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DEVELOPMENTS IN GEOTECHNICAL ENGINEERING 34A

PRACTICAL PROBLEMS IN SOIL MECHANICS AND FOUNDATION ENGINEERING, 1

PHYSICAL CHARACTERISTICS OF SOILS, PLASTICITY, SETTLEMENT CALCULATIONS, INTERPRETATION OF IN-SITU TESTS

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PREFACE

by PROF. Dr. VICTOR F.B. De MELLO

President International Society for Soil Mechanics and Foundation Engineering 1981—1985

In the continuum of persistent change which characterizes the professional quest for scientific and engineering solutions, there is an absolute need for pauses and movement by steps. Such a need is felt all the more intensely as all social and technological factors have made the continuum of change more and more accelerated.

Man, and especially the Engineer, cannot shy away from the discontinuity imposed by a *yes* vs. *no* decision: *maybe* does not exist, because its implementation would be as maybe-yes or maybe-no. Both right and wrong, however arbitrary and nominal, must be allowed to stand long enough to permit the experience cycle to close, starting with a given set of data, hypotheses, calculations and decisions, and reaching a certain set of observations on the constructed product under operational conditions.

Far too much of the modern production of technical literature is conditioned by the eureka complex, especially in the respected advanced technological centers. Yet, Man's and Society's time cycle of experience is still deeply conditioned by an animal life cycle, even if somewhat altered by physiological and social evolutions. A house is intended to be a home, and its life cycle should respect a span roughly between twenty and eighty years; public works should serve a couple of generations. It is not only materially but also socially that from the solutions of one generation or period arise the plagues of a following generation.

The appropriately named book, *Practical Problems in Soil Mechanics and Foundation Engineering* by Sanglerat, Olivari and Cambou, comes to fulfill a very important need of thousands of practicing engineers in the geotechnical profession. It sets a modern, practical milestone for reference, and is almost unique in doing this with its emphasis on calculations, the principal working tool of engineers. The analysis and calculation procedures presented, which encompass the great proportion of geotechnical problems, are simultaneously both the indication of accepted practice and the reminder that such accepted practice is based on hypotheses: both the hypotheses and the rules developed from them must always be clearly stated, not only so that exceptions may be distinguished, but also so that the consequences of a given practice may be used to establish a modicum statistical universe of case histories for judging the results achieved and for subsequent iterative adjustment.

Solutions in engineering are immediately recognized to be wrong if a patent or catastrophic failure ensues. Time, however, reveals the other extreme of the histogram of failures of engineering solutions, when they conceal a condition of being too safe and relatively less economical than desirable or acceptable. The authors are to be thanked for having offered a good up-to-date reference for appraising both ends of the spectrum. Engineers should be enjoined to state clearly the design procedures according to which their projects of a given period were calculated. This book augurs well to stand as a guide for many, many such calculations.

INTRODUCTION

Guy Sanglerat has taught geotechnical engineering at the "Ecole Centrale de Lyon" since 1967. This discipline was introduced there by Jean Costet. Since 1968 and 1970, respectively, Gilbert Olivari and Bernard Cambou actively assisted in this responsibility. They directed laboratory work, outside studies and led special study groups.

In order to master any scientific discipline, it is necessary to apply its theoretical principles to practice and to readily solve its problems. This holds true also for theoretical soil mechanics when applied to geotechnical engineering.

From Costet's and Sanglerat's experiences with their previously published textbooks in geotechnical engineering, which contain example-problems and answers, it became evident that one element was still missing in conveying the understanding of the subject matter to the solution of practical problems: problems apparently needed detailed, step-by-step solutions.

For this reason and at the request of many of their students, Sanglerat, Olivari and Cambou decided to publish problems. Over the years since 1967 the problems in this text have been given to students of the "Ecole Centrale de Lyon" and since 1976 to special geotechnical engineering study groups of the Public Works Department of the National School at Vaulx-en-Velin, where Gilbert Olivari was assigned to teach soil mechanics.

In order to assist the reader of these volumes, it was decided to categorize problems by degrees of solution difficulty. Therefore, easy problems are preceded by one star (★), those considered most difficult by 4 stars (★★★★). Depending on his degree of interest, the reader may choose the types of problems he wishes to solve.

The authors direct the problems not only to students but also to the practicing Civil Engineer and to others who, on occasion, need to solve geotechnical engineering problems. To all, this work offers an easy reference, provided that similarities of actual conditions can be found in one or more of the solutions prescribed herein.

Mainly, the S.I. (Système International) units have been used. But, since practice cannot be ignored, it was deemed necessary to incorporate other widely accepted units. Thus the C.G.S. and English units (inch, foot, pounds per cubic foot, etc.) have been included because a large quantity of literature is based on these units.

The authors are grateful to Mr. Jean Kerisel, past president of the International Society for Soil Mechanics and Foundation Engineering, for having written the Preface to the French edition and allowing the authors to include one of the problems given his students while Professor of Soil Mechanics at the "Ecole Nationale de Ponts et Chaussées" in Paris. Their gratitude also goes to Victor F.B. de Mello, President of the International Society for Soil Mechanics, who had the kindness to preface the English edition.

The first problems were originally prepared by Jean Costet for the course in soil mechanics which he introduced in Lyon.

Thanks are also due to Jean-Claude Rouault of "Air Liquide" and Henri Vidal of "Reinforced Earth" and also to our Brazilian friend Lucien Decourt for contributing problems, and to Thierry Sanglerat for proofreading manuscripts and printed proofs.

NOTATIONS

The following general notations appear in the problems:

A	: Skempton's second coefficient (sometimes A refers also to cross-sectional area).
A_f	: value of A at failure
B	: footing width (sometimes B refers also to Skempton's first coefficient).
c	: soil cohesion (undifferentiated)
c'	: effective cohesion
c''	: reduced cohesion (slope stability)
c_u	: undrained cohesion
c_{cu}	: consolidated-undrained cohesion
C_c	: compression index
C_u	: uniformity coefficient, defined as d_{60}/d_{10}
c_v	: coefficient of consolidation
d	: soil particle diameter (sometimes: horizontal distance between adjacent, similar structures, as in the case of subsurface drains)
d_y	: equivalent diameter of sieve openings in grain-size distribution
D	: depth to bottom of footings (sometimes D refers to depth to hard layer under the toe of a slope).
e	: void ratio (sometimes: e refers to eccentricity of a concentrated force acting on a footing)
e_{max}, e_{min}	: maximum and minimum void ratios
E	: Young's modulus
E_p	: pressuremeter modulus
FR	: friction ratio (static penetrometer test)
g	: acceleration due to gravity (gravie)
G	: shear modulus
h	: hydraulic head
H	: soil layer thickness (or normal cohesion: $H = c \cot \varphi$)
i	: hydraulic gradient
i_c	: critical hydraulic gradient
IP	: plasticity index
k	: coefficient of permeability
$k_{a\gamma}$, k_{aq}, k_{ac}	: active earth pressure coefficients due to overburden, surcharge and cohesion, respectively

$k_{p\gamma}$, k_{pq}, k_{pc}	: passive earth pressure coefficients
$K_{a\gamma}$, K_{aq}, K_{ac}	: active earth pressures perpendicular to a given plane
$K_{p\gamma}$, K_{pq}, K_{pc}	: passive earth pressures perpendicular to a given plane
k_s	: soil reaction modulus
K	: bulk modulus (K_s of soil structure, K_w of water)
K_0	: coefficient of earth pressure at rest
l	: width of an excavation
L	: length of an excavation
m_v	: coefficient of compressibility
M_m	: driving moment
M_R	: resisting moment
M	: bending moment
n	: porosity
n_D	: stability coefficient (slope stability problems)
N_γ, N_q, N_c	: bearing capacity factors for foundation design
P	: concentrated (point) load
p_l	: limit pressure (pressuremeter test)
p_f	: creep pressure (pressuremeter test)
q	: uniformly distributed load (or percolation discharge)
Q	: discharge (or load acting upon a footing)
Q_f	: friction force of pile shaft (total skin friction force)
Q_p	: end-bearing force of pile (total)
q_d	: ultimate bearing capacity of soil under a footing or pile
q_{ad}	: allowable bearing capacity of a footing or pile
R	: radius of a circular footing (or radius of drawdown of a well)
RD	: relative density $(e_{max} - e)/(e_{max} - e_{min})$
r	: well radius (or polar radius in polar coordinate system)
R_p or q_c	: end-bearing on the area of a static penetrometer (cone resistance)
s	: curvilinear abscissa (or cross-sectional area of a thin wall tube, or settlement)
S	: cross-sectional area of a mold or a sample
$S.G.$	: specific gravity
S_t	: degree of saturation
t	: time
T	: shear
T_v	: time factor
u	: porewater pressure
U	: degree of consolidation (or resultant of pore-water pressure forces)
v	: rate of percolation
V	: volume
W	: weight of a given soil volume

w	: water content or settlement
w_l, w_p	: liquid limit, plastic limit
x, y, z	: Cartesian coordinates, with Oz usually considered the vertical, downward axis
α	: angle between orientations, usually reserved for the angle between two crystal faces. Also used to classify soils for the purpose of their compressibility from static cone penetrometer test data C.P.T.
β	: slope of the surface of backfill behind a retaining wall (angle of slope)
γ	: unit weight of soil (unspecified)
γ_s	: soil particles unit weight (specific gravity)
γ_{sat}	: saturated unit weight of soil
γ_h	: wet unit weight of soil
γ_w	: unit weight of water = 9.81 kN/m^3.
γ_d	: dry unit weight of soil
γ'	: effective unit weight of soil
γ_{xy}, γ_{yz}, γ_{zx}	: shear strain, twice the angular deformation in a rectangular, 3-dimensional system
δ	: angle of friction between soil and retaining wall surface in passive or active earth pressure problems, or the angle of inclination of a point load acting on a footing
η	: dynamic viscosity of water
ϵ_x, ϵ_y, ϵ_z	: axial strains in a rectangular, 3-dimensional system
ϵ_1, ϵ_2, ϵ_3	: principal stress
ϵ_v	: volumetric strain
θ	: angle of radius in polar coordinates system (sometimes: temperature)
ν	: Poisson's ratio
σ'	: effective normal stress
σ	: total normal stress
σ_x, σ_y, σ_z	: normal stresses in a rectangular, 3-dimensional system
σ_1, σ_2, σ_3	: major principal stresses
σ_m	: average stress
τ	: shear stress
τ_m	: average shear stress
τ_{xy}, τ_{yz}, τ_{zx}	: shear stresses in a rectangular, 3-dimensional system
φ	: angle of internal friction (undefined)
φ'	: effective angle of internal friction
φ''	: reduced, effective angle of internal friction (slope-stability analyses)
φ_{cu}	: angle of internal friction, consolidated, undrained
λ	: slope of a wall from the vertical
ω_β, ω_δ	: auxiliary angles defined by $\sin \omega_\beta = \sin \beta / \sin \varphi$ and $\sin \omega_\delta = \sin \delta / \sin \varphi$

π	: 3.1416
ρ	: distance from origin to a point in polar coordinate system
ψ	: angle of major principal stress with radius vector (plasticity problems)

ENGINEERING UNITS

It is presently required that all scientific and technical publications resort to the S.I. units (Système International) and their multipliers (deca, hecta, kilo, Mega, Giga). Geotechnical engineering units follow this requirement and most of the problems treated here are in the S.I. system.

Fundamental S.I. units:

length	: meter (m)
mass	: kilogram (kg)
time	: second (s)

S.I. Units derived from the above

surface	: square meter (m^2)
volume	: cubic meter (m^3)
specific mass	: kilogram per cubic meter (kg/m^3)
velocity (permeability)	: meter per second (m/s)
acceleration	: meter per second per second (m/s^2)
discharge	: cubic meter per second (m^3/s)
force (weight)	: Newton (N)
unit weight	: Newton per cubic meter (N/m^3)
pressure, stress	: Pascal (Pa) $1\ Pa = 1\ N/m^2$
work (energy)	: Joule (J) $1\ J = 1\ N \times m$
viscosity	: Pascal-second* $Pa \times s$

However, in practice, other units are encountered frequently. Table A presents correlations between the S.I. and two other unit systems encountered worldwide. This is to familiarize the readers of any publication with the units used therein. For that purpose also, British units have been adopted for some of the presented problems.

Force (pressure) conversions

Force units	: see Table B
Pressure units	: see Table C
Weight unit	: $1\ kN/m^3 = 0.102\ tf/m^3$

*This unit used to be called the "poiseuille", but it has not been officially adopted.

TABLE A
Correlations between most common unit systems

	Système International (S.I.)		Meter-Kilogram system (M.K.)		Centimeter-Gram-Second system (C.G.S.)	
	units	common multiples	units	common multiples	units	common multiples
Length	meter (m)	km	meter (m)	km	cm	m
Mass	kilogram (kg)	tonne (t)	gravie*	—	g	—
Time	second (s)	—	second (s)	—	s	—
Force	Newton (N)	kN	kilogram force (kgf)	tf	dyne	—
Pressure (stress)	Pascal (Pa)	kPa MPa	kilogram force per square meter (kgf/m^2)	t/m^2 kg/cm^2	barye	bar (10^6 baryes)
Work (energy)	Joule (J)	kJ	kilogram meter (kgm)	tf.m	erg	Joule (10^7 ergs)

*Note that 1 gravie = 9.81 kg (in most problems rounded off to 10).

The unit weight of water is: $\gamma_w = 9.81\,kN/m^3$ but it is often rounded off to: $\gamma_w = 10\,kN/m^3$.

Energy units:

1 Joule $= 0.102$ kg.m $= 1.02 \times 10^{-4}$ t.m
1 kgf.m $= 9.81$ Joules
1 tf.m $= 9.81 \times 10^3$ Joules

Dynamic viscosity units:

1 Pascal-second (Pa.s) = 10 poises (Po).

British units:

1 inch	= 0.025 4 m	1 m = 39.370 in.
1 foot	= 0.304 8 m	1 m = 3.280 8 foot
1 square inch	= 6.451 6 cm^2	1 cm^2 = 0.155 sq. in.
1 square foot	= 144 sq. in. = 0.092 9 m^2	
1 m^2	= 10.764 sq. ft.	
1 cubic inch	= 16.387 cm^3	1 cm^3 = 0.061 0 cu. in.
1 cubic foot	= 1728 cu. in. = 0.028 317 m^3	
1 m^3	= 35.314 cu. ft.	
1 pound (lb)	= 4.449 7 Newton = 0.453 59 kgf	
1 Newton	= 0.225 lb = 0.112 4 $\times 10^{-3}$ sh. ton. (1 sh. ton. = 2 kip) = 1.003 $\times 10^{-4}$ ton.	
1 lb/cu. in.	= 270.27 kN/m^3	

1 lb/cu. ft.	= 0.156 99 kN/m^3
1 kN/m^3	= 3.7×10^{-3} lb/cu. in. = 6.37 lb/cu. ft.
1 lb/sq. in. (p.s.i.)	= $6.896\,55 \times 10^3$ Pa
1 Pascal	= 14.50×10^{-5} p.s.i.
100 kPa	= 1 bar = 14.50 p.s.i.

TABLE B

Force units conversions

Value of ↓ / expressed in →	Newton	Decanewton	Kilonewton	Kilogram force	Tonne force	Dyne
Newton	1	10^{-1}	10^{-3}	1.02×10^{-1}	1.02×10^{-4}	10^{5}
Decanewton	10	1	10^{-2}	1.02	1.02×10^{-3}	10^{6}
Kilonewton	10^{3}	10^{2}	1	1.02×10^{2}	1.02×10^{-1}	10^{8}
Kilogram force	9.81	9.81×10^{-1}	9.81×10^{-3}	1	10^{-3}	9.81×10^{5}
Tonne force	9.81×10^{3}	9.81×10^{2}	9.81	10^{3}	1	9.81×10^{8}
Dyne	10^{-5}	10^{-6}	10^{-8}	1.02×10^{-6}	1.02×10^{-9}	1

TABLE C

Pressure units conversions

Value of ↓ / expressed in →	Pa	kPa	bar	hbar	barye	kg/cm^2	kg/mm^2	t/m^2	cm of water	atm.
Pascal	1	10^{-3}	10^{-5}	10^{-7}	10	1.02×10^{-5}	1.02×10^{-7}	1.02×10^{-4}	1.02×10^{-2}	9.869×10^{-6}
Kilopascal	10^{3}	1	10^{-2}	10^{-4}	10^{4}	1.02×10^{-2}	1.02×10^{-4}	1.02×10^{-1}	10.2	9.869×10^{-3}
Bar	10^{5}	10^{2}	1	10^{-2}	10^{6}	1.02	1.02×10^{-2}	10.2	1.02×10^{3}	0.986 9
Hectobar	10^{7}	10^{4}	10^{2}	1	10^{8}	1.02×10^{2}	1.02	1.02×10^{3}	1.02×10^{5}	9.869×10^{1}
Barye	0.1	10^{-4}	10^{-6}	10^{-8}	1	1.02×10^{-6}	1.02×10^{-8}	1.02×10^{-5}	1.02×10^{-3}	9.869×10^{-7}
kg/cm^2	9.81×10^{4}	9.81×10^{1}	0.981	9.81×10^{3}	9.81×10^{5}	1	10^{-2}	10	10^{3}	0.968 1
kg/mm^2	9.81×10^{6}	9.81×10^{-3}	9.81×10^{1}	0.981	9.81×10^{7}	10^{2}	1	10^{3}	10^{5}	9.681×10^{1}
t/m^2	9.81×10^{3}	9.81	9.81×10^{-2}	9.81×10^{-4}	9.81×10^{4}	0.1	10^{-3}	1	10^{2}	9.681×10^{-2}
cm of water	9.81×10^{1}	9.81×10^{-2}	9.81×10^{-4}	9.81×10^{-6}	9.81×10^{2}	10^{-3}	10^{-5}	10^{-2}	1	9.681×10^{-4}
Atmosphere	$1.013\,3 \times 10^{5}$	$1.013\,3 \times 10^{2}$	1.013 3	$1.013\,3 \times 10^{-2}$	$1.013\,3 \times 10^{6}$	1.033	1.033×10^{-2}	1.033×10^{1}	1.033×10^{3}	1

Chapter 1

PHYSICAL CHARACTERISTICS OF SOIL

★*Problem 1.1* Water content

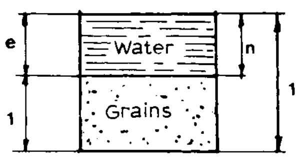
A saturated clay sample has a mass of 1526 g. After drying, its mass is 1053 g. The solid constituant (soil particles) has a specific gravity of 2.7.
Find:
— *water content,* w
— *void ratio,* e
— *porosity,* n
— *wet unit weight,* γ_h
— *wet density,* γ_h/γ_w.
Take $g = 9.81\ m/s^2$.

Solution

The weight of the clay sample is: $1.526 \times 9.81 = 14.97$ N.
The dry weight is: $1.053 \times 9.81 = 10.33$ N.
The weight of water contained in the sample is: $14.97 - 10.33 = 4.64$ N.
The water content: w = weight of water/weight of dry soil = $4.64/10.33 = 0.45$.

The void ratio is: e = volume of water/volume of soil particles.

Since the soil is saturated, the voids between soil particles are filled with water and the volume of voids is equal to the volume of water (Fig. 1.1).



$$\gamma_w = 9.81 \cdot 10^3\ \text{N/m}^3.$$

Fig. 1.1.

The volume of water is:

$$\frac{\text{weight of water}}{\gamma_w} = \frac{4.64}{9.81 \times 10^3} = 0.473 \times 10^{-3}\ \text{m}^3 = 473\ \text{cm}^3.$$

The unit weight of the soil grains is:

$$\gamma_s = \frac{\gamma_s}{\gamma_w} \times \gamma_w = 2.7 \times 9.81 = 26.5 \text{ kN/m}^3 = 26.5 \times 10^3 \text{ N/m}^3.$$

The volume of the soil grains is:

$$\frac{\text{weight of soil grains}}{\gamma_s} = \frac{10.33}{26.5 \times 10^3} = 0.390 \times 10^{-3} \text{ m}^3 = 390 \text{ cm}^3.$$

The void ratio is then: $e = \dfrac{473}{390} = 1.21.$

$$\text{Porosity } n = \frac{\text{volume of voids}}{\text{total volume}} = \frac{473}{473 + 390} = \frac{473}{863} = 0.55.$$

The saturated unit weight γ_h, is:

$$\gamma_h = \frac{\text{weight of saturated sample}}{\text{volume of saturated sample}} = \frac{14.97}{0.863 \times 10^{-3}} = 17.34 \times 10^3 \text{ N/m}^3$$

$$= 17.34 \text{ kN/m}^3.$$

The saturated density $\gamma_h/\gamma_w = 17.34/9.81 = 1.77$

Summary of answers
$w = 0.45$; $e = 1.21$; $n = 0.55$; $\gamma_h = 17.34$ kN/m^3; $\gamma_h/\gamma_w = 1.77$.

★*Problem 1.2* Water content, degree of saturation

A soil sample has a mass of 129.1 g and a volume of 56.4 cm^3. Mass of the soil grains is 121.5 g. The soil grains specific gravity is 2.7. Find:
— the water content, w
— the void ratio, e
— the degree of saturation, S_r.
Take $g = 9.81$ m/s^2.

Solution
The weight of the sample is: $0.1291 \times 9.81 = 1.2665$ N.
The weight of the dry soil (soil grains) is: $0.1215 \times 9.81 = 1.1919$ N.
The weight of the water is the difference between the two calculated weights:
$1.2665 - 1.1919 = 0.0746$ N
and the water content:

w = (weight of water)/(weight of soil) = $0.0746/1.192 \simeq 0.063$, or $w = 6.3\%$.

The void ratio is:

e = volume of voids (water + air)/volume of soil grains = V_v/V_s (Fig. 1.2)

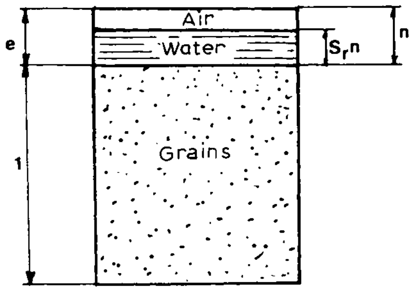


Fig. 1.2.

The volume of voids is equal to the total volume less the volume of grains. The total volume is known: 56.4 cm^3.

The volume of the grains is: $\dfrac{\text{weight of grains}}{\text{unit weight } (\gamma_s)}$

Since specific gravity

S.G. $= \gamma_s/\gamma_w = 2.7$

where $\gamma_w = \rho g = 9.81\ kN/m^3$ and $\gamma_s = 2.7 \times 9.81 \times 10^3\ N/m^3$.

The volume of grains V_s is:

$V_s = 1.1919/2.7 \times 9.81 \times 10^3 = 4.5 \times 10^{-5}\ m^3 = 45\ cm^3$

and the volume of voids is:

$V_v = 56.4 - 45 = 11.4\ cm^3$.

The void ratio: $e = 11.4/45 = 0.253$, say $e = 0.25$.
The degree of saturation S_r is given by:

volume of water/volume of voids

The volume of water: V_w = weight of water/density of water $= 0.0746/9.81 \times 10^3 = 7.6 \times 10^{-6}\ m^3$ or $V_w = 7.6\ cm^3$.
The degree of saturation is: $S_r = 7.6/11.4 = 0.666$, say 67%.

Summary of answers:
$w = 6.3\%$; $e = 0.25$; $S_r = 67\%$.

★*Problem 1.3* **Unit weight and density**

A quartzitic sand weighs, in a dry condition, 15.4 kN/m^3. What is its wet unit weight (γ_h) and its wet density γ_h/γ_w when it is saturated?
Assume: specific gravity of sand: S.G. = 2.66, acceleration due to gravity: $g = 9.81$ m/s^2, unit mass of water: $\rho = 10^3$ kg/m^3.

Solution
The unit weight of the sand grains is:

$$\gamma_s = \text{S.G.} \times \gamma_w = \text{S.G.} \times \rho \times g = 2.66 \cdot 10^3 \cdot 9.81\ \text{N/m}^3 = 26.10\ \text{kN/m}^3.$$

A cubic meter of dry sand contains $15.40/26.10 = 0.59\ \text{m}^3$ of grains and, consequently, $1 - 0.59 = 0.41\ \text{m}^3$ of voids.
When this sand is saturated, the voids are completely filled with water. The weight of the void water is then:

$$0.41 \times \gamma_w = 0.41 \times 9.81 = 4.02\ \text{kN}.$$

The weight of a cubic meter of saturated sand is thus $15.4 + 4.02 = 19.42$ kN or $\gamma_h = 19.42\ \text{kN/m}^3$.
The density of sand $\gamma_h/\gamma_w = 19.42/9.81 = 1.98$.

Summary of answers:
$\gamma_h = 19.42\ \text{kN/m}^3$; $\gamma_h/\gamma_w = 1.98$.

★*Problem 1.4* **Unit weight and density; saturation and water content**

A clay sample is placed in a glass container. Total mass of clay sample and container is 72.49 g. After drying in an oven, the dry mass of the clay and container is 61.28 g. The mass of the container is 32.54 g. A specific gravity test by the picnometer method has determined that S.G. of the soil constituant is 2.69.

(a) Assume the sample to be saturated, find:
- *the water content, w*
- *the porosity, n*
- *the void ratio, e*
- *the wet density (γ_h/γ_w)*
- *the dry density (γ_d/γ_w)*
- *the buoyant density (γ'/γ_w).*

(b) Before drying the sample, its volume V was determined by immersing the soil in mercury ($V = 22.31$ cm^3). What is the actual degree of saturation and what are the new values of the densities determined in (a)?

Solution
(a) The mass of water contained in the sample is: $72.49 - 61.28 = 11.21$ g.

The mass of dry soil particles is: 61.28 − 32.54 = 28.74 g.
The water content w = weight of water/weight of dry soil = mass of water/mass of dry soil = 11.21/28.74 = 0.39, w = 39%.

Porosity n = volume of voids/total volume.

Since the sample is assumed to be saturated, the volume of voids is equal to the volume of water or 11.21 cm^3 (the unit mass of water is $\rho = 1 g/cm^3$).

V_g = mass of dry soil grains/specific gravity of soil = 28.74/2.69 $\simeq$ 10.68 cm^3.

Therefore, $n = (11.21)/(11.21 + 10.68) \simeq 0.512$, say $n = 0.51$.
The void ratio is e = volume of voids/volume of soil grains = 11.21/10.68 = 1.049, say $e \simeq 1.05$.

Since the unit mass of water is 1 g/cm^3, densities are of the same numerical values as the unit masses. The mass of the wet sample is, therefore: 72.49 − 32.54 = 39.95 g.

The total volume of the clay sample is: 11.21 + 10.68 = 21.89 cm^3.

The wet unit mass is: 39.95/21.89 = 1.825 g/cm^3, say 1.83 g/cm^3, and the wet density γ_h/γ_w thus 1.83.

The mass of the dry soil is 28.74 g. Its dry unit mass is: 28.74/21.89 = 1.313 g/cm^3, say 1.31 g/cm^3, the dry density γ_d/γ_w = 1.31.

In order to obtain the buoyant density, the weight of water displaced by the submerged mass of the soil grains must be subtracted from the soil weight. The volume of the grains is 10.68 cm^3. Their mass in water will then be: 28.74 − 10.68 = 18.06 g, and the buoyant unit mass is: 18.06/21.89 = 0.825 g/cm^3; say 0.83 g/cm^3. (Another, most commonly used way of determining buoyant unit mass, is from the relation:
buoyant unit mass = saturated unit mass − unit mass of water).

(b) The volume of the samples being 22.31 cm^3 proves that the clay is not saturated. Part of the voids is filled with air. The air volume is: 22.31 − 21.89 = 0.42 cm^3.
The degree of saturation, S_r = volume of water/total void volume.

The volume of water calculated in (a) is 11.21 cm^3, the volume of void = 11.21 + 0.42 = 11.63 cm^3 then S_r = 11.21/11.63 = 0.963, say S_r = 0.96.

The total soil sample volume is 22.31 cm^3. The corrected densities are thus: for the wet density: γ_h/γ_w = 39.95/22.31 = 1.79 and for the dry density: γ_d/γ_w = 28.74/22.31 = 1.29.

Since the concept of buoyant mass is applicable to saturated soils only, it should not be calculated in this instance.

Summary of answers
(a) w = 39%; n = 0.51; e = 1.05; γ_h/γ_w = 1.83; γ_d/γ_w = 1.31; γ'/γ_w = 0.83.
(b) S_r = 96%; γ_h/γ_w = 1.79; γ_d/γ_w = 1.29. (γ'/γ_w has no meaning in the second part of the problem because the soil is not saturated.)

★*Problem 1.5* **Grain-size distribution: effective diameter and Hazen's coefficient**

A grain-size analysis is performed on 3500 g of dry sand from the Saône valley. No soil is retained on the 12.5-mm openings sieve. A nest of six sieves is subsequently used to separate the various sand sizes. The openings of the sieve meshes are, from top to bottom; 5, 2, 1, 0.5, 0.2 and 0.1 mm. The soil masses remaining on each of the six sieves are 217 g, 868 g, 1095 g, 809 g, 444 g, 39 g, and the amount of soil in the bottom pan is 28 g.

Draw the grain-size distribution curve and find the effective diameter and the uniformity coefficient (Hazen's coefficient) of this sand.

Solution

Drawing the grain-size distribution curve consists of connecting the points on a graph which represent the cumulative mass percentages passing down to the sieves size.

As shown in Fig. 1.3, the soil passing sieve n = soil passing sieve $(n-1)$ minus the soil retained on sieve n, or $T_n = T_{n-1} - R_n$, where T_n is the weight passing sieve n.

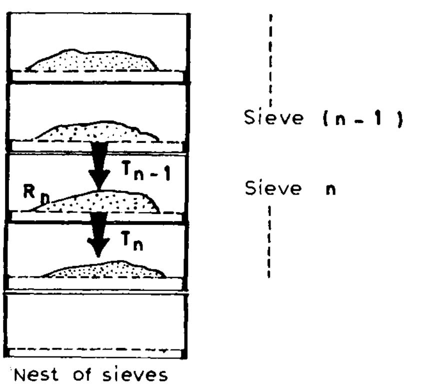


Fig. 1.3. A nest of sieves.

Since the initial sieve (12.5-mm openings) retained no soil, $T_1 = 3500$ g.

The soil retained on the top sieve of the nest is 217 g, therefore $T_2 = 3500 - 217\text{ g} = 3283\text{ g}$, $T_3 = 3283 - 868\text{ g} = 2415$, and so on.

A table, such as the one shown below, is constructed to give the calculated values of the percents passing. The values in the last column are plotted on a semi-log grid (see Fig. 1.4).

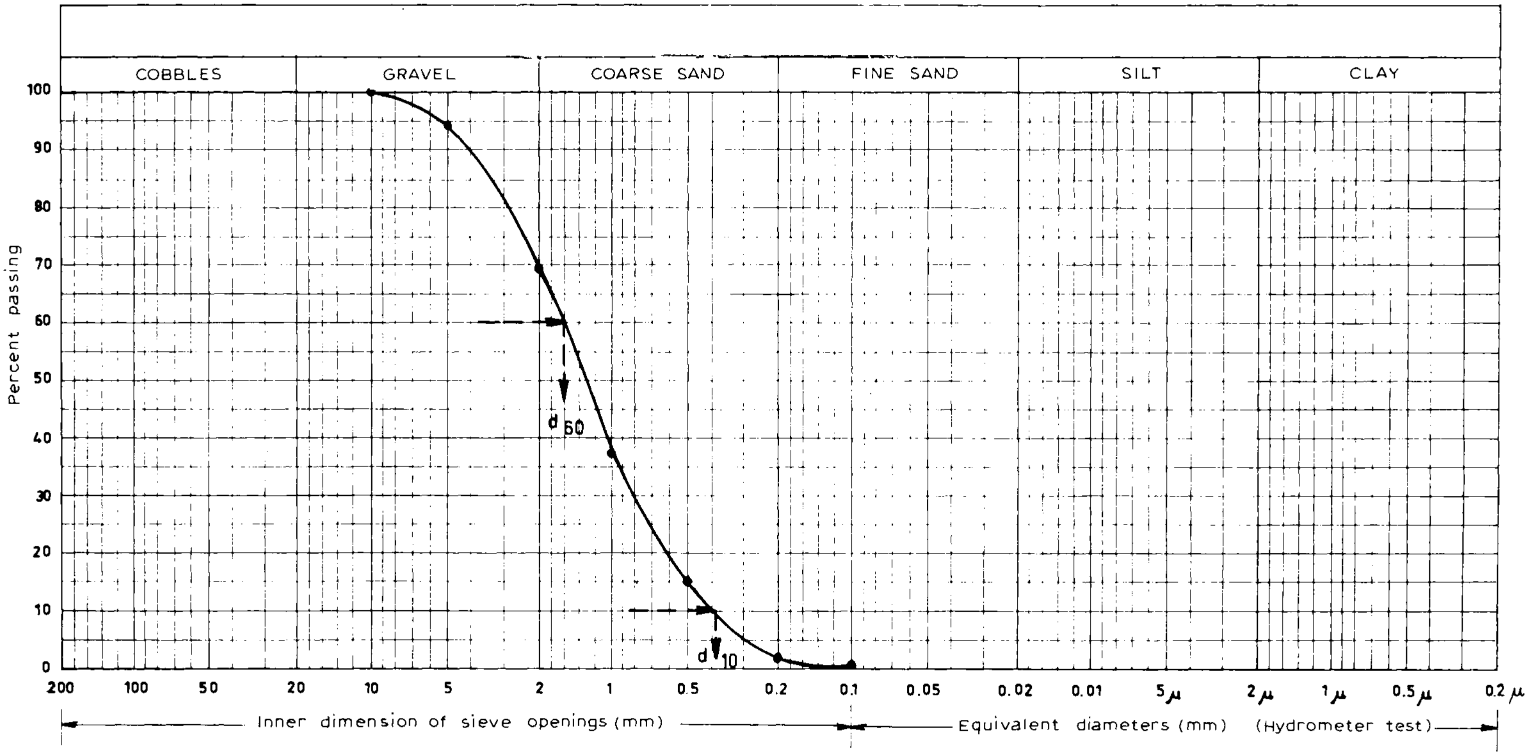


Fig. 1.4. Grain-size distribution curve.

TABLE 1A

Sieve no.	Sieve openings (mm)	Soil retained (g)	Soil passing (s)	Percent passing
1	12.5	0	3500	100
2	5	217	3283	94
3	2	868	2415	69
4	1	1095	1320	37.7
5	0.5	809	511	14.6
6	0.2	444	67	1.92
7	0.1	39	28	0.80
Rest	—	28*	—	

*The masses retained should always (very nearly) add up to the amount of the whole sample tested.

The Hazen coefficient, or uniformity coefficient is, by definition $C_u = d_{60}/d_{10}$. From Fig. 1.4, the 10% passing corresponds to a diameter of $d_{10} = 0.37$ mm (effective diameter) and $d_{60} = 1.60$ mm. Therefore, $d_{60}/d_{10} = 1.60/0.37 = 4.3$.

The sand is well-graded since its coefficient is larger than 2.

Summary of answer
Effective diameter, $d_{10} = 0.37$ mm; Hazen's coefficient $d_{60}/d_{10} = 4.3$.

★*Problem 1.6* Classification H.R.B.

Atterberg limits and sieve tests were performed on five soil samples identified in Table 1B as a through e. Classify the soils according to the Highway Research Board Classification (H.R.B.).

TABLE 1B

Sample	Atterberg limits		Passing (%)		
	w_L	w_P	2 mm	0.4 mm	80 μm
a	—	—	97	59	0.1
b	24	16	99	93	73
c	28	17	99	76	57
d	—	—	84	8	1
e	23	16	100	85	28

Solution

The H.R.B. classification is summarized in Table 1C.
From this data, the soils can be classified as follows.

TABLE 1C

Summarized H.R.B-classification

	Less than 35% passing 80-μ sieve							More than 35% passing 80-μ sieve				
	A_{1a}	A_{1b}	A_3	A_{2-4}	A_{2-5}	A_{2-6}	A_{2-7}	A_4	A_5	A_6	A_{7-5}	A_{7-6}
Percent passing:												
2-mm sieve	⩽50											
0.4-mm sieve	⩽30	⩽50	⩾51									
80-μm sieve	⩽15	⩽25	⩽10	⩽35	⩽35	⩽35	⩽35	⩾36	⩾36	⩾36	⩾36	⩾36
Characteristics of portions passing the 2-mm sieve:												
— plasticity index	<5		no test	⩽10	⩽10	⩾11	⩾11	⩽10	⩽10	⩾11	⩾11	⩾11
— liquid limit	no test		—	⩽40	⩾41	⩽40	⩾41	⩽40	⩾41	⩽40	⩾41	⩾41
— group index	0		0	0		⩽4		⩽8	⩽12	⩽16	⩽20	⩽20
— general name	cobbles gravels sands		fine sand	mixture of silty gravel or clayey gravel with silty or clayey sand				silty soils			clayey soils	

Sample a. (1) The percent passing the 80 μm = 0.1%, the soil must be classified as granular soil. (2) The percent passing 0.4 mm is more than half, it is 59%. The soil is a fine sand of type A_3 (non plastic).

Sample b. (1) The percent passing the 80 μm: 73% > 35%; it is therefore a fine-grained soil. (2) The plasticity index $I_p = w_L - w_p = 24 - 16 = 8 < 10\%$: the soil is silty. (3) The liquid limit $w_L = 24 < 40\%$: the classification is type A_4: a silt.

Sample c. (1) Percent passing 80 μm: 57% > 35%: a fine-grained soil. (2) plasticity index: $I_p = w_L - w_p = 28 - 17 = 11\%$: a clayey soil. (3) $w_L = 28 < 40\%$; this soil is of type A_6, clay.

Sample d. (1) Percent passing 80 μm: 1% < 35%: a coarse-grained soil. (2) Percent passing the 0.4 mm: 8% < 30%. (3) Percent passing the 2 mm: 84% > 50%: this is the type A_{1b} soil, a gravelly sand.

Sample e. (1) Percent passing 80 μm: 28% < 35%: a coarse-grained soil; (2) Plasticity index $I_p = w_L - w_p = 23 - 16 = 7 < 10\%$. (3) Liquid limit $w_L = 23 < 40\%$: this is a type $A_{2\text{-}4}$ soil, a silty sand.

Summary of answers
Samples a: type A_3, b: type A_4, c: type A_6, d: type A_{1b}, e: type $A_{2\text{-}4}$.

★★*Problem 1.7* Atterberg limits

An Atterberg limits test on soil samples gave the results shown in Tables 1D and 1E.

TABLE 1D

Liquid limits (masses in grams)

Number of blows	17		21		26		30		34	
Test nr.	1*a*	1*b*	2*a*	2*b*	3*a*	3*b*	4*a*	4*b*	5*a*	5*b*
Total wet mass (soil + tare)	9.35	9.68	13.69	12.16	10.11	9.27	10.31	11.08	11.50	9.59
Total dry mass (soil + tare)	8.79	9.20	11.35	10.19	8.67	8.02	8.84	9.42	9.78	8.31
Tare mass	7.11	7.77	4.05	4.05	4.10	4.07	4.10	4.10	4.07	4.05

TABLE 1E

Plastic limits (masses in grams)

	1st test		2nd test	
Container nr.	*A*	*B*	*E*	*F*
Total wet mass	6.32	6.56	6.54	6.36
Total dry mass	5.94	6.15	6.12	5.97
Tare mass	4.06	4.10	4.07	4.05

Calculate the liquid limit w_L and the plastic limit w_p of the soil. What is the plasticity index? Compare the results of w_L with those given by the following (approximate) mathematical relation: $w_L = w(N/25)^{0.121}$.

Classify the soil in accordance with Casagrande's A-line.

Solution

By definition,

$$w = \frac{\text{weight of water}}{\text{weight of dry soil}} = \frac{\text{mass of water}}{\text{mass of dry soil}}$$

The mass of water = total wet mass − total dry mass; the mass of dry soil = total dry mass − tare mass.

The average of two values in each column is taken to make a new tabulation as shown in Table 1F.

TABLE 1F

Liquid limits (w_L)

Number of blows	17		21		26		30		34	
Test nr.	1a	1b	2a	2b	3a	3b	4a	4b	5a	5b
Mass of water	0.56	0.48	2.34	1.97	1.44	1.25	1.47	1.66	1.72	1.28
Mass of soil	1.68	1.43	7.30	6.14	4.57	3.95	4.74	5.32	5.71	4.26
Water content	33.30	33.60	32.10	32.10	31.50	31.60	31.00	31.2	30.10	30.00
Averages	≃33.5		32.10		≃31.6		31.1		≃30.1	

The average values of the water contents are plotted against their corresponding numbers of blows on the graph of Fig. 1.5. By definition, the liquid limit w_L is the water content corresponding to 25 blows. So w_L = 31.6%, say 32%.

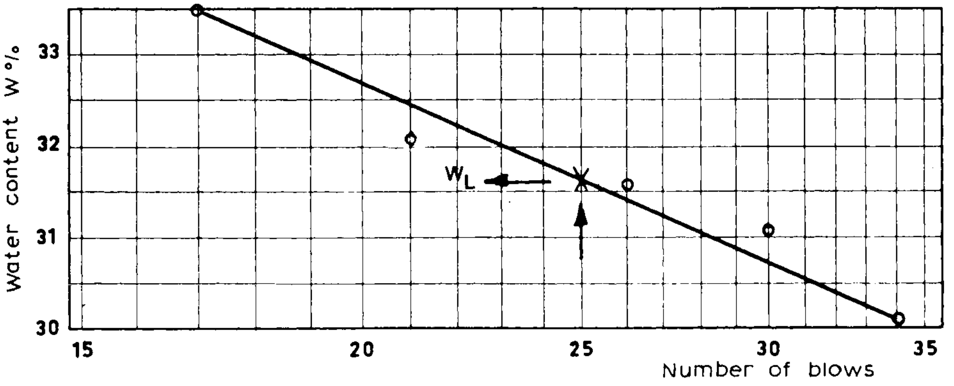


Fig. 1.5. Average water contents plotted against number of blows.

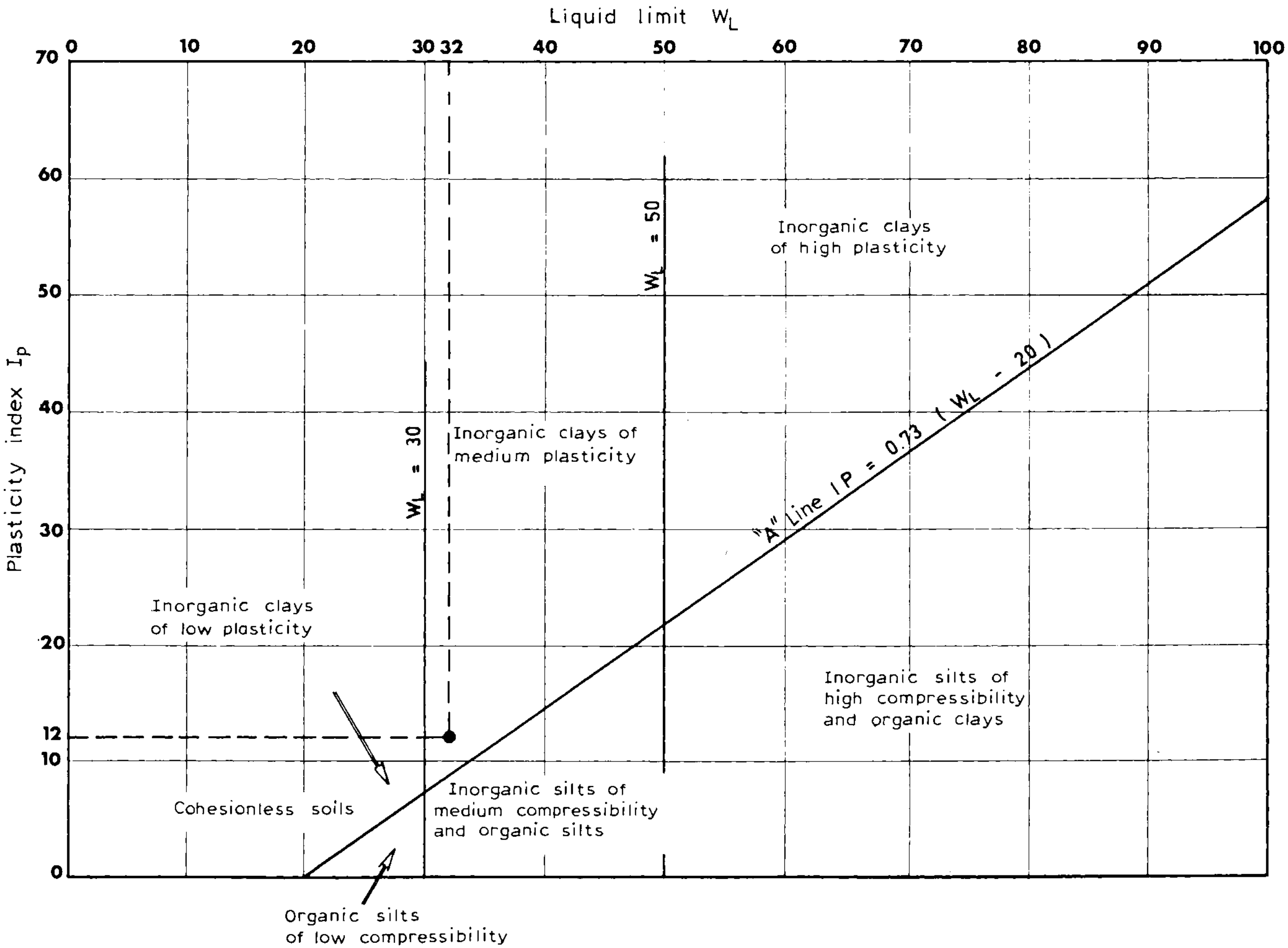


Fig. 1.6. Casagrande's graph.

Table 1G compares the values of w_L obtained from the laboratory test versus those obtained by the use of the empirical formula $w_L = w(N/25)^{0.121}$.

The laboratory determination of w_L entails an error estimated to be half a point of the value of w_l or: $0.5/31.6 = 1.6\%$.

The empirical method yields an average value of 31.6 with a maximum error of 0.4 point. The two methods are acceptable to the same degree of accuracy for this particular soil.

For the plastic limits, w_p, Table 1H, similar to the previous one, can be made up.

TABLE 1G

N	$N/25$	$(N/25)^{0.121}$	w	w_L
17	0.68	0.954	33.5	$\simeq 32$
21	0.84	0.979	32.1	31.4
26	1.04	1.005	31.6	31.7
30	1.20	1.022	31.1	31.8
34	1.36	1.038	30.1	31.2

TABLE 1H

Plastic limits (w_p)

	1st test		2nd test	
Container i.d.	A	B	E	F
Mass of water (g)	0.38	0.41	0.42	0.39
Mass of dry soil (g)	1.88	2.05	2.05	1.92
Water content (%)	20.2	20.0	20.5	20.3
Averages (%)	20.1		20.4	

The plastic limit is 20%, the nearest whole number of the experimental results. Therefore: $w_L = 32\%$, $w_p = 20\%$, $I_p = w_L - w_p = 12\%$.

Casagrande's A-line graph shown in Fig. 1.6 with the results plotted on it indicates that the soil is an inorganic clay of medium plasticity.

★★★*Problem 1.8* Correction of a grain-size distribution curve: scalping and mixing of soils

The sieve analysis of an alluvial gravelly soil sample gave the following size distribution:

$d_{100} = 100$ $d_{75} = 50$ $d_{45} = 20$ $d_{38} = 10$ $d_{34} = 5$

$d_{30} = 2$ $d_{29} = 1$ $d_{25} = 0.5$ $d_{10} = 0.2$ $d_3 = 0.08$

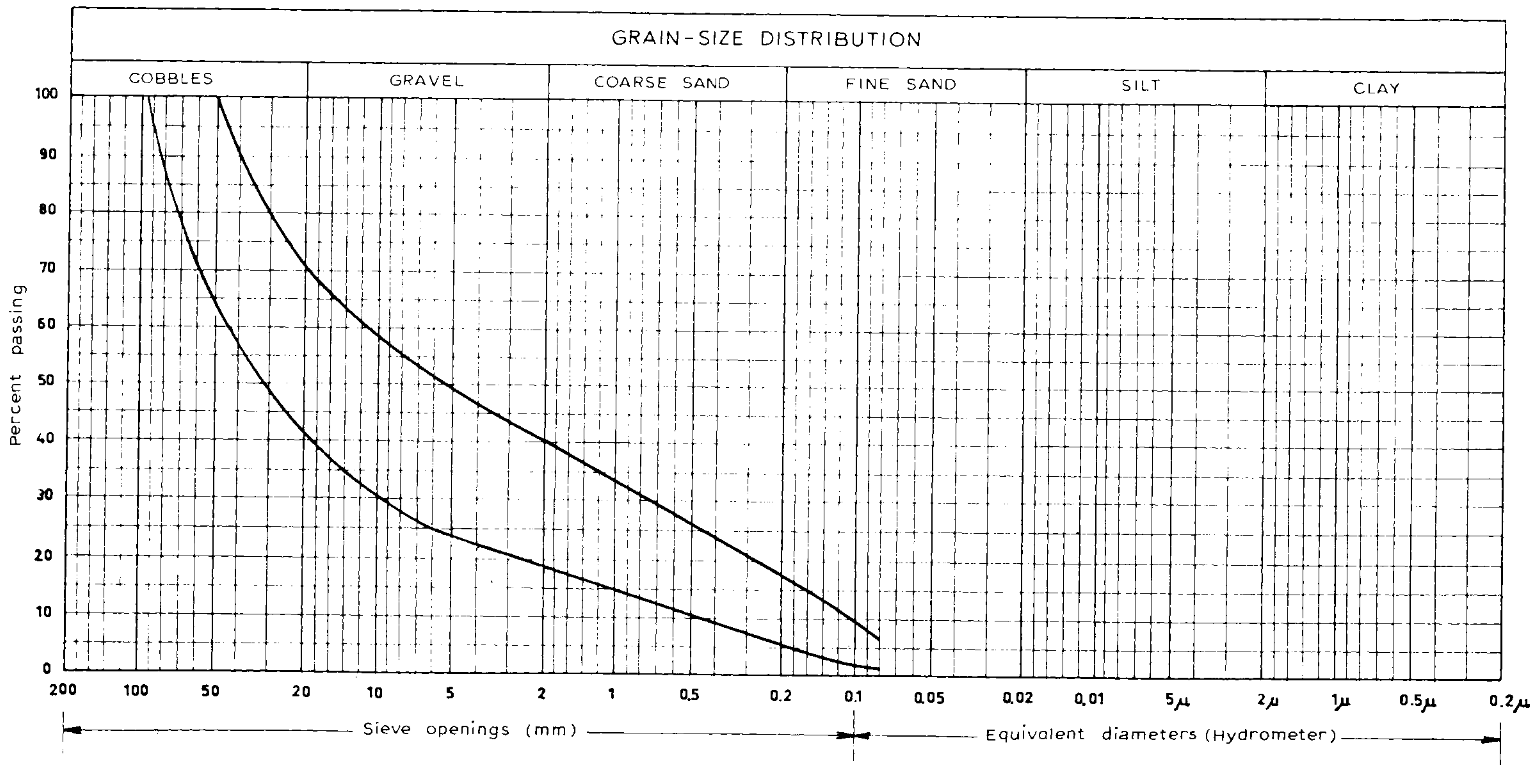


Fig. 1.7. Grain-size distribution for problem 1.8.

(1) Determine whether this material would meet the gradation requirement of an acceptable foundation soil as defined by the limit curves shown in Fig. 1.7.

(2) It is desirable to reduce the sand content by 5% between 0.2 and 0.5, which is, in its present quantity, considered detrimental for achieving proper compaction. However, the percentages of sizes over 10 mm should not be changed. Recommend a procedure to correct the grain-size distribution. All values of d_y are in millimeters.

Solution

(1) The grain-size distribution curves for the upper and lower limits as well as the averages between them and for the soil sample are shown in Fig. 1.8. The curve for the sample is contained entirely within the specified limits.

The soil is an acceptable material for the foundation. It is noticed however, that its grain-size distribution deviates substantially from the average between the upper and lower limits and shows a 'hump' in the sand range between $d = 0.2$ and $d = 2$. This hump is also evident in the histogram plotted in Fig. 1.9, which shows the individual (as opposed to the cumulative) percentages for each consecutive sieve-size opening range. The average-curve histogram is also shown. The 'hump' in the sand fraction is seen to occur more precisely between sieve sizes 0.2 and 0.5.

(2) To reduce the amount of sand in the range 0.2—0.5 by 5% may be interpreted to mean that the quantity of the size corresponding to 15% must be lowered to 10%. Furthermore, there is the requirement not to change the percentages of sizes equal to or greater than 10 mm. In order to achieve this, an amount p of a soil of an as yet undetermined grain-size distribution must be mixed with the alluvial gravelly soil to bring the 0.2—0.5 range of the mixture down to 10%.

For, let us say, 100 kg of gravel G, the weight p to be added, is:

$$15 = 10/100(100 + p) \quad \text{or} \quad p = 50\,\text{kg}.$$

Since all sizes equal to or greater than 10 mm amount to $25 + 30 + 7 = 62\%$ of the weight of the original sample, it is necessary to add a proportionate part, or $0.62 \times 50\,\text{kg} = 31\,\text{kg}$ of material scalped from the gravel retained on sieves 10 mm and above. This only leaves $50 - 31 = 19\,\text{kg}$ to add a material that has the gradation of 'fine gravelly sand' (maximum diameter smaller than 10 mm) but is coarse enough not to have sizes less than 0.5 mm.

From the histogram of Fig. 1.9, it is evident that the amount of gravelly sand in the range 0.5—5.0 mm is below the average gradation of the two allowable limits. One possible solution to lower d_{15} would be to add 19 kg of gravelly sand with a gradation between 0.5 and 5. Such a sand (with a distribution $d_{100} = 5$ mm, $d_{90} = 2$ mm and $d_{30} = 1$ mm) is plotted in Fig. 1.10 as sand S. Hence Table 1I is obtained.

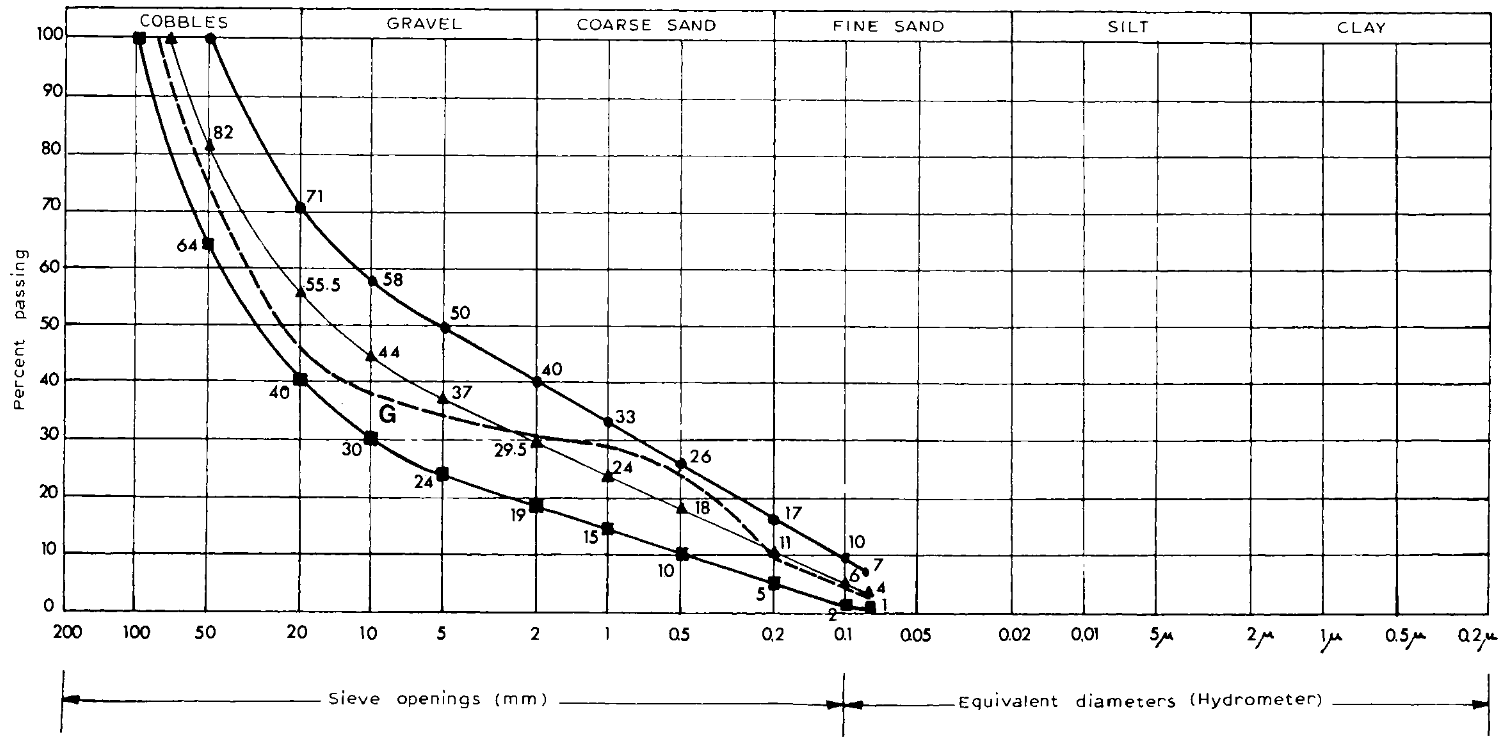


Fig. 1.8. Grain-size distribution of gravel sample G, values of allowable and average limits.

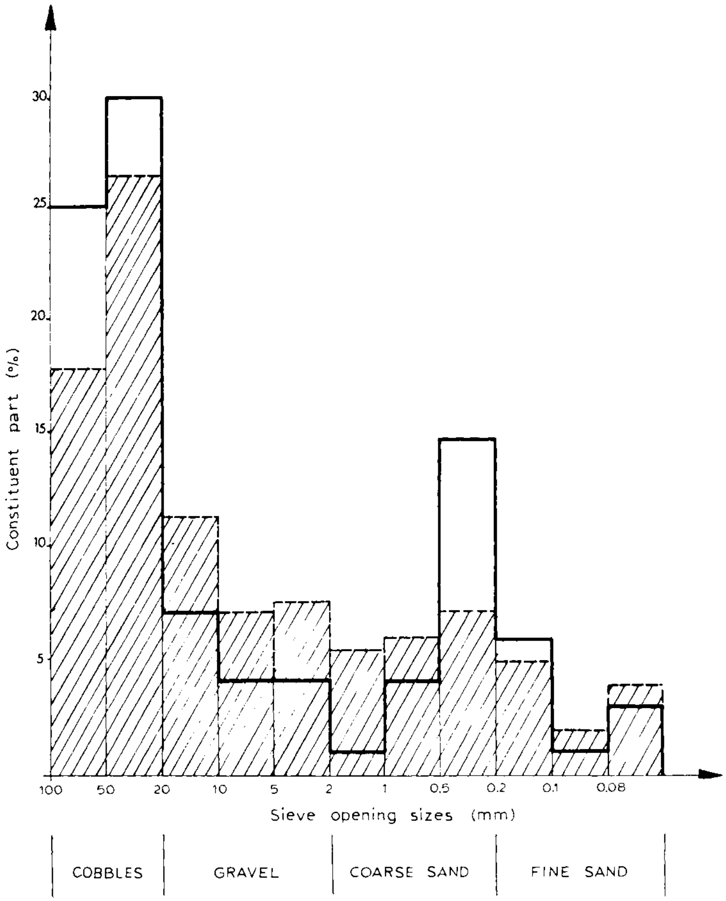


Fig. 1.9. Grain-size histogram of gravel sample G. Cross-hatched areas represent the average curve between allowable limits.

Fig. 1.11 shows the corrected curve G' obtained on the data of Table 1J. The boxed-in figures of that Table indicate that the requirements of the problem have been met.

The corrected curve G' (see Fig. 1.11) is based on the following figures.

TABLE 1I

Sieve size	Percent passing	Constituent part	Weight for 19 kg of sand S
5	100	10	1.9
2	90	60	11.4
1	30	30	5.7
0.5	0		$\Sigma = 19.0$

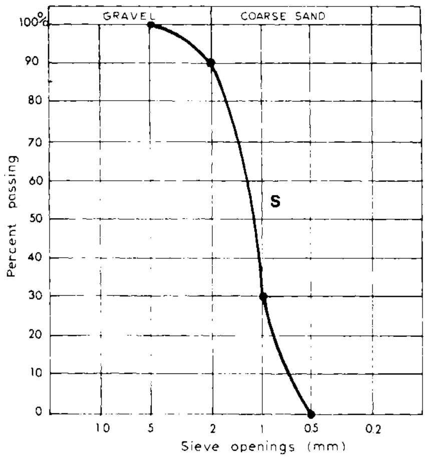


Fig. 1.10. Grain-size distribution of sand S.

TABLE 1J

Sieve sizes	Weight in kg at various constituents for 150 kg of G'		Constituent parts (%)	% passing
100				100
	25 × 1.5 =	37.50	25%	
50				75
	30 × 1.5 =	45.00	30%	
20				45
	7 × 1.5 =	10.50	7%	
10				38
		4.00	2.6%	
5				35.4
	4 + 1.9 =	5.90	3.9%	
2				31.5
	1 + 11.4 =	12.40	8.3%	
1				23.2
	4 + 5.7 =	9.70	6.5%	
0.5				16.7
		15.00	10%	
0.2				6.7
		6.00	4%	
0.1				2.7
		1.00	0.7%	
0.08				2
		3.00	2%	
	$\Sigma = 150$ kg			0

The values in Table 1J show that the imposed conditions are verified:
% of $d_y > 10$ unchanged
% of $0.2 < d_y < 0.5$ decrease to 10%

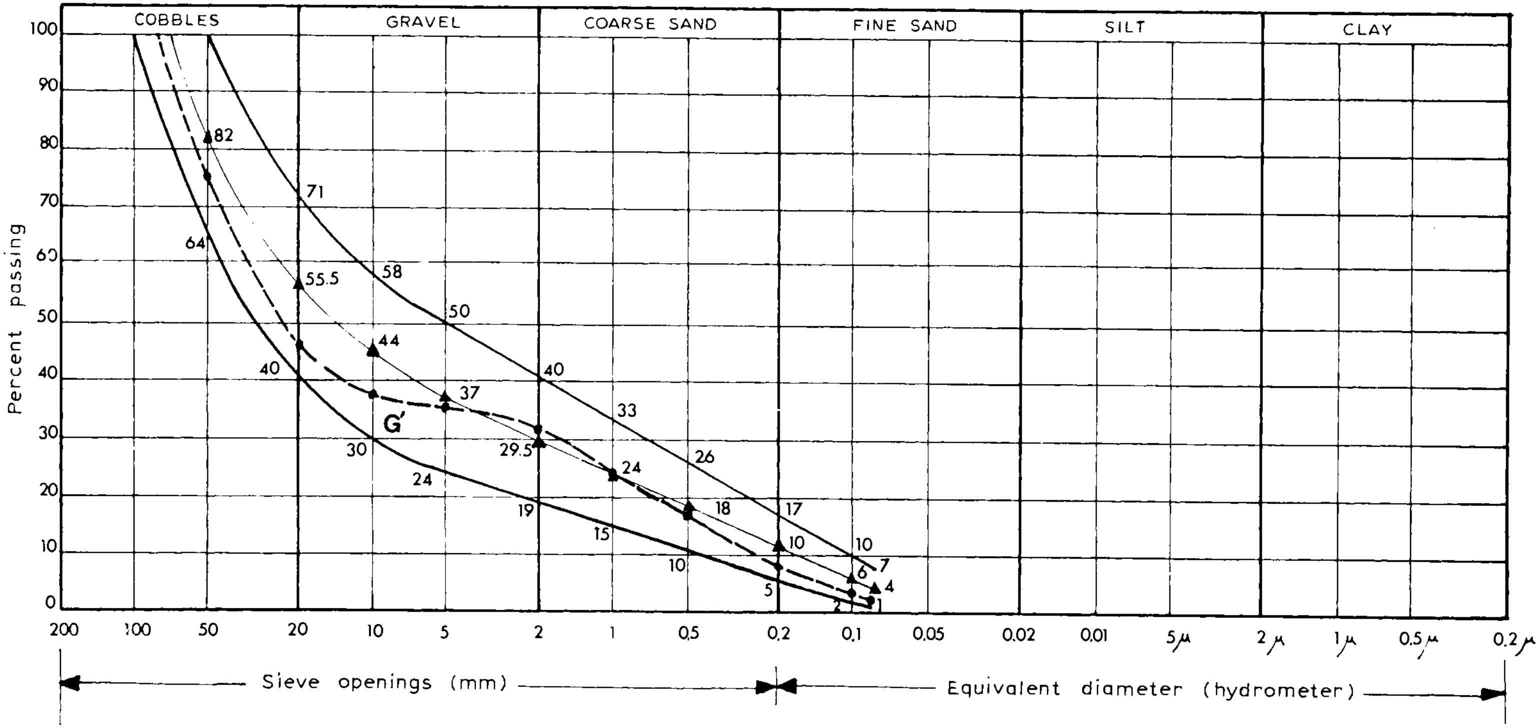


Fig. 1.11. Grain-size distribution of corrected material.

Conclusions. The natural gravel sample has to be scalped on a 10-mm size screen. Retained material must be mixed with a gravelly sand soil meeting the gradation of material S.

For each ton of gravel G, 310 kg of the scalped material and 190 kg of sand S will have to be added, to meet requirements. In actual practice the solution of the problem could read: add 300 kg of scalped material for each ton of gravel and add 200 kg of sand S for each ton of gravel G.

★★★*Problem 1.9* Compaction, Proctor diagram and saturation curve

(a) A modified Proctor-test yielded the following values for water content and densities of a clayey gravel.

$w(\%)$:	3.00	4.45	5.85	6.95	8.05	9.46	9.90
γ_d/γ_w:	1.94	2.01	2.06	2.09	2.08	2.06	2.05

Draw the Proctor compaction curve and determine values at optimum condition. Calculate the degree of saturation corresponding to the optimum condition, assuming the soil specific gravity to be 2.65.

(b) Calculate the percentage of air for a given porosity n and degree of saturation S_r. On the dry density–moisture content graph, find the equation of the curve connecting points of equal degree of saturation (or equal percentage of air voids). From this, determine the equation of the curve for 100% saturation. What are the characteristics of this curve?

(c) Consider an equilateral triangle ASW whose height is Aa or Ss or Ww. Show that the conditions of a soil regarding the volumes of air, of soil grains and of water can be represented by a point M located inside the triangle in such a manner that the perpendicular distances from point M to each of the triangle sides are proportional to three volumes, V_a, V_s and V_w.

— draw in the triangle curves of equal air-void percentage;

— what does the saturation curve of the Proctor-diagram represent?

— show that the set of straight lines from point S correspond to the lines showing the state of soils for a constant degree of saturation;

— draw in this diagram the Modified Proctor-compaction curve of question (a) above;

— on a random curve C, analogous to the test curve, consider two points M_1 and M_2 so that M_1M_2 is parallel to AW. What can be said about the state of soils at points M_1 and M_2?

Solution

(a) The test results may be plotted directly on a graph such as that of Fig. 1.12. With the dry density as ordinate and water content as abscissa. The coordinates at maximum dry density correspond to the optimum dry density and optimum water content (the so-called modified Proctor line), are: $(\gamma_d/\gamma_w)_{opt} = 2.09$, $w_{opt} = 7.5\%$

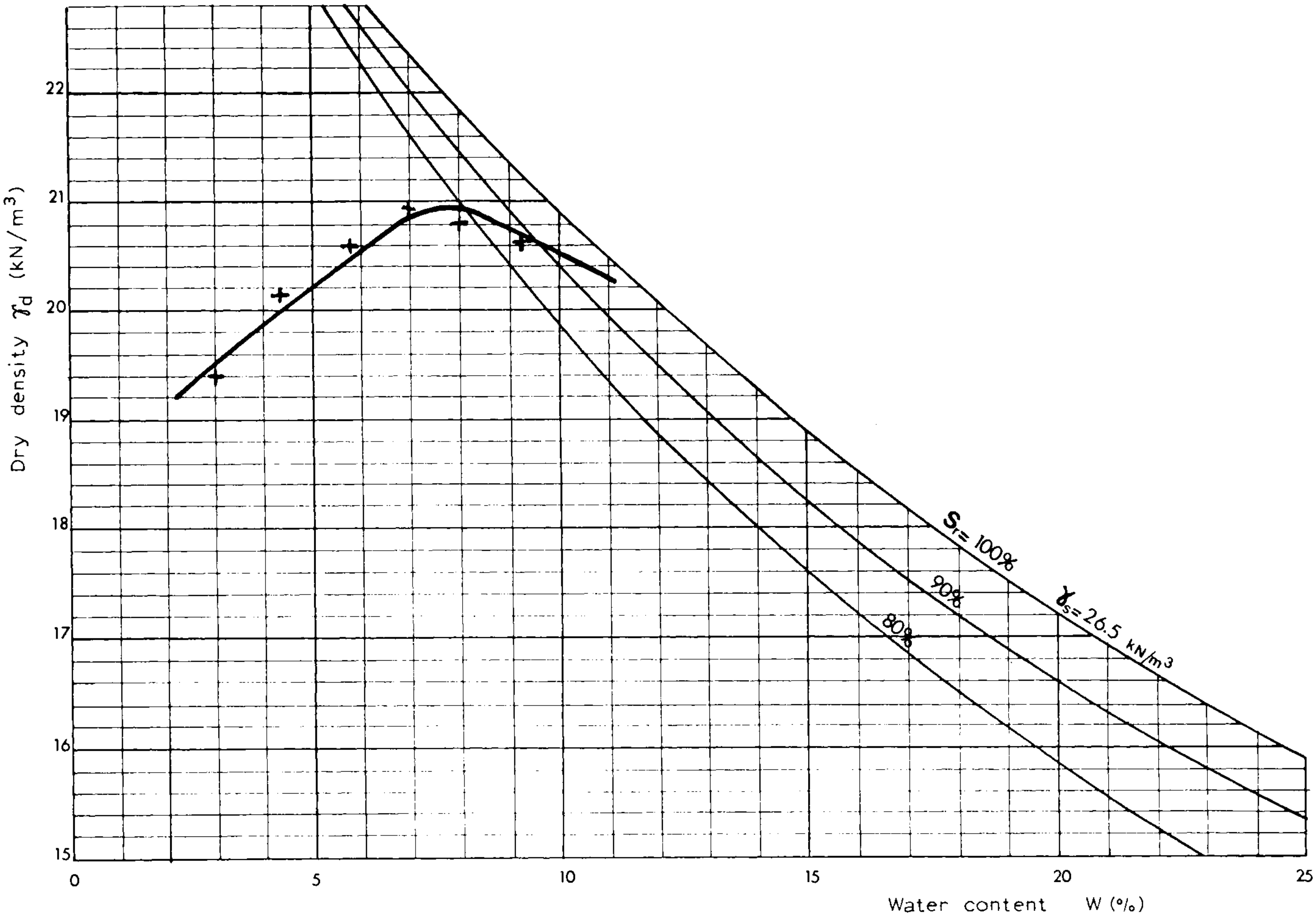


Fig. 1.12. Modified Proctor Test, C.B.R mold. $\gamma_{d\,\mathrm{opt}} = 20.9\ \mathrm{kN/m^3}$. $W_{\mathrm{opt}} = 7.5\%$. Test made on fraction 0—20 mm.

Since $W = \dfrac{eS_r\gamma_w}{\gamma_s}$ and $\gamma_d = \gamma_s/(1+e)$, then $S_r = (w/e) \times (\gamma_s/\gamma_w)$

and $e = (\gamma_s/\gamma_d) - 1$.

Therefore: $\dfrac{1}{e} = \gamma_d/(\gamma_s - \gamma_d)$.

At optimum condition the degree of saturation will be:

$$S_{r(opt)} = w_{opt} \times (\gamma_s/\gamma_w) \times \gamma_{d(opt)}/(\gamma_s - \gamma_{d(opt)}) = 7.5 \times 2.65 \times \frac{2.09}{2.65 - 2.09}$$
$$\simeq 74\%$$

(b) Fig. 1.13 is a graphic representation of a unit soil volume, where a = volume of air $= 1 - nS_r - (1-n)$ or $a = n(1 - S_r)$.

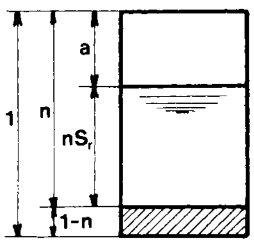


Fig. 1.13.

The following relationships exist: $a = 1 -$volume of grains$-$volume of water;
volume of grains = (weight of grains)/γ_s = γ_d/γ_s;
volume of water = weight of water/γ_w

$$= \text{(weight of water)/(weight of grains)} \times \frac{\text{weight of grains}}{\gamma_w} = w(\gamma_d/\gamma_w)$$

then $a = 1 - \dfrac{\gamma_d}{\gamma_s} - w\dfrac{\gamma_d}{\gamma_w}$.

Since for any soil at a particular moisture content γ_s and γ_w are constants, all points representing states of soil for a given percentage of air voids are on one curve whose equation is $1 - \dfrac{\gamma_d}{\gamma_s} - w\dfrac{\gamma_d}{\gamma_w} = a$ (constant)

or: $\dfrac{\gamma_d}{\gamma_w} = \dfrac{(1-a)\gamma_s}{\gamma_s w + \gamma_w}$.

This is a portion of a hyperbola with $w > 0$, whose asymptote is the w-axis, passing through $w = 0$, such that $\gamma_d = (1 - a)\gamma_s$ (see Fig. 1.14).

If $a = 0$, the volume of air is zero and the soil is saturated. The saturation curve then is represented by the equation: $\dfrac{\gamma_d}{\gamma_w} = \dfrac{\gamma_s}{\gamma_s w + \gamma_w}$.

Note: The same result is obtained by determining the equation of a family of curves of equal saturation S_r:

$$S_r = \frac{\text{volume of water}}{\text{volume of air} + \text{volume of water}}$$

or, for a unit volume:

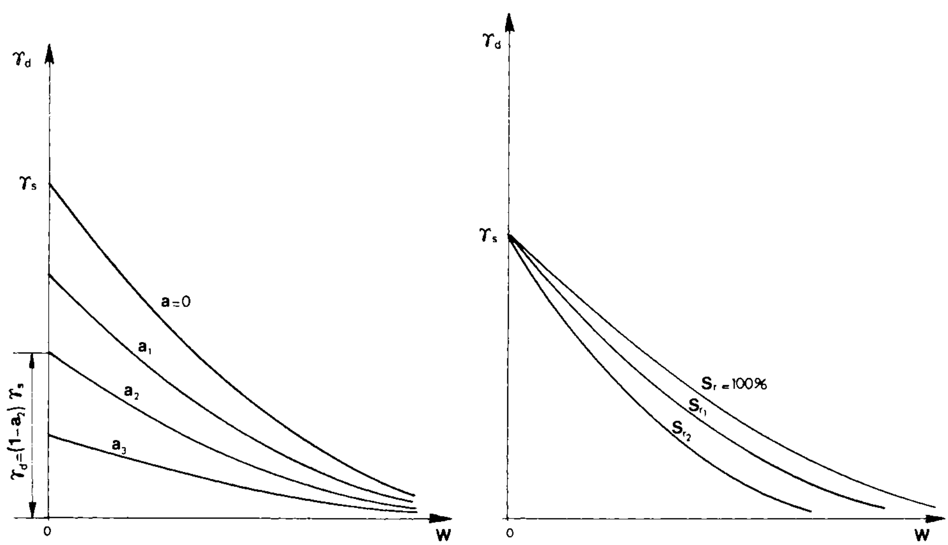
$$S_r = \frac{\text{volume of water}}{1 - \text{volume of soil grains}} = \frac{w(\gamma_d/\gamma_w)}{1 - (\gamma_d/\gamma_s)}$$

or: $\gamma_d\left(\dfrac{w}{\gamma_w} + \dfrac{S_r}{\gamma_s}\right) = S_r$

$$\frac{\gamma_d}{\gamma_w} = \frac{S_r \gamma_s}{w\gamma_s + S_r\gamma_w}$$

The curves are also hyperbolas with the w-axis as an asymptote. Only the sections corresponding to $w > 0$ have a physical meaning.

If $w = 0$, all the curves pass through point γ_s (see Fig. 1.15). If $S_r = 1$ in



Figs. 1.14 and 1.15

the above formula, the full saturation equation is obtained which is identical to the first one.

(c) A soil is a three-phase system defined by the respective fractional volumes of air, water and soil adding up to a unit volume, so that:

$V_a + V_w + V_s = 1$ (Fig. 1.16)

In general: $0 \leqslant V_a \leqslant 1$, $0 \leqslant V_w \leqslant 1$, $0 \leqslant V_s \leqslant 1$.

If we now consider an equilateral triangle and a point M inside that triangle (Fig. 1.17) it can easily be shown that the sum of the perpendiculars from M

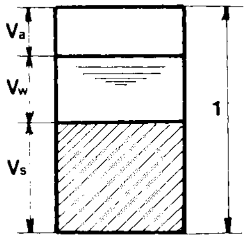


Fig. 1.16

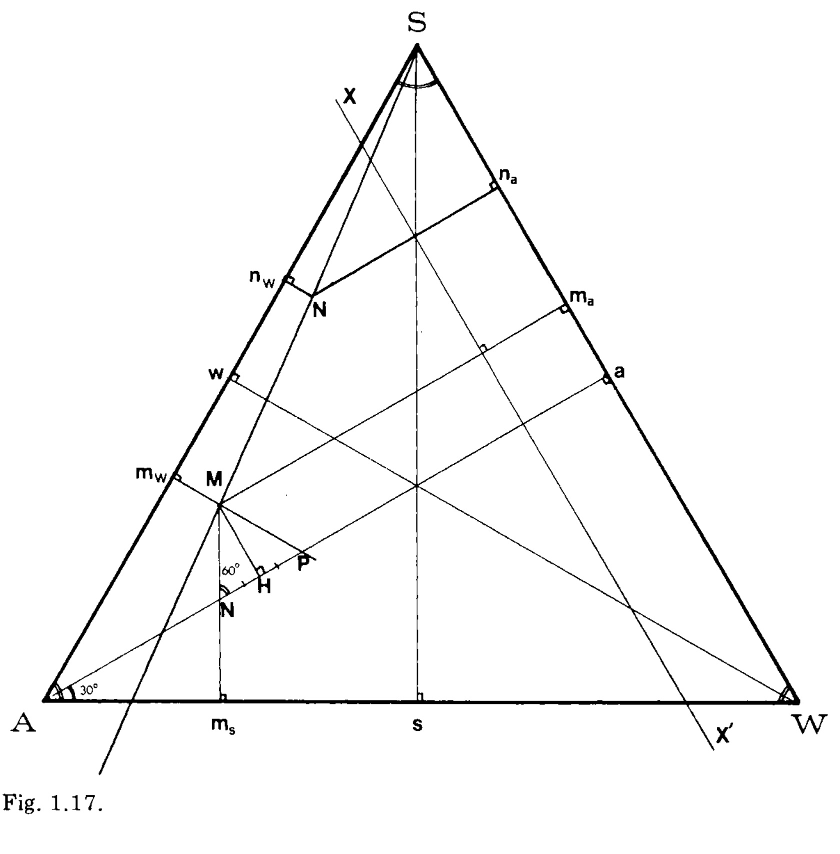


Fig. 1.17.

to the three sides is equal to the height of the triangle. From Fig. 1.17:

$Ha = Mm_a$

$Km_s = \frac{1}{2}AK$

$PM_w = PM + Mm_w = KM + Mm_w = \frac{1}{2}AP$

$KM_s + KM + Mm_w = Mm_s + Mm_w = \frac{1}{2}(AK + AP) = AH$

hence: $Mm_s + Mm_w + Mm_a = AH + Ha = Aa = 1.$

If the height of the equilateral triangle is unity, the perpendiculars from M represent the volumes of the three phases (soil, water and air).
— The curves of equal air void volume (V_a = constant) are straight lines parallel to the side SW (for example $X'\,X$).
— The saturation line of the Proctor-diagram is given by side SW ($V_a = 0$).
— Let N be any point on line SM where the projections on SW and SA are na and n_w, respectively, then the similarity of the triangles SMm_w and $S'Nn_w$ gives:

$$\frac{V_w}{V'_w} = \frac{SM}{SN} \text{ and } \frac{V_a}{V'_a} = \frac{SM}{SN}.$$

By definition $S_r = V_w/(V_a + V_w)$.
The degree of saturation represented by point N is:

$$S'_r = \frac{V'_w}{V'_a + V'_w} = \frac{(SM/SN)\,V_w}{(SM/SN)(V_a + V_w)} = S_r$$

The straight lines from S are therefore lines of equal degree of saturation and we have: $V_s = \gamma_d/\gamma_s$, $V_w = (\gamma_d/\gamma_w)w$, $V_a = 1 - (V_s + V_w)$.
Going back to the Modified Proctor-test results, Table 1K can be made up:

TABLE 1K

w%	3.00	4.45	5.85	6.95	8.05	9.46	9.90
γ_d/γ_w	1.94	2.01	2.06	2.09	2.08	2.06	2.05
V_s	0.732	0.758	0.777	0.789	0.785	0.777	0.774
V_w	0.058	0.089	0.121	0.145	0.167	0.195	0.203
V_a	0.210	0.153	0.102	0.066	0.048	0.028	0.023

Fig. 1.18 shows the modified Proctor-curve in the ASW triangle.

Points M_1 and M_2 correspond to the soil states existing at constant dry unit weight (see Fig. 1.19): $(\gamma_d)_{M_1} = (\gamma_d)_{M_2}$ and, therefore, a constant void volume. The voids are filled in part by incompressible water and compressible air. However, the proportionate parts of water and air at M_1 and M_2 are not the same:

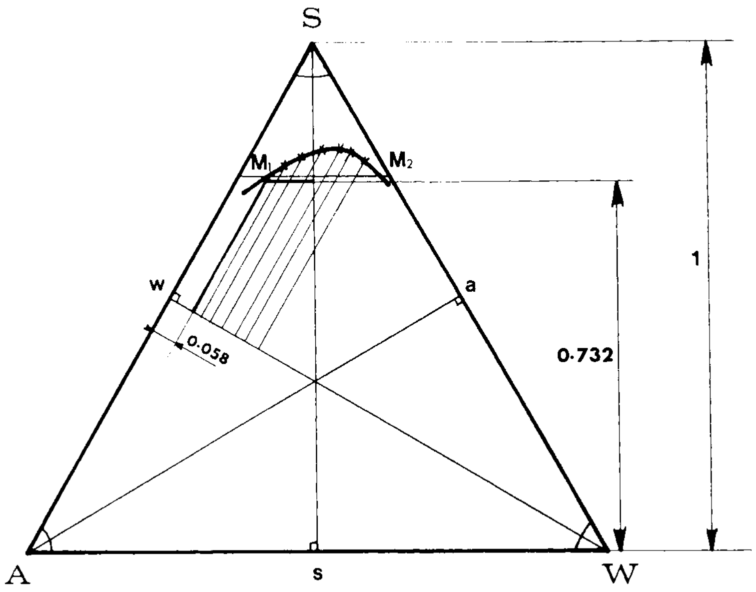


Fig. 1.18.

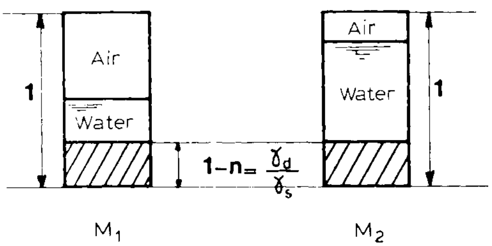


Fig. 1.19.

— in order to improve the mechanical properties of the soil in condition M_1 its water content will have to be increased to within a close range around the optimum moisture content, then the soil will have to be compacted to increase its dry unit weight;

— on the other hand, in order to increase the mechanical properties of the same soil at point M_2, the moisture content should be decreased to a value beneath w_{opt} (by drying) and the soil then compacted.

It will be noticed that the water content at M_2 is near 100% saturation. Compacting this soil at that moisture content would tend to bring the soil close to complete saturation and would likely lead to pumping, causing excessive deformations of the soil. It would not be possible to use the compacted material, for instance, for a stable pavement subgrade.

★★*Problem 1.10* Void ratio of an organic soil

Let us assume that the unit weights of the soil, γ_{sm}, and organic matters, γ_{so} are known. Then:

(1) What is the unit weight of the combined dry organic soil whose organic content is M_0(*)?

(2) What is the void ratio of this soil, if it is known that its water content is w and its degree of saturation is S_r?

Solution

We use the following definitions (Fig. 1.20):

$$\text{for void ratio: } e = \frac{\text{volume of voids}}{\text{volume of soil grains}}, \tag{1}$$

$$\text{for degree of saturation: } S_r = \frac{\text{volume of water}}{\text{volume of voids}}, \tag{2}$$

$$\text{for water content: } w = \frac{\text{weight of water}}{\text{weight of dry soil}}, \tag{3}$$

$$\text{for organic content: } M_0 = \frac{\text{dry organic matter weight}}{\text{total dry sample weight}} \tag{4}$$

for a unit weight of dry soil we have: M_0 = weight of organic matter,
$1 - M_0$ = weight of mineral matter,
M_0/γ_{so} = volume of organic matter,
$(1 - M_0)/\gamma_{sm}$ = volume of mineral matter.

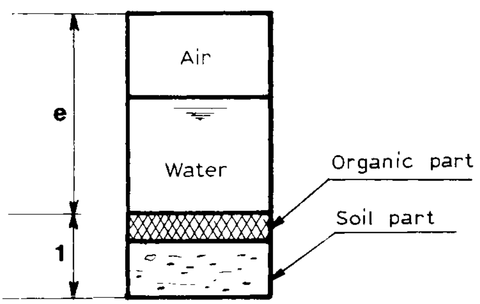


Fig. 1.20.

*Note: The organic content is the percentage by weight, of the dry organic constituent of the total dry weight of sample for a given volume.

The total unit weight, γ_s, of the dry soil is the weight of a unit volume:

$$\gamma_s = \frac{1}{(M_0/\gamma_{so}) + [(1-M_0)/\gamma_{sm}]}$$

or: $$\gamma_s = \frac{\gamma_{so} \times \gamma_{sm}}{M_0(\gamma_{sm} - \gamma_{so}) + V_{so}}$$

γ_s is a function of the form $Y = a/(bx + c)$, meaning that the curve representing Y as a function of M_0 is a part of a hyperbola (see Fig. 1.21). Since γ_{sm} ($= 26.5$ kN/m^3) is always a greater value than γ_{so}, the curve, which is only real for value where $0 \leqslant M_0 \leqslant 100\%$, decreases and varies between the limit values γ_{sm}, for $M_0 = 0$, and γ_{so}, for $M_0 = 100\%$ (see Fig. 1.21).

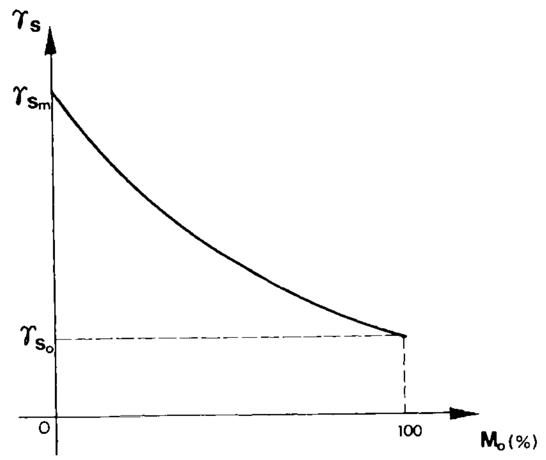


Fig. 1.21.

The expression for S_r can be transformed to give:

$$\text{volume of voids} = \frac{\text{volume of water}}{S_r}$$

but we know also that:

$$\text{volume of water} = \frac{\text{weight of water}}{\gamma_w} = w \times \frac{(\text{weight of dry matter})}{\gamma_w}$$

From (1):

$$e = \frac{w}{S_r \gamma_w} \cdot \frac{1}{(M_0/\gamma_{so}) + [(1-M_0)/\gamma_{sm}]}$$

or:

$$e = \frac{w}{S_r\gamma_w} \cdot \frac{\gamma_{so}\gamma_{sm}}{M_0(\gamma_{sm} - \gamma_{so}) + \gamma_{so}}$$

Summary of answers

$$\gamma_s = \frac{\gamma_{so} \times \gamma_{sm}}{M_0(\gamma_{sm} - \gamma_{so}) + \gamma_{so}}$$

$$e = \frac{w}{S_r\gamma_w} \times \frac{\gamma_{so}\gamma_{sm}}{M_0(\gamma_{sm} - \gamma_{so}) + \gamma_{so}}$$

★★*Problem 1.11* **Hydrometer analysis**

A hydrometer analysis is performed on a 2000 cm³ solution containing 50 g of dry soil. The solution concentration at a depth of 5 cm is 5 g/l after a sedimentation time of 80 minutes. Find:

(a) the maximum diameter d_y of the particles at that depth and time;

(b) the percentage of dry soil of particles having a diameter equal to or smaller than d_y.

Assume $\eta = 1$ cPo (dynamic viscosity of water at 20°C) and $\gamma_s/\gamma_w = 2.65$.

Solution

(a) The maximum diameter d_y is that of the soil particles which at time zero were at the surface of the solution and at time $t = 80$ min., have travelled 5 cm.

$$d_y = \sqrt{\frac{18\,\eta}{\gamma_s - \gamma_w} \cdot \frac{H}{t}}$$

In the C.G.S. system:

$t = 80 \times 60 = 4800$ s

$H = 5$ cm

$\eta = 1\,cPo = 1 \cdot 10^{-2}\,Po$

$\gamma_s = 2.65 \times 9.81$ dynes/cm³

$\gamma_w = 1 \times 9.81$ dynes/cm³

$$d_y = \sqrt{\frac{18 \cdot 10^{-2} \times 5}{9.81(2.65 - 1.00) \times 4.8 \cdot 10^3}} = 3.4 \cdot 10^{-3} \text{ cm (or } 34\,\mu\text{m).}$$

(b) The density of the solution at 5 cm after 80 min. is:

$$r = \frac{\gamma}{\gamma_w} = \frac{5 + \left(1000 - \frac{5}{2.65}\right) \times 1}{1000 \times 1} = 1.003$$

The percentage of weight is:

$$y = \frac{V}{p} \frac{\gamma_s \gamma_w}{\gamma - \gamma_w} (r - 1) = \frac{2000}{50} \times \frac{2.65 \times 1.00}{2.65 - 1.00} (1.003 - 1) = 0.19$$

$y = 19\%$

where V is the volume of the solution and p the weight of the soil.

Summary of answers
$d_y = 3.4 \cdot 10^{-3}$ cm; $y = 19\%$.

★★*Problem 1.12* **Relative density (English units)**

Determine the relative density of a calcareous sand ($\gamma_{s1} = 146.3$ lb/cu. ft) and that of a quartzitic sand ($\gamma_{s2} = 165.6$ lb/cu. ft) whose void ratios are:
— for the calcareous sand: $e_{max} = 0.89$, $e_{min} = 0.62$;
— for the quartzitic sand: $e_{max} = 0.98$, $e_{min} = 0.53$.

The measurement of the above void ratios was made in a Proctor-mold ($\phi = 4$ in., $H = 4.59$ in.) filled with following weights: $P_1 = 2.868$ lb of dry calcareous sand, $P_2 = 3.254$ lb of dry quartzitic sand.

Solution

Relative density is: $D_r = I_d = \dfrac{e_{max} - e}{e_{max} - e_{min}}$.

The void ratio of the calcareous sand is:

$$e_1 = \frac{\pi H \phi^2/4 - P_1/\gamma_{s1}}{P_1/\gamma_{s1}} = \frac{(3.14 \times 4.59 \times 4^2/4) - (2.868/146.3) \times 12^3}{2.868 \times 12^3/146.3} = 0.70$$

and that of the quartzitic sand is:

$$e_2 = \frac{\pi H \phi^2/4 - P_2/\gamma_{s2}}{P_2/\gamma_{s2}} = \frac{(3.14 \times 4.59 \times 4^2/4) - (3.254 \times 12^3/165.6)}{3.254 \times 12^3/165.6} = 0.70.$$

It will be noticed that $e_1 = e_2$.

The relative density for each of the sands is:

$$D_{r1} = \frac{0.89 - 0.70}{0.89 - 0.62} = 0.70 \quad \text{calcareous sand}$$

$$D_{r2} = \frac{0.98 - 0.70}{0.98 - 0.53} = 0.62 \quad \text{quartzitic sand.}$$

The two sands, even though they have the same in place void ratio, have different relative densities, which indicates that the grains of the calcareous sand are more tightly packed than those of the quartzitic sand. This cannot be concluded either from the void ratio or from the dry unit weights.

It is to be noted that the dry unit weight is greater for the soil with lower relative density:

$$(\gamma_d)_1 = \frac{\gamma_{s1}}{1+e_1} = \frac{146.3}{1+0.7} = 86.1 \text{ lb/cu. ft} \quad (D_{r1} = 0.70)$$

$$(\gamma_d)_2 = \frac{\gamma_{s2}}{1+e_2} = \frac{165.6}{1+0.7} = 97.4 \text{ lb/cu. ft} \quad (D_{r2} = 0.62).$$

Summary of answers
$(I_D)_1 = (D_r)_1 = 0.70$; $(I_D)_2 = (D_r)_2 = 0.62$.

★★*Problem 1.13* Design of an optimum grain-size distribution by mixing soils

Three soils are available. One consists of a gravel, another of a sandy gravel and the third of a sand. Their individual grain-size distributions are shown in Fig. 1.22 (curves 1, 2 and 3).

It is desirable to mix the three soils in such a way that the combined grain-size distribution would closely approximate the average theoretical curve representing the mean of the upper and lower acceptable limits (see curve 4 of Fig. 1.22).

Calculate the relative parts, in percent, α, β and γ of the three soils in order to achieve the average size-distribution.

Solution

The method to use is the least-squares method applied to the deviations between randomly mixed material sizes and the average optimal curve for selected grain diameters.

Let T_i, t_{1i}, t_{2i}, t_{3i} be the accumulated amounts of optimal material and materials 1, 2, 3 passing through sieve number i, and α, β, γ the amount by weight of each material in the mix.

The square of the deviation of the optimal curve and that actually obtained, down to sieve size i, is:

$$\Delta_i = [T_i - (\alpha t_{1i} + \beta t_{2i} + \gamma t_{3i})]^2 .$$

The sum of the squares of the deviations (sum over the first 10 sieves) is

$$\Delta = \sum_{i=1}^{i=10} \Delta_i .$$

We also know that $\alpha + \beta + \gamma = 1$, or $\gamma = 1 - (\alpha + \beta)$.

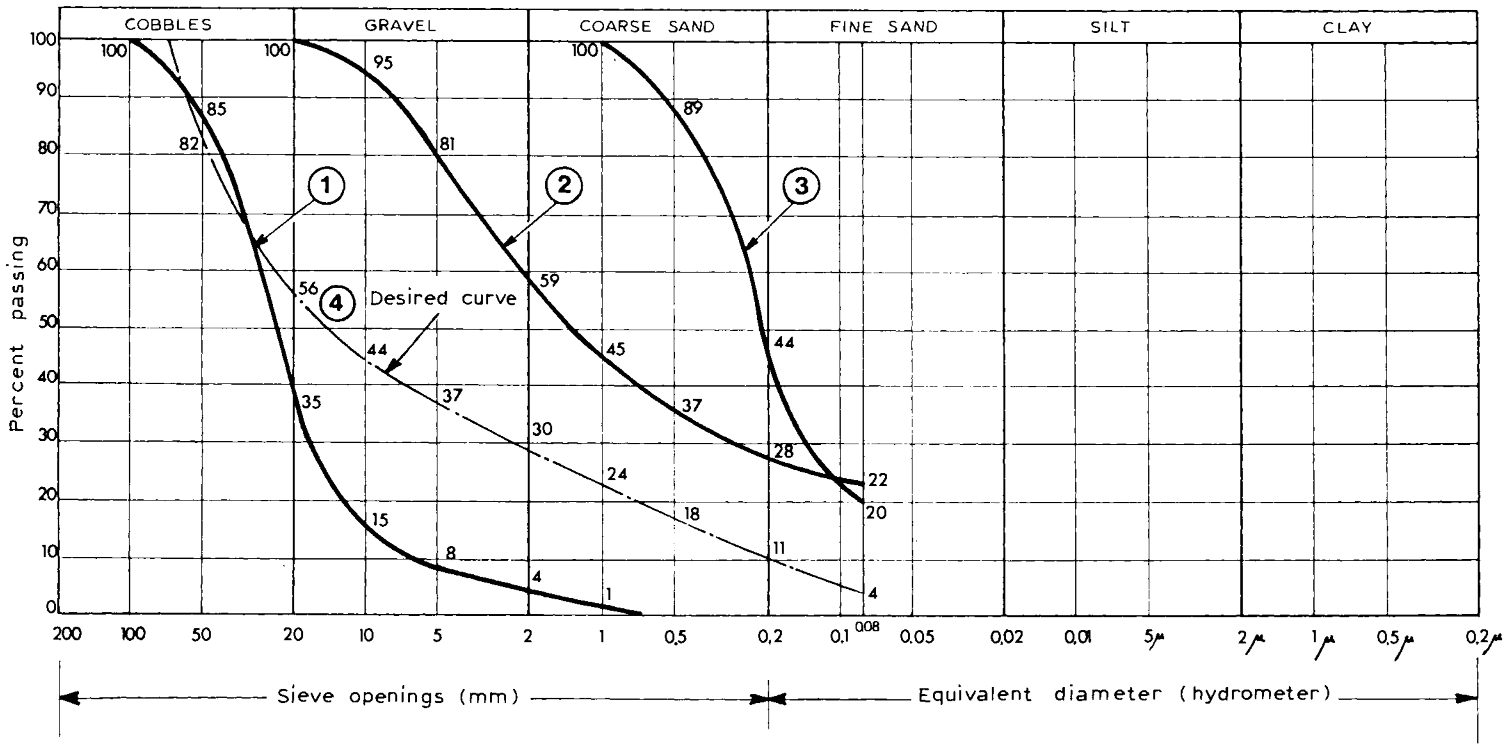


Fig. 1.22. Grain-size distribution curves.

Hence: $\Delta = \sum_{1}^{10} [T_i - (\alpha t_{1i} + \beta t_{2i}) - \{1 - (\alpha + \beta)\} t_{3i}]^2$

$$\Delta = \sum_{1}^{10} [(T_i - t_{3i}) - \alpha(t_{1i} - t_{3i}) - \beta(t_{2i} - t_{3i})]^2 .$$

If we let: $U_i = T_i - t_{3i}$, $u_{1i} = t_{1i} - t_{3i}$, $u_{2i} = t_{2i} - t_{3i}$,

we have: $\Delta = \sum_{1}^{10} [U_i - \alpha u_{1i} - \beta u_{2i}]^2 .$

The sum Δ must be a minimum, or its partial derivative with respect to α and β, must be zero:

$$\frac{\partial \Delta}{\partial \alpha} = \sum_{1}^{10} 2(U_i - \alpha u_{1i} - \beta u_{2i}) \times u_{1i} = 0$$

$$\frac{\partial \Delta}{\partial \beta} = \sum_{1}^{10} 2(U_i - \alpha u_{1i} - \beta u_{2i}) \times u_{2i} = 0$$

or:

$$\alpha \sum_{1}^{10} u_{1i}^2 + \beta \sum_{1}^{10} u_{1i} u_{2i} = \sum_{1}^{10} U_i u_{1i}$$

$$\alpha \sum_{1}^{10} u_{1i} u_{2i} + \beta \sum_{1}^{10} u_{2i}^2 = \sum_{1}^{10} U_i u_{2i}$$

This is a linear set of equations for which α, β, and γ may be determined knowing that: $\gamma = 1 - (\alpha + \beta)$.

Tables 1L and 1M summarize the percentages and sizes of the curves of Fig. 1.22 and the coefficients of the linear set of equations. From these the following equations are obtained: $49{,}413\alpha + 16{,}846\beta = 35{,}321$; $16{,}846\alpha + 8{,}056\beta = 12{,}715$, resulting in $\alpha = 62\%$, $\beta = 29\%$, $\gamma = 9\%$.
Hence the amount of mixed soil passing through sieve number i is:

$$T_i' = \alpha t_{2i} + \beta t_{2i} + \gamma t_{3i}.$$

Table 1N summarizes the calculation for different percentages of soil retained in order to construct the grain-size distribution curve of the mixed soil, curve 5 of Fig. 1.23.

If the three soils are mixed in the proportions shown above, the grain-size distribution (curve 5 of Fig. 1.23) is obtained, which corresponds to the first ten sieve-sizes.

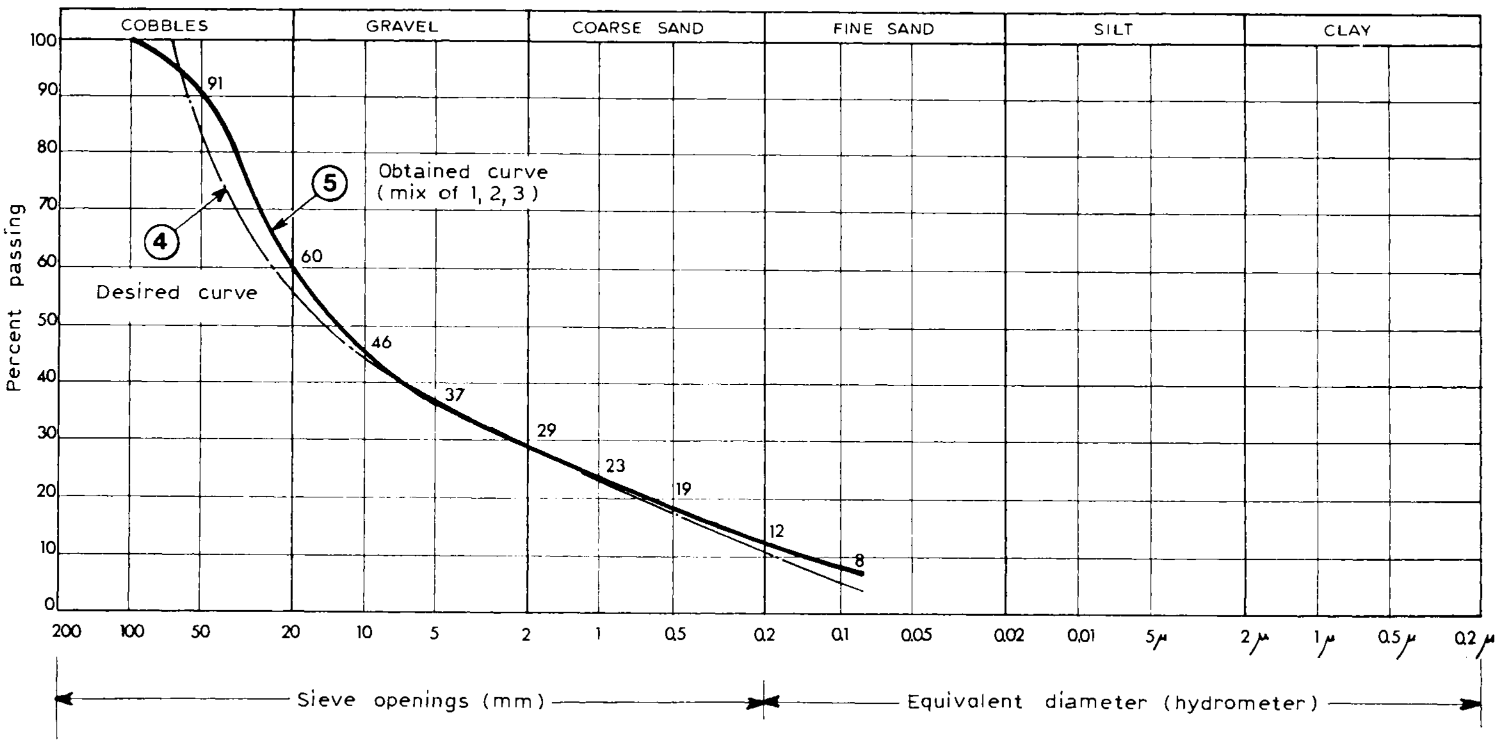


Fig. 1.23. Grain-size distribution curve of the mixture.

TABLE 1L

Sieve number	Opening size (mm)	Percent passing (%)			
		T_i optimum (4)	t_{1i} gravel (1)	t_{2i} gravel (2)	t_{3i} sand (3)
1	100	100	100	100	100
2	50	82	85	100	100
3	20	56	35	100	100
4	10	44	15	95	100
5	5	37	8	81	100
6	2	30	4	59	100
7	1	24	1	45	100
8	0.5	18	0	37	89
9	0.2	11	0	28	44
10	0.08	4	0	22	20

TABLE 1M

i	$U_i = T_i - t_{3i}$	u_{1i}	u_{2i}	$U_i \times u_{1i}$	$U_i \times u_{2i}$	u_{1i}^2	u_{2i}^2	$u_{1i}\, u_{2i}$
1	0	0	0	0	0	0	0	0
2	−18	−15	0	+270	0	+225	0	0
3	−44	−65	0	+2860	0	+4225	0	0
4	−56	−85	−5	+4760	+280	+7225	+25	+425
5	−63	−92	−19	+5796	+1197	+8464	+361	+1748
6	−70	−96	−41	+6720	+2870	+9216	+1681	+3936
7	−76	−99	−55	+7524	+4180	+9801	+3025	+5445
8	−71	−89	−52	+6319	+3692	+7921	+2704	+4628
9	−33	−44	−16	+1452	+528	+1936	+256	+704
10	−16	−20	+2	+320	−32	+400	+4	−10
			$\Sigma =$	35,321	12,715	49,413	8056	16,846

TABLE 1N

i	$\alpha \times t_{1i}$	$\beta \times t_{2i}$	$\gamma \times t_{3i}$	T'_i (5)
1	62	29	9	100
2	52.70	29	9	91
3	21.7	29	9	60
4	9.3	27.55	9	46
5	4.96	23.49	9	37
6	2.48	17.11	9	29
7	0.62	13.05	9	23
8	0	10.73	8.01	19
9	0	8.12	3.96	12
10	0	6.38	1.80	8

★★*Problem 1.14* **Study of a soil structure by means of two-dimensional theoretical packing (small-cylinder analogy)**

Let us consider an analogical model of a soil medium formed by an assembly of thin cylinders. The problems to be solved, are:

(1) What regular, stable packing may be made if all the thin cylinders have the same diameter D? Determine the void ratio and the dry unit weight for these different assemblies ($\gamma_s = 27\ kN/m^3$).

(2) What is the maximum diameter d of other cylinders which could be introduced in the voids of the D-size packing?

For each of the original packing, determine the grain-size distribution of D- and d-sizes leading to maximum densities. Calculate also the void ratios and the dry unit weights of the mixtures at maximum compactness.

Which combination leads to maximum packing? In order to draw the grain-size curves, assume D = 5 mm.

Solution

(1) The two stable, regular packing arrangements correspond to square (2-dimensional) and equilateral triangle configurations as shown in Fig. 1.24. Hexagonal packing is very improbable because it is very unstable.

Fig. 1.24.

The void ratio *for the square arrangement* is:

$$e = \frac{D^2(1-\pi/4)}{\pi D^2/4} = \frac{4-\pi}{\pi} = 0.273.$$

If we assume the unit weight of the cylinders to be $\gamma_s = 27\ \mathrm{kN/m^3}$, we have:

$$\gamma_d = \frac{(\pi D^2/4) \times \gamma_s}{D^2} = 27 \times \frac{\pi}{4} = 21.2\ \mathrm{kN/m^3}.$$

For *the triangular arrangement*, the values of e and γ_d are:

$$e = \frac{(\sqrt{3}\,D^2/4) - (\pi D^2/8)}{\pi D^2/8} = \frac{2\sqrt{3} - \pi}{\pi} = 0.102$$

$$\gamma_d = \frac{(\pi D^2/8)\gamma_s}{(\sqrt{3}/4)D^2} = 27 \times \frac{\pi}{2\sqrt{3}} = 24.5 \text{ kN/m}^3 .$$

(2) *Arrangement for the square packing:* the size of a cylinder that could be introduced in the void will have a diameter $d = D(\sqrt{2} - 1)$.

In order to obtain maximum packing with cylinders D and d, all the elements of the mass should have the shape of that shown in Fig. 1.25, a.

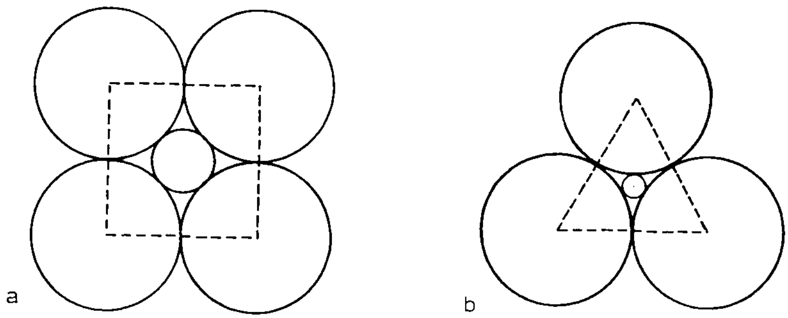


Fig. 1.25. a. Square element of mass. b. Triangular element of mass.

In each element there is one cylinder of diameter D and one of diameter d. This corresponds to weight percentages of:

$$\frac{D^2}{D^2 + d^2} = \frac{D^2}{D^2[(1 + (\sqrt{2} - 1)^2]} = \frac{1}{1 + (\sqrt{2} - 1)^2} = 0.85$$

for particle of diameter D and

$$\frac{d^2}{D^2 + d^2} = \frac{(\sqrt{2} - 1)^2}{1 + (\sqrt{2} - 1)^2} = 0.15$$

for the d-size particles.

The mixture, therefore, will have the grain-size distribution curve as shown in Fig. 1.26. At maximum compaction, the mixture will look exactly like the element of Fig. 1.25, a. Therefore:

$$e = \frac{D^2(1 - \pi/4) - D^2(\sqrt{2} - 1)^2(\pi/4)}{\frac{\pi D^2}{4}[(1 + (\sqrt{2} - 1)^2)]} = 0.087$$

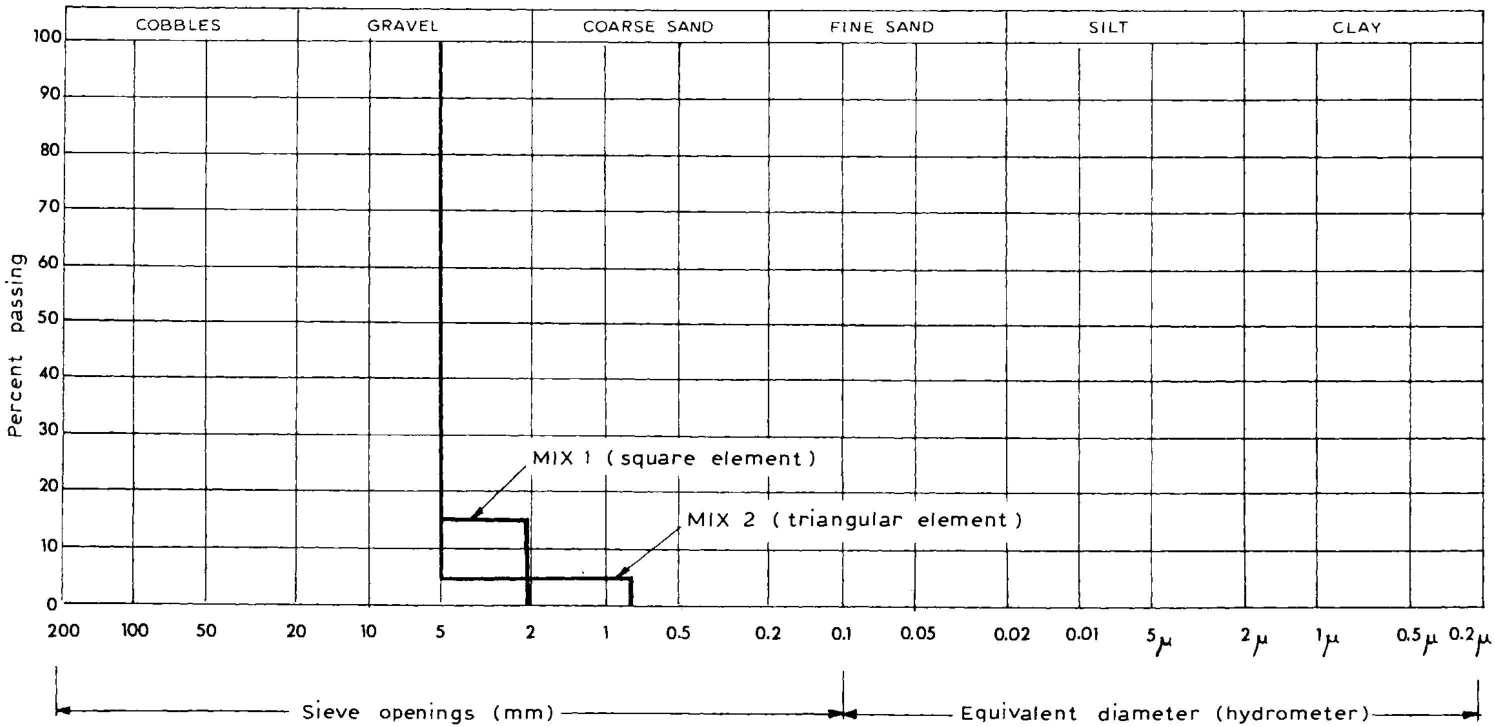


Fig. 1.26. Grain-size distribution.

$$\gamma_d = \frac{\frac{\pi D^2}{4}[(1+(\sqrt{2}-1)^2)]\gamma_s}{D^2} = 24.8\ \mathrm{kN/m^3}.$$

For *the arrangement of triangular packing*, the distance between the center of particles D and d is equal to: $(2D/3)(\sqrt{3}/2) = D/\sqrt{3}$. The size of the cylinder that can be introduced in the central void will have a maximum diameter of $d = 2(D/\sqrt{3} - D/2) = D(2/\sqrt{3} - 1)$.

In order to obtain maximum packing of rods D- and d-sizes, each element of mass will have to look like Fig. 1.25, b. In each element of mass, there will be half a particle of size D and one of size d.

The percentage of weights of the size-D particle is:

$$\frac{D^2}{D^2 + 2d^2} = \frac{1}{1 + 2(2/\sqrt{3} - 1)^2} = 0.956.$$

The percentage by weight of the size-d particles is 0.044. So: 95.6% of D-size and 4.4% of d-size particles make up the mass. The mix will have a grain-size distribution as shown by the curve of Fig. 1.26. The mixture will have maximum compactness when its mass will have elemental sections identical to that of Fig. 1.25, b and

$$e = \frac{(\sqrt{3}D^2/4) - (\pi D^2/8) - \pi D^2/4(2/\sqrt{3} - 1)^2}{(\pi D^2/4)[1/2 + (2/\sqrt{3} - 1)^2]} = 0.052$$

$$\gamma_d = \frac{(\pi D^2/4)[1/2 + (2/\sqrt{3} - 1)^2]\gamma_s}{\sqrt{3}D^2/4} = 25.6\ \mathrm{kN/m^3}.$$

The maximum packing arrangement is that of the equilateral configuration.

Chapter 2

WATER IN THE SOIL

★*Problem 2.1* Permeability of sand

A coarse-sand sample is 15 cm high and has 5.5 cm diameter. It is placed in a constant-head permeameter. Water passes through the sample under a constant head of 40 cm and after 6 sec, 40 g of water has been collected. What is the coefficient of permeability of the sand?

Solution

The flow of water through a soil is governed by Darcy's law

$$v = ki \tag{1}$$

The amount of water percolated is $q = v \times s$, the rate percolation is:

$$v = \frac{q}{s} = \frac{4q}{\pi d^2} = \frac{4 \times 40}{6 \times \pi \times 5.5^2} = 0.28 \text{ cm/s}$$

The hydraulic gradient $i = h/l = 40/15 = 2.66$

From equation (1): $k = v/i = 0.28/2.66 = 0.105$, say 0.11 cm/sec.

Answer

$k = 0.11$ cm/sec.

★*Problem 2.2* Permeability of clay

A clay sample is 2.5 cm high and has a diameter of 6.5 cm. It is placed in an oedometer with a variable-head permeameter. The water percolation through the sample is measured in a standpipe whose inner diameter is 1.7 mm. The tube is graduated in centimeters from the top to the bottom. The top graduation is zero and is located 35 cm above the base of the oedometer. The overflow in the oedometer is 3 cm above its base. At the start of the test, the water level in the tube is at zero; 6 mins and 35 secs later, the water level has dropped to graduation 2. What is the coefficient of permeability of the clay?

Solution

It is assumed that after achieving saturation of the sample, the flow of water is sufficiently slow to apply Darcy's law for each time increment during which the water flows $(t, t + dt)$.

The hydraulic gradient (see Fig. 2.1) is $i = h/l$

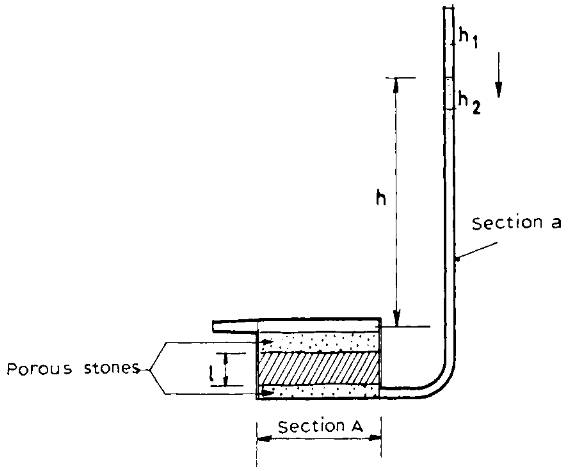


Fig. 2.1.

Since $v = ki$ (Darcy's law), the quantity of water is $q = Akh/l$, where A = cross-sectional area of the clay sample. Since the volume of water permeating through the sample is equal to the volume of water which left the standpipe, we have:

$$q\mathrm{d}t = (Akh/l) = \mathrm{d}t = -a\mathrm{d}h$$

where a = the cross-sectional area of the standpipe.
Then:

$$k\mathrm{d}t = -\frac{a}{A}\, l\, \frac{\mathrm{d}h}{h}$$

Integrating this value between height h_1 and h_2 of the standpipe gives:

$$kT = -\frac{a}{A}\, l \log (h_2)/h_1 \text{ or:}$$

$$k = 2.3\,\frac{a}{A}\,\frac{l}{T}\log\left(\frac{h_1}{h_2}\right), \quad \text{but} \quad a = \frac{\pi d^2}{4} \quad \text{and} \quad A = \frac{\pi D^2}{4}$$

therefore, $k = 2.3(d/D)^2\, \dfrac{l}{T} \log (h_1/h_2)$.

Numerical application: $d = 0.17$ cm, $D = 6.5$ cm, $l = 2.5$ cm, $T = 6$ min 35 s $= 395$ s, $h_1 = 35 - 3 = 32$ cm, $h_2 = 32 - 2 = 30$ cm,

$$\text{so: } k = 2.3 \times \left(\frac{0.17}{6.5}\right)^2 \times \frac{2.5}{395} \log\frac{32}{30} = 2.8 \cdot 10^{-7}\ \text{cm/s}.$$

Answer

$k = 2.8 \cdot 10^{-7}$ cm/s.

★*Problem 2.3* Permeability of sand

A well graded sand sample containing well-rounded grains has a void ratio of 0.62 and a coefficient of permeability of $2.5 \cdot 10^{-2}$ cm/s. Estimate the coefficient of permeability for the same sand with a void ratio of 0.73, using the Casagrande and Terzaghi formulas.

Solution

Casagrande's formula is $k = 1.4\, k_{0.85}\,(e)^2$,

therefore: $k_{0.62} = 1.4 k_{0.85}\, \overline{0.62^2}$ and $k_{0.73} = 1.4 k_{0.85}\, \overline{0.73^2}$

$$\text{Therefore,}\quad k_{0.73} = k_{0.62} \frac{\overline{0.73^2}}{0.62^2} = 2.5 \cdot 10^{-2} \times (1.18)^2$$

$$= 3.48 \cdot 10^{-2} \simeq 3.5 \cdot 10^{-2} \text{ cm/s.}$$

Terzaghi's formula is:

$$k = \frac{C}{\eta} \frac{(n-0.13)^2}{\sqrt[3]{1-n}}\, d_{10}^2.$$

For specific test conditions, the ratio $k_{0.73}/k_{0.62}$ may be calculated because the value of viscosity, η would be the same in both instances.

$$\frac{k_{0.73}}{k_{0.62}} = \frac{(n_{0.73} - 0.13)^2}{(n_{0.62} - 0.13)^2} \times \frac{\sqrt[3]{1 - n_{0.62}}}{\sqrt[3]{1 - n_{0.73}}}$$

where $n = e/(1+e)$, hence $e = 0.62$ corresponds to $n = 0.62/1.62 \simeq 0.38$, $e = 0.73$ corresponds to $n = 0.73/1.73 \simeq 0.42$

$$k_{0.73} = 2.5 \cdot 10^{-2} \times \left(\frac{0.29}{0.25}\right)^2 \cdot \sqrt[3]{\frac{0.62}{0.58}} = 2.5 \cdot 10^{-2} \times 1.346 \times 1.022$$

$$k_{0.73} = 3.44 \cdot 10^{-2} \simeq 3.4 \cdot 10^{-2} \text{cm/s.}$$

Terzaghi's formula gives a value of permeability slightly lower than that of Casagrande.

Summary of answers

Casagrande's formula $k_{0.73} = 3.5 \cdot 10^{-2}$ cm/s, Terzaghi's formula $k_{0.73} = 3.4 \cdot 10^{-2}$ cm/s, then $3.4 \cdot 10^{-2}$ cm/s $< k_{0.73} < 3.5 \cdot 10^{-2}$ cm/s.

★★*Problem 2.4* **Average coefficient of permeability of a layered system**

A sand deposit contains three distinct horizontal layers of equal thicknesses. The coefficient of permeability of the upper and lower sand is 10^{-3} cm/s. That of the middle layer is 10^{-2} cm/s.

What are the horizontal and vertical coefficients of permeability of the three-layered system, and what is their ratio.

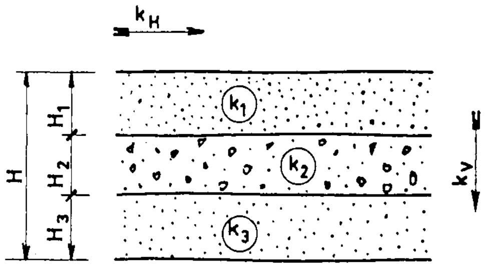


Fig. 2.2.

Solution

Let us consider first an horizontal flow. It is parallel to the layers. We assume that all three layers have the same hydraulic gradient, i, (see Fig. 2.2), then: $v_1 = k_1 i, \quad v_2 = k_2 i, \quad v_3 = k_3 i.$

Let us consider the amount of water passing through an imaginary vertical plane through the three layers, of unit width; it could be seen that the average value of the rate of seepage is:

$$v = \frac{1}{H}(v_1 H_1 + v_2 H_2 + v_3 H_3) = \frac{i}{H}(k_1 H_1 + k_2 H_2 + k_3 H_3).$$

By definition;

$$v = k_H i, \quad \text{and} \quad k_H = \frac{1}{H}(k_1 H_1 + k_2 H_2 + k_3 H_3).$$

For this particular problem $H_1 = H_2 = H_3 = \dfrac{H}{3}$ and $k_1 = k_3$, therefore:

$$k_H = \frac{1}{3}(2k_1 + k_2) \quad \text{or:} \quad k_H = \frac{1}{3}(2 \cdot 10^{-3} + 10^{-2}) = 0.004, \quad \text{or:}$$

$$4 \cdot 10^{-3} \text{ cm/s.}$$

For the vertical flow of water, in the perpendicular direction to the three beds, the principle of continuity requires that the rate of discharge at each layer boundary be the same.

Therefore: $v = k_v i = k_1 i_1 = k_2 i_2 = k_3 i_3$; thus the hydraulic gradient, i, is equal to $\frac{h}{H} = \frac{h_1 + h_2 + h_3}{H}$ where h_1, h_2, and h_3 correspond to the head losses across each of the layers and h is the total head loss.

$$\text{Then: } v = k_v i = k_v \frac{H_1 i_1 + H_2 i_2 + H_3 i_3}{H}$$

$$= k_v \frac{H_1(v/k_1) + H_2(v/k_2) + H_3(v/k_3)}{H}$$

and:

$$k_v = \frac{H}{H_1/k_1 + H_2/k_2 + H_3/k_3}$$

$$\text{Since: } H_2 = H_3 = \frac{H}{3} \text{ and } k_1 = k_3,$$

$$\text{we have: } k_v = \frac{3}{2/k_1 + 1/k_2} = \frac{3k_1k_2}{2k_2 + k_1} = \frac{3 \cdot 10^{-3} \cdot 10^{-2}}{2 \cdot 10^{-2} + 10^{-3}}$$

$$= 1.43 \cdot 10^{-3}\ \text{cm/s} \simeq 1.4 \cdot 10^{-3}\ \text{cm/s}.$$

and:

$$\frac{k_H}{k_v} = \frac{4 \cdot 10^{-3}}{1.40 \cdot 10^{-3}} = 2.86 \simeq 2.9.$$

Summary of answers

$k_H = 4 \cdot 10^{-3}$ cm/s; $k_v = 1.4 \cdot 10^{-3}$ cm/s; $k_H/k_v = 2.9$.

★★*Problem 2.5* Coefficient of permeability determined by pump-out test

A pump-out test is carried out in a perforated well of 30 cm in diameter, extending into an impervious zone, and through an aquifer 17 m thick from the ground surface. The phreatic line is at 4 m below ground-surface level (see Fig. 2.3). After 24 h of continuously pumping water out of the well, equilibrium has been reached. The discharge of the pump is 5.4 m^3/h and the drawdown is 4 m. The effective porosity of the soil tested was estimated to be 0.29.

Determine the radius of influence R of the well and the average coefficient of permeability k.

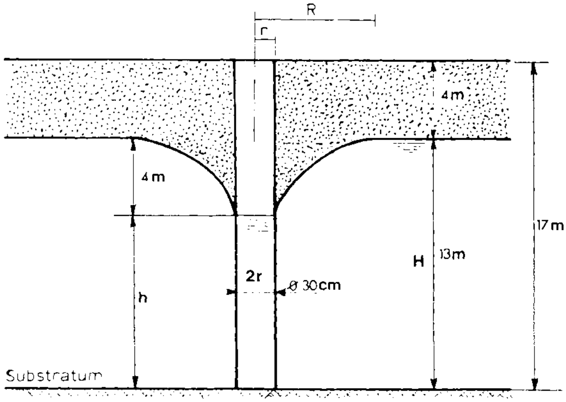


Fig. 2.3.

Solution

Once flow equilibrium is reached, after 24 h of pumping, Dupuit's equation becomes applicable:

$$q = \pi k \frac{H^2 - h^2}{\ln (R/r)} \tag{1}$$

or:

$$q = 1.365 k \frac{H^2 - h^2}{\log (R/r)} \tag{1'}$$

where q = discharge at pump, k = coefficient of permeability of soil mass, H = thickness of the aquifer, h = height of water in the well, R = radius of influence, and r = well radius.

Equation (1) contains two unknowns, namely R and k. It is valid only for $t \geqslant 24$ h. For $t \leqslant 24$ h there is another formula for the radius R which is applicable only to *non-equilibrium* condition. It is:

$$R = 1.5\sqrt{(kH/n)t} \tag{2}$$

where: R = the radius of influence at time t, k = coefficient of permeability of the soil mass, H = aquifer thickness, and n = effective porosity of soil.

For t = 24 h both eqs. 1 and 2 are applicable (see Fig. 2.4) and we dispose now of two equations with two unknown factors, R and k.

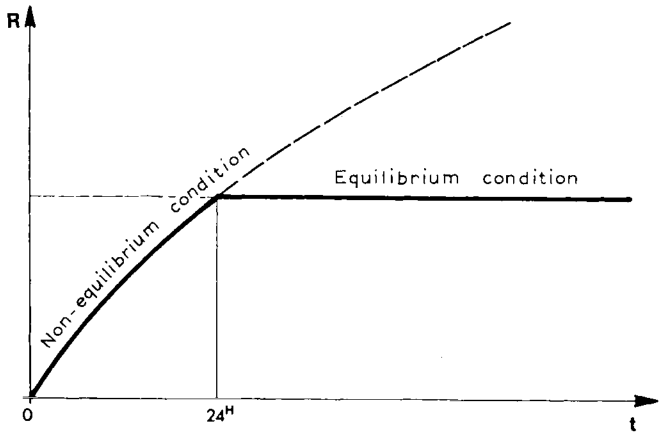


Fig. 2.4.

The solution may be either solved graphically or by numerical iteration, since one of the equations is transcendental. For this example, a graphic solution has been chosen and is accurate for the purpose.

Equation (2) can be written as:

$$k = \frac{nR^2}{2.25\,tH}$$

and eqn. (1′) as:

$$\log R - \log r = 1.365k\,\frac{H^2 - h^2}{q}$$

from which:

$$\log R = 0.607\,\frac{n(H^2 - h^2)}{qtH}\,R^2 + \log r \tag{3}$$

The intersection of the two plotted curves (Fig. 2.5), one the function $\log R$ and the other a parabola with an Oy axis, yields the solution.

Numerical application (see Fig. 2.5); $n = 0.29$, $H = 17 - 4 = 13$ m, $h = 13 - 4 = 9$ m, $q = 5.4\,\text{m}^3/\text{h}$, $t = 24$ h, $r = 0.15$ m, $\log r = \bar{1}.176 = 0.824$. Equation (3) may be written as:

$\log R = 0.009\ 19R^2 - 0.824$. The only realistic solution is: $R = 15$ m. Transposing this value of R to calculate k:

$$k = \frac{q \log (R/r)}{1.365(H^2 - h^2)} \simeq \frac{5.4 \log 100}{1.365 \times 88} = 9 \cdot 10^{-2}\ \text{m/h} = 2.5 \times 10^{-3}\ \text{cm/s}.$$

Summary of answers

$R = 15\,\text{m}; \quad k = 2.5 \cdot 10^{-3}\,\text{cm/s}.$

Remarks

(*a*) R should be expected to fall within the range of $100r$ to $300r$ or 15—45 m, which is the case.

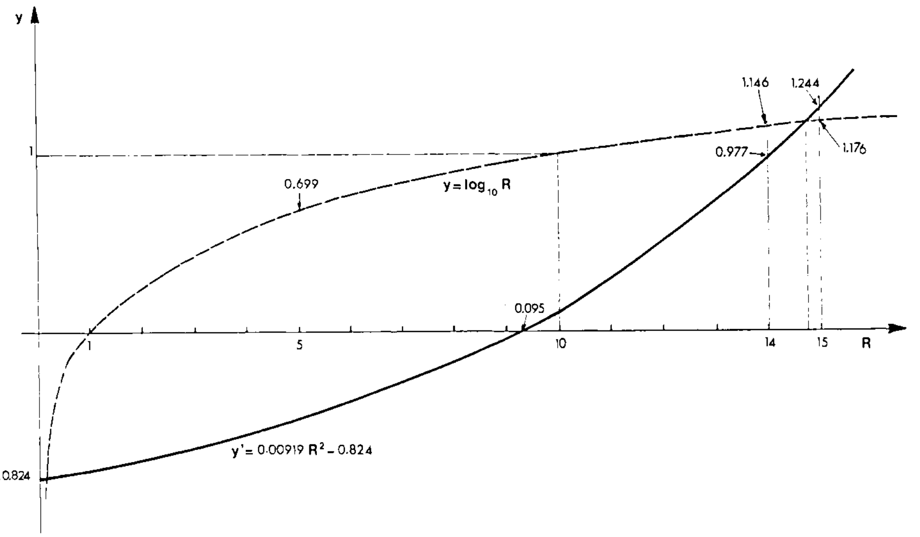


Fig. 2.5.

(*b*) The Sichardt formula for R is: $R = 3000\,(H - h)\sqrt{k}$ and would yield:

$$R = 3 \cdot 10^3 \times (13 - 9)\sqrt{2.5 \cdot 10^{-5}} = 12 \times \sqrt{2.5 \cdot 10^6 \cdot 10^{-5}} = 60\,\text{m}.$$

The radius is 4 times greater. The large differences can be accounted for when considering that the formula $R = 1.5\sqrt{(kH/n)t}$ is only valid for the logarithmic approximation of the solution of the equation. This assumes a relatively small drawdown, *which is not* the case here (drawdown is 4 m for a height of 13 m, or 30%). In practice, the radius of influence would be much greater than found in the problem.

★*Problem 2.6* Effective stress in sand

The ground-water level in a thick, very fine sand deposit is located 1.20 m below ground surface. Above the free ground-water line, the sand is saturated by capillary action. The unit weight of the saturated sand is 20.3 kN/m³.

What is the effective vertical stress on a horizontal plane located 3.60 m below ground surface?

Solution

The pore-water pressure at M (see Fig. 2.6) is:

$$u = \gamma_w (H - h) \quad \text{or} \quad u = 10(3.60 - 1.20) = 24 \text{ kPa}.$$

Since the sand is saturated above the water table, due to capillary action, the total vertical stress at M is: $\sigma_v = 3.60 \times 20.3 \simeq 73$ kPa.
and the effective stress at M is: $\sigma'_v = (\sigma_v - u) = 73 - 24 = 49$ kPa

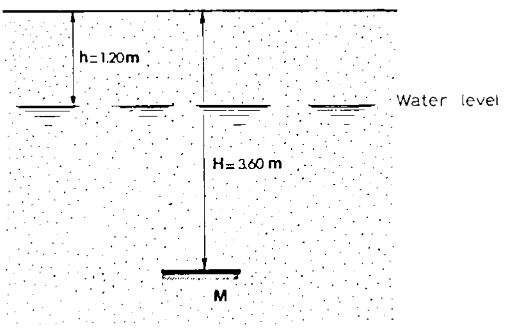


Fig. 2.6.

★*Problem 2.7* Effective stress in a clay

A submerged clay layer is 15 m thick. Its water content is 54%. The density of the soil particles is 2.78. What is the effective vertical stress due to the weight of soil at the base of the layer?

Solution

The effective vertical stress at M (see Fig. 2.7) is: $\sigma' = \gamma' h$. Since the clay is saturated and submerged, the water content is $w = e\gamma_w / \gamma_s$

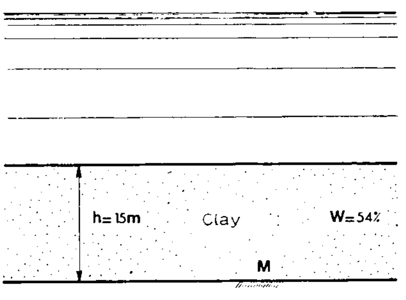


Fig. 2.7.

from which $e = w\dfrac{\gamma_s}{\gamma_w} = 0.54 \times \dfrac{2.78}{1} = 1.50$

We also know that:

$$\gamma' = (\gamma_s - \gamma_w)(1 - n) = \frac{\gamma_s - \gamma_w}{1 + e} \quad \text{and } \gamma_s/\gamma_w = 2.78 \text{ is given,}$$

therefore $\gamma_s = 27.8\,\text{kN/m}^3$ and $\gamma_w = 10\,\text{kN/m}^3$.

Substituting: $\gamma' = \dfrac{27.8 - 10}{1 + 1.50} = 7.12\,\text{kN/m}^3$.

For the total thickness: $\sigma' = 7.12 \times 15 = 107\,\text{kPa}$

Answer

$\sigma' \simeq 107\,\text{kPa}$.

★*Problem 2.8* Critical hydraulic gradient of sands at various densities

The specific gravity of sand particles is 26.6 kN/m³. The porosity of the sand mass in both its least and maximum compactness is 45% and 37%, respectively. What are the critical hydraulic gradients in these two cases?

Solution

The critical hydraulic gradient, i_{crit}, corresponds to a condition wherein the effective vertical stress is zero.

$$\sigma' = (\gamma' - i\gamma_w)z, \quad \text{therefore} \quad i_{\text{crit}} = \frac{\gamma'}{\gamma_w}.$$

We know that $\gamma' = (1 - n)(\gamma_s - \gamma_w)$. For the loose sand $n_1 = 0.45$, $\gamma'_1 = (1 - 0.45)(26.6 - 10) = 9.13 \simeq 9.1\,\text{kN/m}^3$.
For the dense sand: $n_2 = 0.37$, $\gamma'_2 = (1 - 0.37)(26.6 - 10) = 10.45 \simeq 10.5\,\text{kN/m}^3$.
Since $\gamma_w = 10\,\text{kN/m}^3$, we have $i_{1\,\text{crit}} = 0.91$, $i_{2\,\text{crit}} = 1.05$.

★★★*Problem 2.9* Blow out and piping

A large-sized excavation is made in a stiff clay whose saturated density is 1.76. When the depth of the excavation reaches 7.5 m, cracks appear and water begins to flow upward to bring up sand to the surface. Subsequent borings indicate that the clay is underlain by sand at a depth of 11 m below the original ground surface.

What is the depth to the water table outside the excavation below the original ground level?

Solution

Making an excavation in the clay, creates a hydraulic gradient between the top of the sand layer and the bottom of the excavation. As a consequence, water starts seeping in a vertical, upward direction from the sand layer towards the excavation floor. Because the clay has a very low permeability, flow equilibrium can only be reached after a long time period.

The solution must be considered over a short time interval. Two phases appear successively.

(*1*) *First phase: clay fissuration*

The floor of the excavation is stable only if the water pressure p, at the top of the sand layer (see Fig. 2.8) is counterbalanced by the weight P of the clay above it, disregarding shear strength of the clay. Stability condition therefore is:

$$\gamma_h (H - f) > \gamma_w (H - d)$$

or:

$$(\gamma' + \gamma_w)(H - f) > \gamma_w (H - d)$$

or:

$$\gamma'(H - f) > \gamma_w (f - d)$$

When incipient failure conditions occur, then:

$$\gamma'(H - f) = \gamma_w (f - d)$$

from which:

$$d = \frac{f\gamma_w - (H - f)\gamma'}{\gamma_w}$$

or:

$$d = 7.50 \times 1 - (11.00 - 7.50) \times 0.76 = 4.84\ \text{m}.$$

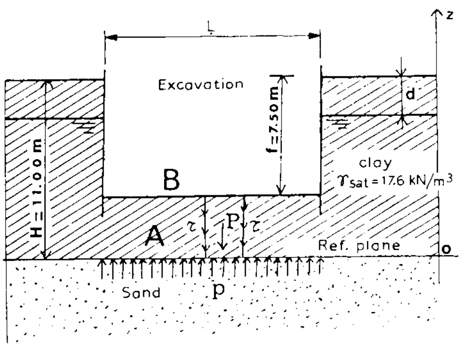


Fig. 2.8.

(2) Second phase: uplift of sand (quick sand)

A steady flow of water rising vertically through the cracks has been reached, the top of the sand layer and the bottom of the excavation being equipotential lines (see Fig. 2.8). Depending on the value of the hydraulic gradient, piping may occur.

The head at point A (Fig. 2.9) is:

$$h_A = \frac{u_A}{\gamma_w} + z_A = \frac{(H-d)\gamma_w}{\gamma_w} = H - d$$

and at point B:

$$h_B = \frac{u_B}{\gamma_w} + z_B = H - f, \text{ because } u_B = 0$$

with reference to the top of the sand layer.

The hydraulic gradient is constant from A to B and is:

$$\frac{h_A - h_B}{AB} = \frac{f-d}{H-f}.$$

Its critical value is $i_{crit} = \gamma'/\gamma_w$ and a blowout would occur when

$$\frac{f-d}{H-f} = \frac{\gamma_{sat} - \gamma_w}{\gamma_w}$$

(γ_{sat} is the unit weight of sand, assumed to be 20 kN/m^3).

or:

$$\frac{7.50 - d}{11.00 - 7.50} = 1, \quad \text{therefore } d = 7.50 - 3.50 \times 1 = 4.00 \text{ m}.$$

In order to bring up sand through the cracks, the depth, d to the water table must be $d = 4.0$ m.

Notes

(1) In either case the results are the same if the unit weights of the clay and the sand are identical.

— for the first phase, the shear strength of the clay was not taken into account;

— for the second phase, shear strength of clay was also implicitly neglected and it was assumed that the cracks in the clay would permit passage of the sand grains.

(2) Neglecting the shear strength of clay is reasonable if the width of excavation is large enough with respect to the thickness of the clay layer at the bottom of the excavation. This layer acts like a slab with a uniform load acting upward from beneath and it is in tension, which results in fissures through the clay. The fissures eliminate shear-strength consideration. (see Fig. 2.9 and 2.10).

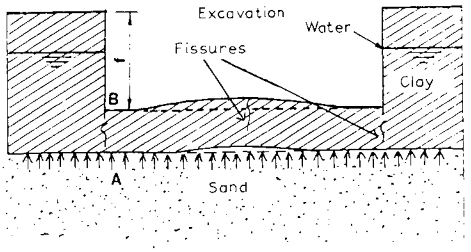


Fig. 2.9.

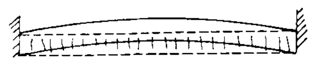


Fig. 2.10

★★*Problem 2.10* Calculate capillary rise, from laboratory test

A soil sample is placed in a "kh mold"[1]*. Its porosity is $n = 0.34$. The volume of water absorbed is measured by weighing the sample. The volume increases are as follows, as a function of time.*

t(h)	1	2	4	7	25	49
V(cm^3)	109	137	176	219	376	509

(1) Calculate kh. Assume the coefficient of permeability of the soil is 10^{-7} cm/s (relative to its capillarity). Calculate the capillary rise, h.

(2) Water had risen to height z just before the sample weighing after 49 hs. What is the relative error ϵ made in the formula: $\left(\ln \dfrac{1}{1 - z/h} - z/h\right)$ in the kh mold theory?

Solution

(1) Plot the experimental data on a graph where the abscissa is the square root of time, in hours, and the ordinate is volume of water absorbed in cm^3 (see Fig. 2.11). Experimental points fall on a straight line and V_0 may be evaluated at 50 cm^3 from the intersection of this line at time 0.

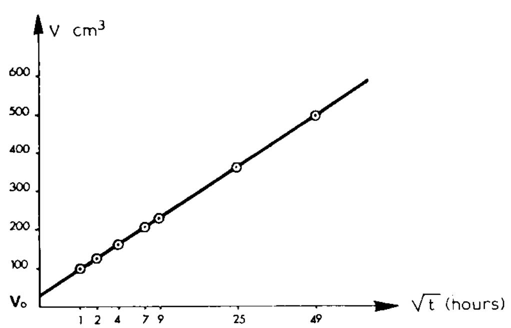


Fig. 2.11

[1] This device is commonly used in France; its diameter is 15 cm.

To obtain a more accurate value, we may write, since the product kh remains constant:

$$\frac{(V_7 - V_0)^2}{A^2} \times \frac{1}{2nt_7} = \frac{(V_1 - V_0)^2}{A^2} \times \frac{1}{2nt_1}$$

or:

$$\frac{V_7 - V_0}{V_1 - V_0} = \sqrt{t_7/t_1} = \sqrt{49/1} = 7$$

or:

$$V_7 - V_0 = 7(V_1 - V_0)$$

then:

$$V_0 = \frac{7V_1 - V_7}{6}$$

Therefore: $V_0 = \dfrac{7 \times 109 - 509}{6} = 42.3 \# 42\,\text{cm}^3.$

Since the diameter of the mold is 15 cm:

$$A = \frac{\pi \times \overline{15}^2}{4}$$

from which:

$$kh = \left(\frac{V_7 - V_0}{A}\right)^2 \times \frac{1}{2\,nt_7}$$

$$kh = \left(\frac{467 \times 4}{\pi \times \overline{15}^2}\right)^2 \times \frac{1}{2 \times 0.34 \times 49} = 0.21\,\text{cm}^2/\text{h}.$$

and therefore: $0.1 < kh < 1$

This value of kh shows that we have, in this case, a soil with an average capillary rise.

If the coefficient of permeability, relative to capillarity, is 10^{-7} cm/s, we then have:

$$h = \frac{0.21}{3600 \times 10^{-7}} = 5.8 \cdot 10^2\,\text{cm}$$

or: $h = 5.80$ m.

(2) Let A be equal to: $\ln \dfrac{1}{1-z/h} - \dfrac{z}{h}$.

We are seeking the relative error $\epsilon = \Delta A/A$ made in the kh mold theory. This theory assumes z/h is small and a niperian logarithm series is developed in which terms over 2 are not taken into consideration. Therefore, the error is of the order of the first term not considered of $(z/h)^3$.

Say that: $z/h = x$

$$1/(1-x) = 1 + x + x^2 + x^3 + 0(x^4)$$

from which:

$$\log \frac{1}{1-x} = x + \frac{x^2}{2} + R_3(x)$$

with:

$$R_3(x) = \frac{x^3}{3} + \frac{x^4}{4} + 0(x^5) = x^3\left[\frac{1}{3} + \frac{x}{4} + 0(x^2)\right]$$

therefore: $R_3(x) < x^3\ [1 + x + 0(x^2)]$ or: $R_3(x) < x^3/(1-x)$.

On the other hand, we have: $R_3(x) > x^3/3$, and $\Delta A = R_3(x)$ and $A \simeq x^2/2$

therefore: $2/3x < \Delta A/A < 2x/(1-x)$.

In the present example we have:

$$z = \sqrt{\frac{2kh}{n}\cdot t} = \sqrt{\frac{2\times 0.21}{0.34}\times 49} \simeq 7.8\,\text{cm}$$

therefore: $x = z/h \simeq 7.8/580 \simeq 1.35\times 10^{-2}$; $0.9\% < \Delta A/A < 2.74\%$

or, rounding off: $1\% < \epsilon < 3\%$.

Summary of answers:

$kh = 0.21\ \text{cm}^2/h$; $h = 5.80$ m; $1\% < \epsilon < 3\%$.

★*Problem 2.11* Hydraulic gradient and discharge of a subsurface toe drain

A homogenous slope of infinite length, making an angle β with the horizontal is composed of a soil with a permeability k. Water seeps through the soil in uniform, rectilinear flow inclined at an angle α to the horizontal near the drainage blanket. The face of the slope intersects the water-free surface at point O_1, which corresponds to a height H above the toe of the slope. A drainage system consists of a sloped drainage blanket and a subsurface drain located at the toe and parallel to the slope face (see Fig. 2.12).

(1) What is the hydraulic gradient of the water seeping in the soil near the drainage blanket? Can this gradient be assumed constant all over the flow net?

(2) What is the discharge of the drain if it is assumed that the length of the slope is 100 meters? (Assume: $H = 10\,m$, $\alpha = 20°$, $\beta = 34°$, (slope $1\frac{1}{2}$ to 1), $k = 5 \times 10^{-4}$ cm/s.)

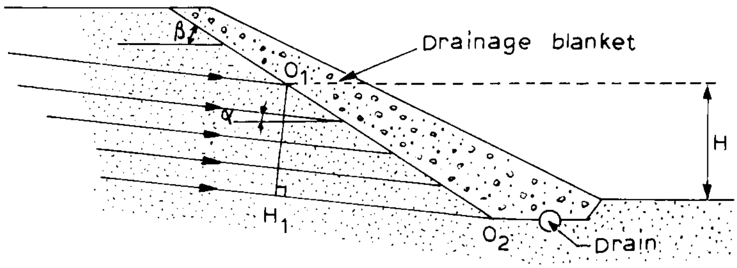


Fig. 2.12.

Solution

(1) Let A be a point in the flow near the drainage blanket which intersects the face of the slope at point B. The equipotential line through A is the line perpendicular to the flow line. It intersects the slope face at point A' (see Fig. 2.13). The permeability of the drainage blanket facing the slope is very large with respect to that of the soil in the slope. We can, therefore, assume that points A' and B are at atmospheric pressure and thus: $u_{A'} = u_B = 0$.

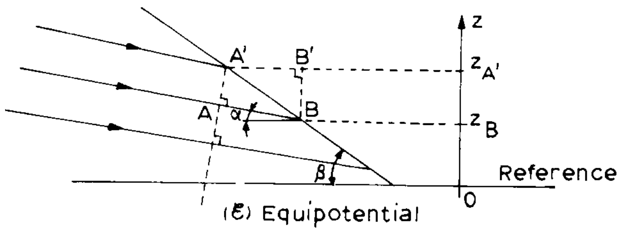


Fig. 2.13.

Since A and A' are on an equipotential line, the total heads are equal and

$$h_{A'} = h_{A'} = \frac{u_{A'}}{\gamma_w} + z_{A'} = z_{A'}, \text{ since } u_{A'} = 0.$$

By the same token: $h_B = u_B/\gamma_w + z_B = z_B$ with respect to an arbitrary horizontal plane through O.

The head loss between A and B is thus equal to the difference in elevation between A' and B: $\quad dh = h_A - h_B = z_{A'} - z_B = B'B$.

The hydraulic gradient at A is therefore:

$$i = -\frac{dh}{dl} = -\frac{B'B}{AB} \quad \text{but} \quad \frac{B'B}{A'B} = \sin\beta \quad \text{and}$$

$$AB/A'B = \cos(\beta - \alpha), \quad \text{and thus} \quad i = -\frac{\sin\beta}{\cos(\beta - \alpha)}.$$

Let us now consider a point A of the flow net at a great distance to the drainage blanket. The equipotential line through A no longer intersects the face of the slope O_1O_2 (see Fig. 2.12), but the top flow line.

A simiiar procedure leads to:

$$i = -\sin\alpha$$

Therefore, it is not possible to assume a uniform, rectilinear flow all over the embankment. The real flow net consists of parabolic shaped flow lines.

(2) From continuity of discharge, the computation can only be made in the area near the drainage blanket, where the gradient is assumed to be constant as a first approximation.

The discharge passing through an imaginary surface perpendicular to the flow lines is: $q = vS = k\,|\,i\,|\,S$.
For a length b of the slope, the surface is $S = b \times O_1H_1$,

but: $\dfrac{O_1H_1}{O_1O_2} = \sin(\beta-\alpha)$ and $\dfrac{H}{O_1O_2} = \sin\beta$

therefore, $O_1H_1 = H\dfrac{\sin(\beta-\alpha)}{\sin\beta}$

The total discharge, Q, which will pass through the toe drain will then be:

$$Q = k\,|\,i\,|\,L \times O_1H_1 = k\frac{\sin\beta}{\cos(\beta-\alpha)}L \times H\frac{\sin(\beta-\alpha)}{\sin\beta}$$

or: $Q = kLH\tan(\beta-\alpha)$.

Numerical application: $k = 5\cdot10^{-4}$ cm/s $= 5\cdot10^{-6}$ m/s, $L = 100$ m, $H = 10$ m, $\beta = 34°$, $\alpha = 20°$, therefore $\tan(\beta-\alpha) = 0.249$ from which $Q = 5\cdot10^{6}\cdot10^{2}\times10\times0.249 = 1.245\times10^{-3}$ m^3/s, say 1.25 l/s.

★★★*Problem 2.12* Flow net and discharge of seepage through a dam on a homogeneous anisotropic soil foundation

A dam has a width B of 50 m. The cross-section is shown on Fig. 2.14. It is supported by an alluvial deposit 6 m thick overlaying on impervious bedrock. The central part of the dam is 12 m wide at an elevation of − 2.00 m below the river bed elevation. Upstream pool elevation is at + 7.00 m and downstream is + 0.50 m.

The alluvial material is anisotropic and its coefficients of permeability in the horizontal and vertical directions are: $k_H = 1.44\cdot10^{-2}$ *cm/s,* $k_v = 1.60\cdot10^{-3}$ *cm/s.*

(1) Referring to typical flow nets of Fig. 2.15 and taking into account boundary conditions, draw an approximate flow net applicable to the given geometry of this problem. The drawing of the flow net can be improved by using the finite difference method.

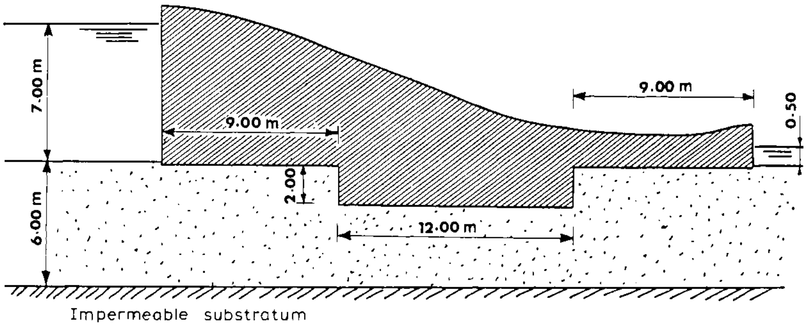


Fig. 2.14. Dam on anisotropic soil

(2) From the flow net, find: (a) the discharge through seepage, and (b) the uplift pressure on the bottom of the dam. What are the consequences of the uplift pressures?

Solution

For a two-dimensional anisotropic medium, Darcy's law is:

$\mathrm{v} = -\bar{\bar{k}}$ grad h, where $\bar{\bar{k}}$ is the tensor of permeability,

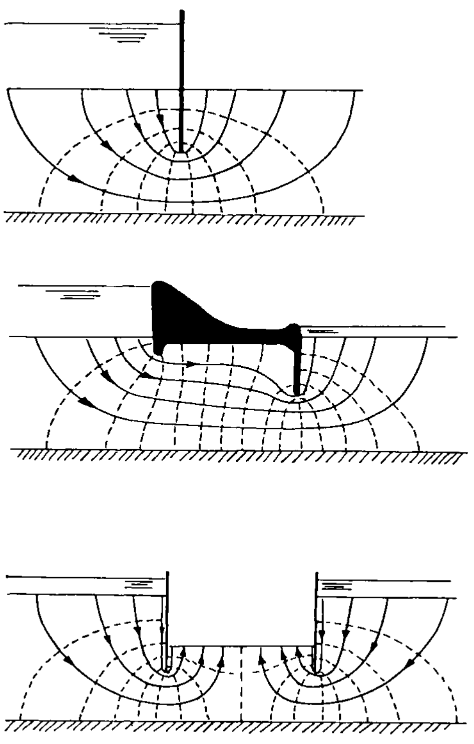


Fig. 2.15. Typical flow nets

therefore $\begin{cases} v_x = -k_h \dfrac{\partial h}{\partial x} \\ v_z = -k_v \dfrac{\partial h}{\partial z} \end{cases}$ (1)

The requirement of the principle of continuity for a two-dimensional system is:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \tag{2}$$

therefore, from eqn. (1) and for an anisotropic soil, we have:

$$k_h \frac{\partial^2 h}{\partial x^2} + k_v \frac{\partial^2 h}{\partial z^2} = 0 \tag{3}$$

It is important to note that the latter equation is not similar to a Laplace equation. The flow net is no longer made up of orthogonal curves. In order to solve the problem, we must resort to a transformed section which is obtained by assuming that the foundation alluvium is isotropic but of a different geometry. In order to accomplish this, the x and y directions must be transformed to new values in accordance with the following relationships:

$$\begin{cases} X = x\sqrt{k_v/k_h} \\ Z = z. \end{cases}$$

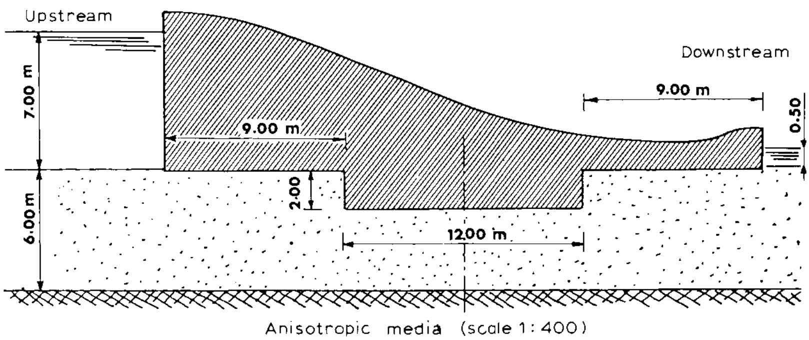
Therefore:

$$\frac{\partial X}{\partial x} = \sqrt{\frac{k_v}{k_h}},$$

from which: $\dfrac{\partial^2 h}{\partial x^2} = \dfrac{k_v}{k_h}\dfrac{\partial^2 h}{\partial X^2}$ and $\dfrac{\partial^2 h}{\partial z^2} = \dfrac{\partial^2 h}{\partial Z^2}$

Equation (3) then becomes:

$$\frac{\partial^2 h}{\partial X^2} + \frac{\partial^2 h}{\partial Z^2} = 0 \tag{3'}$$



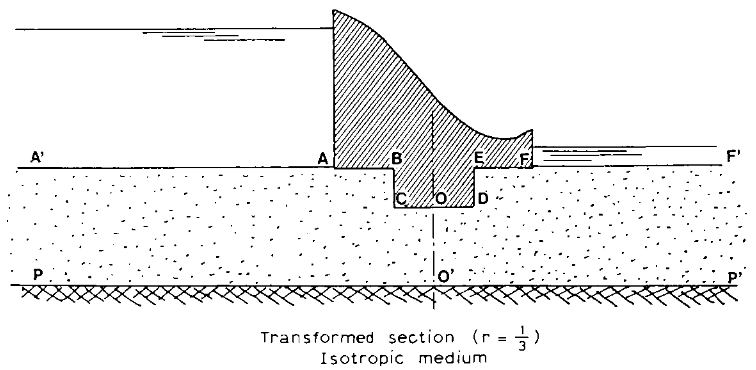


Fig. 2.16.

Conclusion

The transformed section allows one to consider an isotropic condition with the ratio $\sqrt{k_v/k_h}$. In this case, where $k_v = 1.6 \times 10^{-3}$ cm/s, $k_h = 14.4 \times 10^{-3}$ cm/s and $r = \sqrt{k_v/k_h} = \sqrt{1/9} = 1/3$, Fig. 2.16 shows the anisotropic section at a scale of 1 : 400 (axes x, z) and the transformed section with a ratio of 1 : 3 (axes X, Z).

Drawing the flow net

Since the transformed section represents a fictitious isotropic condition as far as permeability is concerned, the orthogonal net exists with equipotential and flow lines.

The net may be drawn by reference to the typical nets given in the problem and by observing the following:

(a) The geometry of the soil foundation is symmetrical with respect to axis OO'. Therefore only half of the need be drawn, say to the left of OO' (see Fig. 2.17).

(b) The interface soil-water upstream, denoted by line AA' on Fig. 2.16 is an equipotential line. The same is true for line FF' downstream. By symmetry, the OO' must also be an equipotential line.

(c) When meeting an impervious boundary, water cannot flow through it and the rate of percolation perpendicular to the boundary is zero. If derivatives are taken in the perpendicular and tangential direction at such a boundary then:

$$\frac{\partial \phi}{\partial n} = 0 = -v_{\mathrm{n}}, \quad \frac{\partial \psi}{\partial t} = 0$$

(ψ: conjugate function of ϕ), therefore an impervious boundary is parallel to the flow direction.

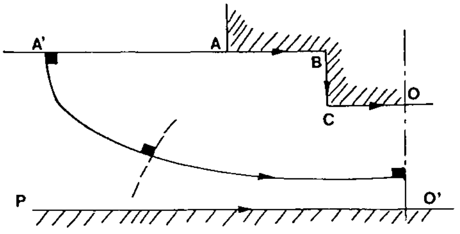


Fig. 2.17.

So, in this problem *ABCODEF* (Fig. 2.16) is a flow line and $PO'P'$ is a flow line. We know that a flow line must be perpendicular to an equipotential line, as AA' and $O'O$. (Fig. 2.17). These flow lines are progressively deformed from the straight line $PO'P'$ to the broken line *ABCO* (Fig. 2.16).

The net should be started to be drawn from the axis of symmetry at OO', choosing an arbitrary number of flow lines (between 5 and 10). It is in this zone that the figure between two adjacent flow lines and two adjacent equipotential lines approximates the most a square (see Fig. 2.18).

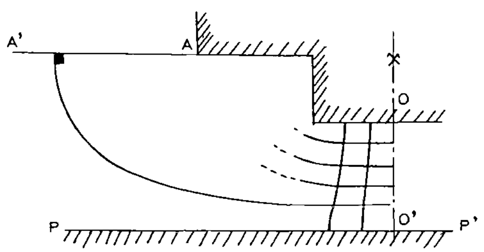


Fig. 2.18.

By using the finite-difference method the flow net can be improved. The trial flow-net (Fig. 2.19) gives the first potential values at the nodes of the grid. The computation is made by a successive relaxation method. Results are given in Fig. 2.19.

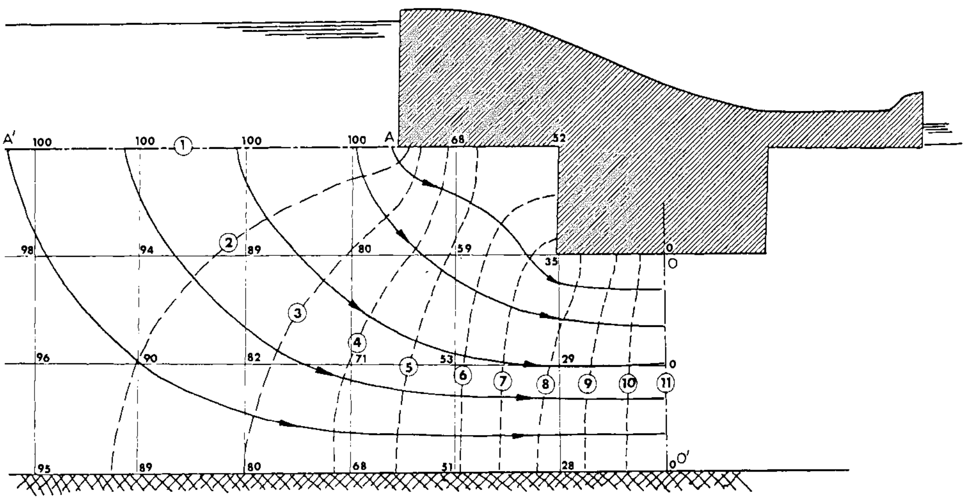


Fig. 2.19. Flow net and grid used for the finite difference method. Note: The potential has been arbitrarily chosen equal to 100 along AA' and to zero along OO'.

Percolation discharge. At any one point of the anisotropic section, we have

$$dq = v_x \, dz \, dy + v_z \, dx \, dy$$

$$\text{but} \begin{cases} v_x = -\sqrt{k_h k_v} \dfrac{\partial h}{\partial X} \\ v_z = -k_v \dfrac{\partial h}{\partial Z} \end{cases}$$

In the plane of symmetry OO' we have $v_z = 0$,
then:

$$dq = -\sqrt{k_h k_v} \frac{\partial h}{\partial X} \cdot dz \times 1$$

and the equivalent permeability k is obtained: $k = \sqrt{k_h k_v}$

Earlier, it was shown that $k_h = 9 k_v$ or $k = 3 k_v$, or $k = 4.8 \cdot 10^{-3}$ cm/s.

The discharge is given by the formula: $q = k \times \Delta H \times N_T / N_H$, where ΔH = total head loss between the upstream and downstream equipotential lines, N_T = number of flow channels, N_H = number of equipotential intervals from which:

$$q = 4.8 \times 10^{-5} \times 6.5 \times (6/20) \times 3.6 \times 10^3 = 3.37 \times 10^{-1} \text{ m}^3/\text{h}$$

or $q = 0.34 \text{ m}^3/\text{h}$ for a 1 meter wide slice through the dam. The dam width is 50 m; therefore, the total percolation loss is $Q = 17 \text{ m}^3/\text{h}$.

Remark. Considering a horizontal plane (at the beginning or the end of the net), we have $v_x = 0$.

Then: $\mathrm{d}q = -k_v \dfrac{\partial h}{\partial Z}\mathrm{d}x \times 1, \quad \dfrac{\partial h}{\partial Z} = \dfrac{\partial h}{\partial z}$

and after changing the variable:

$$\mathrm{d}x = \sqrt{\frac{k_h}{k_v}}\,\mathrm{d}X, \quad \text{then} \quad \mathrm{d}q = -k_v \frac{\partial h}{\partial Z}\sqrt{\frac{k_h}{k_v}}\,\mathrm{d}X \times 1 = -\sqrt{k_h k_v}\,\frac{\partial h}{\partial Z}\mathrm{d}X \times 1$$

Uplift pressures. Let the top of the foundation soil layer be the reference plane of Fig. 2.20. The head at any random point is $h = u/\gamma_w + z$ where z is the elevation of the random point with respect to the reference plane. There are 20 equipotential drops. The total head loss is:

$$\Delta H = h_A - h_F = 7.00 - 0.50 = 6.50\,\mathrm{m}$$

The head drop between any two consecutive equipotential lines is:

$$\Delta h = \Delta H/N = 6.50/20 = 0.325\,\mathrm{m}.$$

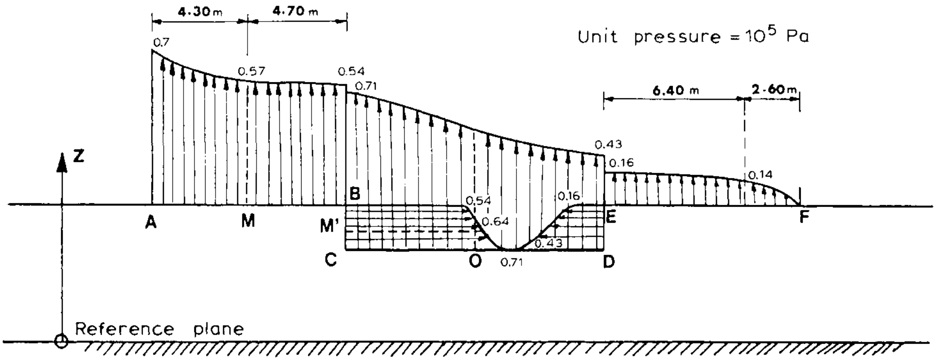


Fig. 2.20. Schematic representation of uplift pressures

At point A:

$$h_A = \frac{u_A}{\gamma_w} + z_A = \frac{7.00}{1} + 6.00 = 13.00\,\mathrm{m}, \quad u_A = 70\,\mathrm{kPa}.$$

At point M (equipotential line number 5, see Figs. 2.19 and 2.20):

$$h_M = \frac{u_M}{\gamma_w} + z_M = \frac{u_M}{\gamma_w} + 6 = h_A - 4\Delta h$$

$$= 13 - 4 \times 0.325 = 11.70\,\mathrm{m}$$

$$\frac{u_M}{\gamma_w} = 11.70 - 6.00 = 5.70\,\mathrm{m}, \qquad u_M = 57\,\mathrm{kPa}.$$

At M' (equipotential line number 6):

$$h_{M'} = \frac{u'_M}{\gamma_w} + z'_M = \frac{u'_M}{\gamma_w} + 5.00 = h_A - 5\,\Delta h = 11.375\,\text{m}$$

$$\frac{u'_M}{\gamma_w} = 11.375 - 5.00 = 6.375\,\text{m}, \quad u'_M = 64\,\text{kPa}.$$

At point O (equipotential line number 11):

$$h_O = \frac{u_O}{\gamma_w} + z_O = \frac{u_O}{\gamma_w} + 4.00 = h_A - 10\,\Delta h = 13.00 - 3.25 = 9.75\,\text{m}$$

$$\frac{u_O}{\gamma_w} = 9.75 - 4.00 = 5.75\,\text{m}, \quad u_O = 57.5\,\text{kPa}.$$

The uplift pressure (see Fig. 2.20) may be computed in this manner for each equipotential line. Planes BC and ED of the dam foundation are under horizontal pressures of opposite directions. The resultant of these horizontal pressures is a net pressure-acting downstream.

The vertical uplift pressure resultant for a 1 m slice of the dam is:
for length AB:

$$10^2 \times \left(4.30 \times 1.00 \times \frac{0.70 + 0.57}{2} + 4.70 \times 1.00 \times \frac{0.57 + 0.54}{2}\right) \simeq 530\,\text{kN}$$

for length CD:

$$10^2 \times \left(12.00 \times 1.00 \times \frac{0.71 + 0.43}{2}\right) \simeq 680\,\text{kN}$$

for length EF:

$$10^2 \times \left(6.40 \times 1.00 \times \frac{0.16 + 0.14}{2} + 2.60 \times 1.00 \times \frac{0.14}{2}\right) \simeq 110\,\text{kN}$$

$$\text{Total} = 1320\,\text{kN}$$

Since the length of dam is $2 \times 9.00 + 12 = 30\,\text{m}$ and assuming a unit weight of concrete of $23\,\text{kN/m}^3$, an average height of at least $\dfrac{1320}{23 \times 1.00 \times 30} = 1.9\,\text{m}$ of concrete is needed to balance the uplift pressure.

★★*Problem 2.13* **Earth dam: quick-sand condition; drainage blanket; discharge by percolation**

Consider an earth fill dam supported on a stratified, heterogeneous foun-

dation whose cross-section is shown on Fig. 2.21. (The grain-size distribution of the sand is shown on Fig. 2.22 by the curve S.)

(a) Show that quicksand occurs at the toe of the dam.

(b) Determine the thickness of a drainage blanket required to be placed over the sand to avoid quicksand. Find also the percolation discharge. Assume the gravel permeability $k_1 = 1$ cm/s and the gravel layer thickness is constant throughout. The unit weight of sand is $\gamma_d = 16\,kN/m^3$ and its permeability is $k_2 = 10^{-2}\,cm^2/s$. The unit weight of the drainage blanket in place is $\gamma_d = 19\,kN/m^3$.

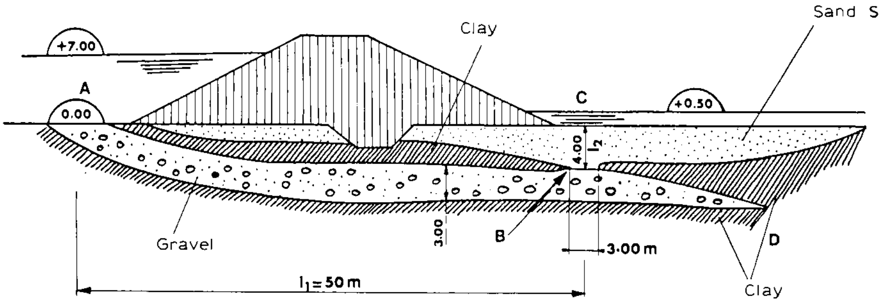


Fig. 2.21 Dam on stratified soil

Solution

(a) The flow net for this problem is entirely different than for the preceding example. The seepage of water is uniform throughout the gravel layer between points A and B. There is also a uniform flow in the upward, vertical direction from point B to point C in the sand layer. In the gravel zone between B and D, no flow occurs although ground water is present. Since the horizontal distance of 3.00 m between the ends of the clay layer is equal to the thickness of the gravel layer, and by virtue of the principle of continuity, the rate of percolation (or discharge) through the sand layer is equal to the rate of percolation of water through the gravel layer. Therefore:

$$v = k_1 i_1 = k_2 i_2, \quad k_1(h_1/l_1) = k_2(h_2/l_2)$$

$$k_1 = 1\text{ cm/s}, \quad l_1 = 50\text{ m}, \quad k_2 = 10^{-2}\text{ cm/s}, \quad l_2 = 4.00\text{ m}$$

and

$$h_1 = h_2 \times \frac{k_2}{k_1} \times \frac{l_1}{l_2} = \frac{10^{-2}}{1} \times \frac{50}{4} h_2 = 0.125\,h_2.$$

The total head loss $H = h_1 + h_2$ is equal to 6.50 m. Therefore: $1.125\,h_2 = 6.50$ and $h_2 = 5.78$ m (head loss through the sand layer). The hydraulic gradient is: $i_2 = \dfrac{h_2}{l_2} = \dfrac{5.78}{4} = 1.45.$

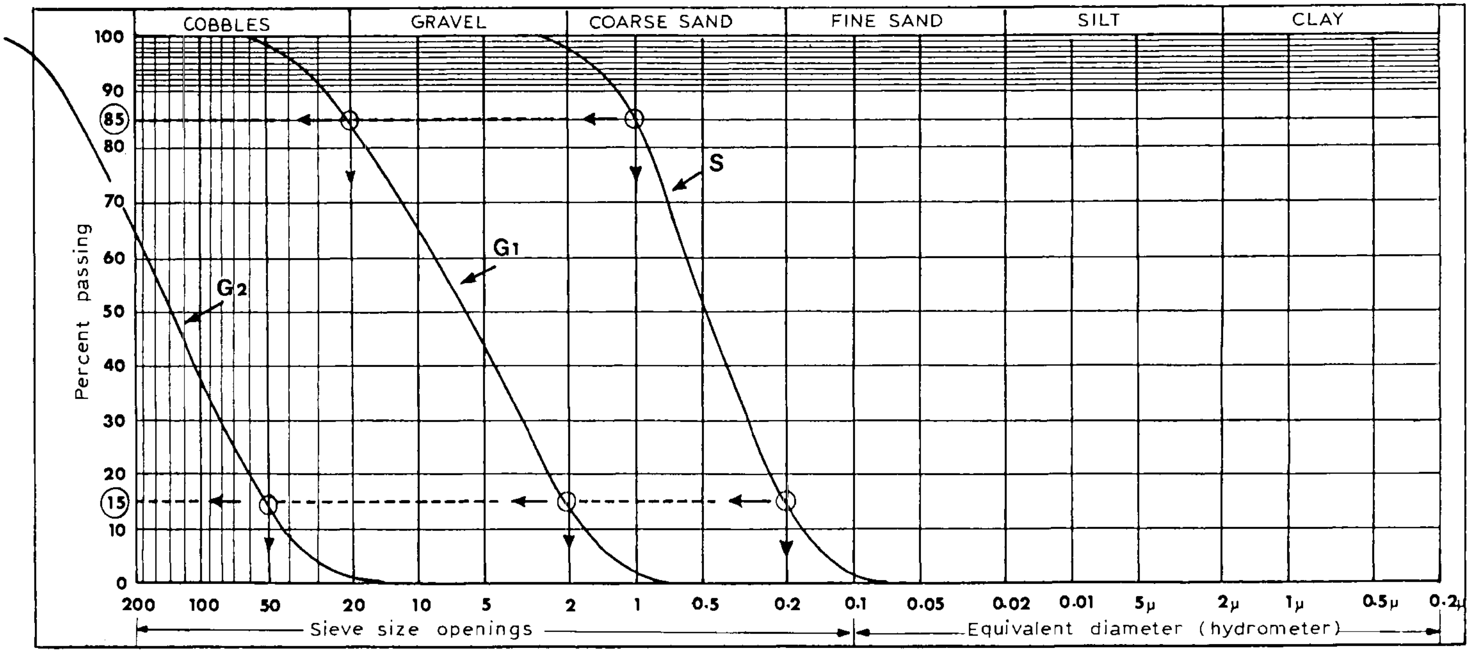


Fig. 2.22 Grain-size distribution curves

Quicksand condition in the sand would be:

$$i \geqslant i_{\text{crit}} = \frac{\gamma'}{\gamma_w} = \frac{\gamma_s - \gamma_w}{\gamma_s}\frac{\gamma_d}{\gamma_w}, \quad \text{but:} \quad \gamma_d = 16\,\text{kN/m}^3, \text{and}$$

$\gamma_s = 27\,\text{kN/m}^3$, hence $i_{\text{crit}} \simeq 1.01$, so $i_2 = 1.45 > 1.01$,

i.e. quicksand would occur.

(b) Consider the sand column element dS of height Z in Fig. 2.23.
This element is being acted upon, downwardly by its buoyant weight $W' = \gamma' Z\,dS$ and upwardly by the percolation water force $F = i.\gamma_w Z\,dS$.
We have shown that quicksand occurs, therefore we know that $F > W'$. In order to avoid quicksand, a weight acting downward must be added to the sand in order to have $q'dS + W' + F \geqslant 0$ so that the resultant is acting downward (positive direction). Therefore:

$$q'dS + \gamma' Z dS - i\gamma_w Z dS > 0, \quad \text{or} \quad q' > (i\gamma_w - \gamma')Z.$$

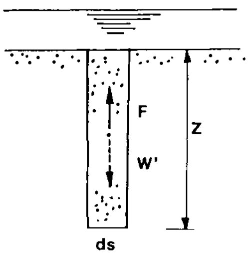


Fig. 2.23

The maximum value of Z must be selected, i.e.: $Z = l_2 = 4$ m, $\gamma_{d\ \text{sand}} = 16\,\text{kN/m}^3$ or $\gamma'_{\text{sand}} = 10.1\,\text{kN/m}^3$.
We need $q' \geqslant (14.5 - 10.1) \times 4.00$ or $q' > 17.6\,\text{kN/m}^3$.
Water will rise up into the filter blanket to level + 0.50.
Therefore: $q' = \gamma'_{\text{filter}} \times 0.50 + (\gamma_d)_{\text{filter}} \times H = 17.6\,\text{kN/m}^2$.

$(\gamma_d)_{\text{filter}} = 19\,\text{kN/m}^3$,

from which: $$\gamma'_{\text{filter}} = \frac{\gamma_s - \gamma_w}{\gamma_s}\gamma_d = \frac{1.7}{2.7} \times 19 = 12\,\text{kPa}$$

and $q = 12 \times 0.50 + 19 \times H = 17.6\,\text{kN/m}^3$. Finally:

$$H = \frac{17.6 - 6}{19} = 0.61\,\text{cm}.$$

In view of the simplifying, conservative assumptions made (constant hydraulic gradient in the sand), a lower safety factor may be accepted and the value of $H = 80$ cm adopted.

The drainage blanket would be composed of two layers in accordance with the design criteria for filters.

$4.5\, d_{15\,\text{subgrade}} \leqslant d_{15\,\text{filter}} \leqslant 4.5\, d_{85\,\text{subgrade}}$

The layer immediately on top of the sand (material G_1 on grain-size curves of Fig. 2.22) would be 50 cm thick, layer G_2 would be 80 cm thick.

From the grain-size curves of the sand we get: $d_{15} = 0.2$ mm and $d_{85} = 1$ mm, giving, for instance, for material G_1 : $(d_{15})_{F_1} = 2$ mm, $(d_{85})_{F_1} = 20$ mm and for G_2 : $9 \text{ mm} \leqslant (d_{15})_{F_2} \leqslant 90$ mm.

The larger sizes should be placed on the upper side of G_2 material (on top of the filter blanket).

Percolation discharge: $q = vS = k_2(h_2/l_2)S$. The cross-sectional area of the gravel layer $S = 3.00 \times 1.00 = 3\,\text{m}^2$ per linear meter of dam length, and $q = 0.44$ l/s per linear meter of length, but $B = 50$ m, hence $Q = 50 \times 4.35 \cdot 10^{-4} \times 3.6 \cdot 10^3 \simeq 78.5\,\text{m}^3/\text{h}$.

★★*Problem 2.14* Capillary rise in a homogeneous soil, effective stresses

A dense silt layer has the following properties: void ratio $e = 0.40$, effective diameter $d_{10} = 10\,\mu m$, capillary constant $C = 0.20\,cm^2$.

Free ground-water level is 8.00 m below surface. Find:

(1) the height of capillary rise in the silt; (2) the vertical effective stress at depths of 5 m and 10 m. Assume $\gamma_s = 26.5\,kN/m^3$ and that the soil above the capillary rise and ground surface is partially saturated at 50%.

Solution

(1) The capillary rise is calculated from the formula $h = C/ed_{10}$ where h and d are in cm and C in cm^2.

Therefore: $d_{10} = 10\,\mu\text{m} = 10^{-3}\,\text{cm}$, and $\text{h} = \dfrac{0.20}{0.40 \times 10^{-3}} = 0.5 \times 10^3\,\text{cm}$,

or $h = 5$ m.

(2) For the vertical effective stress at any depth above the free ground water (see Fig. 2.24), consider a unit area water column. It is in static equilibrium, hence: $u = -\gamma_w Z$.

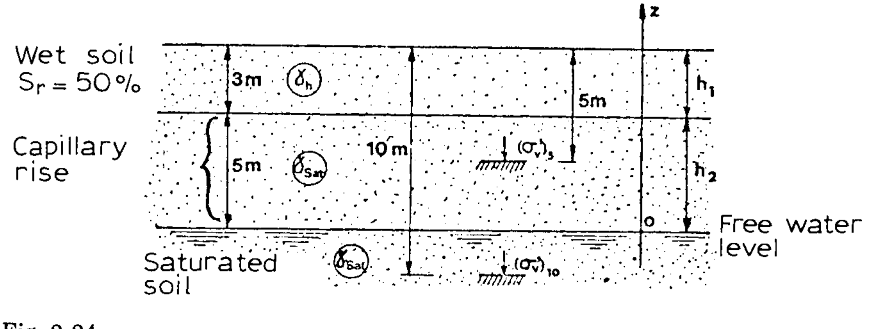


Fig. 2.24.

Pore-water pressure is negative above the free ground water table, in the capillary rise area. Hence Terzaghi's equation $\sigma = \sigma' + u$, can be written as: $\sigma = \sigma' - |u|$ or $\sigma' = \sigma + |u|$.

At a depth of 5 m, the total vertical stress is: $(\sigma_v)_5 = h_1\gamma_h + (h_2 - z)\gamma_{sat}$, but $u = -z.\gamma_w$, from which: $(\sigma'_v)_5 = h_1\gamma_h + (h_2 - z)\gamma_{sat} + z\gamma_w$.

Let us calculate γ_{sat}: $\gamma_{sat} = \gamma_s \dfrac{1+w}{1+e}$; $\quad e = 2.65\,w$

and $\gamma_{sat} = 26.5 \times \dfrac{(2.65 + 0.40)}{2.65 \times 1.40} = 21.8\,\text{kN/m}^3$.

Let us now calculate γ_h. Above ground water, $S_r = 50\%$, so

$$w = e\frac{S_r\gamma_w}{\gamma_s} = \frac{0.40 \times 0.50 \times 10}{26.5} = 7.5\%$$

from which:

$$\gamma_h = \gamma_s \frac{1+w}{1+e} = 26.5 \times \frac{1.075}{1.40} = 20.4\,\text{kN/m}^3$$

and: $(\sigma'_v)_5 = 3.00 \times 20.4 + 2.00 \times 21.8 + 3 \times 10 = 134.8\,\text{kPa}$,

say 135 kPa.

At a depth of 10 m, the effective stress $(\sigma'_v)_{10} = h_1\gamma_h + h_2\gamma_{sat} + |z|\gamma' = 3.00 \times 20.4 + 5.00 \times 21.8 + 2.00 \times 11.8 = 193.8\,\text{kPa}$.

Summary of answers

$h = 5.00\,\text{m}$; $(\sigma'_v)_5 \simeq 135\,\text{kPa}$; $(\sigma'_v)_{10} \simeq 194\,\text{kPa}$.

★★★*Problem 2.15* Capillary rise in an analogous soil model

A poorly graded sand may be idealized by stacking of spheres of identical diameters. Find the capillary rise in this theoretical model assuming that the spheres are stacked in the loosest packing arrangement ($n = 0.48$). The surface tension of water is 75 dynes/cm and assumed acceleration due to gravity is $g = 1000\,\text{cm/s}^2$. Compare the result with the result of the empirical formula $h = C/ed_{10}$.

Solution

In its loosest state, a stacking of spheres looks like the diagram of Fig. 2.25. Let us study the static forces acting on a column of capillary water as shown in Fig. 2.26. Let us first find the maximum capillary tension due to surface tension at the contact between water and a sphere (at the meniscus), as shown in Fig. 2.27. This figure makes clear that the contact curve is made up of four $\frac{1}{4}$ circles of radius r. But $r = R \cos \beta$ where R is the radius of one of the spheres.

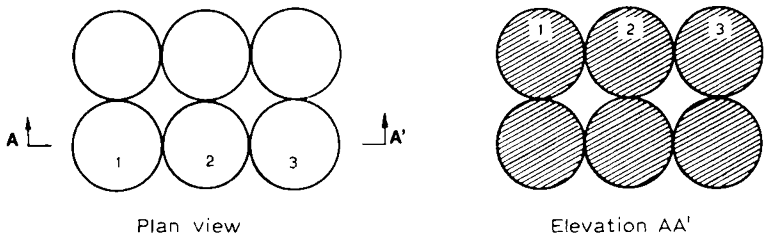


Fig. 2.25.

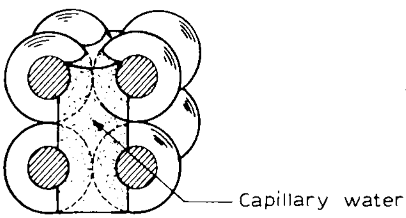


Fig. 2.26.

Let α be the wetting angle (between the two tangents of the meniscus and of the surface particle s at the point of contact), then the vertical component of the capillary force is: $T_v = 2\pi rT \cos(\alpha + \beta) = 2\pi RT \cos\beta \cos(\alpha + \beta)$.

The maximum value will occur when $\beta = -\alpha/2$:
$T_{max} = 2\pi RT \cos^2 \alpha/2$.

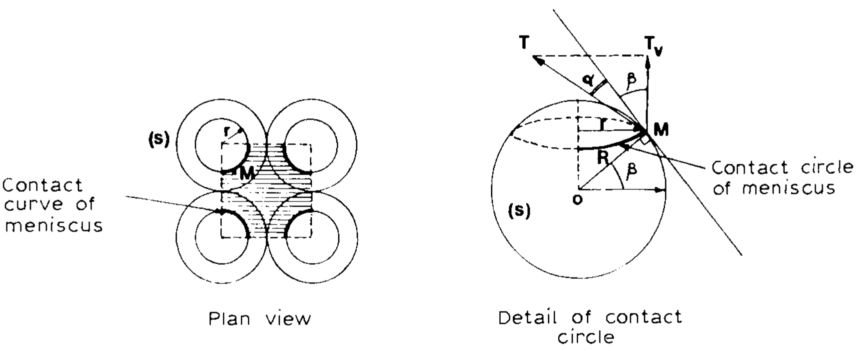


Fig. 2.27.

Let us now assume $\alpha = 0$. In this case the capillary maximum force will occur when the meniscus contacts a row of spheres along a plane through the center of these spheres. The vertical force due to capillary action will be balanced by the weight of a water column of height h, whose cross-sectional area, S_m, is yet to be determined. Let us consider one row of spheres of height $d = 2R$. The void ratio, for one element of mass (4 spheres) is calculated from:

volume of voids: $V_v = S_m \times d$, volume of grains: $V_s = \frac{4}{3}\pi R^3 = \frac{\pi d^3}{6}$,

and: $e = V_v/V_g$, from which: $d \times S_m = e\pi \frac{d^3}{6}$ or $S_m = e\frac{\pi d^2}{6}$

The weight of the water column of height h and of section S_m is then: $W = e\pi(d^2/6)h\gamma_w$. This weight is supported by the capillary force which is, for $\alpha = 0$: $T_v = \pi dT$. Therefore: $\pi dT = e\pi\frac{d^2}{6}h\gamma_w$ or $h = 6T/ed\gamma_w$.

If we take $T = 75$ dynes/cm and $g = 1000\ \text{cm/s}^2$, which gives $\gamma_w =$ 1000 dynes, then:

$$h = \frac{6 \times 75}{1000\, ed} = \frac{0.45}{ed}.$$

Since all the spheres have the same diameter, $d = d_{10}$, the empirical formula is proven: $h = C/ed_{10}$ (h and d_{10} are in cm) in which C varies between 0.1 and 0.5 cm^2, depending on the soil type.

Remark: In reality the wetting of the grain is far from ideal, therefore angle α is different from zero, which yields:

$$h = \frac{6\,T\cos^2(\alpha/2)}{ed}.$$

The corresponding value of C is lower but remains in the range 0.1—0.5 cm^2.

★★★*Problem 2.16* Water table drawdown by vertical drains

(1) Assuming that Dupuit's hypothesis is applicable to the problem, find the discharge and the equation of the free ground-water surface of a drainage trench of infinite length excavated to an impervious substratum through a soil of permeability k (assumed to be isotropic) and in which the static, free ground-water height is H above the substratum. Let R be the horizontal distance from the side of the trench to a location where the drawdown is zero, (analogous to the radius of influence of a pump-out well). Assume that the ground-water table is replenished by two infinite sources on each side of the trench whose horizontal distance from the trench is larger than R.

(2) Vertical drains consist of tubes of small diameter (from 5 to 8 cm) perforated at the base. They are usually installed by jetting, when spaced

about 1 m on center. Assume that one row of drains is equivalent to a drainage trench. The vertical drains are connected at the surface by a collector pipe attached to a pump.

In order to install a sewerline, it is necessary to excavate a 4 m deep trench through a silty sand of permeability $k = 2 \times 10^{-3}$ cm/s. The axis of the trench is located parallel to a river bank at 55 m distance. The impervious substratum is at 7 m below the existing ground surface. A row of vertical drain is installed 5 m away from the axis of the sewer. The free ground-water, prior to excavation of pumping is at 0.50 m below ground surface. See Fig. 2.28.

The vertical drains are jetted to the depth of the substratum. The length of the trench is 150 m. Find the discharge required to draw the water table down to a depth of 0.5 m below the bottom of the trench. Assume R = 50 m.

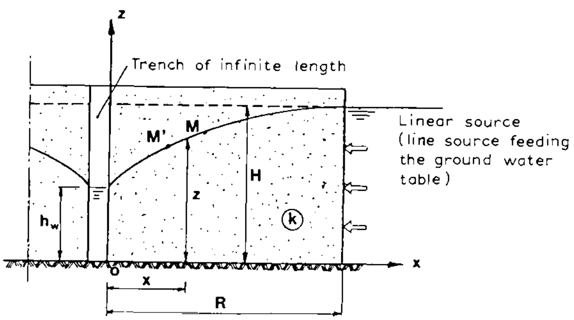


Fig. 2.28

(3) The silty sand through which the water table is drawn has $e = 0.60$, $d_{10} = 15\,\mu m$, $\gamma_{sat} = 20.4\,kN/m^3$. Its wet unit weight, above the capillary rise is estimated to be $\gamma_h = 17.6\,kN/m^3$. Find the change in effective stress during the drawdown, acting on a horizontal surface element located 10 m horizontally from the trench at a depth of 5 m. What is to be concluded? Assume $C = 0.3\,cm^2$ for the capillary rise empirical formula.

Solution

(1) Let M be a point on the free ground-water surface along the drawdown curve. The hydraulic gradient at that point is $i = -\Delta h/\Delta l$.

Since M is at the free water surface, the pore pressure is zero and if M' is located an incremental distance away from M on the free surface, then:

$$\Delta h = h_M - h_{M'} = -dz \quad \text{and} \quad i = -\frac{dz}{ds}$$

where s is the abscissa along the curved line of the free surface oriented from M towards M'. If the direction of orientation of the curvilinear abscissa is changed towards $O_{x'}$ then $i = \mathrm{d}z/\mathrm{d}s$.

The rate of discharge, according to Darcy's Law is $v = k(\mathrm{d}z/\mathrm{d}s)$. The horizontal component of this velocity is (see Fig. 2.29)

$$v_x = v\cos\alpha = v\frac{\mathrm{d}x}{\mathrm{d}s}, \quad v_x = k\frac{\mathrm{d}z}{\mathrm{d}s}\frac{\mathrm{d}x}{\mathrm{d}s} = k\frac{\mathrm{d}z}{\mathrm{d}x}\left(\frac{\mathrm{d}x}{\mathrm{d}s}\right)^2$$

Since the slope of the free water surface is low, Dupuit's hypothesis is applicable. So:

$$\frac{\mathrm{d}x}{\mathrm{d}s} = \cos\alpha \simeq 1 - \frac{\alpha^2}{2} \simeq 1 - \frac{1}{2}\left(\frac{\mathrm{d}z}{\mathrm{d}x}\right)^2 \quad \text{and} \quad \left(\frac{\mathrm{d}x}{\mathrm{d}s}\right)^2 \simeq 1 - \left(\frac{\mathrm{d}z}{\mathrm{d}x}\right)^2$$

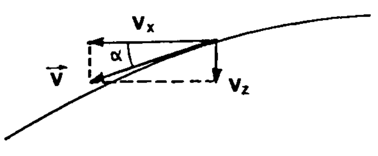


Fig. 2.29.

$$\text{from which: } v_x = k\frac{\mathrm{d}z}{\mathrm{d}x}\left[1 - \left(\frac{\mathrm{d}z}{\mathrm{d}x}\right)^2\right] \simeq k\frac{\mathrm{d}z}{\mathrm{d}x}$$

if we neglect the third order term.

Assuming that the water flow is laminar (Dupuit) v_x is the average value of the discharge rate across a vertical plane located at a distance x. The discharge across that plane passing by point M, parallel to the trench axis over a length b of the trench is:

$$q = v_x . S = v_x bz = kbz\frac{\mathrm{d}z}{\mathrm{d}x}.$$

Since water is incompressible and a steady flow is ultimately obtained, this unit discharge is half that flowing from both sides of the trench. Therefore we have the differential equation: $q\mathrm{d}x = kbz\mathrm{d}z$, and, integrating it between points O and R, we have:

$$qR = kb\frac{H^2 - h_w^2}{2}$$

The discharge pumped out for a unit length of trench ($b = 1$) is:

$$Q = 2q = k\left(\frac{H^2 - h_w^2}{R}\right) \tag{1}$$

Integrating between O and x yields:

$$qx = kb\left(\frac{z^2 - h_w^2}{2}\right) \quad \text{and from eqn. (1):} \quad \frac{1}{k} = \frac{H^2 - h_w^2}{2qR}$$

from which: $z^2 - h_w^2 = \dfrac{2qx}{k}$ (for $b = 1$)

$$\text{then: } z^2 = h_w^2 + \frac{x}{R}(H^2 - h_w^2) \tag{2}$$

(2) *Numerical application: vertical drains:*

$R = 50\,\text{m}$, $k = 2 \times 10^{-3}$ cm/s $= 2 \times 10^{-5}$ m/s, $H = 7 - 0.50 = 6.50$ m.

and along the axis of the sewer trench:

$x = 5.00$ m, $z = 7.00 - 4.50 = 2.50$ m.

Equation (2) gives: $\overline{2.50}^2 = h_w^2 + \dfrac{5}{50}(\overline{6.50}^2 - h_w^2)$.

from which $h_w \simeq 1.50$ m.
Equation (1) yields then:

$$Q = 2 \times 10^{-5} \times \frac{\overline{6.50}^2 - \overline{1.50}^2}{50} = 1.6 \times 10^{-5}\ \text{m}^3/\text{s}$$

per meter length of trench.
For a 150 m long trench,

$Q_T = 1.6 \times 10^{-5} \times 150 \times 3.6 \times 10^3 = 8.64\ \text{m}^3/\text{h}$ (or 2.4 l/s).

(3) The *capillary rise* in the sand, in cm, is calculated from $h = C/ed_{10}$ for: $e = 0.60$, $C = 0.3\ \text{cm}^2$ and $d_{10} = 50\ \mu\text{m} = 50 \times 10^{-3}$ mm $=$ 5×10^{-3} cm, we get: $h = \dfrac{0.3}{0.60 \times 5 \times 10^{-3}} = 100$ cm.

The capillary height above the free ground-water level is 1.00 m. The unit weight of soil to be applicable to this capillary zone is the saturated unit weight $\gamma_{sat} = 20.4\,\text{kN/m}^3$. Above the capillary rise, $\gamma_h = 17.6\,\text{kN/m}^3$ as given in the problem.

Let us calculate the level of the drawdown at equilibrium 10 m away from the trench:

$$z_1^2 = (\overline{1.50^2}) + \frac{10}{50}(\overline{6.50^2} - \overline{1.50^2}) \quad \text{from which } z_1 = 3.20\,\text{m}.$$

The total stress at 5 m depth and 10 m from the trench, before the drawdown, was: $(\sigma_v)_i = 5\gamma_{sat} = 20.4 \times 5 = 102\,\text{kPa}$ and the pore-water pressure was: $(u)_i = 4.5\gamma_w = 45\,\text{kPa}$.
The effective stress, prior to drawdown was: $(\sigma'_v)_i = 102 - 45 = 57\,\text{kPa}$.
After equilibrium has been reached with full drawdown:

$$(\sigma_v)_f = \gamma_{sat}(4.20 - 2) + \gamma_h(7 - 4.20) = 20.4 \times 2.20 + 17.6 \times 2.8 =$$

$$94.2\,\text{kPa}. \quad (u)_f = \gamma_w(3.2 - 2) = 12\,\text{kPa},$$

$$\text{from which } (\sigma'_v)_f = 94.2 - 12 \simeq 82\,\text{kPa}.$$

Drawing the water down causes an increase in effective stresses $\Delta\sigma'_v = 25\,\text{kPa}$. This is a considerable amount as it is greater by:

$$\frac{\Delta\sigma'_v}{(\sigma'_v)_i} = \frac{25}{57} \simeq 44\%.$$

This increase in stress is sufficiently large that consideration would have to be given to settlements resulting from a dewatering operation occurring in compressible soils.

★★*Problem 2.17* Piping conditions in a fractured rock mass

Consider the dam whose cross-section is shown on Fig. 2.30. An unlined drainage gallery was excavated in the rock downstream of the dam. The gallery intersects a fracture filled with pervious soil. The thickness of rock above the center line of the gallery is x, at the location of the fracture. The level difference between the reservoir pool and the grade where the fissure intersects the ground surface is Z. If we know that the unit weight of the fracture fill material is $\gamma_s = 26.5\,kN/m^3$, determine what values of x and z and of porosity n, would cause piping in the fracture.
Calculate x in terms of z, for $n = 40\%$.

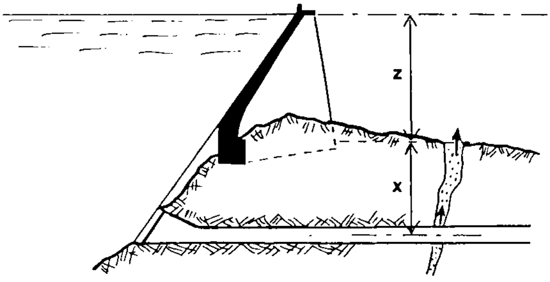


Fig. 2.30. Piping in fractured rocks.

Solution

Piping occurs where soil is carried away by water flowing upward and exerting a force greater than the buoyant weight of the soil particles. The consequence of piping is excessive loss of water and partial caving of foundation materials and possibly disastrous damage to the dam structure. In the case of rock, as in this problem, the only probable consequence would be excessive leakage through the foundation.

Piping would occur when at a point in the fracture, the weight of the soil is balanced by an upward force of the water flow. If it is assumed that the head gradient is constant throughout the length of the fracture, piping will occur when the pressure at the bottom of the fracture is $u = (z+x)\gamma_w$, and equal to the total stress which is $\sigma = x\gamma_{sat}$, therefore when $u = (z+x)\gamma_w = x\gamma_{sat} = x(\gamma' + \gamma_w)$ or when $z\gamma_w = x\gamma'$.

To avoid piping we must maintain the condition $z\gamma_w < x\gamma'$. We know that the buoyant unit weight of the soil material in the fracture is: $\gamma' = (\gamma_s - \gamma_w)(1-n)$ from which: $x/z > \gamma_w/(\gamma_s - \gamma_w)(1-n)$.

Conditions for piping are realized then, when $x/z > 10/16.5(1-0.40)$ or $x/z > 1/0.99 \simeq 1$.

To avoid piping, the rock thickness above the gallery would have to be greater than z.

★★*Problem 2.18* Permeability

A test is set up as shown in Fig. 2.31. A cylindrical mold of 4 in. in diameter ($D = 4$ in.) is filled to height $h_2 = 0.2$ ft with silt whose permeability is $k_2 = 5.3 \cdot 10^{-4}$ ft/min.

A second coaxial mold is placed on top of the first mold whose inside diameter is $d = 1.5$ in. and whose height is $h_1 = 0.3$ ft. Its thickness is negligible. The inside of this second mold is filled with the same silt, but the anular ring outside the small tube and outer tube is filled with sand whose permeability is $k_1 = 2 \cdot 10^{-3}$ ft/min.

The test set-up is a permeameter of constant head. Water is placed in the

mold and maintained at a level $Z = 1.25\,ft$ above the level of the outlet. It may be considered that altogether the system consists of a fictitious soil of thickness $H = h_1 + h_2$ and of permeability k_f.
(1) Find the value of k_f. (2) Determine the volume of water which percolated after 30 min.

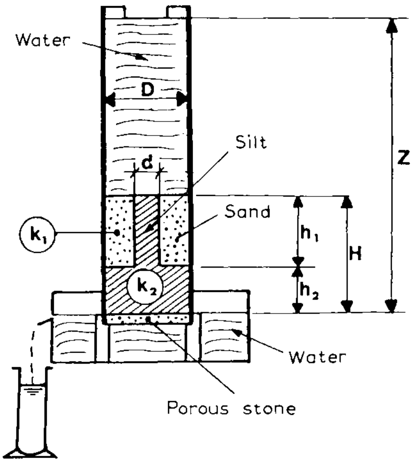


Fig. 2.31.

Solution

(1) The radius of frozen soil R_1 around a liquid pipe can be calculated, through the anular space q_1 and through the inner mold q_2. The hydraulic gradient in both instances is Z/H. Find q_2 from Darcy's law, where $v = ki$:

$$v_2 = k_2 i = k_2 \frac{Z}{H} \quad \text{from which} \quad q_2 = V_2 S_2 = k_2 \frac{Z}{H} \pi \frac{d^2}{4}.$$

The same applies to: $q_1 = k_v \dfrac{Z}{H} \pi \dfrac{D^2 - d^2}{4}$ (1)

but here, k_v is the equivalent permeability corresponding to the 2-layer system, therefore: $\dfrac{H}{k_v} = \dfrac{h_1}{k_1} + \dfrac{h_2}{k_2}$,

from which: $k_v = \dfrac{k_1 k_2 H}{k_1 h_2 + k_2 h_1}$ and $q_1 = \dfrac{k_1 k_2 H}{k_1 h_2 + k_2 h_1} \times \dfrac{Z}{H} \times \pi \dfrac{D^2 - d^2}{4}$. (2)

The total discharge at the outlet is $q = q_1 + q_2$. (3)

We also have: $q = k_f \dfrac{Z}{H} \times \pi \dfrac{D^2}{4}$. (4)

Rearranging equations (1), (2), (3) and (4):

$$k_f = k_2\frac{d^2}{D^2} + \frac{k_1 k_2 H}{k_1 h_2 + k_2 h_1} \times \frac{D^2 - d^2}{D^2}.$$

Therefore:

$$k_f = 5.3\cdot10^{-4} \times \frac{(1.5)^2}{(4)^2} + \frac{2\cdot10^{-3} \times 5.3\cdot10^{-4} \times 0.5}{2\cdot10^{-3} \times 0.2 + 5.3\cdot10^{-4} \times 0.3} \times \frac{4^2 - \overline{1.5}^2}{4^2}.$$

Thus: $k_f = 8.9\cdot10^{-4}$ ft/min or $k_f \simeq 1.06\cdot10^{-2}$ in./min.

(2) The total discharge is: $q = k_f \times \frac{Z}{H} \times \frac{\pi D^2}{4}$; $D = 4$ in. $= \frac{1}{3}$ ft.

$$q = 8.9 \times 10^{-4} \times \frac{1.25}{0.5} \times \frac{\pi}{4} \times \frac{1}{9} \simeq 1.94 \times 10^{-4} \text{ cu. ft/min}$$

or $q \simeq 0.33$ cu. in./min.
The volume of water collected at the bottom is: $V_{30} = 0.33 \times 30 \simeq 9.9$ cu. in.

Summary of answers
$k_f = 8.9\cdot10^{-4}$ ft/min ($\simeq 4.51\cdot10^{-4}$ cm/s); $V_{30} = 9.9$ cu. in. (or 162 cm^3).

★★★*Problem 2.19* **Practical application of soil freezing; design of an ice wall**

Let us assume that it is necessary to construct a 5-m diameter well in sandy soil whose properties are: water content $w = 30\%$, unit wet weight $\gamma_h = 20$ kN/m^3, specific heat $C_p = 1400$ J/kg/^{0}C (assumption B of Fig. 2.32).

Also given is the radius of freezing as determined from the graph of Fig. 2.23. It is assumed that freezing is 1.5 times slower around a nitrogen gas pipe than around a liquid nitrogen pipe.

(1) Determine the pipes spacing if the freezing from one pipe to the next has to occur in 48 h.

(2) Estimate the volume of soil frozen after 3 days.

(3) Using the graph of Fig. 2.33, determine the average temperature of the frozen wall. Evaluate the liquid nitrogen consumption per m^3 of soil frozen, assuming an efficiency of 80%.

(4) Determine the diameter of a circle around which the pipes would have to be installed and calculate the consumption per meter of well. Read the technical note presented at the end of this problem before starting the solution.

Solution

(1) The radius of frozen soil R, around a liquid pipe can be calculated, in cm, from the following equation, derived from Fig. 2.32: $R_1 = 30 + 0.7(t - 20)$ valid for any time t larger than 20 (in hours).

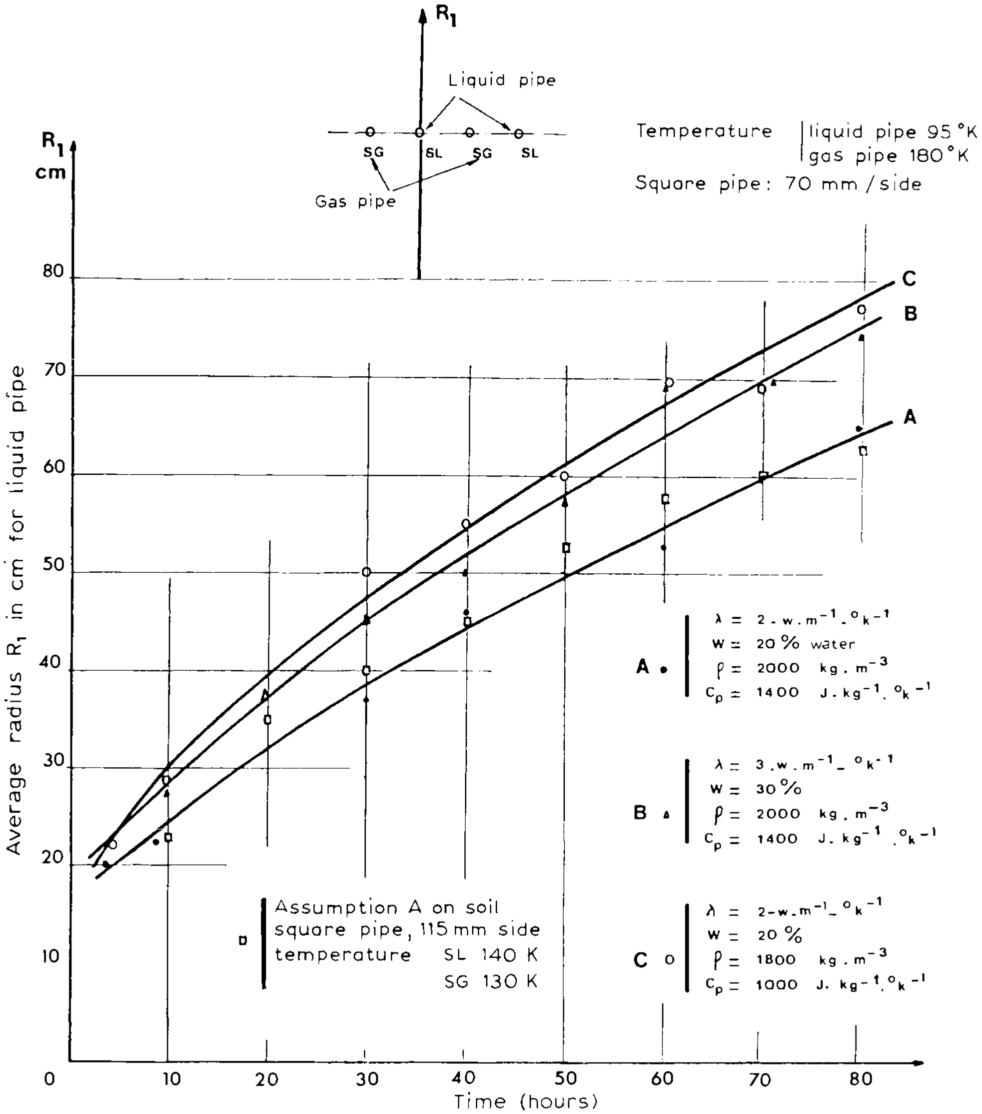


Fig. 2.32. Isotherm 0°C evolution with time along an axis perpendicular to the pipe alignment.

The freezing radius R_2 around a nitrogen gas pipe will be $R_2 = 20 + 0.47(t - 20)$ and at the end of 48 hours: $R_1 + R_2 = 49.6 + 33.2 = 82.8\,\text{cm}$.

This distance corresponds to the theoretical spacing because at the end of this time the two radii would touch and the freezing circles would be tangent.

(2) At the end of 72 h, the respective frozen soil radii will be $R_1 = 66.4\,\text{cm}$ (circle area $= 1.38\,\text{m}^2$), $R_2 = 44.6\,\text{cm}$ (circle area $= 0.62\,\text{m}^2$). If the excess

freezing, occurring due to the overlap of the two circles, is neglected, the total soil volume frozen is $2.00\,m^3/2 = 1\,m^3$ for a length of travel of 0.83 m and for a depth of pipe of 1 m. The average thickness of frozen wall is therefore 1.2 m.

(3) With assumption B, from Fig. 2.33, at the end of 72 h, the average wall temperature is 240° K or −33°C. The amount of liquid nitrogen needed to lower the soil to that temperature can be calculated: the energy required

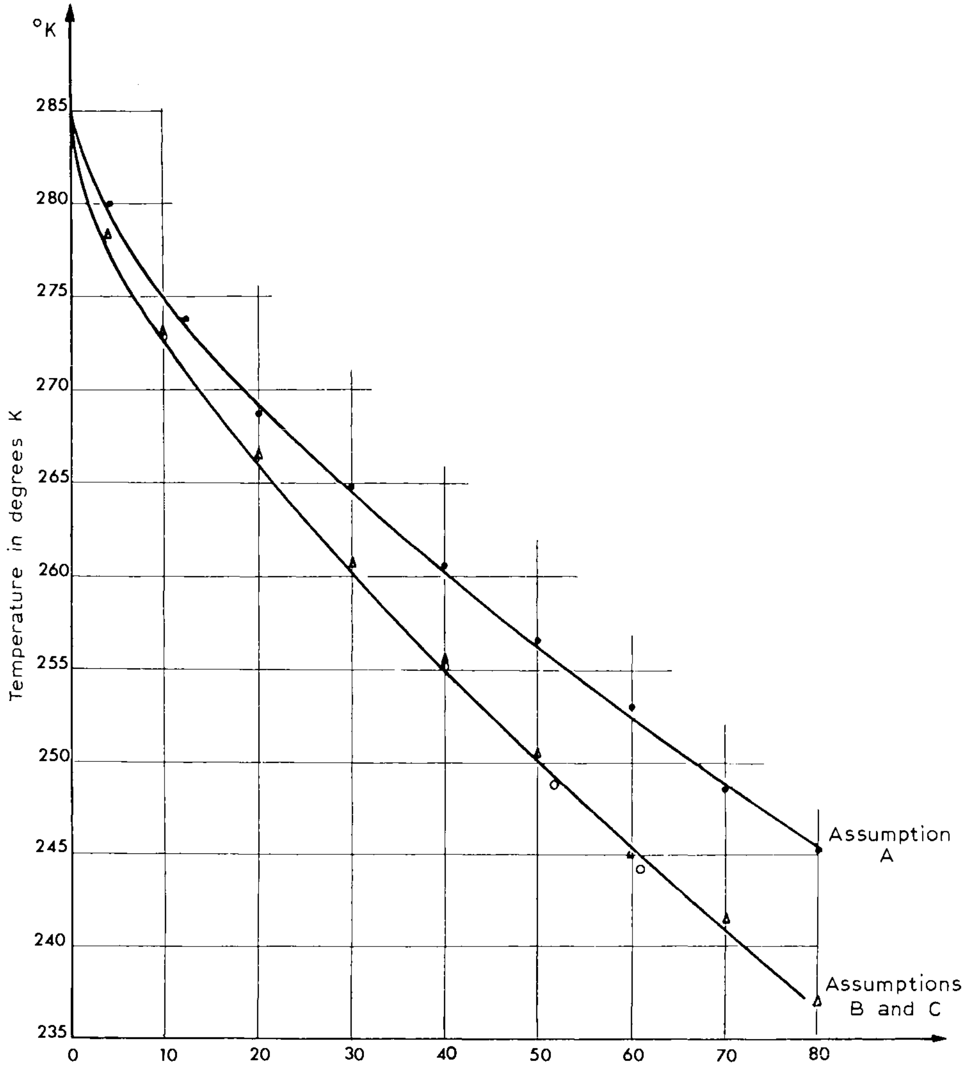


Fig. 2.33. Average wall temperature (°C = 273° K).

to cool 1 m^3 of soil (water excluded) from $+12^\circ C$ to $-33^\circ C$ ($\Delta T = 45^\circ C$) is: 2000 kg $\times$ 1400 J $^\circ C^{-1}$ kg^{-1} $\times$ 45 = 126,000,000 J.

One m^3 of soil with a water content of 30% contains 460 l of water. In order to freeze the water, the energy required will be 80 $\times$ 4.18 $\times$ 460,000 = 153,824,000 J, and the total energy need for both soil and water is: 279,824,000 J.

But 1 l of liquid nitrogen has a mass of 800 g and 1 g provides 400 J/g when changing phase (vaporizing) to raise its temperature to the ambient one. If the efficiency of this process is 80%, then 1 l of liquid nitrogen provides 256,000 J. Therefore, the liquid nitrogen demand is:

$$\frac{279{,}824{,}000}{256{,}000} = 1093 \text{ l liquid nitrogen per m}^3 \text{ of ground to freeze.}$$

(4) It was shown that the radius of the frozen ground around the liquid pipe is 66.4 cm after 72 h. The minimum radius of a circular layout of pipes would then be 5.00 m + 2 $\times$ 0.664 = 6.33 m, giving a perimeter of 19.87 m and a number of pipes of 19.87/0.83 = 23.9, say 24 pipes.

These pipes will be placed along the perimeter of a circle whose diameter is 6.35 m. The frozen ground volume will be, per lineal meter of pipe: 6.35 $\times$ 3.14 $\times$ 1.20 = 23.9 m^3 (thickness of 1.20 m is average frozen thickness already computed).

The construction of the frozen wall will require: 23.9 $\times$ 1093 = 26,123 l of liquid nitrogen per meter of well.

Appendix of Chapter 2

Technical note about freezing of soils

1. *Principle of computation*

Usually, the placement of pipes carrying the cooling agent is determined by the geometry of the area to be excavated. It is necessary to determine the spacing between the pipes. The diameters of the pipes are standard for this type of ground preparation and usually consider the following:

— for large excavations pipe lengths greater than 10 m, the outside pipe diameter is of the order of 3 inches;

— for smaller works, such as in localized areas and for short lengths of pipe, the pipes may be of the order of 2 inches (50 to 60 mm).

The following assumptions can be made: the pipe temperature is fixed at say θ_s, only slightly warmer than the temperature of liquid nitrogen along the liquid pipes. It is variable along the length of the pipe when nitrogen gas is used.

If thermal losses through the pipe walls are omitted, the following equations apply:

$$\frac{\partial \theta}{\partial t} = a_f\left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r}\right) \quad \text{(heat transfer in frozen ground)}$$

$$\left(-\lambda \frac{\partial\theta}{\partial r}\right)_{\mathrm{f}} - \left(-\lambda \frac{\partial\theta}{\partial r}\right)_{\mathrm{nf}} = \frac{\alpha}{\alpha + 100} \cdot \rho \cdot L \cdot \frac{dr_c}{dt} \quad \text{(freezing at isothermal 0°C)}$$

$$\frac{\partial\theta}{\partial t} = a_{\mathrm{nf}}\left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{2}\frac{\partial\theta}{\partial\mathrm{d}r}\right) \quad \text{(heat transfer in unfrozen ground)}$$

where: θ = temperature, t = time, r = radius, r_c = freezing radius, λ = conductivity, $a = \lambda/\rho_c$ = diffusion, ρ = unit mass, c = specific heat, f = frozen, nf = not frozen, L = phase change heat of water (freezing).

The limit conditions are:

$$t = 0 \quad \begin{cases} \theta = \theta_s & \text{(for } r = \text{pipe radius)} \\ \theta = \theta_0 & \text{initial temperature of the ground (for } r > \text{pipe radius)} \end{cases}$$

$$r = r_c \qquad \theta_{\mathrm{f}} = 0^\circ\mathrm{C}$$

$$r = \infty \qquad \theta = \theta_0.$$

A graphic interpretation is best used for the solution of the equations (method of Binder-Schmidt) or by computer.

(a) The following *simplifying assumptions* may be made:

Heat flows according to: $\dfrac{\theta - \theta_s}{\theta_0 - \theta_s} = \dfrac{\ln r/r_s}{\ln r_0/r_s}$;

where s = pipe subscript; r_0 = radius beyond which temperature changes are negligible; $\theta_{r_0} \simeq \theta_0$;

$$r_0 = r_s \exp\left(\frac{\theta_0 - \theta_s}{\theta_f - \theta_s} \ln(r_c/r_s)\right).$$

Instead of mental calculation which would imply too crude an assumption, the previous hypothesis allows us to calculate the average temperature of a frozen block as a function of two parameters: pipe temperature and freezing radius. From there on it is easy to determine the amount of liquid nitrogen which is needed to reach this freezing radius.

Example

Consider a 3-in. outside-diameter pipe (D = 88.9 mm) at a temperature of −133°C (140° K). For a 45-cm freezing radius (r_c/r_s = 10) the average temperature will be −42°C (see Fig. 2.34). If the soil has the following properties: θ_0 = +12°C, w = 30%, then $L = 334 \times \dfrac{30 \cdot 10^3}{130}$ = 77,000 Joules/dm^3, ρ = 1.8 kg/dm^3, C = 1400 J·kg^{-1}·K^{-1}, the energy required to lower the temperature of the ground from +12 to 0°C to freeze it and to decrease the temperature from 0°C to −42°C will be: 1.8[77,000 + 1400(42 + 12)] = 274,000 J per m^3 of ground; this corresponds to a nitrogen demand, at 80% efficiency (θ-outlet = −70°C) of 1.07 liters per m^3 of ground. (See Fig. 2.35). If the nitrogen gas pipe, next in line after a liquid nitrogen pipe, has an average between outlet and inlet temperature of −100°C, the average temperature of the frozen ground bloc of 45 cm of radius will be −32°C (see Fig. 2.33). The amount of energy required then is:
1.8[77,000 + 1400(32 + 12)] = 249,000 J per m^3 of ground, or 0.97 liters of nitrogen per m^3 of ground.

By and large, it is seen that nitrogen consumption will be about 1 m^3 of liquid nitrogen per m^3 of ground to freeze.

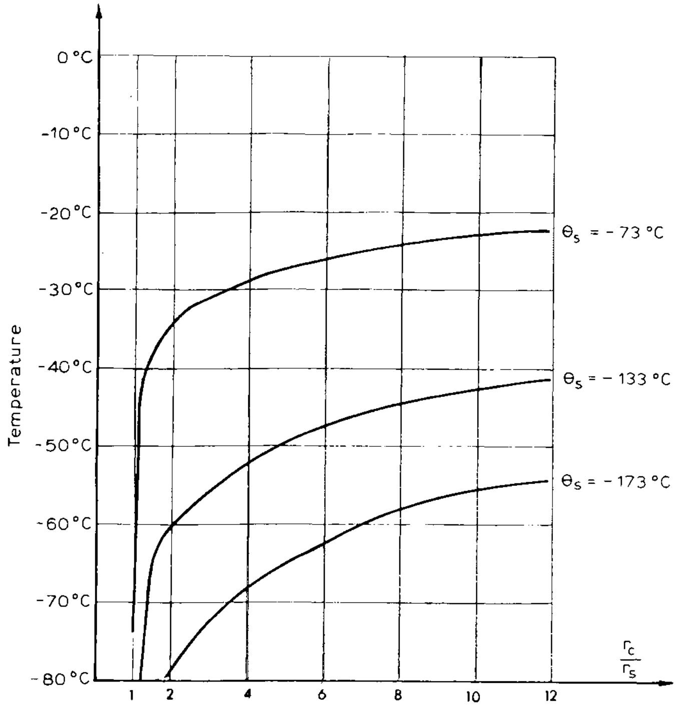


Fig. 2.34. Average temperature of frozen block (assuming a logarithmic distribution of temperatures) as a function of r_c/r_s, where r_c is the freezing radius and r_s is the pipe radius.

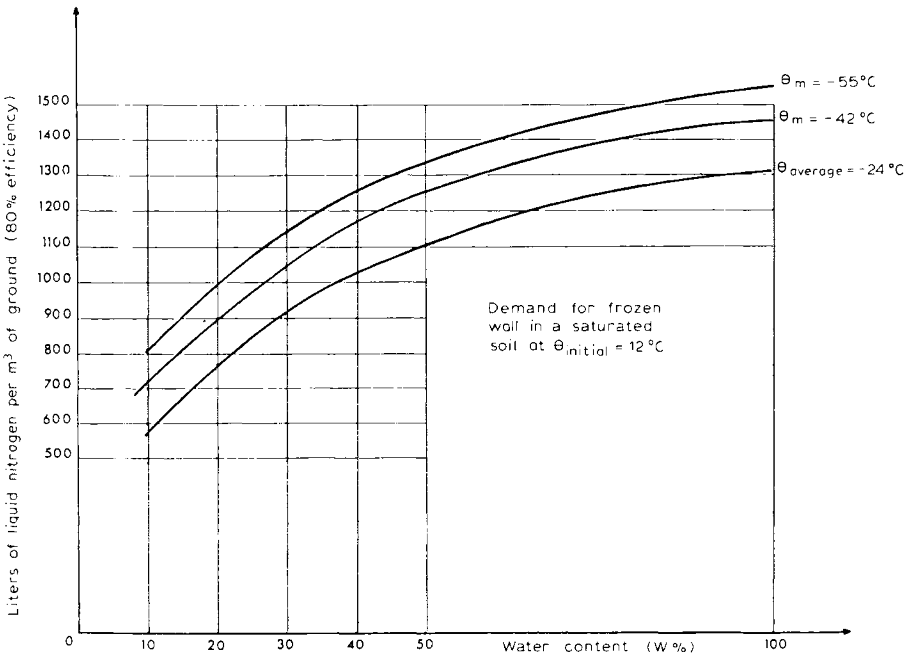


Fig. 2.35.

The simplifying assumptions neglect: (1) Cooling of the ground beyond the frozen radius, which in fact reduces consumption by 3 to 5%; (2) the effect of alternating liquid and gaseous nitrogen pipes which increase nitrogen consumption by about 10% (the average temperature is, in fact, higher).

(b) *Soil information.*

Heat conductivity: Heat conductivity of the soil depends primarily on the following factors: water content (first of all); grain-size distribution, compactness.

The value of λ is different for a frozen and a thawed soil (see Fig. 2.36 for this and for thermal conductivities).

Specific heat: We know that $C_{\text{water}} = 1\ \text{cal}\cdot\text{g}^{-1}\cdot\text{K}^{-1} = 4.18\ \text{J}\cdot\text{g}^{-1}\cdot\text{K}^{-1}$ and $C_{\text{ice}} = 0.5\ \text{cal}\cdot\text{g}^{-1}\cdot\text{K}^{-1} = 2.09\ \text{J}\cdot\text{g}^{-1}\cdot\text{K}^{-1}$. It can be assumed that:

$C_{\text{grain}} = 0.17\ \text{cal}\cdot\text{g}^{-1}\cdot\text{K}^{-1} = 0.76\ \text{J}\cdot\text{g}^{-1}\cdot\text{K}^{-1}$ (average value for any type of soil grains).

The specific heat for *unfrozen soil* is: $C_u = \gamma_d(0.17 + w) = \dfrac{\gamma_s}{\gamma_s w + \gamma_w}(0.17 + w)$

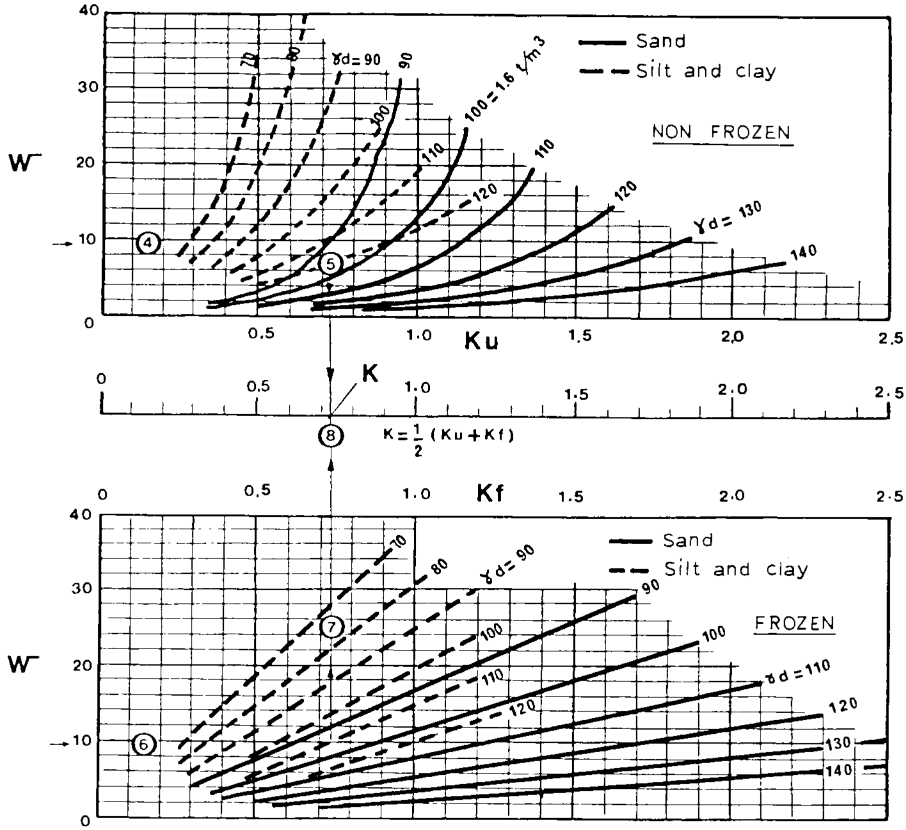


Fig. 2.36. Thermal conductivity of soils (after Kersten). γ_d(lb/cu. ft = 0.016 g km^3); K = λ = conductivity (BTU/ft^{-1} h^{-1} ^{0}F^{-1} = 1.73 w m^{-1}/$^\circ$K^{-1}; Ku = conductivity of non-frozen soil; K$_f$ = conductivity of frozen soil.

and for *frozen soil:* $C_f = \dfrac{\gamma_s}{\gamma_s w + \gamma_w}\,(0.17 + 0.5\,w).$

Specific heat of soil when changing phase (freezing) is $L = \dfrac{\gamma_s w}{\gamma_s w + \gamma_w}\,L_{water}$,

from which Table 2A is made up and on the values of which the curves of Fig. 2.35 were based.

TABLE 2A

Conductivity of soils

w%	c_u		c_g		L	
	cal/cm^3/K	J/cm^3/K	cal/cm^3/K	J/cm^3/K	cal/cm^3	J/cm^3/K
0	0.46	1.92	0.46	1.92	0	0
10	0.57	2.38	0.47	1.96	17	71
20	0.65	2.72	0.47	1.96	28	117
30	0.70	2.93	0.48	2.01	36	150
40	0.74	3.09	0.48	2.01	42	175
50	0.77	3.22	0.48	2.01	46	192
100	0.85	3.55	0.49	2.05	58	242

2. *Example of results from computer application.*

The diameters of the pipes are 80/88.9. Temperatures were chosen to account for the actual performance of the first pipe with liquid nitrogen.

Other givens are: pipe equivalent to square section of 70 mm side; initial temperature 285° K; liquid nitrogen pipe temperature 95° K; gaseous nitrogen pipe temperature 180° K.

Other assumptions regarding the soil:

$$A\begin{cases}\lambda = 2\ \mathrm{w\,m^{-1}\,K^{-1}}\\ w = 20\%\\ \rho = 2000\ \mathrm{kg\,m^{-3}}\\ c_p = 1400\ \mathrm{J\,kg^{-1}\,K^{-1}}\end{cases}$$

$$B\begin{cases}\lambda = 2\ \mathrm{w\,m^{-1}\,K^{-1}}\\ w = 30\%\\ \rho = 2000\ \mathrm{kg\,m^{-3}}\\ c_p = 1400\ \mathrm{J\,kg^{-1}\,K^{-1}}\end{cases}$$

$$C\begin{cases}\lambda = 2\ \mathrm{w\,m^{-1}\,K^{-1}}\\ w = 20\%\\ \rho = 1800\ \mathrm{kg\ m^{-3}}\\ c_p = 1000\ \mathrm{J\,kg\,K^{-1}}.\end{cases}$$

Results are presented on the graphs of Figs. 2.32, 2.33 and 2.35. Fig. 2.32 gives the frozen radius along an axis through a liquid nitrogen pipe and perpendicular to the line of pipes. Not only have the curves the same shape, but it will be noticed that they appear closely grouped. This means that the rate of freezing does not change much for different soil conditions. Also the order of magnitude of R (in cm) is $30 + 0.7t$ $(t - 20)$ for t in hours greater than 20.

Fig. 2.33 shows the average frozen-wall temperature. This result is in good agreement with the curve giving the freezing radius. It should be noted that the average temperature varies little in different soil types. The order of magnitude is from $-20°C$ to $-28°C$ after 60 hours.

Chapter 3

PRACTICAL SETTLEMENT CALCULATIONS — COMPRESSIBILITY AND THEORY OF CONSOLIDATION

★★*Problem 3.1* Oedometer test on sand

An oedometer test performed on a sand sample gave the following results:

loading (in kg) in frame	*0*	*24.52*	*73.56*	*171.70*	*367.90*
settlement (in mm)	*0*	*0.04*	*0.12*	*0.25*	*0.41*

The oedometer ring area is 38.5 cm^2*, the sample has an initial height of 24 mm.*
(a) Draw the compression curve and find the consolidation moduli (in (daN/cm) corresponding to the load intervals.
(b) Calculate the slope of the compression curve for the last two load increments and compare it to the compression moduli.

Solution

The compression curve for the sand is plotted from the strain values $\frac{\Delta h}{h_0} = (h_0 - h)/h_0$ and corresponding load stresses. (Fig. 3.1). These are presented on Table 3A.

The oedometric modulus for a load increment $\Delta\sigma = \sigma_2 - \sigma_1$ by definition is:

$$E' = -\frac{\Delta\sigma}{\Delta h/h}$$

TABLE 3A

Masses (kg)	σ (daN/cm^2)	$h - h_0$ (mm)	$\frac{h - h_0}{h_0}$
0	0	0	0
24.52	0.64	0.04	0.167
73.56	1.91	0.12	0.50
171.70	4.46	0.25	1.04
367.90	9.53	0.41	1.71

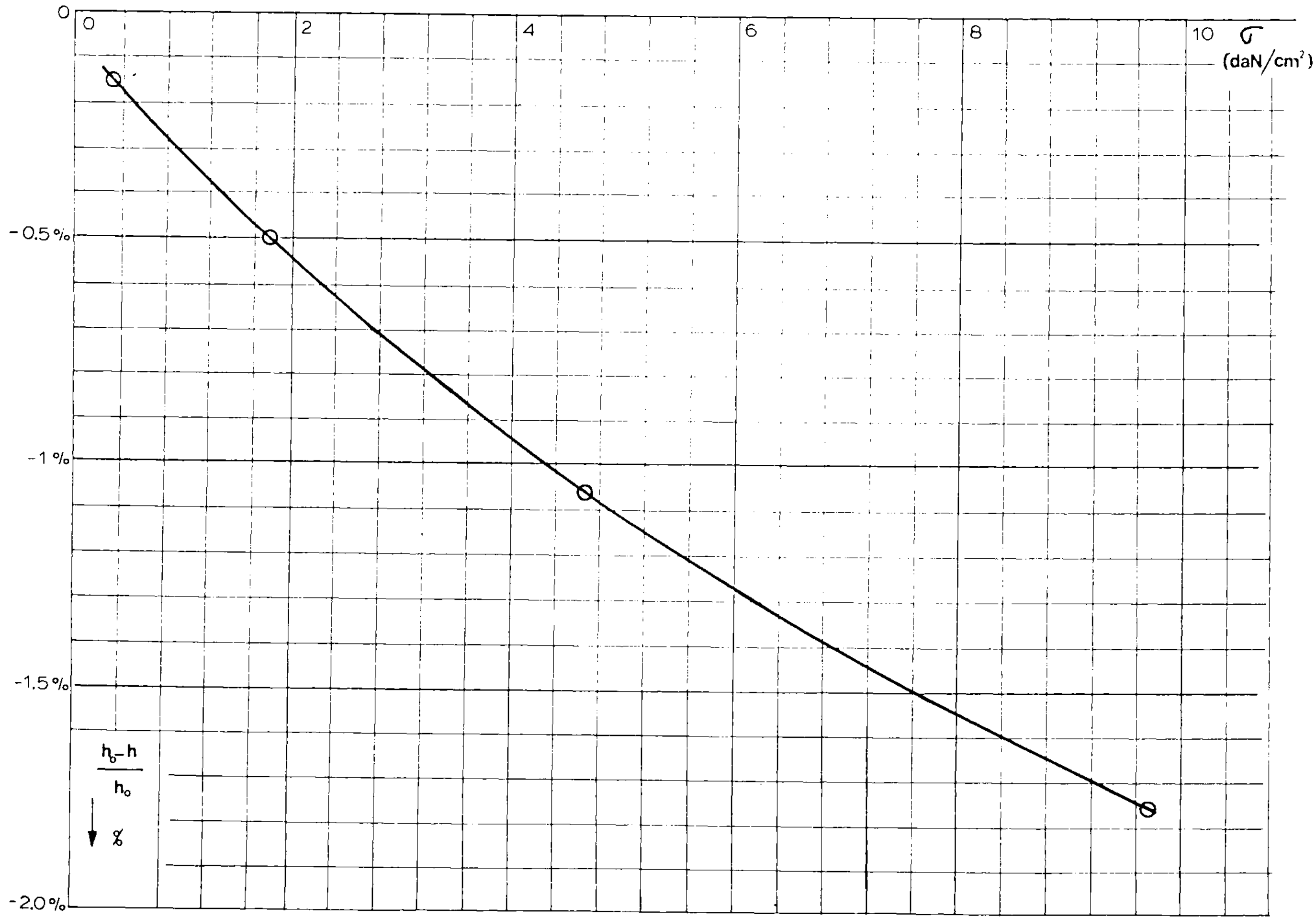


Fig. 3.1. Consolidation test for sand.

where h is the sample height for $\sigma = \sigma_1$, Δh is the settlement of the sample under a load of $\Delta\sigma$.
Table 3B summarizes the oedometric moduli for each load increment

TABLE 3B

σ (daN/cm^2)	$\Delta\sigma$ (daN/cm^2)	h (mm)	Δh	$\Delta h/h$ (%)	E' (daN/cm^2)
0		24.00			
	0.64		0.04	0.167	383
0.64		23.96			
	1.27		0.08	0.334	380
1.91		23.88			
	2.55		0.13	0.545	469
4.46		23.75			
	5.07		0.16	0.675	752
9.53		23.59			

The slope of the compression curve is:

$$-\frac{\Delta\sigma}{\dfrac{h_2-h_0}{h_0}-\dfrac{h_1-h_0}{h_0}} = -\frac{\Delta\sigma}{\dfrac{h_2-h_1}{h_0}} = -\frac{\Delta\sigma}{\dfrac{\Delta h}{h_0}}$$

and since $h_0 > h$, the value of this slope is slightly greater than the oedometric modulus.

For the last two load increments we have:

(a) $1.91 \leqslant \sigma \leqslant 4.46$, $\Delta\sigma = 2.55$,

from Table 3A:

$$\frac{h_2-h_0}{h_0} = 1.04\%, \quad \text{and} \quad \frac{h_1-h_0}{h_0} = 0.50\%$$

and the slope is:

$$-\frac{\Delta\sigma}{\Delta h/h_0} = \frac{2.55}{0.54/100} = \frac{2550}{5.4} = 472 \quad \text{instead of 469 for } E'$$

(b) $4.46 \leqslant \sigma \leqslant 9.53$, $\Delta\sigma = 5.07$.

Similarly: $\dfrac{h_2-h_0}{h_0} = 1.71\%$ and $\dfrac{h_1-h_0}{h_0} = 1.04\%$

from which the slope is: $5070/6.7 = 757$, instead of 752 for E'. It will be noticed that the two values are very close and the difference is less than inaccuracies due to test errors.

★★*Problem 3.2* **Consolidation test on clay**

A clay sample is tested in an oedometer with the following results:

stress (daN/cm^2)	*settlement (mm)*
0	0
0.1	0.2
0.2	0.03
0.4	0.05
0.8	0.10
1.6	0.19
3.2	0.43
6.4	1.09
12.8	1.78
1.6	1.58
0.4	1.43
0.1	1.22

At the start of the test, the sample height was 25 mm and its void ratio was 1.01. Draw the compression curve (e vs 1 g σ) and calculate the compression index C_c and the oedometric modulus corresponding to the load increment 6.4 daN/cm^2 to 12.8 daN/cm^2 (loading). Compare this modulus with the secant modulus for point load at 6.4 daN/cm^2.

The consolidation pressure σ_c may be determined from the classical method on the $e - \log \sigma$ curve and the $\ln(1 + e) - \log \sigma$ diagram as recommended by Butterfield (Fig. 3.2b).

Solution

Since the volume of soil grains remains constant throughout the test, we have:

$$\frac{\Delta h}{h_0} = \frac{\Delta e}{1 + e_0}$$

for $h_0 = 25$ mm and $e_0 = 1.01$, and calculating Δe, if Δh is in mm:

$$\Delta e = \frac{1 + e_0}{h_0} \Delta h = \frac{2.01}{25} \Delta h = 8.04 \times 10^{-2} \times \Delta h.$$

The values of void ratio e change, corresponding to the settlements observed during the test, and are given in Table 3C.

The compression index C_c is the slope of the straight line portion of the compression curve for $\sigma > \sigma_c$ (see Figs. 3.2a and 3.2b) (σ_c = consolidation stress)

$$C_c = -\frac{\Delta e}{\Delta \log \sigma}.$$

The diagram may be approximated to a straight line for the interval

TABLE 3C

σ (daN/cm^2)	Δh (mm)	Δe	e
0	0	0	1.010
0.1	0.02	16.1×10^{-4}	1.008 4
0.2	0.03	24.1×10^{-4}	1.007 6
0.4	0.05	40.2×10^{-4}	1.006
0.8	0.10	80.4×10^{-4}	1.002
1.6	0.19	152.8×10^{-4}	0.995
3.2	0.43	345.7×10^{-4}	0.975
6.4	1.09	876.4×10^{-4}	0.922
12.8	1.78	1431.1×10^{-4}	0.867
1.6	1.58	1270.3×10^{-4}	0.883
0.4	1.43	1149.7×10^{-4}	0.895
0.1	1.22	980.9×10^{-4}	0.912

$3.2 \leqslant \sigma \leqslant 12.8$ daN/cm^2, and:

$$C_c = \frac{0.867 - 0.975}{\log 12.8 - \log 3.2} = \frac{0.108}{\log 4} = \frac{0.108}{0.602} \simeq 0.18.$$

The oedometric modulus E' corresponding to the stress interval 6.4 to 12.8 daN/cm^2 is given by: $E' = -\dfrac{\Delta\sigma}{\Delta h/h}$

where h is the height of the sample when $\sigma = 6.4$ daN/cm^2 and Δh represents the settlement occurring when the load is increased from 6.4 to 12.8 daN/cm^2.
$h = 25.00 - 1.09 = 23.91$ mm, $\Delta h = -(1.78 - 1.09) = -0.69$ mm, $\Delta\sigma = 12.8 - 6.4 = 6.4$ daN/cm^2

from which: $\dfrac{\Delta h}{h} = -\dfrac{0.69}{23.91} = -0.0288$, $\quad E' = \dfrac{6.4}{0.0288} \simeq 222$ daN/cm.

The secant modulus at any one point is: $E'_s = -\dfrac{\sigma}{\Delta h/h_0}$

where h_0 is the initial height of sample, Δh is the settlement under σ and therefore:

$$E'_s = -\frac{6.4}{-(1.09/25)} = \frac{640}{4.36} = 147 \text{ daN/cm}^2.$$

In this instance, the difference between the two moduli is substantial.

The consolidation pressure of about 2.4 daN/cm^2 is obtained by simply

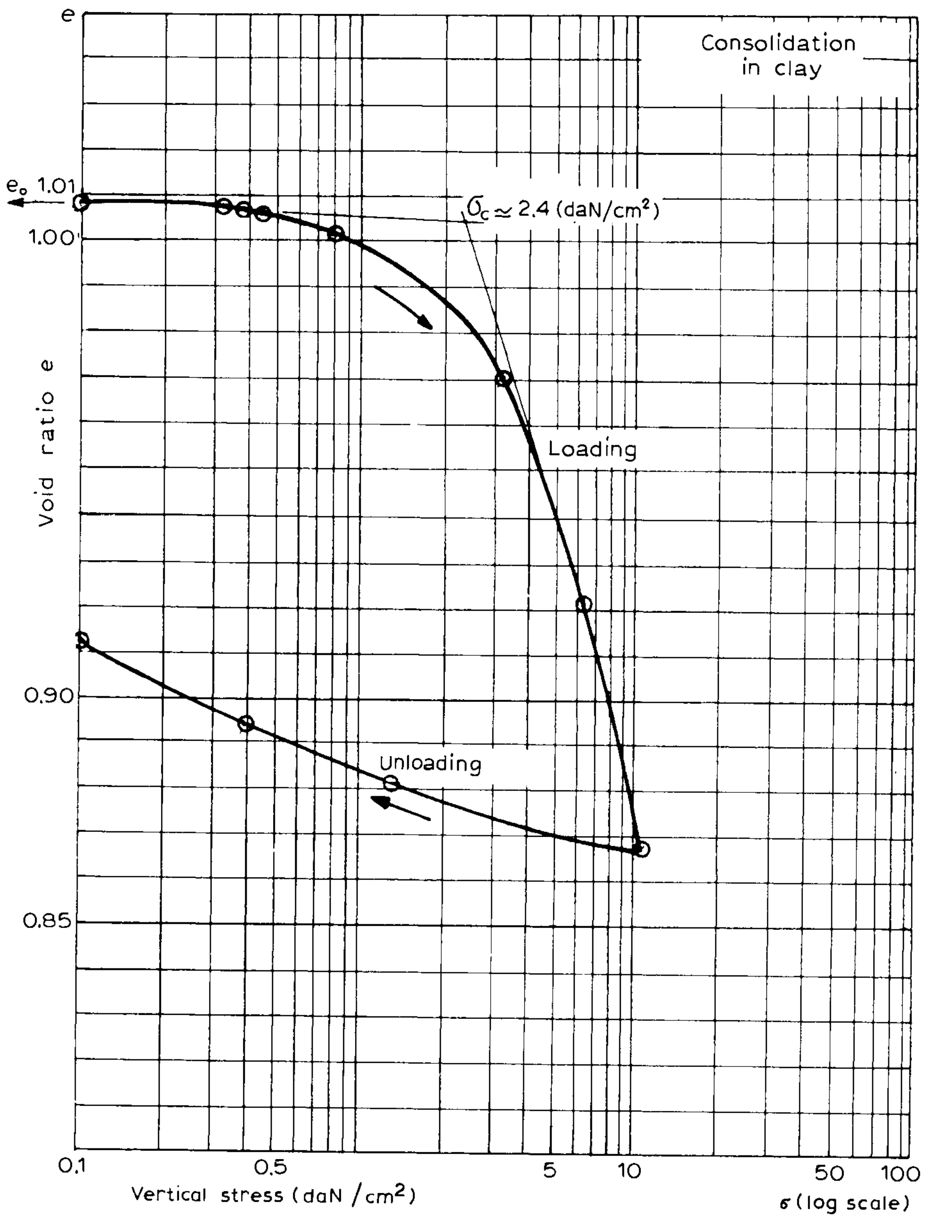


Fig. 3.2a. Curve of compression in clay.

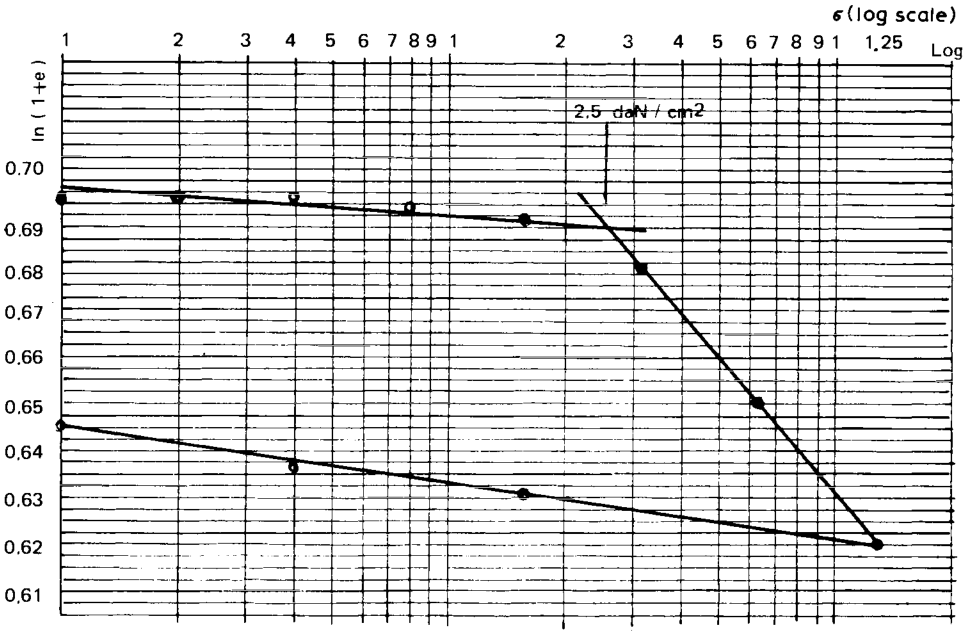


Fig. 3.2b. After Butterfield.

drawing the intersection of the tangents to the consolidation curve on the e — log σ graph, as indicated on Fig. 3.2a. On the other hand, if we use the Butterfield construction (see *Geotechnique*, 29: 469—479), we find a consolidation pressure of 2.5 daN/cm^2 (see Fig. 3.2b). Butterfield's construction appears to be simpler and avoids errors of estimating from the graphical construction.

Summary of answers

$C_c = 0.18, \qquad \sigma_c \simeq 2.5\,\text{daN/cm}^2$

and

$E' = 222\,\text{daN/cm}^2$ for $6.4 \leqslant \sigma \leqslant 12.8\,\text{daN/cm}^2$

$E' = 147\,\text{daN/cm}^2$ for $\sigma = 6.4\,\text{daN/cm}^2$.

★★*Problem 3.3* Approximate evaluation of the compression index C_c and of the settlement of a normally consolidated clay

Borings were made for a construction project. They showed that subsurface soils consist of a layer of fine sand 10.60 m thick, overlaying a soft clay layer 7.60 m thick. The free ground-water table is at 4.60 m below the ground surface (see Fig. 3.3).

The buoyant unit weight is 1.04. The wet sand density is 1.76 above the water table. The water content of the normally consolidated clay is $w = 40\%$, *its liquid limit $w_L = 45\%$ and the soil particles unit weight of the soil constituent is $2.78\,kN/m^3$. The proposed construction will impart a net stress of $1.2\,daN/cm^2$.*

Find the average settlement of the clay layer (determine the compression index C_c by Skempton's formula and start from the initial vertical stress in the middle of the clay layer).

Solution

Skempton's formula for the compression index is $C_c = 0.009(w_L - 10)$, and for this problem, it is: $C_c = 9 \times 10^{-3}(45 - 10) = 9 \times 35 \times 10^{-3} = 315 \times 10^{-3} = 0.32$.

The settlement is given by the formula:

$$\frac{\Delta h}{h} = -\frac{C_c}{1+e} \log\left(1 + \frac{\Delta\sigma}{\sigma'}\right).$$

Since the clay is saturated, its initial void ratio is:

$$e = w\frac{\gamma_s}{\gamma_w} = 0.40 \times 2.78 = 1.11.$$

(This value corresponds to a soft clay.)
The vertical stress acting at mid-height in the clay layer is:

$$\sigma' = 4.60 \times 17.6 + 6.00 \times 10.4 + \frac{7.60}{2} \times \gamma'$$

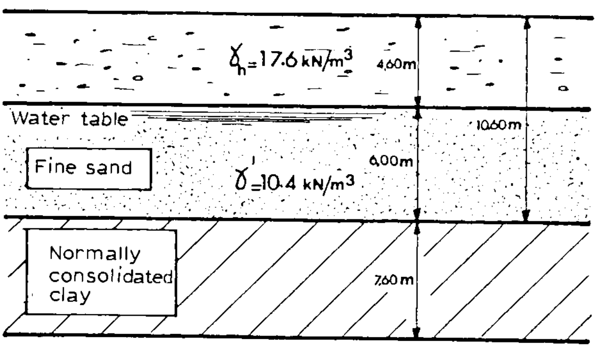


Fig. 3.3.

where γ' is the buoyant weight of the clay:

$$\gamma_h = \frac{\gamma_s + e\gamma_w}{1+e} = \frac{27.8 + 11.1}{21.1} = 18.44 \simeq 18.4, \text{ so } \gamma' = 8.4\,\mathrm{kN/m^3}.$$

The vertical stress then is:

$$\sigma' = 4.60\times17.6 + 6.00\times10.4 + 3.80\times8.4 = 175.3\,\text{kPa}$$

and $\dfrac{\Delta h}{7.60} = -\dfrac{0.32}{2.11}\log\left(1+\dfrac{12}{17.5}\right)$, $\Delta h = -\dfrac{7.60\times0.32}{2.11}\log 1.686 = 0.261$

or $\Delta h \simeq 26\,\text{cm}$.

Summary of answers
$C_c = 0.32$; settlement: 26 cm.

★★*Problem 3.4* Approximate evaluation of settlements and of preconsolidation pressure for an overconsolidated clay

An area under consideration is known to have been a lake at the beginning of the Quaternary era. The lake bottom consisted then of a sand layer 55.70 m thick overlaying a clay layer of 7.6 m in thickness (see Fig. 3.4). With time, the lake disappeared and the lake bottom became a plateau through which a river eventually carved a deep valley. The plateau is now some 45 m above the bottom of this valley. The water in the river is 1.5 m below the level of the valley (see Fig. 3.5).

The sand layer has the following characteristics: buoyant unit density: 1.04, wet density above the water table: 1.76.

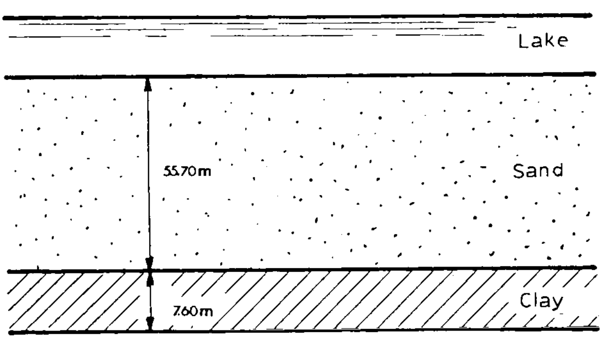


Fig. 3.4.

The clay layer properties are: natural water content 35%, liquid limit 45%, specific gravity of soil particles 2.78.

(a) Find the pre-consolidation pressure (neglect the weight of the clay).

(b) Evaluate the settlement range which could occur due to consolidation of the clay if a building is constructed which would impart a stress of 0.9 daN/cm² to the clay layer.

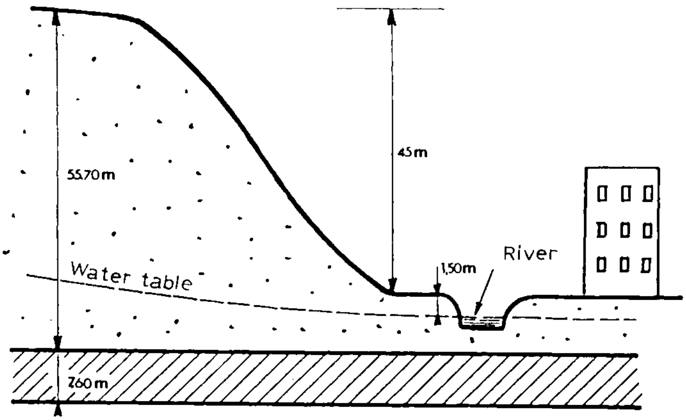


Fig. 3.5.

Solution

(a) By definition, the pre-consolidation pressure is the largest effective vertical stress ever experienced by the soil during its geological history. At the time when the lake existed, the effective overburden pressure was, at the top of the clay layer: $\sigma' = 55.70 \times 10.4 = 579$ kPa.

If the water level remained at the ground surface during the erosional process, the largest overburden pressure was the one calculated above, therefore: $\sigma_c = 580$ kPa, or $\sigma_c = 5.8$ daN/cm^2.

Remarks

For the case corresponding to the conditions summarized in Fig. 3.6, which would correspond to a long dry period before the erosional process started, the stress at the top of the clay layer would be: $\sigma' = (55.70 - H_2)\gamma_h + (H_2 - H_1)\gamma_{sat} + H_1\gamma'$.

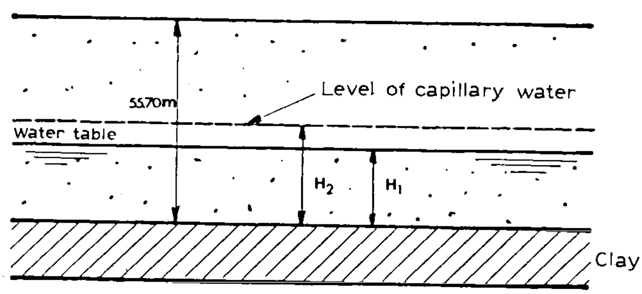


Fig. 3.6.

If we assume the following values: $H_2 = 45.70$ m, $H_1 = 45.00$ m, then $\sigma' = (55.7 - 45.7) \times 17.6 + 0.70 \times 20.4 + 45.00 \times 10.4 = 176 + 14.3 + 468 =$

658.3 kPa = 6.6 daN/cm^2, which is considerably larger than the preconsolidation pressure.

(b) Let us calculate the vertical in-situ stress σ'_0 at the mid-height of the clay layer before construction of the building. We must first know the unit weight of the clay.

·We know:

$$\gamma_h = \gamma_s \frac{1+w}{1+e}, \quad e = w(\gamma_s/\gamma_w) = 0.35 \times 2.78 = 0.973.$$

Because the clay is saturated: $\gamma_h = 27.8(1.35/1.973) \simeq 19\,\text{kN/m}^3$, then $\gamma'/\gamma_w = 0.9$.

The unit weight of the saturated sand in the capillary rise zone is 20.4 kN/m^3. The thickness of this layer is 1.5 m. We then have: $\sigma'_0 = 1.50 \times 20.4 + (55.70 - 46.50) \times 10.4 + 3.80 \times 9 = 160.5$ kPa or $\sigma'_0 = 1.6$ daN/cm^2.

This stress is smaller than the pre-consolidation pressure ($\sigma'_0 < \sigma_c$), so the clay is overconsolidated.

The weight of the structure will increase the stress on the clay layer by an amount of $\Delta\sigma = 0.9$ daN/cm^2, but $\sigma_c - \sigma'_0 = 5.7 - 1.6 = 4.1$ daN/cm^2, therefore: $\Delta\sigma = 0.9\,\text{daN/cm}^2 < \frac{1}{2}(\sigma_c - \sigma'_0)$.

If we let Δh be the settlement of a normally consolidated clay layer of equal thickness and equal liquid limit, we would approximately have, for the settlement, s, of the overconsolidated clay: $\Delta h/10 < s < \Delta h/4$ and for a normally consolidated clay, Skempton's formula for the compression index gives: $C_c = 0.009(w_L - 10) = 0.009 \times 35 = 0.315 \simeq 0.32$.

If the clay were normally consolidated, the settlement would be:

$$\frac{\Delta h}{h} = \frac{C_c}{1+e} \log\left(1 + \frac{\Delta\sigma}{\sigma'_0}\right) = \frac{0.32}{1.973} \log\left(1 + \frac{0.9}{1.56}\right) \simeq \frac{0.32}{1.97} \log 1.58$$

$$\simeq 3.21 \times 10^{-2}$$

$$\Delta h = 760 \times 3.21 \times 10^{-2} = 24.4\,\text{cm}.$$

For the overconsolidated clay then: $\Delta h/10 \simeq 2.4$ cm, $\Delta h/4 = 6$ cm hence $2.4 < s < 6$ cm.

Summary of answers

$\sigma_c \simeq 580$ kPa; $2.4 < s < 6$ cm.

★★*Problem 3.5* Stresses at depth below shallow footings

A shallow footing is 12 m square and 20 cm thick. It supports a load whose intensity is 0.78 daN/cm^2. Assume concrete unit weight to be 2.5.

Determine the vertical stresses due to the footing and its load, at a depth of 24 m below the ground surface and on verticals from points A, C, E and F as shown in Fig. 3.7. Compare these values to those obtained if it were asssumed that all loads are concentrated at point C.

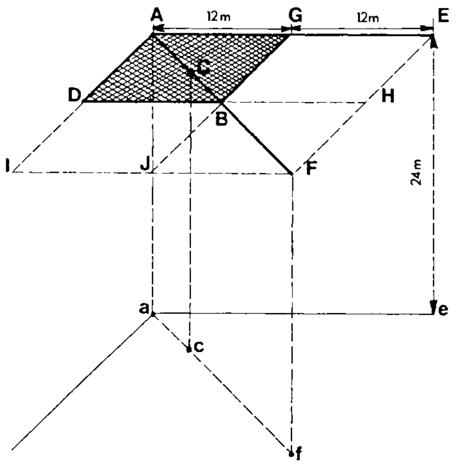


Fig. 3.7.

Solution

The stress due to the footing weight is: $0.20 \times 1.00 \times 25 = 5$ kPa or about $0.05\,\text{daN/cm}^2$. The uniform stress below the footing is: $q = 0.78 + 0.05 = 0.83\,\text{daN/cm}^2$.

(a) Let a, c, e and f (Fig. 3.7) be the orthogonal projections of A, C, E and F on the horizontal plane at 24 m below ground surface. The stress increase at a due to the footing is obtained from the graph of Fig. 3.7a. for $L/B = 1$.

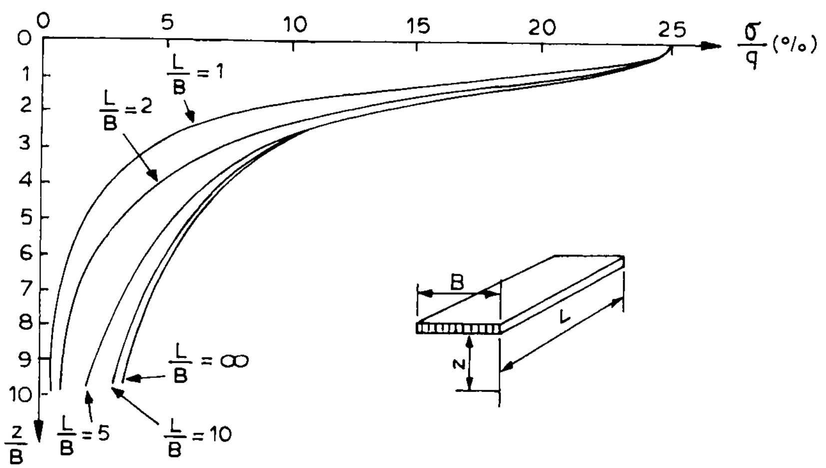


Fig. 3.7a. Stress below a corner of a shallow footing.

We have: $z/B = 24/12 = 2$ and $L/B = 1$, which gives $\Delta\sigma/q = 8.5\%$ therefore $\Delta\sigma = 0.83 \times 8.5\% \simeq 7 \cdot 10^{-2}\,\text{daN/cm}^2$.

The increase in vertical stress at c is obtained by subdividing the footing into four equal auxiliary squares. Using Fig. 3.7a, we have for each of these squares: $z/B = 24/6 = 4$; $L/B = 1$ from which $\Delta\sigma_i/q \simeq 3\%$, hence:

$$\Delta\sigma_i = 0.83 \times 3\% \simeq 2.5 \times 10^{-2} \text{ daN/cm}^2$$

$$\Delta\sigma = q(I_1 + I_2 + I_3 + I_4) = 4qI_1 = 4\Delta\sigma_i$$

$$\Delta\sigma = 4 \times 2.5 \cdot 10^{-2} = 10 \cdot 10^{-2} \text{ daN/cm}^2.$$

The same procedure is followed for stresses at e and f, by using auxiliary rectangles and squares. To determine the increase in vertical stress at e, consider the rectangle $AEHD$ and the square $GEHB$ (Fig. 3.7). For rectangle $AEHD$: $z/B = 24/12 = 2$, $L/B = 24/12 = 2$ from which $\Delta\sigma_i/q = 12\%$ and $\Delta\sigma_i = 0.83 \times 12\% = 10 \cdot 10^{-2}$ daN/cm^2.

For square $AGBD$, using the same method as above, $\Delta\sigma_2 = 7 \cdot 10^{-2}$ daN/cm^2 and $\Delta\sigma = q(I_1 - I_2) = \Delta\sigma_1 - \Delta\sigma_2 = 3 \cdot 10^{-2}$ daN/cm^2.

To determine the increase in stress at f, the following auxiliary surfaces must be considered:

square 1:	$AEFI$	coefficient of influence: I_1
rectangle 2:	$GEFJ$	coefficient of influence: I_2
rectangle 3:	$DHFI$	coefficient of influence: $I_3 = I_2$
square 4:	$BHFJ$	coefficient of influence: I_4

$$\Delta\sigma = q(I_1 - 2I_2 + I_4).$$

To calculate $\Delta\sigma_1$: $z/B = 24/24 = 1$, $L/B = 1$, from which: $I_1 = \Delta\sigma_1/q \simeq 18\%$ and $\Delta\sigma_1 = 0.83 \times 18\% \simeq 15 \cdot 10^{-2}$ daN/cm^2.

For $\Delta\sigma_2$, using the preceding procedures, $I_2 \simeq 12\%$: $\Delta\sigma_2 = 10 \cdot 10^{-2}$ daN/cm^2.

For $\Delta\sigma_4$, $I_4 = 8.5\%$, $\Delta\sigma_4 = 7 \cdot 10^{-2}$ daN/cm^2.

Thus, the total is: $\Delta\sigma = (15 - 20 + 7)10^{-2} = 2 \cdot 10^{-2}$ daN/cm^2.

(b) Assume now that the entire loading is concentrated at point C, the centre of the footing. In this case:

$$\Delta\sigma = \frac{3P}{2\pi z^2}\cos^5\theta, \quad P = 0.83 \times 12 \times 12 = 119.5 \cdot 10^5 \text{ N}, \quad z = 24 \text{ m},$$

from which:

$$\Delta\sigma = \frac{3 \times 119.5 \cdot 10^5}{2\pi \times 24^2}\cos^5\theta \text{ N/m}^2 = 0.99 \cdot 10^4 \times \cos^5\theta \text{ N/m}^2$$

$$= 9.9 \times \cos^5\theta \cdot 10^{-2} \text{ daN/cm}^2.$$

Table 3D summarizes the computations.

TABLE 3D

Point	r	$\cos\theta = z/r$	$\cos^5\theta$	$\Delta\sigma (10^{-2}$ daN/cm$^2)$
A	25.46	0.943	0.745	7.4
C	24	1	1	9.9
E	30.6	0.784	0.296	2.9
F	35	0.686	0.152	1.5

Summary of answers

TABLE 3E

Vertical stress increase (in 10^{-2} daN/cm^2)

Points	Surface loads	Concentrated load
A	7.0	7.4
C	10.0	9.9
E	3.0	2.9
F	2.0	1.5

It is clear that the differences are small. This is due mainly to the fact that the depth at which the stresses are calculated is twice the width of the footing. Had the depth been smaller, the differences would be more significant.

★★*Problem 3.6* Settlement under a point load in a clay layer

A load of 3200 kN bears upon a thick sand layer (assumed incompressible) in which there is a compressible clay layer at a depth of 8 m from the surface (see Fig. 3.8). This clay layer is 4.80 m thick. The oedometric modulus of

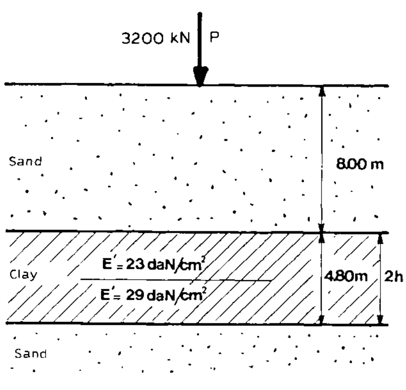


Fig. 3.8.

the clay is estimated to be 23 daN/cm² in the upper half of the clay layer and 29 daN/cm² in the lower half. Determine the vertical settlements of the load assumed to be concentrated at one point.

Solution

Assume that the load of 3200 kN is transmitted to the foundation soil through an isolated footing at the surface of the soil mass. The stress distribution in the soil may then be compared to that of a point load at P acting on a semi-infinite elastic and homogenous mass.

Under these conditions, at depth z (Fig. 3.9) on a vertical from P, the vertical stress on the horizontal plane is determined by Boussinesq's formula: $\sigma = (3P/2\pi z^2)\cos^5\theta$ in which: $\theta = 0$, therefore: $\sigma = 3P/2\pi z^2$.

To determine the settlement, the clay layer may be divided into two sublayers of equal thicknesses. At mid-height of the upper clay layer, σ is:

$$(\sigma)_1 = \frac{3\times 3200}{2\pi\times(8+1.20)^2} = 18.1\,\text{kPa},\quad \text{or } 0.18\,\text{daN/cm}^2.$$

At mid-height of the lower clay layer, σ is:

$$(\sigma)_2 = \frac{3\times 3200}{2\pi\times(8+3.60)^2} = 11.4\,\text{kPa},\quad \text{or } 0.11\,\text{daN/cm}^2.$$

Let $2h$ be the thickness of the clay layer, then the settlement directly under point P will be:

$$s = \int_0^h \frac{(\sigma)_1}{E_1'}\,dz + \int_h^{2h} \frac{(\sigma)_2}{E_2'}\,dz = \frac{(\sigma)_1 h}{E_1'} + \frac{(\sigma)_2 h}{E_2'}$$

or:

$$s = \frac{0.18\times 240}{23} + \frac{0.11\times 240}{29} = 2.79\,\text{cm} \simeq 2.8\,\text{cm}.$$

Remarks

(1) The Boussinesq formula corresponds to a concentration factor of $n = 3$. Fröhlich's formula, with a concentration factor of $n = 4$, gives:

$$\sigma = 4P/2\pi z^2,\quad \text{or:}\quad (\sigma)_{1f} = \tfrac{4}{3}(\sigma)_1,\qquad (\sigma)_{2f} = \tfrac{4}{3}(\sigma)_2$$

and

$$s_f = \tfrac{4}{3}\times s = 3.7\,\text{cm}.$$

(2) In order to evaluate the accuracy of the hypothesis (load assumed to be concentrated at one point), it is necessary to assume certain values as shown below. Let $\gamma = 17\,\text{kN/m}^3$ and $\varphi = 30°$ be the properties of the sand. If the

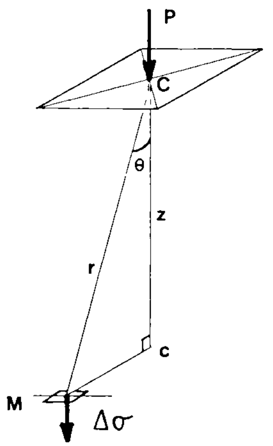


Fig. 3.9.

footing is square, and its side is B, the bearing capacity will be:

$$q_d = 0.8\,\gamma\frac{B}{2}N_\gamma + \gamma D N_q + 1.2\,cN_c = 0.8\,\gamma\frac{B}{2}N_\gamma, \text{ because } D = 0 \text{ and } c = 0.$$

Therefore, $P = q_d B^2/3$ with a safety factor of 3
or: $9600 = 0.4 \times 17 \times 21.8 \times B^3$, since $N_\gamma = 21.8$ for $\varphi = 30°$,
from which $B \simeq 4$ m, and $q_{ad} \simeq 2$ daN/cm^2.

At mid-height of the upper clay layer, σ will be, from Fig. 3.7a:

$$(\sigma)_1 \simeq \frac{6.5}{100}q_{ad} = 0.13\ \text{daN/cm}^2 \qquad \left(\frac{z}{B} = \frac{9.20}{4.00} = 2.3\right).$$

At mid-height of the lower clay layer:

$$(\sigma)_2 \simeq \frac{5}{100}q_{ad} = 0.10\ \text{daN/cm}^2 \qquad \left(\frac{z}{B} = \frac{11.60}{4.00} = 2.9\right).$$

The total settlement will be:

$$s = \frac{0.13 \times 240}{23} + \frac{0.10 \times 240}{29} = 2.18\ \text{cm} \simeq 2.2\ \text{cm}.$$

In this particular instance, the settlement is overestimated if the load is assumed to be concentrated rather than acting as a footing. The overestimate is $\Delta s/s = (2.8 - 2.2)/2.2 \simeq 27\%$.

Since the settlement is overestimated, it is on the safe side.

Remark.

It could be expected that the difference would depend to a great extent on the arbitrarily chosen value of φ of the sand. We know indeed that the coefficient N_γ is very sensitive to changes in φ. However, in the formula for B above, this factor appears as a cubic root and its effect is therefore greatly reduced. As an example, for $\varphi = 25°$, the relative error is $\Delta s/s = 22\%$ and for $\varphi = 35°$ the relative error is 40%. The order of magnitude of about 30% of error is still maintained.

Summary of answer
$s = 2.8$ cm.

★★*Problem 3.7* Determination of the modulus of subgrade reaction, k_s

A plate bearing test was performed on a 45-cm diameter plate. A seating load of 0.7 daN/cm² was applied initially after which the load was lowered to 0.1 daN/cm². The strain gages read 55, 103 and 72 hundreds of a mm. The load was then increased to 0.7 daN/cm² and the gages read 70, 118 and 86 hundreds of mm. Determine the reaction modulus k_s for a plate diameter of 75 cm.

Solution

The soil reaction modulus calculated from Westergaard's equation is:

$$k_s = \frac{\sigma}{s} = \frac{\sigma_1 - \sigma_0}{s_1 - s_0} \quad \text{(Fig. 3.10)}$$

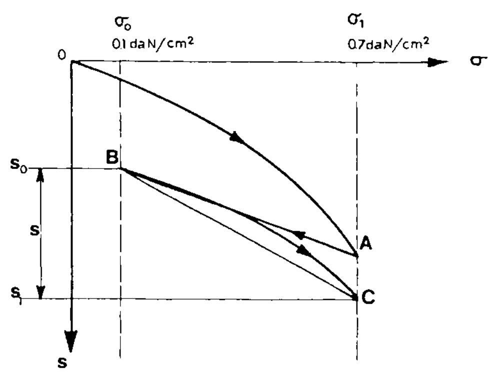


Fig. 3.10.

k_s is inversely proportional to the radius R of the plate. For the 75-cm radius plate, and expressing σ in daN/cm^2, we get:

$$k_{s(\phi 75)} = \frac{0.6}{s_1 - s_0} \times \frac{45}{75} = \frac{0.36}{s_1 - s_0}$$

in which s_1 and s_0 are in cm and k_s in daN/cm^3.

After the first loading cycle, the average of three gage readings is:

$$(s_0)_{av} = \frac{55 + 103 + 72}{3} = 76.7 \text{ hundreds of mm.}$$

After the second load increment, this average is:

$$(s_1)_{av} = \frac{70 + 118 + 86}{3} = 91.3 \text{ hundreds of mm.}$$

The permanent settlement is therefore: $91.3 - 76.7 = 14.6$ hundreds of mm and the modulus is:

$$k_{s(\phi 75)} = \frac{0.36}{\frac{14.6}{100} \times 10^{-1}} = 24.6 \text{ daN/cm}^3\text{, say } 24 \text{ daN/cm}^2.$$

Answer
$k_s = 24$ daN/cm^2.

★*Problem 3.8* Time of consolidation of a clay layer with double drainage

An 8 m thick clay layer, which has two drainage boundaries is being consolidated. The coefficient of permeability of the clay is $3 \cdot 10^{-9}$ cm/s. Its oedometric modulus is 400 daN/cm^2. What is the time required to achieve a degree of consolidation of 40%, and of 80%?

Solution

From the Terzaghi and Fröhlich theories, the degree of consolidation is dependent on the time factor T_v:

$$T_v = (c_v/h^2)t.$$

TABLE 3F: $U(T_v)$

T_v	U	T_v	U	T_v	U
0.02	0.160	0.3	0.613	0.8	0.887
0.06	0.276	0.4	0.697	0.9	0.912
0.10	0.356	0.5	0.764	1	0.931
0.15	0.437	0.6	0.816	2	0.994
0.20	0.504	0.7	0.856	∞	1.000

TABLE 3G: $T_v(U)$

U	10%	20%	30%	40%	60%
T_v	0.008	0.031	0.071	0.127	0.289

The functions $U = f(T_v)$ and $T_v = \phi(U)$ are given in Tables 3F and 3G. From these tables we get $T_v = 0.127$ for $U = 40\%$, but Table 3G does not show a value for $U = 80\%$. We only know that T_v for 80% consolidation is between 0.5 and 0.6. The approximate relationship of Brinch Hansen gives:

$$U = \sqrt[6]{\frac{T_v^3}{T_v^3 + 0.5}} \quad \text{from which: } T_v^3 = \frac{U^6}{2(1 - U^6)} = \frac{\overline{0.8}^6}{2(1 - \overline{0.8}^6)} = 0.178,$$

and $T_v \simeq 0.56$.

Therefore: $t_{40\%} = 0.127 \dfrac{h^2}{c_v}$ and because $c_v = \dfrac{kE'}{\gamma_w}$, $t_{40\%} = 0.127 \dfrac{h^2 \gamma_w}{kE'}$.

By the same token, $t_{80\%} = 0.56 \dfrac{h^2 \gamma_w}{kE'}$.

Numerical application:

h = half thickness of clay layer (open layer) = 4 m

k = 3×10^{-9} cm/s = 3×10^{-11} m/s

E' = 4×10^2 daN/cm^2 = 4×10^7 Pa

γ_w = 9.81×10^3 N/m^3

$$t_{40\%} = \frac{0.127 \times 16 \times 9.81 \times 10^3}{3 \times 10^{-11} \times 4 \times 10^7} = 0.127 \times 13.08 \times 10^7 \text{ sec}$$

$$t_{40\%} = 0.127 \times \frac{13.08}{0.864} \times 10^2 \text{ days} = 192 \text{ days} = 6 \text{ months and } 11 \text{ days}$$

$$t_{80\%} = 0.56 \times \frac{13.08}{0.864} \times 10^2 \text{ days} = 848 \text{ days} = 2 \text{ years, } 3 \text{ months, } 27 \text{ days.}$$

Summary of answers

6 months, 11 days; 2 years, 3 months, 27 days.

★*Problem 3.9* Coefficient of permeability

The void ratio of clay A decreases from 0.581 to 0.512 when the applied stress is changed from 1.1 to 1.7 daN/cm². That of clay B decreases from

0.609 to 0.596 under the same load transfer. The thickness of sample A is 50% greater than that of B. Nevertheless, sample B requires 3 times the amount of time needed for A to reach 50% consolidation under identical boundary drainage conditions. What is the ratio of coefficients of permeability of the two clays?

Solution

From Terzaghi and Fröhlich's theory, the coefficient of consolidation of a clay is:

$$c_v = kE'/\gamma_w \tag{1}$$

the time factor is:

$$T_v = (c_v/H^2)t. \tag{2}$$

For a given degree of consolidation, the time factor is the same for both samples, therefore:

$$\frac{c_{v_A}}{H_A^2} t_{50\,A} = \frac{c_{v_B}}{H_B^2} t_{50\,B}$$

or:

$$\frac{c_{v_A}}{c_{v_B}} = \left(\frac{H_A}{H_B}\right)^2 \times \frac{t_{50\,B}}{t_{50\,A}} = \left(\frac{3}{2}\right)^2 \times 3 = \frac{27}{4}.$$

From eqn. (1), we have: $\dfrac{c_{v_A}}{c_{v_B}} = \dfrac{k_A E'_A}{k_B E'_B}$

The oedometric modulus is a function of void ratio:

$$\frac{\Delta\sigma}{E'} = -\frac{\Delta e}{1+e} \quad \text{or} \quad E' = -\frac{\Delta\sigma(1+e)}{\Delta e}.$$

For a given stress increase, $\Delta\sigma$, of 1.1 to 1.7 daN/cm^2 we have:

$$\frac{E'_B}{E'_A} = \frac{\Delta e_A(1+e_B)}{\Delta e_B(1+e_A)} = \frac{0.069 \times 1.609}{0.013 \times 1.581} = 5.4$$

from which: $\dfrac{k_A}{k_B} = \dfrac{c_{v_A}}{c_{v_B}} \times \dfrac{E'_B}{E'_A} = \dfrac{27}{4} \times 5.4 = 36.4$ or $k_A/k_B = 36.$

Remark

The given of $U = 50\%$ is redundant. Sample B takes 3 times as much time to consolidate than A regardless the value of U.

★*Problem 3.10* Time of consolidation

An oedometer test is performed on a 2-cm thick clay sample. After 5 minutes, 50% of consolidation is reached. After how long a time would the same degree of consolidation be achieved in the field where the clay layer is 3.70 m thick?

Assume the sample and the clay layer have the same drainage boundary conditions.

Solution

The time factor function is: $T_v = c_v \cdot t/h^2$ where $c_v = kE'/\gamma_w$.

For double drainage conditions, h represents half the thickness of the layer (drainage path length). In single drainage condition, h would represent the total clay layer thickness. (see Fig. 3.11). Let h' be the length corresponding to the field condition and t' the time required for the layer to reach U degree of consolidation. Since the clay and drainage conditions are the same as those of the laboratory test, the time factor for both is the same as is the coefficient of consolidation c_v. Therefore:

$$T_v = \frac{c_v t'}{h'^2} = \frac{c_v t}{h^2}, \quad \text{so} \quad t' = t \times \left(\frac{h'}{h}\right)^2.$$

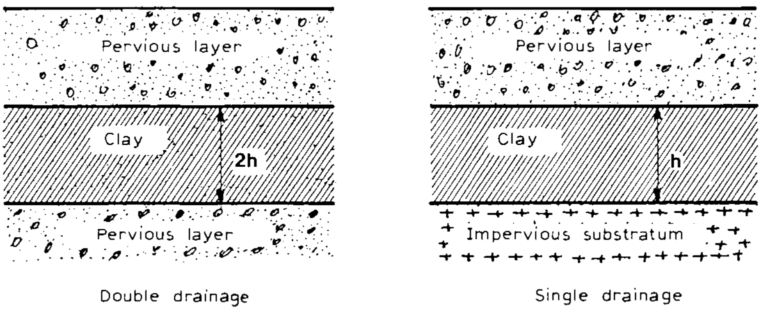


Fig. 3.11.

Numerical application

t = 5 minutes = 5 × 60 seconds, h' = 370 cm, h = 2 cm

1 day = 86,400 seconds and $t = 5 \times 60 \times \left(\frac{370}{2}\right)^2 \times \left(\frac{1}{86400}\right)$

t = 118.8, say 119 days or 4 months.

Remark:

The given $U = 50\%$ is redundant, the answer is independent of U.

Summary of answer

$t = 4$ months.

★★★*Problem 3.11* **Compressibility and consolidation curves; settlement calculation; preloading requirements**

A highway embankment 2.40 m high is to be constructed over a saturated, homogeneous clay of thickness $2H = 4$ m. The clay is underlain by a sandy gravel which, for all practical purposes, is incompressible. The water table is at ground surface. The density of the embankment is 2.1 (see Fig. 3.12).

A sample of clay is recovered from a depth of 2 m and a consolidation test performed whose results are summarized in Tables 3H and 3I. The initial sample thickness is $2h_0 = 24.0$ mm. The initial water content is $w = 69\%$ and the density of the soil particles is 2.7.

(a) Draw the compression ($e - \log \sigma$) curve and the consolidation ($e - \log t$) curve. Determine the approximate value of the preconsolidation pressure, σ_c, and of C_c and c_v.

(b) What is the total settlement of the embankment? After how long a time will this settlement be obtained?

(c) Determine the total thickness of the embankment and surcharge to attain the expected total settlement under the design embankment height after a period of 4 months.

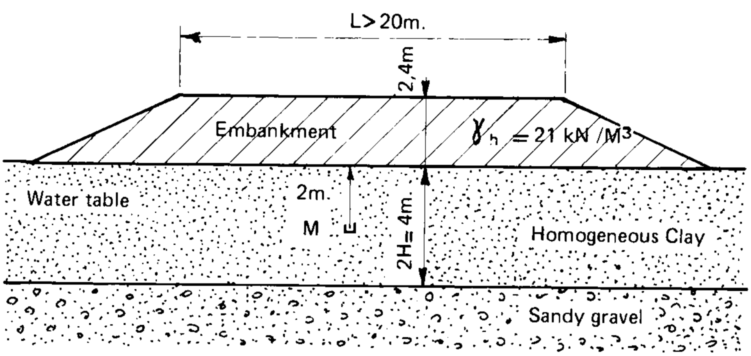


Fig. 3.12.

TABLE 3H: Consolidation
(each load is maintained for 24 h)

Stress (daN/cm^2)	Void ratio
0.05	1.82
0.1	1.81
0.2	1.80
0.4	1.74
0.8	1.40
1.6	0.80
3.2	0.16

TABLE 3I: Compression
(from 0.4 to 0.8 daN/cm^2)

Time (min)	Void ratio
0.1	1.700
0.2	1.690
0.3	1.683
0.5	1.675
1	1.650
2.5	1.600
5	1.550
10	1.504
20	1.451
50	1.432
100	1.421
200	1.418
500	1.409
1400	1.400

Solution

(a) Tables 3H and 3I provide the data to construct the curves. The compression curve is shown on Fig. 3.13. The initial void ratio for a saturated soil is: $e_0 = w(\gamma_s/\gamma_w)$ or $e_0 = 0.69 \times 2.7 = 1.86$.

Table 3I gives the data needed to plot the compression curve as a function of time ($e - \log t$) for a constant stress of 0.8 daN/cm^2, after the sample has consolidated under the stress of 0.4 daN/cm^2. This curve is plotted on Fig. 3.14.

The preconsolidation stress is the maximum effective overburden stress to which the clay was ever submitted. It can be evaluated from the construction shown on the curve of Fig. 3.13; it is $\sigma'_c \simeq 0.50\,daN/cm^2$. The compression index, C_c, is the slope of the straight line BC in Fig. 3.13 and corresponds (after Casagrande's construction) to:

$$C_c = \frac{-\Delta e}{\Delta \log \sigma'}$$

We have:

$$\sigma'_1 = 0.80\,daN/cm^2,\ e_1 = 1.40,\ \sigma'_2 = 3.20\,daN/cm^2,\ e_2 = 0.16.$$

$$\Delta e = 0.16 - 1.40 = -1.24,$$

$$\Delta \log \sigma' = \log \sigma'_2 - \log \sigma'_1 = \log \frac{\sigma'_2}{\sigma'_1} = \log \frac{3.2}{0.8} = \log 4$$

from which: $C_c = -(-1.24/0.602) \simeq 2.06, \qquad C_c = 2.1.$

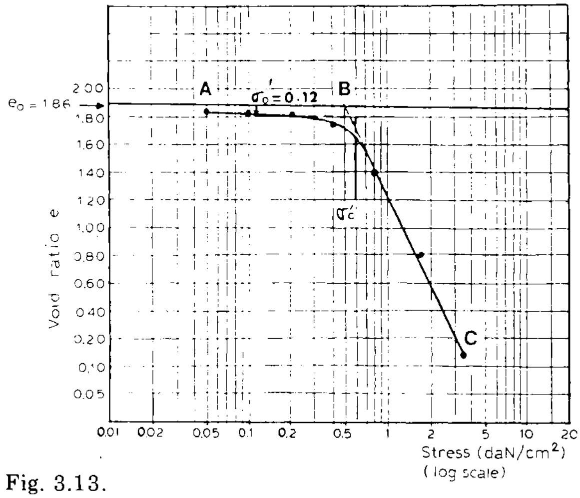


Fig. 3.13.

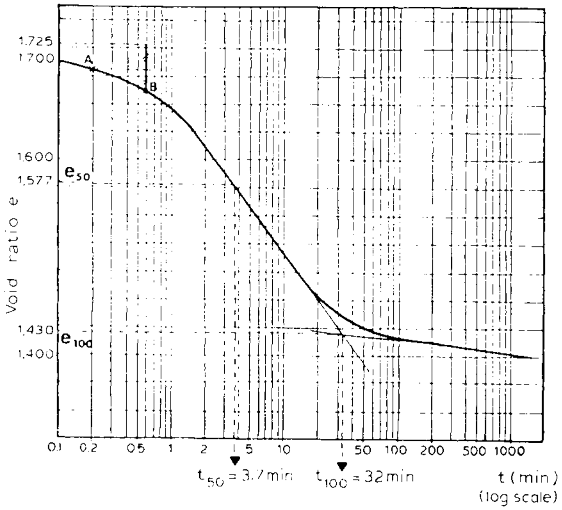


Fig. 3.14.

Consolidation coefficient

The coefficient of consolidation, c_v, is determined by the following method:

(1) Determination of the void ratio at 100% consolidation. The intersection of the two tangents determines e_{100} corresponding to the end of the primary consolidation (Fig. 3.14).

(2) Determination of the initial void ratio e_0. The following procedure is used (see Fig. 3.15): Choose a time near the origin, in the example $t_1 = 0.1$ min, and another time 4 times longer so that $t_2 = 4t_1, = 0.4$ min (in the example).

The consolidation curve is drawn on a t-e graph of arithmetic scale for the time axis.
We have then: $e_1 = 1.700$ for $t = t_1$ (direct reading), $e_2 = 1.678$ for $t = t_2$ (interpolated reading).

By approximating the curve to a parabola with a horizontal axis near the origin and from the geometric properties of a parabola, the following relationships exist: $e_{0c} - e_1 = e_1 - e_2$ from which $e_{0c} = 1.721$.

Fig. 3.14 gives: $e_{100} = 1.430$, $t_{100} \simeq 32.5\,\text{min}$
from which:

$$e_{50} = \frac{e_{0c} + e_{100}}{2} = \frac{1.721 + 1.430}{2} = 1.576, \quad t_{50} \simeq 3.7\,\text{min}.$$

The height of the sample at the start of consolidation under the $0.8\,\text{daN/cm}^2$ load is $2h$ and we have:

$$\frac{2h}{1 + e_{0c}} = \frac{2h_0}{1 + e_0}$$

from which:

$$h = h_0 \frac{1 + e_{0c}}{1 + e_0} = 12.0 \times \frac{2.721}{2.860} = 11.4\,\text{mm}.$$

The coefficient of consolidation c_v is: $0.197\,h^2/t_{50}$,
so: $c_v = 0.197 \times (1.14)^2/(3.7 \times 60) \simeq 1.15 \times 10^{-3}\,\text{cm}^2/\text{s}$.

Total settlement of embankment

Because the thickness of the compressible clay layer is rather small compared to the width of the embankment (for highways, this width is seldom less than 25 m), it may be assumed that the stress distribution in the clay due to the load imposed by the embankment is uniform (Fig. 3.16). We thus have: $\Delta\sigma' = 2.40 \times 21 = 50\,\text{kPa} \approx 0.50\,\text{daN/cm}^2$.

The vertical, initial stress at point M, at mid-height in the clay layer is: $\sigma_0' = H \times \gamma_{\text{clay}}'$.

To find γ_{clay}', we proceed as follows:

$$\gamma' = \frac{\gamma_s - \gamma_w}{\gamma_s} \times \gamma_d, \quad \gamma_d = \frac{\gamma_s}{1 + e_0}.$$

Earlier it was found that $e_0 = 1.86$, therefore $\gamma'/\gamma_w = (\gamma_s - \gamma_w)/(1 + e_0) = (2.70 - 1.00)/2.86 = 0.594$ and $\sigma_0' = 2.00 \times 5.94 = 11.9\,\text{kPa} \# 0.12\,\text{daN/cm}^2$.

We found, in the previous section, that the preconsolidation pressure is

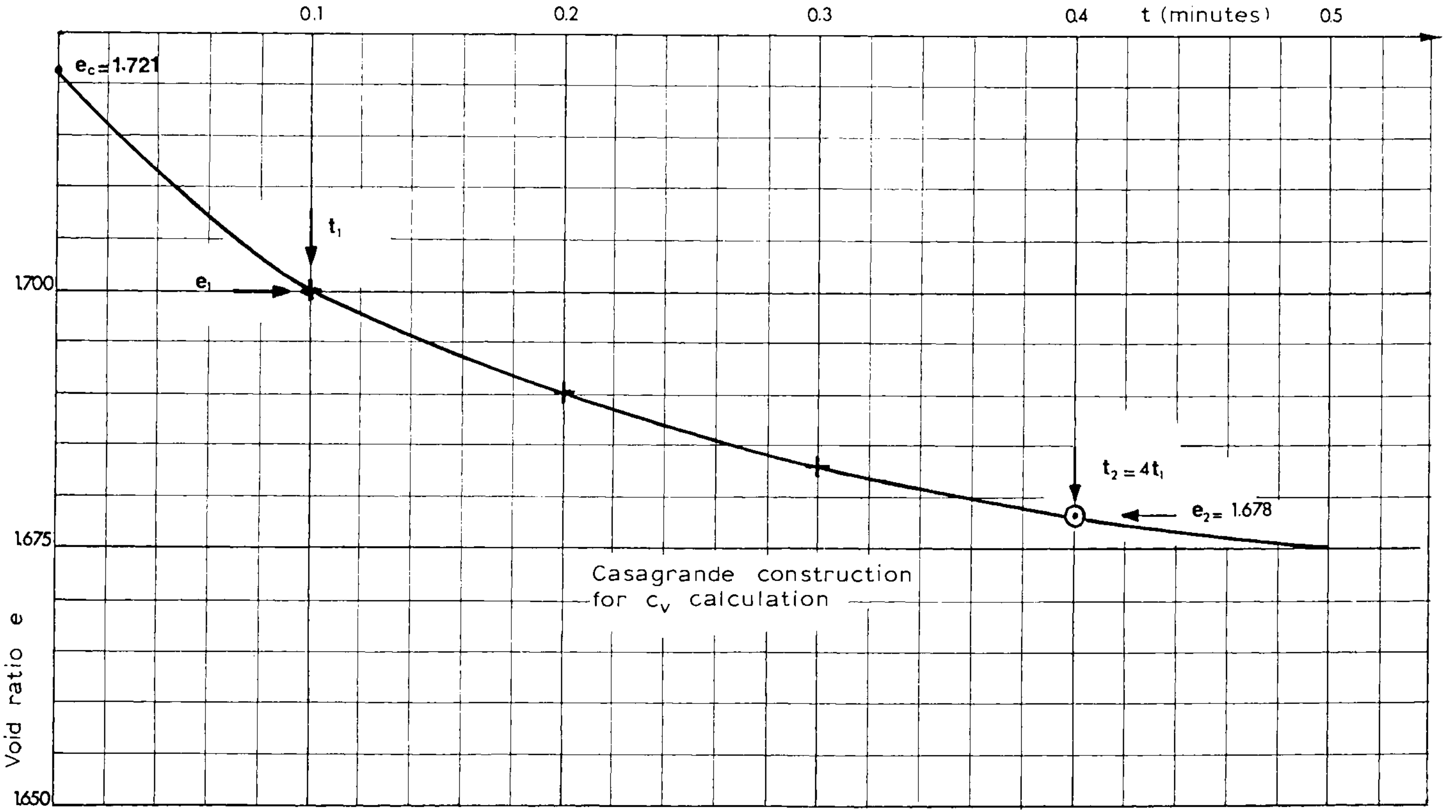

0.1
0.2
0.3
0.4
t (minutes)
0.5
$e_c = 1.721$
t_1
1.700
e_1
$t_2 = 4t_1$
$e_2 = 1.678$
1.675
Casagrande construction for c_v calculation
Void ratio e
1.650

Fig. 3.15.

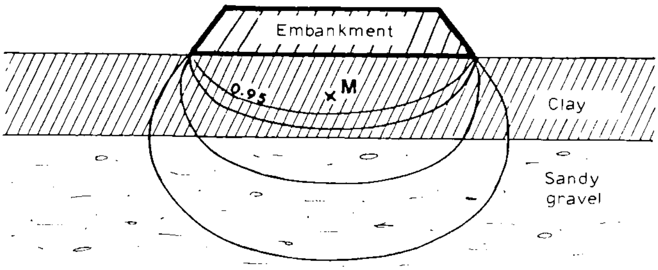


Fig. 3.16.

$\sigma'_c = 0.50$ daN/cm^2. The existing effective vertical stress at the mid-height of the clay layer is less than the preconsolidation pressure: the clay is overconsolidated.

After construction of the embankment, the effective vertical stress at the mid-height of the layer will be (see Fig. 3.17): $\sigma'_0 + \Delta\sigma' = 0.12 + 0.50 = 0.62$ daN/cm^2 which leads to: $\sigma'_0 + \Delta\sigma' > \sigma'_c$.

From the compression curve of Fig. 3.13, we have for:
$\sigma'_0 = 0.12$ daN/cm^2, $e = 1.80$ and: $\sigma'_0 = 0.62$ daN/cm^2, $e = 1.56$ and from: $\Delta h/h = \Delta e/(1 + e)$ we get:

$$\Delta h = 400 \times \frac{1.80 - 1.56}{2.80} = 34.3 \text{ cm}, \quad \text{say } 34 \text{ cm}.$$

It would be necessary, in this case, to check the stability of the embankment for foundation failure. The stability is satisfactory if in this case, $c_u = 40$ kPa.

Thickness of embankment surcharge to reach the total settlement in 4 months

What would be the time required for the clay layer to consolidate 34 cm?

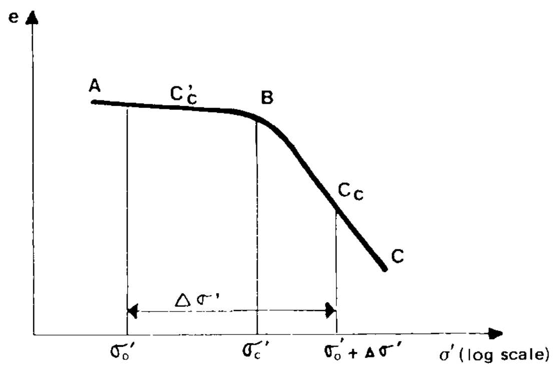


Fig. 3.17.

Theoretically, this time is infinite, but in practice, it is considered that 100 percent of the settlement is obtained when $T_v = 2$, which corresponds to $U = 99.4\%$.

Since $T_v = (c_v/h^2)t$ and since the drainage of the clay layer is in both directions: $h = H/2$

$$\text{hence: } t = \frac{2 \times (200)^2}{1.15 \times 10^{-3}}$$

$$\text{and: } t = \frac{8}{1.15} \times 10^7 \times \frac{1}{8.64 \times 10^4} = 805 \text{ days}$$

or 2 years, 2 months, 15 days.

To decrease the settlement time to 4 months, (120 days), the time factor T_v must be:

$$T_v = \frac{1.15 \times 10^{-3} \times 120 \times 8.64 \times 10^4}{(200)^2} = 0.298 \simeq 0.30$$

The corresponding U value is $U = 0.613$ (see Table 3F). This means that the settlement of 34 cm corresponds to 61.3% of the settlement S that would be obtained under a heavier surcharge, which settlement would be: $0.34/0.613 \simeq 0.55$ m.

The following equation for S gives the surcharge needed:

$$S = H \times \frac{C_c}{1 + e_0} \log \left(\frac{\sigma_0' + \Delta\sigma'^*}{\sigma_c'} \right).$$

$$\text{If we let } x = \frac{\sigma_0' + \Delta\sigma'^*}{\sigma_c'}, \quad \text{then:} \quad \log x = \frac{55 \times 2.86}{400 \times 2.1} = 0.187,$$

and $x = 1.54$, from which

$$\Delta\sigma'^* = 1.54 \times 0.50 - 0.12 \simeq 0.65 \text{ daN/cm}^2$$

$$\Delta\sigma'^* = 65 \text{ kPa } (^*).$$

The corresponding embankment height is: $H' = 65/21 = 3.10$ m, and the surcharge needed would be: $\Delta H = 3.10 - 2.40 = 0.70$ m.

This example shows that it is possible to quickly stabilize an embankment, if the time needed for preconsolidation by surcharge is programmed. A surcharge of 70 cm, left in place for 4 months and then removed would eliminate any significant embankment settlement thereafter.

*This value could also have been obtained from the $e - \log \sigma'$ graph.

Summary of answers
(a) see Figs. 3.13 and 3.14: $\sigma'_c = 0.5\,\text{daN/cm}^2$; $C_c = 2.1$; $c_v = 1.15 \times 10^{-3}$ cm²/s; (b) $s = 34$ cm after 2 years, 2 months and 15 days; (c) about 310 cm.

★★*Problem 3.12* Oedometric moduli: behavior of an overconsolidated clay

From the givens of problem 3.11:

(a) Calculate the oedometric modulus E' of the clay in the section BC of the compression curve corresponding to the stress increase due to the embankment load (Fig. 3.12), at mid-height in the clay layer. Compare the result with the modulus obtained by the approximate formula:

$$E' = 2.3\,\sigma\,\frac{1+e}{C_c}$$

(b) Find the compression index C'_c of the linear portion AB of the compression curve corresponding to a stress level less than 0.20 daN/cm² (Figs. 3.13 and 3.19). Find the corresponding oedometric modulus.

(c) What can be inferred about the clay from this? What are some practical conclusions?

Solution

(a) From the answer of problem 3.11, the initial effective stress at mid-height in the clay layer is: $\sigma'_0 = 0.12\,\text{daN/cm}^2$. The increase in stress due to the weight of the embankment is: $\Delta\sigma' = 0.92\,\text{daN/cm}^2$. The pre-consolidation stress of the clay was found to be $\sigma'_c = 0.5\,\text{daN/cm}^2$.

In order to calculate E', the initial point on the compression curve, to start from, is the pre-consolidation stress σ'_c. Therefore:

$$\Delta\sigma' = 0.92 - 0.50 = 0.42\,\text{daN/cm}^2$$

$$E' = \frac{1+e}{C_c}\,\frac{\Delta\sigma'}{\log(1 + (\Delta\sigma'/\sigma'_c))}$$

where e is the void ratio at σ', or $e = 1.68$ (read on the compression curve), and:

$$E' = \frac{1+1.68}{2.1} \times 0.42/[\log(1 + (0.42/0.50)] = 1.84$$

$$\log 1.84 = 0.265$$

$$E' = (2.68/2.1) \times (0.42/0.265) = 2.02\,\text{daN/cm}^2\,(*) \text{ or: } 2.00\,\text{daN/cm}^2.$$

*E' could also be calculated from the compression curve by writing: $\Delta h/h = \Delta e/(1+e)$ and $E' = \Delta\sigma'/(\Delta h/h)$.

The approximate formula gives: $E' = 2.3 \times 0.5 \times (2.68/2.1) = 1.47\,\mathrm{daN/cm^2}$. This approximation is not very good because $\Delta\sigma'$ is too large in comparison to σ'_c. The value of $E' = 2.00\,\mathrm{daN/cm^2}$ corresponds to that of a soft clay.

(b) The compression index C'_c is found from the relation: $C'_c = -\Delta e/\Delta \log \sigma'$, where: $e_1 = 1.82$, $\sigma'_1 = 0.05\,\mathrm{daN/cm^2}$, $\log \sigma'_1 = \bar{2}.699 = -1.301$; $e_2 = 1.80$, $\sigma'_2 = 0.20\,\mathrm{daN/cm^2}$, $\log \sigma'_2 = \bar{1}.301 = -0.699$ from which:

$$C'_c = \frac{-(1.80 - 1.82)}{-0.699 + 1.301} = \frac{0.02}{0.602} \simeq 0.033.$$

The oedometric modulus is:

$$E' = \frac{1+e}{C_c} \frac{\Delta\sigma'}{\log(1 + (\Delta\sigma'/\sigma'_c))}.$$

As an example, we can take $\Delta\sigma' = 0.10\,\mathrm{daN/cm^2}$ in order to remain on the AB portion of the curve (Fig. 3.18), which corresponds to stresses less than σ'_2.

$\sigma'_0 = 0.10\,\mathrm{daN/cm^2}, e_0 = 1.81$ (from the given)

$$E' = \frac{2.81}{0.033} \times \frac{0.10}{\log 2} = \frac{0.281}{0.033 \times 0.301} = 28.3\,\mathrm{daN/cm^2}.$$

This value corresponds with that of a stiff clay.

(c) The results show that for small increases in stress $\Delta\sigma'$, the clay is stiff (see Fig. 3.18).

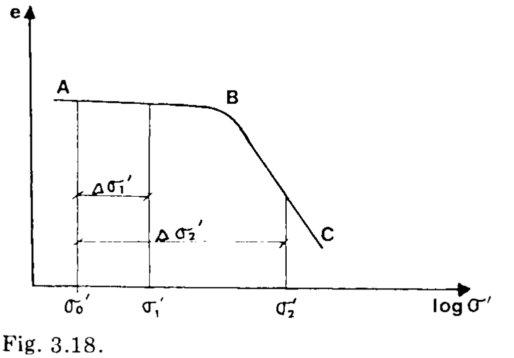


Fig. 3.18.

On the other hand for larger stress increases ($\sigma'_c + \Delta\sigma' > \sigma'_c$) the over-consolidated clay initially behaves as a stiff clay (as long as the stress is smaller than σ'_c), and as a soft clay as soon as the stress level increases beyond the value of σ'_c.

The clay behavior in consolidation therefore depends on the level of stresses. The qualifications 'soft' or 'stiff' must be viewed with respect to the imposed loads and cannot be based only on visual inspection in an open pit.

★★★*Problem 3.13* Consolidation with vertical sand drains

Referring once again to the geometry of problem 3.11, it is now desired to decrease the settlement time by accelerating the clay consolidation with vertical sand drains, going through the compressible layer to the underlaying gravel (shown in Fig. 3.19).

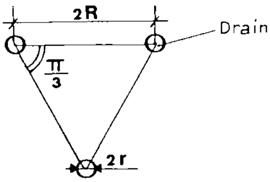


Fig. 3.19. Plan view.

The center to center distance between the two adjacent drains is 2 R, the drain diameter, 2 r. The drains have equal diameters. Under these conditions, it can be assumed that the clay drainage during the consolidation consists of the superposition of a vertical drainage, characterized by c_{vz}, k_v, U_z and T_v and of a horizontal, radial drainage characterized by c_{vr}, k_h, U_r and T_r with: $T_v = (c_{vz}/h^2)t$ and $T_r = [c_{vr}/(2R)^2]\,t$.

The coefficient of radial consolidation c_{vr} is further defined by the equation: $c_{vr}/c_{vz} = k_h/k_v$.

Under these conditions the degree of consolidation U is given by: $1 - U = (1 - U_z)(1 - U_r)$.

The degree of radial consolidation U_r, function of T_r and of the geometry of the drains, is presented in Table 3J.

Assuming that we have a condition where $k_h = 5\,k_v$, recommend drain spacing and drain diameter to obtain all settlements in 4 months, taking into account current practice i.e. drain diameters vary from 0.3 to 0.8 m and spacing goes from 2.5 to 7.5 m.

Solution

Since $k_h = 5\,k_v$, we can determine immediately: $c_{vr} = 5 \times 1.15 \cdot 10^{-3} = 5.75 \cdot 10^{-3}\ \text{cm}^2/\text{s}$.

From the previous solution of problem 3.11 we know that $U_z = 61.3\%$. Since we must realise the settlement in a period of 4 months, we also know $U = 99.4\%$.

Therefore:

$$1 - U_r = \frac{1 - U}{1 - U_z} = \frac{1 - 0.994}{1 - 0.613} \simeq 0.016, \text{ then } U_r = 1 - 0.016$$

$$= 0.984 \simeq 0.99.$$

TABLE 3J

Radial drainage, equal vertical strains (after Leonards)

Degree of consolidation $U_r(\%)$	Time factor										
	$\frac{R}{r}=5$	10	15	20	25	30	40	50	60	80	100
5	0.006	0.010	0.013	0.014	0.016	0.017	0.019	0.020	0.021	0.023	0.025
10	0.012	0.021	0.026	0.030	0.032	0.035	0.039	0.042	0.044	0.048	0.051
15	0.019	0.032	0.040	0.046	0.050	0.054	0.060	0.064	0.068	0.074	0.079
20	0.026	0.044	0.055	0.063	0.069	0.074	0.082	0.088	0.092	0.101	0.107
25	0.034	0.057	0.071	0.081	0.089	0.096	0.106	0.114	0.120	0.131	0.139
30	0.042	0.070	0.088	0.101	0.110	0.118	0.131	0.141	0.149	0.162	0.172
35	0.050	0.085	0.106	0.121	0.133	0.143	0.158	0.170	0.180	0.196	0.208
40	0.060	0.101	0.125	0.144	0.158	0.170	0.188	0.202	0.214	0.232	0.246
45	0.070	0.118	0.147	0.169	0.185	0.198	0.220	0.236	0.250	0.291	0.288
50	0.081	0.137	0.170	0.195	0.214	0.230	0.255	0.274	0.290	0.315	0.334
55	0.094	0.157	0.197	0.225	0.247	0.265	0.294	0.316	0.334	0.363	0.385
60	0.107	0.180	0.226	0.258	0.283	0.304	0.337	0.362	0.383	0.416	0.441
65	0.123	0.207	0.259	0.296	0.325	0.348	0.386	0.415	0.439	0.477	0.506
70	0.137	0.231	0.289	0.330	0.362	0.389	0.431	0.463	0.490	0.532	0.564
75	0.162	0.273	0.342	0.391	0.429	0.460	0.510	0.548	0.579	0.629	0.668
80	0.188	0.317	0.397	0.453	0.498	0.534	0.592	0.636	0.673	0.730	0.775
85	0.222	0.373	0.467	0.534	0.587	0.629	0.697	0.750	0.793	0.861	0.914
90	0.270	0.455	0.567	0.649	0.712	0.764	0.847	0.911	0.963	1.046	1.110
95	0.351	0.590	0.738	0.844	0.926	0.994	1.102	1.185	1.253	1.360	1.444
99	0.539	0.907	1.135	1.298	1.423	1.528	1.693	1.821	1.925	2.091	2.219

Table 3J gives the values of T_r as a function of U_r and of the geometry of the drains (R/r).

On the other hand, we have $T_r = \dfrac{c_{vr}}{(2R)^2}t = \dfrac{5.75\times10^{-3}\times1.2\times8.64\times10^6}{4R^2\times10^4}$.

Therefore: $T_r = 1.49/R^2$.

Now one has to make a choice between various theoretical solutions. Let us say, for instance, that $R/r = 10$. Table 3J gives then $T_r = 0.907$ and therefore: $R^2 = 1.49/0.907 = 1.64$ or $R = 1.28$ m and $r = 0.128$ m.
In this case the distance between two drains is: $D = 2R = 2.56$ m, and drain diameter is: $\phi = 2r = 25.6 \simeq 26$ cm.

If we take $R/r = 5$, then $T_r = 0.539$, therefore $R^2 = 1.49/0.539 = 2.77$, $R = 1.66$ m and $r = 0.33$ m, which corresponds to a distance between drains of $D = 2R \simeq 3.30$ m and a drain diameter of $\phi = 2r \simeq 0.65$ m = 65 cm.

The latter assumption is the better one because it is more in line with common practice as stated in the given of the problem. The geometry is shown in Figs. 3.20 and 3.21.

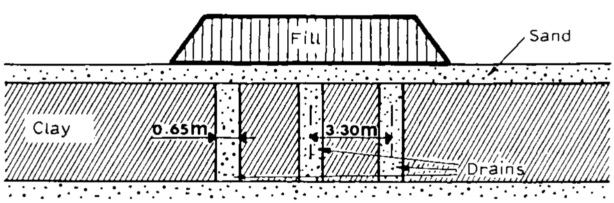


Fig. 3.20.

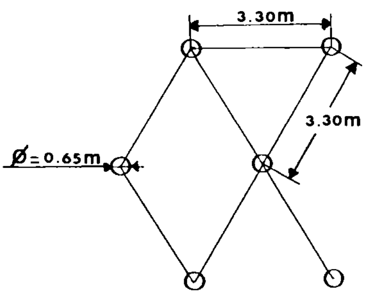


Fig. 3.21. Plan view.

Summary of answers
drain spacing: $D = 3.30$ cm; drain diameter: $\phi = 65$ cm.

★★★*Problem 3.14* Consolidation of a multi-layered system, Absi's theory; surcharging

An embankment similar to the one of problem 3.11 (see Fig. 3.12) has been constructed on a two-layer foundation. The characteristics of the two layers are:
— the upper layer consists of clay, 1.50 m thick having $c_{v1} = 1.15\times10^{-3}$ *cm²/s.*
— the lower layer (also clay) is 2.50 m thick: coefficient of consolidation is $c_{v2} = 4.5\times10^{-4}$ *cm²/s.*
Assuming that both layers have the same oedometric modulus, what is the

difference between this condition and that of problem 3.13. Refer to Absi's theory [1].

Solution

If the oedometric moduli and densities are the same, the total settlement is the same as that of problem 3.11, $s_\infty = 0.34\,\mathrm{m}$. But because of the two-layer system, the time of consolidation will be different.

According to the theory of Absi, the two-layer system may be equated to a system of a single layer of thickness $H' = 4.00\,\mathrm{m}$ ($H' = 2h'$) and with an apparent coefficient of consolidation c_{va} such that:

$$c_{va} = \frac{H^2}{\left[\sum_i \dfrac{h_i}{\sqrt{c_{vi}}}\right]^2} = \frac{H^2}{\left(\dfrac{H_1}{\sqrt{c_{v1}}} + \dfrac{H_2}{\sqrt{c_{v2}}}\right)^2}$$

$$c_{va} = \frac{(4 \times 10^2)^2}{\left(\dfrac{1.50 \times 10^2}{\sqrt{1.15 \times 10^{-3}}} + \dfrac{2.50 \times 10^2}{\sqrt{4.50 \times 10^{-4}}}\right)^2} \approx \frac{1.6}{\dfrac{1.5}{3.39} + \dfrac{2.5}{2.12}} \times 10^{-3}$$

$$c_{va} = \frac{1.6}{0.442 + 1.179} \times 10^{-3} \simeq 0.99 \times 10^{-3}\,\mathrm{cm^2/s}.$$

finally: $H' = 2h' = 4.00\,\mathrm{m}$, $c_{va} \simeq 9.9 \times 10^{-4}\,\mathrm{cm^2/s}$.

To decrease the settlement time to 4 months, a larger surcharge than placed in the case of problem 3.11 will have to be considered here, because:

$$T_v = \frac{c_{va}}{h'^2}\,t = \frac{9.9 \times 10^{-4} \times 1.2 \times 8.64 \times 10^6}{4 \cdot 10^4} = 0.256 \simeq 0.26$$

$T_v = 0.26$ which gives, after Terzaghi's approximation,

$U \simeq 1.128\sqrt{0.26} = 0.575.$

Therefore: $S = s_\infty \times 1/0.575 = 0.34/0.575 = 0.59\,\mathrm{m}$

$$\log x = \frac{59 \times 2.86}{400 \times 2.1} = 0.201, \text{ from which } x = 1.59;$$

$\Delta\sigma' = 0.50 \times 1.59 - 0.12 = 0.68\,\mathrm{daN/cm^2}$.

Thus, the total embankment height is: $68/21 = 3.24\,\mathrm{m}$.

A temporary surcharge, left in place for 4 months, of wet density equal to $\gamma_h = 21\,\mathrm{kN/m^3}$ will have a height of $3.24 - 2.40\,\mathrm{m} = 0.84\,\mathrm{m}$.

Note. The theory of Absi is only an approximation. It does not take into account the continuity across the two-layer faces. A more accurate but also more time-consuming solution would be the use of the finite-difference method (see problem 3.19).

Answer

In order to obtain a total settlement in 4 months, a surcharge of 0.84 m must be placed.

★★*Problem 3.15* **Consolidation test on an overconsolidated clay**

A saturated clay sample obtained from a depth of 4 m is tested in the oedometer. The results are summarized in Table 3K.

At the end of the test, the sample wet weight is 0.8738N and its dry weight 0.7399N.

The density of the soil constituant is 2.65.

(a) What is the final void ratio after testing?

(b) Establish a simple relation between void ratio and sample thickness h.

(c) Draw the compression curve, $e - \log \sigma'$. What is the compression condition of the clay?

What can be inferred as to the maximum loading the clay layer has undergone in the past? Assume that elevation of the water table corresponds to ground surface.

TABLE 3K

Stress (kPa)	Sample height (mm)
100	11.99
200	11.85
400	11.63
800	11.05
1600	10.40
400	10.54
100	10.76

Solution

(a) At the end of the test, the water content of the clay is:

$$w = \frac{W - W_d}{W_d} \doteq \frac{0.8738 - 0.7399}{0.7399} \simeq 0.181 = 18.1\%.$$

The soil, being saturated, we know the void ratio, $e = w(\gamma_s/\gamma_w)$, or $e = 0.181 \times 2.65 \simeq 0.48$. Thus, at the end of the test, the final void ratio is: $e_f = 0.48$.

(b) The volume of the soil grains within the sample for a unit cross-section area has not changed and therefore is $h(1 + e)$. Thus: $h/(1 + e) = h_f/(1 + e_f)$ where h_f = is the height of the sample at the end of the test. The following relation then applies: $1 + e = h(1 + e_f)/h_f = 1.48/10.76h$ or $1 + e = 0.1375h$ from which: $e = 0.1375h - 1$ (h in mm).

(c) This relation is the basis for Table 3L which presents void ratios as a function of the vertical stress applied to the sample. From the data in that Table, it is possible to draw the compression curve ($e - \log \sigma'$) as presented in Fig. 3.22.
The preconsolidation pressure σ_c, estimated from this figure, is about 300 kPa.

Now, we have to calculate the effective vertical stress to which the soil sample was submitted in situ.

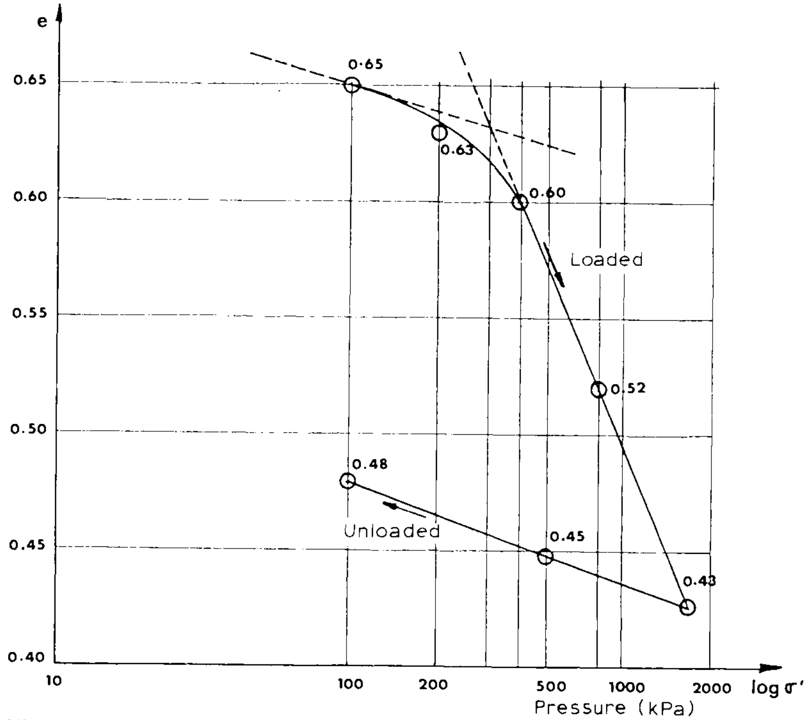


Fig. 3.22.

TABLE 3L

σ (kPa)	e
100	0.65
200	0.63
400	0.60
800	0.52
1600	0.43
400	0.45
100	0.48

We have: $\sigma_0' = \gamma' \times h$ with $\gamma' = \gamma_{sat} - \gamma_w$; $\gamma_{sat} = \gamma_s(1 + w/1 + e)$, and $e = 2.65\,w$.

The in-situ saturated unit weight can be estimated from γ_{sat} at the start of the test, and therefore:

$$\gamma_{sat} = \gamma_s \frac{2.65 + e}{2.65(1 + e)} = 10 \times \frac{2.65 + 0.65}{1.65} = 20\,\text{kN/m}^3$$

and finally: $\gamma' = 20 - 10 = 10\,\text{kN/m}^3$. Then $\sigma_0' = 10 \times 4 = 40\,\text{kPa}$. This stress is considerably less than σ_c': the clay is overconsolidated.

Since the preconsolidation pressure corresponds to the highest effective stress ever applied to the clay throughout its geologic history, this weight may have been caused by an overburden since then eroded, or a glacier.

Because the actual pressure is σ_0', the difference, $\sigma_c' - \sigma_0'$, $= 300 - 40 = 260\,\text{kPa}$ is the weight of that past overburden. In this case, where soil buoyant density $\gamma' = 10\,\text{kN/m}^3$ and that of ice, $\gamma_i = 9.17\,\text{kN/m}^3$, the stress difference may mean a soil erosion thickness of $260/10 = 26\,\text{m}$ or a glacier ice thickness of $260/9.17 = 28\,\text{m}$.

★★*Problem 3.16* Determination of C_c and m_v and the settlements of a saturated clay; degree of consolidation

An undisturbed clay sample of 20 mm thickness is tested in an oedometer and the following results are obtained:

effective stresses applied (kN/m^2):	*50,*	*100,*	*200*
sample thicknesses, h(mm):	*20,*	*19.62,*	*19.24*

The initial water content is 40%, the density of soil grains is 2.7.

(a) What is the compression index C_c and the coefficient of volumetric compressibility m_v for each of the stress increments?

(b) The sample was recovered from a clay layer 4 m thick located over an impervious rock base and overlain by a sand layer. The average vertical effective stress in the clay layer is 75 kN/m^2. This stress is ultimately increased to 150 kN/m^2 as a consequence of a surface load placed at the ground surface. Calculate the total settlement of the clay layer due to this new condition, using the appropriate C_c value.

(c) In practice, some engineers prefer using m_v rather than C_c to calculate the settlements. This is because in reality the e—log σ curve may not be straight. Can you give another reason?

(d) During the consolidation test, 90% of the primary consolidation was obtained after one hour for the stress interval of 100 to 200 kN/m^2. After what time would a degree of 50% consolidation be obtained?

Solution

(a) From the consolidation test it is observed that the 50 kN/m^2 stress did not result in a significant decrease in sample height. Therefore, one may expect that $e_0 \simeq e_{50}$.

Since the soil is saturated, $e_0 = w(\gamma_s/\gamma_w)$ and $e_0 = e_{50} = 0.40 \times 2.7 = 1.08$. The variation of void ratio Δe is proportional to the sample height change Δh. The relation is: $\Delta e/(1 + e_0) = \Delta h/h_0$, from which: $\Delta e = [(1 + e)/h_0]\,\Delta h$ or $\Delta e = [2.08/20]\,\Delta h = 0.104\,\Delta h$, where Δh is in millimeters.

The compression index C_c is given by: $C_c = -\Delta e/\Delta \log \sigma'$. Since the load is doubled for each increment, we have: $\Delta \log \sigma' = \log(100/50) = \log(200/100) = \log 2 = 0.30103$.

Since the sample height change is the same for both load increments, $\Delta h = 19.62 - 20 = 19.24 - 19.62 = -0.38$ mm. The compression index is the same in both cases and is: $C_c = -(0.104 \times 0.38)/0.30103 = 0.131$.

The coefficient of volumetric compressibility is: $m_v = -(\Delta h/h)/\Delta\sigma$. In the first load increment, then: $m_v = -(0.38/20)/(100 - 50) = 3.8 \times 10^{-4}\ \mathrm{m^2/kN} = 3.8 \times 10^{-2}\ \mathrm{cm^2/daN}$ and in the second: $m_v = -(0.38/19.62)/(200 - 100) = 1.9 \times 10^{-4}\ \mathrm{m^2/kN} = 1.9 \times 10^{-2}\ \mathrm{cm^2/daN}$.

(b) The effective vertical stress is within the range 50—200 kN/m^2 for which we have $C_c = 0.133$. This value may be used to calculate the clay layer settlement.

For $\sigma' = 75$ kN/m^2, the value of the void ratio is: $\Delta e = C_c \log[(\sigma' + \Delta\sigma)/\sigma']$ therefore: $e_{75} - e_{50} = -0.131 \log(75/50) = -0.023$ and $e_{75} = 1.08 - 0.023 = 1.057$.

For the increase in stress from 75 to 150 kN/m^2, we have:

$$\Delta e = e_{150} - e_{75} = -0.131 \log(150/75) = -0.0394 \text{ and}$$

$$\Delta h/h = \Delta e/(1 + e_{75}) = -0.0394/(1 + 1.057).$$

The settlement due to the consolidation of the 4-m thick layer will be: $\Delta h = (0.0394 \times 4 \times 10^3)/2.052 = 76.7$, or 77 mm.

(c) If the settlement is calcuated from m_v, it is not necessary to calculate the void ratio.

(d) From the Terzaghi and Fröhlich theories, we have: $T_v = (c_v/h^2)t$. The functions $U = f(T_v)$ and $T_v = \phi(U)$ are given in Tables 3F and 3G [8].

During the laboratory consolidation test, the sample is drained both at the upper and lower faces. The height to consider is that corresponding to the shortest drainage path, or half the sample height. Therefore $h_1 = 10$ mm.

A degree of consolidation $U_1 = 90\%$, gives by interpolation (see Table 3F) a time factor $T_{v1} \simeq 0.85$:

$$T_{v1} = c_v t_1/h_1^2 = 0.85. \tag{1}$$

However, the clay layer in situ is only drained through the top surface, the drainage path therefore corresponds to the clay layer thickness and $h_2 = 4$ m. For a degree of consolidation of 50%, the same source [8] gives a time factor $T_{v2} = 0.20$ and therefore:

$$T_{v2} = c_v t_2/h_2^2 = 0.20. \tag{2}$$

The ratio of eqs. (1) and (2) eliminates c_v which is the coefficient of consolidation and assumed to be the same for both cases. Therefore:

$(t_2/h_2^2)/(t_1/h_1^2) = 0.20/0.85 = 0.235$

from which, expressing h_1 and h_2 in meters,

$t_2 = 0.235 \times t_1 \times (h_2^2/h_1^2) = 0.235 \times 3600 \times (4/10^{-2})^2 = 1.3536 \times 10^8$ sec = 4.29 years.

Thus, over 4 years would be needed to achieve 50% consolidation of the clay layer.

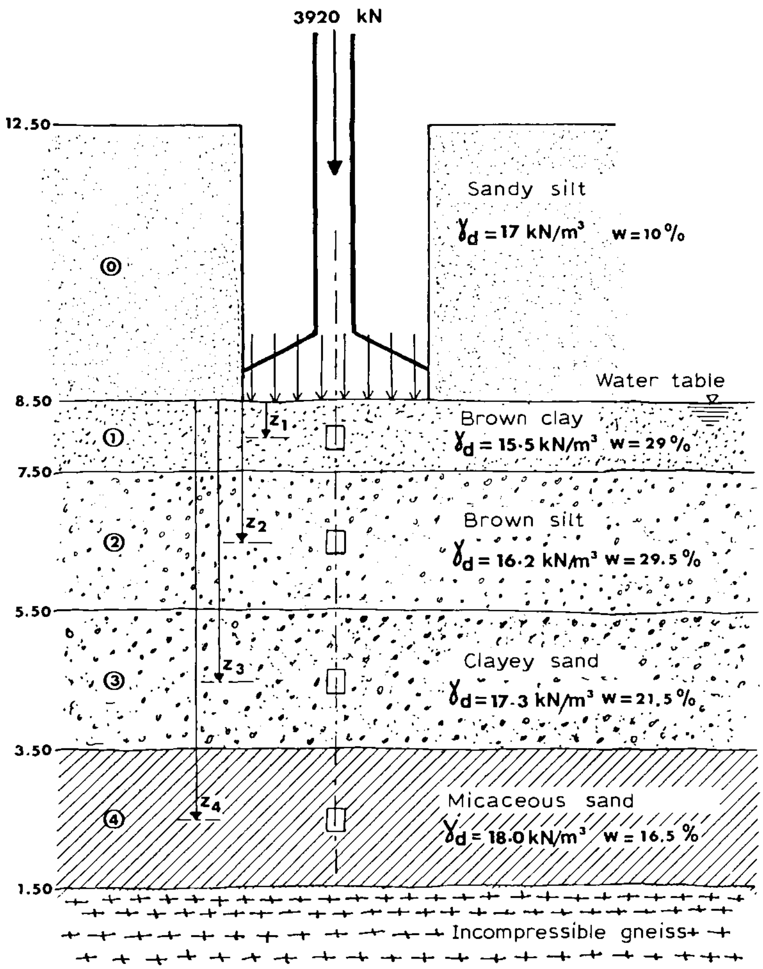


Fig. 3.23.

★★★*Problem 3.17* Settlement of shallow footings

During the preliminary foundation studies for a proposed steel mill plant, it was found necessary to determine the settlement of a typical column of the laminating plant expected to be loaded up to 3920 kN. From the soil investigation, it is known that sound, gneiss bedrock is at an average depth of 11 m. The alluvial soil profile above the rock consists of, from the surface to the gneiss: sandy silt, clay, silt, clayey and micaceous sand layers.

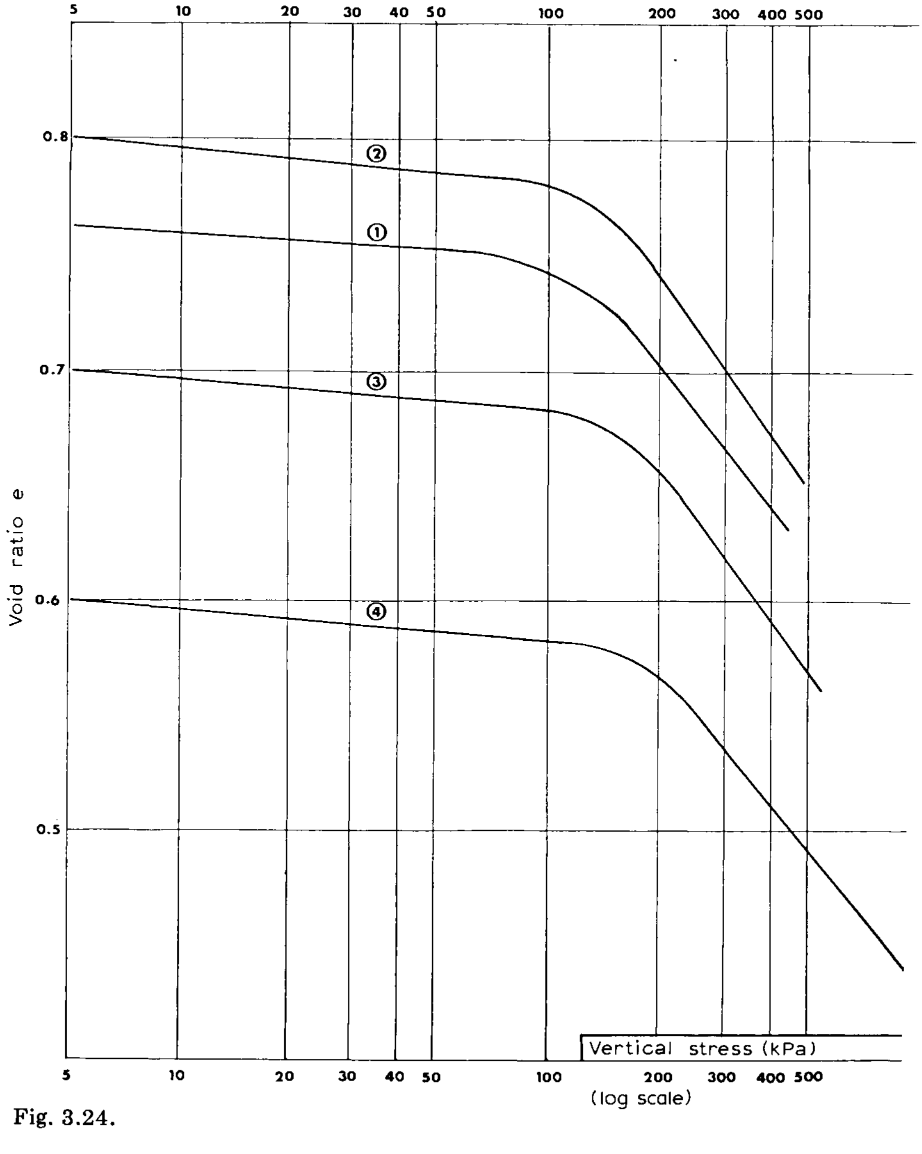


Fig. 3.24.

The average ground surface elevation is +12.5 m. The bottom of a square footing, 4 by 4 m in plan dimension, is proposed at an elevation of 8.50 m. The cross-section and the properties of the soil layers are shown on Fig. 3.23. The ground water table is at the footing level. Four soil layers have been identified which are: $D_1 = 1.00$ *m,* $Z_1 = 0.50$ *m;* $D_2 = 2.00$ *m,* $Z_2 = 2.00$*m;* $D_3 = 2.00$ *m,* $Z_3 = 4.00$ *m;* $D_4 = 2.00$ *m,* $Z_4 = 6.00$ *m.*

Four samples were recovered from depths Z_1*,* Z_2*,* Z_3 *and* Z_4 *and were tested by means of a consolidometer and a triaxial settlement test.*

(1) From the compression curves of the four samples (see Fig. 3.24) define the state of consolidation of the various layers.

(2) To what laboratory load stresses should the samples be reconsolidated before being tested in the triaxial load frame?

(3) Determine the stress increase caused by the column at elevation +8.50, assumed that the footing is flexible. Then, determine the additional stress increases $\Delta\sigma_1$*, and* $\Delta\sigma_3$*, which will have to be applied in the triaxial test, to reproduce the in situ condition, when the column load is applied. Assume* $\nu = 0.25$*.*

(4) With the triaxial test under additional stress tensor (Fig. 3.25) curves giving settlement as a function of load stress were obtained. Determine the total settlement of the column. What can be said about the fact that the footing is well below ground surface? According to the shape of the curves in Fig 3.25 compute the instantaneous (at 10 sec), primary and secondary settlements. Express each settlement as a percentage of the total settlement.

(5) Determine the settlement of the column from the oedometer test results. Do not apply Skempton's correction.

Now go through Skempton's correction for the settlement calculated by the oedometric method. What are the conclusions?

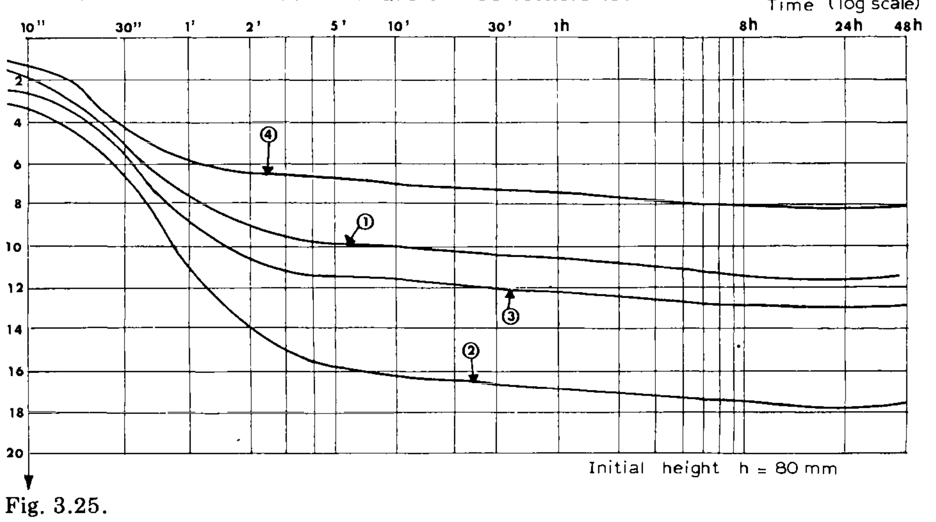


Fig. 3.25.

(6) What can be said regarding the two methods of settlement calculations? Use the data of Fig. 3.26.

Solution

The buoyant wet unit weights of the soils in the four layers (the upper sandy silt layer is not saturated) are determined by: $\gamma_h = \gamma_d(1 + w)$, $\gamma' = \gamma_{sat} - \gamma_w$. When expressed in kN/m^3, they are as tabulated in Table 3M.

Note that for the upper layer, it is not necessary to calculate the buoyant weight. The wet unit weight is used to evaluate the overburden pressure (wet because of the capillary zone).

(1) The effective overburden stresses, acting at mid-height of each of the four layers at depths Z_1, Z_2, Z_3 and Z_4 are:

$\sigma_1' = 4.00 \times 18.7 + 0.50 \times 10.0 = 79.8$ kPa;

$\sigma_2' = \sigma_1' + 0.50 \times 10.0 + 1.00 \times 11.0 = 95.8$ kPa;

$\sigma_3' = \sigma_2' + 2.00 \times 11.0 = 117.8$ kPa;

$\sigma_4' = \sigma_3' + 2.00 \times 11 = 139.8$ kPa.

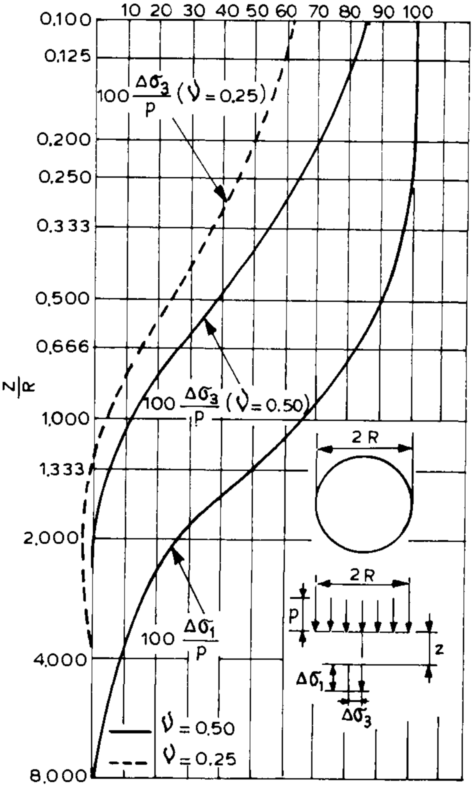


Fig. 3.26.

TABLE 3M

Layers	No.	γ_d	w	γ_h	γ'
Sandy silt	0	17	10%	18.7	—
Brown clay	1	15.5	29%	20.0	10.0
Brown silt	2	16,2	29.5%	21.0	11.0
Clayey sand	3	17.3	21.5%	21.0	11.0
Micaceous sand	4	18.0	16.5%	21.0	11.0

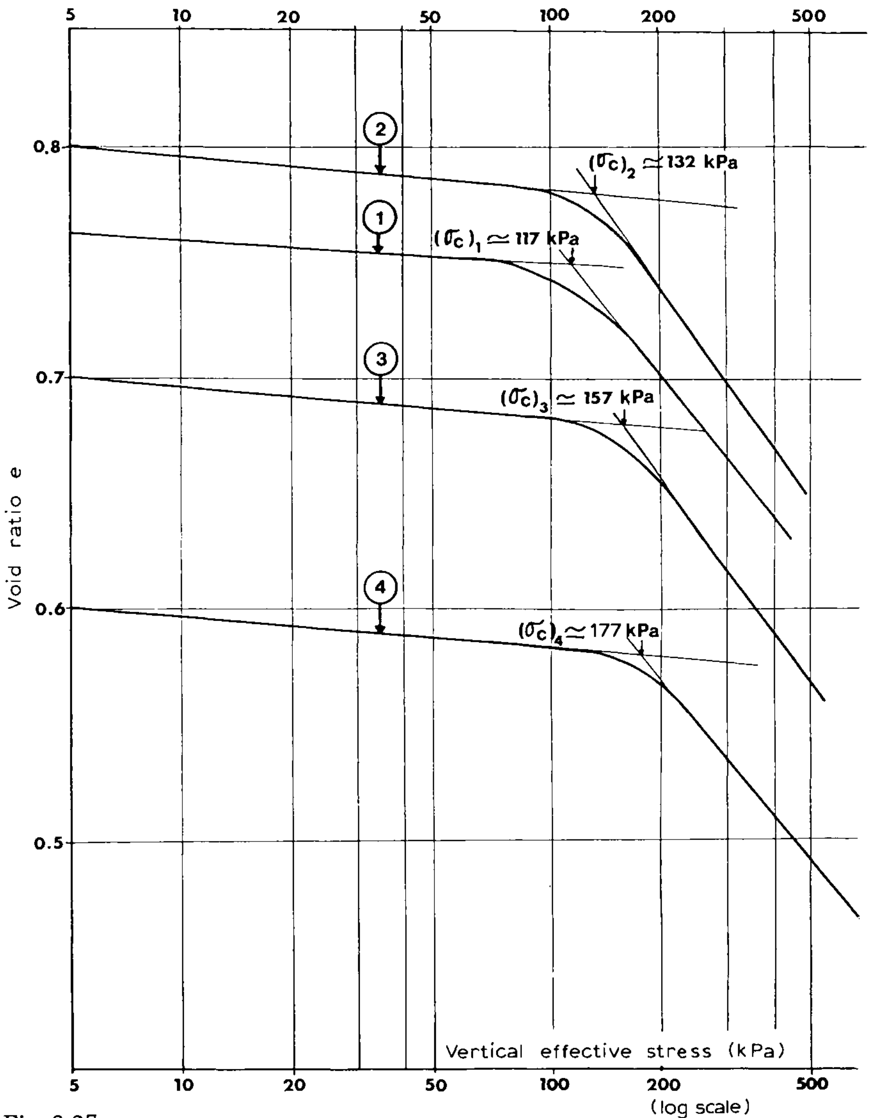


Fig. 3.27.

From Fig. 3.27, the pre-consolidation pressures are:

$(\sigma_c')_1 \simeq 117$ kPa; $(\sigma_c')_2 \simeq 132$ kPa; $(\sigma_c')_3 \simeq 157$ kPa; $(\sigma_c')_4 \simeq 177$ kPa.

By comparing these values with those of the overburden pressure, it is seen that in each instance $\sigma_i' < (\sigma_c')i$. So all the layers are over-consolidated.

(2) Before proceeding with the triaxial testing of the samples, they will have to be reconsolidated under an effective stress equivalent to that under which the soils were subjected in situ. The ideal consolidation condition in the lab would be to apply an anisotropic stress such that $\sigma_1'/\sigma_3' = K_0(\sigma_c')_i$, which would require determining the value of K_0 for each of the layers. In practice an isotropic reconsolidation under a spherical tensor σ_i' could be admitted, so that the following confining pressures should be applied: 80 kPa for layer 1; 96 kPa for layer 2; 118 kPa for layer 3; 140 kPa for layer 4.

(3) The error involved by assuming that the unit weight of concrete (of the footing) and of the fill material used for seating has the same value as the soil in place, is negligible when considering the magnitude of the expected column load.

Since the footing is assumed to be flexible, the vertical stress distribution below it is uniform. The stress increase due to the column load at El. + 8.5 is: $q = 3920/(4 \times 4) = 245$ kPa.

From graphs of Fig. 3.27a the stress increases in the various layers can be estimated, and they are given in Table 3N.

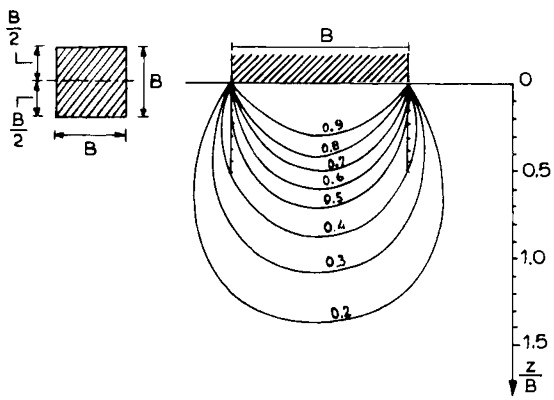


Fig. 3.27a.

To obtain the increments of horizontal stress $\Delta\sigma_3$, corresponding to the increase in vertical stress, it is necessary to resort to Fig. 3.26. It can be used for square footings by letting $R = B/2$. The results are summarized in Table 3O.

(4) The shape of the curves of Fig. 3.25 indicates that there is not much difference between the ultimate settlement at $t = \infty$ and that occurring after 48 h. The secondary compression occurring after the initial 48 h may be

TABLE 3N

Z	Z/B	$\Delta\sigma_1/q$	$\Delta\sigma_1$ (kPa)
0.50	0.125	0.995	244
2.00	0.50	0.70	171
4.00	1.00	0.33	81
6.00	1.50	0.18	44

TABLE 3O

Z	Z/R	$\Delta\sigma_3/q$ ($\nu = 0.25$)	$\Delta\sigma_3$ (kPa)
0.50	0.250	0.45	110
2.00	1.00	0.05	12
4.00	2.00	−0.02 (tension)	0
6.00	3.00	−0.02 (tension)	0

overlooked. Since the sample height in the triaxial is 80 mm, the change in sample thickness for each test is:

sample no. 1: $\Delta h_{t\infty} \simeq 12/10$ mm;

sample no. 2: $\Delta h_{t\infty} \simeq 18/10$ mm;

sample no. 3: $\Delta h_{t\infty} \simeq 13/10$ mm;

sample no. 4: $\Delta h_{t\infty} \simeq 8.3/10$ mm.

Thus the total settlement under the footing will be:

$$\Delta H = \frac{12}{10} \times \frac{100}{8} + \frac{18}{10} \times \frac{200}{8} + \frac{13}{10} \times \frac{200}{8} + \frac{8.3}{10} \times \frac{200}{8} = 113$$

$\Delta H \simeq 113$ mm.

The fact that the footing is 4 m below grade does not greatly influence the amount of total settlement. Contrary to the outdated method of determining the stresses in a semi-infinite body by the Mindlin theory, the more accurate solution today makes use of finite element method which is based more on the real conditions (Fig. 3.28, 3.29). By Mindlin's method, tensil stresses are

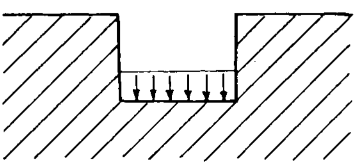


Fig. 3.28.

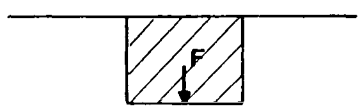


Fig. 3.29.

developed in the shaded area of Fig. 3.29 which, we know, cannot exist in soils. The calculated settlement should not be considered conservative and therefore the value of $\Delta H = 113$ mm should be reported.

Instantaneous settlement

In the given of the problem, it was specified that the settlement at ten seconds should be considered. This is a purely arbitrary consideration. Based on that, however, we have:

sample no. 1: 0.18 mm; sample no. 2: 0.34 mm;

sample no. 3: 0.27 mm; sample no. 4: 0.12 mm.

The initial, instantaneous settlement of all soil layers will be:

$$\Delta H_{\text{initial}} = 0.18 \times \frac{100}{8} + 0.34 \times \frac{200}{8} + 0.27 \times \frac{200}{8} + 0.12 \times \frac{200}{8} \simeq 20 \text{ mm}$$

or: $\Delta H_{\text{initial}} \simeq 18\%$.

The 10 s consideration to estimate instantaneous settlement is completely arbitrary. In fact, it often occurs in the initial two seconds but it is not possible to determine accurately the duration of this phenomenon.

The reasons for this instantaneous settlement are numerous.

In the laboratory they may be due to:

— incomplete saturation of the sample,

— the porous stone of the oedometer is not properly seated on the sample and penetrates in the sample during the first seconds.

In the field, this settlement may be due to:

— disturbance of the soil immediately below the footing bottom;

— a general deformation of the soil mass with no change in volume occurring prior to the consolidation. This type of settlement cannot be estimated from laboratory test results given in this problem.

It should thus be noted that instantaneous settlements in the field and in the laboratory are of different nature, and that the first one cannot be estimated by the second.

Primary settlement

The end of the primary consolidation corresponds to the intersection of the two tangents of the curve of settlement versus time obtained from the laboratory test. The values thus obtained are summarized in Table 3P.

TABLE 3P

Sample no.	ΔH (mm)		Primary settlement (mm)
	Start of primary consolidation	End of primary consolidation	
1	0.18	0.95	0.77
2	0.34	1.59	1.25
3	0.27	1.09	0.82
4	0.12	0.60	0.48

The primary settlement, corresponding to the primary consolidation of all the layers is thus:

$$0.77 \times \frac{100}{8} + 1.25 \times \frac{200}{8} + 0.82 \times \frac{200}{8} + 0.48 \times \frac{200}{8} = 73\,\text{mm}$$

say $\simeq$ 65% of the total settlement.

(5) *Oedometer method*

We must now refer to the oedometer diagrams and the oedometric equation: $\Delta H/H = \Delta e/(1 + e)$, on which is based Table 3Q (stresses in kPA).

TABLE 3Q

σ'	$\sigma' + \Delta\sigma'$	$e(\sigma)$	$e(\sigma' + \Delta\sigma')$	Δe	$\frac{\Delta e}{1+e}$	ΔH
78	323	0.749	0.657	0.092	0.0526	5.26
94	265	0.782	0.710	0.072	0.0404	8.08
115	198	0.681	0.658	0.023	0.0137	2.74
136	180	0.580	0.572	0.008	0.005	1.00

Therefore: $\Delta H_{\text{oedometer}} = 17\,\text{cm}$. $\Sigma = 17.08\,\text{cm}$

From experimental triaxial curves (Fig. 3.25) the following correlations can be anticipated for the oedometer test:

$$s^{\text{primary}}_{\text{oedometer}} \simeq 35\%\ s^{\text{total}}_{\text{oedometer}} \simeq 65\%\ s^{24\text{h}}_{\text{oedometer}}$$

On the other hand, the soils are slightly overconsolidated.
We can estimate A to be 0.4. With $H = 7$ m and $B = 4$ m we get: $H/B = 1.75$, and Skempton and Bjerrum's graph gives: $\mu = 0.55$ to 0.60 or:

$$s_{\text{p}} = 0.55 \times (0.65 \times 170) \simeq 60\,\text{mm}$$

and:

$$s^{24\text{h}} = 60 + 0.35 \times 170 = 119.5 \text{ or } \simeq 120\,\text{mm}$$

We then have:

$\Delta h = 113$ mm (triaxial test method),

$\Delta h = 170$ mm (original oedometer method),

$\Delta h = 120$ mm (Skempton corrected oedometer method).

The agreement between the two methods is acceptable, provided Skempton's correction is made.

(6) The oedometer method leaves something to be desired. It does not account for the horizontal strains. When calculating the settlements of a relatively thin clay layer that is loaded by a large area fill or a wide embankment, this method gives much more satisfactory results.

In this example, however, where the thicknesses of the layers are large with respect to the loaded area (thickness wider than footing dimension) the oedometer method lacks accuracy. Theoretically, the triaxial test method is preferable because it reproduces more realistic the in-situ stress conditions. But the Poisson's ratio of the soil must be known and it is difficult to determine this value accurately. Because of this limitation and the complexity of performing triaxial tests, the theoretical advantage of the method is doubtful.

★★*Problem 3.18* Settlement calculation with Newmark's chart, effect of adjacent footings

A square footing S_1 is 2 m wide and located 2 m below the ground surface in a sandy gravel layer assumed to be incompressible. This layer is 5.5 m

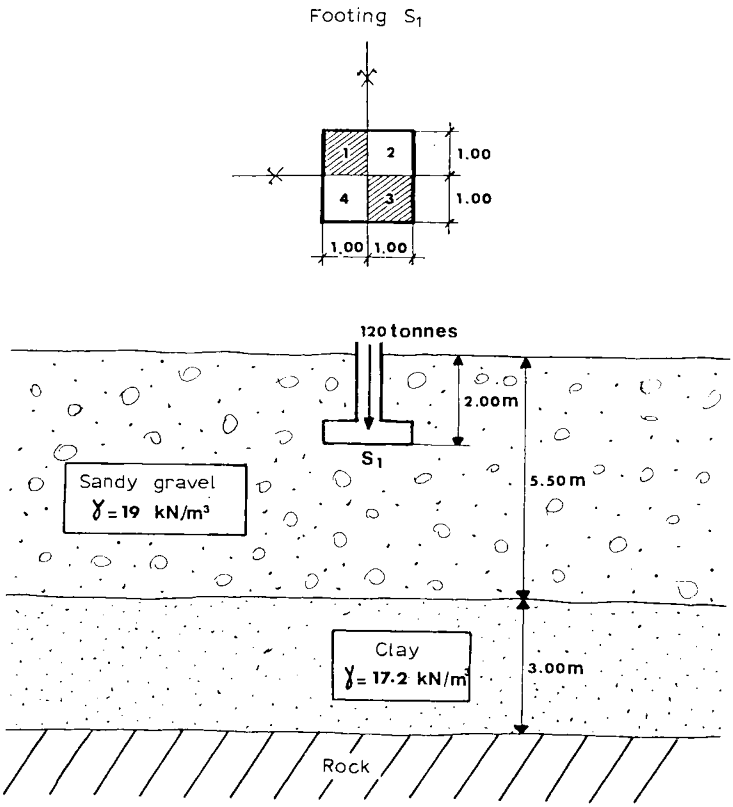


Fig. 3.30.

thick. Below it is a clay layer 3 m thick which overlays a sound bedrock (see Fig. 3.30). The footing supports a column loaded to 1200 kN. The in-place density of the sandy gravel is 1.9 and that of the clay is 1.72.

The preconsolidation pressure of the clay was found to be $\sigma_c = 1.29$ daN/cm^2 from laboratory tests. The oedometric modulus is $E' = 30$ daN/cm^2 for stresses less than 10 daN/cm^2.

(1) Calculate the settlement at the center of the footing.

(2) This footing is surrounded by other footings, S_2, S_3 and S_4 as shown on the plan of Fig. 3.31. All are at the same level and carry loads of 580, 435 and 1085 kN, respectively.

Calculate the settlement of S_1 under the influence of adjacent loads. Assume that the unit weights of concrete and the surrounding soil are the same and that the excavation is filled up after construction. Use Newmarks chart of Fig. 3.32.

Solution

(1) The settlement beneath the center of the square footing is:

$$\Delta h/h = -\Delta\sigma/E'. \tag{1}$$

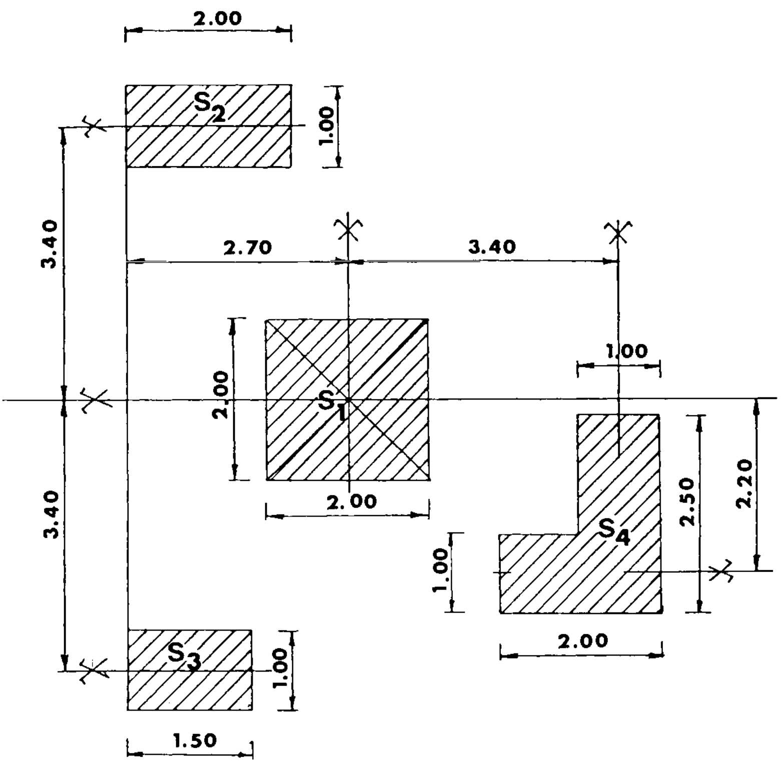


Fig. 3.31.

The stress increase $\Delta\sigma$ corresponds to the increase at mid-height in the clay layer due to the footing load.

To determine $\Delta\sigma$, Fig. 3.26a cannot be used, for the fact that $Z/B = 2.5$ is too large ($Z = 5.50 - 2.00 + 1.50 = 5$ m, $B = 2.00$ m). Therefore, the graph shown in Fig. 3.27a may be used if we consider the four quarters of the square footings as identified in Fig. 3.31.

The stress increase along a vertical passing through the center of the square footing will be determined from the influence factors $I_1 = I_2 = I_3 = I_4 = I$ and $\Delta\sigma = q \times 4I$.

For a quarter, we have: $L/B = 1$ (square footing), $z/B = 5/1 = 5$. We find that $I \simeq 2\%$ and $4I = 8/100$. Also $q = 1200/(2 \times 2) = 300$ kPa $\simeq 3$ daN/cm^2 and $\Delta\sigma = 8 \times 3/100 = 0.24$ daN/cm^2.

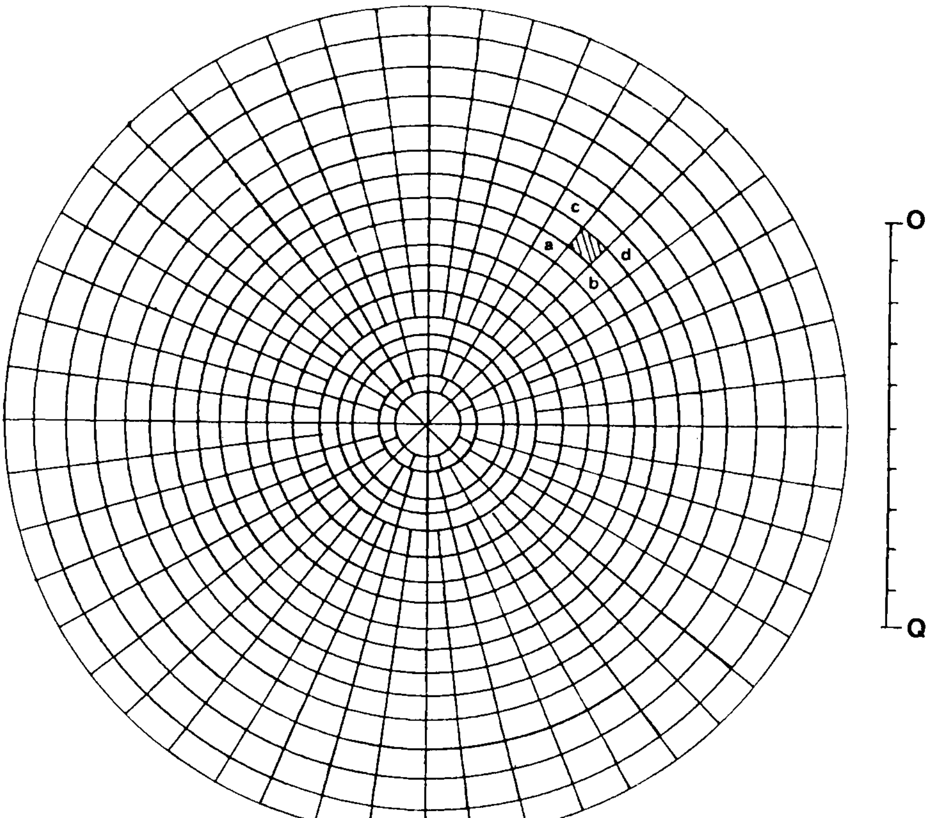


Fig. 3.32. Newman's chart. The influence of each curvilinear square on the chart, such as *abcd* is 0.001. *OQ* (the lateral vertical scale), represents depth Z, at which the stress is to be calculated, so that: $\Delta\sigma = 0.001 \times n \times q$, where q = vertical stress at the surface of the soil below the footing bottom, uniformly distributed, n = number of squares covered by the size of the footing drawn to the same scale as $Z = OQ$.

The overburden pressure at mid-height in the clay, prior to the start of construction, was: $\sigma_0 = 5.50 \times 19 + 1.50 \times 17.2 = 130.3$ kPa ≈ 1.3 daN/cm^2.

We know that the preconsolidation pressure is $\sigma_c = 1.29$ daN/cm^2, therefore we can say that the clay is normally consolidated. Formula (1) may be used with $E' = 30$daN/cm^2 which we were told is valid up to a stress level of 10 daN/cm^2. Therefore: $\Delta h = -h(\Delta\sigma/E')$, $|\Delta h| = 300 \times 0.24/30 = 2.4$ cm.

(2) Additional settlement will occur under the square footing due to the loads imposed by adjacent footings S_2, S_3, S_4 (Fig. 3.31). Newmark's chart is used for calculating these settlements. The size of the footing is drawn on the chart and the squares within the footing are counted. Here we have $OQ = 6.5$ cm which corresponds to $Z = 5.00$ m.

For the number of squares (see Fig. 3.33) we have:

footing S_1: $n_1 = 74$ squares
footing S_2: $n_2 = 11.5$ squares
footing S_3: $n_3 = 8.5$ squares
footing S_4: $n_4 = 22$ squares.

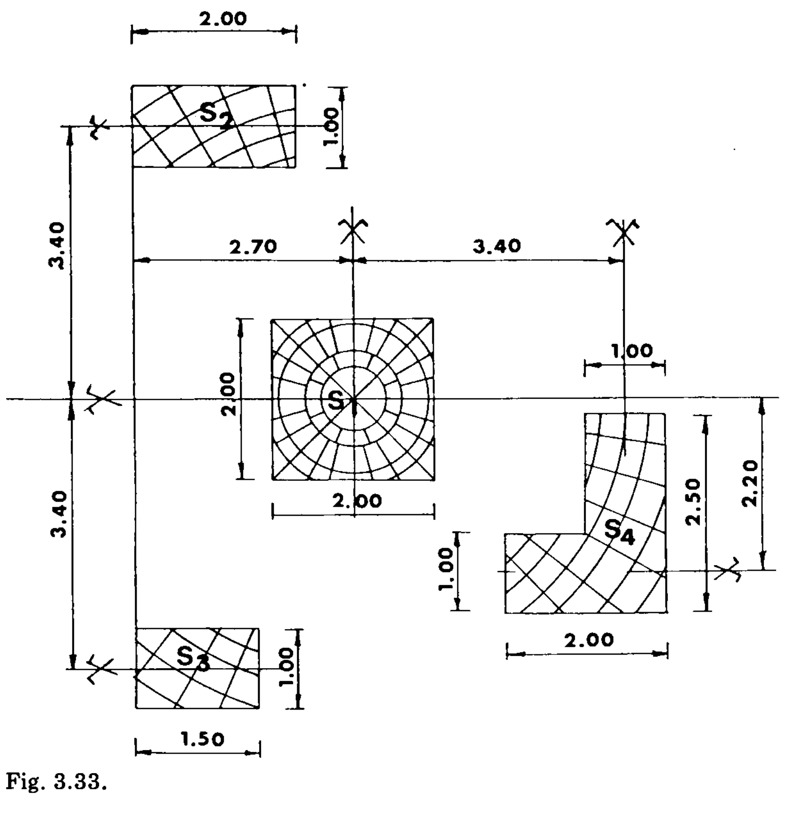


Fig. 3.33.

The respective footing areas are: S_1 : 4.00 m^2, S_2 : 2.00 m^2, S_3 : 1.50 m^2, S_4 : 3.50 m^2, from which:

$q_1 = 1200/4 = 300$ kPa $= 3$ daN/cm^2,

$q_2 = 580/2 = 290$ kPa $= 2.9$ daN/cm^2,

$q_3 = 435/1.5 = 290$ kPa $= 2.9$ daN/cm^2,

$q_4 = 1085/3.5 = 310$ kPa $= 3.1$ daN/cm^2.

The total is: $\Delta\sigma' = 0.001[74 \times 3 + 11.5 \times 2.9 + 8.5 \times 2.9 + 22 \times 3.1] = 0.348 \simeq 0.35$ daN/cm^2.

The settlement below the center of the square footing will therefore be:

$\Delta h = 300 \times 0.35/30 = 3.5$ cm.

The adjacent footings increase the settlement of the square footing by a factor of about 46%.

Remark

Newman's chart may of course also be used to determine the stress for the isolated footing S_1. If used, it gives a value of $\Delta\sigma'' = 0.001 \times 74 \times 3 = 0.22$ daN/cm^2 as opposed to 0.24 daN/cm^2 found in our solution.

★★★*Problem 3.19* **Solution of Terzaghi's consolidation equation by the finite-difference method: application to a two-layered system**

In order to obtain an approximate numerical solution to Terzaghi's consolidation equation, increments to time t and depth Z will be given finite values Δt and ΔZ. Let us adopt the indexes i for depth and j for time; the pore pressure is noted with two indices: u_{ij}, and $u_{i+1,j-1}$ means the pore pressure at depth $Z + \Delta Z$ at time $t - \Delta t$.

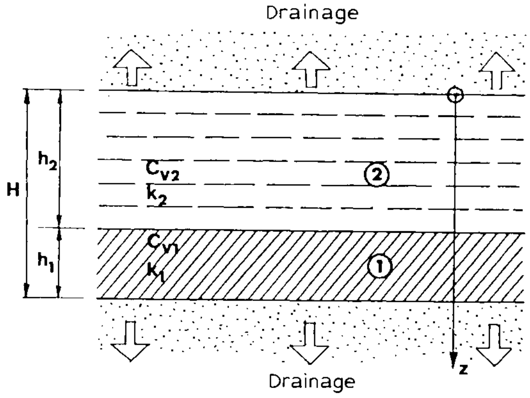


Fig. 3.34.

(1) By incremental changes the two elements of Terzaghi's formula, show that this equation consists of a linear recurrence relation between $u_{i,j}$, $u_{i,j+1}$, $u_{i+1,j}$ *and* $u_{i-1,j}$. *Let* $A = c_v(\Delta t/\Delta Z^2)$. *For which value of A is the relation very simple?*

(2) Consider a two-layered system of compressible soils located between pervious boundaries (Fig. 3.34). To solve Terzaghi's equation, layer 1 is divided into n_1 *sub-layers, each of thickness* ΔZ_1 *and layer 2 is divided in* n_2 *sublayers of thickness* ΔZ_2.

Show that ΔZ_1 *and* ΔZ_2 *are not independent and determine what the relation is between* n_2 *and* n_1.

(3) Find the initial and limit conditions on the upper and lower faces of the two-layered system which indicate that drainage is ideal.

Assume that the initial excess pore-water pressure is 1 over the entire height of the layers, except at the boundaries where $u(0, 0) = u(H, 0) = 0.5$. *At the boundaries in fact, at* $t = 0$, *the excess pore pressure is 1 and become zero at* $t = t + \epsilon$.

Give an equation of continuity at the interface between the two layers using the principle of continuity of the flow.

(4) For the rest of the problem, assume $A = 1/2$ *to simplify the computations.*

Remark

We could also assume $A \neq 1/2$; *the accuracy of the method depends on its value. The smaller the elements, the longer is the computation but the better is the accuracy.*

For computer use, it is generally assumed that $A = 1/6$. *Given are:*

$$h_2 = 2.5h_1; \quad c_{v1} = 1.8 \times 10^{-4}\ \text{cm}^2/\text{s}; \quad c_{v2} = 1.62 \times 10^{-3}\ \text{cm}^2/\text{s};$$

$$k_1 = 1.1 \times 10^{-4}\ \text{cm/s}; \quad k_2 = 2.2 \times 10^{-4}\ \text{cm/s}; \quad H = 7.00\ \text{m}.$$

Based on the equations obtained above, show that it is possible to construct, point by point, an isochrone net. Show the computation in the form of tables to the 10th increment of Δt. *Take the minimum whole values for* n_1 *and* n_2. *Draw the net corresponding to* $t = 0$, $t_1 = 5\ \Delta t$, *and* $t_2 = 10\ \Delta t$.

If we call the degree of consolidation at time t for depth Z by $u(t, Z) = 1 - u(t, Z)/u_0$ (u_0 *= initial excess pore-water pressure) give the formula for the degree of consolidation of the two-layered system and find its value for* $t = t_2$ *and* $t = t_1$.

Solution

(1) For a function $f(x)$ the formula for finite increments is:

$$f(a + h) = f(a) + hf'(a + \theta h) \quad \text{with} \quad 0 < \theta < 1. \tag{1}$$

Porewater pressure is a function of Z and t: $u(Z, t)$. Let's fix $Z = Z_0$,

$u(Z, t)$ then becomes a function of time alone:

$$u(t) = u(Z_0, t),$$
$$u(t + \Delta t) = u(t) + \Delta t . u'(t + \theta\, \Delta t) \qquad 0 < \theta < 1.$$

If Δt is sufficiently small, then $u'(t + \theta\, \Delta t) \simeq u'(t) \simeq u'(t + \Delta t)$ and

$$u'(t) = \frac{\partial u}{\partial t} \simeq \frac{u(t + \Delta t) - u(t)}{\Delta t}$$

or, using the indices: $\dfrac{\partial u}{\partial t} \simeq \dfrac{u_{i,j+1} - u_{i,j}}{\Delta t}$ (2)

Similarly, for $t = t_0$:

$$\frac{\partial u}{\partial Z} \simeq \frac{u_{i+1,j} - u_{i,j}}{\Delta Z}, \quad \frac{\partial^2 u}{\partial Z^2} \simeq \left[\frac{u_{i+1,j} - u_{i,j}}{\Delta Z} - \frac{u_{i,j} - u_{i-1,j}}{\Delta Z}\right] \Big/ \Delta Z$$

or $\dfrac{\partial^2 u}{\partial Z^2} \simeq \dfrac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta Z^2}$. (3)

The consolidation equation $\dfrac{\partial u}{\partial t} = c_v \dfrac{\partial^2 u}{\partial Z^2}$ may be written as:

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = c_v \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta Z^2}$$

letting $A = c_v(\Delta t/\Delta Z^2)$ we get:

$$u_{i,j+1} = (1 - 2A)u_{i,j} + A(u_{i+1,j} + u_{i-1,j}) \tag{4}$$

which is the relation asked for.

This relation may be simplified if $A = 1/2$, it then becomes:

$$u_{i,j+1} = \tfrac{1}{2}(u_{i+1,j} + u_{i-1,j}) \tag{5}$$

(2) We have $A = c_{v1}(\Delta t/\Delta Z_1^2) = c_{v2}(\Delta t/\Delta Z_2^2)$ since the same time increment must be chosen for the two layers. Therefore: $\Delta Z_1/\Delta Z_2 = \sqrt{c_{v1}/c_{v2}}$.

Therefore we have: $\Delta Z_1 = h_1/n_1$ and $\Delta Z_2 = h_2/n_2$, and

$$n_1 = \frac{h_1}{h_2}\sqrt{\frac{c_{v2}}{c_{v1}}} \cdot n_2. \tag{6}$$

(3a) The boundary conditions are:

$$\begin{cases} u(0, t) = 0 \\ u(H, t) = 0 \end{cases} \quad \text{for} \quad t \neq 0 \tag{7}$$

which implies that drainage at the surfaces of the system is ideal.

(b) The initial conditions may be written:

$$\begin{cases} u(Z,0) = 1 \quad \forall Z \neq \begin{cases} 0 \\ H \end{cases} \\ u(0,0) = u(H,0) = 0.5 \end{cases} \tag{8}$$

(c) Let us write the condition of continuity of flow at the interface between the two compressible layers. Let us start by determining the head h at a point M at elevation Z. Say H_0 is the distance to the water table from an arbitrarily chosen reference level (see Fig. 3.35).

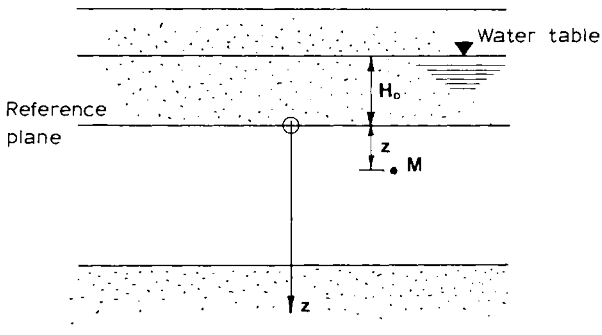


Fig. 3.35.

The pore-water pressure at M at time t is equal to the sum of the excess pore pressure $u(Z,t)$ and of the hydrostatic pressure $(H_0 + Z)\gamma_w$ existing at M prior to the increase $u(Z,t)$. The head h is usually expressed by $h = (u/\gamma_w) + Z$.

The positive vertical direction is downward and $u = (H_0 + Z)\gamma_w$. Therefore:

$$h = \frac{(H_0 + Z)\gamma_w + u(Z,t)}{\gamma_w} - Z = H_0 + u(Z,t). \tag{9}$$

The continuity of flow at the interface may be expressed by the continuity of rate of percolation $v_1 = v_2$ or $k_1 i_1 = k_2 i_2$ or also by taking into account eqn. (9): $k_1(\Delta u/\Delta Z)_1 = k_2(\Delta u/\Delta Z)_2$

therefore by $k_1(u_{k,j} - u_{k-1,j}) = k_2(u_{k+1,j} - u_{k,j})(\Delta Z_1/\Delta Z_2)$ (10)

where index k corresponds the value of index i at the interface, and if we assume that index i increases from bottom to top of the compressible layers.

(4) The numerical applications give:

$$\frac{c_{v2}}{c_{v1}} = \frac{1.62 \times 10^{-3}}{1.8 \times 10^{-4}} = 9$$

from which:

$\Delta Z_1/\Delta Z_2 = \sqrt{1/9} = 1/3$ from which $H = 7.00$ m, $h_2 = 2.5\,h_1, h_1 = 2.00$ m, $h_2 = 5.00$ m,

$$n_1 = \frac{h_1}{h_2}\sqrt{\frac{c_{v2}}{c_{v1}}} \times n_2 = \frac{1}{2.5} \times 3 \times n_2 = 1.2\,n_2$$

or $n_2 = 5$, $n_1 = 6$, finally $k_2/k_1 = 2$.

We then have the following relations:

$$u_{i,j+1} = \tfrac{1}{2}(u_{i+1,j} + u_{i-1,j}) \tag{5}$$

$$u_{k,j} - u_{k-1,j} = 2 \times \tfrac{1}{3}(u_{k-1,j} - u_{k,j}). \tag{10bis}$$

The interface between the two layers corresponds to the value 6 of index k and since equation (10 bis) is valid regardless of t, index j is no longer necessary. Therefore, eqn. (10 bis) may be rewritten as:

$$u_6 - u_5 = \tfrac{2}{3}(u_7 - u_6) \tag{10ter}$$

or $\quad u_6 = \tfrac{1}{5}(3u_5 + 2u_7)$.

Finally, increment Δt is given by: $c_v \Delta t/\Delta Z^2 = 1/2$.
For instance, if values applicable to layer 1 are substituted, we have:

$$\Delta Z_1 = \frac{2 \times 10^2}{6}\ \text{cm} \quad \text{from which} \quad \Delta t = \frac{1}{2} \times \left(\frac{2 \times 10^2}{6}\right)^2 \times \frac{1}{1.8 \times 10^{-4}}$$

$\Delta t \simeq 3.09 \cdot 10^6$ seconds or about 36 days.

The initial and boundary conditions are: $u_{0,0} = 0.5$, $u_{0,j} = 0$, $u_{11,0} = 0.5$, $u_{11,j} = 0$, which allow us to calculate u_{ij} by successive approximations with eqn. (5). When the value of 6 is reached for index i, equation (10 ter) must then be used.

Calculations are made easier if tabulated with double entry. For instance, each column may be made to correspond to a specific index value corresponding to the location of a point in the two-layered system. ($i = 0, 1, 2, \ldots 11$) and each line corresponds to a time increment $0, \Delta t, 2\Delta t, \ldots, n\Delta t$ ($j = 0, 1, 2, \ldots n$). As specified in the given of the problem, the limit of j is 10.

The first line of the table may be filled in immediately since it is assumed that $u = 1$, except at the end surfaces ($i = 0$ and $i = 11$) at which $u = 0.5$.

Columns 0 and 1 can also be filled in immediately since $u_{0,j} = 0$ and $u_{11,j} = 0$, regardless of j. Line 2 is easily filled in for $u_{0,j} = 0$ and $u_{1,1} = \tfrac{1}{2}(1 + 0.5) = 0.75$.

The same procedure is used for $u_{10,1}$, at the right end of the table line 1.

For line $j = 4$, we would have:

$$u_{6,4} = \tfrac{1}{5}(3u_{5,4} + 2u_{7,4}) = \tfrac{1}{5}[(3 \times 1) + (2 \times 0.968)]$$

$$u_{6,4} = 0.987.$$

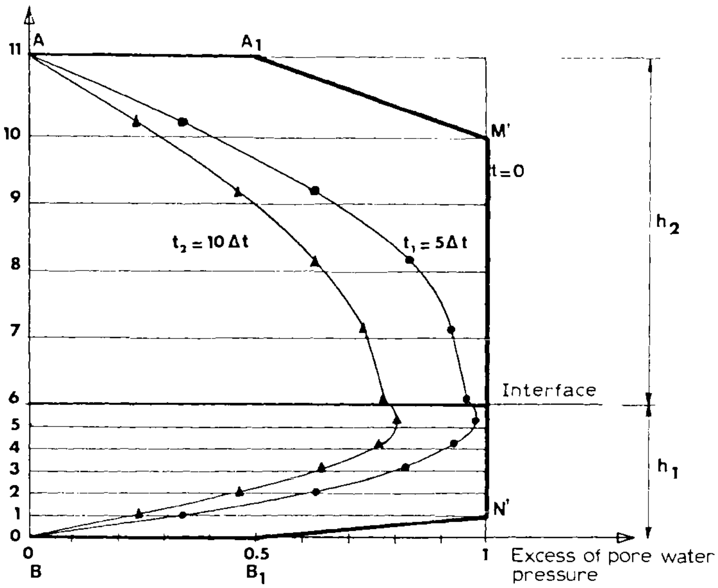


Fig. 3.36.

Table 3Q is filled in from the extremities of each line and to be terminated in column 6 with equation (10ter). Each line of the table thus represents a point-by-point construction of the net.

The isochrons corresponding to $t = 0$, $t_1 = 5\,\Delta t$ and $t_2 = 10\,\Delta t$ are presented in Fig. 3.36.

The degree of consolidation at time t and depth Z are defined by:

$$U(t,Z) = 1 - \frac{u(t,Z)}{u_0} \quad (u_0 = \text{initial excess pore-water pressure}).$$

The degree of consolidation of the two-layered system is

$$U(t) = \frac{1}{H}\int_0^H \left[1 - \frac{u(t,Z)}{u_0}\right] \mathrm{d}Z.$$

which is the ratio of the cross-hatched area ($\mathscr{A}$) shown in Fig. 3.37 and the rectangle $AMNB$, or Hu_0:

$$U(t) = \mathscr{A}/Hu_0.$$

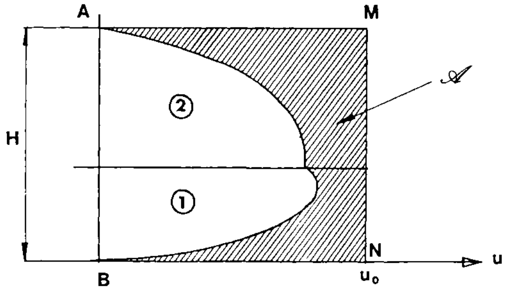


Fig. 3.37.

The area $\mathscr{A}$ may be computed by numerical method (trapezoid method) or by planimeter.

We find: $U(t_1) \simeq 0.34$ and $U(t_2) \simeq 0.49$.

Fig. 3.38. Different settlement below a 16th century old building in Lyons, France.

Chapter 4

PLASTICITY AND SHEAR STRENGTH

★*Problem 4.1* Triaxial test on sand

A dry sample of sand is tested in triaxial compression. It is assumed that the internal angle of friction is $\varphi = 36°$. If the minor principal stress, σ_3, is 300 kPa, at which value of maximum principal stress, σ_1 will the sample fail?

Solution

Failure, or plastic deformation of the sample, will occur when Mohr's circle becomes tangent to the failure envelope. The failure envelope consists of two straight lines which, in this case, make an angle of 36° above and below the principal stress axis Oσ. At the beginning of the test, the sample is subjected to a confining isotropic stress, σ_3. For this condition, Mohr's circle reduces to a single point, which abscissa is σ_3.

When the major principal stress σ_1 is applied, the diameter of Mohr's circle increases by a distance of $\sigma_1 - \sigma_3$ while σ_3 remains constant (see Fig. 4-1). The diameter continues to increase until the circle becomes tangent to the two straight lines.

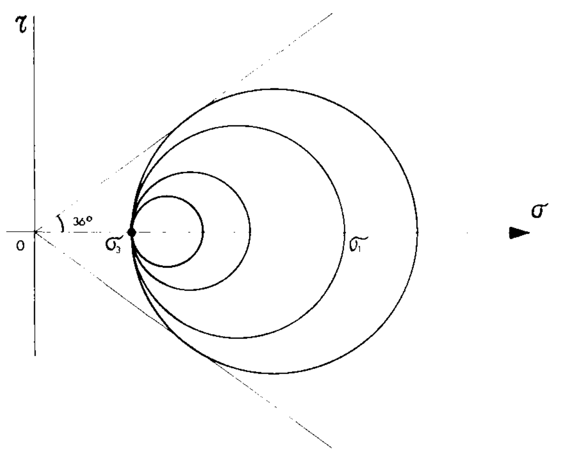


Fig. 4.1.

Let d be the abscissa of the center, and R the radius of Mohr's circle at failure (see Fig. 4.2), then:

$$\frac{R}{d} = \sin\varphi; \quad \sigma_3 = d - R = \frac{R}{\sin\varphi} - R = R\frac{1-\sin\varphi}{\sin\varphi}$$

and:

$$\sigma_{1R} = d + R = \frac{R}{\sin\varphi} + R = R\frac{1+\sin\varphi}{\sin\varphi}$$ Therefore:

$$\frac{\sigma_{1R}}{\sigma_3} = \frac{1+\sin\varphi}{1-\sin\varphi} = \tan^2\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)$$

Numerical application
With $\sigma_3 = 300\,\text{kPa}$, $\varphi = \underline{36^\circ}$. We have: $\pi/4 + \varphi/2 = 45 + 18 = 63^\circ$; $\tan 63^\circ \simeq 1.96$; $\sigma_{1R} = 300 \times 1.96^2 = 1\,155$, say $1160\,\text{kPa}$.

Answer
$\sigma_{1R} = 1\,160\,\text{kPa}$.

★*Problem 4.2* Triaxial test on sand with cohesion

Using the data of problem 4.1, solve for σ_{1R} assuming that the sand has a cohesion of 12 kPa.

Solution
The failure envelope, as in problem 4.1, consists of two lines inclined at 36° above and below the principal stresses axis. However, the two straight lines now no longer cross the axis at the origin but to the left of it at a point whose coordinates are $\sigma = 0$, $\tau = \pm c$ (see Fig. 4.3).

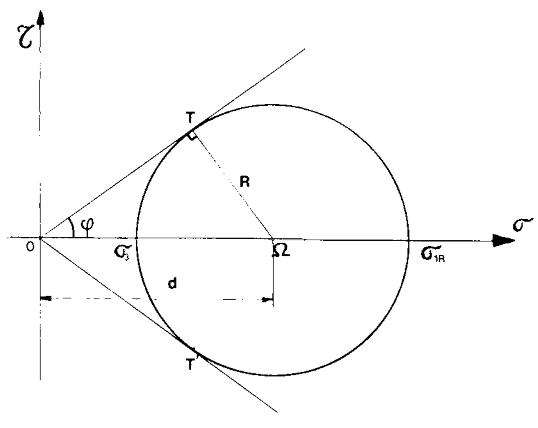


Fig. 4.2.

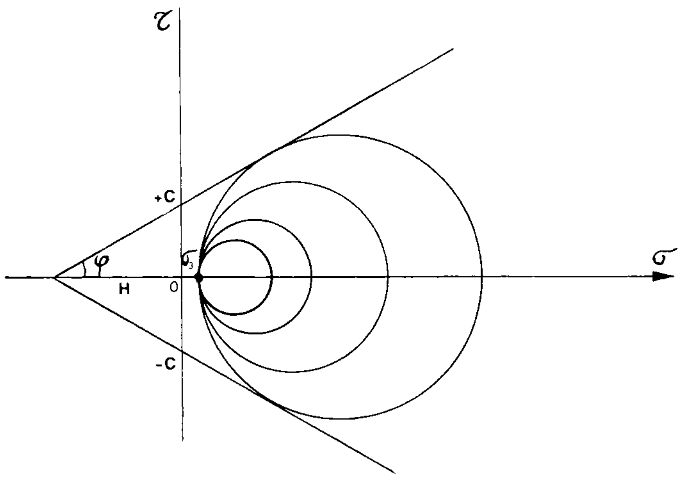


Fig. 4.3.

A computation similar to that of example problem 4.1 gives:
$H = c\ \cot\varphi$ (Fig. 4.4), $\sigma_{1R} + H = (\sigma_3 + H)\tan^2(\pi/4 + \varphi/2)$, $\sigma_{1R} = (\sigma_3 + H)\tan^2(\pi/4 + \varphi/2) - H$.

Numerical application
$\sigma_3 = 300$ kPa, $\varphi = 36°$, $\cot\varphi = 1.376$, $\tan^2(\pi/4 + \varphi/2) = 3.85$, $c = 12$ kPa, $H = c\ \cot\varphi = 16.5$, say 17 kPa from which $\sigma_{1R} = (300 + 17) \times 3.85 - 17 =$ 1 203, say 1 200 kPa.

Answer
$\sigma_{1R} = 1\ 200$ kPa.

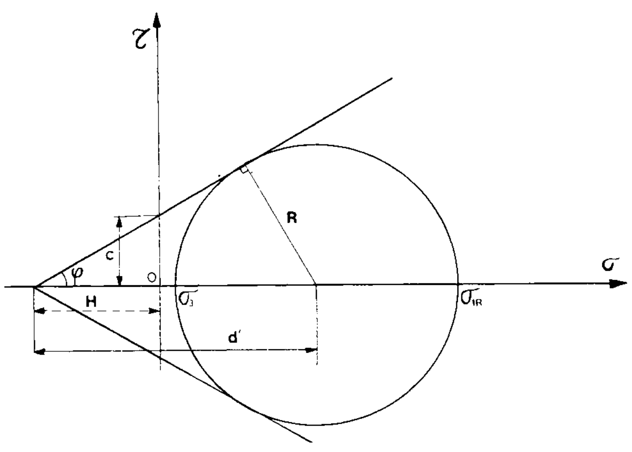


Fig. 4.4.

★*Problem 4.3* Evaluation of c and φ from a triaxial test result

Two triaxial tests are performed on a cohesive soil. In the first test, the confining pressure is 200 kPa and failure occurs when an increase of 600 kPa of the vertical stress is applied.

In the second test, the confining pressure is 300 kPa and failure occurs when an increase of 800 kPa is applied (see Fig. 4.5). What are the values of c and φ for this soil?

Solution

Each of the two tests provides a Mohr's circle tangent to the failure envelope. In the preceding example, the equation shown below was developed, which is:

$$\sigma_{1R} = (\sigma_3 + H)\tan^2(\pi/4 + \varphi/2) - H.$$

Let: $K = \tan^2(\pi/4 + \varphi/2)$.
For the first test: $\sigma_3 = 200$ kPa and $\sigma_{1R} = 200 + 600 = 800$ kPa.
For the second test: $\sigma_3 = 300$ kPa and $\sigma_{1R} = 300 + 800 = 1100$ kPa.

The following relations therefore exist:

$$800 - 200K = H(K-1) \quad (1)$$

$$1100 - 300K = H(K-1) \quad (2)$$

from which $K = 3\tan^2(\pi/4 + \varphi/2) = 3$.

Therefore $\pi/4 + \varphi/2 = 60°$, $\varphi = 30°$.
Equation (1) gives: $H = 800 - 200K/(K-1) = 100$ or $c \cot\varphi = 100$ from which: $c = 100 \tan\varphi = 100 \tan 30° = 100\sqrt{3}/3 = 57.7$ kPa, say 58 kPa.

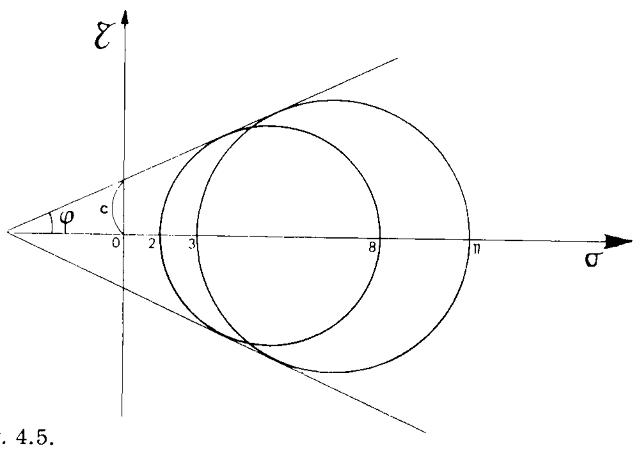


Fig. 4.5.

Summary of answers

$c = 58$ kPa, $\varphi = 30°$.

Remark

Answers can also be obtained graphically from the diagram of Fig. 4.5 showing the two Mohr's circles tangent to the failure envelope.

Problem 4.4. **Triaxial compression tests performed under different draining conditions; Mohr's circles and failure envelopes**

A 40 mm diameter saturated sample is 80 mm high and weighs 1.85 N. It is tested in a triaxial compession test. Specific gravity of the soil constituants is 2.75.

(a) The sample is subjected to a confining pressure of 85 kPa without allowing any drainage to occur. The pore-water pressure is measured at 69 kPa. What was the value of the initial pore-water pressure u_0 *after the sample was removed from the borehole but before it was subjected to the confining pressure in the laboratory? Why is the initial pore pressure,* u_0 *not equal to zero?*

(b) The sample is now allowed to drain until the pore-water pressure is zero, indicating that the sample has been fully consolidated. If the volume of the sample has decreased by 10%, what is the compression index C_c *of the clay?*

(c) The same sample, after consolidation in the triaxial cell, is submitted to an undrained compression test, the confining pressure being 85 kPa. At failure, the pore pressure is 35 kPa and the load on the sample is 123 N (applied to the piston). The height of the resulting sample is 3.6 mm shorter. Assuming that during the test the sample retained the shape of a right cylinder, calculate the deviator stress $(\sigma_1 - \sigma_3)$ *and from the calculated value, the principal effective stresses* σ'_1 *and* σ'_3*.*

(d) Another identical sample is tested in a drained condition with a confining pressure of 85 kPa. At failure, the major effective principal stress σ'_1 *is 255 kPa. Draw the Mohr's circles for the two tests at failure with effective stresses and draw the failure envelope. What are the* c' *and* φ' *values?*

Solution

(a) During sample recovery, a stress relief occurs since the soil sample is no longer subjected to the overburden pressure. Assuming that the laboratory tests are performed a short time after sample recovery, the water losses are negligible. Since water is incompressible with respect to the soil structure, the observed volume change will be nil. Furthermore, the effective stresses remain constant since they are linked to the deformation of soil structure, therefore to volume change.

Under isotropic stress conditions in the triaxial device we have thus $\sigma'_3 = \sigma_3 - u$. At atmospheric pressure, just before the test, $(\sigma'_3)_0 = (\sigma_3)_0 - u_0$. Since atmospheric conditions correspond to the zero total stress condition: $(\sigma_3)_0 = 0$, therefore: $(\sigma'_3)_0 = -u_0$.

The effective stresses are constant, so $(\sigma'_3)_0 = \sigma'_3$ and: $-u_0 = \sigma_3 - u$

$$u_0 = -(\sigma_3 - u) = -(85 - 69) = -16\,\text{kPa}$$

$$u_0 = -16\,\text{kPa}$$

(b) First, let us calculate the wet unit weight of the clay sample. The weight is $P = 1.85\,\text{N}$, its volume is $V = \pi R^2 h = \pi \times \overline{2}^2 \times 8\,\text{cm}^3 \simeq 100.5\,\text{cm}^3$. The unit weight is $P/V = 1.85/100 = 0.0185\,\text{N/cm}^3 = 18.4\,\text{kN/m}^3$.

From the relations between γ_h, γ_s, γ_w and e:

$$(1 + e)\gamma_h = \gamma_s + e\gamma_w$$

$$e = (\gamma_s - \gamma_h)/(\gamma_h - \gamma_w). \qquad \gamma_w = 10\,\text{kN/m}^3.$$

Therefore: $e = (27.5 - 18.4)/(18.4 - 10) \simeq 1.08$.

This is the initial void ratio corresponding to an effective initial stress due to overburden pressure of $\sigma_0 = 16\,\text{kPa}$. After applying the confining pressure of $85\,\text{kN/m}^2$ until the pore pressure is zero, the effective stress is equal to $\sigma' = 85\,\text{kN/m}^2$.

If V is the total volume of the sample and if V_g is the volume of soil particles assumed to be constant (soil grains are assumed incompressible), it is possible to write:

initial state: $V = V_g(1 + e)$,

final state: $V + \Delta V = V_g(1 + e + \Delta e)$

from which: $\Delta V = V_g \Delta e$, or: $\Delta V = V(1 + e)\Delta e$.

$\Delta V/V = \Delta e/(1 + e)$ and $|\Delta e| = 0.10 \times (1 + 1.08) \simeq 0.21$
So $\Delta e = -0.21$.

Stress paths followed during an isotropic triaxial test and that of an oedometer test are very similar. It can, therefore, be assumed that volume changes during the test will be controlled by the same equation:

$$C_c = \frac{-\Delta e}{\Delta \log \sigma'} = \frac{-\Delta e}{\log(\sigma'/\sigma'_0)}. \qquad \text{So: } C_c = \frac{0.21}{\log(85/16)} = 0.290$$

Remark.

The compression index, as defined by an oedometer test was evaluated in this example from a triaxial test. Since the stress paths for the two tests are not identical (only similar) it may be that the calculated value of C_c may vary from that obtained from an oedometer test.

(c) At each point during compression, the deviator stress $\sigma_1 - \sigma_3$ is given by: $\sigma_1 - \sigma_3 = F/S$ in which S is the actual cross-sectional area of the sample. If the sample remained a right cylinder during testing, the following relationship is true since no volume changes occur in an undrained test: $H_o S_o = H_f S_f$, where subscripts o and f represent initial and final values, respectively, from which:

$$S_f = \frac{40}{40 - 3.6} \times \frac{\pi \times 4^2}{4} = 13.81\,\text{cm}^2$$

At failure we have: $(\sigma_1 - \sigma_3)_f = (123 \times 10^{-3})/(13.81 \times 10^{-4}) \simeq 89.1$ kPa.
The minor effective principal stress is:

$\sigma_3' = \sigma_3 - u = 85 - 35 = 50$ kPa. But $\sigma_2' - \sigma_3' = \sigma_1 - \sigma_3$.

Then the major effective principal stress is:

$\sigma_1' = 89.1 + 50 = 139.1$ kPa

Summary

$\sigma_1' = 139.1$ kPa; $\sigma_3' = 50$ kPa.

(d) In the drained condition, $u = 0$ and we have: $\sigma_3' = \sigma_3 = 85\ \text{kN/m}^2$, $\sigma_1' = 255\ \text{kN/m}^2$.

With this set of values and that obtained in the preceding paragraph it is possible to draw the Mohr's circle for the two samples (see Fig. 4.6). The failure enevelope of these circles passes through the origin. The mechanical properties of the soil, in terms of effective stresses, are: cohesion $c' = 0$, angle of internal friction $\varphi = 30°$.

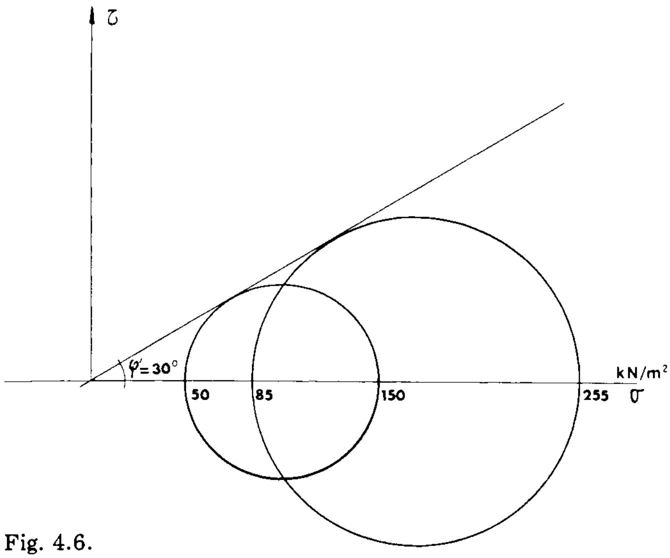


Fig. 4.6.

★★*Problem 4.5* Shear strength of a clay; effective stresses

A clay layer is 20 ft thick and covered by a sandy gravel layer whose porosity $n = 0.30$ and which is 40 ft thick. The water table is at 13 ft below ground surface (see Fig. 4.7). Triaxial tests are performed on the clay samples. These tests are undrained with pore-water pressure measurements. The following results were obtained: cohesion $c' = 2.9\ lb/in^2$, angle of internal friction $\varphi' = 24°$.

The dry unit weight of the sandy gravel is $103\ lb/ft^3$ and the saturated unit weight of the clay is $112\ lb/ft^3$. Find: (1) the soil shear strength at mid-

height of the clay layer, and (2) the effective and total stresses acting on a vertical face of a soil element at mid-height of the clay layer.

Solution

(1) At mid-height in the clay layer, the vertical total stress is (see Fig. 4.7): $\sigma_1 = d\gamma_d + (H_1 - d)\gamma^G_{sat} + (H_2/2)\gamma^A_{sat}$.

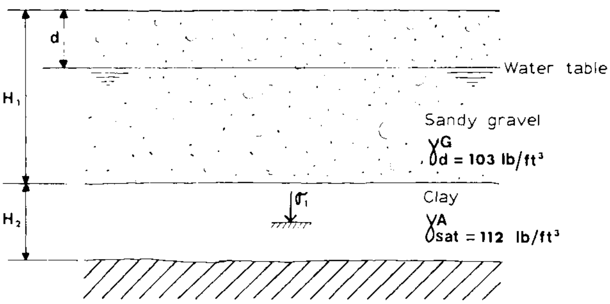


Fig. 4.7.

The saturated unit weight of the sandy gravel is:

$\gamma^G_{sat} = \gamma^G_d + n\gamma_w$, $\gamma_w = 10\,\text{kN/m}^3 = 62.5\,\text{lb/ft}^3$,

$\gamma^G_{sat} = 103 + 0.30 \times 62.5 = 121.75\,\text{lb/ft}^3$,

from which:

$\sigma_1 = 13 \times 103 + (40 - 13) \times 121.75 + 10 \times 112 = 5746.25\,\text{lb/ft}^3 = 38\,\text{lb/in}^2$.

The corresponding effective stress is:

$$\sigma'_1 = \sigma_1 - u = \sigma_1 - \gamma_w(H_2/2 + H_1 - d) = 5746.25 - 62.5\,(10 + 27)$$
$$= 3433.75\,\text{lb/ft}^2 = 24\,\text{lb/in}^2.$$

The shear strength in the middle of the clay layer (Fig. 4.8) will be:

$s = c' + \sigma'_1 \tan\varphi = 2.9 + 24 \tan 24° = 2.9 + 24 \times 0.445 = 13.58\,\text{lb/in}^2$

So $s \simeq 13.6\,\text{lb/in}^2$.

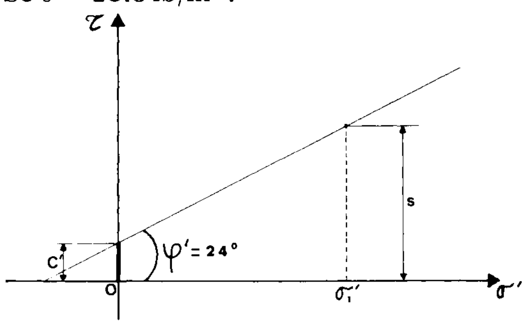


Fig. 4.8.

(2) Since the clay is saturated, the coefficient of earth pressure at rest, K_0, is 0.50. The effective stress acting on a vertical face of a soil element at mid-height in the clay will be: $\sigma_3' = K_0\sigma_1' = 0.50 \times 24 = 12\,\text{lb/in}^2$.

The total stress corresponding to this value is:

$$\sigma_3 = \sigma_3' + u = 12 + \frac{62.5 \times 37}{144} \simeq 28\,\text{lb/in}^2\,(*).$$

Summary of solutions

$s = 13.6\,\text{lb/in}^2$ or 94 kPa, $\sigma_3' = 12\,\text{lb/in}^2$ or 83 kPa, $\sigma_3 = 28\,\text{lb/in}^2$ or 193.5 kPa.

★★★★*Problem 4.6* Interpretation of various types of triaxial tests (drained, undrained, consolidated, unconsolidated)

Several triaxial tests have been performed on identical clay samples. The variation in test procedures for samples X and Y only varied by one factor.

In each of the following tests, draw the Mohr's circles and failure envelopes for both total and effective stresses.

For each of the conditions below, which sample was subjected to the highest value of shear stress?

(1) Consolidated drained test (C.D. test): consolidation pressure σ_3; for sample X, rate of strain is 1 mm/min; for sample Y, rate of strain is 0.005 mm/min.

(2) Consolidated drained test (C.D. test): consolidation pressure: $\sigma_3 =$ 0.3 MPa; sample X was subjected to a pre-consolidation pressure of 0.2 MPa (rate of strain = 0.005 mm/min.); sample Y was subjected to a pre-consolidation pressure of 0.4 MPa (rate of strain = 0.005 mm/min).

(3) For sample Y the test was a consolidated drained (C.D.) test, but for sample X the test was a consolidated, then undrained (C.U.) test. The consolidation pressure σ_3 is the same for both samples.

(4) Both samples are overconsolidated to 0.6 MPa: sample X was drained and consolidated to 0.2 MPa; sample Y was undrained and consolidated to a pressure σ_3 such that the effective stress σ_3' at failure is equal to $\sigma_3' =$ 0.2 MPa.

(5) Consolidated undrained test (C.U. test): the consolidation stress was σ_3', sample Y was not remolded, sample X was remolded and recompacted to the same density as that of Y. The clay was a sensitive clay.

Solution

For all types of triaxial tests of certain states of stresses, the Mohr's circle for effective stresses is obtained from that for total stresses by simply

*Note: $1\,\text{lb/in}^2 = 144\,\text{lb/ft}^2$, $1\,\text{lb/ft}^2 = 48\,\text{Pa}$, $1\,\text{lb/in}^2 = 6.897\,\text{Pa}$.

translating the circle by a value of u (pore-water pressure). If pore-water pressure is zero in all locations in the sample (drained test condition) the circles for both effective and total stresses are superimposed. We also know the maximum shear stress (shear strength) that can be applied to a soil, corresponds to the radius of the Mohr's circle at failure.

(1) In the first condition, a rate of strain of 1 mm/min is considered too fast to represent a drained test condition. Because clays have a very low coefficient of permeability, pore-water pressure would increase during this test. Let that pressure be u at failure at mid-height of the sample. The test on sample X can be assumed to represent drained condition. The higher shear stress will be applied to sample Y because its effective confining pressure will be higher than that of sample X (see Fig. 4.9).

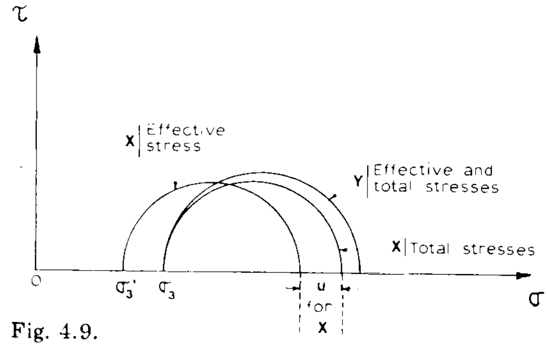


Fig. 4.9.

Then the diameter of the Mohr's circle, representing the stress condition at failure, for sample Y will be larger than that of sample X (see Fig. 4.9).

(2) Since sample Y was subjected to a pre-consolidation pressure greater than that of sample X, its shear strength is greater (see Fig. 4.10).

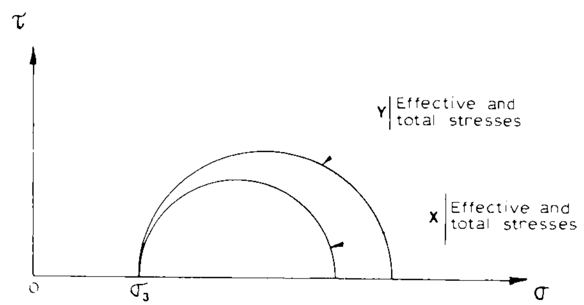


Fig. 4.10.

(3) This is the same test condition as in case (1). The solution is the same.

(4) We agree with Wroth, Roscoe and Schofield [Ref. 33] who have stated that the limit state conditions are the same in their e, p (average stresses) and

q (deviator) space, regardless of the type of test. Sample Y is undrained, its void ratio remains constant at e_0 during testing. Since sample X is drained, its void ratio increases during shearing (the clay is overconsolidated) and its final condition is $e_f > e_0$.

In the space diagram p, q, e, for a constant value of p, q decreases with e [Ref. 33]. The radius of Mohr's circle at failure will therefore be smaller for sample X than for sample Y (see Fig. 4.11).

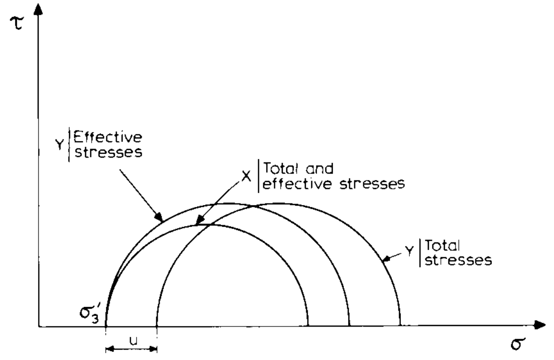


Fig. 4.11.

(5) If the clay is sensitive, its shear strength decreases immediately after it has been remolded. The maximum shear stress for sample Y will be higher than that for sample X (see Fig. 4.12).

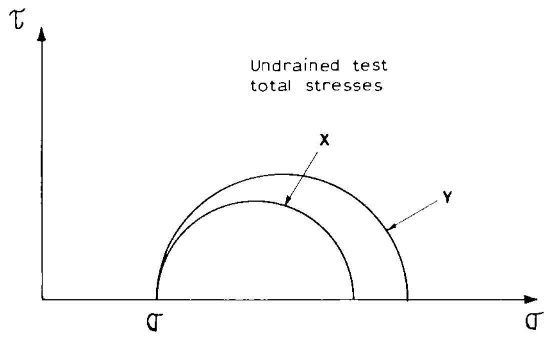


Fig. 4.12.

★★*Problem 4.7* Unconfined compressive strength from a consolidated undrained triaxial test

A consolidated, undrained (C. U.) triaxial test is performed on a clay sample of low plasticity whose dry unit weight is $\gamma_d = 1.7 \times 10^4$ N/m^3. The sample was recovered from a depth of 8.15 m. The porosity of the clay is

$n = 0.35$. The triaxial test results yielded the following values: $\varphi_{cu} = 14°$, $c_{cu} = 4 \times 10^4$ Pa.

Determine the unconfined compressive strength of a saturated sample of this clay from the same depth (assume that 100% saturation was maintained after sample recovery and during handling).

Solution

The unconfined compression shear strength of the saturated clay sample is given by $R_c = 2c_u$. It is necessary to determine c_u from the parameters φ_{cu} and c_{cu}. Let us draw Mohr's circle for total stress conditions (see Fig. 4.13) of the consolidated undrained test. The consolidation pressure σ_0 is the same during the test. At the end of consolidation, $u = 0$ since the drainage occurs then. So $\sigma_0 = \sigma_3$ is one of the effective stresses at the start of the test. At the end of the test, however, since after consolidation of the sample no more drainage was allowed, $u \neq 0$ and $\sigma_3 = \sigma_0$ is a total stress.

It will be noticed that line $O'T$ (see Fig. 4.13) is not a failure envelope during testing, because then two different phases exist, namely a liquid and a solid one.

Let r be the radius of Mohr's circle, then:

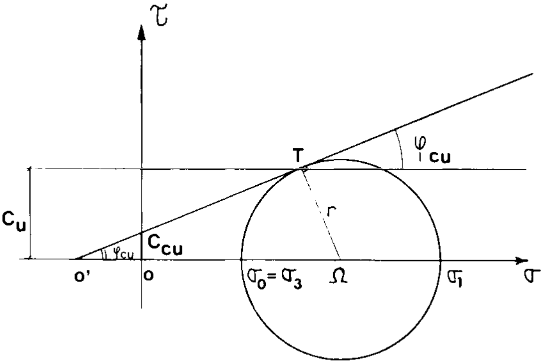


Fig. 4.13.

$$c_u = (\sigma_1 - \sigma_0)/2 = r \quad \text{and} \quad \Omega T/O'\Omega = \sin\varphi_{cu},$$

therefore: $$\frac{r}{\sigma_0 + r + c_{cu}\cot\varphi_{cu}} = \sin\varphi_{cu}$$

or: $c_u = (\sigma_0 + c_u)\sin\varphi_{cu} + c_{cu}\cos\varphi_{cu}$

and thus: $c_u(1 - \sin\varphi_{cu}) = \sigma_0 \sin\varphi_{cu} + c_{cu}\cos\varphi_{cu}$.

and finally: $$c_u = \sigma_0 \frac{\sin\varphi_{cu}}{1-\sin\varphi_{cu}} + c_{cu}\frac{\cos\varphi_{cu}}{1-\sin\varphi_{cu}}$$

The consolidation pressure σ_0 corresponding to the effective overburden pressure σ'_v at depth H of the sample has yet to be determined. It is found as follows:

$$\sigma_0 = \sigma'_v = \gamma' H; \quad \gamma' = \gamma_{sat} - \gamma_w$$

$$\gamma_{sat} = \gamma_d + n\gamma_w = 1.7 + 0.35 \times 1 \times 10^4 = 2.05 \times 10^4 \text{ N/m}^3$$

$$\sigma'_v = 1.05 \times 10^4 \times 8.15 = 8.56 \times 10^4 \text{ Pa.}$$

$$\text{from which: } c_u = 8.56 \times 10^4 \frac{\sin 14^\circ}{1 - \sin 14^\circ} + 4 \times 10^4 \frac{\cos 14^\circ}{1 - \sin 14^\circ}$$

$$= 7.85 \times 10^4 \text{ Pa}$$

or: $R_c \simeq 1.57 \times 10^5$ Pa.

It can be noted that the soil is a stiff clay.

★★*Problem 4.8* Relation between Young's modulus, oedometric modulus and Poisson's ratio

(1) Consider a linearly elastic soil. Find the change in volume of this material when subjected to isotropic stress loading. What can be said about the Poisson's ratio of this soil. For what condition is $\nu = 0.5$?

(2) In an oedometric test, assume that for an increase in stress $\Delta\sigma_v$ vertically applied to the sample, the soil shows a linear, elastic behavior. Determine then the relation between the oedometric modulus E', Young's modulus E and the Poisson ratio ν. What is the relation for $\nu = 0.33$ (usual assumption made for soils in general).

Solution

(1) Let σ_i be the isotropic stress applied to the soil with Young's modulus E and Poisson's ratio ν. Adopting the sign convention usually adopted in soil mechanics, linear elasticity is expressed by:

$$\epsilon_i = -\frac{1}{E}(\sigma_i - 2\nu\sigma_i) = -\frac{\sigma_i(1 - 2\nu)}{E}.$$

The volume change is equal to the first invariant of the strain tensor:

$$\frac{\Delta V}{V} = 3\epsilon_i = -\frac{3\sigma_i(1 - 2\nu)}{E}.$$

If the isotropic stress is compression ($\sigma_i > 0$) the volume change corresponds to a volume decrease $\Delta V/V < 0$. Therefore, the following condition must exist:

$$1 - 2\nu > 0 \quad \text{or:} \quad \nu < 1/2.$$

The case when $\nu = 0.5$ corresponds to $\Delta V/V = 0$, that is to say, to a material that is incompressible.

(2) The oedometric modulus is defined by:

$$E' = -\Delta\sigma_v/(\Delta h/h) \quad \text{or} \quad \Delta\sigma_v = -E'\epsilon_v. \qquad (1)$$

The symbol $\Delta\sigma_v$ means that the linear elastic behavior is only valid for a small range of stress change. In the oedometer, the vertical load corresponds to a major principal stress and axisymmetry implies $\sigma_2 = \sigma_3$. Therefore, (1) we can write:

$$\Delta\sigma_1 = -E'\epsilon_1. \tag{2}$$

Since the elastic deformations of the metal mold holding the sample may be considered negligible compared with those of the soil, we can formulate: $\epsilon_2 = \epsilon_3 = 0$. Linear elasticity condition then means that:

$$\epsilon_1 = -\frac{1}{E}[\Delta\sigma_1 - \nu(\Delta\sigma_2 + \Delta\sigma_3)] = -\frac{1}{E}[\Delta\sigma_1 - 2\nu\Delta\sigma_3] \tag{3}$$

$$\epsilon_2 = \epsilon_3 = -\frac{1}{E}[\Delta\sigma_3 - \nu(\Delta\sigma_1 + \Delta\sigma_3)] = 0 \tag{4}$$

From eqn. (4), we have: $\sigma_3 - \nu(\Delta\sigma_1 + \Delta\sigma_3) = 0$ or $\Delta\sigma_3 = \Delta\sigma_1 \dfrac{\nu}{1-\nu}$ ($\nu \neq 1$ from eqn. 1).

By replacing the value of $\Delta\sigma_3$ in eqn. 3, we obtain:

$$\epsilon_1 = -\frac{\Delta\sigma_1}{E}\left[1 - \frac{2\nu^2}{1-\nu}\right] \tag{5}$$

and by comparing eqn. (2) and (5), we have:

$$E = E'\left[1 - \frac{2\nu^2}{1-\nu}\right]$$

In the particular instance of $\nu = 0.33$ or $1/3$, we have $E = 2/3E'$.

The soil appears to be less compressible than it is in reality.

★★*Problem 4.9* Evaluation of Poisson's ratio from triaxial test

A soil sample is tested in a triaxial compression test at small increments of principal stresses: $\Delta\sigma_1$ and $\Delta\sigma_2 = \Delta\sigma_3$.

Assume that the soil behaves in a linearly elastic manner. Axial strain ($\Delta h/h = \epsilon_1$) and volumetric changes $\Delta V/V$, corresponding to the drainage of pore water during testing, are measured. (see drained test set up Fig. 4.14).

Derive the formula for Poisson's ratio of this soil under the test conditions. For a numerical application, consider the soil to be a loose sand in one case and a normally consolidated clay in another. The test results for both are presented in the graphs of Fig. 4.15.

Solution

In the triaxial cell axisymmetry implies, $\Delta\sigma_2 = \Delta\sigma_3$ and from the elasticity

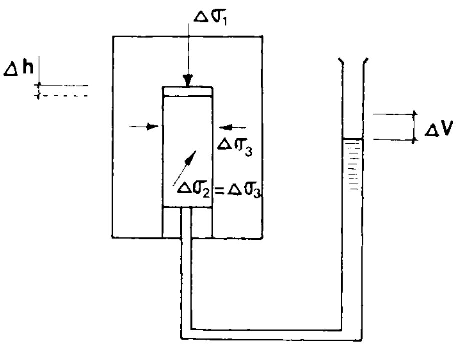


Fig. 4.14.

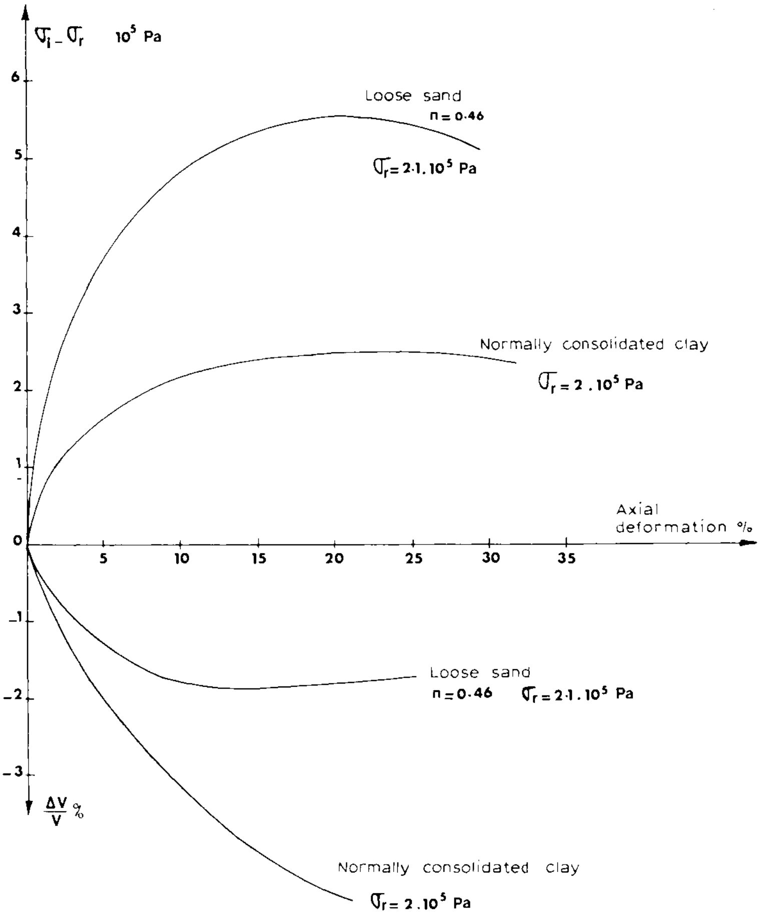


Fig. 4.15. Drained triaxial test results on saturated samples.

equations, we have:

$$\epsilon_1 = -\frac{1}{E}(\Delta\sigma_1 - 2\nu\Delta\sigma_3) \tag{1}$$

$$\epsilon_2 = \epsilon_3 = -\frac{1}{E}[(\Delta\sigma_3 - \nu(\Delta\sigma_1 + \Delta\sigma_3)]. \tag{2}$$

The volumetric strain is equal to the first invariant of the strain tensor from which: $\Delta V/V = \epsilon_1 + \epsilon_2 + \epsilon_3 = \epsilon_1 + 2\epsilon_3$

$$\frac{\Delta V}{V} = -\frac{1}{E}[\Delta\sigma_1 + 2\Delta\sigma_3\,(1 - 2\nu) \tag{3}$$

Solving $1/E$ in eqn. (1) and placing it in eqn. (3) gives:

$$\frac{\Delta V}{V} = \frac{\Delta h}{h}\,\frac{(\Delta\sigma_1 + 2\Delta\sigma_3)(1 - 2\nu)}{\Delta\sigma_1 - 2\nu\Delta\sigma_3}$$

from which:

$$\nu = \frac{1}{2} \times \frac{\Delta\sigma_1(\Delta V/V) - \epsilon_1(\Delta\sigma_1 + 2\Delta\sigma_3)}{\Delta\sigma_3(\Delta V/V) - \epsilon_1(\Delta\sigma_1 + 2\Delta\sigma_3)}.$$

Numerical application

For this loose sand, $n = 0.46$. For small strain increments $|\epsilon_1| = 1\%$, $\epsilon_1 = -1\%$ (shortened)

$\sigma_r (= \sigma_2 = \sigma_3) = 2.1 \times 10^5$ Pa

$\sigma_1 - \sigma_r \simeq 1.6 \times 10^5$ Pa; $\Delta V/V = -0.45\%$

from which: $\Delta\sigma_1 = \sigma_1 - \sigma_r = 1.6 \times 10^5$ Pa, $\Delta\sigma_3 = 0$ and:

$$\nu = \frac{1}{2} \times \frac{-0.0045 + 0.01}{0.01} = 0.275 \simeq 0.28.$$

For normally consolidated clay:

$$\epsilon_1 = -1\%, \quad \sigma_r = 2 \times 10^5 \text{ Pa}, \quad \Delta\sigma_1 = \sigma_1 - \sigma_r = 7 \times 10^4 \text{ Pa},$$

$$\Delta V/V = -0.55\%, \quad \Delta\sigma_3 = 0$$

$$\nu = \frac{1}{2} \times \frac{-0.0055 + 0.01}{0.01} = 0.225 \simeq 0.23.$$

Note. From Fig. 4.15, it is evident that E, just as ν, depends on the state of stress and the stress path.

★★*Problem 4.10* Comparison of the principal stress rotations in a direct shear test and in a triaxial test

Let us assume homogeneous stress and strain conditions. Determine the changes in the Mohr's circle and in the principal stress directions occurring during a direct shear test (Casagrande shear box) and during a triaxial test. What are the conclusions?

Solution

(a) Direct shear test. At the onset of the test, the major principal stress is vertical then $\sigma_1 = \sigma_v$ and the minor principal stress is $\sigma_3 = K_o \sigma_1$ since lateral strains are nil (rigid-sided box). The Mohr's circle corresponding to this state of stress is identified as C_o in Fig. 4.16. At failure, the Mohr's circle C_f is tangent to the failure envelope, the point of tangency then corresponds to the peak of the stress—strain curve. Therefore, at failure, the

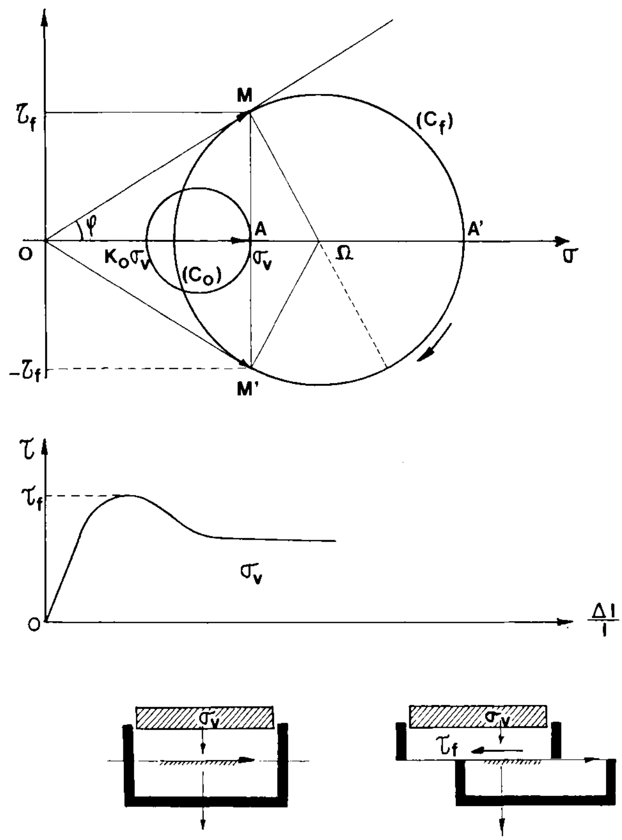


Fig. 4.16. Shear test in Casagrande's shear box

stress vector acting on the horizontal shear plane is **OM** (σ_v, σ_f) whereas initially it was figured by **OA**.

Let Ω be the center of Mohr's circle (C_f). The end of **OM** rotated clockwise at an angle of $\pi/2 + \varphi$ since $(\Omega A', \Omega M') = (\pi/2) + \varphi$. Therefore the face on which the major principal stress **OA**$'$ acts at failure makes an angle of $+ (\pi/4 + \varphi/2)$ with the horizontal.

(b) Triaxial compression test. During the triaxial test, the vertical stress acting on a horizontal plane through the sample center remains the major principal stress throughout. Since the sample is cylindrical, any direction perpendicular to the vertical is also a principal stress direction.

In conclusion then, the shear plane direction in the direct shear device is predetermined by the geometry of the test setup. As a consequence, the orientation of the major principal stresses undergoes a rotation of $\pi/4 + \varphi/2$ during the shearing process. In the case of the triaxial test, however, the orientation of the principal stresses remains the same, the angle of the shear plane is $\alpha = \pi/4 + \varphi/2$ with the horizontal (Fig. 4.17).

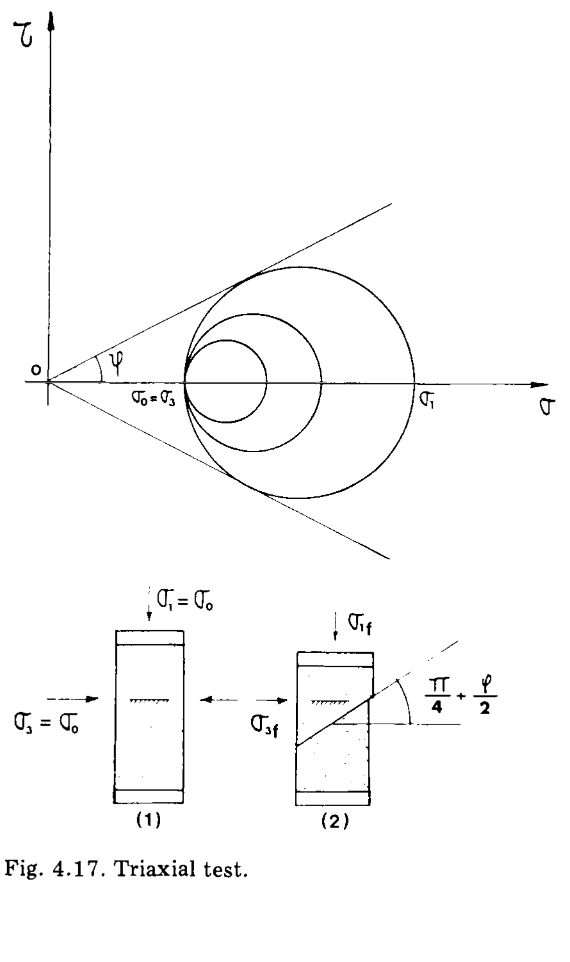


Fig. 4.17. Triaxial test.

★★*Problem 4.11* Triaxial test: determination of the pore-water pressure at failure and of Skempton's A_f coefficient

A mud has the following properties: $\varphi' = 25°$ and $c' = 16$ kPa. A sample of this material is tested in a triaxial compression test (consolidated, undrained). During testing, the pore-water pressure is measured. If the confining pressure is 50 kPa and the deviator stress at failure is 80 kPa, determine the pore-water pressure at the moment of failure.

What is Skempton's A_f coefficient?

Solution

The Mohr's circle for *effective* stress conditions (C') is tangent to the line representing Coulomb's equation, $\tau = c' + \sigma' \tan \varphi'$. Furthermore, from Mohr's circle, the *total* stresses are known by translating the values by $-u$.

From Fig. 4.18, we see:

$$\Omega T / A\Omega = \sin \varphi' \quad \text{or} \quad R(H' + d - u) = \sin \varphi'. \tag{7}$$

If we let d be the abscissa of the center of the total stress circle and R its radius, then we have:

$$R = \frac{\sigma_1 - \sigma_3}{2}, \quad d = \frac{\sigma_1 + \sigma_3}{2} = \frac{1}{2}[(\sigma_1 - \sigma_3) + 2\sigma_3]$$

and, on the other hand, $H' = c' \cot \varphi'$.

From eqn. 1 we have:

$$u = H' + d - \frac{R}{\sin \varphi'} = c' \cot \varphi' + d - \frac{R}{\sin \varphi'}.$$

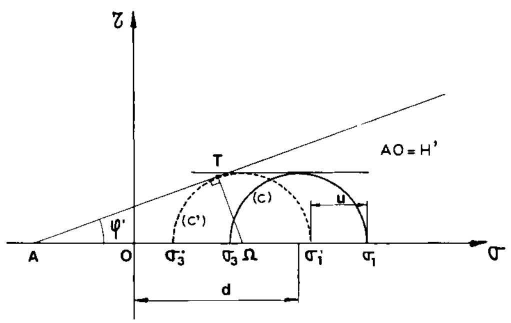


Fig. 4.18. Consolidated, undrained test.

Numerical application

$R = 80/2 = 40$ kPa

$d = 1/2(80 + 2 \times 50) = 90$ kPa

$\sin 25° = 0.423, \quad \cot 25° = 2.145$

from which: $u = 16 \times 2.145 + 90 - 40/0.423 = 29.7$ kPa
or: $u \simeq 30$ kPa.

For a saturated sample, $B = 1$, and Skempton's relation becomes:

$$\Delta u = \Delta\sigma_r + A(\Delta\sigma_1 - \Delta\sigma_r).$$

In the standard triaxial test:

$$\Delta\sigma_r = 0 \quad (\sigma_3 = \text{constant}) \quad \text{and:} \quad A_f = (\Delta u/\Delta\sigma_1)_f.$$

Before starting the compression, the soil sample is consolidated under a confining pressure of 50 kPa (Fig. 4.19).

$$\sigma_3 = \sigma_3' = (\sigma_1)_0 = (\sigma_1')_0 = 50\,\text{kPa}; \quad u = 0.$$

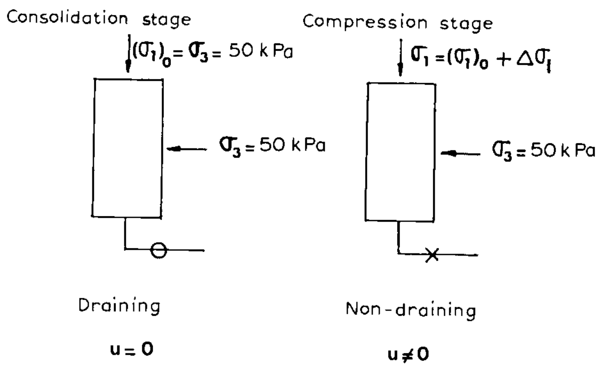


Fig. 4.19.

Then the drainage tube is closed and the compression stage starts by applying a load, whereas $\sigma_3 = 50$ kPa remains constant. Therefore, we have:

$$\Delta u = u_f - 0 = 30\,\text{kPa}$$

$$\Delta\sigma_1 = (\sigma_1)_f - (\sigma_1)_0 = (\sigma_1 - \sigma_3) + \sigma_3 - (\sigma_1)_0 = 80 + 50 - 50 = 80\,\text{kPa}.$$

and finally: $A_f = 30/80 \simeq 0.38$.

Note

This value for A_f corresponds to a saturated clay which is slightly over-consolidated ($0.3 < A_f < 0.7$ after Leonards). This is a reasonable conclusion for a $c' = 16$ kPa (for normally consolidated clay, $c' = 0$).

Summary of answers

$u = 30$ kPa; $A_f = 0.38$.

★★*Problem 4.12* **Stress paths for various test types**

The following tests have been performed:
— consolidation test
— direct shear test (normal stress = σ_N)
— standard C.D. triaxial test ($\sigma_3 = \sigma_2$ = constant)
— isotropic C.D. triaxial test ($\sigma_3 = \sigma_2 = \sigma_1$)
— triaxial constant average stress test.

(1) Draw in σ_1, σ_2 and σ_3 space the stress paths corresponding to these various tests.

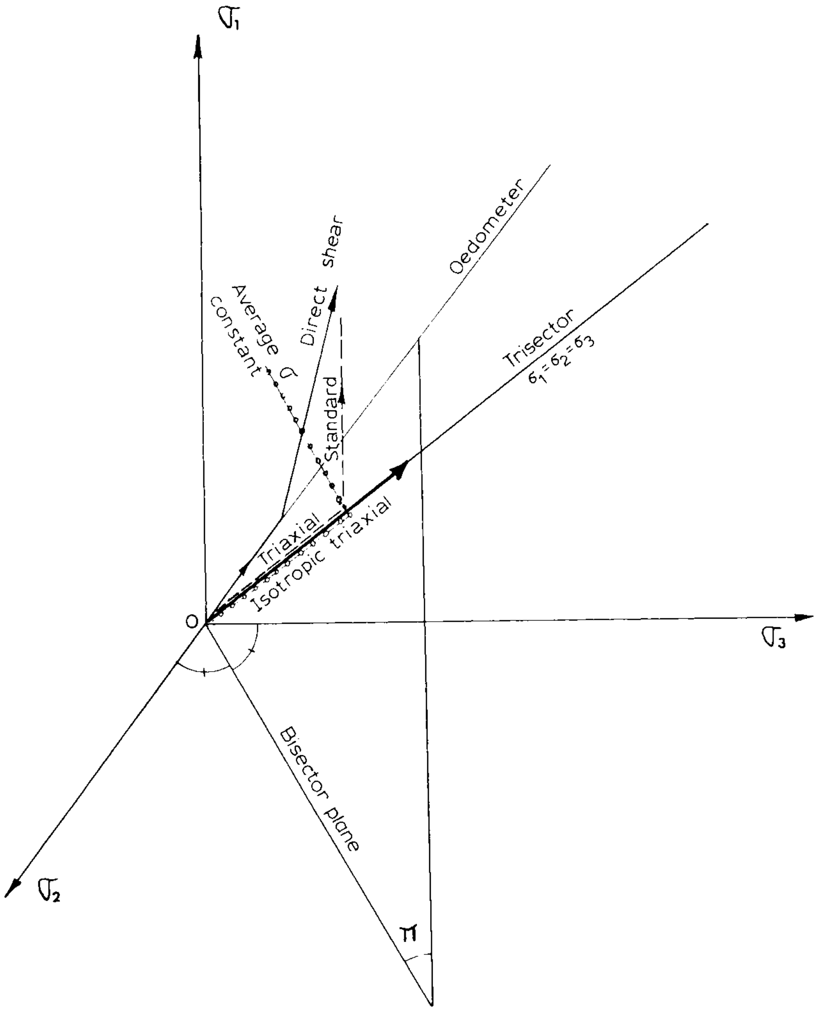


Fig. 4.20. Stress-paths.

(2) Draw in the p, q plane the stress paths of the tests. Let p and q be defined by: $p = 1/3(\sigma_1 + \sigma_2 + \sigma_3)$, $q = \sigma_1 - \sigma_2 = \sigma_1 - \sigma_3$.

(3) Draw on the Mohr-diagram, the circles representing the states of stress at the beginning and end of each test. Determine for each condition, if the direction of the principal stress remains constant during the test.

Solution

(1) Fig. 4.20 gives the answer to part 1, and in Fig. 4.21 each of the test stress paths is shown in the bisector plane π.

(2) In the plane p, q the stress paths are: $p = 1/3(\sigma_1 + \sigma_2 + \sigma_3)$ and $q = \sigma_1 - \sigma_2 = \sigma_1 - \sigma_3$, from which Fig. 4.22 has been drawn.

The only test wherein principal stresses undergo a rotation of orientation during the test is the direct shear test (see Problem 4.10). Figs. 4.23, 4.24

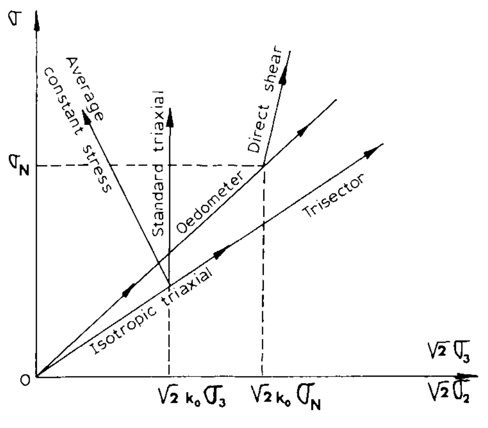


Fig. 4.21. Stress paths in bisector plane π.

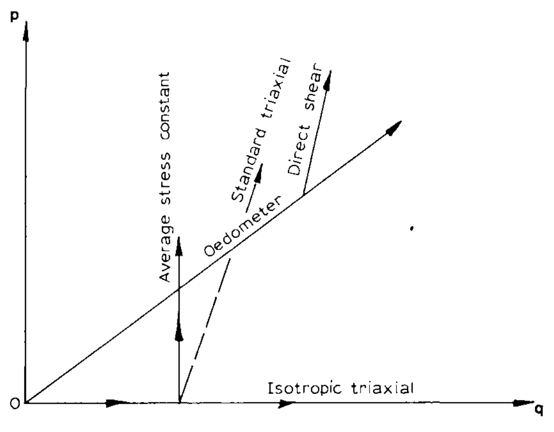


Fig. 4.22. Stress paths in plane p, q.

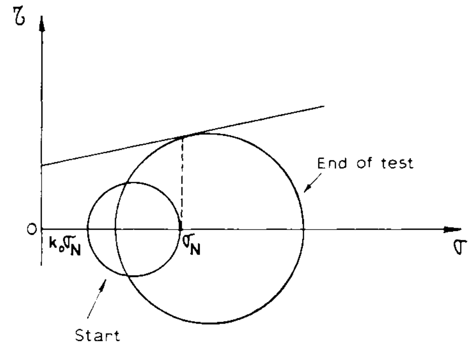


Fig. 4.23. Mohr's circles for the start and the end of the direct shear test.

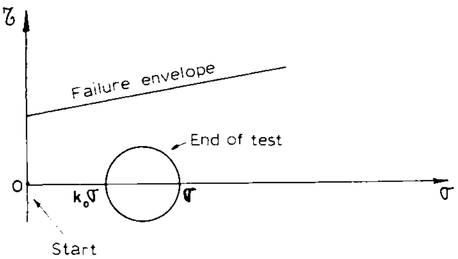
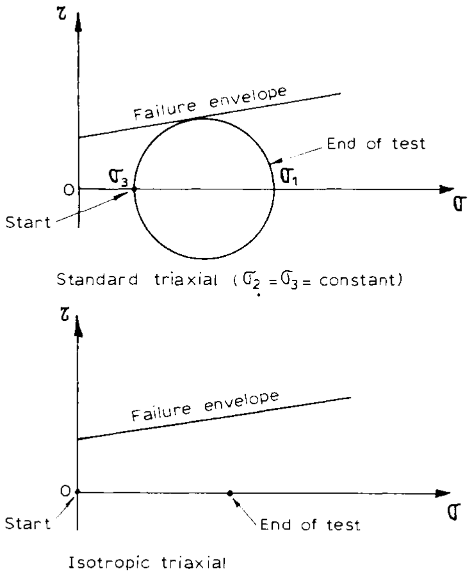


Fig. 4.24. Mohr's circle for the start and end conditions of the consolidation test.

and 4.25 present the Mohr's circles at the start and end of the direct-shear test, the oedometer test and the various triaxial compression tests.



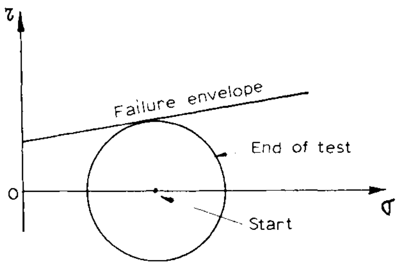


Fig. 4.25. Triaxial constant average stress Mohr's circle at start and end of test.

★★★*Problem 4.13* Spherical tensor, deviatoric tensor and volume change

The following strain tensor is considered for any orthogonal axes system:

$$T = \begin{bmatrix} \epsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \epsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \epsilon_z \end{bmatrix}$$

The average strain is defined by: $\epsilon_m = (\epsilon_x + \epsilon_y + \epsilon_z)/3$.

(1) Show that tensor T is equal to the sum of a spherical tensor

$$S = \begin{bmatrix} \epsilon_m & 0 & 0 \\ 0 & \epsilon_m & 0 \\ 0 & 0 & \epsilon_m \end{bmatrix}$$

and a deviatoric tensor D. Strains may be considered as the superposition of two sets of strains represented by S and D.

(2) Develop the decomposition of T in the principal axes directions and show that D may be subdivided into 3 tensors D_1, D_2 *and* D_3.

(3) Consider an elemental rectangular parallelopiped whose sides are parallel to the principal axes. Formulate an equation for the volume change as a function of invariants of tensor T. As a first approximation, what is the value of the volume change θ*?*

(4) What may be said about the strain represented by S? What is the term for the volume variation corresponding to this deformation? Same question for deviator D.

(5) Apply the above studied decomposition to the stress tensor in the direction of the principal axes $O\sigma_1$, σ_2, σ_3.

Let Δ *be the trisector of the three axes and a state of stress be represented by vector OM. If we let OH be the projection of OM on* Δ *and* **Om** *be its* *projection on plane (II), perpendicular to Δ at 0, analyze the decomposition $OM = OH + Om$.*

Solution

(1) We get immediately:

$$D = \begin{bmatrix} \dfrac{2\epsilon_x - \epsilon_y - \epsilon_z}{3} & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \dfrac{2\epsilon_y - \epsilon_x - \epsilon_z}{3} & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \dfrac{2\epsilon_z - \epsilon_x - \epsilon_y}{3} \end{bmatrix}$$

$$T = S + D$$

(2) For the principal axes:

$$D = \begin{bmatrix} \dfrac{2\epsilon_1 - \epsilon_2 - \epsilon_3}{3} & 0 & 0 \\ 0 & \dfrac{2\epsilon_2 - \epsilon_1 - \epsilon_3}{3} & 0 \\ 0 & 0 & \dfrac{2\epsilon_3 - \epsilon_1 - \epsilon_2}{3} \end{bmatrix}$$

$$S = \begin{bmatrix} \dfrac{\epsilon_1 + \epsilon_2 + \epsilon_3}{3} & 0 & 0 \\ 0 & \dfrac{\epsilon_1 + \epsilon_2 + \epsilon_3}{3} & 0 \\ 0 & 0 & \dfrac{\epsilon_1 + \epsilon_2 + \epsilon_3}{3} \end{bmatrix}$$

Furthermore, it can be stated that: $\quad D = D_1 + D_2 + D_3$

with:

$$D_1 = \begin{bmatrix} \dfrac{\epsilon_1 - \epsilon_2}{3} & 0 & 0 \\ 0 & -\dfrac{\epsilon_1 - \epsilon_2}{3} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \dfrac{\epsilon_2 - \epsilon_3}{3} & 0 \\ 0 & 0 & -\dfrac{\epsilon_2 - \epsilon_3}{3} \end{bmatrix}$$

$$D_3 = \begin{bmatrix} -\dfrac{\epsilon_3 - \epsilon_1}{3} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \dfrac{\epsilon_3 - \epsilon_1}{3} \end{bmatrix}$$

(3) The elemental rectangular parallelopiped has sides ΔX, ΔY and ΔZ which coincide with the principal axes (see Fig. 4.26). After strains occur, the volume remains rectangular. Then, its sides have the following dimensions: $\Delta X(1 + \epsilon_1)$, $\Delta Y(1 + \epsilon_2)$, $\Delta Z(1 + \epsilon_3)$ and its volume becomes: $V_2 = \Delta X \Delta Y \Delta Z(1 + \epsilon_1)(1 + \epsilon_2)(1 + \epsilon_3)$.

The relative volume change θ, is:

$$\theta = \frac{\Delta V}{V} = \frac{V_2 - V_1}{V_1} = \frac{\Delta X \Delta Y \Delta Z(1 + \epsilon_1)(1 + \epsilon_2)(1 + \epsilon_3)}{\Delta X \Delta Y \Delta Z} - 1$$

or, by simplifying the equation:

$$\theta = \epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_1\epsilon_2 + \epsilon_2\epsilon_3 + \epsilon_3\epsilon_1 + \epsilon_1\epsilon_2\epsilon_3$$

the quantities $I_1 = \epsilon_1 + \epsilon_2 + \epsilon_3$, $I_2 = \epsilon_1\epsilon_2 + \epsilon_2\epsilon_3 + \epsilon_3\epsilon_1$, $I_3 = \epsilon_1\epsilon_2\epsilon_3$ are the invariants of the strain tensor.

As a first-order approximation, we have:

$$\theta = \Delta V/V \simeq I_1 = \epsilon_1 + \epsilon_2 + \epsilon_3 = \epsilon_x + \epsilon_y + \epsilon_z.$$

(4) From the above results, it can be seen that the deformation represented by the tensor:

$$S = \begin{bmatrix} \epsilon_m & 0 & 0 \\ 0 & \epsilon_m & 0 \\ 0 & 0 & \epsilon_m \end{bmatrix}$$

corresponds to a volume change of $\Delta V/V = 3\epsilon_m = \epsilon_x + \epsilon_y + \epsilon_z$, which is the volume change expression represented by tensor T. It is characterized by three equal axial strains and no shear strains. The Lamé quadratic is a sphere and is the reason why the tensor S is called spherical tensor.

The deviator **D** corresponds to a deformation where shear strains are

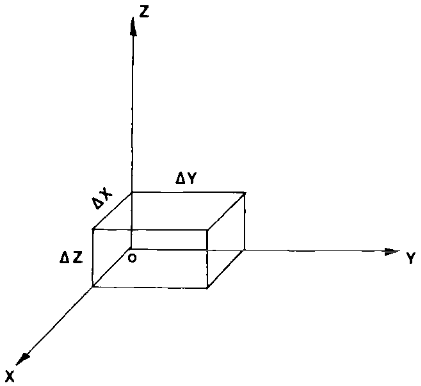


Fig. 4.26. Elementary parallelopiped.

different from zero but where the volume remains constant, since:

$$\theta = I_1 = \frac{2\epsilon_x - \epsilon_y - \epsilon_z}{3} + \frac{2\epsilon_y - \epsilon_z - \epsilon_x}{3} + \frac{2\epsilon_z - \epsilon_x - \epsilon_y}{3} = 0.$$

The deviatoric tensor corresponds to a change of the shape of the sample without volume change.

(5) The same applies to the stress tensors:

$$\sigma = \begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix} + \begin{bmatrix} \frac{2\sigma_1 - \sigma_2 - \sigma_3}{3} & 0 & 0 \\ 0 & \frac{2\sigma_2 - \sigma_3 - \sigma_1}{3} & 0 \\ 0 & 0 & \frac{2\sigma_3 - \sigma_1 - \sigma_2}{3} \end{bmatrix}$$

or: $\sigma = \mathbf{s} + \mathbf{d}$.

Tensor s corresponds to the spherical state of stress (or isotropic, sometimes called hydrostatic state because it corresponds to the stress at any point in a liquid). Tensor **d** corresponds to the state of deviatoric stresses.

In the principal axes of stress $O\sigma_1, \sigma_2, \sigma_3$ (see Fig. 4.27) the trisector of the axes (Δ) has a unit vector $\mathbf{k}'$ $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$. Let $M(\sigma_1, \sigma_2, \sigma_3)$ be the point representing a random stress condition:

$$\mathbf{OM} = \mathbf{OH} + \mathbf{HM} = \mathbf{OH} + \mathbf{Om}$$

$$|\mathbf{OH}| = \mathbf{OM} \cdot \mathbf{k}' = 1/\sqrt{3}(\sigma_1 + \sigma_2 + \sigma_3).$$

or: $\sigma_1 + \sigma_2 + \sigma_3 = 3OH^2$.

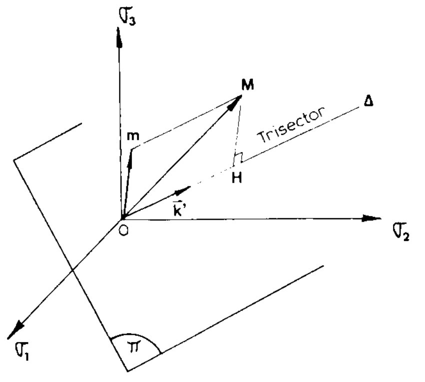


Fig. 4.27. Representation of a random state of stress.

If we let:

$$\mathrm{OH}\begin{cases} u \\ u \\ u \end{cases} \quad \text{then:} \quad (\sigma_1 + \sigma_2 + \sigma_3)^2 = 3 \times 3u^2 = 9u^2$$

from which: $u = (\sigma_1 + \sigma_2 + \sigma_3)/3 = \sigma_m$.

Point m represents the state of stress $(\sigma_1', \sigma_2', \sigma_3')$ so that $(\sigma_1' + \sigma_2' + \sigma_3')/3 = 0$ since the projection of m onto (Δ) is at O. Point m then represents a deviator. This, once again, expresses the geometric interpretation of the analyses found above. Point H (or vector **OH**) represents the isotropic (or spherical) tensors.

Point m (or vector **Om**) represents the deviator.

★★★*Problem 4.14* Determination of Henkel's coefficients

On the principal axes $O\sigma_1$, σ_2, σ_3, consider the decomposition of the stress tensor and its geometric representation of problem 4.12. It is assumed that the stresses vary from M $(\sigma_1, \sigma_2, \sigma_3)$ to N $(\sigma_1 + \Delta\sigma_1, \sigma_2 + \Delta\sigma_2, \sigma_3 + \Delta\sigma_3)$.

(1) What is the geometric representation of the stress increase $\Delta\sigma$?

(2) If the soil is assumed to be isotropic and homogeneous, to which quantities can pore-water pressure increment Δu which occurs, be related with?

What relation similar to that of Bishop and Skempton may be written (introduce two coefficients β and α)?

Solution

From the solution of problem 4.13, and as shown on Fig. 4.28, it is seen that $\Delta\sigma(\Delta\sigma_1, \Delta\sigma_2, \Delta\sigma_3)$ of the stress tensor may be represented by the vector MN(MN = $\mathrm{MM_1} + \mathrm{M_1N}$) which may be divided as:

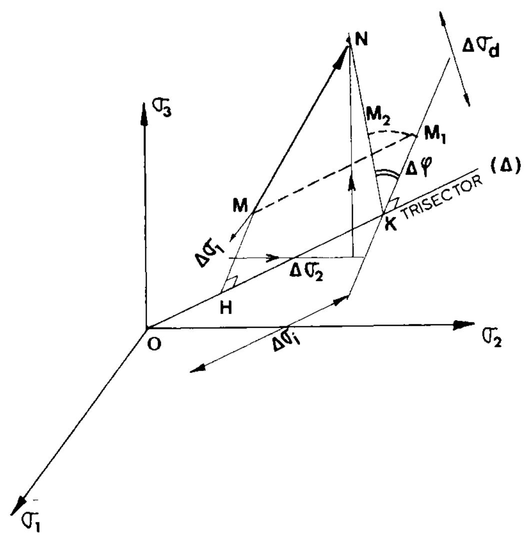


Fig. 4.28.

$\Delta\sigma_i$ = variation of the isotropic part (vector $\mathbf{MM_1}$)

$\Delta\sigma_d$ = variation of the magnitude of the deviator
$\Delta\Phi$ = variation of the direction of the deviator } vector ($\mathbf{MN_1}$).

(2) The soil is assumed to be isotropic and homogeneous. So there are no reasons for $\Delta\Phi$ to affect the variation in pore-water pressure Δu that appears when stress increase $\Delta\sigma$ is applied to the soil. Therefore, Δu is related to $\Delta\sigma_i$ and $\Delta\sigma_d$. By analogy to the formula of Bishop and Skempton, we may write:

$$\Delta u = \beta\Delta\sigma_i + \alpha\Delta\sigma_d, \qquad \text{and} \qquad \Delta\sigma_i = (\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3)/3,$$

we then have: $\beta = B$.

Let us now express $\Delta\sigma_d$:

If we let the increases $\Delta\sigma_{1,2,3}$ be sufficiently small, $\Delta\varphi$ is low and the variation of the deviator magnitude $\Delta\sigma_d$ may be assumed to coincide with M_1N.

The general expression of the deviator magnitude may be derived then as shown below.

For the stress tensor $(\sigma_1, \sigma_2, \sigma_3)$ represented by M, we have:

$$OH = \frac{\sigma_1 + \sigma_2 + \sigma_3}{\sqrt{3}}.$$

The magnitude of the deviator is then:

$$HP^2 = OP^2 - OH^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \frac{(\sigma_1 + \sigma_2 + \sigma_3)^2}{3}$$
$$= \tfrac{2}{3}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$$

but:

$$(\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2$$
$$= 2[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1]$$

from which:

$$HP = \tfrac{1}{3}\sqrt{(\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2}.$$

Let us apply the results to the stress increment vector **MN**:

$$\mathbf{M_1N} = \tfrac{1}{3}\sqrt{(\Delta\sigma_2 - \Delta\sigma_3)^3 + (\Delta\sigma_3 - \Delta\sigma_1)^2 + (\Delta\sigma_1 - \Delta\sigma_2)^2}.$$

if the increments $\Delta\sigma_{1,\,2,\,3}$ are small, then we can write:

$$\Delta u = \beta\Delta\sigma_i + \tfrac{1}{3}\sqrt{(\Delta\sigma_2 - \Delta\sigma_3)^2 + (\Delta\sigma_3 - \Delta\sigma_1)^2 + (\Delta\sigma_1 - \Delta\sigma_2)^2}.$$

This is the Henkel relation, which is a generalized form of that of Bishop and Skempton.

★★★*Problem 4.15* **Coefficients of Henkel. Comparison of two triaxial tests**

A saturated clay is tested in two triaxial compression devices. Both tests are undrained and have the following characteristics:
— in one test, the confining pressure is constant throughout the test and the axial load is increased until failure occurs (standard triaxial test);
— in the other test, the axial stress is kept constant and the confining pressure is increased until failure occurs.

Compare the variations of pore-water pressure occurring during the two types of testing. Use the Henkel coefficients. What is the conclusion?

Solution:

In the first test, we have: $\Delta\sigma_2 = \Delta\sigma_3 = 0, \quad \Delta\sigma_1 = \sigma_L - \sigma_r.$
Henkel's equation (see problem 4.14) gives:

$$\Delta u = \beta\frac{\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3}{3} + \alpha\sqrt{(\Delta\sigma_1 - \Delta\sigma_2)^2 + (\Delta\sigma_2 - \Delta\sigma_3)^2 + (\Delta\sigma_3 - \Delta\sigma_1)^2}$$

from which ($\beta = 1$, for saturated clay):

$$\Delta u = \frac{\sigma_L - \sigma_r}{3} + \alpha\sqrt{2}(\sigma_L - \sigma_r)$$

or:

$$\Delta u = (\sigma_l - \sigma_r)(1/3 + \alpha\sqrt{2}). \qquad (1)$$

In the second test: $\Delta\sigma_1 = \Delta\sigma_2 = \sigma_L - \sigma_r, \quad \Delta\sigma_3 = 0.$

Henkel's equation here gives:

$$\Delta u = \frac{2(\sigma_L - \sigma_r)}{3} + \alpha\sqrt{2}\,(\sigma_L - \sigma_r)$$

or:

$$\Delta u = (\sigma_L - \sigma_r)\,(2/3 + \alpha\sqrt{2}).$$

For the triaxial compression test at constant axial stress the confining pressure increases result in a higher pore-water pressure than in the standard triaxial test.

★★★*Problem 4.16* Influence of loading condition on the behavior of a soil. Bishop and Skempton's coefficients

Consider an elemental volume ΔV of a saturated soil in situ. Assume that the loading conditions are such that the soil is in a plane strain state (usual assumption made in soil mechanics). Within the range of deformations being considered here, the soil structure may be assumed to behave elastically with a Young's modulus of E and Poisson's ratio ν.

(1) Find the relation between the principal stresses σ_1, σ_2, and σ_3 which express the plane-strain condition.

(2) Find the "pseudo" bulk modulus which relates the volume change to the variation of the average stress of σ_1 and σ_3.

(3) What may be concluded about Bishop and Skempton's parameters?

Solution

(1) From elasticity theory, disregarding the second-order terms, we can write:

$$\Delta V/V = \epsilon_1 + \epsilon_2 + \epsilon_3.$$

For plane-strain condition, we have $\epsilon_2 = 0$.

From the generalized Hooke's equation, we have:

$$\epsilon_2 = \sigma_2/E - (\nu/E)(\sigma_1 + \sigma_3).$$

But since the plane-strain condition is that $\epsilon_2 = 0$ we can write:

$$\sigma_2 = \nu(\sigma_1 + \sigma_3) \qquad (1)$$

(2) $\Delta V/V = \epsilon_1 + \epsilon_2 + \epsilon_3 = \epsilon_1 + \epsilon_3$ since $\epsilon_2 = 0$. Furthermore:

$$\begin{cases} \epsilon_1 = (\sigma_1/E) - (\nu/E)(\sigma_2 + \sigma_3) \\ \epsilon_3 = (\sigma_3/E) - (\nu/E)(\sigma_1 + \sigma_2) \end{cases}$$

$$\Delta V/V = (\sigma_1 + \sigma_3)/E - (\nu/E)(2\sigma_2 + \sigma_1 + \sigma_3)$$

from which:

$$\Delta V/V = (2\sigma_m/E) - (\nu/E)[\sigma_1 + 2\nu(\sigma_1 + \sigma_3) + \sigma_3]$$

from eqn. (1) with:

$$\sigma_m = \frac{\sigma_1 + \sigma_3}{2}. \quad \text{Or:}$$

$$\Delta V/V = (2\sigma_m/E) - 2(\nu/E)\sigma_m(1 + 2\nu) = (2\sigma_m/E)[1 - \nu(1 + 2\nu)]$$

$$\Delta V/V = (2\sigma_m/E)(1 + \nu)(1 - 2\nu).$$

So we can write:

$$\sigma_m = K_s'(\Delta V/V) \tag{2}$$

with:

$$K_s' = \frac{E}{2(1 + \nu)(1 - 2\nu)} \qquad \text{“pseudo” bulk modulus}$$

(3) If the soil structure behaves elastically, we have:
— in the triaxial condition (axisymmetry): $\sigma_1 \neq \sigma_2 = \sigma_3$

$$\sigma_m = \sigma_i = (\sigma_1 + \sigma_2 + \sigma_3)/3 \qquad \text{and:}$$

$$K_s = \frac{E}{3(1 - 2\nu)} \quad \text{from which:}$$

$$\Delta u = B\Delta\sigma_i \quad \text{with} \quad B = \frac{1}{1 + n(K_s/K_w)}$$

— in the case of in-situ plane strain: $\sigma_1 \neq \sigma_2 \neq \sigma_3$,

$$\sigma_m = (\sigma_1 + \sigma_3)/2, \qquad \sigma_m \neq \sigma_i \text{ and:}$$

$$K_s' = \frac{E}{2(1 + \nu)(1 - 2\nu)}.$$

The volumetric strain $\Delta V/V$ is independent of σ_2, thus, as a result, Δu depends only on $\Delta\sigma_m$, therefore:

$$\Delta u = B'\Delta\sigma_m \quad \text{with} \quad B' = \frac{1}{1 + n(K_s'/K_w)}.$$

Note that the Bishop and Skempton's parameters, normally measured in a triaxial test, cannot, in principle, be applied to the frequent condition in practice of the plane strain condition.

For a numerical example, let's assume $\nu = 0.33 = 1/3$:

$$K_s = \frac{E}{3(1 - 2 \times 1/3)} = E$$

$$K_s' = \frac{E}{2(1 + 1/3)(1 - 2 \times 1/3)} = \frac{9}{8}E > K_s.$$

Thus for the plane strain condition, usually found in-situ, the soil would deform less than it does in the triaxial test. On the other hand, the values of K_s and K are relatively close for the usual values of ν. This justifies the use of Bishop and Skempton's coefficients in plane strain problems and besides, the assumption that the soil structure behaves elastically, is only an approximation of real conditions.

★★★*Problem 4.17* Measurement of the coefficient of earth pressure at rest K_0 using a triaxial device

A saturated clay sample is recovered from a depth of 12 m. Its unit weight is 19 kN/m^3. The pore-water pressure measured with a piezometer at that depth was 110 kPa. The Atterberg limits of the clay are: $w_L = 52\%$, $w_p = 17\%$.

The sample is tested in a triaxial compression test. Drainage is allowed during compression and zero radial deformation conditions are maintained by adjusting the confining pressure σ_r and the axial load σ_1. When σ_r reaches a value of 50 kPa, the deviator stress is 68 kPa. Calculate the coefficient of earth pressure at rest, K_0. Does this value appear acceptable, knowing that the clay is normally consolidated and that a consolidated drained test yielded values of $\varphi' = 20°$ and $A_f = 1.1$.? What is your estimate of the apparent cohesion for this soil?

Solution

Since no radial deformation is allowed during testing, the sample is in the same condition in the lab and in situ, provided that the same stresses are applied. The test is a drained one, then $u = 0$. Therefore $\sigma_1' = \sigma_1$ and $\sigma_r' = \sigma_r$.

For $\sigma_r' = 50$ kPa, we have $\sigma_1' - \sigma_r' = 68$ kPa or $\sigma_1' = 68 + 50 = 118$ kPa.

The effective vertical stress in situ is:

$$\sigma_v' = \gamma_{sat} H - u = 19 \times 12 - 110 = 118 \text{ kPa}.$$

Since the stress conditions in the test and in situ are the same: $\sigma_r' = K_0 \sigma_1'$, from which $K_0 = \sigma_r'/\sigma_1' = 0.50/1.18 \simeq 0.42$.

We know that for a normally consolidated clay ($c' = 0$) the parameters c_u, σ_c, φ', K_o and A_f are not independent, but are related by the following equation:

$$\frac{c_u}{\sigma_v'} = \frac{\sin\varphi'[K_0 + A_f(1 - K_0)]}{1 + \sin\varphi'(2A_f - 1)}$$

Therefore:

$$\frac{c_u}{\sigma_v'} = \frac{\sin 20°(0.42 + 1.1 \times 0.58)}{1 + \sin 20°(2 \times 1.1 - 1)} \simeq 0.26.$$

Skempton has given a good correlation between the ratio c_u/σ'_v and the plasticity index of a clay, I_p:

$$\frac{c_u}{\sigma'_v} = 0.11 + 0.37 I_p$$

For this problem: $I_p = w_L - w_p = 0.52 - 0.17 = 0.35$.
Therefore: $c_v/\sigma'_v = 0.11 + 0.37 \times 0.35 \simeq 0.24$.

The agreement between the two results is good. Taking the average of the two values, 0.25, we have: $c_u = 0.25 \times 118 \simeq 30$ kPa.

Summary of answers

$K_0 = 0.42$; $c_u = 30$ kPa.

★★★*Problem 4.18* **Stress paths applied on a soil element during the construction of an earth dam**

It is necessary for the design of a clay core earth dam to specify tests on soil samples which would give representative values for the behavior of the clay core not only during construction but also during filling of the reservoir and under rapid drawdown conditions. The clay core would be compacted in place at a moisture content close to the optimum moisture as determined by the Proctor test where the effective vertical compaction pressure is 400 kPa. At optimum, the clay exhibits $\gamma_h = 18$ kN/m^3 and, when saturated, $\gamma_{sat} = 20$ kN/m^3.

When the reservoir is filled, pore-water pressures at two locations (1 and 2) are measured: $u_1 = 30$ kPa and $u_2 = 340$ kPa.

(1) Determine the stress history at points 1 and 2 (Fig. 4.29). Draw the stress path in the principal effective stress σ'_1, σ'_2, σ'_3 space. Draw the stress path in the plane p, q. Find in each case if the clay is overconsolidated or normally consolidated.

(2) Determine also the test procedures to subject the clay sample in the lab to a stress path similar to that it would undergo in the dam.

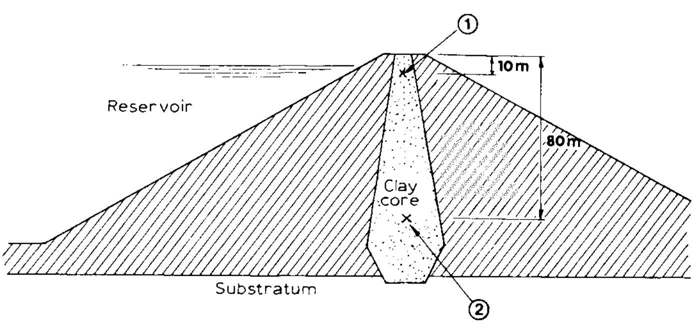


Fig. 4.29.

Solution

(1) Stress history

(a) During compaction $\sigma_1' = \sigma_{1c}' = 400$ kPa.

If K_0 is the coefficient of earth pressure at rest, we have $\sigma_2' = K_0 \sigma_{1c}' = \sigma_3'$. The stress tensor is completely defined by σ_2', σ_3' in the horizontal plane, σ_1' being vertical. Therefore:

$p = \frac{1}{3}(\sigma_1' + \sigma_2' + \sigma_3') = \frac{1}{3}\sigma_1'(1 + 2K_0)$,

$q = \sigma_1' - \sigma_2' = \sigma_1' - \sigma_3' = \sigma_1'(1 - K_0)$.

(b) At the end of construction. Assume that construction progress was sufficiently slow to allow for the full dissipation of pore-water pressures:

$\sigma_1' = \gamma_h z$,

$\sigma_2' = \sigma_3' = K_0 \gamma_h z$,

where z is the depth from the crest of the dam.

$$p = \frac{\sigma_1'}{3}(1 + 2K_0) = \frac{\gamma_h z}{3}(1 + 2K_0),$$

$$q = \sigma_1' - \sigma_2' = \sigma_1' - \sigma_3' = \gamma_h z(1 - K_0).$$

The soil element at location 1 is overconsolidated because at the end of construction, the vertical stress is $\sigma_1' = 180$ kPa and it was previously subjected to a compaction pressure of 400 kPa. The soil element at location 2 is normally consolidated since at the end of construction, it would experience an overburden pressure of $\sigma_1' = 1440$ kPa.

(c) Filling of the reservoir. Seepage occurs through the core. At each of the two locations there will be saturation condition and pore-water pressure u.

We have: $\sigma_1' = \gamma_{sat}(z - u)$ and $\sigma_2' = \sigma_3' = K_0 \gamma_{sat}(z - u)$.

At location 1, the clay will be overconsolidated because $\sigma_1' = 200 - 30 = 170$ kPa.

At location 2, the clay is still overconsolidated because the overburden effective stress is $\sigma_1' = 1600 - 340 = 1260$ kPa whereas at the end of construction, $\sigma_1' = 1440$ kPa.

(d) Rapid drawdown. The seepage in the core reverses its direction after drawdown. The pore-water pressure decreases slowly with time. At the start of the rapid drawdown, the stress conditions are the same as those of the filling of the reservoir, then, as the pore-water pressure decreases, the effective stress increases and when $u = 0$, then:

$\sigma_1' = \gamma_{sat} z = 200$ kPa,

$\sigma_2' = \sigma_3' = K_0 \gamma_{sat} \cdot z = 1600$ kPa.

At location 1, the clay is overconsolidated and at location 2 it is normally consolidated. The various stress paths are presented in Figs. 4.30 and 4.31.

(2) Test procedures to be used in the laboratory. The most versatile equipment to reproduce the stress conditions is that of the triaxial compression test.

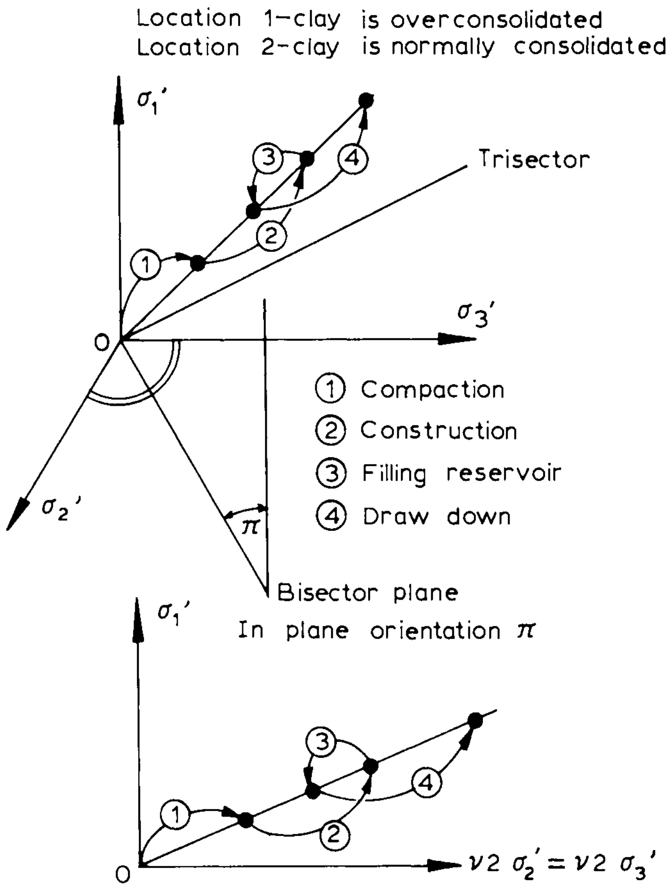


Fig. 4.30. Stress paths.

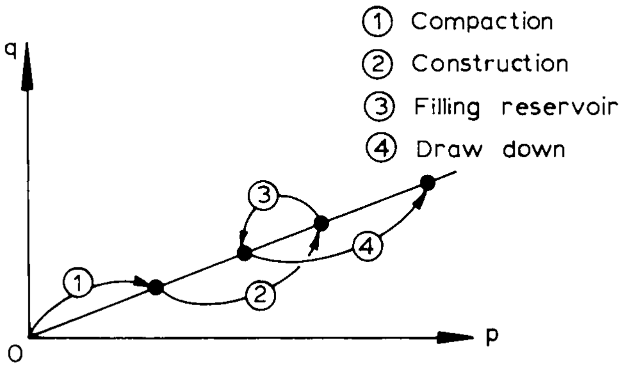


Fig. 4.31. Stress paths in plane p—q.

(a) For compaction conditions. The sample should be anisotropically consolidated (open drainage) under the effective load of $\sigma'_1 = 400\,\text{kPa}$, $\sigma'_2 = \sigma'_3 = K_0\sigma'_1$.

(b) During construction. The stress tensor of consolidation should be increased to the following values:

$$\sigma'_1 = \gamma_h z$$

$$\sigma'_2 = \sigma'_3 = K_0\gamma_h z$$

Depending on the location of the sample, there will either be an increase or a decrease of stress. All variation is done with drainage.

(c) Filling of the reservoir. Then drainage is no more allowed and pore-water pressure u is applied. Seepage is caused to occur in the sample in order to saturate it completely (this is very difficult to do in practice and very time consuming). For the filling condition, the reproduction in the lab of the field condition leaves something to be desired, because in the dam core seepage is not interrupted.

(d) Drawdown. The drainage is once again open and the pore-water pressure allowed to dissipate slowly.

Note

The various stress paths assume that the relation $\sigma'_3 = K_0\sigma'_1$ is always true and that K_0 remains constant. This is an approximation which is most likely not fully representative of the complex actual field conditions.

Chapter 5

PLASTIC EQUILIBRIUM

★★*Problem 5.1* The pole of Mohr's circle

In a two-dimensional space,

(1) show that Mohr's circle does not sufficiently define the stress conditions at a point in a continuous mass;

(2) assume that the stress tensor at a point A in a continuous mass is known and defined by the following data: Mohr's circle and a stress vector **f** *acting on a small face RR' in direction Δ. Let M be the point on Mohr's circle representing* f, *and m be the symmetrical point of M with respect to the principal stress axis, $O\sigma$. From m, a parallel line to the Δ direction is drawn which crosses Mohr's circle at a point P. Show that when the face RR' rotates about A, point P remains fixed. This point is called the pole of the Mohr's circle.*

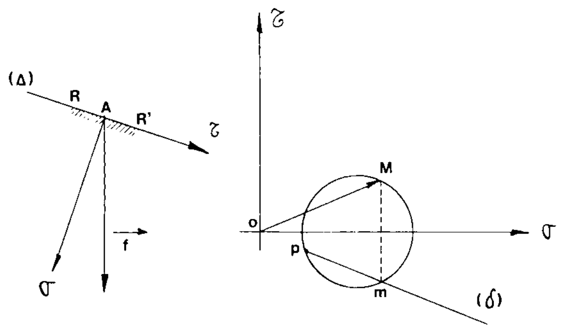


Fig. 5.1.

(3) Show that knowing the pole location enables the determination of the orientation of a face for a given stress vector, and, inversely, the stress vector for a given orientation of the face. Apply this method to find the orientation of the principal stresses.

Solution

(1) The Mohr's circle only gives the magnitude of the principal stresses, but does not define their orientation in space. For any given Mohr's circle, several equal Lamé's ellipses of various orientations (Fig. 5.2).

(2) Let the small face RR' (Fig. 5.3) be rotated counter-clockwise through an angle α. When it reaches the orientation QQ', the stress $\mathbf{f}'$, represented by M' on Mohr's circle, is known to act. From the properties of Mohr's circle,

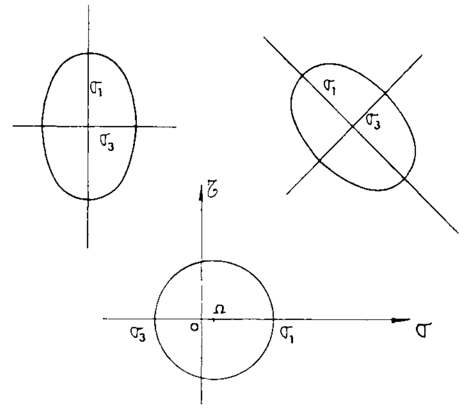


Fig. 5.2. Various Lamé's ellipses for one Mohr's circle.

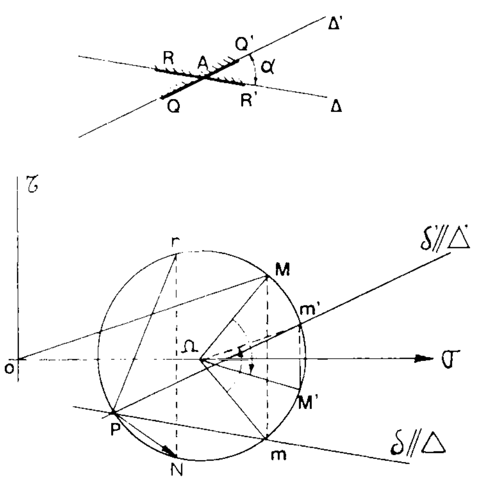


Fig. 5.3.

we have: $\widehat{M\Omega M'} = -2\alpha$. Therefore: $\widehat{m\Omega m'} = +2\alpha$ (symmetry with respect to $O\sigma$ axis).

Connect m' to point P. We now have $\widehat{mPm'} = \alpha$ (inscribed angle). Therefore $m'P$ is parallel to face QQ' and $m'P$ is colinear with (δ').

Consequently, point P is fixed when the face turns about point A.

(3) From the above construction and if we now consider a stress vector of which N is the end, we know immediately the orientation of the face upon which it acts by simply connecting pole P to the symmetrical point of N with respect to the principal stress axis $O\sigma$ (and reciprocally).

Application: By connecting the pole to points B and C of the diameter of the circle on the principal stress axis $O\sigma$, the directions of the faces upon which the principal stresses act are known (see Fig. 5.4).

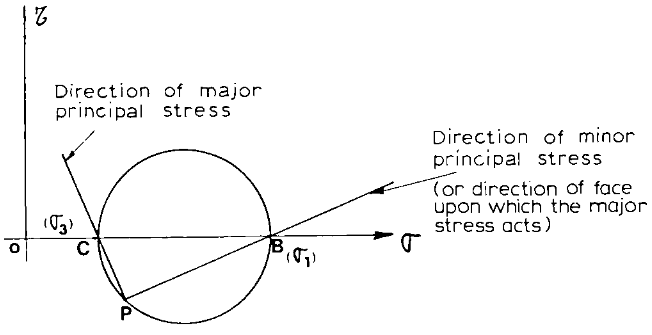


Fig. 5.4.

★★*Problem 5.2* Limit equilibrium of a granular, semi-infinite body

Let us assume a point in a granular, semi-infinite mass in a state of limit equilibrium. Show that if a stress vector is known for the face of a given orientation, two Mohr's circles may be constructed. What conditions do these two circles represent? Find also the orientation of the faces upon which the stress is the least favorable (corresponding to the limit stress vectors).

Solution

The properties of every circle meeting the above conditions are:

— the center is on the axis $O\sigma$;
— it must go through point M;
— it must be tangent to the failure envelope (therefore to two lines D and D' which are symmetrical with respect to $O\sigma$) (see Fig. 5.5).

The geometric solution shows that there are two circles which satisfy the above conditions.

The *construction* of the circles is done in the following manner (see Figs. 5.5. and 5.6).

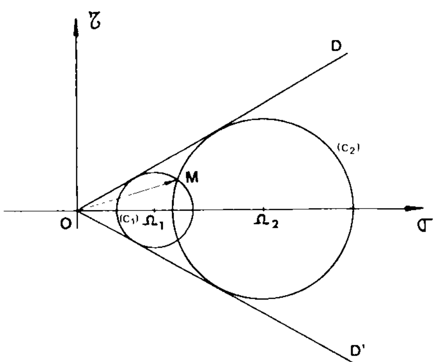


Fig. 5.5.

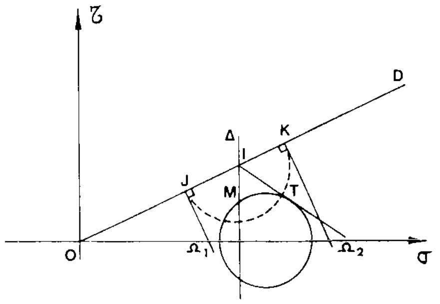


Fig. 5.6.

From point M, draw the perpendicular line to $O\sigma$ which then intersects D at point I. Draw a random circle centered on $O\sigma$ passing through M. From I, draw a tangent IT to this circle. The dashed circle, centered at I the radius of which is equal to IT crosses line D at points J and K which are the points where D contacts the two circles sought. Their centers are at Ω_1 and Ω_2 (Fig. 5.6).

The *interpretation* shows that two limit equilibrium conditions are possible for a given granular mass (superior and inferior Rankine equilibria).

★★*Problem 5.3* Limit equilibrium of semi-infinite granular media with an inclined free surface

Consider a semi-infinite, granular mass, not loaded at the surface which is inclined at an angle β with respect to the horizontal. Let us assume that the limit equilibrium corresponds to the lower Rankine equilibrium. The unit weight of the mass is γ and its angle of internal friction is φ.

(1) Determine the stress at depth z acting on a face parallel to the free surface of the mass, $x'x$. From which find, by graphical solution, the stresses acting at that depth z on a vertical face and on a face at an angle θ with the vertical. What are the stress conditions when $\beta = 0$?

(2) With Mohr's circle, draw the net of slip lines. What is the conclusion? What is the angle between the slip lines and the free surface $x'x$?
NB. The pole construction is assumed to be known (see problem 5.1).

Solution

(1) For a cylindrical soil element with its base on the face under consideration dS (see Fig. 5.7), Rankine's theory states that the forces acting on the vertical surface of the cylinder cancel each other. The stress acting on face dS is therefore:

$$\sigma_{\mathrm{v}} = \frac{\text{cylinder weight}}{\mathrm{d}S} = \frac{\gamma z \,\mathrm{d}S \cos\beta}{\mathrm{d}S} = \gamma z \cos\beta.$$

From Fig. 5.7, it is easily seen that the components of this stress are:

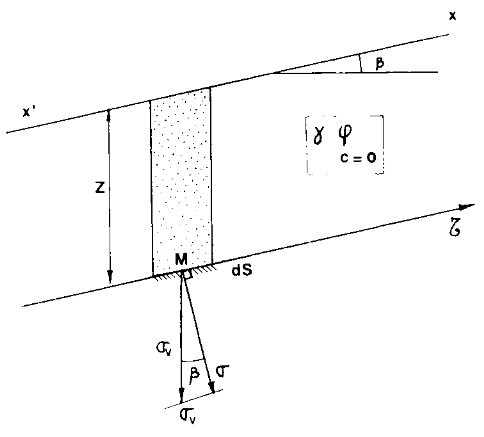


Fig. 5.7.

$$\sigma_v \begin{cases} \sigma = \gamma z \cos^2 \beta \\ \tau = \gamma z \cos\beta \sin\beta . \end{cases}$$

Mohr's circle corresponding to this condition is drawn in Fig. 5.8. It goes through point M, which is the end of the stress vector σ_v and it is tangent to the failure envelope (Coulomb's lines) since the soil mass is at limit equilibrium.

It was shown in problem 5.2. that two circles satisfy the conditions. Since it is assumed that the soil mass is at the lower Rankine limit equilibrium, the circle representing the stresses is the smaller of the two. From the pole construction (see problem 5.1) we know the stress vectors acting on the vertical face **OM′** and on the face making an angle θ with the vertical **OM″**. Note that the point **M′** is the same as the pole of Mohr's circle.

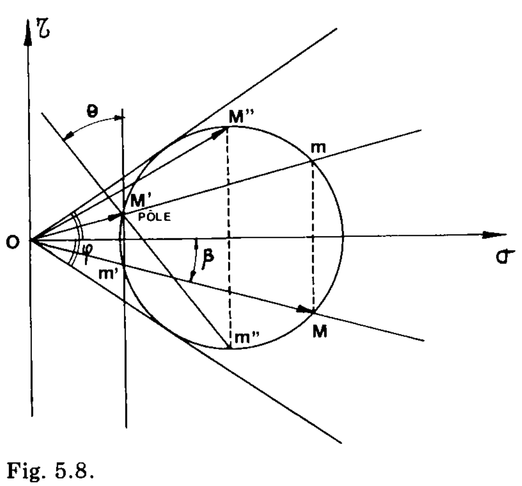


Fig. 5.8.

In the case where $\beta = 0$, the stress σ_v is perpendicular to the face upon which it acts. It is therefore a principal stress. Because we are assuming lower Rankine limit equilibrium it is the major principal stress (Fig. 5.9).

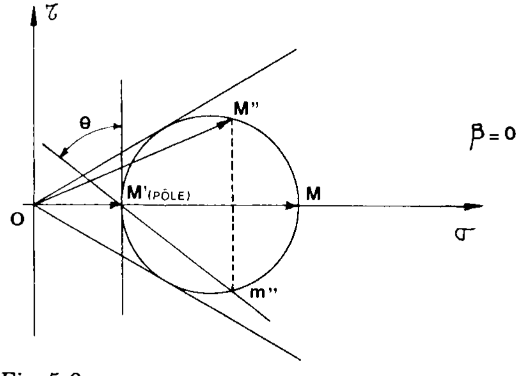


Fig. 5.9.

(2) Net of slip lines. From the construction of the pole, we know the orientation of the faces upon which the limit stresses **OT** and **OT**$'$ acts (these are stresses with maximum inclination, equal to φ with the perpendicular to the face upon which they act). The directions of these faces are obtained by joining the pole to points T and T'. It will then be noticed that directions PA and PB which are the principal directions of the stress vectors, bisect the slip faces (because A and B are at the middle of arcs whose ends are T and T'). As before, $T\Omega A = \pi/2 + \varphi$, and thus the faces make an angle of (1/2) $(T\Omega A) = \pi/4 + \varphi/2$ with the direction of PA, that of the face upon which the major principal stress acts (or by the same token, the direction of the minor principal stress).

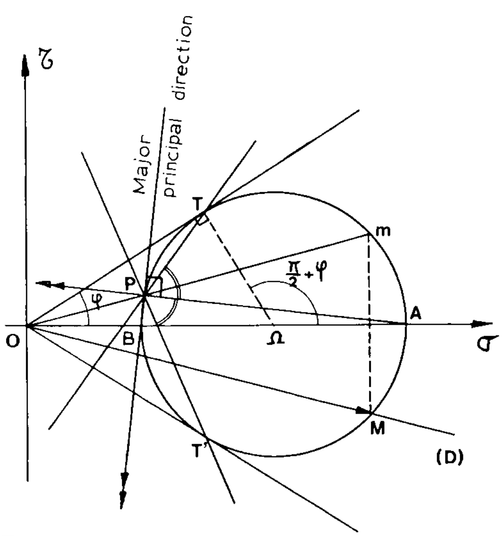


Fig. 5.10.

When considering the effect of depth z, point M describes a straight line D inclined at β on $O\sigma$, since this angle remains constant. The Mohr's circle remains homothetical and the direction of the slip faces remains the same. Therefore the slip lines, which are the envelops of slip faces, are straight lines with an angle of $\pm(\pi/4 + \varphi/2)$ with the direction of the minor principal stress.

It should be noticed that it is not possible to have a condition wherein $\beta > \varphi$ because an equilibrium may not exist in such conditions.

Let us now find the angle between the vertical and the direction of the minor principal stress. Consider angles $OP\Omega$ and $P\Omega A$ (Fig. 5.11). In triangle $OP\Omega$, we have: $\omega_\beta = \beta + 2\gamma$, and in triangle $P\Omega A$, we have: $\alpha = \pi - 2\gamma$, from which $\alpha = \pi - (\omega_\beta - \beta)$.

To define angle ω_β, we have: $\sin \omega_\beta = \Omega H/R$, but $\sin \beta = \Omega H/O\Omega$ and $\sin \varphi = R/O\Omega$. Therefore: $\sin \omega_\beta = \sin \beta/\sin \varphi$, $\omega_\beta = \arcsin(\sin\beta/\sin\varphi)$.

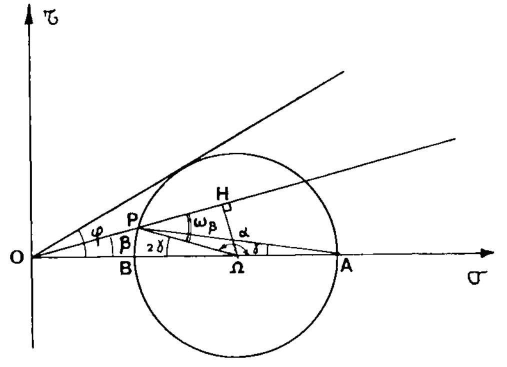


Fig. 5.11.

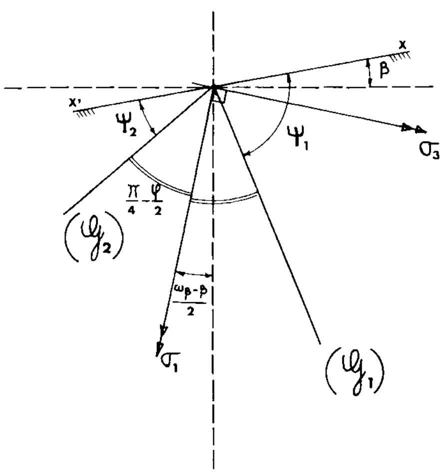


Fig. 5.12.

From the properties of Mohr's circle it is found that σ_3 makes an angle with the vertical equal to $\alpha/2$ or: $\pi/2-(\omega_\beta-\beta)/2$ and that σ_1 is inclined to the vertical by angle $(\omega_\beta-\beta)/2$.

From this, the slip lines $(\mathscr{G}_1)$ and $(\mathscr{G}_2)$ make an angle with the free face of the mass of:

$$\Psi_1=\frac{\pi}{4}+\frac{\varphi}{2}+\frac{\omega_\beta+\beta}{2},\quad \text{and}\quad \Psi_2=\frac{\pi}{4}+\frac{\varphi}{2}-\frac{\omega_\beta+\beta}{2}$$

with $\omega_\beta=\arcsin(\sin\beta/\sin\varphi)$.

These results are summarized in Fig. 5.12.

★★*Problem 5.4* Equilibrium of Rankine

Let us consider a semi-infinite mass with a free face inclined at β with respect to the horizontal. The mass is cohesionless soil, with an angle of internal friction of φ and unit weight of γ. The purpose is to find the stresses acting along a line OL making an angle θ with the vertical. Let α be the angle of inclination of the stress acting at depth h on a face whose center M (θ, r) is on OL (Fig. 5.13).

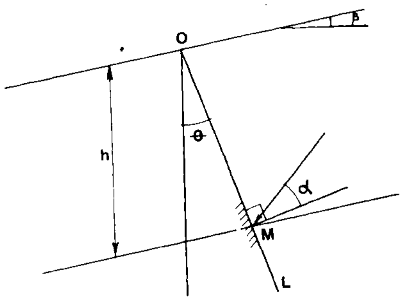


Fig. 5.13.

Assume that the mass is at the lower limit equilibrium of Rankine.

(1) Draw Mohr's circle corresponding to the lower equilibrium at M. Let Ω be the center of this circle and m be the point on the circle representing the stress acting on a face parallel to the free boundary and passing through M. Om crosses the circle at point n. Let ω_β be the angle $\widehat{\Omega nm}$ which must be expressed by one of its trigonometric lines as a function of β and φ (see Fig. 5.15).

Let p and R be the abscissa of the center and the radius respectively of the Mohr's circle. Express R in terms of p and φ and p in terms of β and ω_β.

(2) Construct point m_1 of the Mohr's circle associated with a face whose center is M in which lies OL (Fig. 5.13). Calculate the angle α of the corresponding stress and show it is independent of r.

(3) Find the magnitude of the normal stress σ_θ acting on the face of

center M and direction OL. Show it is proportional to γ and to r and that the equation $\sigma_0 = K\gamma r$ exists.

Write K as a function of θ, β, φ and of angle ω_β. What can be said about the stress distribution on segment OL?

Solution

(1) The stress on face dS at M has a value of (Fig. 5.14):

$$df = \frac{\text{weight of cylinder}}{ds} = \frac{\gamma h \, ds \cos\beta}{ds} = \gamma h \cos\beta.$$

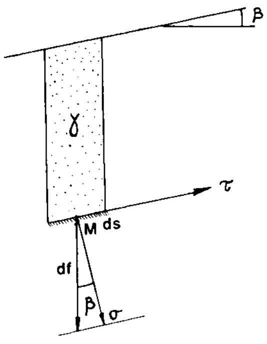


Fig. 5.14.

Hence Mohr's circle passes through point m, the coordinates of which are:

$$m\begin{cases}\gamma h \cos^2\beta \\ \gamma h \cos\beta \sin\beta \text{ (Fig. 5.15)}\end{cases}$$

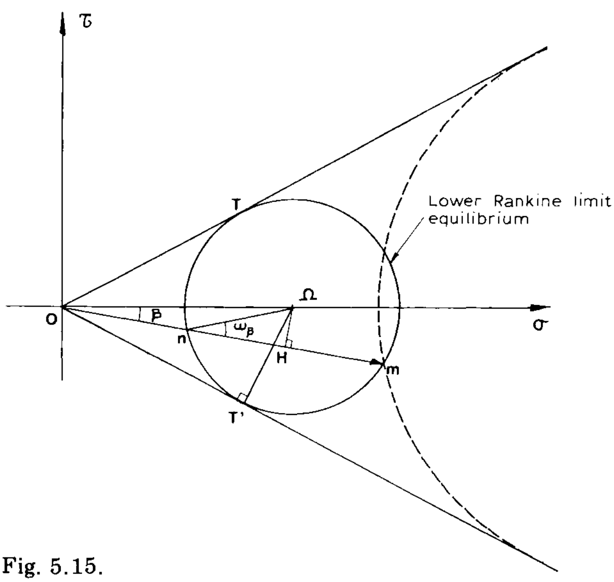


Fig. 5.15.

to find angle ω_β: $\sin \omega_\beta = \Omega H/R$
but: $\sin \beta = \Omega H/O\Omega$ and $\sin \varphi = R/O\Omega$
therefore: $\sin \omega_\beta = (\Omega H/O\Omega) \times (O\Omega/R) = \sin \beta/\sin \varphi$
$\sin \omega_\beta = \sin \beta/\sin \varphi$.
To find R, we have $R = p \sin \varphi$.

To find p, consider triangle $O\Omega m$ (Fig. 5.16). We have:
$p/\sin \omega_\beta = Om/\sin \widehat{O\Omega m}$, but:

$$\widehat{O\Omega m} = \omega_\beta - \beta + (\pi - 2\omega_\beta) = \pi - (\omega_\beta + \beta), \quad \sin \widehat{O\Omega m} = \sin(\omega_\beta + \beta)$$

from which:

$$\frac{p}{\sin \omega_\beta} = \frac{Om}{\sin(\omega_\beta + \beta)}, \quad \text{but:} \quad Om = \gamma h \cos \beta,$$

hence:

$$p = \gamma h \cos \beta \frac{\sin \omega_\beta}{\sin(\omega_\beta + \beta)}.$$

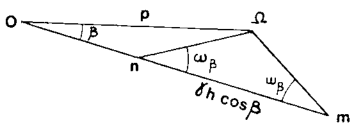


Fig. 5.16.

(2) We must first construct the pole P of Mohr's circle (see problem 5.1 and Fig. 5.17). P is also the representative point of the stress vector acting on a vertical plane through M. Hence for a plane at an angle $+\theta$ with the vertical, the representative point m_1 is obtained by rotating by -2θ on Mohr's circle. Let m' be the projection of m_1 on axis $O\sigma$.

We have then: $\alpha = \widehat{m'Om_1}$ or $\tan \alpha = m_1 m'/Om'$.

Consider now the triangle $\Omega m' m_1$; we have: $\widehat{m'\Omega m_1} = 2\theta + \omega_\beta - \beta$.
Therefore: $m_1 m' = R \sin(2\theta + \omega_\beta - \beta)$, $Om' = p - R\cos(2\theta + \omega_\beta - \beta)$.

From the first part of solution: $R = p \sin \varphi$
$Om' = p[1 - \sin \varphi \cos(2\theta + \omega_\beta - \beta)]$
from which:

$$\tan \alpha = \frac{p \sin \varphi \sin(2\theta + \omega_\beta - \beta)}{p[1 - \sin \varphi \cos(2\theta + \omega_\beta - \beta)]}$$

This equation shows that $\tan \alpha$ is independent of r. Therefore, the inclination α of the stress acting on a face through OL is the same all along the line OL.

(3)

$$\sigma_\theta = Om' = p - R\cos(2\theta + \omega_\beta - \beta)$$

$$\sigma_\theta = p[1 - \sin \varphi \cos(2\theta + \omega_\beta - \beta)].$$

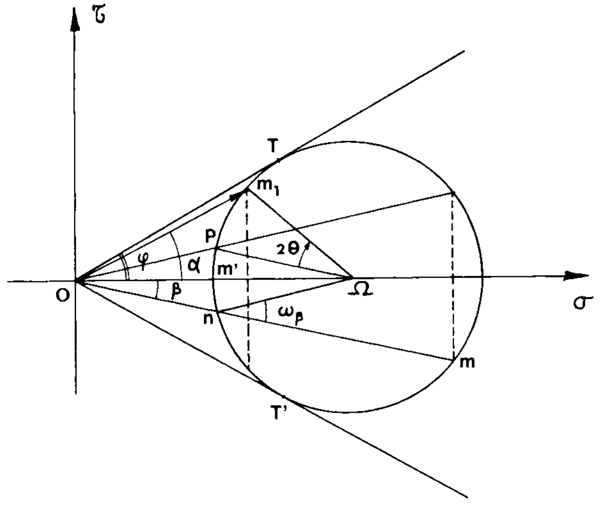


Fig. 5.17.

Then, from the first part of the solution:

$$p = \gamma h \cos\beta \frac{\sin\omega_\beta}{\sin(\omega_\beta + \beta)}.$$

and from triangle *ONM* (Fig. 5.18) we also have:

$$\frac{r}{\sin(\pi/2 - \beta)} = \frac{h}{\sin[\pi - \theta - (\pi/2 - \beta)]}$$

$$\frac{r}{\cos\beta} = \frac{h}{\cos(\beta - \theta)}$$

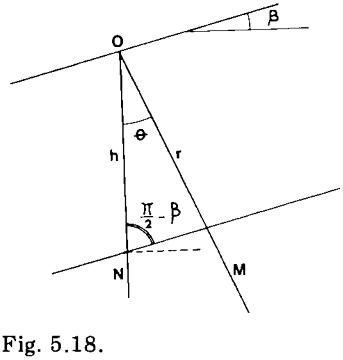


Fig. 5.18.

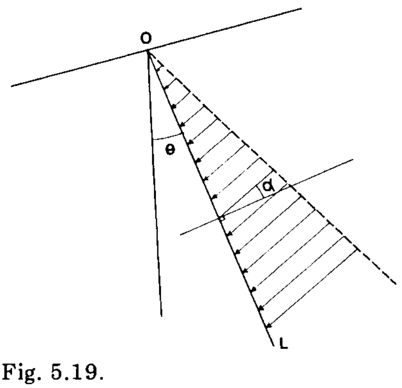


Fig. 5.19.

from which:

$$h = r\frac{\cos(\beta-\theta)}{\cos\beta}.$$

Therefore we have: $\sigma_\theta = K\cdot\gamma\cdot r$ with:

$$K = \frac{\cos(\theta-\beta)\sin\omega_\beta}{\sin(\omega_\beta+\beta)}\,[1-\sin\varphi\cos(2\theta+\omega_\beta-\beta)].$$

The stress distribution along OL is linear (Fig. 5.19) and to summarize: Rankine's equilibrium is characterized by:

— slip lines which are straight lines;
— triangular stress distribution with constant inclination along a straight line through the mass.

★★★*Problem 5.5* **Plasticity; limit equilibrium of a weightless mass loaded at the surface; Prandtl corner**

(See pole construction of problem 5.1)

(1) Study the limit equilibrium of a semi-infinite cohesionless mass having an angle of internal friction of φ, whose free surface is inclined at an angle β with the horizontal. It is subjected to a uniform vertical load of q (Fig. 5.20). What can be said about the stress tensor? Draw the slip lines. What happens when $\beta = 0$?

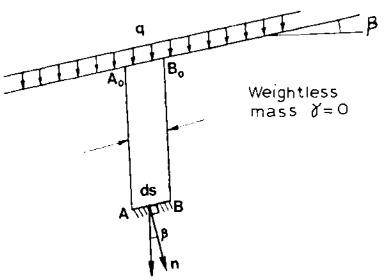


Fig. 5.20.

(2) Consider the same mass with a horizontal free surface and a point A at the surface. Assume that a uniform load p_0 is applied to the surface along a line to the right of point A and a uniform load p_2 is applied along the same line but to the left of point A. Assume that the mass is at limit equilibrium.

(a) Show that it could be assumed that there coexist two limit equilibria zones. What are they and what are the limit equilibria which could exist there? What relation must exist between p_2 and p_0 for these equilibria to exist? Show, considering the slip lines net, that this solution is not kinematically possible.

(b) Which three zones configurations can logically be chosen? What will be the slip lines net in the third zone? What is the relation between p_0 and p_2?

(c) Compute the value of the coefficient N_q and N_c of the bearing capacity formula for shallow footings from the above mentioned result.

Solution

(1) The stresses acting on planes A_0A and B_0B of the elemental prism of base dS cancel two by two. (Fig. 5.20).

Since the mass is assumed weightless, the stress acting on plane AB is equal to q, whatever the depth of the face. *The stress tensor is thus the same at all points.*

The stress vector acting on a face parallel to the free surface is known (Fig. 5.21). Mohr's circle must pass through the M-end of this vector and must be tangent to Coulomb's straight lines. Two solutions correspond to this equilibrium (lower and upper Rankine equilibrium).

Draw line Om parallel to the plane through point m which is the mirror image of M. The line crosses the circles at points P_1 and P_2 (the poles), from which we obtain the slip lines.

Indeed, the stress tensor being the same at all points, the faces upon which the least favorable stresses act (stress vectors on the Coulomb lines) have the same direction at all points of the soil mass. Hence, the slip lines, which are the envelope of these faces, are straight lines.

In Fig. 5.21, it can be seen that the failure planes are straight lines parallel to P_1T_1 and P_1T_1' for the lower equilibrium and to P_2T_2 and P_2T_2' for the upper equilibrium.

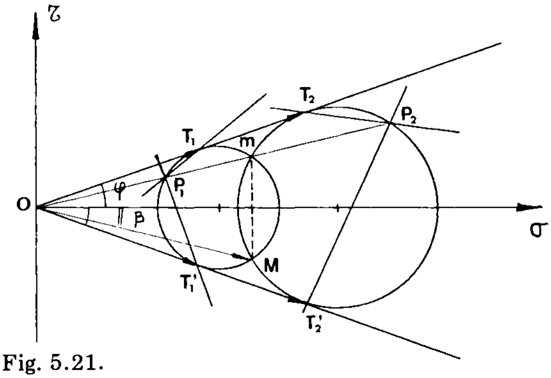


Fig. 5.21.

For the particular case of $\beta = 0$, point M is on the axis $O\sigma$ (M is the end point of the stress vector acting on plane AB), because the vertical stress is a principal stress. The two limit Mohr's circles are tangent at M (Fig. 5.22).

Lower equilibrium. $OM = q$, the major principal stress. From Fig. 5.22, T_1 and T_1' are obtained from rotation of M of $\pm (\pi/2 + \varphi)$. Hence the face on

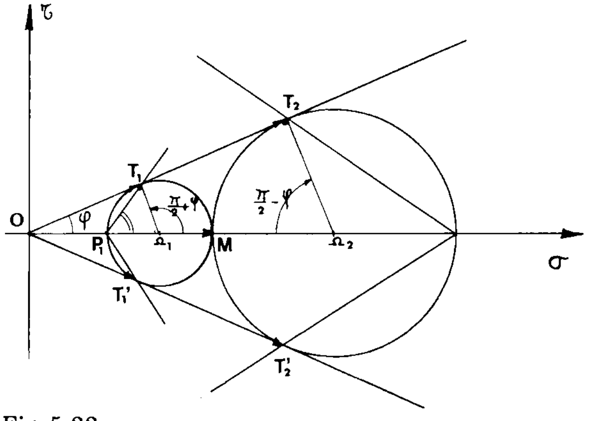


Fig. 5.22.

which OT_1 (or OT'_1) acts and which coincides with the slip plane orientation, is obtained from the horizontal face rotated by an angle of $\pm(\pi/4 + \varphi/2)$. Therefore, the slip lines are straight lines inclined at an angle of $\pm(\pi/4 + \varphi/2)$ with the horizontal, respectively $\pm(\pi/4 - \varphi/2)$ with the vertical.

Upper equilibrium: the results are the same if we change $\pi/4 + \varphi/2$ to $\pi/4 - \varphi/2$.

(2a) At depth Z_0, if we are sufficiently far from the vertical AX passing through A, it may seem acceptable to assume that a limit equilibrium would develop wherein p_2 acts as the load q of the preceding question for points in the mass located to the left (zone II) of A, and that another limit equilibrium condition would exist wherein p_0 would act as load q at any point in the mass located to the right of point A (zone I, see Fig. 5.23).

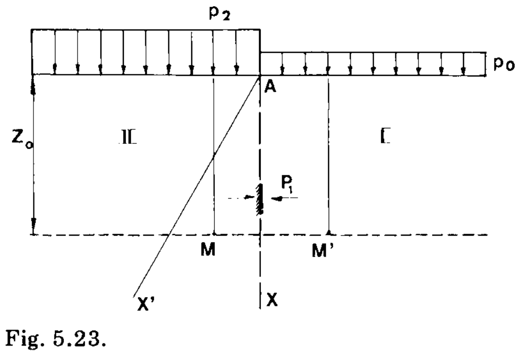


Fig. 5.23.

Let us assume that the boundary between the two zones is a straight line passing through A. If the boundary is not vertical, such as AX is, an impossible condition occurs, because if we consider two points, such as M and M' in the same zone, we find from the above results that the stress tensors at M and M' are not the same. Therefore, if such a boundary exists, it can only be a vertical such as AX.

In order for equilibrium to be realized, however, the stress acting on a vertical face through AX must be the same in the two zones. Let p_1 be that value. It is clear, that we must necessarily have $p_2 > p_1 > p_0$ and that the two equilibrium states are represented by two tangent circles (Fig. 5.24).

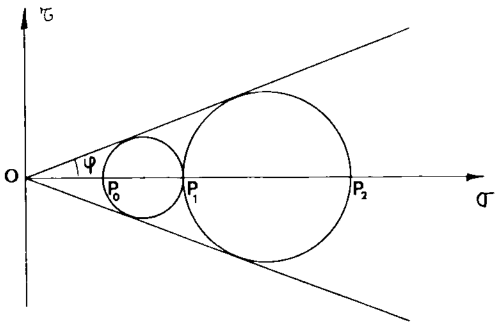


Fig. 5.24.

We have already seen that for a limit equilibrium we have:

$\sigma_3/\sigma_1 \;=\; \tan^2(\pi/4 - \varphi/2)$ and $\sigma_1/\sigma_3 \;=\; \tan^2(\pi/4 + \varphi/2)$.

That means in this case:

$$p_2/p_1 \;=\; \tan^2(\pi/4 + \varphi/2)$$
$$p_0/p_1 \;=\; \tan^2(\pi/4 - \varphi/2)$$

and, therefore:

$$p_2/p_0 = \tan^4(\pi/4 + \varphi/2) \tag{1}$$

Thus, for such a state to exist, p_2 and p_0 cannot have any value but must satisfy the relation (1).

This scheme must also be kinematically acceptable, that is to say, it must be compatible with the continuity of mass and its boundary conditions.

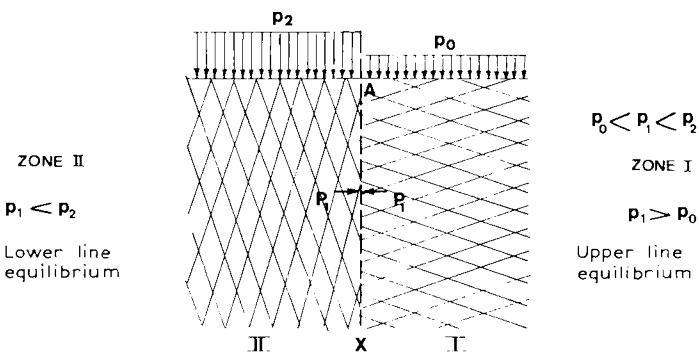


Fig. 5.25.

For the case under consideration (see Fig. 5.25), it is obvious that the failure planes do not have the same inclination with respect to the vertical. Line AX is a line of kinematic discontinuity.* Besides, this line does not coincide with a slip line, then this scheme is not kinematically acceptable. We must therefore consider a three-zone system.

(b) The simple solution which comes to mind in this context, is to introduce a third zone bounded by two straight lines passing through A which are slip lines, respectively AY and AZ, for each of the limit equilibrium states. The second set of slip lines in the third zone must make at any point M, an angle of $(\pi/2 - \varphi)$ with the slip line of the first family, that is with the straight line AM (see Fig. 5.26). But we know that the logarithmic spiral is a curve the tangent of which at any point makes a constant angle with the radius. Therefore, the two zones may be linked by a third zone containing logarithmic spirals with a pole at A and whose tangent makes an angle of $\pi/2 - \varphi$ with the radius.

This 3rd zone is called Prandtl's corner. The slip lines are: —log spirals with a pole *at* A, — straight lines issuing *from* A.

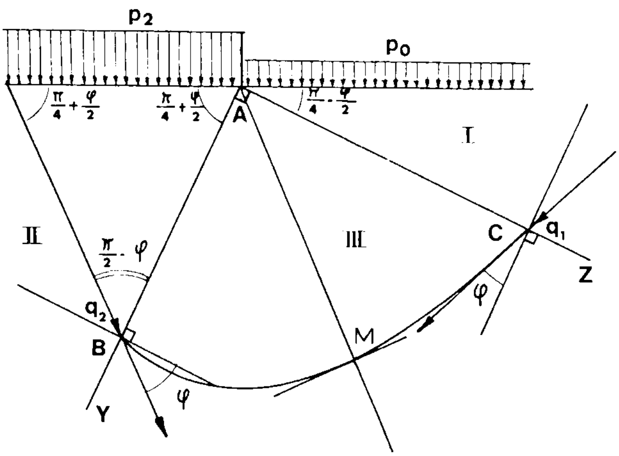


Fig. 5.26.

The 3-zone condition is kinematically acceptable. Now, we must determine the relation between p_0 and p_2 with respect to stresses q_2 and q_1 acting on the faces through AZ and AY. For this purpose, we write that the prism ABD of Fig. 5.27 is in equilibrium, by equating to zero the sum of the moments acting on faces AB, BD and AD. The stresses on the portion BD of the spiral act along the straight slip lines and pass through point A. Their moment arm is zero. Stress q_2 acts on face AB. Stress q_α acts on face AD which makes an angle α with AY. Since the mass is weightless, the stress distribution is uniform along AB and AD. The moment equilibrium with

*Along AX a segment stretches if in Zone I or shrinks if in Zone II.

respect to point A gives:

$$q_2 \cos\varphi \frac{\overline{AB}^2}{2} = q_\alpha \cos\varphi \frac{\overline{AD}^2}{2}$$

from which:

$$\frac{q_\alpha}{q_2} = \frac{\overline{AB}^2}{\overline{AD}^2} = \frac{\rho_0^2\, e^{2(\theta_2-\theta_0)\tan\varphi}}{\rho_0^2\, e^{2(\theta_\alpha-\theta_0)\tan\varphi}}$$

If we let θ_2 and θ_α be the polar angles corresponding to the radii of AB and AD with respect to an arbitrary origin corresponding to a value ρ_0 of the radius then: $q_\alpha = q_2 e^{2(\theta_2-\theta_\alpha)\tan\varphi} = q_2 e^{-2\alpha\tan\varphi}$.

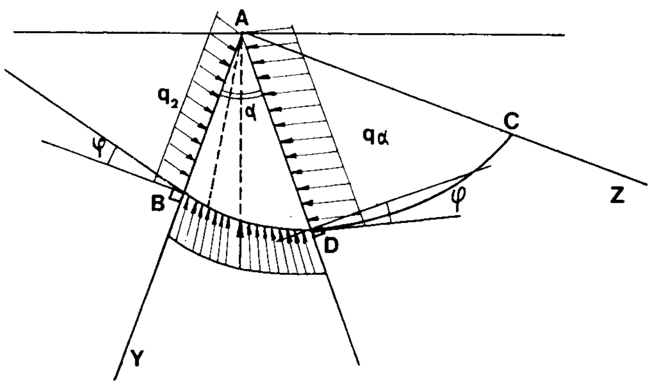


Fig. 5.27.

For the relation between q_2 and p_2, we have to show that $q_2 = p_2 \tan(\pi/4 - \varphi/2)$.

Let us draw Mohr's circle for point B (Fig. 5.28)

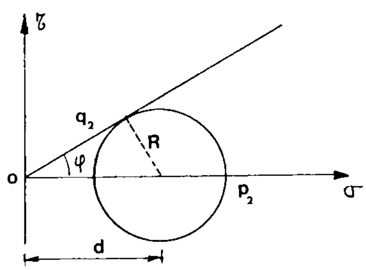


Fig. 5.28.

$$\begin{aligned} p_2 &= d + R \\ R &= d \sin\varphi \\ q_2 &= d\cos\varphi \end{aligned}$$

from which:

$$p_2 = d/(1 + \sin\varphi) \quad \text{and} \quad q_2 = p_2 \frac{\cos\varphi}{1 + \sin\varphi}$$

but:

$$\frac{\cos\varphi}{1+\sin\varphi} = \frac{\sin\left(\dfrac{\pi}{2} - \varphi\right)}{\sin\dfrac{\pi}{2} + \sin\varphi} = \frac{2\sin\left(\dfrac{\pi}{4} - \dfrac{\varphi}{2}\right)\cos\left(\dfrac{\pi}{4} - \dfrac{\varphi}{2}\right)}{2\sin\left(\dfrac{\pi}{4} + \dfrac{\varphi}{2}\right)\cos\left(\dfrac{\pi}{4} - \dfrac{\varphi}{2}\right)}$$

$$= \frac{\sin\left(\dfrac{\pi}{4} - \dfrac{\varphi}{2}\right)}{\cos\left[\dfrac{\pi}{2} - \left(\dfrac{\pi}{4} + \dfrac{\varphi}{2}\right)\right]} = \frac{\sin\left(\dfrac{\pi}{4} - \dfrac{\varphi}{2}\right)}{\cos\left(\dfrac{\pi}{4} - \dfrac{\varphi}{2}\right)} = \tan\left(\frac{\pi}{4} - \frac{\varphi}{2}\right).$$

We finally get: $q_\alpha = p_2 \tan(\pi/4 - \varphi/2)e^{-2\alpha\tan\varphi}$

By drawing Mohr's circle for point C (Fig. 5.29) a similar calculation as above gives: $q_1 = p_0 \tan(\pi/4 + \varphi/2)$.
But: $q_1 = q(\alpha) = p_2 \tan(\pi/4 - \varphi/2)\, e^{-\pi\tan\varphi}$ for $\alpha = \pi/2$.
Hence p_2 and p_0 are related by:

$$p_2 = p_0 \frac{\tan(\pi/4 + \varphi/2)\, e^{\pi\tan\varphi}}{\tan(\pi/4 - \varphi/2)} \quad \text{or} \quad p_2 = p_0 \tan(\pi/4 + \varphi/2)e^{\pi\tan\varphi}.$$

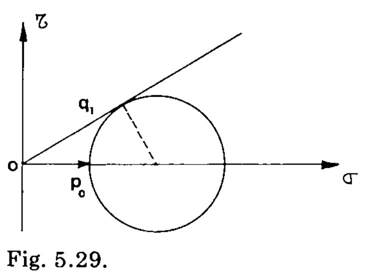


Fig. 5.29.

(c) Coefficients N_q and N_c of the bearing capacity formula for shallow footings. The above calculations are applicable to determine the depth factor N_q in the formula referred to. The assumption is made that the footing does not alter the inclination of the failure lines. Under this condition the stress condition under a footing is as shown in Fig. 5.30.

The distribution p_0 corresponds to the weight of the overburden above the level of the base of the footing $x'x$, therefore to $p_0 = \gamma D$.

We therefore have:

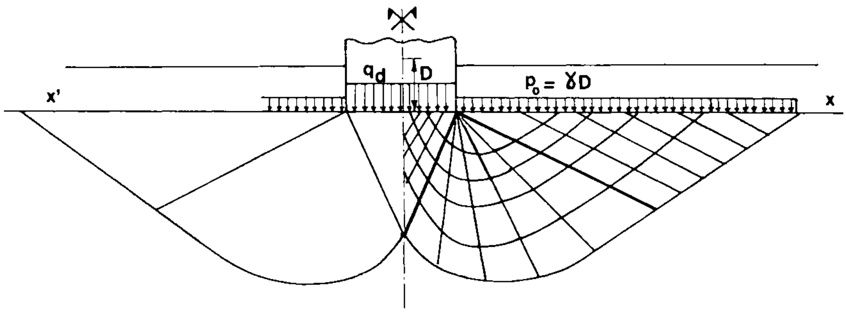


Fig. 5.30.

$$q_d = \gamma D \left(\frac{\pi}{4} + \frac{\varphi}{2}\right) e^{\pi \tan \varphi} \quad \text{from which:}$$

$$N_q = \tan^2 \left(\frac{\pi}{4} + \frac{\varphi}{2}\right) e^{\pi \tan \varphi}.$$

For a cohesive soil with cohesion c, the uniform vertical fictitious stress $H = c \cot \varphi$ is assumed to act at the free face of the weightless mass. The fictitious bearing capacity would be:
$(q_d)' = q_d + H$
and we will also have: $(q_d)' = N_q (H + \gamma D)$
or: $q_d + c \cot \varphi = N_q (\gamma D + c \cot \varphi)$
or: $q_d = \gamma D N_q + c \cot \varphi (N_q - 1)$
and: $q_d = \gamma D N_q + c N_c$ with: $N_c = \cot \varphi (N_q - 1)$.

N_c is the coefficient corresponding to the cohesion factor in the bearing capacity formula. The coefficient N_γ can of course not be calculated by this method since it must consider the weight of the mass.

Note on the stress vector of zone III (Prandtl's corner): When the radius

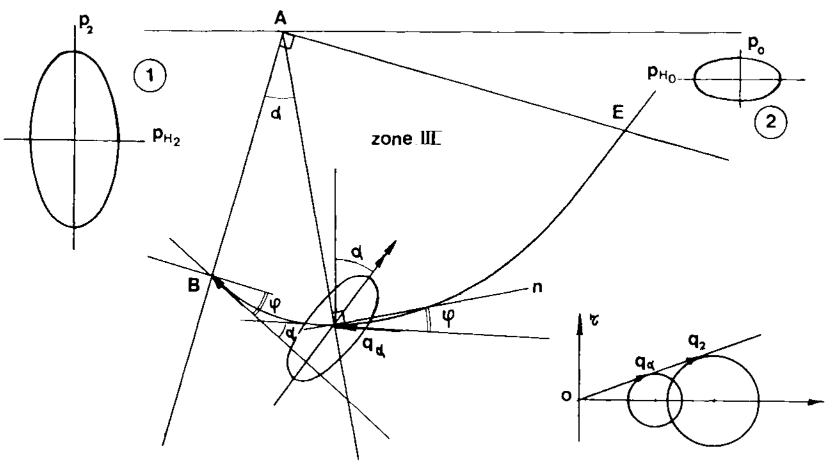


Fig. 5.31.

rotates through an angle α from AB, the direction of the stress q rotates through angle α, but since q makes an angle φ — which remains constant — with the radius, the principal directions also rotate by α. But at B, the major principal stress direction is vertical. Therefore, at D the direction of the major principal stress makes an angle α with the vertical. Lamé's ellipse rotates and the size of the axes decreases to go from ellipse 1 to ellipse 2 (see Fig. 5.31).

★★★*Problem 5.6* Limit equilibrium of a semi-infinite cohesive mass with an inclined free surface

Refer to problem 5.3 and assume that the soil has a cohesion c. Two cases are then considered in analyzing slip lines, depending on the value of β. The inclination angle of the slip lines with the free surface x'x and the orientation of the asymptotes, if any, will be determined. (Refer to problem 5.1 for the construction to determine the pole of the Mohr's circle).

Solution

(1) This problem is similar to problem 5.3, but a distinction must be made between real and fictitious stresses.

Real stress at depth z is: $\sigma_v = \gamma z \cos\beta$ (Fig. 5.7).

It is represented by vector OM of Mohr's diagram (Fig. 5.32) and its components are:

$$\sigma_v \begin{cases} \sigma = \gamma z \cos^2 \beta. \\ \tau = \gamma z \cos\beta \sin\beta. \end{cases}$$

The fictitious stress, represented by vector O′M is:

$$\sigma'_v \begin{cases} \sigma' = H + \gamma z \cos^2\beta = c \cot\varphi + \gamma z \cos^2\beta \\ \tau' = \gamma z \cos\beta \sin\beta \end{cases}$$

therefore: $|\sigma'_v| = \sqrt{\sigma'^2 + \tau'^2}$.

The pole is determined as above (see problem 5.1), from which the real and fictitious stresses (OM′, OM″ and O′M′, O′M″, respectively), acting on a vertical face and an inclined face at angle θ, are determined (Fig. 5.32).

At the free surface, we have:
real stress: $\sigma_v = 0$,
fictitious stress:

$$\sigma'_v = H = c \cot\varphi \begin{cases} \sigma' = \sigma'_v \\ \tau' = 0. \end{cases}$$

The fictitious stress is perpendicular to the free surface.
The condition $\beta = 0$, is represented in Fig. 5.33. In this case, the pole of Mohr's circle is located on $O\sigma$.

(2) Slip lines.

Mohr's diagram (Fig. 5.34) shows that two conditions exist depending on the value of β:

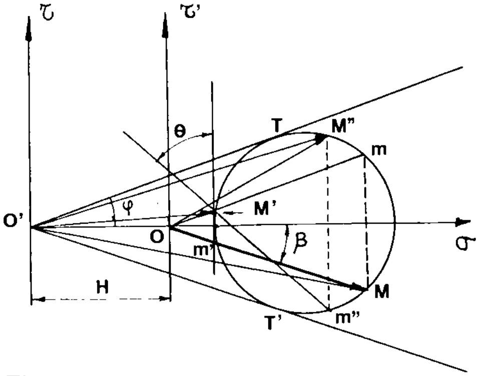


Fig. 5.32.

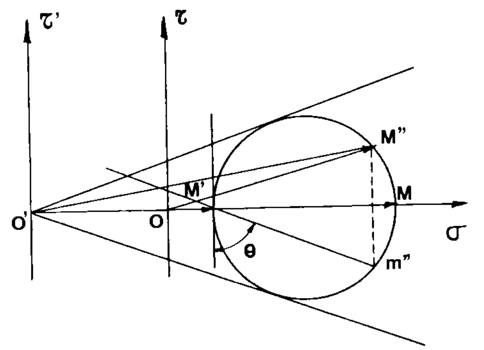


Fig. 5.33.

condition 1: if $\beta < \varphi$, drawing Mohr's circle corresponding to the limit lower equilibrium of Rankine is always possible,
condition 2: if $\beta > \varphi$, Mohr's circle can only be drawn if $|\sigma_v| < OT'$.

Therefore, the magnitude of the stress vector cannot exceed the limit value of OT, otherwise the equilibrium is not possible. Hence there is a limit depth, z_1, for which equilibrium exists (see Fig. 5.34).

(a) Condition of $\beta < \varphi$ (Fig. 5.35)

To determine the orientation of the faces upon which the limit stresses OT and OT$'$ act, the procedure is identical to the one in problem 5.3. It will be noted however, that as z varies, the Mohr's circles are no longer similar because vector OM does not originate from the intersection of O' of Coulomb's lines. The orientation of the faces upon which the limit stresses act, varies with depth z. Therefore the slip envelopes are no longer straight lines. The net of the slip lines will then consist of two families of curves crossing at an angle $\pi/2 + \varphi$, because we have:

$$(\Omega T', \Omega T) = 2(PT', PT)$$

$$(\Omega T', \Omega T) = 2(\Omega A, \Omega T) = 2(\pi/2 + \varphi).$$

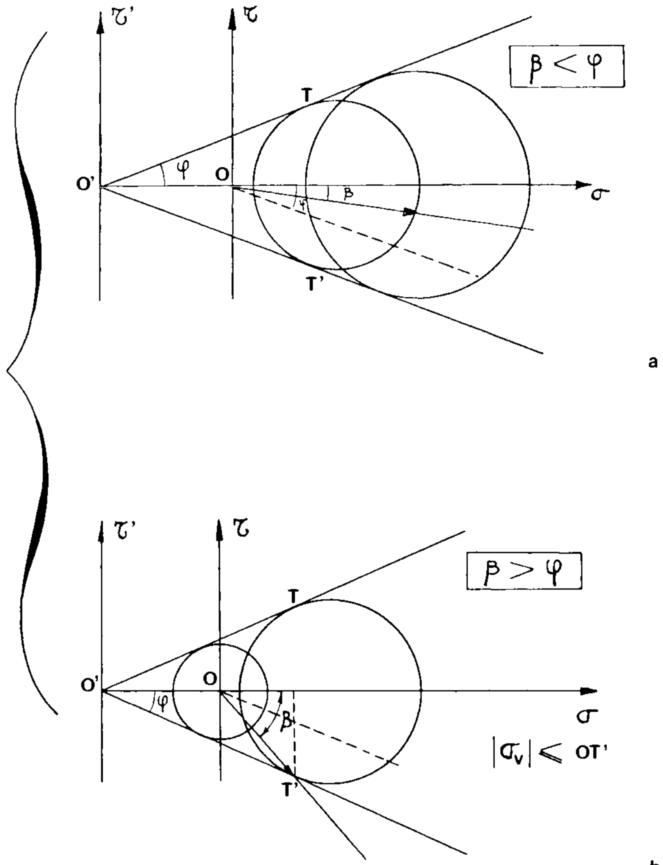


Fig. 5.34.

From which: $(PT', PT) = \pi/2 + \varphi$.

Asymptotic direction: whenever depth z becomes infinite, Fig. 5.35 shows that M tends toward infinite on line D. But, O′M = H + OM and H remains finite, therefore OM → O′M when $z \to \infty$. The asymptotes to the slip lines in the case of a cohesive soil are the straight slip lines of the cohesionless mass. When point M is at the surface, Mohr's circle is tangent at O to OT (Fig. 5.36). We then have:

$$(\Omega O, \Omega T) = \pi(2 + \varphi)$$

$$(\Omega O, \Omega T') = -(\pi/2 + \varphi).$$

Point O corresponds to the end of the stress vector O′O acting on the free surface. Therefore, from the classic property of Mohr's circle, the planes on which OT and OT′ act, are inclined to the slope surface by angles of $\pm(\pi/4 + \varphi/2)$.

Summary: In the case of $\beta < \varphi$ the failure lines belong to two families of

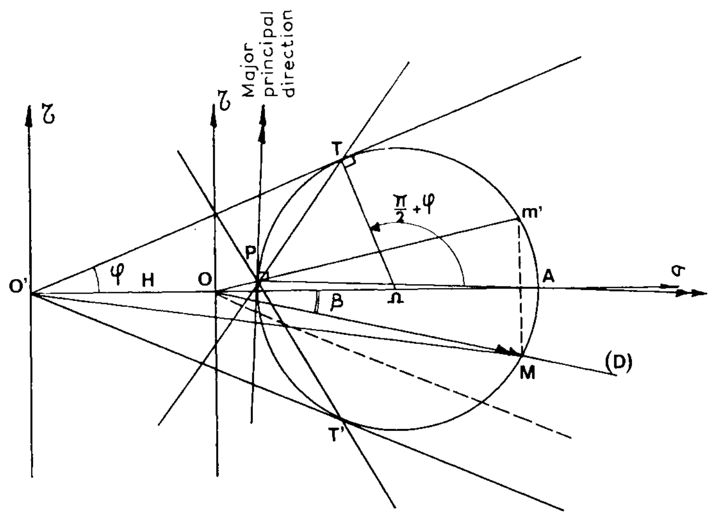


Fig. 5.35.

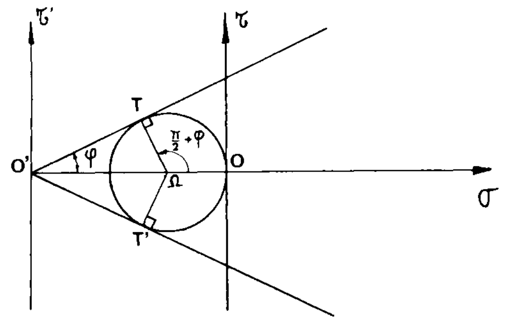


Fig. 5.36.

curves crossing at a constant angle to $(\pi/2 + \varphi)$, the asymptotive directions of which are the straight slip lines of the associated cohesionless mass of internal friction angle φ and crossing the free surface at an angle equal to $[\pm (\pi/4) + \varphi/2]$.

The failure planes orientations are shown in Fig. 5.37.

(b) Condition of $\beta > \varphi$ (Fig. 5.38)

Calculate the value of the limit depth z_l (Fig. 5.38). From the law of sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

applied to triangle $O'OT'$ (Fig. 5.39), we have:

$$\frac{\sigma_v}{\sin \varphi} = \frac{\sigma'}{\sin \beta} = \frac{H}{\sin (\beta - \varphi)}$$

We have also: $OK = \sigma_v \cos^2\beta = \gamma z_l \cos^2\beta$.

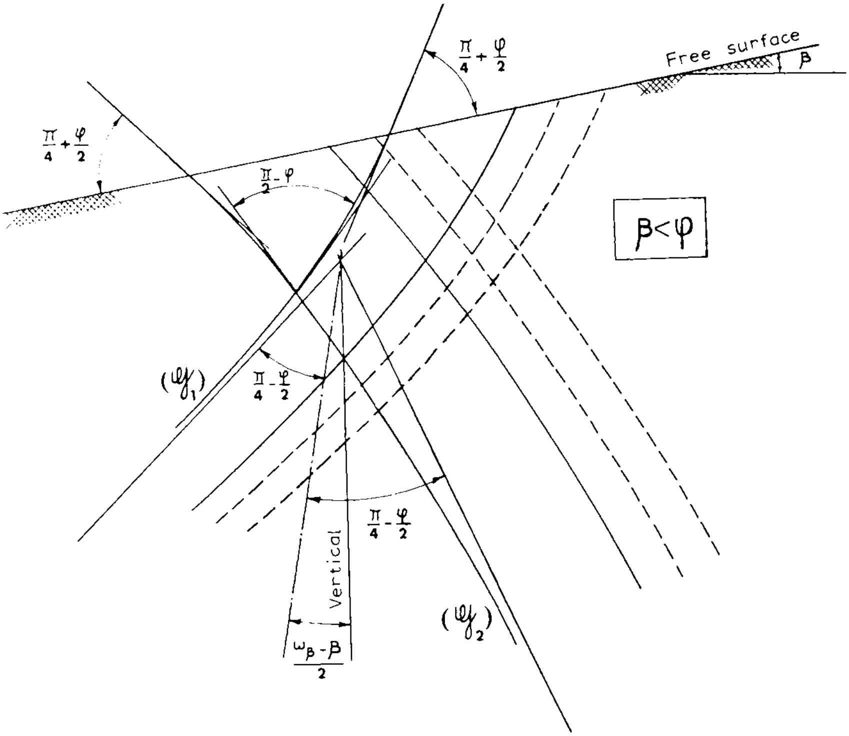


Fig. 5.37.

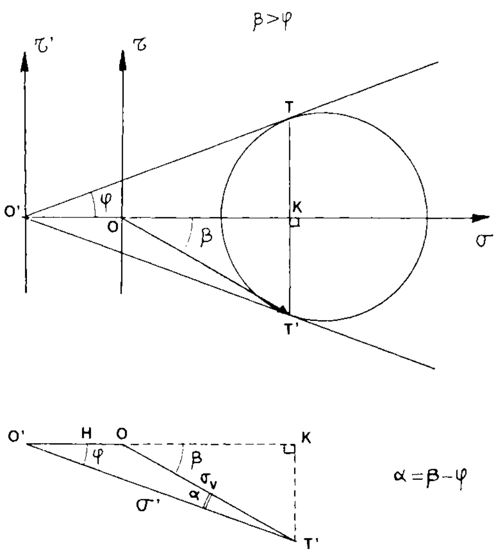


Fig. 5.38.

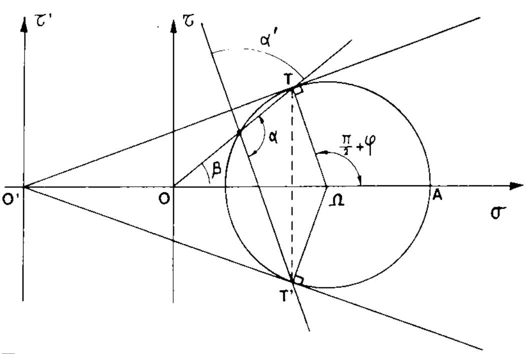


Fig. 5.39

Therefore:

$$\frac{H \sin \varphi}{\sin (\beta - \varphi)} \cos \beta = \gamma z_1 \cos^2 \beta.$$

from which:

$$z_1 = \frac{H \sin \varphi}{\gamma \cos \beta \sin (\beta - \varphi)} = \frac{c \cos \varphi}{\gamma \cos \beta \sin (\beta - \varphi)}.$$

Equilibrium only exists if the depth of the mass is less than or equal to z_1. *Slip lines*: when point M is at the surface, there are no changes from the preceding analysis. Slip lines cross the free surface at angles $\pm (\pi/4 + \varphi/2)$.

When point M is at depth z_1, OT (on Fig. 5.39) makes an angle β with $O\sigma$

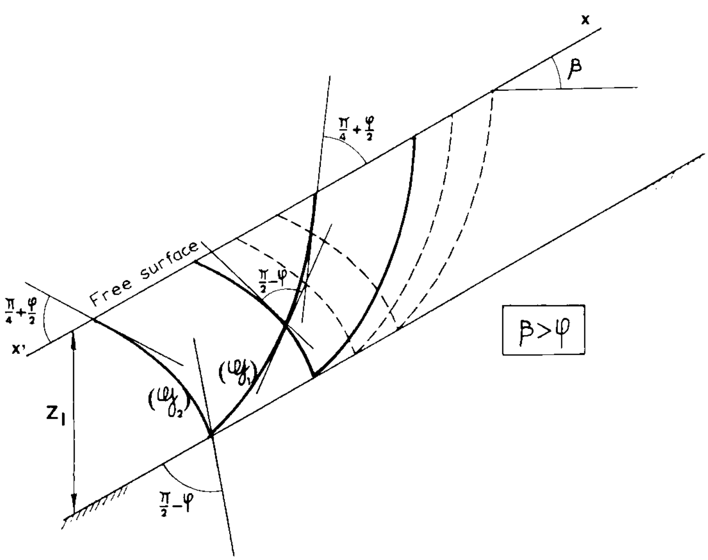


Fig. 5.40.

and one of the two families of curves representing the slip lines is tangent to a line parallel to the free surface at depth z_1.

We also have: $(\Omega T, \Omega T') = 2\alpha = 2(\pi/2 + \varphi) = \pi + 2\varphi$ from which: $\alpha = \pi/2 + \varphi$ and $\alpha' = \pi - (\pi/2 + \varphi) = \pi/2 - \varphi$.

The second family of curves is tangent to the line making an angle of $\pi/2 - \varphi$ with the free surface. This result is readily observed since as before, the slip lines cross each other at an angle of $\pi/(2 - \varphi)$.

The shape of the net of slip lines for the case $\beta > \varphi$ is shown in Fig. 5.40.

★★*Problem 5.7* Maximum height of an excavation in cohesive soil

Find the maximum height of an unsupported vertical cut in a cohesive soil with a horizontal surface which supports no load. Let γ, c and φ be the unit weight, cohesion and angle of internal friction respectively. For a numerical application, assume $\gamma = 20$ kN/m^3, $c = 10^4$ Pa, $\varphi = 10°$.

Solution

At a point M at depth z in the cut, the total vertical stress is equal to the weight of the overburden on a horizontal face through M (Fig. 5.41). The total horizontal stress acting on the vertical face through M is zero since M is at the free face of the cut.

Let us find at what depth z_1, Mohr's circle becomes tangent to the failure envelope at the lower Rankine limit equilibrium (Fig. 5.42).

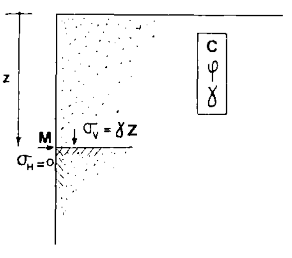


Fig. 5.41.

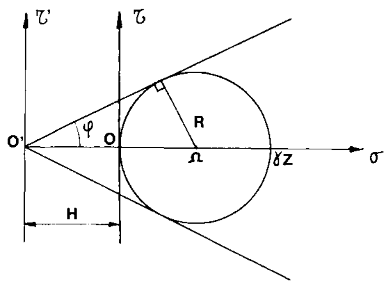


Fig. 5.42.

Let R be the radius of the circle, then:

$$\frac{R}{R+H} = \sin\varphi$$

from which: $R(1 - \sin\varphi) = H\sin\varphi = c \cdot \cot\varphi \sin\varphi = c \cdot \cos\varphi$
but: $R = \sigma_1/2 = \gamma z_1/2$ from which:

$$\frac{\gamma z_1}{2} = c \cdot \frac{\cos\varphi}{1 - \sin\varphi} = c \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right).$$

Therefore:

$$z_1 = \frac{2c}{\gamma} \tan(\pi/4 + \varphi/2). \tag{1}$$

Below depth z_1 the soil becomes in a plastic state. However, because of stress relaxation, the stresses will redistribute themselves in part to adjacent upper zones which are in elastic state. It is therefore possible that the maximum height may be somewhat greater than the calculated value of z_1 by the above formula (1). Terzaghi has proposed: $z_1' = 2.67\,(c/\gamma)\tan(\pi/4 + \varphi/2)$.

Numerical application:

$\gamma = 20\,\text{kN/m}^3 \quad \varphi = 10°, \quad c = 10^4\ \text{Pa} = 10\,\text{kN/m}^2$

$\tan(\pi/4 + \varphi/2) = \tan 50° \simeq 1.192$: $z_1 = 1.19\,\text{m}$ and $z_1' = 1.59\,\text{m}$.

★★★ *Problem 5.8* Superposition of two limit equilibrium states

Assume a two-dimensional configuration. Consider two limit equilibria at point M in a soil mass with an internal angle of φ. The two equilibrium states are defined by the Lamé's ellipses at M.

(1) Show that the superposition of the two-limit states is generally not one limit equilibrium state. Consider the Mohr's circles C_1 and C_2 corresponding to the two limit equilibrium states and Mohr's circle C corresponding to the stresses after superposition.

(2) Evaluate the ratio ρ of the radius of Mohr's circle C to that of Mohr's circle at failure centered at the same point Ω (let λ be the ratio of the radii of the circles C_1 and C_2). What conclusion may be drawn?

(3) Are there particular cases wherein the superposition leads to an incipient failure condition? What other notable conclusions can be drawn?

Solution

(1) Assume a cohesionless soil. The case of a cohesive soil is solved in the same manner but with fictitious stresses.

We may consider the minor principal direction of the first equilibrium state as vertical without changing the generalized solution. The minor principal direction of the second equilibrium state is assumed then to be at an angle α with the vertical. A face of a given orientation Δ will then have an angle α_1 and α_2 with the two minor principal directions (Fig. 5.43).

Consider the Mohr diagram of Fig. 5.44. For equilibrium state 1, the stress acting on the plane oriented in Δ is $\mathbf{OM_1}$ so that $\widehat{x\Omega_1 M_1} = -2\alpha_1$. The equilibrium state 2, $\mathbf{OM_2}$ is such that $\widehat{x\Omega_2 M_2} = -2\alpha_2$ to conform to the properties of Mohr's circle.

Whatever the direction of the plane may be, we always have $\alpha_1 - \alpha_2 =$

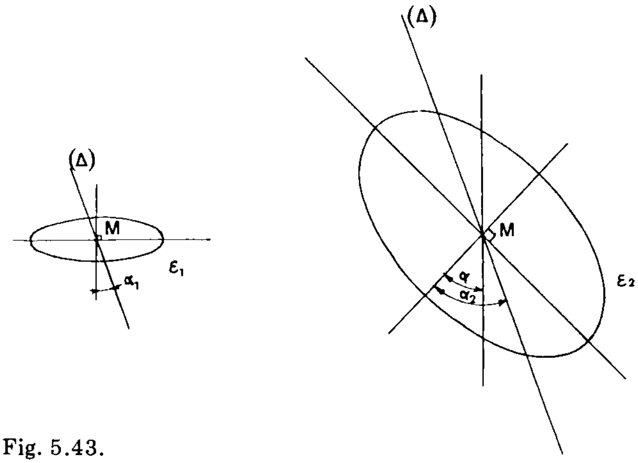


Fig. 5.43.

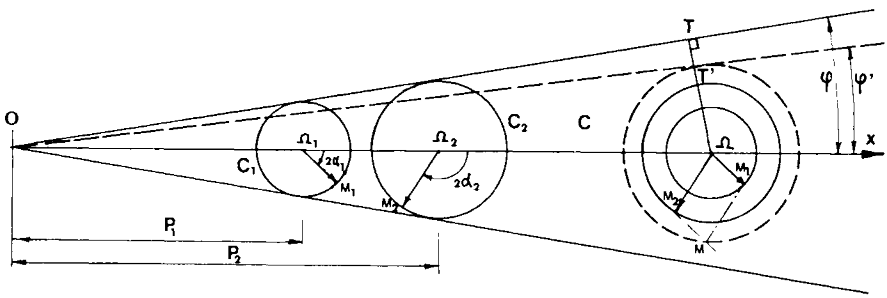


Fig. 5.44.

α = constant (for one given location M) because the angle α is the angle of the principal stresses of the two states at M.

To superimpose the limit equilibrium conditions at M, we add vectorial stresses acting on each plane. The result is:

$\mathrm{OM} = \mathrm{OM}_1 + \mathrm{OM}_2 = \mathrm{O}\Omega_1 + \mathrm{O}\Omega_2 + \Omega_1\mathrm{M}_1 + \Omega_2\mathrm{M}_2 = \mathrm{O}\Omega + \Omega\mathrm{M}$.

When the orientation of plane Δ changes, we have:

$\mathrm{O}\Omega_1 + \mathrm{O}\Omega_2 = \mathrm{O}\Omega =$ constant

and: $\Omega_1\mathrm{M}_1 + \Omega_2\mathrm{M}_2 = \Omega\mathrm{M}$ with $|\Omega\mathrm{M}|$ = constant, since α is constant.

Therefore, the locus of M is a circle C. But this circle is not tangent to the failure envelopes. Therefore: the superposition of the two equilibrium states is generally not an incipient failure condition.

(2) From Fig. 5.44, we see:

$$\rho^2 = \frac{\Omega M^2}{\Omega T^2} = \frac{\Omega M_1'^2 + \Omega M_2'^2 + 2\Omega M_1' \cdot \Omega M_2 \cos 2\alpha}{(\Omega M_1' + \Omega M_2')^2}$$

but:

$$\lambda = \frac{O\Omega_1}{O\Omega_2} = \frac{\Omega M_1'}{\Omega M_2'}, \quad \text{from which:}$$

$$\rho^2 = \frac{\lambda^2 + 1 + 2\lambda \cos 2\alpha}{(\lambda + 1)^2}.$$

We always have $\rho \leqslant 1$; hence Mohr's circle is always inside the failure envelope.

The superposition of the two limit equilibria results in a stress condition which can actually exist but is generally on the safe side because $\rho < 1$.

(3) If $\alpha = 0$, we have $\rho = 1$. The directions of the principal stresses are the same. We then have $\Omega T = \Omega T'$ and the superposition of the two limit states is still a condition of limit equilibrium. In this case, the slip lines net can be superimposed.

If $\alpha = \pi/2$:

$$\rho = \frac{\Omega T'}{\Omega T} = \frac{\Omega_1 M_1 - \Omega_2 M_2}{\Omega_1 M_1 + \Omega M_2} = \frac{p_2 - p_1}{p_2 + p_1}.$$

Furthermore, if $p_2 = p_1$ (see Fig. 5.44), then $\rho = 0$ and the Mohr's circle reduces to a point.

Hence by superimposing two equal stress conditions whose principal directions are at right angle, an isotropic stress condition is obtained.

Chapter 6

INTERPRETATION OF IN-SITU TESTS

★*Problem 6.1* Interpretation of static penetration tests

A soil investigation was performed by the static penetrometer along the Rhône river. Test results were very uniform and are summarized by the diagram of Fig. 6.1.

Determine what the possible types of foundation are and the allowable bearing capacities for a four-storey building with basement requiring a 2 m deep excavation (the sand layer will therefore be excavated).

The building will be 10 by 20 m in plan dimensions. Columns will be 4 m on center. The total weight of the structure will impart an average stress over its plan dimensions equivalent to 40 kPa. The ground-water table is 1.5 m below the surface.

Soil unit densities: $\gamma_h/\gamma_w = 1.8$ *above the water table,* $\gamma'/\gamma_w = 1$ *below the water table.*

Solution

From the penetration diagram of Fig. 6.1 it can be concluded that the soil conditions consist of a shallow surface sand layer, some 2 m thick, underlain by a thick clay layer.

In fact the ratio of R_f/q_c called friction ratio [19], [22], [23] for each of the layers is less than 2% and over 4%.

It is not possible, however, to determine the foundation dimensions because one important given for the problem is missing, namely, the type of penetrometer used in the investigation. It is not known if the cone of the penetrometer is of the Delft type or a simple cone type [22], [23], [29], [30][1].

Without additional information, a dangerously erroneous interpretation could be made due to the low shear strength of the clay.

★★*Problem 6.2* Interpretation of a dynamic penetration test

A dynamic penetration test was performed at a construction site. The test result is shown in Fig. 6.2 in the form of the number of blows counted for 20 cm penetration at the respective depth increments.

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[1] The solutions to this problem are given in problems 6.3 and 6.4.

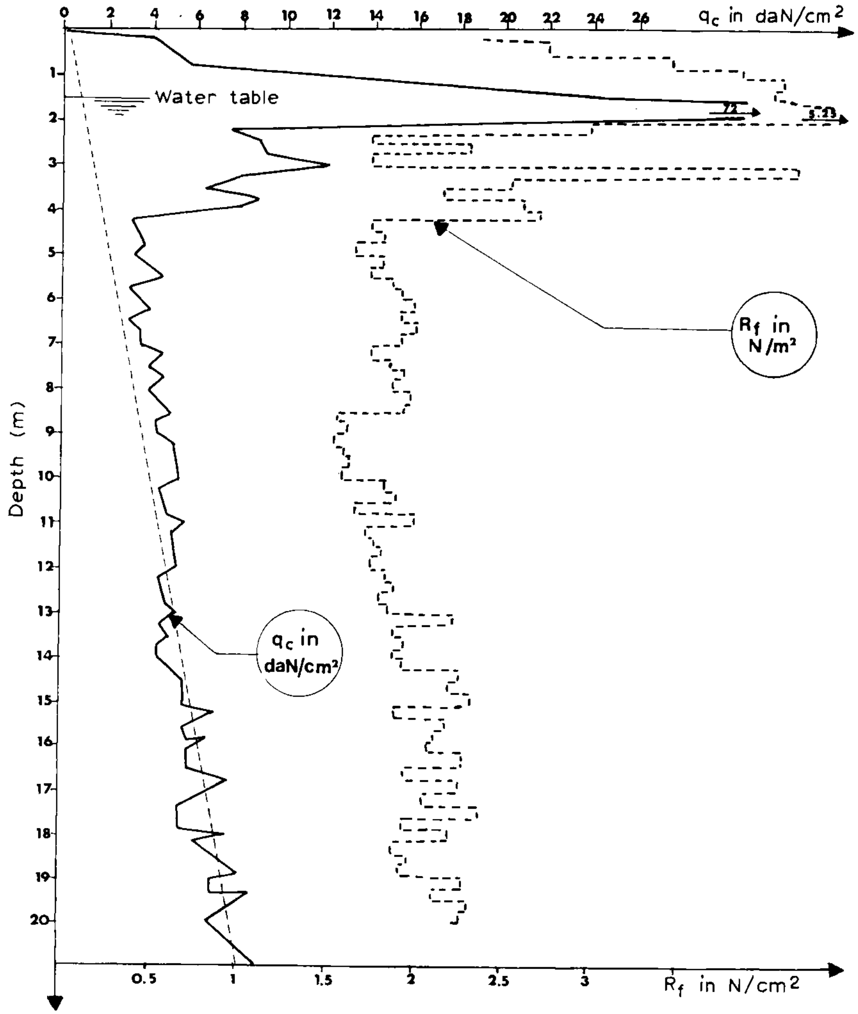


Fig. 6.1. Static penetration test.

What is the allowable bearing capacity of shallow footings at a depth of 3 m for a three-storey structure with basement?

Solution

It is not possible to make a valid evaluation of the diagram of Fig. 6.2 because the following needed information is missing to characterize N_{20} [23], [29]:

— type of penetrometer and height H of fall of the hammer (is the fall height constant?);

— weight of the hammer and of the rods;

— diameter of the rods, and cross sectional area of the point;

— was the hole cased with hollow rods of outer diameter equal to the point diameter to eliminate parasitic side friction?
— was drilling mud used to stabilize the hole?
— type of soil tested;
— location of the ground water table.
On the other hand, one test only has not much meaning. It is necessary to perform several tests at various locations within a site in order to appreciate eventual variations. Finally, it is always preferable, for ultimate users of the diagrams of dynamic penetration tests, to draw the plot values of R_d vs. depth, where the resistance value R_d is computed by the Dutch formula and a safety factor of 1, shown as $R_d = M^2H/(M+P)eA$ where e is defined as the average penetration by blow.

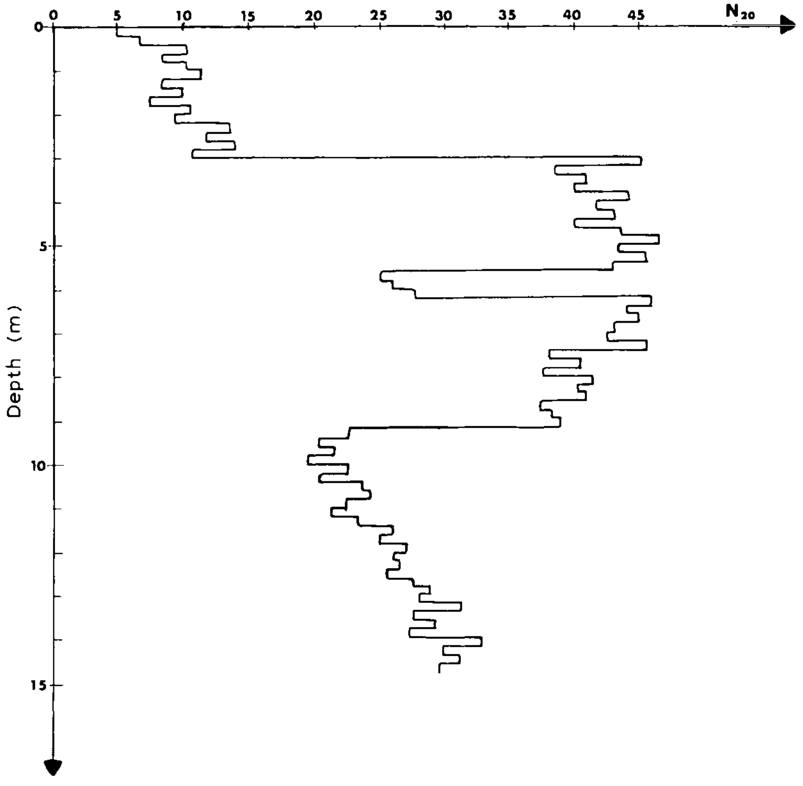


Fig. 6.2.

★★*Problem 6.3* Interpretation of a Delft-type static cone penetrometer test in clay

The givens are similar to those of problem 6.1, with the added information that the cone penetrometer test was performed with a Delft-type cone pushed at a rate of 2 cm/s penetration [19, 22, 23, 29].

Determine the type of foundation and allowable bearing capacity of the soil to support an apartment building, 4 storeys high with basement located 2 m below finished exterior grade. Refer to problem 6.1 for the geometry of the building.

Solution

The clay layer is very thick ($q_c = 400$ kPa, $R_f = 20$ kPa, friction ratio $FR > 4\%$) and contains lenses of sand ($2.4 < q_c < 7.2$ MPa and $FR < 2\%$) [29, 30].

The penetration diagrams of Fig. 6.1 shows that it would not be practical to recommend a semi-deep pier-type foundation or a pile foundation bearing on a very dense layer. The diagram shows no good bearing layer at depths down to 20 m. A pile foundation would have to be designed with friction piles whose performance is sensitive to local soil variations and to differential settlements. The only other solution is to recommend a shallow foundation.

For clays, the apparent cohesion of the soil may be evaluated from the penetration diagram. From this value then, the allowable bearing capacity of the soil may be calculated for different types of footings.

For a Delft-cone, the correlation between cone resistance and cohesion is $c_u = q_c/15$.

From the average value of the end bearing $q_c = 400\text{ kPa} = 4\text{ daN/cm}^2$, we have $c_u = 400/15 = 26.7$ kPa.

Let us now first look at strip footings embedded 1 m below grade. For a purely cohesive soil ($\varphi = 0$) the allowable bearing capacity of the soil is: $q_{ad} = \gamma D + 5.14\,c_u/3$.

For short-term stability, and therefore considering total stresses, and for $\gamma = 18\text{ kN/m}^3$ and $D = 1$ m, we have $q_{ad} = 18 \times 1 + (5.14 \times 26.7)/3 \approx 64$ kPa.

It must be remembered that in most cases, the long-term stability of bearing capacity is higher than that of short-term.

The foundation load imposed by the building of 4 stories is, for a spacing of footings of 4 m, about: $Q = 40\text{ kPa} \times 4 = 160\text{ kN} \simeq 16$ t (per m width). The width of the strip footing would then be: $B = Q/q_{ad} = 160/64 = 2.50$ m.

Once the width of a footing is larger than half the spacing distance, it is generally admitted that a mat-type foundation is more economical. The formula to use then for this type of foundation is:

$$q_{ad} = \gamma D + \frac{5.14(1 + 0.2\,B/L)c_u}{3}$$

where γD corresponds to the overall embedment (2 m), $\gamma D = 18 \times 2 = 36$ kPa, $B = 10$ m, $L = 20$ m.

$$\text{We get:}\quad q_{ad} = 36 + \frac{5.14[1 + (0.2 \times 10/20)] \times 26.7}{3} = 86.3$$

$$\simeq 86\text{ kPa}.$$

But the pressure imparted by the total weight of the building is 40 kPa; furthermore, 2 m of soil were excavated which represent a vertical stress of 36 kPa. The net pressure of the building is only 4 kPa. The raft foundation is the preferred one for this case. (The buoyant force of the ground water is generally not counted upon since the level of the water may vary.)

An additional advantage of the mat-type foundation resides in the ease with which the basement floors may be made impervious.

It should be noted that certain authors use slightly different correlations to estimate c_u from the penetrometer test, such as $q_c/18 < c_u < q_c/15$ when measured with the Delft-type cone pushed at 2 cm/s.

★★*Problem 6.4* Interpretation of a static penetrometer test with simple cone point in the clay

Once again consider the givens of problem 6.1 but assume that the penetration diagram of Fig. 6.1 pertains to a test done with a simple cone (a simple cone is one with a constant cross-sectional circular area above the cone, as opposed to the Delft-cone). Recommend the foundation type and allowable bearing capacity of the clay, for the building under consideration.

Solution

The initial reasoning presented in problem 6.2 is valid here. Therefore only shallow foundations are considered.

The apparent cohesion, c_u is determined to calculate the bearing capacity. Instead of using the correlation $q_c/15$, it is now necessary to use $c_u = q_c/10$.

For an average value of $q_c = 4\,\text{daN/cm}^2$, this gives $c_u = 4/10 = 0.4\,\text{daN/cm}^2 = 40\,\text{kPa}$.

The importance of knowing the type of penetrometer used is well illustrated here if this result is compared with that of problem 6.3.

Let us consider the strip footing with 1 m embedment. From the same formula of problem 6.3, we have:

$$q_{ad} = 18 \times 1 + \frac{5.14 \times 40}{3} = 87\,\text{kPa}.$$

The load carried by interior partition is $Q = 160\,\text{kN}$ (per meter width). The maximum width of the strip footing would then be:

$$B = Q/q_{ad} = 160/87 = 1.84\,\text{m}.$$

In this instance, the width of footing is less than half the spacing between line loads and this type of footing is more economical than would be a raft foundation.

In conclusion, it is seen that depending on the type of penetrometer used, for a given diagram one may recommend differing types of foundation. This illustrates the importance of knowing the type of static-cone penetrometer utilized. The test specification should always be included in the test results [22, 23, 29, 30].

★*Problem 6.5* Interpretation of a dynamic penetration diagram

On a fairly horizontal site, dynamic penetration tests were performed. The results were very closely grouped. The least resistance was encountered at one test location whose diagram is shown in Fig. 6.3.

To develop the site, about 1 m of the upper soil will have to be excavated. Under these conditions, determine the allowable bearing capacity for foundations of an industrial building whose footings bottoms will be 1 m below finish grade (that is 2 m below the 0-level of the penetration diagram). The building will be supported by steel columns, 10 m on center each way and each supporting an axial load of 1000 kN.

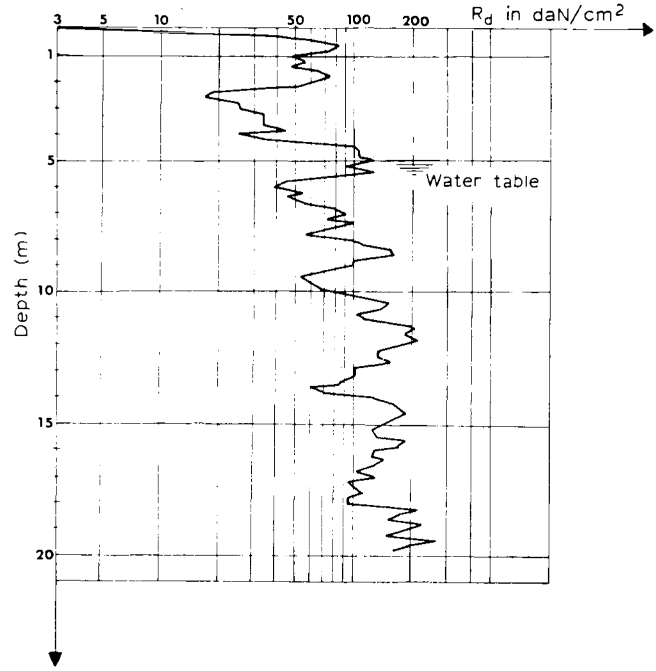


Fig. 6.3. Dynamic penetration test.

Solution

As was the case in problems 6.1 and 6.2, it is not truly possible to solve the problem. From an examination of the diagram, one could conclude that no resistance is expected below a low value of 18 daN/cm^2 and from the usual formula, one would conclude: $q_{ad} = R_d/20$ (for cohesionless soils) or 90 kPa would be an allowable bearing pressure. For a net load of 630 kN at the surface, a 7 m² footing would support the load (or square footings of 2.6 x 2.6 m). Note that the 1000 kN load is not obtained. But it must be remembered that it is not possible to interpret a penetration diagram without knowing the type of equipment used in testing, and the soil type. The givens of problem 6.5 are not sufficient to be of value. If this conclusion appears too critical, consider the diagram of Fig. 6.4 on which, in addition to the dynamic penetrometer diagram, a static-cone diagram obtained with the Andina penetrometer has been drawn for comparison purpose. The dynamic penetration data were in fact obtained with a Durmeyer device.

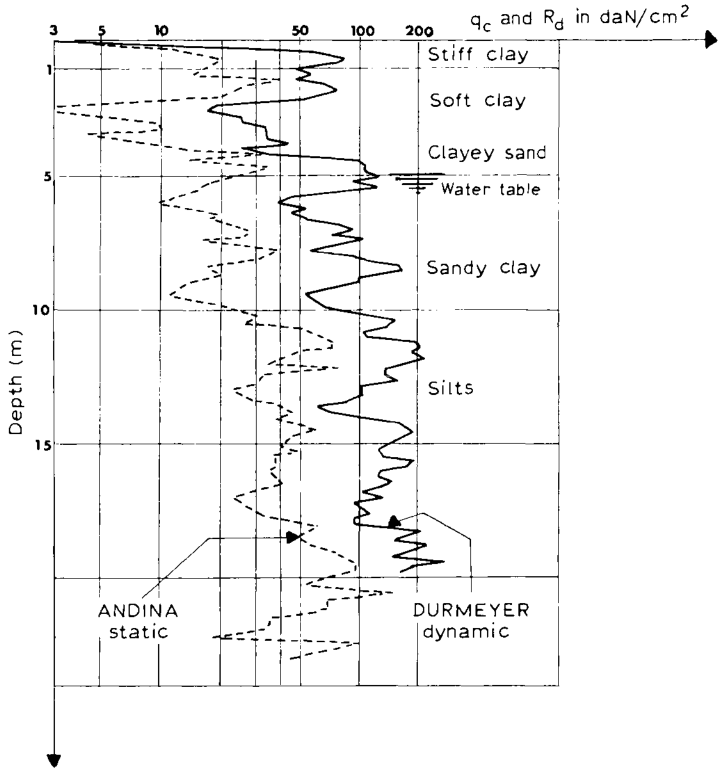


Fig. 6.4. Comparison between static Andina and dynamic Durmeyer penetration tests.

For a foundation level proposed at 2 m, the point bearing resistance of the static-cone penetrometer test is almost zero and the bearing of 90 kPa would not have been acceptable.

The main conclusion to be drawn from this problem is that it is not possible to correctly interpret dynamic penetration tests in soft clays, nor in any clay below the water table [13, 15].

★*Problem 6.6* **Interpretation of a dynamic penetration test of the Sermes type**

Several Sermes-type dynamic penetration tests were performed at a site in Salies-de-Bearn (France). Assume that the results obtained are represented by the diagram of Fig. 6.5, which show conventional resistance, R_d, as a function of depth.

Can this test be interpreted? What would be the allowable bearing pressure for pier foundations located 4 m deep, having a diameter of 1 m and spaced 10 m on center?

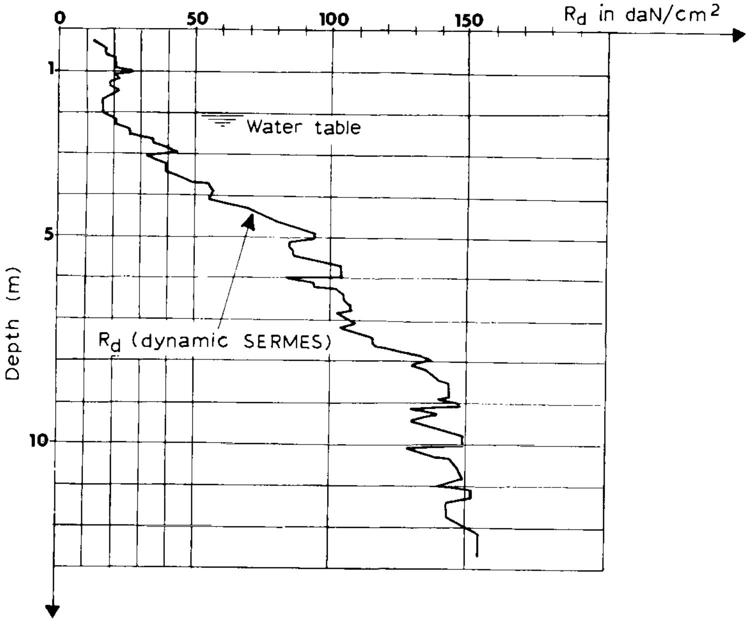


Fig. 6.5. Dynamic penetration test (Sermes).

Solution

It really is not possible to interpret the diagram because we do not know the soil type and there are no indications as to whether the test was performed with or without drilling mud. The solution to this problem is presented in problem 6.7 where all data are provided in the givens.

★★*Problem 6.7* **Interpretation of a dynamic penetration test of the Sermes-type in submerged clay soil**

The conditions are similar to those of problem 6.6. We know the standard procedures for the Sermes-type test were followed, that is, without the use *of bentonite mud. We know that the soil is clay and that the water table is very shallow.*

Interpret the diagram for the foundation type described in problem 6.6.

Solution

It is not always possible to interpret the diagram correctly because the test was performed without mud and in clay soil below the water table. Under such conditions, the dynamic penetrometer often gives erroneous indications because of parasitic friction which develops along the tubes due to the squeezing of the clay around them. At the same site, a static-cone penetrometer test was also performed with the Gouda-type penetrometer. The diagram is shown on Fig. 6.6 along with that of the dynamic test.

The comparison between the two shows the danger associated with interpreting dynamic tests. Note the very large difference between the resistances R_d and q_c (the ratio R_d/q_c is larger than 10 below 5 m depth). The difference is mainly due to the lateral friction of the soil against the rods and to a smaller extent due to the instantaneous increase in pore-water pressure which is greater during dynamic testing than in static testing.

To further illustrate the danger, let us interpret the diagrams and calculate the allowable bearing capacity at a depth of 4 m for a long footing.

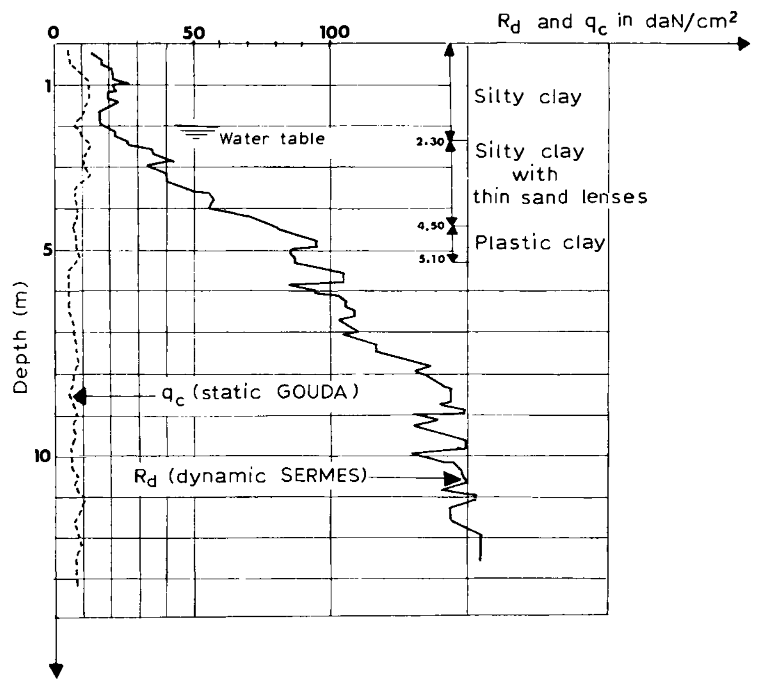


Fig. 6.6. Comparison between the static Delft-cone and the dynamic test in clay.

Dynamic penetration test. A general rule often used to determine allowable bearing pressure for shallow footings or semi-deep foundation is: $R_d/20$. At 4 m, $R_d > 6000$ kPa (60 daN/cm^2) from which:
$q_{ad} = 6000/20 = 300$ kPa (3 daN/cm^2).

Static penetrometer test. The Dutch-type penetrometer (Gouda) has a Delft-type cone. q_c is practically constant and equal to 800 kPa (or 8 daN/cm^2). Cohesion of the clay may be estimated at:

$$c_u = q_c/15 = 800/15 = 53\,\text{kPa}\ (0.53\,\text{daN/cm}^2)$$

and the allowable bearing pressure would then be:

$$q_{ad} = \frac{5.14\,c_u}{3} = \frac{5.14 \times 53}{3} = 90\,\text{kPa}\ (0.9\,\text{daN/cm}^2)$$

neglecting the depth term γD.

The allowable bearing pressure calculated from the dynamic penetrometer is about equal to the ultimate bearing capacity calculated by the static penetrometer. These results speak for themselves.

★*Problem 6.8* Interpretation of a Bevac-type dynamic penetration test in submerged clay soil

The Bevac-type penetrometer was used at a site near Nantua (France), in a very thick layer of soft clay and a water table at about 1 m depth. The results of the test are presented in Fig. 6.7.

Can this diagram be interpreted? If so, give the allowable bearing capacity of a strip footing of 1 m width constructed at 2.5 m depth.

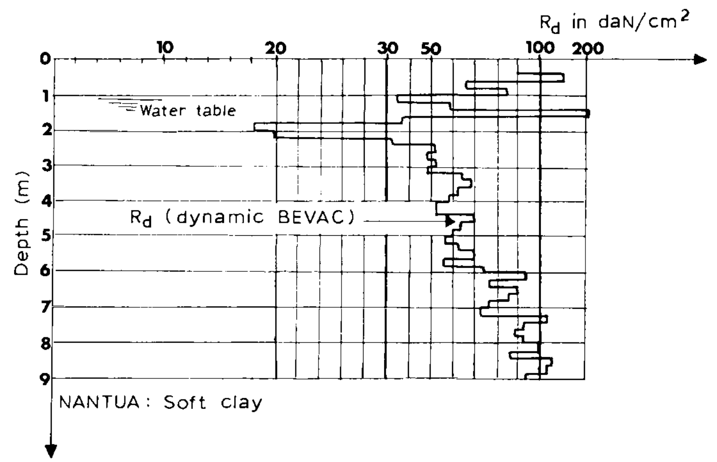


Fig. 6.7. Dynamic penetration test (Bevac).

Solution

This type of dynamic test in clay below the water table cannot be interpreted for the same reasons as cited in problem 6.7 [13, 15].

To confirm this conclusion, two static penetrometer tests were performed at the same site, one with the Gouda- and the other with the Andina-penetrometers, very near to the dynamic penetrometer test. The results are shown in the diagram of Fig. 6.8. Once again a large difference between R_d and q_c is noticeable. The two static-cone tests, however, agree quite well with each other. The difference is due to the same causes as explained in problem 6.7.

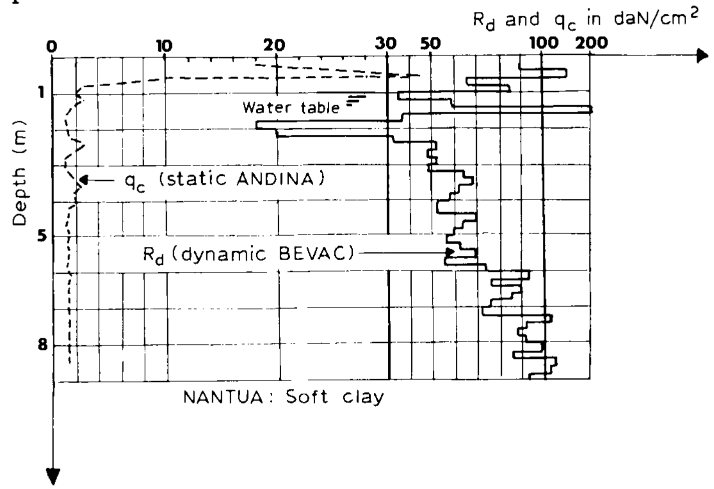


Fig. 6.8. Comparison between static-cone penetrometer and dynamic penetration test.

If we took $R_d/20$ (as for cohesionless soils) to calculate the allowable bearing pressure at 2.5 m depth, we would get: $q_{ad} = 250$ kPa (2.5 daN/cm^2). On the other hand, for the static-cone penetrometer test: $q_c = 180$ kPa (simple cone) or $q_c = 300$ kPa (Delft-cone).

The Andina penetrometer has a simple point, therefore:

$$c_u = q_c/10 = 180/10 = 18\,\text{kPa}\ (0.18\,\text{daN/cm}^2)$$

and the allowable bearing pressure for a smooth strip footing at 2.5 m is:

$$q_{ad} = \frac{5.14\,c_u}{3} = \frac{5.14 \times 18}{3} = 30\,\text{kPa}\ (0.3\,\text{daN/cm}^2)$$

The allowable bearing pressure is 8 times less with the static penetrometer than with the dynamic penetrometer. It should be noted that at this site, a raft-type foundation designed for 30 kPa underwent substantial settlements. See problem 6.16 and Ref. [14].

It should be remembered that when $q_c < 1200$ kPa, it is absolutely necessary to recover undisturbed samples for laboratory testing.

★★★*Problem 6.9* **Settlement calculations from cone penetrometer diagrams in the case of a two-layer system**

A soil investigation was carried out using the static-dynamic Andina-type penetrometer for an industrial complex near Berre Lake, France (cf. [22], [23] and [29]). Assume that the diagram of Fig. 6.9 represents the results of all tests performed. Drilling and sampling made at the site confirmed the soil profile to be (see Fig. 6.10): from 0 to 4 m: cemented, very dense sand and gravel; from 4 to 8.5 m: compressible silt; from 8.5 m: dense silty sand.

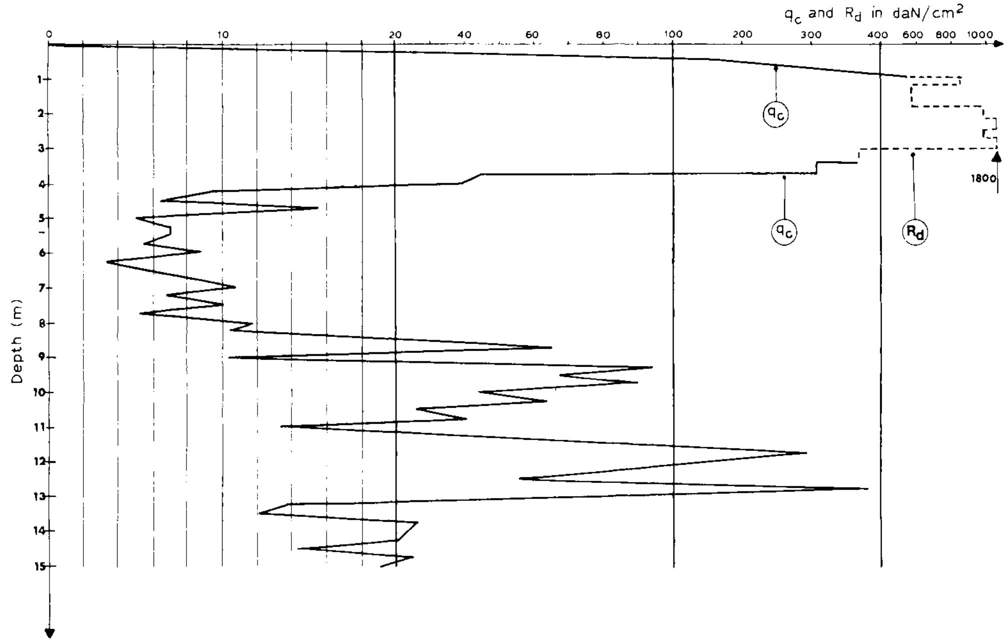


Fig. 6.9. Berre Lake static-dynamic Andina penetrometer test result.

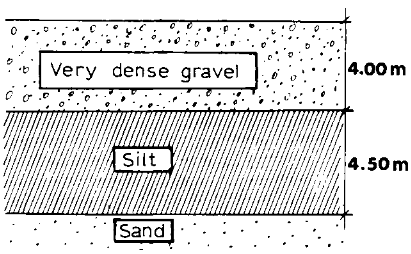


Fig. 6.10.

Consolidation tests were performed on undisturbed samples of the compressible silt recovered at locations very near to those of cone-penetrometer tests.

The observed correlation between the point resistance q_c and oedometric modulus E' indicates that the coefficient α is equal to 3 and is valid for the whole site ($m_v = 1/\alpha q_c$).

Assuming that the upper sand and gravel and lower sand layers are incompressible, calculate the settlement of a strip footing, 2 m wide, imparting a pressure of 400 kPa and located at 0.5 m below existing grade.

Could the settlement be reduced, assuming the same total load if the width of the footing was increased to 4 m, therefore reducing the pressure to 200 kPa?

Note: It may appear not logical to correlate an 'elastic' characteristic, such as the modulus of consolidation to an ultimate strength of the penetrometer. In fact, the soil behavior is a very complex phenomena which always consists of some reversible and other irreversible mechanisms. Perfect elasticity and plasticity are only approximations. During the two types of tests, both phenomena occur, which may account for the possibility of empirical correlation observed between E' and q_c. It should be remembered that the various correlations established are only valid for well defined soil types.

Solution

First method. Settlement calculations from the Giroud-tables. The tables for the calculation of settlement of J. P. Giroud, give formulas which permit direct calculation of settlements for various footing types bearing on homogeneous soils of finite thickness and whose deformation modulus is known and bearing on an unyielding substratum ([10]).

Formulas have been developed for the case of footings bearing directly on the compressible layer. However, at the Berre Lake site, there exists a sand and gravel layer overlaying the compressible layer. The analysis, therefore, may be as follows. Because the sand and gravel layer is very dense and very rigid, the soil stress at the top of the silt layer may be calculated by assuming a stress distribution of 45° through the cemented sand and gravel layer as shown in Fig. 6.11. Normally, such a distribution is assumed to spread over a 30° angle according to the pressure bulb of Boussinesq.

By utilizing the notations of Giroud tables*, the spread at the top of the silt layer is: $2a = 2 + 3.5 \times 2 = 9\,m$ and the stress is: $p = 400 \times 2/9 = 89$ kPa ($0.89\,\text{daN/cm}^2$) with $H/a = 4.50/4.50 = 1$ and $\nu = 1/3$. From the Giroud table, p. 399* we get: $\bar{p}_H = 0.31$.

The settlement is given by the formula $w = p \dfrac{2a}{E} \bar{p}_H$.

*Printed by Dunod, Paris in 1972 and 1973 [10].

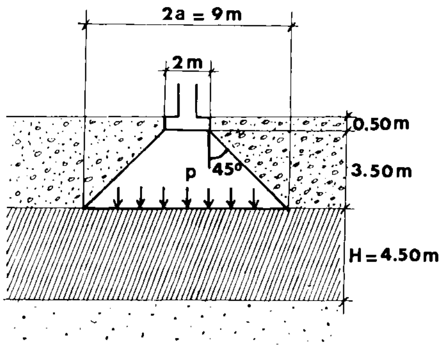


Fig. 6.11.

The oedometric modulus is obtained from the static penetrometer test: $E' = \alpha q_c = 3q_c$.

The average value of q_c in the silt layer is calculated from depth 4–8.5 m, and it is $q_c \simeq 800$ kPa, from which:

$$E' = 3 \times 800 = 2400\,\text{kPa} \qquad (24\,\text{daN/cm}^2)$$

and for $\nu = 1/3$, Young's modulus is $E = 2/3\,E' = 16\,\text{daN/cm}^2$.

Finally, the settlement is $w = (0.89 \times 900/16) \times 0.31 = 15.5$ cm or $w =$ 16 cm.

Let us now consider the case of the 4 m wide footing, imparting a pressure of 200 kPa.

We have $2a = 4 + 2 \times 3.50 = 11$ m, from which $a = 5.50$ m. Therefore, $H/a = 4.50/5.50 = 0.82$ and: $\bar{p}_H = 0.25$, $p = 200 \times 4/11 = 73$ kPa (0.73 daN/cm^2) and the settlement is: $w = (0.73 \times 1.100/16) \times 0.25 = 12.5$ cm say $w \simeq 13$ cm.

The settlement is decreased very little by doubling the width of the footing. This is accounted for by the rigidity of the upper layer. It would therefore not be advisable to recommend an increase of the footing width. The cemented sand and gravel layer acts like a mat foundation.

Second method. A more classical method is to determine the stresses and then calculate the settlement layer by layer, using the formula $\Delta h/h = -\Delta\sigma/E'$.

Let us first consider the stress distribution at depth for a flexible footing and calculate the stress at mid-height in the silt layer as well as at the upper and lower boundaries of this layer.

Once again, the Giroud table can be used, and Table 6A is compiled (results along the footing axis).

The silt layer is divided into two equally thick layers of 2.25 m. The average stress in the upper half-layer is:

$$\Delta\sigma_1 = (140 + 88)/2 = 114\,\text{kPa} \qquad (1.14\,\text{daN/cm}^2)$$

TABLE 6A

z_{Average}	z/a	$\bar{k}_0$	$\Delta\sigma$ (kPa)
3.50	3.50	0.35	140
5.75	5.75	0.22	88
8	8	0.16	64

and in the lower half-layer is:

$\Delta\sigma_2 = (88 + 64)/2 = 76\,\text{kPa} \qquad (0.76\,\text{daN/cm}^2).$

Assuming that the modulus E' is the same for both half layers; the settlement is: $\Sigma_{\Delta h} = \dfrac{h}{E'}(\Delta\sigma_1 + \Delta\sigma_2) = \dfrac{225}{2400}(114 + 76) = 17.8\,\text{cm}.$

If we now assume the footing to be rigid, and using Giroud's table (Vol. II, p. 365) we can make up Table 6B.

The same results are obtained for all practical purposes in assuming the footing to be rigid. This is understandable because for ratios of z/a over 3 the stress distribution for strip footings is practically the same, regardless of the assumption on the rigidity of the footing.

TABLE 6B

z_{Average}	z/a	$\bar{k}_0$	$\Delta\sigma$
3.50	3.50	0.34	136
5.75	5.75	0.22	88
8	8	0.16	64

The conclusion would not have been the same had the footing been resting directly on the silt layer.

Comparison between the two methods. Settlements computed by the second method are somewhat higher than those of the first method. This is because in the second method stress conditions are assumed which correspond to a homogeneous soil and no account is taken of the high rigidity of the cemented sand and gravel layer. The first method therefore is considered more representative of reality.

A third method could be used, by referring to the tables of Bottero and Touzot for settlements on a two-layer system. Assuming a value of 1200 daN/cm^2 for the modulus of the cemented sand and gravel layer, a settlement of 10.3 cm would be obtained for the 2 m wide footing. This is close to the result of the first method.

★★*Problem 6.10* **Settlement calculations based on Schmertmann's method and static penetrometer tests in gravels and sands**

The city hall of Lyons consists mainly of an administration block supported by 4 concrete columns of 10 by 10 m (see Fig. 6.12) containing stair wells, elevators and utility shafts. Spacing between the columns is 20 m. Each column rests on a square footing bearing a net load of 70 MN after considering an excavation of 10 m of soil from ground level.

Over the site of this building, the soil profile is:

— fill, 1 to 2 m thick;

— clayey silt, 1 to 2 m thick;

— sandy gravels to a maximum thickness of 20 m;

— sandstone substratum;

— water table at about 5 m below natural grade.

A penetration test was performed at the location of one of the columns with an Andina static-dynamic penetrometer after excavation. The test results are shown in Fig. 6.13.

Over a depth of about 10 m, dense gravels were encountered which are underlain by incompressible sandstone whose characteristics are well known in the area.

Assuming a coefficient $\alpha = 3$, calculate the settlement of a column from the test result, using Schmertmann's method (cf [35]).

We have: $\gamma/\gamma_w = 2$ for the soil above the water table, and $\gamma'/\gamma_w = 1$ for the soil below the water table.

Solution

Schmertmann's method is valid only for cohesionless soils. It consists of drawing a very simple vertical stress diagram at depths due to a rigid footing.

The stress distribution is assumed to be triangular and the maximum stress occurs at depth $B/2$, where B is the width of the footing, and that below $B/2$, no appreciable vertical stresses exist in the soil. (Fig. 6.14). If Δp is the stress increase due to the footing, according to Schmertmann, at depth $B/2$, the stress is $0.6\,\Delta p$. Correction factors are needed as indicated below.

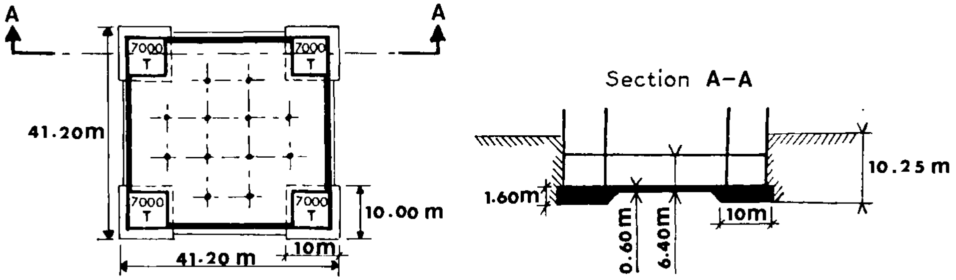


Fig. 6.12. City Hall of Lyons: plan and section.

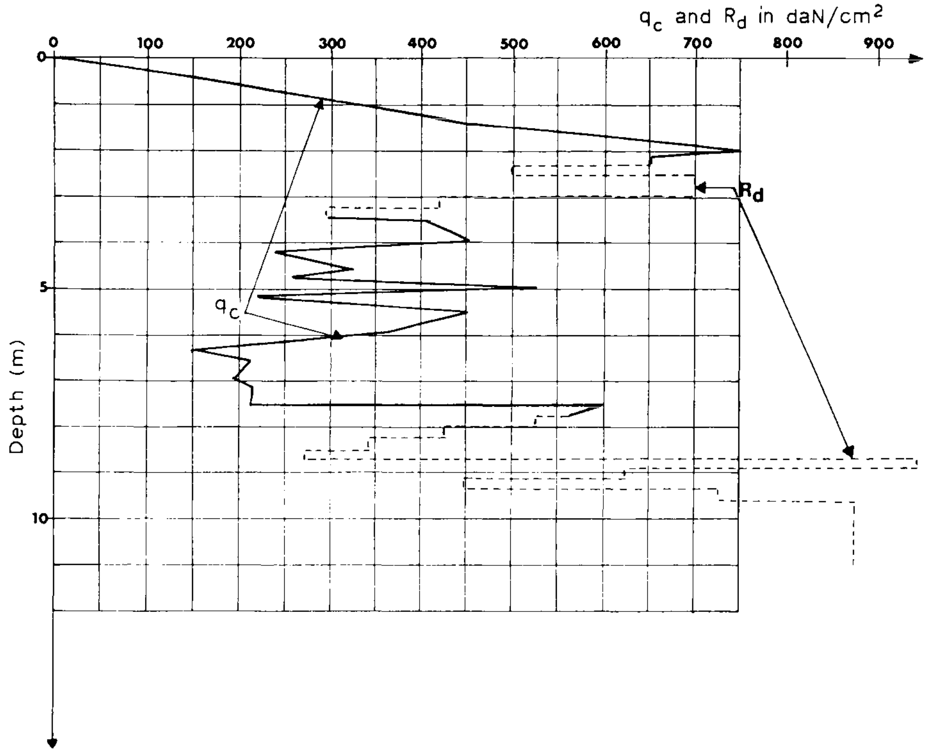


Fig. 6.13. City Hall of Lyons: static-dynamic penetrometer test result.

Influence due to embedment: $C_1 = 1-0.5 \quad (\sigma'_0/\Delta p)$.

σ'_0 effective stress exerted by the weight of overburden above the bottom of the footing, therefore:

$$\sigma'_0 = \underbrace{20 \times 5}_{\substack{\text{soil above}\\ \text{the water}\\ \text{table}}} + \underbrace{10 \times 5}_{\substack{\text{soil below}\\ \text{the water}\\ \text{table}}} = 150\,\text{kPa} = 1.5\,\text{daN/cm}^2$$

$$\Delta p = \sigma' - \sigma'_0$$

where σ' is the stress due to the footing at the level of the footing.

$$\sigma' = \frac{7 \cdot 10^4}{10 \times 10} = 700\,\text{kPa} \simeq 7\,\text{daN/cm}^2, \quad \Delta p = 7 - 1.5 = 5.5\,\text{daN/cm}^2,$$

$$C_1 = 1 - 0.5 \times (1.5/5.5) = 0.86.$$

Time-dependent settlement. Taking into account the very dense state of the sandy gravel, the time-dependent settlement is not considered and $C_2 = 1$ (see [35]).

Interaction between columns. For two identical footings, each 10 m wide, $B_1 = B_2 = 10$ m and the distance between the footing is known to be 20 m.

We have: $0.6(B_1 + B_2) = 0.6 \times 20 = 12$ m, so $L > 0.6(B_1 + B_2)$, and the footings behave as isolated footings.

Stress diagrams. The maximum stress is: $\sigma = C_1 \times 0.6\,\Delta p$ or $\sigma = 0.86 \times 0.6 \times 5.5 \simeq 2.84$ daN/cm^2, and the stress diagram is shown on Fig. 6.14.

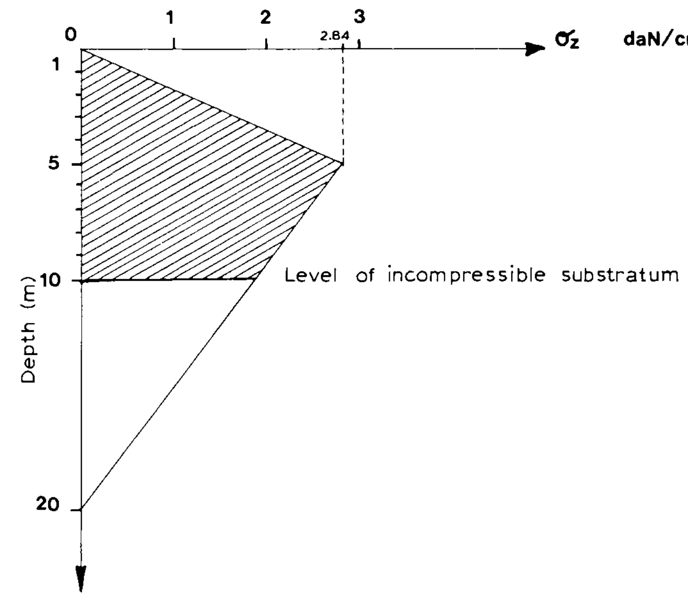


Fig. 6.14.

Settlement calculation

A rigid soil layer is encountered at 10 m below the bottom of the footing, that is at a depth inferior to $2B = 20$ m. The same triangular distribution is assumed down to the level of the rigid soil layer, but settlements are only computed to a depth of 10 m (Fig. 6.14). The sandy gravel layer of 10 m may be divided into 10 layers each 1 m thick. The σ-value is determined for each layer as well as the average cone resistance q_c. Table 6C is then made up. The total settlement is computed to be 17 mm.

Actual settlement measurements made on the structure showed:

5—7 mm for a load of 40 MN, 8—10 mm for a load of 60 MN.

This example confirms the validity of settlement estimates based on cone penetrometer test data.

TABLE 6C

Depth (m)	σ_z (daN/cm^2)	q_c (daN/cm^2)	α	$\sigma_z/3q_c$
0-1	0.28	160	3	0.00058
1-2	0.85	550	3	0.00052
2-3	1.42	650	3	0.00073
3-4	1.99	400	3	0.00166
4-5	2.56	340	3	0.00251
5-6	2.75	330	3	0.00278
6-7	2.56	200	3	0.00427
7-8	2.37	390	3	0.00203
8-9	2.18	500	3	0.00145
9-10	1.99	740	3	0.00090
				0.01743 m
			$w = 17$ mm	

★★Problem 6.11 Settlement calculations of a compressible sloping layer, based on static penetrometer test data

The city of Annecy (France) is built in part on thick lake deposits consisting mainly of compressible silts overlaying a limestone substratum which in places is overlain by dense morraine deposits. The soil profile shown on Fig. 6.15 was established from numerous static-dynamic Andina penetrometer tests performed at a particular site in Annecy. Construction consisted of erecting 2 buildings. Building A was to be constructed over a sloping rock substratum while building B was planned over the horizontal portion of the bedrock.

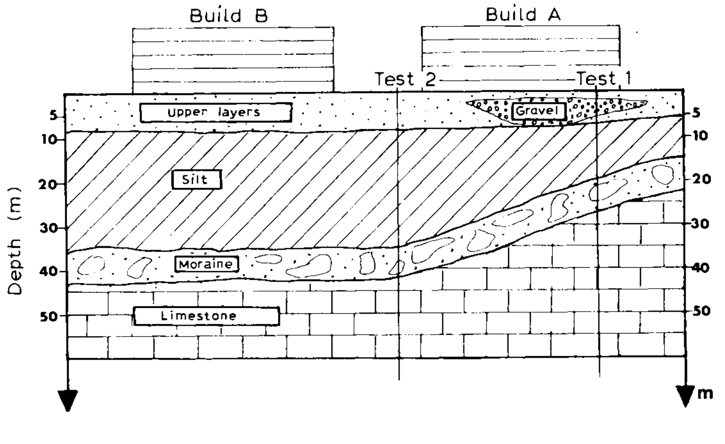


Fig. 6.15. Longitudinal soil sections.

Two test results obtained by the Andina-penetrometer are shown in Figures 6.16 and 6.17. Test 2 is assumed to be representative of the soil conditions throughout the site of building B. The water table is at a depth of 1.5 m. All the buildings have a cellar, excavated to 2.5 m and have 5 stories. The stress may be assumed at 70 kPa. The width of the cellar is 22 m and each building is 80 m in length.

Give recommendations for the foundation of each building. Calculate the settlement of shallow footings at the locations of the penetrometer tests, assuming α in the silt to be $5 < \alpha < 10$ and $\alpha = 3$ in the gravel layer. We also have: $\gamma/\gamma_w = 2$ above the water table, $\gamma'/\gamma_w = 1$ below the water table.
What solution can be chosen for each building?

Solution
(1) Deep footings. Because of the sloping bedrock for building A, and the low consistency of the silt, the first solution that comes to mind is to support the buildings on piles driven to the limestone or to the dense moraine. ($q_c > 10^4$ kPa or 100 daN/cm^2).

From the technical standpoint, this would be the safest solution because total and differential settlement problems would be virtually non-existent [30].

(2) Shallow footings. The deep foundation scheme is very costly because of the great lengths of piles required. It is therefore worthwhile to investigate the possibility of supporting the structures on shallow footings. The allowable bearing capacity of the silt is: taking $q_c = 300$ kPa (3 daN/cm^2), $c_u = q_c/10 = 300/10 = 30$ kPa, from which $q_{ad} = 5.14\,c_u/3 = (5.14 \times 30/3) = 51.4$ kPa (0.5 daN/cm^2).

From the total building weight, the only possible shallow footing type would be a raft-type foundation, as should have been expected. The problem associated with the raft-foundation is that of evaluating settlements.

The net pressure due to the new structure at the foundation level at the bottom of the raft at a depth of 2.5 m is the total building pressure less the weight of the soil excavated to a depth of 2.5 m, or:

$$\underbrace{20 \times 1.50}_{\substack{\text{soil above}\\ \text{the water}\\ \text{table}}} + \underbrace{10 \times 1}_{\substack{\text{soil below}\\ \text{the water}\\ \text{table}}} = 40\,\text{kPa}$$

The net pressure at the bottom of the raft is $\Delta\sigma = 70 - 40 = 30\,\text{kPa} \simeq 0.3\,\text{daN/cm}^2$.

For each penetrometer test, an idealized soil cross section may be drawn into a number of layers which also accounts for the gravel layer encountered in C.P.T. no. 1, as shown in Fig. 6.15.

The vertical stress due to the net building load at mid-height of each layer is calculated. Because of the plan dimensions of the buildings, the stresses

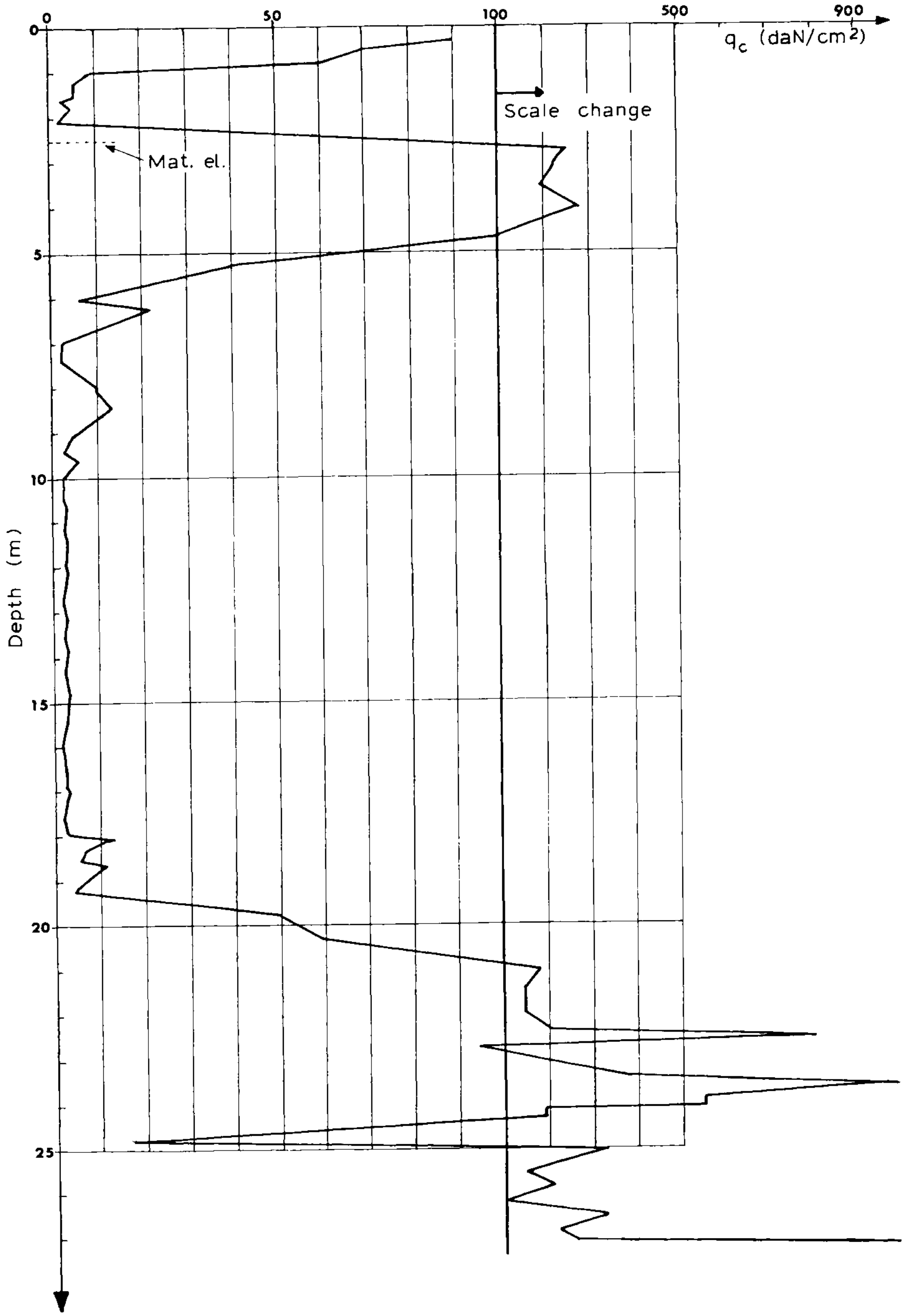


Fig. 6.16. Static-dynamic Andina penetrometer test 1.

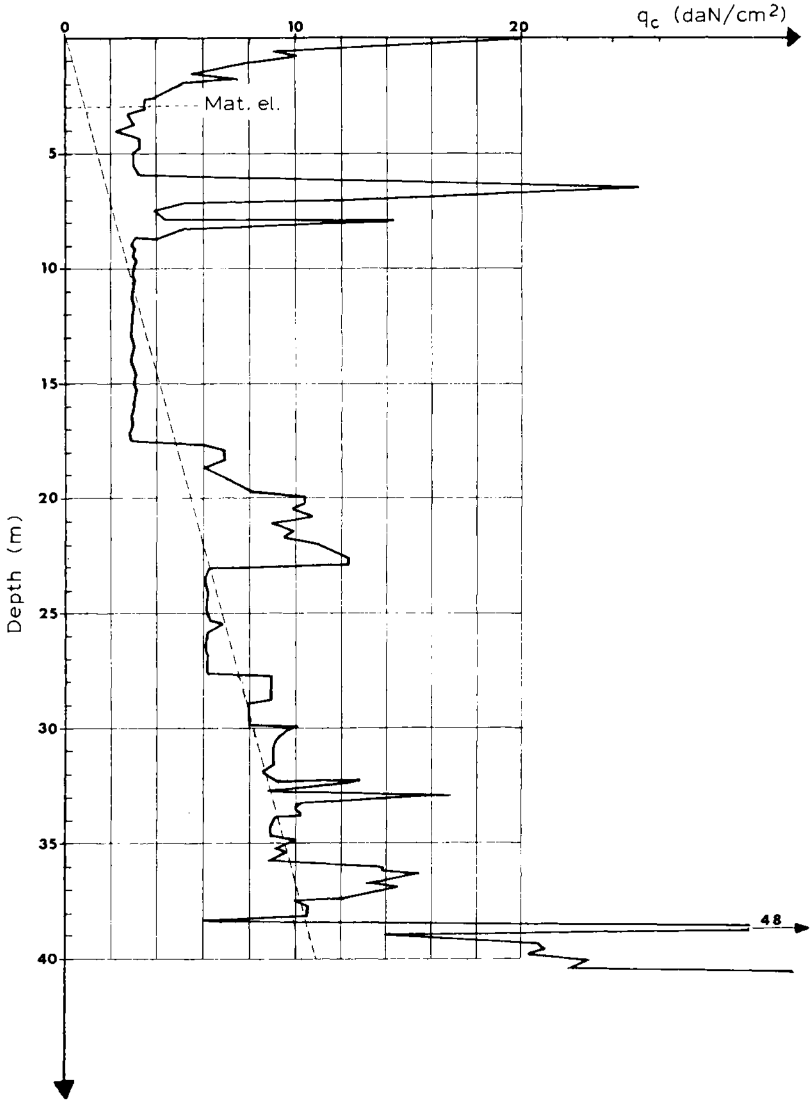


Fig. 6.17. Static Andina penetrometer test 2.

may be equated to those generated by a rigid, long narrow footing of 22 m in width [11]*.

Tables 6D and 6E of vertical stresses are shown below, giving $\Delta\sigma$ the increment of vertical stress, α a dimensionless multiplier, q_c, the average

*It could be taken into account that the footing is rigid and rectangular by using Kerisel's method [6] which gives settlements of 9 and 17.2 cm, respectively for C.P.T. no. 1 and C.P.T. no. 2, i.e. a differential settlement of 8 cm. According to the hypothesis of non-rigid footing (which does not correspond to reality in this case) the settlements would be 3.8 and 7.4 cm respectively.

TABLE 6D

Depth below mat	z (average)	z/a	$\bar{k}_0$	$\Delta\sigma$ (daN/cm^2)	α	q_c (daN/cm^2)	h (cm)	$h\Delta\sigma/\alpha q_c$
0–3	1.5	0.14	0.64	0.19	3	140	300	0.14
3–5	4	0.36	0.67	0.20	10	15	200	0.27
5–10	7.5	0.68	0.69	0.21	10	5	500	2.10
10–15	12.5	1.14	0.66	0.20	10	3	500	3.33
15–17	16	1.45	0.61	0.18	10	8	200	0.45

Total settlement: 6.29 cm

value of the cone resistance and the settlement due to each of the layers: $\Delta h = h\Delta\sigma/\alpha q_c$ (h = one layer thickness).

For C.P.T. no. 1, Table 6D summarizes the above values:

The total settlement, due to all layers at C.P.T. location no. 1 is the total of the last column or 6.29 cm.

Taking $\alpha = 5$ for cohesive soils, we would calculate a total settlement of 12.44 cm, but local experience indicates that actual settlements are better evaluated using a value of $\alpha = 10$.

The same procedure for C.P.T. no. 2 is given in Table 6E.

TABLE 6E

Depth below mat	z (average)	z/a	$\bar{k}_0$	q_c (daN/cm^2)	α	q_c (daN/cm^2)	h (cm)	$h\Delta\sigma/\alpha q_c$ (cm)
0–5	2.5	0.23	0.65	0.20	10	5.6	500	1.79
5–10	7.5	0.68	0.69	0.21	10	3	500	3.50
10–15	12.5	1.14	0.66	0.20	10	3	500	3.33
15–20	17.5	1.59	0.58	0.17	10	9	500	0.94
20–25	22.5	2.05	0.51	0.15	10	6.5	500	1.15
25–30	27.5	2.50	0.45	0.14	10	9	500	0.78
30–35	32.5	2.95	0.38	0.11	10	10	500	0.55

Total settlement: 12.04 cm.

The total estimated settlement is 12.04 cm.

Conclusion

For the first group of buildings, total settlement could be of the order of 6 to 12 cm, from one end of the structures to the other. The resulting differential settlement could present a problem for the proper operation of the buildings. The differential settlement is caused by the sloping bedrock and the presence of a gravel layer. For this reason, it would be recommended to support the structures of group A on piles. For the other group of buildings, differential settlements should be nominal and a raft foundation could be recommended.

★★★*Problem 6.12* **Pile foundation calculation from static cone test in a tri-layer system**

Cone penetrometer test data were performed in the Annecy (France) area at a swampy site, with static-dynamic Andina penetrometer. The penetration record of Fig. 6.18 is representative of the subsurface soil conditions. The cone point used in the test has a diameter of 8 cm.

The soil cross-section is:

— 0 to 8 m, peat and soft clay;
— 8 to 11.5 m, sand and gravel layer;
— from 11.5 m, soft clay.

To support the proposed structures, the obvious solution is to resort to driven pile foundation, penetrating to the upper part of the sand and gravel layer.

Estimate the bearing capacity of 0.6 m and 1.0 m diameter piles whose

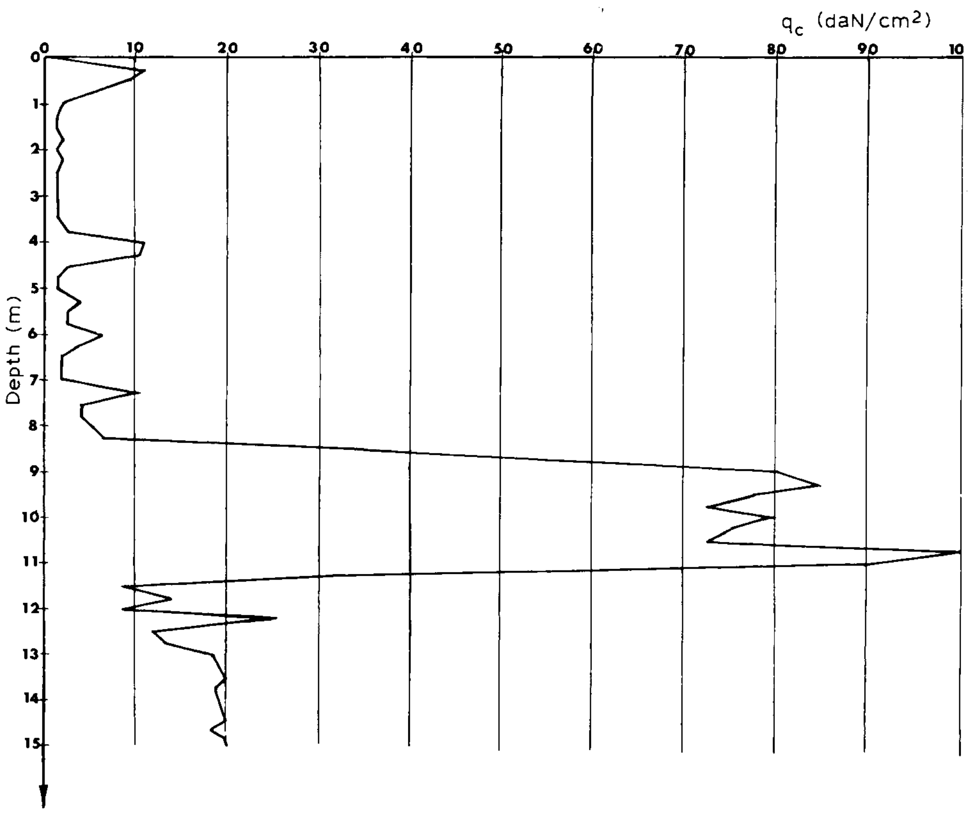


Fig. 6.18. Tri-layer system Andina static penetrometer test.

tips would be driven to a depth of 8.5 m, using the following 3 methods: the Dutch method, Geuze's rule, Meyerhof's method [22, 25].

Compare the results. In addition, for the 0.6-m diameter pile, estimate the load capacity if the pile is driven one additional meter (to -9.50 *m).*

Solution

(1) The Dutch method. This method consists of calculating the average cone bearing over a depth increment of 8 diameters above the pile tip (q_{c_1}) and the average cone bearing resistance to 4 diameters depth below the pile tip q_{c_2} (tip being at -8.5 m).

The ultimate bearing pressure at the pile tip is then: $q_d = (q_{c_1} + q_{c_2})/2$ and the allowable bearing pressure is $q_{ad} = q_d/2 = (q_{c_1} + q_{c_2})/4$ for the safety factor of 2.

If a layer of low cone bearing is encountered, with a minimum value of q_{mini} over a depth of 4 diameters below the pile tip, and if n cone bearing readings were made in this soft layer, the value of q_{p_2} is then:

$$q_{p_2} = \frac{q_1 + q_2 + q_n + n q_{mini}}{2n}.$$

(a) For the 0.6-m diameter pile ($8\phi = 4.8$ m and $4\phi = 2.4$ m) the calculated average cone bearing values from 3.7 to 8.5 m, is $q_{c_1} \simeq 6$ daN/cm^2; for values from 8.5 to 10.9 m $q_{c_2} \simeq 80$ daN/cm^2, and the allowable bearing pressure at the tip of the pile is $q_{ad} = (6 + 80)/4 = 21.5$ daN/cm^2, say 20 daN/cm^2.

(b) For the 1-m diameter pile ($8\phi = 8$ m and $4\phi = 4$ m) the average value of q_{c_1}, between 0.5 and 8.5 m is $q_{c_1} \simeq 4.6$ daN/cm^2.

For the depth 4ϕ below the proposed pile tip elevation the presence of a soft layer is encountered. The bearing capacity of the soil for the 1 m ϕ pile will have to be decreased to take this into account. The Andina-type penetrometer measures q_c every 0.25 m; over 4 m there are 16 readings. The average value of q_{c_2} from 8.5 to 12.5 m is equal to 56 daN/cm^2 and the minimal value of q_{c_2} is 8.5 daN/cm^2. We therefore have:

$$q_{c_2} = \frac{56 \times 16 + 8.5 \times 16}{2 \times 16} \simeq 32 \text{ daN/cm}^2.$$

Hence the allowable bearing pressure is:
$q_{ad} = (4.6 + 32.2)/4 = 9.2$ daN/cm^2, say 10 daN/cm^2.

Remark

Piles of 1 m in diameter have an allowable tip bearing pressure considerably lower than that of the 0.6-m diameter pile (not quite half). Note: The pressure bulb at the tip of the large-diameter pile is larger than that for the small-diameter pile and the bulb extends therefore into the soft underlaying clay. Thus it is advantageous to use smaller-diameter piles. However, it does not appear reasonable to use the 20 daN/cm^2 allowable pressure for the 0.6-m diameter pile because there is the soft clay layer at depth. In the event of accidental overdriving a few of the 0.6-diameter pile below 8.5 m depth, the bearing capacity of the overdriven piles would be greatly reduced.

Let us calculate the allowable bearing pressure at the tip of a 0.60-m diameter pile driven to 9.5 m depth, instead of the 8.5 m.

Over a depth of $8\phi = 4.8$ m above the base, from depth 4.7 to 9.5, we have: $q_{c_1} \simeq 19.5$ daN/cm^2, for the $4\phi = 2.4$ m depth below the tip, from 9.5 to 11.9 m, the soft clay layer must be accounted for with $q_{mini} = 8.5$ daN/cm^2.

For 10 readings of q_c of an average value of 55.5 daN/cm^2, we have: $q_{c_1} = (55.5 \times 10 + 8.5 \times 10)/20 = 32$ daN/cm^2, $q_{ad} = (19.5 + 32)/4 = 12.9$ daN/cm^2.

This is considerably lower than the allowable pressure calculated for a pile tip at 8.5 m.

(2) Geuze's rule. This method is summarized in Fig. 6.19.

(a) The 0.6-m ϕ pile. We have $\tan\alpha \simeq 0.062$ and $\tan\beta \simeq 0.038$ (see Fig.

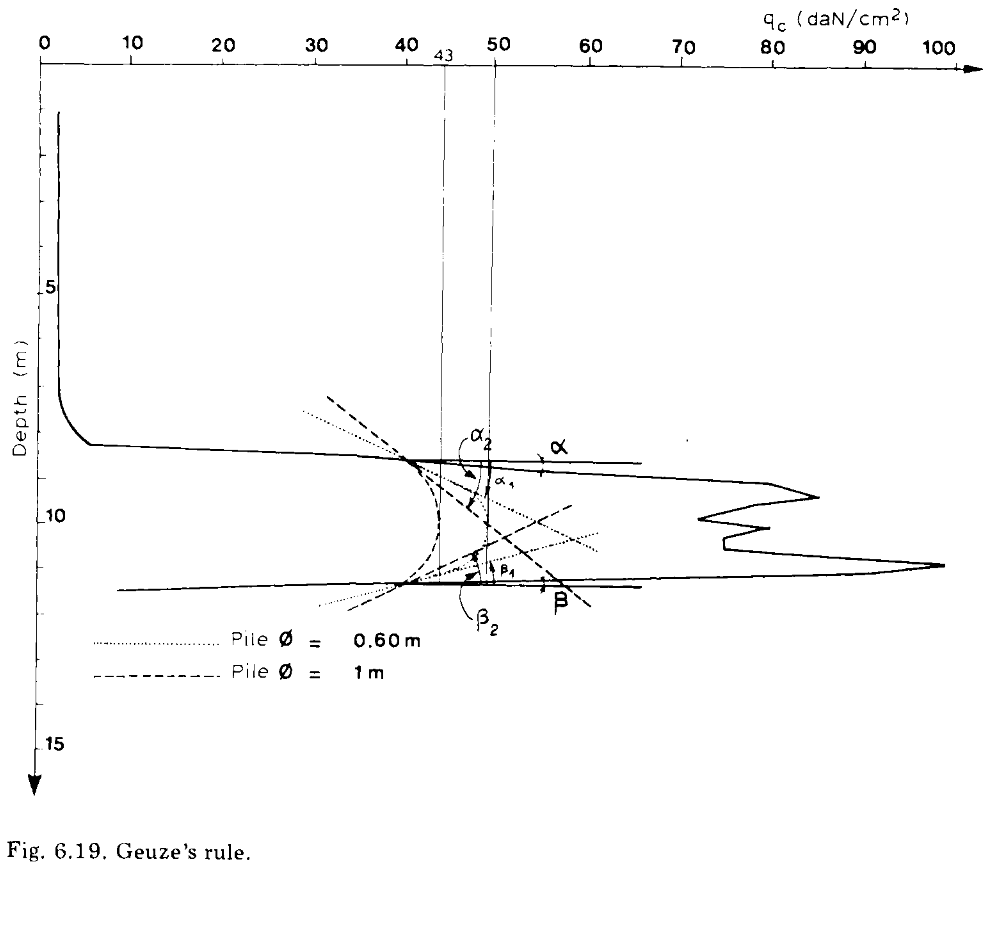


Fig. 6.19. Geuze's rule.

6.19). The diameter of the Andina penetrometer cone is $b = 8\,\text{cm}$, and that of the pile $B = 60$ cm.

Geuze's law states: $\tan\alpha_1 = B/b \tan\alpha = (60/8) \times 0.062 = 0.47$ and $\tan\beta_1 = B/b \tan\beta = (60/8) \times 0.038 = 0.29$.

The actual cone resistance of Fig. 6.19 is deduced graphically to $q_c = 50\,\text{daN/cm}^2$ and, using a safety factor of 4, $q_{ad} = 50/4 = 12.5\,\text{daN/cm}^2$.

(b) The 1.0-m ϕ pile. The same procedure is used for $B = 100$ cm from which $\tan\alpha_1 = 0.062 \times 100/8 = 0.78$ and $\tan\beta_1 = 0.038 \times 100/8 = 0.48$. The corrected cone bearing value is $q_c \simeq 43\,\text{daN/cm}^2$ and $q_{ad} = 43/4 \simeq 11\,\text{daN/cm}^2$. The allowable bearing pressures for the 2 pile sizes are very near the same in this instance.

(3) Meyerhof's method.

(a) The 0.6-m ϕ pile. The tip of the piles at 8.5 m depth is located at a distance H of 3 m above the soft underlaying layer. This is less than the critical distance $10B = 6$ m. The approximate cone bearing value of the soft layer is $q_{c_0} \simeq 10\,\text{daN/cm}^2$ and that of the sand and gravel layer is $q_c \simeq 75\,\text{daN/cm}^2$.

According to Meyerhof, the ultimate bearing capacity of the soil is:

$$q_d = q_{c_0} + \frac{(q_c - q_{c_0})H}{10B} = 10 + \frac{65 \times 3}{6} = 42.5\,\text{daN/cm}^2.$$

and for a safety factor of 3 (because the method is a little pessimistic), we have: $q_{ad} = 42.5/3 = 14\,\text{daN/cm}^2$.

(b) The 1.0-m ϕ pile. $H = 3\,\text{m} < 10B = 10\,\text{m}$, $q_{c_0} = 10\,\text{daN/cm}^2$ and $q_c \simeq 75\,\text{daN/cm}^2$, from which $q_d = 10 + (65 \times 3)/10 = 29.5\,\text{daN/cm}^2$ and $q_{ad} = q_d/3 = 10\,\text{daN/cm}^2$.

Conclusion

The three methods yield closely related results, except for the 0.6-m diameter pile at 8.5 m depth; however, for practical reasons, it was pointed out that the high value of allowable pressure for this instance was not to be recommended.

An average allowable pressure of all the methods may be recommended at 8.5 m. This would be about $13\,\text{daN/cm}^2$ for the 0.6-m diameter pile. No fill should be placed on the ground surface and all ground floors should also be supported on piles to avoid differential settlements between columns, walls and floors.

★★★*Problem 6.13* Settlement estimates of a surcharge fill, from static-cone penetrometer test data

In the Cannes region (France), it is not uncommon to have recent alluvial deposits of clays and silts over depths of 40 m above the marly bedrock. Cone-penetrometer tests performed with the Gouda 100 kN device have *shown that in such soils the cone-bearing value is very constant and of about 1000 kPa for these alluvial deposits. The project consists of the construction of several relatively low buildings one of which of five storeys.*

In order to reduce the total and differential settlements of the structures, it was decided to surcharge the sites and consequently, a 5 m high fill was placed over a plan dimension of about 40 × 130 m. The vertical cross-section of the fill has a trapezoidal shape with a top horizontal dimension of 30 m. The soil density is 1.8.

Calculate the settlement caused by the surcharge fill along its central axis, assuming the following values of α: $3 < \alpha < 5$ and $m_v = 1/\alpha q_c$ [23, 27, 28].

Solution

The most direct method of estimating settlements is to resort to vertical stress distribution and tables of Giroud (vol. II, 6.11, p. 436), formula 2:

$$w = \frac{\gamma h}{E}\frac{a^2}{a-a'}\left[(r_h - (a'/a)^2 r'_h)\right], \quad \gamma = 18\,\text{kN/m}^3,\ h = 5\,\text{m},$$

$$E = 2/3\,E' = 2/3\,\alpha q_c \quad \text{with} \quad \nu = 0.33,\ a = 20\,\text{m},\ a' = 15\,\text{m}.$$

Here, for $3 < \alpha < 6$, we get $3000 < E < 6000$ kPa and we have:

$H/a' = 40/15 = 2.67$ from which $r'_n = 1.142$,

$H/a = 40/20 = 2$ from which $r_h = 0.983$

with: $E' = 3000$ kPa, $E = 2000$ kPa, we get:

$$w = \frac{18 \times 5}{2000} \times \frac{400}{5}(0.983 - \left(\frac{15}{20}\right)^2 \times 1.142) = 1.23\,\text{m},$$

with $E' = 6000$ kPa, $E = 4000$ kPa, we get: $w = 0.62$ m.
Thus, the settlement would be between 62 and 123 cm [32].

Remark

The actual settlement measured at the end of 4 months of preloading was between 30 and 42 cm at different observation stations. These settlements are smaller than the range predicted from penetrometer tests. This may be due to:

— the ultimate settlements have not yet occurred at the end of 4 months;

— no account was taken of the slight increase in cone-bearing value of the silt starting at a depth of 20 m, nor of the presence of a few sand lenses.

The actual settlements, however, validate the method of estimating settlements from static-penetrometer test data.

★★★*Problem 6.14* **Settlement calculation of a surcharge fill from the results of an Andina penetrometer test**

Two cooling structures were constructed near Lyons for the development of a nuclear power plant. Each structure is 127 m high and has a diameter of

102 m at the base. These structures are rigid and do not tolerate large differential settlements. The foundation soil consists of sandy-gravelly alluvium underlain by a thick compressible clay layer over a sandstone bedrock.

Assume that the soil conditions are uniform and are represented by the penetration diagram of Fig. 6.20.

In order to decrease the total and differential settlements that the structures could experience, it was decided to surcharge the areas of the cooling towers. Two preload fills were constructed in the shape of a torus, of trapezoidal cross-section, having an outside diameter of 138 m and an interior diameter of 62 m. The fill height was 10 m and the side slopes inclination of 3 horizontal to 2 vertical (see Fig. 6.21).

Assuming the unit weight of the surcharge fill to be 20 kN/m^3, find the settlement caused by the fill weight, along its axis, at the location of the test whose results are shown in Fig. 6.20.

Take $\alpha = 3$ in the sandy gravel zones where $q_c > 4000$ kPa, and $\alpha = 10$ in the deep clay layer and $\alpha = 5$ in the clay lenses present in the superficial alluvium.

The value of α in the deep clay is high because it has been proven that this clay was overconsolidated by the weight of ancient glaciers at the start of the Quaternary (Fig. 6.20).

Solution

In every settlement problem, the initial calculations consist of determining the vertical pressure distribution variations as a function of depth. This was done in this instance according to Cordary's thesis 'Contribution to the study of settlements of shallow footings', University of Grenoble 1973. It consists of giving the pressure distribution under circular rings.

The calculations apply to the points located vertically below the axis of the surcharge fill. The value of q_c chosen for each 1 m depth increment corresponds to the average q_c values within each interval measured every 0.25 m with the Andina quasi static penetrometer.

The settlement of an element layer of height H is: $\Delta h = H(\Delta_p/\alpha q_c)$ and if $H = 1$ m, total settlement $S = \Sigma(\Delta p/\alpha q_c)$. Table 6F may then be completed, where the obtained settlement is 0.26 m.

Remark

Compare the result with those given by the following two finite element methods:

(1) Calculation of the linear elasticity and elasto-plastic behavior of the soil and assuming that the fill is rectilinear and of infinite length.

(2) Calculation of the linear elasticity of the soil assuming the axisymmetry of the fill. Note in this case the uneven pressure distribution due to the annular configuration of the fill (Fig. 6.23).

Note the uneven pressure distribution due to the annular configuration

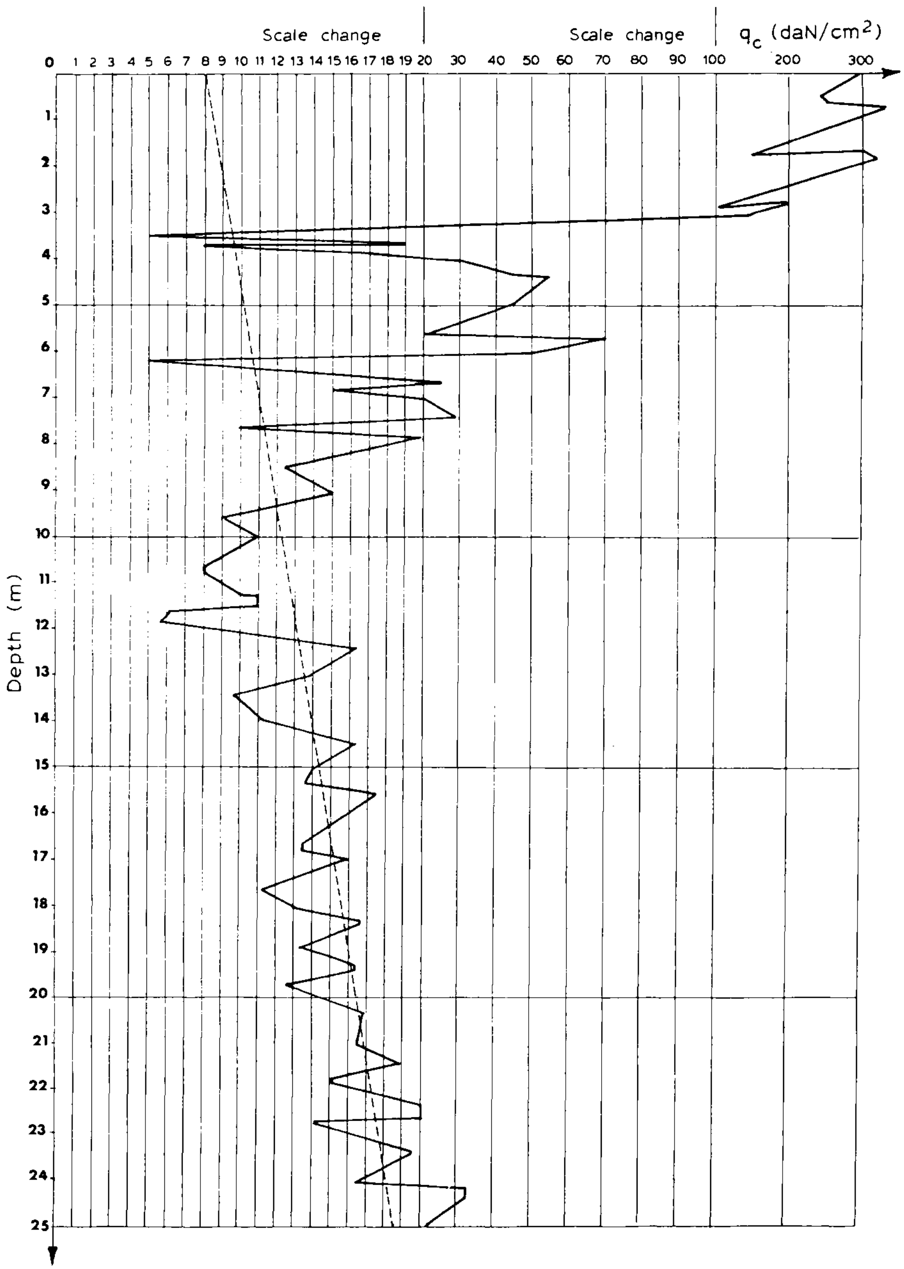


Fig. 6.20. Andina static penetrometer test.

of the fill (Fig. 6.23) in this case. The various computations do indicate that maximum settlement occur between 15 and 30 cm. This is in good agreement with the result calculated above which is based on the penetrometer test result.

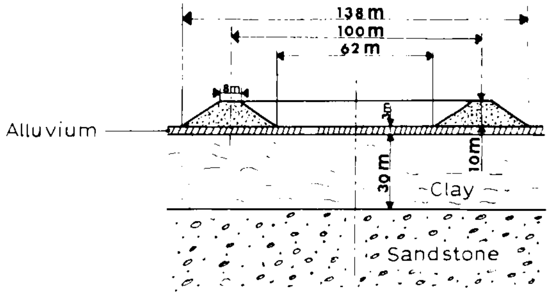


Fig. 6.21. Cross-section of surcharge fill.

TABLE 6F

Depth	Δp (daN/cm^2)	α	q_c (daN/cm^2)	αR_p	$\Delta p/\alpha q_c$
1	1.98	3	300	900	0.0022
2	1.97	3	250	750	0.0026
3	1.95	3	150	450	0.0043
4	1.92	5	30	150	0.0128
5	1.87	3	45	135	0.0139
6	1.83	3	50	150	0.0122
7	1.78	5	20	100	0.0178
8	1.73	5	20	100	0.0173
9	1.67	5	15	75	0.0223
10	1.62	10	11	110	0.0147
11	1.56	10	8.5	85	0.0184
12	1.50	10	9	90	0.0167
13	1.45	10	13	130	0.0112
14	1.41	10	11	110	0.0128
15	1.36	10	14.5	145	0.0094
16	1.31	10	16	160	0.0082
17	1.27	10	15.5	155	0.0082
18	1.23	10	13	130	0.0095
19	1.18	10	14.5	145	0.0081
20	1.14	10	14.5	145	0.0079
21	1.10	10	16.5	165	0.0067
22	1.07	10	17	170	0.0063
23	1.04	10	17.5	175	0.0059
24	1.01	10	18	180	0.0056
25	0.98	10	26	260	0.0038

$\Sigma \Delta p/\alpha q_c = 0.26$ m.

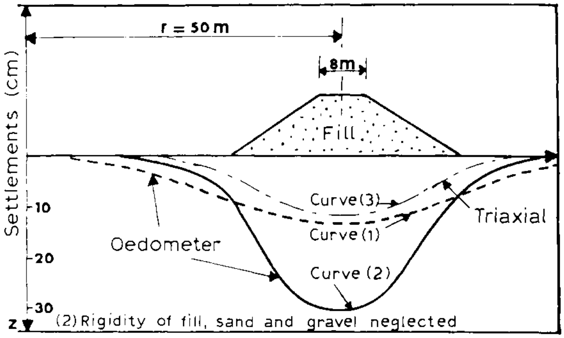


Fig. 6.22. Settlement under surcharge fill if length is unlimited. Curves *1* and *2* correspond to linear elastic behavior and curve *3* to elastoplastic behavior.

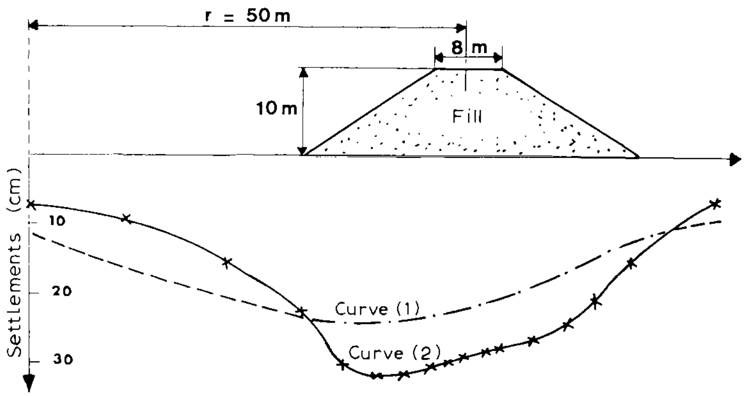


Fig. 6.23. Settlement under surcharge fill in annular configuration (axisymmetry). Curve *2*: rigidity of fill, sand and gravel neglected.

★★★*Problem 6.15* Settlement computations based on static penetrometer test results

Static penetrometer tests of the Parez-type penetrometer [23, 29, 30] were performed at the location of a proposed building. Fig. 6.24 shows a representative penetration record of the tests. The building plan dimensions will be 20 by 20 m.

The substratum, consisting of Plaisancien marns, is overlain by a silty clay alluvium of about 30 m in thickness [31, 37]. This layer is very compressible. It was resolved to proceed in the following manner to support the building:

— excavate the soil to a depth of 2 m;

— backfill with a compacted selected fill and overfill laterally by 2 m in all directions;

— design the shallow footings for an allowable bearing pressure on the engineered fill of 150 kPa.

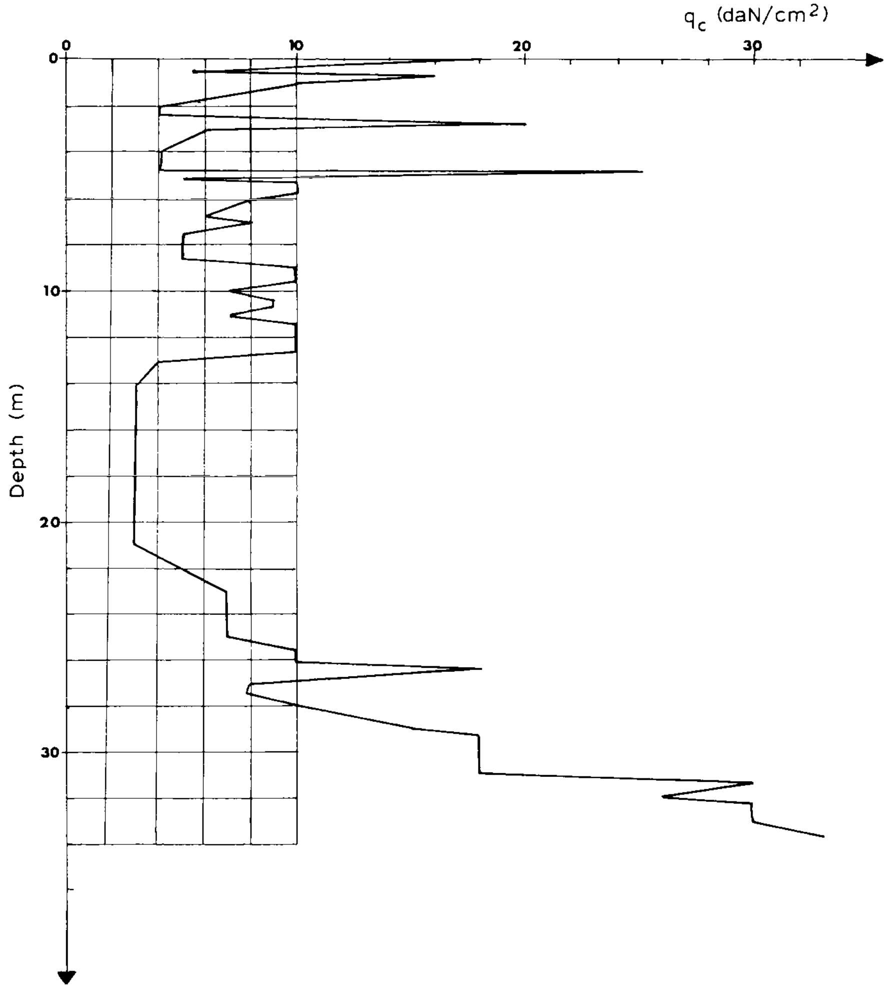


Fig. 6.24. Parez static penetrometer test.

Assume that if the building were to be supported on a mat-foundation, the uniform soil pressure would be about 70 kPa. In fact spread footings are 4 m apart.

Calculate the settlement of the structure. Take $\alpha = 6$ for the silty clay and assume the fill to be incompressible.

Solution

The replacement of the clay by compacted fill is a common method of foundation soil improvement. It does not decrease total settlements but does decrease differential settlements between spread footings.

The load on the footings is: $Q = 70 \times 4 = 280\,\text{kN}$ (per meter length).

The width of the footing should therefore be: $B = Q/q_{ad} = 280/150 = 187$ m, say 1.9 m.

Let us calculate the vertical stress at the bottom of the fill, assuming a distribution of 1 horizontal to 2 vertical slopes, and we obtain the fictitious width at 2 m depth: $B' = 1.9 + 2 = 3.9$ m.

Since the footings are 4 m apart, the load at the bottom of the fill is equal to the load of the building if it were evenly distributed over its entire surface at grade. In effect, the fill acts as a mat-foundation.

The overall settlement of the building can be calculated by assuming a uniform pressure acting over a plan area of 22 by 22 m. The uniform pressure is: $p = \dfrac{70 \times 20 \times 20}{22 \times 22} = 58\,\text{kPa} = 0.58\,\text{daN/cm}^2$.

To calculate the settlement, the subsurface soil is divided into horizontal slices for each one of which an average value of the point resistance would be calculated, and for each of which the increment of vertical load due to the building must be calculated. Since the fill is assumed to be incompressible, the vertical stress increase starts at the bottom of the fill, or 2 m below grade. The results are tabulated as Table 6G. The total settlement is of the order of 20 cm but this is an average value because the assumption of a rigid mat (fill). According to Kerisel ([6], p. 27) the settlement on the axis, assuming a flexible mat, should be increased by the ratio 2/1.57 = 1.274, from which a settlement of 25 cm is obtained.

It should be noted that these two different values — one for a rigid and the other for a flexible foundation — do not give the maximal and minimal boundaries of the solution.

TABLE 6G

Layer thickness (m)	Depth of fill bottom	z/B	p_z^0	$\Delta\sigma$ (daN/cm^2)	α	q_c (daN/cm^2)	$h\Delta\sigma/\alpha q_c$ (cm)
2-5	1.5	0.07	0.50	0.29	6	5	2.90
5-10	5.5	0.25	0.53	0.30	6	7	3.57
10-15	10.5	0.48	0.51	0.29	6	6.5	3.72
15-20	15.5	0.70	0.38	0.22	6	3	6.11
20-25	20.5	0.93	0.32	0.18	6	5	3.00
25-30	25.5	1.16	0.24	0.14	6	15	0.78
						Total settlement	20.08 cm

Note: $B = 22$ m width of the fictitious mat (fill).

Remarks

A direct estimate of settlement could have been performed as a first approximation by the following method, using Giroud's tables. The constant q_c value in the clay is about 5 daN/cm^2. The clay layer thickness is 30 m and the approximate modulus $E' = \alpha q_c = 6 \times 5 = 30$ daN/cm^2. Therefore, $E = 2/3\,E' = 20$ daN/cm^2.

Assuming that the settlement of a rigid foundation is close to the average settlement of a flexible foundation, from Giroud's tables: $w_m = (pB/E)p_{Hm}$, $p = 0.58\,\text{daN/cm}^2$, $L = B = 22\,\text{m}$ from which $L/B = 1$, $H = 30\,\text{m}$, $H/B = 30/22 = 1.36$, if $\nu = 0.33$.

From Giroud's tables (vol. II, p. 163), we get $p_{HM} = 0.6$ and: $w_m = (0.58 \times 2200/20) \times 0.6 = 38\,\text{cm}$.

This approximate method is faster and the result is close to the more rigorous method, only a bit higher.

★★*Problem 6.16* Settlement evaluation in clay soils from static penetrometer tests; influence of fills

A low-rise building was erected some 15 years ago on the shore of the Nantua lake [28]. The building width varies as does its total height along its main axis. The structural frame is reinforced concrete supported on a mat-foundation. Its plan dimensions and loads are shown on Fig. 6.25.

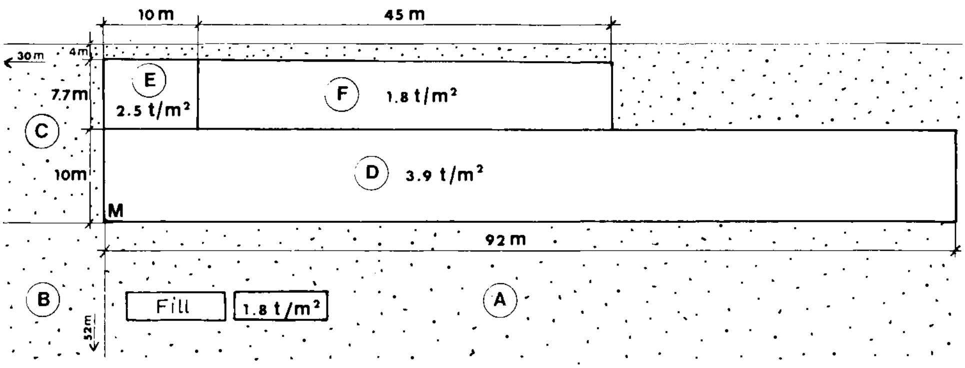


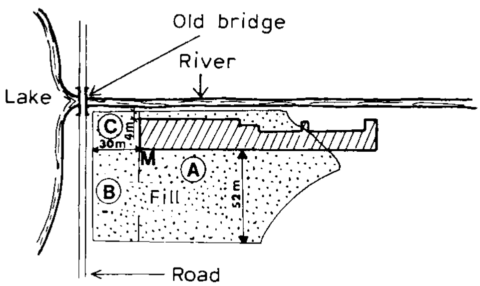
Fig. 6.25. Nantua-plan view of building (France).

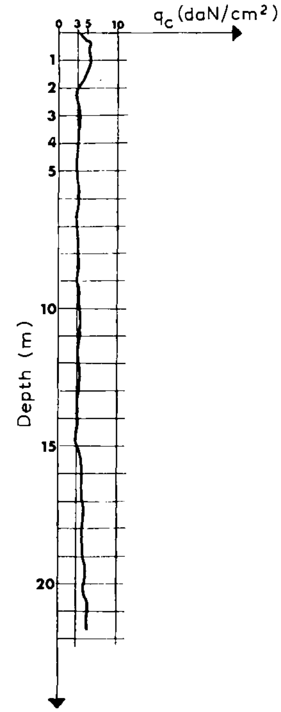
The building is surrounded by fill of about 1 m in thickness over the adjacent natural grade (see Fig. 6.26).

Four static penetrometer tests were performed before the start of construction with a Gouda-penetrometer [19, 22, 23, 29]. The results were very similar and can all be summarized by the diagram of Fig. 6.27. The subsurface soil consisted of a thick layer of soft organic clay.

The structure underwent substantial settlements. Show that the settlement magnitude could have been predicted from the static-penetrometer test data. What is the fraction of the settlement atributable to the weight of the fill?

Calculate the settlement at point M (see Figs. 6.25 and 6.26). Assume the clay layer thickness to be 30.5 m.





Figs. 6.26 and 6.27.

Solution

The vertical stress distributions due to the building and the fill may be evaluated from Newmark's chart [see Table 6H]. Assume that the cone bearing value is constant throughout the clay and equal to 3 daN/cm^2.

TABLE 6H

h	z	$\Delta\sigma$ (total) (daN/cm^2)	q_c	α	$h\Delta\sigma/\alpha q_c$ (cm)	$\Delta\sigma$	$h\Delta\sigma/\alpha q_c$ (cm)
1.50	0.75	0.233	3	1.5	7.77	0.135	4.50
1	2	0.233	3	1.5	5.18	0.135	3
2	3.5	0.230	3	1.5	10.22	0.135	6
2	5.5	0.227	3	1.5	10.09	0.135	6
4	8.5	0.221	3	1.5	19.64	0.132	11.73
6	13.5	0.203	3	1.5	27.07	0.130	17.33
6	19.5	0.184	3	1.5	24.54	0.123	16.40
8	26.5	0.150	3	1.5	26.66	0.110	19.56
					131.17		84.52
				$\alpha = 1.5$	$w = 131$ cm	$\alpha = 1.5$	$w = 85$ cm
				$\alpha = 4$	$w = 49$ cm		

Since the clay is organic and has a high water content, assume $1.5 < \alpha < 4$. The data to calculate the settlement are shown in Table 6H.

The total settlement should be of the order of 1.30m for $\alpha = 1.5$ and of the order of 0.50 m for $\alpha = 4$.

For the former, the settlement due to the fill is 0.85 m or about 65% of the total settlement. This shows that placement of fill on compressible soils around a structure significantly contribute to settlements, in this case, to over 50% of the settlement.

The actual, measured settlement of point M was 1.20 m five years after construction. This confirms the validity of the estimate based on penetrometer test results. However, the soil must be sampled in order to determine its water content when $q_c < 1.200$ kPa.

★★★*Problem 6.17* Dimensioning shallow footings on sand, based on SPT (Standard Penetration Test) results

The results of SPT performed in a sandy layer of 12 m thick and with a water table at a depth of 2 m are shown in Fig. 6.28, as a function of depth, in ft. Determine the allowable bearing values q_{ad} in pounds per square foot (lbs/ft^2) for a strip shallow footing of 3 ft width and embedded a distance D into the sand. Assume D at 0, 1, 5 and 10 ft.

Draw the curve of q_{ad} as a function of D. Use Terzaghi and Peck's graphs [(22), pp. 245–247].

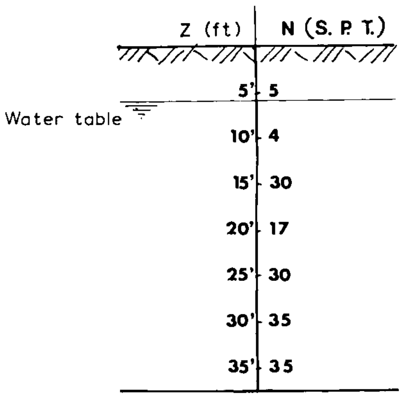


Fig. 6.28.

Solution

The problem may be solved in three different manners.

(1) Evaluate N_γ and N_q from N values of Fig. 6.28 and determine the allowable pressure from the classical formula:

$$q_d = \frac{\gamma B}{2} N_\gamma + \gamma D N_q + c N_c$$

wherein the unit weights of the soil must be assumed, (γ and γ') as a function of N. Because the soil is sand, $c = 0$.

For simplicity, the five Terzaghi and Peck graphs will hereafter be referred to as A, B, C, D and E, to correspond to the order in which they were published [22].

(a) $D = 0$, $N = 5$. For this value of N, the sand is very loose. Because of the proximity of the water table, assume the sand to be saturated by capillarity. We can then assume:

$$\gamma_d = 90\,\text{lb/ft}^3 \quad \text{and} \quad \gamma_h = 100\,\text{lb/ft}^3.$$

Graph A of Terzaghi and Peck gives, for $N = 5$; $N_\gamma = 5$ and $N_q = 8$ from which, with $B = 3$ ft: $q_d = (100 \times 3 \times 5)/2 = 750\,\text{lb/ft}^2$. For a safety factor of 3, $q_{ad} = 250\,\text{lb/ft}^2$ or 12 kPa.

(b) $D = 1$ ft, $N = 5$, we have $N_\gamma = 5$ and $N_q = 8$ from which: $q_d = 750 + 100 \times 1 \times 8 = 1550\,\text{lb/ft}^2$ and $q_{ad} = 517\,\text{lb/ft}^2 = 25$ kPa.

(c) $D = 5$ ft. Because we are now very close to the water table (at 6 ft) its influence on the value of N must be taken into account and corrected by: $N = 15 + \frac{1}{2}(N' - 15)$ where $N' = 4$, no correction is necessary and $N_\gamma = 4$ and $N_q = 7$. N' being the SPT value below water table.

To calculate γN_γ the buoyant weight must be used; we have: $\gamma' = \gamma_h - \gamma_w = (100 - 62.4) = 37.6$ say $38\,\text{lb/ft}^3$.

For the overburden weight, the saturated weight of soil must be considered:

$$q_d = \frac{38 \times 3 \times 4}{2} + 100 \times 5 \times 7 = 228 + 3500 = 3728\,\text{lb/ft}^2,$$

$$q_{ad} = 1242\,\text{lb/ft}^2 = 0.6\,\text{daN/cm}^2.$$

(d) $D = 10$ ft. The average value of N' must be calculated from the value at the level of the bottom of the footing and 5 ft below that. In this case it is: $N' = (4 + 30)/2 = 17$ from which: $N = 15 + \frac{1}{2}(17 - 15) = 16$.

Table A then gives: $N_\gamma = 14$, $N_q = 16$.

N increases between 10 and 15 ft, therefore γ_d and γ_h increase. So it can be assumed that: $\gamma_d = 100\,\text{lb/ft}^3$, $\gamma_h = 115\,\text{lb/ft}^3$ and $\gamma' = 53\,\text{lb/ft}^3$.

For embedment the effective stress must be calculated at the footing level according to the water table (6 ft.). From which:
$\gamma D = 6 \times 100 + 4 \times 53 = 812\,\text{lb/ft}^2$

$$\text{and } q_d = \frac{53 \times 3 \times 14}{2} + 812 \times 16 = 1.113 + 12.992 = 14.105\,\text{lb/ft}^2,$$

$$q_{ad} = 4.700\,\text{lb/ft}^2 = 225\,\text{kPa}.$$

Fig. 6.29 shows the variations of the allowable soil bearing pressure as a function of depth D.

(2) Evaluation of q_{ad} from graphs.

(a) $D = 0$. Graph B gives directly q_{ad} for $D = 0$ and for a safety factor of 3: $q_{ad} = 0.1\,\text{ton/ft}^2 = 11$ kPa. The two methods give identical results.

(b) $D = 1$, $N = 5$. Graph C gives the increase in bearing pressure as a function of depth D: $q_{ad} = 0.1 + 0.13 = 0.23\,\text{tons/ft}^2 = 25$ kPa.

The two methods give identical results.

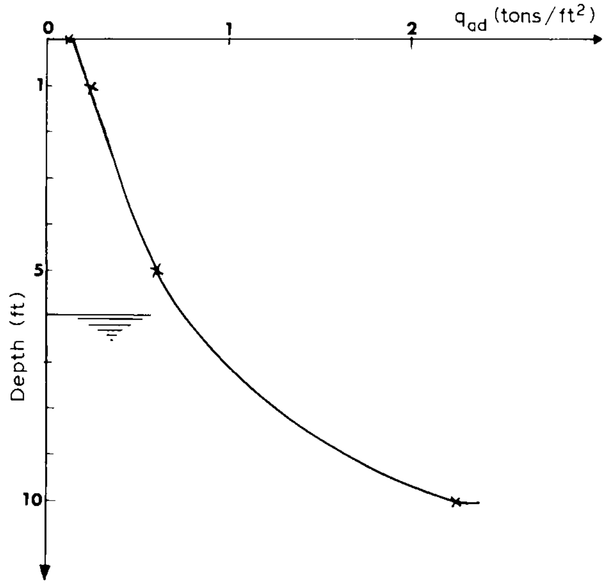


Fig. 6.29.

(c) $D = 5$. If there were no water table, we would have $q_{\text{ad}} = 0.1 + 0.52 = 0.62\,\text{ton/ft}^2 = 66\,\text{kPa}$. Because of the presence of the water-table 1 ft below the footing bottom, the first term must be divided by $(1 + 2/3) = 1.66$. Then $q_{\text{ad}} = 0.06 + 0.52 = 0.58\,\text{ton/ft}^2 = 62\,\text{kPa}$.

(d) $D = 10$ ft. For $N' = 17$, it was shown that $N = 16$. The allowable bearing pressure is, according to graph B: $q_{\text{ad}} = 1\,\text{ton/ft}^2$, but this pressure must be divided by 2 to account for the water table. The increase of the stress due to embedment is $2.3\,\text{tons/ft}^2$ (see graph C).

Since the embedment is 4 ft below the water table, this increase must be divided by $(1 + 4/10) = 1.4$ or: $q_{\text{ad}} = 1/2 + 2.3/1.4 = 0.5 + 1.64 = 2.14\,\text{tons/ft}^2 = 229\,\text{kPa}$.
A good accordance between the two methods can also be noted.

Remark

The allowable bearing pressures are limited further to account for settlements. All the above calculated allowable bearing pressures have a safety factor with respect to the ultimate failure pressure. They must be further reduced to account for the settlements that they could generate. For that reason, graph D must be used which gives q_{ad} for a maximum settlement of 1 inch , assuming the water table at depth B below the footing. If the water table is close to the footing, q_{ad} given by graph D must be divided by 2.

For $B = 3$ ft, graph D gives
for $N = 5$: $q_{\text{ad}} = 0.5\,\text{tons/ft}^2 = 54\,\text{kPa}$,
for $N = 16$: $q_{\text{ad}} = 1.9\,\text{tons/ft}^2 = 203\,\text{kPa}$.

(3) Allowable bearing pressures from Meyerhof's formula [22].

$$q_{ad} = NB(1 + D/B)(1/30) \tag{1}$$

for B, expressed in feet, q_{ad} in tons/ft^2. The above formula is valid only for cohesionless soils above the water table. For clayey sands and submerged sands, q_{ad} must be divided by 2.

Meyerhof also proposed: $q_d = N(B + D)/C$ where $C = 10$ for fine sand, $6 < C < 8$ for coarse sand and gravelly sand, $15 < C < 20$ for silty sand and non-plastic silts.

Equation (1) gives respectively:

(a) $D = 0$, $N = 5$: $q_{ad} = (5 \times 3)/30 = 0.5$ tons/ft^2 = 53 kPa.

(b) $D = 1$, $N = 5$: $q_{ad} = (5 \times 3)(1 + 1/3) \times 1/30 = 0.667$ tons/ft^2 = 71 kPa.

(c) $D = 5$, $N = 4$, if there were no water table: $q_{ad} = 4 \times 3(1 + 5/3) \times 1/30 = 1.07$ tons/ft^2 but because of the water table, the result must be divided by 2 and $q_{ad} = 0.54$ tons/ft^2 = 58 kPa.

(d) $D = 10, N = 16$: $q_{ad} = \frac{1}{2}[16 \times 3(1 + 10/3) \times 1/30] = 3.47$ tons/ft^2 = 370 kPa.

For cases (a) and (b), Meyerhof's formula gives allowable bearing pressures considerably higher than the other two methods. For cases (c) and (d) it gives about the same or slightly higher results. These values should also be decreased to limit the footing settlements to 1 inch or less.

Meyerhof esimates that for $B < 4$ ft, q_{ad} must be limited by $N/8$ or for each of the preceding cases:

case *a*	$q_{ad} \leqslant 0.62$ ton/ft^2 = 66 kPa	$(D = 0)$
case *b*	$q_{ad} \leqslant 0.62$ ton/ft^2 = 66 kPa	$(D = 1')$
case *c*	$q_{ad} \leqslant 0.5$ ton/ft^2 = 54 kPa	$(D = 5')$
case *d*	$q_{ad} \leqslant 2$ ton/ft^2 = 214 kPa	$(D = 10')$

Finally, it can be concluded that all the methods give close results, except for case *a*, which is hardly ever encountered in practice anyway.

★★ *Problem 6.18* Pile capacity determination in sand from SPT

Assume the same givens as in the preceding problem, and assume that the sand layer overlies bedrock. Determine the ultimate bearing capacity, in tons, of a driven pile of 1 ft diameter at a depth of 25 ft.

Solution

The ultimate pile bearing capacity is composed of two components, namely end-bearing and skin frictions: $Q_d = Q_a + Q_f$.

Let A be the end area of the pile of diameter B, from Meyerhof and for the case of sands:

$$Q_d = 4NA + \frac{\pi B \times D}{50} \times \bar{N} \text{ (average over pile shaft).} \qquad (Q_d \text{ in tons})$$

The corrected values of N because of the water table must first be determined

N'	5	4	30	17	30	35	35
N	5	4	22.5	16	22.5	25	25

The $\overline{N}$ average value for lateral friction along the pile shaft is:

$$\overline{N}_{\text{average}} = \frac{5 + 4 + 22.5 + 16 + 22.5}{5} = 70/5 = 14 \text{ from which:}$$

$$Q_d = 4 \times 22.5 \times (\pi/4) \times 1^2 + \pi \times 1 \times 25 \times (14/50) = 70.7 + 22 = 92.7 \text{ tons.}$$

For a safety factor of 2, we have: $Q_{ad} = 46$ tons $= 409$ kN

★*Problem 6.19* **Cost comparison of soil investigations based on different types of in-situ tests**

Determine from the unit prices given below in U.S. dollars 1984 the total costs of soil investigations using different types of in-situ tests, such as pressuremeter tests, static-electric cone-penetrometer tests and S.P.T. Assume sites are readily accessible.

Soil investigation A: for the control of vibroflotation or dynamic compaction (Menard-method) of a fill, it is proposed to perform 10 tests to 3 m depth and 2 to 7 m depth.

Soil investigation B: three tests must be performed to a 20 m depth at the site of a proposed building. Assume that both sites are 150 km from contractor office. The unit rates are given below:

For the pressuremeter:

Mobilization and demobilization	*US \$ 80/hr*
Drilling to 20 m, by linear meter, with a pressuremeter test, each meter	*US \$ 100/hr*

For the static cone-penetrometer:

Mobilization (C.P.T.) demobilization and penetration	*US \$ 120/hr*

For the S.P.T.:

Mobilization and demobilization	*US \$ 80/hr*
Drilling from 0 to 20 m, with a test performed every 1.50 meter	*US \$ 80/hr*

Solution

Soil investigation A:

(1) Pressuremeter tests:

These tests should be performed every meter, therefore:

Mobilization and demobilization, 6 hrs at $80	US $ 480
Drilling and pressuremeter test (44 m) in, say, 44 hrs at $100	US $ 4400
Total	US $ 4880

(2) Cone-penetrometer tests:

Mobilization and demobilization, 6 hrs at $120	US $ 720
Static penetration, assume 12 hrs at $80	US $ 1440
Total	US $ 2160

(3) S.P.T.:

Mobilization and demobilization, 6 hrs at $80	US $ 480
Drilling, 44 m in, say, 28 hrs at $80	US $ 2240
Total	US $ 2720

To summarize, for soil investigation A:

(1) cone-penetrometer	US $ 2160
(2) pressuremeter tests	US $ 4880
(3) S.P.T.	US $ 2720

Because the nature of the fill is known, it is not necessary in this case to include drilling and sampling in the cost estimate.

Soil investigation B

Three tests to 20 m are proposed:

(1) Pressuremeter tests:

Mobilization and demobilization, 6 hrs at $80	US $ 480
Drilling with pressuremeter test, every m, 60 hrs at $100	US $ 6000
Total	US $ 6480

(2) Cone-penetrometer test:

Mobilization and demobilization, 6 hrs at $120	US $ 720
8 hrs of penetration at $120	US $ 960
Total	US $ 1680

(3) S.P.T.:

Mobilization and demobilization, 6 hrs at \$80	US \$ 480
Drilling, 48 hrs (with a test every 1.50 m) at \$80	US \$ 3840
Total	US \$ 4320

For this example, the summary is:

(1) Cone-penetrometer	US \$ 1680
(2) Pressuremeter	US \$ 6480
(3) S.P.T.	US \$ 4320

However, it should be noted that, unless the site is in a very well known geologic and geotechnic area, it would be necessary to include one bore hole with soil sampling to the static-penetrometer investigation. The cost of this bore hole and sampling must be added to the cost of the cone-penetration and this additional cost would still make that investigation the cheapest. Its added cost would be about one-third of the total.

Remarks

Often, in pressuremeter investigations, for the purpose of reducing costs, tests are made only every 2, 3, 4, or even 5 m depth intervals, to the detriment of profile definition.

It is important to underline the fact that the cost of the cone-penetrometer investigation is not high, when considering that it is the only one yielding continuous information throughout the depth of penetration. The more expensive alternate of the pressuremeter investigation yields information at best every meter. More data is made available at a lower total cost with the penetrometer. However, in addition to the p_f and p_l limits obtained by the pressuremeter, the pressuremeter modulus E_p is also determined.

It should also be mentioned that laboratory testing of soils would be more expensive than SPT or pressuremeter measurements because of the cost of recovering undisturbed samples (about US \$30 each) and the added costs of the tests themselves. For each sample, the additional laboratory testing cost would be approximately US \$600, depending on performed tests.

As a general rule, both technical and financial considerations must be evaluated because each type of soil investigation has advantages that the other two do not present. No single method is always the best for all projects.

★★★*Problem 6.20* **Comparison of settlement calculation based on static penetrometer tests and consolidometer tests data; determination of the value of the coefficient α**

The following profile was determined at a site along the Saône River near Lyons (France) with the Andina, static-dynamic penetrometer:

— from 0 to 3 m: old sandy gravel fill;
— from 3 to 6.5 m: clayey silt;
— below 6.5: sandy-gravelly alluvium.

The penetration diagram of Fig. 6.30 is representative of the soil resistances at the site. It is proposed to erect a light building consisting of 2

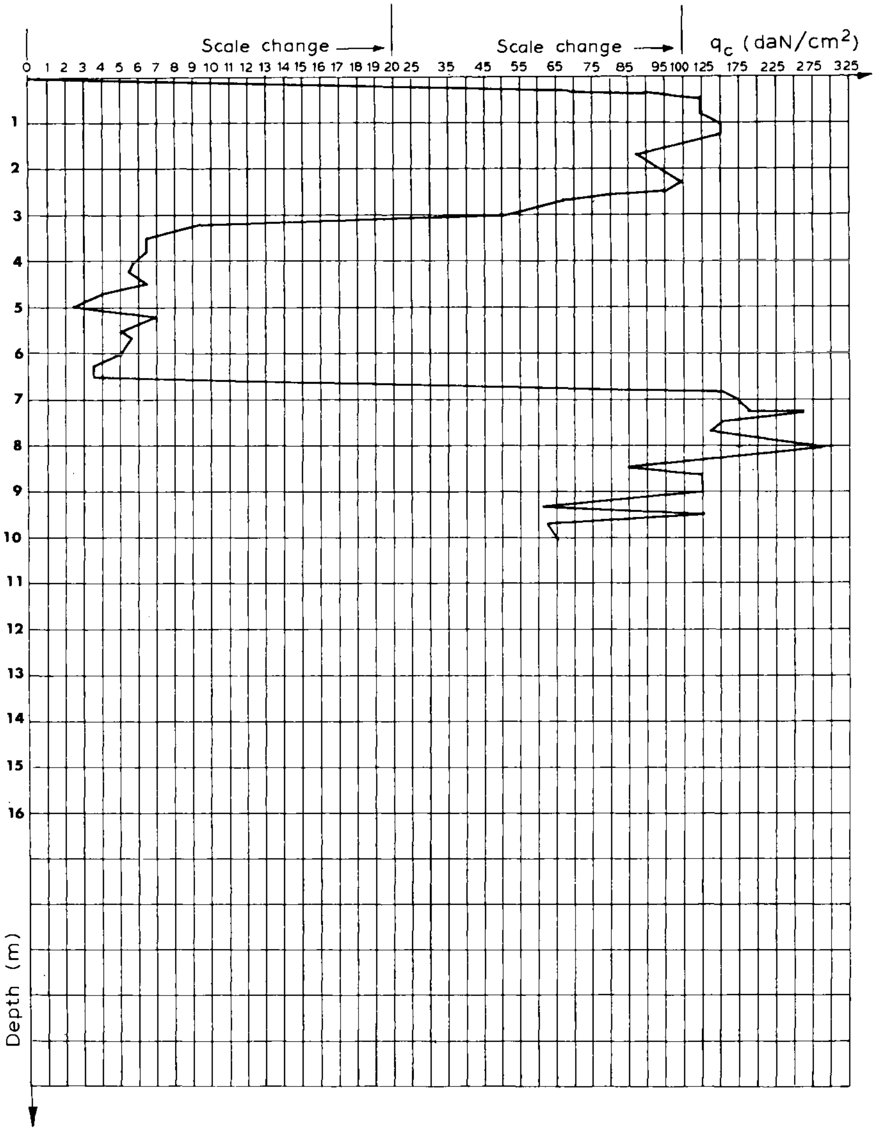


Fig. 6.30. Andina static-dynamic penetrometer result.

stories, which would impart stress of 20 kPa on bearing walls 3 m apart.

Look at a possible shallow footing support with an embedment of 0.5 m into the old fill. From the penetrometer test results determine the settlements expected due to the loading.

An exploration borehole was made to a depth of 4.5 m nearby and an undisturbed sample of silt was recovered at that depth. A consolidation test was performed on the sample in the laboratory and the results are plotted on Fig. 6.31.

Compare the results of the two methods of settlement analysis. Determine the value of the coefficient α by the method suggested by Sanglerat (1972)

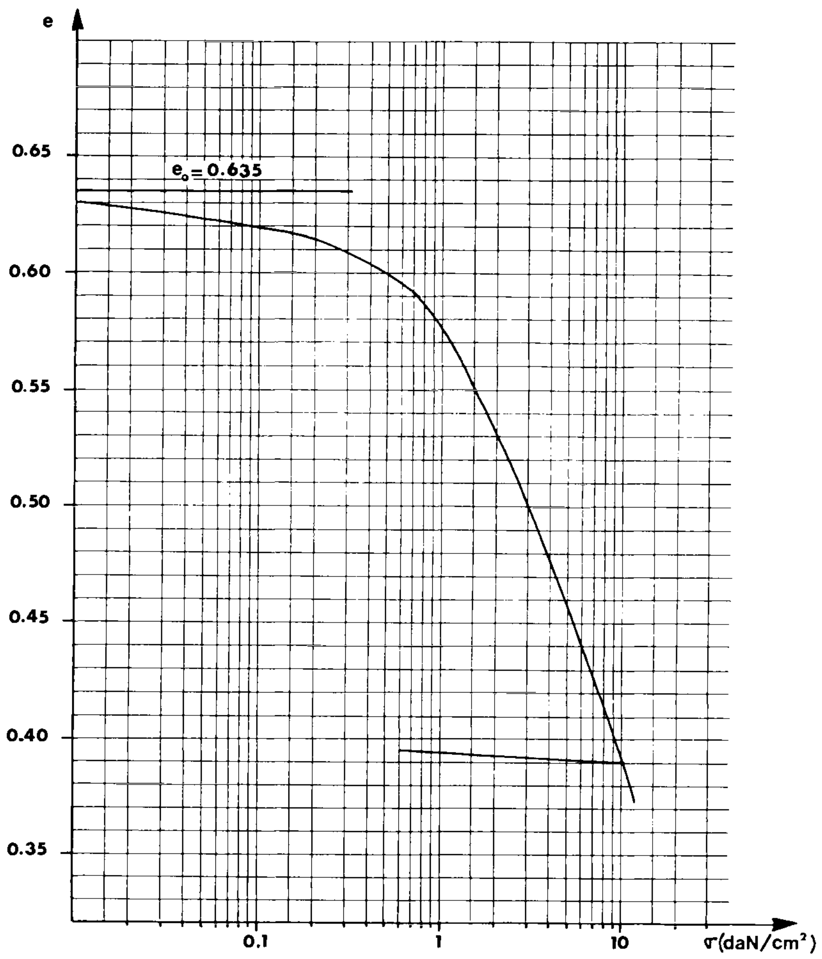


Fig. 6.31. Consolidation test.

[27]. Assume the old fill density and that of the silt to be 1.8. The water table is located at over 6 m depth.

Solution

From the penetrometer test data, it may be concluded that the old fill is quite dense ($50 < q_c < 150\,\mathrm{daN/cm^2}$). Because the building loads are light, the fill is adequate to support the loads. Footings separation, from center to center would be 3 m (that of the bearing walls). The fill thickness below the footings would be 2.5 m. If we assume a load distribution of 2 to 1 through the fill, the stress would be almost uniform at the base of the fill and equal to 20 kPa or $0.2\,\mathrm{daN/cm^2}$. The fill layer acts as a mat-type foundation.

(a) Settlement calculations based on the Andina penetrometer test data.

Settlements would occur in the silt layer. The average cone-bearing value in that layer is $q_c = 5\,\mathrm{daN/cm^2}$. If we assume the silt to have a low plasticity, α will be in the range $3 < \alpha < 6$. $E' = \alpha q_c$ or $15 < E' < 30\,\mathrm{daN/cm^2}$.

The stress increase at the surface of the silt layer is: $\Delta\sigma = 0.2\,\mathrm{daN/cm^2}$ and over the thickness of the silt layer ($H = 3.5$ m) the settlement would be $\Delta h = H\Delta\sigma/\alpha q_c = 350 \times 0.2/15 = 4.7$ cm for $\alpha = 3$ or: $\Delta h = 350 \times 0.2/30 = 2.3$ cm for $\alpha = 6$.

The expected settlement would be between $2\,\mathrm{cm} < \Delta h < 5\,\mathrm{cm}$.

It was implicitly assumed that the fill and underlying sand-gravel substratum are incompressible because of their density and the light surface building loads. An accurate calculation from the static penetrometer would give, for $\alpha = 3$ and $q_c = 100\,\mathrm{daN/cm^2}$ (average value), in the fill a settlement of about $w = 0.2$ cm, which is minimal compared to that caused by the silt layer.

(b) Settlement calculation from the consolidation test result.

The effective overburden pressure at the depth where the sample was recovered must be computed. It is at depth 4.5 m, or just at mid-height of the compressible silt layer.

We have: $\sigma_{c'} = 18 \times 4.5 = 81\,\mathrm{kPa} \simeq 0.8\,\mathrm{daN/cm^2}$.

The compression curve of Fig. 6.31 indicates that the void ratio at that stress level is $e_0 = 0.586$.

The load increment due to the building weight over the thickness of the silt layer is about $\Delta\sigma = 0.2\,\mathrm{daN/cm^2}$. The stress at mid-height would then be: $\sigma = \sigma'_0 + \Delta\sigma = 0.8 + 0.2 = 1\,\mathrm{daN/cm^2}$. The corresponding void ratio for that stress level is: $e = 0.576$.

The void ratio has decreased by $\Delta e = 0.586 - 0.576 = 0.010$. Thus the consolidation of the sample of thickness h is $\Delta h/h = \Delta e/(1 + e_0)$

from which the settlement of the silt layer is: $\Delta h = 350 \times \dfrac{0.010}{1 + 0.586} = 2.2\,\mathrm{cm}$.

(c) Comparison of the results of the two methods.

The settlement computed on the basis of the consolidation test is practically equal to that computed on the basis of the static penetrometer for a value of $\alpha = 6$.

(d) Evaluation of the α coefficient.

The compression curve shows that the preconsolidation stress σ_c is equal to about $0.8\,\text{daN/cm}^2$. Since this corresponds to the effective overburden stress, it is indicative that the silt layer is normally consolidated. It also shows that the consolidation due to the old fill is completed for all practical purposes.

Therefore, the following formula is applicable:

$$\alpha = 2.3\,\frac{1 + e_{co}}{C_{co}}\,\frac{\sigma_0'}{q_c},$$

where σ_0' = effective overburden stress;

e_{co} = void ratio corresponding to σ_0';

$$C_{co} = -\frac{e_c - e_{co}}{\log(1 + 1/\sigma_0')}$$

e_1 = void ratio corresponding to $\sigma_0' + 1$ (daN/cm^2);

q_c = cone bearing value from the penetrometer test at the depth where the sample was recovered, that is from 4.5 to 4.8 m.

In this case we get $q_c = 4\,\text{daN/cm}^2$ at that depth.

From the compression curve we have:
for $\sigma_0' = 0.8\,\text{daN/cm}^2$, $e_{co} = 0.586$
for $\sigma_0' + 1 = 1.8\,\text{daN/cm}^2$, $e_1 = 0.536$

Then:

$$C_{co} = \frac{0.586 - 0.536}{\log(1 + 1/0.8)} = 0.142 \quad \text{and} \quad \alpha = 2.3 \times \frac{1 + 0.586}{0.142} \times \frac{0.8}{4} = 5.14$$

or, say, $\alpha = 5$.

Remark

This value of α is within the range $3 < \alpha < 6$. However, the question arises why the value of $\alpha = 6$ was not obtained since the settlement calculations giving an answer of 2.2 were based on that value.

The difference is due mainly to the following reasons: to calculate α, an increase of stress of $\Delta\sigma = 1\,\text{daN/cm}^2$ from σ_0 was considered. On the compression curve, it can be seen that for this increment, the slope of the curve is steeper than that caused by the building load $\Delta\sigma = 0.2\,\text{daN/cm}^2$. In other words, a value of $\alpha = 5$ would overestimate the settlement.

★★★*Problem 6.21* **Evaluation of the pressuremeter parameters p_l, p_f and E_p for clay**

A pressuremeter test [4, 8] was performed at a depth H of 2.5 m, in clay. The following results were obtained:

Gross pressure P (daN/cm^2)		0.5	1	1.5	2	2.5	3	3.5	4	4.5
Volume change (cm^3)	at 15 s	105	138	154	162	182	192	240	307	555
	at 30 s	107	139	154	163	184	195	245	317	564
	at 60 s	110	140	155	165	185	198	250	325	575

The initial volume of water injected at very low pressure at virgin state was $V_i = 15\,cm^3$ and the probe volume at virgin state was $593\,cm^3$. The probe calibration curve may be plotted from the following table:

Pressure P' (daN/cm^2)	0	0.160	0.230	0.320	0.420	0.510	0.600	0.710	0.750
Volume change in the probe (cm^3) at 60 s	12	50	73	110	163	233	347	503	700

Calculate the pressuremeter parameters assuming that the manometer was located at a height $h = 80\,cm$ above grade at the borehole location (Fig. 6.32).

Solution

To calculate limit pressure p_l, creep pressure p_f and pressuremeter modulus E_p we must consider the actual pressures applied to the borehole wall (Fig. 6.32).

Taking into account the calibration pressure P' at any point, which

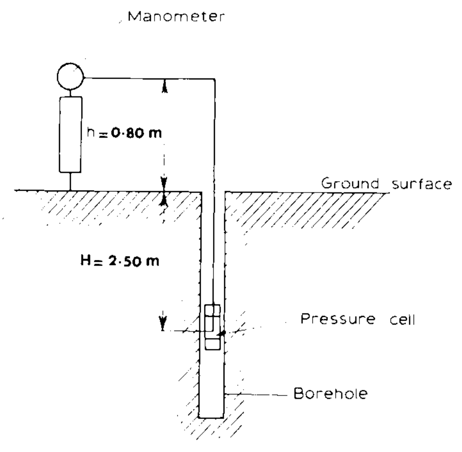


Fig. 6.32.

gives the same volume change as the gross pressure P and the level h of the manometer above the ground, the actual pressure to the soil is given by:

$$P_r = P - P' + (H + h)\gamma_w = P - P' + 0.33\,\text{daN/cm}^2$$

(a) Limit pressure. The gross limit pressure is the asymptote's abscissa of the pressiometric curve (p, V), i.e.: $(p_1)_r = 4.6\,\text{daN/cm}^2$.

But for practical purposes the limit pressure is conventially the point at which the initial borehole volume (at the beginning of the elastic phase) has doubled, i.e.:

$$\Delta V = 593 + 15 + 140 \simeq 750\,\text{cm}^3$$

Therefore:

$$V = 593 + 2(15 + 140) \simeq 903\,\text{cm}^3.$$

At $V = 903\,\text{cm}^3$, the corresponding value of P' is about $0.76\,\text{daN/cm}^2$ on the calibration curve (Fig. 6.34).

From this, the limit pressure is: $p_1 = 4.6 - 0.760 + 0.33 = 4.17\,\text{daN/cm}^2$.

(b) *Creep pressure p_f.* The pressure at the creep point can be determined from either:

— Fig. 6.35, which gives $(p_f)_{r_1} \simeq 2.8\,\text{daN/cm}^2$;

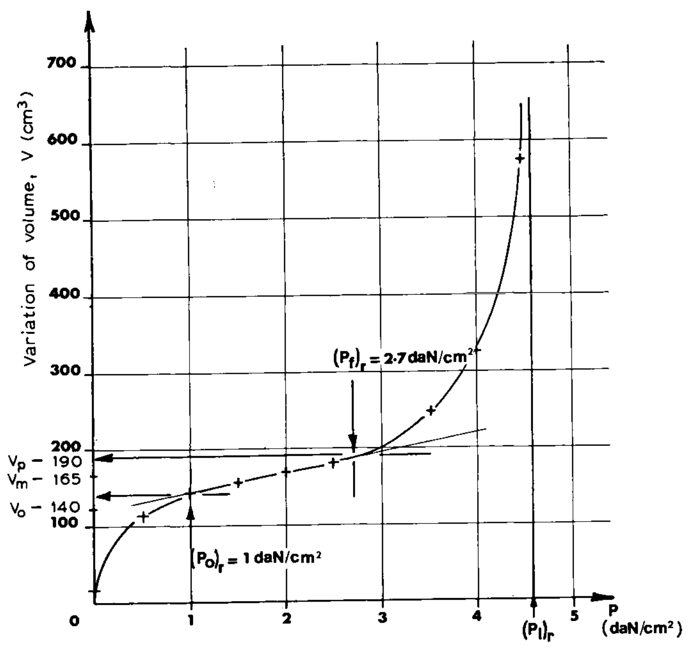


Fig. 6.33. Limit-pressure curve.

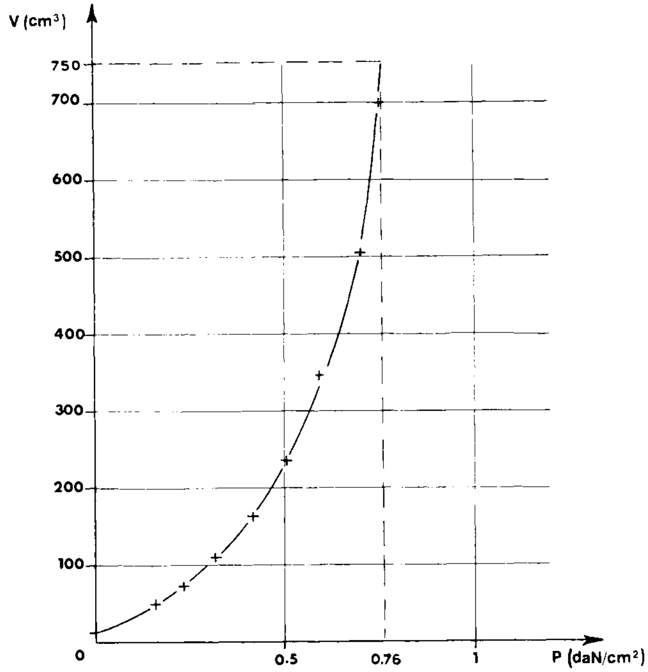


Fig. 6.34. Calibration curve.

— or, more simply, from the pressuremeter curve at the end of the straight line section: $(p_f)_r = 2.7\,\mathrm{daN/cm^2}$.

The corrected creep pressure p_f is obtained from the curve of Fig. 6.35 and from the calibration curve (Fig. 6.34) at $\Delta V = 190\,\mathrm{cm^3}$. From this, $P' = 0.45\,\mathrm{daN/cm^2}$, $p_f = 2.8 - 0.45 + 0.33 = 2.68\,\mathrm{daN/cm^2}$.

(c) *Pressuremetric modulus* E_p. The formula for E_p is $E_p = K \times \Delta P/\Delta V = 2(1+\nu)(V_0 + v_m)\Delta P/\Delta V$.

The values for ΔP and ΔV correspond to the straight line section of the curve on Fig. 6.33.

In this case: $V_0 = 593 + 15 = 608\,\mathrm{cm^3}$ and v_m is the volume of water injected at pressure P at the mid point of the straight-line section of the gross curve, i.e., $v_m = 165\,\mathrm{cm^3}$.

For $\nu = 0.33$, we have:

$$\Delta P = (P_f)_r - (P_0)_r \simeq 2.7 - 1 = 1.7\,\mathrm{daN/cm^2}$$

$$\Delta V = 190 - 140 = 50\,\mathrm{cm^3}$$

$$V_0 + v_m = 608 + 165 = 773\,\mathrm{cm^3}$$

$$E_p = 70\,\mathrm{daN/cm^2}.$$

Remark

The calibration curve should be taken into account:

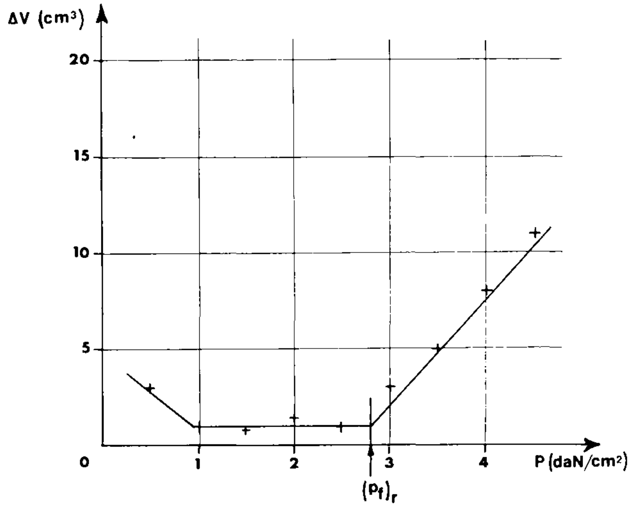


Fig. 6.35. Creep diagram $[\Delta V]_{30\,\text{sec}}^{1\,\text{min}}$.

$$\Delta P = (2.7 - 0.44) - (1 - 0.38) = 1.64\,\text{daN/cm}^2$$

which yields the very close result for $E_p = 67.5\,\text{daN/cm}^2$.

It should be noted that the above method is only correct for the Menard pressuremeter and cannot be used for the self-boring pressuremeter. For this kind of device, the interpretation proposed by Baguelin et al. [2, 3] should be adopted.

★★★*Problem 6.22* Determination of a bridge foundation based on pressuremeter test results

It is proposed to construct a 15 m road bridge across a river. The soil on either bank of the river was investigated by means of boreholes and pressuremeter testing. The results are presented in Fig. 6.36.

Design the foundations scheme using the following criteria
road level: 248 m a.s.l.;
dead load: 200 kN per meter width;
live load: 100 kN per meter width;
A cross-section of the project is shown in Fig. 6.37.

Solution

The retaining walls will be supported on spread footings at the level of the gravelly clay layer. This will eliminate settlements which would occur if the footing was placed on the compressible organic silt (see Fig. 6.37).

The access fills of some 3 m in thickness will settle differentially with respect to the retaining wall since they will cause consolidation of the organic silt layer. It would be advantageous, in this case, to preload the banks. This would require taking undisturbed samples of the silt to determine the optimum time for the application of a preload. The following planning may be adopted:

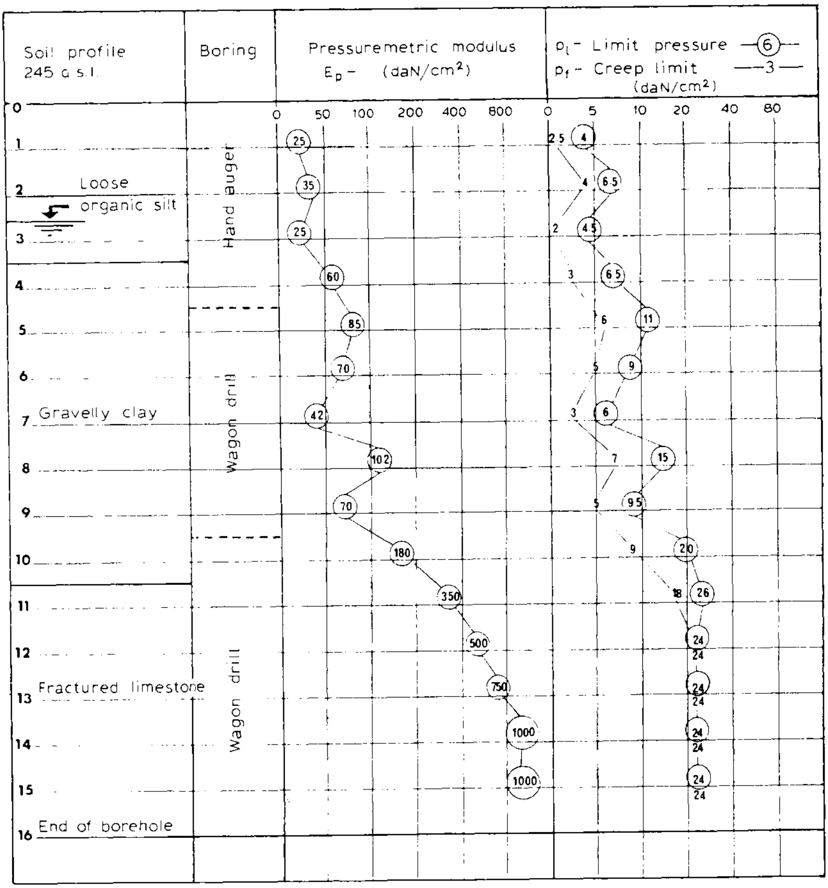


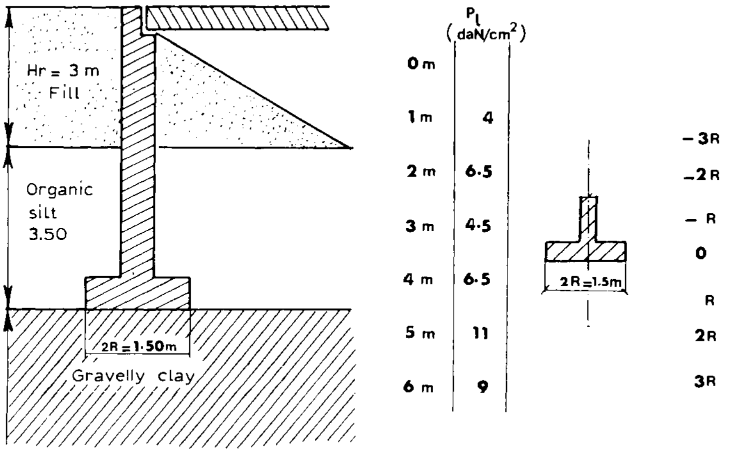
Fig. 6.36.

— construct the footings and the deck columns;
— place a preload fill and allow settlements to occur on the banks;
— build the roadway.

(A) Bearing capacity. To evaluate the allowable soil-bearing pressure, only the vertical loads given will be considered, since the horizontal loads will be nominal because of the construction method (deck columns). The maximum load will be of the order of 300 kN per meter width. The limit pressure is of the order of $(4.5 + 6.5)/2 = 5.5\,\text{daN/cm}^2$ at the proposed foundation level. A foundation whose width will distribute the load so that $q_{ad} \simeq p_l/3 \simeq 200\,\text{kN/m}^2$ must be considered. As a preliminary evaluation, $B = 300/200 = 1.50$ m. The vertical ultimate bearing pressure, q_r, is then:

$$q_r = q_0 + k(p_l - p_0)$$

where:



Figs. 6.37 and 6.38.

• p_0 = total horizontal earth pressure (on a vertical plane) at the time of the test. Assuming a coefficient of earth pressure at rest of $K_0 = 0.5$ and $\gamma_{sat} = \gamma_h = 20\,\mathrm{kN/m^3}$ we have:

$$p_0 = 2.5 \times 20 \times 0.5 + 1 \times 10 \times 0.5 + 10 \times 1 = 40\,\mathrm{kN/m^2} = 0.4\,\mathrm{daN/cm^2}.$$

• q_0 = total vertical overburden stress (on a horizontal plane) at the proposed foundation level after construction of the fill, or:

$$q_0 = \underset{\text{(access fill)}}{3 \times 20} + \underset{\text{(backfill)}}{2.5 \times 20} + 10 \times 2 = 130\,\mathrm{kN/m^2} = 1.3\,\mathrm{daN/cm^2}$$

• p_1 = equivalent limit pressure (the soil at the foundation level is heterogeneous, see Fig. 6.38).

$$p_{le} = \sqrt[7]{p_l(-3R) \times p_l(-2R) \times p_l(-R) \times p_l(0) \times p_l(+R) \times p_l(+2R) \times p_l(+3R)}$$

or: $p_{le} \simeq \sqrt[7]{4 \times 6.5 \times 4.5 \times 5.5 \times 6.5 \times 11 \times 9}$

$$= \sqrt[7]{414{,}092.5} = 6.4\,\mathrm{daN/cm^2}.$$

This value should not be reduced to account for decompression of the soil which will occur due to excavation and groundwater flow, because the proposed method of construction is going to impose a load to reconsolidate the material prior to finalizing the structure.

• k = bearing factor, from Menard's curves (Fig. 6.39) which is a function of embedment, soil type and the footing geometry.

• The embedment ratio: h_e/R

where R = foundation radius = 0.75 m, h_e = embedment:

$$h_e = \frac{1}{p_{le}} \int_0^h p_l(z)\,dz.$$

For a preliminary evaluation, embedment is only considered in the existing soils:

$$h_e = \frac{4 \times 1 + 6.5 \times 1 + 0.5 \times 4.5}{6.4} = 2\,\text{m} \quad \text{and} \quad h_e/R = \frac{2}{0.75} = 2.7.$$

- Soil category

The foundation soil type is a clay, for which the limit pressure is less than 12 daN/cm^2, therefore the soil category is I (see Table 6I).

- Footing geometry

The length of the footing may be assumed equal to the proposed width of the road, or $L = 15$ m, hence: $L/2R = 10$.

The k value is calculated from the graph of Fig. 6.39 for the given example $h_e/R = 2.7$ and $L/2R = 10$; k is found as follows:
Set off k vertical line from $h_e/R = 2.7$, cutting the category I lines for strip and square footings at A and B, respectively. Draw lines A_0A and B_0B to intersect at C. Draw the line $L/2R = 10$ to C which cuts $h_e/2R = 2.7$ at M, then the k value, ordinate of M, is 1.26. The ultimate vertical bearing pressure will then be:

$$q_r = 1.3 + 1.26 \times (6.4 - 0.4) = 8.9\,\text{daN/cm}^2$$

and the allowable bearing pressure then is:

$$q_{ad} = 1.3 + 1.26/3(6.4 - 0.4) = 3.8\,\text{daN/cm}^2.$$

TABLE 6I

Soil type	Nature	Limit pressure p_l (daN/cm^2)
I	clay	0—12
	loam	0—7
II	stiff clay and marl	18—40
	loam	12—30
	loose sand	4—8
	weak soil	10—30
III	sand and gravel	10—20
	rock	40—100
IIIb	very dense sand and gravel	30—60

(After L. Menard)

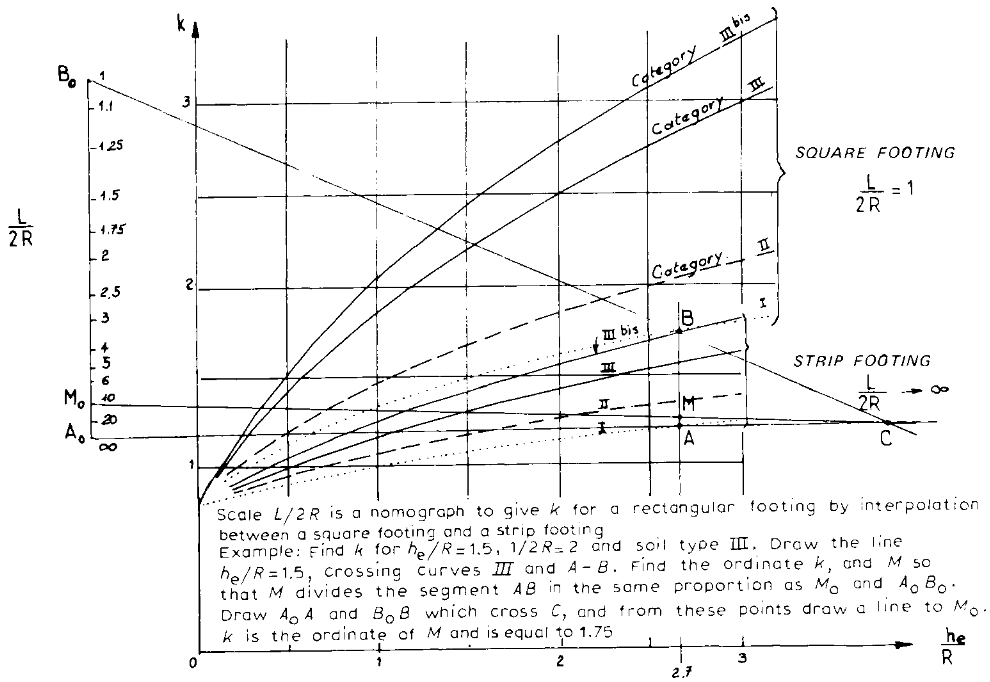


Fig. 6.39. k-values for shallow footings (after Menard).

The allowable load, per meter length of footing, would then be $3.8 \times 150 = 570$ kN, which is appreciably higher than the expected load of 300 kN per meter length.

Remarks

(a) The analysis of the consolidation results of the preload on the silt layer would eventually conclude that the bridge footing could bear on the compacted access fill. This could be checked by a complementary in situ testing investigation with penetrometer tests performed after unloading.

(b) Another construction method would be to remove the organic silt

TABLE 6J

k values (for homogeneous layer)

$\frac{D}{B}$	Square footing				Strip footing			
	I	II	III	IIIb	I	II	III	IIIb
0	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
0.5	1.3	1.5	1.9	2.1	1.0	1.1	1.2	1.3
1.0	1.6	1.8	2.5	2.8	1.2	1.3	1.4	1.6
1.5	1.8	2.1	3.0	3.3	1.2	1.4	1.6	1.8

(After L. Menard).

and replace it by compacted select material to also allow for the construction of shallow footings on the fill.

This method leads to conservatism because:

(1) a surcharge fill would increase the characteristics of the organic silt therefore the value of h_e. This would make us to take this layer into account;

(2) the compacted fill would have a $p_1 > 15\,\mathrm{daN/cm^2}$ which would also lead to a larger h_e;

(b) the allowable bearing capacity will in fact be higher because of the consolidation of the clay due to the surcharge;

(c) it was assumed that the access fills would not impart horizontal pressures.

(B) Settlement evaluations. The final settlement of the footing will be at most equal to the sum of the settlements caused by the fill weight and structural dead loads. Live loads applied only for short durations can be ignored.

(a) Settlement due to fills. The following formula may be used:

$$w_r = \int_0^h \frac{\alpha(z)P(z)\beta(\Gamma)}{E(z)}\,dz$$

where:

- P_z = vertical stress increment at depth z due to the fill.

Because of the large width of the fill at its base (about 25 m) and the thickness of the soil layers (10.25 m) susceptible to consolidation, P_z may be assumed constant with z and equal to:

$$P = h_r \times \gamma_r,$$

where h_r = fill thickness or 3 m, γ_r = unit weight of wet soil of the compacted fill or $\gamma_h = 22\,\mathrm{kN/m^3}$. So $P = 66\,\mathrm{kN/m^2}$.

- $E(z)$ = pressuremeter modulus at depth z.
- $\alpha(z)$ = rheologic coefficient depending on soil type at depth z (see Table 6K): for the organic silt $\alpha = 1/2$, for the gravelly clay $\alpha = 1/2$.
- $\beta(\Gamma) = 1$ if $\Gamma \geqslant 2$ (Γ is the ratio of the limit pressure of the soil to the actual stress at depth z).

For the silt: limit stress is $\simeq 0.8\,p_l$, actual stress is $= 66\,\mathrm{kN/m^2} = 0.66$ $\mathrm{daN/cm^2}$ and $\Gamma = 4/0.66 = 6$.

For the clay, which is stiffer: $\beta(\Gamma) = 2/3 \times \Gamma/(\Gamma - 1) = 0.8$.

Finally:

$$w_r = \alpha \times P \times \beta(\Gamma) \sum_i \frac{h_i}{E(z)}$$

$$= \frac{1}{2} \times 0.66 \times 0.8 \left(\frac{1}{60} + \frac{1}{85} + \frac{1}{70} + \frac{1}{42} + \frac{1}{102} + \frac{1}{70} + \frac{0.75}{180}\right)$$

$$= \frac{0.27}{100} \times (1.7 + 1.17 + 1.43 + 2.4 + 1 + 1.43 + 0.42)$$

$$= \frac{0.27}{100}(9.55), \quad w_r = 2.6\,\text{cm}.$$

TABLE 6K

Soil type	Peat	Clay		Silt		Sand		Sand and gravel		Type	Rock
	α	E/p_l	α	E/p_l	α	E/p_l	α	E/p_l	α		α
Over-consolidated	—	>16	1	>14	2/3	>12	1/2	>10	1/3	slightly fractured	2/3
Normally consolidated	1	9.16	2/3	8.14	1/2	7.12	1/3	6.10	1/4	normal	1/2
Under-consolidated very loose	—	7.9	1/2	5.8	1/2	5.7	1/3		—	highly fractured	1/3
										highly weathered	2/3

(b) *Settlements due to structural dead loads*. Menard proposes the following formula: $W = W_1 + W_2 + W_3$, where W_1 = immediate settlement, not usually considered:

$$W_2 = \frac{1.33}{3E_B} \times p \times R_0 \times \left(\lambda_2 \times \frac{R}{R_0}\right)^{\alpha} \quad \text{(settlement due to deviatoric stress)}$$

$$W_3 = (\alpha/4.5\,E_A) \times p \times \lambda_3 \times R \quad \text{(settlement due to isotropic stress).}$$

where:

- R = half width of a rectangular footing or radius of a circular footing, $R = 75$ cm;
- $R_0 = 30$ cm (reference dimension);
- p = average increase in stress due to the footing, with respect to the natural stress condition, calculated under the permanent structural loads: $p = 200/1.50 = 133$ kPa $= 1.3$ daN/cm^2;

TABLE 6L

λ_2 and λ_3 coefficients in function of $L/2R$

$L/2R$	1 circle	1 square	2	3	5	10	20
λ_2	1	1.12	1.53	1.78	2.14	2.40	2.65
λ_3	1	1.1	1.2	1.3	1.4	1.46	1.5

• λ_2 and λ_3 = shape factors depending on footing dimensions. For $L/2R = 15/1.5 = 10$.
Table 6L gives: $\lambda_2 = 2.40$ and $\lambda_3 = 1.46$.

• E_A and E_B are the pressiometric moduli corresponding to the spherical and deviatoric stresses calculated as shown below.

• α = rheologic coefficient of the soil: $\alpha = 1/2$ (Table 6K).

Determination of E_A:

$E_A = E_1$ harmonic mean of the pressuremeter moduli to a depth R beneath the footing:

$$\frac{1}{E_1} = \frac{1}{2}\left(\frac{1}{E_0} + \frac{1}{E(-R)}\right).$$

In this case:

$$\frac{1}{E_1} = \frac{1}{3}\left[\left(\frac{1}{25} + \frac{1}{60}\right) + \frac{1}{60}\right] = 0.024$$

$$E_1 = E_A = 41\,\mathrm{daN/cm^2}.$$

Determination of E_B:

$$E_B = \frac{4}{\dfrac{1}{E_1} + \dfrac{1}{0.85\,E_2} + \dfrac{1}{E_{3/4/5}} + \dfrac{1}{2.5\,E_{6/7/8}} + \dfrac{1}{2.5\,E_{9\text{ to }16}}}$$

E_2 = harmonic mean of the moduli at depths $-R$ and $-2R$ (Fig. 6.40):

$$\frac{1}{E_2} = \frac{1}{2}\left(\frac{1}{60} + \frac{1}{85}\right) = 0.014.$$

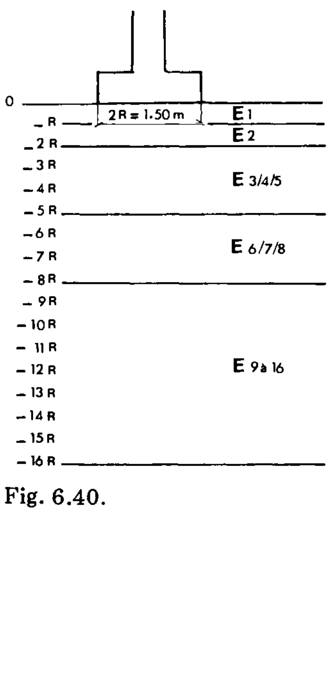


Fig. 6.40.

$E_{3/4/5}$ = harmonic mean of moduli in layers 3 to 5 (Fig. 6.40):

$$\frac{1}{E_{3/4/5}} = \frac{1}{5}\left[\frac{1}{85} + \frac{1}{70} + \left(\frac{1}{70} + \frac{1}{42}\right) + \frac{1}{42}\right] = 0.018.$$

$E_{6/7/8}$ = harmonic mean of moduli of layers 6 to 8.

$$\frac{1}{E_{6/7/8}} = \frac{1}{5}\left[\frac{1}{42} + \frac{1}{102} + \frac{1}{70} + \left(\frac{1}{70} + \frac{1}{180}\right)\right] = 0.014.$$

$E_{9\,\text{to}\,16}$ = harmonic mean of moduli of layers 9—16:

$$\frac{1}{E_{9\,\text{to}\,16}} = \frac{1}{11}\left[\left(\frac{1}{70} + \frac{1}{180}\right) + \frac{1}{180} + \frac{1}{350} + \frac{1}{500} + \left(\frac{1}{500} + \frac{1}{750}\right) + \right.$$

$$\left. + \frac{1}{750} + \frac{1}{1000} + \frac{1}{1000} + \frac{1}{1000}\right] = 0.0035$$

and:

$$E_B = \frac{4}{0.024 + \frac{1}{0.85} \times 0.014 + 0.018 + \frac{1}{2.5}(0.014) + \frac{1}{2.5}(0.003)}$$

or $E_B = 61.3\,\text{daN/cm}^2$.

$$W_2 = \frac{1.33}{3 \times 61.3} \times 1.3 \times 30 \times \left(2.40 \times \frac{75}{30}\right)^{1/2} \simeq 0.7\,\text{cm}$$

$$W_3 = \frac{1}{2 \times 4.5 \times 41} \times 1.3 \times 1.46 \times 75 \simeq 0.4\,\text{cm}.$$

Finally: $W' = W_2 + W_3 = 1.1$ cm.

Note the soft clay layer between 6.5 m and 7.5 m of which the pressuremeter modulus is: $E_c = 42\,\text{daN/cm}^2$.

In this case, Menard's method increases the settlement W' by a value W'' to account for that of the soft layer:

$$W'' = \alpha_c (1/E_c - 1/E_m) \Delta p_c H.$$

where:

- α_c = rheologic coefficient of the soft layer $\alpha_c = 1/2$,
- $E_c = 42\,\text{daN/cm}^2$.
- E_m = mean modulus of deviatoric state calculated without accounting for the presence of the soft layer (substitute to E_c the modulus values E'_c as follows:

$$\frac{1}{E'_c} = \frac{1}{6}\left(\frac{1}{60} + \frac{1}{85} + \frac{1}{70} + \frac{1}{102} + \frac{1}{70} + \frac{1}{180}\right) = 0.012$$

and $E'_c = 83\,\mathrm{daN/cm^2}$.

Going back to the calculations of the pressuremeter moduli in the deviatoric state, as explained for E, we have:

$1/E_1 = 0.024, \quad 1/E_2 = 0.014$

$$\frac{1}{E_{3/4/5}} = \frac{1}{5}\left[\frac{1}{85} + \frac{1}{70} + \frac{1}{70} + \frac{1}{83} + \frac{1}{83}\right] = 0.013$$

$$\frac{1}{E_{6/7/8}} = \frac{1}{5}\left[\frac{1}{83} + \frac{1}{102} + \frac{1}{70} + \frac{1}{70} + \frac{1}{180}\right] = 0.011$$

$$\frac{1}{E_{9\ \mathrm{to}\ 16}} = 0.004$$

$$\text{and} \quad E_m = \frac{4}{0.024 + \dfrac{0.014}{0.85} + 0.013 + \dfrac{0.011}{2.5} + \dfrac{0.004}{2.5}} = 67\,\mathrm{daN/cm^2}.$$

- Δp_c increment of the vertical stress in the middle of the soft layer. The Boussinesq formula estimates this value: $\Delta p_c \simeq 0.3\,\Delta p = 0.3 \times 1.3 = 0.4\,\mathrm{daN/cm^2}$.
- H = thickness of the soft layer: $H = 100$ cm.

$$\text{Finally:} \quad W'' = \frac{1}{2}\left(\frac{1}{42} - \frac{1}{67}\right) \times 0.4 \times 100 = 0.2\,\mathrm{cm}.$$

The expected total settlement of the footing would be:
$W_t = 2.6 + 1.1 + 0.2 = 3.9$ cm.

To take into account the method of construction, the residual footing settlements after completion of construction would be reduced to those caused by the dead loads, or: residual $W = 1.3$ cm. This is tolerable for most bridge structures.

Remark

The total settlement of the fill may be estimated by method B - a.

$$W = \frac{1}{2} \times 0.66 \times 1\left[\frac{1.5}{25} + \frac{1}{35} + \frac{1}{25} + \frac{1}{60} + \frac{1}{85} + \frac{1}{70} + \frac{1}{42}\right] \simeq 6.4\,\mathrm{cm}$$

$W = 6.4$ cm which may be divided into that portion occurring below the footing (2.6 cm) and that occurring above the footing (4.8 cm).

The preloading of foundation soil by placement of the fill will have to be of sufficient time duration to allow for the settlement of 6.4 cm to occur. This may be controlled by field measurement.

★★★*Problem 6.23* Estimation of instantaneous settlement from pressuremeter test results

Assume the pressuremeter modulus of a thick saturated clay layer to be $E_p = 42\,daN/cm^2$. Estimate the immediate settlement of a circular footing of diameter $2R = 2\,m$ supported on the clay and imposing a stress $\sigma = 1.5\,daN/cm^2$.

Solution

The instantaneous settlement corresponds to the initial deformation of the soil and occurs at 'constant volume'. The Poisson's ratio for the undrained condition is $\nu = 0.5$.

The settlement evaluation, taking into account Boussinesq's formula is:

$$s_i = \frac{\pi}{2} \times \frac{1-\nu_u^2}{E_u} \times \sigma \times R.$$

A pressuremeter test performed in a saturated clay may be considered to an undrained test on the soil. Young's modulus must be considered as the constant-volume elastic modulus in the formula for E_p, if the value of $\nu_u = 0.5$. However, $E_p = 2(1+\nu)V(\Delta P/\Delta V)$ is normally calculated for $\nu = 0.33$.

If we take $\nu_u = 0.5$, the corresponding value of E_u is:

$$E_u = \frac{E_p(1+\nu_u)}{1+\nu} = \frac{42(1+0.5)}{1+0.33}$$

or $E_u = 47.4\,\text{daN/cm}^2$, and the instantaneous settlement is:

$$s_i = \frac{3.14}{2} \times \frac{1-(0.5)^2}{47.5} \times 1.5 \times 100 = 3.7\,\text{cm}.$$

Problem 6.24 Design a foundation from static and dynamic penetrometer data and from results of pressuremeter tests

Various other problems covering the interpretation of in situ tests are to be found in Volume II, in particular:

Problems 10.14, 10.15, 10.16: Shallow footings designed from pressuremeter tests.

Problem 11.10: Bearing capacity of a pile from static or dynamic penetrometer or S.P.T. tests.

Problem 11.11: Bearing capacity of a pile from pressuremeter test.

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INDEX

References beginning with numbers 1 to 6 = Volume 1.
References beginning with numbers 7 to 12 = Volume 2.
References correspond to the problem numbers.

Corrigendum

Practical Problems in Soil Mechanics and Foundation Engineering, 1,
by G. Sanglerat, G. Olivari and B. Cambou

p. XV, 6th line, "between two crystal faces" should read "between two soil faces".
p. XX, Table C, 5th column, 7th line should read "9.81×10^{-3}".