SOLUTIONS MANUAL
Bioprocess Engineering Principles

Pauline M. Doran

$$\eta_{i1} = \frac{\tanh \phi_1}{\phi_1}$$

$$\hat{Q} = -kA\frac{dT}{dy}$$

Peclet number

$$\mu = \frac{\mu_{\max}\, s}{K_S + s}$$

Effective diffusivity

Apparent viscosity

$$k = A\, e^{-E/RT}$$

$$\dot{\gamma} = k N_i$$

$$Re_i = \frac{N_i D_i^2 \rho}{K \dot{\gamma}^{n-1}}$$

Specific growth rate

$$\frac{1}{UA} = \frac{1}{h_h A} + \frac{B}{kA} + \frac{1}{h_c A}$$

Thiele modulus

$$\sum_{\substack{\text{input}\\ \text{streams}}} (Mh) - \sum_{\substack{\text{output}\\ \text{streams}}} (Mh) - Q + W_s = \Delta E$$

Fouling factor

Ideal gas law

$$C_{AS}^* = K_F\, C_A^{*\,1/n}$$

Steady-state mass balance

SIGMA FACTOR

$$t_d = \frac{\ln 2}{\mu}$$

Heat of reaction

$$Nu = 0.87\, Re_i^{0.62}\, Pr^{0.33}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

$$\mathcal{D}_{Az}\frac{\partial^2 C_A}{\partial z^2} + u\frac{-\partial C_A}{\partial z} = \frac{\partial C_A}{\partial t} + \left(\frac{1-\varepsilon}{\varepsilon}\right)\frac{\partial C_{AS}}{\partial t}$$

Dimensional homogeneity

$$k_L a\left(C_{AL}^* - C_{AL}\right) = q_O\, x$$

SOLUTIONS MANUAL

Bioprocess Engineering Principles

Pauline M. Doran

University of New South Wales, Sydney, Australia

ISBN 0 7334 1547 4

Table of Contents

Solutions

Page

Chapter 2 *Introduction to Engineering Calculations* 1

Chapter 3 *Presentation and Analysis of Data* 9

Chapter 4 *Material Balances* 17

Chapter 5 *Energy Balances* 41

Chapter 6 *Unsteady-State Material and Energy Balances* 54

Chapter 7 *Fluid Flow and Mixing* 76

Chapter 8 *Heat Transfer* 86

Chapter 9 *Mass Transfer* 98

Chapter 10 *Unit Operations* 106

Chapter 11 *Homogeneous Reactions* 122

Chapter 12 *Heterogeneous Reactions* 139

Chapter 13 *Reactor Engineering* 151

NOTE

All equations, tables, figures, page numbers, etc., mentioned in this manual refer to the textbook, *Bioprocess Engineering Principles.*

2

Introduction to Engineering Calculations

2.1 Unit conversion

(a)

From Table A.9 (Appendix A): 1 cP = 10^{-3} kg m^{-1} s^{-1}
1 m = 100 cm
Therefore:

$$1.5\times 10^{-6}\,\text{cP} = 1.5\times 10^{-6}\,\text{cP}\,.\left|\frac{10^{-3}\,\text{kg m}^{-1}\,\text{s}^{-1}}{1\,\text{cP}}\right|.\left|\frac{1\,\text{m}}{100\,\text{cm}}\right| = 1.5\times 10^{-11}\,\text{kg s}^{-1}\,\text{cm}^{-1}$$

Answer: 1.5×10^{-11} kg s^{-1} cm^{-1}

(b)

From Table A.8 (Appendix A): 1 hp (British) = 42.41 Btu min^{-1}
Therefore:

$$0.122\,\text{hp} = 0.122\,\text{hp}\,.\left|\frac{42.41\,\text{Btu min}^{-1}}{1\,\text{hp}}\right| = 5.17\,\text{Btu min}^{-1}$$

Answer: 5.17 Btu min^{-1}

(c)

From Table A.5 (Appendix A): 1 mmHg = 1.316×10^{-3} atm
From Table A.1 (Appendix A): 1 ft = 0.3048 m
From Table A.7 (Appendix A): 1 l atm = 9.604×10^{-2} Btu
From Table A.8 (Appendix A): 1 Btu min^{-1} = 2.391×10^{-2} metric horsepower
1 m = 100 cm
1 l = 1000 cm^3
1 h = 60 min
Therefore:

$$670\,\text{mmHg ft}^3 = 670\,\text{mmHg ft}^3\,.\left|\frac{1.316\times 10^{-3}\,\text{atm}}{1\,\text{mmHg}}\right|.\left|\frac{9.604\times 10^{-2}\,\text{Btu}}{1\,\text{l atm}}\right|.\left|\frac{0.3048\,\text{m}}{1\,\text{ft}}\right|^3.\left|\frac{100\,\text{cm}}{1\,\text{m}}\right|^3.\left|\frac{1\,\text{l}}{1000\,\text{cm}^3}\right|.$$
$$\left|\frac{2.391\times 10^{-2}\,\text{metric horsepower}}{1\,\text{Btu min}^{-1}}\right|.\left|\frac{1\,\text{h}}{60\,\text{min}}\right| = 9.56\times 10^{-4}\,\text{metric horsepower h}$$

Answer: 9.56×10^{-4} metric horsepower h

(d)

From Table A.7 (Appendix A): 1 Btu = 0.2520 kcal
From Table A.3 (Appendix A): 1 lb = 453.6 g
Therefore:

$$345\,\text{Btu lb}^{-1} = 345\,\text{Btu lb}^{-1}\,.\left|\frac{0.2520\,\text{kcal}}{1\,\text{Btu}}\right|.\left|\frac{1\,\text{lb}}{453.6\,\text{g}}\right| = 0.192\,\text{kcal g}^{-1}$$

Answer: 0.192 kcal g^{-1}

2.2 Unit conversion

Case 1

Convert to units of kg, m, s.
From Table A.3 (Appendix A): 1 lb = 0.4536 kg

From Table A.2 (Appendix A): 1 $\text{ft}^3 = 2.832 \times 10^{-2}\ \text{m}^3$
From Table A.9 (Appendix A): 1 $\text{cP} = 10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}$
1 m = 100 cm = 1000 mm
Therefore, using Eq. (2.1):

$$Re = \frac{D\,u\,\rho}{\mu} = \frac{\left(2\ \text{mm}\,.\left|\frac{1\ \text{m}}{1000\ \text{mm}}\right|\right)\left(3\ \text{cm s}^{-1}\,.\left|\frac{1\ \text{m}}{100\ \text{cm}}\right|\right)\left(25\ \text{lb ft}^{-3}\,.\left|\frac{0.4536\ \text{kg}}{1\ \text{lb}}\right|.\left|\frac{1\ \text{ft}^3}{2.832\times 10^{-2}\ \text{m}^3}\right|\right)}{10^{-6}\ \text{cP}\,.\left|\frac{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}}{1\ \text{cP}}\right|} = 2.4\times 10^{7}$$

Answer: 2.4×10^7

Case 2
Convert to units of kg, m, s.
From Table A.1 (Appendix A): 1 in. = 2.54×10^{-2} m
From Table A.9 (Appendix A): 1 $\text{lb}_m\ \text{ft}^{-1}\ \text{h}^{-1} = 4.134 \times 10^{-4}\ \text{kg m}^{-1}\ \text{s}^{-1}$
1 h = 3600 s
Therefore, using Eq. (2.1):

$$Re = \frac{D\,u\,\rho}{\mu} = \frac{\left(1\ \text{in.}\,.\left|\frac{2.54\times 10^{-2}\ \text{m}}{1\ \text{in.}}\right|\right)\left(1\ \text{m s}^{-1}\right)\left(12.5\ \text{kg m}^{-3}\right)}{0.14\times 10^{-4}\ \text{lb}_m\,\text{s}^{-1}\ \text{ft}^{-1}.\left|\frac{4.134\times 10^{-4}\ \text{kg m}^{-1}\ \text{s}^{-1}}{1\ \text{lb}_m\,\text{ft}^{-1}\ \text{h}^{-1}}\right|.\left|\frac{3600\ \text{s}}{\text{h}}\right|} = 1.5\times 10^{4}$$

Answer: 1.5×10^4

2.3 Dimensionless groups and property data

From the *Chemical Engineers' Handbook*, the diffusivity of oxygen in water at 25°C is $2.5 \times 10^{-5}\ \text{cm}^2\ \text{s}^{-1}$. Assuming this is the same at 28°C, $\mathcal{D} = 2.5 \times 10^{-5}\ \text{cm}^2\ \text{s}^{-1}$. Also, from the *Chemical Engineers' Handbook*, the density of water at 28°C is $\rho_L = 0.9962652\ \text{g cm}^{-3}$, and the viscosity of water at 28°C is $\mu_L = 0.87$ cP. The density of oxygen at 28°C and 1 atm pressure can be calculated using the ideal gas law. As molar density is the same as $^n/_V$, from Eq. (2.32):

$$\rho_G = \frac{n}{V} = \frac{p}{R\,T}$$

Temperature in the ideal gas equation is absolute temperature; therefore, from Eq. (2.24):

$$T = (28 + 273.15)\ \text{K} = 301.15\ \text{K}$$

From Table 2.5, $R = 82.057\ \text{cm}^3\ \text{atm K}^{-1}\ \text{gmol}^{-1}$. Substituting parameter values into the density equation gives:

$$\rho_G = \frac{p}{R\,T} = \frac{1\ \text{atm}}{\left(82.057\ \text{cm}^3\ \text{atm K}^{-1}\ \text{gmol}^{-1}\right)(301.15\ \text{K})} = 4.05\times 10^{-5}\ \text{gmol cm}^{-3}$$

From the atomic weights in Table B.1 (Appendix B), the molecular weight of oxygen is 32.0. Converting the result for ρ_G to mass terms:

$$\rho_G = 4.05\times 10^{-5}\ \text{gmol cm}^{-3}\,.\left|\frac{32.0\ \text{g}}{1\ \text{gmol}}\right| = 1.30\times 10^{-3}\ \text{g cm}^{-3}$$

From Table A.9 (Appendix A): 1 $\text{cP} = 10^{-2}\ \text{g cm}^{-1}\ \text{s}^{-1}$; from Table A.1 (Appendix A): 1 ft = 0.3048 m = 30.48 cm. The parameter values and conversion factors, together with D_b = 2 mm = 0.2 cm, can now be used to calculate the dimensionless groups in the equation for the Sherwood number.

$$Gr = \frac{D_b^3\,\rho_G\,(\rho_L - \rho_G)\,g}{\mu_L^2} = \frac{(0.2\ \text{cm})^3\left(1.30\times 10^{-3}\ \text{g cm}^{-3}\right)\left(0.9962652 - 1.30\times 10^{-3}\right)\text{g cm}^{-3}\left(32.17\ \text{ft s}^{-2}\,.\left|\frac{30.48\ \text{cm}}{1\ \text{ft}}\right|\right)}{\left(0.87\ \text{cP}\,.\left|\frac{10^{-2}\ \text{g cm}^{-1}\ \text{s}^{-1}}{1\ \text{cP}}\right|\right)^2}$$

$$Gr = 134$$

$$Sc = \frac{\mu_L}{\rho_L \mathcal{D}} = \frac{0.87\,\text{cP}\,.\left|\dfrac{10^{-2}\,\text{g cm}^{-1}\,\text{s}^{-1}}{1\,\text{cP}}\right|}{\left(0.9962652\,\text{g cm}^{-3}\right)\left(2.5\times10^{-5}\,\text{cm}^2\,\text{s}^{-1}\right)} = 349$$

Therefore:

$$Sh = 0.31\,Gr^{1/3}\,Sc^{1/3} = 0.31\,(134)^{1/3}\,(349)^{1/3} = 11.2$$

From the equation for *Sh*:

$$k_L = \frac{Sh\,\mathcal{D}}{D_b} = \frac{(11.2)\left(2.5\times10^{-5}\,\text{cm}^2\,\text{s}^{-1}\right)}{0.2\,\text{cm}} = 1.40\times10^{-3}\,\text{cm s}^{-1}$$

Answer: 1.40×10^{-3} cm s^{-1}

2.4 Mass and weight

From the definition of density on p 16, mass is equal to volume multiplied by density. Therefore:

$$\text{Mass of water} = (10\,\text{ft}^3)\,(62.4\,\text{lb}_m\,\text{ft}^{-3}) = 624\,\text{lb}_m$$

From p 16, weight is the force with which a body is attracted to the centre of the earth by gravity. According to Newton's law (p 15), this force is equal to the mass of the body multiplied by the gravitational acceleration.

(a)

From p 15, at sea level and 45° latitude, gravitational acceleration $g = 32.174$ ft s^{-2}. Therefore:

$$\text{Weight} = 624\,\text{lb}_m\,(32.174\,\text{ft s}^{-2}) = 2.008\times10^4\,\text{lb}_m\,\text{ft s}^{-2}$$

Converting these units to lb_f, from Eq. (2.16), 1 $\text{lb}_f = 32.174\,\text{lb}_m$ ft s^{-2}; therefore:

$$\text{Weight} = 2.008\times10^4\,\text{lb}_m\,\text{ft s}^{-2}\,.\left|\frac{1\,\text{lb}_f}{32.174\,\text{lb}_m\,\text{ft s}^{-2}}\right| = 624\,\text{lb}_f$$

Answer: 624 lb_f. *When $g = 32.174$ ft s^{-2}, the number of lb mass is equal to the number of lb force.*

(b)

From Table A.1 (Appendix A): 1 m = 3.281 ft. Using the same procedure as in **(a)**:

$$\text{Weight} = 624\,\text{lb}_m\left(9.76\,\text{m s}^{-2}\,.\left|\frac{3.281\,\text{ft}}{1\,\text{m}}\right|\right) = 1.998\times10^4\,\text{lb}_m\,\text{ft s}^{-2}$$

Converting to lb_f:

$$\text{Weight} = 1.998\times10^4\,\text{lb}_m\,\text{ft s}^{-2}\,.\left|\frac{1\,\text{lb}_f}{32.174\,\text{lb}_m\,\text{ft s}^{-2}}\right| = 621\,\text{lb}_f$$

Answer: 621 lb_f

2.5 Dimensionless numbers

First, evaluate the units of the groups $(^{C_p\,\mu}/_k)$ and $(^{D\,G}/_\mu)$:

$$\text{Units of}\left(\frac{C_p\,\mu}{k}\right) = \frac{\left(\text{Btu lb}^{-1}\,{}^\circ\text{F}^{-1}\right)\text{lb h}^{-1}\,\text{ft}^{-1}}{\text{Btu h}^{-1}\,\text{ft}^{-2}\left({}^\circ\text{F ft}^{-1}\right)^{-1}} = 1$$

$$\text{Units of}\left(\frac{D\,G}{\mu}\right) = \frac{(\text{ft})\,\text{lb h}^{-1}\,\text{ft}^{-2}}{\text{lb h}^{-1}\,\text{ft}^{-1}} = 1$$

Therefore, these groups are dimensionless. For the equation to be dimensionally homogeneous, $(^{h}/C_p\, G)$ must also be dimensionless; the units of h must therefore cancel the units of $C_p\, G$.

$$\text{Units of } h \;=\; \text{units of } C_p\, G \;=\; (\text{Btu lb}^{-1}\,{}^{\circ}\text{F}^{-1})\,(\text{lb h}^{-1}\,\text{ft}^{-2}) \;=\; \text{Btu}\,{}^{\circ}\text{F}^{-1}\,\text{h}^{-1}\,\text{ft}^{-2}$$

The dimensions of h can be deduced from its units. From Table A.7 (Appendix A), Btu is a unit of energy with dimensions = L^2MT^{-2}. °F is a unit of temperature which, from Table 2.1, has the dimensional symbol Θ. h is a unit of time with dimension = T; ft is a unit of length with dimension = L. Therefore:

$$\text{Dimensions of } h \;=\; L^2MT^{-2}\,\Theta^{-1}\,T^{-1}\,L^{-2} \;=\; MT^{-3}\Theta^{-1}$$

Answer: Units = Btu $^{\circ}\text{F}^{-1}$ h^{-1} ft^{-2}; dimensions = $MT^{-3}\Theta^{-1}$

2.6 Dimensional homogeneity and g_c

From Table A.8 (Appendix A), dimensions of $P = L^2MT^{-3}$
Dimensions of $g = LT^{-2}$
Dimensions of $\rho = ML^{-3}$
Dimensions of $D_i = L$
From p 11, the dimensions of rotational speed, $N_i = T^{-1}$; from p 15, the dimensions of $g_c = 1$. Therefore:

$$\text{Dimensions of } \frac{P\,g}{\rho\, N_i^3\, D_i^5} \;=\; \frac{(L^2MT^{-3})(LT^{-2})}{(ML^{-3})(T^{-1})^3\,L^5} \;=\; LT^{-2}$$

As N_P is a dimensionless number, equation (i) is not dimensionally homogeneous and therefore cannot be correct.

$$\text{Dimensions of } \frac{P\,g_c}{\rho\, N_i^3\, D_i^5} \;=\; \frac{(L^2MT^{-3})(1)}{(ML^{-3})(T^{-1})^3\,L^5} \;=\; 1$$

Equation (ii) is dimensionally homogeneous and therefore likely to be correct.

Answer: (ii)

2.7 Molar units

From the atomic weights in Table B.1 (Appendix B), the molecular weight of NaOH is 40.0.

(a)
From Eq. (2.19):

$$\text{lb moles NaOH} \;=\; \frac{20.0\,\text{lb}}{40.0\,\text{lb lbmol}^{-1}} \;=\; 0.50\,\text{lbmol}$$

Answer: 0.50 lbmol

(b)
From Table A.3 (Appendix A): 1 lb = 453.6 g. Therefore:

$$20.0\,\text{lb} \;=\; 20.0\,\text{lb}\,.\left|\frac{453.6\,\text{g}}{1\,\text{lb}}\right| \;=\; 9072\,\text{g}$$

From Eq. (2.18):

$$\text{gram moles NaOH} \;=\; \frac{9072\,\text{g}}{40.0\,\text{g gmol}^{-1}} \;=\; 227\,\text{gmol}$$

Answer: 227 gmol

(c)
From p 16, 1 kgmol = 1000 gmol. Therefore, from **(b)**:

$$\text{kg moles NaOH} = 227\ \text{gmol}\cdot\left|\frac{1\ \text{kgmol}}{1000\ \text{gmol}}\right| = 0.227\ \text{kgmol}$$

Answer: 0.227 kgmol

2.8 Density and specific gravity

(a)

From p 16, the density of water at 4°C can be taken as exactly 1 g cm^{-3}. Therefore, for a substance with specific gravity $1.5129^{20°\text{C}}_{4°\text{C}}$, the density at 20°C is 1.5129 g cm^{-3}.

(i)

1 kg = 1000 g
1 m = 100 cm
Therefore:

$$\text{Density} = 1.5129\ \text{g cm}^{-3}\cdot\left|\frac{1\ \text{kg}}{1000\ \text{g}}\right|\cdot\left|\frac{100\ \text{cm}}{1\ \text{m}}\right|^3 = 1512.9\ \text{kg m}^{-3}$$

Answer: 1512.9 kg m^{-3}

(ii)

From the atomic weights in Table B.1 (Appendix B), the molecular weight of nitric acid (HNO_3) is 63.0. In 1 cm^3 HNO_3, from Eq. (2.18):

$$\text{gram moles} = \frac{1.5129\ \text{g}}{63.0\ \text{g gmol}^{-1}} = 0.0240\ \text{gmol}$$

Therefore, the molar density is 0.0240 gmol cm^{-3}. From the definition of specific volume on p 16:

$$\text{Molar specific volume} = \frac{1}{\text{molar density}} = \frac{1}{0.0240\ \text{gmol cm}^{-3}} = 41.67\ \text{cm}^3\ \text{gmol}^{-1}$$

Answer: 41.67 cm^3 gmol^{-1}

(b)

(i)

From p 16, as density is defined as the mass per unit volume, the mass flow rate is equal to the volumetric flow rate multiplied by the density:

$$\text{Mass flow rate} = \left(50\ \text{cm}^3\ \text{min}^{-1}\right)\left(1.6\ \text{g cm}^{-3}\right) = 80\ \text{g min}^{-1}$$

Answer: 80 g min^{-1}

(ii)

From the atomic weights in Table B.1 (Appendix B), the molecular weight of carbon tetrachloride, CCl_4, is 153.8. Using the mass flow rate from **(a)**:

$$\text{Molar flow rate} = 80\ \text{g min}^{-1}\cdot\left|\frac{1\ \text{gmol}}{153.8\ \text{g}}\right| = 0.52\ \text{gmol min}^{-1}$$

Answer: 0.52 gmol min^{-1}

2.9 Molecular weight

From p 17, the composition of air is close to 21% oxygen and 79% nitrogen. For gases at low pressures, this means 21 mol% O_2 and 79 mol% N_2. Therefore, in 1 gmol air, there are 0.21 gmol O_2 and 0.79 gmol N_2 From the atomic weights in Table B.1 (Appendix B), the molecular weights of O_2 and N_2 are 32.0 and 28.0, respectively. The molecular weight of air is equal to the number of grams in 1 gmol:

$$1\ \text{gmol air} = 0.21\ \text{gmol}\ O_2\cdot\left|\frac{32.0\ \text{g}}{1\ \text{gmol}}\right| + 0.79\ \text{gmol}\ N_2\cdot\left|\frac{28.0\ \text{g}}{1\ \text{gmol}}\right| = 28.8\ \text{g}$$

Answer: 28.8

2.10 Mole fraction

The molecular weights can be obtained from Table B.7 (Appendix B): water 18.0; ethanol 46.1; methanol 32.0; glycerol 92.1; acetic acid 60.1; benzaldehyde 106.1. In 100 g solution, there are 30 g water, 25 g ethanol, 15 g methanol, 12 g glycerol, 10 g acetic acid, 8 g benzaldehyde, and no other components. Therefore:

$$\text{Moles water} = 30\text{ g}\,.\left|\frac{1\text{ gmol}}{18.0\text{ g}}\right| = 1.67\text{ gmol}$$

$$\text{Moles ethanol} = 25\text{ g}\,.\left|\frac{1\text{ gmol}}{46.1\text{ g}}\right| = 0.54\text{ gmol}$$

$$\text{Moles methanol} = 15\text{ g}\,.\left|\frac{1\text{ gmol}}{32.0\text{ g}}\right| = 0.47\text{ gmol}$$

$$\text{Moles glycerol} = 12\text{ g}\,.\left|\frac{1\text{ gmol}}{92.1\text{ g}}\right| = 0.13\text{ gmol}$$

$$\text{Moles acetic acid} = 10\text{ g}\,.\left|\frac{1\text{ gmol}}{60.1\text{ g}}\right| = 0.17\text{ gmol}$$

$$\text{Moles benzaldehyde} = 8\text{ g}\,.\left|\frac{1\text{ gmol}}{106.1\text{ g}}\right| = 0.08\text{ gmol}$$

The total number of moles is 1.67 + 0.54 + 0.47 + 0.13 + 0.17 + 0.08 = 3.06 gmol. From Eq. (2.20):

$$\text{Mole fraction water} = \frac{1.67}{3.06} = 0.55$$

$$\text{Mole fraction ethanol} = \frac{0.54}{3.06} = 0.18$$

$$\text{Mole fraction methanol} = \frac{0.47}{3.06} = 0.15$$

$$\text{Mole fraction glycerol} = \frac{0.13}{3.06} = 0.04$$

$$\text{Mole fraction acetic acid} = \frac{0.17}{3.06} = 0.06$$

$$\text{Mole fraction benzaldehyde} = \frac{0.08}{3.06} = 0.03$$

Answer: 0.55 water; 0.18 ethanol; 0.15 methanol; 0.04 glycerol; 0.06 acetic acid; 0.03 benzaldehyde

2.11 Temperature scales

From Eq. (2.27):

$$\begin{aligned} -40 &= 1.8\,T\,(°\text{C}) + 32 \\ T\,(°\text{C}) &= -40 \end{aligned}$$

From Eq. (2.25):

$$\begin{aligned} T\,(°\text{R}) &= -40 + 459.67 \\ T\,(°\text{R}) &= 420 \end{aligned}$$

From Eq. (2.24) and the result for T (°C):

$$\begin{aligned} T\,(\text{K}) &= -40 + 273.15 \\ T\,(\text{K}) &= 233 \end{aligned}$$

Answer: 40°C; 420°R; 233 K

2.12 Pressure scales

(a)

Assume that the atmospheric pressure is 14.7 psi. From Eq. (2.28):

$$\text{Absolute pressure} = 15\text{ psi} + 14.7\text{ psi} = 29.7\text{ psi}$$

From Table A.5 (Appendix A): 1 psi = 6.805×10^{-2} atm. Therefore:

$$\text{Absolute pressure} = 29.7\ \text{psi} \cdot \left| \frac{6.805 \times 10^{-2}\ \text{atm}}{1\ \text{psi}} \right| = 2.02\ \text{atm}$$

Answer: 29.7 psi; 2.02 atm

(b)
From p 19, vacuum pressure is the pressure below atmospheric. If the atmospheric pressure is 14.7 psi:

$$\text{Absolute pressure} = 14.7\ \text{psi} - 3\ \text{psi} = 11.7\ \text{psi}$$

Answer: 11.7 psi

2.13 Stoichiometry and incomplete reaction

(a)
The molecular weights are calculated from Table B.1 (Appendix B): penicillin = 334.4; glucose = 180.2. The maximum theoretical yield from the stoichiometric equation is 1 gmol of penicillin for every 1.67 gmol of glucose. This is equivalent to 334.4 g penicillin per $1.67 \times 180.2 = 300.9$ g glucose, or 1.1 g $\mathrm{g^{-1}}$.

Answer: 1.1 g $\mathrm{g^{-1}}$

(b)
The maximum theoretical yield in **(a)** is obtained when all the glucose consumed is directed into penicillin production according to the stoichiometric equation. If only 6% of the glucose is used in this way, the actual yield of penicillin from glucose is much lower, at 334.4 g penicillin per ($300.9 \times {}^{100}/_{6}$) g glucose, or 0.067 g $\mathrm{g^{-1}}$.

Answer: 0.067 g $\mathrm{g^{-1}}$

(c)
From the atomic weights in Table B.1 (Appendix B), the molecular weight of phenylacetic acid is 136.2.

(i)
The only possible limiting substrates are glucose and phenylacetic acid. Using a basis of 1 l medium, if $(50 - 5.5) =$ 44.5 g glucose are consumed but only 6% is available for penicillin synthesis, the mass of glucose used in the penicillin reaction is $44.5 \times {}^{6}/_{100} = 2.67$ g. This is equivalent to ${}^{2.67\ \mathrm{g}}/_{180.2\ \mathrm{g\ gmol^{-1}}} = 1.48 \times 10^{-2}$ gmol glucose available for penicillin synthesis. At the same time, 4 g or ${}^{4\ \mathrm{g}}/_{136.2\ \mathrm{g\ gmol^{-1}}} = 2.94 \times 10^{-2}$ gmol phenylacetic acid is available which, according to the stoichiometric equation, requires $1.67 \times 2.94 \times 10^{-2} = 4.91 \times 10^{-2}$ gmol glucose for complete reaction. As the gmol glucose required is greater than the gmol glucose available after growth and maintenance activities, glucose is the limiting substrate.

Answer: Glucose

(ii)
Of the 44.5 g $\mathrm{l^{-1}}$ glucose consumed, 24% or 10.7 g $\mathrm{l^{-1}}$ is used for growth. In a 100-litre tank, the total mass of glucose consumed for growth is therefore 1070 g or 1.07 kg.

Answer: 1.07 kg

(iii)
From (i), 1.48×10^{-2} gmol glucose is used in the penicillin reaction per litre. According to the stoichiometry, this produces ${}^{1.48 \times 10^{-2}}/_{1.67} = 8.86 \times 10^{-3}$ gmol penicillin per litre. Therefore, in a 100-litre tank, 0.886 gmol or 0.886 gmol $\times$ 334.4 g $\mathrm{gmol^{-1}}$ = 296 g penicillin are formed.

Answer: 296 g

(iv)
If, from (i), 1.48×10^{-2} gmol $\mathrm{l^{-1}}$ glucose is used in the penicillin reaction, ${}^{1.48 \times 10^{-2}}/_{1.67} = 8.86 \times 10^{-3}$ gmol $\mathrm{l^{-1}}$ phenylacetic acid must also be used. This is equivalent to 8.86×10^{-3} gmol $\mathrm{l^{-1}} \times 136.2$ g $\mathrm{gmol^{-1}}$ = 1.21 g $\mathrm{l^{-1}}$ phenylacetic acid. As 4 g $\mathrm{l^{-1}}$ are provided, $(4 - 1.21) = 2.79$ g $\mathrm{l^{-1}}$ phenylacetic acid must remain.

Answer: 2.79 g $\mathrm{l^{-1}}$

2.14 Stoichiometry, yield and the ideal gas law

(a)

Adding up the numbers of C, H, O and N atoms on both sides of the equation shows that the equation is balanced.

Answer: Yes

(b)

The molecular weights are calculated from Table B.1 (Appendix B).
Cells: 91.5
Hexadecane: 226.4
From the stoichiometry, as 1 gmol of hexadecane is required to produce 1.65 gmol of cells, the maximum yield is 1.65 gmol × 91.5 g gmol^{-1} = 151 g cells per 226.4 g hexadecane, or 0.67 g g^{-1}.

Answer: 0.67 g g^{-1}

(c)

From the atomic weights in Table B.1 (Appendix B), the molecular weight of oxygen is 32.0. From the stoichiometry, 16.28 gmol of oxygen is required to produce 1.65 gmol of cells which, from **(b)**, is equal to 151 g cells. The maximum yield is therefore 151 g cells per (16.28 gmol × 32.0 g gmol^{-1}) = 521 g oxygen, or 0.29 g g^{-1}.

Answer: 0.29 g g^{-1}

(d)

Production of 2.5 kg cells is equivalent to 2500 g = $^{2500\text{ g}}/_{91.5\text{ g gmol}^{-1}}$ = 27.3 gmol cells. The minimum amounts of substrates are required when 100% of the hexadecane is converted according to the stoichiometric equation.

(i)

From the stoichiometry, production of 27.3 gmol cells requires $^{27.3}/_{1.65}$ = 16.5 gmol = 16.5 gmol × 226.4 g gmol^{-1} = 3736 g = 3.74 kg hexadecane.

Answer: 3.74 kg

(ii)

From the answer in **(d)(i)**, the concentration of hexadecane required is 3.74 kg in 3 m^3, or 1.25 kg m^{-3}.

Answer: 1.25 kg m^{-3}

(iii)

According to the stoichiometric equation, production of 27.3 gmol cells requires 27.3 × 16.28/1.65 = 269.4 gmol oxygen. As air at low pressure contains close to 21 mol% oxygen (p 17), the total moles of air required is $^{269.4}/_{0.21}$ = 1282.9 gmol. The volume of air required can be calculated using the ideal gas law. From Eq. (2.32):

$$V = \frac{n\,R\,T}{p}$$

Temperature in the ideal gas equation is absolute temperature; from Eq. (2.24):

$$T = (20 + 273.15)\text{ K} = 293.15\text{ K}$$

From Table 2.5, R = 82.057 cm^3 atm K^{-1} gmol^{-1}. Substituting these values into the equation for V gives:

$$V = \frac{(1282.9\text{ gmol})\left(82.057\text{ cm}^3\text{ atm K}^{-1}\text{ gmol}^{-1}\right)(293.15\text{ K})}{1\text{ atm}} \cdot \left|\frac{1\text{ m}}{100\text{ cm}}\right|^3 = 31\text{ m}^3$$

Answer: 31 m^3

3

Presentation and Analysis of Data

3.1 Combination of errors

C^*_{AL} = 0.25 mol m^{-3} ± 4% = 0.25 ± 0.010 mol m^{-3}
C_{AL} = 0.183 mol m^{-3} ± 4% = 0.183 ± 0.0073 mol m^{-3}
OTR = 0.011 mol m^{-3} s^{-1} ± 5%
For subtraction, absolute errors are added. Therefore:

$$C^*_{AL} - C_{AL} = (0.25 - 0.183) \pm (0.010 + 0.0073)\ \text{mol m}^{-3} = 0.067 \pm 0.0173\ \text{mol m}^{-3} = 0.067\ \text{mol m}^{-3} \pm 25.8\%$$

For division, relative errors are added. Therefore:

$$k_L a = \frac{0.011\ \text{mol m}^{-3}\ \text{s}^{-1}}{0.067\ \text{mol m}^{-3}} \pm (25.8 + 5)\% = 0.16\ \text{s}^{-1} \pm 31\% = 0.16 \pm 0.05\ \text{s}^{-1}$$

Answer: 31%. *This example illustrates how a combination of small measurement errors can result in a relatively large uncertainty in the final result.*

3.2 Mean and standard deviation

(a)
The best estimate is the mean, $\bar{x}$. From Eq. (3.1):

$$\bar{x} = \frac{5.15 + 5.45 + 5.50 + 5.35}{4} = 5.36$$

Answer: 5.36

(b)
Calculate the standard deviation from Eq. (3.2):

$$\sigma = \sqrt{\frac{(5.15 - 5.36)^2 + (5.45 - 5.36)^2 + (5.50 - 5.36)^2 + (5.35 - 5.36)^2}{4 - 1}} = 0.15$$

Answer: The standard deviation is 0.15. *Note that standard error, which can be calculated from the standard deviation, is a more direct indication of the precision of a mean.*

(c)

$$\bar{x} = \frac{5.15 + 5.45}{2} = 5.30$$

Standard deviation is not appropriate for expressing the accuracy of a mean evaluated using only two samples. In this case the maximum error, i.e. the difference between the mean and either of the two measured values, might be used instead. The maximum error in this example is (5.30 – 5.15) = 0.15.

Answer: 5.30; an indication of the accuracy is ± 0.15

(d)

$$\bar{x} = \frac{5.15 + 5.45 + 5.50 + 5.35 + 5.15 + 5.45 + 5.50 + 5.35}{8} = 5.36$$

$$\sigma = \sqrt{\frac{2\,(5.15-5.36)^2+2\,(5.45-5.36)^2+2\,(5.50-5.36)^2+2\,(5.35-5.36)^2}{8-1}} = 0.14$$

Answer: The best estimate of optimal pH is unchanged at 5.36, but the standard deviation is slightly lower at 0.14. *This example illustrates that although the standard deviation decreases as the number of measurements is increased, σ is not strongly dependent on n. The best way to improve the reliability of the mean is to ensure that the individual measurements are as intrinsically accurate as possible, rather than repeat the measurement many times.*

3.3 Linear and non-linear models

(a)
$x_1 = 1; y_1 = 10$
$x_2 = 8; y_2 = 0.5$
A straight line plot of y versus x on linear coordinates means that the data can be represented using Eq. (3.6). From Eqs (3.7) and (3.8):

$$A = \frac{(y_2-y_1)}{(x_2-x_1)} = \frac{0.5-10}{8-1} = -1.36$$

$$B = y_1 - A\,x_1 = 10-(-1.36)\,1 = 11.4$$

Answer: $y = -1.36\,x + 11.4$

(b)
$x_1 = 3.2; y_1 = 14.5$
$x_2 = 8.9; y_2 = 38.5$
A straight line plot of y versus $x^{1/2}$ on linear coordinates means that the data can be represented using the equation:

$$y = A\,x^{1/2} + B$$

with A and B given by the equations:

$$A = \frac{y_2-y_1}{x_2^{1/2}-x_1^{1/2}} = \frac{38.5-14.5}{8.9^{1/2}-3.2^{1/2}} = 20.1$$

$$B = y_1 - A\,x_1^{1/2} = 14.5-20.1\,(3.2^{1/2}) = -21.5$$

Answer: $y = 20.1\,x^{1/2} - 21.5$

(c)
$x_1 = 5; y_1 = 6$
$x_2 = 1; y_2 = 3$
A straight line plot of $1/y$ versus x^2 on linear coordinates means that the data can be represented using the equation:

$$1/y = A\,x^2 + B$$

with A and B given by the equations:

$$A = \frac{1/y_2 - 1/y_1}{x_2^2 - x_1^2} = \frac{1/3 - 1/6}{1^2-5^2} = -6.9\times 10^{-3}$$

$$B = 1/y_1 - A\,x_1^2 = 1/6 - \left(-6.9\times 10^{-3}\right)(5^2) = 0.34$$

Answer: $1/y = -6.9\times 10^{-3}\,x^2 + 0.34$

(d)
$x_1 = 0.5; y_1 = 25$
$x_2 = 550; y_2 = 2600$
A straight line plot of y versus x on log–log coordinates means that the data can be represented using Eq. (3.10). From Eqs (3.13) and (3.14):

$$A = \frac{(\ln y_2 - \ln y_1)}{(\ln x_2 - \ln x_1)} = \frac{\ln 2600 - \ln 25}{\ln 550 - \ln 0.5} = 0.663$$

$$\ln B = \ln y_1 - A \ln x_1 = \ln 25 - (0.663) \ln 0.5 = 3.678$$

$$B = e^{3.678} = 39.6$$

Answer: $y = 39.6\, x^{0.663}$

(e)
$x_1 = 1.5;\ y_1 = 2.5$
$x_2 = 10;\ y_2 = 0.036$
A straight line plot of y versus x on semi-log coordinates means that the data can be represented using Eq. (3.15). From Eqs (3.17) and (3.18):

$$A = \frac{(\ln y_2 - \ln y_1)}{(x_2 - x_1)} = \frac{\ln 0.036 - \ln 2.5}{10 - 1.5} = -0.50$$

$$\ln B = \ln y_1 - A\, x_1 = \ln 2.5 - (-0.50)\, 1.5 = 1.666$$

$$B = e^{1.666} = 5.29$$

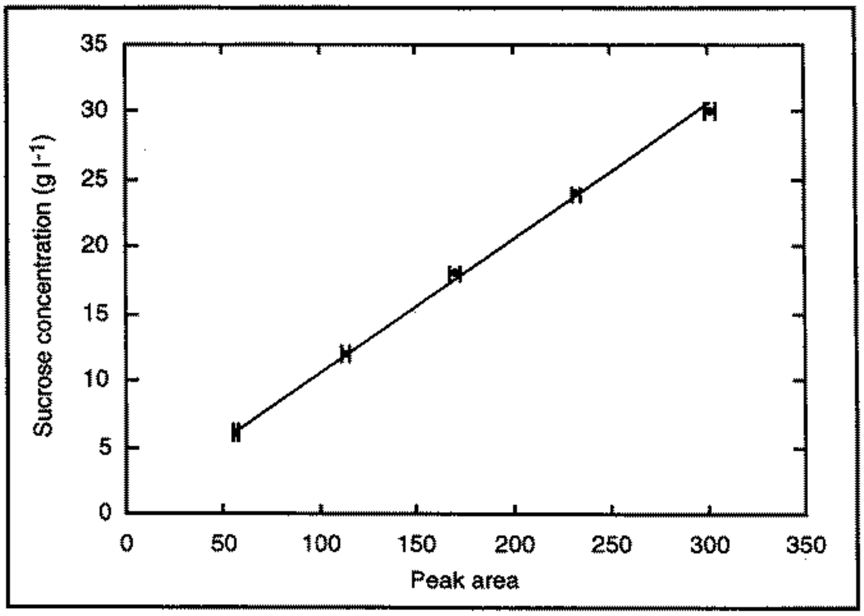
Answer: $y = 5.29\, e^{-0.50\,x}$

3.4 Linear curve fitting

(a)
The results determined using Eqs (3.1) and (3.2) are listed below.

Sucrose concentration (g l^{-1})	Mean peak area	Standard deviation
6.0	56.84	1.21
12.0	112.82	2.06
18.0	170.63	2.54
24.0	232.74	1.80
30.0	302.04	2.21

(b)



(c)
The linear least squares fit of the data is:

$$y = 0.098\,x + 0.83$$

Answer: $y = 0.098\,x + 0.83$, where y is sucrose concentration in g l^{-1} and x is peak area.

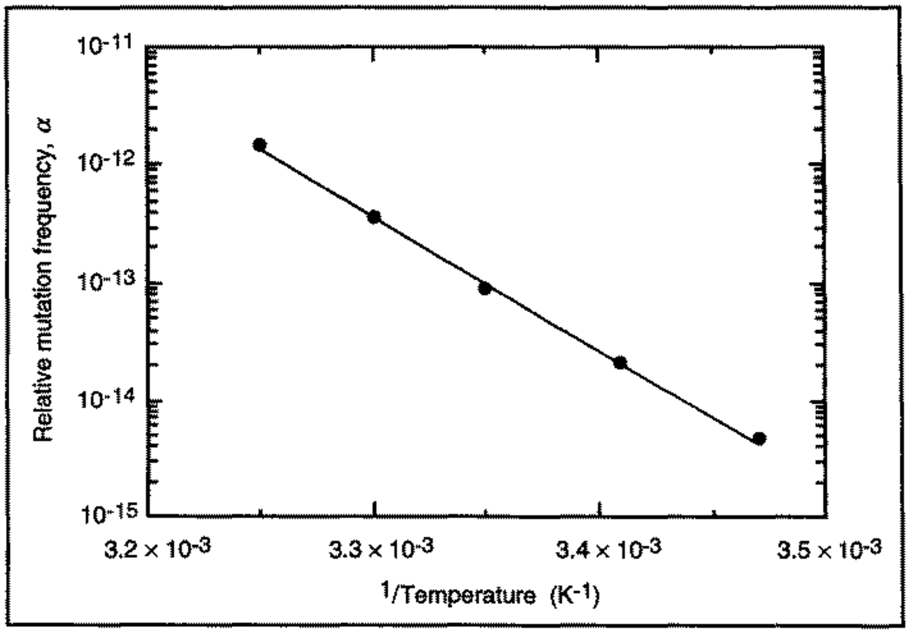
(d)
For $x = 209.86$, the equation in **(c)** gives a sucrose concentration of 21.4 g l^{-1}.

Answer: 21.4 g l^{-1}

3.5 Non-linear model: calculation of parameters

(a)
The proposed model equation has the general form of Eq. (3.15); therefore, if the model is suitable, a plot of α versus $^1/_T$ on semi-logarithmic coordinates will give a straight line. As T in the equation is absolute temperature, °C must first be converted to degrees Kelvin using Eq. (2.24). The data are listed and plotted below.

Temperature (°C)	Temperature (K)	$^1/_T$ (K^{-1})	Relative mutation frequency, α
15	288.15	3.47×10^{-3}	4.4×10^{-15}
20	293.15	3.41×10^{-3}	2.0×10^{-14}
25	298.15	3.35×10^{-3}	8.6×10^{-14}
30	303.15	3.30×10^{-3}	3.5×10^{-13}
35	308.15	3.25×10^{-3}	1.4×10^{-12}



As the data give a straight line on semi-logarithmic coordinates, the model can be considered to fit the data well.

(b)
The equation for the straight line in **(a)** is:

$$y = 9.66 \times 10^{24}\, e^{-26,121x}$$

where y is relative mutation frequency and x is reciprocal temperature in units of K^{-1}. For dimensional homogeneity the exponent must be dimensionless (p 12), so that –26,121 has units of K, and $^E/_R$ in the model equation is equal to 26,121 K. From Table 2.5, $R = 8.3144$ J gmol^{-1} K^{-1}; therefore:

$$E = (26{,}121\text{ K})\,(8.3144\text{ J gmol}^{-1}\text{ K}^{-1}) = 217{,}180.4\text{ J gmol}^{-1} = 217.2\text{ kJ gmol}^{-1}$$

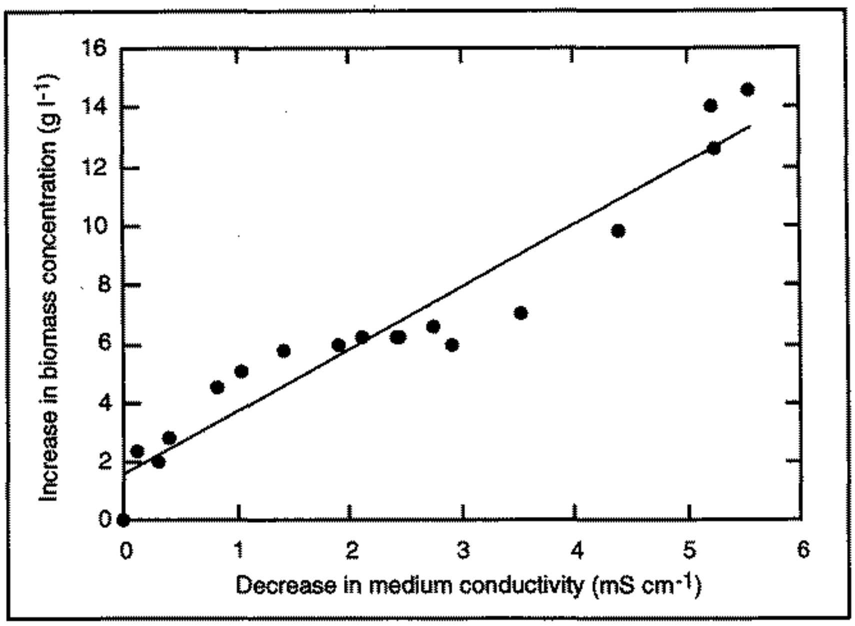
Answer: 217.2 kJ gmol^{-1}

(c)
From the equation in **(b)** for the straight line, α_0 is equal to 9.66×10^{24}.

Answer: 9.66×10^{24}

3.6 Linear regression: distribution of residuals

(a)



The linear least squares fit of the data is:

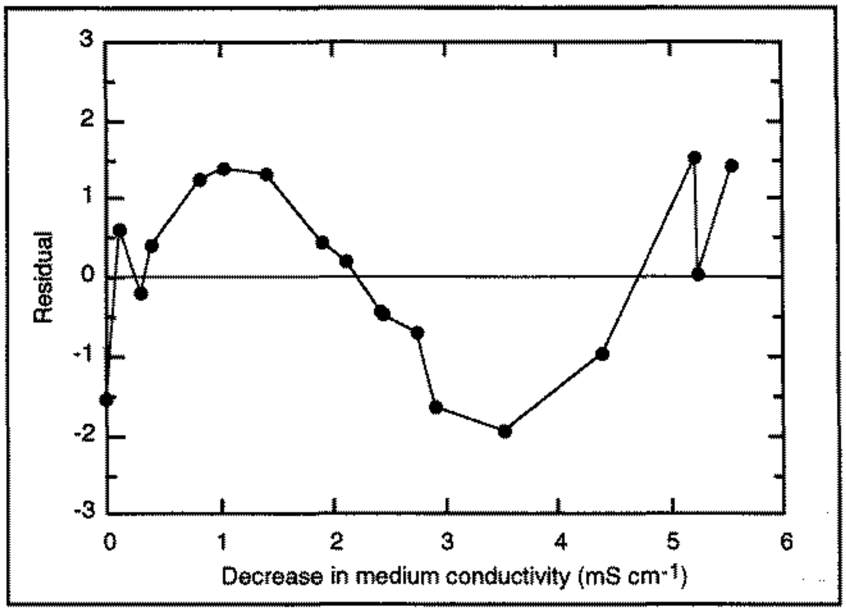
$$y = 1.58 + 2.10\,x$$

where y is increase in biomass concentration in g l^{-1} and x is decrease in medium conductivity in mS cm^{-1}.

(b)
The residuals are calculated as the difference between the measured values for increase in biomass concentration and the values for y obtained from the equation in **(a)**.

Decrease in medium conductivity (mS cm^{-1})	Residual
0	–1.58
0.12	0.57
0.31	–0.23
0.41	0.36
0.82	1.20
1.03	1.36
1.40	1.28
1.91	0.41
2.11	0.19
2.42	–0.46
2.44	–0.50
2.74	–0.73
2.91	–1.69
3.53	–1.99
4.39	–1.00
5.21	1.48
5.24	0.02
5.55	1.37

These results are plotted below.

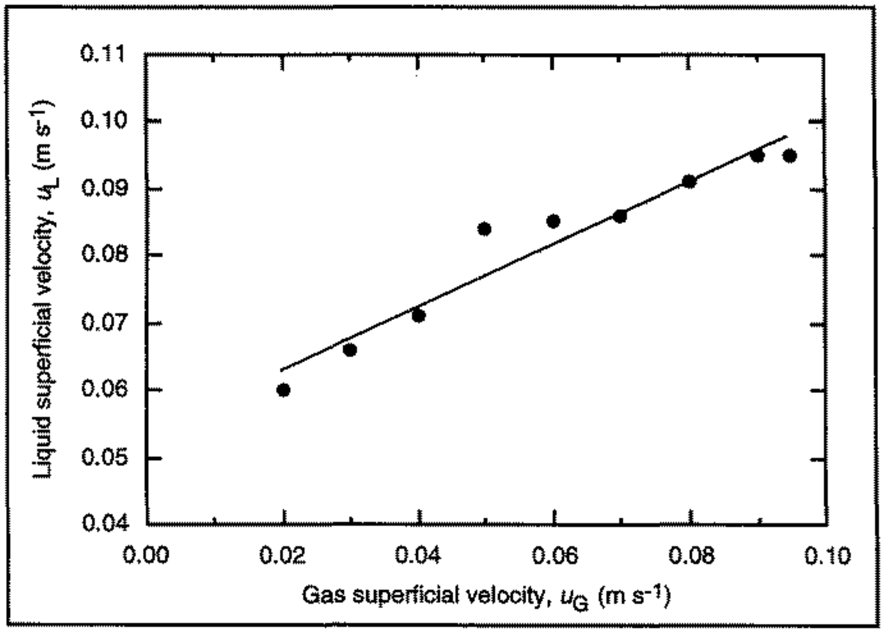


The residuals are not randomly distributed: they are mainly positive at low values of decrease in medium conductivity, then negative, then positive again. Therefore, the straight line fit of the data cannot be considered a very good one.

3.7 Discriminating between rival models

(a)

The results are plotted using linear coordinates below.



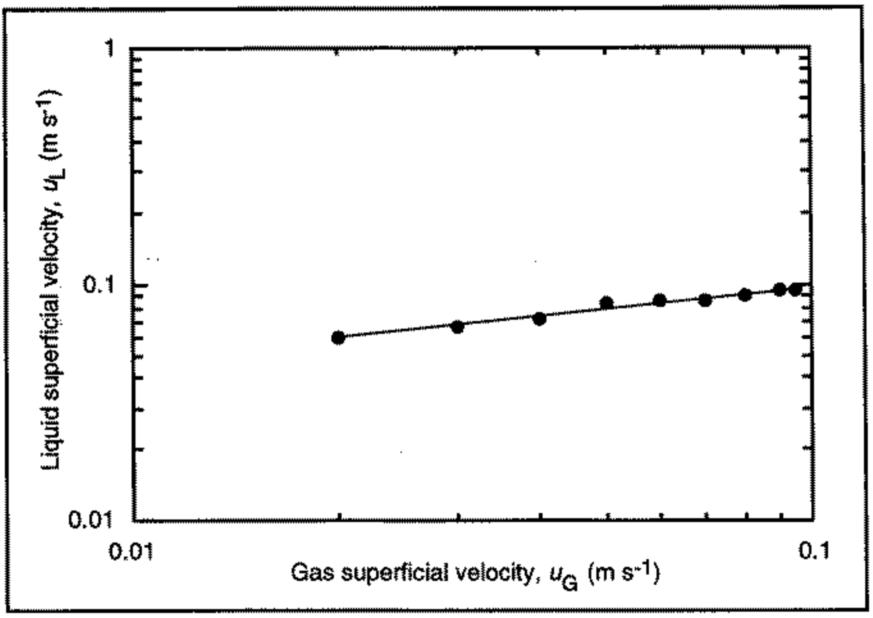
The data are reasonably well fitted using a linear model. The linear least squares equation for the straight line fit is:

$$y = 0.054 + 0.466\,x$$

where y is liquid superficial velocity in m s^{-1} and x is gas superficial velocity in m s^{-1}. The sum of the squares of the residuals between the measured values for liquid superficial velocity and the values for y obtained from the above equation is 8.4×10^{-5}.

(b)

The proposed power law equation has the same form as Eq. (3.10). Therefore, if the power law model is suitable, the data should give a straight line when plotted on log–log coordinates.



The data are reasonably well fitted using a power law model. The equation for the straight line in the plot is:

$$y = 0.199\,x^{0.309}$$

where y is liquid superficial velocity in m s^{-1} and x is gas superficial velocity in m s^{-1}. The sum of the squares of the residuals between the measured values for liquid superficial velocity and the values obtained from the above equation is 4.2×10^{-5}.

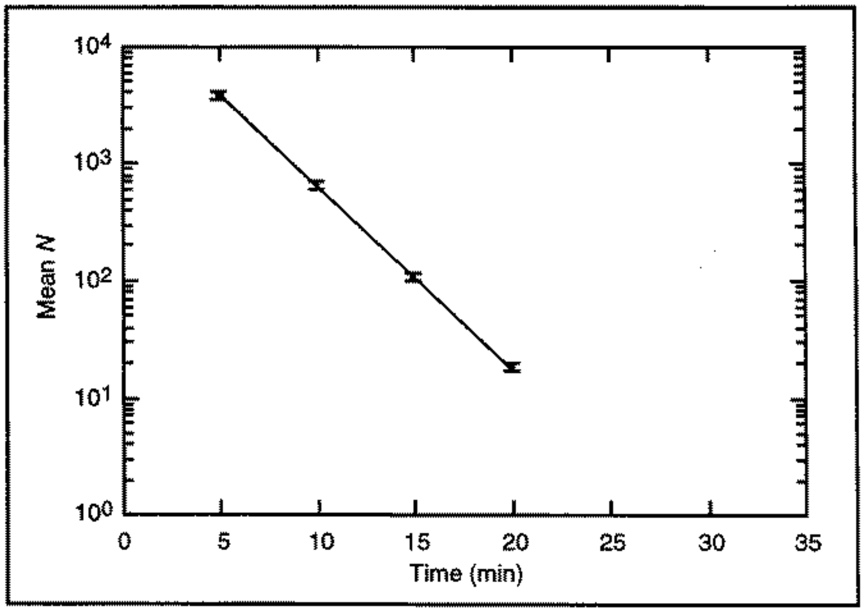
(c)

The non-linear model is better because the sum of squares of the residuals is smaller.

3.8 Non-linear model: calculation of parameters

(a), (b)

The proposed model equation has the same form as Eq. (3.15). Therefore, if the model is suitable, the data should give a straight line when plotted on semi-logarithmic coordinates.



(c), (d)
The equation for the straight line in the figure is:

$$y = 2.13 \times 10^4 \, e^{-0.353x}$$

where y is the number of viable cells and x is time in min. As the exponent must be dimensionless to preserve dimensional homogeneity (p 12), the dimensions of k_d are T^{-1}; therefore $k_d = 0.353 \text{ min}^{-1}$. The dimensions of N_0 are the same as N, i.e. N_0 is dimensionless and equal to 2.13×10^4.

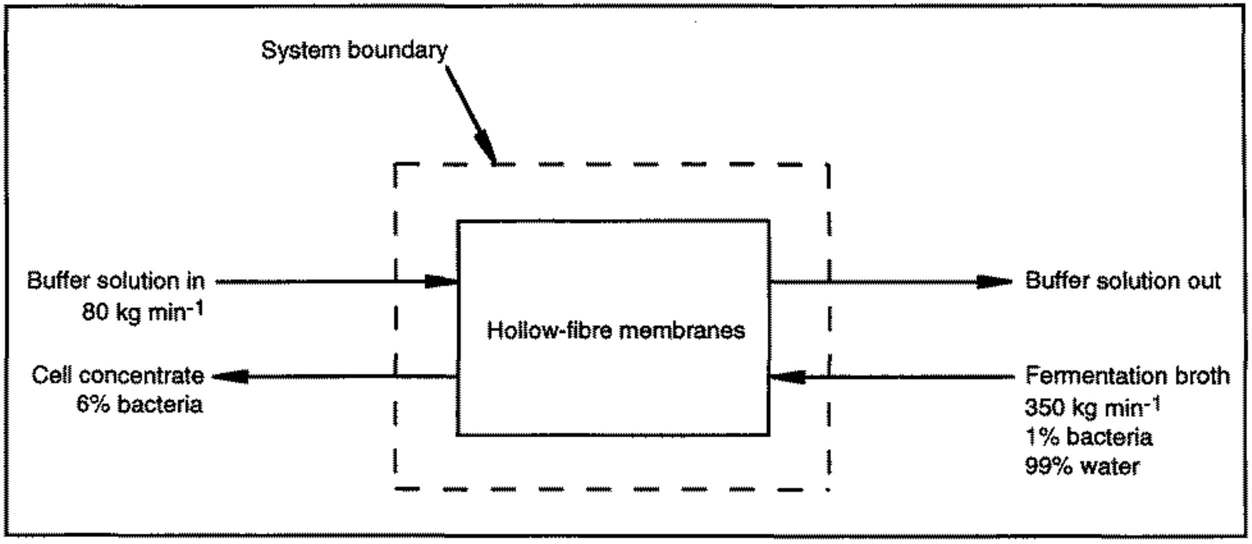
Answer: $k_d = 0.353 \text{ min}^{-1}$, $N_0 = 2.13 \times 10^4$; the dimensions of k_d are T^{-1}, N_0 is dimensionless

4

Material Balances

4.1 Cell concentration using membranes

1. *Assemble*
(i) *Flow sheet*



(ii) *System boundary*
The system boundary is shown on the flow sheet.
(iii) *Reaction equation*
No reaction occurs.

2. *Analyse*
(i) *Assumptions*
- steady state
- no leaks
- only water passes across the membrane

(ii) *Extra data*
No extra data are required.
(iii) *Basis*
1 min, or 350 kg fermentation broth
(iv) *Compounds involved in reaction*
No compounds are involved in reaction.
(v) *Mass-balance equation*
As there is no reaction, the appropriate mass-balance equation is Eq. (4.3):

$$\text{mass in} = \text{mass out}$$

3. *Calculate*
(i) *Calculation table*
The calculation table below shows all given quantities in kg. The total mass of cell concentrate is denoted C; the total mass of buffer out is denoted B. The columns for water refer to water originating in the fermentation broth.

Stream	*In*				*Out*			
	Water	*Bacteria*	*Buffer*	*Total*	*Water*	*Bacteria*	*Buffer*	*Total*
Fermentation broth	346.5	3.5	0	350	–	–	–	–
Buffer solution in	0	0	80	80	–	–	–	–
Cell concentrate	–	–	–	–	?	0.06 C	0	C
Buffer solution out	–	–	–	–	?	0	80	B
Total	346.5	3.5	80	430	?	0.06 C	80	$C + B$

(ii) *Mass-balance calculations*
Bacteria balance

$$3.5 \text{ kg bacteria in } = 0.06\, C \text{ kg bacteria out}$$

$$C = 58.3 \text{ kg}$$

Total mass balance

$$430 \text{ kg total mass in } = (C + B) \text{ kg total mass out}$$

Using the result for C:

$$B = 371.7 \text{ kg}$$

Water balance

$$346.5 \text{ kg water in } = \text{ water out}$$

$$\text{Water out } = 346.5 \text{ kg}$$

These calculations allow completion of the mass-balance table with all quantities in kg.

Stream	*In*				*Out*			
	Water	*Bacteria*	*Buffer*	*Total*	*Water*	*Bacteria*	*Buffer*	*Total*
Fermentation broth	346.5	3.5	0	350	–	–	–	–
Buffer solution in	0	0	80	80	–	–	–	–
Cell concentrate	–	–	–	–	54.8	3.50	0	58.3
Buffer solution out	–	–	–	–	291.7	0	80	371.7
Total	346.5	3.5	80	430	346.5	3.50	80	430

(iii) *Check the results*
All columns and rows of the completed table add up correctly.

4. *Finalise*

(a)
After rounding to three significant figures, the total flow rate of buffer solution out of the annulus is 372 kg min^{-1}.

Answer: 372 kg min^{-1}

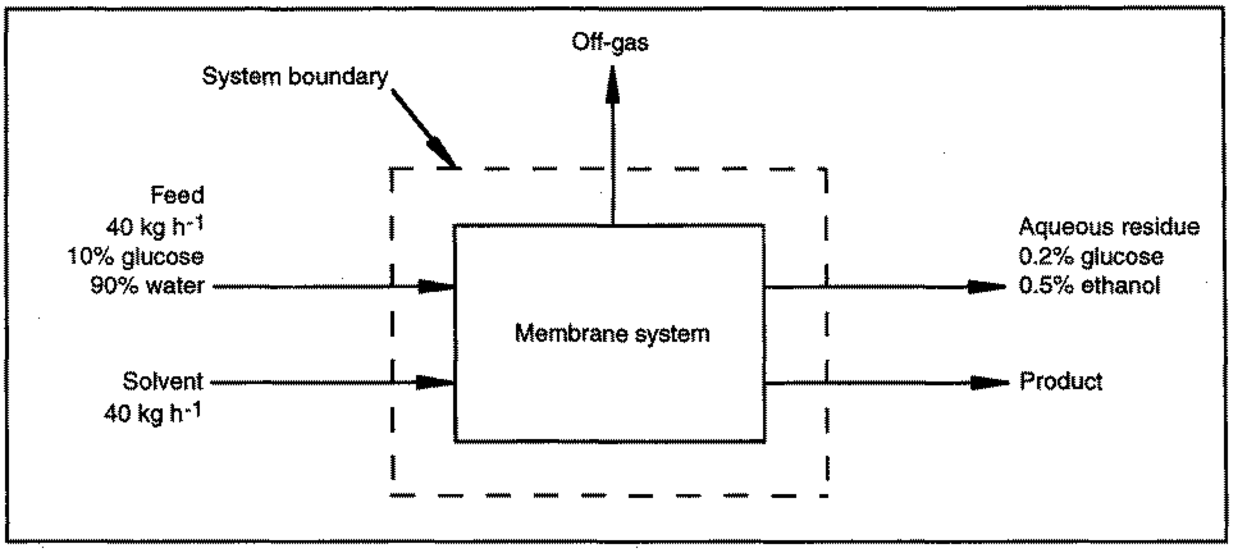
(b)
The total flow rate of cell concentrate from the membrane tubes is 58.3 kg min^{-1}.

Answer: 58.3 kg min^{-1}

4.2 Membrane reactor

1. *Assemble*

(i) *Flow sheet*



(ii) *System boundary*

The system boundary is shown on the flow sheet.

(iii) *Reaction equation*

$$C_6H_{12}O_6 \rightarrow 2\,C_2H_6O + 2\,CO_2$$

2. *Analyse*

(i) *Assumptions*

- steady state
- no leaks
- yeast cells do not grow or dislodge from the membrane
- no evaporation
- all CO_2 produced leaves in the off-gas
- no side reactions

(ii) *Extra data*

Molecular weights (Table B.1, Appendix B): glucose = 180.2
ethanol = 46.1
CO_2 = 44.0

(iii) *Basis*

1 h, or 40 kg feed solution

(iv) *Compounds involved in reaction*

Glucose, ethanol and carbon dioxide are involved in the reaction.

(v) *Mass-balance equations*

For glucose, ethanol and carbon dioxide, the appropriate mass-balance equation is Eq. (4.2):

$$\text{mass in} + \text{mass generated} = \text{mass out} + \text{mass consumed}$$

For water, solvent and total mass, the appropriate mass-balance equation is Eq. (4.3):

$$\text{mass in} = \text{mass out}$$

3. *Calculate*

(i) *Calculation table*

The calculation table below shows all given quantities in kg. The total mass of aqueous residue is denoted *R*; the total mass of product out is denoted *P*; the total mass of carbon dioxide out is denoted *G*.

Stream	*In*						*Out*					
	Glucose	*Ethanol*	*CO_2*	*Solvent*	*H_2O*	*Total*	*Glucose*	*Ethanol*	*CO_2*	*Solvent*	*H_2O*	*Total*
Feed	4	0	0	0	36	40	–	–	–	–	–	–
Solvent	0	0	0	40	0	40	–	–	–	–	–	–
Aqueous residue	–	–	–	–	–	–	$0.002\ R$	$0.005\ R$	0	0	?	R
Product	–	–	–	–	–	–	0	?	0	?	0	P
Off-gas	–	–	–	–	–	–	0	0	?	0	0	G
Total	4	0	0	40	36	80	$0.002\ R$	?	?	?	?	$R + P + G$

(ii) *Mass-balance calculations*

Solvent balance

Solvent is a tie component.

$$40 \text{ kg solvent in } = \text{ solvent out}$$

$$\text{Solvent out } = 40 \text{ kg}$$

Water balance

Water is a tie component.

$$36 \text{ kg water in } = \text{ water out}$$

$$\text{Water out } = 36 \text{ kg}$$

As water appears on the Out side of the table only in the aqueous residue stream:

$$0.002\ R + 0.005\ R + 36 \text{ kg} = R$$

$$R = 36.254 \text{ kg}$$

Therefore, the residual glucose in the aqueous residue stream = $0.002\ R = 0.073$ kg; the ethanol in the aqueous residue stream = $0.005\ R = 0.181$ kg.

Glucose balance

$$4 \text{ kg glucose in} + 0 \text{ kg glucose generated } = 0.073 \text{ kg glucose out} + \text{glucose consumed}$$

$$\text{Glucose consumed } = 3.927 \text{ kg}$$

Converting the glucose consumed to molar terms:

$$3.927 \text{ kg glucose} = 3.927 \text{ kg} \cdot \left| \frac{1 \text{ kgmol}}{180.2 \text{ kg}} \right| = 0.0218 \text{ kgmol}$$

From the reaction stoichiometry, conversion of this amount of glucose generates 2 × 0.0218 = 0.0436 kgmol ethanol and 2 × 0.0218 = 0.0436 kgmol CO_2. Converting these molar quantities to mass:

$$0.0436 \text{ kgmol ethanol} = 0.0436 \text{ kgmol} \cdot \left| \frac{46.1 \text{ kg}}{1 \text{ kgmol}} \right| = 2.010 \text{ kg}$$

$$0.0436 \text{ kgmol } CO_2 = 0.0436 \text{ kgmol} \cdot \left| \frac{44.0 \text{ kg}}{1 \text{ kgmol}} \right| = 1.92 \text{ kg}$$

CO_2 balance

$$0 \text{ kg } CO_2 \text{ in} + 1.92 \text{ kg } CO_2 \text{ generated } = CO_2 \text{ out} + 0 \text{ kg } CO_2 \text{ consumed}$$

$$CO_2 \text{ out } = 1.92 \text{ kg} = G$$

Ethanol balance

$$0 \text{ kg ethanol in} + 2.010 \text{ kg ethanol generated } = \text{ ethanol out} + 0 \text{ kg ethanol consumed}$$

$$\text{Ethanol out } = 2.010 \text{ kg}$$

Ethanol leaves the system only in the product and aqueous residue streams. Therefore:

$$\text{Ethanol out in the product stream} = (2.010 - 0.181)\text{ kg} = 1.829\text{ kg}$$

As the product stream consists of ethanol and solvent only:

$$P = (1.829 + 40)\text{ kg} = 41.829\text{ kg}$$

These calculations allow completion of the mass-balance table with all quantities in kg.

Stream	*In*						*Out*					
	Glucose	*Ethanol*	*CO_2*	*Solvent*	*H_2O*	*Total*	*Glucose*	*Ethanol*	*CO_2*	*Solvent*	*H_2O*	*Total*
Feed	4	0	0	0	36	40	–	–	–	–	–	–
Solvent	0	0	0	40	0	40	–	–	–	–	–	–
Aqueous residue	–	–	–	–	–	–	0.073	0.181	0	0	36	36.254
Product	–	–	–	–	–	–	0	1.829	0	40	0	41.829
Off-gas	–	–	–	–	–	–	0	0	1.92	0	0	1.92
Total	4	0	0	40	36	80	0.073	2.010	1.92	40	36	80.00

(iii) *Check the results*
All columns and rows of the completed table add up correctly.

4. *Finalise*

(a)
1.829 kg ethanol are contained in 41.829 kg of product stream. The ethanol concentration is therefore 1.829/41.829 × 100% = 4.4%.

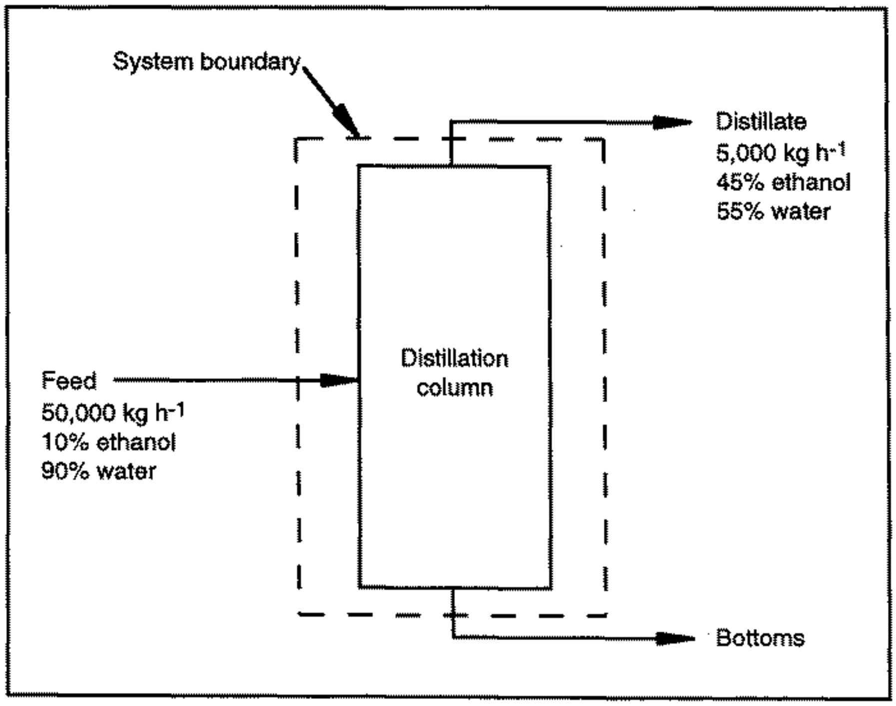
Answer: 4.4%

(b)
The mass flow rate of CO_2 is 1.92 kg h^{-1}.

Answer: 1.92 kg h^{-1}

4.3 Ethanol distillation

1. *Assemble*
(i) *Flow sheet*



(ii) *System boundary*
The system boundary is shown on the flow sheet.
(iii) *Reaction equation*
No reaction occurs.

2. *Analyse*
(i) *Assumptions*
– steady state
– no leaks
(ii) *Extra data*
No extra data are required.
(iii) *Basis*
1 h, or 50,000 kg feed
(iv) *Compounds involved in reaction*
No compounds are involved in reaction.
(v) *Mass-balance equation*
As there is no reaction, the appropriate mass-balance equation is Eq. (4.3):

$$\text{mass in} = \text{mass out}$$

3. *Calculate*
(i) *Calculation table*
The calculation table shows all given quantities in kg.

Stream	*In*			*Out*		
	Ethanol	*Water*	*Total*	*Ethanol*	*Water*	*Total*
Feed	5,000	45,000	50,000	–	–	–
Distillate	–	–	–	2,250	2,750	5,000
Bottoms	–	–	–	?	?	?
Total	5,000	45,000	50,000	?	?	?

(ii) *Mass-balance calculations*
Total mass balance

$$50{,}000 \text{ kg total mass in} = \text{total mass out}$$

$$\text{Total mass out} = 50{,}000 \text{ kg}$$

Therefore, from the total column on the Out side of the table:

$$\text{Bottoms out} = (50{,}000 - 5{,}000) \text{ kg} = 45{,}000 \text{ kg}$$

Ethanol balance

$$5{,}000 \text{ kg ethanol in} = \text{ethanol out}$$

$$\text{Ethanol out} = 5{,}000 \text{ kg}$$

From the ethanol column of the Out side of the table:

$$\text{Ethanol out in the bottoms} = (5{,}000 - 2{,}250) \text{ kg} = 2{,}750 \text{ kg}$$

Water balance

$$45{,}000 \text{ kg water in} = \text{water out}$$

$$\text{Water out} = 45{,}000 \text{ kg}$$

From the water column of the Out side of the table:

$$\text{Water out in the bottoms} = (45{,}000 - 2{,}750) \text{ kg} = 42{,}250 \text{ kg}$$

These calculations allow completion of the mass-balance table with all quantities in kg.

Stream	*In*			*Out*		
	Ethanol	*Water*	*Total*	*Ethanol*	*Water*	*Total*
Feed	5,000	45,000	50,000	–	–	–
Distillate	–	–	–	2,250	2,750	5,000
Bottoms	–	–	–	2,750	42,250	45,000
Total	5,000	45,000	50,000	5,000	45,000	50,000

(iii) *Check the results*
All columns and rows of the completed table add up correctly.

4. *Finalise*
(a)
The bottoms contains 2,750 kg ethanol and 42,250 kg water in a total of 45,000 kg. Therefore, the composition is 2,750/45,000 × 100% = 6.1% ethanol, and 42,250/45,000 × 100% = 93.9% water.

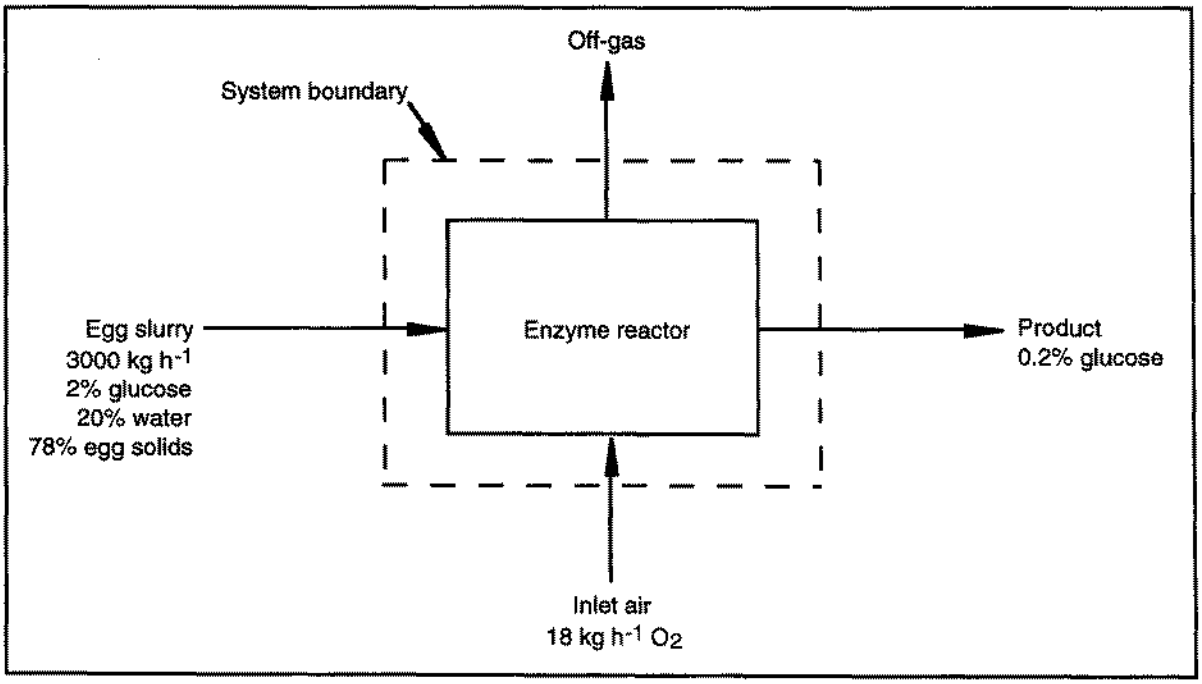
Answer: 6.1% ethanol, 93.9% water

(b)
Directly from the table, the rate of alcohol loss in the bottoms is 2,750 kg h^{-1}.

Answer: 2,750 kg h^{-1}

4.4 Removal of glucose from dried egg

1. *Assemble*
(i) *Flow sheet*



(ii) *System boundary*
The system boundary is shown on the flow sheet.
(iii) *Reaction equation*

$$C_6H_{12}O_6 + O_2 + H_2O \rightarrow C_6H_{12}O_7 + H_2O_2$$

2. *Analyse*
(i) *Assumptions*
– steady state
– no leaks
– air and off-gas are dry
– gases are at low pressure so vol% = mol%
– H_2O_2 remains in the liquid phase

(ii) *Extra data*

Molecular weights (Table B.1, Appendix B): glucose = 180.2
O_2 = 32.0
N_2 = 28.0
H_2O = 18.0
gluconic acid = 196.2
H_2O_2 = 34.0

Composition of air (p 17): 21% O_2, 79% N_2 by volume

(iii) *Basis*

1 h, or 3000 kg egg slurry

(iv) *Compounds involved in reaction*

Glucose, O_2, water, gluconic acid and H_2O_2 are involved in the reaction.

(v) *Mass-balance equations*

For glucose, O_2, water, gluconic acid and H_2O_2, the appropriate mass-balance equation is Eq. (4.2):

$$\text{mass in} + \text{mass generated} = \text{mass out} + \text{mass consumed}$$

For egg solids, N_2 and total mass, the appropriate mass-balance equation is Eq. (4.3):

$$\text{mass in} = \text{mass out}$$

3. *Calculate*

(i) *Calculation table*

The mass of N_2 accompanying 18 kg O_2 in air can be calculated from the known composition of air. Converting 18 kg O_2 to molar units:

$$18\,\text{kg}\,O_2 = 18\,\text{kg}\,O_2 \cdot \left|\frac{1\,\text{kgmol}}{32.0\,\text{kg}}\right| = 0.563\,\text{kgmol}\,O_2$$

Therefore, $79/21 \times 0.563$ kgmol = 2.118 kgmol N_2 enter in the air stream. Converting this to mass units:

$$2.118\,\text{kgmol}\,N_2 = 2.118\,\text{kgmol}\,N_2 \cdot \left|\frac{28.0\,\text{kg}}{1\,\text{kgmol}}\right| = 59.30\,\text{kg}\,N_2$$

The calculation tables below show all known quantities in kg. The total mass of off-gas is denoted *G*; the total mass of product is denoted *P*. The In side of the mass-balance table is complete.

Stream	*In*							
	Glucose	*Water*	*Egg solids*	*O_2*	*N_2*	*Gluconic acid*	*H_2O_2*	*Total*
Egg slurry	60	600	2340	0	0	0	0	3000
Air	0	0	0	18	59.30	0	0	77.3
Off-gas	–	–	–	–	–	–	–	–
Product	–	–	–	–	–	–	–	–
Total	60	600	2340	18	59.30	0	0	3077.3

Stream	*Out*							
	Glucose	*Water*	*Egg solids*	*O_2*	*N_2*	*Gluconic acid*	*H_2O_2*	*Total*
Egg slurry	–	–	–	–	–	–	–	–
Air	–	–	–	–	–	–	–	–
Off-gas	0	0	0	?	?	0	0	*G*
Product	0.002 *P*	?	?	0	0	?	?	*P*
Total	0.002 *P*	?	?	?	?	?	?	*G* + *P*

(ii) *Mass-balance calculations*

Egg solids balance

Egg solids is a tie component.

$$2340\,\text{kg egg solids in} = \text{egg solids out}$$

$$\text{Egg solids out} = 2340\,\text{kg}$$

N_2 balance

N_2 is a tie component.

$$59.30 \text{ kg } N_2 \text{ in } = N_2 \text{ out}$$

$$N_2 \text{ out } = 59.30 \text{ kg}$$

Glucose balance

$$60 \text{ kg glucose in} + 0 \text{ kg glucose generated} = 0.002\, P \text{ kg glucose out} + \text{glucose consumed}$$

$$\text{Glucose consumed} = (60 - 0.002\, P) \text{ kg}$$

Converting the glucose consumed to molar terms:

$$\text{Glucose consumed} = (60 - 0.002\, P) \text{ kg} \cdot \left| \frac{1 \text{ kgmol}}{180.2 \text{ kg}} \right| = \frac{(60 - 0.002\, P)}{180.2} \text{ kgmol}$$

$$\text{Glucose consumed} = \left(0.333 - 1.11 \times 10^{-5}\, P\right) \text{kgmol}$$

From the reaction stoichiometry, conversion of this amount of glucose requires the same number of kgmol O_2. Converting this molar quantity to mass of O_2:

$$\left(0.333 - 1.11 \times 10^{-5}\, P\right) \text{kgmol } O_2 = \left(0.333 - 1.11 \times 10^{-5}\, P\right) \text{kgmol} \cdot \left| \frac{32.0 \text{ kg}}{1 \text{ kgmol}} \right| = \left(10.656 - 3.552 \times 10^{-4}\, P\right) \text{kg } O_2$$

O_2 balance

$$18 \text{ kg } O_2 \text{ in} + 0 \text{ kg } O_2 \text{ generated} = O_2 \text{ out} + (10.656 - 3.552 \times 10^{-4}\, P) \text{ kg } O_2 \text{ consumed}$$

$$O_2 \text{ out} = (18 - (10.656 - 3.552 \times 10^{-4}\, P)) \text{ kg}$$

$$O_2 \text{ out} = (7.344 + 3.552 \times 10^{-4}\, P) \text{ kg}$$

Adding this mass of O_2 to the mass of N_2 in the off-gas:

$$G = 59.30 + (7.344 + 3.552 \times 10^{-4}\, P) \text{ kg}$$

$$G = (66.64 + 3.552 \times 10^{-4}\, P) \text{ kg}$$

Total mass balance

$$3077.3 \text{ kg total mass in} = (G + P) \text{ kg total mass out}$$

Substituting the expression for G into the total mass balance:

$$3077.3 \text{ kg} = (66.64 + 3.552 \times 10^{-4}\, P + P) \text{ kg}$$

$$3010.7 \text{ kg} = 1.0004\, P \text{ kg}$$

$$P = 3009.6 \text{ kg}$$

Therefore, from the oxygen balance:

$$G = (66.64 + 3.552 \times 10^{-4} \times 3009.6) \text{ kg}$$

$$G = 67.71 \text{ kg}$$

and:

$$O_2 \text{ out} = (7.344 + 3.552 \times 10^{-4} \times 3009.6) \text{ kg}$$

$$O_2 \text{ out} = 8.41 \text{ kg}$$

The mass of glucose out is 0.002 × 3009.6 = 6.02 kg. The moles of glucose consumed is:

$$\text{Glucose consumed} = (0.333 - 1.11 \times 10^{-5} \times 3009.6) \text{ kgmol} = 0.300 \text{ kgmol}$$

Therefore, from stoichiometry and the molecular weights:

$$\text{Water consumed} = 0.300 \text{ kgmol} \cdot \left| \frac{18.0 \text{ kg}}{1 \text{ kgmol}} \right| = 5.40 \text{ kg}$$

$$\text{Gluconic acid generated} = 0.300\,\text{kgmol}\,.\left|\frac{196.2\,\text{kg}}{1\,\text{kgmol}}\right| = 58.86\,\text{kg}$$

$$H_2O_2\ \text{generated} = 0.300\,\text{kgmol}\,.\left|\frac{34\,\text{kg}}{1\,\text{kgmol}}\right| = 10.20\,\text{kg}$$

Water balance

$$600\ \text{kg water in} + 0\ \text{kg water generated} = \text{water out} + 5.40\ \text{kg water consumed}$$

$$\text{Water out} = 594.6\ \text{kg}$$

Gluconic acid balance

$$0\ \text{kg gluconic acid in} + 58.86\ \text{kg gluconic acid generated} = \text{gluconic acid out} + 0\ \text{kg gluconic acid consumed}$$

$$\text{Gluconic acid out} = 58.86\ \text{kg}$$

H_2O_2 balance

$$0\ \text{kg}\ H_2O_2\ \text{in} + 10.20\ \text{kg}\ H_2O_2\ \text{generated} = H_2O_2\ \text{out} + 0\ \text{kg}\ H_2O_2\ \text{consumed}$$

$$H_2O_2\ \text{out} = 10.20\ \text{kg}$$

These calculations allow completion of the Out side of the mass-balance table with all quantities in kg.

Stream	*Out*							
	Glucose	*Water*	*Egg solids*	*O_2*	*N_2*	*Gluconic acid*	*H_2O_2*	*Total*
Egg slurry	–	–	–	–	–	–	–	–
Air	–	–	–	–	–	–	–	–
Off-gas	0	0	0	8.41	59.30	0	0	67.71
Product	6.02	594.6	2340	0	0	58.86	10.20	3009.6
Total	6.02	594.6	2340	8.41	59.30	58.86	10.20	3077.3

(iii) *Check the results*
All columns and rows of the completed table add up correctly to within round-off error.

4. *Finalise*

(a)
To determine which is the limiting substrate, the number of moles available of each substrate involved in the reaction must be determined. From the mass-balance table for streams in:

$$\text{Moles glucose} = 60\,\text{kg}\,.\left|\frac{1\,\text{kg mol}}{180.2\,\text{kg}}\right| = 0.333\,\text{kgmol}$$

$$\text{Moles water} = 600\,\text{kg}\,.\left|\frac{1\,\text{kg mol}}{18.0\,\text{kg}}\right| = 33.3\,\text{kgmol}$$

$$\text{Moles}\ O_2 = 18\,\text{kg}\,.\left|\frac{1\,\text{kg mol}}{32.0\,\text{kg}}\right| = 0.563\,\text{kgmol}$$

As the substrates are required in the molar stoichiometric ratio of 1:1:1 and glucose is available in the smallest molar quantity, the extent of the reaction must be limited by glucose.

Answer: Glucose

(b)
Water and O_2 are available in excess. As only 0.333 kgmol of each will be used if the reaction proceeds to completion, from Eq. (2.34):

$$\%\ \text{excess water} = \frac{(33.3-0.333)\,\text{kgmol}}{0.333\,\text{kgmol}} \times 100\% = 9900\%$$

$$\%\ \text{excess}\ O_2 = \frac{(0.563-0.333)\,\text{kgmol}}{0.333\,\text{kgmol}} \times 100\% = 69\%$$

Answer: 9900% excess water; 69% excess O_2

(c)
From the completed mass-balance table, the reactor off-gas contains 8.41 kg O_2 and 59.30 kg N_2. As gas compositions are normally expressed in molar or volumetric terms (p 17), these mass values must be converted to moles:

$$8.41\,\text{kg}\,O_2 = 8.41\,\text{kg}\,O_2 \cdot \left|\frac{1\,\text{kgmol}}{32.0\,\text{kg}}\right| = 0.263\,\text{kgmol}\,O_2$$

As the number of kgmol N_2 was determined in the preliminary calculations to be 2.118, the total number of moles of off-gas is (0.263 + 2.118) = 2.381 kgmol. Therefore, the composition of the off-gas is $0.263/2.381 = 0.11$ O_2, and $2.118/2.381 = 0.89$ N_2.

Answer: 0.11 O_2, 0.89 N_2

(d)
From the completed mass-balance table, the product stream has a total mass of 3009.6 kg and contains 6.02 kg glucose, 594.6 kg water, 2340 kg egg solids, 58.86 kg gluconic acid and 10.20 kg H_2O_2. Therefore, the composition is:

$$\frac{6.02}{3009.6} = 0.002\,\text{glucose}$$

$$\frac{594.6}{3009.6} = 0.198\,\text{water}$$

$$\frac{2340}{3009.6} = 0.778\,\text{egg solids}$$

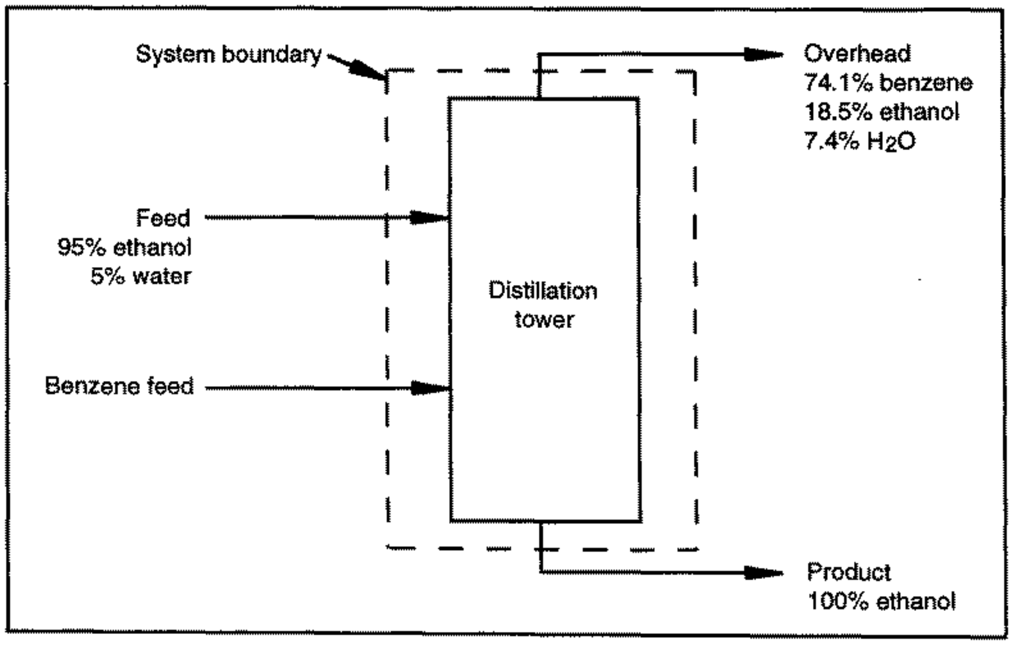
$$\frac{58.86}{3009.6} = 0.020\,\text{gluconic acid}$$

$$\frac{10.20}{3009.6} = 0.003\,H_2O_2$$

Answer: 0.002 glucose, 0.198 water, 0.778 egg solids, 0.020 gluconic acid, 0.003 H_2O_2

4.5 Azeotropic distillation

1. *Assemble*
(i) *Flow sheet*



(ii) *System boundary*
The system boundary is shown on the flow sheet.

(iii) *Reaction equation*
No reaction occurs.

2. *Analyse*
(i) *Assumptions*
- steady state
- no leaks

(ii) *Extra data*
$1000 \text{ cm}^3 = 1 \text{ l}$
$1000 \text{ g} = 1 \text{ kg}$
(iii) *Basis*
250 l ethanol product
(iv) *Compounds involved in reaction*
No compounds are involved in reaction.
(v) *Mass-balance equation*
As there is no reaction, the appropriate mass-balance equation is Eq. (4.3):

$$\text{mass in} = \text{mass out}$$

3. *Calculate*
(i) *Calculation table*
As all quantities in mass-balance calculations must be masses (rather than volumes), 250 l absolute ethanol must first be converted to mass. From the definition of density on p 16, mass is equal to volume multiplied by density:

$$250 \text{ l absolute ethanol} = 250 \text{ l} \times 0.785 \text{ g cm}^{-3} \cdot \left|\frac{1000 \text{ cm}^3}{1 \text{ l}}\right| \cdot \left|\frac{1 \text{ kg}}{1000 \text{ g}}\right| = 196.25 \text{ kg}$$

The calculation table shows all given quantities in kg. The total mass of feed in is denoted F; the total mass of benzene feed in is denoted B; the total mass of overhead out is denoted V.

Stream	*In*				*Out*			
	Ethanol	*Water*	*Benzene*	*Total*	*Ethanol*	*Water*	*Benzene*	*Total*
Feed	0.95 F	0.05 F	0	F	–	–	–	–
Benzene feed	0	0	B	B	–	–	–	–
Product	–	–	–	–	196.25	0	0	196.25
Overhead	–	–	–	–	0.185 V	0.074 V	0.741 V	V
Total	0.95 F	0.05 F	B	$F + B$	196.25 + 0.185 V	0.074 V	0.741 V	196.25 + V

(ii) *Mass-balance calculations*
Ethanol balance

$$0.95\ F \text{ kg ethanol in} = (196.25 + 0.185\ V) \text{ kg ethanol out}$$

$$F = (206.58 + 0.195\ V) \text{ kg}$$

Benzene balance

$$B \text{ kg benzene in} = 0.741\ V \text{ kg benzene out}$$

$$B = 0.741\ V$$

Total mass balance

$$(F + B) \text{ kg total mass in} = (196.25 + V) \text{ kg total mass out}$$

Substituting for F and B from the ethanol and benzene balances:

$$(206.58 + 0.195\ V + 0.741\ V) \text{ kg} = (196.25 + V) \text{ kg}$$

$$10.33 = 0.064\ V$$

$$V = 161.4 \text{ kg}$$

Using this result in the ethanol and benzene balances gives:

$$F = 238.1 \text{ kg}$$

$$B = 119.6 \text{ kg}$$

These calculations allow completion of the mass-balance table with all quantities in kg.

Stream	*In*				*Out*			
	Ethanol	*Water*	*Benzene*	*Total*	*Ethanol*	*Water*	*Benzene*	*Total*
Feed	226.2	11.9	0	238.1	–	–	–	–
Benzene feed	0	0	119.6	119.6	–	–	–	–
Product	–	–	–	–	196.25	0	0	196.25
Overhead	–	–	–	–	29.9	11.9	119.6	161.4
Total	226.2	11.9	119.6	357.7	226.2	11.9	119.6	357.7

(iii) *Check the results*
All columns and rows of the completed table add up correctly to within round-off error.

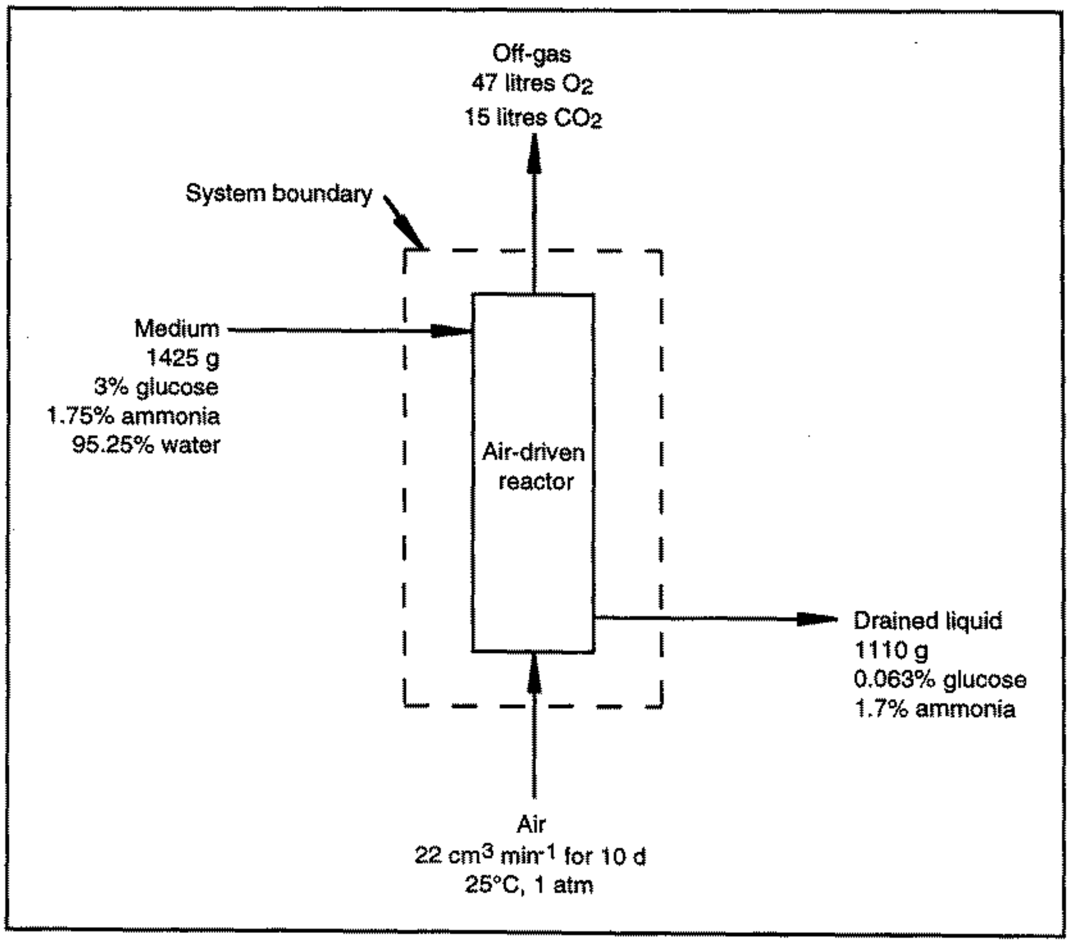
4. *Finalise*
From the completed mass-balance table, the mass of benzene required is 119.6 kg. Using the definition of density on p 16, volume is equal to mass divided by density:

$$\text{Volume of benzene} = \frac{119.6 \text{ kg}}{0.872 \text{ g cm}^{-3}} \cdot \left|\frac{1000 \text{ g}}{1 \text{ kg}}\right| \cdot \left|\frac{1 \text{ l}}{1000 \text{ cm}^3}\right| = 137 \text{ l}$$

Answer: 137 litres

4.6 Culture of plant roots

1. *Assemble*
(i) *Flow sheet*



(ii) *System boundary*
The system boundary is shown on the flow sheet.

(iii) *Reaction equation*
From Table B.2 (Appendix B), the molecular formula for glucose is $C_6H_{12}O_6$. The reaction equation is based on the general stoichiometric equation for aerobic growth, Eq. (4.4):

$$C_6H_{12}O_6 + a\,O_2 + b\,NH_3 \rightarrow c\,CH_\alpha O_\beta N_\delta + d\,CO_2 + e\,H_2O$$

2. *Analyse*
(i) *Assumptions*
- steady state
- no leaks
- air and off-gas are dry
- all the CO_2 produced leaves in the off-gas
- gases are at low pressure so vol% = mol%

(ii) *Extra data*
1 l = 1000 cm^3
Molecular weights (Table B.1, Appendix B): glucose = 180.2
O_2 = 32.0
N_2 = 28.0
NH_3 = 17.0
CO_2 = 44.0
H_2O = 18.0
Composition of air (p 17): 21% O_2, 79% N_2 by volume
Ideal gas constant (Table 2.5): $R = 82.057\ cm^3\ atm\ K^{-1}\ gmol^{-1}$
(iii) *Basis*
10 d, or 1425 g nutrient medium
(iv) *Compounds involved in reaction*
Glucose, O_2, NH_3, biomass, CO_2 and H_2O are involved in the reaction.
(v) *Mass-balance equations*
For glucose, O_2, NH_3, biomass, CO_2 and H_2O, the appropriate mass-balance equation is Eq. (4.2):

$$\text{mass in} + \text{mass generated} = \text{mass out} + \text{mass consumed}$$

For N_2 and total mass, the appropriate mass-balance equation is Eq. (4.3):

$$\text{mass in} = \text{mass out}$$

3. *Calculate*
(i) *Calculation table*
Over 10 d, the volume of air sparged into the fermenter is:

$$\text{Volume of air in} = 22\ cm^3\ min^{-1} \times 10\ d\,.\left|\frac{60\ min}{1\ h}\right|.\left|\frac{24\ h}{1\ d}\right| = 3.168 \times 10^5\ cm^3$$

Converting this gas volume to moles using the ideal gas law, Eq. (2.32), with the temperature converted from °C to degrees Kelvin using Eq. (2.24):

$$\text{Moles of air in} = n = \frac{pV}{RT} = \frac{1\ atm\left(3.168 \times 10^5\ cm^3\right)}{82.057\ cm^3\ atm\ K^{-1}\ gmol^{-1}\ (25 + 273.15)\ K} = 12.95\ gmol$$

From the known composition of air, the moles of O_2 in the incoming air is 0.21 × 12.95 = 2.72 gmol, and the moles of N_2 is 0.79 × 12.95 = 10.23 gmol. Converting these values to masses:

$$\text{Mass of } O_2 \text{ in} = 2.72\ gmol\,.\left|\frac{32.0\ g}{1\ gmol}\right| = 87.04\ g$$

$$\text{Mass of } N_2 \text{ in} = 10.23\ gmol\,.\left|\frac{28.0\ g}{1\ gmol}\right| = 286.4\ g$$

The total mass of air in is therefore (87.04 + 286.4) g = 373.44 g.

The gas volumes in the off-gas must also be converted to masses. First, convert the volumes of O_2 and CO_2 to moles using the ideal gas law, Eq. (2.32), with the temperature converted from °C to degrees Kelvin using Eq. (2.24):

$$\text{Moles of } O_2 \text{ out} = n = \frac{p\,V}{R\,T} = \frac{1\text{ atm}\,(47\text{ litres})\,.\left|\frac{1000\text{ cm}^3}{1\text{ l}}\right|}{82.057\text{ cm}^3\text{ atm K}^{-1}\text{ gmol}^{-1}\,(25+273.15)\text{ K}} = 1.92\text{ gmol}$$

$$\text{Moles of } CO_2 \text{ out} = n = \frac{p\,V}{R\,T} = \frac{1\text{ atm}\,(15\text{ litres})\,.\left|\frac{1000\text{ cm}^3}{1\text{ l}}\right|}{82.057\text{ cm}^3\text{ atm K}^{-1}\text{ gmol}^{-1}\,(25+273.15)\text{ K}} = 0.613\text{ gmol}$$

Calculate the corresponding masses:

$$\text{Mass of } O_2 \text{ out} = 1.92\text{ gmol}\,.\left|\frac{32.0\text{ g}}{1\text{ gmol}}\right| = 61.44\text{ g}$$

$$\text{Mass of } CO_2 \text{ out} = 0.613\text{ gmol}\,.\left|\frac{44.0\text{ g}}{1\text{ gmol}}\right| = 26.97\text{ g}$$

The calculation tables below show all known quantities in g. The In side of the mass-balance table is complete. The total mass of off-gas out is denoted *G*; the total biomass harvested is denoted *B*. As the ratio of biomass fresh weight to dry weight is 14:1, dry biomass comprises $^1/_{15}$ = 0.0667 of the total biomass. Because this problem requires an integral mass balance, the biomass remaining in the fermenter after 10 d culture must also be included in the table even though it is not contained in any of the streams flowing into or out of the vessel.

Stream	*In*							
	Glucose	*O_2*	*N_2*	*NH_3*	*Dry biomass*	*CO_2*	*H_2O*	*Total*
Medium	42.75	0	0	24.94	0	0	1357.31	1425
Air	0	87.04	286.4	0	0	0	0	373.44
Drained liquid	–	–	–	–	–	–	–	–
Off-gas	–	–	–	–	–	–	–	–
Harvested biomass	–	–	–	–	–	–	–	–
Total	42.75	87.04	286.4	24.94	0	0	1357.31	1798.44

Stream	*Out*							
	Glucose	*O_2*	*N_2*	*NH_3*	*Dry biomass*	*CO_2*	*H_2O*	*Total*
Medium	–	–	–	–	–	–	–	–
Air	–	–	–	–	–	–	–	–
Drained liquid	0.699	0	0	18.87	0	0	1090.43	1110
Off-gas	0	61.44	?	0	0	26.97	0	*G*
Harvested biomass	0	0	0	0	0.0667 *B*	0	0.9333 *B*	*B*
Total	0.699	61.44	?	18.87	0.0667 *B*	26.97	1090.43 + 0.9333 *B*	1110 + *G* + *B*

(ii) *Mass-balance calculations*
N_2 balance
N_2 is a tie component.

$$286.4\text{ g } N_2 \text{ in} = N_2 \text{ out}$$

$$N_2 \text{ out} = 286.4\text{ g}$$

Using this result and adding up the row for the off-gas in the out table:

$$G = 61.44 + 286.4 + 26.97 = 374.81\text{ g}$$

Total mass balance

$$1798.44\text{ g total mass in} = (1110 + G + B)\text{ g total mass out}$$

Using the result for *G*:

$$B = 313.63 \text{ g}$$

Therefore, the dry biomass produced is 0.0667 × 313.63 = 20.92 g; the mass of water in the biomass is 0.9333 × 313.63 = 292.71 g.

These calculations allow completion of the Out side of the mass-balance table with all quantities in g.

Stream	*Out*							
	Glucose	*O_2*	*N_2*	*NH_3*	*Dry biomass*	*CO_2*	*H_2O*	*Total*
Medium	–	–	–	–	–	–	–	–
Air	–	–	–	–	–	–	–	–
Drained liquid	0.699	0	0	18.87	0	0	1090.43	1110
Off-gas	0	61.44	286.4	0	0	26.97	0	374.81
Harvested biomass	0	0	0	0	20.92	0	292.71	313.63
Total	0.699	61.44	286.4	18.87	20.92	26.97	1383.14	1798.44

Further mass-balance calculations allow evaluation of the masses of components consumed or generated in the reaction.

Glucose balance

$$42.75 \text{ g glucose in} + 0 \text{ g glucose generated} = 0.699 \text{ g glucose out} + \text{glucose consumed}$$

$$\text{Glucose consumed} = 42.05 \text{ g}$$

O_2 balance

$$87.04 \text{ g } O_2 \text{ in} + 0 \text{ g } O_2 \text{ generated} = 61.44 \text{ g } O_2 \text{ out} + O_2 \text{ consumed}$$

$$O_2 \text{ consumed} = 25.60 \text{ g}$$

NH_3 balance

$$24.94 \text{ g } NH_3 \text{ in} + 0 \text{ g } NH_3 \text{ generated} = 18.87 \text{ g } NH_3 \text{ out} + NH_3 \text{ consumed}$$

$$NH_3 \text{ consumed} = 6.07 \text{ g}$$

CO_2 balance

$$0 \text{ g } CO_2 \text{ in} + CO_2 \text{ generated} = 26.97 \text{ g } CO_2 \text{ out} + 0 \text{ g } CO_2 \text{ consumed}$$

$$CO_2 \text{ generated} = 26.97 \text{ g}$$

H_2O balance

$$1357.31 \text{ g } H_2O \text{ in} + H_2O \text{ generated} = (1090.43 + 0.9333\,B) \text{ g } H_2O \text{ out} + 0 \text{ g } H_2O \text{ consumed}$$

Substituting the value for *B* from the total mass balance:

$$H_2O \text{ generated} = 25.83 \text{ g}$$

(iii) *Check the results*

All columns and rows of the completed mass-balance table add up correctly.

4. *Finalise*

(a)

Rounding to three significant figures from the completed mass-balance table, the mass of dry roots produced is 20.9 g.

Answer: 20.9 g

(b)

To determine the stoichiometry, the calculated masses of components consumed or generated in the reaction must be converted to molar quantities:

$$\text{Moles of glucose consumed} = 42.05\ \text{g}\,.\left|\frac{1\ \text{gmol}}{180.2\ \text{g}}\right| = 0.233\ \text{gmol}$$

$$\text{Moles of } O_2 \text{ consumed} = 25.60\ \text{g}\,.\left|\frac{1\ \text{gmol}}{32.0\ \text{g}}\right| = 0.800\ \text{gmol}$$

$$\text{Moles of } NH_3 \text{ consumed} = 6.07\ \text{g}\,.\left|\frac{1\ \text{gmol}}{17.0\ \text{g}}\right| = 0.357\ \text{gmol}$$

$$\text{Moles of } CO_2 \text{ generated} = 26.97\ \text{g}\,.\left|\frac{1\ \text{gmol}}{44.0\ \text{g}}\right| = 0.613\ \text{gmol}$$

$$\text{Moles of } H_2O \text{ generated} = 25.83\ \text{g}\,.\left|\frac{1\ \text{gmol}}{18.0\ \text{g}}\right| = 1.435\ \text{gmol}$$

$$\text{Moles of biomass generated} = \frac{20.92\ \text{g}}{\text{MW biomass}}$$

The moles of biomass generated is not yet known explicitly because the molecular formula for the dry biomass is unknown. The above molar quantities can be used as coefficients in the reaction equation:

$$0.233\, C_6H_{12}O_6 + 0.800\, O_2 + 0.357\, NH_3 \rightarrow \frac{20.92}{\text{MW biomass}}\, CH_\alpha O_\beta N_\delta + 0.613\, CO_2 + 1.435\, H_2O$$

Dividing each coefficient by 0.233 to obtain the stoichiometry per gmol glucose:

$$C_6H_{12}O_6 + 3.43\, O_2 + 1.53\, NH_3 \rightarrow \frac{89.79}{\text{MW biomass}}\, CH_\alpha O_\beta N_\delta + 2.63\, CO_2 + 6.16\, H_2O$$

The values of α, β and δ and the molecular formula for the biomass can obtained using elemental balances.

C balance: $6 = \dfrac{89.79}{\text{MW biomass}} + 2.63$

Therefore:

$$\frac{89.79}{\text{MW biomass}} = 3.37$$

This result can be used in the remaining elemental balances for completion of the stoichiometric equation.

H balance: $12 + 3 \times 1.53 = 3.37\,\alpha + 2 \times 6.16 \rightarrow \alpha = 1.27$

O balance: $6 + 2 \times 3.43 = 3.37\,\beta + 2 \times 2.63 + 6.16 \rightarrow \beta = 0.43$

N balance: $1.53 = 3.37\,\delta \rightarrow \delta = 0.45$

Answer: The complete stoichiometric equation is:

$$C_6H_{12}O_6 + 3.43\, O_2 + 1.53\, NH_3 \rightarrow 3.37\, CH_{1.27}O_{0.43}N_{0.45} + 2.63\, CO_2 + 6.16\, H_2O$$

The chemical formula for the dry roots is $CH_{1.27}O_{0.43}N_{0.45}$.

(c)

Converting to moles the mass quantities of glucose, O_2 and NH_3 available for reaction on the In side of the mass-balance table:

$$\text{Moles of glucose in} = 42.75\ \text{g}\,.\left|\frac{1\ \text{gmol}}{180.2\ \text{g}}\right| = 0.24\ \text{gmol}$$

$$\text{Moles of } O_2 \text{ in} = 87.04\ \text{g}\,.\left|\frac{1\ \text{gmol}}{32.0\ \text{g}}\right| = 2.72\ \text{gmol}$$

$$\text{Moles of } NH_3 \text{ in} = 24.93\ \text{g}\,.\left|\frac{1\ \text{gmol}}{17.0\ \text{g}}\right| = 1.47\ \text{gmol}$$

From the stoichiometric equation, reaction of 0.24 gmol glucose requires $0.24 \times 3.43 = 0.82$ gmol O_2 and $0.24 \times 1.53 = 0.37$ gmol NH_3. As the molar quantities of O_2 and NH_3 available for reaction are in excess of these values, glucose must be the limiting substrate.

Answer: Glucose

(d)

The mass of glucose consumed is 42.05 g; the mass of dry biomass produced is 20.92 g. Therefore, the biomass yield from glucose is $20.92/42.05 = 0.50$ g g^{-1} dry weight.

Answer: 0.50 g g^{-1} dry weight

4.7 Oxygen requirement for growth on glycerol

From Table B.2 (Appendix B), the molecular formula for glycerol is $C_3H_8O_3$. From Table 4.3, the chemical formula for *Klebsiella aerogenes* can be taken as $CH_{1.75}O_{0.43}N_{0.22}$. Substituting these formulae into the general stoichiometric equation for growth, Eq. (4.4), gives:

$$C_3H_8O_3 + a\,O_2 + b\,NH_3 \rightarrow c\,CH_{1.75}O_{0.43}N_{0.22} + d\,CO_2 + e\,H_2O$$

From Table B.8 (Appendix B), the molecular weight of glycerol is 92.1. The biomass formula weight calculated from the atomic weights in Table B.1 (Appendix B) is 23.74. Taking into account the 8% ash:

$$\text{Biomass molecular weight} = \frac{23.74}{0.92} = 25.8$$

The value of the stoichiometric coefficient c can be determined from the yield $Y_{XS} = 0.4$ g g^{-1} and Eq. (4.12):

$$c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.40\text{ g g}^{-1}\left(92.1\text{ g gmol}^{-1}\right)}{25.8\text{ g gmol}^{-1}} = 1.43\text{ gmol gmol}^{-1}$$

From Table B.2 (Appendix B), the degree of reduction of glycerol relative to NH_3 is $\gamma_S = 4.67$. The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times4 + 1.75\times1 - 0.43\times2 - 0.22\times3}{1} = 4.23$$

which is also listed in Table 4.3. The theoretical oxygen demand can be determined from Eq. (4.16) with $f = 0$; from Eq. (4.13), $w = 3$ for glycerol:

$$a = {}^1\!/_4\,(w\,\gamma_S - c\,\gamma_B) = {}^1\!/_4\,(3\times4.67 - 1.43\times4.23) = 1.99$$

Therefore, 1.99 gmol oxygen are required per gmol glycerol. From the atomic weights in Table B.1 (Appendix B), the molecular weight of oxygen is 32.0. Converting a to mass terms:

$$a = 1.99\text{ gmol gmol}^{-1} = 1.99\text{ gmol gmol}^{-1}\cdot\left|\frac{32.0\text{ g}}{1\text{ gmol}}\right|\cdot\left|\frac{1\text{ gmol}}{92.1\text{ g}}\right| = 0.69\text{ g g}^{-1}$$

Answer: 0.69 g oxygen is required per g glycerol consumed

4.8 Product yield in anaerobic digestion

From Eq. (4.13), the stoichiometric equation for anaerobic growth and product formation by methane bacteria can be written as:

$$CH_3COOH + b\,NH_3 \rightarrow c\,CH_{1.4}O_{0.40}N_{0.20} + d\,CO_2 + e\,H_2O + f\,CH_4$$

From Table B.8 (Appendix B), the molecular weight of acetic acid is 60.1. From Table B.1 (Appendix B), the molecular weight of CO_2 is 44.0. The value of the stoichiometric coefficient d can be determined based on Eq. (4.14) with carbon dioxide as the product and the yield $Y_{PS} = 0.67$ kg kg^{-1} = 0.67 g g^{-1}:

$$d = \frac{Y_{PS}\,(\text{MW substrate})}{\text{MW CO}_2} = \frac{0.67\text{ g g}^{-1}\left(60.1\text{ g gmol}^{-1}\right)}{44.0\text{ g gmol}^{-1}} = 0.915\text{ gmol gmol}^{-1}$$

The other coefficients can be determined using this result and elemental balances.
C balance: $2 = c + d + f = c + 0.915 + f \rightarrow f = 1.085 - c$
H balance: $4 + 3\,b = 1.4\,c + 2\,e + 4\,f$
O balance: $2 = 0.40\,c + 2\,d + e = 0.40\,c + 2\times0.915 + e = 0.40\,c + 1.83 + e \rightarrow e = 0.17 - 0.40\,c$
N balance: $b = 0.20\,c$

Substituting the expressions for f, e and b from the C, O and N balances, respectively, into the H balance:

$$4 + 3 \times 0.20\,c = 1.4\,c + 2 \times (0.17 - 0.40\,c) + 4 \times (1.085 - c)$$

$$4\,c = 0.680$$

$$c = 0.170$$

Substituting this value for c into the expressions for the other coefficients gives $b = 0.034$, $e = 0.102$ and $f = 0.915$. The yield of methane is therefore 0.915 gmol per gmol acetic acid.

The maximum possible methane yield can be calculated using Eq. (4.20). From Eq. (4.13), $w = 2$ for acetic acid and $j = 1$ for methane. From Table B.2 (Appendix B), the degree of reduction of acetic acid relative to NH_3 is $\gamma_S = 4.00$, and the degree of reduction of methane relative to NH_3 is $\gamma_P = 8.00$. Substituting these values into Eq. (4.20) gives:

$$f_{max} = \frac{w\,\gamma_S}{j\,\gamma_P} = \frac{2\,(4.00)}{1\,(8.00)} = 1.0\ \text{gmol gmol}^{-1}$$

The actual methane yield of 0.915 gmol gmol^{-1} therefore represents 91.5% of the theoretical maximum.

Answer: 91.5% of the theoretical maximum

4.9 Stoichiometry of single-cell protein synthesis

(a)

From Table B.2 (Appendix B), the molecular formula for glucose is $C_6H_{12}O_6$. If all carbon in the substrate is converted into biomass, production of carbon dioxide is zero. Therefore, from Eq. (4.4), the stoichiometric equation for anaerobic growth of *Cellulomonas* is:

$$C_6H_{12}O_6 + b\,NH_3 \rightarrow c\,CH_{1.56}O_{0.54}N_{0.16} + e\,H_2O$$

The stoichiometric coefficients can be determined using elemental balances.
C balance: $6 = c$
H balance: $12 + 3\,b = 1.56\,c + 2\,e$
O balance: $6 = 0.54\,c + e$
N balance: $b = 0.16\,c$
Substituting the value for c from the C balance into the O and N balances gives $e = 2.76$ and $b = 0.96$, respectively. The yield of biomass from substrate in molar terms is therefore 6 gmol gmol^{-1}.

Using the atomic weights in Table B.1 (Appendix B), the molecular weight of glucose is 180.2. The biomass formula weight calculated from Table B.1 (Appendix B) is 24.46. Taking into account the 5% ash:

$$\text{Biomass molecular weight} = \frac{24.46}{0.95} = 25.75$$

Therefore, in mass terms, the molar biomass yield of 6 gmol gmol^{-1} = (6 × 25.75) g biomass per 180.2 g substrate = 0.86 g g^{-1}.

The maximum possible biomass yield is calculated using Eq. (4.19). From Eq. (4.13), $w = 6$ for glucose. From Table B.2 (Appendix B), the degree of reduction of glucose relative to NH_3 is $\gamma_S = 4.00$. The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1 \times 4 + 1.56 \times 1 - 0.54 \times 2 - 0.16 \times 3}{1} = 4.00$$

Substituting these values into Eq. (4.19):

$$c_{max} = \frac{w\,\gamma_S}{\gamma_B} = \frac{6\,(4.00)}{4.00} = 6.0\ \text{gmol gmol}^{-1}$$

The theoretical maximum biomass yield is therefore the same as the actual biomass yield.

Answer: The biomass yield from substrate of 0.86 g g^{-1} is 100% of the theoretical maximum. *When there is no product formation and no oxygen for electron transfer, all the available electrons from the substrate must go to the biomass.*

(b)
(i)
From Table B.2 (Appendix B), the molecular formula for methanol is CH_4O and the degree of reduction relative to NH_3 is $\gamma_S = 6.00$. The degree of reduction of *Methylophilus methylotrophus* biomass relative to NH_3 is:

$$\gamma_B = \frac{1 \times 4 + 1.68 \times 1 - 0.36 \times 2 - 0.22 \times 3}{1} = 4.30$$

From Eq. (4.13), $w = 1$ for methanol. Substituting values into Eq. (4.19):

$$c_{\max} = \frac{w\,\gamma_S}{\gamma_B} = \frac{1\,(6.00)}{4.30} = 1.40\,\text{gmol gmol}^{-1}$$

From Table B.8 (Appendix B), the molecular weight of methanol is 32.0. The biomass formula weight calculated from the atomic weights in Table B.1 (Appendix B) is 22.55. With 6% ash:

$$\text{Biomass molecular weight} = \frac{22.55}{0.94} = 23.99$$

In mass terms, the maximum possible molar biomass yield of 1.40 gmol gmol^{-1} is equal to (1.40 × 23.99) g biomass per 32.0 g substrate = 1.05 g g^{-1}.

Answer: The maximum possible biomass yield from methanol is 1.05 g g^{-1}. In terms of C atoms, the biomass yield is 1.40 gmol gmol^{-1} as both biomass and substrate have 1 C atom each. In comparison, the C-atom biomass yield from glucose in **(a)** is 1 gmol gmol^{-1}. The main reason for the increased yield in **(b)** is the high degree of reduction of methanol compared with glucose.

(ii)
The actual yield of biomass from methanol is $c = 0.42 \times 1.40$ gmol $\text{gmol}^{-1} = 0.59$. The oxygen demand can be determined from Eq. (4.16) if biomass remains the only major product so that $f = 0$. Using the parameter values determined in **(b)** **(i)**:

$$a = {}^1/_4\,(w\,\gamma_S - c\,\gamma_B) = {}^1/_4\,(1 \times 6.00 - 0.59 \times 4.30) = 0.87$$

Therefore, 0.87 gmol oxygen is required per gmol methanol. As the molecular weights of methanol and oxygen are the same, the oxygen demand is 0.87 g g^{-1} methanol.

Answer: 0.87 g oxygen is required per g methanol consumed

4.10 Ethanol production by yeast and bacteria

(a)
From Table B.2 (Appendix B), the molecular formula for glucose is $C_6H_{12}O_6$ and the molecular formula for ethanol is C_2H_6O. From Eq. (4.13), the stoichiometric equation for anaerobic growth and product formation is:

$$C_6H_{12}O_6 + b\,NH_3 \rightarrow c\,CH_{1.8}O_{0.5}N_{0.2} + d\,CO_2 + e\,H_2O + f\,C_2H_6O$$

From Table B.8 (Appendix B), the molecular weight of ethanol is 46.1. Using the atomic weights in Table B.1 (Appendix B), the molecular weight of glucose is 180.2 and the biomass molecular weight is 24.6. The values of the stoichiometric coefficients c can be determined from Eq. (4.12) and the yields $Y_{XS} = 0.11$ g g^{-1} for yeast and $Y_{XS} = 0.05$ g g^{-1} for bacteria.

$$\text{For yeast, } c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.11\,\text{g g}^{-1}\left(180.2\,\text{g gmol}^{-1}\right)}{24.6\,\text{g gmol}^{-1}} = 0.81\,\text{gmol gmol}^{-1}$$

$$\text{For bacteria, } c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.05\,\text{g g}^{-1}\left(180.2\,\text{g gmol}^{-1}\right)}{24.6\,\text{g gmol}^{-1}} = 0.37\,\text{gmol gmol}^{-1}$$

The other coefficients can be determined using elemental balances.

Yeast

C balance: $6 = c+d+2f = 0.81+d+2f \rightarrow d = 5.19-2f$

H balance: $12+3b = 1.8c+2e+6f = 1.8\times0.81+2e+6f \rightarrow 10.54+3b = 2e+6f$

O balance: $6 = 0.5c+2d+e+f = 0.5\times0.81+2d+e+f \rightarrow 5.595 = 2d+e+f$

N balance: $b = 0.2c = 0.2\times0.81 = 0.16$

Substituting the expression for b from the N balance into the H balances gives:

$$10.54 + 3\times0.16 = 2e+6f$$

$$e = 5.51-3f$$

Substituting this and the expression for d from the C balance into the O balance gives:

$$5.595 = 2\times(5.19-2f)+(5.51-3f)+f$$

$$6f = 10.295$$

$$f = 1.72$$

Therefore, for yeast, the yield of ethanol from glucose is 1.72 gmol gmol^{-1}.

Bacteria

C balance: $6 = c+d+2f = 0.37+d+2f \rightarrow d = 5.63-2f$

H balance: $12+3b = 1.8c+2e+6f = 1.8\times0.37+2e+6f \rightarrow 11.33+3b = 2e+6f$

O balance: $6 = 0.5c+2d+e+f = 0.5\times0.37+2d+e+f \rightarrow 5.815 = 2d+e+f$

N balance: $b = 0.2c = 0.2\times0.37 = 0.074$

Using the same solution procedure as for yeast, substituting the expression for b from the N balance into the H balances gives:

$$11.33 + 3\times0.074 = 2e+6f$$

$$e = 5.78-3f$$

Substituting this and the expression for d from the C balance into the O balance gives:

$$5.815 = 2\times(5.63-2f)+(5.78-3f)+f$$

$$6f = 11.225$$

$$f = 1.87$$

Therefore, for bacteria, the yield of ethanol from glucose is 1.87 gmol gmol^{-1}.

Answer: 1.72 gmol gmol^{-1} for yeast; 1.87 gmol gmol^{-1} for bacteria

(b)

The maximum possible ethanol yield can be calculated using Eq. (4.20). From Eq. (4.13), $w = 6$ for glucose and $j = 2$ for ethanol. From Table B.2 (Appendix B), the degree of reduction of glucose relative to NH_3 is $\gamma_S = 4.00$, and the degree of reduction of ethanol relative to NH_3 is $\gamma_P = 6.00$. Using these values in Eq. (4.20) gives:

$$f_{\max} = \frac{w\,\gamma_S}{j\,\gamma_P} = \frac{6\,(4.00)}{2\,(6.00)} = 2.0\text{ gmol gmol}^{-1}$$

Therefore, the actual ethanol yield of 1.72 gmol gmol^{-1} for yeast represents 86% of the theoretical maximum; for bacteria, the actual yield of 1.87 gmol gmol^{-1} represents 94% of the theoretical maximum.

Answer: 86% of the theoretical maximum for yeast; 94% of the theoretical maximum for bacteria

4.11 Detecting unknown products

From Table B.2 (Appendix B), the molecular formula for glucose is $C_6H_{12}O_6$. Assuming that no products other than biomass are formed, from Eq. (4.4), the stoichiometric equation for growth is:

$$C_6H_{12}O_6 + a\,O_2 + b\,NH_3 \rightarrow c\,CH_{1.79}O_{0.56}N_{0.17} + d\,CO_2 + e\,H_2O$$

Using the atomic weights in Table B.1 (Appendix B), the molecular weight of glucose is 180.2, the molecular weight of oxygen is 32.0, and the biomass molecular weight is 25.16. The value of the stoichiometric coefficient c can be determined from the yield $Y_{XS} = 0.37$ g g^{-1} and Eq. (4.12):

$$c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.37\text{ g g}^{-1}\left(180.2\text{ g gmol}^{-1}\right)}{25.16\text{ g gmol}^{-1}} = 2.65\text{ gmol gmol}^{-1}$$

Therefore, 2.65 gmol cells are produced per gmol glucose consumed. Converting the oxygen demand to a molar basis:

$$0.88\text{ g O}_2\text{ per g cells} = \frac{0.88\text{ g O}_2}{1\text{ g cells}}\cdot\left|\frac{25.16\text{ g cells}}{1\text{ gmol cells}}\right|\cdot\left|\frac{1\text{ gmol O}_2}{32.0\text{ g O}_2}\right| = 0.69\text{ gmol O}_2\text{ gmol}^{-1}\text{ cells}$$

Combining this with the result for c, the observed oxygen demand a is:

$$a = \frac{0.69\text{ gmol O}_2}{1\text{ gmol cells}}\left(\frac{2.65\text{ gmol cells}}{1\text{ gmol glucose}}\right) = 1.83$$

From Table B.2 (Appendix B), the degree of reduction of glucose relative to NH_3 is $\gamma_S = 4.00$. The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times4 + 1.79\times1 - 0.56\times2 - 0.17\times3}{1} = 4.16$$

If no products are formed other than biomass, the theoretical oxygen demand can be determined from Eq. (4.16) with $f = 0$; from Eq. (4.13), $w = 6$ for glucose:

$$a = {}^1\!/_4\,(w\,\gamma_S - c\,\gamma_B) = {}^1\!/_4\,(6\times4.00 - 2.65\times4.16) = 3.24$$

As the theoretical oxygen demand is significantly higher than that observed, formation of other products acting as electron acceptors is likely to have occurred in the culture.

Answer: Yes

4.12 Medium formulation

Using the atomic weights in Table B.1 (Appendix B), the molecular weight of $(NH_4)_2SO_4$ is 132.1 and the biomass molecular weight is 26.16. Using a basis of 1 litre, production of 25 g cells corresponds to $25/26.16 = 0.956$ gmol cells. As each gmol cells contains 0.25 gmol N, $(0.956 \times 0.25) = 0.239$ gmol N are needed from the medium for biomass synthesis. As $(NH_4)_2SO_4$ is the sole N source and each gmol $(NH_4)_2SO_4$ contains 2 gmol N, $0.239/2 = 0.120$ gmol $(NH_4)_2SO_4$ is required. Multiplying this by the molecular weight, $0.120 \times 132.1 = 15.9$ g $(NH_4)_2SO_4$ are required. The minimum concentration of $(NH_4)_2SO_4$ is therefore 15.9 g l^{-1}.

Answer: 15.9 g l^{-1}

4.13 Oxygen demand for production of recombinant protein

(a)

Recombinant protein can be considered as a product of cell culture even though it is not excreted from the cells; assume that recombinant protein is synthesised in addition to the normal *E. coli* biomass composition. From Table 4.3, the chemical formula for *E. coli* can be taken as $CH_{1.77}O_{0.49}N_{0.24}$; from Table B.2 (Appendix B), the molecular formula for glucose is $C_6H_{12}O_6$. Substituting these formulae into the general stoichiometric equation for growth and product formation, Eq. (4.13), gives:

$$C_6H_{12}O_6 + a\,O_2 + b\,NH_3 \rightarrow c\,CH_{1.77}O_{0.49}N_{0.24} + d\,CO_2 + e\,H_2O + f\,CH_{1.55}O_{0.31}N_{0.25}$$

Using the atomic weights in Table B.1 (Appendix B), the molecular weight of glucose is 180.2, the biomass molecular weight is 25.00, and the recombinant protein molecular weight is 22.03.

Assume that the biomass yield refers to the cell mass without recombinant protein. The value of the stoichiometric coefficient c can be determined from the yield $Y_{XS} = 0.48\ \text{g g}^{-1}$ and Eq. (4.12):

$$c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.48\ \text{g g}^{-1}\left(180.2\ \text{g gmol}^{-1}\right)}{25.00\ \text{g gmol}^{-1}} = 3.46\ \text{gmol gmol}^{-1}$$

The value of the stoichiometric coefficient f can be determined from the yield $Y_{PS} = 0.20 \times 0.48 = 0.096\ \text{g g}^{-1}$ and Eq. (4.14):

$$f = \frac{Y_{PS}\,(\text{MW substrate})}{\text{MW product}} = \frac{0.096\ \text{g g}^{-1}\left(180.2\ \text{g gmol}^{-1}\right)}{22.03\ \text{g gmol}^{-1}} = 0.79\ \text{gmol gmol}^{-1}$$

The ammonia requirement can be determined using an elemental balance for N.
N balance: $b = 0.24\,c + 0.25\,f$
Substituting the above values for c and f into the N balance gives $b = 0.24 \times 3.46 + 0.25 \times 0.79 = 1.03$.

Answer: 1.03 gmol gmol^{-1} glucose

(b)
The oxygen demand can be determined using an electron balance. From Table B.2 (Appendix B), the degree of reduction of glucose relative to NH_3 is $\gamma_S = 4.00$; from Table 4.3, the degree of reduction of *E. coli* relative to NH_3 is $\gamma_B = 4.07$. The degree of reduction of the recombinant protein relative to NH_3 is:

$$\gamma_P = \frac{1 \times 4 + 1.55 \times 1 - 0.31 \times 2 - 0.25 \times 3}{1} = 4.18$$

From Eq. (4.13), $w = 6$ for glucose and $j = 1$ for recombinant protein. Substituting these values into Eq. (4.16) for the theoretical oxygen demand gives:

$$a = {}^{1}\!/_{4}\,(w\,\gamma_S - c\,\gamma_B - f\,j\,\gamma_P) = {}^{1}\!/_{4}\,(6 \times 4.00 - 3.46 \times 4.07 - 0.79 \times 1 \times 4.18) = 1.65$$

Answer: 1.65 gmol per gmol glucose

(c)
If f in the stoichiometric equation is zero but c remains equal to 3.46, the ammonia requirement can be determined using an elemental balance for N as follows:
N balance: $b = 0.24\,c = 0.24 \times 3.46 = 0.83$
Therefore, in wild-type *E. coli* the ammonia requirement is reduced from 1.03 to 0.83 gmol gmol^{-1} glucose, a decrease of 19%. Eq. (4.16) for the oxygen requirement becomes:

$$a = {}^{1}\!/_{4}\,(w\,\gamma_S - c\,\gamma_B) = {}^{1}\!/_{4}\,(6 \times 4.00 - 3.46 \times 4.07) = 2.48$$

Therefore, the oxygen demand is increased from 1.65 to 2.48 gmol gmol^{-1} glucose, a rise of 50%.

Answer: The ammonia and oxygen requirements for wild-type *E. coli* are 0.83 gmol and 2.48 gmol per gmol glucose, respectively. These values represent a 19% reduction and a 50% increase, respectively, compared with the genetically engineered strain.

4.14 Effect of growth on oxygen demand

The stoichiometric equation for acetic acid production using cell culture must include terms for growth. Based on Eq. (4.13), the stoichiometric equation for growth and product formation is:

$$C_2H_6O + a\,O_2 + b\,NH_3 \rightarrow c\,CH_{1.8}O_{0.5}N_{0.2} + d\,CO_2 + e\,H_2O + f\,C_2H_4O_2$$

From Table B.8 (Appendix B), the molecular weights of ethanol and acetic acid are 46.1 and 60.1, respectively. From the atomic weights in Table B.1 (Appendix B), the biomass molecular weight is 24.63. The value of the stoichiometric coefficient c can be determined from the yield $Y_{XS} = 0.14\ \text{g g}^{-1}$ and Eq. (4.12):

$$c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.14\ \text{g g}^{-1}\left(46.1\ \text{g gmol}^{-1}\right)}{24.63\ \text{g gmol}^{-1}} = 0.26\ \text{gmol gmol}^{-1}$$

The value of the stoichiometric coefficient f can be determined from the yield $Y_{PS} = 0.92\ \text{g g}^{-1}$ and Eq. (4.14):

$$f = \frac{Y_{PS}\,(\text{MW substrate})}{\text{MW product}} = \frac{0.92\ \text{g g}^{-1}\left(46.1\ \text{g gmol}^{-1}\right)}{60.1\ \text{g gmol}^{-1}} = 0.71\ \text{gmol gmol}^{-1}$$

From Table B.2 (Appendix B), the degree of reduction of ethanol relative to NH_3 is $\gamma_S = 6.00$, and the degree of reduction of acetic acid relative to NH_3 is $\gamma_P = 4.00$. The degree of reduction of the biomass relative to NH_3 is :

$$\gamma_B = \frac{1 \times 4 + 1.8 \times 1 - 0.5 \times 2 - 0.2 \times 3}{1} = 4.20$$

From Eq. (4.13), $w = 2$ for ethanol and $j = 2$ for acetic acid. Substituting these values into Eq. (4.16) for the theoretical oxygen demand gives:

$$a = {}^{1}\!/_{4}\,(w\,\gamma_S - c\,\gamma_B - f j\,\gamma_P) = {}^{1}\!/_{4}\,(2 \times 6.00 - 0.26 \times 4.20 - 0.71 \times 2 \times 4.00) = 1.31$$

Therefore, with growth, 1.31 gmol oxygen are required per gmol glucose consumed, compared with 1 gmol oxygen per gmol glucose without growth. Therefore, with growth, the oxygen demand for acetic acid production is increased by 31%.

Answer: The oxygen demand is increased by 31%.

5

Energy Balances

5.1 Sensible energy change

(a)

From Table B.5 (Appendix B), C_p for *m*-cresol between 25°C and 100°C is 0.551 cal g^{-1} °C^{-1}. The specific enthalpy change calculated using Eq. (5.13) is:

$$\Delta h = C_p \, \Delta T = 0.551 \text{ cal g}^{-1}\,{}^\circ\text{C}^{-1} \, (100 - 25)^\circ\text{C}$$

$$\Delta h = 41.3 \text{ cal g}^{-1}$$

Answer: 41.3 cal g^{-1}

(b)

From Table B.5 (Appendix B), C_p for ethylene glycol between 10°C and 20°C can be taken as 0.569 cal g^{-1} °C^{-1}. The specific enthalpy change calculated using Eq. (5.13) is:

$$\Delta h = C_p \, \Delta T = 0.569 \text{ cal g}^{-1}\,{}^\circ\text{C}^{-1} \, (10 - 20)^\circ\text{C}$$

$$\Delta h = -5.69 \text{ cal g}^{-1}$$

Answer: –5.69 cal g^{-1}

(c)

From Table B.6 (Appendix B), C_p for succinic acid between 15°C and 120°C is given by the expression 0.248 + 0.00153 T, where T is in °C and C_p is in cal g^{-1} °C^{-1}. The sensible energy change is best determined from the integral of this equation between the limits T = 15°C and T = 120°C:

$$\Delta h = \int_{15^\circ\text{C}}^{120^\circ\text{C}} C_p \, dT = \int_{15^\circ\text{C}}^{120^\circ\text{C}} (0.248 + 0.00153\,T) \, dT \text{ cal g}^{-1}$$

$$\Delta h = \left. \left(0.248\,T + \frac{0.00153}{2}\,T^2 \right) \right|_{15^\circ\text{C}}^{120^\circ\text{C}} \text{ cal g}^{-1}$$

$$\Delta h = 36.9 \text{ cal g}^{-1}$$

Answer: 36.9 cal g^{-1}

(d)

From Table B.3 (Appendix B), the heat capacity of air between 65°C and 150°C is given by the equation:

$$C_p = 28.94 + 0.4147 \times 10^{-2}\,T + 0.3191 \times 10^{-5}\,T^2 - 1.965 \times 10^{-9}\,T^3$$

where C_p is heat capacity in J gmol^{-1} °C^{-1} and T is temperature in °C. The sensible energy change can be determined by evaluating the integral of this expression between the limits T = 150°C and T = 65°C:

$$\Delta h = \int_{150^\circ\text{C}}^{65^\circ\text{C}} C_p \, dT = \int_{150^\circ\text{C}}^{65^\circ\text{C}} \left(28.94 + 0.4147 \times 10^{-2}\,T + 0.3191 \times 10^{-5}\,T^2 - 1.965 \times 10^{-9}\,T^3 \right) dT \text{ J gmol}^{-1}$$

$$\Delta h = \left(28.94\,T + \frac{0.4147\times10^{-2}}{2}T^2 + \frac{0.3191\times10^{-5}}{3}T^3 - \frac{1.965\times10^{-9}}{4}T^4\right)\Bigg|_{150^\circ\mathrm{C}}^{65^\circ\mathrm{C}} \ \mathrm{J\ gmol^{-1}}$$

$$\Delta h = -2500.9\ \mathrm{J\ gmol^{-1}} = -2.50\ \mathrm{kJ\ gmol^{-1}}$$

Answer: –2.50 kJ $\mathrm{gmol^{-1}}$

5.2 Heat of vaporisation

The latent heat of vaporisation of water at 33°C is obtained from Table C.1 (Appendix C). Taking the average of the values at 32°C and 34°C, $\Delta h_v = 2423.55\ \mathrm{kJ\ kg^{-1}}$ at 33°C. From Eq. (5.16):

$$\Delta H = M\,\Delta h_v = 20\ \mathrm{g\ h^{-1}}\left(2423.55\ \mathrm{kJ\ kg^{-1}}\right).\left|\frac{1\ \mathrm{kg}}{1000\ \mathrm{g}}\right|$$

$$\Delta H = 48.5\ \mathrm{kJ\ h^{-1}}$$

Answer: 48.5 kJ $\mathrm{h^{-1}}$

5.3 Steam tables

(a)

The heat of vaporisation of water at 85°C is obtained from Table C.1 (Appendix C). Taking the average of the values at 84°C and 86°C, $\Delta h_v = 2296.05\ \mathrm{kJ\ kg^{-1}}$ at 85°C.

Answer: 2296.05 kJ $\mathrm{kg^{-1}}$

(b)

From Table C.1 (Appendix C), the enthalpy of liquid water at 10°C relative to the triple point is 42.0 kJ $\mathrm{kg^{-1}}$. The enthalpy of liquid water at 35°C relative to the triple point can be estimated as the average of the values in Table C.1 for 34°C and 36°C = 146.55 kJ $\mathrm{kg^{-1}}$. Using the relationship on p 89, the enthalpy of water at 35°C relative to 10°C is therefore (146.55 – 42.0) kJ $\mathrm{kg^{-1}}$ = 104.55 kJ $\mathrm{kg^{-1}}$.

Answer: 104.55 kJ $\mathrm{kg^{-1}}$

(c)

The enthalpy of saturated water vapour at 40°C relative to the triple point can be read directly from Table C.1 (Appendix C) as 2574.4 kJ $\mathrm{kg^{-1}}$.

Answer: 2574.4 kJ $\mathrm{kg^{-1}}$

(d)

The enthalpy of superheated steam at 2.5 atm and 275°C relative to the triple point can be obtained from Table C.3 (Appendix C). To convert the pressure to kPa, from Table A.5 (Appendix A), 1 atm = 1.013×10^5 Pa. Therefore:

$$2.5\ \mathrm{atm} = 2.5\ \mathrm{atm}.\left|\frac{1.013\times10^5\ \mathrm{Pa}}{1\ \mathrm{atm}}\right|.\left|\frac{1\ \mathrm{kPa}}{1000\ \mathrm{Pa}}\right| = 253.3\ \mathrm{kPa}$$

From Table C.3, the enthalpy at 100 kPa and 275°C is 3024 kJ $\mathrm{kg^{-1}}$; the enthalpy at 500 kPa and 275°C is 3013 kJ $\mathrm{kg^{-1}}$. Interpolating between these values gives an enthalpy of 3019.8 kJ $\mathrm{kg^{-1}}$ at 253.3 kPa.

Answer: 3019.8 kJ $\mathrm{kg^{-1}}$

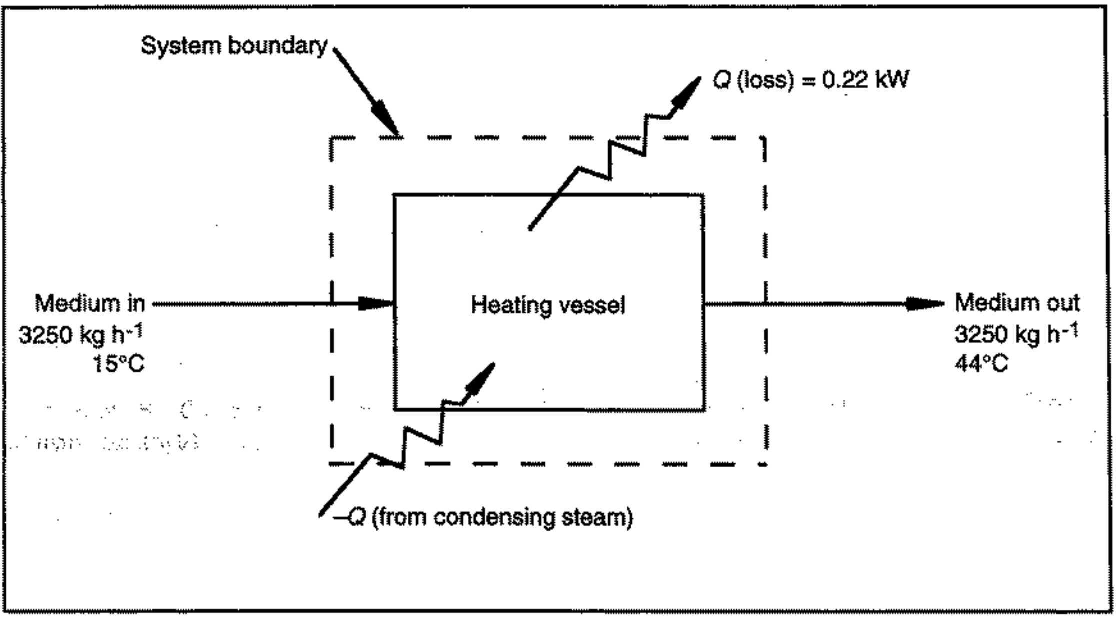
5.4 Pre-heating nutrient medium

1. *Assemble*

(i) Units

kg, h, kJ, °C

(ii) *Flow sheet*



(iii) *System boundary*

The system boundary is shown on the flow sheet.

2. *Analyse*

(i) *Assumptions*

- steady state
- no leaks
- system is homogenous
- condensate temperature is 150°C
- no shaft work

(ii) *Basis*

1 h, or 3250 kg medium in

(iii) *Reference state*

$H = 0$ for water (steam) at its triple point
$H = 0$ for medium at 15°C

(iv) *Extra data*

C_p medium = 0.9 cal g^{-1} °C^{-1} = 0.9 kcal kg^{-1} °C^{-1}
Δh_v water at 150°C = 2113.1 kJ kg^{-1} (Table C.2, Appendix C)
1 kcal = 4.187×10^3 J (Table A.7, Appendix A)
1 W = 1 J s^{-1} (Table A.8, Appendix A); therefore, 1 kW = 1 kJ s^{-1}

(v) *Mass balance*

The mass balance is already complete.

(vi) *Energy-balance equation*

At steady state, Eq. (5.9) applies:

$$\sum_{\substack{\text{input}\\\text{streams}}} (M\,h) - \sum_{\substack{\text{output}\\\text{streams}}} (M\,h) - Q + W_s = 0$$

3. *Calculate*

(i) *Identify terms in the energy-balance equation*

$W_s = 0$. There are two components for the heat term, Q: Q_{loss} and Q from the condensing steam. With symbol MD = medium, the energy-balance equation becomes:

$$(M\,h)_{\text{MD in}} - (M\,h)_{\text{MD out}} - Q - Q_{\text{loss}} = 0$$

$(M\,h)_{\text{MD in}} = 0$ (reference state)
$(M\,h)_{\text{MD out}}$ at 44°C is calculated as a sensible energy change from $H = 0$ at 15°C using Eq. (5.13):

$$(M\,h)_{\text{MD out}} = M\,C_p\,\Delta T = 3250\,\text{kg}\left(0.9\,\text{kcal kg}^{-1}\,°\text{C}^{-1}\right)(44-15)°\text{C} = 8.483 \times 10^4\,\text{kcal}$$

Converting to kJ:

$$(M\,h)_{\mathrm{MD\,out}} = 8.483\times 10^4\,\mathrm{kcal}\,.\left|\frac{4.187\times 10^3\,\mathrm{J}}{1\,\mathrm{kcal}}\right|.\left|\frac{1\,\mathrm{kJ}}{1000\,\mathrm{J}}\right| = 3.55\times 10^5\,\mathrm{kJ}$$

The rate of heat loss is 0.22 kW. Converting to kJ h^{-1}:

$$0.22\,\mathrm{kW} = 0.22\,\mathrm{kW}\,.\left|\frac{1\,\mathrm{kJ\,s^{-1}}}{1\,\mathrm{kW}}\right|.\left|\frac{3600\,\mathrm{s}}{1\,\mathrm{h}}\right| = 792\,\mathrm{kJ\,h^{-1}}$$

Therefore, on the basis of 1 h, $Q_{\mathrm{loss}} = 792$ kJ. Substituting values into the energy-balance equation gives:

$$0 - 3.55\times 10^5\,\mathrm{kJ} - Q - 792\,\mathrm{kJ} = 0$$

$$Q = -3.56\times 10^5\,\mathrm{kJ}$$

Q has a negative value which is consistent with the sign conventions outlined on pp 87–88: heat must be supplied to the system from the surroundings. This heat is provided as the latent heat of vaporisation as saturated steam at 150°C condenses. The enthalpy change from this change of phase is calculated using Eq. (5.16) and must be equal to $-Q$.

$$3.56\times 10^5\,\mathrm{kJ} = M_{\mathrm{steam}}\,\Delta h_{\mathrm{v}} = M_{\mathrm{steam}}\,(2113.1\,\mathrm{kJ\,kg^{-1}})$$

$$M_{\mathrm{steam}} = 168\,\mathrm{kg}$$

4. *Finalise*

Answer: 168 kg

5.5 Production of glutamic acid

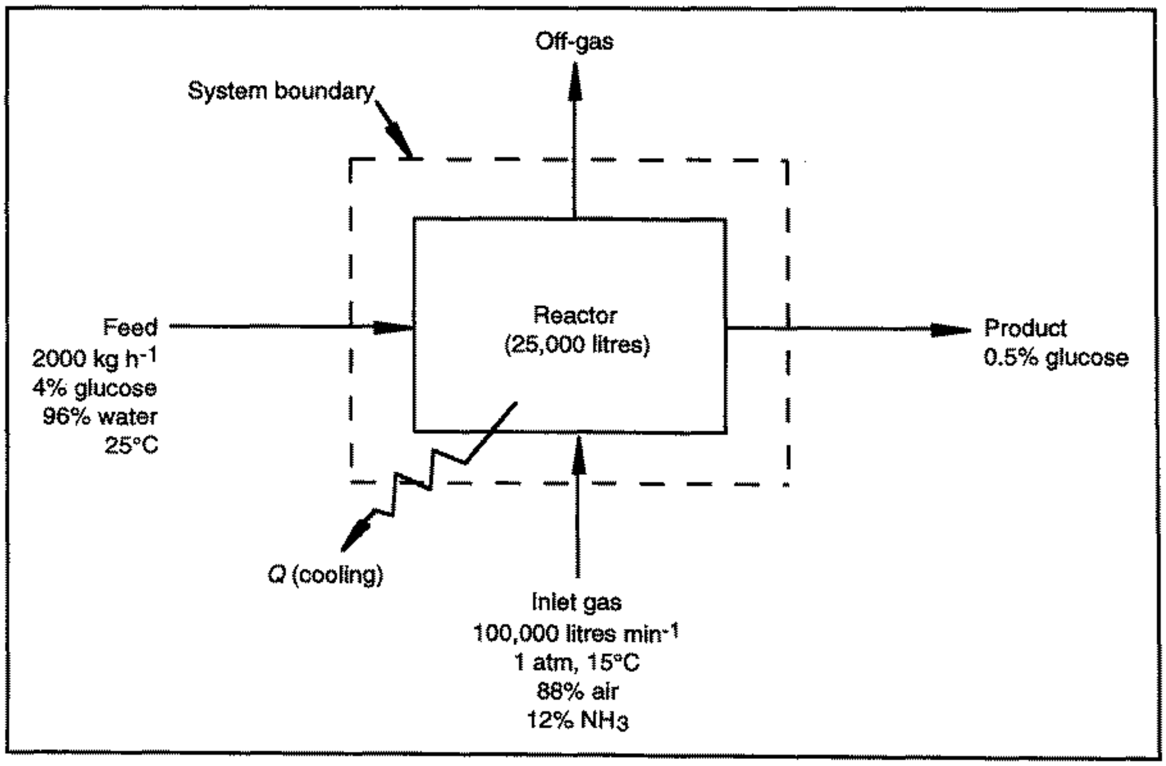
(a)

1. *Assemble*

(i) Units

kg, h, kJ, °C

(ii) *Flow sheet*



(iii) *System boundary*

The system boundary is shown on the flow sheet.

(iv) *Reaction equation*

$$C_6H_{12}O_6 + NH_3 + 1^1/_2\,O_2 \rightarrow C_5H_9NO_4 + CO_2 + 3\,H_2O$$

2. *Analyse*

(i) *Assumptions*

- steady state
- no leaks
- system is homogenous
- solutions are ideal
- inlet air and off-gas are dry
- all excess NH_3 is dissolved in the aqueous phase
- all CO_2 produced leaves in the off-gas
- negligible sensible heat change
- no evaporation
- no shaft work

(ii) *Basis*

1 h, or 2000 kg feed

(iii) *Reference state*

$H = 0$ for water at its triple point
$H = 0$ for feed at 25°C

(iv) *Extra data*

Molecular weights (Table B.1, Appendix B): glucose = 180.2
$NH_3 = 17.0$
$O_2 = 32.0$
$N_2 = 28.0$
glutamic acid = 147.1
$CO_2 = 44.0$
$H_2O = 18.0$
air = 28.8 (see Problem 2.9, Chapter 2)

Ideal gas constant (Table 2.5): $R = 0.082057$ l atm K^{-1} $gmol^{-1}$
Composition of air (p 17): 21% O_2, 79% N_2 by volume
Heats of combustion (Table B.8, Appendix B):

$$\Delta h_c^o \text{ glucose} = -2805.0 \text{ kJ gmol}^{-1}$$

$$\Delta h_c^o \text{ NH}_3 = -382.6 \text{ kJ gmol}^{-1}$$

$$\Delta h_c^o \text{ glutamic acid} = -2244.1 \text{ kJ gmol}^{-1}$$

(v) *Compounds involved in reaction*

Glucose, ammonia, oxygen, glutamic acid, carbon dioxide and water are involved in the reaction.

(vi) *Mass-balance equations*

For glucose, ammonia, oxygen, glutamic acid, carbon dioxide and water, the appropriate mass-balance equation is Eq. (4.2):

$$\text{mass in} + \text{mass generated} = \text{mass out} + \text{mass consumed}$$

For N_2 and total mass, the appropriate mass-balance equation is Eq. (4.3):

$$\text{mass in} = \text{mass out}$$

(vii) *Energy-balance equation*

The modified energy-balance equation, Eq. (5.26), applies:

$$-\Delta H_{rxn} - M_v\,\Delta h_v - Q + W_s = 0$$

3. *Calculate*

(i) *Mass balance*

As gas compositions are normally given in vol%, 88,000 litres air and 12,000 litres NH_3 enter the reactor every min. On a basis of 1 h, the volume of air in is $88{,}000 \times 60 = 5.28 \times 10^6$ litres; the volume of NH_3 in is $12{,}000 \times 60 = 7.20 \times 10^5$ litres. Using the known composition of air, the volume of O_2 in $= 0.21 \times 5.28 \times 10^6 = 1.11 \times 10^6$ litres; the volume of N_2 in $= 0.79 \times 5.28 \times 10^6 = 4.17 \times 10^6$ litres. Converting these gas volumes to moles using the ideal gas law, Eq. (2.32), with the temperature converted from °C to degrees Kelvin using Eq. (2.24):

$$\text{Moles of } O_2 \text{ in} = \frac{p\,V}{R\,T} = \frac{1\text{ atm}\left(1.11\times10^{6}\text{ l}\right)}{0.0820571\text{ atm K}^{-1}\text{ gmol}^{-1}\,(15+273.15)\text{ K}} = 4.69\times10^{4}\text{ gmol}$$

$$\text{Moles of } N_2 \text{ in} = \frac{p\,V}{R\,T} = \frac{1\text{ atm}\left(4.17\times10^{6}\text{ l}\right)}{0.0820571\text{ atm K}^{-1}\text{ gmol}^{-1}\,(15+273.15)\text{ K}} = 1.76\times10^{5}\text{ gmol}$$

$$\text{Moles of } NH_3 \text{ in} = \frac{p\,V}{R\,T} = \frac{1\text{ atm}\left(7.20\times10^{5}\text{ l}\right)}{0.0820571\text{ atm K}^{-1}\text{ gmol}^{-1}\,(15+273.15)\text{ K}} = 3.05\times10^{4}\text{ gmol}$$

These molar quantities can now be converted to masses using the molecular weights:

$$\text{Mass of } O_2 \text{ in} = 4.69\times10^{4}\text{ gmol}\,.\left|\frac{32.0\text{ g}}{1\text{ gmol}}\right|.\left|\frac{1\text{ kg}}{1000\text{ g}}\right| = 1500.8\text{ kg}$$

$$\text{Mass of } N_2 \text{ in} = 1.76\times10^{5}\text{ gmol}\,.\left|\frac{28.0\text{ g}}{1\text{ gmol}}\right|.\left|\frac{1\text{ kg}}{1000\text{ g}}\right| = 4928.0\text{ kg}$$

$$\text{Mass of } NH_3 \text{ in} = 3.05\times10^{4}\text{ gmol}\,.\left|\frac{17.0\text{ g}}{1\text{ gmol}}\right|.\left|\frac{1\text{ kg}}{1000\text{ g}}\right| = 518.5\text{ kg}$$

Therefore, the total mass of inlet gas is 1500.8 + 4928.0 + 518.5 = 6947.3 kg.

The calculation tables below show all known quantities in kg. The total mass of off-gas is denoted *G*; the total mass of product is denoted *P*.

Stream	*In*							
	Glucose	*NH_3*	*O_2*	*N_2*	*Gluconic acid*	*CO_2*	*H_2O*	*Total*
Feed	80	0	0	0	0	0	1920	2000
Inlet gas	0	518.5	1500.8	4928.0	0	0	0	6947.3
Off-gas	–	–	–	–	–	–	–	–
Product	–	–	–	–	–	–	–	–
Total	80	518.5	1500.8	4928.0	0	0	1920	8947.3

Stream	*Out*							
	Glucose	*NH_3*	*O_2*	*N_2*	*Gluconic acid*	*CO_2*	*H_2O*	*Total*
Feed	–	–	–	–	–	–	–	–
Inlet gas	–	–	–	–	–	–	–	–
Off-gas	0	0	?	?	0	?	0	*G*
Product	0.005 *P*	?	0	0	?	0	?	*P*
Total	0.005 *P*	?	?	?	?	?	?	*G* + *P*

N_2 balance
N_2 is a tie component.

$$4928.0\text{ kg } N_2 \text{ in} = N_2 \text{ out}$$

$$N_2 \text{ out} = 4928.0\text{ kg}$$

Glucose balance

$$80\text{ kg glucose in} + 0\text{ kg glucose generated} = 0.005\,P\text{ kg glucose out} + \text{glucose consumed}$$

$$\text{Glucose consumed} = (80 - 0.005\,P)\text{ kg}$$

Converting the glucose consumed to molar terms:

$$\text{Glucose consumed} = (80 - 0.005\,P)\text{ kg}\,.\left|\frac{1\text{ kgmol}}{180.2\text{ kg}}\right| = \left(0.444 - 2.775\times10^{-5}\,P\right)\text{kgmol}$$

From the reaction stoichiometry, conversion of this number of kgmol glucose requires the same number of kgmol NH_3 and 1.5 × the number of kgmol O_2. Converting these molar quantities to masses:

$$\text{NH}_3 \text{ consumed} = \left(0.444 - 2.775\times 10^{-5}\,P\right)\text{kgmol} = \left(0.444 - 2.775\times 10^{-5}\,P\right)\text{kgmol}\,.\left|\frac{17.0\text{ kg}}{1\text{ kgmol}}\right|$$

$$\text{NH}_3 \text{ consumed} = (7.548 - 4.718\times 10^{-4}\,P)\text{ kg}$$

$$\text{O}_2 \text{ consumed} = 1.5\times\left(0.444 - 2.775\times 10^{-5}\,P\right)\text{kgmol} = 1.5\times\left(0.444 - 2.775\times 10^{-5}\,P\right)\text{kgmol}\,.\left|\frac{32.0\text{ kg}}{1\text{ kgmol}}\right|$$

$$\text{O}_2 \text{ consumed} = (21.312 - 1.332\times 10^{-3}\,P)\text{ kg}$$

Similarly, expressions for the masses of glutamic acid, CO_2 and water generated can be determined from the stoichiometry:

$$\text{Glutamic acid generated} = \left(0.444 - 2.775\times 10^{-5}\,P\right)\text{kgmol} = \left(0.444 - 2.775\times 10^{-5}\,P\right)\text{kgmol}\,.\left|\frac{147.1\text{ kg}}{1\text{ kgmol}}\right|$$

$$\text{Glutamic acid generated} = (65.312 - 4.082\times 10^{-3}\,P)\text{ kg}$$

$$\text{CO}_2 \text{ generated} = \left(0.444 - 2.775\times 10^{-5}\,P\right)\text{kgmol} = \left(0.444 - 2.775\times 10^{-5}\,P\right)\text{kgmol}\,.\left|\frac{44.0\text{ kg}}{1\text{ kgmol}}\right|$$

$$\text{CO}_2 \text{ generated} = (19.536 - 1.221\times 10^{-3}\,P)\text{ kg}$$

$$\text{Water generated} = 3\times\left(0.444 - 2.775\times 10^{-5}\,P\right)\text{kgmol} = 3\times\left(0.444 - 2.775\times 10^{-5}\,P\right)\text{kgmol}\,.\left|\frac{18.0\text{ kg}}{1\text{ kgmol}}\right|$$

$$\text{Water generated} = (23.976 - 1.499\times 10^{-3}\,P)\text{ kg}$$

O_2 balance

$$1500.8\text{ kg O}_2\text{ in} + 0\text{ kg O}_2\text{ generated} = \text{O}_2\text{ out} + (21.312 - 1.332\times 10^{-3}\,P)\text{ kg O}_2\text{ consumed}$$

$$\text{O}_2\text{ out} = (1500.8 - (21.312 - 1.332\times 10^{-3}\,P))\text{ kg}$$

$$\text{O}_2\text{ out} = (1479.5 + 1.332\times 10^{-3}\,P)\text{ kg}$$

CO_2 balance

$$0\text{ kg CO}_2\text{ in} + (19.536 - 1.221\times 10^{-3}\,P)\text{ kg CO}_2\text{ generated} = \text{CO}_2\text{ out} + 0\text{ kg CO}_2\text{ consumed}$$

$$\text{CO}_2\text{ out} = (19.536 - 1.221\times 10^{-3}\,P)\text{ kg}$$

Adding together the masses of N_2, O_2 and CO_2 out gives the total mass of off-gas, G:

$$G = 4928.0\text{ kg N}_2 + (1479.5 + 1.332\times 10^{-3}\,P)\text{ kg O}_2 + (19.536 - 1.221\times 10^{-3}\,P)\text{ kg CO}_2$$

$$G = (6427.0 + 1.11\times 10^{-4}\,P)\text{ kg}$$

Total mass balance

$$8947.3\text{ kg total mass in} = (G + P)\text{ kg total mass out}$$

Substituting the above expression for G into the total mass balance:

$$8947.3\text{ kg} = (6427.0 + 1.11\times 10^{-4}\,P + P)\text{ kg}$$

$$2520.3\text{ kg} = 1.000\,P\text{ kg}$$

$$P = 2520.3\text{ kg}$$

Therefore, from the total mass balance:

$$G = (8947.3 - 2520.3)\text{ kg}$$

$$G = 6427.0\text{ kg}$$

Substituting the result for P into the glucose, O_2 and CO_2 balances gives:

$$\text{Glucose consumed} = (80 - 0.005\,P)\text{ kg} = 67.40\text{ kg}$$

$$\text{Glucose out} = 0.005\,P\text{ kg} = 12.60\text{ kg}$$

$$O_2\text{ consumed} = (21.312 - 1.332 \times 10^{-3}\,P)\text{ kg} = 17.95\text{ kg}$$

$$O_2\text{ out} = (1479.5 + 1.332 \times 10^{-3}\,P)\text{ kg} = 1482.9\text{ kg}$$

$$CO_2\text{ out} = CO_2\text{ generated} = (19.536 - 1.221 \times 10^{-3}\,P)\text{ kg} = 16.46\text{ kg}$$

Using the result for P to evaluate the masses of the other reactants and products involved in the reaction:

$$NH_3\text{ consumed} = (7.548 - 4.718 \times 10^{-4}\,P)\text{ kg} = 6.36\text{ kg}$$

$$\text{Glutamic acid generated} = (65.312 - 4.082 \times 10^{-3}\,P)\text{ kg} = 55.02\text{ kg}$$

$$\text{Water generated} = (23.976 - 1.499 \times 10^{-3}\,P)\text{ kg} = 20.20\text{ kg}$$

These results can be used directly in the energy balance for evaluation of the cooling requirements. However, completion of the mass balance allows the calculations to be checked.

NH_3 balance

$$518.5\text{ kg }NH_3\text{ in} + 0\text{ kg }NH_3\text{ generated} = NH_3\text{ out} + 6.36\text{ kg }NH_3\text{ consumed}$$

$$NH_3\text{ out} = 512.14\text{ kg}$$

Water balance

$$1920\text{ kg water in} + 20.20\text{ kg water generated} = \text{water out} + 0\text{ kg water consumed}$$

$$\text{Water out} = 1940.20\text{ kg}$$

Glutamic acid balance

$$0\text{ kg glutamic acid in} + 55.02\text{ kg glutamic acid generated} = \text{glutamic acid out} + 0\text{ kg glutamic acid consumed}$$

$$\text{Glutamic acid out} = 55.02\text{ kg}$$

The Out side of the mass-balance table can now be completed with all quantities in kg.

Stream	*Out*							
	Glucose	*NH_3*	*O_2*	*N_2*	*Gluconic acid*	*CO_2*	*H_2O*	*Total*
Feed	–	–	–	–	–	–	–	–
Inlet gas	–	–	–	–	–	–	–	–
Off-gas	0	0	1482.9	4928.0	0	16.46	0	6427.4
Product	12.60	512.14	0	0	55.02	0	1940.20	2520
Total	12.60	512.14	1482.9	4928.0	55.02	16.46	1940.20	8947.3

All columns and rows of the completed table add up correctly to within round-off error.

(ii) *Energy balance*

$W_s = 0$; $M_v = 0$. Therefore, the energy-balance equation becomes:

$$-\Delta H_{rxn} - Q = 0$$

The heat of reaction is evaluated using Eq. (5.20). As the heat of combustion of CO_2 and H_2O is zero:

$$\Delta H_{rxn} = \left(n\,\Delta h_c^o\right)_G + \left(n\,\Delta h_c^o\right)_A - \left(n\,\Delta h_c^o\right)_{GA}$$

where G = glucose, A = NH_3 and GA = glutamic acid. The n in this equation are the moles of reactant or product involved in the reaction. Converting the masses of reactants and products consumed or generated to moles:

$$\text{Glucose consumed} = 67.40\text{ kg} = 67.40\text{ kg}\cdot\left|\frac{1000\text{ g}}{1\text{ kg}}\right|\cdot\left|\frac{1\text{ gmol}}{180.2\text{ g}}\right| = 374\text{ gmol}$$

As NH_3 and glutamic acid are involved in the reaction in stoichiometric quantities, 374 gmol NH_3 are consumed and 374 gmol glutamic acid are produced. Substituting these quantities into the heat of reaction equation gives:

$$\Delta H_{\mathrm{rxn}} = 374\,\mathrm{gmol}\left(-2805.0\,\mathrm{kJ\,gmol^{-1}}\right) + 374\,\mathrm{gmol}\left(-382.6\,\mathrm{kJ\,gmol^{-1}}\right) - 374\,\mathrm{gmol}\left(-2244.1\,\mathrm{kJ\,gmol^{-1}}\right)$$

$$\Delta H_{\mathrm{rxn}} = -3.53 \times 10^5\,\mathrm{kJ}$$

Substituting this result into the energy-balance equation:

$$3.53 \times 10^5\,\mathrm{kJ} - Q = 0$$

$$Q = 3.53 \times 10^5\,\mathrm{kJ}$$

From the sign conventions outlined on pp 87–88, Q positive indicates that heat must be removed from the system.

Answer: $3.53 \times 10^5\,\mathrm{kJ\,h^{-1}}$

(b)

If cooling were not provided, the heat of reaction would be absorbed as sensible heat by the streams passing through the reactor. For a rough calculation of the effect of this heat on the temperature of the reactor, assume that the $3.53 \times 10^5\,\mathrm{kJ\,h^{-1}}$ is absorbed by 2000 $\mathrm{kg\,h^{-1}}$ aqueous medium and 6947.3 $\mathrm{kg\,h^{-1}}$ gas. Assume that the heat capacity of the aqueous medium is close to that of water = 75.4 $\mathrm{J\,gmol^{-1}\,°C^{-1}}$ (Table B.3, Appendix B) = 75.4 $\mathrm{kJ\,kgmol^{-1}\,°C^{-1}}$, and that the heat capacity of the gas stream is equal to that of air = approx. 29 $\mathrm{J\,gmol^{-1}\,°C^{-1}}$ (Table B.3, Appendix B) = 29 $\mathrm{kJ\,kgmol^{-1}\,°C^{-1}}$. From Eq. (5.12):

$$\Delta T = \frac{\Delta H}{(M\,C_p)_{\mathrm{liquid}} + (M\,C_p)_{\mathrm{gas}}}$$

$$T = \frac{3.53 \times 10^5\,\mathrm{kJ\,h^{-1}}}{\left(2000\,\mathrm{kg\,h^{-1}}\right)\left(75.4\,\mathrm{kJ\,kgmol^{-1}\,°C^{-1}}\right)\cdot\left|\frac{1\,\mathrm{kgmol}}{18.0\,\mathrm{kg}}\right| + \left(6947.3\,\mathrm{kg\,h^{-1}}\right)\left(29\,\mathrm{kJ\,kgmol^{-1}\,°C^{-1}}\right)\cdot\left|\frac{1\,\mathrm{kgmol}}{28.8\,\mathrm{kg}}\right|} = 23°\mathrm{C}$$

As a temperature rise of 23°C in the reactor would not be well tolerated by most commercial organisms, provision of adequate cooling for this reaction is an important consideration. Assuming that the usual temperature for the reaction is 25°C, the temperature without cooling would increase to (25 + 23)°C = 48°C.

Answer: 48°C

5.6 Bacterial production of alginate

Alginate production at a rate of 5 $\mathrm{kg\,h^{-1}}$ requires:

$$\frac{5\,\mathrm{kg\,h^{-1}}}{4\,\mathrm{kg\,kg^{-1}}} = 1.25\,\mathrm{kg\,O_2\,h^{-1}}$$

Converting this quantity to gmol using the molecular weight of O_2 = 32.0 (Table B.1, Appendix B):

$$O_2\ \mathrm{required} = 1.25\,\mathrm{kg\,h^{-1}}\cdot\left|\frac{1000\,\mathrm{g}}{\mathrm{kg}}\right|\cdot\left|\frac{1\,\mathrm{gmol}}{32.0\,\mathrm{g}}\right| = 39.06\,\mathrm{gmol\,h^{-1}}$$

The heat of reaction for aerobic metabolism is approximately –460 $\mathrm{kJ\,gmol^{-1}}$ O_2 (p 100). Therefore, the heat of reaction for alginate production is:

$$\Delta H_{\mathrm{rxn}} = -460\,\mathrm{kJ\,gmol^{-1}}\left(39.06\,\mathrm{gmol\,h^{-1}}\right) = -1.80 \times 10^4\,\mathrm{kJ\,h^{-1}}$$

This result can be used in the modified energy-balance equation, Eq. (5.26):

$$-\Delta H_{\mathrm{rxn}} - M_{\mathrm{v}}\,\Delta h_{\mathrm{v}} - Q + W_{\mathrm{s}} = 0$$

with W_s = 1.5 kW and M_v = 0 (no evaporation). From Table A.8 (Appendix A), 1 W = 1 $\mathrm{J\,s^{-1}}$; therefore 1 kW = 1 $\mathrm{kJ\,s^{-1}}$ and:

$$1.80\times 10^4\ \mathrm{kJ\,h^{-1}} - 0 - Q + 1.5\ \mathrm{kJ\,s^{-1}}\,.\left|\frac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right| = 0$$

$$Q = 2.3\times 10^4\ \mathrm{kJ\,h^{-1}}$$

From the sign conventions outlined on pp 87–88, Q positive indicates that heat must be removed from the system.

Answer: 2.3×10^4 kJ h^{-1}

5.7 Acid fermentation

From Tables B.8 and B.1 (Appendix B), the molecular formulae and molecular weights are:
sucrose = $C_{12}H_{22}O_{11}$: MW = 342.3
propionic acid = $C_3H_6O_2$: MW = 74.1
acetic acid = $C_2H_4O_2$: MW = 60.1
butyric acid = $C_4H_8O_2$: MW = 88.1
lactic acid = $C_3H_6O_3$: MW = 90.1
From p 75, the biomass molecular formula can be taken as the average, $CH_{1.8}O_{0.5}N_{0.2}$. From the end of Table B.8 (Appendix B), the molecular weight of the biomass is 25.9.

The reaction equation can be obtained by modifying Eq. (4.13) for anaerobic growth and product formation:

$$C_{12}H_{22}O_{11} + b\,NH_3 \rightarrow c\,CH_{1.8}O_{0.5}N_{0.2} + d\,CO_2 + e\,H_2O + f_1\,C_3H_6O_2 + f_2\,C_2H_4O_2 + f_3\,C_4H_8O_2 + f_4\,C_3H_6O_3$$

The biomass yield from substrate $Y_{XS} = 0.12$ g g^{-1}. This value can be used to determine the stoichiometric coefficient c using Eq. (4.12):

$$c = \frac{Y_{XS}\,(\text{MW substrate}))}{\text{MW cells}} = \frac{0.12\ \mathrm{g\,g^{-1}}\,(342.3)}{25.9} = 1.59$$

The coefficients f_1, f_2, f_3 and f_4 can be determined similarly using the product yields and Eq. (4.14):

$$f_1 = \frac{Y_{PS}\,(\text{MW substrate}))}{\text{MW propionic acid}} = \frac{0.40\ \mathrm{g\,g^{-1}}\,(342.3)}{74.1} = 1.85$$

$$f_2 = \frac{Y_{PS}\,(\text{MW substrate}))}{\text{MW acetic acid}} = \frac{0.20\ \mathrm{g\,g^{-1}}\,(342.3)}{60.1} = 1.14$$

$$f_3 = \frac{Y_{PS}\,(\text{MW substrate}))}{\text{MW butyric acid}} = \frac{0.05\ \mathrm{g\,g^{-1}}\,(342.3)}{88.1} = 0.19$$

$$f_4 = \frac{Y_{PS}\,(\text{MW substrate}))}{\text{MW lactic acid}} = \frac{0.034\ \mathrm{g\,g^{-1}}\,(342.3)}{90.1} = 0.13$$

Of the remaining coefficients b, d and e, because CO_2 and H_2O do not figure in heat of reaction calculations as their heat of combustion = 0, only b need be determined. This can be done using an elemental balance on N.
N balance: $b = 0.2\,c = 0.2 \times 1.59 = 0.32$.

To calculate the heat of reaction, the heats of combustion of the reactants and products are required from Table B.8 (Appendix B):
Δh_c^o sucrose = -5644.9 kJ gmol^{-1}
Δh_c^o NH_3 = -382.6 kJ gmol^{-1}
Δh_c^o biomass = -552 kJ gmol^{-1}
Δh_c^o propionic acid = -1527.3 kJ gmol^{-1}
Δh_c^o acetic acid = -874.2 kJ gmol^{-1}
Δh_c^o butyric acid = -2183.6 kJ gmol^{-1}
Δh_c^o lactic acid = -1368.3 kJ gmol^{-1}

The heat of reaction is determined using Eq. (5.20). As the heat of combustion of CO_2 and H_2O is zero:

$$\Delta H_{\text{rxn}} = \left(n\,\Delta h_c^o\right)_S + \left(n\,\Delta h_c^o\right)_A - \left(n\,\Delta h_c^o\right)_B - \left(n\,\Delta h_c^o\right)_{PA} - \left(n\,\Delta h_c^o\right)_{AA} - \left(n\,\Delta h_c^o\right)_{BA} - \left(n\,\Delta h_c^o\right)_{LA}$$

where S = sucrose, A = NH_3, B = biomass, PA = propionic acid, AA = acetic acid, BA = butyric acid, and LA = lactic acid. Using a basis of 1 gmol sucrose, the n in this equation are the stoichiometric coefficients. Substituting values:

$$\begin{aligned}\Delta H_{\text{rxn}} = {} & 1\text{ gmol}\left(-5644.9\text{ kJ gmol}^{-1}\right) + 0.32\text{ gmol}\left(-382.6\text{ kJ gmol}^{-1}\right) - 1.59\text{ gmol}\left(-552\text{ kJ gmol}^{-1}\right)\\ & - 1.85\text{ gmol}\left(-1527.3\text{ kJ gmol}^{-1}\right) - 1.14\text{ gmol}\left(-874.2\text{ kJ gmol}^{-1}\right) - 0.19\text{ gmol}\left(-2183.6\text{ kJ gmol}^{-1}\right)\\ & - 0.13\text{ gmol}\left(-1368.3\text{ kJ gmol}^{-1}\right)\end{aligned}$$

$$\Delta H_{\text{rxn}} = -474.8\text{ kJ}$$

This ΔH_{rxn} was determined on the basis of 1 gmol sucrose. For 30 kg sucrose consumed over a period of 10 d:

$$\Delta H_{\text{rxn}} = 30\text{ kg}\cdot\left|\frac{1000\text{ g}}{1\text{ kg}}\right|\cdot\left|\frac{1\text{ gmol}}{342.3\text{ g}}\right|\left(-474.8\text{ kJ gmol}^{-1}\right) = -4.16\times10^4\text{ kJ}$$

The cooling requirements are determined using the modified energy-balance equation, Eq. (5.26). For no evaporation and no shaft work, $M_v = W_s = 0$, so that:

$$Q = -\Delta H_{\text{rxn}} = 4.16\times10^4\text{ kJ}$$

From the sign conventions outlined on pp 87–88, Q positive means that heat must be removed from the system.

Answer: 4.16×10^4 kJ

5.8 Ethanol fermentation

From Table B.2 (Appendix B), the molecular formula for glucose is $C_6H_{12}O_6$ and the molecular formula for ethanol is C_2H_6O. From Eq. (4.13), the stoichiometric equation under anaerobic conditions is:

$$C_6H_{12}O_6 + b\,NH_3 \rightarrow c\,CH_{1.75}O_{0.58}N_{0.18} + d\,CO_2 + e\,H_2O + f\,C_2H_6O$$

From Table B.8 (Appendix B), the molecular weight of ethanol is 46.1. From the atomic weights in Table B.1 (Appendix B), the molecular weight of glucose is 180.2 and the biomass formula weight is 25.58. Taking into account the 8% ash:

$$\text{Biomass molecular weight} = \frac{25.58}{0.92} = 27.80$$

The value of the stoichiometric coefficient f can be determined from the yield $Y_{PS} = 0.45\text{ g g}^{-1}$ and Eq. (4.14):

$$f = \frac{Y_{PS}\,(\text{MW substrate})}{\text{MW product}} = \frac{0.45\text{ g g}^{-1}\left(180.2\text{ g gmol}^{-1}\right)}{46.1\text{ g gmol}^{-1}} = 1.76\text{ gmol gmol}^{-1}$$

The other coefficients can be determined using elemental balances.
C balance: $6 = c + d + 2f = c + d + 2\times1.76 \rightarrow c = 2.48 - d$
H balance: $12 + 3b = 1.75c + 2e + 6f = 1.75c + 2e + 6\times1.76 \rightarrow 1.44 + 3b = 1.75c + 2e$
O balance: $6 = 0.58c + 2d + e + f = 0.58c + 2d + e + 1\times1.76 \rightarrow 4.24 = 0.58c + 2d + e$
N balance: $b = 0.18c$
Substituting the expression for c from the C balance into the N balance:

$$b = 0.18\,(2.48 - d) = 0.45 - 0.18\,d$$

Substituting this and the results from the C and N balances into the H balance:

$$1.44 + 3\,(0.45 - 0.18\,d) = 1.75\,(2.48 - d) + 2\,\text{e}$$

$$1.21\,d - 1.55 = 2\,e$$

$$e = 0.61\,d - 0.78$$

Substituting the expressions for c, b and e into the O balance:

$$4.24 = 0.58\,(2.48 - d) + 2\,d + (0.61\,d - 0.78)$$

$$3.58 = 2.03\,d$$

$$d = 1.76$$

Substituting this value for d into the expressions for the other coefficients gives $c = 0.72$, $b = 0.13$ and $e = 0.29$. The completed stoichiometric equation is therefore:

$$C_6H_{12}O_6 + 0.13\,NH_3 \rightarrow 0.72\,CH_{1.75}O_{0.58}N_{0.18} + 1.76\,CO_2 + 0.29\,H_2O + 1.76\,C_2H_6O$$

Using a basis of 1 h, 0.4 kg ethanol are produced. Converting this to moles:

$$\text{Moles ethanol produced} = 0.4\,\text{kg}\,.\left|\frac{1000\,\text{g}}{1\,\text{kg}}\right|.\left|\frac{1\,\text{gmol}}{46.1\,\text{g}}\right| = 8.68\,\text{gmol}$$

From stoichiometry:

$$\text{Moles glucose consumed} = 8.68\,\text{gmol} \times \frac{1}{1.76} = 4.93\,\text{gmol}$$

$$\text{Moles NH}_3\text{ consumed} = 8.68\,\text{gmol} \times \frac{0.13}{1.76} = 0.64\,\text{gmol}$$

$$\text{Moles biomass produced} = 8.68\,\text{gmol} \times \frac{0.72}{1.76} = 3.55\,\text{gmol}$$

The heats of combustion from Table B.8 (Appendix B) are:

Δh_c^o glucose $= -2805.0$ kJ gmol^{-1}

Δh_c^o $NH_3 = -382.6$ kJ gmol^{-1}

Δh_c^o ethanol $= -1366.8$ kJ gmol^{-1}

From p 101, the heat of combustion of yeast can be taken as -21.2 kJ g^{-1}. The heat of reaction is determined using Eq. (5.20). As the heat of combustion of CO_2 and H_2O is zero:

$$\Delta H_{rxn} = \left(n\,\Delta h_c^o\right)_G + \left(n\,\Delta h_c^o\right)_A - \left(n\,\Delta h_c^o\right)_B - \left(n\,\Delta h_c^o\right)_E$$

where G = glucose, A = NH_3, B = biomass and E = ethanol. The n in this equation are the actual moles of reactants and products consumed or produced. Substituting values gives:

$$\Delta H_{rxn} = 4.93\,\text{gmol}\left(-2805.0\,\text{kJ gmol}^{-1}\right) + 0.64\,\text{gmol}\left(-382.6\,\text{kJ gmol}^{-1}\right) - 3.55\,\text{gmol}\left(-21.2\,\text{kJ g}^{-1}\right).\left|\frac{27.80\,\text{g}}{1\,\text{gmol}}\right| - 8.68\,\text{gmol}\left(-1366.8\,\text{kJ gmol}^{-1}\right)$$

$$\Delta H_{rxn} = -117.5\,\text{kJ}$$

Using the modified energy-balance equation, Eq. (5.26), with $M_v = 0$ and $W_s = 0$:

$$Q = -\Delta H_{rxn} = 117.5\,\text{kJ}$$

From the sign conventions outlined on pp 87–88, Q positive means that heat must be removed from the system; in this case, 117.5 kJ h^{-1} is used to raise the temperature of 2.5 l h^{-1} water from 10°C. The sensible heat change of the water can be calculated from Eq. (5.12). Using a value of 1 kg l^{-1} for the density of water, C_p for water = 75.4 J gmol^{-1} °C^{-1} (Table B.3, Appendix B), and the molecular weight of water = 18.0 (Table B.1, Appendix B):

$$\Delta T = \frac{Q}{M\,C_p} = \frac{117.5\,\text{kJ h}^{-1}}{\left(2.5\,\text{l h}^{-1}.\left|\frac{1\,\text{kg}}{1\,\text{l}}\right|\right)\left(75.4\,\text{J gmol}^{-1}\,{}^\circ\text{C}^{-1}.\left|\frac{1\,\text{gmol}}{18.0\,\text{g}}\right|.\left|\frac{1000\,\text{g}}{1\,\text{kg}}\right|.\left|\frac{1\,\text{kJ}}{1000\,\text{J}}\right|\right)} = 11.2^\circ\text{C}$$

The final temperature of the water is therefore 10°C + 11.2°C = 21.2°C

Answer: 21.2°C

5.9 Production of bakers' yeast

From Table B.8 (Appendix B), the molecular formula for sucrose is $C_{12}H_{22}O_{11}$. Therefore, from Eq. (4.4), the stoichiometric equation for aerobic cell growth is:

$$C_{12}H_{22}O_{11} + a\,O_2 + b\,NH_3 \rightarrow c\,CH_{1.83}O_{0.55}N_{0.17} + d\,CO_2 + e\,H_2O$$

Using the atomic weights in Table B.1 (Appendix B), the molecular weight of sucrose is 342.3 and the biomass formula weight is 25.04. Taking into account the 5% ash:

$$\text{Biomass molecular weight} = \frac{25.04}{0.95} = 26.36$$

The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times 4 + 1.83\times 1 - 0.55\times 2 - 0.17\times 3}{1} = 4.22$$

The degree of reduction of sucrose relative to NH_3 is:

$$\gamma_S = \frac{12\times 4 + 22\times 1 - 11\times 2}{12} = 4.00$$

The value of the stoichiometric coefficient c can be determined from the yield $Y_{XS} = 0.5\ \mathrm{g\,g^{-1}}$ and Eq. (4.12):

$$c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.5\ \mathrm{g\,g^{-1}}\left(342.3\ \mathrm{g\,gmol^{-1}}\right)}{26.36\ \mathrm{g\,gmol^{-1}}} = 6.49\ \mathrm{gmol\,gmol^{-1}}$$

The oxygen requirements can be determined from Eq. (4.16) with $f = 0$; from Eq (4.13), $w = 12$ for sucrose:

$$a = {}^1/_4\,(w\,\gamma_S - c\,\gamma_B) = {}^1/_4\,(12\times 4.00 - 6.49\times 4.22) = 5.15$$

Therefore, 5.15 gmol O_2 are required for each 6.49 gmol biomass produced; this corresponds to 0.79 gmol O_2 per gmol biomass. Converting this oxygen demand to mass terms using the molecular weight of oxygen = 32.0 from Table B.1 (Appendix B):

$$0.79\ \text{gmol } O_2 \text{ per gmol biomass} = \frac{0.79\ \text{gmol } O_2}{1\ \text{gmol biomass}} \cdot \left|\frac{32.0\ \mathrm{g}\ O_2}{1\ \text{gmol } O_2}\right| \cdot \left|\frac{1\ \text{gmol biomass}}{26.36\ \text{g biomass}}\right| = 0.96\ \mathrm{g\,g^{-1}}$$

The specific growth rate represents a rate of growth of 0.45 g biomass produced per g biomass per h. As 0.96 g O_2 are required per g biomass produced, the specific rate of O_2 consumption is $0.45 \times 0.96 = 0.43$ g O_2 per g biomass per h. When the biomass concentration is $10\ \mathrm{g\,l^{-1}}$ in a 50,000 litre fermenter, the mass of cells is $10\ \mathrm{g\,l^{-1}} \times 50{,}000\ \mathrm{l} = 5 \times 10^5$ g. Therefore, the total rate of O_2 consumption is $0.43\ \mathrm{g\,g^{-1}\,h^{-1}} \times 5 \times 10^5\ \mathrm{g} = 2.15 \times 10^5\ \mathrm{g}\ O_2\ \mathrm{h^{-1}}$. Converting this to moles:

$$\text{Rate of } O_2 \text{ consumption} = 2.15\times 10^5\ \mathrm{g\,h^{-1}} \cdot \left|\frac{1\ \mathrm{gmol}}{32.0\ \mathrm{g}}\right| = 6.72\times 10^3\ \mathrm{gmol\,h^{-1}}$$

From p 100, the heat of reaction for aerobic growth is approximately $-460\ \mathrm{kJ\,gmol^{-1}}$ O_2 consumed. Therefore:

$$\Delta H_{rxn} = -460\ \mathrm{kJ\,gmol^{-1}}\,(6.72\times 10^3\ \mathrm{gmol\,h^{-1}}) = -3.09\times 10^6\ \mathrm{kJ\,h^{-1}}$$

The rate of heat removal from the fermenter is determined using the modified energy-balance equation, Eq. (5.26), with $W_s = M_v = 0$:

$$Q = -\Delta H_{rxn} = 3.09\times 10^6\ \mathrm{kJ\,h^{-1}}$$

From the sign conventions outlined on pp 87–88, Q positive confirms that heat is removed from the system.

Answer: $3.09 \times 10^6\ \mathrm{kJ\,h^{-1}}$

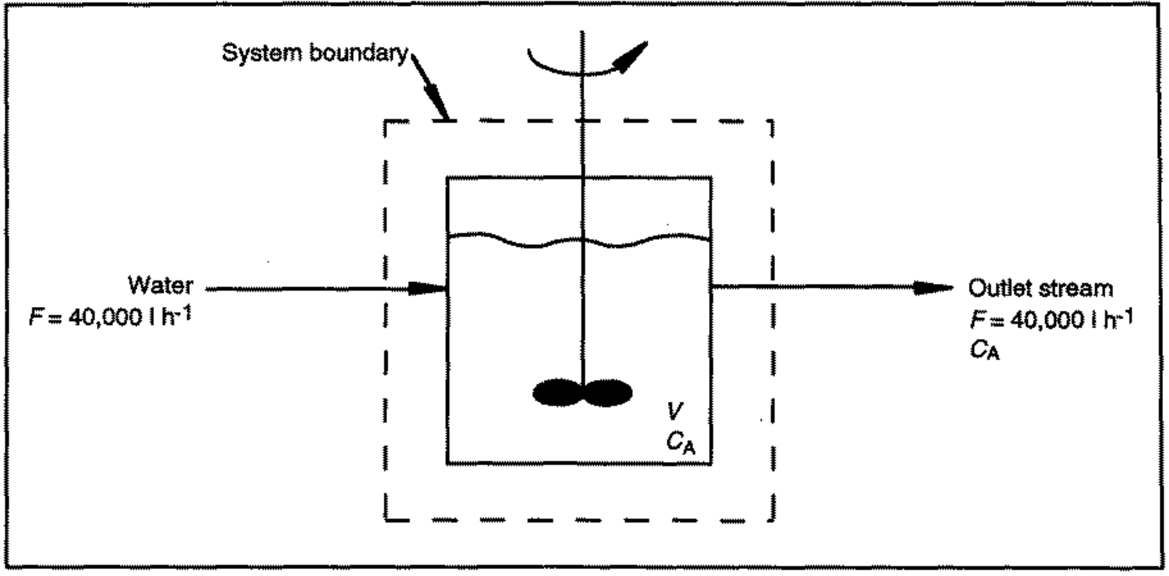
6

Unsteady-State Material and Energy Balances

6.1 Dilution of sewage

(i) *Flow sheet and system boundary*

These are shown in the figure below.



(ii) *Define variables*

V = volume of material in the tank; F = volumetric flow rate into and out of the vessel = 40,000 l $\mathrm{h^{-1}}$; C_A = concentration of suspended solids

(iii) *Assumptions*

- no leaks
- tank is well mixed; therefore C_A in the outlet stream = C_A inside the tank
- density of the solids is the same as that of water = 1 kg $\mathrm{l^{-1}}$

(iv) *Boundary conditions*

At $t = 0$, $V = V_0$ = 440,000 litres + the volume of the solids. The initial mass of solids in the tank is 10,000 kg; therefore, if the density is 1 kg $\mathrm{l^{-1}}$, $V_0 = (440{,}000 + 10{,}000)\ \mathrm{l} = 450{,}000\ \mathrm{l}$. At $t = 0$, $C_A = C_{A0}$:

$$C_{A0} = \frac{10{,}000\ \mathrm{kg}}{450{,}000\ \mathrm{l}} = 0.022\ \mathrm{kg\ l^{-1}}$$

(v) *Total mass balance*

As the volumetric flow rates of material in and out are the same and the density of the inlet and outlet streams are assumed to be equal, the volume of material in the tank is constant (see Example 6.2) so that $V = V_0$ at all times.

(vi) *Solids mass balance*

The general unsteady-state mass-balance equation is Eq. (6.5). As there is no reaction, $R_G = R_C = 0$; in this problem, $\hat{M}_i = 0$, $\hat{M}_o = F\,C_A$ and $M = V\,C_A$. Substituting into Eq. (6.5) gives:

$$\frac{d(V\,C_A)}{dt} = -F\,C_A$$

As V is constant, it can be taken outside of the differential:

$$V\frac{dC_A}{dt} = -F\,C_A$$

As F is also constant, the differential equation contains only two variables, C_A and t. Separating variables gives:

$$\frac{dC_A}{C_A} = \frac{-F}{V}\,dt$$

Integrating:

$$\int \frac{dC_A}{C_A} = \int \frac{-F}{V}\,dt$$

Using integration rules (D.27) and (D.24) from Appendix D and combining the constants of integration:

$$\ln C_A = \frac{-F}{V}\,t + K$$

From the initial condition for C_A, at $t = 0$, $\ln C_{A0} = K$. Substituting this value of K into the equation gives:

$$\ln C_A = \frac{-F}{V}\,t + \ln C_{A0}$$

$$\ln \frac{C_A}{C_{A0}} = \frac{-F}{V}\,t$$

$$C_A = C_{A0}\,e^{(-F/V)\,t}$$

Substituting the known values for C_{A0}, F and V:

$$C_A = 0.022\,e^{-0.089\,t}$$

where C_A has units of kg l^{-1} and t has units of h. From this equation, at $t = 5$ h, $C_A = 0.014$ kg l^{-1}.

Answer: 0.014 kg l^{-1}

6.2 Production of fish-protein concentrate

(i) *System*

The system is the whole gutted fish placed in the batch drier at time zero. During drying, the mass of the system decreases as water is removed.

(ii) *Assumptions*

No additional assumptions are required.

(iii) *Boundary conditions*

At $t = 0$, the mass of water in the fish in the drier is equal to M_0.

(iv) *Water mass balance*

The general unsteady-state mass-balance equation is Eq. (6.5). As there is no reaction, $R_G = R_C = 0$. No water is added during drying; therefore $\hat{M}_i = 0$. At any time during drying, as the rate of water removal $\hat{M}_o$ is proportional to the moisture content M, $\hat{M}_o = k\,M$ where k is a constant. Substituting into Eq. (6.5) gives:

$$\frac{dM}{dt} = -k\,M$$

where M and t are the only variables. Separating variables and integrating:

$$\int \frac{dM}{M} = -\int k\,dt$$

Using integration rules (D.27) and (D.24) from Appendix D and combining the constants of integration:

$$\ln M = -k\,t + K$$

At $t = 0$, $M = M_0$; therefore, $K = \ln M_0$. Substituting this value of K into the equation gives:

$$\ln M = -k\,t + \ln M_0$$

$$\ln \frac{M}{M_0} = -k\,t$$

At $t = 20$ min, $M = 0.5\,M_0$. Substituting these values into the equation:

$$\ln \frac{0.5\,M_0}{M_0} = -k\,(20\,\text{min})$$

$$\ln 0.5 = -k\,(20\,\text{min})$$

$$k = 0.0347\,\text{min}^{-1}$$

Therefore:

$$\ln \frac{M}{M_0} = -0.0347\,t$$

where t has units of min. When 95% of the water has been removed, $M = 0.05\,M_0$. Therefore:

$$\ln \frac{0.05\,M_0}{M_0} = -0.0347\,t$$

$$\ln 0.05 = -0.0347\,t$$

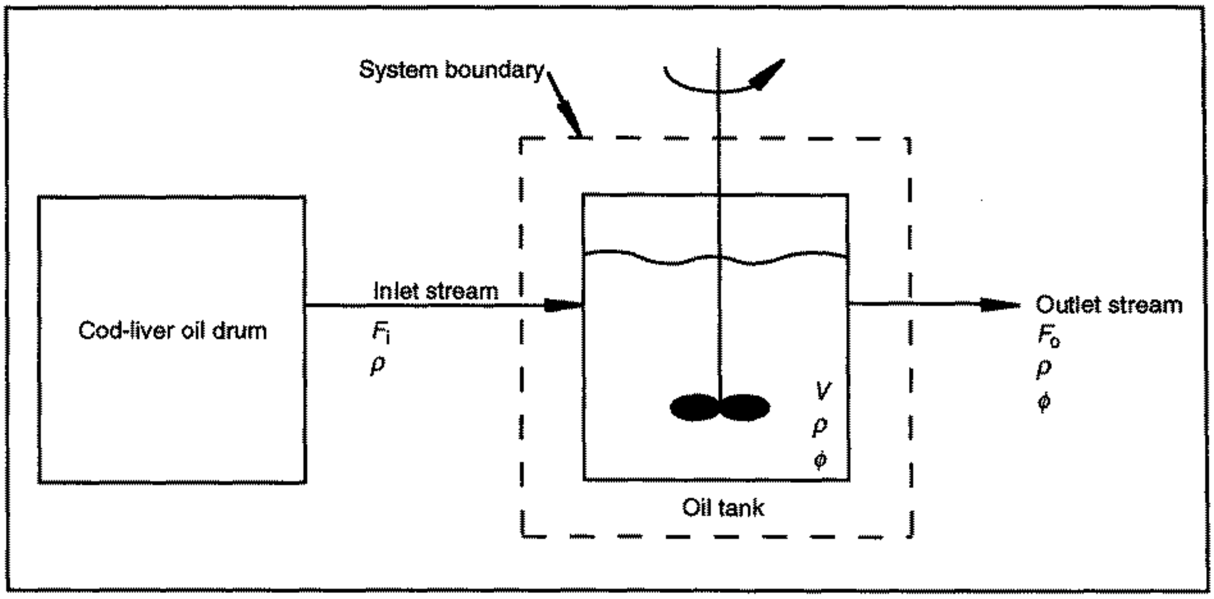
$$t = 86.3\,\text{min} = 86.3\,\text{min}\,.\left|\frac{1\,\text{h}}{60\,\text{min}}\right| = 1.44\,\text{h}$$

Answer: 1.44 h

6.3 Contamination of vegetable oil

(i) *Flow sheet and system boundary*

These are shown in the figure below.



(ii) *Define variables*

V = volume of oil in the tank; F_i = volumetric flow rate of cod-liver oil into the oil tank; F_o = volumetric flow rate of mixed oil out of the oil tank; ϕ = mass fraction of vegetable oil; ρ = density of vegetable and cod-liver oils

(iii) *Assumptions*

- no leaks
- oil tank is well mixed; therefore ϕ in the outlet stream = ϕ inside the tank
- densities of each oil and the oil mixture are the same

(iv) *Boundary conditions*
At $t = 0$ when cod-liver oil first enters the oil tank, $V = V_0 = 60$ litres. At $t = 0$, $\phi = \phi_0 = 1$.

(a)
Total mass balance
The general unsteady-state mass-balance equation is Eq. (6.5). As there is no reaction, $R_G = R_C = 0$. The total mass flow rate into the oil tank is equal to the volumetric flow rate multiplied by the density of the cod-liver oil: $\hat{M}_i = F_i\,\rho$. Similarly, $\hat{M}_o = F_o\,\rho$. The total mass of oil in the tank is equal to the volume of oil multiplied by its density: $M = V\rho$. Substituting these expressions into Eq. (6.5) gives:

$$\frac{\mathrm{d}(V\rho)}{\mathrm{d}t} = F_i\,\rho - F_o\,\rho$$

As ρ is constant it can be taken outside of the differential and cancelled:

$$\rho\frac{\mathrm{d}V}{\mathrm{d}t} = (F_i - F_o)\,\rho$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = (F_i - F_o)$$

The differential equation contains only two variables, V and t. Separating variables and integrating:

$$\mathrm{d}V = (F_i - F_o)\,\mathrm{d}t$$

$$\int \mathrm{d}V = \int (F_i - F_o)\,\mathrm{d}t$$

Using integration rule (D.24) from Appendix D and combining the constants of integration:

$$V = (F_i - F_o)\,t + K$$

From the initial condition for V, at $t = 0$, $K = V_0$. Substituting this value of K into the equation gives:

$$V = (F_i - F_o)\,t + V_0$$

The time between 8 p.m. and 9 a.m. the next morning is 13 h. From the above equation for V, for $t = 13$ h, $F_i = 7.5$ l h^{-1}, $F_o = 4.8$ l h^{-1} and $V_0 = 60$ l, $V = 95.1$ l. As this volume is less than the tank capacity of 100 l, the tank will not overflow.

Answer: No

(b)
Total mass balance
As the volumetric flow rates of oils into and out of the tank are the same and the density of the inlet and outlet streams are assumed to be equal, the volume of oil in the tank is constant (see Example 6.2) so that $V = V_0 = 60$ l at all times.

Vegetable oil mass balance
The general unsteady-state mass-balance equation is Eq. (6.5). As there is no reaction, $R_G = R_C = 0$. No vegetable oil enters the tank; therefore $\hat{M}_i = 0$. The mass flow rate of vegetable oil out is $\hat{M}_o = F_o\,\rho\,\phi$. The mass of vegetable oil in the tank at any time is $M = V\rho\,\phi$. Substituting these expressions into Eq. (6.5) gives:

$$\frac{\mathrm{d}(V\rho\,\phi)}{\mathrm{d}t} = -F_o\,\rho\,\phi$$

As both V and ρ are constants, they can be taken outside of the differential and ρ can be cancelled:

$$V\rho\frac{\mathrm{d}\phi}{\mathrm{d}t} = -F_o\,\rho\,\phi$$

$$V\frac{\mathrm{d}\phi}{\mathrm{d}t} = -F_o\,\phi$$

As F_o is also constant, the differential equation contains only two variables, ϕ and t. Separating variables and integrating:

$$\frac{d\phi}{\phi} = \frac{-F_o}{V}\,dt$$

$$\int \frac{d\phi}{\phi} = \int \frac{-F_o}{V}\,dt$$

Using integration rules (D.27) and (D.24) from Appendix D and combining the constants of integration:

$$\ln \phi = \frac{-F_o}{V}\,t + K$$

From the initial condition for ϕ, at $t = 0$, $\ln \phi_0 = K$. Substituting this value of K into the equation gives:

$$\ln \phi = \frac{-F_o}{V}\,t + \ln \phi_0$$

$$\ln \frac{\phi}{\phi_0} = \frac{-F_o}{V}\,t$$

$$\phi = \phi_0\, e^{(-F_o/V)\,t}$$

In this problem, $F_o = 4.8 \text{ l h}^{-1}$. Substituting the known values for ϕ_0 and V:

$$\phi = 1\, e^{-0.080\,t}$$

where ϕ is dimensionless and t has units of h. The time between 8 p.m. and midnight is 4 h. From the above equation, at $t = 4$ h, $\phi = 0.73$. Therefore, at midnight, the composition of oil in the tank is 73% vegetable oil and 27% cod-liver oil.

Answer: 73% vegetable oil, 27% cod-liver oil

6.4 Batch growth of bacteria

The general unsteady-state mass-balance equation is Eq. (6.5). For a batch culture, $\hat{M}_i = \hat{M}_o = 0$. For a mass balance on cells, assuming there is no loss of cells from the system, e.g. by lysis, $R_C = 0$. The rate of generation of cells R_G is proportional to the concentration of cells present: $R_G = \mu\, x\, V$ where μ is a constant, x is the cell concentration and V is the culture volume. The total mass of cells M is equal to the culture volume V multiplied by the cell concentration x: $M = V x$. Substituting these results into Eq. (6.5) gives:

$$\frac{d(V x)}{dt} = \mu\, x\, V$$

Assuming that V is constant throughout the batch culture, it can be taken outside of the differential:

$$V \frac{dx}{dt} = \mu\, x\, V$$

$$\frac{dx}{dt} = \mu\, x$$

The differential equation contains only two variables, x and t. Separating variables and integrating:

$$\frac{dx}{x} = \mu\, dt$$

$$\int \frac{dx}{x} = \int \mu\, dt$$

Using integration rules (D.27) and (D.24) from Appendix D and combining the constants of integration:

$$\ln x = \mu\, t + K$$

Assume an initial condition: at $t = 0$ at the beginning of exponential growth, $x = x_0$. Therefore, at $t = 0$, $\ln x_0 = K$. Substituting this value of K into the equation gives:

$$\ln x = \mu t + \ln x_0$$

$$\ln \frac{x}{x_0} = \mu t$$

When $t = 45$ min, $x = 2\,x_0$. Substituting these values into the equation:

$$\ln \frac{2\,x_0}{x_0} = \mu\,(45\text{ min})$$

$$\ln 2 = \mu\,(45\text{ min})$$

$$\mu = 0.0154\text{ min}^{-1}$$

Therefore:

$$\ln \frac{x}{x_0} = 0.0154\,t$$

or

$$x = x_0\,e^{0.0154\,t}$$

where t has units of min. For $t = 12\text{ h} = 12 \times 60 = 720$ min:

$$x = 6.54 \times 10^4\,x_0$$

After 12 h, the cell concentration is 6.54×10^4 times the cell concentration at the beginning of exponential growth. If there is a lag phase, the cell concentration at the beginning of exponential growth is usually very close to that at inoculation; therefore, the cell concentration is about 6.54×10^4 times the inoculum level.

Answer: 6.54×10^4 times the inoculum level

6.5 Radioactive decay

The general unsteady-state mass-balance equation is Eq. (6.5). In this problem, $\hat{M}_\mathrm{i} = \hat{M}_\mathrm{o} = 0$. For a mass balance on isotope, $R_\mathrm{G} = 0$. The rate of isotope decay R_C is proportional to the concentration of isotope present: $R_\mathrm{C} = -k_1\,C\,V$, where k_1 is a constant, C is the isotope concentration and V is the solution volume. The total mass of isotope M is equal to the solution volume V multiplied by the isotope concentration C: $M = V\,C$. Substituting these results into Eq. (6.5) gives:

$$\frac{\mathrm{d}(V\,C)}{\mathrm{d}t} = -k_1\,C\,V$$

If we assume that the density of the solution is constant during isotope decay, V is constant and can be taken outside of the differential and cancelled:

$$V\frac{\mathrm{d}C}{\mathrm{d}t} = -k_1\,C\,V$$

$$\frac{\mathrm{d}C}{\mathrm{d}t} = -k_1\,C$$

The differential equation contains only two variables, C and t. Separating variables and integrating:

$$\frac{\mathrm{d}C}{C} = -k_1\,\mathrm{d}t$$

$$\int \frac{\mathrm{d}C}{C} = \int -k_1\,\mathrm{d}t$$

Using integration rules (D.27) and (D.24) from Appendix D and combining the constants of integration:

$$\ln C = -k_1\,t + K$$

Assume an initial condition: at $t = 0$, $C = C_0$. Therefore, at $t = 0$, $\ln C_0 = K$. Substituting this value of K into the equation:

$$\ln C = -k_1 t + \ln C_0$$

$$\ln \frac{C}{C_0} = -k_1 t$$

(a)
When $t = t_h$, the half-life of the isotope, $C = 0.5\,C_0$. Substituting these values into the equation:

$$\ln \frac{0.5\,C_0}{C_0} = -k_1 t_h$$

$$\ln 0.5 = -k_1 t_h$$

$$t_h = \frac{-\ln 0.5}{k_1}$$

Using mathematical rule (D.10) in Appendix D, $-\ln 0.5 = \ln {}^{1}/_{0.5} = \ln 2$. Therefore:

$$t_h = \frac{\ln 2}{k_1}$$

Answer: Q.E.D.

(b)
From the equation derived in **(a)**:

$$k_1 = \frac{\ln 2}{t_h}$$

For $t_h = 14.3$ days:

$$k_1 = 4.85 \times 10^{-2}\ \mathrm{d}^{-1}$$

Substituting this value of k_1 into the general equation for isotope concentration:

$$\ln \frac{C}{C_0} = -4.85 \times 10^{-2}\, t$$

where t has units of days. For $C = 0.01\,C_0$:

$$\ln \frac{0.01\,C_0}{C_0} = -4.85 \times 10^{-2}\, t$$

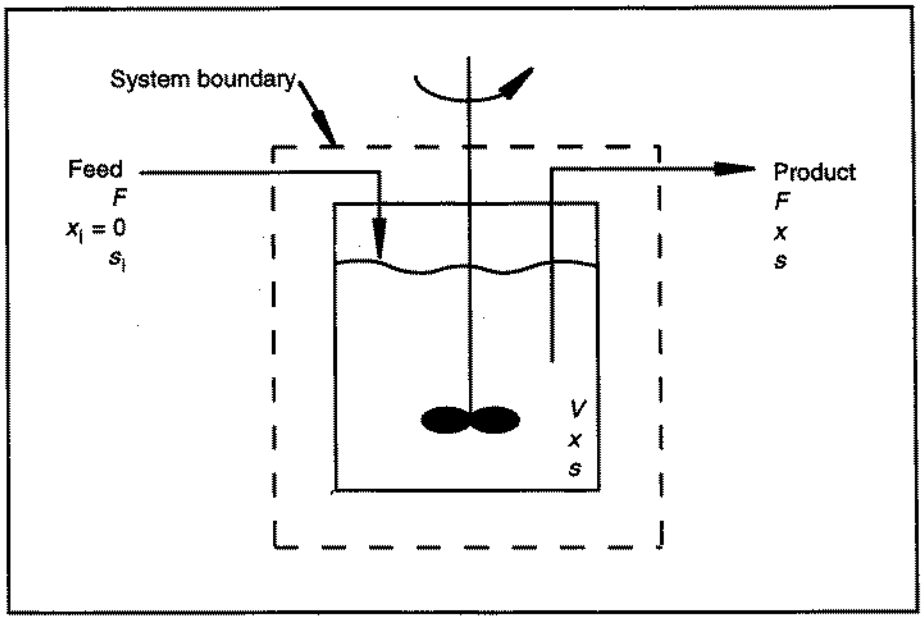
$$\ln 0.01 = -4.85 \times 10^{-2}\, t$$

$$t = 95 \text{ days}$$

Answer: 95 days

6.6 Continuous fermentation

(i) *Flow sheet and system boundary*
These are shown for a continuous fermenter in the figure below.



(ii) *Define variables*
V = volume of broth in the fermenter; F = volumetric flow rate into and out of the vessel; x = concentration of cells; s = concentration of substrate; x_i = concentration of cells in the feed; s_i = concentration of substrate in the feed
(iii) *Assumptions*
- no leaks
- fermenter is well mixed; therefore x and s in the outlet stream = x and s, respectively, inside the fermenter
- density of the fermentation broth is the same as that of the feed

(iv) *Boundary condition*
At $t = 0$, $x = x_0$.
(v) *Total mass balance*
As the volumetric flow rates into and out of the fermenter are the same and the density of the inlet and outlet streams are assumed equal, the volume of broth in the fermenter is constant (see Example 6.2) and equal to V at all times.

(a)
Cell mass balance
The general unsteady-state mass-balance equation is Eq. (6.5). As no cells enter in the feed stream, $\hat{M}_i = 0$. The mass flow rate of cells out $\hat{M}_o = F\,x$. The rate of cell generation $R_G = r_X\,V = k_1\,x\,V$. Assuming that cell lysis is negligible, $R_C = 0$. The mass of cells in the fermenter M is equal to $V\,x$. Substituting these terms into Eq. (6.5) gives:

$$\frac{d(V\,x)}{dt} = -F\,x + k_1\,x\,V$$

As V is constant, it can be taken outside of the differential.

$$V\frac{dx}{dt} = -F\,x + k_1\,x\,V$$

Dividing through by V and grouping terms:

$$\frac{dx}{dt} = x\,(k_1 - \frac{F}{V})$$

Answer: $\frac{dx}{dt} = x\,(k_1 - \frac{F}{V})$

(b)
At steady state, ${}^{dx}/_{dt} = 0$. Therefore, from the equation derived in **(a)**, at steady state k_1 must be equal to ${}^{F}/_{V}$.

Answer: $k_1 = {}^{F}/_{V}$

(c)
As F, V and k_1 are constants, the differential equation derived in **(a)** contains only two variables, x and t. Separating variables and integrating:

$$\frac{dx}{x} = (k_1 - \frac{F}{V})\,dt$$

$$\int \frac{dx}{x} = \int (k_1 - \frac{F}{V})\,dt$$

Using integration rules (D.27) and (D.24) from Appendix D and combining the constants of integration:

$$\ln x = (k_1 - \frac{F}{V})\,t + K$$

From the initial condition for x, at $t = 0$, $\ln x_0 = K$. Substituting this value of K into the equation gives:

$$\ln x = (k_1 - \frac{F}{V})\,t + \ln x_0$$

$$\ln \frac{x}{x_0} = (k_1 - \frac{F}{V})\,t$$

$$x = x_0\, e^{(k_1 - \frac{F}{V})\,t}$$

Answer: $x = x_0\, e^{(k_1 - \frac{F}{V})\,t}$

(d)
Substituting the parameter values into the equation derived in **(c)**:

$$x = (0.5\ \text{g l}^{-1})\, e^{(0.33\ \text{h}^{-1} - \frac{2200\ \text{l h}^{-1}}{10{,}000\ \text{l}})\,t}$$

$$x = 0.5\, e^{0.11\,t}$$

where x has units of g l^{-1} and t has units of h. For $x = 4.0$ g l^{-1}:

$$4 = 0.5\, e^{0.11\,t}$$

$$\ln 8 = 0.11\,t$$

$$t = 18.9\ \text{h}$$

Answer: 18.9 h

(e)
Substrate mass balance
The general unsteady-state mass-balance equation is Eq. (6.5). For substrate, the mass flow rates in and out are $\hat{M}_i = F\,s_i$ and $\hat{M}_o = F\,s$. The rate of substrate generation $R_G = 0$; the rate of substrate consumption $R_C = r_S\,V = k_2\,x\,V$. The mass of substrate in the fermenter M is equal to $V\,s$. Substituting these terms into Eq. (6.5) gives:

$$\frac{d(V\,s)}{dt} = F\,s_i - F\,s - k_2\,x\,V$$

As V is constant, it can be taken outside of the differential:

$$V\frac{ds}{dt} = F\,s_i - F\,s - k_2\,x\,V$$

Dividing through by V and grouping terms:

$$\frac{ds}{dt} = \frac{F}{V}(s_i - s) - k_2\,x$$

Substituting the expression for x from **(c)**:

$$\frac{ds}{dt} = \frac{F}{V}(s_i - s) - k_2 x_0 e^{(k_1 - \frac{F}{V})t}$$

In this equation, F, V, k_2 and x_0 are constants and there are only two variables, s and t. However, the variables cannot be easily separated as in the previous problems, making algebraic solution difficult.

Answer: $\frac{ds}{dt} = \frac{F}{V}(s_i - s) - k_2 x_0 e^{(k_1 - \frac{F}{V})t}$

(f)

An equation for ${}^{ds}/_{dt}$ in terms of x was derived in (e) as:

$$\frac{ds}{dt} = \frac{F}{V}(s_i - s) - k_2 x$$

At steady state, ${}^{ds}/_{dt} = 0$. Therefore:

$$\frac{F}{V}(s_i - s) = k_2 x$$

$$s_i - s = \frac{V k_2}{F} x$$

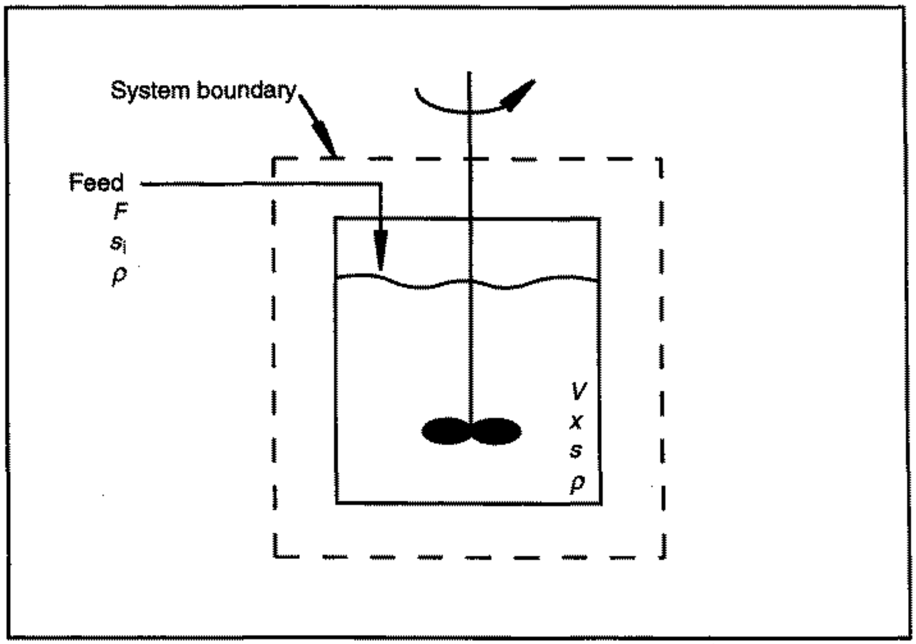
$$s = s_i - \frac{V k_2}{F} x$$

Answer: $s = s_i - \frac{V k_2}{F} x$

6.7 Fed-batch fermentation

(i) *Flow sheet and system boundary*

These are shown for a fed-batch fermenter in the figure below.



(ii) *Define variables*

V = volume of broth in the fermenter; F = volumetric flow rate into the fermenter; x = concentration of cells in the fermenter; s = concentration of substrate in the fermenter; s_i = concentration of substrate in the feed; ρ = density of the feed and fermentation broth

(iii) *Assumptions*

- no leaks
- fermenter is well mixed
- density of the fermentation broth is the same as that of the feed

(iv) *Boundary condition*
At $t = 0$, $V = V_0$.

(a)
Total mass balance
The general unsteady-state mass-balance equation is Eq. (6.5). As total mass cannot be generated or consumed, $R_G = R_C = 0$. No mass leaves the fermenter; therefore $\hat{M}_o = 0$. The mass flow rate in $\hat{M}_i = \rho F$. The total mass in the fermenter M is equal to $V \rho$. Substituting these terms into Eq. (6.5) gives:

$$\frac{d(V\rho)}{dt} = \rho F$$

As ρ is constant, it can be taken outside of the differential and cancelled:

$$\rho \frac{dV}{dt} = \rho F$$

$$\frac{dV}{dt} = F$$

As F is constant, the differential equation contains only two variables, V and t. Separating variables and integrating:

$$dV = F\,dt$$

$$\int dV = \int F\,dt$$

Using integration rule (D.24) from Appendix D and combining the constants of integration:

$$V = F\,t + K$$

From the initial condition for V, at $t = 0$, $V_0 = K$. Substituting this value of K into the equation gives:

$$V = F\,t + V_0$$

Answer: $V = F\,t + V_0$

(b)
Substrate mass balance
The general unsteady-state mass-balance equation is Eq. (6.5). The mass flow rates of substrate in and out are $\hat{M}_i = F s_i$ and $\hat{M}_o = 0$. The rate of substrate generation $R_G = 0$; the rate of substrate consumption $R_C = r_S V = k_1\, s\, V$. The mass of substrate in the fermenter M is equal to $V s$. Substituting these terms into Eq. (6.5) gives:

$$\frac{d(V s)}{dt} = F s_i - k_1 s V$$

As neither V nor s is constant, both must be kept in the differential as a product. Expanding the differential using the product rule (D.22) from Appendix D:

$$V \frac{ds}{dt} + s \frac{dV}{dt} = F s_i - k_1 s V$$

Using the equation $dV/dt = F$ derived in **(a)**:

$$V \frac{ds}{dt} + F s = F s_i - k_1 s V$$

Grouping terms gives:

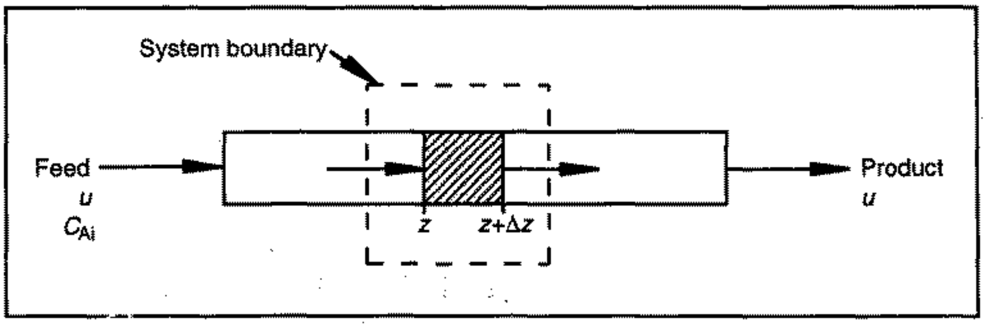
$$\frac{ds}{dt} = \frac{F}{V}(s_i - s) - k_1 s$$

Answer: $\frac{ds}{dt} = \frac{F}{V}(s_i - s) - k_1 s$

6.8 Plug-flow reactor

(i) *Flow sheet and system boundary*

These are shown for a plug-flow reactor in the figure below.



(ii) *Define variables*

u = fluid linear velocity; z = distance along the reactor; C_A = concentration of reactant; C_{Ai} = concentration of reactant in the feed stream; A = reactor cross-sectional area

(iii) *Assumptions*

– no leaks
– plug flow

(a)

Reactant balance

Consider the system to be a small section of the reactor located between z and $z + \Delta z$. The general unsteady-state mass-balance equation is Eq. (6.5). The rate of entry of reactant into the system is:

$$\hat{M}_i = C_A\, u\, A\big|_z$$

where $\big|_z$ means that the parameter values are those at distance z from the front of the reactor. Similarly, the rate at which reactant leaves the system is:

$$\hat{M}_o = C_A\, u\, A\big|_{z+\Delta z}$$

Reactant is not generated; therefore $R_G = 0$. The rate of consumption of reactant is given by the equation:

$$R_C = r_C\, V = r_C\, A\, \Delta z$$

where $A\ \Delta z$ is the volume of the system and r_C is the volumetric rate of reaction. At steady state there is no accumulation in the system and $^{dM}/_{dt} = 0$. Substituting these expressions into Eq. (6.5) gives:

$$0 = C_A\, u\, A\big|_z - C_A\, u\, A\big|_{z+\Delta z} - r_C\, A\, \Delta z$$

As A is constant and does not depend on z, it can be cancelled from each of the terms:

$$0 = C_A\, u\big|_z - C_A\, u\big|_{z+\Delta z} - r_C\, \Delta z$$

Dividing through by Δz:

$$0 = \frac{C_A\, u\big|_z - C_A\, u\big|_{z+\Delta z}}{\Delta z} - r_C$$

Taking the limit as Δz approaches zero and applying the definition of the derivative (D.13) in Appendix D:

$$0 = \frac{-d(C_A\, u)}{dz} - r_C$$

or

$$\frac{d(C_A\, u)}{dz} = -r_C$$

As the fluid velocity is constant throughout the reactor, u can be taken outside of the differential:

$$u \frac{dC_A}{dz} = -r_C$$

Answer: $u \frac{dC_A}{dz} = -r_C$

(b)
Answer: at $z = 0$, $C_A = C_{Ai}$

(c)
If the reaction is first-order, $r_C = k_1 \, C_A$ where k_1 is the first-order rate constant. The differential equation becomes:

$$u \frac{dC_A}{dz} = -k_1 \, C_A$$

As u and k_1 are constants, the differential equation contains only two variables, C_A and z. Separating variables and integrating:

$$\frac{dC_A}{C_A} = \frac{-k_1}{u} dz$$

$$\int \frac{dC_A}{C_A} = \int \frac{-k_1}{u} dz$$

Using integration rules (D.27) and (D.24) from Appendix D and combining the constants of integration:

$$\ln C_A = \frac{-k_1}{u} z + K$$

From the initial condition in **(b)**, at $z = 0$, $\ln C_{Ai} = K$. Substituting this value of K into the equation gives:

$$\ln C_A = \frac{-k_1}{u} z + \ln C_{Ai}$$

$$\ln \frac{C_A}{C_{Ai}} = \frac{-k_1}{u} z$$

$$C_A = C_{Ai} \, e^{(-k_1/u)\, z}$$

Answer: $C_A = C_{Ai} \, e^{(-k_1/u)\, z}$

(d)
The equation derived in **(c)** is directly analogous to the equation for change in reactant concentration in a batch reactor. As $z = u\,t$ where t is the time taken for the fluid to travel distance z, the above equation can be written as:

$$C_A = C_{Ai} \, e^{-k_1 t}$$

which is the same as the equation for reactant concentration in a batch reactor where C_{Ai} is the concentration at time zero.

Answer: Essentially identical

6.9 Boiling water

The system is the beaker containing water.
(i) *Assumptions*
- no evaporation
- water is well mixed
- no shaft work
- heat capacity is independent of temperature
- heat losses are negligible
- the density of water is constant between 18°C and 100°C

(ii) *Extra data*
Density of water = 1 kg l^{-1}
C_p water = 75.4 J $gmol^{-1}$ $°C^{-1}$ (Table B.3, Appendix B) = 75.4 kJ $kgmol^{-1}$ $°C^{-1}$
Molecular weight of water (Table B.1, Appendix B) = 18.0
1 W = 1 J s^{-1} (Table A.8, Appendix A); therefore, 1 kW = 1 kJ s^{-1}
(iii) *Boundary conditions*
At $t = 0$, $T = T_0 = 18°C$

(a)
The general unsteady-state energy-balance equation is Eq. (6.10). For a batch system, $\hat{M}_i = \hat{M}_o = 0$; also $\hat{W}_s = 0$. Energy is accumulated by the system in the form of sensible heat only; therefore:

$$\frac{dE}{dt} = M\,C_p\,\frac{dT}{dt}$$

where M is the mass of water in the beaker and T is its temperature. Substituting these expressions into Eq. (6.10) gives:

$$M\,C_p\,\frac{dT}{dt} = -\hat{Q}$$

Answer: $M\,C_p\,\frac{dT}{dt} = -\hat{Q}$

(b)
If $\hat{Q}$, C_p and M are constant, T and t are the only variables in the differential equation. Separating variables and integrating:

$$dT = \frac{-\hat{Q}}{M\,C_p}\,dt$$

$$\int dT = \int \frac{-\hat{Q}}{M\,C_p}\,dt$$

Using integration rule (D.24) from Appendix D and combining the constants of integration:

$$T = \frac{-\hat{Q}}{M\,C_p}\,t + K$$

From the initial condition for T, at $t = 0$, $T_0 = K$. Substituting this value of K into the equation gives:

$$T = \frac{-\hat{Q}}{M\,C_p}\,t + T_0$$

Using the density of water = 1 kg l^{-1}, the mass of 2 litres of water $M = 2$ kg. Converting the C_p for water to mass terms:

$$C_p = 75.4\,\text{kJ kgmol}^{-1}\,°\text{C}^{-1}\cdot\left|\frac{1\,\text{kgmol}}{18.0\,\text{kg}}\right| = 4.189\,\text{kJ kg}^{-1}\,°\text{C}^{-1}$$

T = the boiling temperature of water = 100°C; t = 11 min. Substituting the parameter values into the equation for T:

$$100°\text{C} = \frac{-\hat{Q}}{2\,\text{kg}\left(4.189\,\text{kJ kg}^{-1}\,°\text{C}^{-1}\right)}(11\,\text{min}) + 18°\text{C}$$

$$\hat{Q} = -62.45\,\text{kJ min}^{-1} = -62.45\,\text{kJ min}^{-1}\cdot\left|\frac{1\,\text{min}}{60\,\text{s}}\right|\cdot\left|\frac{1\,\text{kW}}{1\,\text{kJ s}^{-1}}\right| = -1.04\,\text{kW}$$

From the sign conventions outlined on pp 87–88, the negative value for $\hat{Q}$ confirms that heat is added to the system.

Answer: 1.04 kW

6.10 Heating glycerol solution

The system is the stirred tank containing the solution of glycerol in water.

(i) *Assumptions*

- no leaks
- no evaporation
- tank is well mixed
- no shaft work
- heat capacities are independent of temperature between 15°C and 90°C
- ideal solution
- system is adiabatic; therefore heat losses are negligible

(ii) *Extra data*

C_p glycerol = 0.576 cal g^{-1} °C^{-1} (Table B.5, Appendix B) = 0.576 kcal kg^{-1} °C^{-1}
C_p water = 75.4 J $gmol^{-1}$ °C^{-1} (Table B.3, Appendix B) = 75.4 kJ $kgmol^{-1}$ °C^{-1}
Molecular weight of water (Table B.1, Appendix B) = 18.0
1 kcal = 4.187×10^3 J (Table A.7, Appendix A) = 4.187 kJ
1 W = 1 J s^{-1} (Table A.8, Appendix A); therefore, 1 kW = 1 kJ s^{-1}

(iii) *Boundary conditions*

At $t = 0$, $T = T_0 = 15$°C

(a)

The general unsteady-state energy-balance equation is Eq. (6.10). For a batch system, $\hat{M}_i = \hat{M}_o = 0$; also $\hat{W}_s = 0$. Energy is accumulated by the system in the form of sensible heat only; therefore:

$$\frac{dE}{dt} = M\,C_p\frac{dT}{dt}$$

where M is the mass of glycerol solution in the tank and T is its temperature. Substituting these expressions into Eq. (6.10) gives:

$$M\,C_p\frac{dT}{dt} = -\hat{Q}$$

M has two components, glycerol and water, which have different heat capacities. Therefore, this equation can be written:

$$\left(M_W\,C_{pW} + M_G\,C_{pG}\right)\frac{dT}{dt} = -\hat{Q}$$

where M_W is the mass of water in the tank, M_G is the mass of glycerol, C_{pW} is the heat capacity of water, and C_{pG} is the heat capacity of glycerol.

Answer: $\left(M_W\,C_{pW} + M_G\,C_{pG}\right)\frac{dT}{dt} = -\hat{Q}$

(b)

If $\hat{Q}$, C_{pW}, C_{pG}, M_W and M_G are constant, T and t are the only variables in the differential equation. Separating variables and integrating:

$$dT = \frac{-\hat{Q}}{\left(M_W\,C_{pW} + M_G\,C_{pG}\right)}\,dt$$

$$\int dT = \int \frac{-\hat{Q}}{\left(M_W\,C_{pW} + M_G\,C_{pG}\right)}\,dt$$

Using integration rule (D.24) from Appendix D and combining the constants of integration:

$$T = \frac{-\hat{Q}}{\left(M_W\,C_{pW} + M_G\,C_{pG}\right)}\,t + K$$

From the initial condition for T, at $t = 0$, $T_0 = K$. Substituting this value of K into the equation gives:

$$T = \frac{-\hat{Q}}{\left(M_W\, C_{pW} + M_G\, C_{pG}\right)}\, t + T_0$$

Answer: $T = \frac{-\hat{Q}}{\left(M_W\, C_{pW} + M_G\, C_{pG}\right)}\, t + T_0$

(c)
The mass of glycerol in the tank $M_G = 45$ kg; the mass of water $M_W = 55$ kg. Converting the C_p for water to mass terms:

$$C_{pW} = 75.4\,\text{kJ kgmol}^{-1}\,{}^\circ\text{C}^{-1}\,.\left|\frac{1\,\text{kgmol}}{18.0\,\text{kg}}\right| = 4.189\,\text{kJ kg}^{-1}\,{}^\circ\text{C}^{-1}$$

Converting the C_p for glycerol to kJ:

$$C_{pG} = 0.576\,\text{kcal kg}^{-1}\,{}^\circ\text{C}^{-1}\,.\left|\frac{4.187\,\text{kJ}}{1\,\text{kcal}}\right| = 2.412\,\text{kJ kg}^{-1}\,{}^\circ\text{C}^{-1}$$

The rate of heat input to the system is 0.88 × 2.5 kW = 2.2 kW = 2.2 kJ s^{-1}. From the sign conventions outlined on pp 87–88, $\hat{Q}$ must be negative as heat is added to the system; therefore, $\hat{Q} = -2.2$ kJ s^{-1}. Substituting the parameter values into the equation for T with $T_0 = 15$°C and $T = 90$°C:

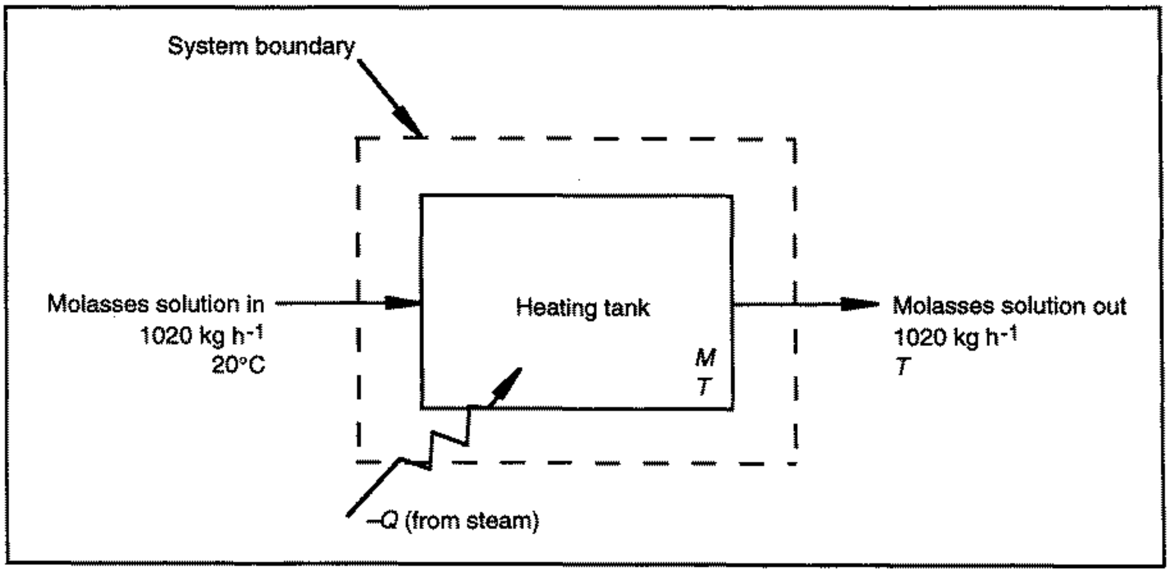
$$90^\circ\text{C} = \frac{2.2\,\text{kJ s}^{-1}}{55\,\text{kg}\left(4.189\,\text{kJ kg}^{-1}\,{}^\circ\text{C}^{-1}\right) + 45\,\text{kg}\left(2.412\,\text{kJ kg}^{-1}\,{}^\circ\text{C}^{-1}\right)}\, t + 15^\circ\text{C}$$

$$t = 1.16\times 10^4\,\text{s} = 1.16\times 10^4\,\text{s}\,.\left|\frac{1\,\text{h}}{3600\,\text{s}}\right| = 3.2\,\text{h}$$

Answer: 3.2 h

6.11 Heating molasses

(i) *Flow sheet and system boundary*
These are shown in the figure below.



(ii) *Assumptions*
- no leaks
- no evaporation
- tank is well mixed; therefore the temperature of the molasses solution out is the same as in the tank
- no shaft work
- heat capacity is independent of temperature
- negligible heat losses

– condensate from the steam leaves at saturation conditions

(iii) *Reference state*

T_{ref} = 20°C; $H = 0$ for molasses solution at 20°C

(iv) *Extra data*

$C_p = 0.85 \text{ kcal kg}^{-1} \,°\text{C}^{-1}$

1 psi = 6.895×10^3 Pa (Table A.5, Appendix A) = 6.895 kPa

Converting 40 psi to kPa:

$$40 \text{ psi} = 40 \text{ psi} \cdot \left| \frac{6.895 \text{ kPa}}{1 \text{ psi}} \right| = 275.8 \text{ kPa}$$

The temperature of saturated steam at 275.8 kPa interpolated from Table C.2 (Appendix C) = 130.7°C.

(v) *Boundary conditions*

At $t = 0$, $T = T_0 = 20°\text{C}$

(vi) *Total mass balance*

As the mass flow rates into and out of the tank are the same, the mass of molasses solution in the tank M is constant and equal to 5000 kg at all times.

(a)

The general unsteady-state energy-balance equation is Eq. (6.10). From the reference state, $h_i = 0$. The value of h_o relative to the reference state is equal to the sensible heat absorbed by the molasses solution between T_{ref} and the exit temperature T. From Eq. (5.13):

$$h_o = \Delta h = C_p (T - T_{ref})$$

$\hat{W}_s = 0$. The rate at which the molasses solution is heated is given by the equation:

$$\hat{Q} = -UA\,(T_{steam} - T)$$

From the sign conventions outlined on pp 87–88, $\hat{Q}$ must be negative as heat is added to the system. Energy is accumulated in the form of sensible heat only; therefore:

$$\frac{dE}{dt} = M\,C_p \frac{dT}{dt}$$

Substituting these expressions into Eq. (6.10) gives:

$$M\,C_p \frac{dT}{dt} = -\hat{M}_o\,C_p\,(T - T_{ref}) + UA\,(T_{steam} - T)$$

After rearranging, the differential equation is:

$$\frac{dT}{dt} = \frac{UA\,T_{steam} + \hat{M}_o\,C_p\,T_{ref}}{M\,C_p} - \left(\frac{\hat{M}_o\,C_p + UA}{M\,C_p}\right) T$$

Answer: $\frac{dT}{dt} = \frac{UA\,T_{steam} + \hat{M}_o\,C_p\,T_{ref}}{M\,C_p} - \left(\frac{\hat{M}_o\,C_p + UA}{M\,C_p}\right) T$

(b)

As U, A, T_{steam}, $\hat{M}_o$, C_p, T_{ref} and M are all constant, T and t are the only variables in the differential equation derived in **(a)**. Substituting the known numerical values for the parameters:

$$\frac{dT}{dt} = \frac{190 \text{ kcal m}^{-2} \text{ h}^{-1} \,°\text{C}^{-1}\,(1.5 \text{ m}^2)\,(130.7°\text{C}) + 1020 \text{ kg h}^{-1}\left(0.85 \text{ kcal kg}^{-1} \,°\text{C}^{-1}\right)(20°\text{C})}{5000 \text{ kg}\left(0.85 \text{ kcal kg}^{-1} \,°\text{C}^{-1}\right)}$$
$$- \left(\frac{1020 \text{ kg h}^{-1}\left(0.85 \text{ kcal kg}^{-1} \,°\text{C}^{-1}\right) + 190 \text{ kcal m}^{-2} \text{ h}^{-1} \,°\text{C}^{-1}\,(1.5 \text{ m}^2)}{5000 \text{ kg}\left(0.85 \text{ kcal kg}^{-1} \,°\text{C}^{-1}\right)}\right) T$$

$$\frac{dT}{dt} = 12.84 - 0.271\,T$$

where T has units °C and t has units h. Separating variables and integrating:

$$\frac{dT}{12.84 - 0.271\,T} = dt$$

$$\int \frac{dT}{12.84 - 0.271\,T} = \int dt$$

Using integration rules (D.28) and (D.24) from Appendix D and combining the constants of integration:

$$\frac{-1}{0.271} \ln(12.84 - 0.271\,T) = t + K$$

From the initial condition for T, at $t = 0$:

$$K = \frac{-1}{0.271} \ln(12.84 - 0.271\,T_0)$$

Applying the numerical value of $T_0 = 20$°C, $K = -7.395$. Substituting this value for K into the equation gives:

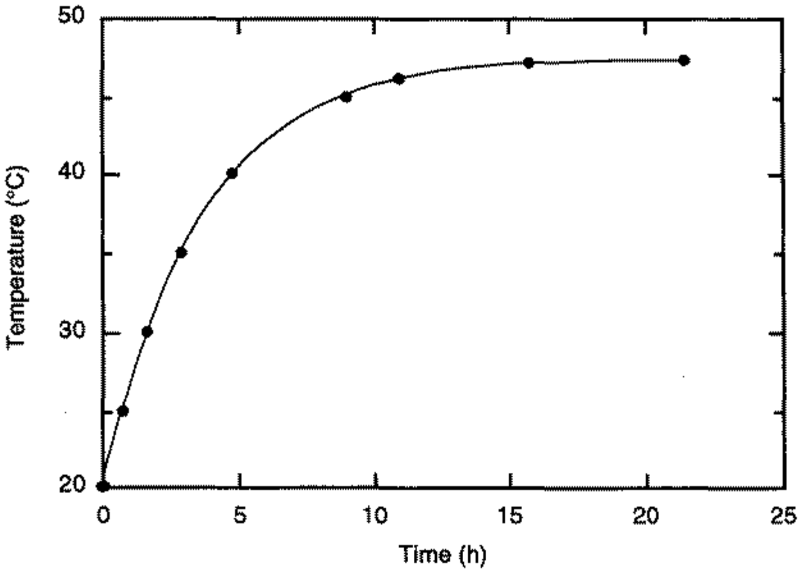
$$\frac{-1}{0.271} \ln(12.84 - 0.271\,T) + 7.395 = t$$

Answer: $\frac{-1}{0.271} \ln(12.84 - 0.271\,T) + 7.395 = t$

(c)
Values of t corresponding to various temperatures in the tank can be calculated from the equation derived in **(b)**.

Temperature, T (°C)	Time, t (h)
20	0.00
25	0.74
30	1.68
35	2.93
40	4.84
45	9.01
46	11.0
47	15.8
47.3	21.5

The results are shown in the figure below.



(d)

From the equation derived in **(b)**, as the logarithm of zero and negative numbers is not defined (p 413), the theoretical maximum temperature that can be achieved in the tank occurs when $12.84 - 0.271\,T = 0$; i.e. when $T = 47.4$°C.

Answer: 47.4°C

(e)

The temperature changes constantly with time; therefore, strictly speaking, there is no steady state. For practical purposes, however, the temperature approaches a constant value after about 16 h.

Answer: About 16 h

(f)

From the calculation table in **(c)**, the temperature reaches 40°C after 4.84 h.

Answer: 4.84 h

6.12 Pre-heating culture medium

The system is the glass fermenter containing nutrient medium.

(i) *Assumptions*

- no evaporation
- fermenter is well mixed
- no shaft work
- heat capacities are independent of temperature between 15°C and 36°C
- heat losses are negligible

(ii) *Extra data*

C_p glass vessel = $0.20\ \text{cal g}^{-1}\,°\text{C}^{-1} = 0.20\ \text{kcal kg}^{-1}\,°\text{C}^{-1}$

C_p medium = $0.92\ \text{cal g}^{-1}\,°\text{C}^{-1} = 0.92\ \text{kcal kg}^{-1}\,°\text{C}^{-1}$

$1\ \text{W} = 1.433 \times 10^{-2}\ \text{kcal min}^{-1}$ (Table A.8, Appendix A)

(iii) *Boundary conditions*

At $t = 0$, $T = T_0 = 15$°C

(iv) *Energy balance*

The general unsteady-state energy-balance equation is Eq. (6.10). For a batch system, $\hat{M}_i = \hat{M}_o = 0$; also $\hat{W}_s = 0$. Energy is accumulated by the system in the form of sensible heat only; therefore:

$$\frac{dE}{dt} = M\,C_p\,\frac{dT}{dt}$$

M has two components, the glass vessel and the medium, which have different heat capacities. Therefore, this equation can be written:

$$\frac{dE}{dt} = \left(M_V\,C_{pV} + M_M\,C_{pM}\right)\frac{dT}{dt}$$

where M_V is the mass of the glass vessel, M_M is the mass of the medium, C_{pV} is the heat capacity of the vessel, and C_{pM} is the heat capacity of the medium. Substituting into Eq. (6.10) gives:

$$\left(M_V\,C_{pV} + M_M\,C_{pM}\right)\frac{dT}{dt} = -\hat{Q}$$

As $\hat{Q}$, C_{pV}, C_{pM}, M_V and M_M are constant, T and t are the only variables in the differential equation. Separating variables and integrating:

$$dT = \frac{-\hat{Q}}{\left(M_V\,C_{pV} + M_M\,C_{pM}\right)}\,dt$$

$$\int dT = \int \frac{-\hat{Q}}{\left(M_V\,C_{pV} + M_M\,C_{pM}\right)}\,dt$$

Using integration rule (D.24) from Appendix D and combining the constants of integration:

$$T = \frac{-\hat{Q}}{\left(M_V\, C_{pV} + M_M\, C_{pM}\right)}\, t + K$$

From the initial condition for T, at $t = 0$, $T_0 = K$. Substituting this value of K into the equation gives:

$$T = \frac{-\hat{Q}}{\left(M_V\, C_{pV} + M_M\, C_{pM}\right)}\, t + T_0$$

The rate of heat input to the system is 450 W. Converting this to kcal min^{-1}:

$$\hat{Q} = -450\,\text{W} = -450\,\text{W}\cdot\left|\frac{1.433\times 10^{-2}\,\text{kcal min}^{-1}}{1\,\text{W}}\right| = -6.45\,\text{kcal min}^{-1}$$

From the sign conventions outlined on pp 87–88, $\hat{Q}$ must be negative as heat is added to the system. Substituting this and the other parameter values into the equation for T with $T_0 = 15°\text{C}$ and $T = 36°\text{C}$:

$$36°\text{C} = \frac{6.45\,\text{kcal min}^{-1}}{12.75\,\text{kg}\left(0.20\,\text{kcal kg}^{-1}\,°\text{C}^{-1}\right) + 7.5\,\text{kg}\left(0.92\,\text{kcal kg}^{-1}\,°\text{C}^{-1}\right)}\, t + 15°\text{C}$$

$$t = 30.8\,\text{min}$$

Answer: 30.8 min

6.13 Water heater

The system is the tank containing the water.

(i) *Assumptions*

- no leaks
- no evaporation
- tank is well mixed
- no shaft work
- heat capacity is independent of temperature
- condensate from the steam leaves at saturation conditions

(ii) *Extra data*

C_p water = 75.4 J gmol^{-1} °C^{-1} (Table B.3, Appendix B) = 75.4 kJ kgmol^{-1} °C^{-1}
Molecular weight of water (Table B.1, Appendix B) = 18.0
1 kcal = 4.187×10^3 J (Table A.7, Appendix A) = 4.187 kJ

(iii) *Boundary conditions*

At $t = 0$, $T = T_0 = 24°\text{C}$

(iv) *Energy balance*

The general unsteady-state energy-balance equation is Eq. (6.10). For a batch system, $\hat{M}_i = \hat{M}_o = 0$; also $\hat{W}_s = 0$. Energy is accumulated by the system in the form of sensible heat only; therefore:

$$\frac{dE}{dt} = M\, C_p \frac{dT}{dt}$$

where M is the mass of water in the tank and T is its temperature. Substituting these expressions into Eq. (6.10) gives:

$$M\, C_p \frac{dT}{dt} = -\hat{Q}$$

(a)

There are two components to $\hat{Q}$: the rate of heating from the steam, and the rate of heat loss to the surrounding air:

$$\hat{Q} = U_1 A_1 (T - T_{air}) - U_2 A_2 (T_{steam} - T)$$

This equation reflects the sign conventions outlined on pp 87–88: the term for the heat loss to the atmosphere is positive to indicate heat removal from the system, while the term for heat input from the steam is negative. Substituting into Eq. (6.10) gives:

$$M\,C_p\frac{dT}{dt} = U_2A_2\,(T_{\text{steam}}-T) - U_1A_1\,(T-T_{\text{air}})$$

$$\frac{dT}{dt} = \frac{U_2A_2\,(T_{\text{steam}}-T) - U_1A_1\,(T-T_{\text{air}})}{M\,C_p}$$

As U_1, A_1, U_2, A_2, T_{steam}, T_{air}, C_p and M are all constant, T and t are the only variables in the differential equation. Substituting the known parameter values:

$$\frac{dT}{dt} = \frac{220\text{ kcal m}^{-2}\text{ h}^{-1}\,°\text{C}^{-1}\,(0.3\text{ m}^2)\,(130-T)°\text{C} - 25\text{ kcal m}^{-2}\text{ h}^{-1}\,°\text{C}^{-1}\,(0.9\text{ m}^2)\,(T-20)°\text{C}}{1000\text{ kg}\left(75.4\text{ kJ kgmol}^{-1}\,°\text{C}^{-1}\cdot\left|\frac{1\text{ kgmol}}{18.0\text{ kg}}\right|\cdot\left|\frac{1\text{ kcal}}{4.187\text{ kJ}}\right|\right)}$$

$$\frac{dT}{dt} = 9.026 - 0.088\,T$$

where T has units of °C and t has units of h. Separating variables and integrating:

$$\frac{dT}{9.026-0.088\,T} = dt$$

$$\int\frac{dT}{9.026-0.088\,T} = \int dt$$

Using integration rules (D.28) and (D.24) from Appendix D and combining the constants of integration:

$$\frac{-1}{0.088}\ln(9.026-0.088\,T) = t+K$$

From the initial condition for T, at $t = 0$:

$$K = \frac{-1}{0.088}\ln(9.026-0.088\,T_0)$$

Applying the numerical value of $T_0 = 24°\text{C}$, $K = -21.97$. Substituting this value for K into the equation gives:

$$\frac{-1}{0.088}\ln(9.026-0.088\,T) + 21.97 = t$$

From this equation, for $T = 80°\text{C}$, $t = 14.2$ h.

Answer: 14.2 h

(b)
If heat losses can be neglected, $\hat{Q}$ has only one component and Eq. (6.10) becomes:

$$M\,C_p\frac{dT}{dt} = U_2A_2\,(T_{\text{steam}}-T)$$

$$\frac{dT}{dt} = \frac{U_2A_2\,(T_{\text{steam}}-T)}{M\,C_p}$$

Substituting the known numerical values:

$$\frac{dT}{dt} = \frac{220\text{ kcal m}^{-2}\text{ h}^{-1}\,°\text{C}^{-1}\,(0.3\text{ m}^2)\,(130-T)°\text{C}}{1000\text{ kg}\left(75.4\text{ kJ kgmol}^{-1}\,°\text{C}^{-1}\cdot\left|\frac{1\text{ kgmol}}{18.0\text{ kg}}\right|\cdot\left|\frac{1\text{ kcal}}{4.187\text{ kJ}}\right|\right)}$$

$$\frac{dT}{dt} = 8.576 - 0.066\,T$$

where T has units of °C and t has units of h. Separating variables and integrating:

$$\frac{dT}{8.576-0.066\,T} = dt$$

$$\int \frac{dT}{8.576-0.066\,T} = \int dt$$

Using integration rules (D.28) and (D.24) from Appendix D and combining the constants of integration:

$$\frac{-1}{0.066} \ln(8.576-0.066\,T) = t + K$$

From the initial condition for T, at $t = 0$:

$$K = \frac{-1}{0.066} \ln(8.576-0.066\,T_0)$$

Applying the numerical value of $T_0 = 24$°C, $K = -29.47$. Substituting this value for K into the equation gives:

$$\frac{-1}{0.066} \ln(8.576-0.066\,T) + 29.47 = t$$

From this equation, for $T = 80$°C, $t = 11.4$ h. Therefore, from (**a**), the time saved is (14.2 − 11.4) h = 2.8 h, which corresponds to 20% of the time required when heat losses occur.

Answer: 2.8 h, or 20% of the time required when heat losses occur

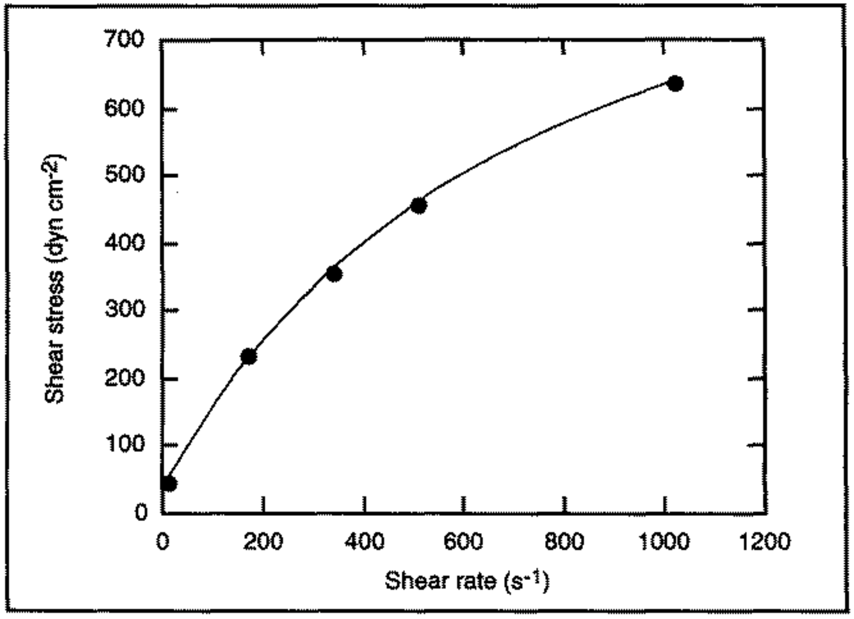
7

Fluid Flow and Mixing

7.1 Rheology of fermentation broth

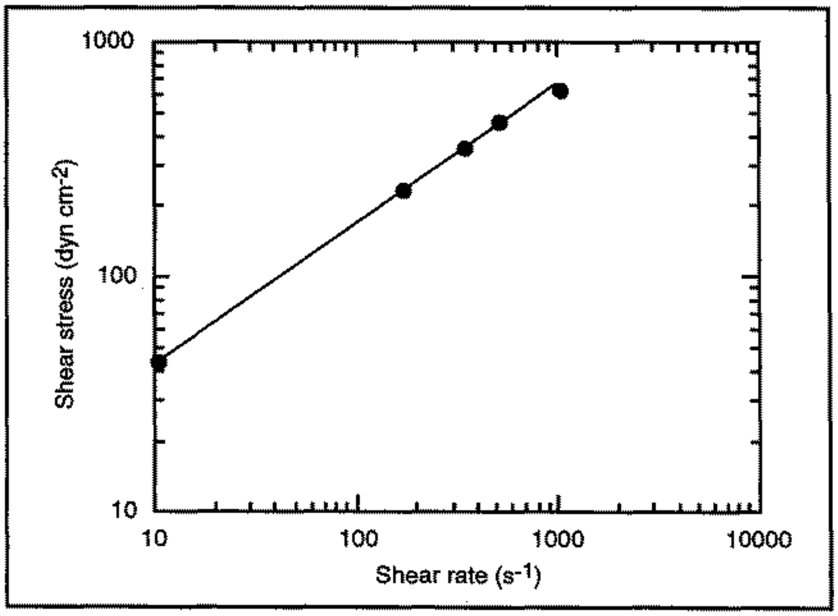
(a)

The rheogram is obtained by plotting shear stress against shear rate.



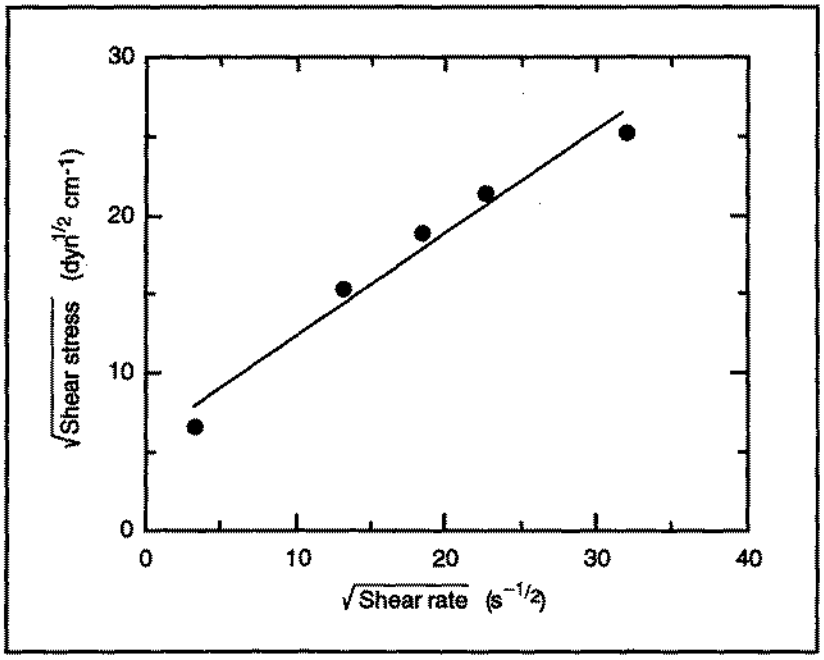
(b)

The rheogram in **(a)** is similar to those for pseudoplastic and Casson plastic fluids in Figure 7.7. It is not clear from the rheogram whether the fluid exhibits a yield stress at $\dot{\gamma} = 0$ or not; therefore, both non-Newtonian models are worth checking. From the equation in Figure 7.7 for pseudoplastic fluids, a plot of shear stress versus shear rate on log–log coordinates would be expected to give a straight line. This plot is shown below.



The equation for the straight line in the log–log plot is $\tau = 11.43\,\dot{\gamma}^{0.587}$. Therefore, the flow behaviour index n = 0.587 and the consistency index K = 11.43 dyn s^n cm^{-2}. The sum of squares of the residuals for this data fit is 1137.

From the equation in Figure 7.7 for Casson plastic fluids, a plot of the square root of shear stress versus the square root of shear rate on linear coordinates would be expected to give a straight line. This plot is shown below.



The equation for the straight line in the plot is $\tau^{1/2} = 5.93 + 0.648\,\dot{\gamma}^{1/2}$. From the equation in Figure 7.7, this means that $K_p = 0.648$ dyn$^{1/2}$ s$^{1/2}$ cm^{-1} and the yield stress τ_0 = 35.2 dyn cm^{-2}. The sum of squares of the residuals for this data fit is 8986.

Comparison of the residuals from the two models suggests that the equation for a pseudoplastic fluid is the better fit.

Answer: Pseudoplastic fluid: $n = 0.587$; K = 11.43 dyn s^n cm^{-2}

(c)
The apparent viscosity for a pseudoplastic fluid is given by Eq. (7.8) and can be calculated using the parameter values determined in **(b)**.
(i)

$$\mu_a = K\,\dot{\gamma}^{n-1} = 11.43\,\text{dyn}\,\text{s}^{0.587}\,\text{cm}^{-2}\,(15\,\text{s}^{-1})^{0.587-1} = 3.7\,\text{dyn}\,\text{s}\,\text{cm}^{-2}$$

Answer: 3.7 dyn s cm^{-2}

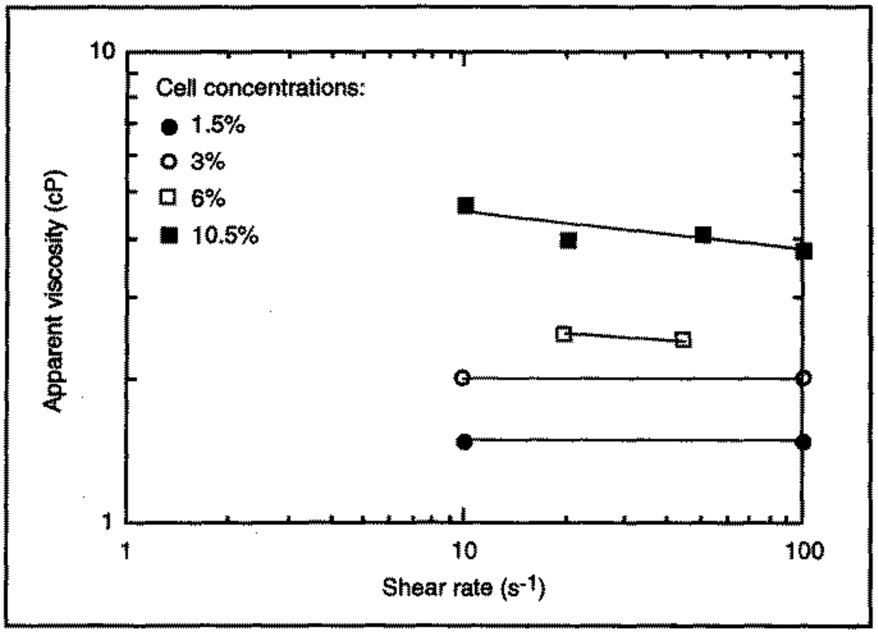
(ii)

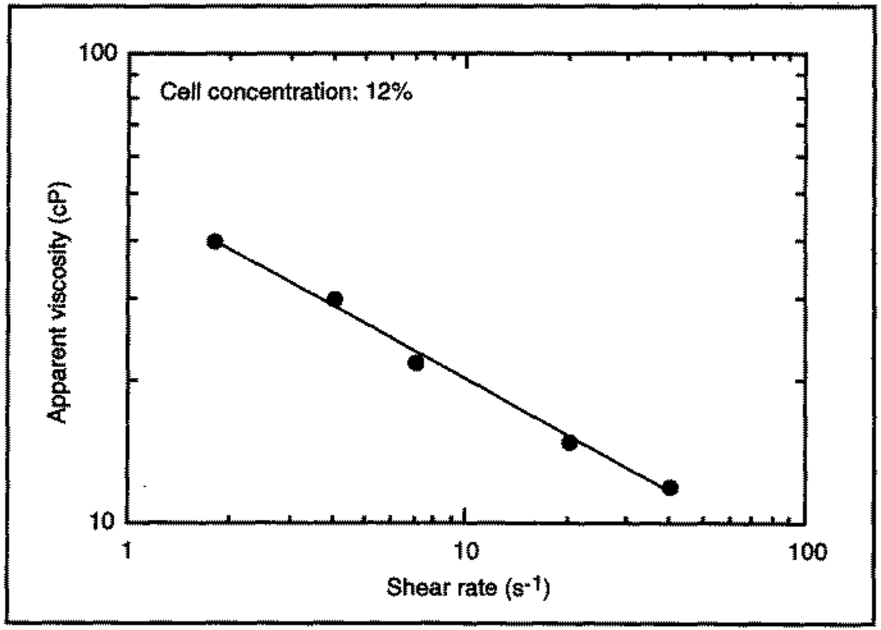
$$\mu_a = K\,\dot{\gamma}^{n-1} = 11.43\,\text{dyn}\,\text{s}^{0.587}\,\text{cm}^{-2}\,(200\,\text{s}^{-1})^{0.587-1} = 1.3\,\text{dyn}\,\text{s}\,\text{cm}^{-2}$$

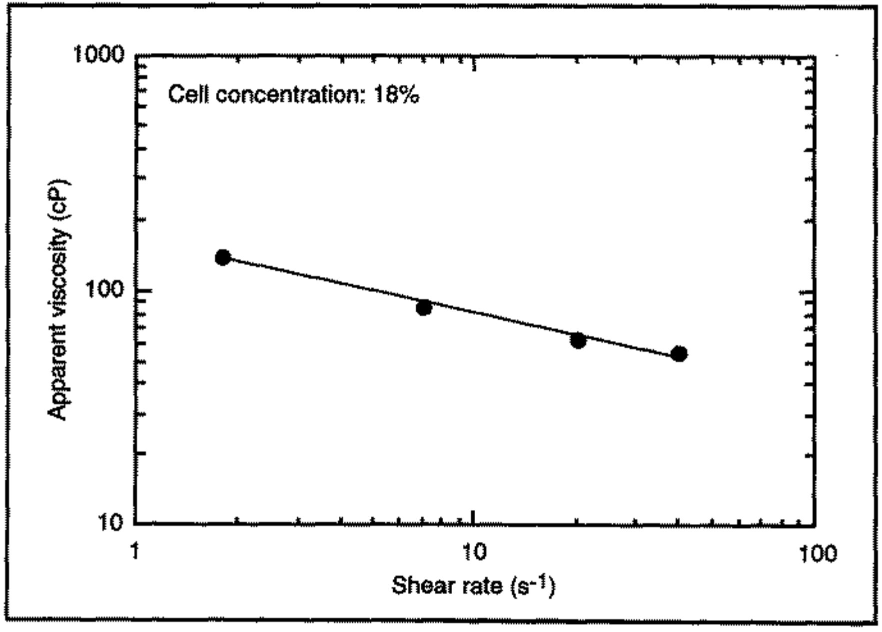
Answer: 1.3 dyn s cm^{-2}

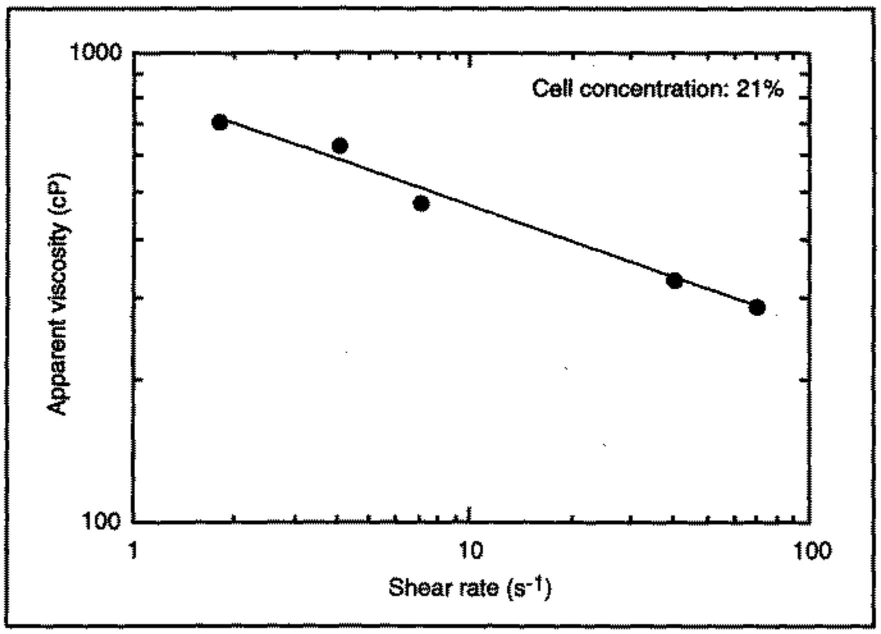
7.2 Rheology of yeast suspensions

From Eq. (7.8), for pseudoplastic fluids, a plot of apparent viscosity versus shear rate on log–log coordinates can be expected to give a straight line. Log–log plots for the different cell concentrations are shown below.


10
1
1
10
100
Apparent viscosity (cP)
Shear rate (s^{-1})
Cell concentrations:
● 1.5%
○ 3%
☐ 6%
■ 10.5%


100
10
1
10
100
Apparent viscosity (cP)
Shear rate (s^{-1})
Cell concentration: 12%

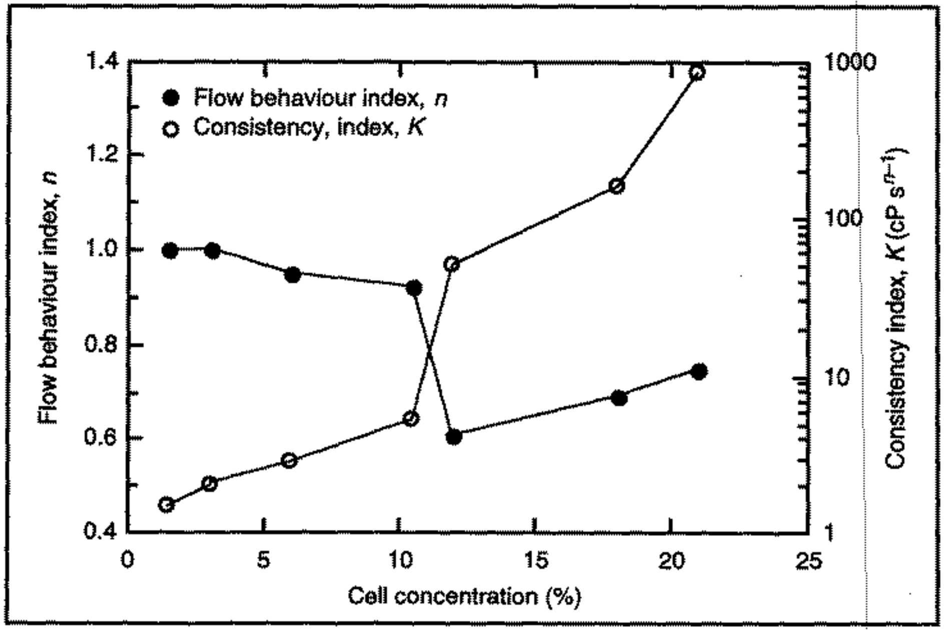

1000
100
10
1
10
100
Apparent viscosity (cP)
Shear rate (s^{-1})
Cell concentration: 18%



The equations and parameter values for the straight lines in each plot are listed below.

Cell concentration (%)	Equation	Flow behaviour index, *n*	Consistency index, *K* ($cP\ s^{n-1}$)
1.5	$\mu_a = 1.5\,\dot{\gamma}^0 = 1.5$	1	1.5
3	$\mu_a = 2.0\,\dot{\gamma}^0 = 2.0$	1	2.0
6	$\mu_a = 2.91\,\dot{\gamma}^{-0.050}$	0.95	2.9
10.5	$\mu_a = 5.38\,\dot{\gamma}^{-0.076}$	0.92	5.4
12	$\mu_a = 50.1\,\dot{\gamma}^{-0.395}$	0.61	50
18	$\mu_a = 162\,\dot{\gamma}^{-0.307}$	0.69	160
21	$\mu_a = 833\,\dot{\gamma}^{-0.251}$	0.75	830

K and *n* are plotted as a function of cell concentration below.



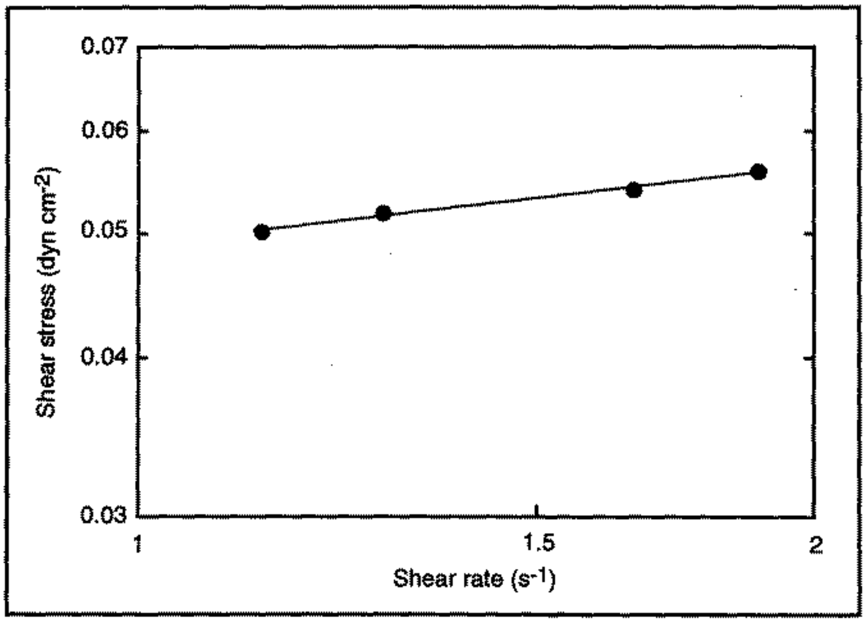
The cell broth is Newtonian up to a cell concentration of about 2%, then becomes pseudoplastic. The flow behaviour index continues to decrease until a cell concentration of about 12% is reached. The consistency index rises throughout the culture with increasing cell concentration.

7.3 Impeller viscometer

(a)

If the rheology can be described using a power-law model, a plot of shear stress versus shear rate on log–log coordinates can be expected to give a straight line. Values of shear stress and shear rate can be determined from torque and stirrer speed data using Eqs (7.11) and (7.12) with $k = 10.2$ and $D_i = 4$ cm $= 4 \times 10^{-2}$ m. The results are listed below and plotted below.

Stirrer speed (s^{-1})	Torque (N m)	Shear stress (N m^{-2})	Shear rate (s^{-1})
0.185	3.57×10^{-6}	0.0559	1.89
0.163	3.45×10^{-6}	0.0540	1.66
0.126	3.31×10^{-6}	0.0518	1.29
0.111	3.20×10^{-6}	0.0501	1.13



The equation for the straight line on the log–log plot is $\tau = 0.049\,\dot{\gamma}^{0.20}$. From Eq. (7.7), this means that the flow behaviour index n is 0.20 and the consistency index K is 0.049 N s^n m^{-2}.

Answer: Yes: $n = 0.20$; $K = 0.049$ N s^n m^{-2}

(b)

The impeller Reynolds number for pseudoplastic fluids is defined in Eq. (7.23). If Re_i at the highest stirrer speed is in the laminar range, conditions at the other stirrer speeds are also laminar. Therefore, for $N_i = 0.185\ s^{-1}$ and using the unit conversion factor 1 N = 1 kg m s^{-2} (Table A.4, Appendix A):

$$Re_i = \frac{N_i^{2-n}\,D_i^2\,\rho}{k^{n-1}\,K} = \frac{\left(0.185\,s^{-1}\right)^{2-0.20}(0.04\,m)^2\,1000\,kg\,m^{-3}}{10.2^{0.20-1}\left(0.049\,N\,s^{0.20}\,m^{-2}\,.\left|\frac{1\,kg\,m\,s^{-2}}{1\,N}\right|\right)} = 10.0$$

This value for Re_i is at the upper limit of the laminar range (p 137); therefore, we can conclude that flow at this and lower stirrer speeds is laminar.

Answer: Flow is laminar.

(c)

From Table 7.4, $k = 30$ for a helical-ribbon impeller. From Figure 7.25, flow with this type of impeller is laminar up to about $Re_i = 10^3$. These conditions allow use of higher stirrer speeds within the laminar region than is possible using a turbine impeller. Solving for N_i from Eq. (7.23) and substituting values of n and K for the fluid from **(a)**, the maximum stirrer speed at $Re_i = 30$ can be calculated as:

$$N_i^{2-n} = \frac{Re_i\, k^{n-1}\, K}{D_i^2\, \rho}$$

$$N_i^{2-0.20} = \frac{10^3\left(30^{0.20-1}\right)\left(0.049\,\mathrm{N\,s^{0.20}\,m^{-2}}\,.\left|\frac{1\,\mathrm{kg\,m\,s^{-2}}}{1\,\mathrm{N}}\right|\right)}{(0.04\,\mathrm{m})^2\, 1000\,\mathrm{kg\,m^{-3}}}$$

$$N_i^{1.8} = 2.02\,\mathrm{s^{-1.8}}$$

$$N_i = 1.48\,\mathrm{s^{-1}}$$

Therefore, from Eq. (7.11), as $\dot{\gamma} = 30\,N_i$ for a helical-ribbon impeller, shear rates up to $30 \times 1.48 = 44\ \mathrm{s^{-1}}$ can be used. This is a considerable improvement on the restricted range of up to about $1.9\ \mathrm{s^{-1}}$ as determined in **(a)** for the upper limit of laminar flow with a Rushton turbine impeller.

Answer: The shear rate range can be extended from $1.9\ \mathrm{s^{-1}}$ with a Rushton turbine impeller to $44\ \mathrm{s^{-1}}$ with a helical-ribbon impeller.

7.4 Particle suspension and gas dispersion

The Zwietering equation is an equation in numerics (p 12); therefore, the parameter values used in the equation must have the units specified. $D_p = 10\ \mu\mathrm{m} = 10 \times 10^{-6}$ m; $g = 9.8\ \mathrm{m\ s^{-2}}$ (p 16); $D_i = 30$ cm $= 0.3$ m; ρ_L = density of water = $1000\ \mathrm{kg\ m^{-3}}$. The dynamic viscosity of water μ_L at 20°C is about 1 cP (p 133); from Table A.9 (Appendix A), this is the same as $\mu_L = 10^{-3}\ \mathrm{kg\ m^{-1}\ s^{-1}}$. Using the definition of kinematic viscosity from p 133:

$$\nu_L = \frac{\mu_L}{\rho_L} = \frac{10^{-3}\,\mathrm{kg\,m^{-1}\,s^{-1}}}{1000\,\mathrm{kg\,m^{-3}}} = 10^{-6}\,\mathrm{m^2\,s^{-1}}$$

The density of the cells ρ_p is $1.04\ \mathrm{g\ cm^{-3}}$. Converting to $\mathrm{kg\ m^{-3}}$:

$$\rho_p = 1.04\,\mathrm{g\,cm^{-3}}\,.\left|\frac{100\,\mathrm{cm}}{1\,\mathrm{m}}\right|^3.\left|\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right| = 1040\,\mathrm{kg\,m^{-3}}$$

Substituting values in the correct units into the Zwietering equation:

$$N_i^* = C\,\nu_L^{0.1}\,D_p^{0.2}\left(\frac{g\,(\rho_p - \rho_L)}{\rho_L}\right)^{0.45} D_i^{-0.85}\,x^{0.13}$$

$$N_i^* = 7.7\left(10^{-6}\right)^{0.1}\left(10 \times 10^{-6}\right)^{0.2}\left(\frac{9.8\,(1040 - 1000)}{1000}\right)^{0.45}(0.3)^{-0.85}\,(40)^{0.13} = 0.57\,\mathrm{s^{-1}}$$

This minimum stirrer speed for suspension of the cells can be compared with the minimum stirrer speed for dispersion of air bubbles. Taking the average minimum tip speed of $2.0\ \mathrm{m\ s^{-1}}$ for bubble dispersion:

$$N_i = \frac{\text{tip speed}}{\pi D_i} = \frac{2.0\,\mathrm{m\,s^{-1}}}{\pi\,(0.3\,\mathrm{m})} = 2.1\,\mathrm{s^{-1}}$$

The stirrer speed for air dispersion is 3.7 times higher than for cell suspension. As power in the turbulent regime is proportional to the stirrer speed cubed, about 50 times more power is required to disperse the air bubbles than to keep the cells in suspension.

Answer: Bubble dispersion requires significantly more power than cell suspension.

7.5 Scale-up of mixing system

(a)

The Reynolds number for a Newtonian fluid in a stirred vessel is defined in Eq. (7.2). Using this equation to evaluate the maximum viscosity μ under turbulent conditions with the density of water $\rho = 1000\ \text{kg m}^{-3}$:

$$\mu = \frac{N_i D_i^2 \rho}{Re_i} = \frac{\left(800\ \text{min}^{-1} . \left|\frac{1\ \text{min}}{60\ \text{s}}\right|\right)\left((5\ \text{cm})^2 . \left|\frac{1\ \text{m}}{100\ \text{cm}}\right|^2\right) 1000\ \text{kg m}^{-3}}{10^4} = 3.3 \times 10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}$$

From Table A.9 (Appendix A), 1 kg $\text{m}^{-1}\ \text{s}^{-1} = 10^3$ cP; therefore:

$$\mu = 3.3 \times 10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1} . \left|\frac{10^3\ \text{cP}}{1\ \text{kg m}^{-1}\ \text{s}^{-1}}\right| = 3.3\ \text{cP}$$

Answer: 3.3 cP

(b)

The tip speed in the laboratory equipment is:

$$\text{Tip speed} = \pi N_i D_i = \pi\left(800\ \text{min}^{-1} . \left|\frac{1\ \text{min}}{60\ \text{s}}\right|\right)\left(5\ \text{cm} . \left|\frac{1\ \text{m}}{100\ \text{cm}}\right|\right) = 2.1\ \text{m s}^{-1}$$

In the large-scale vessel, $D_i = 15 \times 5\ \text{cm} = 75\ \text{cm}$. If the tip speed is kept the same after scale-up:

$$N_i = \frac{\text{tip speed}}{\pi D_i} = \frac{2.1\ \text{m s}^{-1}}{\pi\left(75\ \text{cm} . \left|\frac{1\ \text{m}}{100\ \text{cm}}\right|\right)} = 0.89\ \text{s}^{-1}$$

Using this value of N_i to calculate the maximum viscosity as in **(a)**:

$$\mu = \frac{N_i D_i^2 \rho}{Re_i} = \frac{0.89\ \text{s}^{-1}\left((75\ \text{cm})^2 . \left|\frac{1\ \text{m}}{100\ \text{cm}}\right|^2\right) 1000\ \text{kg m}^{-3}}{10^4} = 0.05\ \text{kg m}^{-1}\ \text{s}^{-1}$$

Converting to cP:

$$\mu = 0.05\ \text{kg m}^{-1}\ \text{s}^{-1} . \left|\frac{10^3\ \text{cP}}{1\ \text{kg m}^{-1}\ \text{s}^{-1}}\right| = 50\ \text{cP}$$

Answer: Scale-up increases the maximum viscosity for turbulent conditions to 50 cP.

7.6 Effect of viscosity on power requirements

(a)

$D_i = 1$ m; $\rho = 1\ \text{g cm}^{-3} = 1000\ \text{kg m}^{-3}$.

(i)

The viscosity of water at 20°C is about 1 cP (p 133); from Table A.9 (Appendix A), this is the same as 10^{-3} kg $\text{m}^{-1}\ \text{s}^{-1}$. Substituting parameter values into Eq. (7.2):

$$Re_i = \frac{N_i D_i^2 \rho}{\mu} = \frac{\left(90\ \text{min}^{-1} . \left|\frac{1\ \text{min}}{60\ \text{s}}\right|\right)(1\ \text{m})^2\ 1000\ \text{kg m}^{-3}}{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}} = 1.5 \times 10^6$$

From Figure 7.24 for a Rushton turbine, this value of Re_i corresponds to turbulent flow and N_P' can be taken as 5.8. The power required is evaluated using Eq. (7.20):

$$P = N_P' \rho N_i^3 D_i^5 = 5.8\left(1000\ \text{kg m}^{-3}\right)\left(90\ \text{min}^{-1} . \left|\frac{1\ \text{min}}{60\ \text{s}}\right|\right)^3 (1\ \text{m})^5 = 1.96 \times 10^4\ \text{kg m}^2\ \text{s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore:

$$P = 1.96 \times 10^4 \text{ W} = 19.6 \text{ kW}$$

Answer: 19.6 kW

(ii)
For a viscosity of 100×10^{-3} kg m^{-1} s^{-1} = 0.1 kg m^{-1} s^{-1}, Re_i is:

$$Re_i = \frac{N_i D_i^2 \rho}{\mu} = \frac{\left(90 \text{ min}^{-1} . \left|\frac{1 \text{ min}}{60 \text{ s}}\right|\right) (1 \text{ m})^2\, 1000 \text{ kg m}^{-3}}{0.1 \text{ kg m}^{-1} \text{ s}^{-1}} = 1.5 \times 10^4$$

From Figure 7.24 for a Rushton turbine, this value of Re_i is still within the turbulent regime so that N_P' is again 5.8. Therefore, the power required is the same as that calculated in **(i)**: P = 19.6 kW.

Answer: 19.6 kW

(iii)
For a viscosity of $10^4 \times 10^{-3}$ kg m^{-1} s^{-1} = 10 kg m^{-1} s^{-1}, Re_i is:

$$Re_i = \frac{N_i D_i^2 \rho}{\mu} = \frac{\left(90 \text{ min}^{-1} . \left|\frac{1 \text{ min}}{60 \text{ s}}\right|\right) (1 \text{ m})^2\, 1000 \text{ kg m}^{-3}}{10 \text{ kg m}^{-1} \text{ s}^{-1}} = 150$$

From Figure 7.24 for a Rushton turbine, this value of Re_i is within the transition regime. N_P read from Figure 7.24 is about 3.5. The power required is evaluated using Eq. (7.18):

$$P = N_P \rho N_i^3 D_i^5 = 3.5\left(1000 \text{ kg m}^{-3}\right)\left(90 \text{ min}^{-1} . \left|\frac{1 \text{ min}}{60 \text{ s}}\right|\right)^3 (1 \text{ m})^5 = 1.18 \times 10^4 \text{ kg m}^2 \text{ s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore:

$$P = 1.18 \times 10^4 \text{ W} = 11.8 \text{ kW}$$

Answer: 11.8 kW

(b)
From Figure 7.24 for a Rushton turbine, turbulence with N_P' = 5.8 is achieved at a minimum Reynolds number of about 10^4. For a viscosity of 1000×10^{-3} kg m^{-1} s^{-1} = 1 kg m^{-1} s^{-1}, the stirrer speed required can be determined from Eq. (7.2):

$$N_i = \frac{Re_i \mu}{D_i^2 \rho} = \frac{10^4\left(1 \text{ kg m}^{-1} \text{ s}^{-1}\right)}{(1 \text{ m})^2\, 1000 \text{ kg m}^{-3}} = 10 \text{ s}^{-1}$$

Using this result in Eq. (7.20):

$$P = N_P' \rho N_i^3 D_i^5 = 5.8\left(1000 \text{ kg m}^{-3}\right)\left(10 \text{ s}^{-1}\right)^3 (1 \text{ m})^5 = 5.80 \times 10^6 \text{ kg m}^2 \text{ s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore:

$$P = 5.80 \times 10^6 \text{ W} = 5.80 \times 10^3 \text{ kW}$$

Answer: 5.80×10^3 kW

7.7 Electrical power required for mixing

D_i = 7 cm = 0.07 m; ρ = 1000 kg m^{-3}. The viscosity of water at 20°C is about 1 cP (p 133); from Table A.9 (Appendix A), this is the same as 10^{-3} kg m^{-1} s^{-1}. Substituting parameter values into Eq. (7.2):

$$Re_i = \frac{N_i D_i^2 \rho}{\mu} = \frac{\left(900\,\text{min}^{-1}\cdot\left|\frac{1\,\text{min}}{60\,\text{s}}\right|\right)(0.07\,\text{m})^2\,1000\,\text{kg m}^{-3}}{10^{-3}\,\text{kg m}^{-1}\,\text{s}^{-1}} = 7.4\times10^4$$

From Figure 7.24 for a Rushton turbine, this value of Re_i corresponds to turbulent flow and N'_P can be taken as 5.8. The power required is evaluated using Eq. (7.20):

$$P = N'_P \rho N_i^3 D_i^5 = 5.8\left(1000\,\text{kg m}^{-3}\right)\left(900\,\text{min}^{-1}\cdot\left|\frac{1\,\text{min}}{60\,\text{s}}\right|\right)^3 (0.07\,\text{m})^5 = 32.9\,\text{kg m}^2\,\text{s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore:

$$P = 32.9\,\text{W}$$

This value is considerably lower that the electrical power consumed by the stirrer motor. Much of the remainder of the electrical power is converted into heat within the motor housing.

Answer: 32.9 W; a significant fraction of the electrical power is dissipated as heat within the motor housing

7.8 Mixing time with aeration

D_i = 0.67 m; ρ = density of water = 1000 kg m^{-3}. From Table A.9 (Appendix A), 1 cP = 10^{-3} kg m^{-1} s^{-1}; therefore, μ = 4 cP = 4×10^{-3} kg m^{-1} s^{-1}. For a cylindrical tank of diameter D_T = 2 m and height H= 2 m, the volume V is:

$$V = \pi\left(\frac{D_T}{2}\right)^2 H = \pi\left(\frac{2\,\text{m}}{2}\right)^2 2\,\text{m} = 6.28\,\text{m}^3$$

(a)
If the maximum specific power consumption is 1.5 kW m^{-3}, the maximum power P is:

$$P = \left(1.5\,\text{kW m}^{-3}\right)6.28\,\text{m}^3 = 9.42\,\text{kW}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore 1 kW = 1000 kg m^2 s^{-3}. Converting P to kg m^2 s^{-3}:

$$P = 9.42\,\text{kW} = 9.42\,\text{kW}\cdot\left|\frac{1000\,\text{kg m}^2\,\text{s}^{-3}}{1\,\text{kW}}\right| = 9.42\times10^3\,\text{kg m}^2\,\text{s}^{-3}$$

Assume for now that the fluid flow is turbulent and N'_P = 5.8. The stirrer speed can be evaluated using Eq. (7.20):

$$N_i^3 = \frac{P}{N'_P \rho D_i^5} = \frac{9.42\times10^3\,\text{kg m}^2\,\text{s}^{-3}}{5.8\left(1000\,\text{kg m}^{-3}\right)(0.67\,\text{m})^5} = 12.0\,\text{s}^{-3}$$

$$N_i = 2.29\,\text{s}^{-1} = 137\,\text{rpm}$$

Check that this stirrer speed provides turbulent mixing conditions by evaluating the Reynolds number. Re_i for Newtonian fluids in a stirred vessel is defined in Eq. (7.2):

$$Re_i = \frac{N_i D_i^2 \rho}{\mu} = \frac{\left(2.29\,\text{s}^{-1}\right)(0.67\,\text{m})^2\,1000\,\text{kg m}^{-3}}{4\times10^{-3}\,\text{kg m}^{-1}\,\text{s}^{-1}} = 2.6\times10^5$$

From Figure 7.24 for a Rushton turbine, this value of Re_i is well into the turbulent regime; therefore the value for N'_P assumed above is valid.

For high Re_i, the mixing time can be calculated using Eq. (7.16):

$$t_m = \frac{1.54\,V}{D_i^3\,N_i} = \frac{1.54\left(6.28\,\text{m}^3\right)}{(0.67\,\text{m})^3\,2.29\,\text{s}^{-1}} = 14\,\text{s}$$

Answer: The maximum allowable stirrer speed is 2.29 s^{-1} or 137 rpm; the mixing time is 14 s.

(b)
The ungassed power number was 5.8; therefore, the power number with gassing $(N_P)_g = 0.5 \times 5.8 = 2.9$. The stirrer speed which delivers the maximum power $P = 9.42 \times 10^3$ kg m^2 s^{-3} can be evaluated using Eq. (7.18):

$$N_i^3 = \frac{P}{(N_P)_g\,\rho\,D_i^5} = \frac{9.42\times 10^3\,\text{kg m}^2\,\text{s}^{-3}}{2.9\left(1000\,\text{kg m}^{-3}\right)(0.67\,\text{m})^5} = 24.1\,\text{s}^{-3}$$

$$N_i = 2.89\,\text{s}^{-1} = 173\,\text{rpm}$$

The mixing time evaluated using Eq. (7.16) is:

$$t_m = \frac{1.54\,V}{D_i^3\,N_i} = \frac{1.54\left(6.28\,\text{m}^3\right)}{(0.67\,\text{m})^3\,2.89\,\text{s}^{-1}} = 11\,\text{s}$$

Answer: The maximum allowable stirrer speed is 2.89 s^{-1} or 173 rpm; the mixing time is 11 s.

8

Heat Transfer

8.1 Rate of conduction

(a)

$B = 15$ cm $= 0.15$ m; $\Delta T = (700 - 80) = 620$°C, which is equal to 620 K as temperature differences are the same on the Celsius and Kelvin scales (p 18). The rate of heat conduction can be calculated using Eq. (8.10):

$$\bar{Q} = \frac{kA}{B}\Delta T = \frac{\left(0.3\ \mathrm{W\,m^{-1}\,K^{-1}}\right)1.5\ \mathrm{m^2}}{0.15\ \mathrm{m}}(620\ \mathrm{K}) = 1860\ \mathrm{W} = 1.86\ \mathrm{kW}$$

Answer: 1.86 kW

(b)

In this case there are two thermal resistances in series. Their magnitudes are calculated using Eq. (8.15). For the firebrick:

$$R_1 = \frac{B_1}{k_1 A} = \frac{0.15\ \mathrm{m}}{\left(0.3\ \mathrm{W\,m^{-1}\,K^{-1}}\right)1.5\ \mathrm{m^2}} = 0.33\ \mathrm{K\,W^{-1}}$$

For the asbestos, $B_2 = 4$ cm $= 0.04$ m, so that:

$$R_2 = \frac{B_2}{k_2 A} = \frac{0.04\ \mathrm{m}}{\left(0.1\ \mathrm{W\,m^{-1}\,K^{-1}}\right)1.5\ \mathrm{m^2}} = 0.27\ \mathrm{K\,W^{-1}}$$

Therefore, the total wall resistance $R_W = R_1 + R_2 = (0.33 + 0.27)\ \mathrm{K\,W^{-1}} = 0.60\ \mathrm{K\,W^{-1}}$. For thermal resistances in series, the rate of heat conduction is calculated using Eq. (8.14):

$$\bar{Q} = \frac{\Delta T}{R_W} = \frac{620\ \mathrm{K}}{0.60\ \mathrm{K\,W^{-1}}} = 1033\ \mathrm{W} = 1.03\ \mathrm{kW}$$

Answer: 1.03 kW

8.2 Overall heat-transfer coefficient

The overall heat-transfer coefficient is calculated using Eq. (8.24) with $h_{fh} = 830\ \mathrm{W\,m^{-2}\,K^{-1}}$, $h_h = 1.2\ \mathrm{kW\,m^{-2}\,K^{-1}}$, $B = 6$ mm $= 0.006$ m, $k = 19\ \mathrm{W\,m^{-1}\,K^{-1}}$, $h_c = 1.7\ \mathrm{kW\,m^{-2}\,K^{-1}}$ and $h_{fc} = 0$.

$$\frac{1}{U} = \frac{1}{h_{fh}} + \frac{1}{h_h} + \frac{B}{k} + \frac{1}{h_c} + \frac{1}{h_{fc}}$$

$$\frac{1}{U} = \frac{1}{830\ \mathrm{W\,m^{-2}\,K^{-1}}} + \frac{1}{1.2\ \mathrm{kW\,m^{-2}\,K^{-1}}\cdot\left|\frac{1000\ \mathrm{W}}{1\ \mathrm{kW}}\right|} + \frac{0.006\ \mathrm{m}}{19\ \mathrm{W\,m^{-1}\,K^{-1}}} + \frac{1}{1.7\ \mathrm{kW\,m^{-2}\,K^{-1}}\cdot\left|\frac{1000\ \mathrm{W}}{1\ \mathrm{kW}}\right|} + 0$$

$$\frac{1}{U} = 2.94\times10^{-3}\ \mathrm{W^{-1}\,m^2\,K}$$

$$U = 340\ \mathrm{W\,m^{-2}\,K^{-1}}$$

Answer: 340 W $\mathrm{m^{-2}\,K^{-1}}$

8.3 Effect of cooling-coil length on coolant requirements

(a)

The steady-state energy-balance equation for the cooling-coil is Eq. (8.32):

$$\hat{Q} = \hat{M}_c \, C_{pc} \left(T_{co} - T_{ci}\right) = \left(0.5 \text{ kg s}^{-1}\right) 4.18 \text{ kJ kg}^{-1} \, ^\circ\text{C}^{-1} \, (15 - 8)^\circ\text{C} = 14.6 \text{ kJ s}^{-1}$$

Answer: 14.6 kJ s^{-1}

(b)

The mean temperature difference between the fermentation fluid and the cooling water is calculated from Eq. (8.35):

$$\Delta T_A = \frac{2\,T_F - \left(T_{ci} + T_{co}\right)}{2} = \frac{(2 \times 35)^\circ\text{C} - (8 + 15)^\circ\text{C}}{2} = 23.5^\circ\text{C}$$

Answer: 23.5°C

(c)

UA is evaluated using Eq. (8.19) with $\Delta T = \Delta T_A$:

$$UA = \frac{\hat{Q}}{\Delta T_A} = \frac{14.6 \text{ kJ s}^{-1}}{23.5^\circ\text{C}} = 0.62 \text{ kJ s}^{-1} \, ^\circ\text{C}^{-1}$$

Answer: 0.62 kJ s^{-1} $^\circ\text{C}^{-1}$

(d)

$$UA' = 1.5\,UA = 1.5 \times 0.62 \text{ kJ s}^{-1} \, ^\circ\text{C}^{-1} = 0.93 \text{ kJ s}^{-1} \, ^\circ\text{C}^{-1}$$

Answer: 0.93 kJ s^{-1} $^\circ\text{C}^{-1}$

(e)

Applying Eq. (8.19) to determine $\Delta T = \Delta T_A$ for the new coil:

$$\Delta T_A = \frac{\hat{Q}}{UA'} = \frac{14.6 \text{ kJ s}^{-1}}{0.93 \text{ kJ s}^{-1} \, ^\circ\text{C}^{-1}} = 15.7^\circ\text{C}$$

The new cooling water outlet temperature T_{co} is determined using Eq. (8.35):

$$T_{co} = 2\,(T_F - \Delta T_A) - T_{ci} = 2\,(35 - 15.7)^\circ\text{C} - 8^\circ\text{C} = 30.6^\circ\text{C}$$

Answer: 30.6°C

(f)

From Eq. (8.32), for the new coil with $\hat{Q} = 14.6$ kJ s^{-1}, $T_{ci} = 8^\circ$C and $T_{co} = 30.6^\circ$C:

$$\hat{M}_c = \frac{\hat{Q}}{C_{pc}\left(T_{co} - T_{ci}\right)} = \frac{14.6 \text{ kJ s}^{-1}}{4.18 \text{ kJ kg}^{-1} \, ^\circ\text{C}^{-1} \, (30.6 - 8)^\circ\text{C}} = 0.15 \text{ kg s}^{-1}$$

Therefore, installation of the new coil allows a 70% reduction in cooling-water requirements.

Answer: 70%

8.4 Calculation of heat-transfer area in fermenter design

$N_i = 80$ rpm $= {}^{80}/_{60} = 1.33$ s^{-1}. C_p culture fluid $= 4.2$ kJ kg^{-1} $^\circ\text{C}^{-1}$ $= 4.2 \times 10^3$ J kg^{-1} $^\circ\text{C}^{-1}$. $C_{pc} = C_p$ water $= 75.4$ J gmol^{-1} $^\circ\text{C}^{-1}$ (Table B.3, Appendix B) $= 75.4$ kJ kgmol^{-1} $^\circ\text{C}^{-1}$. Converting C_{pc} to a mass basis using the molecular weight of water = 18.0 (Table B.1, Appendix B):

$$C_{pc} = 75.4 \text{ kJ kgmol}^{-1} \, ^\circ\text{C}^{-1} = 75.4 \text{ kJ kgmol}^{-1} \, ^\circ\text{C}^{-1} \, . \left|\frac{1 \text{ kgmol}}{18.0 \text{ kg}}\right| = 4.19 \text{ kJ kg}^{-1} \, ^\circ\text{C}^{-1}$$

$\mu_b = 10^{-3}$ N s m^{-2}; from Table A.9 (Appendix A), 1 N s m^{-2} = 1 kg m^{-1} s^{-1}; therefore $\mu_b = 10^{-3}$ kg m^{-1} s^{-1}. $k_{fb} = 0.6$ W m^{-1} $^\circ\text{C}^{-1}$; from Table A.8 (Appendix A), 1 W = 1 J s^{-1}; therefore $k_{fb} = 0.6$ J s^{-1} m^{-1} $^\circ\text{C}^{-1}$. $B = 6$ mm $= 0.006$ m.

From p 97, as the heat of reaction for an exothermic reaction is negative, $\Delta\widetilde{H}_{\rm rxn} = -2500\,{\rm kW}$. From the modified energy-balance equation, Eq. (8.33), when evaporation and shaft work can be neglected, $\widehat{Q} = -\Delta\widetilde{H}_{\rm rxn} = 2500\,{\rm kW} = 2.5 \times 10^6$ W. According to the sign conventions on pp 88–89, $\widehat{Q}$ positive is consistent with heat being removed from the system. From Table A.8 (Appendix A), 1 W = 1 J s^{-1}; therefore $\widehat{Q} = 2.5 \times 10^6\ {\rm J\,s^{-1}} = 2500\ {\rm kJ\,s^{-1}}$.

(a)
The fermenter-side heat-transfer coefficient can be evaluated using the empirical correlation, Eq. (8.45). The dimensionless numbers in this equation are $Re_{\rm i}$, Pr and Nu. $Re_{\rm i}$ is given by Eq. (8.39):

$$Re_{\rm i} = \frac{N_{\rm i}\,D_{\rm i}^2\,\rho}{\mu_{\rm b}} = \frac{1.33\,{\rm s^{-1}}\,(1.7\,{\rm m})^2\,10^3\,{\rm kg\,m^{-3}}}{10^{-3}\,{\rm kg\,m^{-1}\,s^{-1}}} = 3.84 \times 10^6$$

Pr is given by Eq. (8.40):

$$Pr = \frac{C_p\,\mu_{\rm b}}{k_{\rm fb}} = \frac{4.2 \times 10^3\,{\rm J\,kg^{-1}\,{}^\circ C^{-1}}\left(10^{-3}\,{\rm kg\,m^{-1}\,s^{-1}}\right)}{0.6\,{\rm J\,s^{-1}\,m^{-1}\,{}^\circ C^{-1}}} = 7.0$$

From Eq. (8.45):

$$Nu = 0.87\,Re_{\rm i}^{0.62}\,Pr^{0.33}\left(\frac{\mu_{\rm b}}{\mu_{\rm w}}\right)^{0.14} = 0.87\left(3.84 \times 10^6\right)^{0.62}(7.0)^{0.33}(1)^{0.14} = 2.0 \times 10^4$$

From the definition of Nu in Eq. (8.37):

$$h = \frac{Nu\,k_{\rm fb}}{D} = \frac{2.0 \times 10^4\left(0.6\,{\rm W\,m^{-1}\,{}^\circ C^{-1}}\right)}{5\,{\rm m}} = 2.4 \times 10^3\,{\rm W\,m^{-2}\,{}^\circ C^{-1}}$$

Therefore, as the fermenter fluid is the hot fluid, $h_{\rm h} = 2.4 \times 10^3\ {\rm W\,m^{-2}\,{}^\circ C^{-1}}$.

Answer: $2.4 \times 10^3\ {\rm W\,m^{-2}\,{}^\circ C^{-1}}$

(b)
The overall heat-transfer coefficient in the absence of fouling layers can be calculated using Eq. (8.23). Assuming that the heat-transfer coefficient for the cooling water $h_{\rm c}$ can be neglected (p 185):

$$\frac{1}{U} = \frac{1}{h_{\rm h}} + \frac{B}{k} = \frac{1}{2.4 \times 10^3\,{\rm W\,m^{-2}\,{}^\circ C^{-1}}} + \frac{0.006\,{\rm m}}{20\,{\rm W\,m^{-1}\,{}^\circ C^{-1}}} = 7.17 \times 10^{-4}\,{\rm W^{-1}\,m^2\,{}^\circ C}$$

$$U = 1.40 \times 10^3\,{\rm W\,m^{-2}\,{}^\circ C^{-1}} = 1.40\,{\rm kW\,m^{-2}\,{}^\circ C^{-1}}$$

From Eqs (8.21) and (8.22), the heat-transfer resistance due to the pipe wall is $^{B}/_{k\,A}$; from Eq. (8.20), the total resistance to heat transfer is $^{1}/_{U\,A}$. Therefore, the proportion of the total resistance due to the pipe wall is:

$$\frac{^{B}/_{k\,A}}{^{1}/_{U\,A}} = \frac{B\,U}{k} = \frac{0.006\,{\rm m}\left(1.40 \times 10^3\,{\rm W\,m^{-2}\,{}^\circ C^{-1}}\right)}{20\,{\rm W\,m^{-1}\,{}^\circ C^{-1}}} = 0.42$$

Answer: $1.40\ {\rm kW\,m^{-2}\,{}^\circ C^{-1}}$. The pipe wall contributes 42% of the total resistance to heat transfer.

(c)
The energy-balance equation for the cooling water, Eq. (8.32), relates the cooling water flow rate, the inlet and outlet temperatures and the rate of heat transfer. From this equation, an expression for the outlet cooling-water temperature is:

$$T_{\rm co} = \frac{\widehat{Q}}{\widetilde{M}_{\rm c}\,C_{p{\rm c}}} + T_{\rm ci} = \frac{2500\,{\rm kJ\,s^{-1}}}{\widetilde{M}_{\rm c}\cdot\left|\frac{1\,{\rm h}}{3600\,{\rm s}}\right|\left(4.19\,{\rm kJ\,kg^{-1}\,{}^\circ C^{-1}}\right)} + 10{}^\circ{\rm C}$$

$$T_{\rm co} = \frac{2.15 \times 10^6}{\widetilde{M}_{\rm c}} + 10$$

where T_{co} has units of °C and $\hat{M}_c$ has units of kg h^{-1}. An expression for the mean temperature difference between the fermentation fluid and the cooling water can be determined from Eq. (8.35):

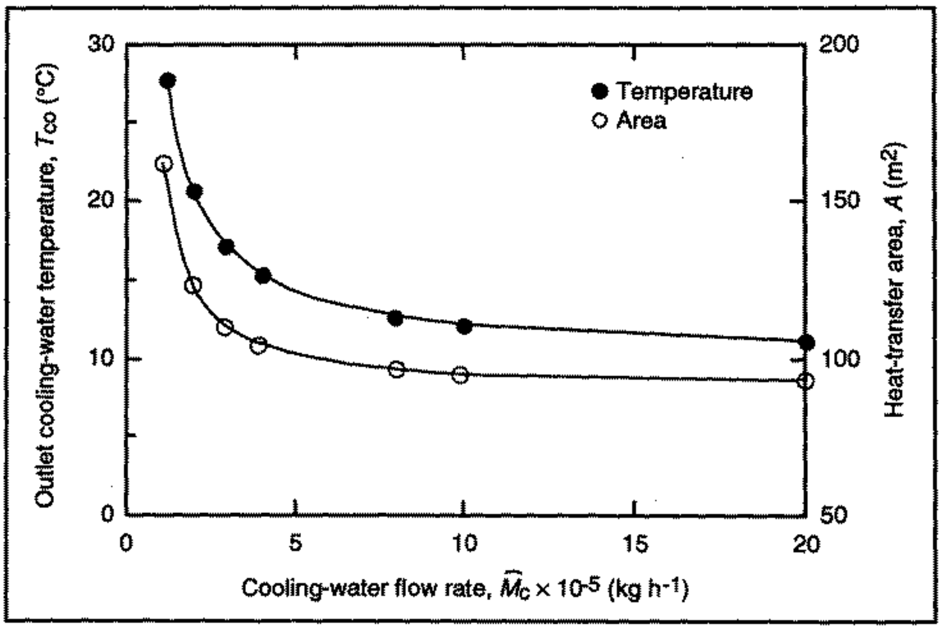
$$\Delta T_A = \frac{2\,T_F - (T_{ci} + T_{co})}{2} = \frac{(2\times 30)°\mathrm{C} - (10 + T_{co})°\mathrm{C}}{2} = \frac{60°\mathrm{C} - 10°\mathrm{C} - T_{co}}{2} = 25 - \frac{T_{co}}{2}$$

where ΔT_A and T_{co} have units of °C. Substituting this expression into Eq. (8.19) for evaluation of A with $\Delta T = \Delta T_A$ and the value of U from **(b)**:

$$A = \frac{\hat{Q}}{U\,\Delta T_A} = \frac{2500\,\mathrm{kW}}{1.40\,\mathrm{kW\,m^{-2}\,°C^{-1}}\left(25 - \frac{T_{co}}{2}\right)°\mathrm{C}} = \frac{1.8\times 10^3}{25 - \frac{T_{co}}{2}}$$

where A has units of m^2 and T_{co} has units of °C. The above equations for T_{co} and A allow evaluation of these parameters as a function of $\hat{M}_c$. The results for several values of $\hat{M}_c$ between 1.2×10^5 kg h^{-1} and 2×10^6 kg h^{-1} are listed and plotted below.

$\hat{M}_c$ (kg h^{-1})	T_{co} (°C)	A (m^2)
1.2×10^5	27.9	163
2.0×10^5	20.8	123
3.0×10^5	17.2	110
4.0×10^5	15.4	104
8.0×10^5	12.7	97
1.0×10^6	12.2	95
2.0×10^6	11.1	93



(d)

From the equations developed in (c), at a cooling-water flow rate of 5×10^5 kg h^{-1}, T_{co} = 14.3°C and A = 101 m^2. The area A of a cylindrical cooling-coil is equal to $2\,\pi\,r\,L$, where r is the cylinder radius and L is its length. For a radius of 5 cm = 0.05 m:

$$L = \frac{A}{2\,\pi\,r} = \frac{101\,\mathrm{m}^2}{2\,\pi\,(0.05\,\mathrm{m})} = 321\,\mathrm{m}$$

Answer: 321 m. *This is a long cooling-coil, representing a considerable expense when fabricated from stainless steel.*

8.5 Effect of fouling on heat-transfer resistance

(a)

$C_{pc} = C_p$ water = 75.4 J gmol^{-1} °C^{-1} (Table B.3, Appendix B) = 75.4 kJ kgmol^{-1} °C^{-1}. Converting to mass terms using the molecular weight of water = 18.0 (Table B.1, Appendix B):

$$C_{pc} = 75.4\,\text{kJ kgmol}^{-1}\,°\text{C}^{-1} = 75.4\,\text{kJ kgmol}^{-1}\,°\text{C}^{-1} \cdot \left|\frac{1\,\text{kgmol}}{18.0\,\text{kg}}\right| = 4.19\,\text{kJ kg}^{-1}\,°\text{C}^{-1}$$

From Eq. (8.32), the rate of heat removal to the cooling water before cleaning is:

$$\hat{Q} = \hat{M}_c\, C_{pc}\left(T_{co} - T_{ci}\right) = 20\,\text{kg s}^{-1}\left(4.19\,\text{kJ kg}^{-1}\,°\text{C}^{-1}\right)(28-12)°\text{C} = 1340.8\,\text{kJ s}^{-1}$$

The mean temperature difference between the fermentation fluid and the cooling water can be calculated from Eq. (8.35):

$$\Delta T_A = \frac{2\,T_F - \left(T_{ci} + T_{co}\right)}{2} = \frac{(2\times 37)°\text{C} - (12+28)°\text{C}}{2} = 17°\text{C}$$

The area A of the cylindrical cooling-coil is equal to $2\,\pi\, r\, L$, where r is the cylinder radius and L is its length. For r = 6 cm = 0.06 m and L = 150 m:

$$A = 2\,\pi\,(0.06\,\text{m})\,(150\,\text{m}) = 56.5\,\text{m}^2$$

Evaluating the overall heat-transfer coefficient U from Eq. (8.19) with $\Delta T = \Delta T_A$:

$$U = \frac{\hat{Q}}{A\,\Delta T_A} = \frac{1340.8\,\text{kJ s}^{-1}}{\left(56.5\,\text{m}^2\right)17°\text{C}} = 1.40\,\text{kJ s}^{-1}\,\text{m}^{-2}\,°\text{C}^{-1}$$

Answer: 1.40 kJ s^{-1} m^{-2} °C^{-1}

(b)

If the fermentation temperature is maintained the same after cleaning the cooling-coil, the rates of heat removal $\hat{Q}$ before and after cleaning must be equal. The new outlet cooling-water temperature can be calculated from Eq. (8.32):

$$T_{co} = \frac{\hat{Q}}{\hat{M}_c\, C_{pc}} + T_{ci} = \frac{1340.8\,\text{kJ s}^{-1}}{13\,\text{kg s}^{-1}\left(4.19\,\text{kJ kg}^{-1}\,°\text{C}^{-1}\right)} + 12°\text{C} = 36.6°\text{C}$$

Answer: 36.6°C

(c)

The mean temperature difference between the fermentation fluid and the cooling water after cleaning is determined from Eq. (8.35):

$$\Delta T_A = \frac{2\,T_F - \left(T_{ci} + T_{co}\right)}{2} = \frac{(2\times 37)°\text{C} - (12+36.6)°\text{C}}{2} = 12.7°\text{C}$$

The heat-transfer area is the same as before cleaning: A = 56.5 m^2. Evaluating U from Eq. (8.19) with $\Delta T = \Delta T_A$:

$$U = \frac{\hat{Q}}{A\,\Delta T_A} = \frac{1340.8\,\text{kJ s}^{-1}}{\left(56.5\,\text{m}^2\right)12.7°\text{C}} = 1.87\,\text{kJ s}^{-1}\,\text{m}^{-2}\,°\text{C}^{-1}$$

The overall heat-transfer resistance is given by Eq. (8.20):

$$R_T = \frac{1}{UA} = \frac{1}{1.87\,\text{kJ s}^{-1}\,\text{m}^{-2}\,°\text{C}^{-1}\left(56.5\,\text{m}^2\right)} = 9.46\times 10^{-3}\,\text{kJ}^{-1}\,\text{s}\,°\text{C}$$

This is the heat-transfer resistance after cleaning the cooling coil. The resistance before cleaning is calculated from Eq. (8.20) using the value of U from **(a)**:

$$R_T = \frac{1}{UA} = \frac{1}{1.40\,\text{kJ s}^{-1}\,\text{m}^{-2}\,°\text{C}^{-1}\left(56.5\,\text{m}^2\right)} = 1.26\times 10^{-2}\,\text{kJ}^{-1}\,\text{s}\,°\text{C}$$

The resistance due to the fouling deposits is equal to the difference between these two overall resistances = ($1.26 \times 10^{-2} - 9.46 \times 10^{-3}$) = 3.14×10^{-3} kJ^{-1} s °C. Therefore, fouling contributes:

$$\frac{3.14 \times 10^{-3}\,\mathrm{kJ^{-1}\,s\,°C}}{1.26 \times 10^{-2}\,\mathrm{kJ^{-1}\,s\,°C}} = 0.25$$

of the total resistance before cleaning.

Answer: 0.25

8.6 Pre-heating of nutrient medium

$C_{ph} = C_{pc} = C_p$ water = 75.4 J gmol^{-1} °C^{-1} (Table B.3, Appendix B) = 75.4 kJ kgmol^{-1} °C^{-1}. Converting to mass terms using the molecular weight of water = 18.0 (Table B.1, Appendix B):

$$C_{ph} = C_{pc} = 75.4\,\mathrm{kJ\,kgmol^{-1}\,°C^{-1}} = 75.4\,\mathrm{kJ\,kgmol^{-1}\,°C^{-1}} \cdot \left|\frac{1\,\mathrm{kgmol}}{18.0\,\mathrm{kg}}\right| = 4.19\,\mathrm{kJ\,kg^{-1}\,°C^{-1}}$$

ρ medium = ρ water = 1000 kg m^{-3}. The viscosity of water at 20°C is about 1 cP (p 133); from Table A.9 (Appendix A), this is the same as 10^{-3} kg m^{-1} s^{-1}; therefore, μ medium = μ water = 10^{-3} kg m^{-1} s^{-1}. k_{fb} medium = 0.54 W m^{-1} °C^{-1}; from Table A.8 (Appendix A), this is the same as 0.54 J s^{-1} m^{-1} °C^{-1}; therefore, k_{fb} medium = 0.54×10^{-3} kJ s^{-1} m^{-1} °C^{-1}. From Table 8.1, k_{fb} water at 303 K = 0.62 W m^{-1} °C^{-1}; from Table A.8 (Appendix A), this is the same as 0.62 J s^{-1} m^{-1} °C^{-1}; therefore, k_{fb} water = 0.62×10^{-3} kJ s^{-1} m^{-1} °C^{-1}. From Eq. (2.24), 303 K = approx. 30°C, which is close enough to the conditions in the heat exchanger for evaluation of thermal conductivity. k pipe wall = 50 W m^{-1} °C^{-1}; from Table A.8 (Appendix A), this is the same as 50 J s^{-1} m^{-1} °C^{-1}; therefore, $k = 50 \times 10^{-3}$ kJ s^{-1} m^{-1} °C^{-1}. B = 5 mm = 0.005 m.

(a)
From the definition of density (p 16), the mass flow rate of the medium is equal to the volumetric flow rate multiplied by the density:

$$\hat{M}_c = 50\,\mathrm{m^3\,h^{-1}}\left(1000\,\mathrm{kg\,m^{-3}}\right) \cdot \left|\frac{1\,\mathrm{h}}{3600\,\mathrm{s}}\right| = 13.9\,\mathrm{kg\,s^{-1}}$$

The steady-state energy-balance equation for medium in the tubes of the heat exchanger is Eq. (8.32); therefore:

$$\hat{Q} = \hat{M}_c\, C_{pc}\left(T_{co} - T_{ci}\right) = 13.9\,\mathrm{kg\,s^{-1}}\left(4.19\,\mathrm{kJ\,kg^{-1}\,°C^{-1}}\right)(28 - 10)°\mathrm{C} = 1048.3\,\mathrm{kJ\,s^{-1}}$$

Answer: 1048.3 kJ s^{-1}

(b)
The heat-transfer coefficient for the medium in the tubes of the heat exchanger can be calculated using the empirical correlation, Eq. (8.42). The parameters in this equation are *Re*, *Pr* and *Nu*. *Re* is given by Eq. (8.38). The linear velocity of the fluid u is equal to the volumetric flow rate per tube divided by the cross-sectional area of the tube. The cross-sectional area of a cylindrical tube = πr^2 where r is the tube radius = 2.5 cm = 0.025 m. Therefore:

$$u = \frac{\text{volumetric flow rate}}{\text{number of tubes}\left(\pi r^2\right)} = \frac{50\,\mathrm{m^3\,h^{-1}} \cdot \left|\frac{1\,\mathrm{h}}{3600\,\mathrm{s}}\right|}{(30)\,\pi\,(0.025\,\mathrm{m})^2} = 0.236\,\mathrm{m\,s^{-1}}$$

Substituting parameter values into Eq. (8.38) with the tube diameter D = 5 cm = 0.05 m:

$$Re = \frac{D\,u\,\rho}{\mu_b} = \frac{0.05\,\mathrm{m}\left(0.236\,\mathrm{m\,s^{-1}}\right)1000\,\mathrm{kg\,m^{-3}}}{10^{-3}\,\mathrm{kg\,m^{-1}\,s^{-1}}} = 1.18 \times 10^4$$

This value of *Re* is within the range of validity of Eq. (8.42). *Pr* is given by Eq. (8.40):

$$Pr = \frac{C_p\,\mu_b}{k_{fb}} = \frac{4.19\,\mathrm{kJ\,kg^{-1}\,°C^{-1}}\left(10^{-3}\,\mathrm{kg\,m^{-1}\,s^{-1}}\right)}{0.54 \times 10^{-3}\,\mathrm{kJ\,s^{-1}\,m^{-1}\,°C^{-1}}} = 7.8$$

This value of *Pr* is also within the range of validity of Eq. (8.42). From Eq. (8.42):

$$Nu = 0.023\,Re^{0.8}\,Pr^{0.4} = 0.023\left(1.18\times 10^{4}\right)^{0.8}(7.8)^{0.4} = 94.6$$

From the definition of Nu in Eq. (8.37):

$$h = \frac{Nu\,k_{\text{fb}}}{D} = \frac{94.6\left(0.54\times 10^{-3}\text{ kJ s}^{-1}\text{ m}^{-1}\,°\text{C}^{-1}\right)}{0.05\text{ m}} = 1.02\text{ kJ s}^{-1}\text{ m}^{-2}\,°\text{C}^{-1}$$

As the medium in the tubes is the cold fluid in this heat exchange system, $h_c = 1.02$ kJ s^{-1} m^{-2} $°C^{-1}$.

The heat-transfer coefficient for the water flowing in the shell of the heat exchanger can be calculated using the empirical correlation, Eq. (8.44). The parameters in this equation are C, Re_{max}, Pr and Nu. As the tubes of the heat exchanger are arranged in line, from p 183, $C = 0.26$. Re_{max} is given by Eq. (8.38) with D equal to the outer tube diameter determined as the sum of the inner tube diameter and the pipe wall thicknesses: $D = 0.05\text{ m} + 2\times 0.005\text{ m} = 0.06\text{ m}$. Substituting parameter values into Eq. (8.38):

$$Re_{\text{max}} = \frac{D\,u\,\rho}{\mu_b} = \frac{0.06\text{ m}\left(0.15\text{ m s}^{-1}\right)1000\text{ kg m}^{-3}}{10^{-3}\text{ kg m}^{-1}\text{ s}^{-1}} = 9.00\times 10^{3}$$

As this value is $> 6\times 10^{3}$, Re_{max} is within the range of validity of Eq. (8.44). Pr for the shell-side water is given by Eq. (8.40):

$$Pr = \frac{C_p\,\mu_b}{k_{\text{fb}}} = \frac{4.19\text{ kJ kg}^{-1}\,°\text{C}^{-1}\left(10^{-3}\text{ kg m}^{-1}\text{ s}^{-1}\right)}{0.62\times 10^{-3}\text{ kJ s}^{-1}\text{ m}^{-1}\,°\text{C}^{-1}} = 6.8$$

Therefore, from Eq. (8.44):

$$Nu = C\,Re_{\text{max}}^{0.6}\,Pr^{0.33} = 0.26\left(9.00\times 10^{3}\right)^{0.6}(6.8)^{0.33} = 115$$

From the definition of Nu in Eq. (8.37) with D the outside tube diameter:

$$h = \frac{Nu\,k_{\text{fb}}}{D} = \frac{115\left(0.62\times 10^{-3}\text{ kJ s}^{-1}\text{ m}^{-1}\,°\text{C}^{-1}\right)}{0.06\text{ m}} = 1.19\text{ kJ s}^{-1}\text{ m}^{-2}\,°\text{C}^{-1}$$

As water is the hot fluid in this heat exchange system, $h_h = 1.19$ kJ s^{-1} m^{-2} $°C^{-1}$.

Answer: $h_c = 1.02$ kJ s^{-1} m^{-2} $°C^{-1}$; $h_h = 1.19$ kJ s^{-1} m^{-2} $°C^{-1}$

(c)

The overall heat-transfer coefficient without fouling factors is calculated using Eq. (8.23):

$$\frac{1}{U} = \frac{1}{h_h} + \frac{B}{k} + \frac{1}{h_c} = \frac{1}{1.19\text{ kJ s}^{-1}\text{ m}^{-2}\,°\text{C}^{-1}} + \frac{0.005\text{ m}}{50\times 10^{-3}\text{ kJ s}^{-1}\text{ m}^{-1}\,°\text{C}^{-1}} + \frac{1}{1.02\text{ kJ s}^{-1}\text{ m}^{-2}\,°\text{C}^{-1}}$$

$$\frac{1}{U} = 1.92\text{ kJ}^{-1}\text{ s m}^{2}\,°\text{C}$$

$$U = 0.52\text{ kJ s}^{-1}\text{ m}^{-2}\,°\text{C}^{-1}$$

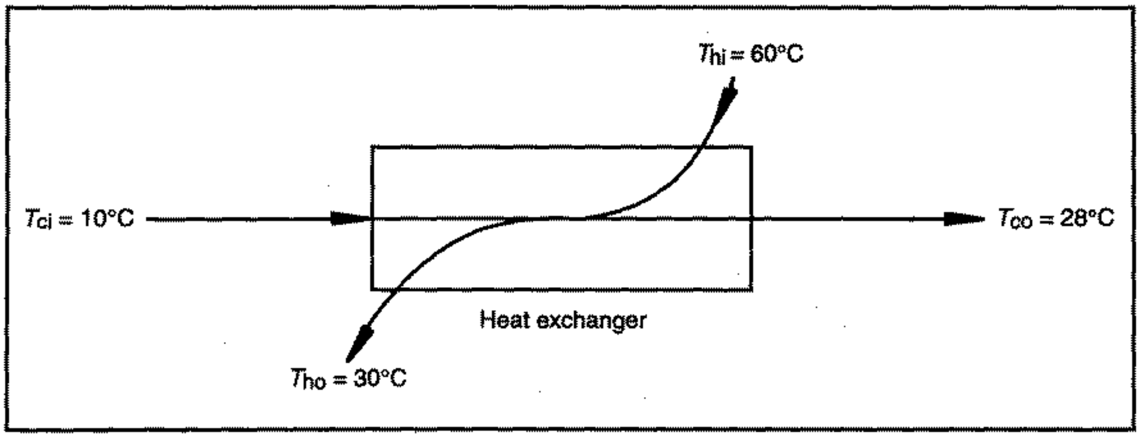
Answer: 0.52 kJ s^{-1} m^{-2} $°C^{-1}$

(d)

The outlet temperature of the water from the shell is determined from Eq. (8.31):

$$T_{\text{ho}} = T_{\text{hi}} - \frac{\hat{Q}}{\hat{M}_h\,C_{ph}} = 60°\text{C} - \frac{1048.3\text{ kJ s}^{-1}}{3\times 10^{4}\text{ kg h}^{-1}\cdot\left|\frac{1\text{ h}}{3600\text{ s}}\right|\left(4.19\text{ kJ kg}^{-1}\,°\text{C}^{-1}\right)} = 30°\text{C}$$

The fluid flow directions and the inlet and outlet temperatures for a single-pass countercurrent shell-and-tube heat exchanger are represented graphically below.



The temperature differences at the two ends of the exchanger are $\Delta T_1 = (30 - 10) = 20°\text{C}$ and $\Delta T_2 = (60 - 28) = 32°\text{C}$. Substituting these values into Eq. (8.34) for the log-mean temperature difference:

$$\Delta T_L = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\Delta T_2/\Delta T_1\right)} = \frac{(32-20)°\text{C}}{\ln\left(32/20\right)} = 25.5°\text{C}$$

Answer: 25.5°C

(e)
The heat-transfer area is determined from Eq. (8.19) with $\Delta T = \Delta T_L$:

$$A = \frac{\hat{Q}}{U\,\Delta T_L} = \frac{1048.3\,\text{kJ s}^{-1}}{0.52\,\text{kJ s}^{-1}\,\text{m}^{-2}\,°\text{C}^{-1}\,(25.5°\text{C})} = 79\,\text{m}^2$$

Answer: 79 m^2

(f)
The total area A of the tubes in a shall-and-tube heat exchanger is equal to $2\,\pi\,r\,L\,N$, where r is the tube radius, L is the length of the tubes and N is the number of tubes. For $r = 2.5$ cm = 0.025 m and $N = 30$:

$$L = \frac{A}{2\,\pi\,r\,N} = \frac{79\,\text{m}^2}{2\,\pi\,(0.025\,\text{m})\,30} = 16.8\,\text{m}$$

From these results, $L/D = 16.8/0.05 = 336$, where D is the tube diameter. As this value is > 60, application of Eq. (8.42) used to determine the tube-side heat-transfer coefficient is valid.

Answer: 16.8 m

8.7 Suitability of an existing cooling-coil

$N_i = 50$ rpm $= 50/60 = 0.83\ \text{s}^{-1}$. $C_{ph} = C_{pc} = C_p$ water = 75.4 J $\text{gmol}^{-1}\ °\text{C}^{-1}$ (Table B.3, Appendix B) = 75.4 kJ $\text{kgmol}^{-1}\ °\text{C}^{-1}$. Converting to mass terms using the molecular weight of water = 18.0 (Table B.1, Appendix B):

$$C_{ph} = C_{pc} = 75.4\,\text{kJ kgmol}^{-1}\,°\text{C}^{-1} = 75.4\,\text{kJ kgmol}^{-1}\,°\text{C}^{-1}\,.\left|\frac{1\,\text{kgmol}}{18.0\,\text{kg}}\right| = 4.19\,\text{kJ kg}^{-1}\,°\text{C}^{-1}$$

k_{fb} fermentation fluid = k_{fb} water. From Table 8.1, k_{fb} water at 303 K = 0.62 W $\text{m}^{-1}\ °\text{C}^{-1}$; from Table A.8 (Appendix A), this is the same as 0.62 J $\text{s}^{-1}\ \text{m}^{-1}\ °\text{C}^{-1}$; therefore, $k_{fb} = 0.62 \times 10^{-3}$ kJ $\text{s}^{-1}\ \text{m}^{-1}\ °\text{C}^{-1}$. From Eq. (2.24), 303 K = approx. 30°C, which is close enough to the conditions in the fermentation system for evaluation of thermal conductivity. ρ fermentation fluid = ρ water = 10^3 kg m^{-3}. μ fermentation fluid = μ water. The viscosity of water at 20°C is about 1 cP (p 133); from Table A.9 (Appendix A), this is the same as 10^{-3} kg $\text{m}^{-1}\ \text{s}^{-1}$. Assume that this value applies under the conditions in this system, and that the fluid viscosity at the wall μ_w is equal to the bulk viscosity μ_b.

The fermenter-side heat-transfer coefficient can be evaluated using the empirical correlation, Eq. (8.45). The dimensionless numbers in this equation are Re_i, Pr and Nu. Re_i is given by Eq. (8.39):

$$Re_i = \frac{N_i D_i^2 \rho}{\mu_b} = \frac{0.83\,\text{s}^{-1}\,(1\,\text{m})^2\,10^3\,\text{kg m}^{-3}}{10^{-3}\,\text{kg m}^{-1}\,\text{s}^{-1}} = 8.3 \times 10^5$$

Pr is given by Eq. (8.40):

$$Pr = \frac{C_p \mu_b}{k_{fb}} = \frac{4.19\,\text{kJ kg}^{-1}\,°\text{C}^{-1}\left(10^{-3}\,\text{kg m}^{-1}\,\text{s}^{-1}\right)}{0.62 \times 10^{-3}\,\text{kJ s}^{-1}\,\text{m}^{-1}\,°\text{C}^{-1}} = 6.8$$

From Eq. (8.45):

$$Nu = 0.87\, Re_i^{0.62}\, Pr^{0.33}\left(\frac{\mu_b}{\mu_w}\right)^{0.14} = 0.87\left(8.3 \times 10^5\right)^{0.62}(6.8)^{0.33}\,(1)^{0.14} = 7.7 \times 10^3$$

From the definition of *Nu* in Eq. (8.37) with D = the tank diameter = 3 m:

$$h = \frac{Nu\, k_{fb}}{D} = \frac{7.7 \times 10^3\left(0.62 \times 10^{-3}\,\text{kJ s}^{-1}\,\text{m}^{-1}\,°\text{C}^{-1}\right)}{3\,\text{m}} = 1.59\,\text{kJ s}^{-1}\,\text{m}^{-2}\,°\text{C}^{-1}$$

Therefore, as the fermentation fluid is the hot fluid, h_h = 1.59 kJ s^{-1} m^{-2} $°\text{C}^{-1}$. The overall heat-transfer coefficient in the absence of fouling layers is calculated using Eq. (8.23). Assuming that the heat-transfer coefficient for the cooling water h_c and the tube wall resistance $^B/_k$ can be neglected (p 185), from Eq. (8.23), $U = h_h$ = 1.59 kJ s^{-1} m^{-2} $°\text{C}^{-1}$.

To maintain the fermentation temperature constant, the cooling-coil must be capable of removing heat from the fermenter at the rate at which it is produced. From p 100, the heat of reaction for aerobic cultures is –460 kJ per gmol oxygen consumed. Therefore, if the maximum oxygen demand = 90 gmol m^{-3} h^{-1} and the fermenter volume = 20 m^3:

$$\Delta\hat{H}_{rxn} = -460\,\text{kJ gmol}^{-1}\left(90\,\text{gmol m}^{-3}\,\text{h}^{-1}\right)\left(20\,\text{m}^3\right).\left|\frac{1\,\text{h}}{3600\,\text{s}}\right| = -230\,\text{kJ s}^{-1}$$

From the modified energy-balance equation, Eq. (8.33), when evaporation and shaft work can be neglected, $\hat{Q} = -\Delta\hat{H}_{rxn}$ = 230 kJ s^{-1}. According to the sign conventions on pp 88–89, $\hat{Q}$ positive is consistent with heat being removed from the system. From the definition of density (p 16), the mass flow rate of the cooling water is equal to the volumetric flow rate multiplied by the density:

$$\hat{M}_c = 20\,\text{m}^3\,\text{h}^{-1}\left(10^3\,\text{kg m}^{-3}\right).\left|\frac{1\,\text{h}}{3600\,\text{s}}\right| = 5.56\,\text{kg s}^{-1}$$

From Eq. (8.32), the outlet cooling-water temperature is:

$$T_{co} = \frac{\hat{Q}}{\hat{M}_c C_{pc}} + T_{ci} = \frac{230\,\text{kJ s}^{-1}}{5.56\,\text{kg s}^{-1}\left(4.19\,\text{kJ kg}^{-1}\,°\text{C}^{-1}\right)} + 12°\text{C} = 21.9°\text{C}$$

The mean temperature difference between the fermentation fluid and the cooling water can be determined from Eq. (8.35):

$$\Delta T_A = \frac{2\,T_F - \left(T_{ci} + T_{co}\right)}{2} = \frac{(2 \times 28)°\text{C} - (12 + 21.9)°\text{C}}{2} = 11.1°\text{C}$$

Substituting the results into Eq. (8.19) for evaluation of A with $\Delta T = \Delta T_A$:

$$A = \frac{\hat{Q}}{U\,\Delta T_A} = \frac{230\,\text{kJ s}^{-1}}{1.59\,\text{kJ s}^{-1}\,\text{m}^{-2}\,°\text{C}^{-1}\,(11.1°\text{C})} = 13\,\text{m}^2$$

The area A of a cylindrical cooling-coil is equal to $2\,\pi\,r\,L$, where r is the cylinder radius and L is its length. For r = $^{7.5}/_2$ = 3.75 cm = 0.0375 m:

$$L = \frac{A}{2\,\pi\,r} = \frac{13\,\text{m}^2}{2\,\pi\,(0.0375\,\text{m})} = 55\,\text{m}$$

As the required length of cooling-coil is considerably longer than the 45 m available in the offered fermenter, the second-hand fermenter without modification is unsuitable for the proposed culture.

Answer: No

8.8 Optimum stirring speed for removal of heat from viscous broth

$\mu_b = 10{,}000$ cP. From Table A.9 (Appendix A), 1 cP = 10^{-3} kg m^{-1} s^{-1}; therefore, $\mu_b = 10{,}000 \times 10^{-3}$ kg m^{-1} s^{-1} = 10 kg m^{-1} s^{-1}. k_{fb} = 2 W m^{-1} °C^{-1}. From Table A.8 (Appendix A), this is the same as 2 J s^{-1} m^{-1} °C^{-1}; therefore, k_{fb} = 2 J s^{-1} m^{-1} °C = 2×10^{-3} kJ s^{-1} m^{-1} °C^{-1}.

(a), (b) and (c)

The fermenter-side heat-transfer coefficient can be evaluated using the empirical correlation, Eq. (8.45). The dimensionless numbers in this equation are *Re*$_i$, *Pr* and *Nu*. The dependence of *Re*$_i$ on stirrer speed is given by Eq. (8.39):

$$Re_i = \frac{N_i D_i^2 \rho}{\mu_b} = \frac{N_i (0.78\,\text{m})^2\, 10^3\,\text{kg m}^{-3}}{10\,\text{kg m}^{-1}\,\text{s}^{-1}} = 60.8\,N_i$$

where N_i has units s^{-1}. *Pr* is given by Eq. (8.40):

$$Pr = \frac{C_p \mu_b}{k_{fb}} = \frac{2\,\text{kJ kg}^{-1}\,°\text{C}^{-1}\left(10\,\text{kg m}^{-1}\,\text{s}^{-1}\right)}{2 \times 10^{-3}\,\text{kJ s}^{-1}\,\text{m}^{-1}\,°\text{C}^{-1}} = 10^4$$

From Eq. (8.45), assuming that the viscosity at the wall is equal to the viscosity of the bulk fluid:

$$Nu = 0.87\,Re_i^{0.62}\,Pr^{0.33}\left(\frac{\mu_b}{\mu_w}\right)^{0.14} = 0.87\left(60.8\,N_i\right)^{0.62}\left(10^4\right)^{0.33}(1)^{0.14} = 232\,N_i^{0.62}$$

From the definition of *Nu* in Eq. (8.37) with *D* = the tank diameter = 2.3 m:

$$h = \frac{Nu\,k_{fb}}{D} = \frac{232\,N_i^{0.62}\left(2 \times 10^{-3}\,\text{kJ s}^{-1}\,\text{m}^{-1}\,°\text{C}^{-1}\right)}{2.3\,\text{m}} = 0.20\,N_i^{0.62}\,\text{kJ s}^{-1}\,\text{m}^{-2}\,°\text{C}^{-1}$$

Therefore, as the fermentation broth is the hot fluid:

$$h_h = 0.20\,N_i^{0.62}\,\text{kJ s}^{-1}\,\text{m}^{-2}\,°\text{C}^{-1}$$

Assuming that the heat-transfer coefficient for the cooling water h_c and the tube wall resistance $^{B}/_{k}$ can be neglected (p 185), from Eq. (8.23):

$$U = h_h = 0.20\,N_i^{0.62}\,\text{kJ s}^{-1}\,\text{m}^{-2}\,°\text{C}^{-1}$$

Substituting the known parameters into Eq. (8.19) with $\Delta T = \Delta T_A$ = 20°C:

$$\hat{Q} = U A\,\Delta T_A = 0.20\,N_i^{0.62}\,\text{kJ s}^{-1}\,\text{m}^{-2}\,°\text{C}^{-1}\left(14\,\text{m}^2\right)20°\text{C} = 56.0\,N_i^{0.62}\,\text{kJ s}^{-1}$$

The power dissipated from the stirrer, $\hat{W}_s$, is equal to the power *P* calculated using Eq. (7.18). In this system, the value of N_P with gassing (N_{Pg}) is 40% lower or 0.6 × the value of N_P read from Figure 7.24. Therefore:

$$\hat{W}_s = P = N_{Pg}\,\rho\,N_i^3\,D_i^5 = 0.6\,N_P\left(10^3\,\text{kg m}^{-3}\right)N_i^3\,(0.78\,\text{m})^5 = 173\,N_P\,N_i^3\;\text{kg m}^2\,\text{s}^{-3}$$

where the value of N_P depends on *Re*$_i$, and N_i has units of s^{-1}. From Table A.8 (Appendix A), 1 J s^{-1} = 1 kg m^2 s^{-3}; therefore, 1 kJ s^{-1} = 10^3 kg m^2 s^{-3} and:

$$\hat{W}_s = 173\,N_P\,N_i^3\;\text{kg m}^2\,\text{s}^{-3}\,.\left|\frac{1\,\text{kJ s}^{-1}}{1000\,\text{kg m}^2\,\text{s}^{-3}}\right| = 0.173\,N_P\,N_i^3\;\text{kJ s}^{-1}$$

From the modified steady-state energy-balance equation, Eq. (8.33), assuming evaporation is negligible:

$$\hat{Q} = -\Delta\hat{H}_{\mathrm{rxn}} + \hat{W}_{\mathrm{s}}$$

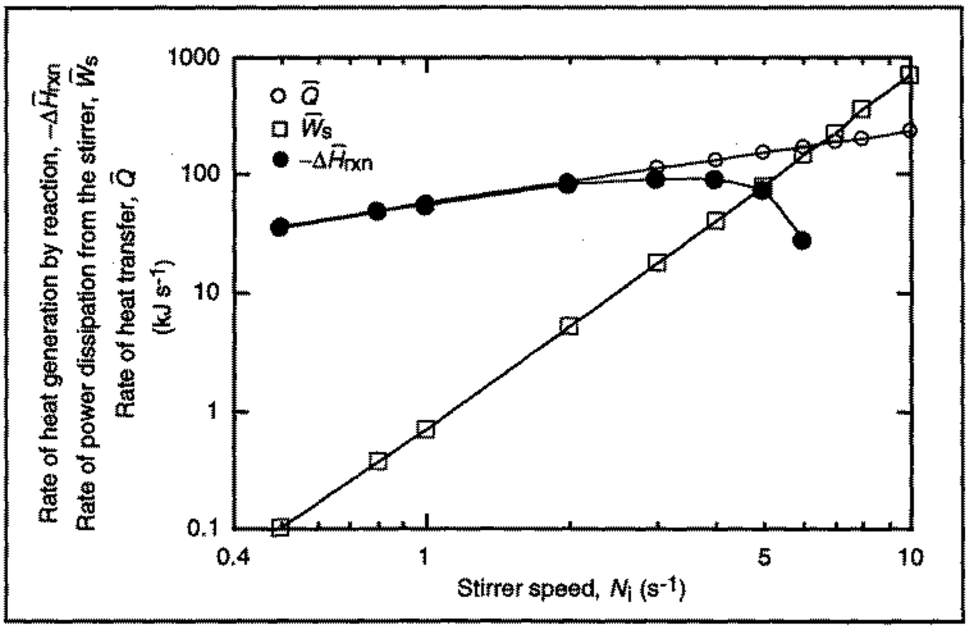
Therefore:

$$-\Delta\hat{H}_{\mathrm{rxn}} = \hat{Q} - \hat{W}_{\mathrm{s}} = \left(56.0\,N_{\mathrm{i}}^{0.62} - 0.173\,N_{\mathrm{P}}\,N_{\mathrm{i}}^{3}\right)\,\mathrm{kJ\,s^{-1}}$$

Values of $\hat{Q}$, $\hat{W}_{\mathrm{s}}$ and $-\Delta\hat{H}_{\mathrm{rxn}}$ calculated using the equations derived above are listed below as a function of N_{i}. Values of N_{P} as a function of Re_{i} were obtained from the original reference (J.H. Rushton, E.W. Costich and H.J. Everett, 1950, Power characteristics of mixing impellers. *Chem. Eng. Prog.* **46**, 467–476) for more accurate interpolation of the power curve in Figure 7.24.

N_{i} ($\mathrm{s^{-1}}$)	Re_{i}	$\hat{Q}$ ($\mathrm{kJ\,s^{-1}}$)	N_{P}	$\hat{W}_{\mathrm{s}}$ ($\mathrm{kJ\,s^{-1}}$)	$-\Delta\hat{H}_{\mathrm{rxn}}$ ($\mathrm{kJ\,s^{-1}}$)
0.5	30.4	36.4	4.5	0.10	36.3
0.8	48.6	48.8	4.1	0.36	48.4
1.0	60.8	56.0	4.0	0.69	55.3
2.0	122	86.1	3.7	5.12	80.9
3.0	182	111	3.7	17.3	93.4
4.0	243	132	3.6	39.9	92.4
5.0	304	152	3.6	77.9	74.0
6.0	365	170	3.8	142	28.1
7.0	426	187	3.8	225	–38
8.0	486	203	4.0	354	–151
10.0	608	233	4.0	692	–459

The values of $\hat{Q}$, $\hat{W}_{\mathrm{s}}$ and $-\Delta\hat{H}_{\mathrm{rxn}}$ are plotted as a function of N_{i} below.



(d)

The rate of removal of metabolic heat (the $-\Delta\hat{H}_{\mathrm{rxn}}$ component of $\hat{Q}$) reaches a maximum at around $N_{\mathrm{i}} = 3\ \mathrm{s^{-1}}$.

Answer: About 3 $\mathrm{s^{-1}}$

(e)

From p 100, the heat of reaction for aerobic cultures is –460 kJ per gmol oxygen consumed. Therefore, if the rate of oxygen consumption per g cells is 6 mmol $\mathrm{g^{-1}\,h^{-1}} = 6 \times 10^{-3}$ gmol $\mathrm{g^{-1}\,h^{-1}}$, when the stirrer speed is 3 $\mathrm{s^{-1}}$ and $\Delta\hat{H}_{\mathrm{rxn}} = -93.4\ \mathrm{kJ\,s^{-1}}$:

$$\text{Biomass} = \frac{-93.4\ \text{kJ s}^{-1}}{6\times10^{-3}\ \text{gmol g}^{-1}\ \text{h}^{-1}\,.\left|\frac{1\ \text{h}}{3600\ \text{s}}\right|\left(-460\ \text{kJ gmol}^{-1}\right)} = 1.22\times10^{5}\ \text{g}$$

The cell concentration is equal to the biomass divided by the fermenter volume:

$$\text{Cell concentration} = \frac{1.22\times10^{5}\ \text{g}}{10\ \text{m}^{3}\,.\left|\frac{1000\ \text{l}}{1\ \text{m}^{3}}\right|} = 12.2\ \text{g l}^{-1}$$

Answer: 12.2 g l^{-1}

(f)
From the graph, as the stirrer speed is raised, the overall heat-transfer coefficient increases as heat transfer through the boundary layer in the fermentation broth is improved. Therefore, the rate at which heat can be removed from the system $\hat{Q}$ is increased. However, the rate at which heat is dissipated by the stirrer $\hat{W}_s$ also increases with stirrer speed. When the curves for $\hat{Q}$ and $\hat{W}_s$ intersect, the entire heat-transfer capacity of the fermenter cooling system is being used just to remove the heat from stirring; the system in unable to remove any of the heat generated from reaction. Accordingly, at high stirrer speeds, the system has limited capacity to handle exothermic reactions.

9

Mass Transfer

9.1 Rate-controlling processes in fermentation

Converting the units of the maximum specific oxygen uptake rate using the molecular weight of oxygen = 32.0 (Table B.1, Appendix B):

$$q_O = 5\,\text{mmol g}^{-1}\,\text{h}^{-1} \cdot \left|\frac{1\,\text{gmol}}{1000\,\text{mmol}}\right| \cdot \left|\frac{32.0\,\text{g}}{1\,\text{gmol}}\right| \cdot \left|\frac{1\,\text{h}}{3600\,\text{s}}\right| = 4.44 \times 10^{-5}\,\text{g g}^{-1}\,\text{s}^{-1}$$

At a cell density of 40 g l^{-1}, the maximum oxygen requirement is:

$$q_O\, x = 4.44 \times 10^{-5}\,\text{g g}^{-1}\,\text{s}^{-1} \left(40\,\text{g l}^{-1}\right) \cdot \left|\frac{1000\,\text{l}}{1\,\text{m}^3}\right| \cdot \left|\frac{1\,\text{kg}}{1000\,\text{g}}\right| = 1.78 \times 10^{-3}\,\text{kg m}^{-3}\,\text{s}^{-1}$$

The rate of oxygen transfer is given by Eq. (9.37); N_A is maximum when $C_{AL} = 0$:

$$k_L a\, C^*_{AL} = 0.15\,\text{s}^{-1}\left(8 \times 10^{-3}\,\text{kg m}^{-3}\right) = 1.20 \times 10^{-3}\,\text{kg m}^{-3}\,\text{s}^{-1}$$

As the maximum oxygen demand of the culture is greater than the maximum oxygen transfer rate in the fermenter, the system will be limited by mass transfer.

Answer: Limited by mass transfer.

9.2 $k_L a$ required to maintain critical oxygen concentration

Converting the units of the oxygen uptake rate using the molecular weight of oxygen = 32.0 (Table B.1, Appendix B):

$$q_O\, x = 80\,\text{mmol l}^{-1}\,\text{h}^{-1} \cdot \left|\frac{1\,\text{gmol}}{1000\,\text{mmol}}\right| \cdot \left|\frac{32.0\,\text{g}}{1\,\text{gmol}}\right| \cdot \left|\frac{1\,\text{kg}}{1000\,\text{g}}\right| \cdot \left|\frac{1000\,\text{l}}{1\,\text{m}^3}\right| \cdot \left|\frac{1\,\text{h}}{3600\,\text{s}}\right| = 7.11 \times 10^{-4}\,\text{kg m}^{-3}\,\text{s}^{-1}$$

Converting the units of the critical oxygen concentration:

$$C_{crit} = 0.004\,\text{mM} = 0.004\,\text{mmol l}^{-1} \cdot \left|\frac{1\,\text{gmol}}{1000\,\text{mmol}}\right| \cdot \left|\frac{32.0\,\text{g}}{1\,\text{gmol}}\right| \cdot \left|\frac{1\,\text{kg}}{1000\,\text{g}}\right| \cdot \left|\frac{1000\,\text{l}}{1\,\text{m}^3}\right| = 1.28 \times 10^{-4}\,\text{kg m}^{-3}$$

(a)

From Table 9.2, the solubility of oxygen in water at 30°C under 1 atm air pressure is 8.05×10^{-3} kg m^{-3}. If the solubility in medium is 10% lower:

$$C^*_{AL} = 0.9 \times (8.05 \times 10^{-3})\,\text{kg m}^{-3} = 7.25 \times 10^{-3}\,\text{kg m}^{-3}$$

To maintain the oxygen concentration in the medium at the critical level, from Eq. (9.41):

$$\left(k_L a\right)_{crit} = \frac{q_O\, x}{\left(C^*_{AL} - C_{crit}\right)} = \frac{7.11 \times 10^{-4}\,\text{kg m}^{-3}\,\text{s}^{-1}}{\left(7.25 \times 10^{-3}\,\text{kg m}^{-3} - 1.28 \times 10^{-4}\,\text{kg m}^{-3}\right)} = 0.10\,\text{s}^{-1}$$

Answer: 0.10 s^{-1}

(b)

From Table 9.1, the solubility of oxygen in water at 30°C under 1 atm oxygen pressure is 3.84×10^{-2} kg m^{-3}. Therefore:

$$C^*_{AL} = 0.9 \times (3.84 \times 10^{-2})\,\text{kg m}^{-3} = 0.0346\,\text{kg m}^{-3}$$

From Eq. (9.41):

$$\left(k_La\right)_{\text{crit}} = \frac{q_O\,x}{\left(C^*_{\text{AL}} - C_{\text{crit}}\right)} = \frac{7.11\times10^{-4}\,\text{kg m}^{-3}\,\text{s}^{-1}}{\left(0.0346\,\text{kg m}^{-3} - 1.28\times10^{-4}\,\text{kg m}^{-3}\right)} = 0.021\,\text{s}^{-1}$$

Answer: 0.021 s^{-1}

9.3 Single-point k_La determination using the oxygen-balance method

(a)

The oxygen transfer rate for k_La determination by the oxygen-balance method is given as Eq. (9.48). From p 17, the partial pressure of oxygen in the inlet air at 1 atm is (0.21 × 1 atm) = 0.21 atm; assuming that the exit gas leaves the fermenter at the fermentation conditions (1 atm pressure and 28°C), the partial pressure of oxygen in the exit gas is (0.201 × 1 atm) = 0.201 atm. Using R = 0.000082057 m^3 atm K^{-1} gmol^{-1} from Table 2.5 and converting the temperatures to degrees Kelvin using Eq. (2.24):

$$N_{\text{A}} = \frac{1}{R\,V_{\text{L}}}\left[\left(\frac{F_g\,p_{\text{AG}}}{T}\right)_{\text{i}} - \left(\frac{F_g\,p_{\text{AG}}}{T}\right)_{\text{o}}\right]$$

$$N_{\text{A}} = \frac{1}{0.000082057\,\text{m}^3\,\text{atm K}^{-1}\,\text{gmol}^{-1}\,(200\,\text{l})}\left[\left(\frac{200\,\text{l min}^{-1}\,.\left|\frac{1\,\text{min}}{60\,\text{s}}\right|0.21\,\text{atm}}{(20+273.15)\,\text{K}}\right) - \left(\frac{189\,\text{l min}^{-1}\,.\left|\frac{1\,\text{min}}{60\,\text{s}}\right|0.201\,\text{atm}}{(28+273.15)\,\text{K}}\right)\right]$$

$$N_{\text{A}} = 0.0174\,\text{gmol m}^{-3}\,\text{s}^{-1}$$

Answer: 0.0174 gmol m^{-3} s^{-1}

(b)

From Eq. (9.37) with the units of N_{A} from **(a)** converted to mass terms using the molecular weight of oxygen = 32.0 (Table B.1, Appendix B):

$$k_La = \frac{N_{\text{A}}}{\left(C^*_{\text{AL}} - C_{\text{AL}}\right)} = \frac{0.0174\,\text{gmol m}^{-3}\,\text{s}^{-1}\,.\left|\frac{32.0\,\text{g}}{1\,\text{gmol}}\right|.\left|\frac{1\,\text{kg}}{1000\,\text{g}}\right|}{\left(7.8\times10^{-3}\,\text{kg m}^{-3} - 0.52\times7.8\times10^{-3}\,\text{kg m}^{-3}\right)} = 0.15\,\text{s}^{-1}$$

Answer: 0.15 s^{-1}

(c)

If the measured exit gas composition of 20.1% O_2 is an overestimation, the actual value is ($^1/_{1.1}$ × 20.1)% = 18.3% O_2. Therefore, the partial pressure of oxygen in the exit gas is (0.183 × 1 atm) = 0.183 atm. From Eq. (9.48):

$$N_{\text{A}} = \frac{1}{R\,V_{\text{L}}}\left[\left(\frac{F_g\,p_{\text{AG}}}{T}\right)_{\text{i}} - \left(\frac{F_g\,p_{\text{AG}}}{T}\right)_{\text{o}}\right]$$

$$N_{\text{A}} = \frac{1}{0.000082057\,\text{m}^3\,\text{atm K}^{-1}\,\text{gmol}^{-1}\,(200\,\text{l})}\left[\left(\frac{200\,\text{l min}^{-1}\,.\left|\frac{1\,\text{min}}{60\,\text{s}}\right|0.21\,\text{atm}}{(20+273.15)\,\text{K}}\right) - \left(\frac{189\,\text{l min}^{-1}\,.\left|\frac{1\,\text{min}}{60\,\text{s}}\right|0.183\,\text{atm}}{(28+273.15)\,\text{K}}\right)\right]$$

$$N_{\text{A}} = 0.0289\,\text{gmol m}^{-3}\,\text{s}^{-1}$$

Therefore, from Eq. (9.37) with N_{A} converted to mass units:

$$k_La = \frac{N_{\text{A}}}{\left(C^*_{\text{AL}} - C_{\text{AL}}\right)} = \frac{0.0289\,\text{gmol m}^{-3}\,\text{s}^{-1}\,.\left|\frac{32.0\,\text{g}}{1\,\text{gmol}}\right|.\left|\frac{1\,\text{kg}}{1000\,\text{g}}\right|}{\left(7.8\times10^{-3}\,\text{kg m}^{-3} - 0.52\times7.8\times10^{-3}\,\text{kg m}^{-3}\right)} = 0.25\,\text{s}^{-1}$$

The k_La value obtained in **(b)** using the incorrectly calibrated oxygen analyser is 60% of the actual k_La value; the error is therefore 40%.

Answer: 40%. *This calculation illustrates the sensitivity of the oxygen balance method to the accuracy of all of the measured parameters used in Eq. (9.48). This sensitivity arises from the subtraction of two similar numbers for the moles of oxygen in and out of the system. When similar errors in both F_g terms are taken into consideration, the error in the final k_La value can be very large.*

9.4 k_La measurement

(a)

From p 16, $g = 9.8 \text{ m s}^{-2}$. Assuming the density of the culture broth to be the same as water, $\rho = 1000 \text{ kg m}^{-3}$. The static pressure at the sparger is:

$$p_s = \rho\, g\, h = 1000 \text{ kg m}^{-3}\,(9.8 \text{ m s}^{-2})\,(3.5 \text{ m}) = 3.43 \times 10^4 \text{ kg m}^{-1}\text{ s}^{-2}$$

From Table A.5 (Appendix A), $1 \text{ kg m}^{-1}\text{ s}^{-2} = 9.869 \times 10^{-6}$ atm; therefore:

$$p_s = 3.43 \times 10^4 \text{ kg m}^{-1}\text{ s}^{-1} = 3.43 \times 10^4 \text{ kg m}^{-1}\text{ s}^{-1} \cdot \left| \frac{9.869 \times 10^{-6} \text{ atm}}{1 \text{ kg m}^{-1}\text{ s}^{-1}} \right| = 0.34 \text{ atm}$$

p_s is the pressure due to the head of liquid above the sparger. The total pressure at the sparger is p_s + atmospheric pressure = 1.34 atm.

Answer: 1.34 atm

(b)

From Tables B.8 and B.1 (Appendix B), the molecular formulae for glucose and sucrose and the molecular weights of the medium components are: glucose ($C_6H_{12}O_6$) = 180.2, sucrose ($C_{12}H_{22}O_{11}$) = 342.3, $CaCO_3$ = 100.1, $(NH_4)_2SO_4$ = 132.1, Na_2HPO_4 = 142.0 and KH_2PO_4 = 136.1.

The parameter values for application in Eq. (9.45) are listed below. Values of H_i and K_j are taken from Table 9.5.

Medium component	H_i or K_j ($\text{m}^3 \text{ mol}^{-1}$)	z_i	C_{iL} or C_{jL} (mol m^{-3})
Glucose	0.119×10^{-3}	–	$20 \text{ g l}^{-1} \cdot \left\lvert \frac{1 \text{ mol}}{180.2 \text{ g}} \right\rvert \cdot \left\lvert \frac{1000 \text{ l}}{1 \text{ m}^3} \right\rvert = 111$
Sucrose	0.149×10^{-3}	–	$8.5 \text{ g l}^{-1} \cdot \left\lvert \frac{1 \text{ mol}}{342.3 \text{ g}} \right\rvert \cdot \left\lvert \frac{1000 \text{ l}}{1 \text{ m}^3} \right\rvert = 24.8$
Ca^{2+}	-0.303×10^{-3}	2	$1.3 \text{ g l}^{-1} \cdot \left\lvert \frac{1 \text{ mol}}{100.1 \text{ g}} \right\rvert \cdot \left\lvert \frac{1000 \text{ l}}{1 \text{ m}^3} \right\rvert = 13.0$
CO_3^{2-}	0.485×10^{-3}	2	$1.3 \text{ g l}^{-1} \cdot \left\lvert \frac{1 \text{ mol}}{100.1 \text{ g}} \right\rvert \cdot \left\lvert \frac{1000 \text{ l}}{1 \text{ m}^3} \right\rvert = 13.0$
NH_4^+	-0.720×10^{-3}	1	$2 \times 1.3 \text{ g l}^{-1} \cdot \left\lvert \frac{1 \text{ mol}}{132.1 \text{ g}} \right\rvert \cdot \left\lvert \frac{1000 \text{ l}}{1 \text{ m}^3} \right\rvert = 19.7$
SO_4^{2-}	0.453×10^{-3}	2	$1.3 \text{ g l}^{-1} \cdot \left\lvert \frac{1 \text{ mol}}{132.1 \text{ g}} \right\rvert \cdot \left\lvert \frac{1000 \text{ l}}{1 \text{ m}^3} \right\rvert = 9.8$
Na^+	-0.550×10^{-3}	1	$2 \times 0.09 \text{ g l}^{-1} \cdot \left\lvert \frac{1 \text{ mol}}{142.0 \text{ g}} \right\rvert \cdot \left\lvert \frac{1000 \text{ l}}{1 \text{ m}^3} \right\rvert = 1.3$
HPO_4^{2-}	0.485×10^{-3}	2	$0.09 \text{ g l}^{-1} \cdot \left\lvert \frac{1 \text{ mol}}{142.0 \text{ g}} \right\rvert \cdot \left\lvert \frac{1000 \text{ l}}{1 \text{ m}^3} \right\rvert = 0.63$
K^+	-0.596×10^{-3}	1	$0.12 \text{ g l}^{-1} \cdot \left\lvert \frac{1 \text{ mol}}{136.1 \text{ g}} \right\rvert \cdot \left\lvert \frac{1000 \text{ l}}{1 \text{ m}^3} \right\rvert = 0.88$
$H_2PO_4^-$	1.037×10^{-3}	1	$0.12 \text{ g l}^{-1} \cdot \left\lvert \frac{1 \text{ mol}}{136.1 \text{ g}} \right\rvert \cdot \left\lvert \frac{1000 \text{ l}}{1 \text{ m}^3} \right\rvert = 0.88$

Substituting these values into Eq. (9.45) gives:

$$\log_{10}\left(\frac{C^*_{\mathrm{AL0}}}{C^*_{\mathrm{AL}}}\right) = 0.5\sum_i H_i z_i^2 C_{i\mathrm{L}} + \sum_j K_j C_{j\mathrm{L}}$$

$$= 0.5\left[\begin{array}{l}\left(-0.303\times10^{-3}\right)(2)^2\,13.0+\left(0.485\times10^{-3}\right)(2)^2\,13.0+\left(-0.720\times10^{-3}\right)(1)^2\,19.7+\left(0.453\times10^{-3}\right)(2)^2\,9.8+\\ \left(-0.550\times10^{-3}\right)(1)^2\,1.3+\left(0.485\times10^{-3}\right)(2)^2\,0.63+\left(-0.596\times10^{-3}\right)(1)^2\,0.88+\left(1.037\times10^{-3}\right)(1)^2\,0.88\end{array}\right]$$
$$+\left[\left(0.119\times10^{-3}\right)111+\left(0.149\times10^{-3}\right)24.8\right]$$

$$\log_{10}\left(\frac{C^*_{\mathrm{AL0}}}{C^*_{\mathrm{AL}}}\right) = 2.39\times10^{-2}$$

$$\frac{C^*_{\mathrm{AL0}}}{C^*_{\mathrm{AL}}} = 1.06$$

$$C^*_{\mathrm{AL}} = 0.95\,C^*_{\mathrm{AL0}}$$

Solutes in the medium reduce the oxygen solubility by about 5%. From Table 9.2, the solubility of oxygen in water at 35°C and 1 atm air pressure is 7.52×10^{-3} kg m^{-3}. Using this value for C^*_{AL0}, $C^*_{\mathrm{AL}} = 0.95 \times 7.52 \times 10^{-3}$ kg m^{-3} = 7.14×10^{-3} kg m^{-3}.

Answer: 7.14×10^{-3} kg m^{-3}

(c)

From Henry's law, Eq. (9.43), the solubility of a gas is directly proportional to the total pressure. From **(a)**, the total pressure at the bottom of the tank is 1.34 atm rather than 1 atm; therefore:

$$C^*_{\mathrm{AL}} = 1.34\left(7.14\times10^{-3}\ \mathrm{kg\ m^{-3}}\right) = 9.57\times10^{-3}\ \mathrm{kg\ m^{-3}}$$

Answer: 9.57×10^{-3} kg m^{-3}

(d)

The logarithmic-mean concentration difference is given by Eq. (9.53). From **(b)**, the solubility of oxygen at the top or outlet end of the vessel is 7.14×10^{-3} kg m^{-3}; the dissolved oxygen concentration at this location is 50% air saturation or $0.50 \times 7.14 \times 10^{-3} = 3.57 \times 10^{-3}$ kg m^{-3}. From **(c)**, the solubility of oxygen at the bottom or inlet end of the vessel is 9.57×10^{-3} kg m^{-3}; the dissolved oxygen concentration at this location is 65% air saturation or $0.65 \times 9.57 \times 10^{-3}$ = 6.22×10^{-3} kg m^{-3}. Substituting these values into Eq. (9.53):

$$\left(C^*_{\mathrm{AL}}-C_{\mathrm{AL}}\right)_{\mathrm{L}} = \frac{\left(C^*_{\mathrm{AL}}-C_{\mathrm{AL}}\right)_{\mathrm{o}}-\left(C^*_{\mathrm{AL}}-C_{\mathrm{AL}}\right)_{\mathrm{i}}}{\ln\left[\dfrac{\left(C^*_{\mathrm{AL}}-C_{\mathrm{AL}}\right)_{\mathrm{o}}}{\left(C^*_{\mathrm{AL}}-C_{\mathrm{AL}}\right)_{\mathrm{i}}}\right]} = \frac{(7.14-3.57)\times10^{-3}\ \mathrm{kg\ m^{-3}}-(9.57-6.22)\times10^{-3}\ \mathrm{kg\ m^{-3}}}{\ln\left[\dfrac{(7.14-3.57)\times10^{-3}\ \mathrm{kg\ m^{-3}}}{(9.57-6.22)\times10^{-3}\ \mathrm{kg\ m^{-3}}}\right]}$$

$$\left(C^*_{\mathrm{AL}}-C_{\mathrm{AL}}\right)_{\mathrm{L}} = 3.46\times10^{-3}\ \mathrm{kg\ m^{-3}}$$

Answer: 3.46×10^{-3} kg m^{-3}

(e)

The oxygen transfer rate for $k_{\mathrm{L}}a$ determination by the oxygen-balance method is given in Eq. (9.48). Assume that the volumetric flow rates of gas into and out of the fermenter are measured under ambient pressure, i.e. at 1 atm. From p 17, the partial pressure of oxygen in the inlet air at 1 atm is (0.21 × 1 atm) = 0.21 atm; the partial pressure of oxygen in the outlet gas is (0.2015 × 1 atm) = 0.2015 atm. Assuming that the exit gas leaves the fermenter at the fermentation temperature (35°C), using R = 0.000082057 m^3 atm K^{-1} gmol^{-1} from Table 2.5 and converting the temperatures to degrees Kelvin using Eq. (2.24):

$$N_{\mathrm{A}} = \frac{1}{R\,V_{\mathrm{L}}}\left[\left(\frac{F_{\mathrm{g}}\,p_{\mathrm{AG}}}{T}\right)_{\mathrm{i}}-\left(\frac{F_{\mathrm{g}}\,p_{\mathrm{AG}}}{T}\right)_{\mathrm{o}}\right]$$

$$N_A = \frac{1}{0.000082057\,\mathrm{m^3\,atm\,K^{-1}\,gmol^{-1}}\left(20\,\mathrm{m^3}\right)}\left[\left(\frac{25\,\mathrm{m^3\,min^{-1}}\cdot\left|\frac{1\,\mathrm{min}}{60\,\mathrm{s}}\right|0.21\,\mathrm{atm}}{(25+273.15)\,\mathrm{K}}\right)-\left(\frac{407\,\mathrm{l\,s^{-1}}\cdot\left|\frac{1\,\mathrm{m^3}}{1000\,\mathrm{l}}\right|0.2015\,\mathrm{atm}}{(35+273.15)\,\mathrm{K}}\right)\right]$$

$$N_A = 0.0167\ \mathrm{gmol\,m^{-3}\,s^{-1}}$$

Answer: 0.0167 gmol $\mathrm{m^{-3}\,s^{-1}}$

(f)

From Eq. (9.37) with $(C^*_{AL} - C_{AL}) = (C^*_{AL} - C_{AL})_L$ from **(d)** and units of N_A converted to mass terms using the molecular weight of oxygen = 32.0 (Table B.1, Appendix B):

$$k_La = \frac{N_A}{\left(C^*_{AL}-C_{AL}\right)_L} = \frac{0.0167\,\mathrm{gmol\,m^{-3}\,s^{-1}}\cdot\left|\frac{32.0\,\mathrm{g}}{1\,\mathrm{gmol}}\right|\cdot\left|\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right|}{3.46\times10^{-3}\,\mathrm{kg\,m^{-3}}} = 0.15\,\mathrm{s^{-1}}$$

Answer: 0.15 $\mathrm{s^{-1}}$

(g)

The maximum cell concentration supported by the fermenter can be calculated using Eq. (9.40) with C^*_{AL} equal to the average solubility of oxygen between the top and bottom of the fermenter:

$$x_{max} = \frac{k_La\left(C^*_{AL}\right)}{q_O} = \frac{0.15\,\mathrm{s^{-1}}\left[\left(\frac{7.14+9.57}{2}\right)\times10^{-3}\right]\mathrm{kg\,m^{-3}}\cdot\left|\frac{1\,\mathrm{m^3}}{1000\,\mathrm{l}}\right|}{7.4\,\mathrm{mmol\,g^{-1}\,h^{-1}}\cdot\left|\frac{1\,\mathrm{gmol}}{1000\,\mathrm{mmol}}\right|\cdot\left|\frac{32.0\,\mathrm{g}}{1\,\mathrm{gmol}}\right|\cdot\left|\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right|\cdot\left|\frac{1\,\mathrm{h}}{3600\,\mathrm{s}}\right|} = 19\,\mathrm{g\,l^{-1}}$$

Answer: 19 g $\mathrm{l^{-1}}$

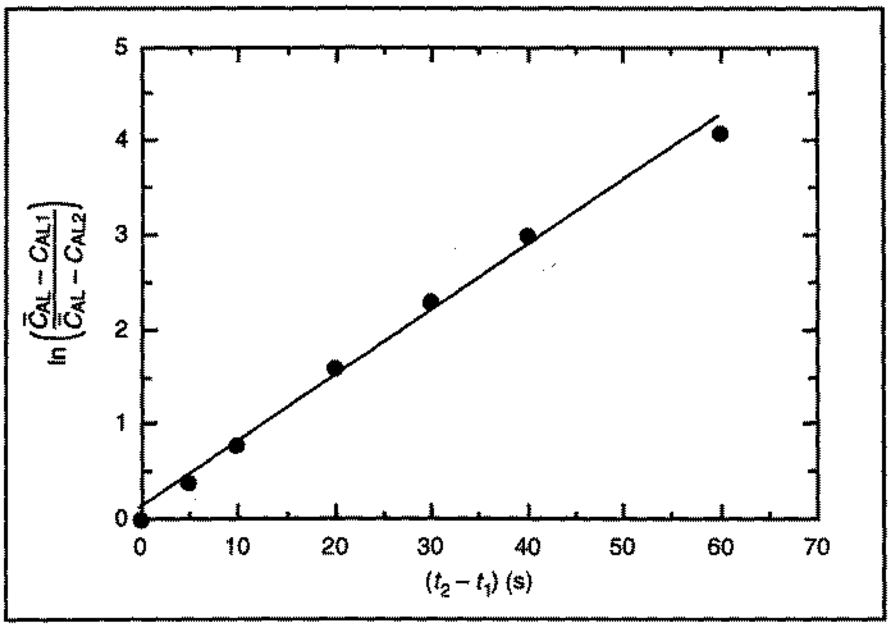
9.5 Dynamic k_La measurement

k_La is determined using Eq. (9.52) and the method described in Section 9.10.2. Take t_1 = 10 s and C_{AL1} = 43.5% air saturation. The steady-state dissolved oxygen tension $\bar{C}_{AL}$ = 73.5%. Calculated values of

$$\ln\left(\frac{\bar{C}_{AL}-C_{AL1}}{\bar{C}_{AL}-C_{AL2}}\right)$$

and $(t_2 - t_1)$ are listed and plotted below.

$\ln\left(\frac{\bar{C}_{AL}-C_{AL1}}{\bar{C}_{AL}-C_{AL2}}\right)$	$(t_2 - t_1)$ (s)
0.00	0
0.41	5
0.80	10
1.6	20
2.3	30
3.0	40
4.1	60



k_La is equal to the slope of the straight line in the plot = 0.069 s^{-1}.

Answer: 0.069 s^{-1}

9.6 Measurement of k_La as a function of stirrer speed: the oxygen-balance method of Mukhopadhyay and Ghose

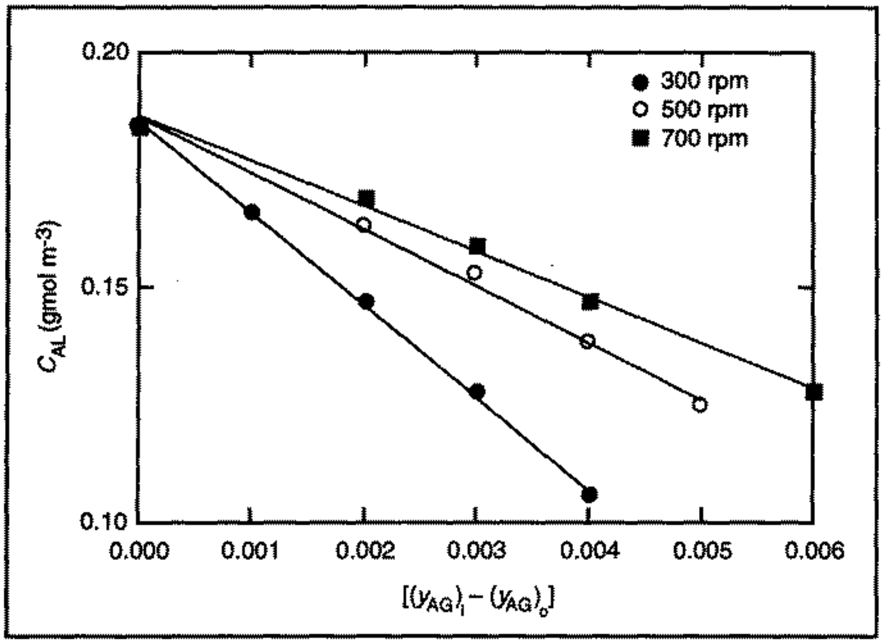
From p 17, the mole fraction of oxygen in the inlet air $(y_{AG})_i = 0.210$. Convert the data for C_{AL} to units of gmol m^{-3} using the molecular weight of oxygen = 32 (Table B.1, Appendix B) and assuming that the density of the fermentation broth is the same as that of water = 1000 kg m^{-3}:

$$1\text{ ppm} = \frac{1\text{ g}}{10^6\text{ g}}\left(1000\text{ kg m}^{-3}\right).\left|\frac{1000\text{ g}}{\text{kg}}\right|.\left|\frac{1\text{ gmol}}{32.0\text{ g}}\right| = 3.125\times 10^{-2}\text{ gmol m}^{-3}$$

Values of C_{AL} and $[(y_{AG})_i - (y_{AG})_o]$ for each stirrer speed are listed below.

Fermentation time (h)	Agitator speed					
	300 rpm		500 rpm		700 rpm	
	C_{AL} (gmol m^{-3})	$[(y_{AG})_i - (y_{AG})_o]$	C_{AL} (gmol m^{-3})	$[(y_{AG})_i - (y_{AG})_o]$	C_{AL} (gmol m^{-3})	$[(y_{AG})_i - (y_{AG})_o]$
0	0.184	0	0.184	0	0.184	0
4	–	–	0.175	0.001	0.178	0.001
5	0.166	0.001	–	–	–	–
6	–	–	0.163	0.002	0.169	0.002
7	0.147	0.002	0.153	0.003	0.159	0.003
8	0.128	0.003	0.138	0.004	0.147	0.004
9	0.106	0.004	0.125	0.005	0.128	0.006
10	0.106	0.004	0.125	0.005	0.128	0.006
11	0.109	0.003	0.131	0.004	0.131	0.005

The tabulated values of C_{AL} and $[(y_{AG})_i - (y_{AG})_o]$ for 0–10 h are plotted below. As C_{AL} increases at 11 h after being constant at 9 and 10 h, the final values in the table for each stirrer speed most likely reflect the decline phase of culture growth and are therefore not included in the k_La analysis.



(a)
The slopes of the straight lines in the plot for 300, 500 and 700 rpm are −19.58, −12.04 and −9.68 gmol m^{-3}, respectively. These values can be used to determine k_La using $R = 0.000082057$ m^3 atm $gmol^{-1}$ K^{-1} from Table 2.5 and converting the temperature to degrees Kelvin using Eq. (2.24):

$$k_La = \frac{-p_T F_g}{R\,T\,V_L \times \text{slope}} = \frac{-3\text{ atm}\left(3\text{ l min}^{-1}\,.\left|\frac{1\text{ min}}{60\text{ s}}\right|\right)}{0.000082057\,\text{m}^3\text{ atm gmol}^{-1}\text{ K}^{-1}\,(29 + 273.15)\text{ K}\,(3\text{ l}) \times \text{slope}} = \frac{-2.02\text{ gmol m}^{-3}\text{ s}^{-1}}{\text{slope}}$$

For 300 rpm:

$$k_La = \frac{-2.02\text{ gmol m}^{-3}\text{ s}^{-1}}{-19.58\text{ gmol m}^{-3}} = 0.10\text{ s}^{-1}$$

For 500 rpm:

$$k_La = \frac{-2.02\text{ gmol m}^{-3}\text{ s}^{-1}}{-12.04\text{ gmol m}^{-3}} = 0.17\text{ s}^{-1}$$

At 700 rpm:

$$k_La = \frac{-2.02\text{ gmol m}^{-3}\text{ s}^{-1}}{-9.68\text{ gmol m}^{-3}} = 0.21\text{ s}^{-1}$$

Answer: 0.10 s^{-1} at 300 rpm; 0.17 s^{-1} at 500 rpm; 0.21 s^{-1} at 700 rpm

(b)
The intercepts of the straight lines in the plot for 300, 500 and 700 rpm are 0.185, 0.186 and 0.187 gmol m^{-3}, respectively. Taking the average, C^*_{AL} is 0.186 gmol m^{-3}. Converting to mass terms using the molecular weight of oxygen = 32.0 (Table B.1, Appendix B):

$$C^*_{AL} = 0.186\text{ gmol m}^{-3} = 0.186\text{ gmol m}^{-3}\,.\left|\frac{32.0\text{ g}}{1\text{ gmol}}\right|.\left|\frac{1\text{ kg}}{1000\text{ g}}\right| = 5.95 \times 10^{-3}\text{ kg m}^{-3}$$

Answer: 5.95×10^{-3} kg m^{-3}

(c)
Equating Eqs (9.39) and (9P6.2):

$$q_O x = \frac{p_T F_g}{R T V_L}\left[(y_{AG})_i - (y_{AG})_o\right]$$

$$q_O x = \frac{3\ \text{atm}\left(3\ \text{l min}^{-1} \cdot \left|\frac{1\ \text{min}}{60\ \text{s}}\right|\right)\left[(y_{AG})_i - (y_{AG})_o\right]}{0.000082057\ \text{m}^3\ \text{atm gmol}^{-1}\ \text{K}^{-1}\ (29 + 273.15)\ \text{K}\ (3\ \text{l})} = 2.02\left[(y_{AG})_i - (y_{AG})_o\right]\ \text{gmol m}^{-3}\ \text{s}^{-1}$$

The maximum rate of oxygen uptake occurs when $[(y_{AG})_i - (y_{AG})_o]$ is maximum. At 300 rpm, the maximum value of $[(y_{AG})_i - (y_{AG})_o]$ is 0.004; therefore, $(q_O\, x)_{max} = 2.02 \times 0.004 = 8.1 \times 10^{-3}$ gmol m^{-3} s^{-1}. At 500 rpm, the maximum value of $[(y_{AG})_i - (y_{AG})_o]$ is 0.005; therefore, $(q_O\, x)_{max} = 2.02 \times 0.005 = 1.0 \times 10^{-2}$ gmol m^{-3} s^{-1}. At 700 rpm, the maximum value of $[(y_{AG})_i - (y_{AG})_o]$ is 0.006; therefore, $(q_O\, x)_{max} = 2.02 \times 0.006 = 1.2 \times 10^{-2}$ gmol m^{-3} s^{-1}.

Answer: 8.1×10^{-3} gmol m^{-3} s^{-1} at 300 rpm; 1.0×10^{-2} gmol m^{-3} s^{-1} at 500 rpm; 1.2×10^{-2} gmol m^{-3} s^{-1} at 700 rpm

10

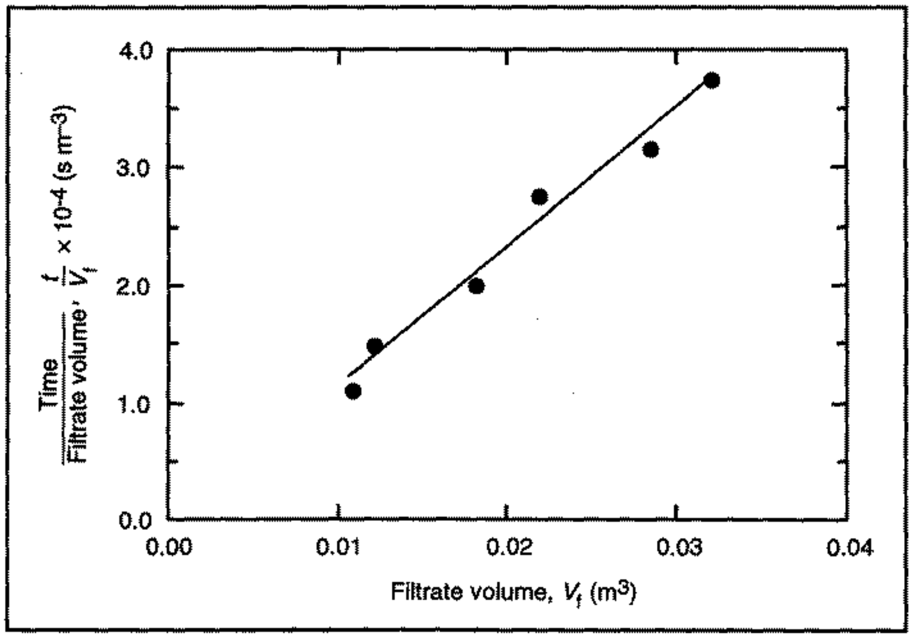
Unit Operations

10.1 Bacterial filtration

(a)

According to Eqs (10.12)–(10.14), a plot of ${}^{t}/_{V_f}$ versus V_f should yield a straight line for determination of the filtration parameters. The data after converting the units to s and m^3 are listed and plotted below.

Time, t (s)	Filtrate volume, V_f (m^3)	${}^{t}/_{V_f}$ (s m^{-3})
120	0.0108	1.11×10^4
180	0.0121	1.49×10^4
360	0.0180	2.00×10^4
600	0.0218	2.75×10^4
900	0.0284	3.17×10^4
1200	0.0320	3.75×10^4



The slope of the straight line in the plot $K_1 = 1.18 \times 10^6$ s m^{-6}; the intercept $K_2 = -363$ s m^{-3}. From Eq. (10.13), using the conversion factors 1 mmHg = 1.333×10^2 kg m^{-1} s^{-2} (Table A.5, Appendix A) and 1 cP = 10^{-3} kg m^{-1} s^{-1} (Table A.9, Appendix A), the specific cake resistance α is:

$$\alpha = \frac{2\,A^2\,\Delta p\,K_1}{\mu_f\,c} = \frac{2\left(0.25\,\text{m}^2\right)^2 360\,\text{mmHg}\,.\left|\dfrac{1.333\times10^2\,\text{kg m}^{-1}\,\text{s}^{-2}}{1\,\text{mmHg}}\right|\left(1.18\times10^6\,\text{s m}^{-6}\right)}{4.0\,\text{cP}\,.\left|\dfrac{10^{-3}\,\text{kg m}^{-1}\,\text{s}^{-1}}{1\,\text{cP}}\right|\left(22\,\text{g l}^{-1}\,.\left|\dfrac{1000\,\text{l}}{1\,\text{m}^3}\right|.\left|\dfrac{1\,\text{kg}}{1000\,\text{g}}\right|\right)}$$

$$\alpha = 8.0 \times 10^{10}\ \text{m kg}^{-1}$$

The intercept K_2 is a relatively small negative number, which may practically be taken as zero. Therefore, from

Eq. (10.14), the filter medium resistance r_m is zero.

Answer: $\alpha = 8.0 \times 10^{10}$ m kg^{-1}; r_m is effectively zero.

(b)

${}^{t}/_{V_f} = 7.5 \times 10^{-3}$ min l^{-1}. From **(a)**, $K_2 = 0$; therefore, from Eq. (10.12):

$$K_1 = \frac{{}^{t}/_{V_f}}{V_f} = \frac{7.5 \times 10^{-3}\ \text{min l}^{-1}}{4000\ \text{l}} \cdot \left|\frac{1000\ \text{l}}{1\ \text{m}^3}\right|^2 \cdot \left|\frac{60\ \text{s}}{1\ \text{min}}\right| = 1.13 \times 10^2\ \text{s m}^{-6}$$

The filter area is obtained from Eq. (10.13) using the result for α from **(a)**:

$$A^2 = \frac{\mu_f\, \alpha\, c}{2\, K_1\, \Delta p} = \frac{4.0\ \text{cP} \cdot \left|\frac{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}}{1\ \text{cP}}\right| 8.0 \times 10^{10}\ \text{m kg}^{-1} \left(22\ \text{g l}^{-1} \cdot \left|\frac{1000\ \text{l}}{1\ \text{m}^3}\right| \cdot \left|\frac{1\ \text{kg}}{1000\ \text{g}}\right|\right)}{2\left(1.13 \times 10^2\ \text{s m}^{-6}\right) 360\ \text{mmHg} \cdot \left|\frac{1.333 \times 10^2\ \text{kg m}^{-1}\ \text{s}^{-2}}{1\ \text{mmHg}}\right|} = 649\ \text{m}^4$$

$$A = 25.5\ \text{m}^2$$

Answer: 25.5 m^2

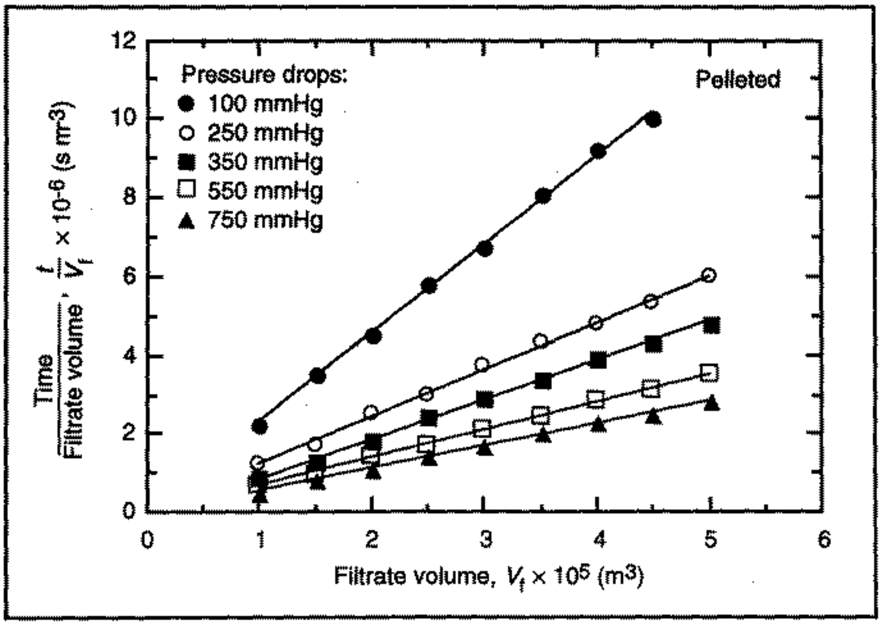
10.2 Filtration of mycelial suspensions

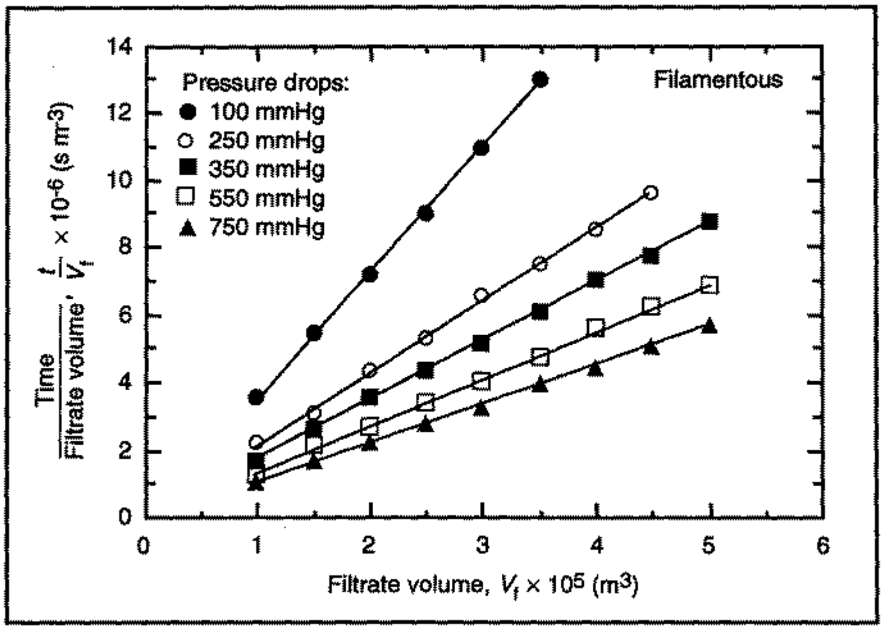
(a)

After converting units using the conversion factor 1 m^3 = 10^6 ml, the data for ${}^{t}/_{V_f}$ and V_f are listed and plotted below.

			Pressure drop (mmHg)		
	100	250	350	550	750
V_f (m^3) pelleted			${}^{t}/_{V_f}$ (s m^{-3})		
1.0×10^{-5}	2.2×10^6	1.2×10^6	9.0×10^5	7.0×10^5	5.0×10^5
1.5×10^{-5}	3.5×10^6	1.7×10^6	1.3×10^6	9.3×10^5	8.0×10^5
2.0×10^{-5}	4.5×10^6	2.5×10^6	1.8×10^6	1.4×10^6	1.1×10^6
2.5×10^{-5}	5.8×10^6	3.0×10^6	2.4×10^6	1.7×10^6	1.4×10^6
3.0×10^{-5}	6.7×10^6	3.7×10^6	2.9×10^6	2.1×10^6	1.7×10^6
3.5×10^{-5}	8.1×10^6	4.3×10^6	3.4×10^6	2.4×10^6	2.0×10^6
4.0×10^{-5}	9.2×10^6	4.8×10^6	3.9×10^6	2.8×10^6	2.3×10^6
4.5×10^{-5}	1.0×10^7	5.3×10^6	4.3×10^6	3.1×10^6	2.5×10^6
5.0×10^{-5}	–	6.0×10^6	4.8×10^6	3.5×10^6	2.8×10^6

			Pressure drop (mmHg)		
	100	250	350	550	750
V_f (m^3) filamentous			${}^{t}/_{V_f}$ (s m^{-3})		
1.0×10^{-5}	3.6×10^6	2.2×10^6	1.7×10^6	1.3×10^6	1.1×10^6
1.5×10^{-5}	5.5×10^6	3.1×10^6	2.7×10^6	2.1×10^6	1.7×10^6
2.0×10^{-5}	7.2×10^6	4.3×10^6	3.6×10^6	2.7×10^6	2.3×10^6
2.5×10^{-5}	9.0×10^6	5.3×10^6	4.4×10^6	3.4×10^6	2.8×10^6
3.0×10^{-5}	1.1×10^7	6.5×10^6	5.2×10^6	4.0×10^6	3.3×10^6
3.5×10^{-5}	1.3×10^7	7.5×10^6	6.1×10^6	4.7×10^6	4.0×10^6
4.0×10^{-5}	–	8.5×10^6	7.1×10^6	5.6×10^6	4.5×10^6
4.5×10^{-5}	–	9.6×10^6	7.8×10^6	6.2×10^6	5.1×10^6
5.0×10^{-5}	–	–	8.8×10^6	6.8×10^6	5.7×10^6





From Eq. (10.12), the slopes of the straight lines in the plots are equal to K_1 for each pressure drop. These values are listed below.

	Pressure drop (mmHg)				
	100	250	350	550	750
Pelleted					
Slope = K_1 (s m^{-6})	2.26×10^{11}	1.20×10^{11}	9.93×10^{10}	7.07×10^{10}	5.77×10^{10}
Filamentous					
Slope = K_1 (s m^{-6})	3.73×10^{11}	2.13×10^{11}	1.75×10^{11}	1.38×10^{11}	1.14×10^{11}

From Eq. (10.13), using the conversion factors 1 mmHg = 1.333×10^2 kg m^{-1} s^{-2} (Table A.5, Appendix A) and 1 cP = 10^{-3} kg m^{-1} s^{-1} (Table A.9, Appendix A), the specific cake resistance α of the pelleted cells is:

$$\alpha = \frac{2A^2 \Delta p\, K_1}{\mu_f c} = \frac{2\left(1.8\,\text{cm}^2 . \left|\frac{1\,\text{m}}{100\,\text{cm}}\right|^2\right)^2 \Delta p . \left|\frac{1.333\times 10^2\,\text{kg m}^{-1}\,\text{s}^{-2}}{1\,\text{mmHg}}\right| K_1}{1.4\,\text{cP} . \left|\frac{10^{-3}\,\text{kg m}^{-1}\,\text{s}^{-1}}{1\,\text{cP}}\right| \left(0.25\,\text{g ml}^{-1} . \left|\frac{10^6\,\text{ml}}{1\,\text{m}^3}\right| . \left|\frac{1\,\text{kg}}{1000\,\text{g}}\right|\right)} = 2.47\times 10^{-5}\,\Delta p\, K_1\ \text{m kg}^{-1}$$

where Δp has units of mmHg and K_1 has units of s m^{-6}. Similarly for the filamentous cells:

$$\alpha = \frac{2A^2 \Delta p\, K_1}{\mu_f c} = \frac{2\left(1.8\,\text{cm}^2 . \left|\frac{1\,\text{m}}{100\,\text{cm}}\right|^2\right)^2 \Delta p . \left|\frac{1.333\times 10^2\,\text{kg m}^{-1}\,\text{s}^{-2}}{1\,\text{mmHg}}\right| K_1}{1.4\,\text{cP} . \left|\frac{10^{-3}\,\text{kg m}^{-1}\,\text{s}^{-1}}{1\,\text{cP}}\right| \left(0.1\,\text{g ml}^{-1} . \left|\frac{10^6\,\text{ml}}{1\,\text{m}^3}\right| . \left|\frac{1\,\text{kg}}{1000\,\text{g}}\right|\right)} = 6.17\times 10^{-5}\,\Delta p\, K_1\ \text{m kg}^{-1}$$

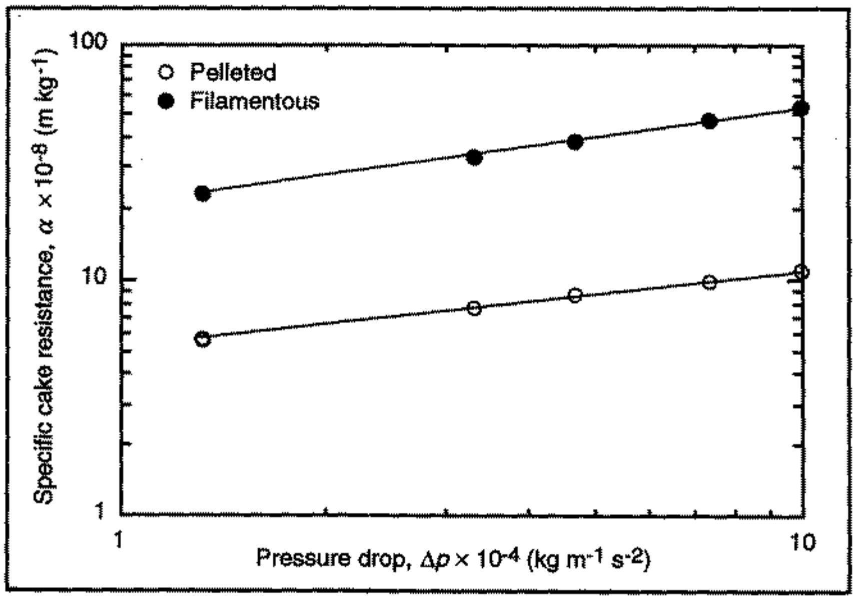
The results for specific cake resistance obtained after substituting the values for Δp and K_1 into the equations are listed below.

	Pressure drop (mmHg)				
	100	250	350	550	750
Pelleted					
α (m kg^{-1})	5.58×10^8	7.41×10^8	8.58×10^8	9.60×10^8	1.07×10^9
Filamentous					
α (m kg^{-1})	2.30×10^9	3.29×10^9	3.78×10^9	4.68×10^9	5.28×10^9

(b)

From Eq. (10.2), the compressibility can be obtained by plotting the specific cake resistance α versus Δp on log–log coordinates. Using the conversion factor 1 mmHg = 1.333×10^2 kg m^{-1} s^{-2} (Table A.5, Appendix A) to convert the units of Δp, the data are listed and plotted below.

	Pressure drop (kg m^{-1} s^{-2})				
	1.33×10^4	3.33×10^4	4.67×10^4	7.33×10^4	1.00×10^5
Pelleted					
α (m kg^{-1})	5.58×10^8	7.41×10^8	8.58×10^8	9.60×10^8	1.07×10^9
Filamentous					
α (m kg^{-1})	2.30×10^9	3.29×10^9	3.78×10^9	4.68×10^9	5.28×10^9



From Eq. (10.2), the equations for the two lines in the figure have the form, $\alpha = \alpha' \, \Delta p^s$. For the pelleted suspension, $\alpha = 2.60 \times 10^7 \, \Delta p^{0.323}$; for the filamentous suspension, $\alpha = 4.42 \times 10^7 \, \Delta p^{0.415}$, where Δp has units kg m^{-1} s^{-2}, α has units m kg^{-1}, and α' has units $\mathrm{kg}^{-(1+s)}\,\mathrm{m}^{1+s}\,\mathrm{s}^{2s}$. Therefore, the compressibility is 0.323 for the pelleted suspension and 0.415 for the filamentous suspension.

Answer: 0.323 for the pelleted suspension; 0.415 for the filamentous suspension

(c)

The filtration equation for a compressible filter cake is Eq. (10.11) with $\alpha = \alpha' \, \Delta p^s$. Assuming that the filter medium resistance r_m is negligible:

$$\frac{t}{V_f} = \frac{\mu_f \, \alpha' \, \Delta p^s \, c}{2 A^2 \, \Delta p} V_f = \frac{\mu_f \, \alpha' \, \Delta p^{s-1} \, c}{2 A^2} V_f$$

Solving for $(\Delta p)^{s-1}$:

$$(\Delta p)^{s-1} = \frac{t}{V_f^2} \frac{2 A^2}{\mu_f \, \alpha' \, c}$$

Substituting the parameter values for the filamentous suspension using consistent units, including the results $\alpha' = 4.42 \times 10^7 \, \mathrm{kg}^{-(1+s)}\,\mathrm{m}^{1+s}\,\mathrm{s}^{2s}$ and $s = 0.415$ from **(b)**:

$$(\Delta p)^{0.415-1} = \frac{1\,\mathrm{h} \cdot \left|\frac{3600\,\mathrm{s}}{1\,\mathrm{h}}\right|}{\left(20\,\mathrm{m}^3\right)^2} \times$$

$$\frac{2\left(15\,\mathrm{m}^2\right)^2}{1.4\,\mathrm{cP} \cdot \left|\frac{10^{-3}\,\mathrm{kg\,m^{-1}\,s^{-1}}}{1\,\mathrm{cP}}\right| 4.42 \times 10^7\,\mathrm{kg}^{-(1+0.415)}\,\mathrm{m}^{1+0.415}\,\mathrm{s}^{2 \times 0.415} \left(0.1\,\mathrm{g\,ml^{-1}} \cdot \left|\frac{10^6\,\mathrm{ml}}{1\,\mathrm{m}^3}\right| \cdot \left|\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right|\right)}$$

$$(\Delta p)^{-0.585} = 6.54 \times 10^{-4}\,\mathrm{kg}^{-0.585}\,\mathrm{m}^{0.585}\,\mathrm{s}^{1.170}$$

$$\Delta p = 2.78 \times 10^5\,\mathrm{kg\,m^{-1}\,s^{-2}}$$

Converting units using the conversion factor 1 mmHg = 1.333×10^2 kg m^{-1} s^{-2} (Table A.5, Appendix A):

$$\Delta p = 2.78 \times 10^5\,\mathrm{kg\,m^{-1}\,s^{-2}} = 2.78 \times 10^5\,\mathrm{kg\,m^{-1}\,s^{-2}} \cdot \left|\frac{1\,\mathrm{mmHg}}{1.333 \times 10^2\,\mathrm{kg\,m^{-1}\,s^{-2}}}\right| = 2086\,\mathrm{mmHg}$$

Answer: 2086 mmHg. *An assumption associated with this answer is that the filter cake characteristics measured at pressures between 100 and 750 mmHg apply at the much higher pressure of 2086 mmHg.*

10.3 Rotary-drum vacuum filtration

(a)

For negligible filter medium resistance, the filtration equation for a compressible filter cake is Eq. (10.11) with $r_m = 0$ and $\alpha = \alpha' \, \Delta p^s$:

$$\frac{t}{V_f} = \frac{\mu_f \, \alpha' \, \Delta p^s \, c}{2 A^2 \, \Delta p} V_f = \frac{\mu_f \, \alpha' \, \Delta p^{s-1} \, c}{2 A^2} V_f$$

Solving for $\mu_f \, \alpha' \, c$:

$$\mu_f \, \alpha' \, c = \frac{t}{V_f^2} \frac{2 A^2}{\Delta p^{s-1}}$$

Substituting the parameter values for the laboratory filter and using the conversion factor 1 psi = 6.895×10^3 kg m^{-1} s^{-2} (Table A.5, Appendix A):

$$\mu_\text{f}\,\alpha'\,c = \frac{23.5\,\text{min}\,.\left|\frac{60\,\text{s}}{1\,\text{min}}\right|}{\left(500\,\text{ml}\,.\left|\frac{1\,\text{m}^3}{10^6\,\text{ml}}\right|\right)^2}\;\frac{2\left(5\,\text{cm}^2\,.\left|\frac{1\,\text{m}}{100\,\text{cm}}\right|^2\right)^2}{\left(12\,\text{psi}\,.\left|\frac{6.895\times10^3\,\text{kg}\,\text{m}^{-1}\,\text{s}^{-2}}{1\,\text{psi}}\right|\right)^{0.57-1}} = 3.67\times10^5\,\text{kg}^{0.43}\,\text{m}^{-2.43}\,\text{s}^{0.14}$$

Answer: $3.67 \times 10^5\,\text{kg}^{0.43}\,\text{m}^{-2.43}\,\text{s}^{0.14}$

(b)

The cycle time is $^1/_N$ hours per revolution.

Answer: $^1/_N$ h

(c)

As 30% of the rotating filter cloth is submerged at any time, each cm^2 of cloth is submerged for 0.3 × the cycle time = $^{0.3}/_N$ h.

Answer: $^{0.3}/_N$ h

(d)

The volume filtered per revolution is $20\,\text{m}^3\,\text{h}^{-1} \times {}^1/_N\,\text{h} = {}^{20}/_N\,\text{m}^3$.

Answer: $^{20}/_N\,\text{m}^3$

(e)

Eq. (10.11) with $r_\text{m} = 0$ and $\alpha = \alpha'\,\Delta p^s$ is:

$$\frac{t}{V_\text{f}} = \frac{\mu_\text{f}\,\alpha'\,\Delta p^s\,c}{2\,A^2\,\Delta p}\,V_\text{f} = \frac{\mu_\text{f}\,\alpha'\,c\,\Delta p^{s-1}}{2\,A^2}\,V_\text{f}$$

Applying this equation to a single revolution of the filter, $t = {}^{0.3}/_N$ h and $V_\text{f} = {}^{20}/_N\,\text{m}^3$. Substituting the filter cake parameter values from **(a)**, evaluating the filter area $A = 2\,\pi\,r\,W$ where r is the drum radius = 0.75 m and W is the drum width = 1.2 m, and using the conversion factor 1 psi = $6.895 \times 10^3\,\text{kg}\,\text{m}^{-1}\,\text{s}^{-2}$ (Table A.5, Appendix A):

$$\frac{\frac{0.3}{N}\,\text{h}\,.\left|\frac{3600\,\text{s}}{1\,\text{h}}\right|}{\frac{20}{N}\,\text{m}^3} = \frac{3.67\times10^5\,\text{kg}^{0.43}\,\text{m}^{-2.43}\,\text{s}^{0.14}\left(4.5\,\text{psi}\,.\left|\frac{6.895\times10^3\,\text{kg}\,\text{m}^{-1}\,\text{s}^{-2}}{1\,\text{psi}}\right|\right)^{0.57-1}}{2\,(2\,\pi\,(0.75\,\text{m})\,(1.2\,\text{m}))^2}\left(\frac{20}{N}\,\text{m}^3\right)$$

where N is the number of revolutions per hour. Calculating both sides of the equation gives:

$$54\,\text{s}\,\text{m}^{-3} = \frac{1343.9}{N}\,\text{s}\,\text{m}^{-3}$$

$$N = 24.9\,\text{rph}$$

Answer: 24.9 revolutions per hour

(f)

The filtration time is increased from $^{0.3}/_N$ h to $^{0.5}/_N$ h. Substituting this value for t into the left-hand-side of the equation in **(e)**:

$$\frac{\frac{0.5}{N}\,\text{h}\,.\left|\frac{3600\,\text{s}}{1\,\text{h}}\right|}{\frac{20}{N}\,\text{m}^3} = \frac{1343.9}{N}\,\text{s}\,\text{m}^{-3}$$

$$N = 14.9\ \mathrm{rph}$$

Answer: 14.9 revolutions per hour

10.4 Centrifugation of yeast

(a)
Convert the parameter values to units of kg, m, s:

$$\rho_\mathrm{p} = 1.06\,\mathrm{g\,cm^{-3}} = 1.06\,\mathrm{g\,cm^{-3}}\,.\left|\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right|.\left|\frac{100\,\mathrm{cm}}{1\,\mathrm{m}}\right|^3 = 1060\,\mathrm{kg\,m^{-3}}$$

$$\rho_\mathrm{f} = 0.997\,\mathrm{g\,cm^{-3}} = 0.997\,\mathrm{g\,cm^{-3}}\,.\left|\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right|.\left|\frac{100\,\mathrm{cm}}{1\,\mathrm{m}}\right|^3 = 997\,\mathrm{kg\,m^{-3}}$$

$$Q = 500\,\mathrm{l\,h^{-1}} = 500\,\mathrm{l\,h^{-1}}\,.\left|\frac{1\,\mathrm{m^3}}{1000\,\mathrm{l}}\right|.\left|\frac{1\,\mathrm{h}}{3600\,\mathrm{s}}\right| = 1.39\times10^{-4}\,\mathrm{m^3\,s^{-1}}$$

From Table A.9 (Appendix A), 1 N s $\mathrm{m^{-2}}$ = 1 kg $\mathrm{m^{-1}\,s^{-1}}$; therefore $\mu = 1.36 \times 10^{-3}\ \mathrm{kg\,m^{-1}\,s^{-1}}$. $D_\mathrm{p} = 5\ \mu\mathrm{m} = 5 \times 10^{-6}$ m. From Eq. (10.15) with $g = 9.8\ \mathrm{m\,s^{-2}}$ (p 16), the sedimentation velocity is:

$$u_\mathrm{g} = \frac{\rho_\mathrm{p}-\rho_\mathrm{f}}{18\,\mu}D_\mathrm{p}^2\,g = \frac{(1060-997)\,\mathrm{kg\,m^{-3}}}{18\left(1.36\times10^{-3}\,\mathrm{kg\,m^{-1}\,s^{-1}}\right)}\left(5\times10^{-6}\,\mathrm{m}\right)^2 9.8\,\mathrm{m\,s^{-2}} = 6.31\times10^{-7}\,\mathrm{m\,s^{-1}}$$

Substituting this result into Eq. (10.18):

$$\Sigma = \frac{Q}{2\,u_\mathrm{g}} = \frac{1.39\times10^{-4}\,\mathrm{m^3\,s^{-1}}}{2\left(6.31\times10^{-7}\,\mathrm{m\,s^{-1}}\right)} = 110\,\mathrm{m^2}$$

Answer: 110 $\mathrm{m^2}$

(b)
From the relationship between specific gravity and density (p 16), as the density of water is close to 1.0000 g $\mathrm{cm^{-3}}$ (p 16):

$$\rho_\mathrm{p} = 2.0\,\mathrm{g\,cm^{-3}} = 2.0\,\mathrm{g\,cm^{-3}}\,.\left|\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right|.\left|\frac{100\,\mathrm{cm}}{1\,\mathrm{m}}\right|^3 = 2000\,\mathrm{kg\,m^{-3}}$$

$D_\mathrm{p} = 0.1\ \mathrm{mm} = 0.1 \times 10^{-3}$ m. From Eq. (10.15), the sedimentation velocity for the quartz particles is:

$$u_\mathrm{g} = \frac{\rho_\mathrm{p}-\rho_\mathrm{f}}{18\,\mu}D_\mathrm{p}^2\,g = \frac{(2000-997)\,\mathrm{kg\,m^{-3}}}{18\left(1.36\times10^{-3}\,\mathrm{kg\,m^{-1}\,s^{-1}}\right)}\left(0.1\times10^{-3}\,\mathrm{m}\right)^2 9.8\,\mathrm{m\,s^{-2}} = 4.02\times10^{-3}\,\mathrm{m\,s^{-1}}$$

From Eq. (10.18), the sigma factor is:

$$\Sigma = \frac{Q}{2\,u_\mathrm{g}} = \frac{1.39\times10^{-4}\,\mathrm{m^3\,s^{-1}}}{2\left(4.02\times10^{-3}\,\mathrm{m\,s^{-1}}\right)} = 0.017\,\mathrm{m^2}$$

From the result in **(a)**, Σ for the yeast cells is $^{110}/_{0.017}$ = 6470 times that for the quartz particles.

Answer: By a factor of 6470.

10.5 Centrifugation of food particles

Convert the parameter values to units of kg, m, s:

$$\rho_\mathrm{p} = 1.03\,\mathrm{g\,cm^{-3}} = 1.03\,\mathrm{g\,cm^{-3}}\,.\left|\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right|.\left|\frac{100\,\mathrm{cm}}{1\,\mathrm{m}}\right|^3 = 1030\,\mathrm{kg\,m^{-3}}$$

$$\rho_f = 1.00\,\text{g cm}^{-3} = 1.00\,\text{g cm}^{-3}\cdot\left|\frac{1\,\text{kg}}{1000\,\text{g}}\right|\cdot\left|\frac{100\,\text{cm}}{1\,\text{m}}\right|^3 = 1000\,\text{kg m}^{-3}$$

From Table A.9 (Appendix A), 1 Pa s = 1 kg $\text{m}^{-1}\,\text{s}^{-1}$; therefore $\mu = 1.25\times10^{-3}$ kg $\text{m}^{-1}\,\text{s}^{-1}$. $D_p = 10^{-2}$ mm $= 10^{-5}$ m; b = 70 cm = 0.70 m; r = 11.5 cm = 0.115 m. As one revolution = 2 π radians where radians in a non-dimensional unit (p 11), converting the centrifuge speed to radians s^{-1}:

$$\omega = 10{,}000\,\text{rpm} = 2\,\pi\,(10{,}000)\,\text{min}^{-1}\cdot\left|\frac{1\,\text{min}}{60\,\text{s}}\right| = 1.047\times10^{3}\,\text{s}^{-1}$$

From Eq. (10.22) with $g = 9.8$ m s^{-2} (p 16), the sigma factor for the tubular-bowl centrifuge is:

$$\Sigma = \frac{2\,\pi\,\omega^2\,b\,r^2}{g} = \frac{2\,\pi\left(1.047\times10^{3}\,\text{s}^{-1}\right)^2 0.70\,\text{m}\,(0.115\,\text{m})^2}{9.8\,\text{m s}^{-2}} = 6.506\times10^{3}\,\text{m}^2$$

From Eq. (10.15):

$$u_g = \frac{\rho_p-\rho_f}{18\,\mu}D_p^2\,g = \frac{(1030-1000)\,\text{kg m}^{-3}}{18\left(1.25\times10^{-3}\,\text{kg m}^{-1}\,\text{s}^{-1}\right)}\left(10^{-5}\,\text{m}\right)^2 9.8\,\text{m s}^{-2} = 1.307\times10^{-6}\,\text{m s}^{-1}$$

From Eq. (10.18):

$$Q = 2\,u_g\,\Sigma = 2\left(1.307\times10^{-6}\,\text{m s}^{-1}\right)6.506\times10^{3}\,\text{m}^2 = 0.017\,\text{m}^3\,\text{s}^{-1}$$

Answer: 0.017 $\text{m}^3\,\text{s}^{-1}$

10.6 Scale-up of disc-stack centrifuge

The sigma factor for the pilot-scale disc-stack bowl centrifuge Σ_1 is calculated using Eq. (10.20) with $g = 9.8$ m s^{-2} (p 16), and ω converted to rad s^{-1} using the conversion factor 1 revolution = 2 π radians where radians is a non-dimensional unit (p 11). For the pilot-scale device, r_2 = 5 cm = 0.05 m and r_1 = 1 cm = 0.01 m:

$$\Sigma_1 = \frac{2\,\pi\,\omega^2\,(N-1)}{3g\,\tan\theta}\left(r_2^3-r_1^3\right) = \frac{2\,\pi\left(2\,\pi\times3000\,\text{min}^{-1}\cdot\left|\frac{1\,\text{min}}{60\,\text{s}}\right|\right)^2(25-1)}{3\left(9.8\,\text{m s}^{-2}\right)(\tan 35°)}\left((0.05\,\text{m})^3-(0.01\,\text{m})^3\right) = 89.6\,\text{m}^2$$

From Eq. (10.19), the sigma factor Σ_2 for the bigger centrifuge is therefore:

$$\Sigma_2 = \frac{Q_2}{Q_1}\Sigma_1 = \frac{80\,\text{l min}^{-1}}{3.5\,\text{l min}^{-1}}\left(89.6\,\text{m}^2\right) = 2048\,\text{m}^2$$

From Eq. (10.20), for the bigger centrifuge with r_2 = 7.5 cm = 0.075 m and r_1 = 2.35 cm = 0.0235 m:

$$\omega^2 = \frac{3g\,\tan\theta\,\Sigma_2}{2\,\pi\,(N-1)\left(r_2^3-r_1^3\right)} = \frac{3\left(9.8\,\text{m s}^{-2}\right)(\tan 45°)\,2048\,\text{m}^2}{2\,\pi\,(55-1)\left((0.075\,\text{m})^3-(0.0235\,\text{m})^3\right)} = 4.34\times10^{5}\,\text{s}^{-2}$$

$$\omega = 659\,\text{rad s}^{-1}\cdot\left|\frac{1\,\text{revolution}}{2\,\pi\,\text{rad}}\right|\cdot\left|\frac{60\,\text{s}}{1\,\text{min}}\right| = 6290\,\text{rpm}$$

Answer: 6290 rpm

10.7 Centrifugation of yeast and cell debris

Combining Eqs (10.15) and (10.18):

$$\Sigma_1 = \frac{18\,\mu_1\,Q_1}{2\left(\rho_p-\rho_f\right)D_{p1}^2\,g}$$

and

$$\Sigma_2 = \frac{18\,\mu_2\,Q_2}{2\left(\rho_p - \rho_f\right) D_{p2}^2\, g}$$

where subscript 1 refers to the centrifugation conditions for the yeast suspension, and subscript 2 refers to the centrifugation conditions for the cell debris. As the same centrifuge operated at the same speed is used in both applications, $\Sigma_1 = \Sigma_2$ and:

$$\frac{18\,\mu_1\,Q_1}{2\left(\rho_p - \rho_f\right) D_{p1}^2\, g} = \frac{18\,\mu_2\,Q_2}{2\left(\rho_p - \rho_f\right) D_{p2}^2\, g}$$

Assuming that the particle and fluid densities are the same in both applications, cancelling terms gives:

$$\frac{\mu_1\,Q_1}{D_{p1}^2} = \frac{\mu_2\,Q_2}{D_{p2}^2}$$

or

$$Q_2 = \frac{D_{p2}^2}{D_{p1}^2}\,\frac{\mu_1}{\mu_2}\,Q_1$$

As $D_{p2} = {}^1\!/_3\, D_{p1}$, $\mu_2 = 5\,\mu_1$ and $Q_1 = 3\ \mathrm{l\ min^{-1}}$:

$$Q_2 = \frac{\left({}^1\!/_3\, D_{p1}\right)^2}{D_{p1}^2}\,\frac{\mu_1}{5\,\mu_1}\left(3\ \mathrm{l\ min^{-1}}\right) = \frac{\left({}^1\!/_3\right)^2}{5}\left(3\ \mathrm{l\ min^{-1}}\right) = 0.067\ \mathrm{l\ min^{-1}}$$

Answer: 0.067 l min^{-1}

10.8 Cell disruption

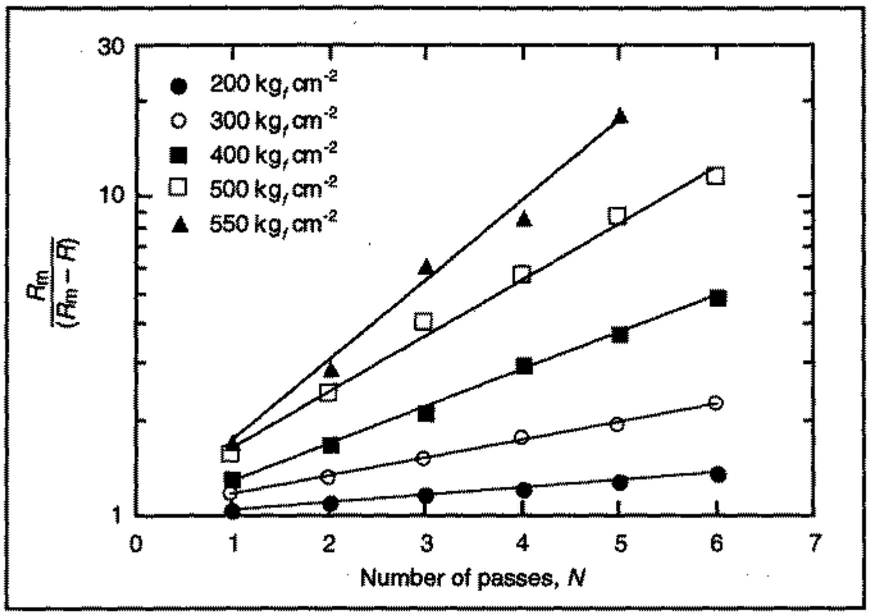
The relationship between pressure and protein release for a Manton-Gaulin homogeniser is given by Eq. (10.23). At constant pressure, a semi-log plot of $R_m/(R_m-R)$ versus N should yield a straight line, so that the value of $k\,p^{\alpha}$ can be determined from the slope.

The data for % protein release represent values of $R/R_m \times 100$. These data can be converted to $R_m/(R_m-R)$ as follows:

$$\frac{R_m}{R_m - R} = \frac{100}{100 - R/R_m \times 100} = \frac{100}{100 - \%\ \text{protein release}}$$

The results are listed and plotted below.

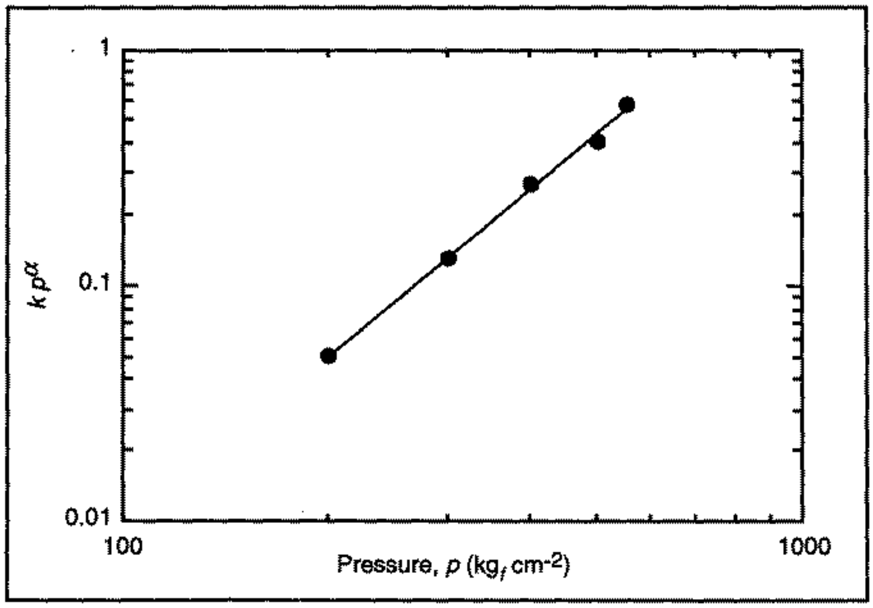
	Pressure drop (kg$_f$ cm^{-2})				
	200	300	400	500	550
N	$\frac{R_m}{R_m-R}$				
1	1.05	1.16	1.30	1.56	1.72
2	1.10	1.31	1.67	2.41	2.94
3	1.16	1.50	2.11	4.00	6.13
4	1.22	1.75	2.99	5.71	8.70
5	1.28	1.90	3.70	8.70	18.2
6	1.35	2.22	4.88	11.5	–



The values of $k\,p^{\alpha}$ obtained from the slopes of the straight lines for each pressure are listed below.

	Pressure drop (kg$_f$ cm^{-2})				
	200	300	400	500	550
$k\,p^{\alpha}$	0.050	0.129	0.267	0.406	0.580

A log–log plot of these values versus p can be expected to give a straight line with the value of α obtained from the slope.



As the equation to the straight line in the plot is $k\,p^{\alpha} = 1.71 \times 10^{-7}\,p^{2.37}$, $\alpha = 2.37$. Therefore, for this system, Eq. (10.23) becomes:

$$\ln\left(\frac{R_m}{R_m - R}\right) = 1.71 \times 10^{-7}\,p^{2.37}\,N$$

where p has units of kg$_f$ cm^{-2}.

(a)
For 80% protein release, $R_m/(R_m-R) = 5.0$. Rearranging the above empirical equation for the homogeniser and substituting $p = 460\ \text{kg}_f\,\text{cm}^{-2}$ gives:

$$N = \frac{\ln\left(\frac{R_m}{R_m - R}\right)}{1.71\times10^{-7}\,p^{2.37}} = \frac{\ln 5.0}{1.71\times10^{-7}\,(460)^{2.37}} = 4.6$$

Therefore, 80% protein release is achieved within 5 passes through the homogeniser.

Answer: 5

(b)
For 70% protein release, $R_m/(R_m-R) = 3.33$. Rearranging the empirical equation for the homogeniser and substituting $N = 2$ gives:

$$p^{2.37} = \frac{\ln\left(\frac{R_m}{R_m - R}\right)}{1.71\times10^{-7}\,N} = \frac{\ln 3.33}{1.71\times10^{-7}\,(2)} = 3.52\times10^{6}$$

$$p = 578\ \text{kg}_f\,\text{cm}^{-2}$$

Answer: 578 $\text{kg}_f\,\text{cm}^{-2}$. *An assumption associated with this answer is that the homogenisation characteristics measured at pressures between 200 and 550* $\text{kg}_f\,\text{cm}^{-2}$ *apply at the higher pressure of 578* $\text{kg}_f\,\text{cm}^{-2}$.

10.9 Enzyme purification using two-phase aqueous partitioning

(a)
From Eq. (10.24), for $K = 3.5$, the product partitions into the upper phase. Eq. (10.27) with $K = 3.5$ and $Y_u = 0.8$ is:

$$0.8 = \frac{V_u}{V_u + \frac{V_l}{3.5}}$$

Rearranging gives:

$$0.8\,V_u + 0.23\,V_l = V_u$$

$$0.23\,V_l = 0.2\,V_u$$

$$\frac{V_u}{V_l} = 1.15$$

Answer: 1.15

(b)
The mass of enzyme in the two phases must be equal to the mass of enzyme in the original homogenate. The mass-balance equation is:

$$C_{Au}\,V_u + C_{Al}\,V_l = C_{A0}\,V_0$$

From Eq. (10.24), for $K = 3.5$, $C_{Au} = 3.5\,C_{Al}$. If $V_l = 100$ l, from **(a)** for 80% recovery, $V_u = 115$ l. Substituting these values into the mass-balance equation with $V_0 = 150$ l and $C_{A0} = 3.2\ \text{u ml}^{-1} = 3.2\times10^3\ \text{u l}^{-1}$:

$$3.5\,C_{Al}\,(115\ \text{l}) + C_{Al}\,(100\ \text{l}) = (3.2\times10^3\ \text{u l}^{-1})\,150\ \text{l}$$

where C_{Al} has units of u l^{-1}. Solving the equation for C_{Al}:

$$502.5\,C_{Al} = 4.8\times10^5$$

$$C_{Al} = 955\ \text{u l}^{-1}$$

As $C_{Au} = 3.5\ C_{Al}$, $C_{Au} = 3.34 \times 10^3\ \text{u l}^{-1}$. From Eq. (10.29), the concentration factor for product partitioning into the upper phase is:

$$\delta_c = \frac{C_{Au}}{C_{A0}} = \frac{3.34 \times 10^3\ \text{u l}^{-1}}{3.2 \times 10^3\ \text{u l}^{-1}} = 1.04$$

Answer: 1.04

10.10 Recovery of viral particles

From Eq. (10.24), for $K = 10^{-2}$, $C_{Au} = 10^{-2}\ C_{Al}$ and the product partitions into the lower phase. If 5 l culture volume is added to 2 l polymer solution and, after phase separation, the volume of the lower phase is 1 l, the volume of the upper phase must be $(5 + 2) - 1 = 6$ l.

(a)

The yield of virus in the lower phase is given by Eq. (10.28) with $V_l = 1$ l, $V_u = 6$ l and $K = 10^{-2}$:

$$Y_l = \frac{V_l}{V_u K + V_l} = \frac{1\ \text{l}}{6\ \text{l}\left(10^{-2}\right) + 1\ \text{l}} = 0.94$$

Answer: 0.94

(b)

The mass of viral particles in the two phases must be equal to the mass of viral particles in the original culture broth. The mass-balance equation is:

$$C_{Au}\ V_u + C_{Al}\ V_l = C_{A0}\ V_0$$

Answer: $C_{Au}\ V_u + C_{Al}\ V_l = C_{A0}\ V_0$

(c)

From Eq. (10.24):

$$C_{Al} = \frac{C_{Au}}{K}$$

Rearranging the mass-balance equation in **(b)** and substituting for C_{Al} gives:

$$C_{A0} = \frac{C_{Au}\ V_u + C_{Al}\ V_l}{V_0} = \frac{C_{Au}\ V_u + \dfrac{C_{Au}}{K}\ V_l}{V_0}$$

The concentration factor for product partitioning into the lower phase is given by Eq. (10.29). Substituting for C_{Al} and C_{A0} in this equation gives:

$$\delta_c = \frac{C_{Al}}{C_{A0}} = \frac{\dfrac{C_{Au}}{K}}{\dfrac{C_{Au}\ V_u + \dfrac{C_{Au}}{K}\ V_l}{V_0}} = \frac{\dfrac{V_0}{K}}{V_u + \dfrac{V_l}{K}} = \frac{V_0}{V_u K + V_l}$$

Answer: $\delta_c = \dfrac{V_0}{V_u K + V_l}$

(d)

Using the equation for δ_c derived in **(c)** with $V_0 = 5$ l, $V_l = 1$ l, $V_u = 6$ l and $K = 10^{-2}$:

$$\delta_c = \frac{5\ \text{l}}{6\ \text{l}\left(10^{-2}\right) + 1\ \text{l}} = 4.7$$

Answer: 4.7

10.11 Gel chromatography scale-up

(a)

The elution volume for the toxoid is lower than for the impurity. Therefore, as the toxoid stays in the column for the shorter time, it must be the larger molecule.

Answer: Toxoid

(b)

The internal pore volume in the gel in the laboratory reactor can be calculated using Eq. (10.42):

$$V_i = a\,W_r = 10\,\text{g}\,.\left|\frac{1\,\text{kg}}{1000\,\text{g}}\right|\left(0.0035\,\text{m}^3\,\text{kg}^{-1}\right).\left|\frac{10^6\,\text{ml}}{1\,\text{m}^3}\right| = 35\,\text{ml}$$

As $V_o = 23$ ml, the partition coefficients for the toxoid and impurity can be determined using the measured elution volumes and Eq. (10.41):

$$\text{Toxoid}\,K_p = \frac{(V_e - V_o)}{V_i} = \frac{(29-23)\,\text{ml}}{35\,\text{ml}} = 0.171$$

$$\text{Impurity}\,K_p = \frac{(V_e - V_o)}{V_i} = \frac{(45-23)\,\text{ml}}{35\,\text{ml}} = 0.629$$

Answer: Toxoid = 0.171; impurity = 0.629

(c)

Let subscripts 1 and 2 denote the small and large columns, respectively. The total volume of the laboratory column of inner diameter $D_{c1} = 1.5$ cm = 0.015 m and height $H_1 = 0.4$ m is:

$$V_{T1} = \pi\left(\frac{D_{c1}}{2}\right)^2 H_1 = \pi\left(\frac{0.015\,\text{m}}{2}\right)^2 0.4\,\text{m} = 7.07\times 10^{-5}\,\text{m}^3$$

The total volume of the large-scale column of diameter $D_{c2} = 0.5$ m and height $H_2 = 0.6$ m is:

$$V_{T2} = \pi\left(\frac{D_{c2}}{2}\right)^2 H_2 = \pi\left(\frac{0.5\,\text{m}}{2}\right)^2 0.6\,\text{m} = 0.118\,\text{m}^3$$

If the void fraction in the large column is the same as in the small column, V_{o2} in the large column is:

$$V_{o2} = \frac{V_{o1}}{V_{T1}}\,V_{T2} = \frac{23\,\text{ml}\,.\left|\frac{1\,\text{m}^3}{10^6\,\text{ml}}\right|}{7.07\times 10^{-5}\,\text{m}^3}\left(0.118\,\text{m}^3\right) = 0.0384\,\text{m}^3$$

If the pore volume fraction is also the same:

$$V_{i2} = \frac{V_{i1}}{V_{T1}}\,V_{T2} = \frac{35\,\text{ml}\,.\left|\frac{1\,\text{m}^3}{10^6\,\text{ml}}\right|}{7.07\times 10^{-5}\,\text{m}^3}\left(0.118\,\text{m}^3\right) = 0.0584\,\text{m}^3$$

If the large-scale column is operated with the same packing and flow conditions, the partition coefficients can be assumed to be the same as those in the laboratory column. Therefore, from Eq. (10.41), for toxoid in the large column with the value of K_p from **(b)**:

$$V_{e2} = K_p\,V_{i2} + V_{o2} = 0.171\,(0.0584\,\text{m}^3) + 0.0384\,\text{m}^3 = 0.0484\,\text{m}^3$$

Similarly for the impurity:

$$V_{e2} = K_p\,V_{i2} + V_{o2} = 0.629\,(0.0584\,\text{m}^3) + 0.0384\,\text{m}^3 = 0.0751\,\text{m}^3$$

Answer: Toxoid = 0.0484 m^3; impurity = 0.0751 m^3

(d)

The liquid flow rate is scaled up in proportion to the column cross-sectional area. As the cross-sectional area = $\pi\,(^{D_c}/_2)^2$, the volumetric flow rate v_2 in the large column is:

$$v_2 = v_1 \frac{\pi\left(\frac{D_{c2}}{2}\right)^2}{\pi\left(\frac{D_{c1}}{2}\right)^2} = v_1\left(\frac{D_{c2}}{D_{c1}}\right)^2 = 14\,\text{ml min}^{-1} . \left|\frac{1\,\text{m}^3}{10^6\,\text{ml}}\right| \left(\frac{0.5\,\text{m}}{0.015\,\text{m}}\right)^2 = 0.0156\,\text{m}^3\,\text{min}^{-1}$$

Answer: 0.0156 $\text{m}^3\ \text{min}^{-1}$

(e)

The retention time t_R is equal to the elution volume divided by the volumetric flow rate. For the toxoid in the large column, using the results from (**c**) and (**d**):

$$t_R = \frac{V_{e2}}{v_2} = \frac{0.0484\,\text{m}^3}{0.0156\,\text{m}^3\,\text{min}^{-1}} = 3.1\,\text{min}$$

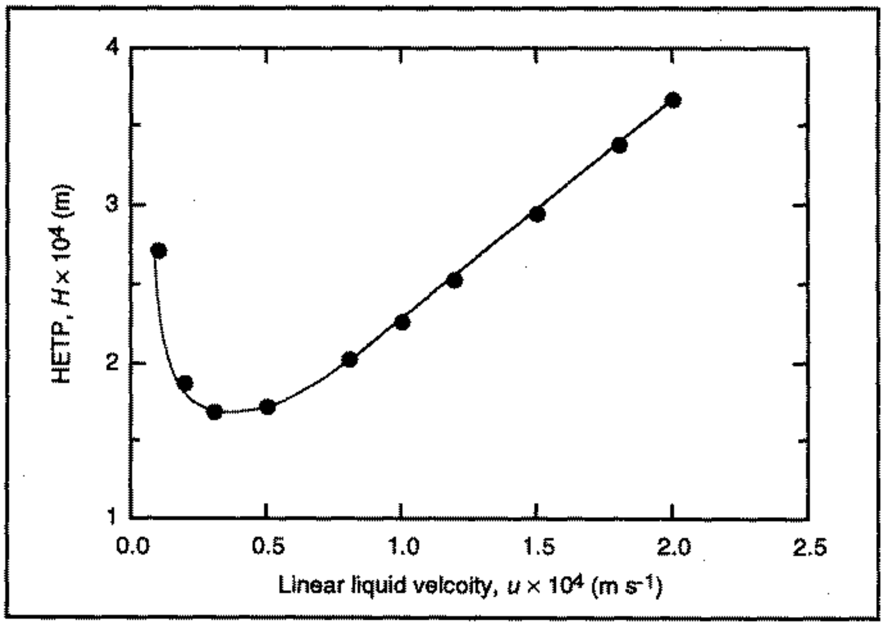
Answer: 3.1 min

10.12 Protein separation using chromatography

(a)

Selected values of *u* and *H* are listed and plotted below.

Linear liquid velocity, *u* (m s^{-1})	HETP, *H* (m)
0.1×10^{-4}	2.72×10^{-4}
0.2×10^{-4}	1.87×10^{-4}
0.3×10^{-4}	1.69×10^{-4}
0.5×10^{-4}	1.72×10^{-4}
0.8×10^{-4}	2.02×10^{-4}
1.0×10^{-4}	2.27×10^{-4}
1.2×10^{-4}	2.54×10^{-4}
1.5×10^{-4}	2.95×10^{-4}
1.8×10^{-4}	3.38×10^{-4}
2.0×10^{-4}	3.67×10^{-4}



(b)

From the graph in **(a)**, the minimum HETP is around 1.7×10^{-4} m. This can be confirmed by differentiating the equation for H and solving for ${}^{\mathrm{d}H}/_{\mathrm{d}u} = 0$:

$$\frac{\mathrm{d}H}{\mathrm{d}u} = \frac{-A}{u^2} + B = 0$$

Therefore:

$$u = \sqrt{\frac{A}{B}} = \sqrt{\frac{2 \times 10^{-9}\,\mathrm{m^2\,s^{-1}}}{1.5\,\mathrm{s}}} = 3.65 \times 10^{-5}\,\mathrm{m\,s^{-1}}$$

The value of H corresponding to this u is:

$$H = \frac{A}{u} + B\,u + C = \frac{2 \times 10^{-9}\,\mathrm{m^2\,s^{-1}}}{3.65 \times 10^{-5}\,\mathrm{m\,s^{-1}}} + 1.5\,\mathrm{s}\left(3.65 \times 10^{-5}\,\mathrm{m\,s^{-1}}\right) + 5.7 \times 10^{-5}\,\mathrm{m} = 1.67 \times 10^{-4}\,\mathrm{m}$$

Answer: The minimum HETP is 1.67×10^{-4} m, obtained at a liquid velocity of 3.65×10^{-5} m s^{-1}.

(c)

The column diameter D_c = 25 cm = 0.25 m. The volumetric flow rate v is 0.31 l min^{-1}. The linear flow rate u is obtained by dividing the volumetric flow rate by the column cross-sectional area:

$$u = \frac{v}{\pi\left(\frac{D_c}{2}\right)^2} = \frac{0.31\,\mathrm{l\,min^{-1}} \cdot \left|\frac{1\,\mathrm{min}}{60\,\mathrm{s}}\right| \cdot \left|\frac{1\,\mathrm{m^3}}{1000\,\mathrm{l}}\right|}{\pi\left(\frac{0.25\,\mathrm{m}}{2}\right)^2} = 1.05 \times 10^{-4}\,\mathrm{m\,s^{-1}}$$

Substituting this result into the equation for H:

$$H = \frac{A}{u} + B\,u + C = \frac{2 \times 10^{-9}\,\mathrm{m^2\,s^{-1}}}{1.05 \times 10^{-4}\,\mathrm{m\,s^{-1}}} + 1.5\,\mathrm{s}\left(1.05 \times 10^{-4}\,\mathrm{m\,s^{-1}}\right) + 5.7 \times 10^{-5}\,\mathrm{m} = 2.34 \times 10^{-4}\,\mathrm{m}$$

The capacity factors for the A and B chains are $k_A = 0.85$ and $k_B = 1.05$, respectively. From Eq. (10.38):

$$\delta = \frac{k_B}{k_A} = \frac{1.05}{0.85} = 1.235$$

Substituting these parameter values into Eq. (10.51) with L = 1 m for the larger column and $k_2 = k_B$:

$$R_N = \frac{1}{4}\sqrt{\frac{L}{H}}\left(\frac{\delta - 1}{\delta}\right)\left(\frac{k_B}{k_B + 1}\right) = \frac{1}{4}\sqrt{\frac{1\,\mathrm{m}}{2.34 \times 10^{-4}\,\mathrm{m}}}\left(\frac{1.235 - 1}{1.235}\right)\left(\frac{1.05}{1.05 + 1}\right) = 1.59$$

From p 248, as $R_N > 1.5$, the two peaks are completely separated.

Answer: Yes

(d)

For L = 0.7 m for the smaller column, the value of H corresponding to R_N = 1.5 or virtual complete separation can be determined using Eq. (10.51):

$$\sqrt{H} = \frac{1}{4}\frac{\sqrt{L}}{R_N}\left(\frac{\delta - 1}{\delta}\right)\left(\frac{k_B}{k_B + 1}\right) = \frac{1}{4}\frac{\sqrt{0.7\,\mathrm{m}}}{1.5}\left(\frac{1.235 - 1}{1.235}\right)\left(\frac{1.05}{1.05 + 1}\right) = 0.0136\sqrt{\mathrm{m}}$$

$$H = 1.85 \times 10^{-4}\,\mathrm{m}$$

The linear liquid velocity at which this value of H is obtained can be determined by rearranging the expression for H as a function of u and solving for u:

$$H\,u = A + B\,u^2 + C\,u$$

$$B\,u^2 + (C-H)\,u + A = 0$$

The solution to this quadratic equation is:

$$u = \frac{-(C-H) \pm \sqrt{(C-H)^2 - 4\,B\,A}}{2\,B} = \frac{H - C \pm \sqrt{(C-H)^2 - 4\,B\,A}}{2\,B}$$

Substituting the parameter values:

$$u = \frac{\left(1.85\times10^{-4} - 5.7\times10^{-5}\right)\mathrm{m} \pm \sqrt{\left(5.7\times10^{-5}\,\mathrm{m} - 1.85\times10^{-4}\,\mathrm{m}\right)^2 - 4\,(1.5\,\mathrm{s})\left(2\times10^{-9}\,\mathrm{m}^2\,\mathrm{s}^{-1}\right)}}{2\,(1.5\,\mathrm{s})}$$

$$u = 4.27\times10^{-5}\,\mathrm{m\,s^{-1}} \pm 2.21\times10^{-5}\,\mathrm{m\,s^{-1}}$$

Therefore:

$$u = 6.48\times10^{-5}\,\mathrm{m\,s^{-1}} \quad \text{or} \quad u = 2.06\times10^{-5}\,\mathrm{m\,s^{-1}}$$

The maximum flow rate for complete separation is $u = 6.48\times10^{-5}$ m s^{-1}. Between $u = 2.06\times10^{-5}$ m s^{-1} and $u = 6.48\times10^{-5}$ m s^{-1}, $H < 1.85\times10^{-4}$ m so that $R_N > 1.5$ and complete separation is maintained. The volumetric flow rate v is equal to u multiplied by the column cross-sectional area:

$$v = u\,\pi\left(\frac{D_c}{2}\right)^2 = \left(6.48\times10^{-5}\,\mathrm{m\,s^{-1}}\right)\pi\left(\frac{0.25\,\mathrm{m}}{2}\right)^2 = 3.18\times10^{-6}\,\mathrm{m^3\,s^{-1}}$$

Converting units:

$$v = 3.18\times10^{-6}\,\mathrm{m^3\,s^{-1}}\,.\left|\frac{1000\,\mathrm{l}}{1\,\mathrm{m^3}}\right|.\left|\frac{60\,\mathrm{s}}{1\,\mathrm{min}}\right| = 0.19\,\mathrm{l\,min^{-1}}$$

Answer: 0.19 l min^{-1}

11

Homogeneous Reactions

11.1 Reaction equilibrium

From p 258, take the standard conditions to be 25°C and 1 atm pressure. The temperature is converted to degrees Kelvin using Eq. (2.24); from Table 2.5, $R = 8.3144\ \text{J K}^{-1}\ \text{gmol}^{-1} = 8.3144 \times 10^{-3}\ \text{kJ K}^{-1}\ \text{gmol}^{-1}$.

(a)
From Eq. (11.3):

$$\ln K = \frac{-\Delta G^{\text{o}}_{\text{rxn}}}{R\,T} = \frac{14.1\ \text{kJ mol}^{-1}}{8.3144 \times 10^{-3}\ \text{kJ K}^{-1}\ \text{gmol}^{-1}\ (25 + 273.15\ \text{K})} = 5.69$$

$$K = 295$$

The large magnitude of K indicates that the reaction can be considered irreversible.

Answer: 295; irreversible

(b)
From Eq. (11.3):

$$\ln K = \frac{-\Delta G^{\text{o}}_{\text{rxn}}}{R\,T} = \frac{-3.2\ \text{kJ mol}^{-1}}{8.3144 \times 10^{-3}\ \text{kJ K}^{-1}\ \text{gmol}^{-1}\ (25 + 273.15\ \text{K})} = -1.29$$

$$K = 0.275$$

The relatively small magnitude of K indicates that the reaction is reversible.

Answer: 0.275; reversible

11.2 Equilibrium yield

(a)
From Eq. (11.2) with G6P and G1P representing glucose 6-phosphate and glucose 1-phosphate, respectively:

$$K = \frac{C_{\text{G6P}}\ \text{at equilibrium}}{C_{\text{G1P}}\ \text{at equilibrium}} = \frac{0.038\ \text{M}}{0.002\ \text{M}} = 19$$

Answer: 19

(b)
From Eq. (11.9) and the reaction stoichiometry:

$$\text{Theoretical yield} = \frac{\text{moles G6P formed}}{\text{moles G1P used to form G6P}} = \frac{1\ \text{mol}}{1\ \text{mol}} = 1\ \text{mol mol}^{-1}$$

Answer: 1 mol mol^{-1}

(c)
From p 260 and using a basis of 1 litre:

$$\text{Gross yield} = \frac{\text{moles G6P formed}}{\text{moles G1P supplied}} = \frac{0.038\ \text{mol}}{0.04\ \text{mol}} = 0.95\ \text{mol mol}^{-1}$$

Answer: 0.95 mol mol^{-1}

11.3 Reaction rate

(a)

Terms used to express reaction rate are outlined on p 261.

(i)

Answer: The volumetric productivity is unaffected by change in volume.

(ii)

Answer: The specific productivity is unaffected by change in volume.

(iii)

Answer: The total productivity is doubled if the fermenter volume is doubled.

(b)

Answer: The volumetric productivity is doubled, the specific productivity is unaffected, and the total productivity is doubled.

(c)

(i)

$R_A = 100\ \text{kg d}^{-1}$; $r_A = 0.8\ \text{g l}^{-1}\ \text{h}^{-1}$. From Eq. (11.16):

$$V = \frac{R_A}{r_A} = \frac{100\ \text{kg d}^{-1}}{0.8\ \text{g l}^{-1}\ \text{h}^{-1} \cdot \left|\frac{24\ \text{h}}{1\ \text{d}}\right| \cdot \left|\frac{1\ \text{kg}}{1000\ \text{g}}\right|} = 5208\ \text{l}$$

Answer: 5208 l

(ii)

From Eq. (11.67):

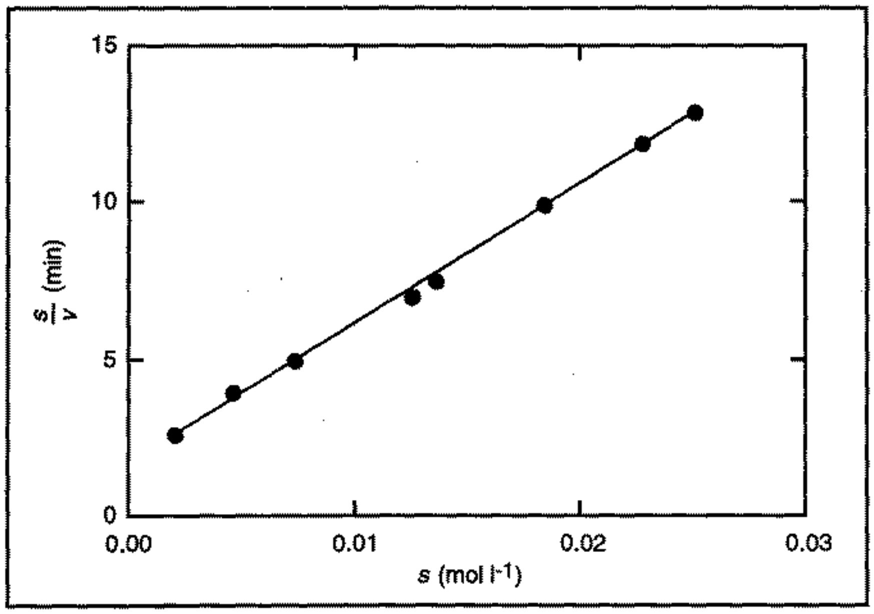
$$q_A = \frac{r_A}{x} = \frac{0.8\ \text{g l}^{-1}\ \text{h}^{-1}}{20\ \text{g l}^{-1}} = 0.04\ \text{g g}^{-1}\ \text{h}^{-1}$$

Answer: 0.4 g g^{-1} h^{-1}

11.4 Enzyme kinetics

Evaluate the enzyme kinetic parameters by plotting s/v versus s as a Langmuir plot (p 272). The values are listed and plotted below.

s (mol l^{-1})	s/v (min)
0.0250	12.89
0.0227	11.88
0.0184	9.95
0.0135	7.50
0.0125	7.02
0.00730	5.00
0.00460	3.93
0.00204	2.62



The equation for the straight line in the plot is $^{s}/_{v} = 1.70 + 445\,s$, where s has units of mol l^{-1} and $^{s}/_{v}$ has units of min. Therefore, from Eq. (11.39), $^{1}/_{v_{max}} = 445$ mol^{-1} l min, so that $v_{max} = 2.25 \times 10^{-3}$ mol l^{-1} min^{-1}. Also from Eq. (11.39), $^{K_m}/_{v_{max}} = 1.70$. Multiplying this value by the result for v_{max}, $K_m = 3.83 \times 10^{-3}$ mol l^{-1}.

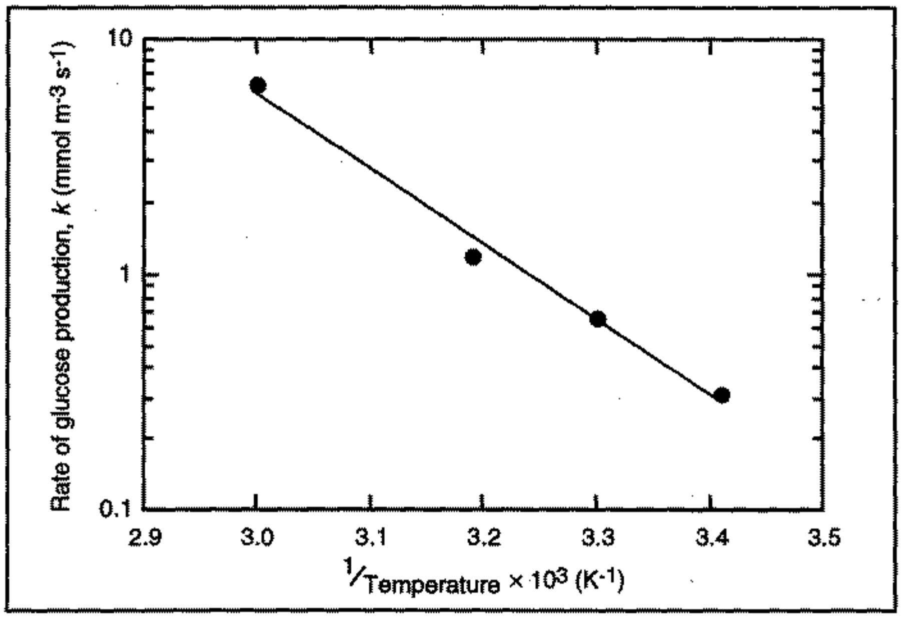
Answer: $v_{max} = 2.25 \times 10^{-3}$ mol l^{-1} min^{-1}; $K_m = 3.83 \times 10^{-3}$ mol l^{-1}

11.5 Effect of temperature on hydrolysis of starch

(a)

The activation energy is determined from the Arrhenius equation, Eq. (11.21), with k equal to the initial rate of glucose production. According to the Arrhenius equation, a plot of k versus $^{1}/_{T}$ on semi-logarithmic coordinates should give a straight line. The parameter values are listed and plotted below; T is converted to degrees Kelvin using Eq. (2.24).

T (°C)	T (K)	$^{1}/_{T}$ (K^{-1})	Rate, k (mmol m^{-3} s^{-1})
20	293.15	3.41×10^{-3}	0.31
30	303.15	3.30×10^{-3}	0.66
40	313.15	3.19×10^{-3}	1.20
60	333.15	3.00×10^{-3}	6.33



The equation for the straight line in the plot is $k = 1.87 \times 10^{10}\, e^{-7300/T}$, where k has units of mmol m^{-3} s^{-1} and T has units of K. Therefore, from Eq. (11.21), $^{E}/_{R} = 7300$ K. From Table 2.5, $R = 8.3144$ J K^{-1} gmol^{-1} = 8.3144×10^{-3} kJ K^{-1} gmol^{-1}; therefore, $E = 7300$ K × 8.3144×10^{-3} kJ K^{-1} gmol^{-1} = 60.7 kJ gmol^{-1}.

Answer: 60.7 kJ gmol^{-1}.

(b)

Converting 55°C to degrees Kelvin using Eq. (2.24), $T = 55 + 273.15 = 328.15$ K. Substituting this value into the equation for k obtained in (**a**):

$$k = 1.87 \times 10^{10}\, e^{-7300/T} = 1.87 \times 10^{10}\, e^{-7300/328.15} = 4.08 \text{ mmol m}^{-3}\text{ s}^{-1}$$

Similarly, for $T = 25°\text{C} = 25 + 273.15 = 298.15$ K:

$$k = 1.87 \times 10^{10}\, e^{-7300/T} = 1.87 \times 10^{10}\, e^{-7300/298.15} = 0.43 \text{ mmol m}^{-3}\text{ s}^{-1}$$

Therefore, the rate at 55°C is $^{4.08}/_{0.43} = 9.5$ times faster than at 25°C.

Answer: The reaction rate at 55°C is 4.08 mmol m^{-3} s^{-1} or 9.5 times faster than the rate of 0.43 mmol m^{-3} s^{-1} at 25°C.

(c)

From Table 2.5, the value of R in the appropriate units is 1.9872 cal K^{-1} gmol^{-1}. At 55°C = 328.15 K:

$$k_\text{d} = 2.25 \times 10^{27}\, e^{-41,630/RT} = 2.25 \times 10^{27}\, e^{-41,630/(1.9872 \times 328.15)} = 0.42\text{ h}^{-1}$$

Therefore, from Eq. (11.45), the half-life of the enzyme at 55°C is:

$$t_\text{h} = \frac{\ln 2}{k_\text{d}} = \frac{\ln 2}{0.42\text{ h}^{-1}} = 1.65\text{ h}$$

At 25°C = 298.15 K:

$$k_\text{d} = 2.25 \times 10^{27}\, e^{-41,630/RT} = 2.25 \times 10^{27}\, e^{-41,630/(1.9872 \times 298.15)} = 6.87 \times 10^{-4}\text{ h}^{-1}$$

and the enzyme half-life is:

$$t_\text{h} = \frac{\ln 2}{k_\text{d}} = \frac{\ln 2}{6.87 \times 10^{-4}\text{ h}^{-1}} = 1009\text{ h}$$

Although the reaction rate is 9.5 times faster at 55°C than at 25°C, the rate of deactivation is $0.42/(6.87 \times 10^{-4}) = 611$ times greater. Therefore, unless there are other considerations, 25°C would probably be the more practical temperature for processing operations.

Answer: The enzyme half-life at 25°C is 1009 h or 611 times longer than the half-life of 1.65 h at 55°C. The more practical operating temperature is probably 25°C.

11.6 Enzyme reaction and deactivation

$K_m = 5\ \text{mM} = 5 \times 10^{-3}\ \text{gmol l}^{-1} = 5 \times 10^{-6}\ \text{gmol m}^{-3}$. The concentration of fat is reduced from 45 gmol m^{-3} to $0.2 \times 45\ \text{gmol m}^{-3} = 9.0\ \text{gmol m}^{-3}$. As this concentration range is well above the value of K_m, $s >> K_m$ and, from p 269, $v \approx v_{\max}$. As $v = {}^{-ds}/_{dt}$, combining Eqs (11.35) and (11.44) gives:

$$\frac{-ds}{dt} = v_{\max 0}\, e^{-k_d t}$$

In this equation, s and t are the only variables. Separating variables and integrating:

$$\int -ds = \int v_{\max 0}\, e^{-k_d t}\, dt$$

Applying calculus rules (D.24) and (D.17) from Appendix D and combining the constants of integration:

$$-s = \frac{-v_{\max 0}}{k_d}\, e^{-k_d t} + K$$

The initial condition is: at $t = 0$, $s = s_0$. Therefore:

$$-s_0 = \frac{-v_{\max 0}}{k_d} + K$$

$$K = \frac{v_{\max 0}}{k_d} - s_0$$

Substituting this expression for K into the equation for $-s$ gives:

$$-s = \frac{-v_{\max 0}}{k_d}\, e^{-k_d t} + \frac{v_{\max 0}}{k_d} - s_0$$

$$s = s_0 - \frac{v_{\max 0}}{k_d}\left(1 - e^{-k_d t}\right)$$

At the beginning of the reaction, $s_0 = 45\ \text{gmol m}^{-3}$. Converting the units of $v_{\max 0}$:

$$v_{\max 0} = 0.07\ \text{mmol l}^{-1}\ \text{s}^{-1} = 0.07\ \text{mmol l}^{-1}\ \text{s}^{-1} \cdot \left|\frac{1\ \text{gmol}}{1000\ \text{mmol}}\right| \cdot \left|\frac{1000\ \text{l}}{1\ \text{m}^3}\right| \cdot \left|\frac{60\ \text{s}}{\text{min}}\right| = 4.2\ \text{gmol m}^{-3}\ \text{min}^{-1}$$

From Eq. (11.45):

$$k_d = \frac{\ln 2}{t_h} = \frac{\ln 2}{8\ \text{min}} = 0.087\ \text{min}^{-1}$$

Substituting parameter values into the equation for s, when 80% of the fat is hydrolysed:

$$0.2 \times 45\ \text{gmol m}^{-3} = 45\ \text{gmol m}^{-3} - \frac{4.2\ \text{gmol m}^{-3}\ \text{min}^{-1}}{0.087\ \text{min}^{-1}}\left(1 - e^{-0.087\,t}\right)$$

where t has units of min. Grouping terms:

$$12.276 = 48.276\, e^{-0.087\,t}$$

$$e^{-0.087\,t} = 0.254$$

$$-0.087\,t = -1.369$$

$$t = 15.7 \text{ min}$$

Answer: 15.7 min

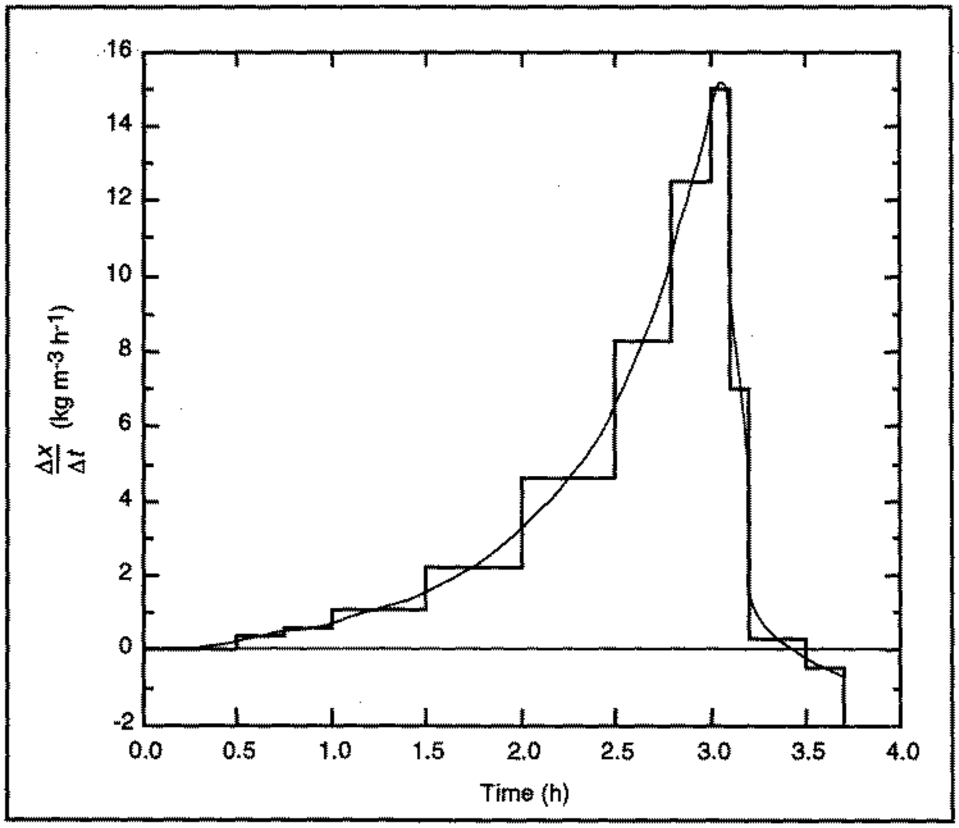
11.7 Growth parameters for recombinant *E. coli*

(a)

The average rate–equal area construction is used to determine growth rates from the concentration data. The data and calculations are tabulated below.

Time, t (h)	x (kg m^{-3})	Δx (kg m^{-3})	Δt (h)	$^{\Delta x}/_{\Delta t}$ (kg m^{-3} h^{-1})
0.0	0.20	0.01	0.33	0.03
0.33	0.21	0.01	0.17	0.06
0.5	0.22	0.10	0.25	0.40
0.75	0.32	0.15	0.25	0.60
1.0	0.47	0.53	0.50	1.06
1.5	1.00	1.10	0.50	2.20
2.0	2.10	2.32	0.50	4.64
2.5	4.42	2.48	0.30	8.27
2.8	6.9	2.50	0.20	12.50
3.0	9.4	1.50	0.10	15.00
3.1	10.9	0.70	0.10	7.00
3.2	11.6	0.10	0.30	0.33
3.5	11.7	–0.10	0.20	–0.50
3.7	11.6			

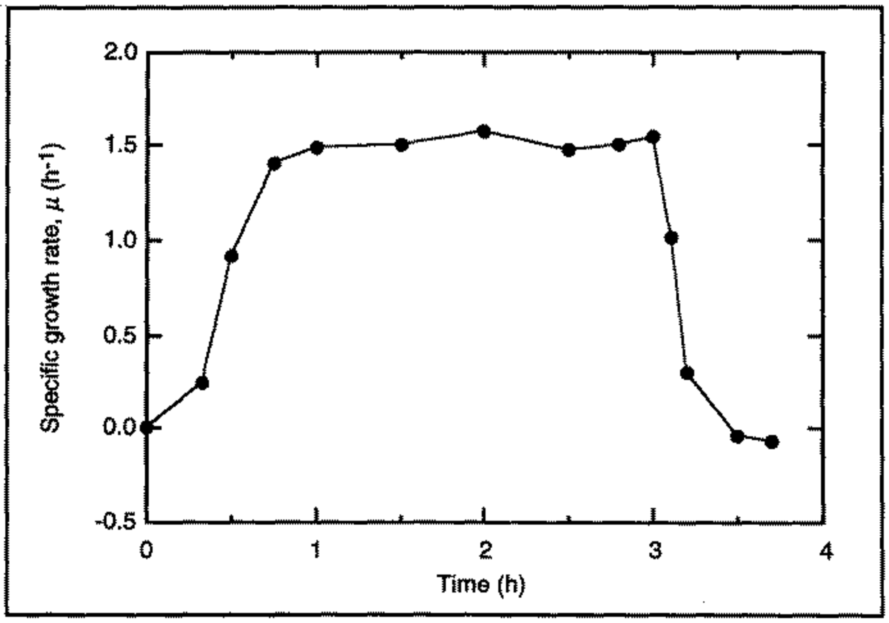
The values of $^{\Delta x}/_{\Delta t}$ are plotted as a function of time below according to the method described on p 263.



Results for $^{dx}/_{dt}$ read from the average rate–equal area curve and the calculated specific growth rates are listed below.

Time, t (h)	$r_X = {}^{dx}/_{dt}$ (kg m^{-3} h^{-1})	$\mu = {}^1/_x\,{}^{dx}/_{dt}$ (h^{-1})
0.0	0	0
0.33	0.05	0.24
0.5	0.20	0.91
0.75	0.45	1.41
1.0	0.70	1.49
1.5	1.5	1.50
2.0	3.3	1.57
2.5	6.5	1.47
2.8	10.4	1.51
3.0	14.5	1.54
3.1	11.0	1.01
3.2	3.5	0.30
3.5	–0.3	–0.03
3.7	–0.8	–0.07

Values of the specific growth rate μ are plotted as a function of time below.



(b)

As expected for most batch cell cultures, growth occurs at around the maximum specific growth rate for most of the culture period. Taking the average of the values of μ between times 0.75 h and 3 h, $\mu_{max} = 1.50 \pm 0.05$ h^{-1}, where 0.05 is the standard deviation.

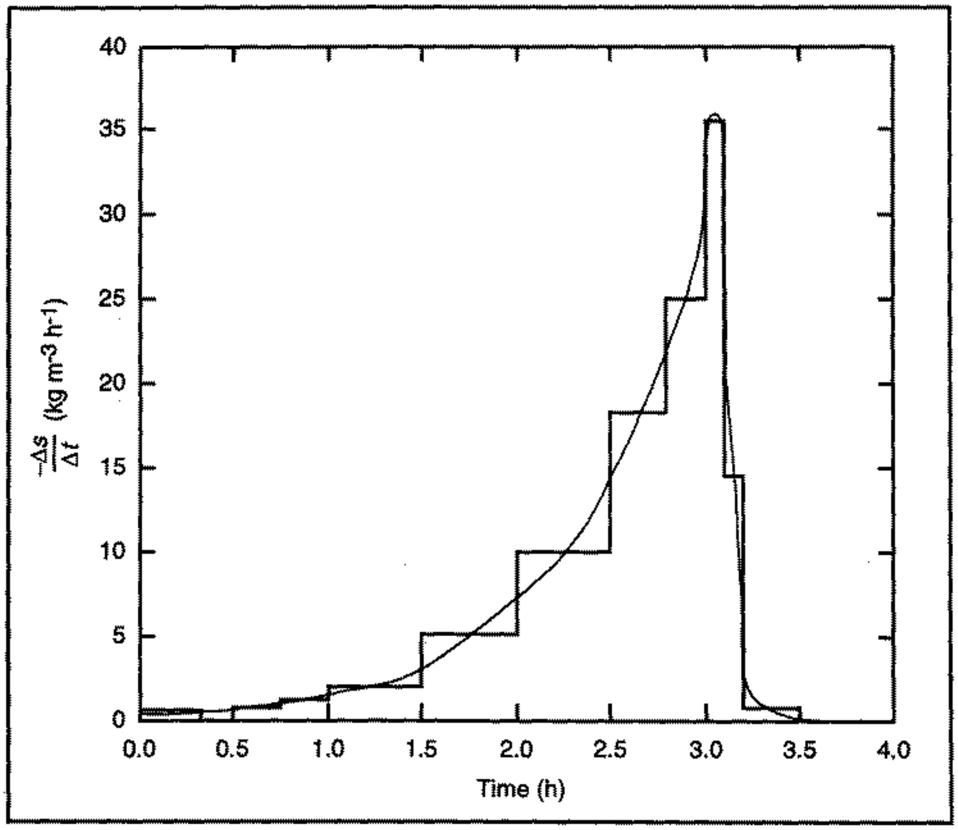
Answer: 1.50 h^{-1}

(c)

From Eq. (11.49), the observed biomass yield Y'_{XS} at any point in time is equal to the ratio of the observed growth and substrate consumption rates. The average rate–equal area construction is used to determine the substrate consumption rates $r_S = {}^{-ds}/_{dt}$ from the concentration data, as shown below.

Time, t (h)	s (kg m^{-3})	Δs (kg m^{-3})	Δt (h)	$-\Delta s/\Delta t$ (kg m^{-3} h^{-1})
0.0	25.0			
		–0.2	0.33	0.61
0.33	24.8			
		0.0	0.17	0.0
0.5	24.8			
		–0.2	0.25	0.80
0.75	24.6			
		–0.3	0.25	1.20
1.0	24.3			
		–1.0	0.50	2.00
1.5	23.3			
		–2.6	0.50	5.20
2.0	20.7			
		–5.0	0.50	10.0
2.5	15.7			
		–5.5	0.30	18.3
2.8	10.2			
		–5.0	0.20	25.0
3.0	5.2			
		–3.55	0.10	35.5
3.1	1.65			
		–1.45	0.10	14.5
3.2	0.2			
		–0.2	0.30	0.67
3.5	0.0			
		0.0	0.20	0.0
3.7	0.0			

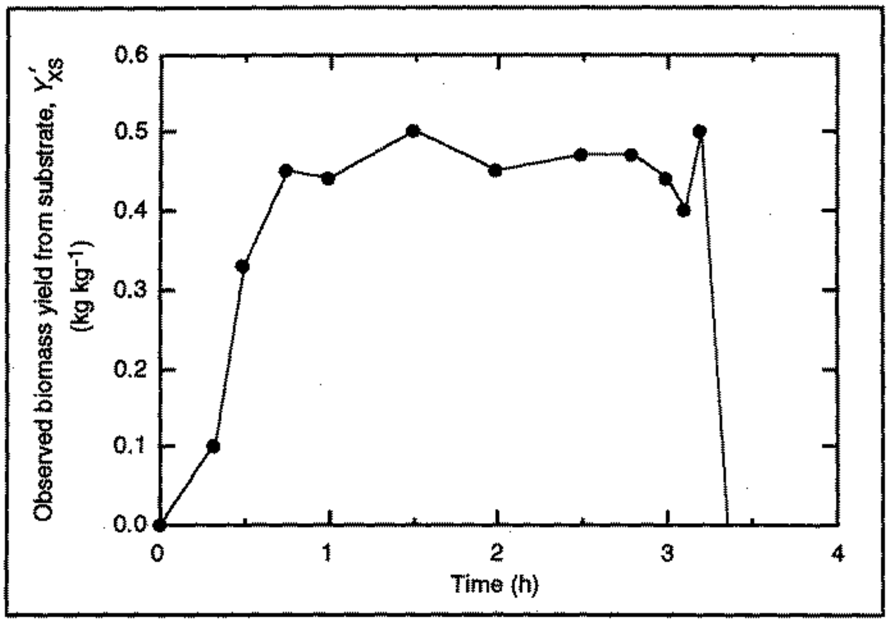
The values of $-\Delta s/\Delta t$ are plotted as a function of time below according to the method described on p 263.



Results for $-\mathrm{d}s/\mathrm{d}t$ read from the average rate–equal area curve are listed below, together with the instantaneous biomass yield coefficients calculated using Eq. (11.49) and the values of r_X from **(a)**.

Time, t (h)	$r_S = -{}^{ds}/_{dt}$ (kg m^{-3} h^{-1})	$Y'_{XS} = {}^{r_X}/_{r_S}$ (kg kg^{-1})
0.0	0.4	0
0.33	0.5	0.10
0.5	0.6	0.33
0.75	1.0	0.45
1.0	1.6	0.44
1.5	3.0	0.50
2.0	7.4	0.45
2.5	13.9	0.47
2.8	22.2	0.47
3.0	33.0	0.44
3.1	27.5	0.40
3.2	7.0	0.50
3.5	0.4	−0.75
3.7	0	–

Values of the observed biomass yield from substrate Y'_{XS} are plotted as a function of time below.



During the exponential growth phase between 0.75 h and 3 h when $\mu = \mu_{max}$, Y'_{XS} is approximately constant with an average value of 0.46 ± 0.02 kg kg^{-1}, where 0.02 is the standard deviation.

Answer: 0.46 kg kg^{-1}; Y'_{XS} is approximately constant during exponential growth

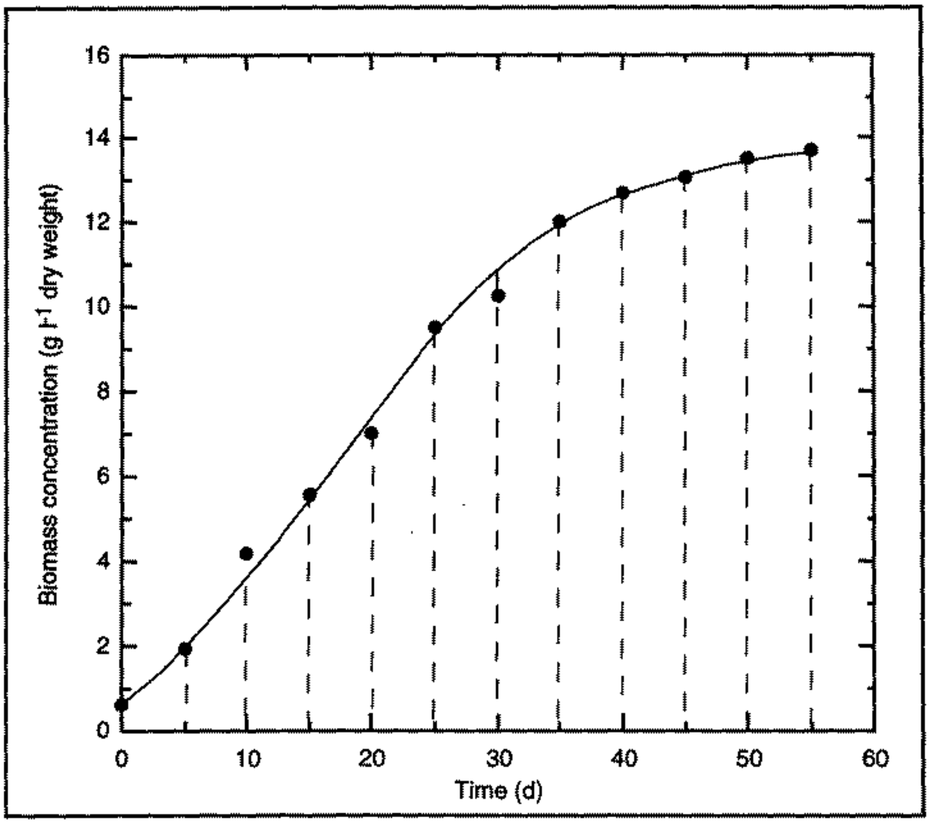
11.8 Growth parameters for hairy roots

(a)

The mid-point slope method is used to determine rates from the concentration data. The growth data are listed and plotted below according to the method described on pp 264–265. Values of $[(x)_{t+\varepsilon} - (x)_{t-\varepsilon}]$ are read from the graph.

Time, t (d)	Biomass concentration, x (g l^{-1})	ε	$[(x)_{t+\varepsilon} - (x)_{t-\varepsilon}]$ (g l^{-1})
0	0.64	5	–
5	1.95	5	3.0
10	4.21	5	3.6
15	5.54	5	3.8

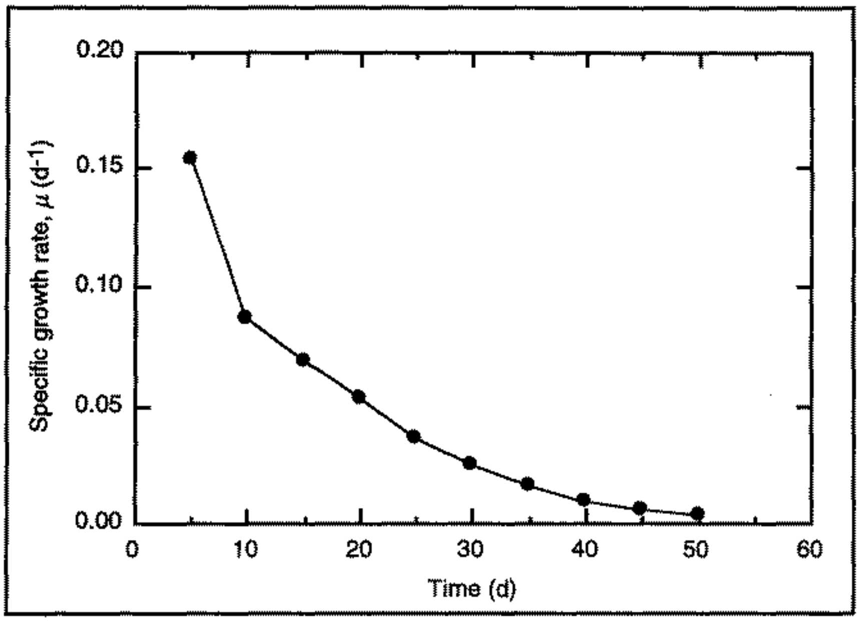
20	6.98	5	3.7
25	9.50	5	3.4
30	10.3	5	2.6
35	12.0	5	1.9
40	12.7	5	1.2
45	13.1	5	0.8
50	13.5	5	0.6
55	13.7	5	–



The growth rate $^{dx}/_{dt}$ is determined using the central-difference formula, Eq. (11.23). These results and values of the specific growth rate μ are listed and plotted below as a function of time.

Time, t (d)	$r_X = {}^{dx}/_{dt}$ (g l^{-1} d^{-1})	$\mu = {}^{1}/_{x}\,{}^{dx}/_{dt}$ (d^{-1})
0	–	–
5	0.30	0.154
10	0.36	0.086
15	0.38	0.069
20	0.37	0.053
25	0.34	0.036
30	0.26	0.025
35	0.19	0.016
40	0.12	0.009
45	0.08	0.006
50	0.06	0.004
55	–	–

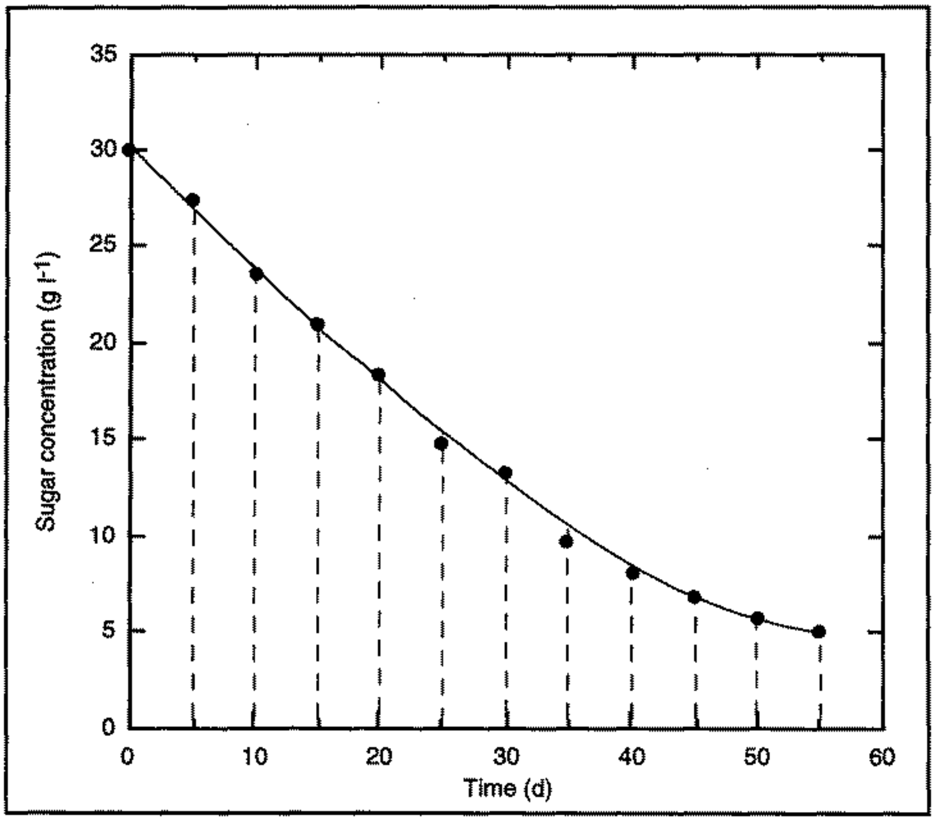
Values of the specific growth rate μ are plotted below.



Answer: Near the beginning of the culture.

(b)

The mid-point slope method is used to determine the rate of substrate uptake as a function of time. The sugar concentration data are plotted below according to the method described on pp 264–265.

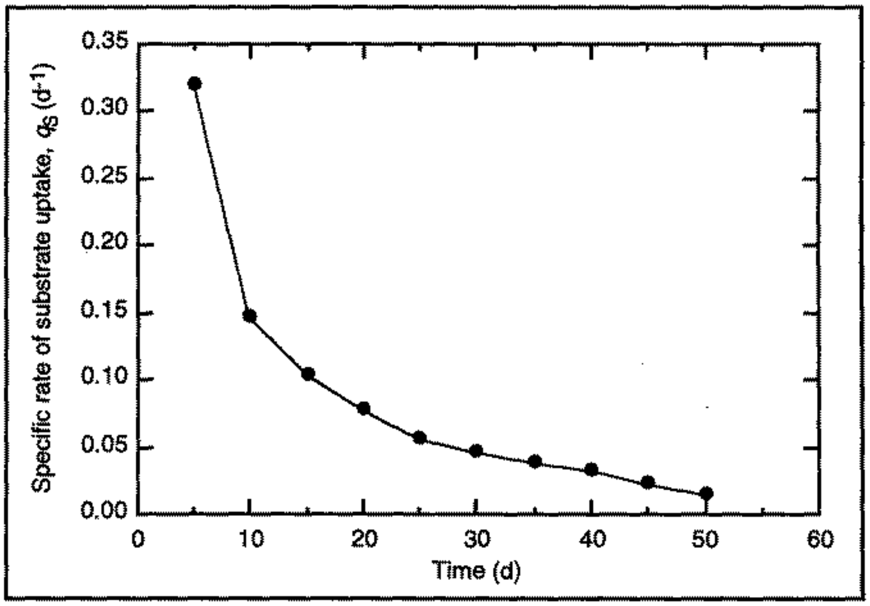


Values of $[(s)_{t+\varepsilon} - (s)_{t-\varepsilon}]$ read from the graph are listed in the table below.

Time, t (d)	Sugar concentration, s (g l^{-1})	ε	$[(s)_{t+\varepsilon} - (s)_{t-\varepsilon}]$ (g l^{-1})
0	30.0	5	–
5	27.4	5	–6.2
10	23.6	5	–6.1
15	21.0	5	–5.7
20	18.4	5	–5.4
25	14.8	5	–5.2
30	13.3	5	–4.6
35	9.7	5	–4.4
40	8.0	5	–3.9
45	6.8	5	–2.7
50	5.7	5	–1.9
55	5.1	5	–

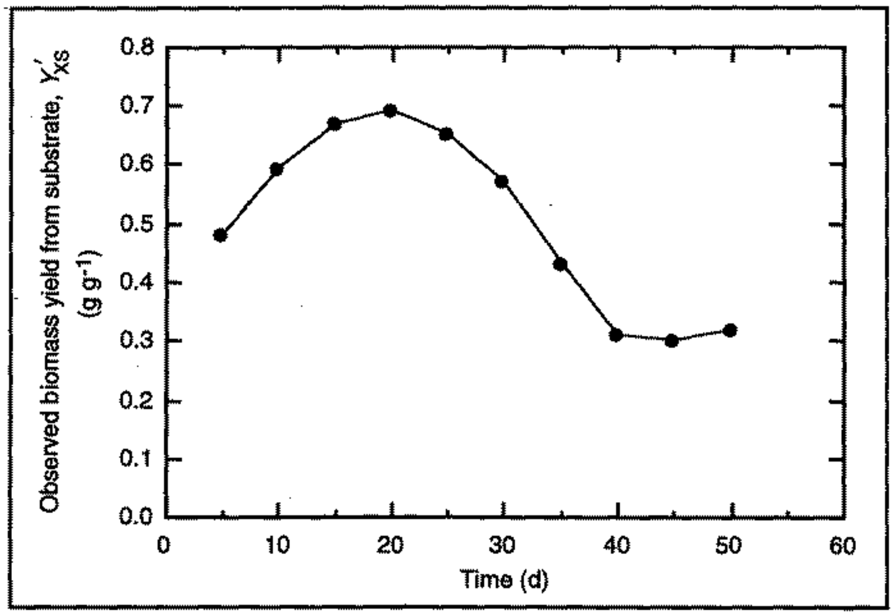
The rate of substrate uptake $-{}^{ds}/_{dt}$ is determined using the central-difference formula, Eq. (11.23). These results and values of the specific rate of substrate uptake q_S are listed and plotted as a function of time below.

Time, t (d)	$r_S = -{}^{ds}/_{dt}$ (g l^{-1} d^{-1})	$q_S = -{}^{1}/_{x}\ {}^{ds}/_{dt}$ (d^{-1})
0	–	–
5	0.62	0.318
10	0.61	0.145
15	0.57	0.103
20	0.54	0.077
25	0.52	0.055
30	0.46	0.045
35	0.44	0.037
40	0.39	0.031
45	0.27	0.021
50	0.19	0.014
55	–	–



(c)
The instantaneous biomass yield coefficient is calculated using Eq. (11.49) and the values of r_X and r_S from **(a)** and **(b)**. The results are listed and plotted below.

Time, t (h)	$Y'_{XS} = r_X/r_S$ (g g^{-1})
0	–
5	0.48
10	0.59
15	0.67
20	0.69
25	0.65
30	0.57
35	0.43
40	0.31
45	0.30
50	0.32
55	–



Answer: Y'_{XS} varies during the culture period within the range 0.30–0.69 g g^{-1} dry weight.

11.9 Ethanol fermentation by yeast and bacteria

(a)

From Table B.8 (Appendix B), the molecular formulae for glucose and ethanol are $C_6H_{12}O_6$ and C_2H_6O, respectively. The reaction equation for fermentation of glucose to ethanol without cell growth is:

$$C_6H_{12}O_6 \rightarrow 2\,C_2H_6O + 2\,CO_2$$

From the stoichiometry, the maximum theoretical yield of ethanol from glucose is 2 gmol gmol^{-1}. From Table B.1 (Appendix B) the molecular weights are: glucose = 180.2; ethanol = 46.1. Therefore:

$$\text{Maximum theoretical yield} = \frac{2\text{ gmol ethanol}}{1\text{ gmol glucose}} = \frac{2\text{ gmol ethanol}\,.\left|\frac{46.1\text{ g}}{1\text{ gmol}}\right|}{1\text{ gmol glucose}\,.\left|\frac{180.2\text{ g}}{1\text{ gmol}}\right|} = 0.51\text{ g g}^{-1}$$

Answer: 0.51 g g^{-1}

(b)

In the absence of growth, μ in Eq. (11.83) is zero and the equation reduces to:

$$Y'_{PS} = \frac{m_P}{m_S}$$

Therefore, for Y'_{PS} equal to the maximum theoretical yield:

$$m_P = 0.51\, m_S$$

For *S. cerevisiae*, $m_S = 0.18$ kg kg^{-1} h^{-1}; for *Z. mobilis*, $m_S = 2.2$ kg kg^{-1} h^{-1}. Applying these parameter values in the above equation gives $m_P = 0.092$ h^{-1} for *S. cerevisiae* and $m_P = 1.12$ h^{-1} for *Z. mobilis*.

Answer: 0.092 h^{-1} for *S. cerevisiae*; 1.12 h^{-1} for *Z. mobilis*

(c)
During batch culture at usual glucose concentrations, $\mu = \mu_{max}$ for most of the culture period (p 279). Eq. (11.83) can therefore be written as:

$$Y'_{PS} = \frac{Y_{PX}\,\mu_{max} + m_P}{\dfrac{\mu_{max}}{Y_{XS}} + m_S}$$

Substituting the parameter values for *S. cerevisiae*, including the result for m_P from **(b)**:

$$Y'_{PS} = \frac{3.9\text{ kg kg}^{-1}\left(0.4\text{ h}^{-1}\right) + 0.092\text{ h}^{-1}}{\dfrac{0.4\text{ h}^{-1}}{0.11\text{ kg kg}^{-1}} + 0.18\text{ kg kg}^{-1}\text{ h}^{-1}} = 0.43\text{ kg kg}^{-1}$$

Similarly for *Z. mobilis*:

$$Y'_{PS} = \frac{7.7\text{ kg kg}^{-1}\left(0.3\text{ h}^{-1}\right) + 1.12\text{ h}^{-1}}{\dfrac{0.3\text{ h}^{-1}}{0.06\text{ kg kg}^{-1}} + 2.2\text{ kg kg}^{-1}\text{ h}^{-1}} = 0.48\text{ kg kg}^{-1}$$

Answer: 0.43 kg kg^{-1} for *S. cerevisiae*; 0.48 kg kg^{-1} for *Z. mobilis*

(d)
Using the results from **(a)** and **(c)**, for *S. cerevisiae*:

$$\text{Efficiency} = \frac{Y'_{PS}}{\text{maximum theoretical yield}} = \frac{0.43\text{ kg kg}^{-1}}{0.51\text{ g g}^{-1}} = 0.84$$

For *Z. mobilis*:

$$\text{Efficiency} = \frac{Y'_{PS}}{\text{maximum theoretical yield}} = \frac{0.48\text{ kg kg}^{-1}}{0.51\text{ g g}^{-1}} = 0.94$$

Answer: 0.84 for *S. cerevisiae*; 0.94 for *Z. mobilis*

(e)
From Eq. (11.70) with $\mu = \mu_{max}$, for *S. cerevisiae*:

$$q_P = Y_{PX}\,\mu_{max} + m_P = 3.9\text{ kg kg}^{-1}\,(0.4\text{ h}^{-1}) + 0.092\text{ h}^{-1} = 1.65\text{ h}^{-1}$$

For *Z. mobilis*:

$$q_P = Y_{PX}\,\mu_{max} + m_P = 7.7\text{ kg kg}^{-1}\,(0.3\text{ h}^{-1}) + 1.12\text{ h}^{-1} = 3.43\text{ h}^{-1}$$

Answer: 1.65 h^{-1} for *S. cerevisiae*; 3.43 h^{-1} for *Z. mobilis*

(f)
In Eq (11.70), the growth-associated term is $Y_{PX}\,\mu_{max}$, the non-growth-associated term is m_P. Therefore, using the result from **(e)**, the proportion of ethanol production from growth for *S. cerevisiae* is:

$$\frac{Y_{PX}\,\mu_{max}}{Y_{PX}\,\mu_{max}+m_P}=\frac{3.9\,\text{kg kg}^{-1}\left(0.4\,\text{h}^{-1}\right)}{3.9\,\text{kg kg}\left(0.4\,\text{h}^{-1}\right)+0.092\,\text{h}^{-1}}=0.94$$

so that the proportion from non-growth metabolism is 0.06. For *Z. mobilis*:

$$\frac{Y_{PX}\,\mu_{max}}{Y_{PX}\,\mu_{max}+m_P}=\frac{7.7\,\text{kg kg}^{-1}\left(0.3\,\text{h}^{-1}\right)}{7.7\,\text{kg kg}^{-1}\left(0.3\,\text{h}^{-1}\right)+1.12\,\text{h}^{-1}}=0.67$$

and the proportion from non-growth metabolism is 0.33. Non-growth-associated ethanol production is more substantial for *Z. mobilis*.

Answer: For *S. cerevisiae*, 0.94 of the ethanol production is growth-associated and 0.06 is non-growth-associated. For *Z. mobilis*, 0.67 is growth-associated and 0.33 is non-growth-associated. *Z. mobilis* produces a more substantial proportion of its ethanol in non-growth-associated metabolism.

(g)
From p 261, volumetric productivity is equal to specific productivity multiplied by cell concentration. From **(e)**, as the specific ethanol productivity q_P for *Z. mobilis* is $^{3.43}/_{1.65}$ = 2.1 times that of *S. cerevisiae*, to achieve the same volumetric productivity, the concentration of yeast must be 2.1 times that of bacteria.

Answer: 2.1 times the concentration of bacteria

(h)
From p 261, total productivity is equal to specific productivity multiplied by cell concentration and fermenter volume. At zero growth, μ in Eq. (11.70) is zero and $q_P = m_P$. Therefore, from **(b)**, the specific productivity q_P for *S. cerevisiae* is 0.092 h^{-1}, and for *Z. mobilis*, 1.12 h^{-1}. As the value for *Z. mobilis* is $^{1.12}/_{0.092}$ = 12.2 times that of *S. cerevisiae*, to achieve the same total productivity at the same cell concentration, the fermenter volume for the yeast culture must be 12.2 times that for the bacteria.

Answer: 12.2 times the volume for the bacterial culture

(i)
From Eq. (11.81) with $\mu = \mu_{max}$, for *S. cerevisiae*:

$$\frac{1}{Y'_{XS}}=\frac{1}{Y_{XS}}+\frac{m_S}{\mu_{max}}=\frac{1}{0.11\,\text{kg kg}^{-1}}+\frac{0.18\,\text{kg kg}^{-1}\,\text{h}^{-1}}{0.4\,\text{h}^{-1}}=9.54\,\text{kg kg}^{-1}$$

$$Y'_{XS}=0.105\,\text{kg kg}^{-1}$$

For *Z. mobilis*:

$$\frac{1}{Y'_{XS}}=\frac{1}{Y_{XS}}+\frac{m_S}{\mu_{max}}=\frac{1}{0.06\,\text{kg kg}^{-1}}+\frac{2.2\,\text{kg kg}^{-1}\,\text{h}^{-1}}{0.3\,\text{h}^{-1}}=24.0\,\text{kg kg}^{-1}$$

$$Y'_{XS}=0.042\,\text{kg kg}^{-1}$$

Answer: 0.105 kg kg^{-1} for *S. cerevisiae*; 0.042 kg kg^{-1} for *Z. mobilis*. As *Z. mobilis* produces less biomass per mass of substrate consumed and per mass of ethanol produced than *S. cerevisiae*, biomass disposal is less of a problem with the bacteria.

(j)
The ethanol yield from substrate is 12% higher using *Z. mobilis* than *S. cerevisiae*, the specific productivity is 2.1 times higher so that a smaller and cheaper fermentation vessel is required to achieve the same rate of ethanol production, and *Z. mobilis* produces less than half the amount of biomass generated by *S. cerevisiae* per mass of glucose consumed. All of these factors mean that *Z. mobilis* performs better than *S. cerevisiae* for ethanol production.

However, other aspects of the cultures and the ethanol production industry also need to be considered. *Z. mobilis* requires a higher pH (5.0) for growth than *S. cerevisiae* (3.5–4.0), and is therefore more susceptible to contamination. *Z. mobilis* also does not grow well on molasses, a common substrate material for fermentations, because of the high salt content. The biomass produced in ethanol fermentation by yeast is often sold for use in animal feeds, whereas application of bacteria for this purpose is not as well accepted in the industry. These are some of the reasons why *Z. mobilis* has not been widely adopted for industrial ethanol production, despite its superior ethanol production characteristics.

11.10 Plasmid loss during culture maintenance

From Eq. (11.65):

$$\alpha = \frac{\mu^-}{\mu^+} = \frac{0.033\,\text{h}^{-1}}{0.025\,\text{h}^{-1}} = 1.32$$

The number of generations of plasmid-containing cells over the 28-day period is, from Eq. (11.66):

$$n = \frac{\mu^+ t}{\ln 2} = \frac{\left(0.025\,\text{h}^{-1}\right) 28\,\text{d}\,.\left|\frac{24\,\text{h}}{1\,\text{d}}\right|}{\ln 2} = 24.2$$

If the fraction of cells containing plasmid is $F = 0.66$, from Eq. (11.64):

$$0.66 = \frac{1-\alpha-p}{1-\alpha-2^{n(\alpha+p-1)}p} = \frac{1-1.32-p}{1-1.32-\left(2^{24.2(1.32+p-1)}\right)p} = \frac{0.32+p}{0.32+\left(2^{24.2(0.32+p)}\right)p} = \frac{0.32+p}{0.32+214.38p\,(2^{24.2p})}$$

Multiplying through by the denominator:

$$0.211 + 141.49p\,(2^{24.2p}) = 0.32 + p$$

$$141.49\,p\,(2^{24.2p}) - 0.109 - p = 0$$

This equation can be solved for p by trial and error. Values of the left-hand-side of the equation for various p are listed below, starting with $p = 0.001$ as the first guess.

p	$141.49\,p\,(2^{24.2p}) - 0.109 - p$
0.001	0.0339
0.002	0.1816
0.0007	–0.0095
0.00076	–0.0008
0.00077	0.0006
0.000766	0.000017

As the value of the expression when $p = 0.000766$ is sufficiently close to zero, the solution can be taken as $p = 0.000766$.

Answer: 0.000766

11.11 Medium sterilisation

(a), (b) and (c)

The specific death constant k_d is evaluated using Eq. (11.46) with $A = 10^{36.2}\,\text{s}^{-1}$ and $E_d = 283$ kJ gmol^{-1}. From Table 2.5, $R = 8.3144$ J K^{-1} gmol^{-1} $= 8.3144 \times 10^{-3}$ kJ K^{-1} gmol^{-1}. Converting the temperatures to degrees Kelvin using Eq. (2.24), 80°C = (80 + 273.15) = 353.15 K, 121°C = (121 + 273.15) = 394.15 K, and 140°C = (140 + 273.15) = 413.15 K. Therefore, at 80°C:

$$k_d = A\,e^{-E_d/R\,T} = 10^{36.2}\,\text{s}^{-1}\,e^{-283\,\text{kJ gmol}^{-1}/(8.3144\times 10^{-3}\,\text{kJ K}^{-1}\,\text{gmol}^{-1}\times 353.15\,\text{K})} = 2.20\times 10^{-6}\,\text{s}^{-1}$$

At 121°C:

$$k_d = A\,e^{-E_d/R\,T} = 10^{36.2}\,\text{s}^{-1}\,e^{-283\,\text{kJ gmol}^{-1}/(8.3144\times 10^{-3}\,\text{kJ K}^{-1}\,\text{gmol}^{-1}\times 394.15\,\text{K})} = 0.0497\,\text{s}^{-1}$$

At 141°C:

$$k_d = A\,e^{-E_d/R\,T} = 10^{36.2}\,\text{s}^{-1}\,e^{-283\,\text{kJ gmol}^{-1}/(8.3144\times 10^{-3}\,\text{kJ K}^{-1}\,\text{gmol}^{-1}\times 413.15\,\text{K})} = 2.63\,\text{s}^{-1}$$

The relationship between the number of viable cells and time is given by Eq. (11.87). Converting the units of the initial concentration of contaminants x_0:

$$x_0 = 10^8\,\text{cells l}^{-1} = 10^8\,\text{cells l}^{-1}\cdot\left|\frac{1000\,\text{l}}{1\,\text{m}^3}\right| = 10^{11}\,\text{cells m}^{-3}$$

Therefore, per m^3, $N_0 = 10^{11}$; $N = 10^{-3}$. Substituting values into Eq. (11.87) gives:

$$10^{-3} = 10^{11}\,e^{-k_d t}$$

$$10^{-14} = e^{-k_d t}$$

$$-32.24 = -k_d\,t$$

$$t = \frac{32.24}{k_d}$$

Therefore, at 80°C:

$$t = \frac{32.24}{k_d} = \frac{32.24}{2.20\times 10^{-6}\,\text{s}^{-1}}\cdot\left|\frac{1\,\text{h}}{3600\,\text{s}}\right| = 4070\,\text{h}$$

At 121°C:

$$t = \frac{32.24}{k_d} = \frac{32.24}{0.0497\,\text{s}^{-1}}\cdot\left|\frac{1\,\text{min}}{60\,\text{s}}\right| = 10.8\,\text{min}$$

At 140°C:

$$t = \frac{32.24}{k_d} = \frac{32.24}{2.63\,\text{s}^{-1}} = 12.3\,\text{s}$$

Answer: 4070 h at 80°C; 10.8 min at 121°C; 12.3 s at 140°C

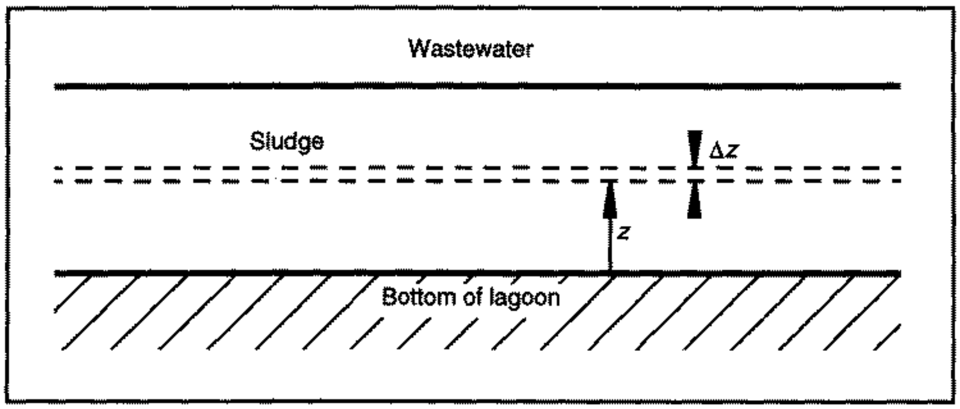
12

Heterogeneous Reactions

12.1 Diffusion and reaction in a waste treatment lagoon

(a)

A substrate shell mass-balance is performed around a thin slice of sludge of area A and thickness Δz located distance z from the bottom of the lagoon, as shown in the diagram below.



Substrate diffuses into the shell across the upper boundary at $z+\Delta z$, and diffuses out across the lower boundary at z. Substrate consumption within the shell follows first-order kinetics. Following the procedure on p 301, from Fick's law, the rate of substrate input by diffusion is:

$$\left(\mathcal{D}_{\mathrm{Se}} A \frac{\mathrm{d}s}{\mathrm{d}z}\right)\bigg|_{z+\Delta z}$$

where $\mathcal{D}_{\mathrm{Se}}$ is the effective diffusivity of substrate in the sludge. Similarly, the rate of substrate output by diffusion is:

$$\left(\mathcal{D}_{\mathrm{Se}} A \frac{\mathrm{d}s}{\mathrm{d}z}\right)\bigg|_{z}$$

The rate of substrate consumption in the shell is $k_1\, s\, A\, \Delta z$. Substituting these expressions into the mass-balance equation, Eq. (4.1), with the rate of substrate generation = 0, at steady state:

$$\left(\mathcal{D}_{\mathrm{Se}} A \frac{\mathrm{d}s}{\mathrm{d}z}\right)\bigg|_{z+\Delta z} - \left(\mathcal{D}_{\mathrm{Se}} A \frac{\mathrm{d}s}{\mathrm{d}z}\right)\bigg|_{z} - k_1\, s\, A\, \Delta z = 0$$

Assuming that the substrate diffusivity and sludge area do not vary with distance z:

$$\mathcal{D}_{\mathrm{Se}} A \left(\frac{\mathrm{d}s}{\mathrm{d}z}\bigg|_{z+\Delta z} - \frac{\mathrm{d}s}{\mathrm{d}z}\bigg|_{z}\right) - k_1\, s\, A\, \Delta z = 0$$

Cancelling A and dividing through by Δz gives:

$$\mathcal{D}_{\mathrm{Se}} \frac{\dfrac{\mathrm{d}s}{\mathrm{d}z}\bigg|_{z+\Delta z} - \dfrac{\mathrm{d}s}{\mathrm{d}z}\bigg|_{z}}{\Delta z} - k_1\, s = 0$$

Taking the limit as $\Delta z \rightarrow 0$ and invoking the definition of the derivative as in Eq. (D.12) in Appendix D:

$$\mathcal{D}_{\mathrm{Se}} \frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\mathrm{d}s}{\mathrm{d}z}\right) - k_1\, s = 0$$

or

$$\mathcal{D}_{\mathrm{Se}} \frac{d^2 s}{dz^2} - k_1 s = 0$$

Answer: $\mathcal{D}_{\mathrm{Se}} \frac{d^2 s}{dz^2} - k_1 s = 0$

(b)

At $z = L$, $s = s_b$. At $z = 0$, we assume that the substrate concentration profile reaches a minimum so that ${}^{ds}/_{dz} = 0$.

Answer: At $z = L$, $s = s_b$; at $z = 0$, ${}^{ds}/_{dz} = 0$.

(c)

(i)

If $s = N\,e^{pz}$, using differentiation rule Eq. (D.17) in Appendix D:

$$\frac{ds}{dz} = p\,N\,e^{pz}$$

and

$$\frac{d^2 s}{dz^2} = p^2\,N\,e^{pz}$$

Substituting the expressions for s and ${}^{d^2s}/_{dz^2}$ into the differential equation in **(a)**:

$$\mathcal{D}_{\mathrm{Se}}\, p^2\, N\, e^{pz} - k_1\, N\, e^{pz} = 0$$

Dividing through by $N\,e^{pz}$ and solving for p^2:

$$p^2 = \frac{k_1}{\mathcal{D}_{\mathrm{Se}}}$$

$$p = \sqrt{k_1/\mathcal{D}_{\mathrm{Se}}} \qquad \text{or} \qquad p = -\sqrt{k_1/\mathcal{D}_{\mathrm{Se}}}$$

Answer: $p = \pm\sqrt{k_1/\mathcal{D}_{\mathrm{Se}}}$

(ii)

If $s = N\,e^{pz} + M\,e^{-pz}$, substituting the values for p from **(i)** above:

$$s = N\,e^{z\sqrt{k_1/\mathcal{D}_{\mathrm{Se}}}} + M\,e^{-z\sqrt{k_1/\mathcal{D}_{\mathrm{Se}}}}$$

Differentiating this equation using differentiation rule Eq. (D.17) in Appendix D:

$$\frac{ds}{dz} = N\sqrt{k_1/\mathcal{D}_{\mathrm{Se}}}\; e^{z\sqrt{k_1/\mathcal{D}_{\mathrm{Se}}}} - M\sqrt{k_1/\mathcal{D}_{\mathrm{Se}}}\; e^{-z\sqrt{k_1/\mathcal{D}_{\mathrm{Se}}}}$$

Applying the boundary condition at $z = 0$:

$$0 = N\sqrt{k_1/\mathcal{D}_{\mathrm{Se}}} - M\sqrt{k_1/\mathcal{D}_{\mathrm{Se}}}$$

Cancelling the square root terms gives:

$$N = M$$

Answer: $N = M$

(iii)

Substituting N for M in the equation for s in **(ii)**:

$$s = N\left(e^{z\sqrt{k_1/\mathcal{D}_{Se}}} + e^{-z\sqrt{k_1/\mathcal{D}_{Se}}}\right)$$

Applying the boundary condition at $z = L$:

$$s_b = N\left(e^{L\sqrt{k_1/\mathcal{D}_{Se}}} + e^{-L\sqrt{k_1/\mathcal{D}_{Se}}}\right)$$

Solving for N:

$$N = \frac{s_b}{e^{L\sqrt{k_1/\mathcal{D}_{Se}}} + e^{-L\sqrt{k_1/\mathcal{D}_{Se}}}}$$

Substituting this result into the equation for s above:

$$s = s_b \frac{e^{z\sqrt{k_1/\mathcal{D}_{Se}}} + e^{-z\sqrt{k_1/\mathcal{D}_{Se}}}}{e^{L\sqrt{k_1/\mathcal{D}_{Se}}} + e^{-L\sqrt{k_1/\mathcal{D}_{Se}}}}$$

Answer: $s = s_b \dfrac{e^{z\sqrt{k_1/\mathcal{D}_{Se}}} + e^{-z\sqrt{k_1/\mathcal{D}_{Se}}}}{e^{L\sqrt{k_1/\mathcal{D}_{Se}}} + e^{-L\sqrt{k_1/\mathcal{D}_{Se}}}}$

(iv)
From the definition of cosh x, the numerator in the equation for s in **(iii)** is equal to $2\cosh\left(z\sqrt{k_1/\mathcal{D}_{Se}}\right)$. Similarly, the denominator is equal to $2\cosh\left(L\sqrt{k_1/\mathcal{D}_{Se}}\right)$. Therefore:

$$s = s_b \frac{2\cosh\left(z\sqrt{k_1/\mathcal{D}_{Se}}\right)}{2\cosh\left(L\sqrt{k_1/\mathcal{D}_{Se}}\right)}$$

or

$$\frac{s}{s_b} = \frac{\cosh\left(z\sqrt{k_1/\mathcal{D}_{Se}}\right)}{\cosh\left(L\sqrt{k_1/\mathcal{D}_{Se}}\right)}$$

Answer: Q.E.D.

(d)
Taking the derivative of s using the equation for s in **(c)** **(iv)**:

$$\frac{ds}{dz} = s_b\sqrt{k_1/\mathcal{D}_{Se}}\,\frac{\sinh\left(z\sqrt{k_1/\mathcal{D}_{Se}}\right)}{\cosh\left(L\sqrt{k_1/\mathcal{D}_{Se}}\right)}$$

Evaluating the derivative at $z = L$:

$$\left.\frac{ds}{dz}\right|_{z=L} = s_b\sqrt{k_1/\mathcal{D}_{Se}}\,\frac{\sinh\left(L\sqrt{k_1/\mathcal{D}_{Se}}\right)}{\cosh\left(L\sqrt{k_1/\mathcal{D}_{Se}}\right)}$$

Substituting this result into the expression for $r_{A,obs}$:

$$r_{A,obs} = \mathcal{D}_{Se}\,A\,s_b\sqrt{k_1/\mathcal{D}_{Se}}\,\frac{\sinh\left(L\sqrt{k_1/\mathcal{D}_{Se}}\right)}{\cosh\left(L\sqrt{k_1/\mathcal{D}_{Se}}\right)}$$

Answer: $r_{A,obs} = \mathcal{D}_{Se}\,A\,s_b\sqrt{k_1/\mathcal{D}_{Se}}\,\dfrac{\sinh\left(L\sqrt{k_1/\mathcal{D}_{Se}}\right)}{\cosh\left(L\sqrt{k_1/\mathcal{D}_{Se}}\right)}$

(e)

The internal effectiveness factor is defined in Eq. (12.26). For first-order reaction kinetics at substrate concentration s_b everywhere in the sludge, the rate of reaction within the entire sludge volume is:

$$\overset{*}{r}_{\mathrm{As}} = k_1\, s_b\, A\, L$$

Substituting this and the expression for $r_{\mathrm{A,obs}}$ from **(d)** into Eq. (12.26) for first-order kinetics:

$$\eta_{i1} = \frac{r_{\mathrm{A,obs}}}{\overset{*}{r}_{\mathrm{As}}} = \frac{\mathcal{D}_{\mathrm{Se}}\, A\, s_b \sqrt{k_1/\mathcal{D}_{\mathrm{Se}}}\; \dfrac{\sinh\left(L\sqrt{k_1/\mathcal{D}_{\mathrm{Se}}}\right)}{\cosh\left(L\sqrt{k_1/\mathcal{D}_{\mathrm{Se}}}\right)}}{k_1\, s_b\, A\, L}$$

Cancelling and grouping terms and applying the definition of tanh x gives:

$$\eta_{i1} = \frac{\tanh\left(L\sqrt{k_1/\mathcal{D}_{\mathrm{Se}}}\right)}{L\sqrt{k_1/\mathcal{D}_{\mathrm{Se}}}}$$

Therefore, applying the definition of ϕ_1:

$$\eta_{i1} = \frac{\tanh \phi_1}{\phi_1}$$

Answer: Q.E.D.

(f)

From the definition of tanh x:

$$\tanh \phi_1 = \frac{e^{\phi_1} - e^{-\phi_1}}{e^{\phi_1} + e^{-\phi_1}}$$

For $L = 2$ cm, the values of ϕ_1, tanh ϕ_1 and η_{i1} evaluated for the three sets of conditions are listed below.

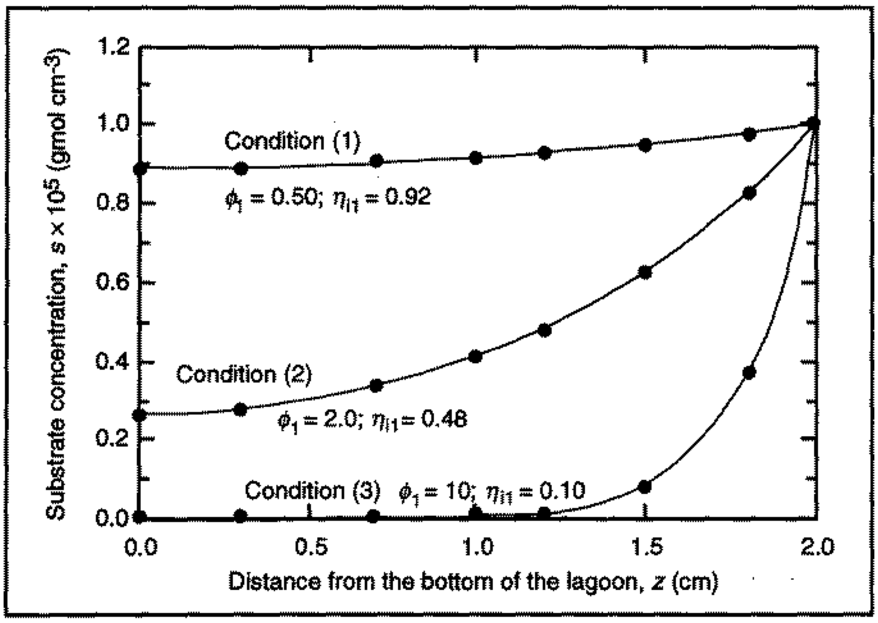
Condition	ϕ_1	tanh ϕ_1	η_{i1}
(1)	0.50	0.46	0.92
(2)	2.0	0.96	0.48
(3)	10	1.0	0.10

The substrate concentration profiles are calculated using the equation for s as a function of z from **(c) (iii)** and applying the definition of ϕ_1:

$$s = s_b\, \frac{e^{z\sqrt{k_1/\mathcal{D}_{\mathrm{Se}}}} + e^{-z\sqrt{k_1/\mathcal{D}_{\mathrm{Se}}}}}{e^{\phi_1} + e^{-\phi_1}}$$

The results for $s_b = 10^{-5}$ gmol cm^{-3} at various values of z are listed and plotted below.

	Substrate concentration, s (gmol cm^{-3})		
Distance, z (cm)	Condition (1)	Condition (2)	Condition (3)
0.0	8.87×10^{-6}	2.66×10^{-6}	9.08×10^{-10}
0.3	8.89×10^{-6}	2.78×10^{-6}	2.14×10^{-9}
0.7	9.00×10^{-6}	3.34×10^{-6}	1.50×10^{-8}
1.0	9.15×10^{-6}	4.10×10^{-6}	6.74×10^{-8}
1.2	9.27×10^{-6}	4.81×10^{-6}	1.83×10^{-7}
1.5	9.50×10^{-6}	6.25×10^{-6}	8.21×10^{-7}
1.8	9.78×10^{-6}	8.26×10^{-6}	3.68×10^{-6}
2.0	1.00×10^{-5}	1.00×10^{-5}	1.00×10^{-5}



As ϕ_1 increases, η_{i1} decreases, the concentration profile becomes steeper, and the minimum substrate concentration in the sludge is reduced.

12.2 Oxygen profile in immobilised-enzyme catalyst

(a)

From p 269, as C_{As} is $^{0.5\ \mathrm{mM}}/_{0.015\ \mathrm{mM}}$ = 33 times the value of K_m, as a first approximation we can consider the kinetics to be effectively zero order with $k_0 = v_{max}$. Converting the units of k_0 to a per volume of gel basis:

$$k_0 = v_{max} = 0.12\,\mathrm{mol\,s^{-1}\,kg^{-1}}\left(0.012\,\mathrm{kg\,m^{-3}}\right).\left|\frac{32\,\mathrm{g}}{1\,\mathrm{mol}}\right|.\left|\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right| = 4.61\times10^{-5}\,\mathrm{kg\,s^{-1}\,m^{-3}}$$

Converting C_{As} to units of kg m^{-3}:

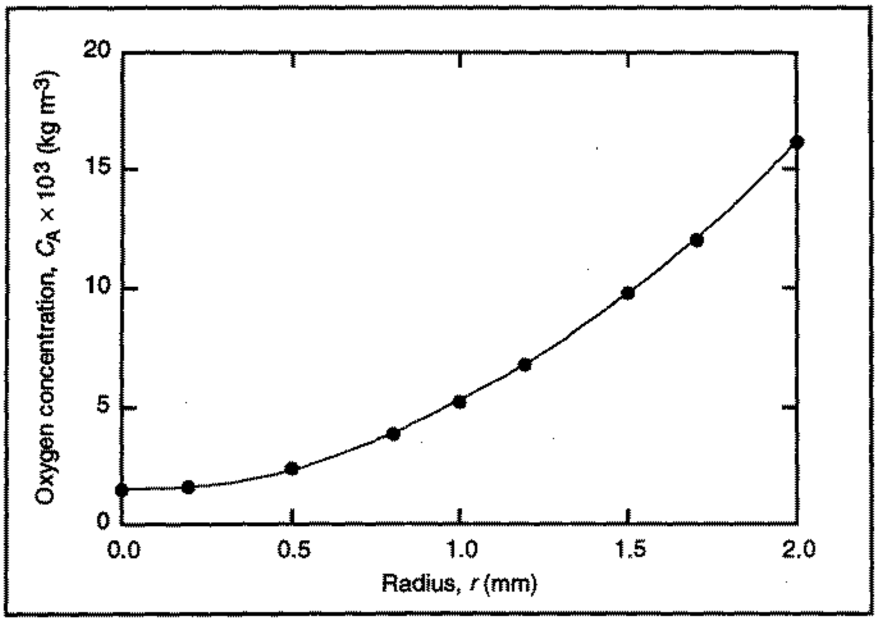
$$C_{As} = 0.5\,\mathrm{mM} = 0.5\times10^{-3}\,\mathrm{gmol\,l^{-1}}.\left|\frac{1000\,\mathrm{l}}{1\,\mathrm{m^3}}\right|.\left|\frac{32\,\mathrm{g}}{1\,\mathrm{gmol}}\right|.\left|\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right| = 0.016\,\mathrm{kg\,m^{-3}}$$

For zero-order reaction, the equation used to determine the substrate concentration inside the beads depends on whether C_A remains > 0 throughout the particle. The maximum particle radius for which this occurs can be calculated using Eq. (12.17):

$$R_{max} = \sqrt{\frac{6\,\mathcal{D}_{Ae}\,C_{As}}{k_0}} = \sqrt{\frac{6\left(2.1\times10^{-9}\,\mathrm{m^2\,s^{-1}}\right)0.016\,\mathrm{kg\,m^{-3}}}{4.61\times10^{-5}\,\mathrm{kg\,s^{-1}\,m^{-3}}}} = 0.0021\,\mathrm{m} = 2.1\,\mathrm{mm}$$

Therefore, the maximum particle diameter for $C_A > 0$ everywhere is 4.2 mm. Because the immobilised-enzyme beads are smaller than this, $C_A > 0$ and the oxygen concentration profile can be calculated using the equation for zero-order reaction and spherical geometry in Table 12.1. Values for C_A as a function of *r* are listed and plotted below.

Radius, *r* (m)	Oxygen concentration, C_A (kg m^{-3})
2.0×10^{-3}	1.60×10^{-2}
1.7×10^{-3}	1.19×10^{-2}
1.5×10^{-3}	9.60×10^{-3}
1.2×10^{-3}	6.63×10^{-3}
1.0×10^{-3}	5.02×10^{-3}
0.8×10^{-3}	3.71×10^{-3}
0.5×10^{-3}	2.28×10^{-3}
0.2×10^{-3}	1.51×10^{-3}
0.0	1.37×10^{-3}



As the minimum value of C_A at the centre of the bead is still about 2.9 times K_m, the assumption of zero-order kinetics is reasonable.

(b)
As $C_A > 0$ everywhere within the bead, for zero-order reaction this means that the entire bead volume is active.

Answer: 1.0

(c)
For zero-order reaction, the maximum conversion rate occurs when the oxygen concentration is greater than zero everywhere in the particle. The largest bead size for this to occur was calculated in **(a)** is 4.2 mm.

Answer: 4.2 mm

12.3 Effect of oxygen transfer on recombinant cells

(a)
Converting the units of k_0 to mass:

$$k_0 = 10^{-3}\,\text{mol}\,\text{s}^{-1}\,\text{m}^{-3} \cdot \left|\frac{32\,\text{g}}{1\,\text{mol}}\right| \cdot \left|\frac{1\,\text{kg}}{1000\,\text{g}}\right| = 3.2 \times 10^{-5}\,\text{kg}\,\text{s}^{-1}\,\text{m}^{-3}$$

The maximum particle radius for which oxygen concentration inside the beads remains greater than zero is calculated using Eq. (12.17):

$$R_{\text{max}} = \sqrt{\frac{6\,\mathcal{D}_{Ae}\,C_{As}}{k_0}} = \sqrt{\frac{6\left(1.4 \times 10^{-9}\,\text{m}^2\,\text{s}^{-1}\right) 8 \times 10^{-3}\,\text{kg}\,\text{m}^{-3}}{3.2 \times 10^{-5}\,\text{kg}\,\text{s}^{-1}\,\text{m}^{-3}}} = 1.45 \times 10^{-3}\,\text{m} = 1.45\,\text{mm}$$

Therefore, the maximum particle diameter for aerobic conditions is 2 × 1.45 = 2.9 mm.

Answer: 2.9 mm

(b)
The particle radius is $^{1.45}/_2 = 0.725$ mm $= 7.25 \times 10^{-4}$ m. As this radius is less than R_{max} determined in **(a)**, oxygen is present everywhere in the particle. Therefore, as the kinetics are zero-order, $\eta_i = 1$. From Eq. (12.26) with $\eta_i = 1$:

$$r_{A,\text{obs}} = r^*_{As} = k_0 = 3.2 \times 10^{-5}\,\text{kg}\,\text{s}^{-1}\,\text{m}^{-3}$$

Substituting parameter values into the equation for the observable Thiele modulus Φ in Table 12.4 for spherical geometry:

$$\Phi = \left(\frac{R}{3}\right)^2 \frac{r_{A,obs}}{\mathscr{D}_{Ae}\, C_{As}} = \left(\frac{7.25 \times 10^{-4}\,\mathrm{m}}{3}\right)^2 \frac{3.2 \times 10^{-5}\,\mathrm{kg\,s^{-1}\,m^{-3}}}{1.4 \times 10^{-9}\,\mathrm{m^2\,s^{-1}}\left(8 \times 10^{-3}\,\mathrm{kg\,m^{-3}}\right)} = 0.167$$

Using this value, the minimum intraparticle oxygen concentration can be calculated from the equation in Table 12.5 for spherical geometry and $\Phi < 0.667$:

$$C_{A,min} = C_{As}\left(1 - \frac{3}{2}\,\Phi\right) = 8 \times 10^{-3}\,\mathrm{kg\,m^{-3}}\left(1 - \frac{3}{2}\,(0.167)\right) = 6.0 \times 10^{-3}\,\mathrm{kg\,m^{-3}}$$

Answer: 6.0×10^{-3} kg m^{-3}

(c)

If the cell density is reduced by a factor of 5, k_0 is reduced to $^1/_5$ its previous value. Therefore, $k_0 = {}^1/_5\,(3.2 \times 10^{-5}$ kg s^{-1} m^{-3}) = 6.4×10^{-6} kg s^{-1} m^{-3}. From Eq. (12.17):

$$R_{max} = \sqrt{\frac{6\,\mathscr{D}_{Ae}\, C_{As}}{k_0}} = \sqrt{\frac{6\left(1.4 \times 10^{-9}\,\mathrm{m^2\,s^{-1}}\right) 8 \times 10^{-3}\,\mathrm{kg\,m^{-3}}}{6.4 \times 10^{-6}\,\mathrm{kg\,s^{-1}\,m^{-3}}}} = 3.24 \times 10^{-3}\,\mathrm{m} = 3.24\,\mathrm{mm}$$

Therefore, the maximum particle diameter for aerobic conditions is 2 × 3.24 = 6.5 mm.

Answer: 6.5 mm diameter

12.4 Ammonia oxidation by immobilised cells

(a)

R = 1.5 mm = 1.5×10^{-3} m. Calculating the observable modulus Ω for spherical geometry from Table 12.6:

$$\Omega = \frac{R}{3}\,\frac{r_{A,obs}}{k_S\, C_{Ab}} = \frac{1.5 \times 10^{-3}\,\mathrm{m}}{3}\;\frac{2.2 \times 10^{-5}\,\mathrm{kg\,s^{-1}\,m^{-3}}}{6 \times 10^{-5}\,\mathrm{m\,s^{-1}}\left(6 \times 10^{-3}\,\mathrm{kg\,m^{-3}}\right)} = 0.031$$

From Eqs (12.43) and (12.44):

$$C_{As} = C_{Ab}\,(1 - \Omega) = C_{Ab}\,(1 - 0.031) = 0.97\, C_{Ab}$$

As $C_{As} \approx C_{Ab}$, external mass-transfer effects are insignificant.

Answer: Insignificant; the surface oxygen concentration is only 3% lower than in the bulk medium.

(b)

From Table 12.7 for zero-order oxygen uptake kinetics, $\eta_{e0} = 1$. The internal effectiveness factor η_{i0} can be determined from Figure 12.11 as a function of the observable Thiele modulus Φ. Evaluating Φ from the equation in Table 12.4 for spherical geometry using the result for C_{As} from **(a)**:

$$\Phi = \left(\frac{R}{3}\right)^2 \frac{r_{A,obs}}{\mathscr{D}_{Ae}\, C_{As}} = \left(\frac{1.5 \times 10^{-3}\,\mathrm{m}}{3}\right)^2 \frac{2.2 \times 10^{-5}\,\mathrm{kg\,s^{-1}\,m^{-3}}}{1.9 \times 10^{-9}\,\mathrm{m^2\,s^{-1}}\left(0.97 \times 6 \times 10^{-3}\,\mathrm{kg\,m^{-3}}\right)} = 0.50$$

From Figure 12.11, at $\Phi = 0.50$, $\eta_{i0} = 1$. Using Eq. (12.46), $\eta_T = \eta_{i0}\,\eta_{e0} = 1 \times 1 = 1$.

Answer: 1

(c)

Using the results from **(a)** and **(b)**, the minimum intraparticle oxygen concentration can be calculated from the equation in Table 12.5 for spherical geometry and $\Phi < 0.667$:

$$C_{A,min} = C_{As}\left(1 - \frac{3}{2}\,\Phi\right) = 0.97 \times 6 \times 10^{-3}\,\mathrm{kg\,m^{-3}}\left(1 - \frac{3}{2}\,(0.50)\right) = 1.5 \times 10^{-3}\,\mathrm{kg\,m^{-3}}$$

This oxygen concentration is greater than the critical level.

Answer: Yes

12.5 Microcarrier culture and external mass transfer

$D_p = 120\ \mu\text{m} = 120 \times 10^{-6}$ m. The external mass-transfer coefficient can be determined using the equations on p 322 for free-moving spheres. The Grashof number is calculated from Eq. (12.51) with $g = 9.8$ m s^{-2} from p 16 and the unit conversion factor 1 N s m^{-2} = 1 kg m^{-1} s^{-1} from Table A.9, Appendix A:

$$Gr = \frac{g\, D_p^3\, \rho_L (\rho_p - \rho_L)}{\mu_L^2} = \frac{9.8\ \text{m s}^{-2} \left(120 \times 10^{-6}\ \text{m}\right)^3\ 10^3\ \text{kg m}^{-3} \left(1.2 \times 10^3\ \text{kg m}^{-3} - 10^3\ \text{kg m}^{-3}\right)}{\left(10^{-3}\ \text{N s m}^{-2} . \left|\frac{1\ \text{kg m}^{-1}\ \text{s}^{-1}}{1\ \text{N s m}^{-2}}\right|\right)^2} = 3.39$$

Therefore, from Eq. (12.52), $Re_p = {}^{Gr}/_{18} = {}^{3.39}/_{18} = 0.188$. The Schmidt number from Eq. (12.49) is:

$$Sc = \frac{\mu_L}{\rho_L\, \mathcal{D}_{AL}} = \frac{10^{-3}\ \text{N s m}^{-2} . \left|\frac{1\ \text{kg m}^{-1}\ \text{s}^{-1}}{1\ \text{N s m}^{-2}}\right|}{10^3\ \text{kg m}^{-3} \left(2.3 \times 10^{-9}\ \text{m}^2\ \text{s}^{-1}\right)} = 435$$

Therefore, $Re_p\, Sc = 0.188 \times 435 = 81.8$. As this value is less than 10^4, the Sherwood number can be evaluated using Eq. (12.55):

$$Sh = \sqrt{4 + 1.21 \left(Re_p\, Sc\right)^{0.67}} = \sqrt{4 + 1.21\, (81.8)^{0.67}} = 5.21$$

From the definition of the Sherwood number in Eq. (12.50):

$$k_S = \frac{Sh\, \mathcal{D}_{AL}}{D_p} = \frac{5.21 \left(2.3 \times 10^{-9}\ \text{m}^2\ \text{s}^{-1}\right)}{120 \times 10^{-6}\ \text{m}} = 9.99 \times 10^{-5}\ \text{m s}^{-1}$$

Using this value of k_S to determine Ω from the equation in Table 12.6 for spherical geometry:

$$\Omega = \frac{R}{3}\, \frac{r_{A,obs}}{k_S\, C_{Ab}} = \frac{\left(\frac{120 \times 10^{-6}\ \text{m}}{2}\right)}{3}\ \frac{0.015\ \text{mol s}^{-1}\ \text{m}^{-3}}{9.99 \times 10^{-5}\ \text{m s}^{-1} \left(0.2\ \text{mol m}^{-3}\right)} = 0.015$$

From Eqs (12.43) and (12.44):

$$C_{As} = C_{Ab}(1 - \Omega) = C_{Ab}(1 - 0.015) = 0.985\, C_{Ab}$$

External mass-transfer effects are insignificant as $C_{As} \approx C_{Ab}$. Because respiration is zero-order and the cells are present only on the surface of the beads, $C_{As} > 0$ is all that is required to ensure maximum reaction rate.

Answer: Negligible

12.6 Immobilised-enzyme reaction kinetics

(a)

$R = 0.8$ mm $= 0.8 \times 10^{-3}$ m. As external boundary-layers have been eliminated, $C_{As} = C_{Ab} = 0.85$ kg m^{-3} and $\eta_e = 1$. The value of β as defined on p 313 is:

$$\beta = \frac{K_m}{C_{As}} = \frac{3.5\ \text{kg m}^{-3}}{0.85\ \text{kg m}^{-3}} = 4.12$$

From Figures 12.10–12.12, this value of β means that the reaction kinetics can be considered effectively first-order. Evaluating the observable Thiele modulus Φ from the equation in Table 12.4 for spherical geometry:

$$\Phi = \left(\frac{R}{3}\right)^2 \frac{r_{A,obs}}{\mathcal{D}_{Ae}\, C_{As}} = \left(\frac{0.8 \times 10^{-3}\ \text{m}}{3}\right)^2 \frac{1.25 \times 10^{-3}\ \text{kg s}^{-1}\ \text{m}^{-3}}{1.3 \times 10^{-11}\ \text{m}^2\ \text{s}^{-1} \left(0.85\ \text{kg m}^{-3}\right)} = 8.0$$

From Figure 12.11, at $\Phi = 8.0$, $\eta_{i1} = 0.12$. Therefore, from Eq. (12.46), $\eta_T = \eta_i\, \eta_e = 0.12 \times 1 = 0.12$.

Answer: 0.12

(b)

From the definition of the effectiveness factor, Eq. (12.26):

$$r^*_{\mathrm{As}} = \frac{r_{\mathrm{A,obs}}}{\eta_{\mathrm{i}}} = \frac{1.25\times10^{-3}\,\mathrm{kg\,s^{-1}\,m^{-3}}}{0.12} = 0.0104\,\mathrm{kg\,s^{-1}\,m^{-3}}$$

For first-order kinetics, $r^*_{\mathrm{As}} = k_1\, C_{\mathrm{As}}$; therefore:

$$k_1 = \frac{r^*_{\mathrm{As}}}{C_{\mathrm{As}}} = \frac{0.0104\,\mathrm{kg\,s^{-1}\,m^{-3}}}{0.85\,\mathrm{kg\,m^{-3}}} = 0.0122\,\mathrm{s^{-1}}$$

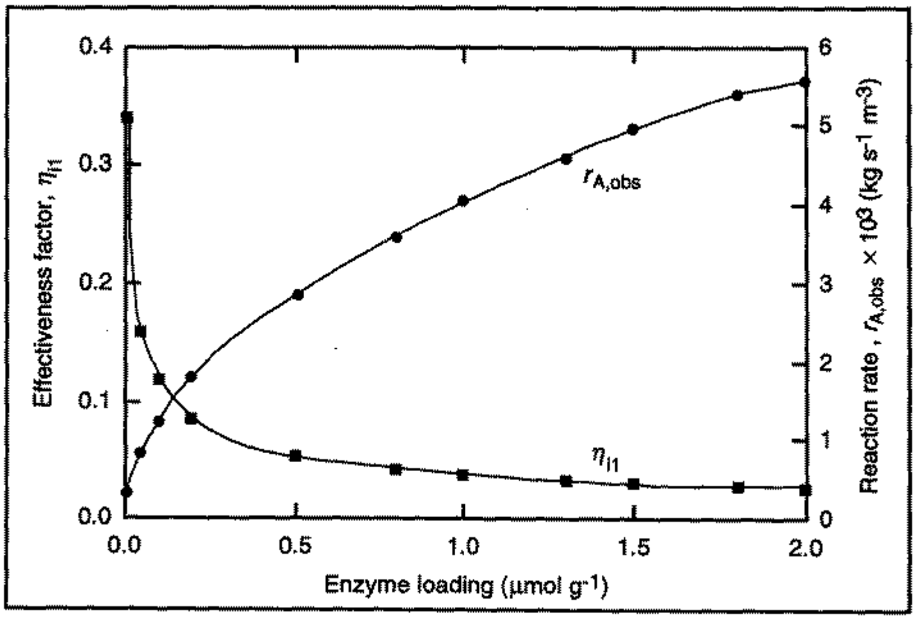
Answer: 0.0122 $\mathrm{s^{-1}}$

(c)

The value of 0.0122 $\mathrm{s^{-1}}$ for k_1 corresponds to an enzyme loading of 0.1 μmol $\mathrm{g^{-1}}$. The Thiele modulus ϕ_1 can be evaluated as a function of enzyme loading using the equation for first-order reaction and spherical geometry from Table 12.2, with k_1 directly proportional to the enzyme loading. For $\phi_1 < 10$, the internal effectiveness factor $\eta_{\mathrm{i}1}$ is determined using the equation in Table 12.3 and the definition of coth x; for $\phi_1 > 10$, from Eq. (12.30) $\eta_{\mathrm{i}1} = {}^1/\phi_1$. For each value of k_1, $r^*_{\mathrm{As}} = k_1\, C_{\mathrm{As}}$, and $r_{\mathrm{A,obs}}$ can be determined from these results and the definition of the effectiveness factor in Eq. (12.26). Calculated values of these parameters for several different enzyme loadings are listed below.

Enzyme loading (μmol $\mathrm{g^{-1}}$)	k_1 ($\mathrm{s^{-1}}$)	ϕ_1	$\eta_{\mathrm{i}1}$	r^*_{As} ($\mathrm{kg\,s^{-1}\,m^{-3}}$)	$r_{\mathrm{A,obs}}$ ($\mathrm{kg\,s^{-1}\,m^{-3}}$)
0.01	0.0012	2.6	0.34	0.0010	3.40×10^{-4}
0.05	0.0061	5.8	0.16	0.0052	8.32×10^{-4}
0.10	0.0122	8.2	0.12	0.0104	1.25×10^{-3}
0.20	0.0244	11.6	0.086	0.021	1.81×10^{-3}
0.50	0.0610	18.3	0.055	0.052	2.86×10^{-3}
0.80	0.0976	23.1	0.043	0.083	3.57×10^{-3}
1.0	0.122	25.8	0.039	0.104	4.06×10^{-3}
1.3	0.159	29.5	0.034	0.135	4.59×10^{-3}
1.5	0.183	31.6	0.032	0.156	4.99×10^{-3}
1.8	0.220	34.7	0.029	0.187	5.42×10^{-3}
2.0	0.244	36.5	0.027	0.207	5.59×10^{-3}

The results for $\eta_{\mathrm{i}1}$ and $r_{\mathrm{A,obs}}$ are plotted below as a function of enzyme loading.



As the enzyme loading is increased from 0.01 μmol g^{-1}, the effectiveness factor drops significantly. Although the reaction rate continues to rise with increasing enzyme loading, at loadings above about 0.5 μmol g^{-1}, less than 5% of the potential activity of the enzyme is being utilised. Further increases in enzyme loading therefore represent an effective waste of more than 95% of that enzyme.

12.7 Mass-transfer effects in plant cell culture

(a)

D_p = 1.5 mm = 1.5×10^{-3} m. The particle Reynolds number is evaluated using Eq. (12.48) with ρ_L = density of water = 10^3 kg m^{-3}, and μ_L = viscosity of water = 1 cP (p 133) = 10^{-3} kg m^{-1} s^{-1} (Table A.9, Appendix A):

$$Re_p = \frac{D_p\, u_{pL}\, \rho_L}{\mu_L} = \frac{1.5\times 10^{-3}\,\text{m}\,(0.83\times 10^{-2}\,\text{m s}^{-1})(10^3\,\text{kg m}^{-3})}{10^{-3}\,\text{kg m}^{-1}\,\text{s}^{-1}} = 12.5$$

As this value is within the range $10 < Re_p < 10^4$, the external mass-transfer coefficient can be determined using Eq. (12.57) for spherical particles in a packed bed. Converting the diffusivity units to m^2 s^{-1}:

$$\mathscr{D}_{Ae} = 9\times 10^{-6}\,\text{cm}^2\,\text{s}^{-1} = 9\times 10^{-6}\,\text{cm}^2\,\text{s}^{-1}\cdot\left|\frac{1\,\text{m}}{100\,\text{cm}}\right|^2 = 9\times 10^{-10}\,\text{m}^2\,\text{s}^{-1}$$

$\mathscr{D}_{AL} = 2 \times \mathscr{D}_{Ae} = 2 \times 9 \times 10^{-10}$ m^2 s^{-1} = 1.8×10^{-9} m^2 s^{-1}. The Schmidt number from Eq. (12.49) is:

$$Sc = \frac{\mu_L}{\rho_L\,\mathscr{D}_{AL}} = \frac{10^{-3}\,\text{kg m}^{-1}\,\text{s}^{-1}}{10^3\,\text{kg m}^{-3}\,(1.8\times 10^{-9}\,\text{m}^2\,\text{s}^{-1})} = 556$$

From Eq. (12.57), the Sherwood number can be evaluated as:

$$Sh = 0.95\,Re_p^{0.5}\,Sc^{0.33} = 0.95\,(12.5)^{0.5}\,(556)^{0.33} = 27.0$$

From the definition of the Sherwood number in Eq. (12.50):

$$k_S = \frac{Sh\,\mathscr{D}_{AL}}{D_p} = \frac{27.0\,(1.8\times 10^{-9}\,\text{m}^2\,\text{s}^{-1})}{1.5\times 10^{-3}\,\text{m}} = 3.24\times 10^{-5}\,\text{m s}^{-1}$$

This value of k_S can be used to determine the observable modulus for external mass-transfer Ω from the equation in Table 12.6 for spherical geometry. As the specific gravity of the wet cells is 1, from p 16, 1 g wet cells occupies a volume of 1 cm^3 and $r_{A,obs}$ = 0.28 mg cm^{-3} h^{-1}. Converting these units to kg s^{-1} m^{-3}:

$$r_{A,obs} = 0.28\,\text{mg cm}^{-3}\,\text{h}^{-1} = 0.28\,\text{mg cm}^{-3}\,\text{h}^{-1}\cdot\left|\frac{1\,\text{kg}}{10^6\,\text{mg}}\right|\cdot\left|\frac{1\,\text{h}}{3600\,\text{s}}\right|\cdot\left|\frac{100\,\text{cm}}{1\,\text{m}}\right|^3 = 7.78\times 10^{-5}\,\text{kg s}^{-1}\,\text{m}^{-3}$$

Substituting this and the other parameter values into the equation for Ω:

$$\Omega = \frac{R}{3}\frac{r_{A,obs}}{k_S\,C_{Ab}} = \frac{\left(\frac{1.5\times 10^{-3}\,\text{m}}{2}\right)}{3}\;\frac{7.78\times 10^{-5}\,\text{kg s}^{-1}\,\text{m}^{-3}}{3.24\times 10^{-5}\,\text{m s}^{-1}\,(8\,\text{mg l}^{-1})\cdot\left|\frac{1\,\text{kg}}{10^6\,\text{mg}}\right|\cdot\left|\frac{1000\,\text{l}}{1\,\text{m}^3}\right|} = 0.075$$

From Eqs (12.43) and (12.44):

$$C_{As} = C_{Ab}\,(1-\Omega) = C_{Ab}\,(1-0.075) = 0.925\,C_{Ab}$$

As C_{As} is close to C_{Ab}, external mass-transfer effects are present but small.

Answer: The effect is small; the surface oxygen concentration is 7.5% lower than in the bulk medium.

(b)

Evaluating the observable Thiele modulus Φ from the equation in Table 12.4 for spherical geometry using the result for C_{As} from (a):

$$\Phi = \left(\frac{R}{3}\right)^2 \frac{r_{\mathrm{A,obs}}}{\mathcal{D}_{\mathrm{Ae}}\, C_{\mathrm{As}}} = \left(\frac{\frac{1.5\times 10^{-3}\,\mathrm{m}}{2}}{3}\right)^2 \frac{7.78\times 10^{-5}\,\mathrm{kg\,s^{-1}\,m^{-3}}}{9\times 10^{-10}\,\mathrm{m^2\,s^{-1}}\left(0.925\times 8\,\mathrm{mg\,l^{-1}}\right).\left|\frac{1\,\mathrm{kg}}{10^6\,\mathrm{mg}}\right|.\left|\frac{1000\,\mathrm{l}}{1\,\mathrm{m^3}}\right|} = 0.73$$

From Figure 12.11 for zero-order reaction, at $\Phi = 0.73$, $\eta_{\mathrm{i}0}$ is very close to but slightly less than 1.0. Therefore, internal mass-transfer effects can be considered negligible.

Answer: Negligible

(c)
$C_{\mathrm{As}} = 0.925 \times 8$ mg $\mathrm{l^{-1}}$ = 7.4 mg $\mathrm{l^{-1}}$. As $\eta_{\mathrm{i}0}$ is only very slightly less than 1.0, it is likely that oxygen is exhausted just close to the centre of the clumps. Taking $r_{\mathrm{A,obs}}$ to be essentially equal to the intrinsic zero-order rate constant k_0, the maximum particle radius for the oxygen concentration to remain greater than zero throughout the clump can be evaluated using Eq. (12.17):

$$R_{\max} = \sqrt{\frac{6\,\mathcal{D}_{\mathrm{Ae}}\,C_{\mathrm{As}}}{k_0}} = \sqrt{\frac{6\left(9\times 10^{-10}\,\mathrm{m^2\,s^{-1}}\right)7.4\,\mathrm{mg\,l^{-1}}.\left|\frac{1\,\mathrm{kg}}{10^6\,\mathrm{mg}}\right|.\left|\frac{1000\,\mathrm{l}}{1\,\mathrm{m^3}}\right|}{7.78\times 10^{-5}\,\mathrm{kg\,s^{-1}\,m^{-3}}}}$$

$$R_{\max} = 7.2\times 10^{-4}\,\mathrm{m} = 0.72\,\mathrm{mm}$$

Therefore, the maximum particle diameter for oxygen through to the centre of the clump is 2 × 0.72 = 1.4 mm, which is only slightly less than the plant cell clump diameter of 1.5 mm.

Answer: The oxygen concentration falls from 7.4 mg $\mathrm{l^{-1}}$ at the external surface to zero just near the centre of the clumps.

12.8 Respiration in mycelial pellets

(a)
R = 2.5 mm = 2.5×10^{-3} m. The presence of external boundary-layers can be checked by calculating the observable modulus for external mass-transfer, Ω. From Table 12.6 for spherical geometry:

$$\Omega = \frac{R}{3}\frac{r_{\mathrm{A,obs}}}{k_{\mathrm{S}}\,C_{\mathrm{Ab}}} = \frac{2.5\times 10^{-3}\,\mathrm{m}}{3}\,\frac{8.7\times 10^{-5}\,\mathrm{kg\,s^{-1}\,m^{-3}}}{3.8\times 10^{-5}\,\mathrm{m\,s^{-1}}\left(8\times 10^{-3}\,\mathrm{kg\,m^{-3}}\right)} = 0.24$$

From Eqs (12.43) and (12.44):

$$C_{\mathrm{As}} = C_{\mathrm{Ab}}(1-\Omega) = C_{\mathrm{Ab}}(1-0.24) = 0.76\,C_{\mathrm{Ab}} = 0.76\,(8\times 10^{-3}\,\mathrm{kg\,m^{-3}}) = 6.1\times 10^{-3}\,\mathrm{kg\,m^{-3}}$$

As C_{As} is significantly less than C_{Ab}, external mass-transfer effects are present.

Answer: Yes

(b)
As oxygen uptake is considered a zero-order reaction, for $C_{\mathrm{As}} > 0$, $\eta_{\mathrm{e}0} = 1$.

Answer: 1

(c)
In the absence of internal and external mass-transfer resistances, the reaction rate is r^*_{Ab} corresponding to $C_{\mathrm{A}} = C_{\mathrm{Ab}}$ throughout the pellets. As r^*_{Ab} is related to $r_{\mathrm{A,obs}}$ by Eq. (12.45), r^*_{Ab} can be determined if we know $\eta_{\mathrm{T}0}$. Evaluating the observable Thiele modulus Φ using the equation in Table 12.4 for spherical geometry and the result for C_{As} from (a):

$$\Phi = \left(\frac{R}{3}\right)^2 \frac{r_{\mathrm{A,obs}}}{\mathcal{D}_{\mathrm{Ae}}\,C_{\mathrm{As}}} = \left(\frac{2.5\times 10^{-3}\,\mathrm{m}}{3}\right)^2 \frac{8.7\times 10^{-5}\,\mathrm{kg\,s^{-1}\,m^{-3}}}{1.75\times 10^{-9}\,\mathrm{m^2\,s^{-1}}\left(6.1\times 10^{-3}\,\mathrm{kg\,m^{-3}}\right)} = 5.66$$

From Figure 12.11 for zero-order reaction, at $\Phi = 5.66$, $\eta_{i0} = 0.30$. Therefore, from Eq. (12.46), as $\eta_{e0} = 1$ from **(b)**, $\eta_{T0} = \eta_{i0}\,\eta_{e0} = 0.30 \times 1 = 0.30$. Using Eq. (12.45):

$$\overset{*}{r}_{Ab} = \frac{r_{A,obs}}{\eta_{T0}} = \frac{8.7\times10^{-5}\ \mathrm{kg\,s^{-1}\,m^{-3}}}{0.30} = 2.9\times10^{-4}\ \mathrm{kg\,s^{-1}\,m^{-3}}$$

Answer: 2.9×10^{-4} kg s^{-1} m^{-3}, or more than three times the rate actually observed

(d)

If external mass-transfer effects were eliminated, $C_{As} = C_{Ab} = 8 \times 10^{-3}$ kg m^{-3}, and the observed reaction rate would be greater than 8.7×10^{-5} kg s^{-1} m^{-3}. Under these conditions, an expression for the observable Thiele modulus Φ from the equation in Table 12.4 for spherical geometry is:

$$\Phi = \left(\frac{R}{3}\right)^2 \frac{r_{A,obs}}{\mathscr{D}_{Ae}\,C_{As}} = \left(\frac{2.5\times10^{-3}\ \mathrm{m}}{3}\right)^2 \frac{r_{A,obs}}{1.75\times10^{-9}\ \mathrm{m^2\,s^{-1}}\left(8\times10^{-3}\ \mathrm{kg\,m^{-3}}\right)} = 4.96\times10^4\, r_{A,obs}$$

where $r_{A,obs}$ has units of kg s^{-1} m^{-3}. Because the reaction is zero-order, $\overset{*}{r}_{As} = \overset{*}{r}_{Ab}$; therefore, from the result in **(c)**, $\overset{*}{r}_{As} = 2.9 \times 10^{-4}$ kg s^{-1} m^{-3}. Using Eq. (12.26):

$$\eta_{i0} = \frac{r_{A,obs}}{\overset{*}{r}_{As}} = \frac{r_{A,obs}}{2.9\times10^{-4}\ \mathrm{kg\,s^{-1}\,m^{-3}}}$$

Φ and η_{i0} are related by the curve in Figure 12.11 for spheres and zero-order reaction. The value of $r_{A,obs}$ can be determined by trial-and-error using Figure 12.11 and the equations derived above. As a first guess, take $r_{A,obs} = 2.0 \times 10^{-4}$ kg s^{-1} m^{-3}. Depending on the difference between the values of η_{i0} obtained from the figure and from the equation with $\overset{*}{r}_{As}$, adjust $r_{A,obs}$ as shown in the table below.

$r_{A,obs}$ (kg s^{-1} m^{-3})	$\eta_{i0} = \dfrac{r_{A,obs}}{\overset{*}{r}_{As}}$	Φ	η_{i0} (from Figure 12.11)
2.0×10^{-4}	0.69	9.92	0.19
1.0×10^{-4}	0.34	4.96	0.34

Since the values for η_{i0} in the last row are as close as practical, $r_{A,obs} = 1.0 \times 10^{-4}$ kg s^{-1} m^{-3}. Therefore, compared with the observed reaction rate of 8.7×10^{-5} kg s^{-1} m^{-3} in the presence of both internal and external mass-transfer resistances, eliminating the external boundary layers increases the reaction rate by about 15%.

Answer: 1.0×10^{-4} kg s^{-1} m^{-3}

13

Reactor Engineering

13.1 Economics of batch enzyme conversion

For 75% conversion, $s_f = 0.25\ s_0$. The batch reaction time for enzyme processes is evaluated using Eq. (13.10):

$$t_b = \frac{K_m}{v_{max}} \ln\frac{s_0}{s_f} + \frac{s_0 - s_f}{v_{max}} = \frac{1.5\ \text{g l}^{-1}}{0.9\ \text{g l}^{-1}\ \text{h}^{-1}} \ln\frac{3\ \text{g l}^{-1}}{0.25\times 3\ \text{g l}^{-1}} + \frac{3\ \text{g l}^{-1} - 0.25\times 3\ \text{g l}^{-1}}{0.9\ \text{g l}^{-1}\ \text{h}^{-1}} = 4.81\ \text{h}$$

The operating cost is therefore:

$$\text{Operating cost} = 4.81\ \text{h}\ .\left|\frac{1\ \text{day}}{24\ \text{h}}\right|.\ \$4800\ \text{day}^{-1} = \$962$$

The cost of downstream processing per kg product is:

$$C = 155 - 0.33\ X = 155 - 0.33\ (75) = \$130.25\ \text{kg}^{-1}$$

The mass of product formed is determined from the mass of substrate consumed, which is equal to the change in substrate concentration multiplied by the volume of the reactor V:

$$\text{Mass of substrate consumed} = (s_0 - s_f)\ V = (3 - 0.25\times 3)\ \text{g l}^{-1}\ (1600\ \text{l}) = 3600\ \text{g}$$

As 1.2 g product are formed per g substrate consumed:

$$\text{Mass of product formed} = 1.2\times 3600\ \text{g} = 4320\ \text{g} = 4.32\ \text{kg}$$

Therefore:

$$\text{Downstream processing cost} = \$130.25\ \text{kg}^{-1}\ (4.32\ \text{kg}) = \$563$$

The revenue from sale of the product is:

$$\text{Revenue} = \$750\ \text{kg}^{-1}\ (4.32\ \text{kg}) = \$3240$$

Therefore the cost benefit at 75% substrate conversion is:

$$\text{Cost benefit} = \text{revenue} - \text{operating cost} - \text{downstream processing cost} = \$3240 - \$962 - \$563 = \$1715$$

Carrying out the calculations for 90% conversion, the batch reaction time for $s_f = 0.10\ s_0$ is:

$$t_b = \frac{K_m}{v_{max}} \ln\frac{s_0}{s_f} + \frac{s_0 - s_f}{v_{max}} = \frac{1.5\ \text{g l}^{-1}}{0.9\ \text{g l}^{-1}\ \text{h}^{-1}} \ln\frac{3\ \text{g l}^{-1}}{0.10\times 3\ \text{g l}^{-1}} + \frac{3\ \text{g l}^{-1} - 0.10\times 3\ \text{g l}^{-1}}{0.9\ \text{g l}^{-1}\ \text{h}^{-1}} = 6.84\ \text{h}$$

At 90% conversion, the operating cost is increased due to the longer reaction time:

$$\text{Operating cost} = 6.84\ \text{h}\ .\left|\frac{1\ \text{day}}{24\ \text{h}}\right|.\ \$4800\ \text{day}^{-1} = \$1368$$

The cost of downstream processing per kg product is:

$$C = 155 - 0.33\ X = 155 - 0.33\ (90) = \$125.30\ \text{kg}^{-1}$$

The mass of substrate consumed is:

$$\text{Mass of substrate consumed} = (s_0 - s_f)\,V = (3 - 0.10 \times 3)\ \text{g l}^{-1}\ (1600\ \text{l}) = 4320\ \text{g}$$

and the mass of product formed is:

$$\text{Mass of product formed} = 1.2 \times 4320\ \text{g} = 5184\ \text{g} = 5.18\ \text{kg}$$

Therefore, the downstream processing cost is:

$$\text{Downstream processing cost} = \$125.30\ \text{kg}^{-1}\ (5.18\ \text{kg}) = \$649$$

The sales revenue is:

$$\text{Revenue} = \$750\ \text{kg}^{-1}\ (5.18\ \text{kg}) = \$3885$$

Therefore, the cost benefit at 90% substrate conversion is:

$$\text{Cost benefit} = \text{revenue} - \text{operating cost} - \text{downstream processing cost} = \$3885 - \$1368 - \$649 = \$1868$$

The gain per batch from increasing the conversion from 75% to 90% is therefore \$1868 − \$1715 = \$153.

Answer: There is a gain of \$153 per batch, representing a 9% increase in the cost benefit at 75% conversion.

13.2 Batch production of aspartic acid using cell-bound enzyme

(a)

The initial concentration of substrate $s_0 = 15\%$ (w/v) = 15 g per 100 ml = 150 g l^{-1}. The final substrate concentration $s_f = 0.15\ s_0 = 0.15 \times 150\ \text{g l}^{-1} = 22.5\ \text{g l}^{-1}$. Calculating the deactivation rate constant at 32°C using Eq. (11.45):

$$k_d = \frac{\ln 2}{t_h} = \frac{\ln 2}{10.5\ \text{d}} \cdot \left|\frac{1\ \text{d}}{24\ \text{h}}\right| = 2.75 \times 10^{-3}\ \text{h}^{-1}$$

For enzyme subject to deactivation, the batch reaction time is evaluated using Eq. (13.13):

$$t_b = \frac{-1}{k_d} \ln\left[1 - k_d\left(\frac{K_m}{v_{\max 0}} \ln\frac{s_0}{s_f} + \frac{s_0 - s_f}{v_{\max 0}}\right)\right]$$

$$= \frac{-1}{2.75 \times 10^{-3}\ \text{h}^{-1}} \ln\left[1 - 2.75 \times 10^{-3}\ \text{h}^{-1}\left(\frac{4.0\ \text{g l}^{-1}}{5.9\ \text{g l}^{-1}\ \text{h}^{-1}} \ln\frac{150\ \text{g l}^{-1}}{22.5\ \text{g l}^{-1}} + \frac{(150 - 22.5)\ \text{g l}^{-1}}{5.9\ \text{g l}^{-1}\ \text{h}^{-1}}\right)\right] = 23.6\ \text{h}$$

At 37°C, the deactivation rate constant is:

$$k_d = \frac{\ln 2}{t_h} = \frac{\ln 2}{2.3\ \text{d}} \cdot \left|\frac{1\ \text{d}}{24\ \text{h}}\right| = 1.26 \times 10^{-2}\ \text{h}^{-1}$$

The batch reaction time is:

$$t_b = \frac{-1}{k_d} \ln\left[1 - k_d\left(\frac{K_m}{v_{\max 0}} \ln\frac{s_0}{s_f} + \frac{s_0 - s_f}{v_{\max 0}}\right)\right]$$

$$= \frac{-1}{1.26 \times 10^{-2}\ \text{h}^{-1}} \ln\left[1 - 1.26 \times 10^{-2}\ \text{h}^{-1}\left(\frac{4.0\ \text{g l}^{-1}}{8.5\ \text{g l}^{-1}\ \text{h}^{-1}} \ln\frac{150\ \text{g l}^{-1}}{22.5\ \text{g l}^{-1}} + \frac{(150 - 22.5)\ \text{g l}^{-1}}{8.5\ \text{g l}^{-1}\ \text{h}^{-1}}\right)\right] = 17.7\ \text{h}$$

As the batch reaction time is lower at 37°C, 37°C is the recommended operating temperature.

Answer: 37°C

(b)

From Eq. (13.33), the total batch reaction time at 37°C is:

$$t_T = t_b + t_{dn} = 17.7\text{ h} + 28\text{ h} = 45.7\text{ h}$$

Therefore, in one year, the number of batches carried out is:

$$\text{Number of batches} = \frac{365\text{ d}\cdot\left|\frac{24\text{ h}}{1\text{ d}}\right|}{45.7\text{ h per batch}} = 192$$

In each batch, the mass of ammonium fumarate converted is 0.85 × 150 g l^{-1} = 127.5 g l^{-1} multiplied by the reactor volume V. Therefore, the mass of substrates converted is 127.5 V g = 0.1275 V kg, where V has units of litres. From the reaction stoichiometry, as the molecular weights of ammonium fumarate and aspartic acid are approximately equal, the mass of aspartic acid produced is also 0.1275 V kg. After one year or 192 batches, the mass of aspartic acid produced is 0.1275 V × 192 = 24.5 V kg. Using the conversion factor 1 tonne = 10^3 kg (Table A.3, Appendix A), the target level of aspartic acid production each year is 5000 × 10^3 = 5 × 10^6 kg. To reach this target level:

$$24.5\,V\text{ kg} = 5\times10^6\text{ kg}$$

$$V = 2.04\times10^5\text{ l} = 204\text{ m}^3$$

Answer: 204 m^3

13.3 Prediction of batch culture time

(a)

The initial cell concentration $x_0 = {}^{12\text{ g}}/_{100\text{ l}} = 0.12$ g l^{-1}. Assume that stationary phase is reached when $s_f = 0$. The batch culture time can be determined using Eq. (13.27):

$$t_b = \frac{1}{\mu_{max}}\ln\left[1+\frac{Y_{XS}}{x_0}(s_0-s_f)\right] = \frac{1}{0.9\text{ h}^{-1}}\ln\left[1+\frac{0.575\text{ g g}^{-1}}{0.12\text{ g l}^{-1}}\left(10\text{ g l}^{-1}-0\right)\right] = 4.3\text{ h}$$

Answer: 4.3 h

(b)

If only 70% of the substrate is consumed, $s_f = 0.3\,s_0 = 0.3\times10$ g l^{-1} = 3 g l^{-1}. From Eq. (13.27):

$$t_b = \frac{1}{\mu_{max}}\ln\left[1+\frac{Y_{XS}}{x_0}(s_0-s_f)\right] = \frac{1}{0.9\text{ h}^{-1}}\ln\left[1+\frac{0.575\text{ g g}^{-1}}{0.12\text{ g l}^{-1}}\left(10\text{ g l}^{-1}-3\text{ g l}^{-1}\right)\right] = 3.9\text{ h}$$

The biomass density at this time can be calculated from Eq. (13.19):

$$x_f = x_0\,e^{\mu_{max}t_b} = 0.12\text{ g l}^{-1}\,e^{(0.9\text{ h}^{-1}\times3.9\text{ h})} = 4.0\text{ g l}^{-1}$$

Answer: 4.0 g l^{-1}

13.4 Fed-batch scheduling

(a)

The initial substrate concentration s_0 = 3% (w/v) = 3 g per 100 ml = 30 g l^{-1}. The batch culture time to achieve $s_f = 0$ is determined using Eq. (13.27):

$$t_b = \frac{1}{\mu_{max}}\ln\left[1+\frac{Y_{XS}}{x_0}(s_0-s_f)\right] = \frac{1}{0.18\text{ d}^{-1}}\ln\left[1+\frac{0.5\text{ g g}^{-1}}{1.5\text{ g l}^{-1}}\left(30\text{ g l}^{-1}-0\right)\right] = 13.3\text{ d}$$

The biomass density at this time can be calculated using Eq. (13.19):

$$x_f = x_0\,e^{\mu_{max}t_b} = 1.5\text{ g l}^{-1}\,e^{(0.18\text{ d}^{-1}\times13.3\text{ d})} = 16.4\text{ g l}^{-1}$$

Answer: The batch culture time is 13.3 days; the final biomass concentration is 16.4 g l^{-1}.

(b)
The mass of cells at the start of fed-batch operation is equal to the final batch cell concentration multiplied by the initial medium volume:

$$X_0 = x_f V = 16.4 \text{ g l}^{-1} (100 \text{ l}) = 1640 \text{ g}$$

The final mass of cells after 40 d fed-batch culture can be determined using Eq. (13.50):

$$X = X_0 + (Y_{XS}\, s_i\, F)\, t_{fb} = 1640 \text{ g} + 0.5 \text{ g g}^{-1} (30 \text{ g l}^{-1}) (4 \text{ l d}^{-1})\, 40 \text{ d} = 4040 \text{ g} = 4.04 \text{ kg}$$

Answer: 4.04 kg

(c)
The mass of cells produced in each reactor run is equal to the final biomass minus the biomass used for inoculation:

$$\text{Biomass produced per run} = 4040 \text{ g} - 1.5 \text{ g l}^{-1} (100 \text{ l}) = 3890 \text{ g} = 3.89 \text{ kg}$$

By analogy with Eq. (13.33), the total reaction time is:

$$t_T = t_b + t_{fb} + t_{dn}$$

where t_b is the batch reaction time and t_{fb} is the fed-batch operation time. Substituting parameter values using the result for t_b from (**a**):

$$t_T = t_b + t_{fb} + t_{dn} = 13.3 \text{ d} + 40 \text{ d} + 1 \text{ d} = 54.3 \text{ d}$$

In one year, the number of runs carried out is:

$$\text{Number of runs} = \frac{275 \text{ d}}{54.3 \text{ d per run}} = 5.06$$

The total biomass produced annually is equal to the biomass produced per run multiplied by the number of runs per year:

$$\text{Biomass produced per year} = 3.89 \text{ kg} \times 5.06 = 19.7 \text{ kg}$$

Answer: 19.7 kg

13.5 Fed-batch production of cheese starter culture

(a)
An expression for the liquid volume as a function of time during fed-batch reactor operation can be derived from an unsteady-state total mass balance as shown in the solution to Problem 6.7a from Chapter 6. Using this expression:

$$V_0 = V - F\,t = 40 \text{ m}^3 - 4 \text{ m}^3 \text{ h}^{-1} (6 \text{ h}) = 16 \text{ m}^3$$

Answer: 16 m^3

(b)
From the definition of the dilution rate in Eq. (13.39), after 6 h of fed-batch operation when $V = 40$ m^3:

$$D = \frac{F}{V} = \frac{4 \text{ m}^3 \text{ h}^{-1}}{40 \text{ m}^3} = 0.10 \text{ h}^{-1}$$

Substituting this value into Eq. (13.45) for the substrate concentration at quasi-steady state:

$$s = \frac{D\,K_S}{\mu_{max} - D} = \frac{0.10 \text{ h}^{-1} \left(0.15 \text{ kg m}^{-3}\right)}{0.35 \text{ h}^{-1} - 0.10 \text{ h}^{-1}} = 0.06 \text{ kg m}^{-3}$$

Answer: 0.06 kg m^{-3}

(c)
Taking maintenance substrate requirements into account, for $q_P = 0$, Eq. (13.43) becomes:

$$\frac{ds}{dt} = D\,(s_i - s) - \left(\frac{\mu}{Y_{XS}} + m_S\right)x$$

At quasi-steady state, ${}^{ds}/_{dt} \approx 0$, $\mu \approx D$, and $s << s_i$. Therefore, the equation reduces to:

$$0 = D\,s_i - \left(\frac{D}{Y_{XS}} + m_S\right)x$$

Solving for x:

$$x = \frac{D\,s_i}{\dfrac{D}{Y_{XS}} + m_S} = \frac{0.10\,\text{h}^{-1}\left(80\,\text{kg m}^{-3}\right)}{\dfrac{0.10\,\text{h}^{-1}}{0.23\,\text{kg kg}^{-1}} + 0.135\,\text{kg kg}^{-1}\,\text{h}^{-1}} = 14.0\,\text{kg m}^{-3}$$

Answer: 14.0 kg m^{-3}

(d)
After 6 h fed-batch operation, the mass of cells is:

$$X = x\,V = 14.0\,\text{kg m}^{-3}\,(40\,\text{m}^3) = 560\,\text{kg}$$

At the start of fed-batch operation when the liquid volume is 16 m^3, if operation is at quasi-steady state, the cell concentration $\approx$ 14.0 kg m^{-3} and:

$$X = x\,V = 14.0\,\text{kg m}^{-3}\,(16\,\text{m}^3) = 224\,\text{kg}$$

Therefore, the mass of cells produced during fed-batch operation is (560 kg − 224 kg) = 336 kg.

Answer: 336 kg

13.6 Continuous enzyme conversion in a fixed-bed reactor

Convert the parameter values to units of kg, m, s. $K_m = 0.54$ g l^{-1} = 0.54 kg m^{-3}. During the reactor operation, s = 0.02 g l^{-1} = 0.02 kg m^{-3}; s_i = 0.42 g l^{-1} = 0.42 kg m^{-3}. R = 1 mm = 10^{-3} m. The active enzyme concentration per unit volume of catalyst e_a is:

$$e_a = \frac{10^{-4}\,\text{g}}{250\,\text{cm}^3} = \frac{10^{-4}\,\text{g}}{250\,\text{cm}^3}\cdot\left|\frac{1\,\text{kg}}{1000\,\text{g}}\right|\cdot\left|\frac{100\,\text{cm}}{1\,\text{m}}\right|^3 = 4\times10^{-4}\,\text{kg m}^{-3}$$

The effective diffusivity of urea in the gel $\mathcal{D}_{Ae}$ is:

$$\mathcal{D}_{Ae} = 7\times10^{-6}\,\text{cm}^2\,\text{s}^{-1} = 7\times10^{-6}\,\text{cm}^2\,\text{s}^{-1}\cdot\left|\frac{1\,\text{m}}{100\,\text{cm}}\right|^2 = 7\times10^{-10}\,\text{m}^2\,\text{s}^{-1}$$

From Table B.1 (Appendix B), the molecular weight of urea is 60.1 and the molecular weight of NH_4^+ is 18.0. Therefore, from the stoichiometry, reaction of 60.1 g urea produces 2 × 18.0 = 36.0 g NH_4^+. Expressing the turnover number k_2 in terms of urea:

$$k_2 = 11{,}000\,\text{g NH}_4^+\,(\text{g enzyme})^{-1}\,\text{s}^{-1} = 11{,}000\,\text{g NH}_4^+\,(\text{g enzyme})^{-1}\,\text{s}^{-1}\cdot\left|\frac{60.1\,\text{g urea}}{36.0\,\text{g NH}_4^+}\right|$$

$$k_2 = 1.84\times10^4\,\text{g urea}\,(\text{g enzyme})^{-1}\,\text{s}^{-1} = 1.84\times10^4\,\text{kg kg}^{-1}\,\text{s}^{-1}$$

From Eq. (11.33), v_{max} expressed on a per volume gel basis is:

$$v_{max} = k_2\,e_a = 1.84\times10^4\,\text{kg kg}^{-1}\,\text{s}^{-1}\,(4\times10^{-4}\,\text{kg m}^{-3}) = 7.36\,\text{kg m}^{-3}\,\text{s}^{-1}$$

As there are 250 cm^3 gel per litre of liquid in the reactor, v_{max} expressed on a per volume liquid basis is:

$$v_{\text{max}} = 7.36\,\text{kg m}^{-3}\,\text{s}^{-1}\left(\frac{250\,\text{cm}^3}{1\,\text{l}}\right)\cdot\left|\frac{1\,\text{m}}{100\,\text{cm}}\right|^3\cdot\left|\frac{1000\,\text{l}}{1\,\text{m}^3}\right| = 1.84\,\text{kg m}^{-3}\,\text{s}^{-1}$$

The rate of reaction can be determined after evaluating the effectiveness factor in the absence of external boundary layers. From the definition of β on p 313 with $C_{\text{As}} = s$:

$$\beta = \frac{K_{\text{m}}}{s} = \frac{0.54\,\text{kg m}^{-3}}{0.02\,\text{kg m}^{-3}} = 27$$

From Figure 12.10, for this value of β the reaction kinetics can be considered first-order. Based on Eq. (11.36), the effective first-order rate constant k_1 is:

$$k_1 = \frac{v_{\text{max}}}{K_{\text{m}}} = \frac{1.84\,\text{kg m}^{-3}\,\text{s}^{-1}}{0.54\,\text{kg m}^{-3}} = 3.41\,\text{s}^{-1}$$

Calculating the Thiele modulus from the equation in Table 12.2 for first-order kinetics and spherical geometry:

$$\phi_1 = \frac{R}{3}\sqrt{\frac{k_1}{\mathcal{D}_{\text{Ae}}}} = \frac{10^{-3}\,\text{m}}{3}\sqrt{\frac{3.41\,\text{s}^{-1}}{7\times 10^{-10}\,\text{m}^2\,\text{s}^{-1}}} = 23.3$$

As $\phi_1 > 10$, from Eq. (12.30):

$$\eta_{\text{i1}} = \frac{1}{\phi_1} = \frac{1}{23.3} = 0.043$$

From Eq. (12.46), as $\eta_{\text{e}} = 1$, $\eta_{\text{T}} = 0.043$. The flow rate of urea solution into and out of the reactor can be determined by evaluating the dilution rate D in the mass-balance equation, Eq. (13.54):

$$D = \frac{\eta_{\text{T}}\,v_{\text{max}}\,s}{(K_{\text{m}} + s)\,(s_{\text{i}} - s)} = \frac{0.043\,(1.84\,\text{kg m}^{-3}\,\text{s}^{-1})\,0.02\,\text{kg m}^{-3}}{(0.54 + 0.02)\,\text{kg m}^{-3}\,(0.42 - 0.02)\,\text{kg m}^{-3}} = 7.06\times 10^{-3}\,\text{s}^{-1}$$

From the definition of the dilution rate, Eq. (13.39):

$$F = D\,V = 7.06\times 10^{-3}\,\text{s}^{-1}\,(1\,\text{l}) = 7.06\times 10^{-3}\,\text{l s}^{-1}$$

In 30 min, the volume of urea solution treated is:

$$\text{Volume treated} = 7.06\times 10^{-3}\,\text{l s}^{-1}\,(30\,\text{min})\cdot\left|\frac{60\,\text{s}}{1\,\text{min}}\right| = 12.7\,\text{l}$$

Answer: 12.7 litres

13.7 Batch and continuous biomass production

$s_0 = s_{\text{i}} = 4\%$ (w/v) = 4 g per 100 ml = 40 g l^{-1} = 40 kg m^{-3}. $s_{\text{f}} = s = 0.02 \times 40$ kg m^{-3} = 0.8 kg m^{-3}. For the batch reactor, $x_0 = 0.01\%$ (w/v) = 0.01 g per 100 ml = 0.1 g l^{-1} = 0.1 kg m^{-3}.

The batch culture time can be determined from Eq. (13.27):

$$t_{\text{b}} = \frac{1}{\mu_{\text{max}}}\ln\left[1 + \frac{Y_{\text{XS}}}{x_0}(s_0 - s_{\text{f}})\right] = \frac{1}{0.44\,\text{h}^{-1}}\ln\left[1 + \frac{0.41\,\text{g g}^{-1}}{0.1\,\text{kg m}^{-3}}(40 - 0.8)\,\text{kg m}^{-3}\right] = 11.6\,\text{h}$$

The biomass density at this time is obtained from Eq. (13.19):

$$x_{\text{f}} = x_0\,e^{\mu_{\text{max}}\,t_{\text{b}}} = 0.1\,\text{kg m}^{-3}\,e^{(0.44\,\text{h}^{-1}\times 11.6\,\text{h})} = 16.5\,\text{kg m}^{-3}$$

Calculating the mass of cells produced per batch:

$$X = (x_{\text{f}} - x_0)\,V = (16.5 - 0.1)\,\text{kg m}^{-3}\,(1000\,\text{m}^3) = 1.64\times 10^4\,\text{kg}$$

If the downtime between batches t_{dn} is 20 h, from Eq. (13.33):

$$t_T = t_b + t_{dn} = 11.6\ \text{h} + 20\ \text{h} = 31.6\ \text{h}$$

Therefore, in one year, the number of batches carried out is:

$$\text{Number of batches} = \frac{365\ \text{d} \cdot \left|\frac{24\ \text{h}}{1\ \text{d}}\right|}{31.6\ \text{h per batch}} = 277$$

The total annual biomass production from batch culture is therefore 1.64×10^4 kg × 277 = 4.54×10^6 kg.

For continuous reactor operation, the steady-state cell concentration is given by Eq. (13.62):

$$x = (s_i - s)\, Y_{XS} = (40 - 0.8)\ \text{kg m}^{-3}\ (0.41\ \text{g g}^{-1}) = 16.1\ \text{kg m}^{-3}$$

The dilution rate D corresponding to $s = 0.8$ kg m^{-3} can be determined using Eqs (13.57) and (11.60):

$$D = \mu = \frac{\mu_{max}\, s}{K_S + s} = \frac{0.44\ \text{h}^{-1}\left(0.8\ \text{kg m}^{-3}\right)}{0.7\ \text{mg l}^{-1} \cdot \left|\frac{1000\ \text{l}}{1\ \text{m}^3}\right| \cdot \left|\frac{1\ \text{kg}}{10^6\ \text{mg}}\right| + 0.8\ \text{kg m}^{-3}} = 0.44\ \text{h}^{-1}$$

From the definition of the dilution rate, Eq. (13.39):

$$F = D\, V = 0.44\ \text{h}^{-1}\ (1000\ \text{m}^3) = 440\ \text{m}^3\ \text{h}^{-1}$$

The rate of biomass production $F x$ = 440 m^3 h^{-1} × 16.1 kg m^{-3} = 7084 kg h^{-1}. The number of days per year available for continuous reactor operation is (365 − 25) = 340 d; this corresponds to 340 d × 24 h d^{-1} = 8160 h. Therefore, the total biomass produced per year is 7084 kg h^{-1} × 8160 h = 5.78×10^7 kg. This production level is $5.78 \times 10^7/4.54 \times 10^6$ = 12.7 times the amount produced using batch culture.

Answer: The annual biomass production using continuous operation is 5.78×10^7 kg, which is 12.7 times the production of 4.54×10^6 kg from batch culture.

13.8 Reactor design for immobilised enzymes

$s_0 = s_i$ = 10% (w/v) = 10 g per 100 ml = 100 g l^{-1} = 100 kg m^{-3}. $s_f = s = 0.01 \times 100$ kg m^{-3} = 1 kg m^{-3}. Based on the unsteady-state mass-balance equation derived in Example 6.1 in Chapter 6 for first-order reaction, the equation for the rate of change of substrate concentration in a batch reactor is:

$$\frac{d(V s)}{dt} = -k_1\, s\, V$$

where V is the reaction volume and k_1 is the reaction rate constant. As V can be considered constant in a batch reactor, this term can be taken outside of the differential and cancelled from both sides of the equation:

$$\frac{ds}{dt} = -k_1\, s$$

The differential equation contains only two variables, s and t. Separating variables and integrating:

$$\frac{ds}{s} = -k_1\, dt$$

$$\int \frac{ds}{s} = \int -k_1\, dt$$

Using integration rules (D.27) and (D.24) from Appendix D and combining the constants of integration:

$$\ln s = -k_1\, t + K$$

The initial condition is: at $t = 0$, $s = s_0$. From the equation, therefore, ln $s_0 = K$. Substituting this value of K into the equation gives:

$$\ln s = -k_1\, t + \ln s_0$$

$$\ln \frac{s}{s_0} = -k_1\, t$$

$$t = \frac{-\ln \dfrac{s}{s_0}}{k_1}$$

The batch culture time t_b is the time required for the substrate concentration to reach s_f:

$$t_b = \frac{-\ln \dfrac{s_f}{s_0}}{k_1} = \frac{-\ln \dfrac{1\ \mathrm{kg\ m^{-3}}}{100\ \mathrm{kg\ m^{-3}}}}{0.8 \times 10^{-4}\ \mathrm{s^{-1}} \cdot \left|\dfrac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right|} = 16.0\ \mathrm{h}$$

If the downtime between batches t_{dn} is 20 h, from Eq. (13.33):

$$t_T = t_b + t_{dn} = 16.0\ \mathrm{h} + 20\ \mathrm{h} = 36\ \mathrm{h}$$

Therefore, in one year, the number of batches carried out is:

$$\text{Number of batches} = \frac{365\ \mathrm{d} \cdot \left|\dfrac{24\ \mathrm{h}}{1\ \mathrm{d}}\right|}{36\ \mathrm{h\ per\ batch}} = 243$$

To treat 400 tonnes penicillin G annually, using the unit conversion factor 1 tonne = 10^3 kg (Table A.3, Appendix A):

$$\text{Mass of penicillin G treated per batch} = \frac{400\ \text{tonnes}}{\text{number of batches per year}} = \frac{400\ \text{tonnes} \cdot \left|\dfrac{10^3\ \mathrm{kg}}{1\ \text{tonne}}\right|}{243} = 1.65 \times 10^3\ \mathrm{kg}$$

As the concentration of penicillin G added to the reactor is 100 kg $\mathrm{m^{-3}}$:

$$\text{Reactor volume} = \frac{1.65 \times 10^3\ \mathrm{kg}}{100\ \mathrm{kg\ m^{-3}}} = 16.5\ \mathrm{m^3}$$

The batch reactor volume required is 16.5 $\mathrm{m^3}$.

For a CSTR operated under steady state conditions, $F_i = F_o = F$, V is constant, and ${}^{ds}/_{dt} = 0$. Therefore, the mass-balance equation for first-order reaction derived in Example 6.1 in Chapter 6 becomes:

$$0 = F s_i - F s - k_1\, s\, V$$

$$0 = {}^{F}/_{V}\, (s_i - s) - k_1\, s\, V$$

Solving for ${}^{F}/_{V}$:

$$^{F}/_{V} = \frac{k_1\, s}{s_i - s} = \frac{0.8 \times 10^{-4}\ \mathrm{s^{-1}} \left(1\ \mathrm{kg\ m^{-3}}\right)}{(100 - 1)\ \mathrm{kg\ m^{-3}}} = 8.08 \times 10^{-7}\ \mathrm{s^{-1}}$$

The flow rate of penicillin G into the CSTR is 400 tonnes per year. Using the unit conversion factor 1 tonne = 10^3 kg (Table A.3, Appendix A) and the concentration of substrate in the feed stream s_i = 100 kg $\mathrm{m^{-3}}$, the total volumetric flow rate of the feed stream F is:

$$F = \frac{400\ \text{tonnes year}^{-1} \cdot \left|\dfrac{10^3\ \mathrm{kg}}{1\ \text{tonne}}\right| \cdot \left|\dfrac{1\ \text{year}}{365\ \mathrm{d}}\right| \cdot \left|\dfrac{1\ \mathrm{d}}{24\ \mathrm{h}}\right| \cdot \left|\dfrac{1\ \mathrm{h}}{3600\ \mathrm{s}}\right|}{100\ \mathrm{kg\ m^{-3}}} = 1.27 \times 10^{-4}\ \mathrm{m^3\ s^{-1}}$$

Applying this with the above result for ${}^{F}/_{V}$:

$$V = \frac{F}{F/V} = \frac{1.27\times10^{-4}\ \mathrm{m^3\,s^{-1}}}{8.08\times10^{-7}\ \mathrm{s^{-1}}} = 157\ \mathrm{m^3}$$

The CSTR reactor volume required is 157 $\mathrm{m^3}$.

For the PFTR, if the density of enzyme beads is four times greater than in the other reactors, $k_1 = 4 \times 0.8 \times 10^{-4}\ \mathrm{s^{-1}} = 3.2 \times 10^{-4}\ \mathrm{s^{-1}}$. By analogy with Eq. (13.83), the differential equation for change in substrate concentration with position in the reactor for first-order kinetics is:

$$u\frac{\mathrm{d}s}{\mathrm{d}z} = -k_1 s$$

The differential equation contains only two variables, s and z. Separating variables and integrating:

$$\frac{\mathrm{d}s}{s} = \frac{-k_1}{u}\,\mathrm{d}z$$

$$\int\frac{\mathrm{d}s}{s} = \int\frac{-k_1}{u}\,\mathrm{d}z$$

Using integration rules (D.27) and (D.24) from Appendix D and combining the constants of integration:

$$\ln s = \frac{-k_1}{u}z + K$$

The boundary condition is: at $z = 0$, $s = s_\mathrm{i}$. From the equation, therefore, $\ln s_\mathrm{i} = K$. Substituting this value of K into the equation gives:

$$\ln s = \frac{-k_1}{u}z + \ln s_\mathrm{i}$$

$$\ln\frac{s}{s_\mathrm{i}} = \frac{-k_1}{u}z$$

At the end of the PFTR, $z = L$ and $s = s_\mathrm{f}$; therefore:

$$\ln\frac{s_\mathrm{f}}{s_\mathrm{i}} = \frac{-k_1}{u}L$$

Applying the definition of the reactor residence time τ from Eq. (13.85):

$$\ln\frac{s_\mathrm{f}}{s_\mathrm{i}} = -k_1\tau$$

Rearranging and solving for τ:

$$\tau = \frac{-\ln\frac{s_\mathrm{f}}{s_\mathrm{i}}}{k_1} = \frac{-\ln\dfrac{1\ \mathrm{kg\,m^{-3}}}{100\ \mathrm{kg\,m^{-3}}}}{3.2\times10^{-4}\ \mathrm{s^{-1}}\cdot\left|\dfrac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right|} = 4.0\ \mathrm{h}$$

Note that this is $^1/_4$ the value obtained for the batch reaction time t_b, as expected from the analogous kinetic characteristics of batch and PFTR reactors and the 4 × higher value of k_1 in the PFTR.

As calculated for the CSTR, $F = 1.27 \times 10^{-4}\ \mathrm{m^3\,s^{-1}}$. Therefore, from the definition of τ in Eq. (13.51):

$$V = \tau F = 4.0\ \mathrm{h}\cdot\left|\frac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right|1.27\times10^{-4}\ \mathrm{m^3\,s^{-1}} = 1.83\ \mathrm{m^3}$$

The PFTR reactor volume required is 1.83 $\mathrm{m^3}$.

Answer: The smallest reactor volume is 1.83 $\mathrm{m^3}$ for a PFTR.

13.9 Two-stage chemostat for secondary metabolite production

(a)

s_i = 10 g l^{-1} = 10 kg m^{-3}. The dilution rate, which is the same for both reactors, is calculated using Eq. (13.39):

$$D = \frac{F}{V} = \frac{50\,\mathrm{l\,h^{-1}}\cdot\left|\frac{1\,\mathrm{m^3}}{1000\,\mathrm{l}}\right|}{0.5\,\mathrm{m^3}} = 0.10\,\mathrm{h^{-1}}$$

The cell and substrate concentrations entering the second reactor are the same as those leaving the first reactor. The substrate concentration can be determined using Eq. (13.58):

$$s = \frac{D\,K_S}{\mu_{max} - D} = \frac{0.10\,\mathrm{h^{-1}}\left(1.0\,\mathrm{kg\,m^{-3}}\right)}{0.12\,\mathrm{h^{-1}} - 0.10\,\mathrm{h^{-1}}} = 5.0\,\mathrm{kg\,m^{-3}}$$

When maintenance requirements are significant, the cell concentration is calculated using Eq. (13.61):

$$x = \frac{D\,(s_i - s)}{\frac{D}{Y_{XS}} + m_S} = \frac{0.10\,\mathrm{h^{-1}}\,(10 - 5.0)\,\mathrm{kg\,m^{-3}}}{\frac{0.10\,\mathrm{h^{-1}}}{0.5\,\mathrm{kg\,kg^{-1}}} + 0.025\,\mathrm{kg\,kg^{-1}\,h^{-1}}} = 2.2\,\mathrm{kg\,m^{-3}}$$

Answer: The cell concentration is 2.2 kg m^{-3}; the substrate concentration is 5.0 kg m^{-3}.

(b)

As growth is negligible in the second reactor, $x = x_i$ = 2.2 kg m^{-3}. The substrate concentration is determined by rearranging Eq. (13.59) and solving for s with $\mu = 0$:

$$s = s_i - \left(\frac{q_P}{Y_{PS}} + m_S\right) x \frac{V}{F}$$

Substituting the parameter values with s_i = 5.0 kg m^{-3}:

$$s = 5.0\,\mathrm{kg\,m^{-3}} - \left(\frac{0.16\,\mathrm{kg\,kg^{-1}\,h^{-1}}}{0.85\,\mathrm{kg\,kg^{-1}}} + 0.025\,\mathrm{kg\,kg^{-1}\,h^{-1}}\right) 2.2\,\mathrm{kg\,m^{-3}} \left(\frac{0.5\,\mathrm{m^3}}{50\,\mathrm{l\,h^{-1}}\cdot\left|\frac{1\,\mathrm{m^3}}{1000\,\mathrm{l}}\right|}\right) = 0.31\,\mathrm{kg\,m^{-3}}$$

For the two reactors together:

$$\text{Overall substrate conversion} = \frac{(s_i - s)}{s_i} \times 100\% = \frac{(10 - 0.31)\,\mathrm{kg\,m^{-3}}}{10\,\mathrm{kg\,m^{-3}}} \times 100\% = 97\%$$

Answer: 97%

(c)

As product is not formed in the first reactor, p_i = 0 for the second reactor. The product concentration is determined by rearranging Eq. (13.64) and solving for p:

$$p = \frac{q_P\,x\,V}{F} = \frac{0.16\,\mathrm{kg\,kg^{-1}\,h^{-1}}\left(2.2\,\mathrm{kg\,m^{-3}}\right)0.5\,\mathrm{m^3}}{50\,\mathrm{l\,h^{-1}}\cdot\left|\frac{1\,\mathrm{m^3}}{1000\,\mathrm{l}}\right|} = 3.5\,\mathrm{kg\,m^{-3}}$$

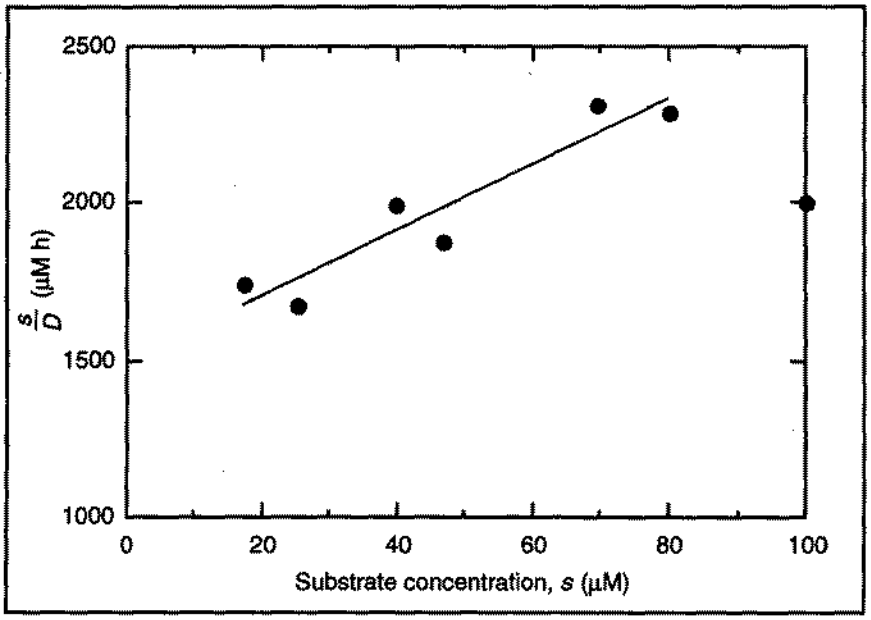
Answer: 3.5 kg m^{-3}

13.10 Kinetic analysis of bioremediating bacteria using a chemostat

(a)

From Eq. (13.92), μ_{max} and K_S can be determined from the slope and intercept of a plot of $^s/_D$ versus s. From the definition of the dilution rate in Eq. (13.39), values of D are evaluated from the experimental flow rates using V = 1 l = 1000 ml. The measured substrate concentration at 50 ml h^{-1} indicates that washout occurs at this flow rate; therefore, this result is not included in the kinetic analysis. The data are listed and plotted below.

Flow rate, F (ml h^{-1})	Dilution rate, D (h^{-1})	Substrate concentration, s (μM)	$^{s}/_{D}$ (μM h)
10	0.010	17.4	1740
15	0.015	25.1	1673
20	0.020	39.8	1990
25	0.025	46.8	1872
30	0.030	69.4	2313
35	0.035	80.1	2289
50	0.050	100	2000



The slope of the straight line in the plot is 10.48 h; the intercept is 1493 μM h. From Eq. (13.92), the slope = $^{1}/_{\mu_{\max}}$; therefore, $\mu_{\max} = {}^{1}/_{10.48\ \mathrm{h}} = 0.095\ \mathrm{h}^{-1}$. The intercept = $^{K_S}/_{\mu_{\max}}$; therefore $K_S = 1493\ \mu\mathrm{M\ h} \times 0.095\ \mathrm{h}^{-1} = 142\ \mu\mathrm{M}$.

Answer: $\mu_{\max} = 0.095\ \mathrm{h}^{-1}$; $K_S = 142\ \mu\mathrm{M}$

(b)
The critical dilution rate D_{crit} is determined using Eq. (13.66):

$$D_{\mathrm{crit}} = \frac{\mu_{\max} s_i}{K_S + s_i} = \frac{0.095\ \mathrm{h}^{-1}\ (100\ \mu\mathrm{M})}{142\ \mu\mathrm{M} + 100\ \mu\mathrm{M}} = 0.039\ \mathrm{h}^{-1}$$

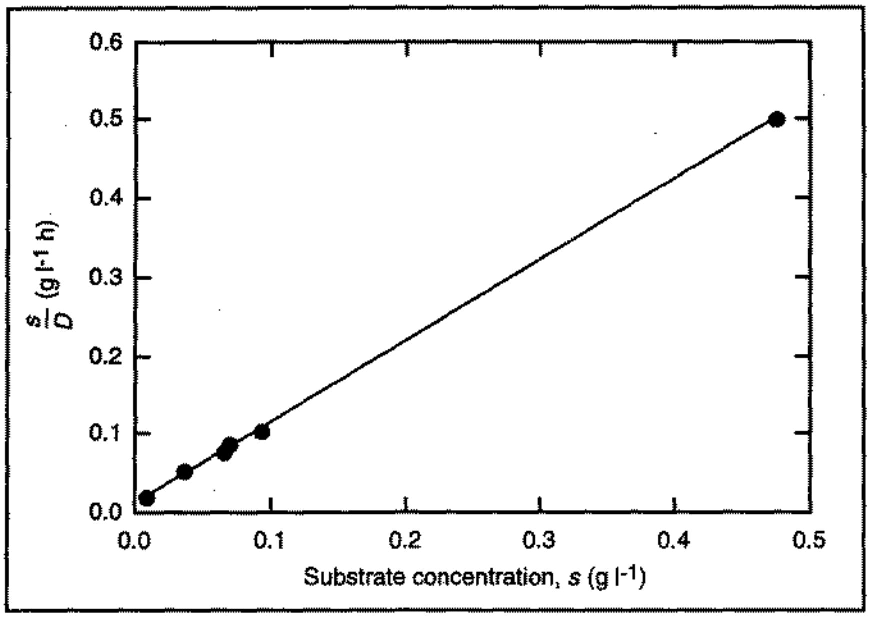
Calculating the flow rate from Eq. (13.39) with $V = 1000$ ml, $F = D\,V = 0.039\ \mathrm{h}^{-1} \times 1000\ \mathrm{ml} = 39\ \mathrm{ml\ h}^{-1}$.

Answer: 39 ml h^{-1}

13.11 Kinetic and yield parameters of an auxotrophic mutant

From Eq. (13.92), $\mu_{\max}$ and K_S can be determined from the slope and intercept of a plot of $^{s}/_{D}$ versus s. From the definition of dilution rate in Eq. (13.39), values of D are evaluated from the experimental flow rates using $V = 2$ l. The relevant data are listed and plotted below.

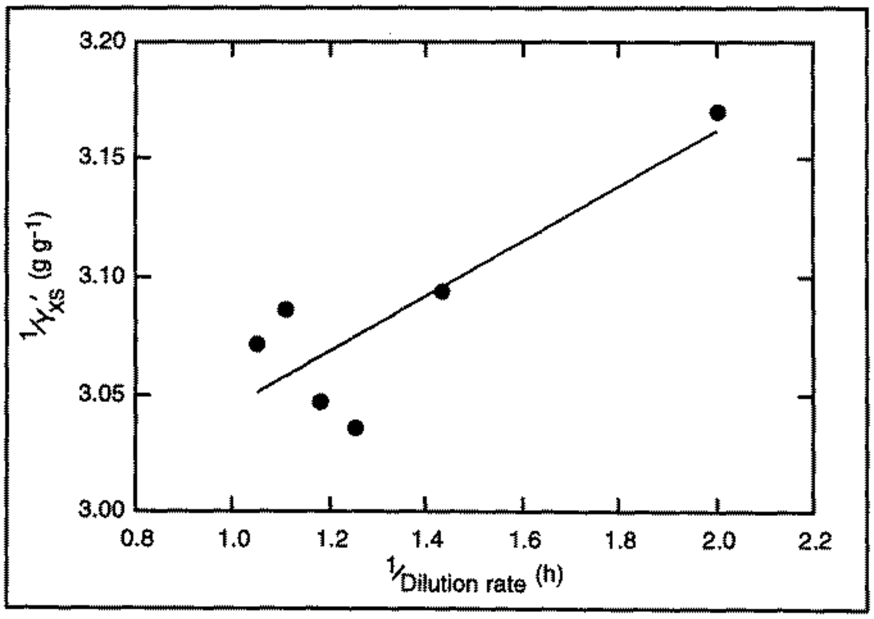
Flow rate, F (l h^{-1})	Dilution rate, D (h^{-1})	Substrate concentration, s (g l^{-1})	$^{s}/_{D}$ (g l^{-1} h)
1.0	0.50	0.010	0.020
1.4	0.70	0.038	0.054
1.6	0.80	0.071	0.089
1.7	0.85	0.066	0.078
1.8	0.90	0.095	0.106
1.9	0.95	0.477	0.502



The slope of the straight line in the plot is 1.027 h; the intercept is 0.0119 g l^{-1} h. From Eq. (13.92), the slope = $^{1}/_{\mu_{max}}$; therefore, $\mu_{max} = {}^{1}/_{1.027\ h} = 0.97\ h^{-1}$. The intercept = $^{K_S}/_{\mu_{max}}$; therefore $K_S = 0.0119\ g\ l^{-1}\ h \times 0.97\ h^{-1} = 0.012\ g\ l^{-1}$.

From Eq. (13.93), Y_{XS} and m_S can be determined from the slope and intercept of a plot of $1/Y'_{XS}$ versus $^{1}/_{D}$, where Y'_{XS} is calculated using Eq. (13.94) with $s_i = 10$ g l^{-1}. The relevant data are listed and plotted below.

Flow rate, F (l h^{-1})	Dilution rate, D (h^{-1})	$^{1}/_{D}$ (h)	Y'_{XS} (g g^{-1})	$1/Y'_{XS}$ (g g^{-1})
1.0	0.50	2.00	0.315	3.175
1.4	0.70	1.43	0.323	3.096
1.6	0.80	1.25	0.329	3.040
1.7	0.85	1.18	0.328	3.049
1.8	0.90	1.11	0.324	3.086
1.9	0.95	1.05	0.326	3.067



The scatter in the plot is typical for measured values of $1/Y'_{XS}$. The slope of the straight line in the plot is 0.12 g g^{-1} h^{-1}; the intercept is 2.93 g g^{-1}. From Eq. (13.93), the slope = m_S; therefore m_S = 0.12 g g^{-1} h^{-1}. The intercept = $^1/Y_{XS}$; therefore $Y_{XS} = {}^1/_{2.93}\ h^{-1}$ = 0.34 g g^{-1}.

Answer: $\mu_{max} = 0.97\ h^{-1}$; $K_S = 0.012\ g\ l^{-1}$; $m_S = 0.12\ g\ g^{-1}\ h^{-1}$; $Y_{XS} = 0.34\ g\ g^{-1}$

13.12 Continuous sterilisation

From the definition of dilution rate in Eq. (13.39), the medium volumetric flow rate $F = D\ V = 0.1\ h^{-1} \times 15\ m^3 = 1.5\ m^3\ h^{-1}$. The linear velocity u in the holding section of the steriliser is determined by dividing F by the pipe cross-sectional area $A = \pi\ r^2$, where r is the pipe radius. For r = 6 cm = 0.06 m:

$$u = \frac{F}{A} = \frac{1.5\,\mathrm{m^3\,h^{-1}}}{\pi\,(0.06\,\mathrm{m})^2} = 132.6\,\mathrm{m\,h^{-1}}$$

The value of the specific death constant is evaluated using Eq. (11.46) with R = 8.3144 J $gmol^{-1}$ K^{-1} from Table 2.5, E_d = 288.5 kJ $gmol^{-1}$ = 2.885 × 10^5 J $gmol^{-1}$, A = 7.5× 10^{39} h^{-1}, and the temperature converted from °C to degrees Kelvin using Eq. (2.24):

$$k_d = A\,e^{-E_d/R\,T} = 7.5\times10^{39}\,\mathrm{h^{-1}}\,e^{-2.885\times10^5\,\mathrm{J\,gmol^{-1}}/\,[(8.3144\,\mathrm{J\,gmol^{-1}\,K^{-1}})\,(130+273.15)\,\mathrm{K}]} = 313.1\,\mathrm{h^{-1}}$$

Within a period of 3 months ≈ 90 d, the number of cells N_1 entering the steriliser is equal to the medium volumetric flow rate F multiplied by the cell concentration and the time:

$$N_1 = 1.5\,\mathrm{m^3\,h^{-1}}\left(10^5\,\mathrm{ml^{-1}}\,.\left|\frac{10^6\,\mathrm{ml}}{1\,\mathrm{m^3}}\right|\right)90\,\mathrm{d}\,.\left|\frac{24\,\mathrm{h}}{1\,\mathrm{d}}\right| = 3.24\times10^{14}$$

Within the same 3-month period, the acceptable number of cells remaining at the end of the sterilisation treatment is N_2 = 1. Therefore:

$$\frac{N_2}{N_1} = \frac{1}{3.24\times10^{14}} = 3.09\times10^{-15}$$

(a)

For perfect plug flow with no axial dispersion, the sterilisation time can be determined using Eq. (13.97):

$$t_{hd} = \frac{\ln\dfrac{N_1}{N_2}}{k_d} = \frac{\ln\dfrac{3.24\times10^{14}}{1}}{313.1\,\mathrm{h^{-1}}} = 0.107\,\mathrm{h}$$

To allow the medium to remain for this period of time in the holding section of the steriliser pipe, the length of pipe required is equal to the linear velocity of the medium u multiplied by t_{hd}:

$$L = u\,t_{hd} = 132.6\,\mathrm{m\,h^{-1}}\times0.107\,\mathrm{h} = 14.2\,\mathrm{m}$$

Answer: 14.2 m

(b)

Calculating the Reynolds number for pipe flow using Eq. (7.1) with pipe diameter D = 12 cm = 0.12 m:

$$Re = \frac{D\,u\,\rho}{\mu} = \frac{0.12\,\mathrm{m}\left(132.6\,\mathrm{m\,h^{-1}}\right)1000\,\mathrm{kg\,m^{-3}}}{4\,\mathrm{kg\,m^{-1}\,h^{-1}}} = 3978$$

The value of $\mathcal{D}_z/u\,D$ corresponding to this Re is found from Figure 13.40. Using the experimental curve as this gives a higher $\mathcal{D}_z$ than the theoretical curve and thus a more conservative design, $\mathcal{D}_z/u\,D \approx 1.5$. Therefore:

$$\mathcal{D}_z = 1.5\,u\,D = 1.5\,(132.6\,\mathrm{m\,h^{-1}})\,0.12\,\mathrm{m} = 23.9\,\mathrm{m^2\,h^{-1}}$$

From Eq. (13.101), an expression for the Peclet number Pe is:

$$Pe = \frac{u\,L}{\mathcal{D}_z} = \frac{\left(132.6\,\mathrm{m\,h^{-1}}\right)L}{23.9\,\mathrm{m^2\,h^{-1}}} = 5.5\,L$$

where L has units of m. Similarly, an expression for the Damköhler number Da from Eq. (13.102) is:

$$Da = \frac{k_d\,L}{u} = \frac{\left(313.1\,\mathrm{h^{-1}}\right)L}{132.6\,\mathrm{m\,h^{-1}}} = 2.36\,L$$

The design problem can be solved from this point using trial-and-error methods and Figure 13.41. As a first guess, try $L = 20$ m. The values for Pe and Da are evaluated using the equations determined above, and the corresponding value for N_2/N_1 read from Figure 13.41. Depending on how this value compares with the target of 3.09×10^{-15}, the value of L is adjusted until the results for N_2/N_1 coincide. The calculations are shown in the table below.

L (m)	Pe	Da	N_2/N_1 (from Figure 13.41)
20	110	47	4×10^{-16}
18	99	42	1×10^{-14}
19	105	45	1.5×10^{-15}

The last value of N_2/N_1 is as close as practicable to 3.09×10^{-15} considering the resolution of Figure 13.41. Therefore, the required length of pipe in the holding section is about 19 m, or 34% longer than that determined for ideal plug flow.

Answer: About 19 m

(c)
For $L = 14.2$ m, from the equations developed in **(b)**, $Pe = 78$ and $Da = 34$. From Figure 13.41, N_2/N_1 is about 5×10^{-12}; therefore, $N_1/N_2 = 2 \times 10^{11}$. As $N_2 = 1$, $N_1 = 2 \times 10^{11}$, i.e. one contaminant enters the fermenter for every 2×10^{11} that enter the steriliser. For $F = 1.5\ \mathrm{m^3\ h^{-1}}$ and an input contaminant concentration of $10^5\ \mathrm{ml^{-1}}$, the time required for 2×10^{11} contaminants to enter the steriliser is:

$$\text{Time} = \frac{2 \times 10^{11}}{1.5\,\mathrm{m^3\,h^{-1}}\left(10^5\,\mathrm{ml^{-1}} \cdot \left|\frac{10^6\,\mathrm{ml}}{1\,\mathrm{m^3}}\right|\right) \cdot \left|\frac{1\,\mathrm{h}}{60\,\mathrm{min}}\right|} = 80\,\mathrm{min}$$

Therefore, contaminants enter the fermenter at a rate of one every 80 min.

Answer: One contaminant enters the fermenter every 80 min.