Solutions Manual for
Fluid Mechanics: Fundamentals and Applications
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CHAPTER 13
OPEN-CHANNEL FLOW

Classification, Froude Number, and Wave Speed

13-1C

Solution We are to discuss differences between open-channel flow and internal flow.

Analysis *Open-channel flow* is the **flow of liquids in channels open to the atmosphere or in partially filled conduits**, and is characterized by the presence of a **liquid-gas interface called the *free surface***, whereas *internal flow* is the **flow of liquids or gases that completely fill a conduit**.

Discussion At a free surface of a liquid, the pressure must be equal to the pressure of the gas above it. This is what controls the pressure in open-channel flow. There is no such restriction in internal (pipe) flow since there is no free surface.

13-2C

Solution We are to discuss the driving force in open-channel flow and how flow rate is determined.

Analysis Flow in a channel is driven **naturally by gravity**. Water flow in a river, for example, is driven by the elevation difference between the source and the sink. The flow rate in an open channel is established by the **dynamic balance between gravity and friction**. Inertia of the flowing fluid also becomes important in unsteady flow.

Discussion In pipe flow, on the other hand, there may be an additional driving force of pressure due to pumps.

13-3C

Solution We are to discuss how pressure changes along the free surface in open-channel flow.

Analysis The free surface **coincides with the hydraulic grade line** (HGL), and the **pressure is constant along the free surface**.

Discussion At a free surface of a liquid, the pressure must be equal to the pressure of the gas above it.

13-4C

Solution We are to determine if the slope of the free surface is equal to the slope of the channel bottom.

Analysis **No in general**. The slope of the free surface is **not necessarily equal** to the slope of the bottom surface even during steady fully developed flow.

Discussion However, there are situations called *uniform flow* in which the conditions here are met.

13-5C

Solution We are to discuss the difference between uniform and nonuniform flow.

Analysis The flow in a channel is said to be *uniform* if the **flow depth (and thus the average velocity) remains constant**. Otherwise, the flow is said to be *nonuniform* or *varied*, indicating that the flow depth varies with distance in the flow direction. Uniform flow conditions are commonly encountered in practice in long straight sections of channels with constant slope and constant cross-section.

Discussion In uniform open-channel flow, the head loss due to frictional effects equals the elevation drop.

13-6C

Solution We are to define normal depth and how it is established.

Analysis In open channels of constant slope and constant cross-section, the fluid accelerates until the head loss due to frictional effects equals the elevation drop. The fluid at this point reaches its terminal velocity, and uniform flow is established. The flow remains uniform as long as the slope, cross-section, and the surface roughness of the channel remain unchanged. **The flow depth in uniform flow** is called the *normal depth* y_n, which is an important characteristic parameter for open-channel flows.

Discussion The normal depth is a fairly strong function of surface roughness.

13-7C

Solution We are to discuss some reasons for nonuniform flow in open channels, and the difference between rapidly varied flow and gradually varied flow.

Analysis The **presence of an obstruction in a channel such as a gate or a change in slope or cross-section** causes the flow depth to vary, and thus the flow to become varied or nonuniform. The varied flow is called *rapidly varied flow* (RVF) if the flow depth changes markedly over a relatively short distance in the flow direction (such as the flow of water past a partially open gate or shortly before a falls), and *gradually varied flow* (GVF) if the flow depth changes gradually over a long distance along the channel.

Discussion The equations of GVF are simplified because of the slow changes in the flow direction.

13-8C

Solution We are to define and discuss hydraulic radius.

Analysis The *hydraulic radius* R_h is defined as **the ratio of the cross-sectional flow area A_c and the wetted perimeter p**. That is, $R_h = A_c/p$. Knowing the hydraulic radius, the hydraulic diameter is determined from $D_h = 4R_h$.

Discussion It is unfortunate (and not our fault!) that hydraulic radius is ¼ rather than ½ of hydraulic diameter.

13-9C

Solution We are to explain how to determine if a flow is tranquil, critical, or rapid.

Analysis Knowing the average flow velocity and flow depth, the Froude number is determined from $\mathrm{Fr} = V/\sqrt{gy}$. Then the flow is classified as

Fr < 1 **Subcritical or tranquil flow**
Fr = 1 **Critical flow**
Fr > 1 **Supercritical or rapid flow**

Discussion The Froude number is the most important parameter in open-channel flow.

13-10C

Solution We are to define and discuss the usefulness of the Froude number.

Analysis *Froude number*, defined as $\boxed{\mathrm{Fr} = V/\sqrt{gy}}$, is a **dimensionless parameter that governs the character of flow in open channels**. Here, g is the gravitational acceleration, V is the mean fluid velocity at a cross-section, and L_c is a characteristic length (L_c = flow depth y for wide rectangular channels). Fr represents the **ratio of inertia forces to viscous forces in open-channel flow**. The Froude number is also the **ratio of the flow speed to wave speed**, $\mathrm{Fr} = V/c_o$.

Discussion The Froude number is the most important parameter in open-channel flow.

13-11C

Solution We are to define critical length, and discuss how it is determined.

Analysis The **flow depth y_c corresponding to a Froude number of Fr = 1** is the *critical depth*, and it is determined from $\boxed{V=\sqrt{gy_c}}$ or $\boxed{y_c = V^2/g}$.

Discussion Critical depth is a useful parameter, even if the depth does not actually equal y_c anywhere in the flow.

13-12C

Solution We are to discuss whether the flow upstream of a hydraulic jump must be supercritical, and whether the flow downstream of a hydraulic jump must be subcritical.

Analysis Upstream of a hydraulic jump, the **upstream flow must be supercritical**. Downstream of a hydraulic jump, the **downstream flow must be subcritical**.

Discussion Otherwise, the second law of thermodynamics would be violated. Note that a hydraulic jump is analogous to a normal shock wave – in that case, the flow upstream must be supersonic and the flow downstream must be subsonic.

13-13

Solution The flow of water in a wide channel is considered. The speed of a small disturbance in flow for two different flow depths is to be determined for both water and oil.

Assumptions The distance across the wave is short and thus friction at the bottom surface and air drag at the top are negligible,

Analysis Surface wave speed can be determined directly from the relation $c_0 = \sqrt{gh}$.

(a) $c_0 = \sqrt{gh} = \sqrt{(9.81\text{ m/s}^2)(0.1\text{ m})} = \mathbf{0.990\ m/s}$

(b) $c_0 = \sqrt{gh} = \sqrt{(9.81\text{ m/s}^2)(0.8\text{ m})} = \mathbf{2.80\ m/s}$

Therefore, a disturbance in the flow will travel at a speed of 0.990 m/s in the first case, and 2.80 m/s in the second case.

Discussion Note that wave speed depends on the water depth, and the wave speed increases as the water depth increases as long as the water remains shallow. Results would not change if the fluid were oil, because the wave speed depends only on the fluid depth.

13-14

Solution Water flows uniformly in a wide rectangular channel. For given flow depth and velocity, it is to be determined whether the flow is laminar or turbulent, and whether it is subcritical or supercritical.

Assumptions The flow is uniform.

Properties The density and dynamic viscosity of water at 20ºC are ρ = 998.0 kg/m^3 and μ = 1.002×10^{-3} kg/m·s.

Analysis (a) The Reynolds number of the flow is $\text{Re} = \dfrac{\rho V y}{\mu} = \dfrac{(998.0\text{ kg/m}^3)(2\text{ m/s})(0.2\text{ m})}{1.002\times 10^{-3}\text{ kg/m}\cdot\text{s}} = 3.984\times 10^5$, which is greater than the critical value of 500. Therefore, the flow is **turbulent**.

(*b*) The Froude number is $\text{Fr} = \dfrac{V}{\sqrt{gy}} = \dfrac{2\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.2\text{ m})}} = 1.43$, which is greater than 1.

Therefore, the flow is **supercritical**.

Discussion The result in (*a*) is expected since almost all open channel flows are turbulent. Also, hydraulic radius for a wide rectangular channel approaches the water depth *y* as the ratio *y*/b approaches zero.

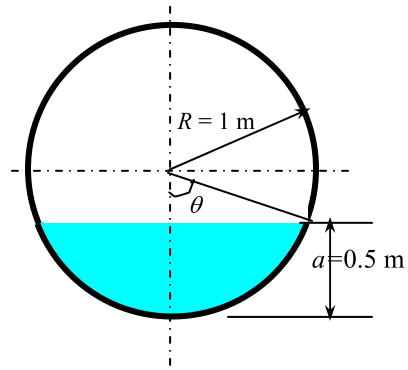
13-15

Solution Water flow in a partially full circular channel is considered. For given water depth and average velocity, the hydraulic radius, Reynolds number, and the flow regime are to be determined.

Assumptions **1** The flow is uniform.

Properties The density and dynamic viscosity of water at 20ºC are $\rho = 998.0$ kg/m^3 and $\mu = 1.002\times10^{-3}$ kg/m·s.

Analysis From geometric considerations,



$$\cos\theta = \frac{R-a}{R} = \frac{1-0.5}{1} = 0.5 \quad\rightarrow\quad \theta = 60° = 60\frac{2\pi}{360} = \frac{\pi}{3}$$

Then the hydraulic radius becomes

$$R_h = \frac{A_c}{p} = \frac{\theta - \sin\theta\cos\theta}{2\theta}R = \frac{\pi/3 - \sin(\pi/3)\cos(\pi/3)}{2\pi/3}(1\text{ m}) = \mathbf{0.293\ m}$$

The Reynolds number of the flow is

$$\text{Re} = \frac{\rho V R_h}{\mu} = \frac{(998.0\text{ kg/m}^3)(2\text{ m/s})(0.293\text{ m})}{1.002\times10^{-3}\text{ kg/m}\cdot\text{s}} = \mathbf{5.84\times10^5}$$

which is greater than the critical value of 500. Therefore, the flow is turbulent.

When calculating the Froude number, the hydraulic depth should be used rather than the maximum depth or the hydraulic radius. For a non-rectangular channel, hydraulic depth is defined as the ratio of the flow area to top width,

$$A_c = R^2(\theta - \sin\theta\cos\theta) = (1\text{ m})^2[\pi/3 - \sin(\pi/3)\cos(\pi/3)] = 0.6142\text{ m}^2$$

$$y_h = \frac{A_c}{\text{Top width}} = \frac{A_c}{2R\sin\theta} = \frac{0.6142\text{ m}^2}{2(1\text{ m})\sin 60°} = 0.3546\text{ m} \quad\rightarrow\quad \text{Fr} = \frac{V}{\sqrt{gy_h}} = \frac{2\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.3546\text{ m})}} = 1.07$$

which is greater than 1. Therefore, the flow is **supercritical** (although, very close to critical).

Discussion Note that if the maximum flow depth were used instead of the hydraulic depth, the result would be subcritical flow, which is not true.

13-16

Solution Water flows uniformly in a wide rectangular channel. For given values of flow depth and velocity, it is to be determined whether the flow is subcritical or supercritical.

Assumptions **1** The flow is uniform. **2** The channel is wide and thus the side wall effects are negligible.

Analysis The Froude number is $\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{4\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.08\text{ m})}} = 4.51$, which is greater than 1.

Therefore, the flow is **supercritical**.

Discussion Note that the Froude Number is not function of any temperature-dependent properties, and thus temperature.

13-17

Solution Rain water flows on a concrete surface. For given values of flow depth and velocity, it is to be determined whether the flow is subcritical or supercritical.

Assumptions **1** The flow is uniform. **2** The thickness of water layer is constant.

Analysis The Froude number is $\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{1.3\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.02\text{ m})}} = 2.93$, which is greater than 1.

Therefore, the flow is **supercritical**.

Discussion This water layer will undergo a hydraulic jump when the ground slope decreases or becomes adverse.

13-18E

Solution Water flows uniformly in a wide rectangular channel. For given flow depth and velocity, it is to be determined whether the flow is laminar or turbulent, and whether it is subcritical or supercritical.

Assumptions The flow is uniform.

Properties The density and dynamic viscosity of water at 70ºF are ρ = 62.30 lbm/ft^3 and μ = 6.556×10^{-4} lbm/ft·s.

Analysis (*a*) The Reynolds number of the flow is $\text{Re} = \frac{\rho V y}{\mu} = \frac{(62.30\text{ lbm/ft}^3)(6\text{ ft/s})(0.5\text{ ft})}{6.556\times10^{-4}\text{ lbm/ft}\cdot\text{s}} = 2.85\times10^5$, which is greater than the critical value of 500. Therefore, the flow is **turbulent**.

(*b*) The Froude number is $\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{6\text{ ft/s}}{\sqrt{(32.2\text{ ft/s}^2)(0.5\text{ ft})}} = 1.50$, which is greater than 1.

Therefore, the flow is **supercritical**.

Discussion The result in (*a*) is expected since almost all open channel flows are turbulent. Also, hydraulic radius for a wide rectangular channel approaches the water depth *y* as the ratio *y*/b approaches zero.

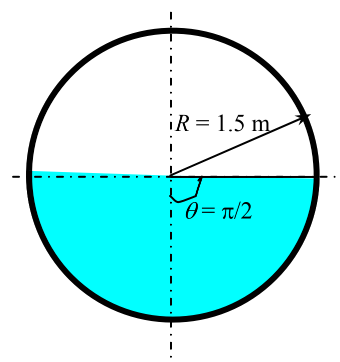
13-19

Solution Water flows uniformly through a half-full circular channel. For a given average velocity, the hydraulic radius, the Reynolds number, and the flow regime are to be determined.

Assumptions The flow is uniform.

Properties The density and dynamic viscosity of water at 10ºC are ρ = 999.7 kg/m^3 and μ = 1.307×10^{-3} kg/m·s.

Analysis From geometric considerations, the hydraulic radius is



$$R_h = \frac{A_c}{p} = \frac{\pi R^2/2}{\pi R} = \frac{R}{2} = \frac{1.5\text{ m}}{2} = \mathbf{0.75\ m}$$

The Reynolds number of the flow is

$$\text{Re} = \frac{\rho V R_h}{\mu} = \frac{(999.7\text{ kg/m}^3)(2.5\text{ m/s})(0.75\text{ m})}{1.307\times10^{-3}\text{ kg/m}\cdot\text{s}} = \mathbf{1.43\times10^6}$$, which is

greater than the critical value of 500. Therefore, **the flow is turbulent**.

When calculating the Froude number, the hydraulic depth should be used rather than the maximum depth or the hydraulic radius. For a non-rectangular channel, hydraulic depth is defined as the ratio of the flow area to top width,

$$y_h = \frac{A_c}{\text{Top width}} = \frac{\pi R^2/2}{2R} = \frac{\pi R}{4} = \frac{\pi(1.5\text{ m})}{4} = 1.178\text{ m}$$

$$\text{Fr} = \frac{V}{\sqrt{g y_h}} = \frac{2.5\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(1.178\text{ m})}} = 0.735$$, which is greater than 1. Therefore, the flow is **subcritical**.

Discussion If the maximum flow depth were used instead of the hydraulic depth, the result would still be subcritical flow, but this is not always the case.

13-20

Solution A single wave is initiated in a sea by a strong jolt during an earthquake. The speed of the resulting wave is to be determined.

Assumptions The depth of water is constant,

Analysis Surface wave speed is determined the wave-speed relation to be

$$c_0 = \sqrt{gh} = \sqrt{(9.81\,\text{m/s}^2)(2000\,\text{m})} = \mathbf{140\ m/s}$$

Discussion Note that wave speed depends on the water depth, and the wave speed increases as the water depth increases. Also, the waves eventually die out because of the viscous effects.

Specific Energy and the Energy Equation

13-21C

Solution We are to define and discuss specific energy.

Analysis The *specific energy* E_s of a fluid flowing in an open channel is the **sum of the pressure and dynamic heads of a fluid**, and is expressed as $\boxed{E_s = y + \dfrac{V^2}{2g}}$.

Discussion Specific energy is very useful when analyzing varied flows.

13-22C

Solution We are to compare the specific energy in two flows – one subcritical and one supercritical.

Analysis A plot of E_s versus y for constant $\dot{V}$ through a rectangular channel of width b reveals that there are two y values corresponding to a fixed value of E_s: one for subcritical flow and one for supercritical flow. Therefore, **the specific energies of water in those two channels can be identical**.

Discussion If the flow is varied (not uniform), however, E_s is not *necessarily* identical in the two channels.

13-23C

Solution We are to examine claims about the minimum value of specific energy.

Analysis The point of minimum specific energy is the critical point, and thus **the first person is correct**.

Discussion The specific energy cannot go below the critical point for a given volume flow rate, as is clear from the plot of specific energy as a function of flow depth.

13-24C

Solution We are to examine a claim about supercritical flow of water in an open channel, namely, that the larger the flow depth, the larger the specific energy.

Analysis **No, the claim is incorrect**. A plot of E_s versus y for constant $\dot{V}$ reveals that the *specific energy decreases as the flow depth increases during supercritical channel flow*.

Discussion This may go against our intuition, since a larger flow depth seems to imply greater energy, but this is not necessarily the case (we cannot always trust our intuition).

13-25C

Solution We are to examine a claim that specific energy remains constant in steady uniform flow.

Analysis The **first person (who claims that specific energy remains constant) is correct** since in uniform flow, the flow depth and the flow velocity, and thus the specific energy, remain constant since $E_s = y + V^2/2g$. The head loss is made up by the decline in elevation (the channel is sloped downward in the flow direction).

Discussion In uniform flow, the flow depth and the average velocity do not change downstream, since the elevation drop exactly overcomes the frictional losses.

13-26C

Solution We are to define and discuss friction slope.

Analysis The *friction slope* is related to head loss h_L, and is defined as $\boxed{S_f = h_L / L}$ where L is the channel length. The **friction slope is equal to the bottom slope when the head loss is equal to the elevation drop**. That is, $S_f = S_0$ when $h_L = z_1 - z_2$.

Discussion Friction slope is a useful concept when analyzing uniform or varied flow in open channels.

13-27C

Solution We are to examine a claim that during steady flow in a wide rectangular channel, the energy line of the flow is parallel to the channel bottom when the frictional losses are negligible.

Analysis **No, the claim is not correct**. The energy line is a distance $E_s = y + V^2/2g$ (total mechanical energy of the fluid) above a horizontal reference datum. When there is no head loss, the energy line is horizontal even when the channel is not. The elevation and velocity heads ($z + y$ and $V^2/2g$) may convert to each other during flow in this case, but their sum remains constant.

Discussion Keep in mind that in real life, there is no such thing as frictionless flow. However, there are situations in which the frictional effects are negligible compared to other effects in the flow.

13-28C

Solution We are to examine a claim that during steady 1-D flow through a wide rectangular channel, the total mechanical energy of the fluid at the free surface is equal to that of the fluid at the channel bottom.

Analysis **Yes, the claim is correct**. During steady one-dimensional flow, the total mechanical energy of a fluid at any point of a cross-section is given by $H = z + y + V^2/2g$.

Discussion The physical elevation of the point under consideration does not appear in the above equation for *H*.

13-29C

Solution We are to express the total mechanical energy in steady 1-D flow in terms of heads.

Analysis The total mechanical energy of a fluid at any point of a cross-section is expressed as $\boxed{H = z + y + V^2/2g}$ where *y* is the flow depth, *z* is the elevation of the channel bottom, and *V* is the average flow velocity. It is related to the specific energy of the fluid by $H = z + E_s$.

Discussion Because of irreversible frictional head losses, *H* must decrease in the flow direction in open-channel flow.

13-30C

Solution We are to express the 1-D energy equation for open-channel flow and discuss head loss.

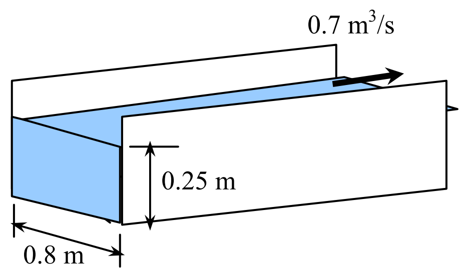
Analysis The one-dimensional energy equation for open channel flow between an upstream section 1 and downstream section 2 is written as $\boxed{z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g} + h_L}$ where y is the flow depth, z is the elevation of the channel bottom, and V is the average flow velocity. The head loss h_L due to frictional effects can be determined from $\boxed{h_L = f\frac{L}{R_h}\frac{V^2}{8g}}$ where f is the average friction factor and L is the length of channel between sections 1 and 2.

Discussion Head loss is always positive – it can never be negative since this would violate the second law of thermodynamics. Thus, the total mechanical energy must decrease downstream in open-channel flow.

13-31

Solution Water flow in a rectangular channel is considered. The character of flow, the flow velocity, and the alternate depth are to be determined.

Assumptions The specific energy is constant.



Analysis The average flow velocity is determined from

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{yb} = \frac{0.7\text{ m}^3/\text{s}}{(0.25\text{ m})(0.8\text{ m})} = \mathbf{3.50\text{ m/s}}$$

The critical depth for this flow is

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(0.7\text{ m}^3/\text{s})^2}{(9.81\text{ m/s}^2)(0.8\text{ m})^2}\right)^{1/3} = 0.427\text{ m}$$

Therefore, the flow is *supercritical* since the actual flow depth is y = 0.25 m, and $y < y_c$. The specific energy for given conditions is

$$E_{s1} = y_1 + \frac{\dot{V}^2}{2gb^2y_1^2} = (0.25\text{ m}) + \frac{(0.7\text{ m}^3/\text{s})^2}{2(9.81\text{ m/s}^2)(0.8\text{ m})^2(0.25\text{ m})^2} = 0.874\text{ m}$$

Then the alternate depth is determined from $E_{s1} = E_{s2}$ to be

$$E_{s2} = y_2 + \frac{\dot{V}^2}{2gb^2y_2^2} \quad\rightarrow\quad 0.874\text{ m} = y_2 + \frac{(0.7\text{ m}^3/\text{s})^2}{2(9.81\text{ m/s}^2)(0.8\text{ m})^2 y_2^2}$$

Solving for y_2 gives the alternate depth to be y_2 = **0.82 m**. There are three roots of this equation; one for subcritical, one for supercritical and third one as a negative root. Therefore, if the character of flow is changed from supercritical to subcritical while holding the specific energy constant, the flow depth will rise from 0.25 m to 0.82 m.

Discussion Two alternate depths show two possible flow conditions for a given specific energy. If the energy is not the minimum specific energy, there are two water depths corresponding to subcritical and supercritical states of flow. As an example, these two depths may be observed before and after a sluice gate as alternate depths, if the losses are disregarded.

13-32

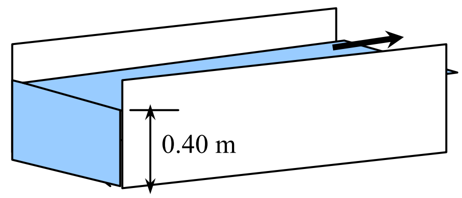
Solution Water flows in a rectangular channel. The specific energy and whether the flow is subcritical or supercritical are to be determined.

Assumptions The flow is uniform and thus the specific energy is constant.

Analysis For convenience, we take the channel width to be b = 1 m. Then the volume flow rate and the critical depth for this flow become

$$\dot{V} = VA_c = Vyb = (6\ \text{m/s})(0.4\ \text{m})(1\ \text{m}) = 2.40\ \text{m}^3/\text{s}$$

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(2.40\ \text{m}^3/\text{s})^2}{(9.81\ \text{m/s}^2)(1\ \text{m})^2}\right)^{1/3} = \mathbf{0.837\ m}$$



The flow is **supercritical** since the actual flow depth is y = 0.4 m, and $y < y_c$. The specific energy for given conditions is

$$E_{s1} = y_1 + \frac{\dot{V}^2}{2gb^2 y_1^2} = y_1 + \frac{V^2}{2g} = (0.4\ \text{m}) + \frac{(6\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = \mathbf{2.23\ m}$$

Discussion Note that the flow may also exist as subcritical flow at the same value of specific energy,

13-33 [*Also solved using EES on enclosed DVD*]

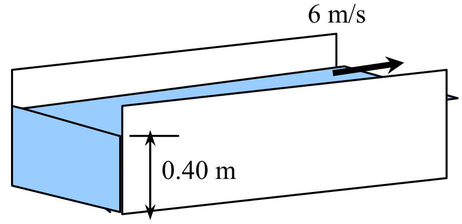
Solution Water flows in a rectangular channel. The critical depth, the alternate depth, and the minimum specific energy are to be determined.

Assumptions The channel is sufficiently wide so that the edge effects are negligible.

Analysis For convenience, we take the channel width to be b = 1 m. Then the volume flow rate and the critical depth for this flow become

$$\dot{V} = VA_c = Vyb = (6\ \text{m/s})(0.4\ \text{m})(1\ \text{m}) = 2.40\ \text{m}^3/\text{s}$$

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(2.40\ \text{m}^3/\text{s})^2}{(9.81\ \text{m/s}^2)(1\ \text{m})^2}\right)^{1/3} = \mathbf{0.837\ m}$$



(b) The flow is *supercritical* since the actual flow depth is y = 0.4 m, and $y < y_c$. The specific energy for given conditions is

$$E_{s1} = y_1 + \frac{\dot{V}^2}{2gb^2 y_1^2} = y_1 + \frac{V^2}{2g} = (0.4\ \text{m}) + \frac{(6\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 2.23\ \text{m}$$

Then the alternate depth is determined from $E_{s1} = E_{s2}$ to be

$$E_{s2} = y_2 + \frac{\dot{V}^2}{2gb^2 y_2^2} \quad \rightarrow \quad 2.23\ \text{m} = y_2 + \frac{0.240\ \text{m}^3/\text{s}}{y_2^2}$$

Solving for y_2 gives the alternate depth to be y_2 = **2.17 m**. Therefore, if the character of flow is changed from supercritical to subcritical while holding the specific energy constant, the flow depth will rise from 0.4 m to 2.17 m.

(c) the minimum specific energy is

$$E_{s,\min} = y_c + \frac{V_c^2}{2g} = y_c + \frac{gy_c}{2g} = \frac{3}{2}y_c = \frac{3}{2}\left(0.837\ \text{m}\right) = \mathbf{1.26\ m}$$

Discussion Note that minimum specific energy is observed when the flow depth is critical.

13-34

Solution Water flows in a rectangular channel. The critical depth, the alternate depth, and whether the flow is subcritical or supercritical are to be determined.

Assumptions The flow is uniform and thus the specific energy is constant.

Analysis (*a*) The critical depth is calculated to be $y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(12\ \text{m}^3/\text{s})^2}{(9.81\ \text{m/s}^2)(6\ \text{m})^2}\right)^{1/3} = 0.742\ \text{m}$

(*b*) The average flow velocity and the Froude number are

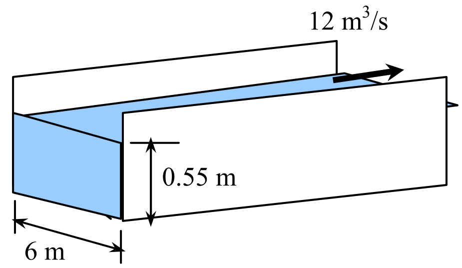
$$V = \frac{\dot{V}}{by} = \frac{12\ \text{m}^3/\text{s}}{(6\ \text{m})(0.55\ \text{m})} = 3.636\ \text{m/s} \quad \text{and} \quad \text{Fr}_1 = \frac{V}{\sqrt{gy}} = \frac{3.636\ \text{m/s}}{\sqrt{(9.81\ \text{m/s}^2)(0.55\ \text{m})}} = 1.565$$

, which is greater than 1.

Therefore, the flow is **supercritical**.

(*c*) Specific energy for this flow is

$$E_{s1} = y_1 + \frac{\dot{V}^2}{2gb^2 y_1^2} = (0.55\ \text{m}) + \frac{(12\ \text{m}^3/\text{s})^2}{2(9.81\ \text{m/s}^2)(6\ \text{m})^2(0.55\ \text{m})^2} = 1.224\ \text{m}$$



Then the alternate depth is determined from $E_{s1}=E_{s2}$,

$$E_{s2} = y_2 + \frac{\dot{V}^2}{2gb^2 y_2^2} \quad \rightarrow \quad 1.224\ \text{m} = y_2 + \frac{(12\ \text{m}^3/\text{s})^2}{2(9.81\ \text{m/s}^2)(6\ \text{m})^2 y_2^2}$$

The alternate depth is calculated to be y_2 = **1.03 m** which is the subcritical depth for the same value of specific energy.

Discussion The depths 0.55 m and 1.03 are alternate depths for the given discharge and specific energy. The flow conditions determine which one is observed.

13-35E

Solution Water flows in a wide rectangular channel. For specified values of flow depth and average velocity, the Froude number, critical depth, and whether the flow is subcritical or supercritical are to be determined.

Assumptions The flow is uniform and thus the specific energy is constant.

Analysis (*a*) The Froude number is $\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{14\ \text{ft/s}}{\sqrt{(32.2\ \text{ft/s}^2)(0.8\ \text{ft})}} = \mathbf{2.76}$

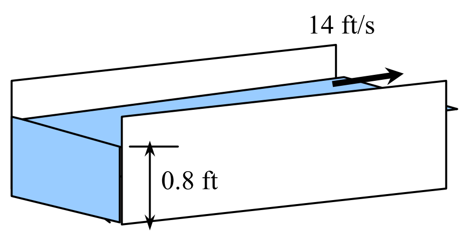
(*b*) The critical depth is calculated to be $y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{V^2 y^2 b^2}{gb^2}\right)^{1/3} = \left(\frac{(14\ \text{ft/s})^2(0.8\ \text{ft})^2}{(32.2\ \text{ft/s}^2)}\right)^{1/3} = \mathbf{1.57\ ft}$

(*c*) The flow is **supercritical** since Fr > 1.

For the case of *y* = 0.2 ft:
Replacing 0.8 ft in above calculations by 0.2 ft gives

$$\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{14\ \text{ft/s}}{\sqrt{(32.2\ \text{ft/s}^2)(0.2\ \text{ft})}} = \mathbf{5.52}$$

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{V^2 y^2 b^2}{gb^2}\right)^{1/3} = \left(\frac{(14\ \text{ft/s})^2(0.2\ \text{ft})^2}{(32.2\ \text{ft/s}^2)}\right)^{1/3} = \mathbf{0.625\ ft}$$



The flow is supercritical in this case also since Fr > 1.

Discussion Note that the value of critical depth depends on flow rate, and it decreases as the flow rate decreases.

13-36E

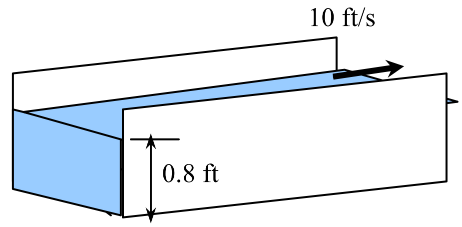
Solution Water flows in a wide rectangular channel. For specified values of flow depth and average velocity, the Froude number, critical depth, and whether the flow is subcritical or supercritical are to be determined.

Assumptions The flow is uniform and thus the specific energy is constant.

Analysis (*a*) The Froude number is $\mathrm{Fr} = \dfrac{V}{\sqrt{gy}} = \dfrac{10\ \mathrm{ft/s}}{\sqrt{(32.2\ \mathrm{ft/s^2})(0.8\ \mathrm{ft})}} = \mathbf{1.97}$

(*b*) The critical depth is calculated to be $y_c = \left(\dfrac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\dfrac{V^2 y^2 b^2}{gb^2}\right)^{1/3} = \left(\dfrac{(10\ \mathrm{ft/s})^2 (0.8\ \mathrm{ft})^2}{(32.2\ \mathrm{ft/s^2})}\right)^{1/3} = \mathbf{1.26\ ft}$

(*c*) The flow is **supercritical** since Fr > 1.



For the case of y = 0.2 ft:
Replacing 0.8 ft in above calculations by 0.2 ft gives

$$\mathrm{Fr} = \frac{V}{\sqrt{gy}} = \frac{10\ \mathrm{ft/s}}{\sqrt{(32.2\ \mathrm{ft/s^2})(0.2\ \mathrm{ft})}} = \mathbf{3.94}$$

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{V^2 y^2 b^2}{gb^2}\right)^{1/3} = \left(\frac{(14\ \mathrm{ft/s})^2 (0.2\ \mathrm{ft})^2}{(32.2\ \mathrm{ft/s^2})}\right)^{1/3} = \mathbf{0.50\ ft}$$

The flow is supercritical in this case also since Fr > 1.

Discussion Note that the value of critical depth depends on flow rate, and it decreases as the flow rate decreases.

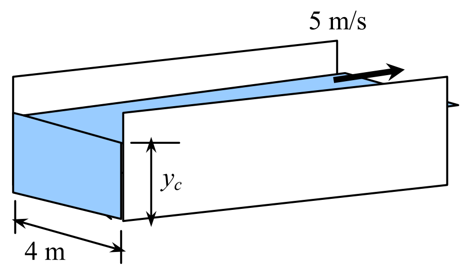
13-37

Solution Critical flow of water in a rectangular channel is considered. For a specified average velocity, the flow rate of water is to be determined.

Assumptions The flow is uniform and thus the specific energy is constant.

Analysis The Froude number must be unity since the flow is critical, and thus $\mathrm{Fr} = V/\sqrt{gy} = 1$. Therefore,

$$y = y_c = \frac{V^2}{g} = \frac{(5\ \mathrm{m/s})^2}{9.81\ \mathrm{m/s^2}} = \mathbf{2.55\ m}$$



Then the flow rate becomes

$$\dot{V} = VA_c = Vby = (5\ \mathrm{m/s})(4\ \mathrm{m})(2.55\ \mathrm{m}) = \mathbf{51.0\ m^3/s}$$

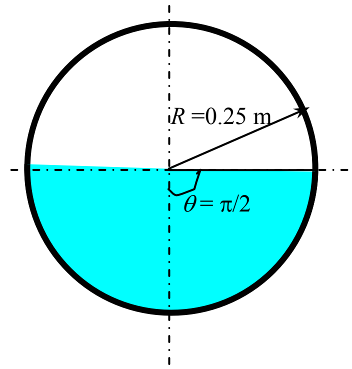
Discussion Critical flow is not a stable type of flow and can be observed for short intervals. Occurrence of critical depth is important as boundary condition most of the time. For example it can be used as a flow rate computation mechanism for a channel ending with a drawdown.

13-38

Solution Water flows uniformly through a half-full circular steel channel. For a given average velocity, the volume flow rate, critical slope, and the critical depth are to be determined.

Assumptions The flow is uniform.

Analysis The volume flow rate is determined from



$$\dot{V} = VA_c = V\frac{\pi R^2}{2} = (2.8\ \text{m/s})\frac{\pi(0.25\ \text{m})^2}{2} = \mathbf{0.275\ m^3/s}$$

When calculating the Froude number, the hydraulic depth should be used rather than the maximum depth or the hydraulic radius. For a non-rectangular channel, hydraulic depth is defined as the ratio of the flow area to top width,

$$y_h = \frac{A_c}{\text{Top width}} = \frac{\pi R^2/2}{2R} = \frac{\pi R}{4} = \frac{\pi(0.25\ \text{m})}{4} = 0.1963\ \text{m}$$

$$\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{2.8\ \text{m/s}}{\sqrt{(9.81\ \text{m/s}^2)(0.1963\ \text{m})}} = 2.02$$

which is greater than 1. Therefore, the flow is **supercritical**.

Discussion Note that if the maximum flow depth were used instead of the hydraulic depth, the result could be different, especially when the Froude number is close to 1.

13-39

Solution Water flows uniformly through a half-full hexagon channel. For a given flow rate, the average velocity and whether the flow is subcritical or supercritical are to be determined.

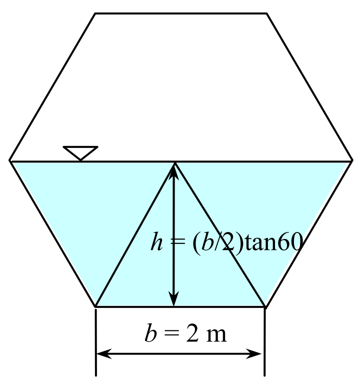
Assumptions The flow is uniform.

Analysis (*a*) The flow area is determined from geometric considerations to be

$$A_c = \frac{(b+2b)}{2}\frac{b}{2}\tan 60° = \frac{(2+2\times 2)\ \text{m}}{2}\frac{2\ \text{m}}{2}\tan 60° = 5.196\ \text{m}^2$$

Then the average velocity becomes

$$V = \frac{\dot{V}}{A_c} = \frac{45\ \text{m}^3/\text{s}}{5.196\ \text{m}^2} = \mathbf{8.66\ m/s}$$



(*b*) When calculating the Froude number, the hydraulic depth should be used rather than the maximum depth or the hydraulic radius. For a non-rectangular channel, hydraulic depth is defined as the ratio of the flow area to top width,

$$y = y_h = \frac{A_c}{\text{Top width}} = \frac{A_c}{2b} = \frac{5.196\ \text{m}}{2\times 2\ \text{m}} = 1.299\ \text{m}$$

Then the Froude number becomes

$$\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{8.66\ \text{m/s}}{\sqrt{(9.81\ \text{m/s}^2)(1.299\ \text{m})}} = 2.43$$

which is greater than 1. Therefore, the flow is **supercritical**.

Discussion The analysis is approximate since the edge effects are significant here compared to a wide rectangular channel, and thus the results should be interpreted accordingly.

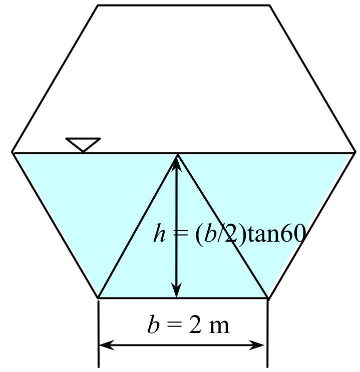
13-40

Solution Water flows uniformly through a half-full hexagon channel. For a given flow rate, the average velocity and whether the flow is subcritical or supercritical are to be determined.

Assumptions The flow is uniform.

Analysis The flow area is determined from geometric considerations to be

$$A_c = \frac{(b+2b)}{2}\frac{b}{2}\tan 60° = \frac{(2+2\times 2)\text{ m}}{2}\frac{2\text{ m}}{2}\tan 60° = 5.196\text{ m}^2$$



Then the average velocity becomes

$$V = \frac{\dot{V}}{A_c} = \frac{30\text{ m}^3/\text{s}}{5.196\text{ m}^2} = \mathbf{5.77\ m/s}$$

When calculating the Froude number, the hydraulic depth should be used rather than the maximum depth or the hydraulic radius. For a non-rectangular channel, hydraulic depth is defined as the ratio of the flow area to top width,

$$y = y_h = \frac{A_c}{\text{Top width}} = \frac{A_c}{2b} = \frac{5.196\text{ m}^2}{2\times 2\text{ m}} = 1.299\text{ m}$$

Then the Froude number becomes

$$\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{5.77\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(1.299\text{ m})}} = 1.62$$

which is greater than 1. Therefore, the flow is **supercritical**.

Discussion The analysis is approximate since the edge effects are significant here compared to a wide rectangular channel, and thus the results should be interpreted accordingly.

Uniform Flow and Best Hydraulic Cross Sections

13-41C

Solution We are to discuss when flow in an open channel is uniform, and how it remains uniform.

Analysis Flow in a channel is called *uniform flow* if the **flow depth (and thus the average flow velocity) remains constant**. **The flow remains uniform as long as the slope, cross-section, and the surface roughness of the channel remain unchanged**.

Discussion Uniform flow in open-channel flow is somewhat analogous to fully developed pipe flow in internal flow.

13-42C

Solution We are to discuss how flow depth changes when the bottom slope is increased.

Analysis The flow depth **decreases** when the bottom slope is increased.

Discussion You can think of it in simple terms this way: As the slope increases, the liquid flows faster, and faster flow requires lower depth.

13-43C

Solution We are to examine a claim that head loss can be determined by multiplying bottom slope by channel length.

Analysis **Yes, the claim is correct**. The head loss in uniform flow is $h_L = S_0 L$ since the head loss must equal elevation loss.

Discussion In uniform flow, frictional head losses are exactly balanced by elevation loss, which is directly proportional to bottom slope.

13-44C

Solution We are to discuss the constants and coefficients in the Manning equation.

Analysis The value of the factor a in SI units is $\boldsymbol{a}$ **= 1 m$^{1/3}$/s**. Combining the relations $C = \sqrt{8g/f}$ and $C = \frac{a}{n} R_h^{1/6}$ and solving them for n gives the desired relation to be $\boxed{n = \frac{a}{\sqrt{8g/f}} R_h^{1/6}}$. In practice, n is usually determined experimentally.

Discussion The value of n varies greatly with surface roughness.

13-45C

Solution It is to be shown that for uniform critical flow, the general critical slope relation $S_c = \frac{gn^2 y_c}{a^2 R_h^{4/3}}$ reduces to $S_c = \frac{gn^2}{a^2 y_c^{1/3}}$ for film flow with $b >> y_c$.

Analysis For critical flow, the flow depth is $y = y_c$. For film flow, the hydraulic radius is $R_h = y = y_c$. Substituting into the critical slope relation gives the desired result, $\boxed{S_c = \frac{gn^2 y_c}{a^2 R_h^{4/3}} = \frac{gn^2 y_c}{a^2 y_c^{4/3}} = \frac{gn^2}{a^2 y_c^{1/3}}}$.

Discussion The reduced equation is valid for film flow only – be careful not to apply it to channels of other shapes.

13-46C

Solution We are to determine which cross section is better – one with a small or large hydraulic radius.

Analysis The best hydraulic cross-section for an open channel is the one with the **maximum hydraulic radius**, or equivalently, the one with the minimum wetted perimeter for a specified cross-sectional area.

Discussion Frictional losses occur at the wetted perimeter walls of the channel, so it makes sense to minimize the wetted perimeter in order to minimize the frictional losses.

13-47C

Solution We are to determine which cross section shape is best for an open channel.

Analysis The best hydraulic cross-section for an open channel is a (*a*) **circular** one.

Discussion Circular channels are often more difficult to construct, however, so they are often not used in practice.

13-48C

Solution We are to determine the best hydraulic cross section for a rectangular channel.

Analysis The best hydraulic cross section for a rectangular channel is one whose fluid height is (*a*) **half the channel width**.

Discussion It turns out that for this case, the wetted perimeter, and thus the frictional losses, are smallest.

13-49C

Solution We are to determine the best hydraulic cross section for a trapezoidal channel.

Analysis The best hydraulic cross section for a trapezoidal channel of base width *b* is (*a*) one for which **the length of the side edge of the flow section is *b***.

Discussion It turns out that for this case, the wetted perimeter, and thus the frictional losses, are smallest.

13-50C

Solution We are to determine how the flow rate changes when the Manning coefficient doubles.

Analysis The flow rate in uniform flow is given as $\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2}$, and thus the flow rate is inversely proportional to the Manning coefficient. Therefore, if the Manning coefficient doubles as a result of some algae growth on surfaces while the flow cross section remains constant, the flow rate will (*d*) **decrease by half**.

Discussion In an actual case, the cross section may also change due to flow depth changes as well.

13-51

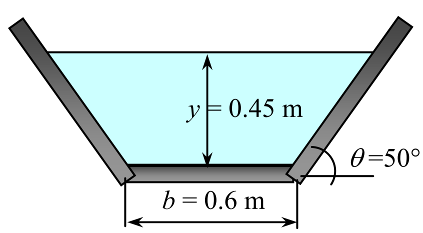
Solution The flow of water in a trapezoidal finished-concrete channel is considered. For a given flow depth and bottom slope, the flow rate is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties Manning coefficient for an open channel of finished concrete is $n = 0.012$ (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are

$$A_c = y\left(b + \frac{y}{\tan\theta}\right) = (0.45\text{ m})\left(0.60\text{ m} + \frac{0.45\text{ m}}{\tan 50°}\right) = 0.3724\ \text{m}^2$$



$$p = b + \frac{2y}{\sin\theta} = 0.6\text{ m} + \frac{2(0.45\text{ m})}{\sin 50°} = 1.775\text{ m}$$

$$R_h = \frac{A_c}{p} = \frac{0.3724\text{ m}^2}{1.775\text{ m}} = 0.2096\ \text{m}$$

Bottom slope of the channel is

$$S_0 = \tan 0.4° = 0.006981$$

Then the flow rate can be determined from Manning's equation to be

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{1\text{ m}^{1/3}/s}{0.012}(0.3724\text{ m}^2)(0.2096\text{ m})^{2/3}(0.006981)^{1/2} = \mathbf{0.915\ m^3/s}$$

Discussion Note that the flow rate in a given channel is a strong function of the bottom slope.

13-52

Solution Water flows uniformly half-full in a circular finished-concrete channel. For a given bottom slope, the flow rate is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

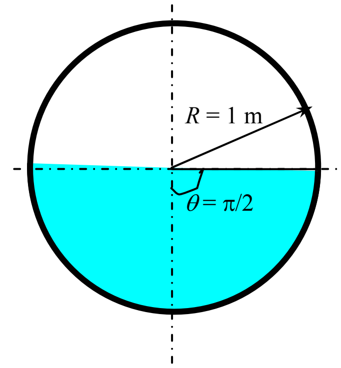
Properties Manning coefficient for an open channel of finished concrete is n = 0.012 (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are

$$A_c = \frac{\pi R^2}{2} = \frac{\pi (1\,\text{m})^2}{2} = 1.571\,\text{m}^2$$

$$p = \frac{2\pi R}{2} = \frac{2\pi(1\,\text{m})}{2} = 3.142\,\text{m}$$

$$R_h = \frac{A_c}{P} = \frac{\pi R^2/2}{\pi R} = \frac{R}{2} = \frac{1\,\text{m}}{2} = 0.50\ \text{m}$$



Then the flow rate can be determined from Manning's equation to be

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{1\,\text{m}^{1/3}/s}{0.012}(1.571\,\text{m}^2)(0.50\,\text{m})^{2/3}(1.5/1000)^{1/2} = \mathbf{3.19\ m^3/s}$$

Discussion Note that the flow rate in a given channel is a strong function of the bottom slope.

13-53E

Solution Water is to be transported uniformly in a full semi-circular unfinished-concrete channel. For a specified flow rate, the elevation difference across the channel is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

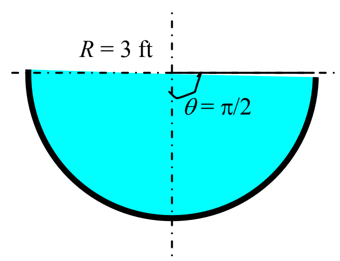
Properties Manning coefficient for an open channel of unfinished concrete is n = 0.014 (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are

$$A_c = \frac{\pi R^2}{2} = \frac{\pi (3\,\text{ft})^2}{2} = 14.14\,\text{ft}^2$$

$$p = \frac{2\pi R}{2} = \frac{2\pi(3\,\text{ft})}{2} = 9.425\,\text{ft}$$

$$R_h = \frac{A_c}{P} = \frac{\pi R^2/2}{\pi R} = \frac{R}{2} = \frac{3\,\text{ft}}{2} = 1.50\ \text{ft}$$



Substituting the given quantities into Manning's equation,

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \rightarrow \quad 150\,\text{ft}^3/\text{s} = \frac{1.486\,\text{ft}^{1/3}/s}{0.014}(14.14\,\text{ft}^2)(1.50\,\text{ft})^{2/3} S_0^{1/2}$$

It gives the slope to be S_0 = 0.005817. Therefore, the *elevation difference* Δz across a pipe length of L = 1 mile = 5280 ft must be

$$\Delta z = S_0 L = 0.005817(5280\,\text{ft}) = \mathbf{30.7\ ft}$$

Discussion Note that when transporting water through a region of fixed elevation drop, the only way to increase the flow rate is to use a channel with a larger cross-section.

13-54

Solution Water is to be transported uniformly in a trapezoidal asphalt-lined channel. For a specified flow rate, the required elevation drop per km channel length is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

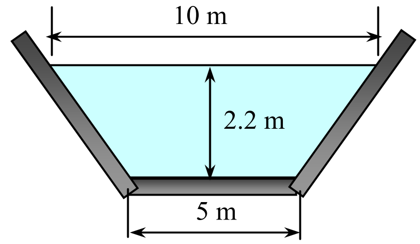
Properties Manning coefficient for an asphalt-lined open channel is n = 0.016 (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are

$$A_c = \frac{10\,\text{m} + 5\,\text{m}}{2}(2.2\,\text{m}) = 16.5\,\text{m}^2$$

$$p = (5\,\text{m}) + 2\sqrt{(2.2\,\text{m})^2 + (2.5\,\text{m})^2} = 11.66\,\text{m}$$

$$R_h = \frac{A_c}{p} = \frac{16.5\,\text{m}^2}{11.66\,\text{m}} = 1.415\,\text{m}$$



Substituting the given quantities into Manning's equation,

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \quad \rightarrow \quad 120\,\text{m}^3/\text{s} = \frac{1\,\text{m}^{1/3}/s}{0.016}(16.5\,\text{m}^2)(1.415\,\text{m})^{2/3} S_0^{1/2}$$

It gives the slope to be S_0 = 0.008524. Therefore, the *elevation drop* Δz across a pipe length of L = 1 km must be

$$\Delta z = S_0 L = 0.008524(1000\,\text{m}) = \mathbf{8.52\,m}$$

Discussion Note that when transporting water through a region of fixed elevation drop, the only way to increase the flow rate is to use a channel with a larger cross-section.

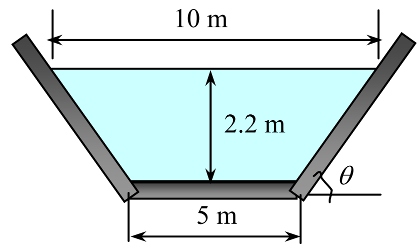
13-55

Solution The flow of water through the trapezoidal asphalt-lined channel in the previous problem is reconsidered. The maximum flow rate corresponding to a given maximum channel height is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Analysis We denote the flow conditions in the previous problem by subscript 1 and the conditions for the maximum case in this problem by subscript 2. Using the Manning's equation $\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2}$ and noting that the Manning coefficient and the channel slope remain constant, the flow rate in case 2 can be expressed in terms of flow rate in case 1 as

$$\frac{\dot{V}_2}{\dot{V}_1} = \frac{(a/n)A_{c2}R_{h2}^{2/3}}{(a/n)A_{c1}R_{h1}^{2/3}} \quad \rightarrow \quad \dot{V}_2 = \frac{A_{c2}}{A_{c1}}\left(\frac{R_{h2}}{R_{h1}}\right)^{2/3}\dot{V}_1$$



The trapezoid angle is $\tan\theta = 2.2/2.5 = 0.88 \;\rightarrow\; \theta = 2.2/2.5 = 41.34°$.

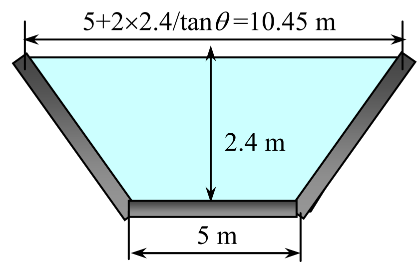
From geometric considerations,

$$A_{c1} = \frac{10\text{ m} + 5\text{ m}}{2}(2.2\text{ m}) = 16.5\text{ m}^2$$

$$p_1 = (5\text{ m}) + 2\sqrt{(2.2\text{ m})^2 + (2.5\text{ m})^2} = 11.66\text{ m}$$

$$R_{h1} = \frac{A_{c1}}{p_1} = \frac{16.5\text{ m}^2}{11.66\text{ m}} = 1.415\text{ m}$$

and



$$A_{c2} = \frac{10.45\text{ m} + 5\text{ m}}{2}(2.4\text{ m}) = 18.54\text{ m}^2$$

$$p_2 = (5\text{ m}) + 2\sqrt{(2.4\text{ m})^2 + (5.45/2\text{ m})^2} = 12.26\text{ m}$$

$$R_{h2} = \frac{A_{c2}}{p_2} = \frac{18.54\text{ m}^2}{12.26\text{ m}} = 1.512\text{ m}$$

Substituting,

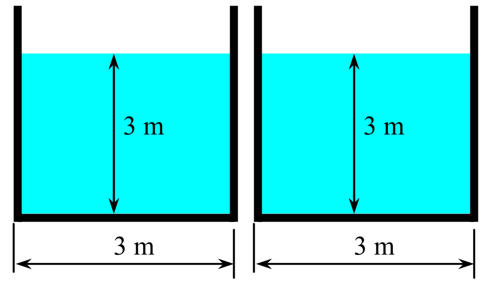
$$\dot{V}_2 = \frac{A_{c2}}{A_{c1}}\left(\frac{R_{h2}}{R_{h1}}\right)^{2/3}\dot{V}_1 = \frac{18.54\text{ m}^2}{16.5\text{ m}^2}\left(\frac{1.512\text{ m}}{1.415\text{ m}}\right)^{2/3}(120\text{ m}^3/\text{s}) = \mathbf{141\ m^3/s}$$

Discussion Note that a 9% increase in flow depth results in an 18% increase in flow rate.

13-56

Solution The flow of water through two identical channels with square flow sections is considered. The percent increase in flow rate as a result of combining the two channels while the flow depth remains constant is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.



Analysis We denote the flow conditions for two separate channels by subscript 1 and the conditions for the combined wide channel by subscript 2. Using the Manning's equation $\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2}$ and noting that the Manning coefficient, channel slope, and the flow area A_c remain constant, the flow rate in case 2 can be expressed in terms of flow rate in case 1 as

$$\frac{\dot{V}_2}{\dot{V}_1} = \frac{(a/n)A_{c2}R_{h2}^{2/3}}{(a/n)A_{c1}R_{h1}^{2/3}} = \left(\frac{R_{h2}}{R_{h1}}\right)^{2/3} = \left(\frac{A_{c2}/p_2}{A_{c1}/p_1}\right)^{2/3} = \left(\frac{p_1}{p_2}\right)^{2/3}$$

where p is the wetted perimeter. Substituting,

$$\frac{\dot{V}_2}{\dot{V}_1} = \left(\frac{p_2}{p_2}\right)^{2/3} = \left(\frac{6\times 3\text{ m}}{4\times 3\text{ m}}\right)^{2/3} = \left(\frac{3}{2}\right)^{2/3} = \mathbf{1.31} \qquad \textbf{(31\% increase)}$$

Discussion This is a very significant increase, and shows the importance of eliminating unnecessary surfaces in flow systems, including pipe flow.

13-57

Solution The flow of water in a trapezoidal channel made of unfinished-concrete is considered. For given flow rate and bottom slope, the flow depth is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

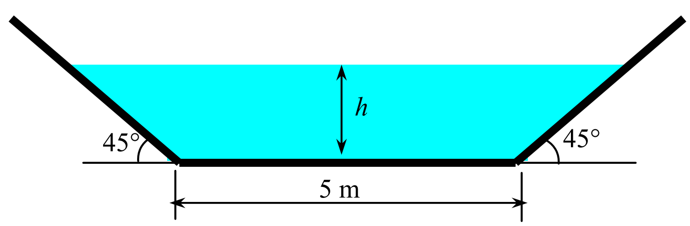
Properties Manning coefficient for an open channel of unfinished concrete is $n = 0.014$ (Table 13-1).

Analysis From geometric considerations, the flow area, wetted perimeter, and hydraulic radius are

$$A_c = \frac{5\text{ m} + 5\text{ m} + 2h}{2} h = (5+h)h$$

$$p = (5\text{ m}) + 2h/\sin 45° = 5 + 2.828h$$

$$R_h = \frac{A_c}{p} = \frac{(5+h)h}{5 + 2h/\sin 45°}$$



Substituting the given quantities into Manning's equation,

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \quad \rightarrow \quad 25\text{ m}^3/\text{s} = \frac{1\text{ m}^{1/3}/s}{0.014}(5+h)h\left(\frac{(5+h)h}{5+2h/\sin 45°}\right)^{2/3}(\tan 1°)^{1/2}$$

It gives the flow depth to be h = **0.685 m**.

Discussion Non-linear equations frequently arise in the solution of open channel flow problems. They are best handled by equation solvers such as EES.

13-58

Solution The flow of water in a weedy excavated trapezoidal channel is considered. For given flow rate and bottom slope, the flow depth is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

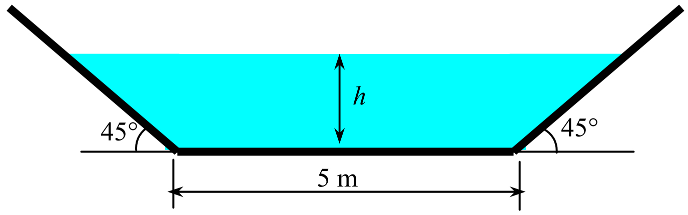
Properties Manning coefficient for the channel is given to be n = 0.030.

Analysis From geometric considerations, the flow area, wetted perimeter, and hydraulic radius are

$$A_c = \frac{5\text{ m} + 5\text{ m} + 2h}{2} h = (5+h)h$$

$$p = (5\text{ m}) + 2h/\sin 45° = 5 + 2.828h$$

$$R_h = \frac{A_c}{p} = \frac{(5+h)h}{5 + 2h/\sin 45°}$$



Substituting the given quantities into Manning's equation,

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \rightarrow 25\text{ m}^3/\text{s} = \frac{1\text{ m}^{1/3}/s}{0.030}(5+h)h\left(\frac{(5+h)h}{5+2h/\sin 45°}\right)^{2/3}(\tan 1°)^{1/2}$$

It gives the flow depth to be y = **1.07 m**.

Discussion Note that as the Manning coefficient increases because of the increased surface roughness of the channel, the flow depth required to maintain the same flow rate also increases.

13-59

Solution The flow of water in a V-shaped cast iron channel is considered. For a given flow depth and bottom slope, the flow rate is to be determined.

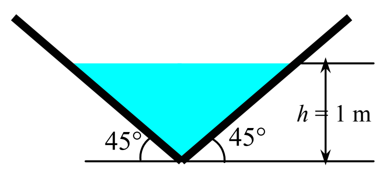
Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties Manning coefficient for an open channel of cast iron is n = 0.013 (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are

$$A_c = \frac{2h \times h}{2} = h^2 = (1\text{ m})^2 = 1\text{ m}^2 \qquad p = 2h/\sin\theta = 2(1\text{ m})/\sin 45° = 2\sqrt{2}\text{ m}$$

$$R_h = \frac{A_c}{p} = \frac{1\text{ m}^2}{2\sqrt{2}\text{ m}} = 0.3536\text{ m}$$



The bottom slope of the channel is $S_0 = \tan 0.5° = 0.008727$.

Then the flow rate can be determined from Manning's equation to be

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{1\text{ m}^{1/3}/s}{0.013}(1\text{ m}^2)(0.3536\text{ m})^{2/3}(0.008727)^{1/2} = \mathbf{3.59\ m^3/s}$$

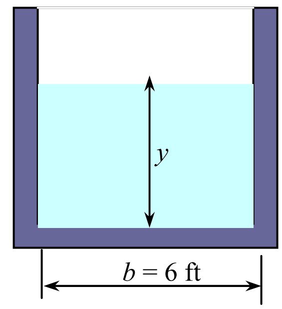
Discussion Note that the flow rate in a given channel is a strong function of the bottom slope.

13-60E

Solution The flow of water in a rectangular cast iron channel is considered. For given flow rate and bottom slope, the flow depth is to be determined.

Assumptions **1** The flow is steady and uniform. **2** The bottom slope is constant. **3** The roughness coefficient is constant.

Properties Manning coefficient for a cast iron open channel is $n = 0.013$ (Table 13-1).



Analysis From the geometry, the flow area, wetted perimeter, and hydraulic radius are

$$A_c = by = (6\text{ ft})y = 6y \qquad p = (6\text{ ft}) + 2y = 6 + 2y \qquad R_h = \frac{A_c}{p} = \frac{6y}{6+2y}$$

The channel bottom slope is $S_0 = 1.5/1000 = 0.0015$.
Substituting the given quantities into Manning's equation,

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \rightarrow 70\text{ ft}^3/\text{s} = \frac{1.486\text{ ft}^{1/3}/s}{0.013}(6y)\left(\frac{6y}{6+2y}\right)^{2/3}(0.0015)^{1/2}$$

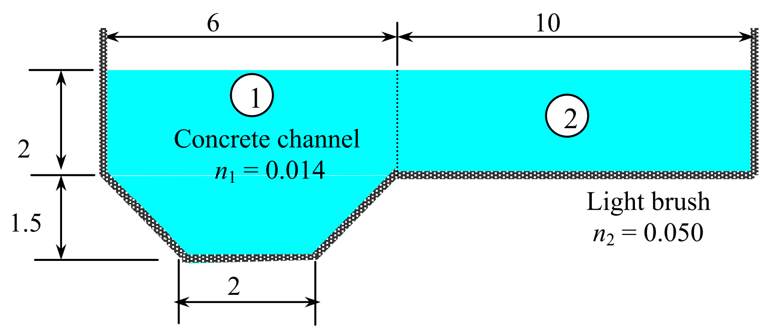
Solution of the above equation gives the flow depth to be h = **2.24 ft**.

Discussion Non-linear equations frequently arise in the solution of open channel flow problems. They are best handled by equation solvers such as EES.

13-61

Solution Water is flowing through a channel with nonuniform surface properties. The flow rate and the effective Manning coefficient are to be determined.

Assumptions **1** The flow is steady and uniform. **2** The bottom slope is constant. **3** The Manning coefficients do not vary along the channel.



Analysis The channel involves two parts with different roughness, and thus it is appropriate to divide the channel into two subsections. The flow rate for each subsection can be determined from the Manning equation, and the total flow rate can be determined by adding them up.
The flow area, perimeter, and hydraulic radius for each subsection and the entire channel are:

Subsection 1: $A_{c1} = 18\text{ m}^2$, $p_1 = 9\text{ m}$, $R_{h1} = \frac{A_{c1}}{p_1} = \frac{18\text{ m}^2}{9\text{ m}} = 2.00\text{ m}$

Subsection 2: $A_{c2} = 20\text{ m}^2$, $p_2 = 12\text{ m}$, $R_{h2} = \frac{A_{c2}}{p_2} = \frac{20\text{ m}^2}{12\text{ m}} = 1.67\text{ m}$

Entire channel: $A_c = 38\text{ m}^2$, $p = 21\text{ m}$, $R_h = \frac{A_c}{p} = \frac{38\text{ m}^2}{21\text{ m}} = 1.81\text{ m}$

Applying the Manning equation to each subsection, the total flow rate through the channel is determined to be

$$\dot{V} = \dot{V}_1 + \dot{V}_2 = \frac{a}{n_1} A_1 R_1^{2/3} S_0^{1/2} + \frac{a}{n_1} A_1 R_1^{2/3} S_0^{1/2} = \left(1\text{ m}^{1/3}/\text{s}\right)\left(\frac{(18\text{ m}^2)\,(2\text{ m})^{2/3}}{0.014} + \frac{(20\text{ m}^2)\,(1.67\text{ m})^{2/3}}{0.05}\right)(0.002)^{1/2} = \mathbf{116\ m^3/s}$$

Knowing the total flow rate, the effective Manning coefficient for the entire channel can be determined from the Manning equation to be

$$n_{\text{eff}} = \frac{aA_c R_h^{2/3} S_0^{1/2}}{\dot{V}} = \frac{(1\text{ m}^{1/3}/\text{s})(38\text{ m}^2)(1.81\text{ m})^{2/3}(0.002)^{1/2}}{116\text{ m}^3/\text{s}} = \mathbf{0.0217}$$

Discussion The effective Manning coefficient n_{eff} lies between the two n values as expected. The weighted average of the Manning coefficient of the channel is $n_{\text{ave}} = (n_1 p_1 + n_2 p_2)/p = 0.035$, which is quite different than n_{eff}. Therefore, using a weighted average Manning coefficient for the entire channel may be tempting, but it would not be accurate.

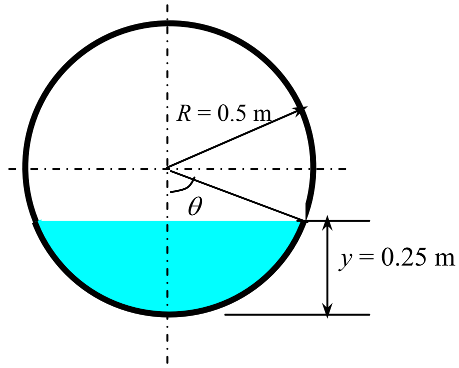
13-62

Solution Waters flows in a partially filled circular channel made of finished concrete. For a given flow depth and bottom slope, the flow rate is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties Manning coefficient for an open channel made of finished concrete is $n = 0.012$ (Table 13-1).

Analysis From geometric considerations,



$$\cos\theta = \frac{R-y}{R} = \frac{0.5-0.25}{0.5} = 0.5 \quad \rightarrow \quad \theta = 60° = 60\frac{2\pi}{360} = \frac{\pi}{3}$$

$$A_c = R^2(\theta - \sin\theta\cos\theta) = (0.5\text{ m})^2[\pi/3 - \sin(\pi/3)\cos(\pi/3)] = 0.1535\text{ m}^2$$

$$R_h = \frac{A_c}{p} = \frac{\theta - \sin\theta\cos\theta}{2\theta}R = \frac{\pi/3 - \sin(\pi/3)\cos(\pi/3)}{2\pi/3}(0.5\text{ m}) = 0.1466\text{ m}$$

Then the flow rate can be determined from Manning's equation to be

$$\dot{V} = \frac{a}{n}A_c R_h^{2/3} S_0^{1/2} = \frac{1\text{ m}^{1/3}/s}{0.012}(0.1535\text{ m}^2)(0.1466\text{ m})^{2/3}(0.002)^{1/2} = \mathbf{0.159\ m^3/s}$$

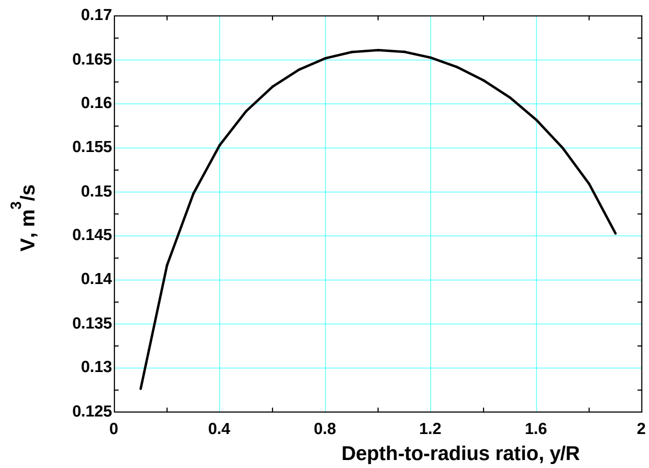
Discussion Note that the flow rate in a given channel is a strong function of the bottom slope.

13-63

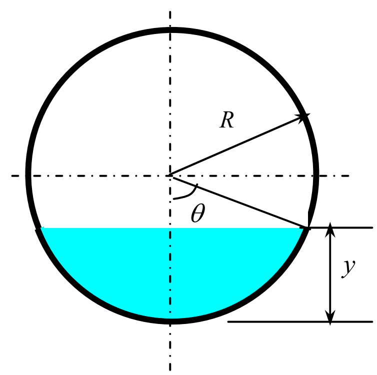
Solution The previous problem is reconsidered. By varying the flow depth-to-radius ratio from 0.1 to 1.9 for a fixed value of flow area, it is the to be shown that the best hydraulic cross section occurs when the circular channel is half-full, and the results are to be plotted.

Analysis The EES *Equations* window is printed below, along with the tabulated and plotted results.

```
a=1
n=0.012
s=0.002
Ac=0.1536 "Flow area kept constant"
ratio=y/R "This ratio is varied from 0.1 to 1.9"
bdeg=arcsin((R-y)/R)
tetadeg=90-bdeg
teta=tetadeg*2*pi/360
Ac=R^2*(teta-sin(tetadeg)*cos(tetadeg))
p=2*teta*R
Rh=Ac/p
Vdot=(a/n)*Ac*Rh^(2/3)*SQRT(s)
```



Depth-to-radius ratio, y/R	Channel radius, R, m	Flow rate, $\dot{V}$, m^3/s
0.1	1.617	0.1276
0.2	0.969	0.1417
0.3	0.721	0.1498
0.4	0.586	0.1553
0.5	0.500	0.1592
0.6	0.440	0.1620
0.7	0.396	0.1639
0.8	0.362	0.1652
0.9	0.335	0.1659
1.0	0.313	**0.1661**
1.1	0.295	0.1659
1.2	0.279	0.1653
1.3	0.267	0.1642
1.4	0.256	0.1627
1.5	0.247	0.1607
1.6	0.239	0.1582
1.7	0.232	0.1550
1.8	0.227	0.1509
1.9	0.223	0.1453



Discussion The depth-to-radius ratio of $y/R = 1$ corresponds to a half-full circular channel, and it is clear from the table and the chart that, for a fixed flow area, the flow rate becomes maximum when the channel is half-full.

13-64

Solution Water is to be transported uniformly in a clean-earth trapezoidal channel. For a specified flow rate, the required elevation drop per km channel length is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

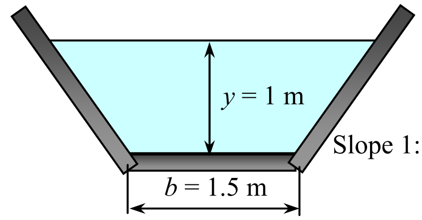
Properties Manning coefficient for the clean-earth lined open channel is $n = 0.022$ (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are

$$A_c = \frac{(1.5+1.5+2)\ \text{m}}{2}(1\,\text{m}) = 2.5\,\text{m}^2$$

$$p = (1.5\,\text{m}) + 2\sqrt{(1\,\text{m})^2 + (1\,\text{m})^2} = 4.328\,\text{m}$$

$$R_h = \frac{A_c}{p} = \frac{2.5\,\text{m}^2}{4.328\,\text{m}} = 0.5776\,\text{m}$$



Substituting the given quantities into Manning's equation,

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \quad \rightarrow \quad 8\,\text{m}^3/\text{s} = \frac{1\,\text{m}^{1/3}/s}{0.022}(2.5\,\text{m}^2)(0.5776\,\text{m})^{2/3} S_0^{1/2}$$

It gives the slope to be $S_0 = 0.0103$. Therefore, the *elevation drop* Δz across a pipe length of $L = 1$ km must be

$$\Delta z = S_0 L = 0.0103(1000\,\text{m}) = \mathbf{10.3\ m}$$

Discussion Note that when transporting water through a region of fixed elevation drop, the only way to increase the flow rate is to use a channel with a larger cross-section.

13-65

Solution A water draining system consists of three circular channels, two of which draining into the third one. If all channels are to run half-full, the diameter of the third channel is to be determined.

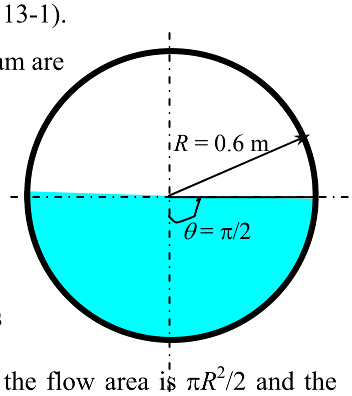
Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel. **4** Losses at the junction are negligible.

Properties The Manning coefficient for asphalt lined open channels is $n = 0.016$ (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the two pipes upstream are

$$A_c = \frac{\pi R^2}{2} = \frac{\pi(0.6\,\text{m})^2}{2} = 0.5655\,\text{m}^2 \qquad p = \frac{2\pi R}{2} = \frac{2\pi(0.6\,\text{m})}{2} = 1.885\,\text{m}$$

$$R_h = \frac{A_c}{P} = \frac{\pi R^2/2}{\pi R} = \frac{R}{2} = \frac{0.6\,\text{m}}{2} = 0.30\ \text{m}$$



Then the flow rate through the 2 pipes becomes, from Manning's equation,

$$\dot{V} = 2\frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = 2\frac{1\,\text{m}^{1/3}/s}{0.016}(0.5655\,\text{m}^2)(0.30\,\text{m})^{2/3}(0.0015)^{1/2} = 1.227\,\text{m}^3/\text{s}$$

The third channel is half-full, and the flow rate through it remains the same. Noting that the flow area is $\pi R^2/2$ and the hydraulic radius is $R/2$, we have

$$1.227\,\text{m}^3/\text{s} = \frac{1\,\text{m}^{1/3}/s}{0.016}(\pi R^2/2\,\text{m}^2)(R/2\,\text{m})^{2/3}(0.0015)^{1/2}$$

Solving for R gives $R = 0.778$ m. Therefore, the diameter of the third channel is D_3 **= 1.56 m** .

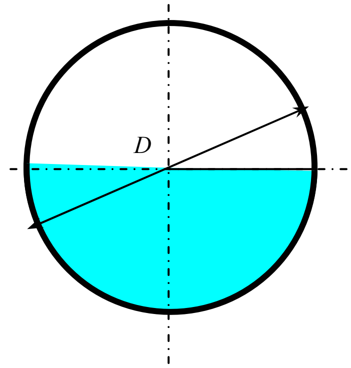
Discussion Note that if the channel diameter were larger, the channel would have been less than half full.

13-66

Solution Water is transported in an asphalt lined open channel at a specified rate. The dimensions of the best cross-section for various geometric shapes are to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

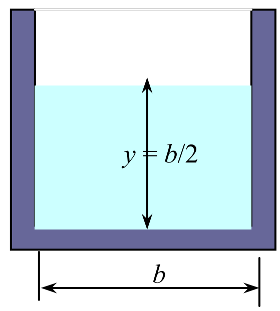
Properties The Manning coefficient for asphalt lined open channels is $n = 0.016$ (Table 13-1).



Analysis (*a*) Circular channel of Diameter *D*: Best cross-section occurs when the channel is half-full, and thus the flow area is $\pi D^2/8$ and the hydraulic radius is $D/4$. Then from Manning's equation, $\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2}$,

$$4\ \text{m}^3/\text{s} = \frac{1\ \text{m}^{1/3}/s}{0.016}(\pi D^2/8\ \text{m}^2)(D/4\ \text{m})^{2/3}(0.0015)^{1/2}$$

which gives D **= 2.42 m.**

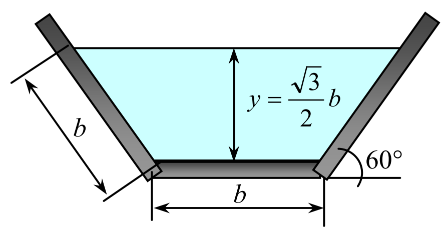


(*b*) Rectangular channel of bottom width *b*: For best cross-section, $y = b/2$. Then $A_c = yb = b^2/2$ and $R_h = b/4$. From the Manning equation,

$$4\ \text{m}^3/\text{s} = \frac{1\ \text{m}^{1/3}/s}{0.016}(b^2/2\ \text{m}^2)(b/4\ \text{m})^{2/3}(0.0015)^{1/2}$$

which gives b **= 2.21 m,** and $y = b/2$ = **1.11 m.**

(*c*) Trapezoidal channel of bottom width *b*: For best cross-section, $\theta = 60°$ and $y = b\sqrt{3}/2$. Then, $A_c = y(b + b\cos\theta) = 0.5\sqrt{3}b^2(1+\cos 60°) = 0.75\sqrt{3}b^2$, $p = 3b$, $R_h = \frac{y}{2} = \frac{\sqrt{3}}{4}b$. From the Manning equation,



$$4\ \text{m}^3/\text{s} = \frac{1\ \text{m}^{1/3}/s}{0.016}(0.75\sqrt{3}b^2\ \text{m}^2)(\sqrt{3}b/4\ \text{m})^{2/3}(0.0015)^{1/2}$$

which gives b **= 1.35 m,** and y = **1.17 m** and θ = **60°.**

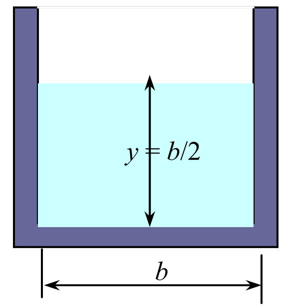
Discussion The perimeters for the circular, rectangular, and trapezoidal channels are 3.80 m, 4.42 m, and 4.05 m, respectively. Therefore, the circular cross-section has the smallest perimeter.

13-67E

Solution Water is to be transported in a rectangular channel at a specified rate. The dimensions for the best cross-section if the channel is made of unfinished and finished concrete are to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties The Manning coefficient is n = 0.012 and n = 0.014 for finished and unfinished concrete, respectively (Table 13-1).



Analysis For best cross-section of a rectangular cross-section, $y = b/2$. Then $A_c = yb = b^2/2$ and $R_h = b/4$. The flow rate is determined from the Manning equation, $\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2}$,

(*a*) Finished concrete, n = 0.012:

$$800 \text{ ft}^3/\text{s} = \frac{1.486 \text{ ft}^{1/3}/s}{0.012}(b^2/2 \text{ ft}^2)(b/4 \text{ ft})^{2/3}(0.0005)^{1/2}$$

It gives b **= 15.4 ft,** and $y = b/2$ = **7.68 ft**

(*b*) Unfinished concrete, n = 0.014:

$$800 \text{ ft}^3/\text{s} = \frac{1.486 \text{ ft}^{1/3}/s}{0.014}(b^2/2 \text{ ft}^2)(b/4 \text{ ft})^{2/3}(0.0005)^{1/2}$$

It gives b **= 16.3 ft,** and $y = b/2$ = **8.13 ft**

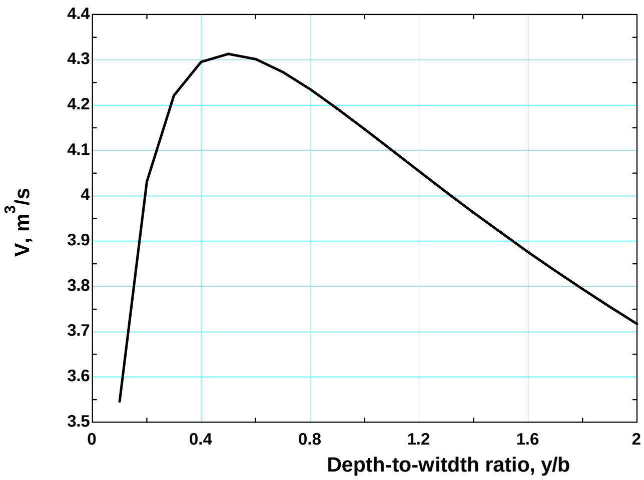
Discussion Note that channels with rough surfaces require a larger cross-section to transport the same amount of water.

13-68

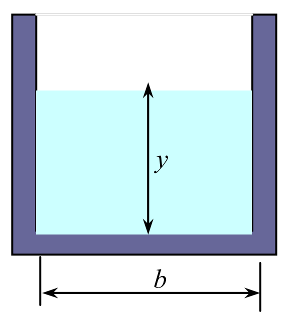
Solution Uniform flow in an asphalt-lined rectangular channel is considered. By varying the depth-to-width ratio from 0.1 to 2 in increments of 0.1 for a fixed value of flow area, it is the to be shown that the best hydraulic cross section occurs when $y/b = 0.5$, and the results are to be plotted.

Analysis The EES *Equations* window is printed below, along with the tabulated and plotted results.

```
a=1
n=0.016 "Manning coefficient"
s=0.003 "Bottom slope is constant"
Ac=2 "Flow area remains constant at 2 m2"
Ratio=y/b
Ac=b*y
p=b+2*y
Rh=Ac/p "Hydraulic radius"
Vdot=(a/n)*Ac*Rh^(2/3)*SQRT(s) "Volume flow rate"
```



Depth-to-width ratio, y/b	Channel width, b, m	Flow rate, $\dot{V}$, m^3/s
0.1	4.47	3.546
0.2	3.16	4.031
0.3	2.58	4.221
0.4	2.24	4.295
0.5	2.00	**4.313**
0.6	1.83	4.301
0.7	1.69	4.273
0.8	1.58	4.235
0.9	1.49	4.192
1.0	1.41	4.147
1.1	1.35	4.101
1.2	1.29	4.054
1.3	1.24	4.008
1.4	1.20	3.963
1.5	1.15	3.919
1.6	1.12	3.876
1.7	1.08	3.834
1.8	1.05	3.794
1.9	1.03	3.755
2.0	1.00	3.717



Discussion It is clear from the table and the chart that the depth-to-width ratio of $y/b = 0.5$ corresponds to the best cross-section for an open channel of rectangular cross-section.

Gradually and Rapidly Varied Flows and Hydraulic Jump

13-69C

Solution We are to discuss the difference between uniform and nonuniform (varied) flow.

Analysis Both uniform and varied flows are steady, and thus neither involves any change with time at a specified location. In *uniform flow*, **the flow depth *y* and the flow velocity *V* remain constant** whereas in *nonuniform* or *varied flow*, **the flow depth and velocity vary in the streamwise direction of the flow**. In *uniform flow*, **the slope of the energy line is equal to the slope of the bottom surface**. Therefore, the friction slope equals the bottom slope, $S_f = S_0$. In *varied flow*, however, **these slopes are different**.

Discussion Varied flows are further classified into gradually varied flow (GVF) and rapidly varied flow (RVF).

13-70C

Solution We are to discuss the differences between GVF and RVF.

Analysis *Gradually varied flow* (GVF) is characterized by **gradual variations in flow depth and velocity** (small slopes and no abrupt changes) and **a free surface that always remains smooth** (no discontinuities or zigzags). *Rapidly varied flow* (RVF) involves **rapid changes in flow depth and velocity**. A change in the bottom slope or cross-section of a channel or an obstruction on the path of flow may cause the uniform flow in a channel to become gradually or rapidly varied flow. **Analytical relations for the profile of the free surface can be obtained in GVF, but this is not the case for RVF because of the intense agitation**.

Discussion In many situations, the shape of the free surface must be solved numerically, even for GVF.

13-71C

Solution We are to analyze a claim that wall shear is negligible in RVF but important in GVF.

Analysis **Yes, we agree with this claim**. Rapidly varied flows occur over a short section of the channel with relatively small surface area, and thus frictional losses associated with wall shear are negligible compared with losses due to intense agitation and turbulence. Losses in GVF, on the other hand, are primarily due to frictional effects along the channel, and should be considered.

Discussion There is somewhat of an analogy here with internal flows. In long pipe sections with entrance lengths and/or gradually changing pipe diameter, wall shear is important. However, in short sections of piping with rapid change of diameter or a blockage or turn, etc (minor loss), friction along the wall is typically negligible compared to other losses.

13-72C

Solution We are to analyze what happens to flow depth in a horizontal rectangular channel during subcritical flow.

Analysis The flow depth *y* must (*c*) ***decrease* in the flow direction**.

Discussion Since the flow is subcritical, there is no possibility of a hydraulic jump.

13-73C

Solution We are to analyze what happens to flow depth in a sloped rectangular channel during subcritical flow.

Analysis The flow depth *y* must (*a*) ***increase* in the flow direction**.

Discussion Since the flow is subcritical, there is no possibility of a hydraulic jump.

13-74C

Solution We are to analyze what happens to flow depth in a horizontal rectangular channel during supercritical flow.

Analysis The flow depth y (a) ***increases* in the flow direction**.

Discussion Since the flow is supercritical, this increase in flow depth may occur via a hydraulic jump.

13-75C

Solution We are to analyze what happens to flow depth in a sloped rectangular channel during subcritical flow.

Analysis The flow depth y (c) ***decreases* in the flow direction**.

Discussion Since the flow is subcritical, there is no possibility of a hydraulic jump.

13-76C

Solution We are to analyze what happens to flow depth in an upward-sloped rectangular channel during supercritical flow.

Analysis The flow depth y (a) ***increases* in the flow direction**.

Discussion Since the flow is supercritical, this increase in flow depth may occur via a hydraulic jump.

13-77C

Solution We are to determine if it is possible for subcritical flow to undergo a hydraulic jump.

Analysis **No. It is impossible for subcritical flow to undergo a hydraulic jump**. Such a process would require the head loss h_L to become negative, which is impossible. It would correspond to negative entropy generation, which would be a violation of the second law of thermodynamics. Therefore, the upstream flow must be supercritical ($Fr_1 > 1$) for a hydraulic jump to occur.

Discussion This is analogous to **normal shock waves** in gases – the only way a shock wave can occur is if the flow upstream of the shock wave is supersonic with $Ma_1 > 1$ (analogous to supercritical in open-channel flow with $Fr_1 > 1$).

13-78C

Solution We are to define the energy dissipation ratio for a hydraulic jump and discuss why a hydraulic jump is sometimes used to dissipate energy.

Analysis Hydraulic jumps are often designed in conjunction with stilling basins and spillways of dams in order to waste as much of the mechanical energy as possible **to minimize the mechanical energy of the fluid and thus its potential to cause damage**. In such cases, a measure of performance of a hydraulic jump is the *energy dissipation ratio*, which is the **fraction of energy dissipated** through a hydraulic jump, defined as

$$\boxed{\text{Dissipation ratio} = \frac{h_L}{E_{s1}} = \frac{h_L}{y_1 + V_1^2/(2g)} = \frac{h_L}{y_1\left(1+\text{Fr}_1^2/2\right)}}.$$

Discussion Since the head loss is always positive, the dissipation ratio is also always positive.

13-79

Solution Water is flowing in an open channel uniformly. It is to be determined whether the channel slope is mild, critical, or steep for this flow.

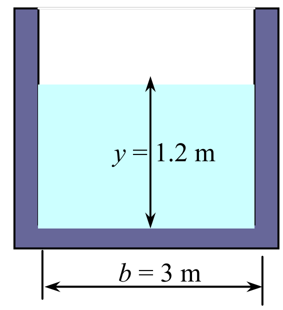
Assumptions **1** The flow is steady and uniform. **2** The bottom slope is constant. **3** The roughness of the wetted surface of the channel and thus the friction coefficient are constant.

Properties The Manning coefficient for an open channel with finished concrete surfaces is $n = 0.012$ (Table 13-1).

Analysis The cross-sectional area, perimeter, and hydraulic radius are

$$A_c = yb = (1.2\ \text{m})(3\ \text{m}) = 3.6\ \text{m}^2 \qquad p = b + 2y = 3\ \text{m} + 2(1.2\ \text{m}) = 5.4\ \text{m}$$

$$R_h = \frac{A_c}{p} = \frac{3.6\ \text{m}^2}{5.4\ \text{m}} = 0.6667\ \text{m}$$



The flow rate is determined from the Manning equation to be

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{1\ \text{m}^{1/3}/\text{s}}{0.012}(3.6\ \text{m}^2)(0.6667\ \text{m})^{2/3}(0.002)^{1/2} = 10.2\ \text{m}^3/\text{s}$$

Noting that the flow is uniform, the specified flow rate is the normal depth and thus $y = y_n = 1.2$ m. The critical depth for this flow is

$$y_c = \left(\frac{\dot{V}^2}{g b^2}\right)^{1/3} = \left(\frac{(10.2\ \text{m}^3/\text{s})^2}{(9.81\ \text{m/s}^2)(3\ \text{m})^2}\right)^{1/3} = 1.06\ \text{m}$$

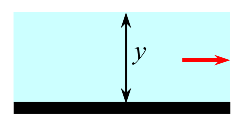
This channel at these flow conditions is classified as **mild** since $y > y_c$, and the flow is subcritical.

Discussion If the flow depth were smaller than 1.06 m, the channel slope would be said to be *steep*. Therefore, the bottom slope alone is not sufficient to classify a downhill channel as being mild, critical, or steep.

13-80

Solution Water is flowing in an wide brick open channel uniformly. The range of flow depth for which the channel can be classified as "steep" is to be determined.

Assumptions **1** The flow is steady and uniform. **2** The bottom slope is constant. **3** The roughness of the wetted surface of the channel and thus the friction coefficient are constant.



Properties The Manning coefficient for a brick open channel is $n = 0.015$ (Table 13-1).

Analysis The slope of the channel is $S_0 = \tan\alpha = \tan 0.4° = 0.006981$.

The hydraulic radius for a wide channel is equal to the flow depth, $R_h = y$. Now assume the flow in the channel to be critical, The channel flow in this case would be critical slope S_c, and the flow depth would be the critical flow depth, which is determined from

$$S_c = \frac{g n^2}{a^2 y_c^{1/3}} \quad \rightarrow \quad y_c = \left(\frac{g n^2}{a^2 S_c}\right)^3$$

Substituting,

$$y_c = \left(\frac{g n^2}{a^2 S_c}\right)^3 = \left(\frac{(9.81\ \text{m/s}^2)(0.015)^2}{(1\ \text{m}^{1/3}/s)^2(0.006981)}\right)^3 = 0.0316\ \text{m}$$

Therefore, this channel can be classified as *steep* for uniform flow depths less than y_c, i.e., ***y* < 0.0316 m**.

Discussion Note that two channels of the same slope can be classified as differently (one mild and the other steep) if they have different roughness and thus different values of n.

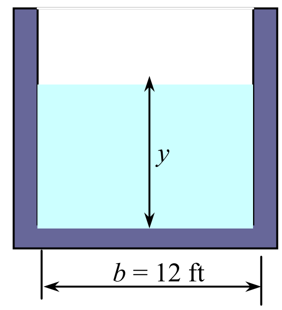
13-81E

Solution Water is flowing in a rectangular open channel with a specified bottom slope at a specified flow rate. It is to be determined whether the slope of this channel should be classified as mild, critical, or steep. The surface profile is also to be classified for a specified flow depth of 2 m.

Assumptions **1** The flow is steady and uniform. **2** The bottom slope is constant. **3** The roughness of the wetted surface of the channel and thus the friction coefficient are constant.

Properties The Manning coefficient of a channel with unfinished concrete surfaces is $n = 0.014$ (Table 13-1).

Analysis The cross-sectional area, perimeter, and hydraulic radius are



$$A_c = yb = y(12\text{ ft}) = 12y\text{ ft}^2 \qquad p = b + 2y = 12\text{ ft} + 2y = 12 + 2y\text{ ft}$$

$$R_h = \frac{A_c}{p} = \frac{12y\text{ ft}^2}{12 + 2\text{y ft}} = 0.6667\text{ m}$$

Substituting the known quantities into the Manning equation,

$$\dot{V} = \frac{a}{n}A_c R_h^{2/3} S_0^{1/2} \quad \rightarrow \quad 300\text{ ft}^3/\text{s} = \frac{1.486\text{ ft}^{1/3}/s}{0.014}(12y)\left(\frac{12y}{12+2y}\right)^{2/3}(\tan 0.5°)^{1/2}$$

Solving for the flow depth y gives $y = 1.95$ ft. The critical depth for this flow is

$$y_c = \frac{\dot{V}^2}{gA_c^2} = \frac{(300\text{ ft}^3/\text{s})^2}{(32.2\text{ ft/s}^2)(12\text{ ft} \times 1.95\text{ ft})^2} = 5.10\text{ ft}$$

This channel at these flow conditions is classified as **steep** since $y < y_c$, and the flow is supercritical. Alternately, we could solve for Froude number and show that Fr > 1 and reach the same conclusion. The given flow is uniform, and thus $y = y_n$ = 1.95 ft. Therefore, the given value of $y = 3$ ft during development is between y_c and y_n, and the **flow profile** is **S2** (Table 13-3).

Discussion If the flow depth were larger than 5.19 ft, the channel slope would be said to be *mild*. Therefore, the bottom slope alone is not sufficient to classify a downhill channel as being mild, critical, or steep.

13-82

Solution Water is flowing in a V-shaped open channel with a specified bottom slope at a specified rate. It is to be determined whether the slope of this channel should be classified as mild, critical, or steep.

Assumptions **1** The flow is steady and uniform. **2** The bottom slope is constant. **3** The roughness of the wetted surface of the channel and thus the friction coefficient are constant.

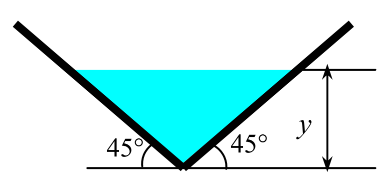
Properties The Manning coefficient for a cast iron channel is $n = 0.013$ (Table 13-1).

Analysis From geometric considerations, the cross-sectional area, perimeter, and hydraulic radius are

$$A_c = y(2y)/2 = y^2 \qquad p = 2\sqrt{y^2 + y^2} = 2\sqrt{2}y \qquad R_h = \frac{A_c}{p} = \frac{y^2}{2\sqrt{2}\text{y}} = \frac{y}{2\sqrt{2}}$$

Substituting the known quantities into the Manning equation,

$$\dot{V} = \frac{a}{n}A_c R_h^{2/3} S_0^{1/2} \quad \rightarrow 3\text{ m}^3/\text{s} = \frac{1\text{ m}^{1/3}/\text{s}}{0.013}(y^2)\left(\frac{\text{y}}{2\sqrt{2}}\right)^{2/3}(0.002)^{1/2}$$



Solving for the flow depth y gives $y = 1.23$ m. The critical depth for this flow is

$$y_c = \frac{\dot{V}^2}{gA_c^2} = \frac{(3\text{ m}^3/\text{s})^2}{(9.81\text{ m/s}^2)(1.23\text{ m})^2} = 0.61\text{ m}$$

This channel at these flow conditions is classified as **mild** since $y > y_c$, and the flow is subcritical.

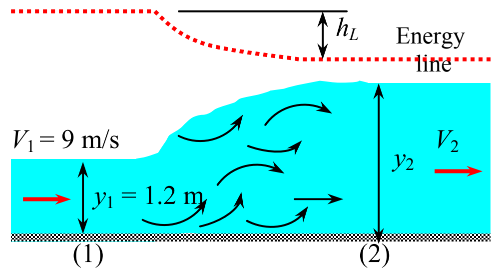
Discussion If the flow depth were smaller than 0.61 m, the channel slope would be said to be *steep*. Therefore, the bottom slope alone is not sufficient to classify a downhill channel as being mild, critical, or steep.

13-83 [*Also solved using EES on enclosed DVD*]

Solution Water at a specified depth and velocity undergoes a hydraulic jump. The depth and Froude number after the jump, the head loss and dissipation ratio, and dissipated mechanical power are to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The channel is horizontal.

Properties The density of water is 1000 kg/m^3.



Analysis (*a*) The Froude number before the hydraulic jump is

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{9\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(1.2\ \mathrm{m})}} = 2.62$$

which is greater than 1. Therefore, the flow is supercritical before the jump. The flow depth, velocity, and Froude number after the jump are

$$y_2 = 0.5y_1\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right) = 0.5(1.2\ \mathrm{m})\left(-1+\sqrt{1+8\times 2.62^2}\right) = \mathbf{3.89\ m}$$

$$V_2 = \frac{y_1}{y_2}V_1 = \frac{1.2\ \mathrm{m}}{3.89\ \mathrm{m}}(9\ \mathrm{m/s}) = 2.78\ \mathrm{m/s}$$

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{2.78\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(3.89\ \mathrm{m})}} = \mathbf{0.449}$$

(*b*) The head loss is determined from the energy equation to be

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (1.2\ \mathrm{m}) - (3.89\ \mathrm{m}) + \frac{(9\ \mathrm{m/s})^2 - (2.78\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} = \mathbf{1.05\ m}$$

The specific energy of water before the jump and the dissipation ratio are

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = (1.2\ \mathrm{m}) + \frac{(9\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} = 5.33\ \mathrm{m}$$

$$\text{Dissipation ratio} = \frac{h_L}{E_{s1}} = \frac{1.04\ \mathrm{m}}{5.33\ \mathrm{m}} = \mathbf{0.195}$$

Therefore, 19.5% of the available head (or mechanical energy) of the liquid is wasted (converted to thermal energy) as a result of frictional effects during this hydraulic jump.

(*c*) The mass flow rate of water is

$$\dot{m} = \rho\dot{V} = \rho b y_1 V_1 = (1000\ \mathrm{kg/m^3})(1.2\ \mathrm{m})(8\ \mathrm{m})(9\ \mathrm{m/s}) = 86{,}400\ \mathrm{kg/s}$$

Then the dissipated mechanical power becomes

$$\dot{E}_{\text{dissipated}} = \dot{m}gh_L = (86{,}400\ \mathrm{kg/s})(9.81\ \mathrm{m/s^2})(1.04\ \mathrm{m})\left(\frac{1\ \mathrm{N}}{1\ \mathrm{kg\cdot m/s^2}}\right) = 881{,}000\ \mathrm{Nm/s} = \mathbf{881\,kW}$$

Discussion The results show that the hydraulic jump is a highly dissipative process, wasting 881 kW of power production potential in this case. That is, if the water is routed to a hydraulic turbine instead of being released from the sluice gate, up to 881 kW of power could be produced.

13-84

Solution Water at a specified depth and velocity undergoes a hydraulic jump. The head loss associated with this process is to be determined.

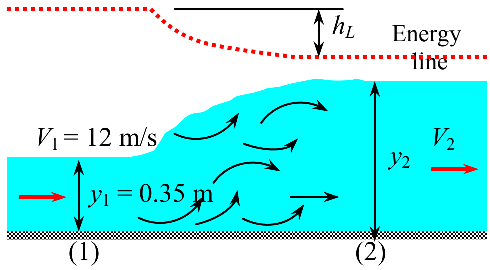
Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The channel is horizontal.

Analysis The Froude number before the hydraulic jump is $\mathrm{Fr}_1 = \dfrac{V_1}{\sqrt{gy_1}} = \dfrac{12\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.35\text{ m})}} = 6.476$, which is greater than 1. Therefore, the flow is indeed supercritical before the jump. The flow depth, velocity, and Froude number after the jump are

$$y_2 = 0.5y_1\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right) = 0.5(0.35\text{ m})\left(-1+\sqrt{1+8\times 6.476^2}\right) = 3.035\text{ m}$$

$$V_2 = \frac{y_1}{y_2}V_1 = \frac{0.35\text{ m}}{3.035\text{ m}}(12\text{ m/s}) = 1.384\text{ m/s}$$

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{1.384\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(3.035\text{ m})}} = 0.2536$$



The head loss is determined from the energy equation to be

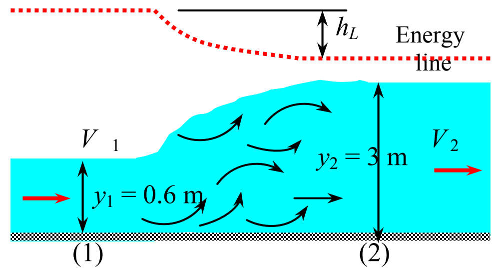
$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (0.35\text{ m}) - (3.035\text{ m}) + \frac{(12\text{ m/s})^2 - (1.384\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = \mathbf{4.56\text{ m}}$$

Discussion The results show that the hydraulic jump is a highly dissipative process, wasting 4.56 m of head in the process.

13-85

Solution The increase in flow depth during a hydraulic jump is given. The velocities and Froude numbers before and after the jump, and the energy dissipation ratio are to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The channel is horizontal.



Analysis The Froude number before the jump is determined from

$$y_2 = 0.5y_1\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right) \rightarrow 3\text{ m} = 0.5\times(0.6\text{ m})\left(-1+\sqrt{1+8\mathrm{Fr}^2}\right)$$

which gives $\mathrm{Fr}_1 =$ **3.873.** Then,

$$V_1 = \mathrm{Fr}_1\sqrt{gy_1} = 3.873\sqrt{(9.81\text{ m/s}^2)(0.6\text{ m})} = \mathbf{9.40\text{ m/s}}$$

$$V_2 = \frac{y_1}{y_2}V_1 = \frac{0.6\text{ m}}{3\text{ m}}(9.40\text{ m/s}) = \mathbf{1.88\text{ m/s}}$$

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{1.88\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(3\text{ m})}} = \mathbf{0.347}$$

The head loss is determined from the energy equation to be

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (0.6\text{ m}) - (3\text{ m}) + \frac{(9.40\text{ m/s})^2 - (1.88\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = \mathbf{1.92\text{ m}}$$

The specific energy of water before the jump and the dissipation ratio are

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = (0.6\text{ m}) + \frac{(9.40\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 5.10\text{ m}$$

$$\text{Dissipation ratio} = \frac{h_L}{E_{s1}} = \frac{1.92\text{ m}}{5.10\text{ m}} = \mathbf{0.376}$$

Therefore, 37.6% of the available head (or mechanical energy) of water is wasted (converted to thermal energy) as a result of frictional effects during this hydraulic jump.

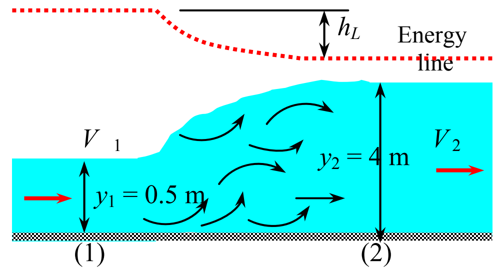
Discussion The results show that the hydraulic jump is a highly dissipative process, wasting over one-third of the available head.

13-86

Solution Water flowing in a wide channel at a specified depth and flow rate undergoes a hydraulic jump. The mechanical power wasted during this process is to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The channel is horizontal.

Properties The density of water is 1000 kg/m^3.



Analysis Average velocities before and after the jump are

$$V_1 = \frac{70\ \text{m}^3/\text{s}}{(10\ \text{m})(0.5\ \text{m})} = 14\ \text{m/s}$$

$$V_2 = \frac{70\ \text{m}^3/\text{s}}{(10\ \text{m})(4\ \text{m})} = 1.75\ \text{m/s}$$

The head loss is determined from the energy equation to be

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (0.5\ \text{m}) - (4\ \text{m}) + \frac{(14\ \text{m/s})^2 - (1.75\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 6.33\ \text{m}$$

The mass flow rate of water is

$$\dot{m} = \rho\dot{V} = (1000\ \text{kg/m}^3)(70\ \text{m}^3/\text{s}) = 70{,}000\ \text{kg/s}$$

Then the dissipated mechanical power becomes

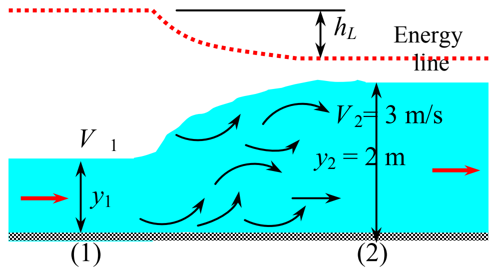
$$\dot{E}_{\text{dissipated}} = \dot{m}gh_L = (70{,}000\ \text{kg/s})(9.81\ \text{m/s}^2)(6.33\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 4350\ \text{kNm/s} = \mathbf{4.35\ MW}$$

Discussion The results show that the hydraulic jump is a highly dissipative process, wasting 4.35 MW of power production potential in this case.

13-87

Solution The flow depth and average velocity of water after a hydraulic jump are measured. The flow depth and velocity before the jump as well as the fraction of mechanical energy dissipated are to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The channel is horizontal.



Analysis The Froude number after the hydraulic jump is

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{3\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(2\ \mathrm{m})}} = 0.6773$$

It can be shown that the subscripts in the relation $y_2 = 0.5y_1\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right)$ are interchangeable. Thus,

$$y_1 = 0.5y_2\left(-1+\sqrt{1+8\mathrm{Fr}_2^2}\right) = 0.5(2\ \mathrm{m})\left(-1+\sqrt{1+8\times 0.6773^2}\right) = 1.16\ \mathrm{m}$$

$$V_1 = \frac{y_2}{y_1}V_2 = \frac{2\ \mathrm{m}}{1.161\ \mathrm{m}}(3\ \mathrm{m/s}) = 5.17\ \mathrm{m/s}$$

The Froude number before the jump is

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{5.17\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(1.16\ \mathrm{m})}} = 1.53$$

which is greater than 1. Therefore, the flow is indeed supercritical before the jump. The head loss is determined from the energy equation to be

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (1.16\ \mathrm{m}) - (2\ \mathrm{m}) + \frac{(5.17\ \mathrm{m/s})^2 - (3\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} = 0.0636\ \mathrm{m}$$

The specific energy of water before the jump and the dissipation ratio is

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = (1.16\ \mathrm{m}) + \frac{(5.17\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} = 2.52\ \mathrm{m}$$

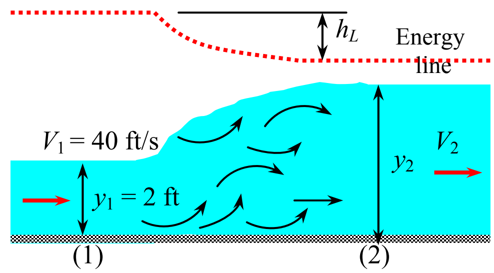
$$\text{Dissipation ratio} = \frac{h_L}{E_{s1}} = \frac{0.0636\ \mathrm{m}}{2.52\ \mathrm{m}} = \mathbf{0.025}$$

Discussion Note that this is a "mild" hydraulic jump, and only 2.5% of the available energy is wasted.

13-88E

Solution Water at a specified depth and velocity undergoes a hydraulic jump, and dissipates a known fraction of its energy. The flow depth, velocity, and Froude number after the jump and the head loss associated with the jump are to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The channel is horizontal.



Analysis The Froude number before the hydraulic jump is

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{40 \text{ ft/s}}{\sqrt{(32.2 \text{ m/s}^2)(2 \text{ ft})}} = 4.984$$

which is greater than 1. Therefore, the flow is indeed supercritical before the jump. The flow depth, velocity, and Froude number after the jump are

$$y_2 = 0.5 y_1\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right) = 0.5(2 \text{ ft})\left(-1+\sqrt{1+8\times 4.984^2}\right) = \mathbf{13.1\ ft}$$

$$V_2 = \frac{y_1}{y_2} V_1 = \frac{2 \text{ ft}}{13.1 \text{ ft}}(40 \text{ ft/s}) = \mathbf{6.09\ ft/s}$$

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{6.091 \text{ ft/s}}{\sqrt{(32.2 \text{ ft/s}^2)(13.13 \text{ m})}} = \mathbf{0.296}$$

The head loss is determined from the energy equation to be

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (2 \text{ ft}) - (13.1 \text{ ft}) + \frac{(40 \text{ ft/s})^2 - (6.09 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = \mathbf{13.2\ ft}$$

Discussion The results show that the hydraulic jump is a highly dissipative process, wasting 13.2 ft of head in the process.

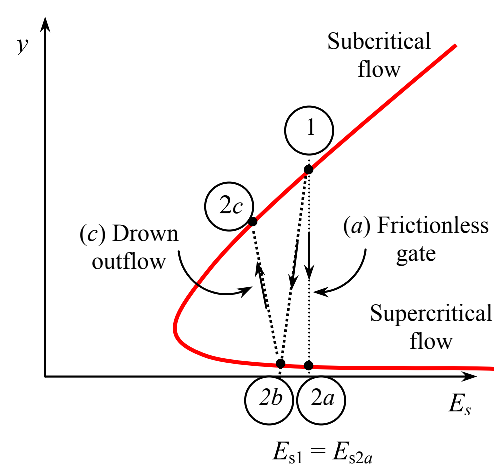
Flow Control and Measurement in Channels

13-89C

Solution We are to draw a flow depth-specific energy diagram for several types of flow.

Analysis On the figure, diagram 1-2a is for frictionless gate, **1-2b is for sluice gate with free outflow**, and **1-2b-2c is for sluice gate with drown outflow, including the hydraulic jump back to subcritical flow**.

Discussion A plot of flow depth as a function of specific energy, as shown here, is quite useful in the analysis of varied open-channel flow because the states upstream and downstream of a change must jump between the two branches.



13-90C

Solution We are to define the discharge coefficient for sluice gates, and discuss some typical values.

Analysis For sluice gates, the *discharge coefficient* C_d is defined as **the ratio of the actual velocity through the gate to the maximum velocity** as determined by the Bernoulli equation for the idealized frictionless flow case. **For ideal flow, C_d = 1**. Typical values of C_d for sluice gates with free outflow are in the range of **0.55 to 0.60**.

Discussion Actual values of the discharge coefficient must be less than one or else the second law would be violated.

13-91C

Solution We are to discuss how flow rate is measured with a broad-crested weir.

Analysis The operation of broad crested weir is based on **blocking the flow in the channel with a rectangular block, and establishing critical flow over the block**. Then the flow rate is determined by measuring flow depths.

Discussion This technique is quite obtrusive, but requires no special measuring equipment or probes.

13-92C

Solution We are to analyze whether the free surface of flow over a bump will increase, decrease, or remain constant.

Analysis In the case of *subcritical flow*, the **flow depth *y* will decrease during flow over the bump**.

Discussion This may be contrary to our intuition at first, but if we think in terms of increasing velocity and decreasing pressure over the bump (a Bernoulli type of analysis), it makes sense that the surface will decrease over the bump.

13-93C

Solution We are to analyze what happens in subcritical flow over a bump when the bump height increases.

Analysis When the specific energy reaches its minimum value, **the flow is critical, and the flow at this point is said to be choked**. If the bump height is increased even further, **the flow remains critical and thus choked. The flow will not become supercritical**.

Discussion This is somewhat analogous to compressible flow in a converging nozzle – the flow cannot become supersonic at the nozzle exit unless there is a diverging section of the nozzle downstream of the throat.

13-94C

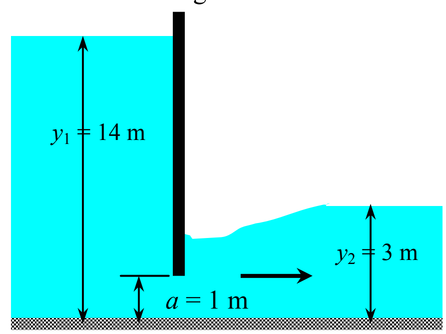
Solution We are to define and classify sharp-crested weirs.

Analysis A *sharp-crested weir* is a **vertical plate placed in a channel that forces the fluid to flow through an opening to measure the flow rate**. They are **characterized by the shape of the opening**. For example, a weir with a triangular opening is referred to as a triangular weir.

Discussion Similar to the broad-crested weir, this type of flow measurement is quite obtrusive, but requires no special measuring equipment or probes.

13-95

Solution Water is released from a reservoir through a sluice gate into an open channel. For specified flow depths, the rate of discharge is to be determined.



Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.

Analysis The depth ratio y_1/a and the contraction coefficient y_2/a are

$$\frac{y_1}{a}=\frac{14\text{ m}}{1\text{ m}}=14 \quad \text{and} \quad \frac{y_2}{a}=\frac{3\text{ m}}{1\text{ m}}=3$$

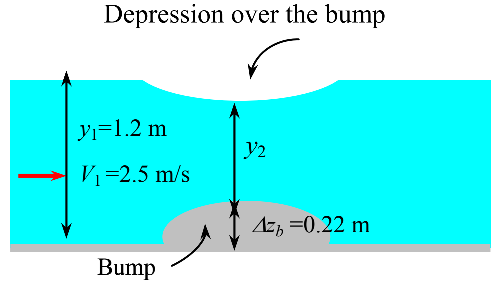
The corresponding discharge coefficient is determined from Fig. 13-38 to be C_d = 0.59. Then the discharge rate becomes

$$\dot{V}=C_d ba\sqrt{2gy_1}=0.59(5\text{ m})(1\text{ m})\sqrt{2(9.81\text{ m/s}^2)(14\text{ m})}=\mathbf{48.9\ m^3/s}$$

Discussion Discharge coefficient is the same as free flow because of small depth ratio after the gate. So, the flow rate would not change if it were not drowned.

13-96

Solution Water flowing in a horizontal open channel encounters a bump. It will be determined if the flow over the bump is choked.



Assumptions **1** The flow is steady. **2** Frictional effects are negligible so that there is no dissipation of mechanical energy. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The upstream Froude number and the critical depth are

$$\text{Fr}_1=\frac{V_1}{\sqrt{gy_1}}=\frac{2.5\text{ m/s}}{\sqrt{(9.81\text{ m}^2/\text{s})(1.2\text{ m})}}=0.729$$

$$y_c=\left(\frac{\dot{V}^2}{gb^2}\right)^{1/3}=\left(\frac{(by_1V_1)^2}{gb^2}\right)^{1/3}=\left(\frac{y_1^2V_1^2}{g}\right)^{1/3}=\left(\frac{(1.2\text{ m})^2(2.5\text{ m/s})^2}{9.81\text{ m/s}^2}\right)^{1/3}=0.972\text{ m}$$

The flow is subcritical since Fr < 1, and the flow depth decreases over the bump. The upstream, over the bump, and critical specific energies are

$$E_{s1}=y_1+\frac{V_1^2}{2g}=(1.2\text{ m})+\frac{(2.5\text{ m/s})^2}{2(9.81\text{ m/s}^2)}=1.52\text{ m}$$

$$E_{s2}=E_{s1}-\Delta z_b=1.52-0.22=1.30\text{ m}$$

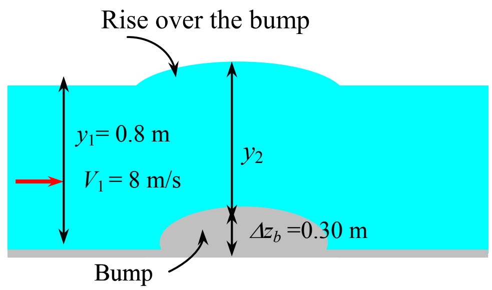
$$E_c=\frac{3}{2}y_c=1.46\text{ m}$$

We have an interesting situation: The calculations show that $E_{s2} < E_c$. That is, the specific energy of the fluid decreases below the level of energy at the critical point, which is the minimum energy, and this is impossible. Therefore, the flow at specified conditions cannot exist. **The flow is choked** when the specific energy drops to the minimum value of 1.46 m, which occurs at a bump-height of $\Delta z_{b,\max}=E_{s1}-E_c=1.52-1.46=0.06\text{ m}$.

Discussion A bump-height over 6 cm results in a reduction in the flow rate of water, or a rise of upstream water level. Therefore, a 22-cm high bump alters the upstream flow. On the other hand, a bump less than 6 cm high will not affect the upstream flow.

13-97

Solution Water flowing in a horizontal open channel encounters a bump. The change in the surface level over the bump and the type of flow (sub- or supercritical) over the bump are to be determined.



Assumptions **1** The flow is steady. **2** Frictional effects are negligible so that there is no dissipation of mechanical energy. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The upstream Froude number and the critical depth are

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{8 \text{ m/s}}{\sqrt{(9.81 \text{ m}^2/\text{s})(0.8 \text{ m})}} = 2.856$$

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(by_1V_1)^2}{gb^2}\right)^{1/3} = \left(\frac{y_1^2V_1^2}{g}\right)^{1/3} = \left(\frac{(0.8 \text{ m})^2(8 \text{ m/s})^2}{9.81 \text{ m/s}^2}\right)^{1/3} = 1.61 \text{ m}$$

The upstream flow is supercritical since Fr > 1, and the flow depth increases over the bump. The upstream, over the bump, and critical specific energies are

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = (0.8 \text{ m}) + \frac{(8 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 4.06 \text{ m}$$

$$E_{s2} = E_{s1} - \Delta z_b = 4.06 - 0.30 = 3.76 \text{ m}$$

$$E_c = \frac{3}{2}y_c = 2.42 \text{ m}$$

The flow depth over the bump is determined from

$$y_2^3 - (E_{s1} - \Delta z_b)y_2^2 + \frac{V_1^2}{2g}y_1^2 = 0 \quad \rightarrow \quad y_2^3 - (4.06 - 0.30 \text{ m})y_2^2 + \frac{(8 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)}(0.80 \text{ m})^2 = 0$$

Using an equation solver, the physically meaningful root of this equation is determined to be 0.846 m. Therefore, there is a rise of

$$\text{Rise over bump} = y_2 - y_1 + \Delta z_b = 0.846 - 0.80 + 0.30 = \mathbf{0.346 \text{ m}}$$

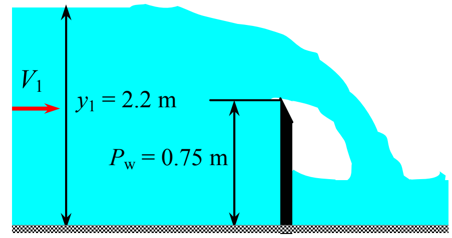
over the surface relative to the upstream water surface. The specific energy decreases over the bump from, 4.06 to 3.76 m, but it is still over the minimum value of 2.42 m. Therefore, the flow over the bump is still **supercritical**.

Discussion The actual value of surface rise may be different than 4.6 cm because of frictional effects that are neglected in this simplified analysis.

13-98

Solution The flow rate in an open channel is to be measured using a sharp-crested rectangular weir. For a measured value of flow depth upstream, the flow rate is to be determined.

Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.



Sharp-crested rectangular weir

Analysis The weir head is

$$H = y_1 - P_w = 2.2 - 0.75 = 1.45 \text{ m}$$

The discharge coefficient of the weir is

$$C_{wd,\text{rec}} = 0.598 + 0.0897\frac{H}{P_w} = 0.598 + 0.0897\frac{1.45 \text{ m}}{0.75 \text{ m}} = 0.771$$

The condition $H/P_w < 2$ is satisfied since 1.45/0.75 = 1.93. Then the water flow rate through the channel becomes

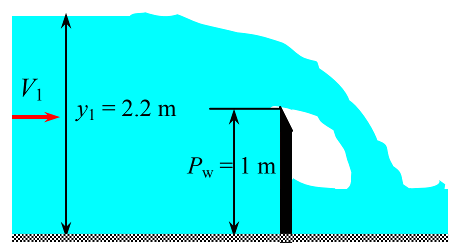
$$\dot{V}_{\text{rec}} = C_{wd,\text{rec}}\frac{2}{3}b\sqrt{2g}H^{3/2} = (0.7714)\frac{2}{3}(4 \text{ m})\sqrt{2(9.81 \text{ m/s}^2)}(1.45 \text{ m})^{3/2} = \mathbf{15.9 \text{ m}^3\text{/s}}$$

Discussion The upstream velocity and the upstream velocity head are $V_1 = \frac{\dot{V}}{by_1} = \frac{15.9 \text{ m}^3\text{/s}}{(4 \text{ m})(2.2 \text{ m})} = 1.81 \text{ m/s}$ and $\frac{V_1^2}{2g} = \frac{(1.81 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 0.167 \text{ m}$ respectively. This is 11.5% of the weir head, which is significant. When the upstream velocity head is considered, the flow rate becomes 18.1 m^3/s, which is about 14 percent higher than the value determined above. Therefore, it is good practice to consider the upstream velocity head unless the weir height P_w is very large relative to the weir head H.

13-99

Solution The flow rate in an open channel is to be measured using a sharp-crested rectangular weir. For a measured value of flow depth upstream, the flow rate is to be determined.

Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.



Sharp-crested rectangular weir

Analysis The weir head is $H = y_1 - P_w = 2.2 - 1.0 = 1.2 \text{ m}$. The discharge coefficient of the weir is

$$C_{wd,\text{rec}} = 0.598 + 0.0897\frac{H}{P_w} = 0.598 + 0.0897\frac{1.2 \text{ m}}{1.0 \text{ m}} = 0.7056$$

The condition $H/P_w < 2$ is satisfied since 1.2/1.0 = 1.20. Then the water flow rate through the channel becomes

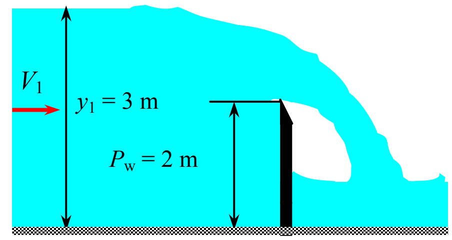
$$\dot{V}_{\text{rec}} = C_{wd,\text{rec}}\frac{2}{3}b\sqrt{2g}H^{3/2} = (0.7056)\frac{2}{3}(4 \text{ m})\sqrt{2(9.81 \text{ m/s}^2)}(1.2 \text{ m})^{3/2} = \mathbf{11.0 \text{ m}^3\text{/s}}$$

Discussion The upstream velocity and the upstream velocity head are $V_1 = \frac{\dot{V}}{by_1} = \frac{11.0 \text{ m}^3\text{/s}}{(4 \text{ m})(2.2 \text{ m})} = 1.25 \text{ m/s}$ and $\frac{V_1^2}{2g} = \frac{(1.25 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 0.079 \text{ m}$, respectively. This is 6.6% of the weir head, which may be significant. When the upstream velocity head is considered, the flow rate becomes 11.9 m^3/s, which is about 8 percent higher than the value determined above. Therefore, it is good practice to consider the upstream velocity head unless the weir height P_w is very large relative to the weir head H.

13-100

Solution Water flowing over a sharp-crested rectangular weir is discharged into a channel where uniform flow conditions are established. The maximum slope of the downstream channel to avoid hydraulic jump is to be determined.

Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.



Sharp-crested rectangular weir

Properties Manning coefficient for an open channel of unfinished concrete is $n = 0.014$ (Table 13-1).

Analysis The weir head is $H = y_1 - P_w = 3.0\text{ m} - 2.0\text{ m} = 1.0\text{ m}$. The condition $H/P_w < 2$ is satisfied since 1.0/2.0 = 0.5. The discharge coefficient of the weir is

$$C_{wd,\text{rec}} = 0.598 + 0.0897\frac{H}{P_w} = 0.598 + 0.0897\frac{1.0\text{ m}}{2.0\text{ m}} = 0.6429$$

Then the water flow rate through the channel per meter width (i.e., taking b = 1 m) becomes

$$\dot{V}_{\text{rec}} = C_{wd,\text{rec}}\frac{2}{3}b\sqrt{2g}H^{3/2} = (0.6429)\frac{2}{3}(1\text{ m})\sqrt{2(9.81\text{ m/s}^2)}(1.0\text{ m})^{3/2} = 1.898\text{ m}^3/\text{s}$$

To avoid hydraulic jump, we must avoid supercritical flow in the channel. Therefore, the bottom slope should not be higher than the critical slope, in which case the flow depth becomes the critical depth,

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(1.898\text{ m}^3/\text{s})^2}{(9.81\text{ m/s}^2)(1\text{ m}^2)}\right)^{1/3} = 0.7162\text{ m}$$

Noting that the hydraulic radius of a wide channel is equal to the flow depth, the bottom slope is determined from the Manning equation to be

$$\dot{V} = \frac{a}{n}A_c R_h^{2/3} S_0^{1/2} \quad\rightarrow\quad 1.898\text{ m}^3/\text{s} = \frac{1\text{ m}^{1/3}/s}{0.014}(0.7162\times 1\text{ m}^2)(0.7162\text{ m})^{2/3} S_0^{1/2}$$

Solution gives the slope to be $S_0 = 0.00215$. Therefore, $S_{0,\max}$ = **0.00215**.

Discussion For a bottom slope smaller than calculated value, downstream channel would have a mild slope, that will force the flow to remain subcritical.

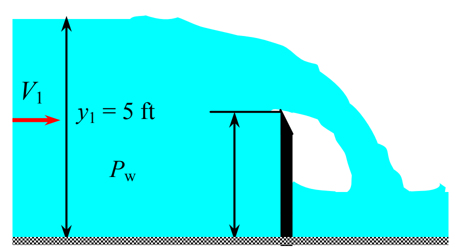
13-101E

Solution The flow rate in an open channel is to be measured using a sharp-crested rectangular weir. For specified upper limits of flow rate and flow depth, the appropriate height of the weir is to be determined.

Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The weir head is $H = y_1 - P_w = 5 - P_w$. The discharge coefficient of the weir is

$$C_{wd,\text{rec}} = 0.598 + 0.0897\frac{H}{P_w} = 0.598 + 0.0897\frac{5 - P_w}{P_w}$$



Sharp-crested rectangular weir

The water flow rate through the channel can be expressed as

$$\dot{V}_{\text{rec}} = C_{wd,\text{rec}}\frac{2}{3}b\sqrt{2g}H^{3/2}$$

Substituting the known quantities,

$$150\text{ ft}^3/\text{s} = \left(0.598 + 0.0897\frac{5 - P_w}{P_w}\right)\frac{2}{3}(10\text{ ft})\sqrt{2(32.2\text{ ft/s}^2)}(5 - P_w)^{3/2}$$

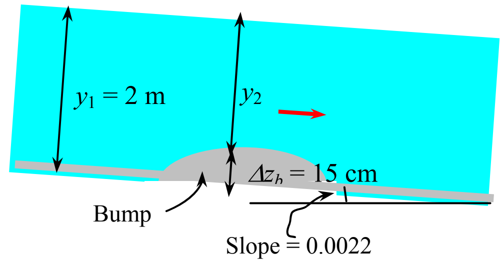
Solution of the above equation yields the weir height as P_w = **2.46 ft**.

Discussion Nonlinear equations of this kind can be solved easily using equation solvers like EES.

13-102

Solution The flow of water in a wide channel with a bump is considered. The flow rate of water without the bump and the effect of the bump on the flow rate for the case of a flat surface are to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel. **4** The channel is sufficiently wide so that the end effects are negligible. **5** Frictional effects during flow over the bump are negligible.



Properties Manning coefficient for an open channel of unfinished concrete is $n = 0.014$ (Table 13-1).

Analysis For a wide channel, the hydraulic radius is equal to the flow depth, and thus $R_h = 2$ m. Then the flow rate *before the bump* per m width (i.e., $b = 1$ m) can be determined from Manning's equation to be

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{1\,\text{m}^{1/3}/s}{0.014}(1\times 2\,\text{m}^2)(2\,\text{m})^{2/3}(0.0022)^{1/2} = 10.64\,\text{m}^3/\text{s}$$

The average flow velocity is $V = \dfrac{\dot{V}}{A_c} = \dfrac{10.64\,\text{m}^3/\text{s}}{1\times 2\,\text{m}^2} = 5.32\,\text{m/s}$.

When a bump is placed, it is said that the flow depth remains the same and there is no rise/drop, and thus $y_2 = y_1 - \Delta z_b$. But the energy equation is given as

$$E_{s2} = E_{s1} - \Delta z_b \quad \rightarrow \quad y_2 + \frac{V_2^2}{2g} = y_1 + \frac{V_1^2}{2g} - \Delta z_b \quad \rightarrow \quad \frac{V_2^2}{2g} = \frac{V_1^2}{2g}$$

since $y_2 = y_1 - \Delta z_b$, and thus $V_1 = V_2$. But from the continuity equation $y_2 V_2 = y_1 V_1$, this is possible only if the flow depth over the bump remains constant, i.e., $y_1 = y_2$, which is a contradiction since y_2 cannot be equal to both y_1 and $y_1 - \Delta z_b$ while Δz_b remains nonzero. Therefore, the second part of the problem can have **no solution** since it is physically impossible.

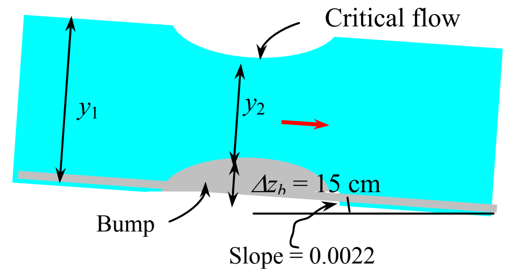
Discussion Note that sometimes it is better to investigate whether there is really a solution before spending a lot of time trying to find a solution.

13-103

Solution Uniform subcritical water flow of water in a wide channel with a bump is considered. For critical flow over the bump, the flow rate of water and the flow depth over the bump are to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel. **4** The channel is sufficiently wide so that the end effects are negligible. **5** Frictional effects during flow over the bump are negligible.

Properties Manning coefficient for an open channel of unfinished concrete is $n = 0.014$ (Table 13-1).



Analysis Let subscript 1 denote the upstream conditions (uniform flow) in the channel, and 2 denote the critical conditions over the bump. For a wide channel, the hydraulic radius is equal to the flow depth, and thus $R_h = y_1$. Then the flow rate per m width (i.e., $b = 1$ m) can be determined from Manning's equation,

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{1\,\text{m}^{1/3}/s}{0.014} y_1 (y_1)^{2/3} (0.0022)^{1/2} = 3.350 y_1^{5/3} \text{ m}^3/\text{s}$$

The critical depth corresponding to this flow rate is (note that $b = 1$ m),

$$y_2 = y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(3.350 y_1^{5/3})^2}{g}\right)^{1/3} = \left(\frac{11.224 y_1^{10/3}}{9.81\,\text{m/s}^2}\right)^{1/3} = 1.046 y_1^{10/9}$$

The average flow velocity is $V_1 = \dot{V}/A_c = 3.350 y_1^{5/3}/y_1 = 3.350 y_1^{2/3}$ m/s . Also,

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{(3.350 y_1^{2/3})^2}{2(9.81\,\text{m/s}^2)} = y_1 + 0.5720 y_1^{4/3}$$

$$E_{s2} = E_c = \frac{3}{2} y_c = \frac{3}{2}(1.046 y_1^{10/9}) = 1.569 y_1^{10/9}$$

Substituting these two relations into $E_{s2} = E_{s1} - \Delta z_b$ where $\Delta z_b = 0.15$ m gives

$$1.569 y_1^{10/9} = y_1 + 0.5720 y_1^{4/3} - 0.15$$

Using an equation solver such as EES or an iterative approach, the flow depth upstream is determined to be

$$y_1 = 2.947 \text{ m}$$

Then the flow rate and the flow depth over the bump becomes

$$\dot{V} = 3.350 y_1^{5/3} = 3.350(2.947)^{5/3} = \mathbf{20.3\ m^3/s}$$

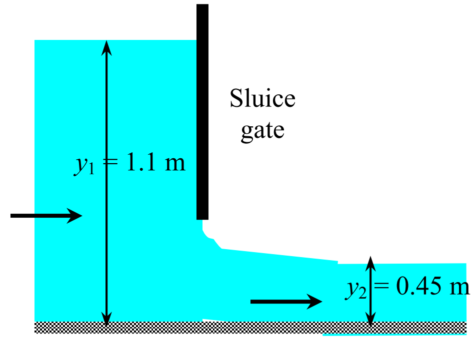
$$y_2 = y_c = 1.046 y_1^{10/9} = 1.046(2.947)^{10/9} = \mathbf{3.48\ m}$$

Discussion Note that when critical flow is established and the flow is "choked", the flow rate calculations become very easy, and it required minimal measurements. Also, $V_1 = 3.350(2.947)^{2/3} = 6.89$ m/s and

$\text{Fr}_1 = V_1/\sqrt{gy_1} = (6.89\text{ m/s})/\sqrt{(9.81\text{ m}^2/\text{s})(2.947\text{ m})} = 1.28$, and thus the upstream flow is supercritical.

13-104

Solution A sluice gate is used to control the flow rate of water in a channel. For specified flow depths upstream and downstream from the gate, the flow rate of water and the downstream Froude number are to be determined.



Assumptions **1** The flow is steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** Frictional effects associated with sluice gate are negligible. **4** The channel is horizontal.

Analysis When frictional effects are negligible and the flow section is horizontal, the specific energy remains constant, $E_{s1} = E_{s2}$. Then,

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} \quad \rightarrow \quad 1.1\,\text{m} + \frac{\dot{V}^2}{2(9.81\,\text{m/s}^2)[(5\,\text{m})(1.1\,\text{m})]^2} = 0.45\,\text{m} + \frac{\dot{V}^2}{2(9.81\,\text{m/s}^2)[(5\,\text{m})(0.45\,\text{m})]^2}$$

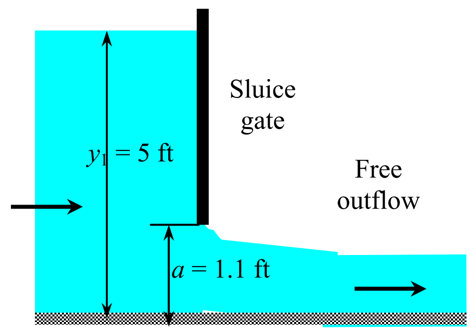
Solving for the flow rate gives $\dot{V} = \mathbf{8.806\ m^3/s}$. The downstream velocity and Froude number are

$$V_2 = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{by_2} = \frac{8.806\,\text{m}^3/\text{s}}{(5\,\text{m})(0.45\,\text{m})} = 3.914\,\text{m/s} \quad \text{and} \quad \text{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{3.914\,\text{m/s}}{\sqrt{(9.81\,\text{m/s}^2)(0.45\,\text{m})}} = \mathbf{1.86}$$

Discussion The actual values will be somewhat different because of frictional effects.

13-105E

Solution Water is released from a reservoir through a sluice gate with free outflow. For specified flow depths, the flow rate per unit width and the downstream Froude number are to be determined.



Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.

Analysis For free outflow, we only need the depth ratio y_1/a to determine the discharge coefficient (for drowned outflow, we also need to know y_2/a and thus the flow depth y_2 downstream the gate)

$$\frac{y_1}{a} = \frac{5\text{ ft}}{1.1\text{ ft}} = 4.55$$

The corresponding discharge coefficient is determined from Fig. 13-38 to be C_d = 0.55. Then the discharge rate becomes

$$\dot{V} = C_d ba\sqrt{2gy_1} = 0.55\,(1\,\text{ft})(1.1\,\text{ft})\sqrt{2\,(32.2\,\text{ft/s}^2)(5\,\text{ft})} = \mathbf{10.9\ \ ft^3/s}$$

The specific energy of a fluid remains constant during horizontal flow when the frictional effects are negligible, $E_{s1} = E_{s2}$. With these approximations, the flow depth past the gate and the Froude number are determined to be

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{\dot{V}^2}{2g(by_1)^2} = 5\,\text{ft} + \frac{(10.9\,\text{ft/s}^2)^2}{2(32.2\,\text{ft/s}^2)[(1\,\text{ft})(5\,\text{ft})]^2} = 5.074\,\text{ft}$$

$$E_{s2} = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{\dot{V}^2}{2g(by_2)^2} = E_{s1} \quad \rightarrow \quad y_2 + \frac{(10.9\,\text{ft/s}^2)^2}{2(32.2\,\text{ft/s}^2)[(1\,\text{ft})(y_2)]^2} = 5.074\,\text{ft}$$

Solution yields y_2 = 0.643 ft as the physically meaningful root (positive and less than 5 ft). Then,

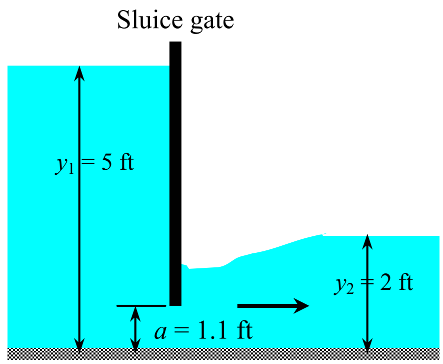
$$V_2 = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{by_2} = \frac{10.9\,\text{ft}^3/\text{s}}{(1\,\text{ft})(0.643\,\text{ft})} = 16.9\,\text{ft/s} \quad \text{and} \quad \text{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{16.9\,\text{ft/s}}{\sqrt{(32.2\,\text{ft/s}^2)(0.643\,\text{ft})}} = \mathbf{3.71}$$

Discussion In actual gates some frictional losses are unavoidable, and thus the actual velocity and Froude number downstream will be lower.

13-106E

Solution Water is released from a reservoir through a drowned sluice gate into an open channel. For specified flow depths, the rate of discharge is to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.



Analysis The depth ratio y_1/a and the contraction coefficient y_2/a are

$$\frac{y_1}{a} = \frac{5\text{ ft}}{1.1\text{ ft}} = 4.55 \quad \text{and} \quad \frac{y_2}{a} = \frac{3.3\text{ ft}}{1.1\text{ ft}} = 3$$

The corresponding discharge coefficient is determined from Fig. 13-38 to be C_d = 0.44. Then the discharge rate becomes

$$\dot{V} = C_d ba\sqrt{2gy_1} = 0.44(1\text{ ft})(1.1\text{ ft})\sqrt{2(32.2\text{ ft/s}^2)(5\text{ ft})} = \mathbf{8.69\ ft^3/s}$$

Then the Froude number downstream the gate becomes

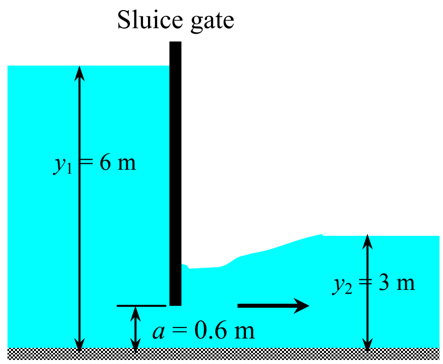
$$V_2 = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{by_2} = \frac{8.69\text{ ft}^3\text{/s}}{(1\text{ ft})(3.3\text{ ft})} = 2.63\text{ ft/s} \quad \rightarrow \quad \text{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{2.63\text{ ft/s}}{\sqrt{(32.2\text{ ft/s}^2)(3.3\text{ ft})}} = \mathbf{0.255}$$

Discussion Note that the flow past the gate becomes subcritical when the outflow is drowned.

13-107

Solution Water is released from a lake through a drowned sluice gate into an open channel. For specified flow depths, the rate of discharge through the gate is to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.



Analysis The depth ratio y_1/a and the contraction coefficient y_2/a are

$$\frac{y_1}{a} = \frac{6\text{ m}}{0.6\text{ m}} = 10 \quad \text{and} \quad \frac{y_2}{a} = \frac{3\text{ m}}{0.6\text{ m}} = 5$$

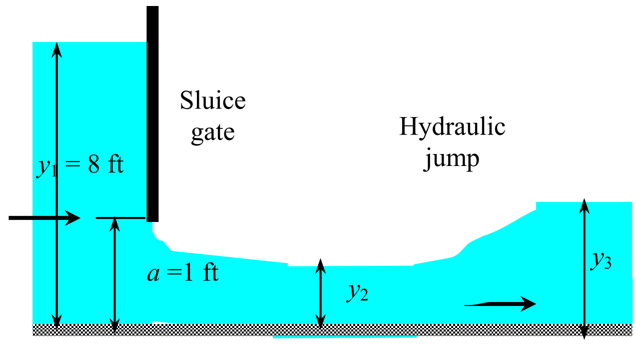
The corresponding discharge coefficient is determined from Fig. 13-38 to be C_d = 0.48. Then the discharge rate becomes

$$\dot{V} = C_d ba\sqrt{2gy_1} = 0.48(5\text{ m})(0.6\text{ m})\sqrt{2(9.81\text{ m/s}^2)(6\text{ m})} = \mathbf{15.6\ m^3/s}$$

Discussion Note that the use of the discharge coefficient enables us to determine the flow rate through sluice gates by measuring 3 flow depths only.

13-108E

Solution Water discharged through a sluice gate undergoes a hydraulic jump. The flow depth and velocities before and after the jump and the fraction of mechanical energy dissipated are to be determined.



Assumptions **1** The flow is steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** Frictional effects associated with sluice gate are negligible. **4** The channel is horizontal.

Analysis For free outflow, we only need the depth ratio y_1/a to determine the discharge coefficient,

$$\frac{y_1}{a} = \frac{8\text{ ft}}{1\text{ ft}} = 8$$

The corresponding discharge coefficient is determined from Fig. 13-38 to be $C_d = 0.58$. Then the discharge rate becomes

$$\dot{V} = C_d ba\sqrt{2gy_1} = 0.58\,(1\text{ ft})(1\text{ ft})\sqrt{2\,(32.2\text{ ft/s}^2)(8\text{ ft})} = \mathbf{13.16\ ft^3/s}$$

The specific energy of a fluid remains constant during horizontal flow when the frictional effects are negligible, $E_{s1} = E_{s2}$. With these approximations, the flow depth past the gate and the Froude number are determined to be

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{\dot{V}^2}{2g(by_1)^2} = 8\text{ ft} + \frac{(13.16\text{ ft}^3/\text{s})^2}{2(32.2\text{ ft/s}^2)[(1\text{ ft})(8\text{ ft})]^2} = 8.042\text{ ft}$$

$$E_{s2} = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{\dot{V}^2}{2g(by_2)^2} = E_{s1} \rightarrow y_2 + \frac{(13.16\text{ ft}^3/\text{s})^2}{2(32.2\text{ ft/s}^2)[(1\text{ ft})(y_2)]^2} = 8.042\text{ ft}$$

It gives $\boldsymbol{y_2} = \mathbf{0.601}$ ft as the physically meaningful root (positive and less than 8 ft). Then,

$$V_2 = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{by_2} = \frac{13.16\text{ ft}^3/\text{s}}{(1\text{ ft})(0.601\text{ ft})} = \mathbf{21.9\ ft/s}$$

$$\text{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{21.9\text{ ft/s}}{\sqrt{(32.2\text{ ft/s}^2)(0.601\text{ ft})}} = 4.97$$

Then the flow depth and velocity after the jump (state 3) become

$$y_3 = 0.5y_2\left(-1+\sqrt{1+8\text{Fr}_2^2}\right) = 0.5(0.601\text{ ft})\left(-1+\sqrt{1+8\times 4.97^2}\right) = \mathbf{3.94\ ft}$$

$$V_3 = \frac{y_2}{y_3}V_2 = \frac{0.601\text{ ft}}{3.94\text{ ft}}(21.9\text{ ft/s}) = \mathbf{3.34\ ft/s}$$

The head loss and the fraction of mechanical energy dissipated during the jump are

$$h_L = y_2 - y_3 + \frac{V_2^2 - V_3^2}{2g} = (0.601\text{ ft}) - (3.94\text{ ft}) + \frac{(21.9\text{ ft/s})^2 - (3.34\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = 3.93\text{ ft}$$

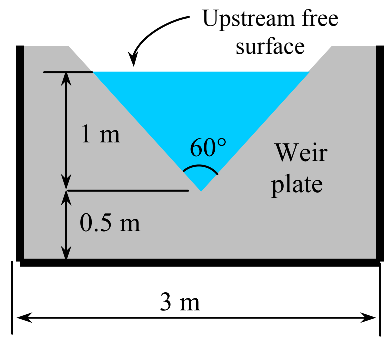
$$\text{Dissipation ratio} = \frac{h_L}{E_{s2}} = \frac{h_L}{y_2(1+\text{Fr}_2^2/2)} = \frac{3.93\text{ ft}}{(0.601\text{ ft})(1+4.97^2/2)} = \mathbf{0.488}$$

Discussion Note that almost half of the mechanical energy of the fluid is dissipated during hydraulic jump.

13-109

Solution The flow rate of water in an open channel is to be measured with a sharp-crested triangular weir. For a given flow depth upstream the weir, the flow rate is to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.



Properties The weir discharge coefficient is given to be 0.60.

Analysis The discharge rate of water is determined directly from

$$\dot{V} = C_{wd,\text{tri}} \frac{8}{15} \tan\left(\frac{\theta}{2}\right)\sqrt{2g}H^{5/2}$$

where C_{wd} = 0.60, θ = 60°, and H = 1 m. Substituting,

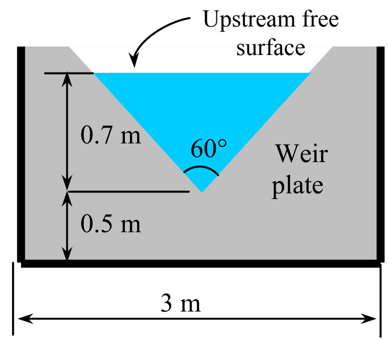
$$\dot{V} = (0.60)\frac{8}{15}\tan\left(\frac{60°}{2}\right)\sqrt{2(9.81\text{ m/s}^2)}(1\text{ m})^{5/2} = \mathbf{0.818\ m^3/s}$$

Discussion Note that the use of the discharge coefficient enables us to determine the flow rate in a channel by measuring a single flow depth. Triangular weirs are best-suited to measure low discharge rates as they are more accurate than the other weirs for small heads.

13-110

Solution The flow rate of water in an open channel is to be measured with a sharp-crested triangular weir. For a given flow depth upstream the weir, the flow rate is to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.



Properties The weir discharge coefficient is given to be 0.60.

Analysis The discharge rate of water is determined directly from

$$\dot{V} = C_{wd,\text{tri}} \frac{8}{15} \tan\left(\frac{\theta}{2}\right)\sqrt{2g}H^{5/2}$$

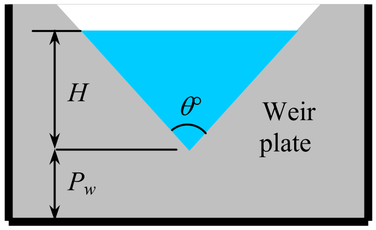
where C_{wd} = 0.60, θ = 60°, and H = 0.7 m. Substituting,

$$\dot{V} = (0.60)\frac{8}{15}\tan\left(\frac{60°}{2}\right)\sqrt{2(9.81\text{ m/s}^2)}(0.7\text{ m})^{5/2} = \mathbf{0.335\ m^3/s}$$

Discussion Note that the use of the discharge coefficient enables us to determine the flow rate in a channel by measuring a single flow depth. Triangular weirs are best-suited to measure low discharge rates as they are more accurate than the other weirs for small heads.

13-111

Solution The notch angle of a sharp-crested triangular weir used to measure the discharge rate of water from a lake is reduced by half. The percent reduction in the discharge rate is to be determined.



Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The water depth in the lake and the weir discharge coefficient remain unchanged.

Analysis The discharge rate through a triangular weir is given as

$$\dot{V} = C_{wd,\text{tri}} \frac{8}{15} \tan\left(\frac{\theta}{2}\right)\sqrt{2g}H^{5/2}$$

Therefore, the discharge rate is proportional to the tangent of the half notch angle, and the ratio of discharge rates is calculated to be

$$\dot{V} = \frac{\dot{V}_{50°}}{\dot{V}_{100°}} = \frac{\tan(50°/2)}{\tan(100°/2)} = 0.391$$

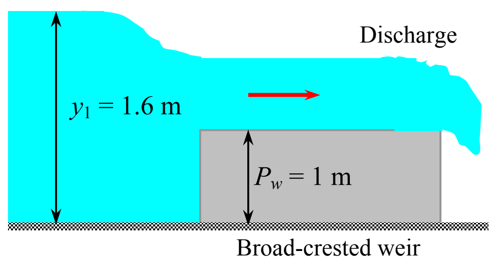
When the notch angle is reduced by half, the discharge rate drops to 39.1% of the original level. Therefore, the percent reduction in the discharge rate is

Percent reduction = 1-0.391 =0.609= **60.9%**

Discussion Note that triangular weirs with small notch angles can be used to measure small discharge rates while weirs with large notch angles can be used to measure for large discharge rates.

13-112

Solution The flow rate in an open channel is to be measured using a broad-crested rectangular weir. For a measured value of flow depth upstream, the flow rate is to be determined.



Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The weir head is $H = y_1 - P_w = 1.6 - 1.0 = 0.6$ m. The discharge coefficient of the weir is

$$C_{wd,\text{broad}} = \frac{0.65}{\sqrt{1+H/P_w}} = \frac{0.65}{\sqrt{1+(0.6\text{ m})/(1.0\text{ m})}} = 0.5139$$

Then the water flow rate through the channel becomes

$$\dot{V}_{\text{rec}} = C_{wd,\text{broad}} b\sqrt{g}\left(\frac{2}{3}\right)^{3/2} H^{3/2} = (0.5139)(5\text{ m})(2/3)^{3/2}\sqrt{9.81\text{ m/s}^2}\,(0.6\text{ m})^{3/2} = \mathbf{2.04\ m^3/s}$$

The minimum flow depth above the weir is the critical depth, which is determined from

$$y_{\min} = y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(2.04\text{ m}^3/\text{s})^2}{(9.81\text{ m/s}^2)(5\text{ m})^2}\right)^{1/3} = \mathbf{0.257\ m}$$

Discussion The upstream velocity and the upstream velocity head are

$$V_1 = \frac{\dot{V}}{by_1} = \frac{2.04\text{ m}^3/\text{s}}{(5\text{ m})(1.6\text{ m})} = 0.255\text{ m/s} \quad \text{and} \quad \frac{V_1^2}{2g} = \frac{(0.255\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.0033\text{ m}$$

This is 0.3% of the weir head, which is negligible. When the upstream velocity head is considered (by replacing H in the flow rate relation by $H + V_1^2/2g$), the flow rate becomes 2.05 m^3/s, which is practically identical to the value determined above.

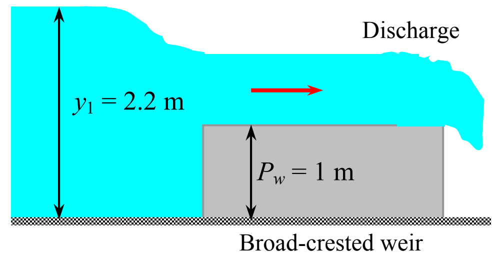
13-113

Solution The flow rate in an open channel is to be measured using a broad-crested rectangular weir. For a measured value of flow depth upstream, the flow rate is to be determined.

Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The weir head is $H = y_1 - P_w = 2.2 - 1.0 = 1.2\text{ m}$. The discharge coefficient of the weir is

$$C_{wd,\text{broad}} = \frac{0.65}{\sqrt{1+H/P_w}} = \frac{0.65}{\sqrt{1+(1.2\text{ m})/(1.0\text{ m})}} = 0.4382$$



Then the water flow rate through the channel becomes

$$\dot{V}_{\text{rec}} = C_{wd,\text{broad}}\, b\sqrt{g}\left(\frac{2}{3}\right)^{3/2} H^{3/2}$$

$$= (0.4382)(5\text{ m})(2/3)^{3/2}\sqrt{9.81\text{ m/s}^2}\,(1.2\text{ m})^{3/2}$$

$$= \mathbf{4.91\ m^3/s}$$

The minimum flow depth above the weir is the critical depth, which is determined from

$$y_{\min} = y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(4.91\text{ m}^3/\text{s})^2}{(9.81\text{ m/s}^2)(5\text{ m})^2}\right)^{1/3} = \mathbf{0.462\ m}$$

Discussion The upstream velocity and the upstream velocity head are

$$V_1 = \frac{\dot{V}}{by_1} = \frac{4.91\text{ m}^3/\text{s}}{(5\text{ m})(2.2\text{ m})} = 0.446\text{ m/s} \quad \text{and} \quad \frac{V_1^2}{2g} = \frac{(0.446\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.010\text{ m}$$

This is 0.8% of the weir head, which is negligible. When the upstream velocity head is considered (by replacing H in the flow rate relation by $H + V_1^2/2g$, the flow rate becomes 4.97 m^3/s, which is practically identical to the value determined above.

13-114

Solution The flow rate in an open channel is measured using a broad-crested rectangular weir. For a measured value of minimum flow depth over the weir, the flow rate and the upstream flow dept are to be determined.

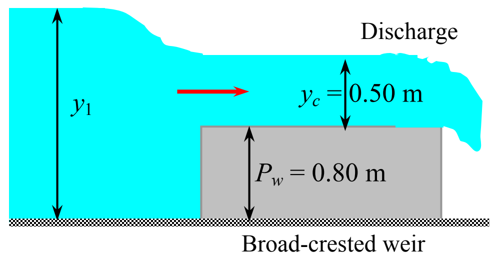
Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The flow depth over the reaches its minimum value when the flow becomes critical. Therefore, the measured minimum depth is the critical depth y_c. Then the flow rate is determined from the critical depth relation to be

$$y_{\min} = y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} \rightarrow \dot{V} = \sqrt{y_c^3 g b^2} = \sqrt{(0.50\text{ m})^3(9.81\text{ m/s}^2)(1\text{ m})^2} = \mathbf{1.11\ m^3/s}$$

This is the flow rate per m width of the channel since we have taken b = 1 m. Disregarding the upstream velocity head and noting that the discharge coefficient of the weir is $C_{wd,\text{broad}} = 0.65/\sqrt{1+H/P_w}$, the flow rate for a broad-crested weir can be expressed as

$$\dot{V}_{\text{rec}} = \frac{0.65}{\sqrt{1+H/P_w}} b\sqrt{g}\left(\frac{2}{3}\right)^{3/2} H^{3/2}$$



Substituting,

$$1.11\text{ m}^3/\text{s} = \frac{0.65\text{ m}}{\sqrt{1+\text{H}/(0.8\text{ m})}}(1\text{ m})(2/3)^{3/2}\sqrt{9.81\text{ m/s}^2}\, H^{3/2}$$
$$= \mathbf{4.91 m^3/s}$$

Its solution is H = 1.40 m. Then the flow depth upstream the weir becomes

$$y_1 = H + P_w = 1.40 + 0.80 = \mathbf{2.20\ m}$$

Discussion The upstream velocity and the upstream velocity head are

$$V_1 = \frac{\dot{V}}{by_1} = \frac{1.11\text{ m}^3/\text{s}}{(1\text{ m})(2.2\text{ m})} = 0.503\text{ m/s} \quad \text{and} \quad \frac{V_1^2}{2g} = \frac{(0.503\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.013\text{ m}$$

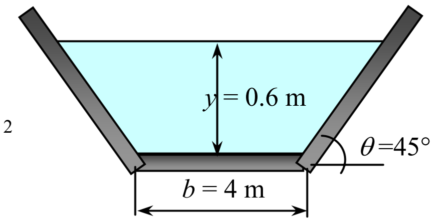
This is 0.9% of the weir head, which is negligible. When the upstream velocity head is considered (by replacing H in the flow rate relation by $H + V_1^2/2g$, the flow rate becomes 1.12 m^3/s, which is practically identical to the value determined above.

Review Problems

13-115

Solution Water flows uniformly in a trapezoidal channel. For a given flow depth, it is to be determined whether the flow is subcritical or supercritical.

Assumptions The flow is uniform.



Analysis The flow area and the average velocity are

$$A_c = y\frac{(b+b+2y/\tan\theta)}{2} = (0.60\text{ m})\frac{[4+4+2(0.60\text{ m})/\tan 45°]\text{ m}}{2} = 2.76\text{ m}^2$$

$$V = \frac{\dot{V}}{A_c} = \frac{18\text{ m}^3/\text{s}}{2.76\text{ m}^2} = 6.522\text{ m/s}$$

When calculating the Froude number, the hydraulic depth should be used rather than the maximum depth or the hydraulic radius. For a non-rectangular channel, hydraulic depth is defined as the ratio of the flow area to top width,

$$y = y_h = \frac{A_c}{\text{Top width}} = \frac{A_c}{b+2y/\tan\theta} = \frac{2.76\text{ m}^2}{(4+2\times 0.60/\tan 45°)\text{ m}} = 0.5308\text{ m}$$

Then the Froude number becomes $\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{6.522\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.5308\text{ m})}} = 2.86$, which is greater than 1.

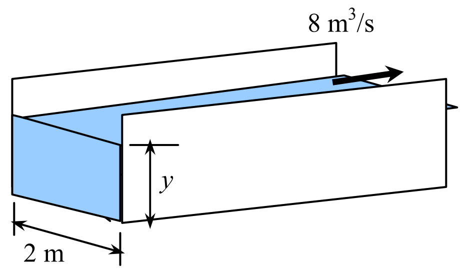
Therefore, the flow is **supercritical**.

Discussion The analysis is approximate since the edge effects are significant here compared to a wide rectangular channel, and thus the results should be interpreted accordingly.

13-116

Solution Water flows in a rectangular channel. The flow depth below which the flow is supercritical is to be determined.

Assumptions The flow is uniform.



Analysis The flow depth below which the flow is super critical is the critical depth y_c determined from

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(8\text{ m}^3/\text{s})^2}{(9.81\text{ m/s}^2)(2\text{ m})^2}\right)^{1/3} = \mathbf{1.18\text{ m}}$$

Therefore, flow is **supercritical** for $y < 1.18$ m.

Discussion Note that a flow is more likely to exist as supercritical when the flow depth is low and thus the flow velocity is high.

13-117

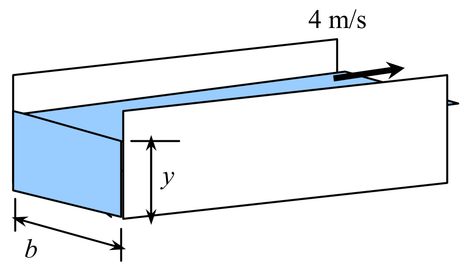
Solution Water flows in a canal at a specified average velocity. For various flow depths, it is to be determined whether the flow is subcritical or supercritical.

Assumptions The flow is uniform.

Analysis For each depth, we determine the Froude number and compare it to the critical value of 1:

(***a***) y = 0.2 m: $$\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{4\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.2\text{ m})}} = 2.86 > 1$$

which is greater than 1. Therefore, the flow is **supercritical**.



(***b***) y = 2 m: $$\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{4\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(2\text{ m})}} = 0.903 < 1$$

which is less than 1. Therefore, the flow is **subcritical**.

(***c***) y = 1.63 m: $$\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{4\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(1.63\text{ m})}} = 1$$

which is equal to 1. Therefore, the flow is **critical**.

Discussion Note that a flow is more likely to exist as supercritical when the flow depth is low and thus the flow velocity is high. Also, the type of flow can be determined easily by checking Froude number.

13-118

Solution The flow of water in a rectangular channel is considered. For a given flow depth and bottom slope, the flow rate is to be determined.

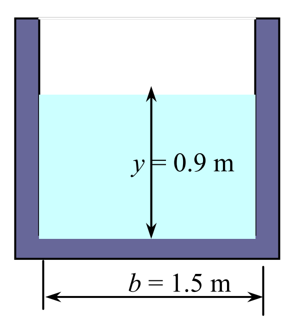
Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties The Manning coefficient is given to be n = 0.012 (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are

$$A_c = by = (1.5\text{ m})(0.9\text{ m}) = 1.35\text{ m}^2 \qquad p = 1.5\text{ m} + 2\times 0.9\text{ m} = 3.3\text{ m}$$

$$R_h = \frac{A_c}{p} = \frac{1.35\text{ m}^2}{3.3\text{ m}} = 0.4091\text{ m}$$



Bottom slope of the channel is

$$S_0 = \tan 0.6^\circ = 0.01047$$

Then the flow rate can be determined from Manning's equation to be

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{1\text{ m}^{1/3}/s}{0.012}(1.35\text{ m}^2)(0.4091\text{ m})^{2/3}(0.01047)^{1/2} = \mathbf{6.34\ m^3/s}$$

Discussion Note that the flow rate in a given channel is a strong function of the bottom slope.

13-119

Solution The flow of water in a rectangular channel is considered. The effect of bottom slope on the flow rate is to be investigated as the bottom angle varies from 0.5 to 10°.

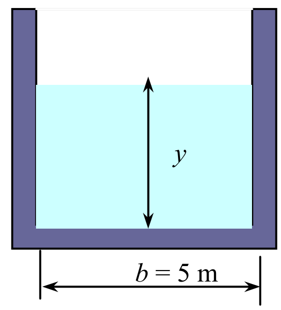
Assumptions **1** The flow is steady and uniform. **2** Roughness coefficient is constant along the channel.

Properties Manning coefficient for an open channel made of finished concrete is n = 0.012 (Table 13-1).

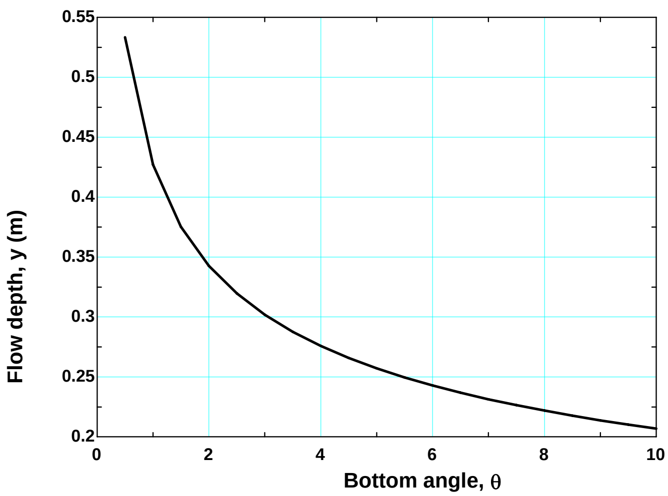
Analysis The EES *Equations* window is printed below, along with the tabulated and plotted results.

```
a=1
b=5
Vdot = 12 "m3/s"
n=0.012

s=tan(teta)
Ac=b*y
p=b+2*y
Rh=Ac/p
Vdot=(a/n)*Ac*Rh^(2/3)*SQRT(s)
```



Bottom angle, θ°	Flow depth, y, m
0.5	0.533
1.0	0.427
1.5	0.375
2.0	0.343
2.5	0.320
3.0	0.302
3.5	0.287
4.0	0.276
4.5	0.266
5.0	0.257
5.5	0.250
6.0	0.243
6.5	0.237
7.0	0.231
7.5	0.226
8.0	0.222
8.5	0.218
9.0	0.214
9.5	0.210
10.0	0.207



Discussion Note that the flow depth decreases as the bottom angle increases, as expected.

13-120

Solution The flow of water in a trapezoidal channel is considered. The effect of bottom slope on the flow rate is to be investigated as the bottom angle varies from 0.5 to 10°.

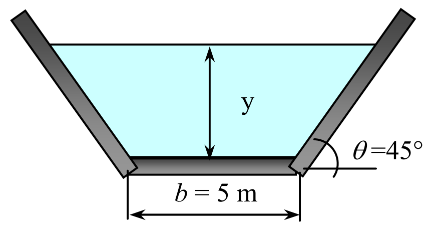
Assumptions **1** The flow is steady and uniform. **2** Roughness coefficient is constant along the channel.

Properties Manning coefficient for an open channel made of finished concrete is n = 0.012 (Table 13-1).

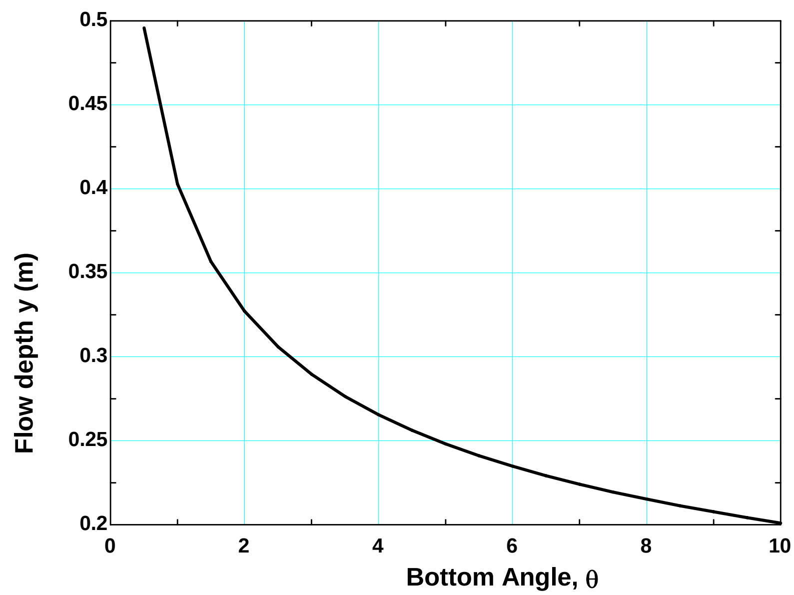
Analysis The EES *Equations* window is printed below, along with the tabulated and plotted results.

```
a=1
b=5
Vdot = 12 "m3/s"
n=0.012

s=tan(teta)
Ac=y*(b+y/tan(45))
p=b+2*y/sin(45)
Rh=Ac/p
Vdot=(a/n)*Ac*Rh^(2/3)*SQRT(s)
```



Bottom angle, $\theta°$	Flow depth, y, m
0.5	0.496
1.0	0.403
1.5	0.357
2.0	0.327
2.5	0.306
3.0	0.290
3.5	0.276
4.0	0.266
4.5	0.256
5.0	0.248
5.5	0.241
6.0	0.235
6.5	0.229
7.0	0.224
7.5	0.220
8.0	0.215
8.5	0.211
9.0	0.208
9.5	0.204
10.0	0.201



Discussion As expected, flow depth decreases with increasing bottom angle, but the relationship is far from linear.

13-121

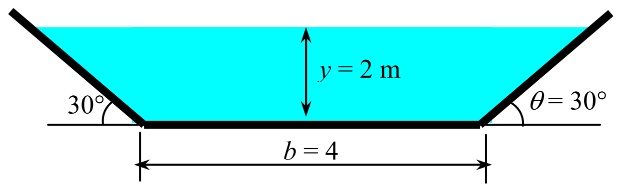
Solution The flow of water in a trapezoidal channel is considered. For a given flow depth and bottom slope, the flow rate is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties The Manning coefficient for a brick-lined open channel is n = 0.015 (Table 13-1).

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are

$$A_c = y\left(b+\frac{y}{\tan\theta}\right) = (2\text{ m})\left(4\text{ m}+\frac{2\text{ m}}{\tan 30°}\right) = 14.93\ \text{m}^2$$



$$p = b+\frac{2y}{\sin\theta} = 4\text{ m}+\frac{2(2\text{ m})}{\sin 30°} = 12\text{ m}$$

$$R_h = \frac{A_c}{p} = \frac{14.93\text{ m}^2}{12\text{ m}} = 1.244\ \text{m}$$

Bottom slope of the channel is S_o = 0.001.Then the flow rate can be determined from Manning's equation to be

$$\dot{V} = \frac{a}{n}A_c R_h^{2/3} S_0^{1/2} = \frac{1\text{ m}^{1/3}/s}{0.015}(14.93\text{ m}^2)(1.244\text{ m})^{2/3}(0.001)^{1/2} = \mathbf{36.4\ m^3/s}$$

Discussion Note that the flow rate in a given channel is a strong function of the bottom slope.

13-122

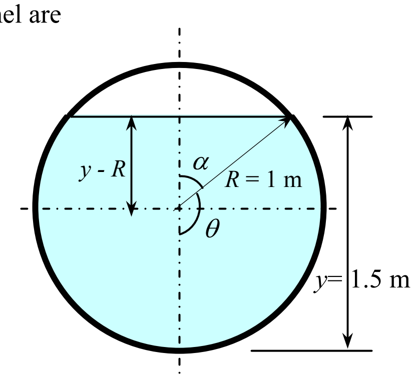
Solution The flow of water in a circular open channel is considered. For given flow depth and flow rate, the elevation drop per km length is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties The Manning coefficient for the steel channel is given to be n = 0.012.

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are

$$\cos\alpha = \frac{y-R}{R} = \frac{1.5-1}{1} = 0.5 \quad\rightarrow\quad \alpha = 60° = 60\frac{2\pi}{360} = \frac{\pi}{3}$$



$$\theta = \pi-\alpha = \pi-\frac{\pi}{3} = \frac{2\pi}{3} = 120°$$

$$A_c = R^2(\theta-\sin\theta\cos\theta) = (1\text{ m})^2[2\pi/3-\sin(2\pi/3)\cos(2\pi/3)] = 2.527\text{ m}^2$$

$$R_h = \frac{A_c}{p} = \frac{\theta-\sin\theta\cos\theta}{2\theta}R = \frac{2\pi/3-\sin(2\pi/3)\cos(2\pi/3)}{2\times 2\pi/3}(1\text{ m}) = 0.6034\text{ m}$$

Substituting the given quantities into Manning's equation,

$$\dot{V} = \frac{a}{n}A_c R_h^{2/3} S_0^{1/2} \quad\rightarrow\quad 12\text{ m}^3/\text{s} = \frac{1\text{ m}^{1/3}/s}{0.012}(2.527\text{ m}^2)(0.6034\text{ m})^{2/3} S_0^{1/2}$$

It gives the slope to be S_0 = 0.00637. Therefore, the *elevation drop* Δz across a pipe length of L = 1 km must be

$$\Delta z = S_0 L = 0.00637(1000\text{ m}) = \mathbf{6.37\ m}$$

Discussion Note that when transporting water through a region of fixed elevation drop, the only way to increase the flow rate is to use a channel with a larger cross-section.

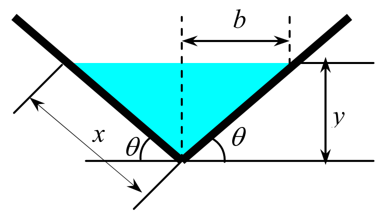
13-123

Solution The flow of water through a V-shaped open channel is considered. The angle θ the channel makes from the horizontal is to be determined for the case of most efficient flow.

Assumptions **1** The flow is steady and uniform. **2** The bottom slope is constant. **3** The roughness coefficient is constant.

Analysis We let the length of the sidewall of the channel be *x*. From trigonometry,

$$\sin\theta = \frac{y}{x} \quad \rightarrow \quad y = x\sin\theta \qquad \cos\theta = \frac{b}{x} \quad \rightarrow \quad b = x\cos\theta$$



Then the cross-sectional area and the perimeter of the flow section become

$$A_c = by = x\cos\theta\sin\theta = \frac{x^2}{2}\sin 2\theta \quad \rightarrow \quad x = \sqrt{\frac{2A_c}{\sin 2\theta}}$$

$$p = 2x = 2\sqrt{\frac{2A_c}{\sin 2\theta}} \quad \rightarrow \quad p = 2\sqrt{2A_c}(\sin 2\theta)^{-1/2}$$

Now we apply the criterion that the best hydraulic cross-section for an open channel is the one with the minimum wetted perimeter for a given cross-section. Taking the derivative of *p* with respect to *θ* while holding A_c constant gives

$$\frac{dp}{d\theta} = 2\sqrt{2A_c}\frac{d[(\sin 2\theta)^{-1/2}]}{d\theta} = 2\sqrt{2A_c}\frac{d[(\sin 2\theta)^{-1/2}]}{d(\sin 2\theta)}\frac{d(\sin 2\theta)}{d\theta} = 2\sqrt{2A_c}\frac{-3}{2(\sin 2\theta)^{3/2}}2\cos 2\theta$$

Setting $dp/d\theta = 0$ gives $\cos 2\theta = 0$, which is satisfied when $2\theta = 90°$. Therefore, the criterion for the best hydraulic cross-section for a triangular channel is determined to be $\boldsymbol{\theta = 45°}$.

Discussion The procedure used here can be used to determine the best hydraulic cross-section for any geometric shape.

13-124E

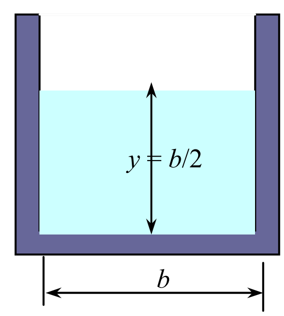
Solution Water is to be transported in a rectangular channel at a specified rate. The dimensions for the best cross-section if the channel is made of unfinished concrete are to be determined.

Assumptions **1** The flow is steady and uniform. **2** The bottom slope is constant. **3** The roughness coefficient is constant.

Properties The Manning coefficient is $n = 0.014$ for channels made of unfinished concrete (Table 13-1).

Analysis For best cross-section of a rectangular cross-section, $y = b/2$. Then $A_c = yb = b^2/2$, and $R_h = b/4$.

The flow rate is determined from the Manning equation, $\dot{V} = \frac{a}{n}A_c R_h^{2/3} S_0^{1/2}$.



(*a*) Bottom drop of 8 ft per mile: $s = (8\text{ ft})/(5280\text{ ft}) = 0.001515$

$$200\text{ ft}^3/\text{s} = \frac{1.486\text{ ft}^{1/3}/s}{0.014}(b^2/2)(b/4)^{2/3}(0.001515)^{1/2}$$

Solving the above equation gives *b* = **7.86 ft,** and *y* = *b*/2 = **3.93 ft.**

(*b*) Bottom drop of 10 ft per mile: $s = (10\text{ ft})/(5280\text{ ft}) = 0.001894$

$$200\text{ ft}^3/\text{s} = \frac{1.486\text{ ft}^{1/3}/s}{0.014}(b^2/2)(b/4)^{2/3}(0.001894)^{1/2}$$

Solving the above equation gives *b* = **7.54 ft,** and *y* = *b*/2 = **3.77 ft.**

Discussion The concept of best cross-section is an important consideration in the design of open channels because it directly affects the construction costs.

13-125E

Solution Water is to be transported in a trapezoidal channel at a specified rate. The dimensions for the best cross-section if the channel is made of unfinished concrete are to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Properties The Manning coefficient is n = 0.014 for channels made of unfinished concrete (Table 13-1).

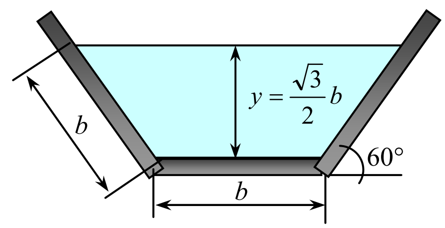
Analysis For best cross-section of a trapezoidal channel of bottom width b, θ = 60° and $y = b\sqrt{3}/2$. Then,

$$A_c = y(b + b\cos\theta) = 0.5\sqrt{3}b^2(1+\cos 60°) = 0.75\sqrt{3}b^2\,,\quad p = 3b\,,\text{ and }\ R_h = \frac{y}{2} = \frac{\sqrt{3}}{4}b\,.$$

The flow rate is determined from the Manning equation, $\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2}$,

(*a*) Bottom drop of 8 ft per mile:

$$s = (8\text{ ft})/(5280\text{ ft}) = 0.001515$$



$$200\text{ ft}^3/\text{s} = \frac{1.486\text{ ft}^{1/3}/s}{0.014}(0.75\sqrt{3}b^2)(\sqrt{3}b/4)^{2/3}(0.001515)^{1/2}$$

It gives b = **4.79 ft,** and y = **4.15 ft.**

(*b*) Bottom drop of 10 ft per mile:

$$s = (10\text{ ft})/(5280\text{ ft}) = 0.001894$$

$$200\text{ ft}^3/\text{s} = \frac{1.486\text{ ft}^{1/3}/s}{0.014}(0.75\sqrt{3}b^2)(\sqrt{3}b/4)^{2/3}(0.001894)^{1/2}$$

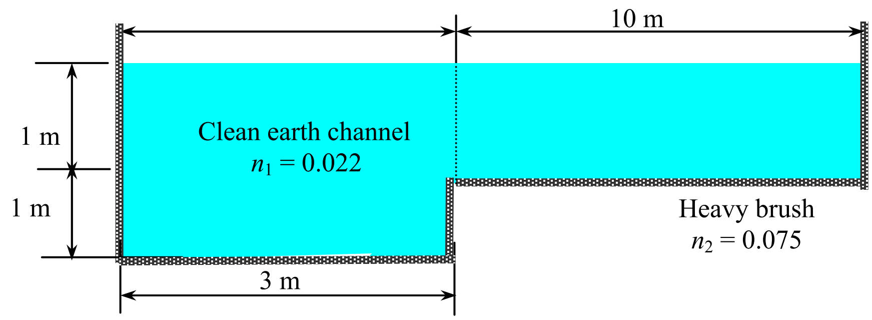
It gives b = **4.59 ft,** and y = **3.98 ft.**

Discussion The concept of best cross-section is an important consideration in the design of open channels because it directly affects the construction costs.

13-126

Solution Water is flowing through a channel with nonuniform surface properties. The flow rate through the channel and the effective Manning coefficient are to be determined.

Assumptions **1** The flow is steady and uniform. **2** The bottom slope is constant. **3** The Manning coefficients do not vary along the channel.



Analysis The channel involves two parts with different roughness, and thus it is appropriate to divide the channel into two subsections. The flow rate for each subsection can be determined from the Manning equation, and the total flow rate can be determined by adding them up.

The flow area, perimeter, and hydraulic radius for each subsection and the entire channel are:

Subsection 1: $A_{c1} = 6\text{ m}^2$, $p_1 = 6\text{ m}$, $R_{h1} = \dfrac{A_{c1}}{p_1} = \dfrac{6\text{ m}^2}{6\text{ m}} = 1.00\text{ m}$

Subsection 2: $A_{c2} = 10\text{ m}^2$, $p_2 = 11\text{ m}$, $R_{h2} = \dfrac{A_{c2}}{p_2} = \dfrac{10\text{ m}^2}{11\text{ m}} = 0.909\text{ m}$

Entire channel: $A_c = 16\text{ m}^2$, $p = 17\text{ m}$, $R_h = \dfrac{A_c}{p} = \dfrac{16\text{ m}^2}{17\text{ m}} = 0.941\text{ m}$

Applying the Manning equation to each subsection, the total flow rate through the channel becomes

$$\dot V = \dot V_1 + \dot V_2 = \frac{a}{n_1}A_1R_1^{2/3}S_0^{1/2} + \frac{a}{n_2}A_2R_2^{2/3}S_0^{1/2}$$

$$= (1\text{ m}^{1/3}/\text{s})\left(\frac{(6\text{ m}^2)\ (1\text{ m})^{2/3}}{0.022} + \frac{(10\text{ m}^2)(0.909\text{ m})^{2/3}}{0.075}\right)(\tan 0.5°)^{1/2}$$

$$= \mathbf{37.2\ m^3/s}$$

Knowing the total flow rate, the effective Manning coefficient for the entire channel can be determined from the Manning equation to be

$$n_{\text{eff}} = \frac{aA_cR_h^{2/3}S_0^{1/2}}{\dot V} = \frac{(1\text{ m}^{1/3}/\text{s})(16\text{ m}^2)(0.941\text{ m})^{2/3}(0.00873)^{1/2}}{37.2\text{ m}^3/\text{s}} = \mathbf{0.0386}$$

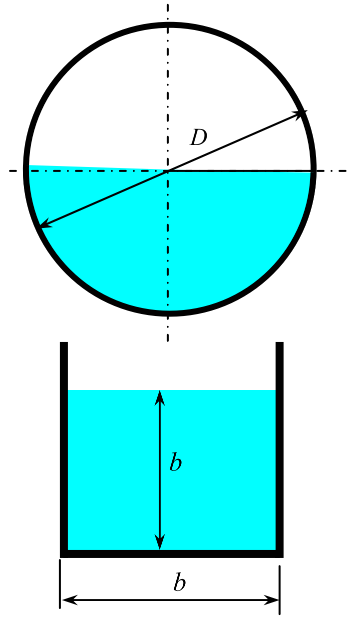
Discussion The effective Manning coefficient n_{eff} of the channel turns out to lie between the two n values, as expected. The weighted average of the Manning coefficient of the channel is $n_{\text{ave}}=(n_1p_1+ n_2p_2)/p$ = 0.056, which is quite different than n_{eff}. Therefore, using a weighted average Manning coefficient for the entire channel may be tempting, but it would not be so accurate.

13-127

Solution Two identical channels, one rectangular of bottom width b and one circular of diameter D, with identical flow rates, bottom slopes, and surface linings are considered. The relation between b and D is to be determined for the case of the flow height $y = b$ and the circular channel is flowing half full.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Analysis The cross-sectional area, perimeter, and hydraulic radius of the rectangular channel are



$$A_c = b^2, \quad p = 3b, \text{ and } R_h = \frac{A_c}{p} = \frac{b^2}{3b} = \frac{b}{3}$$

Then using the Manning equation, the flow rate can be expressed as

$$\dot{V}_{\text{rec}} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{a}{n} b^2 \left(\frac{b}{3}\right)^{2/3} S_0^{1/2} = \frac{a}{n} S_0^{1/2} \frac{b^{8/3}}{3^{2/3}}$$

The corresponding relations for the semi-circular channel are

$$A_c = \frac{\pi D^2}{8}, \quad p = \frac{\pi D}{2}, \text{ and } R_h = \frac{A_c}{p} = \frac{D}{4}$$

and

$$\dot{V}_{\text{cir}} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} = \frac{a}{n} \pi \frac{D^2}{8} \left(\frac{D}{4}\right)^{2/3} S_0^{1/2} = \frac{a}{n} S_0^{1/2} \frac{\pi D^{8/3}}{8 \times 4^{2/3}}$$

Setting the flow rates in the two channels equal to each other $\dot{V}_{\text{cir}} = \dot{V}_{\text{rec}}$ gives

$$\frac{a}{n} S_0^{1/2} \frac{b^{8/3}}{3^{2/3}} = \frac{a}{n} \frac{\pi D^{8/3}}{8 \times 4^{2/3}} S_0^{1/2} \;\rightarrow\; \frac{b^{8/3}}{3^{2/3}} = \frac{\pi D^{8/3}}{8 \times 4^{2/3}} \;\rightarrow\; \frac{b}{D} = \left(\frac{\pi 3^{2/3}}{8 \times 4^{2/3}}\right)^{3/8} = 0.655$$

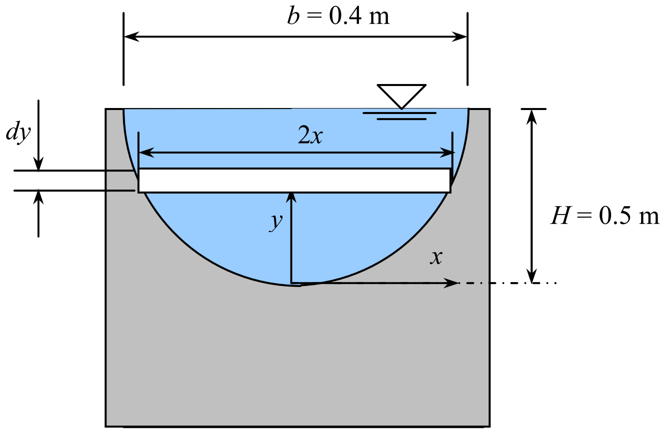
Therefore, the desired relation is **$b = 0.655D$.**

Discussion Note that the wetted perimeters in this case are $p_{\text{rec}} = 3b = 2.0D$ and $p_{\text{cir}} = \pi D/2 = 1.57D$. Therefore, the semi-circular channel is a more efficient channel than the rectangular one.

13-128

Solution The flow of water through a parabolic notch is considered. A relation is to be developed for the flow rate, and its numerical value is to be calculated.

Assumptions **1** The flow is steady. **2** All frictional effects are negligible, and Toricelli's equation can be used for the velocity.



Analysis The notch is parabolic with $y = 0$ at $x = 0$, and thus it can be expressed analytically as $y = Cx^2$. Using the coordinates of the upper right corner, the value of the constant is determined to be

$$C = y/x^2 = H/(b/2)^2 = 4H/b^2 =$$

$$4(0.5 \text{ m})/(0.4 \text{ m})^2 = 12.5 \text{ m}^{-1}.$$

A differential area strip can be expressed as

$$dA = 2xdy = 2\sqrt{y/C}dy$$

Noting that the flow velocity is $V = \sqrt{2g(H-y)}$, the flow rate through this differential area is

$$VdA = V\left(2\sqrt{y/C}dy\right) = \sqrt{2g(H-y)}2\sqrt{y/C}dy = 2\sqrt{2g/C}\sqrt{y(H-y)}dy$$

Then the flow rate through the entire notch is determined by integration to be

$$\dot{V} = \int_A VdA = 2\sqrt{2g/C}\int_{y=0}^{H} \sqrt{y(H-y)}dy$$

where

$$\int_{y=0}^{H} \sqrt{y(H-y)}dy = \left[\frac{1}{4}(2y-H)\sqrt{Hy-y^2} + \frac{H^2}{8} Arc\tan\left(\frac{2y-H}{2\sqrt{Hy-y^2}}\right)\right]_0^H = \frac{\pi}{16}H^2$$

Then the expression for the volume flow rate and its numerical value become

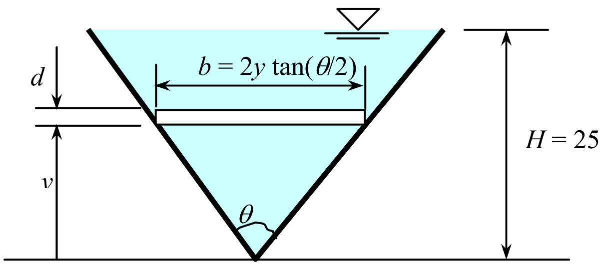
$$\boxed{\dot{V} = \frac{\pi}{8}\sqrt{\frac{2g}{C}}H^2 = \frac{\pi}{8}\sqrt{\frac{2(9.81 \text{ m/s}^2)}{12.5 \text{ m}^{-1}}}H^2 = (0.492 \text{ m/s})H^2 = (0.492 \text{ m/s})(0.5 \text{ m})^2 = 0.123 \text{ m}^3/\text{s}}$$

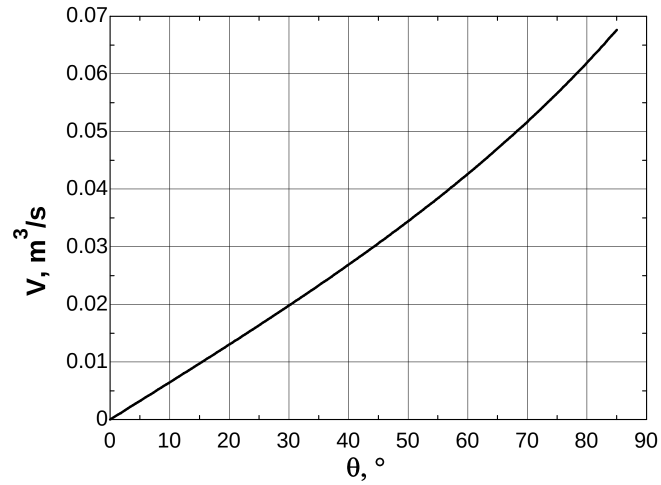
Discussion Note that a general flow rate equation for parabolic notch would be in the form of $\dot{V} = KH^2$, where $K = C_d \frac{\pi}{8}\sqrt{\frac{2g}{C}}$ and C_d is the discharge coefficient whose value is determined experimentally to account for nonideal effects.

13-129

Solution The flow of water through a parabolic notch is considered. A relation is to be developed for the flow rate, and its numerical value is to be calculated.

Assumptions **1** The flow is steady. **2** All frictional effects are negligible, and Toricelli's equation can be used for the velocity.





Analysis Consider a differential strip area shown on the sketch. It can be expressed as

$$dA = bdy = 2y\tan(\theta/2)dy$$

Noting that the flow velocity is $V=\sqrt{2g(H-y)}$, the flow rate through this differential area is

$$VdA = V\big(2y\tan(\theta/2)dy\big) = \sqrt{2g(H-y)}\,2y\tan(\theta/2)dy = 2\sqrt{2g}\tan(\theta/2)y\sqrt{H-y}dy$$

Then the flow rate through the entire notch is determined by integration to be

$$\dot{V} = \int_A VdA = 2\sqrt{2g}\tan(\theta/2)\int_{y=0}^{H} y\sqrt{H-y}dy$$

where

$$\int_{y=0}^{H} y\sqrt{H-y}dy = \left[-\frac{2}{5}y^{5/2}+\frac{2}{3}Hy^{3/2}\right]_0^H = \frac{4}{15}H^{5/2}$$

Then the expression for the volume flow rate and its numerical value become

$$\dot{V} = \frac{8\sqrt{2g}}{15}\tan(\theta/2)H^{5/2} = \frac{8\sqrt{2(9.81\text{ m/s}^2)}}{15}\tan(\theta/2)(0.25)^{5/2} = 0.07382\tan(\theta/2)\quad(\text{m}^3/\text{s})$$

θ= **25°**: $\dot{V} = 0.07382\tan(25°/2) = \mathbf{0.0164\ m^3/s}$

θ= **40°**: $\dot{V} = 0.07382\tan(40°/2) = \mathbf{0.0269\ m^3/s}$

θ= **60°**: $\dot{V} = 0.07382\tan(60°/2) = \mathbf{0.0426\ m^3/s}$

θ= **75°**: $\dot{V} = 0.07382\tan(75°/2) = \mathbf{0.0566\ m^3/s}$

These results are plotted, using EES.

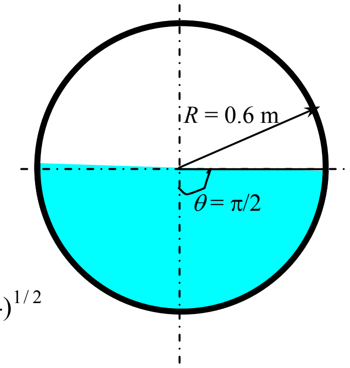
Discussion Note that a general flow rate equation for the V-notch would be in the form of $\dot{V} = K\tan(\theta/2)H^{5/2}$, where $K = C_d 8\sqrt{2g}/15$ and C_d is the discharge coefficient whose value is determined experimentally to account for nonideal effects.

13-130

Solution Water flows uniformly half-full in a circular channel. For specified flow rate and bottom slope, the Manning coefficient is to be determined.

Assumptions **1** The flow is steady and uniform. **2** Bottom slope is constant. **3** Roughness coefficient is constant along the channel.

Analysis The flow area, wetted perimeter, and hydraulic radius of the channel are



$$A_c = \frac{\pi R^2}{2} = \frac{\pi\,(0.6\text{ m})^2}{2} = 0.5655\text{ m}^2$$

$$p = \frac{2\pi R}{2} = \frac{2\pi(0.6\text{ m})}{2} = 1.885\text{ m}$$

$$R_h = \frac{A_c}{P} = \frac{\pi R^2/2}{\pi R} = \frac{R}{2} = \frac{0.60\text{ m}}{2} = 0.30\text{ m}$$

Then the Manning coefficient can be determined from Manning's equation to be

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \quad \rightarrow \quad 1.25\text{ m}^3/\text{s} = \frac{1\text{ m}^{1/3}/s}{n}(0.5655\text{ m}^2)(0.30\text{ m})^{2/3}(0.004)^{1/2}$$

It gives the Manning coefficient to be

$$n = \mathbf{0.013}$$

When calculating the Froude number, the hydraulic depth should be used rather than the maximum depth or the hydraulic radius. For a non-rectangular channel, hydraulic depth is defined as the ratio of the flow area to top width,

$$y_h = \frac{A_c}{\text{Top width}} = \frac{\pi R^2/2}{2R} = \frac{\pi R}{4} = \frac{\pi(0.6\text{ m})}{4} = 0.4712\text{ m}$$

$$V = \frac{\dot{V}}{A_c} = \frac{1.25\text{ m}^3/\text{s}}{0.5655\text{ m}^2} = 2.210\text{ m/s}$$

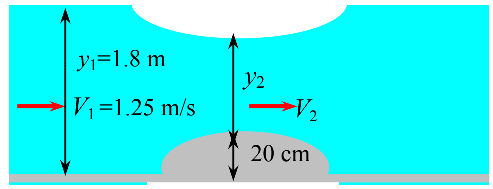
$$\text{Fr} = \frac{V}{\sqrt{gy}} = \frac{2.21\text{m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.4721\text{ m})}} = \mathbf{1.03}$$

which is greater than 1. Therefore, the flow is *supercritical*.

Discussion It appears that this channel is made of cast iron or unplaned wood .

13-131

Solution Water flowing in a horizontal open channel encounters a bump. Flow properties over the bump are to be determined.



Assumptions **1** The flow is steady. **2** Frictional effects are negligible so that there is no dissipation of mechanical energy. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The upstream Froude number and the critical depth are

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{1.25\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m^2/s})(1.8\ \mathrm{m})}} = 0.297$$

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(by_1V_1)^2}{gb^2}\right)^{1/3} = \left(\frac{y_1^2V_1^2}{g}\right)^{1/3} = \left(\frac{(1.8\ \mathrm{m})^2(1.25\ \mathrm{m/s})^2}{9.81\ \mathrm{m/s^2}}\right)^{1/3} = 0.802\ \mathrm{m}$$

The upstream flow is subcritical since Fr < 1, and the flow depth decreases over the bump. The upstream, over the bump, and critical specific energy are

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = (1.80\ \mathrm{m}) + \frac{(1.25\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} = 1.88\ \mathrm{m}$$

The flow depth over the bump can be determined from

$$y_2^3 - (E_{s1} - \Delta z_b)y_2^2 + \frac{V_1^2}{2g}y_1^2 = 0 \quad \rightarrow \quad y_2^3 - (1.88 - 0.20\ \mathrm{m})y_2^2 + \frac{(1.25\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})}(1.80\ \mathrm{m})^2 = 0$$

Using an equation solver, the physically meaningful root of this equation is determined to be y_2 = **1.576 m**. Then,

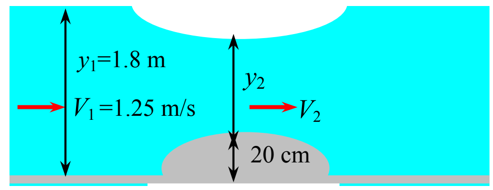
$$V_2 = \frac{y_1}{y_2}V_1 = \frac{1.8\ \mathrm{m}}{1.576\ \mathrm{m}}(1.25\ \mathrm{m/s}) = \mathbf{1.43\ m/s}$$

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{1.428\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(1.576\ \mathrm{m})}} = \mathbf{0.363}$$

Discussion The actual values may be somewhat different than those given above because of the frictional effects that are neglected in the analysis.

13-132

Solution Water flowing in a horizontal open channel encounters a bump. The bump height for which the flow over the bump is critical is to be determined.



Assumptions **1** The flow is steady. **2** Frictional effects are negligible so that there is no dissipation of mechanical energy. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The upstream Froude number and the critical depth are

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{1.25\text{ m/s}}{\sqrt{(9.81\text{ m}^2\text{/s})(1.8\text{ m})}} = 0.297$$

$$y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3} = \left(\frac{(by_1V_1)^2}{gb^2}\right)^{1/3} = \left(\frac{y_1^2V_1^2}{g}\right)^{1/3} = \left(\frac{(1.8\text{ m})^2(1.25\text{ m/s})^2}{9.81\text{ m/s}^2}\right)^{1/3} = 0.802\text{ m}$$

The upstream flow is subcritical since Fr < 1, and the flow depth decreases over the bump. The upstream specific energy is

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = (1.80\text{ m}) + \frac{(1.25\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 1.88\text{ m}$$

Noting that the flow over the bump is critical and that $E_{s2} = E_{s1} - \Delta z_b$,

$$E_{s2} = E_c = \frac{3}{2}y_c = \frac{3}{2}(0.802\text{ m}) = 1.20\text{ m}$$

and

$$\Delta z_b = E_{s1} - E_{s2} = 1.88 - 1.20 = \mathbf{0.68\text{ m}}$$

Discussion If a higher bump is used, the flow will remain critical but the flow rate will decrease (the choking effect).

13-133

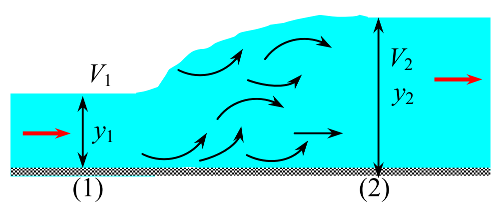
Solution Water flow through a wide rectangular channel undergoing a hydraulic jump is considered. It is to be shown that the ratio of the Froude numbers before and after the jump can be expressed in terms of flow depths y_1 and y_2 before and after the jump, respectively, as $\mathrm{Fr}_1/\mathrm{Fr}_2 = \sqrt{(y_2/y_1)^3}$.

Assumptions **1** The flow is steady. **2** The channel is sufficiently wide so that the end effects are negligible.

Analysis The Froude number for a wide channel of width b and flow depth y is given as

$$\mathrm{Fr} = \frac{V}{\sqrt{gy}} = \frac{\dot{V}/by}{\sqrt{gy}} = \frac{\dot{V}}{by\sqrt{gy}} = \frac{\dot{V}}{b\sqrt{gy^3}}$$

Expressing the Froude number before and after the jump and taking their ratio gives



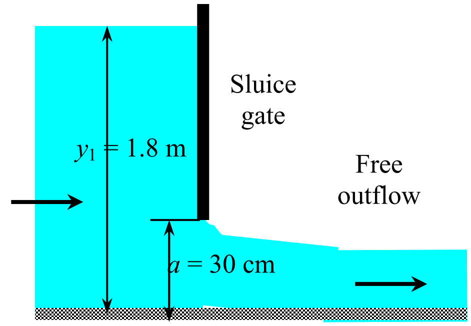
$$\frac{\mathrm{Fr}_1}{\mathrm{Fr}_2} = \frac{\dot{V}/\left(b\sqrt{gy_1^3}\right)}{\dot{V}/\left(b\sqrt{gy_2^3}\right)} = \frac{\sqrt{gy_2^3}}{\sqrt{gy_1^3}} = \sqrt{\left(\frac{y_2}{y_1}\right)^3}$$

which is the desired result.

Discussion Using the momentum equation, other relations such as $y_2 = 0.5y_1\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right)$ can also be developed.

13-134

Solution A sluice gate with free outflow is used to control the flow rate of water. For specified flow depths, the flow rate per unit width and the downstream flow depth and velocity are to be determined.



Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.

Analysis For free outflow, we only need the depth ratio y_1/a to determine the discharge coefficient (for drowned outflow, we also need to know y_2/a and thus the flow depth y_2 downstream the gate),

$$\frac{y_1}{a} = \frac{1.8\text{ m}}{0.30\text{ m}} = 6$$

The corresponding discharge coefficient is determined from Fig. 13-38 to be $C_d = 0.57$. Then the discharge rate per m width becomes

$$\dot{V} = C_d ba\sqrt{2gy_1} = 0.57\,(1\text{ m})(0.30\text{ m})\sqrt{2\,(9.81\text{ m/s}^2)(1.8\text{ m})} = \mathbf{1.02\ m^3/s}$$

The specific energy of a fluid remains constant during horizontal flow when the frictional effects are negligible, $E_{s1} = E_{s2}$. With these approximations, the flow depth and velocity past the gate become

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{\dot{V}^2}{2g(by_1)^2} = 1.8\text{ m} + \frac{(1.02\text{ m/s}^2)^2}{2(9.81\text{ m/s}^2)[(1\text{ m})(1.8\text{ m})]^2} = 1.816\text{ m}$$

$$E_{s2} = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{\dot{V}^2}{2g(by_2)^2} = E_{s1} \;\rightarrow\; y_2 + \frac{(1.02\text{ m/s}^2)^2}{2(9.81\text{ m/s}^2)[(1\text{ m})(y_2)]^2} = 1.816\text{ m}$$

It gives y_2 = **0.179** m for flow depth as the physically meaningful root (positive and less than 1.8 m). Also,

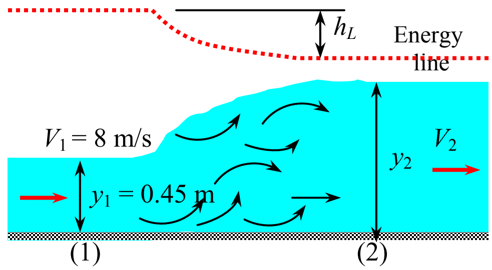
$$V_2 = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{by_2} = \frac{1.02\text{ m}^3/\text{s}}{(1\text{ m})(0.179\text{ m})} = \mathbf{5.67\ m/s}$$

Discussion In actual gates some frictional losses are unavoidable, and thus the actual velocity downstream will be lower.

13-135

Solution Water at a specified depth and velocity undergoes a hydraulic jump. The fraction of mechanical energy dissipated is to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The channel is horizontal.



Analysis The Froude number before the hydraulic jump is

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{8\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(0.45\ \mathrm{m})}} = 3.808$$

which is greater than 1. Therefore, the flow is indeed supercritical before the jump. The flow depth, velocity, and Froude number after the jump are

$$y_2 = 0.5y_1\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right) = 0.5(0.45\ \mathrm{m})\left(-1+\sqrt{1+8\times 3.808^2}\right) = 2.209\ \mathrm{m}$$

$$V_2 = \frac{y_1}{y_2}V_1 = \frac{0.45\ \mathrm{m}}{2.209\ \mathrm{m}}(8\ \mathrm{m/s}) = 1.630\ \mathrm{m/s}$$

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{1.630\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(2.209\ \mathrm{m})}} = 0.350$$

The head loss and the fraction of mechanical energy dissipated during the jump are

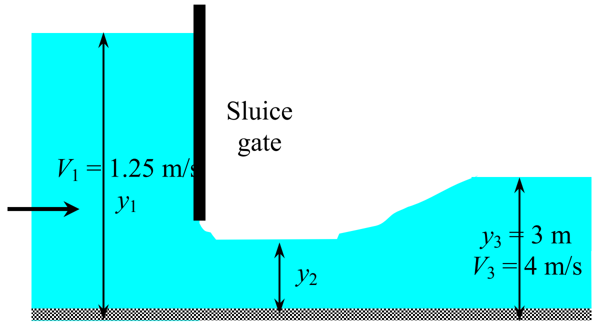
$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (0.45\ \mathrm{m}) - (2.209\ \mathrm{m}) + \frac{(8\ \mathrm{m/s})^2 - (1.63\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} = 1.368\ \mathrm{m}$$

$$\text{Dissipation ratio} = \frac{h_L}{E_{s1}} = \frac{h_L}{y_1(1+\mathrm{Fr}_1^2/2)} = \frac{1.368\ \mathrm{m}}{(0.45\ \mathrm{m})(1+3.808^2/2)} = \mathbf{0.369}$$

Discussion Note that almost over one-third of the mechanical energy of the fluid is dissipated during hydraulic jump.

13-136

Solution The flow depth and average velocity of water after a hydraulic jump together with approach velocity to sluice gate are given. The flow rate per m width, the flow depths before and after the gate, and the energy dissipation ratio are to be determined.



Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.

Analysis The flow rate per m width of channel, flow depth before the sluice gate, and the Froude number after the jump is

$$\dot{V} = V_3 A_{c3} = V_3 b y_3 = (4\text{ m/s})(1\text{ m})(3\text{ m}) = \mathbf{12\ m^3/s}$$

$$y_1 = \frac{V_3}{V_1} y_3 = \frac{4\text{ m/s}}{1.25\text{ m/s}}(3\text{ m}) = \mathbf{9.60\ m}$$

$$\text{Fr}_3 = \frac{V_3}{\sqrt{g y_3}} = \frac{4\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(3\text{ m})}} = 0.7373$$

The flow dept, velocity, and Froude number before the jump are

$$y_2 = 0.5 y_3\left(-1+\sqrt{1+8\text{Fr}_3^2}\right) = 0.5(3\text{ m})\left(-1+\sqrt{1+8\times 0.7373^2}\right) = 1.969\text{ m} \cong \mathbf{1.97\ m}$$

$$V_2 = \frac{y_3}{y_2} V_3 = \frac{3\text{ m}}{1.969\text{ m}}(4\text{ m/s}) = 6.094\text{ m/s}$$

$$\text{Fr}_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{6.094\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(1.969\text{ m})}} = 1.387$$

which is greater than 1, and thus the flow before the jump is indeed supercritical. The head loss and the fraction of mechanical energy dissipated during hydraulic jump are

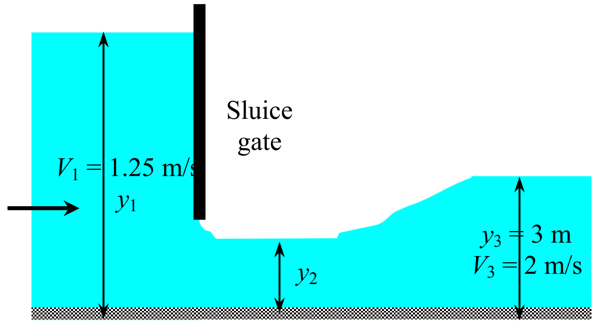
$$h_L = y_2 - y_3 + \frac{V_2^2 - V_3^2}{2g} = (1.969\text{ m}) - (3\text{ m}) + \frac{(6.094\text{ m/s})^2 - (4\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.0463\text{ m}$$

$$\text{Dissipation ratio} = \frac{h_L}{E_{s2}} = \frac{h_L}{y_2(1+\text{Fr}_2^2/2)} = \frac{0.0463\text{ m}}{(1.969\text{ m})(1+1.387^2/2)} = \mathbf{0.0120}$$

Discussion Note that this is a "mild" hydraulic jump, and only 1.2% of the mechanical energy is wasted.

13-137

Solution The flow depth and average velocity of water after a hydraulic jump together with approach velocity to sluice gate are given. The flow rate per m width, the flow depths before and after the gate, and the energy dissipation ratio are to be determined.



Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.

Analysis The flow rate per m width of channel, flow depth before the sluice gate, and the Froude number after the jump is

$$\dot{V} = V_3 A_{c3} = V_3 b y_3 = (2\text{ m/s})(1\text{ m})(3\text{ m}) = \mathbf{6\ m^3/s}$$

$$y_1 = \frac{V_3}{V_1} y_3 = \frac{2\text{ m/s}}{1.25\text{ m/s}}(3\text{ m}) = \mathbf{4.8\ m}$$

$$\text{Fr}_3 = \frac{V_3}{\sqrt{g y_3}} = \frac{2\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(3\text{ m})}} = 0.3687$$

The flow dept, velocity, and Froude number before the jump are

$$y_2 = 0.5 y_3\left(-1+\sqrt{1+8\text{Fr}_3^2}\right) = 0.5(3\text{ m})\left(-1+\sqrt{1+8\times 0.3687^2}\right) = 0.6671\text{ m} \cong \mathbf{0.667\ \ m}$$

$$V_2 = \frac{y_3}{y_2} V_3 = \frac{3\text{ m}}{0.6671\text{ m}}(2\text{ m/s}) = 8.994\text{ m/s}$$

$$\text{Fr}_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{8.994\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.6671\text{ m})}} = 3.516$$

which is greater than 1, and thus the flow before the jump is indeed supercritical. The head loss and the fraction of mechanical energy dissipated during hydraulic jump are

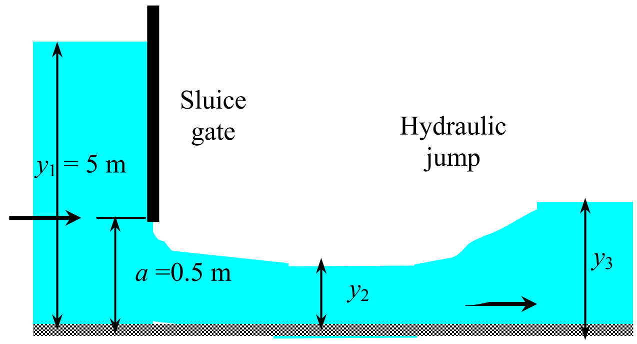
$$h_L = y_2 - y_3 + \frac{V_2^2 - V_3^2}{2g} = (0.6671\text{ m}) - (3\text{ m}) + \frac{(8.994\text{ m/s})^2 - (2\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 1.586\text{ m}$$

$$\text{Dissipation ratio} = \frac{h_L}{E_{s2}} = \frac{h_L}{y_2(1+\text{Fr}_2^2/2)} = \frac{1.586\text{ m}}{(0.6671\text{ m})(1+3.516^2/2)} = \mathbf{0.331}$$

Discussion Note that this is a fairly "strong" hydraulic jump, wasting 33.1% of the mechanical energy of the fluid.

13-138

Solution Water from a lake is discharged through a sluice gate into a channel where uniform flow conditions are established, and then undergoes a hydraulic jump. The flow depth, velocity, and Froude number after the jump are to be determined.



Assumptions **1** The flow is steady. **2** The channel is sufficiently wide so that the end effects are negligible. **3** The effects of channel slope on hydraulic jump are negligible.

Properties The Manning coefficient for an open channel made of finished concrete is n = 0.012 (Table 13-1).

Analysis For free outflow, we only need the depth ratio y_1/a to determine the discharge coefficient,

$$\frac{y_1}{a}=\frac{5\text{ m}}{0.5\text{ m}}=10$$

The corresponding discharge coefficient is determined from Fig. 13-38 to be C_d= 0.58. Then the discharge rate per m width (b = 1 m) becomes

$$\dot{V}=C_d ba\sqrt{2gy_1}=0.58\,(1\text{ m})(0.5\text{ m})\sqrt{2\,(9.81\text{ m/s}^2)(5\text{ m})}=2.872\text{ m}^3/\text{s}$$

For wide channels, hydraulic radius is the flow depth and thus $R_h = y_2$. Then the flow depth in uniform flow after the gate is determined from the Manning's equation to be

$$\dot{V}=\frac{a}{n}A_c R_h^{2/3}S_0^{1/2}\quad\rightarrow\quad 2.872\text{ m}^3/\text{s}=\frac{1\text{ m}^{1/3}/s}{0.012}[(1\text{ m})y_2](y_2)^{2/3}0.004^{1/2}$$

It gives y_2 = 0.6948 m, which is also the flow depth before water undergoes a hydraulic jump. The flow velocity and Froude number in uniform flow are

$$V_2=\frac{\dot{V}}{b\,y_2}=\frac{2.872\text{ m}^3/\text{s}}{(1\text{ m})\,(0.6948\text{ m})}=4.134\text{ m/s}$$

$$\text{Fr}_2=\frac{V_2}{\sqrt{gy_2}}=\frac{4.134\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(0.6948\text{ m})}}=1.584$$

Then the flow depth, velocity, and Froude number after the jump (state 3) become

$$y_3=0.5y_2\left(-1+\sqrt{1+8\text{Fr}_2^2}\right)=0.5(0.6948\text{ m})\left(-1+\sqrt{1+8\times1.584^2}\right)=\mathbf{1.25\text{ m}}$$

$$V_3=\frac{y_2}{y_3}V_2=\frac{0.6948\text{ m}}{1.25\text{ m}}(4.134\text{ m/s})=\mathbf{2.30\text{ m/s}}$$

$$\text{Fr}_3=\frac{V_3}{\sqrt{gy_{23}}}=\frac{2.30\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(1.25\text{ m})}}=\mathbf{0.659}$$

Discussion This is a relatively "mild" jump. It can be shown that the head loss during hydraulic jump is 0.049 m, which corresponds to an energy dissipation ratio of 3.1%.

13-139

Solution Water is discharged from a dam into a wide spillway to reduce the risk of flooding by dissipating a large fraction of mechanical energy via hydraulic jump. For specified flow depths, the velocities before and after the jump, and the mechanical power dissipated per meter with of the spillway are to be determined.

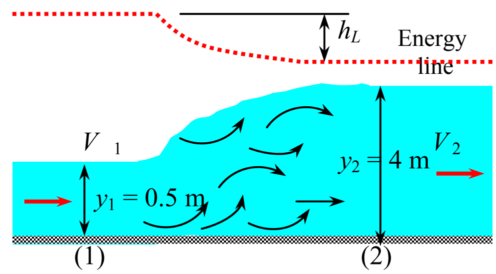
Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.

Properties The density of water is 1000 kg/m^3.

Analysis The Froude number and velocity before the jump are

$$\frac{y_2}{y_1} = 0.5\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right) \rightarrow \frac{4\text{ m}}{0.5\text{ m}} = 0.5\left(-1+\sqrt{1+8\mathrm{Fr}_1^2}\right)$$

which gives $\mathrm{Fr}_1 = 6$. Also, from the definition of Froude number,



$$V_1 = \mathrm{Fr}_1\sqrt{gy_1} = (6)\sqrt{(9.81\text{ m/s}^2)(0.5\text{ m})} = \mathbf{13.3\ m/s}$$

Velocity and Froude number after the jump are

$$V_2 = \frac{y_1}{y_2}V_1 = \frac{0.5\text{ m}}{4\text{ m}}(13.3\text{ m/s}) = \mathbf{1.66\ m/s}$$

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{1.66\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(4\text{ m})}} = 0.265$$

The head loss is determined from the energy equation to be

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (0.5\text{ m}) - (4\text{ m}) + \frac{(13.3\text{ m/s})^2 - (1.66\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 5.36\text{ m}$$

The volume and mass flow rates of water per m width are

$$\dot{V} = V_1 A_{c1} = V_1 b y_1 = (13.3\text{ m/s})(1\text{ m})(0.5\text{ m}) = 6.64\text{ m}^3/\text{s}$$

$$\dot{m} = \rho\dot{V} = (1000\text{ kg/m}^3)(6.64\text{ m}^3/\text{s}) = 6640\text{ kg/s}$$

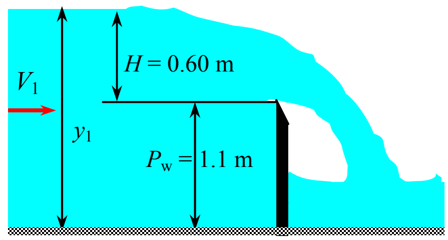
Then the dissipated mechanical power becomes

$$\dot{E}_{\text{dissipated}} = \dot{m}gh_L = (6640\text{ kg/s})(9.81\text{ m/s}^2)(5.36\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 349\text{ kNm/s} = \mathbf{349\ kW}$$

Discussion The results show that the hydraulic jump is a highly dissipative process, wasting 349 kW of power in this case.

13-140

Solution The flow rate in an open channel is to be measured using a sharp-crested rectangular weir. For a measured value of flow depth upstream, the flow rate is to be determined.



Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The weir head is given to be $H = 0.60$ m. The discharge coefficient of the weir is

$$C_{wd,\text{rec}} = 0.598 + 0.0897\frac{H}{P_w} = 0.598 + 0.0897\frac{0.60\text{ m}}{1.1\text{ m}} = 0.6469$$

The condition $H/P_w < 2$ is satisfied since 0.60/1.1 = 0.55. Then the water flow rate through the channel becomes

$$\dot{V} = C_{wd,\text{rec}}\frac{2}{3}b\sqrt{2g}H^{3/2}$$
$$= (0.6469)\frac{2}{3}(6\text{ m})\sqrt{2(9.81\text{ m/s}^2)}(0.60\text{ m})^{3/2}$$
$$= \mathbf{5.33\ m^3/s}$$

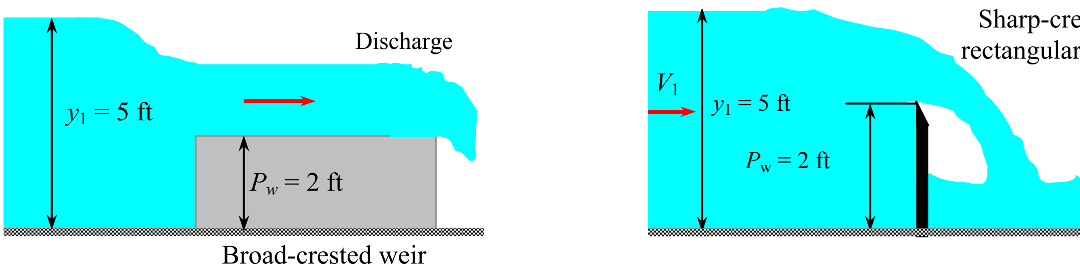
Discussion The upstream velocity and the upstream velocity head are

$$V_1 = \frac{\dot{V}}{by_1} = \frac{5.33\text{ m}^3/\text{s}}{(6\text{ m})(1.70\text{ m})} = 0.522\text{ m/s} \quad \text{and} \quad \frac{V_1^2}{2g} = \frac{(0.522\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 0.014\text{ m}$$

This is 2.3% of the weir head, which is negligible. When the upstream velocity head is considered, the flow rate becomes 5.50 m^3/s, which is about 3 percent higher than the value determined above. Therefore, the assumption of negligible velocity head is reasonable in this case.

13-141E

Solution The flow rates in two open channels are to be measured using a sharp-crested weir in one and a broad-crested rectangular weir in the other. For identical flow depths, the flow rates through both channels are to be determined.



Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The weir head is

$$H = y_1 - P_w = 5.0\text{ ft} - 2.0\text{ ft} = 3.0\text{ ft}$$

The condition $H/P_w < 2$ is satisfied since 3.0/2.0 = 1.5. The discharge coefficients of the weirs are

Sharp-crested weir:

$$C_{wd,\text{sharp}} = 0.598 + 0.0897\frac{H}{P_w} = 0.598 + 0.0897\frac{3.0\text{ ft}}{2.0\text{ ft}} = 0.7326$$

$$\dot{V}_{\text{sharp}} = C_{wd,sharp}\frac{2}{3}b\sqrt{2g}H^{3/2} = (0.7326)\frac{2}{3}(12\text{ ft})\sqrt{2(32.2\text{ ft/s}^2)}(3.0\text{ ft})^{3/2} = \mathbf{244\ m^3/s}$$

Broad-crested weir:

$$C_{wd,\text{broad}} = \frac{0.65}{\sqrt{1+H/P_w}} = \frac{0.65}{\sqrt{1+(3.0\text{ ft})/(2.0\text{ ft})}} = 0.4111$$

$$\dot{V}_{\text{broad}} = C_{wd,\text{broad}}b\sqrt{g}\left(\frac{2}{3}\right)^{3/2}H^{3/2} = (0.4111)(12\text{ ft})(2/3)^{3/2}\sqrt{32.2\text{ ft/s}^2}(3.0\text{ ft})^{3/2} = \mathbf{79.2\ m^3/s}$$

Discussion Note that the flow rate in the channel with the broad-crested weir is much less than the channel with the sharp-crested weir. Also, if the upstream velocity is taken into consideration, the flow rate would be 270 ft^3/s (11% difference) for the channel with the sharp-crested weir, and 80.3 ft^3/s (1% difference) for the one with broad-crested weir. Therefore, the assumption of negligible dynamic head is not quite appropriate for the channel with the sharp-crested weir.

Design and Essay Problems

13-142 to 13-143

Solution Students' essays and designs should be unique and will differ from each other.