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Paolo Bussotti

The Complex Itinerary of Leibniz's Planetary Theory

Physical Convictions, Metaphysical Principles and Keplerian Inspiration

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Foreword

Authors like the late Eric Aiton, Domenico Bertoloni Meli, François Duchesneau, Alexandre Koyré and many others have diligently studied, explained or criticized Leibniz's planetary theory. Leibniz, it is true, changed his relative opinions in many respects in the course of time. But he always adhered to some fundamental convictions, among them being the strong assertion that all hypotheses must be based on mechanical models. This is especially true of his different explanations of gravity that are closely connected with his cosmological considerations. He thus inevitably refused Newton's celestical mechanics because it was based on the unexplained notion of gravity.

Paolo Bussotti makes a new, comprehensive effort to interpret Leibniz's different trials to develop a consistent planetary theory well knowing that "it is difficult to offer a coherent picture of Leibniz's theory of motion". Yet, he rightly emphasizes that Leibniz aimed at a physical-structural theory, not only at a kinematical or dynamical theory in order to understand the world system.

Bussotti presents a subtle analysis of Leibniz's thinking and argumentation. Leibniz's natural inertia is not Newton's inertia. Leibniz had no inertia concept that was comparable to that of Newton. He tried to replace it by means of his forces. Leibniz's main physical quantity was speed, not acceleration. When he elaborated his theory of a harmonic circulation and a paracentric motion as basic ingredients of his planetary theory, he did it with regard to Newton's *Principia mathematica*. He wanted to offer a physical alternative to Newton's physics.

What is more, Bussotti's aim is to explain the internal change of Leibniz's concept of gravity. Leibniz finally came to the conclusion that gravity originates from the circulation of the ether. Yet, the origin of gravity was not certain for him. He continued to write on it up to the end of his life. He attributed to Kepler the idea that gravity is due to the centrifugal force of the fluid. It is worth mentioning that such a fluid is a reminiscence of Ptolemy's cosmology.

Therefore Bussotti justly concludes that a full understanding of Leibniz's planetary theory is not possible without an understanding of its connection with Leibniz's general, physical, and metaphysical principles.

In my eyes Bussotti's last chapter is especially important and original. It analyses Kepler's influence on Leibniz's scientific thinking and planetary theory. Influence does not necessarily mean agreement, though Leibniz himself considered himself as somebody who continued Kepler's work. For example Leibniz did not accept Kepler's planetary souls or magnetic influences. For him even the orbit of the planets might be not an ellipse.

Bussotti demonstrates that Leibniz falsely ascribed the insight to Kepler that in a curvilinear motion a body tends to escape along the tangent. But Leibniz obviously took the idea of the paracentric motion, as well as that of a decomposition of planetary motions, into two components from Kepler. On the other hand, he was not influenced by his countryman when he conceived of the *circulatio harmonica*. Both scientists shared the conviction that harmony determines the structure of the universe.

In spite of many differences between the two thinkers, Bussotti emphasizes the similarity between their ways of thinking, of approaching the problems, and of conceiving of the universe and of its relation with God. Bussotti teaches the reader to see Leibniz's metaphysics under a new perspective, to see Leibniz as a modern Keplerian. Kepler and Leibniz shared indeed a common vision of the universe that was based on harmony, final causes, and on a conception of the world as a true *kosmos*.

Berlin, Germany
June 2015

Eberhard Knobloch

Preface

The genesis of this book begins with an Alexander von Humboldt fellowship that I had achieved in the period 2003–2005 at Ludwig Maximilians University, Munich though in those years I did not focus on Leibniz. Some years later I extended the privilege of this Fellowship during a three month period from December 2013 to February 2014 at the Berlin-Brandenburg Academy of Science, Berlin. The host of my fellowship was Professor Dr. Eberhard Knobloch. In the previous six months, I had frequent e-mail contacts with Professor Knobloch and we shared the idea that, during this time period in Berlin, I would focus my studies on the influence exerted by Kepler on Leibniz's planetary theory. Therefore, I began my research with this clear intention. However, my reading of Leibniz's works and the existing literature on the subject, as well as discussions with Professor Knobloch, convinced me to extend my research beyond this narrow intention. Thus, my aim was widened to frame Leibniz's planetary theory inside his physics and metaphysics. In particular, I wondered if planetary theory was, for Leibniz, something like an academic exercise or, in any case, a secondary part of his general order of ideas, scarcely connected with the whole of his production or if, in contrast, it played an important role in the development of his entire way of thinking. My attempts to answer such questions are the core of this book, inside which, without entering into details, which the reader will control in the running text, it is possible to recognize three main conceptual centres:

1) Description and specification of the details (in particular mathematical and physical details) of Leibniz's planetary theory, also considering its historical evolution. The Chaps. 2 and 4 are dedicated to this problem;
2) Connection between Leibniz's gravity theory—perhaps better to speak of Leibniz's ideas on gravity rather than a theory in a proper sense—and planetary theory. This is the subject of Chap. 5;
3) Kepler's influence on Leibniz. This was my original project. It is developed in Chap. 6, where I show the influence exerted by Kepler on Leibniz's planetary theory, but where I try to extend the argumentation, as I attempt to prove that

Kepler was also influential on Leibniz's metaphysics, in particular as far as the concept of pre-established harmony is concerned.

Chapter 1 is a historical and conceptual introduction to the scenario described in the book, while Chap. 3 has to be interpreted as a brief parenthesis concerning the concept of inertia in Leibniz, especially focusing on the aspects connected to planetary theory. To be clear, my intention has not been to deal with the complex general problems of Leibniz's physics, on which a huge and profound literature exists.

As to the quotations, in the running text I have always offered the English translation from original works or letters, which are almost exclusively written in two languages: Latin (in most cases) and French (in several cases). If not explicitly specified otherwise, the translation is mine.

I wish to express my particular gratitude to Professor Dr. Eberhard Knobloch. He followed my research in Germany and he read the whole of my work, giving me precious advice. Finally, he contacted the publishing house Birkhäuser to propose the publication of this book. Without his collaboration and precious help, this research would have been neither written nor published.

I am also grateful to Professor Danilo Capecchi for his qualified, numerous and profound tips, as to the content and form of my work.

I am indebted with Dr. Raffaele Pisano, with whom I have published several works and who also gave me valuable help.

I wish to thank Professor Niccolò Guicciardini for an important observation concerning Chap. 4 and Dr. Stefano Gattei for some advice regarding Chap. 2.

It is obvious that possible mistakes or imperfections rest entirely upon the author.

I wish to express my gratitude to the Alexander von Humboldt Foundation for having financed my research-period in Berlin.

I am grateful to the Birkhäuser Publishing House for having accepted my book for publication.

Udine, Italy Paolo Bussotti

Contents

Chapter 1
An Introduction: The Historical-Conceptual Reference Frame

Leibniz dealt with planetary theory in three papers written between 1689 and 1706.[1] The first paper, titled *Tentamen de Motuum Coelestium Causis*, is the only one which was published—in the *Acta Eruditorum Lipsiensium*, 1689—during Leibniz's lifetime. In the *Tentamen* Leibniz tried to construct a planetary theory based on a refinement and specification of the vortex theory. In particular, he attempted to supply a series of mathematical considerations, which allowed him to obtain (1) Kepler's area law; (2) the inverse square law; (3) ellipticity of the planetary orbits, without resorting to the Newtonian concept of force. Leibniz developed a second version (*zweite Bearbeitung*) of the *Tentamen* (see note 1), which was not published at that time, but which presents important specifications, in particular as to: (a) the structure and history of vortex theory; (b) the nature of gravity; (c) the completion of mathematical proofs which were only outlined in the published version.

In general, Leibniz's ideas on astronomy were not welcome: Huygens developed a series of criticisms, which were not based on the mathematical treatment, but on some physical concepts introduced by Leibniz, in particular that of *circulatio harmonica*. The correspondence between Huygens and Leibniz is important to understand the nature of Huygens' critics and of Leibniz's point of view.[2] Varignon discovered a mathematical mistake, which could be corrected without compromising the general structure of the theory.[3] However, the campaign against Leibniz

[1] All the mentioned contributions have been published by Gerhardt in Leibniz (1860, 1962), VI. The first one is the *Tentamen*, pp. 144–161; the second one is the *Tentamen (Zweite Bearbeitung)*, pp. 161–187; the third one is *Illustratio Tentaminis de Motuum Coelestium Causis*, parts 1 and 2 plus *Beilage*, pp. 254–280.

[2] For the critics addressed by Huygens to several concepts Leibniz used in his planetary theory, in particular the concept of *circulatio harmonica*, see Chap. 2, where I will deal with this question in detail.

[3] See the letter Varignon sent to Leibniz on the 6th December 1704, in Leibniz ([1849–1863], 1962, IV, pp. 113–127).

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came, basically, from Newton and the Newtonians: Newton himself, Gregory and Keill were the protagonists.[4] Their criticisms were of various kinds:

1. some supposed mathematical mistakes were pointed out;
2. from a physical point of view, the charge was that Leibniz had not taken into account vortices-instability proved by Newton. In particular the movements of the comets would have been inexplicable inside Leibniz's theory;
3. the third Kepler law was not coherent with some of Leibniz's assumptions.

Leibniz wrote one paper titled *Illustratio Tentaminis de Motuum Coelestium Causis* (see note 1) divided into two parts. This contribution, written probably around 1706, was not published in Leibniz's lifetime. In the *Illustratio* Leibniz tried to answer the critics and to better clarify the physical bases of his theory. Other works by Leibniz, written at the end of the seventeenth century do not deal directly with astronomy, but, since they concern—in part or *in toto*—gravity, they get a relevant importance in our context. These works are *De Causa gravitatis, et defensio sententiae Autoris de veris Naturae Legibus contra Cartesianos*, published in the *Acta Eruditorum Lipsiensium*, 1690 and the two parts of the *Specimen Dynamicum*, the former published in *Acta Eruditorum Lipsiensium*, 1695, the latter unpublished in Leibniz's lifetime.[5] Significant references are also present in Leibniz's correspondence and in other published or unpublished works, but the mentioned ones are the most important.

The reasons of interest behind Leibniz's celestial mechanics are numerous:

1. From a historical point of view: why did Leibniz publish a contribution on planetary theory two years after the publication of Newton's *Principia*, in which, for the first time, a complete physical theory of planetary motions was expounded?
2. From a mathematical standpoint: are the mathematical argumentations used by Leibniz correct?
3. In a physical perspective:
   (a) is the physical structure of the world proposed by Leibniz, inside which he tried to explain the movements of the planets, stable?
   (b) Is the use of the physical quantities utilized by Leibniz suitable for an inquiry on the planetary motions? These questions imply that the term *physical* has three meanings:
      (b-i) referred to the supposed *real physical structure of the world* (for example: according to Leibniz the vortices are physically existing entities). I call a theory dealing with this level of reality a *physical-structural theory*.

[4] See D. Gregory (1702, pp. 99–104), Newton (1712?, 1850), Keill (1714).

[5] For the *De causa gravitatis*, see Leibniz (1690, 1860, 1962, VI, pp. 193–203); for the *Specimen Dynamicum* parts 1 and 2, see Leibniz (1695, 1860, 1962, VI, pp. 234–254).

(b-ii) referred to *dynamics* (let us remember that Leibniz was the inventor of this term), that is to explanations of the movements by means of *forces* (whatever the meaning of this word is). This implies not only a kinematical description of the movements, but also the research of the cause/s of the movement or of the change of movement (this last one is Newton's perspective).

(b-iii) referred to *kinematics*: namely a theory can provide a description of certain movements and can be able to foresee the positions of certain bodies without dealing either with the physical reality of the world or with the actions which determine the movements or the change of movement. Only to give an example: Ptolemaic planetary theory expounded in the *Almagest* is merely kinematic.

An explanation can be dynamical, but not physical-structural. For example, gravity theory explained by Newton in the *Principia* is dynamical, but not physical-structural, because Newton deals with gravity as a given force and does not look for its origin in some features of the real physical world. This is the meaning of the famous "Hypotheses non fingo". For, a physical explanation has to provide the structure of the world and the origin of the acting forces in this structure. In the case of Leibniz, the vortices are real entities and gravity action has to be explained in terms of plausible mechanism of the real physical world. Instead, a dynamical explanation can take for granted the origin of a certain force and only propose a model, which is coherent with the phenomena and with the supposed features of the considered actions. This is an explicative level different from a merely kinematical approach—where forces play no role—but which is less demanding than the physical-structural explicative level. The difference between the three meanings of the words *physics*/*physical* is an important topic in history of physics and astronomy. This distinction has not always been given sufficient consideration in the literature, while the difference dynamics/kinematics is well known and explored.

4. As to the relations among the different aspects of Leibniz's thought: which are the connections between Leibniz's physics (at least the physics he developed after the publication of Newton's *Principia*) and his planetary theory? In a more general perspective: how did Leibniz's metaphysical and ontological convictions influence his planetary theory?
5. With regard to Leibniz's sources, one author seems to be particularly significant: Kepler: (i) what are the real connections between Kepler's physical astronomy and Leibniz's physical astronomy? (ii) What did Leibniz think about the relations between his own and Kepler's points of view, that is how did Leibniz interpret the physical parts of Kepler's astronomy? Many other authors influenced Leibniz's celestial mechanics, in particular Descartes, Borelli and

Huygens. However, their influence on Leibniz is clear enough, while this is not always the case with Kepler.

In the literature, there are several contributions on Leibniz's planetary theory, although they are far less numerous than those dedicated to his physics or mathematics or philosophy. Probably the most significant researches are due to three authors: Alexandre Koyré, Eric J. Aiton and Domenico Bertoloni Meli.[6] In the appendix A of his *Newtonian Studies*, Koyré deals with Leibniz's celestial mechanics. Without entering into the general structure of Koyré's reasoning, his judgement on Leibniz's celestial mechanics is extremely negative, basically because of a supposed physical-mathematical mistake: Koyré interprets the locution *velocitas circulandi* used by Leibniz as referring to the module of velocity. If this were the case, the whole theory expounded by Leibniz would have been affected by a mistake, which would have completely compromised it. In a series of four fundamental papers written in the period 1960–1965 and published in *Annals of Science*, Aiton carries out a robust campaign in defence of Leibniz. He begins by interpreting *velocitas circulandi* as *transverse velocity*. If this is true, the critics of Koyré would derive from a serious misunderstanding of Leibniz's concepts. In the first paper, Aiton describes the bases of Leibniz's theory and, despite a general positive judgement, he adheres to some critics by Newton and the Newtonians. However, in the following contributions he changes his mind: these criticisms are due to an incorrect understanding of Leibniz's thought, which—in spite of numerous obscurities in the language—is basically correct. In his paper written in 1965, Aiton explicitly critisizes Koyré's interpretation. Aiton proposes the same picture in his book *The vortex theory of planetary motion*, 1972.

A fundamental contribution is Bertoloni Meli's *Equivalence and Priority: Newton versus Leibniz*, 1993, because Bertoloni Meli: (1) looks for Leibniz's sources; (2) tries to understand the relations between Leibniz's planetary theory and Newton's *Principia*, in particular if the intention to propose a theory alternative to Newton's played a role in the development of Leibniz's concepts; (3) expounds and translates into English the *Tentamen* and a series of Leibniz's unknown manuscripts on celestial mechanics, which were written in the years immediately preceding the publication of the *Tentamen*. In this way the development of Leibniz's thought can be convincingly traced.

Given this picture, many aspects of Leibniz's planetary theory have been clarified. Nevertheless, some of them still remain rather obscure or, at least, not completely clear. In particular:

(A) As to Leibniz's sources, the relations between Kepler's and Leibniz's theories and the interpretation Leibniz gave of Kepler's astronomy and Kepler's concept of inertia;

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[6] The fundamental works on Leibniz's planetary theory are: Aiton (1960, 1962, 1964, 1965, 1972), Bertoloni Meli (1993), Koyré (1965), Appendix A. Important studies are also: Aiton (1984, 1995), Bertoloni Meli (1988a, b, 1990, 2006), Cohen (1962), Hoyer (1979a).

(B) With regard to the physics behind planetary theory, two questions have to be clarified; (a) as to the previous items (3.b-i), till which point did Leibniz think that his vortex theory and his ideas on gravity represented the real structure of the world and the very reason of gravity rather than a model of these phenomena? With regard to (3.b-ii), Leibniz quotes more than once the term inertia, but he seems to think that *forces* should explain the movements and not only the changes of movements (accelerations). This is connected to his concept of inertia, to his idea on Kepler's concept of natural inertia and to his scarce—almost absent—consideration of the initial conditions of a motion (in particular of the initial velocities). Leibniz's critics to the action at a distance and to the concept of absolute space and time are surely the most known against Newton. But, I will try to prove that the different considerations of the initial conditions of the motion are an important aspect of the different approaches of the two scientists.

The aim of this contribution is hence to deal with the questions (1) and (2). The structure of this book is, thus, the following: (1) The historical-conceptual reference frame; (2) description of the most important elements of Leibniz's planetary theory with commentaries on the most problematic aspects; (3) an interlude on the concept of inertia in Leibniz; (4) planetary theory's structure after the critics addressed by Newton and the Newtonians. (5) The problem of gravity in Leibniz; (6) Leibniz and Kepler: a critical comparison; (7) final remarks.

Chapter 2
Description of the Most Important Elements of Leibniz's Planetary Theory

This chapter is divided into four parts according to an ideal division of the *Tentamen*. In the first part Leibniz dealt with harmonic circulation and introduced paracentric motion; in the second one he analysed the properties of paracentric motion; in the third one he dealt with the inverse square law and the elliptic movements of the planets; in the fourth one Leibniz provided a summary of his model. Every paragraph is divided into two subparagraphs: 1. Leibniz's assertions; 2. commentaries.

2.1 Physical Presuppositions, the *Circulatio harmonica* and the *Motus paracentricus*

2.1.1 Leibniz's Assertions

In the published version of the *Tentamen*, Leibniz, after a general historical introduction concerning the development of astronomy and vortex theory, clarified the physical assumption on which his planetary theory is based: the planets are moved by a rotating fluid in which they are situated, because: a) planets' orbits are curved lines; b) each body moving in a curved line is subject to a *conatus* to recede along the tangent, that is a centrifugal force; c) the planets do not recede along the tangent; d) hence it is necessary that something exists allowing the planets to continue their curved paths; e) thus, the only physical possibility to explain this motion is the hypothesis of a moving fluid vortex, which surrounds every planet and transports the planet by means of its motion. The planets are afloat in the vortex which communicates them its movement.

After these physical considerations, Leibniz introduced the definition of *Circulatio Harmonica* (harmonic circulation) like this:

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> I call a *circulation* a *harmonic* one if the velocities of circulation in some body are inversely proportional to the radii or distances from the centre of circulation, or (what is the same) if the velocities of circulation round the centre decrease proportionally as the distances from the centre increase, or most briefly, if the velocities of circulation increase proportionally to the closeness.[1]

According to Leibniz, the harmonic circulation can characterize the arcs of every curve, not only the arcs of a circle.

The next step consists in two different possible decompositions of the curvilinear motion (see Fig. 2.1). Let a body move along a curve $M_1M_2M_3$ describing the elementary arcs M_1M_2 and M_2M_3 in equal time, then its motion can be decomposed into: a) a circular motion around the centre Θ (M_2T_1 and M_3T_2 are, in this case, infinitesimal circular arcs) plus a rectilinear motion as T_1M_1 and T_2M_2; b) the motion of a rigid ruler around the centre Θ plus the rectilinear motion of the body M along the rotating ruler. The motion of M along the ruler was called by Leibniz *motus paracentricus* (paracentric motion). Leibniz adopted this second decomposition of the curvilinear motion. Then, without considering for the moment the paracentric motion, a circulation is harmonic if the infinitesimal circulations M_2T_1 and M_3T_2, completed in equal elements of time, are inversely as the radii ΘM_2 and ΘM_3. Leibniz wrote:

> For since these arcs of elementary circulations are as the times and the speeds combined, and the elements of time are taken to be equal, the circulations will be as the velocities, and consequently the velocities inversely as the radii, and therefore the circulation will be called harmonic.[2]

Leibniz could now prove that the area law is valid for bodies which move according to a harmonic circulation. Actually, rather than a demonstration, the area law is a definitory property of the harmonic circulation, once specified the proportionality between elementary circulations and speeds.

In the sixth paragraph of the *Tentamen* Leibniz claimed that, since the planets move according to the area law and given the logical equivalence between area law and harmonic circulation, the planets move with a harmonic circulation.

The seventh paragraph deals briefly with a problem which is important in order to understand Leibniz's way of reasoning, which runs as follows: a) as already seen a body which is posed in a fluid does not move spontaneously in a curved line, this means that the aether itself is not at rest; b) it is reasonable to think (*rationis est*

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[1] Translation drawn from Bertoloni Meli (1993, pp. 129–130). Original latin text: "Circulationem voco Harmonicam, si velocitates circulandi, quae sunt in aliquo corpore, sint radiis seu distantiis a centro circulationis reciproce proportionales, vel (quod idem) si ea proportione decrescant velocitates circulandi circa centrum, in qua crescunt distantiae a centro, vel brevissime, si crescant velocitates circulandi proportione viciniarum." (Leibniz 1689, 1860, 1962, VI, pp. 149–150).

[2] Translation drawn from Bertoloni Meli (1993, p. 130). Original latin text: "Cum enim arcus isti elementarium circolationum sunt in ratione composita temporum et velocitatum, tempora autem elementaria assumantur equalia, erunt circulationes ut velocitates, itaque et velocitates reciproce ut radii erunt, adeoque circulatio dicetur harmonica." (Leibniz 1689, 1860, 1962, VI, p. 150).

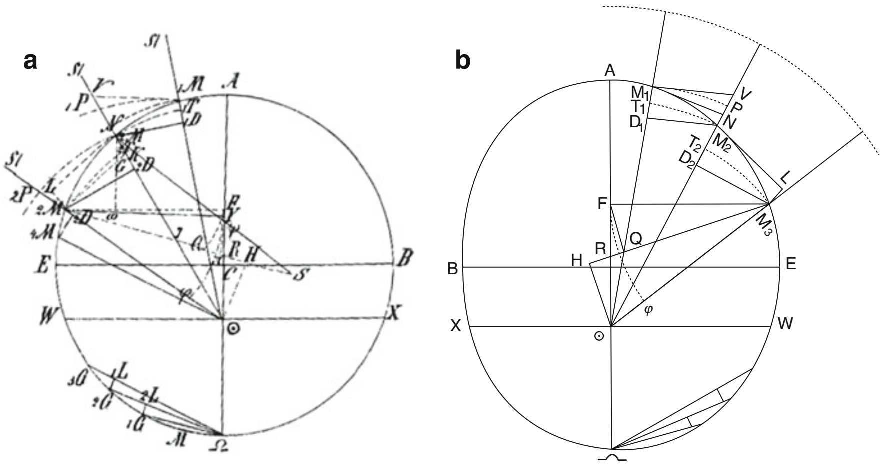

Fig. 2.1 Leibniz's planetary theory model. (**a**) This is Leibniz's original figure posed by Gerhardt at the end of Leibniz 1860, 1962. The diagram is unclear. There are many letters and this makes it difficult to clearly read the diagram. There is a typo because the $_2M$ written immediately over $_4M$ is a mistake. The right form is $_3M$. Furthermore there is the habit to write the index of a letter before the letter, while nowadays we write after the letter. Because of all these reasons—if I do not specify otherwise—I will refer to (**b**), which is written in a more modern form but does not betray Leibniz's thought, at all. This diagram is drawn from Aiton (1960, p. 69)

credere, *ivi*, p. 151) that the movement of the aethereal fluid has the same features as planet's movement, hence, it follows: c) the motion of the fluid itself is harmonic.

Leibniz imagined the situation like this: the planet moves in an ellipsis (he dealt with the properties of the elliptic motion in the next paragraphs of the *Tentamen*) of harmonic circulation. Let us consider the part of aether, which constitutes a ring, whose centre is in the sun, whose major radius is the distance sun-aphelion and whose minor radius is the distance sun-perihelion. This ring can be thought as divided into concentric circumferences of small thickness (*exiguae crassitudinis*, *ivi*, p. 152), centred in the sun with the property that the fluid composing every circumference moves harmonically. Therefore, the planet moves harmonically on an ellipsis, every aethereal fluid's circular section of infinitesimal thickness moves harmonically, this means that the whole aethereal fluid moves harmonically according to a circular motion. Therefore (par. 8), the motion of a planet can be considered as decomposed in the harmonic motion of the fluid plus the paracentric motion along the ruler. When a planet, at the time t, moves in the circumference C of the aethereal fluid, the planet itself does not retain the impetus of circulation (*impetus circulandi*, *ivi*, p. 152) it had got while moving along a different circumference at the time $t_i < t$; rather it assumes immediately the harmonic movement of the circumference in which it is at the time t.

This assertion in paragraph 8 concludes ideally the first part of the *Tentamen*, in which the essential properties of the planetary harmonic circulation are explained. The second part will face the paracentric motion.

2.1.2 Commentaries

In these commentaries I will remain strictly adherent to Leibniz's text, while dealing with more general questions in Chap. 3.

1. The role of harmonic circulation of the aethereal fluid is twofold:
   a) from a kinematical point of view, it has to provide the mean motion of the planet. The deviation from the uniform circular motion is given by the paracentric motion.
   b) from a physical-structural point of view, the aethereal vortex is a real existing entity, according to Leibniz. As we will see, he proposed, at least, two hypotheses on the features of the vortices when he needed to better specify some dynamical properties of gravity or to explain the movements of the comets inside his system, but Leibniz never doubted the physical existence of the vortices and of their harmonic circulation. In this regard, the correspondence with Huygens is significant: it is well known that both Leibniz and Huygens did not accept the idea of action at a distance, both of them sustained vortex theory, but Huygens never accepted the role ascribed by Leibniz to the harmonic motion of the aethereal vortex. He saw harmonic circulation as a useless additional hypothesis, because the area law was given for granted in this hypothesis and, as to gravity, the harmonic circulation—not the vortices in themselves—seemed to play no role. Therefore Huygens was not able to understand the meaning of harmonical vortices.

   In a brief but dense passage of a letter to Leibniz on 11 July 1992, Huygens wrote:

> It is sure that the gravities (*pesanteurs*) of the planets are in inverse double reason as their distances from the sun, which, together with the centrifugal virtue (*vertu*), provides Kepler's eccentrical ellipses. But I was never able to understand, relying upon your explanation given in the *Acta* of Leipzig [the published version of the *Tentamen*], how you deduce the same ellipses, replacing your harmonic circulation and maintaining the same proportions of gravities. I do not see how you find the place for a kind of Descartes' deferent-vortex, which you want to maintain, since the mentioned proportion of gravity, joined with the centrifugal force, produces—by itself—Keplerian ellipses, according to the proof given by Mr. Newton. For a long time, you promised me to clarify this difficulty.[3]

[3] LSB, III, 5, p. 337. Original French text: "Il est certain que les pesanteurs des Planetes estant posees en raison double reciproque de leurs distances du soleil, cela, avec la vertu Centrifuge, donne les Eccentriques Elliptiques de Kepler. Mais comment en substituant vostre Circulation Harmonique, et retenant la mesme proportion des pesanteurs, vous en deduisez les mesmes Ellipses, c'est ce que je n'ay jamais pu comprendre par vostre explication qui est aux *Acta* de Leipsich; ne voiant pas comment vous trouvez place à quelque espece de Tourbillon deferant de des Cartes, que vous voulez conserver; puisque la dite proportion de pesanteur, avec la force Centrifuge produisent elles seules les Ellipses Keplerienes selon la demonstration de Mr Newton. Vous m'aviez promis il y a longtemps d'eclaireir cette difficulté".

Aiton claims:

> Since the harmonic vortex played no part in the motion of a planet in its orbit, this vortex may be left out of account in the analysis of Leibniz's theory.[4]

And again:

> What he [Leibniz] still failed to see clearly was that the harmonic circulation of the planet followed from the attraction, so that his resolution of the orbital motion into transverse and radial components, which gave a correct mathematical representation, had a sufficient physical foundation in the attraction without the addition of the harmonic vortex.[5]

As a matter of fact, Aiton's observation is similar to Huygens': the harmonic hypothesis is useless for the theory,[6] which is certainly true if the aim is a mere mathematical analysis of the paracentric motion. However, from a conceptual point of view the harmonic motion has an important role because it allowed Leibniz to prove the area law without resorting to the immediate action at a distance of a centripetal force. On the other hand, to admit a harmonic circulation means, essentially, to postulate, not to prove, the area law. The situation looks like this: Leibniz was going to provide a theory which described the real structure and functioning of the solar system, not only a kinematical and dynamical model, but a very physical-structural theory. The harmonic vortex has a fundamental role because it describes something really existing, not exclusively a model. Leibniz preferred to sacrifice the empirical content of his theory—because he almost postulated the area law—rather than to admit a Newtonian force, for which no mechanical support had been given. It is necessary to add that a further problem exists: Leibniz condemned the action at a distance and every action which should be immediately transmitted without respecting the principle of continuity. But, if one reflects on the way Leibniz imagined the harmonic motion in the planetary ellipses, one discovers a problem similar to the immediate action (even though not at a distance): we have seen that every circumference of infinitesimal thickness of the aethereal vortex included between aphelion and perihelion moves harmonically and that the planet, while moving from a circumference C to another D assumes *immediately* the motion of D without retaining the one of C. But this is exactly an action which is immediate, though by contact. The principle of continuity is not respected because the motion should instantaneously lose its previous properties. Not only: this immediate action should take place in every instant because the planet changes its distance from the sun in every instant and hence, in a finite time, there should be $2^{\aleph_0}$—to use a Cantorian language—immediate adaptions of the planet to its new condition of harmonic motion. Every point of the space-

[4] Aiton (1964, p. 112).

[5] Aiton (1972, p. 136).

[6] Huygens' and Aiton's aims are, however, different, which is obvious: Huygens seems to invite Leibniz to abandon the harmonic circulation, while Aiton has the intention to prove that the mathematical treatment of the paracentric motion is independent of harmonic circulation.

temporal *continuum* would represent a point of discontinuity in the motion of the planet. Leibniz was against an immediate action in physics, also considering the action with contact: his well known ideas on the collisions—which, according to him, can never be considered as if they took place among perfectly hard bodies—and his oppositions to the existence of the atoms are, in great part, based exactly on the refusal of an immediate action, which changes the condition of the bodies-motion. Whereas the elliptic harmonical circulation of the planets needed more than a denumerable infinity of these immediate changes in a finite time. It seems difficult to conceive a physical mechanism which allows a body to completely cancel its preceding motion-state, at least as far as the transversal direction is concerned and Leibniz was absolutely clear that this is a property of the harmonic circulation shared with no other kind of motion. For, he wrote to Huygens in 1690:

> And the body itself is moved in the aether, as if it tranquilly navigated, without either impetuosity or residue of the preceding impressions. The body only obeys to the aether, which surrounds it. [...] But in each other circulation, excluded that harmonic, the bodies maintain the preceding impression.[7]

Therefore, from a logical point of view the fact that a body does not retain any data of its preceding physical state seems to be in conflict with Leibniz refusal of an immediate action and with his principle of continuity; from a physical standpoint, the one described is a mechanism which is difficult to conceive. Anyway, the harmonic vortices aimed at: a) supplying the real structure of the solar system; b) offering an alternative to Newton's model; c) avoiding the action at a distance.

2. The kind of velocity, of which Leibniz was speaking about while referring to the *velocitas circulandi*.

   There is no doubt after Aiton's contributions: he was considering a reference frame in polar coordinates, whose pole is in the sun and the model applied is that of the rotating-ruler plus paracentric motion. Leibniz considers the situation from the perspective of the rotating planet and analyses, in every moment, the planetary movement in terms of the physical quantities experienced by the planet. The *velocitas circulandi* is the transverse velocity. Under the condition that such a velocity is harmonic, it is trivial to prove that (in modern terms) the angular moment—even though this is not a concept, to which Leibniz explicitly referred—is conserved and that, which is equivalent, the areolar velocity is a constant of the motion, that is the area law.

[7] Leibniz (1690a, 1860, 1962, VI, pp. 189–190). This is a letter written in October 1690 and edited by Gerhardt in *Ivi*, pp. 187–193. This letter was never sent to Huygens. On this see Aiton (1964, p. 114, note 16). Original French text: "El le même corps aussi est mû dans l'ether comme s'il y nageoit tranquillement sans avoir aucune impetuosité propre, *ny aucun reste des impressions precedentes*, et ne faisoit qu'obeïr absolument à l'ether qui l'environne [...] Mais quelque autre circulation qu'on suppose hors l'harmonique, le corps gardant l'impression precedente [...]".

A confirmation that the *velocitas circulandi* is the transverse velocity is given by the above mentioned letter to Huygens, where Leibniz wrote that, if we compare velocities' modules of the different planets in their orbits, then they are as square root of the distance (as Newton had proved in *Principia*, I, prop. IV, cor. 6), but if we consider a single planet in its orbit, then in the different points of the orbit, the *velocitas circulandi* is as the inverse distance from the sun, which supplies the area law. Thence there is no contradiction between the two assertions because they are referred to different kinds of velocity. Leibniz is clear, for he wrote:

> Perhaps, Mister, you will immediately say that the hypothesis of the squares of the velocities equal to the reciprocal of the distaces is not in agreement with the harmonic circulation. But I answer that the harmonic circulation is valid for each singular body, if ones compares its different distances [from the sun], but the harmonic circulation *in potentia* (where the squares of velocities are reciprocal to the distances) is valid when one compares the different bodies, both in the cases in which they describe a circular line, or when one considers their mean movement [. . .] for the circular orbit they describe.[8]

2.2 The *Motus Paracentricus* and Its Properties

2.2.1 Leibniz's Assertions

The *circulatio harmonica* provides the mean motion of the planets, while the *motus paracentricus* is the motion of approaching and moving away of the planet from centre of gravity along the radius-vector. It is the radial motion. The paracentric motion is due to two opposite tendencies: 1) the *impressio excussoria circulationis*; 2) the *attractio solaris* (*ivi*, par. 9, p. 152).

The *impressio excussoria circulationis* (translated by Bertoloni Meli as "outward impression of the circulation", p. 132) is the centrifugal force due to the harmonic circulation. The centrifugal force tends outwards. Leibniz's problem is to find a geometrical representation and an analytical expression for the instantaneous centrifugal acceleration that he called *conatus centrifugus* or *conatus excussorius circulationis*. In paragraph 10 and 11 of the *Tentamen* Leibniz solved the problem to find a geometrical representation of the *conatus centrifugus*. For, he wrote:

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[8] *Ivi*, p. 192. See also Aiton (1964, pp. 113–115). Original French text: "Vous dirés peutestre d'abord, Monsieur que l'hypothese de quarrés des vistesses reciproques aux distances ne s'accorde pas avec la circulation harmonique. Mais la réponse ast aisée: la circulation harmonique se rencontre dans châque corps a part, comparant les distances differentes qu'il a, mais la circulation harmonique en puissance (où le quarrés des velocités sont reciproques aux distances) se rencontre en comparant des differens corps, soit qu'ils décrivent une ligne circulaire, ou qu'on prenne leur moyen movement [. . .] pour l'orbe circulaire qu'ils décrivent".

> This conatus can be measured by the perpendicular from the following point to the tangent at the inassignably distant preceding point.[9]

This means (*Tentamen*, par. 11) that the *conatus excussorius* can be represented by PN (see Fig. 2.1b), namely the versed sine of the angle of circulation $M_1\Theta N$. For, the versed sine—Leibniz continues—"is equal to the perpendicular drawn from one end-point of the arc of a circle to the tangent from the other end-point".[10] The versed sine can be identified with D_1T_1, the inassignable difference between two infinitely near radii-vector. This means that, in general, the *conatus escussorius* can be represented by segments of the type D_iT_i, for every position of the radius vector. It is then easy to prove that the *conatus centrifugus* is equal to PV.[11]

Leibniz is here imagining the trajectory as composed of an infinite number of infinitesimal circular arcs whose radii have infinitesimal differences and are all centrated in the sun. Given this situation, the infinitesimal arcs of circumference can be considered as sides of a polygon. In the commentaries we will see that the consideration of the trajectory as composed of infinitesimal arcs or of infinitesimal sides of a polygon implies a problem as to the concept of tangent, with the consequence that Leibniz wrongly added a factor 2. This mistake did not have remarkable effects on the coherence of Leibniz's theory.

With regard to the analytical expression of the *conatus centrifugus*, if the motion is circular and uniform, than the *conatus* is as V^2, where V is the transverse velocity, since the versed sine is proportional to the square of the chord and the transverse velocity is proportional to the chord. If two or more circles are considered in which the movement is uniform, then the *conatus* are as V^2/R, where R is the radius. From this expression for the centrifugal force, Leibniz deduced another expression which is fundamental in his reasoning: if a body moves with a harmonic circulation, the *conatus centrifugus* is inversely proportional to the radius vector. This happens because of the inverse proportion between transverse velocity and radius vector in the *circulatio harmonica* and because of the relation $c = V^2/R$, where c is the centrifugal conate. From here another expression is possible: Leibniz considered a fixed elementary area, completed by the radius-vector in an infinitesimal time dt (the area law is valid), which he indicated by ϑa and assumed it equal to the double of the elementary triangle $M_2M_3\Theta$, namely equal to $D_2M_3 \cdot \Theta M_2$. The expressions ΘM_n can be indicated by $r =$ radius, because the difference between ΘM_i and ΘM_{i-1} is an infinitesimal, which can be neglected in this calculation. Therefore $D_2M_3 = \vartheta a/r$ and the centrifugal conate $D_2T_2 = (D_2M_3)^2/2\Theta M_3$. Thus, in conclusion

[9] Translation drawn from Bertoloni Meli (1993, p. 132). Original Latin text: "Hunc conatum metiri licebit perpendiculari ex puncto seguenti in tangentem puncti praecedentis inassignabiliter distantis." (Leibniz 1689, 1860, 1962, VI, p. 152).

[10] Translation drawn from Bertoloni Meli (1993, p. 133). Original latin text: " [...] aequatur perpendiculari ex uno extremo arcus circuli puncto in tangentem alterius ductae [...]." (Leibniz 1689, 1860, 1962, VI, p. 153).

[11] See Leibniz (1689, 1860, 1962, VI, paragraph 11, p. 153).

$$D_2T_2 = \frac{\vartheta^2 a^2}{2r^{3.}}$$

That is: the centrifugal conate is as the inverse of the radius-cube.

This means that Leibniz is considering a non-inertial reference frame in polar coordinates, whose origin is posed in the rotating planet. From the point of view of the planet, the acceleration along the radius is given by two components: one outwards, which is the *conatus centrifugus* due to the harmonic circulation; the other one is due to gravity or levity. Leibniz thought that this second component can be either inwards (gravity), which is the normal experienced case, or outwards (levity), which is a theoretical case. The acceleration along the radius is the algebraic sum of the two components, which is an arithmetical difference in case of gravity and an arithmetical sum in case of levity. Considering the case of gravity, if the *conatus centrifugus* prevails,[12] the radial acceleration is directed outwards. While, if the *solicitatio gravitatis* prevails, the radial acceleration is directed inwards.

We have seen how Leibniz represented the *conatus centrifugus*. As to the *solicitatio gravitatis*, Leibniz claimed:

> Paracentric solicitation, whether of gravity or levity is expressed by the straight line M_3L drawn from the point M_3 of the curve to the tangent M_2L (produced to L), of the preceding inassignably distant point M_2 parallel to the preceding radius ΘM_2 (drawn from the centre to the preceding point M_2).[13]

Leibniz imagined hence that, given an infinitesimal arc M_1M_2, which can be approximeted by its chord, the inertial motion of a body moving in such an arc can be approximated by the prolongation of the chord (the *tangent* in the Leibnizian sense) rather than by the Euclidean *tangent* (on this, see the following commentaries) without a detectable mistake. This kind of representation, as well as the idea that the trajectory can be considered a polygon with infinitesimal sides, is evidently the same as the one used by Newton in the proposition I of the first book of his *Principia*.

The section of the *Tentamen*, which concludes the part concerning the general properties of the paracentric motion is the 15th paragraph, where Leibniz determined geometrically the element of the *impetus paracentricus*, that is the instantaneous acceleration along the radius. He claimed that in every harmonic circulation the element of *impetus paracentricus* is the difference or the sum of the paracentric

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[12] Leibniz wrote "[...] differentia vel summa solicitationis paracentricae [...] et *dupli* conatus centrifugi [...]" (my italics, Leibniz 1689, 1860, 1962, VI, p. 154), referring to the double centrifugal conate and not to the simple centrifugal conate. This is a mistake highlighted by Varignon. For an explanation see next Sect. 2.2.2. *Commentaries*.

[13] Translation drawn from Bertoloni Meli (1993, p. 134). Original Latin: "Solicitatio paracentrica, seu gravitatis vel levitatis exprimitur recta M_3L ex puncto curvae M_3 in puncti praecedentis inassignabiliter distantis M_2 tangentem M_2L (productam in L) acta, radio praecedenti ΘM_2 (ex centro Θ in punctum precedens M_2 ducto) parallela". (Leibniz 1689, 1860, 1962, VI, p. 154).

solicitation and of the double centrifugal conatus. We refer to Leibniz reasoning because:

A) it is an example of what one could call *infinitesimal geometry applied to physics*, that is both the finite and the infinitesimal quantities are represented by means of geometrical constructions and, at least in this paragraph, there is not a transcription into analytical terms;
B) it is an example which clearly shows the use of differentials of different degree in a geometrical context (for more details see the next commentaries).

Leibniz reasoned like this:

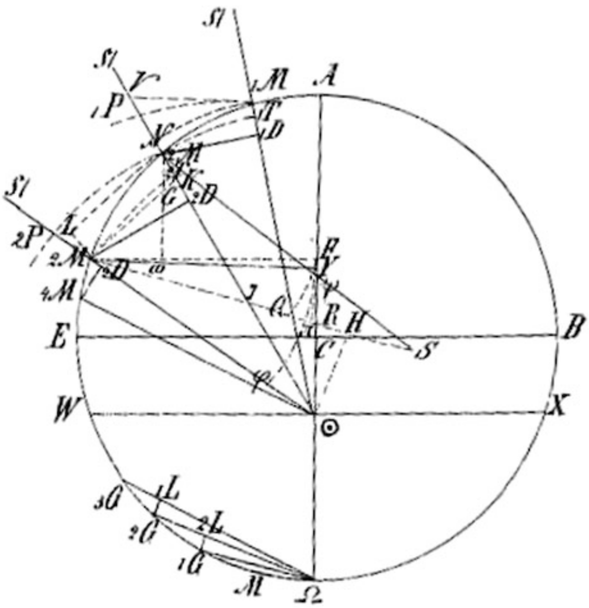

Fig. 2.2 Enlarged imagine of Leibniz's planetary theory-figure. I offer here the reader an enlarged imagine of Leibniz's planetary theory. The imagine is the same as Fig. 2.1a. I present this imagine because in Aiton's the point G is not represented, while it is quite important in the context I am dealing with. I hope this imagine can help the reader to follow the mathematical reasoning developed in the running text. Let us remind the reader that the symbol $_2M$ near $_4M$ has to be replaced with $_3M$ (in my running text M_3)

1. let M_1N and M_3D_2 be the perpendiculars from M_1 and M_3 to ΘM_2.
2. The circulation is harmonic, hence the triangles $M_1M_2\Theta$ and $M_2M_3\Theta$ are congruent. Therefore their altitudes M_1N and M_3D_2 are congruent.
3. Let M_2G be congruent to LM_3 and M_3G parallel to M_2L.
4. The triangles M_1NM_2 and M_3D_2G are congruent.[14] Therefore it is $M_1M_2 = GM_3$ and $NM_2 = GD_2$.
5. Let us assume $\Theta P = \Theta M_1$ and $\Theta T_2 = \Theta M_3$, so.

[14] I remind the reader that the two triangles are congruent because: a) $M_3D_2 = M_1N$; b) they are right triangles; c) For the angles the following identities are valid: $M_1M_2N = D_2M_2L$ and $D_2M_2L = D_2GM_3$, because of the parallels M_3G and M_2L. Thus, $M_1M_2N = D_2GM_3$. Hence, the thesis follows.

6. $PM_2 = \Theta M_1 - \Theta M_2$ and $T_2M_2 = \Theta M_2 - \Theta M_3$.
7. $PM_2 (= NM_2) = GD_2 + NP$ and $T_2M_3 = M_2G + GD_2 - D_2T_2$. Hence.
8. $PM_2 - T_2M_2 = NP + D_2T_2 - M_2G$. But.
9. $NP = D_2T_2$ because they are the versed sines of two angles and radii whose differences are inassignable. Hence.
10. $PM_2 - T_2M_2 = 2D_2T_2 - M_2G$.
11. The difference of the radii expresses the paracentric velocity; the difference of the differences expresses the element of the paracentric velocity (that is the paracentric acceleration). But D_2T_2 or NP is the centrifugal conatus of circulation and M_2G or M_3G is the paracentric solicitation. This proves the theorem.

In this demonstration: the segments, one extremum of which is the centre of gravity Θ are finite; all other elements used in the proof are infinitesimal. The quantities P_2M_2 and T_2M_2 are first differences and represent the instantaneous radial velocity; their difference $PM_2 - T_2M_2$ is a second difference and represents the radial instantaneous acceleration.

With this demonstration, Leibniz completed the description and the explanation of the basic elements of his theory. He then applied these elements to the case of the elliptical orbits, the ones which are relevant for the planetary motions. In particular: at the moment Leibniz has been able to determine both a geometrical and an algebraic-analytical form with regard to the *conatus centrifugus*, while, for the *solicitatio paracentrica*, he has only given the geometrical form. His next step is to prove that such a solicitation is as the inverse of the square distance.

2.2.2 Commentaries

1. Relation between harmonic circulation and paracentric motion.

   Let us summarize the results obtained by Leibniz till the paragraph 17 of the *Tentamen*: Leibniz considered the situation from the point of view of an observer posed in the rotating planet, which is subject to three actions:

   1) the action due to the *circulatio harmonica*, which determines the transverse velocity of the planet;
   2) the centrifugal force due to the rotating vortex. In this case it is necessary to underline that the physical cause of the transverse velocity and of the centrifugal force is the same, that is the harmonic vortex, but, while the area law depends on the fact that the circulation of the vortex is harmonic so that the areal velocity is constant, the centrifugal force depends on the rotation, not on the fact that the rotation is harmonic;
   3) the solicitation of gravity or of levity. In the case of the solar system, the solicitation of gravity due to the sun. Centrifugal force plus solicitation of gravity provide the *paracentric motion*.

A brief physical explanation is maybe useful: from the point of view of an inertial reference frame, the so called centrifugal force is not a really existing force. However, from the point of view of the rotating observer the situation is different: for him the centrifugal force is a real force and depends on the rotation originated—according to Newton–by two physical quantities and situations:

A) *The centripetal force;*
B) *The initial conditions of the motion; basically the initial inertial velocity.*[15]

The conditions A) and B) determine the rotation of the planet and hence the intensity and the direction of the physical quantities in the rotating system, in particular, of the centrifugal force. The physicists call it *fictitious centrifugal force* and we can call *Leibnizian centrifugal force*. This force simply depends on the fact that a system is rotating, independently of the dynamical causes of the rotation, because the rotating observer experiences a centrifugal force in the case of planetary motion (and this, in Newtonian terms, depends on centripetal force plus initial velocity), but also, for example, in a roundabout, where no centripetal force exists. When the intensity of the centripetal force is equal to that of Leibnizian centrifugal force, then the motion is circular and uniform, otherwise it is not.

An explanation in modern terms can be useful for a complete understanding of Leibniz's reasoning. In a rotating reference frame the forces equation can be written, using polar coordinates like this:

$$F(r) = m\left(\left(\ddot{r} - r\dot{\theta}^2\right)\hat{\mathrm{r}}\right) + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\theta}) \tag{2.1}$$

where r is the variable radius vector, θ is the angular distance from an angular position of the radius vector assumed equal to 0, $\hat{r}$ is the radial versor and $\hat{\theta}$ is the versor in the direction perpendicular to $\hat{r}$. Since we are in a field of central forces, the transverse component of the acceleration $r\ddot{\theta} + 2\dot{r}\dot{\theta}$ is zero, the whole acceleration is radial and is expressed by the term $\ddot{r} - r\dot{\theta}^2$. Therefore if one wonders how the acceleration along the radius vector varies, one gets the equation

$$m\ddot{r} = F(r) + r\dot{\theta}^2 . \tag{2.2}$$

Since in a central field the angular moment $L = m\dot{\theta}r^2$ is conserved, Eq. (2.2) gets the form

[15] The explanation of the centrifugal force in terms of A) and B) could be called an inertial interpretation of a non-inertial reference frame. Historically, Leibniz did not resort to it. However, this explanation is useful to catch the situation from a physical point of view and to better understand the correct reasoning of Leibniz as to the centrifugal force.

$$m\ddot{r} = F(r) + \frac{L^2}{mr^3}. \quad (2.3)$$

The term $\frac{L^2}{mr^3}$ is called *centrifugal force*. We have seen that the centrifugal conate is expressed by Leibniz as $D_2T_2 = \frac{\vartheta^2 a^2}{2r^3}$. Since ϑa represents an infinitesimal area, it can be indicated by dA. In modern terms the relation between the infinitesimal area swept by the radius vector and the angular moment is $L = 2m\frac{dA}{dt}$. If one does not take into account the constant factor m and considers (so to say)—as Leibniz did—a unitary infinitesimal time, then the relation becomes $L = 2dA$. Therefore, if we exclude a constant factor 8, Leibniz's result is perfectly correct.[16] This is an important and new result in history of physics. Let us add that, if in Eq. (2.3), we consider *F(r)* acting as gravity acceleration, namely as $-\frac{1}{r^2}$, one gets exactly the situation taken into account by Leibniz.

The structure in terms of forces is now complete, as to its fundamental elements. Leibniz had still to determine the specific expression of the solicitation of gravity. With regard to the physical structure of the world, the harmonic vortex produces the first two actions; as to the mechanical cause of gravity, Leibniz—as we will see—faced the problem in various works, but in the *Tentamen* the question is merely outlined, hence, for the moment, I will not deal with it. The examination of the paracentric motion along the radius vector is basically correct and—from the standpoint of history of physics—is an important contribution. It is however significant that Newton criticized[17] the way in which Leibniz presented the centrifugal force. For Newton wrote, speaking in third person:

> Eleventh proposition of the Tentamen: the centrifugal conate can be expressed by means of circulation angle's versed sine. This proposition is true, when the circulation takes place in a circle, without the paracentric motion. But when the movement takes place in an eccentric orbit, the proposition is not true. The centrifugal conate is always equal to gravity and is directed in the opposite direction, according to the third law of motion of Newton's Principia Mathematica, and the force of gravity cannot be expressed by the versed sine of circulation's angle, but it is reciprocal to the distance square.[18]

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[16] For a slightly different explanation of this result by Leibniz, see Aiton (1960, pp. 61–62; 1964, pp. 117–121).

[17] The documents in which Newton and Keill criticized Leibniz are three: 1) Newton's writing titled "Epistola cujusdam ad amicum", published in Edleston 1850. Edleston claims that, probably this letter was written in 1712; 2) a second document sent by Newton to Keill and titled "Notae in Acta Eruditorum an. 89 p. 84 et sequ", available in the University Library of Cambridge, Add. MS 3985 f. 6; 3) the only published work on this question, that is Keill (1714). Keill's work is almost completely based upon Newton's ideas. For a complete report on these critics, see Aiton (1962).

[18] Newton in Edleston 1850, p. 311. Original latin text: "Undecima Tentaminis Propositio est haec: *Conatus centrifugus exprimi potest per sinum versum anguli circulationis*. Et vera quidem est haec propositio ubi circulatio fit in circulo sine motu paracentrico. Sed ubi fit in Orbe excentrico propositio vera non est. Conatus centrifugus semper equalis est vi gravitatis et in contrarias partes dirigitur per tertiam motus Legem in Principiis Mathematicis Newtoni, et vis gravitatis esprimi

And again:

> Propositions 20th (*sic*) and 25th are false, because they show a centrifugal force which is less than planet's gravity towards the sun. Therefore they are false. The motion of a planet in its orbit does not depend on the excess of gravity upon centrifugal force (as Leibniz believes), but the orbit is incurved only by gravity's action, to which the centrifugal force (as reaction or resistance) is always equal and opposed, as to the direction, according the third law posed by Newton.[19]

The situation is like this: Newton believes that the centrifugal force is a mere reaction to the centripetal force, which is the real force acting on the planets. This is in agreement with the third law. Considering the question under this perspective, one could claim that Newton did not correctly understand Leibniz's way of reasoning, in particular the fact that Leibniz was looking at the situation from the point of view of the rotating planet. This is probably part of the truth. The other part of the truth is that, likely, in Newton's eyes the whole *Tentamen* seemed something odd. We will deal with this general question in the fourth section of this book, while analysing the final version of Leibniz's planetary theory written in 1706, after David Gregory's critics in 1702.[20]

Anyway, according to Leibniz's aims and way of thinking, the correct expression for the movement along the radius vector is an instrument in his hands to present his system of the world. If he had considered such an examination just as a contribution to mathematical-physics, it would have been only a different presentation of results already obtained by Newton—although Newton did not recognize this point—, it would have been something like "some new points of view in Newtonian physics", not certainly a new system of the world alternative to Newton's, whereas Leibniz intended to construct such a system. Because of this it is necessary to follow the way in which Leibniz continued to construct his planetary theory.

2. The concept of tangent and the second order differences.

   In the item 4) of Leibniz's demonstration, the triangles M_1NM_2 and M_3D_2G are congruent, so $NM_2 = GD_2$. Newton criticized this assertion by Leibniz[21]: if M_2L is the Euclidean tangent in the point M_2, the direction is not the same as M_1M_2, therefore GM_3 is not parallel to M_1M_2 and the triangles M_1NM_2 and M_3D_2G are not congruent, hence NM_2 is not equal to GD_2. Aiton provides a

non potest per sinum versum anguli circulationis, sed est reciproce ut quadratum radii". Italics in the text.

19 Ivi, p. 313. Original latin text: "Propositio vigesima (*sic*) prima et vigesima quinta, minorem exhibent vim centrifugam quam gravitatem Planetae in Solem ideoq: falsae sunt. Motus Planetae in orbe non pendet ab excessu gravitatis supra vim centrifugam (ut credit Leibnitius) sed Orbis incurvatur a gravitatis actione sola, cui vis centrifuga (ut reactio vel resistentia) semper est equalis et contraria per motus Legem tertiam a Newtono positam".

20 See Gregory (1702, pp. 99–104).

21 Newton in Edleston 1850, p. 312.

different interpretation[22]: the segment M_2L is not the Euclidean tangent, but the prolongation of the chord M_1M_2, that is the model presented by Leibniz is the "polygonal model", in which the trajectory is interpreted as composed of a polygon with infinitesimal sides.[23] This interpretation is surely the correct one, taking into account that Leibniz in the *Illustratio Tentaminis* explicitly claimed:

> Furthermore, in general, let us consider (Fig. 31) two sides M_1M_2 and M_2M_3 of the polygon which constitutes the curve, and let us prolong one of them, M_1M_2, till L, so that the straight line M_2L represents the velocity, with which the mobile tends to continue its motion along the same line, after having passed through M_1M_2.[24]

Therefore Aiton's interpretation is correct and no mistake is present in this mathematical reasoning by Leibniz.

A further question, connected to the preceding one, concerns the calculation of the centrifugal force: Varignon calculated the centrifugal force according to the concept of Leibniz's tangent and discovered that its value is double that computed by Leibniz. He wrote to Leibniz on 6 December 1704.[25] Leibniz corrected the mistake and expressed his gratitude to Varignon for having discovered and communicated the mistake to him. In paragraph 12 of the *Illustratio Tentaminis*, Leibniz highlighted all the occurrences[26] of the *Tentamen* in which the expression double *conatus centrifugus* has to be replaced with *conatus centrifugus*.

Newton and the Newtonians also criticized Leibniz for the problem of second order differences: Newton and Keill objected that Leibniz's assumption, according to which NP and D_2T_2 are equal (assumption 9) is not correct because

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[22] Aiton (1962, p. 37; 1964, pp. 119, 120; 1972, pp. 138–142), where the most clear explanation is provided. See also Bertoloni Meli (1993, p. 188).

[23] In his work *Nova Methodus pro Maximis et Minimis, itemque tangentibus [...]* (see Leibniz 1684, 1858, 1962, V, p. 223), Leibniz explicitly claimed that the tangent can be considered as the ordinary Euclidean tangent or as the prolongation of the side of the infinitangular polygon which can be thought as equivalent to the curve, at least as far as some mathematical considerations are concerned. For, Leibniz wrote: "to find the tangent is to draw the straight line which joins two points of a curve, whose distance is infinitely small, or the prolonged side of the infinitangular polygon, which, for us, is equivalent to the curve". Original Latin text: "[...]*tangentem* invenire esse rectam ducere, quae duo curvae puncta distantiam infinite parvam habentia jungat, seu latus productum polygoni infinitanguli, quod nobis *curvae* equivalet." (I am grateful to Professor Dr. Eberhard Knobloch for this indication). In the case I am analysing, the two representations of the tangent as ordinary tangent or as prolongation of the infinitangular polygon, are not equivalent as the mathematical consequences are different, according to which representation one uses. However: Leibniz had already spoken of the two representations, as the mentioned passage confirms, hence this makes Aiton's interpretation quite plausible.

[24] Leibniz (1706, 1860, 1962, VI, p. 261). Original latin text: "Porro generatim concipiendo (fig. 31) duo Latera polygon curvam constituentis M_1M_2 et M_2M_3, et unum ex illis M_1M_2 continuando in L ita, ut recta M_2L celeritatem repraesentet, quo mobile post percursam M_1M_2 in eadem recta pergere tendit[...]".

[25] Varignon to Leibniz 6 December 1704 in Leibniz (1859, 1962, IV, pp. 113–127).

[26] Leibniz (1706, 1860, 1962, VI, pp. 264–266).

the two segments differ by a second order infinitesimal and, in the context dealt with by Leibniz, where second differences are taken into account, an error of a second order infinitesimal is not acceptable. Aiton has shown that such a mistake does not exist in Leibniz's theory, if one interprets the word *tangent* as *prolongation of the chord* and that the mistake is a third order infinitesimal. We refer to Aiton's works for this problem.[27]

2.3 Elliptical Motion and Inverse Square Law

2.3.1 Leibniz's Assertions

The two paragraphs of the *Tentamen* in which Leibniz faced the motion on an ellipsis, where both the centres of the harmonic circulation and of the gravitational attraction are in the same focus, are the 18th and the 19th. The form in which Leibniz expounded the results is quite different in the published *Tentamen* and in the unpublished *Zweite Bearteitung* because, in this second work, he added the complete demonstrations of his propositions and a series of further mathematical propositions which allowed him to reach interesting astronomical results, whereas in the published version the demonstrations are only outlined and many results are missing. The literature, whose aim has been to provide the general ideas behind Leibniz's planetary theory and the analysis of the problems connected with Huygens', Newton's and Newtonians' critics, has underestimated the importance of the specific contributions expounded in the *Zweite Bearbeitung*.[28] I will face the results and methods of proof explained in this work, because all the results of the *Tentamen* are included here together with further ones.

Leibniz (see, Fig. 2.3) reminded the reader that the velocity of circulation (transverse velocity) can be expressed by the segments T_2M_3 or D_2M_3, since the difference between these two segments is negligible. The paracentric (radial) velocity is expressed by means of D_2M_2 and the velocity of the body in the orbit, which, Leibniz underlined, is composed of the two, by the segment M_2M_3 (*ivi*, par. 18, p. 172).

[27] Aiton (1962, p. 39; 1972, pp. 144–145).

[28] Up to now, the most complete report of the *Zweite Bearbeitung* is in Bertoloni Meli (1993, pp. 155–161).

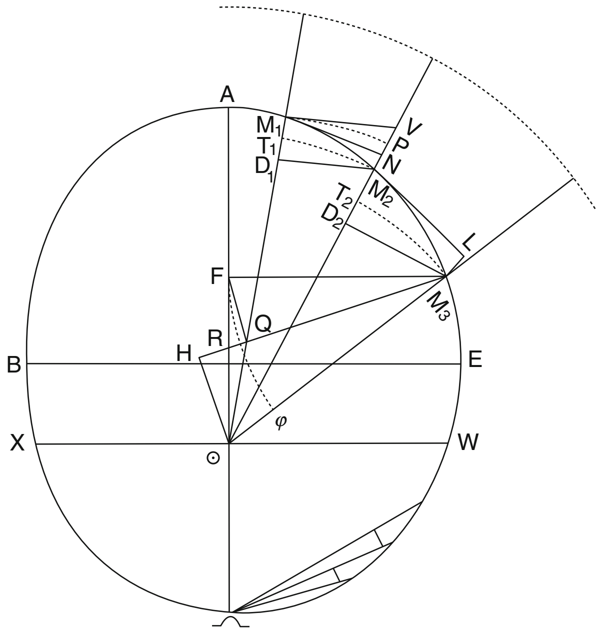


Fig. 2.3 Enlarged view of the Fig. 2.1b. I propose here an enlarged view of the figure Fig. 2.1b, because it can facilitate the reader to follow Leibniz's long reasoning, of which all the steps are explained in the running text.

Leibniz' reasoning (*ivi*, par. 18, pp. 172–174) is developed as follows:
for the previous segments, which represent the three velocities, the proportion

$$D_2M_3 : D_2M_2 : M_2M_3 = BE : \sqrt{(F\Theta + \Theta\varphi)(F\Theta - \Theta\varphi)} : 2\sqrt{\Theta M_3 \cdot FM_3} \quad (2.4a)$$

holds, where F is the focus of the ellipsis in which there is not the sun and $FM_3 = \varphi M_3$.

Leibniz proved easily that the following proportion holds:

$$D_2M_3 : D_2M_2 : M_2M_3 = M_3H : H\Theta : \Theta M_3 \quad (2.4b)$$

where M_3H is the perpendicular to the ellipsis in M_3 and FQ and ΘH the perpendiculars from the foci to M_3H.

Therefore he has to prove

1)
$$M_3H : H\Theta : \Theta M_3 = BE : \sqrt{(F\Theta + \Theta\varphi)(F\Theta - \Theta\varphi)} : 2\sqrt{\Theta M_3 \cdot FM_3}$$

Since M_3H is perpendicular to M_2M_3 (ellipsis' arc), that is to its tangent, in M_3, then the angles HM_3F and $HM_3\Theta$ are equal, as follows from the properties of the tangents to the ellipsis, thence

2) the triangles $M_3H\Theta$ and M_3QF are similar and the angle ΘM_3F is bisected by M_3H.

Therefore, if M_3H saws ΘF in R, it holds, from a theorem of elementary geometry

3) $\Theta R : FR = M_3\Theta : M_3F$;
4) the triangles ΘHR and FQR are similar, hence
5) their homologous sides are as $\Theta R : FR$, that is, from 3), as $M_3\Theta : M_3F$ and hence as the homologous sides of the similar triangles $M_3H\Theta$ and M_3QF.
6) $M_3\Theta + M_3F = A\Omega$ because the figure is an ellipsis.
7) Let $M_3\Theta - M_3F = \Theta\varphi$.
8) from the properties of the ellipsis it is: $A\Omega^2 - \Theta F^2 = EB^2 = A\Omega \cdot XW$, where XW is the *latus rectum*.
9) (my addition) given a triangle abc, let l_a be the bisectrix of the angle in A, it is known that its measure is $l_a = \frac{2\sqrt{bc\,p(p-a)}}{b+c}$, where p is the half-perimeter. This expression can also be written as $\frac{\sqrt{bc\left[(b+c)^2-a^2\right]}}{b+c}$, from which the proportion $l_a : \sqrt{(b+c)^2 - a^2} = \sqrt{bc} : (b+c)$ follows. Leibniz applied this proportion to the triangle $FM_3\Theta$, considering the bisectrix M_3R. Therefore he could write:
10) $M_3R : \sqrt{A\Omega^2 - \Theta\varphi^2} = \sqrt{M_3\Omega \cdot M_3F} : A\Omega$. But, because of 8), $\sqrt{A\Omega^2 - \Theta\varphi^2} = BE$ and, elevating to square the relations 6) and 7), and subtracting the results of 7) from that of 6), one gets $M_3\Theta \cdot M_3F = \frac{1}{4}\left(A\Omega^2 - \Theta\varphi^2\right)$, so that Leibniz can obtain the proportion $M_3R : BE = \frac{1}{2}\sqrt{A\Omega^2 - \Theta\varphi^2} : A\Omega$.
11) $$M_3R(\Theta H + QF) = 2\text{area}\left(\Theta \overset{\Delta}{M_3} F\right)$$
12) $\frac{1}{2}\sqrt{A\Omega^2 - \Theta F^2} \cdot \sqrt{\Theta F^2 - \Theta\varphi^2} = 2\text{area}\left(\Theta \overset{\Delta}{M_3} F\right)$, because of the Heron-formula applied at the triangle ΘM_3F, hence:
13) $M_3R(\Theta H + QF) = \frac{1}{2}\sqrt{A\Omega^2 - \Theta F^2} \cdot \sqrt{\Theta F^2 - \Theta\varphi^2}$, which can be written $M_3R : BE = \frac{1}{2}\sqrt{\Theta F^2 - \Theta\varphi^2} : (\Theta H + QF)$.
14) From 10) and 13) one gets $\sqrt{A\Omega^2 - \Theta\varphi^2} : A\Omega = \sqrt{\Theta F^2 - \Theta\varphi^2} : (\Theta H + QF)$, which can, obviously, be written as $(\Theta H + QF) : A\Omega = \sqrt{\Theta F^2 - \Theta\varphi^2} : \sqrt{A\Omega^2 - \Theta\varphi^2}$.
15) Applying 5) one has: $(\Theta H + QF) : (M_3\Theta + M_3F) = \Theta H : M_3\Theta$. But $M_3\Theta + M_3F = A\Omega$, hence from 14) and 15), Leibniz obtained
16) $\Theta H : M_3\Theta = \sqrt{\Theta F^2 - \Theta\varphi^2} : \sqrt{A\Omega^2 - \Theta\varphi^2}$ and elevating to square and subtracting
17) $\left(M_3\Theta^2 - \Theta H^2\right) : M_3\Theta^2 = \left(A\Omega^2 - \Theta F^2\right) : \left(A\Omega^2 - \Theta\varphi^2\right)$, that is $M_3H^2 : M_3\Theta^2 = BE^2 : \left(A\Omega^2 - \Theta\varphi^2\right)$. And finally, from 16) and 17), it follows
18) $$M_3H : \Theta H : M_3\Theta = BE : \sqrt{\Theta F^2 - \Theta\varphi^2} : \sqrt{A\Omega^2 - \Theta\varphi^2}.$$

At the conclusion of this reasoning, Leibniz can claim:

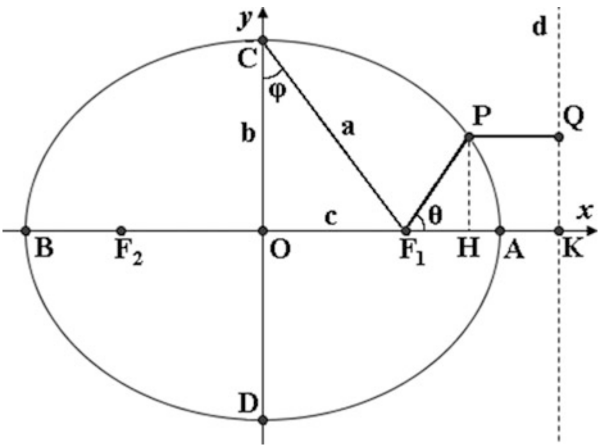
> If a body is moved in an ellipsis, the velocity of circulation around a focus is at the paracentric velocity, that is the velocity with which the body descends towards the focus, as the minor or transverse axis is at the square root of the difference between the square of the focal distance and the square of the difference of the mobile's distances from the two foci.[29]

From this proposition a series of corollaries follow, which describe important properties of the motion in an elliptical orbit in which the centre of the forces is in one of the foci.

The first corollary, which Leibniz deduces easily from the explained reasoning, is: in an ellipsis, given a point P, the ratio between the paracentric (radial) velocity and the velocity of circulation (transverse velocity) is proportional to the ordinate PH, that is: the ratio between the velocity with which the planet approaches to or recedes from the sun is to the velocity of circulation as the distance of the planet from the apses-line.[30]

[29] Leibniz (1790?, 1860, 1962, VI, p. 174). Original latin text: "Si quid moveatur in Ellipsi, velocitas circulandi circa focum est ad velocitatem paracentricam, nempe descendendi ad focum vel a foco recedendi, ut axis minor seu transversus est ad latus differentiae inter potestatem distantiae focorum inter se et potestatem differentiae distantiarum mobilis a focis". At the end of the quotation, Leibniz used the Euclidean language to indicate the segments. I have provided a modern translation of "[...] ad latus differentiae inter potestatem distantiae focorum inter se et potestatem differentiae distantiarum mobilis a foci". It is, obviously, possible to give a translation, which is more faithful to Euclid's tradition: "[...] at the side of the difference between the power of foci's distance and the power of the difference of mobile's distances from the foci".

[30] Leibniz (1790?, 1860, 1962, VI, p. 175). This is an important relation between the radial and transverse velocity, which, in modern terms, can be proved like this:



the radial velocity is $v_r = dr/dt$ and the transverse velocity is $v_\theta = r \cdot d\theta/dt$, therefore $v_r/v_\theta = dr/r \cdot d\theta$. Since the orbit is an ellipsis, its polar equation is $r = \frac{ed}{1+e\cos\theta}$, where e is the eccentricity and d is the distance F_1K of the focus F_1 from the directrix d. Differentiating

Two other corollaries proved by Leibniz are:

1) in an ellipsis, given the mobile point P, the ratio between the velocity in the orbit and the velocity of circulation is as the mean proportional between the distances of P from the foci. (This corollary is a direct consequence of 18).
2) The velocities, with which a point M_3 changes its distance from the minor axis BE, are as the velocities with which it changes its distance from the focus Θ.

All these corollaries are missing in the published version of the *Tentamen*. These sets of results show that Leibniz's knowledge of the kinematical aspects of the planetary motions were profound and that he was an original thinker, as to this subject.

Let us now consider how Leibniz approached the problem of determining gravity attraction. In this case, too, the difference between the published and the unpublished version of the *Tentamen* is conspicuous. In both contributions the following reasoning exists:

Positions (referring to Fig. 2.3):

a) $A\Omega = q$; b) $\Theta F = e$ (eccentricity); c) $BE = b$ (minor axis); d) $\Theta M_2 = r$ (radius vector); e) $\Theta\varphi = OM_2 - FM_3 = 2r - q = p$; f) $WX = a = b^2/q$ (latus rectum); g) double area element= $2M_1M_2\Theta = \vartheta a$, where ϑ is a constant element of time; h) D_2M_2 is the difference between two radii $= dr$; i) $ddr = d^2r$ second difference.

Reasoning:

1) D_2M_3 (=circulation)= $\vartheta a/r$ (for what was proved in paragraph 12);
2) $dr(= D_2M_2) : \vartheta a/r(= D_2M_3) = \sqrt{e^2 - p^2} : b$, for the proved theorem we have seen in details. Therefore
3) $br \cdot dr = \vartheta a\sqrt{e^2 - p^2}$. By differentiating, one gets the second order differences equation
4) $b \cdot dr^2 + br \cdot d^2r = -2pa\vartheta \cdot dr : \sqrt{e^2 - p^2}$. That is, replacing dr with its value deduced from 3), Leibniz got:
5) $d^2r = \left(b^2a^2\vartheta^2 - 2a^2qr\vartheta^2\right)/b^2r^3$.

But: d^2r is the element of paracentric velocity and the first expression in the right-hand member of the equation 5); furthermore $a^2\vartheta^2/r^3$ is the double *conatus centrifugus*. This means that the other expression represents the solicitation of gravity. Since $a = b^2/q$, such expression gets the form $2a\vartheta^2/r^2$. Leibniz multiplies this expression by the constant value $a/2$ and obtains $a^2\vartheta^2/r^2$, that is the square of the circulation. This means that the solicitation of gravity is as the square of the circulation, namely is as the inverse of the radius-square.

This concludes Leibniz's proof, which is explained both in the published and in the unpublished version of the *Tentamen*. However in the unpublished version Leibniz added a series of interesting considerations which do not exist in the

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this expression one gets $\frac{dr}{d\theta} = \frac{r^2 \sin\theta}{d}$; therefore $\frac{v_r}{v_\theta} = \frac{r\mathrm{sen}\theta}{d}$; but d is a constant and $r\mathrm{sen}\theta = PH$; this is the corollary of Leibniz (diagram drawn from www.fmboschetto.it/tde2/gravit4.htm).

published one. First of all he developed some remarks and clarifications as to the differential equation in 3). Actually, what is far more interesting from a physical point of view, is the following long observation, which includes almost three pages in the edition by Gerhardt (pp. 178–180): up to this moment, Leibniz provided a representation of the planetary motions using the concept of *conatus centrifugus*. However the *conatus centrifugus* is referred to the harmonic motion of the vortex, that is to a circular harmonic motion. In fact, the orbit is an ellipsis and the movement in the ellipsis is harmonic, too, as Leibniz underlined. This means that another *conatus centrifugus* exists which depends only *indirectly* on the harmonic circulation of the vortex responsible for the mean motion of the planet and *directly* form the elliptical harmonic circulation, that is, from the true orbit of the planet. To indicate this *conatus* Leibniz used the generic expression *conatus excussorius* (used, in the published version of the *Tentamen*, as a synonymous of *conatus centrifugus*, as we have seen), maintaining the expression *conatus centrifugus* only in the case in which the motion is circular. Since the *conatus excussorius* is not, in general, referred to a circular motion, but to every curvilinear motion, Leibniz was in the need to exploit the concept of osculating circle to get a representation of its, which is useful for a mathematical treatment. The aim of Leibniz is rather interesting: he wanted to prove that, even in the case one adopts the representation through the *conatus excussorius*, one obtains the inverse square law, though by different steps than those used while exploiting the concept of *conatus centrifugus*. In the commentaries, I will deal with the possible reasons which induced Leibniz to deal with two different approaches. Leibniz represented the *conatus excussorius* like this (see Fig. 2.2): he considered in the ellipsis two infinitely near points M_2 and M_3, he drew the perpendiculars to the curve in these two points and indicated by S their intersection. This is the centre of the osculating circle. He drew the straight line M_3G, parallel to the line which is the tangent at the ellipsis in M_2. This line saws perpendicularly M_2S in K. Considering M_2M_3 as an infinitesimal arc of the osculating circle and adopting the same representation for gravity and the *conatus excussorius-centrifugus* as that used up to now, one has that M_2G represents the solicitation of gravity and M_2K the *conatus excussorius*. During the proof, Leibniz demonstrated two interesting theorems as to the kinematics of the elliptical motion considering the osculating circle.[31]

[31] The two theorems which Leibniz proved and used to prove the inverse square law by means of the *conatus excussorius* are: 1) in every straight line the solicitation of gravity M_2G is at the excussorius conate M_2K as M_3G (that is M_2M_3, which is the element of the curve or the orbital velocity) is at the velocity of circulation M_3D_2 (Leibniz 1690?, 1860, 1962, VI, pp. 178–179); 2) in every line of motion, it is $M_2K = \frac{M_3K^2}{SM}$, namely, to tell *à la Leibniz*: the *conatus excussori* are as the duplicate ratio of the orbital velocities directly and the simple ratio of the radii of the osculating circle inversely (*Ivi*, p. 179).

2.3.2 Commentaries: Two Different Models for Planetary Theory

In the *Zweite Berbeitung* of the *Tentamen*, Leibniz proposes, as a matter of fact, two different models to prove the inverse square law:

a) the model already used in the published version, in which the orbit is imagined as a polygon composed of triangles, with one infinitesimal side (that, whose extrema are the points of the trajectory). The infinitesimal sides of all the triangles compose the polygon.
b) the model in which the osculating circle is used and where, so to say, the main point of the reasoning becomes the variable centre *S* of the osculating circle.

Both models are referred to rotating reference frames. Bertoloni Meli underlines that:

> The additions to paragraph 19 consist in an attempt of reformulating the demonstration of the equation of paracentric motion without the differential calculus.[32]

This is true. Anyway some further specifications seem to me necessary: the description of the model a) has two conceptual cores:

i) Leibniz provided the geometrical expressions of his physical—both finite and infinitesimal quantities—one could say *à la Newton*.[33]—;
ii) Calculus is used to differentiate the expression of *dr*, so to get *ddr* as a function of centrifugal force and gravitational attraction.

As Bertoloni Meli rightly highlights, in model b) calculus is not used and Leibniz underlined the difference between the methods a) and b), as he writes:

> "[...] exactly as previously, in this same article we had found our result by means of a different way, that is by resorting to our differential calculus and by the theorem proposed in the article 15.[34]

I think the reasons why Leibniz provided a different proof are three:

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[32] Bertoloni Meli (1993, p. 159).

[33] In Newton's *Principia*, one could speak of "infinitesimal geometry" because Newton needs the instantaneous physical quantities, but his resort to calculus is—at least *explicitly*—limited enough in his masterpiece. He provides geometrical demonstrations in which the infinitesimal segments and areas are described as part of a figure. Since in many cases these segments represent potentially infinite quantities, it is possible to speak of *infinitesimal geometry*. The literature on this subject is conspicuous. I provide here only five references in which the problem is faced and explained: Bussotti and Pisano (2014a), in particular pp. 35–37; Bussotti and Pisano (2014b), in particular p. 435; De Gandt (1995), Guicciardini (1998, 1999, 2009). Leibniz uses here a similar technique.

[34] Leibniz (1790?, 1860, 1962, VI, p. 180). Original latin text: "[...] prorsus ut antea in hoc ipso praesente articulo per viam diversam, nempe ope calculi nostri differentialis et theorematis articulo 15 propositi inveneramus".

1) the one indicated by Bertoloni Meli;
2) every great mathematician is pleased to offer different demonstrations of the same proposition. Strictly connected to our context, let us think of Newton's *Principia*, in which numerous propositions are proved in different manners;
3) this is maybe the most important reason: we have to remember that Leibniz had the intention to provide the real physical-structural system of the world, not just a dynamical model. The planet, in its orbit, as a matter of fact, feels the *conatus excussorius*, not the *conatus centrifugus* because its orbit is not a circumference. This means that the model expressed in terms of *conatus excussorius* is more adherent to the forces really experienced by the planet, although the two models are equivalent from a dynamical point of view. This is the reason why Leibniz felt the need to add these considerations on the *conatus excussorius*. This does not mean that the model of the infinitangular polygon cannot be applied to an eccentric path, too.

2.4 The Final Description of the Solar System in the *Tentamen*

2.4.1 Leibniz's Assertions

Leibniz explained the mean motion of a planet in its orbit as due to the constant transverse velocity of the harmonic aethereal vortex in which the planet is afloat and the deviations from the mean motion in terms of two opposite tendencies: the *conatus excussorius*/*centrifugus*; the *solicitation of gravity*. In the paragraph 27, he supplied a unified vision of his planetary system, also based on two corollaries expounded in the paragraphs 21 and 24. In the former Leibniz proved that the ratio between gravity and centrifugal conate (really the half of the centrifugal conate) are as the distance of the planet from the sun; in the latter that the greatest speed of approaching to or of receding from the sun occurs when the distance of the planet from the sun is equal to ½ *latus rectum* of the ellipsis. This speed is equal to 0 at aphelion and perihelion.

Leibniz summarized his results in this manner: at the aphelion A, gravity is stronger than double centrifugal conate (really centrifugal conate, not double) because of the corollary in paragraph 21, hence the planet approaches the Sun. The speed with which the planet approaches the sun gets a maximum in W (corollary in 24), here the double centrifugal conate (really the simple centrifugal conate) begins to prevail on gravity and the approaching speed diminishes till the perihelion Ω (see Fig. 2.3) where its value is 0 and after Ω, this value becomes negative, this means that the planets begins to recede from the sun till the point X, where the receding velocity has a maximum and where gravity begins to prevail on the double centrifugal conate (really the simple centrifugal conate); the planet continues to recede until the aphelion A, where the receding velocity is null and the cycle begins once again. This is the general mechanism through which the planets rotate around the sun.

Leibniz concluded (paragraph 30) that if the centrifugal conate (really ½ centrifugal conate) is equal to gravity, the trajectory is a parabola; if it is stronger, the trajectory is a hyperbola whose focus is between the sun and the focus of the parabola, if the attraction is an attraction of levity and not of gravity, then the planet is repelled from the sun along the opposite hyperbola.

2.4.2 Commentaries

The description of the planetary motions given by Leibniz in the two versions of the *Tentamen* has its conclusion in the described picture, in which the motion of approaching to or receding from the sun is described as due to the difference between the solar attraction and the centrifugal force, while the deviation from the rectilinear path is due to the harmonic vortex. From a physical point of view, the most interesting aspect is the use made by Leibniz of the initial radial velocities for a given time t. Leibniz is aware that for a time $t_1 > t$ the motion is given by the radial velocity at time t and by the forces acting on the body. For—as we have seen—he underlines that—starting from the aphelion—the approaching velocity of a planet has a maximum when the solar attraction is equal to the *conatus centrifugus*. However, in the moment in which the *conatus* begins to prevail, the velocity of approaching begins to diminish, but this does not mean that the planet begins to recede. This happens only when, at the time t_2, the prevailing *conatus* has produced an effect which is superior to the combined effect of the gravity and of the velocity, which is direct inwards until t_2. This is the case in the perihelion. Therefore Leibniz considered the velocity as an *initial instantaneous datum* for every instant t. This datum changes in every instant. Thence a constant datum as the initial velocity when the elliptic motion is described in terms of centripetal forces *à la Newton* does not exist in Leibniz's description. For every instant the initial velocity changes, but, in that instant, it has to be considered as an initial constant of the motion. It is necessary to highlight that the description of the curvilinear motion using a rotating reference frame is not in contradiction with Newton's work, even if Newton himself thought otherwise, as we have seen. It is a description which uses a different point of view, but there is no contradiction among the two.

However, if the description in kinematical and dynamical terms provided by Leibniz is coherent with Newton's, the situation completely changes when one analyses the physical-structural point of view. In particular: why did Leibniz feel the need to provide such a description of planetary motion? Which are Leibniz's physical convictions and how did they influence his planetary theory? What is the real value of such a theory and in which sense can it represent a real alternative to Newton's conception? Who are the authors who can be considered Leibniz's reference points? The answers to these questions are the subjects of the next chapters.

Chapter 3
An Interlude: Leibniz's Concept of Inertia

In the general context of physics and in the planetary theory, in our specific case, the tendency of a rotating body to recede along the tangent is a fundamental element. We have seen that paracentric motion depends on the two opposite tendencies due to the solicitation of gravity and to the conate to recede. In terms of Newtonian physics the latter is a consequence of the inertia principle. Although Leibniz considered the conate to recede as a pivotal physical feature of curvilinear motions, he never associated it to the word *inertia*. Leibniz spoke of *natural inertia*, but this has nothing to do with the tendency to escape along the tangent. More in general: the *natural inertia* in Leibniz is not Newton's inertia.[1] Therefore some questions arise:

1) what is exactly *natural inertia* for Leibniz?
2) is there, in Leibniz, a concept or a series of concepts which correspond/s to Newton's inertia? And, in the affirmative case, what is their role inside Leibniz theory? More in general: is Leibniz's physics a theory in which Newton's concept of inertia—or an equivalent one—plays a significant role? If the answer to this question is negative, what is then the status of the conate to recede, which actually, is so important in Leibniz's train of thoughts?

In this Chapter I will deal with these two items.

The principle of inertia is fundamental for classical physics. Thence the way in which Leibniz used the word "inertia" and the possible existence or the lack of

[1] It is possible to speak of Galilean inertia or Cartesian inertia or, without referring to the inventor of inertia principle, of rectilinear inertia. I prefer to use the expression Newtonian inertia, as a principle becomes really significant only when it is inserted in a general picture—a theory—where it plays a precise role in the architectonics of the theory and in its deductive structure. This happened with Newton's *Principia*. On this problem, I agree with Garber (see Garber 1992, pp. 200–204). Garber, even avoids using the expression "inertia principle" in reference to Descartes and writes: "[. . .] I have chosen to break the tradition and *not* to use the term 'inertia' in connection with Descartes' first two laws of motion" (*ivi*, p. 203).

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something as the inertia principle in his theory is a significant subject to catch the features of Leibniz's physics, inside which planetary theory is inscribed. Therefore, although this chapter is only in part connected to planetary theory, it can be useful to highlight an important feature regarding the context of which such a theory is a part. I have no claim to offer a final answer to the problem of inertia in Leibniz, but rather to clarify some aspects of this concept and the relation of Leibniz's natural inertia with Kepler's work, which is one of the purposes of my book.

In the literature, the works that are *in toto* or in part dedicated to the concept of inertia in Leibniz are numerous[2] and the subject is quite difficult and tangled because Leibniz was not always clear and because he introduced a conspicuous series of notions, which can be interpreted as connected to the concept of inertia.

Curiously enough Leibniz attributed to Kepler the merit for discovery of the receding tendency in curvilinear motion and for clarification of the natural inertia concept, but there is no explicit link between the two notions.

3.1 Leibniz and Natural Inertia

A premise: it is well known that Leibniz's ontology is stratified. There are many levels of reality, and although their laws and properties are connected, they are different. In the previous sections of this book I have distinguished the physics of Leibniz into three levels: 1) physical-structural; 2) dynamical; 3) kinematical. To deal with Leibniz's concept of natural inertia, it is necessary to refer to his metaphysics. I distinguish Leibniz's metaphysics into two levels:

A) *dynamical-metaphysical*, where Leibniz identifies inherent properties of substance, which are also useful to catch some physical aspects of how substance acts. These aspects are those dynamical of the previous tripartition;
B) *absolute-metaphysical*, where Leibniz explains how, from a very metaphysical point of view, the *supposed* interactions among substances take place. This is the reign of the great principles of sufficient reason and pre-established harmony, with which I will deal in the sixth chapter.

As far as this section is concerned, while speaking of metaphysics, I will refer to the meaning A).

Kepler used the concept of inertia or natural inertia on several occasions in his *Epitome Astronomiae Copernicanae*. For my aims, it is not important what the

[2] Almost in every research on Leibniz's physics there is a section concerning inertia. As studies specifically dedicated to the concept of inertia in Leibniz, I mention, without any pretension of being exhaustive: Bernstein (1981), Gabbey (1971), Ghins (1990), Giorgio (2011), Giulini (2002), Lariviere (1987), Look (2011), Ranea (1986), Woolhouse (2000a). Important references to the concept of inertia in Leibniz are also present in: Arthur (1998), Bertoloni Meli (1993), Bouquiaux (2008), Crockett (2008), Duchesneau (1994), Garber (1994, 2006, 2009), Jauernig (2008), Papineau (1977), Puryear (2012), Roberts (2003), Suisky (2009) (in particular Chap. 2).

origin of this concept is or what its role inside Kepler's theory or its relations with previous scientific and philosophical traditions are. The important aspect concerns the way in which Leibniz interpreted and used it. On such question there is no doubt: Leibniz, who read directly the *Epitome* (on this, see the details in the sixth chapter), thought that, according to Kepler, matter has a natural tendency to oppose the motion. This is the meaning of natural or Keplerian inertia. Indeed, an interpretation as Leibniz's is not certainly a distortion of Kepler's ideas, if we think he wrote sentences as the following ones:

> Third: the earth is surely inert to the motion and, to a certain extent, it resists to that motion brought from another place. But all the bodies have this feature, as far as they are bodies. Thence, the Earth—among other bodies—does not deserve the place of a centre for this inertia.[3]

Here the idea that each body has inertia as a natural characteristic is clearly expressed. Through inertia, the body opposes a resistance to the movement. On the other hand, the inertia does not have a centre, which contradicts—as to this aspect—the Aristotelian idea that the earth were, so to speak, the inertial centre of the world.

Kepler specified and used many times his concept of inertia. Among several possible quotations, the following two seem particularly significant to me:

> The whole earth, as a whole, and in respect to its matter, has, according to its nature, no motion at all. To the matter, of which most of the earth is composed, the inertia is peculiar. The inertia is contrary to the motion. This opposition is the stronger, the greater the quantity of matter (*copia materiae*) pressed in a narrow space is.[4]

Here the concept of *copia materiae* is introduced, which, together with the idea that inertia increases when a major quantity of matter is pressed in the same volume, can induce us to think of a concept similar to that of mass. But for my aims the literal translation of *copia materiae* as quantity of matter is the correct one.

The following quotation seems to summarize the previous two: each body has its own peculiar inertia, whose value is different from the inertia of the other bodies. Kepler wrote:

> Although a celestial globe is not heavy (*gravis*) in the same manner in which a stone is called heavy (*gravis*) on the earth neither it is light, as the fire by us. Nevertheless, it has a natural *adynamia* [lack of power] to move from one place to another place, in proportion to its matter. The globe has a natural inertia or stillness, for which it remains at rest in every place, where it is posed alone.[5]

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[3] KGW, VII, p. 79, lines 31–34. Original Latin text: "Tertiò iners quidem est terra ad motum, eidemque aliunde illato quadamtenus resistit: at talia sunt omnia corpora, quatenus corpora; non meretur igitur Terra prae alijs corporibus locum centri hac inertia".

[4] *Ivi*, p. 88, lines 9–13. Original Latin text: "Terra tota, quatenus tota, et respectu suae materiae, motum planè nullum habet naturaliter: materiae enim, qua plurima Terra constat, propria est inertia, repugnans motui, eaque tanto fortior, quanto major est copia materiae in angustum coacta spacium".

[5] *Ivi*, p. 296, lines 30–33. Original Latin text: "Etsi globus aliquis coelestis non est sic gravis, ut aliquod in Terra saxum grave dicitur, nec sic levis, ut penes nos ignis: habet tamen ratione suae

To conclude: for Kepler: 1) the natural inertia is an essential property of the matter, according to which the matter opposes a resistance to the motion, whatever such a motion is. There is no distinction between rectilinear uniform motion and other kinds of movement; 2) on the other hand, no privileged centre of inertia exists.

How is Leibniz's concept of inertia tied to Kepler's? The answer is not easy, because Leibniz, while speaking of natural inertia, pointed out two properties which, according to his opinion, pertains to matter and bodies: a) matter opposes a resistance to be put in movement; b) the bodies, once the movements have begun, has the tendency to maintain the same speed and direction. However, the problem can, at least in part, be disentangled if one thinks that, in Leibniz's mind, the natural inertia is responsible only for the tendency a), that is the resistance to the movement. In contrast to this, the tendency b) is not given only by the inertia, but by entelechy, too, which is a property shared by each body. Therefore there are two conflictual tendencies: inertia and entelechy. Starting from the middle of the 1690s, Leibniz was clear on this conception. Let us see how he explained this situation.[6]

In a letter to Sturm, dated by the editors of the *Akademie Ausgabe* before 5 July 1697, Leibniz wrote:

> [...] and hence, as the resistance constitutes the general matter of a body, so the *nisus* constitutes the peculiar form of each body or its first activity (as I would translate *entelecheian ten proten*, according to my meaning), since the division itself of the extended [matter] into parts, and hence the form, arises from such *nisus*. And the resistance does not set up only bodies' impenetrability, but also the natural inertia, which is commonly less acknowledged. The natural inertia was called in this manner by Kepler. It is the property for which the matter is not indifferent to motion and rest (as many think), but rather opposes to a new motion in proportion to its mass (*molis*), in the same way we see that a loaded ship is transported more slowly by the same wind.[7]

This quotation immediately clarifies that the natural inertia is not connected, for Leibniz, only with physics, but it is, first of all, a metaphysical property of the matter, which can be reconducted to Leibniz's new application of concepts arising, in part, from Aristotelian tradition. Given a body, the matter is its passive aspect and

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materiae naturalem ἀδυναμίαν transeundi de loco in locum, habet naturalem inertiam seu quietem, qua quiescit in omni loco, ubi solitarius collocatur".

[6] I mention here some passages from Leibniz's epistolary and works I consider particularly significant. Many other quotations could have been chosen, since Leibniz expressed these conceptions on several occasions.

[7] Leibniz for Sturm, before 5 July 1687, LSB, II, 3, pp. 335–346. Quotation pp. 339–340. Original Latin text: "[...] atque adeo ut resistentia generalem corporis materiam, ita nisus peculiarem cujusque corporis formam vel primam activitatem (ut ἐντελέχειαν τήν πρώτην ex meo sensu interpreter) constituit; cum ipsa etiam extensi in partes divisio atque adeo figura, ex ipso oriatur. Et resistentia non tantum facit corporum impenetrabilitatem sed et aliud minus vulgo expensum nempe inertiam naturalem, a Keplero sic appellatam, qua fit ut materia non sit ad motum quietemque (ut plurimi arbitrantur) indifferens sed potius novo motui proportione molis suae repugnet, quemadmodum videmus navem magis oneratam, eodem vento tardius ferri". The concept of Leibniz's entelechy is developed, among other writings, in the *Specimen Dynamicum*.

the inertia is the expression of such an aspect, while form is the active aspect. Entelechy is its expression. In the second part of the quotation, Leibniz clarifies a coherent conclusion of his conception: the matter is not indifferent to motion and rest. This is a mistake of many thinkers. Who are these thinkers? My conviction is that, among them, Leibniz included Newton, too. For, according to Newton—using a Leibnizian language—the matter is indifferent to rest or rectilinear uniform motion, which is not the case for Leibniz. Hence, at a metaphysical level, Newton's principle of inertia is false. The matter tends to rest. Rest is a metaphysical state which is completely different from uniform rectilinear motion. Leibniz continued clarifying that, if a body is in motion, its inertia opposes a new motion. This cannot be interpreted as if Leibniz thought, *a là* Newton, that inertia is essentially connected with non-accelerated motions. Rather his conception seems like this:

1) matter has the tendency to rest, due to its inertia;
2) once a body is in motion, inertia continues to oppose motion, but the entelechy of the body wins the inertia and the body remains in motion;
3) however, the inertia continues to operate and its results on a body in motion is that, if a new action is not exerted, the state of motion does not change. That is: there is an equilibrium between the active principle of the bodies—entelechy—and passive principle—inertia.

This conception seems to make it possible to regain Newton's inertia principle at a physical level. This conclusion is plausible, though it remains an interpretation, given the difference Leibniz indicated between a motion (also a rectilinear one) and rest.

These conceptions are clarified by the following Leibniz's statement. For, we read in a quite significant letter addressed to De Volder on the 24 March (3 April) 1699:

> Since matter in itself therefore resists motion by a general passive force of resistance but it is set in motion by a special force of action, or entelechy, it follows that inertia also constantly resists the entelechy or motive force during its motion.[8]

This quotation confirms *in toto* what was claimed about the role and the behaviour of inertia during the motion, too. Although passive, inertia is a force. This means that Leibniz saw inertia as a continuous action (or better, a continuous passion), which is exerted. The quoted brief passage seems almost a commentary to what Leibniz had been written in the same letter some lines above. He expressed it like this:

> Somewhere in his letters I have observed that Descartes, too, following Kepler's example, has acknowledged that there is inertia in the matter. This you derive from the power which

[8] Leibniz to De Volder, 24 March (3 April) 1699, in LSB, II, 3, pp. 544–551. Quotation p. 547. Translation drawn from Leibniz (1989, p. 517). Original Latin text: "Cum igitur materia motui per se repugnet vi generali passiva resistentiae; at vi speciali actionis seu entelechiae in motum feratur; sequitur ut etiam inertia durante motu Entelechiae seu vi motrici perpetuo resistat". The correspondence Leibniz–De Volder has been edited and translated by P. Lodge, see Leibniz (2013).

> you say everything has to remain in its own state and which is not different from the nature of that thing itself. So, you believe, the simple concept of extension suffices to explain this phenomenon too. But the axiom that a body conserves its own state needs itself to be changed, for a body moving in a curve, for example, does not conserve its curvilinear path but only its direction. But granted that there is in matter a force to maintain its state, this force can certainly not be derived in any way from extension alone. I admit that each thing remains in its state unless there is a reason for change; this is a principle of metaphysical necessity. But it is one thing to retain its state until there is something which changes it, which this may do even though it is in itself indifferent to either state; it is another and far more significant matter if a thing is not indifferent to change but has a force and an inclination, as it were, to retain its state and so to resist motion.[9]

This quotation is extremely dense and useful to clarify Leibniz's conception:

A) The inertia is a force, a passive force, but a force. In representing the passive aspect of the bodies, it tends to maintain a *status*. In this case it seems that Leibniz is thinking of any state, not only the rest. However, to keep the coherence with the previous quotation and with what Leibniz wrote on many other occasions—as we will see—it seems to me necessary to think Leibniz is referring to rest. In my opinion, in this case, Leibniz has contracted his way of expression, so to result elliptical. In the example of the curvilinear motion, Leibniz's complete reasoning should be like this: a) inertia tends to maintain a body at rest; b) once begun the motion, entelechy is subject to modify such a motion; c) the result of these two opposite forces is a motion in which there is the tendency to maintain the direction. In a brachilogic manner one can say that inertia tends to maintain the direction, but this is not the real situation. On the other hand, since the original state of the matter, as far as it is mere matter (we will see this is not the case for the bodies, which are matter plus form), is the state of rest, it is consistent—though not necessary—to think that inertia opposes any movement. Furthermore, if inertia tends to restore the state of rest, its condition of force assumes a well defined character.

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[9] *Ivi*, pp. 546–547. Translation drawn from Leibniz (1989, p. 516). Original Latin text: "Inertiam in materia alicubi, exemplo Kepleri, et Cartesium in Epistolis agnovisse notavi. Hanc deducis ex vi quam quaevis res habeat permanendi in statu suo, quae ab ipsa ejus natura non differat: ita simplicem extensionis conceptum sufficere etiam ad hoc phaenomenon arbitraris. Sed axioma ipsum de conservando statu, modificatione indiget, neque enim (ex. gr.) quod in linea curva movetur curvedinem per se, sed tantum directionem servat. Sed esto, sit in materia vis tuendi statum suum; ea certe vis ex sola extensione duci nullo modo potest. Fateor unumquodque manere in statu suo, donec ratio sit mutationis, quod est metaphysicae necessitatis principium, sed aliud est statum retinere donec sit quod mutet, quod etiam facit per se indifferens ad utrumque, aliud est multoque plus continet rem non esse indifferentem sed vim habere et velut inclinationem ad statum retinendum atque adeo resistere mutanti". I do not enter into the problem of Leibniz's reference to Descartes. It seems that Leibniz—also considering the reference to Descartes' letters—is not referring to the first law of motion posed by Descartes in his *Principia philosophiae*, II, 37 (see *Oeuvres de Descartes*, 8, pp, 62–63), whose formulation is associated with the inertia principle. With regard to Descartes and natural inertia I refer to D.M. Clarke (1982), Appendix 2: *The impact rules of Cartesian dynamics*, in particular note 12, p. 232.

B) The forces belong to a domain, which is different from extension. This is one of the *topoi* of Leibniz's conception.[10] For, it is well known Leibniz's critics to Descartes' idea, according to which the body coincides with extension. According to Leibniz, the *vires* represent an essential aspect of the bodies, which cannot be confused with the mere extension.

   In this context, the letter Leibniz addressed to Denis Papin on 28 February (10 March) 1699 is a fundamental document. Here we read:

> Now, for me, rest is nothing but a privation. It follows that the mass in itself resists to the movement and this is what I call, with Kepler, inertia. But when the body is in movement and resists to rest, then I think there is a force or entelechy, which makes the body to continuously tend to movement. From here, it follows that the mass resists continuously to entelechy. So there is always an action and reaction in the same body.[11]

Not on many occasions was Leibniz so clear as in this letter: 1) inertia is an opposition to the movement, not to the change of movement; 2) entelechy is a force under whose action the body tends to the movement (no specification what kind of movement, this almost surely means *every movement*); 3) there is a conflict between inertia and entelechy, for which Leibniz refers to the action-reaction principle. It is known that Leibniz posed the action and reaction principle as a basis of his dynamics. For example in the second part of the *Specimen Dynamicum*, he wrote:

> It is also understood from what has been said that there is never an action of the bodies without reaction and that both are to each other and in contrary directions.[12]

It is difficult to think that Leibniz did not take the idea of action and reaction from Newton's *Principia*, although he used it in a completely different dynamical context.

In *Essais de Théodicée*, first part, Chap. 30, Leibniz expressed conceptions quite similar to the one already expounded. For he wrote:

> The famous Kepler and, after him Descartes (in his letters) spoke of bodies' natural inertia. This is something which can be considered as a perfect imagine and even as an example of creatures' original limitation [. . .]. Matter is hence originally inclined to slowness, or to

[10] Leibniz expressed several times the idea that the essence of the bodies cannot be reduced to their extension. For example the beginning of the *Specimen Dynamicum* 1, is quite clear as to this problem. See Leibniz (1695, 1860, 1962, VI, pp. 234–236).

[11] Leibniz to Papin 28 February (10 March) 1699, in LSB III, 8, pp. 67–71. Quotation pp. 69–70. Original French text: "Or, selon moy le repos n'estant autre chose qu'une simple privation; il s'ensuit que c'est donc la masse en elle même qui resiste au mouvement, et c'est ce que j'appelle avec Kepler, inertie. Mais quand le corps est en mouvement, et resiste au repos, alors je tiens qu'il a une force ou entelechie, qui le fait tendre à continuer le mouvement. D'où il s'ensuit que la masse resiste continuellement à l'entelechie, et ainsi qu'il y a action et reaction dans le corps même".

[12] Leibniz (1695, 1860, 1962, VI, pp. 251–252). Translation drawn from Leibniz (1989, p. 449). Original Latin text: "Ex dictis etiam intelligitur, actionem corporum nunquam esse sine reactione, et ambas inter se aequales, ac directe contrarias esse".

> privation of speed, not to diminish such speed of itself, when it received the speed, because this would be an action, but to moderate the impulse's effect by its resistance, when it has to receive it.[13]

After a few lines, in the same chapter of *Théodicée*, Leibniz explained once again that matter is not indifferent to movement, but it has a natural inertia, which is a sort of reluctance to be moved.

The following brief quotation from *Specimen Dynamicum* 2, is useful to conclude the reasonings developed until here and to briefly introduce the last question connected to natural inertia. Leibniz wrote:

> Nothing more foreign to nature can be conceived, moreover, than to seek firmness in rest, for, there is never any true rest in bodies, and nothing but rest can arise from rest [. . .].[14]

For my aims, the second part of this quotation is fundamental. First of all: from rest, only rest can be originated. There is an absolute difference between motion (whatever kind of motion one considers) and rest. However, the bodies are never properly at rest. They always move. This is another of the *topoi* of Leibniz's physics and metaphysics. It is connected to his general conception of motion, namely to the fact that, according to Leibniz, it makes sense to speak, in a reasonable meaning, of absolute motion. It is well known that Leibniz had a relativistic conception of space and time. But, for the movement, things are far more complicated. I adhere to the ideas of those scholars who think that, for Leibniz, an absolute motion exists, but we have no method to identify it in any reference frame. This fascinating subject would bring my research far from its aims, hence I refer to the literature.[15] I limit my reference to this quotation by Leibniz, which seems to me particularly significant as a summary of his conception:

> As to the absolute motion, nothing can determine it mechanically, because everything is resolved in ratios. This permits a perfect equivalence of the ratios, as in astronomy where, whatever the number of the considered bodies is, it is always arbitrary to assign the rest or a certain degree of speed to the body we are going to choose. The phenomena of the rectilinear or circular or composed movement cannot contradict this choice. Notwithstanding, it is

[13] Leibniz (1885, 1978, 6, pp. 119–120). Original French text : "Le celebre Kepler et apres luy M. des Cartes (dans ses Lettres) ont parlé de l'inertie naturelle des corps; et c'est quelque chose qu'on peut considerer comme une parfaite image et même comme un echantillon de la limitation originale des creatures [. . .] C'est donc que la matiere est portée originairement à la tardivité, ou à la privation de la vitesse; non pas pour la diminuir par soy même, quand elle a déja reçu cette vitesse, car ce saroit agir, mais pour moderer pas sa receptivité l'effect de l'impression, quand elle le doit recevoir".

[14] Leibniz (1695, 1860, 1962, VI, p. 252). Translation drawn from Leibniz (1989, p. 49). Original Latin text: "Nihil autem potuit magis alienum rebus excogitari, quam firmitatem a quieti peti, nam nulla est unquam quies vera in corporibus, nec a quiete aliud nasci potest quam quies [. . .]".

[15] See, only to provide some significant examples, Bertoloni Meli (1993, pp. 76–78), Garber (2009, p. 173–189), Jauernig (2008, p. 19, 33, note 16), Puryear (2012), Roberts (2003).

reasonable to ascribe real movements to the bodies, according to the hypothesis which allows us to explain the phenomena in the most intelligible way. For, this denomination is consistent with the concept of action we have established.[16]

Thus, from a physical point of view, every motion is relative, but in a very metaphysical perspective the absolute motion exists. The concepts of inertia and entelechy, as far as they are metaphysical truths, that is they express essential ontological properties of the bodies, deal with absolute motion. The *inertia naturalis* is a tendency to rest, which is never completely realized because the entelechy is opposed to it. Inertia represents what can be called the material essence of the bodies, while entelechy represents the formal essence. They are in conflict and no one of the two tendencies wins completely. The bodies move, but inertia is always operating as a tendency to rest. In the physical phenomena, this conflict generates the tendency of the bodies to maintain their state of motion, something which could be compared with Newtonian inertia, but whose origin is completely different. In the following scheme, I will summarize the properties of natural inertia and its relations with entelechy (Table 3.1).

Table 3.1 A comparison between natural inertia and entelechy

Natural inertia	• Metaphysical property of matter. Connected to the material aspect of the bodies. • It tends to maintain the bodies at rest. • It represents the passive aspect of the bodies. • However it is not indifference to movement. It is a passive force, but a force.
Entelechy	• Metaphysical property of form and activity. • Hence it tends to make the bodies to move. • It represents the active aspect of the bodies. It is a force.
Relation inertia-entelechy	• Motion and rest are mutually extraneous states. From rest only rest can be originated. • Natural inertia opposes to entelechy and tries to limit the movement, according to the principle of action-reaction. • A physical consequence of this condition is that a body in motion tends to continue the movement with its instantaneous speed and direction. The result is an almost-Newtonian physical principle of inertia. However, the conception in which rest and uniform rectilinear motion are identified is extraneous and inconsistent with the dialectic natural inertia-entelechy.

[16] Leibniz (1695a, 1875–1890, 1978, IV, pp. 486–487). Original French text: “Et quant au movement absolu, rien ne peut le determiner mathematiquement, puisque tout se termine en rapports: ce qui fait qu’il y a tousjours une parfaite equivalence des Hypotheses, comme dans l’Astronomie, en sorte que quelque nombre de corps qu’on prenne, il est arbitraire d’assigner le repos ou bien un tel degré de vistesse à celuy qu’on en voudra choisir, sans que les phenomenes du mouvement droit, circulaire, ou composé, le puissent refuter. Cependant il est raisonnable d’attribuer aux corps des veritables mouvemens, suivant la supposition qui rend raison des phenomenes, de la maniere la plus intelligible, cette denomination estant conforme à la notion de l’Action, que nous venons d’établir”.

3.2 Leibniz: Newtonian Inertia and Conate to Recede Along the Tangent

In the previous section I have tried to explain how an almost-Newtonian principle of inertia could be the result of Leibniz's concept of entelechy and natural inertia. Nevertheless, at a metaphysical level, Newtonian inertia is inconsistent with the conception of natural inertia-entelechy. However, if we consider the phenomenal and dynamical level, is it possible to find a concept which gets the same role as inertia principle? First of all, it is necessary to remember that Leibniz recognized, as a fundamental property of the curvilinear motion, its tendency to recede along the tangent. We have seen this in the already examined features of the planetary theory and we will see in the next chapters, also referring to Kepler's influence (Chap. 6). With regard to the curvilinear motion in Leibniz, two interpretations are possible: the premise is that Leibniz clearly claimed more than once that the curvilinear motions are composed of rectilinear uniform motions. This is well known. The problem is: at what level does this decomposition take place? One line of interpretation considers that the decomposition is a mathematical fiction. Bertoloni Meli can be taken as an example of this exegesis[17]: to expound his physical theories, Leibniz has considered the curvilinear motions composed of rectilinear uniform motions. This is consistent with the idea—underlined by Bertoloni Meli—that Leibniz tried to develop—at least as far as this was possible—a physics without accelerations (Bertoloni Meli 1993, p. 80). Furthermore, the model in which a curvilinear motion is decomposed in an infinitangular polygon is coherent with Leibniz's mechanistic conviction that a change of movement can take place only by impact. The vertices of the polygon are the impact-points. Although all these advantages exist, Bertolini Meli writes:

> The choice of the specific polygon entails a degree of arbitrariness depending on the progression of variables. In our case *dt* [the time differential] is constant, hence the chords [the sides of the infinitangular polygon] *EA, AG* are equal. However, different progressions of the variables could have been selected. In general, the vertices of the infinitangular polygon cannot be the actual place where impacts occur; Leibniz's mathematical representations of curvilinear motion are fictitious.[18]

On the other hand, Anja Jauernig, in her stimulating paper "Leibniz on motion and the equivalence of hypotheses" claims that all motions are composed of rectilinear uniform motion *in actu* and that, hence, Leibniz's model of decomposition is a copy of physical reality, not a mere mathematical model.[19] Jauernig starts from a wrong presupposition: "All bodies naturally move uniformly and rectilinearly" (*ivi*, p. 21). This is Newton, not Leibniz, for whom: all bodies naturally (if by "naturally" one means "as far as their matter, their inertia, are concerned") *would stay at rest*.

[17] Bertoloni Meli (1993, pp. 75–84), section 4.2: *Mathematical representation of motion*.

[18] *Ivi*, p. 83.

[19] Jauernig (2008, pp. 20–26), section IV. *Proving EH*.

However, once the movement has begun, this presupposition can be accepted, as we have seen in the previous section. If one adopts this interpretation, the conate to recede could be easily explained: what in our eyes is a conate to recede is the mere tendency of the bodies to continue in a straight line their *actual* rectilinear motion. The situation would be quite easy and satisfying from a mechanical point of view: the conate is simply due to the impact of a particle with other particles of the surrounding environment. The particle acts on the environment and produces the conate to recede, at the same time, the reaction of the environment pushes the particle in a curve trajectory. I believe this way of interpreting the decomposition of the curvilinear motion is not touched by Bertoloni Meli's considerations, because one could think that the impacts do not take place in the same element of time and that the sides of the infinitangular polygon are of different lengths. For sure, this would make the mathematical treatment difficult, but the situation is not impossible from a physical point of view. However, there is maybe another problem: the sides of the polygon, which approximates the curve, cannot be interpreted as segments whose length is an actual infinitesimal. Apart from the physical problems deriving from this consideration, it is well known that Leibniz did not believe in the existence of actual infinitesimals either in mathematics or in physics. On the other hand, while speaking of infinitangular polygon—also inside a potential conception of infinity—there is the idea that the sides of the polygon can become less than any given finite segment and, in any case, a limit-process is involved, which seems not suitable to identify a precise physical situation at a given time. Notwithstanding, in my opinion, the idea that the curvilinear motions are composed of segments which, in an intuitive, but reasonable sense, are *quite small* in comparison to the segments of our common experience, is a view, which catches the main features of Leibniz's thought. Obviously it is quite difficult to offer a coherent picture of Leibniz's theory of motion because sometimes—as seen in the previous chapter—it seems that basic principles, as that of continuity, are violated, on other occasions it is difficult to imagine a consistent model of the physical situations and properties described by Leibniz. Finally there is always the difficulty to understand at what level of his extremely stratified ontology Leibniz poses his concepts. But the previous decomposition seems to me to solve more problems than it creates.[20]

Furthermore, the decomposition of a curvilinear motion into rectilinear uniform motions is consistent—as Bertoloni Meli himself points out, see previous lines—with Leibniz's attempt to reduce the role of acceleration inside physics: the train of thought in which Newton inserted the principle of inertia, namely the idea that the instantaneous acceleration was the fundamental quantity involved in the main physical concept—the Newtonian force—is strictly connected with the distinction between rest and uniform rectilinear motion as inertial states and accelerated

[20] It is important to point out that the possible origin of curvilinear motions from rectilinear ones does not mean, obviously, to eliminate the physical differences between the two. The curvilinear motions maintain their own properties. Nevertheless their—so to speak—microscopic structure depends on the rectilinear motion. This structure, in Leibniz's perspectives, allows him to explain, from a physical standpoint, properties as the tendency to recede along the tangent.

motions as non-inertial state. However, if the acceleration is not seen as such a fundamental physical quantity, because there is no identification, as to the inertia, between rest and uniform rectilinear motion, it is consistent to think that the main physical quantity is speed—which separates rest from motion—and hence to develop a physics based on the speed, rather than on accelerations. It is not by chance that the measure of *vis viva*, the most important quantity in Leibniz's dynamics, involves the square of the speed, not the acceleration.

On the relations between Newton's and Leibniz's inertia and on the nature of uniform rectilinear motion and rest in Leibniz, many authors have expressed opinions similar to those expounded in the previous section.

Bertoloni Meli, for example claims that:

> Leibnitian inertia is resistance to impressed motion rather than *vis insita* or the tendency to continue motion uniformly in a straight line.[21]

Garber stresses that, when we consider the dynamical level of force and activity, the very distinction is between motion and rest, not, I add, that between rest-uniform motion and accelerated motion. Garber writes:

> When we consider only the geometrical properties of bodies, all questions of motion and rest are open, and all the hypotheses are equally good. But when we consider force and activity as well, we can actually assign motion and rest.[22]

The ontological difference posed by Leibniz between rest and motion, but, on the other hand, the property that when the motion has begun, a tendency to maintain its velocity and direction exists, has brought Duchesneau to think that, while two physical states existed for Newton, that is: a) rest-uniform rectilinear motion; b) accelerated motion, according to Leibniz there are three: a) rest; b) uniform rectilinear motion; c) accelerated motion.[23] This picture is consistent with the one I have proposed in the previous section.

Is there in Leibniz a concept that can be compared with Newtonian inertia? I think that, from the 1690s onwards the answer is: no. The rectilinear uniform motion and the tendency to maintain this motion was explained by Leibniz as an equilibrium between natural inertia and entelechy once the movement has begun. However, if we think of the early Leibniz's works some doubts can exist. For example in *Theoria motus abstracti* (1671), section *Fundamenta praedemonstrabilia*, Leibniz wrote:

> Indeed, when a thing is at rest in one place, it will remain at rest, unless a new cause of motion occurs. In contrast to this, what is once moved, if left alone, is moved with the same speed and direction.[24]

[21] Bertoloni Meli (1993, p. 31).

[22] Garber (2009, p. 115).

[23] Duchesneau (1994, p. 121).

[24] Leibniz (1671, 1860, 1962, VI, p. 68). Original Latin text: "Nam ubi semel res quieverit, nisi nova motus causa accedat, semper quiescet. Contra, quod semel movetur, quantum in ipso est, semper movetur eadem velocitate et plaga".

In his classical paper "Passivity and inertia in Leibniz's *Dynamics*", Bernstein quotes the previous passage as evidence of the presence of an "inertial mark" in Leibniz's early physics.[25] As explained, I think that a fundamental element is missing: the identification of rest and uniform rectilinear motion as the same inertial state. Certainly Leibniz recognized that, if a body is moved and no action is added during its motion, it will continue to move with rectilinear uniform motion. However, rest is separated from motion in this early work, too. It seems that already in his youth, when the complex apparatus of his dynamics was not yet developed, Leibniz thought of three different physical states: rest; rectilinear uniform motion; other motions. In any case, Bernstein himself clarifies that, in Leibniz, motion is always connected to a form of activity (*ivi*, p. 108), therefore he proposes to identify the inertial motion with a "changelessness" rather than with Newtonian "forcelessness". The prototype of this kind of motion would be the horizontal one (*ivi*, pp. 106–111). The argumentations developed by Bernstein as to the horizontal motion are quite interesting, but the idea that "changelessness"—to speak *à la* Bernstein—has to be identified with an equilibrium between natural inertia and entelechy is missing, while I think this is the fundamental element characterising the uniform rectilinear motion. Bernstein adds that, anyway, the canonical inertia is "[. . .] almost trivial in a Leibnitian framework" (*ivi*, p. 109), which is surely true.

Suisky, who tends to stress the similarities rather than the differences between Newton's and Leibniz's mechanics, notwithstanding points out:

> Later [after the *Theoria motus abstracti*] Leibniz did not explain the conservation of a state as Descartes and Newton by inertia. As a result, instead of simplifying and generalizing the theory, Leibniz was forced to introduce a variety of different forces [. . .] and run in trouble to define analytically the relation between forces.[26]

And again:

> The only candidate for being as necessary as the extension was the inertia which had been rejected by Leibniz. Being aware of this gap Leibniz replaced the previous concept of inertia by the conservation of "living forces" where the shadow of inertia is entering as the numerical value of the masses of bodies involved in the impact.[27]

Here a further interesting aspect is highlighted: Leibniz's physics is without Newtonian inertia; Leibniz tried to replace it by means of his "forces". His mathematical treatment of these forces was unsatisfactory in Leibniz. This does not mean it is impossible: according to Suisky, Euler developed Leibniz's programme in a coherent manner (*ivi*, p. 36).

The scope of this section has been to detect whether something equivalent to Newton inertia can be found in Leibniz's physics. I have expounded the ideas of several scholars. Their different interpretations show how difficult this subject is.

[25] Bernstein (1981, p. 101).

[26] Suisky (2009, p. 55).

[27] *Ivi*, p. 62.

To conclude: Leibniz did not have an inertia concept comparable to that of Newton or of Descartes himself. This would have been inconsistent with his idea that the essence of the substances is their activity, their entelechy, to which a passive force due to matter, the *inertia naturalis*, is opposed. Given this conception, motion can never be confused or compared with rest, whatever this motion is. Hence a physics which, without further explanations, poses uniform rectilinear motion and rest on the same level is wrong. It is true that, from a merely phenomenal point of view, uniform motion tends to prosecute indefinitely if no further action occurs. Leibniz does not deny this statement. But this depends on the equilibrium between natural inertia and entelechy. The state of rest is only an abstraction, which is useful to highlight the features of the natural inertia. But a mere state of rest is unconceivable, because this would mean that entelechy is not acting, which is not possible. Hence there is a conceptual gulf between motion and rest. One could add that, given that the rectilinear uniform motion tends to conserve itself—as, *in abstracto*—is the case for rest, the treatment given by Newton in terms of inertia principle can be accepted as a hypothesis mathematically equivalent to the real treatment of the physical states in terms of Leibnizian forces. But the presuppositions of Newton's mechanics are wrong. This situations has some similarities—not identities—with that concerning gravity, as we will see in the fifth chapter: Leibniz recognized that Newton's mathematical treatment is acceptable to deduce the phenomena, but it is based on false presuppositions, namely action at a distance, absolute space and time and so on. Thence, since Leibniz had the intention to offer the very metaphysical system of the world from which correct physical conceptions had to be deduced, he proposed a theory in which the properties Newton deduced from his inertia principle and from his treatment of gravity are replaced by others based on true metaphysical statements.

Chapter 4
The Final Version of Leibniz's Planetary Theory

Leibniz explained the final version of his theory in the *Illustratio Tentaminis de Motuum Coelestium Causis* (1706). This work, which was not published in Leibniz's lifetime, is divided into two parts. However, with regard to the content, it is possible to identify three conceptual cores:

1) from the point of view of mathematical physics, Leibniz expounded a new way to express the *conatus excussorius*, or better a *conatus* whose kind is *excussorius*, relying upon the model of the infinitangular polygon. We have seen two different representations: one of the *conatus centrifugus* and one of the *conatus excussorius*. The latter exploits the concept of osculating circle. Then why did Leibniz feel the need to add a further representation? He did not answer this question explicitly, but the likely reason is quite interesting and, as I will try to show, it is related to the particular physical conditions of motion Leibniz was considering.
2) In 1702 David Gregory published *Astronomiae physicae et geometricae elementa*. This is an important book, probably used by Gregory in his lessons. He explained physical astronomy basing on Newton's *Principia* and providing a series of useful specifications and explanations which were only implicit in Newton's masterpiece. As to our subject, in the first book, after having introduced the basic propositions of Newtonian astronomy, Gregory presented and underlined the weak points of Kepler's physical astronomy (pp. 78–87) of Descartes' vortex theory (pp. 87–99)—showing the instability of the vortices—and of Leibniz's planetary theory (pp. 99–104). In the second part of the *Illustratio* Leibniz tried to answer Gregory's criticism.
3) The *Illustratio* also contains a series of considerations on gravity and on its origin. They are inserted inside the answers to Gregory's criticisms, but, from a conceptual point of view, they have to be kept separated, because the attempt to clarify the nature of gravity dates to the initial works of Leibniz on physics and depends on his physical and metaphysical convictions. Thence, Leibniz's ideas on gravity are only indirectly connected to the answer to Gregory's criticisms.

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> The problem of gravity is strictly linked to the cosmological ideas expressed by Leibniz from his early work *Hypothesis Physica nova* (1671). Due to the importance of this question and to the strong relations between gravity and planetary theory, I will dedicate the entire Chap. 5 to gravity and cosmology.

From a conceptual point of view, the further specification of the polygonal model for a *conatus*, whose nature is *excussorius*, is remarkable and allows us to deeply enter into Leibniz's ways of conceiving physics and, in particular, movement. The answer to Gregory's criticisms and the ideas on gravity get a completely different tenor because a heavy series of hypotheses is introduced. These hypotheses are quite problematic from a physical standpoint and, after all, it seems difficult to find a convincing justification for their introduction. Therefore items 2) and 3) acquire a relevant historiographical interest, but—conceptually—Leibniz's argumentations seem not sound enough.

This chapter will be divided into two parts, according to items 1) and 2). As in Chap. 2, the reasonings and results by Leibniz will be explained and the commentaries will follow.

4.1 A New Model for the *conatus excussorius*

The new model for the *conatus excussorius*—or better, as Leibniz clarifies—for a *conatus* whose kind is *excussorius*, is expounded in a section, titled *De Vi Centrifuga Circulantis* (pp. 258–266), inserted in the first part (pp. 254–266) of the *Illustratio*. Leibniz's general idea behind the series of considerations explained in *De Vi Centrifuga Circulantis* is to provide a more complete foundation to his planetary theory. This attempt is divided into two phases:

In the former Leibniz tried to connect the movement of a body falling in a gravitational field with that of a body moving along a circle or a curve for which it is possible to consider the osculating circle in its various points. The purpose of this operation consists in determining a relation between gravity, as the force responsible for the fall of the bodies on the Earth, and the conditions of the planetary motion.

In the latter, Leibniz dealt with his new model of *conatus excussorius*. According to his opinion, this should complete the description of the planetary motion. While the physical foundations of his theory will be completed in the answer to Gregory and in the considerations on the origin of gravity.

4.1.1 *Circular Path and Falling Bodies: Leibniz's Assertions*

Before beginning his analysis, Leibniz clarified that he will consider a circle as an infinitangular polygon (see Fig. 4.1a).

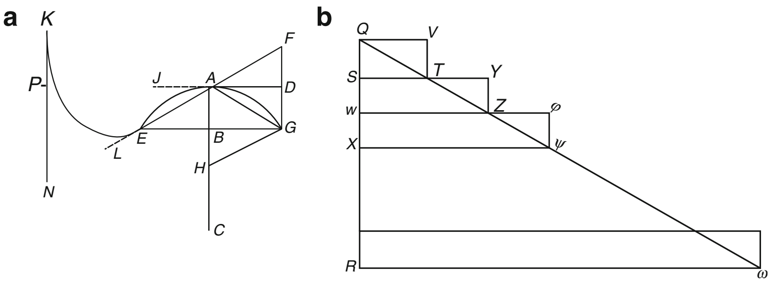

Fig. 4.1 Falling bodies in a gravitational field and *conatus recedendi*. (**a**) Comparison between the movement of a body in a gravitational field and the solicitations to recede from the centre in a body moving in a circular—or, anyway—curved path. Drawn from Leibniz (1860, 1962, VI), final diagrams. (**b**) The geometrical representation of physical quantities: the *vertical lines* represent the times, the *horizontal* ones the velocities and the areas represent the spaces. Drawn from *Ivi*

Leibniz developed the following argumentation:

Positions EAG is an arc of circumference whose centre is C; AC is perpendicular to EG (hence $EB = BG$). Let us complete the rectangle $ABDG$ and let EA and GD be continued until they meet in F. Then $FG = 2AB$ follows.

Reasoning Leibniz considered an infinitesimal element of time in which the motion along the side EA of the infinitangular polygon EAG... can be considered uniform. If there were no attraction nor impediments, a body M moving along EA would reach F from A in the same element of time necessary for it to reach A from E. But M receives a *conatus*, as AH, and hence arrives at G, not at F. The same reasoning can be applied when M is in G, so that M moves with a uniform angular speed (Leibniz spoke of *velocitas circulandi*, p. 259) along the infinitesimal sides of the infinitangular polygon EAG...

After this initial phase of his reasoning, Leibniz explicitly claimed:

> Let us now compare the centrifugal conate with gravity's conate, from which one can consider the velocity of circulation has been produced.[1]

The problem which Leibniz is now posing consists in determining the initial velocity with which the body M begins its circular motion in the circumference EAG. He supposed that, at the beginning, M moves along a path KL and that the only acting force is gravity. Leibniz then assumed that the velocity of the body, when it begins its circular motion in E, is the same one it gets in the point N, while falling along the straight line KN under the only action of gravity. This velocity can be easily calculated. The segment KP represents the infinitesimal distance covered by the falling body, with a motion that can be considered uniform, in the same

[1] Leibniz (1706, 1860, 1962, VI, p. 259). Original Latin text: "Comparemus iam conatum centrifugum cum conatu gravitatis, a quo velocitas circulationis orta intelligi possit".

elementary time in which the body M covers one side of the infinitangular polygon. To obtain his purpose, Leibniz resorts to a geometrical diagram (Fig. 4.1b): he represents the times of falling by means of a vertical line QR, where the segment QS represents the elementary time, in which the speed of the falling body can be considered uniform. The perpendicular line $R\omega$ represents the velocities, hence ST is the elementary speed, which can be considered uniform. Therefore the areas of the scalariform figure $QR\omega\psi\varphi ZYTV$ represents the altitudes of the falling body. In particular the rectangle $QSTV$ represents the descent KP in the elementary time QS. Leibniz proved easily that the altitude can be approximated by the triangle $QR\omega$—rather than by the whole figure $QR\omega\psi\varphi ZYTV$—without a detectable error. This means that—if S(A) indicates the surface of the figure A (Fig. 4.1b)—the following proportion holds

$$KN : KP = \mathrm{S}(QR\omega) : S(QSTV). \tag{4.1}$$

Furthermore

$$\mathrm{S}(QR\omega) : S(QSTV) = \frac{(R\omega)^2}{2} : (ST)^2. \tag{4.2}$$

Now there is an important step in the reasoning: since the uniform velocity with which the elementary segment EA is traversed is the velocity with which the falling body reaches the point N, it is exactly the one represented by $R\omega$. Let us moreover remind the reader that the segments KP, EA, AG,... are traversed in the same elementary time. This means that the proportion

$$AF : KP = R\omega : ST \tag{4.3}$$

holds.

Hence by 1), 2) and 3) Leibniz deduces

$$KN : KP = \frac{(AF)^2}{2} : (KP)^2, \text{ that is } KP = \frac{(AF)^2}{2KN}. \tag{4.4}$$

Considering the circle EAG, the proportion

$$AB : BG = BG : (BC + AC) \tag{4.5}$$

holds.

Since AG is an element of arch (Leibniz speaks of *arcus elementaris*, *ivi*, p. 260), the proportion 5) can be transformed, without a compromising error, into

$$AB : BG = BG : 2AC. \tag{4.6}$$

Therefore it is

$$AB = \frac{(BG)^2}{2AC}, \text{ hence } AH = \frac{(BG)^2}{AC}. \tag{4.7}$$

Since BG and AF have an infinitesimal difference,[2] from 4) it is

$$KP = \frac{(BG)^2}{2KN.} \tag{4.8}$$

Obtaining $(BG)^2$ from the second relation in 7) and from 8), finally one gets:

$$AH : KP = 2KN : AC. \tag{4.9}$$

In this way, Leibniz reached the conclusion that the paracentric solicitation AH is to gravity KP as the altitude KN (at the final point of which, N, the body has the same velocity with which it begins its circular motion) is to the half of the circulation-radius.

In this manner, Leibniz found a link between the motion under the sole action of gravity and the motion of a body along a circle having an initial velocity (given by the described conditions) and moving under the paracentric solicitation.

Leibniz introduced a distinction between the *conatus recedendi* from the centre of the motion in the first element of time in which the body M begins its movement along the circumference and the rest of the time in which M moves along the circumference: a) if M moves maintaining its speed along the line JA[3]; b) if the circular motion begins in the point A, then, after the element of time, the body will reach the point G rather than D. Therefore, in this initial moment in which the body moves along the circumference, the *conatus recedendi* is DG, which is the half of the *conatus* got by the body when its movement in the curve has already begun. For, its *conatus* is $AH = FG$. However, Leibniz underlined (*ivi*, p. 261) that the receding conate at the contact angle DG is only momentaneous and that, hence, only the conates like FG have to be taken into account in the physical analysis of the planetary motion.

4.1.2 Circular Path and Falling Bodies: Commentaries

This first section of *De Vi Centrifuga Circulantis* has the aim to better clarify the foundations of Leibniz's physical concepts applied to planetary theory, according to the expounded line of reasoning. Leibniz recognized that Varignon's observation, according to which the double *conatus centrifugus* has to be replaced with *conatus*

[2] Leibniz writes "differunt incomparabiliter" (*Ivi*, p. 260).

[3] Leibniz uses the expression "continuatoque impetu suo" (*ivi*, p. 261), whose best translation, in this context, seems to me "continuing the motion with a velocity, whose modulus is unmodified".

centrifugus, gave him the occasion to rethink some important concepts of his theory. For, we read:

> The illustrious Varignon gave me the occasion of this consideration, although with a meditation, whose aim was different. I have no doubt that our analysis will obtain many excellent additions from this.[4]

1. Leibniz's sources of inspiration

   The observation of Varignon was likely a direct source of inspiration for the last distinction made by Leibniz, the one between the initial *conatus recedendi* and the *conatus recedendi* after a body has begun its circular motion. This is a specification of the concept of *conatus* and can be directly inspired by Varignon's observation concerning this concept. But the long argumentation through which Leibniz reached the result expressed by proportion (4.9) seems to have a different source of inspiration. In substance Leibniz tried to find a physical link between the motion under the action of gravity and the circular (it seems, uniform) motion. The final result is a relation between gravity force and paracentric solicitation. My opinion is that Leibniz was influenced by the seventh and eighth sections of the first book of Newton's *Principia*. These two sections constitute a long, interconnected conceptual itinerary at the end of which Newton solved (in a geometrical way) the so-called inverse problem of the central forces[5] (proposition XLI—which is the fundamental one—and proposition XLII), namely, given any centripetal force and granted the quadrature of the rectilinear figures, determine the trajectories and the times of the motions along the found trajectories. One of the basic techniques used by Newton is exactly the comparison between a body which falls under the sole action of gravity and a body moving along a curvilinear path. This can already be seen in some theorems and problems explained in the seventh section. For example, in the Proposition XXXIII, Theorem IX, Newton proves that:

> The things above found being supposed, I say that the velocity of a falling body in any place C, is to the velocity of a body, describing a circle about the centre B at the distance BC, in the subduplicate ratio of AC, the distance of the body from the remoter vertex A of the circle or rectangular hyperbola, to $1/2AB$, the principal semi-diameter of the figure.[6]

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[4] *Ivi*, p. 258. Original Latin text: "Hujus autem considerationis occasionem mihi dedit Cl. Varignonius, alterius licet scopi meditatione, a quo non dubito multas egregias accessiones habituram Analysin nostram".

[5] The inverse problem of central forces in fundamental in Newton's physics. Here I provide some references without any claim to be exhaustive: Aiton (1964a, 1988), Bussotti and Pisano (2014b, pp. 425–439), Erlichson (1994), De Gandt (1995), Guicciardini (1995, 1999), Stein (1996).

[6] Newton (1726, 1739–1742, 1822, p. 228). Latin text: "Positis jam inventis, dico quod corporis cadentis velocitas in loco quovis C est ad velocitatem corporis centro B intervallo BC circulum describentis, in subduplicata ratione quam AC distantia corporis a circuli vel hyporbolae rectangulae vertice ulteriore A, habet ad figurae semidiametrum principalem ½ AB". For all Newton's passages, I will refer to Motte's translation, see Newton (1726, 1729). Quotation in the running text, book I, p. 156.

And in the proposition XXXIX, theorem XXVII, which is fundamental in the proof of the proposition XLI, it is demonstrated that:

> Supposing a centripetal force of any kind, and granting the quadratures of curvilinear figures; it is required to find the velocity of a body, ascending or descending in a right line, in the several places through which it passes; as also the time in which it will arrive at any place; And vice versa.[7]

Finally, the proposition XL (section VIII) sounds:

> If a body, acted upon by any centripetal force, is any how moved, and another body ascends or descends in a right line; and their velocities be equal in any one case of equal altitudes, their velocities will be also equal at all equal altitudes.[8]

Furthermore Newton uses the comparison between a body falling in a straight line under the action of a centripetal force and a body describing a curve under the action of another centripetal force in the proof of the propositions XLI and XLII themselves. It is clear that the perspectives of the two authors are different: Newton speaks of centripetal forces, whereas Leibniz speaks of gravity (a centripetal force) and of paracetric solicitation, a kind of solicitation which is equivalent to the algebraic sum of gravity and *conatus excussorius* and for which there is no equivalent in Newton's physics. Furthermore, it is difficult to frame this result in the rest of Leibniz's theory: it looks like an isolated result because its connections with the propositions of the two *Tentamina* and with those expounded in the rest of the *Illustratio* seem rather tenuous. One gets the impression that this proposition could represent the beginning of a possible itinerary towards something equivalent—in Leibniz's mind—to Newton's universal gravitation, because the idea to compare the gravitational force with the force/s responsible for the planetary motion is typical of the universal gravitation. Consideration of the initial velocity in the circular orbit as corresponding to the velocity reached at a certain altitude in the falling motion, seems to prefigure further researches concerning the relation between these two kinds of motion. These researches are missing.

2. The concept of time element

The studies on the potential and actual infinity in Leibniz and, more in general, in mathematical analysis in the 17th and 18th are so abundant and profound that, in this context, it is impossible to deal with such a problem. Nevertheless, a consideration seems appropriate: Leibniz spoke of *element of*

[7] Translation from Newton (1726, 1729, I, p. 163). Latin text: "Posita cujuscumque generis vi centripeta, et concessis figurarum curvilinearum quadraturis, requiritur corporis recta ascendentis vel descendentis tum velocitas in locis singulis, tum tempus quo corpus ad locum quemvis perveniet. Et contra". From Newton (1726, 1739–1742, 1822, p. 236).

[8] Translation from *ivi*, p. 168. Latin text: "Si corpus, cogente vi quacunque centripeta, moveatur utcunque, et corpus aliud recta ascendat vel descendat, sintque eorum velocitates in aliquo aequalium altitudinum casu aequales, velocitates eorum in omnibus aequalibus altitudinis erunt aequales". From Newton (1726, 1739–1742, 1822, p. 241).

time. For example in the two following passages of the *De Vi Centrifuga Circulantis*, we read:

> Let us suppose that a mobile traverses, in an element of time, the side *EA* with uniform speed, and, continuing its motion, tends towards *F*, so that, if nothing prevents it, the body would traverse the segment *AF* with a time element equal to the previous one.[9]

And again:

> Let us suppose that a heavy body, which descends from *K* in the first element of time, equal to the previous elements, has descended of an altitude *KL*.[10]

In this last quotation Leibniz is considering a series of elements of time and the descents in these elements. Therefore the situation is not exactly the same as when the limit of a certain quantity or ratio of quantities is calculated, when one of the quantities tend to 0. In this latter case, whatever the used language is, one deals with the concept of *potential infinity*. But in the case of the time-element, Leibniz needs to consider the element of time as a given quantity in which something happens.

In a passage of the proof of proposition XLI of the first book of the *Principia*, Newton wrote:

> And things remaining as in prop. 39, the lineola *IK*, described in the least given time will be as the velocity, and therefore as the right line whose power is the area *ABFD*, and the triangle *ICK* proportional to this time will be given [. . .].[11]

Very interesting is another passage by Newton: in the section XII of the first book, proposition LXXXIII, Newton was dealing with the problem to find the forces under the effect of which a corpuscle in the centre of a sphere is attracted towards any segment of the sphere. In the course of the reasoning Newton wrote:

> Let us suppose that surface to be not a merely mathematical, but a physical one, having the least possible thickness.[12]

Then, these times and surfaces of which Leibniz and Newton are speaking, are not infinitesimal quantities, in the usual meaning of this terms. They are infinitesimal, but given quantities. What can we conclude: did Leibniz and Newton

[9] Leibniz (1706, 1860, 1962, VI, p. 258). Original Latin text: "Ponamus jam mobile *elemento temporis* aliquo percurrere latus *EA* celeritate uniformi, motuque eodem continuato tendere in *F*, ita ut si nihil impederet, aequali cum priore temporis elemento percursurum sit rectam *AF* [. . .]". The italics are mine.

[10] *Ivi*, p. 259. Original Latin text: "Ponamus autem grave descendens ex *K primo temporis elemento* prioribus aequali descendisse ex altitudine *KL*". The italics are mine.

[11] Translation from Newton (1726, 1729, I, p. 171). Latin text: "Et stantibus quae in Propositione XXXIX lineola *IK*, *dato tempore quam minimo descripta*, erit ut velocitas, atque ideo ut recta quae potest aream *ABFD* et triangulum *ICK* tempori proportionale dabitur [. . .]". From Newton (1726, 1739–1742, 1822, p. 246). The italics are mine.

[12] Latin text: "Sit autem superficies illa non pure mathematica, sed physica, profunditatem habens quam minimam". From *ivi*, p. 385. In this case I have preferred not resort to Motte's translation, because it seems to me not appropriate, as far as this important passage is concerned.

use actual infinitesimal quantities in their physics? Where for "actual infinitesimal quantities" have to be meant a quantity whose measure is a given number which is not 0, but is less than every difference between two real numbers. Certainly this is not the correct answer[13]: Newton and Leibniz are speaking of a given quantity which is smaller than any real quantity or—to say *à la* Newton—the least possible one (*quam minima*), as a *physical fiction*. In a phase of their reasoning it is convenient to consider an infinitesimal given time or an infinitesimal given surface in order to develop the argumentation. This is an expedient, a device, which allows them to develop the proof. This does not mean that Leibniz and Newton believed in the physical existence of an actually infinitesimal time or surface. These are instruments to develop the reasoning. They are useful mathematical and physical devices and hence Newton and Leibniz used them, although they are fictitious entities.[14] Leibniz's pragmatic approach to this problem is confirmed by what he wrote in the published version of the *Tentamen*. After having introduced elements of area and elements of time, he explicitly claimed that a physicist is free to consider these quantities as quantities which are as small as necessary in the reasoning or as actually infinitesimal quantities. This seems to me a further indication that, for Leibniz, these quantities were fictions. He wrote:

> In the demonstrations I have employed *incomparably small quantities*, such as the difference between two finite quantities, incomparable with the quantities themselves. Such matters, if I am not mistaken, can be exposed most lucidly as follows. Thus if someone does not want to employ *infinitely small* quantities, one can take them to be as small as one judges sufficient as to be incomparable, so that they produce an error of no importance and even smaller than allowed.[15]

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[13] With regard to the concept of infinitesimal in Leibniz, there is an abundant literaute. I mention the fundamental Knobloch (2008). In this work a definitive answer to the question is given.

[14] These considerations can be inserted in the vast and profound debate concerning the concept of existence in mathematics and in physics as well as the legitimacy to use certain objects, fictions and methods in given contexts. This is a debate connected to philosophy, epistemology, history and methodology of science and mathematics. I have no room to deal with this problem here, but I think that the physical fictions of Leibniz and Newton have not yet got the importance they deserve inside this debate.

[15] Leibniz (1689, 1860, 1962, VI, pp. 150–151). Original Latin text: "Assumsi inter demonstrandum quantitates incomparabiliter parvas, verbi gratia differentiam duarum quantitatum communium ipsis quantitatibus incomparabilem. Sic enim talia, ni fallor, lucidissime exponi possunt. Itaque si quis nolit adhibere infinite parvas, potest assumere tam parvas quam sufficere judicat, ut sint incomparabiles et errorem nullius momenti, imo dato minorem, producant". Translation drawn from Bertoloni Meli (1993, pp. 130–131). Italics in the translation.

4.1.3 *The New Model for the* conatus excussorius*: Leibniz's Assertions*

The new concept of a *conatus*, whose kind is *excussorius*, was conceived by Leibniz in connection to the representation of the planetary motion by means of an infinitangular polygon: Leibniz considered an infinitesimal part of the trajectory of a body M moving along a curve as represented by the infinitesimal sides of the polygon M_1M_2 and M_2M_3 (see Fig. 4.2).

Then he considered the segment M_2L, such that the points M_1, M_2, L are aligned. Leibniz wrote that the segment M_2L represents the velocity with which the body M tends to prosecute its motion along the straight line M_1M_2L. If the segment M_3L is drawn from the point M_3 to the point L, it represents the *Conatus Declinationis* (*ivi*, p. 261). Leibniz explained that, if the parallelogram M_2LM_3G is drawn, the body M has two tendencies in the point M_2: the former is the tendency to prosecute its motion along M_1M_2; the latter is the tendency to go towards the point Θ, seen as a centre of attraction. The resultant is the segment M_2M_3, which is the trajectory of the body. Leibniz added that, if the elements of time, in which the segments M_1M_2 and M_2M_3 are traversed, are equal and if there is no other cause (for example, a friction) which can diminish the *impetus* along the line M_1M_2L, it is $M_1M_2 = M_2L$ (*ivi*, pp. 261–262).

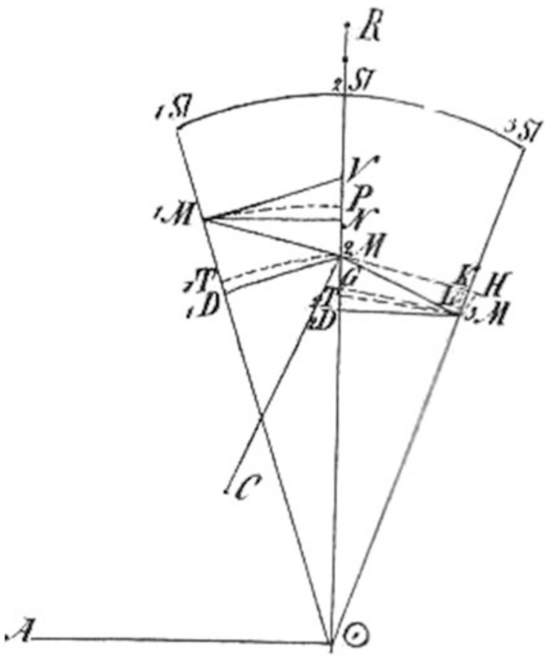

Fig. 4.2 The new model of Leibniz's planetary theory. In this picture all the most important novelties of Leibniz's planetary theory are present. In particular the *conatus declinationis* is the segment $M_3L = M_2G$. Drawn from Leibniz (1860, 1962), final diagrams

For a better understanding, it is maybe appropriate to underline that the whole representation is more clear if one imagines that:

1) the segment M_2L represents the infinitesimal space traversed by the body M in the time-element when no external force influences the motion of M.
2) the segments M_2G or M_3L represent the space traversed by the body M in the same time element under the action of a force that—also according to Leibniz's words (*ivi*, p. 262)—can be something like an attractive gravitational or magnetic force or a repulsive *levitas*.

In Leibniz's parallelogram M_2LM_3G, heterogeneous elements are present because M_2L is an infinitesimal velocity, which, anyway, is constant; M_2G is a *conatus*, which represents an infinitesimal change of velocity, and the diagonal M_2M_3 is an infinitesimal part of trajectory. However, the whole representation works, if we consider that all the segments indicate spaces.

To continue his reasoning, Leibniz traced the perpendicular M_3K from M_3 to the straight line M_1M_2L. The segment M_3K represents the *conatus* which was called *excussorius* by Leibniz in the *Tentamen*. Leibniz claimed that, if the point K coincides with L, the *conatus declinationis* coincides with the *conatus excussorius* (*ivi*, p. 262). Furthermore (as already seen in Sect. 2.3.1), the *conatus excussorius* coincides with the *conatus centrifugus* if the trajectory is the circumference M_1, M_2, M_3.

Once he had introduced the concept of *conatus declinationis*, Leibniz proved (section 11 of the *Illustratio*, pp. 262–263) that the harmonic circulation is coherent with the decomposition of the segment M_2M_3 in the two components M_2L and M_2G (*conatus declinationis*). In particular: the line described by the *circulatio harmonica* is the same as that described by the vectorial sum of M_2L and M_3L. Leibniz writes indeed:

> [...] and thus, the line described with harmonic circulation, of which I have spoken, is also described by the motion composed of the trajectory M_2L, which prosecutes the previous *impetus* M_1M_2, and by attraction's movement M_2G.[16]

The segment M_2G is, thus, an attractive force. Leibniz calls it *solicitatio paracentrica gravitatis*, but he also used the Newtonian expression *vis centripeta*. After a series of reasonings which are similar to some of those expounded in Chap. 2, Leibniz concluded that the planetary motion can be thought as generated:

1) by the harmonic circulation and by the paracentric velocity. He wrote indeed:

> Therefore, whatever we suppose the cause of planetary motion is, at least that motion could be considered as composed of the harmonic circulation T_2M_3 and of the paracentric velocity

[16] Leibniz (1706, 1860, 1962, VI, p. 263). Original Latin text: "[...] adeoque linea quae describitur circulatione harmonica quam dixi, etiam describitur motu composito ex Trajectorio M_2L, impetum priorem M_1M_2 continuante, et motu attractionis M_2G". The proof of this proposition, given by Leibniz at p. 263 is long, but not complicated, at all.

> M_2T_2, because of the difference, which is continuously generated, of the two elements, namely the centripetal conatus M_2G and twice the centrifugal conate D_2T_2.[17] (Reference to Fig. 4.2).

2) By the rectilinear motion and the *conatus centripetus*. Leibniz wrote:

> But the same motion can be considered as composed of the rectilinear motion of translation M_2L (according to the previous *impetus* M_1M_2) and of the same centripetal conate M_2G.[18] (Reference to Fig. 4.2).

This new presentation of planetary motion, based on the concept of *conatus declinationis* and of *conatus centripetus*, could appear difficult to understand: why did Leibniz feel the need to propose a further representation? In the commentaries I will try to answer this question. However, before dealing with it a further fundamental specification is necessary: the segment M_2G is directed towards the centre of the trajectory and its absolute value is the same as the *conatus declinationis* M_3L. Notwithstanding, the *conatus declinations* is a *conatus excussorius*, this means that its tendency is outwards, not inwards. For, Leibniz never calls M_2G *conatus excussorius*, but as seen, *solicitatio paracentrica gravitatis* or *conatus centripetus*. Then, what is exactly the *conatus declinationis* from a physical point of view? I think the only possible answer is the following: given a certain gravity paracentric solicitation at the time t, the planet, according to Newton's principle of *action and reaction*, in which Leibniz believed, experiences an outwards tendency, which is the reaction to the centripetal force. However, differently than in Newton, this reaction is not instantaneous. The reaction subsists after an unitary inifinitesimal time $t + dt$, that is when the planet is in M_3. Therefore M_3L is the outwards tendency, the *conatus declinationis*, which is the reaction to the gravity paracentric solicitation. This is the physical explanation of the *conatus declinationis*. A comparison with *conatus centrifugus* and *conatus excussorius* is developed in the next section.

4.1.4 *The New Model for the* conatus excussorius*: Commentaries*

1. The general meaning of the model with the *conatus declinationis*.

   After having explained the equivalence of the two expounded models for planetary theory, Leibniz wrote:

[17] *Ivi*, p. 264. Original Latin text: "Quamcunque igitur causam Motus planetae esse ponamus, saltem intelligi poterit compositus ex circulatione harmonica T_2M_3 et ex velocitate paracentrica M_2T_2, per Elementorum duorum, nempe conatus centripeti M_2G et centrifugi bis D_2T_2, differentiam continue generata".

[18] *Ivi*, p. 264. Original Latin text: "Sed idem motus simul potest intelligi compositus ex motu trajectionis rectilineo M_2L (secundum impetum priorem M_1M_2) et eodem conatu centripeto M_2G".

> Thence, although at the beginning, the *impetus* of the planet, derived from the already conceived motion, could be not in agreement with the impressions of the deferent fluid, nevertheless, finally, it happens that—mutually agreeing the planet and the fluid—the planet moves, in an absolutely free way, inside the fluid, as if the means [the fluid itself] were not resistant. In its turn, the fluid moves with the planet, so that the planet is not carried by its own *impetus*, but by the tranquil movement of the fluid. I have already proved that the final coincidence of the two motions is the marvellous privilege of the sole Fluids' Harmonic Circulation.[19]

First of all, it is appropriate to point out that the model "rectilinear motion with a given initial velocity + *conatus declinationis*" is—apart from the linguistic differences—the same model used by Newton in the first proposition of the *Principia* to prove the area law. Leibniz's language itself seems to converge towards Newton's way of expression. For, Leibniz, in this contribution, used the term *centripetus*. To make Leibniz's reasoning more explicit, it is possible to argue like this: let us suppose that a body *M* is moving inertially. At a given instant, it enters into the gravitational field of a far more massive body *N* (as the sun in comparison to the planets), then the two bodies begin to rotate around their centre of gravity, which is inside or near *N*. One can say that *M* is attracted by *N*. This is the Newtonian explanation. However, Leibniz claimed: *this is not the only possible physical description*. For, let us suppose that *M* enters into the gravitational field of *N*; since the Newtonian model and the harmonic model are equivalent from a kinematical standpoint—at least as far as this general description of the planetary motion is concerned—, at the end (*tandem* is the term used by Leibniz, see previous quotation) it will be impossible to understand if the motion of the planets arises from a physical situation as that imagined by Newton or as that described by Leibniz. Given the impossibility to conceive an immediate action at a distance and a void space, Leibniz concluded that Newton's is only a *dynamical description* of the planetary motion, whereas his own is the true *physical-structural planetary theory*. This is the *mirabile privilegium* of the *circulatio harmonica*.

This confirms that, from a purely historical point of view,—I mean, if the purpose is to reconstruct Leibniz's way of thinking—it makes no sense to

[19] *Ivi*, p. 264. Original Latin text: "Ideo quanquam initio planetae impetus ex concepto jam motu cum fluidi deferentis impressionibus non consensisset, tandem tamen factum est, ut fluido ac planeta sese accomodantibus, planeta liberrime jam moveatur in fluido tanquam medium resistens nullum esset; et fluidum vicissim ita moveatur cum planeta tanquam planeta nullo proprio impetu sed tranquilla a fluido gestatione deferretur. Quod unius Circulationis Fluidorum Harmonicae mirabile privilegium ex hac ipsa utriusque motuum compositionis coincidentia jam tandem habetur demonstratum".

analyse Leibniz's theory independently of his conception of harmonic circulation. While, it makes perfect sense to separate the single ideas expounded by Leibniz in his planetary theory, if the aim is the reconstruction, the history of some scientific ideas—as, for example, the history of non-inertial reference frames-. Leibniz's planetary theory, as many other theories can, indeed, be faced under different perspectives. This specific theory is particularly significant because of the continuous need Leibniz felt to compare his ideas on planetary motion and on the physics of the solar system with an unavoidable reference point: Newton's *Principia*.

2. A comparison between *conatus excussorius* and *conatus declinationis*

   As seen in Sect. 4.1.3, Leibniz claimed that the *conatus excussorius* is represented, in the planetary model shown in Fig. 4.2, by the segment M_3K. We have seen that in the *Tentamen* Leibniz defined the *conatus excussorius* as the perpendicular drawn from an extremum of an arc AB to the tangent in A.[20] In the model of Fig. 4.2, the segment M_3K is hence the perpendicular to the tangent to the orbit drawn from M_2. The segment M_2K, which is the prolongation of M_1M_2, is, thus, considered by Leibniz the tangent to the curve (which is approximated by the infinitangular polygon) in the point M_2. This is the conclusive confirmation that Leibniz is here using the concept of tangent as *prosecution of an (infinitesimal) chord* and not as the *Euclidean tangent*.

   But why replace the concept of *conatus excussorius* with that of *conatus declinationis*? My interpretation is connected to what is explained in the item 1. of this Sect. 4.1.4: Leibniz needed a physical quantity which allowed him to compare Newton's model explained in the first proposition of the *Principia* with his *circulatio harmonica*. In Newton there is no geometrical-physical equivalent of the *conatus excussorius*, but there is of the *conatus declinationis* (or better, of gravity paracentric solicitation).

   The segment Bc (see Fig. 4.3) is, in Leibniz's language, the tangent to the trajectory in the point B because Bc is the prosecution of the chord AB, so that the segment Cc represents the deviation from the rectilinear path ABc in the arc BC. This is represented in the same way as Leibniz's *conatus declinationis*, not *conatus excussorius*. It is worth remarking that Newton did not give a specific name to this segment, which enters into the proof of: a) Proposition I; b) Proposition I, corollary 3; c) Proposition II of *Principia*'s first book. In Leibniz's perspective this segment plays an important role because it allows him a direct comparison with Newton's

[20] Leibniz (1689, 1860, 1962, VI, p. 153).

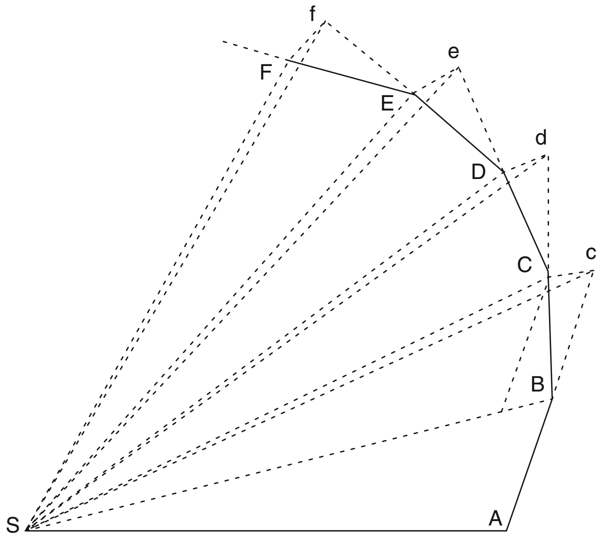

Fig. 4.3 Newton's *Principia*, proposition 1. The first proposition of Newton's *Principia*, with the segments Cc, Dd, Ee, Ff, which have the same physical role as the *conatus declinationis* in Leibniz. Imagine drawn from the site http://www.gutenberg.org/files/28233/28233-h/28233-h.htm, p. 37

conception and—in Leibniz's mind—this should bring to the conclusion that harmonic circulation is equivalent to Newton's model.

If Leibniz's way of proceeding is clear, his representation of the physical quantities seems, once again, not perspicuous because of the parallelogram M_2LM_3G, whose sides represent heterogeneous quantities, while this problem does not exist in Newton representation of Fig. 4.3 because all segments represent infinitesimal spaces. It seems that in Leibniz's representation of the *conatus declinationis* two different physical quantities converge: 1) the instantaneous change of velocity due to gravity; 2) the instantaneous change in the direction of the trajectory; namely a differential of velocity and a differential of space. However, the purposes and the ideas of Leibniz are clear: to show the equivalence between the representation of planetary motion by means of Newtonian concepts and by the harmonic circulation. In the conclusive remarks, I will deal with this subject in a more general manner. Before dealing with Gregory's criticism and Leibniz's answer, I propose a table (Table 4.1) in which I summarize the differences among the three *conatus excussorii* used by Leibniz in the *Tentamen. Erste Bearbeitung* (the published version); *Tentamen. Zweite Bearbeitung*; *Illustratio Tentaminis*.

Table 4.1 The *conatus* used by Leibniz in his planetary theory

Kind of *conatus*	Features
Conatus Centrifugus	In the published version of the *Tentamen* and in the *Zweite Bearbeitung*, too, until Leibniz did not feel the need to analyse the specific properties of the motion in an ellipsis (18th and 19th propositions of the *Zweite Bearbeitung*), the expression *conatus centrifugus* indicates the general instantaneous tendency to recide from the centre in a curvilinear motion. Starting from the 18th proposition of the *Zweite Bearbeitung*, the expression *conatus centrifugus* indicates the instantaneous tendency to recede from the centre in a *circular motion*, not in a general curved motion.
Conatus Excussorius	Until the 18th proposition of the *Zweite Bearbeitung*, this expression is used as a synonymous of *conatus centrifugus*. Starting from such proposition *conatus excussorius* indicates the general instantaneous tendence to recide from the centre in *any curvilinear motion*. As seen, Leibniz studied its features in the elliptical motion. If the trajectory of the planets is considered an infinitangular polygon and the tangent is considered the prosecution of a side, then given two consecutive sides AB, BC, both the *conatus centrifugus* and the *conatus excussorius* are represented by the perpendicular drawn from C to the prolongation of AB (the tangent).
Conatus Declinationis	This is a different segment.[a] Given two consecutive sides AB, BC and the prolongation of AB, the *conatus declinationis* is not the perpendicular to the tangent. Rather: 1) given the infinitesimal side BC: 2) considered it as the diagonal of the parallelogramm $BCDG$, one side of which is given by the velocity in B, represented along the straight line AB by BD, the *conatus declinationis* is the side CD. It represents a tendency to recede from the centre, which, in absolute value, is exactly the same as the inwards (centripetal) tendency experienced by the planet in the point B. This point B is that reached by the planet in an infinitesimal-time-unity before the planet reaches C. Leibniz clarifies when the three conatus coincide.

[a]In this section of the table concerning the *conatus declinationis*, the letter A corresponds to M_1 in Fig. 4.2; B to M_2, C to M_3, D to L and G to itself

4.2 Gregory's Criticism and Leibniz's Answers

This section is divided into three parts: 1) Gregory's criticisms; 2) Leibniz's answers; 3) commentaries. It concerns the second part of the *Illustratio* (pp. 266–276), in which Leibniz faced Gregory's criticisms.

4.2.1 Gregory's Criticisms

In the proposition LXXVII of *Astronomiae physicae et geometricae elementa* (pp. 99–101) Gregory summarized Leibniz's planetary theory. In the proposition LXXVIII (pp. 101–104) he expounded his critical remarks to such theory. In particular he considered three argumentations:

1) The paths of the comets along the Zodiac are quite oblique, sometimes they form right angles with the Zodiac, sometimes they move in the opposite direction to the planets. Since their motion satisfies the area law, each comet should have a vortex. This vortex should interact with the planetary vortices. Here is the question: Newton had proved and Gregory specified that a single vortex cannot be stable.[21] But in case of Leibniz's theory the situation is even more difficult to be imagined from a physical point of view, because one should admit that the cometary vortices and the planetary vortices have no mutual interaction so that the harmonic circulation of each vortex is preserved when the matter composing the vortices are in contact. They do not reciprocally modify their state of motion.
2) The *circulatio harmonica* does not satisfy Kepler's third law because the spaces are as the times directly and the velocities indirectly. In the circular motion the spaces are as the radii directly. In the harmonic circulation the velocities or the circulations[22] are as the radii inversely. Therefore the periodic times are as the squares of the radii, which is in contradiction with Kepler's third law. In this context, Gregory underlined that the velocity is as the square root of the distance from the centre of the forces, mentioning (*ivi*, p. 103) his propositions XXVII and XXVII. This result is, in fact, obtained by Newton in the corollary 6 to the proposition IV of the first book of the *Principia*. In Gregory's conception, Leibniz's idea according to which vortices' harmonic circulation exists only in the ring whose minor radius is the distance sun-perihelion and major radius is the distance sun-aphelion creates more problems than it solves.
3) Gregory claimed that in a system as that of Leibniz, it is impossible to admit the action of gravity. For, if the *circulatio harmonica* has a *conatus excussorius* and if, as Leibniz recognized, a body tends to prosecute its trajectory along the tangent when no force exists, how is it possible that the trajectory is a conic and not a straight line? In practice Gregory claimed: Newton admits gravity and does not explain its physical origin. Leibniz creates a mechanism in which it seems impossible to admit gravity, therefore Leibniz has to necessarily add the physical explanation for gravity. He cannot take for granted this force because in its system gravity could not exist.

The three criticisms of Gregory concern three rather different aspects of Leibniz's theory: the first one is an extension of the criticisms Newton had already addressed to Descartes' vortex theory; the second one regards a specific lack of Leibniz's theory concerning a relation between physical quantities; the third one faces the entire structure of the theory because if gravity were really incompatible with Leibniz's theory, this theory would be clearly wrong.

[21] For references see, in this book, Sect. 4.2.3, item 3.

[22] Gregory writes "[. . .] velocitates sive circulationes [. . .]" (Gregory 1702, p. 102, line 22). In the Sects. 4.2.2 and 4.2.3 I will clarify the importance of the fact that Gregory considered these two words equivalent.

4.2.2 Leibniz's Answers

Let us begin from the answers concerning the problems of the relations planetary vortices-cometary vortices and vortices' instability. Leibniz's line of defence appears divided into two kind of arguments:

1) From a *physical standpoint*, some phenomena provide examples of vortices which do not mutually disturb their motions: a) some little stones thrown in the water form circular vortices which do no interfere each other, although all the vortices belong to the same means: the water; b) the sound-waves permeate the air, but they do not disturb each other. Leibniz also added that comets' motions are not yet well known. In particular—according to his opinion—, it is not yet proved, beyond any doubt, that their motions satisfy area law and hence that their circulation is harmonic.
2) With regard to the *dynamical problem* concerning the fact that the planets would had met a resistance—due to vortex's matter—in their motion inside the vortex, Leibniz provided this answer, which seems an *a posteriori* justification of his theory. He asserted: i) since, as proved at the end of the first part of the *Illustratio*, the harmonic circulation is a model which is mathematically equivalent to the Newtonian model in which the space is supposed to be void, ii) since the planets move as if the space was void and iii) since a void space and an interaction at a distance cannot exist, then one concludes necessarily that, in the *circulatio harmonica*, the planets move as in a void space and the matter of the vortex exerts no resistance. Leibniz wrote these clear words:

> The new and marvellous property of the Harmonic Circulation, already shown at the end of § 12, helps us. This property has the feature that the bodies carried in a resistant means, which, however, circulates harmonically, can move absolutely freely, as if they were moved in a non-resistant means.[23]

This implies that a body, in a vortex with harmonic circulation, moves as if it were afloat, without experiencing any resistance. Furthermore, given the equivalence between the motion in a void space and the motion in a harmonic vortex, if the harmonic circulation is interrupted at a certain instant or point, it will be quickly re-established.

Leibniz did not address the problem of the vortex stability, but it is likely that, according to his opinion, this stability is unquestionable, because if the vortices were not stable, then the harmonic circulation, which is a *dynamical* and *physical-structural truth*, would be destroyed. In practice Leibniz justified the general stability of the vortices with the existence of the harmonical vortices, which—according to his opinion—are a physical necessity.

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[23] Leibniz (1706, 1860, 1962, VI, p. 269). Original Latin text: "Hic ergo succurrit nobis nova et pulcherrima Circulationis Harmonicae jam sub finem § 12 ostensa proprietas qua efficitur, ut quae feruntur in medio resistente [. . .] sed tamen harmonice circulante, moveri possint liberrime, non minus quam si moverentur in medio resistendi non capace".

Leibniz added a further reason in favour of his theory: the planets rotate in an orbit which has a slight inclination on the sun equator, furthermore they rotate around their axes in the same sense of rotation as the sun around its axis. The satellites rotate in a plane whose inclination on the planetary orbit is slight and they rotate around their axes coherently with the axis-movement of the planets. Gravity in itself cannot explain this situation. While a series of vortices moving in the same direction can. Leibniz carried out an interesting observation: the Earth is the only planet whose path is considerably inclined on the plane of sun-equator. The moon has an orbit which is near to the ecliptic, not to the sun equator. Probably, Leibniz added, before being attracted by the earth, the moon orbit was as inclined as those of the other planets. When—for some reasons—the moon has been attracted by the earth, the system of vortices earth-moon became a sole system and the moon begun to rotate according to the earth vortex, that is with an inclination on the sun equator which is far greater than that of the other planetary orbits.

Hence Leibniz also offered here some historical elements concerning the evolution of the solar system. The consideration of this historical aspect testifies Leibniz's broad-mindedness and the clear intention to frame his planetary theory inside his general physical-philosophical conceptions and views. For sure, he did not consider his ideas on the planetary motion as a mere technical result.

Leibniz also briefly mentioned other possible arguments to explain the inclination of the planetary paths and the rotations of the planets around their axes, but excluded them as far less likely than his explanation.

The answer to the objection according to which the *circulatio harmonica* does not satisfy Kepler's third law is based on the following reasoning: Leibniz hypothesized that all planetary vortices had the same *vis viva* or *potentia*, furthermore the matter of the vortices is limited—as already seen—to the tiny ring between aphelion and perihelion. Therefore, on the basis of these two suppositions, one has:

1) The matters—the masses m. By mass, I simply refer to the quantity of matter—of the orbits are as are as the circumferences (the ring is so tiny that can be approximated by a circumference), namely as the radii or the distances from the sun;
2) Therefore, since $vis\ viva = m \cdot v^2$, the distances from the sun are inversely proportional to the squares of the velocities;
3) If the motion is circular and uniform (in this context, this is a reasonable approximation for the planetary motion) the circumferences are as the periodical time by the velocity with which the body moves; therefore the distances from the sun have this same proportion;
4) The velocities are hence as the distances directly and the periodical times inversely;
5) Therefore the squares of the velocities are as the squares of the distances directly and the squares of the periodical times inversely;
6) But, according to 2) the squares of the velocities are inversely proportional to the distances from the sun;
7) Thence, from 5) and 6) it follows that the squares of the periodical times are as the cubes of the distances from the sun.

In the following section I will provide a general commentary to Leibniz's answer. However, a question—if not the answer—cannot be postponed: one of the bases of the harmonic circulation is that the *velocitas circulandi* is inversely as the distance from the body to the centre of the motion, therefore how can Leibniz admit that the velocities are as the square root of such distances? The answer is that, according to Leibniz, there is no contradiction: the velocity with which he is dealing in this context is the modulus of the vector velocity, not the *velocitas circulandi*, that is the tranverse velocity. This is the reason why he did not consider Gregory's objection as if it mined the whole conception of *circulatio harmonica*. Rather he added the series of considerations we have analysed to insert Kepler's third law inside his theory, but, in his perspectives, there is no contradiction between harmonic circulation and Kepler's third law. Further specifications to this problem will be added in the commentaries.

4.2.3 Leibniz's Answers to Gregory's Criticisms: Commentaries

The general answers given by Leibniz against the criticism to the vortices and the harmonic circulation are clearly unsatisfactory to our eyes and would have been unsatisfactory at all for the Newtonians of the early eighteenth century, too:

1) the theory of comets expounded in the first edition of the *Principia* is not as complete as those explained in the second and in the third edition, but the fact that the area law is valid for the comets and that the comets obey to the same rules of motion characterizing the planets was clearly expounded by Newton;
2) the fact that the vortices produced by the stones in the water and by the sound-waves do not interfere needs—first of all—a physical analysis which is missing in Leibniz's work. For example, if two sound-waves have the same frequency they interfere. Furthermore, also admitting these phenomena, there is no guarantee that the possible planetary vortices behave as water and sound-waves vortices. This is a mere analogy, whose justification is far from being acceptable without any argument;
3) to the eyes of the Newtonians, the most inacceptable aspect concerned the existence of the vortices themselves: the section IX of the second book of the *Principia*, titled *De motu Circulari Fluidorum* was present in the first edition of Newton's masterpiece. In this section Newton proved—among other results—that the vortex theory for the planets is not valid because the vortices cannot be stable. Even though, as Aiton claims: "These Newtonian objections did not, as commonly supposed, constitute a decisive refutation of the Cartesian theory of vortices"[24] because they are based on initial hypotheses which are not

[24] Aiton (1972, p. 112).

necessarily valid for the planetary vortices, for sure a defender of the vortex theory would have had to face these profound objections, based on precise mathematical and physical arguments, and to explain why they were not decisive. While in Leibniz there is no trace of such analysis.

4) Leibniz added a series of hypotheses (beyond those which are typical of every theory based on the vortices) which are really strong assertions on the physical structure of the universe. Probably the most extreme is the idea that all the planetary vortices have the same *vis viva*. This assertion is so strong from an epistemological and physical point of view, that probably, no scientist would have considered it as an acceptable initial hypothesis. The famous critics addressed by Cotes against the vortex theory in the preface to the second edition of the *Principia* concern indeed not only the physical aspects of the theory, but also the epistemological ones. Cotes, in particular, underlined a quite important problem: the vortex theory was assuming such strong initial hypotheses that they are more complex to be explained than the phenomena for which they were formulated. Cotes wrote:

> Without any doubt, if these imaginary motions are more complex and difficult to explain than the true motions of the planets and of the comets, it seems to me they have not to be accepted in philosophy: for, each cause must be simpler than its effect.[25]

If we think that, as Bertoloni Meli highlights, Cotes deemed Leibniz's *Tentamen* as a good example of a work which "deserves a censure",[26] it is likely that Leibniz was one of the most important scientists against whom Cotes wrote his preface to the *Principia*. Furthermore, it is necessary to add that, likely, Cotes did not know the extreme positions sustained by Leibniz in the *Illustratio*.

These were the perspectives of the Newtonians, but which were Leibniz's perspectives? They were the perspectives of a scientist for whom the philosophical and metaphysical convictions influenced the way to conceive the physical world and, hence, physics as a discipline. In this specific case: the idea that every vortex of the planets has the same *vis viva* appears bizarre to us, but it was not for Leibniz: as well known, he had dealt with the concept of *vis viva* and with the concept of *vis viva* conservation in numerous contributions. It is hence quite probable that his line of thought was the following one:

A) the principle of *vis viva* conservation exists;
B) the idea that the *vis viva* is the same in every planetary vortex can be interpreted as an extension of the principle of *vis viva* conservation. The latter principle concerns the transition from a physical state to another physical state, whereas

[25] Cotes in Newton (1726, 1739–1742, 1822, p. XXII). In this edition Cotes' preface to the second edition is reported. Latin text: "Sane si motus hi fictitii sunt magis compositi et difficilius explicantur, quam veri illi motus planetarum et cometarum; frustra mihi videntur in philosophiam recipi: omnis enim causa debet esse effectu suo simplicior". My translation from Latin into English.

[26] Bertoloni Meli (1993, p. 207).

> the former regards the situation of physical states and objects in themselves (the movements of the planets, namely a state, and the matter of the vortices, namely an object) which were, in Leibniz's perspective, analogous and hence had to keep, at least, one invariable common physical quantity. He identified this quantity with the *vis viva*.

Obviously the proposition on the *vis viva* of the planetary vortices has no logical connection with the principle of *vis viva*; nevertheless, this proposition belongs to the same conceptual frame, to the same way of thinking and conceiving the world, which is typical of Leibniz and which is based on the idea that both in physical *processes* and *states* some invariable quantities are conserved.[27] The conservation of certain quantities in the physical processes will become one of the bases of modern physics. While the conviction that certain states—because of a supposed physical and metaphysical analogy—share invariable quantities and forms can be interpreted as the heritage of a way of thinking which was typical of Kepler and that characterized Leibniz's approach to science, too. This will be clarified in the chapter on the influence Kepler had on Leibniz and on the way in which Leibniz interpreted Kepler.

Anyway, it is clear that in our perspective—and, in fact, in the perspectives of most physicists of the eighteenth century, too—this assumption was an unacceptable *ad hoc* hypothesis, which, moreover, was based on a falsified theory—the vortex theory—but this was not the case in Leibniz's mind.

In the end, the reasons that induced Leibniz to develop his hypothesis on *vis viva* can be understood by taking into account his general physical and metaphysical principles, whereas the lack of an answer to Newton's proofs against the stability of the vortices is more difficult to explain, also in a Leibnizian perspective. The only possible answer is that, in his opinion, the proofs by Newton did not challenge his theory: as above explained, Leibniz thought to have shown that, under his hypotheses, the *circulatio harmonica* ensures the planets rotate in their vortices without experiencing any friction and that, hence, the matter of the vortices acts, *as far as the planetary movements are concerned*, as a void space, not as a normal fluid matter. Thence—according to Leibniz—Newton's argument cannot be applied to his vortices.

A further interesting consideration concerns the fact that Leibniz ignored Gregory's critics according to which the *circulatio harmonica*, where the velocity of circulation is inversely as the distance from the centre of the forces, does not fit

[27] The idea that a physical state *A* has a certain *vis viva*, which is typical of *A* in every time is connected to the even more basic idea that, according to Leibniz, a substance has a *vis viva*, which connotes it eternally. The whole question is strictly tied to the difference between a phenomenal level, a dynamical level and a metaphysical level. This difference deals with the relation physics-metaphysics in Leibniz and must not be confused with the difference among kinematical level, dynamical level and physical-structural level, which is internal to Leibniz's physics. Beyond what already told in Chap. 3, I will deal with the relations physics-metaphysics in Leibniz in Chaps. 5–7. In Chaps. 5 and 6 as to gravity and pre-established harmony respectively; while in Chap. 7 the final remarks will be proposed.

with Kepler's third law. Indeed, Leibniz considered his ideas on the *vis vis* as an attempt to justify Kepler's third law inside his system, as an addition to complete the system, but he never took into account the idea to renounce harmonic circulation. While, if Gregory's criticism had been right, he would have had completely abandoned this idea. The thing is that, probably, Gregory misunderstood Leibniz's expression *velocitas circulandi*, thinking that, by this expression, Leibniz was referring to the modulus of the vector velocity and not to the transverse velocity. This is the same misunderstanding by Koyré, pointed out by Aiton. Such interpretation is confirmed by Gregory's assertions that the velocities are as the square root of the distances from the sun, which is true for the modulus of the vector velocity. This is the likely reason why Leibniz did not answer this general objection by Gregory.

Actually, the question is complicated by a further more specific consideration by Gregory (*ivi*, p. 103). He underlined that in paragraph 17 of the *Tentamen*, Leibniz wrote:

> [. . .] at double the distance only the fourth part of the angle is covered in the same element of time, at triple the distance only the ninth.[28]

Such assertion derives from the general proposition expounded by Leibniz at the beginning of the 17th paragraph of *Tentamen*, where we read:

> In equal elements of time the increments of the angles of harmonic circulation are inversely as the squares of the radii.[29]

This implies that the times are as the squares of the distances, which is a clear contradiction with Kepler's third law. The proof of this proposition is based—as Gregory pointed out—on an extension of the harmonic circulation to all the planets. To be clearer: given different planets A, B, C, . . . if Leibniz claims to extend the relation between velocity of circulation and distance from the sun to the orbits of these planets so that, if the planets A has the distance r from the sun and the planet B the distance s, then their velocities of circulations are as $1/r$ and $1/s$, it is possible to deduce the wrong proposition of the paragraph 17. The rest of Leibniz's theory is independent of such an extension, which hence mines only this proposition.

[28] Leibniz (1689, 1860, 1962, VI, p. 155). Original Latin text: "[. . .] in distantia dupla tantum quarta pars anguli eodem temporis elemento absolvatur, in tripla tantum nona". Translation from Bertoloni Meli (1993, p. 136).

[29] *Ivi*, p. 155. Original Latin text: "Aequalibus temporum elementis incrementa angulorum circulationis harmonicae sunt in ratione duplicata reciproca radiorum". Translation from Bertoloni Meli (1993, p. 136). Bertoloni Meli underlines that this proposition, beyond the wrong theoretical presuppositions on which it is based, also relies upon wrong empirical assumptions on the length of the day on each planet (see Bertoloni Meli 1993, p. 136, note 16).

On the other hand, it is strange that, at the beginning of the *Tentamen*, Leibniz mentioned Kepler's third law in the correct form. For, we read:

> The same man [Kepler] found that the periodic times of the several planets of the same system are in the sesquialterate ratio of their mean distances from the Sun [. . .].[30]

While in proposition 17 he reached some conclusions which were in contradiction with Kepler's third law. My interpretation is that what Leibniz wrote in proposition 17 of the *Tentamen* has not to be overestimated inside Leibniz's planetary theory. This proposition looks like a parenthesis added by Leibniz only to complete the picture of his theory. As a matter of fact, it plays no role in the rest of Leibniz's conclusions. The very ideas of Leibniz on the deduction of Kepler's third law are those expounded in the *Illustratio*, where the generalised harmonic circulation (which is typical of proposition 17) is replaced with the hypothesis on the *vis viva* of the planetary orbits.

With regard to Leibniz's assertion that many uncertainties existed on comets' paths, it can seem an inappropriate attempt to defend his own theoretical convictions, and in part it was. Nevertheless, a well-pondered historical judgement cannot leave out of consideration that, between the end of the seventeenth and the beginning of the eighteenth centuries, there were some famous astronomers who even doubted planetary orbits' ellipticity, also independently of the area law problem or of the problems connected to vortex theory. The most famous case is that of Gian Domenico Cassini (1625–1712), who proposed the idea that the planetary orbits were those curves of fourth degree, which are nowadays known as Cassini ovals, whose geometrical characteristic is that, for their points, the product—not the sum, as in the ellipsis—of the distance from two given points, called foci, is constant. Leibniz was deeply interested in Cassini's idea. Cohen has dedicated a paper to this subject.[31] He mainly focused on the correspondence between Leibniz and the Académie royale des Sciences around 1700, but it is necessary to point out that in the *Illustratio* itself, Leibniz wrote:

> Actually, before I had had to discuss all these questions, I would have preferred that all of them were submitted to the observations more diligently, especially as the outstanding astronomer Gian Domenico Cassini proposed ovals of a new kind, and the famous De la Hire, excellent in these studies, was satisfied of no hypothesis.[32]

As a matter of fact, Leibniz's interest in Cassini's ovals dated back at least to the beginning of the decade 1690–1700. For, although Leibniz had in general claimed to consider Kepler's ellipses as the true orbits of the planets, he believed this was an

[30] Leibniz (1689, 1860, 1962, VI, p. 148). Original Latin text: "Idem [Keplerus] deprehendit plures planetas ejusdem systematis habere tempora periodica in sesquiplicata ratione distantiarum mediarum a Sole [. . .]". Translation from Bertoloni Meli (1993, p. 127).

[31] Cohen (1962).

[32] Leibniz (1706, 1860, 1962, VI, pp. 254–255). Original Latin text: "Ego vero antequam haec iterum mihi discutienda fuissent, maluissem ad observationes omnia expansa diligentius, praesertim cum Astronomus summus Joh. Dominicus Cassinus novi generi Ovales ex eo attulerit, et nulla hypothesi contentus videatur Cl. Lahirus in his studiis excellens".

open problem. Three interesting documents are three letters Leibniz wrote respectively to Huygens in April 1692, to De La Hire in October 1697 and to Johann Bernoulli in June 1698.[33] From them, a slight change in Leibniz's opinion appears: in the first letter Leibniz is sceptical about Cassini's new curves, while in the letter to De La Hire, he appears more possibilist. What Leibniz wrote to Huygens confirms a relatively early interest of his in Cassini's ideas, furthermore we learn that he knew Cassini's ovals by means of the *Dictionnaire Mathematique* by Ozanam:

> In his *Dictionnaire Mathematique*, mister Ozanam has expounded a hypothesis by mister Cassini, who, instead of Kepler's ellipses, conceives ellipsoidal figures, where the rectangle [product] of the lines drawn from the two foci to the extremities is equal to a given rectangle [product]. I do not know if he will provide some physical reasons. Waiting for this, I am much satisfied with Kepler's ellipses, as they are in such a good agreement with mechanics that the aberrations derive, rather, from the mutual actions of the planets [. . .].[34]

Seven years later Leibniz wrote to De La Hire:

> I imagine it was worked on the new lines, with which Mister Cassini has replaced Kepler's ellipses and for which he will give a physical cause. The great and illuminating contributions that Mr Cassini gives to astronomy, do not allow me to have any doubt he had considerable reasons to introduce these curves and that he has based on long observations.[35]

Therefore, according to Leibniz and to some other scientists, the ellipticity of the orbits was not to be given for granted. Thus, the basic principles and laws of theoretical astronomy were still under discussion at the beginning of the eighteenth century, even though Newton's theory was progressively imposing. Hence Leibniz's consideration on the uncertainty of comets' paths are not such an *ad hoc* defence, as they could seem in our eyes.

[33] For the letter to Huygens, see LSB, III, 5, pp. 287–291. For that to De La Hire LSB, III, 7, pp. 610–618. For that to Johann Bernoulli LSB, III, 7, pp. 792–797.

[34] LSB, III, 5, pp. 288–289. Original French text: "Mons Osannam a mis dans son *Dictionnaire Mathematique* une hypothese de M. Cassini, qui au lieu des Ellipses de Kepler conçoit des figures Ellipsoides, où le rectangle des droits menées des deux foyers aux extremités est egal à un rectangle donné. Je ne sçay s'il en donnera quelque raison physique. En attendant je trouve les Ellipses de Kepler fort à mon gré, puisqu'elles s'accordent si bien avec la Mechanique, et peutestre que les aberrations viennent des actions des Planetes entre elles [. . .]". Leibniz was surely referring to the following passage of Ozanam's *Dictionnaire* (see Ozanam 1691, p. 436): "Mr. Cassini invented a new kind of ellipsis to represent the movements of the planets and the earth around the sun. This ellipsis is a line of second order, as you will learn from what follows". The description of the curve follows. Original French text: "*Monsieur Cassini* a inventé une nouvelle espece d'Ellipse, pour representer les mouvements des Planetes et de la Terre autour du Soleil. Cette Ellipse est une ligne du second genre, comme vous connoîtrez par sa description qui est telle" (Italics in the text). The most famous work in which Cassini expounded his ovals is Cassini (1693). Hence it is likely that Leibniz, after having learnt by Ozanam about the existence of this curve, read directly Cassini's work published in 1693. The ovals are mentioned in Cassini (1693, p. 36).

[35] LSB, III, 7, p. 618. Original French text: "Je m'imagine qu'on aura travaillé sur les nouvelles lignes que Mons. Cassini substitute aux Ellipses de Kepler et qu'il en donnera des causes physiques. Les grandes lumieres que Mons. Cassini a dans l'Astronomie ne me laissent point douter, qu'il n'ait eu des raisons considerable pour les étabilir, et qu'il ne se fonde sur des longues observations".

Chapter 5
Gravity and Cosmology

Leibniz's planetary theory can be analysed without dealing with his conception concerning the origin of gravity and the causes of the planetary motions. It is possible to assume gravity and such causes as given since their origin and nature do not enter *directly* into the structure of Leibniz's planetary theory. This perspective has been—in a sense—encouraged by Leibniz himself who, at the end of the preface in the *Zweite Bearbeitung* of the *Tentamen* wrote:

> What follows is not based on hypotheses, but is deduced from the phenomena by means of the motion laws. For, even though the sun would attract the planets, it is sufficient we calculate that approaching or moving away, namely distance's increment or decrement, which would exist if the planets were attracted by that prescribed law. [. . .] We will leave to everyone's prudence what can be concluded from here as to the causes of the motions. Perhaps the thing has been developed till the point that the intelligent poet was right when he did not dare to speak anymore to the astronomers: "Talia frustra/Quaerite quos agitat mundi labor, at mihi semper/Tu quaecunque paret tam crebros causa meatus/Ut superi voluere late".[1]

Notwithstanding, Leibniz had dedicated the previous four pages of the *Zweite Bearbeitung* exactly to the problem of gravity and to the relations between gravity and the causes determining the planetary motions. He had proposed a series of hypotheses on this question, admitting that there was no certainty on such a subject. However, he believed that some hypotheses were more plausible and complete than others.

[1] Leibniz (1690?, 1860, 1962, VI, p. 166). Original Latin text: "Quae enim sequuntur, non constant Hypothesibus, sed ex phaenomenis per leges motuum concluduntur; sive enim detur sive non detur attractio planetarum ex sole, sufficit a nobis eum colligi accessum et recessum, hoc est distantiae incrementum vel decrementum, quem haberet si praescripta lege attraherentur. [. . .] Quantum autem hinc de ipsis motuum causis sit concludendum, prudentiae cujusque aestimandum relinquemus, fortasse enim eo res jam perducta est, ut Pöeta intelligens non amplius dicere ausit Astronomis: "Talia frustra/Quaerite quos agitat mundi labor, at mihi semper/Tu quaecunque paret tam crebros causa meatus/Ut superi voluere late"".

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In this chapter I am going to show the complete comprehension of Leibniz's planetary theory cannot prescind from the evolution of his ideas on gravity. Whatever the influence of Newton's *Principia* reading on Leibniz was, the way in which he structured his conceptions on the planetary motion has—at least in part—its origin in the series of ideas on gravity he had developed from his youth.

Actually, the research on the cause of gravity characterizes Leibniz's production from his early works and allows us to follow the development of Leibniz's physics. For, starting from his early *Hypothesis Physica nova* (1671) till the correspondence with Clarke (1715–1716), Leibniz dedicated some sections of his works to gravity. He changed his mind on gravity in the course of his scientific career, but his way to frame the problem is—more or less—the same: 1) he presented more than one hypothesis; 2) he recognized that there is no certainty and, after all; 3) he chose the hypothesis he believed to be the most likely. All the hypotheses are based upon mechanical models. The connection between gravity and the causes of the planetary motions was already felt as an important problem in *Hypothesis Physica nova*. Leibniz recognized the *similarity* (in what follows I will clarify the meaning of this word) between these two forces,[2] but did not claim they are the same force.

In part, the interest in gravity was due to the fact that numerous Leibniz's works concerning physics dealt with the foundations of physics, or, at least, part of these works did. Since Leibniz changed often opinion on the foundations of physics and on the nature of the fundamental interactions—gravity, magnetism, elasticity, cohesion—, the same subjects were treated in different perspectives in the course of Leibniz's life, although a basic idea and a basic aim remained unchanged:

1) Idea: these interactions have to be explained by means of the movement of a fluid (aether) which surrounds the bodies and acts with them in different manners so to produce such interactions;
2) Aim: to provide a general theory inside which all the fundamental interactions could be explained. This is the vortex theory, with the necessary and appropriate modifications proposed by Leibniz in his works.

Due to this picture, Leibniz tried to find a series of connections between gravity, magnetism, elasticity and cohesion, interactions which—according to his mind—could be attributed to a common root.

On the other hand—especially after the publication of Newton's *Principia*—Leibniz tried to clarify the connections between gravity and the forces responsible for the planetary motions in a more profound and specific way. This became an almost unavoidable purpose for his research: the action at a distance had to be refused for the well known reasons. However, if no form of the vortex theory—and

[2] In these pages I have referred to "causes of planetary motion" rather than to "forces". My choice depends on the well known ambiguity of the word "force". However, in what follows I will use sometimes the words "force" and "interaction", because, in many circumstances, these words make the meaning of a sentence clearer than the locution "causes of planetary motion". Anyway, in this context, "force" has to be interpreted as cause of movement in a general meaning, hence neither as Newtonian force nor as Leibnitian *vis viva*. Therefore there is no ambiguity.

more in general of any theory in which all interactions are transmitted mechanically—could offer a plausible gravitational theory and an appropriate theory of planetary motions' causes, the vortex theory would have lost its reliability. The physicists would have embraced the action at a distance, or, at least, would have considered the problem of the origin of the interactions as a problem which could not be solved inside physics, or which was not necessary to solve. Therefore, in Leibniz's perspective, the stakes were high: Newton had provided a general theory in which gravity is responsible both for the fall of the bodies on the earth and for the movements of the planets in the skies; furthermore he had proved that—under enough general conditions—the vortices could not be stable. The weak point of his theory was that gravity was not explained, but treated as an action at a distance.[3] It is worth highlighting that, although the problem of action at a distance-gravity was basically faced after the publication of Newton's *Principia*, the idea that gravity could be a mutual attraction between bodies and not an action due to the movement of a fluid surrounding the earth was discussed several years before the publication of Newton's masterpiece. A quite interesting example is offered by what follows: between the 7th August 1669 and the 20th November 1669, a discussion concerning gravity was developed at the Académie Royale des Sciences, Paris. Huygens was the main protagonist of this discussion—to which we will refer several times—. The others were Roberval, Frenicle de Bessy, Mariotte, Hamel, Perrault.[4] As well known, Huygens developed the idea that gravity was due to the centrifugal force produced by the movement of an aether surrounding the earth. But not all the participants shared Huygens' point of view: Roberval mentioned three opinions about gravity: according to the first one, gravity is an inner property of the bodies, according to the second one gravity is *a reciprocal attraction among bodies, which tends to unify them.* The third opinion is that mechanistic proposed by Descartes and shared by Huygens after conspicuous modifications of Descartes' doctrine. Roberval explicitly claimed that, if one admits occulte qualities—namely qualities, which cannot be perceived by our senses—the second opinion is the preferable one because it does not need to postulate the existence of an intangible fluid (*Ivi*, pp. 629). In a further phase of this discussion Roberval and Mariotte claimed that Huygens "excludes the attractive and repulsive qualities from nature without a proof and wants to introduce the sole magnitudes, figures and movements".[5]

[3] I am not claiming Newton considered gravity as an action at a distance during the whole of his scientific career. Newton's conception of gravity is a complex problem on which a huge amount of literature exists. This problem has nothing to do with the context I am dealing with. It is sufficient to say that Newton's treatment of gravity is compatible with the idea gravity is an immediate action at a distance. This was enough to explain Leibniz's attempts to offer a mechanic explanation of gravity.

[4] This debate has been reported in the XIX volume of Huygens' works. See Huygens (1669, 1937, pp. 628–645).

[5] In Huygens (1669, 1937, p. 640). Original French text: "D'abord il exclud de la nature sans preuve les qualites attractives et expulsives et il veut introduire sans fondement les seules grandeurs, les figures et le mouvement".

Therefore, both in case Roberval's and Mariotte's opinions represented an inheritance of the old tradition of the occult qualities or a new conception of the physical qualities—I do not enter into this discussion-, it is clear that a supporter of mechanistic philosophy had to face these standpoints. And Leibniz did. Thus, Leibniz had to prove that gravity and the force or forces responsible for the planetary motions can be treated in a satisfactory manner also inside vortex theory. Not only: vortex theory can offer some plausible—though not definitive—ideas on the origin of such interactions. Therefore, it is comprehensible that Leibniz dealt with these problems also before the publications of the *Principia*. However, Newton's work changed completely the situation. Thus, starting from the *Tentamen*, Leibniz dedicated a long series of considerations to gravity, and above all, to the possible relations between gravity and planetary motion. In the previous contributions, gravity had been profoundly dealt with, but the links between gravity and the causes of the planetary motions were less strong than in the *Tentamen* and in the works written after the *Tentamen*, even though Leibniz was always sensible to this problem, due to his particular interests and to the debate on gravity, which had been developed in the scientific community. For, the question of gravity had been one of the main subjects inside vortex theory starting from Descartes' original formulation. Leibniz was deeply influenced by Descartes and, later on, by Huygens' vortex theory. Especially in *Hypothesis Physica nova*, he was also influenced by Hobbes and other philosophers and physicists. However, my aim, in this context, is not to compare Leibniz's vortex theory with Descartes' or Huygens' or to detect the influence that other authors exerted on Leibniz. My purpose is to give an idea of the internal change of Leibniz's conceptions of gravity in function of the development of his planetary theory conceived as a possible answer to Newton's *Principia*. Therefore I will basically follow the internal development of Leibniz's thought in relation to this problem, referring, for the other questions, to the literature.[6]

5.1 *Hypothesis Physica nova*

The *Theoria motus concreti*, first section of *Hypothesis Physica nova*, presents a series of argumentations which connect gravity and planetery motion. The scenario is the following: Leibniz assumed as given two physical conditions:

[6] I provide here some references concerning the problem of gravity in Leibniz without any claim to be exhaustive: Attfield (2005), Duchesneau (1994), Engfer (1987), Gale (1988), Garber (1994, 2006, 2009), Gregory (2007), Gueroult (1934), Janiak (2007), Koffi (2003), McRae (1994), Suisky (2009). This book is on Euler, but there are many precious indications as to Leibniz's gravitational theory; Vailati (1997), Vincent (2002), Zehe (1980). These indications concern papers or books, part of which regard Leibniz's ideas on gravity. I do not mention the huge amount of publications which refer to the critics addressed by Leibniz to Newton's theory of gravity, but which do not face the content of Leibniz's gravity theory.

1) existence of the aether;
2) motion of the sun and of the earth, as well as of all other possible existing celestial bodies, around an axis passing through their centres.[7]

This motion, Leibniz claimed, is a necessary condition for the existence itself of the bodies. Without this movement, the celestial bodies would have no cohesion ("what is at rest has no cohesion" *Ivi*, § 2, p. 20, Leibniz wrote[8]).

These properties are explained in the first three paragraphs of the *Hypothesis*.

Once introduced the initial hypotheses, the treatment begins with § 4, which is one of the most interesting for our aims. Leibniz reminded the reader that Hobbes and Torricelli had recognized only the motion of the celestial bodies around their axes and tried to explain a series of phenomena by means of this hypothesis.[9] In particular, Leibniz underlined, if their hypotheses were true, the cause of the rotation of the planets around the sun and of the gravity on earth would be the same. He wrote:

> It would follow that, as a stone tends to the Earth, so the earth and all the other planets tend to the sun; and it cannot be told that efficacy is diminished by distance, because, on the contrary, in this hypothesis, efficacy is increased by the rays of the bigger circle because of the bigger circumference.[10]

Therefore, if Hobbes' and Torricelli's considerations worked, a unified theory of many phenomena could be provided; but this is not the case. Leibniz pointed out that every motion, different from a motion which returns on itself, cannot be explained only by the mentioned hypothesis. For Leibniz wrote:

> But in the sun, it is necessary to suppose another motion, too, which gets out from itself. This is the cause from which, in the world, the motion which does not return in itself derives: for, the motion around its centre does not get out from itself.[11]

Leibniz's words can be interpreted as follows: the circular motions can be, in general, explained by supposing a body which rotates around its axis and an aether

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[7] Leibniz speaks of "motum circa proprium centrum" (Leibniz 1671, 1860, 1962, VI, § 3, p. 20), but he is clearly referring to an axis passing through the centre.

[8] Original Latin text: "[. . .] nulla autem sit cohesio quiescentis".

[9] The influence exerted by Hobbes on Leibniz's early speculations on physics is an explored subject in the literature. See, only to give an idea, Duchesneau (1994), where a long series of references are present, too. In this context it is interesting enough Leibniz's reference to Torricelli because in Torricelli's works available to Leibniz there is no passage which could induce Leibniz to express an opinion as the one we read. However, Leibniz himself dispels any doubt. For, he wrote (Leibniz 1689, 1860, 1962, VI, p. 149) to have drawn Torricelli's assertions from the *Journal des vojages* by Balthasar de Monconys (1611–1665) (see de Monconys 1666, posthumous). Monconys claimed to have met Torricelli in Florance in 1646 and, on that occasion, Torricelli expressed him the opinion reported in the *Journal* and to which Leibniz referred.

[10] Leibniz (1671, 1860, 1962, VI, p. 21). Original Latin text: "Sequetur enim ut lapis ad terram, ita terram ceterosque planetas ad solem tendere; nec dici potest distantia minui efficaciam, cum contra in hac hypothesi ob majorem majoris radiis circulum augeatur".

[11] *Ivi*, pp. 20–21. Original Latin text: "Sed in sole simul et alius motus supponendus est, quo agat extra se, unde causa in mundo motus in se non redeuntis derivetur: motus enim circa proprium Centrum extra se non agit."

surrounding the body which moves circularly around the body itself, but the rectilinear motions or the motions in which a rectilinear component exists cannot.

It is worth highlighting this train of though is similar to the one that brough Leibniz to distinguish between *Circulatio harmonica* and *motus paracentricus*. In terms of what expounded in the *Hypothesis*, the *motus paracentricus*, which is a rectilinear motion, cannot be explained only by means of circular motions. Although, in a more mature phase of his thought, Leibniz rejected many of the conceptions to which he had adhered in the *Hypothesis*, the need to consider a motion which is different from the circulation of the aether around the sun and the planets remained a significant point in his theory.

This motion is given by little particles of the sun (or of any radiating body) which leave sun's surface. The motion of these particles is not uniform either as far as its speed or its direction are concerned. If the motions of all the particles were rectilinear, the sun would disappear because would lose all its particles. Hence Leibniz supposed that many of the particles return to the sun surface after having described curved lines with a not uniform motion and with a trajectory which is completely different from aether's. Only a limited number of particles are expelled once and for ever out of the sun through a rectilinear trajectory. Leibniz, after this description, made two assertions whose historical and conceptual interest is really relevant.

First: in every point the aether is agitated by a ray of particles emitted from the sun. This does not mean that the particles of the sun are present *in actu*, but that their action is present. Leibniz wrote:

> And, as no perceptible point around the sun can be given, which arrives until the earth and farther on, unless a sun-ray, namely the movement of the aether caused by a particle emitted by the sun in a straight line (if not the particle itself), reaches that point in any perceptible instant.[12]

Second: the sun cannot shine *ab aeterno*, unless new particles—it is not clear if created or converging to the sun from the external world—compensate for the loss of the particles which leave the sun without coming back on its surface. We read:

> By the way, from these propositions, as I remember incidentally, it can be demonstrated, by necessity, the impossibility sun shines from eternity, unless new material restores it perpetually.[13]

The first quotation is fundamental for the whole theory expounded in the *Hypothesis* because it is the initial assertion by which Leibniz clarified the way in which the earth was formed.[14] The second assertion implies that, according to

[12] *Ivi*, p. 21. Original Latin text: "Et tot quidem, ut non possit dari punctum sensibile circa solem ad tellurem usque et ultra, ad quod non quolibet instanti sensibili radius aliquis solis, id est, aetheris agitatio per emissam a sole recta linea partem (etsi non pars ipsa) perveniat".

[13] *Ivi*, p. 22. Original Latin text: "Ceterum ex his, ut obiter admoneam, necessario demonstrari potest, impossibile esse, ut sol luxerit ab aeterno, nisi sit unde perpetuo reparetur".

[14] The first quotation is also interesting from a historiographic point of view because it can be connected—though not identified—with some conceptions of Kepler, in particular that of *species*

Leibniz, the celestial bodies have a history of which physics has to provide a plausible reconstruction. We have seen that Leibniz, also in further phases of his scientific career, took into account the evolution of the solar system. Hence, the physical time of which Leibniz was thinking, cannot be identified with the reversible time of the Newtonian physics.[15] In the *Hypothesis* things work like this: at the beginning the earth globe was composed of a homogeneous matter. The rays of the sun hit the earth with different angles and intensity so that the aether can penetrate the different parts of the earth in function of such angles and intensities—actually Leibniz did not supply sufficient specifications either a quantitative treatment of these interactions-. The parts in which a little quantity of aether penetrated are the most hard and heavy and form the element "earth", the "water" has a middle quantity of aether's particles and in the "air" the aether particles are abundant.

The interactions between the aether and its circular movement with the movement of the sun light's rays can explain the way in which the other planets have been formed and above all *the movement of the earth around the sun* and *of the moon around the earth*. Leibniz is clear: the main problem of the system of the world is to understand how, from the rotation of the sun around its axis and from the rectilinear action of its particles towards the earth, the movement of the earth around the sun is born. An analogous problem is to explain the rotation of the moon around the earth, also taking into account the reflexion of the sun rays. Leibniz claimed:

> [. . .] this pertains to the doctrine of the system of the world; for, because of the same reason by which, from the rotation of the sun around its centre, which concurs with its rectilinear action towards the earth, the motion of the earth around the sun originates, and from the motion of the earth around its centre, which concurs with its rectlinear action of reflecting solar light towards the moon, moon's motion around the earth originates, it is probably allowed us to claim that, for the other planets, things procede in the same way.[16]

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immateriata, of Descartes and of Huygens. Since in the context of Leibniz's planetary theory, the comparison with Kepler's conception is particularly significant, I will deal with it in the following Chap. 6. With regard to Huygens, between the 7th August 1669 and the 20th November 1669, a discussion concerning gravity was developed at the Académie Royale des Sciences, Paris, as explained in the running text. Huygens expounded a series of ideas on gravity, which are similar to some ones of those explained by Leibniz in the *Hypothesis*, where Leibniz seems to rework, with some modifications, Huygens' conception that the earth is formed by the action of an aethereal fluid matter on a heavier matter, which, because of the fast circular movement of the aethereal fluid, is pushed towards a centre, around which it is condensed, giving origin to the terrestrial globe, see Huygens (1669, 1937, pp. 635–636). Huygens will reformulate the ideas expounded in the late 1660s in a contribution, where his complete theory of gravity is expounded: *Discours de la cause de la pesanteur* (Huygens 1690, 1944, pp. 451–499).

[15] The aim of this work is not to deal with Leibniz's conception of time. However, it is worth highlighting the unitariness of Leibniz's speculation. For, the fact that the time of physics is a historical time is compatible with the idea that no absolute time exists.

[16] *Ivi*, pp. 22–23. Original Latin text: "[. . .] pertinent talia ad doctrinam de systemate mundi; quemadmodum id quoque, qua ratione ex rotatione solis circa proprium centrum concurrente ejus actione rectilinea in terram oriatur motus terrae circa solem, et ex motu terrae circa proprium

This means that Leibniz was interested in the problems of the planetary motions starting from *Hypothesis*. He did not develop his planetary theory on that occasion, but his two main lines of thought can be found in this early work, although in an embryonic form:

1) the rotational motion of the sun will be developed in the *circulatio harmonica*;
2) the actions of the particles responsible for the rectilinear movement correspond to that part of Leibniz's planetary theory concerning the *motus paracentricus*.

This idea to decompose the planetary motion in a rectilinear and a circular component is hence already present in the *Hypothesis*, even though the identification of the cause of the rectilinear motions with the solar light will be modified in the successive works concerning the planetary theory. Furthermore the theory of the *bullae*, expounded in the *Hypothesis* will be abandoned. Nevertheless, at least the initial part of the *Hypothesis* has to be included in a description of Leibniz's ideas on the planetary motions. Some basic conceptions persisted, hence, from Leibniz's early works till the ripest ones.

With regard to the earth, Leibniz hypothesized that it is surrounded by a subtle aether, which, under the action of the sun light, rotates in the opposite direction of the earth movement around its axis. Hence the aether moves from East to West. Under the action of the sun rays the aether enters in contact with the earth primitive matter and forms bubbles (*bullae*) of different density and size according to the different kind of movements impressed by the sun light to the aether. Leibniz (*Ivi*, p. 23) claimed the bubbles are the seeds of all things ("Hae jam bullae sunt semina rerum [...]"). The water and the air are composed of bubbles, which are different according to the elements they compose. Here Leibniz provided an important distinction: the air is heavy and composed of bubbes, whereas the aether is not heavy, rather, with its movement, it is the cause of gravity. Indeed Leibniz wrote:

> I distinguish the air from the aether as the air is heavy, while the aether produces gravity by means of its circulation.[17]

This introduces the problem of gravity on the earth: according to Leibniz, the earth is composed of glass bubbles. He imagined that the light of the sun rays, which arrived directly at the surface of the earth, transformed the original matter of the earth in a series of glass bubbles of different sizes which compose the element "earth". The association of ideas is clear: the glass is formed by the solidification of a liquid material without crystallization. In its turn the liquid material derives from a previously solid one, made liquid by the use of high temperatures. The sun light produces high temperatures, hence it can make liquid a solid material, which is then ready to become a bubble of glass under certain conditions. This is the constitution of the earth. Our planet has what Leibniz called a general affection (*Ivi*, p. 25, §

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centrum, concurrente ejus lucem solarem reflectentis actione rectilinea in lunam, motus lunae circa terram; quae de caeteris planetis eadem probabilitate dicere licet [...]".

[17] *Ivi*, p. 24. Original Latin text: "[...] aërem enim in eo ab aethere distinguo, quod aër est gravis, aether circulatione sua causa gravitatis".

15, line 4): gravity, which is the fundamental cause to understand the motions on the earth. Because of its importance, a duty of the physicists is to provide a mechanical explanation to gravity (*ivi*, p. 25).

Thus, in paragraph 16, Leibniz claimed that gravity originates from the circulation of the aether *around the earth, on the earth and across the earth*, according to the above described mechanism which is based on the interactions between rays of solar light, aether and original undifferentiated matter of the earth. The earth descends in the water as the earth contains less aether, analogously the water in the air. The aether can, hence, be interpreted as a principle of *levitas*, because the lack of aether determines the relative gravity of the bodies. Leibniz wrote:

> Gravity originates from aether's circulation around, inside and trough the earth. The cause of gravity was given in the previous § 9 and 10. Furthermore, aether penetrates water and air because they are more porous. Therefore the earth descends in the water as it contains more aether, which is not in the appropriate place, than water. Due to the same reason, the water descends in the air.[18]

The role of the aether is hence of being the element of *levitas*. Thus, it enters actively into the mechanism which determines the fall of the bodies: in normal conditions the water gravitates on the earth and the air on the water. The circulation of the aether around the earth is not perturbed. Let us suppose that, for any reason, some particles of earth are in the air. What happens? The circulation of the aether tends to expel every object which can perturb it. Leibniz introduced here an implicit principle of minimum: a direction exists in which the initial state without perturbations is restored as quickly as possible with the further condition that the body meets the least possible resistance by the aether. This direction is downwards. This is why every body falls on the earth. The double role of the aether is: 1) to be element of levity; 2) to be cause of the expulsive motion of a body perturbing aether's circulation. Thus, aether determines gravity. We read:

> This is the reason why air, water and earth gravitate in the aether: for, they are pushed down by aether's circulation. Since they would interfere with the circulation, they are expelled; this happens not upwards, as they will interfere even more with this circulation (because the spherical surfaces increase as the squares of the diameters, not in the same ratio as the diameters; thence, the inequality of the sections which act on the same body is bigger), rather downwards, this means they descend.[19]

[18] *Ivi*, p. 25. Original Latin text: "Gravitas oritur ex circulatione aetheris circa terram, in terra, per terram, de cujus causa supra § 9 et 10. Is porro maxime aquam at aërem penetrat, quippe porosiores. Unde terra in aqua, nisi cum plus aetheris superficiarii continet, quam ipsa aqua, aqua in aëre descendit".

[19] *Ivi*, pp. 25–26. Original Latin text: "Haec jam ratio est, cur et aër, et aqua, et terra in aethere gravitent: nam circulatione ejus dejiciuntur. Cum enim turbent circulationem, expelluntur; non sursum, nam eo magis turbabunt (quia superficies sphaericae crescunt in duplicata ratione diametrorum, non in eadem cum diametris ratione; ac proinde sectionum quoque in idem corpus agentium inaequalitas major evenit), ergo deorsum, id est descendent".

The general conception expressed by Leibniz is clear, but the words into the round brackets seems to me not completely clear. Duchesneau interprets these words claiming:

> Then Leibniz proposes a law, which is false but logic, according to which the increment of gravity is as the duplicate reason of ether's circulation-sphere diameter.[20]

In this case, Duchesneau's statement does not seem to fully clarify Leibniz's thought. I think that Leibniz was imagining a situation like this: the aether surrounding earth moves circularly with a constant angular speed. In this case, for any given aether's layer, the centrifugal force F is as its distance from earth's centre. On the other hand, a body in the aether perturbs aether's movement and, hence, it is pushed downwards by a tendency of intensity T. Gravity will be given by the difference T-F. While calculating T, Leibniz supposed that, imagining the aether divided into concentric spheres, a body does not perturb only a point, but a part of the spherical surface, whose area is a function of radius' square. Thence, the perturbing action of a body on aether is as the square of the distance body-earth centre. Therefore, the global action of gravity is as the difference between the radius' square and the radius of each aether's spherical layer. This is the intuitive argument used by Leibniz. I do not enter into the inappropriateness of this reasoning, as it is evident.

Rather, it is worth pointing out the connections between the causes of planetary movements and the causes of gravity:

1) the circulation of the aether around the sun, which seems due to the rotation of the sun around its axis (Leibniz was not explicit on this point, probably it was to give for grated) determines the mean motion of the planets. This idea is expressed *in nuce* in the *Hypothesis* and it is also one of the fundamental bases of the theory explained in the *Tentamen*;
2) the rotation of the sun determines the way in which the light rays leave the sun and have an impact on the terrestrial aether;
3) the interaction between sun rays, terrestrial aether and earth's original matter determines the kind of bubbles of the terrestrial materials;
4) every kind of material has a specific weight which is given by the quantity of aether in its bubbles (here the connections with Archimedes principle of hydrodynamic is evident)[21];
5) gravity depends on the circulation of aether by means of the described mechanism.

[20] Duchesneau (1994, p. 80). Original French text: "Leibniz proposes alors une loi fausse mais logique de l'accroissement de la gravité en raison double du diamètre de la sphère de circulation de l'éther".

[21] Leibniz developed it in the § 24 (*Ivi*, pp. 31–32). He analysed the action of gravity in different terrestrial phenomena and its connection with elasticity. An interesting summary of these problems is offered by § 58 (*ivi*, pp. 50–51). I do not enter into these questions because they are not particularly significant for my aim.

Therefore the motions of the celestial bodies around the sun, the motion of the sun itself around its axis and the problem of gravity are interconnected from the beginning of Leibniz's thought. Perhaps the only question, which Leibniz did not outline in the *Hypothesis* concerns the fact that the distance sun-planet is not constant.

It is well known that Leibniz himself criticized many aspects of his early work. On the other hand, the historical and conceptual bases of the theory are interesting. In part I will deal with them in the next section, in part I refer to the literature, because, as previously pointed out, my aim is, in this section, to follow the internal development of Leibniz's train of thoughts in order to prove that many of the ideas he carried out in the *Tentamen* and specified in numerous letters and in the *Illustratio* were present from the beginning of his speculation, although embryonically. In particular: the links between planetary theory and gravitational theory are so narrow, that it is difficult to believe Leibniz thought to develop the one without taking into account the other, too.

The way in which Leibniz began the *Conclusio* (*ivi*, pp. 58–59) to his work is the most significant manner to clarify the links he saw between gravity and system of the world. For, he wrote: every *globus mundanus* (probably to translate as "celestial body") rotates around its axis; only the sun, with its light rays, exerts a rectilinear action. The movement of the aether allows us to deduce the Copernican system (evidently considering the paths of the planets as circular orbits). Gravity and elastic force depend on the aether. Thence, the inclusion of the system of the world and of gravity in a sole theory was an idea Leibniz cultivated from the initial phases of his speculation.

5.2 1677: Letter to Honoratus Fabri

The letter to Honoratus Fabri written around May 1677,[22] is one of the most interesting documents to understand the connections between Leibniz's planetary theory and gravity as well as to fully catch unitariness of Leibniz's project. This long letter was written six years after the publication of the *Hypothesis*. From a conceptual standpoint, it is divided into three sections: an introduction, 21 propositions and a conclusion. In the initial part, Leibniz claimed that, when he wrote the *Hypothesis* his knowledge of mathematics was not sufficient and that he had assumed possible causes as true causes, whereas the true causes were in front of his eyes (Leibniz 1677, 1860, 1962, VI, p. 85). Leibniz abandoned the bubbles theory, but the general considerations about the system of the world and the way in which he thought the interactions are spread can be interpreted as an evolution of the paradigm expounded in the *Hypothesis*, not as the creation of a new paradigm. For:

[22] This letter can be consulted in Leibniz (1677, 1860, 1962, VI, pp. 81–98) and in LSB, II, 1, pp. 441–466.

1) the fluid which surrounds us is responsible for the action of the planets and for the spread of the solar light;
2) it is moved by motions of various origins. However:
3) all these motions tend to the uniformity;
4) the most important of them is the movement by which the solar light surrounds the earth every day;
5) all these movements have to be explained by mechanical causes;
6) gravity, elasticity and magnetism are direct consequences of such motions.

In the introduction Leibniz clearly expounded this programme, which he tried to develop in the 21 propositions. For, we read:

> And so, every fluid surrounding us is excited by motions, which are caused, first of all, by the action of the wandering celestial bodies and by sun light. They are different as to their origin, but, if considered together, tend to the equality. Among these motions, firstly, that rapid enough motion stands out, by which light turns around the earth every day. By means of laws drawn from mechanics, I wanted to look for the consequences of these causes, which are so powerful and largely spread. It seems to me that, among these consequences, I have also found Gravity, that force which is called elastic, the direction of the magnet, as well as many other natural phenomena.[23]

Unitariness of Leibniz's project is evident from this quotation. He developed his reasoning as follows: in proposition 1, Leibniz claimed that the world surrounding the planets has to be considered full. The argument by which this thesis is proved is interesting. We could summarize like this: we see light everywhere, but light needs an intermediary means to be transmitted, hence the void does not exist. Indeed, Leibniz wrote:

> Everywhere a *lux* can be seen or a *lumen* can pass, a body necessarily exists.[24]

In proposition 2, Leibniz claimed that every motion in a full liquid is transmitted everywhere. For:

1) if the motion is spread by means of closed trajectories, so that the initial point of the trajectory is the same as the final one and the matter of the fluid is transported along the trajectories, then every particle of the fluid pushes the following one, so that the movement is spread along the whole trajectory;

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[23] *Ivi*, p. 85. Original Latin text: "Itaque cum constet astrorum imprimis errantium actione atque luce solis fluidum omne circa nos motibus origine quidem variis, attamen in aequabilitatem compositis cieri, ex quibus ille imprimis motus eminet satis rapidus, quo lux quotidie tellurem ambit; volui harum causarum tam potentium tamque late fusarum consequantias scrutari adhibitis Mechanices legibus. Has inter consequentias visus sum mihi et Gravitatem et vim quam Elasticam vocant et Magnetis directionem, et multa alia naturae phenomena reperisse".

[24] *Ivi*, p. 86. Original Latin text: "Ubicunque autem lux videri vel lumen transire potest, corpus esse necesse est". It is worth underlying the difference between *lux* and *lumen*, which is typical of medieval and early-modern age theory of light and of vision. *Lux* is the light we see, we perceive, connected to the subjective act of the vision, whereas *lumen* is a corporeal entity that nowadays we call light-ray. It is not by chance that Leibniz used the verb *video* for the *lux* and *transeo* for the *lumen*. This subject is well known among the historians of medieval and modern age science.

2) on the other hand, if there is no transport of matter among the different layers of the fluid, but the motion is originated by the rotation around an axis, the motion is spread everywhere because of the tendency of the particles to escape along the tangent. The particles of a layer are prevented to escape along the tangent by the particles of the superior layers, so that there is no translatory motion and the motion is hence circular.

In proposition 3, Leibniz claimed that every liquid or fluid has an internal motion. It is clear that Leibniz is thinking here of the solar aether in which the planets move. For, he wrote, as a brief proof of this proposition:

> Indeed, the planets are moved, and certainly this happens in a *plenum,* for proposition 1, so that their motion is propagated till reaching us, according to proposition 2.[25]

Proposition 6 is fundamental for the scenario I am tracing. Here, Leibniz stated that, if in a homogeneous fluid moving with a uniform circular motion, a body is inserted so that the original motion is perturbed, there is the tendency to reach a new *status* of uniform motion. Leibniz wrote:

> Everywhere the motion is perturbed, there is a tendency to uniformity.[26]

As we have seen, when Leibniz tried to answer Gregory's objections, the fact that the *circulatio harmonica* restored its regular motion quite quickly played an important role (see Sect. 4.2.2, item 2). Proposition 6 of the letter to Fabri expounds the same idea in a slightly different manner. Hence, beyond the way of expression, this concept was already present in Leibniz's argumentation 29 years before the *Illustratio* and exactly in the same context: while referring to the motion of the sun aether, not of a generic fluid, even though the proposition is—according to Leibniz—valid for every fluid. This means that a conspicuous part of the letter to Fabri has strong connections with Leibniz's ideas on planetary theory.

Propositions 7, 8 and 9 are something strange in our perspective. Leibniz explained that if a fluid F is surrounded by a different homogeneous fluid G, then F tends to condensate in circular drops (*guttae*, prop. 7) and the solids surrounded by the fluids tend to assume a spherical form. This should explain the spherical form of the earth (prop. 9). Hence the form of the earth depends on the interaction between its original matter and the solar aethereal vortex. The idea of the drops looks like an attempt to replace the theory of the bubbles (*bullae*) expounded in the *Hypothesis*.

The problem of gravity is dealt with in a brief consideration in proposition 10 and in the propositions 14–17, whose content is quite interesting. In proposition 10, Leibniz supplied an explanation similar to that expounded in the *Hypothesis*: if a body is in the atmosphere, it perturbs the circulation of the aether and, hence, due to the tendency to reach a new equilibrium, the body is pushed down by an action

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[25] *Ivi*, p. 86. Original Latin text: "Nam moventur Planetae et quidem in loco [. . .] pleno per prop. 1, unde eorum motus ad nos propagatur per prop. 2".

[26] *Ivi*, p. 87. Original Latin text : "Ubicunque motus est turbatus, conatus est ad aequabilitatem".

we call gravity. The different specific weights of the materials are also explained in terms of aether: the less aether one body contains, the heavier is the body.[27] However, once given this general explanation, Leibniz posed the problem to make it coherent with a kinematical consideration: if—as Leibniz admitted—the earth rotates around its axis, the bodies on the earth surface have the tendency to escape along the tangent. Each theory—without excluding Copernicus' and Kepler's—which admits the diurnal rotation of the earth has to face this problem. Therefore, it is necessary to explain how gravity can oppose the tendency to recede along the tangent. Leibniz was completely clear on this problem:

> Therefore, in this hypothesis, it is necessary a force (*vim*), which maintains the objects on the earth, exists. This force must be stronger than earth's force of receding.[28]

Leibniz's answer runs like this: if the attractive effect is due to the rotation of the earth, which is responsible for the tendency to escape along the tangent, that is if the attractive effect depends on the diurnal rotation of the earth, it is necessary that little solid and insensible corpuscles exist around the earth. Although insensible, these corpuscles are dense (the adjective is *creber*, *ivi*, p. 90). Given the same volume, their number is bigger in the air than in a stone. These subtle corpuscles tend to be rejected by the earth movement around its axis more strongly than the more solid (the adjective is *crassus*) corpuscles composing the stones or other materials, which are hence pushed downwards and maintained on earth's surface.[29] As a further specification: the subtle corpuscles have a tendency to recede along the tangent which is stronger than that of the bigger corpuscles. Leibniz imagined that this tendency outwards can produce an action directed inwards on the more solid material which is under the insensible corpuscles. This is a possible explanation of how the outwards tendency due to the rotation of the earth around its axis can produce an inwards tendency of the layers, which are nearer to earth's surface. By other words: the centrifugal tendency produces a centripetal action. However, Leibniz added, there are two problems: 1) why do not the subtle corpuscles fly

[27] Leibniz wrote: "[. . .] therefore the bodies which are more solid and which contain a less quantity of suble and ethereal material and a bigger quantity of thick and earthy material, are heavier than the others". Original Latin text: "[. . .] unde et solidiora ac minus subtilis atque aetherei, plus crassi atque terrei continentia, aliis graviora sunt." (*Ivi*, p. 88).

[28] *Ivi*, p. 90. Original Latin text: "Necesse est ergo in ea Hypothesi esse vim retinentem vi terrae rejicientis fortiorem".

[29] I mention Leibniz's quotation because it will be important in an argumentation explained in the following Sect. 5.3: "If this is the same receding force (*vis*) of the earth, then it is necessary we suppose that little solid, insensible, but dense corpuscles exist around the earth, so that in a given space, the density of these little particles is less in a stone than in the air. In this way, it will happen that these subtle, solid parts, rather than the heavy ones, recede. Therefore those heavy will be pushed downwards and kept on the earth". Original Latin text: "Si est ipsa vis terrae rejicens, tunc necesse est, ut ponamus esse circa terram corpuscula solida exigua atque insensibilia sed crebra, ita ut sit minus soliditatis exiguarum partium in lapide, quam in aëre paris spatii; ita enim fiet ut potius solida illa subtilia rejiciantur prae crassis, ac proinde crassa deprimantur et retineantur." (*Ivi*, p. 90).

away along the tangent?; 2) if the only acting force was that due to the diurnal movement of the earth, then gravity should be directed towards the axis and not towards the centre of the earth. The conclusion of this long reasoning is quite interesting: gravity is not due to the local effect of earth's diurnal motion, but to the general effect of the sun light on the aether. Leibniz had already embraced this hypothesis, but now he is sure it is not a hypothesis, but the truth. For, he wrote:

> Therefore, finally, it is necessary to resort to our cause of gravity by means of proposition 10, which is not based on a hypothesis, but on a sure demonstration. This cause not only formed the earth, but also keeps it unified, and constrains the whole material surrounding the earth inside narrow limits and joins it in a whole.[30]

In propositions 16 and 17, Leibniz explained why the action of sun light pushes the bodies towards the centre of the earth. This is a more refined explanation of what he had already outlined in the *Hypothesis*: the sun light reaches the earth quite quickly and with a great force ("vis ejus maxima", *Ivi*, p. 91). Because of this, each sun-light-ray—as far as its effect on the aether surrounding the earth is concerned—can be considered as a solid stick of infinitesimal section which rotates in agreement with sun's rotation around its axis (proposition 16). What is the effect of all the solar rays on the aether surrounding the earth? The answer is in proposition 17, where Leibniz claimed that the global action of light, which hits the earth at the equator and along the parallels is, in fact, directed along the meridians, and hence towards the centre of the earth. The reasoning by Leibniz is divided into two arguments. The former is—as a matter of fact—a hypothesis. The latter is a rather complicated reasoning which deserves an explanation. For, according to Leibniz:

1) *sun light reaches the earth with rays which are parallel to the equator*; but
2) *this does not mean that gravity acts along the parallels*. Indeed, if a body is over earth's surface, aether tends to restore the equilibrium perturbed by the body. This means that aether pushes the body towards the zones where the speed has a minimum. These zones are three: the centre of the earth and the two poles. Since the centre of the earth cannot be reached, while the poles are the two sole points belonging to earth's rotation axis and to earth's surface, then the resultant of the tendencies towards the centre and towards the poles is an action along the meridians. But the resultant of the action along all meridians is an action towards the centre of the earth. Because of this, gravity is a force acting towards earth's centre.

I think this is the only possible interpretation of Leibniz's assertions, which I quote:

> Proposition 17: light's motion at the equator and parallels pushes the solid corpuscles towards the poles in the meridians. For, since a solid body cannot follow the movement of a

[30] *Ivi*, pp. 90–91. Original Latin text: "Itaque ad nostram tandem gravitatis causam confugiendum est per prop. 10, quae non hypothesi, sed certa demonstratione nititur, terramque non formavit tantum, sed et continet et quicquid ei circumfusum est arctis limitibus coercit atque in unum compellit".

> liquid and more subtle body, maintaining its same speed, it will perturb the liquid. As the nature tends to uniformity, the solid body will be pushed in a weaker place, namely where the movement is minor. This means towards the centre or (since the centre is already occupied) towards the poles and this happens through the shortest way in the sphere, that is along the meridians. This motion, since it occurs in the *circulis magnis*, whose common centre is the centre of the earth, has to be considered among those primary motions of the liquid surrounding the earth. From these motions, which conspire in a sole motion and maintain its uniformity, we deduced gravity in the previous sections.[31]

This explanation is nothing but a specification of the train of thought already expounded in *Hypothesis*, once eliminated bubbles theory.

The problem concerning the reason why gravity is directed towards the centre of the earth and not towards earth's axis was considered as one of the most important by Leibniz and by the scholars who claimed gravity was due to the movements of a fluid surrounding the earth. As we will see, Leibniz faced more than once this problem during the course of his speculations on gravity, which is a clear indication that, inside a mechanistic theory of gravity, this question was judged significant.

Once again Huygens explained this problem in a clear manner. If we suppose:

1) the existence of an aether, which surrounds the earth and which is moved in the same direction as earth's axis, but with a major angular speed than axis';
2) the aether is responsible for gravity. Then clearly follows:
3) gravity action should be directed towards the axis and not towards earth's centre. Hence the bodies should fall perpendicularly to earth's axis and not perpendicularly to the horizon (Huygens 1669, 1937, p. 634).

Huygens offered something as a statistical explanation for the action of gravity directed towards the centre of the earth: he supposed a spherical space around the earth. He imagined this space limited by the bodies surrounding the aether.[32] Inside this space an aether composed of little particles, whose motion is quite quick (*agiteé en tous sens avec beaucoup de rapidité*), exists. It will follow that aether's motion was limited inside such a space by the bodies surrounding the earth. In this case, the casual motion of the particles will originate a global motion towards the centre of the earth. For, Huygens claimed that, if a matter is agitated with casual motions and is constrained to develop its motion inside a sphere, the motion resulting from

[31] *Ivi*, p. 91. Original Latin text: "Propositio 17: Motus lucis in aequatore et parallelis rejicit corpuscula solida versus polos in meridianis. Cum enim solidum corpus non possit motum liquidi subtilioris aequis passibus sequi, eum turbabit; quare conante ad uniformitatem natura, rejicietur in locum debiliorem, id est ubi minor est motus, adeoque vel versus centrum vel (cum ille locus jam occupatus est) versus polos et quidem via in sphaera brevissima, id est per meridianos. Hic motus, cum sit in circulis magnis, quorum omnium centrum commune centrum terrae est, inter primarios illos liquidi terram ambientis motus censeri debet, ex quibus in unum conspirantibus et uniformitatem suam tuentibus supra gravitatem deduximus".

[32] Perhaps Huygens was thinking of the moon, the planets and the sun, but it is not easy to give a completely satisfactory explanation to this statement concerning the bodies, which surround the earth. By the way, in the *Discours de la cause de la pesanteur*, there is no further specification on this problem (see Huygens 1690, 1944, p. 455).

mutually opposite rectilinear movements will be circular around the centre of the sphere. To confirm this interpretation of gravity, Huygens provided some analogies drawn from observations of physical phenomena and from experiments (*ivi*, pp. 634–635).

The explanations given by Leibniz and Huygens of the reasons why gravity tends to earth's centre are different, although both of them are mechanical. The differences in these explanations can be considered as a consequence or, anyway as a profound link, connected to the general ideas held by Huygens and Leibniz on the vortices: Huygens thought of a series of vortices surrounding every planet and the sun. These vortices were separated by great distances. Hence, he considered the gravity on the earth exclusively as a local problem. Whereas, Leibniz believed—in this following Descartes—in the existence of a global solar vortex, in which the planets gravitate. Since the solar light plays a role inside Leibniz's ideas on gravity expounded in the letter to Fabri I am analysing, it is necessary that the explanation of the reason why gravity tends towards earth's centre also takes into account the way in which sun-light rays reach the earth. This means that gravity is not a mere local phenomenon.[33] This is fully coherent with the planetary theory developed in the *Tentamen*: since this theory is based on the *circulatio harmonica* of each planet around the sun, this common kind of *circulatio* is far more plausible, from a physical standpoint, if we admit the existence of a huge solar vortex, inside which the planets rotate maintaining the features we have analysed as to their transverse velocity. From a merely logical perspective, it is possible to conceive of the planets rotating with that speed, admitting separate vortices, too. However, this becomes quite unlikely from a physical point of view, and as a sort of miracle, exactly what Leibniz intended to avoid. This is a confirmation of the link between some general physical conceptions of Leibniz concerning the structure of the universe and his future planetary theory.

Thus, the letter to Fabri on May 1677 is a further evidence of the unitariness of Leibniz's project. It is, in fact, possible to speak of a very cosmology in Leibniz, in which the theory of planetary motion is connected to the theory of gravity in an entire cosmological vision, which was specified and modified in the course of the years, but whose bases remained unmodified:

[33] Some questions to clarify: 1) The difference between Huygens' and Leibniz's vortices is well known. See, for example, the "Appendice II" added by the editors of Huygens' works to the *Discours de la cause de la pesanteur* (Huygens 1690, 1944, pp. 494–499). The editors are referring to the discussions between Leibniz and Huygens developed immediately after the publication of Leibniz's *Tentamen*. However, the different conceptions of the vortices sustained by the two scientists were already clear in the period we are dealing with. See also Aiton (1972, pp. 125–127). 2) I do not enter into the discussion of what kind of void Huygens admitted among the planetary vortices. This is an interesting topic, but does not concern my subject. See, i.e., Koyré (1965, pp. 122), where he distinguishes between *vacuum interspersum* or *disseminatum* and *vacuum separatum*; 3) I am not claiming that Leibniz believed only in the existence of a global solar vortex, he also believed in vortices surrounding the planets. The difference with Huygens is that the latter did not admit a global solar vortex.

1) the movements of the planets and gravity depend on the aethereal vortices surrounding the sun and the planets themselves;
2) every action is transmitted mechanically, that is by contact;
3) every action needs a means to be transmitted, which is not the void.

Inside this general scheme, many changes occurred: in this initial, or almost initial, phase of his thought, Leibniz considered a sole aethereal vortex, which surrounds all the bodies of the solar system and which is responsible for the movements. We will see that, some years later, he differentiated two kind of vortices, at least. The intermediary of gravity is—in this phase—the solar light.

No assertion is as clear as the following passage by Leibniz, where he: 1) claimed that all motions depend on the motion of the stars and on light; 2) remarked the fixed stars also have an effect on the earth, even though their action is slow and difficult to be detected because of their distance; 3) asserted that every property concerning the sun and the planets has to be transcribed into properties of light and movements by means of geometry and mechanics.

This is a bold project to construct a system of the world, in which planetary theory and gravity are among the most important subjects. We read:

> I have no doubt that all the motions in the bodies, which are in front of us, derive from stars' movements and light. However, the great distance of the fixed stars is the cause why I believe that their motions, though having some effects, produce ones, which are slow and barely perceptible for us in the course of many centuries. The only possibility remains that every effect is transported by the light and by the movements of the sun and of the planets. These motions are not so numerous and complicated that we cannot hope the most skilled geometers and mechanicians reach to know them precisely enough.[34]

The evolution of Leibniz's thought on gravity is hence connected to the evolution of his cosmology, of which, planetary theory developed in the *Tentamen* and *Illustratio* is a section. To follow Leibniz's cosmological project, I will examine the evolution of his ideas on gravity.

5.3 The *Tentamen*

The published version of the *Tentamen* does not deal mainly with the cause of gravity, but with the mechanism of planetary motion, once taken for granted the existence of gravity. Nevertheless, there is a series of observations on gravity, which are basically—though not exclusively—concentrated in the introduction.

[34] Leibniz (1677, 1860, 1962, VI, p. 93). Original Latin text: "Cum enim ego pro certe habeam, omnes motus in corporibus nobis obviis ab Astrorum motibus atque luce oriri, fixarum autem distantia causa sit cur credam, quae in ipsis fiunt, ea effectus quidem aliquos sed lentos tamen et multorum saeculorum decursu aegre sensibiles apud nos excitare; ideo superest, ut omnia solis et planetarum luci et motibus transscribantur. Hi motus neque tam multi neque tam implicati sunt ut a Geometriae et Mechanices intelligentibus accurate satis cognosci posse sit desperandum".

These considerations provide an interesting and complicated picture, whose interpretation is not easy, from a conceptual standpoint. The general scenario expounded by Leibniz is inserted inside his mechanical conceiving of the interactions. Indeed, also while mentioning the author who is one of his main reference points—Kepler—, Leibniz pointed out that his belief in sympathies and intelligences prevented Kepler from reaching a more refined theory of the interactions, although some of his basic ideas were right (Leibniz 1689, 1860, 1962, pp. 148–149). However, if, from the general scenario, we go into the details, the picture becomes problematic. Leibniz claimed:

1) The true cause of gravity depends on the fact that the rotating matter tends to excape along the tangent. If stalks and straws are afloat on water and water moves in a vortex inside a vassel, then the stalks and straws are pushed towards the centre from the tendency of the water to excape along the tangent, as water, being denser than stalks and straws, is driven out the centre more strongly than them. Leibniz ascribed this conception to Kepler (*Ivi*, p. 148).
2) Each body which describes a curved line has the tendency to recede along the tangent. Hence, if it does not recede, it is necessary that the fluid surrounding it, maintains it in the curved trajectory (*Ivi*, § 1, p. 149).
3) Thence, the planets are moved by their aether (*Ivi*, § 2, p. 149).
4) Solicitation of gravity has a fundamental role in opposing to *conatus centrifugus* and in maintaining a planet in its orbit. We have analysed in depth this mechanism in Sect. 2.

5.3.1 Interpretation of Leibniz's Statements

With regard to item 1), Bertolini Meli reminds the reader that, when Kepler spoke of the argument expounded by Leibniz, he only provided an example of a mechanism which could push a body towards a centre of movement, but that Kepler explicitly denied it could be the mechanism of planetary motion because aether is certainly more tenuous than earth.[35] Leibniz had a different opinion: Following Descartes—as to this question—he thought that aether was denser than the terrestrial bodies, even though it was weightless.[36] In what follows in this chapter, we will see that Leibniz presented a series of mechanisms in which the rotational

[35] Bertoloni Meli (1993, p. 28).

[36] Here I remind the reader of the paragraphs of Descartes' *Principia* where we find the conceptions which inspired Leibniz and, more in general, the supporters of vortex theory, although Leibniz, as well known, was quite critical and sceptical about some aspects of Descartes' vortex theory. In *Principia*, II, §§ 56–62 (*Oeuvres*, VIII, pp. 71–77), Descartes speaks of the movement of a hard body inside a moving fluid. In III, §§ 24 and 25 (*ivi*, p. 89) he addresses the problem of skies' fluidity and explains how the fluid skies carry all the bodies they contain. In III, §§ 58–60 (*ivi*, pp. 96–98), he deals with the tendency of a body moving in a circle to recede from the centre of the movement and applies this property to the skies. In IV, §§ 22 and 23 (*ivi*, pp. 213–214) Descartes

movement of a dense *and heavy* means pushes the solid bodies which are less dense and which have a less specific weight than the means towards the axis or the centre of rotation. However, from what we have seen in the *Hypothesis* and in the letter to Fabri, Leibniz certainly thought that aether has a less specific weight than the terrestrial bodies. To be more precise: Leibniz thought that aether is weightless, but dense. What produces gravity is aether's density and lack of weight. This means that, according to Leibniz, *density and specific weight are not necessarily proportional*: The aether is denser than any material on the earth; its produces gravity, but it is not subject to gravity, hence it makes no sense to ascribe a weight and hence, a specific weight to aether.

The problem of the relation between the density of the aether surrounding the earth and the bodies on the earth is complex and it will play a fundamental role in the explanation of gravity given by Leibniz in *De causa gravitatis, et defensio sententiae autoris de veris naturae legibus contra Cartesianos* (Leibniz 1690, 1860, 1962, VI, pp. 193–203), as we will see. But it is now necessary to point out an important aspect of this problem: when Leibniz used adjectives as *crassus* in reference to the terrestrial bodies which are pushed downwards by the particles of the aether which are *crebra* (dense) but without weight, it is clear that *crassus* does not mean dense because the terrestrial bodies are more *crassi* than aether, but less dense. On the other hand, it makes no sense to translate *crassus* with heavy because to be *crassus* is a property held by a body, which, under the action of the aether, becomes heavy. But this is not yet the heaviness, because to be *crassus* is a property the bodies hold independently of the aether's action, while the heaviness depends on the action of aether. Furthermore, it is a mistake to think that the adjective *crassus* is referred to the mass rather than to the weight (also without entering into the discussion mass-weight before Newton), because it seems to refer to a specific property concerning the way in which the particles of the body are disposed rather than to the dimensions of the body, too. While in the concept of mass the dimensions (volume) are, obviously, a fundamental component. My impression is that—beyond the term used to translate *crassus*—Leibniz had the idea that two different kinds of density existed:

1) density of the aether, that is of a material which is not subject to gravity, and which is the cause of gravity;
2) density of the bodies subject to gravity. This is the *crassitudo*.

We could say: both concepts 1) and 2) indicate the quantity of matter in a certain volume, but 1) concerns the gravific element, 2) the elements subject to gravity. For Leibniz this difference is fundamental, although, admittedly, it is not easy to be explained in a modern perspective.

The further interesting consideration is that in the *Tentamen* Leibniz spoke of gravity as an action which is part of the mechanism responsible for planetary motion, as seen in details in the second chapter. This shift of meaning of the

speaks of the lightness of sky's matter and expounds the way in which this light, but dense matter makes the terrestrial bodies heavy, that is how gravity is produced.

word *gravitas* is important: in his previous works Leibniz had associated gravity and motion of the aether surrounding each planet. In particular, the motion of the aether was responsible for gravity. But there was not a strong similarity between gravity on the earth and force determining the planetary motion. This is a novelty, which was perhaps due to Newton's universal gravitation.

To summarize: from the scarce indications given by Leibniz on gravity in the *Tentamen*, it seems that:

1) The true cause of gravity is due to the tendency of rotating bodies to escape along the tangent;
2) There is no reference to the possible "motor" of the aether, whose role could be compared with the sun light in the letter to Fabri;
3) Gravity on the earth and gravity as a part of the mechanism of planetary motion are similar attractions, or at least, the same word is used without any further specification.

A series of interesting specifications are, in fact, provided by Leibniz in the *Zweite Bearbeitung* of the *Tentamen*.

5.4 The *Tentamen*: *Zweite Bearbeitung*

The *Zweite Bearbeitung* of the *Tentamen* seems to open a new phase in Leibniz's speculation on gravity. For, Leibniz expounded some ideas concerning the origin of gravity. He introduced the hypothesis that gravity is due to a *conatus explosivus*. Thence gravity can be produced by: 1) a centrifugal force; 2) a *conatus explosivus*. However, Leibniz admitted that—in the moment in which he was writing—there was no completely satisfying solution. At the same time, Leibniz was convinced that the solution to the problem concerning gravity's origin could be found inside the mechanisms of the vortex theory. Furthermore, in this paper he tried to better specify the possible relations between the kind of fluid responsible for the harmonic circulation and the one on which gravity on earth depends. After the general introduction, which was already present in the first version of the *Tentamen*, Leibniz began to deal with gravity comparing this attraction with that magnetic and claiming there is no certainty if the two attractions have the same origin. But more interestingly he wrote:

> It results that every body in the world, which is bigger than the others, has the force (*vim*) to attract (at least) the like bodies inside its sphere. As to the terrestrial bodies we call gravity this force, and we are transferring this name to the celestial bodies by means of an analogy.[37]

[37] Leibniz (1690?, 1860, 1862, VI, p. 163). Original Latin text: "Constat [. . .] omne corpus mundanum majus [. . .] vim habere attrahendi cognata (minimum) corpora intra sphaeram suam, quam in terrestribus vocamus gravitatem, et analogia quadam ad sidera transferemus."

Hence:

1) Gravity is an attractive action exerted by a major body on a minor body. This action is effective *at least* among *cognata*[38] (cognate, similar) bodies. Hence, Leibniz seems to think gravity does not necessarily act between two bodies which are not *cognata*. This means that—after all—Leibniz considered the universal character of gravity still an open question, since he referred to a qualitative property as that to be *cognata corpora*. Furthermore, he gave the impression of thinking gravity is not a reciprocal attraction, but is exerted by a major body on a minor one and not viceversa, as well. In this last assertion, the words "gives the impression" are necessary because it is not correct to deduce a whole theoretical idea from an outlined reference.
2) We extend gravity, *by analogy*, to the stars. The locution "by analogy" is meaningful: this means that, according to Leibniz, the attraction exerted by the stars—which, in the case of the sun, plays a quite important role inside Leibniz's planetary theory, because it is responsible for the inwards component of *motus paracentricus*—is analogous to gravity on the earth, but—probably—it is not exactly the same attraction. In these cases the questions are two: a) what does specifically "analogy" mean? b) which are the limits of analogy? We will see that Leibniz was rather obscure on these important questions.

Anyway Leibniz, from the beginning of his considerations, posed a link between gravity and force on which the *motus paracentricus* depends, even though this link was not completely clear.

5.4.1 Leibniz's Assertions and Specific Interpretations

What follows is not easy to be interpreted, or better, it is easy as far as a literal explanation of Leibniz's statements is looked for, but, as soon as we try to catch Leibniz's global aim, the picture becomes less clear. Leibniz's reasoning is characterized by the following steps, in which two hypotheses on gravity are proposed:

1) The attraction of gravity depends on a corporal radiation (*ivi*, p. 163);
2) In an attractive body (supposed to be a sphere) there is a conate which tends to expel far from the sphere (*conatus explosivus*) the matter which perturbs the general motion. Leibniz used the expression "materiae inconvenientis sive perturbantis". These words have to be referred to the fact that such matter perturbs the motion—supposedly of the aether—. This matter cannot hence move freely. Therefore a matter whose motion fits better with the global movement of the sphere and that, hence, perturbs it less, is attracted by an impulse coming from everywhere (we could say circular. The word is

[38] The concept of *cognata corpora* was introduced, in connection with gravity, by Kepler in the *Astronomia Nova*. See, in particular, the subsection of the *Introductio*, entitled *Vera doctrina de gravitate* (KGW, III, pp. 25–26).

circumpulsio). The flame is an example of a physical phenomenon which behaves as described by Leibniz.[39]

3) Matter's tendency to assume a form, which perturbs as less as possible the surrounding environment, can also explain earth's origin. For, Leibniz wondered: how is it possible that a solid globe is formed in a situation, where only liquids exist? He imagined a drop of oil, which is afloat in the water: he thought that at the beginning the matter of the globe was fluid—evidently of a different density than the surrounding fluid—. This matter assumed the form which, according to Leibniz, perturbed the rest of the fluid as little as possible. This is exactly the form of a circular drop, as a drop of oil in the water. Since the drop, due to its suitable form, opposes a scarce resistance to environment, it gained a certain stability and its process of solidification began. However, the solidity is never complete and some passages in the material remained open, which were convenient with the motions of the residue fluid that entered into these passages (*ivi*, p. 164). This is earth's origin.
4) Every fluid has internal motions. If the matter does not fly away from the centre of the fluid, these motions tend to become circular. The circles tend to become as great as possible because, in this way, they can better oppose to the conate to recede. In practice Leibniz is saying: *coeteris paribus*, the smaller the radius, the bigger the centrifugal force (*ivi*, p. 164).
5) If gravity has to be explained by means of the centrifugal force *a là* Kepler (in fact, *a là* Descartes or *à la* Huygens), then there could be the already analysed problem that gravity should act towards the terrestrial axis and not towards earth's centre. It is possible to avoid this obstacle by means of the reasoning already expounded by Leibniz in the letter to Fabri.[40] In this manner all the interactions could be explained. Here Leibniz added: it is possible to think that the trajectories, along which matter opposes the least resistance to aether's movement, are along the meridians. This could explain gravity's tendency towards earth's centre rather than axis.
6) After these rather detailed explanations, Leibniz suddenly changed his tone: whatever the cause of gravity is, it is enough to think that the attracting globe

[39] For completeness, I quote here this not easy passage by Leibniz: "Therefore, it is coherent that, in the sphere, there is a *conatus* to expel the improper or perturbing matter or the matter, which is not posed in a sufficiently suitable place for the motions to be exerted as freely as possible. Thence, other matter, which is arranged to agree with the internal movement [of the aether] or which has a motion such to disturb less [aether's] internal motion is attracted by a circular impulse. The flame offers an example of this mechanism, as the flame, on the basis of the sensible experience itself, expels a matter and attracts another matter". Original Latin text: "Deinde consentaneum est, esse in globi corpore conatum explosivum materiae inconvenientis sive perturbantis seu non satis apto ad motus liberrime excercendos loco positae, unde per circumpulsionem attrahatur alia consentiens seu motum ejusmodi habens, ut motum attrahentis intestinum minus perturbet, exemplo flammae, in qua expulsionem unius et attractionem alterius ipsi sensus docent". (Leibniz 1690?, 1860, 1962, VI, pp. 163–164).

[40] Leibniz refers to his previous contribution with the words: "[. . .] ut jam olim annotare memini [. . .]" (*Ivi*, p. 164).

emits material rays similar to light-rays or impetus-rays emanating in all directions from the centre. This impetus is not necessarily propagated by transport of matter, but by the pressure of contiguous matters, as in the phenomena of light, sound and motions in liquids. It seems that Leibniz is thinking of a wave (*ivi*, p. 164). This means that the *impetus* can be propagated in two manners: a) by the direct moving of matters; b) by a wave.

7) The action is not instantaneous, but a certain time is necessary for it to be propagated (*ivi*, p. 164).
8) This is an important item, because it explains the mechanism of gravity based upon *conatus explosivus*. Though Leibniz gave a sole explanation, I think it is necessary to divide his argument into two hypotheses:

   A) *conatus explosivus* due to a very transport of matter: the rays—which Leibniz calls "ut ita dicam magnetici" (*ivi*, p. 164)—are produced by a recessive conate of an insensible fluid, very subtle and divisible, whose parts are strictly adherent. The terrestrial bodies have—for the same volume—a minor quantity of matter which tends to recede from the centre in comparison with the subtle fluid. When these bodies are posed inside this fluid, it tends to enter and to exit from their pores, but, since they have less conate to recede than the fluid, they have less levity, thus the emitted fluid prevails on the entering fluid and the bodies are pushed downwards. This is similar to a reaction-motor mechanism: the fluid enters inside the terrestrial bodies with a certain speed and is expelled out of them with a major speed, so that the bodies are pushed downwards. This is, in my opinion, the physical meaning of the fact that the emitted fluid prevails on the entering one (*ivi*, pp. 164–165). A further possible interpretation is that, until the bodies are not over earth's surface, the quantity of the fluid which gets out from the bodies is superior to that of the entering fluid. When a body has arrived at the surface, the two quantities reach an equilibrium. When the body is transported once again over the surface, a further quantity of aether penetrates it and so on.

      It seems to me that the interpretation based on the difference of speeds is more plausible from a physical standpoint.
   B) *Conatus explosivus* due to radiation caused by the gravific fluid: in this case the action of the gravitational waves going out from the terrestrial bodies would be more intense than the action of the waves entering into the bodies. This would produce a mechanism analogous to the previous one—although without any transport of matter—. The difference between the intensity of the waves entering into the bodies and that of the waves, which get out, originates gravity.

9) This mechanism can explain gravity. But what about the specific weights of the bodies? Leibniz hypothesized the existence of a further fluid which is responsible for the different specific weights. This fluid does not follow the same motion as that responsible for gravity. The pores of the terrestrial bodies are different as to dimensions and as to their density inside the body. The second

subtle fluid enters easier into those pores of the matter which are smaller, but quite massed. The little dimensions of this kind of pores prevent the aether determining gravity to penetrate inside them, but they do not prevent the subtle aether responsible for specific weight, which, hence, enters and, by means of a mechanism similar to that analysed for gravity, determines the specific weight. Leibniz added a brief consideration on his idea that, in some circumstances, the bodies composed of the same matter have different specific weights. These considerations are not important for the picture I am describing (*ivi*, p. 165).

10) Finally Leibniz showed that the intensity of the light-rays diminishes as the square of the distance from the centre of emanation (this was an already known condition).[41] Hence, if gravity acts with a mechanism similar to light, then it diminishes as the squares of the distance (*ivi*, pp. 165–166).

5.4.2 General Interpretation

What has been expounded in the previous ten items can be divided into five explicative levels:

a) the first level includes items 1)–4). Here a new hypothesis on gravity is proposed: gravity is a radiation, due to a *conatus explosivus*. The conate can be generated by the vortical motions of the fluid which was produced when the earth was created by condensation from an original less dense matter rotating inside the solar vortex. Although its origin could be in the internal vortical motions of the rotating fluid, the action of the conate propagates in straight lines. This is a very strong hypothesis, because it would be necessary to explain that a vortical circular motion can produce a straight conate or impulse. This hypothesis is different from the idea that gravity depends directly on the movement of the fluid surrounding the earth: the fluid produces the conate which, in its turn, produces the expulsion of the fluid from pores of the bodies which are over the earth's surface.
b) Item 5) proposes the idea that gravity is due to the centrifugal force of the fluid. As we have seen, Leibniz attributed this idea to Kepler. In the letter to Fabri he had already hypothesized that this mechanism could push the bodies towards the centre of the earth, although, at a first glance, it seems to push them towards the rotating axis. In this phase of his thought Leibniz seems to prefer the solution with the *conatus explosivus* rather than that based on the centrifugal force. We will see that he will rethink this problem.

[41] The source of inspiration of Leibniz was, also in this case, Kepler, who in his *Ad Vitellionem paralipomena* (work quoted in more than one occasion by Leibniz, see, inside this book, Sect. 6.1.2.2) proved that light's intensity decreases as the inverse of the square distance from the source (see KGW, II, book I, proposition 9—based on the propositions 6–8, pp. 21–22).

c) The items 6)–8) introduce a further hypothesis on the origin of the *conatus explosivus*: it can be originated by an impulse, through which the gravific fluid is pushed out from earth's centre in straight lines towards any direction. Item 8) specifies how such a fluid could act to produce gravity. This can happen by transport of matter or by a wave.
d) Item 9) concerns the specific weight.
e) Item 10) is strictly connected to the hypothesis that gravity acts by straight lines as light. For, if one wishes to reduce the fact that gravity acts with an intensity which decreases as the square of the distance to the fact that light acts in this manner, he has to show that gravity acts with a mechanism similar to light's. This is the reason why Leibniz embraced the hypothesis of the *conatus explosivus*.

Some commentaries:

Aiton stresses that the fluid responsible for gravity is different from that causing the harmonic vortex.[42] He quotes a passage of the already mentioned letter to Huygens in which Leibniz explicitly claimed:

> I distinguish the aether which causes the gravity (*pesanteur*) (and perhaps also the direction of parallelism of the axes) from that which carries the planets, which is rather coarser.[43]

The quotation mentioned by Aiton is perfectly coherent with what Leibniz asserted in the *Zweite Bearbeitung*. For, if the earth is born by means of the condensation of a certain matter, it is quite plausible that the least dense part of the original matter—namely a liquid part—has been pushed outside. It met the original fluid responsible for the harmonic circulation and—being less dense than this fluid—begun to orbit around the earth. The dense harmonic vortex, which surrounded the terrestrial vortex, prevented it to escape along the tangent. Given this picture, it is then necessary to add, that the fluid necessary for Leibniz's theory were, in fact, three and not only two:

a) fluid responsible for the harmonic vortex;
b) fluid responsible for gravity;
c) fluid responsible for the different specific weights of the bodies.

For, in the previous quotation the word *pesanteur* seems clearly to refer to gravity as a general phenomenon, not to the different specific weights of the bodies.

What Leibniz explained in the *Zweite Bearbeitung* is interesting from many aspects and unsatisfactory for other aspects. First of all the argument is complex as far as the line of reasoning is concerned because there is a triple causal level: the

[42] Aiton (1972, p. 131).

[43] Leibniz 1690, in Huygens, 1901, *Oeuvres Complétes*, IX p. 526. See also Leibniz (1690a, 1860, 1962, VI, p. 192). Aiton quotes this passage, translated into English, in Aiton (1972, p. 134). I have quoted Aiton's translation. Original French text: "Cependant je distingue l'aether qui fait la pesanteur (et puetestre aussi la direction ou le parallelisme des axes) de celuy qui defere les planetes, qui est bien plus grossier".

first level of causation is the most general and concerns the way in which gravity is born, starting from the origin of the earth. By the way, this is coherent with Leibniz's idea that the history of a system is important to understand the physics of the system. The second level of causation concerns the way in which gravity acts (items 8) and connected item 10)), independently of its origin. Finally the third level of causation concerns a mathematical analysis of planetary motion, granted that gravity acts in the given manner. Inside this complex context another annotation is necessary: Leibniz tried to justify that gravity acts with the inverse square law *a priori* by means of comparison with light (item 10)), (Leibniz 1690?, 1860, 1692, VI, pp. 165–166) and *a posteriori,*[44] showing that, if the orbit is an ellipsis in which the sun is in one of the two foci, the solicitation of gravity acts as the inverse of the square distance sun-planet (see, in this work Sect. 2.3). The described picture is a further confirmation that Leibniz was going to frame his planetary theory inside a general physical theory, of which cosmology is a part, where planetary theory is inscribed. In this attempt Leibniz was coherent because he tried to unify all aspects of his theory. However, his writings on gravity—and I think on physics in general—seems to have a provisional nature because Leibniz often changed his mind on various questions and, in every contribution, faced the problems from the foundation as if the foundations were not stable. The argumentative level becomes often tortuous and obscure, and it seems incoherent, but it is not. It is possible to find coherent interpretations, as I have tried to show and I will try to do in what follows. The fact is simply that the physical hypothesis of the vortices is wrong and hence, when a refined and specific explanation of the phenomena is needed, it is always necessary to add *ad hoc* hypotheses whose epistemological *status* is often so strong that they appear exactly as *ad hoc* and not framed from the beginning inside a theory. *Mutatis mutandis*, this situation is similar to that of Ptolemaic theory and Copernican theory in which the orbits are circles: given observations with a certain precision, the theories can appear relatively easy, but when the observations become more precise and the theories have to explain a series of particular facts, a relevant number of *ad hoc* hypotheses has to be added, which, in a sense, can save the theory, but that make it absolutely unlikely from a physical point of view. Kepler thought that the orbits were not circles when he realized that, to maintain such hypothesis, the *punctum aequans* had to move to and fro in a segment of straight line, which cannot be explained by any physical cause.[45] Hence he began to think of another kind of orbit. The situation is similar: every attraction can be explained by postulating the existence of an *ad hoc* fluid with specific movements.

[44] *A posteriori* means, in this context, "without considering the origin of gravity".

[45] The concept of *physical cause* in Kepler is fundamental to understand his thought. In this context I can only refer to Kepler's words by which he expressed the idea that Mars' orbit is not an eccentric circle. Kepler wrote: "You see, thoughtful and ingenious lector, that this opinion, according to which the path of a planet is a perfect, eccentric circle implies many incredible things in physical speculations". Original Latin text: "Vides lector considerate et ingeniose, quod haec opinio de perfecto circulo eccentrico itineris Planetarii multa incredibilia in speculationibus Physicis involvat". (KGW, III, p. 262).

But, first of all it is highly implausible the existence of a plurality of insensible fluids which have no interactions with us and which are detectable in no way, but which determine such important attractive actions; secondly, it was difficult to organize the theory based on these fluids in a coherent picture, as Gregory stressed and we have seen. Finally, the model risks becoming more complicated than the phenomena themselves for which it has been created and hence gets a scarce explicative value. But all these observations do not imply that Leibniz was not coherent in his aims and in his approach.

In fact, Leibniz emphasized his belief that the origin of gravity is not certain, but he continued to write about it until the end of his life because it was fundamental to show a plausible origin by means of a mechanical approach. Otherwise Newton's theory, which is incomparably better structured from a physical-mathematical point of view than any vortex theory, would have definitely won, with the unacceptable idea of action at a distance. Thence, Leibniz continued his researches on gravity, as we will see.

A brief summary of the conception expounded in the *Zweite Bearbeitung* is this:

1) Harmonic circulation is due to the aether spread in the whole solar system;
2) Gravity on the earth is due to the aether surrounding our planet. There are two possible hypotheses on how gravity acts;
3) The difference between the specific weights of the materials is due to a third fluid, more tenuous than the second one, which, in its turn, is more tenuous than the fluid responsible for harmonic circulation.

There is no specification of which fluid should be responsible for *motus paracentricus'* inwards component, that is for the gravity of the stars, which is *analogous* to gravity on the planets.

5.5 *De causa gravitatis* and Leibniz's Thought Around 1690

The paper *De causa gravitatis et defensio sententiae autoris de veris naturae legibus contra Cartesianos* appeared in 1690 in the *Acta Eruditorum*. The general frame is given by the polemics of Leibniz against Descartes' and other Cartesians' ideas on the fact that—according to Leibniz—the *vis viva* rather than the quantity of motion is conserved. In this case Leibniz's critics included the paper of Denis Papin *De Gravitatis causa et proprietatibus observationes* (1689), which also appeared in the *Acta Eruditorum* (see Papin 1689), in which Papin entered into the Leibniz-Catalan polemic on the principles of conservation. The other work to take into account in this discussion is the *Discour de la cause de la pesanteur*[46] published by

[46] Huygens (1690, 1944). Aiton reminds us that Huygens sent Leibniz a copy of his *Traité de la lumière*—of which the *Discours* is an appendix—early in 1690 (Aiton 1972, p. 132). Therefore it is almost sure that Leibniz knew the whole of Huygens' ideas on gravity at the beginning of the 1690.

Huygens in 1690, where, in the first part, Huygens expounded ideas he had already explained in discussions at the Académie at the end of the 1660s, while in the second one he added new argumentations.

As a matter of fact, the first half (initial 11 propositions) of Leibniz's *De causa gravitatis* regards gravity, the second one the problem *vis viva*-quantity of motion. I will concentrate on the first part.

The argumentative structure of *De causa* is rather refined and a certain prudence is necessary to avoid possible mistakes in interpretation: Leibniz claimed Galileo had proved, by means of experiments, that in the fall of bodies the speeds are as the times. Huygens, relying: 1) on Cartesian vortices hypothesis; 2) on the hypothesis that the aether responsible for gravity has an infinite speed, if compared to that of the falling bodies; 3) on the hypothesis that gravity is produced by the centrifugal force of the aether, was able, by a series of steps, to determine the relation between the speed of the aether and the speed of a point rotating at the equator for the falling bodies to fulfil Galileo's law (*ivi*, p. 194). This reasoning is essentially the same as the one presented by Huygens in the discussions on gravity developed at the Académie in 1669.[47] As we have seen, Huygens also tried to explain why gravity would be directed towards earth's centre and Leibniz, in the letter to Fabri, tried too. Sturm and Jakob Bernoulli criticized this solution with the already seen objection that, in the hypothesis gravity depends exclusively on the difference between aether's and earth's diurnal rotation centrifugal force, gravity would have been directed towards the axis of the earth and not towards the centre. In particular, Jakob Bernoulli published a note in the *Acta Eruditorum* (see J. Bernoulli 1686), in which he referred to and literally quoted a series of Sturm's considerations on this fact.[48] Sturm's and Bernoulli's arguments were carried out by means of mathematical proofs. Papin proposed a solution to this problem, deemed unsatisfactory by Leibniz. In my opinion, the very conceptual core of *De causa gravitatis* is exactly the problem that gravity, in the hypothesis in which it is supposed to derive only from the centrifugal tendency of a fluid, should be directed towards the rotational axis and not towards a centre. The objections of Sturm and Bernoulli had probably convinced Leibniz of the weakness of Huygens' and his own argumentations to solve this problem:

1) Leibniz criticized Papin's solution because it was based on the idea that the difficulties pointed out by Sturm and Jakob Bernoulli could be overcome by postulating that aether's speed is incomparably superior to the speed with which the earth rotates around its axis. But this idea is wrong (*ivi*, pp. 194–195 and proposition 8, p. 197) because the problem is not connected to a relation between the absolute values of two speeds, but to their directions. As we will see, Leibniz also provided an experiment to prove his thesis;

[47] See Huygens (1669, 1937, pp. 638–640) and Huygens (1690, 1944, pp. 459–461). In the *Discours*, Huygens developed a series of reasonings connected to his calculation of aether's speed at the pages 461–466, which were not present in the discussions at the Académie.

[48] Leibniz explicitly refers to this note (see Leibniz 1690, 1860, 1962, VI, p. 194).

2) Leibniz explicitly claimed that the hypothesis of gravity deriving only from a centrifugal tendency is subject to quite serious doubts. He spoke of *dubitationes gravissimas* (*ivi*, p. 197);
3) It is true that Leibniz mentioned, in three brief lines, his solution to this problem (*ivi*, proposition 8, p. 198), but he did not insist on its application to Sturm's and Bernoulli's critics.

Leibniz gives hence the clear impression that in 1690 he did not think this problem had a convincing solution.

Inside this conceptual reference frame, Leibniz added here a significant epistemological observation, which is consistent with the way in which he dealt with the problem of the origin of gravity along his entire scientific career, even though it is not directly connected to the problem gravity-towards the axis/gravity-towards the centre: the proof of the truth of a statement (in this case, Galileo's law of falling bodies) is a completely different problem from the discovery of the physical situation determining that truth. In the specific case: the reasoning by Huygens concerns Galileo's law and has to be interpreted as a plausible *hypothesis* to explain that the origin of a *truth*, is not necessarily a *truth*. Leibniz is clear when he wrote:

> Our objector [Papin] confuses the proof of a certain truth with the explanation of the cause of this through a hypothesis. Perhaps he did not catch Huygens' advice enough, whose intention (as far as it is possible to claim), in the reasoning we have posed, was not to prove that gravity acceleration has the nature of which we have spoken, but once posed this nature (maybe deduced from the phenomena) to explain, in a plausible manner, how this acceleration can rise.[49]

Leibniz stated that Papin had not understood this fundamental difference. What follows is strictly connected to this epistemological frame: Leibniz asserted—as he did in many other circumstances—that Kepler first suggested the idea that gravity could be produced by a dense rotating fluid, which, thanks to its tendency to recede from the centre, could induce gravity (*ivi*, p. 195). Leibniz also provided a concrete example in which a dense and heavy fluid—mercury—rotating inside a tube can push some solid bodies, which are inside the fluid and which are less dense and less heavy than mercury, towards the *axis*.[50] We have seen that Kepler described a mechanism similar to Leibniz's, but he did not think it could be the mechanism of gravity. Anyway, what is far more important is to understand that Leibniz provided this mechanism as an example of a mechanical (namely based on movement, contact and impacts) device which can push the bodies towards a rotating axis.

[49] Leibniz (1690, 1860, 1962, VI, p. 195). Original Latin text: "Confundit noster Objector demonstrationem veritatis alicujus cum causae redditione per quandam hypothesin, nec fortasse Hugenii consilium satis percepit, cujus institutum (quantum assequi licet) in ea quam posuimus ratiocinatione non fuit demonstrare, eam esse accelerationis gravium naturam quam diximus, sed posito (ex phaenomenis forsan) talem esse, explicare modum verisimilem, quo possit oriri".

[50] I do not enter into the mechanism described by Leibniz because it is known enough and because its specification is not fundamental in the context I am dealing with in the running text. See Leibniz (1690, 1860, 1962, VI, pp. 196–197 and connected Fig. 20).

My interpretation is that he did not propose the mercury tube–model as a model for gravity. Leibniz supplied this example in which a denser means pushes a less dense material towards the axis. The problem is that Leibniz was perfectly aware that mercury also has a major specific weight than the solid bodies posed in the tube. However, in a model for gravity, the material which surrounds the bodies should be—in Leibniz's perspective—denser but lighter than the bodies themselves. Hence he should have proved that mercury pushes the bodies towards the axis because it is denser than these bodies and not because it has a major specific weight. Leibniz did not carry out this operation, because he limited himself to provide an example of the described mechanism, not exactly a precise model for gravity. Rather, he supplied a mechanism that fulfils the conditions under which, according to Papin, the bodies would be pushed towards the centre of rotation. While, they are, in fact, pushed towards the axis. Therefore the mercury experiment is an argument against Papin, not a possible model of Leibniz's gravity. Furthermore, as we have seen, in this phase of his thought he was inclined to think that the model of the centrifugal force was not completely satisfactory to explain the origin of gravity.

On the other hand, in that period, the idea to provide a device, drawn from the common experience or from physical experiments, which could justify gravity from a mechanical point of view, but which cannot exactly be identified with gravity-mechanism, because of the particular nature of the aether, was a common enough idea: Huygens, both in 1669 and in 1690, developed an experiment, in which a mechanism *analogous* to that of gravity pushes the bodies towards the centre of the movement. Nevertheless, he explicitly claimed this mechanism could not be *exactly the same* as gravity because the weight-relation between the rotating fluid and the bodies afloat in the fluid do not reproduce the weight-relation aether-terrestrial bodies.[51] The difference between Huygens and Leibniz is that the former presented a mechanism, in which the bodies tend to the centre, because he was going to provide a plausible—even though not perfectly precise—mechanism for gravity, while the latter expounded, in 1690, a mechanism through which the bodies converge to a rotating axis, as a counter-example to Papin's arguments, not as a possible mechanism for gravity.

It is clear that we are in a minefield: Leibniz was conscious that the supporters of the origin of gravity from the movement of an aethereal vortex had to introduce strong hypotheses concerning the density of the aether, its movements and the means that could interact with aether to produce those movements. The whole picture was complicated—in Leibniz's specific case—by the fact that the fluid responsible for *circulatio harmonica* could not be the same as the one responsible for the inwards component of the *motus paracentricus* and for earth's gravity. Therefore Leibniz chose those hypotheses which could be more compatible with his general view, in which planetary theory has a conspicuous part. Around the 1690s, the most plausible hypothesis was—in his opinion—that of the *conatus explosivus*, although he was always open to the possibility that gravity derives from

[51] Huygens (1669, 1937, pp. 631–633) and Huygens (1690, 1944, pp. 451–454).

aether's centrifugal force. In *De causa* there is an important specification in this regard: first of all Leibniz clearly confirmed that gravity was due to the expulsion of a matter from the centre of the attracting body, which produces a radiation that, by means of the already analysed mechanism, generates gravity. What is very important is that Leibniz considered here the origin of gravity on the earth and the origin of gravity on the other celestial bodies due to the same mechanism. Thence the idea of the *conatus explosivus* could be an explanation not only for terrestrial gravity, but for the attraction of the sun towards the planets, too. Leibniz wrote, as to these questions:

> Another cause can be given of this phenomenon [gravity], which is not involved with this difficulty [the difficulty behind the idea of gravity as due to the centrifugal force]. This can be obtained conceiving the explosion of a matter pushed everywhere from earth's globe or from another celestial body, which produces a certain radiation, analogous to light's radiation. For, in this way, we will have the receding of aethereal matter from the centre, and, since the thicker (*crassiora*) bodies (as I will explain elsewhere) do not have the same force (*vim*) of receding, this mechanism pushes them towards the centre, or make them heavy.[52]

In what follows Leibniz confirmed the idea that the earth and its aether are created with the same mechanism with which oil-drops condense in the water.

In *De causa*, gravity theory and planetary theory are strictly connected once again and more thoroughly than in the previous works because Leibniz explicitly claimed that earth-gravity and sun-gravity are due to the same mechanism. Hence, this hypothesis allowed him to unify all actions responsible for the planetary motions and to provide, in addition, a plausible hypothesis for gravity:

1) *Circulatio harmonica*: due to the direct circulation of the sun aether which transports circularly the planets;
2) *Motus paracentricus*: outwards tendency, due to the fact itself that the motion is circular, hence it has a centrifugal force; inwards tendency, due to gravity of the sun whose mechanism is the same as the gravity on the earth: the mechanism due to the *conatus explosivus*.

However, given the complexity required to explain the origin of gravity and the uncertainty of every hypothesis, in October 1690,[53] Leibniz, in the letter to Huygens I have already mentioned in Sect. 2.2.1, proposed a picture of his thought which is slightly different from that explained in *De causa*. Leibniz confirmed that he believed the hypothesis of the *conatus explosivus* to be plausible because, apart from other questions, the idea that gravity was due to a radiation which acts as light could explain the inverse square law, as we have seen above. But Huygens—in his

[52] *Ivi*, p. 197. Original Latin text: "Alia ejusdem assignari posset causa non obnoxia huic difficultati, concipiendo displosionem materiae cujusdam ex globo telluris aut alterius sideris in omnes partes propulsae, quae radiationem quandam producat, radiationi lucis analogam; ita enim habebimus recessum a centro materiae aethereae, quae corpora crassiora eandem (ut alibi explicabo) vim recedendi non habentia versus centrum depellet, seu gravia reddet".

[53] Leibniz (1690a, 1860, 1962, VI, pp. 187–193).

Discours de la cause de la pesanteur—had tried to explain gravity relying upon the hypothesis that it was due to the centrifugal force of aether. In his letter Leibniz showed that the hypothesis "gravity due to centrifugal force" plus a further hypothesis could explain the inverse square law, as well. On this occasion, he outlined the explanation fully developed in the *Illustratio Tentaminis*: if every orbit has the same *vis viva*, Leibniz showed easily that the speeds are as the square roots of the distance from the sun.[54] As we have already seen in the answer to Gregory's critics, under this condition, the third Kepler law could be explained inside the vortex theory of planetary motion—to use an expression by Aiton. As to gravity, a consequence of this idea is the inverse square law. For, the centrifugal tendencies ("tendencies centrifuges", *Ivi*, p. 192) are as $\frac{v^2}{r}$, where v is the speed of the planet and r its distance from the sun; but the squares of the speeds are as the distances, hence the centrifugal tendencies are as $\frac{k}{r^2}$, being k a constant value.

Table 5.1 Leibniz's two hypotheses on the origin of gravity

First hypothesis: *Centrifugal force* plus *vis viva*	1. *Physical mechanism responsible for gravity*: the difference between the centrifugal force of the aether and the centrifugal force exerted on the bodies by earth's diurnal rotation. 2. *Additional physical hypothesis*: the orbits of the planets have the same *vis viva*. 3. *Features*: a) able to explain the inverse square law; b) compatible with both the ideas that the aether for the harmonic circulation is the same or is different as/from the aether responsible for gravity. Leibniz embraced this second hypothesis. 4. *Problems*: a) the general mechanism has the problem to postulate the existence of a means which is denser, but less heavy than any matter. Even though Leibniz tried to solve it, its explanation remained qualitative; b) without further specifications, gravity should be directed towards the axis of the earth and not towards the centre.
Second hypothesis: *Conatus explosivus*	1. *Physical mechanism responsible for gravity*: the action of the aether coming out from the centre of the bodies, which produces an action propagating in a straight line. 2. *Additional physical hypothesis*: the action is spread as light action, this means with the inverse square law. In this case such hypothesis is hence equivalent, from a dynamical point of view to the inverse square law itself. 3. *Features*: a) gravity is directed towards a centre; b) the fluid for gravity is different from that responsible for harmonic circulation; 4. *Problems*: there are not specific problems as those in item 4) of the previous hypothesis.

[54] This is the right relation obtained by Newton, relying upon completely different principles in the first book of the *Principia*, Proposition IV, corollary 6.

Thus, Leibniz expounded two hypotheses on gravity at the beginning of the 1690s. While the hypothesis based on the *conatus explosivus* is plausible in itself, the one based on the centrifugal force becomes plausible under the further condition that the orbits of the planets have the same *vis viva*.

We can summarize the two hypotheses on gravity by means of a scheme in which I try to clarify, in synthesis, their physical and epistemological features (see Table 5.1).

Both hypotheses rely upon strong assumptions. It is clear that Leibniz resorted to such assumptions in his attempt to provide a general mechanism which allowed him to explain gravity and movements of the planets without admitting action at a distance.

5.6 *Illustratio Tentaminis* and Other Late Works

The described itinerary concerning the relations between gravity, cosmology and planetary theory has its almost natural conclusion in the *Illustratio Tentaminis*, where Leibniz connected, once again, these three parts of his physics showing that they belong to a sole picture.

In the *Illustratio* Leibniz developed further considerations concerning his two hypotheses on gravity. He dealt with the hypothesis of the *conatus explosivus* in § 2 and with the hypothesis of "gravity deriving from centrifugal force" in §§ 16–18. The conclusive paragraphs 19–23 contain a series of physical, astronomical and epistemological considerations, part of which concern gravity.

5.6.1 *Gravity as an Action Deriving from the* conatus explosivus

At this stage of his thought, the identification of terrestrial gravity with the force responsible for the inwards tendency in the *motus paracentricus* is completed, at least as far as the action of this force is concerned (inverse square law). While, with regard to the fluids responsible for gravity in the solar system and gravity on the earth, Leibniz's thought is not completely explicit. Anyway, the analogy has been overcome.

With regard to the hypothesis—addressed in § 2—that gravity is due to a *conatus explosivus*, it is interesting that Leibniz referred to gravity as the solicitation involved in determining the *motus paracentricus*. Leibniz was facing gravity as the force acting between the sun and the planets.

He added further specifications concerning the *conatus explosivus'* hypothesis: Leibniz conceived the rays of attraction as rays of light, thence with a simple reasoning, he was able to prove that gravity, due to such rays, is spread with the inverse square law. However, for an exegesis of his conception, another remark is fundamental: the analogy of gravity with radiation emanating from the explosion of a compressed matter. Leibniz explained that, if the solid[55] bodies surrounding the matter which is exploding have many cavities, so that their specific density is less than that of the exploding matter, a gravitation towards the exploding matter is produced because the tendency (*nisus*, *ivi*, p. 256) to recede from the centre of the explosion of these bodies is less than the receding tendency of the exploding fluid. This fluid is quite dense and subtle (*ivi*, p. 256). In this hypothesis—Leibniz adds—it is not necessary that the particles of the fluid touch directly the bodies, because they produce a sort of shock wave, which is the cause of this attraction. Leibniz wrote:

> In this hypothesis, it is not necessary that the emitted particles reach the heavy body itself.[56]

The interpretation of this relatively brief Leibniz's argumentation is really difficult: Leibniz tried to clarify that the exploding mass is subject to the same mechanism "as the ignitions of the gunpowder or, at least, as the wind-guns, whose nature is not different".[57] Therefore I think that here Leibniz is trying to describe the real mechanism of gravity in the hypothesis of the *conatus explosivus*. That is, as a matter of fact, the described mechanism is more than an analogy. Things should be like this: in the centre of the sun and of the earth there is—so to say—a gravitational core which explodes continuously. This core is composed of the dense and subtle matter of which Leibniz spoke. The radiation deriving from this core produces gravity according to the specified mechanism, which acts with the same mathematical laws as light, but which is different from light. A further specification: the fluid is denser than the surrounding bodies, but it is very subtle. Here we have the already remarked divergence between the concept of density and that of specific weight. The fluid is denser than the terrestrial bodies or than the planets (if we are referring to sun's gravity), but it is so subtle that it is weightless. It is the cause of gravity but it is not subject to gravity. This conception is strange for us, but not for Leibniz. Furthermore, let us take into account that the exploding element is equivalent to the aether in the hypothesis "gravity deriving from centrifugal force", and the aether is the element of "lightness", thence it cannot have a major specific weight than the surrounding bodies, or better, it has no weight.

[55] Here there is an important terminological question which is conceptual, as well and which also concerns all my previous translations of the adjective *crassus* I have translated with "solid". I point out that, in this context Leibniz used the word *densitas*. In the previous pages we have seen that Leibniz used *creber* for dense. This picture is made even more complicated by the fact that Leibniz also used the adjective *gravis* and, in French, *massif*. Therefore, one could think of translating *gravis* by "heavy" and *crassus* by "massive". However, due to what is explained in Sect. 5.3.1 and to the fact that this translation refers to a clear distinction between weight and mass, which is not always present in Leibniz's works, I have preferred to maintain my translation. I have translated *massif* with "massive", but it is necessary to highlight that this translation is also not plane.

[56] *Ivi*, p. 256. Original Latin text: "Neque enim in hac hypothesi necesse est, ut emissa particula ad ipsum usque grave pertingant".

[57] Original Latin text: "quales sunt accensi pulveris vel saltem sclopeti ventanei, cuius non dispar natura" (*ivi*, p. 256).

No weight, but a considerable density. These are additional specifications to what was written on the hypothesis of the *conatus explosivus* in the *Zweite Bearbeitung*.

5.6.2 Gravity as an Action Deriving from the Centrifugal Force

In § 16 of the *Illustratio*, Leibniz focused on gravity as an action deriving from the centrifugal force of an aethereal matter. The context in which Leibniz inscribed his speculations on gravity is the same as where he:

1) starting from the supposition that all planetary orbits have the same *vis viva*, proved that Kepler's third law can be deduced inside his system;
2) showed—at least in his opinion—that the planets moving of harmonic circulation do not perceive any resistance of the means in which they move (for these items, see Sect. 4.2, in particular Sects. 4.2.2 and 4.2.3).

The hypothesis of the orbits having the same *vis viva* allowed Leibniz to propose a unified and simple version of his theory because, by means of this hypothesis, the necessity to postulate the existence of different aethereal matters for gravity disappeared: Leibniz reminded the reader:

A) the idea of gravity could derive from a centrifugal tendency was due to Kepler and developed by Descartes, Huygens and Leibniz himself (*ivi*, proposition 16, p. 268).
B) There was the need to find a mechanism which permitted an action towards the centre and not towards an axis of rotation. This mechanism had been conceived by Huygens in *Traité de la lumière* and its supplement, *Discours de la cause de la pesanteur* to explain gravity in the solar system and by Leibniz himself (*ivi*, p. 268). As we have seen, Huygens had already thought of this mechanism in 1669 to explain gravity on the earth.
C) Since the fluid of the harmonic vortex tends towards an axis, it was necessary for Leibniz to think of a different kind of aether responsible for gravity (*ivi*, p. 268). Nevertheless, Leibniz, as it was also the case in *De causa*, did not insist on this question. Rather, he developed the reasoning I expound in the following two pages, whose basis is the idea that, although the problems of gravity towards the axis rather towards the centre does not yet have a completely satisfying solution, the hypothesis of the centrifugal force to explain gravity is so good that it has to be, anyway, accepted.

On the other hand, the idea that the orbits have the same *vis viva* implies that the inverse square law is valid for gravity without the need to postulate e further aether. This does not exactly mean that Leibniz thought there was a sole kind of aether, but certainly only one kind of aether was sufficient for the reasoning to be correct in mathematical terms.

The argumentation runs like this:

Let us indicate by V the *velocitas circulandi*; by r the distance sun-planet, by c the *conatus centrifugus* and by the symbol "$\approx$" the expression "to be proportional to".

1) $c \approx \frac{V^2}{r}$; 2) $V^2 \approx \frac{1}{r}$, this is a consequence of the hypothesis on the *vis viva* of the orbits (see Sect. 4.2.2, item 2)); 3) hence $c \approx \frac{1}{r^2}$.

A very important annotation is necessary: as seen, Leibniz, while referring to *velocitas circulandi* thought of the vector today we call transverse velocity, while the deduction 2) from the hypothesis on the *vis viva* of the orbits concerns the modulus of the vector velocity, that is: V in 2) is not the transverse velocity. This means that Leibniz's reasoning works if and only if the orbit is a circle and the centre of the forces is in the geometrical centre of the movement so that the radial velocity is null and hence the modulus of velocity coincides with the modulus of the transverse velocity. This is not the case with planetary motion. But the eccentricity is so small, that Leibniz's reasoning—in the case one accepts his strong hypotheses—can be accepted as a good approximation of reality.

Since the centrifugal conate is as the inverse of the distance-square, then gravity, deduced from it, is as the inverse of the distance-square, as well.

Leibniz added that the denser the centrifugal fluid is, the stronger the tendency towards the centre of a less dense mass is.

Leibniz triumphantly concluded this part of his argumentation with these words:

> In this way, the sole hypothesis of the concentric orbits having the same power of circulation, which, in itself, can be considered quite consistent with the reasoning, would provide both gravity law and periodic times' law.[58]

From an epistemological standpoint, the position to which Leibniz's adheres in these pages is quite interesting:

1) he aimed at finding the real mechanism of gravity, not at determining a gravity theory, which works independently of its (at least possible) physical truth. The aethereal vortices offer this opportunity;
2) however, the problem to explain why gravity acts towards a centre rather than towards an axis has not been solved in a satisfactory manner;
3) notwithstanding, since the hypothesis of *vis viva*'s equality among the orbits offers a proof of the inverse squares law and of the third Kepler law, then the problem in 2) can be—at least temporarily—neglected, though it is not solved.

Therefore: while Newton did not deal with the general physical mechanism of gravity—this means that he posed gravity as a given force—Leibniz dealt with this mechanism, but, once he had explained the mechanism, he adopted a hypothetical view as to a sub-mechanism, the one that pushes the bodies towards a centre, rather

[58] Leibniz (1706, 1860, 1962, VI, p. 268). Original Latin text: "Ita unica hypothesis orbium concentricorum potentia circulandi aequalium, quae per se rationi admodum consentanea judicari potest, simul legem gravitationis et legem temporum periodicorum daret".

than towards an axis. For, Leibniz, in this final phase of his scientific speculation on gravity, not in the previous ones, seems to assume this sub-mechanism as given, independently of his physical explanation. The fact that his theory—at least according to his opinion—solves the problems in 3) appeared a sufficient justification for the theory and the sub-mechanism. But the situation is even more serious because Leibniz accepted a mechanism, which—at least if the explanations given by Huygens and by Leibniz himself are deemed unsatisfactory, as Leibniz seems to think since 1690—appears to be in contradiction with the nature of the vortices. Here we are in the presence of a tangible case of one of those characteristics connoting scientific theories identified by Feyerabend in his *Against Method*: a scientist can neglect to accept a series of facts or consequences of a scientific theory if, in most and in the most important cases, theory works.[59] And Leibniz thought his theory worked.

Finally, Leibniz added that other causes of gravity are possible. He mentioned, for example, Kepler's idea that the rotation of the planets around the sun is due to some particles emitted by the sun and forming something like a series of bars from the sun to the planets which—thanks to the rotation of the sun around its axis—determines the mean motion of the planets.[60] However, after all, the hypothesis of vortices, with appropriate corrections, specifications and improvement, carried out by Huygens and by Leibniz himself, is the most plausible and the one which offers the most complete picture to explain the celestial and terrestrial phenomena.

5.7 Final Remarks on Leibniz's Gravity Theory

In this chapter I have tried to show how strong the connection between planetary theory, cosmology and theory of gravity is, and how these three sections of Leibniz's physics were developed jointly, although it is possible to identify the two *Tentamina* and the *Illustratio* as specific works concerning planetary theory, while Leibniz's ideas on cosmology and gravity are disseminated in numerous works. However, it is impossible to get a complete idea of planetary theory without specific references to gravity theory and cosmology. We have also seen the evolution of Leibniz gravitational theory and of his ideas on the origin of gravity. In this context, until the publication of Newton's *Principia*, the main concern of Leibniz was to provide a plausible mechanism for gravity, in the tradition of Descartes, Hobbes and Torricelli, although with personal ideas which, often, did not coincide with those of these other scientists. After publication of the *Principia* and with the full comprehension that a theory of gravity based on mechanical models had to fulfil two requests: 1) to explain from the phenomenological point of view how

[59] As to this question, see the interesting considerations by Feyerabend in Chaps. 5–9 in Feyerabend (1975).

[60] I will face the problem of the influence of Kepler's theory on Leibniz in Chap. 6.

gravity is produced; 2) to explain from a physical-mathematical standpoint the inverse square law, Leibniz's researches became wider and more profound. In this context Huygens' works were fundamental for Leibniz, but the critical comparison with authors such as Papin, Catalan, De Volder, Fatio de Duillier, Johann and Jakob Bernoulli was important, as well.[61] In particular, in the years following the publication of the *Principia*, the problem of guaranteeing the validity of the inverse square law inside the vortex theory took a prominent role in Leibniz's thought. Progressively, Leibniz also adhered to the idea that gravity on the earth was an action quite similar to the force responsible for planetary movements, even though it is difficult to establish whether he had fully realized it was exactly the same force. There are numerous passages in Leibniz's works and letters which testify to this situation. Among them, I have chosen these three which seem to me particularly significant. Many others could be selected.

One of the most interesting documents is the letter to Des Billettes in December 1696, where Leibniz dealt with many questions concerning gravity. In particular he wrote:

> As to gravity (*pesanteur*) Mr. Newton taught us a proportion, of which I knew already something; namely that the planets are such that the gravities or attractions are in inverse proportion as the squares of the distances. [. . .] Now, I have found this agrees with the action of light rays. For, a subtle, but dense (*solide*) fluid will recede from the centre and form something as emission rays. This is also in agreement with the receding from the centre along the tangent, property which Kepler first applied to gravity. In this Mr. Descartes followed him. But the sole instantaneous *conatus* of the centrifugal force is not enough to originate either light or gravity, in the manner imagined by Mr. Descartes. It is necessary that a very movement of emission is produced, as a wind which blows and which requires time. The emission of a more massive or more dense (*serré*) fluid produces necessarily the attractions of the bodies which are less massive or less dense.[62]

[61] As to gravity in Leibniz, my aim has been—at least essentially—to follow the development of Leibniz's ideas and their connections with his planetary theory. A comparison between Leibniz's conceptions of gravity with those of other authors who dealt with this subject in the same years goes beyond my purposes. Fatio de Duillier is particularly significant in this context. The work *De la cause de la pesanteur* by Fatio de Duilliers, edited in 1929 by K. Bopp is now available on the internet in pdf version. Web site: http://www.mahag.com/grav/bopp/fatio-bopp.pdf, see De Duilliers (1690). With regard to Fatio and to his theory of gravity, Zehe (1980) is a fundamental reference point.

[62] Leibniz ([1875–1890], 1978, VII, p. 452). Original French text: "Quant à la pesanteur, Mons. Newton nous a appris une proportion dont je sçavois pourtant dèja quelque chose; c'est que les planetes sont voir que les pesanteurs ou attractions s'y sont en raison reciproque quarrée des distances. [. . .] Or je trouve que cela s'accorde avec l'action des rayons de la lumiere, car encore un fluide mince mais solide, s'eloignant du centre, formera comme des rayons d'emission. Cela s'accorde aussi avec l'eloignement du centre par la tangente, qui Kepler a appliqué le premier à la pesanteur, en quoy M. des Cartes l'a suivi. Mais le seul conatus instantané de la force centrifugue ne suffit pas pour former soit lumiere ou pesanteur comme M. des Cartes a cru: il faut qu'il en naisse un veritable mouvement d'emission, et comme un vent qui souffle et qui demande du temps. Or l'emission d'une fluide plus massif ou plus serré fait necessairement l'attraction des corps qui le sont moins".

This brief quotation seems almost a summary on Leibniz's late conception of gravity:

1) one of the main goals is to prove the inverse square law inside vortex theory;
2) a *subtle* but *dense* fluid emanating from the centre forms a ray of emission which satisfies the same laws as light. Hence, in particular, the inverse square law. This emission is not instantaneous. This is the hypothesis of the *conatus explosivus*;
3) the idea of gravity deriving from centrifugal force is also coherent with the inverse square law. In the central phase of his thought Leibniz seems more favourable to the idea of a *conatus explosivus*, whereas in the *Illustratio*—ten years later—he seems to prefer the hypothesis of a centrifugal force with the further condition on the identity of planetary orbits' *vis viva*.

The fundamental interpretative element is, in my opinion, the last sentence of the quotation: a fluid which is more *massive* (with a bigger specific weight) or more *dense*, this is the translation of *serré*, produces gravity. Thence *specific weight* is not proportional to *density*, according to Leibniz. Certainly—referring to the examined mercury-experiment—the mercury in the tube is heavier and denser than the solid bodies which are in it, but it is also possible that a material *A* is denser but not heavier than another material *B*. Notwithstanding, *A* can be the cause of *B*'s gravity, under certain conditions. This is the conception Leibniz had of aether.

A very interesting letter was addressed by Leibniz to Hartsoeker on 8th February 1712. Here we read:

> At all appearances, the gravitation of the planets towards the sun is due to a cause similar to that producing the gravitation of the terrestrial bodies: now, if one conceives gravity, in an abstract and mathematical way, as a cause, which pushes the heavy bodies towards the centre by means of rays which can be traced by straight lines from the centre towards the heavy bodies, it follows, on the basis of a geometrical reasoning, that the gravities are as the inverse squares of the distances. This demonstration procedes in the same manner as that, by which the opticians prove the bodies are illuminated in inverse proportion to the square distance. I found that, when a planet rotates around the sun in inverse proportion to its distance from the sun, this circulation added to gravity produces perfectly Kepler's planetary laws.[63]

Leibniz underlined once again the importance of the inverse square law and the connection between the way in which light and gravity are spread. He also proposed a physical-mathematical model of gravity which can be visualized as a central mass

[63] Leibniz ([1875–1890], 1978, III, p. 534). Original French text: "Il y a de l'apparence, que la pesanteur des planetes vers le soleil vient d'une cause semblable à celle, qui fait la pesanteur des corps terrestres: or concevans la pesanteur abstraitement et mathematiquement comme une cause qui pousse le corps pesant vers le centre par autant de rayons qu'on peut tirer des lignes droites du centre vers le corps pesant, il en vient par un raisonnement Geometrique, que les pesanteurs sont en raison doublée reciproque des distances, de la même maniere, que les opticiens prouvent que les corps sont illuminés en raison quarrée ou doublée reciproque des mêmes distances. J'ay trouvé, que lorsqu'une planete circule à l'entour du soleil en raison reciproque de sa distance du soleil, cette circulation jointe à la pesanteur, produit parfaitement les loix planetaires de Kepler".

with a star[64] of arrows centred in the centre of the mass and with the points directed to that centre, as well. Thence, Leibniz distinguished between the physical-mathematical model which works independently of the real nature of gravity and the models based on the vortex theory which, actually, are physical-structural models because they have to describe the real situation from which gravity arises. Finally: Leibniz spoke of a "similarity" between gravity on the earth and gravity as cause of the planetary motion. Once again it is difficult to understand if he realized that the two gravities are exactly the same force or if he thought that there was a similarity in a broader sense as the one he had seen, for example, between gravity and magnetism or gravity and elastic forces. To admit they are the same force, he should have developed a theory in which the two gravities are produced by a sole kind of aether. We have seen that in the *Illustratio*—as a matter of fact—a sole aether seems needed, but—I repeat—this does not mean that Leibniz necessarily believed a sole aether exists. However, one fact is clear: in the course of the years he progressively recognized the similarities between the two gravities.

In the correspondence Leibniz-Clarke, the fifth writing by Leibniz is quite important in this sense, as we read:

> For, both mercury and water are masses of heavy material. They are full of holes, across which many non-heavy matters pass, which do not resist in a perceptible manner, as it happens for the matter of the light rays and of other insensible fluids. In particular this is the case for the fluid itself, which causes the gravity of the solid bodies, when they are moved away from the centre, towards which this fluid makes them to move. For, it is a strange fiction to suppose the whole matter is heavy, and that it is heavy with respect to the entire rest of matter, as if each body would attract, in the same manner, each other body, in proportion to the masses and the distances, that is thanks to an attraction in a proper sense, which is not derived from an occult impulse of the bodies. In contrast to this, the gravity of the sensible bodies towards earth's centre has to be produced by the movement of a liquid. For the other gravities, as that of the planets towards the sun, or the mutual attraction of the planets, the situation is the same. A body is never moved in a natural way if not by another body, which hits and, consequently, pushes it. After that, this body will continue its motion until it is not prevented from moving by another body, which touches it.[65]

[64] By the word "star" I mean here the "star of lines" of spatial projective geometry.

[65] Leibniz ([1875–1890], 1978, VII, pp. 397–398). Original French text: "Car tant le vif argent que l'eau, sont des masses de matiere pesante, percées à jour, à travers desquelles passe beaucoup de matiere non pesante, et qui ne resiste point sensiblement, comme est apparemment celle des rayons de lumiere, et d'autres fluides insensibles; tels que celuy sur tout, qui cause luy même la pesanteur des corps grossiers, en s'ecartant du centre où il les fait aller. Car c'est une étrange fiction que de faire toute la matiere pesante; et même vers toute autre matiere, comme si tout corps attiroit egalement tout autre corps selon les masses et les distances; et cela par une attraction proprement dite, qui ne soit point derivée d'une impulsion occulte des corps: au lieu que la pesanteur des corps sensibles vers le centre de la terre, doit étre produite par le mouvement de quelque fluide. Et il en sera de même d'autres pesanteurs, comme celle des planetes vers le soleil, ou entre elles. Un corps n'est jamais mû naturellement, que par un autre corps qui le pousse en le touchant; et apres cela il continue jusqu'à ce qu'i1 soit empeché par un autre corps qui le touche".

This quotation refers to many *topoi* of Leibniz's conception as that according to which the movement can be generated and modified only by contact. There are two interesting observations for the context I am dealing with:

1) Leibniz spoke of the gravity of bodies on earth and of the gravitation of the planets around the sun. Both of them are produced by the movement of "quelque fluide", but, once again, it not possible to deduce if, according to Leibniz, the fluid is the same or is different.
2) not each matter is heavy. Many non-heavy matters pass across the pores of the heavy matters and generate interactions such as light and gravity.

This second issue is remarkable: Leibniz distinguishes between a non-heavy matter which is the active cause of gravity and a heavy matter which is the subject of gravity. This conception is not so far from that expressed by Leibniz in his early work *Hypothesis physica nova*. This means that, despite the numerous and important developments of Leibniz's conception, many of the basic properties of aether he had identified in his early production were considered valid in his late works, too. This letter to Clarke clarifies in a definitive manner Leibniz's thought, but this clarification has been possible only after Leibniz had continuously reworked and tried to improve his gravitational theory in the light of Newton's *Principia* and of his correspondence and polemics with the Newtonians. All the specifications Leibniz added to Cartesian vortex theory, also thanks to the discussions with other supporters of such a theory—mainly Huygens—brought him to the final form of gravity theory I have tried to analyse. The analysis of the gravity mechanisms created by Leibniz is fundamental for a correct interpretation of his general view on physics and on cosmology. In particular, Leibniz's efforts to make the aether-vortex theory coherent with the data and his attempts to satisfy, inside this theory, the mathematical results of Newton's physics probably represent the ripest form of the vortex theory itself and are the results of a continuous work to which Leibniz attended in the course of his life. Nevertheless, given the development of physics between the publication of Newton's *Principia* and Leibniz's death, the opinion of Garber can be shared:

> By the time he [Leibniz] died in 1716, the strict mechanism that had been so modern and daring in his youth, the view around which he built his metaphysical physics, was well on its way to becoming an anachronism.[66]

Finally an observation on the general way in which Leibniz framed gravity theory inside his general thought: in Chap. 3, I have already offered a distinction inside Leibniz's metaphysics. Here I deal with a general tripartition of Leibniz's ontology. It is recognized by several scholars that, after all Leibniz distinguished three levels of reality:

1) the phenomenal level, to which the physical laws based on the efficient causes (typically the principle of cause-effect) correspond;

[66] Garber (1994, 2006, p. 335).

2) the dynamical level, to which the principles of conservation belong. The most important of these principles is the *vis viva* conservation principle. This is the level in which Leibniz introduces the final causes in physics;
3) the metaphysical level, in which, properly speaking, the substances do not interact among them. There is no interaction body-body, body-mind, mind-mind. This level is dominated by the two great principles of sufficient reason and pre-established harmony.[67]

Gravity is connected with all three levels: from the phenomenal point of view, gravity is due to the mechanisms described in the previous pages, based on movements and impacts, according to the mechanistic tradition; from the dynamical point of view, the *vis viva* is an element which connotes the vortices surrounding each planet. For, each planetary aethereal vortex has the same *vis viva*. This is an aspect, which makes sense only inside an ontological conception of *vis viva*, because in our vision it makes no sense to claim that a body or a system of bodies maintains eternally the same *vis viva*. We have a relational conception of energy, and this is true for Leibniz, too, if we refer to the phenomenal level. Namely, it makes sense to claim that, in a given interaction, a body acquires and a body loses part of its *vis viva*. However, from a metaphysical and authentic point of view each body has a quantity of *vis viva*, which connotes it forever. The assertion that all planetary vortices have the same *vis viva* enters hence inside this picture and shows how the dynamical level is in between the phenomenal and the metaphysical one. From a purely metaphysical standpoint gravity, as all physical and mental phenomena, can be explained in terms of pre-established harmony. However, in this regard, Gregory Brown poses a profound question and offers an answer, which is interesting for the picture I am describing[68]: Brown wonders why Leibniz did not connect directly gravity to pre-established harmony, but used all the devices I have described to explain gravity. After all, Leibniz could have thought—coherently with the principle of the pre-established harmony—of an attraction given to the bodies by God from the beginning of the universe. In this way an attraction at a distance would have not existed as a physical mysterious entity; the attraction would have been reconducted once and forever to God. At the same time, this hypothesis would have prevented Leibniz from the enterprise of explaining gravity by means of an intangible, dense, but not heavy and, in fact, problematic aether. To speak *à la* Brown "the *Une-Fois-Pour-Toutes* explanation of gravity" (*ivi*, p. 154) was fully coherent with the metaphysical principle of the pre-established harmony. Why did not Leibniz adopt this argumentative strategy? Brown, relying upon some evidences drawn from Leibniz identifies two reasons:

[67] I will deal with these problems more in depth in the following Chap. 6, especially as to the pre-established harmony. For the moment, I will introduce these three ontological levels only in relation to gravity. I follow the interpretation according to which, from a metaphysical point of view, there is no interaction either among bodies (see, i.e. G. Brown 2007, p. 154).

[68] See G. Brown (2007), in particular pp. 154–157.

1) the *Une-Fois-Pour-Toutes* explanation is nothing but a miracle, because this attraction—although reconducted to a direct decision of God—would have interfered with the way in which the movements can be explained in the phenomenal world, namely by impact. This means that every movement, whatever its real and metaphysical origin is, must not be in contradiction with the physics of the efficient causes, namely with the mechanics based upon laws explained by impacts. If this does not happen, the world becomes unintelligible at all, anything can assume the form of a miracle and, hence, no authentic science could exist;
2) In Leibniz's conception, the *Une-Fois-Pour-Toutes* explanation is not coherent with the *vis viva* conservation principle. For, when two masses are attracted and move the one towards the other because of this attraction, the quantity of *vis viva* in the universe—*coeteris paribus*—increases (*ivi*, pp. 155–156).

These are the reasons why—according to Brown—Leibniz did not connect directly gravity to the metaphysical principle of pre-established harmony. This would have meant denying almost the whole of his physics.

I have mentioned these conceptions by Brown because they seem to me coherent with my line of interpretation: planetary theory is connected to gravity, gravity is connected to general physical and metaphysical conceptions by Leibniz. Thus, we achieve a confirmation that a full understanding of planetary theory cannot be obtained without a profound glance at the way in which it is connected to Leibniz's general principles, both physical and metaphysical. This will be even clearer in the next chapter, while dealing with the influence exerted by Kepler on Leibniz's planetary theory and, more in general, on his system.

Chapter 6
Kepler's Influence on Leibniz's Planetary Theory

Kepler was one of the most important sources of inspiration for Leibniz. The influence of Kepler can be detected in numerous features of Leibniz's planetary theory as well as in other aspects of his thought:

1) Leibniz, as Kepler, was convinced that the analysis of planetary motions had to be inscribed inside a general view of the phenomena in which the reference frame was offered by metaphysics;
2) Mathematics had to play a fundamental role in the astronomical explanations, but astronomy could not be only a device to "save the phenomena". It had to supply a physical explanation of the planetary movements, too;
3) Some Keplerian conceptions such as the "true hypothesis" had a parallel in Leibniz's conceptions as "natural laws";
4) Leibniz, as we have seen in detail, distinguished between two components of planetary motion: a component responsible for the mean motion (*circulatio harmonica*) and a component responsible for the approaching and moving away of a planet to/from the sun (*motus paracentricus*). Kepler distinguished various components of planetary motion, but the most important ones were exactly those to which Leibniz also referred.

Furthermore there are other questions connected to the complex Leibniz-Kepler relation:

5) Leibniz often referred to some Keplerian concepts, as that of *inertia naturalis*, giving the reader to understand his ideas on inertia were drawn from Kepler. I have dealt with this problem in Chap. 3.
6) With respect to the problem of inertia, Leibniz attributed to Kepler the discovery that, in curvilinear motion, bodies tend to escape along the tangent, a discovery that Kepler never claimed.

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7) As Bertoloni Meli underlines,[1] Leibniz did not consider Kepler's laws as fundamental laws of nature until the end of the 1670s. After that, he changed his mind and Kepler's laws play a central role in the *Tentamen* and in the *Illustratio*. Nevertheless, as we have seen, Leibniz seemed open to the possibility that the orbit of a planet was not an ellipsis (see what he claimed on Cassini's ovals) many years after the publication of Newton's *Principia*. This is a further interesting element, which concerns the scientific influence exerted by Kepler on Leibniz, but which also regards important aspects on the way in which Leibniz's thought was developed in the course of the years.

Obviously, it is necessary to take into account that Leibniz wrote his works on planetary motion almost 60 years after Kepler's death and almost 70 years after the publication of the *Epitome*, therefore many new scientific acquisitions had been achieved and some differences between Kepler's and Leibniz's conceptions can be explained in the light of the new scientific gains. Nevertheless, the fact that, after so many years, Leibniz referred to Kepler as one of his most important—perhaps the most important—source on planetary theory makes it clear that the scientific acquisitions between Kepler's *Epitome* and Newton's *Principia* were not sound. This must induce us to appreciate even more Newton's work.

Given the complexity of the scientific Kepler-Leibniz relations and the difficulty to disentangle the seven previous items,[2] it is possible to follow two approaches: (1) either a chronological approach. This means to detect Kepler's influence on Leibniz's works starting from *Dissertatio de Arte Combinatoria* until the correspondence with Clarke; (2) or an approach which starts from a specific subject, and, from this starting point, tries to progressively include all seven previous items. I choose the latter approach because Leibniz's planetary theory is based on the distinction between *circulatio harmonica* and *motus paracentricus* in an essential manner. All the interpretations and the attempts to insert planetary theory inside the general context of Leibniz's thought have to begin with this initial unavoidable datum. Hence, it is first necessary to check what was Kepler's influence on this distinction carried out by Leibniz.

6.1 Kepler/Leibniz: The Division of Orbital Motion into Two Components

In *Astronomia Nova* and, afterwards, in *Epitome Astronomiae Copernicanae*, Kepler recognized four kinds of planetary motions: (a) the mean motion; (b) the approaching and moving away of a planet to/from the sun; (c) the motion in latitude; (d) the motion of the apses-line, which can be interpreted as a motion of the reference frame. I will concentrate on the first two, because they are relevant for Leibniz.

[1] Bertoloni Meli (1993, p. 36).

[2] In this chapter I will not deal with the concept of inertia because this problem has been already addressed in Chap. 3.

6.1.1 The Mean Motion

With regard to the mean motion of the planets, in Kepler it is due to the *virtus motrix* of the sun, which is spread from the sun through a *species immateriata*. The intensity of the *virtus motrix* decreases as the distance from the sun. The *species immateriata* is composed of immaterial rays, whose nature is almost-magnetic, which emanate from the sun's surface. Each ray hits the planets and, because of the daily rotation of the sun—which is a fundamental datum in Kepler's physical explanation of planetary motions—, it induces a movement in the planets in the same direction as the sun's rotation. Because of this Kepler spoke of such a force as a *virtus promotoria*, that is which induces movement, rather than *virtus tractoria*, namely which attracts—as gravity, or magnetism in a proper sense.[3] Kepler claimed the intensity of the *virtus* decreases as the distance from the sun because, although the rays lose nothing of their intensity while moving away from the sun, their density diminishes,[4] hence their global effect diminishes. Kepler, also considering his researches on the way in which light is spread, conceived the idea that intensity of the *virtus motrix* could be spread as the inverse square distance from the sun. However, after all he decided for a linear diminishing. With regard to speed, in Kepler's conception the *virtus motrix* induces velocities and not accelerations, hence the farther a planet from the sun, the slower its motion, because—given the time t—a superior planet is touched by fewer rays of the sun's virtue than an inferior one during the sun's rotation. Actually, the mass also plays a role because—given Kepler's conception of natural inertia—the more massive a planet, the bigger its resistance to the action of the solar virtue is, due to the natural inertia of the masses. Therefore, the speed of a planet depends inversely as its distance from the sun and has an inverse functional link with masses, too, although this link is not a linear one. The *virtus motrix* in isolation can only produce a uniform circular motion in which the sun is in the geometric centre of the orbit, namely it can produce the mean motion of the planets.

The given ones are only the basic features of Kepler's *virtus motrix*, whose full exegesis is rather complicated till the point that there is not a general agreement

[3] The distinction between *virtus promotoria* and *virtus tractoria* is evident in *Astronomia Nova* and in *Epitome*, but these expressions are explicitly used by Kepler in a letter to Maestlin on 5 March 1605 (KGW, XV, pp. 170–176).

[4] Kepler's idea is that every point of the sun's surface emits one ray. The reduction of the ray's density while moving away from the sun is due to this: let us suppose that two points P and Q of the sun's surface are separated by a certain arch which corresponds to an angle at the centre α, with C being the sun's centre. One ray is spread in the direction CP, the other one in the direction CQ. Moving away from the sun's surface, the arch of circumference (that is the distance which separates the two rays) increases. This is why Kepler claimed that the density of the rays diminishes when they go away from the sun's surface.

among the scholars as to some of its characteristics.[5] However, this is enough in the perspective of a research which aims at detecting Kepler's influence on Leibniz.

In *Astronomia Nova*, Kepler dealt with *virtus motrix* in Chaps. XXXII–XXXVI (KGW, III, pp. 233–252). In *Epitome*, Kepler explained the features of *virtus motrix* in Book IV, Part II, Chap. III, *De revolutione corporis Solaris circa suum axem, ejusque effectus in motu planetarum* (KGW, VII, pp. 298–306). There is no doubt that Leibniz knew *Epitome*, as he quoted this work several times. While, it is likely that Leibniz did not read directly *Astronomia Nova*, since he did not mention it.

From a kinematical point of view, Leibniz's *circulatio harmonica* has exactly the same role as Kepler's *virtus motrix*, namely both are responsible for the mean motion of the planets. Even though Leibniz did not explicitly claim that he drew his conception directly from Kepler, it is quite probable he did. There are many evidences in this sense. The first one is the conceptual evidence due to the similarity the two concepts hold inside the systems of the two authors. Furthermore there is also linguistic evidence: in *Epitome*, Kepler used four times the word *vortex* in four circumstances, while referring to *virtus motrix* and to the sun's rotation around its axis: (1) KGW, VII, p. 299, line 44; (2) KGW, VII, p. 307, line 1; (3) KGW, VII, p. 320, line 10; (4) KGW, VII, p. 329, line 40. Leibniz always considered Kepler the inventor of modern vortex theory and—in substance—accused Descartes of plagiarism: Descartes would have developed a conception which was already present in Kepler. This position might seem only an attempt to discredit Descartes, but on second thought a justification can be found, which relies on the most profound convictions of Leibniz: it is true that Descartes developed modern vortex theory—or, probably, from a historiographic point of view, it would be more correct to say he invented vortex theory, as Kepler's considerations are far from being a theory—, but in Descartes' theory, there is no attempt to distinguish between the mean motion of the planets and their motion towards and away from the sun. In Kepler this part of the theory is well developed, though the general statement that a vortex theory existed in Kepler is debatable. In the course of the years, Leibniz's interest in planetary motions increased and he found in Kepler an author who—at least according to Leibniz's mind—shared many of his opinions. In particular they shared the idea on the necessity to separate the planetary motion around the sun into two components. Thence, although Kepler did not develop a complete vortex theory, he had caught—according to Leibniz—the rich conceptual core of the theory, which means the idea that two different causes were responsible for planetary motions. On the contrary, Descartes did not develop this part of the theory and—superficially and with many mistakes, according to Leibniz—tried imprecise generalizations without dealing with the important conceptual core pointed out by Kepler. Because of this, for Leibniz, the very inventor of modern vortex theory is Kepler rather than Descartes. This is confirmed by what Leibniz

[5] The literature on Kepler's *virtus motrix* and, more in general, on the concept of "force" in Kepler is rather conspicuous. For the conceptualization of these problems and for an abundant series of references, see Pisano and Bussotti (2016)—forthcoming.

wrote at the beginning of the *Tentamen*, where he appeared surprised that Descartes, who developed many aspects of vortex theory, did not try to offer explanations of the planetary motions and of their ellipticity, inside vortex theory. Leibniz wrote:

> Further I often marvel that Descartes did not even try to provide reasons for the celestial laws discovered by Kepler, as far as we know, either because he could not reconcile them sufficiently with his own opinion, or because he remained ignorant of the fruitfulness of the discovery and did not consider it to be so accurately followed by nature.[6]

While, Leibniz was going to create a vortex theory in which the planetary movements were explained: the vortex created by the *virtus motrix* in Kepler and the *aethereal harmonic vortex* in Leibniz hold the same role and both of them are the sole causes of the planetary mean motion. Namely: in Leibniz, as in Kepler, the circular planetary mean motion is due to a sole action, not to a combined action as in Newton, where the curvilinear planetary motion is due to an initial velocity plus a centripetal force. Here we have a situation, which is quite interesting from a historical point of view: Kepler thought that the solar virtue moves the planets by the rays of the *species immateriata*; these rays can be thought of as threads which touch a planet and move it. If the threads were cut or if, for some reasons, the sun ceased its rotation, according to the Keplerian concept of *natural inertia*, the planet would stop. Since Kepler did not explicitly deal with this problem, it is not clear if the planet would stop immediately or (which is far more likely) after having exhausted the *impetus* imparted by the solar virtue. For sure, it would have been stopped after a certain time. Leibniz, who wrote after Huygens' researches on centrifugal force and—as far as the *Tentamen* is concerned—after Newton's *Principia*, cannot accept this conception: he is perfectly aware—as we have seen—that, if in a sling the thread is cut, the projectile escapes—indefinitely—along the tangent. This is not and could not be Kepler's conception—given his ideas on inertia (see Chap. 3). Nevertheless, Leibniz ascribed this conception to Kepler, which is a mistake, but it can be explained if one considers that Leibniz drew the idea to decompose the planetary motions in two components by Kepler. Hence he had the tendency to highlight the correspondences between his own and Kepler's ideas, also beyond the true similarities. This tendency was encouraged by the fact that Kepler thought of two different "forces" which produce velocities (mean

[6] Leibniz (1689, 1860, 1962, VI, p. 148). Translation drawn from Bertoloni Meli (1993, p. 128). Original Latin text: "Miratus autem saepe sum, quod Cartesius legum coelestium a Keplero inventarum rationes reddere ne aggressus est quidem, quantum constat, sive quod non satis conciliare posset cum suis placitis, sive quod felicitatem inventi ignoraret nec putaret tam studiose a natura observari". It is difficult to establish whether Descartes did not deal with this problem because the ellipticity was difficult to be explained inside his theory, or because he was afraid of Chatolic Church's censure of Copernicanism and, hence, in his *Principia*, he did not want to enter into many details, or for both reasons. With regard to the style of Descartes' *Principia* and on the omissions of ideas which could be condemned by the Church, see Bussotti and Pisano (2013), in particular the section "Final remarks on Descartes' *physical works*", p. 121. We also provide references on this subject.

motion and approaching to/moving away from the sun) and not modifications of velocities. When Kepler wrote, there was no inquiry on the physical-dynamical causes of the planetary motions, but when Leibniz wrote the *Tentamen*, Newton had clarified his conception of force. Newtonian forces produce accelerations, not velocities. While—as underlined—Leibniz tried to construct a mechanism in which the actions produce velocities, therefore Kepler was a perfect reference point. Thence Leibniz overestimated the similarities between his ideas and the ideas of his source of inspiration. Since Leibniz stressed on several occasions that Kepler reached the idea of the tendency to escape along the tangent in a curvilinear motion, it is likely he really thought this was Kepler's opinion and that hence Leibniz's was not—or, at least, was not exclusively—an operation to find an authoritative predecessor and to separate his own theory both from Descartes' (who—as told—did not distinguish between mean motion and approaching to/moving away of a planet from the sun) and Newton's.

Leibniz—which is significant—refused those aspects of Kepler's theory that seemed to him related to souls, magnetic influences (if not explained mechanically), immaterial entities and everything which was not connected to physically detectable quantities. According to Leibniz, this prevented Kepler from developing a complete and successful theoretical planetary physics. Thence, Leibniz considered himself the very continuer of Kepler's work. To be precise: the scientist who—also thanks to a kind of mathematics, the calculus, which was not available to Kepler and of which he was the inventor—developed and made Kepler's ideas more precise and well defined, by eliminating the recourse to mysterious and not detectable entities. In the introductory section of the *Tentamen*, Leibniz first of all, underlined that Kepler was almost unaware of the richness of his physical ideas. For, he wrote:

> [. . .]insufficiently aware of how many things would follow therefrom in physics and especially in astronomy.[7]

He continued:

> Further, since it seems not at all the province of physics, and indeed unworthy of the admirable workmanship of God, to assign to the stars individual intelligences directing their course, as if He lacked the means for accomplishing the same by laws governing bodies; and to be sure solid orbs have some while now been rejected, while sympathies, magnetisms and other abstruse qualities of that kind are either not understood or, when they are, they are judged to be effects consequent on corporeal impressions-I myself judge there is no alternative left but that the cause of celestial motions should originate in the motions of the aether, or, using astronomical terms, in orbs which are deferent, yet fluid.[8]

[7] Leibniz (1689, 1860, 1962, VI, p. 148). Translation drawn from Bertoloni Meli (1993, p. 128). Original Latin text: "[. . .] nec satis conscius quanta inde sequerentur tum in Physica tum speciatim in Astronomia".

[8] *Ivi*, pp. 148–149. Translation drawn from Bertoloni Meli (1993, p. 128). Original Latin text: "Porro cum minime physicum videatur, imo nec admirandis Dei machinamentis dignum, Intelligentias peculiares itineris directrices assignare sideribus, quasi Deo deessent rationes eadem corporeis legibus perficiendi, et vero orbes solidi dudum sint explosi, sympathiae autem

Leibniz criticized Kepler's use of intelligences, sympathies, magnetism and other abstruse qualities to provide physical explanations. From the point of view of *Kepler Forschung*, the role Kepler ascribed to entities as souls and intelligences inside his physical theory is not easy. In principle, as far as the movements in the skies are concerned, it seems that the rotational movement around the sun's or planetary axis is ascribed by Kepler to the sun's or planets' souls, while, for the movements in which a translatory component exists, Kepler tried to identify merely physical quantities, not tied to vitalistic conceptions. However, in this case, too, there are various interpretations among Kepler's scholars. What is sure—and this is important in our context—is that Leibniz believed these entities had a primary role in Kepler's physical astronomy. Leibniz denied the entities connected both to intelligences and souls. This is fully coherent with Leibniz's general point of view about the physical world: the phenomena have to be explained only by means of the movements of material entities. Therefore, the critical observations to some aspects of Keplerian conception are exactly of the same kind as Leibniz addressed to the Newtonian action at a distance: no physics can exist without a mechanical explanation. This is the very Cartesian heritage in Leibniz's theories.

6.1.2 Approaching to and Moving Away from the Sun: Area Law and the Problem of Ellipticity

Kepler clearly distinguished between the mechanism responsible for the mean motion of the planets and that responsible for the approaching to and moving away from the sun, on which orbits' ellipticity depends. This mechanism is expounded in *Astronomia Nova* (KGW, III, pp. 348–364), but it is explained far more clearly in *Epitome* (KGW, VII, pp. 337–342). This work was Leibniz's reference point.

6.1.2.1 Kepler's Doctrine and Its Interpretation

Kepler imagined the sun as a magnet in which a pole is extended on the whole surface and the other pole is internal. Each planet has a magnetic axis, represented in Fig. 6.1 by an arrow, in which the top identifies the magnetic pole Kepler calls *amicus* of the pole extended on the sun's surface, while the tail of the arrow identifies the pole called *discors*. When the planet rotates around the sun because

et magnetismi aliaeque id genus abstrusae qualitates aut non intelligantur, aut ubi intelliguntur, corporearum impressionum effectus appariturae judicentur; nihil aliud ego quidem superesse judico, quam ut causa motuum coelestium a motibus aetheris, sive ut astronomice loquar, ab orbibus deferentibus quidem, sed fluidis, oriantur".

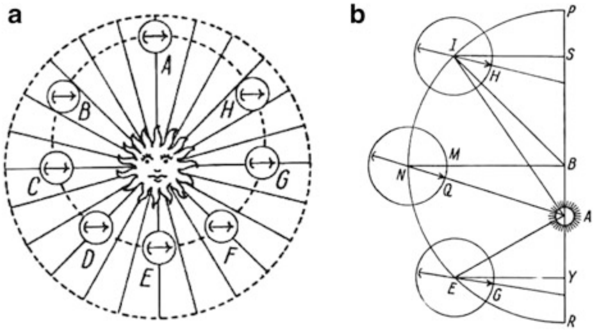

Fig. 6.1 The mechanism of libration in Kepler's *Epitome*. (**a**) The mechanism explaining planetary *libration*. Particular drawn from *Epitome* (KGW, VII, p. 337). (**b**) Specification of the same mechanism. Particular drawn from *Epitome* (KGW, VII, p. 339)

of the *virtus motrix* and the *amicus* pole is directed to the sun, the planet is attracted by the sun; when the *discors* pole is directed towards the sun, the planet is moved away from the sun. This is the basic mechanism, to which Kepler added a series of specifications, but, in our context this is enough.[9] This mechanism, which in itself could also be valid to explain an eccentric circular motion, is applied by Kepler as a physical explanation of the elliptical planetary motion. This is exactly the movement Leibniz called *paracentricus* and that Kepler called *libratio*.[10]

Once again: it is quite probable that Leibniz drew the idea of the paracentric motion from Kepler. The similarities are strong and this makes the hypothesis plausible. However, there is another consideration which, if true, would make the connection Kepler-Leibniz even more profound: we have seen that Aiton has interpreted Leibniz's expression *velocitas circulandi* as transverse velocity. As to Kepler, there is a similar problem: in the *Astronomia Nova*, Kepler claimed that the velocity of a planet in its orbit is as the inverse distance from the sun, which is a serious mistake, as already remarked. While in the *Epitome*, there is a passage in which Kepler gives the impression of thinking of transverse velocity, while referring to the inverse ratio velocity-distance. This question is so important in our context that it is worth being analysed. The reference is to Book V, Part I, Chap. IV of the *Epitome* (KGW, VII, pp. 375–379), where Kepler dealt with the proof of area law. The passage is this:

[9] In this case too, the references are abundant. The possible interpretations of Kepler's mechanism as well as a conspicuous series of references are mentioned in Pisano and Bussotti (2016)—forthcoming.

[10] With regard to the similarity between Kepler's *libratio* and Leibniz's *motus paracentricus* and, more in general, on the influence Kepler exterted on Leibniz as to the decomposition of planetary movements, see: Bertoloni Meli (1993, pp. 27–28), Aiton (1969, p. 77; 1972, p. 128); Hoyer (1979a).

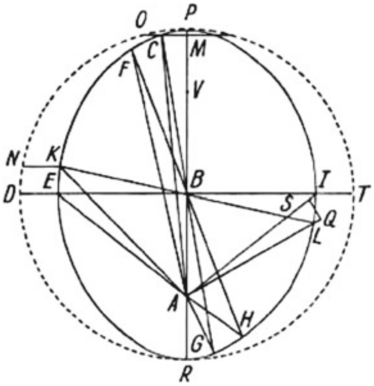	"For it has been said in the above that, if the orbit of the planet is divided into the most minute equal parts, the delays (*morae*) of the planet in them increase in the ratio of the distances between them and the sun. But this is to be understood not of all equal parts as such, but principally of those which are opposite to the sun in a straight line, as PC and RG, where there are the right angles APC and ARG. But in the case of the other parts which face the sun obliquely, this is to be undertood only of that which in any of those parts belongs to the movement around the sun. For since the orbit of the planets is eccentric, therefore in order to form it two elements of movement are mingled together –as has been demonstrated already: one element comes from the revolution around the sun by reason of one solar virtue; the other comes from the libration towards the sun, by reason of another solar virtue distinct from the first". [Translation drawn from Kepler 1995, p. 141.] Original Latin text: "Dictum quidem est in superioribus, divisa orbita in particulas minutissimas aequales: accrescere iis moras planetae per eas, in proportione interuallorum inter eas et Solem. Id verò intelligendum est non de omnimoda portionum aequalitate, sed de iis potissimum, quae recta obiiciuntur Soli, vt de PC. RG. vbi recti sunt anguli APC.ARG. in caeteris verò obliquè obiectis intelligendum est hoc de eo solùm, quod de qualibet illarum portionum competit motui circa Solem. Nam quia orbita planetae est eccentrica, miscentur igitur ad eam efformandam duo motus elementa, vt hactenus fuit demonstratum, alterum est circumlationis circa Solem virtute Solis vna, reliquum librationis versus Solem virtute Solis alia distincta a priori".
The figure used by Kepler in *Epitome* to prove area law, to which the mentioned passage is referred (KGW, VII, p. 376).	The passage in which Kepler seems to refer to the transverse component of velocity (KGW, VII, p. 377, lines 27-36).

Fig. 6.2 The component of velocity perpendicular to the radius-vector and the area law

Kepler is considering *particulas minutissimas aequales* (quite little equal parts [of the orbit]), which are perpendicular to what today we call radius-vector. The *morae*[11] for the planet to complete, with its motion, each particle is as the distance particle of the orbit-sun. Which, according to our modern view, is equivalent to claim that the transverse velocity is as the inverse of the distance planet-sun. Kepler clearly distinguished between the two elements of the planetary motions around the sun. As far as Kepler's exegesis is concerned, the problem is to understand what the importance of this passage inside Kepler's production is.

In the literature there are various interpretations: before Max Caspar this passage did not hold a significant position in *Kepler Forschung*. With Caspar things radically change: in a clear and incisive note to the *Epitome* (KGW, VII, note to p. 377, pp. 597–598) Caspar claims Kepler clearly understood the difference

[11] As to the concept of *mora*, the fundamental reference is Stephenson (1987, 1994), who translates *mora* with "delay" (pp. 13, 62–63, 80–85, 149, 187).

between the radial and tranverse components of velocity. Not only that, Kepler had all instruments to reach the correct law of planetary velocity, stated by Newton: the velocities of a planet in a point P are inversely to the perpendicular drawn from the sun to the tangent in P(Newton, *Principia*, I, III, prop. XVI). Caspar provides a mathematical proof of the equivalence between Kepler's and Newton's statement on velocities.

Koyré in *La révolution astronomique* offers a different interpretation[12]: Kepler understood the difference between the two components of velocity. However, although the mathematical steps developed by Caspar were within the grasp of Kepler, the physical context in which Kepler believed was so different from Newton's that he could not have reached what Caspar proposed. I think Koyré's arguments are convincing as to this problem. More in general, reading Koyré's commentaries on Kepler's distinction between radial and transverse components of velocity, one gets the impression that Koyré does not ascribe to this distinction a decisive importance for Kepler's interpretation.

Aiton also highlights that Kepler, while referring in the *Epitome* to the area law, was considering the transverse component of velocity.[13] He agrees with Koyré on the conceptual differences between Kepler's discovery that the transverse component of velocity is inversely as the distance of the planet from the sun and the mentioned Newton's proposition on the planetary velocities (Aiton 1969, p. 88). Nevertheless, his interpretation of the quoted passage of the *Epitome* and, above all, of the relation Kepler-Leibniz is completely different from Koyré's. Aiton tends to ascribe a relevant importance to what Kepler wrote in the *Epitome* as to the transverse velocity and therefore—at least as to this problem—he seems to credit Kepler's physical astronomy with a more correct and precise view than Koyré did. With regard to the relations Kepler-Leibniz, the views of Koyré and Aiton might not be more different: Aiton is convinced that: (1) Leibniz drew his decomposition of the planetary motion from Kepler and made it perfect; (2) the specific passage I have quoted from the *Epitome* is important for the conception of Leibniz's *velocitas circulandi* as inverse to the distance sun-planet. Aiton wrote:

> Koyré further assertion, that Leibniz supposed the speed of a planet in its orbit to be inversely proportional to the distance from the sun, was an unfortunate misinterpretation that seems to have led him unfairly to denigrate Leibniz's analysis of planetary motion. Elsewhere [Aiton is referring to Aiton 1965] it has been shown that Leibniz did not commit this mistake but followed the corrected form of the distance law, equivalent to the area law, *clearly stated by Kepler in the* Epitome*, a work cited by Leibniz in his "Tentamen de motuum coelestium causis"*.[14]

This quotation clarifies Aiton's ideas on the influence exerted by Kepler on Leibniz: the latter gave a precise mathematical treatment and framed a series of conceptions expressed by the former in a more imprecise manner into a coherent

[12] Koyré (1961, pp. 321–323).

[13] Aiton (1969, 1971, 1973, 1972).

[14] Aiton (1969, p. 76). My italics.

physical picture. The link between the two authors is profound and concerns both the *general* and the *specific* aspects of planetary motions' decomposition.

Hoyer, although detecting a significant difference between *Keplers Himmelsmechanik* and *Leibniz' Himmelsmechanik* (Hoyer 1979a), agrees with Aiton as to the importance of Kepler's statement on transverse velocity (Hoyer 1979b, p. 71) and on the specific influence Kepler exerted on Leibniz (Hoyer 1979a).

Later on, Davis developed the interpretation of the fact that Kepler, in the *Epitome* was referring to the transverse component of velocity for the area law.[15] Nevertheless, Davis deals with an internal interpretation of Kepler's procedure, not with a comparison Kepler-Leibniz.

6.1.2.2 The Influence of Kepler on Leibniz's Concept of *Velocitas Circulandi* and *Circulatio Harmonica*

In this context it is not appropriate to deal with an examination on the importance of the passage mentioned in Fig. 6.2 for an internal analysis of Kepler's work, but it is necessary to detect whether this passage influenced directly Leibniz's idea that *velocitas circulandi* is inverse to the distance sun-planet in the *circulatio harmonica*, or if this idea was Leibniz's original. Kepler influenced Leibniz as to decomposition of velocity in a radial and a transverse component, but what about the more specific mentioned problem? To answer this question, which is significant in an exegesis of Leibniz's thought, due to the importance of the concept of *velocitas circulandi*, I will refer to passages where Leibniz mentioned specifically Kepler's conceptions and works (I avoid Leibniz's generic references to Kepler). This can be useful to give an idea of the knowledge Leibniz got of Kepler's work, which can help us to find an answer for the specific question. For, there is no doubt: in order to note the mentioned passage, the only one in *Epitome* where Kepler spoke of the component of velocity perpendicular to the radius-vector sun-planet as inverse to the distance sun-planet, it is necessary to have a profound knowledge of *Epitome*, also considering that Kepler's language is quite indirect: he did not use the word *velocitas*, but a concept as that of *mora*, whose interpretation is not completely plane. In that context Kepler seems to be more interested in proving area law, than to supply general conceptions as that of Leibniz's *velocitas circulandi*, even though, from a mathematical point of view, the property of *velocitas circulandi* in the *circulatio harmonica* is equivalent to area law. The aim of these pages is to show that Leibniz had a good knowledge of *Paralipomena ad Vitellionem,* of *Harmonice Mundi* and of *Epitome*, but, probably not so profound to note the importance of the mentioned passage. Certainly Leibniz knew the *Epitome* and quoted this work more than once. Nevertheless, it is significant that he never

[15] Davis (1992a), in particular Sect. 7, entitled *Interpretation: 'distance law' v. transverse component of velocity*, pp. 116–120. See also Davis (1992b). Later on, Davis offered an interpretation of Kepler's area-law proof in *Epitome* without resorting to the use of velocity (see Davis 2003).

mentioned specific passages of the *Epitome*, but always general conceptions. Furthermore Leibniz ascribed to Kepler concepts which were not Keplerian, as the idea that in a curvilinear motion there is a tendency to recede along the tangent. Kepler never claimed something like this. More specifically, Leibniz proudly mentioned *circulatio harmonica* as his own discovery by which it is possible to prove Kepler's area law. If he had noted that Kepler developed, although *in nuce*, the inverse proportionality between the component of velocity perpendicular to the radius-vector and the distance from the sun, it does not seem plausible he had written nothing about this. Thence, I think that, as far as the properties of *circulatio harmonica*, are concerned, Leibniz was original, not influenced by Kepler. While the influence of Kepler is detectable for the decompositions of motions and, as we will see, for the general frame in which planetary theory is posed.

In Leibniz's works, the references to Kepler became numerous starting around from 1686, but they were not missing in the previous years, too.

In this section, I will mention Leibniz's references to Kepler with regard to:

1. Centrifugal tendency of curvilinear motion and causes of the planetary motions;
2. Optics;
3. Mathematics;
4. Other subjects.

I will conclude with some remarks by Leibniz on the *circulatio harmonica*. Hence, this section is also going to provide a general view on the knowledge Leibniz got of Kepler.

1. Centrifugal Tendency of Curvilinear Motion and Causes of Planetary Motions

This section also resembles some of the ideas Leibniz ascribed to Kepler with regard to gravity. It is hence connected to Chap. 5, where the problem of gravity in Leibniz has been dealt with.

The *Specimen inventorum de admirabilis naturae generalis arcani* is one of the works where Leibniz ascribed to Kepler the conception according to which in a curvilinear motion a body tends to escape along the tangent. Leibniz wrote:

> All motions can be reciprocally composed. The trajectory is a line that geometry will trace. Thence, a body, whose trajectory is a curved line has the direction to continue along the tangent, unless it is prevented from this tendency, which can be easily proved, as Kepler first observed.[16]

[16] The *Specimen* is not dated. Reasonably it was written around 1686 (see Parkinson 1974). Quotation drawn from Leibniz (1686?, [1875–1890], 1978 VII, p. 317). Original Latin text: "Omnes motus componi possunt inter se, lineaque erit, quam designabit Geometria; unde corpus quod fertur in linea curva directionem habet pergendi in recta tangente nisi impediatur, quemadmodum facile demonstrari potest, quod primus observavit Keplerus".

This idea became one of the most important keys used by Leibniz to show that Descartes' conception is, in fact, a Keplerian conception and that hence, the vortex theory—not only Descartes', but also Huygens' and his own—, which tries to explain the curvilinear movements treating the centrifugal force as a real force, relies upon a Keplerian basis. This is an important exegetical operation tried by Leibniz as to history of physics and can be posed inside his attempt to find an alternative to Newtonian centripetal forces. Kepler represents the authoritative predecessor.

In the *Animadversio in philosophiam Cartesii* written in 1689, in which Leibniz tried to prove that the most important ideas by Descartes are drawn from other authors, Kepler is mentioned as the scholar who first understood the composition of the curve motion in connection with the problem of gravity and the importance of the hyperbola in refraction theory. These ideas were hence not original by Descartes. Leibniz wrote:

> The tendency of the bodies to recede along the tangent in a circular motion and the idea to explain the fall of the bodies towards earth's centre by means of it is due to Kepler. He also first guessed that hyperbola satisfies the refractions[17]

In our context, it is also significant what Leibniz wrote to Vagetius on the 27th September 1693. Leibniz accepted the inverse square law for gravity, but he did not accept Newton's explanation. He claimed that, considering gravity as a magnetic force that is spread like light, it is easy to reach the inverse square law. Kepler is mentioned as the first one who dealt with the problem of centrifugal forces. Leibniz wrote:

> With regard to the cause of gravity you ask for, the thing is not yet completely clear. Newton turns to a certain immaterial reason, to which I cannot assent. Kepler first used the centrifugal force to explain gravity. Descartes developed this idea, but Huygens explained it in the most illuminating way. This statement is quite plausible and opposes to those who usually criticize Descartes; for me there is still specifically a problem: from Newton's and my calculations it appears that the planets gravitate towards the sun in duplicate reciprocal ratio of their distances from the sun, which can be explained quite easily, if we suppose that some attractive or magnetic rays are emitted from the sun, as it is the case for light-rays, whatever the nature of these rays or their way of attraction are. But if we wish that gravity of the planets derives from centrifugal force, the reason of this proportion is more obscure, although I have imagined something to explain gravity in this manner.[18]

[17] Leibniz (1689a), in LSB, VI, 4C, p. 2044. Original Latin text: "Recessum corporum a circulo per tangentem et rationem hinc explicandi detrusionem gravium ad centrum debebat Keplero, idem primus Hyperbolam satisfacere refractionibus subodoratus erat".

[18] Leibniz to Augustinus Vagetius 27 September (7 October) 1693, LSB, III, 5, pp. 640–643. Quotation pp. 641–642. Original Latin text: "De causa gravitatis quod quaeris res nondum plane liquida est. Neutonus ad immaterialem quandam rationem confugit, in quo ipsi assentiri non possum. Vim centrifugam primus huc adhibuit Keplerus, promovit Cartesius, sed maxime illustravit Hugenius. Sententia hujus plurimum habet plausibilitatis, et occurrit iis quae Cartesio objici solent; unum adhuc maxime nos male habet, quod ex Neutoni pariter et meis calculis apparet, planetas gravitare versus solem in ratione duplicata reciproca distantiarum a sole; quae res facillimam habet explicationem, si radios quosdam attractorios sue magneticos a sole ad instar luminis emissos fingamus, quaecunque demum sit horum radiorum natura modusve attractionis; sed si velimus gravitatem planetarum petere a vi centrifuga, obscurior est ratio hujus proportionis, tametsi ego quoque in eam rem aliquid sim commentus".

Here many *topoi* of Leibniz's thought and argumenantive way of proceeding are present: (1) Kepler discovered centrifugal force in each curvilinear motion; (2) Descartes and Huygens developed Kepler's ideas; (3) Newton, but Leibniz, too, have proved the inverse square law; (4) if the attractive-sun rays behave as light rays, it is quite easy to understand inverse square law, because light-intensity attenuates as the inverse square distance from light source; (5) the hypothesis of the *conatus explosivus* seems plausible.

The idea to compare the solar rays which induce movement to magnetism and light is an idea due to Kepler, although Kepler believed that the solar action was a *virtus promotoria* rather than a *virtus tractoria* and, after all, he thought of a simple inverse-relation between intensity of *virtus* and distance from the sun. From this quotation, it is evident that Kepler was the fundamental source of inspiration for Leibniz but that Leibniz's interpretation of Kepler's ideas was not always quite accurate. In particular, many interpretations derived from Leibniz's conviction that Kepler formulated the centrifugal-force law for the curvilinear motion, which is not the case.[19]

In a letter to Des Billettes in December 1696,[20] Leibniz claimed that the square distance law is in agreement with Kepler's ideas concerning the moving away of a body from the centre along the tangent. Kepler first applied this law to gravity.

Specifically, as to the movements of the planets towards or away from the sun, Leibniz wrote:

> So Gilbert who first wrote on the magnet carefully and not without success, suspected magnetism was hidden in many other phenomena. However, afterwards on this idea there were mistakes, as Kepler, outstanding man as to all the other questions, who excogitated some attractive or repelling fibres in the planets.[21]

Certainly Kepler thought of the magnetics fibres to which Leibniz refers and a certain knowledge of Kepler's works is necessary to write what Leibniz claimed, but not a profound one because Kepler often repeated his idea on magnetic fibres and Leibniz gives the impression of mentioning a general idea rather than a specific part of a work by Kepler.

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[19] Without any claim to be exhaustive, I mention here other passages where Leibniz expressed the idea that Kepler was the discoverer of the tendency to recede along the tangent in a curvilinear motion: (1) Leibniz: *Remarques sur la doctrine Cartesienne*, 1689 (Leibniz 1689b), in LSB VI, 4C, pp. 2049–2050; (2) Leibniz: *Notata quaedam G.G.L. circa vitam et doctrinam Cartesii*, 1689 (Leibniz 1689c), pp. 2059–2060; (3) Leibniz (1690?, 1860, 1962, VI, p. 164); (4) Leibniz to Gilles Filleau des Billettes, 4/14 December 1696, in LSB, I, 13, p. 372.

[20] In Leibniz ([1875–1890], 1978, VII, p. 452).

[21] The quotation is drawn from *Antibarbarus physicus pro philosophia reali contra renovationes qualitatum scholasticarum et intellegentiarum chimaericarum*, work whose dating is difficult. The context makes it sure it dates back to the last years of Leibniz's life, between 1710 and 1716. It is impossible to be more precise, see, for example, Smith (2012, p. 106). Quotation drawn from Leibniz (1710–1716, [1875–1890], 1978 VII, p. 341). Original Latin text: "Itaque Gilbertus qui de Magnete primus cum cura nec sine successu scripsit, suspicatus est in multis quoque aliis latere magnetismum. In quo tamen subinde deceptus est, quemadmodum Keplerus, cetera vir summus, qui fibras quasdam magneticas attrahentes aut repellents in planetis excogitavit".

Actually, Leibniz ascribed to Kepler—and specifically in the *Epitome*—the idea of the tendency to recede along the tangent in a curvilinear motion and an idea of gravity, which is extraneous to Kepler. This is expressed in the published version of the *Tentamen*, which reasonably means Leibniz thought that Kepler substained the opinions he ascribed to him. I repeat: it was not only a literary artifice by Leibniz in order to find an authoritative predecessor. Leibniz really thought what he wrote. Although—as already remarked—he added that Kepler did not fully understand all the potentialities of his discoveries, we read:

> For to him [Kepler] we owe the first indication of the true cause of gravity and of this law of nature on which gravity depends, that rotating bodies endeavour to recede from the centre along the tangent; thus if stalks or straws are afloat on water moving in a vortex by the rotation of a vassel, the water being denser than the stalks and therefore being driven out from the centre more strongly than they are, will push them towards the centre. Kepler himself clearly explained this in two or more places in Epitome Astronomiae, though he was still in doubt and ignorant of his own means, and insufficiently aware of how many things would follow therefrom in physics and especially in astronomy.[22]

It is difficult to understand of what passages of the *Epitome* Leibniz was thinking. It seems to me plausible he was referring to the following one, where Kepler dealt with the way in which lighter bodies are pushed towards the centre of a vortex composed of a heavier material. Kepler wrote:

> *Proof that the falling bodies are not pushed towards the centre by the violence of the world's motion*. In the violent circular motion, if something tends toward the centre of the whole mobile environment, it is necessarily lighter than that moving environment, as in the vortices, woods and straws are lighter than the rotating water: for, there, a major impression, due to rotation, takes place in the water's body, which is heavier, so that it acts fast with vehemence, and tends to the direction in straight line tending to the most extreme parts of the circle and as if it emptied the centre. Once made this, the lighter bodies which are afloat, when they are abandoned in the water, because of a minor motion's impression as well as because of a slower motion, and are pushed inwards by the waters, which are faster, then, also due to the declivity of the centre, they go naturally in the middle.[23]

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[22] Leibniz (1689, 1860, 1962, VI, p. 148). Translation drawn from Bertoloni Meli (1993, pp. 127–128). Original Latin text: "Nam ipsi primum iudicium debetur verae causae gravitatis, et huius naturae legis, a quo gravitas pendet, quod corpora rotata conantur a centro recedere per tangentem, et ideo si in aqua festucae vel paleae innatent, rotato vase aqua in verticem acta, festucis densior atque ideo fortius quam ipsae excussa a medio, festucas versus centrum compellit, quemadmodum ipse diserte duabus et amplius loci in Epitome Astronomiae exposuit, quamquam adhuc subdubitabundus et suas ipse opes ignorans, nec satis conscius quanta inde sequerentur tum in Physica tum speciatim in Astronomia."

[23] KGW, VII, Book I, Pars IV, pp. 75–76. Italics in the text. Original Latin text: "*Proba neque violentia Motus Mundani excuti gravia in medium.* In motu circulari violento, si qua petunt medium totius rei mobilis, illa oportet esse leviora re ipsa mota, vt in Vorticibus Ligna et paleae sunt leviora, quam est aqua ipsa rotata in gyrum: ibi namque major à rotatione fit impressio in corpus aquae, quod gravius est, vt impetu ruat, et rectitudinem affectans extima circuli petat, centrumque veluti exhauriat: quo facto, leviora innatantia, cùm propter minorem impressionem motus in ipsa, tardioremque motum, destituuntur, et ab aquis velocioribus introrsum repelluntur, tum etiam propter declivitatem centri, in medium naturaliter influunt".

Kepler is referring to the movement in a water-vortex and to pieces of wood and straw, but there is no reference to any tendency to escape along the tangent. Certainly, Kepler speaks of a tendency to the *rectitudinem*, notwithstanding this cannot be interpreted as a tendency to escape along the tangent or, at least, it is a clear overreading.

It is hence possible to conclude that Leibniz knew the general argumentations of the *Epitome*, that probably he knew some parts better than others, but there is no evidence: (1) to claim that he had a profound knowledge of *Epitome*; (2) to detect what parts he knew better.

2. Optics

Leibniz referred to Kepler's optics basically for three questions:

a) Equality of the angles of incidence and reflexion;
b) refraction law and role of hyperbola as diaclastic curve (that is, role of hyperbola inside refraction theory);
c) Kepler's considerations on the telescope in the *Dissertatio cum Nuncio Sidereo* and Kepler's telescope.

With regard to items (a) and (b) Leibniz was going to show that the law of refraction was known by Snell and by Kepler before Descartes and that Kepler formulated the correct reflexion law. When referring directly to a work by Kepler, Leibniz mentioned the *Paralipomena ad Vitellionem*. Since this subject is not directly connected to planetary theory, I will only mention one passage in which Leibniz's thought is clearly expounded. While, I refer to other passages useful to get a clear idea on this question in the note 25.

In *Remarques sur la doctrine Cartesienne* Leibniz wrote:

> As to dioptrics, he [Descartes] confesses, in his letters, that Kepler was his master in this science, and that he had been the most learned of all men in this discipline. However, Descartes has not mentioned Kepler in his works. The less he mentioned Snell, from whom it seems Descartes learnt the very rule of the refractions, as M. Isaac Vossius has discovered. He also avoided naming Maurolicus and De Dominis, who had opened the way for the discovery of the rainbow. Kepler also found that the dioptric line approaches hyperbola, and such a skilled geometer, as Descartes was, after having learnt the rule from Snell, could easily find it was the hyperbola itself.[24]

[24] Leibniz (1689b), LSB VI, 4C, pp. 2049–2050. Quotation p. 2049. Original French text: "Quant à la dioptrique, il [Descartes] avoue dans ses lettres, que Kepler a esté son maistre dans cette science, et celuy de tous les hommes qui en avoit sçû le plus, cependant il n'avoit garde de le nommer dans ses ouvrages. Et bien moins Snellius dont il paroist avoir appris la veritable regle des refractions comme M. Isaac Vossius a decouvert. Il se donne bien de garde aussi de nommer Maurolycus et de Dominis, qui avoient ouvert le chemin à la decouverte des raisons de l'arc en ciel. C'est Kepler aussi qui avoit trouvé que la ligne dioptrique approchoit de l'Hyperbole, et un aussi habile Geometre que des Cartes, après avoir appris le regle de Snellius, pouvoit trouver aisement que c'estoit l'Hyperbole même". The same concepts are expressed in Leibniz (1689c): Notata quaedam G.G.L. circa vitam et doctrinam Cartesii. LSB VI, 4C, pp. 2059–2060.

In *De causa gravitatis seu defensio sententiae autoris de veris naturae legibus contra Cartesianos*, Leibniz explicitly mentioned the *Paralipomena ad Vitellionem*. For, he wrote:

> Descartes used this idea [gravity deriving from the tendency to recede in a curved motion] of his [Kepler], as other ideas, too, in his own researches, without mentioning the author (according to a regrettable custom of his). In the same manner, he drew the explanation concerning the equality of incidence's and reflection's angle, by means of the decomposition of two motions, from Kepler's *Paralipomena ad Vitellionem*. From Snell, he learnt the very rule of refraction.[25]

Once again: Leibniz shows his knowledge of several aspects of Kepler's optics. He mentioned the *Paralipomena* in more than one occasion and surely he knew the most important results expounded by Kepler through a direct reading, but it is difficult to say how profound Leibniz's knowledge of Kepler's argumenations were.

With regard to the questions *Dissertatio cum Nuncio Sidereo*-*Narratio*-Keplerian telescope, things are easier, because these two works by Kepler are brief and their argumentative structure is far simpler than that of *Paralipomena* or *Epitome*, hence it is likely that Leibniz knew all the details. As to these problems, Leibniz in *Dissertatio exoterica de usu geometriae, et statu praesenti, ac novissimis ejus incrementis* reported that Kepler, in *Dissertatio cum Nuncio Sidereo*, told him that the imperator Rudolph spoke to him of Della Porta's idea of constructing a visual instrument by means of two lenses. Given the obscurity of Della Porta's

[25] Leibniz (1690, 1860, 1962, VI, pp. 195–196). Original Latin text: "Hac eius cogitatione, quemadmodum et aliis pluribus, in rem suam usus est Cartesius, autorem (pro more suo illaudabili) dissimulans, quemadmodum et ex Kepleri Paralipomenis in Vitellionem sumsit explicationem aequalitatis anguli incidentiae et reflexionis, per compositionem duorum motuum, et a Snellio didicit veram regulam refractionis". In a mathematical manuscript concerning *de Arte Combinatoria*, discovered by Couturat and datable around 1680, Leibniz asserted Kepler first understood that hyperbola is important in dioptrics, which was afterwards proved by Descartes. Today see *De arte combinatoria scribenda*, LSB, VII, 4, A p. 425. In the letter to Huygens added by Gerhardt as a *Beilage* to the *Tentamen*, Leibniz wrote that Snellius was the discoverer of refraction law, attributed to Descartes and Kepler (see LSB, III, 4, p. 618). In a letter to Gerhard Meier, October 1690, Leibniz claimed that Kepler, in the *Paralipomena*, opened the way to Descartes as to the equality of the incidence and reflection angles. Kepler also understood the role of hyperbola in refraction theory (see LSB, I, 6, p. 272). In *Dinamica de Potentia et Legibus Naturae corporeae,* pars II, sectio III, *De Concursu corporum*, Leibniz mentioned the *Paralipomena ad Vitellionem*, claiming that Kepler had shown the equality of the angle of incidence and reflection. Descartes drew this law from Kepler, see Leibniz, wd, after 1690, 1860, 1962, VI, p. 514. In an interesting letter to Johann Bernoulli on July 1697, in which the general context concerns the problem of the brachistrochrone, Leibniz proposed an epistemological observation, according to which, it is possible to see the connection between a certain problem and its solution, but that the step towards the full comprehension of the problem can be huge. Kepler, for example, guessed the connection between the diaclastic (anaclastic) curve and the hyperbola, but he did not see that hyperbola is exactly the diaclastic itself (see LSB, III, 7, p. 508). In a letter to Johann Bernoulli on 29 November 1703, which concerns catoptric, Leibniz claimed that Kepler and Descartes knew the right reflection law, while this was not the case for Honoré de Fabri, see Leibniz ([1849–1863], 1962, III, p. 728).

description and the lack of a good theoretical support concerning refraction, Kepler was, in fact, sceptical. Leibniz wrote:

> Kepler in the Epistle to Galileo, with which he answers to *Sidereus Nuncius*, tells that the imperator Rudolph, who, as far as we know, delighted admirably in these studies, had shown him the description of a machine composed of two glasses before than something was heard on the telescope. He had found the description in Della Porta's Collectanea. The description was rather obscure. Kepler said he considered this indication in a superficial manner and, with the usual annoyance of the learned men when some strange and dubious inventions are proposed, he immediately refused it. Now he was regretting his carelessness and precipitate judgement.[26]

In *Remarques sur la doctrine Cartesienne*, Leibniz spoke of dioptrique and, in particular, referred to the Keplerian telescope constructed with two convex lenses (after that Kepler had seen Galileo was able to construct a functioning telescope). We read:

> Kepler, too, has remarked that Della Porta gave some explanations, which were based on reasoning rather than on experience and which were useful to the Netherlandish inventor. Kepler himself, thanks to the force of his genius, has discovered the telescopes whose glasses are convex. They are far more excellent than the others.[27]

References to the posthumous *Somnium seu de astronomia lunari* are not missing. For example in a letter to Claude Nicaise on 27th December 1697 Leibniz referred to Huygens' opinion according to which Descartes would have drawn his theory of rainbow from the posthumous *De Astronomia Lunari* by Kepler.[28]

According to the evidence, we conclude that Leibniz had a good and relatively complete knowledge of Kepler's optics. It is, however, difficult to say how profound this knowledge was. With "profound", I mean how much Leibniz knew the innermost steps of Kepler's argumentations.

3. Mathematics

Leibniz's knowledge of Kepler's mathematics appears to be accurate enough starting from an early phase of his scientific career. In his letters and works Leibniz

[26] Leibniz (1676), LSB, VII, 6, p. 492. Original Latin text: "Keplerus in Epistola ad Galilaeum, qua *Nuntio Sidereo* respondet, haec narrat, Rudolphum Imperatorem, qui ut constat his studiis mire delectabatur, jam dudum antequam de Telescopio quicquam auditum esset, ostendisse sibi descriptionem Machinae duobus vitris instructae, inter Portae collectanea repertam, obscuriuscule traditam; hanc se obiter considerasse, ait Keplerus, et familiari eruditis fastidio, quoties aliena, et suspecta inventa offerentur, statim rejecisse: nunc vero poenas dare temeritatis et juducii praecipitati."

[27] Leibniz (1689b), LSB, VI, 4 C, pp. 2050–2051. Original French text: "Aussi Kepler a remarqué que Porta en avoit donné quelques lumieres qui estoient fondées plustost sur la raison que sur l'expérience et qui ont peutestre servi à l'inventeur Hollandois. Et Kepler luy même par la force de son genie, a decouvert les Telescopes dont tous les verres sont convexes et qui sont bien plus excellens que les autres."

[28] Leibniz, LSB, II, 3, p. 8166.

referred, in particular, to the results obtained by Kepler in the *Nova stereometria doliorum* and in the *Harmonice Mundi*.

In Spring of 1675 Leibniz, in the additions to a letter addressed to Oldenburg, explicitly mentioned the solution proposed by Kepler in the *Stereometria*, even though the details are missing.

With regard to geometry, Leibniz showed a good knowledge of *Harmonice Mundi* starting from the *Dissertatio de Arte Combinatoria*, where we read the following interesting passages: Leibniz is speaking of the way in which Kepler used and constructed the geometrical figures in his *Harmonice Mundi*. He claimed that this procedure gave rich and profound results. He wrote:

> For, the seventh use consists in complicating the geometrical figures, question in which Kepler broke the ice in the second book of the *Harmonice*. By means of these complications, not only geometry can be enriched with an infinite number of new theorems, because a new complication produces a new composed figure, by contemplating the properties of which, we construct new theorems and new demonstrations, but this is also [. . .] the only way to penetrate the mysteries of the nature, since it is said that one knows the thing the more perfectly, the more he perceives the parts of the things and the parts of the parts, as well as their forms and positions.[29]

Probably this quotation is one of those in which Leibniz mentioned one of Kepler's works more in detail. It is almost sure he had got a good knowledge of important sections of *Harmonice Mundi*.

4. Other Subjects

With regard to other subjects and contexts in which Leibniz mentioned Kepler and his works, it is worth reminding the reader of the references to the *De nive sexangula* and to the *Tabulae Rudolphinae*.

As to the *De nive*, in a letter to Nikolaus Hartsoeker, dated 8 February 1712, Leibniz dealt with the relation between the natural laws and possible direct interventions of God in the phenomenic world. Kepler is mentioned as to his researches on the *nive sexangula*.

Leibniz wrote:

> [. . .] we will not say, with the author of *Philosophia Mosaica* (Robert Fludd) that it is not needed to sustain that the thunderbolt derives from some exhalations, which have connections with nitrate and sulphur, either to look for, with Kepler, how the snow hexagons are

[29] Leibniz (1666, 1858, 1962, V, p. 34). Original Latine text: "Nam VIImus est in complicandis figuris geometricis usus, qua in re glaciem fregit Joh. Keplerus lib. 2 Harmonicῶν. Istis complicationibus non solum infinitis novis theorematibus locupletari geometria potest, nova enim complicatio novam figuram compositam efficit, cujus jam contemplando proprietates, nova theoremata, novas demonstrationes fabricamus, sed et [. . .] unica ista via est in arcana naturae penetrandi, quando eo quisque perfectius rem cognoscere dicitur, quo magis rei partes et partium partes, earumque figuras positusque percepit".

formed or what regulates planets' movements; but that we should be satisfied to claim that this is God's will, that God thunders, snows and governs the heavenly bodies.[30]

With regard to the *Tabulae Rudolphinae*, Leibniz mentioned them several times, for example: (1) Leibniz to Johann Andreas Schmidt, January 1700; (2) Leibniz to Johann Andreas Schmidt, 18 March 1700; (3) Leibniz to Christoph Schrader, 27 December 1701; (4) Leibniz to Philipp Ludwig von Sinzendorff, 14 March 1716.[31] In this case, the reference is not particularly significant because the *Tabulae Rudolphinae* were the most modern astronomical tables available at that time and they were hence an obligatory reference point for any astronomer.

The aim of this examination has been to detect how profound the knowledge Leibniz achieved of Kepler's work in the period in which he composed the *Tentamen* and in the following years was. The answer is not easy: from the evidence I have analysed, and part of which I have mentioned here, it is reasonable to think that—in the course of the years—Leibniz acquired a good knowledge of conspicuous parts of Kepler's production, till the point that Kepler became probably his most important source of inspiration. However, Leibniz's knowledge of Kepler's works seems dissimilar, also considering a single work. For example, as seen, in the *Dissertatio de Arte Combinatoria*, Leibniz seems to know some aspects of the *Harmonice Mundi* in a good manner. However, the third Kepler law, which is so important in planetary theory, in substance was dealt with by Leibniz only after 1700, in the epoch in which Leibniz wrote the *Illustratio*. This might depend on a plurality of reasons: (1) the difficulty Leibniz found in framing the third law inside the theory he had developed; but also (2) a not full comprehension of how important this law is in Keplers' theory.

Considering the described picture, the most plausible interpretation as to the influence Kepler had on Leibniz's distinction into two components of the planetary movements is this:

A) Kepler was fundamental for the idea to decompose the movement into a radial and radial-perpendicular component;
B) Kepler did not influence Leibniz's idea of *circulatio harmonica*. For, Kepler expressed the conception according to which the transverse component of velocity is as the inverse-distance sun-planet in one passage of *Epitome* inserted inside the demonstration of area law. This property of velocity is not presented as a proposition in itself, which means it could be noted only reading quite

[30] LSB, I, Transkriptionen 1712, p. 69–70. Original French text: "[. . .] ne dirons nous pas avec l'auteur de la Philosophie Mosaique (Rober Flud) qu'il ne faut point soûtenir que la foudre vient de quelques exhalaisons, qui ayent du rapport au nitre et au soufre, ny chercher avec Kepler, comment se forment les hexagones de la neige ou ce qui peut regler les mouvemens des planetes; mais qu'on doit se contenter de dire que Dieu le veut, et que c'est Dieu qui tonne, qui neige, qui gouverne les astres". Leibniz also referred to the hexagonal form of the snowflakes in *Dissertatio exoterica De usu geometriae, et statu praesenti, ac novissimis ejus incrementis*. Even though Kepler is not mentioned, the reference to his work is clear, see Leibniz (1676), LSB, VII, 6, p. 485, line 7.

[31] See respectively; (1) LSB, I, 18, p. 246; (2) *Ivi*, p. 458; (3) LSB, I, 20, p. 694; (4) LSB, Transkriptionen 1716, p. 161.

carefully the entire proof. It was not easy to realize that the concept of velocity was directly involved because Kepler spoke of *morae* and distancies, not of velocity.

It is, hence, quite probable that the properties of the *velocitas circulandi* were formulated by Leibniz without the direct influence of Kepler. Actually, Leibniz underlined so often the importance of the *circulatio harmonica* and its equivalence to area law, that, if he had noted that a similar proposition existed in Kepler, it is legitimate to think he would have mentioned the passage. Which is not the case and, in fact, the *circulatio harmonica* is seen by Leibniz as a cornerstone of his theory—independently of the fact that, from a physical point of view, this conviction by Leibniz on his own theory is right or wrong—while Kepler's assertion is inserted in a quite specific part of his work and Kepler did not face this problem in other contexts. This indicates his ideas on the properties of the transverse velocity were not as clear as Leibniz's. I mention one passage from which it results, beyond any doubts that, according to Leibniz, he himself first offered a satisfying proof of area's law.

In the unpublished and unfinished dialogue *Phoranomus sue de potentia et legibus naturae*, dated Summer 1689, Leibniz wrote:

> For, I found that this universal planetary motion is explained, in the most beautiful manner, by means of a common vortex of the planets around the sun. I have also found that a mathematical consequence of the motion law itself is that the motion can be decomposed into two components: the harmonic circulation around the sun (namely a circulation whose velocity is the minor, the greater the distance from the sun is), and the rectilinear approaching towards the sun (on the example of gravity and magnetism); I obtained these results after having proved that a universal and reciprocal property of harmonic circulation (in which, when the distances from the centre increase uniformly, the velocities decrease harmonically and viceversa) is that the areas cut from the centre are as the times, whatever the law of paracentric motion is. So it results that we have reduced, by means of geometrical analysis, the primary phenomena of the universe to principles, which are quite simple and easy to understand, that is to an excellent and absolutely true hypothesis.[32]

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[32] Leibniz (1689d, 1903, p. 593). Original Latin text: "Inveni enim motum hunc planetarium universalem pulcherrime explicari per communem planetarum vorticem circa solem, imo ex ipsa motus lege consequi geometrice, ut resolve possit in duos, circulationem harmonicam circa solem <(cujus scilicet proportione minor in majore a sole distantia sit velocitas)>, et accessum rectilineum ad solem <exemplo gravitatis vel magnetismi>; postquam <scilicet> demonstravi eam esse proprietatem universalem et reciprocam circulationis harmonicae (in qua crescentibus uniformiter distantiis a centro, harmonice decrescunt velocitates, et vicissim) ut areae ex centro abscissae sint temporibus proportionales, quaecunque sit lex motus paracentrici. Itaque eo jam res rediit, ut quod veteres vix votis attigisse videntur, primaria Universi phaenomena per Geometricam Analysin ad simplicissima et clarissima intelligendi principia, id est optimam adeoque et verissimam (eo quo diximus sensu) Hypothesin reducta habeamus." Couturat, relying upon Gerhardt, whom he mentions, refers to 1688 as date of composition of the dialogue. Nowadays, we are almost sure that this work dates to the Summer of 1689 (see Duchesneau 1998). I mention here the references to some other passages of Leibniz, where he claimed: (a) to be the discoverer of the *velocitas circulandi* in the *circulatio harmonica*; (b) by means of *circulatio harmonica* he has been able to prove the area law in a rigorous manner. This law was obtained by Kepler only relying upon observations. See: (1) Leibniz to Arnauld, 23 March 1690. LSB, II,

Once clarified the way in which Kepler was a source of inspiration for Leibniz's distinction between *circulatio harmonica* and *motus paracentricus*, it is worth pointing out that—in a sense and in the context of planetary theory—Kepler was used by Leibniz against Newton. I will clarify this in the next section.

6.2 The Physical Support of Kepler's and Leibniz's Planetary Theory

Leibniz was going to explain the planetary motions in the context of the vortex theory, by using only mechanic interactions. In this context Kepler's distinction of planetary motions into two components was precious for Leibniz. Indeed, Kepler's distinction was not only a device to improve and to make deeper a geometrical-kinematical theory of planetary motion. For, it is well known that Kepler was going to find a physical theory of the movements in the skies. This means that for him, it was not enough to have found the form of the orbits and the laws to which the planets obey in their movements. It was necessary to find the causes. The concept of causation is complicated in general and is extremely problematic in Kepler because in his theory, several levels of interrelated causes play important roles: these causes comprehend what we could call *physical forces*, intelligences, souls until reaching God himself, passing through the concept of harmony which was modified in the course of Kepler's scientific career. As to the distinction of the planetary motions into two components, we have seen that the first level of causation is due to two *physical forces*: (1) solar moving virtue for the mean motion, (2) magnetic interaction sun-planets for the approaching and moving away to/from the sun. Therefore two physical causes for two movements. Every scientist wishes to reduce as much as possible the number of the causes, so to provide a more unitary picture of the universe. It is hence evident that Kepler was not able to identify a sole mechanism-cause for the planetary motions. Newton did because—as already highlighted—the planetary movements are based only on centripetal force (plus initial velocity) not on two forces. However, there was the problem of action at a distance. This was unacceptable for Leibniz. Thence he preferred to adopt the Keplerian model, although based on two forces. In this sense Kepler was used by Leibniz against Newton.

With regard to the nature of the two forces, Leibniz replaced the *virtus motrix* with the *circulatio harmonica* and the magnetic attraction sun-planets with the interaction gravity-tendency to recede along the tangent. This tendency is typical of every curvilinear motion. As to the reasons of these replacements, we have seen that

2 (edition 2009), p. 314; (2) Leibniz to Erhard Weigl, September 1690. LSB, II, 2 (edition 2009), p. 347; (3) Leibniz to von Bodenhausen, November 1697, LSB, III, 7, p. 652; (4) Leibniz, *Illustratio Tentaminis* (1706, 1860, 1962, VI, p. 257).

Leibniz thought Kepler's fertile ideas were involved into unacceptable conceptions, according to which the soul of the sun and the intelligences of the sun or of the planets had a role in the planetary movements. When mentioning these criticisms Leibniz seems to refer to the whole conception by Kepler in which non-mechanical entities played a direct role into physical explanations. I cannot enter here into the mined field concerning the role that such entities got in Kepler's thought.[33] For sure the *species immateriata* of the *virtus motrix* could induce the movement thanks to the rotation of the sun, which, in its turn, was ascribed by Kepler to the sun's soul.[34] With regard to the *species immateriata*, Leibniz seems to consider the possibility that its action is mechanic, although the *species* is immaterial. Indeed, we read in the *Illustratio*:

> However, it is maybe possible to imagine something different for this problem. For example, that the sun emits corpuscles which, by means of their impressions, replace the incorporeal forces, by which Kepler thought that the planets, grasped as with chains, were moved around the sun. Indeed (as the projectiles), these corpuscles will have a double *impetus*, one deriving from the emitting force, the other from the circulation of the sun around its axis, by means of which they will try to push also an interposed body towards the same parts.[35]

The metaphor *velut vectibus* is adherent to what Kepler thought with regard to the way in which the *species immateriata* of the sun acts moving the planets. However, according to Leibniz: (1) the idea that a soul moves the sun is unacceptable; (2) the nature of the *species immateriata* is ambiguous; (3) the Keplerian forces in themselves do not offer a unitary picture of the universe. The two mechanisms of the planetary motions are completely separated, whereas the *circulatio harmonica*: (a) relies upon a direct mechanical device produced by the solar vortex; (b) offers a unitary picture of the planetary movements because, although different forces are necessary for the mean motion and the approaching to/moving away from the sun, the centrifugal tendency in the paracentric motion is due to the movement induced by the harmonic vortex. In Leibniz's view this offers

[33] Without any claim to be exhaustive, I mention on this problem: Barker and Goldstein (2001), Boner (2006, 2008, 2013), Escobar (2008), Granada (2009), Jardine (1984), Voelkel (1999).

[34] On this see, for example, *Epitome*, Book IV, pars II, chapter III, in particular the answers to the three questions: "*Habes etiam alia argumenta praeter motum, quibus verisimile fiat in corpore Solis animam inesse?*, „*Num etiam mentem aut intelligentiam addes Solis animae, quae moderetur hunc ejus motum circa axem?*, „*Ergone Sol gyratione sui corporis circumfert planetas? et quomodo hoc potest, cum careat Sol manibus, quibus prenset planetam tanto intervallo absentem, secumque convolutes circumagat?*", in which Kepler explained a moving soul is necessary for the rotation of the sun around its axis, but not an intelligence (KGW, VII, pp. 298–299).

[35] Leibniz (1706, 1860, 1962, VI, p. 272). Original Latin text: "Liceret tamen fortasse nonnulla alia in eam rem comminisci. Exempli gratia dici poterit, Solem emittere corpuscula quae impressionibus suis vices subeant virium incorporearum, quibus velut vectibus Keplerus apprehensos planetas in gyrum a sole agi putavit. Nam (ut projecta) duplicem impetum habebunt, unum a vi emittente, alterum a circulatione solis circa axem, qua et objectum corpus impellere tentabunt in easdem partes."

a unitary and, at the same time, precise vision which is absent in every other planetary theory. A few lines after the previous quotation, Leibniz wrote:

> But, anyway, I would not easily abandon the sole common connection which links the planets, since the vortices (once they have been amended) fit with nature's custom, which does not permit anything torpid, not ordered and not connected, and commands that everything agrees.[36]

Given his conception of vortex's movements, it is comprehensible that Leibniz ascribed to Kepler the idea that, in a curvilinear motion, a body tends to escape along the tangent. The likely Leibniz's trains of thoughts was this: (1) Kepler had first the idea of a vortex to explain planetary mean motion. For, the rotating *species immateriata* of the *solar virtus* moves as a vortex because of the sun's rotations; (2) although he did not develop the theory, the basic elements of the circular motion had to be available to him; (3) the tendency to escape along the tangent is the basic element; (4) hence Kepler knew this tendency. Leibniz—in this case—provided a rational reconstruction of what he believed Kepler's convictions and knowledge on vortex theory and circular motion were. Actually, this interpretation was based on Leibniz's physical convictions rather than on Kepler's.

With regard to the approaching to and moving away from the sun, the critics addressed by Leibniz to the influence of souls, intelligences and not mechanically explained *magnetic influences* were likely referred exactly to this kind of movement. Gravity explained by the movements of a specific vortex eliminates the problems of such non mechanical entities. It is necessary to point out that, in this case, an important distinction has to be highlighted in Kepler's thought: it is true that in Kepler's conception the sun's soul induces the movement of the sun around its axis. However, it seems to me that Koyré's interpretations, expounded in the chapter dedicated to Kepler in *La revolution astronomique*, is correct as far as Keplerian movements in the skies are concerned. Namely: Kepler, already starting from *Astronomia Nova*, attempted to reconduct the movements in the skies to physical reasons, trying to avoid the resort to entities as intelligences and souls. The sole movement explained by means of a soul's action is exactly the rotation of the sun and the planets around their axes. In particular, as to the approaching and moving away of the planets to/from the sun, the two following quotations drawn from *Epitome* leave few doubts on the fact that Kepler believed impossible that souls or intelligences were responsible for such a motion. Kepler wrote:

> For Aristotle will readily grant that a body cannot be transported by its soul from place to place, if the sphere lacks the organ which reaches out through the whole circuit to be traversed, and if there is no immobile body upon which the sphere may rest.[37]

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[36] *Ivi*, p. 272. Original Latin text: "Sed non ideo tamen fluidi communis nexum non unum planetas afficentem facile deserere velim, cum vortices (sed emendati) consuetudini naturae conveniant, quae nihil torpidum inordinatumque aut inconnexum relinquit, omniaque inter se conspirare jubet."

[37] KGW, VII, Book IV, Part II, Chap. II, p. 294, lines 37–41. Translation drawn from Kepler (1995, p. 52). Original Latin text: "Facilè enim hoc concesserit Aristoteles, corpus aliquod ab anima sua

And again:

> There is no need of these intelligences, as will be proved; and it is not possible for the planetary globe to be carried around by intelligence alone. For in the first place, mind is destitute of the animal power sufficient to cause movement, and it does not possess any motor force in its assent alone and it cannot be heard and perceived by the irrational globe; and, even if mind were perceived, the material globe would have no faculty of obeying or moving itself. But before this, it has already been said that no animal force is sufficient for transporting the body from place to place, unless there are organs and some body which is at rest and on which the movement can take place. [. . .] But on the contrary the natural powers which are implanted in the planetary bodies can enable the planet to be transported from place to place.[38]

Finally, with regard to the elliptical form of the orbit, we read:

> For firstly, the orbit of the planet is not a perfect circle. But if mind caused the orbit, it would lay out the orbit in a perfect circle, which has beauty and perfection to mind. On the contrary, the elliptic figure of the route of the planet and the laws of movement whereby such a figure is caused smell of the nature of the balance or of a material necessity rather than of the conception and determination of the mind, as will be shown below.[39]

Therefore, in this case, there is no influence of souls or intelligences. However, what kind of attraction is the magnetic action which ties sun and planets? It is evident that, according to Leibniz it is an action at a distance and it has to be refused exactly as Newton's centripetal forces. It is to underline that Leibniz considered chimerical, miraculous and absurd from a scientific point of view the interventions of souls and intelligences as well as the action at a distance. In Leibniz's perspective there are few—if no—differences between these two conceptions, which, in our eyes are so different. For him, both conceptions needed *ad hoc* interventions of entities, which—in Leibniz's mind—should play no direct role in the physical explanations. According to Leibniz, the physical support of the conceptions involving souls and intelligences was the same one as that involving the action at a distance and it is a wrong support.

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transportari non posse de loco in locum, si destituta fuerit orbis instrumento, qui per totum circuitum absolvendum sit exporrectus, si item absit corpus immobile, cui orbis innitatur."

[38] *Ivi*, p. 295, lines 12–21. Translation drawn from Kepler (1995, p. 52). Original Latin text: "Nec opus est his, ut probabitur, nec fieri potest, ut globus planetarius circumagatur per solam intelligentiam. Nam primò mens destituta potentia animali sufficienti ad motum inferendum, nec possidet ullam vim motricem in solo nutu, nec audiri et percipi à bruto globo potest, nec, si perciperetur, globus materiatus, facultatem haberet obsequendi, seque ipsum movendi. Iam antea verò dictum, nullam sufficere vim animalem transferendi suum corpus, de loco in locum, nisi adsint instrumenta et quiescens aliquod corpus, super quo fiat motus; [. . .]. E contrario verò potentiae naturales, insitae corporibus ipsis planetarum, praestare hoc possunt, ut planeta de loco in locum transferatur."

[39] *Ivi*, p. 295, lines 29–34. Translation drawn from Kepler (1995, pp. 52–53). Original Latin text: "Nam primo; Planetae orbita non est perfectus circulus; at si Mens hanc efficeret: ordinaret utique eam in perfectum circulum, cujus est mentalis pulchritudo et perfectio. Ex adverso figura Elliptica itineris planetarij, legesque motuum, quibus talis efficitur figura, sapiunt potiùs naturam staterae seu necessitatem materialem, quàm conceptum et destinationem mentis, ut infra patebit." For a partial reference to these three last quotations, see also Koyré (1961, pp. 238–248).

The general conceptions Leibniz had of the physical interactions explain hence why he was able to concede to Kepler the discovery of vortex theory and of the tendency to escape along the tangent in the planetary motion, but he was so strong in criticizing him as to ideas as those of the magnetic attraction sun-planets, at least in the form provided by Kepler, namely without an intermediary fluid.

Thus, the influence of Kepler was enormous and the interpretation given by Leibniz of Kepler's thought is extremely useful to go in depth into Leibniz's conceptions.

6.3 The Second Level of Causation: The Concept of Harmony

As to the movements in the skies, the first level of causation is hence represented in Kepler by: (A) the *virtus motrix*, due to the *species immateriata* radiating from the sun and put in action by the rotation of the sun around its axis; (B) the magnetic attraction sun-planets responsible for the elliptical movement. In Leibniz it is provided by the *circulatio harmonica* with its *velocitas circulandi* (mean motion) and by gravity-*motus paracentricus* with their causes.

The second level of causation concerns the cause of the actions, which determine the planetary movements. In Kepler the second level of causation can be divided into two levels:

1. from a *functional point of view*, the cause of the rotation of the sun—which, in its turns, is the cause why the *virtus motrix* is effective—is sun's soul because it allows the sun to rotate and, hence, permits, in fact, the planetary movements. Although, as a series of studies has shown, the souls and the intelligence assumed an important role in Kepler's thought (see note 33), there is no doubt that, as to the movements of the planets, Kepler, starting from *Astronomia Nova*, limited the causal role of the souls to the rotations of the sun and of the planets around their axes. However, this was an important role. He avoided the resort to intelligences in this context.
2. from a *structural point of view*, the second level of causation is given by harmony. Harmony is the principle which determines the structure of the universe: it is well known that, starting from *Mysterium Cosmographicum*, Kepler thought that the universe was structured on the basis of harmonious relations. In the case of his early work, these relations were searched and found in geometry. They were represented by the ratios between the radiuses of the planetary spheres that Kepler imagined circumscribed to and inscribed in the regular convex polyhedron. Spheres and Platonic polyhedron were the "perfect" figures, according to which God had constructed the universe.[40]

[40] Without any claim to be exhaustive, I mention the following literature on the *Mysterium*, in particular as far as the geometrical doctrine expounded in this work is concerned: Aiton (1977), Barker and Goldstein (2001), Di Liscia (2009), Field (1988), Gerdes (1975), Gingerich (2011), Hübner (1975), Jardine (2009), Pisano and Bussotti (2012).

> In the *Harmonice Mundi*, Kepler rethought the structure presented in *Mysterium* and, although he never denied the content of his early work, he inserted the geometrical properties inside a more general perspective which can be called "the musical harmony of the skies" and according to which he tried to explain the relations between all the variables concerning the planetary motions: angular velocities of the planets at aphelion and at perihelion; law of the periodical times (third Kepler law); eccentricities of the planetary orbits; mean, minimal and maximal distance of the planets from the sun, masses of the planets.[41]

With regard to Leibniz, no question that he denied souls and intelligences should get any direct role in physics. This question in not problematic.

Nevertheless, is there in Leibniz a second level of causation which is at the basis of the first causal level (vortex's *circulatio harmonica* and gravity) and which can be connected, at least in part, to Kepler? There is a word which connotes both Kepler's and Leibniz's thought. This word is *harmony*. In Leibniz the concept of harmony is used at least in three different, but connected, meanings:

(a) *circulatio harmonica* in planetary theory;
(b) principle of *pre-established harmony* in metaphysics;
(c) *harmony* in the common sense.

Due to the *circulatio harmonica* and to the principle of pre-established harmony, the universe is harmonious.

We have already seen the role of the *circulatio harmonica*. As to the principle of pre-established harmony, it is known that it gets a fundamental role in Leibniz's philosophy[42] and that it was one of Leibniz's principles which was more difficult to

[41] Once again, without any claim to be exhaustive, I mention here some important works on Kepler's harmony. First of all, I remind the reader of the contributions referred to in note 33, in which the concept of harmony is dealt with under various perspectives. Here I add: Bialas (2003), Bruhn (2005), Dickreiter (1973), Fabbri (2003), Field (2009), Haase (1973, 1998), Juste (2010), Menschl (2003), Stephenson (1994), Voltmer (1998).

[42] Leibniz dealt with the principle of pre-established harmony in numerous works and letters. The two classical works in which the principle is clearly explained are the *Essais de Théodicée* (Leibniz 1710, 1885, 1978) and *La Monadologie* (Leibniz 1714, 1875–1890, 1978, VI, pp. 607–623). The correspondence with Arnauld [see Leibniz (1967) and, for a quite good commentary, see Sleigh (1990a)] and with Malebranche (for complete indications on Leibniz-Malebranche relations and correspondence, see Robinet 1981) is fundamental, too. Among the numerous works in which Leibnz speaks of the pre-established harmony I mention those to which I have referred: Leibniz (1686, 1875–1890, 1978, IV, pp. 427–463); Leibniz (1686?, 1903, pp. 518–523); Leibniz (1686?, 1875–1890, 1978, VII, pp. 309–318); Leibniz (1691, 1860, 1962, VI, pp. 215–230), in particular pp. 228–229; Leibniz (1695a, 1875–1890, 1978, IV, pp. 477–487); Leibniz (1695b, 1875–1890, 1978, IV, pp. 490–493); Leibniz (1694, 1875–1890, 1978, IV, pp. 468–470); Leibniz (1697, 1875–1890, 1978, VII, pp. 302–308); Leibniz (1698a, 1875–1890, 1978, IV, pp. 517–524); Leibniz (1698b, 1875–1890, 1978, IV, pp. 554–571); Leibniz (1705, 1875–1890, 1978, VI, pp. 539–556); Leibniz (1705?, 1875–1890, 1978, VI, pp. 598–606).

The literature on Leibniz's principle of pre-established harmony and on Leibniz's concept of harmony is quite abundant and concerns different aspects of this principle inside Leibniz's production. There are many nuances connected both with the principle in itself and with its be understood by his contemporaries. The argumentative line I will follow here aims at showing:

1. the connection of the principle of pre-established harmony with another basic idea of Leibniz: the efficient causes are not enough to explain the world, either the physical world. The final causes play a fundamental role. The role of final causes is, in its turn, connected to the entelechy which characterizes each substance;
2. the universe of Leibniz is hence a *kosmos*. It cannot be reduced to its mere extension and corporeity. It is subject to metaphysical principles, which determine those that are physical, as far as these latter are based on final causes and laws. The metaphysical and physical principles determine the set of *laws* which govern the universe. However they determine, at least to some extent, the structure of the universe, too. The *kosmos* is ordered;
3. the concepts of harmony and the role of final causes are fundamental in Kepler, too. Harmony is the final and formal cause, according to which the universe is regulated and structured. This means that Kepler's universe is a *kosmos par excellence*;
4. if we imagine being at the beginning of the eighteenth century, Leibniz could be defined as *a modern Keplerian* because the two scholars share a common vision of the universe based on harmony, final causes and the world as a *kosmos*. Therefore the influence of Kepler on Leibniz—or at least a shared conception—goes far beyond planetary theory. Obviously Leibniz lived after Descartes and was a contemporary of Huygens, Newton and other protagonists of the scientific revolution. He saw the way in which Cartesianism was developed. Furthermore Kepler was basically an astronomer and was interested in the study of every aspect of the skies, whereas Leibniz was a philosopher in the full meaning of this word, and he was far inferior to Kepler as an astronomer. Therefore there were numerous and important differences between the two scholars. Nevertheless, the ways of thinking and of approaching the problems and conceiving the universe and its relations with God seem to me similar. In my argumentation, I will follow the four items I have indicated.

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possible origin inside Leibniz's thought. I am trying to highlight the importance of Kepler for the origin of the principle in Leibniz, but, of course, there are many other kinds of influence, as a profound literature has pointed out. As to the meaning of the principle, I stress its general features which are useful in connection to Kepler and to Leibniz's planetary theory. It is only natural that the whole problem of the pre-established harmony principle is far wider. I have focused on one aspect. I mention here some important contributions, again without any claim to be exhaustive: Adams (1994, 1996), Arthur (1998), G. Brown (1987a, b, 1988, 1992, 1994a, b), Fichant (2001), Frankel (1989, 1993), Futch (2008), Glowienka (2011), Knobloch (1994, 1995) (translation into English of Knobloch (1994), written in German); Kulstad (1993a, b, 2000), Lodge (1998b), Maraguat (2010), Mendelson (1995), Menédes Torellas (1999), Moll (1999), Newlands (2010), Orio de Miguel (2008), Phemister (1996), Ramati (1996), Rateau (2011), Riley (2007), Rodriguez-Pereyra (2009), Rozemond (1997), Schadel (1995), Serfati (2006), Smith (2012), Stillfried (2006), Watkins (1998), Wilson (1993), Woolhouse (1994, 2000a, b), Wren (1972).

6.3.1 *Leibniz: Pre-established Harmony, Entelechies, Final Causes*

The principle of pre-established harmony is an *order-principle* of the universe. Leibniz's conception is specified at least starting from the *Discours de Metaphysique* in 1686 and from the letters to Arnauld written between 1686 and 1687: among these letters, the one sent to Arnauld on the 30th April 1687 is particularly significant.[43] Here Leibniz addressed a criticism to Descartes, which he repeated in numerous circumstances: if one admits the existence of an immaterial reality—and in particular of souls—, a possible way to explain the interaction between soul and body is that proposed by Descartes: there is a real influence of the soul on the body and the movements of the body are due to the actions or to the decisions of the soul. The problem—Leibniz claimed—is that the way in which such interactions should be created and transmitted is incomprehensible both from a metaphysical and physical standpoint. It seems that such influence of the soul on the body is due to a continuous miracle because no reasonable explanation is available. The occasionalists (mainly Guelincx and Malebranche) criticized this view by Descartes because the way in which two completely heterogeneous realities, as soul and body, interact was explained in an unsatisfactory way by Descartes. The occasionalists denied a direct and causal influence of the soul on the body and viceversa. They claimed that the interaction between souls and bodies is due to God's will, which coordinates the actions of the souls with the movements of the bodies. This position has to be specified. Leibniz tends to interpret it as if the occasionalists imagined, for every movement, an act of God's will which coordinates soul and body so to create an effective movement or action.[44] However, Arnauld observed that the occasionalists do not claim that, for every action, an act of God's will is necessary, but that, from the Creation, God has synchronized soul's and body's actions so that they fit.[45] Leibniz's answer is quite subtle and it is indicative of his way of thinking. For we read:

> You say, Mister, that, who sustains the hypothesis of the occasional causes, claiming my will is the occasional cause and God the real cause of my arm's movement, does not pretend God makes this in the time by means of a new will-act, which He would produce each time I

[43] See LSB, II, 2B, pp. 174–193.

[44] The literature on Leibniz and his relations with the occasionalism is abundant and, in great part, connected to that concerning the pre-established harmony. Hence the works mentioned in note 42 deal, in part, with the problem of occasionalism. As specific literature—without any claim to be exhaustive—I add: Bernardini (1984), S. Brown (1990), Detlefsen (2002), Gaudemar (1998), Greenberg (2011), Hoskyn (1930, 1992), Jalabert (1981), Jolley (1992, 1998, 2013), Lee (2009), Lennon (1999), Marion (1985), Nadler (1994, 1996, 1997, 2001, 2008), N'Diaye (1996, 1999), Piclin (1971), Remaud (1998), Robinet (1968, 1981a, 1992), Rutherford (1999), Scribano (2003), Sleigh (1990, 1996), Stieler (1930), Vailati (2002), Wahl (2007), Weismann (1895), Woolhouse (1992, 1994a).

[45] I am referring, in particular, to the letter Arnauld sent to Leibniz on the 28th September 1686 and on the 4th March 1687. See respectively: LSB, II, 2B, pp. 93–99 and LSB II, 2B, pp. 150–156.

> want to raise my arm, but by means of that sole act of the eternal will, through which He wanted to do all what He had foreseen it was necessary to do. To this, I answer that, one could say, with the same reason, the miracles themselves do not occur because of a new will-act of God, since they are in accordance with the general plan. I have already remarked that each God's will-act implies all the others, but according to *a certain order of priority*. In effect, if I catch correctly the thought of the occasional causes' authors, they introduce a miracle, which does not cease to be a miracle because it is continuous.[46]

Leibniz's main point is expressed in the words "according to a certain order of priority" ("avec quelque ordre de priorité", my italics): God's will acts according to a *certain order of priority*, this means it is not an indifferentiated will which, from the creation, has posed a mysterious correlation between soul's and body's actions. For, in this manner such a correlation would be a miracle exactly as if God's will operated in occasion of each body's movements and soul's action. While this is not the case: God has created a fundamental intermediate level between his will, from one side, and soul's actions and body's movements, from another side: it is the laws-level. God has produced the laws of the bodies and the laws of the souls. These laws can be grasped by men, they are not a miracle and have a precise formulation. On the bases of these laws, a perfect co-ordination soul-body exists, which is not a cause-effect link. Therefore: the occasionalists are right in claiming that no real causal body-soul law exists, but they are wrong as far as they think that God acts directly on the phenomena and on the coordination of soul-body. God has created the laws, according to which soul and body agree in their actions. In this phase of his thought Leibniz calls this principle "l'Hypothese de la concomitance, ou de l'accord des substances entre elle".[47] This is what later on he called the principle of pre-established harmony. In the letters to Arnauld, an immediate consequence of this principle is deduced: the idea that the nature of every substance implies a general expression of the whole universe.Thus, Leibniz's idea is that if we analyse a single substance—for example a single living being—we do not exactly catch the meaning of his principle: *it is necessary to consider the universe as a whole*. In this case, it will be clear that the soul, due to the pre-established harmony, will express the entire universe—not only its body—, even though the expression of its own body will be more distinct than the expression of other bodies. The following long quotation seems to me paradigmatic of Leibniz's way of thinking:

[46] LSB, II, 2B, pp. 178–179. My italics. Original French text: "Vous dites, Monsieur, que ceux qui soutiennent l'Hypothèse des causes occasionnelles, et disans que ma volonté est la cause occasionnelle, et Dieu la cause reelle du mouvement de mon bras, ne prétendent pas que Dieu fasse cela dans le temps par une nouvelle volonté, qu'il ait chaque fois que je veux lever mon bras, mais par cet acte unique de la volonté eternelle par laquelle il a voulu faire tout ce qu'il a prevu qu'il seroit necessaire qu'il fist. A quoy je reponds, qu'on pourra dire par la même raison, que les miracles mêmes ne se font pas par une nouvelle volonté de Dieu, estant conformes à son dessein general, et j'ay déjà remarqué dans le précédentes que chaque volonté de Dieu enferme les autres mais *avec quelque ordre de priorité*. En effect, si j'entends bien le sentiment des auteurs des causes occasionalles, ils introduisent un miracle, qui ne l'est pas moins pour estre continuel."

[47] Leibniz to Arnauld, 14 July 1686, in LSB, II, 2B, pp. 67–84. Quotation, p. 82.

> But, it is possible to object, how does the soul know this bad disposition of the body? I answer this does not depend upon an impression or an action of the body on the soul, but because the nature of all substances keeps a general expression of the whole universe, and the nature of soul keeps, more particularly, a more distinct expression of what is happening, in that moment, in its body. Because of this, it is natural, for the soul, to mark and to know the accidents of its body by means of its own accidents. The same happens as to the body, when it harmonizes with soul's thoughts. When I want to raise my arm, this happens when everything is disposed in the body for this effect, so that the body moves according to its laws. However, thanks to the admirable, but unfailing accordance of the things, these laws, whatever happens, conspire in the precise moment, in which the will is formed. For, God took this into account when He decided on this sequence of all the things in the universe. All this is a consequence of the notion of an individual substance, which includes all its phenomena, so that nothing could happen to a substance, which does not arise from its own ground, but in conformity to what happens to another substance, although the one acts freely and the other one without choice.[48]

The principle of the pre-established harmony will remain a cornerstone of Leibniz's thought until his last works as *Théodicée*, *Monadologie* and correspondence with Clarke. He will specify and refine it, but, on this subject, his ideas were already formed at mid 1680s. The connection seen by Leibniz between the principle of the pre-established harmony and the natural laws is something quite interesting: the mechanism of Leibniz in physics is well known. If we look for the *efficient causes* of the phenomena and we are going to explain the facts of the physical world by means of these causes, the correct position is to reduce them to movements and the movements to collisions between bodies. However, this conception—which is useful and provides the correct results, if correctly applied, as to physics—is not satisfying from a metaphysical point of view because the true reasons of those movements can be found by the principle of pre-established harmony. Bodies move because of the laws of pre-established harmony, which have nothing to do with efficient causes. This means that collisions among bodies are only epiphenomenal appearances of a more profound ontological level. In fact, as no soul can act on another entity, no body can act on another body. It is only because of the pre-stablished harmony that, when a body hits another body, a movement is

[48] Leibniz to Arnauld 28 November–8 December 1686, in LSB, II, 2B, pp. 116–127. Quotation pp. 118–119. Original Franch text: "Mais (dira-t-on) comment sçait elle [l'ame] cette mauvaise disposition du corps [?] Je reponds, que ce n'est pas par aucune impression ou action des corps sur l'ame, mais parce que la nature de toute substance porte une expression generale de tout l'univers, et que la nature de l'ame porte plus particulierement une expression plus distincte de ce qui arrive maintenant à l'égard de son corps. C'est pourquoy il luy est naturel de marquer et de connoistre les accidens de son corps par les siens. Il en est de même à l'égard du corps, lorsqu'il s'accommode aux pensées de l'ame; et lorsque je veux lever le bras, c'est justement dans le moment que tout est disposé dans le corps pour cet effect; de sorte que le corps se meut en vertu de ses propres loix; quoyqu'il arrive par l'accord admirable mais immanquable des choses entre elles, que ces loix y conspirent justement dans le moment que la volonté s'y porte, Dieu y ayant eu egard par avance, lors qu'il a pris sa resolution sur cette suite de toutes les choses de l'univers. Tout cela ne sont que des consequences de la notion d'une substance individuelle qui enveloppe tous ses phenomenes, en sorte que rien ne sçauroit arriver à une substance, qui ne luy naisse de son propre fonds, mais conformement à ce qui arrive à une autre, quoyque l'une agisse librement et l'autre sans choix".

produced. At a metaphysical level, the impact is not the cause of the movement, but laws coherent with the pre-established harmony are. Leibniz, in *Specimen inventorum de admirandis naturae generalis arcanis*, expressed clearly his opinion as follows:

> This is so true that also in physics, if the things are examined carefully, one discovers that the *impetus* is never transfered from a body to another body. Rather each body is moved by an innate force, which is determined at the occasion of the other body, or in respect to it. For, it has been recognized by outstanding men that the push a body receives from another one is the elasticity of the body itself, which is repelled by the other. The cause of elasticity is the internal movement of elastic bodies' parts [. . .].[49]

While, if one is interested only in studying physics, it is perfectly correct to claim that the impact is the cause of the motion. The following quotation, drawn from the letter to Arnauld on 30 April 1687, is emblematic:

> But I have a different opinion and think what is real in the state we call movement proceeds from the corporeal substance, as thought and will proceed from the spirit. In each substance, everything happens as a consequence of the first state God gave it when he created such a substance. Put aside any extraordinary concourse, his ordinary concourse consists only in the conservation of the substance itself, in conformity with its previous state and of the occurred changes. Nonetheless, it is absolutely correct to say that a body pushes another body, namely one finds that a body begins to have a certain tendency only when another body, which hits it, loses the tendency in agreement with the constant laws, which we observe in the phenomena. In effect, since the movements are real phenomena rather than beings, a movement, as a phenomenon, is, in my spirit, the immediate consequence or the effect of another phenomenon.The same happens in the spirit of the others. But the state of a substance is not the immediate consequence of another particular substance.[50]

The writing *De primae philosophiae Emendatione et de Notione Substantiae* belongs to the conceptual horizon I am analysing. Here Leibniz faced the problem of the relations between phenomena, physical laws and metaphysical principles. His assertions on *vis activa* and on gravity are particularly revealing of his train of

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[49] Leibniz (1686?, 1875–1890, 1978, VII, p. 313). Original Latin text: "Haec adeo vera sunt ut in physicis quoque re accurate inspecta appareat, nullum ab uno corpore impetum in aliud transferri, sed unumquodque a vi insita moveri quae tantum alterius occasione sive respectu determinatur. Jam enim agnitum est a viris egregiis, causam impulsus corporis a corpore esse ipsum corporis Elastrum, quo ab alio resilit. Elastri autem causa est motus partium Elastici corporis intestinus [. . .]".

[50] LSB, II, 2B, pp. 177–178. Original French text: "Mais je suis dans une autre opinion, je tiens que ce qu'il y a de reel dans l'estat qu'on appelle le mouvement procede aussi bien de la substance corporelle, que la pensée et la volonté procedent de l'esprit. Tout arrive dans chaque substance en consequence du premier estat que Dieu luy a donné en la creant, et le concours extraordinaire mis à part, son concours ordinaire ne consiste que dans la conservation de la substance même, conformement à son estat precedent et aux changemens qu'il porte. Cependant on dit fort bien, qu'un corps pousse un autre, c'est à dire qu'il se trouve qu'un corps ne commence jamais d'avoir une certaine tendence, que lorsqu'un autre qui le touche en perd à proportion suivant les loix constantes que nous observons dans les phenomenes. Et en effet les mouvemens estant des phenomenes reels plustost que des estres, un mouvement comme phenomene, est dans mon esprit la suite immediate d'un autre phenomene et de même dans l'esprit des autres, mais l'estat d'une substance n'est pas la suite immediate de l'estat d'une autre substance particuliere. "

thoughts. He claimed that the *vis activa* is "[...] inter facultatem agendi actionemque ipsam media [...]".[51] And, with regard to the important problem of gravity and elastic force, he made an assertion which is fully coherent with the above mentioned one on the relation between impact and motion, because Leibniz wrote:

> Although gravity and elastic force can and must be explained through the movement of aether, nevertheless the final reason of the movement in nature is the force impressed in the creation, which is present in each body, but, in nature, it is variously limited and constrained by the conflict itself of the bodies.[52]

Now, the question is: are there laws explaining the phenomena, which can be directly connected to the principle of the pre-established harmony? These would be the very metaphysical laws of physics. According to Leibniz, such laws can be grasped only if one leaves the research of *efficient causes* and looks for *final causes*.[53] Starting from the *Discours de métaphysique*, Leibniz clearly explained the concept of *complete notion* with its actually infinite quantity of predicates and with its past, present and future history inscribed into the complete notion itself from the beginning of its creation. This history is developed by the actions of the substance-complete notion. The productive action of each substance is its final cause, its entelechy. Like the complete notion, the universe considered as a whole—which does not mean as a living whole—has its history and its actions depending upon the universe's final causes and entelechy. The properties of the entelechy get a precise form which can be expressed in mathematical terms as conservation-laws. The fundamental law of conservation is the law of *vis viva* conservation: the universe has a certain amount of *vis viva* which is conserved from the Creation and every single substance has its own amount of *vis viva* which is conserved and maintained along its history. In this sense the *vis viva* is something absolute, whereas movement, space and time are relative and are entities of reason, according to Leibniz.[54] Therefore, the law of *vis viva* conservation is the instrument used by

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[51] Leibniz (1694, 1875–1890, 1978, IV, p. 469).

[52] *Ivi*, pp. 469–470. Original Latin text: "Etsi enim gravitas aut vis elastica mechanice explicari possint debeantque ex aetheris motu, ultima tamen ratio motus in materia est vis in creatione impressa, quae in unoquoque corpore inest, sed ipso conflictu corporum varie in natura limitatur et coercetur".

[53] The literature on final causes in Leibniz is abundant. Many of the works mentioned in note 44 of this chapter also deal with this problem. Here, in addition, I refer to: Attfield (2005), Begby (2005), Bobro (1996), Carlin (2006), Cox (2002), Di Bella (1995, 2008), Duchesneau (1996), Dumitrescu (2011), Falkenburg (1998), Frankel (1989), Hunter (1988), Jolley (1998, 2013), Knebel (2001), Lagerlund (2011), Look (2011), Lyssy (2010, 2011), Mainzer (1990), Matsuda (2010), Reinhardt (1974), Rozemond (2009), Sleigh (1990), Vailati (2002), Vargas (2001), Vuillemin (1961).

[54] As to this subject, a particularly significant work is the second part of the *Specimen Dynamicum*, see Leibniz (1695, 1860, 1962, VI, pp. 246–254). This conception by Leibniz is problematic because, if space and time are relative, it is difficult to conceive the idea that velocity and its square multiplied by the mass—which is the measure of the *vis viva*—are something absolute. A partial answer to this question can be found in Leibniz's distinction between *vis primitiva*, which is the real active force inherent in the substance and *vis derivativa*, which is born through a limitation of

God for the actions of the corporeal substances mutually to fit. It could be considered as the physical law which is an expression of the metaphysical principle of pre-established harmony. In several circumstances, Leibniz pointed out the importance of final causes in physics. The *Specimen Dynamicum* is one of the works in which he claimed—against Descartes—that the extension cannot be the only property of the corporeal substance, the action (depending on the *vis viva*) is the most important one. The *vis* is an innermost (*intima*) property of the bodies. We read:

> We have suggested elsewhere that there is something besides extension in corporeal things; indeed, that there is something prior to extension, namely, a natural force everywhere implanted by the Author of nature—a force which does not consist merely in a simple faculty such as that with the Scholastics seem to have contented themselves but which is provided besides with a striving of effort [*conatus seu nisus*] which has its full effect unless impeded by a contrary striving [. . .] But if we cannot ascribe it to God by some miracle, it is certainly necessary that this force be produced by him within bodies themselves. Indeed it must constitute the inmost nature of the body, since it is the character of the substances to act, and extension means only the continuation or diffusion of an already presupposed acting or resisting substance. So far is extension itself from comprising substance![55]

The active character of the substance is expounded—among other works—in the *Specimen*, together with the importance of the concept of *vis* and the impossibility to reduct the substance and its actions, that is its movements, to mere logical or mathematical determinations:

> I concluded, therefore, that besides purely mathematical principles subject to the imagination, there must be admitted certain metaphysical principles perceptible only by the mind and that a certain higher and so to speak, formal principle must be added to that of material mass, since all the truths about corporeal things cannot be derived from logical and geometrical axioms alone, namely, those of great and small, whole and part, figure and

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the *vis primitiva* (Leibniz 1695, 1860, 1962, VI, p. 236). The *vis derivativa* is the *vis* whose intensity changes in the interactions among bodies. The measure of the *vis viva* can be interpreted as connected to the *vis derivativa*. However, Leibniz is not explicit on this and the problem is still alive in *Leibniz Forschung*. In this context my aim is to point out and to explain Leibniz's basic ideas which can be useful for a comparison with Kepler. It is not to enter into specific questions of Leibniz's physics, for which I refer to the abundant and profound literature. In general, with regard to the concept of force in Leibniz, see: Allen (1984), Gabbey (1998), Gale (1973, 1984), Garber (1985, 2008, 2009), Glenn (1984), Gueroult (1934), Iltis (1971, 1973), Kneser (1928), Lindsay (1975), Lodge (1997, 2001), Miller (1982), Mormino (2011), Papineau (1977), Rauzy (2005), Reichenberger (2012), Rutherford (2008), Shimony (2010), Stammel (1982, 1984), Stevenson (1997), Vaysse (1995).

[55] Leibniz (1695, 1860, 1962, VI, p. 235). Translation drawn from Leibniz (1989, p. 435). Original Latin text: "In rebus corporeis esse aliquid praeter extensionem, imo extensione prius, alibi admonuimus, nempe ipsam vim naturae ubique ab Autore inditam, quae non in simplici facultate consistit, qua Scholae contentae fuisse videntur, sed praeterea conatu sive nisu instruitur, effectum plenum habituro, nisi contrario conatu impediatur. [. . .]. Quod si jam Deo per miraculum transcribi non debet, certe oportet, ut vis illa in ipsis corporibus ab ipso producatur, imo ut intimam corporum naturam constituat, quando agere est character substantiarum, extensioque nil aliud quam jam praesuppositae nitentis renitentisque id est resistentis substantiae continuationem sive diffusionem dicit, tantum abest ut ipsammet substantiam facere possit."

> situation, but that there must be added those of cause and effect, action and passion, in order to give a reasonable account of the order of things. Whether we call this principle form, entelechy, or force does not matter provided that we remember that it can be explained intelligibly only through the concept of forces.[56]

Leibniz added that the final causes can be concretely exploited in physics. Thence, they do not have only the value to justify the physical laws at a metaphysical level, they also make a new approach to physics possible, by which significant results can be reached. In physics, the laws relying upon efficient causes have to be hence supported by those depending on final causes. With regard to physics as a discipline concerning the phenomena, both approaches are acceptable and useful. Sometimes the approach based on final causes allows us to solve a problem quicker. The paragraphs XXI and XXII of the *Discours de métaphysique* are maybe the most illuminating pieces written by Leibniz on the two possible ways which can be used in physics: the way of efficient causes and the way of final causes. Both of them are legitimate. In some circumstances the one is preferable to the other one. Often the way of final causes is simpler and more immediate than that of efficient causes. The laws of conservations depend on final causes and other principles, in particular the principle of the impossibility of a *mechanic perpetuum mobile*, which plays such an important role in Leibniz' physics.[57]

Therefore Leibniz's physics of the principles is based on metaphysical needs, but also enables us to solve concrete physical problems.

Leibniz wrote:

> [. . .] final causes may be introduced with great fruitfulness even into the special problems of physics, not merely to increase our admiration for the most beautiful works of the supreme Author, but also to help us make predictions by means of them which would not be as apparent, except perhaps hypothetically, through the use of efficient causes.[58]

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[56] Ivi, pp. 241–242. Translation drawn from Leibniz (1989), p. 441. Original Latin text: "Hinc igitur, praeter pure mathematica et imaginationi subjecta, collegi quaedam metaphysica solaque mente perceptibilia esse admittenda, et massae materiali principium quoddam superius, et ut sic dicam formale addendum, quandoquidem omnes veritates rerum corporearum ex solis axiomatibus logisticis et geometricis, nempe de magno et parvo, toto et parte, figura et situ, colligi non possint, sed alia de causa et effectu, actioneque et passione accedere debeant, quibus ordinis rerum rationes salventur. Id principium Formam, an ἐντελέχειαν, an Vim appellemus, non refert, modo meminerimus per solam virium notionem intelligibiliter explicari".

[57] Among the several works in which Leibniz used the principle of *perpetuum mobile* excluded I mention the *Brevis Demonstratio* (Leibniz 1686, 1860, 1962, VI, pp. 117–123); the *Essay de Dynamique sur les loix du mouvement* (Leibniz 1691, 1860, 1962, VI, pp. 215–230) and the *Specimen Dynamicum* (Leibniz 1695, 1860, 1962, VI, pp. 234–254).

[58] Leibniz (1695, 1860, 1962, VI, p. 243). Translation drawn from Leibniz (1989, p. 442). Original Latin text: "Sane et finales causes [. . .] subinde magno cum fructu etiam in physicis specialibus adhibentur, non tantum ut supremi Autoris pulcherrima opera magis admiremur, sed etiam ut divinemus interdum hac via, quae per illam efficientium non aeque aut non nisi hypothetice patent."

The pre-established harmony implies that, not only the absolute quantity of *vis viva* in any substance is conserved, but the direction, too. Leibniz wrote to Arnauld:

> And if one wants to say, as Descartes seems to intend, that the soul or God change, according to the occasion, only the direction or determination of movement and not the force which is in the body, since he does not seem likely to him that God violates in each moment, on the occasion of all the wills of the spirits, this natural and general law of force conservation, I answer it is equally difficult to explain which connection can subsist among soul's thoughts and the sides or the angles of bodies' direction. Moreover there is a further general law in nature, of which Descartes was not aware, and which is not less important than the former, namely that the same sum of direction or determination must always be conserved in nature. For, I find that, if any straight line is traced from a given point, for example from East to West, and if all the directions of all the bodies in the world are calculated, when they progressively move forward or backwards along lines parallel to the given one, the difference among the sums of the quantities of all Western directions and of all Eastern directions would be always the same, both in case we consider only some bodies in particular—in the hypothesis that, at the moment, an action subsists only among them—, and the whole universe, where the difference is always null, because everything is perfectly in equilibrium and because the directions towards East or towards West are exactly equal in the universe. If God acts against this rule, it is a miracle.[59]

Leibniz is here referring to what in *Essay de dynamique sur les loix du movement [. . .]* he called *quantité de progress,*[60] that is the quantity of motion as a vector—to use a modern terminology. In this work Leibniz criticized Descartes because—as well known—Descartes thought that the absolute value of the quantity of motion was conserved, whereas, in absolute value the *vis viva* is conserved, but, if we consider the quantity of motion as a vector, then the law of conservation holds. This is the conservation of the directions. Hence, both the conservation of the *vis visa*

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[59] Leibniz to Arnauld, 30 April 1687. Quotation LSB, II, 2B, pp. 180–182. Original French text: "Et si l'on veut dire, comme il semble que M. Descartes l'entend, que l'ame ou Dieu à son occasion, change seulement la direction ou determination du mouvement et non la force qui est dans les corps, ne luy paroissant pas probable que Dieu viole à tout moment, à l'occasion de toutes les volontés des esprits, cette loy generale de la nature, que la même force doit subsister, je reponds qu'il sera encor assez difficile d'expliquer quelle connexion il y peut avoir entre les pensées, de l'ame et les costés ou angles de la direction des corps, et de plus qu'il y a encor dans la nature une autre loy generale, dont M. des Cartes ne s'est point apperçu, qui n'est pas moins considerable, sçavoir que la même la determination ou direction en somme doit tousjours subsister; car je trouve que si on menoit quelque ligne droite que ce soit, par exemple d'orient en occident par un point donné, et si on calculoit toutes les directions de tous les corps du monde autant qu'ils avancent ou reculent dans les lignes paralleles à cette ligne, la difference entre les sommes des quantités de toutes les directions orientales, et de toutes les directions occidentales se trouveroit tousjours la même, tant entre certains corps particuliers, si on suppose qu'ils ont seuls commerce entre eux maintenant, qu'à l'égard de tout l'univers, où la difference est tousjours nulle, tout etant parfaitement balancé et les directions orientales et occidentales etant parfaitement egales dans l'univers. Si Dieu fait quelque chose contre cette regle, c'est un miracle."

[60] In the *Essay de dynamique* Leibniz wrote: "I call progress the quantity of motion with which one proceeds towards a certain direction, so that, if the body would go in the opposite direction, this progress is negative". Original French text: "J'appelle progrès la Quantité du mouvement avec la quelle on procede vers un certain costé, de sorte que si les corps alloit d'un sens contraire, ce progrès serait une quantité negative" (Leibniz 1691, 1860, 1962, VI. Quotation pp. 216–217).

and of the *quantité de progres* have their origin in the principle of the pre-established harmony as far as it expresses the final causes of each substance.

The principle of pre-established harmony can—not must—be interpreted as the cornerstone of Leibniz's metaphysics. Writing on the universe as the Best of the possible worlds, that is the world where the "quantity of being" reaches its possible maximum, Leibniz wrote in *De rerum originatione radicali*:

> From this, it is possible to understand, in an admirable way, how a certain divine mathematics or metaphysical mechanism is exerted in the origin itself of the things and the determination of the maximum takes place. As in geometry the right angle is determined among all the angles and as the liquids, posed in heterogeneous means, get the figure whose capacity is the maximum one, namely the sphere. An above all, as it is the case in the common mechanics, when, among many falling bodies, which mutually conflict, at the end a motion arises, for which the global descent is the maximum.[61]

The *Mechanismus Metaphysicus*, of which Leibniz speaks, can—and I think should—be interpreted as the principle of pre-established harmony.

Therefore: in Leibniz the universe is considered as a *whole*, as a *kosmos* which obeys metaphysical laws. Each part of the universe is in connection with each other part because the action of each substance, of each monad, can be understood only by taking into account its role inside the general project conceived by God from the Creation, even though, from the very metaphysical standpoint a substance cannot act on any other substance. Thus, the principle of the pre-established harmony is the fundamental law which—as far as the individual level of a single monad is concerned—allows it to coordinate its actions. While, in regard to the whole universe, the principle permits coordination of the different substances which make the universe an ordered *kosmos* with laws to which we give a *physical interpretation*. As Leibniz clarified, the physical laws based on efficient causes—for example the mechanical explanation of gravity—depend on the pre-established harmony and are only useful fictions because, *de facto*, there is no reciprocal influence among bodies. The universe is a "Kingdom of the Ends" and the facts depend on the Ends. The final causes are the very metaphysical causes of the universe's order and the physical principles (conservation of *vis viva*, perpetual motion excluded, principles of minimum and maximum) rely upon the final causes, which can be grasped by means of the pre-established harmony. In the light of these considerations, Leibniz's refusal of action at a distance can also be explained with the fact that action at a distance does not hold the form of a relation cause-effect among bodies because no impact is necessary to justify the action at a distance. Hence, since action at a distance does not belong to the reign of the physical laws

[61] Leibniz ([1875–1890], 1978, VII, p. 304). Original Latin text: "Ex his jam mirifice intelligitur, quomodo in ipsa originatione rerum Mathesis quaedam Divina seu Mechanismus Metaphysicus exerceatur, et maximi determinatio habeat locum. Uti ex omnibus angulis determinatus est rectus in Geometria, et uti liquores in heterogeneis positi sese in capacissimam figuram nempe sphaericam componunt, sed potissimum uti in ipsa Mechanica communi pluribus corporibus gravibus inter se luctantibus talis demum oritur motus, per quem fit maximus descensus in summa."

which can be reconducted to mechanical rules, it should be a physical principle as, for example, the *vis viva* conservation. But this is not the case because the principles of conservation as *vis viva* or *quantité de progress* establish properties of every physical state. While action at a distance says nothing—in itself—on the physical state. It is as a metaphysical presupposition of physics. Something which—at least as far as the physical world is concerned—should operate at the same level as the principle of pre-established harmony. However, if this is the case, action at a distance is contradictory with the principle of the pre-established harmony. To be clearer: it claims exactly the opposite of the pre-established harmony because the action at a distance foresees a direct action of a substance on each other substance. Not only: this direct action is transmitted at a distance and not by contact. Therefore if the action at a distance were true, the whole metaphysics by Leibniz would collapse, not only his physics. Thus, the only manner in which the action at a distance could exist would be that of a miracle, since God can operate miracles beyond the general laws he himself has established. But—according to Leibniz—to explain physics resorting to miracles means to explain nothing.

6.3.2 Connections with Kepler

In this section I will deal with the connections Kepler-Leibniz as to the role of the final causes and the concept of harmony. My idea is that Kepler's approach profoundly influenced Leibniz, beyond the planetary theory. Therefore, Leibniz's planetary theory, the comparison with Kepler and the principle of pre-established harmony—in part explained in reference to Kepler—might represent an access-key to and a perspective on Leibniz's philosophy. It is well known that a plurality of interpretations of Leibniz's metaphysics has been offered, due to the complexity of Leibniz's thought, to its evolutive character—since Leibniz changed his mind on some important questions during his scientific career—, and to the difficulty of interpreting the huge amount of his writings—often letters or little contributions—which, in many cases, were not published during Leibniz's lifetime. Without claiming to provide the reader with a general interpretation of Leibniz's thought—this is far beyond the limit of my work—I wish to propose an access-key, which could be integrated with the known ones to get a more extensive and complete view on Leibniz's philosophy.

6.3.2.1 Final-Formal Causes and Harmony: Analogies Kepler-Leibniz

The fundamental idea behind Kepler's and Leibniz's conception of the universe is that of *kosmos*, where the mathematical laws, by which it is possible to determine the nexuses cause-effect, depend on a more profound metaphysical structure. This structure was created by God, but it cannot be identified with God. Rather, it is identified with a set of laws and rules, which represent the second level of causation.

It can be called the reign of final and formal causes, which, obviously, has similarities and connections with the Aristotelian formal and final causes, but that the two authors developed in a manner such that, in our context, a comparison with Aristotle can be avoided to catch the influence Kepler exerted on Leibniz.

Starting from *Mysterium Cosmographicum* (1596), Kepler was explicit: he claimed that Copernicus looked for the mathematical (kinematical) foundations of his system, while he would have looked also for the physical and metaphysical foundations.[62] When Kepler, in 1621, published the second edition of his early work, he explicitly claimed that the whole development of his thought was based on what he had produced in the *Mysterium*.[63] This is the mere truth. For, Kepler changed his mind on fundamental aspects, but the ontological and gnoseological structure expounded in the *Mysterium* remained at the basis of his following speculation. This means:

1. The kinematical aspect was improved because—as well known—in the *Mysterium*, Kepler believed in the circularity of the orbits, while in *Astronomia Nova* he realized the obits were ellipses. However, from the period in which he wrote the *Mysterium*, Kepler felt the need to go beyond the kinematical standpoint;
2. The physical aspect was already present in the last chapters of the *Mysterium* (in particular Chaps. XX–XXII). Kepler was perfectly aware that his speculations inserted at the end of his early work were only an outline of what the physics of the skies should be. The development of his physical astronomy in the *Astronomia nova* and in the *Epitome* is coherent, at all, with the development of his original plan.
3. The metaphysical aspect is the main subject of the *Mysterium*. The theory of the regular polyhedron inscribed in and circumscribed to the planetary spheres represents, in a sense, a static metaphysical conception because Kepler identified the geometrical structure behind the universe, but, for the moment, he provided the architecture of the world, not the laws on which such metaphysical architecture depended. The structure expounded in the *Mysterium* joins together the final and the formal cause of the universe: *final*, as far as this structure is a manifestation of God's perfection in the world. It is enough to think of what Kepler wrote as to the relations sun-God, fixed stars-Son, space between sun and fixed stars-Holy Spirit[64] and to the perfections of the regular polyhedron, used by

[62] Kepler wrote: "Iamque in eo eram, vt eidem etiam Telluri motum Solarem, vt Copernicus Mathematicis, sic ego Physicis, seu mauis, Metaphysicis rationibus ascriberem", KGW, I, p. 9, lines 17–19.

[63] In the dedicatory epistle of the second edition of *Mysterium*, Kepler clarified that almost all successive works by him could be interpreted as specifications of single chapters of *Mysterium*. See KGW, VIII, p. 9, lines 24–28. With regard to the relations Kepler saw between *Mysterium* and his following contributions—in particular exactly the *Harmonice Mundi*—the "In Titulum Libri Notae Auctoris" is quite significant. See KGW, VIII, p. 15.

[64] KGW, I, pp. 24–25. On the argumentative structure of the *Mysterium*, the reader can consult Pisano and Bussotti (2012, pp. 121–135). We also provide an abundant series of references. As to

God to reach his *final aim*: to provide a perfect structure to the universe. *Formal*, because it is an archetypical structure inside which the efficient causes (physical laws) operate and the phenomena happen. This statical conception was transformed into a relational one in the *Harmonice Mundi* and in the books of the *Epitome* (4th–7th) written after the publication of the *Harmonice*. Kepler reached a general concept of harmony in which the geometrical relations became a part of a very metaphysical theory. Certainly, the musical harmonic relations played a prominent role, but the problems connected to the way in which human knowledge is developed and with the relations between the objective level of the world and the subjective level of knowledge are profoundly analysed, too. In this context, even though it is not possible to speak of pre-established harmony in Kepler, the German astronomer carried out some conceptions which could have directly inspired Leibniz. Due to the importance of the question, I will focus on some aspects of Kepler's metaphysics, in particular those which can be connected to Leibniz.

In the section dedicated to Kepler of his *Das Erkentnisproblem in der Philosophie und Wissenschaft der neuren Zeit*, Ernst Cassirer claims that, in Kepler, the relation between God and the universe is modified in respect to that of Renaissance authors such as Patrizi: God does not enter into nature from outside.[65] Nature, the universe itself tends to the Divine. We add: the interrelations God-universe are determined, in Kepler, by the harmonic laws, which imply precise relations between the angular velocities of the planets, the musical harmony and the geometrical structure of the skies. As above outlined, these laws are metaphysical as far as they do not concern the "forces" acting in the universe and determining the movements of the planets, but represent a higher order of legacy, inside which the physical laws—on which the efficient causes depend—are inscribed. The laws of harmony: (1) are expressed in mathematical terms; (2) exclude that God acts by means of continuous miracles. He acts according to the metaphysical laws established by himself. Thence the laws of harmony are final and formal causes.

As to Leibniz:

1. his opposition to every tendency in which the miracles could—also implicitly—play a role in the explanation of nature is well known. This exists in Kepler, too. On the other hand, as seen, the level of the final causes is important for the development of dynamics. The law of *vis viva* conservation and the principles of minimum and maximum in nature are a manifestation of the final causes in the phenomenal world and are fundamental in Leibniz's physics. Their importance is not inferior to that of the harmonic laws in Kepler, which are a manifestation of the final causes. With regard to Kepler, it is enough to think that his third law—which is so important in astronomy—was introduced on the basis of

the relations sun-God, fixed stars-Son; space between sun and fixed stars-Holy Spirit, see *ivi*, p. 121.

[65] See, in particular, Cassirer (1906, 1922, pp. 365–367).

harmonic considerations in the *Harmonice Mundi*.[66] Thus, the influence of the formal-final causes on the laws of physics and astronomy is a feature shared by Kepler and Leibniz. However, the analogy with Kepler is even more profound: in Kepler's universe, the laws of harmony do not appear only in the relations between the angular velocities of the planets, in the determination of their mutual distances and of their distances from the sun as well as in the periodical times. They also enter into the material composition of the celestial bodies, in particular as far as the density and the mass of the planets is concerned. What Kepler wrote as to this question is worth being reported:

> Furthermore, the division in two of intervals' proportion is established with geometrical elegance: so that, as above, two proportional means, 4 and 16, had to be introduced between the intervals of two planets from the sun (let them be, for example, 1 and 64) to determine the two remaining dimensions of the bodies (so that, given two bodies having the mutual ratio 1 and 64, the globes' surface will be as 1 to 16 or 4 to 64 and their diameters as 1 to 4, or 4 to 16 or 16 to 64) so now, between the intervals of the same two planets from the sun, 1 and 64, a sole proportional means is posed, in order to determine physically, inside the bodies, the structure of their matter, which is a sole thing, so that, once again, if the spaces occupied by these globes [the volumes] are as 1 to 64, the quantity of matter and, at the same time, the rarity in the minor one is to the rarity in the major one as 1 to 8, or 8 to 64, and, inversely the density as 8 to 1, or 64 to 8.[67]

Using the same symbols as Koyré,[68] one could indicate by v the volume of a planet, by r its distance from the sun, by m its mass and by d its density. Kepler is claiming that

$$\frac{v_1}{v_2} = \frac{r_1}{r_2};\ \frac{m_1}{m_2} = \frac{\sqrt{r_1}}{\sqrt{r_2}};\ \frac{d_2}{d_1} = \frac{\sqrt{r_{2.}}}{\sqrt{r_1}}$$

This clearly means that the laws of harmony also determine some aspects of the material structure of the world. In this sense, Leibniz's approach is in general different because he was not inclined to deduce properties of matter from the second level of causation. Nevertheless, this does not mean Leibniz thought that this level has no incidence on the material composition of the universe. For,

[66] See KGW, book III, p. 302, lines 22–24.

[67] This passage is draw from the *Epitome*, fourth book, first part, chapter "De raritate et densitate horum sex globorum, quid tenendum?" (KGW, VII, pp. 283–284, quotation, p. 284, lines 16–27). Original Latin text: "ldem etiam semissis proportionis intervallorum stabilitur concinnitate hac Geometrica: vt sicut superius inter duorum planetarum intervalla à Sole (verbi causa, sint 1. 64. ob facilitatem numerorum) statuenda fuerunt duo media proportionalia 4. 16. quippe ad formandas duas residuas dimensiones corporum, vt ita corpora quidem ipsa globorum mobilium essent inter se etiam vt 1. ad 64, superficies verò globorum, vt 1. ad 16, vel 4. ad 64, diametri denique eorundem, vt 1. ad 4, vel 4. ad 16, vel 16. ad 64: Sic nunc inter eorundem duorum planetarum intervalla à Sole 1. 64 statuatur vnum medium proportionale 8, quippe ad physicè formandam intus corporum materiam, quae est res vnica: vt ita rursum ipsa quidem globorum spacia sint vt 1. ad 64, copia verò materiae, et simul raritas in minori ad illam in majori, sit vt 1. ad 8. vel 8. ad 64: seu contraria densitas, vt 8.ad 1. vel 64. ad 8."

[68] Koyré (1961, p. 386).

when dealing with Gregory's criticism to the fact that Kepler's third law is not satisfied by Leibniz's planetary vortices, Leibniz (see Sect. 4.2.2 of this book) replied with the hypothesis that all the planetary orbits have exactly the same *vis viva*. From here he developed the series of arguments I have explained in that paragraph. The hypothesis on *vis viva* concerns the dynamical structure of the universe, but the material one, too, because the *vis viva*—or, at least its measure—is also a function of the mass. This means: let us assume the *vis viva* is constant; since given two planets, the one farer from the sun is slower, then the mass of a planet is bigger the further from the sun the planet is—even though the mass is not a linear function of the distance.[69] By the way—as seen—Kepler is convinced that the mass of a planet is the bigger the more external the planet is.

There is no reason depending on a mechanical conception such as Leibniz's physical one, or on natural laws not relying upon final causes and harmonic ideas on the *kosmos*, which can justify assertions such as Kepler's or Leibniz's. Their justification can only be found in a global vision of the universe as the reign of harmony and of final and formal causes. This is a further feature shared by Kepler and Leibniz. One could say that Leibniz, to make his vortices theory coherent with Kepler's third law, resorted to a Keplerian-style reasoning[70]: such is the hypothesis of the identity of orbit's *vis viva*.

In conclusion: in Kepler and in Leibniz, the second-level laws of the *kosmos* exert also an influence on the physical structure of the universe.

2. Cassirer points out that Kepler distinguishes two kinds of harmony[71]: a pure harmony and a sensible harmony. The pure harmony is a formal archetype, for example a certain numerical relation which might occur in different fields, as in musical ratios, or in the relations between the dimensions of a figure or between the dimensions of different figure or, again, in some properties of different movements, for example in the ratios between velocities or velocities and the distances from a certain body and so on. The sensible harmony needs four elements: (a) the perceived objects; (b) a conscience which perceives the objects; (c) the activity of the conscience which compares the objects; (d) the capability of the conscience to identify a relation among the compared objects. Cassirer tends to highlight the subjective-relational character of harmony, where the word "subjective", in the case of Cassirer, assumes, in fact, an intersubjective and transcendental character: harmony exists as a "phenomenon" because of an act of the conscience, which is possible because of conscience's transcendental structure. Cassirer mentions a long passage of the *Harmonice Mundi*, in which Kepler stressed the importance of conscience's activity for the concept of

[69] A possible problem to this picture could derive from the consideration that the mass of an orbit does not depend only on the planet, but also on the aether of the orbit. Nevertheless, Leibniz does not seem to take into account this consideration.

[70] Obviously I am referring to the style of Leibniz's reasoning, not to the specific content.

[71] On this subject, see the entire chapter—inside the part dedicated to Kepler—entitled "Der Begriff der Harmonie", Cassirer (1906, 1922, pp. 328–352).

harmony to be reached.[72] In relation to Leibniz two considerations are significant:

(A) the activity of conscience is important in Kepler, but there is no doubt that harmony is not created by the perceiving human conscience, but by God himself. Harmony is not something in itself—this is true—but it is expressed by a set of relations which are posed by God and which are perceived by human conscience as harmonic because God created these relations in a way that men, who—as well known—have a privileged position in Kepler's chain of being, may perceive them in a particularly pleasant manner. This pleasure is connected to the particular mathematical character of the harmonic relations. The idea of harmonical relations created by God is almost exclusively—even though not exclusively, at all—limited to geometry in the *Mysterium* and is extended to the music of the heavens in the *Harmonice*, but the first reference point is, in any case, God's creative activity. Human conscience's perceiving and regulative activity is the feature which allows us to catch and—in a sense—to create harmony. But our creation depends directly on God's creation. In Leibniz's terms: a pre-established harmony between the archetypical laws created by God for the universe and the faculties of our soul-conscience exists. This pre-established harmony allows man to catch the mathematical and the sensible harmony with which Kepler dealt with. My hypothesis is hence that Leibniz found in Kepler an idea of the relations between God-our soul-archetypical laws of the universe which inspired him for the creation of the pre-established harmony principle. Leibniz extended this principle far beyond the relations conscience-archetypical laws of the universe. As we have seen and as well known, the principle of the pre-established harmony became one of the cornerstones of Leibniz's metaphysics and was used by Leibniz to explain the properties of the substance inside a context in which the problems connected to the physical universe and to its possible archetypical laws were secondary in comparison to that of providing an acceptable foundation to ontology. Nevertheless, the form, if not the content of this principle, seems to me based on a Keplerian inspiration. The principle of pre-established harmony is also the basis on which we perceive some relations as harmonious in the intuitive sense of the world. Furthermore, the reference to music is one of the examples to which Leibniz resorted to explain his principle. In a letter to Arnauld, we read:

> Finally, to use a comparison, I will say, as to the concomitance I sustain, it is similar to that, which would subsist among different orchestras or choirs, which carry out their parts separately and are connected, so that they cannot see and hear each other, but, despite this, can reciprocally tune by following the notes—each one its own notes—in a way that the listeners find a marvellous harmony. This harmony is far more surprising than the one which would exist if there were a connection among the choirs. It could also happen that, if

[72] *Ivi*, p. 333.

a person was posed near a choir, from this choir he judged what the other one carries out and he got into the habit (especially in case we suppose he can hear only his choir, without seeing it, and see the other one, without hearing) that—thanks to imagination's help—he did not think anymore of the choir, where he is, but of the other one, or that he considered his own choir as an echo of the other one, only ascribing to his choir some intermezzos, in which, some rules of symphony do not appear, by which it is possible to judge the other choir [. . .].[73]

I think hence that the marvellous harmony, the concomitance of which Leibniz spoke, was—at least in part—inspired by Kepler's idea of a harmony conceived as a relation between—from one side—the archetypical laws, according to which the universe was conceived by God, and—from the other side—the capabilities of our soul.

(B) A further aspect which characterizes both Kepler's and Leibniz's *kosmos* is the universal connection among the entities. For example, in the *Monadologie*, Leibniz wrote:

And consequently every body feels the effect of all that takes place in the universe, so that he who sees all might read in each what is happening everywhere, and even what has happened or shall happen, observing in the present that which is far off as well in time as in place.[74]

Even though:

Thus, although each created Monad represents the whole universe, it represents more distinctly the body which specially pertains to it, and of which it is the entelechy; and as this body expresses the whole universe through the connexion of all matter in the plenum, the soul also represents the whole universe in representing this body, which belongs to it in a special way.[75]

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[73] Leibniz to Arnauld, 30 April 1687, LSB, II, 2B, pp. 182–183. Original French text: "Enfin, pour me servir d'une comparaison, je diray qu'à l'egard de cette concomitance que je soutiens c'est comme à l'egard de plusieurs differentes bandes de musiciens ou choeurs, jouans separement leurs parties, et placés en sorte qu'ils ne se voyent et même ne s'entendent point, qui peuvent neantmoins s'accorder parfaitement en suivant leurs notes, chacun les siennes, de sorte que celuy qui les ecoute tous y trouve une harmonie merveilleuse et bien plus surprenante que s'il y avoit de la connexion entre eux. Il se pourroit même faire que quelqu'un estant du costé de l'un de ces deux choeurs jugeast par l'un ce que fait l'autre, et en prist une telle habitude (particulierement si on supposoit qu'il pust entendre le sien sans le voir, et voir l'autre sans l'entendre) que, son imagination y suppleant, il ne pensât plus au choeur où il est, mais à l'autre, ou ne prit le sien que pour un echo de l'autre, n'attribuant à celuy où il est que certains intermedes, dans lesquels quelques regles de symphonie, par lesquelles il juge de l'autre, ne paroissent point [. . .]".

[74] Leibniz (1714, 1875–1890, 1978, VI, p. 617). Translation by R. Latta drawn from the web site: http://oregonstate.edu/instruct/phl302/texts/leibniz/monadology.html. Original French text: "Et par consequent tout corps se ressent de tout ce qui se fait dans l'univers, tellement que celuy, qui voit tout, pourroit lire dans chacun ce qui se fait partout et même ce qui s'est fait ou se fera, en remarquant dans le present ce qui est éloigné tant selon les temps que selon les lieux[. . .]".

[75] *Ivi*, p. 617. Translation by R. Latta drawn from the web site: http://oregonstate.edu/instruct/phl302/texts/leibniz/monadology.html.Original French text: "62. Ainsi quoyque chaque Monade creée represente tout l'univers, elle represente plus distinctement le corps qui luy est affecté particulierement et dont elle fait l'Entelechie: et comme ce corps exprime tout l'univers par la connexion de toute la matiere dans le plein, l'ame represente aussi tout l'univers en representant ce corps, qui luy appartient d'une maniere particuliere".

> Thus, in Leibniz everything is connected to everything. But this is true for Kepler's *kosmos*, too. The assertion according to which the harmony exists only in the relations among planets (distances from the sun, angular velocities, tones of the notes emitted by the planets, and so on) and not in a single distance or in a single angular velocity, or in a single tone, means exactly that everything is connected to everything and that the meaning of the whole can be grasped only by taking into account the whole. This idea of a universal connection among the entities and the creatures connote the second level of causation, that of the formal and final causes, while it is not necessary in the first level of causation, that of efficient causes. At this stage, according to the circumstances, one can reasonably consider the actions limiting to two bodies: Kepler himself, while dealing with the *virtus motrix* of the sun or the magnetic attraction sun-planet, which determines the ellipticity of the orbits, considered only the relation sun-planet (at the beginning the relations sun-Mars, specifically), not the all relations sun-planets in the solar system. This approach is not possible while facing the second level of causation. Obviously, the idea that all the creatures of the universe influence mutually each other is probably old as the man himself and many Renaissance scholars had developed vitalistic conceptions based on such ideas. However, the possibility to transcribe such general and not always well determined views into a coherent (and not necessarily vitalistic) system, which was based upon mathematical regularities was developed by Kepler in his *Mysterium* and—even more—in *Harmonice Mundi*. As we have seen, there is no doubt that this work impressed Leibniz from his youth. It is hence plausible to suppose that the idea to create a system in which the connection of everything with everything was not a vague and undetermined precept, but became a precise metaphysical principle, was introjected by Leibniz starting from his early reading of *Harmonice* and developed during his scientific and philosophical career.

Connected to this question, there is the relation mathematics-metaphysics. On several occasions Leibniz pointed out:

1. mechanics depends on higher laws;
2. the physical laws cannot be reduced to the rules of the extension (in particular to geometry). The laws of metaphysics have to be added to the rules of the extension;
3. in the origin of the things, a divine mathematics exists.

With regard to the items (1) and (2), it is possible to remind the reader of Chaps. XII, XVIII and XXI of the *Discours de metaphysique*. In the 12th chapter Leibniz wrote that the nature of bodies cannot be reduced to extension and that the existence of a substantial form has to be recognized. The Chap. 18 is quite important: Leibniz in the previous chapter had proved that the "force", and not the Cartesian quantity of motion, is conserved. He claimed in the 18th chapter:

> Now, this force is something different from size, shape, and motion, and this shows us that—contrary to what our moderns have talked themselves into believing—not everything that we can conceive in bodies is a matter of extension and its modifications.[76]

Since *grandeur*, *figure* and *movement* represent the extension and the *force* cannot be reduced to them, the *force* is not a merely mathematical and mechanical entity, rather a metaphysical one. Leibniz's conclusion is really emblematic of his way of thinking, as we read:

> And it becomes more and more apparent that although all particular natural events can be explained mathematically or mechanically by those who understand them, the general principles of corporeal nature and even—the somewhat less general principles—of mechanics belong to metaphysics rather than to geometry, and have to do with certain indivisible forms or natures, as the causes of appearances, rather than with corporeal or extended mass.[77]

In Chap. XXI, Leibniz added that, if the rules of mechanics depended only on mathematics, without metaphysics, the phenomena would be different from how they appear. This chapter is maybe less clear than the previous ones, but I think the interpretation is clear: the principles of conservation do not depend on extension, rather on metaphysical rules, as seen in the part concerning harmony.

In the *Specimen Dynamicum*, Leibniz specified his opinion explicitly adding the principles to which he was referring. Indeed, he wrote:

> Later, however, after I had examined everything more thoroughly, I saw wherein the systematic explanation of things consists and discovered that my earlier hypothesis about the definition of a body was incomplete. In this very fact, along with other arguments, I found a proof that something more than magnitude and impenetrability must be assumed in body, from which an interpretation of the forces may arise. By adding the metaphysical laws of this factor to the laws of extension, there arise those rules of motion which I should call systematic—namely, that all change occurs gradually, that every action involves a reaction, that no new force is produced without diminishing the earlier force, so that a body which carries another is retarded by the body carried away, and that there is neither more nor less power in the effect than in the cause.[78]

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[76] Leibniz (1686, 1875–1890, 1978, IV, p. 444). Translation drawn from the web site: http://www.earlymoderntexts.com/pdfs/leibniz1686d.pdf, copyright by J. Bennett. Original French text: "Or cette force est quelque chose de different de la grandeur, de la figure et du mouvement, et on peut juger par là que tout ce qui est conçû dans les corps ne consiste pas uniquement dans l'étendue et dans ses modifications, comme nos modernes se le persuadent".

[77] *Ivi*, p. 444. Translation drawn from the web site: http://www.earlymoderntexts.com/pdfs/leibniz1686d.pdf, copyright by J. Bennett. Original French text: "Et il paroist de plus en plus quoyque tous les phenomenes particuliers de la nature se puissant expliquer mathematiquement ou mechaniquement par ceux qui les entendent, que neantmois les principes generaux de la nature corporelle et de la mechanique même sont plustot metaphysiques que Geometriques, et appartiennent plustot à qualques formes ou natures indivisibles comme causes des apparences qu'à la masse corporelle ou étendue".

[78] Leibniz (1695, 1860, 1962, VI, p. 241). Translation drawn from Leibniz (1989), pp. 440–441. Original Latin text: "Sed postea omnia altius scrutatus, vidi in quo consisteret systematica rerum explicatio, animadvertique hypothesin illam priorem notionis corporeae non esse completam, et cum aliis argumentis tum etiam hoc ipso comprobari, quod in corpore praeter magnitudinem et impenetrabilitatem poni debeat aliquid, unde virium consideratio oriatur, cujus leges metaphysicas

With regard to item (3), the divine mathematics, which is at the origin of the things, is the *metaphysical mechanism* of which we have already spoken. It can be reconducted to the pre-established harmony and on the basis of this mechanism God chooses, among all possible worlds, the best one, in which the maximum of essence, of being, is realized with the least possible effort.

In Kepler a parallelism with the third item exists:

1. mechanics depends on higher laws: in Leibniz these laws are the metaphysical ones, in particular the principle of the pre-established harmony, and the physical laws which can be reconducted to metaphysics, specifically the law of *vis viva* conservation and the pinciples of maximum and minimum. In Kepler such laws comprehend the geometrical structure expounded in the *Mysterium* and the more general harmonic laws of *Harmonice*.
2. This item is strictly connected to Leibniz's controversy against the Cartesians. Therefore, in this form, such conceptions do not exist in Kepler. Nevertheless, although Kepler pointed out the necessity to treat physics with a mathematical approach, it is evident that, for him, the laws of nature are not the mere laws of extension. Certainly the second level of causation, which is so important in the legal structure of the world, cannot be reduced to extension. Once again: since the archetypical laws are metaphysical laws which have an influence on the corporeal structure of the universe (both in Kepler and in Leibniz), the laws of nature are not only the laws of extension.
3. The aspect of divine mathematics at the origin of things is quite interesting in Leibniz: given the context, in which Leibniz spoke of the best possible world, he seems to interpret divine mathematics as related to a problem of maximum: God has chosen the existing world—with its combination of creatures and with the activity of its creatures—among a series of possible worlds. The present world—as we have seen—satisfies the principle of the maximum of reality with the least possible expense of *vis*. Here Leibniz seems to refer to a function in which the number of the creatures and their activity have to reach a maximum under the constraint that the *vis viva*—which is a measure of the activity of any creature—is constant and is not infinite. This is an interpretation because Leibniz was rather vague on divine mathematics and he did not develop it. Anyway—beyond the content of Leibniz's divine mathematics—it is likely that his sources of inspiration were basically two: Spinoza, who in the *Etica ordine geometrico demonstrata*, thought of and developed ethics on the basis of axioms and deductive reasoning, although in Spinoza's *Etica* there is no calculation or

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extensionis legibus addendo nascantur eae ipsae regulae motus, quas systematicas appelleram, nempe ut omnis mutatio fiat per gradus, et omnis actio sit cum reactione, et nova vis non prodeat sine detrimento prioris, adeoque semper abripiens retardetur ab abrepto, nec plus minusve potentiae in effectu quam in causa contineatur. Quae lex cum non derivetur ex notione molis, necesse est consequi eam ex alia re, quae corporibus insit, nempe ex ipsa vi, quae scilicet eandem semper quantitatem sui tuetur, licet a diversis corporibus exerceatur".

geometrical demonstration. It is well known that Leibniz knew profoundly this work by Spinoza[79]—in whose first part the problem of God is dealt with—and, though he had criticized Spinoza's pantheism, he was impressed by *Etica*. The second source of inspiration is once again Kepler, in which geometry in *Mysterium* and harmony in *Harmonice Mundi* are explicitly presented as a divine mathematics, that is the mathematics according to which God structured the world and its metaphysical-mathematical laws. Therefore: physics cannot be reduced to extension, but mathematics is not only the study of extension, it also includes the physical principles and the divine mathematics.

6.4 Final Remarks

The aims of this chapter have been two:

1. to show that there is a double influence of Kepler on Leibniz. The first one is a direct influence and concerns the theory of planetary motion, in particular the decomposition of planetary motion into two components: one along the radius-vector sun-planet and one perpendicular to this radius. Leibniz drew this general idea from Kepler and, after that, developed independently from Kepler the conception of the *circulatio harmonica*.
2. The second kind of influence regards many aspects of Leibniz metaphysics, assuming as conceptual starting point his planetary theory. Obviously, I am not proposing an interpretation which might replace the traditional ones. Mine is only a point of view to see Leibniz's metaphysics under a new perspective. This can enrich Leibniz *Forschung*, adding a new interpretative element to such a complicated and faceted problem as the exegesis of Leibniz's philosophy is. In this perspective, the principle of pre-established harmony plays a particularly important role. This principle and, more in general, the concept of harmony in Leibniz and his ideas on the possible relationship metaphysics-mathematics, have some significant contact points with Kepler's conception of harmony. My idea is that such contacts are not due to a mere convergence of thoughts, but, given some strong similarities between the two authors and the knowledge

[79] The literature on the influence Spinoza exerted on Leibniz and on Leibniz's critics to Spinoza is really huge. I mention here some important references without any claim to be exhaustive. I remind the reader I do not mention the works on Leibniz-Spinoza which do not deal with the subjects I have faced in the running text (for example political theory, free will, and so on): Bartuschat (1981, 2002), Belaval (1995), Biasutti (1990), Blank (2009), Bouveresse (1988), Curley and Heinekamp (1990), Dascal (1990), Ferry (2013), Friedmann (1946, 1975), Garrett (1990), Goldenbaum (2007, 2011), Griffin (2008, 2013), Hart (1982), Homan (2011), Hubbeling (1983), Iriarte-Agirrezabal (1938), Israel (2014), Kneale (1992), Laerke (2006), Latta (1899), Leinkauf (2010), Malcom (2003), Manzini (2009), McRae (1983), Mercer (1999), Moreau (1981), Morfino (1996), Moya Bedoya (2002, 2003), Nachtomy (2011, 2011a), Newlands (2010), Piro (1994), Robinet (1981b), Seidel (1977), Stewart (2010), Stoichita (2010), Woolhouse (1993), von Zimmermann (1890).

Leibniz gained of Kepler's works—specifically the *Harmonice Mundi*, in this case—, to a direct influence of Kepler's reading on Leibniz.

With regard to the difference Leibniz-Kepler, some of them are obvious: first of all Leibniz intended to develop a complete ontology based on the concept of substance, while all the considerations of Kepler have the physical universe and his laws (more specifically regarding astronomy) as reference points. When Kepler spoke of harmony, he was referring, in any case, to a series of ratios existing among the planets and their properties; while referring to the souls of the planets and of all the celestial bodies, he was dealing with something connected to astronomy (see, for example, the problem of which motions the celestial bodies' souls can be responsible for). This does not mean that Kepler did not develop a series of profound considerations which touch many aspects—for example the concept of scientific hypothesis, or the way in which our conscience reaches the knowledge of physical and harmonic laws—but the main point of Kepler remained astronomy, while this is not the case for Leibniz.

A remarkable difference is that the principle of pre-established harmony implies there is no direct action of the soul on the body and viceversa; while in Kepler this situation is different. Without entering into further details: we have seen that the rotation of the celestial bodies around their axes is due to the action of celestial bodies' soul. Therefore, there is a direct action of the soul on the body.

Connected to this context, we have already spoken of the criticisms addressed by Leibniz to Kepler's conceptions which could be referred to mysterious souls, intelligences and actions, that is to the part of Kepler's thought in which the primary level of causation (that of efficient causes) seems to depend on non-mechanical means and mechanisms.

However, although these differences must be taken into account, the influence of Kepler on Leibniz was enormous: Kepler offered a *kosmos* in which everything was connected to everything in a precise and mathematized form. This is something different from the qualitative idea of *kosmos* characterizing part of the Renaissance science and philosophy as well as from a conception of Pythagorean origin in which the harmony of the *kosmos* depends on merely numerical regularities. First of all Kepler thought that the basis of the archetypical laws of the universe was geometry and not arithmetic, furthermore he tried to offer a complete theory, also including—inside harmony—the physical properties of celestial bodies, as, the *molis* and the kinematical properties, as ratios among periodical times, among angular velocities and, for a planet, between periodical time and distance from the sun. That of Kepler was a very distinct theory, not only an idea of a theory. Leibniz found hence a conspicuous source of inspiration in Kepler for some important concepts of his thought, as that of pre-established harmony, even though—this is clear—the idea of a pre-established harmony in Leibnizian meaning did not exist in Kepler.

Chapter 7
Conclusion

The aim of this book has been to point out the connections between Leibniz's planetary theory and some of his general conceptions concerning physics and metaphysics as well as to identify the reasons why Leibniz felt the need to develop a planetary theory. We can summarize as follows:

1. Leibniz's planetary theory can be interpreted as an attempt to offer an alternative point of view to Newton's theory. It is difficult to think that, without Newton's results, Leibniz would have written a paper and carried out a series of further works on planetary theory. In the published version of the *Tentamen*, Leibniz concentrated almost exclusively on the mechanisms by which the planets rotate around the sun, without dealing with the problem of gravity. Nevertheless, the reasons why Leibniz divided the velocity of the planets into a radial and a transverse component cannot be understood only by analysing the mathematical details of his reasoning: his conviction was that harmonic circulation was a physical-structural reality and that, hence, the centrifugal force due to the harmonic vortex was a real force, not a fictitious one. Since the general plane of Leibniz was to offer a physics alternative to Newton's, he felt, hence, the need to explain planetary theory in terms of a force—the centrifugal one—which, according to his opinion, was real. The other true force acting in opposition to the centrifugal one is the *solicitatio paracentrica* or of gravity. This is one of the reasons why Leibniz, after the *Tentamen*, dedicated a series of studies to the problem of gravity, although—which is quite important to be underlined—he never explicitly referred to the universal character of gravity. This means it is difficult to realize if he fully understood that gravity on the earth is the same force responsible for the inwards tendency of *motus paracentricus*. Leibniz's studies explained in Sect. 4.1.2 seem an initial attempt to seek some relations between circular or, more in general, curved motion and the motion of falling bodies, as Newton had done.

   Leibniz's general problem was to offer not only a functioning dynamical model of the solar system, but to explain the real physical-structural form of

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the system: since a theory based on false principle, as Newton's, gave so convincing provisions both in terrestrial and celestial physics, Leibniz felt as a due of his himself to construct a true theory to explain the phenomena. Thus, his purpose was not only to construct a good dynamical model. By way of doing this, he exploited, although in a different context, many of Newton's concepts and results, such as, for instance, the action and reaction principle and the inverse square law.

2. Although it is possible to speak of a planetary theory in Leibniz only starting from the *Tentamen*, his interests in cosmology date back to his early works. The *Hypothesis physica nova* is interesting in this sense. Leibniz recognized that the problem of the origin of gravity, as well as that of the elastic force, was connected to the way in which the earth had been formed. In these speculations, the sun, and in particular the sun light and the supposed aethereal solar vortex, play a fundamental role. The planetary theory developed in the *Tentamen* and in the following contributions is also based on vortices, which, hence, represent a strong connection between this theory and a part of Leibniz's physics. The vortices are the access key to Leibniz's ideas on gravity, which is connected to planetary theory in the way explained in the fifth chapter. A fundamental dynamical concept in Leibniz's physics—that of *vis viva*—was exploited by Leibniz in an attempt to insert Kepler's third law inside his planetary theory in the *Illustratio tentaminis*. This attempt was based on the hypothesis that the orbits of the planets have the same *vis viva*, which induced Leibniz to prefer, at the end of his scientific career, the gravity theory of the *vis centrifuga*—to summarize—rather than that of the c*onatus explosivus*, though this theory was preferable for other physical reasons. This shows the inner connections between planetary theory-ideas of gravity-concept of *vis viva*.
3. But, the concept of *vis viva* is also tied to the most personal metaphysical convictions by Leibniz because the *vis viva*—of which mv^2 is a measure—indicates the active aspect of the substance and represents a force which each substance maintains from the moment of its creation. This introduces a kind of reasoning which is typical by Leibniz and which could be called the *triple truth*: a) the phenomenal truth; b) the dynamical truth; c) the metaphysical truth. At the phenomenal level, the explanation based on the efficient cause is acceptable and correct. However, these explanations rely upon more profound truths: the one of the dynamics, based on the principles of conservation such as the conservation of the *vis viva* or of the *quantité de progress*. This is the reign of the dynamical-final causes. On the other hand, a further and more profound level exists: the metaphysical one, where the substances do not act among them, but the supposed interactions are determined by the principle of pre-established harmony. The division of the explicative level according to the three mentioned items was used by Leibniz as a weapon to accept from a phenomenal point of view principles which he refused from a dynamical or metaphysical perspective. The case of the Newtonian inertia principle is emblematic. He refused it in dynamical and metaphysical terms, but, in practice, accepted it from a

phenomenal point of view. The action produced by centripetal force is another example: Leibniz refused the idea that action could be transmitted immediately at a distance, but accepted the mathematical results by Newton. The distinction into three levels was in the chords of Leibniz's philosophy but it was also a useful device to accept some results in a phenomenal perspective claiming that, from a authentic metaphysical perspective, these results were false.

4. Finally I have tried to show that Kepler's influence permeates several aspects of Leibniz's physics and metaphysics as far as the concept of harmony is concerned. The reference is to harmonic circulation and to the distinction of planetary velocity into two components as well as to the more general idea that harmony is an intrinsic property of the universe. In this picture, I have attempted to highlight the similarities between the harmony of the worlds in Kepler and the principle of the pre-established harmony in Leibniz.

With this book, I have had the intention to deal with some specific interesting aspects of Leibniz's thought, to provide some interpretations of them and to show that the picture inside which Leibniz worked was unitary. I have developed an exegesis based on planetary theory, being perfectly conscious that it is a partial approach to the complexity of Leibniz's philosophy. I hope this perspective has been interesting for the reader.

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