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Supply Chain Disruption Management Using Stochastic Mixed Integer Programming

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To Bartek,
Siasia,
Kappa,
and Toranoko with love
and to the Memory of my Parents

Preface

Scope

This book deals with stochastic combinatorial optimization problems in supply chain disruption management, with a particular focus on management of disrupted flows in customer-driven supply chains. The problems are modeled using a scenario-based stochastic mixed integer programming to address risk-neutral, risk-averse, and mean-risk decision-making in the presence of supply chain disruption risks. One of the main objectives of this book is to present a computationally efficient portfolio approach to integrated decision-making in global supply chains under disruption risks, where the portfolio is defined as the allocation of demand for parts and finished products, respectively, among suppliers and production facilities. The allocation of demand for parts among suppliers is defined as a supply portfolio, whereas the allocation of demand for products among production facilities (e.g., assembly plants) is defined as a demand or capacity portfolio. Unlike most of reported research on the supply chain risk management which mainly focuses on the risk mitigation decisions taken prior to a disruption, the proposed portfolio approach combines decisions made before, during, and after the disruption. When a disruption occurs, the primary portfolios determined prior to a disruption are replaced by recovery portfolios. The selection of portfolios will be combined with management of disrupted material flows, i.e., supply, production, and distribution scheduling under disruption risks. This book demonstrates that the developed portfolio approach leads to a well-structured decision-making and computationally efficient mathematical formulations, in particular, to stochastic mixed integer programs with a strong LP relaxation. Moreover,

- integrated versus hierarchical decision-making is compared depending on the available information on disruptive events;
- a multi-objective decision-making is analyzed to trade off between: cost versus service level objective functions, fairness versus non-equitability of objective functions, average versus worst-case performance of a supply chain, etc.;

- a multi-period decision-making is modeled to capture dynamics of disruption and recovery processes, i.e., static versus dynamic portfolios, scheduling of supply, production, and distribution operations to control disrupted flows under time-varying conditions such as demands and capacities;
- a multi-level disruption scenarios are modeled to capture partially disrupted flows, partially fulfilled orders, partially recovered facilities, and partially available capacity.

This book also addresses the issue of fundamental understanding of average-case and worst-case performance of a global supply chain in the presence of flow disruption risks as well as understanding of the recovery mechanisms.

A straightforward computational approach used in this book is to solve the deterministic equivalent mixed integer program of a two-stage stochastic mixed integer program with recourse, which allows for a direct application of commercially available software for mixed integer programming. In the computational experiments reported throughout this book, an advanced algebraic modeling language AMPL (see, Fourer et al. 2003) and the CPLEX, Gurobi, and XPRESS solvers have been applied.

Content

This book is divided into an introductory Chap. 1, where an overview of supply chain disruption modeling and management is provided, and the five main parts. Part I addresses selection of a supply portfolio, Part II, integrated selection of supply portfolio and scheduling, Part III, integrated, equitably efficient selection of supply portfolio and scheduling, Part IV, integrated selection of primary and recovery supply (and demand) portfolios and scheduling, and finally, Part V addresses disruption management of information flows in supply chains.

Part I (Chaps. 2–4) introduces the portfolio approach for supplier selection and order quantity allocation in the presence of supply chain disruption risks, i.e., for determining a supply portfolio. The proposed portfolio approach allows the two popular in financial engineering percentile measures of risk, Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), to be applied for managing the risk of supply disruptions. For a finite number of scenarios, CVaR allows the evaluation of worst-case costs (or worst-case service level) and shaping of the resulting cost (service level) distribution through optimal supplier selection and order quantity allocation decisions, i.e., the selection of optimal supply portfolio. Part I is comprised of these chapters:

- Chapter 2, Selection of Static Supply Portfolio. This chapter deals with selection of a static supply portfolio under disruption risks, i.e., for determining a single-period allocation of demand for parts among selected suppliers to minimize expected or expected worst-case cost or maximize expected or expected worst-case service level.

- Chapter 3, Selection of Dynamic Supply Portfolio. In this chapter, the static portfolio approach and the stochastic mixed integer programming formulations presented in Chap. 2 are enhanced for a multi-period supplier selection and order quantity allocation in the presence of both the low-probability and high-impact supply chain disruption risks and the high-probability and low-impact supply chain delay risks. The suppliers are subject to local delivery delay risks, and both local and regional delivery disruption risks. In the delivery scenario analysis, both types of the supply chain risks are simultaneously considered.
- Chapter 4, Selection of Resilient Supply Portfolio. In this chapter, the portfolio approach and SMIP (Stochastic Mixed Integer Programming) models presented in Chap. 2 are enhanced for the combined selection and protection of part suppliers and order quantity allocation in a supply chain with disruption risks. The protection decisions include the selection of suppliers to be protected against disruptions and the allocation of emergency inventory of parts to be prepositioned at the protected suppliers so as to maintain uninterrupted supplies in case of natural or man-made disruptive events.

Part II of this book concerns with integrated selection of supply portfolio and scheduling. The medium- to short-term decisions of the supplier selection and order quantity allocation, driven by the time-varying customer demand, are made along with scheduling of customer orders execution and distribution. The advantage of such a joint decision-making is especially evident in the presence of supply chain disruption risks. Part II has two chapters:

- Chapter 5, Integrated Selection of Supply Portfolio and Scheduling of Production. This chapter proposes a SMIP approach to integrated supplier selection and customer order scheduling in the presence of supply chain disruption risks, for a single, dual, or multiple sourcing strategy. The suppliers are assumed to be located in two or more disjoint geographic regions: in the producer's region (domestic suppliers) and outside the producer's region (foreign suppliers). The supplies are subject to independent random local disruptions that are uniquely associated with a particular supplier and to random regional disruptions that may result in disruption of all suppliers in the same geographic region simultaneously.
- Chapter 6, Integrated Selection of Supply Portfolio and Scheduling of Production and Distribution. The purpose of this chapter is to study the integrated decision-making to simultaneously select suppliers of parts, allocate order quantity, and schedule production and delivery of finished products to customers in a supply chain under disruption risks. In addition to supplier selection, order quantity allocation, and scheduling of customer orders, distribution of finished products to customers is simultaneously considered with different shipping methods to optimize the trade-off between cost and service level. The three different shipping methods will be modeled and compared for the distribution of products: batch shipping with a single shipment of different

customer orders, batch shipping with multiple shipments of different customer orders, and individual shipping of each customer order immediately after its completion.

Part III addresses equitably efficient selection of supply portfolio and scheduling. A fair optimization of an average performance of a supply chain with respect to equally important conflicting objective functions and a fair mean-risk optimization of average-case and worst-case performance are considered in the presence of supply chain disruption risks. The conflicting and equally important objective functions are expected values of cost and customer service level and the corresponding expected and expected worst-case values, respectively. The fairness and the mean-risk fairness reflect the decision maker's common requirement to maintain an equally good performance of a supply chain with respect to equally important objectives and under varying operating conditions. Part III has two chapters:

- Chapter 7, A Fair Decision-Making under Disruption Risks. In this chapter, the two risk-neutral conflicting criteria—expected cost and expected service level—are fairly optimized to achieve an equitably efficient supply portfolio and production schedule in the presence of supply chain disruption risks. In order to obtain an equitably efficient solution, the ordered weighted averaging (OWA) aggregation of the two conflicting objective functions is applied. The equitably efficient solutions obtained for the ordered weighted averaging aggregation of the two conflicting objective functions will be compared with non-dominated solutions obtained using the weighted-sum aggregation approach.
- Chapter 8, A Robust Decision-Making under Disruption Risks. In this chapter, we look for an equitably efficient solution with respect to both average-case and worst-case performance measures of a supply chain. Such an equitably efficient average-case and worst-case solution or equivalently equitably efficient risk-neutral and risk-averse solution will be called a fair mean-risk solution. The solution will equitably focus on the two objective functions: the expected value (average-case performance measure) and the expected worst-case value (worst-case performance measure), i.e., Conditional Value-at-Risk of the selected optimality criterion, cost, or service level. The fair mean-risk decision-making aims at equalizing the distance to optimality both under business-as-usual and under worst-case conditions, which reflects a common requirement to maintain an equally good performance of a supply chain under varying operating conditions. Therefore, the mean-risk fairness, i.e., the equitably efficient performance of a supply chain in the average case as well as in the worst case, in this chapter will be called robustness.

Part IV focuses on selection of primary and recovery portfolios and scheduling. The selection of primary suppliers and order quantity allocation to mitigate the impact of disruption risks is combined with selection of recovery suppliers and assembly plants to optimize the recovery processes. The two decision-making approaches will be considered and compared: an integrated approach with some information about the future potential disruption scenarios available ahead of time and a hierarchical approach with no such information available. In the integrated approach, which may account for all potential disruption scenarios, the primary portfolios that will hedge against all potential disruptive events will be determined along with the recovery portfolios for each scenario. In the hierarchical approach, first the primary portfolios are determined, and then, when the primary portfolios are impacted by a disruptive event, the recovery portfolios are selected to optimize the process of recovery from the disruption. Both the integrated and the hierarchical decision-making account for time and cost of mitigation and recovery processes and aim at optimizing cost and service level as the two equally important, conflicting objective functions. Part IV has two chapters:

- Chapter 9, Selection of Primary and Recovery Supply Portfolios and Scheduling. In this chapter, the portfolio approach presented in the previous chapters for the selection of primary suppliers and order quantity allocation to mitigate the impact of disruption risks is enhanced also for the recovery process, i.e., for the selection of both primary and recovery suppliers and order quantity allocation to mitigate the impact of disruption risks and optimize the recovery process. Unlike most of reported research on the supply chain risk management which focuses on the risk mitigation decisions taken prior to a disruption, this chapter combines decisions made before, during, and after the disruption.
- Chapter 10, Selection of Primary and Recovery Supply and Demand Portfolios and Scheduling. In this chapter, the portfolio approach proposed in Chap. 9 for the selection of primary and recovery suppliers and order quantity allocation to mitigate the impact of disruption risks is enhanced also for the recovery process of the firm's assembly plants for finished products. Unlike most of reported research on supply chain disruption management, a disruptive event is assumed to impact both a primary supplier of parts and the buyer's firm primary assembly plant. Then, the firm may choose alternate (recovery) suppliers and move production to alternate (recovery) plants along with transshipment of parts from the impacted primary plant to the recovery plants. The resulting allocation of unfulfilled demand for parts among recovery suppliers and unfulfilled demand for products among recovery assembly plants determines recovery supply portfolio and recovery demand portfolio, respectively.

Part V deals with disruption management of information flows in supply chains caused by cybersecurity incidents. The supply portfolio approach applied to mitigate the impact of supply disruptions has been modified to select countermeasure portfolio to mitigate the impact of information flow disruptions. Part V has one chapter:

- Chapter 11, Selection of Cybersecurity Safeguards Portfolio. This chapter deals with the selection of countermeasure portfolio in cybersecurity planning to prevent or mitigate the impact of information flow disruptions on a supply chain. A scenario-based bi-objective SMIP approach with CVaR as a risk measure is proposed for the decision-making. Given a set of potential threats and a set of available countermeasures, the decision maker needs to decide which countermeasure to implement under limited budget to minimize potential losses from successful cyberattacks. The selection of countermeasures is based on their effectiveness of blocking different threats, implementation costs, and probability of potential attack scenarios. The bi-objective trade-off model provides the decision maker with a simple tool for balancing expected and worst-case losses and for shaping of the resulting cost distribution through the selection of optimal subset of countermeasures for implementation, i.e., the selection of optimal countermeasure portfolio.

Parts I–IV and the chapters within each part are arranged in the order recommended for reading, while Part V with Chap. 11 can be read independently of the other chapters. Each chapter ends with the end-of-chapter problems to help the reader a self-check of material comprehension and to encourage for a further self-study.

This book can be considered a companion as well as a follower of my previous book on scheduling in supply chains using mixed integer programming (Sawik 2011a), where deterministic MIP approaches were developed for integrated scheduling in customer-driven supply chains, in particular, in the electronics supply chains. The reader interested in knowing more about stochastic programming is referred to the monographs by Birge and Louveaux (2011) or Kall and Mayer (2011). For a general introduction to mixed integer programming models and techniques, the reader is referred to the application-oriented book by Chen et al. (2010) or to the seminal work in the field by Nemhauser and Wolsey (1999). The fundamentals of supply chain theory are well presented by Snyder and Shen (2011), and for an engineering-oriented general reference work on supply chains, the reader is referred to the book by Dolgui and Proth (2010). Finally, some books cover supply chain risk management in general, e.g., Kouvelis et al. (2011), and some of these emphasize supply chain disruption management, e.g., Gurnani et al. (2012).

Audience

This book is addressed to practitioners and researchers on supply chain risk management and disruption management, and to students in management, industrial engineering, operations research, applied mathematics, computer science and the like at masters and Ph.D. levels. It is not necessary to have a detailed knowledge of stochastic programming and integer programming in order to go through this book. The knowledge required corresponds to the level of an introductory course in operations research and supply chain management for engineering, management, and economics students.

Kraków, Poland Tadeusz Sawik
April 2017

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Finally, I wish to express my thanks to Springer Editor, Camille Price, who invited me to prepare this monograph for International Series in Operations Research & Management Science.

Contents

Acronyms

ABC_E	Model for risk-neutral decision-making
ABC_CV	Model for risk-averse decision-making
ABC_ECV	Model for mean-risk decision-making
CVaR	Conditional Value-at-Risk
$CVaR^c$	Conditional Cost-at-Risk
$CVaR^{sl}$	Conditional Service-at-Risk
E^c	Expected cost
E^{sl}	Expected service level
MIP	Mixed Integer Programming
SMIP	Stochastic Mixed Integer Programming
VaR	Value-at-Risk
VaR^c	Cost-at-Risk
VaR^{sl}	Service-at-Risk

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Chapter 1
Introduction

1.1 Overview of Supply Chain Disruption Management

A typical global supply chain in modern industry is a network of multiple part suppliers at different geographical locations and multiple production plants and distribution centers, where supplied parts are assembled into finished products and next distributed to customers. Figure 1.1 shows a schematic diagram of a supply chain network, where each vertical level (suppliers, producers, distribution centers, customers) is called an echelon, and the arcs represent material flows. In order to achieve a high customer service level at a low cost, a variety of complex, interconnected decision-making problems should be solved. The decision-making problems are strictly associated with the control and optimization of material flows (as well as financial and information flows) in the network, in particular, optimization of disrupted flows. Different types of material flows (e.g., flows of parts from suppliers to producers, flows of semi-finished products at producers, flows of finished products from producers to distribution centers and from the distribution centers to customers) should be coordinated in an efficient manner. In global supply chains, the control and optimization of material flows are accomplished by scheduling of manufacturing and supplies of parts, scheduling of production and customer orders for finished products, and scheduling of deliveries to customers. All those scheduling decisions should be efficiently coordinated to fulfill customer demand, especially in the presence of supply chain disruption risks. The schedule of customer orders immediately depends on the schedule of parts supplies, which in turn depends on supplier selection and order quantity allocation, that is, on supply portfolio. On the other hand, the schedule of customer orders implicitly defines the schedule of deliveries of finished products to customers, which in turn depends on customer order allocation among assembly plants, that is, on demand portfolio. In view of the recent trend of outsourcing and globalization, coordinated decision-making, e.g., selection of primary and recovery part suppliers and allocation of order quantities, selection of primary and recovery assembly plants and allocation of customer demand, and scheduling of

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customer orders in the assembly plants may significantly improve performance of a multi-echelon supply chain under disruption risks. However, most work on coordinated supply chain scheduling focuses on coordinating non disrupted flows of supply and demand over a supply chain network to minimize the inventory, transportation and shortage costs. The research on quantitative approaches to coordinated scheduling of disrupted flows in supply chains has not been sufficiently reported in the literature and is mostly limited to separate investigation of supply, production or distribution stage.

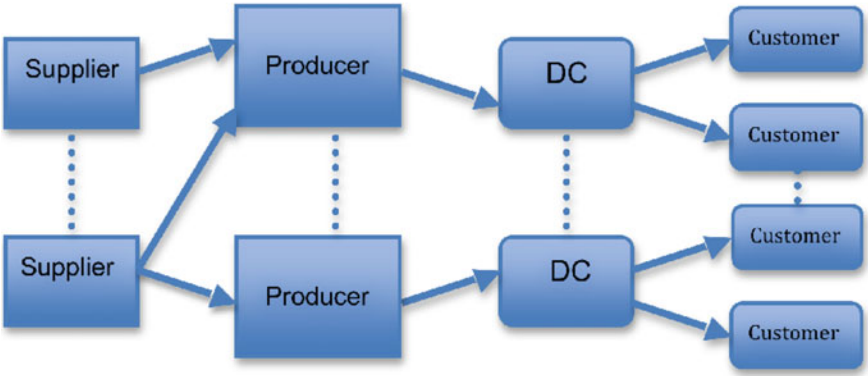

Fig. 1.1 A supply chain network

In modern global supply chains, disruption risk management has become a vital part of supply chain management strategy. Material flows in supply chains can be disrupted by unexpected natural or man-made disasters such as earthquakes, fires, floods, hurricanes or equipment breakdowns, labor strikes, economic crisis, bankruptcy or by a deliberate sabotage or terrorist attack. The low-probability and high-impact flow disruptions and the resulting losses may threaten financial state of firms. For example, the Taiwan earthquake of September 1999 created huge losses for many electronics companies supplied with components by Taiwanese manufacturers, e.g., Apple lost many customer orders due to supply shortage of DRAM chips (Sheffi 2005). The Philips microchip plant fire of March 2000 in New Mexico resulted in 400 million Euros in lost sales by a major cell phone producer, Ericsson (Norrman and Jansson 2004). The disruptions in the automotive and electronics supply chains that occurred in 2011 after the Great East Japan Tohoku earthquake on March 11 and then the Thailand flooding in October, resulted in huge losses of major automakers and electronics manufacturers, e.g., Haraguchi and Lall (2015), Park et al. (2013). Similar effects were observed after the recent Kyushu earthquake in April 2016, e.g., Marszewska (2016). Toyota supply chain has been again severely disrupted when two plants of Aisin Seiki, a key supplier of car body and engine components were destroyed, and Renesas, a key supplier of semiconductors for Toyota engines, had to halt production in Kumamoto plant.

In order to minimize losses caused by the shortage of material supplies, customer companies (firms) apply different disruption management strategies. For example, the firms may participate in supplier's recovery process after disruption to reduce recovery time. When the Tohoku earthquake and tsunami disrupted Toyota supply chain and, in particular, supply chain of automotive semiconductors, Toyota supported recovery of its suppliers, Fujimoto and Park (2013). The automotive semiconductors were manufactured by Renesas Electronics, who shares over 44% of world-wide automobile microcontroller units, and its main plant in Naka was severely damaged by the earthquake. The shipping of automotive semiconductors was expected to be stopped for eight months. In order to shorten the expected recovery time, Toyota and other Japanese automotive, electronics and semiconductor equipment manufacturers sent to Naka over 2500 engineers to support plant recovery. As a result, the recovery time to start shipping of automobile microcontroller units was shortened from eight to five months, Matsuo (2015). Another example of helping by customer companies in supplier's recovery process was the case of Riken Corporation, the largest supplier of piston rings to all Japanese automobile manufacturers. In July 2007, Riken plant in city of Kashiwazaki was hit by a strong earthquake and severely damaged. Immediately after the shutdown, the Japanese automakers coordinated by Toyota, sent over 650 people including many equipment engineers to help its recovery and as a result the stoppage of piston rings production was shortened to two weeks only, Whitney et al. (2014). A similar action has been applied to help recovery of Aisin Seiki plants after the Kyushu earthquake in April 2016, (Marszewska 2016).

The above real-world examples illustrate a well-known disruption management strategy of helping a primary supplier recover more quickly. However, when a primary supplier is hit by disruption, the customer company may choose either to support recovery of disrupted primary supplier, rely on a preselected backup supplier or select an alternate (recovery) supplier, non-disrupted or disrupted less severely than the primary supplier. In a similar way, a firm whose primary plant is hit by disruption may either stop production until recovery process is finished or move production to alternate (recovery) plants along with transshipment of parts to the recovery plants. The complex decision-making that involves various characteristics of a supply chain should be supported by optimization models to minimize cost (or maximize profit) and maximize service level as typical objective functions. The objective of this book is to present such optimization models and to stimulate further research on fundamental understanding of various mitigation and recovery policies in the presence of flow disruption risks in global supply chains.

1.2 Value-at-Risk Versus Conditional Value-at-Risk

A common tool for supply chain optimization under disruption risks is stochastic programming, in particular stochastic mixed integer programming (SMIP), e.g., Heckmann et al. (2015). SMIP is an exact mathematical modeling approach that allows for the inclusion of uncertainty by probabilistic scenarios of disruptive events and for finding the optimal solutions with respect to multiple objective functions. In this book the stochastic combinatorial decision-making problems will be formulated as multi-period, multi-objective stochastic mixed integer programs with expected or expected worst-case performance measures and trade-offs between various objective functions. In order to mitigate the impact of supply chain disruptions, the two popular in financial engineering percentile measures of risk, Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), will be applied for the decision-making, e.g., Sawik (2011, 2012a, b, c, 2016). This section briefly defines and compares VaR and CVaR, e.g., Sarykalin et al. (2008).

Let $F_W(u) = Prob\{W \leq u\}$ be the cumulative distribution function of a random variable W representing cost. The VaR (Value-at-Risk) of W with the confidence level $\alpha \in [0, 1)$ is defined as a lower α-percentile of the random variable W

$$VaR_\alpha(W) = \min\{u : F_W(u) \geq \alpha\}.$$

VaR represents the maximum cost associated with a specified confidence level of outcomes (i.e., the likelihood that a given portfolio's costs will not exceed the amount defined as VaR). However, VaR does not account for properties of the cost distribution beyond the confidence level and hence does not explain the magnitude of the cost when the VaR limit is exceeded. Moreover, VaR is not a coherent measure of risk since it fails to hold the sub-additivity property ($f(x + y) \leq f(x) + f(y)$ where $f(.)$ is the risk measure). VaR of a portfolio can be higher than the sum of VaRs of the individual assets in the portfolio.

On the other hand, CVaR focuses on the tail of the cost distribution, that is, on outcomes with the highest cost. Assuming that $F_W(u)$ is a continuous distribution function, the CVaR of W with the confidence level $\alpha \in [0, 1)$, $CVaR_\alpha(W)$, equals the expectation of W subject to $W \geq VaR_\alpha(W)$. However, in the general case $CVaR_\alpha(W)$ is not equal to an average of outcomes greater than $VaR_\alpha(W)$ and is defined as the mean of the generalized α-tail distribution

$$CVaR_\alpha(W) = \int_{-\infty}^{\infty} u dF_W^\alpha(u),$$

where

$$F_W^\alpha(u) = \begin{cases} 0 & \text{if } u < VaR_\alpha(W) \\ \frac{F_W(u)-\alpha}{1-\alpha} & \text{if } u \geq VaR_\alpha(W). \end{cases}$$

$CVaR_\alpha(W)$ can be considered as a generalization of the expected value, when $\alpha = 0$ they are equivalent. On the other hand, the higher is the confidence level α, the closer are values of $VaR_\alpha(W)$ and $CVaR_\alpha(W)$.

Alternatively, $CVaR_\alpha(W)$ can be defined as the weighted average of $VaR_\alpha(W)$ and the conditional expectation of W subject to $W > VaR_\alpha(W)$.

When the distribution function has a vertical jump at $VaR_\alpha(W)$ (the probability interval of confidence level α with the same VaR), a probability "atom" is said to be present at $VaR_\alpha(W)$. For example, when the distribution is modeled by scenarios, the probability measure is concentrated in finitely many points and the corresponding distribution function is a step function (constant between jumps) with jumps at those points. Since $CVaR_\alpha(W)$ is obtained by averaging a fractional number of scenarios, the $VaR_\alpha(W)$ atom can be split. When $F_W(VaR_\alpha(W)) > \alpha$, then probability $1 - F_W(VaR_\alpha(W))$ of the cost interval $[VaR_\alpha(W), \infty)$ is smaller than $1 - \alpha$.

Note that if $F_W(VaR_\alpha(W)) = 1$, so that $VaR_\alpha(W)$ is the highest cost that may occur, then $CVaR_\alpha(W) = VaR_\alpha(W)$.

Summarizing the above definitions (from now on, VaR and CVaR will be denoted without subscript α, and with superscripts c and sl to denote cost and service level, respectively):

- Cost-at-Risk (VaR^c) at a $100\alpha\%$ confidence level is the targeted cost such that for $100\alpha\%$ of the scenarios, the outcome will not exceed VaR^c. In other words, VaR^c is a decision variable based on the α-percentile of costs, i.e., in $100(1-\alpha)\%$ of the scenarios, the outcome may exceed VaR^c.
- Conditional Cost-at-Risk (CVaR^c) at a $100\alpha\%$ confidence level is the expected cost in the worst $100(1-\alpha)\%$ of the cases. In other words, we allow $100(1-\alpha)\%$ of the outcomes to exceed VaR^c, and the mean value of these outcomes is represented by CVaR^c.

In other words, VaR^c is the acceptable cost level above which we want to minimize the number of outcomes and CVaR^c considers those portfolio outcomes, where costs exceed VaR^c (see, Fig. 1.2).

Generally, confidence level α indicates the level of conservatism that a decision-maker is willing to adopt. As α approaches one, the range of acceptable worst-cases becomes narrower in the corresponding optimization problem. Figure 1.2 clarifies the concept of CVaR and demonstrates that CVaR is the conditional expected value exceeding the VaR.

Similar definitions of VaR and CVaR for service level are given below.

- Service-at-Risk (VaR^{sl}) at a $100\alpha\%$ confidence level is the targeted service level such that for $100\alpha\%$ of the scenarios, the outcome will not be below VaR^{sl}. In other words, VaR^{sl} is a decision variable based on the α-percentile of service level, i.e., in $100(1-\alpha)\%$ of the scenarios, the outcome may be below VaR^{sl}.
- Conditional Service-at-Risk (CVaR^{sl}) at a $100\alpha\%$ confidence level is the expected service level in the worst $100(1-\alpha)\%$ of the cases. In other words, we allow $100(1-\alpha)\%$ of the outcomes to be below VaR^{sl}, and the mean value of these outcomes is represented by CVaR^{sl}.

In other words, VaR^{sl} is the acceptable service level below which we want to maximize the number of outcomes and CVaR^{sl} considers those portfolio outcomes, where service levels do not exceed VaR^{sl} (see, Fig. 1.3).

Since VaR and CVaR measure different parts of the cost (service level) distribution, VaR may be better for optimizing portfolios when good models for tails are not available, otherwise CVaR may be preferred.

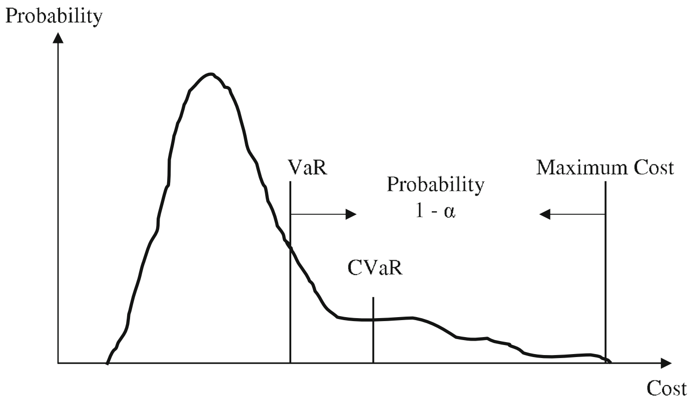

Fig. 1.2 Distribution of cost: VaR^c versus CVaR^c

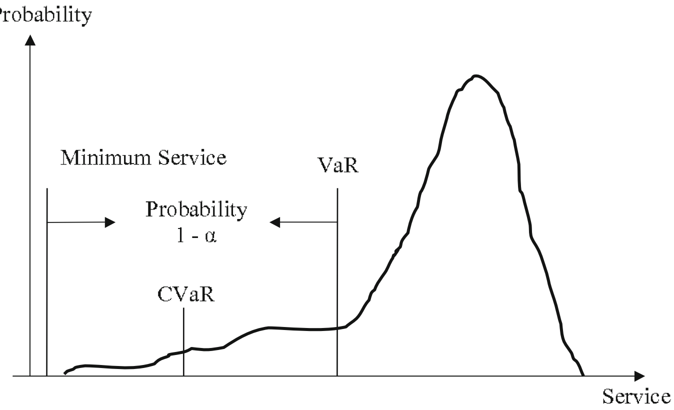

Fig. 1.3 Distribution of service level: VaR^{sl} versus CVaR^{sl}

1.3 Local, Regional and Global Disruptions

Assume that the supply chain consists of $\overline{I}$ interconnected facilities (nodes in this network) that are located in $\overline{R}$ disjoint geographic regions. A facility in a supply chain network may be a supplier of raw material or components, a manufacturer or an assembly plant, a distribution center, or a retailer. Denote by $I^r \subseteq I = \{1, \ldots, \overline{I}\}$ the subset of supply chain facilities in region $r \in R = \{1, \ldots, \overline{R}\}$, where $\bigcup_{r \in R} I^r = I$. The supply chain facilities are subject to random independent local disruptions that are uniquely associated with a particular facility, which may arise from equipment breakdowns, local labour strikes, fires, etc. Denote by p_i the local disruption probability for object $i \in I$, i.e., the state of facility i is "on" (non-disrupted) with probability $(1 - p_i)$, or is "off" (disrupted) with probability p_i.

In addition to independent local disruptions of each facility individually, there are also potential regional disruptive events that may result in correlated regional disruption of all facilities in the same geographic region, and global disruptive super events that may simultaneously impact all the facilities, i.e., the entire supply chain. For example, such regional disruptive events may include, flood, earthquake, regional strike in a transportation sector, whereas global disaster super events may include an economic crisis, widespread labor strike in a transportation sector, etc.

Let p^r and p^* be the probability of correlated regional disruption, simultaneously of all facilities $i \in I^r$ in region $r \in R$, and correlated global disruption, simultaneously of all facilities $i \in I$, respectively. The global disruptive super event, the regional disruptive events in each region and the local disruptive events are assumed to be independent events. Thus, the disruption probability, π_i, of every facility $i \in I^r, r \in R$ is

$$\pi_i = p^* + (1 - p^*)p^r + (1 - p^*)(1 - p^r)p_i; \; i \in I^r, r \in R. \tag{1.1}$$

Denote by P_s the probability that disruption scenario s is realized, where each scenario $s \in S$ is comprised of a unique subset $I_s \subset I$ of facilities that are in state "on" (non-disrupted), and $S = \{1, \ldots, \overline{S}\}$ is the index set of all disruption scenarios. Each scenario $s \in S$ can be represented by a 0–1 vector $\lambda_s = \{\lambda_{1s}, \ldots, \lambda_{\overline{I}s}\}$, where $\lambda_{is} = 0$ denotes disruption of facility $i \in I$ under scenario $s \in S$, and $\lambda_{is} = 1$ denotes a normal, non-disruptive state of facility $i \in I$ under scenario $s \in S$. Given the number of facilities $\overline{I}$, the total number of scenarios in which at least one facility is non-disrupted is given by $\sum_{i=1}^{\overline{I}} \binom{\overline{I}}{i} = 2^{\overline{I}} - 1$. Including the scenario in which all facilities are disrupted, there are a total of $\overline{S} = 2^{\overline{I}}$ potential disruption scenarios. For each scenario $s \in S$, the facilities $i \in I \setminus I_s$, can be disrupted either by a local, regional or global disaster event.

The probability P_s of each disruption scenario $s \in S$ is derived as follows. First, the probability P_s^r of realizing disruption scenario s for suppliers in I^r is determined. If there are non-disrupted suppliers in region r, i.e., $I^r \bigcap I_s \neq \emptyset$, then P_s^r is the product of regional non-disruption probability, $(1 - p^r)$, local probabilities of non-disrupted suppliers, $(1 - p_i)$, $i \in I^r \bigcap I_s$, and local probabilities of disrupted suppliers, p_i, $i \in I^r \setminus I_s$. Otherwise, i.e., if all suppliers in region r are disrupted, $I^r \bigcap I_s = \emptyset$, then either the entire region is disrupted with probability, p^r, or the region is non-disrupted with probability, $(1 - p^r)$, and every supplier $i \in I^r$ is locally disrupted with probability, p_i. Thus, the probability P_s^r is

$$P_s^r = \begin{cases} (1 - p^r) \prod_{i \in I^r \bigcap I_s} (1 - p_i) \prod_{i \in I^r \setminus I_s} p_i & \text{if } I^r \bigcap I_s \neq \emptyset \\ p^r + (1 - p^r) \prod_{i \in I^r} p_i & \text{if } I^r \bigcap I_s = \emptyset. \end{cases} \tag{1.2}$$

The probability P_s for each disruption scenario $s \in S$ with the subset I_s of non-disrupted facilities, and with all possible combinations of different disruptive events considered, is

$$P_s = \begin{cases} (1-p^*)\prod_{r\in R} P_s^r & \text{if } I_s \neq \emptyset \\ p^* + (1-p^*)\prod_{r\in R} P_s^r & \text{if } I_s = \emptyset, \end{cases} \tag{1.3}$$

If global and regional disruption probabilities are negligible, i.e., $p^* = 0$ and $p^r = 0, r \in R$, the probability P_s of disruption scenario s in the presence of independent local disruptive events only, reduces to

$$P_s = \prod_{i\in I_s}(1-p_i) \cdot \prod_{i\notin I_s} p_i. \tag{1.4}$$

1.4 Two-level Versus Multi-level Disruptions

In this section the scenarios with the two-level, all-or-nothing disruptive events considered in Sect. 1.3 are enhanced for the multi-level (partial) disruptive events. In contrast to yield uncertainty (e.g., defective products) that occurs, for instance, when the quantity of supply delivered is a random variable, typically modeled as either a random additive or multiplicative quantity, the multi-level disruptions are modeled as events of different level (e.g., partial capacity available) which occur randomly and may have a random length, e.g., Schmitt and Singh (2012).

Assume that each facility $i \in I$ is subject to random independent local disruptions of different levels, $l \in L_i = \{0, \ldots, \overline{L_i}\}$, where the disruption level refers to the available fraction of full capacity of a facility (e.g., a partial fulfillment of an order by a supplier, a partial fulfillment of a customer order by a producer, etc.).

Level $l = 0$ represents complete shutdown of a facility, i.e., no capacity available, (e.g., no order delivery) while level $l = \overline{L_i}$ represents normal conditions with full capacity available (e.g., full order delivery). The fraction of full capacity of facility i available under disruption level l is described by γ_{il}

$$\gamma_{il} \begin{cases} = 0 & \text{if } l = 0 \\ \in (0,1) & \text{if } l = 1, \ldots, \overline{L_i} - 1 \\ = 1 & \text{if } l = \overline{L_i}. \end{cases} \tag{1.5}$$

Denote by $S = \{1, \ldots, \overline{S}\}$ the index set of all disruption scenarios, and by P_s the probability of disruption scenario $s \in S$. Each scenario $s \in S$ can be represented by an integer-valued vector $\lambda_s = \{\lambda_{1s}, \ldots, \lambda_{\overline{I}s}\}$, where $\lambda_{is} \in L_i$ is the disruption level of facility $i \in I$ under scenario $s \in S$. When all potential disruption scenarios are considered, then $\overline{S} = \prod_{i\in I}(\overline{L_i} + 1)$.

Assume that for each scenario $s \in S$, each facility can be disrupted either by a multi-level local disruptive event or by a two-level regional disruptive event. The probability P_s for disruption scenario $s \in S$ with the subset I_s of non-shutdown facilities is given by (1.3). Now, the probability, P^r_s, of realizing of disruption scenario s for facilities in I^r is (e.g., Sawik 2015b)

$$P^r_s = \begin{cases} (1 - p^r)\prod_{i \in I^r}(p_{i,\lambda_{is}}) & \text{if } I^r \bigcap I_s \neq \emptyset \\ p^r + (1 - p^r)\prod_{i \in I^r} p_{i0} & \text{if } I^r \bigcap I_s = \emptyset, \end{cases} \tag{1.6}$$

where $p_{i,\lambda_{is}}$ is the probability of occurrence the disruption at level $l = \lambda_{is}$ at facility i.

1.5 Risk-Neutral, Risk-Averse and Mean-Risk Decision-Making

In this book we consider three types of decision-making policies.

1. Risk-neutral, that is based on expected value optimization approach, e.g., expected cost minimization or expected service level maximization approach. The risk-neutral policy focuses on an average performance of a supply chain.
2. Risk-averse wherein, rather that optimizing the expected value of an objective function, the decision-maker uses a Conditional Value-at-Risk (CVaR) approach to measure and quantify risk and to define what comprises a worst-case scenario. The CVaR methodology allows the decision-maker to evaluate worst-case values of an objective function, to specify to what extent worst-case scenarios should be avoided and to shape distribution of the resulting objective function values, associated with such a policy. The risk-averse policy focuses on worst-case performance of a supply chain.
3. Mean-risk wherein the decision-maker seeks for a best trade-off between expected value and CVaR of an objective function. The mean-risk policy focuses on both the average and the worst-case performance of a supply chain, simultaneously.

In this book we utilize stochastic mixed integer programming approach, that leads to a two-stage optimization problem. The decisions that are made ahead of time are considered the first stage decisions and are represented by the first stage decision variables.

Typical *first stage decision variables* are

- binary selection variables, such as supplier selection variables, supplier protection variables,
- fractional allocation variables, such as order quantity allocation variables, emergency inventory pre-positioning variables.

In the risk-averse decision-making, VaR (cost-at-risk, service-at-risk, etc.) can also be interpreted as first stage variables.

The second stage decision variables represent decisions that are made after the realization of the random events (e.g., supply disruptions) is known. The second stage variables are dependent on the realized random event.

Typical *second stage decision variables* are

- binary selection variables, such as recovery supplier selection variables, recovery plant selection variables,
- time-indexed binary assignment variables, such as production scheduling variables, distribution scheduling variables,
- fractional allocation variables, such as recovery order quantity allocation variables, recovery demand allocation variables, emergency inventory usage variables, transshipment variables,
- continuous variables, such as tail cost, tail service level.

Risk-Neutral Decision-Making

The stochastic formulation of the risk-neutral decision-making problem aimed at loss (cost) minimization can be written as

$$\min_{\mathbf{x}\in X} \mathbf{c}^T\mathbf{x} + E[Q(\mathbf{x}, \xi^s)], \tag{1.7}$$

where $\mathbf{c}^T\mathbf{x} + E[Q(\mathbf{x}, \xi^s)]$ is the total cost function of the first-stage problem and

$$Q(\mathbf{x}, \xi^s) = \min_{\mathbf{y}^s\in Y^s}\{(\mathbf{q}^s)^T\mathbf{y}^s\} \tag{1.8}$$

is the optimal solution of the second-stage problem corresponding to the first stage decision variables $\mathbf{x}$ and the realization of the random data ξ^s for scenario $s \in S$, denoted by $\xi^s = (\mathbf{q}^s, Y^s)$.

$E[Q(\mathbf{x}, \xi^s)]$ is the expected "cost" taken with respect to random scenario $s \in S$.

The objective function $Q(\mathbf{x}, \xi^s)$ of the second-stage problem (1.8), also known as the recourse (cost) function, is a random variable.

Here $\mathbf{x}$ and $\mathbf{y}^s$ are the vectors of first stage and second stage decision variables, where the first stage decisions are deterministic and the second-stage decisions are dependent on random scenario s. X denotes the feasible set of first stage decisions and Y^s is the feasible set of second stage decisions for random scenario $s \in S$. The second-stage problem (1.8) may be infeasible for some first stage decisions $\mathbf{x} \in X$.

To deal with the uncertainty in the second stage, a scenario-based modeling approach is proposed that has been widely used in stochastic programming. In the second stage, let us consider random scenario $s \in S$ to have a discrete distribution, where P_s is the probability of occurrence for scenario $s \in S$, and S is a finite set of scenarios. Given a finite set of scenarios, S, with associated probabilities, P_s, $s \in S$, the expected value $E[Q(\mathbf{x}, \xi^s)]$ can be evaluated as $E[Q(\mathbf{x}, \xi^s)] = \sum_{s \in S} P_s Q(\mathbf{x}, \xi^s)$. Hence, we can present the deterministic equivalent of the stochastic formulation (1.7).

$$\begin{aligned} \min \quad & \mathbf{c}^T\mathbf{x} + \sum_{s \in S} P_s (\mathbf{q}^s)^T \mathbf{y}^s \\ \text{s.t.} \quad & \mathbf{x} \in X,\ \mathbf{y}^s \in Y^s;\ s \in S. \end{aligned} \tag{1.9}$$

Model (1.9) is also known as the wait-and-see model (e.g., Birge and Louveaux 2011; Kall and Mayer 2011). In contrast to the two-stage approach with the recourse model (1.7), in the wait-and-see approach both, the decisions on the first stage variables $\mathbf{x}$ and the second stage variables $\mathbf{y}^s$, are taken simultaneously only when the outcome of $\xi^s = (\mathbf{q}^s, Y^s)$ is known.

Risk-Averse Decision-Making

In the model proposed below, CVaR is represented by an auxiliary function (1.10) introduced by Rockafellar and Uryasev (2000), for a set of pre-defined scenarios $s \in S$ with corresponding probabilities, P_s. Using the wait-and-see approach, the stochastic formulation of the risk-averse decision-making problem aimed at CVaR of loss (cost) minimization, given confidence level α, can be written as

Minimize

$$CVaR = VaR + (1-\alpha)^{-1} \sum_{s \in S} P_s \tau_s \tag{1.10}$$

subject to

$$\tau_s \geq \mathbf{c}^T\mathbf{x} + (\mathbf{q}^s)^T\mathbf{y}^s - VaR;\ s \in S \tag{1.11}$$

$$\mathbf{x} \in X \tag{1.12}$$

$$\mathbf{y}^s \in Y^s;\ s \in S \tag{1.13}$$

$$\tau_s \geq 0;\ s \in S. \tag{1.14}$$

In the above formulation, constraints (1.11) compute the tail cost, τ_s, for scenario s and condition (1.14) indicates that the scenarios in which the loss exceeds VaR are considered only.

Mean-Risk Decision-Making

In the mean-risk formulation for the wait-and-see approach, λ is a non-negative trade-off coefficient representing the decision-maker risk preference. For a given confidence level α one can construct the mean-risk efficient frontier by the parameterization on λ the weighted-sum program presented below.

Minimize

$$\lambda(\mathbf{c}^T\mathbf{x} + \sum_{s\in S} P_s(\mathbf{q}^s)^T\mathbf{y}^s) + (1-\lambda)(VaR + (1-\alpha)^{-1}\sum_{s\in S} P_s\tau_s), \quad (1.15)$$

where $0 \leq \lambda \leq 1$
subject to

$$\tau_s \geq \mathbf{c}^T\mathbf{x} + (\mathbf{q}^s)^T\mathbf{y}^s - VaR;\ \ s \in S \quad (1.16)$$

$$\mathbf{x} \in X \quad (1.17)$$

$$\mathbf{y}^s \in Y^s;\ \ s \in S \quad (1.18)$$

$$\tau_s \geq 0;\ \ s \in S. \quad (1.19)$$

The resulting decision vector $\mathbf{x}$ is efficient in the mean-risk sense, i.e., it has the lowest possible CVaR for a given expected cost, and for a given CVaR it has the lowest possible expected cost.

Part I
Selection of Supply Portfolio

Chapter 2
Selection of Static Supply Portfolio

2.1 Introduction

In a customer-driven supply chain manufacturers should be prepared to produce different products to meet different customer needs. Each product is typically composed of many common and non-common (custom) parts that can be sourced from different suppliers with different supply capacity. An important issue is how to best allocate the orders for parts among various part suppliers to fulfill all customer orders for products and to achieve a high customer service level at a low cost and, in addition, to mitigate the impact of supply chain disruption risks. Supply chain management, in particular, deals with selection of supply portfolio, i.e., selection of suppliers and order quantity allocation under uncertain quality of supplied materials and reliability of on-time delivery. The decision maker needs to decide from which supplier to purchase parts required to meet customer demand. The decision is based on price, quality and reliability criteria that may conflict each other. For example, the supplier offering the lowest price may not have the best quality or the supplier with the best quality may not deliver on time. In stochastic supply settings, supplier selection allows the producer do decide whether it should cooperate with a low cost, yet risky suppliers over more expensive but possibly more reliable suppliers. A common risk-neutral objective of minimizing expected cost or maximizing expected service level is therefore influenced by uncertainty and risk. As a result, new non-risk-neutral objectives of minimizing and maximizing the number of outcomes that could occur above an acceptable cost level and below an acceptable service level, respectively, are observed in practice. Furthermore, to reduce the fixed ordering costs of creating contracts and maintaining relationships with suppliers, the number of suppliers and the total number of orders should be minimized. On the other hand, however, the selection of more suppliers may divert the risk of unreliable supplies. In global supply chains a multi-regional suppliers base is a frequent solution, where suppliers from different geographic regions are selected. Then, in addition to independent local disruptions (i.e., equipment breakdown, fires, etc.) that are uniquely associated with a

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particular supplier, the supplies of parts are also subject to regional disruptions (e.g., floods, hurricanes, earthquakes, economic crisis, etc.) simultaneously of all suppliers in the same region.

This chapter deals with selection of a static supply portfolio under disruption risks, i.e., for determining a single-period supply portfolio. The proposed portfolio approach allows the two popular in financial engineering percentile measures of risk, Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) to be applied for managing the risk of supply disruptions. For a finite number of scenarios, CVaR allows the evaluation of worst-case costs (or worst-case service level) and shaping of the resulting cost (service level) distribution through optimal supplier selection and order allocation decisions, i.e., the selection of optimal supply portfolio. Since common parts can be efficiently managed by material requirement planning methods, the focus in this chapter is on supplies of custom parts that can be critical in a make-to-order manufacturing.

The following SMIP models are presented in this chapter:

SP_E(c) for risk-neutral selection of supply portfolio to minimize expected cost;
SP_E(sl) for risk-neutral selection of supply portfolio to maximize expected service level;
SP_CV(c) for risk-averse selection of supply portfolio to minimize CVaR of cost;
SP_CV(sl) for risk-averse selection of supply portfolio to maximize CVaR of service level;
SP_ECV(c) for mean-risk selection of supply portfolio to optimize trade-off between expected cost and CVaR of cost;
SP_ECV(sl) for mean-risk selection of supply portfolio to optimize trade-off between expected service level and CVaR of service level.

In the computational experiments described in Sect. 2.6, single-region and multi-region sourcing subject to local and regional disruption risks are illustrated with numerical examples.

In the next chapter the portfolio approach will be to enhanced for a multi-period supplier selection and order allocation in the presence of supply chain disruption and delay risks, where in the scenario analysis the low probability and high impact supply disruptions are combined with the high probability and low impact supply delays. Unlike for a single-period problem considered in this chapter, in a multi-period setting the decision maker needs to decide from which supplier and when to purchase custom parts required for each customer order to meet customer requested due dates at a low cost and a high customer service level and to mitigate the impact of disruption and delay risks.

2.2 Problem Description

In the supply chain under consideration various types of products are assembled by a single producer to satisfy customer orders, using custom parts purchased from multiple suppliers (for notation used, see Table 2.1). Each supplier can provide the producer with custom parts for all customer orders. However, the suppliers have different limited capacity and, in addition, differ in price and quality of offered parts and in reliability of delivery of parts. Let $I = \{1, \ldots, \bar{I}\}$ be the set of $\bar{I}$ suppliers and $J = \{1, \ldots, \bar{J}\}$ the set of $\bar{J}$ customer orders for the products, known ahead of time.

Table 2.1 Notation: static supply portfolio

	Indices
i	= supplier, $i \in I$
j	= customer order, $j \in J$
r	= geographic region, $r \in R$
s	= disruption scenario, $s \in S$
	Input Parameters
c_i	= capacity of supplier i
d_j	= demand for parts required for customer order j
D	$= \sum_{j \in J} d_j$ - total demand for parts
e_i	= cost of ordering parts from supplier i
h_j	= per unit penalty cost of unfulfilled customer order j
o_{ij}	= unit price of parts for customer order j purchased from supplier i
p_i	= local disruption probability for supplier i
p^r	= regional disruption probability for all suppliers in region r
α	= confidence level
ρ_i	= expected defect rate of supplier i

Each order $j \in J$ is described by the quantity d_j of required custom parts. Denote by c_i the capacity of supplier $i \in I$, by e_i the cost of ordering parts from supplier $i \in I$, and by o_{ij} the unit purchasing price of parts for customer order $j \in J$ from supplier $i \in I$.

The ordered parts are dispatched to the producer after the completion time of their manufacturing. For each supplier, however, the quality of delivered part may vary randomly. When the suppliers are selected the risk of defective parts can be considered using past observations. Since quality of parts vary among different suppliers, a different average defect rate can be associated with each supply portfolio. Let ρ_i be the expected defect rate of supplier i.

The suppliers are assumed to be located in $\bar{R}$ disjoint geographic regions. Denote by $I^r \subseteq I$ the subset of suppliers in region $r \in R = \{1, \ldots, \bar{R}\}$, where $\bigcup_{r \in R} I^r = I$. The supplies of parts are subject to random local disruptions that are uniquely associated with a particular supplier, which may arise from equipment breakdowns, local labor strike, fires, etc. Denote by p_i the local disruption probability for supplier i, i.e., the parts ordered from supplier i are delivered without disruptions with probability $(1 - p_i)$, or not at all with probability p_i. In addition to independent local disruptions of each supplier individually, the supplies of parts are also subject to regional correlated disruption of all suppliers in the same region simultaneously, with probability p^r for region $r \in R$.

Denote by π_i the disruption probability of every supplier $i \in I^r, r \in R$

$$\pi_i = p^r + (1 - p^r)p_i; \;\; i \in I^r, r \in R. \tag{2.1}$$

Let $S = \{1, \ldots, \overline{S}\}$ be the index set of $\overline{S} = 2^{\overline{I}}$ disruption scenarios, where each scenario $s \in S$ defines a subset $I_s \subset I$ of non-disrupted suppliers. The supplies from every supplier, $i \in I \setminus I_s$, can be independently disrupted either by a local or by a regional disaster event. The probability P_s of each disruption scenario $s \in S$ is a product over all regions $r \in R$ of probabilities P_s^r of realizing disruption scenario s for suppliers in I^r,

$$P_s = \prod_{r \in R} P_s^r, \tag{2.2}$$

where P_s^r is (cf. Sect. 1.3)

$$P_s^r = \begin{cases} (1 - p^r) \prod_{i \in I^r \bigcap I_s} (1 - p_i) \prod_{i \in I^r \setminus I_s} p_i & \text{if } \; I^r \bigcap I_s \neq \emptyset \\ p^r + (1 - p^r) \prod_{i \in I^r} p_i & \text{if } \; I^r \bigcap I_s = \emptyset. \end{cases} \tag{2.3}$$

The producer does not need to pay for ordered and defective or undelivered parts. However, the producer may be charged with a much higher cost of unfulfilled customer orders for products, caused by the shortage of parts, undelivered due to supply disruptions. Let h_j be the per unit penalty cost of unfulfilled customer order j.

The decision maker needs to decide from which suppliers to purchase custom parts required for each customer order to achieve a minimum cost of ordering, purchasing and shortages and to mitigate the impact of disruption risks by minimizing the potential worst-case cost or maximizing the potential worst-case service level.

2.3 Models for Risk-Neutral Decision-Making

In this section two SMIP models **SP_E(c)** and **SP_E(sl)** are proposed for risk-neutral selection of a static supply portfolio, i.e., for determining a single-period supply portfolio to minimize expected cost and maximize expected service level, respectively. The static supply portfolio is defined below, (for definition of problem variables, see Table 2.2).

$$(V_1, \ldots, V_{\overline{I}}),$$

where

$$\sum_{i \in I} V_i = 1$$

and $0 \leq V_i \leq 1$ is the fraction of the total demand for parts ordered from supplier i, and V_i is determined by the custom parts allocation variables v_{ij}

Table 2.2 Variables: static supply portfolio

		First stage variables
u_i	$=$	1, if an order for parts is placed on supplier i; otherwise $u_i = 0$ (supplier selection)
v_{ij}	$=$	the fraction of demand for parts required for customer order j ordered from supplier i (allocation of demand for custom parts)
		Auxiliary variables
V_i	$=$	the fraction of total demand for parts allocated to supplier i (supply portfolio: allocation of total demand for parts)
VaR^c		Cost-at-Risk, the targeted cost such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is below VaR^c
VaR^{sl}		Service-at-Risk, the targeted service level such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is above VaR^{sl}
$\mathscr{C}_s$	$\geq$	0, the tail cost for scenario s, i.e., the amount by which costs in scenario s exceed VaR^c
$\mathscr{S}_s$	$\geq$	0, the tail service level for scenario s, i.e., the amount by which VaR^{sl} exceeds service level in scenario s

$$V_i = \sum_{j \in J} d_j v_{ij}/D; \;\; i \in I. \tag{2.4}$$

When deciding on a static supply portfolio it is assumed that the orders for all parts are simultaneously placed on selected suppliers (e.g., at time 0), and each supplier delivers all the ordered parts at the earliest possible delivery date. Therefore, the allocation of orders for parts among the suppliers is not combined with the allocation of orders among the planning periods. Nevertheless, the static portfolio should be checked against the risk of supply disruptions across all potential disruption scenarios.

Notice that Table 2.2 does not explicitly define the second stage variables for the SMIP problem considered. The seconds stage variables are simply demand allocation variables for realized disruption scenarios s, $\tilde{v}_{ij}^s;\;\; i \in I, j \in J, s \in S$, defined as follows

$$\tilde{v}_{ij}^s = \begin{cases} v_{ij} & \text{if} \;\; i \in I_s, j \in J, s \in S \\ 0 & \text{if} \;\; i \notin I_s, j \in J, s \in S. \end{cases}$$

In view of the above definition, an explicit introduction of the second stage variables $\tilde{v}_{ij}^s$ into the SMIP model formulations is not required.

In a risk-neutral operating conditions the overall quality of the supply portfolio can be measured by the expected cost per part, E^c, (2.5), or expected service level E^{sl}, (2.6).

$$E^c = \sum_{i \in I} e_i u_i / D + \sum_{i \in I} \sum_{j \in J} o_{ij} d_j v_{ij} / D + \sum_{s \in S} \sum_{i \notin I_s} \sum_{j \in J} P_s (h_j - o_{ij}) d_j v_{ij} / D \tag{2.5}$$

$$E^{sl} = \sum_{s \in S} \sum_{i \in I_s} \sum_{j \in J} P_s d_j v_{ij} / D \tag{2.6}$$

The expected cost E^c includes, cost of ordering,
$\sum_{i \in I} e_i u_i / D$,
cost of purchasing non defective parts,
$\sum_{i \in I} \sum_{j \in J} o_{ij} d_j v_{ij} / D$,
and cost of shortage of parts due to supply disruptions (cost of unfulfilled customer orders less cost of non delivered parts),
$\sum_{s \in S} \sum_{i \notin I_s} \sum_{j \in J} P_s (h_j - o_{ij}) d_j v_{ij} / D$.
The purchase orders for parts are assumed to be inflated by the reject rates ρ_i of defective parts, i.e., are equal to $(1 + \rho_i) d_j v_{ij}$ for all $i \in I, j \in J$. However, since the producer does not need to pay for ordered and defective parts in the amount of $\rho_i d_j v_{ij}$, the corresponding purchasing cost per part for delivered parts is simply given by $\sum_{i \in I} \sum_{j \in J} o_{ij} d_j v_{ij} / D - \sum_{s \in S} \sum_{i \notin I_s} \sum_{j \in J} P_s o_{ij} d_j v_{ij} / D$.

The expected cost per part, E^c, (2.5), can also be written as follows

$$E^c = \sum_{i \in I} e_i u_i / D + \sum_{s \in S} \sum_{i \in I_s} \sum_{j \in J} P_s o_{ij} d_j v_{ij} / D + \sum_{s \in S} \sum_{i \notin I_s} \sum_{j \in J} P_s h_j d_j v_{ij} / D, \tag{2.7}$$

where $\sum_{s \in S} \sum_{i \in I_s} \sum_{j \in J} P_s o_{ij} d_j v_{ij} / D$ is the expected purchasing cost per part for delivered parts.

The expected service level, E^{sl}, (2.6), is a surrogate measure of expected customer demand fulfillment rate and represents the expected fraction of fulfilled demand for required parts.

The SMIP models **SP_E(c)** and **SP_E(sl)** are formulated below. The supply portfolio will be optimized by minimizing expected cost per part, E^c, (2.5) or by maximizing expected service level, E^{sl}, (2.6).

SP_E(c): *Selection of risk-neutral* **S***upply* **P***ortfolio to minimize expected cost*

Minimize (2.5)

subject to

1. Supply portfolio selection constraints:

- the total demand for parts required for each customer order must be fully allocated among suppliers,

- for each selected supplier the total quantity of ordered parts cannot exceed the supplier capacity,
- parts cannot be ordered from non-selected suppliers,
- at least one customer order should be assigned to each selected supplier,

$$\sum_{i \in I} v_{ij} = 1; \ j \in J \tag{2.8}$$

$$\sum_{j \in J} (1 + \rho_i) d_j v_{ij} \leq c_i u_i; \ i \in I \tag{2.9}$$

$$v_{ij} \leq u_i; \ i \in I, j \in J \tag{2.10}$$

$$\sum_{j \in J} v_{ij} \geq u_i; \ i \in I \tag{2.11}$$

2. Non-negativity and integrality conditions

$$u_i \in \{0, 1\}; \ i \in I \tag{2.12}$$

$$v_{ij} \in [0, 1]; \ i \in I, j \in J. \tag{2.13}$$

Notice that if $h_j = h \forall j \in J$, i.e., per unit penalty cost of unfulfilled customer order is identical for all orders j, then E^c, (2.7), can be expressed by the following simplified formula

$$E^c = \sum_{i \in I} e_i u_i / D + \sum_{s \in S} \sum_{i \in I_s} \sum_{j \in J} P_s o_{ij} d_j v_{ij} / D + h(1 - E^{sl}), \tag{2.14}$$

where E^{sl} is the expected service level (2.6).

SP_E(sl): *Selection of risk-neutral supply portfolio to maximize expected service level*

Maximize (2.6)
subject to (2.8)–(2.13).

If total available capacity of all suppliers is less than total demand for required parts, i.e., $\sum_{i \in I} c_i/(1 + \rho_i) \leq \sum_{j \in J} d_j$, then the demand allocation equality constraints (2.8) should be replaced by inequalities

$$\sum_{i \in I} v_{ij} \leq 1; \ j \in J; \tag{2.15}$$

otherwise no feasible solution exists.

A simple upper bound on the expected service level, E^{sl}, (2.6), is derived below.

Proposition 2.1

$$E^{sl} \leq \min\{1, \sum_{r\in R}\sum_{i\in I^r}(1-p^r)(1-p_i)c_i/(1+\rho_i)D\}. \tag{2.16}$$

Proof The supply portfolio selection constraints (2.9) imply that

$$\begin{aligned}
\sum_{s\in S}\sum_{i\in I_s}\sum_{j\in J} P_s d_j v_{ij}/D \leq \\
\sum_{r\in R}\sum_{i\in I^r}\sum_{j\in J}(1-p^r)(1-p_i)d_j v_{ij}/D \leq \\
\sum_{r\in R}\sum_{i\in I^r}(1-p^r)(1-p_i)c_i u_i/(1+\rho_i)D \leq \\
\sum_{r\in R}\sum_{i\in I^r}(1-p^r)(1-p_i)c_i/(1+\rho_i)D,
\end{aligned}$$

where $(1-p^r)(1-p_i) = 1-\pi_i$, (2.1), is non-disruption probability of supplier $i \in I^r$.

Since E^{sl} cannot be greater than 1, its upper bound is 1, if $\sum_{r\in R}\sum_{i\in I^r}(1-p^r)(1-p_i)c_i/(1+\rho_i)D > 1$.

In the proposed models the parts required for each customer order are assumed to be partially provided by one or more suppliers and the customer order allocation variable v_{ij} represents the fraction of all parts required for order j provided by supplier i. In some practical cases, all custom parts of the same type that are required for a customer order are purchased from a single supplier. Then, the corresponding continuous allocation variable v_{ij} should be redefined as a binary assignment variable denoting whether or not all parts required for order j are provided by supplier i. If all v_{ij} are defined to be binary variables, then **SP_E(c)** and **SP_E(sl)** become pure stochastic binary programs.

2.4 Models for Risk-Averse Decision-Making

In the risk-averse selection of supply portfolio under disruption risks, the confidence level α is fixed by the decision maker to control the risk of losses due to supply disruptions. We assume that the decision maker is willing to accept only portfolios for which the total probability of scenarios with costs greater than VaR^c or with service level lower than VaR^{sl} is not greater than $1-\alpha$. Furthermore, a risk aversive decision maker wants to minimize the expected worst-case costs exceeding VaR^c or to maximize the expected worst-case service level below VaR^{sl}.

Define by $\mathscr{C}_s$ the tail cost for scenario s, where tail cost is defined as the amount by which costs in scenario s exceed VaR^c. In a similar way, define by $\mathscr{S}_s$ the tail service level for scenario s, where tail service level is defined as the nonnegative amount by which VaR^{sl} exceeds service level in scenario s.

The portfolio will be optimized by calculating VaR^c and minimizing CVaR^c simultaneously or by calculating VaR^{sl} and maximizing CVaR^{sl}, respectively. By measuring CVaR^c or CVaR^{sl}, the magnitude of the tail costs or the tail service level is considered to achieve a more accurate estimate of the risks of minimizing cost or maximizing service level, respectively. When using CVaR^c to minimize worst-case costs and CVaR^{sl} to maximize worst-case service level, CVaR^c is always not less than VaR^c and CVaR^{sl} is always not greater than VaR^{sl}, respectively.

In the proposed model CVaR is represented by an auxiliary function (2.17) and (2.20) introduced by Rockafellar and Uryasev (2000). The SMIP models **SP_CV(c)** and **SP_CV(sl)** for selection of risk-averse supply portfolio to reduce the risk of high costs and the risk of low service level, respectively, is formulated below.

SP_CV(c): *Selection of risk-averse supply portfolio to minimize CVaR of cost*

Minimize

$$CVaR^c = VaR^c + (1-\alpha)^{-1} \sum_{s \in S} P_s \mathscr{C}_s \tag{2.17}$$

subject to

1. Supply portfolio selection constraints: (2.8)–(2.11)

2. Risk constraints:

- the tail cost for scenario s is defined as the nonnegative amount by which cost per part in scenario s exceeds VaR^c,

$$\mathscr{C}_s \geq \sum_{i \in I} e_i u_i / D + \sum_{i \in I} \sum_{j \in J} o_{ij} d_j v_{ij} / D + \sum_{i \notin I_s} \sum_{j \in J} (h_j - o_{ij}) d_j v_{ij} / D - VaR^c; \ \ s \in S \tag{2.18}$$

3. Non-negativity and integrality conditions: (2.12), (2.13) and

$$\mathscr{C}_s \geq 0; \ \ s \in S. \tag{2.19}$$

SP_CV(sl): *Selection of risk-averse supply portfolio to maximize CVaR of service level*

Maximize

$$CVaR^{sl} = VaR^{sl} - (1-\alpha)^{-1} \sum_{s \in S} P_s \mathscr{S}_s \tag{2.20}$$

subject to

1. Supply portfolio selection constraints: (2.8)–(2.11)

2. Risk constraints:

- the tail service level for scenario *s* is defined as the nonnegative amount by which VaR^{sl} exceeds service level in scenario *s*,

$$\mathscr{S}_s \geq VaR^{sl} - \sum_{i \in I_s} \sum_{j \in J} d_j v_{ij} / D; \;\; s \in S \tag{2.21}$$

3. Non-negativity and integrality conditions: (2.12), (2.13) and

$$\mathscr{S}_s \geq 0; \;\; s \in S. \tag{2.22}$$

Note that as $\mathscr{C}_s$ and $\mathscr{S}_s$ are constrained of being positive, the model **SP_CV(c)** tries to decrease VaR^c and the model **SP_CV(sl)** tries to increase VaR^{sl}, respectively. Hence they positively impact the objective functions. However, large reduction in VaR^c and large increase in VaR^{sl} may result in more scenarios with positive tail costs and with positive tail service levels, respectively.

If for some customer order *j* all required parts must be supplied by a single supplier, then the corresponding nonnegative allocation variable v_{ij} should be redefined as a binary assignment variable denoting whether or not all parts required for order *j* are provided by supplier *i*, similarly as for the risk-neutral models **SP_E(c)** and **SP_E(sl)**.

2.5 Models for Mean-Risk Decision-Making

In the single objective approach the supply portfolio is selected by minimizing either the expected cost per part, E^c, (2.5), the expected service level, E^{sl}, (2.6), the expected worst-case cost per part, $CVaR^c$, (2.17) or the expected worst-case service level, $CVaR^{sl}$, (2.20). In this section the two cost functions and the two service level functions are considered simultaneously, and a bi-objective selection of supply portfolio is presented aimed at minimizing both objective functions to balance expected costs or expected service level with the risk tolerance. This trade-off model is known as the mean-risk model (e.g., Ogryczak and Ruszczynski 2002), formulated as the optimization of a composite objective consisting of the expected cost (service level) and the CVaR as a risk measure.

The nondominated solution set of the bi-objective supply portfolio can be found by the parameterization on λ the weighted-sum programs **SP_ECV(c)** and **SP_ECV(sl)** presented below. The mean-risk program **SP_ECV(c)** is based on model **SP_CV(c)** with the addition of objective (2.5) of model **SP_E(c)**. Similarly, the mean-risk program **SP_ECV(sl)** is based on model **SP_CV(sl)** with the addition of objective (2.6) of model **SP_E(sl)**.

SP_ECV(c): *Selection of mean-risk supply portfolio to minimize weighted sum of expected cost and CVaR of cost*

Minimize

$$\lambda E^c + (1 - \lambda) CVaR^c \tag{2.23}$$

where $0 \leq \lambda \leq 1$,
subject to (2.5), (2.8)–(2.13), (2.17)–(2.19).

SP_ECV(sl): *Selection of mean-risk supply portfolio to maximize weighted sum of expected service level and CVaR of service level*

Maximize

$$\lambda E^{sl} + (1 - \lambda) CVaR^{sl} \tag{2.24}$$

where $0 \leq \lambda \leq 1$,
subject to (2.6), (2.8)–(2.13), (2.20)–(2.22).

Steuer (1996) proved that for mixed integer programs, there may be portions of the nondominated set (nearby weakly nondominated solution) that the above approach is unable to compute, even if the complete parameterization on λ is attempted.

2.6 Computational Examples

In this section some computational examples are presented to illustrate possible applications of the proposed SMIP approach for selection of static supply portfolio under disruption risks. First, a single-region sourcing case will be illustrated, where all suppliers are located in a single geographic region, and then examples of multi-region sourcing with subsets of suppliers in different geographic regions, each subject to different regional disruption risks. For the single-region sourcing, minimization of cost is considered only, whereas for the multi-region sourcing, both minimization of cost and maximization of service level are considered.

2.6.1 Single-Region Sourcing

In this subsection all suppliers are assumed to be located in the same geographic region, and hence the regional disruption can be called a global disruption. Denote by, p^*, the global disruption probability for the entire region. Now, the probability, P_s (2.3), of each disruption scenario s, can be calculated using the following formula

$$P_s = \begin{cases} (1-p^*)\hat{P}_s & \text{if } I_s \neq \emptyset \\ p^* + (1-p^*)\prod_{i\in I} p_i & \text{if } I_s = \emptyset, \end{cases} \tag{2.25}$$

where $\hat{P}_s$ is the probability of disruption scenario s in the presence of independent local disruptive events only

$$\hat{P}_s = \prod_{i\in I_s}(1-p_i)\cdot\prod_{i\notin I_s} p_i. \tag{2.26}$$

If the probability of regional disruption $p^* = 0$, then the probability P_s reduces to $\hat{P}_s$ for independent local disruptive events.

The following parameters have been used for the example problems:

- $\bar{I}$, the number of suppliers, was equal to 7, 10 or 14 and the corresponding number $\bar{S} = 2^{\bar{I}}$ of disruption scenarios, was equal to 128, 1024 or 16384, respectively;
- $\bar{J}$, the number of customer orders, was equal to 50;
- d_j, the numbers of required parts for each customer order, were integers uniformly distributed over [100, 500], i.e., generated from a U[100;500] distribution;
- c_i, the capacity of each supplier i, was equal to $\lceil 2\sum_{j\in J} d_j/\bar{I}\rceil$ ($\lceil\cdot\rceil$ denotes the smallest integer not less than $\cdot$), i.e., the total capacity of all suppliers was equal to the double total demand for parts;
- e_i, the cost of ordering parts from supplier i, was equal to 500 for each supplier i;
- h_j, the per unit shortage cost for customer order j, was equal to 100 for all customer orders j;
- ρ_i, the expected defect rate of each supplier i, was exponentially distributed, ranging from 0.0003 to 0.03;
- o_{ij}, the unit price of parts required for each customer order j purchased from each supplier i, was uniformly distributed over [10,15] (i.e., drawn from U[10;15]) and reduced by the factor $(1-\rho_i)$ to get a lower price for parts from the suppliers with a higher defect rate;
- p_i, the local disruption probability was uniformly distributed over [0,0.06], i.e., the disruption probabilities were drawn independently from U[0;0.06];
- p^*, the global disruption probability was equal to 0.01;
- α, the confidence level, was equal to 0.50, 0.75, 0.90, 0.95 or 0.99.

For the example problems, the total demand for parts is $D = \sum_{j\in J} d_j = 14750$ parts. Solution results for the risk-neutral model **SP_E(c)** are shown in Table 2.3, and for the risk-averse model **SP_CV(c)** with different confidence levels, in Table 2.4. The size of the mixed integer programs for different number $\bar{I}$ of suppliers is represented by the total number of variables, Var., number of binary variables, Bin., number of constraints, Cons, and number of nonzero coefficients in the constraint matrix, Nonz. Table 2.4 also presents the probability $1 - F(VaR^c)$ of outcomes with worst-case cost above VaR^c. Note that the number of variables and constraints in the mixed integer program **SP_CV** grows exponentially in the number $\bar{I}$ of suppliers. The table demonstrate that the number of selected suppliers increases with the confidence level α, which indicates that the impact of disruption risks is mitigated by diversification of the supply portfolio. Note that VaR^c becomes smaller than expected cost when $\alpha = 0.50$ and $\alpha = 0.75$.

Table 2.3 Risk-neutral solutions for model **SP_E(c)**: single-region sourcing

No. of Suppliers	Expected Cost	No. of Selected Suppliers
7	12.38	4
Var. = 364, Bin. = 7, Cons. = 72, Nonz. = 1428 $^{(a)}$		
10	13.18	5
Var. = 520, Bin. = 10, Cons. = 81, Nonz. = 2040 $^{(a)}$		
14	12.03	7
Var. = 728, Bin. = 14, Cons. = 93, Nonz. = 2856 $^{(a)}$		

$^{(a)}$ Var. = no. of variables, Bin. = no. of binary variables, Cons. = no. of constraints, Nonz. = no. of nonzero coefficients

The optimal risk-neutral supply portfolio for model **SP_E(c)** and 10 suppliers is shown in Fig. 2.1. In addition, the figure presents for each supplier i the expected defect rate ρ_i, the average unit price $\sum_{j \in J} o_{ij}/\bar{J}$, and disruption probabilities, π_i, (2.1). For the optimal supply portfolio the total demand was equally allocated among five suppliers with the lowest disruption probabilities.

In the computational experiments the confidence level α is set at five levels of 0.5, 0.75, 0.90, 0.95, and 0.99, which means that focus is on minimizing the highest 50%, 25%, 10%, 5%, and 1% of all scenario outcomes, i.e., costs per part.

Figure 2.2 shows the probability mass functions and the cumulative distribution functions for the optimal risk-averse portfolios with different confidence levels for 10 suppliers. Figure 2.2 indicates that the mass function of cost per part is concentrated in a few points and the resulting cumulative distribution is a discontinues step function with jumps at those points. Such results are typical for the scenario-based optimization under uncertainty, where the probability measure is concentrated in finitely many points. The resulting discontinuity (vertical jumps) of the distribution function leads to probability intervals of confidence level α with the same VaR. The discrete distributions of cost per part for the optimal supply portfolios with four different confidence levels and the corresponding probabilities concentrated at each level of cost are presented also in Table 2.5. The table shows that the probabilities are concentrated at 6, 10, 10, 10 points, respectively for the confidence level $\alpha =$ 0.5, 0.9, 0.989, 0.99. In the examples, a large probability atom is concentrated at the highest cost. As a consequence, a slight increase of the confidence level from

Table 2.4 Risk-averse solutions: single-region sourcing

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **SP_CV(c)**: 7 suppliers					
Var. = 493, Bin. = 7, Cons. = 200, Nonz. = 47380 [a]					
CVaR^c	14.15	17.69	28.21	39.01	100.14
VaR^c	10.61	10.61	13.90	21.19	100.14
E^c	12.38	12.38	12.59	12.94	14.24
$1 - F(VaR^c)$	0.053	0.053	0.039	0.018	0
No. of suppliers selected	4	4	7	7	4
Model **SP_CV(c)**: 10 suppliers					
Var. = 1545, Bin. = 10, Cons. = 1115, Nonz. = 526338 [a]					
CVaR^c	16.02	21.69	31.67	42.05	100.17
VaR^c	10.35	10.35	20.29	23.19	100.17
E^c	13.18	13.18	14.06	13.96	15.89
$1 - F(VaR^c)$	0.113	0.113	0.041	0.032	0
No. of suppliers selected	5	5	9	10	5
Model **SP_CV(c)**: 14 suppliers					
Var. = 17113, Bin. = 14, Cons. = 16477, Nonz. = 11733800 [a]					
CVaR^c	13.60	16.70	25.59	35.94	100.24
VaR^c	10.51	10.51	13.75	16.89	100.24
E^c	12.05	12.05	12.50	12.43	15.50
$1 - F(VaR^c)$	0.115	0.097	0.058	0.029	0
No. of suppliers selected	7	7	13	11	7

[a] Var. = no. of variables, Bin. = no. of binary variables, Cons. = no. of constraints, Nonz. = no. of nonzero coefficients

$\alpha = 0.989$ to $\alpha = 0.99$ results in a significant change in VaR from 37.33 to 100.17, while only a slight increase of CVaR^c from 94.81 to 100.17 is observed. Moreover, the optimal portfolio has been changed significantly; for $\alpha = 0.989$ the total demand has been equally allocated among all ten suppliers ($V_i = 0.1$ forall i), whereas for $\alpha = 0.99$ among five suppliers only ($V_i = 0.2$ for $i = 1, 2, 6, 7, 8$). This degree of instability of the optimal supply portfolio due to the discontinuity in the distribution function may be distressing in practice, when a slightly higher confidence level is required. Despite the limited change in CVaR^c, the above results demonstrate that the well known misbehaviour in the dependence of VaR^c and optimal supply portfolio on the confidence level can as well be encountered when CVaR^c is applied as a risk measure.

The computational results indicate that the smaller is the number of concentration points and the greater are probability atoms concentrated at those points, the greater can be the positive difference $F(VaR^c) - \alpha$, i.e., the smaller than $1 - \alpha$ can be the probability of outcomes with cost higher than VaR^c. For example (see, Table 2.4), for $\bar{I} = 10$ and $\alpha = 0.5$, $VaR^c = 10.35$ and $F(VaR^c) = 0.88666 > 0.5$, which indicates a high concentration of probability measure at point 10.35 for the optimal supply portfolio. Actually, the probability that cost per part is 10.35 is 0.88666 (see, Table 2.5), which indicates that $VaR^c = 10.35$ is the lowest cost that may occur and

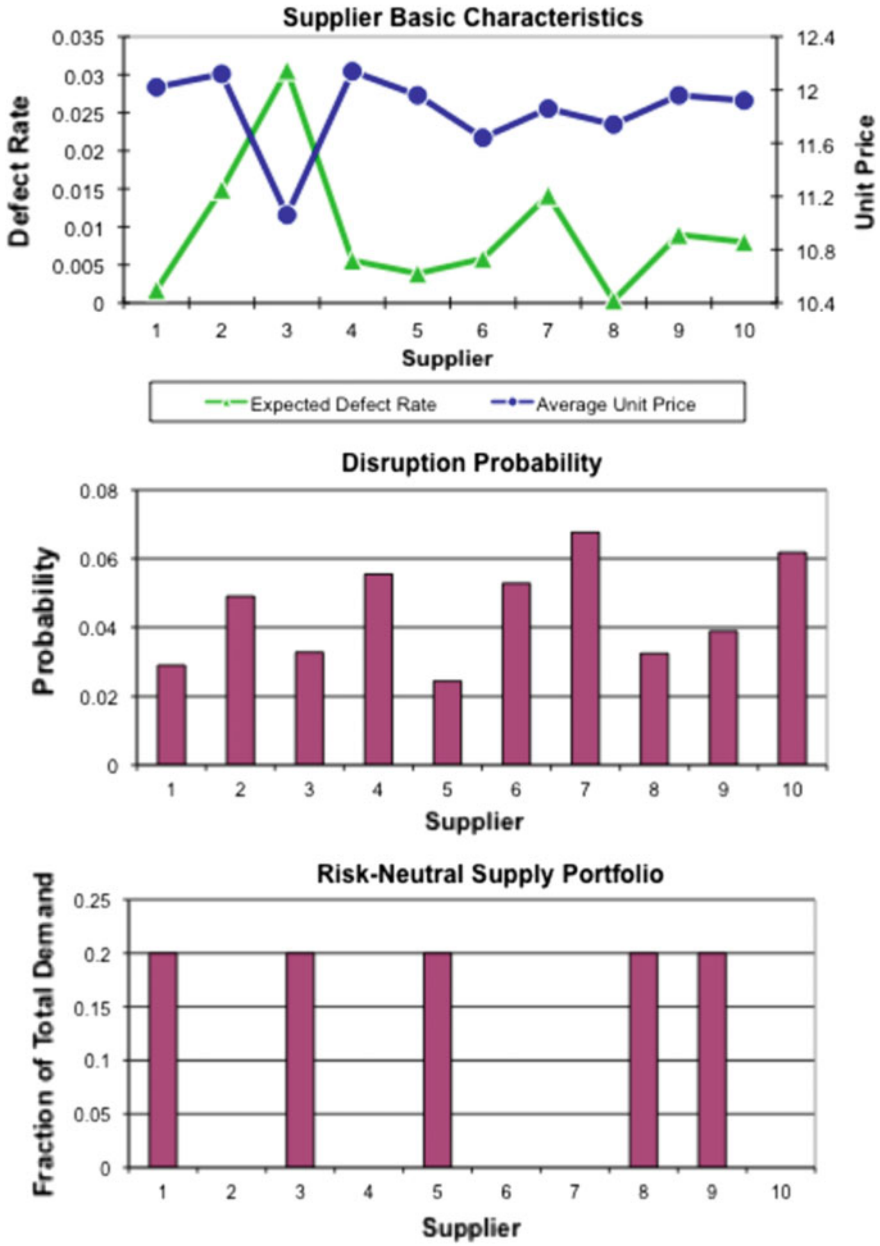

Fig. 2.1 Risk-neutral supply portfolio for model **SP_E(c)**: 10 suppliers

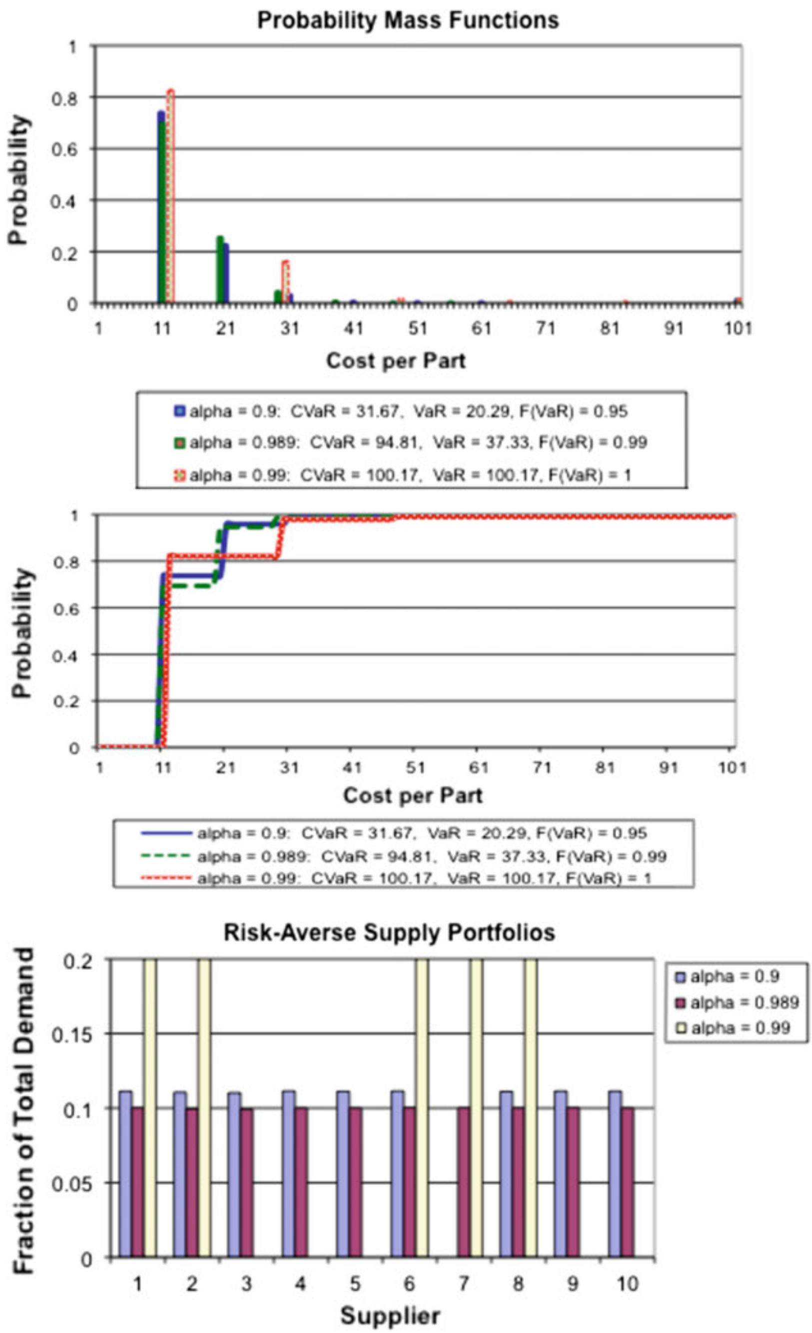

Fig. 2.2 Risk-averse supply portfolios and cost distributions for model **SP_CV(c)**: 10 suppliers

Table 2.5 Probability of cost per part for optimal risk-averse supply portfolios: 10 suppliers

Cost interval	$\alpha = 0.5$	$\alpha = 0.9$	$\alpha = 0.989$	$\alpha = 0.99$
[10, 11)	0.886661618	0.736768317	0.693881575	0
[11, 12)	0	0	0	0.821176973
[19, 20)	0	0	0.251829646	0
[20, 21)	0	0.221857039	0	0
[28, 29)	0.098887038	0	0.040311352	0
[29, 30)	0	0	0	0.156904596
[30, 31)	0	0.029090556	0	0
[37, 38)	0	0	0.003744839	0
[40, 41)	0	0.002178297	0	0
[46, 47)	0.004355651	0	0.000223405	0
[47, 48)	0	0	0	0.011507829
[50, 51)	0	0.000102579	0	0
[55, 56)	0	0	8.94e-06	0
[60, 61)	0	3.15e-06	0	0
[64, 65)	9.47e-05	0	2.43e-07	0.0004038
[70, 71)	0	6.30e-08	0	0
[73, 74)	0	0	4.41e-09	0
[80, 81)	0	7.91e-10	0	0
[82, 83)	1.01e-06	0	5.14e-11	6.76e-06
[90, 91)	0	5.66e-12	0	0
[91, 92)	0	0	3.46e-13	0
[100, 101)	0.010000004	0.01	0.01	0.010000043

that for the confidence level $\alpha = 0.5$, less than 11.33% of the cost outcomes are above VaR^c.

Moreover, if the highest cost probability is greater than $1 - \alpha$, then CVaR^c and VaR^c are identical and both equal to the highest cost. In the example for ten suppliers and $\alpha = 0.99$, the highest cost per part is 100.17 and the probability concentrated at 100.17 is $0.01000004 > 1 - \alpha$, then $VaR^c = 100.17$ is the highest cost per part that may occur and hence $CVaR^c = VaR^c = 100.17$ (see, Table 2.5 and Fig. 2.2). Similar results $CVaR^c = VaR^c = 100.14$ and $CVaR^c = VaR^c = 100.24$ have been obtained for $\alpha = 0.99$, respectively for seven and 14 suppliers (see, Table 2.4), which indicates that the corresponding probabilities of the highest cost per part are greater than $1 - \alpha = 0.01$.

If the probability measure is concentrated at the highest cost and is greater than $1 - \alpha$, so that CVaR^c and VaR^c are identical with the highest cost, then for a higher confidence level α, a smaller number of suppliers are selected, which indicates that diversification of the supply portfolio is not necessary any more. For instance, the optimal risk-averse supply portfolio selected for $\alpha = 0.99$ consists of five suppliers only, the same number as that for a much lower α (cf. Table 2.4, Fig. 2.2).

In the computational experiments the local disruption probabilities p_i were assumed to be very low and were drawn from U[0;0.06]. To study the effect of the increasing range of disruption probabilities, the probabilities have been drawn also from U[0;0.25], U[0;0.5] or U[0;1]. The effect of varying distribution of local disruption probabilities is illustrated in Fig. 2.3, where the optimal risk-averse supply portfolios are presented. Figure 2.3 indicates that for a greater range of disruption probabilities, the suppliers with the highest disruption probabilities are not selected. For example, for U[0;1] and $\alpha = 0.75$, suppliers $i = 4, 6, 7, 10$ with the four highest disruption rates were not selected, whereas for $\alpha = 0.985$, suppliers $i = 7, 10$ with the two highest disruption rates were not selected. Similar results are observed for the other distributions of disruption probability.

Table 2.6 Solutions results for model **SP_CV(c)** with binary assignment variables $v_{ij} \in \{0, 1\}$: 10 suppliers

Confidence level α	0.50	0.75	0.90	0.95	0.99
Var. = 1545, Bin. = 510, Cons. = 1105, Nonz. = 526328 [a]					
CVaR^c	16.08	21.80	31.75	42.08	100.20
VaR^c	10.36	10.40	20.16	23.21	100.20
E^c	13.22	13.25	14.07	13.94	15.74
$1 - F(VaR^c)$	0.155	0.155	0.094	0.053	0
No. of suppliers selected	6	6	9	10	6

[a] Var. = no. of variables, Bin. = no. of binary variables, Cons. = no. of constraints, Nonz. = no. of nonzero coefficients

Finally, Table 2.6 presents solution results for model **SP_CV(c)** applied to optimization of a single sourcing, with binary assignment variables v_{ij}, i.e., when for each customer order, the required parts must be provided by a single supplier only. Comparison of the results shown in Table 2.6 with the corresponding results presented in Table 2.4 for continuous allocation variables v_{ij}, indicates that in the former case both the expected cost per part and CVaR^c were slightly higher and, in addition, for a low α the number of selected suppliers was greater. Such results were expected, since model **SP_CV(c)** with the continuous allocation variables is a partial LP relaxation of that model with the binary assignment variables.

For the mean-risk approach, the subsets of nondominated solutions were computed by parameterization on $\lambda \in \{0.01, 0.10, 0.25, 0.50, 0.75, 0.90, 0.99\}$ the weighted-sum program **SP_ECV(c)**. The subset of nondominated solutions found for the selected seven levels of trade-off parameter λ is: $(E^c, CVaR^c) = (13.18, 36.32)$, (13.33, 33.97), (13.49, 32.65), (13.84, 31.84), (13.86, 31.82), (14.06, 31.67). The trade-off between the expected cost and the expected worst-case cost is clearly shown in Fig. 2.4, where the convex efficient front for the mean-risk model **SP_ECV(c)** with $\alpha = 0.9$ is presented. The results emphasize the effect of varying cost/risk preference of the decision maker.

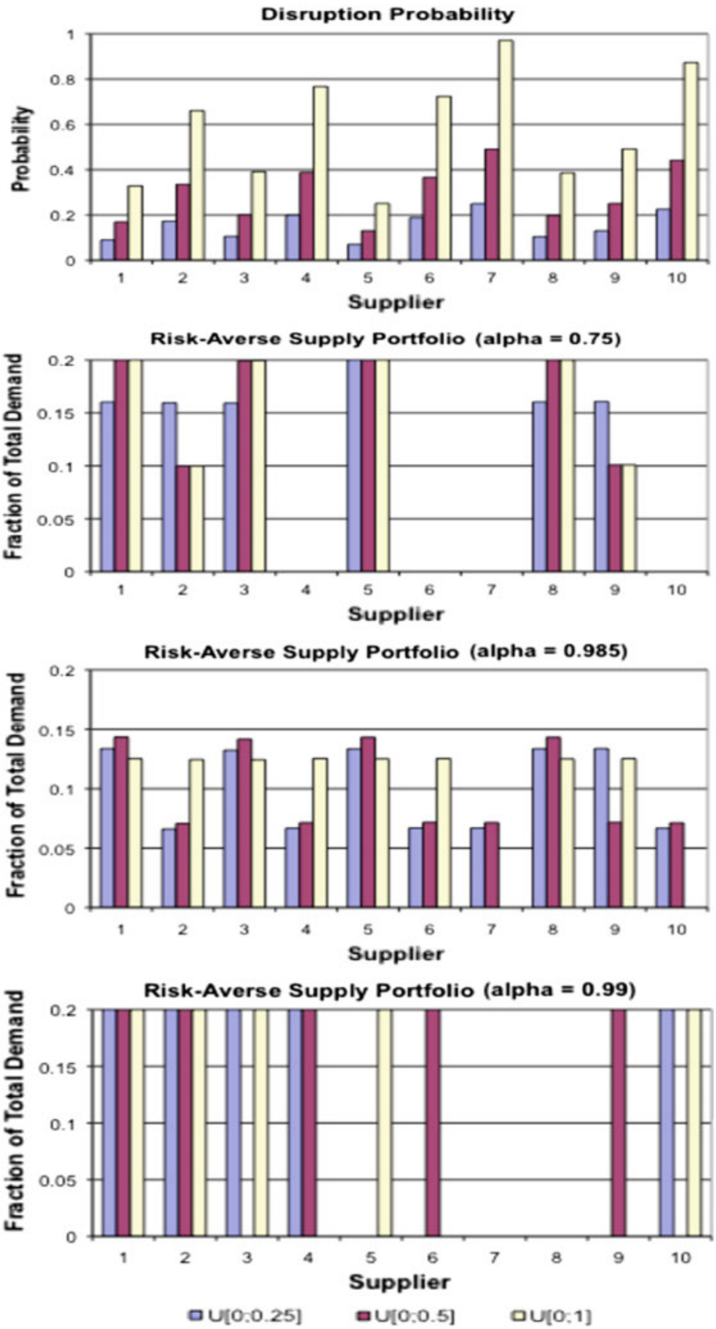

Fig. 2.3 Risk-averse supply portfolios for different local disruption probabilities for model **SP_CV(c)**: 10 suppliers

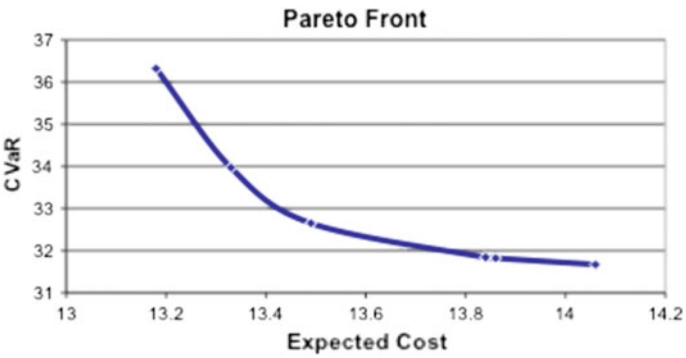

Fig. 2.4 Pareto front for mean-risk model **SP_ECV(c)**: 10 suppliers, $\alpha = 0.9$

Note that solutions to single objective models **SP_E(c)** and **SP_CV(c)** are equivalent to the nondominated solutions of the weighted-sum program **SP_ECV(c)** for $\lambda = 1$ and $\lambda = 0$, respectively.

The computational experiments were performed using the AMPL programming language and the CPLEX solver. The solver was capable of finding proven optimal solutions within CPU seconds for all examples.

2.6.2 *Multi-region Sourcing*

In this subsection the suppliers are assumed to be located in multiple geographic regions subject to different regional disruption risks. The following parameters used for the example problems are different from those in Sect. 2.6.1:

- $\overline{I}$, the number of suppliers, was equal to 10 and the corresponding number $\overline{S} = 2^{\overline{I}}$ of disruption scenarios, was equal to 1024;
- $\overline{R}$, the number of geographic regions, was equal to 3, and the subsets of suppliers were $I^1 = \{1, 2, 3\}$, $I^2 = \{4, 5, 6\}$ and $I^3 = \{7, 8, 9, 10\}$, respectively;
- $\overline{J}$, the number of customer orders, was equal to 25;
- d_j, the numbers of required parts for each customer order, were integers uniformly distributed over [1000, 15000] for all customer orders j. and the resulting total demand for parts was $D = 132500$;
- e_i, the cost of ordering parts, were integers in $\{5000, 6000, \ldots, 10000\}$, $\{10000, 11000, \ldots, 15000\}$ and $\{15000, 16000, \ldots, 30000\}$, respectively for suppliers $i \in I^1$, $i \in I^2$ and $i \in I^3$;
- h_j, the per unit shortage cost for each customer order j, was integer uniformly distributed over $[\max_{i\in I}(o_{ij}), 4\max_{i\in I}(o_{ij})]$, i.e., ranging from one to four times of maximum purchasing cost of required parts;

- o_{ij}, the unit price of parts for customer order j purchased from supplier i, was uniformly distributed over [13,15], [11,13] and [9,11], respectively for suppliers $i \in I^1$, $i \in I^2$ and $i \in I^3$;
- p_i, the local disruption probability was uniformly distributed over [0.005,0.01], [0.01,0.05] and [0.05;0.10], respectively for suppliers $i \in I^1$, $i \in I^2$ and $i \in I^3$, i.e., the disruption probabilities were drawn independently from U[0.005;0.01], U[0.01,0.05] and U[0.05;0.10], respectively;
- p^r, the regional disruption probability was 0.001, 0.005 and 0.01, respectively for region $r = 1$, $r = 2$ and $r = 3$;

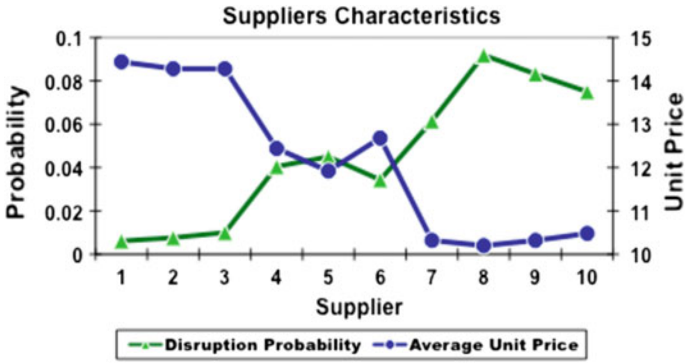

Fig. 2.5 Suppliers

Basic characteristics of suppliers: average unit price $\sum_{j \in J} o_{ij}/\bar{J}$, and disruption probability, π_i, (2.1), are presented in Fig. 2.5. Figure 2.5 indicates that the most reliable and most expensive are suppliers $i = 1, 2, 3$ in region $r = 1$, while suppliers $i = 7, 8, 9, 10$ in region $r = 3$ are most competitive and most unreliable. In particular, supplier $i = 8$ is the cheapest and most unreliable among all suppliers.

The solution results for the risk-averse models **SP_CV(c)** and **SP_CV(sl)** with different confidence levels are shown in Table 2.7. For both models the number of selected suppliers increases with the confidence level. While for model **SP_CV(c)** the cheapest, yet most unreliable supplier $i = 8$ was never selected, for model **SP_CV(sl)** all 10 suppliers are selected for $\alpha = 0.9,\ 0.95$ and $\alpha = 0.99$.

Figure 2.6 shows the optimal risk-averse supply portfolios for models **SP_CV(c)** and **SP_CV(sl)** and the three confidence levels, $\alpha = 0.75,\ 0.9,\ 0.99$. For both models and $\alpha = 0.75$ the most unreliable suppliers $i = 7, 8, 9, 10$ in region $r = 3$ are not selected and for all confidence levels most demand for parts is allocated among the three most reliable, yet most expensive suppliers, $i = 1, 2, 3$, in region $r = 1$, in particular for $\alpha = 0.75, 0.9$. Similar properties of the risk-averse supply portfolios were observed in case of single-region sourcing (see, Fig. 2.3).

Table 2.7 Risk-averse solutions: multi-region sourcing

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **SP_CV(c)**					
Var. = 1285, Bin. = 10, Cons. = 1319, Nonz. = 269558 [(a)]					
CVaR^c	15.35	17.15	20.49	23.90	30.43
VaR^c	13.29	13.68	16.44	17.19	26.21
$1 - F(VaR^c)$	0.149	0.125	0.055	0.036	0.005
E^c	14.32	14.55	14.59	14.51	14.61
E^{sl} [(b)]	97.51	97.98	97.30	97.56	96.15
Suppliers Selected(% of total demand)	1(20)	1(20)	1(20)	1(20)	1(19)
	2(19)	2(19)	2(20)	2(19)	2(13)
	3(20)	3(20)	3(20)	3(20)	3(12)
		4(14)	4(8)	4(8)	4(6)
	5(8)	5(7)	5(6)	5(9)	5(6)
	6(13)	6(20)	6(6)	6(8)	6(8)
	7(20)		7(12)	7(16)	7(9)
			9(8)		9(12)
					10(15)
Model **SP_CV(sl)**					
Var. = 1285, Bin. = 10, Cons. = 1319, Nonz. = 131318 [(a)]					
CVaR^{sl}%	96.01	92.02	85.29	80.12	69.82
VaR^{sl}%	100	100	91.87	87.50	77.77
$1 - F(VaR^{sl})$	0.125	0.125	0.064	0.033	0.007
E^{sl} [(b)]	98.00	98.00	97.29	97.43	97.04
E^c	15.56	15.48	15.86	15.85	15.61
Suppliers Selected(% of total demand)	1(20)	1(20)	1(20)	1(20)	1(20)
	2(19)	2(19)	2(20)	2(20)	2(18)
	3(20)	3(20)	3(20)	3(20)	3(18)
	4(20)	4(20)	4(8)	4(8)	4(10)
	5(1)	5(1)	5(8)	5(9)	5(4)
	6(20)	6(20)	6(8)	6(9)	6(9)
			7(4)	7(3)	7(9)
			8(4)	8(3)	8(4)
			9(4)	9(3)	9(4)
			10(4)	10(3)	10(4)

[(a)] Var. = no. of variables, Bin. = no. of binary variables, Cons. = no. of constraints, Nonz. = no. of nonzero coefficients
[(b)] $(\sum_{s \in S} \sum_{i \in I_s} \sum_{j \in J} P_s d_j v_{ij} / D)100\%$

The computational experiments indicate that

- *probability of disruption a supply is a key determinant in the decision of allocation of demand among the suppliers. In a risk averse model, an order for delivery of* *parts from a particular supplier is selected more on the supply non-disruption likelihood than on its purchasing cost or defect rate.*

- *The suppliers associated with the highest disruption rates are rarely selected.*

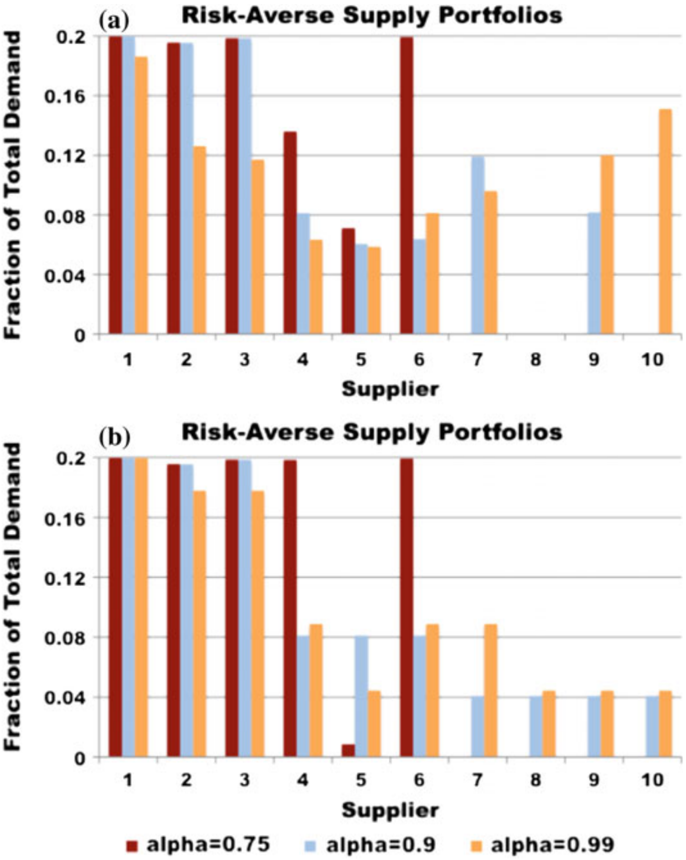

Fig. 2.6 Risk-averse supply portfolios: **a** model **SP_CV(c)**, **b** model **SP_CV(sl)**

- *In most cases the number of selected suppliers increases with the confidence level α, which indicates that the impact of disruption risks is mitigated by diversification of the supply portfolio.*
- *The greater is the range of disruption probabilities, the higher are both expected and worst-case costs.*
- *The closer are disruption rates for different suppliers, the closer are the corresponding quantities of ordered parts in the optimal portfolio.*

The probability measure for the scenario-based optimization under uncertainty is concentrated in finitely many points and the resulting discrete distribution of cost may have different effect on the optimal portfolio. In particular, the well known misbehaviour in the dependence of VaR on the confidence level can as well be encountered when CVaR is applied as a risk measure. For instance, if a large probability atom is concentrated at some cost, a slight increase of the confidence level may results in a significant change in VaR as well as in the optimal portfolio, while only a slight change of CVaR may occur. Such an instability of the optimal portfolio due to the discontinuity in the distribution function may be distressing in practice, when a slightly higher confidence level is required.

On the other hand the computational results indicate that the smaller is the number of concentration points and the greater are probability atoms concentrated at those points, the greater can be the positive difference $F(VaR) - \alpha$, i.e., the smaller than $1 - \alpha$ can be the probability of outcomes with cost higher (service level lower) than VaR.

The computational experiments prove that the proposed exact solution approach based on MIP approach provides the decision maker with a simple tool for evaluating the relationship between expected and worst-case costs. For a finite number of scenarios, the proposed models allow the evaluation of worst-case costs and shaping of the resulting cost distribution through the selection of optimal supply portfolio. The optimal risk averse supply portfolio can be found within CPU seconds for a limited number of scenarios considered, using commercially available solvers for MIP.

2.7 Notes

The supply chain risk management has been extensively studied over the past decade. Research addresses the two risk levels (e.g., Tang 2006): operational risks or disruption risks. Operational risks are referred to the inherent uncertainties arising from the problems of coordinating supply and demand such as uncertain customer demand, uncertain supply, and uncertain cost. Disruption risks are referred to the major disruptions to normal activities caused by natural and man-made disasters such as earthquakes, floods, hurricanes, etc., or equipment breakdowns, economic crises such as currency evaluation, labor strikes, terrorist attacks. In most cases, the business impact associated with disruption risks is much greater than that of the operational risks. In practice four basic approaches can be applied to mitigate the impact of supply chain risks (Tang 2006): supply management, demand management, product management, and information management. In particular, to ensure efficient supply of materials along a supply chain, supply chain management deals with selection of a supply portfolio, i.e., supplier selection and order quantity allocation under uncertain quality of supplied materials and reliability of on-time delivery. The supplier selection and order quantity allocation problem is a complex stochastic combinatorial optimization problem, however the research on supplier selection under disruption risks is limited. For example, chance-constrained programming models were developed by Kasilingam and Lee (1996) to account for stochastic demand and by Wu and Olson (2008) to consider expected losses from quality acceptance inspection or late delivery. Parlar and Perry (1996) present a continuous time model in which the availability of each of the m suppliers is uncertain because of disruptions such as equipment breakdown. By considering the case that each supplier is either "on" or "off", there are 2^m possible number of states for the whole system. For each of these 2^m states, they analyze a state-specific (q, Q) ordering policy so that the buyer would order Q units when the on-hand inventory reaches q. The risks associated with a supplier network was studied by Berger et al. (2004), who considered catastrophic super events that affect all suppliers, as well as unique events that impact only one single supplier, and then a decision-tree based model was presented to help determine the optimal number of suppliers needed for the buying firm. Ruiz-Torres and Mahmoodi (2007) considered unequal failure probabilities for all the suppliers. Berger and Zeng (2006) studied the optimal supply size in a single or multiple sourcing strategy context, under a number of scenarios that are determined by various financial loss functions, the operating cost functions and the probabilities of all the suppliers being down. Yu et al. (2009) considered the impacts of supply disruption risks on the choice between the single and dual sourcing methods in a two-echelon supply chain with a non-stationary and price-sensitive demand. Yue et al. (2010) introduced frontier sourcing portfolios to support manufacturers sourcing decisions, which consider the cost and probability of finishing the order on time. Ravindran et al. (2012) developed multi-criteria supplier selection models incorporating supplier risks. In the multi-objective formulation, price, lead-time, disruption risk due to natural event and quality risk are explicitly considered as four conflicting objectives that have to be minimized simultaneously. Four different variants of goal programming were used to solve the multi-objective optimization problem. Xanthopoulos et al. (2012) developed newsvendor-type inventory models for capturing the trade-off between inventory policies and disruption risks in a dual-sourcing supply chain network, where both supply channels are subject to disruption risks. The models were developed for both risk-neutral and risk-averse decision-making. Li and Zabinsky (2011) developed a two-stage stochastic programming model and a chance-constrained programming model to determine a minimal set of suppliers and optimal order quantities. Both models include several objectives and strive to balance a small number of suppliers with the risk of not being able to meet demand. The stochastic programming model is scenario-based and uses penalty coefficients whereas the chance-constrained programming model assumes a probability distribution and constrains the probability of not meeting demand. Hammami et al. (2014) proposed a scenario-based stochastic model for supplier selection in the presence of uncertain fluctuations of currency exchange rates and price discounts.

The vast majority of the decision models are mathematical programming models either single objective, e.g., Kasilingam and Lee (1996), Basnet and Leung (2005), Sawik (2005) or multiple objectives, e.g., Weber and Current (1993), Xia and Wu (2007), Demirtas and Ustun (2008), Ustun and Demirtas (2008). The models developed for supplier selection and order allocation can be either single-period models (e.g., Weber and Current 1993, Demirtas and Ustun 2008) that do not consider inventory management or multi-period models (e.g., Ghodsypour 2001, Basnet and Leung

2005, Ustun and Demirtas 2008, Che and Wang 2008) which consider the inventory management by lot-sizing and scheduling of orders.

The material presented in this chapter is based on research reported by Sawik (2011b, c), who proposed a portfolio approach for the supplier selection and order quantity allocation under disruption risks and under operational risks, respectively. The author applied the two popular in financial engineering percentile measures of risk, value-at-risk (VaR) and conditional value-at-risk (CVaR) (e.g., Sarykalin et al. 2008) for managing the risk of supply disruptions or supply delays. The proposed models were further enhanced in this chapter for maximization of expected or expected worst-case service level and for a multi-region sourcing subject to regional disruption risks.

Various simplifying assumptions that have been used in the models presented in this chapter can be relaxed. For example, it has been assumed that each supplier is capable of manufacturing all required part types. In a more general setting, each supplier may only be prepared to manufacture a subset of part types and provide with the parts the corresponding subset of customer orders. The proposed models can be enhanced also for a discount environment, where the suppliers offer discounts based on quantity or business volume of ordered parts, e.g., Sawik (2010). A critical issue that need to be considered before any practical application of the proposed models is attempted, however, is the estimation of probabilities and the resulting costs associated with each type of disaster event, for which different approaches are suggested in the literature, such as expert systems, game theory, utilization of large simulation models, etc. (e.g., Knemeyer et al. 2009).

Problems

2.1 Modify the probability for disruption scenarios (2.2) to account for correlated regional disruptions that may affect simultaneously suppliers in different regions.

2.2 Modify the SMIP models presented in this chapter for multiple part types with subsets of part types required for each customer order and subsets of suppliers available for each part type.

2.3 Enhance the SMIP models presented in this chapter for selection of static supply portfolio
(a) with total quantity discount for all ordered parts.
(b) with total business volume discount.
(see, Sawik 2010).

2.4 Mixed mean-risk static supply portfolio
(a) Modify model **SP_ECV(c)** to optimize expected cost and CVaR of service level and model **SP_ECV(sl)** to optimize expected service level and CVaR of cost.
(b) How should the values of the optimized objective functions be scaled into the interval [0,1] to avoid dimensional inconsistency among the two objectives and how

should the trade-off parameter be selected?
(c) How would you interpret the mixed mean-risk supply portfolio?

2.5 Explain why for a greater range of disruption probabilities, both expected and expected worst-case costs are higher.

Chapter 3
Selection of Dynamic Supply Portfolio

3.1 Introduction

In this chapter the static portfolio approach and the SMIP formulations presented in Chap. 2 are enhanced for a multi-period supplier selection and order quantity allocation in the presence of both the low probability and high impact supply chain disruption risks and the high probability and low impact supply chain delay risks. The suppliers are subject to local delivery delay risks, and both local and regional delivery disruption risks. In the delivery scenario analysis, both types of the supply chain risks are simultaneously considered. The high probability supply delays may have a significant impact in a make-to-order manufacturing and just-in-time environment. In particular, when all suppliers are similarly exposed to high impact disruption risks, then the low impact delay risk may predominate the supplier selection decision.

The following SMIP models are presented in this chapter:

DSP_E(c) for risk-neutral selection of dynamic supply portfolio to minimize expected cost;
DSP_E(sl) for risk-neutral selection of dynamic supply portfolio to maximize expected service level;
DSP_CV(c) for risk-averse selection of dynamic supply portfolio to minimize CVaR of cost;
DSP_CV(sl) for risk-averse selection of dynamic supply portfolio to maximize CVaR of service level;
DSP_ECV(c) for mean-risk selection of dynamic supply portfolio to optimize trade-off between expected cost and CVaR of cost;
DSP_ECV(sl) for mean-risk selection of dynamic supply portfolio to optimize trade-off between expected service level and CVaR of service level.

T. Sawik, *Supply Chain Disruption Management Using Stochastic Mixed Integer Programming*, International Series in Operations Research & Management Science 256, DOI 10.1007/978-3-319-58823-0_3

In the computational experiments described in Sect. 3.6, the general ‘on-time, delay or never’ delivery scenarios are illustrated with numerical examples and compared with ‘longest delay or never’ worst-case scenarios.

3.2 Problem Description

Table 3.1 Notation: dynamic supply portfolio

	Indices
i	= supplier, $i \in I$
j	= customer order, $j \in J$
r	= geographic region, $r \in R$
s	= delivery scenario, $s \in S$
t	= planning period, $t \in T$
	Input Parameters
c_{it}	= capacity of supplier i in period t
d_j	= demand for parts required for customer order j
D	$= \sum_{j \in J} d_j$ - total demand for parts
$\underline{\delta}_j$	= the earliest delivery date of parts for customer order j
δ_j	= the latest delivery date of parts for customer order j
e_i	= cost of ordering parts from supplier i
g_j	= per unit and per period penalty cost of delayed customer order j caused by delayed delivery of required parts
h_j	= per unit penalty cost of unfulfilled customer order j caused by shortage of required parts
o_{ij}	= unit price of parts for customer order j purchased from supplier i
ε_{ij}	= per unit price reduction for each delayed part for customer order j from supplier i
$p_{i,\tau}$	= probability of τ periods delay for delivery of parts from supplier i, $\tau \in \{0, \ldots, \overline{\tau}\}$
$p_{i,\overline{\tau}+1}$	= local disruption probability of supplier i
p^r	= regional disruption probability of all suppliers $i \in I^r$ in region r
α	= confidence level
τ	$= 0, \ldots, \overline{\tau}$, delivery delay
ρ_i	= expected defect rate of supplier i

The approach proposed in this chapter can be applied for a very common type of supply chain, with a producer of single product, who obtains raw materials from several different suppliers with limited supply capacity to meet customer orders by customer requested due dates. However, the approach is also applicable to the case of a single producer, who assembles different types of products using product-specific parts purchased from multiple suppliers (for notation used, see Table 3.1). In order to simplify further considerations it is assumed that for each product type, one product-specific part type (e.g. a critical custom part type) needs to be supplied in required amount by custom parts manufacturers. For example, in the electronics industry producer of different electronic devices needs to be supplied by electronics manufacturers with printed wiring boards of different device-specific design. However, the last assumption can be easily relaxed to consider supplies of different product-specific part types required for each product type.

Let $I = \{1, \ldots, \overline{I}\}$ be the set of $\overline{I}$ suppliers and $J = \{1, \ldots, \overline{J}\}$ the set of $\overline{J}$ customer orders for the finished products, known ahead of time. Each customer order $j \in J$ is described by the quantity d_j of required custom parts and their latest delivery date, δ_j, to ensure meeting the customer requested due date for products. The planning horizon consists of $\overline{T}$ periods and denote by $T = \{1, \ldots \overline{T}\}$ the set of planning periods.

Each supplier can provide the producer with custom parts for all customer orders. However, the suppliers have different capacity and, in addition, differ in price and quality of purchased parts and in reliability of delivery. Let c_{it} be the capacity of supplier i in period t, e_i, cost of ordering parts from supplier i, o_{ij}, per unit price of custom parts for customer order j purchased from supplier i and ρ_i, the expected defect rate for supplier i.

The suppliers are assumed to be located in $\overline{R}$ disjoint geographic regions. Denote by $I^r \subseteq I$ the subset of suppliers in region $r \in R = \{1, \ldots, \overline{R}\}$, where $\bigcup_{r \in R} I^r = I$. The supplies of parts are subject to random local delays or disruptions that are uniquely associated with a particular supplier, which may arise from equipment breakdowns, local labor strike, fires, etc. In addition to independent local delays and independent local disruptions of each supplier individually there are also potential regional disasters that may result in correlated regional disruption of all suppliers in the same region simultaneously. For example, such regional disaster events may include floods, hurricanes, earthquakes, widespread labor strikes in a transportation sector, etc.

The delivery of parts from each supplier $i \in I$ is subject to random delay of different length, $\tau \in \mathscr{T} = \{0, \ldots, \overline{\tau}, \overline{\tau} + 1\}$, i.e., parts ordered for period t are delivered in period $t + \tau$, where $\tau = 0$, represents on time delivery and $\tau = \overline{\tau}$, represents maximum delay that may occur. The delivery of parts is also subject to random disruptions, i.e., parts are not delivered at all. By convention, the dummy delay $\tau = \overline{\tau} + 1$, represents disruption of supplies, i.e., no delivery of parts. If the delivery of parts for customer order j occurs in period $t + \tau > \delta_j$, then the delivery is delayed and the producer is charged by the customer with contractual penalty, $g_j d_j (t + \tau - \delta_j)$, where g_j is the per unit and per period penalty cost of delayed customer order j caused by the delayed delivery of required parts. If supplies of parts for customer order j are disrupted (i.e., dummy delay of $\tau = \overline{\tau} + 1$ periods), then the producer is charged with the contractual penalty, $h_j d_j$, for unfulfilled order j, where h_j is the per unit penalty cost of unfulfilled customer order j caused by the shortage of required parts.

Denote by $S = \{1, \ldots, \overline{S}\}$ the index set of all delivery scenarios, and by P_s the probability of delivery scenario $s \in S$. While supply disruptions are typically modelled by binomial random variables, the combined disruptions and different length delays of supply are modelled by multinomial random variables. Each scenario $s \in S$ can be represented by an integer-valued vector $\tau_s = \{\tau_{1s}, \ldots, \tau_{\overline{I}s}\}$, where $\tau_{is} \in \mathscr{T}$ is the length of delay from supplier $i \in I$ under scenario $s \in S$. When all potential delivery scenarios are considered, then $\overline{S} = (\overline{\tau} + 2)^{\overline{I}}$. For each scenario $s \in S$, the supplies from every supplier can be delayed or disrupted either by a local or a regional event. Denote by $I_s \subset I$ the subset of non-shutdown (non-disrupted) suppliers, who can deliver orders on time or delayed under scenario s. The probability P_s for delivery scenario $s \in S$ with the subset I_s of non-shutdown suppliers is

$$P_s = \prod_{r \in R} P_s^r. \tag{3.1}$$

P_s^r is the probability of realizing of delivery scenario s for suppliers in I^r

$$P_s^r = \begin{cases} (1 - p^r) \prod_{i \in I^r} (p_{i,\tau_{is}}) & \text{if } \; I^r \bigcap I_s \neq \emptyset \\ p^r + (1 - p^r) \prod_{i \in I^r} p_{i,\overline{\tau}+1} & \text{if } \; I^r \bigcap I_s = \emptyset, \end{cases} \tag{3.2}$$

where $p_{i,\tau_{is}}$ is the probability of occurrence the delay of length τ_{is} periods from supplier i under scenario s. By convention, $p_{i,\overline{\tau}+1}$, is the probability of local disruption of supplier i. The parts ordered from supplier i can be delivered on-time with probability, p_{i0}, delayed with probability $\sum_{\tau=1}^{\overline{\tau}} p_{i\tau}$, or not delivered at all with probability $p_{i,\overline{\tau}+1} = 1 - \sum_{\tau=0}^{\overline{\tau}} p_{i\tau}$.

Denote by π_i the total disruption (shutdown) probability of every supplier $i \in I^r, r \in R$

$$\pi_i = p^r + (1 - p^r) p_{i,\overline{\tau}+1}; \; i \in I^r, r \in R. \tag{3.3}$$

The decision maker needs to decide on the selection of part suppliers, on quantities of various custom parts to be ordered from each selected supplier and on delivery dates to minimize expected cost or maximize expected service level or to mitigate the impact of disruption risks by minimizing the potential worst-case cost or maximizing the potential worst-case service level. Hence, the decision maker needs to select a risk-neutral or risk-averse dynamic supply portfolio, i.e. the allocation of demand parts among the suppliers and among the planning periods.

3.3 Models for Risk-Neutral Decision-Making

In this section two SMIP models **DSP_E(c)** and **DSP_E(sl)** are presented for selection of a risk-neutral multi-period supply portfolio in the presence of supply delay and disruption risks to minimize expected cost or to maximize expected service level, respectively.

The decision maker needs to select a dynamic supply portfolio, i.e. the allocation of orders for parts among the suppliers and among the planning periods. The dynamic supply portfolio is defined below, (for definition of problem variables, see Table 3.2).

$$\{V_{it} : \; i \in I, t \in T\},$$

Table 3.2 Variables: dynamic supply portfolio

		First stage variables
u_i	=	1, if an order for parts is placed on supplier i; otherwise $u_i = 0$ (supplier selection)
v_{ijt}	=	the fraction of total demand for parts required for customer order j to be delivered by supplier i in period t (allocation of demand for custom parts)
		Auxiliary variables
V_{it}	=	the fraction of total demand for parts ordered from supplier i to be delivered in period t (dynamic supply portfolio: allocation of total demand for parts among suppliers and over time)
VaR^c		Cost-at-Risk, the targeted cost such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is below VaR^c
VaR^{sl}		Service-at-Risk, the targeted service level such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is above VaR^{sl}
$\mathscr{C}_s$	$\geq$	0, the tail cost for scenario s, i.e., the amount by which costs in scenario s exceed VaR^c
$\mathscr{S}_s$	$\geq$	0, the tail service level for scenario s, i.e., the amount by which VaR^{sl} exceeds service level in scenario s

where

$$\sum_{i\in I}\sum_{t\in T} V_{it} = 1$$

and $0 \leq V_{it} \leq 1$ is the fraction of the total demand for parts ordered from supplier i for period t and V_{it} is determined by the custom parts allocation variables v_{ijt}

$$V_{it} = \sum_{j\in J} d_j v_{ijt}/D; \;\; i \in I. \tag{3.4}$$

The supply delays and disruptions may result in the shortage of required parts and the corresponding delay and shortage penalty costs of delayed or unfulfilled customer orders should be incorporated into the model. Clearly, the producer does not need to pay for ordered and defective or undelivered parts, whereas parts delivered late may be paid for at a reduced price. However, the producer can be charged with a much higher penalty cost for delayed or unfulfilled customer orders for products, caused by the shortage of required parts due to defective, delayed or undelivered parts. In a make-to-order manufacturing no inventory of custom parts can be kept on hand and the parts are requisitioned with each customer order. In addition, custom parts required for each customer order j are to be delivered within a time window $[\underline{\delta}_j, \delta_j]$ derived from the customer requested due date to further reduce any inventory holding cost. The delivery cannot be earlier than the earliest date $\underline{\delta}_j$ and the required parts are assumed to be processed as soon as they are delivered. As a result no overage costs can be considered. On the other hand, the required parts can be delivered late, beyond the latest date δ_j, or not delivered at all. Then, delay or disruption penalty costs represent the underage costs.

Notice that Table 3.2 does not explicitly define the second stage variables for the SMIP problem considered. The second stage variables are simply demand allocation variables for realized disruption scenarios s, $\tilde{v}^s_{ijt};\ i \in I, j \in J, t \in T, s \in S$, defined as follows

$$\tilde{v}^s_{ijt} = \begin{cases} v_{ijt-\tau_{is}} & \text{if } i \in I_s, j \in J, t \in T, s \in S \\ 0 & \text{if } i \notin I_s, j \in J, t \in T, s \in S. \end{cases}$$

In view of the above definition, an explicit introduction of the second stage variables $\tilde{v}^s_{ijt}$ into the SMIP model formulations is not required.

For each delivery scenario $s \in S$ and each customer order $j \in J$ to be supplied with parts by non-disrupted supplier $i \in I_s$ in period t (i.e., with $v_{ijt} > 0$), the penalty cost of delivery delayed by $1 \le \tau_{is} \le \overline{\tau}$ periods is given by

$$g_j \max\{0, t + \tau_{is} - \delta_j\} d_j,$$

i.e., deliveries later than δ_j are penalized only.

The resulting total expected delay penalty cost over all delivery scenarios can be expressed by

$$\sum_{s\in S}\sum_{i\in I_s}\sum_{t\in T}\sum_{j\in J:\delta_j<t+\tau_{is}} P_s(g_j(t+\tau_{is}-\delta_j) - \varepsilon_{ij})d_j v_{ijt},$$

where the producer does not need to pay full price for late delivery of parts and hence the purchasing costs of those parts are reduced by ε_{ij} per unit for each delayed part for customer order j from supplier i.

The total expected penalty cost of unfulfilled customer orders due to shortage of required parts caused by supply disruptions is

$$\sum_{s\in S}\sum_{i\notin I_s}\sum_{j\in J}\sum_{t\in T} P_s(h_j - o_{ij})d_j v_{ijt},$$

where the producer does not need to pay for parts ordered and non delivered and hence the purchasing costs of those parts are deducted from the cost of unfulfilled customer orders.

In a risk-neutral operating conditions the overall quality of the supply portfolio can be measured by the expected cost per part, E^c, (3.5), or expected service level E^{sl}, (3.6).

$$\begin{aligned} E^c = \sum_{i\in I} e_i u_i/D + \sum_{i\in I}\sum_{j\in J}\sum_{t\in T} o_{ij} d_j v_{ijt}/D \\ + \sum_{s\in S}\sum_{i\in I_s}\sum_{t\in T}\sum_{j\in J:\delta_j<t+\tau_{is}} P_s(g_j(t+\tau_{is}-\delta_j) - \varepsilon_{ij})d_j v_{ijt}/D \\ + \sum_{s\in S}\sum_{i\notin I_s}\sum_{j\in J}\sum_{t\in T} P_s(h_j - o_{ij})d_j v_{ijt}/D \end{aligned} \tag{3.5}$$

$$E^{sl} = \sum_{s\in S}\sum_{i\in I_s}\sum_{j\in J}\sum_{t\in T} P_s d_j v_{ijt}/D \tag{3.6}$$

The expected cost E^c includes, cost of ordering,
$\sum_{i\in I} e_i u_i/D$,
cost of purchasing non defective parts,
$\sum_{i\in I}\sum_{j\in J}\sum_{t\in T} d_j o_{ij} v_{ijt}/D$,
cost of delayed deliveries of parts,
$\sum_{s\in S}\sum_{i\in I_s}\sum_{t\in T}\sum_{j\in J:\delta_j<t+\tau_{is}} P_s(g_j(t+\tau_{is}-\delta_j)-\varepsilon_{ij})d_j v_{ijt}/D$,
and cost of shortage of parts due to supply disruptions (cost of unfulfilled customer orders less cost of non delivered parts),
$\sum_{s\in S}\sum_{i\notin I_s}\sum_{j\in J}\sum_{t\in T} P_s(h_j-o_{ij})d_j v_{ijt}/D$.

The purchase orders for parts are assumed to be inflated by the reject rates ρ_i, $i\in I$ of defective parts, i.e., are equal to $\sum_{i\in I}\sum_{j\in J}\sum_{t\in T}(1+\rho_i)d_j v_{ijt}$. However, since the producer does not need to pay for ordered and defective parts in the amount of $\sum_{i\in I}\sum_{j\in J}\sum_{t\in T}\rho_i d_j v_{ijt}$, and pays a reduced price for delayed parts in the amount of $\sum_{i\in I}\sum_{t\in T}\sum_{j\in J:\delta_j<t+\tau} d_j v_{ijt}$, the corresponding purchasing cost per part for delivered parts is simply given by

$$\sum_{i\in I}\sum_{j\in J}\sum_{t\in T} o_{ij} d_j v_{ijt}/D$$
$$-\sum_{s\in S}\sum_{i\in I_s}\sum_{t\in T}\sum_{j\in J:\delta_j<t+\tau_{is}} P_s\varepsilon_{ij} d_j v_{ijt}/D-\sum_{s\in S}\sum_{i\notin I_s}\sum_{j\in J}\sum_{t\in T} P_s o_{ij} d_j v_{ijt}/D.$$

The expected service level, E^{sl}, (3.6), is a surrogate measure of expected customer demand fulfillment rate and represents the expected fraction of fulfilled demand for required partss.

The SMIP models **DSP_E(c)** and **DSP_E(sl)** for selection of risk-neutral dynamic supply portfolio to minimize expected cost or maximize expected service level, respectively, are presented below.

DSP_E(c): *Selection of risk-neutral* **D***ynamic* **S***upply* **P***ortfolio to minimize expected cost*

Minimize (3.5)

subject to

1. Dynamic supply portfolio selection constraints:

- for each customer order the required parts must be delivered not earlier than the earliest and not later than the latest delivery date,
- parts cannot be ordered from non-selected suppliers,
- for each selected supplier the total quantity of ordered parts cannot exceed the supplier capacity,
- at least a minimum order size should be assigned to each selected supplier,

$$\sum_{i\in I}\sum_{t\in T:\underline{\delta}_j\le t\le\delta_j} v_{ijt}=1;\ \ j\in J \tag{3.7}$$

$$v_{ijt}\le u_i;\ \ i\in I,\ j\in J,\ t\in T \tag{3.8}$$

$$\sum_{j \in J}(1+\rho_i)d_j v_{ijt} \le c_{it}u_i;\ \ i \in I, t \in T \tag{3.9}$$

$$\sum_{j \in J}\sum_{t \in T} v_{ijt} \ge u_i;\ \ i \in I \tag{3.10}$$

4. Non-negativity and integrality conditions:

$$u_i \in \{0,1\};\ \ i \in I, t \in T \tag{3.11}$$

$$v_{ijt} \in [0,1];\ \ i \in I, j \in J, t \in T. \tag{3.12}$$

DSP_E(sl): *Selection of risk-neutral dynamic supply portfolio to maximize expected service level*

Maximize (3.6)
subject to (3.7)–(3.12).

If total available capacity of all suppliers is less than total demand for required parts, i.e., $\sum_{i\in I}\sum_{t\in T} c_{it}/(1+\rho_i) \le \sum_{j\in J} d_j$, then the demand allocation equality constraints (3.7) should be replaced by inequalities

$$\sum_{i \in I}\ \sum_{t \in T:\underline{\delta}_j \le t \le \delta_j} v_{ijt} \le 1;\ \ j \in J; \tag{3.13}$$

otherwise no feasible solution exists.

A simple upper bound on the expected service level, E^{sl}, (3.6), is derived below.

Proposition 3.1

$$E^{sl} \le \min\{1, \sum_{r\in R}\sum_{i\in I^r}\sum_{t\in T}(1-p^r)(1-p_{i,\overline{\tau}+1})c_{it}/(1+\rho_i)D\}. \tag{3.14}$$

Proof The dynamic supply portfolio selection constraints (3.9) imply that

$$\sum_{s\in S}\sum_{i\in I_s}\sum_{j\in J}\sum_{t\in T} P_s d_j v_{ijt}/D \le$$
$$\sum_{r\in R}\sum_{i\in I^r}\sum_{j\in J}\sum_{t\in T}(1-p^r)(1-p_{i,\overline{\tau}+1})d_j v_{ijt}/D \le$$
$$\sum_{r\in R}\sum_{i\in I^r}\sum_{t\in T}(1-p^r)(1-p_i)c_{it}u_i/(1+\rho_i)D \le$$
$$\sum_{r\in R}\sum_{i\in I^r}\sum_{t\in T}(1-p^r)(1-p_{i,\overline{\tau}+1})c_{it}/(1+\rho_i)D,$$

where $(1-p^r)(1-p_{i,\overline{\tau}+1}) = 1-\pi_i$, (3.3), is a non-disruption probability of supplier $i \in I^r$.

Since E^{sl} cannot be greater than 1, its upper bound is 1, if $\sum_{r\in R}\sum_{i\in I^r}\sum_{t\in T}(1-p^r)(1-p_{i,\overline{\tau}+1})c_{it}/(1+\rho_i)D > 1$.

If for each customer order, all custom parts should be provided by a single delivery from one supplier only, then the continuous demand allocation variables v_{ijt} should be redefined as binary assignment variables denoting whether or not all parts required for customer order j are provided by supplier i in period t, (e.g., Chap. 2).

3.3.1 Dynamic Supply Portfolio for All-or-Nothing Delivery Scenarios

The dynamic supply portfolio approach is particularly useful when supply delay risks need to be considered. Otherwise, the static supply portfolio presented in Chap. 2 may be a more appropriate approach. For the pure all-or-nothing delivery scenarios (i.e., for disruption scenarios), with supply disruptions only and no delays (see Sects. 2.2, 2.3 in Chap. 2), the proposed multi-period models for $\overline{I}$ suppliers and $\overline{T}$ planning periods can be transformed into equivalent single-period models for $m = (\overline{I})(\overline{T})$ suppliers. To this end, each pair of indices $(i,t), i \in I, t \in T$ should be replaced with a single index $k \in K$, where $K = \{1,\ldots,m\}$ represents the set of equivalent m single-period suppliers. In the proposed SMIP models, variables $u_i,\ v_{ijt}$ should be replaced by $u_k,\ v_{kj}$, respectively. Then, the dynamic supply portfolio selection constraints (3.7) are replaced by

$$\sum_{k\in K_j} v_{kj} = 1;\ \ j \in J,$$

where K_j is the subset of equivalent single-period suppliers that are capable of delivering parts required for customer order j and it represents the subset $\{(i,t) : i \in I,\ \underline{\delta}_j \le t \le \delta_j\}$ of pairs (i,t) feasible for customer order j.

3.4 Models for Risk-Averse Decision-Making

In the risk-averse selection of supply portfolio under disruption risks, the confidence level α is fixed by the decision maker to control the risk of losses due to supply disruptions. We assume that the decision maker is willing to accept only portfolios for which the the total probability of scenarios with costs greater than VaR^c or with service level lower than VaR^{sl} is not greater than $1-\alpha$. Furthermore, a risk aversive decision maker wants to minimize the expected worst-case costs exceeding VaR^c or to maximize the expected worst-case service level below VaR^{sl}.

Define by $\mathscr{C}_s$ the tail cost for scenario s, where tail cost is defined as the amount by which costs in scenario s exceed VaR^c. In a similar way, define by $\mathscr{S}_s$ the tail service level for scenario s, where tail service level is defined as the nonnegative amount by which VaR^{sl} exceeds service level in scenario s.

The portfolio will be optimized by calculating VaR^c and minimizing CVaR^c simultaneously or by calculating VaR^{sl} and maximizing CVaR^{sl}, respectively.

In the proposed models CVaR is represented by an auxiliary function (3.15) and (3.18) introduced by Rockafellar and Uryasev (2000). The SMIP models **DSP_CV(c)** and **DSP_CV(sl)** for the risk-averse selection of supply portfolio to reduce the risk of high costs and the risk of low service level, respectively, is formulated below.

DSP_CV(c): *Selection of risk-averse dynamic supply portfolio to minimize CVaR of cost*

Minimize

$$CVaR^c = VaR^c + (1-\alpha)^{-1} \sum_{s \in S} P_s \mathscr{C}_s \tag{3.15}$$

subject to

1. Dynamic supply portfolio selection constraints: (3.7)–(3.10)

2. Risk constraints:

- the tail cost for scenario s is defined as the nonnegative amount by which cost per part in scenario s exceeds VaR^c,

$$\begin{aligned} \mathscr{C}_s \geq & \sum_{i \in I} e_i u_i / D + \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} o_{ij} d_j v_{ijt} / D \\ & + \sum_{i \in I_s} \sum_{t \in T} \sum_{j \in J : \delta_j < t + \tau_{is}} (g_j(t + \tau_{is} - \delta_j) - \varepsilon_{ij}) d_j v_{ijt} / D \\ & + \sum_{i \notin I_s} \sum_{j \in J} \sum_{t \in T} (h_j - o_{ij}) d_j v_{ijt} / D - VaR^c \end{aligned} \tag{3.16}$$

3. Non-negativity and integrality conditions: (3.11), (3.12) and

$$\mathscr{C}_s \geq 0; \;\; s \in S. \tag{3.17}$$

DSP_CV(sl): *Selection of risk-averse dynamic supply portfolio to maximize CVaR of service level*

Maximize

$$CVaR^{sl} = VaR^{sl} - (1-\alpha)^{-1} \sum_{s \in S} P_s \mathscr{S}_s \tag{3.18}$$

subject to

1. Dynamic supply portfolio selection constraints: (3.7)–(3.10)

2. Risk constraints:

- the tail service level for scenario s is defined as the nonnegative amount by which VaR^{sl} exceeds service level in scenario s,

$$\mathscr{S}_s \geq VaR^{sl} - \sum_{i \in I_s} \sum_{j \in J} \sum_{t \in T} d_j v_{ijt}/D; \;\; s \in S \tag{3.19}$$

3. Non-negativity and integrality conditions: (3.11), (3.12) and

$$\mathscr{S}_s \geq 0; \;\; s \in S. \tag{3.20}$$

Note that variables VaR^c and VaR^{sl} do not need to be constrained of being nonnegative. As $\mathscr{C}_s$ and $\mathscr{S}_s$ are constrained of being positive, the model tries to decrease VaR^c and increase VaR^{sl}, respectively, and hence positively impact the objective function. However, large reduction in VaR^c and increase in VaR^{sl} may result in more scenarios with positive tail costs and service levels, respectively.

3.5 Models for Mean-Risk Decision-Making

In the single objective approach the supply portfolio is selected by minimizing either the expected cost per part, E^c, (3.5), or the expected worst-case cost per part, $CVaR^c$, (3.15) or by maximizing either the expected service level, E^{sl}, (3.6), or the expected worst-case service level, $CVaR^{sl}$, (3.18). In this section the two cost functions and the two service level functions are considered simultaneously, and a bi-objective selection of supply portfolio is presented aimed at minimizing both objective functions to balance expected costs or expected service level with the risk tolerance. This trade-off model is known as the mean-risk model, formulated as the optimization of a composite objective consisting of the expected cost (service level) and the CVaR as a risk measure.

A subset of nondominated solutions for the bi-objective dynamic supply portfolio can be found by the parameterization on λ the weighted-sum programs **DSP_ECV(c)** and **DSP_ECV(sl)** presented below. The mean-risk program **DSP_ECV(c)** is based on model **DSP_CV(c)** with the addition of objective (3.5) of model **SP_E(c)**. Similarly, the mean-risk program **DSP_ECV(sl)** is based on model **DSP_CV(sl)** with the addition of objective (3.6) of model **DSP_E(sl)**.

DSP_ECV(c): *Selection of mean-risk dynamic supply portfolio to minimize weighted sum of expected cost and CVaR of cost*

Minimize

$$\lambda E^{c} + (1-\lambda)CVaR^{c}, \tag{3.21}$$

where $0 \le \lambda \le 1$
subject to (3.5), (3.7)–(3.12), (3.15)–(3.17).

DSP_ECV(sl): *Selection of mean-risk dynamic supply portfolio to maximize weighted sum of expected service level and CVaR of service level*
Maximize

$$\lambda E^{sl} + (1-\lambda)CVaR^{sl} \tag{3.22}$$

where $0 \le \lambda \le 1$,
subject to (3.6)–(3.12), (3.18)–(3.20).

3.6 Computational Examples

In this section computational examples are presented to illustrate possible applications of the proposed SMIP models for selection of a dynamic supply portfolio in the presence of delay and disruption risks. Although the input data for the examples are

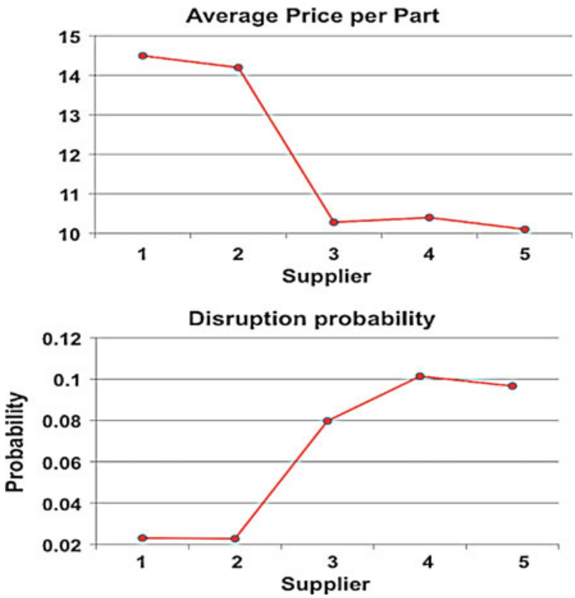

Fig. 3.1 Suppliers

hypothetical, their relations to each other are real. In the cost-oriented selection of supply portfolio a combination of disruption and delay risks simultaneously impacts both shortage and delay penalties. In the service-oriented decisions, demand fulfillment rate (the fraction of demand for parts fulfilled during the planning horizon) is impacted.

The following parameters have been used for the example problems:

- $\overline{I}$, the number of suppliers, was equal to 5.
- $\overline{R}$, the number of geographic regions, was equal to 2, and the subsets of suppliers were $I^1 = \{1, 2\}$ and $I^2 = \{3, 4, 5\}$, respectively.
- $\overline{J}$, the number of customer orders, was equal to 50.
- $\overline{T}$, the planning horizon, was equal to 14 periods.
- $\overline{\tau}$, maximum delivery delay was equal to 4 periods for each supplier, so that the set of all possible delays is $\mathscr{T} = \{0, 1, 2, 3, 4, 5\}$, where a dummy delay, $\overline{\tau} + 1 = 5$, represents local disruption of a supplier. The corresponding number of delivery scenarios, was equal to $\overline{S} = 6^{\overline{I}} = 7776$.
- d_j, the numbers of required parts for each customer order, were integers uniformly distributed over $[500, 5000]$ for all customer orders j and the resulting total demand for parts was $D = 138000$.
- e_i, the cost of ordering parts, were integers in $\{8000, 9000\}$ and $\{12000, 13000, 14000\}$, respectively for suppliers $i \in I^1$ and $i \in I^2$.
- o_{ij}, the unit price of parts for customer order j purchased from supplier i, was uniformly distributed over [13,15] and [9,11], respectively for suppliers $i \in I^1$ and $i \in I^2$.
- ε_{ij}=0.05o_{ij}, per unit price reduction for each delayed part of customer order j from supplier i was 5% of the unit price.
- $g_j = 1$, per unit and per period delay penalty cost was equal to 1 for all customer orders j.
- $h_j = 4 \max_{i \in I}(o_{ij})$, per unit shortage cost for each customer order j, was equal to four times of the maximum purchasing cost of required parts.
- $c_{it} = \lceil 2D/((\overline{T})(\overline{I}))\rceil$, ($\lceil\cdot\rceil$ denotes the smallest integer not less than $\cdot$), i.e., the total capacity of all suppliers was equal to the double total demand for parts;
- p_{i5}, the local disruption probability (dummy delay, $\overline{\tau} + 1 = 5$, represents local disruption) was uniformly distributed over [0.01,0.05] and [0.05;0.10], respectively for suppliers $i \in I^1$ and $i \in I^2$, i.e., the disruption probabilities were drawn independently from U[0.01,0.05] and U[0.05;0.10], respectively.
  Given local disruption probabilities, $p_{i5},\ i \in I$, the probabilities for different delay length $\tau = 0, 1, 2, 3, 4$ were calculated as follows:
  probability of on time delivery ($\tau = 0$), $p_{i0} = 0.3(1 - p_{i5})$;
  probability of 1-period delay ($\tau = 1$), $p_{i1} = 0.25(1 - p_{i5})$;
  probability of 2-period delay ($\tau = 2$), $p_{i2} = 0.2(1 - p_{i5})$;
  probability of 3-period delay ($\tau = 3$), $p_{i3} = 0.15(1 - p_{i5})$;
  probability of maximum, 4-period delay ($\tau = \overline{\tau} = 4$), $p_{i4} = 0.1(1 - p_{i5})$;
  for all suppliers $i \in I$, i.e., the longer delay the lower probability of its occurrence.

Notice that for each supplier i, the expected delay is equal to 2 periods ($\lceil 0.25(1-p_{i5}) + 2(0.2(1-p_{i5})) + 3(0.15(1-p_{i5})) + 4(0.1(1-p_{i5}))/(1-p_{i5})\rceil$=2)

- p^r, the regional disruption probability was 0.005 and 0.01, respectively for region $r=1$ and $r=2$.
- δ_j, the latest delivery date of parts required for customer order j, was integer uniformly distributed over $[3, \overline{T}]$, i.e., generated from a U[3;14] distribution.
- $\underline{\delta}_j = \delta_j - 2$, the earliest delivery date of parts required for each customer order j, was two periods earlier (i.e., the expected delay) than the latest delivery date.
- ρ_i, the expected defect rate of each supplier i was exponentially distributed.
- α, the confidence level, was equal to 0.50, 0.75, 0.90, 0.95 or 0.99.

Note that cost of lost customer orders, h_j, is set to be much higher than the corresponding cost, g_j, of delayed orders, which is typical for industrial practice.

Figure 3.1 shows basic characteristics of each supplier i, average price per part, $\sum_{j\in J} o_{ij}/\overline{J}$, and disruption probability, π_i, (3.3). Suppliers $i=1,2$ in region $r=1$ are most reliable and most expensive, supplier $i=4$ is most unreliable, while supplier $i=5$ is the cheapest one.

The risk-neutral solutions for models **DSP_E(c)** and **DSP_E(sl)** are shown in Table 3.3, and the risk-averse solutions for models **DSP_CV(c)** and **DSP_CV(sl)** with different confidence levels, in Table 3.4. Table 3.3 indicates that for both objective functions, the risk-neutral supply portfolios do not include the most unreliable supplier $i=4$. The corresponding risk-neutral dynamic supply portfolios for each supplier i, $(\sum_{j\in J} d_j v_{ijt}/D, t\in T)$, are shown in Fig. 3.2. The cost-based dynamic supply portfolio is concentrated in a shorter time interval than the supply portfolio for the service level objective, which may reduce delay penalty costs. Both dynamic portfolios are leveled with respect to the three most reliable suppliers $i=1,2,3$, where, supplier $i=3$, is, in addition, one of the cheapest suppliers. The full capacity of these suppliers is fully utilized most of the time.

Figure 3.3 shows distribution of the expected risk-neutral dynamic supply portfolio, $\tilde{V}_{it}, i\in I, t\in\{1,\ldots,\overline{T}+\overline{\tau}\}$, defined below.

$$\tilde{V}_{it} = \sum_{s\in S}\sum_{j\in J}\sum_{t'\in T:t=t'+\tau_{is}} P_s d_j \tilde{v}^s_{ijt'}$$

where

$$\tilde{v}^s_{ijt'} = \begin{cases} v_{ijt'} & \text{if } \tau_{is} \le \overline{\tau} \\ 0 & \text{if } \tau_{is} = \overline{\tau}+1. \end{cases}$$

The expected fraction of total demand for parts is unevenly distributed over the planning horizon. For both objective functions, the highest delivery levels appear in mid-horizon and the lowest levels at the beginning and at the end of the horizon.

In the computational experiments for the risk-averse portfolios, the confidence level α is set at five levels of 0.5, 0.75, 0.90, 0.95, and 0.99, which means that focus is on minimizing the highest 50%, 25%, 10%, 5%, and 1% of all scenario outcomes, i.e., costs per part or service level. When α increases, a more risk-aversive decision-making focuses on a smaller set of outcomes and the number of selected suppliers also is increasing to mitigate the impact of disruptions risks by diversification of the supply portfolio. Table 3.4 demonstrates that supplier $i = 4$ is not selected for the lowest confidence level $\alpha = 0.5$, for which the risk-averse supply portfolio is similar to the risk-neutral portfolio. For larger α, all suppliers are selected, except for the largest $\alpha = 0.99$ and model **DSP_CV(sl)**, for which supplier $i = 4$ is not selected again. This indicates that diversification of the supply portfolio is no longer required to mitigate the impact of disruption risks. Note that for $\alpha = 0.50$ and $\alpha = 0.75$, VaR^c is smaller than expected cost, E^c, whereas VaR^{sl} is greater than expected service level, E^{sl}.

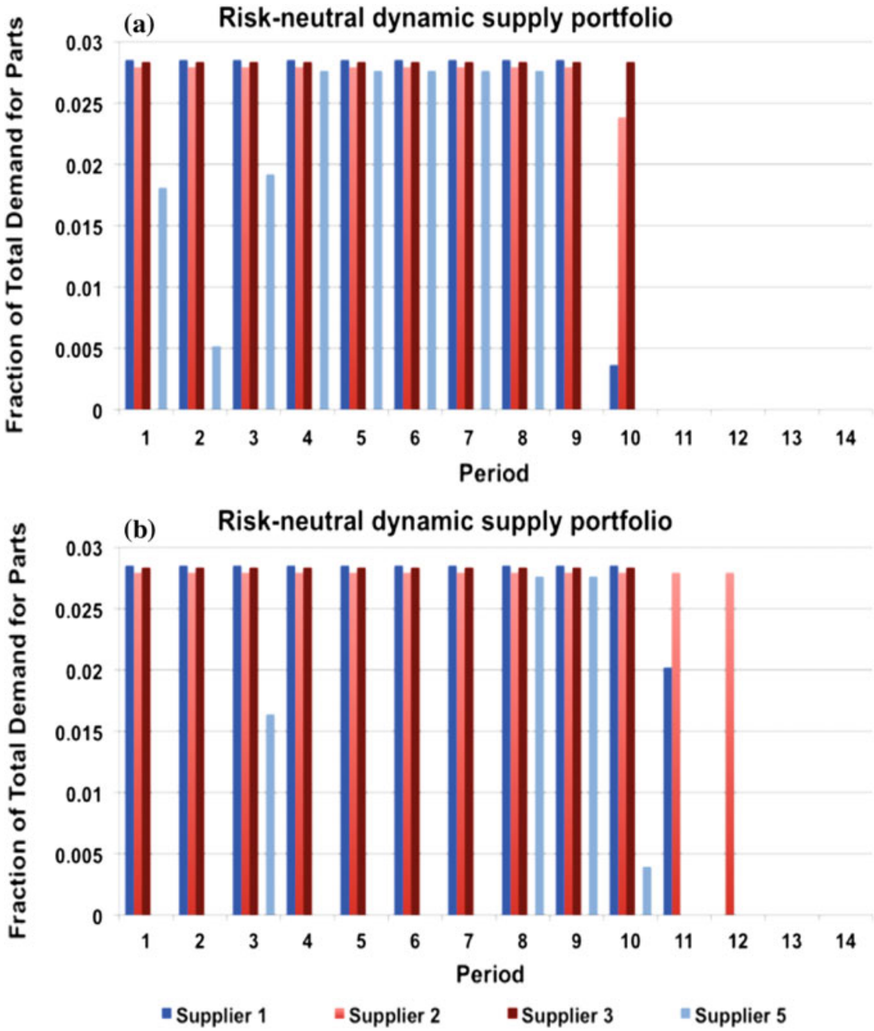

Fig. 3.2 Risk-neutral dynamic supply portfolio: **a** model **DSP_E(c)**, **b** model **DSP_E(sl)**

The risk-averse dynamic supply portfolios for $\alpha = 0.9$ and $\alpha = 0.99$ and the two models are shown in Figs. 3.4 and 3.5.

Unlike the risk-neutral dynamic supply portfolios that are leveled over the planning horizon, in particular with respect to the three most reliable suppliers $i = 1, 2, 3$, the corresponding risk-averse portfolios are more unevenly distributed over the horizon. The most unleveled are the risk-averse supply portfolios for service level objective. The only exception is the risk-averse dynamic supply portfolios for the cost-based objective and the highest confidence level $\alpha = 0.99$, which is very similar to the risk-neutral portfolio (cf. Figs. 3.2a and 3.4b). The main difference is the selection of supplier $i = 4$ as a supportive supplier for the risk-averse portfolio, with a small fraction of total demand allotted.

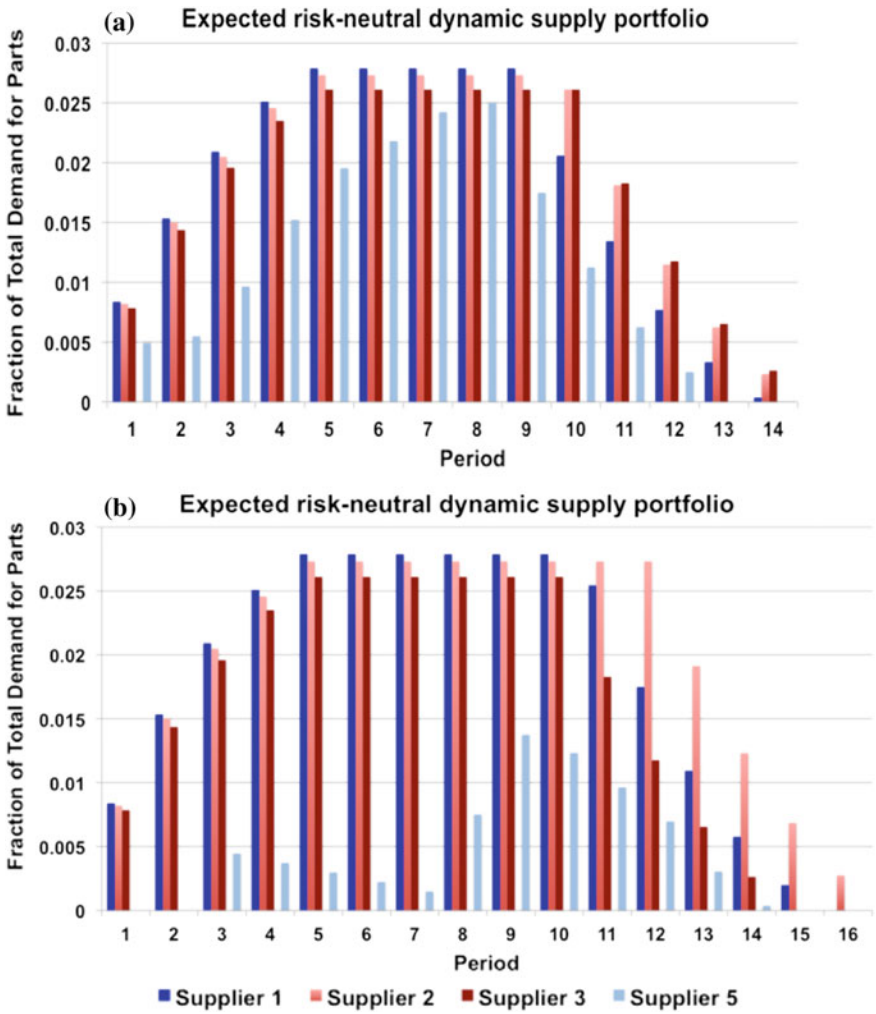

Fig. 3.3 Expected risk-neutral dynamic supply portfolio: **a** model **DSP_E(c)**, **b** model **DSP_E(sl)**

Table 3.3 Risk-neutral solutions

Model **DSP_E(c)**	Model **DSP_E(sl)**
Var. = 3505, Bin. = 5, Cons. = 3675, Nonz. = 18325 [(a)]	
$E^c = 15.76$	$E^{sl} = 95.54$
$E^{sl} = 94.76$	$E^c = 16.51$
Suppliers Selected(% of total demand)[(b)]	
1(26)	1(30)
2(28)	2(34)
3(28)	3(28)
5(18)	5(8)

[(a)] Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients.
[(b)] $1(\sum_{j\in J}\sum_{t\in T} d_j v_{1jt}/D \times 100)$,
$2(\sum_{j\in J}\sum_{t\in T} d_j v_{2jt}/D \times 100)$,
$3(\sum_{j\in J}\sum_{t\in T} d_j v_{3jt}/D \times 100)$,
$4(\sum_{j\in J}\sum_{t\in T} d_j v_{4jt}/D \times 100)$,
$5(\sum_{j\in J}\sum_{t\in T} d_j v_{5jt}/D \times 100)$.

The optimal risk-averse cost distributions for confidence level $\alpha = 0.9$ and $\alpha = 0.99$ are shown in Fig. 3.7. For $\alpha = 0.9$, larger probability atoms are concentrated at higher costs than for $\alpha = 0.99$, i.e., more risk-aversive supply portfolios better mitigate the risk of high costs.

The computational results (e.g., Sawik 2011d) indicate that

- *both cost- and service-oriented dynamic supply portfolios that simultaneously mitigate the impact of low-probability disruption risk and high-probability delay risk lead to a high expected demand fulfillment rate;*
- *as shortage-to-delay unit penalty ratio increases, more demand is moved from unreliable, low-cost suppliers to reliable, high-cost suppliers to minimize shortage cost and better mitigate the impact of disruption risk;*
- *a cost-oriented dynamic supply portfolio is better leveled over the planning horizon and is concentrated in a fewer number of periods;*
- *neglecting potential delay risks in supplier selection may lead to greater supply fluctuations and manufacturing delays,which is particularly harmful in a make-to-order and just-in-time environment.*

The proposed dynamic portfolio approach leads to time-indexed stochastic mixed integer programs with a strong LP relaxation, which has proven to be computationally very efficient.

3.6.1 Worst-case Scenarios with Longest Delays and Disruptions

In this subsection a special subset of delivery scenarios is considered such that delivery of parts from each supplier is either delayed by maximum delay length, $\overline{\tau}$, or parts are not delivered at all, because of supply disruptions. Let us call such delivery scenarios, LDN (Longest Delay-or-Never) scenarios. The total number of LDN scenarios is $2^{\overline{I}} = 32$. Given local disruption probabilities, $p_{i5},\ i \in I$, the probability for the longest delay of delivery is simply $p_{i4} = (1 - p_{i5})$, while the remaining proba-

Table 3.4 Risk-averse solutions

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **DSP_CV(c)**					
Var. = 11282, Bin. = 5, Cons. = 11451, Nonz. = 27288757 [(a)]					
CVaR^c	18.92	24.22	30.29	35.93	45.66
VaR^c	13.10	15.27	21.05	30.14	43.17
E^c	15.88	16.06	16.05	15.95	15.86
E^{sl} [(b)]	95.49	95.52	95.20	94.88	94.53
Suppliers Selected(% of total demand)[(c)]	1(31)	1(31)	1(31)	1(30)	1(26)
	2(33)	2(33)	2(33)	2(29)	2(26)
	3(26)	3(28)	3(12)	3(14)	3(23)
		4(4)	4(12)	4(14)	4(7)
	5(10)	5(4)	5(12)	5(13)	5(18)
Model **DSP_CV(sl)**					
Var. = 11282, Bin. = 5, Cons. = 11451, Nonz. = 22713877 [(a)]					
CVaR^{sl}%	96.01	92.02	85.29	80.12	69.82
VaR^{sl}%	100	100	91.87	87.50	77.77
E^{sl} [(b)]	98.00	98.00	97.29	97.43	97.04
E^c	15.56	15.48	15.86	15.85	15.61
Suppliers Selected(% of total demand)[(c)]	1(30)	1(30)	1(30)	1(29)	1(25)
	2(34)	2(34)	2(34)	2(29)	2(25)
	3(28)	3(28)	3(12)	3(14)	3(25)
		4(4)	4(12)	4(14)	
	5(8)	5(4)	5(12)	5(14)	5(25)

[(a)] Var. = no. of variables, Bin. = no. of binary variables,
Cons. = no. of constraints, Nonz. = no. of nonzero coefficients.
[(b)] $\sum_{s\in S}\sum_{i\in I_s}\sum_{j\in J}\sum_{t\in T} P_s d_j v_{ijt}/D \times 100\%$.
[(c)] $1(\sum_{j\in J}\sum_{t\in T} d_j v_{1jt}/D \times 100)$,
$2(\sum_{j\in J}\sum_{t\in T} d_j v_{2jt}/D \times 100)$,
$3(\sum_{j\in J}\sum_{t\in T} d_j v_{3jt}/D \times 100)$,
$4(\sum_{j\in J}\sum_{t\in T} d_j v_{4jt}/D \times 100)$,
$5(\sum_{j\in J}\sum_{t\in T} d_j v_{5jt}/D \times 100)$.

bilities, $p_{i0}, p_{i1}, p_{i2}, p_{i3}$ are zero for all $i \in I$. Notice that LDN delivery scenarios can be considered as pure all-or-nothing disruption scenarios, where ordered parts are either delivered with a fixed delay, $\overline{\tau}$, or not delivered at all.

The risk-neutral solutions for LDN scenarios and models **DSP_E(c)** and **DSP_E(sl)** are summarized in Table 3.5. The solution results for model **DSP_E(c)** are similar and for model **DSP_E(sl)** are identical with the corresponding results for the general delivery scenarios. The probabilities P_s for scenarios $s \in S$ with non-disrupted subsets of suppliers, I_s, are identical for both general and LDN scenarios and the service level objective is independent of delivery delays. Table 3.5 shows that the expected costs are greater for LDN scenarios (cf. Table 3.3).

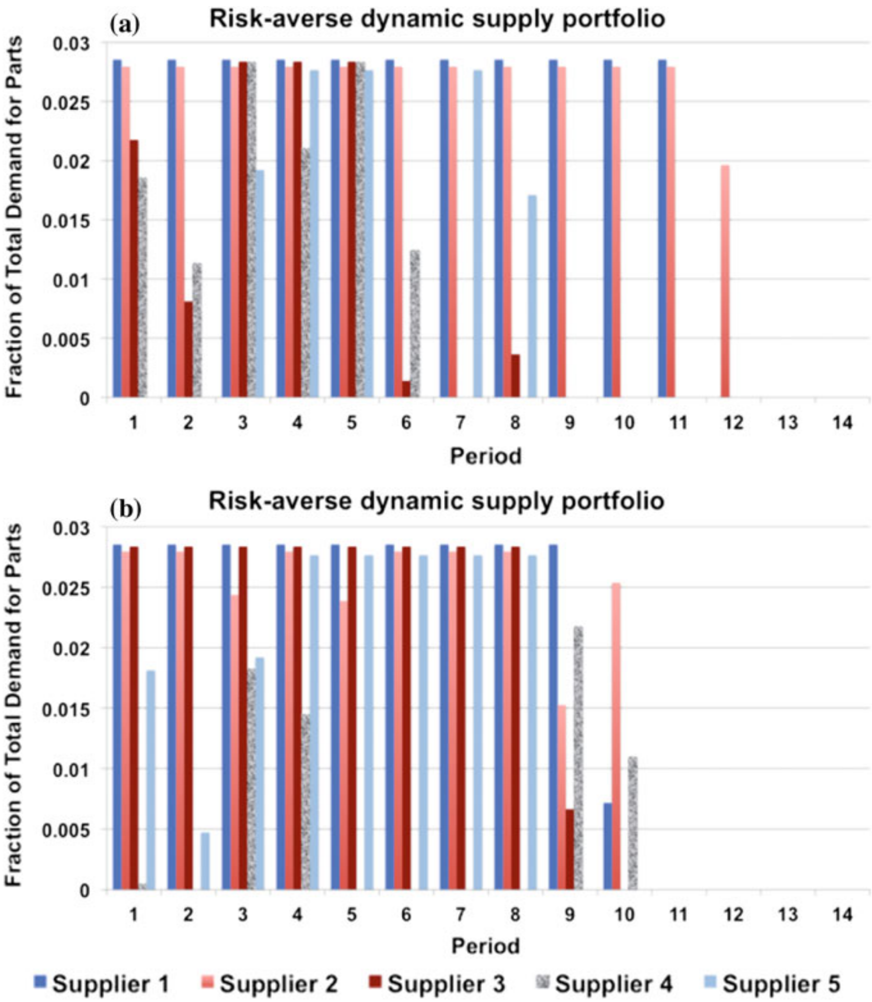

Fig. 3.4 Risk-averse dynamic supply portfolio for model **DSP_CV(c)**: $\alpha = 0.9$ and $\alpha = 0.99$

The risk-averse solutions for LDN scenarios and model **DSP_CV(c)** are summarized in Table 3.6, while Fig. 3.6 presents examples of optimal risk-averse dynamic supply portfolios for confidence levels $\alpha = 0.9$ and $\alpha = 0.99$. Table 3.6 shows that solution values for the LDN delivery scenarios are greater than the corresponding values for the general delivery scenario, while the corresponding expected service level and supply portfolios are similar. The main difference is no selection of most unreliable supplier $i = 4$ for the highest confidence level $\alpha = 0.99$ (cf. Table 3.4). The dynamic risk-averse supply portfolio for the highest confidence level, $\alpha = 0.99$, is again better leveled, however, not as well as for the general delivery scenario.

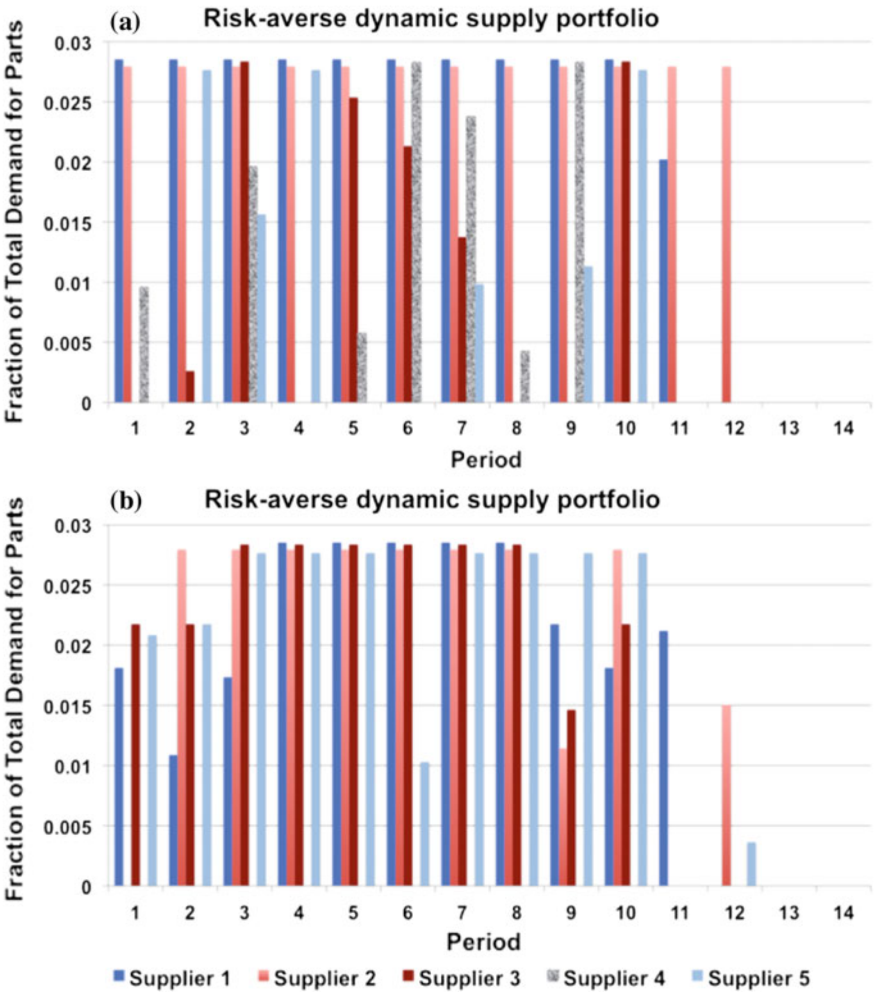

Fig. 3.5 Risk-averse dynamic supply portfolio for model **DSP_CV(sl)**: $\alpha = 0.9$ and $\alpha = 0.99$

Table 3.5 Risk-neutral solutions for LDN scenarios

Model **DSP_E(c)**	Model **DSP_E(sl)**
Var. = 3505, Bin. = 5, Cons. = 3675, Nonz. = 18325 [(a)]	
$E^c = 17.02$	$E^{sl} = 95.54$
$E^{sl} = 94.71$	$E^c = 18.22$
Suppliers Selected(% of total demand)[(b)]	
1(26)	1(30)
2(27)	2(34)
3(27)	3(28)
5(20)	5(8)

[(a)] Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients.
[(b)] $1(\sum_{j\in J}\sum_{t\in T} d_j v_{1jt}/D \times 100)$,
$2(\sum_{j\in J}\sum_{t\in T} d_j v_{2jt}/D \times 100)$,
$3(\sum_{j\in J}\sum_{t\in T} d_j v_{3jt}/D \times 100)$,
$4(\sum_{j\in J}\sum_{t\in T} d_j v_{4jt}/D \times 100)$,
$5(\sum_{j\in J}\sum_{t\in T} d_j v_{5jt}/D \times 100)$.

Table 3.6 Risk-averse solutions for LDN scenarios: model **DSP_CV(c)**

Confidence level α	0.50	0.75	0.90	0.95	0.99
Var. = 3538, Bin. = 5, Cons. = 3707, Nonz. = 130549 [(a)]					
CVaR^c	20.08	25.45	31.18	36.72	46.12
VaR^c	14.49	18.72	22.00	32.22	43.85
E^c	17.28	17.39	17.43	17.25	17.06
E^{sl} [(b)]	95.44	95.41	95.20	94.87	94.56
Suppliers Selected(% of total demand)[(c)]	1(31)	1(31)	1(31)	1(30)	1(26)
	2(33)	2(33)	2(33)	2(29)	2(26)
	3(23)	3(23)	3(12)	3(14)	3(24)
		4(6)	4(12)	4(14)	
	5(13)	5(7)	5(12)	5(13)	5(24)

[(a)] Var. = no. of variables, Bin. = no. of binary variables,
Cons. = no. of constraints, Nonz. = no. of nonzero coefficients.
[(b)] $\sum_{s\in S}\sum_{i\in I_s}\sum_{j\in J}\sum_{t\in T} P_s d_j v_{ijt}/D \times 100\%$.
[(c)] $1(\sum_{j\in J}\sum_{t\in T} d_j v_{1jt}/D \times 100)$,
$2(\sum_{j\in J}\sum_{t\in T} d_j v_{2jt}/D \times 100)$,
$3(\sum_{j\in J}\sum_{t\in T} d_j v_{3jt}/D \times 100)$,
$4(\sum_{j\in J}\sum_{t\in T} d_j v_{4jt}/D \times 100)$,
$5(\sum_{j\in J}\sum_{t\in T} d_j v_{5jt}/D \times 100)$.

The solution results for LDN scenarios and model **DSP_CV(sl)** are not shown, since they are identical with those for the general delivery scenarios presented in Table 3.4.

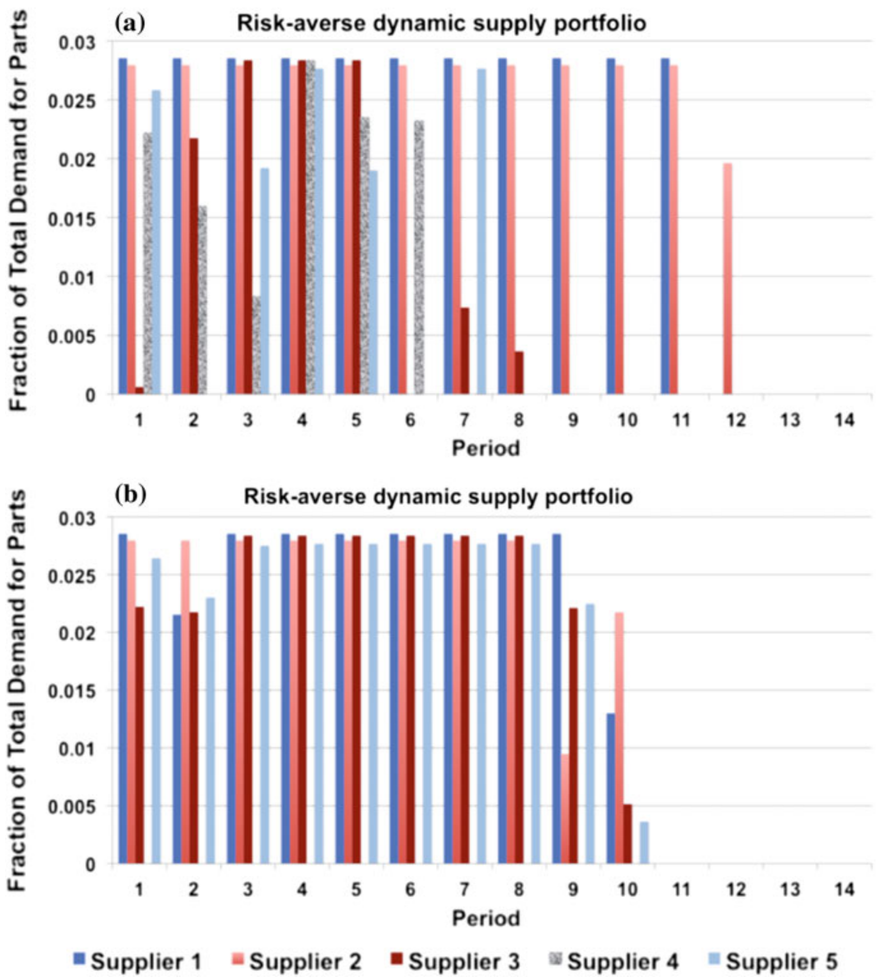

Fig. 3.6 Risk-averse dynamic supply portfolios for model **DSP_CV(c)** and LDN scenarios: **a** $\alpha = 0.9$, **b** $\alpha = 0.99$

The optimal risk-averse cost distributions for confidence level $\alpha = 0.9$ and $\alpha = 0.99$ are shown in Figs. 3.7 and 3.8, respectively for general delivery scenarios and LDN disruption scenario. Figures 3.7 and 3.8 indicate that the probability mass function of cost per part is concentrated in a few points, which is typical for the scenario-based optimization under uncertainty, where the probability measure is concentrated in finitely many points. Comparison of probability mass functions in Figs. 3.7 and 3.8 shows that for the binomial LDN disruption scenarios the probability measure is more concentrated in finitely many points than for the general multinomial delivery scenarios.

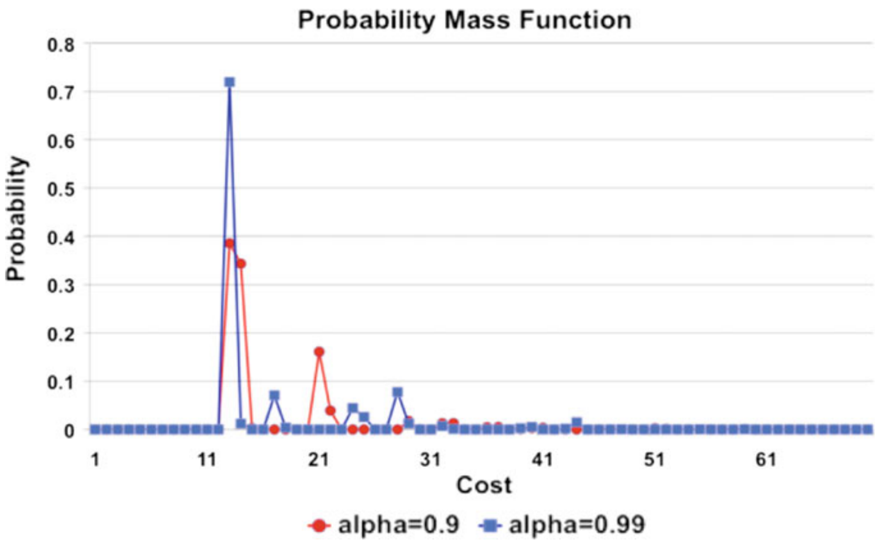

Fig. 3.7 Probability mass function for model **DSP_CV(c)**: $\alpha = 0.9$ and $\alpha = 0.99$

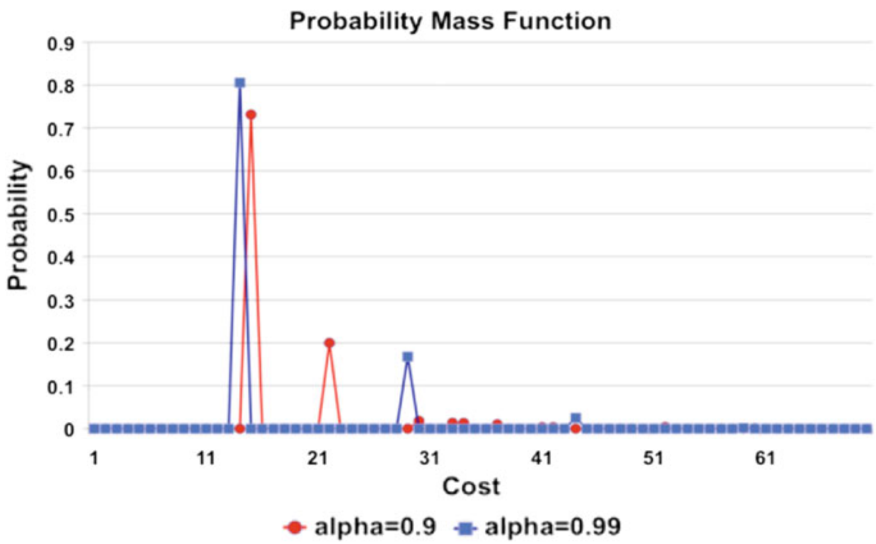

Fig. 3.8 Probability mass function for model **DSP_CV(c)** and LDN scenarios: $\alpha = 0.9$ and $\alpha = 0.99$

The computational experiments were performed using the AMPL programming language and the Gurobi 7.0.0 solver on a MacBookPro laptop with Intel Core i7 processor running at 2.8 GHz and with 16GB RAM. The solver was capable of finding proven optimal solution for all examples with CPU time ranging from fraction of a second for risk-neutral solutions to several seconds for risk-averse solutions.

3.7 Notes

The problem of a multi-period supplier selection and order quantity allocation in the presence of supply chain delay and disruption risks is very rarely reported in the literature. The decision-making requires to simultaneously consider the high probability and low impact supply delays and the low probability and high impact supply disruptions. While the delay risks may frequently arise from problems in coordinating supply and demand, (e.g., Oke and Gopalakrishnan 2009; Sawik 1977, 2011c), the disruption risks are referred to the major disruptions to normal activities caused by natural and man-made disasters. In most cases, the business impact associated with disruption risks is much greater than that of the delay risks. The high probability supply delays may have a significant impact in a make-to-order manufacturing and just-in-time environment. In particular, when all suppliers are similarly exposed to high impact disruption risks, then the low impact delay risk may predominate the supplier selection decision. For example, in Toyota supply chain, many suppliers are similarly exposed to low probability regional disruption risks due to seismic hazard in many Japanese prefectures (Marszewska 2016). Then, the supplier reliability of on-time delivery and the associated delay risk become important criteria for supplier selection in just-in-time environment. For example, to reduce dependency on external suppliers and attain the capacity to absorb supply fluctuations Toyota invested in the technology necessary to produce higher-end electronic components in-house (Ahmadjian and Lincoln 2001). The majority of models developed for supplier selection and order quantity allocation are static (single-period) models (e.g., Weber and Current 1993; Demirtas and Ustun 2008) that do not consider inventory management. The dynamic (multi-period) models are capable of considering the inventory management by lot-sizing and scheduling of orders (e.g., Ghodsypour 2001; Basnet and Leung 2005; Ustun and Demirtas 2008; Che and Wang 2008). For custom-engineered products, however, no inventory of custom parts can be kept on hand. Instead, the custom parts often need to be requisitioned with each customer order and hence the custom parts inventory need not to be considered.

The idea of a dynamic supply portfolio approach in the presence of both the low probability and high impact supply disruptions and the high probability and low impact supply delays was presented by Sawik (2011d) for a multi-period supplier selection and order quantity allocation in a make-to-order environment. For the selection of a dynamic supply portfolio, a SMIP was proposed to incorporate risks via scenario analysis. In the scenario analysis, both types of the supply chain risks are simultaneously considered. The disruption and delay risks were incorporated utilizing the concepts of percentile measures of risk, VaR and CVaR. In Sawik (2011d), the cost-based objective function was considered only and general delivery scenarios with on time, delayed or no supplies. The delays were modeled as statistically independent, discrete multivariate random variables. In addition to the general scenarios, the two special subsets of the scenarios were considered, with at most one disruption of each supplier over the horizon and with multiple consecutive disruptions of each supplier over the horizon since its first disruption. The best-case and worst-case scenarios, respectively with no delays or with the longest delays were analyzed.

In the future research, service level, (3.6), represented by the expected demand for parts fulfilled during the entire planning horizon can be replaced by the expected demand fulfilled within delivery due windows.

Problems

3.1 Modify the probability for delivery scenarios (3.2) to account for regional disruptions that may affect suppliers during a fixed time interval.

3.2 Modify the SMIP models presented in this chapter for multiple part types with subsets of part types required for each customer order and subsets of suppliers available for each part type.

3.3 Modify the SMIP models presented in this chapter to replace multiple deliveries from each supplier by a single delivery at a fixed delivery date or delivery time window.

3.4 Mixed mean-risk dynamic supply portfolio
(a) Modify model **DSP_ECV(c)** to optimize expected cost and CVaR of service level and model **DSP_ECV(sl)** to optimize expected service level and CVaR of cost.
(b) How should the values of the optimized objective functions be scaled into the interval [0,1] to avoid dimensional inconsistency among the two objectives and how should the trade-off parameter be selected?
(c) How would you interpret the mixed mean-risk dynamic supply portfolio?

3.5 In the computational examples in Sect. 3.6, Figs. 3.4 and 3.5 indicate that the cost-optimal risk-averse dynamic supply portfolio for the highest confidence level $\alpha = 0.99$ is better balanced than the corresponding service-optimal portfolio, while for a lower α, both portfolios are more similar. Why do the optimal risk-averse service-based dynamic supply portfolios have such properties for the example problem?

Chapter 4
Selection of Resilient Supply Portfolio

4.1 Introduction

In this chapter, the portfolio approach and SMIP models presented in Chap. 2 are enhanced for the combined selection and protection of part suppliers and order quantity allocation in a supply chain with disruption risks. The protection decisions include the selection of suppliers to be protected against disruptions and the allocation of emergency inventory of parts to be pre-positioned at the protected suppliers so as to maintain uninterrupted supplies in case of natural or man-made disruptive events. The decision maker needs to decide which supplier to select for parts delivery and how to allocate orders quantity among the selected suppliers, and which of the selected suppliers to protect against disruptions and how to allocate emergency inventory among the protected suppliers. The problem objective is to achieve a minimum cost of suppliers protection, emergency inventory pre-positioning, parts ordering, purchasing, transportation and shortage and to mitigate the impact of disruption risks by minimizing the potential worst-case cost. The resulting supply portfolio can be called resilient, with the supplier's resiliency defined as its capability of supplying parts in the face of disruptive events. The portfolio includes protected suppliers that are capable of fully or partially supplying parts in the face of disruptive events as well as the emergency inventory pre-positioned at the protected suppliers. Depending on the level of supplier protection, the capacity of each protected supplier remains fully or partially available under a disruptive event. The emergency inventory is used to compensate for the loss of capacity of suppliers hit by disruptions, unprotected or insufficiently protected, and to partially or fully replace non-delivered parts ordered from the disrupted suppliers. For the selection of risk-neutral, risk-averse or mean-risk supply portfolio, SMIP formulations are developed with single- or multi-level protection of fortified suppliers. In the former case, full remaining capacity of a disrupted supplier is maintained by its fortification (protection against disruptions), whereas in the latter case the remaining fraction of full capacity depends on the protection level applied. A simple protection index is introduced to evaluate the trade-off

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between the cost of suppliers protection and the estimated losses caused by supply disruptions, if no protective countermeasure is applied.

The following SMIP models are presented in this chapter:

RSP_E for risk-neutral selection of resilient supply portfolio to minimize expected cost;
RSP_CV for risk-averse selection of resilient supply portfolio to minimize CVaR of cost;
RSP_ECV for mean-risk selection of resilient supply portfolio to optimize trade-off between expected cost and CVaR of cost.
RSP(mlp)_E model **RSP_E** for multi-level protection;
RSP(mlp)_CV model **RSP_CV** for multi-level protection;
RSP(mlp)_ECV model **RSP_ECV** for multi-level protection.

The resilient supply portfolio is selected ahead of time to optimize average (risk-neutral models, **RSP_E** and **RSP(mlp)_E**), worst-case (risk-averse models, **RSP_CV** and **RSP(mlp)_CV**) or combined, average - worst-case (mean-risk models **RSP_ECV** and **RSP(mlp)_ECV**) performance of a supply chain under disruption risks.

Computational examples are provided in Sect. 4.6 to illustrate the proposed SMIP approach.

4.2 Problem Description: Single-Level Protection

In the supply chain under consideration various types of products are assembled by a single producer to satisfy customer orders, using different part types purchased from multiple suppliers (for notation used, see Table 4.1). The suppliers have different limited capacity and, in addition, differ in price and quality of offered parts. Let $I = \{1, \dots \bar{I}\}$ be the set of $\bar{I}$ suppliers and $J = \{1, \dots \bar{J}\}$ the set of $\bar{J}$ part types required for the products. Denote by d_j the demand for each part type $j \in J$ and assume that d_j is known ahead of time.

The usage per part of supplier's capacity is assumed to be different for different part types and suppliers. Let c_i be the total capacity of supplier $i \in I$ (e.g., the total number of available machine-hours) and a_{ij} the unit capacity consumption of supplier i for part type j, i.e., the amount of capacity of supplier i used to manufacture one part type j (e.g., the unit processing time).

Denote by o_{ij} the unit purchasing price, including shipping cost of part type $j \in J$ from supplier $i \in I$ (assume that for suppliers incapable of providing some part types, the corresponding unit price and unit capacity consumption are very large numbers). Let ρ_{ij} be the expected defect rate (reject rate) of supplier i for part type j. The rate

Table 4.1 Notation: single-level protection

Indices	
i	= supplier, $i \in I$
j	= part type, $j \in J$
s	= disruption scenario, $s \in S$
Input Parameters	
a_{ij}	= per unit capacity consumption of supplier i for part type j
c_i	= capacity of supplier i
d_j	= demand for part type j
D	$= \sum_{j \in J} d_j$ - total demand for parts
e_i	= cost of ordering parts from supplier i
f_i	= protection cost for supplier i
g_j	= per unit shortage cost of part type j
h_{ij}	= per unit cost of pre-positioning emergency inventory of parts type j at supplier i
o_{ij}	= unit price of part type j purchased and shipped from supplier i
p_i	= local disruption probability for supplier i
p^*	= global disruption probability for all suppliers
α	= confidence level
ν	= minimum order size
ρ_{ij}	= expected defect rate of supplier i for part type j

is based on past observations. The fixed cost of ordering parts from supplier $i \in I$ is denoted by e_i.

The supplies of parts are subject to independent random local disruptions that are uniquely associated with a particular supplier, which may arise from equipment breakdowns, local labor strike, bankruptcy, terrorist attack, from local natural disasters such as earthquakes, fires, floods, hurricanes, etc. Denote by p_i the local disruption probability for supplier i, i.e., the parts ordered from supplier i are delivered without disruptions with probability $(1 - p_i)$, or not at all with probability p_i.

In addition to independent local disruptions of each supplier, there are potential global disasters that may result in all suppliers disruption simultaneously. For example, such global super events may include economic crisis, widespread labor strike in a transportation sector, etc. Although the probability of such disaster events usually is very low, its consequences may be very high. Denote by p^* the probability of simultaneous correlated global disruption of all suppliers due to some disaster super event.

Let P_s be the probability that disruption scenario s is realized, where each scenario $s \in S$ is comprised of a unique subset $I_s \subset I$ of suppliers who deliver parts without disruptions, and $S = \{1, \ldots, \overline{S}\}$ is the index set of all scenarios. There are a total of $\overline{S} = 2^{\overline{I}}$ potential disruption scenarios.

The global disaster and the local disruptive events at each supplier are assumed to be independent events, therefore the probability P_s of each disruption scenario $s \in S$ under the risks of both type of events is

$$P_s = \begin{cases} (1-p^*)\hat{P}_s & \text{if } I_s \neq \emptyset \\ p^* + (1-p^*)\prod_{i \in I} p_i & \text{if } I_s = \emptyset, \end{cases}$$

where $\hat{P}_s$ is the probability of disruption scenario s in the presence of independent local disruptive events only

$$\hat{P}_s = \prod_{i \in I_s}(1-p_i) \cdot \prod_{i \notin I_s} p_i.$$

If the probability of global disruption $p^* = 0$, then the probability P_s reduces to $\hat{P}_s$ for independent local disruptive events.

The producer does not need to pay for ordered and undelivered parts. However, the producer is charged with a much higher cost of unfulfilled customer orders for products, caused by the shortage of parts, undelivered due to supply disruptions. Let g_j be the per unit cost of shortage of part type j.

In order to mitigate the impact of disruption risks, managers may consider protective countermeasures to fully fortify suppliers against disruptions and to maintain the suppliers normal capability. The capacity of a protected supplier is assumed to remain unchanged under a disruptive event. Therefore, for a protected supplier i its disruption probability p_i is in practice changed to zero.

The protective countermeasure of each supplier may be combined with prepositioning of emergency inventory of parts manufactured by the protected supplier. The emergency inventory is used to compensate for the loss of capacity of the other suppliers, unprotected and hit by disruptions, and to fulfill non-delivered orders placed on the disrupted suppliers. The emergency inventory pre-positioned at a protected supplier is limited, e.g., by available budget, by available storage space or by its base capacity, if the inventory is an overtime production. The inventory is replenished once it is used. Assume that the inventory quantity is linked to capacity and cannot be greater than the protected supplier normal capacity. As a result each protected supplier is capable of supplying twice as many parts as its base capacity.

Let f_i be the cost required for full protection of supplier i against disruptions, and denote by h_{ij} the per unit cost of pre-positioning the emergency inventory of part type j at the protected supplier i.

The decision maker needs to decide which supplier to select for purchasing the required parts, which of the selected suppliers to protect against disruptive events and how to allocate order quantity among the selected suppliers and emergency inventory among the protected suppliers to achieve a minimum total cost of suppliers protection, emergency inventory pre-positioning, parts ordering, purchasing, transportation, defects and shortage and to mitigate the impact of disruption risks by minimizing the potential worst-case cost.

4.3 Resilient Supply Portfolio with Single-Level Protection

In this section three SMIP models are formulated for supplier selection and protection, and order quantity allocation problem, i.e., for determining a resilient supply portfolio. For definition of problem variables, see Table 4.2.

Table 4.2 Variables: resilient supply portfolio with single-level protection

	First stage variables
q_i	$= 1$, if selected supplier i is protected against disruptions; otherwise $q_i = 0$ (supplier protection variable)
u_i	$= 1$, if an order for parts is placed on supplier i; otherwise $u_i = 0$ (supplier selection variable)
v_{ij}	= the fraction of demand for parts type j ordered from unprotected supplier i (part type demand unprotected allocation variable)
w_{ij}	= the fraction of demand for parts type j ordered from protected supplier i (part type demand protected allocation variable)
x_{ij}	= emergency inventory of parts type j pre-positioned at protected supplier i, in fraction of supplier's capacity c_i (inventory allocation variable)
	Second stage variables
y_{ijs}	= emergency inventory of parts type j pre-positioned at protected supplier i, used under disruption scenario s (inventory usage variable)
z_{js}	= inventory shortage of parts type j under disruption scenario s
	Auxiliary variables
VaR^c	Cost-at-Risk, the targeted cost such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is below VaR^c
$\mathscr{C}_s$	≥ 0, the tail cost for scenario s, i.e., the amount by which costs in scenario s exceed VaR^c

The resilient supply portfolio is selected ahead of time in such a way as to minimize the potential average or worst-case cost (or a combination of both) under different disruption scenarios. The capacity of protected suppliers and the emergency inventory pre-positioned at the protected suppliers are used to reduce the potential highest costs.

When deciding on a supply portfolio it is assumed that the orders for all parts are simultaneously placed on selected suppliers, and each protected and each non-disrupted unprotected supplier delivers all the ordered parts. However, unprotected suppliers hit by disruptions fail to deliver the ordered parts, and then the non delivered orders are partially or fully replaced by the emergency inventory of parts pre-positioned at the protected suppliers. The decision on the protection of selected suppliers and the pro-positioning of emergency inventory at the protected suppliers is a part of the resilient supply portfolio decision-making.

4.3.1 Model for Risk-Neutral Decision-Making

In this subsection a SMIP model **RSP_E** is presented for selection of risk-neutral resilient supply portfolio. In the risk-neutral decision-making the overall quality of the supply portfolio can be measured by the expected cost per part, E^c (4.1), of suppliers protection, $\sum_{i\in I} f_i q_i/D$, and emergency inventory pre-positioning, $\sum_{i\in I}\sum_{j\in J} h_{ij}c_i x_{ij}/a_{ij}D$, of parts ordering, $\sum_{i\in I} e_i u_i/D$, of parts purchasing and shipping, including defects, plus replacement delivery of parts pre-positioned, in which the cost of a defective part is assumed to be identical with its price, $\sum_{i\in I}\sum_{j\in J} o_{ij}(d_j v_{ij}/(1-\rho_{ij}) + d_j w_{ij}/(1-\rho_{ij}) + \sum_{s\in S} P_s c_i y_{ijs}/a_{ij})/D$, and finally cost of shortage of parts due to supply disruptions less cost of non delivered parts, $\sum_{s\in S} P_s(\sum_{j\in J} g_j z_{js} - \sum_{i\notin I_s}\sum_{j\in J} o_{ij} d_j v_{ij}/(1-\rho_{ij}))/D$.

The purchase orders for parts are assumed to be inflated by the reject rates ρ_{ij} of defective parts, i.e., are equal to $d_j v_{ij}/(1-\rho_{ij})$ and $d_j w_{ij}/(1-\rho_{ij})$, for all $i \in I, j \in J$, respectively for unprotected and protected suppliers.

Note, that if an unprotected supplier is subject to a disruptive event and fails to deliver the ordered parts, the non delivered parts can be replaced by emergency inventory pre-positioned at protected suppliers. Then, the amount $d_j v_{ij}/(1-\rho_{ij})$ of part type j ordered from unprotected and failed supplier $i \notin I_s$ can be fully or partially met with the emergency inventory $\sum_{i\in I} c_i x_{ij}/a_{ij}$. The resilient supply portfolio aims at reducing potential losses from non delivered parts through the optimal selection of suppliers for protection, the allocation among them the emergency inventory and the allocation of orders for parts among both the unprotected and protected suppliers.

The SMIP model **RSP_E** for selection of risk-neutral resilient supply portfolio is formulated below. In the proposed model, the portfolio will be optimized by minimizing expected cost per part E^c, (4.1).

RSP_E: *Selection of risk-neutral* **R***esilient* **S***upply* **P***ortfolio to minimize expected cost*

Minimize Expected Cost per Part

$$E^c = \sum_{i\in I} e_i u_i/D + \sum_{i\in I} f_i q_i/D + \sum_{i\in I}\sum_{j\in J} h_{ij} c_i x_{ij}/a_{ij} D + \sum_{i\in I}\sum_{j\in J} o_{ij} d_j (v_{ij} + w_{ij})/(1-\rho_{ij}) D + \sum_{s\in S} P_s (\sum_{j\in J} g_j z_{js} + \sum_{i\in I}\sum_{j\in J} o_{ij} c_i y_{ijs}/a_{ij} - \sum_{i\notin I_s}\sum_{j\in J} o_{ij} d_j v_{ij}/(1-\rho_{ij}))/D \quad (4.1)$$

subject to

1. Supplier selection and protection constraints:

– only selected suppliers can be protected, and emergency inventory can be pre-positioned only at protected suppliers,

– the orders for parts can be allocated among selected suppliers only,

– each selected supplier is either protected or unprotected and so are the corresponding types of orders for parts allocated among the suppliers,

$$\sum_{j \in J} x_{ij} \le q_i \le u_i; \;\; i \in I \tag{4.2}$$

$$v_{ij} \le u_i; \;\; i \in I, j \in J \tag{4.3}$$

$$v_{ij} \le 1 - q_i; \;\; i \in I, j \in J \tag{4.4}$$

$$w_{ij} \le q_i; \;\; i \in I, j \in J \tag{4.5}$$

2. Order quantity and emergency inventory allocation constraints:

– orders for all required parts of each type must be allocated among selected suppliers,

– for each selected supplier (unprotected and protected), the total capacity required to manufacture the ordered quantities of parts cannot exceed available capacity,

– the ordered quantity assigned to each selected supplier (unprotected or protected) cannot be less than the minimum order size,

– the emergency inventory used to replace non-delivered units cannot exceed the pre-positioned emergency inventory,

– for each part type and disruption scenario, non delivered orders can (no inventory shortage, $z_{js} = 0$) or cannot (inventory shortage, $z_{js} > 0$) be fully replaced by using the emergency inventory,

$$\sum_{i \in I} (v_{ij} + w_{ij}) = 1; \;\; j \in J \tag{4.6}$$

$$\sum_{j \in J} a_{ij} d_j v_{ij}/(1 - \rho_{ij}) \le c_i(u_i - q_i); \;\; i \in I \tag{4.7}$$

$$\sum_{j \in J} a_{ij} d_j w_{ij}/(1 - \rho_{ij}) \le c_i q_i; \;\; i \in I \tag{4.8}$$

$$\sum_{j \in J} d_j v_{ij}/(1 - \rho_{ij}) \ge \nu(u_i - q_i); \;\; i \in I \tag{4.9}$$

$$\sum_{j \in J} (d_j w_{ij}/(1 - \rho_{ij}) + c_i y_{ijs}/a_{ij}) \ge \nu q_i; \;\; i \in I, s \in S \tag{4.10}$$

$$y_{ijs} \le x_{ij}; \;\; i \in I, j \in J, s \in S \tag{4.11}$$

$$z_{js} = \sum_{i \notin I_s} d_j v_{ij}/(1 - \rho_{ij}) - \sum_{i \in I} c_i y_{ijs}/a_{ij}; \;\; j \in J, s \in S \tag{4.12}$$

3. Non-negativity and integrality conditions

$$q_i \in \{0, 1\};\ \ i \in I \tag{4.13}$$

$$u_i \in \{0, 1\};\ \ i \in I \tag{4.14}$$

$$v_{ij} \in [0, 1];\ \ i \in I, j \in J \tag{4.15}$$

$$w_{ij} \in [0, 1];\ \ i \in I, j \in J \tag{4.16}$$

$$x_{ij} \in [0, 1];\ \ i \in I, j \in J \tag{4.17}$$

$$y_{ijs} \in [0, 1];\ \ i \in I, j \in J, s \in S \tag{4.18}$$

$$z_{js} \geq 0;\ \ j \in J, s \in S. \tag{4.19}$$

Denote by

$$(V_1, \ldots, V_{\overline{I}}),$$

the supply portfolio, where $\sum_{i \in I} V_i = 1$ and $0 \leq V_i \leq 1$ is the fraction of the total demand for parts ordered from supplier i. The resilient supply portfolio is based on demand allocation, does not account for reject rates and is determined by the order quantity allocation variables v_{ij}, w_{ij}

$$V_i = \sum_{j \in J} d_j (v_{ij} + w_{ij})/D;\ \ i \in I. \tag{4.20}$$

Alternatively, to account for the actual supplies of parts and the reject rates, the supply portfolio $(V_1', \ldots, V_{\overline{I}}')$ can be calculated as below.

$$V_i' = \frac{\sum_{j \in J} d_j (v_{ij} + w_{ij})/(1 - \rho_{ij})}{\sum_{i \in I} \sum_{j \in J} d_j (v_{ij} + w_{ij})/(1 - \rho_{ij})};\ \ i \in I. \tag{4.21}$$

Note that the nonnegative inventory shortage variables z_{js} can be eliminated from the model using Eq. (4.12). Then, variables z_{js} in the objective function (4.1) should be replaced by

$$\sum_{i \notin I_s} d_j v_{ij}/(1 - \rho_{ij}) - \sum_{i \in I} c_i y_{ijs}/a_{ij},$$

and equality constraints (4.12), replaced with inequality constraints

$$\sum_{i \in I} c_i y_{ijs}/a_{ij} \leq \sum_{i \notin I_s} d_j v_{ij}/(1 - \rho_{ij});\ \ j \in J, s \in S.$$

The inventory shortage variables z_{js} are slack variables for the last constraints.

Similarly, slack variables for inequality constraints (4.11) represent surplus of the pre-positioned inventory $(x_{ij} - y_{ijs})$ of each part type j at each supplier i, under each disruption scenario $s \in S$. Given optimal supply portfolio, the actual usage of emergency inventory depends on the realized disruption scenario and the corresponding needs for a replacement of non delivered orders from disrupted suppliers. Depending on the realized disruption scenario, the pre-positioned inventory can be used fully, partially or not at all.

In the proposed model the required parts of each type are assumed to be partially provided by one or more suppliers and the order allocation variables v_{ij} or w_{ij} represent the fraction of all required parts of type j provided, respectively by unprotected or protected supplier i. In some practical cases all parts of the same type are purchased from a single supplier. Then, the corresponding continuous allocation variables v_{ij} and w_{ij} should be redefined as binary assignment variables denoting whether or not all parts of type j are provided, respectively by unprotected or protected supplier i.

4.3.2 Model for Risk-Averse Decision-Making

In the selection of a resilient supply portfolio under disruption risks, the decision maker controls the risk of high losses due to supply disruptions by choosing the confidence level α. We assume that the decision maker is willing to accept only portfolios for which the total probability of scenarios with costs greater than VaR^c is not greater than $1 - \alpha$. The greater the confidence level α, the more risk aversive is the decision maker and the smaller percent of the highest cost outcomes is focused on. Moreover, a risk aversive decision maker wants to minimize the expected worst-case costs exceeding VaR^c, by minimizing CVaR^c. When using CVaR^c to minimize worst-case costs, CVaR^c is always not less than VaR^c.

Define $\mathscr{C}_s$ as the tail cost for scenario s, where tail cost is defined as the amount by which costs in scenario s exceed VaR^c. The portfolio will be optimized by calculating VaR^c and minimizing CVaR^c simultaneously. By measuring CVaR^c, the magnitude of the tail costs is considered to achieve a more accurate estimate of the risks of minimizing cost. In the proposed model, CVaR^c is represented by an auxiliary function (4.22) introduced by Rockafellar and Uryasev (2000). The SMIP model **RSP_CV** for selection of risk-averse resilient supply portfolio to reduce the risk of high costs is formulated below.

RSP_CV: *Selection of risk-averse resilient supply portfolio to minimize CVaR of cost*

Minimize

$$CVaR^c = VaR^c + (1-\alpha)^{-1} \sum_{s \in S} P_s \mathscr{C}_s \tag{4.22}$$

subject to

1. Supplier selection and protection constraints: (4.2)–(4.5)

2. Order quantity and emergency inventory allocation constraints: (4.6)–(4.12)

3. Risk constraints:

– the tail cost for scenario s is defined as the nonnegative amount by which cost per part in scenario s exceeds VaR^c,

$$\begin{aligned}\mathscr{C}_s \geq \sum_{i\in I} e_i u_i/D + \sum_{i\in I} f_i q_i/D + \sum_{i\in I}\sum_{j\in J} h_{ij} c_i x_{ij}/a_{ij} D \\ + \sum_{i\in I}\sum_{j\in J} o_{ij} d_j (v_{ij} + w_{ij})/(1-\rho_{ij}) D \\ +(\sum_{j\in J} g_j z_{js} + \sum_{i\in I}\sum_{j\in J} o_{ij} c_i y_{ijs}/a_{ij} - \sum_{i\notin I_s}\sum_{j\in J} o_{ij} d_j v_{ij}/(1-\rho_{ij}))/D \\ -VaR^c;\ s\in S\end{aligned} \tag{4.23}$$

4. Non-negativity and integrality conditions: (4.13)–(4.19) and

$$\mathscr{C}_s \geq 0;\ s\in S. \tag{4.24}$$

Note that, if for some part type j all required parts must be supplied by a single supplier, then the corresponding nonnegative allocation variables $v_{ij},\ w_{ij},\ i\in I$ should be redefined to be binary assignment variables, similarly as for **RSP_E** model.

4.3.3 Model for Mean-Risk Decision-Making

In the single objective approach the resilient supply portfolio is selected by minimizing either the expected cost per part, E^c, (4.1) or the expected worst-case cost per part, $CVaR^c$, (4.22). In this subsection the two cost functions are considered simultaneously, and a bi-objective selection of resilient supply portfolio is presented aimed at minimizing both objective functions to balance expected costs with the risk tolerance. This type of trade-off model is known as the mean-risk model (e.g. Ogryczak and Ruszczynski 2002), formulated as the optimization of a composite objective consisting of the expected cost and the conditional-cost-at-risk as a risk measure.

The nondominated solution set of the bi-objective resilient supply portfolio can be found by the parameterization on λ the weighted-sum program **RSP_ECV** presented below. The scalarizing SMIP model is based on **RSP_CV** model with the addition of objective (4.1) of model **RSP_E**.

RSP_ECV: *Selection of mean-risk resilient supply portfolio to minimize weighted sum of expected cost and CVaR of cost*

Minimize

$$\lambda E^c + (1-\lambda)CVaR^c \quad (4.25)$$

where $0 \le \lambda \le 1$,

subject to (4.1)–(4.19), (4.22)–(4.24).

4.4 Protection Index

There are many alternative countermeasures that decision maker must consider to manage the risk of supply disruptions. For each protective countermeasure being considered, it is important to determine the cost of the countermeasure and the estimated loss to be incurred if a supply disruption occurs. For example, if a protective wall is built to protect a supplier against flooding, the decision maker must consider the construction cost against the estimated loss caused by potential flooding that may disrupt supplies of materials, if no protective wall is build. The trade-off between the cost of suppliers protection against potential disruptions and the losses caused by supply disruptions can be evaluated, for example, by the following unit protection cost

$$\varphi = \frac{\sum_{i\in I} f_i}{\sum_{j\in J} g_j d_j}. \quad (4.26)$$

The unit protection cost is the ratio of total protection cost to total loss resulted from the shortage of parts caused by the potential supply disruptions. Given demand for parts, the unit protection cost, φ, increases with suppliers protection costs, f_i, $i \in I$, and decreases with parts shortage costs, $g_j, j \in J$.

To ensure delivery of all required parts under disruption risks it is not necessary to protect all suppliers against disruptions and only a subset of suppliers needs to be protected. As a result the actual value of unit protection cost can be less than (4.26), and its optimal value φ^o can be found as a solution to the following mixed integer program.

$$\varphi^o = \min\{\frac{\sum_{i\in I} f_i q_i}{\sum_{j\in J} g_j d_j} : \sum_{j\in J} a_{ij} d_j w_{ij}/(1-\rho_{ij}) \le 2c_i q_i; i \in I,$$
$$\sum_{i\in I} w_{ij} = 1; j \in J,\ w_{ij} \le q_i; i \in I, j \in J,$$
$$q_i \in \{0, 1\}; i \in I,\ w_{ij} \in [0, 1]; i \in I, j \in J\}. \quad (4.27)$$

The optimal value φ^o is calculated under global disruption scenario, assuming no supplies from unprotected suppliers and a full level of emergency inventory pre-positioned at each protected supplier. As a result a double capacity is available at each protected supplier and no capacity at each unprotected supplier.

The unit protection cost, however, does not involve any information on the estimated probability of potential disruptive events. It is intuitively believed, that the higher the probability of a disruptive event and the higher the potential losses, the higher the protection cost, the decision maker is willing to cover. In addition to the above unit protection cost, φ, the following protection index Φ is introduced in which the protection cost of each supplier i, f_i, is additionally divided by its disruption probability, $(p^* + (1-p^*)p_i)$

$$\Phi = \frac{\sum_{i \in I} f_i/(p^* + (1-p^*)p_i)}{\sum_{j \in J} g_j d_j}. \tag{4.28}$$

The protection index is the ratio of total weighted (multiplied by $1/(p^* + (1-p^*)p_i)$) protection cost to total loss resulted from the shortage of parts caused by supply disruptions. Given demand for parts, the protection index, Φ, increases with supplier protection cost per disruption probability, $f_i/(p^* + (1-p^*)p_i)$, $i \in I$, and decreases with parts shortage costs, $g_j, j \in J$.

The optimal value Φ^o of protection index can be found as a solution to the following mixed integer program, in which no supplies from unprotected suppliers and a full level of emergency inventory pre-positioned at each protected supplier are assumed.

$$\begin{aligned} \Phi^o = \min\{ & \frac{\sum_{i \in I} (f_i/(p^* + (1-p^*)p_i))q_i}{\sum_{j \in J} g_j d_j} : \\ & \sum_{j \in J} a_{ij} d_j w_{ij}/(1-\rho_{ij}) \le 2c_i q_i;\ i \in I, \\ & \sum_{i \in I} w_{ij} = 1;\ j \in J,\ w_{ij} \le q_i;\ i \in I, j \in J, \\ & q_i \in \{0,1\};\ i \in I,\ w_{ij} \in [0,1];\ i \in I, j \in J\}. \end{aligned} \tag{4.29}$$

Both the unit protection cost and the protection index can be used for a rough evaluation of different protection strategies.

4.5 Resilient Supply Portfolio with Multi-Level Protection

One of the main assumptions in the models presented in Sect. 4.3 is that the capacity of a fortified supplier remains unchanged after any disruptive event and hence the impacted supplier is capable of full delivering all ordered parts. However, this assumption may not be always realistic in practice. For example, fortifying a supplier against an earthquake may not protect it sufficiently well against a much stronger earthquake or a combined earthquake and flooding, etc., and hence the capacity of an impacted supplier can be actually reduced.

In this section, an enhancement of proposed models is considered assuming that the impact of disruptive events on suppliers capacity may not be fully mitigated by the fortification of a supplier, based on the level of protection investments. In the models presented below, the suppliers can be fortified at different discrete levels of protection, each of which has its own fortification cost and the associated capacity available after a disruptive event. The remaining capacity available depends on the protection level applied. The higher the protection level the higher the remaining capacity in the aftermath of a disruptive event. However, the maximum amount of emergency inventory pre-positioned at a fortified supplier is bounded by its full capacity, independent of the protection level.

The new parameters and variables are introduced in Table 4.3. Denote by $l \in L_i = \{1, \ldots, \overline{L_i}\}$, the protection level of supplier i, which refers to the fortification cost and the fraction of supplier's full capacity available after occurrence of a disruptive event. Level $l = \overline{L_i}$ represents the highest available protection for supplier i against disruptions and refers to the highest fortification cost and the highest fraction of remaining capacity. Let $\hat{f}_{il}$ and γ_{il} be the fortification cost and the remaining fraction of full capacity for supplier i protected at level l, where

$$0 < \hat{f}_{i1} < \hat{f}_{i2} < \cdots < \hat{f}_{i\overline{L_i}}$$

and

$$0 < \gamma_{i1} < \gamma_{i2} < \cdots < \gamma_{i\overline{L_i}} \leq 1.$$

Table 4.3 Notation: multi-level protection

	Indices
l	= protection level, $l \in L_i$, $i \in I$
	Parameters
$\hat{f}_{il}$	= fortification cost for supplier i protected at level l
γ_{il}	= the fraction of full capacity of supplier i protected at level l, available under disruptive event
	First stage variables
Q_{il}	= 1, if selected supplier i is protected at level l; otherwise (supplier protection level variable)
W_{ijl}	= the fraction of demand for parts type j ordered from supplier i protected at level l (part type demand allocation variable)

A supplier i protected at level l delivers fraction γ_{il} of all ordered parts, under a disruptive event. The non delivered parts can be replaced by the emergency inventory pre-positioned at protected suppliers. The non delivered amount, $\sum_{i\in I} d_j(1-\gamma_{il})W_{ijl}/(1-\rho_{ij})$, of each part type j can be fully or partially met with the emergency inventory $\sum_{i\in I} c_i x_{ij}/a_{ij}$.

The enhancement **RSP(mlp)_E** of model **RSP_E** is shown below.

RSP(mlp)_E: *Selection of risk-neutral resilient supply portfolio with multi-level protection*

Minimize

$$\hat{E}^c = \sum_{i\in I} e_i u_i/D + \sum_{i\in I}\sum_{l\in L_i} \hat{f}_{il} Q_{il}/D + \sum_{i\in I}\sum_{j\in J} h_{ij} c_i x_{ij}/a_{ij} D + \sum_{i\in I}\sum_{j\in J} o_{ij} d_j (v_{ij} + \sum_{l\in L_i} W_{ijl})/(1-\rho_{ij})D + \sum_{s\in S} P_s(\sum_{j\in J} g_j z_{js} + \sum_{i\in I}\sum_{j\in J} o_{ij} c_i y_{ijs}/a_{ij} - \sum_{i\notin I_s}\sum_{j\in J} o_{ij} d_j v_{ij}/(1-\rho_{ij}) - \sum_{i\notin I_s}\sum_{j\in J}\sum_{l\in L_i} o_{ij} d_j (1-\gamma_{il}) W_{ijl}/(1-\rho_{ij}))/D \tag{4.30}$$

subject to (4.3), (4.11), (4.14), (4.15), (4.17)–(4.19) and

$$\sum_{j\in J} x_{ij} \le \sum_{l\in L_i} Q_{il} \le u_i;\ i \in I \tag{4.31}$$

$$v_{ij} \le 1 - \sum_{l\in L_i} Q_{il};\ i \in I, j \in J \tag{4.32}$$

$$W_{ijl} \le Q_{il};\ i \in I, j \in J, l \in L_i \tag{4.33}$$

$$\sum_{i\in I}(v_{ij} + \sum_{l\in L_i} W_{ijl}) = 1;\ j \in J \tag{4.34}$$

$$\sum_{j\in J} a_{ij} d_j v_{ij}/(1-\rho_{ij}) \le c_i(u_i - \sum_{l\in L_i} Q_{il});\ i \in I \tag{4.35}$$

$$\sum_{j\in J} a_{ij} d_j W_{ijl}/(1-\rho_{ij}) \le c_i Q_{il};\ i \in I, l \in L_i \tag{4.36}$$

$$\sum_{j\in J} d_j v_{ij}/(1-\rho_{ij}) \ge \nu(u_i - \sum_{l\in L_i} Q_{il});\ i \in I \tag{4.37}$$

$$\sum_{j\in J}\sum_{l\in L_i}(d_j W_{ijl}/(1-\rho_{ij}) + c_i y_{ijs}/a_{ij}) \ge \nu \sum_{l\in L_i} Q_{il};\ i \in I, s \in S \tag{4.38}$$

$$z_{js} = \sum_{i \notin I_s} d_j v_{ij}/(1-\rho_{ij}) + \sum_{i \notin I_s} \sum_{l \in L_i} d_j (1-\gamma_{il}) W_{ijl}/(1-\rho_{ij}) - \sum_{i \in I} c_i y_{ijs}/a_{ij};\ j \in J, s \in S \quad (4.39)$$

$$Q_{il} \in \{0,1\};\ i \in I, l \in L_i \quad (4.40)$$

$$W_{ijl} \in [0,1];\ i \in I, j \in J, l \in L_i. \quad (4.41)$$

In the objective function (4.30), $\sum_{i\in I}\sum_{l\in L_i}\hat{f}_{il}Q_{il}$, is the total fortification cost of protected suppliers, and the last subtracted term, $\sum_{s\in S}P_s\sum_{i\notin I_s}\sum_{j\in J}\sum_{l\in L_i}o_{ij}d_j(1-\gamma_{il})W_{ijl}/(1-\rho_{ij})$, is the expected cost of non-delivered parts by protected suppliers.

In model **RSP(mlp)_E**, constraints (4.31) and (4.33) ensure that each supplier can be protected at most at one level, and the allocation of demand for parts among the fortified suppliers accounts for their protection levels. Since each supplier i can be protected at most at one level l, (4.31), at most one variable W_{ijl}, (4.33), may take on a positive value for each part type j. Equation (4.39) defines shortage of each part type inventory under each disruptive event, due to undelivered parts by both unprotected and protected suppliers. Equation (4.39) is equivalent to (4.12) in model **RSP_E**, where a fortified supplier was capable of full delivering of ordered parts under disruptive event.

Notice that model **RSP(mlp)_E** can be derived from **RSP_E** by replacing variables $q_i, i \in I$ and $w_{ij}, i \in I, j \in J$ by sums over all protection levels $l \in L_i$ of the new variables Q_{il} and W_{ijl}: $q_i = \sum_{l\in L_i} Q_{il};\ i \in I$ and $w_{ij} = \sum_{l\in L_i} W_{ijl};\ i \in I, j \in J$. In addition, the objective function (4.1) and constraint (4.12) have been modified to account for undelivered parts by protected suppliers, and Eqs. (4.5) and (4.8) redefined for each protection level.

The enhancements, **RSP(mlp)_CV** and **RSP(mlp)_ECV**, respectively of risk-averse model **RSP_CV** and mean-risk model **RSP_ECV** are presented below.

RSP(mlp)_CV: *Selection of risk-averse resilient supply portfolio with multi-level protection*

Minimize (4.22)

subject to (4.3), (4.11), (4.14), (4.15), (4.17)–(4.19), (4.22), (4.31)–(4.41) and

$$\mathscr{C}_s \geq \sum_{i\in I} e_i u_i/D + \sum_{i\in I}\sum_{l\in L_i}\hat{f}_{il}Q_{il}/D + \sum_{i\in I}\sum_{j\in J} h_{ij}c_i x_{ij}/a_{ij}D + \sum_{i\in I}\sum_{j\in J} o_{ij}d_j(v_{ij} + \sum_{l\in L_i} W_{ijl})/(1-\rho_{ij})D$$

$$+(\sum_{j \in J} g_j z_{js} + \sum_{i \in I} \sum_{j \in J} o_{ij} c_i y_{ijs}/a_{ij} - \sum_{i \notin I_s} \sum_{j \in J} o_{ij} d_j v_{ij}/(1-\rho_{ij})$$

$$-\sum_{i \notin I_s} \sum_{j \in J} \sum_{l \in L_i} o_{ij} d_j (1-\gamma_{il}) W_{ijl}/(1-\rho_{ij}))/D - VaR^c; \ s \in S. \quad (4.42)$$

RSP(mlp)_ECV: *Selection of mean-risk resilient supply portfolio with multi-level protection*

Minimize

$$\lambda \hat{E}^c + (1-\lambda) CVaR^c \quad (4.43)$$

where $0 \leq \lambda \leq 1$,

subject to (4.3), (4.11), (4.14), (4.15), (4.17)–(4.19), (4.22), (4.24), (4.30)–(4.41).

4.6 Computational Examples

In this section some computational examples are presented to illustrate possible applications of the proposed SMIP approach for the selection and protection of suppliers and order quantity allocation in a supply chain under disruption risks. First, the examples with single protection levels are presented and then, for comparison, applications of models for multi-level protection are illustrated. The following parameters have been used for the example problems:

- $\overline{I}$, the number of suppliers, was equal to 10 and the corresponding number $\overline{S} = 2^{\overline{I}}$ of disruption scenarios, was equal to 1024;
- $\overline{J}$, the number of part types, was equal to 25;
- a_{ij}, the unit capacity consumptions were integers in $\{1, 2, 3\}$ drawn from int(U[1;3]) distribution, for all suppliers i and part types j;
- c_i, the capacity of each supplier i, was integer drawn from $1000\lceil(\sum_{j \in J} a_{ij} d_j / \overline{I}) U[0.75; 1.25]/1000\rceil$ distribution ($\lceil \cdot \rceil$ denotes the smallest integer not less than $\cdot$);
- d_j, the required numbers of parts, were integers in $\{5000, 10000, 15000, 20000, 25000, 30000, 35000, 40000, 45000, 50000\}$ drawn from 5000int(U[1; 10]) distribution, for all part types j;
- e_i, the cost of ordering parts, were integers in $\{500, 600, 700, 800, 900, 1000\}$ drawn from 100int(U[5;10]) distribution, for all suppliers i;

- f_i, the protection costs, were integers in $\{500000, 550000, 600000, 650000,$ $700000, 750000, 800000, 850000, 900000\}$ drawn from 50000int(U[10;18]) distribution, for all suppliers i;
- g_j, the unit shortage costs were integers in $\{70, 80, 90, 100\}$ drawn from 10int(U[7;10]) distribution, for all part types j;
- h_{ij}, the unit cost of emergency inventory of part type j pre-positioned at supplier i, was equal to $0.4o_{ij}$ for all suppliers i and part types j;
- o_{ij}, the unit price (including shipping cost) of part type j purchased and transported from each supplier i, was uniformly distributed over [10,15] (i.e. drawn from U[10;15]) and reduced by the factor $(1 - \rho_{ij})$ to get a lower price for parts from the suppliers with a higher defect rate;
- ρ_{ij}, the expected defect rate of each supplier i for each part type j, was exponentially distributed, ranging from 0.0003 to 0.03;
- ν, the minimum order size, was equal to 500;
- p_i, the local disruption probability was uniformly distributed over [0,0.06] or over [0.06,0.15], i.e., the disruption probabilities were drawn independently from U[0;0.06] or from U[0.06;0.15]. The global disruption probability p^*, was either 0.001 or 0.01, respectively;
- α, the confidence level, was equal to 0.50, 0.75, 0.90, 0.95 or 0.99.

The computational experiments were performed for the same replication of the above input data set. The resulting total demand for parts was $D = \sum_{j \in J} d_j =$ $740,000$, for all test examples. The two different combinations of global and local disruption probabilities were considered: $p^* = 0.001$ and $p_i \in U[0; 0.06]$, $i \in I$ for reliable suppliers and $p^* = 0.01$ and $p_i \in U[0.06; 0.15]$, $i \in I$ for unreliable suppliers.

4.6.1 Single-Level Protection

The solution results are presented in Tables 4.4 and 4.5, respectively for the risk-neutral model **RSP_E** and the risk-averse model **RSP_CV** with different confidence levels. The confidence level α is set at five levels of 0.5, 0.75, 0.90, 0.95, and 0.99, which means that focus is on minimizing the highest 50%, 25%, 10%, 5%, and 1% of all scenario outcomes, i.e., costs per part. The size of the corresponding mixed integer programs is represented by the total number of variables, Var., number of binary variables, Bin., number of constraints, Cons, and number of nonzero coefficients in the constraint matrix, Nonz. Table 4.5 also presents the probability $1 - F(VaR^c)$ of outcomes with worst-case cost above VaR^c as well as the expected cost for the optimal risk-averse supply portfolio.

Table 4.4 Solution results for model **RSP_E**

Disruption probability	$p^* = 0.001$ $p_i \in [0, 0.06]$, $i \in I$	$p^* = 0.01$, $p_i \in [0.06, 0.15]$, $i \in I$
Var. = 282105, Bin. = 20, Cons. = 282151, Nonz. = 924155 †		
Expected Cost	12.63	14.97
No.of Suppliers Selected	9	10
(incl. Suppliers Protected)	(0)	(2)
Pre-Positioned Emergency Inventory	0	141 888
CPU‡	5	3243

† Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients.
‡ CPU seconds for proving optimality on a MacBookPro6.2, Intel Core i7, 2.66 GHz, RAM 8 GB/CPLEX 12.4.

The optimal risk-neutral supply portfolio, $(V_1, \ldots, V_{\bar{I}})$, (4.20), for model **RSP_E** that aims at minimization of total expected cost per part is shown in Fig. 4.1 for two scenarios:
(a) with global disruption probability $p^* = 0.001$ and reliable suppliers with local disruption probabilities $p_i \in [0, 0.06]$, $i \in I$, and
(b) with global disruption probability $p^* = 0.01$ and unreliable suppliers with local disruption probabilities $p_i \in [0.06, 0.15]$, $i \in I$.

In addition, total disruption probability $p^* + (1 - p^*)p_i$ for each supplier i is also presented. The optimal supply portfolio for scenario with reliable suppliers, allocates the total demand for parts among nine unprotected suppliers with the lowest disruption probabilities, except for supplier 4 with the highest disruption probability. The lower disruption probability of a supplier, the higher percentage of the total demand allocated. The above observations indicate that when the shortage cost of parts dominates the purchasing cost, disruption probability becomes a key determinant in the decision of demand allocation among the suppliers to minimize total expected cost. Similar results are observed for scenario with unreliable suppliers. However, the total demand is now allocated among all ten suppliers, with the largest orders placed on protected suppliers 1 and 10, where the emergency inventory was pre-positioned (see, Table 4.4).

The optimal risk-averse supply portfolios for **RSP_CV** model and the three confidence levels: 0.90, 0.95, and 0.99 are shown in Fig. 4.2. Table 4.5 and Fig. 4.2 indicate that when α increases and a more risk-averse decision-making focuses on a smaller set of outcomes, the number of protected suppliers also increases to mitigate the impact of disruption risks. Simultaneously, the pre-positioned emergency inventory at the protected suppliers increases, while the total number of all selected suppliers decreases and more orders are placed on the protected suppliers. This indicates that instead of further diversification of supplies by selecting of more suppliers, the impact of disruption risks is rather mitigated by selecting of less suppliers and by protecting most of the selected suppliers and, in addition, by the pre-positioning of emergency inventory at the protected suppliers.

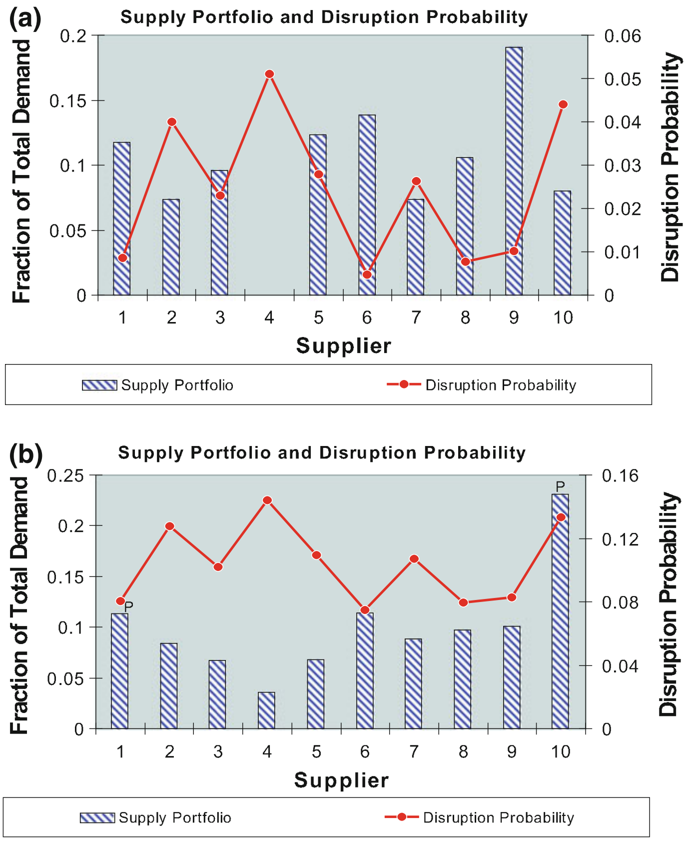

Fig. 4.1 Optimal supply portfolio for model **RSP_E**: **a** $p^* = 0.001$, $p_i \in [0, 0.06]$, $i \in I$, **b** $p^* = 0.01$, $p_i \in [0.06, 0.15]$, $i \in I$ (P - protected supplier)

Unless protected, the suppliers with the highest disruption probability are rarely selected. For example (see, Fig. 4.2), in the optimal supply portfolio for scenario with reliable suppliers and $\alpha = 0.99$ or for scenario with unreliable suppliers with $\alpha = 0.9, 0.95, 0.99$, supplier 4 (cf. Fig. 4.1) is not selected, while supplier 10 is selected and protected.

The discrete distributions of cost per part for the optimal risk-averse supply portfolios (Fig. 4.2a) with three different confidence levels, for global disruption probability $p^* = 0.001$ and reliable suppliers with local disruption probabilities $p_i \in [0, 0.06]$, $i \in I$ are presented in Table 4.6. The table demonstrates that the probability mass function is concentrated in a few points, which is typical for the scenario-based optimization under uncertainty, where the probability measure is concentrated in finitely many points. Large probability atoms are concentrated at eight points, six points, one point, respectively for the confidence level $\alpha = 0.9, 0.95, 0.99$. In particular, for $\alpha = 0.99$ the whole probability is concentrated at one point only: at cost 16.00.

Table 4.5 Solutions results for model **RSP_CV**

Confidence level α	0.50	0.75	0.90	0.95	0.99
Var. = 283130, Bin. = 20, Cons. = 283175, Nonz. = 1 868 008 †					
$p^* = 0.001, p_i \in U[0; 0.06]$					
CVaR^c	13.43	14.02	14.68	15.21	**16.00**♯
VaR^c	12.78	12.93	14.01	14.36	**16.00**
E^c	13.11	13.20	14.08	14.35	**16.00**
$1 - F(VaR^c)$	0.151	0.151	0.033	0.025	0.000
No. of Suppliers Selected (incl. Protected)	10(1)	10(1)	10(2)	10(2)	8(4)
Pre-Positioned Emergency Inventory	70 139	65 652	128 192	133 349	193 950
CPU‡	1037	3079	4770	16752	1852
$p^* = 0.01, p_i \in U[0.06; 0.15]$					
CVaR^c	15.55	15.89	**16.00**	**16.00**	**16.00**
VaR^c	15.02	15.44	**16.00**	**16.00**	**16.00**
E^c	15.28	15.54	**16.00**	**16.00**	**16.00**
$1 - F(VaR^c)$	0.302	0.067	0.000	0.000	0.000
No. of Suppliers Selected (incl. Protected)	10(3)	10(3)	8(4)	8(4)	8(4)
Pre-Positioned Emergency Inventory	167 357	213 402	193 950	193 950	193 950
CPU‡	23931	28856	2354	4030	4357

† Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients.
♯ $VaR^c = CVaR^c = E^c = 16.00$ is the highest cost that may occur
for the optimal supply portfolio, 1-F(16) = 0.
‡ CPU seconds for proving optimality on a MacBookPro 6.2, Intel Core i7, 2.66 GHz,
RAM 8 GB/CPLEX 12.4.

Similar results were obtained for the global disruption probability $p^* = 0.01$ and the unreliable suppliers with local disruption probabilities $p_i \in [0.06, 0.15], i \in I$. For the optimal resilient supply portfolio (Fig. 4.2b), the whole probability is concentrated at a single cost level 16.00 for each confidence level $\alpha = 0.9, 0.95, 0.99$, i.e., the cost distributions were identically shaped by the optimal resilient supply portfolio for the three different confidence levels. The optimal resilient supply portfolios for scenario with reliable suppliers with $\alpha = 0.99$ as well as for scenario with unreliable suppliers with $\alpha = 0.9, 0.95, 0.99$, are identical (see, Table 4.5 and Fig. 4.2).

The computational results indicate that the smaller is the number of concentration points and the greater are probability atoms concentrated at those points, the greater can be the positive difference $F(VaR^c) - \alpha$, i.e., the smaller than $1 - \alpha$ can be the probability $1 - F(VaR^c)$ of outcomes with worst-case cost higher than VaR^c. For example (see, Table 4.5), for $\alpha = 0.9$, VaR^c=14.01 and $1 - F(VaR^c) = 0.033 < 1 - \alpha = 0.1$, which indicates a high concentration of probability measure at point 14.01 for the optimal supply portfolio. Actually, the probability that cost per part is 14.01 is 0.967, which indicates that $VaR^c = 14.01$ is the lowest cost that may occur and

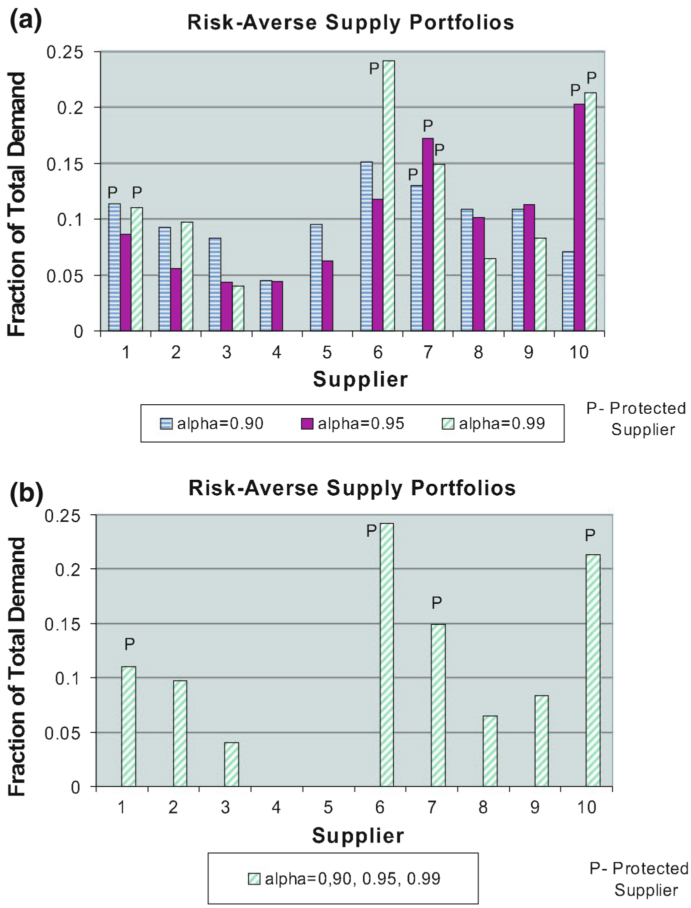

Fig. 4.2 Optimal supply portfolio for model **RSP_CV**: **a** $p^* = 0.001$, $p_i \in [0, 0.06], i \in I$, **b** $p^* = 0.01, p_i \in [0.06, 0.15], i \in I$

that for the confidence level $\alpha = 0.9$, less than 3.3% of the cost outcomes are above VaR^c.

Moreover, if the highest cost probability is greater than $1 - \alpha$, then CVaR^c and VaR^c are identical and both equal to the highest cost. In the example for scenario with reliable suppliers and $\alpha = 0.99$ (see, Table 4.6), the highest cost per part is 16.00 and the probability concentrated at 16.00 is $0.999 > 1 - \alpha = 0.01$, then $VaR^c = 16.00$ is the highest cost per part that may occur ($1 - F(VaR^c) = 0.000$, in Table 4.5) and hence $CVaR^c = VaR^c = 16.00$. Similar results with $CVaR^c = VaR^c = 16.00$ were obtained for the optimal supply portfolios with $\alpha = 0.9, 0.95, 0.99$, for scenarios with unreliable suppliers with global and local disruption probabilities, respectively $p^* = 0.01, p_i \in [0.06, 0.15], i \in I$, (see, Table 4.5). The results presented in Table 4.5 indicate that when for the optimal supply portfolio with a confidence level α, the

Table 4.6 Probability of cost per part for optimal supply portfolios: $p^* = 0.001$, $p_i \in [0, 0.06]$, $i \in I$

Cost interval	$\alpha = 0.90$	$\alpha = 0.95$	$\alpha = 0.99$
[14, 15)	0.9899	0.9950	0
[15, 16)	0.0032	0.0014	1
[16, 17)	0.0040	0.0020	0
[17, 18)	0.0005	0.0003	0
[18, 19)	0.0011	0.0003	0
[19, 20)	0.0002	0	0
[20, 21)	0.0001	0	0
[47, 48)	0	0.0010	0
[53, 54)	0.0010	0	0

probability measure is concentrated at the highest cost and hence is greater than $1 - \alpha$, so that CVaR^c and VaR^c are identical with the highest cost, then the number of selected suppliers is smaller than for a lower confidence level, however more suppliers are protected.

Table 4.5, also demonstrates that the higher the protection index Φ (4.28) (or its optimal value Φ^o (4.29)), the less the number of protected suppliers in the optimal supply portfolio. For example, for scenario with reliable suppliers ($\Phi = 8.353$ and $\Phi^o = 1.033$) only a few of the selected suppliers are protected and only for higher confidence levels α, whereas for scenario with unreliable suppliers ($\Phi = 1.141$ and $\Phi^o = 0.295$) more of the selected suppliers are protected, even for lower confidence levels.

Table 4.7 Nondominated solutions for mean-risk model **RSP_ECV**:$\alpha = 0.99$, $p^* = 0.001$, $p_i \in [0, 0.06]$, $i \in I$

λ	0.01	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99
$p^* = 0.001, p_i \in U[0; 0.06], i \in I$ ($\varphi^o = 0.032$, $\Phi^o = 1.033$)											
E^c	16.00				15.98	15.43	15.03	14.39	14.36	13.15	12.63
CVaR^c	16.00				16.00	16.44	16.89	18.29	18.40	25.71	33.70
VaR^c	16.00				15.99	15.42	15.07	14.55	14.52	19.08	23.77
No. of Suppliers Selected	8				8	10	10	10	10	10	9
(incl. Protected)	(4)				(4)	(3)	(3)	(2)	(2)	(1)	(0)
Emergency Inventory	193950				193997	218988	177867	144381	141565	70594	0
CPU‡	5186	4200	1531	1659	4720	6902	7961	10345	8250	2997	217

121‡ CPU seconds for proving optimality on a MacBookPro 6.2, Intel Core i7, 2.66 GHz, 121RAM 8 GB/CPLEX 12.4.

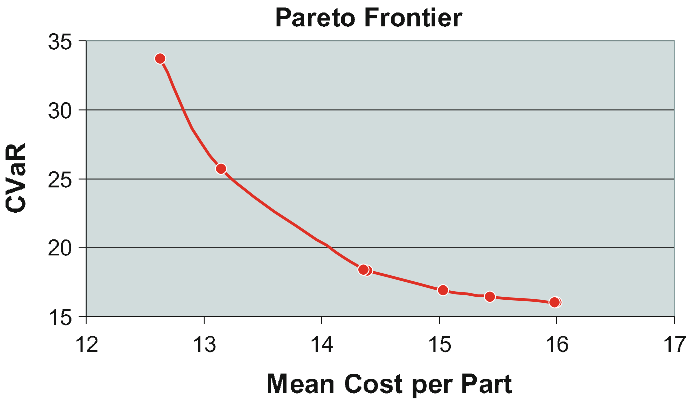

Fig. 4.3 Pareto front for model **RSP_ECV**: $\alpha = 0.99$, $p^* = 0.001$, $p_i \in [0, 0.06]$, $i \in I$

For the bi-objective mean-risk approach, the subsets of nondominated solutions were computed by parameterization on $\lambda \in \{0.01, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 0.99\}$ the weighted-sum program **RSP_ECV**. The results obtained for the confidence level $\alpha = 0.99$ and reliable suppliers with global and local disruption probability, respectively $p^* = 0.001$ and $p_i \in [0, 0.06]$, $i \in I$, are presented in Table 4.7. The subsets of nondominated solutions that were found for the selected 11 levels of weight λ consists of eight different nondominated solutions. The trade-off between the expected cost and the expected worst-case cost is clearly shown in Fig. 4.3, where the convex efficient front of Mean Cost - CVaR, for the confidence level $\alpha = 0.99$ is presented. The results emphasize the effect of varying average/worst-case cost preference of the decision maker; the lower the trade-off parameter λ, the more risk-oriented the decision making. Note that solutions to single objective models **RSP_E** and **RSP_CV** are equivalent to the nondominated solutions of the weighted-sum program **RSP_ECV** with $\lambda = 1$ and $\lambda = 0$, respectively.

Similar computations for the confidence level $\alpha = 0.99$ and unreliable suppliers with global and local disruption probability, respectively $p^* = 0.01$ and $p_i \in [0.06, 0.15]$, $i \in I$, produce the three nondominated solutions only, (E^c, CVaR^c): (16.00,16,00) for $\lambda \in \{0.01, 0.10, 0.20, 0.30, 0.40\}$, (15.98,16.01) for $\lambda \in \{0.50, 0.60, 0.70, 0.80, 0.90\}$ and (14.97,47.25) for $\lambda = 0.99$. For the first two solutions eight suppliers were selected, including two protected, and for the last solution ten suppliers were selected of them two were protected. Figure 4.4 presents the supply portfolio for the last nondominated solution as well as the corresponding optimal cost distribution, which indicates a large probability atom of 0.01 concentrated at the highest cost of 47.25. Note that the risk-averse supply portfolio for $\lambda = 0.99$ is an average performance-oriented portfolio (cf., objective function (4.25)) and virtually neglects the risk of high costs (0.01 weight of CVaR^c in (4.25)). Therefore, it is very close to the risk-neutral portfolio presented in Fig. 4.1b. The cost distribution shaped with a risky supply portfolio that focuses on an average performance of supply chain more often includes probability atoms at high costs.

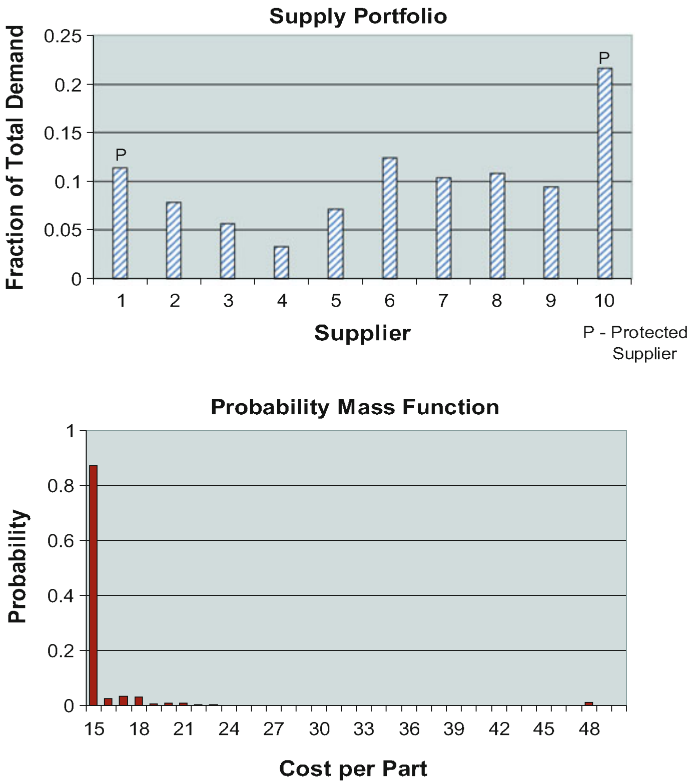

Fig. 4.4 Optimal supply portfolio and probability mass function for model **RSP_ECV**: $p^* = 0.01$, $p_i \in [0.06, 0.15]$, $i \in I$, $\alpha = 0.99$, $\lambda = 0.99$

Comparison of risk-neutral and risk-averse supply portfolios indicates that most risk-averse portfolios include protected suppliers and hence, can be resilient. In contrast, the risk-neutral model that focuses on the expected cost only, rarely selects a resilient portfolio with protected suppliers. Moreover, the mean-risk model that minimizes the weighted sum of the expected cost and the expected worst-case cost produces a subset of nondominated supply portfolios that contains both protected and unprotected suppliers.

The computational experiments with the single-level protection models indicate that:

- *when the shortage cost of parts dominates the purchasing cost, probability of disrupting a supply is a key determinant in the selection of risk-neutral supply portfolio.*
- *a particular supplier is selected more on the supplier non-disruption likelihood than on the other factors such as purchasing cost or defect rate,*

- *the suppliers associated with the highest disruption rates, unless protected, are rarely selected.*
- *in the risk-averse decision-making, the number of protected suppliers increases with the confidence level α, and simultaneously the number of all selected suppliers decreases,*
- *in the risk-averse decision-making instead of diversification of supplies by selecting of more suppliers, the impact of disruption risks is rather mitigated by selecting of less suppliers and by protecting most of the selected suppliers and, in addition, by the pre-positioning of emergency inventory at the protected suppliers.*

The computational experiments were performed using the AMPL programming language and the CPLEX 12.4 solver (with the default traditional branch-and-cut settings) on a laptop MacBookPro 6.2 with Intel Core i7 processor running at 2.66 GHz and with 8GB RAM. The CPLEX solver was capable of finding proven optimal solutions for all examples with CPU time ranging from several to several thousands seconds.

In most cases the CPU time required to find proven optimal solutions for scenario with unreliable suppliers ($p^* = 0.01$ and $p_i \in [0.06, 0.15], i \in I$) was greater than that required for scenario with reliable suppliers ($p^* = 0.001$ and $p_i \in [0, 0.06], i \in I$). For a higher disruption probability, the protection of suppliers and the pre-positioning and usage of emergency inventory occur more frequently. For example, the optimal risk-neutral solution with the protected suppliers and the pre-positioned emergency inventory, required much longer CPU time than the low probability case with no protected suppliers and no emergency inventory (see, Table 4.4).

4.6.2 Multi-level Protection

This subsection presents computational examples to illustrate the SMIP approach to selection of resilient supply portfolio with multi-level protection of fortified suppliers. For comparison, the basic input data used are the same as for the examples with single protection levels. The local disruption probabilities of suppliers, p_i, were distributed exactly the same as for the examples with single protection levels, however slightly different values were generated for the examples in this subsection. Figure 4.5 shows basic characteristics of each supplier i: average price per part, $\sum_{j \in J} o_{ij}/\overline{J}$, and disruption probability, $p^* + (1 - p^*)p_i$, for two scenarios:
(a) with global disruption probability $p^* = 0.001$ and reliable suppliers with local disruption probabilities $p_i \in [0, 0.06], i \in I$, and
(b) with global disruption probability $p^* = 0.01$ and unreliable suppliers with local disruption probabilities $p_i \in [0.06, 0.15], i \in I$.

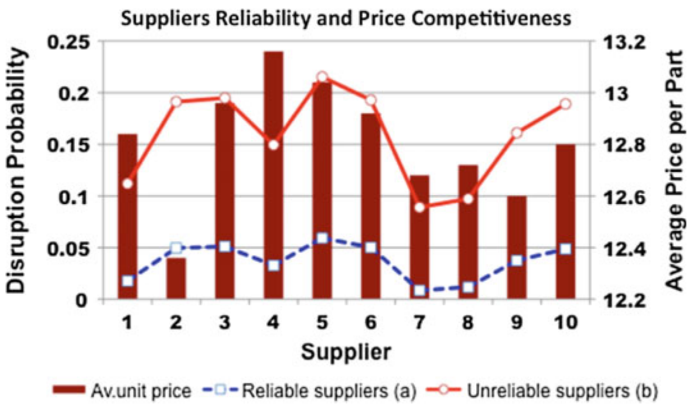

Fig. 4.5 Suppliers

Table 4.8 Solution results for model **RSP(mlp)_E**

Disruption probability	$p^* = 0.001, p_i \in [0, 0.06], i \in I$	$p^* = 0.01, p_i \in [0.06, 0.15], i \in I$
Var. = 282875, Bin. = 50, Cons. = 282920, Nonz. = 1439505 †		
Expected Cost	12.79	14.53
No.of Suppliers Selected	10	10
Protected Suppliers(protection level)	9(1)	1(1), 6(1), 9(1)
Pre-Positioned Emergency Inventory	85030	261 909
CPU‡	17	1044

† Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients.
‡ CPU seconds for proving optimality on a MacBookPro, Intel Core i7, 2.8 GHz, RAM 16 GB/Gurobi 7

The only new parameters, the number of protection levels, $\overline{L}_i$, the fortification cost, $\hat{f}_{il}$, and the remaining fraction of capacity, γ_{il}, for supplier i protected at level l, are shown below.

$\overline{L}_i = 4$ for all suppliers $i \in I$.

$\hat{f}_{i1} = f_i/4,\ \hat{f}_{i2} = f_i/2,\ \hat{f}_{i3} = 3f_i/4,\ \hat{f}_{i4} = f_i$ for all suppliers $i \in I$, where f_i is the fortification cost for the case with single protection levels.

$\gamma_{i1} = 0.2,\ \gamma_{i1} = 0.5,\ \gamma_{i3} = 0.7,\ \gamma_{i4} = 0.9$ for all suppliers $i \in I$.

Thus, the fortification cost for the highest protection level, $l = 4$, is identical with the fortification cost, f_i, for the examples with single protection levels. However, unlike for single protection levels, where 100% of protected supplier's capacity is available after disruption, now only 90% is the highest fraction of remaining capacity.

Table 4.9 Solutions results for model **RSP(mlp)_CV**

Confidence level α	0.50	0.75	0.90	0.95	0.99
Var. = 283900, Bin. = 50, Cons. = 283944, Nonz. = 3 182 078 †					
$p^* = 0.001, p_i \in U[0; 0.06]$					
CVaR^c	13.32	13.77	14.45	14.93	16.17
VaR^c	12.34	13.00	13.80	13.90	16.15
E^c	12.83	13.19	13.87	13.94	16.12
$1 - F(VaR^c)$	0.281	0.162	0.029	0.019	0.006
No. of Suppliers Selected	10	10	9	10	10
Protected Suppliers(protection level)		1(1)	1(1)	1(1)	1(2)
					2(1)
					4(1)
			6(1)	6(1)	
					7(1)
	9(1)	9(1)	9(1)	9(1)	9(2)
		10(1)			
Pre-Positioned Emergency Inventory	91574	173 542	259 787	264 526	551 749
CPU‡	565	1271	6304	4230	2190
$p^* = 0.01, p_i \in U[0.06; 0.15]$					
CVaR^c	15.21	15.99	16.17	16.17	**16.18**
VaR^c	14.34	14.89	16.16	16.18	**16.18**
E^c	14.77	15.16	16.14	16.16	**16.18**
$1 - F(VaR^c)$	0.222	0.134	0.047	0.005	0.000
No. of Suppliers Selected	10	9	9	10	8
Protected Suppliers(protection level)	1(1)	1(1)	1(2)	1(2)	1(2)
			2(1)	2(1)	2(1)
			4(1)	4(1)	4(1)
	6(1)	6(1)			
			7(1)	7(1)	7(1)
	9(1)	9(2)	9(2)	9(2)	9(2)
	10(1)	10(1)			
Pre-Positioned Emergency Inventory	336 809	377 519	543 054	541 012	538 451
CPU‡	6865	11133	1598	4084	6035

† Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients.
♯ $VaR^c = CVaR^c = E^c = 16.18$ is the highest cost that may occur
for the optimal supply portfolio, 1-F(16.18) = 0.
‡ CPU seconds for proving optimality on a MacBookPro, Intel Core i7, 2.8 GHz,
RAM 16 GB/Gurobi 7

The solution results are presented in Tables 4.8 and 4.9, respectively for the risk-neutral model **RSP(mlp)_E** and the risk-averse model **RSP(mlp)_CV** with different confidence levels. The tables show results for two scenarios: (a) with reliable suppliers and (b) with unreliable suppliers.

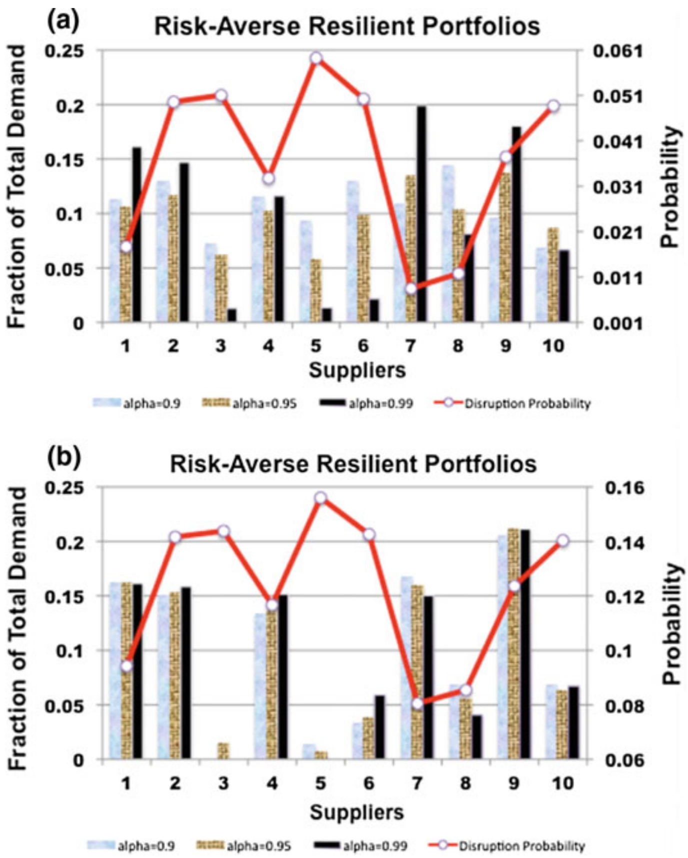

Fig. 4.6 Optimal supply portfolio for model **RSP(mlp)_CV**: **a** $p^* = 0.001$, $p_i \in [0, 0.06]$, $i \in I$, **b** $p^* = 0.01$, $p_i \in [0.06, 0.15]$, $i \in I$

In general, risk-neutral supply portfolios for single- and multi-level protection are similar, (cf. Tables 4.4 and 4.8), as well as the corresponding risk-averse portfolios, (cf. Tables 4.5 and 4.9). For both the risk-neutral and the risk averse portfolio, the number of protected suppliers is greater for scenario (b) with unreliable suppliers, while it additionally increases with the confidence level for the risk-averse portfolio. For example, for scenario (b) with unreliable suppliers, the risk-averse subset of protected suppliers for confidence levels, $\alpha = 0.9, 0.95, 0.99$, is identical with the subset of protected suppliers for scenario (a) with reliable suppliers and the highest confidence level, $\alpha = 0.99$. The corresponding solution values are very close each other (see, Table 4.9). The optimal risk-averse supply portfolios, $(V_1, \ldots, V_{\bar{I}})$, (4.20), for scenario (a) with reliable suppliers and scenario (b) with unreliable suppliers and confidence levels, $\alpha = 0.9, 0.95, 0.99$, as well as the disruption probability, $p^* + (1 - p^*)p_i$, for each supplier i, are shown in Fig. 4.6. The resilient portfolios for scenario (b) and different confidence levels are very close each other. The largest portion of demand for parts has been allocated among the five protected suppliers $i = 1, 2, 4, 7, 9$, while remaining small orders for parts are placed among unprotected suppliers.

Comparison of the corresponding portfolios for single- and multi-level protection demonstrate that only lower protection levels are selected to fortify suppliers. Instead of selection more costly, higher protection levels, the resilient solution rather pre-positions more emergency inventory at fortified suppliers to minimize expected and expected worst-case cost, respectively for model **RSP(mlp)_E** *and* **RSP(mlp)_CV**.

4.7 Notes

Supply chain resilience is a relatively new concept that can be defined as the adaptive capability of the supply chain to prepare for unexpected events, respond to disruptions, and recover from them by maintaining continuity of operations at the desired level of connectedness and control over structure and function (Ponomarov and Holcomb 2009). Falasca et al. (2008) defined resilience as the ability of a supply chain system to reduce the probabilities of disruptions, to reduce the consequences of those disruptions and the time to recover disrupted operations to their normal performance. Also Fiksel (2006) refers resiliency to a firm's capacity to survive, adapt, and grow in the face of change and uncertainty. According to Sheffi (2005), the companies can develop the resilience in three general ways: (1) creating redundancies throughout the supply chain; for example with holding extra inventory, maintain low capacity utilization, and contracting with multiple suppliers, (2) increasing supply chain flexibility; for example with adoption of standardized processes, using concurrent instead of sequential processes, plan to postpone, align procurement strategy with supplier relationships, and (3) changing the corporate culture.

Despite the abundant literature on supplier selection and order quantity allocation problems (e.g., Aissaoui et al. 2007, Ho et al. 2010), the research on quantitative approaches for building a resilient portfolio of suppliers has not often been reported in the literature. Torabi et al. (2015) proposed a bi-objective mixed possibilistic, two-stage stochastic programming model for supplier selection and order allocation problem to build the resilient supply base under operational and disruption risks. The model accounts for uncertainty of critical data and applies several proactive strategies such as suppliers business continuity plans, fortification of suppliers and contracting with backup suppliers to enhance the resilience level of the selected supply base. The issue of linking risk assessment with risk mitigation for low-probability high-consequence events such as disruptions of supplies is discussed by Kleindorfer and Saad (2005) and Cohen and Kunreuther (2007) and the need to build resiliency to disruptive events in supply chains is discussed by Knemeyer et al. (2009), who considered a proactive planning, based on methodology used by the insurance industry. They quantify the risk of multiple types of catastrophic events on key supply chain locations. The proposed proactive planning process involves four critical steps: identification of key supply chain locations and threats, estimation of probabilities and loss for each location, evaluation of alternative countermeasures for each location, and selection of countermeasures that may prevent or mitigate disruption risks. Examples of such countermeasures include relocation of facility away from high risk location, e.g., moving a warehouse to a hurricane-free area, redesign of facility to increase storm preparedness, building storm walls to help protect against flooding, maintaining excess inventory, etc. In practice, the fortification of suppliers to protect them against disruptions is recently observed. For example, to prevent flooding during monsoons, a giant flood wall with sealable aluminum flood barriers across entrance points has been constructed in Thailand, around the perimeter of Nava Nakorn industrial zone, where over 220 factories of electronics and computer components suppliers are located (Fuller 2012).

In a related stream of research, the design of supply chain networks that are resilient to disruptions is considered and the fortification models are developed to improve the reliability of the existing infrastructure systems for which a complete reconfiguration would be cost prohibitive, e.g., Snyder et al. (2005). The objective of such fortification models is to identify optimal strategies for allocating limited resources among possible mitigation investments. For example, Church and Scaparra (2006) introduced the r-interdiction median problem with fortification in a distribution network with p operating facilities and a set of system users who receive service from their nearest facility. The problem objective is to determine the optimal allocation of a limited amount of protective resources in such a way that the accessibility reduction due to a worst-case loss of r unprotected facilities is minimized. They formulate the problem as a mixed integer program. The proposed model presents the limitation of requiring a complete enumeration of all possible ways of interdicting r of the p facilities. In order to alleviate the size restrictions, in a subsequent work Scaparra and Church (2008) presented a bi-level formulation of the r-interdiction median problem with fortification. The top level problem involves the decisions about which facilities to fortify in order to minimize the worst-case efficiency reduction due to the loss of unprotected facilities and worst-case scenario losses are modeled in the lower-level interdiction problem.

In the disaster management literature a limited number of studies focus on the pre-positioning of the different types of emergency supplies in the presence of uncertainty. For example, Rawls and Turnquist (2010) and Noyan (2012) consider the problem of determining the locations of the response facilities and the inventory levels of disaster relief supplies at each facility to effectively manage the response operations when a disaster occurs. There is also a growing body of literature addressing the quantities of pre-positioned emergency inventory, also called, strategic inventory reserves, Schmitt (2011) or just-in-case inventory, Sheffi (2005), Sheffi and Rice (2005), which should be held throughout the supply chain to protect against disruption risks.

The material presented in this chapter is based in part on research reported in Sawik (2013a), where SMIP models were proposed for risk-neutral, risk averse and a bi-objective mean-risk selection of resilient supply portfolio under disruption risks. In the proposed models a single protection level was considered only with a full remaining capacity of a fortified supplier available after disruption. In this chapter, however, the SMIP approach was enhanced for multi-level protection with the amount of capacity remaining after disruption, dependent on the fortification cost and the protection level applied. In Sawik (2013b) a resilient supply portfolio was considered with fortified suppliers and regular inventory pre-positioned at the fortified suppliers. The regular inventory can be fully used under each disruption scenario to fulfill regular orders placed on the protected suppliers.

It is worth to note that the number of variables and constraints in the proposed models grow exponentially in the number $\overline{I}$ of suppliers, if all $\overline{S} = 2^{\overline{I}}$ potential scenarios are considered. The 10-supplier examples for 25-part types have approximately 280,000 variables and 280,000 constraints, while for a 20-supplier problem with all potential disruption scenarios considered and a single part type only, this increases to over 20,000,000 variables and 20,000,000 constraints. Solving the problem with such huge number of scenarios is cumbersome. Even the construction of problems of this size may become intractable, e.g., Chahara and Taaffe (2009). However, the total number of potential disruption scenarios that need to be considered can be reduced by eliminating scenarios that are unlikely to realize. Moreover, using of scenario management approaches (e.g., Jenkins 2000 can also be considered to identify a subset of potential disruption scenarios for which a detailed analysis of their impact on supply chain performance may provide information on the impact of all potential disruption scenarios. Another approach used to reduce the number of random disruption scenarios is the Fuzzy C-Mean clustering technique (e.g., Izakian and Abraham 2011) and the possibilistic scenario-based model (e.g., Torabi et al. 2015). In order to reduce the number of scenarios, the FCM technique is used to cluster possible disruptive events at suppliers to different clusters by which the centers of clusters are used as representatives of disruptive events.

In this chapter, the local and regional supply disruptions are assumed to occur independently. The future research should consider new resilience strategies for dependent disruptive events (e.g., Li et al. 2013) or multiple concurrent disruptions (Zobel and Khansa 2014).

Problems

4.1 Modify the probability, P_s, for disruption scenarios to account for suppliers located in different regions, subject to regional disruptions that may affect all suppliers in the same region simultaneously. Modify the SMIP models presented in this chapter for a joint fortification of all suppliers in one region.

4.2 Define service level for the SMIP models presented in this chapter and modify the models for the service level objective function.

4.3 Limited storage space for emergency inventory
(a) Modify the SMIP models presented in Sect. 4.3 for the resilient supply portfolio with single protection levels to account for limited storage space for emergency inventory at protected suppliers.
(b) Modify the SMIP models presented in Sect. 4.5 for the resilient supply portfolio with multiple protection levels to account for limited storage space for emergency inventory at protected suppliers, dependent on protection level.

4.4 Modify the protection index introduced in Sect. 4.4 for the resilient supply portfolios with multiple protection levels.

4.5 In the computational examples for unreliable suppliers, Table 4.9 and Fig. 4.6b indicate that the risk-averse resilient solutions for confidence levels $\alpha = 0.9,\ 0.95,$ 0.99 are very close each other. How, would you explain the reason for that, and what would be the explanation for the case with identical solutions?
Hint: compare with solution results presented in Table 4.5 for a single protection level.

Part II
Integrated Selection of Supply Portfolio and Scheduling

Chapter 5
Integrated Selection of Supply Portfolio and Scheduling of Production

5.1 Introduction

The supplier selection and order quantity allocation are a medium- to short-term decision, driven by the time-varying customer demand. The scheduling horizon for supplies of parts coincides with the scheduling horizon for customer orders and to achieve the best results the supplier selection and order quantity allocation decisions should also be made for the same time horizon. The advantage of a joint decision making can be shown especially in the presence of supply chain disruption risks. This chapter proposes a SMIP approach to integrated supplier selection and customer order scheduling in the presence of supply chain disruption risks, for a single, dual or multiple sourcing strategy. The suppliers are assumed to be located in two or more disjoint geographic regions: in the producer's region (domestic suppliers) and outside the producer's region (foreign suppliers). The supplies are subject to independent random local disruptions that are uniquely associated with a particular supplier and to random regional disruptions that may result in disruption of all suppliers in the same geographic region simultaneously. The domestic suppliers are relatively reliable but more expensive, while the foreign suppliers offer competitive prices. However the foreign suppliers are more prone to breakdowns and material flows from these suppliers are more exposed to unexpected disruptions due to natural or man made disasters and longer shipping time and distance. Given a set of customer orders for products, the decision maker needs to decide which single supplier, which two suppliers (one from each region) or which multiple suppliers to select for purchasing parts required to complete the customer orders and how to schedule the orders over the planning horizon, to mitigate the impact of disruption risks. The problem objective is either to minimize total cost of ordering and purchasing of parts plus penalty cost of delayed and unfulfilled customer orders due to the parts shortages or to maximize customer service level. The resulting allocation of total demand for parts among the selected suppliers and the schedule of customer orders for every potential disruption

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scenario should be determined ahead of time, either to minimize the average or worst-case cost or to maximize the average or worst-case customer service level.

The following time-indexed SMIP models are presented in this chapter:

SPSm_E(c), SPS2_E(c) and **SPS1_E(c)** for risk-neutral selection of supply portfolio and scheduling of customer orders to minimize expected cost: multiple sourcing, dual sourcing and single sourcing, respectively;

SPSm_E(sl), SPS2_E(sl) and **SPS1_E(sl)** for risk-neutral selection of supply portfolio and scheduling of customer orders to maximize expected service level: multiple sourcing, dual sourcing and single sourcing, respectively;

SPSm_CV(c), SPS2_CV(c) and **SPS1_CV(c)** for risk-averse selection of supply portfolio and scheduling of customer orders to minimize CVaR of cost: multiple sourcing, dual sourcing and single sourcing, respectively;

SPSm_CV(sl), SPS2_CV(sl) and **SPS1_CV(sl)** for risk-averse selection of supply portfolio and scheduling of customer orders to maximize CVaR of service level: multiple sourcing, dual sourcing and single sourcing, respectively;

SPSm_E(c)CV(sl) for mixed mean-risk selection of supply portfolio and scheduling of customer orders to trade-off expected cost and CVaR of service level: multiple sourcing.

For a single, dual or multiple sourcing strategy and for the two different objective functions, the risk-neutral, risk-averse and mean-risk solutions that optimize, respectively average, worst-case and trade-off between average and worst-case performance of a supply chain are illustrated with computational examples and compared in Sect. 5.6. In addition, the two risk-averse service level measures: expected worst-case order fulfillment rate and expected worst-case demand fulfillment rate were computationally compared in Sect. 5.6.3.

5.2 Problem Description

In this section the problem of integrated supplier selection, order quantity allocation and customer orders scheduling in the presence of supply chain disruption risks is described. While the supplier selection is considered to be a long-term strategic decision, the order quantity allocation and customer order scheduling are short- to medium-term tactical decisions. In particular, in a make-to-order manufacturing, all the above decisions can be made for a short- to medium-term planning horizon. Given a set of part suppliers, the supply portfolio determines an allocation of demand for parts among a subset of selected suppliers and simultaneously, for each disruption scenario an assignment of customer orders to time periods over the planning horizon is found. To justify the integration of supplier selection, order quantity allocation and customer order aggregate scheduling, assume that all these decisions are made

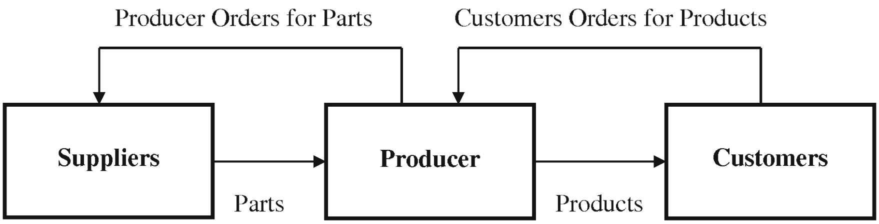

Fig. 5.1 A three-echelon supply chain

for a medium-term planning horizon. However, the order quantity allocation among the suppliers selected for a longer time horizon, in a make-to-order environment can also be made for a short-term horizon with no orders placed on some suppliers, if a shorter planning horizon needs to be considered.

Consider a three-echelon customer driven supply chain (Fig. 5.1) in which various types of products are assembled by a single producer to meet customer demand, using the same critical part type that can be manufactured and provided by many suppliers (for notation used, see Table 5.1).

Let $I = \{1, \dots, \overline{I}\}$ be the set of $\overline{I}$ suppliers, $J = \{1, \dots, \overline{J}\}$ the set of $\overline{J}$ customer orders for products, and $T = \{1, \dots, \overline{T}\}$ the set of $\overline{T}$ planning periods.

Denote by b_j and d_j, respectively the size and the due date of customer order $j \in J$, i.e., the number units of ordered product type and the latest period of their completion required to deliver the products to the customer by requested date. The customer orders are single-period orders such that each order can be completed in one planning period, e.g., Sawik (2007).

Let a_j be the unit requirement for the critical part of each product in customer order $j \in J$. The total demand for all parts is $A = \sum_{j \in J} a_j b_j$ and the total demand for all products is $B = \sum_{j \in J} b_j$.

The orders for parts are assumed to be placed at the start of the planning horizon, when all customer orders for products are known. Let o_i be the unit purchasing price of parts from supplier $i \in I$ and denote by e_i the fixed ordering cost of creating contracts and maintaining relationships with supplier $i \in I$. Each supplier has sufficient capacity to meet total demand for parts and to complete and prepare orders for shipping. Then, all parts ordered from a supplier are shipped together in a single delivery. The order preparation and transportation time of a shipment from supplier $i \in I$ to the producer is constant and equals to τ_i periods so that the parts ordered from supplier $i \in I$ are delivered in period τ_i and then can be used for the assembly of products in period $\tau_i + 1$, at the earliest.

The suppliers are assumed to be located in $\overline{R}$ disjoint geographic regions. Denote by $I^r \subseteq I$ the subset of suppliers in region $r \in R = \{1, \dots, \overline{R}\}$, where $\bigcup_{r \in R} I^r = I$. The supplies of parts are subject to random independent local disruptions that are uniquely associated with a particular supplier, which may arise from equipment breakdowns, local labor strike, fires, etc. Denote by p_i the local disruption probability for supplier i, i.e., the parts ordered from supplier i are delivered without disruptions with probability $(1 - p_i)$, or not at all with probability p_i.

Table 5.1 Notation: selection of supply portfolio and scheduling

Indices	
i	= supplier, $i \in I$
j	= customer order, $j \in J$
r	= geographic region, $r \in R$
s	= disruption scenario, $s \in S$
t	= planning period, $t \in T$
Input Parameters	
a_j	= per unit requirement for parts of each product in customer order j
b_j	= size (number of products) of customer order j
A	= total demand for parts
B	= total demand for products
c_j	= per unit capacity consumption of producer for customer order j
C_t	= capacity of producer in period t
d_j	= due date for customer order j
e_i	= fixed cost of ordering parts from supplier i
g_j	= per unit and per period penalty cost of delayed customer order j
h_j	= per unit penalty cost of unfulfilled customer order j
I^r	= subset of suppliers in geographic region r
o_i	= per unit price of parts purchased from supplier i
p_i	= local disruption probability for supplier i
p^r	= regional disruption probability for all suppliers in region r
p^*	= global disruption probability for all suppliers
α	= confidence level
τ_i	= delivery lead time from supplier i

In addition to independent local disruptions of each supplier individually, there are also potential regional disasters that may result in correlated regional disruption of all suppliers in the same region and global disruptive super events that may simultaneously impact all suppliers. For example, such global disaster super events may include an economic crisis, widespread labor strike in a transportation sector.

Denote by p^r and p^* the probability of regional disruption, simultaneously of all suppliers $i \in I^r$ in region $r \in R$, and global disruption, simultaneously of all suppliers $i \in I$, respectively. The global disaster, the regional disasters in each region and the local disruptive events at each supplier are assumed to be independent events. Let π_i be the disruption probability of every supplier $i \in I^r, r \in R$

$$\pi_i = p^* + (1 - p^*)p^r + (1 - p^*)(1 - p^r)p_i;\ i \in I^r, r \in R. \qquad (5.1)$$

Denote by P_s the probability that disruption scenario s is realized, where each scenario $s \in S$ is comprised of a unique subset $I_s \subset I$ of suppliers who deliver parts without disruptions, and $S = \{1, \ldots, \overline{S}\}$ is the index set of all disruption scenarios (there are a total of $\overline{S} = 2^{\overline{I}}$ potential disruption scenarios). For each scenario $s \in S$, the supplies from every supplier, $i \in I \setminus I_s$, can be disrupted either by a local, regional or global disruptive event.

The probability P_s for each disruption scenario $s \in S$ with the subset I_s of non-disrupted suppliers, and with all possible combinations of different disaster events considered, is (cf. Sect. 1.3)

$$P_s = \begin{cases} (1-p^*)\prod_{r\in R} P_s^r & \text{if } \ I_s \neq \emptyset \\ p^* + (1-p^*)\prod_{r\in R} P_s^r & \text{if } \ I_s = \emptyset, \end{cases} \tag{5.2}$$

where P_s^r is the probability of realizing of disruption scenario s for suppliers in I^r

$$P_s^r = \begin{cases} (1-p^r)\prod_{i\in I^r\bigcap I_s}(1-p_i)\prod_{i\in I^r\setminus I_s} p_i & \text{if } \ I^r \bigcap I_s \neq \emptyset \\ p^r + (1-p^r)\prod_{i\in I^r} p_i & \text{if } \ I^r \bigcap I_s = \emptyset. \end{cases} \tag{5.3}$$

Assume that the producer has limited time-varying capacity, and denote by C_t the producer capacity available in planning period $t \in T$, and by c_j the unit capacity consumption for each product in customer order $j \in J$.

The producer can be charged with a contractual, order specific penalty cost for delayed or unfulfilled customer orders, caused by the shortage of parts, that are delivered late or not at all due to supply disruptions. Let g_j and h_j be, respectively the per unit and per period penalty cost of delayed customer order $j \in J$ and the per unit total penalty cost of unfulfilled customer order $j \in J$. The penalty cost h_j is assumed to also include the producer lost profit. Therefore, the unit penalty costs g_j and h_j are selected in such a way that each product in an unfulfilled order is penalized much higher than the corresponding product in a delayed order, i.e., $h_j \gg g_j, \ j \in J$. In some cases the contractual penalty cost for unfulfilled customer orders can be shared between the supplier and the producer to compensate the latter for the lost profit due to the undelivered parts. This can be modeled by choosing appropriate lower values of the corresponding unit penalty costs, h_j, to reduce the producer direct losses. Then, the remaining part of the producer penalty cost is assumed to be covered by the contractual compensation from the disrupted supplier.

The producer has three sourcing alternatives to select: single, dual or multiple sourcing. The objective of the integrated supplier selection and customer order scheduling is to select a single supplier (single sourcing), two suppliers from two different regions (dual sourcing), or multiple suppliers (multiple sourcing), allocate the total demand for parts among the selected suppliers and schedule the customer orders over the planning horizon to complete the orders and mitigate the impact of disruption risks. The three sourcing strategies will be compared with respect to the two alternative optimality criteria: cost and service level. The objective function is either to minimize expected cost, expected worst-case cost, or maximize expected service level or expected worst-case service level. The supply portfolio (the allocation of total demand for parts among the selected suppliers) and the schedule of customer orders for every potential disruption scenario are determined at the beginning of the planning horizon.

5.3 Models for Risk-Neutral Decision-Making

In this section six time-indexed SMIP models, **SPS1_E(c)**, **SPS2_E(c)**, **SPSm_E(c)**, **SPS1_E(sl)**, **SPS2_E(sl)**, **SPSm_E(sl)**, are proposed for the integrated supplier selection and customer order scheduling to optimize average performance of a supply chain in the presence of disruption risks. The objective of models **SPS1_E(c)**, **SPS2_E(c)**, **SPSm_E(c)** and **SPS1_E(sl)**, **SPS2_E(sl)**, **SPSm_E(sl)** is to minimize the expected cost per product ("E(c)" in the model name) and to maximize the expected service level ("E(sl)" in the model name), respectively.

While the objective of supplier selection is to determine a supply portfolio, i.e., an allocation of demand for parts among the suppliers, the objective of customer orders scheduling is to determine an aggregate production schedule, i.e., an assignment of orders to planning periods over a planning horizon, subject to capacity and parts availability constraints. For the selected supply portfolio and each disruption scenario the optimal schedule of customer orders is determined in such a way as to minimize the penalty cost for delayed and unscheduled (rejected) orders, and by this minimize total cost or maximize service level, respectively.

The following three different sourcing strategies will be considered:

- Single sourcing ("1" in the model name), where the total demand for parts is assigned to exactly one supplier.
- Dual sourcing ("2" in the model name), where the total demand for parts is either assigned to one supplier or allocated between two suppliers, from two different geographic regions. For example, the total demand for parts is split between the two suppliers, one more expensive and more reliable domestic supplier, and one less expensive and less reliable foreign supplier.
- Multiple sourcing ("m" in the model name), where the total demand for parts is allocated among many suppliers.

The problem variables are defined Table 5.2.

5.3.1 Minimization of Cost

In this subsection the three time-indexed SMIP models are presented for the risk-neutral selection of supply portfolio and scheduling of customer orders to minimize expected cost: model **SPSm_E(c)** for multiple sourcing, model **SPS2_E(c)** for dual sourcing and model **SPS1_E(c)** for single sourcing. The single sourcing strategy can be considered to be a special case of a dual sourcing, while the dual sourcing can be considered a special case of a multiple sourcing. Therefore, first model **SPSm_E(c)** is presented, then model **SPS2_E(c)** and finally model **SPS1_E(c)**.

Table 5.2 Variables: selection of supply portfolio and scheduling

	First stage variables
u_i	$= 1$, if supplier i is selected; otherwise $u_i = 0$ (supplier selection)
v_i	$\in [0, 1]$, the fraction of total demand for parts ordered from supplier i (supply portfolio for dual or multiple sourcing)
	Second stage variables
w^s_{jt}	$= 1$, if under disruption scenario s customer order j is scheduled for period t; otherwise $w^s_{jt} = 0$ (production scheduling)
	Auxiliary variables
VaR^c	Cost-at-Risk, the targeted cost such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is below VaR^c
VaR^{sl}	Service-at-Risk, the targeted service level such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is above VaR^{sl}
$\mathscr{C}_s$	≥ 0, the tail cost for scenario s, i.e., the amount by which costs in scenario s exceed VaR^c
$\mathscr{S}_s$	≥ 0, the tail service level for scenario s, i.e., the amount by which VaR^{sl} exceeds service level in scenario s

In the risk-neutral decision-making, the overall quality of the supply portfolio and the schedule of customer orders can be measured by the expected cost per product, (5.4), of parts ordering, $\sum_{i \in I} e_i u_i / B$, and purchasing $(\sum_{s \in S} P_s (\sum_{i \in I_s} A o_i v_i)/B)$, where the producer is not charged with ordered and undelivered parts, plus penalty cost of delayed and unfulfilled (rejected) customer orders due to delays and disruptions of part supplies, $\sum_{s \in S} P_s (\sum_{j \in J} \sum_{t \in T: t > d_j} g_j b_j (t - d_j) w^s_{jt})/B + \sum_{s \in S} P_s (\sum_{j \in J} h_j b_j (1 - \sum_{t \in T} w^s_{jt}))/B$.

The SMIP model **SPSm_E(c)** for a multiple sourcing selection of supply portfolio and scheduling of customer orders to minimize expected cost is formulated below.

SPSm_E(c): *Risk-neutral selection of* **S***upply* **P***ortfolio and* **S***cheduling of customer orders to minimize expected cost:* **m***ultiple sourcing*

Minimize

$$E^c = \sum_{i \in I} e_i u_i / B + \sum_{s \in S} P_s (\sum_{i \in I_s} A o_i v_i)/B$$
$$+ \sum_{s \in S} P_s (\sum_{j \in J} \sum_{t \in T: t > d_j} g_j b_j (t - d_j) w^s_{jt})/B$$
$$+ \sum_{s \in S} P_s (\sum_{j \in J} h_j b_j (1 - \sum_{t \in T} w^s_{jt}))/B \tag{5.4}$$

subject to

Demand allocation constraints:

- the total demand for parts must be fully allocated among the suppliers,

$$\sum_{i \in I} v_i = 1 \tag{5.5}$$

Multiple sourcing strategy constraints:

- demand for parts cannot be assigned to non-selected suppliers,

$$v_i \le u_i; \; i \in I \tag{5.6}$$

Order-to-period assignment constraints:

- for each disruption scenario s, each customer order j is either scheduled during the planning horizon ($\sum_{t \in T} w^s_{jt} = 1$), or unscheduled and rejected ($\sum_{t \in T} w^s_{jt} = 0$),
- for each disruption scenario s and each planning period t, the cumulative demand for parts of all customer orders scheduled in periods 1 through t cannot exceed the cumulative deliveries of parts in periods 1 through $t - 1$, from the non-disrupted suppliers $i \in I_s$,
- for each disruption scenario s, the total requirement for parts of scheduled customer orders is not greater than the total supplies from the non-disrupted suppliers $i \in I_s$,

$$\sum_{t \in T} w^s_{jt} \le 1; \; j \in J, s \in S \tag{5.7}$$

$$\sum_{j \in J} \sum_{t' \in T : t' \le t} a_j b_j w^s_{jt'} \le A \sum_{i \in I_s : \tau_i \le t-1} v_i; \; t \in T, s \in S \tag{5.8}$$

$$\sum_{j \in J} \sum_{t \in T} a_j b_j w^s_{jt} \le A \sum_{i \in I_s} v_i; \; s \in S \tag{5.9}$$

Producer capacity constraints:

- for any period t and each disruption scenario s, the total demand on capacity of all customer orders scheduled in period t must not exceed the producer capacity available in this period,

$$\sum_{j \in J} b_j c_j w^s_{jt} \le C_t; \; t \in T, s \in S \tag{5.10}$$

Non-negativity and integrality conditions:

$$u_i \in \{0, 1\}; \; i \in I \tag{5.11}$$

$$v_i \in [0, 1]; \; i \in I \tag{5.12}$$

$$w^s_{jt} \in \{0, 1\}; \; j \in J, t \in T, s \in S. \tag{5.13}$$

In the SMIP model **SPS2_E(c)** formulated below for a dual sourcing, all suppliers are assumed to be located in two different geographic regions and at most one supplier can be selected from each region.

SPS2_E(c): *Risk-neutral selection of supply portfolio and scheduling of customer orders to minimize expected cost: dual sourcing*

Minimize (5.4)

subject to (5.5)–(5.13) and

Dual sourcing strategy constraints:

- at most one supplier can be selected from each region,

$$\sum_{i \in I^r} u_i \leq 1; \; r = 1, 2. \tag{5.14}$$

Note that dual and multiple sourcing allows, respectively two and more suppliers to be selected, however an optimal solution may assign total demand for parts to a single supplier only, if such a solution optimizes the objective function.

The single-sourcing model **SPS1_E(c)** is a special case of the dual-sourcing model **SPS2_E(c)**. In order to derive **SPS1_E(c)** from **SPS2_E(c)**, it is sufficient to replace inequality $v_i \leq u_i; \; i \in I$, (5.6), with an equality constraint $v_i = u_i; \; i \in I$ and by this eliminate variables $v_i, \; i \in I$. Simultaneously, constraint (5.14) becomes weaker than (5.5) and can be removed, while constraints (5.8) and (5.9) should be replaced by

$$\sum_{j \in J} \sum_{t' \in T: t' \leq t} w^s_{jt'} \leq \overline{J} \sum_{i \in I_s: \tau_i \leq t-1} u_i; \; t \in T, s \in S$$

or

$$\sum_{t' \in T: t' \leq t} w^s_{jt'} \leq \sum_{i \in I_s: \tau_i \leq t-1} u_i; \; j \in J, t \in T, s \in S,$$

and

$$\sum_{j \in J} \sum_{t \in T} w^s_{jt} \leq \overline{J} \sum_{i \in I_s} u_i; \; s \in S$$

or

$$\sum_{t \in T} w^s_{jt} \leq \sum_{i \in I_s} u_i; \; j \in J, s \in S,$$

respectively.

Constraints (5.8) can also be replaced by the following alternative constraints

$$\sum_{t \in T} t w^s_{jt} \geq \sum_{i \in I_s} (\tau_i + 1) u_i; \; j \in J, s \in S,$$

which sometimes may appear to be more efficient computationally.

The stochastic binary program **SPS1_E(c)** for a single sourcing selection of supply portfolio and scheduling of customer orders is formulated below.

SPS1_E(c): *Risk-neutral selection of supply portfolio and scheduling of customer orders to minimize expected cost: single sourcing*

Minimize

$$\sum_{i \in I} e_i u_i / B + \sum_{s \in S} P_s (\sum_{i \in I_s} Ao_i u_i)/B$$
$$+ \sum_{s \in S} P_s (\sum_{j \in J} \sum_{t \in T: t > d_j} g_j b_j (t - d_j) w_{jt}^s)/B$$
$$+ \sum_{s \in S} P_s (\sum_{j \in J} h_j b_j (1 - \sum_{t \in T} w_{jt}^s))/B \tag{5.15}$$

subject to (5.10), (5.11), (5.13) and

Supplier selection constraints:

- exactly one supplier of parts is selected,

$$\sum_{i \in I} u_i = 1 \tag{5.16}$$

Order-to-period assignment constraints: (5.7) and

- for each disruption scenario s, all customer orders can be scheduled only after the delivery of required parts, purchased from a non disrupted supplier $i \in I_s$,

- each customer order j can be scheduled during the planning horizon under disruption scenario s, only if required parts are ordered from a non disrupted supplier $i \in I_s$,

$$\sum_{t' \in T: t' \leq t} w_{jt'}^s \leq \sum_{i \in I_s: \tau_i \leq t-1} u_i; \; j \in J, t \in T, s \in S \tag{5.17}$$

$$\sum_{t \in T} w_{jt}^s \leq \sum_{i \in I_s} u_i; \; j \in J, s \in S. \tag{5.18}$$

The solution to supplier selection and customer order scheduling problem determines for every disruption scenario $s \in S$, the aggregate production schedule $\{\sum_{j \in J} b_j w_{jt}^s; \; t \in T\}$ as well as the corresponding service level, $\sum_{j \in J} \sum_{t \in T: t \leq d_j} w_{jt}^s / \overline{J}$, (the fraction of customer orders fulfilled by their due dates). In addition, for every disruption scenario $s \in S$, the subset of customer orders scheduled by the requested due dates d_j, $\{j \in J : 1 + \min_{i \in I} \tau_i \leq \sum_{t \in T} t w_{jt}^s \leq d_j\}$, the subset of delayed customer orders, $\{j \in J : \sum_{t \in T} t w_{jt}^{s} > d_j\}$, and the subset of rejected (unscheduled) customer orders, $\{j \in J : \sum_{t \in T} w_{jt}^{s} = 0\}$, are determined.

5.3.2 Maximization of Customer Service Level

The customer service level can be measured either by *order fulfillment rate* or by *demand fulfillment rate*, where the order fulfillment rate and the demand fulfillment rate is the fraction of customer orders (irrespective of their size) and the fraction of customer demand, respectively, that is fulfilled by the customer requested due dates. When the focus is on fulfilling customer orders rather than total demanded quantity, the order fulfillment rate would be the preferred service level measure. Otherwise, the demand fulfillment rate would be selected. Since the order fulfillment rate does not account for the size of customer orders, a high service level can be achieved by fulfilling a large number of small size orders, while leaving the unfulfilled demand relatively high. On the other hand, a high demand fulfillment rate can be achieved by fulfilling a few large size orders, while leaving relatively high, the number of unfulfilled small size orders.

The aim of the next three SMIP models is to achieve the best average customer service level by maximizing the expected fraction of customer orders fulfilled by their due dates, i.e., by maximizing the expected order fulfillment rate.

SPSm_E(sl): *Risk-neutral selection of supply portfolio and scheduling of customer orders to maximize expected service level: multiple sourcing*

Maximize

$$\sum_{s \in S} P_s \sum_{j \in J} \sum_{t \in T : t \leq d_j} w_{jt}^{s} / \overline{J} \tag{5.19}$$

subject to (5.5)–(5.13).

SPS2_E(sl): *Risk-neutral selection of supply portfolio and scheduling of customer orders to maximize expected service level: dual sourcing*

Maximize (5.19)

subject to (5.5)–(5.14).

SPS1_E(sl): *Risk-neutral selection of supply portfolio and scheduling of customer orders to maximize expected service level: single sourcing*

Maximize (5.19)

subject to (5.7), (5.10), (5.11), (5.13), (5.16)–(5.18).

Note that maximization of the customer service level simultaneously leads to reduction of the penalty costs for delayed and unfulfilled customer orders, that represent an important part of the total cost structure. However, as the objective function (5.19) does not directly include any cost components, the optimal solution to **SPSm_E(sl)**, **SPS2_E(sl)** and **SPS1_E(sl)** that maximizes the customer service level depends only on the delivery lead time from suppliers and distribution among the suppliers of local and regional disruption probabilities. Therefore, the solution can be considered to be an ideal solution with respect to the customer service level in a risk-neutral decision-making, and may be used for comparisons with the optimal cost-based solutions.

If instead of the number of customer orders fulfilled on time (i.e., the order, fulfillment rate) customer service level is measured by the fraction of customer demand fulfilled on time (i.e., by the demand fulfillment rate), then (5.19) in models **SPSm_E(sl)**, **SPS2_E(sl)** and **SPS1_E(sl)** should be replaced by the following objective function to be maximized

$$\sum_{s \in S} P_s \sum_{j \in J} \sum_{t \in T: t \le d_j} b_j w_{jt}^s / B. \tag{5.20}$$

5.4 Models for Risk-Averse Decision-Making

In the selection of supply portfolio and scheduling of customer orders under disruption risks, the decision maker controls the risk of high losses due to supply disruptions by choosing the confidence level α. For a given confidence level, let VaR^c be the acceptable cost level above which we want to minimize the number of outcomes and CVaR^c considers those portfolio outcomes, where costs exceed VaR^c. In a similar way, denote by VaR^{sl} the acceptable service level below which we want to maximize the number of outcomes and CVaR^{sl} considers those portfolio outcomes, where service levels are below VaR^{sl}. We assume that the decision maker is willing to accept only portfolios for which the total probability of scenarios with costs greater than VaR^c (or with service levels lower than VaR^{sl}) is not greater than $1 - \alpha$. The greater the confidence level α, the more risk aversive is the decision maker and the smaller percent of the highest cost (of the lowest service level, respectively) outcomes is focused on.

In this section the six time-indexed SMIP models: **SPSm_CV(c)**, **SPS2_CV(c)**, **SPS1_CV(c)**, **SPSm_CV(sl)**, **SPS2_CV(sl)**, **SPS1_CV(sl)** are proposed for the integrated supplier selection and customer order scheduling to optimize worst-case performance of a supply chain in the presence of disruption risks. The objective of models **SPSm_CV(c)**, **SPS2_CV(c)**, **SPS1_CV(c)** and **SPSm_CV(sl)**, **SPS2_CV(sl)**, **SPS1_CV(sl)** is to minimize CVaR^c of cost per product ("CV(c)" in the model name) and to maximize CVaR^{sl} of service level ("CV(sl)" in the model name), respectively. The models are risk-averse equivalents, respectively, of **SPSm_E(c)**, **SPS2_E(c)**, **SPS1_E(c)** and **SPSm_E(sl)**, **SPS2_E(sl)**, **SPS1_E(sl)** formulations presented in Sect. 5.3 for the risk-neutral decision-making.

5.4.1 *Minimization of Cost*

Define $\mathscr{C}_s$ as the tail cost for scenario s, where tail cost is defined as the amount by which costs in scenario s exceed VaR^c. The risk-averse supply portfolio and production schedule will be optimized by calculating VaR^c and minimizing CVaR^c simultaneously. In the proposed model, CVaR^c is represented by an auxiliary function (5.21) introduced by Rockafellar and Uryasev (2000).

The SMIP models **SPSm_CV(c)**, **SPS2_CV(c)** and **SPS1_CV(c)** for supplier selection and customer order scheduling to optimize worst-case performance of a supply chain and reduce the risk of high costs, respectively for multiple, dual and single sourcing, are formulated below.

SPSm_CV(c): *Risk-averse selection of supply portfolio and scheduling of customer orders to minimize CVaR of cost: multiple sourcing*

Minimize

$$CVaR^c = VaR^c + (1-\alpha)^{-1}\sum_{s\in S} P_s\mathscr{C}_s \tag{5.21}$$

subject to (5.5)–(5.13) and

Risk constraints:

- the tail cost for scenario s is defined as the nonnegative amount by which cost in scenario s exceeds VaR^c,

$$\mathscr{C}_s \geq \sum_{i\in I} e_i u_i/B + \sum_{i\in I_s} Ao_i v_i/B + \sum_{j\in J}\sum_{t\in T:t>d_j} g_j b_j (t-d_j) w^s_{jt}/B + \sum_{j\in J} h_j b_j (1-\sum_{t\in T} w^s_{jt})/B - VaR^c;\ s\in S \tag{5.22}$$

$$\mathscr{C}_s \geq 0. \tag{5.23}$$

SPS2_CV(c): *Risk-averse selection of supply portfolio and scheduling of customer orders to minimize CVaR of cost: dual sourcing*
Minimize (5.21)
subject to (5.5)–(5.14), (5.22), (5.23).

SPS1_CV(c): *Risk-averse selection of supply portfolio and scheduling of customer orders to minimize CVaR of cost: single sourcing*
Minimize (5.21)
subject to (5.7), (5.10), (5.11), (5.13), (5.16)–(5.18), (5.23) and

Risk constraints:
- the tail cost for scenario s is defined as the nonnegative amount by which cost in scenario s exceeds VaR^c,

$$\mathscr{C}_s \geq \sum_{i \in I} e_i u_i / B + \sum_{i \in I_s} A o_i u_i / B + \sum_{j \in J} \sum_{t \in T: t > d_j} g_j b_j (t - d_j) w^s_{jt} / B + \sum_{j \in J} h_j b_j (1 - \sum_{t \in T} w^s_{jt}) / B - VaR^c; \; s \in S. \quad (5.24)$$

5.4.2 Maximization of Customer Service Level

Define $\mathscr{S}_s$ as the tail service level for scenario s, where tail service level is defined as the amount by which VaR^{sl} exceeds service level in scenario s. The risk-averse supply portfolio and production schedule will be optimized by calculating VaR^{sl} and maximizing CVaR^c simultaneously.

The SMIP models **SPSm_CV(sl)**, **SPS2_CV(sl)** and **SPS1_CV(sl)** for supplier selection and customer order scheduling to optimize worst-case performance of a supply chain and reduce the risk of low service levels, respectively for multiple, dual and single sourcing, are formulated below.

SPSm_CV(sl): *Risk-averse selection of supply portfolio and scheduling of customer orders to maximize CVaR of service level: multiple sourcing*
Maximize

$$CVaR^{sl} = VaR^{sl} - (1 - \alpha)^{-1} \sum_{s \in S} P_s \mathscr{S}_s \quad (5.25)$$

subject to (5.5)–(5.13) and

Risk constraints:

- the tail service level for scenario s is defined as the nonnegative amount by which VaR^{sl} exceeds service level in scenario s,

$$\mathscr{S}_s \geq VaR^{sl} - \sum_{j \in J} \sum_{t \in T: t \leq d_j} w^s_{jt}/\overline{J};\ s \in S \tag{5.26}$$

$$\mathscr{S}_s \geq 0. \tag{5.27}$$

SPS2_CV(sl): *Risk-averse selection of supply portfolio and scheduling of customer orders to maximize CVaR of service level: dual sourcing*

Maximize (5.25)

subject to (5.5)–(5.14), (5.26), (5.27).

SPS1_CV(sl): *Risk-averse selection of supply portfolio and scheduling of customer orders to maximize CVaR of service level: single sourcing*

Maximize (5.25)

subject to (5.7), (5.10), (5.11), (5.13), (5.16)–(5.18), (5.26), (5.27).

If instead of the number of customer orders fulfilled on time (i.e., the order, fulfillment rate) customer service level is measured by the fraction of customer demand fulfilled on time (i.e., by the demand fulfillment rate), then (5.26) in models **SPSm_CV(sl)**, **SPS2_CV(sl)** and **SPS1_CV(sl)** should be replaced by the following constraints

$$\mathscr{S}_s \geq VaR^{sl} - \sum_{j \in J} \sum_{t \in T: t \leq d_j} b_j w^s_{jt}/B;\ s \in S. \tag{5.28}$$

Model Enhancements

The SMIP models **SPSm_CV(sl)**, **SPS2_CV(sl)** and **SPS1_CV(sl)** can be strengthened by the addition of valid inequalities to precisely determine VaR^{sl} of service level for a given confidence level α. As a result, tighter LP relaxations of the corresponding mixed integer programs are achieved.

First, introduce the additional binary variable:

- scenario selection variable: $z_s = 1$, if for scenario s, customer service level, $\sum_{j \in J} \sum_{t \in T: t \leq d_j} w^s_{jt}/\overline{J}$, is not less than VaR^{sl}; otherwise $z_s = 0$.

The valid inequalities that define VaR^{sl} of service level are shown below

$$z_s \geq \sum_{j \in J} \sum_{t \in T: t \leq d_j} w_{jt}^s / \overline{J} - VaR^{sl}; \; s \in S \tag{5.29}$$

$$z_s \leq 1 + \sum_{j \in J} \sum_{t \in T: t \leq d_j} w_{jt}^s / \overline{J} - VaR^{sl}; \; s \in S \tag{5.30}$$

$$\sum_{s \in S} P_s z_s \geq \alpha, \tag{5.31}$$

where (5.29) and (5.30) determine scenarios s with the customer service level not less than VaR^{sl}, and (5.31) ensures that the total probability of all such scenarios is not less than the confidence level α.

The introduction of valid inequalities will increase the size of the mixed integer programs **SPSm_CV(sl)**, **SPS2_CV(sl)** and **SPS1_CV(sl)**. The total number of additional binary variables z_s and constraints (5.29)–(5.31) is respectively, $2^{\overline{I}}$ and $2^{\overline{I}+1} + 1$, and hence they grow exponentially with the number $\overline{I}$ of suppliers.

Note that both VaR^{sl} and $\mathscr{S}_s$ can be restricted to being not greater that one and, in addition, $\mathscr{S}_s \leq 1 - z_s, \forall s \in S$, which is equivalent to (5.29).

Denote by **SPSm_CV(sl)+**, **SPS2_CV(sl)+** and **SPS1_CV(sl)+**, the stochastic mixed integer programs **SPSm_CV(sl)**, **SPS2_CV(sl)** and **SPS1_CV(sl)** strengthened with added valid inequalities (5.29)–(5.31), respectively.

If in models **SPSm_CV(sl)**, **SPS2_CV(sl)** and **SPS1_CV(sl)**, customer service level is measured by the fraction of customer demand fulfilled on time (i.e., by demand fulfillment rate), then in the definition of scenario selection variable z_s and in constraints (5.29), (5.30), the term $\sum_{j \in J} \sum_{t \in T: t \leq d_j} w_{jt}^s / \overline{J}$, should be replaced by $\sum_{j \in J} \sum_{t \in T: t \leq d_j} b_j w_{jt}^s / B$.

5.5 Model for Mixed Mean-Risk Decision-Making

In this section a trade-off model **SPSm_E(c)CV(sl)** for a multiple sourcing strategy is developed to evaluate the impact on average cost of optimization the expected worst-case service level. The expected cost per product, E^c (5.4), and the expected worst-case service level, $CVaR^{sl}$ (5.25), will be simultaneously optimized using the weighted-sum approach.

In order to avoid dimensional inconsistency among the two conflicting objective functions, their values are scaled into the interval [0,1].

Denote by $f^c = \frac{E^c - \underline{E^c}}{\overline{E^c} - \underline{E^c}}$, the normalized expected cost per product ($\underline{E^c}$, $\overline{E^c}$ are the minimum, the maximum values of E^c, respectively), and by $f^{sl} = \frac{\overline{CVaR^{sl}} - CVaR^{sl}}{\overline{CVaR^{sl}} - \underline{CVaR^{sl}}}$, the normalized expected worst-case service level ($\underline{CVaR^{sl}}$, $\overline{CVaR^{sl}}$ are the minimum, the maximum values of $CVaR^{sl}$ for a given confidence level α, respectively).

The normalized objective functions f^c and f^{sl} are defined below

$$f^c = (\sum_{i \in I} e_i u_i + \sum_{s \in S} P_s (\sum_{i \in I_s} Ao_i v_i + \sum_{j \in J} \sum_{t \in T: t > d_j} g_j b_j (t - d_j) w_{jt}^s$$

$$+ \sum_{j \in J} h_j b_j (1 - \sum_{t \in T} w_{jt}^s)))/B - \underline{E^c})/(\overline{E^c} - \underline{E^c}) \quad (5.32)$$

$$f^{sl} = \frac{\overline{CVaR^{sl}} - (VaR - (1-\alpha)^{-1} \sum_{s \in S} P_s \mathscr{S}_s)}{\overline{CVaR^{sl}} - \underline{CVaR^{sl}}}. \quad (5.33)$$

The maximum value of expected worst-case service level (5.25), $\overline{CVaR^{sl}}$, for a given confidence level α, and the associated maximum value of expected cost (5.4), $\overline{E^c}$, are obtained as the optimal solution of **SPSm_CV(sl)** (with constraint (5.26)/(5.28), respectively for order/demand fulfillment rate as a service level measure). The minimum value of expected cost (5.4), $\underline{E^c}$, and the associated minimum value of expected worst-case service level, $\underline{CVaR^{sl}}$, for a given confidence level α are obtained as the optimal solution to the following SMIP problem (with constraint (5.26)/(5.28), respectively for order/demand fulfillment rate as a service level measure).

Minimize $0.99999E^c$-$0.00001CVaR^{sl}$
subject to (5.5)–(5.13), (5.27), (5.26)/(5.28).

The much greater weight assigned to the objective function, E^c, reflects the priority given to minimization of expected cost.

Finally, the trade-off model **SPSm_E(c)CV(sl)** for a mixed mean-risk optimization of expected cost per product and expected worst-case service level is shown below (with constraint (5.26)/(5.28), respectively for order/demand fulfillment rate as a service level measure).

SPSm_E(c)CV(sl): *Mixed mean-risk selection of supply portfolio and scheduling of customer orders to trade-off expected cost and CVaR of service level: multiple sourcing*

Minimize

$$\lambda f^c + (1-\lambda) f^{sl}, \quad (5.34)$$

where $0 \le \lambda \le 1$,
subject to (5.5)–(5.13), (5.27), (5.26)/(5.28), (5.32), (5.33).

5.6 Computational Examples

In this section some computational examples are presented to illustrate possible applications of the proposed SMIP approach for the selection of suppliers, order quantity allocation and customer orders scheduling for a single, dual and multiple sourcing strategy and the two different service level measures: order fulfillment rate and demand fulfillment rate. First, single versus dual sourcing and single versus multiple sourcing strategy are compared, then order fulfillment rate versus demand fulfillment rate are compared as worst-case service level performance measures and finally the impact of optimization the worst-case service on average cost is evaluated.

5.6.1 Single Versus Dual Sourcing

We assume that suppliers are located in two different geographic regions $r = 1, 2$. Suppliers $i \in I^1$ are domestic suppliers, relatively reliable but more expensive. Suppliers $i \in I^2$ are located outside the producer's geographic region and offer competitive prices. For the domestic suppliers $i \in I^1$, the unit prices, o_i, are higher than for the foreign suppliers $i \in I^2$, while the fixed ordering costs e_i are lower. However the foreign suppliers are more prone to breakdowns and material flows from these suppliers are more exposed to unexpected disruptions due to natural or man made disasters and longer shipping times and distance to the producer. The regional disruption probability, p^1, for the producer domestic region $r = 1$, is much lower than the probability, p^2, for the foreign region $r = 2$.

The probability P_s of each disruption scenario $s \in S$ under the risks of local and regional disaster events is

$$P_s = \begin{cases} p^1 p^2 + (1-p^1)(1-p^2)\prod_{i \in I} p_i & \text{if } I_s = \emptyset \\ (1-p^1)p^2 \prod_{i \in I_s}(1-p_i)\prod_{i \in I^1 \setminus I_s} p_i + (1-p^1)(1-p^2)\hat{P}_s & \text{if } I_s \subseteq I^1 \\ p^1(1-p^2)\prod_{i \in I_s}(1-p_i)\prod_{i \in I^2 \setminus I_s} p_i + (1-p^1)(1-p^2)\hat{P}_s & \text{if } I_s \subseteq I^2 \\ (1-p^1)(1-p^2)\hat{P}_s & \text{if } I_s \bigcap I^1 \neq \emptyset,\, I_s \bigcap I^2 \neq \emptyset, \end{cases} \quad (5.35)$$

where $\hat{P}_s$ is the probability of disruption scenario s in the presence of independent local disruptive events only

$$\hat{P}_s = \prod_{i \in I_s}(1-p_i) \cdot \prod_{i \notin I_s} p_i. \quad (5.36)$$

The following parameters have been used for the example problems:

- $\overline{I}$, the number of suppliers, was equal to 10 and the number of disruption scenarios, was equal to the total number of all potential scenarios $\overline{S} = 2^{\overline{I}} = 1024$;
- $\overline{J}$, the number of customer orders, was equal to 25;

- $\overline{R}$, the number of geographic regions, was equal to 2, and the subsets of domestic and foreign suppliers were $I^1 = \{1, 2, 3, 4, 5\}$ and $I^2 = \{6, 7, 8, 9, 10\}$, respectively;
- $\overline{T}$, the number of planning periods, was equal to 10;
- a_j, the unit requirements for parts of products in customer orders were integers in $\{1, 2, 3\}$ drawn from int(U[1;3]) distribution, for all orders j;
- b_j, the size of customer orders (required numbers of products), were integers in $\{500, 1000, \ldots, 5000\}$ drawn from 500int(U[1;10]) distribution, for all customer orders j;
- c_j, the unit capacity consumptions of producer, were integers in $\{1, 2, 3\}$ drawn from int(U[1;3]) distribution, for all customer orders j;
- C_t, the capacity of producer in each period t, was integer drawn from $1000\lceil(2\sum_{j\in J} b_j c_j/(\overline{T} - \max_{i\in I}\tau_i))U[0.75; 1.25]/1000\rceil$ distribution, i.e., in each period the producer capacity was from 75% to 125% of the double capacity required to complete all customer orders during the planning horizon, after the latest delivery of parts;
- d_j, the due dates for customer orders, were integers in $\{1 + \min_{i\in I}(\tau_i), \ldots, \overline{T}\}$ drawn from int(U[2;10]) distribution, for all customer orders j;
- e_i, the cost of ordering parts, were integers in $\{5000, 6000, \ldots, 10000\}$ and integers in $\{15000, 16000, \ldots, 30000\}$, respectively for domestic suppliers $i \in I^1$ and for foreign suppliers $i \in I^2$;
- g_j, the unit daily penalty cost of delayed customer orders, was equal to $\lceil a_j \max_{i\in I}(o_i)/350\rceil$ for all orders j, i.e., was approximately 0.28% of the maximum unit price of required parts;
- h_j, the unit penalty cost of unfulfilled customer orders, was to $2\lceil a_j \max_{i\in I}(o_i)\rceil$ for all orders j, i.e., was approximately twice as large as the maximum unit price of required parts;
- o_i, the unit price of parts purchased from supplier i, was uniformly distributed over [11,16] and over [1,6], respectively for domestic suppliers $i \in I^1$ and for foreign suppliers $i \in I^2$.
- α, the confidence level, was equal to 0.50, 0.75, 0.90, 0.95 or 0.99.
- p_i, the local disruption probability was uniformly distributed over [0.005,0.01] for domestic suppliers $i \in I^1$ and over [0.05,0.10] for foreign suppliers $i \in I^2$, i.e., the disruption probabilities were drawn independently from U[0.005;0.01] and from U[0.05;0.10], respectively for domestic and foreign suppliers.
  The regional disruption probability was $p^1 = 0.001$ for domestic suppliers $i \in I^1$ and $p^2 = 0.01$ for foreign suppliers $i \in I^2$.
  p^*, the global disruption probability was 0, i.e., no global disaster super event is considered.
- τ_i, delivery lead time from domestic suppliers $i \in I^1$, were integers in $\{1, 2\}$ drawn from int(U[1;2]) distribution, and from foreign suppliers $i \in I^2$, were integers in $\{2, 3, 4\}$ drawn from int(U[2;4]) distribution.

The computational experiments were performed for the same replication of the above input data set. For all test examples, the resulting total demand for parts and products is $A = 132500$ and $B = 66000$, respectively. The unit price per part o_i and disruption probability $p^r + (1 - p^r)p_i$ of each supplier $i \in I^r, r = 1, 2$ are shown in Fig. 5.2.

The solution results for the two sourcing strategies and the two objective functions are presented in Tables 5.3, 5.4 and 5.5, respectively for the risk-neutral models and the risk-averse models with different confidence levels, where for the risk-averse maximization of service level the enhanced models **SPS1_CV(sl)+** and **SPS2_CV(sl)+** were applied. The confidence level α is set at five levels of 0.5, 0.75, 0.90, 0.95, and 0.99, which means that focus is on minimizing the highest (maximizing the lowest) 50%, 25%, 10%, 5%, and 1% of all scenario outcomes, i.e., costs per product (service levels, respectively). The size of the corresponding mixed integer programs is represented by the total number of variables, Var., number of binary variables, Bin., number of constraints, Cons, and number of nonzero coefficients in

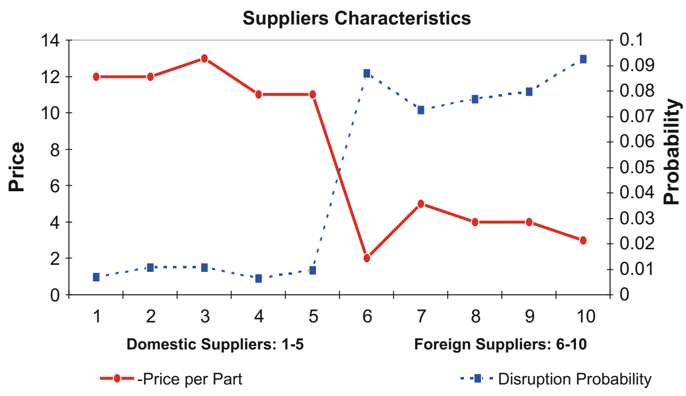

Fig. 5.2 Suppliers

Table 5.3 Risk-neutral solutions: single versus dual sourcing

Sourcing strategy	Single sourcing	Dual sourcing
	Minimization of expected cost (5.4) and (5.15)	
Expected Cost	11.92	11.92
Expected service level $^{(a)}$	61.80	61.80
Selected Supplier	6	6
CPU$^{(b)}$	144	45
	Maximization of expected service level (5.19)	
Expected service level	99.35	99.35
Expected Cost	23.19	23.19
Selected Supplier	4	4
CPU$^{(b)}$	61	89

$^{(a)}$ $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T:t\le d_j} w^s_{jt}/\overline{J})100\%$

$^{(b)}$ CPU seconds for proving optimality on a MacBookPro 6.2, Intel Core i7, 2.66GHz, RAM 8GB/CPLEX 12.5

the constraint matrix, Nonz. In addition to the optimal solution values for the primary objective functions and the allocation of demand among the selected suppliers, Tables 5.3, 5.4 and 5.5 present the associated values of the other objective function.

Table 5.3 indicates that for the risk-neutral models the same optimal solution was found for each sourcing strategy, and the obtained risk-neutral solutions are identical with the corresponding risk-averse solutions for $\alpha = 0.5$ (see, Tables 5.4 and 5.5). Note that a low price, unreliable foreign supplier $i = 6$ was selected to minimize the expected cost per product, while a high price, reliable domestic supplier $i = 4$ was selected to maximize the expected service level.

For the risk-averse cost minimization, Table 5.4 indicates that the solution results for single and dual sourcing are identical for all confidence levels α except for the highest level $\alpha = 0.99$, where for a dual sourcing the total demand is allocated between two suppliers to reduce the worst-case cost outcomes. For the low confidence level $\alpha = 0.5$, a less reliable, low price foreign supplier $i = 6$ is selected, while for a higher α more reliable and more expensive domestic supplier $i = 4$ is chosen. When $\alpha = 0.99$, the expected worst-case cost is lower for a dual sourcing, for which the total demand is allocated between two suppliers.

Table 5.4 Risk-averse minimization of cost: single versus dual sourcing

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **SPS1_CV(c)**: Var. = 230 060, Bin. = 229 035, Cons. = 45 945, Nonz. = 2 115 890 [(a)]					
CVaR^c	16.05	23.78	24.96	26.92	42.64
VaR^c	7.79	22.99	22.99	22.99	22.99
Supplier Selected	6	4	4	4	4
Expected Cost	11.92	23.19	23.19	23.19	23.19
Expected Service Level [(b)]	61.80	99.35	99.35	99.35	99.35
CPU[(c)]	88	103	204	60	880
Model **SPS2_CV(c)**: Var. = 230 070, Bin. = 229 035, Cons. = 45 957, Nonz. = 2 121 040 [(a)]					
CVaR^c	16.05	23.78	24.96	26.92	38.29
VaR^c	7.79	22.99	22.99	22.99	37.20
Suppliers Selected(% of total demand)	6(100)	4(100)	4(100)	4(100)	4(60)
					10(40)
Expected Cost	11.92	23.19	23.19	23.19	23.19
Expected Service Level [(b)]	61.80	99.35	99.35	99.35	55.32
CPU[(c)]	34	363	55	44	3473

[(a)] Var. = number of variables, Bin. = number of binary variables, Cons. = number of constraints, Nonz. = number of nonzero coefficients
[(b)] $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T: t\le d_j} w^s_{jt}/\overline{J})100\%$
[(c)] CPU seconds for proving optimality on a MacBookPro 6.2, Intel Core i7, 2.66GHz, RAM 8GB/CPLEX 12.5

For the risk-averse service level maximization, for which the supplier selection is independent on any cost parameters and the solution depends only on disruption probability, Table 5.5 demonstrates that for a single sourcing, the same expensive but reliable domestic supplier $i = 4$ is selected for all confidence levels. The same results are obtained for a dual sourcing and $\alpha = 0.5, 0.75, 0.9$, while for a higher confidence level, the total demand is allocated between two suppliers to reduce the worst-case service level outcomes. Then, the expected worst-case service level is higher than the corresponding level for a single sourcing. In particular, for the highest confidence level $\alpha = 0.99$, the expected worst-case service level for a dual sourcing is twice as large as that for a single sourcing. Table 5.5 also shows the results for the original models **SPS1_CV(sl)** and **SPS2_CV(sl)**. The comparison of CPU times clearly indicates the advantage of the enhanced models, in particular, model **SPS2_CV(sl)+**.

To identify the possible impact of the basic cost parameter setting on the achieved solution results, the additional computational experiments were performed with the much smaller differences of the fixed ordering cost, e_i, and the unit price, o_i, for the two groups of suppliers. The following values of parameters e_i and o_i, $i = 1, \ldots, 10$, were selected: $e = (6000, 8000, 7000, 8000, 8500, 8500, 9500, 9000, 10000, 9500)$ and $o = (6, 6, 5, 7, 6, 4, 5, 4, 4, 3)$. Since the service level-based objective function is independent on any cost parameters, the optimal solutions have not changed, while

Table 5.5 Risk-averse maximization of service level: single versus dual sourcing

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **SPS1_CV(sl)+**: Var. = 231 082, Bin. = 230 053, Cons. = 49 035, Nonz. = 2 242 779 $^{(a)}$					
CVaR^{sl}%	98.70	97.39	93.48	86.95	34.77
VaR^{sl}%	100	100	100	100	100
Expected Service Level $^{(b)}$	99.35	99.35	99.35	99.35	99.35
Expected Cost	23.19	23.19	23.19	23.19	23.19
Supplier Selected	4	4	4	4	4
CPU$^{(c)}$	10	10	12	14	258
Model **SPS1_CV(sl)**: Var. = 231 060, Bin. = 229 035, Cons. = 46 028, Nonz. = 1 934 798 $^{(a)}$					
CPU$^{(c)}$	19	20	15	18	293
Model **SPS2_CV(sl)+**: Var. = 231 088, Bin. = 230 053, Cons. = 48 017, Nonz. = 2 240 749 $^{(a)}$					
CVaR^{sl}%	98.70	97.39	93.48	87.35	66.17
VaR^{sl}%	100	100	100	96	72
Expected Service Level$^{(b)}$	99.35	99.35	99.35	99.16	95.01
Expected Cost	23.19	23.19	23.19	26.73	25.04
Suppliers Selected(% of total demand)	4(100)	4(100)	4(100)	4(89)	4(45)
				7(11)	7(55)
CPU$^{(c)}$	354	153	192	5900	349
Model **SPS2_CV(sl)**: Var. = 230 070, Bin. = 229 035, Cons. = 45 968, Nonz. = 1 996 339 $^{(a)}$					
CPU$^{(c)}$	465	4380	1409	11920	3725

$^{(a)}$ Var. = number of variables, Bin. = number of binary variables, Cons. = number of constraints, Nonz. = number of nonzero coefficients

$^{(b)}$ $(\sum_{s \in S} P_s \sum_{j \in J} \sum_{t \in T: t \le d_j} w^s_{jt}/\overline{J})100\%$

$^{(c)}$ CPU seconds for proving optimality on a MacBookPro 6.2, Intel Core i7, 2.66GHz, RAM 8GB/CPLEX 12.5

Table 5.6 Risk-averse supply portfolios for suppliers offering similar prices

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **SPS1_CV(c)**					
Supplier Selected	3	3	3	3	3
Model **SPS2_CV(c)**					
Suppliers Selected(% of total demand)	3(100)	3(100)	3(100)	3(100)	3(55%) 10(45)

for the cost-based objective function only slight changes were observed (see, optimal supply portfolios in Table 5.6). The cheapest supplier, $i = 3$, from among the most reliable domestic suppliers is selected as the main one (for a dual sourcing) or the only one (for a single sourcing). The results have indicated that the supplier reliability is a key selection parameter, even for a cost-based objective function.

For the optimal risk-averse supply portfolio with $\alpha = 0.99$ and the two objective functions, Figs. 5.3 and 5.4 show the distribution of cost per product and the

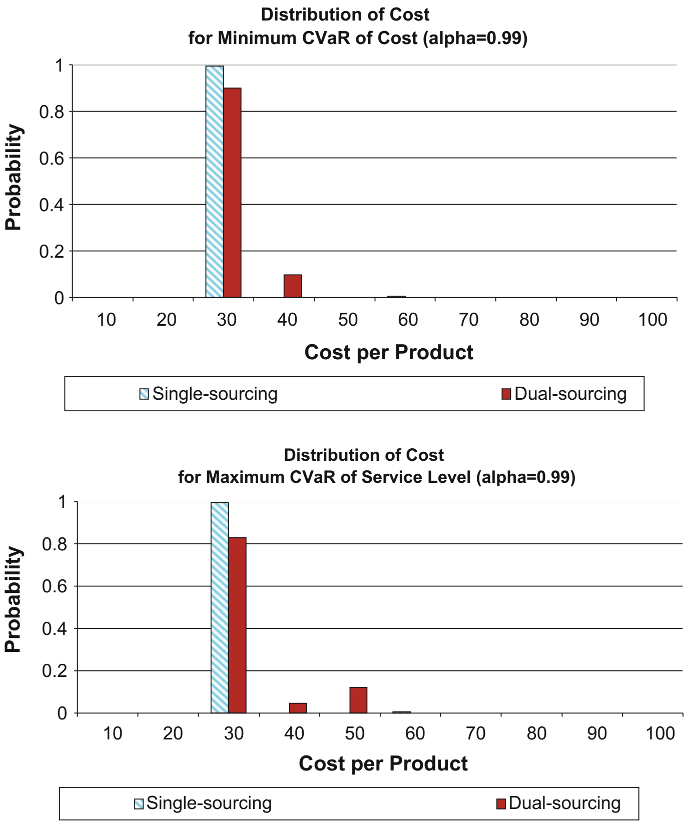

Fig. 5.3 Distribution of cost per product for risk-averse supply portfolio with $\alpha = 0.99$

distribution of customer service level, respectively. Figure 5.3 demonstrates that for both objective functions, the distribution of cost is concentrated at the lowest cost per product for both single and dual sourcing, which indicates that a risk-averse maximization of service level that aims at reducing the expected worst-case fraction of delayed and unscheduled customer orders, implicitly reduces the corresponding expected worst-case penalty costs. Figure 5.4 shows that for a single sourcing, the distribution of service level is concentrated at the highest percent of customer orders fulfilled by their due dates, because for both objective functions the same single, reliable supplier (supplier 4) was selected to minimize the worst-case cost of unfulfilled customer orders or maximize the worst-case customer service level, respectively For a dual sourcing, however, the probability measure is concentrated at the highest service level, only if the CVaR of service level is maximized. When the CVaR of cost is minimized however, the highest probability measure is concentrated

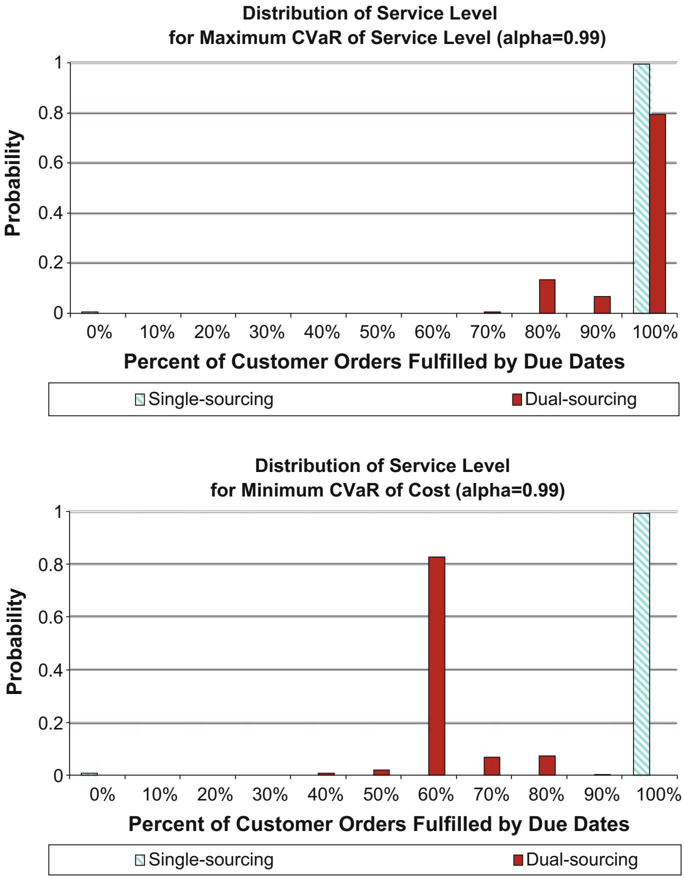

Fig. 5.4 Distribution of customer service level for risk-averse supply portfolio with $\alpha = 0.99$

at a lower service level, which indicates that a risk-averse dual sourcing solution that selects a low price, less reliable supporting supplier (supplier 10 vs. supplier 7, see Fig. 5.2 and Tables 5.3, 5.4) to reduce the worst-case costs, may simultaneously decrease the worst-case service levels. For a single sourcing, however, small probability atoms are concentrated at the highest cost and at the lowest service level (0% - all customer orders delayed or unscheduled), respectively for the risk-averse minimization of cost and maximization of service level. This additionally indicates that for a single sourcing, all customer orders may be either fulfilled by their due dates or delayed/unscheduled, with a small probability for the latter event, whereas for a dual sourcing, the probability measure is distributed over a range of cost or service level outcomes.

Figure 5.5 presents aggregated demand pattern for products, $\sum_{j\in J:d_j=t} b_j$, $t \in T$, and expected aggregated production schedule, $\sum_{s\in S} P_s \sum_{j\in J} b_j w^s_{jt}$, $t \in T$ for the optimal risk-averse supply portfolios, the two objective functions and the two sourcing strategies. For a dual sourcing and maximum CVaR of service level when no cost components are included in the objective function, the expected production

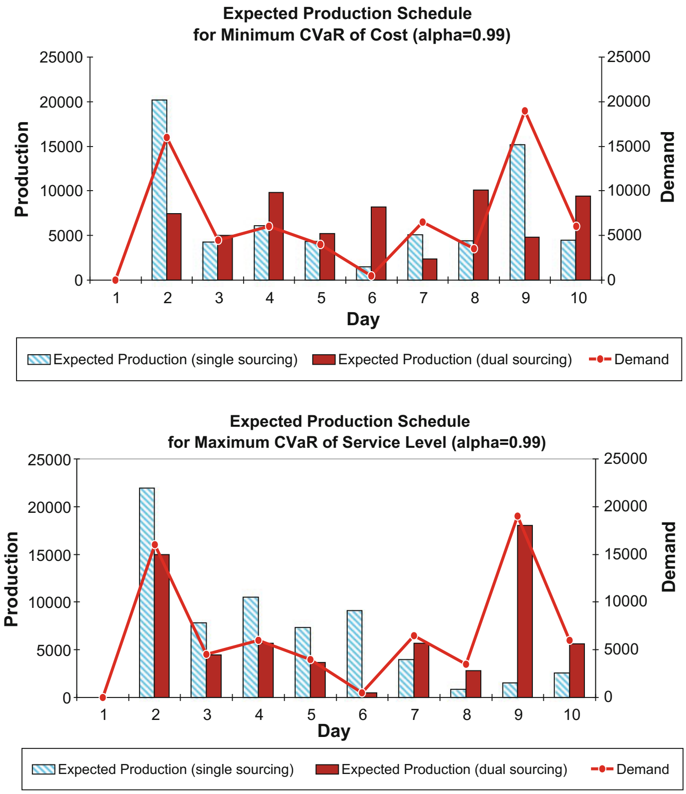

Fig. 5.5 Expected production schedule for risk-averse supply portfolio with $\alpha = 0.99$

schedule approximately follows the aggregated demand and for a single sourcing and both objective functions, the production schedules begin with the highest production level, that may help to reduce the penalty cost or the fraction of delayed customer orders.

Finally, the impact of penalty costs imposed on the producer by the customers is evaluated, assuming that a disrupted supplier is contractually obliged to partially cover the producer charges for unfulfilled customer orders due to undelivered parts. Table 5.7 shows the solution results for the unit penalty cost of unfulfilled customer orders, h_j, reduced by half with respect to their original values, i.e., for h_j approximately as large as the maximum unit price of required parts. Thus, the disrupted suppliers are assumed to cover the remaining 50% of the producer penalty charges. Table 5.7 demonstrates that for a lower producer penalty for unfulfilled customer orders, in a dual sourcing no supporting supplier is needed for the highest confidence level. For both single and dual sourcing, the same single supplier was selected. This indicates that the lower the penalty for unfulfilled customer orders, the less reliable and cheaper supplier can be selected by the producer.

Comparison of single and dual sourcing strategies indicates that for both the risk-neutral and the risk-averse solutions with a low confidence level, the same single supplier is selected only; a low price, risky supplier to minimize cost or an expensive, reliable supplier to maximize customer service level. In order to minimize the expected cost per product or CVaR of cost per product for low confidence levels, the cheapest supplier is usually selected. In contrast, to maximize the expected service level or CVaR of service level for low confidence levels, the most reliable supplier (with the lowest disruption probability) is mostly selected. For a higher confidence level, both single and dual sourcing model selects a single, reliable supplier to minimize worst-case cost of unfulfilled customer orders or maximize worst-case customer service level. A difference between single and dual sourcing solutions arises

Table 5.7 Risk-averse solutions for reduced penalty costs: single versus dual sourcing

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **SPS1_CV(c)**					
CVaR^c	11.52	15.25	23.25	23.52	25.61
VaR^c	7.79	7.79	22.99	22.99	22.99
Suppliers Selected	6	6	4	4	4
Expected Cost	9.65	9.65	23.02	23.02	23.02
Expected Service Level $^{(a)}$	61.62	59.46	99.35	99.35	99.35
Model **SPS2_CV(c)**					
CVaR^c	11.52	15.25	23.25	23.52	25.61
VaR^c	7.79	7.79	22.99	22.99	22.99
Suppliers Selected	6	6	4	4	4
Expected Cost	9.65	9.65	23.02	23.02	23.02
Expected Service Level $^{(a)}$	61.62	59.46	99.35	99.35	99.35

$^{(a)}$ $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T:t\le d_j} w^s_{jt}/\overline{J})100\%$

only for the highest confidence levels. Then, for a dual sourcing and the risk-averse solutions with the highest confidence levels, an expensive, reliable supplier selected for lower confidence levels is additionally supported with a low price, risky supplier to allocate the total demand for parts between the two suppliers. However, the selection of the supporting risky supplier depends on the objective function: a cheaper and less reliable supplier is selected to reduce the risk of high costs and a more expensive and more reliable supplier is selected to reduce the risk of low service level.

In general, the computational results indicate that the supplier reliability is a key selection parameter. In order to maximize service level the most reliable supplier is selected as the main one (for a dual sourcing) or the only one (for a single sourcing), whereas to minimize cost, the cheapest one is selected from among most reliable suppliers, respectively.

For the limited number of scenarios considered, the proven optimal solution can be found, using the CPLEX solver for mixed integer programming. However, in the proposed models the number of scheduling variables w^s_{jt} is $O((\overline{J})(\overline{S})(\overline{T}))$ and the number of constraints is $O((\overline{J}+\overline{T})\overline{S})$, i.e., they grow linearly in the number $\overline{S}$ of disruption scenarios and hence exponentially in the number $\overline{I}$ of suppliers, if all, $\overline{S}=2^{\overline{I}}$, potential scenarios are considered.

The computational experiments were performed using the AMPL programming language and the CPLEX 12.5 solver on a laptop MacBookPro 6.2 with Intel Core i7 processor running at 2.66 GHz and with 8GB RAM. The CPLEX solver was capable of finding proven optimal solutions for all examples with CPU time ranging from several seconds for the cost-based objectives to around 6000 s for the service level objectives. However, the enhanced models **SPS1_CV(sl)+** and **SPS2_CV(sl)+** had to be applied for the risk-averse maximization of service level, otherwise the much longer CPU time (up to 12000 s) was required to prove optimality.

5.6.2 Single Versus Multiple Sourcing

In this subsection the risk-averse strategies are compared for single and multiple sourcing. The following parameters used for the example problems are different from those in Sect. 5.6.1:

- $\overline{R}$, the number of geographic regions, was equal to 3, and the subsets of suppliers were $I^1=\{1,2,3\}$, $I^2=\{4,5,6\}$ and $I^3=\{7,8,9,10\}$, respectively;
- τ_i, the order preparation and shipping times from suppliers were 2, 3 and 4 time periods, respectively for suppliers $i\in I^1, i\in I^2$ and $i\in I^3$;
- d_j, the due dates for customer orders, were integers in $\{3,\ldots,\overline{T}\}$ drawn from int(U[3;10]) distribution, for all customer orders j;
- e_i, the cost of ordering parts, were integers in {5000, 6000, ..., 10000}, {10000, 11000, ..., 15000} and {15000, 16000, ..., 30000}, respectively for suppliers $i\in I^1, i\in I^2$ and $i\in I^3$;

- o_i, the unit price of parts purchased from supplier i, was uniformly distributed over [11,16], [6,11] and [1,6], respectively for suppliers $i \in I^1, i \in I^2$ and $i \in I^3$;
- p_i, the local disruption probability was uniformly distributed over [0.005,0.01], [0.01,0.05] and [0.05;0.10], respectively for suppliers $i \in I^1$, $i \in I^2$ and $i \in I^3$, i.e., the disruption probabilities were drawn independently from U[0.005;0.01], U[0.01,0.05] and U[0.05;0.10], respectively;
- p^r, the regional disruption probability was 0.001, 0.005 and 0.01, respectively for region $r = 1, r = 2$ and $r = 3$;

The computational experiments were performed for the same replication of the above input data set. The following data set was generated for all test examples:
$a = (2, 1, 3, 3, 1, 3, 2, 1, 2, 2, 2, 2, 3, 2, 1, 3, 2, 1, 3, 3, 2, 1, 1, 2, 1)$;
$b = (1, 2, 9, 7, 8, 5, 1, 7, 5, 4, 7, 4, 10, 6, 8, 1, 4, 2, 4, 8, 6, 3, 8, 7, 3) \times 500$;
The resulting total demand for parts and products is $A = 132500$ and $B = 66000$, respectively.
$c = (2, 1, 1, 2, 3, 3, 1, 3, 2, 1, 2, 1, 3, 1, 1, 3, 2, 3, 1, 1, 3, 2, 2, 1, 2)$;
$C_t = 38000, \forall t = 1, \ldots, 10$;
$d = (7, 4, 10, 10, 3, 8, 7, 5, 5, 6, 8, 7, 10, 5, 3, 9, 6, 4, 9, 9, 7, 4, 5, 7, 4)$;
$e = (8, 7, 10, 13, 14, 11, 19, 17, 19, 27) \times 1000$;
$g_j = 1 \forall j = 1, \ldots, 25$;
$h = (52, 26, 78, 78, 26, 78, 52, 26, 52, 52, 52, 52, 78, 52, 26, 78, 52, 26, 78, 78, 52, 26, 26, 52, 26)$;
$o = (13, 12, 12, 8, 6, 6, 2, 5, 4, 4)$;

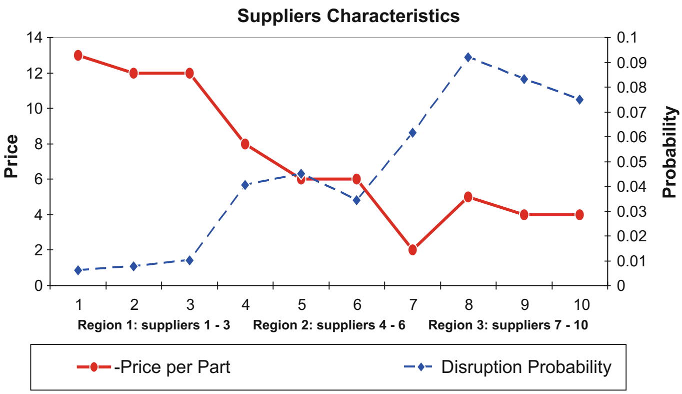

Fig. 5.6 Suppliers

$p = (0.00513571, 0.00666354, 0.00902974, 0.0356206, 0.040175, 0.0294692, 0.0519967, 0.0827215, 0.0739062, 0.0656449)$.

The corresponding disruption probabilities (5.1) of all suppliers are:
$\pi = (0.00613057, 0.00765688, 0.0100207, 0.0404425, 0.0449741, 0.0343219, 0.0614767, 0.0918943, 0.0831672, 0.0749885)$.

All potential disruption scenarios were considered and to calculate the disruption probability for each scenario, the formulae (5.2) and (5.3) were applied. The unit price per part o_i and the disruption probability π_i, (5.1), of each supplier $i \in I^r, r = 1, 2, 3$ are shown in Fig. 5.6. The figure indicates that the most reliable (with the lowest disruption probability) is supplier 1, the least reliable (with the highest disruption probability) is supplier 8, the most expensive (with the highest price per part) is supplier 1, and the cheapest (with the lowest price per part) is supplier 7. Note that geographic regions are numbered in such a way that the unit prices are nonincreasing with r, while the fixed ordering costs and the disruption probabilities are nondecreasing with r, i.e.,

$$o_{i_1} \geq o_{i_2} \geq o_{i_3}, \quad e_{i_1} \leq e_{i_2} \leq e_{i_3} \quad \text{and} \quad \pi_{i_1} \leq \pi_{i_2} \leq \pi_{i_3}; \forall i_1 \in I^1, i_2 \in I^2, i_3 \in I^3.$$

The solution results for the two sourcing strategies and the two objective functions are presented in Tables 5.8 and 5.9, respectively. For the risk-averse maximization of service level, models **SPS1_CV(sl)** and **SPSm_CV(sl)** were enhanced with the valid inequalities (5.29)–(5.31). The confidence level α is set at five levels of 0.5, 0.75, 0.90, 0.95, and 0.99, which means that focus is on minimizing the highest (maximizing the lowest) 50%, 25%, 10%, 5%, and 1% of all scenario outcomes, i.e., costs per product (service levels, respectively). In addition to the optimal solution values for the primary objective functions and the allocation of demand among the selected suppliers, Tables 5.8 and 5.9 present the associated values of the other objective function. In particular, the expected cost and the expected service level associated with the optimal risk-averse solutions are presented to evaluate an average performance of the supply chain.

For the minimization of worst-case cost, Table 5.8 indicates that for low confidence levels α, the solution results for single and multiple sourcing are identical, while for the higher levels a subset of suppliers is selected for the multiple sourcing. For a single sourcing and low confidence level $\alpha = 0.5, 0.75$, the cheapest supplier $i = 7$ is selected, then for a higher α, more reliable and expensive suppliers are chosen and finally, the most reliable and expensive supplier $i = 1$ is selected for $\alpha = 0.99$. For a multiple sourcing and the high confidence levels $\alpha = 0.9, 0.95, 0.99$, the total demand is allocated among five or six suppliers, including the cheapest one.

For the maximization of worst-case service level, where the supplier selection is independent on any cost parameters and the solution depends only on the distribution of disruption probabilities, Table 5.9 demonstrates that for a single sourcing, the same most reliable supplier $i = 1$ is selected for all confidence levels. For a multiple sourcing, however, and the confidence level $\alpha = 0.5, 0.75, 0.9$, similar optimal solutions were found with the total demand for parts allocated among the three most reliable and most expensive suppliers $i = 1, 2, 3$ in region $r = 1$, while to reduce

Table 5.8 Risk-averse minimization of cost: single versus multiple sourcing

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **SPS1_CV(c)**: Var. = 202095, Bin. = 201060, Cons. = 43717, Nonz. = 1754761 [(a)]					
CVaR^c	10.60	16.47	26.07	28.50	42.22
VaR^c	4.73	4.73	12.33	24.20	26.22
Supplier Selected	7	7	6	2	1
Expected Cost	7.66	7.66	13.70	24.42	26.38
Expected Service Level [(b)]	67.55	67.55	88.84	99.23	99.39
CPU[(c)]	969	416	1143	613	2456
Model **SPSm_CV(c)**: Var. = 202095, Bin. = 201060, Cons. = 43717, Nonz. = 1754761 [(a)]					
CVaR^c	10.60	16.47	23.53	26.51	30.74
VaR^c	4.73	4.73	19.00	23.33	29.27
Suppliers Selected(% of total demand)	7	7		2(26.04)	2(21.89)
					3(20.38)
			4(17.35)	4(20.38)	
			5(17.73)	5(18.87)	5(15.47)
			6(18.12)	6(18.87)	6(15.47)
			7(14.72)	7(15.85)	7(12.83)
			9(16.23)		
			10(15.85)		10(13.96)
Expected Cost	7.66	7.66	16.60	22.21	26.65
Expected Service Level [(b)]	67.55	67.55	76.03	38.22	31.43
CPU[(c)]	183	1035	2600	1965	13640

[(a)] Var. = number of variables, Bin. = number of binary variables, Cons. = number of constraints, Nonz. = number of nonzero coefficients
[(b)] $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T:t\le d_j} w^s_{jt}/\overline{J})100\%$
[(c)] CPU seconds for proving optimality on a MacBookPro 6.2, Intel Core i7, 2.66GHz, RAM 8GB/CPLEX 12.5

the worst-case service level outcomes for the high confidence level $\alpha = 0.95$ and $\alpha = 0.99$, total demand is allocated among seven and ten suppliers, respectively, including the most reliable $i = 1, 2, 3$.

Figures 5.7 and 5.8 present the distribution of cost per product and the distribution of customer service level for the two sourcing strategies and the confidence level $\alpha = 0.99$. For a single sourcing and the highest confidence level $\alpha = 0.99$, the highest probability measure of 0.9977 is concentrated at 100% of customer orders fulfilled by their due dates (see, Fig. 5.8), because for both objective functions, the same, most reliable supplier $i = 1$ was selected to minimize the worst-case cost of unfulfilled customer orders or maximize the worst-case customer service level, respectively. Both the distribution of cost and the distribution of service level contain also large probability atoms of 0.0061 at the highest cost, 52.32 (Fig. 5.7) and at the lowest service level, 0% (Fig. 5.8). Similar results are not observed for a multiple sourcing, which eliminates the probability atoms at the highest cost and at the lowest service level. As

Table 5.9 Risk-averse maximization of service level: single versus multiple sourcing

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **SPS1_CV(sl)**: Var. = 202095, Bin. = 201060, Cons. = 43727, Nonz. = 1649427 [(a)]					
$\text{CVaR}^{sl}\%$	98.76	97.54	93.86	87.73	38.68
$\text{VaR}^{sl}\%$	100	100	100	100	100
Supplier Selected	1	1	1	1	1
Expected Service Level [(b)]	99.39	99.39	99.39	99.39	99.39
Expected Cost	26.38	26.38	26.38	26.38	26.38
Model **SPSm_CV(sl)**: Var. = 202095, Bin. = 201060, Cons. = 43727, Nonz. = 1649427 [(a)]					
$\text{CVaR}^{sl}\%$	99.21	98.47	96.22	92.45	86.19
$\text{VaR}^{sl}\%$	100	100	100	100	92
Suppliers Selected(% of total demand)	1(44.15)	1(47.92)	1(47.92)	1(47.92)	1(21.51)
	2(37.74)	2(30.57)	2(30.57)	2(30.57)	2(21.51)
	3(18.11)	3(21.51)	3(21.51)	3(21.51)	3(14.34)
					4(6.79)
					5(7.17)
					6(7.17)
					7(7.17)
					9(7.17)
					10(7.17)
Expected Service Level [(b)]	99.61	99.62	99.62	99.62	95.64
Expected Cost	25.68	25.64	25.64	25.66	22.76

[(a)] Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients
[(b)] $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T: t\le d_j} w^s_{jt}/\bar{J})100\%$

a consequence, the corresponding optimal solution values are much better than for a single sourcing (worst-case cost 30.74 vs. 42.22 and worst-case service level 86.19% vs. 38.68%). On the other hand, for a multiple sourcing and the highest confidence level $\alpha = 0.99$, the optimal solution with the minimum, $CVaR^c = 30.74$, simultaneously produces the lowest expected service level, $E^{sl} = 31.43\%$, (see, Table 5.8), while the optimal solution with the maximum, $CVaR^{sl} = 86.19\%$, generates the average expected cost $E^c = 22.76$ (see, Table 5.9). The above conflicting results for a multiple sourcing are not observed for lower confidence levels α, for which additional supporting suppliers are not selected.

It should be pointed out that the cost and the service level objectives are in conflict, which is clearly indicated by the results in Tables 5.8 and 5.9. The expected service level is much lower for the cost-based objective function (Table 5.8), and vice versa the expected cost is much higher for the service level-based objective function (Table 5.9). The low-cost and low-reliability suppliers dominate the cost-based supply portfolios, while the high-reliability and high-cost suppliers dominate the service level-based portfolios. Furthermore, the average and the worst-case performance measures and the corresponding optimal solutions are also in conflict. The

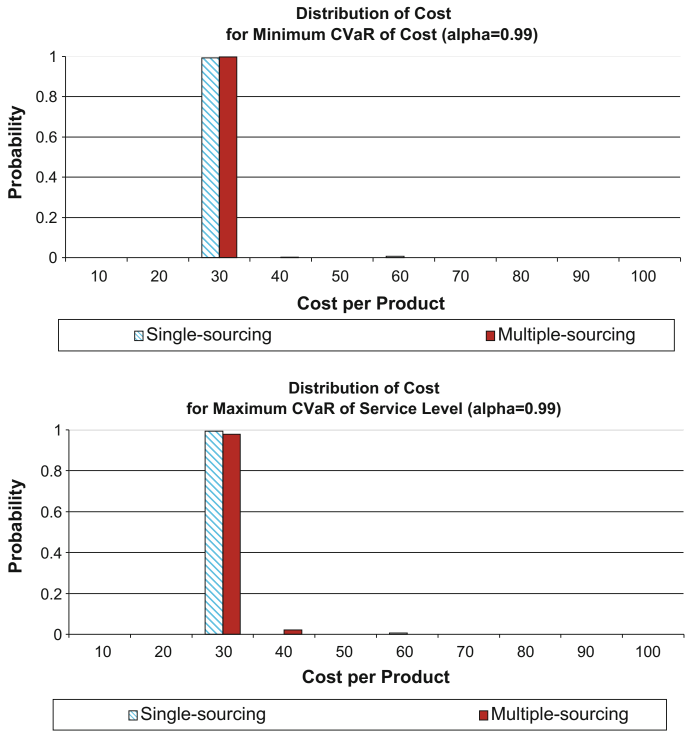

Fig. 5.7 Distribution of cost per product for risk-averse solution with $\alpha = 0.99$

higher the confidence level α, the more risk aversive the decision-making and the smaller percent of the highest cost (or the lowest service level) outcomes is focused on. As a result, the average outcomes are getting neglectful and the average performance associated with the optimal risk-averse solution may become worse. The optimal risk-averse solution with a low worst-case cost may produce a high average cost. Similarly, the optimal risk-averse solution with a high worst-case service level, may yield a low average service level. For a high confidence level α, the impact of disruption risks is usually mitigated by diversification of the supply portfolio, i.e., by selecting of more suppliers. In particular, more expensive or for maximization of demand fulfillment rate less reliable supporting suppliers may be added to the risk-averse supply portfolio, which deteriorates the associated average performance measures, i.e., the expected values of cost or service level.

As an illustrative example, Fig. 5.9 presents the cumulative demand pattern for products, $\sum_{t' \in T: t' \le t} \sum_{j \in J: d_j = t'} b_j, t \in T$, and the expected cumulative production schedules, $\sum_{s \in S} P_s \sum_{t' \in T: t' \le t} \sum_{j \in J} b_j w^s_{jt'}, t \in T$ for the optimal risk-averse solutions with the confidence level $\alpha = 0.9$. Figure 5.9 indicates that for a minimum worst-case cost objective, the demand of customers is met with a small expected fraction of the rejected demand (0.034) for the single sourcing, while for the mul-

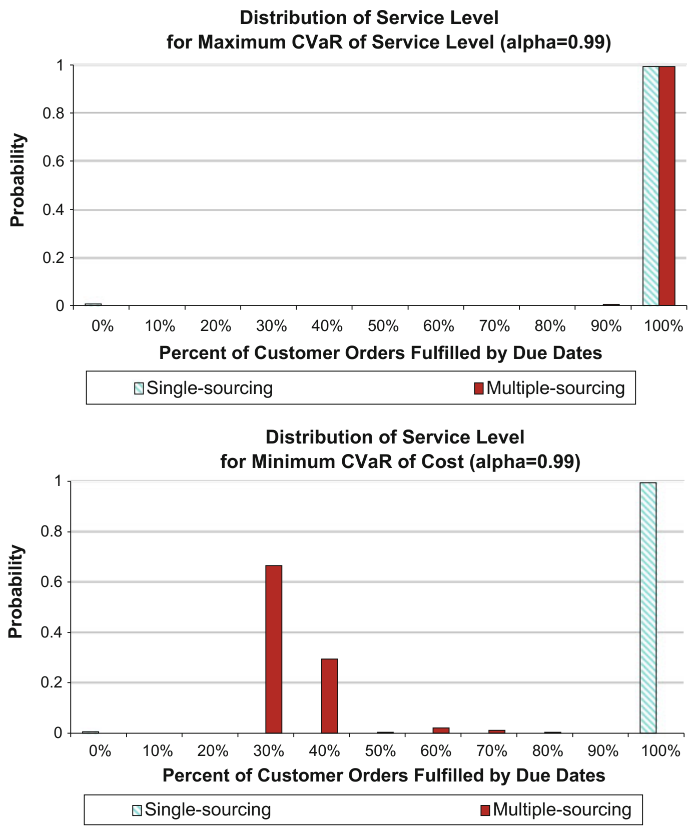

Fig. 5.8 Distribution of customer service level for risk-averse solution with $\alpha = 0.99$

tiple sourcing, the expected fraction is much greater (0.122). The expected rejected demand, however, is much smaller for the worst-case service level objective and both sourcing strategies, when no cost components are included in the objective function and more reliable suppliers are selected. Then, the expected production follows the demand pattern with a very small expected fraction of the rejected demand, 0.0061 and 0.0038, respectively for single and multiple sourcing.

The risk-averse solutions have been compared for the two sourcing strategies and the two objective functions. Some of the basic features of the obtained solutions are discussed below (see, Table 5.10).

For the worst-case cost objective function and a single sourcing, the higher the confidence level, the more reliable supplier is selected to reduce a higher risk of penalty cost for unfulfilled customer orders. In particular, to minimize the expected worst-case cost per product for low confidence levels, the cheapest suppliers are usually selected. For the worst-case cost objective function and low confidence levels, the optimal solutions for a single and a multiple sourcing are identical. A difference

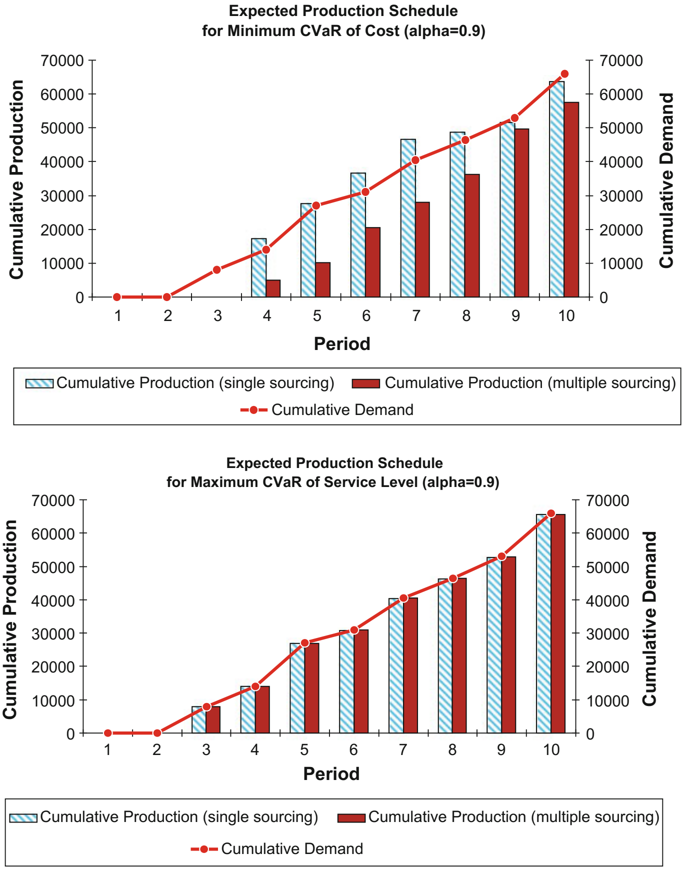

Fig. 5.9 Expected cumulative production for risk-averse solution with $\alpha = 0.9$

Table 5.10 Single versus multiple sourcing solution

Objective function	Single sourcing	Multiple sourcing
Worst-case cost	The higher is the confidence level, the more reliable supplier is selected	For low confidence levels, the solution for a single sourcing
Worst-case service level	The most reliable supplier is selected for all confidence levels	The higher is the confidence level the more suppliers are selected

between single and multiple sourcing solutions arises only for high confidence levels, whereas for the worst-case service level objective, the corresponding single and multiple sourcing solutions are different for all confidence levels.

In order to maximize the expected worst-case service level when no cost components are considered in the objective function, the most reliable supplier (with the lowest disruption probability) is selected for a single sourcing, even for low confidence levels. For a multiple sourcing and the worst-case service level, the total demand for parts is allocated among more suppliers for each confidence level to reduce the risk of unfulfilled customer orders. For high confidence levels and both cost and service level objectives, more suppliers are selected to mitigate the impact of disruption risks by diversification of the supply portfolio. The most reliable suppliers are selected to minimize the expected worst-case cost of unfulfilled customer orders or to maximize the expected worst-case customer service level. Furthermore, for the expected worst-case service level objective and both sourcing strategies, the expected production schedule approximately follows the product demand pattern with a very small expected fraction of rejected demand.

Comparison of the probability mass functions for the two sourcing strategies and the two objective functions has indicated that a multiple sourcing strategy better shapes the distribution of cost or customer service level. The multiple sourcing eliminates large probability atoms at the highest cost or at the lowest service level that otherwise may occur, if a single sourcing strategy is applied. Thus, a multiple sourcing strategy better mitigates the risk of high costs or low service levels.

The computational experiments were performed using the AMPL programming language and the CPLEX 12.5 and Gurobi 5.1 solvers on a laptop MacBookPro 6.2 with Intel Core i7 processor running at 2.66 GHz and with 8GB RAM. The CPLEX solver outperformed the Gurobi solver when the worst-case cost was minimized, while the Gurobi solver outperformed the CPLEX solver when the worst-case service level was maximized. The solvers were capable of finding proven optimal solutions for all examples with CPU time ranging from several seconds to over four hours. However, if valid inequalities (5.29)–(5.31) were not added to tighten models **SPS1_CV(sl)** and **SPSm_CV(sl)** for optimization of the worst-case service level, then CPU time required to find proven optimal solutions for the example problems was up to 15% and up to 6% longer, respectively.

5.6.3 Order Versus Demand Fulfillment Rate

This subsection focuses on comparison of the two alternative risk-averse service level measures: the expected worst-case order fulfillment rate and the expected worst-case demand fulfillment rate. In the risk-averse models for maximization of service level, CVaR^{sl}, for a given confidence level, α, is represented by an auxiliary function (5.25) introduced by Rockafellar and Uryasev (2000), where the probability of tail distribution (i.e., the probability of outcomes with worst-case service level below VaR^{sl}) is fixed to $(1-\alpha)$. However, the actual probability of the tail distribution of service level, $\sum_{s\in S:\mathscr{S}_s>0} P_s$, can be less than $(1-\alpha)$. As a result, the optimized value of CVaR^{sl} (5.25) can be greater than:

- the actual expected worst-case fraction of customer orders fulfilled on time (e.g., model **SPSm_CV(sl)**)

$$\frac{\sum_{s\in S:\mathscr{S}_s>0} P_s \sum_{j\in J} \sum_{t\in T:t\le d_j} w^s_{jt}/\overline{J}}{\sum_{s\in S:\mathscr{S}_s>0} P_s}, \tag{5.37}$$

or

- the actual expected worst-case fraction of demand fulfilled on time (e.g., model **SPSm_CV(sl)** with (5.26) replaced by (5.28))

$$\frac{\sum_{s\in S:\mathscr{S}_s>0} P_s \sum_{j\in J} \sum_{t\in T:t\le d_j} b_j w^s_{jt}/B}{\sum_{s\in S:\mathscr{S}_s>0} P_s}. \tag{5.38}$$

The above results are typical for the scenario-based optimization under uncertainty, where the probability measure is concentrated in finitely many points, called "probability atoms", i.e., singletons which have positive probability measure. The smaller is the number of concentration points and the greater are probability atoms, the smaller than $1-\alpha$ can be the probability, $\sum_{s\in S:\mathscr{S}_s>0} P_s$, of outcomes with service level below VaR, e.g., Sawik (2011b), see also Chap. 2.

The solution results for the two models **SPSm_CV(sl)** and **SPSm_CV(sl)** with (5.26) replaced by (5.28) are compared in Table 5.11. For the service level defined by order fulfillment rate (model **SPSm_CV(sl)**), Table 5.11 demonstrates that for the confidence level $\alpha = 0.5, 0.75, 0.9, 0.95$, similar optimal solutions were found with the total demand for parts allocated among the three most reliable suppliers $i = 1, 2, 3$ in region $r = 1$, and for the highest confidence level $\alpha = 0.99$, total demand is allocated among nine suppliers, except for the least reliable supplier $i = 8$. For the service level defined by demand fulfillment rate (model **SPSm_CV(sl)** with (5.26) replaced by (5.28)), Table 5.11 demonstrates that for the confidence level $\alpha = 0.5, 0.75, 0.9, 0.95$, similar optimal solutions were found with the total demand for parts allocated among the two most reliable suppliers $i = 1, 2$ in region $r = 1$, while to reduce the worst-case service level outcomes for the highest confidence level $\alpha = 0.99$, total demand is allocated among all ten suppliers.

Figure 5.10 presents the distribution of demand fulfillment rate, for the two objective functions of model **SPSm_CV(sl)** and the two confidence levels $\alpha = 0.9$ and $\alpha = 0.99$. The probability mass functions are concentrated in a few points, which is typical for the scenario-based optimization under uncertainty.

For $\alpha = 0.9$, the distribution of service level is similar for both objective functions. The highest probability measure, 0.987, is concentrated at 100% of customer demand fulfilled by due dates. The most reliable suppliers $i = 1, 2$ and $i = 1, 2, 3$ were selected to maximize the expected worst-case service level, for maximization of demand fulfillment rate and order fulfillment rate, respectively. However, the distribution of service level contains also large probability atoms at lower service levels: 0.012 at 70% and 0.001 at the lowest service level, 0%, for maximization of demand fulfillment rate, and 0.009 at 90% 0.006 at 80%, 0.005 at 70% and 0.001 at 0%, for

Table 5.11 Order versus demand fulfillment rate: risk-averse multiple sourcing

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **SPSm_CV(sl)** for order fulfillment rate					
$CVaR^{sl}$%	99.21	98.47	96.22	92.45	86.19
VaR^{sl}%	100	100	100	100	92
Suppliers Selected(% of total demand)	1(44.15)	1(47.92)	1(47.92)	1(47.92)	1(21.51)
	2(37.74)	2(30.57)	2(30.57)	2(30.57)	2(21.51)
	3(18.11)	3(21.51)	3(21.51)	3(21.51)	3(14.34)
					4(6.79)
					5(7.17)
					6(7.17)
					7(7.17)
					9(7.17)
					10(7.17)
Expected Service Level [(a)]	99.61	99.62	99.62	99.62	95.64
Expected Cost	25.68	25.64	25.64	25.66	22.76
Expected Fulfilled Demand [(c)]	99.44	99.45	99.45	99.45	95.86
Expected Worst-Case Fulfilled Orders (5.37)	81.93	82.45	82.49	82.58	83.35
Model **SPSm_CV(sl)** for demand fulfillment rate					
$CVaR^{sl}$%	98.97	97.98	95.00	90.05	78.82
VaR^{sl}%	100	100	100	100	84.85
Suppliers Selected(% of total demand)	1(50.19)	1(50.19)	1(51.32)	1(51.32)	1(21.13)
	2(49.81)	2(49.81)	2(48.68)	2(48.68)	2(16.98)
					3(16.60)
					4(5.66)
					5(5.66)
					6(11.32)
					7(5.66)
					8(5.66)
					9(5.66)
					10(5.66)
Expected Service Level [(b)]	99.49	99.49	99.50	99.50	87.90
Expected Cost	25.52	25.52	25.52	25.54	24.90
Expected Fulfilled Demand [(c)]	99.49	99.49	99.50	99.50	87.90
Expected Worst-Case Fulfilled Demand (5.38)	60.70	60.84	60.94	61.03	75.56

[(a)] $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T: t\le d_j} w^s_{jt}/\overline{J})100\%$

[(b)] $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T: t\le d_j} b_j w^s_{jt}/B)100\%$

[(c)] $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T} b_j w^s_{jt}/B)100\%$

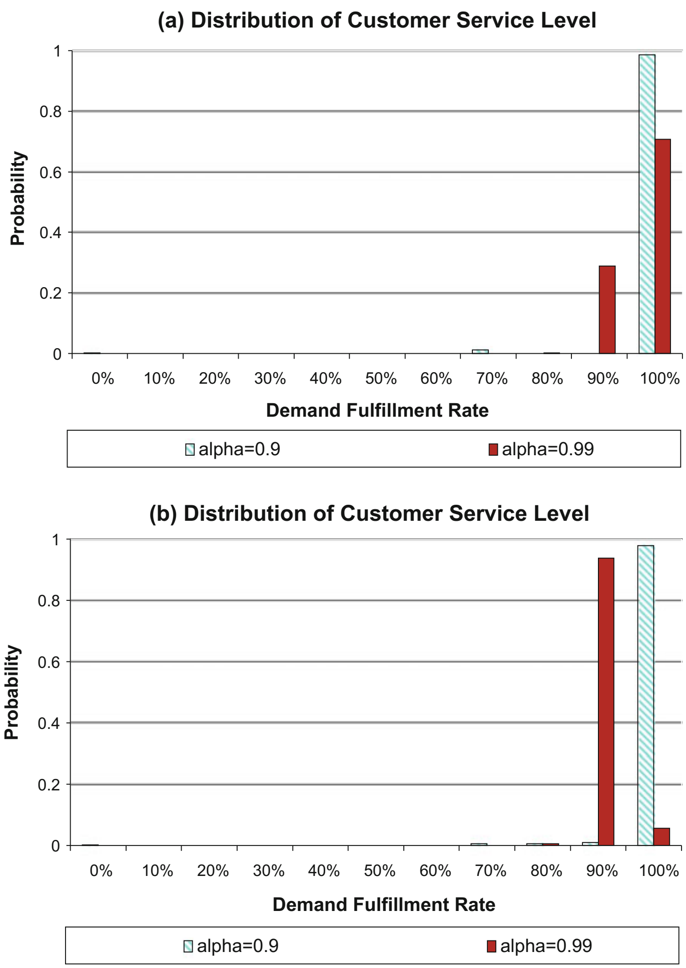

Fig. 5.10 Distribution of demand fulfillment rate: **a** model **SPSm_CV(sl)** for demand fulfillment rate, **b** model **SPSm_CV(sl)** for order fulfillment rate

maximization of order fulfillment rate. Thus, for $\alpha = 0.9$, the solution maximizing demand fulfillment rate with a greater probability atom at 70% may be outperformed by the solution maximizing order fulfillment rate.

For $\alpha = 0.99$, the distribution of service level depends on the objective function selected. For maximization of demand fulfillment rate, the highest probability measure of 0.71 is concentrated at 100% and 0.29 at 90% of customer demand fulfilled by their due dates, while for maximization of order fulfillment rate, 0.938 is concentrated at 90%, 0.0562 at 100% and 0.005 at 80% of customer demand fulfilled by due dates. However, for $\alpha = 0.99$, no probability atoms occur at lower service levels.

For a high confidence level α, the impact of disruption risks is usually mitigated by diversification of the supply portfolio, i.e., by selecting more suppliers. In particular, for maximization of demand fulfillment rate less reliable supporting suppliers may be added to the risk-averse supply portfolio, which deteriorates the associated average performance measure, i.e., the expected value of service level.

5.6.4 Conditional Service-At Risk Versus Expected Cost

In this subsection the impact on the cost in the process of optimization of worst-case service level is illustrated with additional computational examples. The cost includes the cost of ordering and purchasing of parts plus penalty cost of delayed and unfulfilled customer orders due to parts shortages. For example, the impact on the cost in the process of optimization of worst-case service level is illustrated in Table 5.11 with the expected cost per product, E^c, associated with the risk-averse solution for model **SPSm_CV(sl)**. The expected costs associated with the risk-averse solutions for $\alpha = 0.5,\ 0.75,\ 0.9,\ 0.95$ are similar. To maximize the worst-case service level, the parts are purchased from high-reliability and high-cost suppliers only. As a result the purchasing cost dominates, while the cost of parts shortage is not significant. However, for the highest confidence level $\alpha = 0.99$, when the supply portfolio is most diversified to mitigate the impact of worst 1% of all scenarios outcomes, the orders for parts are also placed on low-cost and low-reliability suppliers. The resulting total purchasing cost decreases as well as the resulting total expected cost per product, $E^c = 22.76$ for model **SPSm_CV(sl)** and $E^c = 24.90$ for model **SPSm_CV(sl)** with (5.26) replaced by (5.28).

Table 5.12 presents a subset of nondominated solutions for the mixed mean-risk model **SPSm_E(c)CV(sl)** and confidence level $\alpha = 0.9$, obtained for a subset of trade-off parameter, $\lambda = 0, 0.01, 0.25, 0.5, 0.75, 0.99, 1$. Note that a single-objective solution with maximum conditional service-at-risk (cf. Table 5.11) and with minimum expected cost is obtained for $\lambda = 0$ and $\lambda = 1$, respectively.

Table 5.12 clearly shows that the two objective functions: expected worst-case service level and expected cost are in conflict. The higher the trade-off parameter λ, the more cost-oriented the decision-making and the lower the expected cost, E^c, the lower the conditional service-at-risk, $CVaR^{sl}$, as well the associated expected service level for both measures: order fulfillment rate and demand fulfillment rate. As λ increases from 0 to 1, the supply portfolio is changing from the one based on a subset of most reliable and expensive suppliers, $i = 1, 2, 3$ (model **SPSm_E(c)CV(sl)**) and $i = 1, 2$ (model **SPSm_E(c)CV(sl)** with (5.26) replaced by (5.28)) for $\lambda = 0, 0.01$, followed with more diversified subsets of less reliable and cheaper suppliers selected for $\lambda = 0.25, 0.5, 0.75$, through the single, cheapest supplier, $i = 7$, for $\lambda = 0.99, 1$. While CVaR^{sl} and expected service level, E^{sl}, are significantly decreasing with the trade-off parameter λ, the associated expected fraction of fulfilled (on time or delayed) demand decreases at the much lower rate. This indicates that as λ

Table 5.12 Nondominated solutions for mean-risk model **SPSm_E(c)CV(sl)**: $\alpha = 0.9$

Trade-off parameter λ	0	0.01	0.25	0.50	0.75	0.99	1
Model **SPSm_E(c)CV(sl)** for order fulfillment rate							
E^c	25.64	25.50	14.55	12.94	8.49	7.77	7.66
$CVaR^{sl}\%$	96.22	96.16	88.19	84.88	53.63	28.94	27.66
$VaR^{sl}\%$	100	100	92	92	92	76	72
Suppliers Selected(% of total demand)	1(47.92)	1(40.75)					
	2(30.57)	2(37.74)	2(7.55)	2(6.04)			
	3(21.51)	3(21.51)	3(6.41)				
			5(21.51)	5(20.38)	5(4.90)		
			6(21.51)	6(21.51)	6(5.67)		
			7(21.51)	7(30.56)	7(89.43)	7(100)	7(100)
			10(21.51)	10(21.51)			
Expected Service Level [(a)]	99.62	99.62	98.02	97.74	88.16	71.29	67.56
Expected Fulfilled Demand [(c)]	99.45	99.44	96.70	96.31	93.94	93.82	93.85
Model **SPSm_E(c)CV(sl)** for demand fulfillment rate							
E^c	25.52	25.52	18.52	13.11	8.23	7.67	7.66
$CVaR^{sl}\%$	95.00	95.00	85.79	77.44	43.02	29.91	28.08
$VaR^{sl}\%$	100	100	90.91	84.85	87.88	78.79	73.48
Suppliers Selected(% of total demand)	1(51.32)	1(50.19)	1(4.91)				
	2(48.68)	2(49.81)	2(16.98)	2(6.04)			
			3(13.58)				
			4(13.21)				
			5(13.58)	5(22.64)			
			6(13.58)	6(21.51)	6(10.57)		
			7(13.21)	7(27.17)	7(89.43)	7(100)	7(100)
			9(10.95)	9(22.64)			
Expected Service Level [(b)]	99.50	99.50	97.27	96.26	83.39	73.90	68.96
Expected Fulfilled Demand [(c)]	99.50	99.50	97.27	96.26	94.38	93.84	93.85

[(a)] $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T:t\le d_j} w^s_{jt}/\overline{J})100\%$
[(b)] $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T:t\le d_j} b_j w^s_{jt}/B)100\%$
[(c)] $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T} b_j w^s_{jt}/B)100\%$

approaches 1, the fulfilled demand becomes more delayed and the resulting service level decreases.

Examples of nondominated schedules for the confidence level $\alpha = 0.9$ and the trade-off parameter $\lambda = 0.5$ are shown in Fig. 5.11. Figure 5.11 compares the expected cumulative production with the expected worst-case cumulative production for model **SPSm_E(c)CV(sl)** maximizing either demand fulfillment rate or order fulfillment rate. The corresponding production schedules are similar for both objectives with a slightly greater expected and expected worst-case cumulative production for maximization of demand fulfillment rate.

Overall, the obtained solution results for the service level objectives are in line with the other approaches used in the area of supply chain risk management. For

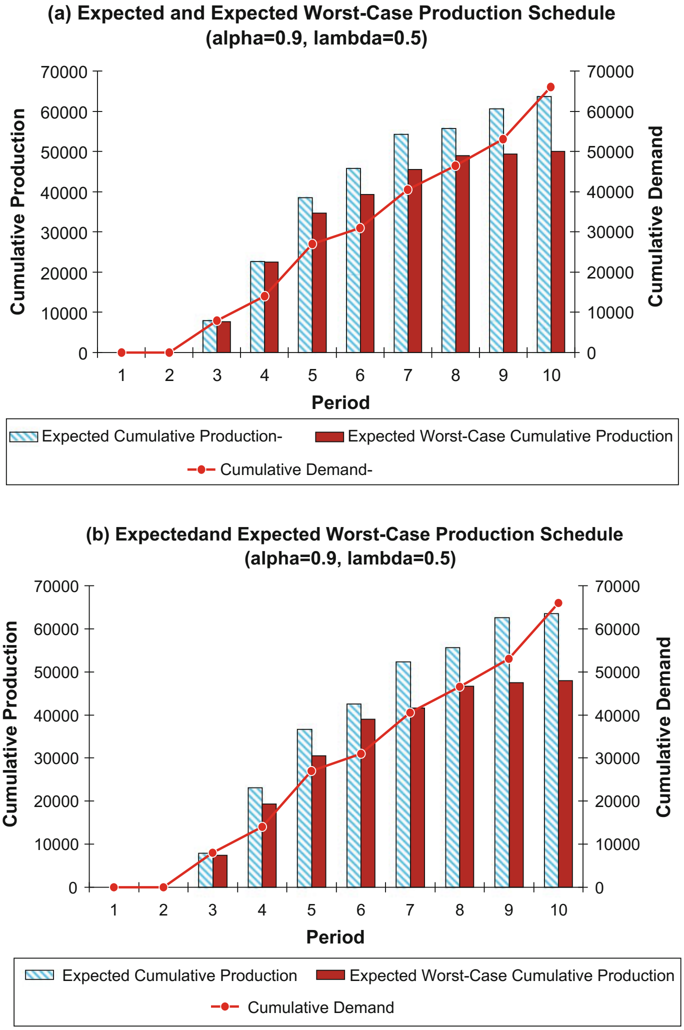

Fig. 5.11 Expected and expected worst-case cumulative production for $\alpha = 0.9$ and $\lambda = 0.5$: **a** model **SPSm_E(c)CV(sl)** for demand fulfillment rate, **b** model **SPSm_E(c)CV(sl)** for order fulfillment rate

example, the higher the confidence level, the more diversified the supply portfolio and the smaller the probability atoms at the lowest service level, i.e., the risk of low service level is more efficiently mitigated. Furthermore, the two service level measures are related in magnitude, while the worst-case service level and the expected cost are in opposition. Some additional properties of the obtained optimal risk-averse solutions are listed and discussed below.

- *Given confidence level α, the worst-case order fulfillment rate shows a higher service performance than the worst-case demand fulfillment rate. The supply portfolio is more diversified and the expected worst-case fraction of fulfilled orders is greater for most confidence levels.*
- *The expected service level exceeds the corresponding expected worst-case service level (conditional service-at-risk), i.e., the average performance is better than the worst-case performance of a supply chain.*
- *For most confidence levels α, the total demand for parts is allocated among the most reliable suppliers only. For the highest level $\alpha = 0.99$, however, the impact of disruption risks is usually mitigated by diversification of the supply portfolio, i.e., by selecting more suppliers. In particular, less reliable supporting suppliers may be added to the risk-averse supply portfolio, which deteriorates the associated average performance measure, i.e., the expected service level.*
- *For both service level measures, the expected production schedule approximately follows the product demand pattern, with a relatively small fraction of unfulfilled customer demand for low confidence levels and greater for the highest confidence level. However, the expected worst-case production depends on the optimized service level measure and the confidence level. The largest unfulfilled demand has been observed for low confidence levels and model* **SPSm_CV(sl)** *for maximization of demand fulfillment rate, for which the supply portfolio is less diversified than the corresponding portfolio for maximization of order fulfillment rate.*
- *Comparison of solution results for the two service level measures indicates that maximization of the expected worst-case order fulfillment rate may better mitigate the impact of disruption risks. For most confidence levels, the expected worst-case order fulfillment rate is greater than the expected worst-case demand fulfillment rate.*

Comparison of optimal CVaR^{sl} (5.25), with the actual expected worst-case service level, (5.37) and (5.38), clearly demonstrates that the optimal value of the auxiliary objective function (5.25) used to maximize conditional service-at-risk, overestimates the actual value of CVaR^{sl} calculated using (5.37) and (5.38), which is typical for the scenario-based optimization under uncertainty, when the probability measure is concentrated in finitely many points.

The computational experiments were performed using the AMPL programming language and Gurobi 6.0 solver on a laptop MacBookPro with Intel Core i7 processor running at 2.8GHz and with 16GB RAM. The solver was capable of finding proven optimal solutions for all examples with CPU time ranging from a few seconds to around one hour. CPU time increases with the confidence level α and approaches one hour for $\alpha = 0.99$, when the supply portfolio is most diversified to mitigate the impact of worst 1% of all scenarios outcomes. Given confidence level, the computational effort required to find proven optimal solution is smaller for maximization of demand fulfillment rate than the order fulfillment rate, for which the size of customer order is not considered and hence all orders are equally important. For model

SPSm_E(c)CV(sl), given confidence level, the greatest CPU time is required for medium values of the trade-off parameter $\lambda = 0.25, 0.5, 0.75$, when both criteria are significant.

5.7 Notes

As firms expand their business globally, their supply chains involve more global partners. According to an empirical study conducted by Shin et al. (2000) and Sounderpandian et al. (2008) dual or multiple sourcing is a common business practice to mitigate the impact of various operational and disruption risks. In view of the recent trend of outsourcing and globalization, integrated selection of part suppliers and allocation of order quantities and scheduling of customer orders may significantly improve performance of a multi-echelon supply chain under disruption risks. However, the research on quantitative approaches to the integrated supplier selection and customer order scheduling in the presence of supply chain disruption risks has not been often reported in the literature. Most work on integrated supply chain scheduling focuses on coordinating the flows of supply and demand over a supply chain network to minimize the inventory, transportation and shortage costs. For example, Chen and Vairaktarakis (2005), Chen and Pundoor (2006) and Pundoor and Chen (2009) studied simplified models for integrated scheduling of production and distribution operations. The authors have analyzed computational complexity of various cases of the problem and have developed heuristics for NP-hard cases. Lei et al. (2006) considered an integrated production, inventory and distribution routing problem involving heterogeneous transporters with non-instantaneous traveling times and many capacitated customer demand centers. A MIP approach combined with a heuristic routing algorithm was proposed to coordinate the production, inventory and transportation operations. Bard and Nananukul (2009) developed a MIP model and a reactive tabu search-based algorithm for a transportation scheduling problem that included a single production facility, a set of customers with time-varying demand and a fleet of vehicles. Wang and Lei (2012) considered the problem of operations scheduling for a capacitated multi-echelon shipping network with delivery deadlines, where semi-finished goods are shipped from suppliers to customers through processing centers, with the objective of minimizing the shipping and penalty cost. The three polynomial-time solvable cases of this problem were reported: with identical order quantities; with designated suppliers; and with divisible customer order sizes. Liu and Papageorgiou (2013) developed a multi objective MIP approach to address production, distribution and capacity planning of global supply chains considering cost, responsiveness and customer service level simultaneously. An integrated approach to deterministic coordinated supply chain scheduling was proposed by Sawik (2007) to simultaneously schedule manufacturing and supply of parts and assembly of finished products. Given a set of part suppliers and a set of customer orders for finished products, the problem objective was to determine which orders were provided with parts by each supplier, to schedule manufacturing of parts at each supplier and delivery of parts from each supplier to the producer, and to schedule customer orders at the producer, such that a high customer service level was achieved and the total cost was minimized. The selection of part supplier for each customer order was combined with a due date setting for some orders to maximize the number of orders that can be completed by customer requested due dates. A monolithic MIP model was presented and compared with a hierarchy of mixed integer programs for a sequential selection of suppliers and scheduling of manufacturing and delivery of parts and assembly of products. Different enhancements of the above MIP approach for the coordinated scheduling in multi-echelon supply chains were presented in Sawik (2009a). Various perspectives on supply chain coordination issues were reported and reviewed by Arshinder (2008) and the gaps existing in the literature were identified. Li and Wang (2007) reviewed coordination mechanisms of supply chain systems in a framework that was based on supply chain decision structure and nature of demand and a review of methods and literature on supply chain coordination through contracts was provided by Hezarkhani and Kubiak (2010).

The customer service level in a make-to-order environment was studied by Altendorfer and Jodlbauer (2011). Shao and Dong (2012) compared and analyzed order fulfillment performance measures for two different production control systems: make-to-order versus make-to-stock. They formulated service-maximization models with inventory cost budget constraints. For the make-to-stock production, Larsen and Thorstenson (2008, 2014) differentiated between an order fill rate and a volume fill rate and specified their performance for different inventory control systems. They showed how the order and volume fill rates are related in magnitude. Bijulal et al. (2011) analyzed the production-inventory system performance in terms of order fill rate and average system costs, affected by the control parameters.

The material presented in this chapter is based on the research results reported in Sawik (2013c, 2014a, b) where single, dual and multiple sourcing strategies were compared for risk-neutral and risk-averse decision-making. In addition, Sawik (2016b) compared the two risk-averse service level measures: expected worst-case order fulfillment rate and expected worst-case demand fulfillment rate. The former corresponds to order fill rate and the latter to volume fill rate in the make-to-stock production (Larsen and Thorstenson 2008, 2014). The future research should focus on comparison of the proposed integrated supplier selection and customer order scheduling with a common hierarchical approach, where first the supplier selection and order quantity allocation subject to disruption risks is accomplished and then, given a schedule of part supplies, the optimal schedule of customer orders is determined for each disruption scenario, subject to parts availability constraints.

Problems

5.1 Modify the SMIP models presented in this chapter for multiple part types with subsets of part types required for each customer order and subsets of suppliers available for each part type.

5.2 Modify the SMIP models presented in this chapter for finite capacity suppliers.

5.3 Mixed mean-risk supply portfolio and scheduling
(a) Formulate model **SPSm_E(sl)CV(c)** for a mixed mean-risk selection of supply portfolio and scheduling of customer orders to trade-off expected service level and CVaR of cost.
(b) How should the values of the optimized objective functions be scaled into the interval [0,1] to avoid dimensional inconsistency among the two objectives and how should the trade-off parameter be selected?
(c) How would you interpret the mixed mean-risk solution?

5.4 Explain why order fulfillment rate shows a higher service performance than demand fulfillment rate.

5.5 Looking into the computational examples try to compare dual versus multiple solution results.

Chapter 6
Integrated Selection of Supply Portfolio and Scheduling of Production and Distribution

6.1 Introduction

The key operational functions in a supply chain are supply, production and distribution operations. To achieve a high performance of supply chain, it is crucial to integrate these three functions and jointly schedule supply, production and distribution in a coordinated manner. For example, in customer-driven supply chains, where customer orders are executed immediately or shortly after arrival of material supplies and the ordered products are delivered to customers immediately or shortly after their completion, the impact of disruption risks can be best mitigated when an integrated decision-making is applied. At the same time, the integrated decision-making allows reaching various conflicting objectives, such as reduction in total cost and increase in service level. The purpose of this chapter is to study the integrated decision-making to simultaneously select suppliers of parts, allocate order quantity and schedule production and delivery of finished products to customers in a supply chain under disruption risks. In addition to supplier selection, order quantity allocation and scheduling of customer orders, distribution of finished products to customers is simultaneously considered with different shipping methods to optimize the trade-off between cost and service level. The service level in this chapter denotes demand fulfillment rate (see, Sect. 5.3.2). The three different shipping methods will be modelled and compared for the distribution of products: batch shipping with a single shipment of different customer orders, batch shipping with multiple shipments of different customer orders and individual shipping of each customer order immediately after its completion. In addition, the SMIP formulation based on the wait-and-see approach to optimize the trade-off between expected cost and expected service level will be compared with a deterministic MIP model based on the expected value approach, in which random parameters are replaced by their expected values.

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The following time-indexed SMIP and MIP models are presented in this chapter:

SCS1_E for risk-neutral scheduling in a supply chain with single batch shipping of products to distribution centers;
SCS2_E for risk-neutral scheduling in a supply chain with individual shipping of products for each customer order to distribution centers;
SCS3_E for risk-neutral scheduling in a supply chain with multiple batch shipping of products to distribution centers;
SCS1_CV for risk-averse scheduling in a supply chain with single batch shipping of products to distribution centers;
SCS2_CV for risk-averse scheduling in a supply chain with individual shipping of products for each customer order to distribution centers;
SCS3_CV for risk-averse scheduling in a supply chain with multiple batch shipping of products to distribution centers;
ESCS1 for the expected value-based supply chain scheduling, corresponding to model **SCS1_E**.

The models incorporate supply-production, production-distribution and supply-distribution coordinating constraints to efficiently coordinate supply, production and distribution schedules.

Numerical examples and computational results are reported in Sect. 6.6 for risk-neutral, risk-averse and expected value - based decision-making.

6.2 Problem Description

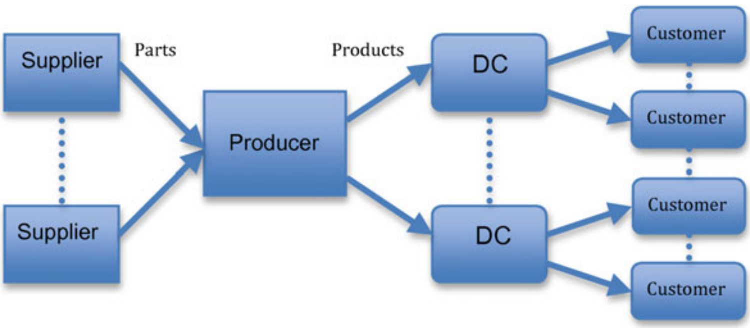

Fig. 6.1 A multi-echelon supply chain

A multi-echelon supply chain under disruption risk is studied with multiple suppliers of a critical part type, producer of one product type, and multiple distribution centers that provide with products a set of customers (see, Fig. 6.1).

Let $I = \{1, \ldots, \overline{I}\}$ be the set of $\overline{I}$ suppliers, $J = \{1, \ldots, \overline{J}\}$ the set of $\overline{J}$ customers, $K = \{1, \ldots, \overline{K}\}$ the set of $\overline{K}$ distribution centers, and $T = \{1, \ldots, \overline{T}\}$ the set of $\overline{T}$ planning periods (for notation used, see Table 6.1).

Table 6.1 Notation: supply chain scheduling

	Indices
i	= supplier, $i \in I$
j	= customer order, $j \in J$
k	= distribution center, $k \in K$
r	= region, $r \in R$
s	= disruption scenario, $s \in S$
t	= planning period, $t \in T$
	Input Parameters
a_k	= fixed transportation cost of each shipment to distribution center $k \in K$
b_j	= size (number of products) of customer order j
B	= $\sum_{j \in J} b_j$, total demand for products
C_P	= per period capacity of producer
C_V	= vehicle capacity
d_j	= due date for customer order j
e_i	= fixed cost of ordering parts from supplier i
g_j	= per unit and per period penalty cost of delayed customer order j
h_j	= per unit penalty cost of unfulfilled customer order j
I^r	= subset of suppliers in region r
J_k	= subset of customers (customer orders) served by distribution center k
o_i	= per unit price of parts purchased from supplier i
p_i	= local disruption probability for supplier i
p^r	= regional disruption probability for all suppliers in region r
τ_i	= delivery lead time from supplier i
σ_k	= transportation time to distribution center k

Denote by $J_k \subset J$ the subset of customers (or equivalently customer orders) served by the distribution center $k \in K$. The demand of each customer $j \in J_k$ is defined by the number of products, b_j and the latest period of their delivery to the distribution center k, d_j. The total demand for parts is identical with the total demand for products, $B = \sum_{j \in J} b_j$. For each customer order $j \in J$, denote by g_j and h_j, the unitary delay penalty cost and unfulfillment penalty cost, respectively. The producer aggregate capacity in each planning period is denoted by C_P. The capacity depends on machine configuration of production facility and the processing time available on each machine in a single planning period, e.g., Sawik (2009c, 2011a). (In the computational examples provided in Sect. 6.6, the production facility is a flexible flow shop with parallel batch machines.)

Each supplier $i \in I$ is characterized by the four parameters: the unit purchasing price, o_i; the fixed ordering cost, e_i; the constant lead time (manufacturing and transportation time to producer), τ_i; and the local disruption probability, p_i, i.e., the probability that parts ordered from supplier i are not delivered. The parts ordered from supplier i can be used by the producer not earlier than in period $\tau_i + 1$.

Three distribution strategies of customer orders will be considered:

- The finished products for all customers $j \in J_k$ are delivered to distribution center k in a single batch shipment, after completion of all scheduled customer orders $j \in J_k$.
- The finished products for each customer order $j \in J_k$ are individually shipped to distribution center k, immediately after its completion.
- The finished products for customers $j \in J_k$ are delivered to distribution center k in multiple batch shipments, subject to limited transportation capacity.

The transportation time to distribution center k is constant and equals to σ_k periods so that the products shipped in period t are delivered in period $t + \sigma_k - 1$.

For the multiple batch shipping strategy, additionally denote by C_V, vehicle limited capacity defined as the maximum number of finished products that can be shipped together, and by a_k, a fixed transportation cost of each shipment to distribution center $k \in K$.

The above shipping methods can be easily justified from a practical point of view. The individual and immediate shipping are commonly used for distribution of time-sensitive products to achieve a fast delivery. The single and multiple batch shipping methods are used for orders going to the same distribution center, The shipping methods with unlimited transportation capacity, e.g., infinitely many vehicles, are applicable when the delivery is handled by a third-party logistics provider that typically owns a large number of vehicles. On the other hand, the multiple batch shipping with limited transportation capacity accounts for limited capacity of a single vehicle. The shipping methods considered do not account for transportation time and costs to customers since routing (e.g., vehicle routing) is not a part of the decision-making, and the customers are assumed to be individually served by the corresponding distribution centers. Alternatively, for each distribution center $k \in K$, all customers $j \in J_k$ can be assumed to be co-located.

The suppliers are assumed to be located in $\overline{R}$ disjoint geographic regions. Denote by $I^r \subseteq I$ the subset of suppliers in region $r \in R = \{1, \ldots, \overline{R}\}$, where $\bigcup_{r \in R} I^r = I$. The supplies of parts are subject to random local disruptions that are uniquely associated with a particular supplier, which may arise from equipment breakdowns, local labor strike, fires, etc. Denote by p_i the local disruption probability for supplier i, i.e., the parts ordered from supplier i are delivered without disruptions with probability $(1 - p_i)$, or not at all with probability p_i. In addition to independent local disruptions of each supplier, the supplies of parts are also subject to correlated regional disruption of all suppliers in the same region simultaneously, with probability p^r for region $r \in R$.

Let $S = \{1, \ldots, \overline{S}\}$ be the index set of $\overline{S} = 2^{\overline{I}}$ disruption scenarios, where each scenario $s \in S$ defines a subset $I_s \subset I$ of non-disrupted suppliers. The supplies from every supplier, $i \in I \setminus I_s$, can be independently disrupted either by a local or by a regional disaster event. The probability P_s of each disruption scenario $s \in S$ is a product over all regions $r \in R$ of probabilities P_s^r of realizing disruption scenario s for suppliers in I^r,

$$P_s = \prod_{r \in R} P_s^r,$$

where P_s^r is (cf. Sect. 1.3)

$$P_s^r = \begin{cases} (1 - p^r) \prod_{i \in I^r \bigcap I_s} (1 - p_i) \prod_{i \in I^r \setminus I_s} p_i & \text{if } I^r \bigcap I_s \neq \emptyset \\ p^r + (1 - p^r) \prod_{i \in I^r} p_i & \text{if } I^r \bigcap I_s = \emptyset. \end{cases}$$

The problem of the integrated supply, production and distribution scheduling will be formulated under the following simplified assumptions:

- orders for parts are placed at the start of the planning horizon, when all customer orders for products are known;
- each supplier capacity is sufficient to meet total demand for parts and to complete and prepare orders for shipping in a single planning period;
- all parts ordered from a supplier are shipped together in a single delivery;
- customers are assigned to distribution centers ahead of time and each customer is served by exactly one distribution center;
- each customer order can be completed by the producer in a single planning period;
- customer requested due dates are replaced with delivery due dates for distribution centers;
- transportation times to distribution centers are constant, and the delivery dates are determined by the shipping dates to distribution centers;
- transportation times and costs to individual customers are not considered and vehicle routing is not a part of the decision-making;
- inventory of parts and products are not considered;
- three shipping methods: single batch shipping, multiple batch shipping and individual and immediate shipping are considered separately.

Some of the above assumptions can be easily relaxed, while others need more advanced models to be developed.

6.3 Models for Risk-Neutral Decision-Making

In this section three SMIP models **SCS1_E**, **SCS2_E** and **SCS3_E** are presented for the integrated supplier selection, order quantity allocation and scheduling production of finished products and distribution to customers to minimize expected cost per product and maximize expected service level (expected demand fulfillment rate). Model **SCS1_E** is formulated for supply chain scheduling with a single batch shipping of finished products to each distribution center for all customers served by that center. Model **SCS2_E** accounts for individual and immediate shipment of products to distribution centers for each customer. In models **SCS1_E** and **SCS2_E** transportation cost and capacity are not considered. Model **SCS3_E** is formulated for multiple batch shipping of finished products to each distribution center and accounts for transportation cost and capacity.

The problem variables are defined in Table 6.2.

Table 6.2 Variables: supply chain scheduling

		First stage variables
u_i	$=$	1, if supplier i is selected; otherwise $u_i = 0$ (supplier selection)
v_i	$\in$	[0, 1], the fraction of total demand for parts ordered from supplier i (supply portfolio)
		Second stage variables
w^s_{jt}	$=$	1, if under disruption scenario s customer order j is scheduled for period t; otherwise $w^s_{jt} = 0$ (production scheduling, single and multiple batch shipping)
	$=$	1, if under disruption scenario s customer order j is scheduled for period t and shipped to distribution center in period $t+1$; otherwise $w^s_{jt} = 0$ (production and distribution scheduling, individual shipping)
x^s_{kt}	$=$	1, if under disruption scenario s, batch shipment of products to distribution center k is scheduled for period t; otherwise $x^s_{kt} = 0$ (distribution scheduling, single and multiple batch shipping)
y^s_j	$=$	1, if under disruption scenario s customer order j is delivered by its due date; otherwise $y^s_j = 0$ (customer order non-delay delivery, single batch shipping)
z^s_{jt}	$=$	1, if under disruption scenario s customer order j is shipped to distribution center in period t; otherwise $z^s_{jt} = 0$ (distribution scheduling, multiple batch shipping)
		Auxiliary variables
VaR^c		Cost-at-Risk, the targeted cost such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is below VaR^c
VaR^{sl}		Service-at-Risk, the targeted service level such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is above VaR^{sl}
$\mathscr{C}_s$	$\geq$	0, the tail cost for scenario s, i.e., the amount by which costs in scenario s exceed VaR^c
$\mathscr{S}_s$	$\geq$	0, the tail service level for scenario s, i.e., the amount by which VaR^{sl} exceeds service level in scenario s

6.3.1 Scheduling with Single Batch Shipping

The total customer demand, $\sum_{j\in J} b_j$, under each disruption scenario $s \in S$, can be split into the three satisfaction levels: non-delayed, delayed or unscheduled (rejected). For a single batch shipping, these portions of the customer demand are defined below.

- Non-delayed, $\sum_{j\in J} b_j y_j^s$,
- Delayed, $\sum_{j\in J} b_j(\sum_{t\in T} w_{jt}^s - y_j^s)$,
- Unscheduled (rejected), $\sum_{j\in J} b_j(1-\sum_{t\in T} w_{jt}^s)$.

The expected cost per product, E^c, and service level, E^{sl}, as well their normalized values, respectively f^c and f^{sl}, are defined below.

$$E^c = (\sum_{i\in I} e_i u_i + \sum_{s\in S} P_s(\sum_{i\in I_s} Bo_i v_i + \sum_{j\in J} g_j b_j(\sum_{t\in T} w_{jt}^s - y_j^s) + \sum_{j\in J} h_j b_j(1-\sum_{t\in T} w_{jt}^s)))/B. \quad (6.1)$$

$$E^{sl} = \sum_{s\in S}\sum_{j\in J} P_s b_j y_j^s/B. \quad (6.2)$$

$$f^c = \frac{E^c - \underline{E}^c}{\overline{E}^c - \underline{E}^c}, \quad (6.3)$$

where $\underline{E}^c$ and $\overline{E}^c$ are the minimum and the maximum values of E^c, respectively.

$$f^{sl} = \frac{\overline{E}^{sl} - E^{sl}}{\overline{E}^{sl} - \underline{E}^{sl}}, \quad (6.4)$$

where $\underline{E}^{sl}$ and $\overline{E}^{sl}$ are the minimum and the maximum values of E^{sl}, respectively.

A subset of nondominated solutions can be obtained by the parameterization on λ, the model presented below.

SCS1_E: *Risk-neutral* **S**upply **C**hain **S**cheduling*: single batch shipping*

Minimize

$$\lambda f^c + (1-\lambda) f^{sl}, \quad (6.5)$$

where $0 \le \lambda \le 1$,

subject to (6.1)–(6.4) and

Supply portfolio selection constraints:

$$\sum_{i\in I} v_i = 1 \quad (6.6)$$

$$v_i \le u_i; \ i \in I \quad (6.7)$$

Customer order scheduling constraints:

$$\sum_{t \in T} w_{jt}^s \le 1; \;\; j \in J, s \in S \tag{6.8}$$

$$\sum_{j \in J} b_j w_{jt}^s \le C_P; \;\; t \in T, s \in S \tag{6.9}$$

Supply-production coordinating constraints:

$$\sum_{j \in J} \sum_{t' \in T : t' \le t} b_j w_{jt'}^s \le B \sum_{i \in I_s : \tau_i \le t-1} v_i; \;\; t \in T, s \in S \tag{6.10}$$

Single batch shipping constraints:

$$\sum_{t \in TK} x_{kt}^s \le 1; \;\; k \in K, s \in S \tag{6.11}$$

$$\sum_{t \in TK} x_{kt}^s \ge \sum_{t \in T} w_{jt}^s; \;\; k \in K, j \in J_k, s \in S \tag{6.12}$$

$$\sum_{t \in TK} x_{kt}^s \le \sum_{j \in J_k} \sum_{t \in T} w_{jt}^s; \;\; k \in K, s \in S, \tag{6.13}$$

where $TK = \{\min_{i \in I} \tau_i + 2, \ldots, \overline{T} + 1\}$ is the set of shipping periods. *Production-distribution coordinating constraints*:

$$\sum_{t \in TK} t x_{kt}^s \ge \sum_{t \in T} (t+1) w_{jt}^s; \;\; k \in K, j \in J_k, s \in S \tag{6.14}$$

Non-delayed delivery constraints:

$$y_j^s \le \sum_{t \in T : t \le d_j - \sigma_k} w_{jt}^s; \;\; k \in K, j \in J_k, s \in S \tag{6.15}$$

$$y_j^s \le \sum_{t \in TK : t \le d_j - \sigma_k + 1} x_{kt}^s; \;\; k \in K, j \in J_k, s \in S \tag{6.16}$$

$$\sum_{t \in T : t \le d_j - \sigma_k} w_{jt}^s + \sum_{t \in TK : t \le d_j - \sigma_k + 1} x_{kt}^s - 1 \le y_j^s; \;\; k \in K, j \in J_k, s \in S \tag{6.17}$$

Non-negativity and integrality conditions:

$$u_i \in \{0, 1\}; \;\; i \in I \tag{6.18}$$

$$v_i \in [0, 1]; \;\; i \in I \tag{6.19}$$

$$w_{jt}^s \in \{0, 1\}; \; j \in J, t \in T, s \in S \tag{6.20}$$

$$x_{kt}^s \in \{0, 1\}; \; k \in K, t \in TK, s \in S \tag{6.21}$$

$$y_j^s \geq 0; \; j \in J, s \in S. \tag{6.22}$$

Constraints (6.6)–(6.7) define a feasible supply portfolio, and (6.8)–(6.9), a feasible assignment of customer orders to planning periods for each disruption scenario. The supply-production coordinating constraints (6.10) ensure that for each disruption scenario s, the cumulative demand for required parts of all customer orders scheduled by period t is not greater than the cumulative supplies by period $t-1$ from the non-disrupted suppliers $i \in I_s$. Equation (6.11) ensures that for each disruption scenario, at most one batch shipment can be scheduled to each distribution center. Constraint (6.12) denotes that for each disruption scenario, shipment to distribution center k is scheduled, only if at least one customer order $j \in J_k$ is completed, and Eq. (6.13) that no shipment to distribution center k is scheduled, if no customer order $j \in J_k$ is completed. The production-distribution coordinating constraints (6.14) ensure that for each disruption scenario, a shipment to distribution center k can be scheduled only after the latest completion period of scheduled customer orders $j \in J_k$. Finally, constraints (6.15)–(6.17) denote that for each disruption scenario $s \in S$, customer order $j \in J_k$ can be delivered without delay (i.e., $y_j^s = 1$), unless it is scheduled not later than $d_j - \sigma_k$ and shipped to distribution center k not later than $d_j - \sigma_k + 1$; otherwise the customer order is delayed or unscheduled (i.e., $y_j^s = 0$),

In the above formulation y_j^s is not restricted to being binary. However, for any feasible solution satisfying customer due-date meeting constraints (6.15)–(6.17), $y_j^s \in \{0, 1\}; \; j \in J, s \in S$. Note that $y_j^s = 1$ only if both $\sum_{t \in T: t \leq d_j - \sigma_k} w_{jt}^s = 1; \; j \in J_k$ and $\sum_{t \in TK: t \leq d_j - \sigma_k + 1} x_{kt}^s = 1; \; j \in J_k$. Then both constraints (6.15) and (6.16) become $y_j^s \leq 1$, while (6.17) becomes $y_j^s \geq 1$, hence $y_j^s = 1$. Otherwise; i.e., if right-hand side of either (6.15) or (6.16), or of both (6.15) and (6.16) is 0, then $y_j^s = 0$.

Since the cost and the service level objectives are in conflict, the minimum and maximum values of expected cost $\underline{E}^c$, $\overline{E}^c$, and expected customer service level, $\underline{E}^{sl}$, $\overline{E}^{sl}$, are obtained by solving the two mixed integer programs presented below.

Minimize E^c, (6.1), subject to (6.6)–(6.22).

Maximize E^{sl}, (6.2), subject to (6.6)–(6.22).

The values $\underline{E}^c$ and $\underline{E}^{sl}$ are determined by solving the first problem, and $\overline{E}^{sl}$ and $\overline{E}^c$, by solving the second problem.

6.3.2 Scheduling with Individual and Immediate Shipping

In model **SCS2_E** presented below the variables, $x^s_{kt} \in \{0, 1\};\ k \in K, t \in TK, s \in S$ and $y^s_j \in \{0, 1\};\ j \in J, s \in S$ are not required any more, however the production scheduling variable, $w^s_{jt} \in \{0, 1\};\ j \in J, t \in T, s \in S$, now becomes a joint production and distribution scheduling variable, redefined as follows:

$w^s_{jt} = 1$, if under disruption scenario s customer order j is processed in period t and shipped to the distribution center in period $t + 1$; otherwise $w^s_{jt} = 0$.

Accordingly, the objective functions E^c, (6.1) and E^{sl}, (6.2) are rewritten as below.

$$E^c = (\sum_{i \in I} e_i u_i + \sum_{s \in S} P_s (\sum_{i \in I_s} Bo_i v_i + \sum_{j \in J} g_j b_j (\sum_{t \in T} w^s_{jt} - \sum_{t \in T: t \le d_j - \sigma_{k_j}} w^s_{jt}) + \sum_{j \in J} h_j b_j (1 - \sum_{t \in T} w^s_{jt})))/B \quad (6.23)$$

$$E^{sl} = \sum_{s \in S} \sum_{j \in J} \sum_{t \in T: t \le d_j - \sigma_{k_j}} P_s b_j w^s_{jt}/B, \quad (6.24)$$

where σ_{k_j} is the transportation time to the distribution center k that serves customer $j \in J_k$

In model **SCS2_E** presented below the objective functions E^c, (6.1) and E^{sl}, (6.2) have been replaced with (6.23) and (6.24), respectively.

> **SCS2_E**: *Risk-neutral supply chain scheduling: individual shipping*
> Minimize (6.5)
> subject to (6.3), (6.4), (6.6)–(6.10), (6.18)–(6.20), (6.23)–(6.24).

For model **SCS2_E**, the values of $\underline{E}^c$, $\overline{E}^c$, and $\underline{E}^{sl}$, $\overline{E}^{sl}$, are obtained by solving the two mixed integer programs:

> Minimize E^c, (6.23), subject to (6.6)–(6.10), (6.18)–(6.20),

> Maximize E^{sl}, (6.24), subject to (6.6)–(6.10), (6.18)–(6.20).

6.3.3 *Scheduling with Multiple Batch Shipping and Limited Transportation Capacity*

In this subsection a multiple batch shipping method is modelled, where each batch size is limited by vehicle capacity C_V and, in addition, a fixed transportation cost, a_k, is incurred for each shipment to distribution center $k \in K$.

In model **SCS3_E** presented below the variables, $y_j^s \in \{0, 1\};\ j \in J, s \in S$, are not required any more. Instead a new customer order shipping variable is introduced, $z_{jt}^s \in \{0, 1\};\ j \in J, t \in TK, s \in S$, defined as follows:

$z_{jt}^s = 1$, if under disruption scenario s customer order j is shipped to the distribution center in period t; otherwise $z_{jt}^s = 0$.

The objective functions E^c and E^{sl} are defined below.

$$E^c = (\sum_{i \in I} e_i u_i + \sum_{s \in S} P_s (\sum_{i \in I_s} Bo_i v_i + \sum_{k \in K} \sum_{t \in TK} a_k x_{kt}^s$$

$$+ \sum_{j \in J} g_j b_j (\sum_{t \in T} w_{jt}^s - \sum_{t \in TK: t \le d_j - \sigma_{k_j} + 1} z_{jt}^s) + \sum_{j \in J} h_j b_j (1 - \sum_{t \in T} w_{jt}^s)))/B \quad (6.25)$$

$$E^{sl} = \sum_{s \in S} \sum_{j \in J} \sum_{t \in TK: t \le d_j - \sigma_{k_j} + 1} P_s b_j z_{jt}^s / B, \quad (6.26)$$

where $\sum_{s \in S} \sum_{k \in K} \sum_{t \in TK} P_s a_k x_{kt}^s$ is the expected transportation cost of shipping to distribution centers and $\sum_{s \in S} \sum_{j \in J} P_s g_j b_j (\sum_{t \in T} w_{jt}^s - \sum_{t \in TK: t \le d_j - \sigma_{k_j} + 1} z_{jt}^s)$ is the expected penalty for delayed orders.

SCS3_E: *Risk-neutral supply chain scheduling: multiple batch shipping*

Minimize (6.5)

subject to (6.3), (6.4), (6.6)–(6.10), (6.18)–(6.21), (6.25)–(6.26) and

Multiple batch shipping constraints:

$$\sum_{t \in TK} z_{jt}^s = \sum_{t \in T} w_{jt}^s;\ j \in J, s \in S \quad (6.27)$$

$$\sum_{j \in J_k} b_j z_{jt}^s \le C_V x_{kt}^s;\ k \in K, t \in TK, s \in S \quad (6.28)$$

$$\sum_{t \in TK} x_{kt}^s \ge \sum_{j \in J_k} \sum_{t \in T} b_j w_{jt}^s / C_V;\ k \in K, s \in S \quad (6.29)$$

$$\sum_{j \in J_k} z_{jt}^s \ge x_{kt}^s;\ k \in K, t \in TK, s \in S \quad (6.30)$$

Production-distribution coordinating constraints:

$$\sum_{t \in TK} t z^s_{jt} \geq \sum_{t \in T} (t+1) w^s_{jt}; \;\; j \in J, s \in S \tag{6.31}$$

Supply-distribution coordinating constraints:

$$\sum_{j \in J} \sum_{t' \in TK: t' \leq t} b_j z^s_{jt'} \leq B \sum_{i \in I_s: \tau_i \leq t-2} v_i; \;\; t \in TK, s \in S \tag{6.32}$$

$$z^s_{jt} \in \{0, 1\}; \;\; j \in J, t \in TK, s \in S. \tag{6.33}$$

For each disruption scenario, Eq. (6.27) ensures that each completed order is shipped to distribution center. Constraint (6.28) ensures that each batch shipment size cannot exceed vehicle limited capacity, and Eq. (6.29) determines minimum number of batch shipments to each distribution center. Eq. (6.30) guarantees that at least one order is shipped in every batch. The production-distribution coordinating constraints (6.31) ensure that each order can be shipped only after its completion. Finally, the supply-distribution coordinating constraints (6.32) ensure that for each disruption scenario s, the cumulative distribution of products shipped to customers by period t is not greater than the cumulative supplies of required parts delivered by the non-disrupted suppliers $i \in I_s$ by period $t-2$. (Note that parts delivered by supplier $i \in I_s$ in period τ_i can be used for production not earlier than in period $\tau_i + 1$, and then the completed products can be shipped to distribution center not earlier than in period $\tau_i + 2$.)

For model **SCS3_E**, the values of $\underline{E}^c$, $\overline{E}^c$, and $\underline{E}^{sl}$, $\overline{E}^{sl}$, are obtained by solving the two mixed integer programs:

Minimize E^c, (6.25), subject to (6.6)–(6.10), (6.18)–(6.21), (6.27)–(6.33).

Maximize E^{sl}, (6.26), subject to (6.6)–(6.10), (6.18)–(6.21), (6.27)–(6.33).

6.4 Models for Risk-Averse Decision-Making

In this section, three time-indexed SMIP models **SCS1_CV**, **SCS2_CV** and **SCS3_CV** are proposed for risk-averse supply chain scheduling to optimize the weighted-sum of worst-case cost and worst-case service level under disruption risks. The models are based on the risk-neutral models **SCS1_E**, **SCS2_E** and **SCS3_E**, respectively.

Let VaR^c be the Value-at-Risk of cost per product, i.e., the targeted cost such that for a given confidence level, α, for $100\alpha\%$ of disruption scenarios, the outcome is below VaR^c and let CVaR^c be Conditional Value-at-Risk of cost per product, i.e., the expected cost in the worst $100(1-\alpha)\%$ of the scenarios with the cost above VaR^c

$$CVaR^c = VaR^c + (1-\alpha)^{-1}\sum_{s\in S} P_s \mathscr{C}_s. \tag{6.34}$$

In a similar way, define by VaR^{sl} the Value-at-Risk of service level, i.e., the targeted service level such that for a given confidence level α, for $100\alpha\%$ of disruption scenarios, the outcome is above VaR^{sl}, and by CVaR^{sl} the Conditional Value-at-Risk of service level, i.e., the expected service level in the worst $100(1-\alpha)\%$ of scenarios with the service level below VaR^{sl}

$$CVaR^{sl} = VaR^{sl} - (1-\alpha)^{-1}\sum_{s\in S} P_s \mathscr{S}_s. \tag{6.35}$$

The risk-averse integrated supply, production and distribution schedule will be optimized by simultaneously calculating VaR^c and VaR^{sl} and minimizing weighted difference of CVaR^c and CVaR^{sl}. Model **SCS1_CV** is presented below.

SCS1_CV: *Risk-averse supply chain scheduling to minimize trade-off between CVaR of cost and CVaR of service level: single batch shipping*
Minimize

$$\lambda CVaR^c - (1-\lambda)CVaR^{sl}, \tag{6.36}$$

where $0 \le \lambda \le 1$,
subject to (6.6)–(6.22), (6.34), (6.35) and

Risk constraints:
– the tail cost for scenario s is defined as the nonnegative amount by which cost in scenario s exceeds VaR^c,
– the tail service level for scenario s is defined as the nonnegative amount by which VaR^{sl} exceeds service level in scenario s,

$$\begin{aligned}\mathscr{C}_s \ge (\sum_{i\in I} e_i u_i + \sum_{i\in I_s} Bo_i v_i \\ + \sum_{j\in J} g_j b_j (\sum_{t\in T} w^s_{jt} - y^s_j) \\ + \sum_{j\in J} h_j b_j (1 - \sum_{t\in T} w^s_{jt}))/B - VaR^c;\ s\in S\end{aligned} \tag{6.37}$$

$$\mathscr{S}_s \geq VaR^{sl} - \sum_{j \in J} b_j y_j^s / B; \;\; s \in S \tag{6.38}$$

$$\mathscr{C}_s \geq 0; \;\; s \in S \tag{6.39}$$

$$\mathscr{S}_s \geq 0; \;\; s \in S, \tag{6.40}$$

where $\mathscr{C}_s$ is the tail cost and $\mathscr{S}_s$ is the tail service level, for scenario s.

In a similar way we can formulate the risk-averse versions **SCS2_CV** and **SCS3_CV** of the risk-neutral models **SCS2_E** and **SCS3_E**. The models are presented below.

SCS2_CV: *Risk-averse supply chain scheduling to minimize trade-off between CVaR of cost and CVaR of service level: individual shipping*

Minimize (6.36)

subject to (6.6)–(6.10), (6.18)–(6.20), (6.23), (6.24), (6.34), (6.35), (6.39), (6.40) and

Risk constraints:

$$\mathscr{C}_s \geq (\sum_{i \in I} e_i u_i + \sum_{i \in I_s} Bo_i v_i$$

$$+ \sum_{k \in K} \sum_{t \in TK} a_k x_{kt}^s + \sum_{j \in J} g_j b_j (\sum_{t \in T} w_{jt}^s - \sum_{t \in TK: t \leq d_j - \sigma_{k_j} + 1} z_{jt}^s)$$

$$+ \sum_{j \in J} h_j b_j (1 - \sum_{t \in T} w_{jt}^s))/B - VaR^c; \;\; s \in S \tag{6.41}$$

$$\mathscr{S}_s \geq VaR^{sl} - \sum_{j \in J} \sum_{t \in T: t \leq d_j - \sigma_{k_j}} b_j w_{jt}^s / B; \;\; s \in S. \tag{6.42}$$

SCS3_CV: *Risk-averse supply chain scheduling to minimize trade-off between CVaR of cost and CVaR of service level: multiple batch shipping*

Minimize (6.36)

subject to (6.6)–(6.10), (6.18)–(6.21), (6.27)–(6.35), (6.39), (6.40) and

Risk constraints:

$$\mathscr{C}_s \geq (\sum_{i \in I} e_i u_i + \sum_{i \in I_s} Bo_i v_i$$

$$+ \sum_{k \in K} \sum_{t \in TK} a_k x_{kt}^s + \sum_{j \in J} g_j b_j (\sum_{t \in T} w_{jt}^s - \sum_{t \in TK: t \leq d_j - \sigma_{k_j} + 1} z_{jt}^s)$$

$$+\sum_{j\in J} h_j b_j (1-\sum_{t\in T} w^s_{jt}))/B - VaR^c;\ s\in S \quad (6.43)$$

$$\mathscr{S}_s \geq VaR^{sl} - \sum_{j\in J}\sum_{t\in TK: t\leq d_j-\sigma_{k_j}+1} b_j z^s_{jt}/B;\ s\in S. \quad (6.44)$$

6.5 Expected Value Problem

Stochastic mixed integer programs are usually hard to solve because they are large-scale optimization problems when applied to real-world problems. A simplified approach, which may sometimes be useful in practice, is to consider a simpler deterministic program, known as expected value problem, in which the random parameters are replaced by their expected values (e.g., Birge and Louveaux 2011, Kall and Mayer 2011, Sawik 2016d). As an example of such approach this section presents the expected value problem corresponding to the risk-neutral scheduling in supply chain with single batch shipping of products to distribution centers, described by SMIP model **SCS1_E**. In model **SCS1_E**, where the randomness is characterized by a set of disruption scenarios, the only random parameters are suppliers all-or-nothing fulfillment rates, which appear both in the objective function (6.1) and in constraints (6.10). Denote by **ESCS1**, a deterministic MIP model of the expected value problem corresponding to model **SCS1_E**. In model **ESCS1**, suppliers probabilistic all-or-nothing fulfillment rates (1, for a non disrupted supplier and 0, for a disrupted supplier) defined for each disruption scenario are replaced by the expected fulfillment rates of each supplier

$$1-\pi_i = (1-p^r)(1-p_i);\ i\in I^r, r\in R,$$

where

$$\pi_i = p^r + (1-p^r)p_i,\ i\in I^r, r\in R \quad (6.45)$$

is total disruption probability of supplier i.

Notice that the expected fulfillment rate of a supplier is identical with his non-disruption probability.

Accordingly, stochastic binary decision variables, $w^s_{jt},\ x^s_{kt},\ y^s_j$, (6.20)–(6.22), defined for each disruption scenario $s\in S$ are replaced by their deterministic equivalents $W_{jt},\ X_{kt},\ Y_j$ (see, Table 6.3)

Table 6.3 Notation: expected value problem

Parameters	
π_i	$= p^r + (1 - p^r)p_i$, total disruption probability of supplier $i \in I^r, r \in R$
$1 - \pi_i$	$= (1 - p^r)(1 - p_i)$, the expected fraction of an order delivered by supplier $i \in I^r, r \in R$ (supplier expected fulfillment rate)
Variables	
W_{jt}	= 1, if customer order j is scheduled for period t; otherwise $W_{jt} = 0$ (production scheduling)
X_{kt}	= 1, if batch shipment of products to distribution center k is scheduled for period t; otherwise $X_{kt} = 0$ (distribution scheduling)
Y_j	= 1, if customer order j is delivered by its due date; otherwise $Y_j = 0$ (customer order non-delay delivery)

Now, the expected cost per product, E^c, and the expected service level, E^{sl}, are defined as follows

$$E^c = (\sum_{i \in I} e_i u_i + \sum_{i \in I} Bo_i(1 - \pi_i)v_i$$

$$+ \sum_{j \in J} g_j b_j (\sum_{t \in T} W_{jt} - Y_j) + \sum_{j \in J} h_j b_j (1 - \sum_{t \in T} W_{jt}))/B \tag{6.46}$$

$$E^{sl} = \sum_{j \in J} b_j Y_j / B \tag{6.47}$$

Model **ESCS1** is presented below.

Model ESCS1

Minimize (6.5)
subject to (6.3), (6.4), (6.6), (6.7), (6.18), (6.19), (6.46), (6.47) and

$$\sum_{t \in T} W_{jt} \le 1; \ \ j \in J \tag{6.48}$$

$$\sum_{j \in J} b_j W_{jt} \le C; \ \ t \in T \tag{6.49}$$

$$\sum_{j \in J} \sum_{t' \in T : t' \le t} b_j W_{jt'} \le B \sum_{i \in I : \tau_i \le t-1} (1 - \pi_i)v_i; \ \ t \in T \tag{6.50}$$

$$\sum_{t \in TK} t X_{kt} \ge \sum_{t \in T} (t+1) W_{jt}; \ \ k \in K, j \in J_k \tag{6.51}$$

$$\sum_{t \in TK} X_{kt} \le 1; \ \ k \in K \tag{6.52}$$

$$Y_j \le \sum_{t\in T: t\le d_j-\sigma_k} W_{jt};\;\; k\in K, j\in J_k \quad (6.53)$$

$$Y_j \le \sum_{t\in TK: t\le d_j-\sigma_k+1} X_{kt};\;\; k\in K, j\in J_k \quad (6.54)$$

$$\sum_{t\in T: t\le d_j-\sigma_k} W_{jt} + \sum_{t\in TK: t\le d_j-\sigma_k+1} X_{kt} - 1 \le Y_j;\, k\in K, j\in J_k \quad (6.55)$$

$$W_{jt}\in\{0,1\};\;\; j\in J, t\in T \quad (6.56)$$

$$X_{kt}\in\{0,1\};\;\; k\in K, t\in TK \quad (6.57)$$

$$Y_j \ge 0;\;\; j\in J. \quad (6.58)$$

Notice that unlike SMIP model **SCS1_E** which is formulated to determine optimal schedules for all potential disruption scenarios, model **ESCS1** accounts for a single scenario only, representing the expected supplies. Except for the expected values of the random parameters, this model does not take into account any distribution information and the solution remains the same as long as the expectations do not change. In contrast to model **SCS1_E**, where the selection of supply portfolio is combined with supply chain scheduling for all disruption scenarios considered, now the portfolio is determined along with a single schedule.

6.6 Computational Examples

The examples presented in this section are modeled after a real world electronics supply chain (e.g., Sawik 2011a). The supply chain consists of multiple manufacturers/suppliers of electronic components, a single producer where finished products (e.g., cellular phones) are assembled to meet customer orders and a set of distribution centers that deliver the products to customers (e.g., carriers) who generate final demand for products. The completed customer orders are shipped to the distribution centers either in batches of different customer orders or each customer order is shipped individually, immediately after its completion using a direct shipping line. The line consists of a surface mount technology line, where printed wiring boards are assembled, material preparation stage, where all materials required for each product are prepared, postponement stage, where products for some orders are customized, flashing/flexing stations, where required software is downloaded and packing stations, where products and required accessories are packed for shipping. The input data for the example problems were prepared considering monthly production.

The following basic parameters have been used for the example problems:

- $\overline{I} = 9$, $\overline{J} = 25$, $\overline{K} = 3$, $\overline{R} = 3$, $\overline{T} = 10$, and $\overline{S} = 2^{\overline{I}} = 512$;
- $K = \{1, 2, 3\}$, and $J_1 = \{1, \dots, 10\}$, $J_2 = \{11, \dots, 20\}$, $J_3 = \{21, \dots, 25\}$;

- $R = \{1, 2, 3\}$, and $I^1 = \{1, 2, 3\}$, $I^2 = \{4, 5, 6\}$, $I^3 = \{7, 8, 9\}$;
- $b_j \in \{500, 1000, \ldots, 5000\}\ \forall j = 1, \ldots, 25$, $B = 66000$;
- $C_P = 20000$;
- $C_V = \max_{j \in J} b_j = 5000$;
- $e = (8, 7, 10, 13, 14, 11, 19, 17, 19) \times 1000$;
- $g_j = 1, h_j = 26\ \forall j = 1, \ldots, 25$;
- $o = (13, 12, 12, 8, 6, 6, 2, 5, 4)$;
- τ_i, was 2, 3 and 4 time periods, respectively for suppliers $i \in I^1, i \in I^2$ and $i \in I^3$;
- σ_k, was 1, 2 and 3 time periods, respectively for transportation to distribution center $k = 1, 2$ and 3;
- $a_k = C_V \sigma_k = 5000, 10000$ and 15000, respectively for transportation to distribution center $k = 1, 2$ and 3;
- $d_j \in \{2 + \min_{i \in I}(\tau_i) + \min_{k \in K}(\sigma_k), \ldots, \overline{T} + \max_{k \in K}(\sigma_k)\}\ \forall j = 1, \ldots, 25$;

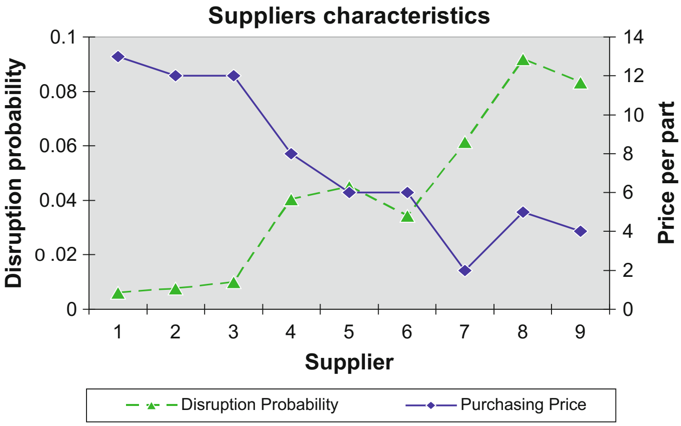

Fig. 6.2 Suppliers

The local and regional disruption probabilities, $p_i, i \in I$ and $p^r, r \in R$, were: p = (0.00513571, 0.00666354, 0.00902974, 0.0356206, 0.040175, 0.0294692, 0.0519967, 0.0827215, 0.0739062) and $p^1 = 0.001$, $p^2 = 0.005$, $p^3 = 0.01$.

The corresponding total disruption probabilities, $\pi_i = p^r + (1 - p^r)p_i,\ i \in I^r$, $r \in R$, (6.45), of all suppliers were: π=(0.00613057, 0.00765688, 0.0100207, 0.0404425, 0.0449741, 0.0343219, 0.0614767, 0.0918943, 0.0831672).

The unit price per part, o_i, and the disruption probability, π_i, of each supplier $i \in I$ are shown in Fig. 6.2. The figure indicates that the most reliable (with the lowest disruption probability) is supplier 1, the least reliable (with the highest disruption probability) is supplier 8, the most expensive (with the highest price per part) is supplier 1, and the cheapest (with the lowest price per part) is supplier 7. Note that geographic regions are numbered in such a way that the unit prices are nonincreasing with r, while the fixed ordering costs and the disruption probabilities are nondecreasing with r, i.e.,

$$o_{i_1} \geq o_{i_2} \geq o_{i_3}, \quad e_{i_1} \leq e_{i_2} \leq e_{i_3} \quad \text{and} \quad \pi_{i_1} \leq \pi_{i_2} \leq \pi_{i_3}; \forall i_1 \in I^1, i_2 \in I^2, i_3 \in I^3.$$

6.6.1 Risk-Neutral Decision-Making

Table 6.4 Risk-neutral solutions: single batch shipping

λ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Var. = 124207, Bin. = 112723, Cons. = 84406, Nonz. = 1628119 [(a)]											
Exp.Cost	16.42($\overline{E}^c$)	10.96	10.67	10.40	7.73	7.72	7.71	5.25	4.70	4.30	4.28($\underline{E}^c$)
Exp.Service Level [(b)]	78.65 ($\overline{E}^{sl}$)	78.36	78.30	78.04	74.30	74.29	74.17	56.67	51.71	41.96	41.94($\underline{E}^{sl}$)
Exp.Fulfilled Demand [(c)]	78.65	88.82	88.85	88.63	96.17	96.19	95.83	94.56	94.10	93.78	93.85
Suppliers Selected	1(21)										
(% of total demand)	2(21)	2(37)	2(30)	2(30)	2(29)	2(29)	2(29)				
	3(18)										
	4(9)										
	5(5)	5(21)	5(20)	5(21)							
	6(8)	6(21)	6(20)	6(21)	6(30)	6(30)	6(30)	6(29)	6(11)		
	7(12)	7(21)	7(21)	7(28)	7(41)	7(41)	7(41)	7(71)	7(89)	7(100)	7(100)
	8(2)										
	9(4)		9(9)								
CPU [sec] [(d)]	15285	74074	21819	60464	31004	42468	14903	37542	26009	69686	113

[(a)] Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients.
[(b)] $\sum_{s \in S} \sum_{j \in J} P_s b_j y_j^s / B \times 100\%$.
[(c)] $\sum_{s \in S} \sum_{j \in J} \sum_{t \in T} P_s b_j w_{jt}^s / B \times 100\%$.
[(d)] CPU seconds for proven optimal solutions, except for $\lambda = 0.1, 0.9$
with GAP<1%

The solution results for the risk-neutral decision-making are presented in Tables 6.4, 6.5 and 6.6, respectively for single batch shipping, individual shipping and multiple batch shipping with limited transportation capacity. For comparison, the expected cost per product in Table 6.6 does not include the expected transportation cost per product, which is shown separately. The results indicate that for $\lambda = 1$ (minimization of cost) the cheapest supplier $i = 7$ is selected only, while for $\lambda = 0$ (maximization of customer service level) the total demand for parts is allocated among all nine suppliers for batch shipping and eight suppliers, except of the least reliable supplier 8, for individual shipping. The highest proportion of demand is allotted among the three most reliable and most expensive suppliers $i = 1, 2, 3$. As λ increases from 0 to 1, i.e., the decision maker preference shifts from customer service level to cost, more demand is moved from expensive and reliable suppliers to low-cost,

Table 6.5 Risk-neutral solutions: individual shipping

λ	0	0.1	0.2	0.3	0.4, 0.5, 0.6, 0.7	0.8, 0.9	1
Var. = 100470, Bin. = 100459, Cons. = 21343, Nonz. = 909247 [(a)]							
Exp.Cost	$15.38(\overline{E}^c)$	10.59	7.87	6.19	5.57	4.77	$4.15(\underline{E}^c)$
Exp.Service Level [(b)]	$87.56(\overline{E}^{sl})$	87.36	85.98	84.52	83.48	74.45	$55.45(\underline{E}^{sl})$
Exp.Fulfilled Demand [(c)]	87.56	97.36	95.74	95.06	94.86	94.37	93.85
Suppliers Selected	1(52)						
(% of total demand)	2(12)	2(40)	2(9)	2(9)	2(9)		
	3(12)	3(12)					
	4(4)	4(12)					
	5(4)	5(12)	5(12)	5(12)			
	6(4)	6(12)	6(55)	6(20)	6(20)	6(20)	
	7(8)	7(12)	7(12)	7(59)	7(71)	7(80)	7(100)
	9(4)		9(12)				
CPU [sec]	1	79	61	409	112, 8, 10, 12	11, 26	3

[(a)] Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients.
[(b)] $\sum_{s\in S}\sum_{j\in J}\sum_{t\in T:t\le d_j-\sigma_{k_j}} P_s b_j w^s_{jt}/B \times 100\%$.
[(c)] $\sum_{s\in S}\sum_{j\in J}\sum_{t\in T} P_s b_j w^s_{jt}/B \times 100\%$

unreliable suppliers. The results for individual and multiple batch shipping, where more shipments are scheduled, are similar.

For all the shipping methods and most λ, the total demand for parts is allocated among four suppliers $i = 2, 5, 6, 7$, where supplier $i = 2$ is the second most reliable and $i = 6$ is the most reliable in regions $r = 2, 3$, while $i = 7$ is the cheapest among all suppliers. For small λ, however, more suppliers are selected to improve individual service level.

In general, for all shipping methods the service-oriented supply portfolio ($\lambda \le 0.3$) is more diversified than the cost-oriented portfolio ($\lambda \ge 0.7$).

In addition to the expected customer service level, i.e., the expected fraction of customer demand fulfilled on time, Tables 6.4, 6.5 and 6.6 also show the expected fraction of fulfilled demand, $\sum_{s\in S}\sum_{j\in J}\sum_{t\in T} P_s b_j w^s_{jt}/B$, i.e., demand fulfilled on time or delayed. The solution results demonstrate that for the maximum service level objective, which is independent of any cost parameters, the largest expected fraction of non-delayed customer demand is associated with the smallest expected fraction of fulfilled demand. This indicates that to maximize expected service level, customer orders that cannot be fulfilled by requested due dates are rejected. Note that the difference between the expected fraction of fulfilled demand and the expected customer service level is the expected fraction of delayed customer demand. The latter measure is equal to zero for the maximum service level objective, increases

Table 6.6 Risk-neutral solutions: multiple batch shipping

λ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7, 0.8	0.9	1
Var. = 124207, Bin. = 112723, Cons. = 84406, Nonz. = 1628119 [(a)]										
Exp.Cost [(b)]	$17.85\overline{E}^c$)	10.35	8.95	7.10	6.58	6.55	5.97	5.41	4.78	6.07($\underline{E}^c$)
Exp.Transportation Cost [(c)]	2.04	2.00	1.97	1.96	1.95	1.95	1.93	1.93	1.92	1.92
Exp.Service Level [(d)]	85.31($\overline{E}^{sl}$)	85.23	84.66	83.11	82.46	82.40	80.42	77.73	70.96	55.44($\underline{E}^{sl}$)
Exp.Fulfilled Demand [(e)]	85.31	97.04	95.78	95.56	95.44	95.43	95.07	94.69	94.36	93.85
Suppliers Selected	1(42)	1(14)								
(% of total demand)	2(14)	2(14)	2(21)	2(21)	2(20)	2(20)	2(14)	2(7)		
	3(14)	3(14)								
	4(4)	4(15)	4(15)							
	5(5)	5(14)	5(14)	5(14)						
	6(6)	6(14)	6(21)	6(14)	6(20)	6(19)	6(19)	6(19)	6(19)	
	7(8)	7(15)	7(15)	7(51)	7(60)	7(61)	7(67)	7(74)	7(81)	7(100)
	8(2)									
	9(5)		9(14)							
CPU [sec]	36	24354	22107	32958	15961	14689	24447	4560, 14392	25220	315

[(a)] Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients.
[(b)] without expected transportation cost, except for $\lambda = 0$ and $\lambda = 1$.
[(c)] $\sum_{s\in S}\sum_{k\in K}\sum_{t\in TK} P_s a_k x^s_{kt}/B$.
[(d)] $\sum_{s\in S}\sum_{j\in J}\sum_{t\in TK:t\le d_j-\sigma_{k_j}+1} P_s b_j z^s_{jt}/B \times 100\%$.
[(e)] $\sum_{s\in S}\sum_{j\in J}\sum_{t\in T} P_s b_j w^s_{jt}/B \times 100\%$

with the trade-off parameter λ, and reaches its maximum for the minimum cost objective.

The results presented for the three shipping methods in Tables 6.4, 6.5 and 6.6 are illustrated and compared in Figs. 6.3 and 6.4. Figure 6.3 compares the expected values of cost per product, service level and fulfilled demand for 11 levels of trade-off parameter λ, while Fig. 6.4 compares the obtained subsets of nondominated supply portfolios (the allocation of total demand for parts among selected suppliers), with the four major suppliers $i = 2, 5, 6, 7$ indicated. The corresponding portfolios for different shipping methods are similar, with the cheapest suppliers dominating in the individual shipping. The results point out that individual and multiple batch shipping lead to better solution values, i.e., a higher expected service level and a lower expected ordering, purchasing and shortage cost.

For single batch shipping, Figs. 6.5, 6.6 and 6.7 show the expected supply, production and delivery schedules, respectively for $\lambda = 0$ (i.e., for the maximum expected service), $\lambda = 0.5$ and $\lambda = 1$ (i.e., for the minimum expected cost). The expected schedules were computed using the formulae presented below:

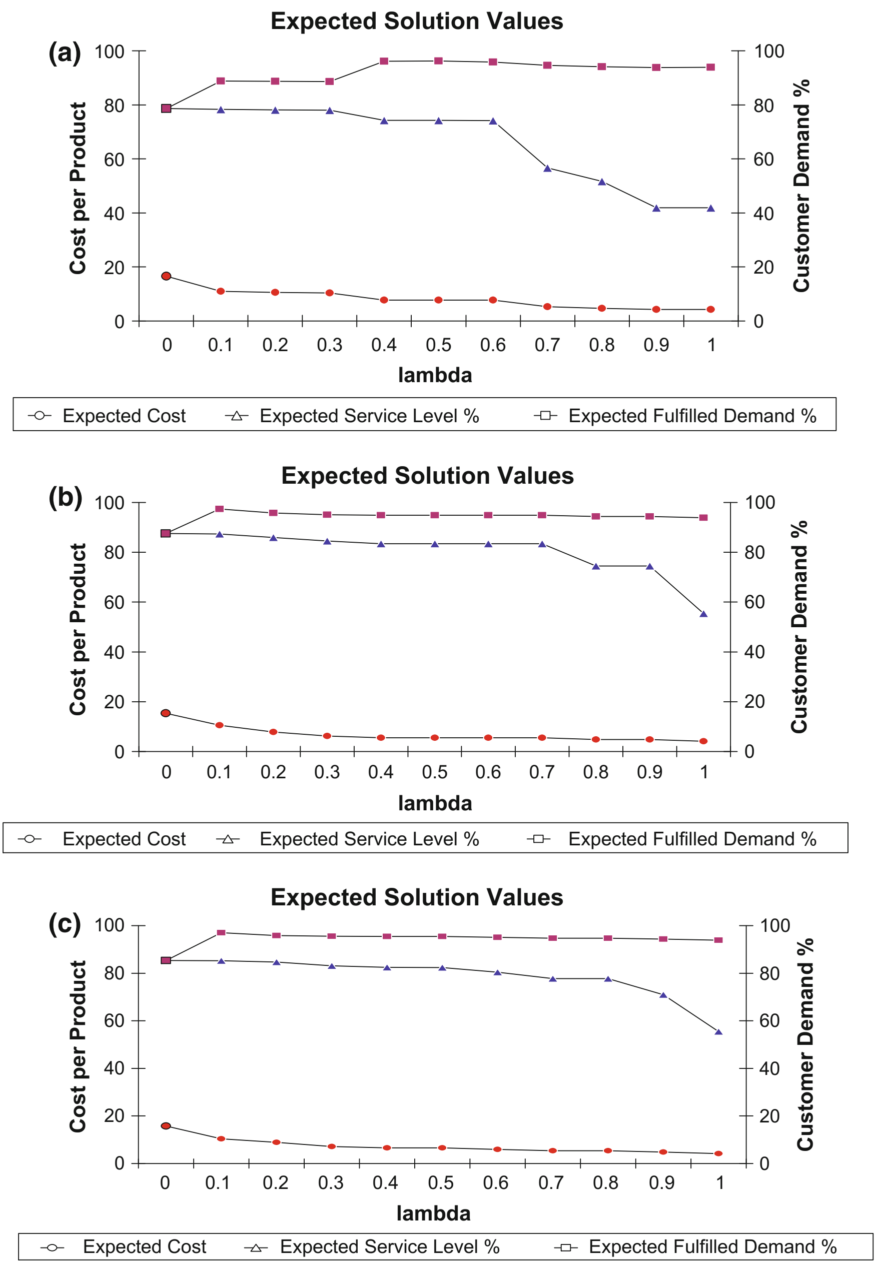

Fig. 6.3 Expected solution values: **a** single batch shipping, **b** individual shipping, **c** multiple batch shipping

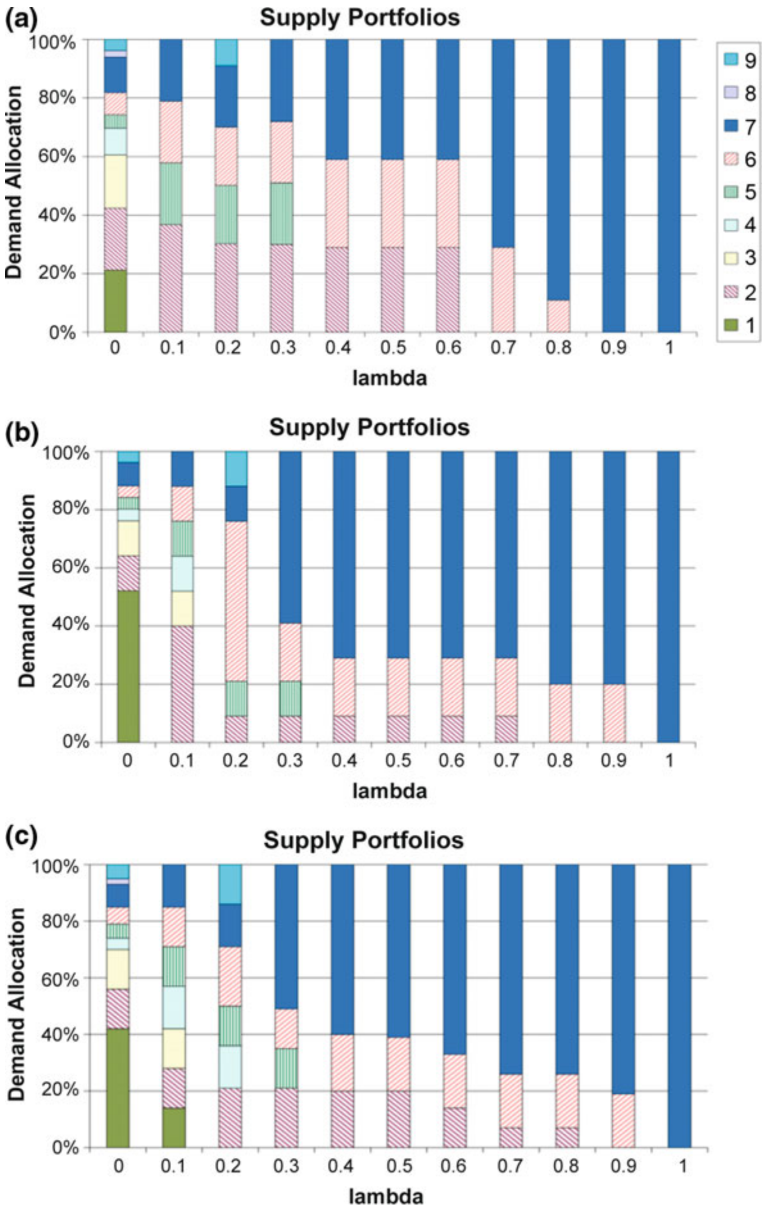

Fig. 6.4 Nondominated supply portfolios: **a** single batch shipping, **b** individual shipping, **c** multiple batch shipping

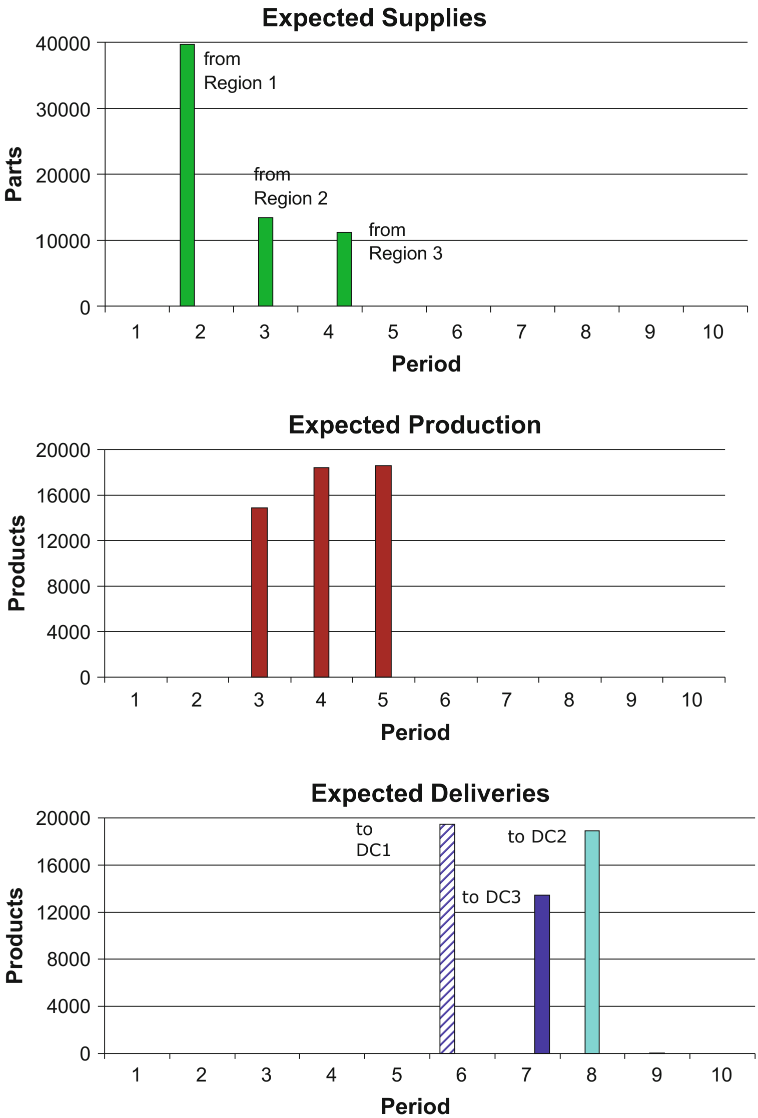

Fig. 6.5 Expected supplies, production and deliveries for maximum service level ($\lambda = 0$): single batch shipping

- Expected schedule of supplies of parts to the producer

$$\sum_{s \in S} \sum_{i \in I_s : \tau_i = t} P_s B v_i; \ t \in T \tag{6.59}$$

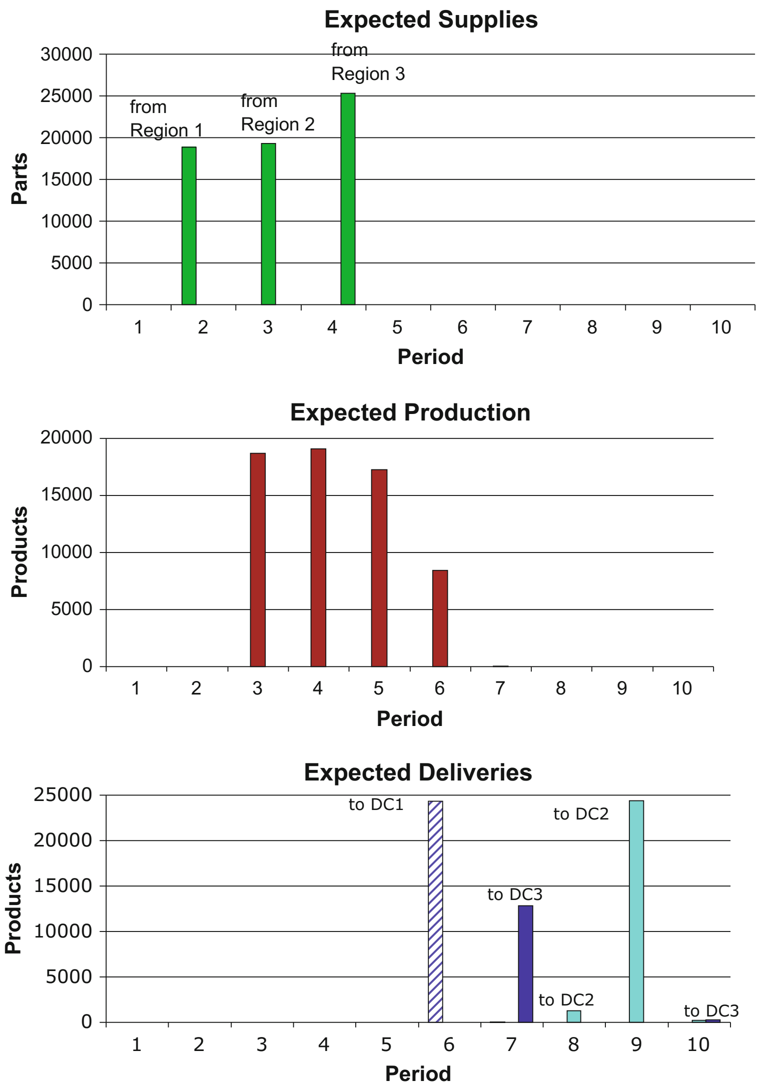

Fig. 6.6 Expected supplies, production and deliveries for $\lambda = 0.5$: single batch shipping

- Expected production schedule

$$\sum_{s \in S} \sum_{j \in J} P_s b_j w^s_{jt}; \ \ t \in T \tag{6.60}$$

- Expected schedule of shipping of products from the producer to the distribution centers

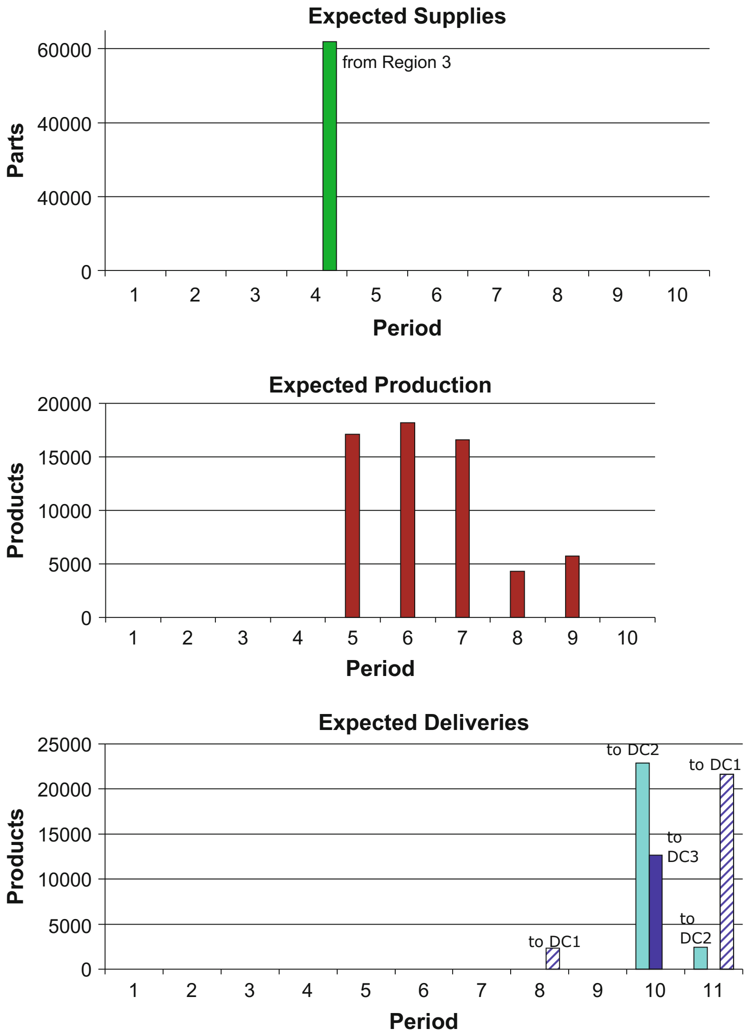

Fig. 6.7 Expected supplies, production and deliveries for minimum cost (λ = 1): single batch shipping

$$\sum_{s\in S}\sum_{j\in J_k} P_s b_j \Big(\sum_{t'\in T: t'<t} w^s_{jt'}\Big) x^s_{kt};\ \ k\in K, t\in TK, \tag{6.61}$$

where $TK = \{\min_{i\in I} \tau_i + 2, \ldots, \overline{T} + 1\}$ is the set of shipping periods.
Note that the delivery schedule for the distribution center k is the shipping schedule delayed by σ_k periods.

The expected supply schedules shown in Figs. 6.5, 6.6 and 6.7 are marked with the supplier region, while the expected delivery schedules indicate the distribution center; DC1 ($k = 1$), DC2 ($k = 2$) and DC3 ($k = 3$).

As λ increases, i.e., the decision maker priority shifts from the maximum service level to minimum cost and more parts are ordered from less reliable and lower cost suppliers, the expected supply schedules and the corresponding production schedules are more delayed as well as the delivery of products to the customers. Note that despite constraint (6.11) for single batch shipping that ensures at most one shipment of products to each distribution center for each disruption scenario, the expected shipping schedule (6.61) may be split into more smaller size shipments, (cf. Figs. 6.6, 6.7 and 6.8).

For individual and multiple batch shipping, Figs. 6.8 and 6.9 show examples of expected supply, production and delivery schedule for $\lambda = 0.5$. Comparison of Fig. 6.6 for single batch shipping, with Fig. 6.8 and with Fig. 6.9 shows that the expected supplies from different regions are more diversified, and the expected production as well as the shipping schedule are distributed among more periods. The results indicate that for individual and multiple batch shipping, where more shipments are scheduled, supply, production and delivery schedules are more unlevelled. On the other hand, comparison of the solution results shown in Tables 6.4, 6.5 and 6.6 indicate that individual and multiple batch shipping may lead to higher expected service level and lower expected ordering, purchasing and shortage cost.

The solution results obtained for numerical examples modelled after realistic problems in the electronics supply chain are in line with other approaches used in the area of supply chain risk management. The main findings are listed below.

- *The supply portfolios for different shipping methods are similar. For all shipping methods, the service-oriented supply portfolio* ($\lambda \leq 0.3$) *is more diversified than the cost-oriented portfolio* ($\lambda \geq 0.7$).
- *For the minimum cost objective the cheapest supplier is usually selected, while for the maximum service level objective the most reliable and most expensive suppliers are dominating in the supply portfolio.*
- *For the maximum service level objective, which is independent of any cost parameters, the largest expected fraction of non-delayed customer demand simultaneously leads to the largest expected fraction of rejected demand. This indicates that in order to maximize expected service level, customer orders that cannot be fulfilled by requested due dates are rejected.*
- *The more cost-oriented decision-making, the more delayed the expected supply, production and distribution schedules.*
- *The individual and multiple batch shipping lead to higher expected service level and lower expected ordering, purchasing, and shortage cost.*

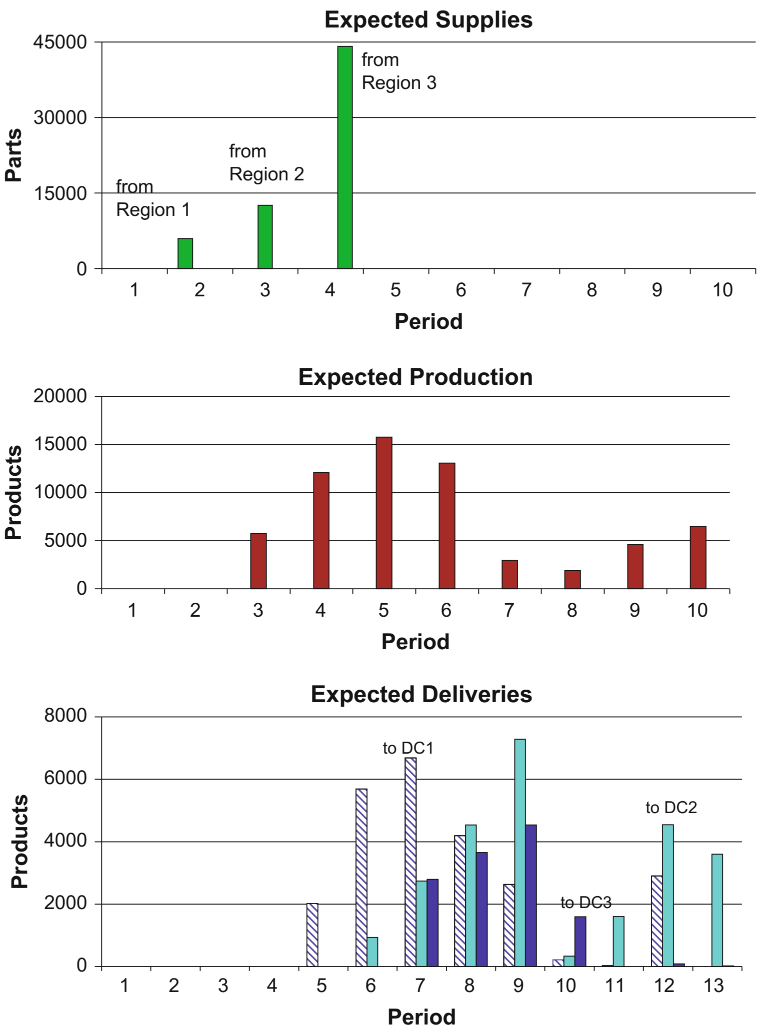

Fig. 6.8 Expected supplies, production and deliveries for $\lambda = 0.5$: individual shipping

The computational experiments were performed using the AMPL programming language and the XPRESS 27.01 solver on a MacBookPro laptop with Intel Core i7 processor running at 2.8 GHz and with 16GB RAM. (For comparison, the computational experiments have been also performed using CPLEX 12.6.2 and Gurobi 6.0.2 solvers, however they were outperformed by the XPRESS 27.01 solver.) The MIP problem sizes (the numbers of variables, constraints and nonzero coefficients) and CPU seconds required to find optimal solutions (or with GAP less than 1%) for the examples are presented in Tables 6.4, 6.5 and 6.6. The solver was capable of finding proven optimal solution for all examples for models **SCS2_E** and **SCS3_E**,

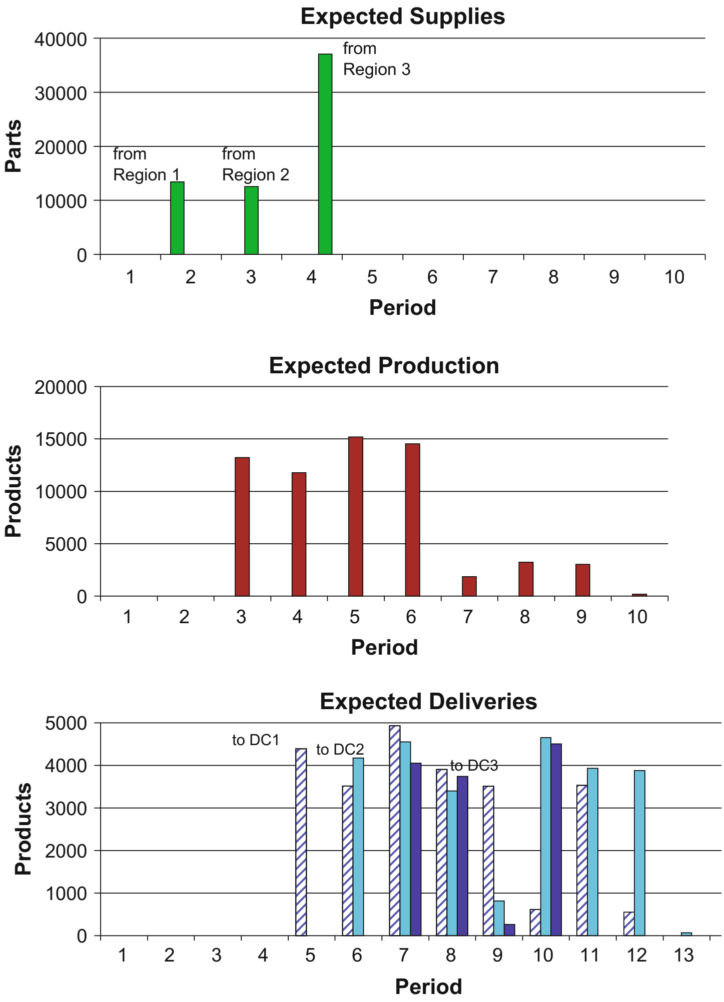

Fig. 6.9 Expected supplies, production and deliveries for $\lambda = 0.5$: multiple batch shipping

with CPU time ranging from several seconds to several hours. Proven optimal solutions were also found for most examples for single batch shipping model **SCS1_E**, except for $\lambda = 0.1, 0.9$, where the solution with GAP less than 1% was found and no improvements were observed within next 3600 CPU seconds. The smallest CPU time was required for the individual shipping model **SCS2_E**. Since model **SCS2** is much simpler than **SCS1_E** and **SCS3_E**, the latter result is obvious.

6.6.2 Risk-Averse Decision-Making

In this subsection some computational examples are presented to illustrate the risk-averse scheduling of supply, production and distribution using model **SCS1_CV**. In the computational experiments data sets provided at the beginning of this section were used. Table 6.7 shows solution results for the objective function (6.36) with $\lambda = 0$ (maximization of CVaR of service level), $\lambda = 1$ (minimization of CVaR of cost) and $\lambda = 0.5$ (balanced trade-off between CVaR of cost and CVaR of service level). The risk-averse supply portfolios and solution values of VaR^c, $CVaR^c$ and VaR^{sl}, $CVaR^{sl}$ and the associated expected values E^c and E^{sl}, respectively of cost and service level are presented for a subset of confidence levels $\alpha = 0.5, 0.75, 0.9, 0.95, 0.99$.

As α increases, the $CVaR^c$ of cost and the associated expected cost, E^c, increase, while expected service level, E^{sl}, decreases. Similarly, the $CVaR^{sl}$ of service level decreases and the associated expected cost, E^c, increases with confidence level α. However, VaR^{sl}, E^{sl} and risk-averse supply portfolios are nearly independent of confidence level, except for the highest $\alpha = 0.99$. At the same time, $CVaR^{sl}$ and E^{sl} are very close to the corresponding VaR^{sl}. For the $CVaR^c$ objective function and increasing confidence level, a greater diversification of supply portfolios is observed, with more demand shifted from the cheapest supplier 7 to more reliable and more expensive suppliers 2, 3, 5, 6 and 9. A low-cost and most unreliable supplier 8 was never selected (cf. Fig. 6.2). In contrast, for the $CVaR^{sl}$ objective function the supply portfolios are diversified, even for small confidence levels.

Comparison of solution results for models **SCS1_E** and **SCS1_CV** (cf. Tables 6.4 and 6.7), demonstrates that *the risk-neutral solutions for maximization of expected service level (that is, for* $\lambda = 0$*) and minimization of expected cost (that is, for* $\lambda = 1$*) are very close to the corresponding risk-averse solutions with confidence level* $\alpha = 0.5$*. Moreover, for the cost objective function, the corresponding risk-neutral and the risk-averse supply portfolios are identical, while for the service level objective they are very close each other.*

The computational experiments were performed using the AMPL programming language and the Gurobi 7.0.0 solver on a MacBookPro laptop with Intel Core i7 processor running at 2.8 GHz and with 16GB RAM. The solver was incapable of finding proven optimal solutions within 3600 CPU seconds. However, for all examples with $\alpha = 0.5$ and $\alpha = 0.75$, the solution with GAP less than 1% was found much earlier, while for the higher confidence levels, more than 3600 CPU seconds were required to achieve GAP less than 1%.

Table 6.7 Risk-averse solutions for model **SCS1_CV**

Confidence level α	0.50	0.75	0.90	0.95	0.99
$\lambda = 0$ (maximization of $CVaR^{sl}$)					
Var. = 125233, Bin. = 112723, Cons. = 85430, Nonz. = 1638087 [(a)]					
$CVaR^{sl}$ 100%	78.58	78.40	77.84	76.89	68.83
VaR^{sl} 100%			78.79		75.76
E^{sl} 100%			78.69		75.88
E^c	16.54	16.56	16.72	16.64	17.40
Supply Portfolio:	1(21)	1(21)	1(21)	1(21)	1(21)
Supplier (% of total demand) [(b)]	2(21)	2(21)	2(21)	2(21)	2(21)
	3(21)	3(21)	3(20)	3(20)	3(18)
	4(8)	4(7)	4(7)	4(8)	4(8)
	5(7)	5(7)	5(7)	5(7)	5(7)
	6(7)	6(8)	6(8)	6(6)	6(8)
	7(14)	7(11)	7(14)	7(13)	7(11)
					8(3)
	9(1)	9(4)	9(2)	9(4)	9(3)
$\lambda = 1$ (minimization of $CVaR^c$)					
$CVaR^c$	5.66	8.56	12.78	13.82	16.10
VaR^c	2.77	2.77	11.25	12.32	15.30
E^c	4.22	4.22	10.17	11.27	13.05
E^{sl} 100%	48.35	48.35	15.57	11.26	5.75
Supply Portfolio:					
Supplier (% of total demand) [(b)]			2(6)	2(26)	2(24)
					3(21)
			4(14)	4(19)	
			5(22)	5(18)	5(15)
			6(21)	6(20)	6(15)
	7(100)	7(100)	7(18)	7(17)	7(12)
			9(19)		9(13)
$\lambda = 0.5$					
$CVaR^c$	5.66	8.74	12.88	13.89	16.17
VaR^c	2.77	3.12	10.79	12.47	15.36
E^c	4.22	4.53	10.73	11.72	14.89
$CVaR^{sl}$ 100%	45.18	42.36	55.40	72.28	59.06
VaR^{sl} 100%	51.52	54.55	56.82	78.79	62.12
E^{sl} 100%	48.35	51.50	57.75	78.46	67.85

(continued)

Table 6.7 (continued)

Confidence level α	0.50	0.75	0.90	0.95	0.99
Supply Portfolio:					
Supplier (% of total demand) $^{(b)}$				2(23)	2(22)
				3(13)	3(20)
			4(15)	4(17)	4(17)
			5(20)	5(17)	
		6(5)	6(20)	6(17)	6(15)
	7(100)	7(95)	7(16)	7(13)	7(13)
			8(15)		
			9(14)		9(13)

$^{(a)}$ Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients.

6.6.3 Expected Value - Based Decision-Making

For the example input data, the suppliers expected fulfillment rates, $1 - \pi_i; i \in I^r, r \in R$, are (0.993869, 0.992343, 0.989979, 0.959558, 0.955026, 0.965678, 0.938523, 0.908106, 0.916833), that is, supplier 1 is most reliable and supplier 8 most unreliable.

Table 6.8 Solution results for the expected value model **ESCS1**

λ	0	0.1 and 0.2 and 0.3	0.4 and 0.5 and 0.6 and 0.7	0.8 and 0.9	1
Var. = 265, Bin. = 233, Cons. = 157, Nonz. = 2249 $^{(a)}$					
Exp.Cost	17.88($\overline{E}^c$)	9.33	7.65	4.49	4.39($\underline{E}^c$)
Exp.Service Level $^{(b)}$	78.78($\overline{E}^{sl}$)	78.03	75.76	54.55	47.73($\underline{E}^{sl}$)
Exp.Fulfilled Demand $^{(c)}$	83.33	90.15	95.45	93.94	93.18
Suppliers Selected	1(97.6)				
(% of total demand)	2(0.8)	2(30)	2(26)		
	5(0.8)				
		6(31)	6(31)	6(5)	
	7(0.8)	7(39)	7(43)	7(95)	7(100)

$^{(a)}$ Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients.
$^{(b)}$ $\sum_{j \in J} b_j Y_j / B \times 100\%$.
$^{(c)}$ $\sum_{j \in J} \sum_{t \in T} b_j W_{jt} / B \times 100\%$.

Table 6.8 presents a subset of nondominated solutions obtained for the expected value problem **ESCS1**, and Fig. 6.10 shows supply, production and shipping sched-

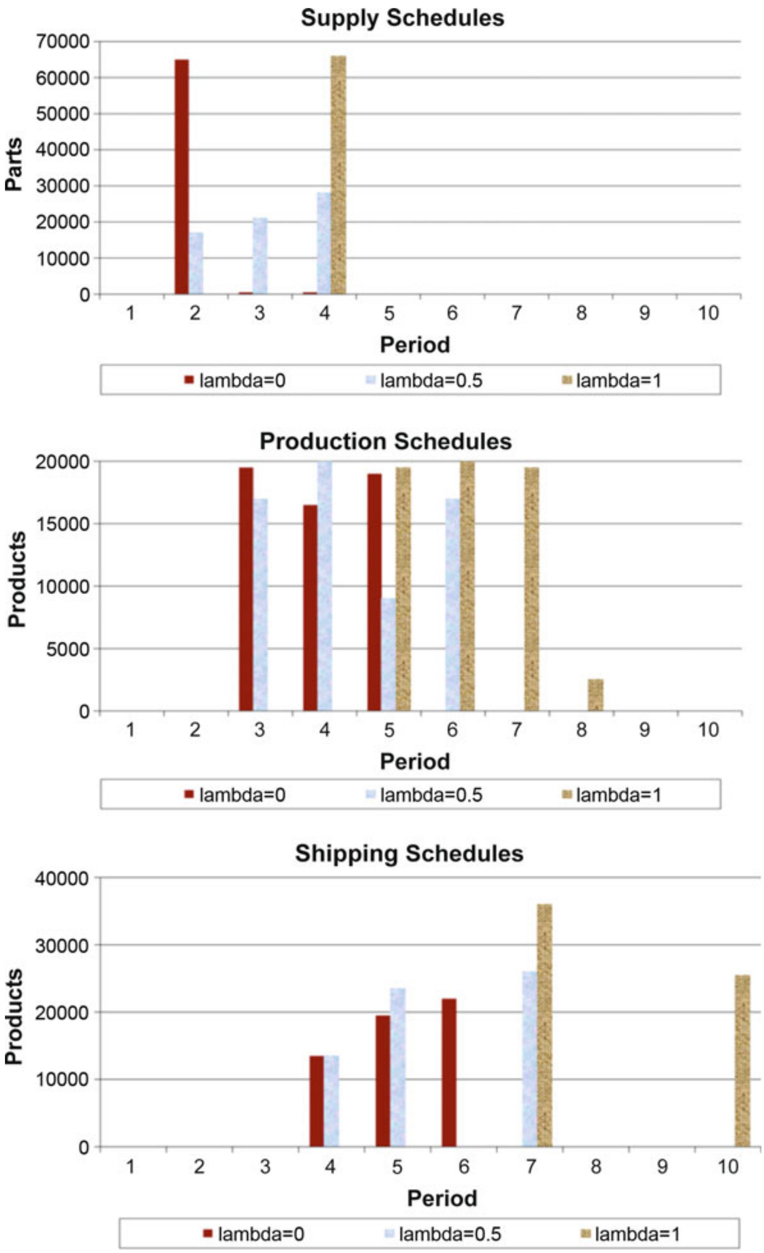

Fig. 6.10 Supplies, production and deliveries for the expected value problem

ules. Unlike the stochastic programming approach which accounts for all potential disruption scenarios to optimize an expected performance of the supply chain, the solution obtained using the deterministic approach is based on an aggregate information on suppliers expected fulfillments.

The solution results are similar for both **SCS1_E ESCS1** *models (cf. Tables 6.4 and 6.8) and the corresponding optimal solution values are close to each other, which indicates that the expected value problem can be used in practice, when stochastic mixed integer programs are hard to solve. However, the stochastic wait-and-see approach leads to a more diversified supply portfolio that will hedge against a variety of scenarios. In particular, the service-oriented supply portfolio is more diversified and may combine both high-cost, reliable suppliers and low-cost unreliable suppliers, while the cost-oriented portfolio depends mainly on low-cost and less reliable suppliers.*

For the deterministic, expected value approach, most of nondominated supply portfolios consist of three suppliers only: $i = 2, 6, 7$, while suppliers $i = 1, 2, 5, 7$ are selected only for the maximum service level objective (that is, for $\lambda = 0$). In addition, the expected schedules are more delayed for the stochastic approach. Comparison of Figs. 6.5, 6.6 6.7 and 6.10 indicates that the expected schedules for model **SCS1_E**, computed as expectations over all schedules for all potential disruption scenarios, (6.59)–(6.61), are similar to the corresponding schedules determined by model **ESCS1**. The main difference is a more delayed expected production and shipping schedule for model **SCS1_E** when minimization of cost is considered (that is, for $\lambda = 1$).

The computational experiments were performed using the AMPL programming language and the Gurobi 7.0.0 solver on a MacBookPro laptop with Intel Core i7 processor running at 2.8 GHz and with 16GB RAM. The solver was capable of finding proven optimal solution for all examples with CPU time ranging from fraction of a second to several seconds. Since model **ESCS1** deals with a single scenario only, the low computational effort required is obvious.

6.7 Notes

There is a growing body of literature on deterministic models on integrated production-distribution planning and scheduling, however, without the supply operations, which are mostly considered separately, e.g., Erenguc et al. (1999). For example, Li et al. (2005) investigated an integrated scheduling of assembly and multi-destination air-transportation in a consumer electronics supply chain. The problem was divided into two sub-problems. The air transportation allocation was formulated and solved using a MIP approach, and two heuristics were proposed for the assembly scheduling problem. Lei et al. (2006) studied an integrated production, inventory and distribution routing problem and proposed a MIP approach combined with a heuristic routing algorithm to coordinate the production, inventory and transportation operations. A comprehensive review and classification of existing deterministic models that integrate production and outbound distribution operations at the detailed scheduling level was presented in Chen (2010). The variety of models were classified by shipping and delivery methods. In particular, models with individual and immediate delivery of each order and models with batch delivery and routing of orders to different customers delivered together in the same shipment were considered. Such models attempt to optimize detailed order-by-order production and delivery scheduling jointly by taking into account relevant revenues, costs, and customer service levels at the individual order level. In a more recent work on deterministic approaches, Cakici et al. (2012) proposed a MIP approach for multi-objective scheduling of customer orders in an integrated production and distribution system. The problem objective was to optimize the trade-off between total weighted tardiness as a service level measure and total distribution costs. Liu and Papageorgiou (2013) and Liu et al. (2014) developed a multi objective MIP approach to address production, distribution and capacity planning of global supply chains considering cost, responsiveness and customer service level simultaneously to achieve an equitably efficient or Pareto optimal solution. An integrated production and distribution scheduling problem in a make-to-order supply chain with limited production and distribution capacity was considered by Viergutz and Knust (2014). The problem consists in finding a selection of customers to be supplied such that the total satisfied demand is maximized. There are also some reported studies on joint supplier selection and production and distribution planning. Sawik (2009a) proposed a MIP approach for integrated scheduling of material manufacturing, supply and product assembly in a customer-driven supply chain. A monolithic approach, where the manufacturing, supply and assembly schedules are determined simultaneously, was compared with a hierarchical approach, see also Sawik (2006). Numerical examples modeled after a real-world scheduling in the electronics supply chain were reported. A monolithic and a hierarchical approach to multi-objective, integrated supply chain scheduling were also compared in Sawik (2009b). A decomposition of the complex multi-objective production, manufacturing and supply scheduling into a hierarchy of much simpler decision-making problems was proposed and simple MIP formulations were provided. The objective functions integrated both the total cost and the customer service level and the scheduling was combined with the selection of part suppliers for each customer order and due date setting for some orders. In Cui (2014) a MIP model was proposed for joint optimization problem of production planning and supplier selection. The objective was to maximize the manufacturers total profit subject to various operating constraints of the supply chain. Gao et al. (2015) developed a MIP formulation and a heuristic with a guaranteed worst-case bound for integrated production and distribution problem in which orders are processed and delivered in batches with limited vehicle capacity. Cheng et al. (2015) considered an integrated scheduling of production and distribution to minimize total cost of production and distribution for the manufacturer. A MIP model was developed and an improved ant colony method was proposed to solve the production scheduling and the First-Fit-Decreasing heuristic used in the bin-packing problem, for the distribution scheduling.

The material presented in this chapter is based on research reported in Sawik (2016a), where bi-objective MIP models were proposed for an integrated selection of supply portfolio and scheduling of production and distribution in the presence of supply chain disruption risks. The models incorporate supply-production, production-distribution and supply-distribution coordinating constraints to efficiently coordinate supply, production and distribution schedules. However, in Sawik (2016a) only a risk-neutral trade-off between expected cost and expected service level has been considered to optimize an overall performance of a supply chain. In this chapter, the MIP approach and the risk-neutral models have been enhanced for the risk-averse decision-making, using CVaR of cost and CVaR of service level as risk measures. Moreover, the risk-neutral SMIP formulation based on the wait-and-see approach is compared with a deterministic MIP model based on the expected value approach, in which random parameters are replaced by their expected values. The expected value problem is often used in practice as the related stochastic mixed integer program is in general much harder to solve since it considers multiple scenarios, e.g., Durbach and Stewart (2009), Maggioni and Wallace (2012).

The future research should concentrate on relaxations of the various simplified assumptions used to formulate the problem (see, Sect. 6.2). For example, multiple supplies of parts to the producer, e.g., partially disrupted supplies (e.g., Sawik (2015b)), and multiple deliveries of finished products from distribution centers to customers, along with an assignment of customers to distribution centers should be considered. In addition, more shipping methods can be modelled such as batch shipping with vehicle routing, methods with fixed shipping or delivery dates or with shipping or delivery time windows, e.g., Chen et al. (2010).

Problems

6.1 Modify the SMIP models presented in this chapter for multiple part types with subsets of part types required for each customer order and subsets of suppliers available for each part type.

6.2 Modify the SMIP models with single and multiple batch shipping of products to distribution centers to account for a limited storage space for products waiting for shipments.

6.3 Mean-risk scheduling in a supply chain
(a) Modify the SMIP models presented in this chapter to minimize expected cost and CVaR of cost or maximize expected service level and CVaR of service level.
(b) How should the values of the optimized objective functions be scaled into the interval [0,1] to avoid dimensional inconsistency among the two objectives and how should the trade-off parameter be selected?
(c) How would you interpret the mean-risk supply chain schedules?

6.4 Mixed mean-risk scheduling in a supply chain
(a) Modify the SMIP models presented in this chapter to optimize expected cost and CVaR of service level or optimize expected service level and CVaR of cost.

(b) How should the values of the optimized objective functions be scaled into the interval [0,1] to avoid dimensional inconsistency among the two objectives and how should the trade-off parameter be selected?
(c) How would you interpret the mixed mean-risk supply chain schedules?

6.5 Explain why the supply portfolios for different shipping methods of products to distribution centers are similar and the service-oriented portfolios are more diversified than the cost-oriented portfolios.

Part III
Equitably Efficient Selection of Supply Portfolio and Scheduling

Chapter 7
A Fair Decision-Making Under Disruption Risks

7.1 Introduction

In global supply chains the optimization of material flows subject to unexpected disruptive events, focuses on a variety of different optimality criteria. The most commonly used criteria are minimization of cost and maximization of service level that measures either fraction of customer orders or fraction of customer demand satisfied on time. The cost and service level are in conflict and, in addition, the decision makers often do not have preference to any objective, i.e., the two objectives are equally important. Then, an equitably efficient solution should be generated, in which the two normalized objective function values are as much close to each other as possible. In this chapter, the two risk-neutral conflicting criteria: expected cost and expected service level are fairly optimized to achieve an equitably efficient supply portfolio and production schedule in the presence of supply chain disruption risks. The supplies of parts are subject to independent random local and correlated regional disruptions. The cost includes the cost of ordering, purchasing and shortage of parts, while the service level is independent of any cost parameters. The two alternative service level measures are compared: the expected order fulfillment rate and the expected demand fulfillment rate (see, Chap. 5). In order to obtain an equitably efficient solution to the combinatorial stochastic optimization problem, the Ordered Weighted Averaging (OWA) aggregation of the two conflicting objective functions is applied, e.g., Yager (1988). The equitably efficient solutions obtained for the ordered weighted averaging aggregation of the two conflicting objective functions will be compared with nondominated solutions obtained using the weighted-sum aggregation approach.

The following time-indexed SMIP models are presented in this chapter:

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ESPS_E for risk-neutral selection of equitably efficient supply portfolio and scheduling of customer orders to minimize expected cost and maximize expected service level;
SPS_E(c) for risk-neutral selection of supply portfolio and scheduling of customer orders to minimize expected cost;
SPS_E(sl) for risk-neutral selection of supply portfolio and scheduling of customer orders to maximize expected service level;
WSPS_E for risk-neutral selection of supply portfolio and scheduling of customer orders to minimize weighted-sum of normalized expected cost and service level.

Numerical examples and some computational results are provided in Sect. 7.5, in particular, comparison of OWA aggregation with weighted-sum aggregation of the two objective functions is discussed.

7.2 The Lexicographic Minimax Optimization and Equitable Aggregation

This section provides an overview of the lexicographic minimax optimization and equitable aggregation.

Consider an optimization problem with m objective functions $f_k(x)$, $k = 1, \ldots m$, that are to be minimized. The problem can be formulated as follows:

$$\min\{f_1(x), f_2(x), \ldots, f_m(x) : x \in Q\}, \tag{7.1}$$

where $f(x) = (f_1(x), f_2(x), \ldots, f_m(x))$ is a vector-function that maps the decision space $X = \mathscr{R}^n$ into the criterion space $Y = \mathscr{R}^m$, $Q \subset X$ denotes the feasible set, and $x \in X$ denotes the vector of decision variables. The elements of the criterion space are referred to as outcome vectors.

Model (7.1) specifies that the objective is to minimize of all outcomes y_k, $k = 1, \ldots m$, however the solution concepts are defined by properties of the corresponding preference model within the outcome space. If all the objective functions are equally important to the decision makers, an equitable solution is sought, in which all normalized objective function values should be as much close to each other as possible.

The relation of equitable dominance can be expressed as a vector inequality on the cumulative ordered outcomes. Let $\Theta : \mathscr{R}^m \to \mathscr{R}^m$ be the ordering map such that $\Theta(y) = (\theta_1(y), \theta_2(y), \ldots \theta_m(y))$, where $\theta_1(y) \geq \theta_2(y) \geq \ldots \geq \theta_m(y))$ and there exists a permutation ϖ of set $\{1, \ldots, m\}$ such that $\theta_k(y) = y_{\varpi(k)}$ for $k = 1, \ldots m$. Apply to ordered outcomes $\Theta(y)$, a linear cumulative map that results in the cumulative ordering map $\overline{\Theta}(y) = (\overline{\theta}_1(y), \overline{\theta}_2(y), \ldots \overline{\theta}_m(y))$, defined as $\overline{\theta}_l(y) = \sum_{k=1}^{l} \theta_k(y)$

for $l = 1, \ldots, m$. The coefficients of vector $\overline{\Theta}(y)$ express: the largest outcome, the total of the two largest outcomes,...,the total of the m outcomes, respectively.

The outcome vector $y' \in Y$ equitably dominates $y'' \in Y$, if and only if $\overline{\theta}_l(y') \leq \overline{\theta}_l(y'')$ for all $l = 1, \ldots, m$ where at least one strict inequality holds.

The following relationship between equitable efficiency and Pareto-optimality can be provided (e.g., Kostreva et al. 2004).

Theorem 1 *A feasible solution $x \in Q$ is an equitably efficient solution of the multiple criteria problem (7.1), iff it is a Pareto-optimal solution of the multiple criteria problem with objectives $\overline{\Theta}(f(x))$:*

$$\min\{(\overline{\theta}_1(f(x)), \overline{\theta}_2(f(x)), \ldots \overline{\theta}_m(f(x))) : x \in Q\}. \tag{7.2}$$

Note that minimization of the first objective in (7.2) corresponds to minimization of the worst outcome, while minimization of the last (mth) objective in (2) aims at minimizing the sum of outcomes.

The quantities $\overline{\theta}_l(y)$ can be modeled with simple auxiliary variables and constraints. For a given outcome vector y, the total of the l largest outcomes, $\overline{\theta}_l(y)$, may be found by solving the following linear program (e.g., Ogryczak and Tamir 2003):

$$\overline{\theta}_l(y) = \min\{l\lambda + \sum_{k=1}^{m} \delta_k : \lambda + \delta_k \geq y_k,\ \delta_k \geq 0;\ k = 1, \ldots m\} \tag{7.3}$$

where λ is an unrestricted variable while nonnegative variables δ_k represent, for several outcome values y_k, their upside deviations from the value of λ.

Using the linear program (7.3), problem (7.2) can be formulated as the following multiple criteria optimization problem:

$$\begin{aligned}\min\{(\lambda_1 + \sum_{k=1}^{m} \delta_{k1}, 2\lambda_2 + \sum_{k=1}^{m} \delta_{k2}, \ldots, m\lambda_m + \sum_{k=1}^{m} \delta_{km}) :\\ x \in Q,\ \lambda_l + \delta_{kl} \geq f_k(x),\ \delta_{kl} \geq 0;\ k, l = 1, \ldots m\}\end{aligned} \tag{7.4}$$

In order to find an equitable efficient solution, the following lexicographic minimax problem can be solved

$$\operatorname{lex}\min\{\Theta(f(x)) : x \in Q\}. \tag{7.5}$$

Unlike in the standard minimax problem where only the worst objective value is minimized, in the lexicographic minimax problem (7.5), first the worst objective value is minimized, then the second worst objective value is minimized (provided that the worst one is as small as possible), the third worst objective value is minimized (provided that the two worst remain as small as possible), and so on. The lexicographic minimax solution can be considered in some sense the "most equitable solution" (Kostreva et al. 2004).

The multiple criteria problem (7.2) can be replaced with the minimization problem of an equitably aggregated objective function. An equitable aggregation can be based on the use of the cumulative ordered outcomes $\overline{\theta}_k(y)$, for example, the weighted sum of $\overline{\theta}_k(y)$,

$$\sum_{k=1}^{m} \nu_k \overline{\theta}_k(y), \tag{7.6}$$

The above aggregation is the so-called ordered weighted averaging (OWA) aggregation, was introduced by Yager (1988). If the weights ν_k are positive, then applying OWA aggregation to the multiple criteria problem (7.1) yields an equitably efficient solution of the latter problem.

Note that in the OWA aggregation the weights are assigned to the ordered values (i.e., to the largest value, the two largest values, etc.) rather than to the specific criteria, like in the commonly used weighted sum aggregation.

By applying the OWA aggregation (7.6) to (7.4), the following OWA problem can be formulated

$$\min\{\sum_{l=1}^{m} \nu_l (l\lambda_l + \sum_{k=1}^{m} \delta_{kl}) : x \in Q,\ \lambda_l + \delta_{kl} \geq f_k(x),\ \delta_{kl} \geq 0;\ k, l = 1, \ldots m\}. \tag{7.7}$$

Liu and Papageorgiou (2013) considered the optimization problem (7.7) with equal weights $\nu_l = 1$ for all $l = 1, \ldots, m$ and $m = 2$ objective functions

$$\min\{\sum_{l=1}^{2} (l\lambda_l + \sum_{k=1}^{2} \delta_{kl}) : x \in Q,\ \lambda_l + \delta_{kl} \geq f_k(x),\ \delta_{kl} \geq 0;\ k, l = 1, 2\}, \tag{7.8}$$

and in a recent paper Liu et al. (2014) proved the following new theorem.

Theorem 2 *If there exists an optimal solution $x^* \in Q$ of the optimization problem (7.7) with equal weights $\nu_l = 1$ for all $l = 1, \ldots, m$, which also satisfies perfect equity (i.e. $f_1(x^*) = f_2(x^*) = \ldots = f_m(x^*)$), then it is the optimal solution of the lexicographic minimax problem (7.5).*

In addition, the following theorem was introduced in Liu et al. (2014), according to Theorem 1 (Kostreva et al. 2004).

Theorem 3 *If $x^* \in Q$ is an optimal solution of the optimization problem (7.7), with equal weights $\nu_l = 1$ for all $l = 1, \ldots, m$, then it is an equitably efficient solution, as well as a Pareto-optimal solution, of the multiobjective optimization problem (7.1).*

The results of this section will be used to formulate and solve the fair, risk-neutral optimization problem in this chapter and the fair, mean-risk (robust) optimization problem in Chap. 8, for a bi-objective selection of supply portfolio and scheduling of customer orders in the presence of supply chain disruption risks.

7.3 Problem Description

Consider a three-echelon customer-driven supply chain in which various types of products are assembled by a single producer to meet customer orders, using the same critical part type that can be manufactured and provided by many suppliers (for notation used, see Table 7.1).

Table 7.1 Notation: selection of equitably efficient supply portfolio and scheduling

	Indices
i	= supplier, $i \in I$
j	= customer order, $j \in J$
r	= region, $r \in R$
s	= disruption scenario, $s \in S$
t	= planning period, $t \in T$
	Input Parameters
a_j	= per unit requirement for parts of each product in customer order j
b_j	= size (number of products) of customer order j
A	$= \sum_{j \in J} a_j b_j$, total demand for parts
B	$= \sum_{j \in J} b_j$, total demand for products
c_j	= per unit capacity consumption of producer for customer order j
C_t	= capacity of producer in period t
d_j	= due date for customer order j
e_i	= fixed cost of ordering parts from supplier i
g_j	= per unit and per period penalty cost of delayed customer order j
h_j	= per unit penalty cost of unfulfilled customer order j
I^r	= subset of suppliers in region r
o_i	= per unit price of parts purchased from supplier i
p_i	= local disruption probability for supplier i
p^r	= regional disruption probability for all suppliers in region r
τ_i	= delivery lead time from supplier i

Let $I = \{1, \ldots, \overline{I}\}$ be the set of $\overline{I}$ suppliers, $J = \{1, \ldots, \overline{J}\}$ the set of $\overline{J}$ customer orders for products, and $T = \{1, \ldots, \overline{T}\}$ the set of $\overline{T}$ planning periods (for notation used, see Table 7.1).

Denote by b_j and d_j, respectively the size and the due date of customer order $j \in J$, i.e., the number of units of ordered product type and the latest period of their completion required to deliver the products to the customer by requested date. Let a_j be the unit requirement for the critical part of each product in customer order $j \in J$. The total demand for all parts is $A = \sum_{j \in J} a_j b_j$ and the total demand for all products is $B = \sum_{j \in J} b_j$.

The customer orders are single-period orders such that each order must be completed in one planning period, e.g., Sawik (2011a). Assume that the producer has limited time-varying capacity, and denote by C_t the producer capacity available in planning period $t \in T$, and by c_j the unit capacity consumption for each product in customer order $j \in J$.

The orders for parts are assumed to be placed at the start of the planning horizon, when all customer orders for products are known. Let o_i be the unit purchasing price of parts from supplier $i \in I$ and denote by e_i the fixed ordering cost of creating contracts and maintaining relationships with supplier $i \in I$. Each supplier have sufficient capacity to meet total demand for parts. The order preparation and transportation time of a shipment from supplier $i \in I$ to the producer is constant and equals to τ_i periods so that the parts ordered from supplier $i \in I$ are delivered in period τ_i and then can be used for the assembly of products in period $\tau_i + 1$, at the earliest.

Assume that the suppliers are located in a number of disjoint geographic regions and denote by $I^r \subseteq I$ the subset of suppliers in region $r \in R$, where $\bigcup_{r \in R} I^r = I$.

The supplies of parts are subject to random independent local disruptions of each supplier individually, which may arise from equipment breakdowns, local labor strike, fires, etc. Denote by p_i the local disruption probability for supplier i.

In addition to independent local disruptions of each supplier, there are potential correlated regional disasters that may result in disruption of all suppliers in the same region simultaneously. For example, such regional disaster events may include an earthquake, flooding, etc. Denote by p^r the probability of correlated regional disruption of all suppliers $i \in I^r$ in region $r \in R$. The regional disasters in each region and the local disasters at each supplier are assumed to be independent events. Let π_i be the disruption probability of every supplier $i \in I^r$, $r \in R$

$$\pi_i = p^r + (1 - p^r)p_i; \;\; i \in I^r, r \in R. \tag{7.9}$$

Denote by $S = \{1, \ldots, \overline{S}\}$ the index set of all disruption scenarios, where each scenario $s \in S$ is comprised of a unique subset $I_s \subset I$ of suppliers who deliver parts without disruptions. All potential disruption scenarios will be considered, that is $\overline{S} = 2^{\overline{I}}$. For each scenario $s \in S$, the supplies from every supplier, $i \in I \setminus I_s$, can be disrupted either by a local or a regional disaster event. The probability P_s for disruption scenario $s \in S$ with the subset I_s of non-disrupted suppliers, and with all possible combinations of different disaster events considered, is (cf. Sect. 1.3)

$$P_s = \prod_{r \in R} P_s^r, \tag{7.10}$$

where P_s^r is the probability of realizing of disruption scenario s for suppliers in I^r

$$P_s^r = \begin{cases} (1 - p^r) \prod_{i \in I^r \bigcap I_s} (1 - p_i) \prod_{i \in I^r \setminus I_s} p_i & \text{if } \; I^r \bigcap I_s \neq \emptyset \\ p^r + (1 - p^r) \prod_{i \in I^r} p_i & \text{if } \; I^r \bigcap I_s = \emptyset. \end{cases} \tag{7.11}$$

The producer can be charged with a contractual, order specific penalty cost for delayed or unfulfilled customer orders, caused by the shortage of parts, that are delivered late or not at all due to supply disruptions. Let g_j and h_j be, respectively the per unit and per period penalty cost of delayed customer order $j \in J$ and the per unit total penalty cost of unfulfilled customer order $j \in J$.

The objective of the equitable optimization of a supply chain under disruption risks is to allocate the total demand for parts among a subset of selected suppliers and to schedule the customer orders for products over the planning horizon to equitably minimize expected cost of ordering, purchasing and shortage of parts and maximize expected customer service level, i.e., the fraction of customer orders (order fulfillment rate) or customer demand (demand fulfillment rate) filled on or before their due dates. The resulting equitably efficient supply portfolio (the allocation of total demand for parts among the selected suppliers) is determined ahead of time as well as the equitably efficient schedule of customer orders for every potential disruption scenario.

7.4 Problem Formulation

In this section the time-indexed SMIP model **ESPS_E** is presented for selection of equitably efficient supply portfolio and scheduling of customer orders to fairly minimize expected cost per product and maximize expected service level, the expected fraction of customer orders filled on or before their due dates (i.e., expected order fulfillment rate). The decision variables thatare jointly determined using the proposed model are defined in Table 7.2.

Table 7.2 Variables: selection of equitably efficient supply portfolio and scheduling

		First stage variables
u_i	$=$	1, if supplier i is selected; otherwise $u_i = 0$ (supplier selection)
v_i	$\in$	[0, 1], the fraction of total demand for parts ordered from supplier i (supply portfolio)
		Auxiliary variables
λ_l		unrestricted auxiliary variable, $l = 1, 2$
δ_{kl}	$\geq$	0, upside deviation of outcome value f_k, $k = 1, 2$ from the value of λ_l, $l = 1, 2$
		Second stage variables
w_{jt}^s	$=$	1, if under disruption scenario s customer order j is scheduled for period t; otherwise $w_{jt}^s = 0$ (production scheduling)

Let E^c be the expected cost per product to be minimized and E^{sl}, the expected service level to be maximized

$$E^c = (\sum_{i \in I} e_i u_i + \sum_{s \in S} P_s (\sum_{i \in I_s} Ao_i v_i$$

$$+ \sum_{j \in J} \sum_{t \in T: t > d_j} g_j b_j (t - d_j) w_{jt}^s + \sum_{j \in J} h_j b_j (1 - \sum_{t \in T} w_{jt}^s)))/B \quad (7.12)$$

$$E^{sl} = \sum_{j \in J} \sum_{t \in T: t \leq d_j} \sum_{s \in S} P_s w_{jt}^s / \bar{J}. \quad (7.13)$$

In order to avoid dimensional inconsistency among the two objectives, the values of the optimized objective functions are scaled into the interval [0,1]. Denote by $f_1 = \frac{E^c - \underline{E}^c}{\overline{E}^c - \underline{E}^c}$, the normalized expected cost per product ($\underline{E}^c, \overline{E}^c$ are the minimum and the maximum values of E^c, respectively), and by $f_2 = \frac{\overline{E}^{sl} - E^{sl}}{\overline{E}^{sl} - \underline{E}^{sl}}$, the normalized expected customer service level ($\underline{E}^{sl}, \overline{E}^{sl}$ are the minimum and the maximum values of E^{sl}, respectively).

The normalized objective functions f_1 and f_2 are defined below

$$f_1 = (\sum_{i \in I} e_i u_i + \sum_{s \in S} P_s (\sum_{i \in I_s} Ao_i v_i + \sum_{j \in J} \sum_{t \in T: t > d_j} g_j b_j (t - d_j) w_{jt}^s$$

$$+ \sum_{j \in J} h_j b_j (1 - \sum_{t \in T} w_{jt}^s)))/B - \underline{E}^c)/(\overline{E}^c - \underline{E}^c) \quad (7.14)$$

$$f_2 = \frac{\overline{E}^{sl} - \sum_{j \in J} \sum_{t \in T: t \leq d_j} \sum_{s \in S} P_s w_{jt}^s / \bar{J}}{(\overline{E}^{sl} - \underline{E}^{sl})}. \quad (7.15)$$

The SMIP model **ESPS_E** is formulated below. The model is based on the SMIP formulations **SPSm_E(c)** and **SPSm_E(sl)** presented in Chap. 5. The objective function (7.16) subject to constraints (7.17) represent the so-called ordered weighted averaging aggregation of the two conflicting criteria with equal weights assigned to each criterion, see, OWA aggregation in Sect. 7.2. Applying OWA aggregation to the bi-objective problem yields an equitably efficient solution to the problem. In the model presented below λ_l are unrestricted variables, while nonnegative variables δ_{kl} represent, for outcome values f_k, their upside deviations from the value of λ_l (see, Sect. 7.2).

ESPS_E: *Risk-neutral selection of* **E***quitably efficient* **S***upply* **P***ortfolio and* **S***cheduling of customer orders to minimize expected cost and maximize expected service level*

Minimize

$$\sum_{l=1}^{2} (l\lambda_l + \sum_{k=1}^{2} \delta_{kl}) \quad (7.16)$$

subject to (7.14), (7.15) and

$$\lambda_l + \delta_{kl} \geq f_k;\ k, l = 1, 2 \tag{7.17}$$

Demand allocation constraints:

– the total demand for parts must be fully allocated among the selected suppliers,

– demand for parts cannot be assigned to non-selected suppliers,

$$\sum_{i \in I} v_i = 1 \tag{7.18}$$

$$v_i \leq u_i;\ \ i \in I \tag{7.19}$$

Order-to-period assignment constraints:

– for each disruption scenario s, each customer order j is either scheduled during the planning horizon ($\sum_{t \in T} w_{jt}^s = 1$), or unscheduled and rejected ($\sum_{t \in T} w_{jt}^s = 0$),

– for each disruption scenario s and each planning period t, the cumulative demand for parts of all customer orders scheduled in periods 1 through t cannot exceed the cumulative deliveries of parts in periods 1 through $t - 1$, from the non-disrupted suppliers $i \in I_s$,

– for each disruption scenario s, the total requirement for parts of scheduled customer orders is not greater than the total supplies from the non-disrupted suppliers $i \in I_s$,

$$\sum_{t \in T} w_{jt}^s \leq 1;\ \ j \in J, s \in S \tag{7.20}$$

$$\sum_{j \in J} \sum_{t' \in T: t' \leq t} a_j b_j w_{jt'}^s \leq A \sum_{i \in I_s: \tau_i \leq t-1} v_i;\ \ t \in T, s \in S \tag{7.21}$$

$$\sum_{j \in J} \sum_{t \in T} a_j b_j w_{jt}^s \leq A \sum_{i \in I_s} v_i;\ \ s \in S \tag{7.22}$$

Producer capacity constraints:

– for any period t and each disruption scenario s, the total demand on capacity of all customer orders scheduled in period t must not exceed the producer capacity available in this period,

$$\sum_{j \in J} b_j c_j w_{jt}^s \leq C_t;\ \ t \in T, s \in S \tag{7.23}$$

Non-negativity and integrality conditions:

$$\delta_{kl} \geq 0; k, l = 1, 2 \tag{7.24}$$

$$u_i \in \{0, 1\}; \ \ i \in I \tag{7.25}$$

$$v_i \in [0, 1]; \ \ i \in I \tag{7.26}$$

$$w^s_{jt} \in \{0, 1\}; \ \ j \in J, t \in T, s \in S. \tag{7.27}$$

In the above model, each supplier is assumed to have sufficient capacity to meet total demand for parts. Such an assumption allows the decision maker to select a single sourcing solution, if such a portfolio is an equitably efficient supply portfolio. However, the assumption can be easily relaxed to account for multiple capacitated suppliers, see Sect. 7.4.2.

7.4.1 Minimum and Maximum Values of the Objective Functions

In this subsection the minimum and maximum values for all objective functions are calculated to determine the normalized values of the objective functions, f_1, (7.14), f_2, (7.15), that is, the values of the optimized objective functions scaled into the interval [0,1]. Note that the cost and the service level objectives are in conflict. Therefore, the minimum and maximum values of expected cost $\underline{E}^c$, $\overline{E}^c$, and expected customer service level, $\underline{E}^{sl}$, $\overline{E}^{sl}$, are obtained by solving the following stochastic mixed integer programs:

SPS_E(c): *Risk-neutral selection of supply portfolio and scheduling of customer orders to minimize expected cost per product*

Minimize E^c, (7.12),

subject to (7.18)–(7.23), (7.25)–(7.27).

SPS_E(sl): *Risk-neutral selection of supply portfolio and scheduling of customer orders to maximize expected service level*

Maximize E^{sl}, (7.13)

subject to (7.18)–(7.23), (7.25)–(7.27).

In problem **SPS_E(c)**, E^c is the minimized objective function, while E^{sl} is not considered. In problem **SPS_E(sl)**, E^{sl} is the maximized objective function, while E^c is not considered. Thus, by solving problem **SPS_E(c)**, the minimum value $\underline{E}^c$ of E^c and the minimum value $\underline{E}^{sl}$ of E^{sl} are determined. Similarly, by solving problem

SPS_E(sl), the maximum value $\overline{E}^{sl}$ of E^{sl} and the maximum value $\overline{E}^{c}$ of E^c are determined.

So far, in the proposed models the customer service level is measured by order fulfillment rate, i.e., the number of customer orders fulfilled by their due dates, with no account for the size of each customer order. For example, a high customer service level can be achieved by fulfilling a large number of small-size orders, while leaving the unfulfilled demand relatively high. To avoid such a solution, in particular when the customer orders of different size are simultaneously considered, the service level can be measured by demand fulfillment rate, i.e., the fraction of total customer demand fulfilled by the requested due dates.

If the customer service level is defined as demand fulfillment rate, then E^{sl}, (7.13) and f_2, (7.15) should be replaced with the following formulae, (7.28) and (7.29), respectively.

$$E^{sl} = \sum_{j \in J} \sum_{t \in T: t \leq d_j} \sum_{s \in S} P_s b_j w^s_{jt} / B \tag{7.28}$$

$$f_2 = \frac{\overline{E}^{sl} - \sum_{j \in J} \sum_{t \in T: t \leq d_j} \sum_{s \in S} P_s b_j w^s_{jt} / B}{(\overline{E}^{sl} - \underline{E}^{sl})}, \tag{7.29}$$

where $\underline{E}^{sl}$, $\overline{E}^{sl}$ are the minimum and the maximum values of E^{sl}, (7.28), respectively. The values of $\underline{E}^{sl}$, $\overline{E}^{sl}$ can be determined using the models **SPS_E(c)** and **SPS_E(sl)**.

In the computational examples presented in the next section, the two metrics of the customer service level will be considered and compared against each other.

7.4.2 Model Limitations and Possible Enhancements

The proposed model has been developed to support decision-making in a make-to-order environment under disruption risks. The model, however, has been formulated under various simplified assumptions that may limit its practical usefulness. The basic assumptions are listed below.

1. A single critical part type is required to fulfill all customer orders for products.
2. The orders for parts are placed at the start of the planning horizon, when all customer orders for products are known.
3. Each supplier have sufficient capacity to meet the total demand for parts.
4. The order preparation time at each supplier is constant, independent of order size, and all parts ordered from a supplier are delivered during a fixed transportation time.
5. Transportation costs are not explicitly considered and the unit purchasing price from each supplier is constant, independent of total volume or value of order for parts, i.e., no quantity or business volume discounts are considered.

6. The customer orders are single-period orders such that each order must be completed in one planning period.
7. The penalty costs for delayed or unfulfilled customer demand are linear.
8. The inventory of parts and products are not considered.
9. The two conflicting objectives: reduction of expected cost and increase of expected customer service level are equally important for the decision maker.

Some of the above assumptions can be easily relaxed, while the other needs a more advanced model to be developed. Possible relaxations of the corresponding assumptions and the model enhancements are listed below.

1. The model can be easily enhanced for multiple part types required to fulfill all customer orders with different subsets of part types needed for different product types and different subsets of capacitated suppliers capable of providing different subsets of part types, e.g., Sawik (2013c).
2. A rolling planning horizon approach can be used to account for a dynamic arrival and scheduling of customer orders as well as the corresponding supply portfolio. In practice, however, the supply portfolio needs to be decided at the start of the planning horizon, based on a forecast of the customer demand.
3. The model can be easily enhanced for multiple capacitated suppliers by the addition of suppliers capacity constraints.
4. The more advanced model can be developed to consider order-dependent processing and transportation times to better coordinate manufacturing and transportation of parts and production of finished products.
5. The model can be easily enhanced to account for quantity or business volume discounts (e.g., Sawik 2010) and the unit purchasing price from each supplier can include unit transportation cost.
6. The model can be easily enhanced to account for large, multi-period customer orders that cannot be completed in one period and must be split into single-period portions to be processed in consecutive planning periods, e.g., Sawik (2011a).
7. The introduction of non-linear penalty costs may lead to a non-linear SMIP model. The model, however, can be linearized in some cases, e.g., by using a piecewise linear representation of the non-linear penalty cost.
8. The model can be enhanced to account for the output inventory of parts at suppliers, the input inventory of parts at the producer and the output inventory of products waiting for shipment to customers. The inventory balance constraints can be added to the model and the producer inventory holding costs, to the cost-based objective function.
9. The number of conflicting and equally important objectives can be increased, for example by the addition of responsiveness (e.g., Liu and Papageorgiou 2013) as another objective function.

7.5 Computational Examples

In this section the proposed SMIP approach for selection of equitably efficient supply portfolio and scheduling of customer orders in a supply chain under disruption risks is compared with the weighted-sum approach and illustrated with computational examples. The following parameters have been used for the example problems (cf. Sect. 6.6):

- $\overline{I} = 9, \overline{J} = 25, \overline{T} = 10$, and $\overline{S} = 2^{\overline{I}} = 512$;
- $R = \{1, 2, 3\}$, and $I^1 = \{1, 2, 3\}, I^2 = \{4, 5, 6\}, I^3 = \{7, 8, 9\}$;
- τ_i, the order preparation and shipping times from suppliers were 2, 3 and 4 time periods, respectively for suppliers $i \in I^1, i \in I^2$ and $i \in I^3$;
- $a_j \in \{1, 2, 3\}, \; b_j \in \{500, 1000, \ldots, 5000\}, \; c_j \in \{1, 2, 3\}, \; d_j \in \{1 + \min_{i \in I}(\tau_i), \ldots, \overline{T}\}$;
- C_t, the capacity of producer in each period t, was integer drawn from $1000\lceil (2 \sum_{j \in J} b_j c_j / (\overline{T} - \max_{i \in I} \tau_i)) U[0.75; 1.25]/1000 \rceil$ distribution, i.e., in each period the producer capacity was from 75% to 125% of the double capacity required to complete all customer orders during the planning horizon, after the latest delivery of parts;
- $e_i \in \{5000, 6000, \ldots, 10000\}, \; i \in I^1, \quad e_i \in \{10000, 11000, \ldots, 15000\}, \; i \in I^2$ and $e_i \in \{15000, 11000, \ldots, 30000\}, \; i \in I^3$;
- o_i, the unit price of parts purchased from supplier i, was uniformly distributed over [11,16], [6,11] and [1,6], respectively for suppliers $i \in I^1, i \in I^2$ and $i \in I^3$;
- $g_j = \lceil a_j \max_{i \in I}(o_i)/350 \rceil, \; j \in J$, i.e., the unit penalty cost per period of each delayed customer order j was approximately 0.28% of the maximum unit price of required parts;
- $h_j = 2\lceil a_j \max_{i \in I}(o_i) \rceil, \; j \in J$, i.e., the unit penalty cost of each unfulfilled customer order j was approximately twice as large as the maximum unit price of required parts;
- p_i, the local disruption probability was uniformly distributed over [0.005,0.01], [0.01,0.05] and [0.05;0.10], respectively for suppliers $i \in I^1, i \in I^2$ and $i \in I^3$;
- $p^1 = 0.001, p^2 = 0.005$ and $p^3 = 0.01$.

The detailed data set was based on the example presented in Sawik (2013c), e.g.,: unit requirements for parts, $a = (2, 1, 3, 3, 1, 3, 2, 1, 2, 2, 2, 2, 3, 2, 1, 3, 2, 1, 3, 3, 2, 1, 1, 2, 1)$;
size of customer orders, $b = (1, 2, 9, 7, 8, 5, 1, 7, 5, 4, 7, 4, 10, 6, 8, 1, 4, 2, 4, 8, 6, 3, 8, 7, 3) \times 500$,
(the resulting total demand for parts and products is $A = 132500$ and $B = 66000$, respectively);
unit capacity consumption, $c = (2, 1, 1, 2, 3, 3, 1, 3, 2, 1, 2, 1, 3, 1, 1, 3, 2, 3, 1, 1, 3, 2, 2, 1, 2)$;
producer available capacity, $C_t = C = 38000, \forall t = 1, \ldots, 10$;
unit prices, $o = (13, 12, 12, 8, 6, 6, 2, 5, 4)$;
local disruption probabilities, $p = (0.00513571, 0.00666354, 0.00902974, 0.0356206,$

0.040175, 0.0294692, 0.0519967, 0.0827215, 0.0739062);
and the corresponding disruption probabilities (7.9), π = (0.00613057, 0.00765688, 0.0100207, 0.0404425, 0.0449741, 0.0343219, 0.0614767, 0.0918943, 0.0831672).

Note that the constant producer capacity, $C = 38000$, allows for completing all customer orders in at most $\lceil \sum_{j \in J} b_j c_j / C \rceil = \lceil 3.26 \rceil = 4$ planning periods, that is, in less than $(\overline{T} - \max_{i \in I} \tau_i) = 6$ periods remaining in the planning horizon after the latest delivery of parts.

In the computational experiments all potential disruption scenarios, $\overline{S} = 2^{\overline{I}} = 512$, and all possible combinations of local and regional disaster events were considered. Each scenario $s \in S$ with the subset I_s of non-disrupted suppliers is represented by an $\overline{I}$-dimensional 0-1 vector with 1, if $i \in I_s$, i.e., if supplier i is not disrupted, and 0; otherwise. The corresponding disruption probability, P_s, for each scenario $s \in S$ was calculated using formulae (7.10) and (7.11).

The unit price per part o_i and the disruption probability π_i, (7.9), of each supplier $i \in I$ are shown in Fig. 6.2. The figure indicates that the most reliable (with the lowest disruption probability, $\pi_1 = 0.00613057$) is supplier 1, the least reliable (with the highest disruption probability, $\pi_8 = 0.0918943$) is supplier 8, the most expensive (with the highest price per part, $o_1 = 13$) is supplier 1, and the cheapest (with the lowest price per part, $o_7 = 2$) is supplier 7. Note that the geographic regions are numbered in such a way that the unit prices are nonincreasing with r, while the fixed ordering costs and the disruption probabilities are nondecreasing with r.

The solution results are presented in Table 7.3. In addition to the optimal absolute and normalized solution values for the primary objective functions and the allocation of demand among the selected suppliers, Table 7.3 presents the expected values of the associated objective function, i.e., the minimum expected service level, $\underline{E}^{sl}$, for model **SPS_E(c)** and the maximum expected cost per product, $\overline{E}^{c}$, for model **SPS_E(sl)**. Table 7.3 indicates that for the cost-based objective (model **SPS_E(c)**) the cheapest supplier $i = 7$ is selected only, while for the customer service level (model **SPS_E(sl)**), the total demand for parts is allocated among the three most reliable and most expensive suppliers $i = 1, 2, 3$ for objective (7.13) and among the two suppliers $i = 1, 2$ for objective (7.28). For the equitable solution (model **ESPS_E**), the supply portfolio contains one reliable and expensive supplier $i = 2$ and two low-cost and unreliable suppliers $i = 6, 7$ for both (7.15) and (7.29), service level objectives.

As an illustrative example, Fig. 7.1 presents the demand for products, $\sum_{j \in J: d_j = t} b_j$, $t \in T$, and the expected production schedules, $\sum_{s \in S} P_s \sum_{j \in J} b_j w_{jt}^s$, $t \in T$ for the optimal cost, optimal customer service level and for the equitably efficient solution. Figure 7.1 compares the expected production schedules for the two different metrics of customer service level: (a) order fulfillment rate (i.e., fraction of customer orders fulfilled on time), (7.13), and (b) demand fulfillment rate (i.e., fraction of customer demand fulfilled on time), (7.28). While for the two different service level metrics, the corresponding supply portfolios are very similar (cf. Table 7.3), and the expected production schedules with respect to the two service level objectives are also very similar, the corresponding schedules for the equitably efficient solutions are different.

Table 7.3 Solution results

Model **SPS_E(c)**: Var. = 100468, Bin. = 100459, Cons. = 21341, Nonz. = 765902 [a]	
Expected Cost ($\underline{E}^c$)	7.66
Suppliers Selected (% of total demand)	7(100)
Expected Service Level ($\underline{E}^{sl}$)	67.60 [b], 66.32 [c]
Model **SPS_E(sl)**: Var. = 100468, Bin. = 100459, Cons. = 21341, Nonz. = 765902 [a]	
Expected Service Level ($\overline{E}^{sl}$)	99.62 [b]
Suppliers Selected (% of total demand)	1(48), 2(31), 3(21)
Expected Cost ($\overline{E}^c$)	25.64
Model **ESPS_E**: Var. = 100476, Bin. = 100459, Cons. = 21347, Nonz. = 921880 [a]	
Expected Cost	9.31
Expected Service Level	96.06 [b]
Normalized Expected Cost	0.098
Normalized Expected Service Level	0.111
Suppliers Selected (% of total demand)	2(6), 6(13), 7(81)
Model **SPS_E(sl)** with (7.13) replaced by (7.28)	
Expected Service Level ($\overline{E}^{sl}$)	99.49 [c]
Suppliers Selected (% of total demand)	1(55), 2(45)
Expected Cost ($\overline{E}^c$)	25.61
Model **ESPS_E** with (7.15) replaced by (7.29)	
Expected Cost	9.25
Expected Service Level	95.29 [c]
Normalized Expected Cost	0.088
Normalized Expected Service Level	0.127
Suppliers Selected (% of total demand)	2(6), 6(10), 7(84)

[a] Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients.
[b] $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T: t\le d_j} w_{jt}^s/\bar{J})100\%$.
[c] $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T: t\le d_j} b_j w_{jt}^s/B)100\%$.

In general, the service level-based solution, when no cost components are included in the objective function, better meets the customer demand, with the smallest fraction of unfulfilled demand. The total customer demand is met with only a small fraction of the expected unfulfilled demand: 0.0615 for the cost-based solution, 0.0055 for the service level-based solution (a), 0.0051 for the service level-based solution (b), 0.0468 for the equitably efficient solution (a), and 0.0470 for the equitably efficient solution (b). In addition, *for the service level-based objective functions, the expected production schedule approximately follows the customer demand pattern, while for the minimum cost objective function the most unbalanced production schedule is achieved.*

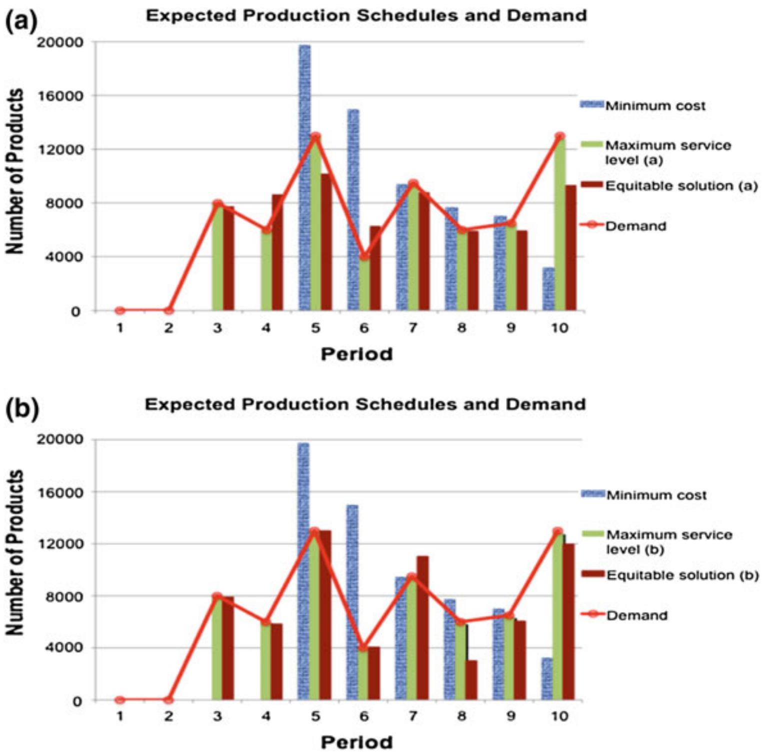

Fig. 7.1 Expected production schedules: **a** order fulfillment rate; **b** demand fulfillment rate

In order to compare the solution results for the two different service level-based objective functions (7.13) and (7.28), the computational experiments were repeated for another example with more diversified size of customer orders, $b_j \in \{500, 1000, \ldots, 12000\}$, i.e., from 500 to 12000 products, for the same total demand for parts and products, $A = 132500$ and $B = 66000$. The solution results are presented in Table 7.4, and Fig. 7.2 compares the expected production schedules for the two different metrics of customer service level. In general, the results presented in Table 7.4 are similar to those in Table 7.3, in particular the supply portfolios are very similar. However, Fig. 7.2 shows that for the more diversified customer orders, the less smoothed expected production schedules are obtained for the equitably efficient solutions. On the other hand, Table 7.4 shows that the equitably efficient solutions with a perfect equity were found (i.e., with identical values of normalized expected cost and normalized expected customer service level), which indicates that the obtained supply portfolios and schedules of customer orders are also the lexicographic minimax optimal solutions as well as the Pareto-optimal solutions (see, Liu and Papageorgiou 2013, Liu et al. 2014).

Table 7.4 Solution results: diversified customer orders

Model **SPS_E(c)**	
Expected Cost ($\underline{E}^c$)	7.44
Suppliers Selected (% of total demand)	7(100)
Expected Service Level ($\underline{E}^{sl}$)	71.32 $^{(a)}$, 78.83 $^{(b)}$
Model **SPS_E(sl)**	
Expected Service Level ($\overline{E}^{sl}$)	99.77 $^{(a)}$
Suppliers Selected (% of total demand)	1(42), 2(41), 3(17)
Expected Cost ($\overline{E}^c$)	25.15
Model **ESPS_E**	
Expected Cost	9.38
Expected Service Level	97.48 $^{(a)}$
Normalized Expected Cost	0.110
Normalized Expected Service Level	0.110
Suppliers Selected (% of total demand)	2(3), 6(22), 7(75)
Model **SPS_E(sl)** with (7.13) replaced by (7.28)	
Expected Service Level ($\overline{E}^{sl}$)	99.47 $^{(b)}$
Suppliers Selected (% of total demand)	1(65), 2(35)
Expected Cost ($\overline{E}^c$)	25.43
Model **ESPS_E** with (7.15) replaced by (7.29)	
Expected Cost	10.44
Expected Service Level	95.99 $^{(b)}$
Normalized Expected Cost	0.168
Normalized Expected Service Level	0.168
Suppliers Selected (% of total demand)	2(3), 5(5), 6(30), 7(62)

$^{(a)}$ $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T: t\le d_j} w_{jt}^s/n)100\%$

$^{(b)}$ $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T: t\le d_j} b_j w_{jt}^s/B)100\%$

The solution results demonstrate that for the minimum cost objective the cheapest supplier is usually selected, for the maximum service level objective a subset of most reliable and most expensive suppliers is usually chosen, whereas the equitably efficient supply portfolio usually combines the two types of suppliers.

Weighted-Sum Approach

The equitably efficient solutions obtained using model **ESPS_E** have been compared with the nondominated solutions obtained by minimizing the weighted-sum aggregation of the two normalized objective functions, f_1, (7.14) and f_2, (7.29), i.e., the weighted-sum of expected cost per product and expected fraction of customer demand fulfilled on time. The weighted-sum model **WSPS_E** is shown below and the solution results for the example with similar and diversified customer orders are presented in Tables 7.5 and 7.6, respectively.

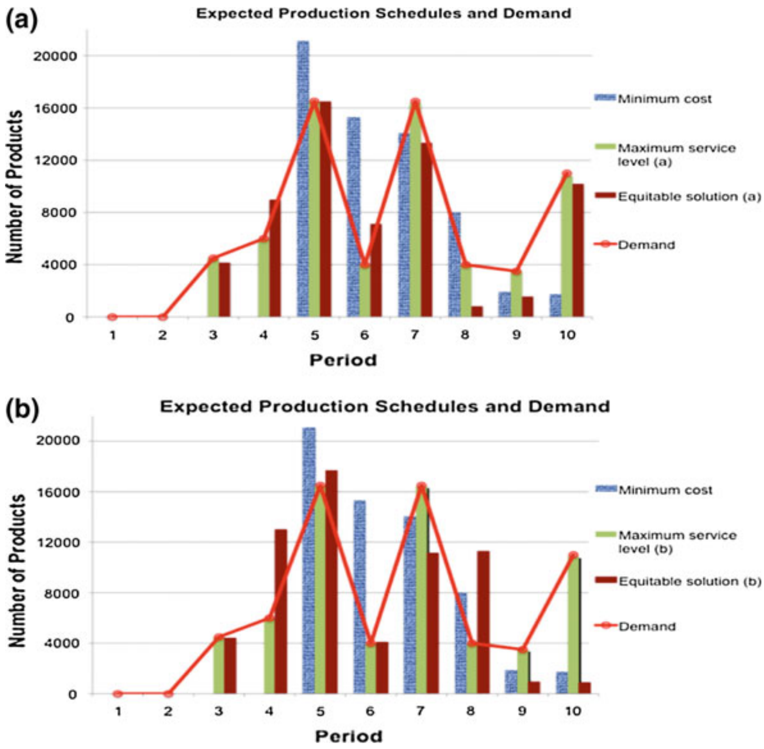

Fig. 7.2 Expected production schedules for diversified customer orders: **a** order fulfillment rate; **b** demand fulfillment rate

WSPS_E: *Risk-neutral selection of supply portfolio and scheduling of customer orders to minimize weighted-sum of normalized expected cost and service level*

Minimize

$$\lambda f_1 + (1 - \lambda) f_2, \tag{7.30}$$

where $0 \leq \lambda \leq 1$,

subject to (7.14), (7.18)–(7.23), (7.25)–(7.27), (7.29).

Tables 7.5 and 7.6 indicate that for $\lambda = 1$ (minimization of cost) the cheapest supplier $i = 7$ is selected only, while for $\lambda = 0$ (maximization of customer service level) the total demand for parts is allocated among the two most reliable and most expensive suppliers $i = 1, 2$. As λ increases from 0 to 1, i.e., the decision maker preference shifts from customer service level to cost, more demand is moved from

Table 7.5 Nondominated solutions

λ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Var. = 100470, Bin. = 100459, Cons. = 21343, Nonz. = 921868 [(a)]											
Exp.Cost	25.61	19.40	11.93	9.25	9.25	8.88	8.88	8.88	8.88	8.01	7.66
Exp.Service Level[(b)]	99.48	98.57	96.86	95.29	95.29	94.73	94.73	94.73	94.73	82.69	66.32
Normalized Exp.Cost	1	0.654	0.238	0.088	0.088	0.068	0.068	0.068	0.068	0.020	0
Normalized Exp.Service Level	0	0.028	0.079	0.127	0.127	0.143	0.143	0.143	0.143	0.506	1
Suppliers Selected	1(55)	1(6)									
(% of total demand)	2(45)	2(44)	2(10)	2(6)	2(6)	2(6)	2(6)	2(6)	2(6)		
		6(50)	6(40)	6(10)	6(10)	6(5)	6(5)	6(5)	6(5)	6(5)	
			7(50)	7(84)	7(84)	7(89)	7(89)	7(89)	7(89)	7(95)	7(100)

[(a)] Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients
[(b)] $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T:t\le d_j} b_j w^s_{jt}/B)100\%$

expensive and reliable suppliers to low-cost, unreliable suppliers. For the example with similar customer orders, the subset of nondominated solutions contains (for $\lambda = 0.3,\ 0.4$) the optimal solution obtained for model **ESPS_E** with constraint (7.21) (cf., Tables 7.3 and 7.5). In contrast to the example with diversified customer orders (cf., Tables 7.4 and 7.6). For diversified orders, Fig. 7.3 shows the nondominated supply portfolios (the allocation of total demand for parts among selected suppliers) for 11 levels of trade-off parameter λ. The subset of selected suppliers consists of four suppliers $i = 1, 2, 6, 7$ of which suppliers $i = 1, 2$ are most reliable and suppliers $i = 6, 7$ are the cheapest suppliers in region $r = 2, 3$, respectively.

For the example with diversified customer orders, the trade-off between the expected cost and the expected customer service level is clearly shown in Fig. 7.4, where the efficient frontier is presented. The results emphasize the effect of varying service level/cost preference of the decision maker; the higher the trade-off parameter λ, the more cost-oriented the decision-making.

Table 7.6 Nondominated solutions: diversified customer orders

λ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Var. = 100470, Bin. = 100459, Cons. = 21343, Nonz. = 921868 [(a)]											
Exp.Cost	25.43	24.81	12.95	11.00	8.61	8.61	8.31	8.31	8.31	8.31	7.44
Exp.Service Level[(b)]	99.47	99.42	97.07	96.30	94.90	94.90	94.45	94.45	94.45	94.45	78.83
Normalized Exp.Cost	1	0.9655	0.3062	0.1977	0.0650	0.0650	0.0486	0.0486	0.0486	0.0486	0
Normalized Exp.Service Level	0	0.0025	0.1169	0.1532	0.2226	0.2224	0.2439	0.2440	0.2442	0.2446	1
Suppliers Selected	1(65)	1(33)									
(% of total demand)	2(35)	2(67)	2(13)	2(8)	2(4)	2(4)	2(3)	2(3)	2(3)	2(3)	
			6(53)	6(35)	6(9)	6(9)	6(5)	6(5)	6(5)	6(5)	
			7(34)	7(57)	7(87)	7(87)	7(92)	7(92)	7(92)	7(92)	7(100)

[(a)] Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients
[(b)] $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T:t\le d_j} b_j w^s_{jt}/B)100\%$

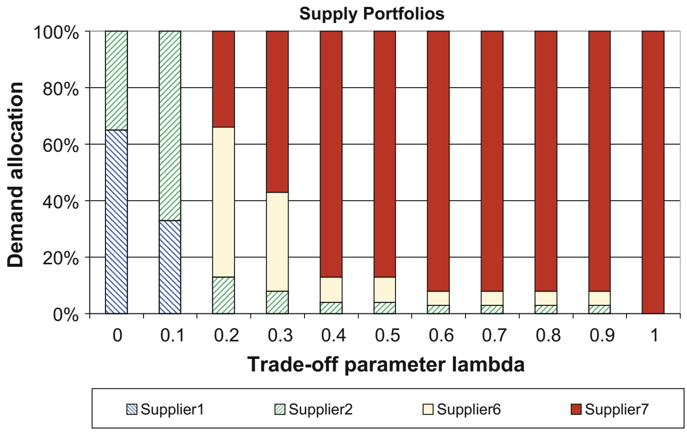

Fig. 7.3 Nondominated supply portfolios: diversified customer orders

The computational experiments were performed using the AMPL programming language and the CPLEX 12.5 solver on a MacBookPro laptop with Intel Core i7 processor running at 2.8 GHz and with 16GB RAM. The solver was capable of finding proven optimal solution for all examples with CPU time ranging from several seconds to a few hours.

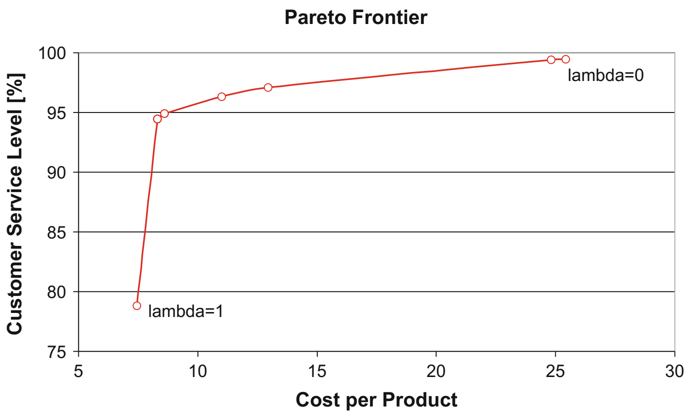

Fig. 7.4 Pareto front for model **WSPS_E**: diversified customer orders

7.6 Notes

The research on equitably efficient multiple objective optimization of a supply chain under disruption risks is rarely reported in the literature. In order to generate an equitably efficient solution with respect to multiple, equally important objective functions, the lexicographic minimax method, e.g., Kostreva et al. (2004), as a special case of the ordered weighted averaging aggregation introduced Yager (1988), can be applied. The lexicographic minimax problem can be transferred to a lexicographic minimization problem, e.g., Erkut et al. (2008), Ogryczak et al. (2008). Recently Liu and Papageorgiou (2013) developed an approach to transfer the lexicographic minimax problem to a minimization optimization problem, instead of a lexicographic minimization problem, which needs to solve a sequence of optimization problems iteratively. The recent approach, however, is restricted to some special cases of a multiple objective problem (Liu et al. 2014).

The material presented in this chapter is based on research reported by Sawik (2014d, 2015a), where a SMIP approach was proposed for the integrated selection of supply portfolio and scheduling of customer orders in a supply chain under disruption risks. In order to equitably optimize expected cost and expected service level, the ordered weighted averaging aggregation of the two conflicting objective functions was applied. The equitably efficient solutions obtained for the ordered weighted averaging aggregation were compared with nondominated solutions obtained using the weighted-sum aggregation approach.

In the proposed model, each supplier is assumed to have sufficient capacity to meet total demand for parts, which allows the decision maker to select a single sourcing type of a supply portfolio, if it is an equitably efficient solution. In the future research, however, that assumption can be easily relaxed to account for multiple capacitated suppliers. Furthermore, the other assumptions can also be relaxed to develop a more advanced model (for possible model relaxations and enhancements, see Sect. 7.4.2). In particular, the future research should focus on equitably efficient decision-making with respect to more equally important and conflicting objectives such as responsiveness, robustness, competitiveness, etc.

Problems

7.1 Modify the SMIP models presented in this chapter for multiple part types with subsets of part types required for each customer order and subsets of capacitated suppliers available for each part type.

7.2 Enhance the SMIP models presented in this chapter for multi-period customer orders that cannot be completed in one period and must be split into single-period portions to be processed in consecutive planning periods (see, Sawik 2011a).

7.3 Limited storage space for parts and products
Enhance the SMIP models presented in this chapter:
(a) for the limited input inventory of parts at the producer;
(b) for the limited output inventory of products at the producer.
Formulate the inventory balance constraints that should be added to the SMIP models.

7.4 In addition to cost and service level, what are the other conflicting and equally important objective functions that can be considered when selecting supply portfolio and scheduling customer orders in the presence of supply chain disruption risks?

7.5 How would you select a nondominated solution obtained using the weighted-sum approach that is as close as possible to the equitably efficient solution?

Chapter 8
A Robust Decision-Making Under Disruption Risks

8.1 Introduction

In this chapter we look for an equitably efficient solution with respect to both average-case and worst-case performance measures of a supply chain. Such an equitably efficient average and worst-case solution, or equivalently equitably efficient risk-neutral and risk-averse solution will be called a fair mean-risk solution. The solution will equitably focus on the two objective functions: the expected value (average-case performance measure) and the expected worst-case value (worst-case performance measure), i.e., Conditional Value-at-Risk of the selected optimality criterion, cost or service level. The fair mean-risk decision-making aims at achieving the normalized expected and expected worst-case values of the selected objective function as much close to each other as possible, that is, the decision-making aims at equalizing the distance to optimality both under business-as-usual and under worst-case conditions. The mean-risk fairness reflects the decision makers common requirement to maintain an equally good performance of a supply chain under varying operating conditions, which is close to the idea of robustness of a supply chain. Therefore, the mean-risk fairness, i.e., the equitably efficient performance of a supply chain in the average-case as well as in the worst-case, in this chapter will be called robustness.

In a make-to-order environment, the choice between high-cost, reliable suppliers and low-cost, unreliable suppliers has a direct impact on both cost and service level. The more reliable and expensive suppliers selected, the higher the service level and the purchasing costs, and the lower penalties for delayed or unfulfilled customer orders. While the risk-neutral solution for business-as-usual conditions does not account for high impact disruptions and hence may imply poor results when such disruptions occur, the risk-averse solution that focuses on potential worst-case losses may not perform well under business-as-usual conditions. It is often most desirable to equitably consider these conflicting objectives, that is, the expected value and the expected worst-case value of cost or customer service level. The models presented

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in this chapter provide such a mean-risk fairness in decision-making to equitably optimize average and worst-case performance of a customer-driven supply chain.

The following time-indexed SMIP models are presented in this chapter:

RSPS_ECV(c) for selection of robust supply portfolio and scheduling of customer orders to equitably optimize expected cost and CVaR of cost;
RSPS_ECV(sl) for selection of robust supply portfolio and scheduling of customer orders to equitably optimize expected service level and CVaR of service level;
SPS_E(c) for risk-neutral selection of supply portfolio and scheduling of customer orders to minimize expected cost;
SPS_E(sl) for risk-neutral selection of supply portfolio and scheduling of customer orders to maximize expected service level;
SPS_CV(c) for risk-averse selection of supply portfolio and scheduling of customer orders to minimize CVaR of cost;
SPS_CV(sl) for risk-averse selection of supply portfolio and scheduling of customer orders to maximize CVaR of service level;
SPS_E(c,α), enhanced model **SPS_E(c)** for computing the maximum CVaR of cost, for a given confidence level α;
SPS_E(sl,α), enhanced model **SPS_E(sl)** for computing the minimum CVaR of service level, for a given confidence level α;
SPS_ECV(c) for mean-risk minimization of expected cost and CVaR of cost;
SPS_ECV(sl) for mean-risk maximization of expected service level and CVaR of service level.

Numerical examples and some computational results, in particular comparison of the mean-risk with the fair mean-risk approach, are provided in Sect. 8.4.

8.2 Problem Description

In a supply chain under consideration various types of products are assembled over a planning horizon by a single producer to meet customer demand, using the same critical part type that can be manufactured and provided by different suppliers. For detailed problem description see Sect. 7.3, and the notation used is introduced in Table 8.1.

The objective of the equitably efficient optimization of average and worst-case performance of a supply chain under disruption risks is to allocate the total demand for parts among a subset of selected suppliers and to schedule the customer orders for products over the planning horizon to equitably minimize expected cost and expected worst-case cost or equitably maximize expected service level and expected worst-case service level. The cost includes the cost of ordering and purchasing of parts plus penalty cost of delayed and unfulfilled customer orders due to the parts shortages,

Table 8.1 Notation: selection of robust supply portfolio and scheduling

	Indices
i	= supplier, $i \in I$
j	= customer order, $j \in J$
r	= region, $r \in R$
s	= disruption scenario, $s \in S$
t	= planning period, $t \in T$
	Input Parameters
a_j	= per unit requirement for parts of each product in customer order j
b_j	= size (number of products) of customer order j
A	$= \sum_{j \in J} a_j b_j$, total demand for parts
B	$= \sum_{j \in J} b_j$, total demand for products
c_j	= per unit capacity consumption of producer for customer order j
C_t	= capacity of producer in period t
d_j	= due date for customer order j
e_i	= fixed cost of ordering parts from supplier i
g_j	= per unit and per period penalty cost of delayed customer order j
h_j	= per unit penalty cost of unfulfilled customer order j
I^r	= subset of suppliers in region r
o_i	= per unit price of parts purchased from supplier i
p_i	= local disruption probability for supplier i
p^r	= regional disruption probability for all suppliers in region r
τ_i	= delivery lead time from supplier i

while the customer service level is a performance measure independent of any cost parameters, defined as the fraction of customer orders filled on or before their due dates (i.e., order fulfillment rate). The equitable solution means an equitably efficient risk-neutral and risk-averse solution. In this chapter such an equitably efficient solution will be called a robust solution. The robust solution is capable of equitably optimizing the performance of a supply chain in the average-case as well as in the worst-case. The robust solution (the supply portfolio and the schedule of customer orders) aims at achieving the normalized expected and expected worst-case values of the selected objective function as much close to each other as possible. To this end, the ordered weighted averaging aggregation of the expected value and conditional value-at-risk of the selected objective function will be applied. The resulting robust supply portfolio (the allocation of total demand for parts among the selected suppliers) is determined ahead of time as well as the equitably efficient schedule of customer orders for every potential disruption scenario.

8.3 Problem Formulation

In this section the two time-indexed SMIP models **RSPS_ECV(c)** and **RSPS_ECV (sl)** are proposed for selection of robust supply portfolio and customer order scheduling to equitably optimize average and worst-case performance of a supply chain in the presence of disruption risks. The models are based on the bi-objective optimization problem formulation (7.8). The objective of model **RSPS_ECV(c)** is to equitably minimize expected cost per product and expected worst-case cost per product and the objective of model **RSPS_ECV(sl)** is to equitably maximize expected service level and expected worst-case service level. The problem variables are introduced in Table 8.2.

Table 8.2 Variables: selection of robust supply portfolio and scheduling

	First stage variables
u_i	$= 1$, if supplier i is selected; otherwise $u_i = 0$ (supplier selection)
v_i	$\in [0, 1]$, the fraction of total demand for parts ordered from supplier i (supply portfolio)
	Auxiliary variables
λ_l	unrestricted auxiliary variable, $l = 1, 2$
δ_{kl}	≥ 0, upside deviation of outcome value $f_k, k = 1, 2$ from the value of $\lambda_l, l = 1, 2$
	Second stage variables
w^s_{jt}	$= 1$, if under disruption scenario s customer order j is scheduled for period t; otherwise $w^s_{jt} = 0$ (production scheduling)
	Auxiliary variables
VaR^c	Cost-at-Risk, the targeted cost such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is below VaR^c
VaR^{sl}	Service-at-Risk, the targeted service level such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is above VaR^{sl}
$\mathscr{C}_s$	≥ 0, the tail cost for scenario s, i.e., the amount by which costs in scenario s exceed VaR^c
$\mathscr{S}_s$	≥ 0, the tail service level for scenario s, i.e., the amount by which VaR^{sl} exceeds service level in scenario s
z_s	$= 1$, if for scenario s, cost per product is not less than VaR^c; otherwise $z_s = 0$
ζ_s	$= 1$, if for scenario s, service level is not less than VaR^{sl}; otherwise $\zeta_s = 0$

To control the risk of supply disruptions, the two percentile measures of risks will be applied: Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR). In the selection of supply portfolio and scheduling of customer orders under disruption risks, the decision maker controls the risk of high losses due to supply disruptions by choosing the confidence level α. The greater the confidence level α, the more risk aversive is the decision maker and the smaller percent of the highest cost (or the lowest service level) outcomes is focused on.

8.3.1 Equitable Minimization of Average and Worst-Case Costs

Let VaR^c be the targeted cost such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is below VaR^c, and let CVaR^c be the expected cost in the worst $100(1-\alpha)\%$ of the cases with the cost above VaR^c. Define $\mathscr{C}_s$ as the tail cost for scenario s, where tail cost is defined as the amount by which costs in scenario s exceed VaR^c. The risk-averse supply portfolio and the production schedule will be optimized by calculating VaR^c and minimizing CVaR^c simultaneously.

Let E^c be the minimized expected cost per product and $CVaR^c$, the minimized expected worst-case cost per product

$$E^c = (\sum_{i \in I} e_i u_i + \sum_{s \in S} P_s (\sum_{i \in I_s} Ao_i v_i$$
$$+ \sum_{j \in J} \sum_{t \in T: t > d_j} g_j b_j (t - d_j) w^s_{jt} + \sum_{j \in J} h_j b_j (1 - \sum_{t \in T} w^s_{jt})))/B \tag{8.1}$$

$$CVaR^c = VaR^c + (1-\alpha)^{-1} \sum_{s \in S} P_s \mathscr{C}_s. \tag{8.2}$$

In order to avoid dimensional inconsistency among various objectives, the values of the optimized objective functions are scaled into the interval [0,1]. Denote by $f_1^c = \frac{E^c - \underline{E}^c}{\overline{E}^c - \underline{E}^c}$, the normalized expected cost per product ($\underline{E}^c$, $\overline{E}^c$ are the minimum and the maximum values of E^c, respectively), and by $f_2^c = \frac{CVaR^c - \underline{CVaR}^c}{\overline{CVaR}^c - \underline{CVaR}^c}$, the normalized expected worst-case cost per product ($\underline{CVaR}^c$, $\overline{CVaR}^c$ are the minimum and the maximum values of $CVaR^c$ for a given confidence level α, respectively).

The normalized cost objective functions f_1^c and f_2^c are defined below

$$f_1^c = (\sum_{i \in I} e_i u_i + \sum_{s \in S} P_s (\sum_{i \in I_s} Ao_i v_i + \sum_{j \in J} \sum_{t \in T: t > d_j} g_j b_j (t - d_j) w^s_{jt}$$
$$+ \sum_{j \in J} h_j b_j (1 - \sum_{t \in T} w^s_{jt})))/B - \underline{E}^c)/(\overline{E}^c - \underline{E}^c) \tag{8.3}$$

$$f_2^c = (VaR^c + (1-\alpha)^{-1} \sum_{s \in S} P_s \mathscr{C}_s - \underline{CVaR}^c)/(\overline{CVaR}^c - \underline{CVaR}^c) \tag{8.4}$$

The SMIP model **RSPS_ECV(c)** for selection of robust supply portfolio and customer order scheduling to equitably minimize expected and expected worst-case cost per product is formulated below.

RSPS_ECV(c): *Selection of* **R***obust* **S***upply* **P***ortfolio and* **S***cheduling of customer orders to equitably optimize expected cost and CVaR of cost*

Minimize

$$\sum_{l=1}^{2}(l\lambda_l + \sum_{k=1}^{2}\delta_{kl}) \tag{8.5}$$

subject to (8.3), (8.4) and

$$\lambda_l + \delta_{kl} \geq f_k^c;\, k, l = 1, 2 \tag{8.6}$$

Demand allocation constraints:

– the total demand for parts must be fully allocated among the suppliers,

– demand for parts cannot be assigned to non-selected suppliers,

$$\sum_{i \in I} v_i = 1 \tag{8.7}$$

$$v_i \leq u_i;\ i \in I \tag{8.8}$$

Order-to-period assignment constraints:

– for each disruption scenario s, each customer order j is either scheduled during the planning horizon ($\sum_{t \in T} w_{jt}^s = 1$), or unscheduled and rejected ($\sum_{t \in T} w_{jt}^s = 0$),

– for each disruption scenario s and each planning period t, the cumulative demand for parts of all customer orders scheduled in periods 1 through t cannot exceed the cumulative deliveries of parts in periods 1 through $t-1$, from the non-disrupted suppliers $i \in I_s$,

– for each disruption scenario s, the total requirement for parts of scheduled customer orders is not greater than the total supplies from the non-disrupted suppliers $i \in I_s$,

$$\sum_{t \in T} w_{jt}^s \leq 1;\ j \in J, s \in S \tag{8.9}$$

$$\sum_{j \in J} \sum_{\tau \in T: \tau \leq t} a_j b_j w_{j\tau}^s \leq A \sum_{i \in I_s: \tau_i \leq t-1} v_i;\ t \in T, s \in S \tag{8.10}$$

$$\sum_{j \in J} \sum_{t \in T} a_j b_j w_{jt}^s \leq A \sum_{i \in I_s} v_i;\ s \in S \tag{8.11}$$

Producer capacity constraints:

– for any period t and each disruption scenario s, the total demand on capacity of all customer orders scheduled in period t must not exceed the producer capacity available in this period,

$$\sum_{j \in J} b_j c_j w_{jt}^s \le C_t; \; t \in T, s \in S \tag{8.12}$$

Risk constraints:

– the tail cost for scenario s is defined as the nonnegative amount by which cost in scenario s exceeds VaR^c,

$$\begin{aligned} \mathscr{C}_s \ge \sum_{i \in I} e_i u_i / B + \sum_{i \in I_s} A o_i v_i / B \\ + \sum_{j \in J} \sum_{t \in T : t > d_j} g_j b_j (t - d_j) w_{jt}^s / B + \sum_{j \in J} h_j b_j (1 - \sum_{t \in T} w_{jt}^s) / B \\ - VaR^c; \; s \in S \end{aligned} \tag{8.13}$$

Non-negativity and integrality conditions:

$$\delta_{kl} \ge 0; k, l = 1, 2 \tag{8.14}$$

$$u_i \in \{0, 1\}; \; i \in I \tag{8.15}$$

$$v_i \in [0, 1]; \; i \in I \tag{8.16}$$

$$w_{jt}^s \in \{0, 1\}; \; j \in J, t \in T, s \in S \tag{8.17}$$

$$\mathscr{C}_s \ge 0; \; s \in S. \tag{8.18}$$

8.3.2 Equitable Maximization of Average and Worst-Case Service Level

In the next model, VaR^{sl} is the targeted customer service level such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is above VaR^{sl}, while CVaR^{sl} is the expected service level in the worst $100(1 - \alpha)\%$ of the cases with the service level below VaR^{sl}. Let $\mathscr{S}_s$ be the tail service level for scenario s, where tail service level is defined as the amount by which VaR^{sl} exceeds service level in scenario s. The aim of the following SMIP model is to maximize average customer service level as well as to reduce the outcomes below the service level represented by VaR^{sl}, and by this to equitably optimize average and worst-case customer service level. The solutions equitably maximize the expected and the expected worst-case fraction of customer orders filled on or before their due dates.

Let E^{sl} be the maximized expected service level and $CVaR^{sl}$, the maximized expected worst-case service level

$$E^{sl} = \sum_{j\in J} \sum_{t\in T:t\le d_j} \sum_{s\in S} P_s w^s_{jt}/\overline{J} \tag{8.19}$$

$$CVaR^{sl} = VaR^{sl} - (1-\alpha)^{-1} \sum_{s\in S} P_s \mathscr{S}_s. \tag{8.20}$$

Denote by $f_1^{sl} = \frac{\overline{E}^{sl} - E^{sl}}{\overline{E}^{sl} - \underline{E}^{sl}}$, the normalized expected service level ($\underline{E}^{sl}$, $\overline{E}^{sl}$ are the minimum and the maximum values of E^{sl}, respectively), and by $f_2^{sl} = \frac{\overline{CVaR}^{sl} - CVaR^{sl}}{\overline{CVaR}^{sl} - \underline{CVaR}^{sl}}$, the normalized expected worst-case service level ($\underline{CVaR}^{sl}$, $\overline{CVaR}^{sl}$ are the minimum and the maximum values of $CVaR^{sl}$ for a given confidence level α, respectively).

The normalized service level objective functions f_1^{sl} and f_2^{sl} are defined below

$$f_1^{sl} = \frac{\overline{E}^{sl} - \sum_{j\in J} \sum_{t\in T:t\le d_j} \sum_{s\in S} P_s w^s_{jt}/\overline{J}}{(\overline{E}^{sl} - \underline{E}^{sl})} \tag{8.21}$$

$$f_2^{sl} = \frac{\overline{CVaR}^{sl} - (VaR^{sl} - (1-\alpha)^{-1} \sum_{s\in S} P_s \mathscr{S}_s)}{\overline{CVaR}^{sl} - \underline{CVaR}^{sl}} \tag{8.22}$$

The SMIP model **RSPS_ECV(sl)** for selection of robust supply portfolio and customer order scheduling to equitably maximize expected and expected worst-case service level is formulated below.

RSPS_ECV(sl): *Selection of robust supply portfolio and scheduling of customer orders to equitably optimize expected service level and CVaR of service level*

Minimize (8.5)
subject to (8.6)–(8.12), (8.14)–(8.17), (8.21), (8.22) and
Risk constraints:
– the tail service level for scenario s is defined as the nonnegative amount by which VaR^{sl} exceeds service level in scenario s,

$$\mathscr{S}_s \ge VaR^{sl} - \sum_{j\in J} \sum_{t\in T:t\le d_j} w^s_{jt}/\overline{J};\; s\in S \tag{8.23}$$

$$\mathscr{S}_s \ge 0. \tag{8.24}$$

Note that the objective functions (8.19) and (8.20) (or normalized objective functions, (8.21) and (8.22)) do not directly account for any cost components. As a result, the optimal solution to **RSPS_ECV(sl)** that aims at equitable optimization of average and worst-case service level depends mainly on the distribution of disruption probabilities among the suppliers. However, the optimization of service level implicitly reduces the penalty costs of delayed and unfulfilled customer orders, that represent an important part of the total cost structure.

8.3.3 Minimum and Maximum Values of the Objective Functions

In this subsection the minimum and maximum values for all objective functions are calculated to determine the normalized values of all objective functions, f_1^c (8.3), f_2^c (8.4), f_1^{sl} (8.21) and f_2^{sl} (8.22), that is, the values of the optimized objective functions scaled into the interval [0,1]. Note that the cost and the service level objectives are in conflict. Similarly, the expected and the expected worst-case values of a given objective function are also in conflict.

Minimum and Maximum Values of Expected Values

The minimum and maximum values of expected cost $\underline{E}^c$, $\overline{E}^c$, and expected service level, $\underline{E}^{sl}$, $\overline{E}^{sl}$, are obtained by solving the following stochastic mixed integer programs:

SPS_E(c): *Risk-neutral selection of supply portfolio and scheduling of customer orders to minimize expected cost*

Minimize E^c, (8.1),
subject to (8.7)–(8.12), (8.15)–(8.17).

SPS_E(sl): *Risk-neutral selection of supply portfolio and scheduling of customer orders to maximize expected service level*

Maximize E^{sl}, (8.19)
subject to (8.7)–(8.12), (8.15)–(8.17).

In problem **SPS_E(c)**, E^c is the minimized objective function, while E^{sl} is not considered. In problem **SPS_E(sl)**, E^{sl} is the maximized objective function, while E^c is not considered. Thus, by solving problem **SPS_E(c)**, the minimum value $\underline{E}^c$ of E^c and the minimum value $\underline{E}^{sl}$ of E^{sl} are determined. Similarly, by solving problem **SPS_E(sl)**, the maximum value $\overline{E}^{sl}$ of E^{sl} and the maximum value $\overline{E}^c$ of E^c are determined.

Minimum and Maximum Values of CVaR

The minimum value of expected worst-case cost, $\underline{CVaR}^c$, and the maximum value of expected worst-case service level, $\overline{CVaR}^{sl}$, for a given confidence level α, are obtained as the optimal solution of problem **SPS_CV(c)** and **SPS_CV(sl)**, respectively. The SMIP models **SPS_CV(c)** and **SPS_CV(sl)** are shown below.

SPS_CV(c): *Risk-averse selection of supply portfolio and scheduling of customer orders to minimize CVaR of cost*

Minimize (8.2)
subject to (8.7)–(8.13), (8.15)–(8.18).

SPS_CV(sl): *Risk-averse selection of supply portfolio and scheduling of customer orders to maximize CVaR of service level*

Maximize (8.20)
subject to (8.7)–(8.12), (8.15)–(8.17), (8.23), (8.24).

Since the expected and the expected worst-case values of a given objective function are in conflict, the maximum value of expected worst-case cost $\overline{CVaR}^c$, and the minimum value of expected worst-case service level, $\underline{CVaR}^{sl}$ for a given confidence level α, are associated with the optimal solutions of SMIP problem **SPS_E(c)** and **SPS_E(sl)**, respectively. In order to compute the associated values of CVaR, the stochastic mixed integer programs are enhanced as shown below.

SPS_E(c,α)

Minimize (8.1),
subject to (8.2), (8.7)–(8.13), (8.15)–(8.18) and

$$z_s \geq (\sum_{i \in I} e_i u_i / B + \sum_{i \in I_s} Ao_i v_i / B + \sum_{j \in J} \sum_{t \in T: t > d_j} g_j b_j (t - d_j) w^s_{jt} / B + \sum_{j \in J} h_j b_j (1 - \sum_{t \in T} w^s_{jt}) / B - VaR^c) / C_{max}; \; s \in S \quad (8.25)$$

$$z_s \leq 1 + (\sum_{i \in I} e_i u_i / B + \sum_{i \in I_s} Ao_i v_i / B$$

$$+\sum_{j\in J}\sum_{t\in T:t>d_j} g_j b_j (t-d_j) w^s_{jt}/B + \sum_{j\in J} h_j b_j (1-\sum_{t\in T} w^s_{jt})/B$$

$$-VaR^c)/C_{max};\ s\in S \quad (8.26)$$

$$\sum_{s\in S} P_s z_s \le 1-\alpha \quad (8.27)$$

$$z_s \in \{0,1\};\ s\in S, \quad (8.28)$$

where (8.25) and (8.26) determine scenarios s with the cost per product not less than VaR^c, and (8.27) ensures that the total probability of all such scenarios is not greater than $1-\alpha$.
C_{max} is an upper bound on cost per product, and the additional binary variable is defined as follows: $z_s = 1$, if for scenario s, cost per product,

$$\sum_{i\in I} e_i u_i/B + \sum_{i\in I_s} Ao_i v_i/B + \sum_{j\in J}\sum_{t\in T:t>d_j} g_j b_j (t-d_j) w^s_{jt}/B + \sum_{j\in J} h_j b_j (1-\sum_{t\in T} w^s_{jt})/B$$

is not less than VaR^c; otherwise $z_s = 0$.

SPS_E(sl,α)

Maximize (8.19)
subject to (8.7)–(8.12), (8.15)–(8.17), (8.20), (8.23), (8.24) and

$$\zeta_s \ge \sum_{j\in J}\sum_{t\in T:t\le d_j} w^s_{jt}/\overline{J} - VaR^{sl};\ s\in S \quad (8.29)$$

$$\zeta_s \le 1+\sum_{j\in J}\sum_{t\in T:t\le d_j} w^s_{jt}/\overline{J} - VaR^{sl};\ s\in S \quad (8.30)$$

$$\sum_{s\in S} P_s \zeta_s \ge \alpha \quad (8.31)$$

$$\zeta_s \in \{0,1\};\ s\in S, \quad (8.32)$$

where (8.29) and (8.30) determine scenarios s with the customer service level not less than VaR^{sl}, and (8.31) ensures that the total probability of all such scenarios is not less than the confidence level α. The additional binary variable is defined as follows: $\zeta_s = 1$, if for scenario s, customer service level, $\sum_{j\in J}\sum_{t\in T:t\le d_j} w^s_{jt}/\overline{J}$, is not less than VaR^{sl}; otherwise $\zeta_s = 0$.

For a given confidence level α, the maximum value of expected worst-case cost, $\overline{CVaR}^{c}$, is obtained by solving problem **SPS_CV(c)**, and the minimum value of expected worst-case service level, $\underline{CVaR}^{sl}$ by solving problem **SPS_CV(sl)**.

8.4 Computational Examples

In this section some computational examples are presented to illustrate possible applications of the proposed SMIP approach for the robust supplier selection, order quantity allocation and customer orders scheduling in a supply chain under disruption risks. The following parameters have been used for the example problems:

- $\overline{I} = 9$, $\overline{J} = 25$, $\overline{K} = 3$, $\overline{R} = 3$, $\overline{T} = 10$, and $\overline{S} = 2^{\overline{I}} = 512$;
- $K = \{1, 2, 3\}$, and $J_1 = \{1, \ldots, 10\}$, $J_2 = \{11, \ldots, 20\}$, $J_3 = \{21, \ldots, 25\}$;
- $R = \{1, 2, 3\}$, and $I^1 = \{1, 2, 3\}$, $I^2 = \{4, 5, 6\}$, $I^3 = \{7, 8, 9\}$;
- $a_j \in \{1, 2, 3\}, \forall j = 1, \ldots, 25$;
- $b_j \in \{500, 1000, \ldots, 5000\}\ \forall j = 1, \ldots, 25$;
- $c_j \in \{1, 2, 3\}, \forall j = 1, \ldots, 25$;
- $C_t = C = 38000, \forall t = 1, \ldots, 10$;
- $e = (8, 7, 10, 13, 14, 11, 19, 17, 19) \times 1000$;
- $g_j = 1, h_j = 26\ \forall j = 1, \ldots, 25$;
- $o = (13, 12, 12, 8, 6, 6, 2, 5, 4)$;
- τ_i, was 2, 3 and 4 time periods, respectively for suppliers $i \in I^1, i \in I^2$ and $i \in I^3$;
- $d_j \in \{1 + \min_{i \in I}(\tau_i), \ldots, \overline{T}\}\ \forall j = 1, \ldots, 25$;
- p_i, the local disruption probability was uniformly distributed over [0.005,0.01], [0.01,0.05] and [0.05;0.10], respectively for suppliers $i \in I^1$, $i \in I^2$ and $i \in I^3$;
- p^r, the regional disruption probability was 0.001, 0.005 and 0.01, respectively for region $r = 1$, $r = 2$ and $r = 3$;
- α, the confidence level, was equal to 0.50, 0.75, 0.90, 0.95 or 0.99.

The computational experiments were performed for the same replication of the above input data set. All potential disruption scenarios were considered and to calculate the corresponding disruption probabilities, formulae (7.10) and (7.11) were applied. For all test examples, the resulting total demand for parts and products is $A = 132500$ and $B = 66000$, respectively. The unit price per part o_i and the disruption probability π_i, (7.9), of each supplier $i \in I$ are shown in Fig. 6.1.

The solution results for the risk-neutral, risk-averse and robust decision-making are presented in Tables 8.3, 8.4 and 8.5, respectively. The minimum and maximum values of all considered objective functions are presented in Table 8.6. In addition to the optimal absolute and normalized solution values for the primary objective functions and the allocation of demand among the selected suppliers, Table 8.5 presents also the expected values of the associated objective function, i.e., the expected service level for model **RSPS_ECV(c)** and the expected cost per product for model **RSPS_ECV(sl)**.

Table 8.3 Risk-neutral solutions

Model **SPS_E(c)**: Var.=100468, Bin.=100459, Cons.=21341, Nonz.=765902 [a]	
Expected Cost	7.66
Suppliers Selected(% of total demand)	7(100)
Model **SPS_E(sl)**: Var.=100468, Bin.=100459, Cons.=21341, Nonz.=765902 [a]	
Expected Service Level(%)	99.62
Suppliers Selected(% of total demand)	1(48), 2(31), 3(21)

[a] Var.=number of variables, Bin.=number of binary variables,
Cons.=number of constraints, Nonz.= number of nonzero coefficients

For the risk-neutral minimization of cost, Table 8.3 indicates that the cheapest supplier $i = 7$ is selected only, while for the risk-neutral maximization of service level, the total demand is allocated among the three most reliable and most expensive suppliers $i = 1, 2, 3$.

For the risk-averse minimization of cost, Table 8.4 indicates that for the lowest confidence levels $\alpha = 0.5, 0.75$, the cheapest supplier $i = 7$ is selected only, while for a higher α, more suppliers are selected, including the reliable but expensive suppliers: $i = 2$ for $\alpha = 0.95$ and $i = 2, 3$ for $\alpha = 0.99$.

Table 8.4 Risk-averse solutions

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **SPS_CV(c)**: Var.=100981, Bin.=100459, Cons.=21853, Nonz.=874288 [a]					
CVaR^c	10.60	16.47	23.75	26.66	30.90
Suppliers Selected(% of total demand)	7(100)	7(100)	4(23)	2(30)	2(22)
			5(21)	4(16)	3(22)
			6(21)	5(19)	5(15)
			7(17)	6(19)	6(16)
			9(18)	7(16)	7(12)
					9(13)
Model **SPS_CV(sl)**: Var.=100981, Bin.=100459, Cons.=21853, Nonz.=822422 [a]					
$\text{CVaR}^{sl}(\%)$	99.25	98.49	96.23	92.46	85.73
Suppliers Selected(% of total demand)	1(48)	1(48)	1(48)	1(48)	1(21)
	2(31)	2(31)	2(31)	2(31)	2(21)
	3(21)	3(21)	3(21)	3(21)	3(14)
					4(7)
					5(7)
					6(8)
					7(8)
					8(7)
					9(7)

[a] Var.=number of variables, Bin.=number of binary variables,
Cons.=number of constraints, Nonz.= number of nonzero coefficients

For the risk-averse maximization of service level, where the supplier selection is independent of any cost parameters and the solution mainly depends on the distribution of disruption probabilities, Table 8.4 demonstrates that the most reliable (and most expensive) suppliers $i = 1, 2, 3$ are selected for all confidence levels, except for the highest confidence level, $\alpha = 0.99$, for which all nine suppliers are selected.

Table 8.5 indicates that for minimization of cost and the low confidence levels, $\alpha = 0.5, 0.75$, the robust supply portfolio is identical with the risk-neutral portfolio. Similarly, for maximization of service level and the confidence levels, $\alpha = 0.5, 0.75, 0.9, 0.95$, the robust supply portfolio is identical with the risk-neutral portfolio, while for $\alpha = 0.99$, the robust supply portfolio is similar to the risk-averse portfolio. For the service level objective function, the robust solutions with a perfect equity were found for all confidence levels, except for the highest $\alpha = 0.99$, which indicates that the obtained robust solutions are also the lexicographic minimax opti-

Table 8.5 Robust solutions

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **RSPS_ECV(c)**: Var.=100989, Bin.=100459, Cons.=21859, Nonz.=975283 $^{(a)}$					
E^c	7.66	7.66	10.67	11.75	11.04
CVaR^c	10.60	16.47	28.55	29.05	35.44
VaR^c	4.73	4.73	18.51	23.70	30.83
Normalized E^c	0	0	0.167	0.227	0.188
Normalized CVaR^c	0	0	0.167	0.093	0.210
Suppliers Selected(% of total demand)	7(100)	7(100)	6(26)	6(37)	6(54)
			7(50)	7(30)	7(46)
			9(24)	9(33)	
E^{sl} $^{(b)}$	67.46	67.56	85.37	84.79	87.18
Model **RSPS_ECV(sl)**: Var.=100989, Bin.=100459, Cons.=21859, Nonz.=878445 $^{(a)}$					
E^{sl} $^{(b)}$	99.62	99.62	99.62	99.62	98.91
CVaR^{sl}(%)	99.25	98.49	96.23	92.46	85.61
VaR^{sl}(%)	100	100	100	100	92.00
Normalized E^{sl}	0	0	0	0	0.022
Normalized CVaR^{sl}	0	0	0	0	0.002
Suppliers Selected(% of total demand)	1(48)	1(48)	1(48)	1(48)	1(22)
	2(31)	2(31)	2(31)	2(31)	2(22)
	3(21)	3(21)	3(21)	3(21)	3(19)
					4(11)
					6(10)
					7(9)
					9(7)
E^c	25.64	25.64	25.64	25.64	21.84

$^{(a)}$ Var.=number of variables, Bin.=number of binary variables,
Cons.=number of constraints, Nonz.= number of nonzero coefficients.
$^{(b)}$ $(\sum_{s \in S} P_s \sum_{j \in J} \sum_{t \in T: t \le d_j} w^s_{jt} / \overline{J})100\%$

Table 8.6 Minimum and maximum values of objective functions

Expected values					
$\underline{E}^c$	7.66				
$\overline{E}^c$	25.64				
(associated with Maximum Expected Service Level)					
$\overline{E}^{sl}(\%)$	99.62				
$\underline{E}^{sl}(\%)$	67.60				
(associated with Minimum Expected Cost)					
Expected worst-case values, CVaR					
Confidence level α	0.50	0.75	0.90	0.95	0.99
$\underline{CVaR}^c$	10.60	16.47	23.75	26.66	30.91
$\overline{CVaR}^c$	52.48 for all confidence levels				
(associated with Minimum Expected Cost)					
$\overline{CVaR}^{sl}(\%)$	99.25	98.49	96.23	92.46	85.73
$\underline{CVaR}^{sl}(\%)$	67.86	67.72	71.26	74.43	39.58
(associated with Maximum Expected Service Level)					

mal solutions (see, Theorem 2 in Sect. 7.2). As a result, the obtained expected service level and the expected worst-case service level are equally very close to their best available values. However, the associated expected cost is much higher than its best value (recall that the service level-based solution is independent of any cost parameters). For the cost-based objective function, the solutions with perfect equity were found only for $\alpha = 0.5, 0.75, 0.9$.

For the cost-based objective function, Fig. 8.1 compares the distribution of cost per product for the risk-neutral, risk-averse and robust decision-making for the two confidence levels $\alpha = 0.75$ and $\alpha = 0.99$. The probability mass functions are concentrated in a few points, which is typical for the scenario-based optimization under uncertainty, where the probability measure is concentrated in finitely many points. For $\alpha = 0.75$, the risk-averse and robust solution are identical with the risk-neutral solution, and so are the corresponding probability mass functions. A large probability atom 0.06148 is concentrated at the highest cost of 52.49. For $\alpha = 0.99$, different solutions are obtained for different types of the decision-making. For the risk-averse solution, the total probability measure of costs between 24 and 29 is 0.9977, while the probability of costs greater than 40 is 0.0023. For the robust solution, the largest probability measure of 0.906 is concentrated at the lowest cost of 8.95 and the probability of the highest cost, 52.65, is only 0.002.

For the service level-based objective function, Fig. 8.2 compares the distribution of customer service level for the risk-neutral, risk-averse and robust decision-making, for the two confidence levels $\alpha = 0.75$ and $\alpha = 0.99$. The solution results for the three types of decision making are identical for $\alpha = 0.75$ and very similar for $\alpha = 0.99$, with the most reliable and most expensive suppliers $i = 1, 2, 3$ predominating the

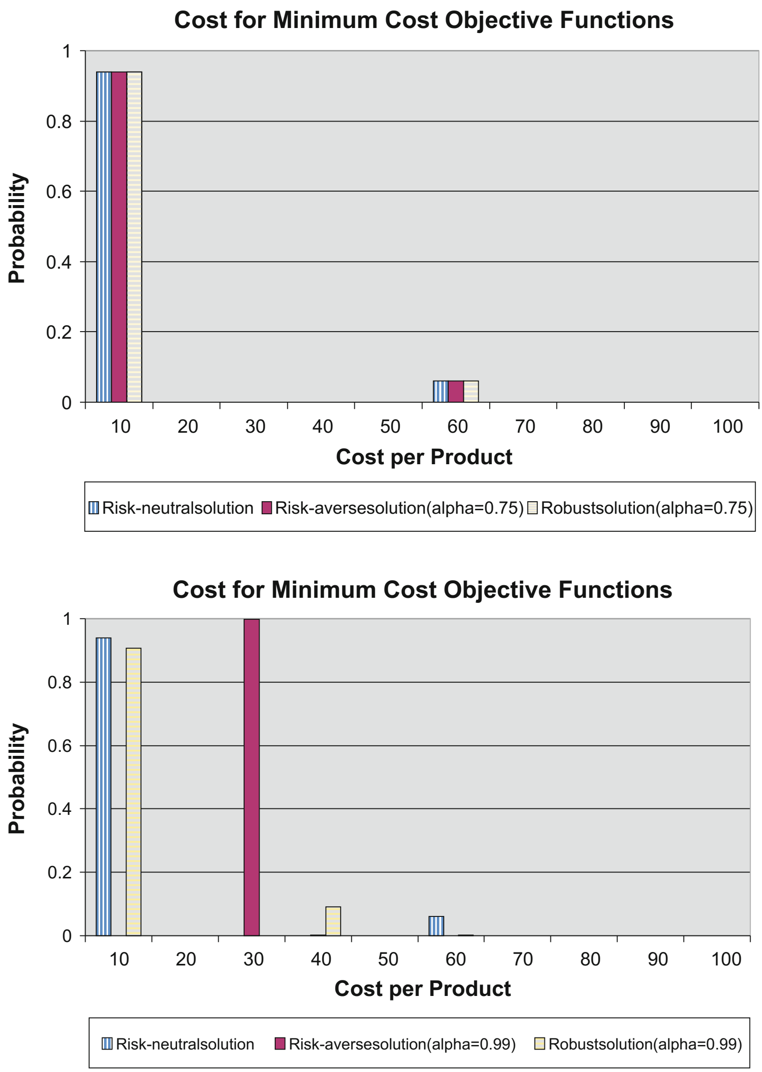

Fig. 8.1 Distribution of cost per product for $\alpha = 0.75$ and $\alpha = 0.99$

supply portfolio. As a result, the probability mass functions presented in Fig. 8.2 are identical or very similar, respectively.

As an illustrative example, Fig. 8.3 presents the demand for products, $\sum_{j\in J:d_j=t} b_j$, $t \in T$, and the expected production schedules, $\sum_{s\in S} P_s \sum_{j\in J} b_j w^s_{jt}$, $t \in T$ for the optimal cost- and service level-based, risk-neutral, risk-averse and robust solutions with the confidence level $\alpha = 0.99$. The total customer demand is met with a small fraction of the expected rejected demand (from 0.0038 for the risk-neutral, service level-based solution up to 0.2407 for the risk-averse, cost-based

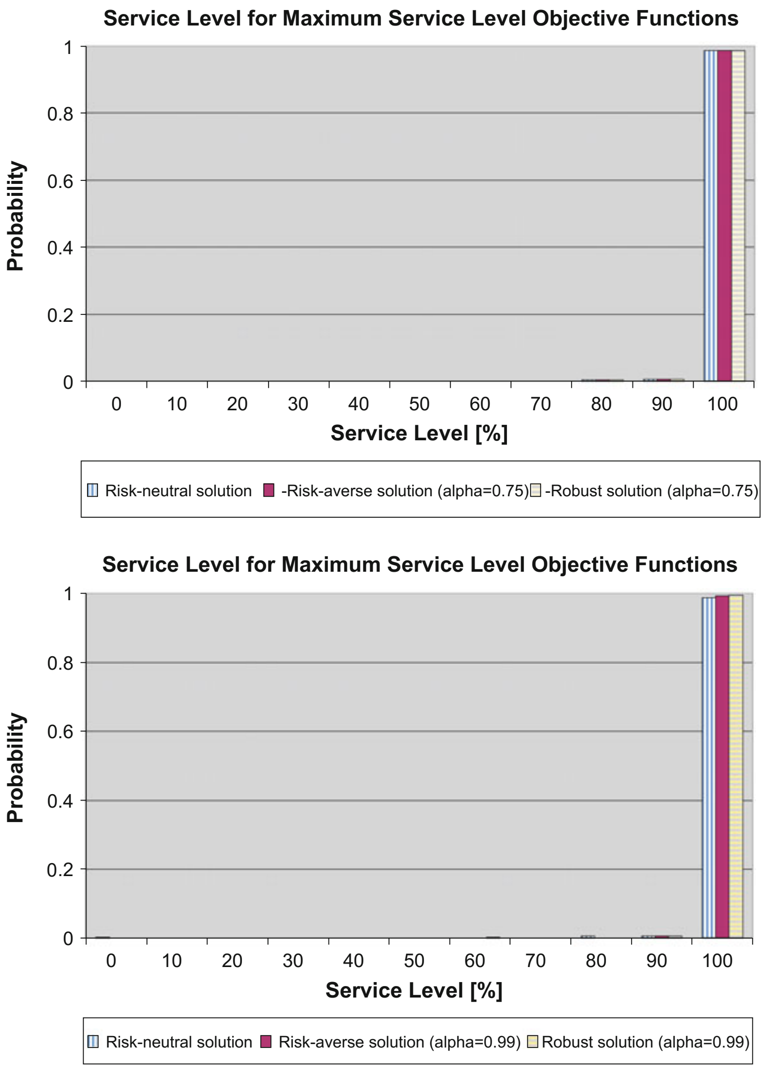

Fig. 8.2 Distribution of service level for $\alpha = 0.75$ and $\alpha = 0.99$

solution). In general, the service level-based solution, when no cost components are included in the objective function, better meets the customer demand, with a smaller fraction of unfulfilled demand. For the service level objective, the expected production approximately follows the demand pattern. In addition, the expected production schedules for the risk-neutral and the robust decision-making are very close to each other, which indicates that in the average-case, the robust solution is nearly as good as the risk-neutral solution.

Finally, Figs. 8.4 and 8.5 present the expected worst-case production schedules for the optimal risk-averse and the robust solutions with the confidence level $\alpha = 0.99$,

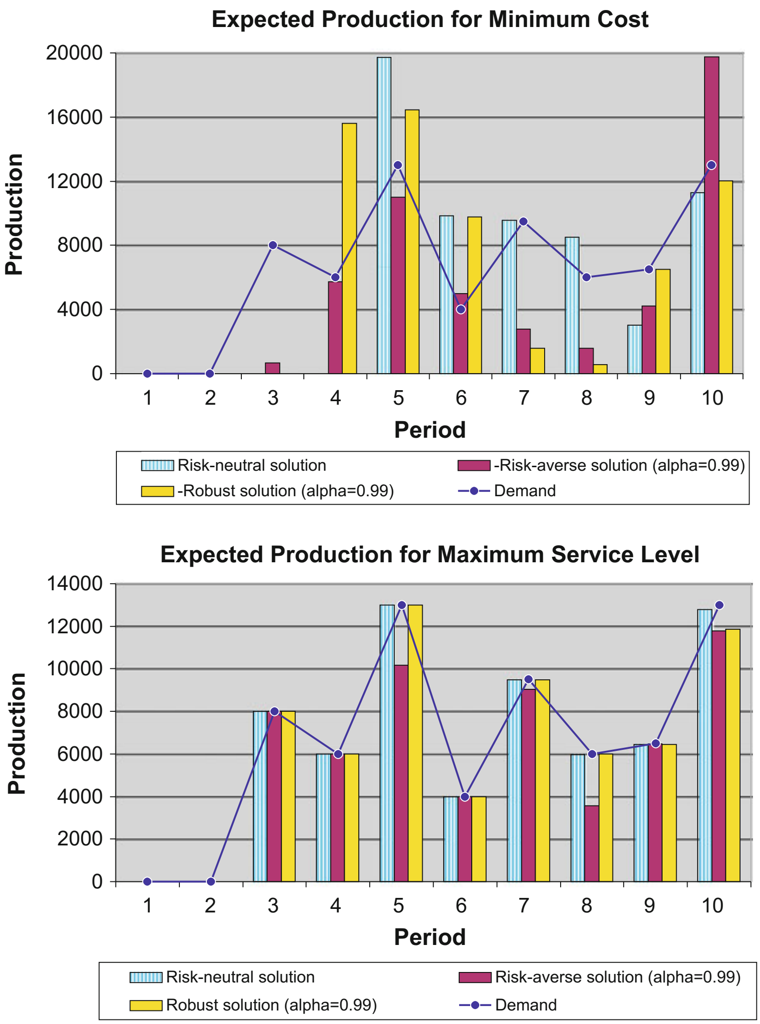

Fig. 8.3 Expected production schedules for $\alpha = 0.99$

for maximization of service level (Fig. 8.4),

$$\frac{\sum_{s \in S: \mathscr{S}_s > 0} (P_s \sum_{j \in J} b_j w_{jt}^s)}{\sum_{s \in S: \mathscr{S}_s > 0} P_s}, t \in T,$$

and for minimization of cost (Fig. 8.5),

$$\frac{\sum_{s \in S: \mathscr{C}_s > 0} (P_s \sum_{j \in J} b_j w_{jt}^s)}{\sum_{s \in S: \mathscr{C}_s > 0} P_s}, t \in T,$$

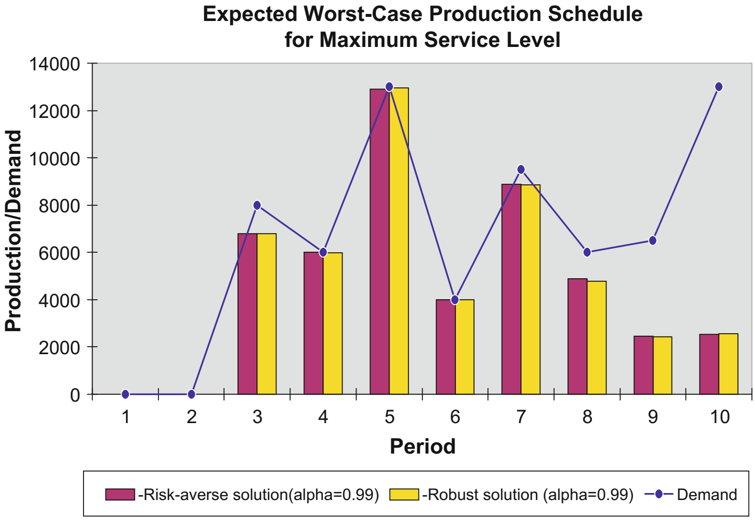

Fig. 8.4 Expected worst-case production schedules for maximum service level, $\alpha = 0.99$

respectively.

The risk-averse and the robust expected worst-case schedules for maximization of service level are very close to each other (cf. similar supply portfolios in Tables 8.4 and 8.5), which indicates that in the worst-case the robust solution is nearly as good as the risk-averse solution. The expected worst-case fractions of fulfilled customer demand for products are similar and close to 75%.

For the minimization of cost, however, the expected worst-case schedules for the risk-averse and the robust solutions are different (cf. different supply portfolios in Tables 8.4 and 8.5). For the risk-averse solution, above 55% of the customer demand is fulfilled in the expected worst-case. However, the expected worst-case production for the robust solution is virtually negligible (see, Fig. 8.5(a)). The corresponding expected worst-case fraction of fulfilled customer demand for products is nearly equal to zero. Such a result may be due to the too low unit penalty cost, $(h_j = 2\lceil a_j \max_{i \in I}(o_i)\rceil,\ j \in J)$, for unfulfilled customer demand, with respect to the purchasing cost of required parts, i.e., the unit penalty cost is approximately twice as large as the maximum unit price of required parts. The resulting robust supply portfolio is based on the low cost, unreliable suppliers $i = 6, 7$, similarly to the risk-neutral portfolio. In contrast, the pure risk-averse portfolio contains the most reliable suppliers $i = 2, 3$.

In order to emphasize the impact of higher penalty costs, the computational experiments were repeated with a double unit penalty cost, $h_j = 4\lceil a_j \max_{i \in I}(o_i)\rceil,\ j \in J$, i.e., with the unit penalty cost for unfulfilled customer demand, approximately four times as large as the maximum unit price of required parts. The solution results for the minimization of cost with a double penalty are presented in Table 8.7. The table shows that for the low confidence levels, $\alpha = 0.5, 0.75$, the risk-averse and

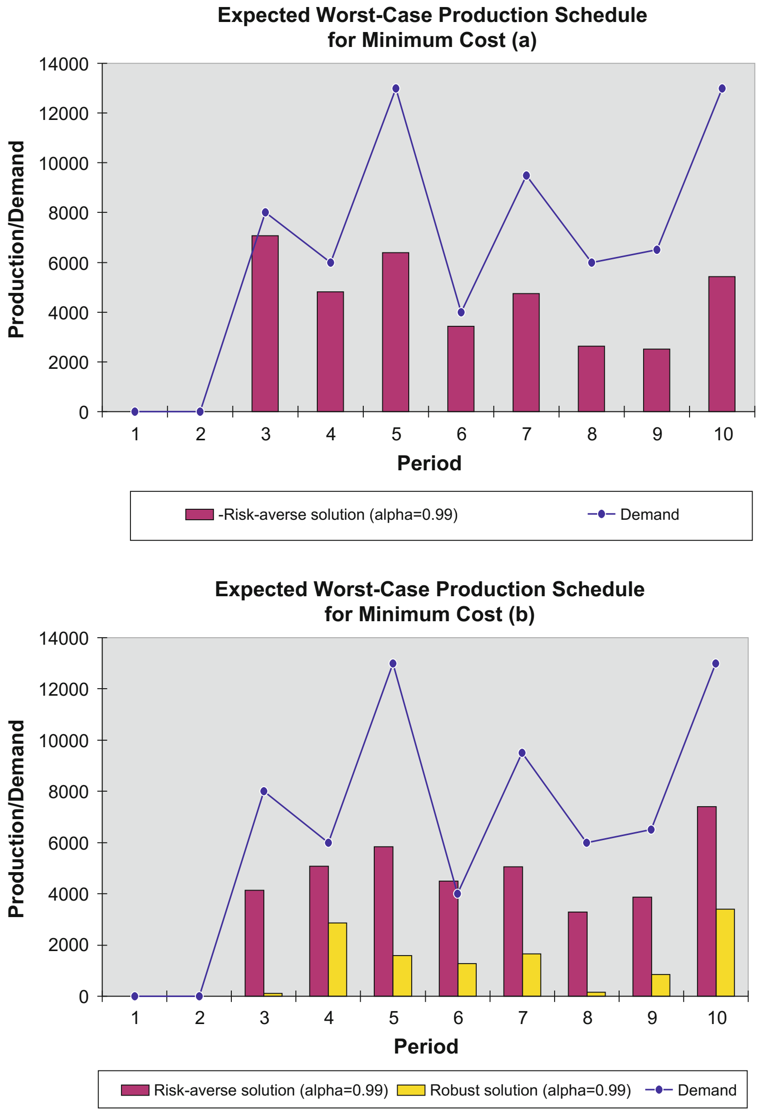

Fig. 8.5 Expected worst-case production schedules for minimum cost, $\alpha = 0.99$: **a** $h_j = 2\lceil a_j \max_{i \in I}(o_i)\rceil$, **b** $h_j = 4\lceil a_j \max_{i \in I}(o_i)\rceil$

the robust supply portfolios are identical with the risk-neutral portfolio that assigns total demand for parts to a single, cheapest supplier $i = 7$. For a higher confidence level, the risk-averse portfolio contains a single, reliable supplier ($i = 2$, for $\alpha = 0.9$, and $i = 1$, for $\alpha = 0.95$), whereas the robust portfolio contains the reliable supplier, $i = 2$, only for the highest confidence level, $\alpha = 0.99$. This simultaneously leads to the highest expected service level, 94.03%. Table 8.7 shows that for most confidence levels, the robust solution with a perfect equity was obtained, which indicates that the obtained robust solutions are also the lexicographic minimax optimal solutions (see, Theorem 2 in Sect. 7.2).

The expected worst-case schedules for a double unit penalty cost, $h_j = 4\lceil a_j \max_{i \in I}(o_i) \rceil,\ j \in J$, is shown in Fig. 8.5(b). The obtained expected worst-case production for the robust solution is no longer negligible, however the production is still smaller than that for a pure risk-averse solution. Now, the robust supply portfolio has been enforced with a reliable supplier $i = 2$, while the pure risk-averse portfolio contains all nine suppliers (see, Table 8.7). For the robust solution only 18% of the total customer demand is fulfilled in the expected worst-case, while for the risk-averse solution, over 60% of the customer demand is fulfilled. On the other hand, the expected percentage of fulfilled customer demand is over 95% for the robust solution and only 75% for the risk-averse solution. Thus, the risk-averse solution outperforms the robust solution in the worst-case, and vice-versa the robust solution outperforms the risk-averse solution in the average-case. The above results demonstrate that the robust solution for the cost-based objective functions is strongly dependent on the cost parameters.

The equitably efficient solutions obtained using models **RSPS_ECV(c)** and **RSPS_ECV(sl)** have been compared with the nondominated solutions minimizing the weighted-sum of the normalized objective functions, respectively f_1^c, f_2^c and f_1^{sl}, f_2^{sl}, with equal weights of each objective. In the literature this type of trade-off model (with a varying trade-off parameter) is known as the mean-risk model (e.g., Ogryczak and Ruszczynski 2002) with weighted-sum objective consisting of the expected value and the CVaR as a risk measure. The weighted-sum programs (with equal weights of each objective function), **SPS_ECV(c)**, (8.33), and **SPS_ECV(sl)**, (8.34), are presented below and the solution results are shown in Table 8.8.

SPS_ECV(c) (8.33)

Minimize $f_1^c + f_2^c$
subject to (8.3), (8.4), (8.7)–(8.13), (8.15)–(8.18).

SPS_ECV(sl) (8.34)

Maximize $f_1^{sl} + f_2^{sl}$
subject to (8.7)–(8.12), (8.15)–(8.17), (8.21)–(8.24).

Table 8.8 indicates that the mean-risk solutions are close to the robust solutions presented in Tables 8.5 and 8.7, while the perfect equity is very rarely achieved. In particular, for the service level objective function the robust and the mean-risk solu-

Table 8.7 Solution results for a double unit penalty cost, $h_j = 4\lceil a_j \max_{i\in I}(o_i)\rceil$

Risk-neutral solution: model **SPS_E(c)**					
E^c			10.87		
Suppliers Selected(% of total demand)			7(100)		
Confidence level α	0.50	0.75	0.90	0.95	0.99
Risk-averse solutions: model **SPS_CV(c)**					
CVaR^c	17.02	24.99	30.35	35.82	46.97
Suppliers Selected(% of total demand)	7(100)	7(100)	2(100)	1(100)	1(19)
					2(19)
					3(20)
					4(6)
					5(7)
					6(9)
					7(9)
					8(5)
					9(6)
Minimum and maximum values of objective functions.					
$\underline{E}^c$			10.87		
$\overline{E}^c$			26.03		
$\underline{CVaR}^c$	17.02	24.99	30.35	35.82	46.97
$\overline{CVaR}^c$			104.68 for all confidence levels		
Robust solutions: model **RSPS_ECV(c)**					
E^c	10.87	10.87	14.35	14.41	14.92
CVaR^c	17.02	29.31	47.38	51.56	62.38
VaR^c	4.73	4.73	37.85	46.17	52.88
Normalized E^c	0	0	0.23	0.23	0.27
Normalized CVaR^c	0	0.05	0.23	0.23	0.27
Suppliers Selected(% of total demand)	7(100)	7(100)	5(30)	5(25)	2(13)
			6(31)	6(39)	6(45)
			7(39)	7(36)	7(42)
$E^{sl\ (b)}$	67.38	67.46	86.55	87.12	94.03

$^{(b)}$ $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T:t\le d_j} w^s_{jt}/\overline{J})100\%$

tions are nearly identical for most confidence levels. However, for the cost objective function, the robust solutions based on the ordered weighted averaging aggregation approach, in most cases are different from the nondominated solutions obtained using the mean-risk approach.

The robust solution that equitably optimizes both the average and the worst-case performance of a supply chain has been compared with the risk-neutral solution that focuses on the average performance only, and with the risk-averse solution that focuses on the worst-case performance only. The computational experiments demonstrate that *the robust solution may be outperformed by an optimal risk-neutral*

Table 8.8 Mean-risk solutions

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **SPS_ECV(c)** (8.33)					
E^c	7.67	7.67	12.74	12.74	11.04
CVaR^c	10.61	16.49	24.17	27.36	35.44
VaR^c	4.73	4.73	20.97	21.00	30.83
Normalized E^c	0.0005	0.0004	0.2827	0.2828	0.1880
Normalized CVaR^c	0.0002	0.0005	0.0146	0.0269	0.2100
Suppliers Selected(% of total demand)	7(100)	7(100)	5(27)	5(27)	6(54)
			6(27)	6(27)	7(46)
			7(22)	7(22)	
			9(24)	9(24)	
E^{sl}(%)	67.01	67.80	84.32	85.15	87.18
Model **SPS_ECV(c)** (8.33) for a double unit penalty cost, $h_j = 4\lceil a_j \max_{i \in I}(o_i)\rceil$.					
E^c	10.87	10.87	15.53	14.73	13.37
CVaR^c	17.02	29.31	41.13	48.70	66.94
VaR^c	4.73	4.73	33.74	41.98	56.76
Normalized E^c	0	0	0.307	0.255	0.165
Normalized CVaR^c	0	0.05	0.145	0.187	0.346
Suppliers Selected(% of total demand)	7(100)	7(100)	5(25)	5(34)	
			6(26)	6(34)	6(52)
			7(24)	7(32)	7(48)
			9(25)		
E^{sl}(%)	67.05	67.50	85.06	86.59	87.13
Model **SPS_ECV(sl)** (8.34)					
E^{sl}(%)	99.62	99.62	99.62	99.62	99.07
CVaR^{sl}(%)	99.25	98.49	96.20	92.40	84.92
VaR^{sl}(%)	100	100	100	100	92.00
Normalized E^{sl}	0	0	0	0	0.0172
Normalized CVaR^{sl}	0	0	0.001	0.003	0.0176
Suppliers Selected(% of total demand)	1(48)	1(48)	1(48)	1(48)	1(21)
	2(31)	2(31)	2(31)	2(31)	2(21)
	3(21)	3(21)	3(21)	3(21)	3(18)
					4(10)
					5(10)
					6(10)
					7(10)
E^c	25.64	25.64	26.03	25.64	21.44

solution in the average-case and by an optimal risk-averse solution in the worst-case. However, the robust solution outperforms the risk-averse solution in the average-case and the risk-neutral solution in the worst-case.

The following basic insights can be derived from the computational experiments.

- *The robust solutions are frequently also the lexicographic minimax optimal solutions as well as the Pareto-optimal solutions.*
  For both cost-based and service-based objective functions, the solutions with a perfect equity are frequently found (see, Theorems 2 and 3 in Sect. 7.2).
- *In average-case the robust solution may perform as well as the risk-neutral solution.*
  For both cost-based and service-based objective functions, the robust solution for a low confidence level is identical with the risk-neutral solution, i.e., the supply portfolio is based on low-cost, unreliable suppliers and on high-cost, reliable suppliers, respectively. Moreover, the expected production schedules for the robust and for the risk-neutral solutions are very close to each other.
- *In worst-case the robust solution for the maximum service level objective is nearly as good as the risk-averse solution.*
  For the maximum service level objective and a high confidence level, the robust solution is very close to the risk-averse solution. Moreover, the expected worst-case schedules for the robust and for the risk-averse solutions are very close to each other.
- *For the minimum cost objective and a high confidence level, the robust solution is strongly dependent on the cost parameters.*
  For example, if unit penalty cost for unfulfilled demand for products is comparable with purchasing cost of required parts, the robust solution for a high confidence level may more resemble the risk-neutral solution, while for a high penalty cost the robust solution may be closer to the risk-averse solution.
- *The mean-risk solutions are rarely the lexicographic minimax optimal solutions.*
  The mean-risk solutions that minimize the sum of the normalized expected value and the normalized CVaR are close to the robust solutions, while the perfect equity is very rarely achieved. For the service level objective function the robust and the mean-risk solutions are nearly identical for most confidence levels. However, for the cost objective function, the robust solution and the mean-risk solutions are different in most cases.
- *The service level and the cost objective functions are in conflict.*
  The best service level-based solutions may perform poorly with respect to the cost-based metrics, and vice versa.

The computational experiments were performed using the AMPL programming language and the CPLEX 12.5 solver on a MacBookPro laptop with Intel Core i7 processor running at 2.8 GHz and with 16 GB RAM. The solver was capable of finding proven optimal solutions for all examples with CPU time ranging from several seconds for the risk-neutral solutions to several hours for the robust or mean-risk solutions and cost-based objectives.

8.5 Notes

In the literature on supply chain risk management, robustness implies that the supply chain is strong enough to be unaffected by disturbances, while retaining its original structure, e.g., Klibi et al. (2010), Gabrel et al. (2014). In contrast to resiliency (see, Chap. 4) that implies that the supply chain needs to adapt (reconfigure) its structure to survive and grow in the face of change and uncertainty (e.g., Fiksel 2006; Klibi and Martel 2012).

The major contribution of this chapter is that it proposes a simple approach for the robust decision-making associated with supplies of parts and deliveries of finished products in a customer-driven supply chain under disruption risks. Here, the robustness is defined as the mean-risk fairness and refers to an equitably efficient performance of a supply chain in the average-case as well as in the worst-case. The fair mean-risk decision-making aims at equalizing the distance to optimality both under business-as-usual and under worst-case conditions, which reflects the decision makers common requirement to maintain an equally good performance of a supply chain under varying operating conditions. The robust decision-making equitably focuses on the two objective functions: the expected value and the expected worst-case value (i.e., Conditional Value-at-Risk) of the selected criterion, cost or service level.

The material presented in this chapter is based on results presented in Sawik (2014c, 2016c), where SMIP models were developed. The models were the enhancements of formulations proposed in Sawik (2013c, 2014a, 2014b, see also Chap. 5) for a single-objective decision-making. In contrast, the stochastic optimization problems considered in this chapter are formulated as bi-objective SMIP models with the two conflicting objective functions: expected cost and expected worst-case cost (Conditional Cost-at-Risk) or expected service level and expected worst-case service level (Conditional Service-at-Risk). In order to obtain an equitably efficient solution to the combinatorial stochastic optimization problem, the ordered weighted averaging aggregation (Yager 1988) of the expected and the expected worst-case value of the selected objective function has been applied. The equitable optimization of the supply chain network under disruption risks and the associated coordinated scheduling of the disrupted material flows are rarely considered. However, another type of a trade-off model is well-known in the literature: the mean-risk model (e.g., Ogryczak and Ruszczynski 2002). The mean-risk model is formulated as the optimization of weighted-sum objective consisting of the expected value and the CVaR as a risk measure. The mean-risk approach aims at balancing the expected value with the risk tolerance, however, the mean-risk model is not designed to achieve an equitably efficient solution.

Problems

8.1 Modify the SMIP models presented in this chapter for multiple part types with subsets of part types required for each customer order and subsets of suppliers available for each part type.

8.2 How would you determine an upper bound, C_{max}, on the cost per product for model **SPS_E(c,α)**?

8.3 Mixed mean-risk decision-making
(a) Modify models **SPS_ECV(sl)** and **SPS_ECV(c)** to optimize expected cost and CVaR of service level and optimize expected service level and CVaR of cost, respectively.
(b) How should the values of the optimized objective functions be scaled into the interval [0,1] to avoid dimensional inconsistency among the two objectives and how should the trade-off parameter be selected?
(c) How would you interpret the mixed mean-risk solutions?

8.4 Explain why the robust supply portfolios for the cost-based objective is strongly dependent on the cost parameters and which parameters are the most influential?

8.5 Explain why the expected worst-case production for the robust solution in Fig. 8.5(a) is nearly negligible?

Part IV
Selection of Primary and Recovery Portfolios and Scheduling

Chapter 9
Selection of Primary and Recovery Supply Portfolios and Scheduling

9.1 Introduction

In this chapter the portfolio approach presented in previous chapters for the selection of primary suppliers and order quantity allocation to mitigate the impact of disruption risks is enhanced also for the recovery process, i.e., for the selection of both primary and recovery suppliers and order quantity allocation to mitigate the impact of disruption risks and optimize the recovery process. Unlike most of reported research on the supply chain risk management which focuses on the risk mitigation decisions taken prior to a disruption, this chapter combines decisions made before, during and after the disruption. The two decision-making approaches will be considered: integrated approach with the perfect information about the future disruption scenarios, and hierarchical approach with no such information available ahead of time. In the integrated approach, which accounts for all potential disruption scenarios, the primary supply portfolio that will hedge against all scenarios is determined along with the recovery supply portfolio and production schedule of finished products for each scenario, to minimize expected cost or CVaR of cost and maximize expected service level or CVaR of service level over all scenarios. In the hierarchical approach first the primary supply portfolio is selected, and then, when a primary supplier is hit by a disruption, the recovery supply portfolio is selected to optimize the process of recovery from the disruption.

The following time-indexed SMIP and MIP models are presented in this chapter:

Support_E for risk-neutral selection of primary and recovery supply portfolios and production scheduling;
SupportMP_E model **Support_E** for multiple part types and product types;
PSupport for selection of primary supply portfolio and production scheduling under deterministic conditions;

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RSupport(s) for selection of recovery supply portfolio and production scheduling, for predetermined primary supply portfolio and the realized disruption scenario;

RSupport_E model **Support_E** for predetermined primary supply portfolio;

Support_CV(c) for risk-averse selection of primary and recovery supply portfolios and production scheduling to minimize CVaR of cost;

Support_CV(sl) for risk-averse selection of primary and recovery supply portfolios and production scheduling to maximize CVaR of service level.

Numerical examples and computational results are reported in Sects. 9.5.1 and 9.5.2, respectively for risk-neutral and risk-averse decision-making.

In the next chapter, the portfolio approach will be further enhanced to simultaneously select supply and demand portfolios, when a disruption impacts both a primary supplier of parts and the buyer's firm primary assembly plant. Then, in addition to determining the primary and recovery supply portfolios, the firm may also choose to move production to alternate (recovery) plants along with transshipment of parts from the impacted primary assembly plant to the recovery plants. The resulting allocation of unfulfilled demand for products among recovery assembly plants will determine a recovery, demand or capacity portfolio.

9.2 Problem Description

Consider a supply chain in which a single producer of one product type, assembles products to meet customer demand, using a critical part type that can be manufactured and provided by several suppliers.

Let $I = \{1, \ldots, \overline{I}\}$ be the set of $\overline{I}$ suppliers, $T = \{1, \ldots, \overline{T}\}$ the set of $\overline{T}$ planning periods, and let d_t be the demand for products in period $t \in T$ (for notation, see Table 9.1).

The orders for parts are assumed to be placed at the beginning of the planning horizon, and under normal conditions the parts ordered from supplier i are delivered in period τ_i, where τ_i represents total of manufacturing lead time and transportation time. If production does not meet the demand, the producer is charged with penalty cost for unfulfilled demand for products.

The suppliers of parts are located in $\overline{R}$ geographic regions, subject to potential regional disasters that may result in complete shutdown of all suppliers in the same region simultaneously. In addition to correlated regional disruptions, each supplier $i \in I$ is subject to random independent local disruptions of different levels, $l \in L_i = \{0, \ldots, \overline{L_i}\}$, where the disruption level refers to the fraction of an order that can be delivered (supplier fulfillment rate). Level $l = 0$ represents complete shutdown of a supplier, i.e., no order delivery, while level $l = \overline{L_i}$ represents normal conditions with no disruption, i.e., full order delivery. The fraction of an order that can be delivered by supplier i under disruption level l is described by the associated supplier fulfillment rate, γ_{il}

Table 9.1 Notation: selection of primary and recovery supply portfolios and scheduling

Indices	
i	= supplier, $i \in I$
l	= disruption level, $l \in L_i$, $i \in I$
r	= region, $r \in R$
s	= disruption scenario, $s \in S$
t	= planning period, $t \in T$
Input Parameters	
c	= per period capacity of producer
C	= total available capacity of producer
d_t	= demand for products in period t
D	= total demand for parts/products
e_i	= fixed ordering cost of creating contracts and maintaining relationships with supplier i
g	= per unit penalty cost of unfulfilled demand for products
o_i	= per unit price of parts purchased from supplier i
p_{il}	= probability of disruption level l for supplier i
p^r	= regional disruption probability for region r
t_s	= start time period of disruptive event s
γ_{il}	= fraction of an order delivered by supplier i under disruption level l (supplier fulfillment rate)
τ_i	= delivery lead time from supplier i
ρ_{is}	= firm's portion of supplier i cost-to-recover from disruption under scenario s
θ_{is}	= time-to-recover of supplier i from disruption under scenario s
$CTR(i, l)$	= firm's portion of cost-to-recover for supplier i hit by disruption at level l
$TTR(i, l)$	= time-to-recover for supplier i hit by disruption at level l

$$\gamma_{il} \begin{cases} = 0 & \text{if } l = 0 \\ \in (0, 1) & \text{if } l = 1, \dots, \overline{L_i} - 1 \\ = 1 & \text{if } l = \overline{L_i}. \end{cases} \tag{9.1}$$

Denote by $S = \{1, \dots, \overline{S}\}$ the index set of all disruption scenarios, and by P_s the probability of disruption scenario $s \in S$. Each scenario $s \in S$ can be represented by an integer-valued vector $\lambda_s = \{\lambda_{1s}, \dots, \lambda_{\overline{I}s}\}$, where $\lambda_{is} \in L_i$ is the disruption level of an order delivery from supplier $i \in I$ under scenario $s \in S$. When all potential disruption scenarios are considered, then $\overline{S} = \prod_{i \in I}(\overline{L_i} + 1)$.

For each scenario $s \in S$, the supplies from every supplier can be disrupted either by a local or a regional disaster event. Denote by $I_s \subset I$ the subset of non-shutdown suppliers, who can deliver parts under scenario s. The probability P_s for disruption scenario $s \in S$ with the subset I_s of non-shutdown suppliers is (cf. Sect. 1.3)

$$P_s = \prod_{r \in R} P_s^r.$$

P_s^r is the probability of realizing of disruption scenario s for suppliers in I^r (cf. Sect. 1.4)

$$P_s^r = \begin{cases} (1-p^r)\prod_{i\in I^r}(p_{i,\lambda_{is}}) & \text{if } I^r \bigcap I_s \neq \emptyset \\ p^r + (1-p^r)\prod_{i\in I^r} p_{i0} & \text{if } I^r \bigcap I_s = \emptyset, \end{cases}$$

where $p_{i,\lambda_{is}}$ is the probability of occurrence the disruption at level $l = \lambda_{is}$ of an order delivery from supplier i under scenario s.

A disruptive event under scenario $s \in S$ is assumed to occur in period t_s. When supplier i is hit by disruption at level l, its recovery process to normal conditions starts in period $t_s + 1$ and takes $TTR(i, l)$ time periods (Time-To-Recover), so that the supplier i recovers to its full pre-disruption capacity in period $t = t_s + TTR(i, l) + 1$. The cost of recovery process can be shared between the supplier of parts and the firm (producer of products). Let $CTR(i, l)$ be the firm's portion of Cost-To-Recover. Note that $CTR(i, \overline{L}) = TTR(i, \overline{L}) = 0$, i.e., no recovery is required for a non-disrupted supplier.

For each supplier i, denote by θ_{is} and ρ_{is}, respectively time-to-recover and firm's portion of cost-to recover from disruption under scenario s

$$\theta_{is} = TTR(i, l);\ \ i \in I, s \in S : l = \lambda_{is} \tag{9.2}$$

$$\rho_{is} = CTR(i, l);\ \ i \in I, s \in S : l = \lambda_{is}. \tag{9.3}$$

The following assumptions are made to formulate the problem.

- Each supplier has sufficient capacity to meet total demand for parts.
- A single recovery mode is considered for each supplier and each disruption level.
- Disruption start times are constant parameters.
- Time-to recover and the associated cost-to-recover are constant parameters that represent recovery of a disrupted supplier to its full capacity.
- The buyer firm may participate in supplier's cost-to-recover.
- Disruption to primary supplier under scenario $s \in S$ occurs in period t_s and recovery process starts in period $t_s + 1$, so that the disrupted supplier $i \in I$ returns to its full capacity in period $t = t_s + \theta_{is} + 1$.
- Multiple disruptions, one after the other in a series, during the recovery process are not considered.
- A recovery supplier can be a disrupted primary supplier after its recovery to full capacity or a new supplier.
- Transition time required for switching to recovery supplier different from the primary suppliers is negligible.
- Each product requires one unit of a critical part type.
- A penalty cost is charged for the demand for products unfulfilled by the end of the planning horizon.

Many of the above assumptions can be easily relaxed. For example, the assumption that each supplier has sufficient capacity to meet total demand for parts, which allows for a single sourcing, often met in practice. Some of them, however, would require development of more advanced models to be relaxed, e.g., random disruption start times, durations and suppliers cost-to-recover.

Notice that no recovery supply portfolio needs to be selected if a disruptive event occurs after the latest delivery lead time from primary suppliers, i.e., when $t_s > \max_{i \in I}(\tau_i)$.

In the sequel two decision-making approaches will be considered: an integrated approach with the perfect information about the future disruption scenarios, and a hierarchical approach with no such information available ahead of time. In the integrated approach, which accounts for all potential disruption scenarios, decisions are made prior to a disruption. The primary supply portfolio that will hedge against all scenarios is determined along with the recovery supply portfolio and production schedule for each scenario. The problem objective is to minimize expected cost or maximize expected service level over all scenarios, for risk-neutral models, and minimize CVaR of cost or maximize CVaR of service level for risk-averse models. In the hierarchical approach, first prior to a disruption, the primary supply portfolio is determined to minimize cost or maximize service level, and then, when a primary supplier is hit by a disruption, the recovery supply portfolio is selected to optimize the process of recovery from the disruption.

9.3 Models for Risk-Neutral Decision-Making

9.3.1 Integrated Selection of Primary and Recovery Supply Portfolios

In this subsection a time-indexed SMIP model **Support_E** is presented for the integrated risk-neutral decision making in the presence of supply chain disruption risks.

The objective of the integrated decision making is to jointly:

- determine the primary supply portfolio, i.e., to allocate the total demand for parts among a subset of selected primary suppliers,
- determine the recovery supply portfolio for each disruption scenario, i.e., to allocate the unfulfilled demand for parts among a subset of selected recovery suppliers, when primary suppliers are disrupted,
- schedule production over the planning horizon for each disruption scenario,

to optimize the expected cost. The cost includes ordering and purchasing cost of parts, recovery cost of disrupted suppliers and penalty cost for unfulfilled demand for products.

The variables used to formulate the SMIP model are defined in Table 9.2.

The primary supply portfolio, $(v_1, \ldots, v_{\bar{I}})$, where $\sum_{i \in I} v_i = 1$ and $0 \le v_i \le 1, i \in I$, is the initial allocation of total demand for parts among primary suppliers.

Table 9.2 Variables: selection of primary and recovery supply portfolios and scheduling

	First stage variables
u_i	$= 1$, if supplier i is selected as a primary supplier; otherwise $u_i = 0$ (primary supplier selection)
v_i	$\in$ [0, 1], the fraction of total demand for parts ordered from primary supplier i (primary supply portfolio)
	Second stage variables
U_i^s	$= 1$, if supplier i is selected as a recovery supplier under scenario s; otherwise $U_i^s = 0$ (recovery supplier selection)
V_i^s	$\in$ [0, 1], the fraction of total demand for parts ordered from recovery supplier i under scenario s (recovery supply portfolio)
x_t^s	≥ 0, production in period t under scenario s (production scheduling)
	Auxiliary variables
q_i^s	$= 1$, if $u_i = U_i^s = 1$; otherwise $q_i^s = 0$ (elimination of double fixed ordering costs)
VaR^c	Cost-at-Risk, the targeted cost such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is below VaR^c
VaR^{sl}	Service-at-Risk, the targeted service level such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is above VaR^{sl}
$\mathscr{C}_s$	≥ 0, the tail cost for scenario s, i.e., the amount by which costs in scenario s exceed VaR^c
$\mathscr{S}_s$	≥ 0, the tail service level for scenario s, i.e., the amount by which VaR^{sl} exceeds service level in scenario s

The recovery supply portfolio for each disruption scenario s, $(V_1^s, \ldots, V_I^s)$, where $\sum_{i \in I}(\gamma_{i,\lambda_{is}} v_i + V_i^s) = 1$ and $0 \leq V_i^s \leq 1, i \in I$, is the allocation among recovery suppliers of unfulfilled demand for parts caused by supply disruptions under scenario s.

Notice that the actual quantity ordered from each supplier i can be determined by multiplying total demand for parts, $(D = \sum_{t \in T} d_t)$, by fractional variable v_i or V_i^s, i.e., by the fraction of total demand allotted to supplier i.

In addition, the following auxiliary binary variable, q_i^s, is introduced in model **Support_E**: $q_i^s = 1$, if $u_i = U_i^s = 1$; otherwise $q_i^s = 0$.

This variable eliminates double charging with fixed ordering cost e_i of each supplier i, who is selected both as primary and recovery supplier.

Let E^c be the expected cost per product to be minimized

$$E^c = \sum_{s \in S} P_s(\sum_{i \in I} e_i(u_i + U_i^s - q_i^s)/D$$
$$+ \sum_{i \in I} \rho_{is} U_i^s/D + \sum_{i \in I} o_i(\gamma_{i,\lambda_{is}} v_i + V_i^s)) + g(1 - E^{sl}), \quad (9.4)$$

where λ_{is} is disruption level of supplier i under scenario s, and $\gamma_{i,\lambda_{is}}$ is the corresponding fulfillment rate, i.e., the fraction of an order delivered by supplier i under disruption scenario s.

$$E^{sl} = \sum_{s \in S} \sum_{t \in T} P_s x_t^s / D \tag{9.5}$$

is the expected service level, i.e., the expected fraction of the total fulfilled demand for products (expected demand fulfillment rate).

The expected cost per product, E^c, (9.4), constitutes of expected ordering cost per product,

$\sum_{s \in S} P_s \sum_{i \in I} e_i(u_i + U_i^s - q_i^s)/D = \sum_{i \in I} e_i u_i/D + \sum_{s \in S} P_s \sum_{i \in I} e_i (U_i^s - q_i^s)/D$,

firm's expected portion of suppliers cost-to-recover from disruptions, per product, $\sum_{s \in S} P_s \sum_{i \in I} \rho_{is} U_i^s/D$,

expected purchasing cost per product for delivered parts,

$\sum_{s \in S} P_s \sum_{i \in I} o_i(\gamma_{i,\lambda_{is}} v_i + V_i^s) = \sum_{i \in I} o_i \Gamma_i v_i + \sum_{s \in S} P_s \sum_{i \in I} o_i V_i^s$,

and expected penalty per product for the unfulfilled demand for products,

$g(1 - \sum_{s \in S} \sum_{t \in T} P_s x_t^s/D) = g(1 - E^{sl})$,

where Γ_i is the expected fulfillment rate of supplier i

$$\Gamma_i = \sum_{s \in S} P_s \gamma_{i,\lambda_{is}}; \; i \in I.$$

Support_E: *Selection of primary and recovery* **Sup***ply* **port***folios and scheduling of production*

Minimize (9.4)

subject to

Primary supply portfolio selection constraints

- the total demand for parts must be fully allocated among the selected primary suppliers,
- demand for parts cannot be assigned to non-selected primary suppliers,

$$\sum_{i \in I} v_i = 1 \tag{9.6}$$

$$v_i \le u_i; \; i \in I \tag{9.7}$$

Recovery supply portfolio selection constraints

- the unfulfilled demand for parts caused by supply disruptions under scenario s must be fully allocated among the selected recovery suppliers,
- the unfulfilled demand for parts caused by supply disruptions under scenario s cannot be assigned to non-selected recovery suppliers,
- each supplier selected to both primary and recovery portfolio is charged exactly once with fixed ordering cost in the objective function (9.4),

$$\sum_{i \in I} (\gamma_{i,\lambda_{is}} v_i + V_i^s) = 1; \; s \in S \tag{9.8}$$

$$V_i^s \le U_i^s; \;\; i \in I, s \in S \tag{9.9}$$

$$q_i^s \le (u_i + U_i^s)/2; \;\; i \in I, s \in S \tag{9.10}$$

Supply-production coordinating constraints

- for each disruption scenario s and each period t, the cumulative demand for parts of production scheduled in periods 1 through t cannot exceed the initial inventory of parts and the cumulative deliveries by period $t-1$ (delivery in period $\tau_i \le t-1$ from each primary supplier i and in period $t_s + \theta_{is} + \tau_i \le t-1$ from each recovery supplier i),

$$\sum_{t' \in T: t' \le t} x_{t'}^s / D \le$$

$$V_0 + \sum_{i \in I: \tau_i \le t-1} \gamma_{i,\lambda_{is}} v_i + \sum_{i \in I: t_s + \theta_{is} + \tau_i \le t-1} V_i^s; \;\; t \in T, s \in S, \tag{9.11}$$

where DV_0 is the initial inventory of parts

Production capacity constraints

- for each disruption scenario s and each period t, production cannot exceed the producer capacity,

$$x_t^s \le c; \;\; t \in T, s \in S \tag{9.12}$$

Non-negativity and integrality conditions

$$u_i \in \{0, 1\}; \;\; i \in I \tag{9.13}$$

$$v_i \in [0, 1]; \;\; i \in I \tag{9.14}$$

$$U_i^s \in \{0, 1\}; \;\; i \in I, s \in S \tag{9.15}$$

$$V_i^s \in [0, 1]; \;\; i \in I, s \in S \tag{9.16}$$

$$q_i^s \in \{0, 1\}; \;\; i \in I, s \in S \tag{9.17}$$

$$x_t^s \ge 0; \;\; t \in T, s \in S. \tag{9.18}$$

A simple upper bound on the expected service level (9.5) is derived below.

Proposition 9.1

$$E^{sl} \le \min\{1, C/D\}, \tag{9.19}$$

where $C = \min\{DV_0, c\tau_{min}\} + c(\overline{T} - \tau_{min})$, *is the total available capacity of producer,* $D = \sum_{t \in T} d_t$, *is the total demand for products, and* $\tau_{min} = \min_{i \in I}(\tau_i)$ *is the minimum delivery lead time from suppliers.*

Proof Supply-production coordinating constraints (9.11) and production capacity constraints (9.12) imply that

$$\sum_{t \in T: t \le \tau_{min}} x_t^s \le \min\{DV_0, c\tau_{min}\}; \;\; s \in S$$

$$\sum_{t \in T: t > \tau_{min}} x_t^s \le c(\overline{T} - \tau_{min}); \;\; s \in S.$$

Thus

$$E^{sl} = \sum_{s \in S} P_s \sum_{t \in T} x_t^s / D \le \sum_{s \in S} P_s C/D = C/D.$$

Since E^{sl} cannot be greater than 1, its upper bound is 1, if $C/D > 1$.

Notice that worst-case disruption scenarios can be identified by the lowest service level, $\underline{SL}$. When there is no initial inventory of parts (i.e., $V_0 = 0$), $\underline{SL}$ can be calculated as below.

$$\underline{SL} = (\overline{T} - \max_{s \in S}\{t_s + \min_{i \in I}\{\theta_{is} + \tau_i\}\})c/D. \tag{9.20}$$

Model **Support_E** is a deterministic equivalent mixed integer program of a two-stage stochastic mixed integer program with recourse. The primary portfolio selection variables, $u_i, \; v_i$, are referred to as first-stage decisions, and the recovery portfolio selection variables, $U_i^s, \; V_i^s$ and production scheduling variables, x_t^s, are referred to as recourse or second-stage decisions (cf. Table 9.2). Unlike the first-stage decisions, the latter variables are dependent on disruption scenario $s \in S$. Model **Support_E** illustrates the wait-and-see approach. Basically, this approach is based on the perfect information about the future. In contrast to the two-stage stochastic program with recourse, in the wait-and-see approach, both the first stage and the second stage decisions are made simultaneously only when disruption scenario is known. Model **Support_E** is capable of simultaneously determining both stage variables to minimize expected cost over all disruption scenarios. When this problem is solved a recommendation is obtained for selection of primary supply portfolio $(u_i, \; v_i)$ that will hedge against a variety of disruption scenarios in which fulfillment rates of certain suppliers are not sufficient to satisfy demand for parts. The recovery supply portfolio selection and production scheduling ($U_i^s, \; V_i^s$ and x_t^s) are decisions that will be implemented in the future, when scenario $s \in S$ is finally realized.

Even though the recovery portfolio selection, $U_i^s, \; V_i^s$, and production scheduling, x_t^s, decisions are associated with each disruption scenario, they can be viewed as recourse decisions during disruption: each scenario $s \in S$ is associated with time period of disruption occurrence, t_s, as well as disruption duration, θ_{is}, (i.e., time-to-recover from disruption of each supplier i). Moreover, a recovery supplier selection decision is interrelated with production scheduling during supply disruption.

9.3.2 Multiple Part Types and Product Types

In this subsection an enhancement of model **Support_E** is described for multiple types of parts and products. Let H and K be, respectively, the set of part types and set of product types, and denote by $a_{hk},\ h \in H, k \in K$ the number of parts type h required to produce one unit of product type k.

If we denote by D_k the total demand for products type k, then $D = \sum_{k\in K} D_k$ is the total demand for all products and $A_h = \sum_{k\in K} a_{hk}D_k$ is the total demand for parts type h.

Now, the portfolio decision variables $v_i, V_i^s,\ i \in I, s \in S$ are replaced by v_{ih}, $V_{ih}^s;\ i \in I, h \in H, s \in S$, defined as fractions of total demand A_h for parts type h, ordered from primary supplier i, recovery supplier i under disruption scenario s, respectively. In addition, the production scheduling variable $x_t^s,\ t \in T, s \in S$ is replaced by $x_{kt}^s,\ k \in K, t \in T, s \in S$ - the production of product type k in period t under disruption scenario s.

Model **SupportMP_E** for multiple part and product types is presented below.

SupportMP

Minimize

$$E^c = \sum_{s\in S} P_s(\sum_{i\in I} e_i(u_i + U_i^s - q_i^s)$$

$$+\sum_{i\in I} \rho_{is}U_i^s + \sum_{i\in I}\sum_{h\in H} o_{ih}A_h(\gamma_{i,\lambda_{is}}v_{ih} + V_{ih}^s)$$

$$+\sum_{k\in K} g_k(D_k - \sum_{t\in T} x_{kt}^s))/D,$$

where o_{ih} is unit purchasing price of part type h from supplier i, and g_k is unit penalty cost of unfulfilled demand for product type k,

subject to

$$\sum_{i\in I} v_{ih} = 1;\ h \in H$$

$$v_{ih} \le u_i;\ i \in I, h \in H$$

$$\sum_{i\in I}(\gamma_{i,\lambda_{is}}v_{ih} + V_{ih}^s) = 1;\ h \in H, s \in S$$

$$V_{ih}^s \le U_i^s;\ i \in I, h \in H, s \in S$$

$$q_i^s \le (u_i + U_i^s)/2;\ i \in I, s \in S$$

$$\sum_{k\in K}\sum_{t'\in T:t'\le t} a_{hk}x_{kt'}^s/A_h \le V_{h0} + \sum_{i\in I:\tau_i\le t-1} \gamma_{i,\lambda_{is}}v_{ih}$$

$$+ \sum_{i \in I: t_s + \theta_{is} + \tau_i \le t-1} V_{ih}^s; \;\; h \in H, t \in T, s \in S$$

$$\sum_{k \in K} x_{kt}^s \le c; \;\; t \in T, s \in S$$

$$u_i \in \{0, 1\}; \;\; i \in I$$

$$v_{ih} \in [0, 1]; \;\; i \in I, h \in H$$

$$U_i^s \in \{0, 1\}; \;\; i \in I, s \in S$$

$$V_{ih}^s \in [0, 1]; \;\; i \in I, h \in H, s \in S$$

$$q_i^s \in \{0, 1\}; \;\; i \in I, s \in S$$

$$x_{kt}^s \ge 0; \;\; k \in K, t \in T, s \in S,$$

where $A_h V_{h0}$ is the initial inventory of parts type h.

Notice that the supply-production coordinating constraints (9.11) are also bill-of-material constraints in model **SupportMP_E**.

The above model can be further enhanced, for example by introducing subsets $I_h \subset I$ of suppliers for each part type h, unit capacity consumption for each product type k in the left-hand side of production capacity constraints, etc.

9.3.3 Hierarchical Selection of Primary and Recovery Supply Portfolios

In this subsection two deterministic MIP models **PSupport** and **RSupport(s)** are presented for the hierarchical decision making in the presence of supply chain disruption risks. The two-stage decision making is described below (Fig. 9.1).

1. Selection of primary supply portfolio for deterministic environment.
   The primary suppliers are determined ahead of time with no disruption scenarios considered, using deterministic MIP model **PSupport**.
2. Selection of recovery supply portfolio, after disruption of primary suppliers.
   The recovery suppliers are determined after a primary supplier was hit by a disruption to optimize the process of recovery from the disruption, using MIP model **RSupport(s)**.

The hierarchical approach is a purely top-down approach.

In the deterministic MIP model **PSupport**, stochastic variable, x_t^s, (9.18), defined in model **Support** for each disruption scenario $s \in S$ has been replaced by its deterministic equivalent X_t.

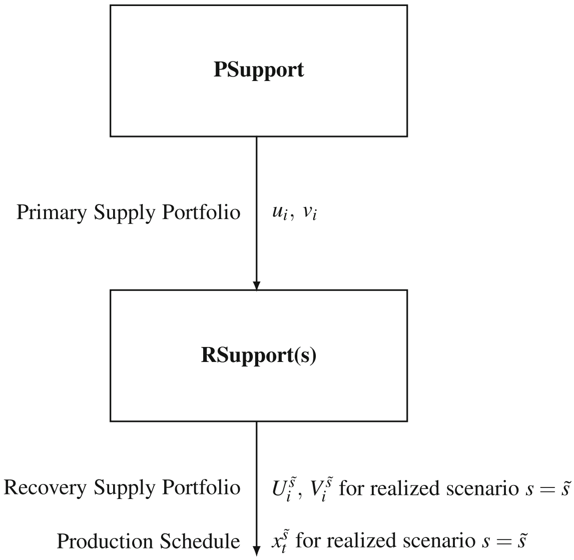

Fig. 9.1 Hierarchical selection of supply portfolios

PSupport: *Primary supply portfolio selection and production scheduling*

Minimize

$$P^c = \sum_{i \in I} (e_i u_i / D + o_i v_i) + g(1 - \sum_{t \in T} X_t / D) \tag{9.21}$$

subject to

$$\sum_{i \in I} v_i = 1 \tag{9.22}$$

$$v_i \le u_i;\ \ i \in I \tag{9.23}$$

$$\sum_{t' \in T : t' \le t} X_{t'} / D \le V_0 + \sum_{i \in I : \tau_i \le t-1} v_i;\ \ t \in T \tag{9.24}$$

$$X_t \le c;\ \ t \in T \tag{9.25}$$

$$u_i \in \{0, 1\};\ \ i \in I \tag{9.26}$$

$$v_i \in [0, 1];\ \ i \in I \tag{9.27}$$

$$X_t \ge 0;\ \ t \in T. \tag{9.28}$$

The solution to **PSupport** is primary supply portfolio: $u_i^*,\ v_i^*;\ i \in I$, and the associated production schedule, X_t^*.

A simple lower bound on the objective function P^c, (9.21), of the top level problem **PSupport** is derived below.

Proposition 9.2

$$P^c \geq \min_{i \in I}(e_i/D + o_i) + g \max\{0, 1 - C/D\}. \tag{9.29}$$

Proof Primary supply portfolio selection constraints (9.22), (9.23) and **Proposition** 9.2, (9.19), imply that

$$P^c = \sum_{i \in I}(e_i u_i/D + o_i v_i) + g(1 - \sum_{t \in T} X_t/D) \geq$$
$$\sum_{i \in I}(e_i/D + o_i)v_i + g \max\{0, 1 - C/D\} \geq$$
$$\min_{i \in I}(e_i/D + o_i) + g \max\{0, 1 - C/D\}.$$

In the deterministic MIP model **RSupport(s)**, stochastic variables, $U_i^s,\ V_i^s,\ x_t^s$, (9.15), (9.16), (9.18), defined in model **Support_E** for each disruption scenario $s \in S$ have been replaced by their deterministic equivalents, $U_i^{\tilde{s}},\ V_i^{\tilde{s}},\ \ x_t^{\tilde{s}}$ for the realized disruption scenario $s = \tilde{s}$.

RSupport(s): *Recovery supply portfolio selection and production scheduling for predetermined primary supply portfolio and the realized disruption scenario*

Minimize

$$R_{\tilde{s}}^c = \sum_{i \in I}(e_i(1 - u_i^*)U_i^{\tilde{s}}/D + \rho_{i\tilde{s}}U_i^{\tilde{s}}/D + o_i V_i^{\tilde{s}}) + g(1 - \sum_{t \in T} x_t^{\tilde{s}}/D) + \sum_{i \in I}(e_i u_i^*/D + o_i \gamma_{i,\lambda_{i\tilde{s}}} v_i^*) \tag{9.30}$$

subject to

$$\sum_{i \in I} V_i^{\tilde{s}} = 1 - \sum_{i \in I} \gamma_{i,\lambda_{i\tilde{s}}} v_i^* \tag{9.31}$$

$$V_i^{\tilde{s}} \leq U_i^{\tilde{s}};\ i \in I \tag{9.32}$$

$$\sum_{t' \in T: t' \leq t} x_{t'}^{\tilde{s}}/D - \sum_{i \in I: t_{\tilde{s}} + \theta_{i\tilde{s}} + \tau_i \leq t-1} V_i^{\tilde{s}} \leq V_0 + \sum_{i \in I: \tau_i \leq t-1} \gamma_{i,\lambda_{i\tilde{s}}} v_i^*;\ t \in T \tag{9.33}$$

$$x_t^{\tilde{s}} \leq c;\ t \in T \tag{9.34}$$

$$U_i^{\tilde{s}} \in \{0, 1\};\ i \in I \tag{9.35}$$

$$V_i^{\tilde{s}} \in [0, 1];\ i \in I \tag{9.36}$$

$$x_t^{\tilde{s}} \geq 0;\ t \in T. \tag{9.37}$$

Since the primary supply portfolio has been predetermined, the last summation term in Eq. (9.30) as well as the right-hand sides of Eqs. (9.31) and (9.33) are constant. The solution to **RSupport(s)** is the recovery supply portfolio, $U_i^{\tilde{s}},\ V_i^{\tilde{s}};\ i \in I$, and the production schedule, $x_t^{\tilde{s}};\ t \in T$, for the realized disruption scenario $s = \tilde{s}$:

Notice that no recovery supply portfolios, $V_i^s = 0,\ \forall i \in I$, will be determined for scenarios s with non disrupted primary suppliers, i.e., for scenarios $s \in S$ such that $\lambda_{is} = \overline{L_i}$ if $v_i^* > 0$. Since $\gamma_{i,\overline{L_i}} = 1$, $\sum_{i \in I} \gamma_{i,\overline{L_i}} v_i^* = 1$, and hence the right-hand side of (9.31) is equal to zero.

Model **Support_E** for the predetermined primary supply portfolio is separable with respect to disruption scenarios $s \in S$, since the objective function (9.4) is additive and separable with respect to s and all constraints (9.8)–(9.12) are also separable with respect to s. Thus, given the primary supply portfolio, the recovery supply portfolios, $U_i^s,\ V_i^s;\ i \in I$, and production schedules, x_t^s, can be found simultaneously for all potential disruption scenarios $s \in S$, by solving

RSupport_E = **Support_E** for predetermined primary supply portfolio: $u_i = u_i^*,\ v_i = v_i^*;\ i \in I$.

RSupport_E: *Recovery supply portfolio selection and production scheduling for predetermined primary supply portfolio*

Minimize (9.4)

subject to (9.8)–(9.12), (9.15)–(9.18) and

$$u_i = u_i^*;\ i \in I \tag{9.38}$$

$$v_i = v_i^*;\ i \in I. \tag{9.39}$$

9.4 Models for Risk-Averse Decision-Making

In this section the two time-indexed SMIP models **Support_CV(c)** and **Support_CV(sl)** are proposed for the integrated, risk-averse selection of primary and recovery supply portfolios and production scheduling to optimize, respectively CVaR of cost and CVaR of service level under disruption risks. The models are based on the risk-neutral model **Support_E**.

Let VaR^c be the Value-at Risk of cost per product, i.e., the targeted cost such that for a given confidence level α, for $100\alpha\%$ of disruption scenarios, the outcome is below VaR^c. Accordingly, let CVaR^c be Conditional Value-at-Risk of cost per product, i.e., the expected cost in the worst $100(1-\alpha)\%$ of the scenarios with the cost above VaR^c.

$$CVaR^c = VaR^c + (1-\alpha)^{-1} \sum_{s \in S} P_s \mathscr{C}_s. \qquad (9.40)$$

The risk-averse primary and recovery supply portfolio and the production schedule will be optimized by calculating VaR^c and minimizing CVaR^c simultaneously. Model **Support_CV(c)** is presented below.

Support_CV(c): *Selection of primary and recovery supply portfolios and scheduling of production to minimize CVaR of cost*

Minimize (9.40)

subject to (9.6)–(9.18) and

Risk constraints:

- the tail cost for scenario s is defined as the nonnegative amount by which cost in scenario s exceeds VaR^c,

$$\mathscr{C}_s \geq \sum_{i \in I} e_i(u_i + U_i^s - q_i^s)/D + \sum_{i \in I} \rho_{is} U_i^s / D$$
$$+ \sum_{i \in I} o_i(\gamma_{i,\lambda_{is}} v_i + V_i^s) + g(1 - \sum_{t \in T} x_t^s / D) - VaR^c; \ s \in S \qquad (9.41)$$
$$\mathscr{C}_s \geq 0; \ s \in S, \qquad (9.42)$$

where $\mathscr{C}_s$ is the tail cost for scenario s.

In the next model, VaR^{sl} is the Value-at Risk of service level, i.e., the targeted service level such that for a given confidence level α, for $100\alpha\%$ of disruption scenarios, the outcome is above VaR^{sl}, while CVaR^{sl} is the Conditional Value-at-Risk of service level, i.e., the expected service level in the worst $100(1-\alpha)\%$ of scenarios with the service level below VaR^{sl}.

$$CVaR^{sl} = VaR^{sl} - (1-\alpha)^{-1} \sum_{s \in S} P_s \mathscr{S}_s. \qquad (9.43)$$

The risk-averse primary and recovery supply portfolio and the production schedule will be optimized by calculating VaR^{sl} and maximizing CVaR^{sl} simultaneously. Model **Support_CV(sl)** is presented below.

Support_CV(sl): *Selection of primary and recovery supply portfolios and scheduling of production to maximize CVaR of service level*

Maximize (9.43)

subject to (9.6)–(9.18) and

Risk constraints:

- the tail service level for scenario s is defined as the nonnegative amount by which VaR^{sl} exceeds service level in scenario s,

$$\mathscr{S}_s \geq VaR^{sl} - \sum_{t \in T} x_t^s / D;\ \ s \in S \tag{9.44}$$

$$\mathscr{S}_s \geq 0;\ \ s \in S, \tag{9.45}$$

where $\mathscr{S}_s$ is the tail service level for scenario s.

9.5 Computational Examples

In this section some computational examples are presented to illustrate the proposed portfolio approach for selection of primary and recovery supply portfolios. The problem of joint selection of primary and recovery supply portfolios and production scheduling under disruption risks considered here is different from the existing literature in many different ways. Although the input data for the examples are hypothetical, their relations to each other are real and in part they have been taken from real case studies. In particular, the case studies of Toyota supply chain disruption and recovery after the Great East Japan earthquake and tsunami of March 11, 2011 (Fujimoto and Park 2013; Park et al. 2013; MacKenzie et al. 2014; Matsuo 2015) and a case study of Thailands floods in 2011 (Haraguchi and Lall 2015) have been analyzed. The following parameters have been selected for the computational examples.

$\overline{I} = 4$ suppliers, $\overline{L_i} = 3$ partial disruption levels for all $i \in I$, $\overline{R} = 2$ geographic regions, $\overline{T} = 30$ planning periods.

$I^1 = \{1, 2\},\ I^2 = \{3, 4\}$.

Delivery lead times from suppliers: $\tau = (2, 2, 4, 4)$.

The initial inventories of parts: $V_0 = 0$.

Customer demand: $d_t = 10000$ for all $t \in T$,

and total demand for parts/products: $D = \sum_{t \in T} d_t = 300000$.

Fixed ordering costs for suppliers: $e = (8000, 6000, 12000, 13000)$.

Unit penalties for unfulfilled demand:

$g \in \{1, 10, 100, 1000, 10000, 100000, \infty\}$, where $g = \infty$ denotes maximization of service level.

Unit purchasing prices from suppliers: $o = (14, 12, 8, 9)$.

Local disruption levels and the associated supplier fulfillment rates (the percentage of an order that can be delivered) are shown below.

$L_i = L = \{0, 1, 2, 3\}$ for all $i \in I$, where $l = 0$, complete shutdown, $\gamma_{i0} = 0\ \forall i \in I$, i.e., 0% of an order delivered; $l = 1$, major disruption, $\gamma_{i1} \in [0.01, 0.50]\ \forall i \in I^1$ and $\gamma_{i1} \in [0.01, 0.30]\ \forall i \in I^2$, i.e., 1% to 50% and 1% to 30% of an order delivered, respectively; $l = 2$, minor disruption, $\gamma_{i2} \in [0, 51, 0.99]\ \forall i \in I^1$ and $\gamma_{i2} \in$

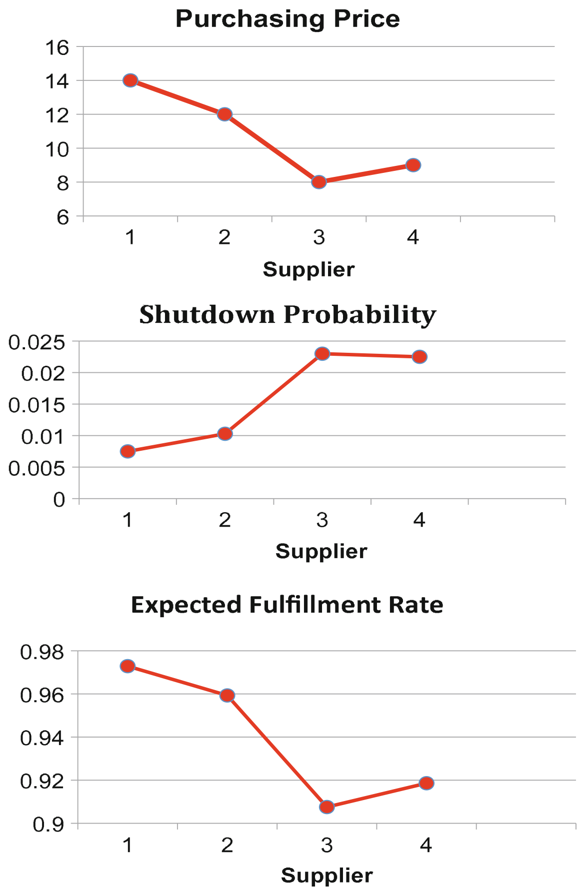

Fig. 9.2 Suppliers

[0, 31, 0.99] $\forall i \in I^2$, i.e., 51% to 99% and 31% to 99% of an order delivered, respectively; $l = \overline{L} = 3$, no disruption, $\gamma_{i3} = 1\ \forall i \in I$, i.e., 100% of an order delivered.

The total number of all potential scenarios is $\overline{S} = (\overline{L} + 1)^{\overline{I}} = 4^4 = 256$ scenarios, where each scenario $s \in S$ is represented by vector $\lambda_s = \{\lambda_{1s}, \ldots, \lambda_{4s}\}$, where $\lambda_{is} \in L,\ i \in I$, see Table 9.3.

The probability of realizing of disruption scenario $s \in S$ for suppliers in region $r = 1, 2$ is calculated as follows (see, Sect. 9.2)

$$P_s^r = \begin{cases} (1-p^r)(\prod_{i \in I^r:\lambda_{is}=0} 0.1(1-p_{i3}))(\prod_{i \in I^r:\lambda_{is}=1} 0.3(1-p_{i3})) & \\ \times(\prod_{i \in I^r:\lambda_{is}=2} 0.6(1-p_{i3}))(\prod_{i \in I^r:\lambda_{is}=3} p_{i3}) & \text{if } \sum_{i \in I^r} \lambda_{is} > 0 \\ p^r + (1-p^r)\prod_{i \in I^r} 0.1(1-p_{i3}) & \text{if } \sum_{i \in I^r} \lambda_{is} = 0, \end{cases}$$

and the probability for disruption scenario $s \in S$ is given by $P_s = P_s^1 P_s^2$.

In the above formula, p_{i3} denotes the local probability of non-disruptive operation (level $l = 3$) for supplier i and p^r denotes the regional disruption probability for all suppliers in region r. The probability p_{i3} was uniformly distributed over [0.89,0.99] and [0.79,0.89], respectively for suppliers $i \in I^1$, and $i \in I^2$.

Given local non disruption probabilities, $p_{i3},\ i \in I$, the probabilities for the remaining local disruption levels $l = 0, 1, 2$ were calculated as follows:
probability of complete shutdown (level $l = 0$), $p_{i0} = 0.1(1 - p_{i3})$;
probability of major disruption (level $l = 1$), $p_{i1} = 0.3(1 - p_{i3})$;
probability of minor disruption (level $l = 2$), $p_{i2} = 0.6(1 - p_{i3})$ for all suppliers $i \in I$.

Notice that, $p_{i0} \le p_{i1} \le p_{i2} \le p_{i3}$, i.e., the probability of disruption level of a supplier increases with the level such that the smallest probability was assigned to complete shutdown (level 0) and the largest to non-disruptive operation (level 3).

The regional disruption probabilities are $p^1 = 0.001$ and $p^2 = 0.01$.

The regional disruption probabilities are chosen to be lower than local shutdown probabilities of suppliers in that region. In addition, the two regional disruption probabilities are significantly different to represent two geographic regions differently exposed to disruptive events.

Cost-to-recover and time-to-recover are defined below.

$CTR(i, l)$=if $l = 0$ then $100000e_i$; if $l = 1$ then $10000e_i$; if $l = 2$ then $1000e_i$ $\forall i \in I$,

$TTR(i, l)$=if $l = 0$ then 12; if $l = 1$ then 10; if $l = 2$ then 8 $\forall i \in I$.

Figure 9.2 presents basic characteristics of all suppliers: purchasing price, $o_i,\ i \in I$, probability of complete shutdown, $p^r + (1 - p^r)p_{i0},\ i \in I^r,\ r \in R$, and expected fulfillment rate, $\Gamma_i = \sum_{l=1,2,3}(1 - p^r)p_{il}\gamma_{il},\ i \in I^r,\ r \in R$.

The three levels of producer per period capacity will be considered: $c \in \{5000, 10000, 15000\}$,

and total available producer capacity, $C = c(\overline{T} - \tau_{min})$, respectively: $C \in \{140000, 280000, 420000\}$.

The corresponding capacity-to-demand ratio is $C/D = 0.47$, 0.93 and 1.4.

9.5.1 *Risk-Neutral Decision-Making*

Scenarios with a Common Disruption Start Time

In this subsection each disruption $s \in S$ to primary suppliers is assumed to occur in the same period $t_s = 1$, before the earliest delivery lead time, $\min_{i \in I}(\tau_i) = 2$, and the recovery process starts in period $t = 2$ so that the disrupted supplier i returns to its full capacity in period $t = TTR(i, l) + 1$.

The optimal solutions for both the integrated and the hierarchical approach are summarized in Tables 9.4, 9.5, and 9.6, respectively for capacity-to-demand ratio, $C/D = 0.47$, 0.93 and 1.4.

For different values of unit penalty cost g, the tables show:

- optimal primary supply portfolio, $(v_1, v_2, v_3.v_4)$, indicating percent of total demand for parts ordered from each supplier;

Table 9.3 Multi-level disruption scenarios: $\lambda_{1s}, \lambda_{2s}, \lambda_{3s}, \lambda_{4s}$; $s = 1, \ldots, 256$

s	i=1	2	3	4	s	i=1	2	3	4	s	i=1	2	3	4	s	i=1	2	3	4
1	0	0	0	0	65	1	0	0	0	129	2	0	0	0	193	3	0	0	0
2	0	0	0	1	66	1	0	0	1	130	2	0	0	1	194	3	0	0	1
3	0	0	0	2	67	1	0	0	2	131	2	0	0	2	195	3	0	0	2
4	0	0	0	3	68	1	0	0	3	132	2	0	0	3	196	3	0	0	3
5	0	0	1	0	69	1	0	1	0	133	2	0	1	0	197	3	0	1	0
6	0	0	1	1	70	1	0	1	1	134	2	0	1	1	198	3	0	1	1
7	0	0	1	2	71	1	0	1	2	135	2	0	1	2	199	3	0	1	2
8	0	0	1	3	72	1	0	1	3	136	2	0	1	3	200	3	0	1	3
9	0	0	2	0	73	1	0	2	0	137	2	0	2	0	201	3	0	2	0
10	0	0	2	1	74	1	0	2	1	138	2	0	2	1	202	3	0	2	1
11	0	0	2	2	75	1	0	2	2	139	2	0	2	2	203	3	0	2	2
12	0	0	2	3	76	1	0	2	3	140	2	0	2	3	204	3	0	2	3
13	0	0	3	0	77	1	0	3	0	141	2	0	3	0	205	3	0	3	0
14	0	0	3	1	78	1	0	3	1	142	2	0	3	1	206	3	0	3	1
15	0	0	3	2	79	1	0	3	2	143	2	0	3	2	207	3	0	3	2
16	0	0	3	3	80	1	0	3	3	144	2	0	3	3	208	3	0	3	3
17	0	1	0	0	81	1	1	0	0	145	2	1	0	0	209	3	1	0	0
18	0	1	0	1	82	1	1	0	1	146	2	1	0	1	210	3	1	0	1
19	0	1	0	2	83	1	1	0	2	147	2	1	0	2	211	3	1	0	2
20	0	1	0	3	84	1	1	0	3	148	2	1	0	3	212	3	1	0	3
21	0	1	1	0	85	1	1	1	0	149	2	1	1	0	213	3	1	1	0
22	0	1	1	1	86	1	1	1	1	150	2	1	1	1	214	3	1	1	1
23	0	1	1	2	87	1	1	1	2	151	2	1	1	2	215	3	1	1	2
24	0	1	1	3	88	1	1	1	3	152	2	1	1	3	216	3	1	1	3
25	0	1	2	0	89	1	1	2	0	153	2	1	2	0	217	3	1	2	0
26	0	1	2	1	90	1	1	2	1	154	2	1	2	1	218	3	1	2	1
27	0	1	2	2	91	1	1	2	2	155	2	1	2	2	219	3	1	2	2
28	0	1	2	3	92	1	1	2	3	156	2	1	2	3	220	3	1	2	3
29	0	1	3	0	93	1	1	3	0	157	2	1	3	0	221	3	1	3	0
30	0	1	3	1	94	1	1	3	1	158	2	1	3	1	222	3	1	3	1
31	0	1	3	2	95	1	1	3	2	159	2	1	3	2	223	3	1	3	2
32	0	1	3	3	96	1	1	3	3	160	2	1	3	3	224	3	1	3	3
33	0	2	0	0	97	1	2	0	0	161	2	2	0	0	225	3	2	0	0
34	0	2	0	1	98	1	2	0	1	162	2	2	0	1	226	3	2	0	1
35	0	2	0	2	99	1	2	0	2	163	2	2	0	2	227	3	2	0	2
36	0	2	0	3	100	1	2	0	3	164	2	2	0	3	228	3	2	0	3
37	0	2	1	0	101	1	2	1	0	165	2	2	1	0	229	3	2	1	0
38	0	2	1	1	102	1	2	1	1	166	2	2	1	1	230	3	2	1	1
39	0	2	1	2	103	1	2	1	2	167	2	2	1	2	231	3	2	1	2
40	0	2	1	3	104	1	2	1	3	168	2	2	1	3	232	3	2	1	3
41	0	2	2	0	105	1	2	2	0	169	2	2	2	0	233	3	2	2	0
42	0	2	2	1	106	1	2	2	1	170	2	2	2	1	234	3	2	2	1
43	0	2	2	2	107	1	2	2	2	171	2	2	2	2	235	3	2	2	2
44	0	2	2	3	108	1	2	2	3	172	2	2	2	3	236	3	2	2	3
45	0	2	3	0	109	1	2	3	0	173	2	2	3	0	237	3	2	3	0
46	0	2	3	1	110	1	2	3	1	174	2	2	3	1	238	3	2	3	1
47	0	2	3	2	111	1	2	3	2	175	2	2	3	2	239	3	2	3	2
48	0	2	3	3	112	1	2	3	3	176	2	2	3	3	240	3	2	3	3
49	0	3	0	0	113	1	3	0	0	177	2	3	0	0	241	3	3	0	0
50	0	3	0	1	114	1	3	0	1	178	2	3	0	1	242	3	3	0	1
51	0	3	0	2	115	1	3	0	2	179	2	3	0	2	243	3	3	0	2
52	0	3	0	3	116	1	3	0	3	180	2	3	0	3	244	3	3	0	3
53	0	3	1	0	117	1	3	1	0	181	2	3	1	0	245	3	3	1	0
54	0	3	1	1	118	1	3	1	1	182	2	3	1	1	246	3	3	1	1
55	0	3	1	2	119	1	3	1	2	183	2	3	1	2	247	3	3	1	2
56	0	3	1	3	120	1	3	1	3	184	2	3	1	3	248	3	3	1	3
57	0	3	2	0	121	1	3	2	0	185	2	3	2	0	249	3	3	2	0
58	0	3	2	1	122	1	3	2	1	186	2	3	2	1	250	3	3	2	1
59	0	3	2	2	123	1	3	2	2	187	2	3	2	2	251	3	3	2	2
60	0	3	2	3	124	1	3	2	3	188	2	3	2	3	252	3	3	2	3
61	0	3	3	0	125	1	3	3	0	189	2	3	3	0	253	3	3	3	0
62	0	3	3	1	126	1	3	3	1	190	2	3	3	1	254	3	3	3	1
63	0	3	3	2	127	1	3	3	2	191	2	3	3	2	255	3	3	3	2
64	0	3	3	3	128	1	3	3	3	192	2	3	3	3	256	3	3	3	3

Table 9.4 Solution results for a common disruption start time: $C/D = 0.47$

g	1	10	10^2	10^3	10^4	10^5	∞ [(a)]
Integrated approach: model **Support_E**							
Var. = 10205, Bin. = 2052, Cons. = 10462, Nonz. = 10205 [(e)]							
Exp.Cost E^c, (9.4)	8.80	13.72	61.76	542	5343	53337	–
Exp.Service E^{sl} × 100%, (9.5)	43	47	47	47	47	47	47
Primary Portfolio: Supplier(% of total demand) [(b)]	1(0)	1(0)		1(2)	1(4)	1(9)	1(31)
	2(0)	2(3)		2(2)	2(4)	2(10)	2(19)
	3(100)	3(97)		3(96)	3(92)	3(79)	3(26)
	4(0)	4(0)		4(0)	4(0)	4(2)	4(24)
Exp.Recovery Portfolio: Supplier(% of total demand) [(c)]	1(0.18)	1(0.27)		1(0.22)	1(0.17)	1(0.16)	1(1.82)
	2(1.86)	2(1.81)		2(1.82)	2(1.71)	2(1.51)	2(1.46)
	3(0)	3(0.04)		3(0.06)	3(0.25)	3(0.7)	3(1.25)
	4(7.21)	4(6.95)		4(6.93)	4(6.63)	4(5.80)	4(1.43)
Hierarchical approach: models **PSupport** and **RSupport**							
PSupport: Var. = 36, Bin. = 4, Cons. = 37, Nonz. = 534							
Primary Portfolio: Supplier(% of total demand) [(b)]	1(0)	1(0)					1(0)
	2(0)	2(3)					2(100)
	3(100)	3(97)					3(0)
	4(0)	4(0)					4(0)
Cost P_c, (9.21)	8.61	13.53	61.53	542	5342	53342	–
Service [(d)]	43	47	47	47	47	47	47
RSupport_E: Var. = 8050, Bin. = 768, Cons. = 6250, Nonz. = 8050							
Exp.Recovery Portfolio: Supplier(% of total demand) [(c)]	1(0.18)	1(0.22)					1(0.83)
	2(1.86)	2(1.73)					2(3.14)
	3(0)	3(0.20)					3(0.07)
	4(7.21)	4(6.75)					4(0.04)
Exp.Cost E^c, (9.4)	8.80	13.84	61.85	542	5344	53365	–
Exp.Service E^{sl} × 100%, (9.5)	43	47	47	47	47	47	47

[(a)] Maximization of service level
[(b)] $1(v_1 \times 100), 2(v_2 \times 100), 3(v_3 \times 100\%), 4(v_4 \times 100)$
[(c)] $1(\sum_{s\in S} P_s V_1^s \times 100), 2(\sum_{s\in S} P_s V_2^s \times 100), 3(\sum_{s\in S} P_s V_3^s \times 100), 4(\sum_{s\in S} P_s V_4^s \times 100)$
[(d)] $\sum_{t\in T} X_t/D \times 100$
[(e)] Var. = number of variables, Bin. = number of binary variables, Cons. = number of constraints, Nonz. = number of nonzero coefficients

- optimal expected recovery supply portfolio, $(\sum_{s\in S} P_s V_1^s, \sum_{s\in S} P_s V_2^s, \sum_{s\in S} P_s V_3^s, \sum_{s\in S} P_s V_4^s)$, indicating expected percent of total demand for parts ordered from each supplier.

For the integrated approach and the bottom level problem **RSupport_E** of the hierarchical approach, expected cost E^c, (9.4), is shown along with the associated expected service level E^{sl}, (9.5). Similarly, for the top level problem **PSupport** of the hierarchical approach, cost P^c, (9.21), is shown along with the associated service level, $\sum_{t\in T} X_t/D$. In addition, Table 9.4 presents the size of each MIP model, **Support_E**, **PSupport** and **RSupport_E**. The results indicate that for most examples the expected service level E^{sl}, (9.5), and the service level for **PSupport** problem,

Table 9.5 Solution results for a common disruption start time: $C/D = 0.93$

g	1	10	10^2	10^3	10^4	10^5	∞ $^{(a)}$
Integrated approach: model **Support_E**							
Exp.Cost E^c, (9.4)	8.36	9.19	15.26	75.53	677	6685	–
Exp.Service $E^{sl} \times 100\%$, (9.5)	87	93	93	93	93	93	93
Primary Portfolio: Supplier(% of total demand) $^{(b)}$	1(0) 2(0) 3(100) 4(0)	1(0) 2(7) 3(93) 4(0)		1(3) 2(4) 3(93) 4(0)	1(8) 2(9) 3(83) 4(0)	1(18) 2(20) 3(57) 4(5)	1(27) 2(25) 3(7) 4(41)
Exp.Recovery Portfolio: Supplier(% of total demand) $^{(c)}$	1(0.18) 2(1.86) 3(0) 4(7.21)	1(0.36) 2(1.77) 3(0.07) 4(6.70)		1(0.25) 2(1.78) 3(0.13) 4(6.65)	1(0.16) 2(1.55) 3(0.49) 4(6.06)	1(0.13) 2(1.16) 3(1.44) 4(4.26)	1(1.99) 2(1.99) 3(1.37) 4(0.40)
Hierarchical approach: models **PSupport** and **RSupport_E**							
Primary Portfolio: Supplier(% of total demand) $^{(b)}$	1(0) 2(0) 3(100) 4(0)	1(0) 2(7) 3(93) 4(0)					1(0) 2(100) 3(0) 4(0)
Cost P^c, (9.21)	8.17	9	15	75	675	6675	–
Service $^{(d)}$	87	93					93
Exp. Recovery Portfolio: Supplier(% of total demand) $^{(c)}$	1(0.18) 2(1.86) 3(0) 4(7.21)	1(0.36) 2(1.77) 3(0.07) 4(6.70)					1(1.18) 2(1.27) 3(1.55) 4(0.07)
Exp.Cost E^c, (9.4)	8.36	9.19	15.26	76.02	684	6759	–
Exp.Service $E^{sl} \times 100\%$, (9.5)	87	93					93

$^{(a)}$ Maximization of service level
$^{(b)}$ $1(v_1 \times 100), 2(v_2 \times 100), 3(v_3 \times 100), 4(v_4 \times 100)$
$^{(c)}$ $1(\sum_{s \in S} P_s V_1^s \times 100), 2(\sum_{s \in S} P_s V_2^s \times 100), 3(\sum_{s \in S} P_s V_3^s \times 100), 4(\sum_{s \in S} P_s V_4^s \times 100)$
$^{(d)}$ $\sum_{t \in T} X_t / D \times 100$

$\sum_{t \in T} X_t/D$, attain their upper bounds C/D, (9.19), whereas the cost P^c, (9.21), remains at its lower bound 8.04, (9.29), only for $C/D = 1.4$.

Tables 9.4 and 9.5 indicate that for $C/D < 1$ and small unit penalty cost g, the cheapest supply portfolio is selected and the achieved service level is less than C/D. The same solution results are obtained for both the integrated and the hierarchical approach. As g increases to reduce the unfulfilled demand, more expensive and diversified supply portfolios are selected and the highest service level C/D, (9.19), is attained. In particular, for integrated approach the more diversified primary supply portfolios are selected for $g > 1$ to hedge against all disruption scenarios. In contrast, for hierarchical approach the results for $g > 1$ are independent of g. When the objective is to maximize service level ($g \to \infty$), the hierarchical approach selects the most reliable supplier as the only primary supplier. In addition, Table 9.6 demonstrates that for $C/D > 1$ and both approaches, the primary portfolio and the expected

Table 9.6 Solution results for a common disruption start time: $C/D = 1.4$

g	1	10	10^2	10^3	10^4	10^5	∞ [(a)]
Integrated approach: model **Support_E**							
Exp.Cost E^c, (9.4)	8.23	8.23	8.23	8.24	8.31	9.05	–
Exp.Service $E^{sl} \times 100\%$, (9.5)	100						100
Primary Portfolio: Supplier(% of total demand) [(b)]	1(0) 2(0) 3(100) 4(0)					1(0) 2(6) 3(94) 4(0)	1(40) 2(10) 3(26) 4(24)
Exp.Recovery Portfolio: Supplier(% of total demand) [(c)]	1(0.18) 2(1.86) 3(0) 4(7.21)					1(0.17) 2(1.74) 3(0.22) 4(6.78)	1(0.43) 2(2.10) 3(1.73) 4(1.62)
Hierarchical approach: models **PSupport** and **RSupport_E**:							
Primary Portfolio: Supplier(% of total demand) [(b)]	1(0) 2(0) 3(100) 4(0)						1(100) 2(0) 3(0) 4(0)
Cost P^c, (9.21)	8.04						–
Service [(d)]	100						100
Exp.Recovery Portfolio: Supplier(% of total demand) [(c)]	1(0.18) 2(1.86) 3(0) 4(7.21)						1(1.45) 2(1.04) 3(0.13) 4(0.08)
Exp.Cost E^c, (9.4)	8.23	8.23	8.23	8.24	8.31	9.05	–
Exp.Service $E^{sl} \times 100\%$, (9.5)	100						100

[(a)] Maximization of service level
[(b)] $1(v_1 \times 100), 2(v_2 \times 100), 3(v_3 \times 100), 4(v_4 \times 100)$
[(c)] $1(\sum_{s \in S} P_s V_1^s \times 100), 2(\sum_{s \in S} P_s V_2^s \times 100), 3(\sum_{s \in S} P_s V_3^s \times 100), 4(\sum_{s \in S} P_s V_4^s \times 100)$
[(d)] $\sum_{t \in T} X_t / D \times 100$

recovery portfolio are independent of g, since for $C/D = 1.4$, the total demand for products is fulfilled for all g and hence no penalty cost appears.

Table 9.7 shows an example of optimal primary supply portfolio v_1, v_2, v_3, v_4 and optimal recovery supply portfolios $V_1^s, V_2^s, V_3^s, V_4^s, \forall s \in S$ for $C/D = 1.4$ and unit penalty $g = 1, 10, 100, 1000, 10000$. Both the integrated approach and the hierarchical approach yield the same solution results (cf. Table 9.6). The table shows the optimal primary supply portfolio, $v_1 = v_2 = 0, v_3 = 1, v_4 = 0$, and the associated cost P^c, (9.21), as well as the optimal recovery supply portfolios and costs R_s^c, (9.30) for each disruption scenarios $s \in S$. The results demonstrate that no recovery supply portfolio is selected, i.e., $V_1^s = V_2^s = V_3^s = V_4^s = 0$, for scenarios $s \in S$ with undisrupted primary supplier, $i = 3$, i.e., for $\{s \in S : \lambda_{3s} = 3\} = \{13 - 16, 29 - 32, 45 - 48, 61 - 64, 77 - 80, 93 - 96, 109 - 112, 125 - 128, 141 - 144, 157 - 160, 173 - 176, 189 - 192, 205 - 208, 221 - 224, 237 - 240, 253 - 256\}$.

Table 9.7 Optimal recovery supply portfolios for a common disruption start time, $C/D = 1.4$, $g = 1, 10, 10^2, 10^3, 10^4$: integrated approach/hierarchical approach†

Scenario: s Optimal recovery supply portfolio: V_1^s V_2^s V_3^s V_4^s Optimal expected cost: R_s^c, (9.30)

s	i=1	2	3	4	Cost	s	i=1	2	3	4	Cost
1	0	1	0	0	2037.06	65	1	0	0	0	295.73
2	0	0	0	1	467.42	66	1	0	0	0	295.73
3	0	0	0	1	67.42	67	0	0	0	1	67.42
4	0	0	0	1	9.08	68	0	0	0	1	9.08
5	0	0	0.9	0	422.72	69	0.9	0	0	0	284.8
6	0	0	0.9	0	422.72	70	0.9	0	0	0	284.8
7	0	0	0	0.9	56.99	71	0	0	0	0.9	56.99
8	0	0	0	0.9	8.98	72	0	0	0	0.9	8.98
9	0	0	0.44	0	48.04	73	0	0	0.44	0	48.04
10	0	0	0.44	0	48.04	74	0	0	0.44	0	48.04
11	0	0	0.44	0	48.04	75	0	0	0.44	0	48.04
12	0	0	0	0.44	8.53	76	0	0	0	0.44	8.53
13	0	0	0	0	8.04	77	0	0	0	0	8.04
14	0	0	0	0	8.04	78	0	0	0	0	8.04
15	0	0	0	0	8.04	79	0	0	0	0	8.04
16	0	0	0	0	8.04	80	0	0	0	0	8.04
17	0	1	0	0	227.06	81	0	1	0	0	227.06
18	0	1	0	0	227.06	82	0	1	0	0	227.06
19	0	0	0	1	67.42	83	0	0	0	1	67.42
20	0	0	0	1	9.08	84	0	0	0	1	9.08
21	0	0.9	0	0	216.33	85	0	0.9	0	0	216.33
22	0	0.9	0	0	216.33	86	0	0.9	0	0	216.33
23	0	0	0	0.9	56.99	87	0	0	0	0.9	56.99
24	0	0	0	0.9	8.98	88	0	0	0	0.9	8.98
25	0	0	0.44	0	48.04	89	0	0	0.44	0	48.04
26	0	0	0.44	0	48.04	90	0	0	0.44	0	48.04
27	0	0	0.44	0	48.04	91	0	0	0.44	0	48.04
28	0	0	0	0.44	8.53	92	0	0	0	0.44	8.53
29	0	0	0	0	8.04	93	0	0	0	0	8.04
30	0	0	0	0	8.04	94	0	0	0	0	8.04
31	0	0	0	0	8.04	95	0	0	0	0	8.04
32	0	0	0	0	8.04	96	0	0	0	0	8.04
33	0	1	0	0	37.06	97	0	1	0	0	37.06
34	0	1	0	0	37.06	98	0	1	0	0	37.06
35	0	1	0	0	37.06	99	0	1	0	0	37.06
36	0	0	0	1	9.08	100	0	0	0	1	9.08
37	0	0.9	0	0	31.65	101	0	0.9	0	0	31.65
38	0	0.9	0	0	31.65	102	0	0.9	0	0	31.65
39	0	0.9	0	0	31.65	103	0	0.9	0	0	31.65
40	0	0	0	0.9	8.98	104	0	0	0	0.9	8.98
41	0	0.44	0	0	29.83	105	0	0.44	0	0	29.83
42	0	0.44	0	0	29.83	106	0	0.44	0	0	29.83
43	0	0.44	0	0	29.83	107	0	0.44	0	0	29.83
44	0	0	0	0.44	8.53	108	0	0	0	0.44	8.53
45	0	0	0	0	8.04	109	0	0	0	0	8.04
46	0	0	0	0	8.04	110	0	0	0	0	8.04
47	0	0	0	0	8.04	111	0	0	0	0	8.04
48	0	0	0	0	8.04	112	0	0	0	0	8.04
49	0	1	0	0	12.06	113	0	1	0	0	12.06
50	0	1	0	0	12.06	114	0	1	0	0	12.06
51	0	1	0	0	12.06	115	0	1	0	0	12.06
52	0	0	0	1	9.08	116	0	0	0	1	9.08
53	0	0.9	0	0	11.65	117	0	0.9	0	0	11.65
54	0	0.9	0	0	11.65	118	0	0.9	0	0	11.65
55	0	0.9	0	0	11.65	119	0	0.9	0	0	11.65
56	0	0	0	0.9	8.98	120	0	0	0	0.9	8.98
57	0	0.44	0	0	9.83	121	0	0.44	0	0	9.83
58	0	0.44	0	0	9.83	122	0	0.44	0	0	9.83
59	0	0.44	0	0	9.83	123	0	0.44	0	0	9.83
60	0	0	0	0.44	8.53	124	0	0	0	0.44	8.53
61	0	0	0	0	8.04	125	0	0	0	0	8.04
62	0	0	0	0	8.04	126	0	0	0	0	8.04
63	0	0	0	0	8.04	127	0	0	0	0	8.04
64	0	0	0	0	8.04	128	0	0	0	0	8.04

(continued)

Table 9.7 (continued)

Scenario: s Optimal recovery supply portfolio: V_1^s V_2^s V_3^s V_4^s Optimal expected cost: R_s^c, (9.30)

s	i=1	2	3	4	Cost	s	i=1	2	3	4	Cost
129	1	0	0	0	45.73	193	1	0	0	0	14.07
130	1	0	0	0	45.73	194	1	0	0	0	14.07
131	1	0	0	0	45.73	195	1	0	0	0	14.07
132	0	0	0	1	9.08	196	0	0	0	1	9.08
133	0.9	0	0	0	40.11	197	0.9	0	0	0	13.45
134	0.9	0	0	0	40.11	198	0.9	0	0	0	13.45
135	0.9	0	0	0	40.11	199	0.9	0	0	0	13.45
136	0	0	0	0.9	8.98	200	0	0	0	0.9	8.98
137	0.44	0	0	0	37.39	201	0.44	0	0	0	10.72
138	0.44	0	0	0	37.39	202	0.44	0	0	0	10.72
139	0.44	0	0	0	37.39	203	0.44	0	0	0	10.72
140	0	0	0	0.44	8.53	204	0	0	0	0.44	8.53
141	0	0	0	0	8.04	205	0	0	0	0	8.04
142	0	0	0	0	8.04	206	0	0	0	0	8.04
143	0	0	0	0	8.04	207	0	0	0	0	8.04
144	0	0	0	0	8.04	208	0	0	0	0	8.04
145	1	0	0	0	45.73	209	1	0	0	0	14.07
146	1	0	0	0	45.73	210	1	0	0	0	14.07
147	1	0	0	0	45.73	211	1	0	0	0	14.07
148	0	0	0	1	9.08	212	0	0	0	1	9.08
149	0.9	0	0	0	40.11	213	0.9	0	0	0	13.45
150	0.9	0	0	0	40.11	214	0.9	0	0	0	13.45
151	0.9	0	0	0	40.11	215	0.9	0	0	0	13.45
152	0	0	0	0.9	8.98	216	0	0	0	0.9	8.98
153	0.44	0	0	0	37.39	217	0.44	0	0	0	10.72
154	0.44	0	0	0	37.39	218	0.44	0	0	0	10.72
155	0.44	0	0	0	37.39	219	0.44	0	0	0	10.72
156	0	0	0	0.44	8.53	220	0	0	0	0.44	8.53
157	0	0	0	0	8.04	221	0	0	0	0	8.04
158	0	0	0	0	8.04	222	0	0	0	0	8.04
159	0	0	0	0	8.04	223	0	0	0	0	8.04
160	0	0	0	0	8.04	224	0	0	0	0	8.04
161	0	1	0	0	37.06	225	1	0	0	0	14.07
162	0	1	0	0	37.06	226	1	0	0	0	14.07
163	0	1	0	0	37.06	227	1	0	0	0	14.07
164	0	0	0	1	9.08	228	0	0	0	1	9.08
165	0	0.9	0	0	31.65	229	0.9	0	0	0	13.45
166	0	0.9	0	0	31.65	230	0.9	0	0	0	13.45
167	0	0.9	0	0	31.65	231	0.9	0	0	0	13.45
168	0	0	0	0.9	8.98	232	0	0	0	0.9	8.98
169	0	0.44	0	0	29.83	233	0.44	0	0	0	10.72
170	0	0.44	0	0	29.83	234	0.44	0	0	0	10.72
171	0	0.44	0	0	29.83	235	0.44	0	0	0	10.72
172	0	0	0	0.44	8.53	236	0	0	0	0.44	8.53
173	0	0	0	0	8.04	237	0	0	0	0	8.04
174	0	0	0	0	8.04	238	0	0	0	0	8.04
175	0	0	0	0	8.04	239	0	0	0	0	8.04
176	0	0	0	0	8.04	240	0	0	0	0	8.04
177	0	1	0	0	12.06	241	0	1	0	0	12.06
178	0	1	0	0	12.06	242	0	1	0	0	12.06
179	0	1	0	0	12.06	243	0	1	0	0	12.06
180	0	0	0	1	9.08	244	0	0	0	1	9.08
181	0	0.9	0	0	11.65	245	0	0.9	0	0	11.65
182	0	0.9	0	0	11.65	246	0	0.9	0	0	11.65
183	0	0.9	0	0	11.65	247	0	0.9	0	0	11.65
184	0	0	0	0.9	8.98	248	0	0	0	0.9	8.98
185	0	0.44	0	0	9.83	249	0	0.44	0	0	9.83
186	0	0.44	0	0	9.83	250	0	0.44	0	0	9.83
187	0	0.44	0	0	9.83	251	0	0.44	0	0	9.83
188	0	0	0	0.44	8.53	252	0	0	0	0.44	8.53
189	0	0	0	0	8.04	253	0	0	0	0	8.04
190	0	0	0	0	8.04	254	0	0	0	0	8.04
191	0	0	0	0	8.04	255	0	0	0	0	8.04
192	0	0	0	0	8.04	256	0	0	0	0	8.04

† Optimal primary supply portfolio: $v_1 = v_2 = 0$, $v_3 = 1$, $v_4 = 0$, optimal expected cost: 8.04, (9.21)

Table 9.8 Solution results for different disruption start times: $C/D = 0.93$

g	1	10	10^2	10^3	10^4	10^5	∞ [(a)]
Integrated approach: model **Support_E**							
Var. = 30596, Bin. = 6148, Cons. = 31365, Nonz. = 451299 [(e)]							
Exp.Cost E^c, (9.4)	8.37	9.20	15.43	75.73	677	6686	–
Exp.Service E^{sl} × 100%, (9.5)	87	93	93	93	93	93	93
Primary portfolio: supplier(% of total demand) [(b)]	1(0)	1(0)	1(0)	1(7)	1(8)	1(17)	1(33)
	2(0)	2(7)	2(8)	2(7)	2(9)	2(20)	2(31)
	3(100)	3(93)	3(92)	3(86)	3(80)	3(51)	3(10)
	4(0)	4(0)	4(0)	4(0)	4(3)	4(12)	4(26)
Exp. recovery portfolio: supplier(% of total demand) [(c)]	1(0.18)	1(0.20)	1(0.19)	1(0.17)	1(0.16)	1(1.13)	1(2.90)
	2(1.86)	2(1.81)	2(1.76)	2(1.64)	2(1.55)	2(1.16)	2(7.70)
	3(0)	3(0.22)	3(0.29)	3(0.39)	3(0.7)	3(1.84)	3(10.39)
	4(7.21)	4(6.67)	4(6.57)	4(6.27)	4(5.82)	4(3.80)	4(5.22)
Hierarchical approach: models **PSupport** and **RSupport_E**							
PSupport: Var. = 36, Bin. = 4, Cons. = 37, Nonz. = 534							
Primary Portfolio: Supplier(% of total demand) [(b)]	1(0)	1(0)					1(0)
	2(0)	2(7)					2(100)
	3(100)	3(93)					3(0)
	4(0)	4(0)					4(0)
Cost P^c, (21)	8.17	9	15	75	675	6675	–
Service [(d)]	87	93	93	93	93	93	93
RSupport_E: Var. = 23846, Bin. = 2304, Cons. = 15838, Nonz. = 203756							
Exp.Recovery Portfolio: Supplier(% of total demand) [(c)]	1(0.18)	1(0.20)					1(1.17)
	2(1.86)	2(1.81)					2(1.27)
	3(0)	3(0.22)					3(1.55)
	4(7.21)	4(6.67)					4(0.07)
Exp.Cost E^c, (9.4)	8.37	9.20	15.48	78.26	706	6985	–
Exp.Service E^{sl} × 100%, (9.5)	87	93	93	93	93	93	93

[(a)] Maximization of service level
[(b)] $1(v_1 \times 100), 2(v_2 \times 100), 3(v_3 \times 100), 4(v_4 \times 100)$
[(c)] $1(\sum_{s\in S} P_s V_1^s \times 100), 2(\sum_{s\in S} P_s V_2^s \times 100), 3(\sum_{s\in S} P_s V_3^s \times 100), 4(\sum_{s\in S} P_s V_4^s \times 100)$
[(d)] $\sum_{t\in T} X_t/D \times 100$
[(e)] Var. = number of variables, Bin. = number of binary variables, Cons. = number of constraints, Nonz. = number of nonzero coefficients

Scenarios with Different Disruption Start Times

In real-life cases, supply disruptions may occur at any time and for any supplier. In this subsection, disruption scenario is defined as a combination of disruptive event and its start time. The start time t_s of each disruptive event $s \in S$ cannot be greater than the maximum delivery lead time, $\max_{i\in I}(\tau_i) = 4$. In the computational examples, $t_s \in \{1, 2, 3\}$, and the total number of all potential scenarios to be considered is 256 $\times\, 3 = 768$, i.e., $S = \{1, \ldots, 768\}$. Now, each disruption scenario $s \in S$ is represented by vector $\lambda_s = \{\lambda_{1s}, \ldots, \lambda_{4s}\}$, (see Table 9.3), where $\lambda_s = \lambda_{s+256} = \lambda_{s+512};\ s \leq 256$, and its start time $t_s = 1$ for $s \leq 256$, $t_s = 2$ for $257 \leq s \leq 512$ and $t_s = 3$ for $513 \leq s \leq 768$.

The probability P_s of realizing each disruption scenario $s \in S$ is calculated as follows:
$P_s = \beta_1 P_s^1 P_s^2$ for $s \leq 256$,
$P_s = \beta_2 P_{s-256}^1 P_{s-256}^2$ for $257 \leq s \leq 512$,
$P_s = \beta_3 P_{s-512}^1 P_{s-512}^2$ for $513 \leq s \leq 768$,
where probabilities $P_s^r;\ r = 1, 2,\ s \leq 256$ are defined at the beginning of this section, and β_1, β_2 and β_3 are nonnegative constants such that: $\beta_1 + \beta_2 + \beta_3 = 1$.

Notice that for $\beta_1 = 1$, P_s reduces to probability of realizing scenario s with a common disruption start time $t_s = 1$, from Sect. 9.5.1.

As illustrative examples, the optimal solutions for $\beta_1 = 0.1$, $\beta_2 = 0.2$, $\beta_3 = 0.7$, and capacity-to-demand ratio, $C/D = 0.93$, are summarized in Tables 9.8, for both integrated and hierarchical approach. In contrast to examples in Sect. 9.5.1, now the probabilities for disruptions with later start times are higher.

Comparison of solution results in Tables 9.5 and 9.8 indicate for the two types of disruption scenarios, nearly identical optimal values of expected cost are achieved for the integrated approach and slightly worse for the hierarchical approach. However, the optimal supply portfolios for the corresponding penalty, g, for unfulfilled demand for products, are not identical (except for the smallest, $g = 1$). This indicates that for both types of scenarios the impact of disruption risks can be similarly mitigated by best selection of primary and recovery portfolios, using the developed approach.

Table 9.9 Worst-case scenarios: $C/D = 0.93,\ g = 100$

Disruption scenario $(\lambda_{1s}, \lambda_{2s}, \lambda_{3s}, \lambda_{4s})$, t_s	Recovery portfolio $(V_1^s, V_2^s, V_3^s, V_4^s)$	Cost [(a)]	Service level [(b)]
Integrated approach: primary portfolio $(v_1, v_2, v_3, v_4) = (0,0.08,0.92,0)$			
Hierarchical approach: primary portfolio $(v_1, v_2, v_3, v_4) = (0,0.07,0.93,0)$			
(0,0,0,0), 1	(0,1,0,0)	2062	50
(0,0,0,0), 2	(0,1,0,0)	2065	47
(0,0,0,0), 3	(0,1,0,0)	2069	43 [(c)]
(0,0,0,1), 1	(0,0,0,1)	492	50
(0,0,0,1), 2	(0,0,0,1)	496	47
(0,0,0,1), 3	(0,0,0,1)	499	43 [(c)]
(1,0,0,0), 3	(1,0,0,0)	331	50
(1,0,0,1), 3	(1,0,0,0)	331	50
(0,0,0,2), 3	(0,0,0,1)	102	50
(1,0,0,2), 3	(0,0,0,1)	102	50

[(a)] $\sum_{i\in I} e_i(u_i + U_i^s - q_i^s)/D + \sum_{i\in I} \rho_{is} U_i^s/D + \sum_{i\in I} o_i(\gamma_{i,\lambda_{is}} v_i + V_i^s) + g(1 - \sum_{t\in T} x_t^s/D)$
[(b)] $\sum_{t\in T} x_t^s/D \times 100$
[(c)] $\underline{SL} \times 100$, (9.20)

9.5.1.1 Best-Case and Worst-Case Analysis

In order to provide more insights, best-case and worst-case scenarios were identified such that service level associated with optimal solution is not less than capacity-to-demand ratio C/D and not greater than some fraction of C/D, respectively. For example, the total number of best-case scenarios for $C/D = 0.93$ and unit penalty $g = 100$, with service level $\sum_{t\in T} x_t^s/D = 0.93$ is 298 and 192, respectively for integrated and hierarchical approach. The corresponding cost per product is ranging, respectively from 15 to 36.69 and from 15 to 20.65. The best-case scenarios for hierarchical approach are a subset of those for the integrated approach. In contrast, the number of worst-case scenarios is much lower. Table 9.9 presents all worst-case scenarios for unit penalty $g = 100$, with service level $\sum_{t\in T} x_t^s/D \le 0.5$. For both integrated and hierarchical approach the same set of 10 worst-case scenarios were identified. The table shows disruption scenario, $(\lambda_{1s}, \lambda_{2s}, \lambda_{3s}, \lambda_{4s})$ along with its start time, t_s, and the corresponding optimal recovery supply portfolio, $(V_1^s, V_2^s, V_3^s, V_4^s)$, cost per product, $\sum_{i\in I} e_i(u_i + U_i^s - q_i^s)/D + \sum_{i\in I} \rho_{is}U_i^s/D + \sum_{i\in I} o_i(\gamma_{i,\lambda_{is}} v_i + V_i^s) + g(1 - \sum_{t\in T} x_t^s/D)$, and service level $\sum_{t\in T} x_t^s/D$. Notice that the lowest service level, $\underline{SL} = 0.43$, (9.20), is attained for two scenarios only.

Table 9.9 indicates that for worst-case scenarios all primary suppliers are shutdown as well as the other suppliers are hit by disruptions. When all suppliers were shutdown, then a single sourcing recovery portfolio is selected with one of primary suppliers chosen as recovery supplier. Otherwise, a single recovery supplier is selected from among those less severely disrupted.

Similar results were obtained for examples with other ratios C/D and unit penalty g as well as for scenarios with a common disruption start time.

Overall, the main results of computational study for the risk-neutral models are in line with other research and indicate that:

- *for both cost and service level objective function, the integrated decision-making selects a more diversified primary supply portfolio, that will hedge against all potential disruption scenarios,*
- *the primary supply portfolio for the hierarchical approach is made up of cheapest suppliers or a single, most reliable primary supplier only, to minimize expected cost or maximize expected service level, respectively,*
- *a single sourcing recovery supply portfolio is usually selected when all primary suppliers are shutdown by disruption.*

The computational experiments were performed using the AMPL programming language and the Gurobi 7.0.0 solver on a MacBookPro laptop with Intel Core i7 processor running at 2.8 GHz and with 16GB RAM. The portfolio approach leads to SMIP formulations with a strong LP relaxation and proves to be computationally very efficient. The solver was capable of finding proven optimal solution for all examples with CPU time ranging from fraction of a second for MIP models **PSupport** and **RSupport(s)** to a few seconds for SMIP models **Support_E** and **RSupport_E**.

9.5.2 *Risk-Averse Decision-Making*

In this subsection some computational examples are presented to illustrate the risk-averse selection of primary and recovery supply portfolios to minimize CVaR of cost or maximize CVaR of service level. In the computational experiments data sets provided at the beginning of this section were used with producer per period capacity fixed to $c = 10000$ and hence $C/D = 0.93$. In addition, for minimization of CVaR of cost, unit penalty was fixed to $g = 100$. The solution results are shown in Tables 9.10 and 9.11, respectively for scenarios with a common disruption start

Table 9.10 Risk-averse solutions for a common disruption start time: $C/D = 0.93$

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **Support_CV(c)**, $g = 100$:					
Var. = 10462, Bin. = 2052, Cons. = 10718, Nonz. = 163986 [a]					
$CVaR^c$	15.50	15.92	16.93	18.03	22.37
VaR^c	15.05	15.09	15.57	16.09	18.82
E^c	15.28	15.30	15.71	15.65	17.70
$E^{sl}100\%$	93.29	93.31	92.92	93.13	93.21
Primary Portfolio: Supplier(% of total demand) [b]	1(2)	1(3)	1(3)	1(3)	1(4)
	2(5)	2(4)	2(5)	2(7)	2(64)
	3(93)	3(93)	3(92)	3(90)	3(31)
	4(0)	4(0)	4(0)	4(0)	4(0)
Exp.Recovery Portfolio: Supplier(% of total demand) [c]	1(0.34)	1(0.39)	1(0.31)	1(0.76)	1(0.29)
	2(1.81)	2(1.85)	2(1.78)	2(1.82)	2(2.34)
	3(0.03)	3(0.02)	3(0.07)	3(0.14)	3(1.67)
	4(6.69)	4(6.69)	4(6.64)	4(5.93)	4(1.39)
Model **Support_CV(sl)**:					
Var. = 10462, Bin. = 2052, Cons. = 10718, Nonz. = 159122 [a]					
$CVaR^{sl}100\%$	93.32	93.30	93.25	93.17	92.53
$VaR^{sl}100\%$	93.33				
$E^{sl}100\%$	93.32				
E^c	19.04	19.06	19.06	19.02	19.06
Primary Portfolio: Supplier(% of total demand) [b]	1(27)				
	2(25)				
	3(7)				
	4(41)				
Exp.Recovery Portfolio: Supplier(% of total demand) [c]	1(1.81)	1(1.14)	1(1.14)	1(1.80)	1(1.52)
	2(0.86)	2(1.55)	2(1.55)	2(0.86)	2(1.16)
	3(2.00)	3(2.41)	3(2.41)	3(2.54)	3(2.25)
	4(1.07)	4(0.64)	4(0.64)	4(0.53)	4(0.81)

[a] Var. = number of variables, Bin. = number of binary variables, Cons. = number of constraints, Nonz. = number of nonzero coefficients
[b] $1(v_1 \times 100), 2(v_2 \times 100), 3(v_3 \times 100), 4(v_4 \times 100)$
[c] $1(\sum_{s\in S} P_s V_1^s \times 100), 2(\sum_{s\in S} P_s V_2^s \times 100), 3(\sum_{s\in S} P_s V_3^s \times 100), 4(\sum_{s\in S} P_s V_4^s \times 100)$

Table 9.11 Risk-averse solutions for different disruption start times: $C/D = 0.93$

Confidence level α	0.50	0.75	0.90	0.95	0.99
Model **Support_CV(c)**, $g = 100$:					
Var. = 31365, Bin. = 6148, Cons. = 32133, Nonz. = 488799 [(a)]					
$CVaR^c$	15.68	16.11	17.07	18.13	22.49
VaR^c	15.25	15.28	15.81	16.22	18.90
E^c	15.47	15.49	15.64	15.80	17.43
$E^{sl}100\%$	93.28	93.29	93.29	93.22	93.08
Primary Portfolio: Supplier(% of total demand) [(b)]	1(5)	1(4)	1(6)	1(6)	1(8)
	2(5)	2(6)	2(6)	2(7)	2(46)
	3(90)	3(90)	3(83)	3(80)	3(46)
	4(0)	4(0)	4(5)	4(7)	4(0)
Exp.Recovery Portfolio: Supplier(% of total demand) [(c)]	1(0.21)	1(0.18)	1(0.31)	1(0.45)	1(0.62)
	2(1.76)	2(1.71)	2(1.80)	2(2.45)	2(1.19)
	3(0.23)	3(0.13)	3(0.26)	3(0.38)	3(1.09)
	4(6.46)	4(6.60)	4(6.12)	4(5.13)	4(2.44)
Model **Support_CV(sl)**:					
Var. = 31365, Bin. = 6148, Cons. = 32133, Nonz. = 474207 [(a)]					
$CVaR^{sl}100\%$	93.32	93.30	93.25	93.17	92.50
$VaR^{sl}100\%$			93.33		
$E^{sl}100\%$			93.32		
E^c	20.52	20.48	20.50	20.45	20.47
Primary portfolio: supplier(% of total demand) [(b)]			1(33)		
			2(31)		
			3(10)		
			4(26)		
Exp. recovery portfolio: supplier(% of total demand) [(c)]	1(2.23)	1(2.35)	1(2.26)	1(1.79)	1(2.31)
	2(0.89)	2(0.81)	2(0.75)	2(0.59)	2(0.54)
	3(1.27)	3(1.16)	3(1.30)	3(1.77)	3(0.97)
	4(0.84)	4(0.90)	4(0.92)	4(1.07)	4(1.41)

[(a)] Var. = number of variables, Bin. = number of binary variables, Cons. = number of constraints, Nonz. = number of nonzero coefficients
[(b)] $1(v_1 \times 100), 2(v_2 \times 100), 3(v_3 \times 100), 4(v_4 \times 100)$
[(c)] $1(\sum_{s\in S} P_s V_1^s \times 100), 2(\sum_{s\in S} P_s V_2^s \times 100), 3(\sum_{s\in S} P_s V_3^s \times 100), 4(\sum_{s\in S} P_s V_4^s \times 100)$

time and with different disruption start times. For the risk-averse solutions, VaR, CVaR and the associated expected values of cost and service level are presented for a subset of confidence levels $\alpha = 0.5, 0.75, 0.9, 0.95, 0.99$. The risk-averse portfolio selected using model **Support_CV(sl)** is more diversified than the risk-averse portfolio determined by model **Support_CV(c)**. As α increases, $CVaR^c$ of cost increases and $CVaR^{sl}$ of service level decreases. At the same, a greater diversification of the primary supply portfolio is observed for model **SupportCV(c)**, with more demand shifted from the cheapest supplier 3 to more reliable supplier 2. For model **Sup-**

port_CV(sl), however, VaR^{sl}, E^{sl} and primary supply portfolio are independent of α. $CVaR^{sl}$ and E^{sl} are very close to VaR^{sl}, while the latter is equal to $C/D = 0.93$. Comparison of solution results in Tables 9.10 and 9.11, for scenarios with a common and different disruption start times indicate that while the latter scenarios lead to higher values of cost, the corresponding values of service level are nearly identical.

Comparison of solution results for models **Support_E** and **Support_CV(c)**, **Support_CV(sl)** (cf. Tables 9.5, 9.10 and 9.8, 9.11), demonstrates that the risk-neutral solutions for minimization of expected cost are very close to the corresponding risk-averse solutions with confidence level $\alpha = 0.5$. Moreover, for the service level objective, the corresponding risk-neutral and the risk-averse primary supply portfolios are identical.

All the above results indicate that *for the service level objective and the example problem parameters, the impact of the worst-case disruption scenarios can be fully mitigated by the optimal selection of primary and recovery supply portfolios.*

The computational experiments were performed using the AMPL programming language and the Gurobi 7.0.0 solver on a MacBookPro laptop with Intel Core i7 processor running at 2.8 GHz and with 16GB RAM. Proven optimal solutions were found for all examples with CPU time ranging from fraction of a second for model **Support_CV(sl)** to a few seconds for model **Support_CV(c)**.

9.6 Notes

The low-probability and high-impact disruptions of material flows in global supply chains and the resulting losses may threaten financial state of firms. For example, the disruptive events that occurred in 2011 in the automotive and electronics supply chains (the Great East Japan earthquake and tsunami in March and Thailand's floods in October) resulted in huge losses of major automakers and electronics manufacturers, e.g., Park et al. (2013), Haraguchi and Lall (2015). In order to minimize losses caused by the shortage of material supplies, customer companies (firms) apply different disruption management strategies, such as maintaining inventory, buying from an alternate supplier or helping a primary supplier recover more quickly. When the latter strategy is applied, the firm participates in supplier's recovery process after disruption to reduce recovery time, which means that the firm may participate in suppliers cost-to-recover. The real-world examples presented in Sect. 1.1 illustrate a typical disruption management strategy. Whenever a primary supplier is hit by disruption, the customer company needs to choose whether to support recovery of disrupted primary supplier or select an alternate (recovery) supplier, non-disrupted or disrupted less severely than the primary supplier. The recovery suppliers are selected in such a way that the recovery process is optimized with respect to recovery time and cost. However, the literature on mitigation the impact of disruption risks and optimization of a recovery process in supply chains is limited (e.g., Tomlin 2006; Ruiz-Torres et al. 2013; MacKenzie et al. 2014; Zeng and Xia 2015; Hamdi et al. 2015; Ivanov et al. 2016). A recent literature review on Operations Research/Management Science models for supply chain disruptions was presented by Snyder et al. (2016). They discussed 180 scholarly works on the topic, organized into six categories: evaluating supply disruptions; strategic decisions; sourcing decisions; contracts and incentives; inventory; and facility location. Paul et al. (2016a) reviewed literature on managing risk and disruption in production-inventory systems and supply chains. They considered four categories of disruptions: disruption in production, disruption in supply, disruption in transportation, and fluctuation in demand. The authors focused on reviewing the mathematical models and the solution approaches used in solving the models using both hypothetical and real-world problem scenarios.

In practice, the probability and magnitude of low-probability and high-impact disruptive events is difficult and often impossible to be estimated. The classical risk assessment and mitigation methods that require such knowledge at an early stage of the risk analysis may fail to prepare supply chains for such disruptive events. A novel risk-exposure model for analyzing operational-disruption risk with no need to estimate the probability of any specific disruptive events, was proposed by Simchi-Levi et al. (2015). The model is capable of assessing the impact of a disruption originating anywhere in a supply chain. The model has been applied by Ford Motor Company to identify risk exposures, evaluate risk mitigation actions, and develop optimal contingency plans in the automotive supply chain.

In another stream of research Paul et al. (2014a, b, 2015a, b) developed mathematical models and solution algorithms for disruption management in production–inventory systems, under single or multiple disruptions. The authors considered back orders, lost sales and/or outsourcing options as recovery strategies.

The material presented in this chapter is based on results achieved by Sawik (2017), where a computationally efficient portfolio approach developed in Sawik (2011a, b, 2013a, b, c, d) was enhanced for simultaneous or sequential selection of primary and recovery supply portfolios under local and regional disruptions risks. However, in Sawik (2017) only a risk-neutral, expected cost (or expected service level) objective has been considered to optimize an overall performance of a supply chain. Minimizing expected cost (or maximizing expected service level) under disruption risks may sometimes be impractical in the long run, especially when large losses could threaten financial state of firms. Then, a downside risk measure such as expected worst-case cost, may be more appropriate. In this chapter, the portfolio approach and the risk-neutral models developed by Sawik (2017) have been enhanced to assess the effect of risk-averse decision making on the selection of primary and recovery supply portfolios, using CVaR as a risk measure.

In the literature on supply uncertainty, the supply is either subject to complete disruptions or yield uncertainty. Yield uncertainty occurs when the quantity of supply delivered is a random variable, modeled as either a random additive or multiplicative quantity, whereas disruptions occur when supply is subject to partial or complete failure. Typically disruptions are modeled as events which occur randomly and may have a random length. Schmitt and Snyder (2012) considered inventory systems subject to both supply disruptions and yield uncertainty. They compared single-period versus multi-period models and showed that the former can lead to selecting the wrong strategy for mitigating supply risk.

The future research should concentrate on relaxations of the various simplified assumptions used to formulate the problem. For example, the assumption that each supplier has sufficient capacity to meet total demand for parts that allows for a single sourcing, can be easily relaxed. The developed models can be enhanced for finite capacity suppliers. A single recovery mode defined by constant recovery time, $TTR(i, l)$ and the associated recovery cost, $CTR(i, l)$ for each supplier i and each disruption level l, can be replaced by a number of available recovery modes, each represented by different value of time, $TTR(i, l)$ and the associated cost, $CTR(i, l)$. Then, a new recovery mode selection variable can enhance the problem formulation. In a more general setting, both disruption start time, recovery time and recovery cost can be modeled as random parameters, e.g., Schmitt (2011), Schmitt and Singh (2012), Paul et al. (2016c).

Problems

9.1 In the SMIP models presented in this chapter introduce delay penalty when demand d_t for period t is not met by that period.

9.2 In the SMIP models presented in this chapter introduce transition time required for switching to recovery supplier different from the primary supplier.

9.3 Enhance the SMIP models presented in this chapter for the limited output inventory of products at the producer. Formulate the inventory balance constraints that should be added to the models.

9.4 Formulate the mean-risk models **Support_ECV(c)** and **Support_ECV(sl)** to minimize expected cost and CVaR of cost or maximize expected service level and CVaR of service level, respectively.

9.5 Enhance model **Support_E** for multiple recovery modes defined by different recovery time, $TTR(i, l)$ and the associated recovery cost, $CTR(i, l)$ for each supplier i and each disruption level l.

Chapter 10
Selection of Primary and Recovery Supply and Demand Portfolios and Scheduling

10.1 Introduction

In this chapter the portfolio approach proposed in Chap. 9 for the selection of primary and recovery suppliers and order quantity allocation to mitigate the impact of disruption risks is enhanced also for the recovery process of the firm's assembly plants for finished products. Unlike most of reported research on supply chain disruption management a disruptive event is assumed to impact both a primary supplier of parts and the buyer's firm primary assembly plant. Then the firm may choose alternate (recovery) suppliers and move production to alternate (recovery) plants along with transshipment of parts from the impacted primary plant to the recovery plants. The resulting allocation of unfulfilled demand for parts among recovery suppliers and unfulfilled demand for products among recovery assembly plants determines recovery supply and demand portfolio, respectively. The enhanced portfolio approach and SMIP formulations with an embedded network flow problem developed in this chapter are capable of selecting primary suppliers, the decision to be implemented before a disruption and of selecting recovery suppliers and recovery assembly plants, the decision to be implemented during and after the disruption. The supply and demand portfolios are determined along with production scheduling in assembly plants. Multi-level disruptions of suppliers and assembly plants will be considered. The objective of selection primary and recovery suppliers and assembly plants and allocation of order quantity for parts and demand for products is to mitigate the impact of disruption risks and optimize the recovery process. The two decision making approaches will be considered: an integrated approach with the perfect information about the future disruption scenarios, and a hierarchical approach with no such information available ahead of time. In the integrated approach, which accounts for all potential disruption scenarios, the primary supply portfolio that will hedge against all scenarios is determined along with the recovery supply and demand portfolios and production schedule of finished products for each scenario, to minimize expected cost (or CVaR of cost) or maximize expected service level (or CVaR

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of service level) over all scenarios. In the hierarchical approach first the primary supply portfolio is selected, and then, when a primary supplier or primary assembly plant is hit by a disruption, the recovery supply and demand portfolios are selected to optimize the process of recovery from the disruption.

The following time-indexed SMIP and MIP models are presented in this chapter:

DSupport_E for risk-neutral selection of primary and recovery supply and demand portfolios and production scheduling to minimize expected cost;

DSupportMP_E model **DSupport_E** for multiple part types and product types;

PSupport for selection of primary supply portfolio and production scheduling to minimize cost under deterministic conditions;

RDSupport(s) for selection of recovery supply and demand portfolios and production scheduling to minimize cost, for predetermined primary portfolios and the realized disruption scenario;

RDSupport_E model **DSupport_E** for predetermined primary portfolios.

DSupport_CV(c) for risk-averse selection of primary and recovery supply and demand portfolios and production scheduling to minimize CVaR of cost;

DSupport_CV(sl) for risk-averse selection of primary and recovery supply and demand portfolios and production scheduling to maximize CVaR of service level;

RDSupport_CV(c) model **DSupport_CV(c)** for predetermined primary portfolios;

RDSupport_CV(sl) model **DSupport_CV(sl)** for predetermined primary portfolios.

Numerical examples, computational results, best-case and worst-case analysis, and some comparison of the two approaches for the selection of primary and recovery supply and demand portfolios are provided in Sects. 10.5.1 and 10.5.2, respectively for risk-neutral and risk-averse decision-making.

10.2 Problem Description

In this section the problem of simultaneous selection of primary and recovery supply and demand portfolios is considered.

Consider a supply chain in which a single producer of one product type, assembles products in several assembly plants to meet customer demand, using a critical part type that can be manufactured and provided by several suppliers (for notation used, see Table 10.1).

Let $I = \{1, \ldots, \overline{I}\}$ be the set of $\overline{I}$ suppliers, $J = \{1, \ldots, \overline{J}\}$ be the set of $\overline{J}$ assembly plants, $T = \{1, \ldots, \overline{T}\}$ the set of $\overline{T}$ planning periods, and denote by D the

Table 10.1 Notation: selection of primary and recovery supply and demand portfolios and scheduling

Indices	
i	= supplier, $i \in I$
j	= assembly plant, $j \in J$
l	= disruption level, $l \in L_i, i \in I, l \in L^j, j \in J$
r	= region, $r \in R$
s	= disruption scenario, $s \in S$
t	= planning period, $t \in T$
Input Parameters	
c_j	= per period capacity of plant j
D	= total demand for products
ε_j	= additional per unit production cost at recovery plant $j > 1$
e_i	= fixed ordering cost for supplier i
φ_j	= fixed production setup cost at recovery plant $j > 1$
ψ_j	= per unit transshipment cost from primary plant $j = 1$ to plant $j > 1$
g	= per unit penalty cost for unfulfilled demand for products
o_i	= per unit price of parts purchased from supplier i
p_{il}	= probability of disruption level l for supplier i
π_{jl}	= probability of disruption level l for plant j
p^r	= regional disruption probability for region r
t_s	= start time period of disruptive event s
γ_{il}	= fraction of an order delivered by supplier i under disruption level l (supplier fulfillment rate)
δ_{jl}	= fraction of capacity of plant j available under disruption level l
τ_{ij}	= delivery lead time from supplier i to plant j
σ_j	= transshipment time from primary plant $j = 1$ to plant $j > 1$
θ_{is}	= time-to-recover of supplier i from disruption under scenario s
ϑ_{js}	= time-to-recover of plant j from disruption under scenario s
ρ_{is}	= cost-to-recover of supplier i from disruption under scenario s
ρ_{js}	= cost-to-recover of plant j from disruption under scenario s

total demand for products. Let $j = 1$ be the primary plant, where the total demand for products, D, is initially assigned.

The suppliers of parts and assembly plants are located in $\overline{R}$ geographic regions, subject to potential regional disasters that may result in complete shutdown of all suppliers and plants in the same region simultaneously. Denote by I^r and J^r, respectively the subsets of suppliers and plants in region $r \in R$, and by p^r, the regional disruption probability for region r.

In addition to correlated regional disruptions, each supplier $i \in I$ is subject to independent local disruptions of different levels, $l \in L_i = \{0, \ldots, \overline{L_i}\}$, where disruption level refers to the fraction of an order that can be delivered, see, Sect. 9.2.

Level $l = 0$ represents complete shutdown of a supplier, i.e., no order delivery, while level $l = \overline{L_i}$ represents normal conditions with no disruption, i.e., full order delivery. Denote by p_{il}, the probability of disruption level l for supplier i, and by γ_{il}, the fraction of an order that can be delivered by supplier i under disruption level l (supplier fulfillment rate),

$$\gamma_{il} \begin{cases} = 0 & \text{if } \ l = 0 \\ \in (0, 1) & \text{if } \ l = 1, \ldots, \overline{L_i} - 1 \\ = 1 & \text{if } \ l = \overline{L_i}. \end{cases} \tag{10.1}$$

Similarly to suppliers, each plant $j \in J$ is subject to random local disruptions of different levels, $l \in L^j = \{0, \ldots, \overline{L^j}\}$, where disruption level refers to available fraction of full capacity, c_j, available per period under normal conditions. Level $l = 0$ represents complete shutdown of an assembly plant, while level $l = \overline{L^j}$ represents normal conditions, i.e., full capacity, c_j, available. Denote by π_{jl}, the probability of disruption level l for plant j, and by δ_{jl}, the fraction of available capacity of plant j under disruption level l.

$$\delta_{jl} \begin{cases} = 0 & \text{if } \ l = 0 \\ \in (0, 1) & \text{if } \ l = 1, \ldots, \overline{L^j} - 1 \\ = 1 & \text{if } \ l = \overline{L^j}. \end{cases} \tag{10.2}$$

The total number of all potential scenarios is $\overline{S} = \prod_{i \in I}(\overline{L_i} + 1) \prod_{j \in J} (\overline{L^j} + 1)$. Each scenario $s \in S$ is represented by an $(\bar{I} + \bar{J})$- dimensional vector $\lambda_s = \{\lambda_{1s}, \ldots, \lambda_{\bar{I}s}, \lambda_{\bar{I}+1,s}, \ldots, \lambda_{\bar{I}+\bar{J},s}\}$, where $\lambda_{is} \in L_i$ is the disruption level of supplier $i \in I$ and $\lambda_{\bar{I}+j,s} \in L^j$ is the disruption level of plant $j \in J$, under scenario $s \in S$.

The probability P_s of disruption scenario $s \in S$ is $P_s = \prod_{r \in R} P_s^r$, where P_s^r is the probability of realizing disruption scenario s in region r

$$P_s^r = \begin{cases} (1 - p^r)(\prod_{i \in I^r} \prod_{l \in L_i : \lambda_{is} = l} p_{il})(\prod_{j \in J^r} \prod_{l \in L^j : \lambda_{\bar{I}+j,s} = l} \pi_{jl}), \\ \text{if } \ \sum_{i \in I^r} \lambda_{is} + \sum_{j \in J^r} \lambda_{\bar{I}+j,s} > 0 \\ p^r + (1 - p^r)(\prod_{i \in I^r} p_{i0})(\prod_{j \in J^r} \pi_{j0}), \\ \text{if } \ \sum_{i \in I^r} \lambda_{is} + \sum_{j \in J^r} \lambda_{\bar{I}+j,s} = 0. \end{cases} \tag{10.3}$$

When supplier i is hit by disruption at level l, its recovery process to normal conditions takes $TTR(i, l)$ time periods (Time-To-Recover) and let $CTR(i, l)$ be the firm's portion of Cost-To-Recover. For each supplier i, denote by θ_{is} and ρ_{is}, respectively time-to-recover and firm's portion of cost-to recover from disruption under scenario s

$$\theta_{is} = TTR(i, l); \ \ i \in I, s \in S : l = \lambda_{is} \tag{10.4}$$

$$\rho_{is} = CTR(i, l); \ \ i \in I, s \in S : l = \lambda_{is}. \tag{10.5}$$

Similarly, when plant j is hit by disruption at level l, its recovery process to normal conditions takes $PRT(j, l)$ time periods (Plant Recovery Time) and cost $PRC(j, l)$ (Plant Recovery Cost). For each plant j, denote by ϑ_{js} and ρ_{js}, respectively time-to-recover and cost-to-recover from disruption under scenario s

$$\vartheta_{js} = PRT(j, l);\ j \in J, s \in S : l = \lambda_{\bar{I}+j,s} \tag{10.6}$$

$$\rho_{js} = PRC(j, l);\ j \in J, s \in S : l = \lambda_{\bar{I}+j,s}. \tag{10.7}$$

The orders for parts are assumed to be placed at the beginning of the planning horizon, and under normal conditions the parts ordered from supplier i are delivered to assembly plant j in period τ_{ij}, where τ_{ij} is the total of manufacturing lead time and transportation time. Denote by σ_j the transshipment time from primary plant $j = 1$ to plant j.

The firm who moves production to an alternate assembly plant $j \in J$ incurs a fixed cost φ_j and encounters additional per unit cost of production ε_j, and per unit cost, ψ_j, of transshipment of parts from the primary plant, where $\varphi_1 = 0$, $\varepsilon_1 = 0$ and $\psi_1 = 0$. A recovery plant can be a disrupted primary plant $j = 1$ with reduced capacity during recovery process and then with its full capacity or a new plant, non-disrupted or disrupted less severely than the primary plant.

The following assumptions are made to formulate the problem.

- Each supplier has sufficient capacity to meet total demand for parts.
- A single disruptive event is assumed to occur over the entire planning horizon. Multiple disruptions, one after the other in a series, during the recovery process are not considered.
- Partial recovery of a disrupted supplier to its partial capacity as well as partial recovery of a disrupted assembly plant to its partial capacity are not considered.
- Time-to recover and the associated cost-to-recover are constant parameters that represent recovery of a disrupted supplier or a disrupted assembly plant to its full capacity.
- A single recovery mode is considered for each supplier, each assembly plant and each disruption level.
- Disruption to primary supplier (primary assembly plant) under scenario $s \in S$ occurs in period t_s and recovery process starts in period $t_s + 1$, so that the disrupted supplier $i \in I$ (disrupted plant $j \in J$) returns to its full capacity in period $t = t_s + \theta_{is}$ $(t = t_s + \vartheta_{js})$.
- A recovery supplier can be a disrupted primary supplier after its recovery to full capacity or a new supplier.
- A recovery assembly plant can be a disrupted primary assembly plant during and after its recovery to full capacity or a new assembly plant.
- Transshipment of parts from the impacted primary assembly plant to recovery plants starts along with the primary plant recovery process.
- Transition time required for switching to recovery supplier different from the primary suppliers is negligible.
- The buyer firm participates in supplier's cost-to-recover.

Table 10.2 Variables: selection of primary and recovery supply and demand portfolios and scheduling

	First stage variables
u_i	$=1$, if supplier i is selected as a primary supplier; otherwise $u_i=0$ (primary supplier selection)
v_i	$\in[0,1]$, the fraction of total demand for parts ordered from primary supplier i to primary plant $j=1$ (primary supply portfolio)
	Second stage variables
U_i^s	$=1$, if supplier i is selected as a recovery supplier under disruption scenario s; otherwise $U_i^s=0$ (recovery supplier selection)
V_{ij}^s	$\in[0,1]$, the fraction of total demand for parts ordered from recovery supplier i to recovery plant j, under disruption scenario s (recovery supply portfolio)
w_j^s	$\in[0,1]$, the fraction of total demand for parts, transshipped from the primary plant $j=1$ to recovery plant j, under disruption scenario s, where w_1^s represents the parts that remain in the primary plant $j=1$ (transshipment variables)
x_{jt}^s	≥ 0, production in plant j in period t under disruption scenario s (production scheduling)
y_j^s	$=1$, if assembly plant j is selected as a recovery plant under disruption scenario s; otherwise $y_j^s=0$ (recovery plant selection)
z_j^s	$\in[0,1]$ the fraction of total demand for products to be completed by recovery plant j under disruption scenario s (recovery demand portfolio).
	Auxiliary variables
q_i^s	$=1$, if $u_i=U_i^s=1$; otherwise $q_i^s=0$ (elimination of double fixed ordering costs)
VaR^c	Cost-at-Risk, the targeted cost such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is below VaR^c
VaR^{sl}	Service-at-Risk, the targeted service level such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is above VaR^{sl}
$\mathscr{C}_s$	≥ 0, the tail cost for scenario s, i.e., the amount by which costs in scenario s exceed VaR^c
$\mathscr{S}_s$	≥ 0, the tail service level for scenario s, i.e., the amount by which VaR^{sl} exceeds service level in scenario s

- A penalty cost is charged for the demand for products unfulfilled by the end of the planning horizon.

10.3 Models for Risk-Neutral Decision-Making

10.3.1 Integrated Selection of Primary and Recovery Supply and Demand Portfolios

In this subsection a SMIP model **DSupport_E** is presented for the integrated selection of supply and demand portfolios under disruption risks.

The following primary and recovery portfolios are jointly determined using the proposed model (for definitions of first and second stage variables, see Table 10.2):

- Primary supply portfolio: $v_i \in [0, 1]$ - the fraction of total demand for parts ordered from primary supplier i to be delivered to primary plant $j = 1$.
- Recovery supply portfolio: $V_{ij}^s \in [0, 1]$ - the fraction of total demand for parts ordered from recovery supplier i to recovery assembly plant j, under disruption scenario s.
- Recovery demand portfolio: $z_j^s \in [0, 1]$ is the fraction of total demand for products to be completed by recovery plant j under disruption scenario s.

Since the total demand for products has been assigned to the single primary assembly plant $j = 1$ and all parts from the primary suppliers are ordered for that plant, the primary demand portfolio is not considered.

The primary supply portfolio, $(v_1, \dots, v_{\bar{I}})$, which determines supply of parts to the primary assembly plant $j = 1$ is selected along with the recovery supply portfolio for each disruption scenario. The recovery supply portfolio for scenario s is determined by supplies of parts, DV_{ij}^s, from recovery suppliers, $i \in I$, to recovery assembly plants, $j \in J$, where $\sum_{i\in I}(\gamma_i^s v_i + \sum_{j\in J} V_{ij}^s) = 1$ and $0 \le V_{ij}^s \le 1, i \in I, j \in J$. The selection of recovery supply and demand portfolios may be combined with transshipment of parts from the primary plant to recovery plants.

Let E^c be the expected cost per product to be minimized, and E^{sl}, the expected service level, i.e., the expected fraction of the total fulfilled demand for products (i.e., the expected demand fulfillment rate) to be maximized.

$$E^c = \sum_{s\in S} P_s \sum_{i\in I}(e_i(u_i + U_i^s - q_i^s)/D + \rho_{is}U_i^s/D + o_i(\gamma_i^s v_i + \sum_{j\in J} V_{ij}^s))$$
$$+ \sum_{s\in S} P_s \sum_{j\in J}(\psi_j w_j^s + (\varphi_j + \rho_{js})y_j^s/D + \varepsilon_j \sum_{t\in T} x_{jt}^s/D) + g(1 - E^{sl}), \quad (10.8)$$

$$E^{sl} = \sum_{s\in S}\sum_{j\in J}\sum_{t\in T} P_s x_{jt}^s/D. \quad (10.9)$$

The auxiliary variable, q_i^s, is introduced to eliminate double charging with fixed ordering cost e_i of each supplier i, who is selected both as primary and recovery supplier: $q_i^s = 1$, if $u_i = U_i^s = 1$; otherwise $q_i^s = 0$.

The expected cost per product, E^c, (10.8), constitutes of different fixed and variable cost per product. The fixed cost per product includes expected cost: of ordering parts from primary suppliers and from recovery suppliers different from primary suppliers, $\sum_{i\in I} e_i u_i/D + \sum_{s\in S}\sum_{i\in I} P_s e_i(U_i^s - q_i^s)/D$, of recovery process for impacted suppliers selected as recovery suppliers, $\sum_{s\in S}\sum_{i\in I} P_s \rho_{is} U_i^s/D$, of moving production from primary assembly plant to recovery assembly plants, $\sum_{s\in S}\sum_{j\in J} P_s \varphi_j y_j^s/D$, and of recovery process for impacted assembly plants selected as recovery plants, $\sum_{s\in S}\sum_{j\in J} P_s \rho_{js} y_j^s/D$. The variable cost per product includes cost: of purchasing parts for partially fulfilled deliveries from primary suppliers, $\sum_{s\in S}\sum_{i\in I} P_s o_i \gamma_i^s v_i$, of purchasing parts from recovery suppliers, $\sum_{s\in S}\sum_{i\in I}\sum_{j\in J} P_s o_i V_{ij}^s$, of transshipment of parts from primary plant to recovery plants, $\sum_{s\in S}\sum_{j\in J} P_s \psi_j w_j^s$, of additional production cost in recovery plants different from primary plant, $\sum_{s \in S} \sum_{j \in J} \sum_{t \in T} \varepsilon_j x^s_{jt}/D$, and of penalty for unfulfilled demand, $g(1 - \sum_{s \in S} \sum_{j \in J} \sum_{t \in T} P_s x^s_{jt}/D)$.

DSupport_E: *Risk-neutral selection of primary and recovery* **D***emand and* **Sup***ply* **port***folios and production scheduling*

Minimize (10.8)

subject to

Primary supply portfolio selection constraints

- the total demand for parts must be fully allocated among the selected primary suppliers (i.e., the primary supply portfolio must be selected),

- demand for parts cannot be assigned to non-selected primary suppliers,

$$\sum_{i \in I} v_i = 1 \tag{10.10}$$

$$v_i \le u_i; \; i \in I \tag{10.11}$$

Recovery supply and demand portfolio selection constraints

- the unfulfilled demand for parts must be fully allocated among the selected recovery suppliers (i.e., the recovery supply portfolio must be selected),

- the unfulfilled demand for parts cannot be assigned to non-selected recovery suppliers,

- the unfulfilled demand for products must be fully allocated among the selected plants (i.e., the recovery demand portfolio must be selected),

- the unfulfilled demand for products cannot be assigned to non-selected recovery plants,

- the supply and demand portfolio flow conservation constraints ensure that for each recovery plant, the recovery supplies and transshipment of parts are in balance with the demand for products to be fulfilled by that plant,

- the balance constraints for parts ensure that the total transshipment of parts from primary plant is equal to the initial inventory of parts and partially fulfilled supplies from primary suppliers less the usage of parts for production before a disruptive event,

- each supplier selected to both primary and recovery portfolio is charged exactly once with fixed ordering cost in the objective function,

$$\sum_{i \in I} (\gamma^s_i v_i + \sum_{j \in J} V^s_{ij}) = 1 - V_0; \; s \in S \tag{10.12}$$

$$V^s_{ij} \le U^s_i; \; i \in I, j \in J, s \in S \tag{10.13}$$

$$\sum_{t \in T: t < t_s} x^s_{1t}/D + \sum_{j \in J} z^s_j = 1; \; s \in S \tag{10.14}$$

$$z^s_j \le y^s_j; \; j \in J, s \in S \tag{10.15}$$

$$\sum_{i\in I} V_{ij}^s + w_j^s = z_j^s;\ j \in J, s \in S \tag{10.16}$$

$$\sum_{j\in J} w_j^s = V_0 + \sum_{i\in I} \gamma_i^s v_i - \sum_{t\in T: t<t_s} x_{1t}^s/D;\ s \in S \tag{10.17}$$

$$q_i^s \le (u_i + U_i^s)/2;\ i \in I, s \in S, \tag{10.18}$$

where DV_0 is the initial inventory of parts at primary assembly plant $j = 1$.

Production capacity constraints

- before a disruptive event, the production at primary plant in every period cannot exceed its capacity,
- after a disruptive event, the production at each selected recovery plant in every period cannot exceed the plant available capacity,
- the total production at each recovery plant cannot exceed the assigned portion of total demand for products,

$$x_{1t}^s \le c_1;\ t \in T, s \in S : t < t_s \tag{10.19}$$

$$x_{jt}^s \le c_{jt}^s y_j^s;\ j \in J, t \in T, s \in S : t \ge t_s \tag{10.20}$$

$$\sum_{t\in T: t\ge t_s} x_{jt}^s/D \le z_j^s;\ j \in J, s \in S \tag{10.21}$$

Supply-transshipment-production coordinating constraints

- for each disruption scenario s and each period t, the cumulative demand for parts of production in primary assembly plant $j = 1$, scheduled in periods 1 through t cannot exceed the initial inventory of parts and the cumulative deliveries by period $t - 1$ (delivery in period $\tau_{i1} \le t - 1$ from each primary supplier i and in period $t_s + \theta_{is} + \tau_{i1} \le t - 1$ from each recovery supplier i), less transhipment of parts to recovery assembly plants $j > 1$,
- for each disruption scenario s and each period $t \le \sigma_j$, the cumulative demand for parts of production scheduled in periods 1 through t in recovery assembly plant $j > 1$, cannot exceed the cumulative deliveries by period $t - 1$ (delivery in period $t_s + \theta_{is} + \tau_{ij} \le t - 1$ from each recovery supplier i, that can be used for production in period $t_s + \theta_{is} + \tau_{ij} + 1$, at the earliest),
- for each disruption scenario s and each period $t \ge \sigma_j + 1$, the cumulative demand for parts of production scheduled in periods 1 through t in recovery assembly plant $j > 1$, cannot exceed the cumulative deliveries by period $t - 1$ (delivery in period $t_s + \theta_{is} + \tau_{ij} \le t - 1$ from each recovery supplier i, that can be used for production in period $t_s + \theta_{is} + \tau_{ij} + 1$, at the earliest), plus transhipment of parts from primary assembly plant $j = 1$,

$$\sum_{t'\in T: t'\le t} x_{1t'}^s/D \le$$

$$V_0 + \sum_{i\in I: \tau_{i1}\le t-1} \gamma_i^s v_i + \sum_{i\in I: t_s+\theta_{is}+\tau_{i1}\le t-1} V_{ij}^s - \sum_{j\in J: j>1} w_j^s;$$

$$t \in T, s \in S \quad (10.22)$$

$$\sum_{t' \in T: t' \le t} x^s_{jt'}/D \le \sum_{i \in I: t_s + \theta_{is} + \tau_{ij} \le t-1} V^s_{ij};$$

$$j \in J, t \in T, s \in S : j > 1, t < \sigma_j + 1 \quad (10.23)$$

$$\sum_{t' \in T: t' \le t} x^s_{jt'}/D \le \sum_{i \in I: t_s + \theta_{is} + \tau_{ij} \le t-1} V^s_{ij} + w^s_j;$$

$$j \in J, t \in T, s \in S : j > 1, t \ge \sigma_j + 1 \quad (10.24)$$

Non-negativity and integrality conditions

$$q^s_i \in \{0, 1\};\ i \in I, s \in S \quad (10.25)$$

$$u_i \in \{0, 1\};\ i \in I \quad (10.26)$$

$$v_i \in [0, 1];\ i \in I \quad (10.27)$$

$$U^s_i \in \{0, 1\};\ i \in I, s \in S \quad (10.28)$$

$$V^s_{ij} \in [0, 1];\ i \in I, j \in J, s \in S \quad (10.29)$$

$$w^s_j \in [0, 1];\ j \in J, s \in S \quad (10.30)$$

$$x^s_{jt} \ge 0;\ j \in J, t \in T, s \in S \quad (10.31)$$

$$y^s_j \in \{0, 1\};\ j \in J, s \in S \quad (10.32)$$

$$z^s_j \in [0, 1];\ j \in J, s \in S, \quad (10.33)$$

where c^s_{jt} is the capacity in period t of plant j under disruption scenario s

$$c^s_{jt} = \begin{cases} \delta^s_j c_j, & \text{if } t_s \le t \le t_s + \vartheta_{js} - 1 \\ c_j, & \text{if } t \le t_s - 1, t \ge t_s + \vartheta_{js}, \end{cases} \quad (10.34)$$

where $\delta^s_j = \delta_{j,\lambda_{\bar{I}+j,s}}$ is the fraction of capacity available at assembly plant j under scenario s, and $\lambda_{\bar{I}+j,s}$ is disruption level of plant j under scenario s.

The supply and demand portfolio selection constraints form an embedded network flow problem. In particular, Eqs. (10.16) and (10.17) are flow conservation constraints for each node j (assembly plant) and node $j = 1$ (primary assembly plant), respectively. Notice that v_i represents flow of parts from supplier i (source node) to primary plant $j = 1$ (sink/transshipment node) and w_j represents flow of parts from primary plant $j = 1$ (transshipment node) to plant j (sink node).

Model **DSupport_E** is a deterministic equivalent mixed integer program of a two-stage stochastic mixed integer program with recourse and illustrates the wait-and-see approach. The primary supply portfolio selection variables, $u_i,\ v_i$, are referred to as first-stage decisions, and the recovery supply portfolio and recovery demand portfolio selection variables, $U_i^s,\ V_{ij}^s$ and $y_j^s,\ z_j^s$, as well as transshipment and production scheduling variables, w_j^s and x_{jt}^s, are referred to as recourse or second-stage decisions (see, Table 10.2). Unlike the first-stage decisions, the latter variables are dependent on disruption scenario $s \in S$.

Worst-case service level

Proposition 10.1

If there is no initial inventory of parts (i.e., $V_0 = 0$), the lowest service level, $\underline{SL}$, can be calculated as below.

$$\underline{SL} = \max_{j \in J}\{c_j[\overline{T} - \max_{s \in S}\{t_s + \max(\min_{i \in I}\{\theta_{is} + \tau_{ij}\}, \vartheta_{js}, \sigma_j)]\}/D. \tag{10.35}$$

Proof.

The lowest service level is associated with worst-case disruption scenario $s \in S$ for which primary supplier and primary assembly plant are both hit by disruption at time t_s and then the plant j with maximum capacity available after its full recovery is selected as a recovery plant. The maximum available capacity of the recovery plant is based on the number of periods remaining for production after its full recovery (after $t_s + \vartheta_{js}$ periods), after the earliest delivery of parts by a recovery supplier (i.e., after $t_s + \min_{i \in I}\{\theta_{is} + \tau_{ij}\}$ periods) and after transshippment of parts from the primary plant (i.e., after $t_s + \sigma_j$ periods), whichever occurs later.

In order to strengthen MIP model **DSupport_E** the following constraint can be added

$$\underline{SL} \le \sum_{t \in T: t < t_s} x_{1t}^s/D + \sum_{j \in J} \sum_{t \in T: t \ge t_s} x_{jt}^s/D \le 1;\ \ s \in S, \tag{10.36}$$

where the right-hand side of (10.36) is implied by Eqs. (10.14) and (10.21).

10.3.2 Multiple Part Types and Product Types

In this subsection an enhancement of model **DSupport_E** is described for multiple types of parts and products. Let H and K be, respectively, the set of part types and the set of product types, and denote by $a_{hk},\ \ h \in H, k \in K$ the number of parts type h required to produce one unit of product type k (for notation used and definition of problem variables, see Table 10.3).

If we denote by D_k the total demand for products type k, then $D = \sum_{k \in K} D_k$ is the total demand for all products and $A_h = \sum_{k \in K} a_{hk} D_k$ is the total demand for parts type h.

Table 10.3 Notation and variables: multiple part types and product types

		Indices
h	=	part type, $h \in H$
k	=	product type, $k \in K$
		Input Parameters
a_{hk}	=	Unit requirement for part type h of product type k
A_h	=	$\sum_{k \in K} a_{hk} D_k$ total demand for parts type h
D_k	=	Total demand for products type k
ε_{jk}	=	Additional per unit production cost for product type k at recovery plant $j > 1$
g_k	=	Per unit penalty cost for unfulfilled demand for products type k
o_{ih}	=	Per unit price of parts type h purchased from supplier i
		First stage variables
v_{ih}	$\in$	[0, 1], the fraction of total demand for parts type h ordered from primary supplier i to primary plant $j = 1$ (primary supply portfolio)
		Second stage variables
V^s_{ijh}	$\in$	[0, 1], the fraction of total demand for parts type h ordered from recovery supplier i to recovery plant j, under disruption scenario s (recovery supply portfolio)
w^s_{jh}	$\in$	[0, 1], the fraction of total demand for parts type h, transshipped from the primary plant $j = 1$ to recovery plant j, under disruption scenario s, where w^s_{1h} represents the parts type h that remain in the primary plant $j = 1$ (transshipment variables)
x^s_{jkt}	$\geq$	0, production in plant j of product type k in period t under disruption scenario s (production scheduling)
z^s_{jk}	$\in$	[0, 1] the fraction of total demand for products type k to be completed by recovery plant j under disruption scenario s (recovery demand portfolio).

Now, the portfolio decision variables $v_i, V^s_{ij},\ i \in I, j \in J, s \in S$ are replaced by $v_{ih}, V^s_{ijh};\ h \in H, i \in I, j \in J, s \in S$, defined as fractions of total demand A_h for parts type h, ordered from primary supplier i, recovery supplier i for recovery plant j under disruption scenario s, respectively. Accordingly, the new transshipment variable w^s_{jh} denotes the fraction of total demand A_h for parts type h transshipped from the primary plant $j = 1$ to recovery plant j, under scenario s. In addition, the production scheduling variable $x^s_{jt},\ j \in J, t \in T, s \in S$ is replaced by $x^s_{jkt},\ j \in J, k \in K, t \in T, s \in S$ - the production in plant j of product type k in period t under disruption scenario s, and $z^s_j,\ j \in J, s \in S$ by $z^s_{jk},\ j \in J, k \in K, s \in S$ - the fraction of demand for products type k to be completed in recovery plant j under disruption scenario s.

Model **DSupportMP_E** for multiple part and product types is presented below.

DSupportMP

Minimize

$$E^c = \sum_{s \in S} P_s \sum_{i \in I} (e_i(u_i + U^s_i - q^s_i) + \rho_{is} U^s_i + \sum_{h \in H} o_{ih} A_h (\gamma^s_i v_{ih} + \sum_{j \in J} V^s_{ijh}))/D$$

$$+\sum_{s\in S}P_s\sum_{j\in J}(\sum_{h\in H}\psi_j A_h w^s_{jh}+(\varphi_j+\rho_{js})y^s_j+\sum_{k\in K}\sum_{t\in T}\varepsilon_{jk}x^s_{jkt})/D$$
$$+\sum_{k\in K}g_k(1-\sum_{s\in S}P_s\sum_{j\in J}\sum_{t\in T}x^s_{jkt}/D_k),$$

subject to

$$\sum_{i\in I}v_{ih}=1;\ h\in H$$
$$v_{ih}\le u_i;\ h\in H, i\in I$$
$$\sum_{i\in I}(\gamma^s_i v_{ih}+\sum_{j\in J}V^s_{ijh})=1-V_{h0};\ h\in H, s\in S$$
$$V^s_{ijh}\le U^s_i;\ h\in H, i\in I, j\in J, s\in S$$
$$\sum_{t\in T:t<t_s}x^s_{1kt}/D_k+\sum_{j\in J}z^s_{jk}=1;\ k\in K, s\in S$$
$$z^s_{jk}\le y^s_j;\ j\in J, k\in K, s\in S$$
$$\sum_{i\in I}V^s_{ijh}+w^s_{jh}=\sum_{k\in K}a_{hk}D_k z^s_{jk}/A_h;\ h\in H, j\in J, s\in S$$
$$\sum_{j\in J}w^s_{jh}=V_{h0}+\sum_{i\in I}\gamma^s_i v_{ih}-\sum_{k\in K}\sum_{t\in T:t<t_s}a_{hk}x^s_{1kt}/A_h;\ h\in H, s\in S$$
$$q^s_i\le(u_i+U^s_i)/2;\ i\in I, s\in S$$
$$\sum_{k\in K}x^s_{1kt}\le c_1;\ t\in T, s\in S: t<t_s$$
$$\sum_{k\in K}x^s_{jkt}\le c^s_{jt}y^s_j;\ j\in J, t\in T, s\in S: t\ge t_s$$
$$\sum_{t\in T:t\ge t_s}x^s_{jkt}/D_k\le z^s_{jk};\ j\in J, k\in K, s\in S$$
$$\sum_{k\in K}\sum_{t'\in T:t'\le t}a_{hk}x^s_{1kt'}/A_h\le V_{h0}+\sum_{i\in I:\tau_{i1}\le t-1}\gamma^s_i v_{ih}+\sum_{i\in I:t_s+\theta_{is}+\tau_{i1}\le t-1}V^s_{ijh}-\sum_{j\in J:j>1}w^s_{jh};$$
$$h\in H, t\in T, s\in S$$
$$\sum_{k\in K}\sum_{t'\in T:t'\le t}a_{hk}x^s_{jkt'}/A_h\le\sum_{i\in I:t_s+\theta_{is}+\tau_{ij}\le t-1}V^s_{ijh};$$
$$h\in H, j\in J, t\in T, s\in S: j>1, t<\sigma_j+1$$
$$\sum_{k\in K}\sum_{t'\in T:t'\le t}a_{hk}x^s_{jkt'}/A_h\le\sum_{i\in I:t_s+\theta_{is}+\tau_{ij}\le t-1}V^s_{ijh}+w^s_{jh};$$
$$h\in H, j\in J, t\in T, s\in S: j>1, t\ge\sigma_j+1$$
$$q^s_i\in\{0,1\};\ i\in I, s\in S$$
$$u_i\in\{0,1\};\ i\in I$$
$$v_{ih}\in[0,1];\ i\in I, h\in H$$
$$U^s_i\in\{0,1\};\ i\in I, s\in S$$
$$V^s_{ijh}\in[0,1];\ i\in I, j\in J, h\in H, s\in S$$

$$w_{jh}^s \in [0, 1];\ j \in J, h \in H, s \in S$$
$$x_{jkt}^s \geq 0;\ j \in J, k \in K, t \in T, s \in S$$
$$y_j^s \in \{0, 1\};\ j \in J, s \in S$$
$$z_{jk}^s \in [0, 1];\ j \in J, k \in K, s \in S,$$

where o_{ih} is unit purchasing price of part type h from supplier i, ε_{jk} is the additional per unit production cost for product type k at recovery plant $j > 1$, g_k is unit penalty cost of unfulfilled demand for product type k, and $A_h V_{h0}$ is the initial inventory of parts type h at primary assembly plant.

Notice that problem **DSupportMP_E** is a multi-portfolio selection problem, since supply and demand portfolios are simultaneously selected for each part type and each product type. Now, the supply and demand portfolio selection constraints form an embedded multicommodity network flow problem. Moreover, the supply and demand portfolio balance constraints (10.16) and the supply-transshipment-production coordinating constraints (10.22)–(10.24) are also bill-of-material constraints in model **DSupportMP_E**.

The demand fulfillment constraints for each product type $k \in K$,

$$\sum_{t \in T: t < t_s} x_{1kt}^s / D_k + \sum_{j \in J} z_{jk}^s = 1;\ k \in K, s \in S,$$

imply the following requirement constraints for each part type $h \in H$,

$$\sum_{k \in K} \sum_{t \in T: t < t_s} a_{hk} x_{1kt}^s / A_h + \sum_{j \in J} \sum_{k \in K} a_{hk} D_k z_{jk}^s / A_h = 1;\ h \in H, s \in S.$$

The latter constraints can be obtained from the previous ones by multiplying both sides by $a_{hk} D_k$ and summing over all $k \in K$, and then replacing the right-hand side, $\sum_{k \in K} a_{hk} D_k$, by A_h. The requirement constraints for part types can be added to model **DSupportMP_E** to strengthen MIP formulation.

The above model can be further enhanced, for example by introducing subsets $I_h \subset I$ of suppliers for each part type h, unit capacity consumption for each product type k and each plant j in the left-hand side of production capacity constraints, etc.

10.3.3 Hierarchical Selection of Primary and Recovery Supply and Demand Portfolios

In this subsection two deterministic MIP models **PSupport** and **RDSupport(s)** are presented for the hierarchical decision making in the presence of supply chain disruption risks. The two-stage decision making is described below (Fig. 10.1).

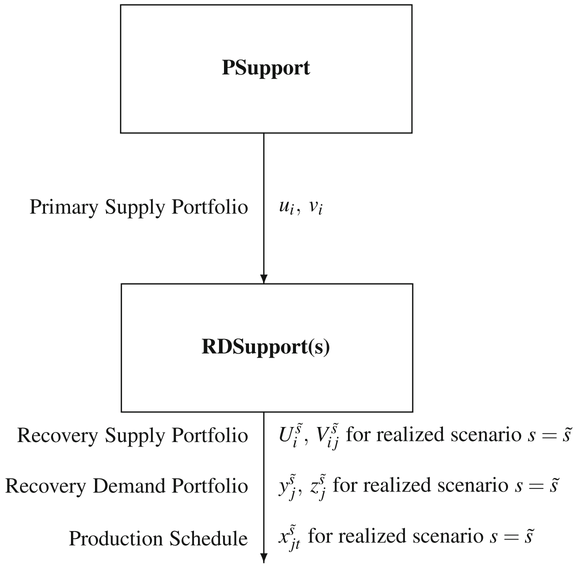

Fig. 10.1 Hierarchical selection of supply and demand portfolios

1. Selection of primary supply portfolio for deterministic environment.
   The primary suppliers are determined ahead of time with no disruption scenarios considered, using deterministic MIP model **PSupport**. The primary assembly plant is plant $j = 1$, by default.
2. Selection of recovery supply and demand portfolios, after disruption of a primary supplier and/or the primary assembly plant.
   The recovery portfolios are determined to optimize the process of recovery from the disruption, using MIP model **RDSupport(s)**.

In the deterministic MIP model **PSupport**, stochastic variable, x_{1t}^s, (10.31), defined in model **DSupport_E** for each disruption scenario $s \in S$ has been replaced by its deterministic equivalent X_t.

PSupport: *Primary supply portfolio selection and production scheduling*

Minimize

$$P^c = \sum_{i \in I} (e_i u_i / D + o_i v_i) + g(1 - \sum_{t \in T} X_t / D) \tag{10.37}$$

subject to

$$\sum_{i \in I} v_i = 1 \tag{10.38}$$

$$v_i \leq u_i; \; i \in I \tag{10.39}$$

$$\sum_{t' \in T: t' \le t} X_{t'}/D \le V_0 + \sum_{i \in I: \tau_{i1} \le t-1} v_i; \; t \in T \tag{10.40}$$

$$X_t \le c_1; \; t \in T \tag{10.41}$$

$$u_i \in \{0, 1\}; \; i \in I \tag{10.42}$$

$$v_i \in [0, 1]; \; i \in I \tag{10.43}$$

$$X_t \ge 0; \; t \in T. \tag{10.44}$$

The solution to **PSupport** is primary supply portfolio: $u_i^*, \; v_i^*; \; i \in I$, and the associated production schedule, X_t^*.

A disruptive event under scenario $s \in S$ is assumed to occur in period t_s and it may impact a primary supplier (as well as the other suppliers) and the primary assembly plant $j = 1$ (as well as the other assembly plants). Then, the recovery process starts in period $t_s + 1$ so that the disrupted supplier $i \in I$ returns to its full capacity in period $t = t_s + \theta_{is}$ and disrupted plant $j \in J$, in period $t = t_s + \vartheta_{js}$.

Denote by D^s and V_0^s respectively, the unfulfilled demand for products in period t_s and the inventory of parts at plant $j = 1$ in period t_s, expressed by the fraction of parts required to complete the unfulfilled demand D^s

$$D^s = D - \sum_{t \in T: t < t_s} x_{1t}^s; \; s \in S \tag{10.45}$$

$$V_0^s = D(V_0 + \sum_{i \in I: \tau_i \le t_s} \gamma_i^s v_i^*)/D^s - \sum_{t \in T: t < t_s} x_{1t}^s/D^s; \; s \in S, \tag{10.46}$$

where $v_i^*, i \in I$, is a predetermined primary supply portfolio.

If $V_0^s > 0$, then selection of recovery portfolios may be combined with transshipment of parts from the primary plant to recovery plants.

When a disruption to primary assembly plant occurs after the latest delivery lead time from primary suppliers of parts, $t_s > \max_{i \in I}(\tau_{i1} u_i^*)$, i.e., after all required parts are delivered to the primary plant, then all parts can be transshipped to recovery plants and no recovery supply portfolio needs to be selected.

In model **RDSupport(s)** presented below, the recovery supply and demand portfolios are selected whenever a disruption occurs to a primary supplier or the primary assembly plant $j = 1$, given a primary supply portfolio and the realized disruption scenario $s = \tilde{s}$.

RDSupport(s): *Recovery supply and demand portfolio selection and production scheduling for predetermined primary portfolios and the realized disruption scenario*

Minimize

$$R^c_{\tilde{s}} = \sum_{i \in I} e_i(1-u^*_i)U^{\tilde{s}}_i/D^{\tilde{s}} + \sum_{i \in I} \rho_{i\tilde{s}} U^{\tilde{s}}_i/D^{\tilde{s}} + \sum_{j \in J}(\psi_j w^{\tilde{s}}_j + \sum_{i \in I} o_i V^{\tilde{s}}_{ij})$$
$$+ \sum_{j \in J}((\varphi_j + \rho_{j\tilde{s}})y^{\tilde{s}}_j + (\varepsilon_j - g)\sum_{t \in T^{\tilde{s}}} x^{\tilde{s}}_{jt})/D^{\tilde{s}} + g$$
$$+ \sum_{i \in I} e_i u^*_i/D^{\tilde{s}} + \sum_{i \in I} o_i \gamma^s_i v^*_i \qquad (10.47)$$

subject to

$$\sum_{i \in I}\sum_{j \in J} V^{\tilde{s}}_{ij} = 1 - V^{\tilde{s}}_0 \qquad (10.48)$$
$$V^{\tilde{s}}_{ij} \le U^{\tilde{s}}_i;\ i \in I, j \in J \qquad (10.49)$$
$$\sum_{j \in J} z^{\tilde{s}}_j = 1 \qquad (10.50)$$
$$z^{\tilde{s}}_j \le y^{\tilde{s}}_j;\ j \in J \qquad (10.51)$$
$$\sum_{i \in I} V^{\tilde{s}}_{ij} + w^{\tilde{s}}_j = z^{\tilde{s}}_j;\ j \in J \qquad (10.52)$$
$$\sum_{j \in J} w^{\tilde{s}}_j = V^{\tilde{s}}_0 \qquad (10.53)$$
$$\sum_{t' \in T^{\tilde{s}}: t' \le t} x^{\tilde{s}}_{1t'}/D^{\tilde{s}} \le V^{\tilde{s}}_0 + \sum_{i \in I: t_{\tilde{s}} + \theta_{i\tilde{s}} + \tau_{i1} \le t-1} V^{\tilde{s}}_{i1} - \sum_{j \in J: j>1} w^{\tilde{s}}_j;\ t \in T^{\tilde{s}} \qquad (10.54)$$
$$\sum_{t' \in T^{\tilde{s}}: t' \le t} x^{\tilde{s}}_{jt'}/D^{\tilde{s}} \le \sum_{i \in I: t_{\tilde{s}} + \theta_{i\tilde{s}} + \tau_{ij} \le t-1} V^{\tilde{s}}_{ij};$$
$$j \in J, t \in T^{\tilde{s}} : j > 1, t < t_{\tilde{s}} + 1 + \sigma_j \qquad (10.55)$$
$$\sum_{t' \in T^{\tilde{s}}: t' \le t} x^{\tilde{s}}_{jt'}/D^{\tilde{s}} \le \sum_{i \in I: t_{\tilde{s}} + \theta_{i\tilde{s}} + \tau_{ij} \le t-1} V^{\tilde{s}}_{ij} + w^{\tilde{s}}_j;$$
$$j \in J, t \in T^{\tilde{s}} : j > 1, t \ge t_{\tilde{s}} + 1 + \sigma_j \qquad (10.56)$$
$$\sum_{t \in T^{\tilde{s}}} x^{\tilde{s}}_{jt}/D^{\tilde{s}} \le z^{\tilde{s}}_j;\ j \in J \qquad (10.57)$$
$$x^{\tilde{s}}_{jt} \le c^{\tilde{s}}_{jt} y^{\tilde{s}}_j;\ j \in J, t \in T^{\tilde{s}} \qquad (10.58)$$
$$U^{\tilde{s}}_i \in \{0,1\};\ i \in I \qquad (10.59)$$
$$V^{\tilde{s}}_{ij} \in [0,1];\ i \in I, j \in J \qquad (10.60)$$
$$w^{\tilde{s}}_j \in [0,1];\ j \in J \qquad (10.61)$$
$$x^{\tilde{s}}_{jt} \ge 0;\ j \in J, t \in T^{\tilde{s}} \qquad (10.62)$$
$$y^{\tilde{s}}_j \in \{0,1\};\ j \in J \qquad (10.63)$$
$$z^{\tilde{s}}_j \in [0,1];\ j \in J, \qquad (10.64)$$

where $T^s = \{t_s + 1, \ldots, \overline{T}\}$ and
$c^{\tilde{s}}_{jt}$ is the capacity in period t of plant j under disruption scenario $\tilde{s}$

$$c^{\tilde{s}}_{jt} = \begin{cases} \delta^{\tilde{s}}_j c_j, & \text{if } t_{\tilde{s}} \le t \le t_{\tilde{s}} + \vartheta_{j\tilde{s}} - 1 \\ c_j, & \text{if } t \ge t_{\tilde{s}} + \vartheta_{j\tilde{s}}. \end{cases} \qquad (10.65)$$

The objective function (10.47) constitutes of different fixed and variable cost per product (cf. Eq.(10.8)), and the last two constant components of (10.47) represent per product costs incurred by predetermined primary supply portfolio.

Notice that model **DSupport_E** for the predetermined primary supply portfolio is separable with respect to disruption scenarios $s \in S$, since the objective function (10.8) is additive and separable with respect to s as well as all constraints (10.12) - (10.24) are separable with respect to s. Thus, for a given primary supply portfolio and start time t_s of disruption s, the recovery supply portfolios, $U_i^s,\ V_{ij}^s;\ i \in I, j \in J$, the recovery demand portfolios, $y_j^s,\ z_j^s;\ j \in J$, the transshipments of parts, $w_j^s,\ j \in J$, and the production schedules, $x_{jt}^s, j \in J$, can be found simultaneously for all potential disruption scenarios $s \in S$, by solving

RDSupport_E = **DSupport_E** for predetermined primary supply portfolio: $u_i = u_i^*,\ v_i = v_i^*;\ i \in I$.

RDSupport_E: *Risk-neutral selection of recovery supply and demand portfolios and production scheduling for predetermined primary portfolios*

Minimize (10.8)

subject to (10.12)–(10.25), (10.28)–(10.34) and

$$u_i = u_i^*;\ i \in I \tag{10.66}$$

$$v_i = v_i^*;\ i \in I. \tag{10.67}$$

Worst-case cost per product

A simple upper bound on cost per product, R_s^c, for scenario s, of model **RDSupport_E** for predetermined primary supply and demand portfolios is derived below.

Proposition 10.2

$$R_s^c \le \overline{R^c};\ s \in S, \tag{10.68}$$

where

$$\begin{aligned} R_s^c = &\sum_{i \in I} e_i(1 - u_i^*)U_i^s/D + \sum_{i \in I} \rho_{is} U_i^s/D + \sum_{j \in J} (\varphi_j + \rho_{js}) y_j^s/D \\ &+ \sum_{j \in J} (\psi_j w_j^s + \sum_{i \in I} o_i V_{ij}^s) + \sum_{j \in J} \varepsilon_j \sum_{t \in T} x_{jt}^s/D + g(1 - \sum_{t \in T} x_{jt}^s/D) \\ &+ \sum_{i \in I} e_i u_i^*/D + \sum_{i \in I} o_i \gamma_i^s v_i^*, \end{aligned} \tag{10.69}$$

and $\overline{R^c}$ is the worst-case cost per product associated with worst-case disruption scenario with respect to cost

$$\overline{R^c} = \sum_{i \in I} e_i/D + g(1 - \underline{SL}) + \psi_{max} + \varepsilon_{max}$$
$$+ \max_{s \in S}\{\sum_{i \in I} \rho_{is} u_i^*/D + \max_{j \in J}(\varphi_j + \rho_{js})/D$$
$$+(o_{max} - \psi_{min})(1 - \sum_{i \in I} \gamma_i^s v_i^*) + \sum_{i \in I} o_i \gamma_i^s v_i^*\}. \quad (10.70)$$

Proof.

Without loss of generality, assume that $V_0 = 0$ and $\sum_{t \in T: t < t_s} x_{1t}^s/D = 0$. Recovery supply and demand portfolio selection constraints (10.12), (10.14), (10.16) and minimization of cost objective function, imply

$$\sum_{i \in I} \rho_{is} U_i^s/D + \sum_{j \in J}(\varphi_j + \rho_{js}) y_j^s/D + \sum_{j \in J} \psi_j w_j^s + \sum_{i \in I}\sum_{j \in J} o_i V_{ij}^s + \sum_{i \in I} o_i \gamma_i^s v_i^* =$$
$$\sum_{i \in I} \rho_{is} U_i^s/D + \sum_{j \in J}(\varphi_j + \rho_{js}) y_j^s/D + \sum_{j \in J} \psi_j (z_j^s - \sum_{i \in I} V_{ij}^s) + \sum_{i \in I}\sum_{j \in J} o_i V_{ij}^s + \sum_{i \in I} o_i \gamma_i^s v_i^* =$$
$$\sum_{i \in I} \rho_{is} U_i^s/D + \sum_{j \in J}(\varphi_j + \rho_{js}) y_j^s/D + \sum_{j \in J} \psi_j z_j^s - \sum_{i \in I}\sum_{j \in J} \psi_j V_{ij}^s + \sum_{i \in I}\sum_{j \in J} o_i V_{ij}^s + \sum_{i \in I} o_i \gamma_i^s v_i^* \le$$
$$\sum_{i \in I} \rho_{is} U_i^s/D + \sum_{j \in J}(\varphi_j + \rho_{js}) y_j^s/D + \psi_{max} + (o_{max} - \psi_{min}) \sum_{i \in I}\sum_{j \in J} V_{ij}^s + \sum_{i \in I} o_i \gamma_i^s v_i^* \le$$
$$\psi_{max} + \max_{s \in S}\{\sum_{i \in I} \rho_{is} u_i^*/D + \max_{j \in J}(\varphi_j + \rho_{js})/D + (o_{max} - \psi_{min})(1 - \sum_{i \in I} \gamma_i^s v_i^*) + \sum_{i \in I} o_i \gamma_i^s v_i^*\},$$

where $o_{max} = \max_{i \in I} o_i$, $\psi_{max} = \max_{j \in J} \psi_j$, $\psi_{min} = \min_{j \in J} \psi_j$.

In addition, Eqs. (10.14), (10.21) and (10.36) imply

$$\sum_{i \in I} e_i(1 - u_i^*) U_i^s/D + \sum_{i \in I} e_i u_i^*/D + \sum_{j \in J} \varepsilon_j \sum_{t \in T} x_{jt}^s/D + g(1 - \sum_{t \in T} x_{jt}^s/D) \le$$
$$\sum_{i \in I} e_i/D + \varepsilon_{max} \sum_{j \in J}\sum_{t \in T} x_{jt}^s/D + g(1 - \underline{SL}) \le$$
$$\sum_{i \in I} e_i/D + \varepsilon_{max} \sum_{j \in J} z_j^s + g(1 - \underline{SL}) \le$$
$$\sum_{i \in I} e_i/D + \varepsilon_{max} + g(1 - \underline{SL}),$$

where $\varepsilon_{max} = max_{j \in J} \varepsilon_j$.

By summing up the final parts of the above two expressions, formula (10.70) for $\overline{R^c}$ is achieved.

10.4 Models for Risk-Averse Decision-Making

10.4.1 Integrated Approach

In this subsection the two time-indexed SMIP models **DSupport_CV(c)** and **DSupport_CV(sl)** are proposed for the integrated, risk-averse selection of primary and recovery supply and demand portfolios and production scheduling to optimize, respectively expected worst-case cost and expected worst-case service level under disruption risks. The models are based on the risk-neutral model **DSupport_E**.

When the worst-case cost is focused on, the risk-averse primary and recovery supply and demand portfolios and the production schedule will be optimized by calculating VaR^c and minimizing CVaR^c simultaneously. Model **DSupport_CV(c)** is presented below.

DSupport_CV(c): *Risk-averse selection of primary and recovery supply and demand portfolios and scheduling of production to minimize CVaR of cost*

Minimize

$$CVaR^c = VaR^c + (1-\alpha)^{-1}\sum_{s\in S} P_s \mathscr{C}_s \tag{10.71}$$

subject to (10.10)–(10.34) and

Risk constraints:

- the tail cost for scenario s is defined as the nonnegative amount by which cost in scenario s exceeds VaR^c,

$$\begin{aligned}\mathscr{C}_s \geq \sum_{i\in I}(e_i(u_i + U_i^s - q_i^s)/D + \rho_{is}U_i^s/D + o_i(\gamma_i^s v_i + \sum_{j\in J} V_{ij}^s))\\ + \sum_{j\in J}(\psi_j w_j^s + (\varphi_j + \rho_{js})y_j^s/D + \varepsilon_j \sum_{t\in T} x_{jt}^s/D)\\ + g(1 - \sum_{j\in J}\sum_{t\in T} x_{jt}^s/D) - VaR^c;\ s\in S\end{aligned} \tag{10.72}$$

$$\mathscr{C}_s \geq 0;\ s\in S, \tag{10.73}$$

where $\mathscr{C}_s$ is the tail cost for scenario s.

If worst-case service level is to be maximized, the risk-averse primary and recovery supply portfolio and the production schedule will be optimized by calculating VaR^{sl} and maximizing CVaR^{sl} simultaneously. Model **DSupport_CV(sl)** is presented below.

DSupport_CV(sl): *Risk-averse selection of primary and recovery supply and demand portfolios and scheduling of production to maximize CVaR of service level*

Maximize

$$CVaR^{sl} = VaR^{sl} - (1-\alpha)^{-1}\sum_{s\in S} P_s \mathscr{S}_s \tag{10.74}$$

subject to (10.10)–(10.34) and

Risk constraints:

- the tail service level for scenario s is defined as the nonnegative amount by which VaR^{sl} exceeds service level in scenario s,

$$\mathscr{S}_s \geq VaR^{sl} - \sum_{j\in J}\sum_{t\in T} x^s_{jt}/D; \;\; s\in S \tag{10.75}$$

$$\mathscr{S}_s \geq 0; \;\; s\in S, \tag{10.76}$$

where $\mathscr{S}_s$ is the tail service level for scenario s.

10.4.2 Hierarchical Approach

In this subsection the two time-indexed SMIP models **RDSupport_CV(c)** and **RDSupport_CV(sl)** are presented for the hierarchical, risk-averse selection of recovery supply and demand portfolios and production scheduling to optimize, respectively expected worst-case cost and expected worst-case service level, given primary supply portfolio.

The models are based on the risk-averse decision-making models, **DSupport_CV(c)** and **DSupport_CV(sl)**, respectively:

- **RDSupport_CV(c)** = **DSupport_CV(c)** for predetermined primary supply portfolio: $u_i = u_i^*, \; v_i = v_i^*; \; i\in I$;
- **RDSupport_CV(sl)** = **DSupport_CV(sl)** for predetermined primary supply portfolio: $u_i = u_i^*, \; v_i = v_i^*; \; i\in I$.

For model **RDSupport_CV(c)**, the primary supply portfolio, $u_i^*, v_i^*; i \in I$, is determined using model **PSupport** with cost-based objective function (10.37). For model **RDSupport_CV(sl)**, however, the cost-based objective function (10.37) is replaced by the following service-based objective function

$$\textit{Maximize} \qquad \sum_{t\in T} X_t/D. \tag{10.77}$$

RDSupport_CV(c): *Risk-averse selection of recovery supply and demand portfolios and production scheduling to minimize CVaR of cost for predetermined primary portfolios.*

Minimize (10.71)
subject to (10.10)–(10.34), (10.66), (10.67), (10.72), (10.73).

RDSupport_CV(sl): *Risk-averse selection of recovery supply and demand portfolios and production scheduling to minimize CVaR of service level for predetermined primary portfolios.*

Minimize (10.74)
subject to (10.10)–(10.34), (10.66), (10.67), (10.75), (10.76).

10.5 Computational Examples

In this section some computational examples are presented to illustrate the proposed portfolio approach for selection of primary and recovery supply and demand portfolios and production scheduling. The input data for the examples are hypothetical, however their relations to each other are real and in part they have been taken from a real case study (e.g., Fujimoto and Park 2013, Park et al. 2013, MacKenzie et al. 2014, Matsuo 2015, Haraguchi and Lall 2015). The basic input parameters are taken from the computational examples presented in Sect. 9.5.

$\overline{I} = 4$ suppliers, $\overline{L_i} = 3$, i.e., four disruption levels for each supplier $i \in I$,

$\overline{J} = 2$ assembly plants, $\overline{L^j} = 1$, i.e., two (all-or-nothing) disruption levels for each plant $j \in J$,

$\overline{R} = 2$ geographic regions, $\overline{T} = 30$ planning periods.

$I^1 = \{1, 2\}$, $I^2 = \{3, 4\}$, $J^1 = \{1, 2\}$, i.e., two suppliers and two plants are located in region $r = 1$, and two suppliers in region $r = 2$.

The initial inventories of parts: $V_0 = 0$.

Total demand for parts/products: $D = 300000$.

Delivery lead times from suppliers: $\tau_{1j} = \tau_{2j} = 2$, $\tau_{3j} = \tau_{4j} = 4, \forall j \in J$.

Fixed ordering costs for suppliers: $e = (8000, 6000, 12000, 13000)$.

Unit purchasing prices from suppliers: $o = (14, 12, 8, 9)$.

Plant capacity: $c_1 = 10000$, $c_2 = 5000$.

Production costs: $\varepsilon_2 = 1$, $\varphi_2 = 100$.

Transshipment cost and time: $\psi_2 = 0.1$, $\sigma_2 = 2$.

Unit penalties for unfulfilled demand:

$g \in \{1, 10, 100, 1000, 10000, 100000, \infty\}$, where $g = \infty$ denotes maximization of service level.

Supplier cost-to-recover and time-to-recover are defined below.

$CTR(i, l)$=if $l = 0$ then $100000e_i$; if $l = 1$ then $10000e_i$; if $l = 2$ then $1000e_i$ $\forall i \in I$,

$TTR(i, l)$=if $l = 0$ then 12; if $l = 1$ then 10; if $l = 2$ then 8 $\forall i \in I$.

Plant recovery cost, $PRC(j, 0)$ and recovery time, $PRT(j, 0)$, are

$PRC(1, 0) = PRC(2, 0) = 10000$ and $PRT(1, 0) = 10,\ PRT(2, 0) = 5$.

Local disruption levels of suppliers and the associated fulfillment rates are shown below.

$L_i = L = \{0, 1, 2, 3\}$ for all $i \in I$, where $l = 0$, complete shutdown, $\gamma_{i0} = 0\ \forall i \in I$, i.e., 0% of an order delivered; $l = 1$, major disruption, $\gamma_{i1} \in [0.01, 0.50]\ \forall i \in I^1$ and $\gamma_{i1} \in [0.01, 0.30]\ \forall i \in I^2$, i.e., 1–50% and 1–30% of an order delivered, respectively; $l = 2$, minor disruption, $\gamma_{i2} \in [0, 51, 0.99]\ \forall i \in I^1$ and $\gamma_{i2} \in [0, 31, 0.99]\ \forall i \in I^2$, i.e., 51–99% and 31–99% of an order delivered, respectively; $l = \overline{L} = 3$, no disruption, $\gamma_{i3} = 1\ \forall i \in I$, i.e., 100% of an order delivered.

The total number of all potential scenarios is $\overline{S} = (\overline{L} + 1)^{\overline{I}}(\overline{L^1} + 1)(\overline{L^2} + 1) = (4^4)(2^2) = 1024$ scenarios. Each scenario $s \in S$ is represented by a 6-dimensional vector $\lambda_s = (\lambda_{1s}, \ldots, \lambda_{4s}, \lambda_{5s}, \lambda_{6s})$, where $\lambda_{is} \in L_i,\ i \in I$, and $\lambda_{5s}, \lambda_{6s} \in \{0, 1\}$, where λ_{5s} and λ_{6s} represent disruption pattern under scenario s of assembly plant $j = 1$ and $j = 2$, respectively. The selected supply and production disruption scenarios are presented in Tables 10.5 and 10.6.

The local probability of suppliers non-disruptive operation (level $l = 3$), p_{i3}, was uniformly distributed over [0.89, 0.99] and [0.79, 0.89], respectively for suppliers $i \in I^1$, and $i \in I^2$.

Given local non disruption probabilities of suppliers, $p_{i3},\ i \in I$, the probabilities for the remaining local disruption levels $l = 0, 1, 2$ were calculated as follows:
probability of complete shutdown (level $l = 0$), $p_{i0} = 0.1(1 - p_{i3})$;
probability of major disruption (level $l = 1$), $p_{i1} = 0.3(1 - p_{i3})$;
probability of minor disruption (level $l = 2$), $p_{i2} = 0.6(1 - p_{i3})$ for all suppliers $i \in I$.

The local probabilities of non-disruptive operation of assembly plants were $\pi_1 = 0.85$ and $\pi_2 = 0.95$, i.e., the primary assembly plant $j = 1$ was modeled to be less reliable to emphasize the impact of its disruption.

The regional disruption probabilities are $p^1 = 0.001$ and $p^2 = 0.01$.

The probability P_s^1 of realizing disruption scenario $s \in S$ for suppliers and assembly plants in region $r = 1$, and P_s^2 of realizing disruption scenario $s \in S$ for suppliers in region $r = 2$ are calculated as follows

$$P_s^1 = \begin{cases} (1-p^1)(\prod_{i\in I^1:\lambda_{is}=0} 0.1(1-p_{i3}))(\prod_{i\in I^1:\lambda_{is}=1} 0.3(1-p_{i3})) \\ \times(\prod_{i\in I^1:\lambda_{is}=2} 0.6(1-p_{i3}))(\prod_{i\in I^1:\lambda_{is}=3} p_{i3}) \\ \times(\prod_{j\in J:\lambda_{\bar{I}+j,s}=0}(1-\pi_j))(\prod_{j\in J:\lambda_{\bar{I}+j,s}=1}\pi_j) \\ \text{if } \sum_{i\in I^1\bigcup\{5,6\}}\lambda_{is} > 0 \\ p^1 + (1-p^1)\prod_{i\in I^1} 0.1(1-p_{i3})\prod_{j\in J}(1-\pi_j) \\ \text{if } \sum_{i\in I^1\bigcup\{5,6\}}\lambda_{is} = 0. \end{cases}$$

$$P_s^2 = \begin{cases} (1-p^2)(\prod_{i\in I^2:\lambda_{is}=0} 0.1(1-p_{i3}))(\prod_{i\in I^2:\lambda_{is}=1} 0.3(1-p_{i3})) & \\ \times(\prod_{i\in I^2:\lambda_{is}=2} 0.6(1-p_{i3}))(\prod_{i\in I^2:\lambda_{is}=3} p_{i3}) & \text{if } \sum_{i\in I^2}\lambda_{is} > 0 \\ p^2 + (1-p^2)\prod_{i\in I^2} 0.1(1-p_{i3}) & \text{if } \sum_{i\in I^2}\lambda_{is} = 0. \end{cases}$$

The probability for disruption scenario $s \in S$ is given by $P_s = P_s^1 P_s^2$.

10.5.1 Risk-Neutral Decision-Making

In the computational examples presented in this subsection each disruption $s \in S$ to primary suppliers (and possibly other suppliers) and/or to primary plant $j = 1$ (and possibly other plants) is assumed to occur in the same period $t_s = 1$, before the earliest delivery lead time, $\min_{i\in I}(\tau_{i1}) = 2$, and the recovery process starts in period $t = 2$ so that the disrupted supplier i and disrupted plant j return to its full capacity in period $t = \theta_{is} + 1$ and $t = \vartheta_{js} + 1$, respectively. The assumption of a common disruption start time of all disruptive events will be relaxed in Sect. 10.5.1.1 to provide more insights by best-case and worst-case analysis, and in Sect. 10.5.2 to illustrate the risk-averse decision-making.

The solution results for both the integrated and the hierarchical approach are summarized in Table 10.4. The table shows primary supply portfolio and expected recovery supply portfolio and recovery demand portfolio for different values of unit penalty cost g. Notice that for each disruption scenario, the recovery supply portfolio fulfills the remaining demand for parts, unmet by partially disrupted primary suppliers (see, (10.12) and Table 10.5). Thus, the numbers representing the expected fraction, over all scenarios, of total demand to be fulfilled by recovery suppliers do not need to add up to 100%. In contrast to expected recovery demand portfolio, that sums up to 100%, since all demand for products is assigned to a single primary plant that is shutdown before any production can be started, if a disruptive event occurs at time $t_s = 1$ (see, (10.14) and Table 10.5). For the integrated approach, **DSupport_E**, and the bottom level problem **RDSupport** of the hierarchical approach, expected cost E^c, (10.8), is shown along with the associated expected service level E^{sl}, (10.9). Similarly, for the top level problem **PSupport** of the hierarchical approach, cost P^c, (10.37), is shown along with the associated service level, $\sum_{t\in T} X_t/D$. In addition, Table 10.4 presents the size of each MIP model, **DSupport_E**, **PSupport** and **RDSupport_E**.

As g increases to reduce the unfulfilled demand, more expensive and diversified supply portfolios are selected and a higher service level is attained. In particular, for integrated approach more diversified primary supply portfolios are selected for $g > 1$ to hedge against all disruption scenarios. In contrast, for hierarchical approach the results for $g > 1$ are independent of g. When the objective is to maximize service level ($g \to \infty$), the hierarchical approach selects the most reliable supplier ($i = 2$) as the only primary supplier. Table 10.4 demonstrates that for low values of g, the two approaches yield identical solution values. This is due to negligible penalty cost for unfulfilled demand for products with respect to other cost components. However, as g increases, the integrated approach outperforms the purely deterministic, top-down hierarchical approach. Both, expected cost over all scenarios is lower and expected service level is higher.

Overall, the computational results demonstrate that:

- *the integrated approach selects for both objectives a more diversified primary supply portfolio that will hedge against all potential disruption scenarios,*
- *the hierarchical approach selects the primary supply portfolio that is made up of cheapest suppliers or a single, most reliable supplier only, to minimize expected cost or maximize expected service level, respectively.*

For the integrated approach (model **DSupport_E**) and selected supply and production disruption scenarios $s \in S$, Table 10.5 shows examples of recovery supply and demand portfolios ($V_{11}^s + V_{12}^s$, $V_{21}^s + V_{22}^s$, $V_{31}^s + V_{32}^s$, $V_{41}^s + V_{42}^s$, z_1^s, z_2^s) along with the associated cost,

$$\sum_{i \in I}(e_i(u_i + U_i^s - q_i^s)/D + \rho_{is}U_i^s/D + o_i(\gamma_i^s v_i + \sum_{j \in J} V_{ij}^s))$$
$$+ \sum_{j \in J}(\psi_j w_j^s + (\varphi_j + \rho_{js})y_j^s/D + \varepsilon_j \sum_{t \in T} x_{jt}^s/D) + g(1 - \sum_{j \in J}\sum_{t \in T} x_{jt}^s/D),$$

and service level,

$$\sum_{j \in J}\sum_{t \in T} x_{jt}^s/D,$$

for a unit penalty $g = 100$. The primary supply portfolio for the example is (see, Table 10.4): $v_1 = 0,\ v_2 = 0.08,\ v_3 = 0.74,\ v_4 = 0.18$. The results indicate that the cost is ranging from 8.57 for scenario $s = 845$ to 2095.44 for scenario $s = 1$ with all suppliers and assembly plants completely shutdown. The corresponding service level is ranging from 100% to 50%. Notice that the objective of model **DSupport_E** is to select primary supply portfolio to hedge against all potential disruption scenarios and to select recovery supply and demand portfolios for each scenario to minimize expected cost over all scenarios.

Table 10.4 Supply and demand portfolios

g	1	10	10^2	10^3	10^4	10^5	∞ [(a)]
Integrated approach: model **DSupport_E**							
Var.=69772, Bin.=10244, Cons.=118037, Nonz.=1036518[(c)]							
Exp.Cost, E^c, (10.8)	13.31	14.52	22.47	96.08	817.53	8031.02	-
Exp.Service, E^{sl}, (10.9)×100%	83	91	92	92	92	92	92
Primary	1(0)	1(0)	1(0)	1(7)	1(11)		1(11)
Supply Portfolio	2(0)	2(7)	2(8)	2(31)	2(31)		2(31)
(% of total demand)	3(100)	3(93)	3(74)	3(37)	3(33)		3(33)
	4(0)	4(0)	4(18)	4(25)	4(25)		4(25)
Exp.Recovery	1(0.16)	1(0.27)	1(0.16)	1(0.13)	1(0.12)		1(2.90)
Supply Portfolio	2(1.68)	2(1.57)	2(1.53)	2(1.03)	2(0.96)		2(2.90)
(% of total demand) [(d)]	3(0)	3(0.12)	3(1.30)	3(2.63)	3(2.69)		3(0.10)
	4(6.56)	4(6.15)	4(4.87)	4(2.58)	4(2.34)		4(0.20)
Exp.Recovery Demand	1(99.70)	1(98,60)	1(97.77)	1(97.30)			1(94.60)
Portfolio (% of total demand) [(e)]	2(0.30)	2(1.40)	2(2.23)	2(2.70)			2(5.40)
Hierarchical approach: models **PSupport** and **RDSupport_E**							
PSupport: Var.=64, Bin.=4, Cons.=65, Nonz.=968							
Primary	1(0)	1(0)					1(0)
Supply Portfolio	2(0)	2(7)					2(100)
(% of total demand)	3(100)	3(93)					3(0)
	4(0)	4(0)					4(0)
Cost, P^c, (10.37)	8.17	9	15	75	675	6675	-
Service[(b)]	87	93					93
RDSupport_E: Var.=65394, Bin.=7232, Cons.=79058, Nonz.=738494							
Exp.Recovery	1(0.16)	1(0.28)	1(0.22)		1(0.21)		1(2.90)
Supply Portfolio	2(1.68)	2(1.57)					2(0.02)
(% of total demand) [(d)]	3(0)	3(0.12)	3(0.17)		3(0.18)		3(0.77)
	4(6.56)	4(6.15)					4(0.30)
Exp.Recovery Demand	1(99.69)	1(98,59)	1(98.50)	1(98.42)	1(98.41)		1(99.05)
Portfolio (% of total demand) [(e)]	2(0.31)	2(1.41)	2(1.50)	2(1.58)	2(1.59)		2(0.95)
Exp.Cost, E^c, (10.8)	13.31	14.52	22.96	106.83	943.55	9310.39	-
Exp.Service, E^{sl}, (10.9)×100%	84	91					90

[(a)] Maximization of service level.
[(b)] $\sum_{t \in T} X_t / D \times 100\%$.
[(c)] Var.=number of variables, Bin.=number of binary variables, Cons.=number of constraints, Nonz.= number of nonzero coefficients.
[(d)] $1(\sum_{j \in J, s \in S} P_s V_{1j}^s \times 100), 2(\sum_{j \in J, s \in S} P_s V_{2j}^s \times 100), 3(\sum_{j \in J, s \in S} P_s V_{3j}^s \times 100), 4(\sum_{j \in J, s \in S} P_s V_{4j}^s \times 100)$.
[(e)] $1(\sum_{s \in S} P_s z_1^s \times 100), 2(\sum_{s \in S} P_s z_2^s \times 100)$.

The corresponding results for the hierarchical approach (models **PSupport** and **RDSupport_E**) are presented in Table 10.6. The primary supply portfolio obtained to minimize purchasing cost for parts and penalty for unfulfilled demand for products in a deterministic operating environment (model **PSupport**) is (see, Table 10.4): $v_1 = 0, \ v_2 = 0.07, \ v_3 = 0.93, \ v_4 = 0$. Then, given the primary supply and demand portfolios and the realized disruption scenario, the recovery supply and demand portfolios are selected to minimize total cost, including cost of recovery from the disruption. Now, the cost is ranging from 9.05 for scenario $s = 844$ to

Table 10.5 Supply and demand recovery portfolios, $(V_{11}^s + V_{12}^s, V_{21}^s + V_{22}^s, V_{31}^s + V_{32}^s, V_{41}^s + V_{42}^s, z_1^s, z_2^s)$ and associated cost and service, for selected supply and production disruption scenarios $s \in S$: integrated approach, $g = 100$

Disruption scenario							Supply portfolio				Demand portfolio		Solution values	
s	λ_{1s}	λ_{2s}	λ_{3s}	λ_{4s}	λ_{5s}	λ_{6s}	i = 1	i = 2	i = 3	i = 4	j = 1	j = 2	Cost	Service %
1	0	0	0	0	0	0	0	1	0	0	1	0	2095.44	50
7	0	0	1	2	0	0	0	0	0	0.81	1	0	119.03	66.67
17	0	1	0	0	0	0	0	0.97	0	0	1	0	285.92	59.51
18	0	1	0	1	0	0	0	0.94	0	0	1	0	282.62	62.72
19	0	1	0	2	0	0	0	0	0	0.86	1	0	119.19	66.67
26	0	1	2	1	0	0	0	0	0.53	0	0.67	0.33	120.53	95
28	0	1	2	3	0	0	0	0	0	0.38	1	0	75.44	66.67
35	0	2	0	2	0	0	0	0.82	0	0	0.67	0.33	100.76	98.33
36	0	2	0	3	0	0	0	0	0	0.76	1	0	75.97	66.67
42	0	2	2	1	0	0	0	0.49	0	0	0.67	0.33	99.35	98.33
44	0	2	2	3	0	0	0	0	0	0.34	1	0	75.56	66.67
49	0	3	0	0	0	0	0	0.92	0	0	0.67	0.33	79.44	100
50	0	3	0	1	0	0	0	0.88	0	0	1	0	78.67	66.67
321	1	0	0	0	0	1	1	0	0	0	0.57	0.43	329.75	85
322	1	0	0	1	0	1	0.1	0	0	0.87	0.6	0.4	755.37	88.21
323	1	0	0	2	0	1	0	0	0	0.89	0.72	0.28	91.39	95
324	1	0	0	3	0	1	0	0	0	0.82	0.67	0.33	43.1	100
325	1	0	1	0	0	1	0.92	0	0	0	0.64	0.36	321.67	92.62
326	1	0	1	1	0	1	0.89	0	0	0	0.72	0.28	319.13	95
328	1	0	1	3	0	1	0	0	0	0.75	0.67	0.33	43.03	100
329	1	0	2	0	0	1	0	0	0.59	0	0.72	0.28	87.05	95
332	1	0	2	3	0	1	0	0	0	0.4	1 0.67	0.33	42.69	100
339	1	1	0	2	0	1	0	0	0	0.86	0.7	2 0.28	91.47	95
340	1	1	0	3	0	1	0	0	0	0.79	0.67	0.33	43.19	100
341	1	1	1	0	0	1	0	0.9	0	0	0.67	0.33	250.75	95
344	1	1	1	3	0	1	0	0	0	0.72	0.67	0.33	43.11	100
353	1	2	0	0	0	1	0	0.93	0	0	0.68	0.32	67.75	98.33
356	1	2	0	3	0	1	0	0	0	0.76	0.67	0.33	43.3	100
357	1	2	1	0	0	1	0	0.86	0	0	0.67	0.33	67.45	98.33
360	1	2	1	3	0	1	0	0	0	0.68	0.67	0.33	43.23	100
361	1	2	2	0	0	1	0	0.52	0	0	0.68	0.32	66.11	98.33
381	1	3	3	0	0	1	0	0	0.18	0	0.82	0.18	57.9	84.39
382	1	3	3	1	0	1	0	0	0.15	0	0.85	0.15	61.1	81.18
383	1	3	3	2	0	1	0	0	0.07	0	0.93	0.07	68.95	73.33
384	1	3	3	3	0	1	0	0	0	0	1	0	75.29	66.67
640	1	3	3	3	1	0	0	0	0	0	1	0	15.29	93.33

(continued)

Table 10.5 (continued)

Disruption scenario							Supply portfolio				Demand portfolio		Solution values	
s	λ_{1s}	λ_{2s}	λ_{3s}	λ_{4s}	λ_{5s}	λ_{6s}	i = 1	i = 2	i = 3	i = 4	j = 1	j = 2	Cost	Service %
646	2	0	1	1	1	0	0.89	0	0	0	1	0	66.03	74.15
647	2	0	1	2	1	0	0.81	0	0	0	0.68	0.32	73.77	100
648	2	0	1	3	1	0	0	0	0	0.75	1	0	22.36	86.67
657	2	1	0	0	1	0	0.97	0	0	0	1	0	74.56	66.18
658	2	1	0	1	1	0	0.94	0	0	0	1	0	71.19	69.39
659	2	1	0	2	1	0	0.86	0	0	0	1	0	62.95	77.24
660	2	1	0	3	1	0	0	0	0	0.79	1	0	19.67	89.51
661	2	1	1	0	1	0	0.9	0	0	0	1	0	66.49	73.8
662	2	1	1	1	1	0	0.86	0	0	0	1	0	63.12	77
663	2	1	1	2	1	0	0.78	0	0	0	1	0	54.88	84.85
664	2	1	1	3	1	0	0	0	0	0.72	1	0	19.6	89.51
673	2	2	0	0	1	0	0	0.93	0	0	1	0	62.1	70
676	2	2	0	3	1	0	0	0	0	0.76	1	0	15.97	93.33
677	2	2	1	0	1	0	0	0.86	0	0	1	0	54.18	77.62
844	1	0	2	3	1	1	0	0	0	0.41	0.87	0.13	9.16	100
845	1	0	3	0	1	1	0	0	0.26	0	0.87	0.13	8.57	100
961	3	0	0	0	1	1	1	0	0	0	0.9	0.1	14.56	100
976	3	0	3	3	1	1	0	0	0.08	0	0.92	0.08	13.55	95.14
977	3	1	0	0	1	1	0.97	0	0	0	0.93	0.07	14.48	100
992	3	1	3	3	1	1	0	0	0.06	0	0.94	0.06	13.64	95.14
1009	3	3	0	0	1	1	0	0.92	0	0	0.93	0.07	12.5	100
1024	3	3	3	3	1	1	0	0	0	0	1	0	15.29	93.33

2095.39 for scenario $s = 1$ with complete shutdown of all suppliers and assembly plants. The corresponding service level is ranging from 100% to 50%.

Observe that no recovery supply or demand portfolio is selected, i.e., $V_{11}^s + V_{12}^s = V_{21}^s + V_{22}^s = V_{31}^s + V_{32}^s = V_{41}^s + V_{42}^s = 0, z_1^s = 1, z_2^s = 0$, for scenarios $s \in S$ with non disrupted all primary suppliers or primary assembly plant, respectively, e.g., for scenarios $s = 640, 1024$.

10.5.1.1 Best-Case and Worst-Case Analysis

In order to provide more insights, in this subsection best-case and worst-case disruption scenarios with respect to cost and service level were identified. Since supply disruptions can happen at any time and for any supplier and assembly plant, from now on a disruption scenario is defined as a combination of disruptive event and its start time. The start time t_s of each disruptive event $s \in S$ is assumed to be not greater

Table 10.6 Supply and demand recovery portfolios, $(V^s_{11} + V^s_{12}, V^s_{21} + V^s_{22}, V^s_{31} + V^s_{32}, V^s_{41} + V^s_{42}, z^s_1, z^s_2)$ and associated cost and service, for selected supply and production disruption scenarios $s \in S$: hierarchical approach, $g = 100$

Disruption scenario							Supply portfolio				Demand portfolio		Solution values	
s	λ_{1s}	λ_{2s}	λ_{3s}	λ_{4s}	λ_{5s}	λ_{6s}	i = 1	i = 2	i = 3	i = 4	j = 1	j = 2	Cost	Service %
1	0	0	0	0	0	0	0	1	0	0	1	0	2095.39	50
7	0	0	1	2	0	0	0	0	0	0.9	1	0	119.38	66.3
17	0	1	0	0	0	0	0	0.98	0	0	1	0	286.49	58.91
19	0	1	0	2	0	0	0	0	0	0.98	1	0	126.93	58.91
26	0	1	2	1	0	0	0	0	0.46	0	1	0	114.82	66.67
28	0	1	2	3	0	0	0	0	0	0.46	1	0	75.32	66.67
35	0	2	0	2	0	0	0	0.95	0	0	1	0	98.73	66.67
36	0	2	0	3	0	0	0	0	0	0.95	1	0	75.93	66.67
42	0	2	2	1	0	0	0	0.43	0	0	1	0	96.65	66.67
44	0	2	2	3	0	0	0	0	0	0.43	1	0	75.41	66.67
49	0	3	0	0	0	0	0	0.93	0	0	1	0	78.73	66.67
50	0	3	0	1	0	0	0	0.93	0	0	1	0	78.73	66.67
321	1	0	0	0	0	1	1	0	0	0	0.57	0.43	329.7	85
322	1	0	0	1	0	1	1	0	0	0	0.72	0.28	329.7	85
323	1	0	0	2	0	1	0	0	0	1	0.57	0.43	101.39	85
324	1	0	0	3	0	1	0	0	0	1	0.67	0.33	43.1	100
325	1	0	1	0	0	1	0.9	0	0	0	0.66	0.34	319.49	94.63
326	1	0	1	1	0	1	0.9	0	0	0	0.66	0.34	319.49	94.63
328	1	0	1	3	0	1	0	0	0	0.9	0.67	0.33	43.01	100
329	1	0	2	0	0	1	0	0	0.48	0	0.72	0.28	87.01	95
332	1	0	2	3	0	1	0	0	0	0.48	0.67	0.33	42.58	100
339	1	1	0	2	0	1	0	0	0	0.98	0.72	0.28	99.21	87.24
340	1	1	0	3	0	1	0	0	0	0.98	0.67	0.33	43.17	100
341	1	1	1	0	0	1	0	0.88	0	0	0.67	0.33	250.62	95
344	1	1	1	3	0	1	0	0	0	0.88	0.67	0.33	43.07	100
353	1	2	0	0	0	1	0	0.95	0	0	0.68	0.32	67.71	98.33
356	1	2	0	3	0	1	0	0	0	0.95	0.67	0.33	43.26	100
357	1	2	1	0	0	1	0	0.85	0	0	0.68	0.32	67.32	98.33
360	1	2	1	3	0	1	0	0	0	0.85	0.67	0.33	43.16	100
361	1	2	2	0	0	1	0	0.43	0	0	0.67	0.33	65.63	98.33
381	1	3	3	0	0	1	0	0	0	0	1	0	74.99	66.67
382	1	3	3	1	0	1	0	0	0	0	1	0	74.99	66.67
383	1	3	3	2	0	1	0	0	0	0	1	0	74.99	66.67
384	1	3	3	3	0	1	0	0	0	0	1	0	74.99	66.67

(continued)

Table 10.6 (continued)

Disruption scenario							Supply portfolio				Demand portfolio		Solution values	
s	λ_{1s}	λ_{2s}	λ_{3s}	λ_{4s}	λ_{5s}	λ_{6s}	i = 1	i = 2	i = 3	i = 4	j = 1	j = 2	Cost	Service %
640	1	3	3	3	1	0	0	0	0	0	1	0	14.99	93.33
646	2	0	1	1	1	0	0.9	0	0	0	1	0	67.21	72.96
647	2	0	1	2	1	0	0.9	0	0	0	1	0	67.21	72.96
648	2	0	1	3	1	0	0	0	0	0.9	1	0	22.34	86.67
657	2	1	0	0	1	0	0.98	0	0	0	1	0	75.14	65.57
658	2	1	0	1	1	0	0.98	0	0	0	1	0	75.14	65.57
659	2	1	0	2	1	0	0.98	0	0	0	1	0	75.14	65.57
660	2	1	0	3	1	0	0	0	0	0.98	1	0	23.6	85.57
661	2	1	1	0	1	0	0.88	0	0	0	1	0	64.93	75.2
662	2	1	1	1	1	0	0.88	0	0	0	1	0	64.93	75.2
663	2	1	1	2	1	0	0.88	0	0	0	1	0	64.93	75.2
664	2	1	1	3	1	0	0	0	0	0.88	1	0	20.17	88.91
673	2	2	0	0	1	0	0	0.95	0	0	1	0	63.48	68.58
676	2	2	0	3	1	0	0	0	0	0.95	1	0	20.68	88.58
677	2	2	1	0	1	0	0	0.85	0	0	1	0	53.47	78.21
844	1	0	2	3	1	1	0	0	0	0.48	0.87	0.13	9.05	100
845	1	0	3	0	1	1	0	0	0.07	0	0.93	0.07	15.13	93.33
961	3	0	0	0	1	1	1	0	0	0	0.9	0.1	14.52	100
976	3	0	3	3	1	1	0	0	0.07	0	0.93	0.07	15.13	93.33
977	3	1	0	0	1	1	0.98	0	0	0	0.92	0.08	14.45	100
992	3	1	3	3	1	1	0	0	0.04	0	0.96	0.04	15.19	93.33
1009	3	3	0	0	1	1	0	0.93	0	0	0.93	0.07	12.46	100
1024	3	3	3	3	1	1	0	0	0	0	1	0	14.99	93.33

than the maximum delivery lead time, $\max_{i\in I, j\in J}(\tau_{ij}) = 4$. Thus $t_s \in \{1, 2, 3, 4\}$, and the total number of all potential scenarios to be considered is 1024×4=4096, i.e., $S = \{1, \ldots, 4096\}$. Now, each disruption scenario $s \in S$ is represented by vector $\lambda_s = (\lambda_{1s}, \ldots, \lambda_{6s})$, where $\lambda_s = \lambda_{s+1024} = \lambda_{s+2048} = \lambda_{s+3072};\ s \le 1024$, and its start time $t_s = 1$ for $s \le 1024$, $t_s = 2$ for $1025 \le s \le 2048$, $t_s = 3$ for $2049 \le s \le 3072$ and $t_s = 4$ for $3073 \le s \le 4096$.

The probability P_s of realizing each disruption scenario $s \in S$ is calculated as follows:

$P_s = \beta_1 P_s^1 P_s^2$ for $s \le 1024$,

$P_s = \beta_2 P_{s-1024}^1 P_{s-1024}^2$ for $1025 \le s \le 2048$,

$P_s = \beta_3 P_{s-2048}^1 P_{s-2048}^2$ for $2049 \le s \le 3072$,

$P_s = \beta_4 P_{s-3072}^1 P_{s-3072}^2$ for $3073 \le s \le 4096$,

where probabilities $P_s^r;\ r = 1, 2,\ s \le 1024$ are defined at the beginning of this section, and β_1, β_2, β_3 and β_4 are nonnegative constants such that: $\beta_1+\beta_2+\beta_3+\beta_4 = 1$. In the computational examples, optimal solutions were determined for $\beta_1 = 0.1$,

Table 10.7 Worst-case disruption scenarios: $g = 100$, integrated approach

Disruption scenario $(\lambda_{1s}, \lambda_{2s}, \lambda_{3s}, \lambda_{4s}, \lambda_{5s}, \lambda_{6s}), t_s$	Recovery portfolio $(V^s_{11} + V^s_{12}, V^s_{21} + V^s_{22}, V^s_{31} + V^s_{32}, V^s_{41} + V^s_{42}, z^s_1, z^s_2)$	Cost [(a)]	Service Level [(b)]
Primary supply portfolio (v_1, v_2, v_3, v_4)= (0,0.04,0.78,0.18)			
Worst-case disruption scenarios with respect to cost			
(0,0,0,0,0,0), 1	(0,1,0,0,1,0)	2095	50
(0,0,0,0,0,1), 1	(0,1,0,0,0.5,0.5)	2071	75
(0,0,0,0,1,0), 1	(0,1,0,0,0.4,0.6)	2081	65
(0,0,0,0,1,1), 1	(0,1,0,0,0.75,0.25)	2038	75
(0,0,0,0,0,0), 2	(0,1,0,0,1,0)	2099	47
(0,0,0,0,0,1), 2	(0,1,0,0,0.77,0.23)	2076	70
(0,0,0,0,1,0), 2	(0,1,0,0,1,0)	2065	47
(0,0,0,0,1,1), 2	(0,1,0,0,0.47,0.53)	2043	70
(0,0,0,0,0,0), 3	(0,1,0,0,1,0)	2102	43
(0,0,0,0,0,1), 3	(0,1,0,0,0.78,0.22)	2081	65
(0,0,0,0,1,0), 3	(0,1,0,0,1,0)	2069	43
(0,0,0,0,1,1), 3	(0,1,0,0,0.43,0.57)	2048	65
(0,0,0,0,0,0), 4	(0,1,0,0,1,0)	2105	40[(c)]
(0,0,0,0,0,1), 4	(0,1,0,0,0.4,0.6)	2086	60
(0,0,0,0,1,0), 4	(0,1,0,0,1,0)	2072	40[(c)]
(0,0,0,0,1,1), 4	(0,1,0,0,0.8,0.2)	2052	60
Selected worst-case disruption scenarios with respect to service level			
(0,0,0,0,0,0), 4	(0,1,0,0,1,0)	2105	40[(c)]
(0,0,0,0,1,0), 4	(0,1,0,0,1,0)	2072	40[(c)]

[(a)] $\sum_{i \in I}(e_i(u_i + U^s_i - q^s_i)/D + \rho_{is}U^s_i/D + o_i(\gamma^s_i v_i + \sum_{j \in J} V^s_{ij})) + \sum_{j \in J}(\psi_j w^s_j + (\varphi_j + \rho_{js})y^s_j/D + \varepsilon_j \sum_{t \in T} x^s_{jt}/D) + g(1 - \sum_{j \in J}\sum_{t \in T} x^s_{jt}/D)$.

[(b)] $\sum_{j \in J}\sum_{t \in T} x^s_{jt}/D \times 100$.

[(c)] $\underline{SL} \times 100$, (10.35).

$\beta_2 = 0.2$, $\beta_3 = 0.3$, $\beta_4 = 0.4$, i.e., disruptive events with later start times are modelled to be more likely.

In order to identify the cost best-case and worst-case disruption scenarios, the cost per product associated with optimal solution of **DSupport_E** for the integrated approach (or **PSupport** and **RDSupport_E** for the hierarchical approach) is chosen to be not greater than 10 and not less than 2000, respectively. For the examples with unit penalty $g = 100$, the total number of cost best-case scenarios is 384 and 171, respectively for the integrated and the hierarchical approach. The maximum service level, $\sum_{j \in J}\sum_{t \in T} x^s_{jt}/D = 1$, was achieved for all best-case scenarios with respect to cost, which clearly shows that for the maximum service level, no penalty cost for unfulfilled customer demand is incurred. The cost best-case disruption scenarios for the hierarchical approach are a subset of those for the integrated approach.

In contrast, the number of cost worst-case disruption scenarios is much smaller, 16 and 53 scenarios, respectively for the integrated and the hierarchical approach.

Table 10.8 Selected worst-case disruption scenarios with respect to cost and service level: $g = 100$, hierarchical approach

Disruption scenario $(\lambda_{1s}, \lambda_{2s}, \lambda_{3s}, \lambda_{4s}, \lambda_{5s}, \lambda_{6s}), t_s$	Recovery portfolio $(V^s_{11} + V^s_{12}, V^s_{21} + V^s_{22}, V^s_{31} + V^s_{32}, V^s_{41} + V^s_{42}, z^s_1, z^s_2)$	Cost [(a)]	Service Level [(b)]
Primary supply portfolio: (v_1, v_2, v_3, v_4)=(0,0.07,0.93,0)			
(0,1,1,0,0,0), 3	(0,0,0,0.88,0.98,0.02)	4469	40
(1,0,0,2,0,0), 3	(0,0.03,0.97,0,1,0)	6102	40
(1,0,1,0,0,0), 3	(0,0,0,0.90,0.38,0.62)	4469	40
(2,0,0,1,0,0), 3	(0,0,0.97,0.03,1,0)	4534	40
(0,1,1,0,0,0), 4	(0,0,0,0.88,0.98,0.02)	4469	40
(1,0,1,0,0,0), 4	(0,0,0,0.90,0.98,0.02)	4469	40

[(a)] $\sum_{i\in I}(e_i(u_i + U^s_i - q^s_i)/D + \rho_{is}U^s_i/D + o_i(\gamma^s_i v_i + \sum_{j\in J} V^s_{ij})) + \sum_{j\in J}(\psi_j w^s_j + (\varphi_j + \rho_{js})y^s_j/D + \varepsilon_j \sum_{t\in T} x^s_{jt}/D) + g(1 - \sum_{j\in J}\sum_{t\in T} x^s_{jt}/D)$.
[(b)] $\underline{SL} \times 100$, (10.35).

The corresponding service level is ranging, respectively from 0.40 to 0.75 and from 0.40 to 0.65. The cost worst-case disruption scenarios for the integrated approach are a subset of those for the hierarchical approach.

In order to identify the service level best-case and worst-case disruption scenarios, the service level associated with optimal solution of **DSupport_E** for the integrated approach (or **PSupport** and **RDSupport_E** for the hierarchical approach) is chosen to be not less than 1 and not greater than $\underline{SL} = 0.40$, (10.35), respectively. For the examples with unit penalty $g = 100$, the total number of best-case scenarios with service level $\sum_{j\in J}\sum_{t\in T} x^s_{jt}/D = 1$ is 1290 and 720, respectively for the integrated and the hierarchical approach. The corresponding cost per product is ranging, respectively from 8.57 to 102.60 and from 9.05 to 81.44. The best-case scenarios for the hierarchical approach are a subset of those for the integrated approach. In contrast, the number of service level worst-case scenarios for the integrated approach is much smaller, 71 scenarios, while for the hierarchical approach there are 573 scenarios. The corresponding cost per product is ranging, respectively from 124 to 2105 and from 68 to 6102. The worst-case scenarios for the integrated approach are a subset of those for the hierarchical approach.

For the examples with unit penalty $g = 100$, Tables 10.7 and 10.8 present worst-case disruption scenarios with respect to cost and service level, respectively for the integrated and the hierarchical approach. For the integrated approach, Table 10.7 presents all 16 worst-case scenarios with cost per product not less than 2000, and a subset of two worst-case scenarios with service level $\sum_{j\in J}\sum_{t\in T} x^s_{jt}/D = \underline{SL} = 0.4$ and cost per product not less than 2000. For the hierarchical approach, Table 10.8 presents a subset of 6 worst-case scenarios with respect to cost such that cost per product is not less than $0.7\overline{R^c}$ (for the example problems, $\overline{R^c} = 6108$, (10.70)) and with respect to service level such that service level is $\sum_{j\in J}\sum_{t\in T} x^s_{jt}/D = \underline{SL} = 0.4$.

Tables 10.7 and 10.8 show disruption scenario, $(\lambda_{1s}, \lambda_{2s}, \lambda_{3s}, \lambda_{4s}, \lambda_{5s}, \lambda_{6s})$, along with its start time, t_s, and the corresponding optimal recovery supply and demand portfolio, $(V^s_{11} + V^s_{12}, V^s_{21} + V^s_{22}, V^s_{31} + V^s_{32}, V^s_{41} + V^s_{42}, z^s_1, z^s_2)$, cost per product,

$$\sum_{i \in I}(e_i(u_i + U^s_i - q^s_i)/D + \rho_{is}U^s_i/D + o_i(\gamma^s_i v_i + \sum_{j \in J} V^s_{ij}))$$
$$+ \sum_{j \in J}(\psi_j w^s_j + (\varphi_j + \rho_{js})y^s_j/D + \varepsilon_j \sum_{t \in T} x^s_{jt}/D)$$
$$+g(1 - \sum_{j \in J}\sum_{t \in T} x^s_{jt}/D),$$

and service level $\sum_{j \in J}\sum_{t \in T} x^s_{jt}/D$.

The results in Table 10.7 demonstrate that for the worst-case scenarios with respect to cost, all primary suppliers are completely shutdown and then the one with shorter delivery lead time is selected as a single sourcing recovery portfolio. When both assembly plants are shutdown, the primary plant is selected as a single recovery plant. Otherwise, the recovery demand portfolio may include two plants. The results also indicate that for a given disruption pattern, $(\lambda_{1s}, \lambda_{2s}, \lambda_{3s}, \lambda_{4s}, \lambda_{5s}, \lambda_{6s})$, both cost per product and service level are deteriorating with disruption start time, t_s. For example (see Table 10.7), for disruption pattern (0, 0, 0, 0, 0, 0), cost per product is increasing, 2095, 2099, 2102, 2105, and service level is decreasing, 50%, 47%, 43%, 40%, respectively with start times, $t_s = 1, 2, 3, 4$. For the hierarchical approach, Table 10.8 shows that for the worst-case scenario with respect to both service level and cost, all assembly plants are shutdown, while primary suppliers are fully or partially disrupted. The highest cost per product, 6102, is connected with scenario, (1, 0, 0, 2, 0, 0) and start time $t_s = 3$, where the primary suppliers were first completely shutdown and then recovered and selected as recovery suppliers. The recovery supply portfolio is very close to the primary supply portfolio and the recovery demand portfolio is identical with the primary demand portfolio.

The computational experiments for the risk-neutral decision-making demonstrate that:

- *when all primary suppliers are completely shutdown, a single sourcing recovery supply portfolio is usually selected,*
- *if all assembly plants are shutdown, the integrated approach selects the primary plant as a single recovery plant, whereas the hierarchical approach may choose multiple recovery plants,*
- *both cost per product and service level are deteriorating with disruption start time,*
- *the best-case and worst-case disruption scenarios for the hierarchical approach are, respectively subsets and supersets of the corresponding scenarios for the integrated approach.*

The computational experiments were performed using the AMPL programming language and the Gurobi 7.0.0 solver on a MacBookPro laptop with Intel Core i7 processor running at 2.8 GHz and with 16GB RAM. The solver was capable of finding proven optimal solution for all examples with CPU time ranging from fraction of a second for deterministic models **PSupport** and **RDSupport(s)** to a few seconds for stochastic models **DSupport_E** and **RDSupport_E**.

10.5.2 Risk-Averse Decision-Making

In this subsection some computational examples are presented to illustrate the risk-averse selection of primary and recovery supply and demand portfolios to minimize CVaR of cost or maximize CVaR of service level. In the computational experiments the data sets provided at the beginning of this section were used, except for disruption scenarios that are described in Sect. 10.5.1.1.

In addition, for minimization of CVaR of cost, unit penalty was fixed to $g = 100$. In order to eliminate recovery cost of impacted suppliers that are not selected to recovery supply portfolio, additional constraint was introduced into the models to assign at least 5% of the total demand of parts to each recovery supplier under each scenario, that is,

$$\sum_{j \in J} V_{ij}^s \geq 0.05 U_i^s; \;\; i \in I, s \in S.$$

The solution results for the integrated approach are summarized in Tables 10.9 and 10.10, respectively for models **DSupport_CV(c)** and **DSupport_CV(sl)**. The tables show primary supply portfolio and expected recovery supply and demand portfolios for different values of confidence level $\alpha = 0.5, 0.75, 0.9, 0.95, 0.99$. The solution values, $CVaR^c$ and $CVaR^{sl}$, are presented along with the associated expected values of cost, E^c, and service level, E^{sl}. While $CVaR^c$ and VaR^c increase, and $CVaR^{sl}$ and VaR^{sl} decrease with the confidence level α, the associated expected values, E^c and E^{sl}, are not varying monotonously.

In addition, Tables 10.9 and 10.10 show best-case and worst-case scenarios with respect to cost, such that cost per product associated with optimal solution is not greater than 10 and not less than 2000, respectively. The tables also show best-case and worst-case scenarios with respect to service such that service level is equal to 1 and to $\underline{SL} = 0.40$, (10.35), respectively. Tables 10.9 and 10.10, clearly indicate that the best-case scenarios with respect to cost are associated with the maximum service level 100%, i.e., with no penalty cost for unfulfilled demand. On the other hand, however, the best case scenarios with respect to service, for which service level also attains 100%, can be associated with a higher cost, e.g., due to required recovery processes. The results indicate that *the best case scenarios with respect to cost are a subset of the best case scenarios with respect to service.*

For model **DSupport_CV(sl)**, where the objective function does not account for any cost parameters, the worst-case scenarios with respect to service are a subset of the worst-case scenarios with respect to cost, see Table 10.10.

In addition, Tables 10.9 and 10.10 show the worst-case scenarios with respect to both cost and service level such that cost per product is not less than 2000 and service level is equal to $\underline{SL} = 0.40$. These scenarios are a subset of maximum cost

Table 10.9 Risk-averse solutions for model **DSupport_CV(c)**, $g = 100$

Confidence level α	0.50	0.75	0.90	0.95	0.99
Var. = 274413, Bin. = 40964, Cons. = 491761, Nonz. = 4451648 [(a)]					
$CVaR^c$	33.98	52.07	81.92	82.84	84.86
VaR^c	15.66	16.03	77.84	82.33	82.33
E^c	24.56	24.80	39.76	33.14	34.93
E^{sl} 100%	88.72	89.91	76.74	81.77	79.86
Primary Supply Portfolio: Supplier(% of total demand) [(b)]	2(23)	2(23)	2(24)	2(23)	2(23)
	3(77)	3(45)	3(63)	3(77)	3(77)
		4(32)	4(13)		
Exp.Recovery Supply Portfolio: Supplier(% of total demand) [(c)]	1(0.98)	1(1.51)	1(1.61)	1(3.34)	1(1.97)
	2(1.68)	2(2.70)	2(1.61)	2(1.46)	2(2.56)
	3(0.67)	3(1.59)	3(2.04)	3(0.48)	3(0.56)
	4(4.04)	4(1.26)	4(1.98)	4(2.09)	4(2.27)
Exp.Recovery Demand Portfolio: Supplier(% of total demand) [(d)]	1(95.80)	1(94.65)	1(97.53)	1(98.03)	1(97.65)
	2(2.93)	2(4.12)	2(2.21)	2(1.46)	2(1.86)
Best-Case and Worst-Case Scenarios					
Number of Cost Best-Case Scenarios (Cost≤ 10)	15	112	1	0	4
Minimum service level %	100	100	100		100
Maximum service level %	100	100	100		100
Number of Service Best-Case Scenarios (Service = 100%)	706	970	63	26	39
Minimum cost	8.53	8.57	10	33.25	8.90
Maximum cost	212	212	212	82.73	212
Number of Cost Worst-Case Scenarios (Cost≥ 2000)	18	16	16	16	16
Minimum service %			40		
Maximum service %			75		
Number of Service Worst-Case Scenarios (Service=$\underline{SL}$)	73	2	787	392	389
Minimum cost	124	2072	68	68	68
Maximum cost	4337	2105	2105	2105	2105
Number of Cost and Service Worst-Case Scenarios	4	2	2	2	3
(Cost≥ 2000 and Service = $\underline{SL}$)					
Maximum Cost	4337	2105	2105	2105	2105

[(a)] Var. = number of variables, Bin. = number of binary variables, Cons.=number of constraints, Nonz. = number of nonzero coefficients.
[(b)] $1(v_1 \times 100), 2(v_2 \times 100), 3(v_3 \times 100), 4(v_4 \times 100)$.
[(c)] $1(\sum_{j\in J, s\in S} P_s V_{1j}^s \times 100), 2(\sum_{j\in J, s\in S} P_s V_{2j}^s \times 100), 3(\sum_{j\in J, s\in S} P_s V_{3j}^s \times 100), 4(\sum_{j\in J, s\in S} P_s V_{4j}^s \times 100)$.
[(d)] $1(\sum_{s\in S} P_s z_1^s \times 100), 2(\sum_{s\in S} P_s z_2^s \times 100)$.

worst-case scenarios with respect to service. Such worst-case scenarios for model **DSupport_CV(c)** are presented in more details in Table 10.11.

Table 10.11 indicates that the worst-case scenarios with respect to both cost and service level are nearly identical for all confidence level, α, except for the lowest $\alpha = 0.5$. Under the worst-case scenario, all suppliers and assembly plants are either completely shutdown or hit by major disruption. When all primary suppliers are shutdown, only one of them is selected for recovery to reduce recovery cost. In

Table 10.10 Risk-averse solutions for model **DSupport_CV(sl)**

Confidence level α	0.50	0.75	0.90	0.95	0.99
Var. = 274413, Bin. = 40964, Cons. = 491761, Nonz. = 4451648 [(a)]					
$CVaR^{sl}$100%	85.53	77.73	61.32	60	60
VaR^{sl}100%	93.33	93.33	66.67	60	60
E^{sl}100%	89.82	89.74	77.50	75.29	75.44
E^{c}	127	95	137	149	177
Primary Supply Portfolio: Supplier(% of total demand) [(b)]			1(34)	1(26)	1(26)
	2(23)	2(23)	2(31)	2(23)	2(23)
	3(37)	3(37)	3(15)	3(22)	3(22)
	4(40)	4(40)	4(20)	4(29)	4(29)
Exp.Recovery Supply Portfolio: Supplier(% of total demand) [(c)]	1(2.19)	1(2.30)	1(1.86)	1(1.67)	1(1.21)
	2(1.88)	2(1.33)	2(0.96)	2(1.25)	2(1.08)
	3(1.82)	3(1.62)	3(1.09)	3(1.76)	3(2.22)
	4(1.11)	4(1.75)	4(0.75)	4(0.79)	4(0.96)
Exp.Recovery Demand Portfolio: Supplier(% of total demand) [(d)]	1(94.13)	1(94.16)	1(96.10)	1(95.63)	1(95.98)
	2(4.68)	2(4.61)	2(3.51)	2(4.03)	2(3.58)
Best-Case and Worst-Case Scenarios					
Number of Cost Best-Case Scenarios (Cost$\leq$ 10)	12	3	1	0	0
Minimum service level %	100	100	100		
Maximum service level %	100	100	100		
Number of Service Best-Case Scenarios (Service = 100%)	312	262	22	23	19
Minimum cost	8.71	9.27	9.71	11.04	46
Maximum cost	8676	7044	6379	7044	4478
Number of Cost Worst-Case Scenarios (Cost$\geq$ 2000)	1602	1589	1425	1642	1987
Minimum service %			40		
Maximum service %			100		
Number of Service Worst-Case Scenarios (Service = $\underline{SL}$)	367	472	581	478	260
Minimum cost	103	102	126	126	159
Maximum cost	10436	8469	7139	8470	7104
Number of Cost and Service Worst-Case Scenarios	192	260	281	257	183
(Cost$\geq$ 2000 and Service = $\underline{SL}$)					
Maximum Cost	10436	8469	7139	8470	7104

[(a)] Var. = number of variables, Bin. = number of binary variables, Cons. = number of constraints, Nonz. = number of nonzero coefficients.
[(b)] $1(v_1 \times 100), 2(v_2 \times 100), 3(v_3 \times 100), 4(v_4 \times 100)$.
[(c)] $1(\sum_{j\in J, s\in S} P_s V_{1j}^s \times 100), 2(\sum_{j\in J, s\in S} P_s V_{2j}^s \times 100), 3(\sum_{j\in J, s\in S} P_s V_{3j}^s \times 100), 4(\sum_{j\in J, s\in S} P_s V_{4j}^s \times 100)$.
[(d)] $1(\sum_{s\in S} P_s z_1^s \times 100), 2(\sum_{s\in S} P_s z_2^s \times 100)$.

Table 10.11 Worst-case scenarios with respect to cost and service level: model **DSupport_CV(c)**, $g = 100$

Disruption scenario $(\lambda_{1s}, \lambda_{2s}, \lambda_{3s}, \lambda_{4s}, \lambda_{5s}, \lambda_{6s}), t_s$	Recovery portfolio $(V^s_{11} + V^s_{12}, V^s_{21} + V^s_{22}, V^s_{31} + V^s_{32}, V^s_{41} + V^s_{42}, z^s_1, z^s_2)$	Cost [a]	Service Level [b]
Confidence level $\alpha = 0.5$, primary suppliers $i = 2, 3$			
(0,1,0,1,0,0), 1	(0,0.32,0.6,0,0.4,0.6)	4337	40[c]
(1,0,0,1,0,0), 1	(0,0,1,0,0.18,0.82)	4135	40
(0,0,0,0,0,0), 4	(0,1,0,0,1,0)	2105	40
(0,0,0,0,1,0), 4	(0,1,0,0,1,0)	2072	40
Confidence level $\alpha = 0.75, 0.90$, primary suppliers $i = 2, 3, 4$			
(0,0,0,0,0,0), 4	(0,1,0,0,1,0)	2105	40
(0,0,0,0,1,0), 4	(0,1,0,0,1,0)	2072	40
Confidence level $\alpha = 0.95$, primary suppliers $i = 2, 3$			
(0,0,0,0,0,0), 4	(0,1,0,0,1,0)	2105	40
(0,0,0,0,1,0), 4	(0,1,0,0,1,0)	2072	40
Confidence level $\alpha = 0.99$, primary suppliers $i = 2, 3$			
(0,0,0,0,0,1), 1	(0,1,0,0,0.4,0.6)	2105	40
(0,0,0,0,0,0), 4	(0,1,0,0,1,0)	2105	40
(0,0,0,0,1,0), 4	(0,1,0,0,1,0)	2072	40

[a] $\sum_{i\in I}(e_i(u_i + U^s_i - q^s_i)/D + \rho_{is}U^s_i/D + o_i(\gamma^s_i v_i + \sum_{j\in J} V^s_{ij})) + \sum_{j\in J}(\psi_j w^s_j + (\varphi_j + \rho_{js})y^s_j/D + \varepsilon_j \sum_{t\in T} x^s_{jt}/D) + g(1 - \sum_{j\in J}\sum_{t\in T} x^s_{jt}/D)$.

[b] $\sum_{j\in J}\sum_{t\in T} x^s_{jt}/D \times 100$.

[c] $\underline{SL} \times 100$, (10.35).

contrast to selection of recovery plants, where both shutdown plants may be selected for recovery. When both plants are selected as recovery plants, a larger fraction of demand for products is assigned to plant $j = 2$. While per period capacity, $c_2 = 5000$ of plant $j = 2$ is half of the capacity, $c_1 = 10000$ of plant $j = 1$, its recovery time $PRT(2, 0) = 5$ is half of the recovery time $PRT(1, 0) = 10$. In the examples, however, the largest portion of total cost is recovery cost of impacted suppliers, $\sum_{i\in I} \rho_{is}U^s_i/D$.

Tables 10.12 and 10.13 provide risk-averse solutions for the hierarchical approach and models **RDSupport_CV(c)** and **RDSupport_CV(sl)**, respectively, with the primary supply portfolio determined using model **PSupport**. A general comparison of solution results in Tables 10.9 and 10.12 and in Tables 10.10 and 10.13 demonstrates that both the integrated and the hierarchical approach lead to similar risk-averse solutions. The main differences are less diversified primary supply portfolios determined by the deterministic upper level model **PSupport**: a dual sourcing for cost-based objective function (10.37) and a single sourcing for service-based objective function (10.77).

The computational experiments for the risk-averse decision-making demonstrate that:

- *the integrated decision-making selects a more diversified primary supply portfolio to hedge against all potential disruption scenarios,*

Table 10.12 Risk-averse solutions for model **RDSupport_CV(c)**, $g = 100$

Primary supply portfolio: $(v_1, v_2, v_3, v_4) = (0,0.07,0.93,0)$

Confidence level α	0.50	0.75	0.90	0.95	0.99
Var. = 247105, Bin. = 27456, Cons. = 310590, Nonz. = 3050044 [(a)]					
$CVaR^c$	33.43	51.84	81.51	82.18	84.22
VaR^c	15.01	15.01	78.34	81.67	81.67
E^c	24.00	24.08	32.16	32.19	32.63
E^{sl} 100%	88.83	89.61	82.46	83.07	81.54
Exp.Recovery Supply Portfolio: Supplier(% of total demand) [(b)]	1(0.32)	1(0.37)	1(1.44)	1(2.18)	1(1.64)
	2(2.04)	2(2.56)	2(2.44)	2(2.07)	2(3.11)
	3(0.38)	3(2.32)	3(0.87)	3(1.48)	3(0.46)
	4(0.13)	4(4.93)	4(3.67)	4(2.36)	4(2.88)
Exp.Recovery Demand Portfolio: Supplier(% of total demand) [(c)]	1(97.56)	1(95.50)	1(98.01)	1(96.41)	1(97.06)
	2(1.15)	2(3.31)	2(0.84)	2(2.40)	2(1.84)
Best-Case and Worst-Case Scenarios					
Number of Cost Best-Case Scenarios (Cost$\leq$ 10)	11	5	1	3	3
Minimum service level %			100		
Maximum service level %			100		
Number of Service Best-Case Scenarios (Service = 100%)	555	744	146	121	106
Minimum cost	9.19	9.32	9.48	9.72	9.62
Maximum cost	82.05	210	87	86.54	125
Number of Cost Worst-Case Scenarios (Cost$\geq$ 2000)	33	16	16	17	16
Minimum service %			40		
Maximum service %	47	75	75	95	75
Number of Service Worst-Case Scenarios (Service = $\underline{SL}$)	1038	9	670	541	578
Minimum cost	68	146	68	68	68
Maximum cost	4135	2105	2105	2105	2105
Number of Cost and Service Worst-Case Scenarios	31	2	2	2	2
(Cost$\geq$ 2000 and Service = $\underline{SL}$)					
Maximum Cost	4135	2105	2105	2105	2105

[(a)] Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients.
[(b)] $1(\sum_{j\in J, s\in S} P_s V_{1j}^s \times 100), 2(\sum_{j\in J, s\in S} P_s V_{2j}^s \times 100), 3(\sum_{j\in J, s\in S} P_s V_{3j}^s \times 100), 4(\sum_{j\in J, s\in S} P_s V_{4j}^s \times 100)$.
[(c)] $1(\sum_{s\in S} P_s z_1^s \times 100), 2(\sum_{s\in S} P_s z_2^s \times 100)$.

- *when all primary suppliers are completely shutdown, a single sourcing recovery supply portfolio is usually selected,*
- *multiple recovery plants may be selected even if all assembly plants are shutdown, in particular for a low confidence level,*
- *the best-case and worst-case disruption scenarios for the hierarchical approach are, respectively subsets and supersets of the corresponding scenarios for the integrated approach.*

Notice that very similar findings were identified for the risk-neutral decision-making in Sect. 10.5.1.

Table 10.13 Risk-averse solutions for model **RDSupport_CV(sl)**

Primary supply portfolio: $(v_1, v_2, v_3, v_4) = (0,1,0,0)$

Confidence level α	0.50	0.75	0.90	0.95	0.99
Var. = 210553, Bin. = 19456, Cons. = 254876, Nonz. = 2338786 [(a)]					
$CVaR^{sl}$100%	84.32	75.36	60.15	60	60
VaR^{sl}100%	93.33	93.33	63.33	60	60
E^{sl}100%	88.83	88.84	86.23	85.93	85.95
E^c	62.34	62.40	61.38	62.64	61.57
Exp.Recovery Supply Portfolio: Supplier(% of total demand) [(b)]	1(1.44)	1(1.43)	1(1.24)	1(0.80)	1(1.41)
	2(2.04)	2(2.04)	2(1.89)	2(2.14)	2(1.54)
	3(0.38)	3(0.38)	3(0.42)	3(0.52)	3(0.59)
	4(0.13)	4(0.13)	4(0.43)	4(0.53)	4(0.44)
Exp.Recovery Demand Portfolio: Supplier(% of total demand) [(c)]	1(97.56)	1(97.55)	1(96.73)	1(96.75)	1(97.27)
	2(1.15)	2(1.16)	2(2.04)	2(2.02)	2(1.50)
Best-Case and Worst-Case Scenarios					
Number of Cost Best-Case Scenarios (Cost$\leq$ 10)			0		
Minimum service level %					
Maximum service level %					
Number of Service Best-Case Scenarios (Service = 100%)	2	2	9	3	7
Minimum cost	2037	2037	44	12	474
Maximum cost	2071	2071	4478	6376	6342
Number of Cost Worst-Case Scenarios (Cost$\geq$ 2000)	1652	1703	1374	1595	1604
Minimum service %			40		
Maximum service %			100		
Number of Service Worst-Case Scenarios (Service = $\underline{SL}$)	1848	1711	538	458	257
Minimum cost	99	99	101	134	167
Maximum cost	6773	6773	8469	8469	7104
Number of Cost and Service Worst-Case Scenarios	1158	1135	338	344	196
(Cost$\geq$ 2000 and Service=$\underline{SL}$)					
Maximum Cost	6773	6773	8469	8469	7104

[(a)] Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints, Nonz. = number of nonzero coefficients.

[(b)] $1(\sum_{j\in J,s\in S} P_s V_{1j}^s \times 100), 2(\sum_{j\in J,s\in S} P_s V_{2j}^s \times 100), 3(\sum_{j\in J,s\in S} P_s V_{3j}^s \times 100), 4(\sum_{j\in J,s\in S} P_s V_{4j}^s \times 100)$.

[(c)] $1(\sum_{s\in S} P_s z_1^s \times 100), 2(\sum_{s\in S} P_s z_2^s \times 100)$.

Overall, the results of computational experiments indicate that the proposed portfolio approach and developed SMIP models with an embedded network flow problem is a flexible and efficient tool for supply chain disruption management. The approach leads to SMIP formulations with a strong LP relaxation and has proven to be computationally very efficient. CPU time required to find proven optimal solutions for all examples, using commercially available software for MIP, was ranging from fraction of a second for model **RDSupport_CV(sl)** to less than an hour for model **DSupport_CV(c)**.

10.6 Notes

The literature on mitigation the impact of disruption risks and optimization of a recovery process in supply chains is limited, e.g., Gurnani et al. (2012). For example, mitigation and contingency/recovery actions were studied by Tomlin (2006) in a dual-sourcing setting, one unreliable supplier and another reliable and more expensive. A buyer that suffers a supply shortage can buy from a more expensive alternate supplier or produce less, and its decision depends on its inventory. If reliable supplier has volume flexibility, contingent rerouting by temporarily increasing its production may prove to be an effective way to speed up recovery process. The author established that, along with cost, percentage of supplier uptime, disruption length, capacity, and flexibility, play an important role in determining a buyers disruption management strategy. MacKenzie et al. (2014) proposed a model for severe disruptions in which a disruption simultaneously impacts several suppliers, while the suppliers' customers (firms) may face supply shortages. The model incorporated decisions made by both suppliers and firms during the disruption of random duration. The decisions may include whether or not suppliers move production to an alternate facility, hold parts inventory; a firm purchases parts from alternate suppliers that are not impacted, helps a primary supplier recover more quickly or holds finished products inventory. The supplier and the firm optimal decisions are expressed in terms of model parameters, e.g., the cost of each strategy, the chances of losing business, and the probability of suppliers recovery. The impact of a disruption was evaluated based on the optimal response of an existing network once that disruption has occurred. The authors derived threshold parameters to support decisions whether and when a supplier moves production to an alternate facility and how much the firm should produce during the disruption to trade off between two conflicting objectives: maximizing profit and service level. They applied model to a simulation based on the Great East Japan earthquake and tsunami in March 2011. Ivanov et al. (2016) analyzed seven proactive supply chain structures and computed recovery policies to re-direct material flows in the case of two disruption scenarios. They assessed the performance impact of the duration of disruptions and the costs of recovery for both service level and costs with the help of a supply chain (re)planning model containing elements of system dynamics and linear programming. A review of literature on Operations Research/Management Science models for supply chain disruptions was presented by Snyder et al. (2016). They discussed 180 scholarly works on the topic, organised into six categories: evaluating supply disruptions; strategic decisions; sourcing decisions; contracts and incentives; inventory; and facility location.

The future research should concentrate on relaxations of the various simplified assumptions used to formulate the problem (see, Sect. 10.2). For example, a single recovery mode defined by constant recovery times, $TTR(i,l)$, $PRT(j,l)$, and the associated recovery costs, $CTR(i,l)$, $PRC(j,l)$, for each supplier i, each assembly plant j and each disruption level l, can be replaced by a number of available recovery modes, each represented by different values of recovery time and the associated recovery cost. Then, introduction of new recovery mode selection variables for each supplier and each assembly plant can enhance the problem formulation. In addition, the inventory of parts at the impacted primary assembly plant may be non available for transshipment or the transshipment itself can be non available. Then, the recovery supply portfolio should be selected, even after full delivery of all required parts to the impacted primary assembly plant. In a more general setting, both recovery time and recovery cost can be modeled as random parameters, e.g., Schmitt (2011),Schmitt and Singh (2012).

Problems

10.1 In the SMIP models presented in this chapter introduce delay penalty when demand d_t for period t is not met by that period.

10.2 Enhance the SMIP models presented in this chapter for the case in which the inventory of parts at the impacted primary assembly plant is not available for transshipment.

10.3 Formulate the mean-risk models **DSupport_ECV(c)** and **DSupport_ECV(sl)** to minimize expected cost and CVaR of cost or maximize expected service level and CVaR of service level, respectively.

10.4 Enhance model **DSupport_E** for multiple recovery modes defined by different recovery time, $TTR(i,l)$, $PRT(j,l)$, and the associated recovery cost, $CTR(i,l)$, $PRC(j,l)$, for each supplier i, each assembly plant j and each disruption level l.

10.5 Explain why the best-case and worst-case disruption scenarios for the hierarchical approach are, respectively subsets and supersets of the corresponding scenarios for the integrated approach.

Part V
Information Flow Disruption Management

Chapter 11
Selection of Cybersecurity Safequards Portfolio

11.1 Introduction

Nowadays information technology (IT) becomes one of the core elements of global supply chains that are increasingly at risk of a disruption of their information systems. IT may improve a supply chain robustness and resilience, e.g., Pereira (2009). For example, an important determinant of supply chain competitiveness is supply chain visibility defined as "the extent to which actors within a supply chain have access to or share information which they consider as key or useful to their operations and which they consider will be of mutual benefit" (Lee et al. 2014). The supply chain visibility is accomplished through inter organizational information systems, which should be very well protected against cyber-attacks. This chapter deals with the selection of cybersecurity safeguards portfolio to prevent and mitigate the impact of information flow disruptions in a supply chain. The objective of IT security planning in supply chains is to protect information flows against disruptions and the supply chain assets against a compromise in the area of confidentiality, integrity or availability, where asset types may include systems and applications, networks, end-user systems, and off line media and devices. The various actions developed to prevent intrusions or to mitigate the impact of successful breaches and information flow disruptions are called security safeguards, controls or countermeasures. In view of the variety of methods used by attackers to infiltrate a supply chain IT infrastructure and disrupt operations, a wide range of different countermeasures are developed. Some countermeasures are used to limit physical access to an IT infrastructure, (e.g., key entry systems, retinal or finger print scans), other block access or protect privacy over networks (e.g., firewalls, data encryption, or virus and spyware scanners), while additional countermeasures are designed to permit recovery after a successful intrusion. A common practice is information redundancy. Firms back up supplier data, customer data, bill-of-material data, etc. However, backup of hard drives may not be sufficient for a supply chain to continue its operations, more often backup of the system itself

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is required. For example, if a widespread power outage severely impacts business continuity by compromising information systems.

In practice, even the most sophisticated countermeasures cannot be expected to completely block attacks as new attack profiles proliferate and the time and cost required to adapt to them the implemented countermeasures are not negligible. The potential threats, however, cannot be ignored and without countermeasures the losses resulting from many more breaches and disruptions of information flows could easily excess the costs of countermeasure implementations. In this chapter a scenario-based SMIP approach with CVaR as a risk measure is proposed for the decision-making. Given a set of potential threats and a set of available countermeasures, the decision maker needs to decide which countermeasure to implement under limited budget to minimize potential losses from successful cyber-attacks. The selection of countermeasures is based on their effectiveness of blocking different threats, implementation costs and probability of potential attack scenarios. The risk-neutral, risk-averse models and, in particular, mean-risk trade-off model provide the decision maker with a simple tool for balancing expected and worst-case losses and for shaping of the resulting cost distribution through the selection of optimal subset of countermeasures for implementation, i.e., the selection of optimal cybersecurity safeguards portfolio.

The following SMIP models are presented in this chapter:

NCP_E for risk-neutral selection of cybersecurity safeguards portfolio to minimize expected loss (nonlinear model);
CP_E for risk-neutral selection of cybersecurity safeguards portfolio to minimize expected loss (linear model);
CP_CV for risk-averse selection of cybersecurity safeguards portfolio to minimize CVaR of loss;
CP_EB for risk-neutral selection of cybersecurity safeguards portfolio to minimize expected loss and required budget;
CP_CVB for risk-averse selection of cybersecurity safeguards portfolio to minimize CVaR of loss and required budget;
CP_EBCV for mean-risk selection of cybersecurity safeguards portfolio to optimize trade-off between expected loss plus required budget and CVaR of loss.

In the computational experiments described in Sect. 11.6, an application of the developed models is illustrated with numerical examples.

11.2 Problem Description

Let $I = \{1, \ldots m\}$ be the set of m threats and $J = \{1, \ldots n\}$ the set of n countermeasures (for notation used, see Table 11.1). Denote by p_i the probability of threat i, i.e., attack episode of threat i occurs with probability p_i, or not at all with probability

Table 11.1 Notation: selection of cybersecurity safeguards portfolio

Indices	
i	= threat, $i \in I$
j	= countermeasure, $j \in J$
s	= attack scenario, $s \in S$
l	= countermeasure implementation level, $l \in L = \{0, 1\}$ ($l = 0$ - off, $l = 1$ - on)
Input Parameters	
a_i	= loss caused by a successful attack episode of threat i
B	= available budget for countermeasures
c_j	= cost of countermeasure j
I_s	= subset of threats in scenario s
p_i	= probability of threat i
q_{ijl}	= $\max\{1 - l, r_{ij}\}$ - proportion of threats i that survive if countermeasure j is implemented at level l ($q_{ij0} = 1$ and $q_{ij1} = r_{ij}$)
r_{ij}	= proportion of threats i that survive if countermeasure j is implemented
α	= confidence level

$(1 - p_i)$. Let P_s be the probability that attack scenario s is realized, where each scenario $s \in S$ is comprised of a unique subset $I_s \subset I$ of threats that appear in the cyberattack, and $S = \{1, \ldots, \overline{S}\}$ is the index set of all scenarios (note that there are a total of $\overline{S} = 2^m$ potential attack scenarios). The probability of attack scenario s in the presence of independent threat events is

$$P_s = \prod_{i \in I_s} p_i \cdot \prod_{i \notin I_s} (1 - p_i). \tag{11.1}$$

Let $r_{ij} \in [0, 1]$ be the proportion of threats i that survive if countermeasure j is implemented, where $r_{ij} = 0$ indicates that countermeasure j totally prevents successful attacks of threat i, whereas $r_{ij} = 1$ denotes that countermeasure j is totally incapable of mitigating threat i.
Note, that even the most sophisticated countermeasures, in practice are rarely expected to completely block attacks, and hence the survival proportions r_{ij} are rarely equal to 0.

The blocking effectiveness of each countermeasure is assumed to be independent whether or not it is used alone or together with other countermeasures, and the proportion of successful attacks of threats type i that survive all countermeasures in the subset $JS \subseteq J$ of selected countermeasures is a multiplication of proportions r_{ij}, $j \in JS$. Let $\prod_{j \in JS} r_{ij}$ be the proportion of threats i that survive if subset JS of countermeasures is implemented. Note that the expected proportion of successful attacks of threat type i for the subset JS of selected countermeasures is $p_i \prod_{j \in JS} r_{ij}$.

Denote by c_j the cost of implementing countermeasure j, and by a_i the cost of (loss from) a successful attack episode of threat i. The available budget B for selected countermeasures implementation is limited. The subset of selected countermeasures $JS \subseteq J$ must satisfy the available budget constraint, $\sum_{j \in JS} c_j \leq B$, i.e., the total expenditures on the selected countermeasures cannot exceed the available budget B. Furthermore, the selected countermeasures are assumed to be feasible with respect to various additional constraints, e.g., they cannot contain mutually exclusive countermeasures or they must contain all countermeasures contingent on each other, etc.

The decision maker needs to decide which countermeasures to select to minimize losses from surviving occurrences of threats under limited budget for countermeasures implementation.

11.3 Models for Risk-Neutral Decision-Making

In this section SMIP models are proposed for a risk-neutral selection of optimal cybersecurity safeguards portfolio, i.e., the risk-neutral selection of optimal subset of countermeasures for implementation. The problem variables are defined in Table 11.2.

Let $L = \{0, 1\}$ be the set of implementation levels of each countermeasure, where $l = 0$ denotes that a particular countermeasure is not selected for implementation, otherwise $l = 1$. The decision whether or not to select a particular countermeasure will be represented by a binary variable u_{jl}, $j \in J, l \in L$, where $u_{jl} = 1$, if countermeasure j is implemented at level l, otherwise $u_{jl} = 0$, that is, countermeasure j is selected for implementation if $u_{j1} = 1$ and $u_{j0} = 0$, otherwise $u_{j1} = 0$ and $u_{j0} = 1$. The above definition implies that each countermeasure j is selected at exactly one level, i.e., $\sum_{l \in L} u_{jl} = 1$.

Denote by $q_{ijl} = \max\{1 - l, r_{ij}\}$ the proportion of threats i that survive if countermeasure j is implemented at level l, that is $q_{ij0} = 1$ and $q_{ij1} = r_{ij}$.

11.3.1 Nonlinear Model for Risk-Neutral Decision-Making

For a risk-neutral decision-making the overall quality of the selected countermeasure portfolio can be measured by the expected cost of losses from successful attacks.

The proportion of successful attacks of threats type i that survive all selected countermeasures is a multiplication of individual proportions of the selected countermeasures and can be expressed as

$$\prod_{j \in J} (\sum_{l \in L} q_{ijl} u_{jl}).$$

As a result, the expected loss from successful attacks is given by a nonlinear formula

$$\sum_{s\in S}\sum_{i\in I_s} P_s a_i (\prod_{j\in J}(\sum_{l\in L} q_{ijl}u_{jl})). \tag{11.2}$$

Table 11.2 Variables: selection of cybersecurity safeguards portfolio

	First stage variables
u_{jl}	$=$ 1, if countermeasure j is implemented at level l, otherwise $u_{jl} = 0$ (countermeasure selection)
v_{ijl}	$=$ proportion of surviving occurrences of threat i to be addressed by countermeasure j at level l
w_{ij}	$=$ proportion of surviving occurrences of threat i that passed countermeasures 1 through j
W_i	$=$ proportion of successful attacks of type i
	Auxiliary variables
VaR	Loss-at-Risk, the targeted loss such that for a given confidence level α, for $100\alpha\%$ of the scenarios, the outcome is below VaR
$\mathscr{C}_s$	$\geq$ 0, the tail loss for attack scenario s, i.e., the amount by which losses in scenario s exceed VaR

The 0–1 nonlinear stochastic programming program **NCP_E** for risk-neutral selection of optimal subset of security safeguards is formulated below.

NCP_E: **N***onlinear model for risk-neutral selection of* **C***ybersecurity safeguards* **P***ortfolio to minimize expected loss*

Minimize (11.2)

subject to

1. Countermeasure selection constraints:

– each countermeasure is selected at exactly one level (i.e., implemented or not implemented),

– the expenditures on selected countermeasures cannot exceed available budget,

$$\sum_{l\in L} u_{jl} = 1;\ j \in J \tag{11.3}$$

$$\sum_{j\in J} c_j u_{j1} \leq B \tag{11.4}$$

3. Integrality conditions:

$$u_{jl} \in \{0, 1\};\ \ j \in J, l \in L. \tag{11.5}$$

The 0–1 nonlinear stochastic program **NCP_E** is computationally hard for solving, even for small size instances of the problem. The next subsection describes a recursive procedure that is capable of computing the nonlinear objective function (11.2) using a set of linear equations.

11.3.2 Linear Model for Risk-Neutral Decision-Making

The nonlinear objective function (11.2) can be replaced with a formula

$$E^c = \sum_{s \in S} \sum_{i \in I_s} P_s a_i W_i, \tag{11.6}$$

where

$$W_i = \prod_{j \in J} (\sum_{l \in L} q_{ijl} u_{jl}); \; i \in I, \tag{11.7}$$

is the proportion of successful attacks of type i.

In order to compute W_i for each threat i, a recursive procedure is proposed below.

Denote by

$$w_{ij} = \prod_{k \in J: k \le j} (\sum_{l \in L} q_{ikl} u_{kl}); \; i \in I, j \in J, \tag{11.8}$$

the proportion of surviving occurrences of threat i that passed countermeasures 1 through j. Note that (cf. (11.7), (11.8))

$$W_i = w_{in}; \; i \in I. \tag{11.9}$$

For each threat $i \in I$ and countermeasure $j \in J$, w_{ij} can be calculated recursively as follows.

The initial condition that explicitly prescribes the first term is

$$w_{i1} = \sum_{l \in L} q_{i1l} u_{1l}; \; i \in I. \tag{11.10}$$

The recurrence formula by means of which the remaining terms are determined inductively is

$$w_{ij} = (\sum_{l \in L} q_{ijl} u_{jl}) w_{i,j-1}, \; i \in I, j \in J. \tag{11.11}$$

In order to eliminate nonlinear terms in the right-hand side of Eq. (11.11), define an auxiliary variable

$$v_{ijl} = u_{jl} w_{ij-1};\ i \in I, j \in J, l \in L, \tag{11.12}$$

where $w_{i0} = 1$ for all i, i.e., all threat events of each type i are capable of attacking supply chain IT infrastructure. Note that v_{ijl} represents the proportion of surviving occurences of threat i to be addressed by countermeasure j at level l

Substituting $v_{ijl} = u_{jl} w_{ij-1}$ into Eq. (11.11) yields

$$w_{ij} = \sum_{l \in L} q_{ijl} v_{ijl};\ i \in I, j \in J, \tag{11.13}$$

and, in particular, for $j = n$,

$$w_{in} = \sum_{l \in L} q_{inl} v_{inl};\ i \in I, \tag{11.14}$$

where $w_{in} = W_i$ for all $i \in I$, (11.9).

On the other hand, replacing j with $j + 1$ in Eq. (11.12) gives

$$v_{i,j+1,l} = u_{j+1,l} w_{ij}; i \in I, j \in J : j < n, \tag{11.15}$$

Summing both sides of Eq. (11.15) on l for each i and j, and using the fact that $\sum_{l \in L} u_{j+1,l} = 1;\ j \in J$, (11.3), yields

$$\sum_{l \in L} v_{i,j+1,l} = w_{ij};\ i \in I, j \in J : j < n. \tag{11.16}$$

Comparison of Eqs. (11.13) and (11.16) produces to the following relation

$$\sum_{l \in L} q_{ijl} v_{ijl} = \sum_{l \in L} v_{i,j+1,l};\ i \in I, j \in J : j < n, \tag{11.17}$$

that is, for each threat i, the proportion of occurrences which survive countermeasure j are addressed by countermeasure $j + 1$.

Finally, setting $j = 1$ and summing both sides of Eq. (11.12) on l, using the fact that $w_{i0} = 1$ for all i and $\sum_{l \in L} u_{1l} = 1$, (11.3), yields

$$\sum_{l \in L} v_{i1l} = 1;\ i \in I, \tag{11.18}$$

that is, all threat events of each type i are addressed by countermeasure $j = 1$.

The above procedure eliminates all variables w_{ij} for each i, except for w_{in}. Summarizing, the proportion of successful attacks $W_i = w_{in}$ for each threat i can be calculated recursively, using Eqs. (11.18), (11.17) and (11.14) with w_{in} replaced by W_i.

Note that the linearizing procedure would not be possible if the implementation level $l \in L = \{0, 1\}$ of each countermeasure $j \in J$ was not introduced and the variable u_{jl} was replaced with a simple binary selection variable $u_j \in \{0, 1\}$, denoting whether or not countermeasure j is selected. In the latter case the proportion of threats i that survive whether or not countermeasure j is selected, is $\max\{1 - u_j, r_{ij}\}$, i.e., the survival proportions would depend nonlinearly on the selection variables u_j. In contrast to constant survival proportions $q_{ijl} = \max\{1 - l, r_{ij}\}$ for variables u_{jl}, the survival proportions for variables u_j could not be expressed by constant coefficients in the constraints.

The linearized version **CP_E** of model **NCP_E** is shown below (see also, network flow model in Deane et al. 2009; Rakes et al. 2012).

CP_E: *Risk-neutral selection of cybersecurity safeguards portfolio to minimize expected loss*

Minimize (11.6)

subject to

1. Countermeasure selection constraints: (11.3), (11.4)

2. Surviving threats balance constraints:

$$\sum_{l \in L} v_{i1l} = 1; \ i \in I \tag{11.19}$$

$$\sum_{l \in L} q_{ijl} v_{ij1} = \sum_{l \in L} v_{ij+1l}; \ i \in I, j \in J : j < n \tag{11.20}$$

$$\sum_{l \in L} q_{inl} v_{inl} = W_i; \ i \in I \tag{11.21}$$

$$v_{ijl} \le u_{jl}; \ i \in I, j \in J, l \in L \tag{11.22}$$

3. Non-negativity and integrality conditions:

$$u_{jl} \in \{0, 1\}; \ j \in J, l \in L \tag{11.23}$$

$$v_{ijl} \ge 0; \ i \in I, j \in J, l \in L \tag{11.24}$$

$$W_i \ge 0; \ i \in I. \tag{11.25}$$

Constraint (11.22) (cf. (11.12)) ensures that threats can only by addressed by the implemented countermeasures.

Note that the expected proportion of successful attacks of threat type i is $p_i W_i$.

Model **CP_E** will be used to compare the risk-neutral results with those obtained by applying a risk aversive decision-making model described in the next section.

11.4 Model for Risk-Averse Decision-Making

In this section a risk-averse selection of optimal cybersecurity safeguards portfolio is considered with the two popular in financial engineering percentile measures of risk, VaR and CVaR, applied to control the risk of high losses.

When selecting the optimal risk-averse countermeasure portfolio, the decision maker controls the risk of high losses caused by operational disruptions by choosing the confidence level α. The greater the confidence level α, the more risk aversive is the decision maker and the smaller percent of the highest loss outcomes is focused on. The risk aversive decision maker wants to minimize the expected worst-case losses exceeding VaR, by minimizing CVaR, given available budget B for selected countermeasures.

Define $\mathscr{C}_s$ as the tail loss for scenario s, where tail loss is defined as the amount by which losses in scenario s exceed VaR. The cybersecurity safeguards portfolio will be optimized by calculating VaR and minimizing CVaR simultaneously. By measuring CVaR, the magnitude of the tail loss is considered to achieve a more accurate estimate of the risks of minimizing loss. In the proposed model, CVaR is represented by an auxiliary function (11.26) introduced by Rockafellar and Uryasev (2000). The SMIP model **CP_CV** for selection of risk-averse cybersecurity safeguards portfolio to reduce the risk of high losses is formulated below.

CP_CV: *Risk-averse selection of cybersecurity safeguards portfolio to minimize CVaR of loss*

Minimize

$$CVaR = VaR + (1-\alpha)^{-1} \sum_{s \in S} P_s \mathscr{C}_s \tag{11.26}$$

subject to

1. Countermeasure selection constraints: (11.3)–(11.4)

2. Surviving threats balance constraints: (11.19)–(11.22)

2. Risk constraints:

– the tail loss for scenario s is defined as the nonnegative amount by which losses in scenario s exceed VaR,

$$\mathscr{C}_s \geq \sum_{i \in I_s} a_i W_i - VaR; \;\; s \in S \tag{11.27}$$

3. Non-negativity and integrality conditions: (11.23)–(11.25).

$$\mathscr{C}_s \geq 0; \;\; s \in S. \tag{11.28}$$

Models **CP_E** and **CP_CV** can be enhanced for simultaneous optimization of the expenditures on countermeasures and the loss from successful attacks. Then, the budget constraints (11.4) should be removed from the models and the fixed cost of selected countermeasures, $\sum_{j \in J} c_j u_{j1}$, added to the objective function.

Define the fixed cost of selected countermeasures as the required budget, $B(u)$,

$$B(u) = \sum_{j \in J} c_j u_{j1}. \tag{11.29}$$

The enhanced models **CP_EB** and **CP_CVB** for simultaneous optimization of fixed cost of countermeasures and variable loss from successful attacks are presented below.

CP_EB: *Risk-neutral selection of cybersecurity safeguards portfolio to minimize expected loss and required budget*

Minimize

$$E^c + B(u) \tag{11.30}$$

subject to (11.3), (11.6), (11.19)–(11.25), (11.29).

CP_CVB: *Risk-averse selection of cybersecurity safeguards portfolio to minimize CVaR of loss and required budget*

Minimize

$$CVaR + B(u) \tag{11.31}$$

subject to (11.3), (11.19)–(11.29).

11.5 Models for Mean-Risk Decision-Making

In the single objective approach the countermeasure portfolio is selected by minimizing either the expected loss (plus the required budget) or the expected worst-case loss (plus the required budget). Since the probability of worst-case loss outcomes is usually very low, the expected cost function that aims at optimizing an average performance of IT security system, virtually neglects the worst-case loss. In contrast, CVaR that aims at optimizing worst-case performance, focuses on the low probability, high loss outcomes, and as the confidence level α increases a more risk-averse decision-making focuses on a smaller set of the highest loss outcomes.

In this section the two conflicting objectives are considered simultaneously, and a bi-objective selection of countermeasures is presented aimed at minimizing both objective functions to balance the required budget and expected cost with the risk tolerance. This trade-off model, known as the mean-risk model, is formulated as the optimization of a convex combination of the expected loss and the CVaR of loss as a risk measure.

The nondominated solution set of the bi-objective countermeasure portfolio can be found by the parameterization on λ the weighted-sum program **CP_EBCV**. The scalarizing mixed integer program is based on **CP_CVB** model with the addition of objective (11.6) of model **CP_EB**.

CP_EBCV: *Mean-risk selection of cybersecurity safeguards portfolio to optimize trade-off between expected loss plus required budget and CVaR of loss*

Minimize

$$\lambda(E^c + B(u)) + (1 - \lambda)CVaR \tag{11.32}$$

where $0 \leq \lambda \leq 1$,
subject to (11.3), (11.6), (11.19)–(11.29).

Now, the decision maker controls both the risk of high losses from successful cyber-attacks by choosing the confidence level α as well as the trade-off between expected and worst-case losses by choosing the trade-off parameter λ. Using the latter parameter, the level of risk allowed into the solution can be controlled. In particular, a decision maker may decide to minimize expected loss while requiring the percentile worst-case loss to be not greater than some parameter. In other words, model **CP_EBCV** with $\lambda = 1$ may include an upper bound on CVaR. However, when α increases so that the worst possible outcomes are considered only, imposing a strict bound on CVaR may result in an infeasible solution, e.g., Chahara and Taaffe (2009).

11.6 Computational Examples

In this section some computational examples are presented to illustrate possible applications of the proposed portfolio approach for the selection of countermeasures to mitigate the impact of information flow disruptions caused by IT security incidents. The following parameters have been used for the example problems:

- $m = n$, the number of threats and the number of countermeasures, were equal to 10, and the corresponding number $\overline{S} = 2^m$ of potential attack scenarios, was equal to 1024;
- a_i, loss from a successful attack of each threat i (in \$1000): $a_1 = 24,\ a_2 = 122,\ a_3 = 350,\ a_4 = 5,\ a_5 = 250,\ a_6 = 20,\ a_7 = 20,\ a_8 = 25,\ a_9 = 30,\ a_{10} = 10000$;
- c_j, cost of each countermeasure j (in \$1000): $c_1 = 40,\ c_2 = 28,\ c_3 = 80,\ c_4 = 24,\ c_5 = 70,\ c_6 = 50,\ c_7 = 40,\ c_8 = 45,\ c_9 = 50,\ c_{10} = 80$;
- p_i, probability of each threat i: $p_1 = 0.35,\ p_2 = 0.25,\ p_3 = 0.15,\ p_4 = 0.25,\ p_5 = 0.20,\ p_6 = 0.25,\ p_7 = 0.50,\ p_8 = 0.35,\ p_9 = 0.40,\ p_{10} = 0.003$;
- r_{ij}, surviving proportions for each threat i and countermeasure j:

i/j	1	2	3	4	5	6	7	8	9	10
1	0.01	0.5	1	1	1	1	1	1	1	1
2	1	0.04	1	1	0.6	1	1	1	1	1
3	1	1	0.01	0.8	1	0.8	0.9	0.95	0.9	1
4	1	1	1	0.25	1	0.8	0.9	0.95	0.9	0.8
5	1	0.5	1	0.8	0.02	0.8	0.9	0.95	0.9	1
6	1	0.6	1	1	1	0.1	0.6	0.65	0.6	1
7	1	0.5	1	1	1	0.5	0.15	0.2	0.25	1
8	1	0.5	1	1	1	0.55	0.2	0.25	0.3	1
9	1	0.5	1	1	1	0.5	0.15	0.2	0.35	1
10	1	1	1	1	1	1	1	1	1	0.2;

- α, the confidence level, was equal to 0.50, 0.75, 0.90, 0.95 or 0.99;
- B, available budget for countermeasures (in \$1000) was equal to 150,300 or $\sum_{j \in J} c_j = 507$.

The above data set is similar to the one presented in Rakes et al. (2012), which was based on the threat set reported on IT security forum EndpointSecurity.org. Note that threat $i = 10$ is a high-impact and low-probability threat with the loss from successful attack equal to $a_{10} = \$10$M and the probability of occurrence equal to $p_{10} = 0.003$. Such a serious and rarely occurring event might be a compromise of some important business data. The remaining threats are more typical such as viruses, data damages, or information disclosures, occurring more frequently and resulting in lower loss from successful attacks. The off-diagonal surviving proportions r_{ij}, $i \neq j$, less than one, indicate that some countermeasures work against their primary threat as well as against other threats. For example, countermeasures $j = 2, 6, 7, 8, 9$ may represent strong security policies focused on particular threats and simultaneously showing benefits across the other threats. In contrast, countermeasures $j = 1, 3$ reduce the survival proportions of a single threat only.

For the risk-neutral models **CP_E** and **CP_EB**, solution results are shown in Table 11.3, and for the risk-averse models **CP_CV** and **CP_CVB** with different confidence levels, in Tables 11.4 and 11.5. In addition to VaR and CVaR, for comparison with risk-neutral solutions, Tables 11.4 and 11.5 show the corresponding expected loss for the optimal risk-averse countermeasure portfolios. In the tables all costs are expressed in thousands of dollars.

For the risk-neutral models, Table 11.3 indicates that when **CP_E** model is applied, the greater the available budget, the more countermeasures are selected to minimize expected loss. If, however, model **CP_EB** is used, a single countermeasure is selected only to minimize expenditures on countermeasures and expected loss from successful attacks. For **CP_E** model, Fig. 11.1 shows the probability mass function of optimal cost of losses for the available budget $B = 150$, with the tail of cost distribution presented in a separate chart. Similarly, for **CP_EB** model with a variable budget, the probability mass function of optimal cost is shown in Fig. 11.2, where the tail of cost distribution is also presented in a separate chart.

For the risk-averse models, Tables 11.4 and 11.5 demonstrate that the number of selected countermeasures increases with the available budget, which indicates that the impact of disruption risks is mitigated by providing a more secure environment. When α increases, a more risk-averse decision-making focuses on a smaller set of outcomes, however the number of selected countermeasures is not increasing. For a limited budget, when α increases, the more expensive countermeasures against high-impact, low probability threats are more frequently selected. As a result the total number of all selected countermeasures decreases with α. The latter result is partly due to the parameter settings for which the countermeasure implementation costs do not always dominate the losses from successful cyber-threats. For example, costs of countermeasures 1, 4, 6, 7, 8 and 9 are higher than the losses from a single successful attack of the corresponding threats.

Note that for **SP_CV** model, VaR becomes smaller than expected loss for all budget levels B, when $\alpha = 0.50$ and $\alpha = 0.75$, and for all confidence levels α, when the highest budget level B =$507,000 allows for the selection of all countermeasures, (see, Table 11.4).

Table 11.3 Solution results for risk-neutral models **CP_E** and **CP_EB**

Model	SP_E			CP_EB
Budget B	150	300	507	28
Expected cost	63.842	17.079	7.589	132.545
Selected countermeasures	2, 3, 7	2, 3, 5, 7, 10	1–10	2

Table 11.4 Solutions results for risk-averse model **CP_CV**

Confidence level α	0.50			0.75		
Budget B	150	300	507	150	300	507
CVaR	121.130	29.154	14.839	224.294	44.849	27.798
VaR	13.500	10.128	1.078	23.780	16.428	2.965
Expected cost	63.842	17.079	7.589	63.842	17.079	7.589
Selected countermeasures	2, 3, 7	2, 3, 5, 7, 10	1–10	2, 3, 7	2, 3, 5, 7, 10	1–10

Table 11.4 (continued)

Confidence level α	0.90			0.95			0.99		
Budget B	150	300	507	150	300	507	150	300	507
CVaR	393.775	84.185	64.597	478.204	145.744	124.985	921.449	624.627	604.858
VaR	302.500	21.450	3.769	318.880	24.178	4.663	414.500	29.028	5.882
Expected cost	92.045	17.079	7.589	92.045	17.079	7.589	92.045	17.079	7.589
Selected countermeasures	2, 4, 10	2, 3, 5, 7, 10	1–10	2, 4, 10	2, 3, 5, 7, 10	1–10	2, 4, 10	2, 3, 5, 7, 10	1–10

Table 11.5 Solutions results for risk-averse model **CP_CVB**

Confidence level α	0.50	0.75	0.90	0.95	0.99
CVaR	144.008	67.089	109.323	172.808	652.214
VaR	29.380	34.500	42.928	49.500	59.000
Expected cost	80.570	31.332	31.332	31.332	31.332
Required budget	108	258	258	258	258
Selected countermeasures	2, 3	2, 3, 5, 10	2, 3, 5, 10	2, 3, 5, 10	2, 3, 5, 10

In the computational experiments with risk-averse models, the confidence level α is set at five levels of 0.5, 0.75, 0.90, 0.95, and 0.99, which means that focus is on minimizing the highest 50%, 25%, 10%, 5%, and 1% of all scenario outcomes, i.e., losses from successful attacks. For **SP_CV** model, Fig. 11.3 shows the probability mass function of optimal cost of losses for the available budget $B = 150$ and confidence level $\alpha = 0.99$, with the tail of loss distribution presented in a separate chart. For **SP_CVB** model with a variable budget and confidence level $\alpha = 0.99$ the probability mass function of optimal cost is shown in Fig. 11.4, where the tail of cost distribution is also presented in a separate chart. Figures 11.1, 11.2, 11.3 and 11.4 indicate that the probability measure is concentrated in finitely many points, which is typical for the scenario-based optimization under uncertainty, (e.g., in Fig. 11.1, the probability measure is concentrated in four points). Moreover, the tail of cost distribution shows that the high loss probability is very low. As a result the expected loss for the optimal risk-neutral portfolio and CVaR for the optimal risk-averse portfolio are much lower than the corresponding worst-case loss outcomes.

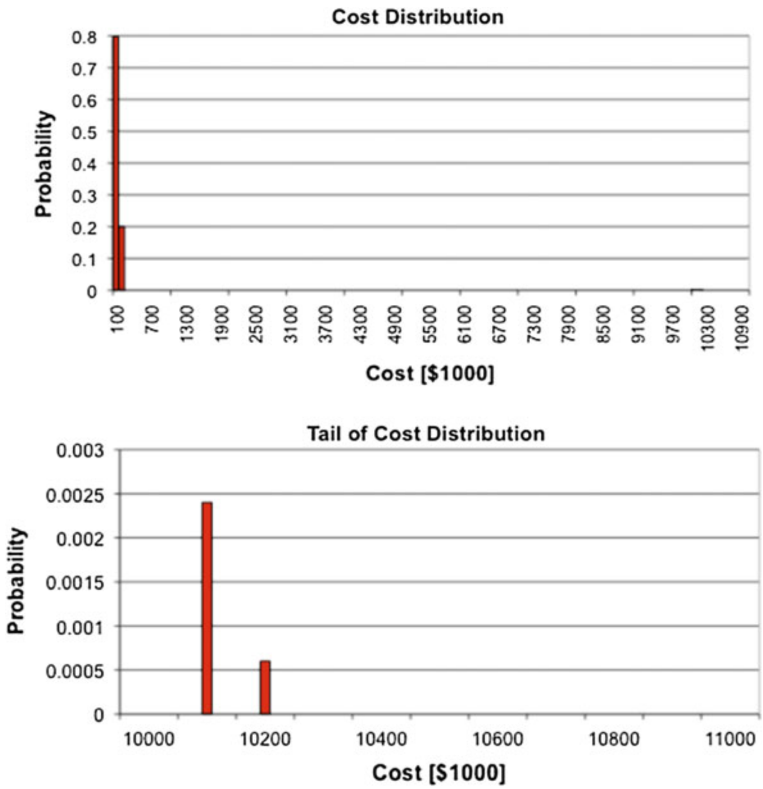
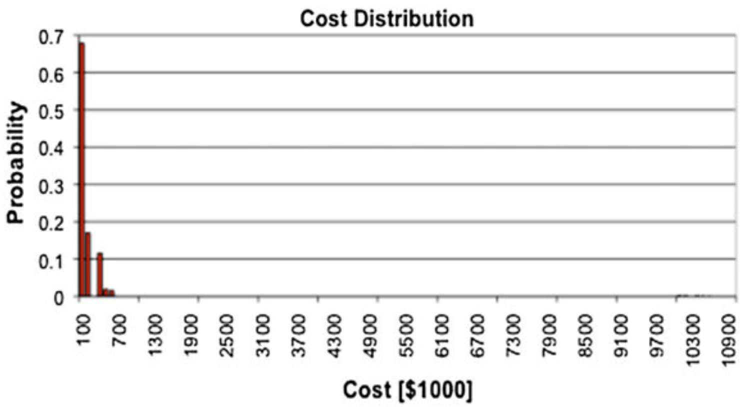


Fig. 11.1 Probability mass function for risk-neutral model **CP_E** with B =$150,000: Expected loss = $63,842



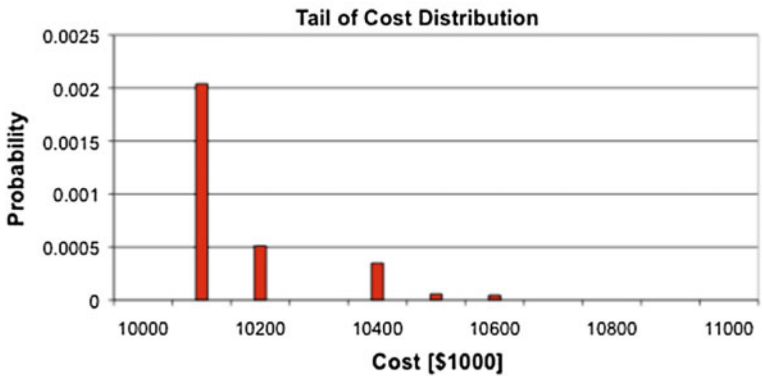


Fig. 11.2 Probability mass function for risk-neutral model **CP_EB**: Required budget = \$28,000, Expected cost = \$132,545

Comparison of tails of the cost distributions for the corresponding risk-neutral and risk-averse solutions (i.e., Figs. 11.1 vs. 11.3 and 11.2 vs. 11.4) indicates that the optimal risk-averse countermeasure portfolio positively shapes the cost distribution, i.e., significantly reduces the worst-case cost outcomes. For example, the worst-case cost of \$10.6M for the risk-neutral portfolio determined using **CP_EB** model is reduced to \$2.06 M, when the risk-averse portfolio is applied, determined using **CP_CVB** model.

Table 11.6 Nondominated solutions for mean-risk model **CP_ECV**: $\alpha = 0.9$

λ	0.01	0.10	0.25	0.50	0.75	0.90	0.99
CVaR	65.049	68.145	84.185	109.323	218.193	633.842	710.691
Expected cost	7.908	9.718	17.079	31.332	56.320	116.108	132.545
Required budget	457	388	298	258	188	52	28
Selected countermeasures	1–8, 10	1–3, 5–7, 10	2, 3, 5, 7, 10	2, 3, 5, 10	2, 3, 10	2, 4	2

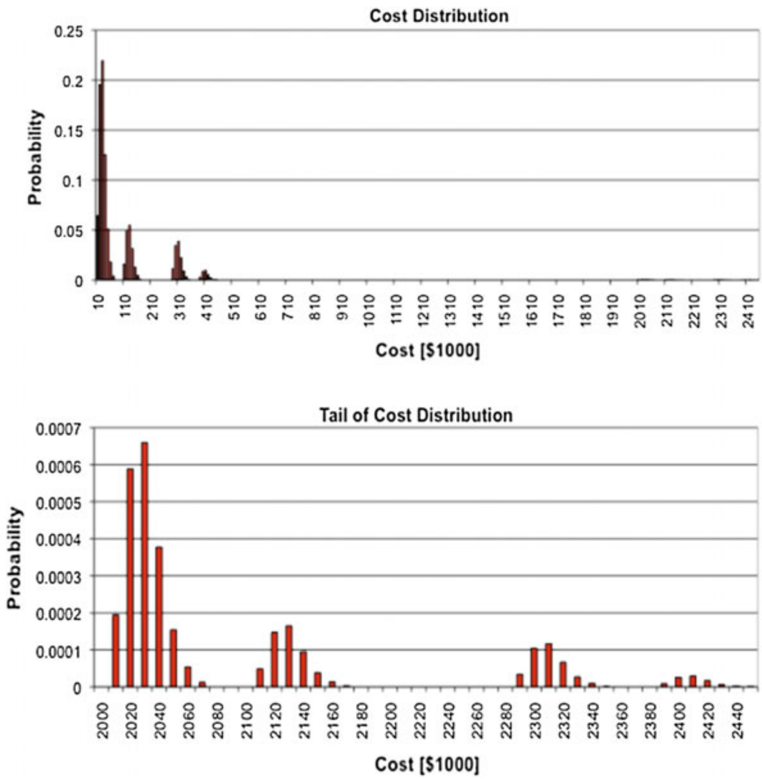


Fig. 11.3 Probability mass function for risk-averse model **CP_CV** with B =\$150,000: $\alpha = 0.99$, CVaR = \$921,449, VaR = \$414,500, Expected cost = \$92,045

For the bi-objective approach, the subsets of nondominated solutions were computed by parameterization on $\lambda \in \{0.01, 0.10, 0.25, 0.50, 0.75, 0.90, 0.99\}$ the weighted-sum program **CP_EBCV**. The results obtained for the confidence level $\alpha = 0.9$ are presented in Table 11.6. The trade-off between the required budget and expected cost, and CVaR is clearly shown in Fig. 11.5, where the convex efficient frontier of the mean-risk model with $\alpha = 0.9$ is presented. The results emphasize the effect of varying cost/risk preference of the decision maker. The higher the weight λ for the required budget and expected cost, the smaller the number of selected countermeasures. When λ increases from 0.01 to 0.99, the size of the optimal countermeasure portfolio decreases from nine to one selected countermeasure. At the same time the required budget and expected cost decreases from \$464,908 to \$160,545, while CVaR increases from \$65,049 to \$710,691.

Note that the nondominated solutions of the weighted-sum program **CP_EBCV** with $\lambda = 1$ and $\lambda = 0$ are identical with the optimal solutions to single objective models **CP_EB** (Table 11.3) and **CP_CV** with unlimited (the highest) budget B =\$507,000 (Table 11.4), respectively.

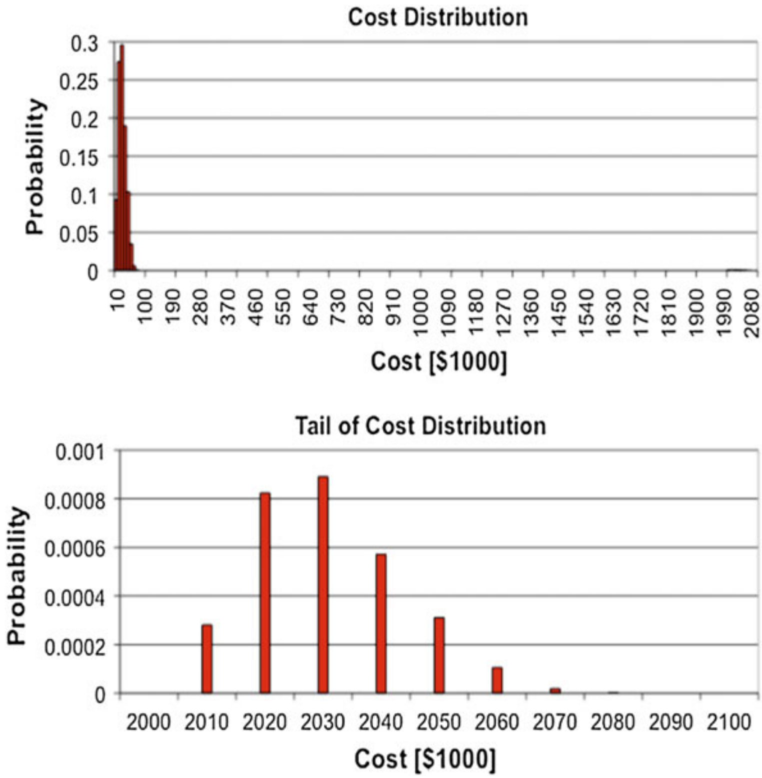


Fig. 11.4 Probability mass function for risk-averse model **CP_CVB**: $\alpha = 0.99$, CVaR = $652,214, VaR = $59,000, Required budget = $258,000, Expected cost = $31,332

The computational experiments were performed using the AMPL programming language and Gurobi 5.0.1 solver on a laptop MacBookPro 6.2 with Intel Core i7 processor running at 2.66 GHz and with 8 GB RAM. The Gurobi solver was capable of finding proven optimal solutions within CPU seconds for all examples.

Examples of the proposed mixed integer programs size for different numbers $m = n$ of threats and countermeasures are shown in Table 11.7. The size of the risk-neutral model **CP_EB** and the risk-averse model **CP_CVB** is represented by the total number of variables, Var., number of binary variables, Bin., and number of constraints, Cons. The CPU time in seconds required to prove optimality of the solution was ranging from fraction of a second to several hundred seconds. Note that the number of variables and constraints in the risk-averse model **CP_CVB** grows exponentially in the number m of threats, when all potential attack scenarios are considered. As a result, CPU time increases rapidly when the number of threats increases from $m = 10$ to $m = 20$.

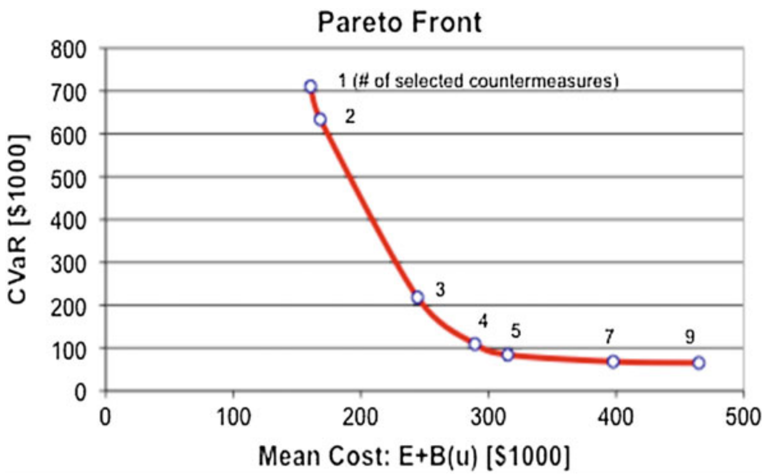


Fig. 11.5 Pareto front for mean-risk model **CP_EBCV**: $\alpha = 0.9$

Table 11.7 Examples of mixed integer program size and CPU time

m	$2^{m(a)}$	Var.$^{(b)}$	Bin.	Cons.	CPU$^{(c)}$
		Risk-neutral model **CP_EB**			
10	1024	231	20	321	<1
15	32768	496	30	766	<1
20	1 048 576	861	40	1241	<1
		Risk-averse model **CP_CVB**			
10	1024	1256	20	1345	<1
15	32768	33265	30	33474	5
20	1 048 576	1 049 438	40	1 049 817	416

$^{(a)}m$ = number of threats (number of countermeasures),
2^m = number of attack scenarios.
$^{(b)}$ Var. = number of variables, Bin. = number of binary variables,
Cons. = number of constraints.
$^{(c)}$ CPU seconds for proving optimality on a MacBookPro 6.2, Intel Core i7, 2.66 GHz,
RAM 8 GB/Gurobi 5.0.1

- *The higher the budget available and the higher the confidence level, the more risk-oriented is the countermeasure portfolio selected. In most cases the number of selected countermeasures increases with the available budget, and for a limited budget decreases with the confidence level.*
- *For a limited budget and lower confidence level α, the expensive countermeasures against low-probability, high-impact threats are rarely selected, and as a result the total number of all selected countermeasures is usually greater than that for a higher α, when the more expensive countermeasures against high-impact threats are chosen.*

- *The trade-off mean-risk model provides the decision maker with a simple tool for balancing expected and worst-case losses and for shaping of the resulting cost distribution through the selection of optimal countermeasure portfolio. The decision maker is capable of controlling both the risk of high losses from successful cyber-attacks by choosing the confidence level α as well as the trade-off between expected and worst-case losses by choosing the trade-off parameter λ that represents cost versus risk preference. Using the latter parameter, the level of risk allowed into the solution can be controlled.*

The computational experiments prove that for a limited number of attack scenarios considered, the optimal risk-averse portfolio can be found within CPU seconds, using the Gurobi MIP solver.

11.7 Notes

The problem of a right choice of countermeasures for implementation under limited budget is not an easy task. The choice depends not only on reliable data on potential cyber-threats and losses but also requires a good security risk planning tool. The National Institute of Standards and Technology classifies information security controls (NIST SP800-53) into the three categories: technical, operational, and management controls (e.g., Viduto et al. 2012), in which the last two categories can become as critical as the first one. For example, operational countermeasures include monitoring and logging procedures, business continuity/incident response procedures, backup and recovery procedures, while management countermeasures include periodic employee training, periodic testing and review procedures, network auditing, and protection for all sensitive informational assets.

Most of IT security planning models in the literature are qualitative. For example, Bojanc and Jerman-Blazic (2008) introduced methods for identification of the assets, the threats, the vulnerabilities of the ICT systems and proposed a procedure that enables selection of the optimal investment of the necessary security technology based on the quantification of the values of the protected systems. In Egan (2005) a checklist in table form was developed to help decision maker planning a coverage strategy. Chen et al. (2011) discussed current research findings in enterprise risk and security management using web mining techniques.

A study by Gordon and Loeb (2002) uses risk analysis to suggest an optimal budget for a risk-neutral decision maker. In their approach, they compared the loss caused by security incidents to the investment required to reduce the related vulnerability. Based on two general classes of security breach functions, they state that the amount to invest is considerably lower than the expected loss caused by an incident. In fact, they find that the amount to invest in security never exceeds 37% of the expected loss and in most cases will be substantially less. However, these observations only hold true if the security breach functions meet the condition of decreasing marginal returns in case of security investments. Hausken (2006) examined four additional types of security breach functions with different shapes and found that the amount to invest is no longer limited by 37% and different investment strategies should be applied in each case. Wang et al. (2008) proposed a more detailed analysis which makes use of security incident data and value-at-risk to support decision-making.

In contrast to qualitative approaches, the literature on quantitative methods for selection of countermeasures to block or mitigate security attacks is very limited. Based on NIST SP800-30 guidelines, Viduto et al. (2012) developed a risk assessment and optimization model to satisfy organizational security needs in a cost-effective manner. The security countermeasure selection problem was formulated as a multi-objective optimization problem, where variables such as financial cost and risk may affect the final solution. A tailored multi-objective tabu search-based heuristic approach was constructed to solve the proposed multi-objective optimization problem and asses the qualities of its solutions with respect to optimal ones. Deane et al. (2009) developed a linear generalized network flow model that quantifies IT security risk in the supply chain. It was shown how to find solutions for optimal risk reduction under several definitions of optimality: minimizing upstream risk, minimizing downstream risk, and minimizing global supply chain risk. Then, following the mathematical models proposed by Deane et al. (2009), an integer programming model was developed by Rakes et al. (2012), for optimally choosing a subset of countermeasures to block or mitigate security attacks in the presence of a given threat level profile. The model was used to examine the two different types of scenarios: under expected threat levels and under worst-case levels. The authors illustrated the tradeoffs in optimal security planning when expected threats are used to parameterize the model versus worst-case values for threat outcomes. To demonstrate the trade-off which occurs if decision makers divert budgets away from planning for ordinary risk in an effort to mitigate the effects of potential high-impact outcomes, budget-dependent risk curves were developed. Schilling and Werners (2016) proposed a combinatorial optimization model to efficiently select security safeguards in order to protect IT infrastructures and systems. The approach is designed to provide decision support for an organization as a whole or separately for specific systems.

The material presented in this chapter is based on research reported in Sawik (2013d), where portfolio approach with CVaR as a risk measure was proposed for the selection of countermeasures in IT security planning to prevent and mitigate the impact of cyber-attacks. A critical issue that need to be considered before any practical application of the proposed models is attempted, however, is the estimation of probabilities and the resulting losses associated with each type of threats and countermeasures. In practice, threat likelihood estimates are provided by security experts (e.g., Ryan et al. 2012) and complete distributional information is not available. However, the proposed scenario-based approach does not require such a complete information to be available and only assumes independence of different threat events. In many cases they are independent of each other, but in principle might have a joint probability distribution. The future research should also consider selection of a countermeasure portfolio under the less restrictive assumptions on a joint probability of different cyber-threats. However, relaxation of the assumption of independent threat events may significantly complicate the problem of building potential attack scenarios. For example, modeling of attack scenarios with correlated threat events represented by Bernoulli random variables may require constructing of the corresponding correlated probability distribution, i.e., a correlated binomial distribution instead of the binomial distribution used in this chapter.

Problems

11.1 Design attack scenarios for correlated threat events.

11.2 Enhance the SMIP models presented in this chapter for multiple implementation levels of countermeasures, where different levels are associated with different implementation costs and different effectiveness of blocking different threats.

11.3 Modify the SMIP models presented in this chapter for selection of cybersecurity portfolio such that cannot contain mutually exclusive countermeasures or such that must contain subsets of countermeasures contingent on each other.

11.4 Relax the assumption that the blocking effectiveness of each countermeasure is independent whether or not it is used alone or together with other countermeasures. What would be the implication of that relaxation on the developed models?

11.5 Identify the best-case and the worst-case attack scenarios such that the CVaR of loss is not greater and not less than than a fixed threshold loss, respectively.

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