Solutions Manual for

Fluid Mechanics: Fundamentals and Applications

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CHAPTER 2
PROPERTIES OF FLUIDS

Density and Specific Gravity

2-1C

Solution We are to discuss the difference between intensive and extensive properties.

Analysis *Intensive properties* **do *not* depend on the size (extent) of the system** but *extensive properties* ***do*** **depend on the size (extent) of the system**.

Discussion An example of an intensive property is temperature. An example of an extensive property is mass.

2-2C

Solution We are to define specific gravity and discuss its relationship to density.

Analysis The *specific gravity***,** or *relative density***,** is defined as **the ratio of the density of a substance to the density of some standard substance at a specified temperature** (the standard is water at 4°C, for which ρ_{H2O} = 1000 kg/m^3). That is, $\boxed{SG = \rho / \rho_{H2O}}$. When specific gravity is known, density is determined from $\boxed{\rho = SG \times \rho_{H2O}}$.

Discussion Specific gravity is dimensionless and unitless [it is just a number without dimensions or units].

2-3C

Solution We are to discuss the applicability of the ideal gas law.

Analysis A gas can be treated as an *ideal gas* when it is **at a *high temperature* and/or a *low pressure* relative to its critical temperature and pressure**.

Discussion Air and many other gases at room temperature and pressure can be approximated as ideal gases without any significant loss of accuracy.

2-4C

Solution We are to discuss the difference between R and R_u.

Analysis R_u is the ***universal gas constant*** that is the **same for all gases**, whereas R is the ***specific gas constant*** that is **different for different gases**. These two are related to each other by $\boxed{R = R_u / M}$, where M is the *molar mass* (also called the *molecular weight*) of the gas.

Discussion Since molar mass has dimensions of mass per mole, R and R_u do not have the same dimensions or units.

2-5

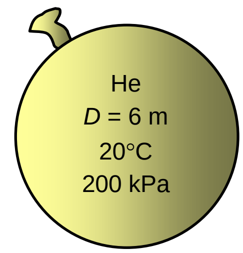
Solution A balloon is filled with helium gas. The number of moles and the mass of helium are to be determined.

Assumptions At specified conditions, helium behaves as an ideal gas.

Properties The universal gas constant is R_u = 8.314 kPa.m^3/kmol.K. The molar mass of helium is 4.0 kg/kmol.

Analysis The volume of the sphere is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3\text{ m})^3 = 113.1\text{ m}^3$$



Assuming ideal gas behavior, the number of moles of He is determined from

$$N = \frac{PV}{R_u T} = \frac{(200\text{ kPa})(113.1\text{ m}^3)}{(8.314\text{ kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(293\text{ K})} = \mathbf{9.286\text{ kmol}}$$

Then the mass of He is determined from

$$m = NM = (9.286\text{ kmol})(4.0\text{ kg/kmol}) = \mathbf{37.1\text{ kg}}$$

Discussion Although the helium mass may seem large (about half the mass of an adult man!), it is much smaller than that of the air it displaces, and that is why helium balloons rise in the air.

2-6

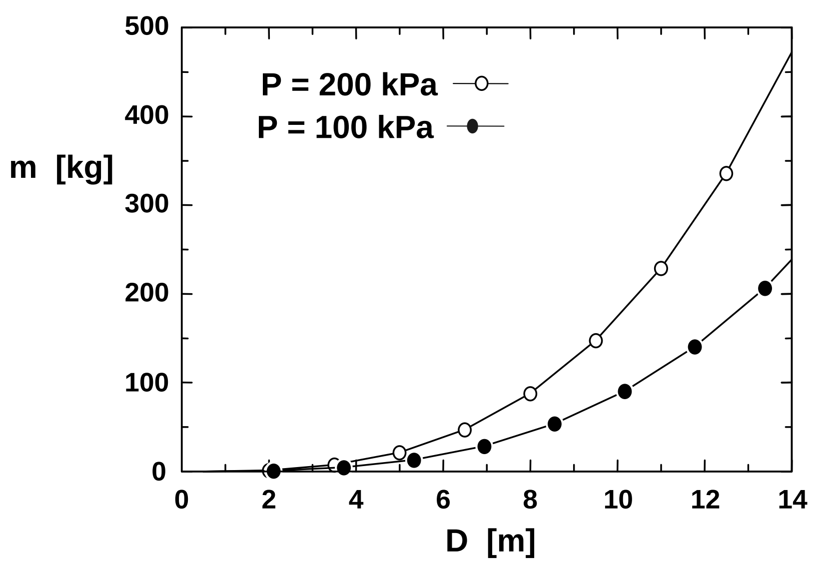
Solution A balloon is filled with helium gas. The effect of the balloon diameter on the mass of helium is to be investigated, and the results are to be tabulated and plotted.

Analysis The EES *Equations* window is shown below, followed by the *Solution* window and the parametric table.

```
"Given Data"
{D=6"[m]"}
{P=200"[kPa]"}
T=20"[C]"
P=100"[kPa]"
R_u=8.314"[kJ/kmol*K]"

"Solution"
P*V=N*R_u*(T+273)
V=4*pi*(D/2)^3/3"[m^3]"
m=N*MOLARMASS(Helium)"[kg]"
```

D [m]	m [kg]
0.5	0.01075
2.111	0.8095
3.722	4.437
5.333	13.05
6.944	28.81
8.556	53.88
10.17	90.41
11.78	140.6
13.39	206.5
15	290.4



Mass of Helium in Balloon as function of Diameter

Discussion Mass increases with diameter as expected, but not linearly since volume is proportional to D^3.

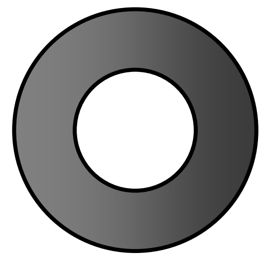
2-7

Solution An automobile tire is inflated with air. The pressure rise of air in the tire when the tire is heated and the amount of air that must be bled off to reduce the temperature to the original value are to be determined.

Assumptions **1** At specified conditions, air behaves as an ideal gas. **2** The volume of the tire remains constant.

Properties The gas constant of air is R = 0.287 kPa·m^3/kg·K.

Analysis Initially, the absolute pressure in the tire is



Tire
25°C
210 kPa

$$P_1 = P_g + P_{atm} = 210 + 100 = 310 \text{ kPa}$$

Treating air as an ideal gas and assuming the volume of the tire to remain constant, the final pressure in the tire is determined from

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \longrightarrow P_2 = \frac{T_2}{T_1} P_1 = \frac{323\text{K}}{298\text{K}}(310\text{kPa}) = 336\text{kPa}$$

Thus the pressure rise is

$$\Delta P = P_2 - P_1 = 336 - 310 = \mathbf{26.0\ kPa}$$

The amount of air that needs to be bled off to restore pressure to its original value is

$$m_1 = \frac{P_1 V}{RT_1} = \frac{(310\text{kPa})(0.025\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298\text{K})} = 0.0906\text{kg}$$

$$m_2 = \frac{P_2 V}{RT_2} = \frac{(310\text{kPa})(0.025\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(323\text{K})} = 0.0836\text{kg}$$

$$\Delta m = m_1 - m_2 = 0.0906 - 0.0836 = \mathbf{0.0070\ kg}$$

Discussion Notice that *absolute* rather than gage pressure must be used in calculations with the ideal gas law.

2-8E

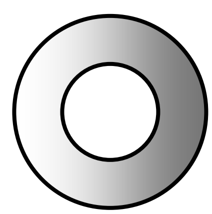
Solution An automobile tire is under-inflated with air. The amount of air that needs to be added to the tire to raise its pressure to the recommended value is to be determined.

Assumptions **1** At specified conditions, air behaves as an ideal gas. **2** The volume of the tire remains constant.

Properties The gas constant of air is R = 0.3704 psia·ft^3/lbm·R.

Analysis The initial and final absolute pressures in the tire are

Tire
0.53 ft^3
90°F
20 psia



$$P_1 = P_{g1} + P_{atm} = 20 + 14.6 = 34.6 \text{ psia}$$

$$P_2 = P_{g2} + P_{atm} = 30 + 14.6 = 44.6 \text{ psia}$$

Treating air as an ideal gas, the initial mass in the tire is

$$m_1 = \frac{P_1 V}{RT_1} = \frac{(34.6 \text{ psia})(0.53 \text{ ft}^3)}{(0.3704 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(550 \text{ R})} = 0.0900 \text{ lbm}$$

Noting that the temperature and the volume of the tire remain constant, the final mass in the tire becomes

$$m_2 = \frac{P_2 V}{RT_2} = \frac{(44.6 \text{ psia})(0.53 \text{ ft}^3)}{(0.3704 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(550 \text{ R})} = 0.1160 \text{ lbm}$$

Thus the amount of air that needs to be added is $\Delta m = m_2 - m_1 = 0.1160 - 0.0900 = \mathbf{0.0260\ lbm}$

Discussion Notice that *absolute* rather than gage pressure must be used in calculations with the ideal gas law.

2-9E

Solution A rigid tank contains slightly pressurized air. The amount of air that needs to be added to the tank to raise its pressure and temperature to the recommended values is to be determined.

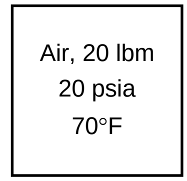
Assumptions **1** At specified conditions, air behaves as an ideal gas. **2** The volume of the tank remains constant.

Properties The gas constant of air is R = 0.3704 psia·ft^3/lbm·R.

Analysis Treating air as an ideal gas, the initial volume and the final mass in the tank are determined to be

$$V = \frac{m_1 R T_1}{P_1} = \frac{(20\text{ lbm})(0.3704\text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(530\text{ R})}{20\text{ psia}} = 196.3\text{ ft}^3$$

$$m_2 = \frac{P_2 V}{R T_2} = \frac{(35\text{ psia})(196.3\text{ ft}^3)}{(0.3704\text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(550\text{ R})} = 33.73\text{ lbm}$$



Thus the amount of air added is

$$\Delta m = m_2 - m_1 = 33.73 - 20.0 = \mathbf{13.7\ lbm}$$

Discussion As the temperature slowly decreases due to heat transfer, the pressure will also decrease.

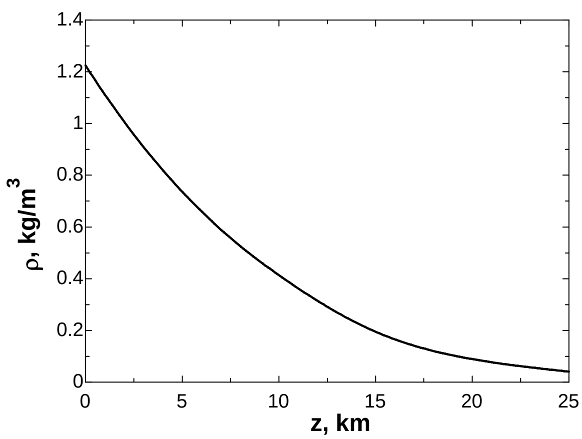
2-10

Solution A relation for the variation of density with elevation is to be obtained, the density at 7 km elevation is to be calculated, and the mass of the atmosphere using the correlation is to be estimated.

Assumptions **1** Atmospheric air behaves as an ideal gas. **2** The earth is perfectly spherical with a radius of 6377 km at sea level, and the thickness of the atmosphere is 25 km.

Properties The density data are given in tabular form as a function of radius and elevation, where $r = z + 6377$ km:

r, km	z, km	ρ, kg/m^3
6377	0	1.225
6378	1	1.112
6379	2	1.007
6380	3	0.9093
6381	4	0.8194
6382	5	0.7364
6383	6	0.6601
6385	8	0.5258
6387	10	0.4135
6392	15	0.1948
6397	20	0.08891
6402	25	0.04008



Analysis Using EES, (1) Define a trivial function "rho= a+z" in the Equation window, (2) select new parametric table from Tables, and type the data in a two-column table, (3) select Plot and plot the data, and (4) select Plot and click on curve fit to get curve fit window. Then specify 2nd order polynomial and enter/edit equation. The results are:

$\rho(z) = a + bz + cz^2 = 1.20252 - 0.101674z + 0.0022375z^2$ for the unit of kg/m^3,
(or, $\rho(z) = (1.20252 - 0.101674z + 0.0022375z^2)\times 10^9$ for the unit of kg/km^3)

where z is the vertical distance from the earth surface at sea level. At z = 7 km, the equation gives $\boldsymbol{\rho}$ **= 0.600 kg/m^3**.

(b) The mass of atmosphere is evaluated by integration to be

$$m = \int_V \rho dV = \int_{z=0}^{h} (a+bz+cz^2)4\pi(r_0+z)^2 dz = 4\pi\int_{z=0}^{h} (a+bz+cz^2)(r_0^2+2r_0z+z^2)dz$$
$$= 4\pi\left[ar_0^2h + r_0(2a+br_0)h^2/2 + (a+2br_0+cr_0^2)h^3/3 + (b+2cr_0)h^4/4 + ch^5/5\right]$$

where r_0 = 6377 km is the radius of the earth, h = 25 km is the thickness of the atmosphere. Also, a = 1.20252, b = -0.101674, and c = 0.0022375 are the constants in the density function. Substituting and multiplying by the factor 10^9 to convert the density from units of kg/km^3 to kg/m^3, the mass of the atmosphere is determined to be approximately

m = **5.09×10^{18} kg**

EES Solution for final result:

```
a=1.2025166
b=-0.10167
c=0.0022375
r=6377
h=25
m=4*pi*(a*r^2*h+r*(2*a+b*r)*h^2/2+(a+2*b*r+c*r^2)*h^3/3+(b+2*c*r)*h^4/4+c*h^5/5)*1E+9
```

Discussion At 7 km, the density of the air is approximately half of its value at sea level.

Vapor Pressure and Cavitation

2-11C

Solution We are to define vapor pressure and discuss its relationship to saturation pressure.

Analysis The *vapor pressure* P_v of a pure substance is defined as **the pressure exerted by a vapor in phase equilibrium with its liquid at a given temperature**. In general, the pressure of a vapor or gas, whether it exists alone or in a mixture with other gases, is called the *partial pressure*. During phase change processes between the liquid and vapor phases of a pure substance, the **saturation pressure and the vapor pressure are equivalent** since the vapor is pure.

Discussion Partial pressure is not necessarily equal to vapor pressure. For example, on a dry day (low relative humidity), the partial pressure of water vapor in the air is less than the vapor pressure of water. If, however, the relative humidity is 100%, the partial pressure and the vapor pressure are equal.

2-12C

Solution We are to discuss whether the boiling temperature of water increases as pressure increases.

Analysis **Yes**. The saturation temperature of a pure substance depends on pressure; in fact, it *increases* with pressure. The higher the pressure, the higher the saturation or boiling temperature.

Discussion This fact is easily seen by looking at the saturated water property tables. Note that boiling temperature and saturation pressure at a given pressure are equivalent.

2-13C

Solution We are to determine if temperature increases or remains constant when the pressure of a boiling substance increases.

Analysis If the pressure of a substance increases during a boiling process, **the temperature also increases** since the boiling (or saturation) temperature of a pure substance depends on pressure and increases with it.

Discussion We are assuming that the liquid will continue to boil. If the pressure is increased fast enough, boiling may stop until the temperature has time to reach its new (higher) boiling temperature. A pressure cooker uses this principle.

2-14C

Solution We are to define and discuss cavitation.

Analysis In the flow of a liquid, *cavitation* is the **vaporization that may occur at locations where the pressure drops below the vapor pressure**. The vapor bubbles collapse as they are swept away from the low pressure regions, generating highly destructive, extremely high-pressure waves. This phenomenon is a common cause for drop in performance and even the erosion of impeller blades.

Discussion The word "cavitation" comes from the fact that a vapor bubble or "cavity" appears in the liquid. Not all cavitation is undesirable. It turns out that some underwater vehicles employ "super cavitation" on purpose to reduce drag.

2-15

Solution The minimum pressure in a piping system to avoid cavitation is to be determined.

Properties The vapor pressure of water at 40°C is 7.38 kPa.

Analysis To avoid cavitation, the pressure anywhere in the flow should not be allowed to drop below the vapor (or saturation) pressure at the given temperature. That is,

$$P_{\min} = P_{\text{sat @ 40°C}} = \mathbf{7.38\ kPa}$$

Therefore, the pressure should be maintained above 7.38 kPa everywhere in flow.

Discussion Note that the vapor pressure increases with increasing temperature, and thus the risk of cavitation is greater at higher fluid temperatures.

2-16

Solution The minimum pressure in a pump is given. It is to be determined if there is a danger of cavitation.

Properties The vapor pressure of water at 20°C is 2.339 kPa.

Analysis To avoid cavitation, the pressure everywhere in the flow should remain above the vapor (or saturation) pressure at the given temperature, which is

$$P_v = P_{\text{sat @ 20°C}} = 2.339 \text{ kPa}$$

The minimum pressure in the pump is 2 kPa, which is less than the vapor pressure. Therefore, a **there is danger of cavitation in the pump**.

Discussion Note that the vapor pressure increases with increasing temperature, and thus there is a greater danger of cavitation at higher fluid temperatures.

2-17E

Solution The minimum pressure in a pump is given. It is to be determined if there is a danger of cavitation.

Properties The vapor pressure of water at 70°F is 0.3632 psia.

Analysis To avoid cavitation, the pressure everywhere in the flow should remain above the vapor (or saturation) pressure at the given temperature, which is

$$P_v = P_{\text{sat @ 70°F}} = 0.3632 \text{ psia}$$

The minimum pressure in the pump is 0.1 psia, which is less than the vapor pressure. Therefore, **there is danger of cavitation in the pump**.

Discussion Note that the vapor pressure increases with increasing temperature, and the danger of cavitation increases at higher fluid temperatures.

2-18

Solution The minimum pressure in a pump to avoid cavitation is to be determined.

Properties The vapor pressure of water at 25°C is 3.17 kPa.

Analysis To avoid cavitation, the pressure anywhere in the system should not be allowed to drop below the vapor (or saturation) pressure at the given temperature. That is,

$$P_{\min} = P_{\text{sat @ 25°C}} = \mathbf{3.17\ kPa}$$

Therefore, the lowest pressure that can exist in the pump is 3.17 kPa.

Discussion Note that the vapor pressure increases with increasing temperature, and thus the risk of cavitation is greater at higher fluid temperatures.

Energy and Specific Heats

2-19C

Solution We are to discuss the difference between macroscopic and microscopic forms of energy.

Analysis The *macroscopic* forms of energy are those **a system possesses as a whole** with respect to some outside reference frame. The *microscopic* forms of energy, on the other hand, are those **related to the molecular structure of a system and the degree of the molecular activity**, and are independent of outside reference frames.

Discussion We mostly deal with macroscopic forms of energy in fluid mechanics.

2-20C

Solution We are to define total energy and identify its constituents.

Analysis The **sum of all forms of the energy a system possesses** is called *total energy*. In the absence of magnetic, electrical, and surface tension effects, **the total energy of a system consists of the kinetic, potential, and internal energies**.

Discussion All three constituents of total energy (kinetic, potential, and internal) need to be considered in an analysis of a general fluid flow.

2-21C

Solution We are to list the forms of energy that contribute to the internal energy of a system.

Analysis The *internal energy* of a system is made up of **sensible, latent, chemical, and nuclear energies**. The sensible internal energy is due to translational, rotational, and vibrational effects.

Discussion We deal with the flow of a single phase fluid in most problems in this textbook; therefore, latent, chemical, and nuclear energies do not need to be considered.

2-22C

Solution We are to discuss the relationship between heat, internal energy, and thermal energy.

Analysis *Thermal energy* is the **sensible and latent forms of internal energy**. It does not include chemical or nuclear forms of energy. In common terminology, thermal energy is referred to as *heat*. However, like work, heat is not a property, whereas thermal energy is a property.

Discussion Technically speaking, "heat" is defined only when there is *heat transfer*, whereas the energy state of a substance can always be defined, even if no heat transfer is taking place.

2-23C

Solution We are to define and discuss flow energy.

Analysis *Flow energy* or *flow work* is the **energy needed to push a fluid into or out of a control volume**. Fluids at rest do not possess any flow energy.

Discussion Flow energy is not a fundamental quantity, like kinetic or potential energy. However, it is a useful concept in fluid mechanics since fluids are often forced into and out of control volumes in practice.

2-24C

Solution We are to compare the energies of flowing and non-flowing fluids.

Analysis **A flowing fluid possesses *flow energy*, which is the energy needed to push a fluid into or out of a control volume**, in addition to the forms of energy possessed by a non-flowing fluid. The total energy of a non-flowing fluid consists of internal and potential energies. If the fluid is moving as a rigid body, but not *flowing*, it may also have kinetic energy (e.g., gasoline in a tank truck moving down the highway at constant speed with no sloshing). The total energy of a flowing fluid consists of internal, kinetic, potential, *and* flow energies.

Discussion Flow energy is not to be confused with kinetic energy, even though both are zero when the fluid is at rest.

2-25C

Solution We are to explain how changes in internal energy can be determined.

Analysis Using specific heat values at the average temperature, the changes in the specific internal energy of ideal gases can be determined from $\boxed{\Delta u = c_{v,avg}\Delta T}$. For incompressible substances, $c_p \cong c_v \cong c$ and $\boxed{\Delta u = c_{avg}\Delta T}$.

Discussion If the fluid can be treated as neither incompressible nor an ideal gas, property tables must be used.

2-26C

Solution We are to explain how changes in enthalpy can be determined.

Analysis Using specific heat values at the average temperature, the changes in specific enthalpy of ideal gases can be determined from $\boxed{\Delta h = c_{p,avg}\Delta T}$. For incompressible substances, $c_p \cong c_v \cong c$ and $\boxed{\Delta h = \Delta u + v\Delta P \cong c_{avg}\Delta T + v\Delta P}$.

Discussion If the fluid can be treated as neither incompressible nor an ideal gas, property tables must be used.

Coefficient of Compressibility

2-27C

Solution We are to discuss the coefficient of compressibility and the isothermal compressibility.

Analysis The *coefficient of compressibility* represents the **variation of pressure of a fluid with volume or density at constant temperature**. *Isothermal compressibility* is the inverse of the coefficient of compressibility, and it represents the **fractional change in volume or density corresponding to a change in pressure**.

Discussion The coefficient of compressibility of an ideal gas is equal to its absolute pressure.

2-28C

Solution We are to define the coefficient of volume expansion.

Analysis The *coefficient of volume expansion* represents the **variation of the *density* of a fluid with temperature at constant pressure**. It differs from the coefficient of compressibility in that the latter represents the variation of *pressure* of a fluid with density at constant temperature.

Discussion The coefficient of volume expansion of an ideal gas is equal to the inverse of its absolute temperature.

2-29C

Solution We are to discuss the sign of the coefficient of compressibility and the coefficient of volume expansion.

Analysis The *coefficient of compressibility* of a fluid **cannot be negative**, but the *coefficient of volume expansion* **can be negative** (e.g., liquid water below 4°C).

Discussion This is the reason that ice floats on water.

2-30

Solution The percent increase in the density of an ideal gas is given for a moderate pressure. The percent increase in density of the gas when compressed at a higher pressure is to be determined.

Assumptions The gas behaves an ideal gas.

Analysis For an ideal gas, $P = \rho RT$ and $(\partial P / \partial \rho)_T = RT = P / \rho$, and thus $\kappa_{\text{ideal gas}} = P$. Therefore, the coefficient of compressibility of an ideal gas is equal to its absolute pressure, and the coefficient of compressibility of the gas increases with increasing pressure.

Substituting $\kappa = P$ into the definition of the coefficient of compressibility $\kappa \cong -\dfrac{\Delta P}{\Delta v / v} \cong \dfrac{\Delta P}{\Delta \rho / \rho}$ and rearranging gives

$$\frac{\Delta \rho}{\rho} = \frac{\Delta P}{P}$$

Therefore, **the percent increase of density of an ideal gas during isothermal compression is equal to the percent increase in pressure**.

At 10 atm: $\dfrac{\Delta \rho}{\rho} = \dfrac{\Delta P}{P} = \dfrac{11-10}{10} = 10\%$

At 100 atm: $\dfrac{\Delta \rho}{\rho} = \dfrac{\Delta P}{P} = \dfrac{101-100}{100} = 1\%$

Therefore, a pressure change of 1 atm causes a density change of 10% at 10 atm and a density change of 1% at 100 atm.

Discussion If temperature were also allowed to change, the relationship would not be so simple.

2-31

Solution Using the definition of the coefficient of volume expansion and the expression $\beta_{\text{ideal gas}} = 1/T$, it is to be shown that the percent increase in the specific volume of an ideal gas during isobaric expansion is equal to the percent increase in absolute temperature.

Assumptions The gas behaves an ideal gas.

Analysis The coefficient of volume expansion β can be expressed as $\beta = \dfrac{1}{v}\left(\dfrac{\partial v}{\partial T}\right)_P \approx \dfrac{\Delta v / v}{\Delta T}$.

Noting that $\beta_{\text{ideal gas}} = 1/T$ for an ideal gas and rearranging give

$$\frac{\Delta v}{v} = \frac{\Delta T}{T}$$

Therefore, **the percent increase in the specific volume of an ideal gas during isobaric expansion is equal to the percent increase in absolute temperature**.

Discussion We must be careful to use *absolute* temperature (K or R), not relative temperature (^{o}C or ^{o}F).

2-32

Solution Water at a given temperature and pressure is compressed to a high pressure isothermally. The increase in the density of water is to be determined.

Assumptions **1** The isothermal compressibility is constant in the given pressure range. **2** An approximate analysis is performed by replacing differential changes by finite changes.

Properties The density of water at 20°C and 1 atm pressure is ρ_1 = 998 kg/m^3. The isothermal compressibility of water is given to be α = 4.80 × 10^{-5} atm^{-1}.

Analysis When differential quantities are replaced by differences and the properties α and β are assumed to be constant, the change in density in terms of the changes in pressure and temperature is expressed approximately as

$$\Delta\rho = \alpha\rho\Delta P - \beta\rho\Delta T$$

The change in density due to a change of pressure from 1 atm to 800 atm at constant temperature is

$$\Delta\rho = \alpha\rho\Delta P = (4.80\times10^{-5}\text{ atm}^{-1})(998\text{ kg/m}^3)(800-1)\text{atm} = \mathbf{38.3\ kg/m^3}$$

Discussion Note that the density of water increases from 998 to 1036.3 kg/m^3 while being compressed, as expected. This problem can be solved more accurately using differential analysis when functional forms of properties are available.

2-33

Solution Water at a given temperature and pressure is heated to a higher temperature at constant pressure. The change in the density of water is to be determined.

Assumptions **1** The coefficient of volume expansion is constant in the given temperature range. **2** An approximate analysis is performed by replacing differential changes in quantities by finite changes.

Properties The density of water at 15°C and 1 atm pressure is ρ_1 = 999.1 kg/m^3. The coefficient of volume expansion at the average temperature of (15+95)/2 = 55°C is β = 0.484 × 10^{-3} K^{-1}.

Analysis When differential quantities are replaced by differences and the properties α and β are assumed to be constant, the change in density in terms of the changes in pressure and temperature is expressed approximately as

$$\Delta\rho = \alpha\rho\Delta P - \beta\rho\Delta T$$

The change in density due to the change of temperature from 15°C to 95°C at constant pressure is

$$\Delta\rho = -\beta\rho\Delta T = -(0.484\times10^{-3}\text{ K}^{-1})(999.1\text{ kg/m}^3)(95-15)\text{K} = \mathbf{-38.7\ kg/m^3}$$

Discussion Noting that $\Delta\rho = \rho_2 - \rho_1$, the density of water at 95°C and 1 atm is

$$\rho_2 = \rho_1 + \Delta\rho = 999.1 + (-38.7) = 960.4\text{ kg/m}^3$$

which is very close to the listed value of 961.5 kg/m^3 at 95°C in water table in the Appendix. This is mostly due to β varying with temperature almost linearly. Note that the density of water decreases while being heated, as expected. This problem can be solved more accurately using differential analysis when functional forms of properties are available.

2-34

Solution Saturated refrigerant-134a at a given temperature is cooled at constant pressure. The change in the density of the refrigerant is to be determined.

Assumptions **1** The coefficient of volume expansion is constant in the given temperature range. **2** An approximate analysis is performed by replacing differential changes in quantities by finite changes.

Properties The density of saturated liquid R-134a at 10°C is ρ_1 =1261 kg/m^3. The coefficient of volume expansion at the average temperature of (10+0)/2 = 5°C is β = 0.00269 K^{-1}.

Analysis When differential quantities are replaced by differences and the properties α and β are assumed to be constant, the change in density in terms of the changes in pressure and temperature is expressed approximately as

$$\Delta\rho = \alpha\rho\Delta P - \beta\rho\Delta T$$

The change in density due to the change of temperature from 10°C to 0°C at constant pressure is

$$\Delta\rho = -\beta\rho\Delta T = -(0.00269\,\text{K}^{-1})(1261\,\text{kg/m}^3)(0-10)\text{K} = \mathbf{33.9\ kg/m^3}$$

Discussion Noting that $\Delta\rho = \rho_2 - \rho_1$, the density of R-134a at 0°C is

$$\rho_2 = \rho_1 + \Delta\rho = 1261 + 33.9 = 1294.9\ \text{kg/m}^3$$

which is almost identical to the listed value of 1295 kg/m^3 at 0°C in R-134a table in the Appendix. This is mostly due to β varying with temperature almost linearly. Note that the density increases during cooling, as expected.

2-35

Solution A water tank completely filled with water can withstand tension caused by a volume expansion of 2%. The maximum temperature rise allowed in the tank without jeopardizing safety is to be determined.

Assumptions **1** The coefficient of volume expansion is constant. **2** An approximate analysis is performed by replacing differential changes in quantities by finite changes. **3** The effect of pressure is disregarded.

Properties The average volume expansion coefficient is given to be $\beta = 0.377 \times 10^{-3}$ K^{-1}.

Analysis When differential quantities are replaced by differences and the properties α and β are assumed to be constant, the change in density in terms of the changes in pressure and temperature is expressed approximately as

$$\Delta\rho = \alpha\rho\Delta P - \beta\rho\Delta T$$

A volume increase of 2% corresponds to a density decrease of 2%, which can be expressed as $\Delta\rho = -0.02\rho$. Then the decrease in density due to a temperature rise of ΔT at constant pressure is

$$-0.02\rho = -\beta\rho\Delta T$$

Solving for ΔT and substituting, the maximum temperature rise is determined to be

$$\Delta T = \frac{0.02}{\beta} = \frac{0.02}{0.377\times10^{-3}\ \text{K}^{-1}} = \mathbf{53.0\ K} = \mathbf{53.0°C}$$

Discussion This result is conservative since in reality the increasing pressure will tend to compress the water and increase its density.

2-36

Solution A water tank completely filled with water can withstand tension caused by a volume expansion of 1%. The maximum temperature rise allowed in the tank without jeopardizing safety is to be determined.

Assumptions **1** The coefficient of volume expansion is constant. **2** An approximate analysis is performed by replacing differential changes in quantities by finite changes. **3** The effect of pressure is disregarded.

Properties The average volume expansion coefficient is given to be $\beta = 0.377 \times 10^{-3}$ K^{-1}.

Analysis When differential quantities are replaced by differences and the properties α and β are assumed to be constant, the change in density in terms of the changes in pressure and temperature is expressed approximately as

$$\Delta\rho = \alpha\rho\Delta P - \beta\rho\Delta T$$

A volume increase of 1% corresponds to a density decrease of 1%, which can be expressed as $\Delta\rho = -0.01\rho$. Then the decrease in density due to a temperature rise of ΔT at constant pressure is

$$-0.01\rho = -\beta\rho\Delta T$$

Solving for ΔT and substituting, the maximum temperature rise is determined to be

$$\Delta T = \frac{0.01}{\beta} = \frac{0.01}{0.377 \times 10^{-3}\ \text{K}^{-1}} = \mathbf{26.5\ K} = \mathbf{26.5^\circ C}$$

Discussion This result is conservative since in reality the increasing pressure will tend to compress the water and increase its density. The change in temperature is exactly half of that of the previous problem, as expected.

2-37

Solution The density of seawater at the free surface and the bulk modulus of elasticity are given. The density and pressure at a depth of 2500 m are to be determined.

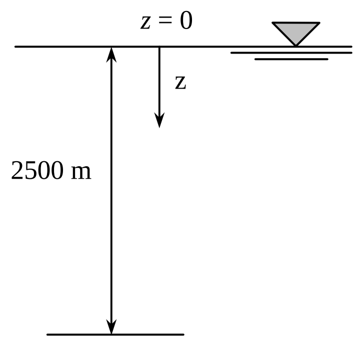
Assumptions **1** The temperature and the bulk modulus of elasticity of seawater is constant. **2** The gravitational acceleration remains constant.

Properties The density of seawater at free surface where the pressure is given to be 1030 kg/m^3, and the bulk modulus of elasticity of seawater is given to be 2.34×10^9 N/m^2.

Analysis The coefficient of compressibility or the bulk modulus of elasticity of fluids is expressed as

$$\kappa = \rho\left(\frac{\partial P}{\partial \rho}\right)_T \qquad \text{or} \qquad \kappa = \rho\frac{dP}{d\rho} \qquad \text{(at constant } T\text{)}$$

The differential pressure change across a differential fluid height of dz is given as



$$dP = \rho g dz$$

Combining the two relations above and rearranging,

$$\kappa = \rho\frac{\rho g dz}{d\rho} = g\rho^2\frac{dz}{d\rho} \quad \rightarrow \quad \frac{d\rho}{\rho^2} = \frac{gdz}{\kappa}$$

Integrating from $z = 0$ where $\rho = \rho_0 = 1030\ \text{kg/m}^3$ to $z = z$ where $\rho = \rho$ gives

$$\int_{\rho_0}^{\rho}\frac{d\rho}{\rho^2} = \frac{g}{\kappa}\int_0^z dz \quad \rightarrow \quad \frac{1}{\rho_0} - \frac{1}{\rho} = \frac{gz}{\kappa}$$

Solving for ρ gives the variation of density with depth as

$$\rho = \frac{1}{(1/\rho_0) - (gz/\kappa)}$$

Substituting into the pressure change relation $dP = \rho g dz$ and integrating from $z = 0$ where $P = P_0 = 98\ \text{kPa}$ to $z = z$ where $P = P$ gives

$$\int_{P_0}^{P} dP = \int_0^z \frac{gdz}{(1/\rho_0) - (gz/\kappa)} \quad \rightarrow \quad P = P_0 + \kappa\ln\left(\frac{1}{1-(\rho_0 gz/\kappa)}\right)$$

which is the desired relation for the variation of pressure in seawater with depth. At z = 2500 m, the values of density and pressure are determined by substitution to be

$$\rho = \frac{1}{1/(1030\ \text{kg/m}^3) - (9.81\ \text{m/s}^2)(2500\ \text{m})/(2.34\times10^9\ \text{N/m}^2)} = \mathbf{1041\ kg/m^3}$$

$$P = (98{,}000\ \text{Pa}) + (2.34\times10^9\ \text{N/m}^2)\ln\left(\frac{1}{1-(1030\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2500\ \text{m})/(2.34\times10^9\ \text{N/m}^2)}\right)$$

$$= 2.550\times10^7\ \text{Pa}$$

$$= \mathbf{25.50\ MPa}$$

since 1 Pa = 1 N/m^2 = 1 kg/m·s^2 and 1 kPa = 1000 Pa.

Discussion Note that if we assumed $\rho = \rho_o$ = constant at 1030 kg/m^3, the pressure at 2500 m would be $P = P_0 + \rho g z$ = 0.098 + 25.26 = 25.36 MPa. Then the density at 2500 m is estimated to be

$$\Delta\rho = \rho\alpha\Delta P = (1030)(2340\ \text{MPa})^{-1}(25.26\ \text{MPa}) = 11.1\ \text{kg/m}^3 \text{ and thus } \rho = 1041\ \text{kg/m}^3$$

Viscosity

2-38C

Solution We are to define and discuss viscosity.

Analysis *Viscosity* is **a measure of the "stickiness" or "resistance to deformation" of a fluid**. It is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other. Viscosity is caused by the cohesive forces between the molecules in liquids, and by the molecular collisions in gases. In general, **liquids have higher dynamic viscosities than gases**.

Discussion The ratio of viscosity μ to density ρ often appears in the equations of fluid mechanics, and is defined as the *kinematic viscosity*, $\nu = \mu/\rho$.

2-39C

Solution We are to discuss Newtonian fluids.

Analysis **Fluids whose shear stress is linearly proportional to the velocity gradient** (shear strain) are called *Newtonian fluids*. Most common fluids such as water, air, gasoline, and oils are Newtonian fluids.

Discussion In the differential analysis of fluid flow, only Newtonian fluids are considered in this textbook.

2-40C

Solution We are to compare the settling speed of balls dropped in water and oil; namely, we are to determine which will reach the bottom of the container first.

Analysis When two identical small glass balls are dropped into two identical containers, one filled with water and the other with oil, **the ball dropped in water will reach the bottom of the container first** because of the much lower viscosity of water relative to oil.

Discussion Oil is very viscous, with typical values of viscosity approximately 800 times greater than that of water at room temperature.

2-41C

Solution We are to discuss how dynamic viscosity varies with temperature in liquids and gases.

Analysis (*a*) The **dynamic viscosity of liquids decreases with temperature**. (*b*) The **dynamic viscosity of gases increases with temperature**.

Discussion A good way to remember this is that a car engine is much harder to start in the winter because the oil in the engine has a higher viscosity at low temperatures.

2-42C

Solution We are to discuss how kinematic viscosity varies with temperature in liquids and gases.

Analysis (*a*) **For *liquids*, the kinematic viscosity decreases with temperature**. (*b*) **For *gases*, the kinematic viscosity increases with temperature**.

Discussion You can easily verify this by looking at the appendices.

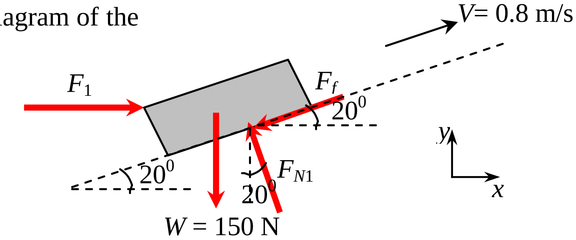
2-43

Solution A block is moved at constant velocity on an inclined surface. The force that needs to be applied in the horizontal direction when the block is dry, and the percent reduction in the required force when an oil film is applied on the surface are to be determined.

Assumptions **1** The inclined surface is plane (perfectly flat, although tilted). **2** The friction coefficient and the oil film thickness are uniform. **3** The weight of the oil layer is negligible.

Properties The absolute viscosity of oil is given to be $\mu = 0.012$ Pa·s = 0.012 N·s/m^2.

Analysis (*a*) The velocity of the block is constant, and thus its acceleration and the net force acting on it are zero. A free body diagram of the block is given. Then the force balance gives



$$\sum F_x = 0: \quad F_1 - F_f \cos 20° - F_{N1} \sin 20° = 0 \quad (1)$$

$$\sum F_y = 0: \quad F_{N1} \cos 20° - F_f \sin 20° - W = 0 \quad (2)$$

Friction force: $F_f = fF_{N1}$ (3)

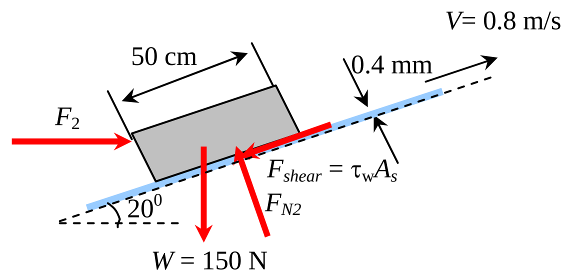
Substituting Eq. (3) into Eq. (2) and solving for F_{N1} gives

$$F_{N1} = \frac{W}{\cos 20° - f \sin 20°} = \frac{150\text{ N}}{\cos 20° - 0.27 \sin 20°} = 177.0\text{ N}$$

Then from Eq. (1):

$$F_1 = F_f \cos 20° + F_{N1} \sin 20° = (0.27 \times 177\text{ N}) \cos 20° + (177\text{ N}) \sin 20° = \mathbf{105.5\ N}$$

(*b*) In this case, the friction force is replaced by the shear force applied on the bottom surface of the block due to the oil. Because of the no-slip condition, the oil film sticks to the inclined surface at the bottom and the lower surface of the block at the top. Then the shear force is expressed as



$$\begin{aligned} F_{shear} &= \tau_w A_s \\ &= \mu A_s \frac{V}{h} \\ &= \left(0.012\text{ N}\cdot\text{s/m}^2\right)\left(0.5 \times 0.2\text{ m}^2\right) \frac{0.8\text{ m/s}}{4 \times 10^{-4}\text{ m}} \\ &= 2.4\text{ N} \end{aligned}$$

Replacing the friction force by the shear force in part (*a*),

$$\sum F_x = 0: \quad F_2 - F_{shear} \cos 20° - F_{N2} \sin 20° = 0 \quad (4)$$

$$\sum F_y = 0: \quad F_{N2} \cos 20° - F_{shear} \sin 20° - W = 0 \quad (5)$$

Eq. (5) gives $F_{N2} = (F_{shear} \sin 20° + W)/\cos 20° = [(2.4\text{ N}) \sin 20° + (150\text{ N})]/\cos 20° = 160.5\text{ N}$

Substituting into Eq. (4), the required horizontal force is determined to be

$$F_2 = F_{shear} \cos 20° + F_{N2} \sin 20° = (2.4\text{ N}) \cos 20° + (160.5\text{ N}) \sin 20° = 57.2\text{ N}$$

Then, our final result is expressed as

$$\text{Percentage reduction in required force} = \frac{F_1 - F_2}{F_1} \times 100\% = \frac{105.5 - 57.2}{105.5} \times 100\% = \mathbf{45.8\%}$$

Discussion Note that the force required to push the block on the inclined surface reduces significantly by oiling the surface.

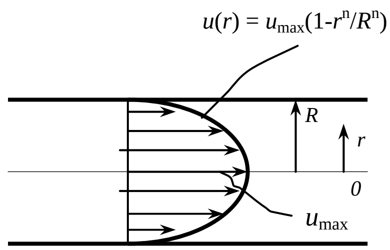
2-44

Solution The velocity profile of a fluid flowing though a circular pipe is given. The friction drag force exerted on the pipe by the fluid in the flow direction per unit length of the pipe is to be determined.

Assumptions The viscosity of the fluid is constant.

Analysis The wall shear stress is determined from its definition to be

$$\tau_w = -\mu \left.\frac{du}{dr}\right|_{r=R} = -\mu u_{\max} \frac{d}{dr}\left(1 - \frac{r^n}{R^n}\right)_{r=R} = -\mu u_{\max} \left.\frac{-nr^{n-1}}{R^n}\right|_{r=R} = \frac{n\mu u_{\max}}{R}$$



Note that the quantity du/dr is negative in pipe flow, and the negative sign is added to the τ_w relation for pipes to make shear stress in the positive (flow) direction a positive quantity. (Or, $du/dr = - du/dy$ since $y = R - r$). Then the friction drag force exerted by the fluid on the inner surface of the pipe becomes

$$F = \tau_w A_w = \frac{n\mu u_{\max}}{R}(2\pi R)L = 2n\pi\mu u_{\max} L$$

Therefore, the drag force per unit length of the pipe is

$$\boxed{F / L = 2n\pi\mu u_{\max}}.$$

Discussion Note that the drag force acting on the pipe in this case is independent of the pipe diameter.

2-45

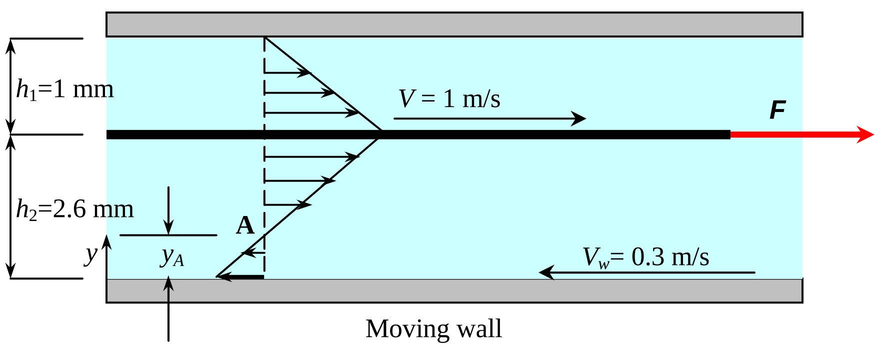
Solution A thin flat plate is pulled horizontally through an oil layer sandwiched between two plates, one stationary and the other moving at a constant velocity. The location in oil where the velocity is zero and the force that needs to be applied on the plate are to be determined.

Assumptions **1** The thickness of the plate is negligible. **2** The velocity profile in each oil layer is linear.

Properties The absolute viscosity of oil is given to be $\mu = 0.027$ Pa·s = 0.027 N·s/m^2.

Analysis (*a*) The velocity profile in each oil layer relative to the fixed wall is as shown in the figure below. The point of zero velocity is indicated by point *A*, and its distance from the lower plate is determined from geometric considerations (the similarity of the two triangles in the lower oil layer) to be

$$\frac{2.6 - y_A}{y_A} = \frac{1}{0.3} \quad \rightarrow \quad y_A = \mathbf{0.60\ mm}$$



(*b*) The magnitudes of shear forces acting on the upper and lower surfaces of the plate are

$$F_{\text{shear, upper}} = \tau_{w,\text{ upper}} A_s = \mu A_s \left|\frac{du}{dy}\right| = \mu A_s \frac{V-0}{h_1} = (0.027\text{ N}\cdot\text{s/m}^2)(0.2\times 0.2\text{ m}^2)\frac{1\text{ m/s}}{1.0\times 10^{-3}\text{ m}} = 1.08\text{ N}$$

$$F_{\text{shear, lower}} = \tau_{w,\text{ lower}} A_s = \mu A_s \left|\frac{du}{dy}\right| = \mu A_s \frac{V-V_w}{h_2} = (0.027\text{ N}\cdot\text{s/m}^2)(0.2\times 0.2\text{ m}^2)\frac{[1-(-0.3)]\text{ m/s}}{2.6\times 10^{-3}\text{ m}} = 0.54\text{ N}$$

Noting that both shear forces are in the opposite direction of motion of the plate, the force *F* is determined from a force balance on the plate to be

$$F = F_{\text{shear,upper}} + F_{\text{shear,lower}} = 1.08 + 0.54 = \mathbf{1.62\ N}$$

Discussion Note that wall shear is a friction force between a solid and a liquid, and it acts in the opposite direction of motion.

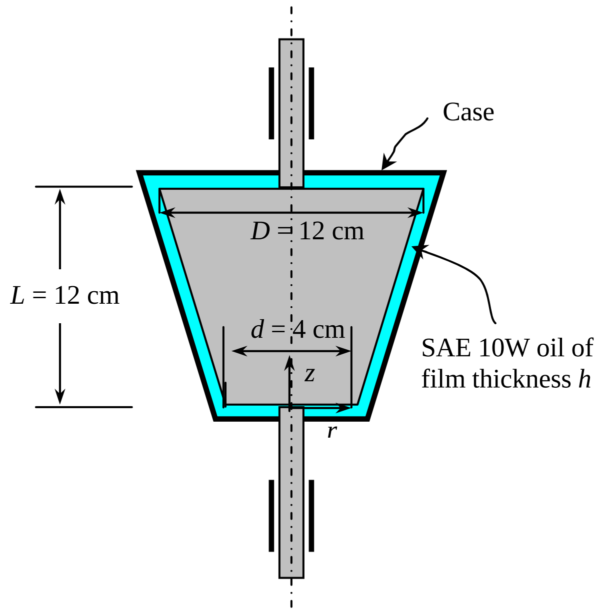
2-46

Solution A frustum shaped body is rotating at a constant angular speed in an oil container. The power required to maintain this motion and the reduction in the required power input when the oil temperature rises are to be determined.

Assumptions The thickness of the oil layer remains constant.

Properties The absolute viscosity of oil is given to be μ = 0.1 Pa·s = 0.1 N·s/m^2 at 20°C and 0.0078 Pa·s at 80°C.

Analysis The velocity gradient anywhere in the oil of film thickness h is V/h where $V = \omega r$ is the tangential velocity. Then the wall shear stress anywhere on the surface of the frustum at a distance r from the axis of rotation is



$$\tau_w = \mu \frac{du}{dr} = \mu \frac{V}{h} = \mu \frac{\omega r}{h}$$

The shear force acting on differential area dA on the surface, the torque it generates, and the shaft power associated with it are expressed as

$$dF = \tau_w dA = \mu \frac{\omega r}{h} dA \qquad d\mathrm{T} = r dF = \mu \frac{\omega r^2}{h} dA$$

$$\mathrm{T} = \frac{\mu\omega}{h} \int_A r^2 dA \qquad \dot{W}_{\text{sh}} = \omega \mathrm{T} = \frac{\mu\omega^2}{h} \int_A r^2 dA$$

Top surface: For the top surface, $dA = 2\pi r dr$. Substituting and integrating,

$$\dot{W}_{\text{sh, top}} = \frac{\mu\omega^2}{h} \int_{r=0}^{D/2} r^2 (2\pi r) dr = \frac{2\pi\mu\omega^2}{h} \int_{r=0}^{D/2} r^3 dr = \frac{2\pi\mu\omega^2}{h} \left. \frac{r^4}{4} \right|_{r=0}^{D/2} = \frac{\pi\mu\omega^2 D^4}{32h}$$

Bottom surface: A relation for the bottom surface is obtained by replacing D by d, $\dot{W}_{\text{sh, bottom}} = \dfrac{\pi\mu\omega^2 d^4}{32h}$

Side surface: The differential area for the side surface can be expressed as $dA = 2\pi r dz$. From geometric considerations, the variation of radius with axial distance is expressed as $r = \dfrac{d}{2} + \dfrac{D-d}{2L} z$.

Differentiating gives $dr = \dfrac{D-d}{2L} dz$ or $dz = \dfrac{2L}{D-d} dr$. Therefore, $dA = 2\pi dz = \dfrac{4\pi L}{D-d} r dr$. Substituting and integrating,

$$\dot{W}_{\text{sh, top}} = \frac{\mu\omega^2}{h} \int_{r=0}^{D/2} r^2 \frac{4\pi L}{D-d} r dr = \frac{4\pi\mu\omega^2 L}{h(D-d)} \int_{r=d/2}^{D/2} r^3 dr = \frac{4\pi\mu\omega^2 L}{h(D-d)} \left. \frac{r^4}{4} \right|_{r=d/2}^{D/2} = \frac{\pi\mu\omega^2 L(D^2-d^2)}{16h(D-d)}$$

Then the total power required becomes

$$\dot{W}_{\text{sh, total}} = \dot{W}_{\text{sh, top}} + \dot{W}_{\text{sh, bottom}} + \dot{W}_{\text{sh, side}} = \frac{\pi\mu\omega^2 D^4}{32h} \left[1 + (d/D)^4 + \frac{2L[1-(d/D)^4)]}{D-d} \right],$$

where d/D = 4/12 = 1/3. Substituting,

$$\dot{W}_{\text{sh, total}} = \frac{\pi(0.1\ \text{N}\cdot\text{s/m}^2)(200\ /\text{s})^2 (0.12\ \text{m})^4}{32(0.0012\ \text{m})} \left[1 + (1/3)^4 + \frac{2(0.12\ \text{m})[1-(1/3)^4)]}{(0.12-0.04)\ \text{m}} \right] \left(\frac{1\ \text{W}}{1\ \text{Nm/s}} \right) = \mathbf{270\ W}$$

Noting that power is proportional to viscosity, the power required at 80°C is

$$\dot{W}_{\text{sh, total, 80°C}} = \frac{\mu_{80°C}}{\mu_{20°C}} \dot{W}_{\text{sh, total, 20°C}} = \frac{0.0078\ \text{N}\cdot\text{s/m}^2}{0.1\ \text{N}\cdot\text{s/m}^2} (270\ \text{W}) = 21.1\ \text{W}$$

Therefore, the reduction in the requires power input at 80°C is $\text{Reduction} = \dot{W}_{\text{sh, total, 20°C}} - \dot{W}_{\text{sh, total, 80°C}} = 270 - 21.1 = \mathbf{249\ W}$, which is about 92%.

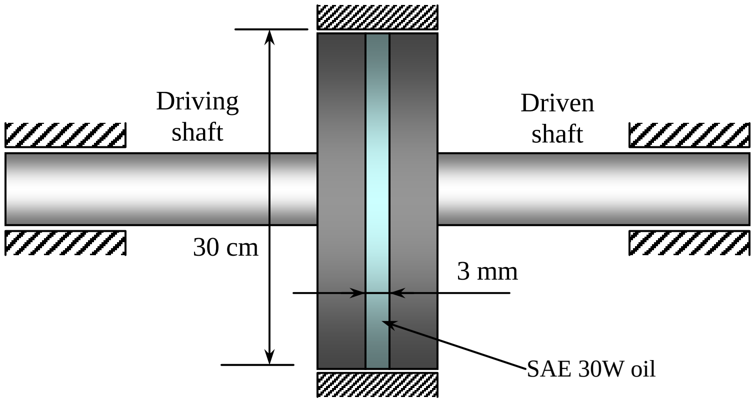
Discussion Note that the power required to overcome shear forces in a viscous fluid greatly depends on temperature.

2-47

Solution A clutch system is used to transmit torque through an oil film between two identical disks. For specified rotational speeds, the transmitted torque is to be determined.

Assumptions **1** The thickness of the oil film is uniform. **2** The rotational speeds of the disks remain constant.

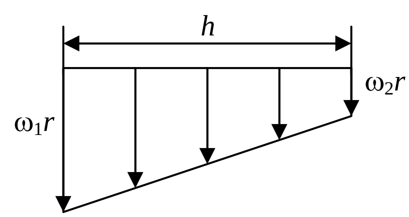
Properties The absolute viscosity of oil is given to be $\mu = 0.38$ N·s/m^2.



Analysis The disks are rotting in the same direction at different angular speeds of ω_1 and of ω_2 . Therefore, we can assume one of the disks to be stationary and the other to be rotating at an angular speed of $\omega_1 - \omega_2$. The velocity gradient anywhere in the oil of film thickness h is $V\ /h$ where $V = (\omega_1 - \omega_2)r$ is the tangential velocity. Then the wall shear stress anywhere on the surface of the faster disk at a distance r from the axis of rotation can be expressed as

$$\tau_w = \mu\frac{du}{dr} = \mu\frac{V}{h} = \mu\frac{(\omega_1 - \omega_2)r}{h}$$

Then the shear force acting on a differential area dA on the surface and the torque generation associated with it can be expressed as



$$dF = \tau_w dA = \mu\frac{(\omega_1 - \omega_2)r}{h}(2\pi r)dr$$

$$d\mathrm{T} = rdF = \mu\frac{(\omega_1 - \omega_2)r^2}{h}(2\pi r)dr = \frac{2\pi\mu(\omega_1 - \omega_2)}{h}r^3dr$$

Integrating,

$$\mathrm{T} = \frac{2\pi\mu(\omega_1 - \omega_2)}{h}\int_{r=0}^{D/2} r^3dr = \frac{2\pi\mu(\omega_1 - \omega_2)}{h}\left.\frac{r^4}{4}\right|_{r=0}^{D/2} = \frac{\pi\mu(\omega_1 - \omega_2)D^4}{32h}$$

Noting that $\omega = 2\pi\ \dot{n}$, the relative angular speed is

$$\omega_1 - \omega_2 = 2\pi(\dot{n}_1 - \dot{n}_2) = (2\pi\ \text{rad/rev})\left[(1450 - 1398)\ \text{rev/min}\right]\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 5.445\ \text{rad/s}\ ,$$

Substituting, the torque transmitted is determined to be

$$\mathrm{T} = \frac{\pi(0.38\ \text{N}\cdot\text{s/m}^2)(5.445\ \text{/s})(0.30\ \text{m})^4}{32(0.003\ \text{m})} = \mathbf{0.55\ N \cdot m}$$

Discussion Note that the torque transmitted is proportional to the fourth power of disk diameter, and is inversely proportional to the thickness of the oil film.

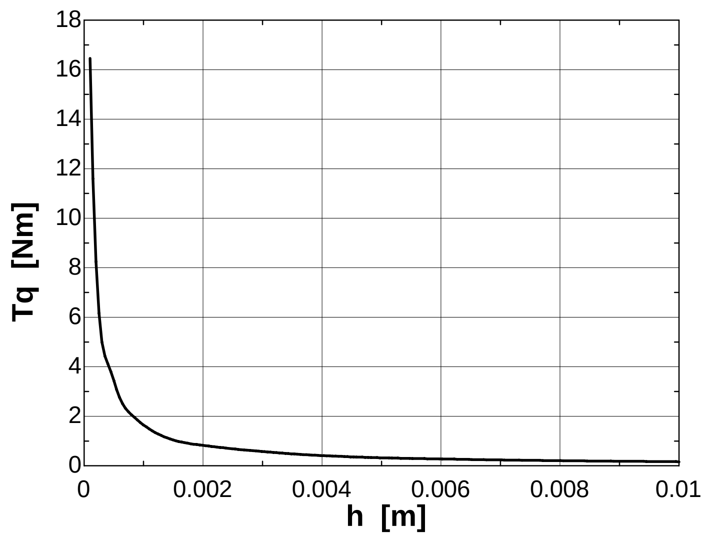
2-48 

Solution We are to investigate the effect of oil film thickness on the transmitted torque.

Analysis The previous problem is reconsidered. Using EES software, the effect of oil film thickness on the torque transmitted is investigated. Film thickness varied from 0.1 mm to 10 mm, and the results are tabulated and plotted. The relation used is $T = \frac{\pi\mu(\omega_1 - \omega_2)D^4}{32h}$. The EES *Equations* window is printed below, followed by the tabulated and plotted results.

```
mu=0.38
n1=1450 "rpm"
w1=2*pi*n1/60 "rad/s"
n2=1398 "rpm"
w2=2*pi*n2/60 "rad/s"
D=0.3 "m"
Tq=pi*mu*(w1-w2)*(D^4)/(32*h)
```

Film thickness *h*, mm	**Torque transmitted T, Nm**
0.1	16.46
0.2	8.23
0.4	4.11
0.6	2.74
0.8	2.06
1	1.65
2	0.82
4	0.41
6	0.27
8	0.21
10	0.16



Conclusion Torque transmitted is inversely proportional to oil film thickness, and the film thickness should be as small as possible to maximize the transmitted torque.

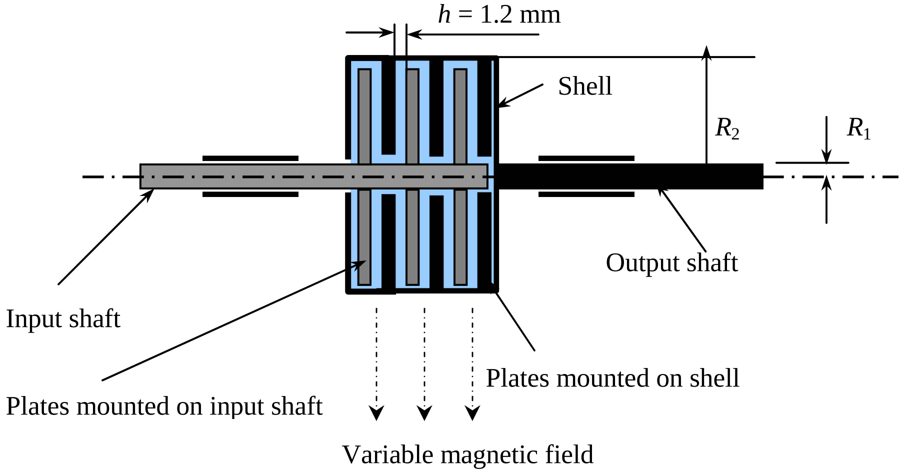
Discussion To obtain the solution in EES, we set up a parametric table, specify *h*, and let EES calculate T for each value of *h*.

2-49

Solution A multi-disk Electro-rheological "ER" clutch is considered. The ER fluid has a shear stress that is expressed as $\tau = \tau_y + \mu(du/dy)$. A relationship for the torque transmitted by the clutch is to be obtained, and the numerical value of the torque is to be calculated.

Assumptions **1** The thickness of the oil layer between the disks is constant. **2** The Bingham plastic model for shear stress expressed as $\tau = \tau_y + \mu(du/dy)$ is valid.

Properties The constants in shear stress relation are given to be μ = 0.1 Pa·s and τ_y = 2.5 kPa.



Analysis (*a*) The velocity gradient anywhere in the oil of film thickness h is V/h where $V = \omega r$ is the tangential velocity relative to plates mounted on the shell. Then the wall shear stress anywhere on the surface of a plate mounted on the input shaft at a distance r from the axis of rotation is expressed as

$$\tau_w = \tau_y + \mu\frac{du}{dr} = \tau_y + \mu\frac{V}{h} = \tau_y + \mu\frac{\omega r}{h}$$

Then the shear force acting on a differential area dA on the surface of a disk and the torque generation associated with it are expressed as

$$dF = \tau_w dA = \left(\tau_y + \mu\frac{\omega r}{h}\right)(2\pi r)dr$$

$$d\mathrm{T} = r dF = r\left(\tau_y + \mu\frac{\omega r}{h}\right)(2\pi r)dr = 2\pi\left(\tau_y r^2 + \mu\frac{\omega r^3}{h}\right)dr$$

Integrating,

$$\mathrm{T} = 2\pi\int_{r=R_1}^{R_2}\left(\tau_y r^2 + \mu\frac{\omega r^3}{h}\right)dr = 2\pi\left[\tau_y\frac{r^3}{3} + \frac{\mu\omega r^4}{4h}\right]_{r=R_1}^{R_2} = 2\pi\left[\frac{\tau_y}{3}(R_2^3 - R_1^3) + \frac{\mu\omega}{4h}(R_2^4 - R_1^4)\right]$$

This is the torque transmitted by one surface of a plate mounted on the input shaft. Then the torque transmitted by both surfaces of N plates attached to input shaft in the clutch becomes

$$\mathrm{T} = 4\pi N\left[\frac{\tau_y}{3}(R_2^3 - R_1^3) + \frac{\mu\omega}{4h}(R_2^4 - R_1^4)\right]$$

(*b*) Noting that $\omega = 2\pi\dot{n} = 2\pi(2400\text{ rev/min}) = 15{,}080\text{ rad/min} = 251.3\text{ rad/s}$ and substituting,

$$\mathrm{T} = (4\pi)(11)\left[\frac{2500\text{ N/m}^2}{3}[(0.20\text{ m})^3 - (0.05\text{ m})^3] + \frac{(0.1\text{ N}\cdot\text{s/m}^2)(251.3\text{ /s})}{4(0.0012\text{ m})}[(0.20\text{ m})^4 - (0.05\text{ m})^4]\right] = \mathbf{2060\ N\cdot m}$$

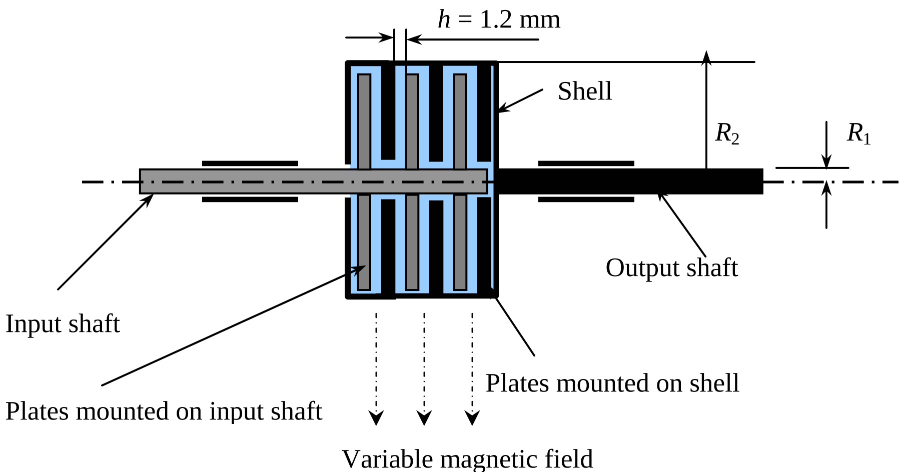
Discussion Can you think of some other potential applications for this kind of fluid?

2-50

Solution A multi-disk magnetorheological "MR" clutch is considered The MR fluid has a shear stress that is expressed as $\tau = \tau_y + K(du/dy)^m$. A relationship for the torque transmitted by the clutch is to be obtained, and the numerical value of the torque is to be calculated.

Assumptions **1** The thickness of the oil layer between the disks is constant. **2** The Herschel-Bulkley model for shear stress expressed as $\tau = \tau_y + K(du/dy)^m$ is valid.

Properties The constants in shear stress relation are given to be τ_y = 900 Pa, K = 58 Pa·s^m, and m = 0.82.



Analysis (*a*) The velocity gradient anywhere in the oil of film thickness *h* is *V*/*h* where $V = \omega r$ is the tangential velocity relative to plates mounted on the shell. Then the wall shear stress anywhere on the surface of a plate mounted on the input shaft at a distance *r* from the axis of rotation is expressed as

$$\tau_w = \tau_y + K\left(\frac{du}{dr}\right)^m = \tau_y + K\left(\frac{V}{h}\right)^m = \tau_y + K\left(\frac{\omega r}{h}\right)^m$$

Then the shear force acting on a differential area *dA* on the surface of a disk and the torque generation associated with it are expressed as

$$dF = \tau_w dA = \left(\tau_y + K\left(\frac{\omega r}{h}\right)^m\right)(2\pi r)dr \quad \text{and} \quad d\mathrm{T} = r dF = r\left(\tau_y + K\left(\frac{\omega r}{h}\right)^m\right)(2\pi r)dr = 2\pi\left(\tau_y r^2 + K\frac{\omega^m r^{m+2}}{h^m}\right)dr$$

Integrating,

$$\mathrm{T} = 2\pi\int_{R_1}^{R_2}\left(\tau_y r^2 + K\frac{\omega^m r^{m+2}}{h^m}\right)dr = 2\pi\left[\tau_y\frac{r^3}{3} + \frac{K\omega^m r^{m+3}}{(m+3)h^m}\right]_{R_1}^{R_2} = 2\pi\left[\frac{\tau_y}{3}(R_2^3 - R_1^3) + \frac{K\omega^m}{(m+3)h^m}(R_2^{m+3} - R_1^{m+3})\right]$$

This is the torque transmitted by one surface of a plate mounted on the input shaft. Then the torque transmitted by both surfaces of *N* plates attached to input shaft in the clutch becomes

$$\mathrm{T} = 4\pi N\left[\frac{\tau_y}{3}(R_2^3 - R_1^3) + \frac{K\omega^m}{(m+3)h^m}(R_2^{m+3} - R_1^{m+3})\right]$$

(*b*) Noting that $\omega = 2\pi\dot{n} = 2\pi(2400 \text{ rev/min}) = 15{,}080 \text{ rad/min} = 251.3 \text{ rad/s}$ and substituting,

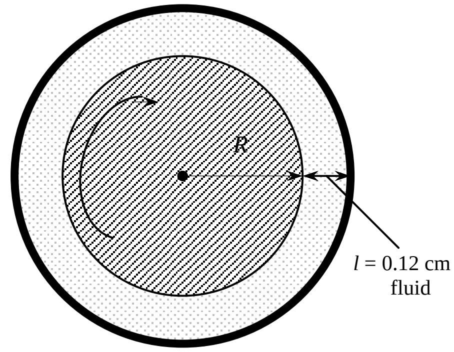
$$\mathrm{T} = (4\pi)(11)\left[\frac{900 \text{ N/m}^2}{3}\left[(0.20 \text{ m})^3 - (0.05 \text{ m})^3\right] + \frac{(58 \text{ N}\cdot\text{s}^{0.82}/\text{m}^2)(251.3 \text{ /s})^{0.82}}{(0.82+3)(0.0012 \text{ m})^{0.82}}\left[(0.20 \text{ m})^{3.82} - (0.05 \text{ m})^{3.82}\right]\right]$$

$= 103.4$ N $\cong$ **103 kN·m**

Discussion Can you think of some other potential applications for this kind of fluid?

2-51

Solution The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.



Assumptions **1** The inner cylinder is completely submerged in oil. **2** The viscous effects on the two ends of the inner cylinder are negligible. **3** The fluid is Newtonian.

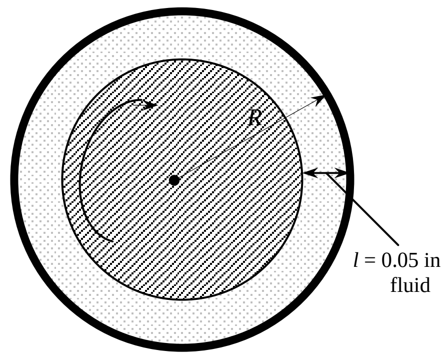
Analysis Substituting the given values, the viscosity of the fluid is determined to be

$$\mu = \frac{\mathbf{T}\ell}{4\pi^2 R^3 \dot{n} L} = \frac{(0.8\ \text{N}\cdot\text{m})(0.0012\ \text{m})}{4\pi^2 (0.075\ \text{m})^3 (200/60\ \text{s}^{-1})(0.75\ \text{m})} = \mathbf{0.0231\ N\cdot s/m^2}$$

Discussion This is the viscosity value at the temperature that existed during the experiment. Viscosity is a strong function of temperature, and the values can be significantly different at different temperatures.

2-52E

Solution The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.



Assumptions **1** The inner cylinder is completely submerged in the fluid. **2** The viscous effects on the two ends of the inner cylinder are negligible. **3** The fluid is Newtonian.

Analysis Substituting the given values, the viscosity of the fluid is determined to be

$$\mu = \frac{\mathbf{T}\ell}{4\pi^2 R^3 \dot{n} L} = \frac{(1.2\ \text{lbf}\cdot\text{ft})(0.05/12\ \text{ft})}{4\pi^2 (5.6/12\ \text{ft})^3 (250/60\ \text{s}^{-1})(3\ \text{ft})} = \mathbf{9.97 \times 10^{-5}\ lbf\cdot s/ft^2}$$

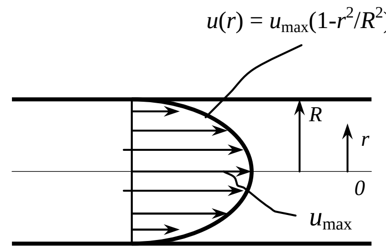
Discussion This is the viscosity value at temperature that existed during the experiment. Viscosity is a strong function of temperature, and the values can be significantly different at different temperatures.

2-53

Solution The velocity profile for laminar one-dimensional flow through a circular pipe is given. A relation for friction drag force exerted on the pipe and its numerical value for water are to be determined.

Assumptions **1** The flow through the circular pipe is one-dimensional. **2** The fluid is Newtonian.

Properties The viscosity of water at 20°C is given to be 0.0010 kg/m·s.



Analysis The velocity profile is given by $u(r) = u_{max}\left(1-\frac{r^2}{R^2}\right)$

where R is the radius of the pipe, r is the radial distance from the center of the pipe, and u_{max} is the maximum flow velocity, which occurs at the center, r = 0. The shear stress at the pipe surface is expressed as

$$\tau_w = -\mu\left.\frac{du}{dr}\right|_{r=R} = -\mu u_{max}\frac{d}{dr}\left(1-\frac{r^2}{R^2}\right)_{r=R} = -\mu u_{max}\left.\frac{-2r}{R^2}\right|_{r=R} = \frac{2\mu u_{max}}{R}$$

Note that the quantity *du/dr* is negative in pipe flow, and the negative sign is added to the τ_w relation for pipes to make shear stress in the positive (flow) direction a positive quantity. (Or, *du/dr* = –*du/dy* since *y* = *R* – *r*). Then the friction drag force exerted by the fluid on the inner surface of the pipe becomes

$$F_D = \tau_w A_s = \frac{2\mu u_{max}}{R}(2\pi RL) = 4\pi\mu L u_{max}$$

Substituting we get $F_D = 4\pi\mu L u_{max} = 4\pi(0.0010\text{ kg/m}\cdot\text{s})(15\text{ m})(3\text{ m/s})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{0.565\ N}$

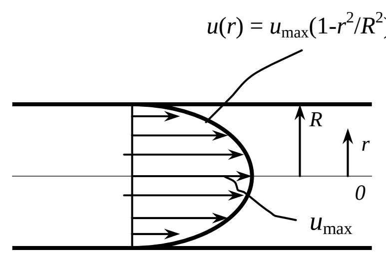
Discussion In the entrance region and during turbulent flow, the velocity gradient is greater near the wall, and thus the drag force in such cases will be greater.

2-54

Solution The velocity profile for laminar one-dimensional flow through a circular pipe is given. A relation for friction drag force exerted on the pipe and its numerical value for water are to be determined.

Assumptions **1** The flow through the circular pipe is one-dimensional. **2** The fluid is Newtonian.

Properties The viscosity of water at 20°C is given to be 0.0010 kg/m·s.



Analysis The velocity profile is given by $u(r) = u_{max}\left(1-\frac{r^2}{R^2}\right)$

where R is the radius of the pipe, r is the radial distance from the center of the pipe, and u_{max} is the maximum flow velocity, which occurs at the center, r = 0. The shear stress at the pipe surface can be expressed as

$$\tau_w = -\mu\left.\frac{du}{dr}\right|_{r=R} = -\mu u_{max}\frac{d}{dr}\left(1-\frac{r^2}{R^2}\right)_{r=R} = -\mu u_{max}\left.\frac{-2r}{R^2}\right|_{r=R} = \frac{2\mu u_{max}}{R}$$

Note that the quantity *du/dr* is negative in pipe flow, and the negative sign is added to the τ_w relation for pipes to make shear stress in the positive (flow) direction a positive quantity. (Or, *du/dr* = –*du/dy* since *y* = *R* – *r*). Then the friction drag force exerted by the fluid on the inner surface of the pipe becomes

$$F_D = \tau_w A_s = \frac{2\mu u_{max}}{R}(2\pi RL) = 4\pi\mu L u_{max}$$

Substituting, we get $F_D = 4\pi\mu L u_{max} = 4\pi(0.0010\text{ kg/m}\cdot\text{s})(15\text{ m})(5\text{ m/s})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{0.942\ N}$

Discussion In the entrance region and during turbulent flow, the velocity gradient is greater near the wall, and thus the drag force in such cases will be larger.

Surface Tension and Capillary Effect

2-55C

Solution We are to define and discuss surface tension.

Analysis The **magnitude of the pulling force at the surface of a liquid per unit length** is called *surface tension* σ_s. It is caused by the attractive forces between the molecules. The surface tension is also surface energy (per unit area) since it represents the stretching work that needs to be done to increase the surface area of the liquid by a unit amount.

Discussion Surface tension is the cause of some very interesting phenomena such as capillary rise and insects that can walk on water.

2-56C

Solution We are to analyze the pressure difference between inside and outside of a soap bubble.

Analysis The **pressure inside a soap bubble is greater than the pressure outside**, as evidenced by the stretch of the soap film.

Discussion You can make an analogy between the soap film and the skin of a balloon.

2-57C

Solution We are to define and discuss the capillary effect.

Analysis The *capillary effect* is the **rise or fall of a liquid in a small-diameter tube inserted into the liquid**. It is caused by the net effect of the *cohesive forces* (the forces between like molecules, like water) and *adhesive forces* (the forces between unlike molecules, like water and glass). **The capillary effect is proportional to the cosine of the *contact angle***, which is the angle that the tangent to the liquid surface makes with the solid surface at the point of contact.

Discussion The contact angle determines whether the *meniscus* at the top of the column is concave or convex.

2-58C

Solution We are to determine whether the level of liquid in a tube will rise or fall due to the capillary effect.

Analysis The **liquid level in the tube will drop** since the contact angle is greater than 90°, and cos(110°) < 0.

Discussion This liquid must be a non-wetting liquid when in contact with the tube material. Mercury is an example of a non-wetting liquid with a contact angle (with glass) that is greater than 90^o.

2-59C

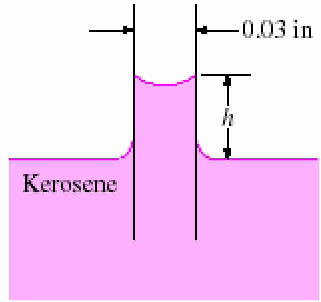
Solution We are to compare the capillary rise in small and large diameter tubes.

Analysis The capillary rise is inversely proportional to the diameter of the tube, and thus **capillary rise is greater in the smaller-diameter tube**.

Discussion Note however, that if the tube diameter is large enough, there is no capillary rise (or fall) at all. Rather, the upward (or downward) rise of the liquid occurs only near the tube walls; the elevation of the middle portion of the liquid in the tube does not change for large diameter tubes.

2-60E

Solution A slender glass tube is inserted into kerosene. The capillary rise of kerosene in the tube is to be determined.



Assumptions **1** There are no impurities in the kerosene, and no contamination on the surfaces of the glass tube. **2** The kerosene is open to the atmospheric air.

Properties The surface tension of kerosene-glass at 68°F (20°C) is σ_s = 0.028×0.06852 = 0.00192 lbf/ft. The density of kerosene at 68°F is ρ = 51.2 lbm/ft^3. The contact angle of kerosene with the glass surface is given to be 26°.

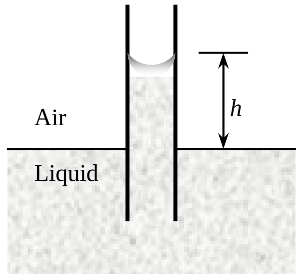
Analysis Substituting the numerical values, the capillary rise is determined to be

$$h = \frac{2\sigma_s \cos\phi}{\rho g R} = \frac{2(0.00192\ \text{lbf/ft})(\cos 26°)}{(51.2\ \text{lbm/ft}^3)(32.2\ \text{ft/s}^2)(0.015/12\ \text{ft})}\left(\frac{32.2\ \text{lbm}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right)$$
$$= 0.0539\ \text{ft} = \mathbf{0.650\ in}$$

Discussion The capillary rise in this case more than half of an inch, and thus it is clearly noticeable.

2-61

Solution A glass tube is inserted into a liquid, and the capillary rise is measured. The surface tension of the liquid is to be determined.



Assumptions **1** There are no impurities in the liquid, and no contamination on the surfaces of the glass tube. **2** The liquid is open to the atmospheric air.

Properties The density of the liquid is given to be 960 kg/m^3. The contact angle is given to be 15°.

Analysis Substituting the numerical values, the surface tension is determined from the capillary rise relation to be

$$\sigma_s = \frac{\rho g R h}{2\cos\phi} = \frac{(960\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.0019/2\ \text{m})(0.005\ \text{m})}{2(\cos 15°)}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{0.0232\ N/m}$$

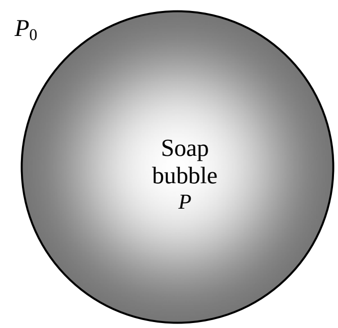
Discussion Since surface tension depends on temperature, the value determined is valid at the liquid's temperature.

2-62

Solution The diameter of a soap bubble is given. The gage pressure inside the bubble is to be determined.

Assumptions The soap bubble is in atmospheric air.

Properties The surface tension of soap water at 20°C is σ_s = 0.025 N/m.



Analysis The pressure difference between the inside and the outside of a bubble is given by

$$\Delta P_{\text{bubble}} = P_i - P_0 = \frac{4\sigma_s}{R}$$

In the open atmosphere P_0 = P_{atm}, and thus ΔP_{bubble} is equivalent to the gage pressure. Substituting,

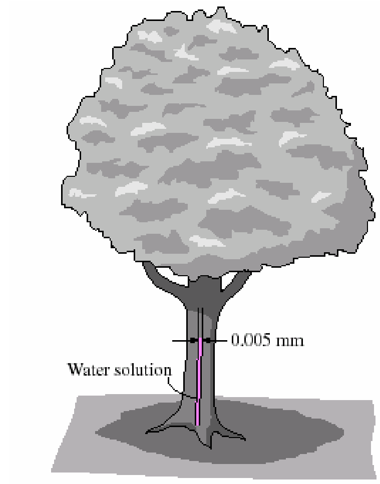
$$P_{i,gage} = \Delta P_{\text{bubble}} = \frac{4(0.025\ \text{N/m})}{0.002/2\ \text{m}} = 100\ \text{N/m}^2 = 100\ \text{Pa}$$

$$P_{i,gage} = \Delta P_{\text{bubble}} = \frac{4(0.025\ \text{N/m})}{0.05/2\ \text{m}} = 4\ \text{N/m}^2 = 4\ \text{Pa}$$

Discussion Note that the gage pressure in a soap bubble is inversely proportional to the radius. Therefore, the excess pressure is larger in smaller bubbles.

2-63

Solution Nutrients dissolved in water are carried to upper parts of plants. The height to which the water solution rises in a tree as a result of the capillary effect is to be determined.



Assumptions **1** The solution can be treated as water with a contact angle of 15°. **2** The diameter of the tube is constant. **3** The temperature of the water solution is 20°C.

Properties The surface tension of water at 20°C is σ_s = 0.073 N/m. The density of water solution can be taken to be 1000 kg/m^3. The contact angle is given to be 15°.

Analysis Substituting the numerical values, the capillary rise is determined to be

$$h = \frac{2\sigma_s \cos\phi}{\rho g R} = \frac{2(0.073\ \text{N/m})(\cos 15°)}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.5\times10^{-6}\ \text{m})}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = \mathbf{5.75\ m}$$

Discussion Other effects such as the chemical potential difference also cause the fluid to rise in trees.

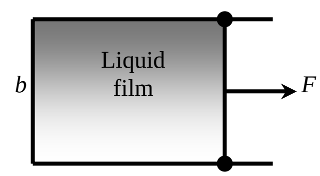
2-64

Solution The force acting on the movable wire of a liquid film suspended on a U-shaped wire frame is measured. The surface tension of the liquid in the air is to be determined.

Assumptions **1** There are no impurities in the liquid, and no contamination on the surfaces of the wire frame. **2** The liquid is open to the atmospheric air.

Analysis Substituting the numerical values, the surface tension is determined from the surface tension force relation to be

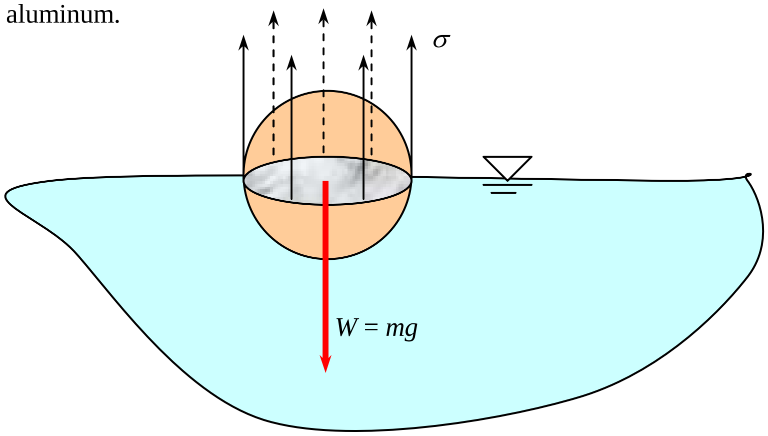
$$\sigma_s = \frac{F}{2b} = \frac{0.012\ \text{N}}{2(0.08\ \text{m})} = \mathbf{0.075\ N/m}$$



Discussion The surface tension depends on temperature. Therefore, the value determined is valid at the temperature of the liquid.

2-65

Solution A steel ball floats on water due to the surface tension effect. The maximum diameter of the ball is to be determined, and the calculations are to be repeated for aluminum.



Assumptions **1** The water is pure, and its temperature is constant. **2** The ball is dropped on water slowly so that the inertial effects are negligible. **3** The contact angle is taken to be 0° for maximum diameter.

Properties The surface tension of water at 20°C is σ_s = 0.073 N/m. The contact angle is taken to be 0°. The densities of steel and aluminum are given to be ρ_{steel} = 7800 kg/m^3 and ρ_{Al} = 2700 kg/m^3.

Analysis The surface tension force and the weight of the ball can be expressed as

$$F_s = \pi D\sigma_s \quad \text{and} \quad W = mg = \rho g V = \rho g \pi D^3/6$$

When the ball floats, the net force acting on the ball in the vertical direction is zero. Therefore, setting $F_s = W$ and solving for diameter D gives $D = \sqrt{\dfrac{6\sigma_s}{\rho g}}$. Substititing the known quantities, the maximum diameters for the steel and aluminum balls become

$$D_{steel} = \sqrt{\frac{6\sigma_s}{\rho g}} = \sqrt{\frac{6(0.073\text{ N/m})}{(7800\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)} = 2.4\times10^{-3}\text{ m} = \mathbf{2.4\ mm}$$

$$D_{Al} = \sqrt{\frac{6\sigma_s}{\rho g}} = \sqrt{\frac{6(0.073\text{ N/m})}{(2700\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)} = 4.1\times10^{-3}\text{ m} = \mathbf{4.1mm}$$

Discussion Note that the ball diameter is inversely proportional to the square root of density, and thus for a given material, the smaller balls are more likely to float.

Review Problems

2-66

Solution The pressure in an automobile tire increases during a trip while its volume remains constant. The percent increase in the absolute temperature of the air in the tire is to be determined.

Assumptions **1** The volume of the tire remains constant. **2** Air is an ideal gas.

Analysis Noting that air is an ideal gas and the volume is constant, the ratio of absolute temperatures after and before the trip are

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \rightarrow \frac{T_2}{T_1} = \frac{P_2}{P_1} = \frac{310\text{ kPa}}{290\text{ kPa}} = 1.069$$

Therefore, the absolute temperature of air in the tire will increase by **6.9%** during this trip.

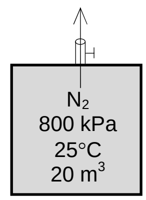
Discussion This may not seem like a large temperature increase, but if the tire is originally at 20°C (293.15 K), the temperature increases to 1.069(293.15 K) = 313.38 K or about 40.2°C.

2-67

Solution A large tank contains nitrogen at a specified temperature and pressure. Now some nitrogen is allowed to escape, and the temperature and pressure of nitrogen drop to new values. The amount of nitrogen that has escaped is to be determined.

Assumptions The tank is insulated so that no heat is transferred.

Analysis Treating N_2 as an ideal gas, the initial and the final masses in the tank are determined to be



$$m_1 = \frac{P_1 V}{RT_1} = \frac{(800\text{kPa})(20\text{m}^3)}{(0.2968\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(298\text{K})} = 180.9\text{kg}$$

$$m_2 = \frac{P_2 V}{RT_2} = \frac{(600\text{kPa})(20\text{m}^3)}{(0.2968\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\text{K})} = 138.0\text{kg}$$

Thus the amount of N_2 that escaped is $\Delta m = m_1 - m_2 = 180.9 - 138.0 =$ **42.9 kg**

Discussion Gas expansion generally causes the temperature to drop. This principle is used in some types of refrigeration.

2-68

Solution Suspended solid particles in water are considered. A relation is to be developed for the specific gravity of the suspension in terms of the mass fraction $C_{s,\text{ mass}}$ and volume fraction $C_{s,\text{ vol}}$ of the particles.

Assumptions **1** The solid particles are distributed uniformly in water so that the solution is homogeneous. **2** The effect of dissimilar molecules on each other is negligible.

Analysis Consider solid particles of mass m_s and volume V_s dissolved in a fluid of mass m_f and volume V_m. The total volume of the suspension (or mixture) is

$$V_m = V_s + V_f$$

Dividing by V_m and using the definition $C_{s,\text{ vol}} = V_s / V_m$ give

$$1 = C_{s,vol} + \frac{V_f}{V_m} \quad \rightarrow \quad \frac{V_f}{V_m} = 1 - C_{s,vol} \qquad (1)$$

The total mass of the suspension (or mixture) is

$$m_m = m_s + m_f$$

Dividing by m_m and using the definition $C_{s,\text{ mass}} = m_s / m_m$ give

$$1 = C_{s,mass} + \frac{m_f}{m_m} = C_{s,mass} + \frac{\rho_f V_f}{\rho_m V_m} \quad \rightarrow \quad \frac{\rho_f}{\rho_m} = (1 - C_{s,mass})\frac{V_m}{V_f} \qquad (2)$$

Combining equations 1 and 2 gives

$$\frac{\rho_f}{\rho_m} = \frac{1 - C_{s,mass}}{1 - C_{s,vol}}$$

When the fluid is water, the ratio ρ_f / ρ_m is the inverse of the definition of specific gravity. Therefore, the desired relation for the specific gravity of the mixture is

$$\boxed{\text{SG}_m = \frac{\rho_m}{\rho_f} = \frac{1 - C_{s,vol}}{1 - C_{s,mass}}}$$

which is the desired result.

Discussion As a quick check, if there were no particles at all, $\text{SG}_m = 0$, as expected.

2-69

Solution The specific gravities of solid particles and carrier fluids of a slurry are given. The relation for the specific gravity of the slurry is to be obtained in terms of the mass fraction $C_{s,\text{ mass}}$ and the specific gravity SG_s of solid particles.

Assumptions **1** The solid particles are distributed uniformly in water so that the solution is homogeneous. **2** The effect of dissimilar molecules on each other is negligible.

Analysis Consider solid particles of mass m_s and volume V_s dissolved in a fluid of mass m_f and volume V_m. The total volume of the suspension (or mixture) is $V_m = V_s + V_f$.

Dividing by V_m gives

$$1 = \frac{V_s}{V_m} + \frac{V_f}{V_m} \quad \rightarrow \quad \frac{V_f}{V_m} = 1 - \frac{V_s}{V_m} = 1 - \frac{m_s/\rho_s}{m_m/\rho_m} = 1 - \frac{m_s}{m_m}\frac{\rho_m}{\rho_s} = 1 - C_{s,\text{mass}}\frac{\text{SG}_m}{\text{SG}_s} \qquad (1)$$

since ratio of densities is equal two the ratio of specific gravities, and $m_s / m_m = C_{\text{s, mass}}$. The total mass of the suspension (or mixture) is $m_m = m_s + m_f$. Dividing by m_m and using the definition $C_{\text{s, mass}} = m_s / m_m$ give

$$1 = C_{s,\text{mass}} + \frac{m_f}{m_m} = C_{s,\text{mass}} + \frac{\rho_f V_f}{\rho_m V_m} \qquad \rightarrow \qquad \frac{\rho_m}{\rho_f} = \frac{V_f}{(1 - C_{s,mass})V_m} \qquad (2)$$

Taking the fluid to be water so that $\rho_m / \rho_f = \text{SG}_m$ and combining equations 1 and 2 give

$$\text{SG}_m = \frac{1 - C_{s,\text{mass}}\text{SG}_m / \text{SG}_s}{1 - C_{s,\text{mass}}}$$

Solving for SG_m and rearranging gives

$$\boxed{\text{SG}_m = \frac{1}{1 + C_{\text{s, mass}}(1/\text{SG}_s - 1)}}$$

which is the desired result.

Discussion As a quick check, if there were no particles at all, $\text{SG}_m = 0$, as expected.

2-70E

Solution The minimum pressure on the suction side of a water pump is given. The maximum water temperature to avoid the danger of cavitation is to be determined.

Properties The saturation temperature of water at 0.95 psia is 100°F.

Analysis To avoid cavitation at a specified pressure, the fluid temperature everywhere in the flow should remain below the saturation temperature at the given pressure, which is

$$T_{\text{max}} = T_{\text{sat @ 0.95 psia}} = \mathbf{100°F}$$

Therefore, ***T*** **must remain below 100°F to avoid the possibility of cavitation**.

Discussion Note that saturation temperature increases with pressure, and thus cavitation may occur at higher pressure at locations with higher fluid temperatures.

2-71

Solution Air in a partially filled closed water tank is evacuated. The absolute pressure in the evacuated space is to be determined.

Properties The saturation pressure of water at 60°C is 19.94 kPa.

Analysis When air is completely evacuated, the vacated space is filled with water vapor, and the tank contains a saturated water-vapor mixture at the given pressure. Since we have a two-phase mixture of a pure substance at a specified temperature, the vapor pressure must be the saturation pressure at this temperature. That is,

$$P_v = P_{\text{sat @ 60°C}} = 19.94 \text{ kPa} \cong \mathbf{19.9 \text{ kPa}}$$

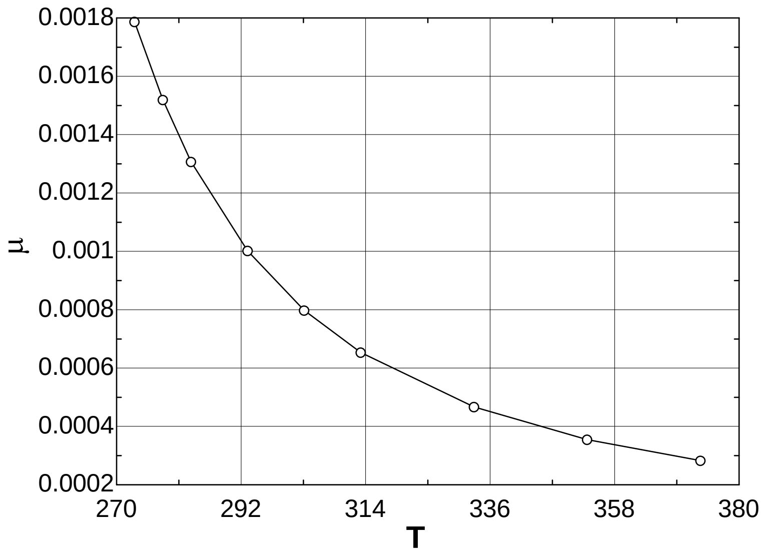
Discussion If there is any air left in the container, the vapor pressure will be less. In that case the sum of the component pressures of vapor and air would equal 19.94 kPa.

2-72

Solution The variation of the dynamic viscosity of water with absolute temperature is given. Using tabular data, a relation is to be obtained for viscosity as a 4$^{\text{th}}$-order polynomial. The result is to be compared to Andrade's equation in the form of $\mu = D \cdot e^{B/T}$.

Properties The viscosity data are given in tabular form as

T (K)	μ (Pa·s)
273.15	1.787×10^{-3}
278.15	1.519×10^{-3}
283.15	1.307×10^{-3}
293.15	1.002×10^{-3}
303.15	7.975×10^{-4}
313.15	6.529×10^{-4}
333.15	4.665×10^{-4}
353.15	3.547×10^{-4}
373.15	2.828×10^{-4}



Analysis Using EES, (1) Define a trivial function "a=mu+T" in the equation window, (2) select new parametric table from Tables, and type the data in a two-column table, (3) select Plot and plot the data, and (4) select plot and click on "curve fit" to get curve fit window. Then specify polynomial and enter/edit equation. The equations and plot are shown here.

μ = 0.489291758 - 0.00568904387T + 0.0000249152104T^2 - 4.86155745×10$^{-8}T^3$ + 3.56198079×10$^{-11}T^4$
μ = 0.000001475*EXP(1926.5/T) [used initial guess of a0=1.8×10^{-6} and a1=1800 in mu=a0*exp(a1/T)]

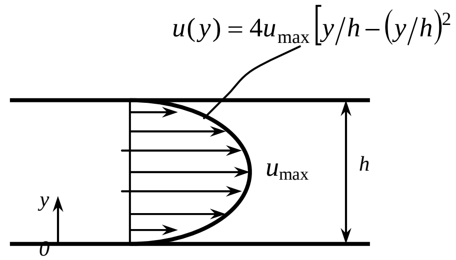
At T = 323.15 K, the polynomial and exponential curve fits give

Polynomial: μ(323.15 K) = 0.0005529 Pa·s (1.1% error, relative to 0.0005468 Pa·s)
Exponential: μ(323.15 K) = 0.0005726 Pa·s (4.7% error, relative to 0.0005468 Pa·s)

Discussion This problem can also be solved using an Excel worksheet, with the following results:
Polynomial: **A = 0.4893, B = -0.005689, C = 0.00002492, D = -0.000000048612, and E = 0.00000000003562**
Andrade's equation: $\boxed{\mu = 1.807952E-6 * e^{1864.06/T}}$

2-73

Solution The velocity profile for laminar one-dimensional flow between two parallel plates is given. A relation for friction drag force exerted on the plates per unit area of the plates is to be obtained.



Assumptions **1** The flow between the plates is one-dimensional. **2** The fluid is Newtonian.

Analysis The velocity profile is given by $u(y) = 4u_{max}\left[y/h - (y/h)^2\right]$

where h is the distance between the two plates, y is the vertical distance from the bottom plate, and u_{max} is the maximum flow velocity that occurs at midplane. The shear stress at the bottom surface can be expressed as

$$\tau_w = \mu \left.\frac{du}{dy}\right|_{y=0} = 4\mu u_{max} \frac{d}{dy}\left(\frac{y}{h} - \frac{y^2}{h^2}\right)_{y=0} = 4\mu u_{max}\left.\left(\frac{1}{h} - \frac{2y}{h^2}\right)\right|_{y=0} = \frac{4\mu u_{max}}{h}$$

Because of symmetry, the wall shear stress is identical at both bottom and top plates. Then the friction drag force exerted by the fluid on the inner surface of the plates becomes

$$F_D = 2\tau_w A_{plate} = \frac{8\mu u_{max}}{h} A_{plate}$$

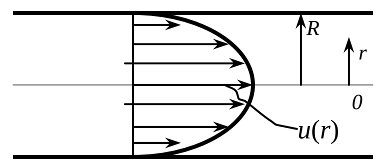
Therefore, the friction drag per unit plate area is

$$\boxed{F_D / A_{plate} = \frac{8\mu u_{max}}{h}}$$

Discussion Note that the friction drag force acting on the plates is inversely proportional to the distance between plates.

2-74

Solution The laminar flow of a Bingham plastic fluid in a horizontal pipe of radius R is considered. The shear stress at the pipe wall and the friction drag force acting on a pipe section of length L are to be determined.



Assumptions **1** The fluid is a Bingham plastic with $\tau = \tau_y + \mu(du/dr)$ where τ_y is the yield stress. **2** The flow through the pipe is one-dimensional.

Analysis The velocity profile is given by $u(r) = \frac{\Delta P}{4\mu L}(r^2 - R^2) + \frac{\tau_y}{\mu}(r - R)$ where $\Delta P/L$ is the pressure drop along the pipe per unit length, μ is the dynamic viscosity, r is the radial distance from the centerline. Its gradient at the pipe wall ($r = R$) is

$$\left.\frac{du}{dr}\right|_{r=R} = \frac{d}{dr}\left.\left(\frac{\Delta P}{4\mu L}(r^2 - R^2) + \frac{\tau_y}{\mu}(r - R)\right)\right|_{r=R} = \left(2r\frac{\Delta P}{4\mu L} + \frac{\tau_y}{\mu}\right)_{r=R} = \frac{1}{\mu}\left(\frac{\Delta P}{2L}R + \tau_y\right)$$

Substituing into $\tau = \tau_y + \mu(du/dr)$, the wall shear stress at the pipe surface becomes

$$\tau_w = \tau_y + \mu\left.\frac{du}{dr}\right|_{r=R} = \tau_y + \frac{\Delta P}{2L}R + \tau_y = 2\tau_y + \frac{\Delta P}{2L}R$$

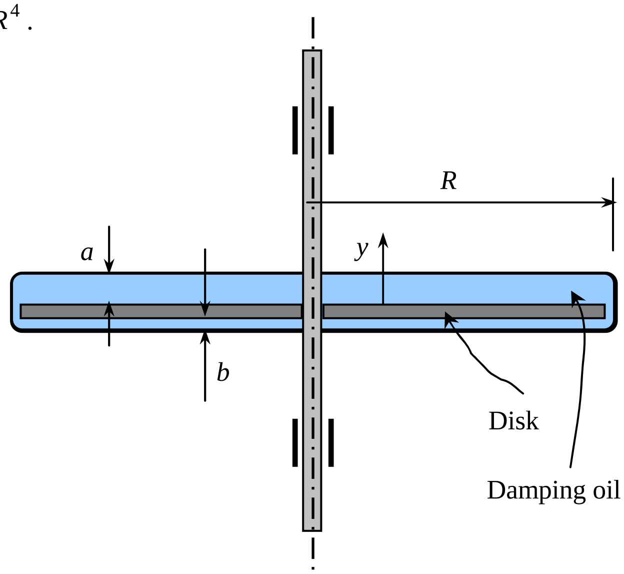
Then the friction drag force exerted by the fluid on the inner surface of the pipe becomes

$$\boxed{F_D = \tau_w A_s = \left(2\tau_y + \frac{\Delta P}{2L}R\right)(2\pi RL) = 2\pi RL\left(2\tau_y + \frac{\Delta P}{2L}R\right) = 4\pi RL\tau_y + \pi R^2 \Delta P}$$

Discussion Note that the total friction drag is proportional to yield shear stress and the pressure drop.

2-75

Solution A circular disk immersed in oil is used as a damper, as shown in the figure. It is to be shown that the damping torque is $\mathrm{T}_{\text{damping}} = C\omega$ where $C = 0.5\pi\mu(1/a + 1/b)R^4$.



Assumptions **1** The thickness of the oil layer on each side remains constant. **2** The velocity profiles are linear on both sides of the disk. **3** The tip effects are negligible. **4** The effect of the shaft is negligible.

Analysis The velocity gradient anywhere in the oil of film thickness *a* is *V*/*a* where *V* = ω*r* is the tangential velocity. Then the wall shear stress anywhere on the upper surface of the disk at a distance *r* from the axis of rotation can be expressed as

$$\tau_w = \mu\frac{du}{dy} = \mu\frac{V}{a} = \mu\frac{\omega r}{a}$$

Then the shear force acting on a differential area *dA* on the surface and the torque it generates can be expressed as

$$dF = \tau_w dA = \mu\frac{\omega r}{a}dA$$

$$d\mathrm{T} = r dF = \mu\frac{\omega r^2}{a}dA$$

Noting that $dA = 2\pi r dr$ and integrating, the torque on the top surface is determined to be

$$\mathrm{T}_{\text{top}} = \frac{\mu\omega}{a}\int_A r^2 dA = \frac{\mu\omega}{a}\int_{r=0}^{R} r^2(2\pi r)dr = \frac{2\pi\mu\omega}{a}\int_{r=0}^{R} r^3 dr = \frac{2\pi\mu\omega}{a}\frac{r^4}{4}\bigg|_{r=0}^{R} = \frac{\pi\mu\omega R^4}{2a}$$

The torque on the bottom surface is obtained by replaying *a* by *b*,

$$\mathrm{T}_{\text{bottom}} = \frac{\pi\mu\omega R^4}{2b}$$

The total torque acting on the disk is the sum of the torques acting on the top and bottom surfaces,

$$\mathrm{T}_{\text{damping, total}} = \mathrm{T}_{\text{bottom}} + \mathrm{T}_{\text{top}} = \frac{\pi\mu\omega R^4}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$$

or,

$$\mathrm{T}_{\text{damping,total}} = C\omega \quad \text{where} \quad C = \frac{\pi\mu R^4}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$$

This completes the proof.

Discussion Note that the damping torque (and thus damping power) is inversely proportional to the thickness of oil films on either side, and it is proportional to the 4th power of the radius of the damper disk.

2-76E

Solution A glass tube is inserted into mercury. The capillary drop of mercury in the tube is to be determined.

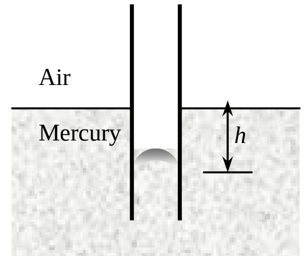
Assumptions **1** There are no impurities in mercury, and no contamination on the surfaces of the glass tube. **2** The mercury is open to the atmospheric air.

Properties The surface tension of mercury-glass in atmospheric air at 68°F (20°C) is σ_s = 0.440×0.06852 = 0.03015 lbf/ft. The density of mercury is ρ = 847 lbm/ft^3 at 77°F, but we can also use this value at 68°F. The contact angle is given to be 140°.

Analysis Substituting the numerical values, the capillary drop is determined to be

$$h = \frac{2\sigma_s \cos\phi}{\rho g R} = \frac{2(0.03015\text{ lbf/ft})(\cos 140°)}{(847\text{ lbm/ft}^3)(32.2\text{ ft/s}^2)(0.45/12\text{ ft})}\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)$$

$$= -0.00145\text{ ft} = \mathbf{-0.0175\ in}$$



Discussion The negative sign indicates capillary drop instead of rise. The drop is very small in this case because of the large diameter of the tube.

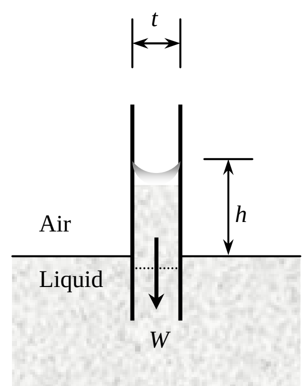
2-77

Solution A relation is to be derived for the capillary rise of a liquid between two large parallel plates a distance t apart inserted into a liquid vertically. The contact angle is given to be ϕ.

Assumptions There are no impurities in the liquid, and no contamination on the surfaces of the plates.

Analysis The magnitude of the capillary rise between two large parallel plates can be determined from a force balance on the rectangular liquid column of height h and width w between the plates. The bottom of the liquid column is at the same level as the free surface of the liquid reservoir, and thus the pressure there must be atmospheric pressure. This will balance the atmospheric pressure acting from the top surface, and thus these two effects will cancel each other. The weight of the liquid column is

$$W = mg = \rho g V = \rho g(w \times t \times h)$$



Equating the vertical component of the surface tension force to the weight gives

$$W = F_{surface} \quad \rightarrow \quad \rho g(w \times t \times h) = 2w\sigma_s \cos\phi$$

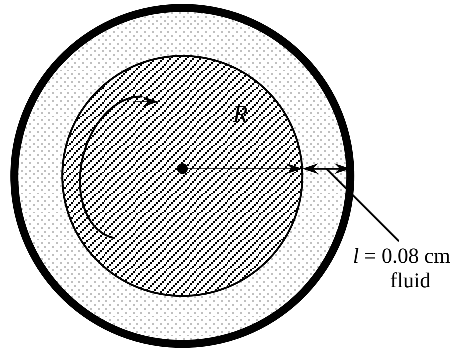
Canceling w and solving for h gives the capillary rise to be

Capillary rise: $$h = \frac{2\sigma_s \cos\phi}{\rho g t}$$

Discussion The relation above is also valid for non-wetting liquids (such as mercury in glass), and gives a capillary drop instead of a capillary rise.

2-78

Solution A journal bearing is lubricated with oil whose viscosity is known. The torques needed to overcome the bearing friction during start-up and steady operation are to be determined.



Assumptions **1** The gap is uniform, and is completely filled with oil. **2** The end effects on the sides of the bearing are negligible. **3** The fluid is Newtonian.

Properties The viscosity of oil is given to be 0.1 kg/m·s at 20°C, and 0.008 kg/m·s at 80°C.

Analysis The radius of the shaft is R = 0.04 m. Substituting the given values, the torque is determined to be

At start up at 20°C:

$$\mathbf{T}=\mu\frac{4\pi^2R^3\dot{n}L}{\ell}=(0.1\,\text{kg/m}\cdot\text{s})\frac{4\pi^2(0.04\,\text{m})^3(500/60\,\text{s}^{-1})(0.30\,\text{m})}{0.0008\,\text{m}}=\mathbf{0.79\,N\cdot m}$$

During steady operation at 80°C:

$$\mathbf{T}=\mu\frac{4\pi^2R^3\dot{n}L}{\ell}=(0.008\,\text{kg/m}\cdot\text{s})\frac{4\pi^2(0.04\,\text{m})^3(500/60\,\text{s}^{-1})(0.30\,\text{m})}{0.0008\,\text{m}}=\mathbf{0.063\,N\cdot m}$$

Discussion Note that the torque needed to overcome friction reduces considerably due to the decrease in the viscosity of oil at higher temperature.

Design and Essay Problems

2-79 to 2-81

Solution Students' essays and designs should be unique and will differ from each other.