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Black Holes: Thermodynamics, Information, and Firewalls

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Black Holes: Thermodynamics, Information, and Firewalls

Abstract Black holes have presented us with some of the most baffling paradoxes in physics. From their original conception as dark stars, they have come to be understood as physical systems with their own thermodynamic behaviour. This same behaviour leads to paradoxical conflicts between some of the basic principles of physics whose resolution is not straightforward and that suggest a new structure—known as a firewall—may be present. I shall review the origins and interconnections between black hole thermodynamics, quantum fields in curved spacetime, the information paradox, and firewalls.

1 Introduction

Black Holes have presented us with paradoxical situations ever since their conceptualization in 1783 by the Reverend Michell [1]. Originally seeking a means for determining stellar masses by measuring the reduction in the speed of corpuscular light due to a given star's gravitational pull, Michell reasoned that the maximal effect measurable would be limited by the escape velocity from the star. This would have to be the speed of light, at that time most recently measured by Bradley to be 301,000 km/s [2]. Any star more massive than this upper bound (500 times the mass of the sun assuming the same average density) would not permit light to escape from its surface. While no theoretical constraints for objects having speeds greater than c were known at the time, there were no empirical measurements indicating such objects existed either. Paradoxically, such stars would be *dark stars*, invisible to an outside observer, though they could be indirectly inferred from their gravitational influence on nearby luminous objects. The relationship between their mass and radius is given by the same relativistic value $R = \sqrt{2GM/c^2}$ for Schwarzschild black holes. Ironically, the method fails because light moves through space at constant speed regardless of the local strength of gravity.

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It would take nearly two centuries before the paradoxes associated with dark stars—now referred to as black holes—would dawn upon the physics community at large. Their inexorable gravitational chokehold on matter turns from puzzle to paradox once the information content of the matter is taken into account. At this point in time there is no consistent understanding of how quantum physics allows information to either be retained in or escape from a black hole.

The purpose of this book is to present the historical and conceptual origins of this problem, making connections with the latest research in the subject. Beginning with the notion of a black hole in Sect. 2, I will review the laws of black hole mechanics and their relationship with the laws of thermodynamics in Sect. 3. These laws in turn depend upon our understanding of quantum field theory in curved space-time (discussed in Sect. 4) and of pair creation (Sect. 5). From this emerges our basic understanding of black hole radiation (Sect. 6). This confluence of ideas led to what became known as the information paradox: the puzzle of how a thermally radiating black hole can be consistent with the unitary evolution quantum physics requires, discussed in Sect. 7. It was generally thought for a time that recent conjectures about duality between gravitational physics and gauge theories straightforwardly resolve the problem (at least in principle). However more detailed study of the information paradox indicates that the resolution of this problem is not at all straightforward, and that a new structure—known as a firewall—may be present. This strange phenomenon is discussed in Sect. 8, along with responses to this new perspective on black holes. An overview of the current situation is the subject of the Sect. 9.

2 Black Holes

The physical notion of a black hole is essentially the same as that contemplated by Michell: a region of space where the gravity is so strong that nothing can escape from it. From a non-relativistic perspective this would imply that the escape velocity from this kind of region is infinite. But we don't need such a strong requirement. It is enough for an object to be a black hole if nothing luminous can escape from it. This will be the case if the escape velocity is greater than the speed of light c.

If the region is spherical, then a particle will be trapped in the region if its kinetic energy is less than its gravitational potential energy

$$\frac{1}{2}mv^2 - \frac{GMm}{r} < \frac{1}{2}mc^2 - \frac{GMm}{r} < 0 \Rightarrow r < \frac{2GM}{c^2} \equiv r_h \tag{1}$$

and so if the mass M is concentrated within a region smaller than r_h it will trap all particles moving at subliminal speed—the object will be a black hole.

Relativistic considerations imply that this is a firm limit: the invariance of the speed of light for all observers indicates that all matter travels at subliminal speed. Hence (without taking quantum effects into account) a black hole will absorb all matter and emit nothing. It is a perfect absorber, whose physical temperature is zero. The earliest and best known example of a black hole is the Schwarzschild solution

$$ds^2 = -c^2\left(1 - \frac{r_h}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{r_h}{r}} + r^2 d\Omega_2^2 \tag{2}$$

where $d\Omega_2^2 = d\theta^2 + (\sin\theta d\phi)^2$ is the standard line element on the sphere S^2. Curiously, the quantity r_h plays the same limiting role as in Newtonian theory.

The metric appears to be singular (i.e. some components becoming infinite) at both $r = r_h$ and $r = 0$, but the former singularity is due simply to a coordinate choice. Writing

$$t = t_* \qquad r = r_h\left[W\left(\exp\left(\frac{r_*}{r_h} - 1\right)\right) + 1\right] \tag{3}$$

yields from (2)

$$\begin{aligned} ds^2 &= \frac{W\left(\exp\left(\frac{r_*}{r_h} - 1\right)\right)}{W\left(\exp\left(\frac{r_*}{r_h} - 1\right)\right) + 1}\left(-dt_*^2 + dr_*^2\right) + r_h^2\left[W\left(\exp\left(\frac{r_*}{r_h} - 1\right)\right) + 1\right]^2 d\Omega_2^2 \\ &= -\frac{W\left(\exp\left(\frac{u-v}{2r_h} - 1\right)\right)}{W\left(\exp\left(\frac{u-v}{2r_h} - 1\right)\right) + 1} du dv + r_h^2\left[W\left(\exp\left(\frac{u-v}{2r_h} - 1\right)\right) + 1\right]^2 d\Omega_2^2 \end{aligned} \tag{4}$$

where W is the Lambert-W function, defined via $W(y)\exp(W(y)) = y$ and $(u, v) = ct_* \pm r_*$. The horizon $r = r_h$ is at $r_* = -\infty$. The space-time smoothly continues through $r = r_h$.

A particle moving on a radial trajectory will have $d\theta/ds = d\phi/ds = 0$; if the particle moves at the speed of light (e.g. a photon) then $ds^2 = 0$. Hence from (4) it is easy to see that ingoing (outgoing) radial light rays follow lines $du/ds = 0$ $(dv/ds = 0)$ or u = constant (v = constant). The metric (2) can be extended across $r = r_h$ along either of these null lines. Writing $(u', v') = (\exp(u/2r_h), -\exp(-v/2r_h)) = \sqrt{r/r_h - 1}$ $(\exp((r + t)/2r_h), -\exp((r - t)/2r_h))$ transforms (2) to

$$ds^2 = -\frac{4r_h^2 e^{-\left[W\left(-\left(\frac{u'v'}{e}\right)\right)+1\right]}}{W\left(-\left(\frac{u'v'}{e}\right)\right) + 1} du' dv' + r_h^2\left[W\left(-\left(\frac{u'v'}{e}\right)\right) + 1\right]^2 d\Omega_2^2 \tag{5}$$

which are referred to as Kruskal coordinates. A plot of the function $W(-x/e)$ indicates that it monotonically increases with increasing negative x and diverges at $x = 1$. Hence the metric is finite at $u'v' = 0$ (corresponding to $r = r_h$) but diverges

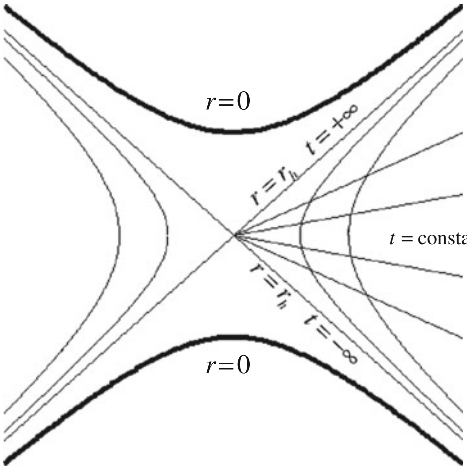

Fig. 1 Kruskal diagram: the structure of the Schwarzschild space-time in Kruskal coordinates, with the event horizon at $r = r_h$ and the singularity at $r = 0$ indicated

at $u'v' = -1$ or $r = 0$. The Kretschmann scalar $R_{abcd}R^{abcd}$ diverges at this point and so this is a genuine curvature singularity. All geodesics either meet this singularity or else extend to infinite affine parameter—in this sense Kruskal coordinates are maximal.

Figure 1 illustrates the nature of the Kruskal description of the Schwarzschild solution (2). The outside of the black hole is generally regarded to be the region at the right—this is the region described by the coordinates in (2). However the coordinates (4) describe a much larger space-time, one that includes the entire region bounded within the thick black lines at $r = 0$. These thick lines denote the curvature singularity. Kruskal coordinates (4) are maximal: all geodesics either extend to infinite affine parameter without leaving this chart or meet the singularity at $r = 0$. The surface $r = r_h$ maps to $u'v' = 0$ and is non-singular.

The chief advantage of the Kruskal diagram is that all light rays (or null rays) are at $\pm 45°$. This makes it easy to discern which regions of spacetime are in causal contact with each other, since all massive particles must move on trajectories whose slopes are steeper than 45°.

The causal structure is more easily shown in a Penrose diagram, which maps the entire space-time to a finite region. For flat Minkowski space-time the metric becomes

$$\begin{aligned} ds^2 &= -c^2dt^2 + dr^2 + r^2 d\Omega_2^2 \\ &= (\sec^2 X_+ \sec^2 X_-)\left[dUdV + \left(\frac{\tan X_+ - \tan X_-}{2\sec X_+ \sec X_-}\right)^2 d\Omega_2^2\right] \\ &= (\sec^2 X_+ \sec^2 X_-)d\tilde{s}^2 \end{aligned} \tag{6}$$

upon setting $\tan X_\pm = ct \pm r$. Since light rays obey $ds^2 = 0 = d\tilde{s}^2$ the causal relations between various regions are preserved in going from ds^2 to $d\tilde{s}^2$. The range

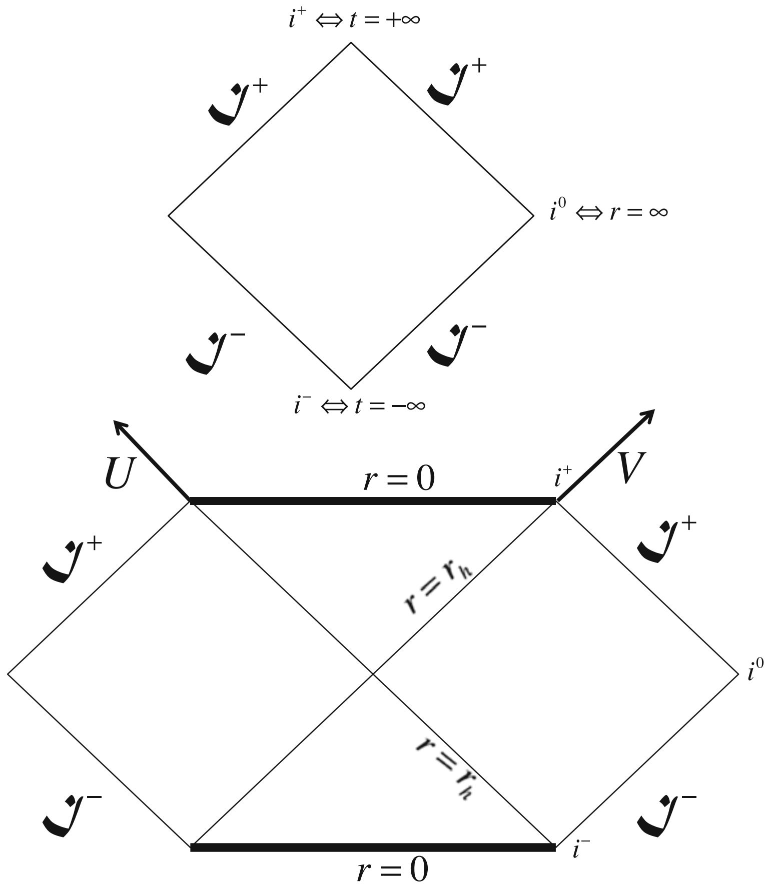

Fig. 2 Penrose diagram: the causal structure of Minkowski space-time (*top*) and Schwarzschild space-time (*bottom*). The coordinates U and V are depicted as well

of $X_\pm$ is between $\pm\frac{\pi}{2}$, and the entire space-time is mapped into a finite region, as shown at the top of Fig. 2.

More generally one maps a given spacetime manifold $\mathbb{M}$ with metric g_{ab} into a subset of a manifold $\tilde{\mathbb{M}}$ with metric $\tilde{g}_{ab}$. The conformal relation between the metrics is $\tilde{g}_{ab} = \Omega^2 g_{ab}$. The boundary of the image of $\mathbb{M}$ in $\tilde{\mathbb{M}}$ represents the ‘points at infinity’ in the original spacetime.

Returning to the Schwarzschild metric, writing

$$\tan U = u' \qquad \tan V = v' \tag{7}$$

yields a metric conformal to (5) (and therefore also to (4)). The coordinates (U, V) have finite range, and one can map the diagram in Fig. 1 to that of the bottom diagram in Fig. 2.

We see from Fig. 2 that in Minkowski space-time all future-directed light rays can reach infinity $\mathcal{I}^+$ ('scri-plus') whereas in Schwarzschild space-time any future-directed light rays that cross $r = r_h$ will encounter the singularity, and hence so will all future-directed timeline curves. This is the idea of the trapped region a black hole induces. To make this notion more precise, we need to define a region to which particles are able to escape. From 2 this region should be the portion 'near infinity', i.e. at $\mathcal{I}^+$. So a black hole region $\mathbb{B}$, in mathematical terms, is defined as

$$\mathbb{B} = \mathbb{M} - \mathbb{I}^-(\mathcal{I}^+) \tag{8}$$

where $\mathbb{I}^-(A)$ denotes the chronological past of a region A. Hence a black hole is that part of space-time not in the past of the escape-region of light rays (not in the past of $\mathcal{I}^+$ or future null infinity).

The event horizon $\mathbb{H}$ of the black hole is the boundary of $\mathbb{B}$. It is a null hypersurface (generally assumed to be at least once-differentiable) composed of future null geodesics without caustics that cannot be extended. In other words the expansion of the null geodesics comprising the horizon cannot become negatively infinite. In physical terms, it is the bounds the region of no-escape for both matter and light.

2.1 Gravitational Collapse

The Schwarzschild black hole (2) is very instructive for understanding some basic properties of black holes, but is physically unrealistic. Due to time-reversal symmetry, the singularity at $r = 0$ in the future has a counterpart in the past, yielding a 'white hole' structure at the bottom of the Penrose diagram in Fig. 2. The white hole $\mathbb{W}$ is defined as $\mathbb{W} = \mathbb{M} - \mathbb{I}^+(\mathcal{I}^-)$: it is the part of the manifold not in the future of the distant past of a time-reversed escape region. Just as a black hole is a total absorber, a white hole is a total emitter: nothing can enter it but anything can leave. Such objects are presumably physically unrealistic—they certainly do not conform to the more intuitive notion of a black hole being the result of the collapse of some form of matter into an ultra-dense no-escape region.

In fact, there is a solution to Einstein's equations describing such a collapsed object: the Oppenheimer and Snyder solution [3]. This solution matched a collapsing ball of dust (a form of stress-energy with density but no pressure) onto the metric (2), yielding a space-time that modelled the collapse of a star into a black hole. Since then many other collapse solutions have been obtained. The general form of the Penrose diagrams for such spacetimes is given in Fig. 3. We see that the left and bottom parts of the original space-time shown at the bottom of Fig. 2 are no longer present. However the future event horizon remains. There is a point in time at which the fluid collapses beyond which nothing can escape, even though the singularity has yet to form. This is given by the intersection of the diagonal line

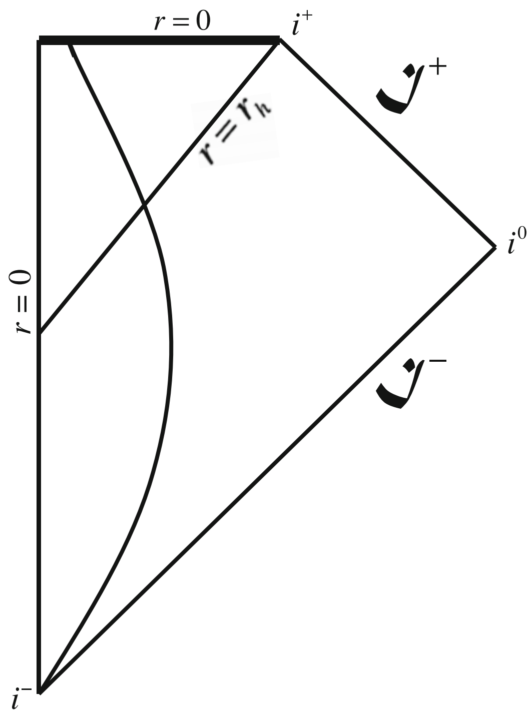

Fig. 3 Fluid collapse: the Penrose diagram of the collapse of a ball of dust. The boundary of the dust is given by the *curved line*

from i^+ with the vertical line in Fig. 3. This is the kind of black hole relevant to astrophysics. A depiction of the process in more familiar coordinates is given in Fig. 4.

It is also relevant to considerations of black hole thermodynamics. The vacuum solution (2) is applicable everywhere outside of the fluid. Since it has an event horizon, the general properties of black hole radiation—and the conundrums they introduce—that are deduced from (2) will also be present for the collapse solution shown in Fig. 4.

2.2 Anti de Sitter Black Holes

An important class of solutions of particular relevance to string theory are solutions in which the space-time is asymptotic to a space-time of constant negative curvature. This latter space-time is known as anti de Sitter (AdS) spacetime, and is a solution to the Einstein equations

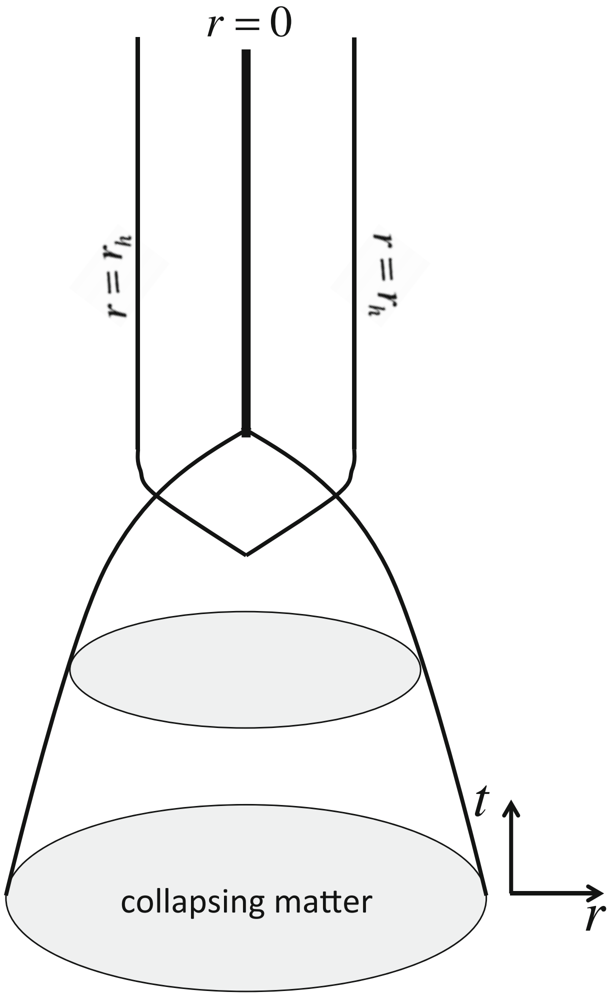

Fig. 4 Gravitational collapse: gravitational collapse of matter shown in more familiar coordinates

$$R_{ab} - \frac{1}{2} g_{ab} R - \Lambda g_{ab} = T_{ab} \tag{9}$$

with matter stress-energy tensor $T_{ab} = 0$ and cosmological constant $\Lambda = -(D-1)(D-2)/2\ell^2 < 0$ in D-dimensions. The most general vacuum metric in the static spherically symmetric case is

$$ds^2 = -c^2\left(\frac{r^2}{\ell^2} + k - \left(\frac{r_0}{r}\right)^{D-3}\right)dt^2 + \frac{dr^2}{\frac{r^2}{\ell^2} + k - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_k^2 \tag{10}$$

where $d\Omega_k^2$ is the metric on a compact $(D-2)$-dimensional space Σ_k of constant curvature with sign k. It can be written as

$$d\Omega_k^2 = d\theta^2 + \frac{\sin^2(\sqrt{k}\theta)}{k} d\Omega_{D-3} \tag{11}$$

with $k = 1$ being the $(D-2)$-sphere, $k = 0$ being a torus, and $k = -1$ being a compact hyperbolic space [4, 5], where $d\Omega_{D-3}$ is the metric of a $(D-2)$-sphere.

The metric (10) describes what is called a Schwarzschild-AdS black hole, with the constant of integration r_0 given by

$$r_0^{D-3} = \frac{16\pi GM}{(D-2)\mathcal{V}(\Sigma_k)} \tag{12}$$

where M is the mass of the black hole and $\mathcal{V}(\Sigma_k)$ is the volume of Σ_k (4π for a two-dimensional sphere). When $M = 0$ the metric (10) is that of anti de Sitter spacetime.

Light cone (u, v) and Kruskal (u', v') coordinates are defined by

$$u, v = t \pm r_* = t \pm \int \frac{dr}{\frac{r^2}{\ell^2} + k - \left(\frac{r_0}{r}\right)^{D-3}} \qquad u' = e^{\kappa u} \quad v' = -e^{-\kappa v} \tag{13}$$

(where κ is a constant whose significance will be discussed later), and repeating the procedure for the Schwarzschild case yields the diagrams illustrated in Figs. 5 and 6. There is no choice of conformal factor that allows both the asymptotic boundaries $\mathcal{I}$ and the singularity at $r = 0$ to be represented as straight lines [6], though it is common in the literature to do so.

Note that asymptotic infinity is timelike in the Schwarzschild-AdS case. Physically this means that any massive object projected away from the black hole will inevitably return to its starting point. The cosmological constant induces a confining potential for any massive particle, as an analysis of the geodesic equation indicates. Light rays, however, can reach $r = \infty$ in finite time. It is common to put reflecting boundary conditions at $r = \infty$ so that light rays are ‘confined’ like massive objects. The negative cosmological constant thus prevents radiation emitted by the black hole from escaping to infinity, allowing the black hole to reach equilibrium with its Hawking radiation, provided it is large enough. In this sense the eternal black hole (10) described by Figs. 5 and 6 is more physically relevant than its asymptotically flat counterpart in Fig. 2.

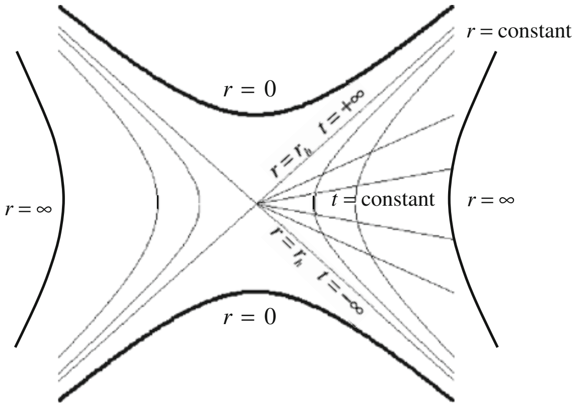

Fig. 5 Kruskal Diagram for Schwarzschild-AdS: the causal structure of the space-time described by the Schwarzschild-AdS metric (10)

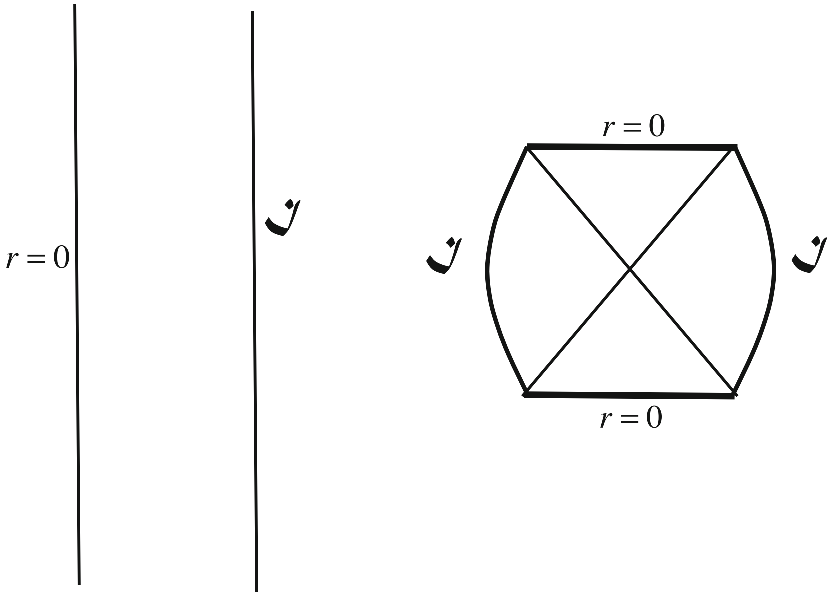

Fig. 6 Penrose diagram for Schwarzschild-AdS: Penrose diagrams for AdS (*left*) and Schwarzschild-AdS (*right*) space times

3 Black Hole Thermodynamics

The first hints of a fundamental relationship between gravitation, thermodynamics, and quantum theory came from studying black holes. For over 40 years a wealth of evidence has been acquired indicating that the laws of black hole mechanics are equivalent to the ordinary laws of thermodynamics applied to a system containing a black hole. The subject inextricably blends both classical thermodynamics and quantum physics, yielding deep insights into both the nature of quantum phenomena occurring in strong gravitational fields and some of the most baffling paradoxes in physics.

In what follows, for simplicity the constants G, c, and $\hbar$ will be set equal to unity; departures from this will be stated as appropriate.

3.1 Black Hole Mechanics

The first realization that black holes are thermodynamic systems came when Bekenstein noted [7] that thermodynamic entropy was closely analogous to the area of a black hole event horizon. Due to a theorem of Hawking [8], implying this area never decreases (provided the energy of matter is positive and space-time is regular), Bekenstein proposed that black holes should indeed be assigned an entropy that was proportional to the horizon area. Shortly afterward four laws of black hole mechanics were formulated [9], recapitulating the area law (or 2nd law of thermodynamics), and adding three others. These were

0th Law Surface gravity (denoted by κ) is constant over the event horizon

1st Law Differences in mass between nearby solutions are equal to differences in area times the surface gravity plus additional work-type terms

2st Law The area of the event horizon never decreases in any physical process provided the energy of matter is positive and space-time is regular

3rd Law No procedure can reduce the surface gravity to zero by a finite number of steps

The analogy between surface gravity and temperature was noted, but a formal equivalence had to await the inclusion of quantum effects. This relationship was first explored by Hawking [10], who made use of the formalism developed by Parker [11] for calculating particle production in curved spacetimes. This is foundational in understanding both black hole radiation and the resultant paradoxes that ensue, as we shall see.

The parallel with standard thermodynamics was therefore proposed to be

Energy	E	$\leftrightarrow$	M	Mass
Temperature	T	$\leftrightarrow$	$\frac{\kappa}{2\pi}$	Surface Gravity
Entropy	S	$\leftrightarrow$	$\frac{A}{4}$	Horizon Area

in units where $G = c = \hbar = k_B = 1$, where on the left-hand side we see the basic thermodynamic quantities of a physical system, and on the right their counterparts in black hole physics. The first law of mechanics for black holes [9] reads

$$dM = \frac{\kappa}{8\pi} dA + \Omega dJ + \Phi dQ \tag{14}$$

where J and Q are respectively the angular momentum and charge of the black hole. Their respective thermodynamic conjugates are the angular velocity Ω and the electromagnetic potential Φ, both evaluated at the horizon. The ΩdJ and ΦdQ terms are understood as thermodynamic work terms. Using the above identifications the correspondence with thermodynamics would be appear to be straightforward:

$$dE = TdS + VdP + \text{work terms} \tag{15}$$

provided one ignores the "pressure-volume" term. Pretty much all investigations into black hole thermodynamics regarded this term as irrelevant. However recent studies have suggested it might play an important role [12].

To see this, consider the application of the Hamiltonian formalism [13–15] to derive the first law of black hole mechanics. Suppose one has a solution to Einstein's equations that describes a black hole with a Killing field. Solutions perturbatively close to this (background) solution, but which do not necessarily have the same symmetries, can be obtained from solving the linearized equations. The linearized constraint equations on a hypersurface can be expressed in the form of a Gauss-type law [13] that relates a boundary integral at the horizon to a boundary integral at infinity. Both the choice of hypersurface and the choice of Killing field will determine the physical interpretation of this Gauss law relation. The first law of black hole mechanics for asymptotically flat or anti de Sitter (AdS) black hole spacetimes follows from taking the Killing vector l^a to be the generator of a Killing horizon and choosing an appropriate spacelike hypersurface [14].

Denoting by Σ the foliation of spacetime by a family of hypersurfaces, the metric can be written as

$$g_{ab} = s_{ab} - n_a n_b \tag{16}$$

where n^a is the unit timelike normal to the hypersurfaces, so that $n \cdot n = -1$. The tensor s_{ab} is the induced metric on the hypersurfaces Σ and satisfies $s_a{}^b n_b = 0$. In the Hamiltonian formalism, it comprises half of the dynamical variables, the other half being its canonically conjugate momentum $\pi^{ab} = -\sqrt{s}(K^{ab} - Ks^{ab})$, where $K_{ab} = s_a{}^c \nabla_c n_b$ is the extrinsic curvature of Σ, with $K = K^a{}_a$ its trace. It is convenient to likewise define $\pi = \pi^a{}_a$ and $s = s^a{}_a$.

Consider evolving the system along the vector field

$$\xi^a = N n^a + N^a, \tag{17}$$

with $N = -\xi \cdot n$ denoting the lapse function and N^a the shift, which is tangential to Σ. Contracting the Einstein tensor with the unit normal yields (in D space-time dimensions)

$$H \equiv -2G_{ab}n^a n^b = -R^{(D-1)} + \frac{1}{|s|}\left(\frac{\pi^2}{D-2} - \pi^{ab}\pi_{ab}\right),$$
$$H_b \equiv -2G_{ac}n^a s^c_b = -2D_a(|s|^{-\frac{1}{2}}\pi^{ab}). \tag{18}$$

and the full gravitational Hamiltonian is then given by $\mathcal{H} = NH + N^a H_a$. The quantity D_a is the covariant derivative operator with respect to s_{ab} on Σ, and $R^{(D-1)}$ its scalar curvature. Regarding the cosmological constant as a form of stress energy implies $8\pi T^a_b = -\Lambda g^a_b$, yielding

$$H = -2\Lambda, \quad H_b = 0 \tag{19}$$

for the constraint equations.

Suppose g_{ab} is a solution to the field equations with Killing vector ξ^a. Writing $\tilde{g}_{ab} = g_{ab} + \delta g_{ab}$ as a linear approximation to another solution, the corresponding perturbations are given by $h_{ab} = \delta s_{ab}$ and $p^{ab} = \delta\pi^{ab}$, where s_{ab} and π^{ab} can be regarded as the initial data for the original (background) solution g_{ab}. The linearized Hamiltonian and momentum constraints will be δH and δH_a and the perturbed solution is assumed to have cosmological constant $\tilde{\Lambda} = \Lambda + \delta\Lambda$. It is straightforward to show [13–15] that

$$D_a B^a = N\delta H + N^a \delta H_a = -2N\delta\Lambda \tag{20}$$

where

$$B^a[\xi] = N(D^a h - D_b h^{ab}) - hD^a N + h^{ab}D_b N + \frac{1}{\sqrt{|s|}}N^b(\pi^{cd}h_{cd}s^a_b - 2\pi^{ac}h_{bc} - 2p^a_b) \tag{21}$$

with the latter equality in (20) following from (19). Equation (20) is a Gauss' law relation with source proportional to $N\delta\Lambda$. Since $N = \xi^a n_a$, with ξ^a a Killing vector, this source itself may be written as a total derivative, yielding from (20) [16, 17]

$$D_a(B^a - 2\delta\Lambda\omega^{ab}n_b) = 0 \tag{22}$$

where $N = -D_c(\omega^{cb}n_b)$. The antisymmetric tensor ω^{ab} satisfies

$$\nabla_c\omega^{cb} = \xi^b \tag{23}$$

and is referred to as the Killing potential.

Integrating (22) over a volume $\hat{V}$ contained in Σ gives

$$\int_{\partial\hat{V}_{out}} d\mathcal{S}r_c\left(B^c[\xi] - 2\delta\Lambda\omega^{cb}n_b\right) = \int_{\partial\hat{V}_{in}} d\mathcal{S}r_c\left(B^c[\xi] - 2\delta\Lambda\omega^{cb}n_b\right) \tag{24}$$

where $\partial\hat{V}_{in,out}$ are the respective inner and outer boundaries of $\hat{V}$. The unit normal on each boundary is denoted as r_c, where r^c points into $\hat{V}$ on the inner boundary and out of $\hat{V}$ on the outer boundary. These different boundary integrals have distinct geometrical and physical interpretations.

The Killing potential is not unique; it is only defined up to a divergenceless term. If ω_{ab} solves $\nabla^a\omega_{ab} = \xi_b$ then so does $\omega'_{ab} = \omega_{ab} + \zeta_{ab}$ where $\nabla^a\zeta_{ab} = 0$. If $\zeta_{ab} = \nabla^c\eta_{[cab]}$ (the square brackets denoting antisymmetry) then the value of (24) remains unchanged. However if ζ_{ab} cannot be written this form, then the values of the integrals on the outer and inner boundaries will change by equal and opposite amounts. Consequently they cannot be given separate interpretations; only their difference is meaningful. It is therefore useful to write $\omega^{cb} = \omega^{cb} - \omega^{cb}_{AdS} + \omega^{cb}_{AdS}$ for the $\partial\hat{V}_{out}$ integral, where ω^{ab}_{AdS} is the Killing potential of the background anti de Sitter spacetime, giving

$$\int_{\partial\hat{V}_{out}} d\mathcal{S}r_c\left(B^c[\xi] - 2\delta\Lambda\omega^{cb}_{AdS}n_b\right) = \int_{\partial\hat{V}_{out}} d\mathcal{S}r_c\left(2\delta\Lambda(\omega^{cb} - \omega^{cb}_{AdS})n_b\right) + \int_{\partial\hat{V}_{in}} d\mathcal{S}r_c\left(B^c[\xi] - 2\delta\Lambda\omega^{cb}n_b\right) \tag{25}$$

Setting the outer boundary at spatial infinity for asymptotically flat or AdS spacetimes, the respective variations in the total mass $\mathcal{M}$ and angular momentum $\mathcal{J}$ of the space-time are defined as

$$16\pi\delta\mathcal{M} = -\int_{\infty} d\mathcal{S}r_c\left(B^c[\partial/\partial t] - 2\delta\Lambda\omega^{cb}_{AdS}n_b\right) \tag{26}$$

$$16\pi\delta\mathcal{J} = \int_{\infty} d\mathcal{S}r_c B^c[\partial/\partial\varphi] \tag{27}$$

and are obtained by respectively setting $\xi^a = (\partial/\partial t)^a$ (time translations) and $\xi^a = (\partial/\partial\varphi)^a$ (rotations). The ω^{cb}_{AdS} term renders δM finite [17]. Setting the inner boundary to be the horizon gives

$$2\kappa_h\delta\mathcal{A}_h = -\int_h d\mathcal{S}r_c B^c[\partial/\partial t + \Omega_h\partial/\partial\varphi] \tag{28}$$

where ξ^a is taken to be the horizon generating Killing vector, with $\kappa_h = \sqrt{-\frac{1}{2}\nabla^a\xi^b\nabla_a\xi_b}\Big|_{r=r_h}$ the surface gravity and $\mathcal{A}_h$ the area of the black hole horizon. The above presumes that this horizon is a bifurcate Killing horizon whose Killing generator vanishes on the bifurcation sphere.

3.2 Enthalpy, Pressure, and Volume

Hence from (24) we obtain a generalization of the first law of black hole mechanics (14)

$$\delta\mathcal{M} = \kappa_h \frac{\delta\mathcal{A}_h}{8\pi} + \Omega_h \delta\mathcal{J} + V\delta P \tag{29}$$

upon taking $\hat{V}_{in}$ to be the black hole horizon and $\hat{V}_{out}$ the boundary at infinity. The last term is the analogue of the familiar pressure-volume term from thermodynamics, with

$$P = -\frac{\Lambda}{8\pi} = \frac{(D-1)(D-2)}{16\pi l^2} \tag{30}$$

interpreted as thermodynamic pressure (the latter relation being the definition of l) and

$$V = \left(\int_\infty dS r_c n_b\left(\omega^{cb} - \omega^{cb}_{AdS}\right) - \int_h dS r_c n_b \omega^{cb}\right) \tag{31}$$

interpreted as its conjugate thermodynamic volume, and is finite because of the presence of the ω^{cb}_{AdS} term.

The preceding argument can be generalized to black holes with multiple rotations and $U(1)$ charges, yielding [18, 19]

$$\delta\mathcal{M} = \kappa_h \frac{\delta\mathcal{A}_h}{8\pi} + \sum_i^{\left[\frac{D-2}{2}\right]} (\Omega^i_h - \Omega^i_\infty)\delta\mathcal{J}^i + V\delta P + \sum_j \Phi^j_h \delta Q^j \tag{32}$$

where $\left[\frac{D-2}{2}\right]$ is the smallest integer greater than $\frac{D-2}{2}$. The quantities Ω^i_∞ allow for the possibility of a rotating frame at infinity [20] and the Φ^i_h are the potentials for the electric (and magnetic) $U(1)$ charges evaluated at the black hole horizon. We see from the first law (32) that the thermodynamic volume V may be interpreted as the change in the mass under variations in the cosmological constant with the black hole horizon area and angular momentum held fixed.

Since a negative cosmological constant induces a vacuum pressure, P is naturally regarded as a thermodynamic pressure. The mass M is then understood as a gravitational version of chemical enthalpy. This is the total energy of a system including both its internal energy E and the energy PV required to "make room for it" by displacing its (vacuum energy) environment: $M = E + PV$. In other words, M is the total energy required to "create a black hole and place it in a cosmological environment".

Another reason for including the pressure volume term is connected with the Smarr formula [21], which relates the various thermodynamic quantities to each other from scaling relations. The scaling dimensions of $\mathcal{A}$ and $\mathcal{J}$ are $D-2$, while Λ has dimension -2. Regarding $\mathcal{M} = \mathcal{M}(\mathcal{A}, \mathcal{J}, \Lambda)$, an application of Euler's formula for homogeneous functions yields [17, 22]

$$\frac{D-3}{D-2}M = \kappa_h \frac{\mathcal{A}_h}{8\pi} + \sum_i (\Omega_h^i - \Omega_\infty^i)\mathcal{J}^i - \frac{2}{D-2}PV_h + \frac{D-3}{D-2}\sum_j \Phi_h^j Q^j \tag{33}$$

since the scaling dimensions of $\mathcal{M}$ and Q are both $D-3$.

Consider the Reissner-Nordstrom anti de Sitter (SAdS) solution as an example. The metric, field strength F and gauge potential A in $d > 3$ spacetime dimensions are

$$\begin{aligned} ds^2 &= -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{D-2}^2\,, \\ F &= dA\,, \quad A = -\sqrt{\frac{D-2}{2(D-3)}}\frac{q}{r^{D-3}}dt \end{aligned} \tag{34}$$

where $f(r)$ is given by

$$f = 1 - \frac{m}{r^{D-3}} + \frac{q^2}{r^{2(D-3)}} + \frac{r^2}{l^2} \tag{35}$$

and $d\Omega_d$ is the metric for the standard element on S^d. The relevant parameters are [23]

$$\mathcal{M} = \frac{D-2}{16\pi}\omega_{D-2}m \qquad\qquad Q = \frac{\sqrt{2(D-2)(D-3)}}{8\pi}\omega_{D-2}q \tag{36}$$

$$\kappa_h = \frac{f'(r_+)}{2} = \frac{D-3}{2r_+}\left(1 - \frac{q^2}{r_+^{2(D-3)}} + \frac{D-1}{D-3}\frac{r_+^2}{l^2}\right) \tag{37}$$

$$\mathcal{A}_h = \omega_{D-2} r_+^{D-2} \qquad\qquad \Phi = \sqrt{\frac{D-2}{2(D-3)}}\frac{q}{r_+^{D-3}} \tag{38}$$

with $\omega_n = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma\left(\frac{n+1}{2}\right)}$ the volume of the unit n-sphere and the location of the event horizon r_+ determined from $f(r_+) = 0$. It is convenient to use this latter relation to obtain

$$m = r_+^{D-3} + \frac{q^2}{r_+^{D-3}} + \frac{r_+^{D-1}}{l^2}$$

thereby yielding

$$\begin{aligned} d\mathcal{M} &= \frac{(D-2)\omega_{D-2}}{16\pi}\left[(D-3)\left(r_+^{D-2} - \frac{q^2}{r_+^{D-2}}\right) + (D-1)\frac{r_+^{D-2}}{l^2}\right]dr_+ \\ &\quad - \frac{(D-2)\omega_{D-2}}{8\pi}\frac{r_+^{D-1}}{l^3}dl + \frac{(D-2)\omega_{D-2}}{8\pi}\frac{q}{r_+^{D-3}}dq \\ dQ &= \frac{\sqrt{2(D-2)(D-3)}}{8\pi}\omega_{D-2}dq \qquad d\mathcal{A} = \omega_{D-2}(D-2)r_+^{D-3}dr_+ \end{aligned} \tag{39}$$

so that

$$\begin{aligned} \kappa\frac{d\mathcal{A}}{8\pi} + \Phi dQ &= \frac{D-3}{16\pi r_+}\left(1 - \frac{q^2}{r_+^{2(D-3)}} + \frac{D-1}{D-3}\frac{r_+^2}{l^2}\right)\omega_{D-2}(D-2)r_+^{D-3}dr_+ \\ &\quad + \frac{(D-2)q}{8\pi r_+^{D-3}}\omega_{D-2}dq \\ &= \frac{D-2}{16\pi}\omega_{D-2}\left[(D-3)\left(r_+^{D-2} - \frac{q^2}{r_+^{D-2}}\right) + (D-1)\frac{r_+^{D-2}}{l^2}\right]dr_+ \\ &\quad + \frac{(D-2)q}{8\pi r_+^{D-3}}\omega_{D-2}dq \\ &= dM + \frac{(D-2)\omega_{D-2}}{8\pi}\frac{r_+^{D-1}}{l^3}dl = dM - VdP \end{aligned} \tag{40}$$

where P is given by (30) and

$$V = \frac{\omega_{D-2} r_+^{D-1}}{D-1}\,. \tag{41}$$

as inferred from the Smarr relation (33) or from (31). Note that the Smarr relation would not hold if the VP term in (33) were absent.

3.3 Black Holes as Chemical Systems

A new perspective on black hole thermodynamics thus emerges, leading to a different understanding of known processes and to the discovery of new phenomena. The thermodynamic correspondence with black hole mechanics is completed to include the familiar pressure/volume terms:

Thermodynamics		Black Hole Mechanics	
Enthalpy	H	Mass	M
Temperature	T	Surface Gravity	$\frac{\kappa}{2\pi}$
Entropy	S	Horizon Area	$\frac{A}{4}$
Pressure	P	Cosmological Constant	$-\frac{\Lambda}{8\pi}$
First Law	$dH = TdS + VdP + \ldots$	First Law	$dM = \frac{\kappa}{8\pi}dA + VdP + \ldots$

(42)

where the black hole work terms are $\sum_i \Omega_i dJ_i + \Phi dQ$ for multiply rotating and charged black holes.

One of the first implications of this new perspective was the realization that charged black holes behave as Van der Waals fluids [24]. The Van der Waals equation modifies the equation of state for an ideal gas to one that approximates the behaviour of real fluids [25]

$$\left(P + \frac{a}{v^2}\right)(v - b) = T \quad \Leftrightarrow \quad P = \frac{T}{v-b} - \frac{a}{v^2}\,, \tag{43}$$

taking into account the attraction between the molecules and their finite size, respectively parameterized by the constants (b, a). Here $v = V/N$ is the specific volume of the fluid, P its pressure, and T its temperature. Critical points occur at isotherms $T = T_c$, where $P = P(v)$ has an inflection point at $P = P_c$ and $v = v_c$, and obey the universal relation $\frac{P_c v_c}{kT_c} = \frac{3}{8}$ for any such fluid. A liquid/gas phase transition takes place at temperatures $T < T_c$, and is governed by Maxwell's equal area law, which states that the two phases coexist when the areas above and below a line of constant pressure drawn through a P-v curve are equal.

Surprisingly, charged AdS black holes obey the same basic relationships.[1] For example, the Reissner-Nordstrom-AdS metric in $D = 4$ is given by (34), with $f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}$. Its thermodynamic volume is given by the "Euclidean

[1] Although the notion that a charged black hole could be considered as a Van der Waals fluid had been noted before [23], the absence of the pressure and volume terms led to a mismatch of intensive and extensive thermodynamic variables [24].

relationship" $V = \frac{4}{3}\pi r_+^3$, where r_+ is the location of the event horizon. The relation $T = \frac{f'(r_+)}{4\pi}$ along with the above identification of pressure and volume yield the equation of state

$$P = \frac{T}{v} - \frac{1}{2\pi v^2} + \frac{2Q^2}{\pi v^4}, \qquad v = 2r_+ l_P^2 = 2\left(\frac{3V}{4\pi}\right)^{1/3}, \tag{44}$$

where the 'specific volume' $v = 6V/N$, with $N = A/l_P^2$ counting the number of degrees of freedom associated with the black hole horizon, $l_P = \sqrt{G\hbar/c^3}$. The P-v and P-T diagrams, Fig. 7, mimic the behavior of a Van der Waals fluid for any fixed Q, with the liquid/gas phase transition replaced by the (first-order) small/large black hole phase transition, still described by the equal-area law. Curiously the parameters at the critical point obey the relation $\frac{P_c v_c}{T_c} = 3/8$, precisely as in the Van der Waals case, and the critical exponents are the same as those for a Van der Waals fluid [24]. So far no black holes have been found in Einstein gravity that have different critical exponents.

Additional insights emerge. The well-known first-order phase transition between radiation and large AdS black holes [26] can be understood as a "solid/liquid" phase transition: the coexistence line in the P-T diagram has no terminal point and is present for all values of Λ. As for new phenomena, both *reentrant phase transitions* [19] and *triple points* [27] have been found for Kerr-AdS black holes.

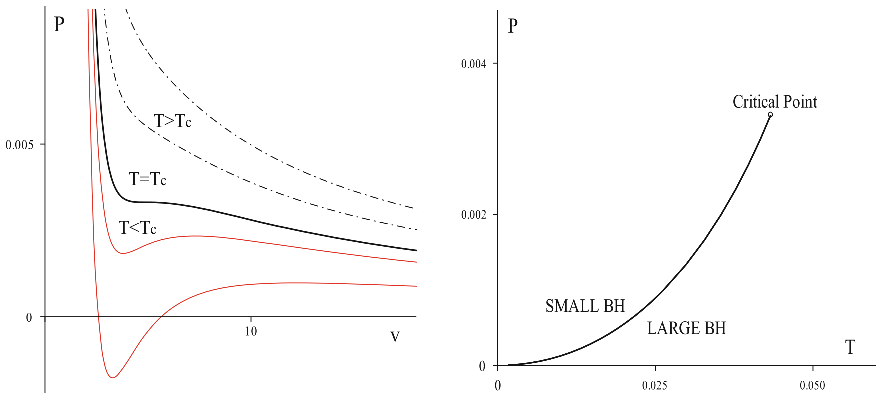

Fig. 7 Fluid behavior of charged AdS black holes: *left* the P-v diagram. Isotherms decrease in temperature from *top* to *bottom*. The two upper *dark curves* correspond to the "ideal gas" one-phase behaviour for $T > T_c$, the critical isotherm $T = T_c$ is denoted by the *thick dark line*, and the lower (*red*) *solid lines* correspond to a two-phase state occurring for $T < T_c$ for which the oscillatory part of the isotherm has to be replaced according to the Maxwell's equal area law. *Right* the P-T diagram. The coexistence line of the two black hole phases terminates at a critical point characterized by the Van der Waals mean field theory critical exponents. In both figures $Q = 1$

The former refers to a situation in which a system can undergo a transition from one phase to another and then back to the first by continuously changing one thermodynamic variable, all others being held constant; it was first observed in a nicotine/water mixture [28] and is present in multicomponent fluid systems, gels, ferroelectrics, liquid crystals, and binary gases [29]. In the case of black holes, three separate phases emerge, referred to as small, intermediate, and large black holes. A standard first order phase transition separates the small and large black holes, but the intermediate and small ones (for a small range of pressures, $P \in (P_t, P_z)$) are separated by a finite jump in the Gibbs free energy G, shown in Fig. 8 for $D = 6$. For any given pressure in this range, the black hole will change from large to small to large (labeled intermediate) upon lowering the temperature—giving the reentrant phase transition. Similar behaviour has been observed for Born–Infeld black holes [30].

Once two rotation parameters are included, a gravitational *triple point* can also emerge [27]. The simplest example is that of a doubly-rotating black hole in $D = 6$, with rotation parameters J_1 and J_2. If the ratio $J_2/J_1 \ll 1$, a new branch of (locally) stable tiny cold black holes appears, with both the $J_2 = 0$ 'no black hole region' and the unstable branch of tiny hot black holes disappearing. The zeroth-order phase

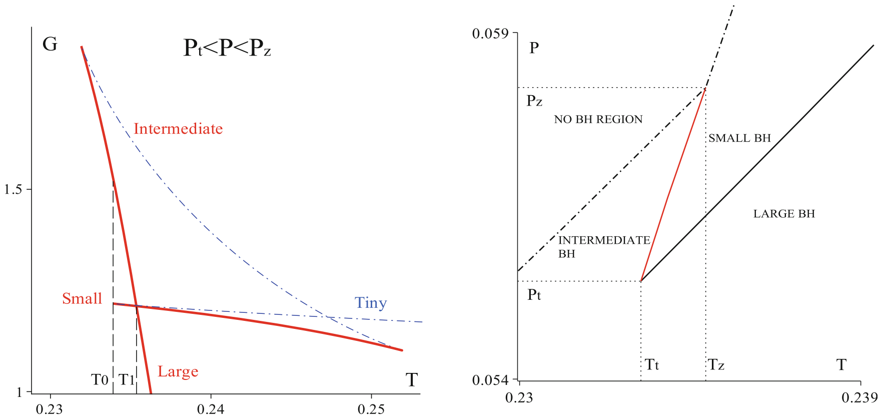

Fig. 8 Reentrant phase transition: *left* the Gibbs free energy G of a singly-spinning black hole in $D = 6$, displayed for fixed pressure $P \in (P_t, P_z)$ and $J = 1$. There is a finite jump in its global minimum indicating the presence of the zeroth-order phase transition. As T decreases, the system follows the lower vertical solid red curve, until at $T = T_1$ it joins the upper horizontal *solid red curve* corresponding to small stable black holes and undergoes a first order large/small black hole phase transition. As T continues to decrease the system follows this upper curve until $T = T_0$, where G has a discontinuity at its global minimum. Further decreasing T, the system jumps to the uppermost *vertical red line* of large stable black holes. This corresponds to the "zeroth order" phase transition between small and large black holes and completes the reentrant phase transition. *Right* the three-phase coexistence diagram displayed in the P-T plane. The first-order phase transition between small/large black holes is displayed by *thick solid black curve*. It emerges from (T_t, P_t) and terminates at a critical point (not displayed) at (T_c, P_c). The *solid red curve* indicates the zeroth-order phase transition between intermediate/small black holes; the *dashed black curve* delineates the 'no black hole region'

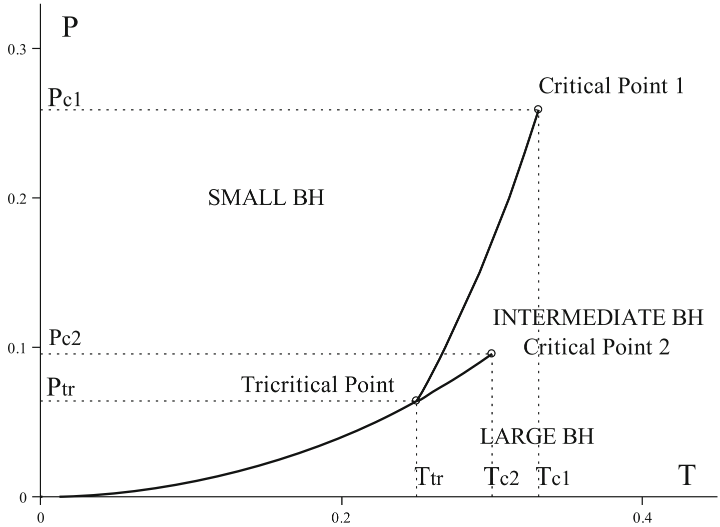

Fig. 9 Triple point: various black hole phases are displayed for a doubly-spinning Kerr-AdS black hole in $D = 6$ and the angular momenta ratio $J_2/J_1 = 0.05$ in the P-T plane. The diagram is in many ways analogous to the solid/liquid/gas phase diagram, including the existence of a triple point where three coexistence lines merge together. Note however that there are two critical points. That is, the small/intermediate black hole coexistence line does not extend to infinity (as in the solid/liquid case) but rather terminates, similar to the "liquid/gas" coexistence line, in a critical point

transition is 'replaced' by a 'solid/liquid'-like phase transition of small to large black holes. Once J_2 becomes sufficiently large a second critical point and the *triple point* both emerge, replicating the behaviour of a solid/liquid/gas system as shown in Fig. 9. Two distinct first order small/intermediate and intermediate/large black hole phase transitions are possible as the temperature increases for fixed pressure in a certain range—the two transitions terminate at critical points on one end and merge at the other end to form a triple point where the three phases coexist. This kind of behaviour has more recently been found for charged AdS black holes in Gauss–Bonnet gravity [31]. Even more interesting phenomena can take place for higher-dimensional rotating black holes; a review of the current situation is given in Ref. [19].

More generally, indications are emerging that every dimensionful parameter has a thermodynamic interpretation. The insight here is from Lovelock gravity, which in any given dimension has a finite number of dimensional coefficients. Each coefficient in the Lovelock action has a thermodynamic interpretation, modifying both the Smarr formula and the first law of thermodynamics [32, 33]. A similar situation takes place in Born-Infeld electrodynamics in which the Born-Infeld coupling constant has a thermodynamic conjugate interpreted as vacuum polarization [30].

The above relations transform from mechanical relations into thermodynamic ones upon making the identifications $S \rightarrow A/4$ and $\frac{\kappa}{2\pi} \rightarrow T$. This latter relation cannot be obtained without invoking quantum physics.

4 Field Quantization in Curved Spacetime

The striking parallel between black hole mechanics and black hole thermodynamics raises the question as to the origin of this correspondence. Pivotal to this relationship is the derivation of black hole temperature, which necessarily relies on the incorporation of quantum physics into curved space-time settings [34].

4.1 Quantum Field Theory in Curved Spacetime

Quantization of matter fields on a fixed spacetime background is analogous to quantizing charged fields (fields describing charged particles) in a fixed classical electromagnetic field background. The presumption is that the gravitational field of the quantized matter and the quantum properties of the background space-time can be neglected. Given the enormous success of quantum field theory in the earth's feeble gravitational field, we have good empirical grounds for accepting this premise of semiclassicality. It is crucial in obtaining black hole radiation.

For field quantization in general spacetime backgrounds it is not possible to use an approach based on a global symmetry, such as that of Poincare invariance used in flat-space quantum field theory. There is no preferred vacuum and no natural Fock space construction of the Hilbert space [35]. Instead, several key assumptions must be made.

1. All quantum states are defined on a spacelike slice Σ of 4-dimensional space-time, whose intrinsic curvature $R^{(3)}_{abcd}$ and extrinsic curvature K_{ab} are both everywhere small compared to the Planck length: $|R^{(3)}_{abcd}| \ll 1/l_p^2, \ |K_{ab}| \ll 1/l_p^2$.
2. There is some spatio-temporal neighbourhood of Σ where the full space-time curvature R_{abcd} is also small: $|R_{abcd}| \ll 1/l_p^2$.
3. The wavelength λ of any quanta on Σ is much longer than the Planck length $\lambda \gg l_p$.
4. The stress-energy of all matter obeys positive energy conditions and the energy and momentum densities of the matter are small compared to the Planck density $(\tilde{1}/l_p^4)$.
5. For a least a certain interval of proper time τ, Σ evolves sufficiently smoothly (so that $|dN/d\tau| \ll 1/l_p$ and $|dN^a/d\tau| \ll 1/l_p$) into future slices that respect the preceding four properties.

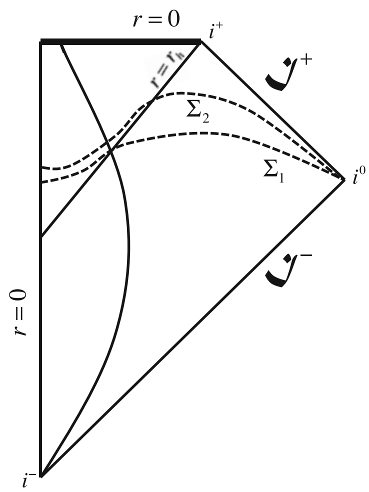

Fig. 10 Nice slices: two different slices for the collapse diagram that satisfy the niceness conditions

The preceding conditions are sometimes referred to as the 'niceness' conditions [36], and are regarded as ensuring that semiclassical physics is valid (Fig. 10).

4.2 Scalars

To see how this works in practice, it is useful to review the method for quantizing a free scalar field φ in a curved space-time, whose action is

$$I = -\frac{1}{2}\int d^D x\sqrt{-g}\left(g^{ab}\nabla_a\varphi\nabla_b\varphi + m^2\varphi^2\right) \tag{45}$$

in D spacetime dimensions, with

$$\nabla^2\varphi - m^2\varphi = 0 \tag{46}$$

being its equation of motion. Its energy-momentum tensor is

$$T_{ab} = \nabla_a\varphi(x)\nabla_b\varphi(x) - \frac{g_{ab}}{2}\left[\nabla^c\varphi(x)\nabla_c\varphi(x) + m^2(\varphi(x))^2\right]$$

obtained by varying the action with respect to the metric. Its conjugate momentum is

$$\pi(x) = \frac{\partial\mathcal{L}_{\text{KG}}}{\partial(n^a\partial_a\varphi(x))} = \sqrt{-g(x)}n^a\nabla_a\varphi(x), \tag{47}$$

with n^a the timelike unit vector normal to Σ.

Taking Σ to be a $(D-1)$ dimensional manifold with a product embedding in D-dimensional space-time (so that the full manifold is $\mathbb{R}\times\Sigma$) then the Hamiltonian is

$$H_{\mathrm{KG}} = \int d^3x\sqrt{-g(x)}T_{00}$$
$$= \int d^3x\sqrt{-g(x)}\left\{\nabla_0\varphi(x)\nabla_0\varphi(x) - g_{00}\frac{1}{2}\left[\nabla^c\varphi(x)\nabla_c\varphi(x) + m^2(\varphi(x))^2\right]\right\}$$

and one can canonically quantize the field by imposing

$$[\varphi(x^0,\vec{x}),\pi(x^0,\vec{y})] = i\delta(\vec{x},\vec{y}) \tag{48}$$

using a coordinate chart $(x^0,\vec{x})$ at some $t = x^0$ (so that $n^a = (1,0,0,0)$), where $\pi(x^0,\vec{y}) = -\sqrt{s}n^a\nabla_a\varphi|_{x=(x^0,\vec{y})}$ is the conjugate momentum to φ and where

$$\int d^{D-1}x\, w(\vec{x})\,\delta(\vec{x},\vec{y}) = w(\vec{y}) \tag{49}$$

for some function $w(\vec{x})$ on Σ. The relation (48) can be obtained from the covariant commutator

$$[\varphi(x),\varphi(y)] = i\Delta(x-y) = \int \frac{d^Dk}{(2\pi)^{D-1}}\delta(k^2-m^2)\frac{k^0}{|k^0|}\exp(-ik\cdot(x-y)) \tag{50}$$

which is independent of the particular choice of slice. If the space-time points (x,y) are spacelike separated, then $\Delta(x-y) = 0$, ensuring that measurements at such separations commute, a consequence of locality and causality.

The current

$$j^a(\chi,\phi) = -i\sqrt{g}g^{ab}(\chi^*\nabla_b\phi - (\nabla_b\chi^*)\phi) \tag{51}$$

is conserved ($\nabla_a j^a = 0$) provided both (χ,ϕ) are solutions to (46). This can in turn be used to define an inner product

$$\langle\langle\chi,\phi\rangle\rangle = \int_\Sigma d\Sigma_a j^a(\chi,\phi) \tag{52}$$

where Σ_a is the volume element on Σ with unit normal n^a. The conservation of j^a ensures that the inner product is independent of the choice of slice (or of x^0 in particular). It is straightforward to show that

$$\langle\langle \chi, \chi^* \rangle\rangle = 0 \qquad \langle\langle \chi, \phi \rangle\rangle^* = -\langle\langle \chi^*, \phi^* \rangle\rangle = \langle\langle \phi, \chi \rangle\rangle \tag{53}$$

The next step is to specify a notion of positive frequency solutions from the full solution space $\mathcal{Q}$, from which a vacuum and the Hilbert space of states could be constructed by restricting the inner product (52) to this subspace. We can write

$$\mathcal{Q} = \mathcal{Q}_+ \oplus \mathcal{Q}_- \qquad \text{where} \quad \forall\, \chi, \phi \in \mathcal{Q}_+ \quad \langle\langle \chi, \chi \rangle\rangle > 0 \quad \text{and} \quad \langle\langle \chi, \phi^* \rangle\rangle = 0 \tag{54}$$

and use the states in $\mathcal{Q}_+$ to define creation and annihilation operators via

$$\hat{a}(\chi) = \langle\langle \chi, \varphi \rangle\rangle \quad \hat{a}^\dagger(\chi) = -\langle\langle \chi^*, \varphi \rangle\rangle = -\hat{a}(\chi^*) \tag{55}$$

which in turn obey the relations

$$[\hat{a}(\chi), \hat{a}^\dagger(\phi)] = \langle\langle \chi, \phi \rangle\rangle \qquad [\hat{a}(\chi), \hat{a}(\phi)] = [\hat{a}^\dagger(\chi), \hat{a}^\dagger(\phi)] = 0 \tag{56}$$

as a consequence of (48). The state $|0\rangle$ obeying

$$\hat{a}(\chi)|0\rangle = 0 \qquad \forall \chi \in \mathcal{Q}_+ \tag{57}$$

is the vacuum, and by acting repeatedly on $|0\rangle$ with creation operators

$$\prod_{n=1}^{N} \hat{a}^\dagger(\chi_n)|0\rangle = \hat{a}^\dagger(\chi_1)\hat{a}^\dagger(\chi_2)\cdots\hat{a}^\dagger(\chi_N)|0\rangle = 0 \qquad \forall \chi_j \in \mathcal{Q}_+ \tag{58}$$

one can construct the Fock space of states as the span of all states of the above form for all N. Constructing an orthonormal basis $\phi_{\boldsymbol{k}}$ for $\mathcal{Q}_+$ we can define

$$\hat{a}_{\boldsymbol{k}} = \langle\langle \phi_{\boldsymbol{k}}, \varphi \rangle\rangle \quad \hat{a}^\dagger_{\boldsymbol{k}} = -\langle\langle \phi^*_{\boldsymbol{k}}, \varphi \rangle\rangle \tag{59}$$

and decompose the field operator φ

$$\varphi(x) = \int d^3\boldsymbol{k} \left[\phi_{\boldsymbol{k}}(x)\hat{a}_{\boldsymbol{k}} + \phi^*_{\boldsymbol{k}}(x)\hat{a}^\dagger_{\boldsymbol{k}} \right]. \tag{60}$$

which is called the *mode expansion*. The functions $\phi_{\boldsymbol{k}}(x)$ are called *mode functions* and the operators $\hat{a}_{\boldsymbol{k}}$ are called *mode operators*.

In spacetimes that possess time translation symmetry, a preferred set of modes can be defined in terms of the eigenfunctions of the timelike Killing vector ξ

$$\xi^a \partial_a \phi_{\boldsymbol{k}}(x) = -i\omega_{\boldsymbol{k}} \phi_{\boldsymbol{k}}(x), \qquad \xi^a \partial_a \phi^*_{\boldsymbol{k}}(x) = i\omega_{\boldsymbol{k}} \phi^*_{\boldsymbol{k}}(x), \tag{61}$$

where $\omega_{\boldsymbol{k}} > 0$. The modes with eigenvalues $\mp i\omega_{\boldsymbol{k}}$ are respectively the *positive/negative frequency* modes. In a coordinate system where $K^a\partial_a = \partial_t$, the Klein-Gordon equation implies that the $\phi_{\boldsymbol{k}}(x)$ are eigenfunctions of ∇_i with eigenvalue $i\boldsymbol{k}$, the magnitude of which is

$$|\boldsymbol{k}|^2 = \omega_{\boldsymbol{k}}^2 - m^2.$$

The mode expansion reduces the problem of solving the quantum field theory to that of finding a complete set of solutions to the $\mathbb{C}$-valued Klein-Gordon equation. It satisfies Hermiticity by construction, and each mode function satisfies (46). If the set of mode functions is complete and normalized to

$$\begin{aligned} \langle\langle \phi_{\boldsymbol{k}}(x), \phi_{\boldsymbol{k}'}(x)\rangle\rangle = -\langle\langle \phi_{\boldsymbol{k}}(x)^*, \phi_{\boldsymbol{k}'}(x)^*\rangle\rangle \; &= \delta^3(\boldsymbol{k}-\boldsymbol{k}'), \\ \langle\langle \phi_{\boldsymbol{k}}(x), \phi_{\boldsymbol{k}'}(x)^*\rangle\rangle = 0, & \end{aligned} \tag{62}$$

then the canonical commutation relations (48) are satisfied if the mode operators satisfy

$$[\hat{a}_{\boldsymbol{k}}, \hat{a}^\dagger_{\boldsymbol{k}'}] = \delta^3(\boldsymbol{k}-\boldsymbol{k}') \tag{63}$$

which is a specification of (56) to this basis.

In the mode expansion the momentum operator is

$$\hat{\boldsymbol{P}} = \int d^3x \hat{T}_{0i} = \int d^3\boldsymbol{k}\boldsymbol{k}\hat{a}^\dagger_{\boldsymbol{k}}\hat{a}_{\boldsymbol{k}},$$

which is a weighted sum of the number operators $N_{\boldsymbol{k}} = \hat{a}^\dagger_{\boldsymbol{k}}\hat{a}_{\boldsymbol{k}}$ for each mode. Its eigenstates are the Fock states, with eigenvalues obtained by solving

$$\hat{\boldsymbol{P}}|n_{\boldsymbol{k}_1}, n_{\boldsymbol{k}_2}, \ldots\rangle = \left[\sum_i n_{\boldsymbol{k}_i}\boldsymbol{k}_i\right]|n_{\boldsymbol{k}_1}, n_{\boldsymbol{k}_2}, \ldots\rangle$$

and it is easy to see that the vacuum state $|0\rangle$ has no momentum. The Hamiltonian is

$$\hat{H}_{\mathrm{KG}} = \int d^3x \hat{T}_{00} = \int d^3\boldsymbol{k}\,\frac{\omega_{\boldsymbol{k}}}{2}\left[\hat{a}_{\boldsymbol{k}}\hat{a}^\dagger_{\boldsymbol{k}} + \hat{a}^\dagger_{\boldsymbol{k}}\hat{a}_{\boldsymbol{k}}\right] = \int d^3\boldsymbol{k}\,\omega_{\boldsymbol{k}}\left[\hat{a}^\dagger_{\boldsymbol{k}}\hat{a}_{\boldsymbol{k}} + \frac{1}{2}[a_{\boldsymbol{k}}, a^\dagger_{\boldsymbol{k}}]\right] \tag{64}$$

in the mode expansion. The latter term is divergent due to (63), but since it is constant the energy scale can be shifted by subtracting it off. Its removal will only affect the value of the cosmological constant, which must be set from observation. This yields

$$\hat{H}_{\mathrm{KG}} = \int d^3\boldsymbol{k}\, \omega_{\boldsymbol{k}} \hat{a}^{\dagger}_{\boldsymbol{k}} \hat{a}_{\boldsymbol{k}}$$

and the energy of a given state is obtained from solving the eigenvalue equation

$$\hat{H}_{\mathrm{KG}} |n_{\boldsymbol{k}_1}, n_{\boldsymbol{k}_2}, \ldots\rangle = \left[\sum_i n_{\boldsymbol{k}_i} \omega_{\boldsymbol{k}_i}\right] |n_{\boldsymbol{k}_1}, n_{\boldsymbol{k}_2}, \ldots\rangle$$

with the zero eigenvalue case being the vacuum, with zero energy.

Since

$$\omega_{\boldsymbol{k}}^2 = |\boldsymbol{k}|^2 + m^2,$$

the Fock states are interpreted as describing particles of mass m. The state $|n_{\boldsymbol{k}}\rangle$ describes a state of $n_{\boldsymbol{k}}$ particles, each with energy $\omega_{\boldsymbol{k}}$, momentum $\boldsymbol{k}$, and mass m. Since they have definite momentum these particles are completely de-localized, unlike classical particles, which are localized objects. Each of these particles is identical, with

$$|n_{\boldsymbol{k}_1}, n_{\boldsymbol{k}_2}\rangle = |n_{\boldsymbol{k}_2}, n_{\boldsymbol{k}_1}\rangle$$

since the mode operators commute. In other words, they obey Bose-Einstein statistics. The Klein-Gordon field φ describes bosonic particles of spin-0.

The problem with this construction is that in a general curved spacetime there are many ways of doing this, with no particular subspace singled out as a natural choice for the positive frequency space [37]. Different notions of positive frequency will yield different Fock space constructions that are unitarily inequivalent [38], and the vacuum state $|0\rangle$ with respect to one choice of $\tilde{\mathcal{Q}}_+$ will not necessarily be in the Fock space constructed from the vacuum $|\tilde{0}\rangle$ with respect to another choice $\tilde{\mathcal{Q}}_+$.

However there is a linear relation between different choices. For another choice $\tilde{\mathcal{Q}}_+$ any $\tilde{\phi} \in \tilde{\mathcal{Q}}_+$ is a linear combination $\tilde{\phi} = \phi + \chi^*$ for some $(\chi, \phi) \in \mathcal{Q}_+$. Consequently two complete sets of modes $\{\phi_{\boldsymbol{k}}, \phi^*_{\boldsymbol{k}}\}$ and $\{\chi_{\boldsymbol{k}}, \chi^*_{\boldsymbol{k}}\}$ with associated mode operators $\hat{a}_{\boldsymbol{k}}$ and $\hat{\mathsf{a}}_{\boldsymbol{k}}$ are, by completeness and orthonormality, related by

$$\chi_{\boldsymbol{k}}(x) = \int d^3\boldsymbol{k}' \left[\alpha_{\boldsymbol{k}\boldsymbol{k}'} \phi_{\boldsymbol{k}'}(x) + \beta_{\boldsymbol{k}\boldsymbol{k}'} \phi^*_{\boldsymbol{k}'}(x)\right],$$

where $\alpha_{\boldsymbol{k}\boldsymbol{k}'} = \langle\langle \chi_{\boldsymbol{k}}(x), \phi_{\boldsymbol{k}'}(x)\rangle\rangle$ and $\beta_{\boldsymbol{k}\boldsymbol{k}'} = -\langle\langle \chi_{\boldsymbol{k}}(x), \phi^*_{\boldsymbol{k}'}(x)\rangle\rangle$. This is called a *Bogoliubov transformation* [34] and the complex numbers $\alpha_{\boldsymbol{k}\boldsymbol{k}'}$ and $\beta_{\boldsymbol{k}\boldsymbol{k}'}$ are called *Bogoliubov coefficients*. Inverting this transformation yields

$$\phi_{\boldsymbol{k}} = \int d^3\boldsymbol{k}' \left[\alpha^*_{\boldsymbol{k}'\boldsymbol{k}}\chi_{\boldsymbol{k}'}(x) - \beta_{\boldsymbol{k}'\boldsymbol{k}}\chi^*_{\boldsymbol{k}'}(x)\right]$$

inducing in turn the following transformations

$$\hat{a}_{\boldsymbol{k}} = \int d^3\boldsymbol{k}' \left[\alpha_{\boldsymbol{k}'\boldsymbol{k}}\hat{\mathsf{a}}_{\boldsymbol{k}'} + \beta^*_{\boldsymbol{k}'\boldsymbol{k}}\hat{\mathsf{a}}^\dagger_{\boldsymbol{k}'}\right] \qquad \hat{\mathsf{a}}_{\boldsymbol{k}} = \int d^3\boldsymbol{k}' \left[\alpha^*_{\boldsymbol{k}\boldsymbol{k}'}\hat{a}_{\boldsymbol{k}'} - \beta^*_{\boldsymbol{k}\boldsymbol{k}'}\hat{a}^\dagger_{\boldsymbol{k}'}\right] \tag{65}$$

on the mode operators.

The commutation relations of the mode operators imply that the Bogoliubov coefficients must satisfy

$$\begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix}\begin{pmatrix} \alpha^\dagger & -\beta^{\mathrm{T}} \\ -\beta^\dagger & \alpha^{\mathrm{T}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

where the individual entries are to be interpreted as block matrices.

From (65) we see that the Fock bases associated with these two mode expansions differ, leading to two different particle interpretations of the field excitations. Whenever any of the β-coefficients are nonzero, positive frequency states get transformed into a combination of positive and negative frequency states, leading to particle production. Specifically, according to the particle interpretation based on the $\phi_{\boldsymbol{k}}(x)$ modes, particles are present in the vacuum of the $\chi_{\boldsymbol{k}}(x)$ mode expansion $|0\rangle_\chi$. The average number of particles present in mode $\boldsymbol{k}$ is given by

$${}_\chi\langle 0|N_k|0\rangle_\chi = {}_\chi\langle 0|\hat{a}^\dagger_{\boldsymbol{k}}\hat{a}_{\boldsymbol{k}}|0\rangle_\chi = \int d^3\boldsymbol{k}' \, |\beta_{\boldsymbol{k}\boldsymbol{k}'}|^2.$$

In this sense, there is no invariant notion of particles in quantum field theory: as with simultaneity, particle interpretations are observer-dependent.

4.3 Spinors

Spin-1/2 particles likewise have observer-dependent interpretations. A Dirac field $\Psi(x)$ of mass m (with minimal coupling to gravity) is described by an action

$$S = \int d^Dx\sqrt{-g(x)}\left\{\frac{i}{2}\left[\bar{\Psi}(x)\gamma^a(\nabla_a\Psi(x)) - (\nabla^a\bar{\Psi}(x))\gamma^a\Psi(x)\right] - m\bar{\Psi}(x)\Psi(x)\right\},$$

where the covariant derivative $\nabla_a = \partial_a = \Gamma_a$, and the *spin connection* Γ_a is

$$\Gamma_a = \frac{1}{4}\gamma_b(\partial_a\gamma^b + \Gamma^b_{ca}\gamma^c)$$

with Γ^b_{ca} the usual Christoffel symbols associated with the metric g_{ab}. The Dirac matrices γ_a satisfy

$$\{\gamma_a, \gamma_b\} = 2g_{ab}$$

and an overbar denotes the Dirac adjoint, defined as

$$\bar{\Psi}(x) = \Psi^{\dagger}(x)\tilde{\gamma}^0$$

with $\tilde{\gamma}^0$ the zeroth component of $\tilde{\gamma}^{\rm A}$, defined by $\gamma^{\rm A} = V^{\rm A}_{\alpha}\tilde{\gamma}^b$, where

$$\{\tilde{\gamma}_{\rm A}, \tilde{\gamma}_{\rm B}\} = 2\eta_{\rm AB}$$

and where $V^{\rm A}_{\alpha}$ is referred to as the vielbein field. The set of $V^{\rm A}_{\alpha}$ comprise a set of local basis vectors for the metric, and are related to it via the equation

$$g_{ab} = \eta_{\rm AB}V_{\alpha}{}^{\rm A}V_{\alpha}{}^{\rm B} \tag{66}$$

with $\eta_{\rm AB}$ the metric on flat Minkowski space-time [34].

Variation of the action with respect to Ψ yields the equation of motion

$$[i\gamma^a\nabla_a - m]\Psi(x) = 0 \tag{67}$$

and varying the action with respect to the metric results in

$$T_{ab} = \frac{i}{2}\left[\bar{\Psi}(x)\gamma_{(a}(\nabla_{b)}\Psi(x)) - (\nabla_{(a}\bar{\Psi}(x))\gamma_{b)}\Psi(x)\right] \tag{68}$$

which is the energy-momentum tensor for the Dirac field.

As with the scalar case, there is an inner product

$$\langle\langle\Psi_1(x), \Psi_2(x)\rangle\rangle = \int d\Sigma_a\bar{\Psi}_1(x)\gamma^a\Psi_2(x)$$

where the integration is over a spacelike hypersurface Σ with unit normal n^a. The current $\bar{\Psi}_1(x)\gamma^a\Psi_2(x)$ is conserved as a consequence of (67). The field Ψ can likewise be expanded in a complete set modes

$$\hat{\Psi}(x) = \sum_s\int d^3\boldsymbol{k}\left[\hat{a}_{\boldsymbol{k},s}\psi^+_{\boldsymbol{k},s}(x) + \hat{b}^{\dagger}_{\boldsymbol{k},s}\psi^-_{\boldsymbol{k},s}(x)\right],$$

where $s \in \{\uparrow, \downarrow\}$ is an index related to the spinor nature of the field. The mode functions $\psi^{\pm}_{\boldsymbol{k},s}(x)$ satisfy the spinor valued Dirac equation and are normalized to

$$\langle\langle \psi^{+}_{\boldsymbol{k},s}(x), \psi^{+}_{\boldsymbol{k}',s'}(x)\rangle\rangle = -\langle\langle \psi^{-}_{\boldsymbol{k},s}(x), \psi^{-}_{\boldsymbol{k}',s'}(x)\rangle\rangle = \delta_{ss'}\delta^3(\boldsymbol{k}-\boldsymbol{k}'),$$
$$\langle\langle \psi^{\pm}_{\boldsymbol{k},s}(x), \psi^{\mp}_{\boldsymbol{k}',s'}(x)\rangle\rangle = 0.$$

Unlike the Klein-Gordon scalar field, the mode operators for the Dirac spinor satisfy the anticommutation relations

$$\{\hat{a}_{\boldsymbol{k},s}, \hat{a}^{\dagger}_{\boldsymbol{k}',s'}\} = \{\hat{b}_{\boldsymbol{k},s}, \hat{b}^{\dagger}_{\boldsymbol{k}',s'}\} = \delta_{ss'}\delta^3(\boldsymbol{k}-\boldsymbol{k}'), \tag{69}$$

with all other anticommutators vanishing. This ensures that the energy of the field has a lower bound and that causality is obeyed [39].

Apart from this, the construction of the Fock basis follows the same procedure as in the scalar case. Excitations of the vacuum state, defined via

$$\hat{a}_{\boldsymbol{k},s}|0\rangle = \hat{b}_{\boldsymbol{k},s}|0\rangle = 0$$

for all $\boldsymbol{k}$ and s, have two different types of quanta. Upon partitioning the mode functions into positive and negative frequency with the timelike Killing vector ξ_a, states created by $a^{\dagger}_{\boldsymbol{k},s}$ can interpreted as *particles*, whereas states created from $b^{\dagger}_{\boldsymbol{k},s}$ can be interpreted as *antiparticles*. Since mode operators anticommute,

$$(a^{\dagger}_{\boldsymbol{k},s})^2 = (b^{\dagger}_{\boldsymbol{k},s})^2 = 0$$

there is only one excitation per mode, yielding a realization of the *Pauli Exclusion Principle*. Particles have mass m and spin-1/2, with s labelling the spin state as being up or down with respect to some axis. Its antiparticle counterpart has the same mass but with the spin state pointing in the opposite direction.

The mode anticommutation relations imply the Fock states are antisymmetric

$$|1_{\boldsymbol{k}_1,s_1}, 1_{\boldsymbol{k}_2,s_2}\rangle = -|1_{\boldsymbol{k}_2,s_2}, 1_{\boldsymbol{k}_1,s_1}\rangle$$

and so the quanta of the Dirac field obey *Fermi-Dirac statistics*.

In a general curved space-time there is no preferred set of modes. But, as with the Klein-Gordon field, all possible sets of modes are related by a Bogoliubov transformation. For example, two sets of modes $\{\psi^{\pm}_{\boldsymbol{k},s}(x)\}$ and $\{\vartheta^{\pm}_{\boldsymbol{k},s}(x)\}$ with corresponding mode operators $\{\hat{a}_{\boldsymbol{k},s}, \hat{b}^{\dagger}_{\boldsymbol{k},s}\}$ and $\{\hat{c}_{\boldsymbol{k},s}, \hat{d}^{\dagger}_{\boldsymbol{k},s}\}$, are related by

$$\hat{a}_{\boldsymbol{k}} = \int d^3\boldsymbol{k}' \left[\alpha_{\boldsymbol{k}\boldsymbol{k}'}\hat{c}_{\boldsymbol{k}'} + \beta^*_{\boldsymbol{k}\boldsymbol{k}'}\hat{d}^\dagger_{\boldsymbol{k}'}\right] \quad \hat{b}_{\boldsymbol{k}} = \int d^3\boldsymbol{k}' \left[\alpha_{\boldsymbol{k}\boldsymbol{k}'}\hat{d}_{\boldsymbol{k}'} + \beta^*_{\boldsymbol{k}\boldsymbol{k}'}\hat{c}^\dagger_{\boldsymbol{k}'}\right]$$
$$\hat{c}_{\boldsymbol{k}} = \int d^3\boldsymbol{k}' \left[\alpha^*_{\boldsymbol{k}\boldsymbol{k}'}\hat{a}_{\boldsymbol{k}'} + \beta^*_{\boldsymbol{k}\boldsymbol{k}'}\hat{b}^\dagger_{\boldsymbol{k}'}\right] \quad \hat{d}_{\boldsymbol{k}} = \int d^3\boldsymbol{k}' \left[\alpha^*_{\boldsymbol{k}\boldsymbol{k}'}\hat{b}_{\boldsymbol{k}'} + \beta^*_{\boldsymbol{k}\boldsymbol{k}'}\hat{a}^\dagger_{\boldsymbol{k}'}\right]$$

where $\alpha_{\boldsymbol{k}\boldsymbol{k}'} = \langle\langle\psi^+_{\boldsymbol{k}}(x), \vartheta^+_{\boldsymbol{k}'}(x)\rangle\rangle$ and $\beta_{\boldsymbol{k}\boldsymbol{k}'} = \langle\langle\psi^+_{\boldsymbol{k}}(x), \vartheta^-_{\boldsymbol{k}'}(x)\rangle\rangle$ suppressing the spin degree of freedom for ease of notation. However, the fermionic Bogoliubov coefficients must satisfy

$$\begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix}\begin{pmatrix} \alpha^\dagger & \beta^{\mathrm{T}} \\ \beta^\dagger & \alpha^{\mathrm{T}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

to be consistent with the anticommutation relations.

4.4 Hadamard States

Intuitively the vacuum is regarded as a state of no particles, and an excited state is (at the least) a state in which the particle number (or more generally the number of quanta) $N = \sum_n \hat{a}^\dagger_n \hat{a}_n$ has some nonzero value. However without an unambiguous definition of positive frequency, as is the case in curved space-time, there is no invariant notion of particle number, even at a fixed instant of time (since that notion itself depends on the choice of slice).

The notion of a state is therefore more usefully given by computing time-ordered expectation values of field operators, such as $\langle 0|\phi(x)|0\rangle \equiv \langle\phi(x)\rangle$, $\langle 0|\phi(x)\phi(y)|0\rangle \equiv \langle\phi(x)\phi(y)\rangle$, $\langle 0|\phi(x)\phi(y)\phi(z)|0\rangle \equiv \langle\phi(x)\phi(y)\phi(z)\rangle$, etc. Restricting attention to situations in which $\langle\phi(x)\rangle = 0$ and for which knowledge of

$$G(x, y) = \langle\phi(x)\phi(y)\rangle \tag{70}$$

(a quantity known as the Wightman function) is sufficient to uniquely determine the n-point function $\langle\phi(x_1)\cdots\phi(x_n)\rangle$ (called quasi-free states [38]), will allow for a labelling of states with values of physical observables, since they can all in principle be determined from (70). One obstacle to overcome here is the determination of observables for which two space-time points coincide: these will contain products of field operators at the same space-time point and so are ill-defined due to the distributional character of the field operators. The most pertinent example here is the expectation value of the stress-energy tensor $\langle T_{ab}(x)\rangle$, which sources the gravitational field in the semiclassical approximation. Normal ordering of creation and annihilation operators—defining the expectation value of an observable so that all creation operators are at the left—yields the difference between the value of the observable for the state of interest and its value in the vacuum. However this difference is not invariant in curved space-time for the same reasons given above for the number operator.

There is one set of states, known as Hadamard states, for which $\langle T_{ab}(x)\rangle$ can be constructed so that it remains both local and conserved. They are defined by the requirement

$$\lim_{x\to y} G(x,y) = \frac{U_1(x,y)}{\sigma(x,y)} + U_2(x,y)\ln\sigma + U_3(x,y) \tag{71}$$

where the smooth functions $U_i(x,y)$ all obey $U(x,x)=1$ and where $\sigma(x,y)$ is the geodesic distance from x to y. The functions U_1 and U_2 are determined by requiring that $G(x,y)$ obey Eq. (46) in terms of x. A state that is initially Hadamard in some suitable neighbourhood of a Cauchy slice will remain Hadamard throughout the spacetime [40], and any Hadamard state can be constructed as a state in the Fock space defined on any other Hadamard state if the slices are spatially compact [38]. The Hadamard condition (71) essentially states that the short-distance (or ultraviolet) behaviour of a state is similar to that of the two-point function $\langle\phi(x)\phi(x')\rangle$ in flat spacetime. States not satisfying the condition (71) have divergent stress-energy tensors, and so are regarded as physically singular.

Hence restriction to the set of Hadamard states is compatible with the semi-classical niceness conditions, namely that on scales much smaller than any curvature scale, flat-space quantum field theory is a good approximation. Futhermore, any state in a Fock space constructed from the usual flat Minkowski vacuum space obeys (71) and so Hadamard states are conceptually the most reasonable generalization of such Minkowski-Fock states to curved space-time [38].

Note that the short-distance behaviour of Hadamard states is determined by $\sigma(x,y)$, in other words by the local space-time geometry. The short-distance (UV) divergences that ensue can be removed by a renormalization procedure that is local and state-independent, yielding a unique result for $\langle T_{ab}(x)\rangle$ upon to the addition of local curvature terms that arise due to a renormalization of gravitational couplings. All observers will see a finite density of particles in any Hadamard state.

5 Particle Creation and Observer-Dependent Radiation

The possibility that temperature and acceleration are related to one another in quantum field theory first came from studying reflecting barriers in flat space [41], and was firmly established by Unruh [42], who investigated how positive-frequency normal modes for uniformly accelerated observers are related to the positive-frequency modes for inertial observers, who use the standard quantization of the free field in Minkowski space. The use of the KMS condition to characterize thermal states for accelerated observers was developed by a number of authors [43–49], responding to analogous developments in the theory of black holes.

Acceleration radiation was perhaps as big a surprise as black hole radiation. The flat space Minkowski metric has, as noted above, a preferred state for a quantum field, namely the Minkowski vacuum. Yet it appears to be thermal to accelerating observers. How can this be, since the space-time has no curvature? Indeed what is the meaning of a thermal state?

5.1 Thermality

The notion of thermality is well defined in a system with finitely many degrees of freedom. The expectation value of any operator O in a canonical ensemble at temperature $\beta^{-1} = T$ is

$$\langle \mathcal{O} \rangle_\beta = \langle e^{-\beta H} \mathcal{O} \rangle = \frac{\mathrm{tr}\left[e^{-\beta H} \mathcal{O}\right]}{Z} \tag{72}$$

where $Z = \mathrm{tr}\left[e^{-\beta H}\right]$ and the trace is over the mircrostates of the system with Hamiltonian H. A quantum field has infinitely many degrees of freedom and so some care must taken in its definition, since the trace will diverge. The relevant notion here is that of a KMS state, which is defined via the condition

$$\langle \mathcal{A}(-i\beta)\mathcal{B} \rangle_\beta = \langle \mathcal{B}\mathcal{A} \rangle_\beta \tag{73}$$

for any operators $(\mathcal{A}, \mathcal{B})$, and where

$$\langle \mathcal{A}(t)\mathcal{B} \rangle \equiv \langle e^{itH} \mathcal{A} e^{-itH} \mathcal{B} \rangle \tag{74}$$

with $\mathcal{A}(t)$ the time-evolution of $\mathcal{A}$. In other words the time coordinate t is transformed ('Wick-rotated') to imaginary values, and

$$\langle \mathcal{A}(-i\beta)\mathcal{B} \rangle_\beta = \langle e^{-\beta H} e^{\beta H} \mathcal{A} e^{-\beta H} \mathcal{B} \rangle = \langle \mathcal{A} e^{-\beta H} \mathcal{B} \rangle = \langle e^{-\beta H} \mathcal{B}\mathcal{A} \rangle = \langle \mathcal{B}\mathcal{A} \rangle_\beta \tag{75}$$

using (72) and (74).

The condition (73) is known as the Kubo-Martin-Schwinger (KMS) condition [50, 51]. It characterizes a thermal state in curved space-time, being equivalent to (72) for finite systems. Systems satisfying the KMS condition locally minimize the free energy, and any finite system coupled to an infinite systems obeying (73) will come into thermal equilibrium with that system, respecting (72) [52].

For a model scalar field system (45) in thermal equilibrium, there must be a timelike Killing vector since an equilibrium state is a stationary state by definition. Indeed, thermal equilibrium is defined with respect to measurements made by observers following orbits of this Killing vector. This symmetry implies that the Wightman function $G(x, y) = \langle \phi(x)\phi(y) \rangle$ from (70) depends on the temporal difference $\Delta t = x^0 - y^0$ of the two scalar fields. Setting $\Delta t = -i\Delta\tau$ the KMS condition is [52]

$$G^\beta(\Delta\tau - i\beta, \vec{x}, \vec{y}) = G^\beta(\Delta\tau, \vec{y}, \vec{x}) \tag{76}$$

where $G^\beta(x,y) \equiv \langle\phi(x)\phi(y)\rangle_\beta$. This relationship can be extended into a single function on the complex-time plane obeying

$$\mathcal{G}^\beta(z, \vec{x}, \vec{y}) = \mathcal{G}^\beta(z + in\beta, \vec{x}, \vec{y}) \tag{77}$$

for integer n. The function $\mathcal{G}^\beta$ is holomorphic in the complex z plane apart from branch cuts along lines $(|\Re(z)| > |\vec{x} - \vec{y}|, \Im(z) = n\beta)$ ending in poles at $\Re(z) = |\vec{x} - \vec{y}|$.

A free scalar field whose Wightman function $\mathcal{G}^\beta(z, \vec{x}, \vec{y})$ obeys (77) on a stationary space-time is defined to be in a thermal state. If the space-time is static then the function G^β periodic in imaginary time τ with period β ($\tau \to \tau + \beta$) is related to thermal states on the original (Lorentzian) space-time by analytic continuation.

5.2 Acceleration Radiation

The metric for flat Minkowski space

$$ds^2 = dx^a dx^a = -dt^2 + dr^2 + r^2 d\Omega^2_{D-2} = -dt^2 + dx^2 + \sum_{i=2}^{D} dw^i dw^i \tag{78}$$

(written above in both spherical and Cartesian coordinates) can also be written as

$$ds^2 = -\kappa^2 \mathsf{X}^2 d\mathsf{T}^2 + d\mathsf{X}^2 + \sum_{i=2}^{d} dW^i dW^i \tag{79}$$

$$= e^{2\kappa\zeta}(d\zeta^2 - d\eta^2) + \sum_{i=2}^{d} dW^i dW^i \tag{80}$$

$$= -e^{\kappa(U-V)} dU\, dV + \sum_{i=2}^{d} dW^i dW^i \tag{81}$$

where these various forms follow from the coordinate transformations

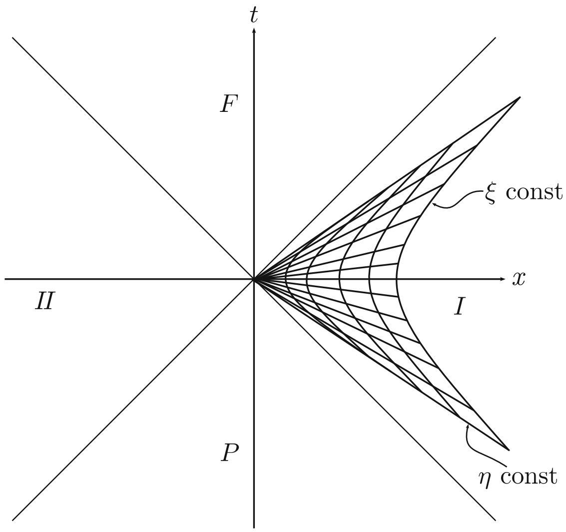

Fig. 11 The Rindler coordinate system in the right Rindler wedge (*I*). The lines of constant ξ are the worldlines of uniformly accelerating observers. Also shown are the *left* Rindler wedge (*II*), the future wedge (*F*), and the past wedge (*P*)

$$t = \mathsf{X}\sinh(\kappa\mathsf{T}) = \frac{1}{\kappa}e^{\kappa\zeta}\sinh(\kappa\eta) = \frac{1}{2\kappa}\left(e^{\kappa U} - e^{-\kappa V}\right) \tag{82}$$

$$x = \mathsf{X}\cosh(\kappa\mathsf{T}) = \frac{1}{\kappa}e^{\kappa\zeta}\cosh(\kappa\eta) = \frac{1}{2\kappa}\left(e^{\kappa U} + e^{-\kappa V}\right) \tag{83}$$

and (for notational convenience) $\vec{w} = \vec{W}$.

The (η, ζ) coordinates take values in the ranges $-\infty < \eta, \zeta < \infty$ and cover the region $x > |t|$, which is called the *right Rindler wedge* and is denoted *I*, as depicted in Fig. 11. The inverse transformation is

$$\eta = \mathsf{T} = \frac{1}{\kappa}\operatorname{arctanh}(t/x),$$

$$\zeta = \frac{1}{\kappa}\ln(\kappa\mathsf{X}) = \frac{1}{2\kappa}\ln[\kappa^2(x^2 - t^2)].$$

The region $-x > |t|$ (or *left Rindler wedge*, denoted *II*) requires another Rindler coordinate system to cover it, one that differs from (82) and (83) by an overall minus sign. In region *II* this implies that η is a decreasing function of t: time "flows backwards" in this wedge. Although the left Rindler wedge is casually disconnected from the right Rindler wedge, both can be influenced by events in the *past wedge* $-t > |x|$, (denoted *P*) and can in turn influence events in the *future wedge* $t > |x|$ (denoted *F*).

Observers located at constant $\zeta = \zeta_0$ (or constant X) are referred to as *Rindler observers*, in contrast to Minkowski observers who are located at $x = x_0$. These observers experience a time translation symmetry $\mathsf{T} \to \mathsf{T} + \mathsf{T}_0$, with Killing vector

$\xi = \partial_T$. The surface at $\mathsf{X} = 0$ in the Rindler coordinates is a Killing horizon for this ξ.

Rindler observers located at $\vec{w} = \vec{w}_0$ have the trajectory

$$z^a(\eta) = \frac{1}{\kappa} e^{\kappa\zeta_0}[\sinh(\kappa\eta), \cosh(\kappa\eta), \vec{w}_0]$$

in terms of Minkowski coordinates, and are confined to the right Rindler wedge. The Killing horizons at $\mathsf{X} = 0$ ($t = \pm x$) are the future/past communication horizons for this observer. The Killing vector ξ generates a boost symmetry along this observer's worldline, whose proper time is $e^{\kappa\zeta_0}\eta$, and whose proper velocity v^a and proper acceleration a^a are

$$\begin{aligned} v^a(\eta) &= e^{-\kappa\zeta_0}\frac{\partial z^a}{\partial \eta} = \left[\cosh(\kappa\eta), \sinh(\kappa\eta), \vec{0}\right] \\ a^a(\eta) &= e^{-\kappa\zeta_0}\frac{\partial v^a}{\partial \eta} = \kappa e^{-\kappa\zeta_0}\left[\sinh(\kappa\eta), \cosh(\kappa\eta), \vec{0}\right] \end{aligned} \tag{84}$$

respectively. The magnitude of the acceleration is

$$a_b a^b = \kappa^2$$

and we see that Rindler observers move with uniform acceleration κ. The line $\zeta = 0$ is the worldline of an observer moving with (eternal) uniform acceleration κ, and who measures proper time η, crossing the $t = 0$ line at $\eta = 0$ and $x = 1/\kappa$. There are analogous observers in region II, whose trajectories are likewise lines of constant ζ in the left Rindler wedge.

The 2-point function in the Minkowski vacuum

$$G_M(x, y) = G_M(\Delta t, \vec{x}, \vec{y}) = \langle 0|\phi(x)\phi(y)|0\rangle \tag{85}$$

satisfies

$$(\nabla_x^2 - m^2)G_M(x, y) = \delta^D(x - y) \tag{86}$$

where $\Delta t = x^0 - y^0 = t_x - t_y$ using the Cartesian coordinates (78). Writing $t = i\tau$ yields the Euclidean metric

$$ds_E^2 = d\tau^2 + dr^2 + r^2 d\Omega_{D-2}^2 = d\tau^2 + dx^2 + \sum_{i=2}^{d} dw^i dw^i \tag{87}$$

and the Euclidean 2-point function

$$G_M(i\Delta\tau, \vec{x}, \vec{y}) = G_E(\Delta\tau, \vec{x}, \vec{y}) = G_E(x, y) \tag{88}$$

(where $\vec{x} = (x, \vec{w})$) is uniquely defined by requiring

$$(\nabla_E^2 - m^2)G_E(x, y) = \delta^D(x - y) \tag{89}$$

where the derivative operator acts on the x coordinates, analogous to (86). If we begin with the Euclidean G_E defined from (89), then the resultant Minkowksi G_M will depend on how the real t-axis is approached due to the branch cuts in G_M.

One could apply the KMS condition (76) at any desired value of β to obtain thermal states in the Minkowski space. However for a Rindler observer the value of β is set by the state of motion. Indeed, writing $(\tau, x, \vec{w}) = (\mathsf{X}\sin\frac{2\pi}{\beta}\Upsilon, \mathsf{X}\cos\frac{2\pi}{\beta}\Upsilon, \vec{W})$ yields $G_R(\Delta\Upsilon, \vec{\mathsf{X}}, \vec{\mathsf{Y}})$, which is periodic in $\Delta\Upsilon$ with period β (with $\vec{\mathsf{X}} = (\mathsf{X}, \vec{W})$). The metric (87) becomes

$$ds^2 = \left(\frac{2\pi}{\beta}\right)^2 X^2 d\Upsilon^2 + d\mathsf{X}^2 + \sum_{i=2}^{d} dW^i dW^i \tag{90}$$

which is identical to the metric (79) upon setting $\mathsf{T} = i\Upsilon$ and identifying $\beta = \frac{2\pi}{\kappa}$. The boost symmetry generated by ∂_T becomes a rotation symmetry generated by ∂_Υ.

Analytically continuing $G_E(\Delta\Upsilon, \vec{\mathsf{X}}, \vec{\mathsf{Y}})$ to a holomorphic function of complex ΔT thus yields

$$G_M(\Delta t, \vec{x}, \vec{y}) \to G_E(\Delta\tau, \vec{x}, \vec{y}) = G_R(\Delta\Upsilon, \vec{\mathsf{X}}, \vec{\mathsf{Y}}) = G_R^{2\pi/\kappa}(\Delta\tau, \vec{\mathsf{Y}}, \vec{\mathsf{X}}) \tag{91}$$

and so the Minkowski vacuum with respect to uniformly accelerating observers (those using Rindler time T) is indeed a thermal state with temperature $T = \beta^{-1} = \frac{\kappa}{2\pi}$.

The thermality experienced by an accelerated detector is a consequence of its causal disconnection with certain regions of space-time. A uniformly accelerated observer in region I (one that follows an orbit of $\partial/\partial\mathsf{T}$) is causally disconnected from region II, and so any state (such as the vacuum state) detected by this observer is obtained by tracing over the degrees of freedom in region II. This yields a mixed state from the perspective of the accelerated observer, with an entropy coming from entanglement between modes from regions I and II.

To see this relationship explicitly, consider the expansion (60), which is

$$\varphi(x) = \int d^3\boldsymbol{k}\left[\phi_{\boldsymbol{k}}(x)\hat{a}_{\boldsymbol{k}} + \phi^*_{\boldsymbol{k}}(x)\hat{a}^\dagger_{\boldsymbol{k}}\right]$$

where Eq. (46) implies that

$$\phi_{\boldsymbol{k}}(x) = \phi_{\boldsymbol{k}}(\vec{x})e^{-i\omega_k t} \tag{92}$$

is a plane-wave mode of positive frequency in the coordinates (78). Note that it is analytic and bounded in the lower-half complex-t plane. However in the Rindler coordinates (79) the modes are

$$^{I}\phi_k(\mathsf{X}) = \begin{cases} \tilde{\phi}_k(\vec{\mathsf{X}})e^{-i\omega T} & \text{region } I \\ 0 & \text{region } II \end{cases} \quad {}^{II}\phi_k(\mathsf{X}) = \begin{cases} 0 & \text{region } I \\ \tilde{\phi}_k(\vec{\mathsf{X}})e^{i\omega T} & \text{region } II \end{cases} \tag{93}$$

in regions I and II respectively. For a massless scalar the positive and negative frequency modes in each region are

$$^{I}\phi^{\pm}_{\omega}(\mathsf{X}) = \begin{cases} \tilde{\phi}_{\omega}(\vec{W})e^{\mp i\omega(T-\epsilon\ln(X)/\kappa)} = \tilde{\phi}_{\omega}(\vec{W})\left(\frac{x-\epsilon t}{L}\right)^{\pm i\epsilon\omega/\kappa} & \text{region } I \\ 0 & \text{region } II \end{cases} \tag{94}$$

$$^{II}\phi^{\pm}_{\omega}(\mathsf{X}) = \begin{pmatrix} 0 & \text{region } I \\ \tilde{\phi}_{\omega}(\vec{W})e^{\pm i\omega\{T-\epsilon\ln(-X)/\kappa)} = \tilde{\phi}_{\omega}(\vec{W})\left(\frac{\epsilon t-x}{L}\right)^{\mp i\epsilon\omega/\kappa} & \text{region } II \end{pmatrix} \tag{95}$$

where the sign flip in region II occurs because the future directed Killing vector is $\xi = \partial_{-\mathrm{T}} = -\partial_{\mathrm{T}}$ in that region. Note that $\phi^{\pm *} = \phi^{\mp}$ in each region. The frequency $\omega > 0$ and the constant $\epsilon = \pm 1$ corresponds to the mode propagating in the positive/negative X-direction respectively. Specification of the constant L is equivalent to specifying the phase of the Rindler modes; this choice is purely a matter of convention.

Note that $^{I}\phi^{+}$ and $^{II}\phi^{-} = ^{II}\phi^{+*}$ are both proportional to $(x - \varepsilon t)^{i\varepsilon\omega/\kappa}$, since the latter is

$$\begin{aligned} ^{II}\phi^{-} \propto \left(\frac{\epsilon t - x}{L}\right)^{i\epsilon\omega/\kappa} = \left(\frac{-(x-\epsilon t)}{L}\right)^{i\epsilon\omega/\kappa} = \left(\frac{(x-\epsilon t)}{L}\right)^{i\epsilon\omega/\kappa}\left(e^{i\pi}\right)^{i\epsilon\omega/\kappa} \\ = \left(\frac{(x-\epsilon t)}{L}\right)^{i\epsilon\omega/\kappa} e^{-\pi\omega/\kappa} \end{aligned} \tag{96}$$

with a similar relation for $^{II}\phi^{+}$. Hence the combination

$$\phi^{p}_{\omega} = ^{I}\phi^{+}_{\omega} + ^{II}\phi^{-}_{\omega} \tag{97}$$

is a positive frequency mode with respect to the Minkowski time t since it is analytic in the lower-half complex plane of $(x - \epsilon t)$. It must therefore share a common vacuum state with the Minkowski modes. The associated annihilation operator is

$$\begin{aligned}\hat{a}_{p,\omega} &= \langle\langle \phi^p_\omega, \varphi \rangle\rangle \\ &= \langle\langle \left({}^I\phi^+_\omega + {}^{II}\phi^-_\omega\right), \varphi \rangle\rangle \\ &= \langle\langle {}^I\phi^+_\omega, \varphi \rangle\rangle + \langle\langle {}^{II}\phi^-_\omega, \varphi \rangle\rangle \\ &= \hat{a}_\omega + e^{-\pi\omega/\kappa}\langle\langle {}^{II}\phi^{+*}_\omega, \varphi \rangle\rangle \\ &= \hat{a}_{\omega,I} - e^{-\pi\omega/\kappa}\hat{a}^\dagger_{\omega,II}\end{aligned} \tag{98}$$

using (59). Since $\hat{a}_p$ is an annihilation operator for positive frequency states, it must act on the Minkowski vacuum as $\hat{a}_p|0\rangle = 0$, and so

$$\hat{a}_{\omega,I}|0\rangle = e^{-\pi\omega/\kappa}\hat{a}^\dagger_{\omega,II}|0\rangle \tag{99}$$

indicating correlations between the modes in regions I and II. Similarly, the combination

$$\phi^{p'}_\omega = {}^{II}\phi^+_\omega + {}^{I}\phi^-_\omega \tag{100}$$

is also a positive frequency mode with respect to the Minkowski time t. Its associated $\hat{a}_{p',\omega} = \langle\langle \phi^{p'}_\omega, \varphi \rangle\rangle$ will also annihilate the Minkowski vacuum and so

$$\hat{a}_{\omega,II}|0\rangle = e^{-\pi\omega/\kappa}\hat{a}^\dagger_{\omega,I}|0\rangle \tag{101}$$

Since $\hat{a}_{\omega,I}$ and $\hat{a}_{\omega,II}$ respectively annihilate the vacua $|0\rangle_I$ and $|0\rangle_{II}$ in regions I and II, we can write

$$|0\rangle \propto \prod_\omega f(\hat{a}^\dagger_{\omega,I}, \hat{a}^\dagger_{\omega,II})|0\rangle_I|0\rangle_{II} \tag{102}$$

and so obtain for any given ω

$$\hat{a}_{\omega,I}|0\rangle = \hat{a}_{\omega,I}f(\hat{a}^\dagger_{\omega,I}, \hat{a}^\dagger_{\omega,II})|0\rangle_I|0\rangle_{II} = [\hat{a}_{\omega,I}, f]|0\rangle_I|0\rangle_{II} = e^{-\pi\omega/\kappa}\hat{a}^\dagger_{\omega,II}|0\rangle_I|0\rangle_{II} \tag{103}$$

$$\hat{a}_{\omega,II}|0\rangle = \hat{a}_{\omega,II}f(\hat{a}^\dagger_{\omega,I}, \hat{a}^\dagger_{\omega,II})|0\rangle_I|0\rangle_{II} = [\hat{a}_{\omega,II}, f]|0\rangle_I|0\rangle_{II} = e^{-\pi\omega/\kappa}\hat{a}^\dagger_{\omega,I}|0\rangle_I|0\rangle_{II} \tag{104}$$

implying

$$f(\hat{a}^\dagger_{\omega,I}, \hat{a}^\dagger_{\omega,II}) = \exp\left[e^{-\pi\omega/\kappa}\hat{a}^\dagger_{\omega,I}\hat{a}^\dagger_{\omega,II}\right] \tag{105}$$

using (56). The operators $\hat{a}_{p,\omega}$ and $\hat{a}_{p',\omega}$ are not properly normalized. Rescaling them to be $\hat{a}_{R,\omega} = \lambda \hat{a}_{p,\omega}$ and $\hat{a}_{L,\omega} = \tilde{\lambda} \hat{a}_{p',\omega}$ so that their commutators are analogous to (63) yields $\lambda = \tilde{\lambda} = 1/\sqrt{1 - e^{-2\pi\omega/\kappa}}$ and so

$$\begin{aligned} \hat{a}_{R,\omega} &= \frac{1}{\sqrt{1 - e^{-2\pi\omega/\kappa}}} \left(\hat{a}_{\omega,I} - e^{-\pi\omega/\kappa} \hat{a}^{\dagger}_{\omega,II} \right) = \cosh r_\omega \, \hat{a}_{\omega,I} - \sinh r_\omega \, \hat{a}^{\dagger}_{\omega,II} \\ \hat{a}_{L,\omega} &= \frac{1}{\sqrt{1 - e^{-2\pi\omega/\kappa}}} \left(\hat{a}_{\omega,II} - e^{-\pi\omega/\kappa} \hat{a}^{\dagger}_{\omega,I} \right) = \cosh r_\omega \, \hat{a}_{\omega,II} - \sinh r_\omega \, \hat{a}^{\dagger}_{\omega,I} \end{aligned} \tag{106}$$

where $\tanh r_\omega \equiv e^{-\pi\omega/\kappa}$. Using the above so that the normalization of both sides of (102) is consistent yields

$$|0\rangle = \prod_\omega \frac{1}{\cosh r_\omega} \sum_{n=0}^{\infty} \tanh^n(r_\omega) |n_\omega\rangle_I |n_\omega\rangle_{II} \tag{107}$$

We see that from the perspective of a uniformly accelerated observer, the Minkowski vacuum $|0\rangle$ consists of correlated pairs of particles. It is a squeezed state with respect to the Rindler basis. Note that the Minkowski vacuum is not in the same Hilbert space as the Rindler vacuum and so the expression (107) is just a formal expression.

A Minkowski observer detects a state absent of particles. However a uniformly accelerated observer detects the state

$$\rho_k^I = \mathrm{tr}_{II}(|0\rangle\langle 0|),$$

where the trace over all states in region II is carried out since the Rindler observer has no causal contact with this region. The expectation value of the particle number operator $N_k = \hat{a}_k^{\dagger} \hat{a}_k$ is then

$$\mathrm{tr}_I(N_k \rho_k^I) = \mathrm{tr}_I(\hat{a}_k^{\dagger} \hat{a}_k \rho_k^I) = \sinh^2(r_k) = \frac{1}{e^{2\pi\omega_k/\kappa} - 1},$$

which is a Bose-Einstein distribution at temperature

$$T = \frac{\kappa}{2\pi k_B}$$

where k_B is Boltzmann's constant.

A similar situation holds from spinors. A Dirac spinor field $\Psi(x)$ of mass m can be quantized in a straightforward manner by expanding it in terms of a complete set of positive and negative frequency modes in Minkowski space-time

$$\hat{\Psi}(x) = \int dk\,(\hat{a}_k \psi_k^+(x) + \hat{b}_k^\dagger \psi_k^-(x)),$$

partitioned according to the eigenvalues Minkowski timelike Killing vector ∂_t. The operators $\hat{a}_k$ and $\hat{b}_k^\dagger$ are then interpreted as particle annihilation operators and antiparticle creation operators, and they obey the canonical anticommutation relations $\{\hat{a}_k, \hat{a}_{k'}^\dagger\} = \{\hat{b}_k, \hat{b}_{k'}^\dagger\} = \delta(k - k')$, with all other anticommutators vanishing. The modes are labelled by $\boldsymbol{k}$ and for notational ease the spin degree of freedom has been suppressed.

From the perspective of a uniformly accelerated observer, the expansion would be in terms of the complete set of positive and negative frequency Rindler modes

$$\hat{\Psi}(x) = \int dk(\hat{c}_k^I \psi_k^{I+}(x) + \hat{d}_k^{I\dagger} \psi_k^{I-}(x) + \hat{c}_k^{II} \psi_k^{II+} + \hat{d}_k^{II\dagger} \psi_k^{II-}(x)),$$

where the canonical anticommutation relations

$$\{\hat{c}_k^I, \hat{c}_{k'}^{I\dagger}\} = \{\hat{d}_k^I, \hat{d}_{k'}^{I\dagger}\} = \delta(k - k'),$$
$$\{\hat{c}_k^{II}, \hat{c}_{k'}^{II\dagger}\} = \{\hat{d}_k^{II}, \hat{d}_{k'}^{II\dagger}\} = \delta(k - k'),$$

hold, with all other anticommutators vanishing. As with the scalar case, the modes $\psi_k^{I\pm}(x)$ have support in region *I*, whereas the modes $\psi_k^{II\pm}(x)$ have support in region *II*, each type divided into positive and negative frequency according to the region-specific Rindler timelike Killing vector. In region *I*/*II* this is given by $\partial_{\pm T}$, the minus sign ensuring it is future pointing in region *II*. The operators $\hat{c}_k^I$ and $\hat{d}_k^{I\dagger}$ respectively annihilate a particle and create an antiparticle in region *I* while their counterparts $\hat{c}_k^{II}$ and $\hat{d}_k^{II\dagger}$ do the same in region *II*.

Arguments similar to those in the scalar case yield the Bogoliubov transformation

$$\begin{pmatrix} \hat{a}_k \\ \hat{b}_{-k}^\dagger \end{pmatrix} = \begin{pmatrix} \cos r_k & -e^{-i\phi_k} \sin r_k \\ e^{i\phi_k} \sin r_k & \cos r_k \end{pmatrix} \begin{pmatrix} \hat{c}_k^I \\ \hat{d}_{-k}^{II\dagger} \end{pmatrix} \tag{108}$$

where

$$\cos r_k = [2\cosh(\pi\omega_k/\kappa)]^{-1/2} \exp(\pi\omega_k/2\kappa), \tag{109}$$

and $\omega_k = \sqrt{k^2 + m^2}$ is the frequency of the mode. This transformation can also be expressed as

$$\begin{pmatrix} \hat{a}_k \\ \hat{b}^\dagger_{-k} \end{pmatrix} = S_+(r_k) \begin{pmatrix} \hat{c}^I_k \\ \hat{d}^{II\dagger}_{-k} \end{pmatrix} S^\dagger_+(r_k), \tag{110}$$

where the unitary operator $S_+(r_k)$

$$S_+(r_k) = \exp\left[r_k(\hat{c}^{I\dagger}_k \hat{d}^{II\dagger}_{-k} e^{-i\phi_k} + \hat{c}^I_k \hat{d}^{II}_{-k} e^{i\phi_k})\right]$$

is the *two mode squeezing operator* [53].

Similarly, if the Rindler observer's antiparticle detector is sensitive to a narrow bandwidth centered about the wavevector $-\boldsymbol{k}_\perp$ then

$$\begin{pmatrix} \hat{b}_k \\ \hat{a}^\dagger_{-k} \end{pmatrix} = \begin{pmatrix} \cos r_k & e^{-i\phi_k} \sin r_k \\ -e^{i\phi_k} \sin r_k & \cos r_k \end{pmatrix} \begin{pmatrix} \hat{d}^I_k \\ \hat{c}^{II\dagger}_{-k} \end{pmatrix}. \tag{111}$$

or

$$\begin{pmatrix} \hat{b}_k \\ \hat{a}^\dagger_{-k} \end{pmatrix} = S_-(r_k) \begin{pmatrix} \hat{d}^I_k \\ \hat{c}^{II\dagger}_{-k} \end{pmatrix} S^\dagger_-(r_k), \tag{112}$$

expressed as a squeezing transformation, where

$$S_-(r_k) = \exp\left[-r_k(\hat{d}^{I\dagger}_k \hat{c}^{II\dagger}_{-k} e^{-i\phi_k} + \hat{d}^I_k \hat{c}^{II}_{-k} e^{i\phi_k})\right].$$

The phase ϕ_k can be absorbed into the definitions of the mode operators.

The operators $\hat{a}_k$ and $\hat{b}_{-k}$, respectively, annihilate the single mode particle and anti-particle Minkowski vacua

$$a_k|0_k\rangle^+ = 0 \qquad \text{and} \qquad b_{-k}|0_{-k}\rangle^- = 0$$

and so it is reasonable to postulate that the Minkowski particle vacuum for mode k in terms of Rindler Fock states is given by

$$|0_k\rangle^+ = \sum_{n=0}^{1} A_n \, |n_k\rangle^+_I \, |n_{-k}\rangle^-_{II}, \tag{113}$$

where

$$c^I_k|0_k\rangle^+_I = 0 \qquad d^{II}_{-k}|0_{-k}\rangle^-_{II} = 0 \qquad c^{I\dagger}_k|0_k\rangle^+_I = |1_k\rangle^+_I \qquad d^{II\dagger}_{-k}|0_{-k}\rangle^-_{II} = |1_{-k}\rangle^-_{II} \tag{114}$$

since $\hat{a}_k$ mixes particles in region I and anti-particles in region II. Using (111) yields

$$\begin{aligned} 0 = a_k|0_k\rangle^+ &= \left(\cos r_k\, c_k^I - e^{-i\phi}\sin r_k\, d_{-k}^{II\dagger}\right)\sum_{n=0}^{1} A_n\,|n_k\rangle_I^+\,|n_{-k}\rangle_{II}^- \\ &= (A_1\,\cos r_k - A_0\,e^{-i\phi}\,\sin r_k)\,|0_k\rangle_I^+\,|1_{-k}\rangle_{II}^- \Rightarrow A_1 = A_0\,e^{-i\phi}\,\tan r_k \end{aligned} \tag{115}$$

and normalization ${}^+\langle 0_k|0_k\rangle^+ = |A_0|^2 + |A_1|^2 = 1$ gives

$$\begin{aligned} |0_k\rangle^+ &= \cos r_k\,|0_k\rangle_I^+\,|0_{-k}\rangle_{II}^- + e^{-i\phi}\,\sin r_k\,|1_k\rangle_I^+\,|1_{-k}\rangle_{II}^- \\ &= \cos r_k\,\exp(\tan(r_k)\,\hat{c}_k^{I\dagger}\,\hat{d}_{-k}^{II\dagger})|0_k\rangle_I^+\,|0_{-k}\rangle_{II}^- \end{aligned} \tag{116}$$

for the mode-k Minkowski particle vacuum. For all modes the formal expression is

$$|0\rangle^+ = \prod_k \cos r_k\,\exp(\tan(r_k)\,\hat{c}_k^{I\dagger}\,\hat{d}_{-k}^{II\dagger})|0_k\rangle_I^+|0_{-k}\rangle_{II}^- \tag{117}$$

for the total Minkowski particle vacuum.

In a similar manner, the Minkowski antiparticle vacuum in mode k can be expressed in the Rindler Fock basis as

$$|0_k\rangle^- = \cos r_k\,\exp(-\tan(r_k)\,\hat{d}_k^{I\dagger}\,\hat{c}_{-k}^{II\dagger})|0_k\rangle_I^-\,|0_{-k}\rangle_{II}^+$$

for the kth mode, so giving

$$|0\rangle^- = \prod_k \cos r_k\,\exp(-\tan(r_k)\,\hat{d}_k^{I\dagger}\,\hat{c}_{-k}^{II\dagger},)|0_k\rangle_I^-\,|0_{-k}\rangle_{II}^+$$

as the formal expression for the total Minkowski antiparticle vacuum.

As with the scalar case, while a Minkowski observer detects only a vacuum absent of particles, the Rindler observer detects the state

$$\rho_k^I = \mathrm{tr}_{II}(|0\rangle^{++}\langle 0|),$$

where the trace over all states in region II is carried out as before. The expectation value of the particle number operator $N_k = \hat{c}_k^{I\dagger}\hat{c}_k^I$ is then

$$\mathrm{tr}_I(N_k\rho_k^I) = \mathrm{tr}_I(\hat{c}_k^{I\dagger}\hat{c}_k^I\rho_k^I) = \sin^2(r_k) = \frac{1}{e^{2\pi\omega_k/\kappa}+1},$$

which is a Fermi-Dirac distribution at temperature

$$T = \frac{\kappa}{2\pi k_B}$$

Note that in both cases that while the frequency with respect to the Rindler time coordinate T is ω, the physical energy of the mode depends on the location of the accelerating observer, with increasing redshift as X increases. The energy measured by a given observer is $E_{local} = \omega/\kappa X$ and an outgoing wave packet moving along lines of constant $t + x$ will redshift as it gets to larger X.

5.3 Pair Creation

An intuitive argument illustrating how particle creation can take place was recently given by Mathur [36]. Consider a choice of spacelike slices in which one spatial region evolves further in time than another, as shown in Fig. 12. This situation is permitted in generally covariant theories of gravity such as general relativity. Both slices satisfy the semiclassical (niceness) conditions. No particle creation occurs classically, but if a quantum field is in the vacuum state ((b) in Fig. 12) then the created-pair state will be in the state

$$|\psi\rangle = \psi_0 e^{\sigma c^\dagger b^\dagger} |0\rangle_c |0\rangle_b = \left(\alpha|0\rangle_c|0\rangle_b + \beta|1\rangle_c|1\rangle_b\right) + \cdots \qquad (118)$$

where ψ_0 and σ are coefficients noted above for scalars and spinors, and where $|\alpha|^2 + |\beta|^2 = 1$; multi-pair creation has been neglected in the above. In Hawking

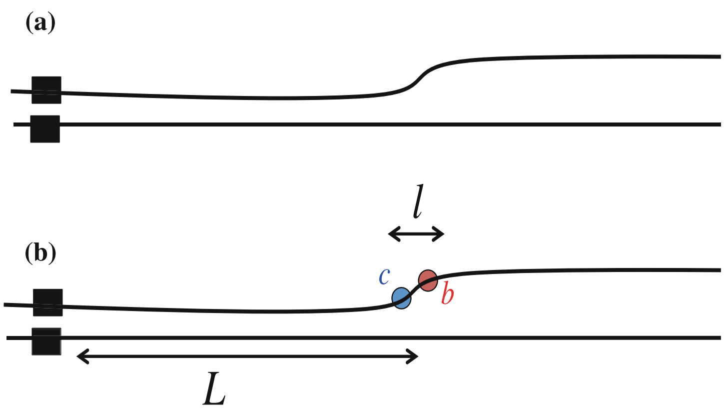

Fig. 12 Particle creation due to spacetime distortion: the intrinsic geometry of the initial spatial slice (*horizontal line*) evolves forward in time differently on the *left* than on the *right*, with a concentration of (classical) matter symbolized by the *box* at the *left*. **a** the evolution is fully classical and no pairs are created. **b** the quantum field on the initial slice is in the vacuum, with the space-time distortion on the next slice creating a pair of quanta

radiation the created pair is maximally entangled, with $|\alpha| = |\beta| = 1/\sqrt{2}$, but we can understand what is going on for arbitrary entanglement. The quantum state of the entire system is

$$|\Psi\rangle \approx |\Phi\rangle_M \otimes (\alpha|0\rangle_c|0\rangle_b + \beta|1\rangle_c|1\rangle_b) \tag{119}$$

where $|\Phi_0\rangle_M$ is the quantum state of the matter, symbolized by the box at the left of Fig. 12. If $l \ll L$ then the influence of the matter on the created pair can be neglected (though in principle there is always some influence), and the full state is well approximated by a tensor product.

The locality assumption ensures that the state Ψ is a tensor product. Small departures from whatever physics yields the (α, β) coefficients are permitted

$$|\Psi\rangle \approx (\tilde{\alpha}|\Phi_0\rangle_M + \tilde{\beta}|\Phi_1\rangle_M) \otimes ((\alpha + \epsilon)|0\rangle_c|0\rangle_b + (\beta - \epsilon)|1\rangle_c|1\rangle_b) \tag{120}$$

but not states of the form

$$|\Psi\rangle \approx ((\tilde{\alpha} + \epsilon)|\Phi_0\rangle_M|0\rangle_c + (\tilde{\beta} - \epsilon)|\Phi_1\rangle_M|1\rangle 1_c) \otimes (\alpha|0\rangle_b + \beta|1\rangle_b) \tag{121}$$

where for simplicity the matter is assumed to be a single qubit in one of two possible states $|\Phi_0\rangle_M$ or $|\Phi_1\rangle_M$. For the former case (120) the entanglement entropy is, upon tracing over the matter and state c,

$$\begin{aligned} S_{\text{ent}} &= -\text{tr}_{c,M}[\rho \ln \rho] \\ &= -(|\alpha|^2 \ln|\alpha| + |\beta|^2 \ln|\beta|) + 2\epsilon(|\beta|\ln(2|\beta|^2) - |\alpha|\ln(2|\alpha|^2)) \\ &\quad - \epsilon^2(6 + 2\ln(|\alpha||\beta|)) \end{aligned} \tag{122}$$

which has the value $S_{\text{ent}} = \ln 2 - \varepsilon^2(6 - 2\ln 2) \approx \ln 2$ for maximal entanglement. However for the latter case (121) the entanglement entropy is

$$S_{\text{ent}} = -\text{tr}_{c,M}[\rho \ln \rho] = 0 \tag{123}$$

since the state b is a direct product with the remaining states. So for $l_p \ll l \ll L$, the entanglement entropy is

$$\left|\frac{S_{\text{ent}}}{\ln 2} - 1\right| \ll 1 \tag{124}$$

from pair creation due to space-time distortion when the semiclassical assumptions are valid.

5.4 Accelerating Detectors

The arguments underlying the Unruh effect [42] given above depend on the idealization of eternal uniform acceleration. The Rindler horizons are always present in the spacetime, allowing for a clean transformation of creation and annihilation operators as in (106) and (108). Of course this idealization cannot be realized in practice—it is not possible to construct a detector that has been in constant acceleration for all time. In general neither a Rindler-like spacetime structure nor the presence of an event horizon is available for physical systems of interest, such as detectors place in cavities or launched in orbit, and so neither thermality arguments nor geometric properties are of much use in understanding the behaviour of an accelerating probe. Furthermore, for non-uniform acceleration there is no timelike Killing vector nor equilibrium condition to define an Unruh temperature for all times.

So do accelerated detectors get hot? This question was raised over 20 years ago [54] and has been a subject of increasing interest ever since [55–75]. Physically there should be no fundamental distinction between nonuniform and uniform acceleration, and so one expects acceleration radiation, though perhaps not in a strictly thermal form. The more general question is that of how a detector responds to changes in its motion, for example from an inertial state to a uniformly accelerated state and back again. This more generic case is what one encounters in the everyday experience of driving a car. However this simple intuitive perspective is not easy to formulate, because of the lack of an event horizon (from the geometric viewpoint) and of an equilibrium condition (from the field theoretic viewpoint). Methods from stochastic field theory indicate that observers experience nonthermal radiance [55] and recently new methods have been developed that are probing the details of how accelerating detectors behave in more physically realistic situations [61, 62, 65, 68, 70, 72, 73]. The emerging results indicate that the temperature dependence on the changing motion of accelerating detectors contains rather subtle and interesting features. A recent review of this subject appears in Ref. [76].

6 Black Hole Radiation

The key distinction between radiation emitted from a black hole and that emitted from a hot material object (such as a lump of coal) is in how the emitted quanta are generated. A lump of coal emits radiation because the atoms near its surface are excited, and emit quanta as they fall to states of lower energy. A black hole, however, emits quanta that arise due to entangled pair creation from the distortion of space-time, with one partner in the pair remaining inside the black hole and thus inaccessible to observers outside. Put another way, hot material bodies emit radiation from their constituents whereas black holes pull entangled pairs of quanta out

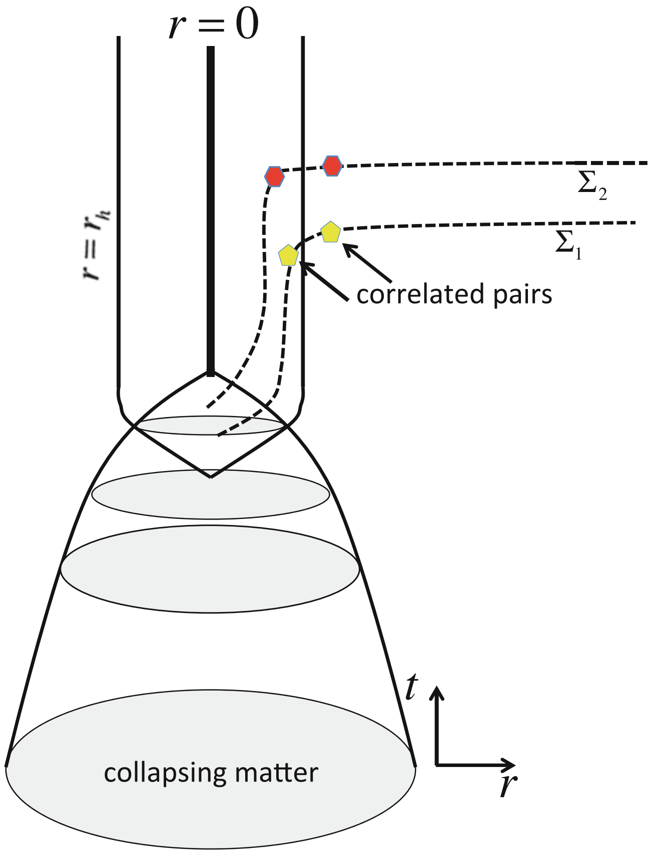

Fig. 13 Creation of correlated pairs outside a black hole: pairs of quanta (symbolized by the shapes) are created near the event horizon. The collapsing matter on the same corresponding spatial slices is very far from these pairs

of the vacuum as the result of 'stretching' a region of a space-like slice. The situation is illustrated in Fig. 13.

The Schwarzschild black hole metric (2) is

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega_2^2 \tag{125}$$

upon setting $r_h = 2M$. The space-time (125) is static, and has a Killing vector $\xi = \partial_t$. There is Killing horizon at $\xi^a \xi_a = g_{tt} = 0$ (which is $r = 2M$) so one might expect some generalization of the situation for uniformly accelerated observers to apply. This is indeed the case. Analytically continuing $t \to i\tau$ yields the Euclidean metric

$$ds^2 = +\left(1 - \frac{2M}{r}\right)d\tau^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega_2^2 \tag{126}$$

which appears to diverge at $r = 2M$. This divergence is actually a bit subtle. Writing

$$R = \int \frac{dr}{1 - \frac{2M}{r}} = r\sqrt{1 - \frac{2M}{r}} + M \ln\left(\frac{r}{M}\left(1 + \sqrt{1 - \frac{2M}{r}}\right) - 1\right) \tag{127}$$

gives

$$ds^2 = +\left(1 - \frac{2M}{r(R)}\right)d\tau^2 + dR^2 + r(R)^2 d\Omega_2^2 \tag{128}$$

from (126). The function $r(R)$ is implicitly defined, but can be expanded near $r = 2M$, yielding $r = 2M + \frac{R^2}{8M} + \cdots$ and

$$ds^2 \approx +\left(\frac{R}{4M}\right)^2 d\tau^2 + dR^2 + (2M)^2 d\Omega_2^2 + \cdots \tag{129}$$

where terms of order R^4/M^4 are neglected. We see that the structure of the metric (128) near $r = 2M$ is actually that of the product of a 2-sphere with a plane written in polar coordinates whose origin is at $r = 2M$. The actual geometry of the entire space is like the product of 2-sphere with a (semi-infinite) cigar whose tip is at $r = 2M$, shown in Fig. 14. The singularity at $r = 2M$ is actually the singularity at the tip of a cone if τ is an angular coordinate (as we anticipate for describing thermal states).

If we are to apply the KMS condition, we must eliminate this singularity. The only way to do so is to set the period of τ so that $r = 2M$ is indeed the origin of a plane and not the tip of a cone. Hence the quantity $\tau/4M$ must have period 2π, or τ must have period $8\pi M$. Taking the Euclidean Green's function $G(\Delta\tau, \vec{x}, \vec{y})$ to define

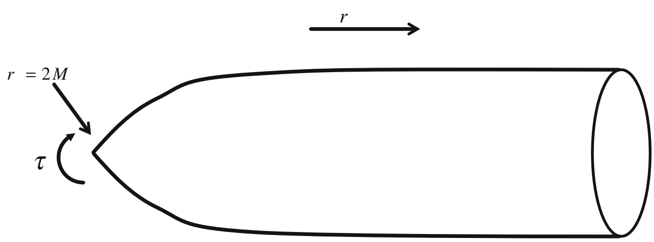

Fig. 14 Euclidean schwarzschild solution: a depiction of the 'cigar' shape of the Schwarzschild space-time. The conical singularity at the $r = 2M$ tip is removed only for the particular choice $8\pi M$ for the period of the Euclidean time τ

a quantum state on this space and then analytically continuing it to obtain a two-point function on the original Lorentzian black hole spacetime, (2), the resulting two-point function $G^{8\pi M}(\Delta t, \vec{x}, \vec{y})$ will satisfy the KMS condition for the same reasons as in the Rindler case. This state, defined by analytic continuation from the Euclidean black hole, is known as the Hartle-Hawking state. It is a thermal state at temperature $T = \kappa/2\pi$, where $\kappa = 1/4M$ is the surface gravity of the black hole.

The temperature is set by the requirement that the Euclidean counterpart of the black hole space-time is smooth at the horizon. This requirement is quite general and holds for a broad variety of black holes. The metric of any static black hole can be written as

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{g(r)} + r^2 d\Omega_2^2 \rightarrow +f(r)d\tau^2 + \frac{dr^2}{g(r)} + r^2 d\Omega_2^2 \tag{130}$$

where the latter relation follows the analytic continuation $t \rightarrow i\tau$. If there is a black hole, then f and g both vanish linearly at the same point $r = r_+$, which is the event horizon. Hence $f(r) = (r - r_+)f'_+ + \cdots,\ g(r) = (r - r_+)g'_+ + \cdots$, and

$$ds^2 = R^2 \frac{g'_+ f'_+}{4} d\tau^2 + dR^2 + r_+^2 d\Omega_2^2 + \cdots \tag{131}$$

where $R = \int dr/\sqrt{g(r)}$ and the prime denotes an r-derivative. The metric (131) will be smooth at the horizon provided $\sqrt{g'_+ f'_+}\tau/2$ has period 2π. Applying the KMS criterion implies $\beta = 2\pi/\kappa$, or that the Hartle-Hawking state has temperature $T = \kappa/2\pi$, where

$$\kappa^2 = -12\nabla^a \xi^b \nabla_a \xi_b|_{r=r_+} = \frac{g(r)(f'(r))^2}{4f(r)}|_{r=r_+} = \frac{g'_+ f'_+}{4} \tag{132}$$

yields the surface gravity at the horizon.

The Hartle-Hawking state functions as the vacuum state of the Schwarzschild black hole (2). In flat Minkowski space there was a natural choice of vacuum since the Killing vector $\xi = \partial_t$ was globally defined. No such globally defined timelike Killing vector exists for the metric (2), or for any metric in the class (130). However it can be shown [40] that the unique Hadamard state invariant under the action of $\xi = \partial_t$ for a free scalar field on the Schwarzschild geometry is the thermal state at temperature $T = \kappa/2\pi$. This singles out the Hartle-Hawking state as the preferred vacuum, and only scalar thermal radiation at this temperature can be in equilibrium with the black hole.

Unfortunately this is an unstable equilibrium. The specific heat $C_V = \partial M/\partial T = -8\pi M^2$ of the black hole is negative: as it radiates its temperature rises, and it emits more radiation than it absorbs. Furthermore, since the Hartle-Hawking state is symmetric under time-reversal, its boundary conditions are such that it describes equal fluxes of ingoing and outgoing radiation, a rather unrealistic situation for an actual black hole even at late times.

So the black hole solution (2) is illustratively useful in describing black hole radiation, but is limited in its physical relevance. Fortunately other options are available for understanding this phenomenon. Hawking's original calculation [10] was for a free scalar field propagating in a classical background spacetime describing gravitational collapse of matter to a Schwarzschild black hole. Prior to collapse the scalar is initially in its vacuum state. At late times, long after the black hole has formed, the positive frequency mode function corresponding to a particle state is traced backwards in time to determine its positive and negative frequency parts in the asymptotic past. The expected number of particles at infinity corresponds to emission from a perfect black body (of finite size) at the temperature $T = \kappa/2\pi$. Other than in justifying use of the background space-time, nowhere are any gravitational field equations employed in this calculation. Furthermore, the results are mathematically well defined: no divergences arise and so there is no need for regularization or renormalization. A collapse situation is well approximated by the eternal black hole (2) but with boundary conditions in which the field is Hadamard only on the future horizon. This is called the Unruh state.

The calculation is quite robust, and can be generalized in a number of ways. First, once the collapsing matter has passed through the event horizon, the black hole will "settle down" to a stationary final state satisfying the 0th law of black hole mechanics. The geometry of any small neighbourhood of the future event horizon will therefore approximate flat space, stationary observers outside the black hole will approximate flat-space Rindler observers, and the result will again follow. Second, the initial state of the field need not be the vacuum state, but can be any Hadamard state. Indeed, if an invariant Hadamard state exists on a stationary spacetime with a bifurcate Killing horizon, it is a KMS state with temperature $T = \kappa/2\pi$ [40]. Third, not only does the expected particle number per mode agree with the black body prediction at temperature T, but *all* aspects of the final state at late times correspond to (possibly rotating) black body thermal radiation emanating from the black hole at temperature T [77].

While the preceding considerations are quite promising, there are a number of caveats and assumptions underlying the calculations of black hole temperature [35, 36].

A1 Invariant Hadamard states do not exist for all stationary black holes. The Kerr solution, describing a rotating black hole, is one such example, and as a consequence has a super-radiant instability [78–80].

A2 Asymptotically flat black holes will lose mass as they radiate, invalidating the late-time stationarity assumption. However the outgoing radiation will only carry an appreciable fraction of the mass over timescales $t \sim (M/M_P)^3$ [81].

A3 One of the most crucial assumptions is that the quantum state of the field is regular (Hadamard) at the horizon: its local behaviour at the horizon is the same as it would be in the Minkowski vacuum. This is an application of the equivalence principle, that locally (on sufficiently short time and distance scales) gravity and acceleration are indistinguishable. Freely-falling observers near the horizon should not see any unusual behaviour in high-energy processes. This is sometimes call the "no drama" assumption.

The no-drama assumption is somewhat paradoxical. An observer distant from a black hole formed from collapse who detects a mode at any finite frequency ω_f will realize that it has been redshifted. This must mean that it had a very large frequency in the past when it was propagating near the event horizon, of order $\omega = e^{\kappa t}\omega_f$ where t is the time it takes for the mode to reach the distant observer: the mode is blueshifted in the past. There is no past horizon (or other obvious physical effect) to provide a compensating effect for a mode propagating through the collapsing matter from the asymptotic past. Within a time scale of order $1/\kappa$ of the black hole's formation, the intermediate steps of the derivation implicitly involve propagation of trans-Planckian modes, modes that are much higher that the Planck frequency $\omega_P = \sqrt{c^5/\hbar G} \sim 10^{43}\,\mathrm{s}^{-1}$. This suggests lots of drama, since it is difficult to believe in the reliability of free-field theory (or any other known physical theory) at such high frequencies/energies [82].

Extensive study of this trans-Planckian problem [83–91] suggests that, despite the above, Hawking radiation is actually a low-energy phenomenon. Studies of quantized sound waves in a fluid undergoing supersonic flow indicate that a sonic analogue of the Hawking effect is present here as well. There is also a past-blueshift effect that renders invalid the continuum fluid equations that described the situation in the first place. However modifying these equations to yield a dispersion relation that is altered at ultra-high frequencies to alleviate this problem still yields sonic Hawking radiation. A variety of alternative models that significantly modify the

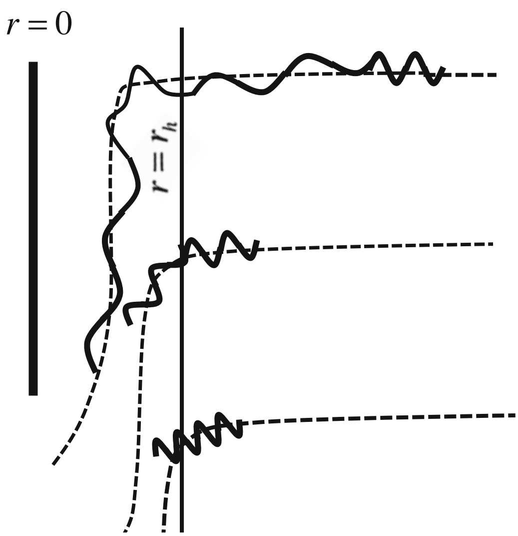

Fig. 15 Mode stretching: a Fourier mode created on some slice is stretched as it evolves to later slices. This eventually leads to a distorted waveform, resulting in particle creation

continuum fluid equations at high energies have verified this, and a recent experiment with water waves [92] is in accord with the basic theoretical predictions.

The low-energy character of Hawking radiation appears to emerge from the behaviour of the field in the WKB regime [93]. Moving backward in time, the past-blueshift effect will bring the field into the WKB regime before it enters the trans-Planckian regime. The WKB approximation remains valid throughout the evolution provided the Hawking temperature is much smaller than the trans-Planckian scale (the scale at which the modified dispersion relation is relevant). The outgoing radiation does not come about because of any interaction with other degrees of freedom but rather is a consequence of the tidal disruption (or space-time distortion) of free-field evolution by stretching the wavelengths. The major ingredient of Hawking radiation appears to be a "tearing apart" of the waves into an outgoing (positive norm) component and its infalling (negative norm) partner (Fig. 15).

6.1 Tunnelling

An intuitive picture, developed in recent years, was the notion that black hole radiation can be understood as a quantum tunnelling process [94]. Unlike other

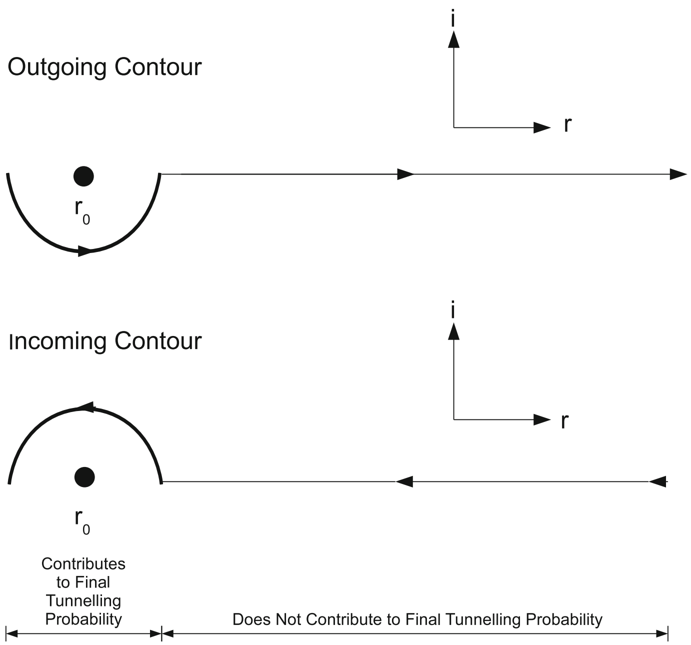

Fig. 16 Diagram of contours between black hole and observer for both outgoing and incoming trajectories

tunnelling processes, in which two separated classical turning points are joined by a trajectory in imaginary time, the outgoing particle itself creates the tunnelling barrier. Its trajectory is from the inside of the black hole to the outside, a classically forbidden process. Many papers have been written on this subject and a review recently appeared describing the state of research in this area [95].

The simplest case for exemplifying this process is to consider scalar particles. The idea is to apply the WKB approximation to the Klein-Gordon equation, solving it with the ansatz $\varphi \propto \exp(\frac{iI}{\hbar})$ and then integrating the action I along the classically forbidden trajectory, which starts inside the horizon and finishes at the outside observer (usually at infinity) [96, 97]. The equation will have a simple pole located at the horizon (since the trajectory is classically forbidden), and complex path analysis can be applied to do the integration [98], appropriately avoiding the pole [99]. The part of the trajectory that starts outside the black hole and continues to the observer will not contribute to the final tunnelling probability and can be safely ignored (shown in Fig. 16). This finally gives

$$\Gamma \propto |\varphi|^2 = \exp\left[-\frac{2\Im[I]}{\hbar}\right] = \exp(-\beta E) \tag{133}$$

for the semi-classical tunnelling probability for the emitted particle, the form of the latter part following from the assumed stationarity of the space-time. The parameter β is interpreted as the inverse temperature of the black hole, and $\Im[I]$ is the imaginary part of the action.

Consider a general (non-extremal) black hole metric of the form

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{g(r)} + C(r)h_{ij}dx^i dx^j \tag{134}$$

Applying the WKB approximation via the ansatz

$$\varphi(t, r, x^i) = \exp[\frac{i}{\hbar}I(t, r, x^i) + I_1(t, r, x^i) + O(\hbar)]$$

yields from the Klein Gordon equation (46)

$$g^{ab}\partial_a\partial_b\varphi - \frac{m^2}{\hbar^2}\varphi = 0 \tag{135}$$

(restoring the factors of $\hbar$) the Hamilton-Jacobi equation

$$-\left[g^{ab}\partial_a I\partial_b I + m^2\right] + O(\hbar) = 0 \tag{136}$$

to lowest order in $\hbar$. Explicitly this equation is

$$-\frac{(\partial_t I)^2}{f(r)} + g(r)(\partial_r I)^2 + \frac{h^{ij}}{C(r)}\partial_i I \partial_j I + m^2 = 0 \quad (137)$$

and has a solution of the form

$$I = -Et + W(r) + J(x^i) + K \quad (138)$$

where

$$\partial_t I = -E, \quad \partial_r I = W'(r), \quad \partial_i I = J_i$$

and K and the J_i's are constant (K can be complex). Since $\xi = \partial_t$ is the timelike killing vector with unit norm at infinity, E is the energy of the particle as detected by an observer at infinity. Solving (137) for $W(r)$ yields

$$W_{\pm}(r) = \pm \int \frac{dr}{\sqrt{f(r)g(r)}} \sqrt{E^2 - f(r)\left(m^2 + \frac{h^{ij}J_iJ_j}{C(r)}\right)} \quad (139)$$

as the two possible solutions. One solution corresponds to scalar particles moving away from the black hole (i.e. +outgoing) and the other solution corresponds to particles moving toward the black hole (i.e. –incoming).

The imaginary parts of the action arise either because of the pole at the horizon or from the imaginary part of K. The probabilities of crossing the horizon each way are proportional to

$$\text{Prob}[out] \propto \exp[-\frac{2}{\hbar}\text{Im}I] = \exp[-\frac{2}{\hbar}(\text{Im}W_+ + \text{Im}K)] \quad (140)$$

$$\text{Prob}[in] \propto \exp[-\frac{2}{\hbar}\text{Im}I] = \exp[-\frac{2}{\hbar}(\text{Im}W_- + \text{Im}K)] \quad (141)$$

Ensuring that any incoming particles crossing the horizon have a 100 % chance of entering the black hole, we set $\text{Im}K = -\text{Im}W_-$. Since $W_+ = -W_-$, the probability of a particle tunnelling from inside to outside the horizon is:

$$\Gamma \propto \exp[-\frac{4}{\hbar}\text{Im}W_+] \quad (142)$$

If one starts with an ansatz that does not contain the constant K, then the ratio of outgoing and incoming rates (140) and (141) gives the correct net tunnelling rate (142).

Integrating (139) for W_+ around the pole at the horizon (and setting $\hbar = 1$) yields [97]

$$W_+ = \frac{\pi i E}{\sqrt{g'(r_0)f'(r_0)}} \tag{143}$$

where the imaginary part of W_+ is now manifest. This leads to the tunnelling probability

$$\Gamma = \exp[-\frac{4\pi}{\sqrt{f'(r_0)g'(r_0)}}E] \tag{144}$$

implying upon using (133)

$$T_H = \frac{\sqrt{f'(r_0)g'(r_0)}}{4\pi}\frac{\kappa}{2\pi} \tag{145}$$

the usual Hawking temperature consistent with (132).

One of the more useful features of the tunnelling approach is that it can be extended straightforwardly to particles of any spin [100–102]. A black hole is expected to radiate all types of particles as though it were a black body at temperature $t = \kappa/2\pi$, and so the emission spectrum of a black hole is expected to contain particles of all spins [103, 104]. Prior to the advent of the tunnelling method there were almost no calculations for fermion emission from black holes, apart from a model in 2 dimensions [105, 106] and an approach using a generalized tortoise coordinate transformation (GTCT) [107–110].

Consider first Rindler spacetime. Instead of the metric (79), set $\kappa^2\mathsf{X}^2 = \kappa^2\mathsf{Z}^2 - 1$ to obtain

$$\begin{aligned} ds^2 &= -f(\mathsf{Z})d\mathsf{T}^2 + \frac{d\mathsf{Z}^2}{g(\mathsf{Z})} + \sum_{i=x}^{y} dW^i dW^i \qquad f(\mathsf{Z}) = \kappa^2\mathsf{Z}^2 - 1 \\ g(z) &= \frac{\kappa^2\mathsf{Z}^2 - 1}{\kappa^2\mathsf{Z}^2} \end{aligned} \tag{146}$$

so chosen for its convenience in extending the technique to non-rotating black holes. The Dirac equation is:

$$i\gamma^a D_a\psi + \frac{m}{\hbar}\psi = 0 \tag{147}$$

where:

$$D_a = \partial_a + \Omega_a \qquad \Omega_a = \frac{1}{2}i\Gamma^{\alpha\ \beta}_{\ a}\Sigma_{\alpha\beta} \qquad \Sigma_{\alpha\beta} = \frac{1}{4}i[\gamma^\alpha, \gamma^\beta] \tag{148}$$

and the γ^a matrices satisfy $\{\gamma^a, \gamma^b\} = 2g^{ab} \times \mathbb{I}$ and can be chosen as

$$\gamma^{\mathrm{t}} = \frac{1}{\sqrt{f(\mathsf{Z})}}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \gamma^{\mathrm{W}_1} = \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix} \qquad \gamma^{\mathrm{W}_2} = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}$$
$$\gamma^{\mathrm{Z}} = \sqrt{g(\mathsf{Z})}\begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix} \tag{149}$$

and the $\sigma' s$ are the Pauli Sigma Matrices.

If the detector measures spin along the Z direction (the direction of the accelerating observer) then the ansatz

$$\psi_{\uparrow}(\mathsf{T}, \mathsf{W}_1, \mathsf{W}_2, \mathsf{Z}) = \begin{bmatrix} \mathsf{A}(\mathsf{T}, \mathsf{W}_1, \mathsf{W}_2, \mathsf{Z})\xi_{\uparrow} \\ B(\mathsf{T}, \mathsf{W}_1, \mathsf{W}_2, \mathsf{Z})\xi_{\uparrow} \end{bmatrix} \exp\left[\frac{i}{\hbar} I_{\uparrow}(\mathsf{T}, \mathsf{W}_1, \mathsf{W}_2, \mathsf{Z})\right] \tag{150}$$

$$\psi_{\downarrow}(\mathsf{T}, \mathsf{W}_1, \mathsf{W}_2, \mathsf{Z}) = \begin{bmatrix} C(\mathsf{T}, \mathsf{W}_1, \mathsf{W}_2, \mathsf{Z})\xi_{\downarrow} \\ D(\mathsf{T}, \mathsf{W}_1, \mathsf{W}_2, \mathsf{Z})\xi_{\downarrow} \end{bmatrix} \exp\left[\frac{i}{\hbar} I_{\downarrow}(\mathsf{T}, \mathsf{W}_1, \mathsf{W}_2, \mathsf{Z})\right] \tag{151}$$

will ensure that $\psi_{\uparrow/\downarrow}$ are eigenvectors

$$S^Z = e^z_i S^i = \sqrt{g(\mathsf{Z})}\frac{\hbar}{2}\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}$$

where

$$S^i = \frac{\hbar}{2}\begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}$$

and $\xi_{\uparrow/\downarrow}$ are the eigenvectors of σ^3. At the location of the Rindler observer $g(\mathsf{Z}) = 1$ so the spin in the Z-direction (and thus the helicity) will be measured as $\pm\hbar/2$.

Applying the WKB approximation (150), the Dirac Equation becomes

$$-B\left(\frac{1}{\sqrt{f(\mathsf{Z})}}\partial_T I_{\uparrow} + \sqrt{g(\mathsf{Z})}\partial_{\mathsf{Z}} I_{\uparrow}\right) + Am = 0 \qquad B\left(\partial_{\mathrm{W}_1} I_{\uparrow} + i\partial_{\mathrm{W}_2} I_{\uparrow}\right) = 0 \tag{152}$$

$$A\left(\frac{1}{\sqrt{f(\mathsf{Z})}}\partial_{\mathrm{T}} I_{\uparrow} - \sqrt{g(\mathsf{Z})}\partial_{\mathsf{Z}} I_{\uparrow}\right) + Bm = 0 \qquad A\left(\partial_{\mathrm{W}_1} I_{\uparrow} + i\partial_{\mathrm{W}_2} I_{\uparrow}\right) = 0 \tag{153}$$

to leading order in $\hbar$. Although A, B are not constant, their derivatives (and the components Ω_a) are all of order $O(\hbar)$ and so can be neglected to lowest order in the WKB approximation we employ.

When $m \neq 0$ the first of Eqs. (152) and (153) couple whereas they decouple when $m = 0$. Using an ansatz similar to that of (138)

$$I_{\uparrow} = -E\mathsf{T} + W(\mathsf{Z}) + P(\vec{\mathsf{W}}) \tag{154}$$

gives

$$-B\left(\frac{-E}{\sqrt{f(\mathsf{Z})}} + \sqrt{g(\mathsf{Z})}W'(\mathsf{Z})\right) + mA = 0 \qquad -B(P_{W_1} + iP_{W_1}) = 0 \tag{155}$$

$$-A\left(\frac{E}{\sqrt{f(\mathsf{Z})}} + \sqrt{g(\mathsf{Z})}W'(\mathsf{Z})\right) + mB = 0 \qquad -A(P_{W_1} + iP_{W_2}) = 0 \tag{156}$$

upon insertion into Eqs. (152, 153), considering only positive frequency contributions without loss of generality. Equations (155) and (156) both yield $(P_{W_1} + iP_{W_2}) = 0$ regardless of A or B, implying

$$P(\vec{\mathsf{W}}) = h(\mathsf{W}_1 + i\mathsf{W}_2) \tag{157}$$

where h is some arbitrary function.

Consider first $m = 0$. The reminder of Eqs. (155) and (156) then have two possible solutions

$$A = 0 \text{ and } W'(\mathsf{Z}) = W'_{+}(\mathsf{Z}) = \frac{E}{\sqrt{f(\mathsf{Z})g(\mathsf{Z})}} \tag{158}$$

$$B = 0 \text{ and } W'(\mathsf{Z}) = W'_{-}(\mathsf{Z}) = \frac{-E}{\sqrt{f(\mathsf{Z})g(\mathsf{Z})}} \tag{159}$$

corresponding to motion away from (+) and toward (−) the horizon, chosen to be at $\mathsf{Z} = 1/\kappa$.

It is straightforward to show that the solution $[A, 0, 0, 0]$ is an eigenvector of $\gamma^5 = i\gamma^T\gamma^{W_1}\gamma^{W_2}\gamma^Z$ with negative eigenvalue. Its spin and momentum vectors are therefore opposite, consistent with the fact that the particle is moving toward the horizon with spin up. It will also have negative helicity since $\hat{p}_r$ is in the negative r-direction, consistent with left-handed chirality. The solution $[0, 0, B, 0]$ is also an eigenvector of γ^5 with positive eigenvalue; its spin and momentum vectors are therefore in the same direction, consistent with the particle being spin up and moving away from the horizon, and with positive helicity (right handed chirality).

Hence with the Rindler horizon at $\mathsf{Z} = 1/\kappa$, the $(\pm)$ cases correspond to outgoing/incoming solutions of the same spin. Neither of these cases is an antiparticle solution since positive frequency modes have been assumed in the ansatz. The ratio of emission probabilities is

$$\Gamma \propto \frac{\text{Prob}[out]}{\text{Prob}[in]} = \frac{\exp[-2(\text{Im}W_+ + \text{Im}\,h)]}{\exp[-2(\text{Im}W_- + \text{Im}\,h)]} = \exp[-2(\text{Im}\,W_+ - \text{Im}W_-)$$
$$= \exp[-4\text{Im}W_+] \tag{160}$$

using reasoning similar to the scalar case. Note that, other than the trivial $P = 0$ solution, $P(\vec{\text{W}})$ must be complex and so yields a contribution for both incoming and outgoing solutions. However these are the same and so cancel out in the ratio.

Solving for W_+ by integrating around the pole

$$W_+(\mathsf{Z}) = \int \frac{Ed\mathsf{Z}}{\sqrt{f(\mathsf{Z})g(\mathsf{Z})}} = \frac{\pi i E}{\sqrt{g'(\mathsf{Z}_0)f'(\mathsf{Z}_0)}} = \frac{\pi E}{2\kappa} \tag{161}$$

yields the resulting tunnelling probability $\Gamma = \exp[-\frac{2\pi}{\kappa}E]$ and so recovering

$$T_H = \frac{\kappa}{2\pi} \tag{162}$$

which is the Unruh temperature.

For the massive case the first of Eqs. (155) and (156) no longer decouple. Eliminating the function $W'(\mathsf{Z})$ from the two equations gives

$$\frac{A}{B} = \frac{-E \pm \sqrt{E^2 + m^2 f(\mathsf{Z})}}{m\sqrt{f(\mathsf{Z})}}$$

where

$$\lim_{Z\to Z_0}\left(\frac{-E + \sqrt{E^2 + m^2 f(\mathsf{Z})}}{m\sqrt{f(\mathsf{Z})}}\right) = 0 \qquad \lim_{Z\to Z_0}\left(\frac{-E - \sqrt{E^2 + m^2 f(\mathsf{Z})}}{m\sqrt{f(\mathsf{Z})}}\right) = -\infty \tag{163}$$

after some manipulation. Hence either $\frac{A}{B} \to 0$ or $\frac{A}{B} \to -\infty$ at the horizon, implying either $A \to 0$ or $B \to 0$.

Solving the first of Eqs. (155, 156) for each of these possibilities yields

$$W'(\mathsf{Z}) = \begin{cases} W'_+(\mathsf{Z}) = \frac{E}{\sqrt{f(\mathsf{Z})g(\mathsf{Z})}}\frac{\left(1+\frac{A^2}{B^2}\right)}{\left(1-\frac{A^2}{B^2}\right)} & A \to 0 \\ W'_-(\mathsf{Z}) = \frac{-E}{\sqrt{f(\mathsf{Z})g(\mathsf{Z})}}\frac{\left(1+\frac{B^2}{A^2}\right)}{\left(1-\frac{B^2}{A^2}\right)} & B \to 0 \end{cases} \tag{164}$$

Since the extra contributions vanish at the horizon, the result of integrating around the pole for W in the massive case is the same as the massless case and the Unruh temperature is recovered for the fermionic Rindler vacuum.

The spin-down case proceeds in a manner fully analogous to the spin-up case discussed above, apart from some changes of sign. Due to the statistical nature of the heat bath there should be no net angular momentum is imparted to the accelerating detector—on average there are as many spin up particles as spin down particles detected. For both the massive and massless cases the Unruh temperature (162) is obtained, implying that both spin up and spin down particles are emitted at the same rate.

The black hole case follows the same procedure as the Rindler case. It is instructive to see how this works in Painlevé coordinates that are not singular at the horizon, indicating that the tunnelling emission rate is not an artifact of a bad choice of coordinates.

The generic static black hole metric

$$ds^2 = -f(r)d\tilde{t}^2 + \frac{dr^2}{g(r)} + r^2(d\theta^2 + \sin^2(\theta)d\phi^2) \tag{165}$$

under the transformation

$$\tilde{t} = t - \int \sqrt{\frac{1-g(r)}{f(r)g(r)}}dr \tag{166}$$

becomes

$$ds^2 = -f(r)dt^2 + 2\sqrt{\frac{f(r)}{g(r)}}\sqrt{1-g(r)}drdt + dr^2 + r^2d\Omega^2 \tag{167}$$

which is the Painlevé form of a spherically symmetric metric. In these coordinates the spatial geometry is flat at any fixed time, and at any fixed radius the boundary geometry for the Painlevé metric is exactly the same as that of the unaltered black hole metric (165). This form of the metric is a very convenient to employ in black hole tunnelling calculations, since the imaginary part of the action for the incoming solution is zero.

Choosing

$$\gamma^t = \frac{1}{\sqrt{f(r)}}\begin{pmatrix} 0 & 1+\sqrt{1-g(r)}\sigma^3 \\ -1+\sqrt{1-g(r)}\sigma^3 & 0 \end{pmatrix}$$

$$\gamma^r = \sqrt{g(r)}\begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix} \qquad \gamma^\theta = \frac{1}{r}\begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix} \qquad \gamma^\phi = \frac{1}{r\sin\theta}\begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}$$

as the representation for the γ matrices, it is easy to show that

$$\gamma^5 = i\gamma^t\gamma^r\gamma^\theta\gamma^\phi = \sqrt{\frac{g(r)}{f(r)}}\frac{1}{r^2\sin\theta}\begin{pmatrix} -1-\sqrt{1-g(r)}\sigma^3 & 0 \\ 0 & +1-\sqrt{1-g(r)}\sigma^3 \end{pmatrix}$$

is the expression for γ^5.

Analogous to the Rindler case, spin will be measured along the r-direction—positive/negative-spin is along the positive/negative r-directions respectively. This suggests the ansatz

$$\begin{aligned} \psi_\uparrow(t,r,\theta,\phi) &= \begin{bmatrix} A(t,r,\theta,\phi)\xi_\uparrow \\ B(t,r,\theta,\phi)\xi_\uparrow \end{bmatrix} \exp\left[\frac{i}{\hbar}I_\uparrow(t,r,\theta,\phi)\right] \\ \psi_\downarrow(t,x,y,z) &= \begin{bmatrix} C(t,r,\theta,\phi)\xi_\downarrow \\ D(t,r,\theta,\phi)\xi_\downarrow \end{bmatrix} \exp\left[\frac{i}{\hbar}I_\downarrow(t,r,\theta,\phi)\right] \end{aligned} \tag{168}$$

and $\xi_{\uparrow/\downarrow}$ the eigenvectors of σ^3 as before. Employing further the ansatz

$$I_\uparrow = -Et + W(r) + J(\theta,\phi) \tag{169}$$

yields from the Dirac equation

$$B\left(\frac{1}{\sqrt{f(r)}}\left(1+\sqrt{1-g(r)}\right)E - \sqrt{g(r)}W'(r)\right) + Am = 0 \tag{170}$$

$$-A\left(\frac{1}{\sqrt{f(r)}}\left(1-\sqrt{1-g(r)}\right)E + \sqrt{g(r)}W'(r)\right) + Bm = 0 \tag{171}$$

along with the equation

$$\left(J_\theta + \frac{1}{\sin\theta}iJ_\phi\right) = 0 \tag{172}$$

implying that $J(\theta,\phi)$ must be a complex function. The same solution for J is obtained for both the outgoing and incoming cases, analogous to the Rindler case.

For $m = 0$ Eqs. (169) and (17) have two possible solutions:

$$W'(r) = W'_\pm(r) = \frac{\pm E\left(1 \pm \sqrt{1-g(r)}\right)}{\sqrt{f(r)g(r)}} \tag{173}$$

for $A = 0$ and $B = 0$ respectively with $W_{+/-}$ corresponding to outward/inward solutions. In these coordinates W'_+ has a pole at the horizon whereas but W'_- does not. Instead it has a well defined limit at the horizon, implying its imaginary part is zero, yielding Prob $[in] = 1$. The overall tunnelling probability is thus

$$\Gamma \propto \text{Prob}\,[out] \propto \exp[-2\Im W_+] = \exp\left[-\frac{4\pi E}{\sqrt{g'(r_0)f'(r_0)}}\right] \tag{174}$$

and so the Hawking Temperature

$$T_H = \frac{\sqrt{f'(r_0)g'(r_0)}}{4\pi} \tag{175}$$

is recovered. The same result holds for $m \neq 0$, as can be seen by solving (169) and (170) for A and B: either $A \to 0$ as $r \to r_0$ or $B \to 0$ as $r \to r_0$, and the manipulations then follow the same path as for the $m \neq 0$ Rindler case.

The tunnelling approach is one of the few methods that can actually calculate spin-1/2 fermion radiation [100, 101], and can be extended to particles of higher-spin [102]. In this sense it is superior to the Wick rotation methods employed above, which can only model a black hole at equilibrium with a scalar particle heat bath. However one of the assumptions of this semi-classical calculation is to neglect any change of angular momentum of the black hole due to the spin of the emitted particle. This should be a good approximation for black holes masses much larger than the Planck mass. Statistically, particles of opposite spin will be emitted in equal numbers, yielding no net change in the angular momentum of the black hole, though second-order statistical fluctuations will be present.

6.2 Black Hole Entropy

One of the puzzles of black hole thermodynamics is the meaning of black hole entropy. In non-gravitational thermodynamic systems (with which we have 'everyday' experience), entropy refers to the degree of disorder in a system. It is a measure of the number of specific ways in which a thermodynamic system may be arranged, or the degree to which the probability of a given system is distributed over its different possible microstates; the more states available, the greater the entropy of the system. As noted above in Eq. (122), it a logarithmic measure of the number of states:

$$S_{\text{ent}} = -k_B \sum_i p_i \ln(p_i) \tag{176}$$

where k_B is Boltzmann's constant and p_i is the probability that the system is in the i-th microstate [111]. If occupation of any state is equally probable, then $p_i = 1/N$ for each i, and so

$$S_{\text{ent}} = -k_B \ln(N) \tag{177}$$

which is the more familiar definition from introductory textbooks.

For non-gravitational systems the mircostates are understood in terms of the degrees of freedom of the system: the positions and velocities of the molecules in a gas, for example. But for a black hole it is not at all clear what the degrees of freedom are, and a huge literature has been devoted to this subject. Indeed, for approximately two decades after the introduction of black hole entropy the relation $S_{bh} = A/4$ from the laws of black hole mechanics was derivable only by integrating the formula $dE = TdS$ from the first law of thermodynamics (32) using the temperature-mass relation $T = \kappa/2\pi$. This only gave the entropy up to an additive constant, which could be fixed by relating it to the black hole instanton in the Euclidean path integral [112], itself a rather mysterious procedure.

Further clues came upon studying the pair production of black holes [113–116]. If indeed black hole entropy counts degrees of freedom, then the number of quantum states of a black hole is given by

$$N_{bh} = \exp(S_{bh}) \tag{178}$$

and is finite. In the pair-production of black holes due to some process (e.g. cosmological expansion [115], one would expect the production rate to be proportional to the number of independent black hole states produced, and so the pair production amplitude should be multiplied by the factor N_{bh}, just as particle/antiparticle pairs produced in a background electromagnetic field have a production rate that grows as the number of particle species produced via the Schwinger process [117]. This was shown to take place in a broad variety of circumstances for a wide range of black holes [5, 113–116, 118–120], providing strong circumstantial evidence for the validity of (177).

However there is still no consensus as to what the physical degrees of freedom are for a black hole, with a number of interpretations remaining open.

Internal states of matter and gravity The laws of black hole mechanics suggest that a black hole with particular values of M, J, and Q can be formed in many ways, and that the entropy of the black hole is given by the internal states of the various matter and gravitational configurations that could have formed it [121].

Brick Wall Model The idea here is that there is a gas of quanta forming a thermal atmosphere near the black hole, whose entropy is the entropy of the black hole [122]. This atmosphere is prevented from making contact with the horizon via a boundary condition referred to as a "brick wall" [123]. The entropy is found to be proportional to the horizon area, but with infinite coefficient, and so a renormalization of sorts is required here as well. This approach works in any dimension [124].

Entropy of entanglement An external observer cannot access the interior state of the black hole, so the only meaningful exterior quantum states are reduced states for which the internal degrees of freedom have been traced out. In general the degrees of any quantum field external to the horizon should be entangled with those inside the black hole, and so even if the global state of the field was pure (and therefore of zero entropy), such reduced states will be mixed, with non-zero entropy. This entanglement entropy is indeed proportional to the horizon area (as required), but with a coefficient that diverges at short distance, requiring renormalization by physical arguments [125, 126].

Noether Charge A geometric argument, valid in theories of gravity more general than general relativity (e.g. that include higher-curvature terms in the action), has been put forward that indicates black hole entropy is the Noether charge of the diffeomorphism symmetry. When the gravitational action is of first order in the curvature (as in general relativity) the relation $S = A/4$ is reproduced, but in higher-curvature theories this formula is modified [127].

Horizon gravitational states Perhaps the degrees of freedom are not inside the black hole, but instead reside on its horizon. The algebra of constraints in general relativity at a black hole horizon, regarded as a boundary, acquires a central extension that contains a natural Virasoro sub-algebra; this can be used to obtain the correct entropy of a black hole in any dimension via Cardy's formula [128].

String Excitations The idea here is that string theory is the underlying theory of quantum gravity and that black hole entropy counts the number of states or excitations of a fundamental string. The first direct demonstration of this idea was in the context of five-dimensional extreme black holes [129]. The approach is to begin with an extremal black hole and compute its entropy. In string theory, extremal black holes are strong coupling analogs of BPS states, and by reducing the string coupling g one obtains a weakly coupled system of strings (and branes) with the same charge. A count of the number of BPS states in the system at this weak coupling yields $N_{BPS} = \exp(S_{bh})$, providing (albeit in this restricted context) a microscopic derivation of (177). Unlike previous attempts to explain black hole entropy, the counting of states is carried out in flat spacetime, where there is no horizon. This calculation was subsequently extended to include both rotation and near-extremal black holes [130], and to include quantum corrections [131].

Quantum Geometric Excitations In the context of Loop Quantum Gravity [132, 133] the entropy of a black hole can be understood as being related to the number of quantum black hole degrees of freedom described by a Chern-Simons field theory on the horizon [134]. The approach here is to isolate a sector of the classical theory corresponding to black holes and find its associated phase space description. Upon quantizing this phase space, the quantum states describing the horizon geometry can be identified; their statistical mechanical entropy yields the expected entropy of the black hole. This approach is not restricted to extremal or near-extremal black holes, though it does require choosing a particular parameter (the Immirzi parameter) appropriately.

It should be clear from the above that there is no consensus as to what the underlying degrees of freedom are for a black hole. Our present understanding of this circumstantial evidence has yet to lead to a resolution of the most baffling of paradoxes associated with black holes: the Information Paradox, and its descendant, the Firewall problem.

7 The Information Paradox

The no-drama assumption is tantamount to assuming the horizon of the black hole is 'information-free': that field modes with wavelengths $l_p \ll \lambda \lesssim M$ are described by curved space-time quantum field theory on the black hole background. We have seen that the notion of a particle is contingent on what the vacuum, or 'empty space' is taken to be. Modes with wavelengths λ smaller than the curvature scale R will yield differing definitions of particle quanta, but the difference between definitions will consist of about 1 quanta for wavelengths as large as the curvature scale, $\lambda \sim R$. For black holes $R \sim M$ and so different particle-definitions will differ by about 1 quanta for wavelength $\lambda \sim M$, but by a negligible number of quanta for wavelengths $\lambda < kM$, where $k \sim 10^{-1}$. Hence a robust notion of vacuum, or empty space, is well defined for such wavelengths: no modes are present for $l_p \ll \lambda < kM$.

In illustrating the information paradox, the assumptions **A1**–**A3** will be assumed to hold, and so it will be sufficient to employ the metric (2). At late times this can describe a collapsing black hole, formed perhaps by a shell of matter or a ball of dust. Since this metric has a singularity at $r = 0$, spacelike slices must avoid this singularity or else the niceness conditions will not hold, undermining the calculations yielding black hole radiation.

These slices will be chosen to obey the following criteria [36]

Outside Σ_O Slices are at t = constant for $r > 4M$; this portion is outside the black hole

Inside Σ_I Slices are at r = constant for $M/2 < r < 3M/2$; this portion is inside the black hole. This segment will be smoothly extended to $r = 0$ at early times before the singularity has formed

Connect Σ_C The preceding two segments are joined by a smooth connecting segment $\mathcal{C}$; this portion crosses the event horizon. The spatio-temporal dimensions of this segment are both of size $\sim M$

thereby respecting the niceness conditions. Nowhere does the slice $\Sigma(t, r, \mathcal{C}) = \Sigma_O \cup \Sigma_I \cup \Sigma_C$ go near the singularity, and the appropriate connecting segment can always be appropriately chosen.

Each slice is contingent on the choice of time, and it is essential that the slices smoothly evolve into each other (not merging or crossing). If $\Sigma_0 = \Sigma(t_0, r_0, \mathcal{C}_0)$ describes an initial slice, then a subsequent slice is $\Sigma_1 = \Sigma(t_1, r_1, \mathcal{C}_1) = \Sigma(t_0 + \delta t, r_0 + \delta r, \mathcal{C}_0 + \delta\mathcal{C})$. Increasing t_0 and r_0 respectively correspond to forward evolution outside and inside the black hole. The geometry of the connecting segments can be taken to be the same for all slices provided $\delta r \ll M$ is sufficiently small. This has the consequence that the constant-r segments of the slices become increasingly longer since the constant-t parts outside the horizon are further in the future.

Foliating the space-time with these slices $\Sigma(t, r, \mathcal{C})$ along a unit timelike normal u^a (with zero shift vector) indicates that only the connecting segment C becomes stretched. The segment Σ_O advances forward in time with lapse function $N = \sqrt{1 - 2M/r}$, and the segment Σ_I will remain unchanged in its intrinsic geometry provided δr is sufficiently small, though its length increases. The connecting segment Σ_C must therefore stretch since it has to evolve to cover the additional part of Σ_I and the connecting segment of the next slice.

The slicing is therefore time-dependent since it must cover both the outside and inside of the black hole; it therefore depends on the temporal coordinate r on the inside of the hole. The Kretschmann scalar and all other measures of curvature are small for all slices in this evolution. It is the time-dependent stretching of Σ_C, and not large-curvature effects, that yields particle production. This choice of slicing (and its resultant stretching) is a necessary consequence of the existence of the black hole. It is not an option in flat space-time: any such choice will necessarily force the slices to eventually become null and then timeline at some point in the evolution. Since spatial and temporal directions interchange roles for a black hole, the slices always remain space like. The stretching of the slices is localized to a region in the vicinity of the horizon: a field mode in this region will become increasingly stretched to longer wavelengths, generating particles for as long as the assumptions concerning the use of (2) are valid.

The particle creation scenario proceeds along the following lines. The quantum state on the initial slice Σ_0 is that of the matter field $|\Phi(t, r)\rangle$ that will later form the black hole. This can be taken to be a sharply-peaked wavepacket that in the classical limit describes a shell of collapsing matter. Prior to formation of the black hole no particles are created since the entire slice is outside of the black hole. Upon formation of the black hole the quantum state is $|\Phi\rangle$. On some subsequent slice there will be sufficient stretching of region Σ_C to create a pair of quanta of sufficiently short wavelength $\lambda = 2\pi/k$. The matter state $|\Phi\rangle$ will be localized in Σ_I and so will be negligibly affected by this process. The quantum state on this slice will therefore be

$$|\Psi\rangle_1 \approx \frac{1}{\sqrt{2}} |\Phi\rangle_{\mathrm{I}_1} \otimes \left(|0_k\rangle^+_{\mathrm{I}_1} |0_{-k}\rangle^-_{\mathrm{O}_1} + |1_k\rangle^+_{\mathrm{I}_1} |1_{-k}\rangle^-_{\mathrm{O}_1} \right) \tag{179}$$

using, for example, (117) near the horizon, where the subscripts I/O refer to inside/outside the horizon. An outside observer has no access to the states inside, and so must employ the reduced density matrix

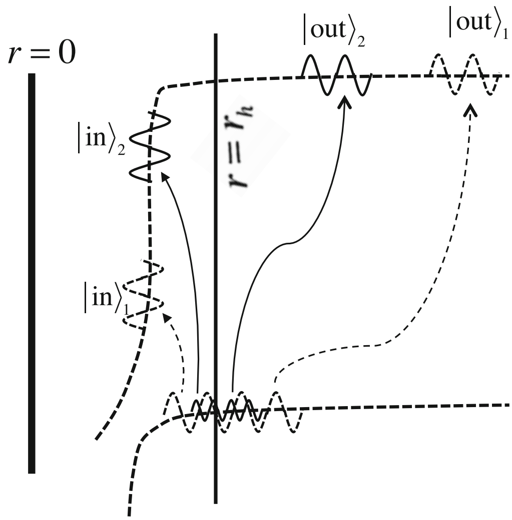

Fig. 17 Mode evolution: Long wavelength modes (*dashed*) and short wavelength modes (*solid*) created on the same slice generate created pairs differently. The long wavelength modes get distorted earlier, creating an entangled pair first ($|\text{outside}\rangle_1, |\text{inside}\rangle_1$). The short wavelength modes take longer to distort, and so the pairs they create ($|\text{outside}\rangle_2, |\text{inside}\rangle_2$) emerge later

$$\rho_{O_1} = \text{tr}_I[|\Psi\rangle\langle\Psi|] = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \tag{180}$$

in describing the physics of the outside state. The entanglement entropy of the outside state with the state inside the black hole is

$$S_{\text{ent}} = -\text{tr}\left[\rho_{O_1} \ln \rho_{O_1}\right] = 2 \times \frac{1}{2} \ln 2 = 2 \ln 2 \tag{181}$$

On the next slice the process repeats. The inside state $|\psi\rangle_I$ moves into region Σ_I and the matter state $|\Phi\rangle_{I_1}$ moves deeper into this region, becoming the state $|\Phi\rangle_{I_2}$. The outside state $|\psi\rangle_O$ moves into region Σ_O. The additional stretching of region Σ_C creates a new pair, yielding the state

$$\begin{aligned} |\Psi\rangle_2 \approx \frac{1}{\sqrt{2}} |\Phi\rangle_{I_2} \otimes (|0_k\rangle^+_{I_1} |0_{-k}\rangle^-_{O_1} + |1_k\rangle^+_{I_1} |1_{-k}\rangle^-_{O_1}) \otimes (|0_k\rangle^+_{I_2} |0_{-k}\rangle^-_{O_2} \\ + |1_k\rangle^+_{I_2} |1_{-k}\rangle^-_{O_2}) \end{aligned} \tag{182}$$

assuming the stretching region has negligible influence on the first pair of states. The reduced density matrix is now

$$\rho_{O_2} = \mathrm{tr_I}[|\Psi\rangle\langle\Psi|] = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{pmatrix} \tag{183}$$

and

$$S_{ent} = -\mathrm{tr}\left[\rho_{O_2} \ln \rho_{O_2}\right] = 4 \times \frac{1}{4} \ln 4 = 4 \ln 2 \tag{184}$$

is entanglement entropy of the outside state on this next slice (Fig. 17).

After n steps the quantum state is

$$|\Psi\rangle_n \approx \frac{1}{\sqrt{2}} |\Phi\rangle_{I_n} \prod_{m=1}^{n} \left[\otimes (|0_k\rangle^+_{I_m} |0_{-k}\rangle^-_{O_m} + |1_k\rangle^+_{I_m} |1_{-k}\rangle^-_{O_m}) \right] \tag{185}$$

and the reduced density matrix is $\rho_{O_n} = \mathrm{diag}(2^{-n}, 2^{-n}, \ldots, 2^{-n})$, yielding

$$S_{ent} = -\mathrm{tr}\left[\rho_{O_n} \ln \rho_{O_n}\right] = -2^n \times \frac{1}{2^n} \ln 2^{-n} = n \ln 2 \tag{186}$$

for the entanglement entropy.

The quantity n is an enormously large number, as can be seen from energy conservation. Suppose each quanta contains the same amount of energy in units of the Planck mass. For a mass M black hole, the energy per quanta is $E_Q = \sigma(M_p/M)M_p \ll M$ where σ is a parameter of order unity. The total mass of the black hole is $nE_Q = M$ implying $n = (M/M_P)^2/\sigma$. For a solar mass black hole $n \sim (2 \times 10^{30}/(2 \times 10^{-8}))^2 = 10^{76}$. There is no upper bound on n since in principle the mass of the black hole can be arbitrarily large, though one might argue that the largest black hole possible is constrained by the mass of the universe, $M_U = 10^{52}$ kg, giving $n \leq 10^{120}$.

Expression (186) contains the nub of the information paradox: the entanglement of the radiation state $|\psi\rangle_O$ grows without bound as more pairs are created. Eventually it must terminate, of course, as the radiation cannot contain more energy than was in the initial quantum state $|\Phi\rangle$ (or the mass M of the black hole). Indeed the niceness conditions will (at least) fail to hold once $M \sim M_p$, since the Kretschmann scalar (for example) $K = 48M^2/r^6 \to M_p^2/l_p^6 = l_p^4$ becomes too large for semi-classicality to hold. But this situation evidently leaves us with one of three unpleasant choices [135].

Remnants Something terminates the evolution once $M \sim M_r \gtrsim M_p$. The object of mass M_r is called a remnant. Its number of possible states must be at least as large as the unbounded number n, since its entanglement with the final state of radiation is $n \ln 2$. Hence it is an n-fold degenerate state of finite size and energy.

Apart from this being a quantum state unlike any normal quantum state we know of, since n is arbitrarily large, it will generate divergent loop corrections in particle physics. No matter how small the coupling of the remnant to normal matter, as long as it is finite, each state of the remnant gives a finite loop correction to any scattering process, and the sum over n yields a divergence. If some other physics yields an upper bound to n (say $n \leq 10^{120}$) then the couplings of all remnant states to normal matter must be kept extremely tiny so as not to significantly modify known scattering process in particle physics.

Mixedness The black hole completely evaporates away, with all of its energy contained in the radiation. However the radiation has entanglement entropy $n \ln 2$, but there is no quantum state with which it is entangled. Hence the evolution has taken us from the pure state $|\Phi\rangle$ to a mixed state described only by the density matrix ρ_{O_n} and not by any quantum wave function. There is no unitary matrix evolving $\rho_{\Phi} = |\Phi\rangle\langle\Phi|$ to ρ_{O_n} or vice versa, so this option violates unitarity.

Bleaching Some process prevents the information associated with the state $|\Phi\rangle$ from entering the black hole or even from forming it in the first place. This begs the question as to what process this might be. It seems straightforward that an astrophysical black hole can absorb an electron, so if this option holds then some new kind of physics—some kind of drama—must be present at the horizon to either prevent this from happening or to decouple the information in this state from its energy and angular momentum (whatever that means). Of course if black holes never form, then the putative astrophysical black hole is not really a black hole, but rather some new form of dark matter whose interactions contain some repulsive effect to counteract any possible gravitational pull toward collapse.

Of course the assumptions leading to (186) do not hold in full generality. The mass of the black hole is not constant, the created pairs might interact with each other, and the state $|\Phi\rangle$ could have some interaction with the created pairs. Could these small corrections invalidate the argument in some way, allowing some escape from one of the these three unpleasant choices?

Before proceeding it is important to note that this resolution itself will involve some strange physics. The state $|\Phi\rangle$ moves a distance M deeper into region Σ_I as each pair is created. This is not too far at first, perhaps, but after $n/2$ pairs, the matter is located at a distance $GM(M/M_P)^2/c^2$ which is about 3×10^{76} km for a solar mass black hole from the pair-creation region. Likewise the earlier created quanta move away at the same rate, so most of them are very distant from this region as well. Of course any given created pair will be most influenced by its recent predecessor pair. But it is the stretching of modes that creates a pair, and this stretching also pushes away the predecessors, minimizing their influence.

However there will still be some correction effects, if only due to gravitational instantons. Corrections due to instantons are to leading order proportional to $\exp\{-S_{\text{inst}}[g]\}$ where $S[g] \sim (M/M_P)^2$ is the Euclidean action of the metric (126).

This is a very tiny effect, but it could perhaps diminish the entanglement (186) between the outside quanta and the inside state. If it could do so completely, then the arguments above are invalidated, and the outside state would be a pure state containing the information in the original $|\Phi\rangle$ and the inside created quanta.

Unfortunately this proves not to be the case [36]. Suppose at the j-th step of the process, the full quantum state is not of the form (185) but is rather

$$|\tilde{\Psi}\rangle_j = |\tilde{\Psi}\rangle^{(+)}_{j-1}|\Xi\rangle^{(+)}_j + |\tilde{\Psi}\rangle^{(-)}_{j-1}|\Xi\rangle^{(-)}_j \tag{187}$$

where

$$|\Xi\rangle^{(\pm)}_j = \frac{1}{\sqrt{2}}\left(|0_k\rangle^+_{\mathrm{I}_j}|0_{-k}\rangle^-_{\mathrm{O}_j} \pm |1_k\rangle^+_{\mathrm{I}_j}|1_{-k}\rangle^-_{\mathrm{O}_j}\right) \tag{188}$$

and the state $|\tilde{\Psi}\rangle^{(+)}_{j-1}$ can be expressed as

$$|\tilde{\Psi}\rangle^{(\pm)}_{j-1} = \sum_{l,m} \alpha_{l,m}|\tilde{\tilde{\psi}}^{\pm}_l(\Phi,\mathrm{I})\rangle|\chi_m(\mathrm{O})\rangle = \sum_m \gamma_m|\tilde{\psi}^{\pm}_m(\Phi,\mathrm{I})\rangle|\chi_m(\mathrm{O})\rangle \tag{189}$$

where $|\tilde{\psi}^{\pm}_m(\Phi,\mathrm{I})\rangle$ and $|\chi_m(\mathrm{O})\rangle$ are orthonormal bases for the respective inside and outside states, and a unitary transformation has been applied to obtain the second equality. The newly created state is spanned by $\Xi^{(\pm)}_j$. Assuming locality, the outside states $|\psi\rangle^-_{\mathrm{O}_j}$ generated in earlier stages of the evolution are not affected by the current proposed step (187); hence the basis $|\chi_m(\mathrm{O})\rangle$ remains unchanged.

The proposed correction (187) to the Hawking process is a deformation of the original process (185), in which $|\tilde{\Psi}\rangle^{(-)}_{j-1} = 0$. Unitarity implies that

$$|\,{}^{(+)}_{j-1}\langle\tilde{\Psi}|\tilde{\Psi}\rangle^{(+)}_{j-1}\,|^2 + |\,{}^{(-)}_{j-1}\langle\tilde{\Psi}|\tilde{\Psi}\rangle^{(-)}_{j-1}\,|^2 = 1 \tag{190}$$

since the $\Xi^{(\pm)}_j$ are orthonormal. If the corrections to (185) due to (187) are small, then

$$|\,{}^{(-)}_{j-1}\langle\tilde{\Psi}|\tilde{\Psi}\rangle^{(-)}_{j-1}\,| < \epsilon \tag{191}$$

where $\epsilon \ll 1$. To say that corrections to the Hawking radiation process are small is to say that one obtains the state $\Xi^{(+)}_j$ with high probability when the pair is created, and that the orthogonal state $\Xi^{(-)}_j$ is observed with low probability.

There are thus three subsystems at stage-j of the evolution: (i) the outside state O_{j-1} of all previously emitted outside quanta, (ii) the inside state $|\tilde{\psi}(\Phi,\mathrm{I})\rangle_{j-1}$

consisting of the original matter state and previously-created inside partner quanta, modified perhaps by previous interactions, and (iii) the newly created pair described by the state $|\Xi\rangle_j$, spanned by the basis $|\Xi\rangle_j^{(\pm)}$. The reduced density matrix for this newly created pair is

$$\begin{aligned}\rho_{\Xi_j} &= \mathrm{tr}_{|\tilde{\psi}(\Phi,\mathrm{I})\rangle_{j-1},\mathrm{O}_{j-1}}\left[|\tilde{\Psi}\rangle_{j\ j}\langle\tilde{\Psi}|\right] = \begin{pmatrix} {}^{(+)}_{j-1}\langle\tilde{\Psi}|\tilde{\Psi}\rangle^{(+)}_{j-1} & {}^{(+)}_{j-1}\langle\tilde{\Psi}|\tilde{\Psi}\rangle^{(-)}_{j-1} \\ {}^{(-)}_{j-1}\langle\tilde{\Psi}|\tilde{\Psi}\rangle^{(+)}_{j-1} & {}^{(-)}_{j-1}\langle\tilde{\Psi}|\tilde{\Psi}\rangle^{(-)}_{j-1}\end{pmatrix} \\ &= \begin{pmatrix} 1-\epsilon_-^2 & \epsilon_{+-} \\ \epsilon_{+-}^* & \epsilon_-^2 \end{pmatrix}\end{aligned} \tag{192}$$

where by (191), $|\epsilon_{+-}|<\epsilon$ and $|\epsilon_-|<\epsilon$. The eigenvalues of this matrix are $\frac{1}{2}(1\pm\sqrt{1+4(|\epsilon_{+-}|^2-\epsilon_-^2)}$ to this order, yielding

$$S(\Xi_j)_{ent} = \left(|\epsilon_{+-}|^2-\epsilon_-^2\right)\ln\left(\frac{e}{|\epsilon_{+-}|^2-\epsilon_-^2}\right)+\cdots<\epsilon^2\ln\epsilon<\epsilon \tag{193}$$

for the entanglement entropy of the newly created pair. The entropy of the joint subsystem $\{\mathrm{O}_{j-1},\Xi_j\}$ is therefore

$$S(\{\mathrm{O}_{j-1},\Xi_j\})\geq|S(\{\mathrm{O}_{j-1}\})+S(\Xi_j\})|=S(\{\mathrm{O}_{j-1}\})-\epsilon \tag{194}$$

using the strong subadditivity property of entropy.

Next, tracing over everything except the inside state I_j yields

$$\begin{aligned}\rho_{\mathrm{I}_j} &= \mathrm{tr}_{|\tilde{\psi}(\Phi,\mathrm{I})\rangle_{j-1},\mathrm{O}_j}\left[|\tilde{\Psi}\rangle_{j\ j}\langle\tilde{\Psi}|\right] = \frac{1}{2}\begin{pmatrix}\left({}^{(+)}_{j-1}\langle\tilde{\Psi}|+{}^{(-)}_{j-1}\langle\tilde{\Psi}|\right)\left(|\tilde{\Psi}\rangle^{(+)}_{j-1}+|\tilde{\Psi}\rangle^{(-)}_{j-1}\right) & 0 \\ 0 & \left({}^{(+)}_{j-1}\langle\tilde{\Psi}|-{}^{(-)}_{j-1}\langle\tilde{\Psi}|\right)\left(|\tilde{\Psi}\rangle^{(+)}_{j-1}-|\tilde{\Psi}\rangle^{(-)}_{j-1}\right)\end{pmatrix} \\ &= \frac{1}{2}\begin{pmatrix}1+\Re(\epsilon_{+-}) & 0 \\ 0 & 1-\Re(\epsilon_{+-})\end{pmatrix}\end{aligned} \tag{195}$$

and so

$$S(\mathrm{I}_j)_{ent} = \ln 2-2[\Re(\epsilon_{+-})]^2 > \ln 2-2\epsilon^2 > \ln 2-2\epsilon \tag{196}$$

for the entanglement entropy of the inside partner of the newly created pair. Applying strong subadditivity to the system $\{\mathrm{O}_{j-1},\mathrm{O}_j,\mathrm{I}_j\}$ gives

$$S(\{\mathrm{O}_j\})_{ent}+S(\Xi_j)_{ent} = S(\{\mathrm{O}_{j-1}\},\mathrm{O}_j)_{ent}+S(\mathrm{O}_j,\mathrm{I}_j)_{ent} > S(\{\mathrm{O}_{j-1}\})+S(\mathrm{I}_j)_{ent} \tag{197}$$

or

$$S(\{\mathrm{O}_j\})_{ent} > S(\{\mathrm{O}_{j-1}\}) + S(\mathrm{I}_j)_{ent} - S(\Xi_j)_{ent} = S(\{\mathrm{O}_{j-1}\}) + \ln 2 - 2\epsilon \tag{198}$$

using (194), (196).

The relation (198) is very important for the information paradox. It demonstrates that the entanglement entropy of the outgoing radiation always increases by at least $\ln\ 2 - 2\epsilon$ as each new pair is created. In other words, the increase of entanglement entropy is stable, and small corrections cannot accumulate to invalidate the result (186) [36].

This is unlike the situation for radiation emitted from normal matter, in which the matter/radiation interaction necessarily increases the dimensionality of the space of entangled states to leading order in the interaction. Each emission of radiation can be entangled with the emitting atom(s) in the matter in any one of a number of orthogonal states. The data of the state of the hot matter is shared amongst many quanta of radiation, making its original state difficult to extract; the actual correlations themselves change radically from emission to emission. In contrast to this, the stretching of space-time requires that the outgoing radiation is always entangled in the same way to leading order regardless of the state of the black hole. The actual correlations themselves do not change radically from emission to emission, but at best receive only small corrections, assuming semiclassical physics is valid at and near the horizon.

7.1 Implications of the Information Paradox

To summarize: if (a) the niceness conditions admit local Hamiltonian evolution and (b) the event horizon of the black hole is information-free (or alternatively freely-falling observers do not see any unusual behaviour in high-energy processes) then the end state of evaporation of the black hole is that of a remnant or a mixed state. These two conditions imply that any outgoing mode $|\vartheta\rangle$ whose wavelength λ is within the range $l_p \ll \lambda \lesssim M$ will predominantly be a vacuum state when expanded in a Fock basis near the horizon

$$|\vartheta\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle + \cdots \tag{199}$$

with $\sum_{j>0} |\alpha_j|^2 = \varepsilon \ll 1$, since otherwise the state at the horizon would not be a vacuum state. The evolution of $|\vartheta\rangle$ must therefore agree with the standard vacuum evolution to leading order such that any corrections are constrained by (191). The entanglement entropy S_{ent} therefore increases by $\ln 2 - 2\epsilon$ (with $\epsilon \sim \varepsilon$) during each step of the evolution. After n steps $S_{ent} > n/2\ \ln\ 2$ since $\epsilon \ll 1$. This process will continue until $n \sim (M/M_P)^2$, when the size of the black hole is about a Planck length L_p. At this point either the process stops, leaving behind a highly-degenerate remnant or the hole fully evaporates, leaving the outgoing radiation in a mixed state (it is entangled with nothing) violating unitarity.

What might resolve the information problem? Clearly what is needed is that the outgoing radiation at least contain the information of the matter state that forms the black hole. To see what this means in a simple example, consider the matter to be a shell collapsing to a black hole that is initially in the state $|\Phi\rangle = \alpha|\Phi_0\rangle + \beta|\Phi_1\rangle$ where $\{|\Phi_0\rangle, |\Phi_1\rangle\}$ are two possible orthogonal states of the shell. The information would escape the black hole if the pairs were created such that the evolution of the state were

$$|\Phi\rangle = \alpha|\Phi_0\rangle + \beta|\Phi_1\rangle \rightarrow \frac{1}{\sqrt{2}}(|\Phi_0\rangle|1_k\rangle^+_{\mathrm{I}_1} + |\Phi_1\rangle|0_k\rangle^+_{\mathrm{I}_1}) \otimes (\alpha|0_{-k}\rangle^-_{\mathrm{O}_1} + \beta|1_{-k}\rangle^-_{\mathrm{O}_1}) \tag{200}$$

since tracing over the inside will yield the density matrix of the pure state $\alpha|0_{-k}\rangle^-_{\mathrm{O}_1} + \beta|1_{-k}\rangle^-_{\mathrm{O}_1}$, which has all the information of the infalling matter. A resolution of the information paradox must implement this kind of evolution. However there are significant obstacles to overcome.

First, the proposed evolution (200) is a radical departure from that in (179) or even (187). It is *not* a small correction to the standard pair-creation process at the horizon, and so its implementation is not obvious. It would represent a radical departure from our understanding of the behaviour of quantum field theory in curved space-time as described in preceding sections, at least near the event horizon.

Second, there can be no other pair-creation processes accompanying (200). To see why, suppose that there are further steps to the evolution (prior and/or afterward) such that for the k-th mode

$$\begin{aligned}|\Phi\rangle = \alpha|\Phi_0\rangle + \beta|\Phi_1\rangle \rightarrow \frac{1}{\sqrt{2}}(|\Phi_0\rangle|1_k\rangle^+_{\mathrm{I}_1} + |\Phi_0\rangle|0_k\rangle^+_{\mathrm{I}_1}) \otimes (\alpha|0_{-k}\rangle^-_{\mathrm{O}_1} + \beta|1_{-k}\rangle^-_{\mathrm{O}_1}) \\ \left[\otimes(|0_k\rangle^+_{I_1}|0_{-k}\rangle^-_{\mathrm{O}_1} + |1_k\rangle^+_{\mathrm{I}_1}|1_{-k}\rangle^-_{\mathrm{O}_1})\right]^n\end{aligned} \tag{201}$$

after n steps. Even though the information forming the black hole comes out, the end point of the evolution is still either a mixed state or a remnant due to all of the other created pairs. It is not sufficient to modify the evolution so that the information comes out. It must be modified to prevent the growth in entanglement entropy from the extra created pairs. The number of quanta emitted from the black hole is somewhat larger than the number needed to retain the information of the infalling matter (the entropy of the emitted radiation is about 30 % larger than the horizon entropy $A/4$ [136]), and it is just as important that these additional quanta do not yield remnants or mixed states.

Third, purity of the outside state is not sufficient. An evolution of the form

$$|\Phi\rangle = \alpha|\Phi_0\rangle + \beta|\Phi_1\rangle \to \frac{1}{\sqrt{2}}(\alpha|\Phi_0\rangle|1_k\rangle^+_{\mathrm{I}_1} + \beta|\Phi_0\rangle|0_k\rangle^+_{\mathrm{I}_1}) \otimes (|0_{-k}\rangle^-_{\mathrm{O}_1} + |1_{-k}\rangle^-_{\mathrm{O}_1}) \tag{202}$$

yields a pure outside state, but this state retains no information about the (α, β) coefficients of the infalling matter. The challenge of resolving the information paradox is to construct an evolution in which the final state of the outside radiation is both pure and information-retaining. Pair-creation at the horizon does neither.

7.2 Remedies for the Information Paradox

In attempting to resolve the information paradox, the obvious thing to try is to modify the assumptions that yielded the unbounded growth in entanglement entropy. Indeed, this has been the approach of many papers on the subject: change one of the assumptions undergirding the pair-creation scenario described above and see if this modification can resolve the problem.

Insufficiency of the Niceness Conditions Perhaps the Niceness conditions don't ensure local Hamiltonian evolution. The large distance $GM(M/M_P)^2/c^2$ along the slice between the matter state and most of the pairs created near the horizon is huge compared to the size $2GM/c^2$ seen by an external observer. This rather strange situation occurs because the spatial slices going from the centre of the collapsing matter prior to singularity formation (but inside the horizon) can be very close to null, greatly stretching their length; indeed it is possible to make the slice arbitrarily long inside the hole for any constant r. But perhaps the actual physics involves the entire horizon region within $2GM/c^2$ such that some non-perturbative physical process yields large corrections to the pair-creation process at the horizon.

The challenge here is to propose a plausible model. An instanton might be expected to suffice, but as noted above it yields corrections of order $\exp[-(M/M_P)^2]$—far too small to do the job. The nonlocal physics proposed must (a) be operative only inside event horizons (or, more generously, only in regimes for which we so far have no experiments), (b) modify the pair creation process at the horizon so the outside radiation is pure and information-retaining, and (c) apply to all possible black holes; if not then the paradox will not be fully solved.

Hairy Black Holes Perhaps when a black hole forms the horizon becomes distorted in a way that retains the information of the infalling matter. Such distortions are called 'hair', and typically have divergent stress-energy at the event horizon. Furthermore there are theorems that indicate the geometry of a black hole is characterized only by its global conserved quantum numbers: its mass, charge and angular momentum. Any attempts to resolve the problem along these lines must both evade the constraints of the no-hair theorems and ensure the hair properly transfers information to the outgoing radiation to ensure purity.

Exotic End states Perhaps there are no black holes, and something halts the gravitational collapse of any form of matter. One might think of some kind of universal “quantum pressure”, analogous to what prevents electrons from spiralling into atoms, generating effective Bohr orbits. As noted above, this would mean that any supposed black hole is really some kind of dark object with a considerable degree of structure. All collapsing matter would have to form such objects without exception, no matter how large the mass of the infalling matter. If they radiate, then they could presumably ‘burn’ the way normal matter does. If these objects do not radiate they would be “hairy remnants”, and it would be necessary to bound the number of microscopic types so as to avoid the problems with divergences in scattering amplitudes noted above.

One of the more concrete ideas along these lines is that of the Fuzzball [137]. This approach grew out of several results that emerged from string theory that suggest the end-state of gravitationally collapsing matter is not a traditional black hole because the degrees of freedom of the hole distribute themselves throughout a horizon sized object referred to as a fuzzball. Particular examples of this kind of structure were obtained by considering various extremal black brane solutions to the low-energy string equations with multiple charges [138–151]. The basic idea is that as matter undergoes gravitational collapse, its (presumed) fundamentally stringy degrees of freedom distribute their momenta in such a way that the final solution has neither a horizon nor a singularity [142–144]. Instead of a black hole, matter undergoing gravitational collapse will quantum tunnel to a fuzzball: a complicated “hairy” structure that contains all of the degrees of freedom of the black hole. Hawking radiation would be due to emission from an ergoregion near the fuzzball. This radiation can carry information about the original state of matter because it is not entangled with any states inside a horizon as no such horizon exists [137].

The problem with the fuzzball proposal at the moment is its lack of generality. Notwithstanding the fact that first-order corrections suggest that perhaps fuzzballs can form from generic collapse [152, 153], the proposal only appears to work for particular brane configurations. However to resolve the information paradox *all* possible matter configurations must form a fuzzball structure.

7.3 Complementarity

One idea that emerged as a means of reconciling black hole radiation with known quantum physics was complementarity [154, 155]. The idea here is that one cannot ask a physical theory to yield descriptions of observers that cannot exist—specifically observers that can make measurements both inside and outside of the black hole.

Consider a choice of spacelike slices describing an evaporating black hole. The original slice Σ contains matter that will go into forming the black hole, and the slices that go through the event horizon obey the niceness conditions. This breaks down at the point P, where the black hole finally evaporates (no remnant assumed), but one might expect that a proper understanding of quantum gravity will ameliorate this, leading to a well-defined physical description of the end point of evaporation. After evaporation, the slices Σ' obey the niceness conditions.

The problem presented by black hole evaporation is that the quantum state on Σ' must be described by unitary evolution from the quantum state on Σ. However the only way this can happen is if the quantum state on the part of the slice inside the black hole has no dependence on the initial state. This is tantamount to bleaching: all distinctions between the initial states of infalling matter must be expunged before the state crosses the global event horizon. In other words, the evolution of the quantum state must proceed as follows

$$\begin{gathered} |\Psi(\Sigma)\rangle \to |\Psi(\Sigma_P)\rangle = |\Xi(\Sigma_{\mathrm{IN}})\rangle \otimes |\Upsilon(\Sigma_{\mathrm{OUT}})\rangle \to |\Psi(\Sigma')\rangle \\ \text{where } |\Psi(\Sigma')\rangle = U_2|\Upsilon(\Sigma_{\mathrm{OUT}})\rangle = U_1|\Psi(\Sigma)\rangle \end{gathered} \tag{203}$$

with U_1 and U_2 unitary operators and with the Hilbert space of states on Σ_P likewise decomposing into a tensor product $\mathcal{H}_P = \mathcal{H}_{\mathrm{IN}} \otimes \mathcal{H}_{\mathrm{OUT}}$ with $\Xi(\Sigma_{\mathrm{IN}}) \subset \mathcal{H}_{\mathrm{BH}}$ and $\Upsilon(\Sigma_{\mathrm{OUT}}) \subset \mathcal{H}_{\mathrm{OUT}}$. The evolution between $|\Psi(\Sigma')\rangle$ and $|\Psi(\Sigma)\rangle$ is fully unitary and reversible, uninfluenced by $|\Xi(\Sigma_{\mathrm{IN}})\rangle$.

Complementarity posits that the flaw in the above argument is in the assumption of the existence of $|\Psi(\Sigma_P)\rangle$. This is a quantum state that simultaneously describes both the interior and the exterior of a black hole. The claim is that any state of this nature has no operational meaning, since no "super-observers" exist that compare measurements both inside and outside the black hole. Rather any observer must choose a basis in which to work: either one describes particles beyond the horizon or the particles in the Hawking radiation, but not both. Indeed, the trans-Planckian problem suggests that large non-vanishing commutators exist between operators describing ingoing material and those describing outgoing Hawking radiation, and so correlations between inside and outside the hole lose any operational meaning.

The advent of the Anti de Sitter/ Conformal Field Theory correspondence provided further confidence for this perspective, suggesting that all information is indeed carried away by the Hawking radiation. The idea here is that the quantum states (and their evolution) of any gravity theory whose solutions are asymptotic to anti de Sitter (AdS) space-time are in 1-1 correspondence with those of a Conformal Field Theory (CFT). This conjecture has more recently has broadened to a proposed duality between gravitational and gauge theories (gauge/gravity duality) under more general asymptotic conditions and symmetries. The CFT (or dual gauge theory) is unitary and so cannot admit any information loss, and its duality with gravity indicates that the same must be true there as well. There is strong circumstantial evidence in favour of this kind of duality, and hence of the purity of radiation emitted by a black hole.

Of course gauge/gravity duality does not prove that information loss cannot occur. Rather it provides a new paradigm by which one might seek to understand the process of black hole formation and evaporation. If indeed a dual gauge theory can describe this process, then the onus is on this theory to explain either (a) which conditions are modified so that mixedness is avoided, (b) the formation of remnants, or (c) new physics in the gravity theory that either prevents black hole formation or modifies the state near the horizon. So far such a description has yet to be given.

So complementarity asserts that there is no logical contradiction in assuming that a distant outside observer sees all infalling information returned in Hawking-like radiation, and that the infalling observer experiences nothing unusual before or during horizon crossing. The thermality of Hawking radiation will be affected by interactions very near the horizon, and these presumably ensure that the net emission process is pure as seen by the outside observer. The physical model an outside observer employs will therefore postulate a boundary condition for all fields a few Planck distances away from the horizon. These include the brick wall model [123, 124], bounce models, and stretched horizon models [154]. A full quantum theory of gravity is expected to set this distance, but it can be input into the theory for the purpose of doing phenomenological calculations. This membrane/wall distance is dependent on the matter content of the theory (the number of fields, for example), and so constraining this distance to be consistent with observation will constrain the matter content of the theory, providing (in principle) an additional degree of falsifiability. From the perspective of the outside observer the membrane/wall absorbs infalling matter and then thermalizes it, unitarily re-radiating it as Hawking radiation via a process similar to the manner in which a normal body radiates. Complementarity implies that an observer falling into the black hole will see no such membrane or brick wall, in contrast to the outside observer for whom this virtual structure is quite real [154].

So the postulates of black hole complementarity are as follows:

1. **Unitarity** The process of formation and evaporation of a black hole, as viewed by a distant observer, can be described entirely within the context of standard quantum theory. In particular, there exists a unitary S-matrix which describes the evolution from infalling matter to outgoing Hawking-like radiation.
2. **Semi-Classicality** Outside the stretched horizon of a massive black hole, physics can be described to good approximation by a set of semi-classical field equations. The semi-classical field equations are those of a low energy effective field theory with local Lorentz invariance.
3. **Placidity** A freely falling observer experiences nothing out of the ordinary when crossing the horizon, as expected from the equivalence principle—gravity is locally indistinguishable from acceleration. This is basically the 'information-free' condition mentioned earlier: it is exponentially unlikely for infalling observer to measure a quantum of energy $E \gg 1/r_+$.
4. **Thermality** To a distant observer, a black hole appears to be a quantum system with discrete energy levels. The dimension of the subspace of states describing a black hole of mass M, angular momentum J, and charge Q is the exponential of the Bekenstein entropy $S\ (M, J, Q)$, so that the standard black hole thermodynamic relations are obeyed.

These postulates have been slightly modified from their original form, but contain the essential aspects of what black hole complementarity is based on. Black hole no-hair theorems implied that the membrane/wall must be virtual, as noted above; no known physics at the time complementarity was proposed could generate such a structure, though new ideas have been put forward recently [156]. But even though the degrees of freedom of the membrane/wall are virtual, they must be generated by some nonlocal effect. The reason is that if normal semiclassical physics is valid for small curvatures, then pair creation takes place via the Hawking process described above and not due to reflection/generation from a wall or membrane. This is a consequence of the niceness conditions, namely the assumption that one can always choose a set of slices through the black hole where curvatures are everywhere small and the vacuum is well-defined. If complementarity is valid then some new, nonlocal, physics must dominate—over scales of the horizon size—since semi-classical physics yielding pair creation is valid along the slices [157].

8 Firewalls

The Firewall argument asserts that the postulates of black hole complementarity are not self-consistent [158, 159]. Specifically one of the first three postulates must be incorrect, since assuming all three together yield a contradiction.

8.1 The Firewall Argument

This rather surprising claim follows from a fairly straightforward argument, put forward by Almheiri, Marolf, Polchinski and Sully (known as AMPS) [159]. The unitarity postulate # 1 implies that radiation emitted from the black hole must not be in a mixed state, and so some process must convert the state (185) to

$$|\Psi\rangle_n \approx \frac{1}{\sqrt{2}}|\Phi\rangle_{\mathrm{I}_n}\prod_{m=1}^{n}\Big[\otimes(|0_k\rangle^+_{\mathrm{I}_m}|0_{-k}\rangle^-_{\mathrm{O}_m}+|1_k\rangle^+_{\mathrm{I}_m}|1_{-k}\rangle^-_{\mathrm{O}_m})\Big]\rightarrow|\tilde{\Phi}\rangle_{\mathrm{I}_n}|\tilde{\Xi}\rangle_{\mathrm{O}_n} \tag{204}$$

after some large number n of steps, where the outside radiation state $|\tilde{\Xi}\rangle_{\mathrm{O}_n}$ is pure. At some point in time (called the Page time [81]) the entanglement entropy of the emitted radiation must reach a maximum, after which point there is more entropy in the radiation than there is in the black hole. The black hole continues to shrink in size and entropy, emitting successively fewer quanta. Consequently the number of states N_L accessible after the Page time [160] (the ‘late’ subspace) will be much smaller than the number N_E in the space of states prior to this (the ‘early’ subspace):

$N_L \ll N_E$. Expanding the full outside radiation state $|\tilde{\Xi}\rangle_{\mathrm{O}_n}$ in an orthonormal basis $\{|\mathrm{j}\rangle_L\}$ of the late subspace yields

$$|\tilde{\Xi}\rangle_{\mathrm{O}_n} = \sum_{\mathrm{k}}^{N_L} |\psi_{\mathrm{k}}\rangle_E \otimes |\mathrm{k}\rangle_L \tag{205}$$

where $\{|\psi_1\rangle_E, |\psi_2\rangle_E, \ldots, |\psi_{N_E}\rangle_E\}$ span the early subspace. Consider the norm of the state $\mathcal{P}^{\mathrm{j}}|\tilde{\Xi}\rangle_{\mathrm{O}_n}$, where the operator $\mathcal{P}^{\mathrm{j}} \equiv P^{\mathrm{j}} - \hat{P}^{\mathrm{j}} = |\mathrm{j}\rangle_{L\,L}\langle\mathrm{j}| - N_L|\psi_{\mathrm{j}}\rangle_{E\,E}\langle\psi_{\mathrm{j}}|$. Expanding this out yields

$$\begin{aligned}
||\mathcal{P}^{\mathrm{j}}|\tilde{\Xi}\rangle_{\mathrm{O}_n}||^2 &= ||\,|\mathrm{j}\rangle_{L\,L}\langle\mathrm{j}|\tilde{\Xi}\rangle_{\mathrm{O}_n} - N_L|\psi_{\mathrm{j}}\rangle_{E\,E}\langle\psi_{\mathrm{j}}|\tilde{\Xi}\rangle_{\mathrm{O}_n}||^2 \\
&= ||\,|\mathrm{j}\rangle_L\,|\psi_{\mathrm{j}}\rangle_E - N_L|\psi_{\mathrm{j}}\rangle_E \sum_{\mathrm{k}}^{N_L} {}_E\langle\psi_{\mathrm{j}}|\psi_{\mathrm{k}}\rangle_E \otimes |\mathrm{k}\rangle_L||^2 \\
&= |||\psi_{\mathrm{j}}\rangle_E||^2\,||\,|\mathrm{j}\rangle_L - N_L \sum_{\mathrm{k}}^{N_L} {}_E\langle\psi_{\mathrm{j}}|\psi_{\mathrm{k}}\rangle_E \otimes |\mathrm{k}\rangle_L||^2 \\
&= |||\psi_{\mathrm{j}}\rangle_E||^2\left[\left(1 - N_{L\,E}\langle\psi_{\mathrm{j}}|\psi_{\mathrm{j}}\rangle_E\right)^2 + N_L^2 \sum_{\mathrm{k}\neq\mathrm{j}}^{N_L} |{}_E\langle\psi_{\mathrm{j}}|\psi_{\mathrm{k}}\rangle_E|^2\right]
\end{aligned} \tag{206}$$

Expanding the state $|\psi_{\mathrm{k}}\rangle_E = \sum_{\mathrm{a}=1}^{N_E} c_{\mathrm{ka}}|\mathrm{a}\rangle$ in an orthonormal basis $|\mathrm{a}\rangle$ of the early states yields

$$\overline{c_{\mathrm{ja}}c^*_{\mathrm{kb}}} = \frac{1}{N_L N_E}\delta_{\mathrm{jk}}\delta_{\mathrm{ab}} \qquad \overline{c_{\mathrm{ja}}c^*_{\mathrm{kb}}c_{\mathrm{ic}}c^*_{\mathrm{ld}}} = \frac{1}{N_L^2 N_E^2}\left(\delta_{\mathrm{jk}}\delta_{\mathrm{ab}}\delta_{\mathrm{il}}\delta_{\mathrm{cd}} + \delta_{\mathrm{jl}}\delta_{\mathrm{ad}}\delta_{\mathrm{ik}}\delta_{\mathrm{bc}}\right) \tag{207}$$

upon averaging over $|\tilde{\Xi}\rangle_{\mathrm{O_n}}$, assuming a uniform measure for the outside state. Consequently $\overline{{}_E\langle\psi_j|\psi_k\rangle_E} = \delta_{jk}/N_L$ and $\overline{{}_E\langle\psi_{\mathrm{j}}|\psi_{\mathrm{k}}\rangle_{EE}\langle\psi_{\mathrm{i}}|\psi_{\mathrm{l}}\rangle_E} = \delta_{\mathrm{jk}}\delta_{\mathrm{il}}/N_L^2 + \delta_{\mathrm{jl}}\delta_{\mathrm{ik}}/(N_L^2 N_E)$, yielding

$$\begin{aligned}
\overline{\mathcal{E}} = \frac{\overline{||\mathcal{P}^{\mathrm{j}}|\tilde{\Xi}\rangle_{\mathrm{O_n}}||^2}}{|||\psi_{\mathrm{j}}\rangle_E||^2} &= 1 - 2N_L\,\overline{{}_E\langle\psi_{\mathrm{j}}|\psi_{\mathrm{j}}\rangle_E} + N_L^2 \sum_{k}^{N_L} \overline{{}_E\langle\psi_{\mathrm{j}}|\psi_{\mathrm{k}}\rangle_{EE}\langle\psi_{\mathrm{j}}|\psi_{\mathrm{k}}\rangle^*_E} \\
&= 1 - 2N_L\frac{\delta_{\mathrm{jj}}}{N_L} + \frac{N_L^2}{N_L^2 N_E^2}\left(N_E^2\delta_{\mathrm{jj}} + N_L N_E \delta_{\mathrm{jj}}\right) \\
&= \frac{N_L}{N_E}
\end{aligned} \tag{208}$$

in the limit $N_E \gg N_L \gg 1$. Hence

$$P^{\mathrm{j}}|\tilde{\Xi}\rangle_{\mathrm{O}_n} \approx \hat{P}^{\mathrm{j}}|\tilde{\Xi}\rangle_{\mathrm{O}_n} = |\psi_{\mathrm{j}}\rangle_E \otimes |\mathrm{j}\rangle_L \tag{209}$$

and so it is possible to project onto any given subspace of the late radiation, up to a relative error of order N_L/N_E. The argument is essentially the same if grey-body factors are taken into account.

So for a distant observer after n steps the radiation $|\tilde{\Xi}\rangle_{\mathrm{O}_n}$ is near infinity and can be decomposed into a set of modes $\{|\mathrm{j}\rangle\}$. In particular, it is possible to project onto eigenspaces of the number operator in an observer-independent way, according to the semi-classicality assumption. These modes can be evolved backward in time toward the horizon—they will be of much higher frequency at these earlier times, but can be kept to the sub-Planckian regime if one does not evolve too far back. However the placidity assumption #3 implies that an infalling observer sees the vacuum near the horizon, and so the number operator of the radiation $|\tilde{\Xi}\rangle_{\mathrm{O}_n}$ must be zero, in contradiction to what the observer at infinity measures. This contradiction can be avoided if the infalling observer does not see a vacuum, but instead encounters a large number of high-energy modes: in other words, a firewall.

A regular horizon implies increasing entanglement, as shown in (198). Conversely, if entanglement is to decrease, then the state at the horizon cannot be the vacuum. This is the firewall argument in a nutshell.

An alternative version of the argument employs the strong subadditivity condition [156]. The radiation after n steps is $|\tilde{\Xi}\rangle_{\mathrm{O}_n}$, with the next mode $|\mathrm{n}+1\rangle_{\mathrm{O}}$ emitted near the horizon. The former can be evolved backward in time near the horizon. Semiclassicality implies that

$$|\tilde{\Xi}\rangle'_{\mathrm{O}_n} = |\tilde{\Xi}\rangle_{\mathrm{O}_n} \qquad |\mathrm{n}+1\rangle'_{\mathrm{O}} = |\mathrm{n}+1\rangle_{\mathrm{O}} \tag{210}$$

where the primes refer to the states measured by the infalling observer. Since this observer sees a vacuum at the horizon, the state $|\mathrm{n}+1\rangle'_{\mathrm{O}}$ must be entangled with some state $|\mathrm{n}+1\rangle'_{\mathrm{I}}$ inside the horizon. Strong subadditivity then implies that (198) holds

$$S'(\mathsf{n}+1)_{ent} > S'(\mathsf{n})_{ent} + \ln 2 - 2\epsilon \tag{211}$$

and the equivalence (210) implies

$$S(\mathsf{n}+1)_{ent} > S(\mathsf{n})_{ent} + \ln 2 - 2\epsilon \tag{212}$$

which means the entropy in the radiation cannot decrease, in contradiction with the unitarity postulate, which implies that it must decrease after the halfway point.

So complementarity is incompatible with the local evolution that creates the pairs of Hawking quanta. Even though complementarity invoked non-locality to argue that the slices permitted by the niceness conditions are not valid, it requires that local semiclassical physics applies outside the membrane or stretched horizon.

Non-local physics is therefore constrained to be inside the horizon. The firewall argument rules out this kind of complementarity and hence this kind of sharply-limited non-locality.

8.2 Responses to the Firewall Argument

The response of the physics community to the firewall argument was rapid, intense, and diverse, ranging from skepticism, to ambivalence, to endorsement.

Those endorsing the firewall argument have emphasized that standard arguments from quantum field theory in curved spacetime and quantum information should lead one to expect this result [161–173]. Standard semi-classical methods analyzing causal patches [169], string-creation [173], and freely-falling observers [170] have each been used to buttress the argument. Indeed numerical analysis of a particular class of models suggests a breakdown of effective field theory, in turn implying the existence of firewalls on black hole horizons [171]. It has even been suggested that alternatives to firewalls may suffer contradictions similar to those associated with time travel [162]. Rindler horizons have been argued to be immune (or at least not necessarily susceptible) to the firewall argument [174].

Nevertheless initial skepticism [175] was soon followed by a number of challenging responses to the firewall argument. A number were rebutted by Almheiri and collaborators [176]. Let's consider some of the main objections and their corresponding responses.

8.2.1 Absorbing the Interior Hilbert Space

Since the outside modes must be entangled with both the early outside modes and with their inside pair-created partners (violating quantum limits on entanglement) then perhaps the interior Hilbert space of an old black hole is embedded in the larger Hilbert space of the early radiation. The claim is that the firewall phenomenon can occur only for an exponentially fine-tuned (and intrinsically quantum mechanical) initial state, analogous to an entropy decreasing process in a system with large degrees of freedom [177–182]. Alternatively, quantum computations required to carry out the thought experiments undergirding the firewall argument take so long (a time exponential in the entropy of the black hole) that this prevents the experiments from being done [183, 184]. In other words, excitations exist at the horizon only if such quantum computations have been performed.

Both considerations run afoul of standard quantum mechanics. Assuming unitarity, an observer outside the black hole (Charlie) can extract a bit of information that will be entangled with a later outside pair-created bit. Another spacelike separated observer (Alice, say) can jump into the black hole and extract information about both the inside and outside created pair, whilst Charlie can send the quantum state of the early bit to Alice. Alice will then possess information concerning three quantum bits, two of which are maximally entangled with the third, which violates quantum mechanics.

More generally, operators associated with the early radiation will generically not commute with operators associated with the Hilbert space of an infalling observer if the interior Hilbert space is embedded in the early radiation Hilbert space [176]. Consider the parity operator $(-1)^{N_e}$ of an early outside bit $e \subset E$

$$(-1)^{N_e} = \sigma^z \otimes I \tag{213}$$

written above in a basis factorized into the measured parity and everything else. Since the interior Hilbert space is a subset of the outside early radiation space, we can expand the parity operator of an inside bit $i \subset I$

$$(-1)^{N_i} = I \otimes S^0 + \sigma^x \otimes S^x + \sigma^y \otimes S^y + \sigma^z \otimes S^z \tag{214}$$

where the matrices S^λ are constrained by the requirement $(-1)^{N_i}(-1)^{N_i} = 1$. Suppose the parity of an early state $|\psi\rangle$ is positive, so that $(-1)^{N_e}|\psi\rangle = +|\psi\rangle$. Then the expectation value of $(-1)^{N_e}$ for the state $(-1)^{N_i}|\psi\rangle$ is

$$\begin{aligned}
&\langle\psi|(-1)^{N_i}(-1)^{N_e}(-1)^{N_i}|\psi\rangle \\
&= \langle\psi|\sigma^z \otimes (S^0)^2 + \sigma^x\sigma^z\sigma^x \otimes (S^x)^2 + \sigma^y\sigma^z\sigma^y \otimes (S^y)^2 + (\sigma^z)^3 \otimes (S^z)^2|\psi\rangle + \text{cross terms} \\
&= \langle\psi|\sigma^z \otimes \left((S^0)^2 - (S^x)^2 - (S^y)^2 + (S^z)^2\right)|\psi\rangle + \text{cross terms}
\end{aligned}$$

Upon averaging over all possible operators S^λ (requiring $(-1)^{N_i}(-1)^{N_i} = 1$) the cross-terms will average to zero since independent sign flips in parity are allowed. Each $(S^\lambda)^2$ term will average to the same value since these operators are generic and so their eigenvalues will be comparable in size. Hence $\langle\psi|(-1)^{N_i}(-1)^{N_e}(-1)^{N_i}|\psi\rangle$ averages to zero.

So if we start with an eigenstate of $(-1)^{N_e}$ and measure the parity $(-1)^{N_i}$, the expectation value flips from 1 to 0. Hence the eigenvalue changes with near-unit probability, implying the commutator of $(-1)^{N_i}$ and $(-1)^{N_e}$ is of order unity, and hence the commutator of early and interior operators is also of order unity. Hence if an infalling observer sees a vacuum (so that $\mathrm{a}|\psi\rangle = 0$ where a is an annihilation operator in the Hilbert space of the infalling observer), then since the interior operators can be expanded in terms of the infalling operators, the early creation/annihilation operators will not commute with any of the operators a, strongly perturbing the infalling vacuum and creating a firewall. This abolishes (or at least renders highly problematic) the notion that infalling observers see no firewall because the deviation from thermality is too small to detect [185].

8.2.2 Broadening Complementarity

One recent proposal posits that each observer has their own Hilbert space, with suitable overlap conditions [186–196]. This broadens the notion of complementarity insofar as there is no global Hilbert space. The idea is that space-time physics is described in terms of an infinite number of quantum systems, each of which encodes the physics as seen along a particular time-like trajectory, in a proper time dependent Hamiltonian [186]. Extending these ideas to a matrix theory model of black holes suggests that there are no high energy particles available that could constitute the firewall [189]. The vacuum entanglement that is a crucial feature of Hawking radiation is claimed not to be a feature of the physics described by matrix theory. Whether or not this proposal can be fully consistently implemented remains to be seen.

8.2.3 Non-locality

Prior to the advent of the Firewall argument, the idea that non-locality can and should play a role in resolving the information paradox was already being actively explored [197, 198]. Although generic nonlocality leads to causality paradoxes, perhaps there are regions (near a black hole, for example) where locality is not exact but only approximate. The idea here is to weaken the assumptions of the semi-classicality postulate #2 and introduce some form of mild (or non-violent) non-local physics [199–204].

Since if locality is exact outside the horizon any information transfer from the black hole to the radiation produces singular behaviour at the horizon [36, 158], it is necessary to weaken locality outside of the black hole. Each black hole would therefore have a “nonlocal zone” (about the size of the black hole itself) within which information transfer from the black hole to the outgoing radiation takes place. This information transfer is a transfer of the entanglement between the early radiation and black hole interior to entanglement between the early and late radiation. It requires an additional energy flux beyond that of the Hawking radiation [201, 202], and modulates the Hawking radiation in a sufficiently fine-grained manner so as to preserve the average properties of the Hawking flux. Leading order calculations in a model in which nonlocal metric perturbations couple to the stress tensor suggest in a two-dimensional model this might be possible [203].

The challenge such proposals face is that any scheme that physically separates transfer of energy from transfer of information runs into conflict with the Bekenstein-Hawking density of states $\exp[S]$ of the black hole. As noted previously, there is strong circumstantial evidence from black hole pair creation to support this notion [113–116], and so modifying it would necessitate at the least a revision of how this process works.

Consider a process that transports quanta behind the horizon where they become outgoing quanta. Suppose a pair is created outside the black hole as in Eq. (179).

$$\frac{1}{\sqrt{2}}|\Phi\rangle_{\mathrm{I}} \otimes (|0_k\rangle_{\mathrm{I}}^{+}|0_{-k}\rangle_{\mathrm{O}}^{-} + |1_k\rangle_{\mathrm{I}}^{+}|1_{-k}\rangle_{\mathrm{O}}^{-}) = \frac{1}{\sqrt{2}}(|\Phi_0\rangle + |\Phi_1\rangle) \otimes (|0_k\rangle_{\mathrm{I}}^{+}|0_{-k}\rangle_{\mathrm{O}}^{-} + |1_k\rangle_{I}^{+}|1_{-k}\rangle_{\mathrm{O}}^{-}) \tag{215}$$

where the Hilbert space has been separated into orthogonal states as in Eq. (200). As the outgoing mode move through the nonlocal zone, some process will cause it to exchange information with a state in the interior so that

$$\frac{1}{\sqrt{2}}|\Phi\rangle_{\mathrm{I}} \otimes (|0_k\rangle_{\mathrm{I}}^{+}|0_{-k}\rangle_{\mathrm{O}}^{-} + |1_k\rangle_{\mathrm{I}}^{+}|1_{-k}\rangle_{\mathrm{O}}^{-}) \rightarrow \frac{1}{\sqrt{2}}(|\Phi_0\rangle|1_k\rangle_{\mathrm{I}_1}^{+} + |\Phi_0\rangle|0_k\rangle_{\mathrm{I}_1}^{+}) \otimes (\alpha|0_{-k}\rangle_{\mathrm{O}_1}^{-} + \beta|1_{-k}\rangle_{\mathrm{O}_1}^{-}) \tag{216}$$

This would avoid the firewall problem just as it resolved the information paradox problem.

However this approach will also entail the same difficulties noted in the previous section, and will allow the number of internal states of the black hole to exceed the entropy $S = A/4$ discussed in Sect. 3. The reason is that it is possible to interact with the outgoing bit as it moves through the nonlocal zone, say by introducing a phase

$$\frac{1}{\sqrt{2}}|\Phi\rangle_{\mathrm{I}} \otimes (|0_k\rangle_{\mathrm{I}}^{+}|0_{-k}\rangle_{\mathrm{O}}^{-} + |1_k\rangle_{\mathrm{I}}^{+}|1_{-k}\rangle_{\mathrm{O}}^{-}) \rightarrow \frac{1}{\sqrt{2}}(|\Phi_0\rangle|1_k\rangle_{\mathrm{I}_1}^{+} - |\Phi_0\rangle|0_k\rangle_{\mathrm{I}_1}^{+}) \otimes (\alpha|0_{-k}\rangle_{\mathrm{O}_1}^{-} + \beta|1_{-k}\rangle_{\mathrm{O}_1}^{-}) \tag{217}$$

and so a larger set of interior states, beyond that given by exp[S] has been accessed by this process.

Another recent proposal involves using wormholes to transfer information beyond the horizon [205–209]. The idea is that the Einstein Rosen bridge connecting the right side of Fig. 2 ("our universe") and its left side is created by quantum correlations between the microstates of the black holes on each side. The conjecture is that any entangled pair of quantum states is connected by a similar sort of space-time bridge or wormhole. These wormholes non-locally connect quantum states inside and outside of the horizon, allowing for information from the black hole to escape.

8.2.4 Exotic Objects

Of course if an event horizon never forms then a firewall can be avoided [210–212]. The fuzzball proposal [137] fits into this class, but there have been several other recent speculative ideas along these lines, including Grireballs [213], leaky horizons [214], and aether-like fluids whose atmosphere mimics Hawking radiation [215, 216]. All of these ideas must universally replace the generic collapse of matter into a black hole if they are to be viable candidates for eliminating the firewall. Of course if a remnant forms, this could also avoid a firewall; an explicit example in two dimensions was recently given [217].

8.2.5 Additional Degrees of Freedom

Some responses to the firewall argument have suggested it is lacking because additional degrees of freedom are present in quantum gravity that are otherwise unaccounted for [218–220] or not properly treated.

One such approach involves distinguishing virtual qubits (the entangled created pairs) from real qubits (that store the information inside the black hole) [221, 222]. The idea is that black hole information is stored both inside and outside the stretched horizon, yielding twice the usual black hole entropy and therefore extra room to arrange the quantum degrees of freedom so that paradoxical results are avoided. The apparent firewall obstruction can be removed, via a universal entanglement swap operation that transports all free quantum information from the interior of the black hole to its exterior. This swap can be created locally and in the near horizon region; however this firewall-removing operation cannot be used to transfer information from an infalling state into the outgoing radiation [223, 224].

8.2.6 Loopholes

A number of papers have been written contending that the existence of firewalls depends on a chain of reasoning that is incomplete, and that one or more loopholes exist that allow one to escape the conclusions of the argument.

It has been suggested that the space of physical quantum gravity states does not factorize into a tensor product of localized degrees of freedom, invalidating one of the assumptions of the firewall argument [225]. The idea here is that in any diffeomorphism invariant ultra-violet complete theory with an asymptotic region in which an algebra of observables can be defined (which presumably is a feature of quantum gravity), the Hamiltonian is a surface integral in this asymptotic region (or boundary). The boundary encodes all degrees of freedom, including those inside the horizon, and the algebra of boundary observables evolves into itself unitarily over time. Hence no boundary information can ever be lost, not even temporarily. This is argued to invalidate a key assumption of the firewall argument, which is that the early time Hawking radiation is in a mixed quantum state and gets purified later by the late time Hawking radiation to preserve unitarity. Rather there must be continuous purity, with the Hawking quanta always entangled with exterior degrees of freedom and never with interior ones. The Hilbert space does not factorize into exterior and interior state spaces, and so the ‘partner behind the horizon’ does not actually exist in a full quantum theory of gravity. Of course for this picture to be accepted, the details of the physical states and how they are encoded into the boundary needs to be made explicit.

Some effort has been put into seeing what happens if the horizon geometry undergoes quantum fluctuations [226]. The claim is that both the black hole information paradox and firewalls originate from treating the geometry as strictly classical, and that it is an ill-posed problem to employ quantum fields in a classical curved space with a horizon. Instead, one should first integrate out fluctuations of the background geometry and then evaluate matter observables. Some models of shell collapse indicate that a firewall may or may not form depending on the ratio of the black hole entropy to the square of the number of coherently emitted particles [227, 228].

Additional evasive tactics have been proposed. Some have proposed that the firewall issue is purely quantum information theoretic and so we should have an answer once we know exactly what computation we need to do [229]. Another argument posits that the firewall paradox is likely to be an artifact of using an effective theory beyond its domain of validity [230]. It has been suggested that a distillation-like process for extracting information needs to be clarified before one can conclude that black hole complementarity is not valid [231]; indeed this distillation process may back-react on the black hole, breaking cross-horizon entanglement and removing the firewall [232].

Another suggestion is that a firewall will not emerge if the energy cost of measurement on the early states (yielding information about the late states) is much smaller than the ultraviolet cutoff scale [233]. Perhaps it is necessary to modify the expected entanglement of states near a horizon [234] or to take macroscopic superpositions of black holes [235–237], or to introduce new causality requirements into physics [238].

The final state-proposal in which a generalization of quantum mechanics allowing postselection on a final state at the black hole singularity, has been suggested as a resolution for the black hole information paradox [239] and for firewalls [240]. The idea here is that quantum information can escape from the black hole interior via postselected quantum teleportation [241]. The information moves forward in time to the singularity, backward in time from the singularity to the horizon, then forward in time from the horizon to future infinity, but if suitable dynamical constraints are satisfied, this is equivalent to a causally ordered flow of information moving unitarily forward in time. However these constraints appear to be rigorously fulfilled only by fine tuning [240]. Furthermore, the final state projection postulate has been shown to be inadequate for abolishing firewalls [242].

Some have contented that the firewall follows from making assumptions about physics inside the stretched horizon that do not follow from the semiclassicality postulate #2. One claim [243, 244] is that firewalls are avoided if the degrees of freedom of the stretched horizon retain information for a sufficiently long time known as the scrambling time (the minimum time required for the information about the initial state to be lost without measuring a large fraction of all the degrees of freedom). Alternatively, if the semiclassicality postulate holds, firewalls are avoided, but at the price of introducing remnants [245].

Finally it was recently shown that Einstein's field equations do not admit a solution in which a Planck-density, Planck-scale firewall is just outside the event horizon [246]. Any shell located at the horizon of an astrophysical black hole must necessarily have a density many orders of magnitude lower than the Planck density. A recent analysis of the behaviour of photons from the cosmic microwave background falling into a black hole indicates that they form a “classical firewall” in the frame of a static observer near the horizon, but that this firewall has negligible effects on both freely infalling observers and the evolution of the black hole [247].

8.3 A Toy Firewall Model

A recent development in exploring the Firewall problem involves modelling it via severing quantum correlations across a Rindler horizon [248]. The rationale behind this is that the firewall argument implies that unitary evolution of the full black hole system cannot permit strong quantum correlations between the black hole interior and exterior to exist. So the toy Rindler firewall is constructed by “by hand”, breaking the quantum correlations across the horizon with no deeper explanation as to how this might come about.

For a conformal scalar field in two flat space-time dimensions, the required quantum state can be written down. A 2-level Unruh de Witt detector crossing the Rindler firewall was found, within first-order perturbation theory of the coupling, to have a sudden but finite effect on the detector’s transition probability [248]. Adding in Rindler excitations behind the horizon so as to model an actual firewall also yields a finite detector response, albeit one that can be made increasingly large by adjusting the associated temperature parameter. These results hold for both linear and derivative couplings of the detector to the scalar field; the latter type of coupling is regarded as being more similar to that of a detector in (3 + 1) dimensions [60, 74, 249].

9 Summary

Black holes retain a powerful grip on both the physical universe and the human imagination. At a classical level they absorb all matter and energy they encounter, growing ever larger in the process. Our best understanding of quantum physics indicates that they thermally radiate like black bodies, undergoing phase transitions into other forms and eventually evaporating away.

But away to what? We have no self-consistent description of this process. Our present understanding suggests that either a radiating black hole eventually either cools down into an information-rich nugget or erects a firewall around itself. Neither scenario appears to be compatible with our understanding of physics. The problem is not so much with particular models of black hole radiation but rather with a clash of the basic principles of relativity and quantum physics.

It appears we must either give up the predictive power of quantum mechanics (unitarity), or the notion that gravity is locally indistinguishable from acceleration (the equivalence principle), or the view that a physical phenomenon is influenced directly only by its immediate surroundings (locality). Each of these principles is supported—directly and indirectly—by a wealth of experimentation. The physics community at the moment is quite divided on the resolution to this problem, and may be for some time to come.

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