An Introduction to Engineering Economics

Kal Renganathan Sharma

MOMENTUM PRESS ENGINEERING

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Prairie View A&M University

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A publication in the Momentum Press Engineering Management
collection

Cover and interior design by S4Carlisle Publishing Services Private Ltd.,
Chennai, India

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Dedication

*This book is dedicated
to my eldest son
R Hari Subrahmanyan Sharma
(alias Ramkishan)
who turned nine on Aug-13-2010.*

About the Author

Kal Renganathan Sharma serves as Adjunct Professor of Chemical Engineering at the Roy G. Perry College of Engineering at Prairie View A & M University, Prairie View, TX. He instructs CHEG 2003 Economic Analysis and Technical Applications. He received his B.Tech. from Indian Institute of Technology, Chennai, India, in 1985, MS and Ph.D degrees in 1987 and 1990 from West Virginia University; Morgantown, WV, all three degrees in chemical engineering. He received the once in a school time prize from nursery to higher secondary school for the Best All Round Student of the Penultimate year from the Rev. Brothers of St. Gabriel at RSK Higher Secondary School, Trichy; India. He has held a number of high level academic positions in India and USA.

Dr. Sharma, KR, is the author of 10 books, 4 Book-Chapters, 19 journal articles, 503 conference papers and 108 other presentations. He has instructed over 2500 students in 83 semesterwise courses in India and USA over the past 15 years.

Honors received include Honorary Fellowship, Australian Institute of High Energetic Materials, Melbourne, Australia, 2010, Fellow, Indian Chemical Society; Kolkata, India, 2007, Who's Who in America, 2009–2010, Marquis Who's Who, Upper Saddle River, NJ, International Biographical Center, Cambridge, UK, 2007, member, New York Academy of Sciences, New York, NY, 1996, Phi Kappa Phi, 1990, Sigma Xi, 1995, cash award, SASTRA University, Thanjavur, India, 2005, cash award, Monsanto, Indian Orchad, MA, 1990, Who's Who in Plastics and Polymers, CRC Press, Baco Raton, FL, 2000. He is a naturalized citizen of USA.

Preface

The economy has changed rapidly. Both the nation's economy and the world economy has undergone changes since the World War II. The end of cold war has given impetus to rise of globalization. China and India are now invited to attend G20 meetings. Engineering education imparts a variety of skills to the student. Skills from economics can be synergistically applied. The engineering economy is a field of endeavor that explains different methods to evaluate alternates available to the business owner. Engineering Economy is the study of the feasibility and evaluation of the cost of possible solutions to engineering problems. When benefits outweigh costs the alternate becomes a acceptable one. The lowest cost among alternates can be selected by using different methods discussed in detail in the textbook. This is calculated at a certain interest rate over a certain prescribed period of time.

The inverse problem of determining the interest rate, i, given the F, A and N is explained in detail in this work. The binomial series expansion of $(1 + i)^N$ is used to obtain mathematical expressions. N can be estimated given the future amount, F, uniform series of payments, A and interest rate i by obtaining the logarithms where necessary.

The fundamental principles, concepts and methods of engineering economy are provided. Current trends and issues are reflected in the worked examples and end of chapter exercises. Alnernates are drawn from case studies. These include the Keystone pipeline project, CW of World Trade Center, six different methods to prepare bioethanol, biodiesel plant in Taiwan, micro power plant, combined cycle power plant, meglumine antimonoate drug from supercritical extraction, on-shore oil well, solar hats, stirling dish solar power plant, photovoltaic panel solar power plant, liquefaction plants, copper chlorine thermochemical cycle for hydrogen, energy efficient windows, continuous mass polymerization process to manufacture ABS, renovation of Macy's department store, heap leaching and agitation leaching of gold, rechargeable batteries for electric car, PEVs

and HEV, hybrid electric vehicles, four different cooling tower designs, oil refinery, economics of life insurance, social security, printing press vs. e-book, anti-allergic cream, sequestration by dimethyl carbonate formation, microfiltration, SWRO, sea water reverse osmosis plant, catalytic process to manufacture CNTs, carbon nanotubes, hexane extraction of rice bran oil, municipal garbage collection truck, activated carbon from bamboo, polypropylene gas manufacture, custom foam fabricator, car rental, coffee house, frozen yoghurt parlor, short path distillation, olive oil business, bonds from private sector, Chevy volt, public internet, gas sales and convenience store, condominiums at Galveston beach, outpatient drug treatment center, overseas travel ticket, snack factory. Break-even points in flower shop, soda beverage, cars, quandratic price-demand relations for tomatoes, peaches, inelastic supply of common salt, dualistic relations of price and demand, Le-Chatellier-Samuelson principle are discussed in detail. G20 nations and GDP estimates by PPP, purchasing power parity and the changing world order in economies is touched upon. Life-cycle costs of aviation lighting, LED, photocopier, HVAC systems are discussed. Depreciation, taxes, replacement analysis are discussed.

Unique Features

Optimization concepts are discussed in detail. Brainstorming can be used to develop alternates for a given problem.

- The five methods of analysis, PW, FW; AW, IRR and ERR can be used to evaluate the alternates. Evaluation of alternates can be used to invest in the more profitable process, selection of the lower cost alternate, quantitate the savings accrued from implementation of a process improvement and assessment of environmental impact of chemical processes. Alternates can be mutually exclusive, independent projects and screening by other criteria.
- When the PWs of both alternates are within 20% of each other the projects can be adjudged as too close to call within the sensitivity of the estimates of the capital and operating expenses and expected revenues.

- Rudiments of household finance are discussed.
- The capitalization worth factors and sinking fund factors are calculated and presented as annuity tables as well as in graphical form.
- In addition to P/A, F/A, NF, NP, F/P, P/F closed form analytical expressions to calculate the interest rate, i and pay period, N has been developed.
- Benefit/Cost Ratio of public sector projects are discussed.

CHAPTER 1

Fundamentals
of Engineering Economy

Main Topics Discussed in this Chapter
- Definitions of Engineering, Economy
- Alternates and Rules of Brainstorming
- Economic Merits of Alternates
- Deviation Analysis, Decision Analysis
- Action Plan
- Principles of Engineering Economy
- Steps of Problem Solving

1.1   What Is Engineering, What Is Economy, and What Is Engineering Economy?

Technological advances in machines, materials, computers, structures, and electronics have changed the job description of the engineer rapidly. Economic merits of the different alternatives available to the solution of a given problem can be evaluated in a systematic manner. The dollars-and-cents side of decision making is discussed in detail in this book. According to an estimate in 2005, there are about 240 million vehicles on the road in the United States. The economy fueled by the automotive sector is large.

Engineering is defined by ABET (Accreditation for Engineering and Technology) as the profession in which the knowledge gained in physics, chemistry, life sciences, and mathematics is applied to make products in large scale that increase the prosperity of man. This must be achieved

with a judicious choice of materials, at the lowest cost in a manner that is benign to the environment, and that keeps all the stakeholders safe. Examples of engineering achievements can be seen in the top engineering inventions of the 20th century. According to the National Academy of Engineering, the top 20 engineering achievements of the 20th century are listed in Table 1.1 (1).

Economy consists of the sum total of all income from goods produced and services offered in a state or nation. It deals with the interactions between people and wealth. The size of the United States' economy can be measured using parameters such as GDP (Gross Domestic Product) GNP (Gross National Product) is different from GDP with respect to inclusion of income earned abroad. GDP sticks to within the domicile of the nation. The sizes of the economies of 20 nations that meet regularly as G20 are listed in Table 1.2. The 19 nations, other than the European Union, are listed according to the sizes of their economies. The numbers in Table 1.2 were developed by the prices laid down by the IMF, the International Monetary Fund, for the year 2008.

The combined GDP of 19 nations listed in Table 1.2 is about 51 trillion US dollars. This is a significant portion of the *world economy*.

Currency exchange rates used to be based upon the wealth of nations as measured by the amount of gold each nation possessed. The wealth of

Table 1.1 Top 20 Engineering Achievements of the 20th Century

#	Feat	#	Feat
1	Electrification	11	Highways
2	Automobile	12	Spacecraft
3	Airplane	13	Internet
4	Water Supply and Distribution	14	Imaging
5	Electronics	15	Household Appliances
6	Radio and Television	16	Health Technologies
7	Agricultural Mechanization	17	Petroleum and Petrochemical Technologies
8	Computers	18	Laser and Fiber Optics
9	Telephone	19	Nuclear Technologies
10	Air Conditioning and Refrigeration	20	High Performance Materials

Table 1.2 GDP of G20 Nations (Purchasing Power Parity or PPP)

#	Nation	GDP (in Billions of US$) PPP	#	Nation	GDP (in Billions of US$) PPP
1	United States	14,256	11	Mexico	1466
2	China	7903	12	South Korea	1364
3	Japan	4159	13	Canada	1281
4	India	3526	14	Indonesia	962
5	Germany	2806	15	Turkey	880
6	United Kingdom	2139	16	Australia	851
7	Italy	2118	17	Argentina	584
8	Russia	2110	18	South Africa	493
9	France	2108	19	Saudi Arabia	370
10	Brazil	2013	20	European Union+	14,793

a nation is not the same as the nation's economy. The calculation of the size of economy varies among one method to another. The PPP method used to calculate the size of the G20 economies listed in Table 1.2 is called the *Purchasing Power Parity* method. This idea can be traced back to the School of Salamanca in the 16th century. The concept stems from the development of exchange rates between two countries based on long-term considerations of relative price levels between the two countries. Identical goods are expected to have the same price and cannot vary with markets. According to Wikipedia (2), more haircuts can be purchased in Thanjavur, India, for 1 US dollar than in San Francisco, California. The PPP method provides for cost-of-living differences between different regions in the world. In 2003, 1 US$ was equivalent to 1.8 yuan (Chinese) using PPP method. The nominal exchange rate is 1 US$ exchanged at 7.6 yuan (Chinese). Based on PPP, China is the world's second largest economy and India is the world's fourth largest economy. India recently added 100 million cellular phones. It has the second largest sugar production by the sugarcane industry. The cement and paper industries are large in India. The population of both China and India are more than 1 billion each. When the economic activities of *all people* are taken into account, the sizes of their economies go through a sudden jump in value!

According to Mahatma Gandhi, India consisted of 300,000 villages during the freedom struggle from Great Britain. According to the 2001 India census, 74% of Indians live in 638,365 different villages. The middle-class wallet in India is bulging. The world's least expensive car was launched recently in India: Tata Nano can be picked up for Rs. 1 lakh (nominally about 2105 US$). The discussions of G20 and a PPP method of calculation are illustrative of the need to be better informed about the evaluation methods of economic alternatives offered by engineering solutions. The *basis* used for the calculations is important.

Engineering Economy is the study of the feasibility and evaluation of the cost of possible solutions to engineering problems. When benefits outweigh costs, the alternate becomes an *acceptable* one. The lowest cost among alternates can be selected by using different methods discussed in detail in the textbook. This is calculated at a certain interest rate over a certain prescribed period of time. Here are some alternates that can be resolved into one selected for implementation by using one of the methods explained in later chapters:

(i) The inclusion of ER fluids (electro rheological fluids) as automatic transmission fluids by Chrysler Corp. is going to increase the mpg of the new line called Dodge Cognizance by 15%. It extends the life of the transmission by 20,000 miles. How much can the company afford to spend in order to implement this invention?

(ii) You are hired by a leading credit card company, Ford Chase Bank. One of their directors, Julie Hollas, wants you to perform a small calculation for her. A customer wants to charge a Japanese robot to his credit card. The price of the robot, including sales tax, is $850.50. How many years will it take to pay off the credit card balance if the monthly payment is $16.70 and the annualized credit card interest rate is 23%?

(iii) Gasoline prices have skyrocketed in the nation. The national average price per gallon of gasoline has risen to $4. The president is considering increasing off-shore drilling to get more oil. Each oil rig would cost $10 million. It would require 70,000 oil rigs to deliver 0.5 billion gallons of gas for automobiles every week. The utility costs run to $120,000 per year. The situation is expected to

continue for the next four years. Calculate the IRR for this investment. The MARR can be taken as 8.0%.

(iv) Calculate the monthly payment on a 2007 Toyota Corolla costing $12,999. Another $1000 was needed to obtain the title, license plate, and taxes. After cash down of $1500, the number of pay periods was 72 months. The interest rate per annum is 7%. A $2500 one-time charge for power train warranty up to 100,000 miles was levied up front.

(v) Alice Jones operates a flower shop in downtown Houston. It costs her $5 per day to pay the rent, $1 per day for the water used to preserve the freshness of the flowers, and $2.5 per day for air conditioning the flowers per day. The cost to pluck and bring the flowers from the garden in rural Texas to Houston city runs her a quarter per flower. The price-demand relation for the flowers takes the form of:

$$p = 0.65 - 0.001D$$

Calculate the break-even points. Construct the total revenue and total cost as a function of demand, D in the attached graph paper. What is the demand at maximum profit to Alice?

(vi) K. R. Sridhar, CEO of Bloom Box, was featured on CBS's *60 Minutes*. Bloom Box makes *micro power plants* using SOFC, solid oxide fuel cell technology. A bloom box can be purchased for $3000. It consists of miniaturized fuel cells made of silica. The cost of electricity to power up a new home would be 8.0 cents per kWh. Should the existing utility company charge 12.5 cents per kWh, calculate the number of years it would take for a bloom box to be a good investment for new homeowner Jack Tripper. Jack Tripper is the head of a family of 6 members with 4 children. Use the present worth method to evaluate the single project of micro power plants for individual homes. Assume that 600 kWh of electricity is consumed every month at the Tripper residence. (i=10%).

(vii) You are hired by Cowboy Investments upon graduation. They are developing a new investment plan for customers named "U-Pay-Now & I-Pay-Later." The deal is that the client pays $500 every month to Cowboy Investments. After a said period in a number

of years, Cowboy Investments will have the client cease payments and begin to receive $500 every month forever. At 10% annualized interest rate (the historic performance of Dow Jones), how many years would you recommend be the said period?

(viii) Furniture-to-Stay offers you the following deal: A sofa, loveseat, and chair are available for $1250. A mattress is built in with the sofa, which can be pulled out when you have guests over. This is at an additional cost of $200. The delivery charge is $100 and a 3-year warranty for stain resistance is available for $200. For the first 3 years there is no interest rate charged (i.e., "No Interest 'Til 2012"). For the following three years, an annualized interest rate of 12% is charged. Calculate the monthly payment over 6 years.

(ix) Sam Malone is considering whether to invest in solar panels. This homeowner has gathered the following information for his town:
   (a) Average cost of residential energy is 12 cents per kWh
   (b) Government subsidy runs to 60% of investment by homeowner
   (c) Each square foot of panel produces 18 kWh every year and the cost of installation per square foot of solar panel is $99.

Sam has 385 sq. ft. of roof space. What is the break-even point in years for the installation of solar panels? Assume an interest rate of 5% per year.

(x) Should overnight delivery courier mail be conducted using airplanes flown by pilots or personal rapid transit systems without onboard operators?

(xi) Should assembly lines for putting together desktop computers be filled with workers or by automated robots?

(xii) Should college computer labs be serviced using a hotline or by a manned computer help desk?

Example 1.0 Start of Technocrats of Texas

A team of alumni and faculty from a leading university in Texas conceived a business plan to develop a consulting service on how to assess, prevent, and remediate the damage caused by oil spills such as the recent one by BP America in Houston. One hundred professionals and

technicians are going to join the venture. A new building is obtained at lease in Hempstead, Texas. How would you develop the alternates for setting up the utilities, such as electrification of the building?

Alternate A: To sign up with companies such as Reliant Energy under one of their plans.

Alternate B: To install solar panels and generate all the electricity needs of the enterprise.

Alternate C: To buy enough fuel cells and install a micro power plant to meet all the electricity needs of the enterprise.

1.2   Seven Principles of Engineering Economy

The foundation of the discipline of Engineering Economy can be seen in terms of seven principles (3). These seven principles are:

1.0   Make a List of Alternates: Plan A, Plan B, etc.
2.0   What Is Different among the Alternates?
3.0   Be Clear on What You Want.
4.0   Develop Common Performance Measures.
5.0   Meet ALL Relevant Criteria.
6.0   Weigh the Risk Against the Projected Rewards.
7.0   Check the Results of Action Plan and Revise Plan If/When Necessary.

The decision has to come from the alternates developed. The alternates can be developed using a *brainstorming session*. The more creative and resourceful the team members are, the better the selection of alternates available for a go/no-go decision would be. The problem can be reformulated and restated to ease the flow of alternates. An open mind is recommended during the development of alternates. The method may not be used to justify a certain course of action. Four rules of brainstorming were dictated by Osborn (4). These are as follows:

(a) Negative feedback can become overly critical and stifle creative juices and alternates from flowing.

(b) Poet-like free flow of ideas is welcome.
(c) Quantity breeds quality.
(d) Improvement can be aimed. Combinations can work out.

The basic steps of brainstorming include:

(a) Preliminary Discussion: Problem is stated and participants get warmed up. This acts as a prelude to the main session.
(b) Brainstorming Main Session: An unrelated problem is first brainstormed. Then the assigned problem is brainstormed. Ideas are allowed to flow and are recorded objectively.
(c) The ideas are evaluated for possible flowering into alternates.

In Principle 2, deviation analysis is used and the focus is on the differences. The future outcomes from the alternates are set as higher priority. Based on differences among alternates, a future course of action is selected. Goal setting is extremely important as emphasized in Principle 3. A common unit of measure (such as dollars and cents) totals costs or total profits that can be generated from the alternate considered is captured in Principle 4. All relevant criteria as mentioned in Principle 5 are important. For instance, retail giant Wal-Mart posts low prices for its customers. However, the company had to pay several million dollars as compensation for discrimination based on the skin color of an African American woman. She was apprehended for forging her money order. Later, she was cleared of any wrongdoing. The Civil Rights Act of 1964, signed into law by President Lyndon B. Johnson, must be followed in all activities of the enterprise. The risks assumed in a potential venture must be identified and understood. The payoff or the reward-to-risk ratio can be considered before taking a decision. One can follow the *Plan-Do-Check*. The results from implementation of the plan can be checked against goals identified. If needed, the plan can be revised.

1.3   Summary

Engineering is defined by ABET (Accreditation for Engineering and Technology) as the profession in which the knowledge gained in physics,

chemistry, life sciences, and mathematics is applied to make products in large scale that increase the prosperity of man. This must be achieved with a judicious choice of materials, at the lowest cost in a manner that is benign to the environment, and that keeps all the stakeholders safe. *Economy* consists of the sum total of all income from goods produced and services offered in a state or nation. It deals with the interactions between people and wealth. *Engineering Economy* is the study of the feasibility and evaluation of the cost of possible solutions to engineering problems. When benefits outweigh costs, the alternate becomes an *acceptable* one. The lowest cost among alternates can be selected by using different methods discussed in detail in the textbook.

Examples of alternates are given such as electro rheological fluids for automatic transmission fluid, a pay-off period for a Japanese robot using a credit card, off-shore drilling, monthly car payment, flower shop, micro power plant, pay-now-cash-in-later schemes, deferred interest, solar panels, overnight mail delivery, use of robots in assembly lines in place of labor, college computer lab help desk, etc. The GDP of nations is discussed. The top engineering achievements of the 20th century are listed.

The seven principles of engineering economy are: (i) develop alternates; (ii) differences among alternates; (iii) what do you want?; (iv) develop common performance measures; (v) meet all relevant criteria; (vi) weigh the risks against the projected rewards; (vii) check the results of action plan and revise the plan if necessary. Brainstorming sessions can be used to develop alternates. The four rules of brainstorming are not to give negative feedback; encourage poet-like free flow of ideas; quantity breeds quality; and improvements and combinations are good. The main steps of brainstorming include: (i) preliminary session; (ii) brainstorming session; (iii) evaluation of ideas and development of alternates.

1.4 References

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CHAPTER 2

Variable and Fixed Costs

Topics Covered
- Delineate Direct and Indirect Costs
- Familiarity with Cash Cost, Book Cost, Sunk Cost, and Opportunity Cost
- Fixed, Variable, and Incremental Costs
- Life Cycle of Enterprise
- Total Cost, Total Revenue, and Profit
- Break-Even Analysis
- Second Break-Even Point—What Does This Mean?

2.1   One-Time and Recurring Costs

An eminent expert in the field of business administration defines the ABCs of the MBA program as: (i) Accounting; (ii) Finance; and (iii) Economics. Executives tending to do well with limited resources usually thank accountants for where they derived their critical information. Sound accounting practices need a good understanding of costing principles. The types of costs can be delineated based on the frequency of their occurrence. For instance, *fixed costs* are usually one-time occurrences. *Variable costs* occur more frequently, usually once every billing period. *Incremental costs* are those that are incurred proportionally to the number of goods produced. Instances of fixed costs, variable costs, and incremental costs can be seen in Example 2.1.

In the Garlic Bread Nibbler snack factory as discussed below in Example 2.1, the following are examples of fixed costs:

- Property Tax
- Equipment Cost of Kneader
- Equipment Cost of Mixer
- Equipment Cost of Chopper
- Equipment Cost of Roaster
- Equipment Cost of Coloring Bowl
- Insurance Fees
- Interest on Borrowed Capital
- General Management & Administration Salaries.

Examples of variable costs are:

- Utility Costs: Gas Heating, Telephone Wires, Electric Utilities, Air Conditioning
- Labor Benefits.

Examples of incremental costs:

- Wheat Flour
- Maize Flour
- Butter
- Salt.

The planned costs per unit of output, anticipated labor hours, materials, and overhead categories are called *standard costs*. *Direct costs* can be measured and attributed to a specific activity or output. *Indirect costs* cannot be attributed to a single activity. Secretarial staff in an office building is an example of overhead costs. In addition to the above discussion, the following costs are also explained:

Cash Cost:	Transactions that involve payment of cash.
Book Cost:	This is a noncash cost such as depreciation on machinery, etc.

Sunk Cost:	Cost that has occurred in the past and has no relevance to estimates of future costs and revenues. It is a nonrefundable cash outlay. Examples of sunk cost are money spent on a passport or an earnest money deposit on a new home purchase.
Opportunity Cost:	Incurred because of the use of limited resources. For example, consider a recent graduate from a BS program at an Ivy League institution. When he signs up for graduate school, the salary he may have made from joining the work force is foregone. The difference in the salary in work force and the stipend received during graduate studies, say for five years, is called the opportunity cost.
Investment Cost:	The capital required in the acquisition phase is called investment cost.
O & M Cost:	Operation and Maintenance cost. This can be incurred by hiring people, purchasing machines, materials, information, and energy.
Working Capital:	These are funds required for the day-to-day operation of the enterprise.
Salvage Value:	The trade-in value of equipment after its full use is called salvage value.

A state of perfect competition is said to exist when there is more than one supplier of a product in demand. Some goods are *necessities* and some are *luxuries*. A necessity is a good that is vital to the function of the enterprise. For example, cash supply for the snack factory to pay its employees their wages is a necessity. Luxury is an item that is in surplus of need. For example, any guest in a five-star hotel is asked to pay a luxury tax. Staying in a five-star hotel is taxable as it is a surplus of need. Luxury is an item that is not needed for the survival of the enterprise. A fur coat is an example of luxury.

2.2   The Life Cycle of An Enterprise

A life cycle exists for every business enterprise. The life cycle of an enterprise consists of the following stages:

(i) Identification of Need
(ii) Conceptual Preliminary Design
(iii) Bench Unit Trial or Prototype Testing
(iv) Detailed Design
(v) Production or Construction

  (vi) Operation
 (vii) Maintenance
(viii) Customer Use
  (ix) Retirement and Disposal
   (x) Recycling.

The cost associated with each step during the life cycle of an enterprise can be categorized according to the step in the cycle.

2.3   Total Revenue, Total Cost, and Profitable Region

The total cost incurred during the operation of an enterprise can be written as follows:

$$C_T = C_F + DC_V \qquad (2.1)$$

where C_V is the variable cost, D is the demand in number of units, and C_F is the fixed cost. The price of the product usually falls with increase in demand in a linear manner.

$$p = a - bD \qquad (2.2)$$

where p is the price and D is the demand in number of units; a is the price and zero demand and b is the rate of fall of price with increase in demand. It has been found that price is determined by supply and demand of goods in a market with perfect competition. The price changes by either falling or rising to equalize the goods produced and goods consumed. The price moves in such a fashion that some *equilibrium* is maintained. Other factors that may affect demand and supply—such as income of customers, price of raw materials that went into the manufacture of the product—do not figure into the price determination directly. Supply is usually driven by marginal cost. Additional output is determined by marginal cost.

The *law of demand* (*ceteris paribus*) states that the customers will be ready to buy more goods as the price decreases [1]. Eq. (2.2) is congruent with the law of demand. It is also consistent with the *elasticity* of supply and demand. Elasticity of demand is measured by the change in demand in response to a change in price. Elasticity of supply is measured by the change in supply in response to a change in price. If the quantity of demand changes

considerably with changes in unit price, the price-demand curve is said to be *elastic*. If the change in quantity was small, then the price-demand curve is said to be *inelastic*. Income elasticity of demand is not considered here. These are the changes that come about in supply and demand in response to change in income levels of the customers. There can be other factors that are not considered, such as cross elasticity of demand.

The total revenue generated by the business—TR—can be calculated as

$$TR = pD \tag{2.3}$$

$$TR = D(a - bD) \tag{2.4}$$

The total revenue reaches a maximum at a certain demand, D″. This can be calculated as follows:

$$\frac{dTR}{dD} = 0$$

$$a = 2bD''$$

$$D'' = \frac{2b}{a}$$

The break-even point is one where the total revenue, TR, exceeds total cost, C_T. This happens when

$$TR = C_T \tag{2.5}$$

Combining Eq. (2.1), Eq. (2.4), and Eq. (2.5):

$$0 = bD^2 + D(C_V - a) + C_F \tag{2.6}$$

From Eq. (2.6), it can be realized that there can be *more than one* break-even point. The number of break-even points that will occur during a given enterprise depends on the order of the algebraic equation given by Eq. (2.6). For the price-demand relation given by Eq. (2.2), it can be seen from Eq. (2.6) that two break-even points can be expected. For enterprises where the price falls with demand in a nonlinear manner, more than two break-even points can be expected. The break-even points occur at D' which can be calculated as

$$D' = \frac{(a - C_V) \pm \sqrt{(a - C_V)^2 - 4bC_F}}{2b} \tag{2.7}$$

The first break-even point occurs when the revenue from the sale of goods produced exceeds the costs incurred. The second break-even point occurs when there is overproduction. From the second break-even point forward, it is *counterproductive* to increase further the demand, D. Here the costs rise at a steady rate and the revenue reaches maxima and begins to fall with further increases in demand. At some point a *loss* is incurred due to excess production. The region in the revenue-demand graph where total revenue exceeds total cost is called a *profitable* region.

Example 2.1   Garlic Bread Nibbler Snack Factory

A team of professors and alumni got together to form a venture to make the Garlic Bread Nibbler Snack Factory. The raw materials procured were wheat, maize and corn flour, butter, and salt. The unit operations used in the preparation were kneader, mixer, baking oven, chopper, roaster, coloring, and packaging. The fixed costs that include leasing the building and land, insurance, taxes, and franchise fees to Snyder's of Hanover Pretzel Makers can be estimated at 1 million US\$. The variable costs, including electric utilities, air conditioning, gas heating, telephone wires, etc. run the enterprise roughly 2 US\$ per bag of snacks. The price-demand relation can be given by a *linear* relation such as

$$p = a - bD \qquad (2.8)$$

When p is the price per bag and D is the number of bags of Nibblers produced, a and b can be given as \$10 and $5* 10^{-6}$ (\$/bag). Find the two break-even points. Sketch the total revenue and total cost as a function of demand.

$$\text{Total Revenue, } TR = pD = aD - bD^2 \qquad (2.9)$$

$$\text{Total Cost, } C_T = C_F + C_v D \qquad (2.10)$$

$$C_F = 1 \text{ million US\$} \qquad (2.11)$$

$$C_v = 2 \text{ US\$ per bag} \qquad (2.12)$$

The total revenue and total cost as a function of bags of Nibbler produced is shown in Figure 2.1. At optimal production, the rate is about 114 bags per hour, assuming that the factory operates 24 hours, 7 days a

Table 2.1 Total Revenue, Total Cost for Garlic Bread Nibbler Snack Factory

# Bags	Price, p (per bag $)	TR, Total Revenue (Thousands of US$)	C$_T$, Total Cost (Thousands of US$)
0	10	0	1000
100,000	9.5	950	1200
200,000	9	1800	1400
300,000	8.5	2550	1600
400,000	8	3200	1800
500,000	7.5	3750	2000
600,000	7	4200	2200
700,000	6.5	4550	2400
800,000	6	4800	2600
900,000	5.5	4950	2800
1,000,000	5	5000	3000
1,100,000	4.5	4950	3200
1,200,000	4	4800	3400
1,300,000	3.5	4550	3600
1,400,000	3	4200	3800
1,500,000	2.5	3750	4000
1,600,000	2	3200	4200
1,700,000	1.5	2550	4400
1,800,000	1	1800	4600
1,900,000	0.5	950	4800
2,000,000	0	0	5000

week, and 52 weeks in a given calendar year. At this point, the revenue is a maximum. This can be seen in Figure 2.1 at 1 million bags produced in a year. The same can be obtained by the use of calculus as follows:

$$\frac{d(TR)}{dD} = 0 = a - 2bD \qquad (2.13)$$

At maximum revenue,

$$D_{TR} = \frac{a}{2b} = \frac{10}{10 * 10^{-6}} = 10^6 \; bags \qquad (2.14)$$
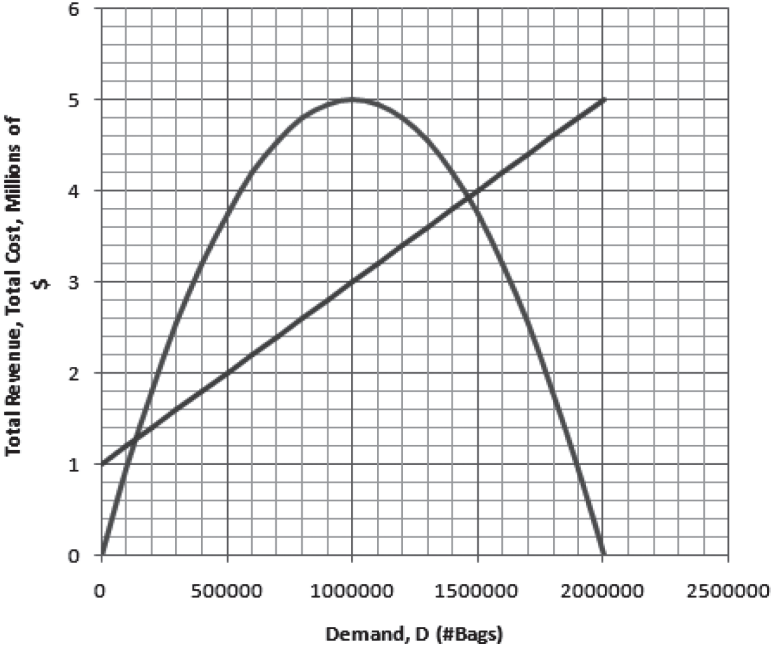
***Figure 2.1 Total Revenue and Total Cost vs. Demand for Garlic
Bread Nibbler Snack Factory***

The profit generated can be calculated as

$$\text{Profit} = P = TR - C_T = aD - bD^2 - C_F - DC_v$$
$$= -bD^2 + D(a\text{-}C_v) - C_F \qquad (2.15)$$

For maximum profit,

$$\frac{dP}{dD} = 0 = -2bD + (a - C_v) \qquad (2.16)$$

At maximum profit, the demand would be calculated as:

$$D_P = \frac{(a - C_V)}{2b} = \frac{(10 - 2)}{2 * 5 * 10^{-6}} = 800,000 \; bags \qquad (2.17)$$

800,000 bags checks out in the graphical solution or by generation
of columns using a Microsoft Excel spreadsheet on a desktop computer.

The break-even points are when the total revenue equals the total cost.
The first break-even point is when, for additional units produced, the

total revenue exceeds the total cost incurred. The second break-even point indicates a point of overproduction to a point of being counterproductive. Here, for additional units produced, the total revenue falls below the total cost.

$$TR = C_T \tag{2.18}$$

$$aD - bD^2 = C_F + DC_v \tag{2.19}$$

The break-even points can be obtained by the solution of the following quadratic equation:

$$bD^2 - D(a - C_v) + C_F = 0 \tag{2.20}$$

The first BEV (break-even) point occurs at a demand of 136,000–137,000 bags of Nibbler bags per year. The second BEV point occurs between 1,460,000 and 1,470,000 Nibbler bags per year. Further production of Nibbler bags would be counterproductive for revenue. The reason for the occurrence of the second break-even point is because the price falls inversely with the demand. When prices are low, the total revenue generated is not sufficient to exceed total cost. Total cost increases with demand in a linear manner.

2.4   Giffen and Veblen Goods

Not all goods obey the law of demand. Examples of *Veblen goods* are diamonds, luxury cars, fur coats, etc. These are status symbols. Families with higher incomes tend to buy them to show their higher status in society. The price of these goods rises with increase in demand. *Giffen goods* experience an increase in demand as the price rises. This does not obey the law of demand. During a potato famine, the price rises. Rather than less demand, poor families buy more potatoes to show their status.

An *elasticity of supply* can be defined as follows:

$$E_{sup} = \frac{\Delta S}{\Delta p} \tag{2.21}$$

Elasticity of supply as defined by Eq. (2.21) denotes the change in supply that can be expected for a small change in price. E_{sup} in most cases is positive, as stated by the law of supply. The supply curve is *concave*

curvature facing upward. The elasticity of supply for a considered good depends on factors such as availability of raw materials, length and complexity of production, time to respond, excess capacity, inventories, etc. The elasticity for gasoline is reported as 1.61 and that of cotton as 0.3. One of the assumptions of the law of demand is that the supply and demand are independent of each other.

An *elasticity of demand* can be defined in a similar manner to Eq. (2.21) as

$$E_{dem} = \frac{\Delta D}{\Delta p} \tag{2.22}$$

Elasticity of demand as defined by Eq. (2.22) denotes the change in demand that can be expected for a small change in price. E_{dem} in most cases is *negative* as stated by the law of demand. Price changes come about to equalize the product of price and demand. As demand increases, price decreases and vice versa. The demand y curve is *concave curvature* facing upward.

Example 2.2   Japanese Robots

The price-demand relation for a certain Japanese robot is given by

$$p = a - bD + cD^2 \tag{2.23}$$

How many break-even points do you expect? What is the demand at

  (i) Maximum revenue?
 (ii) Maximum profit?
(iii) Break-even point(s)?

$$\text{Total Revenue, TR} = pD = aD - bD^2 + cD^3 \tag{2.24}$$

The maximum revenue occurs at a demand that can be calculated as follows:

$$\frac{d(TR)}{dD} = 0 = a - 2bD + 3cD^2 \tag{2.25}$$

The extremas occur at

$$D' = \frac{2b \pm \sqrt{4b^2 - 12ac}}{6c} \qquad (2.26)$$

For maxima, the second derivative of the TR (total revenue function) has to be negative, and evaluated at the extremum values of demand. The second derivative of TR is seen to be

$$\frac{d^2(TR)}{dD^2} = -2b + 6cD \qquad (2.27)$$

This depends on the actual values of a, b, and c.

The cost function remains the same as

$$C_T = C_F + DC_V \qquad (2.28)$$

The total profit can be calculated as:

$$\text{Total Profit} = (a\text{-}C_V)D - bD^2 + cD^3 - C_F$$

The demand at maximum profit can be estimated as follows:

$$\frac{dP}{dD} = 0 = 3cD^2 - 2bD + (a - C_V) \qquad (2.29)$$

The maxima have to be confirmed by ensuring the second derivative of the profit function is positive evaluated at one of the extremum demand values. The second derivative of the profit function is given by

$$\frac{d^2P}{dD^2} = 6cD - 2b \qquad (2.30)$$

The break-even points can be estimated when the total revenue and total cost are equal to each other.

$$(a - C_V)D - bD^2 + cD^3 - C_F = 0 \qquad (2.31)$$

There are three break-even points that can be expected. These are obtained by the solution to the above cubic equation. The solution to cubic equations can be solved by two methods:

(i) Vieta's Substitution to Depressed Cubic Equation
(ii) and Numerical Solution.
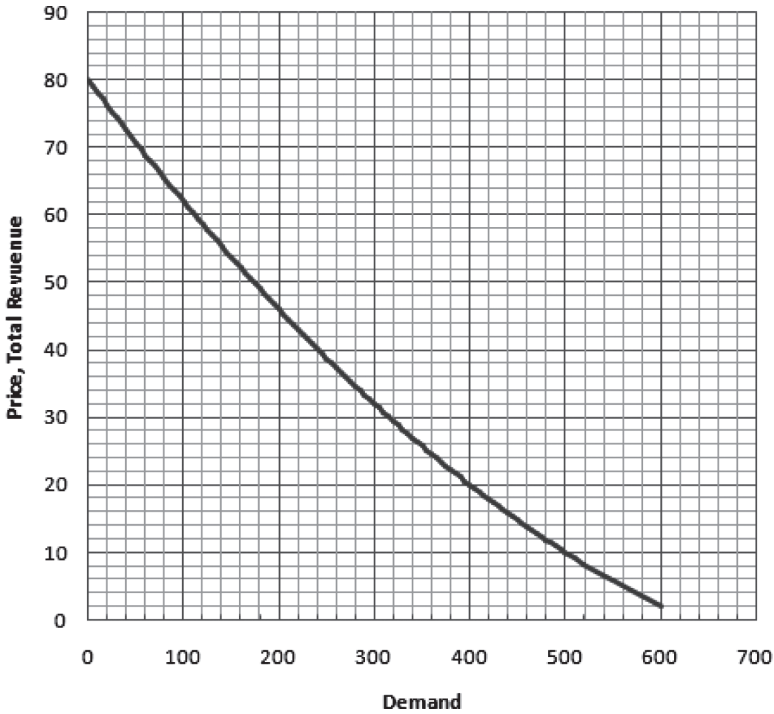
*Figure 2.2 Price-Demand Curve for Chinese Lawn Mowers
in Example 2.3*

Example 2.3   Lawn Mowers from China

The price-demand relation for lawn mowers made by a Chinese manufac-
turer can be given by the following relation:

$$p = 80 - 0.19D + 10^{-4}(D^2) \qquad (2.32)$$

This is shown in Figure 2.2.

The fixed cost to produce the lawn mowers, including taxes, license
fees, and insurance is about $2000. The variable cost that includes elec-
tric, gas, and water utilities can be taken as $10 per lawn mower.

Total Revenue, $TR = pD = aD - bD^2 + cD^3$
$$= 80D - 0.19D^2 + 1E\text{-}4D^2 \qquad (2.33)$$

The maximum revenue occurs at a demand that can be calculated as
follows:

$$\frac{d(TR)}{dD} = 0 = a - 2bD + 3cD^2 \tag{2.34}$$

The extremas occur at

$$D' = \frac{2b \pm \sqrt{4b^2 - 12ac}}{6c} = \frac{+0.38 \pm 0.22}{0.0006} = 266.67 \,\&\, 1000 \tag{2.35}$$

For maxima, the second derivative of the TR, total revenue function, has to be negative, evaluated at the extremum values of demand. The second derivative of TR would be seen to be

$$\frac{d^2(TR)}{dD^2} = -2b + 6cD \tag{2.36}$$

Eq. (2.36) evaluated at the two roots for demand that causes an extremum value in the TR, total revenue function, are as follows:

$$D' = 266.67, \text{ Eq. } (2.36) = -0.160002 \tag{2.37}$$

$$D' \text{ at } 1000, \text{ Eq. } (2.36) = 0.22 \tag{2.38}$$

Thus the extremum of TR function at $D' = 1000$ corresponds to a minima. The other demand, $D' = 266.7$, corresponds to a maxima. The TR, total revenue function, is plotted using Microsoft Excel spreadsheets and shown in Figure 2.3. The maxima can be confirmed from the figure corresponding to a demand, $D' = 266.7$.

The cost function remains the same as

$$C_T = C_F + DC_V \tag{2.39}$$

The total profit can be calculated as

$$\text{Total Profit} = (a\text{-}C_V)D - bD^2 + cD^3 - C_F$$

The demand at maximum profit can be estimated as follows:

$$\frac{dP}{dD} = 0 = 3cD^2 - 2bD + (a - C_V) \tag{2.40}$$

The maxima have to be confirmed by ensuring the second derivative of the profit function is positive evaluated at one of the extremum demand values. The second derivative of the profit function is given by
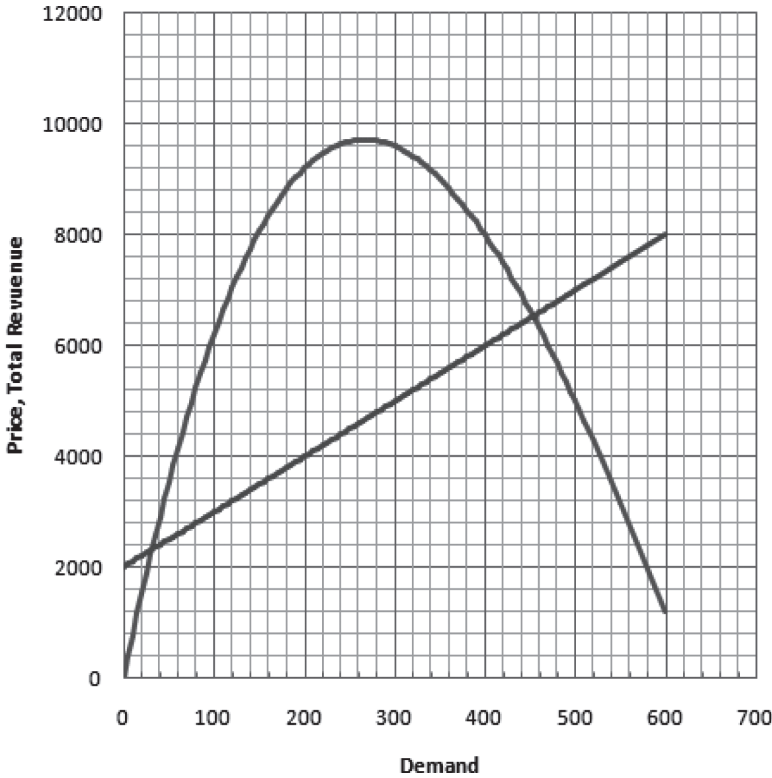
Figure 2-3 Maximum Total Revenue, Break-Even Points for Chinese Lawn Mowers in Example 2.3

$$\frac{d^2P}{dD^2} = 6cD - 2b \tag{2.41}$$

The break-even points can be estimated when the total revenue and total cost are equal to each other.

$$(a\text{-}C_V)D - bD^2 + cD^3 - C_F = 0 \tag{2.42}$$

The demand corresponding to maximum profit can be obtained by inspecting the table generated by an MS Excel spreadsheet. This can be seen to come about at a demand of about 225 lawn mowers. The demand, price, total cost, total revenue, and profit for 0–600 lawn mowers is shown in Table 2.2. The two break-even points can be seen from Table 2.2 to occur at 35 and 450 units.

A third break-even point can be expected, according to Eq. (2.42). In order to search for the third break-even point, the TR function was

Table 2.2 Total Revenue and Profit for Chinese Lawn Mowers

Law of Demand						
		Demand	Price	Total Revenue	Total Cost	Profit
a	80	D	p	TR	CT	P
b	0.19	0	80	0	2000	−2000
c	0.0001	20	76.24	1524.8	2200	−675.2
		40	72.56	2902.4	2400	502.4
C_v	10	60	68.96	4137.6	2600	1537.6
C_F	2000	80	65.44	5235.2	2800	2435.2
		100	62	6200	3000	3200
		120	58.64	7036.8	3200	3836.8
		140	55.36	7750.4	3400	4350.4
		160	52.16	8345.6	3600	4745.6
		180	49.04	8827.2	3800	5027.2
		200	46	9200	4000	5200
		220	43.04	9468.8	4200	5268.8
		240	40.16	9638.4	4400	5238.4
		260	37.36	9713.6	4600	5113.6
		280	34.64	9699.2	4800	4899.2
		300	32	9600	5000	4600
		320	29.44	9420.8	5200	4220.8
		340	26.96	9166.4	5400	3766.4
		360	24.56	8841.6	5600	3241.6
		380	22.24	8451.2	5800	2651.2
		400	20	8000	6000	2000
		420	17.84	7492.8	6200	1292.8
		440	15.76	6934.4	6400	534.4
		460	13.76	6329.6	6600	−270.4
		480	11.84	5683.2	6800	−1116.8
		500	10	5000	7000	−2000
		520	8.24	4284.8	7200	−2915.2
		540	6.56	3542.4	7400	−3857.6
		560	4.96	2777.6	7600	−4822.4
		580	3.44	1995.2	7800	−5804.8
		600	2	1200	8000	−6800

calculated for demands greater than 600 and plotted using an MS Excel spreadsheet and shown in Figure 2.4. A *third break-even point* can be seen at a demand corresponding to about 1420 units, and can be identified from Figure 2.4. This is subject to the applicability of the law of demand given by Eq. (2.32). For demand units greater than 629 lawn mowers, it can be seen from Eq. (2.32) that a "negative" price is associated with the lawn mowers. What is going on in the market is not clear in this regime. The Chinese manufacturer has to pay money to have customers haul away their product. This is also not that uncommon given the cash-down and free promotional offers, etc. advertised time and again on TV commercials. The price comes back again to positive territory for demand greater than 1270 lawn mowers. Maybe at this point the advertisement program and customer reports have worked in favor of the Chinese manufacturer. From this point onward the price increases (!), with increase in further demand. Thus the profits increase with further increase in demand after the third break-even point. It does not decline into a nonprofitable region like in the Garlic Nibbler Snack Factory in Example 2.1. Then again, in
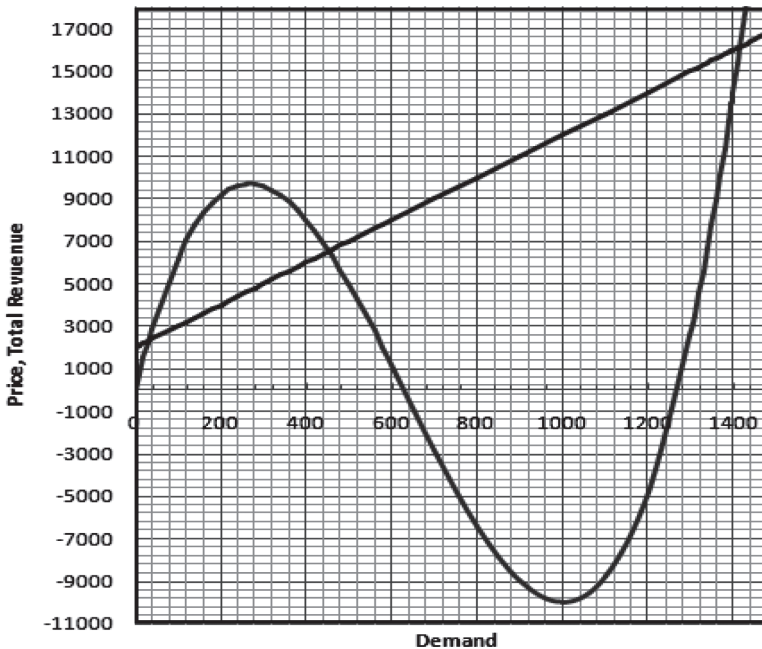

Figure 2.4 Third Break-Even Point for Manufacture of Chinese Lawn Mowers in Example 2.3

that example, the price-demand relation was linear. It obeys the law of demand and price falls with increase in demand.

The break-even points, the demand at which the TR (total revenue) reaches a maximum value, the demand at maximum profit needs to be calculated for every enterprise. The methodology is explained in the above sections. Much depends on the price-demand relation. If it is linear, then enterprise fortunes will conform to behavior as explained in Example 2.1. If the price-demand relation follows a quadratic or quadrilinear relation, the enterprise fortunes will behave in a fashion as explained in Example 2.3.

2.5   Dualistic Relations

Before the dawn of the industrial revolution and scientific progress, nations were affected by periodic famines. The marriage rate was inversely proportional to the price of bread. Floods, droughts, hurricanes, tsunamis, earthquakes, volcano eruptions, and other natural disasters resulted in the deaths of millions of the populace. Natural disasters were known; not much was done about them. The 21st millennium is a period of abundant supply of goods and services. The boom-and-bust cycles of economies sometimes leads to periods of depression. Present-day marriage and divorce rates rise and fall as a function of job opportunities rather than the cost of bread. Famine due to crop failure is averted by trade and the exchange of food supplies from other parts of the globe. The economic policy of the government is to foster economic progress, minimize damage from the bust cycles of the economy, and improve and maintain high standards of living.

Paul Samuelson [2] presented the first lesson in economics: Things are often not what they seem. For instance, if all farmers work hard and a bumper crop is realized, the total farm income falls! In his Nobel lecture in 1970, Samuelson drew an analogy from optics and thermodynamics to economics. He considered a profit-maximizing firm that sells its output a long a demand curve, in which price received is a nonincreasing function of the amount sold. Furthermore, suppose that the output can be generated from 2, 3, or 99 different inputs. He assumed that the production function relating outputs to inputs is smooth and concave. These time and age neural networks can be designed based on this example. Storing the 99 inputs and outputs on the computer is an interesting problem in

data management and storage. It is difficult to find 99 independent surfaces. Knowledge of a single parent surface is sufficient to calculate exact information about 99 children. Observed demand curves are solutions to the maximum-profit problem.

Simple regularity conditions of calculus can be used to show that the demand curves are inverse functions of a family of partial derivatives of the TR (total revenue function). When smooth and *strongly concave*, the parent TR function has as its children 99 × 99 matrix of second partial derivatives. This matrix is symmetric and negative definite. Some algebraic manipulation can be used to show that these functions can be uniquely inverted to form a new family of children with the same properties—99 such children must have a parent function. The parent function may have been formed in a Pygmalion fashion.

$$\text{Max } (\text{TR}(v_1, v_2, \ldots, v_{99})) - \sum_{1}^{99} p_k v_k$$

$$= \text{TR}(v_1{}^*, v_2{}^*, \ldots, v_{99}{}^*) - \sum_{1}^{99} p_k v_k^* \tag{2.43}$$

The TR function can be written in terms of a Hessian matrix. The necessary conditions of maxima can be derived as

$$\frac{\partial TR}{\partial v_i} = p_i \tag{2.44}$$

The Hessian matrix is negative definite. Eq. (2.44) contains necessary and sufficient conditions for maxima. A Hotelling-Roy dual function can be written as

$$v_i^* = \frac{\partial H}{\partial p_i} \qquad (i = 1, \ldots 99) \tag{2.45}$$

Samuelson showed that it follows that

$$\sum \Delta v_i^2 \neq \sum \Delta p_i^2 \neq 0 \tag{2.46}$$

He arrived at the inequality

$$\Delta p_1 \Delta v_1 + \Delta p_2 \Delta v_2 + \ldots + \Delta p_{99} \Delta v_{99} < 0 \tag{2.47}$$

Samuelson went on to combine common-sense and advanced mathematics and revisited the Giffen goods. When the price of potatoes sold

to Irish peasants was raised, the amount of potatoes purchased was less. This is consistent with Eq. (2.47), or $\dfrac{\partial v_i}{\partial p_i} < 0$. This relation is analogous to the Le Chatelier principle: when pressure on a balloon is increased, the volume of the balloon decreases. The pressure volume path taken by an ideal gas is less steep for an isothermal process and a steeper change for an adiabatic process. Or,

$$\left(\frac{\partial v}{\partial p}\right)_T \leq \left(\frac{\partial v}{\partial P}\right)_S \leq 0 \tag{2.48}$$

The rate of change of volume with respect to pressure at constant temperature is less than the rate of change of volume with respect to pressure at constant entropy for an ideal gas. This relation can be used as an analogy in economics. By analogy, wage rate and labor have a *dualistic relation*. For example, as the number of laborers available increases, the wage rate decreases. As the number of laborers is low, or should there be a shortage of labor, the wage rate is bound to increase. The land rent and acres of land also can be seen to have a dualistic relation. The more land available, the lower the rent. Thus land in Australia or rural Texas is available at lower rent and the scarce availability of land in New York City can be used to collect larger rent. These dualistic relations are shown in Figure 2.4.

Two paths are taken in Figure 2.4. The first path is raising price p_1 of the first input labor, while holding constant the second input quantity such as land or insurance. The rise in price lowers the labor availability as indicated by the negative slope of the pv diagram. In the second example, the price p_1 is increased by the same amount holding the v_2, p_2 constant. In order to maximize profit, less of v_1 will be purchased. This is indicated by the negative slope of the heavy curve in Figure 2.4. These observations are captured in the *Le Chatelier-Samuelson principle*. The heavy curve of long-term correction with other quantities constant must be less steep or *more elastic* compared with the light curve that is used to capture demand relation when the other input is held constant. Thus, quantitatively

$$\left(\frac{\partial v_1}{\partial p_1}\right)_{p_2} \leq \left(\frac{\partial v_1}{\partial p_1}\right)_{v_2} \leq 0 \tag{2.49}$$
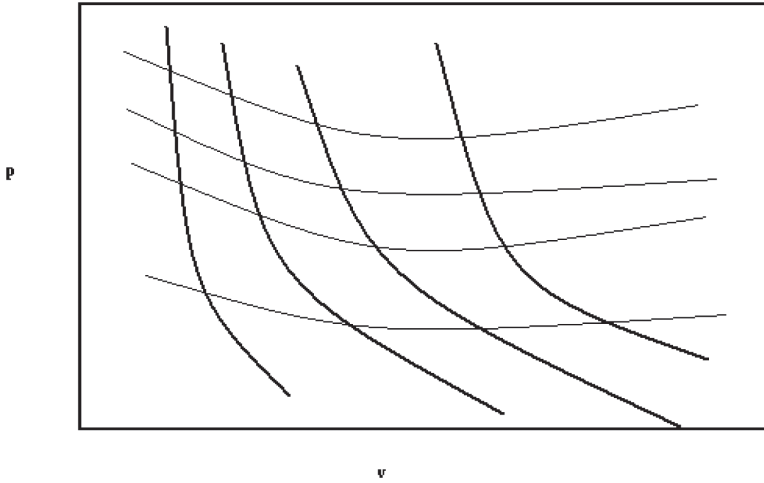
P

v

Figure 2.4 Dualistic Relations of Price and Its Variables Such as Demand

Thus,

$$\frac{\partial D}{\partial p} \leq 0 \qquad (2.50)$$

where D is demand and p is price of a unit as discussed in the previous sections. When the price-demand relation is

$$p = a - bD \qquad (2.51)$$

$$\frac{\partial D}{\partial p} = -\frac{1}{b} \leq 0 \qquad (2.52)$$

a and b are usually constants and positive. Hence the Le Chatelier-Samuelson principle can be verified for the price-demand relation discussed in Section 2.3.

The two inputs shown in Eq. (2.49) can be independent of each other. Samuelson [2] notes that the Le Chatelier-Samuelson inequalities will remain good whether the two inputs are complements of each other (such as pumps and insecticides), or they are substitutes (such as organic and inorganic fertilizers). The Le Chatelier-Samuelson principle has several applications in economy, and is consistent with the theory of production and the general theory of constrained rationing.

2.6   Summary

Fixed costs are usually one-time occurrences. *Variable costs* occur more frequently, usually once every billing period. *Incremental costs* are those costs that are incurred proportionally to the number of goods produced. The planned costs per unit of output, anticipated labor hours, materials, and overhead categories are called *standard costs*. *Direct costs* can be measured and attributed to a specific activity or output. *Indirect* costs cannot be attributed to a single activity. Secretarial staff in an office building is an example of overhead costs. Cash cost, book cost, sunk cost, opportunity cost, investment cost, salvage value, and O & M costs are defined. A state of perfect competition is said to exist when there is more than one supplier of a product in demand. Some goods are *necessities* and some are *luxuries.*

The different stages during the life cycle of an enterprise are discussed. The total costs incurred during the operation of a business enterprise can be expressed in terms of the fixed costs, variable costs, and demand, D (Eq. (2.1). The price, p, is assumed to vary inversely with demand, D, and given by Eq. (2.2). The *law of demand* (*ceteris paribus*) states that customers will be ready to buy more goods as the price decreases [1]. Elastic and inelastic price-demand curves are distinguished from each other. The TR, total revenue, generated by the business enterprise is expressed as a product of price and demand and given by Eq. (2-3). The location where the TR reaches a maxima is found to be at demand $D'' = 2b/a$, where a and b are the constants in Eq. (2.2) used to express the price vs. demand relation. The break-even points are determined when the TR equals the total cost. There are two break-even points. The first break-even point occurs when the revenue from the sale of goods produced exceeds the costs incurred. The second break-even point occurs when there is overproduction. From the second break-even point forward, it is *counterproductive* to increase further the demand, D.

Worked examples on the Garlic Nibbler Snack Factory, Japanese robots, and Chinese lawn mowers are discussed. Not all goods obey the law of demand. Examples of Veblen goods are diamonds, fur coats, etc. The price of Veblen goods increases with an increase in demand. Elasticity of supply and elasticity of demand are defined. When the price of Giffen goods increases, the demand also increases and does not fall.

2.7   References

[1] P. A. Samuelson, *Economics: An Introductory Analysis*. McGraw-Hill (1948), New York, NY.

[2] P. A. Samuelson, "Maximum Principles in Analytical Economics," Nobel Prize Lecture (1970), http://nobel.se, Stockholm, Sweden.

CHAPTER 3

Time Value of Money

Topics Discussed
- Simple and Compound Interest
- Future Worth of Series of Uniform Payments, A
- Present Worth of Series of Uniform Payments, A
- Inverse Problem of Finding i
- Inverse Problem of Finding N
- Gradient Series
- Variable Interest Rate
- Deferred Payments

3.1   Simple and Compound Interest

Capital is often needed for starting new business enterprises. Capital can be dollar currency or other assets such as stocks, bonds, and money market accounts owned in the form of a diversified portfolio. The worth of money can be calculated to the nearest dollar and cent. This chapter is devoted to calculations of the cost of money. It is built on the fundamental notion of interest charge on capital.

The simple interest, I on a principal P, can be calculated as follows at a given interest rate i and pay period N:

$$I = P \, i \, N \qquad\qquad (3.1)$$

where I is the simple interest amount, P the principal amount, i the interest rate expressed as a fraction, and N is the pay period. Over the same pay period, the compound interest and amount owed can be calculated

as follows. At the end of the first pay period, the future amount, F, or amount owed is given by

$$F = P + iP = P(1 + i) \tag{3.2}$$

At the end of the second pay period, the future amount is given by

$$F = P(1 + i) + iP(1 + i) = (1 + i)(P + iP) = P(1 + i)^2 \tag{3.3}$$

In a similar manner, at the end of pay period N the future amount is given by

$$F = P(1 + 1)^N \tag{3.4}$$

The principal, P, is multiplied with the algebraic expression given by Eq. (3.4), in order to obtain the future amount F. This algebraic expression is called a *single payment compound factor*. Eq. (3.4) can be read from Figure 3.1 for interest rates, i ranging from 0 to 30% and pay period, N, ranging from 2 through 12. The single payment compound factor, P/F, given by Eq. (3.4) can be read from Figure 3.2 for interest rates i ranging from 0 to 30% and pay period, N, ranging from 15 through 100. The P/F axis is in logarithmic scale to accommodate the larger P/F values associated with larger pay period N at larger interest rates, i.

Eq. (3.4) may be rearranged to given P given F, i and N as

$$P = \frac{F}{(1 + i)^N} \tag{3.5}$$

The future amount, F, is multiplied with the algebraic expression given by Eq. (3.5), in order to obtain the present amount P. This algebraic expression is called a *single payment present worth factor*.

The pay period N can be calculated given the F, P, and i as follows:

$$N = \frac{\log\left(\dfrac{F}{P}\right)}{\log(1 + i)} \tag{3.6}$$

The interest rate i can be calculated given F, P, and N as follows:

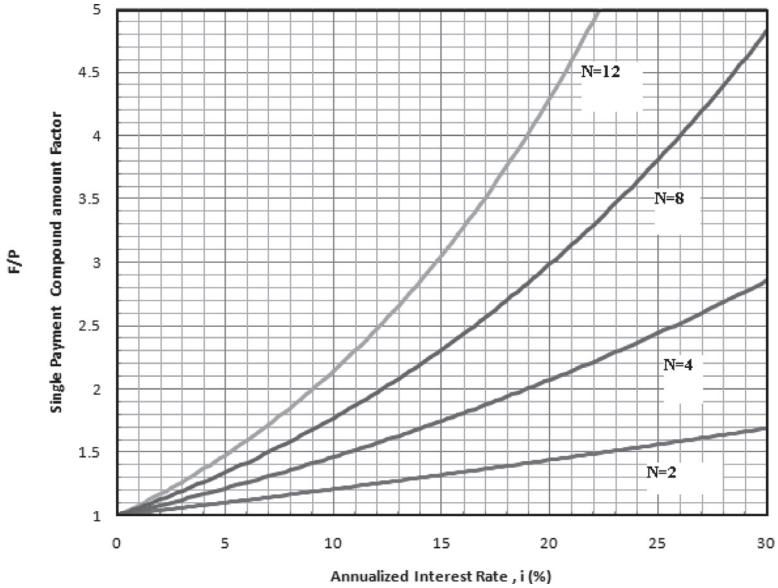$$i = \left(\frac{F}{P}\right)^{\frac{1}{N}} - 1 \tag{3.7}$$

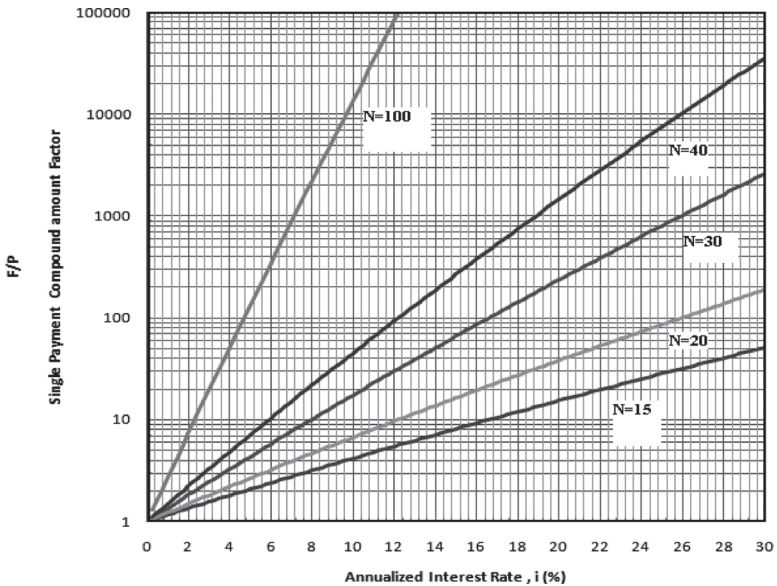**Figure 3.1 Single Payment Compound Amount Factor, F/P for
0 ≤ i ≤ 30% and 2 ≤ N ≤ 12**

**Figure 3.2 Single Payment Compound Amount Factor, P/F for
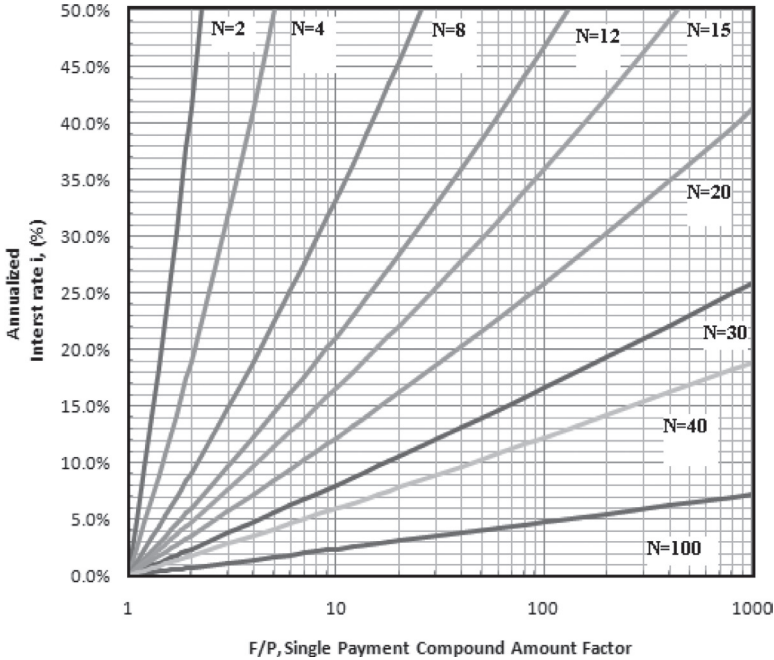Interest Rate 0 ≤ i ≤ 30% and Pay Period 15 ≤ N ≤ 100**

Figure 3.3 Interest Rate, i (%) vs. Single Payment Compound Amount Factor for Different Pay Periods, N

Eq. (3.7) is shown in an easily readable chart in Figure 3.3. The interest rates can be read from Figure 3.3 when F/P, the single payment compound factor and pay period N, are known. Different lines in the figure are for different pay periods N ranging from 2 through 100 years.

The compound interest can be calculated by subtracting P from the future amount given by Eq. (3.4).

$$1 = P\left(\left(1 + 1i\right)^N - 1\right) \qquad (3.8)$$

Example 3.1   Compound Interest Calculated on an Everyday Basis

Your local bank branch recently announced a new savings plan with an interest rate of 0.0274% compounded daily. What is the interest payment on $52,000 at the end of one year?

$$F = P(1 + i')^N = 50{,}000^*(1.000274)^{365} = \$55258.34 \qquad (3.9)$$

From Eq. (3.8), the interest payment, I = $55,258.34-$50,000 = $5,258.34.

The history of mankind has seen the *uses* and *abuses* of compound interest. The farmers in India were persecuted by the feudal lords, or *Zamindars*, for late payment or nonpayment of loans. This was called bonded labor. Here the sons inherited the debts of their fathers and had to work them off. Over time, the compound interest ballooned in payment until the laborers could not repay the debt. Upon India's declaration of independence from the British Empire, the first prime minister, Jawaharlal Nehru, along with another prime minister who smiled on farmers, Mrs. Indira Gandhi, used to forgive the debts of farmers. The two political heads abolished bonded labor.

Bonded labor is an arrangement by which a person pays off his loan with direct labor in lieu of currency over a long period, sometimes even lifetimes. The term *debt slavery* is reserved for arrangements where the value of the work done is significantly greater than the original sum of money borrowed.

Peons are laborers who are bound in servitude until their debts are paid in full. Employers are allowed to extend credit to laborers to buy goods and services from employer-owned stores at exorbitant prices. This was used in the South after the Civil War. Poor African American and Caucasian farmers were called *sharecroppers*. They were given credit to purchase seed, fertilizer, and oxen from the landlord and made payments to him via a share of the crops.

3.1.1 Effective Interest Rate

Effective interest rate is the interest rate that gets compounded on an annual basis. Not all applications of interest rate are charged on an annual basis. Other applications need interest rate compounded on different bases, such as

  (i) Daily
 (ii) Weekly
(iii) Monthly
(iv) Quarterly
 (v) Semiannual.

The interest rate compounded on any pay period is called the nominal interest rate. Federal laws such as the *truth-in-lending* law passed in Congress in 1969 require a statement regarding the APR (annual

percentage rate) being charged on promissory notes and contracts involv-
ing monetary transactions. Compounding is not included.

An *effective interest rate*, i can be defined as a function of the nominal
interest rate as follows [1]:

$$i = \left(1 + \frac{i'}{P}\right)^P - 1 \tag{3.10}$$

where i' is the *nominal interest rate* and P is the number of compound-
ing periods per year. This is the interest earned on the principal on one
year. Effective interest rates are expressed on an annual basis. For example,
the effective interest rate for a nominal rate of 12% compounded on a
monthly basis would be

$$i = \left(1 + \frac{0.12}{12}\right)^{12} - 1 = 0.12683 \tag{3.11}$$

A number of banks post their interest on savings accounts on a
monthly basis and credit card companies calculate the interest charges
due on the unpaid balance on a daily basis. A summary of effective inter-
est rate i and nominal rate i' for different compounding periods is calcu-
lated using Eq. (3.10) and summarized below in Table 3.1.

3.2   Uniform Series of Payments

Cash Flow Diagrams are illustrations that show all the monetary transactions
during the time of an enterprise. They can be used to calculate the equiva-
lence of a series of uniform payments in terms of its present worth, P, or in
terms of its future worth, F. For example, consider the uniform payments A
made as shown in Figure 3.4:

3.2.1   Given A, I, and N Find F

The equivalence of the uniform series of payments A for a said period N
at a fixed interest rate i as shown in Figure 3.1 can be established. The
future equivalent worth of the uniform series of payments can be esti-
mated as follows: The payment A at time period 1 will grow after N
years on account of compound interest using Eq. (3.4) to $A(1 + i)^N$. In
a similar manner, the payment A at time period 2 will grow on account

Table 3.1 Effective Interest Rate i, as a Function of Nominal Interest Rate i'

		Nominal Rates							
	P	28%	24%	21%	18%	15%	12%	9%	6%
Daily	365	32.30%	27.11%	23.36%	19.72%	16.18%	12.75%	9.42%	6.18%
Weekly	52	32.21%	27.05%	23.32%	19.68%	16.16%	12.73%	9.41%	6.18%
Monthly	12	31.89%	26.82%	23.14%	19.56%	16.08%	12.68%	9.38%	6.17%
Quarterly	4	31.08%	26.25%	22.71%	19.25%	15.87%	12.55%	9.31%	6.14%
Semiannual	2	29.96%	25.44%	22.10%	18.81%	15.56%	12.36%	9.20%	6.09%
Annual	1	28.00%	24.00%	21.00%	18.00%	15.00%	12.00%	9.00%	6.00%

of compound interest accrued to $A(1+i)^{N-1}$ at the end of the pay period. Thus, the future equivalent worth of the uniform series of payments A can be given by

$$F = A(1 + i)^N + A(1 + i)^{N-1} + A(1 + i)^{N-2}$$
$$+ \cdots + \cdots A(1 + i) + A \qquad (3.12)$$

Eq. (3.12) can be seen to be a geometric series. This can be summed up by multiplying Eq. (3.12) by the ratio of the series, $(1 + i)$ as follows:

$$F(1 + i) = A(1 + i)^N + A(1 + i)^N + A(1 + i)^{N-1} + \cdots$$
$$+ A(1 + i)^2 + A(1 + i) \qquad (3.13)$$

Subtracting Eq. (3.12) from Eq. (3.13), it can be seen that all but the term A in Eq. (3.10) and $A(1 + i)^N$ in Eq. (3.13) gets canceled, and
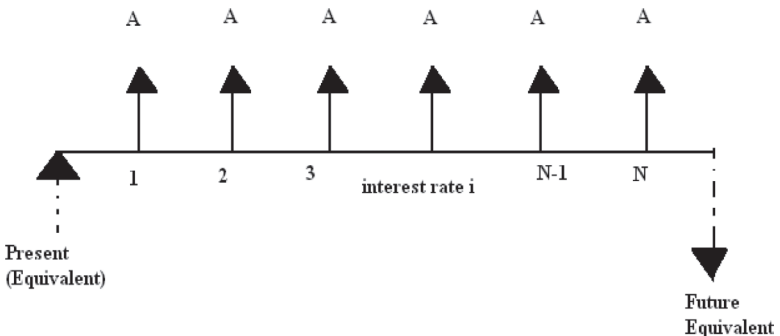

Figure 3.4 Cash Flow Diagram for Uniform Series of Payments A

$$F(1 + i - 1) = A(1 + i)^N - A \tag{3.14}$$

Or
$$F = \frac{A\left((1 + i)^N - 1\right)}{i} \tag{3.15}$$

The uniform series payment A needs to be multiplied with the algebraic expression given by Eq. (3.13) in order to obtain the future worth, F. This algebraic expression is called the *uniform series compound amount factor*. Given the future amount F, interest rate I, and pay period N, the uniform series payment A can be calculated by rearranging Eq. (3.15) as

$$A = \frac{iF}{(1 + i)^N - 1} \tag{3.16}$$

The future equivalent worth, F, needs to be multiplied with the algebraic expression given by Eq. (3.16) in order to obtain the uniform series payment A. This algebraic expression is called a *sinking fund factor*. The sinking fund factor given by Eq. (3.16) is shown in Figure 3.5 for values of interest rates, i (%) ranging from 0 to 30% and pay period N, ranging from 12 to 100. The sinking fund factor, A/F, given by Eq. (3.16) is shown as charts in Figure 3.6 for values of interest rates i(%) ranging from 0 to 30% and pay period N ranging from 2 to 8. It can be seen from Figures 3.5 and 3.6 that the nature of the function A/F vs. interest rate changes about N = 10. The power ponentiated expression $(1 + i)^N$ after N = 10 changes in a steeper manner. The curvature of the sinking fund factor vs. interest rate changes more rapidly at higher pay period values of N.

Example 3.2   Doubling Time of Investment

An investor deposits $1000 every month into his mutual fund with a savvy investment banker. The money was invested into stocks of companies that were marked as aggressively growing in the coming years. In how many pay periods would the future worth of the investments double in value?

In order for the Future worth, F, to double in value after pay period N,
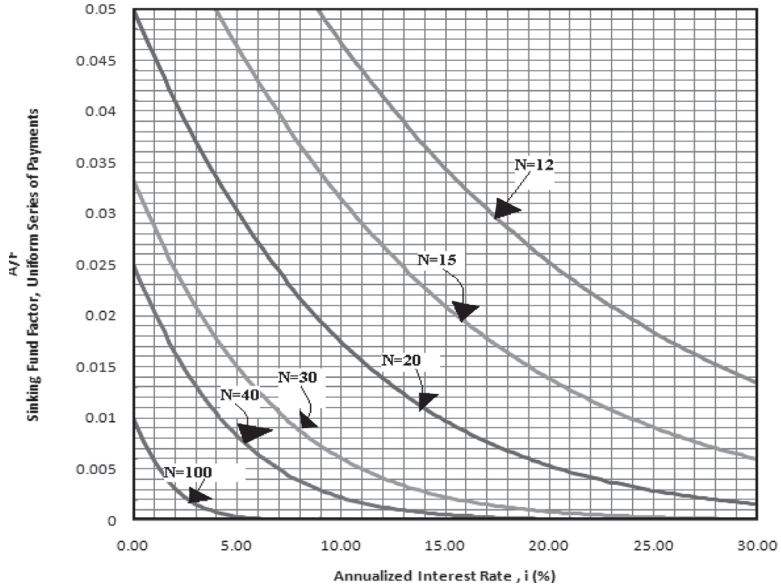F = 2 NA (NA would be the value of the fund if the interest rate was 0%)
Using Eq. (3.13)

Figure 3.5 *Sinking Fund Factor, A/F for Interest Rates,* $0 \leq i \leq 30\%$ *and Pay Period* $12 \leq N \leq 100$
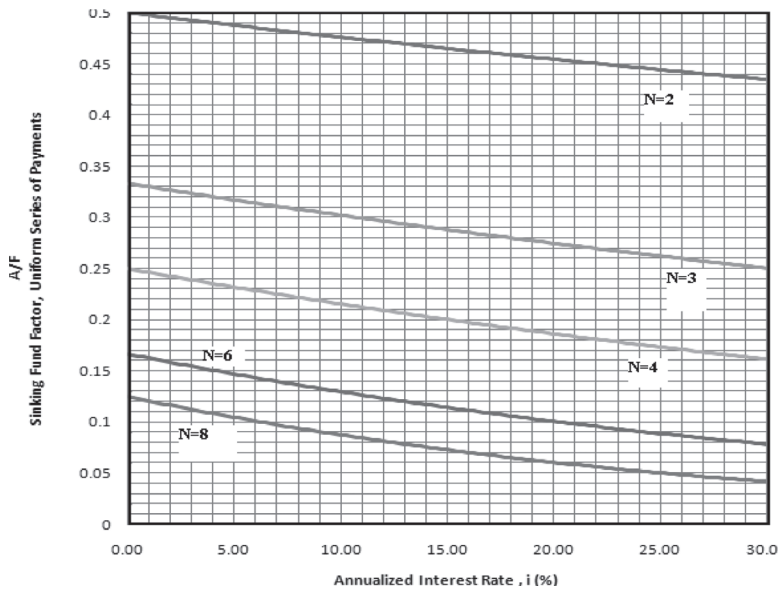

Figure 3.6 *Sinking Fund Factor, A/F for Interest Rates* $0 \leq i \leq 30\%$ *and Pay Periods,* $2 \leq N \leq 8$

$$F/A = 2N = \frac{F}{A} = 2N = \frac{\left((1 + i)^N - 1\right)}{i} \qquad (3.17)$$

Expressing $(1+i)^N$ as a binomial series such as $1 + Ni + N(N-1)i^2/2!$ at a truncation error of the order of magnitude of $O(i^3/3!)$, the above expression can be seen to be:

$$i = \frac{2}{(N-1)} \qquad (3.18)$$

The annualized interest rate i needed for the series of uniform payments A to double in value on account of compound interest paid— i.e., $F = 2AN$ is shown in Table 3.2.

3.2.2   Given A, I, and N, Find P

The present worth, P, of the uniform series of payments, A, for the cash flow diagram shown in Figure 3.1 can be calculated by combining Eq.(3.15) and Eq. (3.4) to yield

Table 3.2 Interest Rate vs. Doubling Time of Investment(s)

End of Year #	Interest Rate Annualized, i
2	24%
3	12%
4	8%
5	6%
6	4.8%
7	4%
8	3.4%
9	3%
10	2.67%
11	2.4%
12	2.18%
13	2%
14	1.85%
15	1.71%

$$P = \frac{A\left((1 + i)^N - 1\right)}{i(1 + i)^N} \qquad (3.19)$$

The algebraic expression given by Eq. (3.10) that the uniform series payment, A, needs to be multiplied with in order to obtain present worth, P, is called the *uniform series present worth factor*. Given the present amount P, interest rate I, and pay period N, the uniform series payment A can be calculated by rearranging Eq. (3.19) as

$$A = \frac{i(1 + i)^N P}{(1 + i)^N - 1} \qquad (3.20)$$

The present worth P needs to be multiplied with the algebraic expression given by Eq. (3.20) in order to obtain the uniform series payment A. This algebraic expression is called a *capital recovery factor*. The capital recovery factor A/P for interest rates $0 \le i \le 30\%$ for pay periods $0 \le N \le \infty$ is shown as easily usable charts in Figure 3.8. It can be seen from the figure even for $N = 12$ the capital recovery factor is only 4% from the values at perpetual payment for N is ∞, infinity for higher values of interest rate.

Perpetual payment plan is one when the pay period N becomes infinity. The limit when N tends to infinity in Eq. (3.20) can be calculated as

$$A = \underset{N \to \infty}{Lt} \frac{iP}{1 - \dfrac{1}{(1 + i)^N}} = iP \qquad (3.21)$$

In Appendix A annuity tables are provided to read F/P, P/F, F/A, A/F, P/A, and A/P values for various values of interest rates ranging from 1/8% to 30% and pay periods N ranging from 1 to 100. In addition to the annuity tables for the uniform series of payments A, annuity tables for uniform gradient series of payments G are also provided in Appendix A.

Example 3.3   Perpetual Payment and Father's Day Prize

The general manager of a factory that makes oil-fired steam boilers wants to establish a prize in a high school that caters to the children of the employees of the boiler plant factory. The prize is forever and for the best all-around student in the penultimate year of schooling. It is awarded as a once-in-a-school-time prize, from nursery school to graduating high

school. Calculate the present amount that needs to be paid at an annualized interest rate of 10% for the prize amount to be $150 during Father's Day celebrations. Father's Day occurs once a year on June 21st, when daytime hours and nighttime hours are equal to each other.

$$\text{From Eq. (3.17)}, P = \frac{A}{i} = \frac{150}{0.1} = 1500 \qquad (3.22)$$

Example 3.4   0% Interest Rate

Calculate the present worth of a series of uniform payments of $300 per month for 72 months. This is for leasing a car from the car dealer near you. The interest rate has dropped to near zero, such that the effective interest rate i can be considered with little error to be zero.

Use of Eq. (3.19) for this example may seem like P would be infinity! This is because i is zero. On careful examination of Eq. (3.19), it can be seen that when i becomes zero, Eq. (3.19) takes on the $\frac{0}{0}$ indeterminate form. The present worth can be re-derived from Figure 3.1 when interest rate is 0. When interest rate is 0, there is no interest charged or compound interest levied. Therefore,

$$P = A + A + A + \cdots\cdots + A = AN \qquad (3.23)$$

Thus, when i=0, Eq. (3.19) is not applicable to calculate P. But it can be seen to be equal to AN, where N is the pay period and A is the uniform series payment. This is the *asymptotic limit* of the principal amount when the interest rate i tends to 0%. The federal fund rates are 0.0% recently in the year 2010. During years when there is *deflation* instead of inflation, this may be more applicable.

Example 3.5   Nest Egg of Alice Smith

Alice Smith signed up with her company's 401(k) retirement savings plan. She pays $330 per month. Her employer matches dollar for dollar and pays as much per month into her retirement savings account. She is 26 years old. When she retires at 65, she hopes to obtain two lump-sum distributions. She can expect a return of 10% compounded annually as interest for her plan. Calculate the future equivalent amount of Alice Smith's retirement savings plan.

Eq. (3.15) can be used to calculate F.

A = 2*$330 = $660

N = 39 years

i = 10%

$$F = \frac{A\left((1+i)^N - 1\right)}{i} = \frac{660((1.1)^{39} - 1)}{0.1} = \$264{,}956 \quad (3.24)$$

Example 3.6   Diversified Portfolio

Darlene Shuster was the sole beneficiary of a life insurance benefit payment of $200,000 when her husband died. She invested the face amount as follows:

Stocks	$50,000
Bonds	$50,000
Money Market	$100,000.

The Dow Jones Industrial Average has performed at an average of 10% growth in value every year for the past 100 years, including the Great Depression and the market corrections of 1987 and 2007. The bond funds' yield is about 6% per year. The money market rates are 2.5% for the years 2010–2015; 5.0% for the years 2015–2020; and 8% for the years 2020–2030. After 20 years, what would the future worth of Ms. Shuster's inheritance be?

By Eq. (3.4), the investment in stocks would grow to

$$= (F_s/50{,}000, 10\%, 20)$$

$$= 50{,}000(1+0.1)^{20} \quad (3.25)$$

$$= \$336{,}375$$

By Eq. (3.4), the investment in bonds would grow to

$$= (F_B/50{,}000, 6\%, 20)$$

$$= 50{,}000(1+.06)^{20} \quad (3.26)$$

$$= \$160{,}357$$

From the derivation of Eq. (3.4) from Eq. (3.3), it can be seen that the money market funds would grow to

$$100,000(1 + 0.025)^5(1 + 0.05)^5(1 + 0.08)^{10} = \$311,748 \quad (3.27)$$

Adding the results from Eqs. (3.20–3.22), the future worth of Darlene's investment portfolio would be

$$336,375 + 160,357 + 311,748 = \$808,480$$

Example 3.7   George Tataseo's Royalty Payments and Investment

George Tataseo writes books. He receives royalty payments ranging from 8%–15% of the cover price of every book sold in the market. The twice-yearly payment is roughly about $8000. The expenses incurred every month during the preparation of the book are about $400. George invests the balance completely into a stock-based mutual fund. The return is about 9.5% on an annualized basis. After 10 years, what would the future worth of George's investments be?

Effective interest rate for six months for return on stock fund

$$= 9.5\%/2 = 4.75\% \quad (3.28)$$

Number of six-month periods in 10 years $= 10 * 2 = 20$

Future Worth $=$ (F/8000, 4.75%, 20) $-$ (F/400,9.5/12,120) (3.29)

$$= 8000 * \frac{(1.0475)^{20} - 1}{0.0475} - 400 * \frac{(1.0475)^{120} - 1}{0.0475}$$

$$= 8000 * 32.2056 - 400 * 12.4027 \quad (3.30)$$

$$= \$252,684$$

3.2.3   Given F, I, and A, Find N

Current books on the subject refer to numerical solutions or trial-and-error methods for obtaining the pay period N given the future equivalent worth F, interest rate i, and uniform series payment A. Here, a closed-form analytical solution is obtained to calculate the pay period N. Eq. (3.15) can be rearranged as follows:
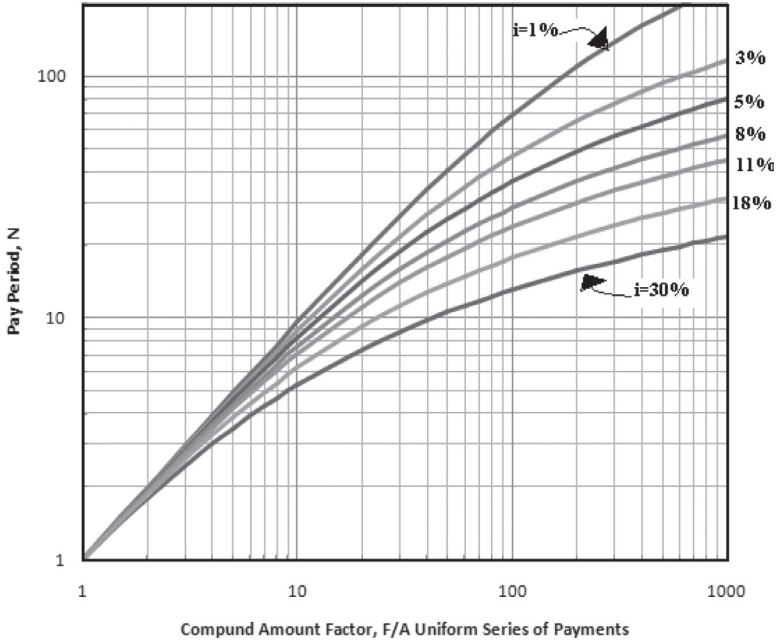
$$\frac{iF}{A} + 1 = (1 + i)^N \quad (3.31)$$

Figure 3.7 Pay Period N vs. Compound Amount Factor, F/A for Interest Rates 1% ≤ i ≤ 30%

$$N = \frac{\log\left(\dfrac{iF}{A} + 1\right)}{\log(1 + i)}$$

Eq. (3.31) is shown in Figure 3.7 as an easily readable chart. The pay period N needed to achieve a certain investment goal F using a uniform series of payments A for various interest rates ranging from 1% ≤ i ≤ 30% can be read from Figure 3.6. Both the axes are logarithmic. This scale captures the functionality of N with respect to F/A and i as shown in Eq. (3.31).

3.2.4   Given P, I, and A, Find N

Current books on the subject refer to numerical solutions or trial-and-error methods for obtaining the pay period N given the present equivalent worth P, interest rate i, and uniform series payment A. Here, a

closed-form analytical solution is obtained to calculate the pay period N. Eq. (3.19) can be rearranged as follows:

$$1 - \frac{iP}{A} = \frac{1}{(1 + i)^N} \tag{3.32}$$

$$N = \frac{\log\left(\dfrac{A}{A - iP}\right)}{\log(1 + i)}$$

3.2.5   Given F, A, and N, Find I

Current books on the subject refer to numerical solutions or trial-and-error methods for obtaining the interest rate i, given the pay period N and given the future equivalent worth F, and uniform series payment A. Here, a closed-form analytical expression is obtained to calculate the interest rate i. It can be noted that the i in the $(1 + i)^N$ is a small number. Given
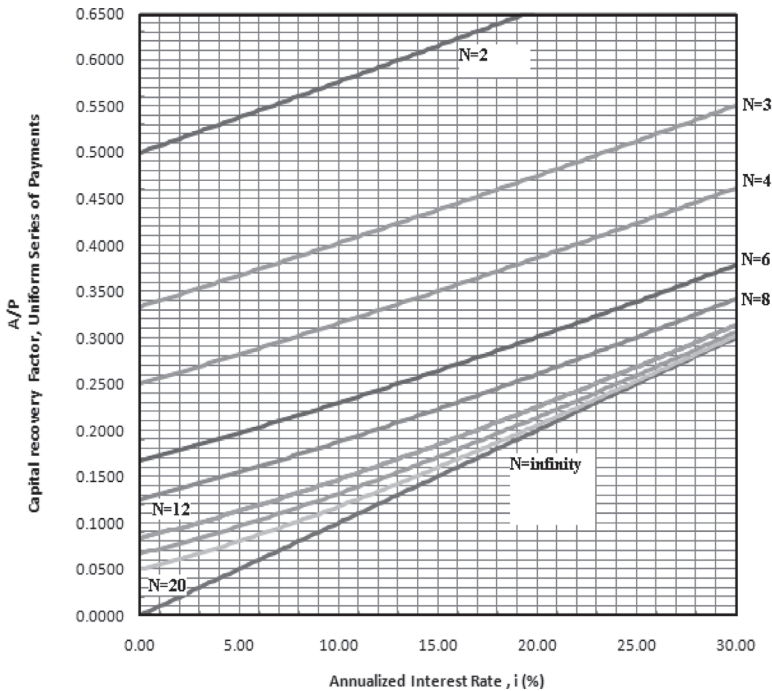

Figure 3.8 Capital Recovery Factor, A/P for Interest Rates
$1 \leq i \leq 30\%$ *and Pay Period* $2 \leq N \leq \infty$

1 is present, the expression is better suited for expansion in the form of a binomial infinite series as follows:

$$(1 + i)^N = 1 + Ni + \frac{N(N - 1)i^2}{2!} + \cdots \qquad (3.33)$$

The terms with i^3 and higher order terms are neglected. The truncation error involved is of the order of magnitude of $O(i^3/3!)$

Combining Eq. (3.33) and Eq. (3.15)

$$\frac{iF}{A} = Ni + \frac{N(N - 1)i^2}{2!} \qquad (3.34)$$

$$\text{or, } i = \frac{2}{(N - 1)}\left(\frac{F}{AN} - 1\right)$$

3.2.6 Given P, A, and N, Find I

Numerical solutions or trial-and-error methods are needed for obtaining the interest rate i, given the pay period N, and given the present equivalent worth P, and uniform series payment A. Here, a closed-form analytical expression is obtained to calculate the interest rate i. It can be noted that the i in the $(1 + i)^N$ is a small number. Given 1 is present, the expression is better suited for expansion in the form of a binomial infinite series as follows:

$$(1 + i)N = 1 + Ni + \frac{N(N - 1)i^2}{2!} + \cdots \qquad (3.35)$$

The terms with i^3 and higher order terms are neglected. The truncation error involved is of the order of magnitude of $O(i^3/3!)$.

Combining Eq. (3.24) and Eq. (3.15)

$$\frac{P}{AN} = \frac{2 + (N - 1)i}{2 + 2i + N(N - 1)i^2} \qquad (3.36)$$

$$\frac{P(N - 1)}{A}i^2 + i\left(\frac{2P}{AN} - (N - 1)\right) + \frac{2P}{AN} - 2 = 0$$

The interest rate i can be calculated by solution of the quadratic equation arrived at in Eq. (3.25).

Thus,

$$ai^2 + bi + c = 0 \qquad\qquad (3.37)$$

where,

$$a = \frac{P(N-1)}{A}; \quad b = \frac{2P}{AN} + 1 - N$$

$$c = \frac{2P}{AN} - 2$$

Example 3.8   U-Pay-Now & I-Pay-Later

You are hired by Cowboy Investments upon graduation. They are developing a new investment plan called "U-Pay-Now & I-Pay-Later" for customers. The deal is that the client pays $500 every month to Cowboy Investments. After a said period in a certain number of years, Cowboy Investments will have the client cease payments; the client then receives $500 every month forever. At a 10% annualized interest rate (the historic performance of Dow Jones), how many years would you recommend the said period be?

Let the said period of investments = N years (Figure 3.9)

$$(F/A,I,N) = (F/6000,10\%,N)$$

$$F = A(\frac{\left(1+i\right)^N - 1}{i})$$

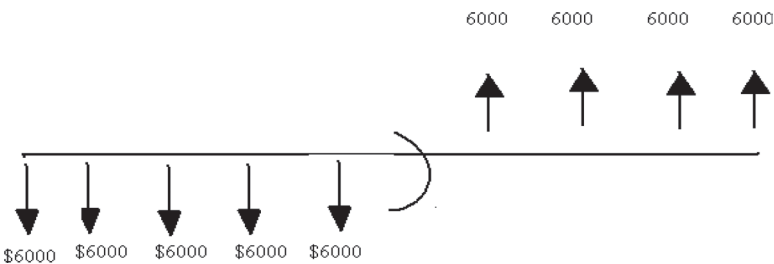The future worth after N years is invested until perpetuity

Figure 3.9 Cash Flow Diagram for "U-Pay-Now & I-Pay-Later"

(P/A, 10%, ∞)

$$\frac{P}{A} = \frac{(1+i)^N - 1}{i(1+i)^N}$$

$$\underset{N \to \infty}{Lt} \; (\frac{iP}{A}) = 1 - \frac{1}{(1+i)^N} = 1$$

$$P = \frac{A}{i} \tag{3.38}$$

It can be seen that from the "U-Pay-Now & I-Pay-Later" plan

$$F = A(\frac{(1+i)^N - 1}{i}) = P = \frac{A}{i}$$

$$(1+i)^N = 2 \tag{3.39}$$

$$N = \frac{\ln(2)}{\ln(1.1)} = 7.3 \; years$$

Example 3.8   When Does a Micro-Power Plant Become More Attractive Investment?

K. R. Sridhar, CEO of Bloom Box, was featured on CBS's *60 Minutes*. Bloom Box makes micro power plants using SOFC, solid oxide fuel cell technology. A bloom box can be purchased for $3000. It consists of miniaturized fuel cells made of silica. The cost of electricity to power up a new home would be 8.0 cents per kWh. Should the existing utility company charge 12.5 cents per kWh, calculate the number of years it would take for a bloom box to be a good investment for new homeowner Jack Tripper. Jack heads a family of six members with four children. Assume that 600 kWh of electricity is consumed every month at the Tripper residence. (i = 10%).

At the point when a Bloom Box will become lower cost compared with existing utility company rates, i.e., after N years

$3000 + 600*.08*12(P/A,10%,N)

$$= 600*0.125*12(P/A,10\%,N) \tag{3.40}$$

$$3000 = 324\left(\frac{1.1^N - 1}{0.1(1.1)^N}\right) \tag{3.41}$$

$$0.9259 = 1 - \frac{1}{1.1^N}$$

$$\frac{1}{1.1^N} = 0.07407$$

$$N = \frac{\ln(13.5)}{\ln(1.1)} = 27.3 \text{ years}$$

3.3   Uniform Gradient Series of Payments

Under certain circumstances, the receipts and expenses of an enterprise are projected to change by a uniform amount each period. A portion of the periodic payment *multiplies* each pay period. These sequences of cash flows are called uniform gradient series of cash flows.

The present equivalent worth of the uniform gradient series of cash payments shown in Figure 3.10 can be calculated as follows:

$$P = \frac{G}{(1+i)} + \frac{G(2)}{(1+i)^2} + \frac{G(3)}{(1+i)^3} + \cdots$$

$$+ \frac{G(N-1)}{(1+i)^{N-1}} + \frac{G(N)}{(1+i)^N} \tag{3.42}$$

$$P = G\sum_{n=1}^{N}\frac{n}{(1+i)^n} \tag{3.43}$$

Figure 3.10 Cash Flow Diagram of Uniform Gradient Series of Payments

Multiply Eq.(3.42) on both sides by $(1 + i)^N$

$$P(1 + i)^N = G(1 + i)^{N-1} + 2G(1 + i)^{N-2}$$
$$+ 3G(1 + i)^{N-3} + \cdots + (N - 1)G(1 + i) + NG \qquad (3.44)$$

The right-hand side, RHS of Eq. (3.44), can be split as follows:

$$RHS = G\left[(1+i)^{N-1} + (1+i)^{N-2} + (1+i)^{N-3} + (1+i) + 1 \right]$$
$$+ G\left[(1 + i)^{N-2} + (1 + i)^{N-3} + \dots (1 + i)) + 1 \right]$$
$$+ G\left[(1 + i)^{N-3} + \cdots + (1 + i) + 1 \right] + \cdots G \qquad (3.45)$$

Each of the terms within the square brackets in Eq. (3.45) is a geometric series and can be summed up as

$$RHS = G\frac{(1 + i)^N - 1}{i} + G\frac{(1 + i)^{N-1} - 1}{i} + G\frac{(1 + i)^{N-2} - 1}{i}$$

$$+ G\frac{(1 + i)^{N-3} - 1}{i} + \cdots + G\frac{(1 + i) - 1}{i} \qquad (3.46)$$

Combining Eq. (3.46) and Eq. (3.44) and simplifying

$$iP(1 + i)^N = G\left(\frac{(1 + i)^{N+1} - 1}{i} - N \right) \qquad (3.47)$$

Thus, P/G can be seen to be

$$P = \frac{G}{i(1 + i)^N}\left(\frac{(1 + i)^{N+1} - 1}{i} - N \right) \qquad (3.48)$$

A *gradient to uniform series conversion factor* can also be defined by combining Eq. (3.48) and that for A/P, Eq. (3.20). Thus,

$$A = \frac{G}{(1 + i)^N - 1}\left(\frac{(1 + i)^{N+1} - 1}{i} - N \right) \qquad (3.49)$$

The future worth of the uniform gradient series of payments can be calculated by combining Eq. (3.48) and Eq. (3.4). Thus,

$$F = \frac{G}{i}\left(\frac{(1 + i)^{N+1} - 1}{i} - N \right) \qquad (3.50)$$

3.4   Continuous Compounding

As discussed in Section 3.1.2, compounding periods may vary from daily, weekly, monthly, quarterly, semiannually, yearly, etc. All these are discrete compounding periods. The interest gets calculated after discrete periods of time. Eq. (3.10) gives the relation between the nominal interest rate per compounding period and the effective interest rate per year.

$$i = \left(1 + \frac{i'}{P}\right)^{P} - 1 \tag{3.51}$$

Let $P/i' = m$, Eq. (3.51) then becomes

$$i = \left(1 + \frac{1}{m}\right)^{mi'} - 1 \tag{3.52}$$

When the compounding period is changed from discrete to continuous, m in Eq. (3.52) becomes ∞, infinity. Further,

$$\underset{m \to \infty}{Lt} \left(1 + \frac{1}{m}\right)^{m} = 1^m + m(1)^{m-1}\frac{1}{m} + \frac{m(m-1)}{2!m^2}1^{m-2}$$

$$+ \frac{m(m-1)(m-2)}{3!m^3}1^{m-3} + \cdots$$

$$= \underset{m \to \infty}{Lt}\ 1 + 1 + \frac{1}{2!}(1 - \frac{1}{m}) + \frac{1}{3!}(1 - \frac{1}{m})^2 + \cdots$$

$$= 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots \tag{3.53}$$

By Taylor series expansion, e^x evaluated at $x = 1$ can be seen to be

$$e^1 = 1 + 1 + \frac{1^2}{2!} + \frac{1^3}{3!} + \cdots = 2.71828182845905 \tag{3.54}$$

Comparing Eq. (3.53) and Eq. (3.54)

$$\underset{m \to \infty}{Lt} \left(1 + \frac{1}{m}\right)^{\infty} = e \tag{3.55}$$

Combining Eq. (3.52) and Eq. (3.55)

$$i = e^{i'} - 1 \tag{3.56}$$

The effective interest rate for *continuous compounding* is given by Eq. (3.56).

3.4.1 *F/P, F/A, P/A, A/F, and A/P for Continuous Compounding*

The corresponding formulae for F/P, F/A, and P/A with continuous compounding that were presented for discrete compounding earlier are given side by side in Table 3.3.

Example 3.10   Continuous Compounding

Show the effect of continuous compounding by comparing the future worth of the nest egg of Alice Smith as discussed in Example 3.4. Sketch the future worth of Alice Smith's nest egg as a function of number of years N. Use the same interest rate for discrete as well as continuous compounding.

$$\text{Discrete Compounding: } F_D = \$660^*(F/A,10\%,N)$$
$$= \frac{(1 + i)^N - 1}{i} = \frac{1.1^N - 1}{0.1} \quad (3.57)$$

Table 3.3 *Factors Used to Calculate Discrete and Continuous Compound Interest*

To Find	Given	Discrete Factor by Which to Multiply	Compounding Factor Name	Factor Functional Symbol	Continuous Factor by Which to Multiply	Compounding Functional Symbol
F	P	$(1 + i)^N$	Single Payment Compound Amount	(F/P,i%,N)	$e^{i'N}$	(F/P,i'%,N)
F	A	$\dfrac{(1 + i)^N - 1}{i}$	Uniform Series Compound Amount	(F/A,i%,N)	$\dfrac{e^{i'N} - 1}{e^{i'} - 1}$	(F/A,i'%,N)
P	A	$\dfrac{(1 + i)^N - 1}{i(1 + i)^N}$	Uniform Series Present Worth	(P/A,i%,N)	$\dfrac{e^{i'N} - 1}{e^{i'N}(e^{i'} - 1)}$	(P/A,i'%,N)
A	F	$\dfrac{i}{(1 + i)^N - 1}$	Sinking Fund	(A/F,i%,N)	$\dfrac{e^{i'} - 1}{e^{i'N} - 1}$	(A/F,i'%,N)
A	P	$\dfrac{i(1 + i)^N}{(1 + i)^N - 1}$	Capital Recovery	(A/P,i%,N)	$\dfrac{e^{i'N}(e^{i'} - 1)}{e^{i'N} - 1}$	(A/P,i'%,N)

The interest rate used is 10%, which is the average performance of the Dow Jones over the past 100 years.

Continuous Compounding:

The nominal interest rate i' is taken as 10%.

Continuous Compounding: $F_C = \$660*(F/A,i'\%,N)$

$$= \frac{e^{i'N} - 1}{e^{i'} - 1} = \frac{e^{0.1N} - 1}{e^{0.1} - 1} \qquad (3.58)$$

Eq. (3.58) and Eq. (3.57) were calculated in an MS Excel spreadsheet and plotted in Figure 3.3. It can be seen from Figure 3.11 that the effect of continuous compounding is only seen after 25 years!

3.4.2   Given F, A, and I', Find N During Continuous Compounding

Current books on the subject refer to numerical solutions or trial-and error-methods for obtaining the pay period N given the future equiva-lent worth F, interest rate for continuous compounding, i'% per year
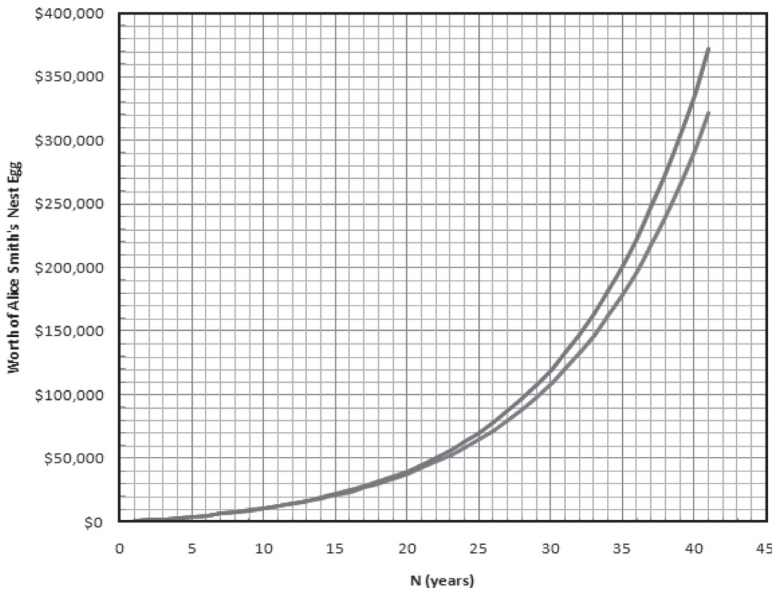

Figure 3.11 Worth of Alice Smith's Nest Egg (F/A) for Continuous & Discrete Compounding

and uniform series payment A. Here, a closed-form analytical solution is obtained to calculate the pay period N. The factor for obtaining F/A for continuous compounding given in Table 3.3 can be seen to be

$$F/A = \frac{e^{i'N} - 1}{e^{i'} - 1} \qquad (3.59)$$

Rearranging Eq. (3.50)

$$\left(e^{i'} - 1\right)\frac{F}{A} + 1 = e^{i'N} \qquad (3.60)$$

Obtaining the natural logarithms on either sides of Eq. (3.51), the period N can be calculated as

$$N = \frac{1}{i'}\ln\left(1 + \frac{F(e^{i'} - 1)}{A}\right) \qquad (3.61)$$

Example 3.11   Time Taken to Achieve Investment Goal

Adam Jackson met with his financial planner and came up with his financial goal of making 1 million dollars. He was willing to deposit $1050 every month with the financial planner, who was going to invest Mr. Jackson's monies into stocks, bonds, money market, and gold. The financial planner projected a return of a nominal rate of 12% compounded on a monthly basis. How many years will it take for

(i) Adam to achieve his financial goal using discrete compounding
(ii) Adam to achieve his financial goal using continuous compounding?

$$F/A = 10^6/1000 = 1000 \qquad (3.62)$$

The effective interest rate for a nominal interest rate of
12% per month is 12.68%   (3.63)

By Eq. (3.31) for discrete compounding (natural logarithms may be used) the time taken to attain the investment goal may be estimated as

$$N = \frac{\log\left(\dfrac{iF}{A} + 1\right)}{\log(1 + i)} = \frac{\ln(0.1268*1000 + 1)}{\ln(1.1268)} = 40.63 \; years \quad (3.64)$$

By Eq. (3.61) for continuous compounding, the time taken to attain the investment goal may be estimated as

$$N = \frac{1}{i'} \ln \left(1 + \frac{F(e^{i'} - 1)}{A} \right) = \frac{1}{0.1268} \ln \left(1 + 1000*(e^{0.1268} - 1) \right)$$
$$= 38.75 \; years \qquad (3.65)$$

The time taken to achieve the investment goal of Adam Jackson changes only by about 1.88 years, even when continuous compounding is considered.

Example 3.12   Biodiesel Production from Waste Cooking Oil

The study of alternate fuel sources to gasoline and coal is of national importance, given the supply-and-demand characteristics of fuels that are used extensively. Biodiesel is derived from vegetable oil or animal fats. It is recommended for use as a substitute for petroleum-based diesel because it is renewable and biodegradable. The common method of preparation of biodiesel is the transesterification of triacylglycerols in vegetable oil or animal fat with an alcohol such as methanol in the presence of an alkali or acid catalyst. The products are FAME (fatty acid methyl esters) and are called biodiesel. Glycerin is formed as a byproduct. Sodium hydroxide or potassium hydroxide is used as an alkali catalyst. A recent study [2] evaluated the economic feasibility of a plant producing approximately 22 million pounds per year of biodiesel. Tallow was transesterified with methanol in the presence of an alkali catalyst. A second plant is based on canola seed used as the raw material. A by-product of credit can be awarded for glycerin produced from seed crushing. A summary of the capital investment, process cost, and revenue accrued of the three different plants is shown in Table 3.4. The capital cost can be assumed to be paid off over a 30-year period at an interest charge of 6% per year.

Which process is the most profitable?

It is assumed that the capital cost is amortized at an interest rate of 6% per year for 30 years.

Plant I

(A/P, 6%, 30) from Table A-10, the capital recovery factor can be seen to be 0.072649

Table 3.4 Economic Evaluations for Biodiesel Production Plants

	Plant I Alkali-Catalyzed Continuous Process	Plant II Alkali-Catalyzed Batch Process	Plant III Alkali-Catalyzed Continuous Process
Plant Capacity	22 million pounds per year	1.7 million pounds per year	2.2 million pounds per year
Raw Material Used	Beef Tallow	Canola Oilseed	Animal Fats
Total Capital Cost	$12 million	$1 million	$3.12 million
Total Manufacturing Cost	$34 million	$5.95 million	$3.4 million
Glycerin Credit	$6 million	$0.9 million	$1.2 million
Price	$2/lb	$4/lb	$3/lb

Profit = 2*22 − 12*(A/P, 6%,30) − 34 + 6 = 15.128 million

Revenue − capital recovery − cost of production
+ by-product credit = profit   (3.66)

Plant II

(A/P, 6%, 30) from Table A-10, the capital recovery factor can be seen to be 0.072649

Profit = 4*1.7 − 1*(A/P, 6%,30) − 5.95 + 0.9 = 1.68 million

Revenue − capital recovery − cost of production
+ by-product credit = profit   (3.67)

Plant III

(A/P, 6%, 30) from Table A-10 the capital recovery factor can be seen to be 0.072649

Profit = 3*2.2 − 3.12*(A/P, 6%,30) − 3.4 + 1.2 = 4.17 million

Revenue − capital recovery − cost of production
+ by-product credit = profit   (3.68)

Thus, Plant I is the most profitable, followed by Plant III and then Plant II.

Example 3.13   Credit Card Debt of U.S. Households

A recent survey shows there are about 114 million households in the United States. The average credit card debt carried per household is $15,519. There are about 576.4 million credit cards in circulation. At an average credit card interest rate of 21% per year, what would the debt be if not paid back in another 10 years?

$$P = 576.4*15,519 = \$8.9451516 \text{ trillion} \qquad (3.69)$$

$$(F/P,21\%,10) = 6.73 \text{ (From Table A-21)} \qquad (3.70)$$

$$F = 6.73*8.9451516 = \$60.2 \text{ trillion} \qquad (3.71)$$

Example 3.14   Time Taken to Pay Off Credit Card Debt

In Example 3.11, how many years will it take for U.S. households to pay off their current household debts at a rate of 21% per year? Assume that the monthly payment per credit card is $300 per month.

$$P/A = 15,519/3600 = 4.3108 \qquad (3.72)$$

$$\text{From Table A-21, N can be seen to be } 12 < N < 14 \qquad (3.73)$$

Another way to calculate the number of years to pay off the debt is by applying Eq. (3.32)

$$N = \frac{\log\left(\dfrac{A}{A - iP}\right)}{\log(1 + i)} = \frac{\ln\left(\dfrac{3600}{3600 - 3259}\right)}{\ln(1.21)} = 12.36 \; years \quad (3.74)$$

Example 3.14   Deferred Annuity

Katy Gerstner, a widow, was planning for her son Joe's college education. What principal amount should Katy set aside now to allow for annual withdrawals of $4500 each on Joe's 18th, 19th, 20th, and 21st birthdays? The interest rate is taken at 11% per year.

Four withdrawals are to be made of $4500 each on Joe's 18th, 19th, 20th, and 21st birthdays, as shown in Figure 3.12. The present equivalent worth of this annuity on Joe's 17th birthday is P_{17}.
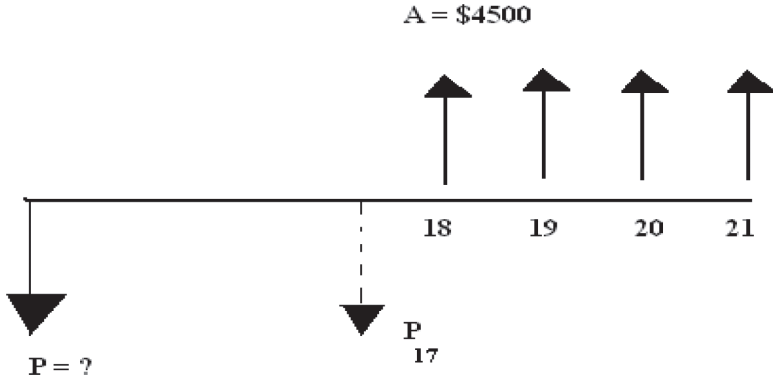
Figure 3.12 Deferred Annuity Payments for Joe Gerstner's College Education

$P_{17} = \$4500(P_{17}/A, 11\%, 4)$

$(P_{17}/A, 11\%, 4)$ from Table A-15 can be seen to be 3.1024   (3.75)

$P_{17} = \$4500*3.1024 = \$13{,}961$

The present equivalent worth of P_{17} can be calculated from a single payment compound factor as follows:

$$P = P_{17}(1 + i)^{-17} \qquad (3.76)$$

(P/F, 11%, 17) from Table A-15 can be seen to be 0.170557 by interpolating between N= 16 and N = 18

$$P = P_{17}*0.170557 = \$13{,}961*0.170557 = \$2381.15 \qquad (3.77)$$

3.5   Summary

Expressions to calculate simple interest and compound interest given the principal, P, interest rate i%, and pay period N are provided. The single payment compound amount factor, F/P, as a function of an annualized interest rate, i, is plotted in Figures 3.1 and 3.2 for different pay periods. The characteristics of the curve change for N > 12. Expressions for pay period N and interest rate i% in terms of the F and P are given in Eq. (3.6) and Eq. (3.7). The interest rate vs. F/P for different N is shown

in Figure 3.3. *Nominal interest rates* get compounded on a different basis other than annual. The relation between nominal interest rate and *effective interest rate* is summarized in Table 3.1.

Cash Flow Diagrams are illustrations that show all the monetary transactions during the time of an enterprise. They can be used to calculate the equivalence of a series of uniform payments in terms of its present worth, P, or in terms of its future worth, F. The sum of a geometric series is used to develop an expression for F/A in terms of interest rate, i%, and pay period N for a uniform series of constant payments, A. F/A is called the *uniform series compound amount factor*. A/F is the *sinking fund factor*. Sinking fund factor is plotted vs. effective interest rate for different pay periods in Figures 3.5 and 3.6. In Appendix A, annuity tables are provided to read F/P, P/F, F/A, A/F, P/A, and A/P values for various values of interest rates ranging from 1/8% to 30%, and pay periods N ranging from 1 to 100. In addition to the annuity tables for the uniform series of payments A, annuity tables for uniform gradient series of payments G are also provided in Appendix A. Expressions for *uniform series present worth factor* are also developed. When N becomes infinity, perpetual payment schemes can be developed. Expressions for *capital recovery factor*, A/P are provided. Cash flow diagrams with a uniform gradient series of payments are discussed. Closed-form analytical expressions for P/G, F/G, and A/G are developed.

The inverse problem of finding N given F, A, and i% or P, A and i% is discussed. Closed-form analytical expressions for N are developed using logarithms. Graphical methods and interpolation in annuity tables may also be used. The inverse problem of finding interest rate i% given F, A, and N or P, A, and N are discussed. In addition to the use of annuity tables, graphical methods, a closed-form analytical expression is developed. This is achieved via the use of binomial expansion of $(1 + i)^N$. As many terms in the infinite series as needed for desirable accuracy can be taken.

Worked examples on doubling time of investment, nominal interest rate, perpetual payment for college days prize, 0% interest rate, nest egg estimates, diversification of portfolio, royalty payment and investment, payment and reimbursement schemes, micro power plant, biodiesel production from waste cooking oil, credit card debt for all U.S. households, deferred annuity, time taken to pay off credit card debt, are provided. Exercises on monthly car payments, number of years taken to pay off the

credit card debt, IRR of off-shore drilling, deferred interest rate charges, federal government debt, a Versailles-type home, variable interest rate, deferred payments, payment amount of a uniform series of payments, 0% interest, perpetual payment, design of a social security system for a town, and hydrogen from biomass are developed.

Continuous compounding methods, as opposed to discrete compounding methods, are also developed. The corresponding formulae for F/P, F/A, and P/A with continuous compounding that were presented for discrete compounding earlier are given side by side in Table 3.3. Appendix B provides the annuity tables for continuous compounding.

3.6   References

[1] W. G. Sullivan, E. M. Wicks, and C. P. Koelling, *Engineering Economy*, 14th Edition. Pearson Prentice Hall (2009), Upper Saddle River, NJ.

[2] Y. Zhang, M. A. Dube, D. D. McLean, and M. Kates, "Biodiesel Production from Waste Cooking Oil: Economic Assessment and Sensitivity Analysis," *Bioresource Technology*, Vol. 90, 3 (2003), 229–240.

[3] P. L. Spath, J. M. Lane, M. K. Mann, and W. A. Amos, "Update of Hydrogen from Biomass-Determination of the Delivered Cost of Hydrogen," Milestone Report for U.S. Department of Energy's Hydrogen Program (2001), NREL Laboratory, Golden, CO.

CHAPTER 4

Methods for Evaluation of Capital Projects

Topics Covered
- Present Worth Analysis
- Future Worth Analysis
- Annual Worth Analysis
- Internal Rate of Return, IRR
- External Rate of Return, ERR
- Payback Period

4.1   Overview

The *profitability* of projects can be evaluated in terms of dollars and cents. In this chapter, five different methods are presented that can be used to determine the worthiness of undertaking an enterprise or project. The time taken for recovery of investment can also be calculated. The five different methods are:

  (i) Present Worth, PW;
 (ii) Future Worth, FW;
(iii) Annual Worth, AW;
 (iv) Internal Rate of Return, IRR;
  (v) External Rate of Return, ERR.

In addition to these five methods, the *payback period* is a method to estimate the time period over which the monies invested can be recovered.

This is not recommended as a primary decision rule. Hence, it is not listed in the five methods mentioned above. The above methods can be used to make decisions based on dollars and cents.

A MARR, minimum acceptable rate of return, can be specified. In order to be selected as a project, a return from the business enterprise has to be established that exceeds a minimum specified level. This minimum level can be called a MARR. A number of factors can go into the determination of a MARR. These include, but are not limited to:

a) Amount of Monies;
b) Source of Monies;
c) Cost of Monies;
d) Number of Feasible Projects;
e) Purpose of Feasible Projects;
f) Risk of Investment Opportunities;
g) Organizational Ethos.

The cost of monies can be calculated using the methods discussed in Chapter 3.0.

4.2   Present Worth Analysis (PW)

The PW, present worth of a project, can be calculated by the equivalence of all cash inflows (such as receipts) and all cash outflows (such as expenses) to the current time at an interest rate that is a MARR. When the calculated PW is positive, the project is acceptable for further pursuits. The period of time, N, can be obtained suitably from the cash flow diagram. The factors considered during PW analysis are as follows:

 (i) Purchase Price of Equipment Needed;
 (ii) Estimated Useful Life;
(iii) Operational and Maintenance Costs;
(iv) Energy and Other Utility Costs;
 (v) Salvage Value;
(vi) Revenue Accrued;
(vii) Interest Rate, i (%).

The effect of *inflation* and *deflation* on raw materials and energy costs can be taken into account as well, using inflation factors. This will be illustrated in Worked Example 4.4. The inflation factor gives the rate at which the prices go up every year. This information can be obtained from the Bureau of Labor Statistics. Consumer Price Indices are reported and available readily via Internet in the public literature.

When N becomes infinity, PW of such projects can also be called CW, the *capitalized worth* of the projects. The CW method is popular when endowments are established and public projects with indefinite lives are pursued.

Example 4.1   Hydrogen from Fast Pyrolysis
and Steam Re-Forming [1]

Hydrogen is seriously considered as an alternative fuel in the transportation sector. In one process, the biomass is pyrolyzed and then steam re-formed to yield hydrogen. Biomass is dried and then converted to oil by very quick exposure to heated particles in a fluidized bed. The char and gases produced are combusted to supply heat to the reactor. The product oils are cooled and condensed. Bio-oil is then shipped by truck from the plant location(s) to the hydrogen production facility. By shipping the bio-oil, which has a higher energy density than the biomass, transportation costs are reduced. On arrival, the bio-oil is fractionated using water extraction into carbohydrate fraction and lignin fractions. The carbohydrate fraction is steam re-formed and the hydrogen that is formed is separated using PSA, pressure swing adsorption. Consider a plant that produces 75,790 kg/day of hydrogen by the pyrolysis process. The biomass feed rate is about 1806 Mg/day. Capital investment fixed costs are $59.4 million. The selling price of hydrogen is $10/GJ. The storage and transportation cost of hydrogen through railroad is $0.94/GJ. The calorific value of hydrogen is 142 MJ/kg. At a MARR of 11%, calculate the PW of the project. The plant has operated for five years. The biomass feedstock cost was $16.5/Mg. The plant operated at 80% time during the year. The costs of utility and wages of laborers can be lumped into 15c/kg of hydrogen produced.
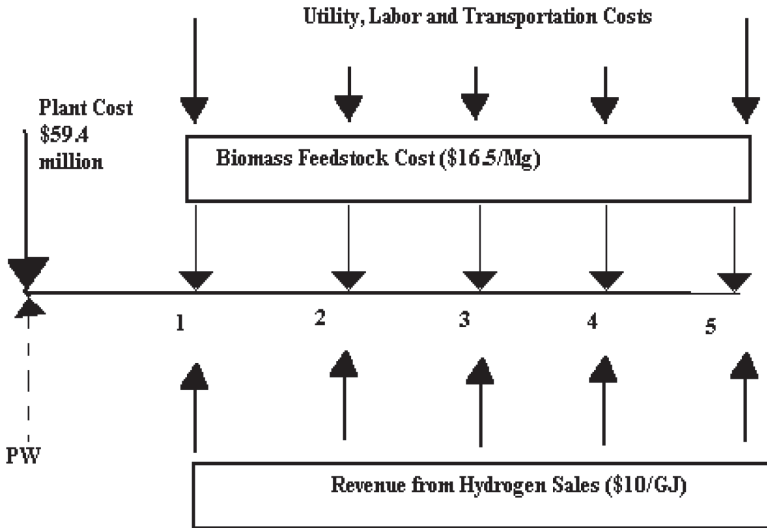
Figure 4.1 Cash Flow Diagram for Revenue and Costs for Hydrogen Production from Pyrolysis and Steam Re-Forming

PW = $-\$59.4$ million (capital cost)

  $-16.5*1806*365*0.8$ (P/A,11%,5) (raw material cost)

  $-365*0.8*75,790*142/1000*0.94$ (P/A,11%,5)

  (transportation cost)

  $-0.15*75790*365*0.8/10^6$ (P/A, 11%,5) (utility and wages)

  $+10*75,790*365*0.8*142/1000$ (P/A,11%,5)

  (revenue from sales of hydrogen)   (4.1)

  (P/A,11%,5) = 3.6959 (From Table A-15)   (4.2)

  PW $=-\$59.4$ million $- 3.6959*8.7 - 3.6959*2.9603$

    $- 3.6959*3.31 + 3.6959*31.43$

    $= \$1.433405$ million $= \$1,433,405$ (4.3)

A present worth of \$1.433 million indicates that the project can be selected at 11% MARR. It can be seen that as the interest rate or MARR is increased, at some interest rate the PW will become 0. This interest rate later will be seen to be the IRR, the internal rate of return. For interest rates less than the IRR, the project will be profitable.

Example 4.2   Replacement for the World Trade Center (WTC)

One proposal to replace the World Trade Center (WTC) came from an architect and real estate tycoon named Donald Stone. He proposed another high-rise to replace the WTC, which was destroyed by terrorists on September 11, 2001. In a typical week, 50,000 people are expected to work in these towers and 200,000 visitors are expected to go the top floor to obtain a glimpse of New York City from the 110th floor. The cost of constructing the building is estimated at $5 billion. Assuming a net profit of collecting rent from the occupants is $250 per person per week and a visitor fee of $10, how many years would it take to see a profit? The interest rate can be taken to be 4%.

$$\text{Rental Fee collected per year} = 250*50{,}000*52/10^6$$
$$= \$ \, 650 \text{ million} \tag{4.4}$$

$$\text{Visitor Fee collected per year} = 200{,}000*10*52/10^6$$
$$= \$104 \text{ million} \tag{4.5}$$

$$\text{A/P} = (650+104)/5000 = 0.1508 \tag{4.6}$$

From Table A-8, (4% annuity table) for A/P = 0.1508, N can be seen to be a little less than eight years.

In a little less than eight years, the venture of replacing the WTC will be profitable.

Example 4.3   Capitalized Worth of Stone's
Replacement for the WTC

In Example 4.2, calculate the capitalized worth of Stone's proposed replacement for the WTC destroyed by terrorists in the 2001 bombings. The interest rate is 4% and the revenues can be taken as the same in Example 4.2.

$$\text{CW} = (\text{P/A}, 4\%, \infty) \tag{4.7}$$

By Eq. (3.19)

$$P = \frac{A\left((1 + i)^N - 1\right)}{i(1 + i)^N} \tag{4.8}$$

When N→∞, Eq. (4.8) becomes

$$P = \frac{A\left(1 - \dfrac{1}{(1+i)^N}\right)}{i} = \frac{A}{i} \qquad (4.9)$$

Thus, CW = ($650 + 104)/0.04 = $18.850 billion dollars.

Thus, the capitalized worth of the replacement for the WTC proposed by architect Stone would be $18.5 billion dollars.

4.3   Future Worth Analysis (FW)

The FW (future worth) of a project can be calculated by the equivalence of all cash inflows such as receipts and all cash outflows such as expenses to a future time at an interest rate that is a MARR. When the calculated FW is positive, the project is acceptable for further pursuits. The period of time, N, can be obtained suitably from the cash flow diagram.

Example 4.4   University Apartments

A group of university alumni and professors invests $25 million to build 250 new student apartments near a state university. The money is borrowed at the prime rate of 3.25% calculated on a yearly basis. The period of the loan is for 30 years. The variable expenses that include maintenance and operation run the group $3000 per apartment per year. The rental fees collected are about $12000 per apartment per year. The occupancy rate is about 70%. What is the future worth of the enterprise, should the salvage value of the apartments be $5 million at the end of 30 years?

$$(F/P,3.25\%,30) = \text{Interpolated from Table A-8 and}$$
$$\text{Table A-7 for 4\% \& 3\%}$$

$$= 2.427262 + 0.25^*(3.243398 - 2.427262) = 2.6313 \quad (4.10)$$

$$(F/A,3.25\%,20) = \text{Interpolated from Table A-8 and}$$
$$\text{Table A-7 for 4\% \& 3\%}$$

$$= 47.575416 + 0.25(56.084938 - 47.575516) = 49.7028 \quad (4.11)$$

F W = −25(F/P,3.25%,30) (principal invested)
3000*250*(F/A,3.25%,30) (annual operational expenses)
+12000*250*0.70(F/A,3.25%,30) (rental revenue)
+5 (salvage value)   (4.12)

FW = −65.7825 − 37.2771 + 104.37588 + 5
= $6.31628 million.   (4.13)

It is profitable to invest in the apartments. The FW of the project would be $6.163 million.

Example 4.5   Present Worth of a Sugar Mill

A business plan was presented to get a sugar mill installed in rural Texas. The cost of raw materials such as sugarcane and utility costs rise every year due to inflation. The revenue from the sale of sugar rises and falls in synchrony with the boom-and-bust cycles of the economy. The EOY (end of year) revenues and costs are shown in Table 4.1.

The capital cost of the sugar mill, including land and all the necessary equipment, is estimated at $8 million. What is the PW of the sugar business at an interest rate of 3%?

Another table is generated using an MS Excel spreadsheet on a desktop computer. Two more columns are added to Table 4.1. One is the

Table 4.1 EOY Receipts and Costs

Year	Receipts Millions $	Cost Millions $
1	2.3	1.00
2	2.5	1.10
3	2	1.21
4	1.5	1.33
5	2	1.46
6	2.5	1.61
7	3	1.77
8	3.5	1.95
9	4	2.14
10	3.5	2.36
11	3	2.59

profit, Pr, obtained each year. This is determined by subtracting the costs from the receipts. The contribution of each cell in the next column to the present worth is obtained using the relation for compound interest given by Eq. (3.5)

$$PW_J = \frac{Pr_J}{(1 + i)^J} \qquad (4.14)$$

where Pr_J is the profit obtained in year J. J varies from 1 to 11 as given in column 1.

$$PW = -8.0 + SUM(P/F,3\%,Year)$$

$$= -8.0 + 9.451591 = +1.451591. \qquad (4.15)$$

Thus, the PW of the sugar business is $1 million and 451,591. It is profitable to start the sugar from sugarcane business.

Example 4.6   Invest in Gold or Stock Market

The gold prices and DJIA (Dow Jones Industrial Average) current, 50, and 100 years ago are given in Table 4.3.

Table 4.2 Present Worth Analysis of EOY Cash Flows (Millions $) in Table 4.1

Year	Receipts	Cost	Profit	(P/F,3%,Year)
1	2.3	1.00	1.30	1.262136
2	2.5	1.10	1.40	1.319634
3	2	1.21	0.79	0.722962
4	1.5	1.33	0.17	0.150154
5	2	1.46	0.54	0.462272
6	2.5	1.61	0.89	0.744934
7	3	1.77	1.23	0.998833
8	3.5	1.95	1.55	1.224597
9	4	2.14	1.86	1.422785
10	3.5	2.36	1.14	0.849794
11	3	2.59	0.41	0.293489
			Total	9.451591

Table 4.3 Gold and DJIA Current, 50, and 100 Years Ago

Year	Gold (per ounce)	Stock (DJIA Dow Jones Industrial Average)
1909	$20.67	70
1959	$35.25	600
2009	$1087.50	10,000

Which would be a better investment of $50,000 today in another 30 years, gold or stocks?

Gold has performed at an annualized rate of

$$F/P = 1087.5/20.67 = 52.61 \tag{4.16}$$

For N = 100 the interest rate can be seen from Appendix A as roughly 4%

$$FW \text{ (gold)} = 50,000(F/P, 4\%, 30) \tag{4.17}$$

From Table A-8, (F/P, 4%, 30) = 3.243398*50,000 = $162,115

DJIA has performed at an annualized rate of

$$F/P = 10,000/70 = 142.86 \tag{4.18}$$

From Appendix A for N = 100, F/P = 142.86 the interest rate can be seen to be 5%

$$\begin{aligned} \text{From Table A-9 (F/P,5\%,30)} &= 4.321942*50,000 \\ &= \$216,097 \end{aligned} \tag{4.19}$$

$219,097 is greater than $162,115. Hence, it is better to invest in the U.S. stock market rather than in gold. The 5% is the historic performance of DJIA. notwithstanding the Great Depression and stock market corrections in 1987 and 2007. The gold price has increased at 4% despite the government changes in the early 1970s.

Example 4.7   Electric/Gas Hybrid Vehicle

A hybrid electric/gas car such as the 2010 Toyota Prius costs about $29,000. The achievable fuel economy using the vehicle is 50 miles per

gallon. A comparable all-gasoline vehicle such as the 2010 Toyota Corolla costs $21,000, and the fuel economy for the Corolla is 32 miles per gallon. At what price of gasoline is it more attractive to purchase the Toyota Prius? It is estimated that the average passenger travels 15,000 miles per year and the life of an automobile is 100,000 miles. The interest rate may be taken as 11%.

The cash flow diagram for Example 4.7 is shown in Figure 4.2.

Let the price of gasoline per gallon be Z $/gallon. Assume that the lives of both the Toyota Corolla and the Toyota Prius are seven years and the passenger drives either vehicle at 15,000 miles per year.

PW of all payments in order to own the Toyota Prius, PW_{prius}, is;

$$PW_{prius} = -\$29,000 - Z*(15,000/50)*(P/A,11\%,7) \qquad (4.20)$$

From Table A-15

$$(P/A,11\%,7) = \tfrac{1}{2}*(4.2305+5.1461) = 4.6883$$

$$PW_{prius} = -29,000 - 1407Z \qquad (4.21)$$

PW of all payments in order to own the Toyota Corolla, $PW_{corolla}$ is

$$PW_{corolla} = -\$21,000 - Z*(15,000/32)*(P/A,11\%,7) \qquad (4.22)$$

From Table A-15

$$(P/A,11\%,7) = \tfrac{1}{2}*(4.2305 + 5.1461) = 4.6883$$

$$PW_{corolla} = -21,000 - 2198Z \qquad (4.23)$$
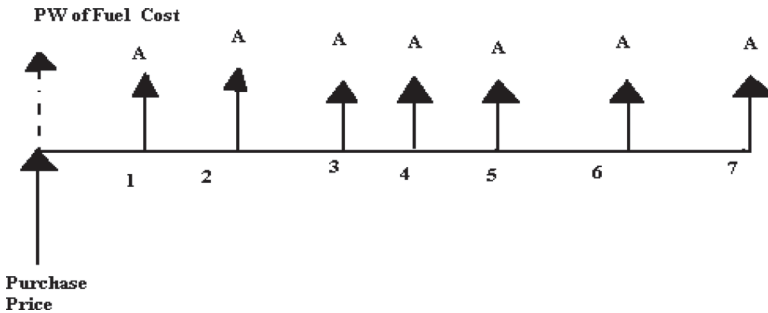
Figure 4.2 Cash Flow Diagram of Fuel Cost for Toyota Prius and Toyota Corolla

At a certain gasoline price, Z, the PW_{prius} will become equal with that of $PW_{corolla}$. Thus,

Eq. (4.23) and Eq. (4.21) can be set equal to each other.

$$-29{,}000 - 1407Z = -21{,}000 - 2198Z \qquad (4.24)$$

Or, $Z = \dfrac{(29{,}000 - 21{,}000)}{(2198 - 1407)} = 10.13$

For gasoline prices greater than $10.13, the Toyota Prius is a lower-cost alternative compared with the Toyota Corolla. For gasoline prices less than $10.13, the Corolla is a better alternative compared to the Prius.

Example 4.8   Effect of Inflation on PW

The inflation rates for the past 10 years (1999–2009) are given below in Table 4.4. This table was prepared using the information provided by the Bureau of Labor Statistics.

In 1999 dollars, calculate the PW (present worth) of an ABS (acrylonitrile butadiene styrene) engineering thermoplastic continuous polymerization plant. The capital cost of a new plant in 1999 dollars was $80 million. The selling price of ABS is $4 per pound. The raw materials cost rises every

Table 4.4 Inflation Rates for 1999–2009

Year	(%) Inflation
1999	2.19
2000	3.38
2001	2.83
2002	1.59
2003	2.27
2004	2.68
2005	3.39
2006	3.24
2007	2.85
2008	3.85
2009	−0.34

year on account of inflation. The price in year 1998 was $2.8 per pound of ABS sold. The process cost also goes up with inflation on account of increase in utility costs and wages to workers. The process cost can be taken as 30 cents per pound of ABS produced in 1998. The production rate from the two CSTR and falling strand devolatilizer plant is 10,000 lb/hr. The plant can be expected to run about 85% of the time. The selling price of ABS remains flat during the 10-year period. An interest rate of 11% may be assumed.

The inflation rates given in Table 4.4 were used to calculate the materials cost and process cost. This is shown in the column in Table 4.5 under Cost–Materials and Cost–Process. Each row was $(1 + f_i)$ times the previous row, where fi is the inflation rate. Inflation rate was calculated by dividing the values provided under the column (%) inflation by 100. The revenue was calculated at a flat price of $4 per lb. of ABS multiplied with the quantity of ABS produced, i.e., 74.46 million lbs. This was arrived at from the production rate of 10,000 lb/hr multiplied with the operating time of the plant 24*365*0.85, where 0.85 is the utilization factor. The inflation effects on the materials and process costs resulted in loss in the years 2008 and 2009. In other years there was a profit. The profit was obtained by subtracting the total expenses of materials and process from the revenue from sales of ABS. The PW contribution of each cell in the profit column was calculated using the single payment compound amount factor of $\dfrac{Pr_J}{(1 + i)^{Year - 1999}}$ where J varied from 0 to 10 and Pr_J was the profit earned in each year. All the PW contributions from 1999 to 2009 were added using the SUM command in MS Excel spreadsheets on a desktop computer. The capital cost of $80 million was also subtracted as a one-time charge. No salvage value was attributed in 2009. The PW of the ABS plant in 1999 dollars was seen to be $157.07 million.

Example 4.9   Life-Cycle Cost Analysis of HVAC Systems

Life-cycle cost analysis has been found to be advantageous over impressions created from one-time sticker-price numbers. A piece of equipment that may appear initially attractive due to its lower cost may "nickel and dime" you in costs during its operation. Maintenance costs may

Table 4.5 Present Worth Analysis with Inflation Effects Accounted for Effect of Inflation

				Year	(%) Inflation	Cost Materials	Cost Process	Revenue Millions $	Profit Million $	PW Million $
				1999	2.19	2.861	0.3066	297.84	61.95891	$61.96
Product Rate	10000	lb/hr		2000	3.38	2.958	0.3169	297.84	53.98613	$48.64
ABS Price	4	$/lb		2001	2.83	3.042	0.3259	297.84	47.08507	$38.22
Materials	2.8	$/lb		2002	1.59	3.090	0.3311	297.84	43.09806	$31.51
Process Cost	0.3	$/lb		2003	2.27	3.160	0.3386	297.84	37.31542	$24.58
				2004	2.68	3.245	0.3477	297.84	30.33336	$18.00
Production (lbs)	7E+07			2005	3.39	3.355	0.3595	297.84	21.26489	$11.37
	74.46	million lb		2006	3.24	3.464	0.3711	297.84	12.30385	$5.93
i	11	%		2007	2.85	3.562	0.3817	297.84	4.16607	$1.81
				2008	3.85	3.700	0.3964	297.84	−7.14037	($2.79)
				2009	−0.34	3.687	0.3950	297.84	−6.10344	($2.15)
Capital Cost of Plant										($80)
									Total	$157.07

mushroom and repair costs may make another alternative a better prospect. So how do you arrive at which equipment to select? This can be best accomplished by a life-cycle cost analysis. Here all the relevant costs are considered by the use of PW analysis.

Consider the residence of Sharron Stone. A new HVAC (heating, ventilation, and air conditioning) system was installed in her residence. The estimated useful life is 15 years. A novel high-efficiency gas furnace with add-on air conditioner costs $11,183. The energy costs per year are $928 and maintenance cost of $60. The salvage value of the equipment is $60. Use PW analysis and calculate its life-cycle cost. An interest rate for funds can be taken as 5%.

$$\text{Life Cycle Cost of HVAC, } C_L = \$11,183 - 60*(P/F,5\%,15)$$
$$+ (\$928+\$60)*(P/A,5\%,15) \qquad (4.25)$$

From Table A-9,

$$(P/F,5\%,15) = \tfrac{1}{2}*(0.505068 + 0.458112) = 0.48159$$

$$(P/A, 5\%, 15) = 1/2*(9.8986 + 10.8378) = 10.3682$$

Eq. (4.25) becomes

$$C_L = \$11,183 - \$60*0.48149 + \$988*10.3682$$
$$= \$21,397.89 \qquad (4.26)$$

The life-cycle costs associated using PW analysis of the HVAC equipment in Stone's residence was found to be $21,398.

Example 4.10   Municipal Garbage Collection Truck

The cost for the truck used for collecting garbage from residences is about $81,450. It can be used to serve 1000 households. A weekly charge per household of $5 can be collected. The gasoline costs run $150 per month. The labor costs for two drivers are $3200 per month. A one-time tax credit of $5000 from the federal government for recycling is given. Is it profitable to run the garbage business at a MARR of 11%. The life of the truck is expected to be eight years.

$$PW = -\$81,450 \text{ (purchase price of truck)}$$
$$+ \$5000 \text{ (tax credit for recycling)}$$

+ 1000*$5*4*12(P/A,11%,8) (revenue from garbage
  collection fees per year)

– 3200*12(P/A,11%,8) (salary for two drivers per year)

$$- \$150*12(P/A,11\%,8) \text{ (gasoline cost of truck per year)} \qquad (4.27)$$

(P/A,11%,8) = 5.1461 (from Table A-15)

PW = −$81,450+$5,000 + $240,000*(5.1461)
     − $36,000*(5.1461) − $1,800*(5.1461)

$$= -\$81,450 + \$5000 + \$1,235,064 - \$197,610.24$$
$$- \$9262.98 = \$951,741 \qquad (4.28)$$

It would be profitable to run the garbage business at a MARR of 11%.

Example 4.11   Hexane Extraction of Rice-Bran Oil

Rice-bran lipids can be extracted using hexane solvent from rice bran at a yield of about 26%. Rice is milled to produce the white long-grain rice sold in supermarkets. The outer layers of the rice kernel are removed. These layers include the hull, the germ, and the bran. The bran includes the testa, peripcarp, nucellus, and aleurone layers. Rice bran becomes rancid and has to be discarded in landfills. Parboiled rice bran has higher lipid levels than unstabilized rice bran. Rice-bran oil consists largely of saponifiable compounds. Hexane costs about $1.15 per gallon. It is an excellent solvent for nonpolar lipids. Calculate the PW of a hexane extraction unit. The following information was obtained from the *Journal of the American Oil Chemists' Association* [2] and modified suitably for the PW analysis in Example 4.11.

Annual Labor Cost = 8332h*$9.62 = $80,154

Rice Bran Processed = 30.96*3.7854*365/0.26 = 164,525 kg/year

$$\text{Annual Revenue} = 30.96*3.7854*365*\$8.08$$
$$= \$345,634 \text{ (from rice-bran oil)}$$
$$+ \$4.928*164525*0.146 = \$118,374 \qquad (4.29)$$

$$\text{Annual Materials \& Labor Cost} = 164,525*(5.18+0.22)$$
$$= \$888,436 \qquad (4.30)$$

Table 4.6 Cost Data for Hexane Extraction Plant for Rice-Bran Oil

Description	Hexane Extraction Unit
Capital Investment	$220,000
Manufacturing Cost	$5.18 /year/kg of rice bran
Materials Cost	$0.22/year/kg of rice bran
Labor Cost	$9.62/hr
Revenue from Rice-Bran Oil	$8.08/kg
Revenue from Rice Protein	$4.928/kg
Production Rate	30.96 gal rice-bran oil/day
Useful Life of Equipment	10 years
Operational Hours per Year	8332 hrs
% Protein in Rice Bran	14.6%
Interest Rate, i	3.0%

$(P/A,3\%,10) = 8.530203$ (From Table A-7)

$$PW = -\$220{,}000 + (-888{,}436 + 345{,}634 + 118{,}374$$
$$- 80{,}154)\, 8.530203 = -\$4.52418 \text{ million} \qquad (4.31)$$

The hexane extraction of rice-bran oil is not a profitable venture.

4.4 Annual Worth Analysis (AW)

Another method of evaluating a business can be completed by reducing all cash flows, including the capital investment, and salvage value in terms of annualized cash flow at a certain interest rate, i. Thus, the AW (annual worth) of a project may be estimated as follows:

$$AW = R_k - E_k - \text{Capital Recovery Factor}$$
$$+ \text{Sinking Fund Factor} \qquad (4.32)$$

Eq. (4.32) is calculated at a certain annualized interest rate i%. The capital recovery factor can be estimated from Eq. (3.20) and looked up in the annuity tables given in Appendix A or read from Figure 3.8. The capital recovery factor $(A/P,i\%,N)$ distributes all the capital investments into an annualized equivalent uniform series of payments. In a similar manner, the salvage value can be distributed using the sinking fund factor $(A/F,i\%,N)$ over a uniform series of payments.

Example 4.12   Annual Worth (AW) of a Biodiesel Plant in Taiwan [3]

Biodiesel is produced by transesterification reactions. Triglycerides present in virgin soybean oil are reacted with anhydrous alcohol such as methanol, ethanol, propanal, etc. to form FAME, fatty acid methyl esters, or biodiesel and glycerol. The alkali catalyst used was sodium hydroxide, NaOH. It can potentially be used as an alternative fuel to the ones currently in vogue that tend to be culprits of pollution causing global warming, acid rain, greenhouse gas emissions, etc. Biodiesel is an attractive fuel due to the environmental benefits it has to offer. It is prepared from renewable energy resources such as vegetable oil. Using the information provided in Table 4.7, calculate the AW of the biodiesel plant. The plant is expected to last for 20 years. The interest rate for equivalence calculations can be taken as 3%. The main costs in these plants are the raw material costs.

Table 4.7 Cost and Revenue Data for Biodiesel Production in Taiwan

Biodiesel Production in Taiwan			
#	Description	Cost	Quantity
1	Transesterification Reactor	$335,000	
2	Neutralization Reactor	$25,000	
3	Washing Column	$115,000	
4	FAME Distillation Column	$181,000	
5	Heat Exchangers	$4,000	
6	Pumps	$53,000	
7	Separator, Vacuum Systems	$52,000	
	Production Rate Materials		800,000 kg/year
8	Soybean Oil Feedstock	$6,234,000	
9	Methanol	$196,000	
10	Catalyst and Solvent	$368,000	
	Labor Cost	$564,000	
	Utilities	$124,000	
	Overhead	$431,000	
	Revenue from Biodiesel	$6,845,000	
	Glycerin Credit	$3,038,000	

The AW analysis is completed using an MS Excel spreadsheet as shown in Table 4.8. The capital cost is obtained by using the SUM command from cells D4 to D10. These are the costs of reactors, washing column, distillation column, heat exchangers, pumps, and vacuum

Table 4.8 AW Analysis of Biodiesel Plant in Taiwan

#	Description	Cost	Quantity	
1	Transesterification Reactor	$335,000		
2	Neutralization Reactor	$25,000		
3	Washing Column	$115,000		
4	FAME Distillation Column	$181,000		
5	Heat Exchangers	$4,000		
6	Pumps	$53,000		
7	Separator, Vacuum Systems	$52,000		
	Production Rate Materials		800,000	kg/year
8	Soybean Oil Feedstock	$6,234,000		
9	Methanol	$196,000		
10	Catalyst and Solvent	$368,000		
	Labor Cost	$564,000		
	Utilities	$124,000		
	Overhead	$431,000		
	Revenue from Biodiesel	$6,845,000		
	Glycerin Credit	$3,038,000		
	Capital Cost	$765,000		SUM(D4:D10)
	Materials Cost	$6,798,000	per year	SUM(D13:D15)
	Labor, Utility, and Overhead	$1,119,000		SUM(D16:D18)
	Total Revenue	$9,883,000	per year	SUM(D19:D20)
	(A/P,3%,20)	0.067216	(From Table A-7)	0.067216
	Capital Recovery	$51,420.24	per year	D26*D22
	AW	$1,914,579.76		D25-D24-D23-D27

systems. The capital cost of $765,000 is amortized over a 20-year period by using the capital recovery factor. The capital recovery factor is obtained from Table A-7 in Appendix A for (A/P,3%,20). This was found to be 0.067216. The materials costs include the soybean oil, methanol, catalyst, and solvent. The annual costs are obtained by adding the materials costs, labor costs, utility costs, and overhead costs. The annual revenue is obtained by adding the sales of biodiesel and glycerin. The AW was calculated as shown by plugging the information available into Eq. (4.32). The AW is about $1.915 million. Thus, it is profitable to operate the biodiesel plant as described in Taiwan.

Example 4.13   Debt Consolidation

A debt consolidation company worked out a payment plan for Mr. Dan Wardrop. Calculate the monthly payment at an annualized interest rate of 11%, should the loan be repaid in 26 years. Mr. Wardrop's credit card balances carried are shown in Table 4.9.

$$AW = P/12 \ (A/P,11\%,26) \tag{4.33}$$

$$(A/P,11\%,26) = 0.117813 \ (\text{Table A-})$$

$$P = SUM(\text{all balances in third Column in Table 4.9}) = \$25,700$$

$$AW \ 1/12 \ (3027.70) = \$252.32 \tag{4.34}$$

Table 4.9 Credit Card Balances of Mr. Dan Wardrop

#	Credit Card Company	Balance	Annualized Interest Rate (%)
1	Wachovia Platinum Visa	$11,500	19%
2	Sears MasterCard	$3,500	21%
3	Amazon Visa	$2,400	17%
4	Rooms-to-Go Furniture	$5000	15%
5	Macy's Department Store	$2000	25%
6	Gap Apparel	$400	19%
7	JC Penney Retail	$900	21%

Example 4.14   Solar Panel

Radio Shack has advertised Sunforce 130 W Solar panels with a Sharp module for $999. Consider purchase of five of these panels for a new home. The monthly utility bill in the new home runs about 750 kWh. The current utility company charges 12 cents for 1 kWh. How many years will it take for the new solar panels to be cost effective if purchased to replace the existing utility? The interest rate is about 11%.

$$\text{Annual Cost of Utilities: } 750*0.12*12 = \$90*12 = \$1008 \quad (4.35)$$

$$\text{Cost of Solar Panels} = 20*\$999 = \$4995$$

$$P/A = 4.9553$$

From Table A-15, N is about eight years.

It would take eight years for the investment in solar panels to be more attractive compared with the current utility company.

Another method to calculate the pay period is from Eq. (3.32)

$$1 - \frac{iP}{A} = \frac{1}{(1 + i)^N}$$

$$N = \frac{\log\left(\frac{A}{A - iP}\right)}{\log(1 + i)} = \frac{\log\left(\frac{1008}{1008 - 0.11*4995}\right)}{\log(1.11)} = 7.55 \text{ years} \quad (4.36)$$

N is plotted as a function of P/A for interest rate, $i = 11\%$ in Figure 4.3. It can be seen in Figure 4.3 that there is an abrupt change in curvature at about P/A = 8.0. The significance of this is that the annualized payment must be above a threshold amount to keep the repayment time within a reasonable timeframe. Should A be lowered any further, the time taken for repayment is large.

4.5   IRR (Internal Rate of Return)

The most widely used method of evaluation of a capital project is the Internal Rate of Return Method, or IRR. Other phrases used for the same method are investor's method, discounted cash flow method, profitability interest, etc. An interest rate is calculated, at which point the PW of the

project goes from negative territory to positive territory. This interest rate is called the IRR of the project. In addition, the IRR rate has to be greater than the MARR rate in order for the project to be considered acceptable. The IRR rate is the interest rate at which point the equivalent worth of all cash outflows and equivalent worth of cash inflows are equated to each other. It can be viewed as another *break-even* point. At the IRR

$$\sum_{j=0}^{N} R_j\left(P/F, i''\%, j\right) \;=\; \sum_{j=0} E_j\left(P/F, i'\%, j\right) \qquad (4.37)$$

The IRR can be calculated using graph paper. The PW of the project can be evaluated at various interest rates such as 2%, 4%, 6%, 8%, 10%, 12%, …, etc. The PW is plotted as a function of interest rate. The *x-intercept* or the interest rate at which the PW emerges into the positive territory from negative territory is called the IRR. Software is available to perform the trial-and-error calculations. For example, the MS Excel Spreadsheet has an IRR (range, guess) function call for obtaining the IRR for a set of cash flows.

Although a popular method, some pitfalls of an IRR include the occurrence of multiple values for the IRR. The variation of PW as a function of interest rate, i, is a function of several factors and may vary from
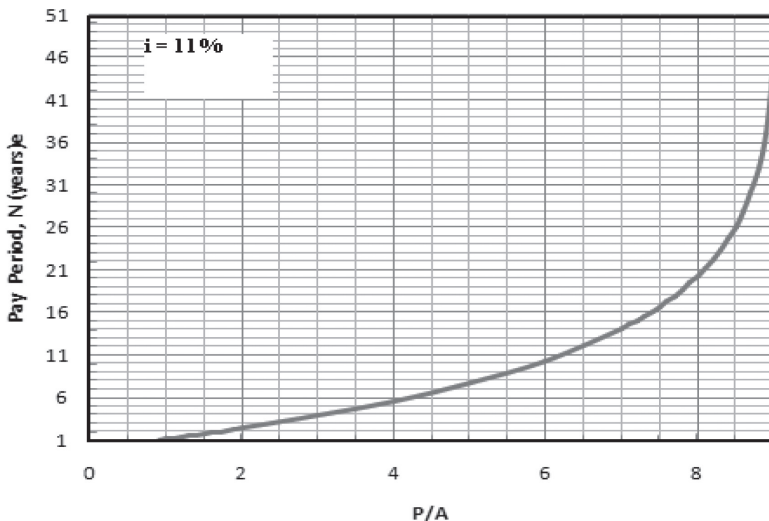
Figure 4.3 Pay Period N for Solar Panels to be Attractive Compared with Current Utility

project to project. When multiple values of an IRR result, one of the other methods of evaluation of the project such as FW, AW, payback period, etc. can be used instead of the IRR method. An IRR method also assumes that the revenues generated are reinvested back into the business. The ERR method does not assume such a thing.

Example 4.15   Profitability of Liquefaction Plants

The conversion of natural gas to liquid (GTL) fuel using the Fischer-Tropsch process was first developed in 1923. Syngas ($CO + H_2$) was converted into synthetic fuels. At a Society of Petroleum Engineers symposium, a novel method of evaluation of the economics of GTL plants was presented [4]. The capital expenses (CAPEX) are pegged to the production of one barrel of hydrocarbon liquid per day (BLPD). Annual operational expenses (OPEX) are expressed as a certain percentage of CAPEX. It is assumed that the overall thermal efficiency of GTL plants is 60% and the plant operates 330 days per annum. Consider one instance of CAPEX of \$40,000/BLPD and OPEX of 7% of CAPEX. Plant is considered to be in existence for eight years. The price of crude oil is \$37.65/bbl. Calculate the IRR for the project.

Basis of Production: 1 bbl a day of liquid fuel/oil

Annual Revenue = \$37.65*330*1 = \$12,424.5

Capital Expenses = \$40,000

Annual Expenses = 0.07*\$40,000 = \$2800

PW = −\$40,000 + \$12,424.5*(P/A,i%,8) − 2800*(P/A,i%,8)

The PW for different interest rates was calculated using an MS Excel Spreadsheet on a desktop computer. The P/A values for the interest rates used in the simulation were read from the annuity tables given in Appendix A. The values are given in Table 4.10 below. The PW as a function of interest rate is plotted in Figure 4.4. The IRR can be read from Figure 4.4 as 17.5%. Should the MARR be less than the IRR, the project is considered *profitable*.

Table 4.10 *IRR Analysis for GTL Plant at 1 Bbl per Day*

N	8		CAPEX	−$40,000
P/A	i (%)	PW	Revenue	$12,424.50
7.325481	2	$30,504.09	OPEX	−$2800
6.732745	4	$24,799.30		
6.2098	6	$19,766.22		
5.7466	8	$15,308.15		
5.3349	10	$11,345.75		
4.9676	12	$7,810.67		
4.4873	15	$3,188.02		
3.9544	19	−$1,940.88		
3.7256	21	−$4,142.96		
3.3289	25	−$7,961.00		
2.9247	30	−$11,851.22		
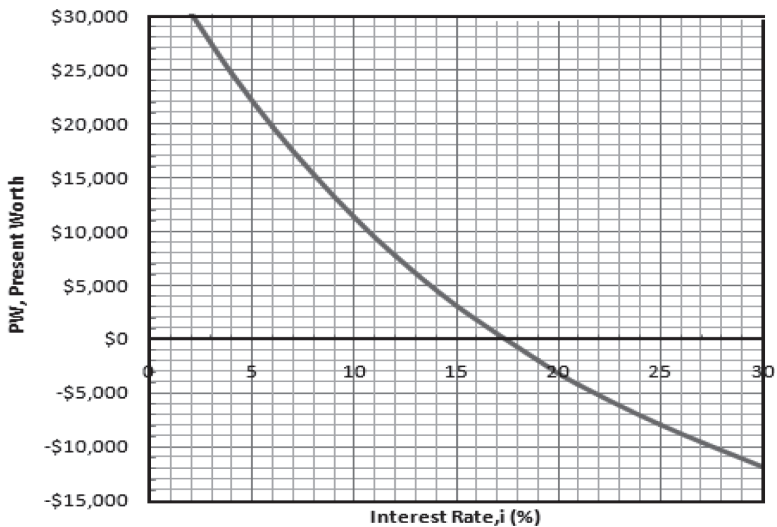
Figure 4.4 *IRR Calculations for GTL Plant*

Example 4.16   Ethanol Production by Fermentation

Bioethanol can be produced by the fermentation of sugars obtained from saccharine biomass such as sugarcane or sugar beets, starchy biomass such

Table 4.11 Cost Information for Six Methods

(M = million)

#	Description	Steam Stripping Process	Flash Fermentation	Single Column Distillation	Two Column Distillation	Distillation with Heat Pump	Flash and Vacuum Distillation
1	Capital Equipment Cost	$6.0M	$18.0M	$14M	$6M	$10M	$25M
2	Utility Cost	$5.5M/year	$7.5M/year	$6.5M/year	$3.5M/year	$3.6M/year	$5.5M/year
3	Feedstock Cost	$10.6M	$14M	$39.1M	$41.7M	$7.9M	$25.6M
4	Revenue	$50M	$50M	$50M	$50M	$50M	$50M
5	Plant Life	10 years	10 years	10 years	10 years	10 years	10 years
6	IRR	565%	157%	28%	77%	385%	77%

as corn, or cellulosic biomass such as agricultural wastes. Bioethanol, which comes from renewable resources, can be used as an alternative to fossil fuel resources such as coal and petroleum. Six different methods have been identified for recovering ethanol from fermentation broth. These methods are:

  (i) Steam Stripping and Distillation;
 (ii) Flash Fermentation and Distillation;
(iii) Single Column Distillation;
(iv) Two Column Distillation;
 (v) Distillation with Heat Pump;
(vi) Flash and Vacuum Distillation.

The ethanol production rate was set at 100 million liters/year in the process simulation study [5]. The cost data for cases where the ethanol concentration in the fermenter was 40 g/l is shown in Table 4.11. At an ethanol price of 50 cents per gallon, calculate the IRR for the six methods.

The IRR in row 6 of Table 4.11 was calculated using an MS Excel spreadsheet on a desktop computer. The PW as a function of interest rate for the six methods of recovery of ethanol from biofeedstock is shown in Figure 4.5.

4.6   ERR (External Rate of Return)

Some of the pitfalls encountered in the IRR method of evaluation of the capital projects are overcome in the ERR method. In the IRR method, the revenues generated are assumed to be reinvested at the IRR. An external interest rate, e, external to the project at which net cash flows required by a project over its life can be borrowed is identified. This is MARR, minimum acceptable rate of return. Identical results are generated from the methods of IRR and ERR when the ERR happens to equal the project's IRR.

During the ERR procedure, all net cash outflows to time 0 at $\varepsilon\%$ per compounding period are discounted. Then all net cash inflows are compounded to period N at e%. The ERR is obtained by establishment of equivalence of the two quantities.
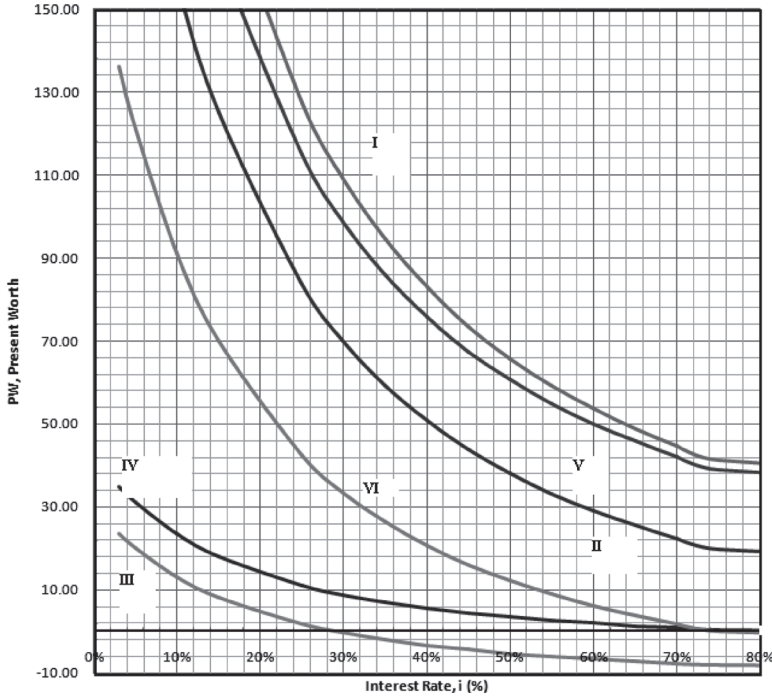
Figure 4.5 IRR Analysis for Six Methods of Recovery of Ethanol from Biomass

$$\sum_{k=0}^{N} E_k\left(P/F, \varepsilon\%, k\right)\left(F/P, i''\%, N\right) = \sum_{k=0}^{N} R_k\left(F/P, \varepsilon\%, N - k\right)$$

Where R_k = excess of receipts over expensed in period k

E_k – excess of expensed over receipts in period k

N – project period

ε - external reinvestment rate per period

4.7   Payback Period

Payback period is the number of years for the cash inflows to equal the cash outflows. It is a measure of the liquidity. It is a simple method. A number of years for break-even point attainment is obtained. But the cash flows occurring after the payback period q is not available using this method. The project desirability is not clear. The speed at which the investment

can be recovered is calculated. It can be used as supplemental information to one of the other five methods discussed earlier in this chapter.

Example 4.17   Copper-Chlorine Thermochemical Cycles for Hydrogen Production

The hybrid copper-chlorine (CuCl) cycle is one promising method of a thermochemical cycle that can be used for hydrogen production with nuclear or solar heat. A CuCl cycle plant producing 125 MT H_2/day required 210 MW of thermal energy and 87.8 MW of electrical energy. The total capital investment for the CuCl plant using Turton et al.'s software package [6] can be estimated at $131 million. Estimated price of hydrogen is $5.0/kg. Calculate the payback period at an interest rate of 5%. The cost of electricity can be taken as $60 per 1000 kWh. Bulk CuCl costs $7200 per pound and bulk HCl costs $241/metric ton. A 50% yield can be assumed for the three major reactions that are needed in the process: (i) electrolysis reaction, by which cupric chloride, $CuCl_2$, is produced at the anode and H_2 at the cathode; (ii) hydrolysis of cupric chloride leads to the formation of copper oxychloride; and (iii) molten cuprous chloride is produced from the decomposition reaction. Thermal utility is $20 per 1000 kWh. Assume 1% loss of HCl and 1 ppm loss of CuCl during the operation of the cycle.

$$2CuCl + 2HCl + 4H_2O \rightarrow 2CuCl_2.2H_2O + H_2$$

$$2CuCl_2 + H_2O \rightarrow CuOCl_2 + 2HCl$$

$$CuOCl_2 \rightarrow \tfrac{1}{2} O_2 + 2CuCl$$
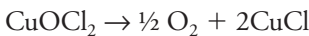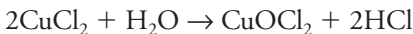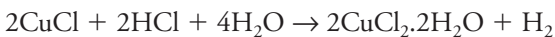
Revenues from H_2 Sales per Year = $(125)*(100)*(\$5)*365/10^6$ = $22.8125 million

Material Cost–HCl per Year = $(125)*(\$241)*(365)*(18.25)*(0.01)/10^6$ = $2.0067 million

Material Cost–CuCl per Year = $(125)*(100)*(\$7200*2.2)*(99)*(365)*(10^{-12})$ = $7.1547 million

Thermal Energy Cost per Year = $(\$20)*(210)*(365)/10^6$ = $1.533 million

Electric Energy Cost per Year = $(\$60)^*(87.8)^*(365)/10^6$

= $1.92282 million

(Revenue–Expenses) per year = $22.8125 − $2.0067−$7.1547

−$1.533-$1.92282

= $10.1953 million

Capital Cost = $131 million

$$\frac{P}{A} = \frac{\$131}{\$10.1952} = 12.8491$$

From Table A-9, N for P/A = 12.8491 1t 5% interest rate can be seen to be 21 years.

4.6   Summary

There are five methods to evaluate a single project. These are: (i) PW (present worth) analysis; (ii) FW (future worth) analysis; (iii) AW (annual worth) analysis; (iv) IRR (internal rate of return) analysis; and (v) ERR (external rate of return) analysis. In addition to these five methods, a *payback period* can be calculated that denotes the time taken over which the monies invested can be recovered back. The MARR is the minimum acceptable rate of return. For a project to be selected, it has to have a rate of return (ROR) greater than that of a MARR.

The PW of a project can be estimated by finding out the equivalence of all the cash inflows (such as receipts) and cash outflows (such as expenses) to the present time at a specified interest rate that is usually greater than the MARR. When PW falls in positive territory, the project is considered to be acceptable. The project study period, N, may be determined after establishing the useful lives of critical equipment used in the execution of the project. The factors considered during PW analysis are capital costs incurred in the purchase of equipment, estimated useful life, variable costs of labor, materials, and utilities, salvage value, revenues accrued from sales of product and the interest rate. Inflation factors may be used to account for the variability in materials costs incurred. When the study period reached infinity, the PW becomes the CW, capitalized worth

of the project. Examples of PW analysis were developed using hydrogen production from fast pyrolysis of biomass and steam re-forming, replacement for the World Trade Center (WTC), sugar mill, hybrid gas/electric vehicle, inflation rates seen in the U.S. economy, life-cycle cost analysis of HVAC systems, municipal garbage collection truck, and hexane extraction of rice-bran oil.

The FW of a project can be estimated by finding out the equivalence of all the cash inflows, such as receipts and all the cash outflows, such as expenses to a future time. Worked examples were developed on a university apartment business and investment in gold or stock.

The AW of a project can be estimated by reducing all cash flows including the capital investment, salvage value in terms of annualized cash flows at an interest rate i. Worked examples on AW analysis were implemented from case studies of the manufacture of a biodiesel plant in Taiwan and debt consolidation.

Worked examples on calculation of payback period were created for solar panels for homes and hydrogen from the thermochemical cycle of copper-chlorine.

The IRR is the most widely used method of evaluation of capital projects. The PW of the project is calculated as a function of interest rate i (%). The point in the graph of PW vs. interest rate where the PW goes from positive territory to negative territory is called the IRR. Worked examples on IRR analysis have been constructed for GTL (gas to liquefaction) plants and ethanol production via fermentation by six different methods.

There are some deficiencies in the IRR method of analysis. Results of multiple IRR is one of them. The ERR method is developed to overcome these deficiencies. The revenues generated are not assumed to be reinvested in this method. An external interest rate external to the project at which net cash flows required by a project over its life can be borrowed is used.

End-of-chapter exercises have been developed on the life-cycle costs of HVAC systems, hybrid electric vehicle, PEV (plug-in-electric vehicle), time taken to repay credit card balances, capitalized worth of a vacation condominium in Galveston, Texas, the AW of energy efficient windows, deferred interest charges, profitability of outpatient drug treatment center, PW of a convenience store and gas station complex, AW of public

Internet, video and printing services, bonds, profitability of an olive oil business, IRR for the short path distillation method of extraction of paraffin wax, car rental business, payback period of a frozen yogurt parlor, FW of a coffeehouse, turnaround business, PW of an onshore oil well, a custom foam fabricator, polypropylene bag manufacture, activated carbon from bamboo, meglumine antimonite drug manufacture, ABS engineering, thermoplastic manufacture, hydrogen from biomass, use of additional wells in an oil field, long-term parking lot, recycling of plastics using closed-loop granulation, solar hats, savings from using PFR (plug flow reactor) over CSTRs, and the use of gradient series.

4.7   References

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CHAPTER 5

Decision Analysis and Comparison of Alternates

Topics Covered
- Develop Alternatives by Brainstorming
- Study Period—Don't Compare an Apple and an Orange
- Useful Lives—Repeatability and Co-terminated Basis
- Rate of Return and Project Ranking
- EW, Equivalent Worth Method
- Optimal EW
- Household Finances

5.1   Overview

As discussed in Chapter 1.0, engineering economy is about the development of alternatives for a given problem. Brainstorming may be used to develop alternatives. All the alternatives selected have to be viable. Decision analysis is to rank the alternatives with the most profitable or lowest cost alternative being given the highest rank. Different alternatives can have different cash flow diagrams. The revenues may be different: costs incurred such as materials, labor, and utility may vary and the interest rate and periods may not be the same. The five methods of evaluation of capital projects discussed in Chapter 4.0 shall be used here to select the best possible solution. These are the PW, present worth analysis, FW, future worth analysis, IRR, internal rate of return, AW, annual worth analysis, ERR, external rate of return. A variation of the IRR method is the payback period analysis. Factors are introduced to take inflation into account.
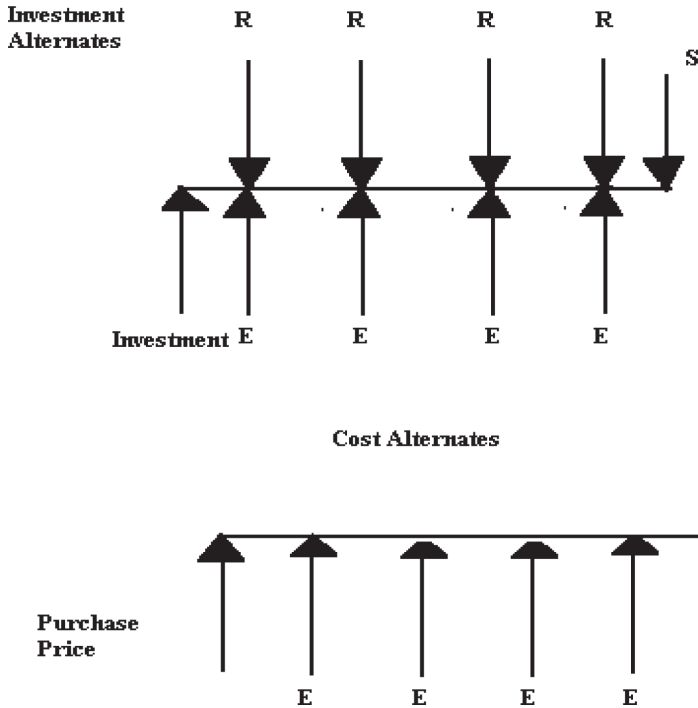
Figure 5.1 Cash Flow Diagrams of Investment and Cost Alternatives

The right questions must be posed: (i) what are the differences among the alternatives; (ii) what is a common unit of measure; (iii) are all the relevant criteria met; (iv) what is the risk-reward ratio; (v) who pays for the capital investments—government or private companies; (vi) what are the local, state, and federal regulations; (vii) is the product for domestic consumption or for export; (viii) what are the exchange rates for the $; (ix) what are the inflation rates and deflation rates; (x) what are the government incentives given; (xii) is the business capital intensive like a nuclear power plant or cash flow intensive such as the gas station and convenience store complex studied in the end of Chapter 4.0 exercises; (xiii) what is the role of assets and liabilities of the enterprise; and (xiv) what is the MARR (%) (minimum acceptable rate of return).

Public sector projects are evaluated slightly differently compared with private sector projects. A benefit-cost analysis will be unveiled at a later chapter. In addition to costs and benefits, the disbenefits are also taken into account in cases of public sector projects.

 (i) Comparison of alternatives can be undertaken for the purpose of:
 (ii) Invest in the more profitable process—Profit Based;
 (iii) The cash flow diagrams have a capital investment and have net cash inflows over the study period as shown in Figure 5.1;
 (iv) Select the lower life-cycle cost alternative—Cost Based;
 (v) The cash flow diagrams have cash outflows only as shown in Figure 5.1;
 (vi) Quantitating the savings accrued from implementation of a process improvement;
 (vii) Assessment of environmental impact of different chemical processes—Other Criteria.

Alternatives can be different kinds:

Mutually Exclusive Alternatives: In this category, only one viable alternative can be selected from all the options available. For example, an engineer has to select the right equipment such as CSTR (continuous stirred tank reactor) or PFR (plug flow reactor) in order to accomplish a given process. When one is selected, the other is rejected.

Independent Projects: Here one or more of the available projects may be selected for implementation. The projects can be evaluated using PW, FW, IRR, AW, or ERR methods and rank ordered according to the profitability. For example, a savvy investor with $1 million dollars may put $300,000 in stocks, $200,000 in bonds; $100,000 in FDIC-insured CDs; $100,000 in money market funds; and $300,000 in gold. Here, the different options were rank ordered and monies allocated according to the rank.

Screening by Other Criteria: In some cases, dollars and cents cannot be the sole determining criteria to evaluate alternatives and come to the most profitable decision. All the relevant criteria as discussed in the seven principles in chapter 1.0 have to be met. (Criteria such as all humans are created equally and laborers cannot be discriminated against based on the color of their skin, or national origin, age, gender, etc.) The environmental impact is also rapidly becoming an added criterion for selection of the best possible alternative.

The PW of the identified alternatives are calculated from their respective cash flow diagrams. The project with the highest PW in dollar

amount is selected as the best possible alternative. Special cases can be identified when there are two alternatives, A and B, considered.

Case A: PW_A, present worth of Project A is in positive territory and PW_B, present worth of Project B is negative territory. $PW_A > 0$ & $PW_B < 0$. This means that Project A would fetch a profit and Project B would result in a loss when undertaken. In such cases, Project A may be selected.

Case B: Both PW_A and PW_B are in positive territory. Both projects can result in a profit. In such cases, the projects may be *rank ordered*. Sometimes the project with the maximum PW may be selected. $PW_A > 0$ & $PW_B > 0$.

Case C: The dollar value of PW_A and PW_B are within 20% of each other. Such cases may be determined as *too close to call*. This is so because of the uncertainty involved in the estimates of revenues, costs, capital investment and salvage values, interest rates, lives of equipment, inflation rates, etc. In a later chapter, sensitivity analysis and spider plots will be discussed in greater detail.

Any of the five methods such as PW, FW, AW, IRR, and ERR may be used in order to evaluate the alternatives prior to arriving at the decision. In cases of PW, FW, AW, the alternative with the higher dollar value can be selected. In the case of IRR, the project with the higher IRR is not selected. IRR between projects should not be compared. The IRR of a project is compared with MARR. If IRR > MARR, the project is acceptable. The business significance of comparison of IRRs from different projects is low.

Example 5.1   Best Process to Manufacture CNTs
(Carbon Nanotubes)

CNTs are one of the four allotropes of carbon: (i) diamond; (ii) graphite; (iii) C_{60} fullerenes; and (iv) CNTs, carbon nanotubes. Graphene sheets of carbon rolled around the needle axis were discovered by Ijima in 1991. The diameter of these tubes is 1–2 nm and length is a few microns. CNTs offer attractive properties ranging from 1 TPa of modulus of elasticity to 2000 w/m/k thermal conductivity, etc. They can be used as a blend substituent to make novel nanocomposites.

There are two processes used to manufacture CNTs. Find out which is more profitable at 12% effective interest rate.

Process A: The HiPCO reactor is used. HiPCO is the high-pressure carbon monoxide process to manufacture CNT using the Boudard reaction [1]. A PFR (plug flow reactor) designed at Rice University in Houston is used at high pressures of 30–50 bar and at temperatures 1273–1473 K. CNTs and CO_2 are produced from CO, carbon monoxide, and iron pent carbonyl catalyst precursor. The conversion was 20%.

Process B: The CoMoCAT reactor is used. CO disproportionation is effected over a mixed cobalt-molybdenum catalyst on silica support in a fluidized bed reactor developed at the University of Oklahoma. The reaction forms CNTs and CO_2-at temperatures between 973–1223 K and total pressure ranging from 15–50 psia. The CO selectivity is 80% and conversion about 20%.

The information needed for economic analysis on the processes is given in Table 5.1 from Agboola et al. [2].

Annual Materials Expenses for Process A = $140 million   (5.1)

Annual Electricity Expenses for Process A
= (1,056)*(24)*(330)*($0.125) = 1.05 million   (5.2)

Annual Gas Expenses for Process A
= (486)*(24)*(330)*($0.50)/1E6 = $1.925 million   (5.3)

Annual Steam Expenses for Process A
= (2,565)*(24)*(330)*($0.125)/1E6 = $2.54 million   (5.4)

Annual Sales of CNTs from Process A
= (595)*(24)*(330)*($38)/1E6 = $179.1 million   (5.5)

Annual Profit (R − E) = ($179.1) − ($140)
−SUM(Gas, Steam, Electricity) = $33.562 million   (5.6)

Present Worth Factor, (P/A,12%,10) = 5.6502   (5.7)

PW_A = −($4.6) + (R − E)*(P/A,12%,10)
= −$4.6 + 5.6502*$33.56185 = $189.63 million   (5.8)

Table 5.1 Two Processes Used to Manufacture CNTs

Item	Process A (Amount)	Process A (Description)	Process B (Amount)	Process B (Description)
Catalyst		Fe		Co-Mo
Reactants		CO & Fe(CO)$_5$		CO
Reactor		Plug Flow Reactor		Fluidized Bed
Temperature		1050°C		950°C
Pressure		450 psia		150 psia
Selectivity		90%		80%
Purification		Oxidation, Filtration		Leaching, Froth Flotation
Product Rate		595 kg/h		595 kg/h
Steam	12.5 cents/kg	2,565 kg/h		2,885 kg/h
Gas	50 cents /kg	486 kg/h		486 kg/h
Electricity	12.5 cents/kWh	1,056 kW		387 kW
Capital Cost of Plant	$4.6 million		$4.4 million	
Price	$38/kg		$25/kg	
Materials Cost	$140 million		$84 million	
Plant Life		10 years		10 years
Operation		330 days/year		330 days/year

Annual Materials Expenses for Process B = $84 million   (5.9)

Annual Electricity Expenses for Process B
= (387)*(24)*(330)*($0.125)= 0.383 million   (5.10)

Annual Gas Expenses for Process B
= (486)*(24)*(330)*($0.50) = $1.925 million   (5.11)

Annual Steam Expenses - Process B
= (2,885)*(24)*(330)*($0.125) = $2.856 million   (5.12)

Annual Sales of CNTs from Process B
= (595)*(24)*(330)*($25) = $117.81 million   (5.13)

Annual Profit (R − E) = ($117.81) − ($84)
−SUM(Gas, Steam, Electricity) = $28.646 million   (5.14)

Present Worth Factor, (P/A,12%,10) = 5.6502   (5.15)

$$PW_B = -(\$4.4) + (R - E)^*(P/A,12\%,10)$$
$$= -\$4.4 + 5.6502^*\$28.64616 = \$157.4565 \text{ million} \qquad (5.16)$$

Comparing Eq. (5.8) and Eq. (5.16), Process A has a higher present worth compared with Process B. Hence, Process A is the more profitable route to prepare CNTs.

Example 5.2   Cost Savings by Using Microfiltration Pretreatment During SWRO (Sea Water Reverse Osmosis)

Pretreatment has been used to improve the performance and lower the overall cost of obtaining water using SWRO (sea water reverse osmosis plants). Control of membrane fouling is agreed upon as a problem encountered during the operation of the SWRO plants. The conventional pretreatment method controls and minimizes the plugging potential of the feed water by coagulating, flocculating, and filtering out colloidal and suspended solids and by achievement of reduction in SDI (silt density index) of feed water. MF (microfiltration) pretreatment appears to be an attractive method. Turbidities of water can be lowered less than 0.1 NTU regardless of inflow turbidity level. UF pretreatment results in removal of TC (total coli) from bacteria *Giardia* and *Cryptosporidium.* Viruses are also removed. The novel pretreatment strategy leads to reduction in capital costs, reduction in operating costs due to lower membrane replacement, less chemical consumption, elimination of cartridge filters, less downtime, less maintenance costs, less labor costs, etc. The MF unit is skid mounted and impurities larger than 200 nm are removed using this device. The machine consists of polypropylene filtration modules, circulation pump, associated valves, piping, and instrumentation and control systems.

The data needed to perform comparison of alternatives is given below in Table 5.2 and taken from [3]. The capital expenses such as treatment plants, pumps, etc. are financed over a 30-year period at 5% interest rate.

Cost of Water (Conventional)

Annual Variable Expenses = $9.648 + $4.662 + $3.478
$$= \$17.788 \text{ million} \qquad (5.17)$$

Capital Recovery Factor (A/P,5%,30) = 0.065051 $\qquad$ (5.18)

Table 5.2 Economic Data for Evaluation of MF Pretreatment

	Conventional			Microfiltration	
Description	$ millions/ year	$ (millions)		$ millions/ year	$ (millions)
Plant		145.3777			40.64489
Pumps		1.829328			1.829328
Dosing Plant		0.230588			0
Structural		11.89832			11.89832
Subtotal		159.336			54.37253
Electricity	9.648			7.09	
Chemicals	4.662			1.637	
Filters	3.478				
Water Rate			27,276 m³/day		

Capital Recovery Amount = $159.336*(0.065051)

= $10.365   (5.19)

Plant operates 330 days in the year

Cost of Water =

$$\frac{(\$17.788 + \$10.365)}{27,276*330} = \$3.13/m^3 = 0.313c/liter \qquad (5.20)$$

Cost of Water (MF Pretreated)

Annual Variable Expenses = $7.09 + $1.637

= $8.727 million   (5.21)

Capital Recovery Factor (A/P,5%,30) = 0.065051   (5.22)

Capital Recovery Amount = $54.37253*(0.065051)

= $3.537   (5.23)

Plant operates 330 days in the year

Cost of Water =

$$\frac{(\$8.727 + \$3.537)}{27,276*330} = \$1.363/m^3 = 0.1363c/liter \qquad (5.24)$$

Comparing Eq. (5.20) and (5.24) the microfiltration pretreatment process modification would lower the cost of producing water by $1.77/m^3$. Hence, the microfiltration pretreatment alternative is preferred over the conventional method of pretreatment.

Example 5.3   Bioethanol from Sugarcane Bagasse

Renewable energy sources are sought the world over in order to provide energy security for our nation and the world. According to a recent Goldman Sachs report, Brazil, Russia, India, and China, BRIC nations, are expected to emerge as the world's largest economies. In Brazil, 387 million tons of sugarcane was produced in 2005/2006. Additionally, 104 million tons of bagasse came from the sugar mills. From a ton of sugarcane, 73 kg of fermentable sugars and 27 kg of dry bagasse can be produced. The composition of bagasse is as follows:

Cellulose	43%
Xylans	25%
Lignin	23%

Hemicellulose is a polymer of xylose and cellulose is a polymer of glucose. The sugarcane bagasse can be burned in boilers to generate electricity. Sugarcane bagasse can be converted to bioethanol and furfural by a dilute acid hydrolysis process. The acid hydrolysis process is a two-stage operation. In the first-stage, hemicellulose is converted into sugar using dilute acid and steam. In the second stage, cellulose is converted to sugar by hydrolysis. Upon hydrolysis, the sugar can be processed in order to produce ethanol, furfural, and other products. Furfural can either be used alone or combined with phenol, acetone, or urea to make solid resins. Such resins are used to make fiberglass, aircraft components, and automotive brakes. It can also be used as a hydrocarbon solvent. Coproduction of ethanol and furfural can make the process more profitable. The acid hydrolysis plant consists of the following equipment:

Pump and Heater;
Reactor to Produce Furfural and Glucose;
Purification and Separation.

After passing through the heater, the feedstock is treated with dilute sulfuric acid solution (0.5 wt%) at high temperature for a short time. Cellulose and pentose are converted to glucose and furfural, respectively. The hydrolyzed stream is then separated using a distillation column. The hydrolyzed material is then neutralized with lime. The upper stream is passed through a condenser leading to the formation of furfural. The un-reacted cellulosic residue and lignin from stream can be burned to gener-ate electricity or sold as a by-product. The total capital investment needed for the acid hydrolysis plant is about $4.016 million [4]. The furfural can be sold for $1.91/kg and ethanol can be sold for $1.37/kg. The bagasse suppliers are paid at 18 cents per kg of feedstock. A dilute acid hydrolysis plant can operate at 4000 kg/h of sugarcane bagasse. The utilities and labor costs were estimated and found to be $0.80 per kg of ethanol and $1.11/kg of furfural. The plant utilization factor is 80%. The yield of ethanol is 15% and that of furfural is 7% from the bagasse.

Which would be a better alternative: (i) start a dilute acid hydrolysis plant with an estimated life of 15 years; or (ii) generate electricity from bagasse using a power plant? The cost of the steam boiler and turbine is about $1 million each. The electricity can be sold at 12.5 cents per kWh. The calorific value of bagasse is 8.0 MJ/kg. The power plant thermal efficiency can be taken as 30%. The interest rate is 6%. For purposes of analysis, assume that the power plant life is equal to that of the acid hydrolysis plant and is 15 years. The operating expenses of the power plant are about 4 cents per kWh.

Dilute Acid Hydrolysis Process

Annual Sales of Furfural
$$= (4000)*(24*365*0.8)*(0.07)*(\$1.91) = \$3.748 \text{ million} \qquad (5.25)$$

Annual Sales of Ethanol
$$= (4000)*(24*365*0.8)*(0.15)*(\$1.37) = \$5.761 \text{ million} \qquad (5.26)$$

Annual Expenses of Furfural
$$= (4000)*(24*365*0.8)*(0.07)*(\$1.11) = \$2.178 \text{ million} \qquad (5.27)$$

Annual Expenses of Ethanol
$$= (4000)*(24*365*0.8)*(0.15)*(\$0.80) = \$3.364 \text{ million} \qquad (5.28)$$

(P/A,6%,15) = 0.5*(9.295 + 10.1059) = 9.723 (from Table A-10)

PW_{acid} = −4.016 +P/A*(R − E) = −4.016
+ 9.723*($3.748 + $5.761 − $2.178 − $3.364)
= $34.55 million   (5.29)

Power Plant Option

Annual Sales of Electricity
= (4000)*(0.3)*(8 E6)*(0.8)*($0.125) = $0.96 million   (5.30)

Annual Cost of Power Plant Operation
= (4000)*(0.3)*(17 E6)*(0.8)*($0.04) = $0.6528 million

PW_{plant} = −2 + 9.723*($0.96 − 0.6528) = $0.987 million (5.31)

Comparing Eq. (5.29) and Eq. (5.31), the present worth of the acid hydrolysis plant is at $34.55 million and is greater than the present worth of the power plant at $0.987 million. Hence, it is better to invest in the acid hydrolysis plant compared with the steam power plant.

Example 5.4   Sequestration by Dimethyl Carbonate Formation

Two routes for chemical reuse of CO_2, carbon dioxide, are considered. Interest in sequestration projects is increasing due to the concerns about global warming [5]. Studies have shown that a 10% dimethyl carbonate (DMC) addition to diesel leads to 20% smoke reduction. Both plants operate at 7,200 h/year. CO_2 is available at no cost. Each ton of CO_2 sequestered leads to carbon credits of $19.02. Electric utilities cost 43 cents per kWh. Which is a better alternative at an interest rate of 10% and a plant life of 14 years?

Route A: Production from Ethylene Oxide: Ethylene oxide and CO_2 are allowed to react to form ethylene carbonate. Ethylene carbonate produced is reacted with methanol in a second PFR (plug flow reactor), leading to the production of DMC and ethylene glycol. The capital cost can be taken as $80 million; 36,732 tons/year of CO_2 are sequestered. The energy used in this process is 216 TJ/year. Revenue from the plant is $2500 per ton of CO_2 sequestered.

Route B: Production from Urea: Urea is produced by reacting ammonia and CO_2. DMC is produced by urea methanolysis. The capital cost for

this route is given as \$100 million; 27,129 tons/year of CO_2 are seques-
tered. The energy used in this process is 415 TJ/year. The revenue from
the plant is \$3800 per ton of CO_2 sequestered.

(P/A,10%,15) = 7.3667 (from Table A-14)

$$\text{Annual Revenue—Route A} = (36,732)*(2,500)$$
$$= \$91.83 \text{ million} \tag{5.32}$$

$$\text{Annual Carbon Credit—Route A} = (36,732)*(\$19.02)$$
$$= \$0.698,643 \text{ million} \tag{5.33}$$

$$\text{Annual Energy Expenses—Route A} =$$
$$(216 \text{ E}12)(\$0.43)/1000/7200 = \$12.94 \text{ million} \tag{5.34}$$

$$PW_A = \text{-}\$80 + 7.3367*(\$91.83 + \$0.698,643 - \$12.94)$$
$$= \$506.292 \text{ million} \tag{5.35}$$

$$\text{Annual Revenue—Route B} = (27,129)*(3,800)$$
$$= \$103.09 \text{ million} \tag{5.36}$$

$$\text{Annual Carbon Credit—Route A} = (27,129)*(\$19.02)$$
$$= \$0.515,994 \text{ million} \tag{5.37}$$

$$\text{Annual Energy Expenses—Route A}$$
$$= (414.99 \text{ E}12)(\$0.43)/1000/7200 = \$24.78 \text{ million} \tag{5.38}$$

$$PW_B = -\$100 + 7.3367*(\$91.83$$
$$+ \$0.698,643 - \$12.94) = \$480.66 \text{ million} \tag{5.39}$$

Comparing Eq. (5.35) and Eq. (5.39), it can be seen that the present
worth (PW) of the plant A for sequestration is \$506.292 million and the
PW of the plant B is \$480.66 million. In this case, it is too close to call
one better than the other. However, Route A can be ranked higher than
Route B, and Route A can be selected as the better alternative.

5.2   Study Period—Don't Compare
an Apple and an Orange

When two alternatives are compared, the life of the equipment used by
either process may be different from each other. When this happens,

comparisons have to be made on a comparable basis. In order to make more meaningful comparisons, the following quantities are defined:

Study Period: The study period is the duration over which the comparisons are drawn. This can be lower, equal to, or greater than the useful life of the equipment used.

Useful Life: The duration over which the equipment is functioning is called the useful life of the equipment.

Repeatability Assumption: This can be used when the useful lives of equipment in Alternative A and Alternative B are different from each other. The study period is selected as the lowest common multiple of the two useful lives. For example, say the decision analysis is to select the best photocopier for the department. Copier A is imported from Japan and has a useful life of nine years. Copier B is from a domestic supplier with a useful life of six years. Then the study period is set at 18 years; 18 is the LCM of (6,9). Two A copiers are allowed to function in series in time, thus lasting 2*9 = 18 years. In a similar manner, three copiers of B are allowed to function one after the next, thus lasting 3*6 = 18 years. The study period of 18 years is now the same for both copiers A and B. The purchase price, annual supplies cost, maintenance cost, utility cost, labor cost, and salvage value obtained at the end of use of the machine can all be accounted for two copiers of A and three copiers of B. In some cases, the quantity may have to be multiplied by the number of copiers—it depends on the given application. Sometimes the LCM may have to be the multiple of the two useful lives. For example, should copier A have a life of five years and copier B have a life of three years, the study period can be selected as LCM(3,5) = 15 years. In this case, five copiers A have to be considered as a basis and three copiers of B have to be considered as a basis. In such cases, projections into the future may cause errors such as changes in interest rates, inflation rates, etc. This method can also be used when the study period tends to infinity or is indefinitely long.

Co-terminated Assumption: The study period is truncated from the useful life of equipment in one alternative in order to have equal

the useful life of the equipment in the second alternative. This can be used when the useful life of alternative A, NA is greater than useful life of alternative B, NB—i.e., NA > NB. The study period can be selected as NB. All the revenues and expenses for alternative A are then reevaluated for a co-terminated period of NB. This is feasible because NB < NA.

Example 5.5   Life-Cycle Cost of Photocopier

The Xerox Work Center can handle up to 200,000 pages per month and has a copy speed of up to 45 pages per minute. Its costs about $2,200 and weighs 80 lbs. The toner cartridge costs about $90 per cartridge. It can be used to print 2000 pages. This machine has a useful life of four years. Toshiba's e-STUDIO 455 can copy up to 45 pages per minute and the base unit costs about $10,800. The toner yield is 30,000 pages and cost is $190 each. The useful life of the e-STUDIO 455 is about 16 years. Which copier has the lower life cycle cost: the Xerox Work Center or the Toshiba e-STUDIO? The interest rate is 8.0%. Assume that 1 million pages are copied every year.

The two machines have unequal lives. The repeatability assumption is used. Four Xerox Work Centers are needed for a common study period of 16 years. They are assumed to work one after the other as shown in the cash flow diagram in Figure 5.2. The PW analysis is used in order to calculate the life-cycle cost of the two different photocopiers.

$$\text{Annual Toner Cost for Xerox Work Center}$$
$$= 1 \text{ E6}/2000*(\$90) = \$45,000 \tag{5.40}$$

$$\text{Annual Toner Cost for Toshiba's e-STUDIO}$$
$$= 1\text{E6}/30,000*(\$190) = \$6333.33 \tag{5.41}$$

$$(P/A,8\%,16) = 8.8514 \text{ (From Table A-12)} \tag{5.42}$$

$$(P/F,8\%,4) = 0.735030 \text{ (From Table A-12)} \tag{5.43}$$

$$(P/F,8\%,8) = 0.540269 \text{ (From Table A-12)} \tag{5.44}$$

$$(P/F,8\%,12) = 0.397114 \text{ (From Table A-12)} \tag{5.45}$$
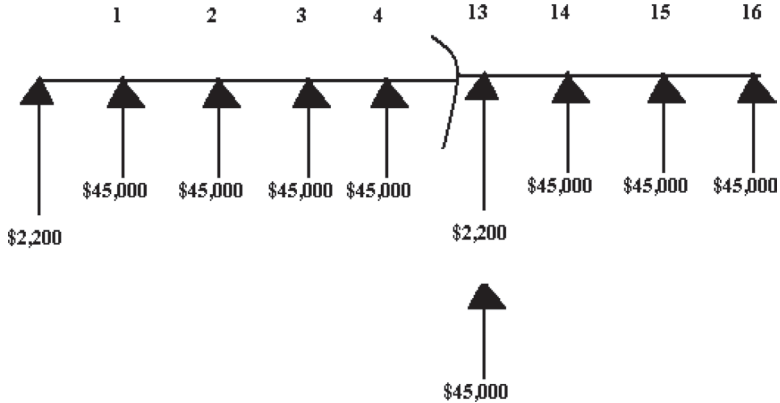
Figure 5.2 Cash Flow Diagram Adjusted for Repeatability Assumption for Xerox Work Center with Useful Life of 4 Years

Life-Cycle Cost of Xerox Work Center
$$= CX + (P/A,8\%,16)*(-\$45,000) + (P/F,8\%,4)*C_X$$
$$+ (P/F,8\%,8)*CX + (P/F,8\%,12)*C_X \qquad (5.46)$$

$$= -\$2,200 - (8.8514)(\$45,000) - 0.735030*(\$2,200)$$
$$- 0.540269*(\$2,200) - 0.397114*(\$2,200) = \$404,192 \quad (5.47)$$

The e-STUDIO was purchased at a one-time charge of $10,800. Over 16 years, the toner costs were found to be $6,333.33 every year. The cash flow diagram for expenses for Toshiba's e-STUDIO is shown in Figure 5.3.
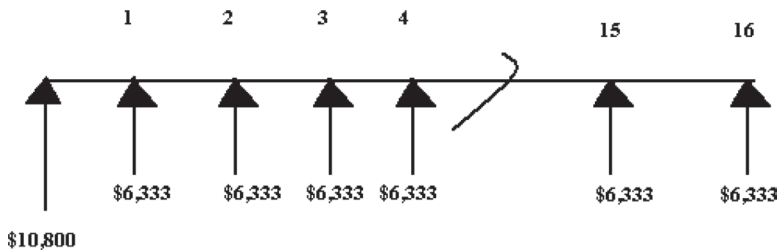
Figure 5.3 Cash Flow Diagram for Toshiba's e-STUDIO with a Useful Life = 16 years

Life-Cycle Cost of Toshiba e-STUDIO
$$= -\$10,800 - 8.8514*(\$6333.33) = \$66,859 \qquad (5.48)$$

It can be seen that by comparison of Eq. (5.47) and Eq. (5.48), the Toshiba e-STUDIO has the lowest life-cycle cost. This is contrary to the purchase price. The purchase price of the Xerox Work Center is lower at $2,200, compared with Toshiba's $10,800. When taking into account the toner expenses, the Toshiba e-STUDIO turns out to be the lower total cost at $66,859 to that of the Xerox Work Center's $404,192. In both cases, the basis was 1 million copies per year made.

Example 5.6   ED Lighting

Light-emitting diode works on the principle of *electroluminescence*, which is the release of photon energy from electron-hole recombinations within the device. The color of the light depends on the wavelength of the light corresponding to the energy released. LEDs have lower energy consumption and longer lives compared with incandescent lamps. Boeing wants you to calculate the life-cycle cost of LEDs and incandescent lamps for its new aircraft. They need 420 lights at 100 W each. The incandescent lamps have 100 hrs/life and cost $1.36 each. The illumination received is 1680 lumens per bulb. Mercury LED lamps of 400 W cost $32.73 per lamp and can last for 24,000 hours at an illumination of 36,000 lumens [6]. Assume that the Boeing flight is in operation for 200 days in the year. The cost of electricity is 25 cents per kWh. The interest rate is 5%.

$$(P/A,5\%,5) = 4.3295 \qquad (5.49)$$

Number of hours of in-flight service per year
$$= 200*24 = 4800 \text{ hrs} \qquad (5.50)$$

The LED lamps and incandescent bulbs have unequal lives. The repeatability assumption can be used. The study period is selected as five years.

The number of incandescent bulbs needed for a five-year
$$\text{period} = (5)*(4800/100)*420 = 100,800 \qquad (5.51)$$

The cost of purchasing the incandescent bulbs
$$= 100,800*(\$1.36) = \$137,88$$

Annual Electric Utility costs
$$= (100)*(420)*(4800)/1000*(\$0.25) = \$50,400/\text{year} \tag{5.52}$$

Total cost of purchase and operation of incandescent bulbs
$$= -\$137,88 - (4.3295)*(\$50,400) = \$319,007 \tag{5.53}$$

Number of LED (mercury lamps) needed for a five-year period at the same illumination

$$= (4800/24,000*420*5)*(1680/36,000) = 19.6 = 20 \text{ lamps} \tag{5.54}$$

Cost of purchasing the mercury lamps
$$= 20*(\$32.73) = \$654.60 \tag{5.55}$$

Annual Electric Utility Costs
$$= (400)*(4)/1000((\$0.25) = \$0.40 \tag{5.56}$$

Total cost of purchase and operation of mercury lamps
$$= -\$654.60 - 4.3295*(\$0.40) = -\$656.33 \tag{5.57}$$

It can be seen that by comparisons of Eq. (5.53) and Eq. (5.57), it is lower in total cost to choose LED mercury lamps as aviation lighting compared with incandescent lamps.

5.3   Equivalent Worth Method

When two alternatives are evaluated and when the critical equipment has unequal lives, other methods of evaluation are considered. One is the *rate of return* method. The rate of return can be calculated as

$$ROR = \frac{(R - E)}{C_I} \tag{5.58}$$

Where R is the revenues generated in a prototypical year, E is the expenses generated in a prototypical year and C_I is the capital investment made less the salvage value received to start the business. The project with the higher rate of return can be selected. The IRRs of two projects should not be compared. The IRR has to be compared with the MARR. IRR comparisons are meaningless because the PW changes with the interest rate depending on the project, particularly the revenue function, cost
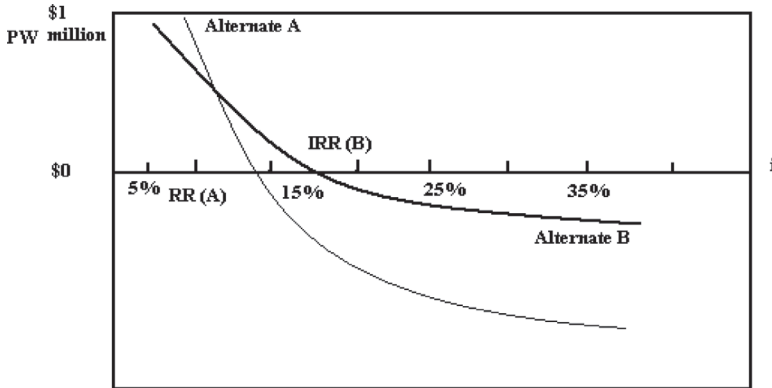
Figure 5.4 Comparison of IRRs Can Lead to Erroneous Conclusions

function changes with the interest rate. Under some conditions, the PW of Project A may be greater than the PW of Project B, although the IRR of Project B may be greater than the IRR of Project A. Under those conditions, Project A has to be selected. Selection is based on the value of PW and not IRR. This is shown in Figure 5.4. As can be seen in Figure 5.4, for interest rates less than 12% $PW_A > PW_B$ and can be selected although the $IRR_B > IRR_A$. Only when the interest rates are high does Project B become more attractive compared with Project A.

AW method analysis may also produce good results. This can be the case when the study period is not equal to useful life.

Example 5.7   Anti-Allergic Cream

A pharmaceutical plant is proposed to manufacture anti-allergic cream. The plant consists of six unit operations: (i) melting; (ii) blending; (iii) filling; (iv) cartoning; and (v) packaging, both shrink wrapping and for shipment. Batches of emulsifying wax and foam stable are independently heated in a jacketed heater. Then emulsifier and ointment are added. Excipient 3 is dissolved in hot deionized water DI and then blended with the other ingredients. The active ingredient is finally added. Cooling procedures are instituted that last for 19 hours. The blended product is filled in tubes of 15 g each. The filling machine can fill 30 tubes per minute. Five sealed boxes per minute are prepared.

Table 5.3 Cost and Revenue Information for Pharmaceutical Plant

	Plan A Manual Cartoning	Plan B Automatic Cartoning
Batches	72/year	195/year
Production	959,976 tubes	2,599,936
Capital Cost (Equipment)	–	$900,000
Operating Cost	$1,437,631	$3,847,157
Revenue	$4,001,400	$10,835,500

In a second method, a new multifunctional blending tank and automatic cartoning machine with line speed of 60 tubes per minute are introduced. The cost is about $900,000. The data needed to evaluate the two alternatives is given in Table 5.3. The interest rate may be taken as 8%. A 20-year life of the modification can be assumed.

The AW method may be used to evaluate the advantages of using automatic cartoning.

The planning horizon is taken as 20 years. The study period is thus the same for Plan A and Plan B.

Plan A—Manual Cartoning

$$AW_A = R_k - E_k = \$4,001,400 - \$1,437,631$$
$$= \$2,563,769 \qquad (5.59)$$

Plan B—Automatic Cartoning

$$(A/P, 8\%, 20) = 0.101852$$

$$AW_B = \$10,835,500 - \$3,847,500 - (A/P,8\%,20)*\$900,000$$
$$= \$6,896,333 \qquad (5.60)$$

It can be seen from comparisons of Eq. (5.59) and (5.60) that Plan B with automatic cartoning has a higher AW compared with Plan A. Plan B with automatic cartoning has been selected.

5.4   Optimization

Often the PW of a business can be *optimized*. There is a trade-off between alternatives with different variable costs and fixed costs. For example, a

number of unit operations in chemical processes and the biotechnology industry are operated stagewise and in a continuous manner. Consider a distillation column that can be used to accomplish the given task, for instance the separation of ethyl alcohol from water. The use of an over-head condenser and reflux can improve the performance of the distillation column. At minimum reflux, the number of plates or stages needed to achieve the desired level of purity in the product would be larger. At a larger reflux ratio, the number of stages required to accomplish the given task would be lower. Consider the two alternatives:

Alternative A: Distillation column with lower reflux ratio and larger number of plates needed to achieve the desired level of consistency of alcohol. This route will require a larger capital cost and lower energy or variable cost.

Alternative B: Distillation column with higher reflux ratio and lower number of plates needed to achieve the desired level of consistency of alcohol. This route will require larger energy or variable costs, but should result in lower capital cost.

Which of the two alternatives should be selected? When capital is lower in cost, a low reflux ratio with a higher number of stages can be selected. When energy is cheap, a higher reflux ratio and lower number of stages can be selected. Let the useful life of the distillation column be N years and the interest rate be i(%). The PW for any distillation column can be written as

$$PW = -N_P C_F' - R C_V' * (P/A, i\%, N) \qquad (5.61)$$

Where the fixed costs increases linearly with the number of stages and the annual utility costs increase linearly with the reflux ratio needed R. The variable costs will increase with the reflux ratio used. The number of stages and reflux ratio can be related using expressions such as the *Kremser Equation*, which is more applicable when the thermodynamic equilibrium relation between the components separated is linear. The Kremser Equation can be written as

$$N_p = \frac{\ln (Purity)}{\ln \left(\dfrac{R}{m(R + 1)} \right)} \qquad (5.62)$$

An optimal PW of the distillation column can be identified when

$$\frac{d(PW)}{dN_p} = 0 \qquad (5.63)$$

At the optimal value, the total life-cycle cost of operation of the distillation column in order to achieve a desired level of purity can be *minimized*.

There is increased interest in recent years to perform what is called a *techno-economic analysis*. Here, the technical feasibility of the process and the EW of the process are evaluated together. Ramifications of such analysis can be interesting. For example, there is identified an optimal number of fermenters that would result in a maximum present worth. A battery of fermenters can be used to generate biotechnology products. The batch fill and emptying times are such that the PW of the plant reaches a maximum at a certain number of fermenters.

At a certain number of effects in multiple effect evaporators, the total cost of the process reaches a minimum. The energy cost and additional capital cost as a function of number of effects are calculated. With more effects, more efficiency is reached in the evaporation, and hence the revenue generated from more effects would be greater. It is not clear at what number of effects maximum profit can be generated. The techno-economic analysis can be performed to arrive at a maximum profitable process. This may work out to be three or four. It depends on the additional efficiency that can be generated in the technology used for more effects. It also depends on the interest rates and revenue that can be accrued for better quality.

A number of discussions in the literature are about the golden triangle of *production, quality, and profit*. When quality required goes up, usually production goes down. More care is needed to produce a product with better quality. The product with better quality can be *priced* higher. Usually this results in better revenue. A techno-economic analysis may be conducted and conditions where profitability can be maximized can be identified.

Example 5.8   Optimal Number of Effects in Multiple Effect Evaporator

One of the critical unit operations used to manufacture *tomato paste* is the use of a multiple effect evaporator. With the use of more effects,
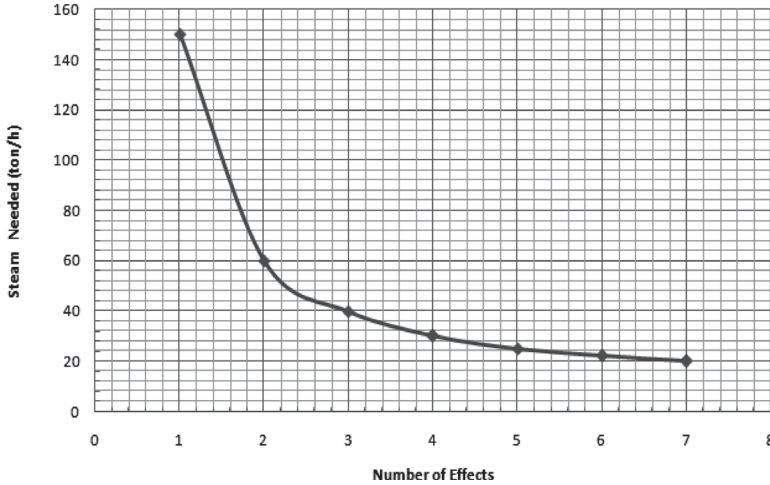
Figure 5.5 Steam Needed as a Function of Number of Effects in a Multiple Effect Evaporator

the capital cost increases linearly and the steam needed decreases in a nonlinear manner. This is shown for a multiple effect evaporator in New Zealand [7] in Figure 5.5 (Steam Flow Rate vs. Number of Effects) and Figure 5.6 (Heat Exchange Area vs. Number of Effects). The cost and economic data to evaluate the multiple effect evaporator needed to concentrate tomato paste is provided in Table 5.4. Calculate the number of effects of the evaporator when the PW of the operation is maximized. The interest rate is 10% and the study period is 13 years. Assume that the plant is operational 225 days per year at 10 tons/h tomato paste production rate.

The quality of the tomato paste from evaporators increases with the number of effects used. Hence, the price of tomato paste from a multiple effect evaporator can be charged higher compared with the product from a single pass evaporator. It can be seen from Figures 5.4 and 5.5 that with more numbers of effects, the capital cost increased but the steam requirement decreased. An *optimal* number of effects can be expected when the PW of the operation is maximized.

The PW of manufacturing tomato paste as a function of number of effects of the evaporator was calculated using an MS Excel spreadsheet using the data given in Table 5.4. The results are shown in Figure 5.7.
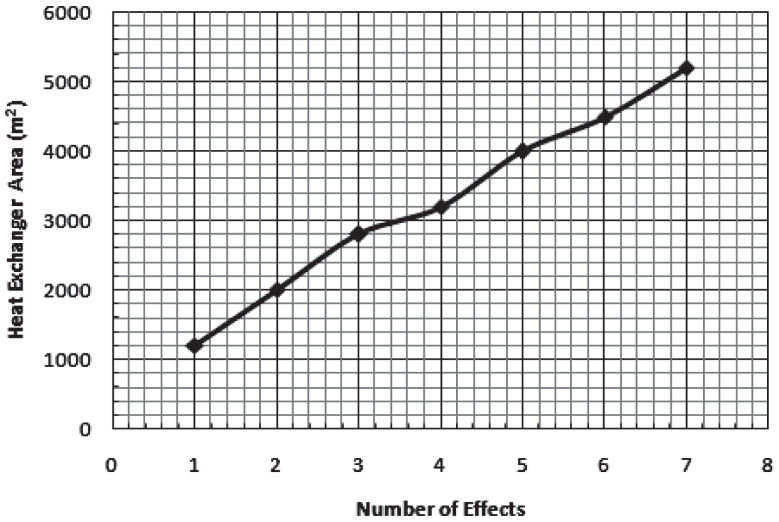
Figure 5.6 Heat Exchanger Area vs. Number of Effects

It can be seen from Figure 5.7 that the optimal number of effects is about 3 for a maximum PW of the manufacture of tomato paste. The results are shown in Table 5.5.

5.5   Household Finances

The head of a household in the 21st century must make a variety of financial decisions. Sometimes the value of money principles and decision analyses explained in the earlier chapters may be applicable to personal

Table 5.4 Cost and Economic Data for Tomato Paste Factory

Number of Effects, N_E	Capital Cost (Millions, $)	Steam Utilities Cost (Millions, $ per year)	Price $ per kg
1	92	8.1	4
2	108	3.24	4.07
3	115	2.16	4.14
4	122	1.62	4.21
5	129	1.35	4.28
6	136	1.188	4.35
7	143	1.08	4.42
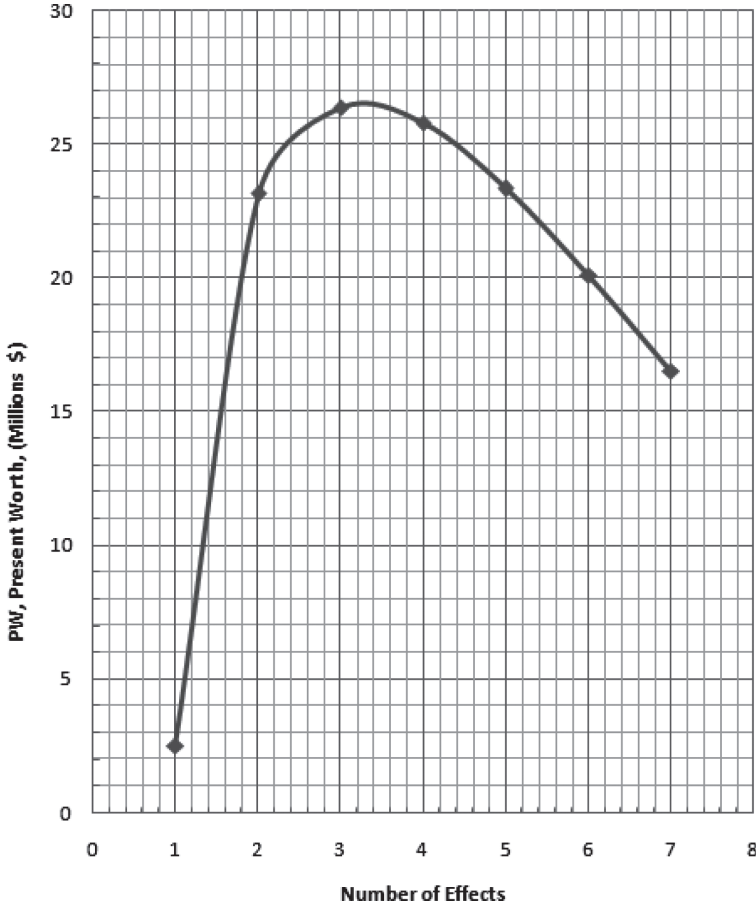
Figure 5.7 PW of the Multiple Effect Evaporator Operation as a Function of Number of Effects

finances. The five largest financial decisions that every individual can be expected to make during their earning years are as follows:

Home Ownership and Mortgage;
Car Ownership and Financing;
Insurance and Taxes;
Credit Card Application and Credit Score;
Retirement Planning—Nest Egg.

The time period associated with home mortgages is usually 30 years. New cars are financed over 60 or 72 months. Income tax to the federal

Table 5.5 PW Analysis for Concentration of Tomato Paste

Number of Effects, N_E	Capital Cost (Millions, $)	Steam Utilities Cost (Millions, $ per year)	Price $ per kg	Revenue (Millions, $)	PW (Millions, $)
1	92	8.1	4	21.6	2.5
2	108	3.24	4.07	21.98	23.2
3	115	2.16	4.14	22.4	26.4
4	122	1.62	4.21	22.7	25.8
5	129	1.35	4.28	23.1	23.3
6	136	1.188	4.35	23.5	20.1
7	143	1.08	4.42	23.9	16.5

government, state government, and city government must be paid every year. Life insurance, health insurance, auto collision and liability insurance, and home owners' or renters' insurance payments are expected once every six months or per quarter. A college graduate starting his career in this day and age would be looking to establish his or her line of credit. Credit card companies loan the principal when one of their customers wants to make a purchase anywhere, anyplace, anytime in the world. In return, the customer pays them every month at an interest rate a lot higher than the prime lending rate to banks and other financial institutions. In the new millennium, social security payments are not sufficient for the elderly after their retirement. Inflation eats into their peace of mind during retirement. IRA, Keogh accounts, and 401(k) savings plan have to be opened and maintained for a secure retirement by middle-class citizens.

The total household wealth in our nation is about $54.2 trillion. According to a recent survey, there are 114 million households in the United States. The number of millionaires has tripled in recent decades.

Home ownership is a means for less-wealthy individuals to attain net worth that will place them in the top quartile of the wealthiest people in America. New homes are built in neighborhoods within commuting distance from major cities. The purchase price of a home ranges from $50,000–$1 million. The purchase price is paid for first by banks that let the individual take out a mortgage. Home mortgages are repaid over a time period of 30 years. As can be seen from the annuity tables in

Appendix A, the multipliers for 30 years are not all that different from the multipliers at infinite time. This is a good break for the borrowers and serves as an incentive to purchase mortgages. Banks borrow money from the Federal Bank Reserve at the prime lending rate. Part of these monies is given out as home mortgages at a slightly higher rate compared with the prime rate. The banks operate within the mortgage and prime rates. All their employees' salaries, profits, and building costs come from these two numbers. For example, in 2010 at the time of this writing, the prime rate is 3.25%. The home mortgage rates from leading lenders are about 4.5%. Mortgage rates can be fixed or a mortgage can be an ARM (adjustable rate mortgage). The interest rates over a 30-year period may change. For example, on December 19, 1980, the prime rate was at a record high of 21.5%. During the course of the time period of a 30-year mortgage, a home owner can do what is called *refinancing*. Here the balance of the loan is paid off by the bank in full and a fresh loan is issued at the new refinanced rate.

When the amount owed on the house is close to the value of the house, leading banks would expect payment of a PMI (mortgage insurance). The monthly payment the individual makes every month consists of three or more components: (i) principal and interest; (ii) real estate taxes; and (iii) mortgage insurance. The principal and interest go toward repayment of the loan at the agreed-upon interest rate. The taxes are paid to the state and local governments. They use these monies for several things, including running schools, etc. Often, politicians run and win elections and lower taxes. Some of the leaders feel that government is the problem. Others, equally popular, feel that government can do more. According to them, there is nothing wrong about America that cannot be corrected with what is right about America. The monthly mortgage payment for an individual depends on the price of the home and the individual's income. The three C's of finance are character, capacity, and collateral. The character is the individual's past credit history. Have they issued bounced checks? Have they paid all their bills and credit card payments on time? Capacity is the individual's income. Is he a physician drawing $200,000 in annual salary? Is he a rock star? Collateral is the other assets that the individual has against which the bank can issue the loan. Rules of thumb such as no more than 28% of monthly income can

go toward mortgage payment is common in parts of Texas. Some banks in New England require the purchase price to be within two times the borrower's annual salary.

A short sale occurs when the selling price of the house is lower than the amount owed on the house. Foreclosure happens after the individual defaults on mortgage payments and the bank takes possession of the house (and usually sells it). Reverse mortgage is applicable to the elderly who have gained equity on their homes. They can take back their equity in the form of uniform monthly payments as fixed income. This is usually used for expenses incurred during retirement.

Example 5.8   Refinancing o.f Steve Gandhi

Steve Gandhi purchased a home in New England in 1980 at a mortgage rate of 21% for 30 years. The price of the home was $150,000. In 1992 the mortgage rate fell to 6.5%. He went for refinancing. With closing costs of $3000, by how much did the monthly payment come down for Steve after the refinancing?

(A/P,21%,30) = 0.210692 (from Appendix A)

Monthly Principal and Interest Payment at 21% mortgage rate
= $150,000*0.210692/12 = $2,633.65

In 1992 the balance owed by Steve = X

X = PW($2633.65,21%,18) = (P/A,21%,18)*2,633.65*12
= 4.6079*$2,633.5*12 = $145,627.2

Number of years left in the 30 year mortgage
= 30 − 12 = 18 years

(A/P,6.5%,18) = 0.092357 (from Appendix A)

Monthly Payment after Refinancing for Steve
= (145,627.2+3000)*0.092357/12 = $1,143.90

Reduction in Monthly Payment of Principal and Interest
= $2,633 − $1,143.90 = $1,489.75

Example 5.9   Toyota National Clearance

The 2010 Toyota Corolla can be purchased for $18,890. There is a rebate of $1,500 or 0% financing for 60 months. The finance rate, should the rebate be selected, is 7.0%. Which option has a lower monthly payment, the cash rebate or 0% financing for 60 months? The life of a 4-cylinder-engine car is around 105,000 miles. With an average driving of 15,000 miles per commuter, this can be taken as seven years.

Option A—0% financing for 60 months

Let the annual payment for 7 years be uniform and = X

For the first 5 years the interest rate is 0%

Hence the amount owed at end of year 5 = $18,890 − 5X

For the 6th and 7th year the interest rate = 7.0%

(A/P,7%,2) = 0.553092

X = 0.553092*($18,890 − 5X)

$$X = \frac{0.553092*\$18,89}{(1 + 0.553092*5)} = \$2774.67$$

Monthly Payment = X/12 = $231.22

Option B—$1500 Rebate

(A/P,7%,7) = 0.5*(0.20976 + 0.167468) = 0.188632

Monthly Payment = (A/P,7%,7)*($18,890 − $1,500)/12 = $273.36

The option with 0% finance rate for 60 months will result in a lower monthly payment compared with the $1500 cash rebate option.

The wealth of households has to come from savings at some point. Japan, the European Union countries, and China have higher savings rates compared to us. Wealth building can come from saving money and investing in assets such as

Stocks;
Bonds;

Money Market;
CDs;
Gold.

Individuals can purchase shares from companies listed in the NYSE (New York Stock Exchange), NASDAQ (National Association of Securities Dealers Automated Quotations), etc. A broker is needed. Leading brokerage firms such as Quick and Reilly allow clients to trade online and brokerage fees are charged as a percentage of the value of the trade. Often individuals are busy making money and do not have time to follow the stock markets and budget deficits of the federal government on a day-to-day basis. There are investment bankers who are willing to take care of the savings of an individual. They operate through *mutual funds*. The front-end and back-end fees of the mutual fund must be considered prior to the individual investing in mutual funds. These days, the mutual fund units are transferrable. Investment bankers let the principle of *diversification* work for their clients. There is a certain *risk* involved in buying stocks or bonds. Rather than the individual assuming the entire risk, the burden of risk is shared by several mutual fund investors. The investment banker may allocate a certain percentage of the funds to stocks, bonds, and money market accounts. He may trade in foreign markets such as Tokyo, the European Union, Bombay, etc. Examples of mutual funds are given in Table 5.4. The banker may buy more stocks and bonds when the interest rates are *low* and buy more money market funds and gold when interest rates are *high*.

The number of credit cards has increased in the last three decades. There are 576.4 million credit cards with an average balance of $15,519 active today, per a recent survey among U.S. households. There is a website, www.creditcards.com, where comparison shopping of credit cards can be done. There are different types of credit cards. These are

Low-Interest Credit Cards;
Instant Approval Credit Cards;
Balance Transfer Cards;
Cash Back Credit Cards;
Travel and Airline Credit Cards;
Business Credit Cards;

Student Credit Cards;

Prepaid and Debit Cards.

Some of these credit cards can be used at locations all over the world. Many hotels and car rental companies will not transact business with customers without a credit card. Credit cards are used as deposits at several places of business. Some of these cards, such as a Macy's Department Store card, can be used only within the retail chain. An Exxon gasoline card can be used to buy gasoline at Exxon gas stations. Most general-purpose cards

Table 5.6 Top Mutual Funds (US News & World Report)

Name of Mutual Fund	Value of Fund	Last 1-Year Return (August 2010)	Stocks, Bonds, Money Market Mix of Holdings
Invesco van Kampen Equity and Income Fund	$12.07 billion	14.8%	Stocks—68.45% Bonds—15.4%
Yachtman Fund	$2.1 billion	22.5%	Stocks—85.11% Cash—14.5%
FMI Large Cap Fund	$2.7 billion	17.0%	Stocks—88.96% Cash—11.04%
Templeton Global Bond Fund	$34.6 billion	14.0%	Bonds—80.42% Cash—16.68%
Eaton Vance Tax Managed Emerging Markets Fund	$1.8 billion	24.7%	Stocks—92.74% Bonds—0.01% Cash—0.85%
TCW Total Return Bond Fund	$5.0 billion	16.1%	Bonds—94.3% Cash—5.68%
Appleseed Fund	$139.6 million	20.3%	Stocks—71.1% Cash—14.1%
Monetta Young Investor	$6.7 million	26.3%	Stocks—85.1% Cash—14.9%
Marsico Flexible Capital Fund	$61.7 million	33.7%	Stocks—84.9% Bonds—6.9% Cash—8.2%
Boston Trust Small Cap Fund	$156.2 million	24.5%	Stocks—97.6% Cash—2.4%
Tilson Dividend Fund	$16.6 million	25.4%	Stocks—86.5% Cash—9.1%

are unsecured. The credit line is extended based on credit history. Secured cards are issued after backed up by opening a savings account. Secured cards cater to a market of individuals with challenged credit or little or no credit history. Some universities have a course on *consumer finance*. The average consumer has nine credit cards.

Credit card interest rates can change dramatically over the pay period of a purchase. It can range from 0% limited balance transfer offers to as high as 30%. The prime rate these days is low at 3.25%. The record high prime rate was 21.5% in the last quarter of 1980. The credit card companies are allowed to charge 20% above the prime rate. So should the prime rate reach 21.5% again, the credit card interest rate would be as high as 41.5%. This will increase the minimum monthly payment and the time taken to pay the credit card back. The APR set for a customer by the credit card issuing bank depends on several factors such as credit score, income, assets, current debt load, credit inquiries, payment history, and economic conditions. The lowest rates are given to customers with positive and proven credit histories.

Banks, credit unions, retailers, and credit card companies are allowed to issue credit cards. Someone who wants to issue a credit card may do so after getting the necessary approvals from state officials. The department that issues the sole proprietorship, partnership, and C corporation can permit people who want to issue credit cards. Visa and MasterCard are companies that help process payments. American Express started like Diners Club as a restaurant card. It used to be that all charges had to be paid at the end of the month when using an American Express card. Discover Card pays some cash back. A small percentage of the purchase amounts are refunded to the customer. Every credit card issued has a signed agreement that has to be followed. The contract is binding. The agreement discloses the credit limit available, APR (annual percentage rate), interest calculation method—such as by daily balance, fixed or variable APR—grace period, fees, etc.

The Credit Card Act of 2009 mandated that credit card bills contain warnings about the consequences of paying only the minimum balance. This act, signed into law by President Obama, limits when issuers of consumer credit cards can increase interest rates and bans billing and payment practices that the Federal Reserve calls "unfair or deceptive."

5.6   Summary

Brainstorming can be used to develop alternatives for a given problem. The five methods of analysis, PW, FW, AW, IRR, and ERR can be used to evaluate the alternatives. Evaluation of alternatives can be used to invest in the more profitable process, selection of the lower cost alternative, quantitate the savings accrued from implementation of a process improvement, and assessment of environmental impact of chemical processes. Alternatives can be mutually exclusive, independent projects, and screening by other criteria.

The PW (present worth) of identified alternatives can be calculated and compared with each other. Three cases are identified. When one PW is greater than zero and the second PW is less than zero, the one greater than zero can be selected over the one that is less than zero. When both PWs are greater than zero, the alternatives are rank ordered. When the PWs of both alternatives are within 20% of each other, the projects can be adjudged as too close to call within the sensitivity of the estimates of the capital and operating expenses and expected revenues.

Worked examples have been developed to select the better process to manufacture CNTs (carbon nanotubes), cost savings by using microfiltration in pretreatment during SWRO (sea water reverse osmosis), evaluation of the process to manufacture bioethanol from sugarcane bagasse, and better routes for sequestration.

When the useful lives of critical equipment are different for different alternatives, two methods for comparison analysis are developed. In the repeatability assumption the study period is selected as a lowest common multiple between the two useful lives. In the co-terminated assumption, the alternative with greater useful life is truncated at the useful life of the second alternative. Expenses and revenues are reevaluated after the truncation. For example, the salvage value can be moved up. Worked examples on comparison on fair basis were discussed on life-cycle costs of copiers with different initial fixed costs and variable costs; life-cycle costs of aviation lighting using LEDs or incandescent bulbs; and evaluation of automatic cartoning vs. manual cartoning in a pharmaceutical plant. The rate of return concept was introduced.

PW can be optimized. This is especially so when there exists a trade-off between variable and incremental capital costs. For example, when tomato paste is manufactured with more effects in a multiple effect evaporator, the energy costs decrease and the capital costs increase. The PW of the plant to manufacture tomato paste reaches a maximum value at an intermediate value of number of effects as the number of effects is increased. There exists a golden triangle of production, quality, and profit. A worked example on optimal number of effects in a multiple effect evaporator was created.

The rudiments of household finances are discussed. These include home mortgage, car ownership and financing, insurance and taxes, credit card application and credit score, and retirement planning and nest eggs. Home mortgages are loans provided for home purchase with a 30-year payback period. During that period, the lender holds the lien on the house. The payback period is nearly infinity. Auto financing is a loan given to the buyer with a six- or seven-year payback period. During that time, the title of the car belongs to the financial service provider. The nest eggs of individuals for retirement can be created with a diversified portfolio of stocks, bonds, money market accounts, and gold. There are many different kinds of credit cards.

End-of-chapter exercises based on case studies were developed. These include evaluation of manufacturing processes for ABS engineering thermoplastic; polystyrene production; snack factory; transcontinental oil pipeline; Stirling dish solar power plant; refinancing scheme; analysis for renting or buying a home; business plan for a software development service; life-cycle costs of a PEV (plug-in electric vehicle); largest solar thermal power plant; and wind energy production in Saudi Arabia. Other examples explored include printing books in hard copy form or e-book form; blow-molding press; life insurance by direct marketing vs. life insurance via the Internet; emissions reduction in an oil refinery; improvements to combined cycle power plants; four different cooling tower designs; rechargeable batteries; and heap leaching and agitation leaching processes for gold recovery.

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An Introduction to Engineering Economics

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