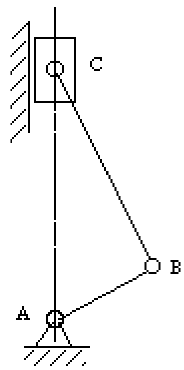

Figure 2-2 Skeleton outline

2.1 The Inclined Plane

Figure 2-3a shows an ***inclined plane***, AB is the base, BC is the height and AC the *inclined plane*. With the use of the inclined plane a given resistance can be overcome with a smaller force than if the plane is not used. For example, in Figure 2-3b, suppose we wish to raise a weight of 1000 lb. through the vertical distance BC = 2 ft. If this weight were raised vertically and without the use of the inclined plane the force 1000 lb. would have to be exerted through the distance BC. If, however, the inclined plane is used and the weight is moved over its inclined plane AC, a force of only 2/3 of 1000 lb. or 667 lb. is necessary, although this force is exerted through a distance AC which is greater than distance BC.

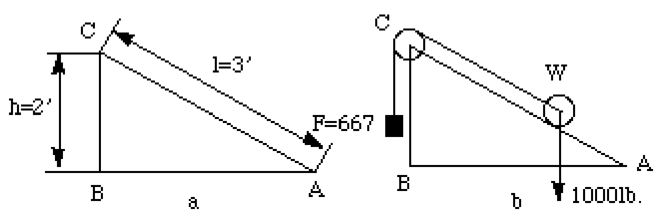

Figure 2-3 Inclined plane

Using an *inclined plane* requires a smaller force exerted through a greater distance to do a certain amount of work.

Letting **F** represent the force required to raise a given weight on the inclined plane, and W the weight to be raised, we have the proportion:

$$\frac{F}{W} = \frac{h}{l}$$

(2-1)

2.1.1 Screw Jack

One of the most common application of the principle of the *inclined plane* is in the ***screw jack*** which is used to overcome a heavy pressure or raise a heavy weight of *W* by a much smaller force *F* applied at the handle. *R* represents the length of the handle and *P* the *pitch* of the screw, or the distance advances in one complete turn.

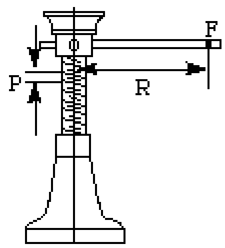

Figure 2-4 The screw jack

Neglecting the friction the following rule is used: The force *F* multiplied by the distance through which it moves in one complete turn is equal to the weight lifted times the distance through which it is lifted in the same time. In one complete turn the end of the handle describes a circle of circumference $2\pi R$. This is the distance through which the force *F* is exerted.

Therefore from the rule above

$$F \times 2\pi R = W \times P$$

(2-2)

and

$$F = \frac{W \times P}{2\pi R}$$

(2-3)

Suppose *R* equals 18 in., *P* equals 1/8 in. and the weight to be lifted equals 100,000 lb., then the force required at *F* is then 110 lb. This means that, neglecting friction, 110 lb. at *F* will raise 100,000 lb. at *W*, but the weight lifted moves much slower than the force applied at *F*.

2.2 Gears

A gear, or toothed wheel, when in operation, may actually be considered as a lever with the additional feature that it can be rotated continuously, instead of rocking back and forth through a short distance. One of the basic relationships for a gear is the number of teeth, the diameter, and the rotary velocity of gears. Figure 2-5 shows the ends of two shafts A and B connected by 2 gears of 24 and 48 teeth respectively. Notice that the larger gear will make only one-half turn while the smaller makes a complete turn. That is, the ratio of speeds (velocity ratio) of the large to the smaller is as 1 to 2.

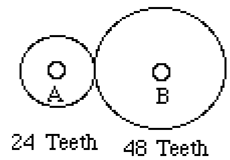

Figure 2-5 Gears

The gear that is closer to the source of power is called the *driver*, and the gear that receives power from the driver is called the *driven gear*.

2.2.1 Gear Trains

A *gear train* may have several drivers and several driven gears.

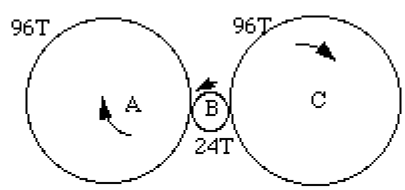

Figure 2-6 Gear train

When gear A turns once clockwise, gear B turns 4 times counter-clockwise and gear C turns once clockwise. Hence gear B does not change the speed of C from what it would have been if geared directly to gear A, but it changes its direction from counterclockwise to clockwise.

The velocity ratio of the first and last gears in a train of simple gears dose not changed by putting any number of gears between them.

Figure 2-7 shows *compound gears* in which two gears are on the middle shaft. Gears B and D rotate at the same speed since they are keyed (fixed) to the same shaft. The number of teeth on each gear is given in the figure. Given these numbers, if gear A rotates at 100 r.p.m. clockwise, gear B turns 400 r.p.m. (rotations per minute) counterclockwise and gear C turns 1200 r.p.m. clockwise.

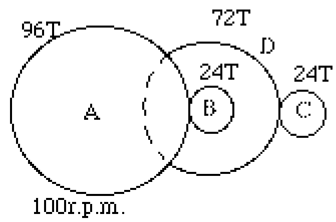

Figure 2-7 Compound gears

2.2.2 Gear Ratios

It is important when working with gears to know what number of teeth the gears should have so that they can mesh properly in a gear train. The size of the teeth for connecting gears must be match properly.

2.3 Belts and Pulleys

Belts and ***pulleys*** are an important part of most machines. ***Pulleys*** are nothing but gears without teeth and instead of running together directly they are made to drive one another by cords, ropes, cables, or belting of some kinds.

As with gears, the velocities of pulleys are inversely proportional to their diameters.

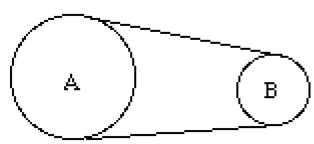

Figure 2-8 Belts and pulleys

Pulleys can also be arranged as a block and tackle.

2.4 Lever

2.5 Wheel and Axle

2.6 Wedge

2.7 Efficiency of Machines

In working out the problems on *levers*, *belts and pulleys*, *inclined planes* and so forth, we have not taken account of friction or other sources of energy loss. In other words, we have supposed them to be perfect, when in fact they are not. To measure the performance of a machine, we often find its ***efficiency***, which is defined as

$$\eta = \frac{W_{out}}{W_{in}}$$

(2-4)

where

η = the efficiency of a machine,
W_{in} = the input work to a machine, and
W_{out} = the output work of a machine.

---

3 More on Machines and Mechanisms

3.1 Planar and Spatial Mechanisms

Mechanisms can be divided into ***planar mechanisms*** and ***spatial mechanisms***, according to the relative motion of the rigid bodies. In a ***planar mechanisms***, all of the relative motions of the rigid bodies are in one plane or in parallel planes. If there is any relative motion that is not in the same plane or in parallel planes, the mechanism is called the ***spatial mechanism***. In other words, *planar mechanisms* are essentially two dimensional while *spatial mechanisms* are three dimensional. This tutorial only covers planar mechanisms.

3.2 Kinematics and Dynamics of Mechanisms

Kinematics of mechanisms is concerned with the motion of the parts without considering how the influencing factors (force and mass) affect the motion. Therefore, kinematics deals with the fundamental concepts of space and time and the quantities velocity and acceleration derived there from.

Kinetics deals with action of forces on bodies. This is where the the effects of gravity come into play.

Dynamics is the combination of *kinematics* and *kinetics*.

Dynamics of mechanisms concerns the forces that act on the parts -- both balanced and unbalanced forces, taking into account the masses and accelerations of the parts as well as the external forces.

3.3 Links, Frames and Kinematic Chains

A ***link*** is defined as a rigid body having two or more pairing elements which connect it to other bodies for the purpose of transmitting force or motion (Ham *et al.* 58).

In every machine, at least one link either occupies a fixed position relative to the earth or carries the machine as a whole along with it during motion. This link is the ***frame*** of the machine and is called the *fixed link.*

The combination of links and pairs without a fixed link is not a mechanism but a ***kinematic chain.***

3.4 Skeleton Outline

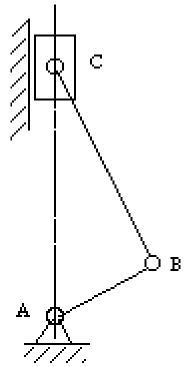

Figure 3-1 Skeleton outline

For the purpose of kinematic analysis, a mechanism may be represented in an abbreviated, or skeleton, form called the *skeleton outline* of the mechanism. The skeleton outline gives all the geometrical information necessary for determining the relative motions of the links. In Figure 3-1, the skeleton outline has been drawn for the engine shown in Figure 2-1. This skeleton contains all necessary information to determine the relative motions of the main links, namely, the length AB of the crank; the length BC of the connecting rod; A the location of the axis of the main bearing; and the path AC of point C, which represents the wrist-pin axis.

3.5 Pairs, Higher Pairs, Lower Pairs and Linkages

A ***pair*** is a joint between the surfaces of two rigid bodies that keeps them in contact and relatively movable. For example, in Figure 3-2, a door jointed to the frame with hinges makes ***revolute joint*** (***pin joint***), allowing the door to be turned around its axis. Figure 3-2b and c show skeletons of a revolute joint. Figure 3-2b is used when both links joined by the pair can turn. Figure 3-2c is used when one of the link jointed by the pair is the frame.

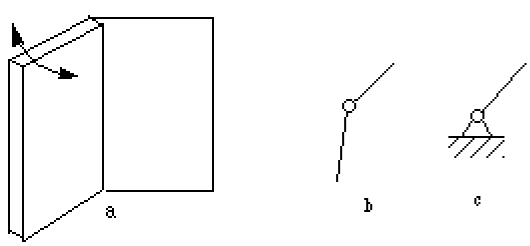

Figure 3-2 Revolute pair

In Figure 3-3a a sash window can be translated relative to the sash. This kind of relative motion is called a ***prismatic pair***. Its skeleton outlines are shown in *b*, *c* and *d*. *c* and *d* are used when one of the links is the *frame*.

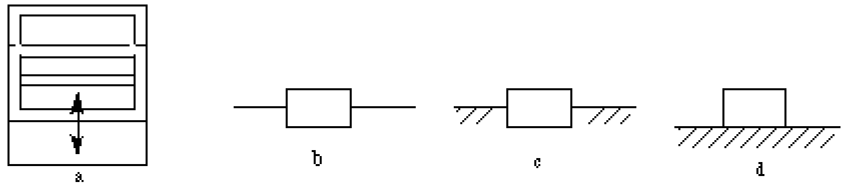

Figure 3-3 Prismatic pair

Generally, there are two kinds of *pairs* in mechanisms, ***lower pairs*** and ***higher pairs***. What differentiates them is the type of contact between the two bodies of the pair. Surface-contact pairs are called ***lower pairs***. In planar (2D) mechanisms, there are two subcategories of lower pairs -- *revolute pairs* and *prismatic pairs*, as shown in Figures 3-2 and 3-3, respectively. Point-, line-, or curve-contact pairs are called ***higher pairs***. Figure 3-4 shows some examples of *higher pairs* Mechanisms composed of rigid bodies and lower pairs are called ***linkages***.

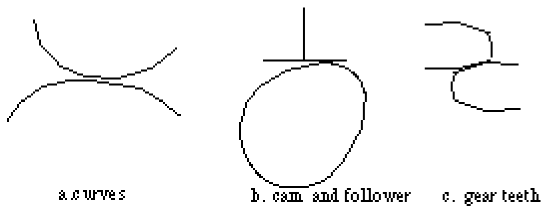

Figure 3-4 Higher pairs

3.6 Kinematic Analysis and Synthesis

In ***kinematic analysis***, a particular given mechanism is investigated based on the mechanism geometry plus other known characteristics (such as input angular velocity, angular acceleration, *etc.*). ***Kinematic synthesis***, on the other hand, is the process of designing a mechanism to accomplish a desired task. Here, both choosing the types as well as the dimensions of the new mechanism can be part of kinematic synthesis. (Sandor & Erdman 84)

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4 Basic Kinematics of Constrained Rigid Bodies

4.1 Degrees of Freedom of a Rigid Body

4.1.1 Degrees of Freedom of a Rigid Body in a Plane

The ***degrees of freedom*** (DOF) of a rigid body is defined as the number of independent movements it has. Figure 4-1 shows a rigid body in a plane. To determine the DOF of this body we must consider how many distinct ways the bar can be moved. In a two dimensional plane such as this computer screen, there are 3 DOF. The bar can be *translated* along the *x* axis, translated along the *y* axis, and *rotated* about its centroid.

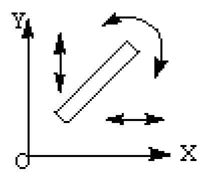


Figure 4-1 Degrees of freedom of a rigid body in a plane

4.1.2 Degrees of Freedom of a Rigid Body in Space

An unrestrained rigid body in space has six degrees of freedom: three translating motions along the *x*, *y* and *z* axes and three rotary motions around the *x*, *y* and *z* axes respectively.

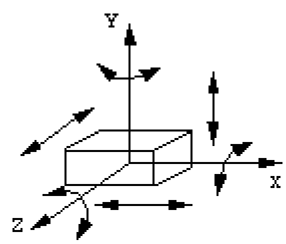


Figure 4-2 Degrees of freedom of a rigid body in space

4.2 Kinematic Constraints

Two or more rigid bodies in space are collectively called a *rigid body system*. We can hinder the motion of these independent rigid bodies with ***kinematic constraints***. *Kinematic constraints* are constraints between rigid bodies that result in the decrease of the degrees of freedom of rigid body system.

The term kinematic pairs actually refers to *kinematic constraints* between rigid bodies. The kinematic pairs are divided into lower pairs and higher pairs, depending on how the two bodies are in contact.

4.2.1 Lower Pairs in Planar Mechanisms

There are two kinds of lower pairs in planar mechanisms: revolute pairs and prismatic pairs.

A rigid body in a plane has only three independent motions -- two translational and one rotary -- so introducing either a revolute pair or a prismatic pair between two rigid bodies removes two degrees of freedom.

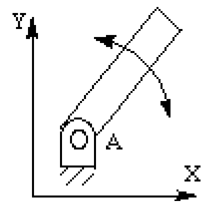


Figure 4-3 A planar revolute pair (R-pair)

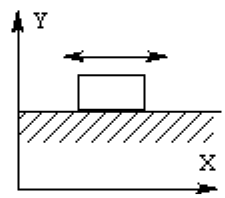


Figure 4-4 A planar prismatic pair (P-pair)

4.2.2 Lower Pairs in Spatial Mechanisms

There are six kinds of lower pairs under the category of spatial mechanisms. The types are: spherical pair, plane pair, cylindrical pair, revolute pair, prismatic pair, and screw pair.

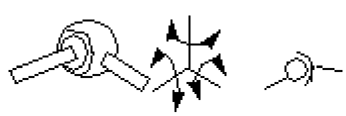


Figure 4-5 A spherical pair (S-pair)

A *spherical pair* keeps two spherical centers together. Two rigid bodies connected by this constraint will be able to *rotate* relatively around *x*, *y* and *z* axes, but there will be no relative translation along any of these axes. Therefore, a spherical pair removes three degrees of freedom in spatial mechanism. **DOF = 3**.

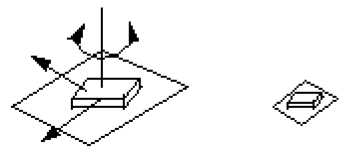


Figure 4-6 A planar pair (E-pair)

A *plane pair* keeps the surfaces of two rigid bodies together. To visualize this, imagine a book lying on a table where is can move in any direction except off the table. Two rigid bodies connected by this kind of pair will have two independent translational motions in the plane, and a rotary motion around the axis that is perpendicular to the plane. Therefore, a plane pair removes three degrees of freedom in spatial mechanism. In our example, the book would not be able to raise off the table or to rotate into the table. **DOF = 3.**

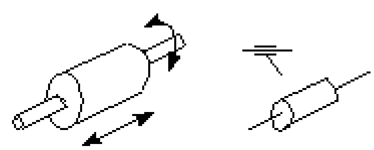


Figure 4-7 A cylindrical pair (C-pair)

A *cylindrical pair* keeps two axes of two rigid bodies aligned. Two rigid bodies that are part of this kind of system will have an independent translational motion along the axis and a relative rotary motion around the axis. Therefore, a cylindrical pair removes four degrees of freedom from spatial mechanism. **DOF = 2.**

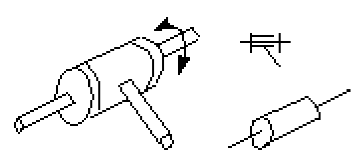


Figure 4-8 A revolute pair (R-pair)

A *revolute pair* keeps the axes of two rigid bodies together. Two rigid bodies constrained by a revolute pair have an independent rotary motion around their common axis. Therefore, a revolute pair removes five degrees of freedom in spatial mechanism. **DOF = 1.**

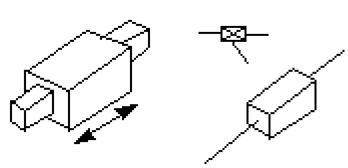


Figure 4-9 A prismatic pair (P-pair)

A *prismatic pair* keeps two axes of two rigid bodies align and allow no relative rotation. Two rigid bodies constrained by this kind of constraint will be able to have an independent translational motion along the axis. Therefore, a prismatic pair removes five degrees of freedom in spatial mechanism. **DOF = 1.**

Figure 4-10 A screw pair (H-pair)

The *screw pair* keeps two axes of two rigid bodies aligned and allows a relative screw motion. Two rigid bodies constrained by a screw pair a motion which is a composition of a translational motion along the axis and a corresponding rotary motion around the axis. Therefore, a screw pair removes five degrees of freedom in spatial mechanism.

4.3 Constrained Rigid Bodies

Rigid bodies and kinematic constraints are the basic components of mechanisms. A constrained rigid body system can be a kinematic chain, a mechanism, a structure, or none of these. The influence of kinematic constraints in the motion of rigid bodies has two intrinsic aspects, which are the geometrical and physical aspects. In other words, we can analyze the motion of the constrained rigid bodies from their geometrical relationships or using Newton's Second Law.

A mechanism is a constrained rigid body system in which one of the bodies is the frame. The degrees of freedom are important when considering a constrained rigid body system that is a mechanism. It is less crucial when the system is a structure or when it does not have definite motion.

Calculating the degrees of freedom of a rigid body system is straight forward. Any unconstrained rigid body has six degrees of freedom in space and three degrees of freedom in a plane. Adding kinematic constraints between rigid bodies will correspondingly decrease the degrees of freedom of the rigid body system. We will discuss more on this topic for planar mechanisms in the next section.

4.4 Degrees of Freedom of Planar Mechanisms

4.4.1 Gruebler's Equation

The definition of the ***degrees of freedom*** of a mechanism is the number of independent relative motions among the rigid bodies. For example, Figure 4-11 shows several cases of a rigid body constrained by different kinds of pairs.

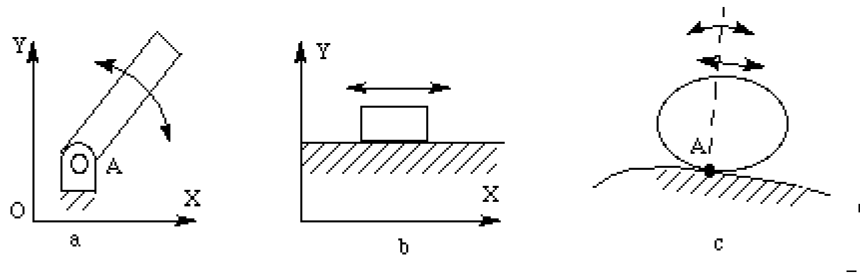


Figure 4-11 Rigid bodies constrained by different kinds of planar pairs

In Figure 4-11a, a rigid body is constrained by a revolute pair which allows only rotational movement around an axis. It has one degree of freedom, turning around point A. The two lost degrees of freedom are translational movements along the *x* and *y* axes. The only way the rigid body can move is to rotate about the fixed point A.

In Figure 4-11b, a rigid body is constrained by a prismatic pair which allows only translational motion. In two dimensions, it has one degree of freedom, translating along the *x* axis. In this example, the body has lost the ability to rotate about any axis, and it cannot move along the *y* axis.

In Figure 4-11c, a rigid body is constrained by a higher pair. It has two degrees of freedom: translating along the curved surface and turning about the instantaneous contact point.

In general, a rigid body in a plane has three degrees of freedom. Kinematic pairs are constraints on rigid bodies that reduce the degrees of freedom of a mechanism. Figure 4-11 shows the three kinds of pairs in planar mechanisms. These pairs reduce the number of the degrees of freedom. If we create a lower pair (Figure 4-11a,b), the degrees of freedom are reduced to 2. Similarly, if we create a higher pair (Figure 4-11c), the degrees of freedom are reduced to 1.

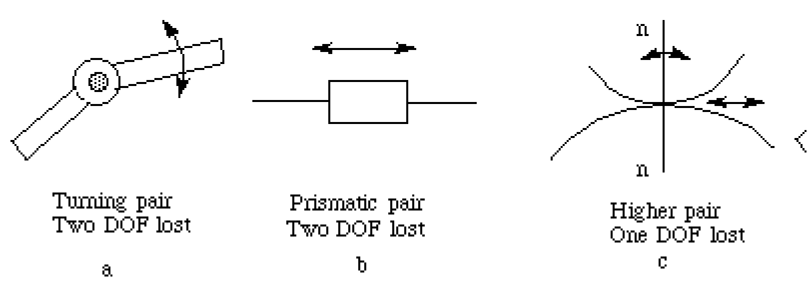


Figure 4-12 Kinematic Pairs in Planar Mechanisms

Therefore, we can write the following equation:

$$F = 3(n-1) - 2l - h$$

(4-1)

Where

F = total degrees of freedom in the mechanism
n = number of links (including the frame)
l = number of lower pairs (one degree of freedom)
h = number of higher pairs (two degrees of freedom)

This equation is also known as ***Gruebler's equation***.

Example 1

Look at the transom above the door in Figure 4-13a. The opening and closing mechanism is shown in Figure 4-13b. Let's calculate its degree of freedom.

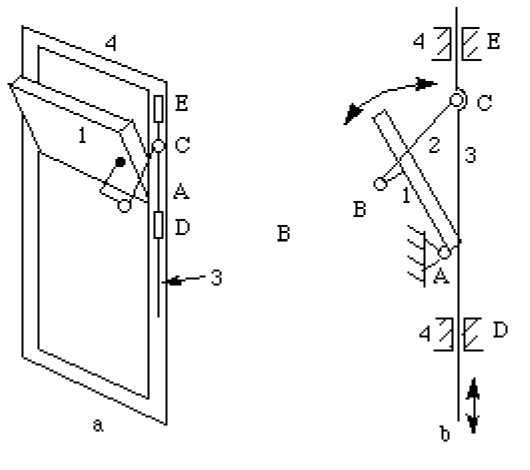


Figure 4-13 Transom mechanism

n = 4 (link 1,3,3 and frame 4), l = 4 (at A, B, C, D), h = 0

$$F = 3(4-1) - 2\times 4 - 1\times 0 = 1$$

(4-2)

Note: D and E function as a same prismatic pair, so they only count as one lower pair.

Example 2

Calculate the degrees of freedom of the mechanisms shown in Figure 4-14b. Figure 4-14a is an application of the mechanism.

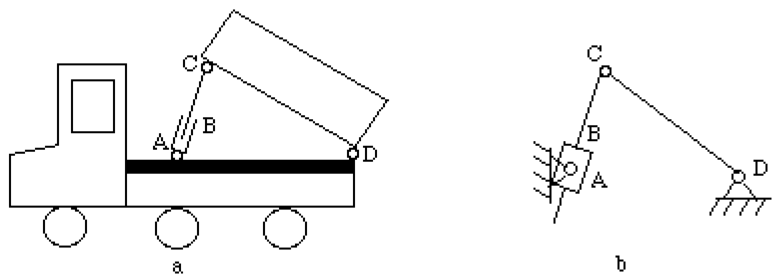


Figure 4-14 Dump truck

n = 4, l = 4 (at A, B, C, D), h = 0

$$F = 3(4-1) - 2\times 4 - 1\times 0 = 1$$
(4-3)

Example 3

Calculate the degrees of freedom of the mechanisms shown in Figure 4-15.

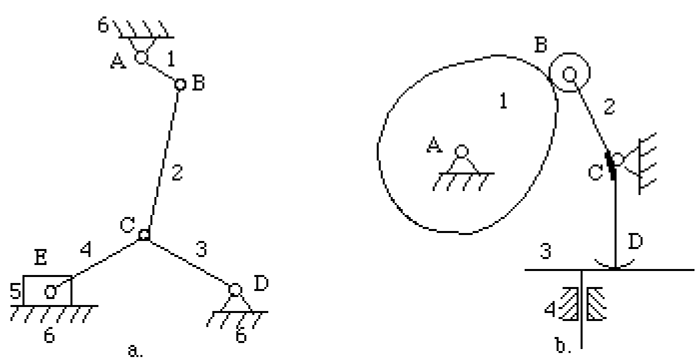


Figure 4-15 Degrees of freedom calculation

For the mechanism in Figure 4-15a

n = 6, l = 7, h = 0

$$F = 3(6-1) - 2\times 7 - 1\times 0 = 1$$
(4-4)

For the mechanism in Figure 4-15b

n = 4, l = 3, h = 2

$$F = 3(4-1) - 2\times 3 - 1\times 2 = 1$$
(4-5)

Note: The rotation of the roller does not influence the relationship of the input and output motion of the mechanism. Hence, the freedom of the roller will not be considered; It is called a ***passive*** or ***redundant*** degree of freedom. Imagine that the roller is welded to link 2 when counting the degrees of freedom for the mechanism.

4.4.2 Kutzbach Criterion

The number of degrees of freedom of a mechanism is also called the *mobility* of the device. The ***mobility*** is the number of input parameters (usually pair variables) that must be independently controlled to bring the device into a particular position. The ***Kutzbach criterion***, which is similar to Gruebler's equation, calculates the *mobility*.

In order to control a mechanism, the number of independent input motions must equal the number of degrees of freedom of the mechanism. For example, the transom in Figure 4-13a has a single degree of freedom, so it needs one independent input motion to open or close the window. That is, you just push or pull rod 3 to operate the window.

To see another example, the mechanism in Figure 4-15a also has 1 degree of freedom. If an independent input is applied to link 1 (*e.g.*, a motor is mounted on joint A to drive link 1), the mechanism will have the a prescribed motion.

4.5 Finite Transformation

Finite transformation is used to describe the motion of a point on rigid body and the motion of the rigid body itself.

4.5.1 Finite Planar Rotational Transformation

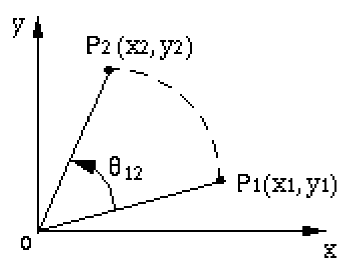


Figure 4-16 Point on a planar rigid body rotated through an angle

Suppose that a point P on a rigid body goes through a rotation describing a circular path from P_1 to P_2 around the origin of a coordinate system. We can describe this motion with a ***rotation operator*** R_{12}:

$$\begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} = R_{12} \begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix}$$

(4-6)

where

$$R_{12} = \begin{bmatrix} \cos\theta_{12} & -\sin\theta_{12} & 0 \\ \sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(4-7)

4.5.2 Finite Planar Translational Transformation

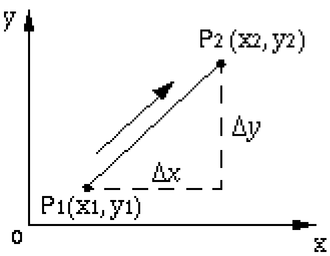


Figure 4-17 Point on a planar rigid body translated through a distance

Suppose that a point *P* on a rigid body goes through a translation describing a straight path from P_1 to P_2 with a change of coordinates of (Δx, Δy). We can describe this motion with a ***translation operator*** T_{12}:

$$\begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} = T_{12} \begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix}$$

(4-8)

where

$$T_{12} = \begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{bmatrix}$$

(4-9)

4.5.3 Concatenation of Finite Planar Displacements

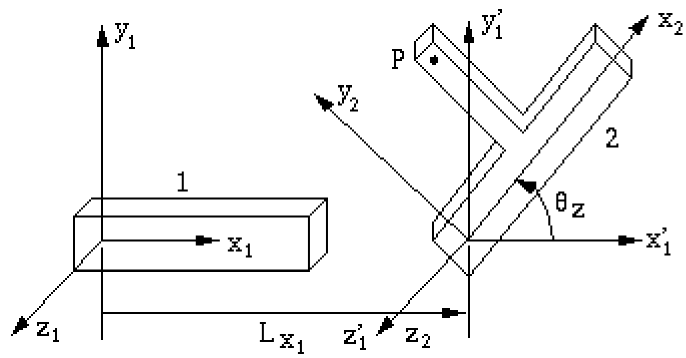


Figure 4-18 Concatenation of finite planar displacements in space

Suppose that a point *P* on a rigid body goes through a rotation describing a circular path from P_1 to P_2' around the origin of a coordinate system, then a translation describing a straight path from P_2' to P_2. We can represent these two steps by

$$\begin{bmatrix} X_2' \\ Y_2' \\ 1 \end{bmatrix} = R_{12} \begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix}$$

(4-10)

and

$$\begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} = T_{12} \begin{bmatrix} X_2' \\ Y_2' \\ 1 \end{bmatrix}$$

(4-11)

We can concatenate these motions to get

$$\begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} = D_{12} \begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix}$$

(4-12)

where D_{12} is the ***planar general displacement operator*** :

$$D_{12} = T_{12} R_{12} = \begin{bmatrix} \cos\theta_{12} & -\sin\theta_{12} & 0 \\ \sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(4-13)

4.5.4 Planar Rigid-Body Transformation

We have discussed various transformations to describe the displacements of a point on rigid body. Can these operators be applied to the displacements of a system of points such as a rigid body?

We used a 3 x 1 homogeneous column matrix to describe a vector representing a single point. A beneficial feature of the planar 3 x 3 translational, rotational, and general displacement matrix operators is that they can easily be programmed on a computer to manipulate a 3 x n matrix of n column vectors representing n points of a rigid body. Since the distance of each particle of a rigid body from every other point of the rigid body is constant, the vectors locating each point of a rigid body must undergo the same transformation when the rigid body moves and the proper axis, angle, and/or translation is specified to represent its motion. (Sandor & Erdman 84). For example, the general planar transformation for the three points *A*, *B*, *C* on a rigid body can be represented by

$$D_{12} = T_{12} R_{12} = \begin{bmatrix} \cos\theta_{12} & -\sin\theta_{12} & \Delta x \\ \sin\theta_{12} & \cos\theta_{12} & \Delta y \\ 0 & 0 & 1 \end{bmatrix}$$

(4-14)

4.5.5 Spatial Rotational Transformation

We can describe a spatial rotation operator for the rotational transformation of a point about an unit axis **u** passing through the origin of the coordinate system. Suppose the rotational angle of the point about **u** is θ, the ***rotation operator*** will be expressed by

$$R_{\theta,u} = \begin{bmatrix} u_x^2 v\theta + c\theta & u_x u_y v\theta - u_z s\theta & u_x u_z v\theta + u_y s\theta & 0 \\ u_x u_y v\theta + u_z s\theta & u_y^2 v\theta + c\theta & u_y u_z v\theta - u_x s\theta & 0 \\ u_x u_z v\theta - u_y s\theta & u_y u_z v\theta + u_x s\theta & u_z^2 v\theta + c\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4-15)

where

u_x, u_y, u_z are the othographical projection of the unit axis **u** on *x*, *y*, and *z* axes, respectively.
$s\theta = \sin\theta$
$c\theta = \cos\theta$
$v\theta = 1 - \cos\theta$

4.5.6 Spatial Translational Transformation

Suppose that a point *P* on a rigid body goes through a translation describing a straight path from P_1 to P_2 with a change of coordinates of (Δx, Δy, Δz), we can describe this motion with a ***translation operator*** *T*:

$$T = \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4-16)

4.5.7 Spatial Translation and Rotation Matrix for Axis Through the Origin

Suppose a point *P* on a rigid body rotates with an angular displacement about an unit axis **u** passing through the origin of the coordinate system at first, and then followed by a translation *D***u** along **u**. This composition of this rotational transformation and this translational transformation is a screw motion. Its corresponding matrix operator, the ***screw operator***, is a concatenation of the translation operator in Equation 4-7 and the rotation operator in Equation 4-9.

$$S = \begin{bmatrix} u_x^2 v\theta + c\theta & u_x u_y v\theta - u_z s\theta & u_x u_z v\theta + u_y s\theta & Du_x \\ u_x u_y v\theta + u_z s\theta & u_y^2 v\theta + c\theta & u_y u_z v\theta - u_x s\theta & Du_y \\ u_x u_z v\theta - u_y s\theta & u_y u_z v\theta + u_x s\theta & u_z^2 v\theta + c\theta & Du_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4-17)

4.6 Transformation Matrix Between Rigid Bodies

4.6.1 Transformation Matrix Between two Arbitray Rigid Bodies

For a system of rigid bodies, we can establish a local Cartesian coordinate system for each rigid body. Transformation matrices are used to describe the relative motion between rigid bodies.

For example, two rigid bodies in a space each have local coordinate systems $x_1y_1z_1$ and $x_2y_2z_2$. Let point *P* be attached to body 2 at location (x_2, y_2, z_2) in body 2's local coordinate system. To find the location of *P* with respect to body 1's local coordinate system, we know that that the point $x_2y_2z_2$ can be obtained from $x_1y_1z_1$ by combining translation L_{x1} along the *x* axis and rotation θz about z axis. We can derive the transformation matrix as follows:

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = T_{12} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & Lx_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_Z & -\sin\theta_Z & 0 & 0 \\ \sin\theta_Z & \cos\theta_Z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_Z & -\sin\theta_Z & 0 & Lx_1 \\ \sin\theta_Z & \cos\theta_Z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}$$

(4-18)

If rigid body 1 is fixed as a frame, a global coordinate system can be created on this body. Therefore, the above transformation can be used to map the local coordinates of a point into the global coordinates.

4.6.2 Kinematic Constraints Between Two Rigid Bodies

The transformation matrix above is a specific example for two unconstrained rigid bodies. The transformation matrix depends on the relative position of the two rigid bodies. If we connect two rigid bodies with a kinematic constraint, their degrees of freedom will be decreased. In other words, their relative motion will be specified in some extent.

Suppose we constrain the two rigid bodies above with a revolute pair as shown in Figure 4-19. We can still write the transformation matrix in the same form as Equation 4-18.

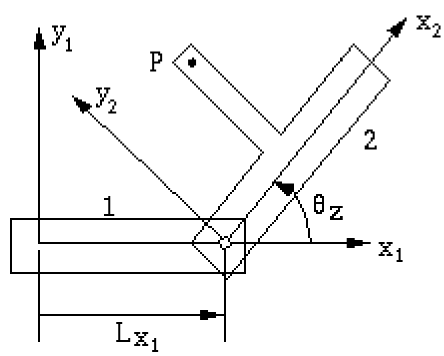


Figure 4-19 Relative position of points on constrained bodies

The difference is that the L_{x1} is a constant now, because the revolute pair fixes the origin of coordinate system $x_2y_2z_2$ with respect to coordinate system $x_1y_1z_1$. However, the rotation θz is still a variable. Therefore, kinematic constraints specify the transformation matrix to some extent.

4.6.3 Denavit-Hartenberg Notation

Denavit-Hartenberg notation (Denavit & Hartenberg 55) is widely used in the transformation of coordinate systems of linkages and robot mechanisms. It can be used to represent the transformation matrix between links as shown in the Figure 4-20.

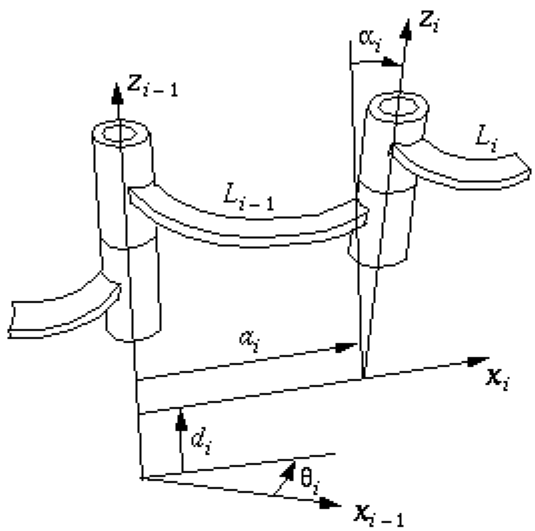


Figure 4-20 Denavit-Hartenberg Notation

In this figure,

- z_{i-1} and z_i are the axes of two revolute pairs;
- θ_i is the included angle of axes x_{i-1} and x_i;
- d_i is the distance between the origin of the coordinate system $x_{i-1}y_{i-1}z_{i-1}$ and the foot of the common perpendicular;
- a_i is the distance between two feet of the common perpendicular;
- α_i is the included angle of axes z_{i-1} and z_i;

The transformation matrix will be $T_{(i-1)i}$

$$T_{(i-1)i} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4-19)

The above transformation matrix can be denoted as $T(a_i, \alpha_i, \theta_i, d_i)$ for convenience.

4.6.4 Application of Transformation Matrices to Linkages

A linkage is composed of several constrained rigid bodies. Like a mechanism, a linkage should have a frame. The matrix method can be used to derive the kinematic equations of the linkage. If all the links form a closed loop, the concatenation of all of the transformation matrices will be an identity matrix. If the mechanism has *n* links, we will have:

$$T_{12}T_{23}...T_{(n-1)n} = I$$

5 Planar Linkages

5.1 Introduction

5.1.1 What are Linkage Mechanisms?

Have you ever wondered what kind of mechanism causes the wind shield wiper on the front widow of car to oscillate (Figure 5-1a)? The mechanism, shown in Figure 5-1b, transforms the rotary motion of the motor into an oscillating motion of the windshield wiper.

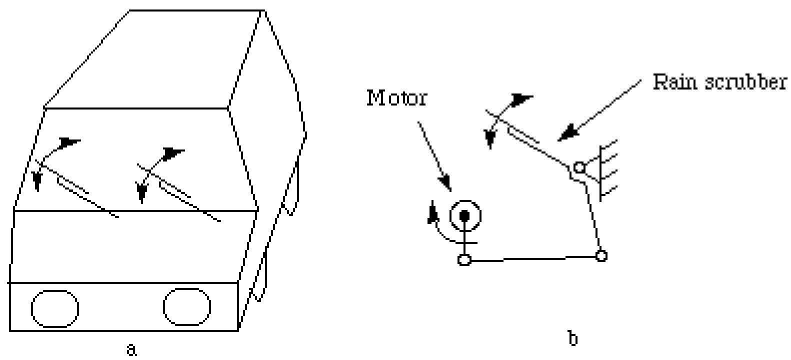


Figure 5-1 Windshield wiper

Let's make a simple mechanism with similar behavior. Take some cardboard and make four strips as shown in Figure 5-2a.

Take 4 pins and assemble them as shown in Figure 5-2b.

Now, hold the 6in. strip so it can't move and turn the 3in. strip. You will see that the 4in. strip oscillates.

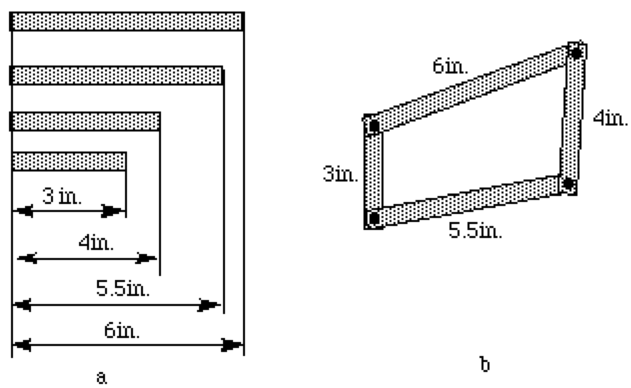


Figure 5-2 Do-it-yourself four bar linkage mechanism

The four bar linkage is the simplest and often times, the most useful mechanism. As we mentioned before, a mechanism composed of rigid bodies and lower pairs is called a linkage (Hunt 78). In planar mechanisms, there are only two kinds of lower pairs --- revolute pairs and prismatic pairs.

The simplest closed-loop linkage is the four bar linkage which has four members, three moving links, one fixed link and four pin joints. A linkage that has at least one fixed link is a mechanism. The following example of a four bar linkage was created in SimDesign in `simdesign/fourbar.sim`

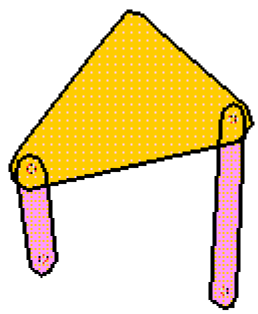


Figure 5-3 Four bar linkage in SimDesign

This mechanism has three moving links. Two of the links are pinned to the frame which is not shown in this picture. In SimDesign, links can be nailed to the background thereby making them into the frame.

How many DOF does this mechanism have? If we want it to have just one, we can impose one constraint on the linkage and it will have a definite motion. The four bar linkage is the simplest and the most useful mechanism.

Reminder: A mechanism is composed of rigid bodies and lower pairs called linkages (Hunt 78). In planar mechanisms there are only two kinds of lower pairs: turning pairs and prismatic pairs.

5.1.2 Functions of Linkages

The function of a link mechanism is to produce rotating, oscillating, or reciprocating motion from the rotation of a crank or *vice versa* (Ham *et al.* 58). Stated more specifically linkages may be used to convert:

1. Continuous rotation into continuous rotation, with a constant or variable angular velocity ratio.
2. Continuous rotation into oscillation or reciprocation (or the reverse), with a constant or variable velocity ratio.
3. Oscillation into oscillation, or reciprocation into reciprocation, with a constant or variable velocity ratio.

Linkages have many different functions, which can be classified according on the primary goal of the mechanism:

- **Function generation**: the relative motion between the links connected to the frame,
- **Path generation**: the path of a tracer point, or
- **Motion generation**: the motion of the coupler link.

5.2 Four Link Mechanisms

One of the simplest examples of a constrained linkage is the ***four-link mechanism***. A variety of useful mechanisms can be formed from a four-link mechanism through slight variations, such as changing the character of the pairs, proportions of links, *etc*. Furthermore, many complex link mechanisms are combinations of two or more such mechanisms. The majority of four-link mechanisms fall into one of the following two classes:

1. the four-bar linkage mechanism, and
2. the slider-crank mechanism.

5.2.1 Examples

Parallelogram Mechanism

In a parallelogram four-bar linkage, the orientation of the coupler does not change during the motion. The figure illustrates a loader. Obvioulsy the behavior of maintaining parallelism is important in a loader. The bucket should not rotate as it is raised and lowered. The corresponding SimDesign file is `simdesign/loader.sim`.

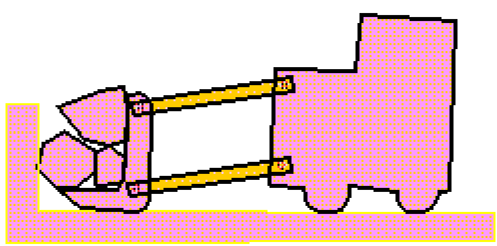


Figure 5-4 Front loader mechanism

Slider-Crank Mechanism

The four-bar mechanism has some special configurations created by making one or more links infinite in length. The slider-crank (or crank and slider) mechanism shown below is a four-bar linkage with the slider replacing an infinitely long output link. The corresponding SimDesign file is `simdesign/slider.crank.sim`.

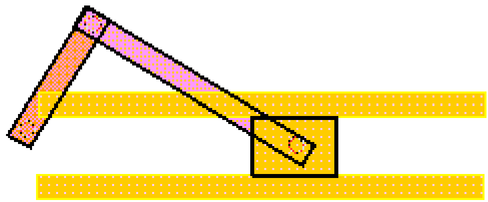


Figure 5-5 Crank and Slider Mechanism

This configuration translates a rotational motion into a translational one. Most mechanisms are driven by motors, and slider-cranks are often used to transform rotary motion into linear motion.

Crank and Piston

You can also use the slider as the input link and the crank as the output link. In this case, the mechanism transfers translational motion into rotary motion. The pistons and crank in an internal combustion engine are an example of this type of mechanism. The corresponding SimDesign file is `simdesign/combustion.sim`.

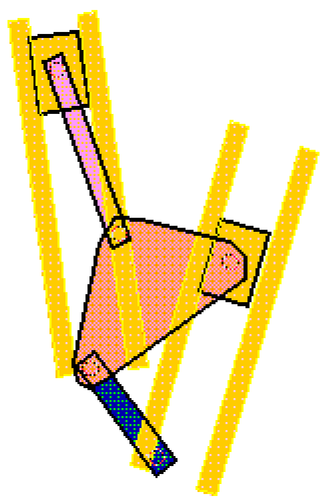


Figure 5-6 Crank and Piston

You might wonder why there is another slider and a link on the left. This mechanism has two dead points. The slider and link on the left help the mechanism to overcome these dead points.

Block Feeder

One interesting application of slider-crank is the block feeder. The SimDesign file can be found in `simdesign/block-feeder.sim`

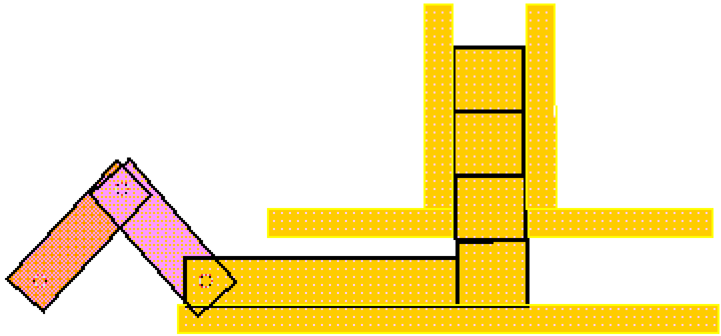


Figure 5-7 Block Feeder

5.2.2 Definitions

In the range of planar mechanisms, the simplest group of lower pair mechanisms are four bar linkages. A ***four bar linkage*** comprises four bar-shaped links and four turning pairs as shown in Figure 5-8.

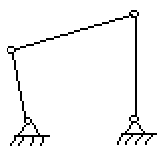


Figure 5-8 Four bar linkage

The link opposite the frame is called the **coupler link**, and the links which are hinged to the frame are called **side links**. A link which is free to rotate through 360 degree with respect to a second link will be said to **revolve** relative to the second link (not necessarily a frame). If it is possible for all four bars to become simultaneously aligned, such a state is called a **change point**.

Some important concepts in link mechanisms are:

1. **Crank**: A side link which revolves relative to the frame is called a *crank*.
2. **Rocker**: Any link which does not revolve is called a *rocker*.
3. **Crank-rocker mechanism**: In a four bar linkage, if the shorter side link revolves and the other one rocks (*i.e.*, oscillates), it is called a *crank-rocker mechanism*.
4. **Double-crank mechanism**: In a four bar linkage, if both of the side links revolve, it is called a *double-crank mechanism*.
5. **Double-rocker mechanism**: In a four bar linkage, if both of the side links rock, it is called a *double-rocker mechanism*.

5.2.3 Classification

Before classifying four-bar linkages, we need to introduce some basic nomenclature.

In a four-bar linkage, we refer to the *line segment between hinges* on a given link as a **bar** where:

- *s* = length of shortest bar
- *l* = length of longest bar
- *p, q* = lengths of intermediate bar

Grashof's theorem states that a four-bar mechanism has *at least* one revolving link if

$$s + l \le p + q \tag{5-1}$$

and all three mobile links will rock if

$$s + l > p + q \tag{5-2}$$

The inequality 5-1 is **Grashof's criterion**.

All four-bar mechanisms fall into one of the four categories listed in Table 5-1:

Case	l + s vers. p + q	Shortest Bar	Type
1	<	Frame	Double-crank
2	<	Side	Rocker-crank
3	<	Coupler	Doubl rocker
4	=	Any	Change point
5	>	Any	Double-rocker

Table 5-1 Classification of Four-Bar Mechanisms

From Table 5-1 we can see that for a mechanism to have a crank, the sum of the length of its shortest and longest links must be less than or equal to the sum of the length of the other two links. However, this condition is necessary but not sufficient. Mechanisms satisfying this condition fall into the following three categories:

1. When the shortest link is a side link, the mechanism is a crank-rocker mechanism. The shortest link is the crank in the mechanism.
2. When the shortest link is the frame of the mechanism, the mechanism is a double-crank mechanism.
3. When the shortest link is the coupler link, the mechanism is a double-rocker mechanism.

5.2.4 Transmission Angle

In Figure 5-11, if *AB* is the input link, the force applied to the output link, *CD*, is transmitted through the coupler link *BC*. (That is, pushing on the link *CD* imposes a force on the link *AB*, which is transmitted through the link *BC*.) For sufficiently slow motions (negligible inertia forces), the force in the coupler link is pure tension or compression (negligible bending action) and is directed along *BC*. For a given force in the coupler link, the torque transmitted to the output bar (about point *D*) is maximum when the angle β between coupler bar *BC* and output bar *CD* is $\pi/2$. Therefore, angle BCD is called **transmission angle**.

$$\alpha_{max} = |90^\circ - \beta|_{min} < 50^\circ \tag{5-3}$$

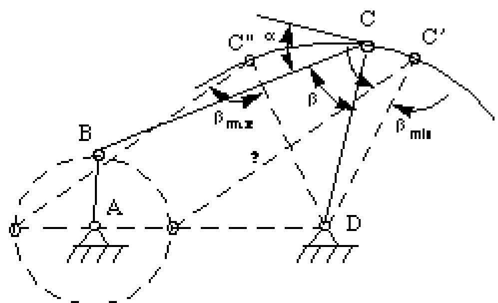


Figure 5-11 Transmission angle

When the *transmission angle* deviates significantly from $\pi/2$, the torque on the output bar decreases and may not be sufficient to overcome the friction in the system. For this reason, the **deviation angle** $\alpha=|\pi/2-\beta|$ should not be too great. In practice, there is no definite upper limit for α, because the existence of the inertia forces may eliminate the undesirable force relationships that is present under static conditions. Nevertheless, the following criterion can be followed.

5.2.5 Dead Point

When a side link such as *AB* in Figure 5-10, becomes aligned with the coupler link *BC*, it can only be compressed or extended by the coupler. In this configuration, a torque applied to the link on the other side, *CD*, cannot induce rotation in link *AB*. This link is therefore said to be at a **dead point** (sometimes called a **toggle point**).

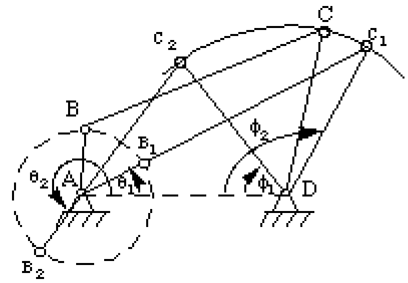


Figure 5-10 Dead point

In Figure 5-11, if *AB* is a crank, it can become aligned with BC in full extension along the line AB_1C_1 or in flexion with AB_2 folded over B_2C_2. We denote the angle *ADC* by ϕ and the angle DAB by θ. We use the subscript 1 to denote the extended state and 2 to denote the flexed state of links *AB* and *BC*. In the extended state, link *CD* cannot rotate clockwise without stretching or compressing the theoretically rigid line AC_1. Therefore, link *CD* cannot move into the *forbidden zone* below C_1D, and ϕ must be at one of its two extreme positions; in other words, link *CD* is at an extremum. A second extremum of link *CD* occurs with $\phi = \phi_1$.

Note that the extreme positions of a side link occur simultaneously with the dead points of the opposite link.

In some cases, the dead point can be useful for tasks such as work fixturing (Figure 5-11).

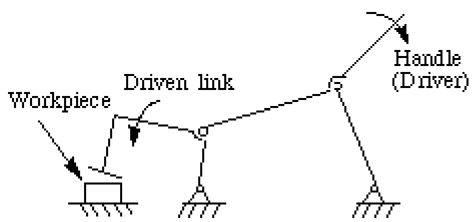


Figure 5-11 Work fixturing

In other cases, dead point should be and can be overcome with the moment of inertia of links or with the asymmetrical deployment of the mechanism (Figure 5-12).

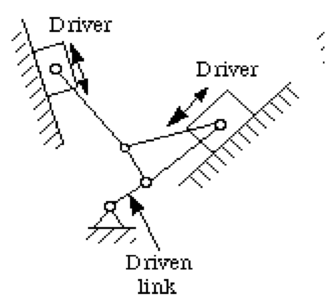


Figure 5-12 Overcoming the dead point by asymmetrical deployment (V engine)

5.2.6 Slider-Crank Mechanism

The slider-crank mechanism, which has a well-known application in engines, is a special case of the crank-rocker mechanism. Notice that if rocker 3 in Figure 5-13a is very long, it can be replaced by a block sliding in a curved slot or guide as shown. If the length of the rocker is infinite, the guide and block are no longer curved. Rather, they are apparently straight, as shown in Figure 5-13b, and the linkage takes the form of the ordinary **slider-crank mechanism**.

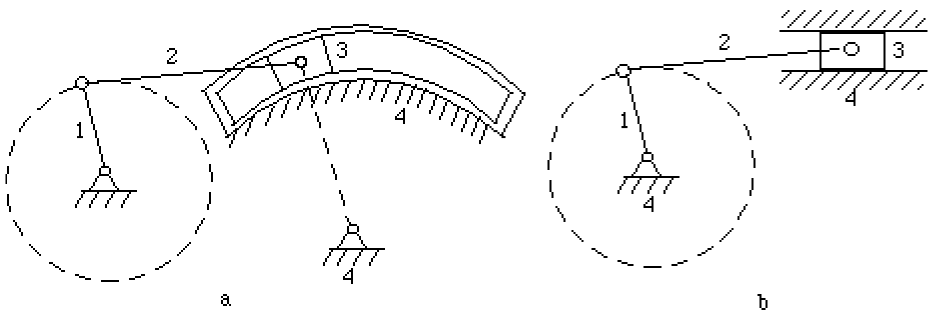


Figure 5-13 Slider-Crank mechanism

5.2.7 Inversion of the Slider-Crank Mechanism

Inversion is a term used in kinematics for a reversal or interchange of form or function as applied to kinematic chains and mechanisms. For example, taking a different link as the fixed link, the slider-crank mechanism shown in Figure 5-14a can be inverted into the mechanisms shown in Figure 5-14b, c, and d. Different examples can be found in the application of these mechanisms. For example, the mechanism of the pump device in Figure 5-15 is the same as that in Figure 5-14b.

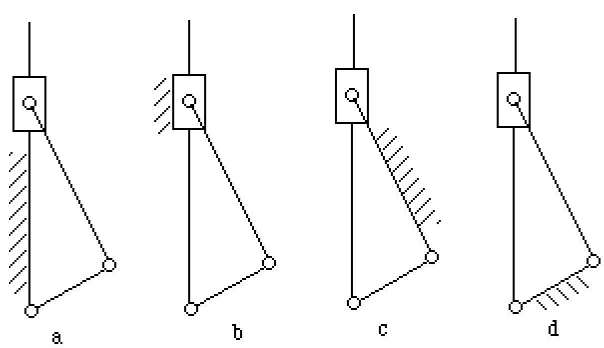


Figure 5-14 Inversions of the crank-slide mechanism

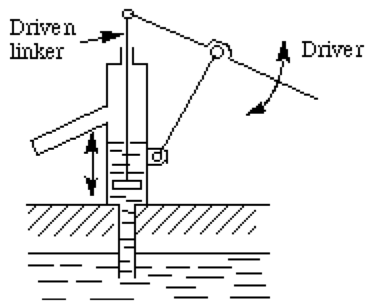


Figure 5-15 A pump device

Keep in mind that the inversion of a mechanism does not change the motions of its links relative to each other but does change their absolute motions.

6 Cams

6.1 Introduction

6.1.1 A Simple Experiment: What is a Cam?

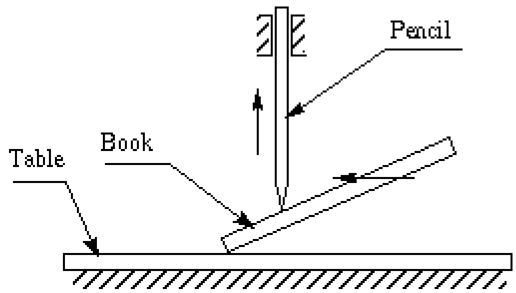


Figure 6-1 Simple Cam experiment

Take a pencil and a book to do an experiment as shown above. Make the book an inclined plane and use the pencil as a slider (use your hand as a guide). When you move the book smoothly upward, what happens to the pencil? It will be pushed up along the guide. By this method, you have transformed one motion into another motion by a very simple device. This is the basic idea of a cam. By rotating the cams in the figure below, the bars will have either translational or oscillatory motion.

6.1.2 Cam Mechanisms

The transformation of one of the simple motions, such as rotation, into any other motions is often conveniently accomplished by means of a **cam mechanism** A cam mechanism usually consists of two moving elements, the cam and the follower, mounted on a fixed frame. Cam devices are versatile, and almost any arbitrarily-specified motion can be obtained. In some instances, they offer the simplest and most compact way to transform motions.

A ***cam*** may be defined as a machine element having a curved outline or a curved groove, which, by its oscillation or rotation motion, gives a predetermined specified motion to another element called the ***follower*** . The cam has a very important function in the operation of many classes of machines, especially those of the automatic type, such as printing presses, shoe machinery, textile machinery, gear-cutting machines, and screw machines. In any class of machinery in which automatic control and accurate timing are paramount, the cam is an indispensable part of mechanism. The possible applications of cams are unlimited, and their shapes occur in great variety. Some of the most common forms will be considered in this chapter.

6.2 Classification of Cam Mechanisms

We can classify cam mechanisms by the modes of input/output motion, the configuration and arrangement of the follower, and the shape of the cam. We can also classify cams by the different types of motion events of the follower and by means of a great variety of the motion characteristics of the cam profile. (Chen 82)

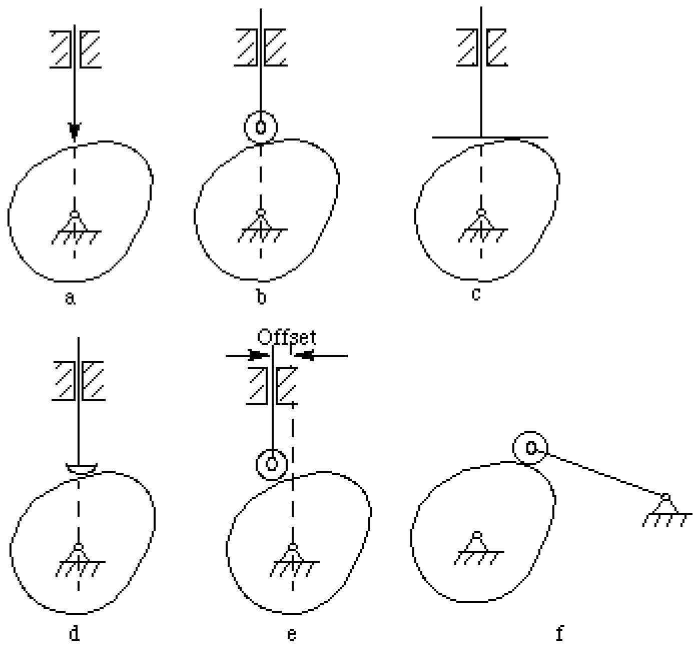


Figure 6-2 Classification of cam mechanisms

4.2.1 Modes of Input/Output Motion

1. Rotating cam-translating follower. (Figure 6-2a,b,c,d,e)
2. Rotating follower (Figure 6-2f):
   The follower arm swings or oscillates in a circular arc with respect to the follower pivot.
3. Translating cam-translating follower (Figure 6-3).
4. Stationary cam-rotating follower:
   The follower system revolves with respect to the center line of the vertical shaft.

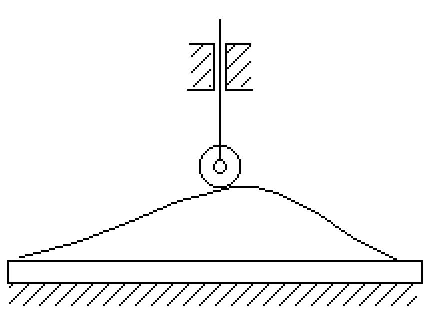


Figure 6-3 Translating cam - translating follower

6.2.1 Follower Configuration

1. **Knife-edge follower** (Figure 6-2a)
2. **Roller follower** (Figure 6-2b,e,f)
3. **Flat-faced follower** (Figure 6-2c)
4. **Oblique flat-faced follower**
5. **Spherical-faced follower** (Figure 6-2d)

6.2.2 Follower Arrangement

1. In-line follower:
   The center line of the follower passes through the center line of the camshaft.
2. Offset follower:
   The center line of the follower does not pass through the center line of the cam shaft. The amount of **offset** is the distance between these two center lines. The offset causes a reduction of the side thrust present in the roller follower.

6.2.3 Cam Shape

1. Plate cam or **disk cam**:
   The follower moves in a plane perpendicular to the axis of rotation of the camshaft. A translating or a swing arm follower must be constrained to maintain contact with the cam profile.
2. **Grooved cam** or closed cam (Figure 6-4):
   This is a plate cam with the follower riding in a groove in the face of the cam.

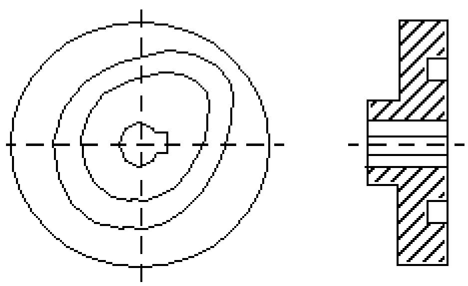


Figure 6-4 Grooved cam

3. **Cylindrical cam** or barrel cam (Figure 6-5a):
   The roller follower operates in a groove cut on the periphery of a cylinder. The follower may translate or oscillate. If the cylindrical surface is replaced by a conical one, a conical cam results.
4. **End cam** (Figure 6-5b):
   This cam has a rotating portion of a cylinder. The follower translates or oscillates, whereas the cam usually rotates. The end cam is rarely used because of the cost and the difficulty in cutting its contour.

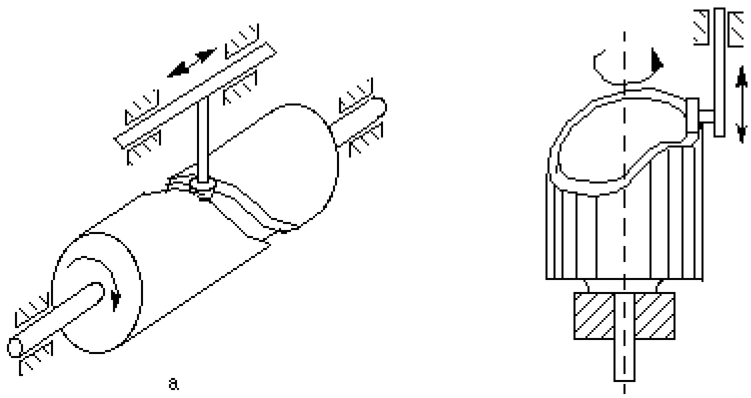


Figure 6-5 Cylindrical cam and end cam

6.2.4 Constraints on the Follower

1. Gravity constraint:
   The weight of the follower system is sufficient to maintain contact.
2. Spring constraint:
   The spring must be properly designed to maintain contact.
3. Positive mechanical constraint:
   A groove maintains positive action. (Figure 6-4 and Figure 6-5a) For the cam in Figure 6-6, the follower has two rollers, separated by a fixed distance, which act as the constraint; the mating cam in such an arrangement is often called a *constant-diameter cam*.

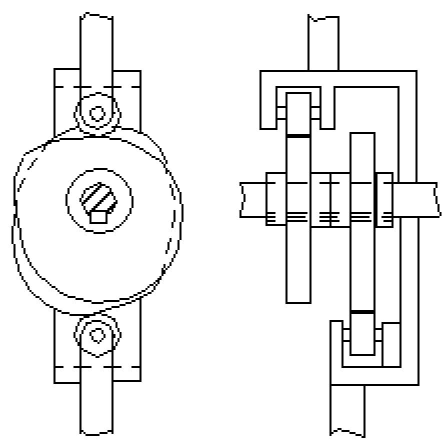


Figure 6-6 Constant diameter cam

A mechanical constraint cam also be introduced by employing a dual or conjugate cam in arrangement similar to what shown in Figure 6-7. Each cam has its own roller, but the rollers are mounted on the same reciprocating or oscillating follower.

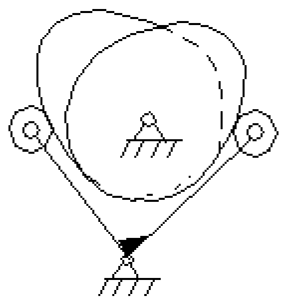


Figure 6-7 Dual cam

6.2.5 Examples in SimDesign

Rotating Cam, Translating Follower

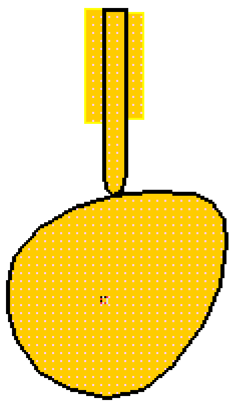


Figure 6-8 SimDesign translating cam

Load the SimDesign file `simdesign/cam.translating.sim`. If you turn the cam, the follower will move. The weight of the follower keeps them in contact. This is called a *gravity constraint* cam.

Rotating Cam/Rotating Follower

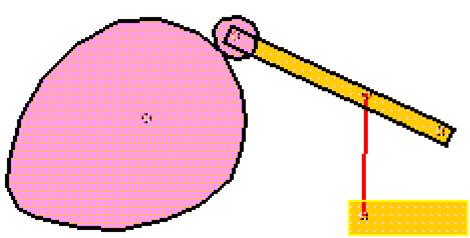


Figure 6-9 SimDesign oscillating cam

The SimDesign file is `simdesign/cam.oscillating.sim`. Notice that a roller is used at the end of the follower. In addition, a spring is used to maintain the contact of the cam and the roller.

If you try to calculate the degrees of freedom (DOF) of the mechanism, you must imagine that the roller is welded onto the follower because turning the roller does not influence the motion of the follower.

6.3 Cam Nomenclature

Figure 6-10 illustrates some cam nomenclature:

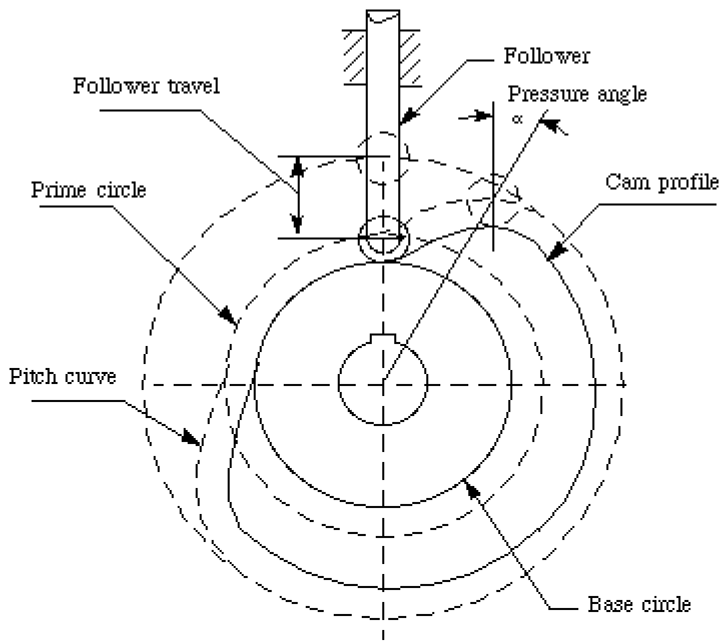


Figure 6-10 Cam nomenclature

- **Trace point**: A theoretical point on the follower, corresponding to the point of a fictitious *knife-edge follower*. It is used to generate the *pitch curve*. In the case of a *roller follower*, the trace point is at the center of the roller.
- **Pitch curve**: The path generated by the trace point at the follower is rotated about a stationary cam.
- **Working curve**: The working surface of a cam in contact with the follower. For the *knife-edge follower* of the plate cam, the *pitch curve* and the *working curves* coincide. In a *close or grooved cam* there is an *inner profile* and an *outer working curve*.
- **Pitch circle**: A circle from the cam center through the pitch point. The pitch circle radius is used to calculate a cam of minimum size for a given *pressure angle*.

- **Prime circle** (**reference circle**): The smallest circle from the cam center through the pitch curve.
- **Base circle**: The smallest circle from the cam center through the cam profile curve.
- **Stroke** or **throw**:The greatest distance or angle through which the follower moves or rotates.
- **Follower displacement**: The position of the follower from a specific zero or rest position (usually its the position when the *f* ollower contacts with the *base circle* of the cam) in relation to time or the rotary angle of the cam.
- **Pressure angle**: The angle at any point between the normal to the pitch curve and the instantaneous direction of the follower motion. This angle is important in cam design because it represents the steepness of the cam profile.

6.4 Motion events

When the cam turns through one motion cycle, the follower executes a series of events consisting of rises, dwells and returns. **Rise** is the motion of the follower away from the cam center, **dwell** is the motion during which the follower is at rest; and **return** is the motion of the follower toward the cam center.

There are many follower motions that can be used for the rises and the returns. In this chapter, we describe a number of basic curves.

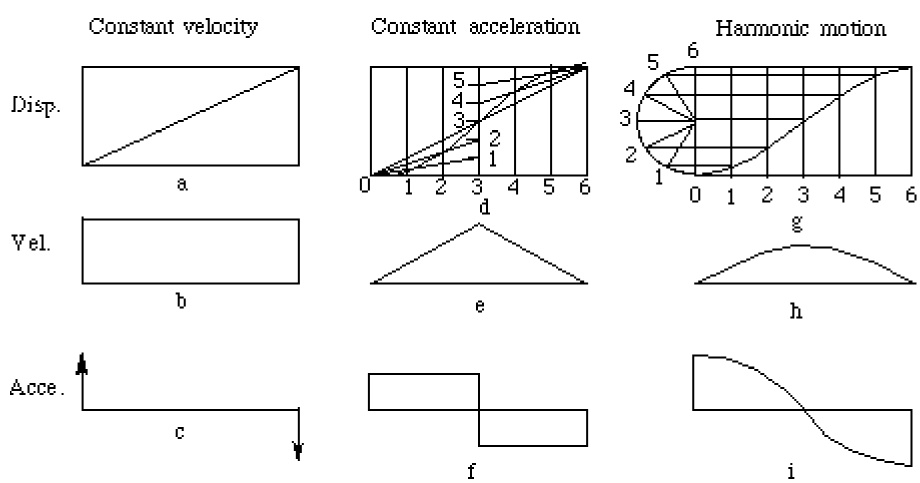


Figure 6-11 Motion events

Notation

Φ: The rotary angle of the cam, measured from the beginning of the motion event;
β: The range of the rotary angle corresponding to the motion event;
h : The stoke of the motion event of the follower;
S : Displacement of the follower;
V : Velocity of the follower;
A : Acceleration of the follower.

6.4.1 Constant Velocity Motion

If the motion of the follower were a straight line, Figure 6-11a,b,c, it would have equal displacements in equal units of time, *i.e.*, uniform velocity from the beginning to the end of the stroke, as shown in b. The acceleration, except at the end of the stroke would be zero, as shown in c. The diagrams show abrupt changes of velocity, which result in large forces at the beginning and the end of the stroke. These forces are undesirable, especially when the cam rotates at high velocity. The *constant velocity motion* is therefore only of theoretical interest.

$$S(\varphi) = h\frac{\varphi}{\beta}$$
$$V(\varphi) = \frac{h}{\beta}$$
$$A(\varphi) = 0$$

(6-1)

6.4.2 Constant Acceleration Motion

Constant acceleration motion is shown in Figure 6-11d, e, f. As indicated in e, the velocity increases at a uniform rate during the first half of the motion and decreases at a uniform rate during the second half of the motion. The acceleration is constant and positive throughout the first half of the motion, as shown in f, and is constant and negative throughout the second half. This type of motion gives the follower the smallest value of maximum acceleration along the path of motion. In high-speed machinery this is particularly important because of the forces that are required to produce the accelerations.

When

$0 \le \varphi \le \frac{\beta}{2}$,

$$S(\varphi) = 2h\frac{\varphi^2}{\beta^2}$$
$$V(\varphi) = \frac{4h}{\beta^2}\varphi$$
$$A(\varphi) = \frac{4h}{\beta^2}$$

(6-2)

When

$\frac{\beta}{2} \le \varphi \le \beta$,

$$S(\varphi) = h - \frac{2h}{\beta^2}(\beta - \varphi)^2$$
$$V(\varphi) = \frac{4h}{\beta}\left(1 - \frac{\varphi}{\beta}\right)$$
$$A(\varphi) = -\frac{4h}{\beta^2}$$

(6-3)

6.4.3 Harmonic Motion

A cam mechanism with the basic curve like g in Figure 6-7g will impart *simple harmonic motion* to the follower. The velocity diagram at h indicates smooth action. The acceleration, as shown at i, is maximum at the initial position, zero at the mid-position, and negative maximum at the final position.

$$S(\varphi) = \frac{h}{2}\left(1 - \cos\frac{\pi\varphi}{\beta}\right)$$
$$V(\varphi) = \frac{h\pi}{2\beta}\sin\frac{\pi\varphi}{\beta}$$
$$A(\varphi) = \frac{h\pi^2}{2\beta^2}\cos\frac{\pi\varphi}{\beta}$$

(6-4)

6.5 Cam Design

The translational or rotational displacement of the follower is a function of the rotary angle of the cam. A designer can define the function according to the specific requirements in the design. The motion requirements, listed below, are commonly used in cam profile design.

6.5.1 Disk Cam with Knife-Edge Translating Follower

Figure 6-12 is a skeleton diagram of a disk cam with a knife-edge translating follower. We assume that the cam mechanism will be used to realize the displacement relationship between the rotation of the cam and the translation of the follower.

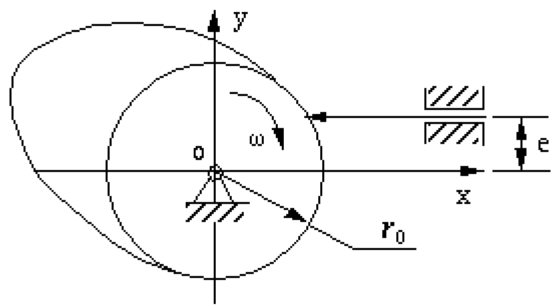


Figure 6-12 A Skeleton Diagram of disk cam with knife-edge translation

Below is a list of the essential parameters for the evaluation of these types of cam mechanisms. However, these parameters are adequate only to define a knife-edge follower and a translating follower cam mechanism.

Parameters:

r_o: The radius of the base circle;
e: The offset of the follower from the rotary center of the cam. Notice: it could be negative.
s: The displacement of the follower which is a function of the rotary angle of the cam -- Φ.
IW: A parameter whose absolute value is 1. It represents the turning direction of the cam. When the cam turns clockwise: *IW=+1*, otherwise: *IW=-1*.

Cam profile design principle:

The method termed inversion is commonly used in cam profile design. For example, in a disk cam with translating follower mechanism, the follower translates when the cam turns. This means that the relative motion between them is a combination of a relative turning motion and a relative translating motion. Without changing this feature of their relative motion, imagine that the cam remains fixed. Now the follower performs both the relative turning and translating motions. We have inverted the mechanism.

Furthermore, imagine that the knife-edge of the follower moves along the fixed cam profile in the inverted mechanism. In other words, the knife edge of the follower draws the profile of the cam. Thus, the problem of designing the cam profile becomes a problem of calculating the trace of the knife edge of the follower whose motion is the combination of the relative turning and the relative translating.

Design equations:

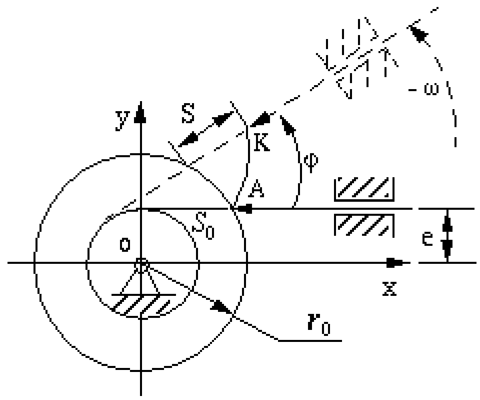


Figure 6-13 Profile design of translating cam follower

In Figure 6-13, only part of the cam profile *AK* is displayed. Assume the cam turns clockwise. At the beginning of motion, the knife edge of the follower contacts the point of intersection *A* of the base circle and the cam profile. The coordinates of *A* are (*So*, *e*), and *So* can be calculated from equation $S_0 = \sqrt{r_0^2 - e^2}$

Suppose the displacement of the follower is *S* when the angular displacement of the cam is Φ. At this moment, the coordinates of the knife edge of the follower should be (*So* + *S*, *e*).

To get the corresponding position of the knife edge of the follower in the inverted mechanism, turn the follower around the center of the cam in the reverse direction through an angle of Φ. The knife edge will be inverted to point *K*, which corresponds to the point on the cam profile in the inverted mechanism. Therefore, the coordinates of point *K* can be calculated with the following equation:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(IW \cdot \varphi) & -\sin(IW \cdot \varphi) \\ \sin(IW \cdot \varphi) & \cos(IW \cdot \varphi) \end{bmatrix} \begin{bmatrix} S_0 + S \\ e \end{bmatrix}$$

(6-5)

Note:

- The offset *e* is negative if the follower is located below the *x* axis.
- When the rotational direction of the cam is clockwise: *IW = +1*, otherwise: *IW = -1*.

6.5.2 Disk Cam with Oscillating Knife-Edge Follower

Suppose the cam mechanism will be used to make the knife edge oscillate. We need to compute the coordinates of the cam profile that results in the required motion of the follower.

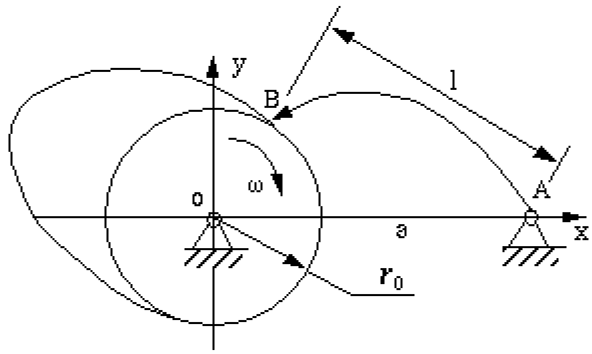


Figure 6-14 Disk cam with knife-edge oscillating follower

The essential parameters in this kind of cam mechanisms are given below.

r_o: The radius of the base circle;
a: The distance between the pivot of the cam and the pivot of the follower.
l: The length of the follower which is a distance from its pivot to its knife edge.
Ψ: The angular displacement of the follower which is a function of the rotary angle of the cam -- Φ.
IP: A parameter whose absolute value is 1. It represents the location of the follower. When the follower is located above the *x* axis: *IP=+1*, otherwise: *IP=-1*.
IW: A parameter whose absolute value is 1. It represents the turning direction of the cam. When the cam turns clockwise: *IW=+1*, otherwise: *IW=-1*.

Cam profile design principle

The fundamental principle in designing the cam profiles is still inversion, similar to that that for designing other cam mechanisms, (*e.g.*, the translating follower cam mechanism). Normally, the follower oscillates when the cam turns. This means that the relative motion between them is a combination of a relative turning motion and a relative oscillating motion. Without changing this feature of their relative motion, let the cam remain fixed and the follower performs both the relative turning motion and oscillating motion. By imagining in this way, we have actually inverted the mechanism.

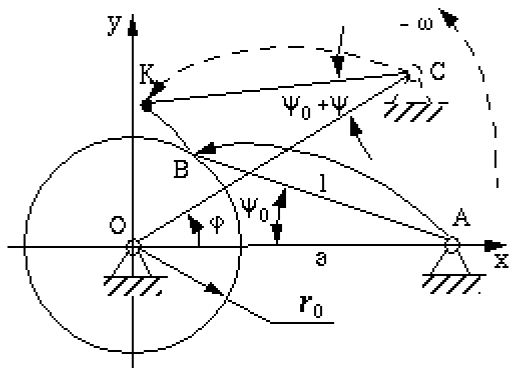


Figure 6-15 Cam profile design for a rotating follower

In Figure 6-15, only part of the cam profile *BK* is shown. We assume that the cam turns clockwise.

At the beginning of motion, the knife edge of the follower contacts the point of intersection (*B*) of the base circle and the cam profile. The initial angle between the follower (*AB*) and the line of two pivots (*AO*) is $\Psi 0$. It can be calculated from the triangle *OAB*.

When the angular displacement of the cam is Φ, the oscillating displacement of the follower is Ψ which measures from its own initial position. At this moment, the angle between the follower and the line passes through two pivots should be $\Psi + \Psi 0$.

The coordinates of the knife edge at this moment will be

$$(a - l\cos[IP(\psi + \psi_0)], l\sin[IP(\psi + \psi_0)])$$

(6-6)

To get the corresponding knife-edge of the follower in the inverted mechanism, simply turn the follower around the center of the cam in the reverse direction of the cam rotation through an angle of Φ. The knife edge will be inverted to point *K* which corresponds to the point on the cam profile in the inverted mechanism. Therefore, the coordinates of point *K* can be calculated with the following equation:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(IW \cdot \varphi) & -\sin(IW \cdot \varphi) \\ \sin(IW \cdot \varphi) & \cos(IW \cdot \varphi) \end{bmatrix} \begin{bmatrix} a - l\cos[IP(\psi + \psi_0)] \\ l\sin[IP(\psi + \psi_0)] \end{bmatrix}$$

(6-7)

Note:

- When the initial position of the follower is above the *x* axis, *IP = +1*, otherwise: *IP = -1*.

- When the rotary direction of the cam is clockwise: *IW = +1*, otherwise: *IW = -1*.

6.5.3 Disk Cam with Roller Follower

Additional parameters:

- *r*: the radius of the roller.
- *IM*: a parameter whose absolute value is 1, indicating which envelope curve will be adopted.
- *RM*: inner or outer envelope curve. When it is an inner envelope curve: *RM=+1*, otherwise: *RM=-1*.

Design principle:

The basic principle of designing a cam profile with the inversion method is still used. However, the curve is not directly generated by inversion. This procedure has two steps:

1. Imagine the center of the roller as a knife edge. This concept is important in cam profile design and is called the trace point) of follower. Calculate the pitch curve *aa*, that is, the trace of the pitch point in the inverted mechanism.
2. The cam profile *bb* is a product of the enveloping motion of a series of rollers.

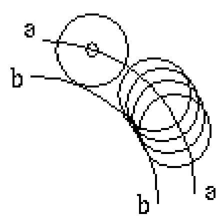


Figure 6-16 The trace point of the follower on a disk cam

Design equations:

The problem of calculating the coordinates of the cam profile is the problem of calculating the tangent points of a sequence of rollers in the inverted mechanism. At the moment shown Figure 6-17, the tangent point is *P* on the cam profile.

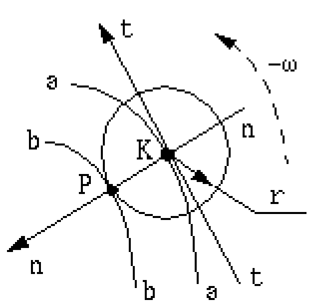


Figure 6-17 The tangent point, P, of a roller to the disk cam

The calculation of the coordinates of the point *P* has two steps:

1. Calculate the slope of the tangent *tt* of point *K* on pitch curve, *aa*.
2. Calculate the slope of the normal *nn* of the curve *aa* at point *K*.

Since we have already have the coordinates of point *K:* (*x, y*), we can express the coordinates of point *P* as

$$\begin{cases} x_p = x - IW \cdot RM \cdot r \cdot \dfrac{dy/d\varphi}{\sqrt{(dx/d\varphi)^2 + (dy/d\varphi)^2}} \\ y_p = y + IW \cdot RM \cdot r \cdot \dfrac{dx/d\varphi}{\sqrt{(dx/d\varphi)^2 + (dy/d\varphi)^2}} \end{cases}$$

(6-8)

Note:

- When the rotary direction of the cam is clockwise: *IW = +1*, otherwise: *IW = -1*.
- when the envelope curve (cam profile) lies inside the pitch curve: *RM = +1*, otherwise: *RM = -1*.

7 Gears

Gears are machine elements that transmit motion by means of successively engaging teeth. The gear teeth act like small levers.

7.1 Gear Classification

Gears may be classified according to the relative position of the axes of revolution. The axes may be

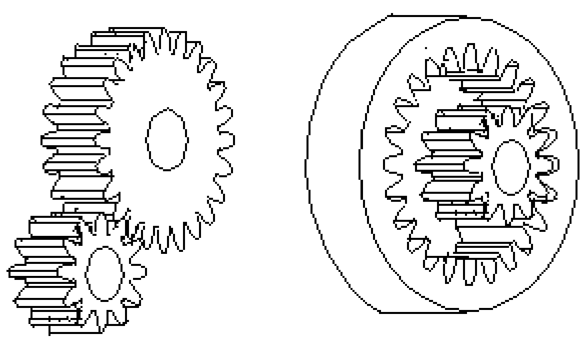
1. parallel,
2. intersecting,
3. neither parallel nor intersecting.

Here is a brief list of the common forms. We will discuss each in more detail later.

- Gears for connecting parallel shafts
- Gears for connecting intersecting shafts
- Neither parallel nor intersecting shafts

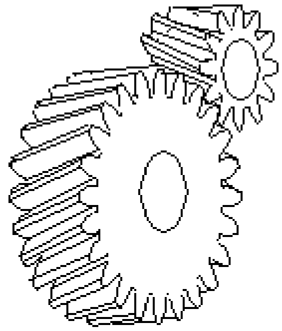
Gears for connecting parallel shafts

1. *Spur gears*

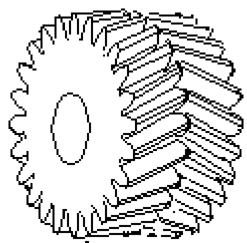


The left pair of gears makes **external contact**, and the right pair of gears makes **internal contact**

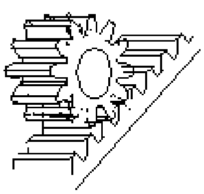
2. *Parallel helical gears*



2. *Herringbone gears* (or double-helical gears)

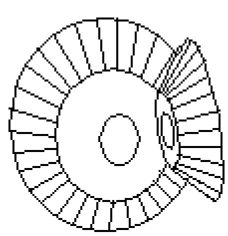


3. *Rack* and *pinion* (The rack is like a gear whose axis is at infinity.)



Gears for connecting intersecting shafts

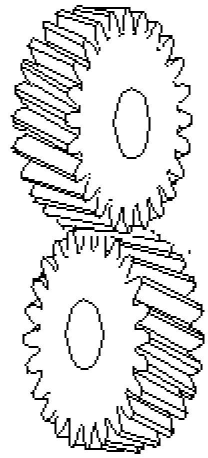
1. *Straight bevel gears*



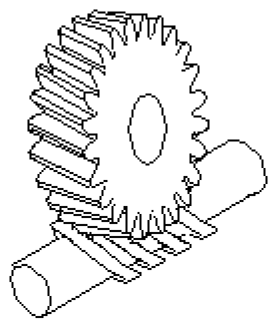
2. *Spiral bevel gears*

Neither parallel nor intersecting shafts

1. *Crossed-helical gears*



2. *Hypoid gears*
3. *Worm and wormgear*



7.2 Gear-Tooth Action

7.2.1 Fundamental Law of Gear-Tooth Action

Figure 7-2 shows two mating gear teeth, in which

- Tooth profile 1 drives tooth profile 2 by acting at the instantaneous contact point *K*.
- N_1N_2 is the common normal of the two profiles.
- N_1 is the foot of the perpendicular from O_1 to N_1N_2
- N_2 is the foot of the perpendicular from O_2 to N_1N_2.

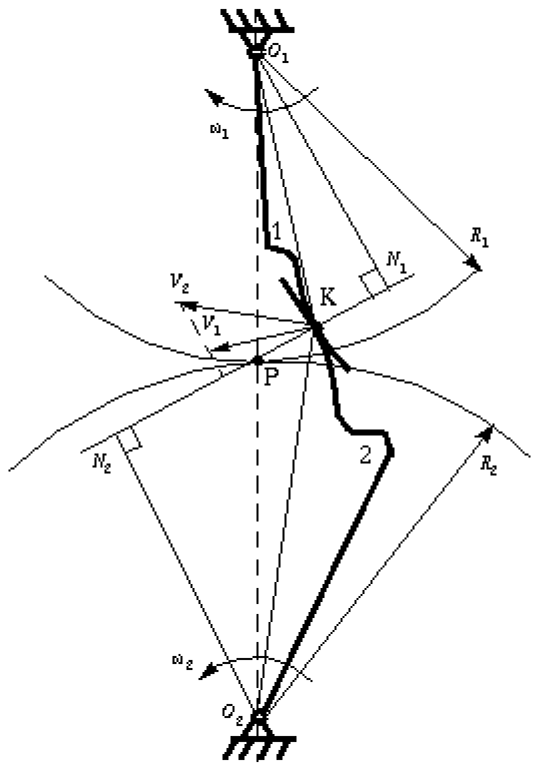


Figure 7-2 Two gearing tooth profiles

Although the two profiles have different velocities $\boldsymbol{V_1}$ and $\boldsymbol{V_2}$ at point *K*, their velocities along N_1N_2 are equal in both magnitude and direction. Otherwise the two tooth profiles would separate from each other. Therefore, we have

$$O_1N_1 \cdot \omega_1 = O_2N_2 \cdot \omega_2$$

(7-1)

or

$$\frac{\omega_1}{\omega_2} = \frac{O_2N_2}{O_1N_1}$$

(7-2)

We notice that the intersection of the tangency N_1N_2 and the line of center O_1O_2 is point *P*, and

$$\Delta O_1N_1P \sim \Delta O_2N_2P$$

(7-3)

Thus, the relationship between the angular velocities of the driving gear to the driven gear, or **velocity ratio**, of a pair of mating teeth is

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P}$$

(7-4)

Point *P* is very important to the velocity ratio, and it is called the **pitch point**. Pitch point divides the line between the line of centers and its position decides the velocity ratio of the two teeth. The above expression is the **fundamental law of gear-tooth action**.

7.2.2 Constant Velocity Ratio

For a constant velocity ratio, the position of *P* should remain unchanged. In this case, the motion transmission between two gears is equivalent to the motion transmission between two imagined slipless cylinders with radius R_1 and R_2 or diameter D_1 and D_2. We can get two circles whose centers are at O_1 and O_2, and through pitch point *P*. These two circle are termed **pitch circles**. The velocity ratio is equal to the inverse ratio of the diameters of pitch circles. This is the fundamental law of gear-tooth action.

The **fundamental law of gear-tooth action** may now also be stated as follow (for gears with fixed center distance) (Ham 58):

The common normal to the tooth profiles at the point of contact must always pass through a fixed point (the pitch point) on the line of centers (to get a constant velocity ration).

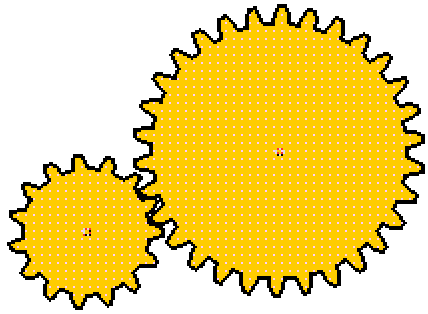
7.2.3 Conjugate Profiles

To obtain the expected *velocity ratio* of two tooth profiles, the normal line of their profiles must pass through the corresponding pitch point, which is decided by the *velocity ratio*. The two profiles which satisfy this requirement are called **conjugate profiles**. Sometimes, we simply termed the tooth profiles which satisfy the *fundamental law of gear-tooth action* the *conjugate profiles*.

Although many tooth shapes are possible for which a mating tooth could be designed to satisfy the fundamental law, only two are in general use: the *cycloidal* and *involute* profiles. The involute has important advantages -- it is easy to manufacture and the center distance between a pair of involute gears can be varied without changing the velocity ratio. Thus close tolerances between shaft locations are not required when using the involute profile. The most commonly used *conjugate* tooth curve is the *involute curve* (Erdman & Sandor 84).

7.3 Involute Curve

The following examples are involute spur gears. We use the word *involute* because the contour of gear teeth curves inward. Gears have many terminologies, parameters and principles. One of the important concepts is the *velocity ratio*, which is the ratio of the rotary velocity of the driver gear to that of the driven gears.



The SimDesign file for these gears is `simdesign/gear15.30.sim`. The number of teeth in these gears are 15 and 30, respectively. If the 15-tooth gear is the driving gear and the 30-teeth gear is the driven gear, their velocity ratio is 2.

Other examples of gears are in `simdesign/gear10.30.sim` and `simdesign/gear20.30.sim`

7.3.1 Generation of the Involute Curve

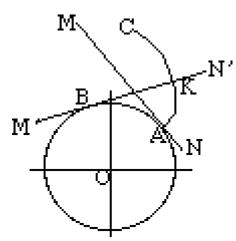


Figure 7-3 Involute curve

The curve most commonly used for gear-tooth profiles is the involute of a circle. This **involute curve** is the path traced by a point on a line as the line rolls without slipping on the circumference of a circle. It may also be defined as a path traced by the end of a string which is originally wrapped on a circle when the string is unwrapped from the circle. The circle from which the involute is derived is called the **base circle**.

In Figure 7-3, let line *MN* roll in the counterclockwise direction on the circumference of a circle without slipping. When the line has reached the position *M'N'*, its original point of tangent *A* has reached the position *K*, having traced the involute curve *AK* during the motion. As the motion continues, the point *A* will trace the involute curve *AKC*.

7.3.2 Properties of Involute Curves

1. The distance *BK* is equal to the arc *AB*, because link *MN* rolls without slipping on the circle.
2. For any instant, the *instantaneous center* of the motion of the line is its point of tangent with the circle.

Note: We have not defined the term *instantaneous center* previously. The **instantaneous center** or **instant center** is defined in two ways (Bradford & Guillet 43):

1. When two bodies have planar relative motion, the instant center is a point on one body about which the other rotates at the instant considered.
2. When two bodies have planar relative motion, the instant center is the point at which the bodies are relatively at rest at the instant considered.

3. The normal at any point of an involute is tangent to the base circle. Because of the property (2) of the involute curve, the motion of the point that is tracing the involute is perpendicular to the line at any instant, and hence the curve traced will also be perpendicular to the line at any instant.
4. There is no involute curve within the base circle.

7.4 Terminology for Spur Gears

Figure 7-4 shows some of the terms for gears.

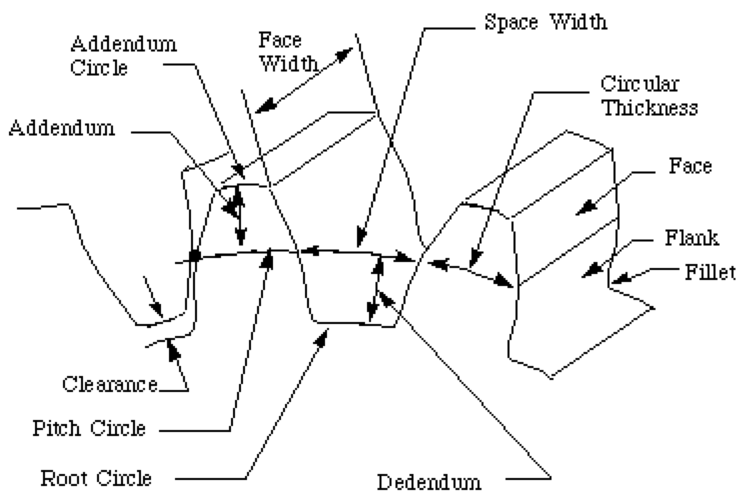


Figure 7-4 Spur Gear

In the following section, we define many of the terms used in the analysis of spur gears. Some of the terminology has been defined previously but we include them here for completeness. (See (Ham 58) for more details.)

- **Pitch surface** : The surface of the imaginary rolling cylinder (cone, etc.) that the toothed gear may be considered to replace.
- **Pitch circle**: A right section of the pitch surface.
- **Addendum circle**: A circle bounding the ends of the teeth, in a right section of the gear.
- **Root (or dedendum) circle**: The circle bounding the spaces between the teeth, in a right section of the gear.
- **Addendum**: The radial distance between the pitch circle and the addendum circle.
- **Dedendum**: The radial distance between the pitch circle and the root circle.

- **Clearance**: The difference between the dedendum of one gear and the addendum of the mating gear.
- **Face of a tooth**: That part of the tooth surface lying outside the pitch surface.
- **Flank of a tooth**: The part of the tooth surface lying inside the pitch surface.
- **Circular thickness** (also called the **tooth thickness**) : The thickness of the tooth measured on the pitch circle. It is the length of an arc and not the length of a straight line.
- **Tooth space**: The distance between adjacent teeth measured on the pitch circle.
- **Backlash**: The difference between the circle thickness of one gear and the tooth space of the mating gear.
- **Circular pitch** p: The width of a tooth and a space, measured on the pitch circle.
- **Diametral pitch** P: The number of teeth of a gear per inch of its pitch diameter. A toothed gear must have an integral number of teeth. The *circular pitch*, therefore, equals the pitch circumference divided by the number of teeth. The *diametral pitch* is, by definition, the number of teeth divided by the *pitch diameter*. That is,

  $$p = \frac{\pi D}{N}$$

  (7-5)

  and

  $$P = \frac{N}{D}$$

  (7-6)

  Hence

  $$pP = \pi$$

  (7-7)

  where

  p = circular pitch
  P = diametral pitch
  N = number of teeth
  D = pitch diameter

  That is, the product of the diametral pitch and the circular pitch equals π.

- **Module** m: Pitch diameter divided by number of teeth. The pitch diameter is usually specified in inches or millimeters; in the former case the module is the inverse of diametral pitch.
- **Fillet** : The small radius that connects the profile of a tooth to the root circle.

- **Pinion**: The smaller of any pair of mating gears. The larger of the pair is called simply the gear.
- **Velocity ratio**: The ratio of the number of revolutions of the driving (or input) gear to the number of revolutions of the driven (or output) gear, in a unit of time.
- **Pitch point**: The point of tangency of the pitch circles of a pair of mating gears.
- **Common tangent**: The line tangent to the pitch circle at the pitch point.
- **Line of action**: A line normal to a pair of mating tooth profiles at their point of contact.
- **Path of contact**: The path traced by the contact point of a pair of tooth profiles.
- **Pressure angle** α: The angle between the common normal at the point of tooth contact and the common tangent to the pitch circles. It is also the angle between the line of action and the common tangent.
- **Base circle** :An imaginary circle used in involute gearing to generate the involutes that form the tooth profiles.

Table 7-1 lists the standard tooth system for spur gears. (Shigley & Uicker 80)

Tooth system	Pressure angle α ,deg	Addendum a	Dedendum
Full depth	20	$\frac{1}{P_d}$ or 1.0m	$\frac{1.25}{P_d}$ or 1.25m $\frac{1.35}{P_d}$ or 1.35m
	$22\frac{1}{2}$	$\frac{1}{P_d}$ or 1.0m	$\frac{1.25}{P_d}$ or 1.25m $\frac{1.35}{P_d}$ or 1.35m
	25	$\frac{1}{P_d}$ or 1.0m	$\frac{1.25}{P_d}$ or 1.25m $\frac{1.35}{P_d}$ or 1.35m
Stub	20	$\frac{0.8}{P_d}$ or 0.8m	$\frac{1}{P_d}$ or 1m

Table 7-1 Standard tooth systems for spur gears

Table 7-2 lists the commonly used diametral pitches.

Coarse pitch	2	2.25	2.5	3	4	6	8	10	12	16
Fine pitch	20	24	32	40	48	64	96	120	150	200

Table 7-2 Commonly used diametral pitches

Instead of using the theoretical pitch circle as an index of tooth size, the base circle, which is a more fundamental circle, can be used. The result is called the **base pitch** p_b, and it is related to the circular pitch p by the equation

$$p_b = p \cos\alpha$$

(7-8)

7.5 Condition for Correct Meshing

Figure 7-5 shows two meshing gears contacting at point K_1 and K_2.

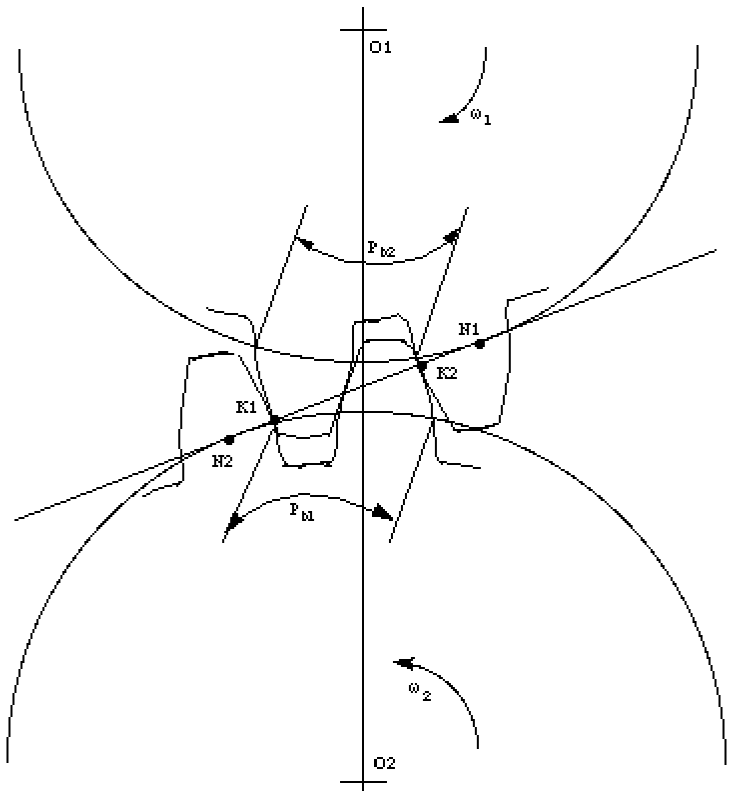


Figure 7-5 Two meshing gears

To get a correct meshing, the distance of K_1K_2 on gear 1 should be the same as the distance of K_1K_2 on gear 2. As K_1K_2 on both gears are equal to the base pitch of their gears, respectively. Hence

$$p_{b1} = p_{b2}$$

(7-9)

Since

$$p_{b1} = p_1 \cos\alpha_1 = \frac{\pi}{P_1} \cos\alpha_1$$

(7-10)

and

$$p_{b2} = p_2 \cos\alpha_2 = \frac{\pi}{P_2} \cos\alpha_2$$

(7-11)

Thus

$$\frac{1}{P_1}\cos\alpha_1 = \frac{1}{P_2}\cos\alpha_2$$

(7-12)

To satisfy the above equation, the pair of meshing gears must satisfy the following condition:

$$\begin{cases} P_1 = P_2 \\ \alpha_1 = \alpha_2 \end{cases}$$

(7-13)

7.6 Ordinary Gear Trains

Gear trains consist of two or more gears for the purpose of transmitting motion from one axis to another. **Ordinary gear trains** have axes, relative to the frame, for all gears comprising the train. Figure 7-6a shows a **simple ordinary train** in which there is only one gear for each axis. In Figure 7-6b a **compound ordinary train** is seen to be one in which two or more gears may rotate about a single axis.

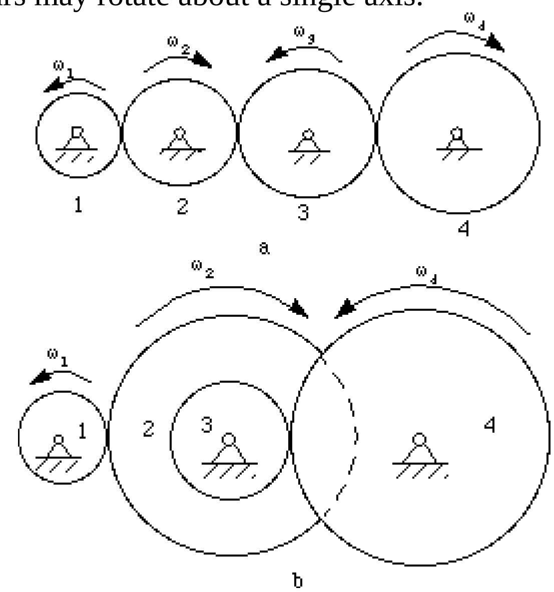


Figure 7-6 Ordinary gear trains

7.6.1 Velocity Ratio

We know that the **velocity ratio** of a pair of gears is the inverse proportion of the diameters of their pitch circle, and the diameter of the pitch circle equals to the number of teeth divided by the diametral pitch. Also, we know that it is necessary for the to mating gears to have the same diametral pitch so that to satisfy the condition of correct meshing.

Thus, we infer that the **velocity ratio** of a pair of gears is the inverse ratio of their number of teeth.

For the ordinary gear trains in Figure 7-6a, we have

$$\frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} \qquad \frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} \qquad \frac{\omega_3}{\omega_4} = \frac{N_4}{N_3}$$

(7-14)

These equations can be combined to give the velocity ratio of the first gear in the train to the last gear:

$$\frac{\omega_1}{\omega_4} = \frac{N_2 N_3 N_4}{N_1 N_2 N_3} = \frac{N_4}{N_1}$$

(7-15)

Note:

- The tooth number in the numerator are those of the driven gears, and the tooth numbers in the denominator belong to the driver gears.
- Gear 2 and 3 both drive and are, in turn, driven. Thus, they are called **idler gears**. Since their tooth numbers cancel, idler gears do not affect the magnitude of the input-output ratio, but they do change the directions of rotation. Note the directional arrows in the figure. Idler gears can also constitute a saving of space and money (If gear 1 and 4 meshes directly across a long center distance, their pitch circle will be much larger.)
- There are two ways to determine the direction of the rotary direction. The first way is to label arrows for each gear as in Figure 7-6. The second way is to multiple *m*th power of "*-1*" to the general velocity ratio. Where *m* is the number of pairs of external contact gears (internal contact gear pairs do not change the rotary direction). However, the second method cannot be applied to the spatial gear trains.

Thus, it is not difficult to get the velocity ratio of the gear train in Figure 7-6b:

$$\frac{\omega_1}{\omega_4} = (-1)^2 \frac{N_2 N_4}{N_1 N_3}$$

(7-16)

7.7 Planetary gear trains

Planetary gear trains, also referred to as **epicyclic gear trains**, are those in which one or more gears orbit about the central axis of the train. Thus, they differ from an ordinary train by having a moving axis or axes. Figure 7-8 shows a basic arrangement that is functional by itself or when used as a part of a more complex system. Gear 1 is called a **sun gear** , gear 2 is a **planet**, link H is an **arm**, or **planet carrier**.

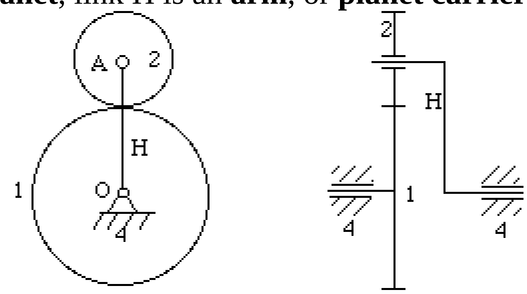


Figure 7-8 Planetary gear trains

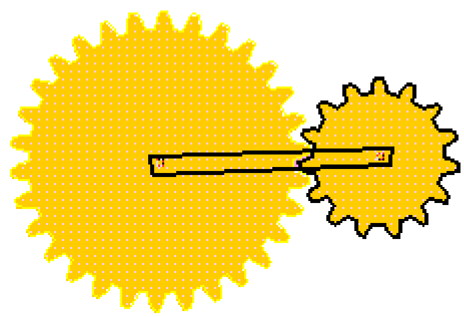


Figure 7-7 Planetary gears modeled using SimDesign

The SimDesign file is `simdesign/gear.planet.sim`. Since the sun gear (the largest gear) is fixed, the DOF of the above mechanism is one. When you pull the arm or the planet, the mechanism has a definite motion. If the sun gear isn't frozen, the relative motion is difficult to control.

7.7.1 Velocity Ratio

To determine the velocity ratio of the planetary gear trains is slightly more complex an analysis than that required for ordinary gear trains. We will follow the procedure:

1. Invert the planetary gear train mechanism by imagining the application a rotary motion with an angular velocity of ω_H to the mechanism. Let's analyse the motion before and after the inversion with Table 7-3:

	Before inversion (Original mechanism)	After inversion (Imagined mechanism)
Arm	ω_H	$\omega_H - \omega_H = 0$
Frame	0	$0 - \omega_H = -\omega_H$
Sun	ω_1	$\omega_1 - \omega_H = \omega_1^H$
Planet	ω_2	$\omega_2 - \omega_H = \omega_2^H$

Table 7-3 Inversion of planetary gear trains.

Note: ω_H *is the rotary velocity of gear i in the imagined mechanism.*

Notice that in the imagined mechanism, the arm *H* is stationary and functions as a frame. No axis of gear moves any more. Hence, the imagined mechanism is an ordinary gear train.

2. Apply the equation of velocity ratio of the ordinary gear trains to the imagined mechanism. We get

$$\frac{\omega_1^H}{\omega_2^H} = -\frac{N_2}{N_1}$$

(7-17)

or

$$\frac{\omega_1 - \omega_H}{\omega_2 - \omega_H} = -\frac{N_2}{N_1}$$

(7-18)

7.7.2 Example

Take the planetary gearing train in Figure 7-8 as an example. Suppose $N_1 = 36$, $N_2 = 18$, $\omega_1 = 0$, $\omega_2 = 30$. What is the value of ω_N?

With the application of the velocity ratio equation for the planetary gearing trains, we have the following equation:

$$\frac{\omega_1}{\omega_2} = \frac{\omega_1 - \omega_H}{\omega_2 - \omega_H} = -\frac{N_2}{N_1}$$

(7-19)

From the equation and the given conditions, we can get the answer: $\omega_N = 10$.

Chapter 8. Other Mechanisms

8.1 Ratchet Mechanisms

A wheel provided with suitably shaped teeth, receiving an intermittent circular motion from an oscillating or reciprocating member, is called a **ratchet wheel**. A simple form of ratchet mechanism is shown in Figure 8-1.

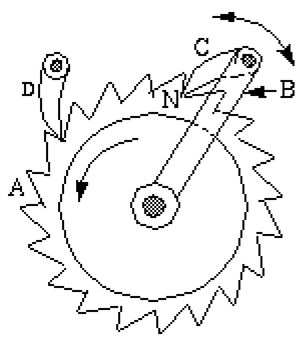


Figure 8-1 Ratchet

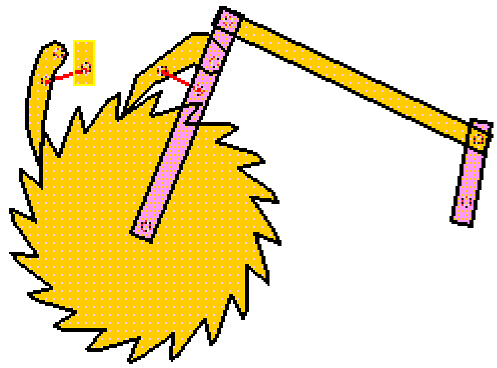
A is the **ratchet wheel**, and *B* is an oscillating lever carrying the **driving pawl**, *C*. A **supplementary pawl** at *D* prevents backward motion of the wheel.

When arm *B* moves counterclockwise, pawl *C* will force the wheel through a fractional part of a revolution dependent upon the motion of *B*. When the arm moves back (clockwise), pawl *C* will slide over the points of the teeth while the wheel remains at rest because of fixed pawl *D*, and will be ready to push the wheel on its forward (counterclockwise) motion as before.

The amount of backward motion possible varies with the pitch of the teeth. This motion could be reduced by using small teeth, and the expedient is sometimes used by placing several pawls side by side on the same axis, the pawls being of different lengths.

The contact surfaces of wheel and pawl should be inclined so that they will not tend to disengage under pressure. This means that the common normal at *N* should pass between the pawl and the ratchet-wheel centers. If this common normal should pass outside these limits, the pawl would be forced out of contact under load unless held by friction. In many ratchet mechanisms the pawl is held against the wheel during motion by the action of a spring.

The usual form of the teeth of a ratchet wheel is that shown in the above Figure, but in feed mechanisms such as used on many machine tools it is necessary to modify the tooth shape for a reversible pawl so that the drive can be in either direction. The following SimDesign example of a ratchet also includes a four bar linkage.



If you try this mechanism, you may turn the crank of the link mechanism. The rocker will drive the driving pawl to drive the ratchet wheel. The corresponding SimDesign data file is:

```
/afs/andrew.cmu.edu/cit/ce/rapidproto/simdesign/ratchet.sim
```

8.2 Overrunning Clutch

A special form of a ratchet is the **overrunning clutch**. Have you ever thought about what kind of mechanism drives the rear axle of bicycle? It is a free-wheel mechanism which is an overrunning clutch. Figure 8-2 illustrates a simplified model. As the driver delivers torque to the driven member, the rollers or balls are wedged into the tapered recesses. This is what gives the positive drive. Should the driven member attempt to drive the driver in the directions shown, the rollers or balls become free and no torque is transmitted.

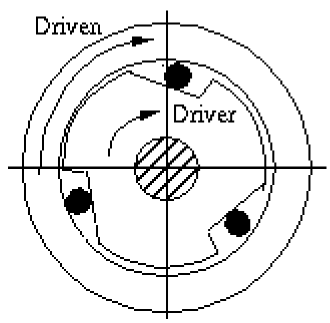


Figure 8-2 Overrunning clutch

8.3 Intermittent Gearing

A pair of rotating members may be designed so that, for continuous rotation of the driver, the follower will alternately roll with the driver and remain stationary. This type of arrangement is know by the general term **intermittent gearing**. This type of gearing occurs in some counting mechanisms, motion-picture machines, feed mechanisms, as well as others.

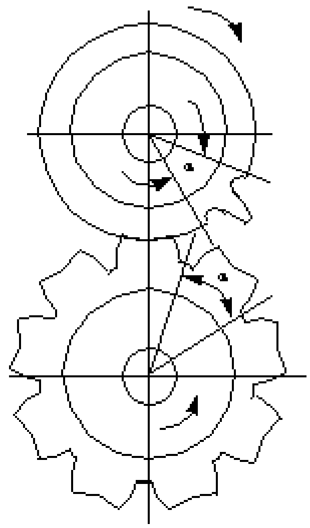


Figure 8-3 Intermittent gearing

The simplest form of intermittent gearing, as illustrated in Figure 8-3 has the same kind of teeth as ordinary gears designed for continuous rotation. This example is a pair of 18-tooth gears modified to meet the requirement that the follower advance one-ninth of a turn for each turn of the driver. The interval of action is the two-pitch angle (indicated on both gears). The single tooth on the driver engages with each space on the follower to produce the required motion of a one-ninth turn of the follower. During the remainder of a driver turn, the follower is locked against rotation in the manner shown in the figure.

To vary the relative movements of the driver and follower, the meshing teeth can be arranged in various ways to suit requirements. For example, the driver may have more than one tooth, and the periods of rest of the follower may be uniform or may vary considerably. Counting mechanisms are often equipped with gearing of this type.

8.4 The Geneva Wheel

An interesting example of intermittent gearing is the **Geneva Wheel** shown in Figure 8-4. In this case the **driven wheel**, *B*, makes one fourth of a turn for one turn of the **driver**, *A*, the **pin**, *a*, working in the **slots**, *b*, causing the motion of *B*. The circular portion of the driver, coming in contact with the corresponding hollow circular parts of the driven wheel, retains it in position when the pin or tooth *a* is out of action. The wheel *A* is cut away near the pin *a* as shown, to provide clearance for wheel *B* in its motion.

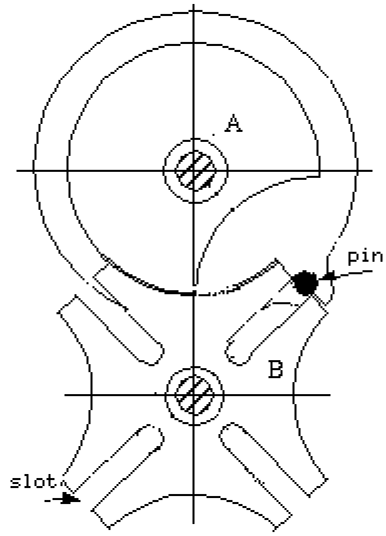
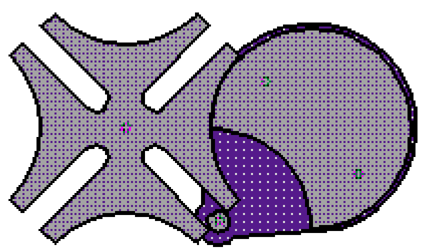


Figure 8-4 Geneva wheel

If one of the slots is closed, *A* can only move through part of the revolution in either direction before pin *a* strikes the closed slot and thus stops the motion. The device in this modified form was used in watches, music boxes, *etc.*, to prevent overwinding. From this application it received the name **Geneva stop**. Arranged as a stop, wheel *A* is secured to the spring shaft, and *B* turns on the axis of the spring barrel. The number of slots or interval units in *B* depends upon the desired number of turns for the spring shaft.

An example of this mechanism has been made in SimDesign, as in the following picture.



The corresponding SimDesign data file is:

`/afs/andrew.cmu.edu/cit/ce/rapidproto/simdesign/geneva.sim`

8.5 The Universal Joint

The engine of an automobile is usually located in front part. How does it connect to the rear axle of the automobile? In this case, universal joints are used to transmit the motion.

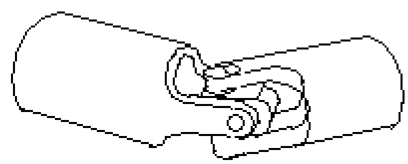


Figure 8-5 Universal joint

The **universal joint** as shown in Figure 8-5 is also known in the older literature as **Hooke's coupling**. Regardless of how it is constructed or proportioned, for practical use it has essentially the form shown in Figure 8-6, consisting of two semicircular forks 2 and 4, pin-jointed to a right -angle cross 3.

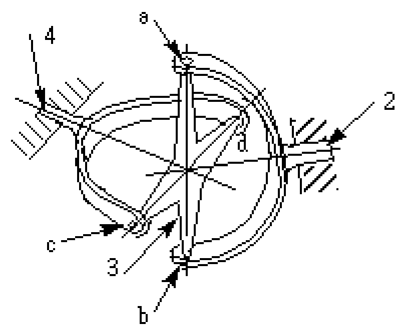


Figure 8-6 General form for a universal joint

The driver 2 and the follower 4 make the complete revolution at the same time, but the velocity ratio is not constant throughout the revolution. The following analysis will show how complete information as to the relative motions of driver and follower may be obtained for any phase of the motion.

8.5.1 Analysis of a Universal Joint

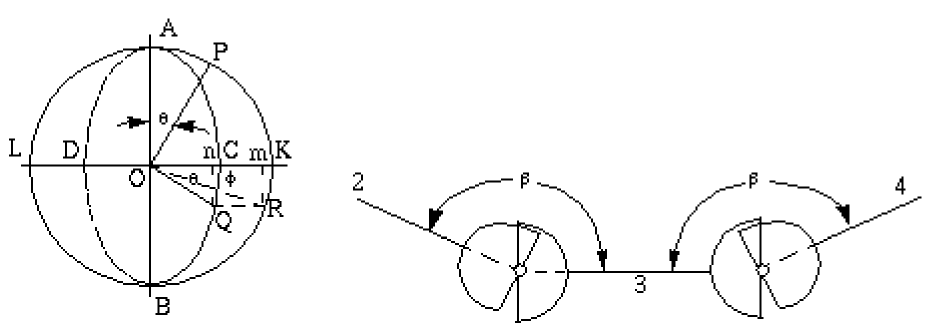


Figure 8-7 Analysis of a universal joint

If the plane of projection is taken perpendicular to the axis of 2, the path of *a* and *b* will be a circle *AKBL* as shown in Figure 8-7.

If the angle between the shafts is β, the path of *c* and *d* will be a circle that is projected as the ellipse *ACBD*, in which

$$OC = OD = OK\cos\beta = OA\cos\beta$$
(8-1)

If one of the arms of the driver is at *A*, an arm of the follower will be at *C*. If the driver arm moves through the angle θ to *P*, the follower arm will move to *Q*. *OQ* will be perpendicular to *OP*; hence: angle *COQ* = θ. But angle *COQ* is the projection of the real angle describes by the follower. *Qn* is the real component of the motion of the follower in a direction parallel to *AB*, and line *A*B is the intersection of the planes of the driver's and the follower's planes. The true angle Φ described by the follower, while the driver describes the angle θ, can be found by revolving *OQ* about AB as an axis into the plane of the circle *AKBL*. Then *OR* = the true length of *OQ*, and *ROK* = Φ= the true angle that is projected as angle *COQ* = θ.

Now

$$\tan\Phi = Rm/Om$$

and

$$\tan\theta = Qn/On$$

But

$$Qn = Rm$$

Hence

$$\frac{\tan\theta}{\tan\phi} = \frac{Om}{On} = \frac{OK}{OC} = \frac{1}{\cos\beta}$$

Therefore

$$\tan\Phi = \cos\beta\tan\theta$$

The ratio of the angular motion of the follower to that of the driver is found as follower, by differentiating above equation, remembering that β is constant

$$\frac{\omega_4}{\omega_2} = \frac{d\phi}{d\theta} = \frac{\cos\beta\sec^2\theta}{\sec^2\phi} = \frac{\cos\beta\sec^2\theta}{1 + \tan^2\phi}$$

Eliminating Φ:

$$\frac{\omega_4}{\omega_2} = \frac{\cos\beta \sec^2\theta}{1 + \cos^2\beta \tan^2\theta} = \frac{\cos\beta}{1 - \sin^2\beta \sin^2\theta}$$

Similarly, θ can be eliminated:

$$\frac{\omega_4}{\omega_2} = \frac{1 - \cos^2\phi \sin^2\beta}{\cos\beta}$$

According to the above equations, when the driver has a uniform angular velocity, the ratio of angular velocities varies between extremes of $\cos\beta$ and $1/\cos\beta$. These variations in velocity give rise to inertia forces, torques, noise, and vibration which would not be present if the velocity ratio were constant.

8.5.2 Double Universal Joint

By using a double joint shown on the right in Figure 8-7, the variation of angular motion between driver and follower can be entirely avoided. This compensating arrangement is to place an intermediate shaft 3 between the driver and follower shafts. The two forks of this intermediate shaft must lie in the same plane, and the angle between the first shaft and the intermediate shaft must exactly be the same with that between the intermediate shaft and the last shaft. If the first shaft rotates uniformly, the angular motion of the intermediate shaft will vary according to the result deduced above. This variation is exactly the same as if the last shaft rotated uniformly, driving the intermediate shaft. Therefore, the variable motion transmitted to the intermediate shaft by the uniform rotation of the first shaft is exactly compensated for by the motion transmitted from the intermediate to the last shaft, the uniform motion of either of these shafts will impart, through the intermediate shaft, uniform motion to the other.

Universal joints, particularly in pairs, are used in many machines. One common application is in the drive shaft which connects the engine of an automobiles to the axle.

Index of Mechanism Terminologies

- pinion
- pitch circle
- pitch point
- pitch surface
- pressure angle
- root circle
- tooth space
- tooth thickness
- velocity ratio
-

gear train (1)
gear train (2)
Geneva Wheel
Geneva stop

- driven wheel
- driver
- pin
- slot

- ordinary gear train
  - simple ordinary train
  - compound ordinary train
- planetary gear train

Grashof's criterion
Grashof's theorem
Gruebler's equation

[h]
harmonic motion
higher pair
Hook's coupling

[i]
idler gear
impulse
inertial mass
inclined plane
instantaneous center
instant center
intermittent gearing
internal contact gears
inversion
involute curve

[j]

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