Solutions Manual for
Fluid Mechanics: Fundamentals and Applications
by Çengel & Cimbala

CHAPTER 3
PRESSURE AND FLUID STATICS

Pressure, Manometer, and Barometer

3-1C

Solution We are to discuss the difference between gage pressure and absolute pressure.

Analysis The **pressure relative to the atmospheric pressure** is called the *gage pressure*, and the **pressure relative to an absolute vacuum** is called *absolute pressure*.

Discussion Most pressure gages (like your bicycle tire gage) read relative to atmospheric pressure, and therefore read the gage pressure.

3-2C

Solution We are to explain nose bleeding and shortness of breath at high elevation.

Analysis Atmospheric air pressure which is the external pressure exerted on the skin decreases with increasing elevation. Therefore, **the pressure is lower at higher elevations. As a result, the difference between the blood pressure in the veins and the air pressure outside increases**. **This pressure imbalance may cause some thin-walled veins such as the ones in the nose to burst, causing bleeding**. The shortness of breath is caused by the lower air density at higher elevations, and thus lower amount of oxygen per unit volume.

Discussion People who climb high mountains like Mt. Everest suffer other physical problems due to the low pressure.

3-3C

Solution We are to examine a claim about absolute pressure.

Analysis **No, the absolute pressure in a liquid of constant density does not double when the depth is doubled**. It is the *gage pressure* that doubles when the depth is doubled.

Discussion This is analogous to temperature scales – when performing analysis using something like the ideal gas law, you *must* use absolute temperature (K), not relative temperature (^{o}C), or you will run into the same kind of problem.

3-4C

Solution We are to compare the pressure on the surfaces of a cube.

Analysis Since pressure increases with depth, **the pressure on the bottom face of the cube is higher than that on the top. The pressure varies linearly along the side faces**. However, if the lengths of the sides of the tiny cube suspended in water by a string are very small, the magnitudes of the pressures on all sides of the cube are nearly the same.

Discussion In the limit of an "infinitesimal cube", we have a fluid particle, with pressure P defined at a "point".

3-5C

Solution We are to define Pascal's law and give an example.

Analysis *Pascal's law* states that **the pressure applied to a confined fluid increases the pressure throughout by the same amount**. This is a consequence of the pressure in a fluid remaining constant in the horizontal direction. An example of Pascal's principle is the operation of the hydraulic car jack.

Discussion The above discussion applies to fluids at rest (hydrostatics). When fluids are in motion, Pascal's principle does not necessarily apply. However, as we shall see in later chapters, the differential equations of incompressible fluid flow contain only pressure *gradients*, and thus an increase in pressure *in the whole system* does *not* affect fluid motion.

3-6C

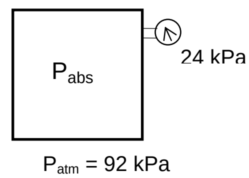
Solution We are to compare the volume and mass flow rates of two fans at different elevations.

Analysis The density of air at sea level is higher than the density of air on top of a high mountain. Therefore, the volume flow rates of the two fans running at identical speeds will be the same, but the mass flow rate of the fan at sea level will be higher.

Discussion In reality, the fan blades on the high mountain would experience less frictional drag, and hence the fan motor would not have as much resistance – the rotational speed of the fan on the mountain would be slightly higher than that at sea level.

3-7

Solution The pressure in a vacuum chamber is measured by a vacuum gage. The absolute pressure in the chamber is to be determined.



Analysis The absolute pressure in the chamber is determined from

$$P_{abs} = P_{atm} - P_{vac} = 92 - 24 = \mathbf{68\ kPa}$$

Discussion We must remember that "vacuum pressure" is the negative of gage pressure – hence the negative sign.

3-8E

Solution The pressure in a tank is measured with a manometer by measuring the differential height of the manometer fluid. The absolute pressure in the tank is to be determined for two cases: the manometer arm with the (*a*) higher and (*b*) lower fluid level being attached to the tank.

Assumptions The fluid in the manometer is incompressible.

Properties The specific gravity of the fluid is given to be SG = 1.25. The density of water at 32°F is 62.4 lbm/ft^3.

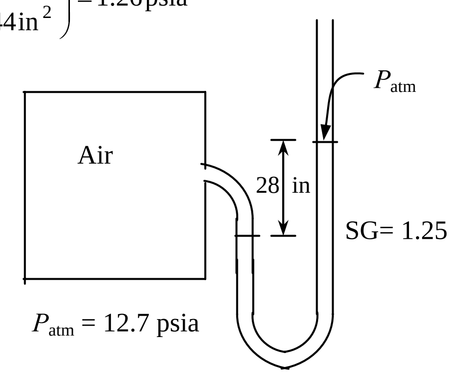
Analysis The density of the fluid is obtained by multiplying its specific gravity by the density of water,

$$\rho = \text{SG} \times \rho_{H_2O} = (1.25)(62.4\ \text{lbm/ft}^3) = 78.0\ \ \text{lbm/ft}^3$$

The pressure difference corresponding to a differential height of 28 in between the two arms of the manometer is

$$\Delta P = \rho g h = (78\text{lbm/ft}^3)(32.174\text{ft/s}^2)(28/12\text{ft})\left(\frac{1\text{lbf}}{32.174\text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\text{ft}^2}{144\text{in}^2}\right) = 1.26\text{psia}$$

Then the absolute pressures in the tank for the two cases become:



(*a*) The fluid level in the arm attached to the tank is higher (vacuum):

$$P_{abs} = P_{atm} - P_{vac} = 12.7 - 1.26 = 11.44\ \text{psia} \cong \mathbf{11.4\ psia}$$

(*b*) The fluid level in the arm attached to the tank is lower:

$$P_{abs} = P_{gage} + P_{atm} = 12.7 + 1.26 = 13.96\ \text{psia} \cong \mathbf{14.0\ \ psia}$$

Discussion The final results are reported to three significant digits. Note that we can determine whether the pressure in a tank is above or below atmospheric pressure by simply observing the side of the manometer arm with the higher fluid level.

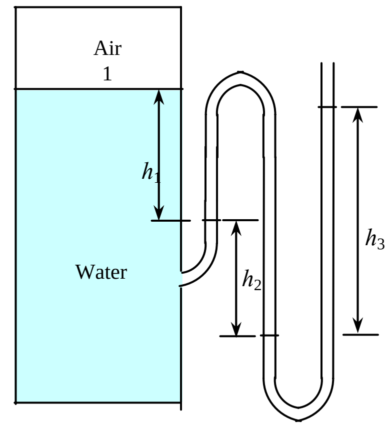
3-9

Solution The pressure in a pressurized water tank is measured by a multi-fluid manometer. The gage pressure of air in the tank is to be determined.

Assumptions The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus we can determine the pressure at the air-water interface.

Properties The densities of mercury, water, and oil are given to be 13,600, 1000, and 850 kg/m^3, respectively.

Analysis Starting with the pressure at point 1 at the air-water interface, and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach point 2, and setting the result equal to P_{atm} since the tube is open to the atmosphere gives



$$P_1 + \rho_{water} g h_1 + \rho_{oil} g h_2 - \rho_{mercury} g h_3 = P_{atm}$$

Solving for $P_{1,}$

$$P_1 = P_{atm} - \rho_{water} g h_1 - \rho_{oil} g h_2 + \rho_{mercury} g h_3$$

or,

$$P_1 - P_{atm} = g(\rho_{mercury} h_3 - \rho_{water} h_1 - \rho_{oil} h_2)$$

Noting that $P_{1,gage} = P_1 - P_{atm}$ and substituting,

$$P_{1,gage} = (9.81\ \text{m/s}^2)[(13{,}600\ \text{kg/m}^3)(0.46\ \text{m}) - (1000\ \text{kg/m}^3)(0.2\ \text{m})$$

$$-(850\ \text{kg/m}^3)(0.3\ \text{m})]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right)$$

$$= \mathbf{56.9\ kPa}$$

Discussion Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly.

3-10

Solution The barometric reading at a location is given in height of mercury column. The atmospheric pressure is to be determined.

Properties The density of mercury is given to be 13,600 kg/m^3.

Analysis The atmospheric pressure is determined directly from

$$P_{atm} = \rho g h = (13{,}600\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.750\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right)$$

$$= 100.1\ \text{kPa} \cong \mathbf{100\ \ kPa}$$

Discussion We round off the final answer to three significant digits. 100 kPa is a fairly typical value of atmospheric pressure on land slightly above sea level.

3-11

Solution The gage pressure in a liquid at a certain depth is given. The gage pressure in the same liquid at a different depth is to be determined.

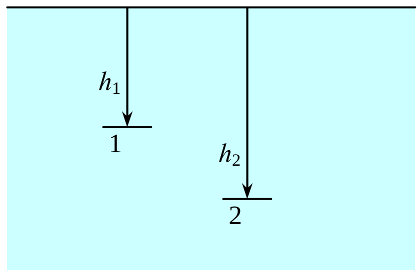
Assumptions The variation of the density of the liquid with depth is negligible.

Analysis The gage pressure at two different depths of a liquid can be expressed as $P_1 = \rho g h_1$ and $P_2 = \rho g h_2$. Taking their ratio,

$$\frac{P_2}{P_1} = \frac{\rho g h_2}{\rho g h_1} = \frac{h_2}{h_1}$$

Solving for P_2 and substituting gives

$$P_2 = \frac{h_2}{h_1} P_1 = \frac{12 \text{ m}}{3 \text{ m}} (28 \text{ kPa}) = \mathbf{112\ kPa}$$



Discussion Note that the gage pressure in a given fluid is proportional to depth.

3-12

Solution The absolute pressure in water at a specified depth is given. The local atmospheric pressure and the absolute pressure at the same depth in a different liquid are to be determined.

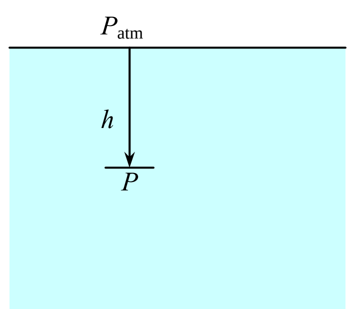
Assumptions The liquid and water are incompressible.

Properties The specific gravity of the fluid is given to be SG = 0.85. We take the density of water to be 1000 kg/m^3. Then density of the liquid is obtained by multiplying its specific gravity by the density of water,

$$\rho = \text{SG} \times \rho_{H_2O} = (0.85)(1000 \text{ kg/m}^3) = 850 \text{ kg/m}^3$$

Analysis (*a*) Knowing the absolute pressure, the atmospheric pressure can be determined from

$$P_{atm} = P - \rho g h$$
$$= (145 \text{ kPa}) - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(5 \text{ m})\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right) = \mathbf{96.0\ kPa}$$



(*b*) The absolute pressure at a depth of 5 m in the other liquid is

$$P = P_{atm} + \rho g h$$
$$= (96.0 \text{ kPa}) + (850 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(5 \text{ m})\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right)$$
$$= 137.7 \text{ kPa} \cong \mathbf{138\ kPa}$$

Discussion Note that at a given depth, the pressure in the lighter fluid is lower, as expected.

3-13E

Solution It is to be shown that 1 kgf/cm^2 = 14.223 psi.

Analysis Noting that 1 kgf = 9.80665 N, 1 N = 0.22481 lbf, and 1 in = 2.54 cm, we have

$$1 \text{ kgf} = 9.80665 \text{ N} = (9.80665 \text{ N})\left(\frac{0.22481 \text{ lbf}}{1 \text{ N}}\right) = 2.20463 \text{ lbf}$$

and $$1 \text{ kgf/cm}^2 = 2.20463 \text{ lbf/cm}^2 = (2.20463 \text{ lbf/cm}^2)\left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)^2 = 14.223 \text{ lbf/in}^2 = \mathbf{14.223\ psi}$$

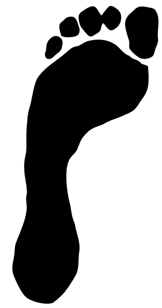
Discussion This relationship may be used as a conversion factor.

3-14E

Solution The weight and the foot imprint area of a person are given. The pressures this man exerts on the ground when he stands on one and on both feet are to be determined.

Assumptions The weight of the person is distributed uniformly on foot imprint area.

Analysis The weight of the man is given to be 200 lbf. Noting that pressure is force per unit area, the pressure this man exerts on the ground is



(*a*) On one foot: $$P = \frac{W}{A} = \frac{200\ \text{lbf}}{36\ \text{in}^2} = 5.56\ \text{lbf/in}^2 = \mathbf{5.56\ psi}$$

(*a*) On both feet: $$P = \frac{W}{2A} = \frac{200\ \text{lbf}}{2\times 36\ \text{in}^2} = 2.78\ \text{lbf/in}^2 = \mathbf{2.78\ psi}$$

Discussion Note that the pressure exerted on the ground (and on the feet) is reduced by half when the person stands on both feet.

3-15

Solution The mass of a woman is given. The minimum imprint area per shoe needed to enable her to walk on the snow without sinking is to be determined.

Assumptions **1** The weight of the person is distributed uniformly on the imprint area of the shoes. **2** One foot carries the entire weight of a person during walking, and the shoe is sized for walking conditions (rather than standing). **3** The weight of the shoes is negligible.

Analysis The mass of the woman is given to be 70 kg. For a pressure of 0.5 kPa on the snow, the imprint area of one shoe must be

$$A = \frac{W}{P} = \frac{mg}{P} = \frac{(70\ \text{kg})(9.81\ \text{m/s}^2)}{0.5\ \text{kPa}}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) = \mathbf{1.37\ m^2}$$

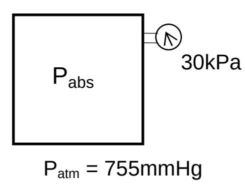
Discussion This is a very large area for a shoe, and such shoes would be impractical to use. Therefore, some sinking of the snow should be allowed to have shoes of reasonable size.

3-16

Solution The vacuum pressure reading of a tank is given. The absolute pressure in the tank is to be determined.

Properties The density of mercury is given to be ρ = 13,590 kg/m^3.

Analysis The atmospheric (or barometric) pressure can be expressed as



$$\begin{aligned} P_{atm} &= \rho g h \\ &= (13{,}590\ \text{kg/m}^3)(9.807\ \text{m/s}^2)(0.755\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= 100.6\ \text{kPa} \end{aligned}$$

Then the absolute pressure in the tank becomes

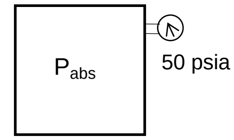
$$P_{abs} = P_{atm} - P_{vac} = 100.6 - 30 = \mathbf{70.6\ \ kPa}$$

Discussion The gage pressure in the tank is the negative of the vacuum pressure, i.e., P_{gage} = –30.0 kPa.

3-17E

Solution A pressure gage connected to a tank reads 50 psi. The absolute pressure in the tank is to be determined.

Properties The density of mercury is given to be ρ = 848.4 lbm/ft^3.



Analysis The atmospheric (or barometric) pressure can be expressed as

$$\begin{aligned} P_{atm} &= \rho g h \\ &= (848.4\ \text{lbm/ft}^3)(32.174\ \text{ft/s}^2)(29.1/12\ \text{ft})\left(\frac{1\ \text{lbf}}{32.174\ \text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right) \\ &= 14.29\ \text{psia} \end{aligned}$$

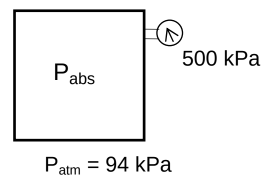
Then the absolute pressure in the tank is

$$P_{abs} = P_{gage} + P_{atm} = 50 + 14.29 = 64.29\ \text{psia} \cong \mathbf{64.3\ psia}$$

Discussion This pressure is more than four times as much as standard atmospheric pressure.

3-18

Solution A pressure gage connected to a tank reads 500 kPa. The absolute pressure in the tank is to be determined.



Analysis The absolute pressure in the tank is determined from

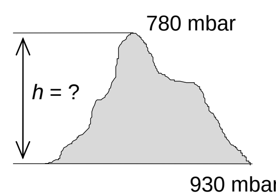
$$P_{\text{abs}} = P_{\text{gage}} + P_{\text{atm}} = 500 + 94 = \mathbf{594\ kPa}$$

Discussion This pressure is almost six times greater than standard atmospheric pressure.

3-19

Solution A mountain hiker records the barometric reading before and after a hiking trip. The vertical distance climbed is to be determined.

Assumptions The variation of air density and the gravitational acceleration with altitude is negligible.



Properties The density of air is given to be ρ = 1.20 kg/m^3.

Analysis Taking an air column between the top and the bottom of the mountain and writing a force balance per unit base area, we obtain

$$W_{\text{air}} / A = P_{\text{bottom}} - P_{\text{top}}$$

$$(\rho g h)_{\text{air}} = P_{\text{bottom}} - P_{\text{top}}$$

$$(1.20\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{bar}}{100{,}000\ \text{N/m}^2}\right) = (0.930 - 0.780)\ \text{bar}$$

It yields h = 1274 m ≅ **1270 m** (to 3 significant digits), which is also the distance climbed.

Discussion A similar principle is used in some aircraft instruments to measure elevation.

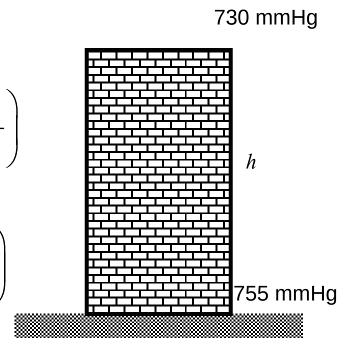
3-20

Solution A barometer is used to measure the height of a building by recording reading at the bottom and at the top of the building. The height of the building is to be determined.

Assumptions The variation of air density with altitude is negligible.

Properties The density of air is given to be ρ = 1.18 kg/m^3. The density of mercury is 13,600 kg/m^3.

Analysis Atmospheric pressures at the top and at the bottom of the building are



$$\begin{aligned} P_{\text{top}} &= (\rho g h)_{\text{top}} \\ &= (13{,}600 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(0.730 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right) \\ &= 97.36 \text{ kPa} \end{aligned}$$

$$\begin{aligned} P_{\text{bottom}} &= (\rho g h)_{\text{bottom}} \\ &= (13{,}600 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(0.755 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right) \\ &= 100.70 \text{ kPa} \end{aligned}$$

Taking an air column between the top and the bottom of the building, we write a force balance per unit base area,

$$W_{\text{air}} / A = P_{\text{bottom}} - P_{\text{top}} \quad \text{and} \quad (\rho g h)_{\text{air}} = P_{\text{bottom}} - P_{\text{top}}$$

$$(1.18 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(h)\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right) = (100.70 - 97.36) \text{ kPa}$$

which yields h = 288.6 m ≅ **289 m**, which is also the height of the building.

Discussion There are more accurate ways to measure the height of a building, but this method is quite simple.

3-21

Solution The previous problem is reconsidered. The EES solution is to be printed out, including proper units.

Analysis The EES *Equations* window is printed below, followed by the *Solution* window.

```
P_bottom=755"[mmHg]"
P_top=730"[mmHg]"
g=9.807    "[m/s^2]"    "local acceleration of gravity at sea level"
rho=1.18"[kg/m^3]"
DELTAP_abs=(P_bottom-P_top)*CONVERT('mmHg','kPa')"[kPa]"    "Delta P reading from the
barometers, converted from mmHg to kPa."
DELTAP_h =rho*g*h/1000 "[kPa]"        "Equ. 1-16. Delta P due to the air fluid column height, h,
between the top and bottom of the building."
"Instead of dividing by 1000 Pa/kPa we could have multiplied rho*g*h by the EES function,
CONVERT('Pa','kPa')"
DELTAP_abs=DELTAP_h
```

SOLUTION

Variables in Main

DELTAP_abs=3.333 [kPa]	DELTAP_h=3.333 [kPa]
g=9.807 [m/s^2]	h=288 [m]
P_bottom=755 [mmHg]	P_top=730 [mmHg]
rho=1.18 [kg/m^3]	

Discussion To obtain the solution in EES, simply click on the icon that looks like a calculator, or Calculate-Solve.

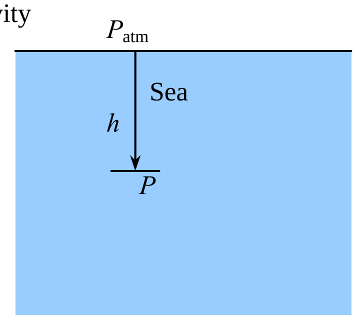
3-22

Solution A diver is moving at a specified depth from the water surface. The pressure exerted on the surface of the diver by the water is to be determined.

Assumptions The variation of the density of water with depth is negligible.

Properties The specific gravity of sea water is given to be SG = 1.03. We take the density of water to be 1000 kg/m^3.

Analysis The density of the sea water is obtained by multiplying its specific gravity by the density of water which is taken to be 1000 kg/m^3:



$$\rho = \text{SG} \times \rho_{H_2O} = (1.03)(1000\ \text{kg/m}^3) = 1030\ \text{kg/m}^3$$

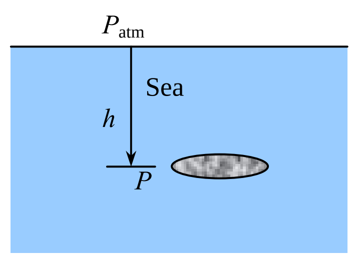
The pressure exerted on a diver at 30 m below the free surface of the sea is the absolute pressure at that location:

$$\begin{aligned} P &= P_{atm} + \rho gh \\ &= (101\,\text{kPa}) + (1030\ \text{kg/m}^3)(9.807\ \text{m/s}^2)(30\ \text{m})\left(\frac{1\,\text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= \mathbf{404\ kPa} \end{aligned}$$

Discussion This is about 4 times the normal sea level value of atmospheric pressure.

3-23E

Solution A submarine is cruising at a specified depth from the water surface. The pressure exerted on the surface of the submarine by water is to be determined.



Assumptions The variation of the density of water with depth is negligible.

Properties The specific gravity of sea water is given to be SG = 1.03. The density of water at 32°F is 62.4 lbm/ft^3.

Analysis The density of the seawater is obtained by multiplying its specific gravity by the density of water,

$$\rho = \text{SG} \times \rho_{H_2O} = (1.03)(62.4\,\text{lbm/ft}^3) = 64.27\,\text{lbm/ft}^3$$

The pressure exerted on the surface of the submarine cruising 300 ft below the free surface of the sea is the absolute pressure at that location:

$$\begin{aligned} P &= P_{atm} + \rho gh \\ &= (14.7\ \text{psia}) + (64.27\ \text{lbm/ft}^3)(32.174\ \text{ft/s}^2)(300\ \text{ft})\left(\frac{1\ \text{lbf}}{32.174\ \text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right) \\ &= 148.6\ \text{psia} \cong \mathbf{149\ psia} \end{aligned}$$

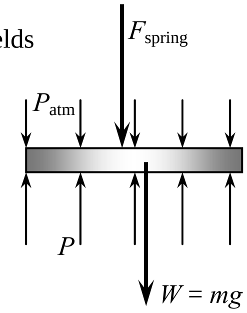
where we have rounded the final answer to three significant digits.

Discussion This is more than 10 times the value of atmospheric pressure at sea level.

3-24

Solution A gas contained in a vertical piston-cylinder device is pressurized by a spring and by the weight of the piston. The pressure of the gas is to be determined.

Analysis Drawing the free body diagram of the piston and balancing the vertical forces yields



$$PA = P_{atm}A + W + F_{spring}$$

Thus,

$$P = P_{atm} + \frac{mg + F_{spring}}{A}$$

$$= (95 \text{ kPa}) + \frac{(4 \text{ kg})(9.807 \text{ m/s}^2) + 60 \text{ N}}{35\times10^{-4} \text{ m}^2}\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right) = 123.4 \text{ kPa} \cong \mathbf{123 \text{ kPa}}$$

Discussion This setup represents a crude but functional way to control the pressure in a tank.

3-25

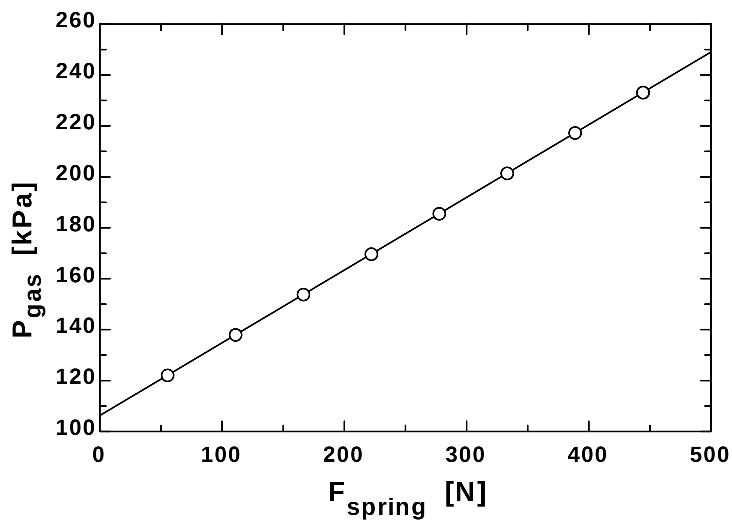
Solution The previous problem is reconsidered. The effect of the spring force in the range of 0 to 500 N on the pressure inside the cylinder is to be investigated. The pressure against the spring force is to be plotted, and results are to be discussed.

Analysis The EES *Equations* window is printed below, followed by the tabulated and plotted results.

```
g=9.807"[m/s^2]"
P_atm= 95"[kPa]"
m_piston=4"[kg]"
{F_spring=60"[N]"}
A=35*CONVERT('cm^2','m^2')"[m^2]"
W_piston=m_piston*g"[N]"
F_atm=P_atm*A*CONVERT('kPa','N/m^2')"[N]"
"From the free body diagram of the piston, the balancing vertical forces yield:"
F_gas= F_atm+F_spring+W_piston"[N]"
P_gas=F_gas/A*CONVERT('N/m^2','kPa')"[kPa]"
```

Results:

F_{spring} [N]	P_{gas} [kPa]
0	106.2
55.56	122.1
111.1	138
166.7	153.8
222.2	169.7
277.8	185.6
333.3	201.4
388.9	217.3
444.4	233.2
500	249.1



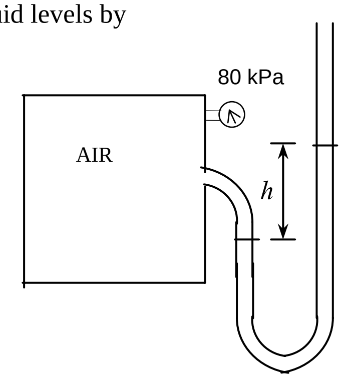
Discussion The relationship is linear, as expected.

3-26 **[*Also solved using EES on enclosed DVD*]**

Solution Both a pressure gage and a manometer are attached to a tank of gas to measure its pressure. For a specified reading of gage pressure, the difference between the fluid levels of the two arms of the manometer is to be determined for mercury and water.

Properties The densities of water and mercury are given to be ρ_{water} = 1000 kg/m^3 and be ρ_{Hg} = 13,600 kg/m^3.

Analysis The gage pressure is related to the vertical distance h between the two fluid levels by



$$P_{\text{gage}} = \rho g h \quad \longrightarrow \quad h = \frac{P_{\text{gage}}}{\rho g}$$

(*a*) For mercury,

$$h = \frac{P_{\text{gage}}}{\rho_{Hg} g} = \frac{80 \text{ kPa}}{(13600 \text{ kg/m}^3)(9.807 \text{ m/s}^2)}\left(\frac{1 \text{kN/m}^2}{1 \text{kPa}}\right)\left(\frac{1000 \text{ kg/m}\cdot\text{s}^2}{1 \text{kN}}\right) = \mathbf{0.60\ m}$$

(*b*) For water,

$$h = \frac{P_{\text{gage}}}{\rho_{H_2O} g} = \frac{80 \text{ kPa}}{(1000 \text{ kg/m}^3)(9.807\text{m/s}^2)}\left(\frac{1 \text{kN/m}^2}{1 \text{kPa}}\right)\left(\frac{1000 \text{ kg/m}\cdot\text{s}^2}{1 \text{kN}}\right) = \mathbf{8.16\ m}$$

Discussion The manometer with water is more precise since the column height is bigger (better resolution). However, a column of water more than 8 meters high would be impractical, so mercury is the better choice of manometer fluid here.

3-27

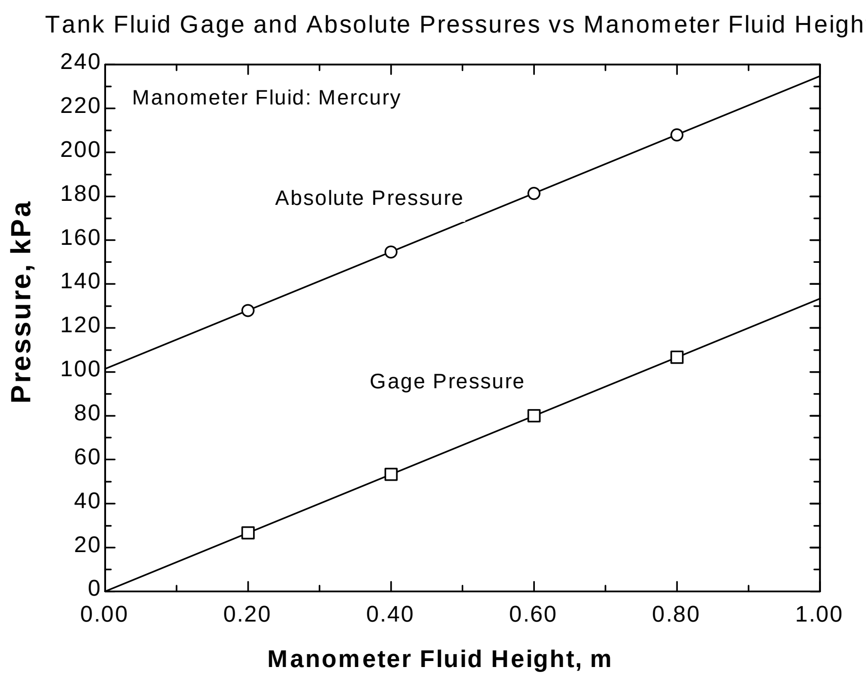
Solution The previous problem is reconsidered. The effect of the manometer fluid density in the range of 800 to 13,000 kg/m^3 on the differential fluid height of the manometer is to be investigated. Differential fluid height is to be plotted as a function of the density, and the results are to be discussed.

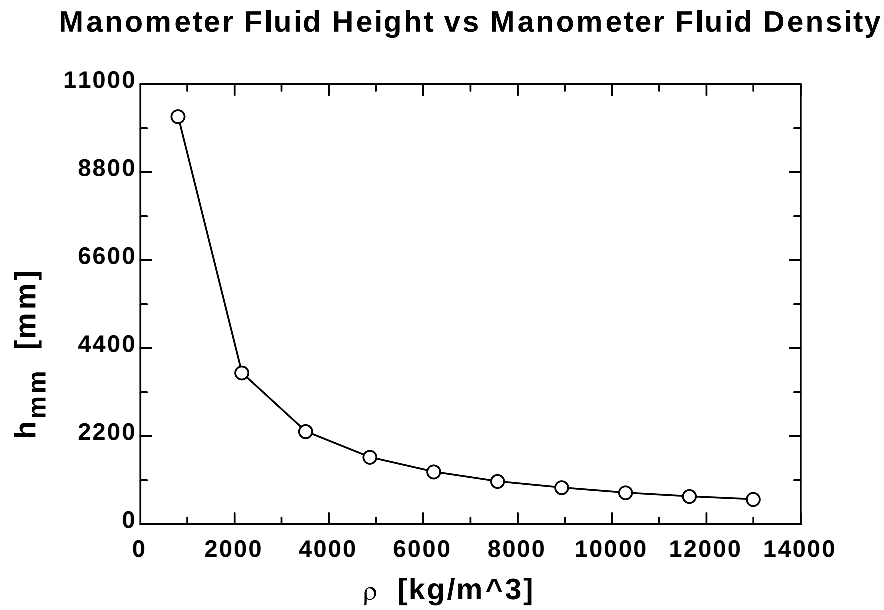
Analysis The EES *Equations* window is printed below, followed by the tabulated and plotted results.

```
Function fluid_density(Fluid$)
    If fluid$='Mercury' then  fluid_density=13600 else fluid_density=1000
end

{Input from the diagram window.  If the diagram window is hidden, then all of the input must come from the
equations window.  Also note that brackets can also denote comments - but these comments do not appear
in the formatted equations window.}

{Fluid$='Mercury'
P_atm = 101.325                    "kpa"
DELTAP=80              "kPa  Note how DELTAP is displayed on the Formatted Equations Window."}

g=9.807                                 "m/s2, local acceleration of gravity at sea level"
rho=Fluid_density(Fluid$)       "Get the fluid density, either Hg or H2O, from the function"
"To plot fluid height against density place {} around the above equation.  Then set up the parametric table
and solve."
DELTAP = RHO*g*h/1000
"Instead of dividing by 1000 Pa/kPa we could have multiplied by the EES function, CONVERT('Pa','kPa')"
h_mm=h*convert('m','mm')       "The fluid height in mm is found using the built-in CONVERT function."
P_abs= P_atm + DELTAP

"To make the graph, hide the diagram window and remove the {}brackets from Fluid$ and from P_atm.
Select New Parametric Table from the Tables menu.  Choose P_abs, DELTAP and h to be in the table.
Choose Alter Values from the Tables menu.  Set values of h to range from 0 to 1 in steps of 0.2.  Choose
Solve Table (or press F3) from the Calculate menu.  Choose New Plot Window from the Plot menu.
Choose to plot P_abs vs h and then choose Overlay Plot from the Plot menu and plot DELTAP on the same
scale."
```

Results:

h_{mm} [mm]	ρ [kg/m^3]
10197	800
3784	2156
2323	3511
1676	4867
1311	6222
1076	7578
913.1	8933
792.8	10289
700.5	11644
627.5	13000

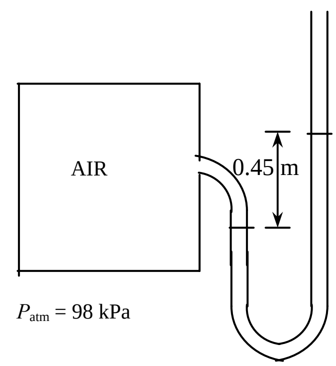




Discussion Many comments are provided in the Equation window above to help you learn some of the features of EES.

3-28

Solution The air pressure in a tank is measured by an oil manometer. For a given oil-level difference between the two columns, the absolute pressure in the tank is to be determined.



Properties The density of oil is given to be $\rho = 850\ \text{kg/m}^3$.

Analysis The absolute pressure in the tank is determined from

$$\begin{aligned} P &= P_{atm} + \rho g h \\ &= (98\ \text{kPa}) + (850\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.45\,\text{m})\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= 101.75\ \text{kPa} \cong \mathbf{102\ kPa} \end{aligned}$$

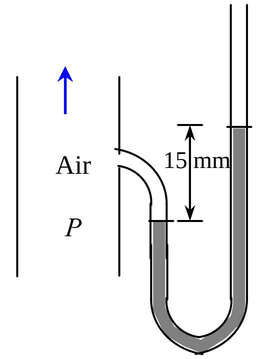
Discussion If a heavier liquid, such as water, were used for the manometer fluid, the column height would be smaller, and thus the reading would be less precise (lower resolution).

3-29

Solution The air pressure in a duct is measured by a mercury manometer. For a given mercury-level difference between the two columns, the absolute pressure in the duct is to be determined.

Properties The density of mercury is given to be ρ = 13,600 kg/m^3.

Analysis (*a*) The pressure in the duct is above atmospheric pressure since the fluid column on the duct side is at a lower level.



(*b*) The absolute pressure in the duct is determined from

$$\begin{aligned} P &= P_{atm} + \rho gh \\ &= (100\ \text{kPa}) + (13{,}600\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.015\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= 102.00\ \text{kPa} \cong \mathbf{102\ kPa} \end{aligned}$$

Discussion When measuring pressures in a fluid flow, the *difference* between two pressures is usually desired. In this case, the difference is between the measurement point and atmospheric pressure.

3-30

Solution The air pressure in a duct is measured by a mercury manometer. For a given mercury-level difference between the two columns, the absolute pressure in the duct is to be determined.

Properties The density of mercury is given to be ρ = 13,600 kg/m^3.

Analysis (*a*) The pressure in the duct is above atmospheric pressure since the fluid column on the duct side is at a lower level.

(*b*) The absolute pressure in the duct is determined from

$$\begin{aligned} P &= P_{atm} + \rho gh \\ &= (100\ \text{kPa}) + (13{,}600\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.030\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= 104.00\ \text{kPa} \cong \mathbf{104\ kPa} \end{aligned}$$

Discussion The final result is given to three significant digits.

3-31

Solution The systolic and diastolic pressures of a healthy person are given in mm of Hg. These pressures are to be expressed in kPa, psi, and meters of water column.

Assumptions Both mercury and water are incompressible substances.

Properties We take the densities of water and mercury to be 1000 kg/m^3 and 13,600 kg/m^3, respectively.

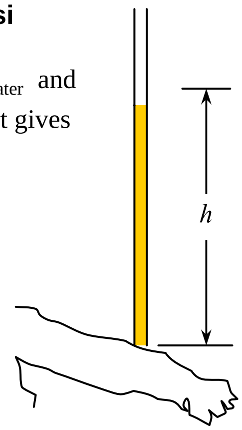
Analysis Using the relation $P = \rho gh$ for gage pressure, the high and low pressures are expressed as

$$P_{\text{high}} = \rho g h_{\text{high}} = (13{,}600\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.12\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) = \mathbf{16.0\ kPa}$$

$$P_{\text{low}} = \rho g h_{\text{low}} = (13{,}600\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.08\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) = \mathbf{10.7\ kPa}$$

Noting that 1 psi = 6.895 kPa,

$$P_{\text{high}} = (16.0\ \text{kPa})\left(\frac{1\ \text{psi}}{6.895\,\text{kPa}}\right) = \mathbf{2.32\ psi} \quad \text{and} \quad P_{\text{low}} = (10.7\ \text{kPa})\left(\frac{1\ \text{psi}}{6.895\,\text{kPa}}\right) = \mathbf{1.55\ psi}$$



For a given pressure, the relation $P = \rho gh$ is expressed for mercury and water as $P = \rho_{\text{water}} g h_{\text{water}}$ and $P = \rho_{\text{mercury}} g h_{\text{mercury}}$. Setting these two relations equal to each other and solving for water height gives

$$P = \rho_{\text{water}} g h_{\text{water}} = \rho_{\text{mercury}} g h_{\text{mercury}} \quad \rightarrow \quad h_{\text{water}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} h_{\text{mercury}}$$

Therefore,

$$h_{\text{water, high}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} h_{\text{mercury, high}} = \frac{13{,}600\ \text{kg/m}^3}{1000\ \text{kg/m}^3}(0.12\ \text{m}) = \mathbf{1.63\ m}$$

$$h_{\text{water, low}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} h_{\text{mercury, low}} = \frac{13{,}600\ \text{kg/m}^3}{1000\ \text{kg/m}^3}(0.08\ \text{m}) = \mathbf{1.09\ m}$$

Discussion Note that measuring blood pressure with a water monometer would involve water column heights higher than the person's height, and thus it is impractical. This problem shows why mercury is a suitable fluid for blood pressure measurement devices.

3-32

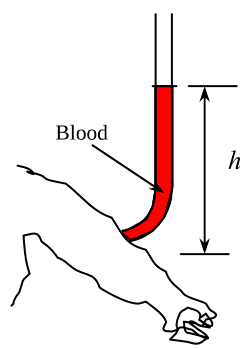
Solution A vertical tube open to the atmosphere is connected to the vein in the arm of a person. The height that the blood rises in the tube is to be determined.

Assumptions **1** The density of blood is constant. **2** The gage pressure of blood is 120 mmHg.

Properties The density of blood is given to be ρ = 1050 kg/m^3.

Analysis For a given gage pressure, the relation $P = \rho gh$ can be expressed for mercury and blood as $P = \rho_{\text{blood}} g h_{\text{blood}}$ and $P = \rho_{\text{mercury}} g h_{\text{mercury}}$. Setting these two relations equal to each other we get

$$P = \rho_{\text{blood}} g h_{\text{blood}} = \rho_{\text{mercury}} g h_{\text{mercury}}$$



Solving for blood height and substituting gives

$$h_{\text{blood}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{blood}}} h_{\text{mercury}} = \frac{13{,}600\ \text{kg/m}^3}{1050\ \text{kg/m}^3}(0.12\ \text{m}) = \mathbf{1.55\ m}$$

Discussion Note that the blood can rise about one and a half meters in a tube connected to the vein. This explains why IV tubes must be placed high to force a fluid into the vein of a patient.

3-33

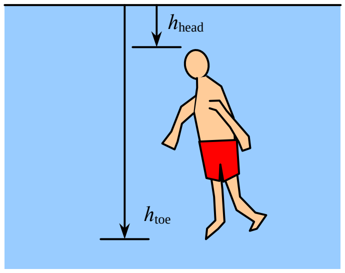
Solution A man is standing in water vertically while being completely submerged. The difference between the pressure acting on his head and the pressure acting on his toes is to be determined.

Assumptions Water is an incompressible substance, and thus the density does not change with depth.

Properties We take the density of water to be $\rho = 1000 \text{ kg/m}^3$.

Analysis The pressures at the head and toes of the person can be expressed as

$$P_{\text{head}} = P_{\text{atm}} + \rho g h_{\text{head}} \quad \text{and} \quad P_{\text{toe}} = P_{\text{atm}} + \rho g h_{\text{toe}}$$



where h is the vertical distance of the location in water from the free surface. The pressure difference between the toes and the head is determined by subtracting the first relation above from the second,

$$P_{\text{toe}} - P_{\text{head}} = \rho g h_{\text{toe}} - \rho g h_{\text{head}} = \rho g (h_{\text{toe}} - h_{\text{head}})$$

Substituting,

$$P_{\text{toe}} - P_{\text{head}} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.80 \text{ m - } 0)\left(\frac{1\text{N}}{1\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{kPa}}{1000\text{ N/m}^2}\right) = \mathbf{17.7 \text{ kPa}}$$

Discussion This problem can also be solved by noting that the atmospheric pressure (1 atm = 101.325 kPa) is equivalent to 10.3-m of water height, and finding the pressure that corresponds to a water height of 1.8 m.

3-34

Solution Water is poured into the U-tube from one arm and oil from the other arm. The water column height in one arm and the ratio of the heights of the two fluids in the other arm are given. The height of each fluid in that arm is to be determined.

Assumptions Both water and oil are incompressible substances.

Properties The density of oil is given to be $\rho_{\text{oil}} = 790 \text{ kg/m}^3$. We take the density of water to be $\rho_{\text{w}} = 1000 \text{ kg/m}^3$.

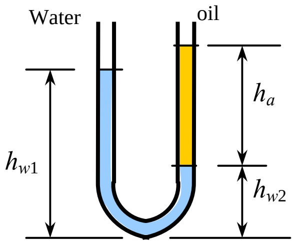
Analysis The height of water column in the left arm of the manometer is given to be $h_{\text{w1}} = 0.70$ m. We let the height of water and oil in the right arm to be h_{w2} and h_{a}, respectively. Then, $h_{\text{a}} = 6h_{\text{w2}}$. Noting that both arms are open to the atmosphere, the pressure at the bottom of the U-tube can be expressed as

$$P_{\text{bottom}} = P_{\text{atm}} + \rho_{\text{w}} g h_{\text{w1}} \quad \text{and} \quad P_{\text{bottom}} = P_{\text{atm}} + \rho_{\text{w}} g h_{\text{w2}} + \rho_{\text{a}} g h_{\text{a}}$$

Setting them equal to each other and simplifying,

$$\rho_{\text{w}} g h_{\text{w1}} = \rho_{\text{w}} g h_{\text{w2}} + \rho_{\text{a}} g h_{\text{a}} \quad \rightarrow \quad \rho_{\text{w}} h_{\text{w1}} = \rho_{\text{w}} h_{\text{w2}} + \rho_{\text{a}} h_{\text{a}} \quad \rightarrow \quad h_{\text{w1}} = h_{\text{w2}} + (\rho_{\text{a}} / \rho_w) h_{\text{a}}$$

Noting that $h_{\text{a}} = 6h_{\text{w2}}$ and we take $\rho_{\text{a}} = \rho_{\text{oil}}$, the water and oil column heights in the second arm are determined to be



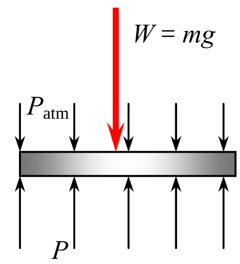
$$0.7 \text{ m} = h_{w2} + (790/1000)6h_{w2} \quad \rightarrow \quad h_{w2} = \mathbf{0.122 \text{ m}}$$

$$0.7 \text{ m} = 0.122 \text{ m} + (790/1000)h_a \quad \rightarrow \quad h_a = \mathbf{0.732 \text{ m}}$$

Discussion Note that the fluid height in the arm that contains oil is higher. This is expected since oil is lighter than water.

3-35

Solution The hydraulic lift in a car repair shop is to lift cars. The fluid gage pressure that must be maintained in the reservoir is to be determined.



Assumptions The weight of the piston of the lift is negligible.

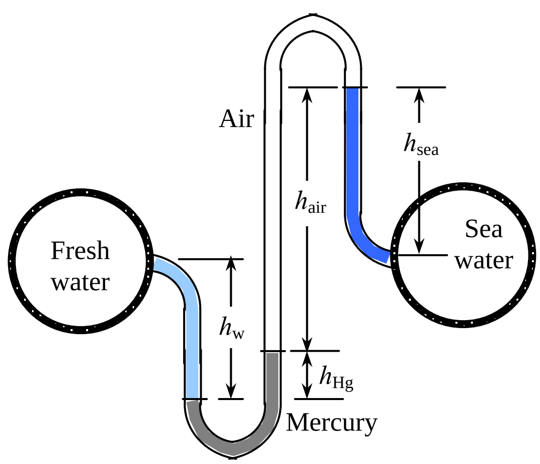
Analysis Pressure is force per unit area, and thus the gage pressure required is simply the ratio of the weight of the car to the area of the lift,

$$P_{\text{gage}} = \frac{W}{A} = \frac{mg}{\pi D^2/4} = \frac{(2000\text{ kg})(9.81\text{ m/s}^2)}{\pi(0.30\text{ m})^2/4}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 278\text{ kN/m}^2 = \mathbf{278\text{ kPa}}$$

Discussion Note that the pressure level in the reservoir can be reduced by using a piston with a larger area.

3-36

Solution Fresh and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer. The pressure difference between the two pipelines is to be determined.



Assumptions **1** All the liquids are incompressible. **2** The effect of air column on pressure is negligible.

Properties The densities of seawater and mercury are given to be ρ_{sea} = 1035 kg/m^3 and ρ_{Hg} = 13,600 kg/m^3. We take the density of water to be ρ_{w} =1000 kg/m^3.

Analysis Starting with the pressure in the fresh water pipe (point 1) and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the sea water pipe (point 2), and setting the result equal to P_2 gives

$$P_1 + \rho_{\text{w}} gh_w - \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_{\text{air}} gh_{\text{air}} + \rho_{\text{sea}} gh_{\text{sea}} = P_2$$

Rearranging and neglecting the effect of air column on pressure,

$$P_1 - P_2 = -\rho_{\text{w}} gh_w + \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_{\text{sea}} gh_{\text{sea}} = g(\rho_{\text{Hg}} h_{\text{Hg}} - \rho_{\text{w}} h_w - \rho_{\text{sea}} h_{\text{sea}})$$

Substituting,

$$\begin{aligned} P_1 - P_2 &= (9.81\text{ m/s}^2)[(13600\text{ kg/m}^3)(0.1\text{ m}) \\ &\quad -(1000\text{ kg/m}^3)(0.6\text{ m}) - (1035\text{ kg/m}^3)(0.4\text{ m})]\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) \\ &= 3.39\text{ kN/m}^2 = \mathbf{3.39\text{ kPa}} \end{aligned}$$

Therefore, the pressure in the fresh water pipe is 3.39 kPa higher than the pressure in the sea water pipe.

Discussion A 0.70-m high air column with a density of 1.2 kg/m^3 corresponds to a pressure difference of 0.008 kPa. Therefore, its effect on the pressure difference between the two pipes is negligible.

3-37

Solution Fresh and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer. The pressure difference between the two pipelines is to be determined.

Assumptions All the liquids are incompressible.

Properties The densities of seawater and mercury are given to be ρ_{sea} = 1035 kg/m^3 and ρ_{Hg} = 13,600 kg/m^3. We take the density of water to be ρ_w =1000 kg/m^3. The specific gravity of oil is given to be 0.72, and thus its density is 720 kg/m^3.

Analysis Starting with the pressure in the fresh water pipe (point 1) and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the sea water pipe (point 2), and setting the result equal to P_2 gives

$$P_1 + \rho_w g h_w - \rho_{Hg} g h_{Hg} - \rho_{oil} g h_{oil} + \rho_{sea} g h_{sea} = P_2$$

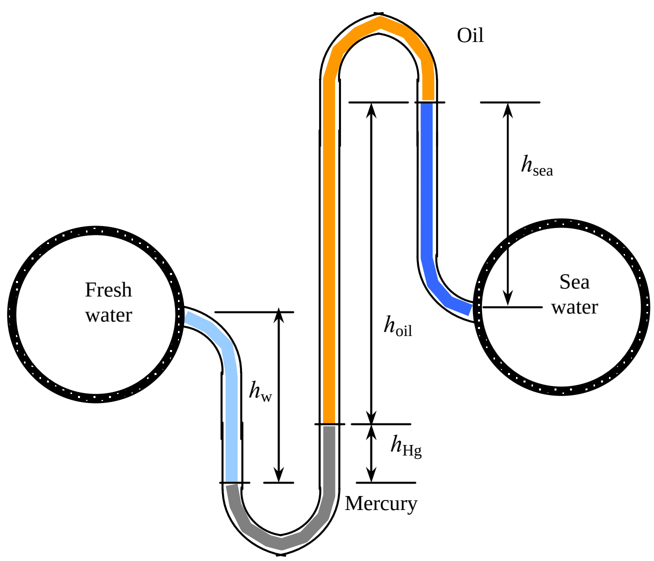
Rearranging,

$$P_1 - P_2 = -\rho_w g h_w + \rho_{Hg} g h_{Hg} + \rho_{oil} g h_{oil} - \rho_{sea} g h_{sea}$$
$$= g(\rho_{Hg} h_{Hg} + \rho_{oil} h_{oil} - \rho_w h_w - \rho_{sea} h_{sea})$$

Substituting,

$$P_1 - P_2 = (9.81\,\text{m/s}^2)[(13600\,\text{kg/m}^3)(0.1\,\text{m}) + (720\,\text{kg/m}^3)(0.7\,\text{m}) - (1000\,\text{kg/m}^3)(0.6\,\text{m})$$
$$-(1035\,\text{kg/m}^3)(0.4\,\text{m})]\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)$$
$$= 8.34\,\text{kN/m}^2 = \mathbf{8.34\,kPa}$$

Therefore, the pressure in the fresh water pipe is 8.34 kPa higher than the pressure in the sea water pipe.



Discussion The result is greater than that of the previous problem since the oil is heavier than the air.

3-38E

Solution The pressure in a natural gas pipeline is measured by a double U-tube manometer with one of the arms open to the atmosphere. The absolute pressure in the pipeline is to be determined.

Assumptions **1** All the liquids are incompressible. **2** The effect of air column on pressure is negligible. **3** The pressure throughout the natural gas (including the tube) is uniform since its density is low.

Properties We take the density of water to be ρ_w = 62.4 lbm/ft^3. The specific gravity of mercury is given to be 13.6, and thus its density is ρ_{Hg} = 13.6×62.4 = 848.6 lbm/ft^3.

Analysis Starting with the pressure at point 1 in the natural gas pipeline, and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to P_{atm} gives

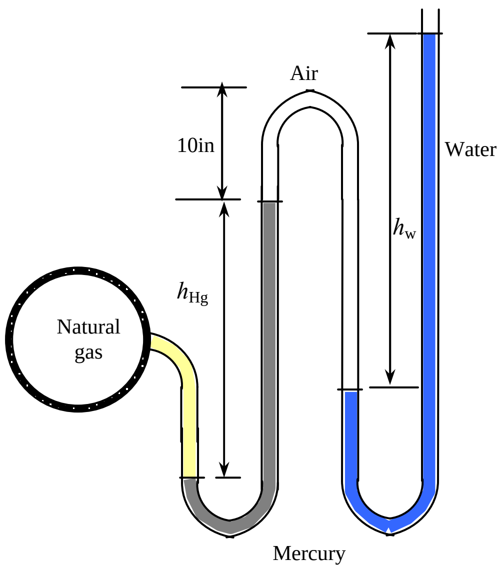
$$P_1 - \rho_{Hg} g h_{Hg} - \rho_{water} g h_{water} = P_{atm}$$

Solving for P_1,

$$P_1 = P_{atm} + \rho_{Hg} g h_{Hg} + \rho_{water} g h_1$$

Substituting,

$$P = 14.2 \text{ psia} + (32.2 \text{ ft/s}^2)[(848.6 \text{ lbm/ft}^3)(6/12 \text{ ft}) + (62.4 \text{lbm/ft}^3)(27/12 \text{ ft})]\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right)$$
$$= \mathbf{18.1 \text{ psia}}$$



Discussion Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly. Also, it can be shown that the 15-in high air column with a density of 0.075 lbm/ft^3 corresponds to a pressure difference of 0.00065 psi. Therefore, its effect on the pressure difference between the two pipes is negligible.

3-39E

Solution The pressure in a natural gas pipeline is measured by a double U-tube manometer with one of the arms open to the atmosphere. The absolute pressure in the pipeline is to be determined.

Assumptions **1** All the liquids are incompressible. **2** The pressure throughout the natural gas (including the tube) is uniform since its density is low.

Properties We take the density of water to be ρ_w = 62.4 lbm/ft^3. The specific gravity of mercury is given to be 13.6, and thus its density is ρ_{Hg} = 13.6×62.4 = 848.6 lbm/ft^3. The specific gravity of oil is given to be 0.69, and thus its density is ρ_{oil} = 0.69×62.4 = 43.1 lbm/ft^3.

Analysis Starting with the pressure at point 1 in the natural gas pipeline, and moving along the tube by adding (as we go down) or subtracting (as we go up) the $\rho g h$ terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to P_{atm} gives

$$P_1 - \rho_{Hg} g h_{Hg} + \rho_{oil} g h_{oil} - \rho_{water} g h_{water} = P_{atm}$$

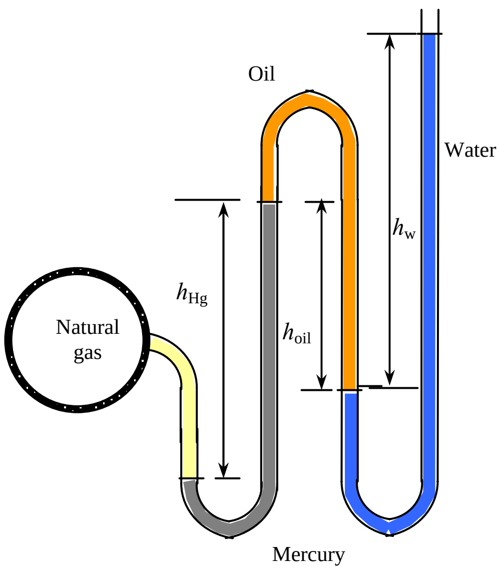
Solving for P_1,

$$P_1 = P_{atm} + \rho_{Hg} g h_{Hg} + \rho_{water} g h_1 - \rho_{oil} g h_{oil}$$

Substituting,

$$P_1 = 14.2\,\text{psia} + (32.2\,\text{ft/s}^2)[(848.6\,\text{lbm/ft}^3)(6/12\,\text{ft}) + (62.4\,\text{lbm/ft}^3)(27/12\,\text{ft}) - (43.1\,\text{lbm/ft}^3)(15/12\,\text{ft})]\left(\frac{1\,\text{lbf}}{32.2\,\text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\,\text{ft}^2}{144\,\text{in}^2}\right)$$

$$= \mathbf{17.7\,psia}$$



Discussion Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly.

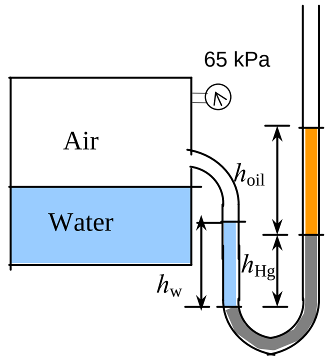
3-40

Solution The gage pressure of air in a pressurized water tank is measured simultaneously by both a pressure gage and a manometer. The differential height *h* of the mercury column is to be determined.

Assumptions The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus the pressure at the air-water interface is the same as the indicated gage pressure.

Properties We take the density of water to be ρ_w =1000 kg/m^3. The specific gravities of oil and mercury are given to be 0.72 and 13.6, respectively.

Analysis Starting with the pressure of air in the tank (point 1), and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to P_{atm} gives



$$P_1 + \rho_w g h_w - \rho_{Hg} g h_{Hg} - \rho_{oil} g h_{oil} = P_{atm}$$

Rearranging,

$$P_1 - P_{atm} = \rho_{oil} g h_{oil} + \rho_{Hg} g h_{Hg} - \rho_w g h_w$$

or,

$$\frac{P_{1,gage}}{\rho_w g} = \rho_{s,oil} h_{oil} + \rho_{s,Hg} h_{Hg} - h_w$$

Substituting,

$$\left(\frac{65\text{ kPa}}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}\right)\left(\frac{1000\ \text{ kg}\cdot\text{m/s}^2}{1\text{ kPa.}\cdot\text{m}^2}\right) = 0.72\times(0.75\text{ m}) + 13.6\times h_{Hg} - 0.3\text{ m}$$

Solving for h_{Hg} gives h_{Hg} = **0.47 m**. Therefore, the differential height of the mercury column must be 47 cm.

Discussion Double instrumentation like this allows one to verify the measurement of one of the instruments by the measurement of another instrument.

3-41

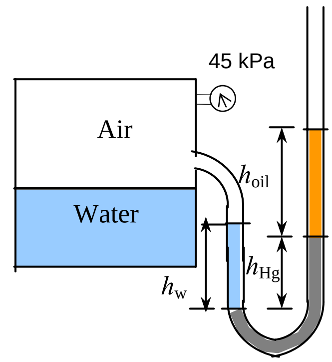
Solution The gage pressure of air in a pressurized water tank is measured simultaneously by both a pressure gage and a manometer. The differential height *h* of the mercury column is to be determined.

Assumptions The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus the pressure at the air-water interface is the same as the indicated gage pressure.

Properties We take the density of water to be ρ_w =1000 kg/m^3. The specific gravities of oil and mercury are given to be 0.72 and 13.6, respectively.

Analysis Starting with the pressure of air in the tank (point 1), and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to P_{atm} gives

$$P_1 + \rho_w g h_w - \rho_{Hg} g h_{Hg} - \rho_{oil} g h_{oil} = P_{atm}$$



Rearranging,

$$P_1 - P_{atm} = \rho_{oil} g h_{oil} + \rho_{Hg} g h_{Hg} - \rho_w g h_w$$

or,

$$\frac{P_{1,gage}}{\rho_w g} = SG_{oil} h_{oil} + SG_{Hg} h_{Hg} - h_w$$

Substituting,

$$\frac{45\text{ kPa}}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}]\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kPa.}\cdot\text{m}^2}\right) = 0.72\times(0.75\text{ m}) + 13.6\times h_{Hg} - 0.3\text{ m}$$

Solving for h_{Hg} gives h_{Hg} = **0.32 m**. Therefore, **the differential height of the mercury column must be 32 cm**.

Discussion Double instrumentation like this allows one to verify the measurement of one of the instruments by the measurement of another instrument.

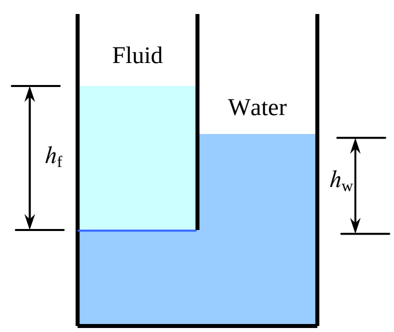
3-42

Solution The top part of a water tank is divided into two compartments, and a fluid with an unknown density is poured into one side. The levels of the water and the liquid are measured. The density of the fluid is to be determined.

Assumptions **1** Both water and the added liquid are incompressible substances. **2** The added liquid does not mix with water.

Properties We take the density of water to be ρ =1000 kg/m^3.

Analysis Both fluids are open to the atmosphere. Noting that the pressure of both water and the added fluid is the same at the contact surface, the pressure at this surface can be expressed as



$$P_{contact} = P_{atm} + \rho_f g h_f = P_{atm} + \rho_w g h_w$$

Simplifying, we have $\rho_f g h_f = \rho_w g h_w$. Solving for ρ_f gives

$$\rho_f = \frac{h_w}{h_f}\rho_w = \frac{45\text{ cm}}{80\text{ cm}}(1000\text{ kg/m}^3) = 562.5\text{ kg/m}^3 \cong \mathbf{563\ kg/m^3}$$

Discussion Note that the added fluid is lighter than water as expected (a heavier fluid would sink in water).

3-43

Solution A load on a hydraulic lift is to be raised by pouring oil from a thin tube. The height of oil in the tube required in order to raise that weight is to be determined.

Assumptions **1** The cylinders of the lift are vertical. **2** There are no leaks. **3** Atmospheric pressure act on both sides, and thus it can be disregarded.

Properties The density of oil is given to be ρ =780 kg/m^3.

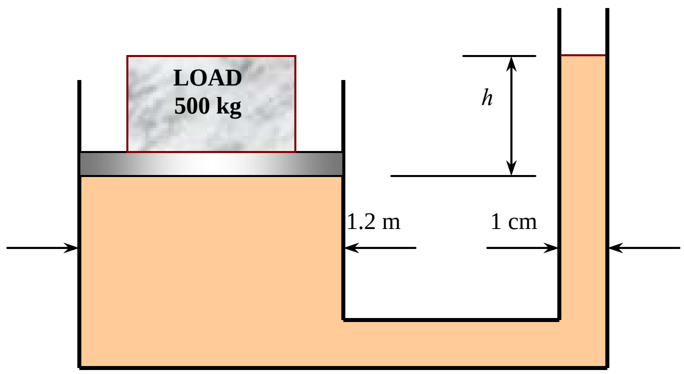
Analysis Noting that pressure is force per unit area, the gage pressure in the fluid under the load is simply the ratio of the weight to the area of the lift,

$$P_{\text{gage}} = \frac{W}{A} = \frac{mg}{\pi D^2/4} = \frac{(500\text{ kg})(9.81\text{ m/s}^2)}{\pi(1.20\text{ m})^2/4}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 4.34\text{ kN/m}^2 = 4.34\text{ kPa}$$

The required oil height that will cause 4.34 kPa of pressure rise is

$$P_{\text{gage}} = \rho g h \quad\rightarrow\quad h = \frac{P_{\text{gage}}}{\rho g} = \frac{4.34\text{ kN/m}^2}{(780\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{kN/m}^2}\right) = \mathbf{0.567\ m}$$

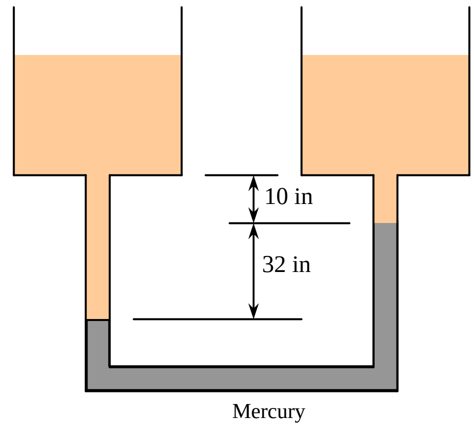
Therefore, a 500 kg load can be raised by this hydraulic lift by simply raising the oil level in the tube by 56.7 cm.



Discussion Note that large weights can be raised by little effort in hydraulic lift by making use of Pascal's principle.

3-44E

Solution Two oil tanks are connected to each other through a mercury manometer. For a given differential height, the pressure difference between the two tanks is to be determined.



Assumptions **1** Both the oil and mercury are incompressible fluids. **2** The oils in both tanks have the same density.

Properties The densities of oil and mercury are given to be ρ_{oil} = 45 lbm/ft^3 and ρ_{Hg} = 848 lbm/ft^3.

Analysis Starting with the pressure at the bottom of tank 1 (where pressure is P_1) and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the bottom of tank 2 (where pressure is P_2) gives

$$P_1 + \rho_{oil} g(h_1 + h_2) - \rho_{Hg} gh_2 - \rho_{oil} gh_1 = P_2$$

where h_1 = 10 in and h_2 = 32 in. Rearranging and simplifying,

$$P_1 - P_2 = \rho_{Hg} gh_2 - \rho_{oil} gh_2 = (\rho_{Hg} - \rho_{oil}) gh_2$$

Substituting,

$$\Delta P = P_1 - P_2 = (848 - 45 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(32/12 \text{ ft})\left(\frac{1 \text{lbf}}{32.2 \text{lbm} \cdot \text{ft/s}^2}\right)\left(\frac{1 \text{ft}^2}{144 \text{in}^2}\right) = \mathbf{14.9 \ psia}$$

Therefore, the pressure in the left oil tank is 14.9 psia higher than the pressure in the right oil tank.

Discussion Note that large pressure differences can be measured conveniently by mercury manometers. If a water manometer were used in this case, the differential height would be over 30 ft.

3-45

Solution The standard atmospheric pressure is expressed in terms of mercury, water, and glycerin columns.

Assumptions The densities of fluids are constant.

Properties The specific gravities are given to be SG = 13.6 for mercury, SG = 1.0 for water, and SG = 1.26 for glycerin. The standard density of water is 1000 kg/m^3, and the standard atmospheric pressure is 101,325 Pa.

Analysis The atmospheric pressure is expressed in terms of a fluid column height as

$$P_{atm} = \rho gh = SG\rho_w gh \quad \rightarrow \quad h = \frac{P_{atm}}{SG\rho_w g}$$

Substituting,

(*a*) Mercury: $$h = \frac{P_{atm}}{SG\rho_w g} = \frac{101{,}325 \text{ N/m}^2}{13.6(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N/m}^2}\right) = \mathbf{0.759 \ m}$$

(*b*) Water: $$h = \frac{P_{atm}}{SG\rho_w g} = \frac{101{,}325 \text{ N/m}^2}{1(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N/m}^2}\right) = \mathbf{10.3 \ m}$$

(*c*) Glycerin: $$h = \frac{P_{atm}}{SG\rho_w g} = \frac{101{,}325 \text{ N/m}^2}{1.26(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N/m}^2}\right) = \mathbf{8.20 \ m}$$

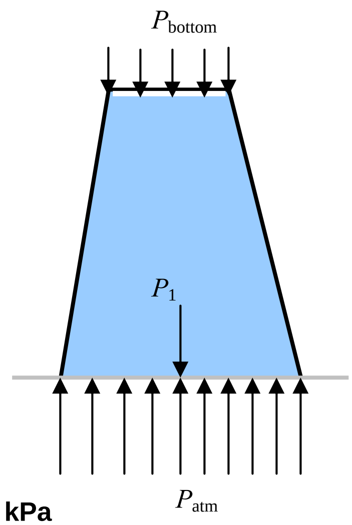
Discussion Using water or glycerin to measure atmospheric pressure requires very long vertical tubes (over 10 m for water), which is not practical. This explains why mercury is used instead of water or a light fluid.

3-46

Solution A glass filled with water and covered with a thin paper is inverted. The pressure at the bottom of the glass is to be determined.

Assumptions **1** Water is an incompressible substance. **2** The weight of the paper is negligible. **3** The atmospheric pressure is 100 kPa.

Properties We take the density of water to be ρ =1000 kg/m^3.



Analysis The paper is in equilibrium, and thus the net force acting on the paper must be zero. A vertical force balance on the paper involves the pressure forces on both sides, and yields

$$P_1 A_{glass} = P_{atm} A_{glass} \quad \rightarrow \quad P_1 = P_{atm}$$

That is, the pressures on both sides of the paper must be the same.
The pressure at the bottom of the glass is determined from the hydrostatic pressure relation to be

$$P_{atm} = P_{bottom} + \rho g h_{glass} \quad \rightarrow \quad P_{bottom} = P_{atm} - \rho g h_{glass}$$

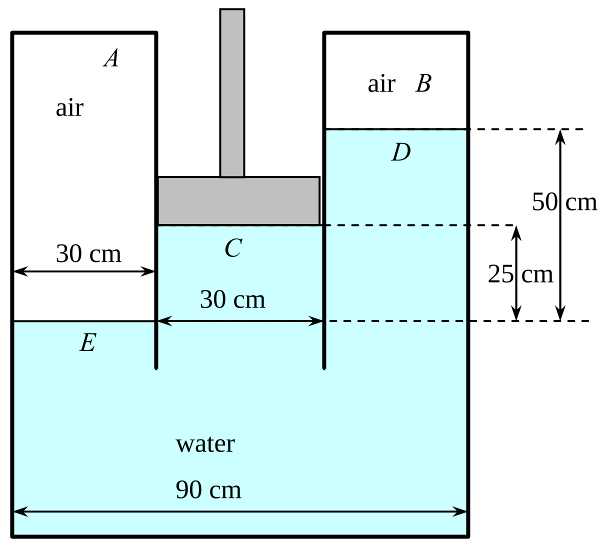
Substituting,

$$P_{bottom} = (100\text{ kPa}) - (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.1\text{ m})\left(\frac{1\text{N}}{1\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{kPa}}{1000\text{ N/m}^2}\right) = \mathbf{99.0\ kPa}$$

Discussion Note that there is a vacuum of 1 kPa at the bottom of the glass, and thus there is an upward pressure force acting on the water body, which balanced by the weight of water. As a result, the net downward force on water is zero, and thus water does not flow down.

3-47

Solution Two chambers with the same fluid at their base are separated by a piston. The gage pressure in each air chamber is to be determined.



Assumptions **1** Water is an incompressible substance. **2** The variation of pressure with elevation in each air chamber is negligible because of the low density of air.

Properties We take the density of water to be ρ =1000 kg/m^3.

Analysis The piston is in equilibrium, and thus the net force acting on the piston must be zero. A vertical force balance on the piston involves the pressure force exerted by water on the piston face, the atmospheric pressure force, and the piston weight, and yields

$$P_C A_{\text{piston}} = P_{\text{atm}} A_{\text{piston}} + W_{\text{piston}} \quad \rightarrow \quad P_C = P_{\text{atm}} + \frac{W_{\text{piston}}}{A_{\text{piston}}}$$

The pressure at the bottom of each air chamber is determined from the hydrostatic pressure relation to be

$$P_{\text{air A}} = P_E = P_C + \rho g \overline{CE} = P_{\text{atm}} + \frac{W_{\text{piston}}}{A_{\text{piston}}} + \rho g \overline{CE} \quad \rightarrow \quad P_{\text{air A, gage}} = \frac{W_{\text{piston}}}{A_{\text{piston}}} + \rho g \overline{CE}$$

$$P_{\text{air B}} = P_D = P_C - \rho g \overline{CD} = P_{\text{atm}} + \frac{W_{\text{piston}}}{A_{\text{piston}}} - \rho g \overline{CD} \quad \rightarrow \quad P_{\text{air B, gage}} = \frac{W_{\text{piston}}}{A_{\text{piston}}} - \rho g \overline{CD}$$

Substituting,

$$P_{\text{air A, gage}} = \frac{25 \text{ N}}{\pi(0.3 \text{ m})^2/4} + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.25 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right) = 2806 \text{ N/m}^2 = \mathbf{2.81 \text{ kPa}}$$

$$P_{\text{air B, gage}} = \frac{25 \text{ N}}{\pi(0.3 \text{ m})^2/4} - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.25 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right) = -2099 \text{ N/m}^2 = \mathbf{-2.10 \text{ kPa}}$$

Discussion Note that there is a vacuum of about 2 kPa in tank B which pulls the water up.

3-48

Solution A double-fluid manometer attached to an air pipe is considered. The specific gravity of one fluid is known, and the specific gravity of the other fluid is to be determined.

Assumptions **1** Densities of liquids are constant. **2** The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus the pressure at the air-water interface is the same as the indicated gage pressure.

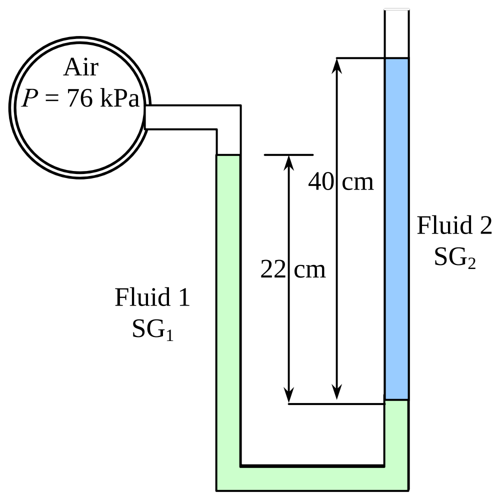
Properties The specific gravity of one fluid is given to be 13.55. We take the standard density of water to be 1000 kg/m^3.

Analysis Starting with the pressure of air in the tank, and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the free surface where the oil tube is exposed to the atmosphere, and setting the result equal to P_{atm} give

$$P_{air} + \rho_1 gh_1 - \rho_2 gh_2 = P_{atm} \quad \rightarrow \quad P_{air} - P_{atm} = SG_2 \rho_w gh_2 - SG_1 \rho_w gh_1$$

Rearranging and solving for SG_2,

$$SG_2 = SG_1 \frac{h_1}{h_2} + \frac{P_{air} - P_{atm}}{\rho_w gh_2} = 13.55 \frac{0.22 \text{ m}}{0.40 \text{ m}} + \left(\frac{(76 - 100) \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.40 \text{ m})} \right) \left(\frac{1000 \text{ kg} \cdot \text{m/s}^2}{1 \text{ kPa} \cdot \text{m}^2} \right) = \mathbf{1.34}$$



Discussion Note that the right fluid column is higher than the left, and this would imply above atmospheric pressure in the pipe for a single-fluid manometer.

3-49

Solution The pressure difference between two pipes is measured by a double-fluid manometer. For given fluid heights and specific gravities, the pressure difference between the pipes is to be calculated.

Assumptions All the liquids are incompressible.

Properties The specific gravities are given to be 13.5 for mercury, 1.26 for glycerin, and 0.88 for oil. We take the standard density of water to be ρ_w =1000 kg/m^3.

Analysis Starting with the pressure in the water pipe (point A) and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the oil pipe (point B), and setting the result equal to P_B give

$$P_A + \rho_{\text{w}} gh_w + \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_{\text{gly}} gh_{\text{gly}} + \rho_{\text{pil}} gh_{\text{oil}} = P_B$$

Rearranging and using the definition of specific gravity,

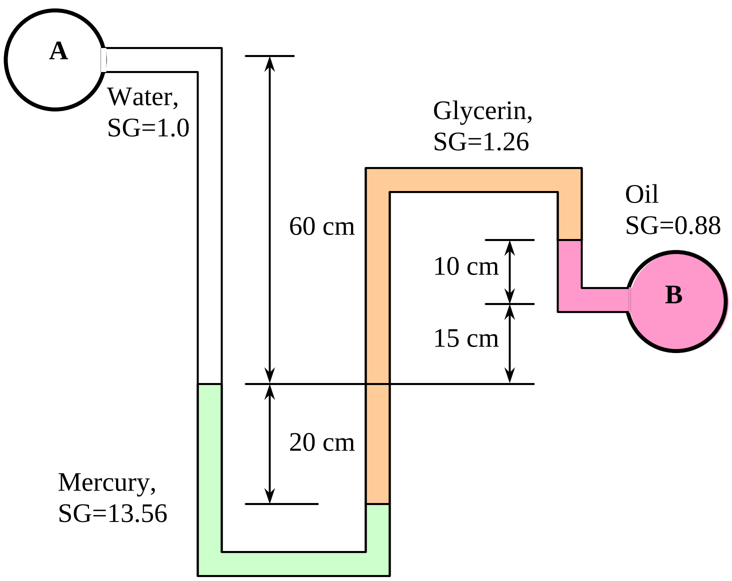
$$\begin{aligned} P_B - P_A &= \text{SG}_w \rho_w gh_w + \text{SG}_{Hg} \rho_w gh_{\text{Hg}} - \text{SG}_{gly} \rho_w gh_{\text{gly}} + \text{SG}_{oil} \rho_w gh_{\text{oil}} \\ &= g\rho_w \left(\text{SG}_w h_w + \text{SG}_{Hg} h_{\text{Hg}} - \text{SG}_{gly} h_{\text{gly}} + \text{SG}_{oil} h_{\text{oil}} \right) \end{aligned}$$

Substituting,

$$P_B - P_A = (9.81\,\text{m/s}^2)(1000\,\text{kg/m}^3)[1(0.6\,\text{m}) + 13.5(0.2\,\text{m}) - 1.26(0.45\,\text{m}) + 0.88(0.1\,\text{m})]\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 27.7\,\text{kN/m}^2 = \mathbf{27.7\,kPa}$$

Therefore, the pressure in the oil pipe is 27.7 kPa higher than the pressure in the water pipe.



Discussion Using a manometer between two pipes is not recommended unless the pressures in the two pipes are relatively constant. Otherwise, an over-rise of pressure in one pipe can push the manometer fluid into the other pipe, creating a short circuit.

3-50

Solution The fluid levels in a multi-fluid U-tube manometer change as a result of a pressure drop in the trapped air space. For a given pressure drop and brine level change, the area ratio is to be determined.

Assumptions **1** All the liquids are incompressible. **2** Pressure in the brine pipe remains constant. **3** The variation of pressure in the trapped air space is negligible.

Properties The specific gravities are given to be 13.56 for mercury and 1.1 for brine. We take the standard density of water to be ρ_w =1000 kg/m^3.

Analysis It is clear from the problem statement and the figure that the brine pressure is much higher than the air pressure, and when the air pressure drops by 0.7 kPa, the pressure difference between the brine and the air space also increases by the same amount. Starting with the air pressure (point A) and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the brine pipe (point B), and setting the result equal to P_B before and after the pressure change of air give

Before: $P_{A1} + \rho_w g h_w + \rho_{Hg} g h_{Hg,1} - \rho_{br} g h_{br,1} = P_B$

After: $P_{A2} + \rho_w g h_w + \rho_{Hg} g h_{Hg,2} - \rho_{br} g h_{br,2} = P_B$

Subtracting,

$$P_{A2} - P_{A1} + \rho_{Hg} g \Delta h_{Hg} - \rho_{br} g \Delta h_{br} = 0 \quad \rightarrow \quad \frac{P_{A1} - P_{A2}}{\rho_w g} = SG_{Hg} \Delta h_{Hg} - SG_{br} \Delta h_{br} = 0 \qquad (1)$$

where Δh_{Hg} and Δh_{br} are the changes in the differential mercury and brine column heights, respectively, due to the drop in air pressure. Both of these are positive quantities since as the mercury-brine interface drops, the differential fluid heights for both mercury and brine increase. Noting also that the volume of mercury is constant, we have $A_1 \Delta h_{Hg,left} = A_2 \Delta h_{Hg,right}$ and

$$P_{A2} - P_{A1} = -0.7 \text{ kPa} = -700 \text{ N/m}^2 = -700 \text{ kg/m} \cdot \text{s}^2$$

$$\Delta h_{br} = 0.005 \text{ m}$$

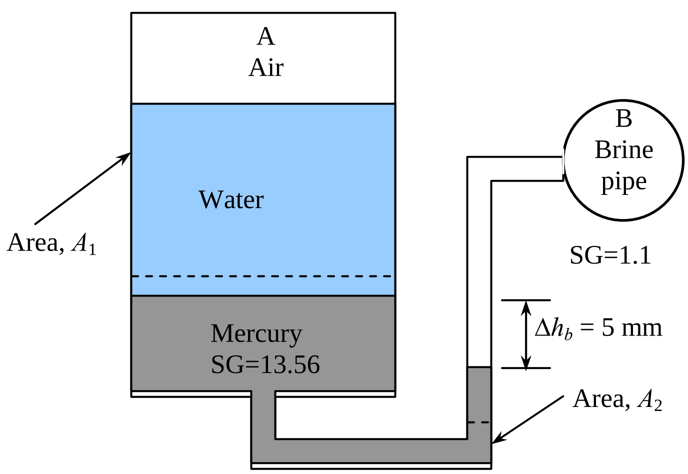
$$\Delta h_{Hg} = \Delta h_{Hg,right} + \Delta h_{Hg,left} = \Delta h_{br} + \Delta h_{br} A_2/\text{A}_1 = \Delta h_{br}(1 + A_2/\text{A}_1)$$

Substituting,

$$\frac{700 \text{ kg/m} \cdot \text{s}^2}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = [13.56 \times 0.005(1 + A_2/A_1) \text{ -.} 1.1 \times 0.005] \text{ m}$$

It gives

A_2/A_1 = **0.134**



Discussion In addition to the equations of hydrostatics, we also utilize conservation of mass in this problem.

3-51

Solution Two water tanks are connected to each other through a mercury manometer with inclined tubes. For a given pressure difference between the two tanks, the parameters a and θ are to be determined.

Assumptions Both water and mercury are incompressible liquids.

Properties The specific gravity of mercury is given to be 13.6. We take the standard density of water to be ρ_w =1000 kg/m^3.

Analysis Starting with the pressure in the tank A and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach tank B, and setting the result equal to P_B give

$$P_A + \rho_{\rm w} ga + \rho_{\rm Hg} g 2a - \rho_{\rm w} ga = P_B \quad \rightarrow \quad 2\rho_{\rm Hg} ga = P_B - P_A$$

Rearranging and substituting the known values,

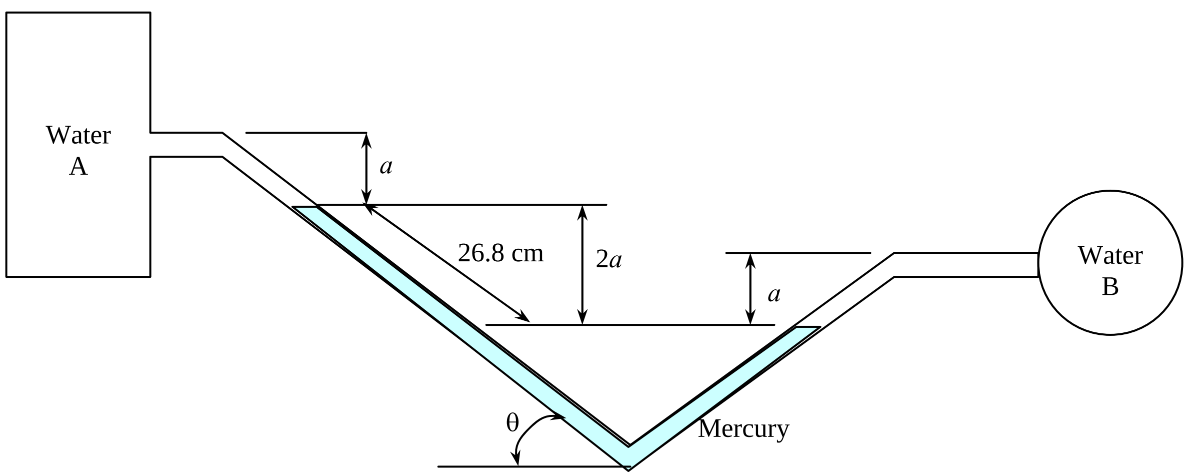
$$a = \frac{P_B - P_A}{2\rho_{Hg} g} = \frac{20 \text{ kN/m}^2}{2(13.6)(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{1000 \text{ kg}\cdot\text{m/s}^2}{1 \text{ kN}}\right) = 0.0750 \text{ m} = \mathbf{7.50 \text{ cm}}$$

From geometric considerations,

$$26.8 \sin\theta = 2a \quad \text{(cm)}$$

Therefore,

$$\sin\theta = \frac{2a}{26.8} = \frac{2\times 7.50}{26.8} = 0.560 \quad \rightarrow \quad \theta = \mathbf{34.0°}$$



Discussion Note that vertical distances are used in manometer analysis. Horizontal distances are of no consequence.

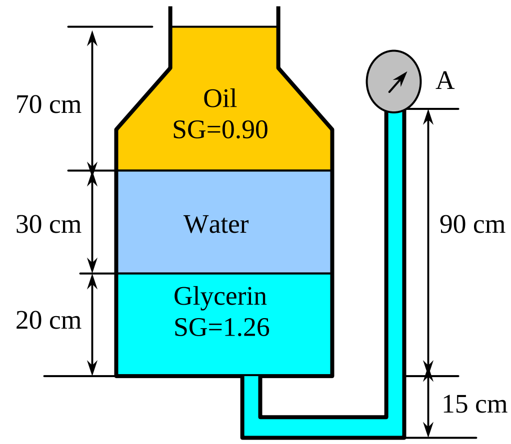
3-52

Solution A multi-fluid container is connected to a U-tube. For the given specific gravities and fluid column heights, the gage pressure at A and the height of a mercury column that would create the same pressure at A are to be determined.

Assumptions **1** All the liquids are incompressible. **2** The multi-fluid container is open to the atmosphere.

Properties The specific gravities are given to be 1.26 for glycerin and 0.90 for oil. We take the standard density of water to be ρ_w =1000 kg/m^3, and the specific gravity of mercury to be 13.6.

Analysis Starting with the atmospheric pressure on the top surface of the container and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach point A, and setting the result equal to P_A give



$$P_{atm} + \rho_{\text{oil}} g h_{\text{oil}} + \rho_{\text{w}} g h_w - \rho_{\text{gly}} g h_{\text{gly}} = P_A$$

Rearranging and using the definition of specific gravity,

$$P_A - P_{atm} = \text{SG}_{oil}\rho_w g h_{\text{oil}} + \text{SG}_w \rho_w g h_w - \text{SG}_{gly}\rho_w g h_{\text{gly}}$$

or

$$P_{A,\text{gage}} = g\rho_w\left(\text{SG}_{\text{oil}} h_{\text{oil}} + \text{SG}_w h_w - \text{SG}_{\text{gly}} h_{\text{gly}}\right)$$

Substituting,

$$P_{A,gage} = (9.81\,\text{m/s}^2)(1000\,\text{kg/m}^3)[0.90(0.70\,\text{m}) + 1(0.3\,\text{m}) - 1.26(0.70\,\text{m})]\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 0.471\,\text{kN/m}^2 = \mathbf{0.471\,kPa}$$

The equivalent mercury column height is

$$h_{Hg} = \frac{P_{A,gage}}{\rho_{Hg}\, g} = \frac{0.471\,\text{kN/m}^2}{(13.6)(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)}\left(\frac{1000\,\text{kg}\cdot\text{m/s}^2}{1\,\text{kN}}\right) = 0.00353\,\text{m} = \mathbf{0.353\,cm}$$

Discussion Note that the high density of mercury makes it a very suitable fluid for measuring high pressures in manometers.

Fluid Statics: Hydrostatic Forces on Plane and Curved Surfaces

3-53C

Solution We are to define resultant force and center of pressure.

Analysis The *resultant hydrostatic force* acting on a submerged surface is the **resultant of the pressure forces acting on the surface**. The **point of application of this resultant force** is called the *center of pressure*.

Discussion The center of pressure is generally not at the center of the body, due to hydrostatic pressure variation.

3-54C

Solution We are to examine a claim about hydrostatic force.

Analysis **Yes**, because the magnitude of the resultant force acting on a plane surface of a completely submerged body in a homogeneous fluid is equal to the product of the pressure P_C at the centroid of the surface and the area A of the surface. The pressure at the centroid of the surface is $P_C = P_0 + \rho g h_C$ where h_C is the vertical distance of the centroid from the free surface of the liquid.

Discussion We have assumed that we also know the pressure at the liquid surface.

3-55C

Solution We are to consider the effect of plate rotation on the hydrostatic force on the plate surface.

Analysis There will be **no change** on the hydrostatic force acting on the top surface of this submerged horizontal flat plate as a result of this rotation since the magnitude of the resultant force acting on a plane surface of a completely submerged body in a homogeneous fluid is equal to the product of the pressure P_C at the centroid of the surface and the area A of the surface.

Discussion If the rotation were not around the centroid, there *would* be a change in the force.

3-56C

Solution We are to explain why dams are bigger at the bottom than at the top.

Analysis Dams are built much thicker at the bottom because **the pressure force increases with depth, and the bottom part of dams are subjected to largest forces**.

Discussion Dam construction requires an enormous amount of concrete, so tapering the dam in this way saves a lot of concrete, and therefore a lot of money.

3-57C

Solution We are to explain how to determine the horizontal component of hydrostatic force on a curved surface.

Analysis The horizontal component of the hydrostatic force acting on a curved surface is equal (in both magnitude and the line of action) to **the hydrostatic force acting on the vertical projection of the curved surface**.

Discussion We could also integrate pressure along the surface, but the method discussed here is much simpler and yields the same answer.

3-58C

Solution We are to explain how to determine the vertical component of hydrostatic force on a curved surface.

Analysis The vertical component of the hydrostatic force acting on a curved surface is equal to **the hydrostatic force acting on the horizontal projection of the curved surface**, **plus** (minus, if acting in the opposite direction) **the weight of the fluid block**.

Discussion We could also integrate pressure along the surface, but the method discussed here is much simpler and yields the same answer.

3-59C

Solution We are to explain how to determine the line of action on a circular surface.

Analysis The resultant hydrostatic force acting on a circular surface always passes through **the center of the circle** since the pressure forces are normal to the surface, and all lines normal to the surface of a circle pass through the center of the circle. Thus the pressure forces form a concurrent force system at the center, which can be reduced to a single equivalent force at that point. If the magnitudes of the horizontal and vertical components of the resultant hydrostatic force are known, the tangent of the angle the resultant hydrostatic force makes with the horizontal is $\boxed{\tan\alpha = F_V / F_H}$.

Discussion This fact makes analysis of circular-shaped surfaces simple. There is no corresponding simplification for shapes other than circular, unfortunately.

3-60

Solution A car is submerged in water. The hydrostatic force on the door and its line of action are to be determined for the cases of the car containing atmospheric air and the car is filled with water.

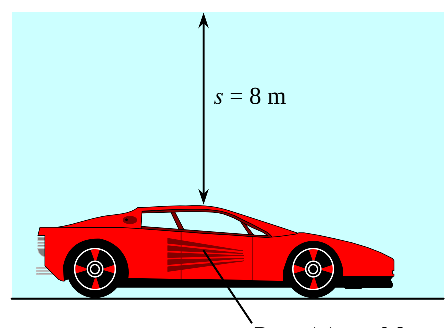
Assumptions **1** The bottom surface of the lake is horizontal. **2** The door can be approximated as a vertical rectangular plate. **3** The pressure in the car remains at atmospheric value since there is no water leaking in, and thus no compression of the air inside. Therefore, we can ignore the atmospheric pressure in calculations since it acts on both sides of the door.

Properties We take the density of lake water to be 1000 kg/m^3 throughout.

Analysis (*a*) When the car is well-sealed and thus the pressure inside the car is the atmospheric pressure, the average pressure on the outer surface of the door is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{\text{avg}} = P_C = \rho g h_C = \rho g\left(s + b/2\right)$$

$$= \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(8 + 1.1/2\ \text{m}\right)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 83.88\ \text{kN/m}^2$$



Then the resultant hydrostatic force on the door becomes

$$F_R = P_{ave} A = (83.88\ \text{kN/m}^2)(0.9\ \text{m} \times 1.1\ \text{m}) = \mathbf{83.0\ \ kN}$$

The pressure center is directly under the midpoint of the plate, and its distance from the surface of the lake is determined to be

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} = 8 + \frac{1.1}{2} + \frac{1.1^2}{12(8 + 1.1/2)} = \mathbf{8.56\ m}$$

(*b*) When the car is filled with water, the net force normal to the surface of the door is **zero** since the pressure on both sides of the door will be the same.

Discussion Note that it is impossible for a person to open the door of the car when it is filled with atmospheric air. But it takes little effort to open the door when car is filled with water, because then the pressure on each side of the door is the same.

3-61E

Solution The height of a water reservoir is controlled by a cylindrical gate hinged to the reservoir. The hydrostatic force on the cylinder and the weight of the cylinder per ft length are to be determined.

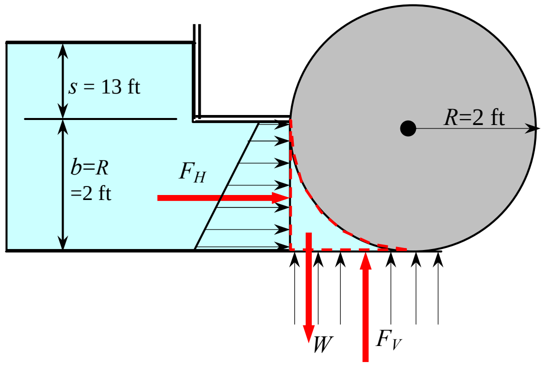
Assumptions **1** The hinge is frictionless. **2** Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 62.4 lbm/ft^3 throughout.

Analysis (*a*) We consider the free body diagram of the liquid block enclosed by the circular surface of the cylinder and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block per ft length of the cylinder are:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g(s + R/2)A$$
$$= (62.4\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)(13 + 2/2\text{ ft})(2\text{ ft} \times 1\text{ ft})\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)$$
$$= 1747\text{ lbf}$$



Vertical force on horizontal surface (*upward*):

$$F_y = P_{avg}A = \rho g h_C A = \rho g h_{bottom} A$$
$$= \left(62.4\text{ lbm/ft}^3\right)\left(32.2\text{ ft/s}^2\right)\left(15\text{ ft}\right)\left(2\text{ ft} \times 1\text{ ft}\right)\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)$$
$$= 1872\text{ lbf}$$

Weight of fluid block per ft length *(downward):*

$$W = mg = \rho g V = \rho g(R^2 - \pi R^2/4)(1\text{ ft}) = \rho g R^2(1 - \pi/4)(1\text{ ft})$$
$$= (62.4\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)(2\text{ ft})^2(1\text{-}\pi/4)(1\text{ ft})\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)$$
$$= 54\text{ lbf}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 1872 - 54 = 1818\text{ lbf}$$

Then the magnitude and direction of the hydrostatic force acting on the cylindrical surface become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{1747^2 + 1818^2} = 2521\text{ lbf} \cong \mathbf{2520\ lbf}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{1818\text{ lbf}}{1747\text{ lbf}} = 1.041 \quad \to \quad \theta = 46.1°$$

Therefore, the magnitude of the hydrostatic force acting on the cylinder is 2521 lbf per ft length of the cylinder, and its line of action passes through the center of the cylinder making an angle 46.1° upwards from the horizontal.

(*b*) When the water level is 15-ft high, the gate opens and the reaction force at the bottom of the cylinder becomes zero. Then the forces other than those at the hinge acting on the cylinder are its weight, acting through the center, and the hydrostatic force exerted by water. Taking a moment about the point *A* where the hinge is and equating it to zero gives

$$F_R R\sin\theta - W_{cyl}R = 0 \quad \to \quad W_{cyl} = F_R\sin\theta = (2521\ \text{ lbf})\sin 46.1° = 1817\text{ lbf} \cong \mathbf{1820\ lbf} \quad \text{(per ft)}$$

Discussion The weight of the cylinder per ft length is determined to be 1820 lbf, which corresponds to a mass of 1820 lbm, and to a density of 145 lbm/ft^3 for the material of the cylinder.

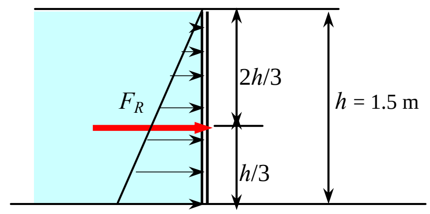
3-62

Solution An above the ground swimming pool is filled with water. The hydrostatic force on each wall and the distance of the line of action from the ground are to be determined, and the effect of doubling the wall height on the hydrostatic force is to be assessed.

Assumptions Atmospheric pressure acts on both sides of the wall of the pool, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be



$$P_{avg} = P_C = \rho g h_C = \rho g(h/2)$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.5/2 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 7357.5 \text{ N/m}^2$$

Then the resultant hydrostatic force on each wall becomes

$$F_R = P_{avg} A = (7357.5 \text{ N/m}^2)(4 \text{ m} \times 1.5 \text{ m}) = 44{,}145 \text{ N} \cong \mathbf{44.1 \text{ kN}}$$

The line of action of the force passes through the pressure center, which is $2h/3$ from the free surface and $h/3$ from the bottom of the pool. Therefore, the distance of the line of action from the ground is

$$y_P = \frac{h}{3} = \frac{1.5}{3} = \mathbf{0.50 \text{ m}} \quad \text{(from the bottom)}$$

If the height of the walls of the pool is doubled, the hydrostatic force **quadruples** since

$$F_R = \rho g h_C A = \rho g(h/2)(h \times w) = \rho g w h^2 / 2$$

and thus the hydrostatic force is proportional to the square of the wall height, h^2.

Discussion This is one reason why above-ground swimming pools are not very deep, whereas in-ground swimming pools can be quite deep.

3-63E

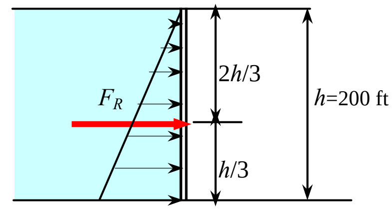
Solution A dam is filled to capacity. The total hydrostatic force on the dam, and the pressures at the top and the bottom are to be determined.

Assumptions Atmospheric pressure acts on both sides of the dam, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 62.4 lbm/ft^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{avg} = \rho g h_C = \rho g (h/2)$$
$$= (62.4\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(200/2\ \text{ft})\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right)$$
$$= 6240\ \text{lbf/ft}^2$$



Then the resultant hydrostatic force acting on the dam becomes

$$F_R = P_{ave} A = (6240\ \text{lbf/ft}^2)(200\ \text{ft} \times 1200\ \text{ft}) = \mathbf{1.50 \times 10^9\ lbf}$$

Resultant force per unit area is pressure, and its value at the top and the bottom of the dam becomes

$$P_{top} = \rho g h_{top} = \mathbf{0\ lbf/ft^2}$$

$$P_{bottom} = \rho g h_{bottom} = (62.4\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(200\ \text{ft})\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) = 12{,}480\ \text{lbf/ft}^2 \cong \mathbf{12{,}500\ lbf/ft^2}$$

Discussion The values above are gave pressures, of course. The gage pressure at the bottom of the dam is about 86.6 psig, or 101.4 psia, which is almost seven times greater than standard atmospheric pressure.

3-64

Solution A room in the lower level of a cruise ship is considered. The hydrostatic force acting on the window and the pressure center are to be determined.

Assumptions Atmospheric pressure acts on both sides of the window, and thus it can be ignored in calculations for convenience.

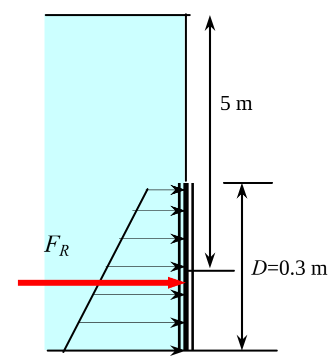
Properties The specific gravity of sea water is given to be 1.025, and thus its density is 1025 kg/m^3.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{avg} = P_C = \rho g h_C = (1025\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 50{,}276\ \text{N/m}^2$$

Then the resultant hydrostatic force on each wall becomes

$$F_R = P_{avg} A = P_{avg}[\pi D^2/4] = (50{,}276\ \text{N/m}^2)[\pi(0.3\ \text{m})^2/4] = 3554\ \text{N} \cong \mathbf{3550\ N}$$



The line of action of the force passes through the pressure center, whose vertical distance from the free surface is determined from

$$y_P = y_C + \frac{I_{xx,C}}{y_C A} = y_C + \frac{\pi R^4/4}{y_C \pi R^2} = y_C + \frac{R^2}{4 y_C} = 5 + \frac{(0.15\ \text{m})^2}{4(5\ \text{m})} = 5.0011\ \text{m} \cong \mathbf{5.00\ m}$$

Discussion For small surfaces deep in a liquid, the pressure center nearly coincides with the centroid of the surface. Here, in fact, to three significant digits in the final answer, the center of pressure and centroid are coincident.

3-65

Solution The cross-section of a dam is a quarter-circle. The hydrostatic force on the dam and its line of action are to be determined.

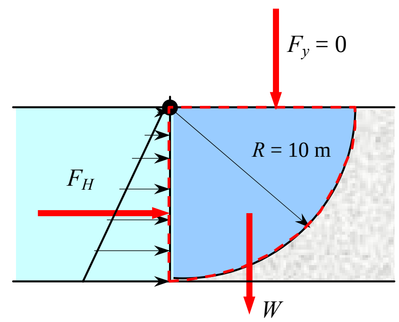
Assumptions Atmospheric pressure acts on both sides of the dam, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis We consider the free body diagram of the liquid block enclosed by the circular surface of the dam and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are:

Horizontal force on vertical surface:

$$F_H = F_x = P_{avg}A = \rho g h_C A = \rho g(R/2)A$$
$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(10/2 \text{ m})(10 \text{ m} \times 100 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)$$
$$= 4.905\times10^7 \text{ N}$$



Vertical force on horizontal surface is zero since it coincides with the free surface of water. The weight of fluid block per m length is

$$F_V = W = \rho g V = \rho g[w\times \pi R^2/4]$$
$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(100 \text{ m})\pi(10 \text{ m})^2/4]\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)$$
$$= 7.705\times10^7 \text{ N}$$

Then the magnitude and direction of the hydrostatic force acting on the surface of the dam become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(4.905\times10^7 \text{ N})^2 + (7.705\times10^7 \text{ N})^2} = 9.134\times10^7 \text{ N} \cong \mathbf{9.13\times10^7 \text{ N}}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{7.705\times10^7 \text{ N}}{4.905\times10^7 \text{ N}} = 1.571 \quad \rightarrow \quad \theta = \mathbf{57.5°}$$

Therefore, the line of action of the hydrostatic force passes through the center of the curvature of the dam, making 57.5° downwards from the horizontal.

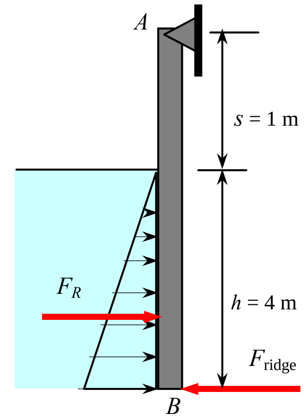
Discussion If the shape were not circular, it would be more difficult to determine the line of action.

3-66

Solution A rectangular plate hinged about a horizontal axis along its upper edge blocks a fresh water channel. The plate is restrained from opening by a fixed ridge at a point *B*. The force exerted to the plate by the ridge is to be determined.

Assumptions Atmospheric pressure acts on both sides of the plate, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 1000 kg/m^3 throughout.



Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{avg} = P_C = \rho g h_C = \rho g (h/2)$$

$$= \left(1000 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)\left(4/2 \text{ m}\right)\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) = 19.62 \text{ kN/m}^2$$

Then the resultant hydrostatic force on each wall becomes

$$F_R = P_{avg} A = \left(19.62 \text{ kN/m}^2\right)\left(4 \text{ m} \times 5 \text{ m}\right) = 392 \text{ kN}$$

The line of action of the force passes through the pressure center, which is $2h/3$ from the free surface,

$$y_P = \frac{2h}{3} = \frac{2 \times (4 \text{ m})}{3} = 2.667 \text{ m}$$

Taking the moment about point *A* and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad F_R (s + y_P) = F_{\text{ridge}} \overline{AB}$$

Solving for F_{ridge} and substituting, the reaction force is determined to be

$$F_{\text{ridge}} = \frac{s + y_P}{\overline{AB}} F_R = \frac{(1 + 2.667) \text{ m}}{5 \text{ m}} (392 \text{ kN}) = \mathbf{288 \text{ kN}}$$

Discussion The difference between F_R and F_{ridge} is the force acting on the hinge at point *A*.

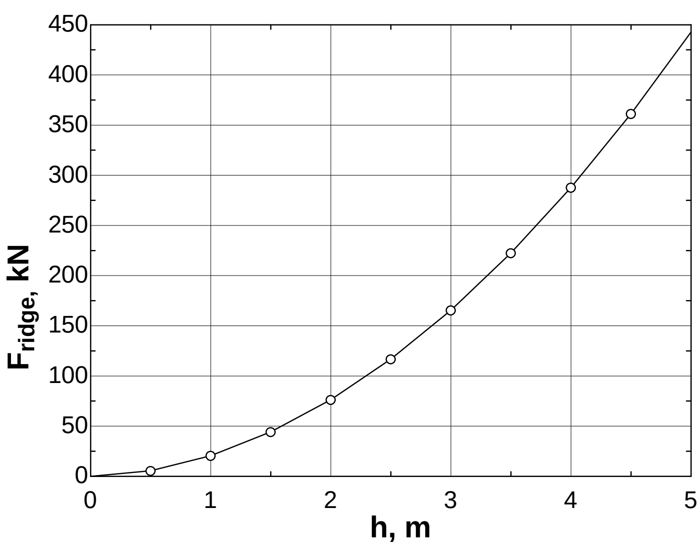
3-67

Solution The previous problem is reconsidered. The effect of water depth on the force exerted on the plate by the ridge as the water depth varies from 0 to 5 m in increments of 0.5 m is to be investigated.

Analysis The EES *Equations* window is printed below, followed by the tabulated and plotted results.

```
g=9.81 "m/s2"
rho=1000 "kg/m3"
s=1"m"

w=5 "m"
A=w*h
P_ave=rho*g*h/2000 "kPa"
F_R=P_ave*A "kN"
y_p=2*h/3
F_ridge=(s+y_p)*F_R/(s+h)
```

Dept h, m	P_{ave}, kPa	F_R kN	y_p m	F_{ridge} kN
0.0	0	0.0	0.00	0
0.5	2.453	6.1	0.33	5
1.0	4.905	24.5	0.67	20
1.5	7.358	55.2	1.00	44
2.0	9.81	98.1	1.33	76
2.5	12.26	153.3	1.67	117
3.0	14.72	220.7	2.00	166
3.5	17.17	300.4	2.33	223
4.0	19.62	392.4	2.67	288
4.5	22.07	496.6	3.00	361
5.0	24.53	613.1	3.33	443



Discussion The force on the ridge does not increase linearly, as we may have suspected.

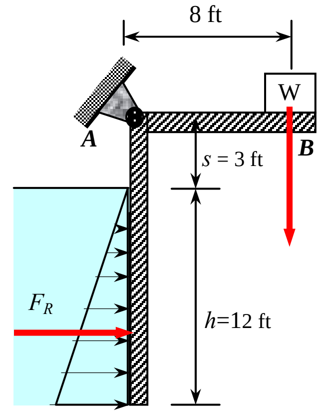
3-68E

Solution The flow of water from a reservoir is controlled by an L-shaped gate hinged at a point A. The required weight W for the gate to open at a specified water height is to be determined.

Assumptions **1** Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **2** The weight of the gate is negligible.

Properties We take the density of water to be 62.4 lbm/ft^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be



$$P_{avg} = \rho g h_C = \rho g (h/2)$$
$$= (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(12/2 \text{ ft})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right)$$
$$= 374.4 \text{ lbf/ft}^2$$

Then the resultant hydrostatic force acting on the dam becomes

$$F_R = P_{avg}A = (374.4 \text{ lbf/ft}^2)(12 \text{ ft} \times 5 \text{ ft}) = 22{,}464 \text{ lbf}$$

The line of action of the force passes through the pressure center, which is $2h/3$ from the free surface,

$$y_P = \frac{2h}{3} = \frac{2\times(12 \text{ ft})}{3} = 8 \text{ ft}$$

Taking the moment about point A and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad F_R(s + y_P) = W\overline{AB}$$

Solving for W and substituting, the required weight is determined to be

$$W = \frac{s + y_P}{\overline{AB}} F_R = \frac{(3+8) \text{ ft}}{8 \text{ ft}}(22{,}464 \text{ lbf}) = \mathbf{30{,}900 \text{ lbf}}$$

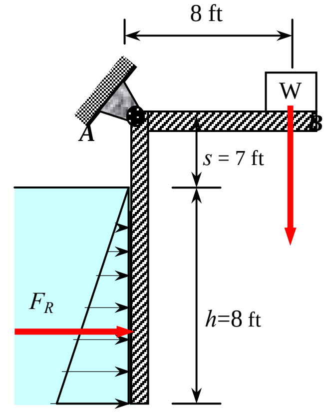
Discussion Note that the required weight is inversely proportional to the distance of the weight from the hinge.

3-69E

Solution The flow of water from a reservoir is controlled by an L-shaped gate hinged at a point *A*. The required weight *W* for the gate to open at a specified water height is to be determined.

Assumptions **1** Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **2** The weight of the gate is negligible.

Properties We take the density of water to be 62.4 lbm/ft³ throughout.



Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be

$$P_{\text{avg}} = \rho g h_C = \rho g (h/2)$$
$$= (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(8/2 \text{ ft})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right)$$
$$= 249.6 \text{ lbf/ft}^2$$

Then the resultant hydrostatic force acting on the dam becomes

$$F_R = P_{\text{avg}} A = (249.6 \text{ lbf/ft}^2)(8 \text{ ft} \times 5 \text{ ft}) = 9984 \text{ lbf}$$

The line of action of the force passes through the pressure center, which is $2h/3$ from the free surface,

$$y_P = \frac{2h}{3} = \frac{2\times(8 \text{ ft})}{3} = 5.333 \text{ ft}$$

Taking the moment about point *A* and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad F_R(s + y_P) = W\overline{AB}$$

Solving for *W* and substituting, the required weight is determined to be

$$W = \frac{s + y_P}{\overline{AB}} F_R = \frac{(7+5.333) \text{ ft}}{8 \text{ ft}}(9984 \text{ lbf}) = 15{,}390 \text{ lbf} \cong \mathbf{15{,}400 \text{ lbf}}$$

Discussion Note that the required weight is inversely proportional to the distance of the weight from the hinge.

3-70

Solution Two parts of a water trough of semi-circular cross-section are held together by cables placed along the length of the trough. The tension T in each cable when the trough is full is to be determined.

Assumptions **1** Atmospheric pressure acts on both sides of the trough wall, and thus it can be ignored in calculations for convenience. **2** The weight of the trough is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

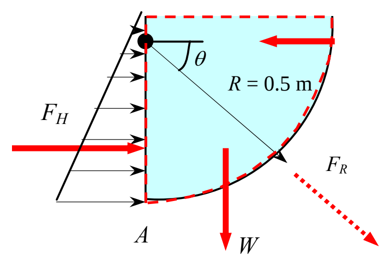
Analysis To expose the cable tension, we consider half of the trough whose cross-section is quarter-circle. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are:

Horizontal force on vertical surface:

$$F_H = F_x = P_{avg} A = \rho g h_C A = \rho g (R/2) A$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.5/2 \text{ m})(0.5 \text{ m} \times 3 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)$$

$$= 3679 \text{ N}$$



The vertical force on the horizontal surface is zero, since it coincides with the free surface of water. The weight of fluid block per 3-m length is

$$F_V = W = \rho g V = \rho g [w \times \pi R^2 / 4]$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(3 \text{ m})\pi(0.5 \text{ m})^2/4]\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)$$

$$= 5779 \text{ N}$$

Then the magnitude and direction of the hydrostatic force acting on the surface of the 3-m long section of the trough become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(3679 \text{ N})^2 + (5779 \text{ N})^2} = 6851 \text{ N}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{5779 \text{ N}}{3679 \text{ N}} = 1.571 \quad \rightarrow \quad \theta = 57.5°$$

Therefore, the line of action passes through the center of the curvature of the trough, making 57.5° downwards from the horizontal. Taking the moment about point A where the two parts are hinged and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad F_R R \sin(90 - 57.5)° = TR$$

Solving for T and substituting, the tension in the cable is determined to be

$$T = F_R \sin(90 - 57.5)° = (6851 \text{ N}) \sin(90 - 57.5)° = 3681 \text{ N} \cong \mathbf{3680 \text{ N}}$$

Discussion This problem can also be solved without finding F_R by finding the lines of action of the horizontal hydrostatic force and the weight.

3-71

Solution Two parts of a water trough of triangular cross-section are held together by cables placed along the length of the trough. The tension T in each cable when the trough is filled to the rim is to be determined.

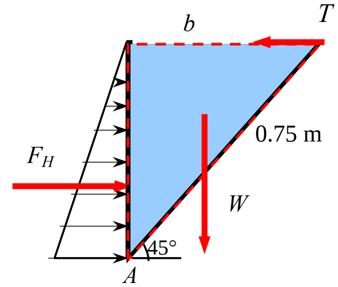
Assumptions **1** Atmospheric pressure acts on both sides of the trough wall, and thus it can be ignored in calculations for convenience. **2** The weight of the trough is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis To expose the cable tension, we consider half of the trough whose cross-section is triangular. The water height h at the midsection of the trough and width of the free surface are

$$h = L\sin\theta = (0.75\text{ m})\sin 45° = 0.530\text{ m}$$

$$b = L\cos\theta = (0.75\text{ m})\cos 45° = 0.530\text{ m}$$



The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

Horizontal force on vertical surface:

$$F_H = F_x = P_{avg}A = \rho g h_C A = \rho g (h/2) A$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.530/2\text{ m})(0.530\text{ m} \times 6\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 8267\text{ N}$$

The vertical force on the horizontal surface is zero since it coincides with the free surface of water. The weight of fluid block per 6-m length is

$$F_V = W = \rho g V = \rho g[w \times bh/2]$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)[(6\text{ m})(0.530\text{ m})(0.530\text{ m})/2]\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 8267\text{ N}$$

The distance of the centroid of a triangle from a side is 1/3 of the height of the triangle for that side. Taking the moment about point A where the two parts are hinged and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad W\frac{b}{3} + F_H\frac{h}{3} = Th$$

Solving for T and substituting, and noting that $h = b$, the tension in the cable is determined to be

$$T = \frac{F_H + W}{3} = \frac{(8267 + 8267)\text{ N}}{3} = 5511\text{ N} \cong \mathbf{5510\ N}$$

Discussion The analysis is simplified because of the symmetry of the trough.

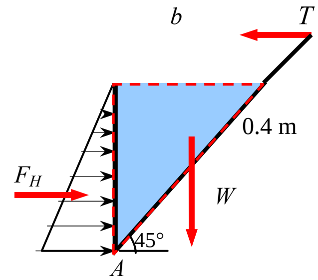
3-72

Solution Two parts of a water trough of triangular cross-section are held together by cables placed along the length of the trough. The tension T in each cable when the trough is filled to the rim is to be determined.

Assumptions **1** Atmospheric pressure acts on both sides of the trough wall, and thus it can be ignored in calculations for convenience. **2** The weight of the trough is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis To expose the cable tension, we consider half of the trough whose cross-section is triangular. The water height is given to be h = 0.4 m at the midsection of the trough, which is equivalent to the width of the free surface b since tan 45° = b/h = 1. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:



Horizontal force on vertical surface:

$$F_H = F_x = P_{avg}A = \rho g h_C A = \rho g (h/2) A$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.4/2 \text{ m})(0.4 \text{ m} \times 3 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 2354 \text{ N}$$

The vertical force on the horizontal surface is zero since it coincides with the free surface of water. The weight of fluid block per 3-m length is

$$F_V = W = \rho g V = \rho g[w \times bh/2]$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(3 \text{ m})(0.4 \text{ m})(0.4 \text{ m})/2]\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 2354 \text{ N}$$

The distance of the centroid of a triangle from a side is 1/3 of the height of the triangle for that side. Taking the moment about point A where the two parts are hinged and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad W\frac{b}{3} + F_H\frac{h}{3} = Th$$

Solving for T and substituting, and noting that $h = b$, the tension in the cable is determined to be

$$T = \frac{F_H + W}{3} = \frac{(2354 + 2354) \text{ N}}{3} = 1569 \text{ N} \cong \mathbf{1570 \text{ N}}$$

Discussion The tension force here is a factor of about 3.5 smaller than that of the previous problem, even though the trough is more than half full.

3-73

Solution A retaining wall against mud slide is to be constructed by rectangular concrete blocks. The mud height at which the blocks will start sliding, and the blocks will tip over are to be determined.

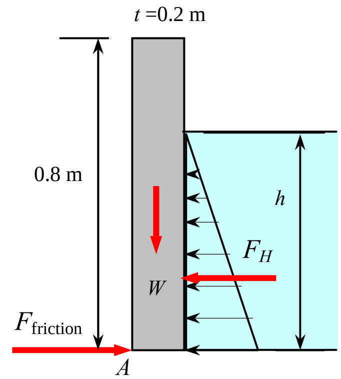
Assumptions Atmospheric pressure acts on both sides of the wall, and thus it can be ignored in calculations for convenience.

Properties The density is given to be 1800 kg/m^3 for the mud, and 2700 kg/m^3 for concrete blocks.

Analysis (*a*) The weight of the concrete wall per unit length (L = 1 m) and the friction force between the wall and the ground are

$$W_{\text{block}} = \rho g V = (2700\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[0.2\times 0.8\times 1\ \text{m}^3]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 4238\ \text{N}$$

$$F_{\text{friction}} = \mu W_{\text{block}} = 0.3(4238\ \text{N}) = 1271\ \text{N}$$



The hydrostatic force exerted by the mud to the wall is

$$F_H = F_x = P_{\text{avg}} A = \rho g h_C A = \rho g (h/2) A$$

$$= (1800\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h/2)(1\times h)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 8829h^2\ \text{N}$$

Setting the hydrostatic and friction forces equal to each other gives

$$F_H = F_{\text{friction}} \quad\rightarrow\quad 8829\,h^2 = 1271 \quad\rightarrow\quad h = \mathbf{0.38\ m}$$

(*b*) The line of action of the hydrostatic force passes through the pressure center, which is 2*h*/3 from the free surface. The line of action of the weight of the wall passes through the midplane of the wall. Taking the moment about point *A* and setting it equal to zero gives

$$\sum M_A = 0 \quad\rightarrow\quad W_{\text{block}}(t/2) = F_H(h/3) \quad\rightarrow\quad W_{\text{block}}(t/2) = 8829h^3/3$$

Solving for *h* and substituting, the mud height for tip over is determined to be

$$h = \left(\frac{3W_{\text{block}}t}{2\times 8829}\right)^{1/3} = \left(\frac{3\times 4238\times 0.2}{2\times 8829}\right)^{1/3} = \mathbf{0.52\ m}$$

Discussion The concrete wall will slide before tipping. Therefore, sliding is more critical than tipping in this case.

3-74

Solution A retaining wall against mud slide is to be constructed by rectangular concrete blocks. The mud height at which the blocks will start sliding, and the blocks will tip over are to be determined.

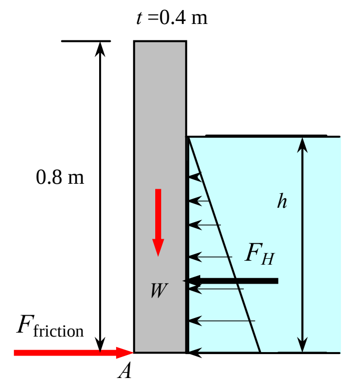
Assumptions Atmospheric pressure acts on both sides of the wall, and thus it can be ignored in calculations for convenience.

Properties The density is given to be 1800 kg/m^3 for the mud, and 2700 kg/m^3 for concrete blocks.

Analysis (*a*) The weight of the concrete wall per unit length (L = 1 m) and the friction force between the wall and the ground are

$$W_{\text{block}} = \rho g V = (2700\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[0.4\times 0.8\times 1\ \text{m}^3]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 8476\ \text{N}$$

$$F_{\text{friction}} = \mu W_{\text{block}} = 0.3(8476\ \text{N}) = 2543\ \text{N}$$



The hydrostatic force exerted by the mud to the wall is

$$F_H = F_x = P_{\text{avg}}A = \rho g h_C A = \rho g\left(h/2\right)A$$

$$= \left(1800\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(h/2\right)(1\times h)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 8829h^2\ \text{N}$$

Setting the hydrostatic and friction forces equal to each other gives

$$F_H = F_{\text{friction}} \quad\rightarrow\quad 8829\,h^2 = 2543 \quad\rightarrow\quad h = \mathbf{0.54\ m}$$

(*b*) The line of action of the hydrostatic force passes through the pressure center, which is 2*h*/3 from the free surface. The line of action of the weight of the wall passes through the midplane of the wall. Taking the moment about point *A* and setting it equal to zero gives

$$\sum M_A = 0 \quad\rightarrow\quad W_{\text{block}}(t/2) = F_H(h/3) \quad\rightarrow\quad W_{\text{block}}(t/2) = 8829h^3/3$$

Solving for *h* and substituting, the mud height for tip over is determined to be

$$h = \left(\frac{3W_{\text{block}}t}{2\times 8829}\right)^{1/3} = \left(\frac{3\times 8476\times 0.3}{2\times 8829}\right)^{1/3} = \mathbf{0.76\ m}$$

Discussion Note that the concrete wall will slide before tipping. Therefore, sliding is more critical than tipping in this case.

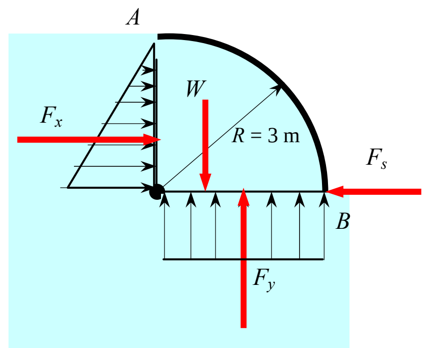
3-75 **[*Also solved using EES on enclosed DVD*]**

Solution A quarter-circular gate hinged about its upper edge controls the flow of water over the ledge at *B* where the gate is pressed by a spring. The minimum spring force required to keep the gate closed when the water level rises to *A* at the upper edge of the gate is to be determined.

Assumptions **1** The hinge is frictionless. **2** Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **3** The weight of the gate is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis We consider the free body diagram of the liquid block enclosed by the circular surface of the gate and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:



Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g(R/2)A$$
$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3/2\ \text{m})(4\ \text{m}\times 3\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$
$$= 176.6\ \text{kN}$$

Vertical force on horizontal surface (upward):

$$F_y = P_{avg}A = \rho g h_C A = \rho g h_{bottom} A$$
$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(4\ \text{m}\times 3\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 353.2\ \text{kN}$$

The weight of fluid block per 4-m length (downwards):

$$W = \rho g V = \rho g\left[w\times \pi R^2/4\right]$$
$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[(4\ \text{m})\pi(3\ \text{m})^2/4\right]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 277.4\ \text{kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 353.2 - 277.4 = 75.8\ \text{kN}$$

Then the magnitude and direction of the hydrostatic force acting on the surface of the 4-m long quarter-circular section of the gate become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(176.6\ \text{kN})^2 + (75.8\ \text{kN})^2} = 192.2\ \text{kN}$$
$$\tan\theta = \frac{F_V}{F_H} = \frac{75.8\ \text{kN}}{176.6\ \text{kN}} = 0.429 \quad\rightarrow\quad \theta = 23.2°$$

Therefore, the magnitude of the hydrostatic force acting on the gate is 192.2 kN, and its line of action passes through the center of the quarter-circular gate making an angle 23.2° upwards from the horizontal.

The minimum spring force needed is determined by taking a moment about the point *A* where the hinge is, and setting it equal to zero,

$$\sum M_A = 0 \quad\rightarrow\quad F_R R\sin(90-\theta) - F_{spring}R = 0$$

Solving for F_{spring} and substituting, the spring force is determined to be

$$F_{spring} = F_R\sin(90\text{-}\theta) = (192.2\ \text{kN})\sin(90° - 23.2°) = \mathbf{177\ kN}$$

Discussion Several variations of this design are possible. Can you think of some of them?

3-76

Solution A quarter-circular gate hinged about its upper edge controls the flow of water over the ledge at *B* where the gate is pressed by a spring. The minimum spring force required to keep the gate closed when the water level rises to *A* at the upper edge of the gate is to be determined.

Assumptions **1** The hinge is frictionless. **2** Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **3** The weight of the gate is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

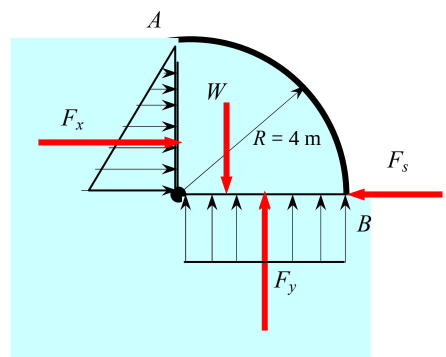
Analysis We consider the free body diagram of the liquid block enclosed by the circular surface of the gate and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g (R/2) A$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(4/2\text{ m})(4\text{ m}\times 4\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 313.9\text{ kN}$$



Vertical force on horizontal surface (upward):

$$F_y = P_{ave}A = \rho g h_C A = \rho g h_{\text{bottom}} A$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(4\text{ m})(4\text{ m}\times 4\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 627.8\text{ kN}$$

The weight of fluid block per 4-m length (downwards):

$$W = \rho g V = \rho g[w \times \pi R^2/4]$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)[(4\text{ m})\pi(4\text{ m})^2/4]\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 493.1\text{ kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 627.8 - 493.1 = 134.7\text{ kN}$$

Then the magnitude and direction of the hydrostatic force acting on the surface of the 4-m long quarter-circular section of the gate become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(313.9\text{ kN})^2 + (134.7\text{ kN})^2} = 341.6\text{ kN}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{134.7\text{kN}}{313.9\text{ kN}} = 0.429 \quad \rightarrow \quad \theta = 23.2°$$

Therefore, the magnitude of the hydrostatic force acting on the gate is 341.6 kN, and its line of action passes through the center of the quarter-circular gate making an angle 23.2° upwards from the horizontal.

The minimum spring force needed is determined by taking a moment about the point *A* where the hinge is, and setting it equal to zero,

$$\sum M_A = 0 \quad \rightarrow \quad F_R R\sin(90-\theta) - F_{\text{spring}} R = 0$$

Solving for F_{spring} and substituting, the spring force is determined to be

$$F_{\text{spring}} = F_R \sin(90 - \theta) = (341.6\text{ kN})\sin(90° - 23.2°) = \mathbf{314\ kN}$$

Discussion If the previous problem is solved using a program like EES, it is simple to repeat with different values.

Buoyancy

3-77C

Solution We are to define and discuss the buoyant force.

Analysis The **upward force a fluid exerts on an immersed body** is called the *buoyant force*. The buoyant force is **caused by the increase of pressure in a fluid with depth**. The *magnitude* of the buoyant force acting on a submerged body whose volume is V is expressed as $\boxed{F_B = \rho_f g V}$. The *direction* of the buoyant force is **upwards**, and its *line of action* **passes through the centroid of the displaced volume**.

Discussion If the buoyant force is greater than the body's weight, it floats.

3-78C

Solution We are to compare the buoyant force on two spheres.

Analysis The magnitude of the buoyant force acting on a submerged body whose volume is V is expressed as $F_B = \rho_f g V$, which is independent of depth. Therefore, **the buoyant forces acting on two identical spherical balls submerged in water at different depths is the same**.

Discussion Buoyant force depends only on the volume of the object, not its density.

3-79C

Solution We are to compare the buoyant force on two spheres.

Analysis The magnitude of the buoyant force acting on a submerged body whose volume is V is expressed as $F_B = \rho_f g V$, which is independent of the density of the body (ρ_f is the fluid density). Therefore, **the buoyant forces acting on the 5-cm diameter aluminum and iron balls submerged in water is the same**.

Discussion Buoyant force depends only on the volume of the object, not its density.

3-80C

Solution We are to compare the buoyant forces on a cube and a sphere.

Analysis The magnitude of the buoyant force acting on a submerged body whose volume is V is expressed as $F_B = \rho_f g V$, which is independent of the shape of the body. Therefore, **the buoyant forces acting on the cube and sphere made of copper submerged in water are the same since they have the same volume**.

Discussion The two objects have the same volume because they have the same mass *and* density.

3-81C

Solution We are to discuss the stability of a submerged and a floating body.

Analysis A *submerged body* whose **center of gravity *G* is above the center of buoyancy *B*, which is the centroid of the displaced volume, is *unstable***. But a *floating body* **may still be stable when *G* is above *B* since the centroid of the displaced volume shifts to the side to a point *B'* during a rotational disturbance while the center of gravity *G* of the body remains unchanged**. If the point *B'* is sufficiently far, these two forces create a restoring moment, and return the body to the original position.

Discussion Stability analysis like this is critical in the design of ship hulls, so that they are least likely to capsize.

3-82

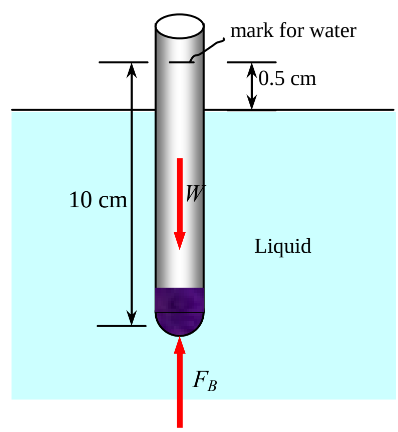
Solution The density of a liquid is to be determined by a hydrometer by establishing division marks in water and in the liquid, and measuring the distance between these marks.

Properties We take the density of pure water to be 1000 kg/m^3.

Analysis A hydrometer floating in water is in static equilibrium, and the buoyant force F_B exerted by the liquid must always be equal to the weight W of the hydrometer, $F_B = W$.

$$F_B = \rho g V_{sub} = \rho g h A_c$$

where h is the height of the submerged portion of the hydrometer and A_c is the cross-sectional area which is constant.



In pure water: $W = \rho_w g h_w A_c$

In the liquid: $W = \rho_{liquid} g h_{liquid} A_c$

Setting the relations above equal to each other (since both equal the weight of the hydrometer) gives

$$\rho_w g h_w A_c = \rho_{liquid} g h_{liquid} A_c$$

Solving for the liquid density and substituting,

$$\rho_{liquid} = \frac{h_{water}}{h_{liquid}} \rho_{water} = \frac{10 \text{ cm}}{(10-0.5) \text{ cm}} (1000 \text{ kg/m}^3) = 1053 \text{ kg/m}^3 \cong \mathbf{1050 \text{ kg/m}^3}$$

Discussion Note that for a given cylindrical hydrometer, the product of the fluid density and the height of the submerged portion of the hydrometer is constant in any fluid.

3-83E

Solution A concrete block is lowered into the sea. The tension in the rope is to be determined before and after the block is immersed in water.

Assumptions **1** The buoyancy force in air is negligible. **2** The weight of the rope is negligible.

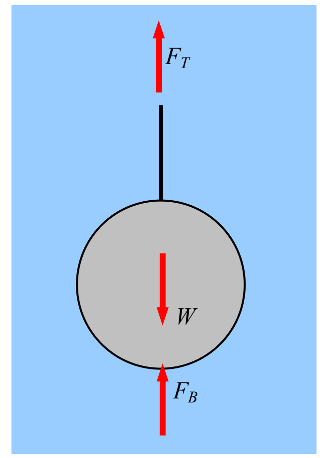
Properties The density of steel block is given to be 494 lbm/ft^3.

Analysis (*a*) The forces acting on the concrete block in air are its downward weight and the upward pull action (tension) by the rope. These two forces must balance each other, and thus the tension in the rope must be equal to the weight of the block:

$$V = 4\pi R^3/3 = 4\pi(1.5\text{ ft})^3/3 = 14.137\text{ ft}^3$$

$$F_T = W = \rho_{\text{concrete}} gV$$

$$= (494\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)(14.137\text{ ft}^3)\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 6984\text{ lbf} \cong \mathbf{6980\ lbf}$$



(*b*) When the block is immersed in water, there is the additional force of buoyancy acting upwards. The force balance in this case gives

$$F_B = \rho_f gV = (62.4\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)(14.137\text{ ft}^3)\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = 882\text{ lbf}$$

$$F_{\text{T,water}} = W - F_B = 6984 - 882 = 6102\text{ lbf} \cong \mathbf{6100\ lbf}$$

Discussion Note that the weight of the concrete block and thus the tension of the rope decreases by (6984 – 6102)/6984 = 12.6% in water.

3-84

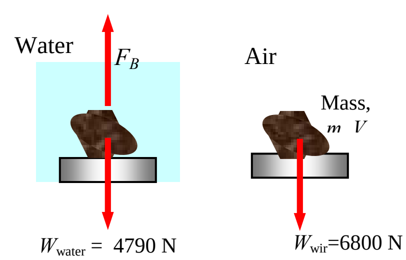
Solution An irregularly shaped body is weighed in air and then in water with a spring scale. The volume and the average density of the body are to be determined.

Properties We take the density of water to be 1000 kg/m^3.

Assumptions **1** The buoyancy force in air is negligible. **2** The body is completely submerged in water.

Analysis The mass of the body is

$$m = \frac{W_{\text{air}}}{g} = \frac{7200\text{ N}}{9.81\text{ m/s}^2}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = 733.9\text{ kg}$$



The difference between the weights in air and in water is due to the buoyancy force in water,

$$F_B = W_{\text{air}} - W_{\text{water}} = 7200 - 4790 = 2410\text{ N}$$

Noting that $F_B = \rho_{\text{water}} gV$, the volume of the body is determined to be

$$V = \frac{F_B}{\rho_{\text{water}} g} = \frac{2410\text{ N}}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)} = 0.2457\text{ m}^3 \cong \mathbf{0.246\ m^3}$$

Then the density of the body becomes

$$\rho = \frac{m}{V} = \frac{733.9\text{ kg}}{0.2457\text{ m}^3} = 2987\text{ kg/m}^3 \cong \mathbf{2990\ kg/m^3}$$

Discussion The volume of the body can also be measured by observing the change in the volume of the container when the body is dropped in it (assuming the body is not porous).

3-85

Solution The height of the portion of a cubic ice block that extends above the water surface is measured. The height of the ice block below the surface is to be determined.

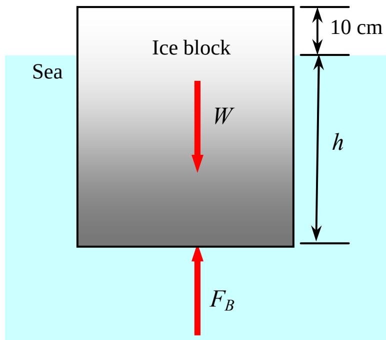
Assumptions **1** The buoyancy force in air is negligible. **2** The top surface of the ice block is parallel to the surface of the sea.

Properties The specific gravities of ice and seawater are given to be 0.92 and 1.025, respectively, and thus the corresponding densities are 920 kg/m^3 and 1025 kg/m^3.

Analysis The weight of a body floating in a fluid is equal to the buoyant force acting on it (a consequence of vertical force balance from static equilibrium). Therefore, in this case the average density of the body must be equal to the density of the fluid since

$$W = F_B \quad \rightarrow \quad \rho_{\text{body}} g V_{\text{total}} = \rho_{\text{fluid}} g V_{\text{submerged}}$$

$$\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{fluid}}}$$



The cross-sectional of a cube is constant, and thus the "volume ratio" can be replaced by "height ratio". Then,

$$\frac{h_{\text{submerged}}}{h_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{fluid}}} \quad \rightarrow \quad \frac{h}{h+0.10} = \frac{\rho_{\text{ice}}}{\rho_{\text{water}}} \quad \rightarrow \quad \frac{h}{h+0.10} = \frac{0.92}{1.025}$$

where h is the height of the ice block below the surface. Solving for h gives

$$h = 0.876 \text{ m} = \mathbf{87.6 \text{ cm}}$$

Discussion Note that the 0.92/1.025 = 90% of the volume of an ice block remains under water. For symmetrical ice blocks this also represents the fraction of height that remains under water.

3-86

Solution A man dives into a lake and tries to lift a large rock. The force that the man needs to apply to lift it from the bottom of the lake is to be determined.

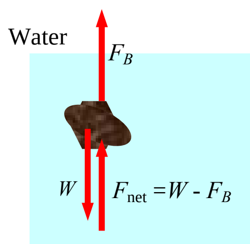
Assumptions **1** The rock is c completely submerged in water. **2** The buoyancy force in air is negligible.

Properties The density of granite rock is given to be 2700 kg/m^3. We take the density of water to be 1000 kg/m^3.

Analysis The weight and volume of the rock are

$$W = mg = (170 \text{ kg})(9.81 \text{m/s}^2)\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right) = 1668 \text{ N}$$

$$V = \frac{m}{\rho} = \frac{170 \text{ kg}}{2700 \text{ kg/m}^3} = 0.06296 \text{ m}^3$$



The buoyancy force acting on the rock is

$$F_B = \rho_{\text{water}} g V = \left(1000 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)\left(0.06296 \text{ m}^3\right)\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right) = 618 \text{ N}$$

The weight of a body submerged in water is equal to the weigh of the body in air minus the buoyancy force,

$$W_{\text{in water}} = W_{\text{in air}} - F_B = 1668 - 618 = \mathbf{1050 \text{ N}}$$

Discussion This force corresponds to a mass of $m = \dfrac{W_{\text{in water}}}{g} = \dfrac{1050 \text{ N}}{9.81 \text{ m/s}^2}\left(\dfrac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right) = 107 \text{ kg}$. Therefore, a person who can lift 107 kg on earth can lift this rock in water.

3-87

Solution An irregularly shaped crown is weighed in air and then in water with a spring scale. It is to be determined if the crown is made of pure gold.

Assumptions **1** The buoyancy force in air is negligible. **2** The crown is completely submerged in water.

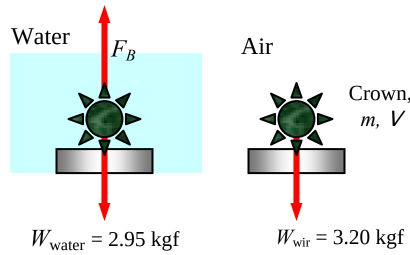
Properties We take the density of water to be 1000 kg/m^3. The density of gold is given to be 19300 kg/m^3.

Analysis The mass of the crown is

$$m = \frac{W_{\text{air}}}{g} = \frac{31.4 \text{ N}}{9.81 \text{ m/s}^2}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right) = 3.20 \text{ kg}$$

The difference between the weights in air and in water is due to the buoyancy force in water, and thus

$$F_B = W_{\text{air}} - W_{\text{water}} = 31.4 - 28.9 = 2.50 \text{ N}$$



Noting that $F_B = \rho_{\text{water}} g V$, the volume of the crown is determined to be

$$V = \frac{F_B}{\rho_{\text{water}} g} = \frac{2.50 \text{ N}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = 2.548\times10^{-4} \text{ m}^3$$

Then the density of the crown becomes

$$\rho = \frac{m}{V} = \frac{3.20 \text{ kg}}{2.548\times10^{-4} \text{ m}^3} = 12{,}560 \text{ kg/m}^3$$

which is considerably less than the density of gold. Therefore, **the crown is NOT made of pure gold**.

Discussion This problem can also be solved without doing any under-water weighing as follows: We would weigh a bucket half-filled with water, and drop the crown into it. After marking the new water level, we would take the crown out, and add water to the bucket until the water level rises to the mark. We would weigh the bucket again. Dividing the weight difference by the density of water and g will give the volume of the crown. Knowing both the weight and the volume of the crown, the density can easily be determined.

3-88

Solution The average density of a person is determined by weighing the person in air and then in water. A relation is to be obtained for the volume fraction of body fat in terms of densities.

Assumptions **1** The buoyancy force in air is negligible. **2** The body is considered to consist of fat and muscle only. **3** The body is completely submerged in water, and the air volume in the lungs is negligible.

Analysis The difference between the weights of the person in air and in water is due to the buoyancy force in water. Therefore,

$$F_B = W_{\text{air}} - W_{\text{water}} \quad \rightarrow \quad \rho_{\text{water}} g V = W_{\text{air}} - W_{\text{water}}$$

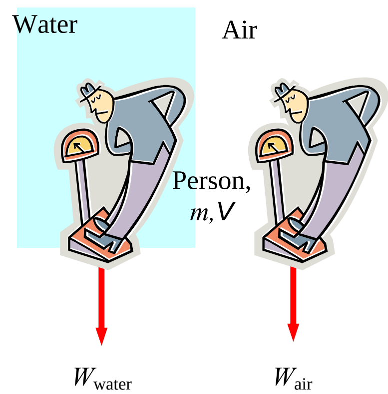
Knowing the weights and the density of water, the relation above gives the volume of the person. Then the average density of the person can be determined from

$$\rho_{\text{ave}} = \frac{m}{V} = \frac{W_{\text{air}}/g}{V}$$

Under assumption #2, the total mass of a person is equal to the sum of the masses of the fat and muscle tissues, and the total volume of a person is equal to the sum of the volumes of the fat and muscle tissues. The volume fraction of body fat is the ratio of the fat volume to the total volume of the person. Therefore,

$$V = V_{\text{fat}} + V_{\text{muscle}} \quad \text{where} \quad V_{\text{fat}} = x_{\text{fat}} V \quad \text{and} \quad V_{\text{muscle}} = x_{\text{muscle}} V = (1 - x_{\text{fat}}) V$$

$$m = m_{\text{fat}} + m_{\text{muscle}}$$



Noting that mass is density times volume, the last relation can be written as

$$\rho_{\text{ave}} V = \rho_{\text{fat}} V_{\text{fat}} + \rho_{\text{muscle}} V_{\text{muscle}}$$

$$\rho_{\text{ave}} V = \rho_{\text{fat}} x_{\text{fat}} V + \rho_{\text{muscle}} (1 - x_{\text{fat}}) V$$

Canceling the V and solving for x_{fat} gives the desired relation,

$$\boxed{x_{\text{fat}} = \frac{\rho_{\text{muscle}} - \rho_{\text{avg}}}{\rho_{\text{muscle}} - \rho_{\text{fat}}}}$$

Discussion Weighing a person in water in order to determine its volume is not practical. A more practical way is to use a large container, and measuring the change in volume when the person is completely submerged in it.

3-89

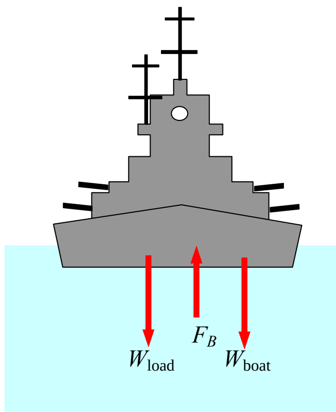
Solution The volume of the hull of a boat is given. The amounts of load the boat can carry in a lake and in the sea are to be determined.

Assumptions **1** The dynamic effects of the waves are disregarded. **2** The buoyancy force in air is negligible.

Properties The density of sea water is given to be 1.03×1000 = 1030 kg/m^3. We take the density of water to be 1000 kg/m^3.

Analysis The weight of the unloaded boat is

$$W_{\text{boat}} = mg = (8560\text{ kg})(9.81\text{ m/s}^2)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 84.0\text{ kN}$$



The buoyancy force becomes a maximum when the entire hull of the boat is submerged in water, and is determined to be

$$F_{B,\text{lake}} = \rho_{\text{lake}} gV = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(150\text{ m}^3)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 1472\text{ kN}$$

$$F_{B,\text{sea}} = \rho_{\text{sea}} gV = (1030\text{ kg/m}^3)(9.81\text{ m/s}^2)(150\text{ m}^3)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 1516\text{ kN T}$$

he total weight of a floating boat (load + boat itself) is equal to the buoyancy force. Therefore, the weight of the maximum load is

$$W_{\text{load, lake}} = F_B\text{, lake} - W_{\text{boat}} = 1472 - 84 = 1388\text{ kN}$$
$$W_{\text{load, sea}} = F_{B,\text{sea}} - W_{\text{boat}} = 1516 - 84 = 1432\text{ kN}$$

The corresponding masses of load are

$$m_{\text{load,lake}} = \frac{W_{\text{load,lake}}}{g} = \frac{1388\text{ kN}}{9.81\text{ m/s}^2}\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kN}}\right) = 141{,}500\text{ kg} \cong \mathbf{142{,}000\text{ kg}}$$

$$m_{\text{load,sea}} = \frac{W_{\text{load,lsea}}}{g} = \frac{1432\text{ kN}}{9.81\text{ m/s}^2}\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kN}}\right) = 145{,}970\text{ kg} \cong \mathbf{146{,}000\text{ kg}}$$

Discussion Note that this boat can carry nearly 4500 kg more load in the sea than it can in fresh water. Fully-loaded boats in sea water should expect to sink into water deeper when they enter fresh water, such as a river where the port may be.

Fluids in Rigid-Body Motion

3-90C

Solution We are to discuss when a fluid can be treated as a rigid body.

Analysis A moving body of fluid can be treated as a rigid body **when there are no shear stresses (i.e., no motion between fluid layers relative to each other) in the fluid body**.

Discussion When there is no relative motion between fluid particles, there are no viscous stresses, and pressure (normal stress) is the only stress.

3-91C

Solution We are to compare the pressure at the bottom of a glass of water moving at various velocities.

Analysis The water pressure at the bottom surface is **the same for all cases** since the acceleration for all four cases is zero.

Discussion When any body, fluid or solid, moves at constant velocity, there is no acceleration, regardless of the direction of the movement.

3-92C

Solution We are to compare the pressure in a glass of water for stationary and accelerating conditions.

Analysis The pressure at the bottom surface is constant when the glass is stationary. For a glass moving on a horizontal plane with constant acceleration, water will collect at the back but the water depth will remain constant at the center. Therefore, the pressure at the midpoint will be the same for both glasses. But **the bottom pressure will be low at the front relative to the stationary glass, and high at the back** (again relative to the stationary glass). Note that the pressure in all cases is the hydrostatic pressure, which is directly proportional to the fluid height.

Discussion We ignore any sloshing of the water.

3-93C

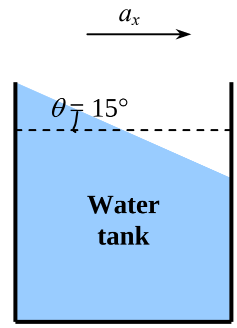
Solution We are to analyze the pressure in a glass of water that is rotating.

Analysis When a vertical cylindrical container partially filled with water is rotated about its axis and rigid body motion is established, the fluid level will drop at the center and rise towards the edges. Noting that hydrostatic pressure is proportional to fluid depth, **the pressure at the mid point will drop and the pressure at the edges of the bottom surface will rise due to the rotation**.

Discussion The highest pressure occurs at the bottom corners of the container.

3-94

Solution A water tank is being towed by a truck on a level road, and the angle the free surface makes with the horizontal is measured. The acceleration of the truck is to be determined.



Assumptions **1** The road is horizontal so that acceleration has no vertical component (a_z = 0). **2** Effects of splashing, breaking, driving over bumps, and climbing hills are assumed to be secondary, and are not considered. **3** The acceleration remains constant.

Analysis We take the x-axis to be the direction of motion, the z-axis to be the upward vertical direction. The tangent of the angle the free surface makes with the horizontal is

$$\tan\theta = \frac{a_x}{g + a_z}$$

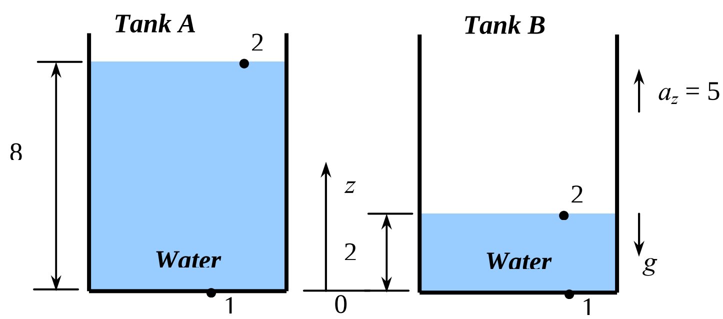
Solving for a_x and substituting,

$$a_x = (g + a_z)\tan\theta = (9.81\ \text{m/s}^2 + 0)\tan 15° = \mathbf{2.63\ m/s^2}$$

Discussion Note that the analysis is valid for any fluid with constant density since we used no information that pertains to fluid properties in the solution.

3-95

Solution Two water tanks filled with water, one stationary and the other moving upwards at constant acceleration. The tank with the higher pressure at the bottom is to be determined.



Assumptions **1** The acceleration remains constant. **2** Water is an incompressible substance.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The pressure difference between two points 1 and 2 in an incompressible fluid is given by

$$P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho(g + a_z)(z_2 - z_1) \qquad \text{or} \qquad P_1 - P_2 = \rho(g + a_z)(z_2 - z_1)$$

since a_x = 0. Taking point 2 at the free surface and point 1 at the tank bottom, we have $P_2 = P_{atm}$ and $z_2 - z_1 = h$ and thus

$$P_{1,\text{ gage}} = P_{\text{bottom}} = \rho(g + a_z)h$$

Tank A: We have a_z = 0, and thus the pressure at the bottom is

$$P_{A,\text{ bottom}} = \rho g h_A = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(8\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 78.5\ \text{kN/m}^2$$

Tank B: We have a_z = +5 m/s^2, and thus the pressure at the bottom is

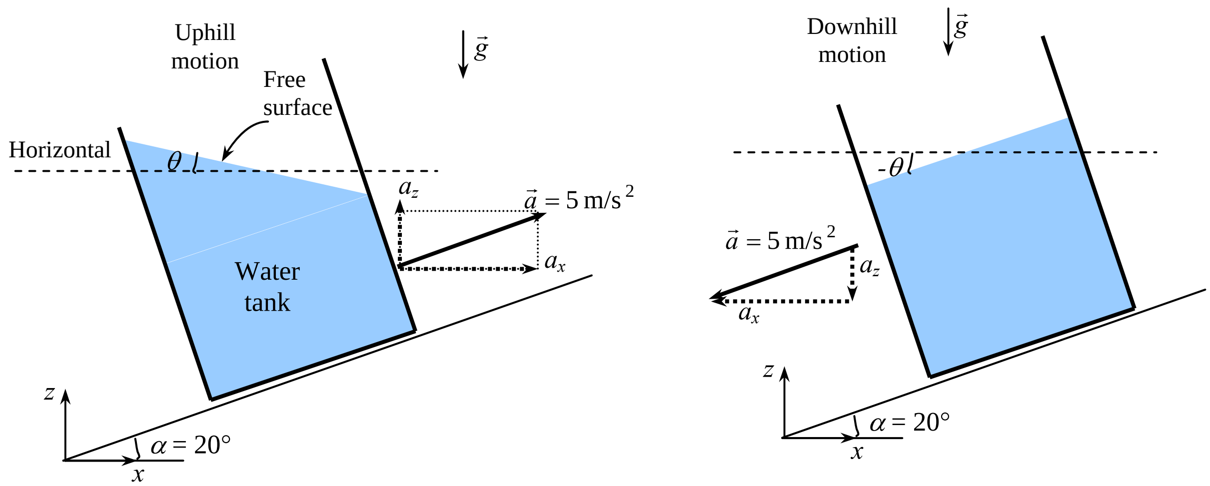
$$P_{B,\text{ bottom}} = \rho(g + a_z)h_B = (1000\ \text{kg/m}^3)(9.81 + 5\ \text{m/s}^2)(2\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 29.6\ \text{kN/m}^2$$

Therefore, **tank A has a higher pressure at the bottom**.

Discussion We can also solve this problem quickly by examining the relation $P_{\text{bottom}} = \rho(g + a_z)h$. Acceleration for tank B is about 1.5 times that of Tank A (14.81 vs 9.81 m/s^2), but the fluid depth for tank A is 4 times that of tank B (8 m vs 2 m). Therefore, the tank with the larger acceleration-fluid height product (tank A in this case) will have a higher pressure at the bottom.

3-96

Solution A water tank is being towed on an uphill road at constant acceleration. The angle the free surface of water makes with the horizontal is to be determined, and the solution is to be repeated for the downhill motion case.



Assumptions **1** Effects of splashing, breaking, driving over bumps, and climbing hills are assumed to be secondary, and are not considered. **2** The acceleration remains constant.

Analysis We take the *x*- and *z*-axes as shown in the figure. From geometrical considerations, the horizontal and vertical components of acceleration are

$$a_x = a\cos\alpha$$
$$a_z = a\sin\alpha$$

The tangent of the angle the free surface makes with the horizontal is

$$\tan\theta = \frac{a_x}{g + a_z} = \frac{a\cos\alpha}{g + a\sin\alpha} = \frac{(5\text{ m/s}^2)\cos 20°}{9.81\text{ m/s}^2 + (5\text{ m/s}^2)\sin 20°} = 0.4078 \quad \rightarrow \quad \theta = \mathbf{22.2°}$$

When the direction of motion is reversed, both a_x and a_z are in negative *x*- and *z*-direction, respectively, and thus become negative quantities,

$$a_x = -a\cos\alpha$$
$$a_z = -a\sin\alpha$$

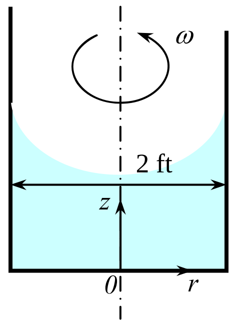
Then the tangent of the angle the free surface makes with the horizontal becomes

$$\tan\theta = \frac{a_x}{g + a_z} = \frac{a\cos\alpha}{g + a\sin\alpha} = \frac{-(5\text{ m/s}^2)\cos 20°}{9.81\text{ m/s}^2 - (5\text{ m/s}^2)\sin 20°} = -0.5801 \quad \rightarrow \quad \theta = \mathbf{-\ 30.1°}$$

Discussion Note that the analysis is valid for any fluid with constant density, not just water, since we used no information that pertains to water in the solution.

3-97E

Solution A vertical cylindrical tank open to the atmosphere is rotated about the centerline. The angular velocity at which the bottom of the tank will first be exposed, and the maximum water height at this moment are to be determined.



Assumptions **1** The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. **2** Water is an incompressible fluid.

Analysis Taking the center of the bottom surface of the rotating vertical cylinder as the origin (r = 0, z = 0), the equation for the free surface of the liquid is given as

$$z_s(r) = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

where h_0 = 1 ft is the original height of the liquid before rotation. Just before dry spot appear at the center of bottom surface, the height of the liquid at the center equals zero, and thus $z_s(0)$ = 0. Solving the equation above for ω and substituting,

$$\omega = \sqrt{\frac{4gh_0}{R^2}} = \sqrt{\frac{4\left(32.2 \text{ ft/s}^2\right)\left(1 \text{ ft}\right)}{\left(1 \text{ ft}\right)^2}} = 11.35 \text{ rad/s} \cong \mathbf{11.4 \text{ rad/s}}$$

Noting that one complete revolution corresponds to 2π radians, the rotational speed of the container can also be expressed in terms of revolutions per minute (rpm) as

$$\dot{n} = \frac{\omega}{2\pi} = \frac{11.35 \text{ rad/s}}{2\pi \text{ rad/rev}}\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \mathbf{108 \text{ rpm}}$$

Therefore, the rotational speed of this container should be limited to 108 rpm to avoid any dry spots at the bottom surface of the tank.

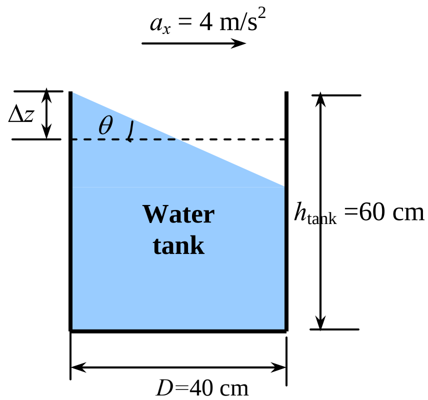
The maximum vertical height of the liquid occurs a the edges of the tank ($r = R$ = 1 ft), and it is

$$z_s(R) = h_0 + \frac{\omega^2 R^2}{4g} = (1 \text{ ft}) + \frac{(11.35 \text{ rad/s})^2 (1 \text{ ft})^2}{4(32.2 \text{ ft/s}^2)} = \mathbf{2.00 \text{ ft}}$$

Discussion Note that the analysis is valid for any liquid since the result is independent of density or any other fluid property.

3-98

Solution A cylindrical tank is being transported on a level road at constant acceleration. The allowable water height to avoid spill of water during acceleration is to be determined.



Assumptions **1** The road is horizontal during acceleration so that acceleration has no vertical component (a_z = 0). **2** Effects of splashing, breaking, driving over bumps, and climbing hills are assumed to be secondary, and are not considered. **3** The acceleration remains constant.

Analysis We take the *x*-axis to be the direction of motion, the *z*-axis to be the upward vertical direction, and the origin to be the midpoint of the tank bottom. The tangent of the angle the free surface makes with the horizontal is

$$\tan\theta = \frac{a_x}{g + a_z} = \frac{4}{9.81 + 0} = 0.4077 \quad \text{(and thus } \theta = 22.2°)$$

The maximum vertical rise of the free surface occurs at the back of the tank, and the vertical midplane experiences no rise or drop during acceleration. Then the maximum vertical rise at the back of the tank relative to the midplane is

$$\Delta z_{\text{max}} = (D/2)\tan\theta = [(0.40 \text{ m})/2] \times 0.4077 = 0.082 \text{ m} = 8.2 \text{ cm}$$

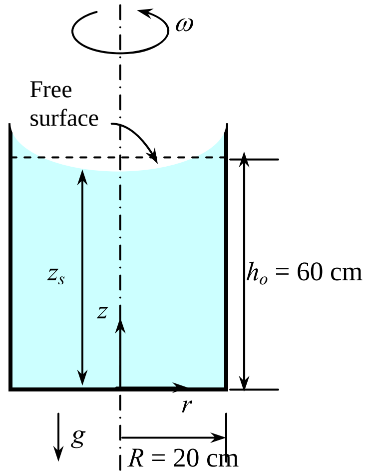
Therefore, the maximum initial water height in the tank to avoid spilling is

$$h_{\text{max}} = h_{\text{tank}} - \Delta z_{\text{max}} = 60 - 8.2 = \mathbf{51.8\ cm}$$

Discussion Note that the analysis is valid for any fluid with constant density, not just water, since we used no information that pertains to water in the solution.

3-99

Solution A vertical cylindrical container partially filled with a liquid is rotated at constant speed. The drop in the liquid level at the center of the cylinder is to be determined.



Assumptions **1** The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. **2** The bottom surface of the container remains covered with liquid during rotation (no dry spots).

Analysis Taking the center of the bottom surface of the rotating vertical cylinder as the origin (r = 0, z = 0), the equation for the free surface of the liquid is given as

$$z_s(r) = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

where h_0 = 0.6 m is the original height of the liquid before rotation, and

$$\omega = 2\pi\dot{n} = 2\pi(120 \text{ rev/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 12.57 \text{ rad/s}$$

Then the vertical height of the liquid at the center of the container where r = 0 becomes

$$z_s(0) = h_0 - \frac{\omega^2 R^2}{4g} = (0.60 \text{ m}) - \frac{(12.57 \text{ rad/s})^2 (0.20 \text{ m})^2}{4(9.81 \text{ m/s}^2)} = 0.44 \text{ m}$$

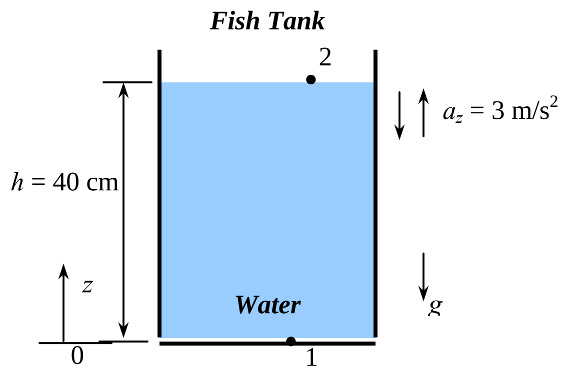
Therefore, the drop in the liquid level at the center of the cylinder is

$$\Delta h_{\text{drop, center}} = h_0 - z_s(0) = 0.60 - 0.44 = \mathbf{0.16 \text{ m}}$$

Discussion Note that the analysis is valid for any liquid since the result is independent of density or any other fluid property. Also, our assumption of no dry spots is validated since $z_0(0)$ is positive.

3-100

Solution The motion of a fish tank in the cabin of an elevator is considered. The pressure at the bottom of the tank when the elevator is stationary, moving up with a specified acceleration, and moving down with a specified acceleration is to be determined.



Assumptions **1** The acceleration remains constant. **2** Water is an incompressible substance.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The pressure difference between two points 1 and 2 in an incompressible fluid is given by

$$P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho(g + a_z)(z_2 - z_1) \quad \text{or} \quad P_1 - P_2 = \rho(g + a_z)(z_2 - z_1)$$

since $a_x = 0$. Taking point 2 at the free surface and point 1 at the tank bottom, we have $P_2 = P_{atm}$ and $z_2 - z_1 = h$ and thus

$$P_{1,\text{ gage}} = P_{\text{bottom}} = \rho(g + a_z)h$$

(*a*) Tank stationary: We have $a_z = 0$, and thus the gage pressure at the tank bottom is

$$P_{\text{bottom}} = \rho g h = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.4\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 3.92\text{ kN/m}^2 = \mathbf{3.92\ kPa}$$

(*b*) Tank moving up: We have $a_z = +3$ m/s^2, and thus the gage pressure at the tank bottom is

$$P_{\text{bottom}} = \rho(g + a_z)h_B = (1000\text{ kg/m}^3)(9.81 + 3\text{ m/s}^2)(0.4\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 5.12\text{ kN/m}^2 = \mathbf{5.12\ kPa}$$

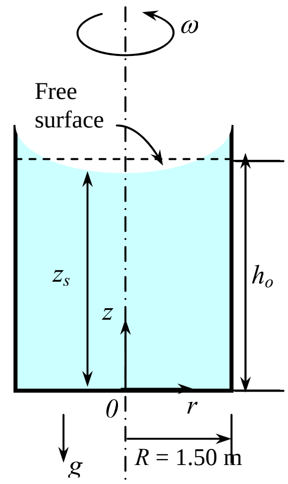
(*c*) Tank moving down: We have $a_z = -3$ m/s^2, and thus the gage pressure at the tank bottom is

$$P_{\text{bottom}} = \rho(g + a_z)h_B = (1000\text{ kg/m}^3)(9.81 - 3\text{ m/s}^2)(0.4\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 2.72\text{ kN/m}^2 = \mathbf{2.72\ kPa}$$

Discussion Note that the pressure at the tank bottom while moving up in an elevator is almost twice that while moving down, and thus the tank is under much greater stress during upward acceleration.

3-101

Solution A vertical cylindrical milk tank is rotated at constant speed, and the pressure at the center of the bottom surface is measured. The pressure at the edge of the bottom surface is to be determined.



Assumptions **1** The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. **2** Milk is an incompressible substance.

Properties The density of the milk is given to be 1030 kg/m^3.

Analysis Taking the center of the bottom surface of the rotating vertical cylinder as the origin (r = 0, z = 0), the equation for the free surface of the liquid is given as

$$z_s(r) = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

where R = 1.5 m is the radius, and

$$\omega = 2\pi\dot{n} = 2\pi(12\text{ rev/min})\left(\frac{1\text{ min}}{60\text{ s}}\right) = 1.2566\text{ rad/s}$$

The fluid rise at the edge relative to the center of the tank is

$$\Delta h = z_s(R) - z_s(0) = \left(h_0 + \frac{\omega^2 R^2}{4g}\right) - \left(h_0 - \frac{\omega^2 R^2}{4g}\right) = \frac{\omega^2 R^2}{2g} = \frac{(1.2566\text{ rad/s})^2(1.50\text{ m})^2}{2(9.81\text{ m/s}^2)} = 1.1811\text{ m}$$

The pressure difference corresponding to this fluid height difference is

$$\Delta P_{\text{bottom}} = \rho g \Delta h = (1030\text{ kg/m}^3)(9.81\text{ m/s}^2)(1.1811\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 1.83\text{ kN/m}^2 = 1.83\text{ kPa}$$

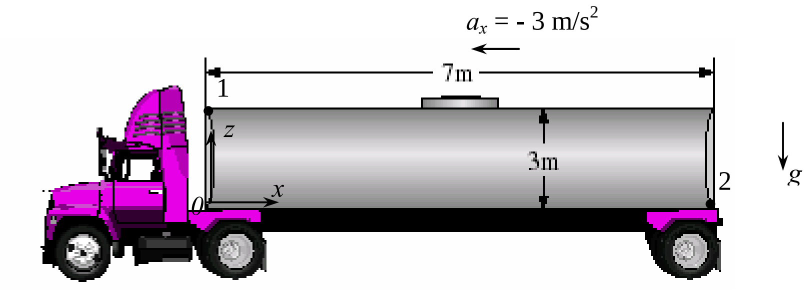
Then the pressure at the edge of the bottom surface becomes

$$P_{\text{bottom, edge}} = P_{\text{bottom, center}} + \Delta P_{\text{bottom}} = 130 + 1.83 = 131.83\text{ kPa} \cong \mathbf{132\text{ kPa}}$$

Discussion Note that the pressure is 1.4% higher at the edge relative to the center of the tank, and there is a fluid level difference of 1.18 m between the edge and center of the tank, and these differences should be considered when designing rotating fluid tanks.

3-102

Solution Milk is transported in a completely filled horizontal cylindrical tank accelerating at a specified rate. The maximum pressure difference in the tanker is to be determined.



Assumptions **1** The acceleration remains constant. **2** Milk is an incompressible substance.

Properties The density of the milk is given to be 1020 kg/m^3.

Analysis We take the *x*- and *z*- axes as shown. The horizontal acceleration is in the negative *x* direction, and thus a_x is negative. Also, there is no acceleration in the vertical direction, and thus a_z = 0. The pressure difference between two points 1 and 2 in an incompressible fluid in linear rigid body motion is given by

$$P_2 - P_1 = -\rho a_x(x_2 - x_1) - \rho(g + a_z)(z_2 - z_1) \quad \rightarrow \quad P_2 - P_1 = -\rho a_x(x_2 - x_1) - \rho g(z_2 - z_1)$$

The first term is due to acceleration in the horizontal direction and the resulting compression effect towards the back of the tanker, while the second term is simply the hydrostatic pressure that increases with depth. Therefore, we reason that the lowest pressure in the tank will occur at point 1 (upper front corner), and the higher pressure at point 2 (the lower rear corner). Therefore, the maximum pressure difference in the tank is

$$\Delta P_{\max} = P_2 - P_1 = -\rho a_x(x_2 - x_1) - \rho g(z_2 - z_1) = -[a_x(x_2 - x_1) + g(z_2 - z_1)]$$

$$= -(1020 \text{ kg/m}^3)\left[(-2.5 \text{ m/s}^2)(7 \text{ m}) + (9.81 \text{ m/s}^2)(-3 \text{ m})\right]\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right)$$

$$= (17.9 + 30.0) \text{ kN/m}^2 = \mathbf{47.9\ kPa}$$

since x_1 = 0, x_2 = 7 m, z_1 = 3 m, and z_2 = 0.

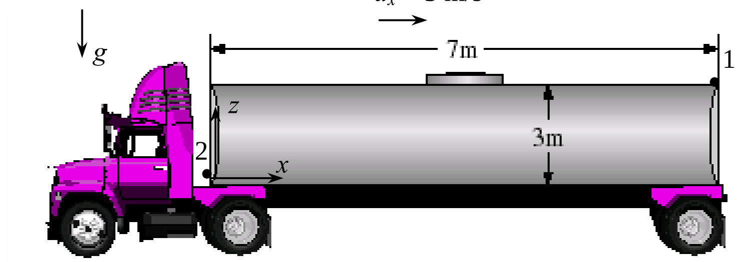
Discussion Note that the variation of pressure along a horizontal line is due to acceleration in the horizontal direction while the variation of pressure in the vertical direction is due to the effects of gravity and acceleration in the vertical direction (which is zero in this case).

3-103

Solution Milk is transported in a completely filled horizontal cylindrical tank decelerating at a specified rate. The maximum pressure difference in the tanker is to be determined.

Assumptions **1** The acceleration remains constant. **2** Milk is an incompressible substance.

Properties The density of the milk is given to be 1020 kg/m^3.



Analysis We take the x- and z- axes as shown. The horizontal deceleration is in the x direction, and thus a_x is positive. Also, there is no acceleration in the vertical direction, and thus a_z = 0. The pressure difference between two points 1 and 2 in an incompressible fluid in linear rigid body motion is given by

$$P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho(g + a_z)(z_2 - z_1) \quad \rightarrow \quad P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho g(z_2 - z_1)$$

The first term is due to deceleration in the horizontal direction and the resulting compression effect towards the front of the tanker, while the second term is simply the hydrostatic pressure that increases with depth. Therefore, we reason that the lowest pressure in the tank will occur at point 1 (upper front corner), and the higher pressure at point 2 (the lower rear corner). Therefore, the maximum pressure difference in the tank is

$$\Delta P_{\max} = P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho g(z_2 - z_1) = -[a_x (x_2 - x_1) + g(z_2 - z_1)]$$

$$= -(1020\ \text{kg/m}^3)\left[(2.5\ \text{m/s}^2)(-7\ \text{m}) + (9.81\ \text{m/s}^2)(-3\ \text{m})\right]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

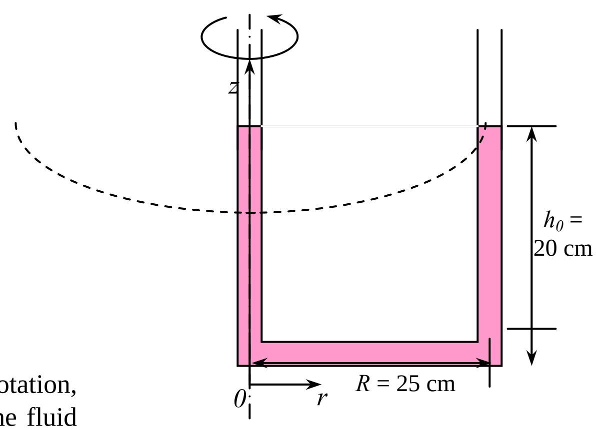
$$= (17.9 + 30.0)\ \text{kN/m}^2 = \mathbf{47.9\ \ kPa}$$

since x_1 = 7 m, x_2 = 0, z_1 = 3 m, and z_2 = 0.

Discussion Note that the variation of pressure along a horizontal line is due to acceleration in the horizontal direction while the variation of pressure in the vertical direction is due to the effects of gravity and acceleration in the vertical direction (which is zero in this case).

3-104

Solution A vertical U-tube partially filled with alcohol is rotated at a specified rate about one of its arms. The elevation difference between the fluid levels in the two arms is to be determined.



Assumptions **1** Alcohol is an incompressible fluid.

Analysis Taking the base of the left arm of the U-tube as the origin (r = 0, z = 0), the equation for the free surface of the liquid is given as

$$z_s(r) = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

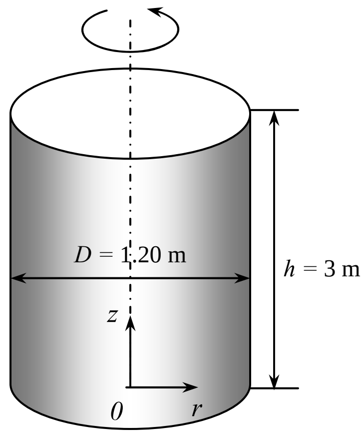
where h_0 = 0.20 m is the original height of the liquid before rotation, and ω = 4.2 rad/s. The fluid rise at the right arm relative to the fluid level in the left arm (the center of rotation) is

$$\Delta h = z_s(R) - z_s(0) = \left(h_0 + \frac{\omega^2 R^2}{4g}\right) - \left(h_0 - \frac{\omega^2 R^2}{4g}\right) = \frac{\omega^2 R^2}{2g} = \frac{(4.2\ \text{rad/s})^2 (0.25\ \text{m})^2}{2(9.81\ \text{m/s}^2)} = \mathbf{0.056\ m}$$

Discussion The analysis is valid for any liquid since the result is independent of density or any other fluid property.

3-105

Solution A vertical cylindrical tank is completely filled with gasoline, and the tank is rotated about its vertical axis at a specified rate. The pressures difference between the centers of the bottom and top surfaces, and the pressures difference between the center and the edge of the bottom surface are to be determined.



Assumptions **1** The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. **2** Gasoline is an incompressible substance.

Properties The density of the gasoline is given to be 740 kg/m^3.

Analysis The pressure difference between two points 1 and 2 in an incompressible fluid rotating in rigid body motion is given by

$$P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1)$$

where R = 0.60 m is the radius, and

$$\omega = 2\pi\dot{n} = 2\pi(70\ \text{rev/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 7.330\ \text{rad/s}$$

(*a*) Taking points 1 and 2 to be the centers of the bottom and top surfaces, respectively, we have $r_1 = r_2 = 0$ and $z_2 - z_1 = h = 3\ \text{m}$. Then,

$$P_{\text{center, top}} - P_{\text{center, bottom}} = 0 - \rho g(z_2 - z_1) = -\rho g h$$

$$= -(740\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 21.8\ \text{kN/m}^2 = \mathbf{21.8\ kPa}$$

(*b*) Taking points 1 and 2 to be the center and edge of the bottom surface, respectively, we have $r_1 = 0$, $r_2 = R$, and $z_2 = z_1 = 0$. Then,

$$P_{\text{edge, bottom}} - P_{\text{center, bottom}} = \frac{\rho\omega^2}{2}(R_2^2 - 0) - 0 = \frac{\rho\omega^2 R^2}{2}$$

$$= \frac{(740\ \text{kg/m}^3)(7.33\ \text{rad/s})^2(0.60\ \text{m})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 7.16\ \text{kN/m}^2 = \mathbf{7.16\ kPa}$$

Discussion Note that the rotation of the tank does not affect the pressure difference along the axis of the tank. But the pressure difference between the edge and the center of the bottom surface (or any other horizontal plane) is due entirely to the rotation of the tank.

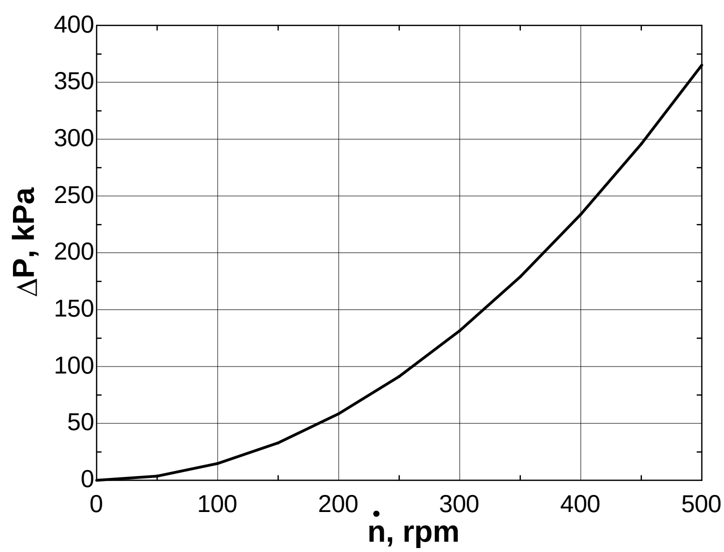
3-106

Solution The previous problem is reconsidered. The effect of rotational speed on the pressure difference between the center and the edge of the bottom surface of the cylinder as the rotational speed varies from 0 to 500 rpm in increments of 50 rpm is to be investigated.

Analysis The EES *Equations* window is printed below, followed by the tabulated and plotted results.

```
g=9.81 "m/s2"
rho=740 "kg/m3"
R=0.6 "m"
h=3 "m"

omega=2*pi*n_dot/60 "rad/s"
DeltaP_axis=rho*g*h/1000 "kPa"
DeltaP_bottom=rho*omega^2*R^2/2000 "kPa"
```

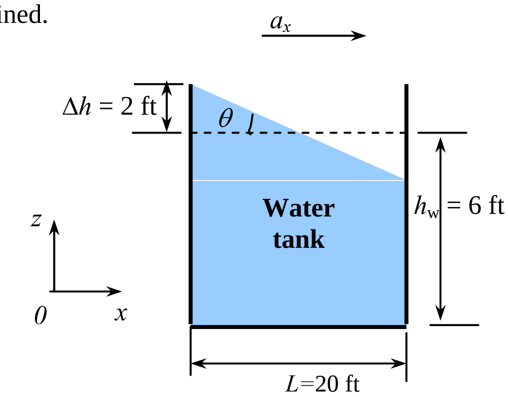
Rotation rate $\dot{n}$, rpm	Angular speed ω, rad/s	$\Delta P_{\text{center-edge}}$ kPa
0	0.0	0.0
50	5.2	3.7
100	10.5	14.6
150	15.7	32.9
200	20.9	58.4
250	26.2	91.3
300	31.4	131.5
350	36.7	178.9
400	41.9	233.7
450	47.1	295.8
500	52.4	365.2



Discussion The pressure rise with rotation rate is not linear, but rather quadratic.

3-107

Solution A water tank partially filled with water is being towed by a truck on a level road. The maximum acceleration (or deceleration) of the truck to avoid spilling is to be determined.



Assumptions **1** The road is horizontal so that acceleration has no vertical component (a_z = 0). **2** Effects of splashing, breaking, driving over bumps, and climbing hills are assumed to be secondary, and are not considered. **3** The acceleration remains constant.

Analysis We take the *x*-axis to be the direction of motion, the *z*-axis to be the upward vertical direction. The shape of the free surface just before spilling is shown in figure. The tangent of the angle the free surface makes with the horizontal is given by

$$\tan\theta = \frac{a_x}{g + a_z} \qquad \rightarrow \qquad a_x = g\tan\theta$$

where a_z = 0 and, from geometric considerations, $\tan\theta$ is $\tan\theta = \frac{\Delta h}{L/2}$. Substituting, we get

$$a_x = g\tan\theta = g\frac{\Delta h}{L/2} = (32.2\text{ ft/s}^2)\frac{2\text{ ft}}{(20\text{ ft})/2} = \mathbf{6.44\ m/s^2}$$

The solution can be repeated for deceleration by replacing a_x by $-a_x$. We obtain a_x = **-6.44 m/s²**.

Discussion Note that the analysis is valid for any fluid with constant density since we used no information that pertains to fluid properties in the solution.

3-108E

Solution A water tank partially filled with water is being towed by a truck on a level road. The maximum acceleration (or deceleration) of the truck to avoid spilling is to be determined.

Assumptions **1** The road is horizontal so that deceleration has no vertical component (a_z = 0). **2** Effects of splashing and driving over bumps are assumed to be secondary, and are not considered. **3** The deceleration remains constant.

Analysis We take the *x*-axis to be the direction of motion, the *z*-axis to be the upward vertical direction. The shape of the free surface just before spilling is shown in figure. The tangent of the angle the free surface makes with the horizontal is given by

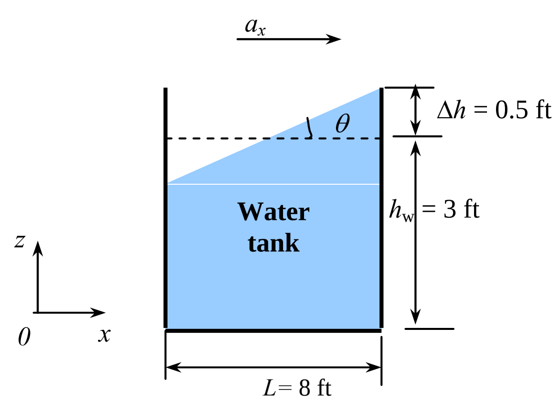
$$\tan\theta = \frac{-a_x}{g + a_z} \qquad \rightarrow \qquad a_x = -g\tan\theta$$

where a_z = 0 and, from geometric considerations, $\tan\theta$ is

$$\tan\theta = \frac{\Delta h}{L/2}$$

Substituting,

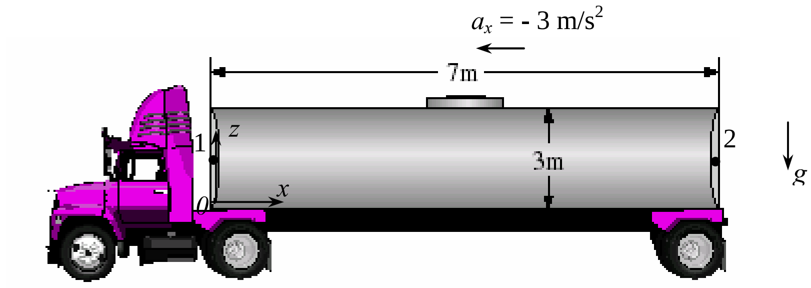
$$a_x = -g\tan\theta = -g\frac{\Delta h}{L/2} = -(32.2\text{ ft/s}^2)\frac{0.5\text{ ft}}{(8\text{ ft})/2} = \mathbf{-4.08\ ft/s^2}$$



Discussion Note that the analysis is valid for any fluid with constant density since we used no information that pertains to fluid properties in the solution.

3-109

Solution Water is transported in a completely filled horizontal cylindrical tanker accelerating at a specified rate. The pressure difference between the front and back ends of the tank along a horizontal line when the truck accelerates and decelerates at specified rates.



Assumptions **1** The acceleration remains constant. **2** Water is an incompressible substance.

Properties We take the density of the water to be 1000 kg/m^3.

Analysis (*a*) We take the *x*- and *z*- axes as shown. The horizontal acceleration is in the negative *x* direction, and thus a_x is negative. Also, there is no acceleration in the vertical direction, and thus a_z = 0. The pressure difference between two points 1 and 2 in an incompressible fluid in linear rigid body motion is given by

$$P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho (g + a_z)(z_2 - z_1) \quad \rightarrow \quad P_2 - P_1 = -\rho a_x (x_2 - x_1)$$

since z_2 - z_1 = 0 along a horizontal line. Therefore, the pressure difference between the front and back of the tank is due to acceleration in the horizontal direction and the resulting compression effect towards the back of the tank. Then the pressure difference along a horizontal line becomes

$$\Delta P = P_2 - P_1 = -\rho a_x (x_2 - x_1) = -(1000 \text{ kg/m}^3)(-3 \text{ m/s}^2)(7 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) = 21 \text{ kN/m}^2 = \mathbf{21\ kPa}$$

since x_1 = 0 and x_2 = 7 m.

(*b*) The pressure difference during deceleration is determined the way, but a_x = 4 m/s^2 in this case,

$$\Delta P = P_2 - P_1 = -\rho a_x (x_2 - x_1) = -(1000 \text{ kg/m}^3)(4 \text{ m/s}^2)(7 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) = -28 \text{ kN/m}^2 = \mathbf{-28\ kPa}$$

Discussion Note that the pressure is higher at the back end of the tank during acceleration, but at the front end during deceleration (during breaking, for example) as expected.

Review Problems

3-110

Solution One section of the duct of an air-conditioning system is laid underwater. The upward force the water exerts on the duct is to be determined.

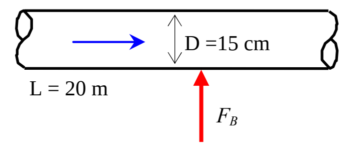
Assumptions **1** The diameter given is the outer diameter of the duct (or, the thickness of the duct material is negligible). **2** The weight of the duct and the air in is negligible.

Properties The density of air is given to be $\rho = 1.30$ kg/m^3. We take the density of water to be 1000 kg/m^3.

Analysis Noting that the weight of the duct and the air in it is negligible, the net upward force acting on the duct is the buoyancy force exerted by water. The volume of the underground section of the duct is

$$V = AL = (\pi D^2/4)L = [\pi(0.15\text{ m})^2/4](20\text{ m}) = 0.3534\text{ m}^3$$

Then the buoyancy force becomes



$$F_B = \rho g V = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.3534\text{ m}^3)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{3.47\ kN}$$

Discussion The upward force exerted by water on the duct is 3.47 kN, which is equivalent to the weight of a mass of 354 kg. Therefore, this force must be treated seriously.

3-111

Solution A helium balloon tied to the ground carries 2 people. The acceleration of the balloon when it is first released is to be determined.

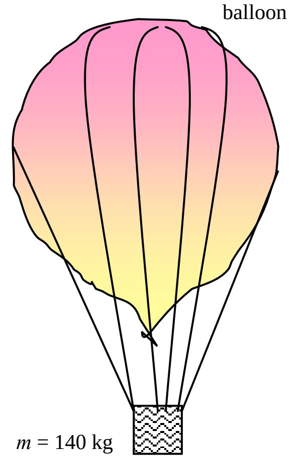
Assumptions The weight of the cage and the ropes of the balloon is negligible.

Properties The density of air is given to be $\rho = 1.16$ kg/m^3. The density of helium gas is 1/7th of this.

Analysis The buoyancy force acting on the balloon is

$$V_{\text{balloon}} = 4\pi r^3/3 = 4\pi(5\text{ m})^3/3 = 523.6\text{ m}^3$$

$$F_B = \rho_{\text{air}} g V_{\text{balloon}} = (1.16\text{ kg/m}^3)(9.81\text{ m/s}^2)(523.6\text{ m}^3)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 5958.4\text{ N}$$



The total mass is

$$m_{\text{He}} = \rho_{\text{He}} V = \left(\frac{1.16}{7}\text{ kg/m}^3\right)(523.6\text{ m}^3) = 86.8\text{ kg}$$

$$m_{\text{total}} = m_{\text{He}} + m_{\text{people}} = 86.8 + 2\times 70 = 226.8\text{ kg}$$

The total weight is

$$W = m_{\text{total}} g = (226.8\text{ kg})(9.81\text{ m/s}^2)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 2224.9\text{ N}$$

Thus the net force acting on the balloon is

$$F_{\text{net}} = F_B - W = 5958.6 - 2224.9 = 3733.5\text{ N}$$

Then the acceleration becomes

$$a = \frac{F_{\text{net}}}{m_{\text{total}}} = \frac{3733.5\text{ N}}{226.8\text{ kg}}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = \mathbf{16.5\ m/s^2}$$

Discussion This is almost twice the acceleration of gravity – aerodynamic drag on the balloon acts quickly to slow down the acceleration.

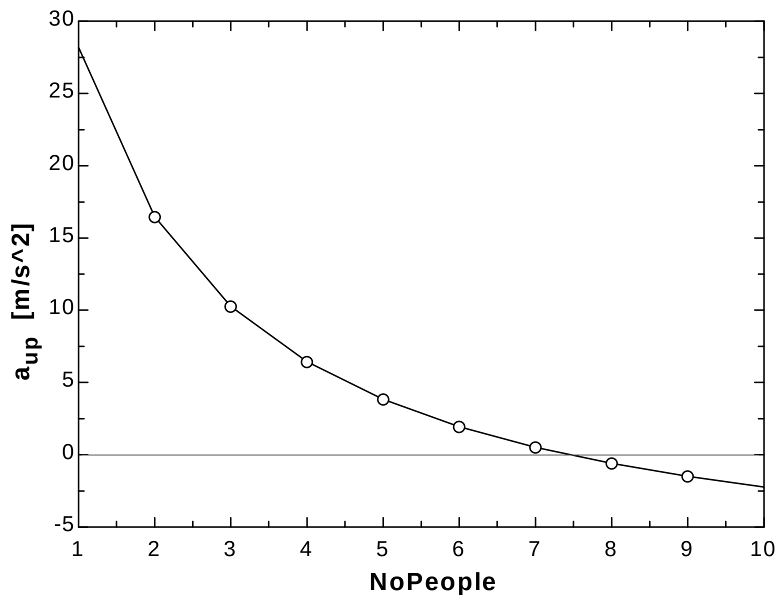
3-112

Solution The previous problem is reconsidered. The effect of the number of people carried in the balloon on acceleration is to be investigated. Acceleration is to be plotted against the number of people, and the results are to be discussed.

Analysis The EES *Equations* window is printed below, followed by the tabulated and plotted results.

```
"Given Data:"
rho_air=1.16"[kg/m^3]" "density of air"
g=9.807"[m/s^2]"
d_balloon=10"[m]"
m_1person=70"[kg]"
{NoPeople = 2}  "Data suppied in Parametric Table"

"Calculated values:"
rho_He=rho_air/7"[kg/m^3]" "density of helium"
r_balloon=d_balloon/2"[m]"
V_balloon=4*pi*r_balloon^3/3"[m^3]"
m_people=NoPeople*m_1person"[kg]"
m_He=rho_He*V_balloon"[kg]"
m_total=m_He+m_people"[kg]"
"The total weight of balloon and people is:"
W_total=m_total*g"[N]"
"The buoyancy force acting on the balloon, F_b, is equal to the weight of the air displaced by the
balloon."
F_b=rho_air*V_balloon*g"[N]"
"From the free body diagram of the balloon, the balancing vertical forces must equal the product of the
total mass and the vertical acceleration:"
F_b- W_total=m_total*a_up
```

A_{up} [m/s^2]	No. People
28.19	1
16.46	2
10.26	3
6.434	4
3.831	5
1.947	6
0.5204	7
-0.5973	8
-1.497	9
-2.236	10



Discussion As expected, the more people, the slower the acceleration. In fact, if more than 7 people are on board, the balloon does not rise at all.

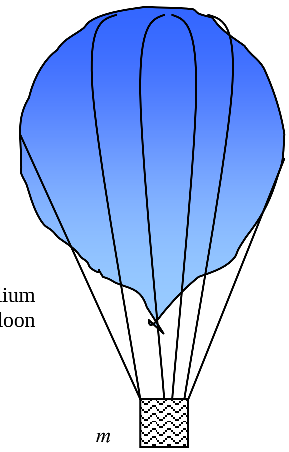
3-113

Solution A balloon is filled with helium gas. The maximum amount of load the balloon can carry is to be determined.

Assumptions The weight of the cage and the ropes of the balloon is negligible.

Properties The density of air is given to be ρ = 1.16 kg/m^3. The density of helium gas is 1/7th of this.

Analysis In the limiting case, the net force acting on the balloon will be zero. That is, the buoyancy force and the weight will balance each other:



$$W = mg = F_B$$

$$m_{total} = \frac{F_B}{g} = \frac{5958.4 \text{ N}}{9.81 \text{ m/s}^2} = 607.4 \text{ kg}$$

Thus,

$$m_{\text{people}} = m_{\text{total}} - m_{\text{He}} = 607.4 - 86.8 = 520.6 \text{ kg} \cong \mathbf{521 \text{ kg}}$$

Discussion When the net weight of the balloon and its cargo exceeds the weight of the air it displaces, the balloon/cargo is no longer "lighter than air", and therefore cannot rise.

3-114E

Solution The pressure in a steam boiler is given in kgf/cm^2. It is to be expressed in psi, kPa, atm, and bars.

Analysis We note that 1 atm = 1.03323 kgf/cm^2, 1 atm = 14.696 psi, 1 atm = 101.325 kPa, and 1 atm = 1.01325 bar (inner cover page of text). Then the desired conversions become:

In atm: $$P = (75 \text{ kgf/cm}^2)\left(\frac{1 \text{ atm}}{1.03323 \text{ kgf/cm}^2}\right) = 72.6 \text{ atm}$$

In psi: $$P = (75 \text{ kgf/cm}^2)\left(\frac{1 \text{ atm}}{1.03323 \text{ kgf/cm}^2}\right)\left(\frac{14.696 \text{ psi}}{1 \text{ atm}}\right) = 1067 \text{ psi} \cong \mathbf{1070 \text{ psi}}$$

In kPa: $$P = (75 \text{ kgf/cm}^2)\left(\frac{1 \text{ atm}}{1.03323 \text{ kgf/cm}^2}\right)\left(\frac{101.325 \text{ kPa}}{1 \text{ atm}}\right) = 7355 \text{ kPa} \cong \mathbf{7360 \text{ kPa}}$$

In bars: $$P = (75 \text{ kgf/cm}^2)\left(\frac{1 \text{ atm}}{1.03323 \text{ kgf/cm}^2}\right)\left(\frac{1.01325 \text{ bar}}{1 \text{ atm}}\right) = 73.55 \text{ bar} \cong \mathbf{73.6 \text{ bar}}$$

Discussion Note that the units atm, kgf/cm^2, and bar are almost identical to each other. All final results are given to three significant digits, but conversion ratios are typically precise to at least five significant digits.

3-115

Solution A barometer is used to measure the altitude of a plane relative to the ground. The barometric readings at the ground and in the plane are given. The altitude of the plane is to be determined.

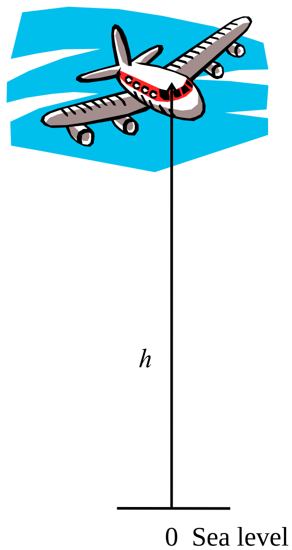
Assumptions The variation of air density with altitude is negligible.

Properties The densities of air and mercury are given to be ρ_{air} = 1.20 kg/m^3 and $\rho_{mercury}$ = 13,600 kg/m^3.

Analysis Atmospheric pressures at the location of the plane and the ground level are

$$\begin{aligned} P_{\text{plane}} &= (\rho g h)_{\text{plane}} \\ &= (13{,}600\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.690\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kPa}}{1000\text{ N/m}^2}\right) \\ &= 92.06\text{ kPa} \end{aligned}$$

$$\begin{aligned} P_{\text{ground}} &= (\rho g h)_{\text{ground}} \\ &= (13{,}600\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.753\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kPa}}{1000\text{ N/m}^2}\right) \\ &= 100.46\text{ kPa} \end{aligned}$$



Taking an air column between the airplane and the ground and writing a force balance per unit base area, we obtain

$$W_{\text{air}} / A = P_{\text{ground}} - P_{\text{plane}}$$

$$(\rho g h)_{\text{air}} = P_{\text{ground}} - P_{\text{plane}}$$

$$(1.20\text{ kg/m}^3)(9.81\text{ m/s}^2)(h)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kPa}}{1000\text{ N/m}^2}\right) = (100.46 - 92.06)\text{ kPa}$$

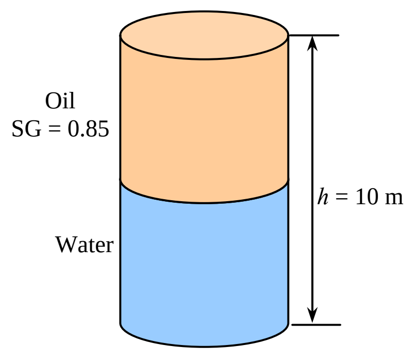
It yields h = **714 m,** which is also the altitude of the airplane.

Discussion Obviously, a mercury barometer is not practical on an airplane – an electronic barometer is used instead.

3-116

Solution A 10-m high cylindrical container is filled with equal volumes of water and oil. The pressure difference between the top and the bottom of the container is to be determined.

Properties The density of water is given to be ρ = 1000 kg/m^3. The specific gravity of oil is given to be 0.85.



Analysis The density of the oil is obtained by multiplying its specific gravity by the density of water,

$$\rho = \text{SG} \times \rho_{H_2O} = (0.85)(1000\text{ kg/m}^3) = 850\text{ kg/m}^3$$

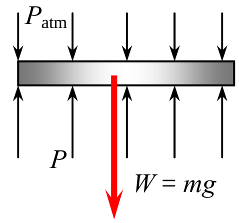
The pressure difference between the top and the bottom of the cylinder is the sum of the pressure differences across the two fluids,

$$\begin{aligned} \Delta P_{\text{total}} &= \Delta P_{\text{oil}} + \Delta P_{\text{water}} = (\rho g h)_{\text{oil}} + (\rho g h)_{\text{water}} \\ &= \left[(850\text{ kg/m}^3)(9.81\text{ m/s}^2)(5\text{ m}) + (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(5\text{ m})\right]\left(\frac{1\text{ kPa}}{1000\text{ N/m}^2}\right) \\ &= \mathbf{90.7\ kPa} \end{aligned}$$

Discussion The pressure at the interface must be the same in the oil and the water. Therefore, we can use the rules for hydrostatics across the two fluids, since they are at rest and there are no appreciable surface tension effects.

3-117

Solution The pressure of a gas contained in a vertical piston-cylinder device is measured to be 500 kPa. The mass of the piston is to be determined.



Assumptions There is no friction between the piston and the cylinder.

Analysis Drawing the free body diagram of the piston and balancing the vertical forces yield

$$W = PA - P_{atm}A$$

$$mg = (P - P_{atm})A$$

$$(m)(9.81 \text{ m/s}^2) = (500 - 100 \text{ kPa})(30\times10^{-4} \text{ m}^2)\left(\frac{1000 \text{ kg/m}\cdot\text{s}^2}{1 \text{ kPa}}\right)$$

Solution of the above equation yields m = **122 kg.**

Discussion The gas cannot distinguish between pressure due to the piston weight and atmospheric pressure – both “feel” like a higher pressure acting on the top of the gas in the cylinder.

3-118

Solution The gage pressure in a pressure cooker is maintained constant at 100 kPa by a petcock. The mass of the petcock is to be determined.

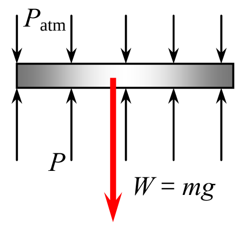
Assumptions There is no blockage of the pressure release valve.

Analysis Atmospheric pressure is acting on all surfaces of the petcock, which balances itself out. Therefore, it can be disregarded in calculations if we use the gage pressure as the cooker pressure. A force balance on the petcock ($\Sigma F_y = 0$) yields

$$W = P_{gage}A$$

$$m = \frac{P_{gage}A}{g} = \frac{(100 \text{ kPa})(4\times10^{-6} \text{ m}^2)}{9.81 \text{ m/s}^2}\left(\frac{1000 \text{ kg/m}\cdot\text{s}^2}{1 \text{ kPa}}\right)$$

$$= \mathbf{0.0408 \text{ kg} = 40.8 \text{ g}}$$



Discussion The higher pressure causes water in the cooker to boil at a higher temperature.

3-119

Solution A glass tube open to the atmosphere is attached to a water pipe, and the pressure at the bottom of the tube is measured. It is to be determined how high the water will rise in the tube.

Properties The density of water is given to be ρ = 1000 kg/m^3.

Analysis The pressure at the bottom of the tube can be expressed as

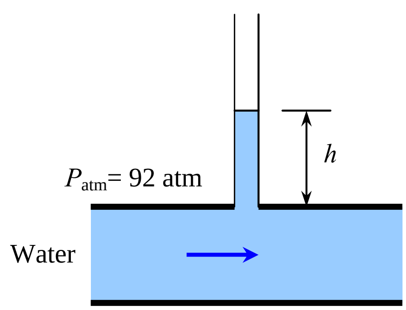
$$P = P_{atm} + (\rho g h)_{tube}$$

Solving for h,

$$h = \frac{P - P_{atm}}{\rho g}$$

$$= \frac{(115 - 92) \text{ kPa}}{(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right)\left(\frac{1000 \text{ N/m}^2}{1 \text{ kPa}}\right)$$

$$= \mathbf{2.35 \text{ m}}$$



Discussion Even though the water is flowing, the water in the tube itself is at rest. If the pressure at the tube bottom had been given in terms of gage pressure, we would not have had to take into account the atmospheric pressure term.

3-120

Solution The average atmospheric pressure is given as $P_{atm} = 101.325(1 - 0.02256\,z)^{5.256}$ where z is the altitude in km. The atmospheric pressures at various locations are to be determined.

Analysis Atmospheric pressure at various locations is obtained by substituting the altitude z values in km into the relation $P_{atm} = 101.325(1 - 0.02256z)^{5.256}$. The results are tabulated below.

Atlanta:	(z = 0.306 km): P_{atm} = 101.325(1 - 0.02256×0.306)$^{5.256}$ = **97.7 kPa**
Denver:	(z = 1.610 km): P_{atm} = 101.325(1 - 0.02256×1.610)$^{5.256}$ = **83.4 kPa**
M. City:	(z = 2.309 km): P_{atm} = 101.325(1 - 0.02256×2.309)$^{5.256}$ = **76.5 kPa**
Mt. Ev.:	(z = 8.848 km): P_{atm} = 101.325(1 - 0.02256×8.848)$^{5.256}$ = **31.4 kPa**

Discussion It may be surprising, but the atmospheric pressure on Mt. Everest is less than 1/3 that at sea level!

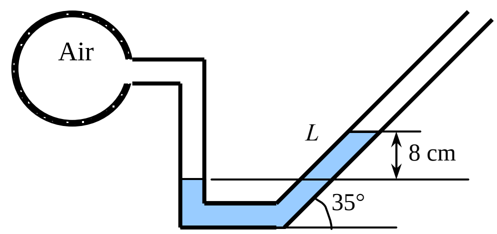
3-121

Solution The air pressure in a duct is measured by an inclined manometer. For a given vertical level difference, the gage pressure in the duct and the length of the differential fluid column are to be determined.

Assumptions The manometer fluid is an incompressible substance.

Properties The density of the liquid is given to be ρ = 0.81 kg/L = 810 kg/m^3.

Analysis The gage pressure in the duct is determined from



$$P_{gage} = P_{abs} - P_{atm} = \rho g h$$
$$= (810\,\text{kg/m}^3)(9.81\ \text{m/s}^2)(0.08\,\text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{Pa}}{1\,\text{N/m}^2}\right)$$
$$= \mathbf{636\ \ Pa}$$

The length of the differential fluid column is

$$L = h/\sin\theta = (8\ \text{cm})/\sin 35° = \mathbf{13.9\ \ cm}$$

Discussion Note that the length of the differential fluid column is extended considerably by inclining the manometer arm for better readability (and therefore higher *precision*).

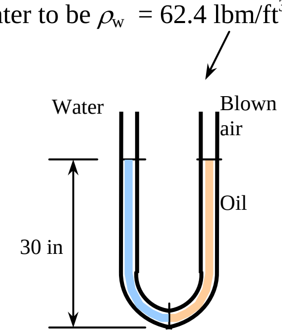
3-122E

Solution Equal volumes of water and oil are poured into a U-tube from different arms, and the oil side is pressurized until the contact surface of the two fluids moves to the bottom and the liquid levels in both arms become the same. The excess pressure applied on the oil side is to be determined.

Assumptions **1** Both water and oil are incompressible substances. **2** Oil does not mix with water. **3** The cross-sectional area of the U-tube is constant.

Properties The density of oil is given to be ρ_{oil} = 49.3 lbm/ft^3. We take the density of water to be ρ_w = 62.4 lbm/ft^3.

Analysis Noting that the pressure of both the water and the oil is the same at the contact surface, the pressure at this surface can be expressed as



$$P_{contact} = P_{blow} + \rho_a g h_a = P_{atm} + \rho_w g h_w$$

Noting that $h_a = h_w$ and rearranging,

$$P_{gage,\,blow} = P_{blow} - P_{atm} = (\rho_w - \rho_{oil})gh$$
$$= (62.4\text{- }49.3\ \ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(30/12\ \text{ft})\left(\frac{1\ \ \text{lbf}}{32.2\ \ \text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\ \ \text{ft}^2}{144\ \ \text{in}^2}\right)$$
$$= \mathbf{0.227\ \ psi}$$

Discussion When the person stops blowing, the oil rises and some water flows into the right arm. It can be shown that when the curvature effects of the tube are disregarded, the differential height of water is 23.7 in to balance 30-in of oil.

3-123

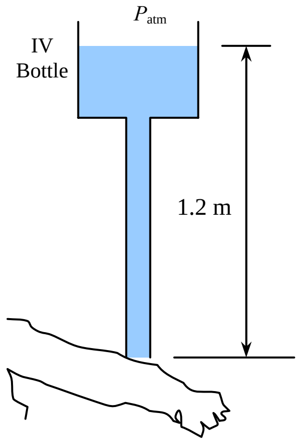
Solution It is given that an IV fluid and the blood pressures balance each other when the bottle is at a certain height, and a certain gage pressure at the arm level is needed for sufficient flow rate. The gage pressure of the blood and elevation of the bottle required to maintain flow at the desired rate are to be determined.

Assumptions **1** The IV fluid is incompressible. **2** The IV bottle is open to the atmosphere.

Properties The density of the IV fluid is given to be $\rho = 1020$ kg/m^3.

Analysis (*a*) Noting that the IV fluid and the blood pressures balance each other when the bottle is 1.2 m above the arm level, the gage pressure of the blood in the arm is simply equal to the gage pressure of the IV fluid at a depth of 1.2 m,

$$P_{\text{gage, arm}} = P_{\text{abs}} - P_{\text{atm}} = \rho g h_{\text{arm-bottle}}$$
$$= (1020\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.20\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right)$$
$$= \mathbf{12.0\ kPa}$$



(*b*) To provide a gage pressure of 20 kPa at the arm level, the height of the bottle from the arm level is again determined from $P_{\text{gage, arm}} = \rho g h_{\text{arm-bottle}}$ to be

$$h_{\text{arm-bottle}} = \frac{P_{\text{gage, arm}}}{\rho g}$$
$$= \frac{20\ \text{kPa}}{(1020\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kN}}\right)\left(\frac{1\ \text{kN/m}^2}{1\ \text{kPa}}\right) = \mathbf{2.0\ m}$$

Discussion Note that the height of the reservoir can be used to control flow rates in gravity driven flows. When there is flow, the pressure drop in the tube due to friction should also be considered. This will result in raising the bottle a little higher to overcome pressure drop.

3-124

Solution A gasoline line is connected to a pressure gage through a double-U manometer. For a given reading of the pressure gage, the gage pressure of the gasoline line is to be determined.

Assumptions **1** All the liquids are incompressible. **2** The effect of air column on pressure is negligible.

Properties The specific gravities of oil, mercury, and gasoline are given to be 0.79, 13.6, and 0.70, respectively. We take the density of water to be ρ_w = 1000 kg/m^3.

Analysis Starting with the pressure indicated by the pressure gage and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the gasoline pipe, and setting the result equal to P_{gasoline} gives

$$P_{\text{gage}} - \rho_w g h_w + \rho_{\text{oil}} g h_{\text{oil}} - \rho_{\text{Hg}} g h_{\text{Hg}} - \rho_{\text{gasoline}} g h_{\text{gasoline}} = P_{\text{gasoline}}$$

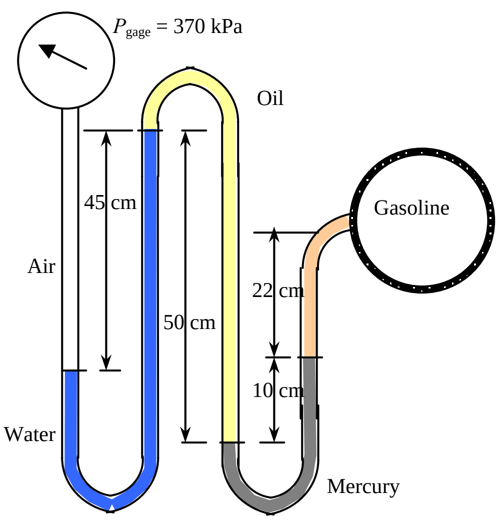
Rearranging,

$$P_{\text{gasoline}} = P_{\text{gage}} - \rho_w g(h_w - SG_{\text{oil}} h_{\text{oil}} + SG_{\text{Hg}} h_{\text{Hg}} + SG_{\text{gasoline}} h_{\text{gasoline}})$$

Substituting,

$$P_{\text{gasoline}} = 370 \text{ kPa -}(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(0.45 \text{ m}) - 0.79(0.5 \text{ m}) + 13.6(0.1 \text{ m}) + 0.70(0.22 \text{ m})]$$
$$\times\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1 \text{ kN/m}^2}\right)$$
$$= 354.6 \text{ kPa} \cong \mathbf{355 \text{ kPa}}$$

Therefore, the pressure in the gasoline pipe is 15.4 kPa lower than the pressure reading of the pressure gage.



Discussion Note that sometimes the use of specific gravity offers great convenience in the solution of problems that involve several fluids.

3-125

Solution A gasoline line is connected to a pressure gage through a double-U manometer. For a given reading of the pressure gage, the gage pressure of the gasoline line is to be determined.

Assumptions **1** All the liquids are incompressible. **2** The effect of air column on pressure is negligible.

Properties The specific gravities of oil, mercury, and gasoline are given to be 0.79, 13.6, and 0.70, respectively. We take the density of water to be $\rho_w = 1000$ kg/m^3.

Analysis Starting with the pressure indicated by the pressure gage and moving along the tube by adding (as we go down) or subtracting (as we go up) the $\rho g h$ terms until we reach the gasoline pipe, and setting the result equal to P_{gasoline} gives

$$P_{gage} - \rho_{\text{w}} g h_w + \rho_{\text{alcohol}} g h_{\text{alcohol}} - \rho_{\text{Hg}} g h_{\text{Hg}} - \rho_{\text{gasoline}} g h_{\text{gasoline}} = P_{\text{gasoline}}$$

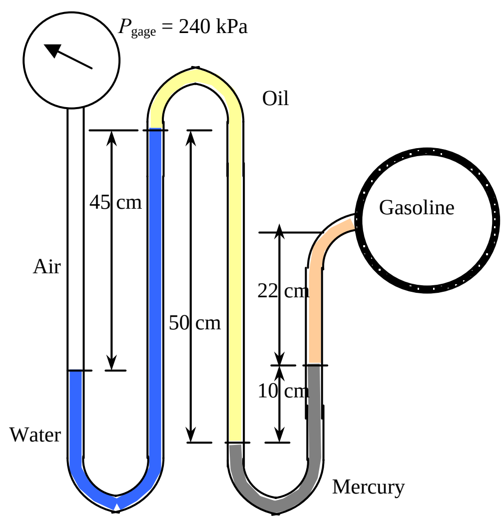
Rearranging,

$$P_{\text{gasoline}} = P_{\text{gage}} - \rho_{\text{w}} g(h_w - SG_{\text{alcohol}} h_{\text{s,alcohol}} + SG_{\text{Hg}} h_{\text{Hg}} + SG_{\text{gasoline}} h_{\text{s,gasoline}})$$

Substituting,

$$P_{\text{gasoline}} = 240 \text{ kPa -}(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(0.45 \text{ m}) - 0.79(0.5 \text{ m}) + 13.6(0.1 \text{ m}) + 0.70(0.22 \text{ m})]$$
$$\times \left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1 \text{ kN/m}^2}\right)$$
$$= 224.6 \text{ kPa} \cong \mathbf{225 \text{ kPa}}$$

Therefore, the pressure in the gasoline pipe is 15.4 kPa lower than the pressure reading of the pressure gage.



Discussion Note that sometimes the use of specific gravity offers great convenience in the solution of problems that involve several fluids.

3-126E

Solution A water pipe is connected to a double-U manometer whose free arm is open to the atmosphere. The absolute pressure at the center of the pipe is to be determined.

Assumptions **1** All the liquids are incompressible. **2** The solubility of the liquids in each other is negligible.

Properties The specific gravities of mercury and oil are given to be 13.6 and 0.80, respectively. We take the density of water to be ρ_w = 62.4 lbm/ft^3.

Analysis Starting with the pressure at the center of the water pipe, and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to P_{atm} gives

$$P_{\text{water pipe}} - \rho_{\text{water}} g h_{\text{water}} + \rho_{\text{alcohol}} g h_{\text{alcohol}} - \rho_{\text{Hg}} g h_{\text{Hg}} - \rho_{\text{oil}} g h_{\text{oil}} = P_{atm}$$

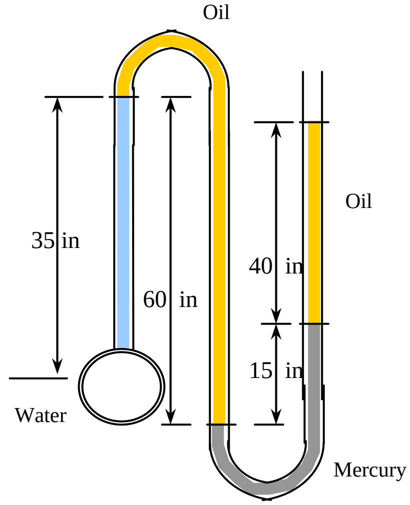
Solving for $P_{\text{water pipe}}$,

$$P_{\text{water pipe}} = P_{atm} + \rho_{\text{water}} g (h_{\text{water}} - SG_{\text{oil}} h_{\text{alcohol}} + SG_{\text{Hg}} h_{\text{Hg}} + SG_{\text{oil}} h_{\text{oil}})$$

Substituting,

$$\begin{aligned} P_{\text{water pipe}} &= 14.2\,\text{psia} + (62.4\,\text{lbm/ft}^3)(32.2\,\text{ft/s}^2)[(35/12\,\text{ft}) - 0.80(60/12\,\text{ ft}) + 13.6(15/12\,\text{ft}) \\ &\quad + 0.8(40/12\,\text{ft})] \times \left(\frac{1\,\text{lbf}}{32.2\,\text{lbm}\cdot\text{ft/s}^2}\right)\left(\frac{1\,\text{ft}^2}{144\,\text{in}^2}\right) \\ &= \mathbf{22.3\,psia} \end{aligned}$$

Therefore, the absolute pressure in the water pipe is 22.3 psia.



Discussion Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly.

3-127

Solution The pressure of water flowing through a pipe is measured by an arrangement that involves both a pressure gage and a manometer. For the values given, the pressure in the pipe is to be determined.

Assumptions **1** All the liquids are incompressible. **2** The effect of air column on pressure is negligible.

Properties The specific gravity of gage fluid is given to be 2.4. We take the standard density of water to be ρ_w = 1000 kg/m^3.

Analysis Starting with the pressure indicated by the pressure gage and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the water pipe, and setting the result equal to P_{water} give

$$P_{gage} + \rho_w g h_{w1} - \rho_{gage} g h_{gage} - \rho_w g h_{w2} = P_{water}$$

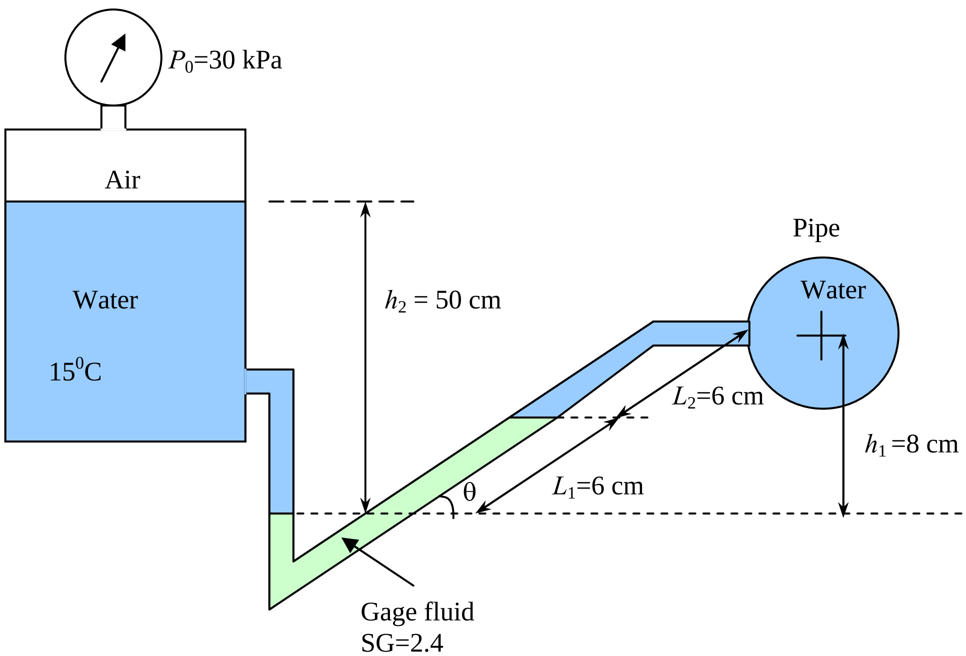
Rearranging,

$$P_{water} = P_{gage} + \rho_w g\left(h_{w1} - SG_{gage} h_{gage} - h_{w2}\right) = P_{gage} + \rho_w g\left(h_2 - SG_{gage} L_1 \sin\theta - L_2 \sin\theta\right)$$

Noting that $\sin\theta = 8/12 = 0.6667$ and substituting,

$$P_{water} = 30\text{ kPa} + (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)[(0.50\text{ m}) - 2.4(0.06\text{ m})0.6667 - (0.06\text{ m})0.6667]$$
$$\times\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)$$
$$= \mathbf{33.6\ kPa}$$

Therefore, the pressure in the gasoline pipe is 3.6 kPa over the reading of the pressure gage.



Discussion Note that even without a manometer, the reading of a pressure gage can be in error if it is not placed at the same level as the pipe when the fluid is a liquid.

3-128

Solution A U-tube filled with mercury except the 18-cm high portion at the top. Oil is poured into the left arm, forcing some mercury from the left arm into the right one. The maximum amount of oil that can be added into the left arm is to be determined.

Assumptions **1** Both liquids are incompressible. **2** The U-tube is perfectly vertical.

Properties The specific gravities are given to be 2.72 for oil and 13.6 for mercury.

Analysis Initially, the mercury levels in both tubes are the same. When oil is poured into the left arm, it will push the mercury in the left down, which will cause the mercury level in the right arm to rise. Noting that the volume of mercury is constant, the decrease in the mercury volume in left column must be equal to the increase in the mercury volume in the right arm. Therefore, if the drop in mercury level in the left arm is x, the rise in the mercury level in the right arm h corresponding to a drop of x in the left arm is

$$V_{\text{left}} = V_{\text{right}} \quad \rightarrow \quad \pi(2d)^2 x = \pi d^2 h \quad \rightarrow \quad h = 4x$$

The pressures at points A and B are equal $P_A = P_B$ and thus

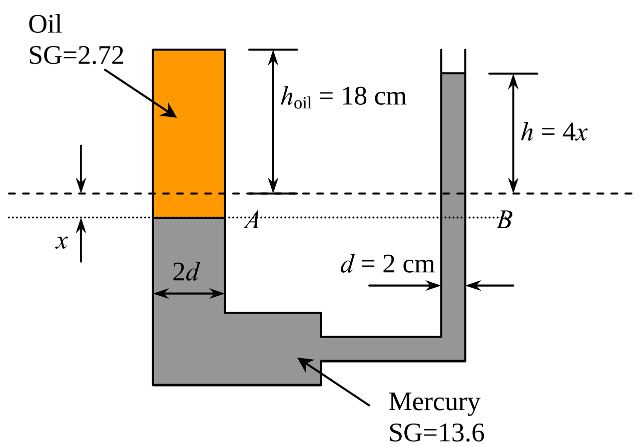
$$P_{atm} + \rho_{\text{oil}} g(h_{oil} + x) = P_{atm} + \rho_{Hg} g h_{Hg} \quad \rightarrow \quad \text{SG}_{\text{oil}}\rho_w g\left(h_{\text{oil}} + x\right) = \text{SG}_{\text{Hg}}\rho_w g\left(5x\right)$$

Solving for x and substituting,

$$x = \frac{\text{SG}_{\text{oil}} h_{\text{oil}}}{5\text{SG}_{\text{Hg}} - \text{SG}_{\text{oil}}} = \frac{2.72\left(18 \text{ cm}\right)}{5\times 13.6 - 2.72} = 0.75 \text{ cm}$$

Therefore, the maximum amount of oil that can be added into the left arm is

$$V_{\text{oil, max}} = \pi(2d/2)^2(h_{\text{oil}} + x) = \pi(2 \text{ cm})^2(18 + 0.75 \text{ cm}) = \mathbf{236\ cm^3} = \mathbf{0.236\ L}$$



Discussion Note that the fluid levels in the two arms of a U-tube can be different when two different fluids are involved.

3-129

Solution The pressure buildup in a teapot may cause the water to overflow through the service tube. The maximum cold-water height to avoid overflow under a specified gage pressure is to be determined.

Assumptions **1** Water is incompressible. **2** Thermal expansion and the amount of water in the service tube are negligible. **3** The cold water temperature is 20°C.

Properties The density of water at 20°C is ρ_w = 998.0 kg/m^3.

Analysis From geometric considerations, the vertical distance between the bottom of the teapot and the tip of the service tube is

$$h_{\text{tip}} = 4 + 12\cos 40° = 13.2 \text{ cm}$$

This would be the maximum water height if there were no pressure build-up inside by the steam. The steam pressure inside the teapot above the atmospheric pressure must be balanced by the water column inside the service tube,

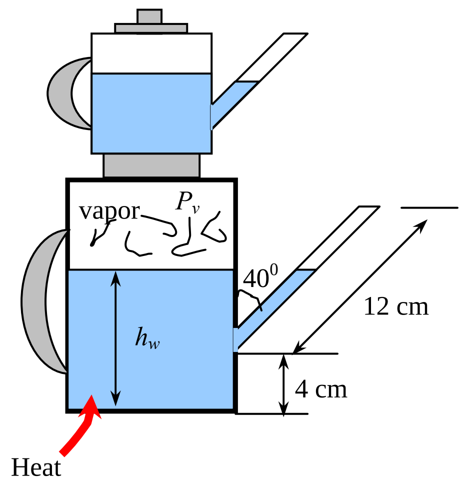
$$P_{\text{v, gage}} = \rho_w g \Delta h_w$$

or,

$$\Delta h_{\text{w}} = \frac{P_{\text{v, gage}}}{\rho_{\text{w}} g} = \frac{0.32 \text{ kPa}}{(998.0 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{1000 \text{ kg}\cdot\text{m/s}^2}{1 \text{ kN}}\right)\left(\frac{1 \text{ kN/m}^2}{1 \text{ kPa}}\right) = 0.033 \text{ m} = 3.3 \text{ cm}$$

Therefore, the water level inside the teapot must be 3.3 cm below the tip of the service tube. Then the maximum initial water height inside the teapot to avoid overflow becomes

$$h_{\text{w, max}} = h_{\text{tip}} - \Delta h_w = 13.2 - 3.3 = \mathbf{9.9 \text{ cm}}$$



Discussion We can obtain the same result formally by starting with the vapor pressure in the teapot and moving along the service tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the atmosphere, and setting the result equal to P_{atm}:

$$P_{atm} + P_{v,gage} - \rho_{\text{w}} g h_w = P_{atm} \quad \rightarrow \quad P_{v,gage} = \rho_{\text{w}} g h_w$$

3-130

Solution The pressure buildup in a teapot may cause the water to overflow through the service tube. The maximum cold-water height to avoid overflow under a specified gage pressure is to be determined by considering the effect of thermal expansion.

Assumptions **1** The amount of water in the service tube is negligible. **3** The cold water temperature is 20°C.

Properties The density of water is ρ_w = 998.0 kg/m^3 at 20°C, and ρ_w = 957.9 kg/m^3 at 100°C

Analysis From geometric considerations, the vertical distance between the bottom of the teapot and the tip of the service tube is

$$h_{\text{tip}} = 4 + 12\cos 40° = 13.2 \text{ cm}$$

This would be the maximum water height if there were no pressure build-up inside by the steam. The steam pressure inside the teapot above the atmospheric pressure must be balanced by the water column inside the service tube,

$$P_{\text{v, gage}} = \rho_w g \Delta h_w$$

or,

$$\Delta h_{\text{w}} = \frac{P_{\text{v, gage}}}{\rho_{\text{w}} g} = \frac{0.32 \text{ kPa}}{(998.0 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{1000 \text{ kg}\cdot\text{m/s}^2}{1 \text{ kN}}\right)\left(\frac{1 \text{ kN/m}^2}{1 \text{ kPa}}\right) = 0.033 \text{ m} = 3.3 \text{ cm}$$

Therefore, the water level inside the teapot must be 3.4 cm below the tip of the service tube. Then the height of hot water inside the teapot to avoid overflow becomes

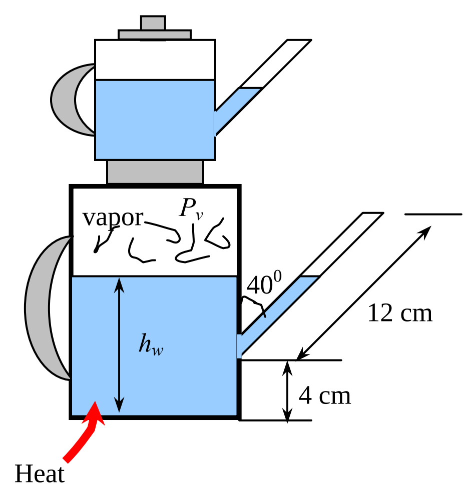
$$h_{\text{w}} = h_{\text{tip}} - \Delta h_w = 13.2 - 3.4 = 9.8 \text{ cm}$$

The specific volume of water is 1/998 m^3/kg at 20°C and 1/957.9 m^3/kg at 100°C. Then the percent drop in the volume of water as it cools from 100°C to 20°C is

$$\text{Volume reduction} = \frac{v_{100°C} - v_{20°C}}{v_{100°C}} = \frac{1/957.9 - 1/998.0}{1/957.9} = 0.040 \quad \text{or } 4.0\%$$

Volume is proportional to water height, and to allow for thermal expansion, the volume of cold water should be 4% less. Therefore, the maximum initial water height to avoid overflow should be

$$h_{\text{w, max}} = (1 - 0.040)h_{\text{w}} = 0.96 \times 9.8 \text{ cm} = \mathbf{9.4 \text{ cm}}$$



Discussion Note that the effect of thermal expansion can be quite significant.

3-131

Solution The temperature of the atmosphere varies with altitude z as $T = T_0 - \beta z$, while the gravitational acceleration varies by $g(z) = g_0/(1 + z/6{,}370{,}320)^2$. Relations for the variation of pressure in atmosphere are to be obtained (*a*) by ignoring and (*b*) by considering the variation of *g* with altitude.

Assumptions The air in the troposphere behaves as an ideal gas.

Analysis (*a*) Pressure change across a differential fluid layer of thickness *dz* in the vertical *z* direction is

$$dP = -\rho g dz$$

From the ideal gas relation, the air density can be expressed as $\rho = \dfrac{P}{RT} = \dfrac{P}{R(T_0 - \beta z)}$. Then,

$$dP = -\frac{P}{R(T_0 - \beta z)} g dz$$

Separating variables and integrating from $z = 0$ where $P = P_0$ to $z = z$ where $P = P$,

$$\int_{P_0}^{P} \frac{dP}{P} = -\int_0^z \frac{gdz}{R(T_0 - \beta z)}$$

Performing the integrations.

$$\ln \frac{P}{P_0} = \frac{g}{R\beta} \ln \frac{T_0 - \beta z}{T_0}$$

Rearranging, the desired relation for atmospheric pressure for the case of constant *g* becomes

$$\boxed{P = P_0 \left(1 - \frac{\beta z}{T_0}\right)^{\frac{g}{\beta R}}}$$

(*b*) When the variation of *g* with altitude is considered, the procedure remains the same but the expressions become more complicated,

$$dP = -\frac{P}{R(T_0 - \beta z)} \frac{g_0}{(1 + z/6{,}370{,}320)^2} dz$$

Separating variables and integrating from $z = 0$ where $P = P_0$ to $z = z$ where $P = P$,

$$\int_{P_0}^{P} \frac{dP}{P} = -\int_0^z \frac{g_0 dz}{R(T_0 - \beta z)(1 + z/6{,}370{,}320)^2}$$

Performing the integrations,

$$\ln P \Big|_{P_0}^{P} = \frac{g_0}{R\beta} \left| \frac{1}{(1 + kT_0/\beta)(1 + kz)} - \frac{1}{(1 + kT_0/\beta)^2} \ln \frac{1 + kz}{T_0 - \beta z} \right|_0^z$$

where R = 287 J/kg·K = 287 m^2/s^2·K is the gas constant of air. After some manipulations, we obtain

$$\boxed{P = P_0 \exp\left[-\frac{g_0}{R(\beta + kT_0)} \left(\frac{1}{1 + 1/kz} + \frac{1}{1 + kT_0/\beta} \ln \frac{1 + kz}{1 - \beta z/T_0} \right) \right]}$$

where T_0 = 288.15 K, β = 0.0065 K/m, g_0 = 9.807 m/s^2, k = 1/6,370,320 m^{-1}, and *z* is the elevation in m..

Discussion When performing the integration in part (*b*), the following expression from integral tables is used, together with a transformation of variable $x = T_0 - \beta z$,

$$\int \frac{dx}{x(a + bx)^2} = \frac{1}{a(a + bx)} - \frac{1}{a^2} \ln \frac{a + bx}{x}$$

Also, for *z* = 11,000 m, for example, the relations in (*a*) and (*b*) give 22.62 and 22.69 kPa, respectively.

3-132

Solution The variation of pressure with density in a thick gas layer is given. A relation is to be obtained for pressure as a function of elevation *z*.

Assumptions The property relation $P = C\rho^n$ is valid over the entire region considered.

Analysis The pressure change across a differential fluid layer of thickness *dz* in the vertical *z* direction is given as,

$$dP = -\rho g dz$$

Also, the relation $P = C\rho^n$ can be expressed as $C = P/\rho^n = P_0/\rho_0^n$, and thus

$$\rho = \rho_0 (P/P_0)^{1/n}$$

Substituting,

$$dP = -g\rho_0 (P/P_0)^{1/n} dz$$

Separating variables and integrating from $z = 0$ where $P = P_0 = C\rho_0^n$ to $z = z$ where $P = P$,

$$\int_{P_0}^{P} (P/P_0)^{-1/n} dP = -\rho_0 g \int_0^z dz$$

Performing the integrations.

$$P_0 \left. \frac{(P/P_0)^{-1/n+1}}{-1/n+1} \right|_{P_0}^{P} = -\rho_0 gz \quad \rightarrow \quad \left(\frac{P}{P_0}\right)^{(n-1)/n} - 1 = -\frac{n-1}{n}\frac{\rho_0 gz}{P_0}$$

Solving for *P*,

$$\boxed{P = P_0 \left(1 - \frac{n-1}{n}\frac{\rho_0 gz}{P_0}\right)^{n/(n-1)}}$$

which is the desired relation.

Discussion The final result could be expressed in various forms. The form given is very convenient for calculations as it facilitates unit cancellations and reduces the chance of error.

3-133

Solution A pressure transducer is used to measure pressure by generating analogue signals, and it is to be calibrated by measuring both the pressure and the electric current simultaneously for various settings, and the results are to be tabulated. A calibration curve in the form of $P = aI + b$ is to be obtained, and the pressure corresponding to a signal of 10 mA is to be calculated.

Assumptions Mercury is an incompressible liquid.

Properties The specific gravity of mercury is given to be 13.56, and thus its density is 13,560 kg/m^3.

Analysis For a given differential height, the pressure can be calculated from

$$P = \rho g \Delta h$$

For Δh = 28.0 mm = 0.0280 m, for example,

$$P = 13.56(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.0280\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) = 3.72\ \text{kPa}$$

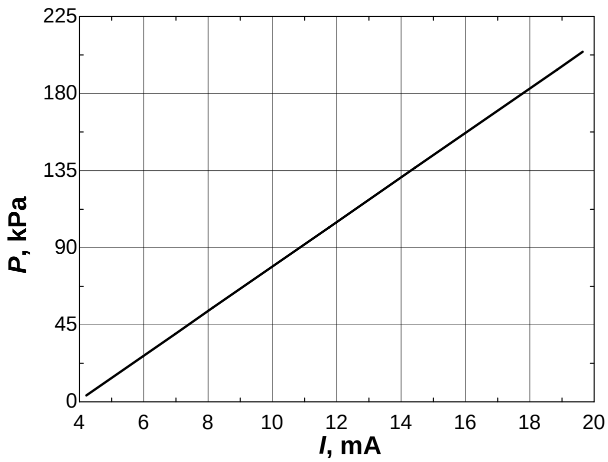
Repeating the calculations and tabulating, we have

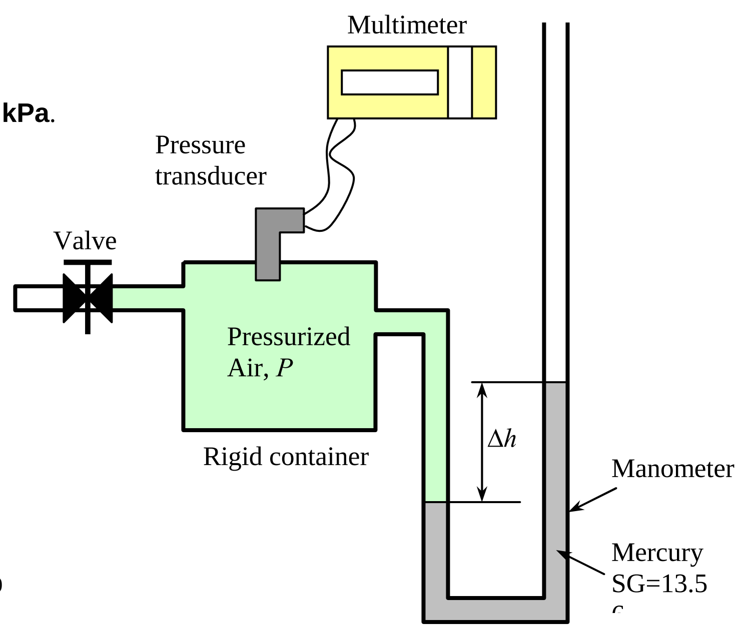
Δh(mm)	28.0	181.5	297.8	413.1	765.9	1027	1149	1362	1458	1536
***P*(kPa)**	**3.72**	**24.14**	**39.61**	**54.95**	**101.9**	**136.6**	**152.8**	**181.2**	**193.9**	**204.3**
I (mA)	4.21	5.78	6.97	8.15	11.76	14.43	15.68	17.86	18.84	19.64

A plot of P versus I is given below. It is clear that the pressure varies linearly with the current, and using EES, the best curve fit is obtained to be

$P = 13.00I - 51.00$ (kPa) for $4.21 \le I \le 19.64$.

For I = 10 mA, for example, we would get P = **79.0 kPa**.





Discussion Note that the calibration relation is valid in the specified range of currents or pressures.

3-134

Solution A system is equipped with two pressure gages and a manometer. For a given differential fluid height, the pressure difference $\Delta P = P_2 - P_1$ is to be determined.

Assumptions **1** All the liquids are incompressible. **2** The effect of air column on pressure is negligible.

Properties The specific gravities are given tone 2.67 for the gage fluid and 0.87 for oil. We take the standard density of water to be ρ_w = 1000 kg/m^3.

Analysis Starting with the pressure indicated by the pressure gage 2 and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms and ignoring the air spaces until we reach the pressure gage 1, and setting the result equal to P_1 give

$$P_2 - \rho_{\text{gage}} g h_{\text{gage}} + \rho_{\text{oil}} g h_{\text{oil}} = P_1$$

Rearranging,

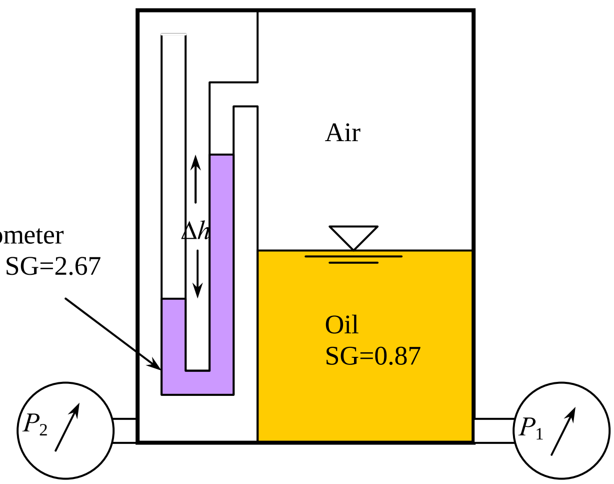
$$P_2 - P_1 = \rho_w g \left(\text{SG}_{\text{gage}} h_{\text{gage}} - \text{SG}_{\text{oil}} h_{\text{oil}} \right)$$

Substituting,

$$P_2 - P_1 = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[2.67(0.08\ \text{m}) - 0.87(0.65\ \text{m})]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right)$$

$$= \mathbf{-3.45\ kPa}$$

Therefore, the pressure reading of the left gage is 3.45 kPa lower than that of the right gage.



Discussion The negative pressure difference indicates that the pressure differential across the oil level is greater than the pressure differential corresponding to the differential height of the manometer fluid.

3-135

Solution An oil pipeline and a rigid air tank are connected to each other by a manometer. The pressure in the pipeline and the change in the level of manometer fluid due to a air temperature drop are to be determined.

Assumptions **1** All the liquids are incompressible. **2** The effect of air column on pressure is negligible. **3** The air volume in the manometer is negligible compared with the volume of the tank.

Properties The specific gravities are given to be 2.68 for oil and 13.6 for mercury. We take the standard density of water to be ρ_w = 1000 kg/m^3. The gas constant of air is 0.287 kPa·m^3/kg·K.

Analysis (*a*) Starting with the oil pipe and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the air tank, and setting the result equal to P_{air} give

$$P_{\text{oil}} + \rho_{\text{oil}} g h_{oil} + \rho_{\text{Hg}} g h_{\text{Hg}} = P_{\text{air}}$$

The absolute pressure in the air tank is determined from the ideal-gas relation $PV = mRT$ to be

$$P_{air} = \frac{mRT}{V} = \frac{(15\text{ kg})(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(80+273)\text{K}}{1.3\text{ m}^3} = 1169\text{ kPa}$$

Then the absolute pressure in the oil pipe becomes

$$P_{\text{oil}} = P_{\text{air}} - \rho_{\text{oil}} g h_{oil} - \rho_{\text{Hg}} g h_{\text{Hg}}$$

$$= 1169\text{ kPa} - \left(1000\text{ kg/m}^3\right)(9.81\text{ m/s}^2)\left[2.68(0.75\text{ m}) + 13.6\left(0.20\text{ m}\right)\right]\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)$$

$$= 1123\text{ kPa} \cong \mathbf{1120\ kPa}$$

(*b*) The pressure in the air tank when the temperature drops to 20°C becomes

$$P_{air} = \frac{mRT}{V} = \frac{(15\text{ kg})(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20+273)\text{K}}{1.3\text{ m}^3} = 970\text{ kPa}$$

When the mercury level in the left arm drops a distance x, the rise in the mercury level in the right arm y becomes

$$V_{\text{left}} = V_{\text{right}} \quad \rightarrow \quad \pi(3d)^2 x = \pi d^2 y \quad \rightarrow \quad y = 9x \quad \text{and} \quad y_{vert} = 9x \sin 50°$$

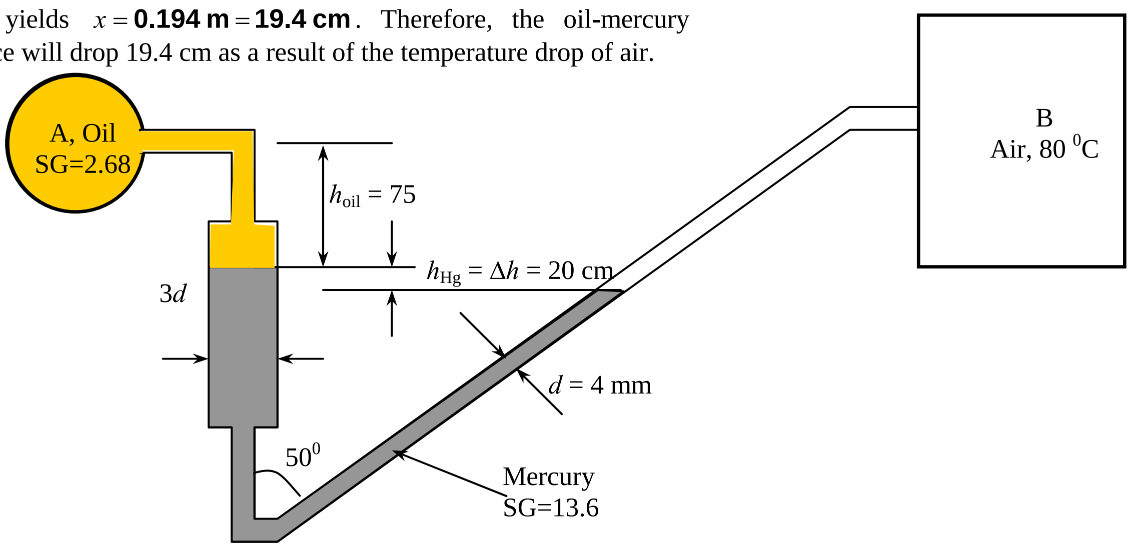
and the mercury fluid height will change by $x + 9x\sin 50°$ or 7.894x. Then,

$$P_{\text{oil}} + \rho_{\text{oil}} g(h_{oil} + x) + \rho_{\text{Hg}} g(h_{\text{Hg}} - 7.894x) = P_{\text{air}} \quad \rightarrow \quad \text{SG}_{\text{oil}}\left(h_{\text{oil}} + x\right) + \text{SG}_{\text{Hg}}\left(h_{\text{Hg}} - 7.894x\right) = \frac{P_{\text{air}} - P_{\text{oil}}}{\rho_w g}$$

Substituting,

$$2.68(0.75 + x) + 13.6(0.20 - 7.894x) = \frac{(970 - 1123)\text{ kPa}}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{1000\text{ kg}\cdot\text{m/s}^2}{1\text{ kN}}\right)\left(\frac{1\text{ kN/m}^2}{1\text{ kPa}}\right)$$

which yields $x = \mathbf{0.194\ m} = \mathbf{19.4\ cm}$. Therefore, the oil-mercury interface will drop 19.4 cm as a result of the temperature drop of air.



Discussion Note that the pressure in constant-volume gas chambers is very sensitive to temperature changes.

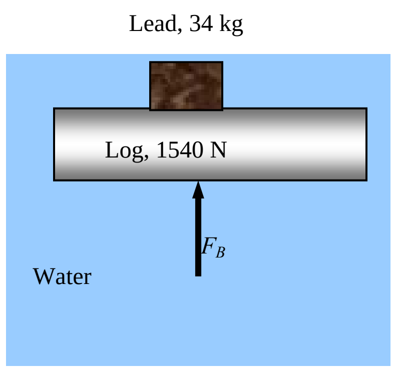
3-136

Solution The density of a wood log is to be measured by tying lead weights to it until both the log and the weights are completely submerged, and then weighing them separately in air. The average density of a given log is to be determined by this approach.

Properties The density of lead weights is given to be 11,300 kg/m^3. We take the density of water to be 1000 kg/m^3.

Analysis The weight of a body is equal to the buoyant force when the body is floating in a fluid while being completely submerged in it (a consequence of vertical force balance from static equilibrium). In this case the average density of the body must be equal to the density of the fluid since

$$W = F_B \quad \rightarrow \quad \rho_{\text{body}} gV = \rho_{\text{fluid}} gV \quad \rightarrow \quad \rho_{\text{body}} = \rho_{\text{fluid}}$$



Therefore,

$$\rho_{ave} = \frac{m_{\text{total}}}{V_{\text{total}}} = \frac{m_{\text{lead}} + m_{\text{log}}}{V_{\text{lead}} + V_{\text{log}}} = \rho_{\text{water}} \quad \rightarrow \quad V_{\text{log}} = V_{\text{lead}} + \frac{m_{\text{lead}} + m_{\text{log}}}{\rho_{\text{water}}}$$

where

$$V_{\text{lead}} = \frac{m_{\text{lead}}}{\rho_{\text{lead}}} = \frac{34 \text{ kg}}{11{,}300 \text{ kg/m}^3} = 3.01\times10^{-3} \text{ m}^3$$

$$m_{\text{log}} = \frac{W_{\text{log}}}{g} = \frac{1540 \text{ N}}{9.81 \text{ m/s}^2}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right) = 157.0 \text{ kg}$$

Substituting, the volume and density of the log are determined to be

$$V_{\text{log}} = V_{\text{lead}} + \frac{m_{\text{lead}} + m_{\text{log}}}{\rho_{\text{water}}} = 3.01\times10^{-3} \text{ m}^3 + \frac{(34+157) \text{ kg}}{1000 \text{ kg/m}^3} = \mathbf{0.194 \text{ m}^3}$$

$$\rho_{\text{log}} = \frac{m_{\text{log}}}{V_{\text{log}}} = \frac{157 \text{ kg}}{0.194 \text{ m}^3} = \mathbf{809 \text{ kg/m}^3}$$

Discussion Note that the log must be completely submerged for this analysis to be valid. Ideally, the lead weights must also be completely submerged, but this is not very critical because of the small volume of the lead weights.

3-137 **[***Also solved using EES on enclosed DVD***]**

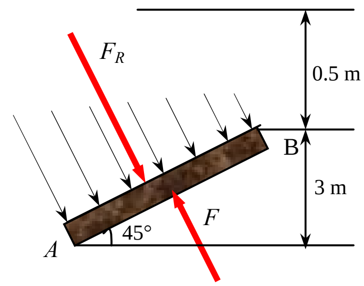
Solution A rectangular gate that leans against the floor with an angle of 45° with the horizontal is to be opened from its lower edge by applying a normal force at its center. The minimum force F required to open the water gate is to be determined.

Assumptions **1** Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **2** Friction at the hinge is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis The length of the gate and the distance of the upper edge of the gate (point B) from the free surface in the plane of the gate are

$$b = \frac{3\text{ m}}{\sin 45°} = 4.243\text{ m} \quad \text{and} \quad s = \frac{0.5\text{ m}}{\sin 45°} = 0.7071\text{ m}$$



The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic on the surface,

$$F_R = P_{avg}A = \rho g h_C A$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(2\text{ m})(5\times 4.243\text{ m}^2)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 416\text{ kN}$$

The distance of the pressure center from the free surface of water along the plane of the gate is

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s+b/2)} = 0.7071 + \frac{4.243}{2} + \frac{4.243^2}{12(0.7071+4.243/2)} = 3.359\text{ m}$$

The distance of the pressure center from the hinge at point B is

$$L_P = y_P - s = 3.359 - 0.7071 = 2.652\text{ m}$$

Taking the moment about point B and setting it equal to zero gives

$$\sum M_B = 0 \quad \rightarrow \quad F_R L_P = Fb/2$$

Solving for F and substituting, the required force is determined to be

$$F = \frac{2F_R L_P}{b} = \frac{2(416\text{ kN})(2.652\text{ m})}{4.243\text{ m}} = \mathbf{520\text{ kN}}$$

Discussion The applied force is inversely proportional to the distance of the point of application from the hinge, and the required force can be reduced by applying the force at a lower point on the gate.

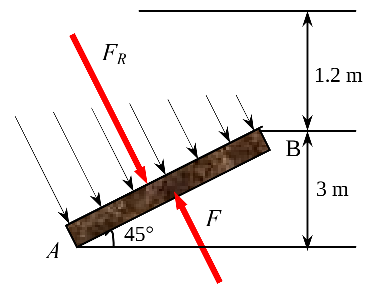
3-138

Solution A rectangular gate that leans against the floor with an angle of 45° with the horizontal is to be opened from its lower edge by applying a normal force at its center. The minimum force F required to open the water gate is to be determined.

Assumptions **1** Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **2** Friction at the hinge is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis The length of the gate and the distance of the upper edge of the gate (point B) from the free surface in the plane of the gate are



$$b = \frac{3\,\text{m}}{\sin 45°} = 4.243\,\text{m} \quad \text{and} \quad s = \frac{1.2\,\text{m}}{\sin 45°} = 1.697\,\text{m}$$

The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic on the surface,

$$F_R = P_{\text{avg}} A = \rho g h_C A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.7\ \text{m})(5 \times 4.243\ \text{m}^2)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 562\ \text{kN}$$

The distance of the pressure center from the free surface of water along the plane of the gate is

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} = 1.697 + \frac{4.243}{2} + \frac{4.243^2}{12(1.697 + 4.243/2)} = 4.211\ \text{m}$$

The distance of the pressure center from the hinge at point B is

$$L_P = y_P - s = 4.211 - 1.697 = 2.514\,\text{m}$$

Taking the moment about point B and setting it equal to zero gives

$$\sum M_B = 0 \quad \rightarrow \quad F_R L_P = Fb/2$$

Solving for F and substituting, the required force is determined to be

$$F = \frac{2F_R L_P}{b} = \frac{2(562\ \text{N})(2.514\ \text{m})}{4.243\,\text{m}} = \mathbf{666\ \ kN}$$

Discussion The applied force is inversely proportional to the distance of the point of application from the hinge, and the required force can be reduced by applying the force at a lower point on the gate.

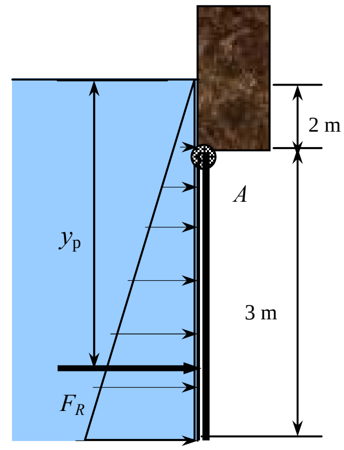
3-139

Solution A rectangular gate hinged about a horizontal axis along its upper edge is restrained by a fixed ridge at point *B*. The force exerted to the plate by the ridge is to be determined.

Assumptions Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic force on the gate,



$$F_R = P_{avg}A = \rho g h_C A$$
$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3.5 \text{ m})(3\times 6 \text{ m}^2)\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right)$$
$$= \mathbf{618 \text{ kN}}$$

The vertical distance of the pressure center from the free surface of water is

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s+b/2)} = 2 + \frac{3}{2} + \frac{3^2}{12(2+3/2)} = \mathbf{3.71 \text{ m}}$$

Discussion You can calculate the force at point *B* required to hold back the gate by setting the net moment around hinge point *A* to zero.

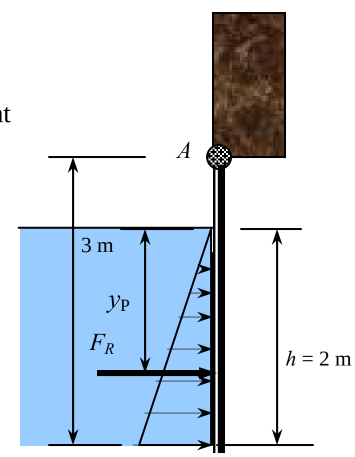
3-140

Solution A rectangular gate hinged about a horizontal axis along its upper edge is restrained by a fixed ridge at point *B*. The force exerted to the plate by the ridge is to be determined.

Assumptions Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the wetted plate area gives the resultant hydrostatic force on the gate,



$$F_R = P_{ave}A = \rho g h_C A$$
$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1 \text{ m})[2\times 6 \text{ m}^2]\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right)$$
$$= \mathbf{118 \text{ kN}}$$

The vertical distance of the pressure center from the free surface of water is

$$y_P = \frac{2h}{3} = \frac{2(2 \text{ m})}{3} = \mathbf{1.33 \text{ m}}$$

Discussion Compared to the previous problem (with higher water depth), the force is much smaller, as expected. Also, the center of pressure on the gate is much lower (closer to the ground) for the case with the lower water depth.

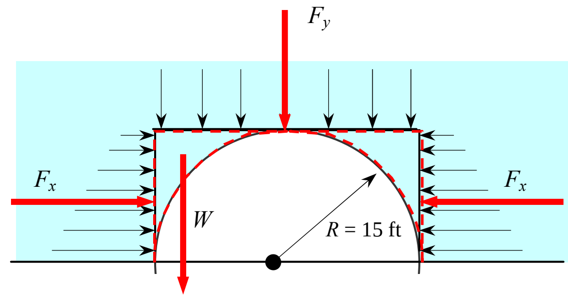
3-141E

Solution A semicircular tunnel is to be built under a lake. The total hydrostatic force acting on the roof of the tunnel is to be determined.

Assumptions Atmospheric pressure acts on both sides of the tunnel, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 62.4 lbm/ft^3 throughout.

Analysis We consider the free body diagram of the liquid block enclosed by the circular surface of the tunnel and its vertical (on both sides) and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:



Horizontal force on vertical surface (each side):

$$F_H = F_x = P_{avg}A = \rho g h_C A = \rho g\left(s + R/2\right)A$$
$$= \left(62.4 \text{ lbm/ft}^3\right)\left(32.2 \text{ ft/s}^2\right)\left(135 + 15/2 \text{ ft}\right)\left(15 \text{ ft } \times 800 \text{ ft}\right)\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right)$$
$$= 1.067\times10^8 \text{ lbf (on each side of the tunnel)}$$

Vertical force on horizontal surface (*downward*):

$$F_y = P_{avg}A = \rho g h_C A = \rho g h_{top} A$$
$$= \left(62.4 \text{ lbm/ft}^3\right)\left(32.2 \text{ ft/s}^2\right)\left(135 \text{ ft}\right)\left(30 \text{ ft } \times 800 \text{ ft}\right)\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right)$$
$$= 2.022\times10^8 \text{ lbf}$$

Weight of fluid block on each side within the control volume *(downward):*

$$W = mg = \rho g V = \rho g(R^2 - \pi R^2/4)(2000 \text{ ft})$$
$$= (62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(15 \text{ ft})^2(1\text{-}\pi/4)(800 \text{ ft})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm}\cdot\text{ft/s}^2}\right)$$
$$= 2.410\times10^6 \text{ lbf (on each side)}$$

Therefore, the net downward vertical force is

$$F_V = F_y + 2W = 2.022\times10^8 + 2\times0.02410\times10^6 = \mathbf{2.07\times10^8 \text{ lbf}}$$

This is also the **net force** acting on the tunnel since the horizontal forces acting on the right and left side of the tunnel cancel each other since they are equal and opposite.

Discussion The weight of the two water bocks on the sides represents only about 2.4% of the total vertical force on the tunnel. Therefore, to obtain a reasonable first approximation for deep tunnels, these volumes can be neglected, yielding F_V = 2.02 × 10^8 lbf. A more conservative approximation would be to estimate the force on the *bottom* of the lake if the tunnel were not there. This yields F_V = 2.25 × 10^8 lbf. The actual force is between these two estimates, as expected.

3-142

Solution A hemispherical dome on a level surface filled with water is to be lifted by attaching a long tube to the top and filling it with water. The required height of water in the tube to lift the dome is to be determined.

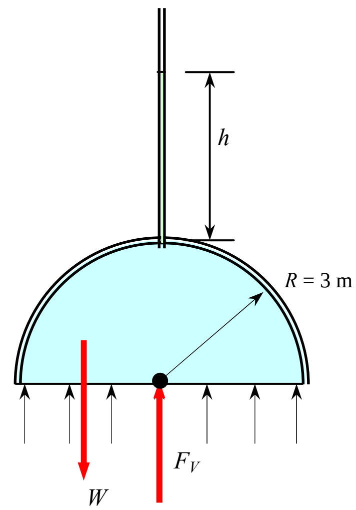
Assumptions **1** Atmospheric pressure acts on both sides of the dome, and thus it can be ignored in calculations for convenience. **2** The weight of the tube and the water in it is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis We take the dome and the water in it as the system. When the dome is about to rise, the reaction force between the dome and the ground becomes zero. Then the free body diagram of this system involves the weights of the dome and the water, balanced by the hydrostatic pressure force from below. Setting these forces equal to each other gives

$$\sum F_y = 0: \quad F_V = W_{dome} + W_{water}$$

$$\rho g(h+R)\pi R^2 = m_{dome} g + m_{water} g$$



Solving for h gives

$$h = \frac{m_{dome} + m_{water}}{\rho \pi R^2} - R = \frac{m_{dome} + \rho[4\pi R^3/6]}{\rho \pi R^2} - R$$

Substituting,

$$h = \frac{(50{,}000\text{ kg}) + 4\pi(1000\text{ kg/m}^3)(3\text{ m})^3/6}{(1000\text{ kg/m}^3)\pi(3\text{ m})^2} - (3\text{ m}) = \mathbf{0.77\ m}$$

Therefore, this dome can be lifted by attaching a tube which is 77 cm long.

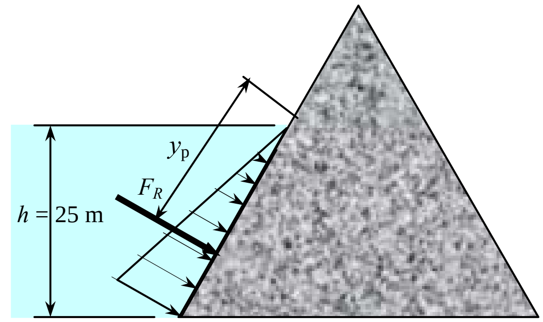
Discussion Note that the water pressure in the dome can be changed greatly by a small amount of water in the vertical tube.

3-143

Solution The water in a reservoir is restrained by a triangular wall. The total force (hydrostatic + atmospheric) acting on the inner surface of the wall and the horizontal component of this force are to be determined.

Assumptions **1** Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **2** Friction at the hinge is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.



Analysis The length of the wall surface underwater is

$$b = \frac{25\text{ m}}{\sin 60°} = 28.87\text{ m}$$

The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and multiplying it by the plate area gives the resultant hydrostatic force on the surface,

$$F_R = P_{avg}A = (P_{atm} + \rho g h_C)A$$

$$= \left[100{,}000\text{ N/m}^2 + (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(12.5\text{ m})\right](150\times 28.87\text{ m}^2)\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= \mathbf{9.64\times 10^8\ N}$$

Noting that

$$\frac{P_0}{\rho g \sin 60°} = \frac{100{,}000\text{ N/m}^2}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)\sin 60°}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = 11.77\text{ m}$$

the distance of the pressure center from the free surface of water along the wall surface is

$$y_p = s + \frac{b}{2} + \frac{b^2}{12\left(s + \frac{b}{2} + \frac{P_0}{\rho g \sin\theta}\right)} = 0 + \frac{28.87\text{ m}}{2} + \frac{(28.87\text{ m})^2}{12\left(0 + \frac{28.87\text{ m}}{2} + 11.77\text{ m}\right)} = \mathbf{17.1\ m}$$

The magnitude of the horizontal component of the hydrostatic force is simply $F_R \sin\theta$,

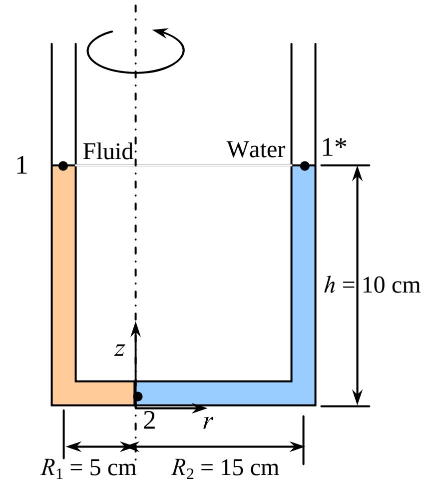
$$F_H = F_R\sin\theta = (9.64\times 10^8\text{ N})\sin 60° = \mathbf{8.35\times 10^8\ N}$$

Discussion Atmospheric pressure is usually ignored in the analysis for convenience since it acts on both sides of the walls.

3-144

Solution A U-tube that contains water in its right arm and another liquid in its left arm is rotated about an axis closer to the left arm. For a known rotation rate at which the liquid levels in both arms are the same, the density of the fluid in the left arm is to be determined.

Assumptions **1** Both the fluid and the water are incompressible fluids. **2** The two fluids meet at the axis of rotation, and thus there is only water to the right of the axis of rotation.



Properties We take the density of water to be 1000 kg/m³.

Analysis The pressure difference between two points 1 and 2 in an incompressible fluid rotating in rigid body motion (the *same* fluid) is given by

$$P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1)$$

where

$$\omega = 2\pi\dot{n} = 2\pi(30\ \text{rev/min})\left(\frac{1\,\text{min}}{60\,\text{s}}\right) = 3.14\ \text{rad/s}$$

(for both arms of the U-tube).

The pressure at point 2 is the same for both fluids, so are the pressures at points 1 and 1* ($P_1 = P_1{}^* = P_{atm}$). Therefore, $P_2 - P_1$ is the same for both fluids. Noting that $z_2 - z_1 = -h$ for both fluids and expressing $P_2 - P_1$ for each fluid,

$$\textit{Water}:\quad P_2 - P_1{}^* = \frac{\rho_w\omega^2}{2}(0 - R_2^2) - \rho_w g(-h) = \rho_w(-\omega^2 R_2^2/2 + gh)$$

$$\textit{Fluid}:\quad P_2 - P_1 = \frac{\rho_f\omega^2}{2}(0 - R_1^2) - \rho_f g(-h) = \rho_f(-\omega^2 R_1^2/2 + gh)$$

Setting them equal to each other and solving for ρ_f gives

$$\rho_f = \frac{-\omega^2 R_2^2/2 + gh}{-\omega^2 R_1^2/2 + gh}\rho_w = \frac{-(3.14\ \text{rad/s})^2(0.15\ \text{m})^2 + (9.81\ \text{m/s}^2)(0.10\ \text{m})}{-(3.14\ \text{rad/s})^2(0.05\ \text{m})^2 + (9.81\ \text{m/s}^2)(0.10\ \text{m})}(1000\ \text{kg/m}^3) = \mathbf{794\ kg/m^3}$$

Discussion Note that this device can be used to determine relative densities, though it wouldn't be very practical.

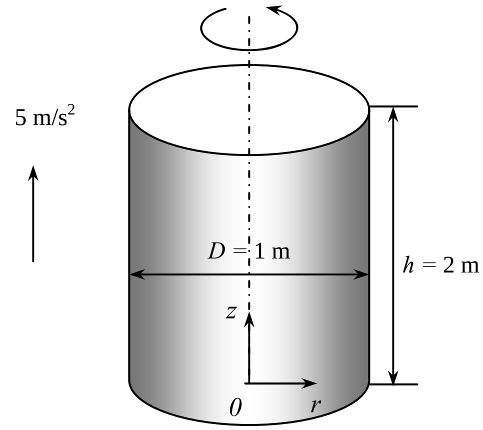
3-145

Solution A vertical cylindrical tank is completely filled with gasoline, and the tank is rotated about its vertical axis at a specified rate while being accelerated upward. The pressures difference between the centers of the bottom and top surfaces, and the pressures difference between the center and the edge of the bottom surface are to be determined.

Assumptions **1** The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. **2** Gasoline is an incompressible substance.

Properties The density of the gasoline is given to be 740 kg/m^3.

Analysis The pressure difference between two points 1 and 2 in an incompressible fluid rotating in rigid body motion is given by $P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1)$. The effect of linear acceleration in the vertical direction is accounted for by replacing g by $g + a_z$. Then,



$$P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho(g + a_z)(z_2 - z_1)$$

where R = 0.50 m is the radius, and

$$\omega = 2\pi\dot{n} = 2\pi(90 \text{ rev/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 9.425 \text{ rad/s}$$

(*a*) Taking points 1 and 2 to be the centers of the bottom and top surfaces, respectively, we have $r_1 = r_2 = 0$ and $z_2 - z_1 = h = 3 \text{ m}$. Then,

$$P_{\text{center, top}} - P_{\text{center, bottom}} = 0 - \rho(g + a_z)(z_2 - z_1) = -\rho(g + a_z)h$$

$$= -(740 \text{ kg/m}^3)(9.81 \text{ m/s}^2 + 5)(2 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) = 21.8 \text{ kN/m}^2 = \mathbf{21.9 \ kPa}$$

(*b*) Taking points 1 and 2 to be the center and edge of the bottom surface, respectively, we have $r_1 = 0$, $r_2 = R$, and $z_2 = z_1 = 0$. Then,

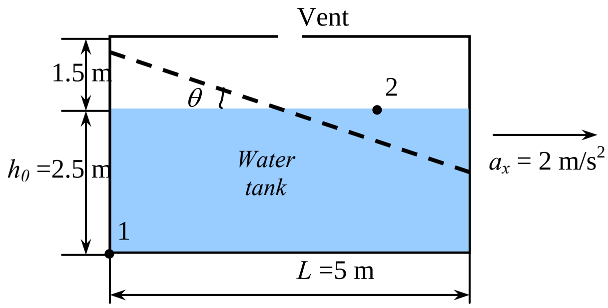
$$P_{\text{edge, bottom}} - P_{\text{center, bottom}} = \frac{\rho\omega^2}{2}(R_2^2 - 0) - 0 = \frac{\rho\omega^2 R^2}{2}$$

$$= \frac{(740 \text{ kg/m}^3)(9.425 \text{ rad/s})^2(0.50 \text{ m})^2}{2}\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) = 8.22 \text{kN/m}^2 = \mathbf{8.22 \ kPa}$$

Discussion Note that the rotation of the tank does not affect the pressure difference along the axis of the tank. Likewise, the vertical acceleration does not affect the pressure difference between the edge and the center of the bottom surface (or any other horizontal plane).

3-146

Solution A rectangular water tank open to the atmosphere is accelerated to the right on a level surface at a specified rate. The maximum pressure in the tank above the atmospheric level is to be determined.



Assumptions **1** The road is horizontal during acceleration so that acceleration has no vertical component (a_z = 0). **2** Effects of splashing, breaking and driving over bumps are assumed to be secondary, and are not considered. **3** The vent is never blocked, and thus the minimum pressure is the atmospheric pressure.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take the *x*-axis to be the direction of motion, the *z*-axis to be the upward vertical direction. The tangent of the angle the free surface makes with the horizontal is

$$\tan\theta = \frac{a_x}{g + a_z} = \frac{2}{9.81 + 0} = 0.2039 \quad \text{(and thus } \theta = 11.5°\text{)}$$

The maximum vertical rise of the free surface occurs at the back of the tank, and the vertical midsection experiences no rise or drop during acceleration. Then the maximum vertical rise at the back of the tank relative to the neutral midplane is

$$\Delta z_{max} = (L/2)\tan\theta = [(5\text{ m})/2] \times 0.2039 = 0.510\text{ m}$$

which is less than 1.5 m high air space. Therefore, water never reaches the ceiling, and the maximum water height and the corresponding maximum pressure are

$$h_{max} = h_0 + \Delta z_{max} = 2.50 + 0.510 = 3.01\text{ m}$$

$$P_{max} = P_1 = \rho g h_{max} = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(3.01\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 29.5\text{ kN/m}^2 = \mathbf{29.5\ kPa}$$

Discussion It can be shown that the gage pressure at the bottom of the tank varies from 29.5 kPa at the back of the tank to 24.5 kPa at the midsection and 19.5 kPa at the front of the tank.

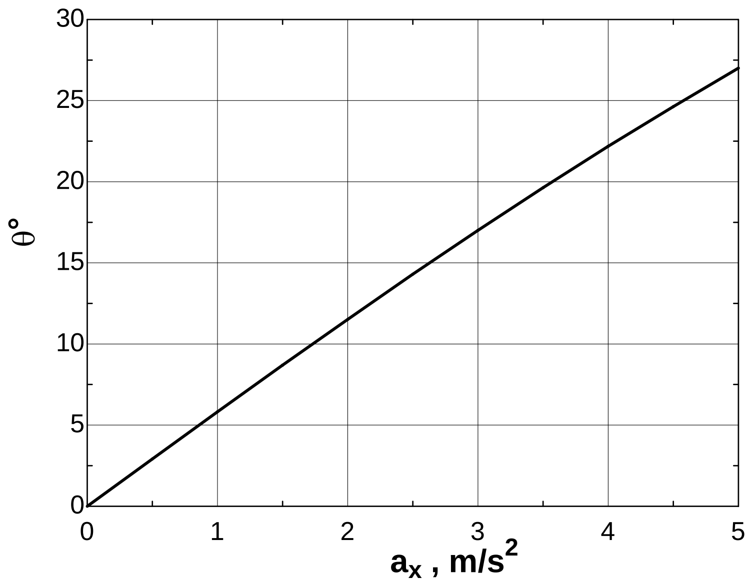
3-147

Solution The previous problem is reconsidered. The effect of acceleration on the slope of the free surface of water in the tank as the acceleration varies from 0 to 5 m/s^2 in increments of 0.5 m/s^2 is to be investigated.

Analysis The EES *Equations* window is printed below, followed by the tabulated and plotted results.

```
g=9.81 "m/s2"
rho=1000 "kg/m3"
L=5 "m"
h0=2.5 "m"

a_z=0
tan(theta)=a_x/(g+a_z)
h_max=h0+(L/2)*tan(theta)
P_max=rho*g*h_max/1000 "kPa"
```

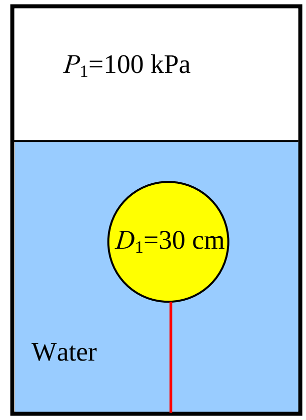
Acceleration a_x, m/s^2	Free surface angle, θ°	Maximum height h_{max}, m	Maximum pressure P_{max}, kPa
0.0	0.0	2.50	24.5
0.5	2.9	2.63	25.8
1.0	5.8	2.75	27.0
1.5	8.7	2.88	28.3
2.0	11.5	3.01	29.5
2.5	14.3	3.14	30.8
3.0	17.0	3.26	32.0
3.5	19.6	3.39	33.3
4.0	22.2	3.52	34.5
4.5	24.6	3.65	35.8
5.0	27.0	3.77	37.0



Discussion Note that water never reaches the ceiling, and a full free surface is formed in the tank.

3-148

Solution An elastic air balloon submerged in water is attached to the base of the tank. The change in the tension force of the cable is to be determined when the tank pressure is increased and the balloon diameter is decreased in accordance with the relation $P = CD^{-2}$.



Assumptions **1** Atmospheric pressure acts on all surfaces, and thus it can be ignored in calculations for convenience. **2** Water is an incompressible fluid. **3** The weight of the balloon and the air in it is negligible.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The tension force on the cable holding the balloon is determined from a force balance on the balloon to be

$$F_{cable} = F_B - W_{balloon} \cong F_B$$

The buoyancy force acting on the balloon initially is

$$F_{B,1} = \rho_w g V_{balloon,1} = \rho_w g \frac{\pi D_1^3}{6} = (1000\ \text{kg/m}^3)(9.81\,\text{m/s}^2)\frac{\pi(0.30\,\text{m})^3}{6}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 138.7\ \text{N}$$

The variation of pressure with diameter is given as $P = CD^{-2}$, which is equivalent to $D = \sqrt{C/P}$. Then the final diameter of the ball becomes

$$\frac{D_2}{D_1} = \frac{\sqrt{C/P_2}}{\sqrt{C/P_1}} = \sqrt{\frac{P_1}{P_2}} \quad \rightarrow \quad D_2 = D_1\sqrt{\frac{P_1}{P_2}} = (0.30\,\text{m})\sqrt{\frac{0.1\,\text{MPa}}{1.6\,\text{MPa}}} = 0.075\,\text{m}$$

The buoyancy force acting on the balloon in this case is

$$F_{B,2} = \rho_w g V_{balloon,2} = \rho_w g \frac{\pi D_2^3}{6} = (1000\ \text{kg/m}^3)(9.81\,\text{m/s}^2)\frac{\pi(0.075\,\text{m})^3}{6}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 2.2\ \text{N}$$

Then the percent change in the cable for becomes

$$Change\% = \frac{F_{cable,1} - F_{cable,2}}{F_{cable,1}} * 100 = \frac{F_{B,1} - F_{B,2}}{F_{B,1}} * 100 = \frac{138.7 - 2.2}{138.7} * 100 = \mathbf{98.4\%}.$$

Therefore, increasing the tank pressure in this case results in 98.4% reduction in cable tension.

Discussion We can obtain a relation for the change in cable tension as follows:

$$Change\% = \frac{F_{B,1} - F_{B,2}}{F_{B,1}} * 100 = \frac{\rho_w g V_{\text{balloon},1} - \rho_w g V_{\text{balloon},2}}{\rho_w g V_{\text{balloon},1}} * 100$$

$$= 100\left(1 - \frac{V_{\text{balloon},2}}{V_{\text{balloon},1}}\right) = 100\left(1 - \frac{D_2^3}{D_1^3}\right) = 100\left(1 - \left(\frac{P_1}{P_2}\right)^{3/2}\right)$$

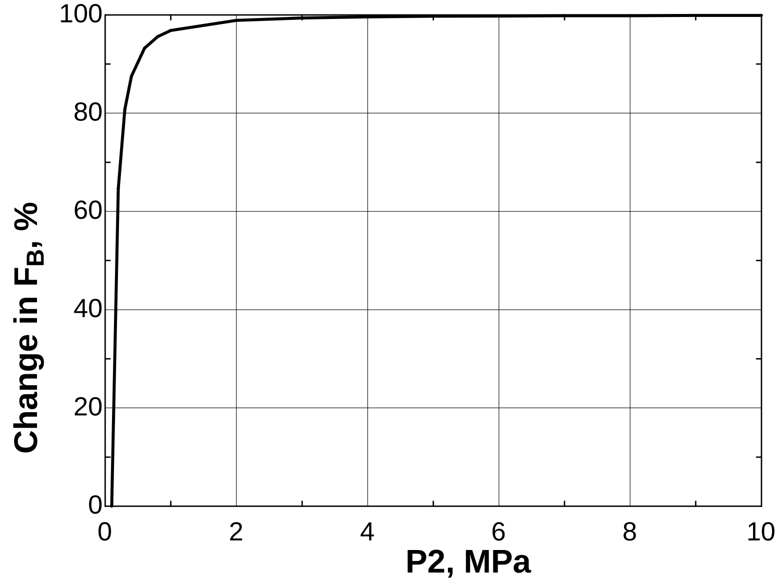
3-149

Solution The previous problem is reconsidered. The effect of the air pressure above the water on the cable force as the pressure varies from 0.1 MPa to 10 MPa is to be investigated.

Analysis The EES *Equations* window is printed below, followed by the tabulated and plotted results.

```
P1=0.1 "MPa"
Change=100*(1-(P1/P2)^1.5)
```

Tank pressure P_2, MPa	%Change in cable tension
0.1	0.0
0.2	64.6
0.3	80.8
0.4	87.5
0.6	93.2
0.8	95.6
1	96.8
2	98.9
3	99.4
4	99.6
5	99.7
6	99.8
7	99.8
8	99.9
9	99.9
10	99.9



Discussion The change in cable tension is at first very rapid, but levels off as the balloon shrinks to nearly zero diameter at high pressure.

3-150

Solution An iceberg floating in seawater is considered. The volume fraction of the iceberg submerged in seawater is to be determined, and the reason for their turnover is to be explained.

Assumptions **1** The buoyancy force in air is negligible. **2** The density of iceberg and seawater are uniform.

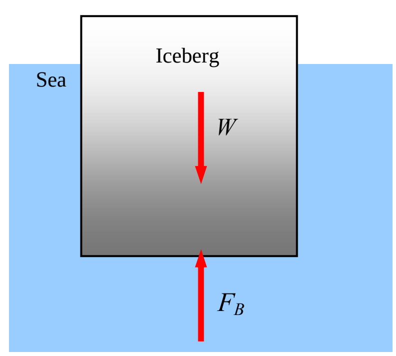
Properties The densities of iceberg and seawater are given to be 917 kg/m^3 and 1042 kg/m^3, respectively.

Analysis (*a*) The weight of a body floating in a fluid is equal to the buoyant force acting on it (a consequence of vertical force balance from static equilibrium). Therefore,

$$W = F_B$$

$$\rho_{\text{body}}\, g V_{\text{total}} = \rho_{\text{fluid}}\, g V_{\text{submerged}}$$

$$\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{fluid}}} = \frac{\rho_{\text{iceberg}}}{\rho_{\text{seawater}}} = \frac{917}{1042} = 0.880 \quad \text{or} \quad \mathbf{88\%}$$



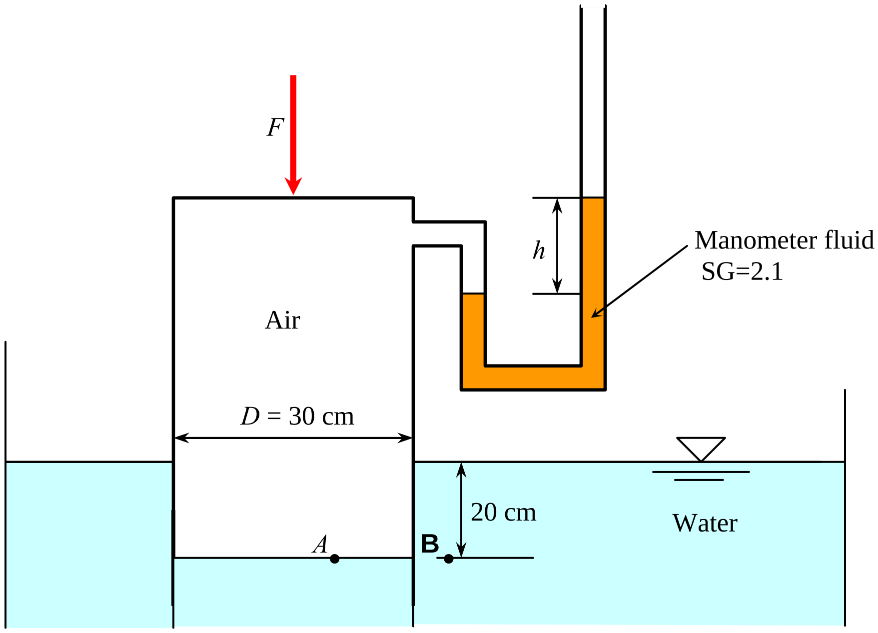
Therefore, 88% of the volume of the iceberg is submerged in this case.

(*b*) Heat transfer to the iceberg due to the temperature difference between the seawater and an iceberg causes uneven melting of the irregularly shaped iceberg. The resulting **shift in the center of mass causes the iceberg to turn over**.

Discussion The submerged fraction depends on the density of seawater, and this fraction can differ in different seas.

3-151

Solution A cylindrical container equipped with a manometer is inverted and pressed into water. The differential height of the manometer and the force needed to hold the container in place are to be determined.



Assumptions **1** Atmospheric pressure acts on all surfaces, and thus it can be ignored in calculations for convenience. **2** The variation of air pressure inside cylinder is negligible.

Properties We take the density of water to be 1000 kg/m^3. The density of the manometer fluid is

$$\rho_{\text{mano}} = \text{SG} \times \rho_w = 2.1\left(1000 \text{ kg/m}^3\right) = 2100 \text{ kg/m}^3$$

Analysis The pressures at point A and B must be the same since they are on the same horizontal line in the same fluid. Then the gage pressure in the cylinder becomes

$$P_{\text{air, gage}} = \rho_w g h_w = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.20 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right) = 1962 \text{ N/m}^2 = 1962 \text{ Pa}$$

The manometer also indicates the gage pressure in the cylinder. Therefore,

$$P_{\text{air, gage}} = (\rho g h)_{\text{mano}} \rightarrow h = \frac{P_{\text{air, gage}}}{\rho_{\text{mano}} g} = \frac{1962 \text{ N/m}^2}{(2100 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ kN/m}^2}\right) = 0.0950 \text{ m} = \mathbf{9.50 \text{ cm}}$$

A force balance on the cylinder in the vertical direction yields

$$F + W = P_{\text{air, gage}} A_c$$

Solving for F and substituting,

$$F = P_{\text{air, gage}} \frac{\pi D^2}{4} - W = \left(1962 \text{ N/m}^2\right)\frac{\pi (0.30 \text{ m})^2}{4} - 79 \text{ N} = \mathbf{59.7 \text{ N}}$$

Discussion We could also solve this problem by considering the atmospheric pressure, but we would obtain the same result since atmospheric pressure would cancel out.

Design and Essay Problems

3-152

Solution We are to discuss the design of shoes that enable people to walk on water.

Discussion Students' discussions should be unique and will differ from each other.

3-153

Solution We are to discuss how to measure the volume of a rock without using any volume measurement devices.

Analysis The volume of a rock can be determined without using any volume measurement devices as follows: We weigh the rock in the air and then in the water. The difference between the two weights is due to the buoyancy force, which is equal to $F_B = \rho_{\text{water}} g V_{\text{body}}$. Solving this relation for V_{body} gives the volume of the rock.

Discussion Since this is an open-ended design problem, students may come up with different, but equally valid techniques.