# The Pyramid Builder's <br> Handbook 

by

Derek K Hitchins


## Dedication

To my darling wife, Trish, who sustained me, body and $k a$, as we explored the magic of ancient Egypt


## TABLE OF CONTENTS

Preface ..... xV
CHAPTER 1. EMERGENCE .....  1
The True Classical Period .....  1
Environment and Society .....  1
The Gift of the Nile .....  .1
Societal Structure .....  7
Joining of The Two Lands .....  7
Kings, Pharaohs and Dynasties .....  9
Feudal hierarchy .....  9
Bureaucracy ..... 10
Energy work and Organization ..... 10
Social Psyche ..... 12
The Ancient Egyptian Psyche ..... 12
Societal Individuation ..... 20
The Rôle and Purpose of The Pyramids. ..... 22
The Evolving Rôle of the Pharaoh ..... 22
Resurrection Machines ..... 24
Stability and Change ..... 25
CHAPTER 2. PYRAMIDS IN PERSPECTIVE ..... 29
Bird's Eye View ..... 29
4- ${ }^{\text {TH }}$ Dynasty Necropolis-Artist's View ..... 30
Pyramid Evolution ..... 31
Evolving precision in Stone ..... 34
Snefru's Pyramids. ..... 34
Khufu's Great Pyramid at Giza - "Akhet Khufu" ..... 39
Djedefre's Pyramid - "Djedefre is a Sehed Star" ..... 46
Khafre's Pyramid - "Great is Khafre" ..... 46
Menkaure's Pyramid - "Menkaure is Divine" ..... 48
Nebka's Pyramid at Zawiyet el-Aryan ..... 50
Shepseskaf's Mastaba el-Fara'un (Pharaoh's Bench) ..... 50
$5^{\text {TH }}$ and $6^{\text {TH }}$ Dynasty PyRamids ..... 51
Necropolis Plans and Layouts. ..... 53
The Orion Hypothesis ..... 53
Necropolis Inter-visibility ..... 54
Giza Pyramid Alignment and Distribution . ..... 56
Pyramid Slopes - the Search for Perfection ..... 58
Is "Hidden" in the Pyramid Slopes? ..... 61
Deductions ..... 62
Why the Pyramid Shape? ..... 64
Comparative Neolithic Stone Building - Gozo and Malta. ..... 66
CHAPTER 3. OF STARS, SPELLS, SHAFTS AND SPECULATIONS ..... 71
The Ancient Night Sky ..... 71
The Pole Star. ..... 71
The Imperishable Stars ..... 73
The Legendary Stars ..... 73
Pyramid Texts ..... 74
Using the Stars ..... 77
Predicting The Inundation ..... 77
Stellar Coincidences ..... 78
Finding North ..... 79
Viewing Proportions? ..... 81
Cattle culture ..... 82
Ancient Egyptian Constellations ..... 84
Identifying the Enduring Bull of the Sky ..... 86
Navigating the Skies by the Bright Stars ..... 87
Symmetry and Sequential Integers in Architecture ..... 88
Integer Succession ..... 88
Queen's chamber ..... 88
The Angle 26.5 ..... 89
The Angle $21^{\circ} 41^{\prime}$ ..... 90
King's Chamber Dimensions. ..... 90
Purpose of the Shafts in the Great Pyramid ..... 91
Ventilation Theory ..... 91
Starshaft Theory ..... 91
Rituals and Ceremonies ..... 94
Limitations in the Starshaft Theory ..... 97
The Stone Amulet Theory ..... 99
Is there a pattern in the Shaft Angles/Slopes ..... 99
KC ( $N$ ) and the Pyramid Slope ..... 100
QC Shafts ..... 101
Looking at the Overall Pattern ..... 101
Two Primes Interplay ..... 102
The Stone Amulet Concept ..... 103
BuILDING PYRAMIDS. ..... 107
CHAPTER 4. PYRAMID LOGISTICS ..... 109
Try a Little Science ..... 109
The Pyramid Calculator ..... 109
Using the Pyramid Calculator ..... 111
How much Stone? ..... 111
How many Men? ..... 112
Time to Build the Red Pyramid ..... 117
Logistics and Feeding the Workforce ..... 117
APPENDIX 1 TO CHAPTER 6: WORK NEEDED TO RAISE KHUFU's PYRAMID ..... 123
Appendix 2 to Chapter 6. Work Output Per Ancient Egyptian Builder ..... 125
Research in Present Day India ..... 125
UK MAFF Values ..... 125
Treadmill Experiments ..... 126
CHAPTER 5. PYRAMID CONSTRUCTION ..... 127
True Pyramid Design ..... 127
Some Methods and Tools ..... 128
Building Slopes at 5150'40" - i.e. Khufu's Pyramid Slope ..... 128
Found in QC (N) Shaft ..... 129
Area of a circle. ..... 131
Constructing Shafts ..... 133
CONSTRUCTING BURIAL CHAMBERS ..... 133
Levelling the Foundations ..... 134
Pyramid Architecture ..... 135
Engineering the Pyramid. ..... 137
The Evidence in the Stones ..... 138
The Workforce ..... 138
Ramps and Ramp Theories ..... 141
Desert Ramps ..... 141
Helical and Zigzag Ramps ..... 142
Ramp Capacity ..... 143
"Rocking" Methods ..... 146
Rocking ..... 146
Rocking Statistics ..... 147
Rocker Sledges. ..... 148
PARALLEL WORKING OPPORTUNITIES ..... 149
CHAPTER 6. SYSTEMS ENGINEERING-EGYPTIAN STYLE ..... 153
Without Reductionism ..... 153
Planning the Great Pyramid of Khufu ..... 154
The Pyramid Project Bar Chart ..... 155
Behaviour Diagram ..... 156
Interpreting Khufu's Requirement ..... 157
Emergent Properties from Khufu's Vision ..... 158
PyRamid Models ..... 160
Model Building $26.5^{\circ}$ Shafts ..... 162
Sliding Plug Stones ..... 162
Constructing a Grand Gallery ..... 163
CHAPTER 7. NAISSANCE AND RENAISSANCE ..... 169
Past Pyramids ..... 169
Silbury Hill. ..... 170
Ancient Pyramidal Structures ..... 173
Middle Ages and Renaissance Pyramids ..... 177
Contemporary Pyramids ..... 179
Future Pyramids? ..... 183
Index ..... 185
Bibliography ..... 189
References ..... 191

## Table of Figures

Figure 1. Nilometer at Aswan .....  3
Figure 2. The First Flowering of Culture .....  4
Figure 3. The Inundation, Lower Egypt .....  5
Figure 4. By the Nile, Upper Egypt .....  6
Figure 5. Modern Hamlet Scene in Upper Egypt .....  6
Figure 6. Houses and Farming Models, Louvre .....  7
Figure 7. The Narmer Palette. .....  8
Figure 8. Pyramid Building Job Trees ..... 11
Figure 9. Ancient Egyptian Psyche ..... 13
Figure 10. Osiris, Tomb of Senedjem, Deir el Medina ..... 14
Figure 11. Self-Reinforcing Beliefs ..... 17
Figure 12. Sixth dynasty Rock-cut Tombs at Aswan ..... 18
Figure 13. Khasekhemwy's Serekh showing both Horus and Seth, Cairo Museum ..... 23
Figure 14. Decline of the Golden Age ..... 26
Figure 15. Famine Amongst Bedouin, Old Kingdom. Louvre ..... 27
Figure 16. N. Egypt showing the Principal Pyramids of $3^{\text {RD }}-6^{\text {TH }}$ Dynasties ..... 29
Figure 17. Artist's View of a Typical Necropolis ..... 30
Figure 18. Djoser's Stepped Pyramid, SaQQara ..... 31
Figure 19. Collage of Features in Djoser's Pyramid Complex ..... 32
Figure 20. King Djoser, Cairo Museum ..... 33
Figure 21. Djoser's Pyramid Construction ..... 33
Figure 22. DJoser's "Brickwork" and Casing Stones ..... 34
Figure 23. Meidum Pyramid Schematic ..... 35
Figure 24. Snefru's Bent Pyramid at Dahshur, North Side. ..... 36
Figure 25. Bent Pyramid: Construction and Ka Pyramid ..... 37
Figure 26. Snefru's Red Pyramid, Dahshur. West Face. ..... 38
Figure 27. Red Pyramid: Reconstructed Pyramidion, Stonework and Casing ..... 38
Figure 28. Khufu's Necropolis - Principal Components ..... 40
Figure 29. Khufu Pyramid Precision ..... 41
Figure 30. Structures in Khufu's Great Pyramid ..... 42
Figure 31. Virtual Views of the Inner Structures in the Great Pyramid. ..... 43
Figure 32. The so-called Relieving Chambers above the King's Chamber, Great Pyramid ..... 44
Figure 33. Relieving Chambers Construction ..... 44
Figure 34. Khafre's Pyramid, North Face ..... 46
Figure 35. Khafre's Valley Temple Construction ..... 47
Figure 36 Menkaure's (Unfinished) Casing and Mortuary Temple ..... 48
Figure 37. Menkaure's Pyramid and Queen's Pyramids ..... 49
Figure 38. Collage of $4^{\text {Th }}$ Dynasty Statuary ..... 51
Figure 39. Giza Pyramid Alignment ..... 53
Figure 40. Djoser's Stepped Pyramid at SaqQara Snefru's Red Pyramid, Dahshur ..... 54
Figure 41. Location of Necropolis Sites on their Mutual Horizons ..... 54
Figure 42. Terrain Simulation, Archaic Memphis to Giza ..... 55
Figure 43. Views of the Giza Pyramids ..... 57
Figure 44. Teti's Pyramid Construction, SaQQara ..... 63
Figure 45. Naturally Occurring Pyramids, Abu Simbal ..... 64
Figure 46. From Stepped to Perfect Pyramid. ..... 65
Figure 47. Conjectural View of a Pyramid as a Mound of Creation ..... 66
Figure 48. The world's First free-Standing stone structure at Ggantija, gozo, ~3600BC. ..... 67
Figure 49. Hagar Qim, Malta, $\sim 3000-2500 \mathrm{BC}$. ..... 67
Figure 50. Oracle Room, Mnajdra Temple, Malta, $\sim 3300-3000 \mathrm{BC}$ ..... 68
Figure 51. The Imperishable Stars, Giza, 28 December BC2575, 06:49 ..... 72
Figure 52. The Orion Constellation and Sirius ..... 73
Figure 53. Pyramid Texts, Unas and Teti Pyramids. ..... 75
Figure 54. Simultaneous rising of the Sun and Sirius, $6^{\text {Th }}$ July 2575BC, 04:44 ..... 77
Figure 55 Dawn Transit of Sirius, $11^{\text {Th }}$ September 2575, 05:25 ..... 78
Figure 56. Simultaneous Dawn Rising of Alnitak and Dawn Transit of Kocab, 2575BC ..... 79
Figure 57. Finding North with Kocab and Mizar ..... 80
Figure 58. Finding North ..... 81
Figure 59. Viewing and Setting Proportions ..... 82
Figure 60. Herding in the New Kingdom, British Museum ..... 83
Figure 61. Ivory Label showing Den, 1st Dynasty, Smiting an Asiatic Foe. British Museum. 83 ..... 83
Figure 62. Bull Palette. Louvre ..... 84
Figure 63. Ancient Egyptian Constellations, New Kingdom ..... 84
Figure 64. The Constellations at Denderah, Ptolemaic Period ..... 85
Figure 65. Ursa Major and Ursa Minor as the Bull of Heaven? ..... 85
Figure 66. Hieroglyph for Bull ..... 86
Figure 67. Enduring Bull of the Sky? ..... 87
Figure 68. Navigating by the Bright Stars. ..... 88
Figure 69. Queen's Chamber Dimensions in Cubits 1 Royal cubit $=52.3$ cm ..... 89
Figure 70. Forming the Angle $26.5^{\circ}$ ..... 89
Figure 71. The Angle $21^{\circ} 41^{\prime}$ ..... 90
Figure 72. Floor Plan of King's Chamber ..... 90
Figure 73. Khufu's Shafts Dates ..... 92
Figure 74 Khufu's Shafts and Annual Dawn Transit Dates, 2575BC ..... 93
Figure 75. Queen's Chamber Niche with Superimposed Statuette of Khufu ..... 95
Figure 76. The Spirit of Osiris Activates Khufu ..... 96
Figure 77. KC (N) and the Pyramid Slope ..... 100
Figure 78. Queen's Chamber Shafts Viewed as Proportions ..... 101
Figure 79. Slope and Proportion Pattern ..... 102
Figure 80. Southern Shaft Opening, Queen's Chamber, Great Pyramid ..... 103
Figure 81. Stone Amulet Concept, King's Chamber ..... 104
Figure 82. The Pyramid Calculator ..... 110
Figure 83. Granite Quarry, Aswan ..... 112
Figure 84. Bored Man Waits for Bread to Prove, $4{ }^{\text {Th }}$ Dynasty, Cairo Museum ..... 118
Figure 85. Model of a Royal Bakery, British Museum ..... 119
Figure 86. Work and Energy Parameters - Khufu's Great Pyramid. ..... 123
Figure 87. Archetypal Old Kingdom Pyramid Design ..... 127
Figure 88. A Working Platform for Constructing 26.5 degree Passages. ..... 128
Figure 89. Set Square for Aligning to Khufu's Pyramid Slope ..... 129
Figure 90. Diorite Hammer and Copper Device found in QC (S) ..... 130
Figure 91. Star-Sight Simulation ..... 131
Figure 92. Competing ideas for the "horned" tool ..... 131
Figure 93. Introducing the Method of Squares ..... 132
Figure 94. Finding the Area of a Circle using the Method of Squares ..... 132
Figure 95. Prefabricating Shafts ..... 133
Figure 96. Burial Chambers ..... 134
Figure 97. Levelling and Foundation ..... 135
Figure 98. Pyramid Construction Sequence ..... 136
Figure 99. Finishing Packing and Casing using Rocker Methods ..... 137
Figure 100. Ramps and Ramp Theories ..... 143
Figure 101. Raising Stones by Rocking ..... 146
Figure 102. Mechanics of Stone Rocking ..... 147
Figure 103. Raising Stones Using Rocker Sledges ..... 148
Figure 104. Parallel Teams Raising Stones ..... 149
Figure 105. Parallel Working - Great Pyramid ..... 150
Figure 106. Synthesis without Reduction ..... 154
Figure 107. Behaviour Diagram for Designing and Planning the Great Pyramid. ..... 157
Figure 108. Entrance to the Queen's Chamber, Great Pyramid ..... 161
Figure 109. Building a Model of a $26.5^{\circ}$ Passage ..... 162
Figure 110. Full-scale Model of Sliding Plug Stones ..... 163
Figure 111. Rules for Corbelling ..... 164
Figure 112. Grand Gallery showing Corbels ..... 164
Figure 113. Grand Gallery Virtual Model ..... 165
Figure 114. Starting to Build the Grand Gallery ..... 165
Figure 115. Grand Gallery VR Construction Seouence ..... 166
Figure 116. Unassigned Old Kingdom Statue with "Pyramid" Skirt, Cairo Museum ..... 169
Figure 117. The Construction of Silbury Hill, c.2660BC. ..... 170
Figure 118. Silbury Hill Today from the Southeast ..... 171
Figure 119. Silbury by Moonlight - Simulation ..... 172
Figure 120. The Great Temple at Chichen-Itza, ~AD1000. Simulation. ..... 173
Figure 121. Middle Kingdom Pyramid of Amenemhat III, 1842 - 1797BC ..... 174
Figure 122. Mountain Overlooking the Valley of the Kings. ..... 174
Figure 123. Frieze from the Egyptian Gallery, British Museum ..... 175
Figure 124. Pyramid Tomb Entrance, Valley of the Workers, New Kingdom ..... 175
Figure 125. The Mappa Mundi, Hereford Cathedral, England, ~AD1290 ..... 176
Figure 126. Kircher's Pyramids ..... 177
Figure 127. Noah's Ark, Gates of Paradise, 1378 - 1455, Florence ..... 178
Figure 128. Renaissance Pyramid Tombs ..... 178
Figure 129. Memorial of the Unknown Soldier in Nasr City, Cairo ..... 179
Figure 130. Egyptian Street Art. ..... 179
Figure 131. Pei's Pyramids at the Louvre. ..... 180
Figure 132. Pyramid Pair ..... 180
Figure 133. Pei Pyramid from Underneath, showing Construction ..... 181
Figure 134. Obelisk, Place de la Concorde, Paris ..... 181
Figure 135. Glass Pyramid Atop Canary Wharf, London ..... 182
Figure 136. Finishing Off Square Towers, England ..... 182
Figure 137. Pyramids of the Future? ..... 183

## Table of Graphs

Graph 1. Variations in the Height of the Annual Inundation, ~3050BC to ~2467BC .....  3
Graph 2. Evolution in Pyramid Chamber Construction ..... 52
Graph 3. Progressive Pyramid Slope by Angle and Ratio. ..... 60
Graph 4. Precessional Wander of Thuban's Dawn Transit ..... 98
Graph 5. Culmination Dates by Shaft in the Great Pyramid ..... 99
Graph 6. Accumulated Weight of the Great Pyramid in Tonnes ..... 111
Graph 7. Work Needed to Raise Khufu's Pyramid by Tier ..... 113
Graph 8. Manpower to Build Khufu's Pyramid ..... 114
Graph 9. Work Associated with Other Necropolis Features ..... 114
Graph 10. Great Pyramid Complex - Annual Manpower Estimates ..... 115
Graph 11. Estimated time to Complete Khufu's Great Pyramid ..... 116
Graph 12. Red Pyramid Build Rate - Theory Vs. Practice ..... 117
Graph 13. Bread and Beer Logistics, Great Pyramid, Proportional Manpower ..... 118
Graph 14. Grain Boat and Stone Boat Deliveries per day - Great Pyramid ..... 119
Graph 15. Acres of Emmer Wheat to be Cultivated, Tier by Tier ..... 120
Graph 16. Calculated Time to Raise Tiers of the Great Pyramid. ..... 138
Graph 17. Comparative Rates of Setting Stones - Great Pyramid ..... 139
Graph 18. Instantaneous and Cumulative Rates of Setting Stone - Proportional Manpower, Great Pyramid ..... 140
Graph 19. Instantaneous and Cumulative Rates of Setting Stone - Constant Manpower, Great Pyramid ..... 141
Graph 20. Volumes of Truncated Pyramid and Notional Desert Ramp ..... 142
Graph 21. Parallel Paths needed on Ramps - Great Pyramid ..... 144
Graph 22. Comparing the Volumes of a Truncated Pyramid and a Helical or Zigzag Ramp ..... 145
Graph 23. Bar Chart for Constructing the Great Pyramid Complex ..... 156
Table of Tables
Table 1. Early Pharaohs, Dates and Titles ..... 22
Table 2. Comparing Pyramid Slopes ..... 58
Table 3. Calculating PI for Old Kingdom Pyramids ..... 62
Table 4. Putative Annual Ceremonies, $\sim 2450 \mathrm{BC}$ ..... 94
Table 5. Elevation Angles of Stars, ~2575BC ..... 98
Table 6. Patterns in Khufu's Shafts ..... 100
Table 7. Energy Utilization Data ..... 125
Table 8. Designing the Pyramid Complex akhet Khufu ..... 158

# The Pyramid Builder's Handbook 

## Preface

Inever intended to write a book about ancient Egypt. It just happened. I happened to be attending a symposium in London. Along with many others, I gave a lecture on systems engineering which was unremarkable except that I included a picture showing four pyramids that had been built over some 100 years. I suggested to the audience that this was an early example of prototyping, and thought nothing more of it.

A couple of months later I got a call from a chap in Houston, Texas, asking if I could go there to talk to a group of scientists. They were working on the then-fledgling Orbiting Space Platform, and they wondered if there were lessons to be learned from the way that the ancient Egyptians had organized the building of the pyramids. I had agreed to go before I realized the enormity of what I had taken on - nobody knows how the ancient Egyptians organized themselves.

I went into a brown stew for a month, before coming up with what might be the way in which they organized and controlled the building programme. I say might, because of course we really do not know, but the method that I came up with is plausible, within the then limits of technology, and would work. (See Pyramid Models on page 149). Was it of use to the scientists? I fear not. But it made me think.

Systems engineering was going through one of its periodic upsurges in popularity at the time, and I was keen to propagate the message to hard-line conventional engineers who couldn't see the point of looking at the big picture - essentially one of the central planks of systems engineering. I hit on the idea of presenting systems engineering in more palatable form - systems engineering the pyramids. I called the lecture, which was to become a long-running series, Understanding the Pyramids, and it was an immediate hit.

I recruited a willing partner in crime, Paul Budgen; we were both members of the UK's Institution of Electrical Engineers. The presentation became a play. He was Khufu, the tyrant king, and I was Hemon, his architect for the Great Pyramid. I wrote a script that had Khufu saying unspeakably rude things to the snivelling excrescence (me) that was trying to build his pyramid. In this way we showed both good systems engineering and bad management at the same time - and the audiences loved it.
Only, the audiences started to change in their nature. Engineers started to bring first their spouses and then their children. Egyptologists came too. We found ourselves invited to lecture not only across industry in the UK and Europe, but also in such places as International Schools. And we found ourselves facing questions from the alternative fringe that follows the pyramids - askance at suggestions that the pyramids were built by ancient Egyptians, not men from Mars, Atlantis, etc.

In response to questions of all sorts, the Egyptology content grew, and I found myself creating dynamic virtual reality (VR) models of pyramid construction, so that I could
prove to audiences - as well as myself - just how pyramids, passages, galleries, etc., were constructed.

In building the models, a kind of understanding did, indeed, develop. I could not say for certain precisely how the ancient builders worked, but I could certainly eliminate a whole bunch of options that constructing the VR models showed to be quite impractical.

In response to other questions I had to show how the stars looked over ancient Egypt, how they measured their time and how they knew when the Inundation of the Nile was due. And it was about then that cracks started to become visible in some of the cherished beliefs held by many in our audiences. Theories that I, too, had found attractive about the pyramids turned out, on proper investigation, to be invalid.

Just as I never intended to write a book, I never intended it to debunk the theories of others. Any debunking, and there is quite a lot in the following pages, came about simply by trying to check the facts and finding that they didn't - check, that is.

For instance, everyone assumes that the ancient Egyptians used a ramp to drag heavy stones up the pyramid. It is straightforward to check the amount of work involved and the number of men needed to execute that work. When you also work out, very simply, how many men are dragging stones on the supposed ramp at the same time, surprise - the ramp becomes a enormous multilane highway screwing its way into the sky! Had nobody done the simple sums before?

Throughout all of the lectures, and the series extended over 5 years, I spent much of my time wondering about the pyramid builders. For an engineer, I suppose it was natural. How many, how to feed them all, shift work, changeovers, etc. But most of all, what was going on in their minds? How did they manage, 4,500 years ago, to build edifices that we just simply could not emulate today?

So, although this book is about stars, mythology, pyramid construction, ancient Egyptian mathematics and a host of other things besides, it is mostly about trying to glimpse the psyche of those alien ancient Egyptians. I hope you have as much fun exploring its pages as I did learning what to write in them. And may you, too, glimpse that alien culture from 4,500 years ago.

## Part 1

## Pyramids:

Environment and Culture

# The Pyramid Builder's Handbook <br> Chapter 1. Emergence 

## The True Classical Period

It seems to be an accident of the Renaissance that Europeans rediscovered Greece and Rome, while remaining unaware of the glory that was Egypt. Open any book on the so-called Classical Period and it will refer largely to Greek and Roman history, art, architecture, society, law, medicine, or philosophy. Yet the ancient Greeks and Romans learnt much from the Egyptians. In looking to ancient Greece and Rome for inspiration and understanding, Renaissance scholars were seemingly unaware that Egypt was the primary source.
Ancient Egypt endured as a civilization and as a culture longer than any other on Earth. The religion of Ancient Egypt, peculiar though it might seem to our modern eyes, endured for at least 3000 years, much longer than either Christianity or Islam so far. The sheer durability of the culture seems astonishing in today's volatile world. But then, the conditions that gave rise to Egypt were unique.

In Mesopotamia and later, in Greece, civilization centred on warring city-states. Compared with Egypt, they did not last long. While Egypt seems to have had its share of early "tribal" conflict, co-operation appears to have broken out along the banks of the Nile to preserve the precious water of the Inundation in irrigation channels. A uniquely spiritual culture evolved there that encouraged further co-operation and mutual helping.

## Environment and Society

## The Gift of the Nile

In 440BC, Herodotus of Halicarnassus ${ }^{1}$ wrote: "For any one who sees Egypt, without having heard a word about it before, must perceive, if he has only common powers of observation, that the Egypt to which the Greeks go in their ships is an acquired country, the gift of the river."

That Egypt is the gift of the Nile has become a cliché. Herodotus meant it in a quite literal sense, however. The very land upon which the Egyptians lived had been washed down from the mountains; without this alluvial deposit there would be no fertile land of Egypt, only bare rock and sand. Herodotus observed that there was very little sand in Egypt.
"... I have seen that...shells appear in the mountains, that salt forms crusts on the surface of the ground and corrodes even the pyramids, and that the only sandy mountain range in Egypt is the one overlooking Memphis...Egyptian soil is black and friable, which suggests that it was once mud and silt carried down from Ethiopia by the river ${ }^{2}$ "

The annual Inundation of the Nile, the longest river in the world, resulted from the spring rains and the melting of snows in the mountains far to the south. Because of the considerable distances involved, the melt did not reach Lower Egypt until the beginning of July, at the height of the Sun's effect. By this time, it carried a wealth of
black silt, picked up and carried by the fast river flow in the mountains. As the water slowed coming into Egypt, only the finest particles remained in suspension to be carried by the waters as they overflowed the banks, covering fields and filling ditches and canals. The combination of heat and summer moisture offered unique fertility, with farmers using irrigation channels to trap and divert precious water. The barren desert to either side of the narrow strip oasis along the banks of the Nile was called the Red Land. The narrow strip of very fertile land along the Nile was called the Black Land, $K M T$, which also came to mean Egypt.

The Nile valley was quite isolated in early, pre-dynastic times. Migrations of early man from East Africa up the Great Rift Valley appear to have followed the route of the Nile before continuing northwards towards present day Israel and beyond. Such migrations were long over, perhaps leaving pockets of early man along the way.

In Mesopotamia, the Euphrates and the Indus rivers flowed from north to south at such rates that they were often non-conducive to transport in both directions by simple rafts and boats. In contrast, the Nile flowed from south to north, with flow rates that allowed travel both up and down the river at most times of year. In addition, the prevailing wind direction was from north to south, helping sailboats to sail south up the Nile, against the flow.

## Inundation - Scourge and Benefactor

Just how variable the Inundation could be during the Pyramid Age is recorded, Graph 1. The Palermo Stone* was carved in $\sim 2400 B C$, and provides the oldest significant extant chronology in history ${ }^{\dagger}$. Of particular interest is the recording on the Palermo Stone of the annual Inundation height ${ }^{3}$. While the record is not continuous, it nonetheless enables the general trend in Inundation height at the time of pyramid construction to be seen.

The graph shows plots the Inundation heights in metres ${ }^{\ddagger}$ over a period of nearly 600 years, taken from the Palermo Stone. The height is clearly highly variable, even allowing for the many gaps in the annual records. The trend line also shows that, over the period in question there was a gradual, but undeniable, diminution in the average level. Weather patterns changed around that time, and it may be significant that the quality of pyramid construction tails off towards the end of the same period.
The high variability in the height of the annual Inundation was of great concern to the people of the Nile. A high Inundation threatened their lives, their livestock, their homes and their fields. A low Inundation threatened poor crops and potential famine. Small wonder that the ancient Egyptians became obsessed with anticipating both the timing and the height of the Inundation, on which they severally and jointly depended for existence.

[^0]

The ancient Egyptians measured the Inundation height using so-called Nilometers, strategically placed up and down the Nile. The best known of these, today, is that at the Island of Elephantine, Aswan, Figure 1, where the Inundation rose some 20 cubits", whereas the figures recorded on the Palermo Stone average some 4 cubits. This suggests that the Nilometer that provided figures for the Palermo Stone was situated further down river, possibly near the Delta region. Here, even slight variations in Nile height could lead to extensive flooding, as the land was flat.

The Palermo Stone and its many inscriptions tell us that the ancient Egyptians were adept with measuring and number systems as well as writing and recording. More, they became adept at predicting the eventual magnitude of each Inundation by examining its early rates of rise using the Nilometer: a rapid rise forecast a higher peak Inundation.

The ancient Egyptians also learned how to forecast the date of the Inundation using the stars. Herodotus recorded their use of the stars:


Figure 1. Nilometer at Aswan

[^1]"The Egyptians...were the first to discover the year, and to distribute throughout the year the twelve parts into which they divide the seasons. They said they discovered this from the stars. It seems to me that the Egyptian monthly system is cleverer than the Greek one; the progress of the seasons forces the Greeks to insert an intercalary month every other year, whereas because the Egyptians have twelve months of thirty days and add five extra days to every year, the seasonal cycle comes round to the same point in their calendar each time ${ }^{4,}$

Additionally, the ancient Egyptians had developed a cunning method for establishing annual dates using the stars. They observed when a star came over the horizon in the early morning at just the same instant as tip of the sun appeared. These so-called heliacal risings* occur on the same day of the year for any particular star. In particular, the ancient astronomers noted that the brightest star in the sky, Sirius, rose with the Sun just at the time of the Inundation on the $5^{\text {th }}$ of July each year. So Sirius heralded the start of the Inundation, which in its turn marked the ancient Egyptian New Year.

Cooperation - Need and Opportunity


The relatively benevolent environment contributed to the first flowerings of a unique culture, Figure 2. The Inundation not only provided the basis for survival and food production, it also encouraged the development of simple technology for irrigation. The Nile provided the ideal transport medium to propagate this co-operation up and down its length. Such propagation was important, since irrigation that ensured food for the many could be damaged by the incompetence or neglect of the few. Moreover, and important in such a hothouse climate, the annual Inundation cleaned the valley,

[^2]sweeping before it all detritus and rubbish, leaving the valley refreshed and revitalized.

The Inundation provided one other essential - spare time. The waters of the Inundation covered the farmland to either side of the Nile, obliging the farmers to retreat with their families to raised ground. There they might stay in cramped surroundings for anything up to three months, with little farming work to do, waiting for the waters to recede. This enforced leisure may have been a two-edged sword. On the one hand, it gave opportunity for intellectual developments, thoughts and pursuits, for the development of skills, and for observations and teaching. On the other hand, it left much of the working population, the farmers, with time on their hands.
Concerned as they were with the unpredictability of the Inundation, the people were concerned to ensure a continual, ample, but steady Inundation through prayer, supplication, lustration and dedication - see the dotted line, which "closes the loop" in the figure above. The Inundation and the lush vegetation it supported created a land that must have seemed like heaven to its fortunate people, Figure 3.


Figure 3. The Inundation, Lower Egypt
In the present day scene beside the river Nile, Upper Egypt, Figure 4, little will have changed since the Old Kingdom. People are harvesting palm fronds, using the shade of the trees to shelter from the sun.
One feature will have changed: the ancient River Nile would not have had such a well-defined, shallow bank. The present bank is determined by the modern Aswan Dam, which regulates the steady flow of the Nile. In ancient times, with the Inundation's rise and fall, the banks would have been alternately submerged and exposed. Depending upon location, the banks may have been occasionally much higher than the river, necessitating docks and landing stages. These in their turn would run the risk of being swept away by the next Inundation.


Figure 4. By the Nile, Upper Egypt
So, there would have been constant, if steady, activity along the Nile: farming the land left fertile by the retreating waters with their rich silt; irrigating to direct and retain the waters as they became available and leeched away each year; moving shelters, cattle, storage, etc. to higher ground, and then back again; and continually rebuilding shelters, dykes, canals, landing stages and docks damaged by the Inundation.


Figure 5. Modern Hamlet Scene in Upper Egypt

High ground was in short supply near the water's uncertain edge. Archaeological excavations have shown that village was built on village, upon village...to such an extent that it is difficult to unravel the many-layered sequence. With so little change, a recent view, Figure 5, suggests how a typical village or hamlet might have appeared over 4,500 years ago.

There is a pond, with ducks and geese. The animals live in and around the village. Donkeys, cows and goats graze, while people watch and go about their business. Even today the houses in such hamlets and villages are built very close to each other to economize in the use of land that could otherwise be put to cultivation. In contrast, the cultivated fields were, and still are, laid out with neatness and precision from water's edge to desert's edge. Irrigation water is allocated to each farmer according to an ageold formula ${ }^{5}$, to just meet his needs.
Farming methods were simple, Figure 6, largely due to the friable black soil. The cattle are shown pulling a simple ard to score the soil, rather than a plough to turn the soil, which was unnecessary. Housing was made from sun-dried Nile mud, with the corners turned up for added strength ${ }^{*}$.


Figure 6. Houses and Farming Models, Louvre

Herodotus records ${ }^{6}$ that mosquitoes were a problem. "Here is how they cope with the huge numbers of mosquitoes there. Those who live inland from the marshes (Delta) have the advantage of living in tall buildings, (top of Figure 6) which helps because the winds stop the mosquitoes flying high in the air. Marsh-dwellers, however, have come up with an alternative. Each man there has a net which he uses in the daytime for fishing, but at night...he drapes it over the bed...Mosquitoes...do not even try to bite through the net at all."

## Societal Structure

## Joining of The Two Lands

It was in the Nature of the way that ancient Egypt had formed that it divided naturally into two lands. The mouth of the Nile had formed into a large, flat delta region, with channels and rivulets crossing a constantly changing marshy expanse. This area became Lower Egypt.

[^3]Upper Egypt, by contrast, was a narrow strip oasis formed between two mountainous ridges. In places, the width of the fertile strip could be as little as a few hundred metres.

As population grew along the Nile , communities formed. Largely self-sustaining, these communities developed their own societies and organizations, beliefs and gods. There is some evidence in early ceremonial stone palettes of tribal conflict, but by about $3000-2900 \mathrm{BC}$ a number of autonomous confederations appears to have developed along the Nile. Several of these confederations had even joined to create a pre-dynastic kingdom ${ }^{7}$ in Upper Egypt.


Around this time, the confederations of Southern Egypt joined to create a single group so powerful it could dominate the north as well, creating one country, with one king. This seems likely to have been a process, taking several years or even decades, rather than a single cataclysmic event. The famous Narmer Palette, Figure 7, offers a cultural milestone of the period. King Narmer is identified wearing the Red Crown of Lower Egypt on the obverse and the White Crown of Upper Egypt on the reverse. The god Horus supports him, presenting some 6,000 prisoners led by a rope through the nose, and there is a record on the palette of many killed ${ }^{\dagger}$.

[^4]
## Kings, Pharaohs and Dynasties

The first king of Upper and Lower Egypt is said to be Menes. There is uncertainty as to whether or not Menes and Narmer were one and the same. Be that as it may, the Palermo Stone lists a number of predynastic kings, and the kings of Egypt up to the $5^{\text {th }}$ Dynasty Pharaoh Neferirkare. In the $3^{\text {rd }}$ Century BC Manetho, a high priest of Heliopolis*, divided the king lists into sets or dynasties; his divisions are still in use today. The various dynasties often reflect the area of origin of the kings or pharaohs ${ }^{\dagger}$. Kings of the fourth dynasty, for instance, are called Memphite because they came from the area of Memphis.
Groups of dynasties are organized into periods. So, Dynasties I and II form the Archaic Period. Dynasties III to VI form the Old Kingdom, the principle period of Pyramid building.

## Feudal hierarchy

Areas of population called nomes formed along the Nile, analogous to counties or cantons. Nomes were governed by nomarchs. The nomarchs that governed each nome would have been feudal lords, controlling and supporting their respective populations, while owing allegiance during pharaonic times to the king in Memphis, the Old Kingdom capital. Nomarchs may have contributed some equivalent of a tax to the central organization, not in money (which had yet to be invented), but in resources such as food, clothing and perhaps work "volunteers" for major national projects such as dams, canals, temples, royal mastabas ${ }^{\ddagger}$ or pyramids. The gathering together of a national workforce has been dubbed the corvée. There does not appear to be any direct evidence for a formal tax system, but it is not unreasonable to assume that it existed in some form, even if the participants did not view it as taxation.

Within each nome there would have been a small elite, the ruling class, each with its own provincial culture, beliefs and practices. They may have had their own schools; they would have had scribes of their own, and may have accommodated viziers, scribes and accountants from the king. The elite would have been wealthy and powerful in their nomes, and would have had their own necropolises, although many may have aspired to being buried in the central necropolis for Memphis at Saqqara (named for Sokar, the god of the dead), or to the traditional site at Abydos in Upper Egypt. For the elite, life would have been good.
For the farm worker or cattle herder, quality of life would have been less assured, depending largely on the Inundation, which could be too high or too low. The land was so fertile that up to 3 crops per year could be grown, implying a steady work pattern for the farmer. Generally, however, it seems likely that the farmers and their families would have been simple and reasonably contented, since many seemed to be aware of their incredible good fortune, living in the uniquely fertile Nile valley.

[^5]So, there emerges by the time of the Old Kingdom a picture of a largely agrarian society living up and down the Nile in the many nomes, but nonetheless acting as a unified society under the direction and control of the king in Memphis. Kings travelled the Nile each year, taking their palace entourage with them. They visited and worshipped at temples and shrines along the Nile; they also initiated, and sometimes personally supervised, important building projects. In any event, the pharaoh would not be a stranger to his people, many of whom would inevitably see him and his ships as they passed by on the Nile.

## Bureaucracy

Viziers also travelled the land, supervising and reporting on projects, dispensing the king's justice and communicating between the nomarchs and the king. An extensive and growing bureaucracy is also evident from such records as the Palermo Stone and, later, from an archive of papyrus scrolls ${ }^{*}$ in the tomb of Raneferef, a short-lived $5^{\text {th }}$ Dynasty pharaoh whose pyramid was never finished, which show an obsession ${ }^{\dagger}$ with record keeping.

With kings, nomarchs, viziers and scribes to keep records and to create and convey orders, instructions, policies and plans, Egypt evidently had the means of establishing a bureaucracy. That bureaucracy, the first such in history, effectively held the disparate parts of the fledgling nation state together, and was essential as a foundation upon which to launch and manage major building projects.

## Energy work and Organization

The River Nile was the highway for trade and commerce. In contrast, then, to the gentle agrarian pace, there would have been riverside centres of hustle and bustle, with shelters, markets, street traders, children stealing food, men drinking together while women carried produce and animals wandered everywhere. As it has been ever since, there would have been the inevitable banter between the farmer and the "townie". The whole network of activities and interactions would have been regulated and punctuated by the seasons.
In addition to this bucolic scene, there was a complex interwoven fabric of commerce and construction, with potters, boat builders, stonemasons, carpenters, sail makers, rope makers, papyrus gatherers, thatchers, brick makers, tool makers, and many more.

There were literally hundreds of jobs to be done, and people that would specialize in those kinds of job. Many professions would be family affairs, with skills handed down from father to son, mother to daughter. Others would be more specialized. A child that showed special talent for drawing might be taught in a special school devoted to that skill. This would be necessary for any rôle where the skill was born to a person rather than learned.
There would be boatyards, docks, potteries, mills, granaries, quarries, jewellerymakers, bakeries, papyrus "factories", chandlers...indeed all the features necessary to

[^6]

Figure 8. Pyramid Building Job Trees
support thriving, creative, energetic communities up and down the Nile. Moreover, these many features were integrated and networked into a social structure, so that the stonemasons could get bread and the sail makers could get ropes. Since money had not yet been invented, barter was the order of the day, which would have invoked markets with vendors and buyers trying to reach acceptable deals.
Figure 8 shows some simple job trees. These connect together the many and various jobs and skills that must have both existed and interacted for the pyramids to have been conceived, designed and constructed. Probably the most important tree, since without it nothing else would have happened, is at lower left in the figure. There must have been tools for farming, canal digging and irrigation, and so there must have been toolmakers. Given irrigation, there would then have been farming, including flax for linen clothing, cattle, grain and breweries for food and drink, papyrus collection for rope and paper manufacture, and so on.
Looking at the simplified figure, it can be seen that sustained building programmes entailed organization, networking and integration on a grand scale. It is no exaggeration to say that Egypt could not have built the pyramids without national mobilization, which in turn depended upon central management and control. Pyramid building made Egypt and bound the many nomes into one nation - the first nation state.

One of the diversions to prevent men turning to infighting, during the enforced inactivity of the Inundation, may have been the corvée, the calling up of men to help
build the pyramid. Herodotus was told by priests that to build the Great Pyramid of Khufu (Cheops ${ }^{*}$ ):

> "They worked in gangs of 100,000 men for three months at a time ${ }^{8} . "$

Such large numbers, if indeed they were correct, suggest that a corvée would have been necessary. Even then, it seems unlikely that the figure would be as high as 100,000 out of a population of only some 1.5 million. A figure of $20-25,000$ seems much more likely, and calculations (q.v.) suggest that Khufu's Pyramid may have taken some 23-25 years, rather than the 20 years that Herodotus was told, some 2000 years after the event.

The wonder of pyramid building is in organizing so many people to work together and retain their focus on such a complex task over such a long period. We could not do it today, not because we lack the technology, but because we lack the motivation, dedication and determination: quite evidently, the ancient Egyptians of the Old Kingdom did not. And here we come to the nub of the matter; just how did the, then, society that was ancient Egypt view and tackle the building of pyramids? To understand that, we will need to look deeper into the processes of social evolution.

## Social Psyche

The ability of the ancient Egyptians to construct massive civil engineering projects taking many years is indicative of a unified strength of purpose and dedication that is difficult to explain in terms of our modern social and behavioural ethics.

Our current incomprehension leads some to suggest that the Ancient Egyptians did not build the pyramids at all, but that some other mystery race was responsible. Such ideas may be fanciful ${ }^{\dagger}$, but unless and until we can explain how it was done, there will always be doubts and sceptics.

Hollywood, on the other hand, has suggested that the ancient Egyptians used slave labour. There is no evidence for this. On the contrary, archaeological evidence from the Giza Plateau shows that the pyramid builders were cared for during life, and honoured in death ${ }^{9}$. It seems, then, that the ancient Egyptian people built the pyramids willingly, even enthusiastically, even though some of them took over 20 years of sustained, high level effort.
If we could explore the social psyche of the people of ancient Egypt at the time when they built the pyramids, it might lead us to understand how they came to perform a feat that we would be unable to match today-notwithstanding our modern technology.

## The Ancient Egyptian Psyche

Jungian philosophy seems singularly apposite to delve into the depths of the ancient Egyptian psyche ${ }^{10}$. Carl Gustav Jung ${ }^{11}$, the Swiss psychologist, famously developed the idea of the collective unconscious and of archetypes.

[^7]
## Archetypes and Duality

Jung identified three main "strata" in the mind: the conscious, the personal unconscious and the species-wide collective unconscious. Archetypes and instincts are the patterns of thought and mythical images that go to make up the collective unconscious. Jung held that ${ }^{12}$ :-
"The collective unconscious can be negatively distinguished from the personal unconscious by the fact that it does not, like the latter, owe its existence to personal experience, and consequently is not a personal acquisition...
"The contents of the collective unconscious have never been in consciousness, and have never been individually acquired, but owe their existence exclusively to heredity.
"Whereas the personal unconscious consists for the most part of complexes, the contents of the collective unconscious are made up essentially of archetypes."


## Figure 9. Ancient Egyptian Psyche

These ideas have been both admired and ridiculed since Jung expounded them. Suggestions that human patterns of thought and behaviour can be inherited are an affront to some, smacking of Lamarckism*. More recently, research into identical twins separated at birth is indicating that much of our behaviour, the clothes we like, the music we play, the partners we choose, even our politics, are "genetically predisposed."
Could it be that the collective unconscious of the Egyptians bubbled up into their personal subconscious and hence, in turn, to their conscious, more than is the case

[^8]with adults today? That would go to explain the outpouring of zoomorphs, with so many animal and animal-headed gods, Figure 9.
Archetypes of thought, belief and behaviour abounded in ancient Egypt. One such was the archetype of the Divine King, the king as both man and as a living god*. Another was of the King as Shepherd. In the same way as a shepherd defends his flock, the king is perceived, and perceives himself, as responsible for the people, although not of the people ${ }^{\dagger}$.
Duality is a powerful psychological influence. In ancient Egypt, duality was an obsession, perhaps accentuated by their rectilinear environment. The Nile flowed from south to north. The prevailing wind blew from north to south. The sun went from east to west. The Moon balanced the Sun. There were two terrains - black, fertile Nile valley and dangerous red desert. There were two lands - north and south, represented by the sedge and the bee, the red crown and the white crown respectively. The king or pharaoh was "neb tawi", "lord of the two lands"; it was his rôle to reconcile the north and the south, Lower and Upper Egypt, at all times. The king's placenta was mummified and used as a totem; seemingly, it served as a placeholder for his "twin" that is for his position in the afterlife.
Duality even extended to the afterlife. For many people, it seemed that their view of the afterlife was as a replica of their earthly existence, but with all life's problems and difficulties removed, with ample servants, rich crops, and comfortable leisure.


Figure 10. Osiris, Tomb of Senedjem, Deir el Medina
Duality was epitomized in the Osirian legend. Osiris, Figure 10, originally an earthly king, was god of the underworld or afterlife. Osiris had a brother, Seth. Osiris and Seth fought. Seth won, and cut Osiris into 15 parts, scattering them around Egypt. Isis, wife and sister to Osiris, found 14 parts, bound them together with help from Anubis, making the first mummy. The crucial part was missing, so Isis made a phallus from Nile mud in place of the missing piece. Isis opened Osiris’ mouth, breathed life

[^9]into him long enough to take his seed. He returned to the Other World*. Nine months later, Isis bore Horus, who grew up to avenge his father, Osiris. Horus and Seth fought; Horus lost an eye, Seth his testicles, making Horus in effect the winner.

This simplified account, of which there are many variants, has sufficient detail to explain many of the otherwise obscure ideas. The jackal-headed god Anubis (represented in Figure 10 by the two animal shapes on the poles) for instance, wears winding tapes about his neck because he is the god of funerary rites, having helped Isis create the first ever mummy by binding the parts of Osiris together.

Seth represents the forces of chaos, while Osiris represents those of order. By extension, Seth represents the desert and its dangers, while Osiris represents the lush vegetation of the strip oasis that was the safe environment of ancient Egypt. The phallus made from Nile mud symbolizes the source of fertility. The whole is a story of continued fertility in the face of obstacles combined with the eventual triumph of good over evil ${ }^{\dagger}$.
Ancient Egyptians sought order, Ma'at, which represented harmony, truth, justice and cosmic order. There appeared to be a sense that there was a proper way for the world to work, for people to behave and for events to unfold. The sense of order evident in Ma'at resonates with duality, since there was both a proper way to behave, and an improper way, there was cosmic order and worldly disorder, and because a parallel was seen between good behaviour in this life and enjoyment of the life thereafter.
Ma'at was an especially remarkable feature of ancient Egyptian society, one that marked it out from other emerging societies. Ultimate responsibility for ensuring and administering Ma'at lay with the Pharaoh. The rule of Ma'at spread throughout Egypt and persisted for thousands of years. It dictated "proper" behaviour, courtesy, honesty, justice and co-operation. It encouraged tolerant and sympathetic behaviour by the rich and powerful towards the poor and weak.
Under the rule of Ma'at, the people saw themselves on a par with the land, objects, animals and artefacts, all of which could house spirits. The ancient Egyptians had a reverence for their environment that would prove strikingly at odds with that of later cultures and theologies, where humans would see themselves as somehow above animals. Ma'at also promoted an attitude of tolerance and openness; there is little evidence of ethnic hostility, colour prejudice, religious intolerance or female subordination in ancient Egypt. At some stage, not entirely certain, Ma'at became projected as a goddess, and symbolized by an ostrich feather - the "Feather of Truth," which would figure in the Hall of Judgement from the Book of the Dead ${ }^{13}$. All of that was, however, for the future.

[^10]
## Emerging Belief Systems

The ancient Egyptians developed a sophisticated spirit world, not least in respect of themselves. There could be a variety of spirits associated with an individual, but three seemed most important.

The $K a$ represented the life essence of an individual. $K a$ came from the ancestors, to whom it essentially belonged. $K a$ was similar in many ways to the Chinese concept of life essence, the $c h i$, and to Jung's libido. In statuary, the $K a$ was represented by a human figure, not indicative of the particular person, with two arms mounted on the head in prayer and supplication. The $K a$ could travel, perhaps even to the stars.

The $B a$ was a bird-like creature with wings and a head representing the person, which could fly and which visited places that the person had visited, and enjoyed, during life. The $B a$ could travel up and down the length of the Nile, and might be able to reach the stars, too. In later times, the $B a$ was to be seen in the Hall of Judgement ${ }^{14}$, so was evidently able to travel there, although that may have been a later development in the theology

The Akh-"shining one" was pure spirit of light, the final state of spiritual existence. The $K a$ would eventually transmogrify into the $A k h$.

Together, they form an interesting triad*. It seems that the $K a$ and the $B a$ required the body remains to be in good order for their continued existence. Mummification was intended to provide and maintain this good order, as was protecting the final resting place of the mummy.
The Osirian Legend indicated a reverence for tradition and for ancestors. Spirits infused throughout their beliefs. Inanimate as well as animate objects could have kas, and gods could not just inhabit mountains or rivers, but could be those mountains or rivers. In a world with no science, beliefs would emerge simply on the basis of their believability and effectiveness, Figure 11. The upper loop in the figure refers to individuals, while the other loops show the effects of beliefs on societies:

A belief system gives to a believer a straightforward, easily understood view of the world. He or she no longer has to work everything out from first principles, and no longer has to seek all the evidence. Beliefs provide believers with a simplified World View, enabling them to interpret everyday events and situations with ease and speed. This reduces psychological uncertainty, so reinforcing confidence in the belief and the concomitant World View. Beliefs, then, need have no relevance to "ground truth", but will persist just so long as they work. If worship and sacrifice intended to make the sun rise are successful, and the sun does, indeed rise, then belief in worship and sacrifice are reinforced.

The lower loops work over a wider scale. Belief systems generally propagate ideas of what is "good" and "bad". Associated with these are ideas of reward and punishment. So, it is good to pray and give offerings for the Inundation. To do so will be recognized and confer status on the persons so doing. Not to do so will detract from the person's standing. Belief systems also frequently generate some icon of the belief, such as the Pharaoh. This icon is not the Pharaoh as an individual, but an awesome,

[^11]
archetypal image of what a Pharaoh is like, his pervading power, godliness, humanity and majesty. These aspects of belief combine to encourage co-operative social behaviour, social cohesion and the gradual development of power structures, often devoted to maintaining the icon. With power structures there generally arises some means of education or indoctrination into the rituals and tenets of the belief system, so that the young, or outsiders may be brought into the ranks of the true believers.

Taken together, the sets of loops suggest how beliefs, no matter how irrational they might seem outside the group or society, can spread widely and become set in place. There is survival advantage in holding to some beliefs; going into a forbidden area may be unwise because it is infested with venomous snakes. The area becomes taboo, and soon the specific reason for the taboo is forgotten; only the belief remains. Shared beliefs, although perhaps without any rational substance, serve to bind a society together, instilling a sense of belonging.
One of the curious features of the ancient Egyptians is that their beliefs and practices appear to have been generally benign, at least towards their own peoples. Greek and Roman gods were separate from the people and delighted from time to time in visiting plagues, famine and pestilence on the population. For the ancient Egyptians, the gods were much more at hand and around, pervading the environment and the atmosphere. Most of the gods seemed either indifferent to humanity or, if not kindly disposed, could be persuaded to look favourably.
It seems likely that most ancient Egyptians believed in an afterlife, and that they had a chance of enjoying one. During the $4^{\text {th }}$ Dynasty, the afterlife for the Pharaoh appears to have been much more certain than for everyone else, but the building of tombs and mastabas into "cities of the dead" around pyramids such as we see at Giza suggest that the nobles certainly believed they were "in with a good chance". Recent discoveries of tombs for the pyramid builders at Giza suggest that they, too, felt they would be blessed. Perhaps this potential afterlife was one of the rewards for devotion to building the pyramids.

Beliefs and practices were widespread; Figure 12 shows the $6^{\text {th }}$ Dynasty tombs of the Governors of Kush: Mekhu, Sabni, Khufhr-Hor and Heqaib, leaders of the time. The tombs are cut into the side of a hill, the "Top of the Winds", on the west bank of the


Figure 12. Sixth dynasty Rock-cut Tombs at Aswan
Nile, forming the necropolis of the princes of Aswan. Heqaib was, in life, Pepynakh ${ }^{15}$, Governor of Elephantine in the reign of Pepi II of the $6{ }^{\text {th }}$ Dynasty. Pharaoh Pepi, who reigned longer ( 94 years) than any other king in recorded history, was so impressed with Pepynakht that he was considered to be the ideal, loyal servant. After Pepynakht died, he was deified and called Heqa-ib (literally "ruler-heart", translated loosely as "king's close friend", or "king's confidante"). Heqaib was worshipped at his shrine until the end of the Middle Kingdom.

As the ancient Egyptians had emerged from a local, tribal background with inter-tribal conflict, it is likely that old tribal loyalties persisted long after the first pharaohs had imposed peace. Such loyalties transcend time. Nomes and their boundaries would most probably have reflected old tribal areas, and nomarchs may well have been chosen from tribal elites. Each nome would have its own local gods and demons. Ancient Egypt was unique in accepting and incorporating these chthonic deities and icons into an overall theological fabric, in which there were both local and state theologies.
There appears to have been a hierarchical structure to the various families of deities, with sacred triads of deities representing the archetypal family (father, mother, child) and enneads of nine deities representing creation/cosmology sets. The Theban Triad of the New Kingdom was Amun, Mut and Khonsu, for example. Amun, "the hidden one", first appeared in the Pyramid Texts ${ }^{16}$, carved on pyramid chamber walls in the $5^{\text {th }}$ and $6^{\text {th }}$ Dynasties - although these texts appear to be copies from much earlier
works. Mut was a sky goddess, in the form of a cow. She adopted Khonsu, the ancient Egyptian god of healing, who later became a war god.
The most famous Ennead would be that based at Heliopolis: Ra-Atum (combined in the $2^{\text {nd }}$ Dynasty as the sun-and-creator god); Shu (air or void); Tefnut (moisture); Geb (earth); Nut (sky); Osiris; Isis; Nephthys (sister to Osiris and Isis, wife to Seth); and Seth (chaos). The sun-cult taught that the primeval mound rose in the pyramidal shape of the sacred ben-ben stone, out of the waters of Chaos. Atum, the Creator, first manifested himself there in the form of the sacred heron-like phoenix, or as Atum in human form. Atum then created Shu and Tefnut, forming a triad. Shu and Tefnut then procreated Geb, the earth and Nut the sky. Shu, the void, separated Geb from Nut. Every evening, Shu lowered the goddess Nut on to the recumbent, ithyphallic Geb, procreating Osiris, Seth, Isis and Nephthys.

Note that Geb, the earth, is male unlike most creation myths, where earth is female. Note also that Geb needs Tefnut, as the earth needs moisture to be fertile - something the Nile valley dwellers would well appreciate. The mysterious origins of life, over which we still ponder today, were explained in the ennead myth. The relationships between the members of the ennead were explained in terms of simple family relationships, providing order and creation sequence in a way understandable to anyone.

In addition to triads and enneads, there was a host of other gods. Some were household gods, such as Bes. He was shown to women suffering the pain of labour and was so ugly that they supposedly forgot their pain in the fright of his appearance.

There were also important and powerful goddesses. Hathor was an ancient goddess, revered in the early dynastic period as the consort of the "Bull of Amenti", the first deity of the necropolis. Later she became the goddess of the sky, daughter of Ra and wife of Horus. She was represented either as a cow, or as a woman with cows ears and/or horns with the solar disc between them. In later periods she became fused with Isis. Hathor means "the temple of Horus", deriving from the belief that Horus entered her mouth each night, emerging as the Sun on the next morning.

Somehow, the ancient Egyptians managed to blend all of these local, regional and state beliefs, gods and spirits, many with quite different mythologies and legendary associations, into one diverse set. There is no evidence of the many inconsistencies creating any difficulties or misunderstandings amongst the people, suggesting that the general population was unaware of the inconsistencies or accepted the various myths without question. Today, children accept fairy stories, tales of magic and Father Christmas. Many adults accept astrology, and either worry about or relish, a black cat crossing their path. Not much has changed.
The Pyramid Texts do not describe the general theology or belief system, apparently assuming that the reader knows it. Instead, they tell us of the Pharaoh's passage to the afterlife, of the perils of the trip and of the spells, incantations and knowledge he must possess in order to pass safely to his other world. In so doing, they illustrate belief in life after death, in resurrection, and in the ideal that being good in this life is a necessary passport to the next. Not surprisingly, they also reveal many concerns and puzzlement over just how the pharaoh reaches his home in the stars.

The ancient Egyptians fascination with the heavens is well known, and is paralleled by many other early civilizations: where information is in short supply, perhaps one looks wherever one can. The stars were mysterious, enigmatic, seemingly permanent
and eternal: small wonder astronomy/astrology was important to the ancient Egyptians.

## Societal Individuation

The ancient Egyptians were as intelligent as we are today; there has been insufficient time for biological evolution to change our mental capacities. Social evolution is another matter altogether; ancient Egyptian society was in its infancy, and few of the great social ideas had been conceived: no democracy, communism or socialism, no work ethic, no Cartesian reductionism, no concept of labour, no concept of withholding labour, etc. People in a society adopt roles and exhibit behaviours according to the niches and rules afforded by that society.
It is probably impossible for us today to "get inside the mind" of a typical ancient Egyptian of the Old Kingdom: indeed, there may not have been a typical ancient Egyptian. Besides, we have so little to go on, and our own cultural perceptions so cloud our viewpoints and judgements, that we are ill equipped to address such a monumental task. Neither Freud nor Jung would have claimed to achieve an in-depth understanding of the ancient Egyptian psyche.

Psychologists observe that we all go through a process of becoming discrete individuals as we develop through childhood into adulthood. They call the process individuation ${ }^{17}$; characterized by our gradually becoming distinct individuals, with our own, established personalities, more or less standing out from the crowd.
Societies seem to individuate ${ }^{18}$, too. While societies may not go through exactly the same stages, and certainly do not invoke the same time scales as individuals, nevertheless it is possible to observe evolution and development in the behaviour of societies and groups as though the whole society behaved as an entity.
It seems that nascent groups operate initially, and for a while, without individuals dominating social behaviour, and without the need for much structure. How long it is before structure and power groups emerge seems to relate to the numbers involved. A large group will take longer than a small committee or team. And, of course, the composition and culture of the social group is important.

This nascent behaviour is similar to shoaling. Large shoals of fish, made up from individuals, nonetheless behave as a single body, moving together in splendid dynamic harmony. As they change direction, different fish appear in the lead; they change again, with a new leader. The leader could be any fish, it is unimportant, transitory, and has no lasting impact on group behaviour.
Analogous behaviour is to be observed amongst large flocks of birds, and on the annual migration of the wildebeest on the Serengeti. Shoals, flocks and herds are nascent groups. They come together naturally, forming and reforming without control and with an observable periodicity. The only requirement seems to be that the individuals interact with each other.

To understand the cultural and societal behaviour of the Egyptians, it would seem sensible to see it in the context of the degree of societal individuation. If, as our modern examples suggest, there is extended group behaviour in the early stages, with little overt personality, then that might go part way to explaining how the society was able to motivate and sustain itself over extended periods with massive effort in the building of many pyramids.

We can only try to imagine a population that was cohesive, enthusiastic, energetic, single minded. In addition to shoaling behaviour, in which people would follow a changing lead unquestioningly and in synchronization, we would expect to see other characteristics in such a nascent society.
Idealism would be evident in the idea that it is right, proper and good to build a pyramid to preserve the pharaoh so that he may continue to look after us, his people, and ensure the Inundation after death. Naïveté and uncritical belief would be necessary to accept that such rewards would accrue. Enthusiasm would be present, to get on with the job to the glory of the pharaoh. Unawareness of time, another characteristic of youth, would allow us to be unconcerned at the awesome size of the task in hand. And, finally, innocence, would let us accept the world of spirits and tales of magic as reality, neither questioning nor challenging.

## Relevance to Pyramid Building

Above is hardly a scientific approach to assessing social individuation, but it does, at least, begin to explain how the people may have viewed the world about them and their rôle in building the pyramid. If so, then young farmers called up by the annual corvée may have viewed the annual event rather like going to a summer camp; they may have looked forward to the team games and competitions, not to mention the rewards. They would have anticipated the fine food, beer and clothing with which they were to be paid. They might have had no more fear of death on the building site than would a child on a climbing frame. And when death came, as it must have done, sorrow and grieving may have been sharp but short-lived.
The team leaders may have turned the whole exercise into a series of competitive games; certainly, some of the surviving graffiti suggests a less than serious-minded approach. Competing work crews had names such as "Friends of Khufu" and "Drunkards of Menkaure", the latter being particularly playful.

There must have been a core of professional builders concerned with the more demanding and intricate aspects. While it is possible to imagine enthusiastic and energetic farmers being employed to drag or raise stone, working on chambers and passages would have called for sterner performers. Perhaps these more demanding and precision tasks were undertaken by an all-the-year round core team of professional builders. After all, the internal structures in each of the pyramids may be complex, but it forms but a tiny part of the overall massive structure; the bulk of each pyramid is solid stone.
The logistics of feeding and supporting this influx of men and, presumably, their families, too, is quite awe inspiring. As this organization effectively maintained the building workers, and their families over several generations of pharaohs and their pyramids, it must have evolved to become a well-oiled organizational machine. Farmers would have to grow sufficient fresh food to be available at the right time (not easy during the Inundation) and sufficient grain to pre-fill the granaries with the staples for beer and bread. Fishermen and their boats must have been active catching boatloads of fish and transporting it to the building sites where it was cleaned and salted. Salt makers would have experienced a boom in demand, too.
Fresh water would have been hauled to the kitchens/galleys associated with the building sites for the brewing of beer - drunk instead of water to neutralize contamination. Brewing vats would have been covered for several days while the yeast did its work. Large earthenware vessels would have mixed and proved bread
several times per day, some of the bread being used instead of yeast to activate the beer brew.

The organization of workers, families and support logistics would be a major task today. Then, it would have been the making of Egypt. Creating the organization and operating it repeatedly over extended periods of time effectively pulled Egypt up by its bootstraps, creating a nation-wide organizational infrastructure peopled with administrators, viziers, managers, coordinators, accountants and civil servants. The ancient Egyptian civil service may be said to have originated at this time.

## The Rôle and Purpose of The Pyramids

To understand the purpose of the pyramids to the ancient Egyptians, it is helpful to appreciate first the rôle of the pharaoh.

## The Evolving Rôle of the Pharaoh

The influence and rôle of the pharaoh changed over time. Taken together with the evidence of the Narmer palette (and other finds), it is reasonable to deduce that civil discord was not unusual right up to the end of the $2^{\text {nd }}$ Dynasty. The titles taken by successive pharaohs give a clue:

Table 1. Early Pharaohs ${ }^{*}$, Dates and Titles ${ }^{19}$

| Dynasty | Years BC | Pharaoh | Horus or Birth Title $^{\dagger}$ | Observation |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $3150-3050$ | Scorpion |  | $\begin{array}{l}\text { The early titles are } \\ \text { generally warlike, } \\ \text { suggesting that the } \\ \text { pharaohs of that time } \\ \text { felt a need to present a } \\ \text { threatening image. } \\ \text { This in turn suggests a }\end{array}$ |
|  |  | Narmer | Striking Catfish |  |
|  |  |  |  |  |
| peaceful nation. |  |  |  |  |$\}$

[^12]| 2 | 2890-Precise <br> dates <br> Unknown | Seth Peribsen | Powerful in Heart (Horus <br> Name) <br> Hope of all hearts <br> (Seth Name) | Horus name then a <br> End of <br> Archaic <br> Period |
| :---: | :--- | :---: | :---: | :--- |

By the $3^{\text {rd }}$ Dynasty, peace appears to have broken out. Khasekhemwy and Djoser were both able to undertake building programmes that would most probably have involved mobilizing a nationwide workforce. Both Pharaohs were able to undertake massive


Figure 13. Khasekhemwy's Serekh showing both Horus and Seth, Cairo Museum building works in stone at Saqqara, near Memphis, at the junction between the two lands. A serekh ${ }^{*}$ showing both the gods Horus and Seth, and hence uniquely associated with Khasekhemwy, is shown in Figure 13.
By the time of Khufu, builder of the Great Pyramid at Giza, the pharaoh was believed to live after death amongst the imperishable (i.e. circumpolar) stars ${ }^{20}$. From there, he would look over Egypt and use his powers to ensure the annual Inundation.

The ancient Egyptians obsession with the Inundation, on which all their lives so visibly depended, thus emerges as the driving force behind pyramid building. Gods determined the Inundation. Only gods could interact with gods. Pharaoh was a god; therefore pharaoh alone could intercede with the other gods on behalf of the ancient Egyptian people. Therefore it was vital to preserve pharaoh's mortal remains upon his

[^13]death, and to project his essence into the circumpolar stars, where he could continue his work of intercession and control of the Inundation for all time.

So, when the ancient Egyptians built the pyramids, they may indeed have gone to work with joy and enthusiasm. Although they were working for their pharaoh, they would perceive that they were also working for themselves - surely an admirable case of highly motivated self-interest.
Khufu's reign may have been the last associated with a largely stellar cult, in which the deities of the Osirian legends were identified with stars, their perceived relationships in the heavens, and with their annual appearances, disappearances and pathways across the celestial sphere. Up until this point, afterlife (at least amongst the stars) was principally, even exclusively, for the pharaoh. During life he was identified with the falcon-god Horus. Upon death he became Osiris, and passed to the afterlife, while his son became Horus. This procedure ensured the sanctity of the new pharaoh and preserved the line of succession. Pharaoh's unique place in the afterlife, and his supreme godliness were not to last unchallenged, however.
Immediately after Khufu, the sun god Re started to appear in the titles of the pharaoh, and "sa Ra" (son of Ra) would become one of his standard titles. By the end of the $4^{\text {th }}$ Dynasty, pharaoh was considered to be the son of Re, or Ra, the sun god. Ra sailed across the heavens by day in a solar barque, bringing light and life to a dormant world. At night, Ra sailed through the underworld, finally emerging in the east again on the next morning. In the solar theology, one of the pharaoh's privileges, upon death, was to help to row Ra's barque every day.
So, pharaoh was no longer the supreme entity, but became responsible after death for maintaining light and life, while the myths explained the effects of the earth's rotation in a way that was consistent with observations. A tangible mark of this theological change was the appearance of sun temples as part of the pharaoh's pyramid complex.

## Resurrection Machines

A means was needed to project pharaoh's essence to the stars. And a means was needed to preserve and maintain the dead pharaoh's essence for all time, so that he could continue in his rôle for eternity.
Pyramids were not, then, just for the pharaoh. They were for the people, to preserve their existence and way of life into the future. The pyramid was a machine, using the power of prayers, offerings and lustrations as its motive force to maintain the pharaoh's $k a$, to project that $k a$ towards the circumpolar stars, and to maintain and nourish the $k a$ for all time. Since it was believed that the $k a$ and $b a$ must re-associate with the preserved mortal remains, then it may be that the pyramid machine also enable the pharaoh's spirit to return from the stars to his beloved Egypt, and to use the boats (which were generally buried as part of the pyramid complex) to sail the Nile, perhaps to the holy place of Abydos.
One problem with pyramids as resurrection machines emerges: no confirmed remains have ever been found of a royal mummy in a sarcophagus. In fact, the location of any interment site for royal mummies of the Old Kingdom is something of an enigma., This need not invalidate the use of the pyramid complex as a resurrection machine, however; in some ways it may even reinforce it.

The pyramid may have been viewed as a spirit "transporter". If we suppose, as seems possible, that a pharaoh's mortal remains were interred elsewhere, then the pharaoh's $k a$ could still travel from the burial site to its respective pyramid, there to be projected to the stars. This underlines the value of the boats buried at the pyramid sites as transport for the pharaoh's $k a$ to and from the burial site, wherever that might be.
Pyramids generally had causeways associated with them, too, reaching from a valley temple by the river up to the mortuary, or pyramid, temple by the pyramid. These causeways would have been used for hauling stone from the river. They were then covered over with stone walls and a stone ceiling, creating a substantial structure. This would surely have been used to convey the pharaoh's mummy from the royal barge to the pyramid as part of the ceremonies associated with his death*. The covered causeway, lit only by sunlight through clerestories, would have created a numinous passageway for the priests carrying the mummy.
The causeway was, however, a tremendous undertaking. Of the Great Pyramid, Herodotus wrote ${ }^{21}$ : "They said it took ten years of hard labour for the people to
construct the causeway along which they hauled the blocks of stone, which I should think involved not much less work than building the Pyramid...It is made of polished stone with figures carved on it." Why cover the causeway in stone unless it was meant to be as permanent as the other stone elements of the pyramid complex?

On the face of it, once interment had been completed, the causeway would not be needed. That the causeway was carved with figures is also difficult to explain, especially since the pyramid itself is devoid of any carvings, unless one invokes the idea that the covered causeway, with its enclosed secrecy and carvings, was an essential part of the dead pharaoh's celestial commutation, and so intended to continue for all time.

One speculative suggestion for this paradox, then, is that the ancient Egyptians may have expected the pharaoh's spirit to pass to and fro using the pyramid as a gateway from, as well as to, the stars, and using the covered-in causeway as a secure guide or path to the Nile, where he would have his boat magically assembled and ready to sail. If this speculation were correct, then the covered-in causeway would need to be permanent, and the notion that the pharaoh was interred elsewhere than his pyramid would be reinforced.

What would have been the purpose of the pyramid sarcophagus, then? If it did not contain the royal mummy, perhaps it served as a temporary resting place for the royal $k a$. For the ancient Egyptians to have had two sarcophagi, one for actual interment and a second for ritual interment of the $k a$, would not be inconsistent with their world of symbolic dualities.

## Stability and Change

Two separate, but linked phenomena can be seen at work in ancient Egypt, during the time of the Old Kingdom. First, social evolution had created a youthful, energetic, integrated nation-state that was becoming at peace with itself and able, therefore, to act an integrated whole. Second, the Inundation, although gradually reducing, was

[^14]still ample to support that evolving society. Together, these two created a window in time, during which occurred the most astounding, matchless outpourings of human creativity. It could not last.
Later generations of ancient Egyptians would look back on the Old Kingdom as the Golden Age. It was certainly a creative and spiritual time, but like all "golden ages", it had to come to an end. Figure 14 shows diagrammatically how this end might have come about. There seem to have been two main threads to the decline: the changing social hierarchy, and the failing Inundations which occurred towards the end of the $6^{\text {th }}$ dynasty. The first of these set the stage for the second.

Early in the Old Kingdom, the pharaoh was regarded as supreme, a wholly divine being, upon whose continued existence depended the future of Egypt. Initially, it seems that only the pharaoh would live after death; then pharaohs, it seems, bestowed the gift of afterlife on close associates. These associates of the pharaoh would need priests to attend their tombs for all time, just as priests attended the royal pyramids and the tombs at Abydos. Gradually, bodies of priests emerged, bent on ensuring the continued existence of their respective charges.
At the same time, the pharaoh granted lands to close associates who would become nomarchs, ruling in his name up and down the country. Once established in their nomes, nomarchs elected to hand on the leadership to their sons, so creating hereditary dynasties, with their own necropolises, priests, etc.

This developing situation created priesthoods with increasing power and, since lands assigned to the priests supported them, independence. This seems to have lead to the emergence of the solar theology in place of the stellar theology, during the $4^{\text {th }}$ Dynasty. The solar theology was more accessible, and it reduced the exclusive pre-


Figure 14. Decline of the Golden Age
eminence of the pharaoh.
These progressive developments gradually took wealth and power away from pharaoh and from Memphis, as evidenced by the falling standards in pyramid building. The trigger for breakdown, however, came from the series of poor Inundations at the end of the $6^{\text {th }}$ Dynasty. This led to shortage of food and some famine, Figure 15. Nomarchs, no longer subservient to central power, fell to internecine struggle over the dwindling food resources, and civil war broke out. It was the end of the Golden Age of the Old Kingdom, and the start of the first Intermediate Period of relative disorder.
Although the Golden Age was irretrievably lost, it left its heritage, not only in the memory of it, but also in the extensive social infrastructure that the pyramid-building era had invoked. That infrastructure, and the ideas of its content and effectiveness,


Figure 15. Famine Amongst Bedouin, Old Kingdom. Louvre
restoring future kingdoms to new glories.

## Chapter 2. Pyramids in Perspective



Figure 16. N. Egypt showing the Principal Pyramids of $3^{\text {rd }}-6^{\text {th }}$ Dynasties

## Bird's Eye View

The map, Figure 16, shows northern Egypt, where most of the pyramids were built. The three pyramids of Giza are at top left. From right to left, in chronological order, they are:

1. Khufu's Great Pyramid

## 2. Khafre's Pyramid

3. Menkaure's Pyramid

The $4^{\text {th }}$ Dynasty Giza Pyramids were built on the edge of a plateau overlooking the Nile Valley. The present course of the Nile is visible. To right and left of the Nile is the Nile flood plain, although the Nile no longer "inundates" north of the modern Aswan Dam.
Further south are: Abusir, where there are four $5^{\text {th }}$ Dynasty Pyramids, housing Pharaohs from Elephantine; Saqqara the principal Necropolis of the Old Kingdom, where Djoser built the first pyramid; Dahshur, where the $4^{\text {th }}$ Dynasty Bent and Red (North) Pyramids are located; Meidum - further south still, and not shown - which houses the $4^{\text {th }}$ Dynasty Fallen Pyramid

The precise location of the ancient capital, Memphis, is not certain. It is possible that there was line of sight between Archaic Memphis and Khufu's Great Pyramid at Giza.

## 4-6 ${ }^{\text {th }}$ Dynasty Necropolis-Artist's View

The figure shows an Old Kingdom necropolis. There are several pyramids; each has a mortuary temple abutting its east wall. There is a covered causeway leading from the mortuary temple down to a valley temple, which acts as a landing stage. A causeway connects the river to the valley temple.
The Necropolis would invariably be sited on the west bank of the Nile, and each pyramid would be set with the four faces pointing towards the 4 points of the compass. So, the valley and mortuary temples face the rising sun, while the opposite side of the pyramid faces the sun setting in the West. The right face, as shown, faces North, towards Lower Egypt, the celestial pole and the circumpolar stars. The


Figure 17. Artist's View of a Typical Necropolis
opposite, South face, points towards Upper Egypt, and Abydos, the holy city: as the Nile unusually flows from south to north, up-river - and hence Upper Egypt - are south, while down-river and Lower Egypt are to the north.
In addition to the main pharaohs' pyramids, there might be smaller pyramids for their queens, a small ka pyramid, a Temenos Wall, boat pits, and a city of the dead. The city, comprising many mastabas and tombs for dead nobles and others of high status, would be laid out like a living city, with rows of streets, junctions, etc.

## Pyramid Evolution

Rather than provide detailed descriptions of each and every pyramid ${ }^{22}$, the objective of this chapter is identify the particular characteristics of pyramids that varied over time, as both theological ideas and constructional capabilities evolved.


Figure 18. Djoser's Stepped Pyramid, Saqqara
The first pyramid was built in the $3^{\text {rd }}$ Dynasty for Pharaoh Djoser (2668-2649) at Saqqara by his vizier, Imhotep, perhaps the first genius* in history. As Figure 18 shows, the pyramid was built up from a series of layers. At that time, subterranean burial chambers were covered with mastabas to cover the shaft leading down to underground burial chamber(s). It seems that Imhotep may have decided to set mastaba upon mastaba.

A wall surrounded Djoser's pyramid complex to enclose the sacred area. It may have been the inclusion of this wall that caused the multi-layered structure. Visibility of the

[^15]pharaoh's tomb seems to have been important (q.v.) and it may have been built up to appear large and impressive over the wall. Originally covered in light stone "cladding", the first pyramid must have presented an awesome sight to the nascent society travelling up and down the river, and gathered in Memphis
The orientation of the pyramid is not precisely towards the cardinal points: this feature appears only in later pyramids.


Figure 19. Collage of Features in Djoser's Pyramid Complex
Figure 19 shows a collage of features peculiar to Djoser's Pyramid complex. At top left is shown one of several "mock" buildings, probably intended as dummy shrines. Bottom left is the ceiling of the entrance through the wall into the complex. Note the construction that, although in stone, takes the form of wooden logs. Bottom centre is a view of the entrance passage showing an avenue of pillars. The pillars are ribbed, reminiscent of Greek Doric pillars, but some 2000 years before their invention. Interestingly, the pillars are not freestanding, but are formed at the end of short walls. It seems that the architect was not confident of building freestanding pillars at this early stage.
Bottom right is a view through one of two eyeholes in Djoser's serdab, or kiosk. This was placed against the north face of the pyramid, so that Djoser, seen inside the serdab, could stare out towards the North. It is possible that he was gazing over a replica of Egypt, keeping eternal vigil over his land and subjects. He may have been gazing towards the circumpolar stars, resting place of dead pharaohs. He might simply have been positioned to observe offerings being made to him. In any event, the statue in the serdab is a replica: the original is in the Cairo Museum, Figure 20.

The eyes have been gouged out in the past, but it is nonetheless an imposing statue. After 4,500 years, Djoser still presents as a powerful, dominating character. Yet, the stonework is relatively primitive. Note the folds of cloth around the arms and legs; in neither case do they conform to the body, or reveal any underlying body features. Similarly, the body is only marginally differentiated from the seat on which the subject sits: also, the fingers of both hands are only partly differentiated.

The right hand, which is across the chest, appears to have held some badge or staff of office at one time. This may be deduced both from its position, with the closed fist, and from the similar pose of later pharaohs. $4^{\text {th }}$ Dynasty pharaohs all appeared in statues and figurines holding some staff of office. It was neither the crook nor the flail, however, so familiar from New Kingdom statues and paintings.

Detail of Djoser's Pyramid construction, Figure 21, also reveals the relatively unpractised approach to working with stone. Each stone was shaped like a bolt of squaredoff wood. These bolts were laid against the core to form columns that sloped inwards towards the centre
The whole was finally finished off with finely jointed ashlars or finishing stones, creating an impressive, white, shining sight, visible on the horizon for many miles.
Two further views appear at Figure 22. At left, the development of the steps can be clearly seen. At right, a few of the remaining ashlars are visible at ground level. The ashlars would have followed the contours of the steps in the pyramid. Although the pyramid was constructed in various stages, the ashlars would have


Figure 20. King Djoser, Cairo Museum given the impression of a single, coherent structure.


Figure 21. Djoser's Pyramid Construction

Djoser's Pyramid marks a significant stage in social development. Building in stone shows intent to endure and, consequently, indicates confidence in the future. It seems that building stone monuments may be a feature common to nascent, developing human societies the world over. Such monuments may be pyramids, but might equally be steeples, towers or other highly visible constructions. During the Middle Ages, Gothic Cathedrals were in vogue. New York, Chicago and Kuala Lumpur skyscrapers are modern examples.


Figure 22. Djoser's "Brickwork" and Casing Stones

## Evolving precision in Stone

Djoser's Pyramid was to be the first of a long line in ancient Egypt. Pyramid construction and architecture went through an intense period of evolution, with apparent trial and error fuelling an evolution towards a remarkable capability, not only in stone construction, but also in organization of manpower on a national scale.

The rapid evolution in Egypt's ability to work with stone took place in just about 100 years, from $3^{\text {rd }}$ Dynasty Djoser (reigned $\sim 2668 \sim 2649$ ) to $4^{\text {th }}$ Dynasty Khufu (reigned ~2589 ~ 2566), whose architect, Hemon, constructed the Great Pyramid. That such evolution occurred indicates a degree of continuity: lessons learned were passed on to successive generations of designers, architects, organizers and managers. By tracking the evolution in stonework, it is possible to glimpse the concomitant development in social structure.

After Djoser there were two further attempts at pyramid construction in the $3^{\text {rd }}$ dynasty. Sekhemkhet and Khaba both attempted to build stepped pyramids but, in both cases, the attempts were left unfinished.
The $4^{\text {th }}$ Dynasty started with Pharaoh Snefru, who was to build 3 pyramids.

## Snefru's Pyramids

Snefru (reigned $\sim 2613$ - 2589) was the premier pyramid builder. Together, his three pyramids are much larger than Khufu's Great Pyramid, yet to be built. Snefru was also a great innovator, attempting on 3 occasions to produce a perfect pyramid, as opposed to a stepped pyramid. On the third attempt he succeeded, but his previous attempts show the methods being used, why they failed, how the limitations were overcome and, not least, an indefatigable drive to succeed.

## Snefru's Meidum, or Fallen, Pyramid - "Snefru Endures"

Snefru's first pyramid was built at Meidum, south of Memphis and Saqqara. Figure 23 shows a schematic diagram of the pyramid. The intention appears to have been to build a true pyramid of slope $51^{\circ} 50^{\prime} 35^{\prime \prime}$. At some point, the outer covering layers slipped and now appear as debris surrounding the base of the exposed core, revealing this to be stepped.

There is some controversy about the time when this slippage occurred. One view ${ }^{23}$ is that it slipped during building, causing Snefru to build more pyramids. Another view is that it slipped much later, and was in use for some time in pristine form. $18^{\text {th }}$ Dynasty graffiti in the east face mortuary temple supports this latter view.
There is also controversy about the original shape. A recent view ${ }^{24}$ suggests that the Meidum Pyramid was always intended to be a stepped pyramid. However, the general consensus suggests that the pyramid was constructed first as a stepped pyramid, then extended, still as a stepped pyramid and that finally an outer casing was built up to create the first true pyramid shape.

The Meidum Pyramid provides insights into the building philosophy at that early stage. As Figure 23 shows, the "shoulders" of the stepped pyramid core were not horizontal. Instead, they sloped downwards and outwards from the core. Building a superstructure above the sloping shoulder encouraged that superstructure to slide off, which was what apparently happened.
Whether the Meidum pyramid fell during the Old Kingdom or not, stability was evidently an issue for the Egyptian architects. Individual stones had to be laid square and horizontal, or side forces would be exerted. On the other hand, dressing each and every stone to be precisely rectangular and horizontal would have been prohibitively time consuming. The general solution to this problem, visible at Meidum and still in


Figure 23. Meidum Pyramid Schematic
use during medieval European cathedral construction, was to build accurate outer walls and to infill them with crudely formed stones, even rubble. As the figure shows, Egyptian pyramids were formed from concentric "shells", with stones at the faces being well finished, while between the faces they were less well finished. This resulted, inter alia, in pyramids being resistant to earthquakes.
The inward leaning shell walls also diverted downwards thrust away from chambers within the body of the pyramid. Early chambers were generally not roofed with horizontal stone beams; instead the ceilings were corbelled. Corbelled ceilings also diverted downwards thrust into the masonry and walls around the chamber. The corbelled chamber in the Meidum Pyramid was formed from unfinished stone, suggesting that the chamber was never finished and fuelling speculation of early abandonment - at least for burial purposes.
Whatever the truth about the fall of the Meidum Pyramid, it seems reasonable to assume that Pharaoh Snefru was not satisfied, since he went on to build two more pyramids at Dahshur. Meidum was abandoned as a site.

## Snefru's Rhomboidal, or Bent, Pyramid - "The Southern Shining Pyramid"

The first of Snefru's pyramids at Dahshur was the Bent, or Rhomboidal Pyramid, Figure 24, which shows clearly how it attracted its epithet. The pyramid rises at a relatively steep angle of $54^{\circ} 27^{\prime}$ and then changes slope to $43^{\circ}$, reaching a height of 105 m . The reason for the change in slope appears to be the development of cracks in the wall near the base. These cracks, covered by the largely extant casing from the outside, are nonetheless visible from within. The architect's strategy may have been to


Figure 24. Snefru's Bent Pyramid at Dahshur, North Side.
reduce the weight of the superstructure by introducing the bend, to reduce the stress and strain on the lowest courses.

One suggestion ${ }^{25}$ sets the initial slope even higher, at some $60^{\circ}$. This slope was reduced to $54^{\circ} 27^{\prime}$ early during building by adding extra masonry on the outside. The trials and tribulations of building the Bent Pyramid were provoked by subsidence in the underlying sand and shale, which provided less than ideal foundations.
These same difficulties suggest that there was a desire to build steeper pyramids. Evidently, the slope of the Meidum Pyramid was to be exceeded, at least. Also, instead of one, unfinished corbelled chamber, there were now three chambers: an underground antechamber, a largely underground "first" chamber and a burial chamber built fully in the masonry. The chambers were roofed with horizontal corbel stones in two orthogonal directions, forming an internal "stepped pyramid" shape above the chamber, although oblong rather than square. The roof and walls of the antechamber remain rough and unfinished.
The reasons for the three chambers are not evident. It would have been difficult, even impossible, to have moved a heavy stone sarcophagus from chamber to chamber as part of some ceremonial burial, for instance. Unusually, the pyramid also had two entrances, one in the West wall, and the other in the more conventional North wall, compounding difficulty in conceiving burial rituals. In any event, the two portcullis stones, which would have sealed the burial chamber after the Pharaoh's interment, had not been activated, suggesting there had been no burial.

Figure 25 shows, at left, the construction at the corner of the Bent Pyramid. Note the downwards and inwards sloping casing stones, still largely intact, covering rough, poorly squared packing stones laid in horizontal layers. At right, on the North side of the bent Pyramid is a small satellite, or $k a$, pyramid, seemingly unremarkable except that it contains a prototype of the Grand Gallery for which the Great Pyramid of Khufu is so justly renowned. The purpose of the ka pyramid is not clear, but may have been associated with ancestor reverence/worship.


Figure 25. Bent Pyramid: Construction and Ka Pyramid

## Snefru's North, or Red, Pyramid - "The Shining Pyramid"



Figure 26. Snefru's Red Pyramid, Dahshur. West Face.
Having appreciated the difficulties of building the Bent Pyramid, it is perhaps understandable why Snefru went on to construct a third pyramid, the so-called Red Pyramid, also at Dahshur. The Red Pyramid, named for the colour of its stone, is the first true pyramid to survive, Figure 26. The Red Pyramid has a slope of some $43^{\circ}$ throughout, making it a "safe" construction, but perhaps falling short of being "high".


Figure 27. Red Pyramid: Reconstructed Pyramidion, Stonework and Casing
Figure 27 shows the reconstructed Pyramidion from the Red Pyramid. At right can be seen the construction which, when compared with the Bent Pyramid, shows packing stones that are slightly larger and distinctly squarer, with courses laid horizontally.
Like the Bent Pyramid, the Red Pyramid also has three internal chambers, each roofed with corbels. The first two chambers were constructed by digging a trench at ground level, before the superstructure was raised. These two chambers, on the same level, are connected by a horizontal passage. Entrance to the third chamber, believed to be the burial chamber, is some 25 feet above the floor of the middle chamber in the north
wall. Here, there is a passageway some 23 feet long, opening out into the final chamber.

Access to the pyramid is via an entrance on the north face, 94 feet above ground level. A passageway descends at $26.5^{\circ}$ (q.v.) for 206 feet. As with the Bent Pyramid, this difficulty of access raises the issue of burial ritual. Why three chambers, why such an elevated access point to the pyramid, and why the elevated and difficult access to the burial chamber? Unlike the Bent Pyramid, the corbelled ceilings in the Red Pyramid chambers are well finished, showing significant advance over the Meidum Pyramid and the Bent Pyramid.

The difficulty of access may have been associated with making robbery difficult - this is the usual suggestion. If so, the pyramid architects must have had an unusually naïve view of tomb robbers' capabilities. The three chambers may have been intended to provide the Pharaoh's spirit with some space. Djoser had a serdab from which he could look out over Egypt, or gaze towards the circumpolar stars, so perhaps one of the chambers was intended as a serdab too. The third chamber may have been intended to store essentials for the journey to afterlife, or for eternal life once there.
If Snefru were buried in any of his three pyramids, it would have most probably been in the Red Pyramid. No sarcophagus was found, and human remains found in the burial chamber could not be confirmed as coming from a royal mummy.

## Comparison of Bent and Red Dimensions

Although superficially quite different, Snefru's Bent and Red pyramids bear striking similarities. Both are 105 m high, and the slopes of the upper half of the Bent Pyramid and the whole of the Red Pyramid are also identical. The usual explanation for this is that the Red Pyramid design would have been considered "safe", and this may have some validity. After all, Snefru would have been getting on in years by the time he started the Red Pyramid and he would have been conscious of the need to have his final resting place complete and ready.

Were speed the driving force, however, Snefru might have considered simply making a smaller pyramid; that would have been safe and swift. There may be more to these striking similarities, as will be discussed later.

The stage was now set for the greatest pyramid of all: Khufu's Great Pyramid

## Khufu's Great Pyramid at Giza - "Akhet Khufu"

While Snefru was the premier builder in terms of the total amount of pyramid construction in one reign, his son Khufu (2589 - 2566) holds the record for the single biggest stone building of all time. Nothing can prepare the first time visitor for the awe-inspiring size of the structure.

The building follows the by-now classical form, Figure 28. In addition to the pyramid, there were 3 Queen's Pyramids, a $k a$ pyramid, a Temenos Wall surrounding the pyramid and enclosing a paved courtyard, at least 3 boat pits, a mortuary temple abutting the pyramid's east face, a covered causeway leading from the mortuary temple down to a valley temple by the edge of the Nile and two areas filled with mastabas and tombs. These last were laid out like cities, with rows of mastabas or small temples, cross roads and junctions. It seemed to be important for high status officials and nobles to be interred near the pharaoh and/or his pyramid.


Figure 28. Khufu's Necropolis* - Principal Components
For Khufu's Great Pyramid, the mortuary temple was significantly larger than previous mortuary temples. In fact, everything appeared to be on a grand scale. The area covered by the pyramid alone covers $53,000 \mathrm{~m}^{2}$ - enough for the cathedrals of Florence, Milan, St Peters, Westminster Abbey and St Paul's Cathedral added together.
The overall weight of the pyramid is some 7 million tonnes. It is 147 m ( 481 ft ) high. There are some 2.3 million limestone blocks in the structure, of mean weight 2.5 tonnes, and maximum weight c .75 tons. Accuracy of alignment north/south and east/ west is a phenomenal 3 minutes of arc, or $0.06 \%$ accuracy. The base size is 230.25 mN by 230.44 S by 230.38 E by 230.35 W , which is accurate to $0.08 \%$
A current view of the exterior of Khufu's north face, Figure 29, highlights comparisons with previous pyramids. The packing stones are very much larger. While their vertical sides may not be precise, their horizontal layers display almost surgical precision. Lower centre, the few remaining casing stones show vividly how accurate the casing was constructed. It would be difficult, if not impossible, to slide a visiting card between casing stones. Also visible in the figure are the remnants of the paved peripheral area. It is possible that this area would have been covered with polished marble, as is the Valley temple of Khafre's Pyramid to this day.

[^16]

Figure 29. Khufu Pyramid Precision

## Akhet Khufu - Internal Layout

Like the other pyramids at Giza, Akhet Khufu (Khufu's Horizon) is built on the limestone plateau overlooking the Nile, Figure 30. The limestone forms a sound foundation, as well as providing the bulk of the building material.
The Great Pyramid is built over a knoll. Knolls and mounds are important in Egyptian mythology, since one of their principal creation stories included a primeval mound emerging from primeval waters of chaos. The mound at Giza may have been instrumental in its choice as a site.

Like Snefru's Bent and Red pyramids before it, the Great Pyramid has 3 chambers. The lowest chamber is underground and is reached by a passage descending for 109 m at $26.5^{\circ}$ from the sole entrance on the North face. The underground chamber is unfinished, leading to suggestions that it was abandoned as part of a change of plan during construction. The only way to light a chamber so far underground would be by using oil or rush lamps. In either case, the lamps would consume oxygen and there would be no way to expel the smoke and fumes. It is possible, therefore, that the chamber was deserted because it proved too dangerous to complete.
The so-called Queen's Chamber* sits right on the vertical centre line. It is reached by a rising passage, again at $26.5^{\circ}$. The first few yards of this passage are cut through pre-laid masonry, supporting the notion of a change of plan upon desertion of the lower chamber. Thereafter, the rising passage walls show evidence that the masonry was properly finished as part of planned construction. At the top of the passage the

[^17]

Figure 30. Structures in Khufu's Great Pyramid
Grand Gallery opens up, still rising at $26.5^{\circ}$. A horizontal passage leads to the Queen's Chamber, through the central floor of the Grand Gallery
The Queen's Chamber has a sloping (gable) roof. There is a tall, corbelled niche in the east wall and two apertures in both the North and South walls at about head height. These apertures, and two similar ones in the King's Chamber, lead to small passages that, after traversing horizontally for a short distance, slant upwards at different angles. For many years these passages have been referred to as ventilation shafts and, if the theory about the reason for deserting the underground chamber is true, the builders might have considered ventilation important.
The shaft in the north wall of the Queen's Chamber is so located that it emerges directly towards the underside of the Grand Gallery, and the shaft appears to be bent around the Grand Gallery to avoid collision. If this was a mistake by the architect, then it was rather an obvious mistake for one who, in so many other respects, had shown great capability. Further, the shaft emerging from the north side of the King's Chamber is similarly located and bent. It is unlikely that, having made a mistake once, the architect would then repeat it. A more reasoned deduction would be that the shafts, at least in the respective north faces of the two chambers, were located for reasons so important that, although severe, bending the shafts was an acceptable building penalty.
The Grand Gallery is 8.5 m high and 47 m long. Civil engineers rate it as the masterpiece of the stone mason's art in this, or any other, age. Apparently, it was used, inter alia, to store granite stone plugs needed to fill rising entrance passage; this was to be the principal protection for the Pharaoh's tomb.

Looking at the Grand Gallery through modern eyes, it seems to be a truly enormous effort to achieve such a purpose. It is not apparent how the stones were stored, or how they were manoeuvred from their stored positions into the descending entrance passage. Once in this passage, they had to slide down without jamming. The first plug stone had to slide down and then stop in the ascending passage so that it did not overrun into, and accidentally seal, the descending passage. The lowest part of the ascending passage is slightly narrower, in order to stop and seal the first plug stone. Subsequent plug stones also had to slide down the ascending passage and, presumably, run into previous stones without snagging and without shunting or damaging the first plug stone.

At the top of the Grand Gallery is the King's Chamber, containing the remnants of the granite sarcophagus. It is slightly larger than the entrance to the chamber, and was evidently introduced during the building, before the roof was completed. Unusually, the Chamber is roofed with horizontal granite beams. Above the King's Chamber are the so-called Relieving Chambers.
It has been suggested that, like the underground chamber, the Queen's Chamber was also deserted, this time in favour of the King's Chamber. Gantenbrink's discovery (q.v.) of the portcullis at the top of the QC (S) shaft, predicates that this notion is incorrect. Figure 30 shows that the portcullis is well above the King's Chamber and so must have been installed after it was completed. There would have been no reason to finish off the southern shaft if the Queen's Chamber had already been deserted. Besides, Snefru had established a tradition of three chambers.


Figure 31. Virtual Views of the Inner Structures in the Great Pyramid
Figure 31 shows two frames taken from a virtual reality flythrough of the Great Pyramid, showing the inner structures, i.e. with pyramid masonry made transparent. At left, the picture shows a perspective view looking upwards from the northwest corner of the Great Pyramid. The passage descending vertically from the start of the Grand Gallery is a narrow escape tunnel for those builders who lowered the plug stones into place, sealing the ascending passage.
At right is a view looking down from above the southeast corner of the pyramid. The Relieving Chambers and the Grand Gallery Corbels are shown. The east and west walls of the Queen's Chamber have been left open, as has the upper part of the Grand Gallery. The curves in the north-going shafts from the King's and Queen's chambers show in both figures. How far the QC (N) shaft goes into the body of the Pyramid is speculative.

Another simulation, Figure 32, shows the so-called Relieving Chambers in more detail (socalled, because they do not appear to relieve stress at all").
The chambers consist of five layers of granite beams weighing some 2,500 tonnes in total, and a final sixth layer of pitched or gabled slabs, with spaces between each layer. From the bottom, the spaces are called: Davidson's, Wellington's, Nelson's, Lady Arbuthnot's and, in the pitched roof space, Campbell's Chambers. Only this last, uppermost, pitched roof serves to relieve the stress on the King's Chamber roofing beams, which are showing signs of cracking.

The whole construction of the Relieving Chambers is quite


Figure 32. The so-called Relieving Chambers above the King's Chamber, Great Pyramid (simulation) remarkable. Each stone beam weighed some $60-75$ tonnes, and had to be raised a considerable height above ground. As the figure shows, the granite beams were dressed at sides and bottom but, curiously, the tops were left rough and unfinished; this is atypical of work on the Great Pyramid.


Figure 33. Relieving Chambers Construction

[^18]The (simulated) construction process is best seen in perspective, Figure 33, with the surrounding masonry, construction ramps, etc., (if indeed there were any) removed. At left, the first three layers of granite beams have been placed with the bottom layer forming the roof of the King's Chamber. The granite beams run across the narrower section of the King's Chamber. The visible surface formed by the beams will become the floor of Nelson's Chamber. Running above the longer walls, two smaller, rectangular cross-section beams are shown, although these may not be single stones. These beams act as spacers and transmit the thrust from the granite beams down on to the walls of the King's Chamber. None of the layers of beams could have served as a platform during construction; they are too uneven. The various layers even have different numbers of beams in them. The layer roofing the King's Chamber has 9 beams. Thereafter, the numbers, going upwards are: $8 ; 9 ; 9$ and 8 . This is accounted for by varying beam widths, visible in the picture at right.
The whole is capped with dressed limestone blocks, which do not bear down on the granite beams but which divert superstructure thrust sideways into masonry not shown in the pictures.
Considering the great insight that the architect, Hemon, evidently displayed, it seems unreasonable to assume that the Relieving Chambers were some kind of mistake. Hemon apparently knew how to divert the down thrust from the mass of masonry above, and had already used a double-pitched roof over the Queen's Chamber. This suggests that the first four layers of the Relieving Chambers had some other purpose, perhaps fulfilling some mythological or ritual purpose.
One possible explanation for this massive, yet apparently purposeless, structure concerns interment of the king's sarcophagus. Originally, the sarcophagus may have been intended for burial deep underground, where it would rest protected by layers of stone through which interlopers would have to dig, or spirits would have to pass. The underground chamber was never finished, leaving open the suggestion that the layers of granite beams above the King's Chamber might have been intended to represent the layers of rock under which the Pharaoh had hoped to be buried. This would explain, too, why the granite beams were undressed; if they represented the underlying rock strata of the plateau, dressing the stones may have seemed inappropriate. Their sides and bottoms were dressed sufficiently to enable them to be manoeuvred into place only.

Running with this notion suggests that the north-pointing ventilation shaft into the King's Chamber might replicate the entrance passageway, which descends to the deserted underground chamber: the short horizontal section of the ventilation shaft would then represent the horizontal entrance. With such a narrow cross-section, the shaft would then either be symbolic, or was perhaps intended only for the Pharaoh's $k a$ to pass through. This suggestion does not, however, explain the other King's Chamber shaft pointing southwards; unless, that is, another passage had been originally intended, leading out of the unfinished subterranean chamber.

Be that conjecture as it may, the construction of the Great Pyramid is remarkable. Even more remarkable must have been the reasons for its construction, for the chambers with their unique shafts, and for the sheer size of the Pyramid. These will be explored later.

## Djedefre's Pyramid - "Djedefre is a Sehed Star"

Khufu's son, Djedefre (2566-2558) built the next pyramid at Abu Roash, some 8 km north of Giza. Today it is in ruins.

It is not clear why Djedefre chose to build at Abu Roash. It has been suggested that he wished to build near the Sun Temple at Heliopolis, and certainly he is the first pharaoh with Re, the sun god, in his name. His pyramid seems to have been reminiscent of Djoser's stepped pyramid, with a large underground chamber

## Khafre's Pyramid - "Great is Khafre"

After Djedefre, Khafre (2558-2532) returned to Giza. Khafre was also Khufu's son,


Figure 34. Khafre's Pyramid, North Face
by a different wife. Khafre's Pyramid, Figure 34, is the second largest of the Egyptian pyramids; at 143.5 m , it is only some 3 m shorter than Khufu's Great Pyramid. It appears to be much taller because it is built on higher ground.
Khafre's Pyramid is, in some respects, retrograde. It has only one chamber, and that appears to have been dug into the surface before the superstructure was erected. Curiously, there are two entrances, both from the north face. One descends at $26.5^{\circ}$ from the north face until it reaches ground level and then continues horizontally with its upper surface at ground level, as though a channel were dug into the surface extending northwards from the chamber. A second passage descends from the north edge of the pyramid at ground level at about $21^{\circ} 41^{\prime}$. This passage then levels out underground, where there is a small, subsidiary chamber off to the west (right hand) side, finally rising to join the horizontal section from the first descending passage.
The reason for the choice of angles will be explained later, but the curious arrangement of descending passages leads to suggestions that either an error or a change of mind occurred. Possible explanations are that the pyramid was reduced in its overall size, had its centre relocated, that there was simply a change of mind, or that the two passages were necessary to fulfil some ritual or mythological purpose.
Figure 34 shows a view of the north face of Khafre's Pyramid. The size of the people on the pyramid gives some idea of scale. The packing stones are large and rectangular, but lack the precision of Khufu's Pyramid; they also diminish in size rising up the pyramid. At the top can be seen the remaining casing stones, made from Tura limestone and once pristine white. The quoins are accurately aligned, as with the Bent Pyramid at Dahshur.

Khafre's valley temple is quite well preserved, at least sufficiently well to appreciate the craftsmanship of the building, see Figure 35.


Figure 35. Khafre's Valley Temple Construction

At left, the fine cut ashlars rise vertically above the polished marble paving. At right, the massive stone lintels create an avenue of shadows from the sun in the southeast.

## Menkaure's Pyramid - "Menkaure is Divine"

The third pyramid at Giza was built for Menkaure (2532-2504), offspring of Khafre and hence grandson to Khufu. Obviously much smaller than the other two Giza pyramids, it nonetheless continues a trend evident with Khafre's Pyramid that, as the pyramid reduces in size, the associated temples became larger and grander. While only 66 m high and less than a quarter of the volume of Khufu's pyramid, Menkaure's pyramid has an impressive underground complex.
A passage descends south at $26^{\circ} 1^{\prime}$ for 31 m from 3 m above the floor of the north face to a panelled chamber, which is followed by the conventional triple portcullis door set at the far (south) end. A horizontal passage leads further south to an antechamber, oriented east west. From the west end, a passage descends westward to a burial chamber. The burial chamber is roofed with granite slabs raised to a point, gable style; these slabs are carved on the inside to form a curved arch.

Another passage, starting in the floor of the antechamber slopes down towards the west and then moves horizontally into the burial chamber, entering underneath the first passage. Off to the right, of the second, lower passage, going northeast, is a chamber with 6 rough-hewn niches. The whole underground structure is unexpectedly complex.
One suggested purpose for the second passage is to convey the large granite slabs into the burial chamber to form its roof. The room with the 6 niches may be a storeroom to provide the pharaoh with afterlife needs.

## Pattern Shift

The two passages, one above the other, are reminiscent of the two entrance passages to Khafre's Pyramid, also one above the other and with a chamber off to the right. Taken together with Djedefre's Pyramid at Abu Roash, Khafre's and Menkaure's pyramids show an evolutionary shift. Instead of building ever more sophisticated vaulted chambers progressively higher up in the masonry of the pyramid (Meidum, Bent, Red and Great pyramids) there is a sudden reversion to underground chambers. The niches in Menkaure's lowest chamber seem most likely to have been intended for storage, probably for the pharaoh's essential afterlife needs. Taken together with the


Figure 36 Menkaure's (Unfinished) Casing and Mortuary Temple
introduction of ' $r e$ ' (the Sun god) into the pharaohs' names post Khufu, there is an indication here of a change in theology/mythology. If pharaohs lived among the imperishable stars, perhaps there was limited need for worldly storage in the pyramid. If, however, pharaohs lived with Osiris in some earth-like paradise, or even beyond the black mountains of the western horizon where the sun descended each night, then earthly provisions would make more sense. If such an explanation were plausible, then the chamber off to the right of Khafre's lower access tunnel may have been intended for storage, and both it and the lower tunnel were intended, not a mistake.

Externally, Menkaure's pyramid is unfinished. The bottom 16 courses are cased in red granite, which has not been finished, Figure 36, which also shows, at right, the remains of the fine mortuary temple abutting the east wall of the pyramid.


Figure 37. Menkaure's Pyramid and Queen's Pyramids
The three queens' pyramids on the south side of Menkaure's Pyramid are shown at Figure 37. The image at left shows the increasing slope of the ground. The Queen's Pyramids are cut into the side of the plateau scarp, the Mokattam Formation. The image at right shows the three satellite pyramids, with their different constructions. The satellite to the left, east, is constructed as a true pyramid, and sits on the northsouth centre line through Menkaure's Pyramid. The other two pyramids appear to be built as stepped pyramids. Whether this is because they, like the main pyramid, are not finished is unclear. Note, on the image at right, that the ground between the satellites falls away rapidly, and that the satellites are built into the slope.
The reason for Menkaure's Pyramid being much smaller than the other two Giza Pyramids has puzzled people for many years. The slope of the surface offers one explanation: there was insufficient room on the plateau for a third full large pyramid.
Suggestions for the smaller size of Menkaure's Pyramid include economy, that the costs of building large pyramids were becoming too great: Menkaure's Pyramid may be smaller, but economy is not in evidence. Casing the lower courses in red granite involved shipping that stone from Aswan and it is, of course, more difficult to work than limestone. The underground complex is more elaborate than for the other two Giza pyramids, too. The large mortuary temple does not suggest economy, either.
Another suggestion is that the three pyramids were located and aligned to represent the three stars in Orion's Belt ${ }^{26}$. The third of these stars is of lower magnitude than the other two, suggesting that Menkaure's Pyramid size was the analogue of perceived brightness. This fascinating and romantic notion will be addressed later.

Finally a movement is suggested away from the cult of the all-powerful stellar godpharaoh, towards a solar theology dominated by solar temples and priests, who wanted more dominance for Sun Temples, less for pyramids.

## Nebka's Pyramid at Zawiyet el-Aryan

There is an unfinished pyramid, seemingly of $4^{\text {th }}$ Dynasty construction at Zawiyet elAryan, some 5 km up river from Giza. It is suggested that this pyramid belonged to a short-lived pharaoh who reigned briefly between Khafre and Menkaure, and whose name consequently does not appear on the ancient lists of kings. The unfinished structure would have been nearly the size of Khafre's Pyramid, i.e. much greater than Menkaure's. The construction included a large pit, like Djedefre's Pyramid at Abu Roash.
If this is correct, it suggests that the neat sequence of pyramids at Giza is an illusion, not just in terms of chronology, but also in terms of construction. It is not difficult to see some sort of struggle or contest between competing entombment philosophies, which could have resulted either from different sides of the family following different paths, perhaps, from diverging theologies. The last pharaonic tomb of the $4^{\text {th }}$ Dynasty reinforces this suggestion.

## Shepseskaf's Mastaba el-Fara'un (Pharaoh's Bench)

Menkaure's successor, Shepseskaf, returned to Saqqara for burial. Instead of a pyramid, he also returned to the mastaba, His mastaba was, however, huge at 99.6 m long and 74.4 m wide.

Many of the features were reminiscent of Menkaure's pyramid. The mastaba was originally cased in fine limestone, but the bottom course was of red granite. A passage descended (at $23^{\circ} 30^{\prime}$ ) for 20.95 m to a corridor chamber, followed by 3 portcullis doors. The burial chamber ceiling was sculpted into a vault. The sarcophagus was sculpted like Menkaure's (which was unfortunately lost at sea on a voyage from Egypt to England). From the southeast of the burial chamber a narrow passage lead to 6 niches

Acollage of $4{ }^{\text {th }}$ Dynasty pharaonic statues, Figure 38, draws comparison with Figure 20, which showed Djoser in the $3^{\text {rd }}$ Dynasty some 100 years earlier. Evidently the Egyptian sculptors had greatly improved their mastery of stone carving. In the collage, the figure of Khafre, left and top left, in particular is a masterpiece. The face and the eyes show a man of confidence and vision, gazing confidently into his eternal future. Djedefre, bottom centre, presents a petulant appearance, as though to confirm his supposed unwillingness to follow in the steps his father, Khufu, top centre-right.

Menkaure, at right, had a number of similar statues sculpted, one for each nome or province. In each he stands, again supremely confident, with the goddess Hathor on his right and the particular nome representative on his left. Above the latter's head is the nome totem.

Both Khafre and Khufu were evidently holding something, Khafre in his right hand, and Khufu possibly in both; neither appears to be displaying the hook and flail.


Figure 38. Collage of $4^{\text {th }}$ Dynasty Statuary. Left is Khafre, plus a profile showing the hawk, Horus at his neck. Bottom centre is Djedefre. Top centre is a small figurine of Khufu. Right is Menkaure. All figures Cairo Museum except Djedefre, Louvre.

Djedefre and Khafre both sport the serpent uraeus, while Menkaure wears the white crown of Upper Egypt. Khufu's headdress is unusual, even unique.

The most noticeable feature of these statues, other than their fine quality, is the absence of any standard representation of the pharaoh. Such a standard would be very evident in later times. The lack of standard, notable in the $4^{\text {th }}$ Dynasty in the Old Kingdom, could have arisen simply because the idea of a standard had not arisen. On the other hand, it could represent each pharaoh trying to be different, to set the contemporary style, even perhaps to set himself apart as an individual, not just a figurehead.

## $5^{\text {th }}$ and $6^{\text {th }}$ Dynasty Pyramids

In retrospect, Khufu's Great Pyramid was the apotheosis of pyramid building. Standards were slipping even during the latter part of the $4^{\text {th }}$ Dynasty. The first pharaoh of the $5^{\text {th }}$ Dynasty, Userkaf, built his pyramid at Saqqara on one corner of Djoser's complex. (The last pharaoh, Unas, would build his pyramid on the opposite corner.) This return to Djoser and Saqqara could be read as a pointed return to orthodoxy, or perhaps a retrospective approach. Userkaf's Pyramid was built above a deep open pit from which a complex substructure was constructed. The pyramid above ground contained no chambers: it is now dilapidated.

The underground burial chamber had a pitched rood comprised of enormous limestone slabs leaning against each other in an inverted ' V '. Future pyramids would repeat this roofing arrangement, except that a second inverted V would be laid upon the first. The designs of future pyramids of the $5^{\text {th }}$ and $6^{\text {th }}$ dynasty generally conform to the standard plan, with a burial chamber more or less underground, covered by a
double-pitched roof. New features began to appear, however, including sun temples as part of the pyramid complex and Pyramid Texts carved into the walls, notably, of Unas' and Teti's underground chambers.

Trends in pyramid chamber construction, in terms of numbers and positions, are shown in Graph 2; there are also variations in construction technique, outlined above. The graph shows the respective pharaoh along the x -axis, while the y axis shows above ground chambers as positive numbers, and below ground chambers as negative numbers. For example, Khufu's Great Pyramid has one underground chamber and two above ground within the masonry.
Some underground chambers were relatively deep underground: others were created by digging a trench at the surface of the ground and roofing it before the pyramid was constructed over the top. These two categories are shown as "underground" and "dug from surface" respectively. Djoser's Stepped Pyramid is shown as having one principal chamber underground although there is also an extensive 4.5 km complex of tunnels and passages. The principal chamber is reached via a central shaft, 28 m deep, so is categorized as "underground." This central shaft construction appears again with Djedefre.
Khafre is shown as having both, since the main burial chamber appears to have been dug from the surface, while the subsidiary chamber is reached via an underground tunnel and must therefore have been cut out from the rock. Towards the end of the $6^{\text {th }}$ Dynasty, chambers were dug into the surface rock, and their roofs fitted at, or just below, the surface; these are categorized as "dug from surface."

Looking at the evolutionary patterns, Graph 2, it seems that there were several phases

or fashions, with two phases of building chambers up in the masonry. The first of these two phases occurred in the $4^{\text {th }}$ Memphite Dynasty under Snefru and Khufu, while the second occurred at Abydos during the $5^{\text {th }}$ Elephantine Dynasty, so called since the pharaohs were understood to have originated from the Aswan area ${ }^{27}$. It is not impossible that the $5^{\text {th }}$ Dynasty kings were trying to emulate their $4^{\text {th }}$ Dynasty predecessors, perhaps to show that they, too, could build superb monuments, or even perhaps to emulate and so recapture the magic of the Giza necropolis.
The graph shows the final phase of Old Kingdom pyramid building relapsing into the repetition of a standard formula, with each pyramid chamber design looking much like its predecessor, and each burial chamber being dug down from the surface.

## Necropolis Plans and Layouts

Looking at individual pyramids, above, has highlighted some trends, competing styles and disjunctions in the evolving architecture and construction. There appears to be much more to the evolving design, however.

## The Orion Hypothesis

Robert Bauval suggested, in his book "the Orion Mystery" that the three pyramids of Giza were configured on the ground to represent the layout and magnitude of the three stars in Orion's Belt. He had noticed that, while the diagonals of Khufu's and Khafre's pyramids were aligned along a single northeast-southwest line, Menkaure's pyramid diagonal was offset to the southeast. He felt that this had to have been deliberate, since it would have been logical, he supposed, to align the pyramids on one common diagonal axis. The three pyramids sat beside the Nile, as the three stars of Orion's Belt existed beside the Milky Way. Bauval went on to identify several other pyramids as members of the Orion constellation. Delightful and seductive though Bauval's theory is, there are one or two difficulties to address.
Orion was a character in Greek mythology and had not been conceived at this time, some 2000 years earlier. Modern translations ${ }^{28}$ of the Pyramid Texts refer to the constellation of Orion, unknown to the Egyptians. While they, undoubtedly, grouped stars into constellations, there is no reason to believe that their groupings matched ours today. It is known that they did not. So, identifying Egyptian pyramids as belonging to today's Orion constellation is an anachronism.

The three stars in Orion's Belt, when viewed from Giza, are broadly aligned with Alnitak, at the southern end. (Alnitak is the left hand star of the belt.) Looking at the three Giza Pyramids, the pyramid equating to Alnitak, Khufu's Pyramid, is at the northern end, Figure 52 on page 73. So, the representation of Orion's Belt would seem to be $180^{\circ}$ out -


Figure 39. Giza Pyramid Alignment. an unlikely mistake.

Significantly, there is no representation of Sirius, the brightest star in the night sky and quite close to Orion's Belt. Sirius is known to have figured in the star mythology
as the goddess Isis, alongside Alnitak, which seems to have represented the god Osiris. If any star were to be represented, Sirius would surely have been foremost.

## Necropolis Intervisibility

Bauval based his


Figure 40. Djoser's Stepped Pyramid at Saqqara through the haze from Snefru's Red Pyramid, Dahshur fascinating theory, which one would love to be right, on an expectation that all three pyramids should have their diagonals on one straight line. In fact, the three pyramids at Giza are aligned, but along their southeast corners, Figure 39. One possible reason for this alignment will be explored below.

The location of the various pyramid fields of the $3^{\text {rd }}$ to $6^{\text {th }}$ Dynasties appears to be concerned with rather more than simply finding suitable rock foundations for building. There is a pattern to the various sites that, together, form an extended necropolis.
The clue is to be seen in the photograph, Figure 40, taken while standing at the north face of Snefru's Red, or North, Pyramid at Dahshur, looking north. A variety of buildings and ruins is visible through the haze, which seems to emanate from the modern traffic and industry in Cairo. The buildings in the foreground are obviously


Figure 41. Location of Necropolis Sites on their Mutual Horizons
modern. Beyond them is a wealth of pyramids and mastabas in various states of decay. Djoser's stepped pyramid, the very first pyramid, can be seen precisely on the horizon. That location provides a clue. Almost invariably, each pyramid site is located precisely on the horizon when viewed from any other.
Figure 41 shows more instances. The image at top left shows the $5{ }^{\text {th }}$ Dynasty Abusir pyramids viewed from Saqqara, the necropolis site nearest to Memphis, the archaic capital. Note their state of decay compared with the earlier Giza pyramids
Top right, is the reverse view to Figure 40, i.e. from Saqqara to Dahshur. Ignoring the modern buildings, both the Bent and Red pyramids are precisely on the horizon, even although the Red Pyramid is the closer of the two by about 1 mile. Perhaps the reason for building both pyramids 105 m high and with the same slope of $43^{\circ}$ (at least for the upper half of the Bent Pyramid) was to create a twin spectacle on the horizon. If that was the intent, it was successful

Bottom left can be seen the view from Djedefre's tomb at Abu Roash, north of Giza, with the Giza Pyramids visible precisely on the skyline.
Finally, bottom right are the Abusir Pyramids again, but this time showing the Giza Pyramids on the horizon, and vice versa.
Not all necropolis sites fit this pattern. Meidum is not visible from any of the other sites; it is too far south; it seems to have been deserted as a pyramid-building site in ancient times, however. Importantly, Giza is not visible from Saqqara; there is a massive limestone mound in the line of sight. Since the $4^{\text {th }}$ dynasty Giza pyramids were erected before the $5^{\text {th }}$ Dynasty Abusir pyramids, these latter cannot be considered as a "stepping stone" of horizon-to-horizon inter-visibility.


Figure 42. Terrain Simulation, Archaic Memphis to Giza

A possible solution to the "horizon visibility" imperative, if indeed there was one, is indicated in Figure 16 on page 29 above. Saqqara was the original necropolis site, on the plateau above archaic Memphis. Quite where archaic Memphis was located seems uncertain, as the path of the River Nile has moved over the millennia. However, as the figure shows, it may have been possible to see the Giza plateau from Memphis - just. There are one or two promontories that might just have impeded the view.
One way to test out this theory is to create a 3D terrain simulation, Figure 42. Top left is the 3D contour derived from a conventional modern contour map. Top right, added to the same contour map, are the Nile water during Inundation, and the pyramids - in the distance, as the view is, supposedly, from Memphis. As the simulation suggests, if Memphis were sited to the left of the viewpoint, some of the promontories visible on the edge of the plateau might begin to obscure the line of sight to the Giza pyramids. Bottom left is a view about halfway from Memphis to Giza. Bottom right is a view from the river at Giza, showing the Sun setting in the west and casting pyramid shadows into the water.
There is no way of proving that the modern topographical map used as the basis for the terrain simulation represents the terrain as it was in the Old Kingdom. Erosion may have changed the outline over the millennia. However, there is margin for error in the simulation, and it is therefore not unreasonable to suggest that the ancient rulers could, indeed, see Giza from their palaces at Memphis - weather permitting. It might even be feasible, reversing this technique, to limit the bounds of the search for archaic Memphis to the area from which Giza would have been visible.
The name of Khufu's Pyramid has been mentioned - Akhet Khufu, or Khufu's Horizon. For Khufu to refer to his tomb as his horizon might seem a little restrictive if he was anticipating living among the imperishable (circumpolar) stars. On the other hand, to refer to Khufu's pyramid as his horizon would have been rather sensible if, indeed, the Great Pyramid were really located on his visual horizon, as seen from Memphis. This would have enabled Khufu to see his pyramid every day, and to monitor the progress of its construction continuously.
Overall, it seems reasonable to suppose that one of the driving influences between the choice of Necropolis sites was simply one of visibility - that the pyramids should be strikingly visible and imposing on the horizon as viewed from any other site. The horizon was important to the ancient Egyptians. It swallowed the Sun each night and gave birth to the Sun each morning. The horizon was, then, the instrument of resurrection. Akhet Khufu might well be translated, not as "Khufu's Horizon" but as "Khufu's Resurrection", or even "Khufu's Resurrection Machine".

## Giza Pyramid Alignment and Distribution

The simulation of Figure 42 raises another issue, that of the alignment of the Giza pyramids. As the simulation shows, the separation between the three pyramids appears to be constant, even although the viewing angle is changing. This is true only because their southeast corners, those nearest the river Nile, are aligned.

Figure 43 illustrates the point. Top left is a contemporary view showing that the pyramid pinnacles appear evenly distributed. Top right is an old aerial photograph
illustrating the same point. Bottom left, another old photograph showing sunset* over the Inundation. Finally, and to emphasise the point about even distribution, the bottom right view shows the pyramids from the Libyan desert; from this standpoint the pyramids are anything but evenly distributed - but then, they were not intended to be viewed from this direction, only from the River Nile, the arterial highway of ancient Egypt.


Figure 43. Views of the Giza Pyramids
Putting the evidence together suggests that the Giza pyramids were aligned along their southeast corners to make them appear evenly distributed over a variety of viewing angles as seen from Memphis and the Nile. This layout prevented any one pyramid from obscuring any other. Menkaure's pyramid was made smaller that the others because it was right on the edge of the plateau. With the southeast corner alignment in mind, it would have been impossible to build it bigger. The three Queen's Pyramids were cut into the edge of the scarp to ensure that they did not obscure the view of Menkaure's Pyramid, especially from Memphis and the Nile approaches to Giza. The three Queens Pyramids were built to the west of the north-south centre line through the pyramid, away from the river, for the same reason. Emphasizing his mortuary temple, carving the underground complex into the bedrock, and covering the lower courses of the pyramid with ostentatious red granite would then have compensated for the size of the pyramid, limited as it was simply by the topography ${ }^{\dagger}$.
Like Khufu, both Khafre and Menkaure would probably have been able to see their pyramids being constructed from Memphis, and Menkaure would have been able to see his Queens' Pyramids against the skyline, because of their offset to the west.

[^19]Taking the pyramid site locations on mutual horizons together with pyramid alignments seems to confirm that the driving force throughout was the creation of spectacle. Each pyramid had to be clearly visible on the horizon and, on drawing closer, no pyramid should obscure any other when viewed from the Nile.
There are four $5^{\text {th }}$ dynasty pyramids at Abusir ("place of Osiris"): Sahure; Neferirkare; Raneferef; and Niuserre, in order of construction. The first three pyramids are aligned, this time on their northwest corners: this alignment makes them appear equally spaced both from the Nile and from Giza, reinforcing the driving need for discrete horizon visibility. The last, Niuserre's, breaks the alignment, but is located near the river to maintain visibility.

## Pyramid Slopes - the Search for Perfection

At first glance, it may seem that the pyramids at Giza all have the same slope. That is not the case, although casual examination of Figure 43 might suggest otherwise.

Table 2. Comparing Pyramid Slopes


Ancient Egyptians did not use angles to measure slopes. Instead they used proportions, typically between the horizontal distance and the vertical rise or fall over
that distance ${ }^{*}$. In geometrical terms, they used the opposite to adjacent proportion. How can we be sure of that?
Table 2 records the $4^{\text {th }}$ to $6^{\text {th }}$ Dynasty pyramids by pharaoh, ignoring tombs that were not pyramids. The second column the conventional modern way of representing slope, as an angle. Angles are a relatively modern invention. Dividing circles into $360^{\circ}$ degrees does not appear to have been an Egyptian practice ${ }^{\dagger}$.
In the third column, the numbers represent the tangent of the angle, derived from tables or a calculator.

In contrast, the numbers in columns 4 and 5 are found by trial and error, searching for two integers that, when divided, give the same answer as the tangent. The point of the exercise is to see if the ancient Egyptians used simple ratios and proportions to determine the slope of each pyramid. So, for the first pyramid, Opposite is 14 units, Adjacent is 11 units; these represent the height of the pyramid and the distance from the base to the centre, starting from base centre of any face. $14 \div 11$ is 1.2727 , which is almost exactly the same as the tangent of 51.84 degrees, with $0.001 \%$ accuracy.
One example might be coincidence. The table, however, presents 18 examples, all offering great accuracy. The greatest error arises with Khafre's Pyramid, which has an error of $0.133 \%$, i.e. 1.3 parts in 1000 ; there appears to have been some problems in the construction of Khafre's pyramid, which shows evidence of twisting near the top, which may account for this minor discrepancy.
Overall, using the ratio between two integers represents pyramid slopes to an average accuracy of 1 part in 10,000 , with a standard deviation of 31 parts in 10,000 . It is reasonable to take this as evidence that the ancient Egyptians used the ratio of two integers to describe, design and control the building of their pyramids. It is also reasonable to deduce that the ancient Egyptians used the particular ratios shown in the table.
However, we have not addressed the last column, headed "seked". The ancient Egyptians manner of specifying slopes was unusual. One seked measured the horizontal distance travelled for a drop of 7 palms, i.e. one royal cubit. In effect, this uses the cotangent of the slope rather than, as we use today, the tangent. As a result, the number of seked decreased with increasing steepness.
Parts of a seked have been calculated using fractions, to keep within the spirit of the time, although some fractions would have to be expanded into a series to fully satisfy their canon ${ }^{\S}$. The results are interesting. Snefru 's Bent Pyramid had a lower slope of

[^20]precisely 5 seked, explaining the use of an inclination which looks almost randomly chosen as an angle of 54 $27^{\prime} 44^{\prime \prime}$. Snefru's Bent (upper) and Red Pyramids measure 7$1 / 4$ seked, showing the introduction of a simple unitary fraction. Khufu and Khafre, adjacent on the Giza Plateau, and to the casual glance having the same slope, measure $5-1 / 2$ and $5-1 / 4$ respectively in seked. The pyramid series ended with four pyramids set at 5-1/4 seked inclination

This way of measuring slope explains why, today, we find the pyramids to have their slopes forming the ratio of two integers, and the use of unitary fractions of a seked results in the seeming preference for successive integers. It is not clear whether the architects worked in proportions, such as $14: 11$ or in seked. For visualisation of proportions, it is more likely that they kept to proportions (as it is believed did medieval builders), while for specifying or describing slope, the use of a measure might have been preferred. The clumsy ratio for Djedkare-Isesi, 32:25, suggests that the ancient Egyptians architects might have worked with 4:3-1/8, i.e. smaller integers and unitary fractions
Much of our information about unitary fractions and measurement units comes from the Rhind Papyrus*, in the British Museum . The Rhind Papyrus ${ }^{29}$ contains

$=1 / 4+1 / 28$, i.e. two different unitary fractions. Similarly, $2 / 5=1 / 3+1 / 15,9 / 10=1 / 2+1 / 3+1 / 15$, and so on. They appeared to avoid symmetry in this context.

* A Scottish collector, Rhind, acquired the Rhind Papyrus , also called the Ahmes Papyrus, in 1858 in Thebes. The papyrus was copied by a scribe, Ahmes (or Ahmos), ( $\sim 1650 \mathrm{BC}$ ) from another document written $\sim 2000 \mathrm{BC}$, that was in its turn copied from a document from $\sim 2650 \mathrm{BC}$ (the time of Imhotep).
mathematical problems and solutions. The papyrus is a veritable cornucopia of ancient mathematical ideas and methods, including fractions, geometric progressions, summing series, and much more.
The Rhind Papyrus calculates pyramid slopes using the seked. These calculations generally employed examples of 5-1/2 seked. As Table 2 shows, this value was preferred earlier on, particularly during the $4^{\text {th }}$ Dynasty. Djoser's Pyramid would have sloped at 5-1/2 seked, for instance, if its sides were filled in. This is one of the reasons for supposing that the content in the Rhind Papyrus originated in the $4^{\text {th }}$ Dynasty.
Graph 3 shows the result of graphing each pyramid angle in chronological order, together with the integer ratios that the Egyptians seem to have used to describe slopes. The progression shows that, after initial wide variations in slope angle, a pattern emerged in which there was a general tendency to increasing slope
There seems to have been a preference for the slope to be determined by the ratio of two consecutive integers, e.g. 17:18, 4:3, 3:2. Only when whole integers did not give sufficiently fine control, may they have resorted to unitary fractions (q.v.) e.g. 1/8, 1/2
Some slope sequences were repeated. From the graph, Khufu's and Djedefre's particular pyramid slopes were repeated in sequence by Niuserre and Djedkare-Ises. The reason for this is unknown, but see Graph 2 and text above. Particular slopes were repeated at intervals, e.g. Snefru at Meidum, Khufu at Giza, Niuserre at Abusir. Again, the reason is unknown, although Meidum, being the first true pyramid, may have served as a model or icon. The commonest slope was $4: 3$


## Is $\pi$ "Hidden" in the Pyramid Slopes?

The evolution in Old Kingdom pyramid slopes can be used to dispel some myths. One of the most enduring anachronisms about ancient Egyptians is that they used the transcendental number $\pi^{*}$ in their constructions. Several fortunes have been made, based on this "discovery".

The reason for the belief seems to reside in the Great Pyramid which, as we have seen, includes opposite-over-adjacent proportions of 14:11. Using those proportions, means that one side would "measure" 22, and the four sides would sum to 88. Dividing 88 by 14 gives $44 / 7$, or $2 \times 22 / 7$. As any $21^{\text {st }}$ century school child will confirm, $22 / 7$ is a reasonable approximation for $\pi$; hence the conclusion that the value of $\pi$ is "hidden" in the Great Pyramid. Quite why the ancient Egyptians would have arranged such a convoluted approach to hiding a circular ratio in a square-based pyramid is never asked.
For a square-based pyramid shape to look "normal", i.e. neither too tall nor too squat, the result of dividing the twice the base length by the height will always turn out to be about 3 . This arises simply as follows. Opposite is greater than adjacent if the slope is greater than $45^{\circ}$ - the usual case. One side, the base length, is twice the adjacent length: two sides are 4 times the adjacent length. So, dividing 4 times the adjacent length by the opposite, which is slightly greater than the adjacent, must give a number less than 4 , i.e. about 3 . This works for cones, too.

[^21]Table 3 shows the results of calculating " $\pi$ " by the same method as used for Khufu's Great Pyramid. Khufu's Pyramid gives a result close to $\pi$, but then so do Snefru's Pyramid at Meidum and Djedefre's Pyramid at Abu Roash. Overall, however, the results are poor. Looking at the table, we would have to believe the ancient Egyptians knew $\pi$ during the construction of the Fallen Pyramid at Meidum, and for Khufu, but conveniently forgot it - twice - the second time, permanently. In the face of such contrary evidence, a reasonable person would conclude that they did not know about $\pi$.

Table 3. Calculating $\boldsymbol{\pi}$ for Old Kingdom Pyramids

| Pharaoh |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: |
| Base $(\mathrm{m})$ | Height $(\mathrm{m})$ | 2xBase $\div$ height | Error on pi |  |
| Snefru (Meidum) | 144 | 92 | 3.13 | $-0.36 \%$ |
| Snefru (Bent A)* | 188 | 105 | 3.58 | $13.99 \%$ |
| Snefru (Bent B) | 188 | 105 | 3.58 | $13.99 \%$ |
| Snefru (Red) | 220 | 105 | 4.19 | $33.39 \%$ |
| Khufu | 230 | 147 | 3.14 | $0.03 \%$ |
| Djedefre | 106 | 67 | 3.16 | $0.72 \%$ |
| Khafre | 215 | 144 | 3.00 | $-4.62 \%$ |
| Menkaure | 103 | 65 | 3.18 | $1.27 \%$ |
| Userkaf | 73 | 49 | 2.99 | $-4.77 \%$ |
| Sahure | 79 | 47 | 3.35 | $6.67 \%$ |
| Neferirkare | 105 | 72 | 2.92 | $-7.16 \%$ |
| Niuserre | 79 | 52 | 3.05 | $-2.81 \%$ |
| Djedkare-Isesi | 79 | 53 | 3.00 | $-4.51 \%$ |
| Unas | 58 | 43 | 2.69 | $-14.50 \%$ |
| Teti | 79 | 53 | 3.00 | $-4.51 \%$ |
| Pepi I | 79 | 53 | 3.00 | $-4.51 \%$ |
| Merenre | 79 | 53 | 3.00 | $-4.51 \%$ |
| Pepi II | 79 | 53 | 3.00 | $-4.51 \%$ |
| rrern |  |  |  |  |

So obsessed have some people become with the belief that the ancient Egyptians knew and used $\pi$, that they have developed theories as to how it came about "by accident." One theory ${ }^{30}$ suggests that, while vertical measures used a ruler marked off in cubits, horizontal measures were made using a roller of 1 cubit diameter. Each revolution of the roller would then measure $\pi$ cubits. Dividing horizontal distance by vertical distance would, then, inevitably include $\pi$ in the result without the Egyptians really being aware of it.

Why the ancient Egyptians would be so stupid is not explained. How sloping distances would be measured is not explained. And how did the ancient Egyptians come to use a roller when the wheel had not been invented?

## Deductions

From the foregoing it is evident that pyramid building was not simply an outburst of architectural exuberance, such as we might see with any emerging society. Evolution in architecture and in construction techniques is evident. In addition, there were themes running through the pyramid sequence, and these themes were sustained over the whole of the $4^{\text {th }}$ to $6^{\text {th }}$ dynasties.
The control and evolution of pyramid slopes could have come about in several ways. Architects for each new pyramid could have investigated previous pyramids and
chosen proportions for the new pyramid based on their findings. Each new generation would learn by observation and investigation, but without there being an accumulated body of knowledge.
On the other hand, there might have been an architectural/theological college in which builders with experience passed on their knowledge, the college gradually building up a body of knowledge which was passed on to successive generations. There could have been theological significance in the choice of proportions, somehow favouring a successful resurrection perhaps. Perhaps it was simply this theological notion that was passed down from generation to generation. In much the same way, certain number combinations might have been considered "magical."


Figure 44. Teti's Pyramid Construction, Saqqara
At the same time as the slopes were developing a pattern over time, the building standards reached a zenith with Khufu, tapered off and then fall away. By the time of Pharaoh Teti (2345-2333), first pharaoh of the $6^{\text {th }}$ dynasty, the standard had dropped considerably.
The limestone is in small, crumbling, rough blocks or slabs, Figure 44, more reminiscent of dry-stone walling than of the glory days of $4^{\text {th }}$ Dynasty. No doubt, Teti's Pyramid looked grand at the time, as it would have been finished off with smooth limestone ashlars, but the underlying building quality had gone.
On the one hand, then, the themes of slope control and evolution held sway to the end of the $6^{\text {th }}$ dynasty, while on the other hand the methods of construction degraded. This suggests that there was, indeed, some institution vested with responsibility over an extended period for the design of pyramids, to ensure that all theological and magical features were properly accommodated. It suggests, too, that the above ground pyramid construction was in the hands of another group, perhaps separately funded by the pharaoh. In support of this separation of responsibilities, compare and contrast Teti's poor pyramid construction, Figure 44, with the great care taken in carving the Pyramid Texts in his tomb, Figure 53, and in covering the vault of the burial chamber with carved stars. No expense seems to have been spared there. The balance of expenditure, then, may be viewed as having shifted in favour of the features designed to ensure resurrection, and away from features principally designed to impress those remaining on earth.

## Why the Pyramid Shape?

Finally, consider the basis for the pyramid shape. There have been many theories, including: the pyramid looks like the sun breaking through cloud, Figure 43; it provides a staircase* for the Pharaoh to climb up to the sky and the stars; and it mimics naturally occurring features, Figure 45.
Each of these theories is plausible, but there is no evidence to support or disprove them. There is, however, evidence to show an evolution from the first, steppedmastaba pyramid of Djoser on to the "pure" Pyramid, which might have been at Meidum, but could also have been the Red Pyramid at Dahshur. It seems likely that, even then, the stepped pyramid remained at the structural heart of the pure pyramid.


Figure 45. Naturally Occurring Pyramids, Abu Simbal
Given a stepped pyramid, covered in Tura limestone, it seems only a short step to the perfect pyramid - see Figure 46. Why would the ancient Egyptians have made that step? One possibility is concerned with security. The stepped pyramid shape lends itself to being climbed, not only by the curious, but also by would-be tomb robbers. Each step is a ledge upon which to rest and work, trying to gain entry. The step even provides convenient corners upon which to start the break-in. Fill in the steps and the perfect pyramid shape, covered with close fitting, smooth Tura limestone casing blocks, offers no purchase, presents a major obstacle to climbing, and neatly conceals the entrance(s).
So, could the smooth, pure pyramid shape have been inspired by enhanced security? It is probably impossible to be sure, but the notion is as plausible as other theories, and it is consistent with the hard evidence. Inside the pyramids, security was evidently an

[^22]issue, with multiple portcullis doors and sliding stones guarding chamber entrances, false and concealed passages, and plug-stones filling complete passageways along their length. Concealing the entrance would surely have been a major issue for the architects, too. Having concealed the entrance under a large expanse of smooth limestone, it would be important not to impede the spirit of the dead pharaoh, hence the provision of a false door from the mortuary temple into the pyramid.


Figure 46. From Stepped to Perfect Pyramid
Security was unlikely to have been the only reason for the pure pyramid shape. It is likely that the ancient Egyptians were concerned with symmetry and duality. Perfect symmetry may have been reserved for the god Osiris, so that that the four equal sides of the pyramid base would have had theological significance - hence the great accuracy of base construction. The reason for the sides facing the four cardinal points is surely concerned with the daily apparent motion of the stars and the Sun from east to west, and the flow of the Nile from south to north.

Pyramids may also have been viewed as "creation mounds." The Great Pyramid seems to have been built over an existing mound on the plateau, which may have had some symbolic significance. The idea of representing, or replicating, the original mound of creation, which first broke through the all-encompassing chaotic waters, permeated ancient Egyptian culture. If the pyramid were viewed as an upward extension of the land then other features of Old Kingdom pyramids may be seen in a different light - see Figure 47, which illustrates a concept, rather than any real pyramid.

Inward sloping columns from which the pyramids are constructed may have been viewed as a continuation of the various rock strata, bent upwards on all four sides towards the peak of the pyramid. The internal structure of a true pyramid was not quite like this - see Figure 23 above - as the various strata were truncated to form steps, but the similarity is nonetheless marked
The formation of a pitched roof above the sunken burial chamber would then emerge naturally from the peaked stratum. The pattern to which pyramids eventually evolved by the $6^{\text {th }}$ Dynasty looks singularly like Figure 47 , with no chambers in the superstructure, a burial chamber dug down from the surface, and a pitched roof over the chamber, which just protruded into the body of the superstructure

The white surface of the pyramid would then be associated with the primeval purity of the Creation Mound, while its external smoothness would represent the natural
continuity of the underlying, primeval strata, now thrust upwards to form a perfect, pristine shape
The concept of the pyramid as the Mound of Creation could possibly explain other puzzling factors. Pyramid construction, with the inward sloping columns, has been remarkably successful at resisting the ravages of time, and in particular of earthquakes. Other monuments have not escaped so lightly, which has raised the question: did the ancient Egyptians consciously design the internal structure to resist earthquakes, or was this simply fortuitous? The Creation Mound concept suggests that the strength inherent in the design may have been fortuitous. It is even possible that the use of pitched roofs above chambers was designed, not as we might perceive


Figure 47. Conjectural View of a Pyramid as a Mound of Creation
today, to divert down thrust from the superstructure, but to represent terrain strata formed into a peak under the Creation Mound. We may never know.

## Comparative Neolithic Stone Building - Gozo and Malta

The ancient Egyptians were not alone. Standing stones and stone constructions in France, Scotland ${ }^{31}$, Orkney, Salisbury Plain, Malta, and many other places precede Pyramids. The oldest unsupported stone structure in the world is claimed to be at Ggantija on the Mediterranean island of Gozo, Homer's Calypso. A community of temple-builders thrived on Gozo and its larger sister island Malta from about 3600BC to 2500 BC .
The temple at Ggantija (Jgant-ee-ah), Gozo, Figure 48, is believed to be the first unsupported stone structure in the world. It is described as "apsidal", since it comprises a number of interconnected semicircular chambers ${ }^{32}$. One reason suggested for this curvilinear construction concerns earlier burial practices on the island.
Earlier inhabitants had dug into the limestone bedrock to create tombs. After making an initial entrance, the diggers dug out sideways to make kidney-shaped underground


Figure 48. The world's first free-standing stone structure at Ggantija, Gozo, ~3600BC (Model, Gozo Museum)
burial chambers, leaving a vertical wall between the chambers to prevent the roof from caving in. The result was a pair of apse-like tombs. The model of the first freestanding construction at Ggantija, ("Giant's Tower") imitates this underground shape, and it seems likely that the apses at Ggantija were roofed over, possibly in the style of an igloo. Subsequent temples followed the same general curvilinear plan.


Figure 49. Hagar Qim, Malta, ~3000-2500BC
Figure 49 shows only a small part of the temple complex at Hagar Q'im (Hadjar'eem) one of the many temples on Malta ${ }^{33}$; the entrance is about 2 m high Evidently, these
people were able to quarry, move, dress and raise large limestone blocks. They also new how to roof over their structures with stone, by corbelling in successively smaller circular tiers, and by sloping stones inwards, igloo-style.
Although they left no written word, it is evident from the construction of hidden chambers, concealed passages and doors, that the temples were used, at least in part, as oracles. Figure 50 shows the inside of the Oracle Room in the Mnajdra Temple, some 500 m towards the sea from Hagar Q'im ("standing stones"). The picture shows the room's curved apse form. The large, butted wall stones are surmounted with horizontal stones, overlapped in the style of a modern bricklayer's Dutch bond, to give strength and stability. There is a hidden chamber behind the wall, accessed through a secret passage entering from behind the temple. At left and right are two precision-cut holes. It seems highly likely that these holes were used either to project the voice of a hidden priest or leader, or to pass objects/offerings, or both. The hole at left has two depths, as can be seen, suggesting that there may have been a cover which could be set in place, or that items could be stood on the ledge either from the apse, or from the hidden chamber.


Figure 50. Oracle Room, Mnajdra Temple, Malta, $\sim 3300-3000 B C$
The designs and apparent purpose of these first, freestanding stone structures offer stark contrast to that of the ancient Egyptians. The Gozo and Malta temples were determinedly curvilinear, whereas those of ancient Egyptian were firmly rectilinear. The purpose of the Gozo and Malta temples seems to have been concerned with answering questions, perhaps with predicting the future and forecasting events, weather, etc. If so, then this amounted to humans manipulating other humans, after the style of the much later Greek Oracle. Again, such a purpose would seem to be quite different from the purposes of ancient Egyptians stone temples and pyramids.
Limestone balls about the size of a man's head have also been found at the Tarxien Temple, Malta, leading to suggestions that these people used the balls to manoeuvre
the blocks during construction. If correct, that would be a very early instance of the wheel indeed; an alternative view is that the balls were used in a divination process. If, when rolled or thrown, the stones fell into stone recesses, then some event or situation was favoured - or not, as the case may be.
In any event, many of these edifices have been eroded by unfavourable climate, and their builders left little in the way of inscriptions, and nothing in writing, to indicate purpose. Only in Egypt is the magnificent flowering of stone building in the late Neolithic both reasonably preserved and accompanied by sufficient written word to support sensible interpretation as to purpose.
Looking at the major stone works that are in evidence across the Mediterranean and Europe, we may have to revise our idea about Neolithic man across that broad area. They appear to have been more capable and organized than we are today!

# Chapter 3. Of Stars, Spells, Shafts and Speculations 

Philosophers and scientists have been puzzled through the ages by the enigma of the pyramids. Some have tried to uncover the alleged "secrets" of the pyramids using copious measurements. Others have tried to interpret the arcane Pyramid Texts carved on the walls of $5^{\text {th }}$ and $6^{\text {th }}$ Dynasty Pyramid tombs. Yet more have tried to work out the meaning of the shafts in the Great Pyramid, which emerge from the King's and Queen's chambers, Figure 30.
With so little hard evidence upon which to draw, it is perhaps not surprising that some investigators have seen what they wanted to see. The Great Pyramid in particular has been the subject of more extravagant and outlandish theories than perhaps any other structure on the planet.
Some of that scant evidence, and what might reasonably - or perhaps unreasonably be deduced from it, will be presented in this chapter.

## The Ancient Night Sky

The ancient night sky was different from today's. Precession, the rotation of the Earth's spin axis on a 26,000 -year cycle, means that the various star formations were seen in different directions during the Old Kingdom. Additionally, certain stars that are relatively close to Earth, notably Sirius, will have moved relative to their background of more distant stars*. Using computer generated star maps, it is straightforward to see how the heavens would have looked during the Old Kingdom by effectively "winding back" both precession and proper motion.

## The Pole Star

One particularly noticeable effect of precession is that it changes the star we observe as the Pole Star. As the Earth spins on its axis, the heavens appear to rotate about two fixed points, the celestial poles. Today, in the Northern Hemisphere, the star nearest to the celestial North Pole is Polaris. During the Old Kingdom, the nearest star to the celestial North Pole was Thuban, underlined at the centre of Figure 51.
As a fixed point in the night sky, Thuban would have been useful, as is Polaris today, for orientation and navigation at night. Thuban would have been visible at night all the year round, always in the same place - a dependable guide.
The elevation of the celestial pole above the horizon is equal to the latitude of observation. In the case of Giza, for example, that is about 30 degrees North, using modern measures and units. Thuban was not precisely at the celestial pole ${ }^{\dagger}$, but was close enough for most practical purposes.

[^23]

Figure 51. The Imperishable Stars, Giza, 28 December BC2575, 06:49
To the ancient Egyptians, it may well have appeared as an essential star. They were concerned about how the pharaoh's spirit would reach heaven to sit among the Imperishable stars. Apart from flying, suggestions included ropes and rope ladders so that the pharaoh could climb there. If a rope were to be hung from anywhere, it would have to be the celestial pole, since that was the only point in the star-field that did not move. Thuban might not have been right on the pole, but it was close enough, and there appear to have been no other choices. Flying and using staircases were always alternatives, of course:
"A stairway to the sky is set up for me so that I may ascend on it to the sky, and I ascend on the smoke of the great censing. I fly up as a bird and alight as a beetle"."

Utterance 267, Pyramid Texts
"A ladder is knotted ${ }^{\dagger}$ together by Re before Osiris , a ladder is knotted together by Horus before his father Osiris when he goes to his spirit..."

Utterance 305, Pyramid Texts

[^24]
## The Imperishable Stars

As stars appear to rotate about the celestial pole, they trace out a circle. Those stars, which did not dip below the horizon on their apparent journey, were called Imperishable Stars, since they were always visible and since dipping below the horizon was equated with dying as the Sun died in the West each night, only to be resurrected in the East each morning. As Giza is latitude 30 degrees North, a circle drawn at 30 degrees elevation and of radius 30 degrees, Figure 51, encompasses the, then, Imperishable Stars. Today's constellations of Ursa Major and Ursa Minor are shown, although the ancient Egyptians would not have recognized them.

## The Legendary Stars

Some stars became associated with the Osirian legends, Figure 52. The figure shows the group of stars that we call the constellation of Orion today. Lines join the stars, supposedly to represent a warrior with a raised sword in one hand and a shield in the other - the Greek warrior Orion. The red giant star, Betelgeuse has a name that means "the armpit" in Arabic.


Figure 52. The Orion Constellation and Sirius
At the centre of the constellation is Orion's Belt, made up of 3 stars: Alnitak ("The Girdle"), Alnilam ("The String of Pearls") and Mintaka ("Belt"). These stars, together with many others, were given Arabic names by the Arab astronomers who were preeminent in the Middle Ages.

There is no single star called Orion. The ancient Egyptians had a star grouping that they referred to as Sah, or Sahu, translated as Orion by Faulkner ${ }^{34}$. Which stars of those in the today's Orion constellation corresponded to the stars forming Sah is not certain. The Pyramid Texts in Unas Pyramid at Saqqara show three stars in a vertical
line when referring to Sah, which might suggest that Sah was just the belt stars; on the other hand, that could be shorthand for a group.
Looking at the night sky today, the formation of Betelgeuse, Bellatrix, Saiph, Rigel and the 3 belt stars stands out as a spectacular group, i.e. without any need to add the "sword arm " and "shield". Late Period representations of the heavens, Figure 64 on page 85 , showed a king wearing the white crown of Southern Egypt and carrying a staff, in the appropriate area of the sky for Orion; there were no indications of individual stars, shields or raised arms.

The three stars of the belt point at the brightest star in the sky, Sirius; today's young astronomers are taught to find Sirius using this line as a pointer. The ancient Egyptians called Sirius "Sothis", or more strictly, they called the star's goddess Sothis. Some of the so-called utterances in the Pyramid Texts may be interpreted as connecting Sirius with the goddess Isis from the Osirian Legend:
"The sky is clear, Sothis lives", because I am a living one, the son of Sothis..."
Utterance 302, Pyramid Texts
"Your sister Isis comes to you rejoicing for love of you. You have placed her on your phallus and your seed issues into her, she being ready as Sothis and Har-Sopd has come forth from you as Horus who is in Sothis ${ }^{\dagger}$."

Utterance 366, Pyramid Texts
Taken together, these two utterances state that the king is the son of Sothis, and suggest that Isis identifies with Sothis. In the first utterance, the king identifies with Horus since Isis/Sothis is Horus' mother, while in the second the king identifies with Osiris, since Isis is Osiris' sister. These are not necessarily inconsistent, since the king transmogrifies from the son Horus into the father Osiris as part of the ascension process. The connection between Sothis and Isis appears to be reasonably based.

## Pyramid Texts

The style of the Pyramid Texts suggests to scholars that some parts were written well before the $4^{\text {th }}$ Dynasty, making them one of the oldest sources of literature in the world. Recent finds at Abydos ${ }^{35}$ indicate that Egyptians may have invented writing before the Sumerians, making Egypt truly the oldest civilization.
Translations of the Pyramid Texts comprise 759 "utterances ${ }^{36 "}$. Utterance 538, for instance, is a protective spell. "Get back, you needy long-horn! Your head is in the hand of Horus, your tail is in the hand of Isis, and the fingers of Atum are on your horns." Although suggestive of cattle or bull imagery, the significance of this and many similar utterances is unclear without an awareness of the theology and context of the time.

[^25]

Figure 53. Pyramid Texts, Unas* and Teti Pyramids.
The pyramid texts were evidently written over a considerable period of time, with new spells and incantations being added to earlier ones. While many of the Utterances are couched in a stellar theology ${ }^{\dagger}$, others clearly reflect the later solar theology. The panel ${ }^{\ddagger}$ shows Utterance 442 from Faulkner's Translation. The context is clearly that of a stellar theology, and the Field of Rushes may be reasonably taken to refer to the Milky Way.


#### Abstract

"Behold, he has come as Orion, behold Osiris has come as Orion, Lord of Wine in the festival...O King, the sky conceives you with Orion, the dawn light bears you with Orion...You will regularly ascend with Orion from the eastern region of the sky, you will regularly descend with Orion into the Western region of the sky, your third is Sothis (Sirius) pure of thrones and it is she who will guide you both on the goodly roads which are in the sky in the Field of Rushes."


(Faulkner's Translation, 1969, Utterance 442)

A later text states:
"I am pure, I take my oar to myself, I occupy my seat, I sit in the bow of the ship, ...I row Re to the West..."
This is clearly a solar context. Pharaoh is no longer a supreme, sole God, but one among many, subservient to Re, the Sun god. Evidently, there has been a shift from a pharaoh-as-sole-god stellar cult to a priest-supported, Re-is-supreme, solar cult.
Ceremonies appear in many passages throughout the Pyramid Texts. Utterance 669 states: "The prince ascends in a great storm from the inner horizon; he sees the preparation for the festival, the making of the braziers, the birth of the gods before

[^26]you in the five epagomenal days..." This passage indicates that annual ceremonies were held, in this instance on the five special, epagomenal days, which made up the full year.
Prayers were held for the pyramid and for its priesthood. Utterance 599 states: "A boon which the King grants and Geb grants of these choice joints, invocation offerings for all the gods who shall bring into being all good things for the King, and who shall cause to endure this construction and this pyramid of the King, in accordance with what the King wishes in the matter, for ever and ever." This implies that those looking after the pyramid would get the best food forever, which in turn implies an endowed priesthood per pyramid.

Utterance 466, again one of many, states: "O King, you are this great star, companion of Orion, who traverses the sky with Orion, who navigates the Netherworld with Osiris; you ascend from the east of the sky, being renewed in your due season and rejuvenated at your due time. The sky has borne you with Orion, the year had put a fillet * on you, the dance has gone down to you, a food-offering is given to you, the Great Mooring-post cries out to you as to Osiris in his suffering. O King, navigate and arrive, but beware of the Great Lake!" The Netherworld refers to the place where stars go to as the Sun rises and they become invisible by day. The Great Mooring-post may be the celestial pole, marked by the then Pole Star, Thuban.

Utterance 509: "...I ascend the sky among the Imperishable Stars, my sister is Sothis (Sirius), my guide is the Morning Star (Venus), and they grasp my hand at the Field of Offerings..."

Utterance 503: "The sky is opened, the earth is opened, the apertures of the celestial windows are opened and the movements of the Abyss are revealed, the movements of the sunlight are released by the One who endures every day...I ascend to the sky...I set myself upon the throne 'She who preserves Justice.' I am back to back with those gods in the north of the sky, the Imperishable Stars; I will not perish - the Inexhaustibles, I will not become exhausted...When Montju ${ }^{\dagger}$ is high, I will be high with him; when Montju runs, I will run with him."

There is an underlying thread woven through all of the Pyramid Texts. Upon death, pharaoh climbs, flies, or is lifted up to the heavens to take his place among the Imperishable Stars. He then either wanders the starry skies using celestial boats, or joins others in the barque of Re , the sun god, ensuring that it travels across the sky each day. In his travels by night he is clearly associated with Sothis/Sirius and Osiris.

Sky directions are confusing and inconsistent. Some suggest that the dead pharaoh is closely associated with Sirius and Osiris. Most suggest he is firmly among the other gods in the Imperishable Stars. Utterance 466 above suggests that the dead pharaoh travels with Orion, and so is not circumpolar but visible only part of the year. Utterance 509, also above, places the pharaoh's $k a$ in the Imperishable Stars (circumpolar) guided by the Morning Star (always in the east) and brother to Sirius (always distant from the Circumpolar Stars.)

[^27]Some of the texts also seem to hark back to primitive burial practices, before the introduction of the mummy. Utterance 373, a so-called resurrection text, states: "Raise yourself O King; receive your head, collect your bones, gather your limbs together, throw off the earth from your flesh, receive your bread, which does not grow mouldy and your beer which does not grow sour and stand at the doors which keep out the plebs...Barley is threshed for you, emmer is reaped for you and offering is made thereof at your monthly festivals, offering is made thereof at your half-monthly festivals...Rise up O King, for you have not died."
Neolithic people in Europe are known to have left dead bodies out to be dismembered and picked clean by wild animals and birds, before gathering the skeletal remains together for entombment. Utterance 373 et al could be read as referring to that practice. Since there appeared to be no such practice in the Old Kingdom, this either refers to an ancient practice - suggesting great age indeed for the Pyramid Texts - or it is some figurative reference to the various parts of the dead body reactivating, sinews tightening, blood flowing, etc.

## Using the Stars

The ancient Egyptians were assiduous and capable astronomers. They evidently used the stars for a variety of purposes, from finding the four cardinal directions with singular accuracy to predicting events.

Predicting The Inundation


Figure 54. Simultaneous rising of the Sun and Sirius, $\boldsymbol{6}^{\text {th }}$ July 2575BC, 04:44
The Sun and Sirius dawned at just the same moment on one day only each year during the Old Kingdom, Figure 54. This occurrence is called the heliacal rising of Sirius, or more simply, the dawn rising. In the figure, the constellation of Orion is shown, with the three belt stars pointing at Sirius, which is precisely on the horizon. The Sun at extreme left, is also precisely on the horizon. The accuracy afforded by this coincidence is remarkable; it identifies the particular minute on the particular day.

For ancient Egypt, the dawn rising of Sirius heralded the Inundation, making Sirius a star of great significance and importance. The dawn rising of other stars could be used similarly to identify other dates.

Additionally, the ancient Egyptians recorded the dawn culmination, or transit of stars. A dawn transit is the time at which a star reaches its highest point, due south or north, just as the sun rises. The dawn transit of Sirius, Figure 55, would have held special significance.
In the figure, note that the Sun is just rising on the horizon at left, and that Sirius is on the 180degree azimuth line, indicating that it was due south. All transits, dawn or otherwise, must be either due south or due north by definition.


Figure 55 Dawn Transit of Sirius, $11^{\text {th }}$ September 2575, 05:25

## Stellar Coincidences

Sometimes dawn transits and dawn risings could coincide. One notable coincidence concerned the stars Kocab, in today's Ursa Minor (Little Bear), and Alnitak, the left hand star in Orion's Belt; Alnitak is the tip, or arrowhead of the pointer towards Sirius.

Alnitak rose at dawn on $8^{\text {th }}$ June each year, Figure 56, at $04: 52$ By a curious coincidence, the star Kocab also happened to culminate at just about the same moment.

What would the ancient Egyptians have made of this curious coincidence? What were the stars telling them? Was Kocab, perhaps, intended to be pharaoh's particular star amongst the Imperishable Stars? Such an annual replay of Pharaoh's transmogrification from Horus to Osiris is indicated in the Pyramid Texts:
"O King, the sky conceives you with Orion (Alnitak?), the dawn light bears you with Orion..." see page 76. "Dawn light" indicates that the star rose in the east.

Another stellar coincidence could be observed with Alnitak: not only does it share a coincidence with Kocab, but it also had a unique elevation angle of 45 degrees at transit.

Remembering that the ancient Egyptians did not use angles but measured elevation by proportions, they would have no doubt seen great significance in the 45 degree elevation of this particular star, which they would have seen as 1:1, i.e. perfect symmetry. Symmetry seemed to hold very special significance to the ancient Egyptians; they went to great lengths in their system of fractions to avoid symmetry, and 45 degrees is conspicuous by its absence from the slopes of pyramids.

Special significance in symmetry may have preceded the measurement of star elevation. The discovery of a star with such a symmetrical elevation that also happened to be the pointer star to Sirius may have suggested that the Osiris to Isis relationship could be seen in the stars. For this reason if no other, it may be suspected that Alnitak was associated with the God Osiris, as Sirius was associated with his wife and sister Isis.


Figure 56. Simultaneous Dawn Rising of Alnitak and Dawn Transit of Kocab, $8^{\text {th }}$ June, 2575BC, 04:52

## Finding North

One last stellar coincidence is particularly worthy of note. One of the features of the Old Kingdom pyramids is the great accuracy with which they face the cardinal points. Kate Spence of Cambridge University has noted that, although the northern face of a pyramid does face generally north, the precise direction varies over time from earlier to later pyramids. "This suggests that their method for finding north may have had a stellar basis, and that that basis was subject to precession wander

If the star Kocab is vertically aligned with the star Mizar, Figure 57, a naked eye double star ${ }^{\dagger}$ in (our) Ursa Major , then the vertical line passes through the celestial pole. As the figure shows, this was precisely true during 2450BC, but give slight errors before and after that year. Kate Spence has found that those errors correlate

[^28]with the errors in facing pyramids to the north over the same period, suggesting that this was how the ancient Egyptians found north, at least when laying out pyramid foundations.


Figure 57. Finding North with Kocab and Mizar
This means of deriving due north would have proved convenient for the ancient Egyptians; they would have been able to judge vertical alignment using a simple plumb bob, and marked the north line against some distant object on their horizon. This suggests that the method would be of most use at dawn or dusk, so that stars, plumb line and horizon were all visible. The method also works, of course, when Mizar is above Kocab, so giving more opportunities.
The method of aligning these two stars would have been valuable once the relationship had been discovered. The method actually determines when Kocab culminates. It follows, therefore, that to conceive the method, the ancient Egyptians must first have been able to establish that Kocab was at its highest point, and hence due north, before then observing that Mizar was fortuitously directly below it at just the right moment. Once the discovery was made, it would have enabled almost anyone to find north easily.

There are other ways that the Egyptians might have employed to find north. David Macaulay ${ }^{37}$ suggests another way, illustrated in Figure 58. A low circular wall is constructed, and the top is made perfectly flat and level. This is achieved by constructing a water channel in the top, and levelling to the watermark. There is an opening at the mouth to the north, and beyond the opening is a number of poles.
To use the apparatus, an astronomer would sit at the focus of the circle, ensuring consistent $\mathrm{x}, \mathrm{y}$ and z positions by aligning the line of vision with suitable poles and markers - or, possibly, by using a head harness. At dusk, when a suitable star appeared over the eastern rim of the wall, a mark would be made on the top of the wall to show its rising angle. As the star fell towards the western horizon, a second
mark would be made on the top of that wall. The two marks give two angles from the eye of the seated observer. Bisecting the angle would give north, and this result would be recorded using vertical poles aligned in a row going north from the circle.


One star would not give a very accurate result. This method lends itself, however, to following several stars each night, and for repetition over many nights. Averaging the results would provide a successive improvement in the estimation of north.
It is possible to simulate this approach using astronomy simulators and making visual judgements about azimuth angles; if these are anything to go by, the method would have worked well.

## Viewing Proportions?

Old Kingdom astronomers and architects were evidently able to set up and measure slopes using their method of proportions; astronomers needed such a means for measuring star elevations, while architects needed it for pyramid, passageway and shaft slopes.
One possible method ${ }^{*}$ for observing, measuring and presenting slopes is shown in Figure 59. At the bottom is a plan view of a simple pegboard, with holes measured off at one-eighth cubit lengths as shown. Above the plan view is a side elevation, showing the side of the pegboard with a vertical pole projecting through the end hole. The pole is marked off in one-quarter cubit lengths. A string is suspended from the top of the pole, and it may be pegged into any of the holes in the pegboard.

[^29]Using such a device, any slope may be set up for passageway or pyramid. Cloth may be suspended over the string to represent a sloping pyramid face, or even a complete pyramid. Measuring off the peg holes in unitary fractions of a cubit would have been an elementary and obvious thing to do. A pharaoh, requiring his pyramid to be steeper or shallower, would have been limited in his choice by the divisions on the pegboard and the vertical pole. While this does not explain the choice of successive integers in the first place, it does offer an explanation for the unitary fractions added to those integers in some instances - such as the example shown in the final column of Table 2 on page 58.


Astronomers would have fitted a fork- or ring-sight on top of the pole to look upwards at objects, e.g. stars, and measure their slope by looking along the string, through the sight, and so deriving an accurate slope.
Such a viewing and measuring device would have enabled ancient Egyptian astronomers to observe when any star reached the highest elevation in its nightly traverse across the heavens. Since such transits were always either due south or due north, astronomers would have had yet another means of marking the cardinal points.

## Cattle culture

Ancient Egypt exhibited many features suggesting it was founded, at least in part, in a cattle culture, i.e. its ideas and practices were based in the land and particularly in the herding of cattle on that land, Figure $60^{*}$. Cattle would represent wealth, and status, of course, but more. The ancient Egyptians employed symbols to an extent probably

[^30]

Figure 60. Herding in the New Kingdom, British Museum
never paralleled by any other culture, and many of their symbols and icons were cattle based, including those in the stars.

Utterance 306 of the Pyramid Texts, one of many such ascension texts, states: "...when you ascend to the sky with the power upon you, your terror about you, and your magic at your feet...Endure, endure, O enduring Bull, that you may be enduring at the head of them and at the head of the spirits for ever." 'Them' may refer to the Imperishable Stars and/or the gods of the sky.
Utterance 480 reiterates: "...Behold, this which has been said to you, you gods, lest the King be not at your head; behold, the king is established at your head as the enduring bull of the Wild Bulls."

An ivory label found at Abydos, Figure 61, shows Pharaoh Den smiting ${ }^{*}$ an Asiatic foe - a scene to be repeated literally hundreds of times up into the New Kingdom.

The pharaoh is shown wearing a bull's tail, evidence of the influence of cattle culture. Bulls were seen to have great creative power, and the pharaoh must be seen in a similar vein. Wearing the bull's tail presented the


Figure 61. Ivory Label showing Den, 1st Dynasty, Smiting an Asiatic Foe. British Museum pharaoh as having the power of

[^31]the bull, lord and master of the herd, prepared to do battle to protect the herd*.


Figure 62. Bull Palette. Louvre

Figure $62, \sim 3200 \mathrm{BC}$, shows an even earlier, and more explicit, reference to the bull. The bull, representing the pharaoh is conquering an Asiatic who struggles, face down and helpless. A rope extends down the palette, to which are attached a number of tribal totems: an Anubis-like figure, facing a placenta as in Figure 61; a second Anubis-like figure; an Ibis; a fourth figure, seemingly long-necked; the fifth figure is missing. Together, they suggest a group of tribes fighting together under the leadership of the pharaoh to rid themselves of the Asiatics.

## Ancient Egyptian Constellations

Like the Chinese, the ancient Egyptians had their own constellations, represented by a menagerie of animals. Along with the crocodile can be seen the lion and the bull. It is tempting to equate these with Leo and Taurus of our present day zodiac, but that would be premature. Little is known of the
early constellations, some of which appear in Figure 63. The stars appear to be distributed more with a view to art than to represent stars in the sky. The lion, for example, is outlined in stars like no known constellation, and today we know of no crocodile constellation.

One constellation is reasonably certain, however. The Bull in the figure seems to have been associated with the circumpolar stars, those that today we call Ursa Major and Ursa Minor. That association precludes it from being Taurus the Bull of today's zodiac.

The temple of Hathor at Denderah was built during the late


Figure 63. Ancient Egyptian Constellations, New Kingdom

[^32]

Figure 64. The Constellations at Denderah, Ptolemaic Period
Ptolemaic Dynasty; Queen Cleopatra is prominent on the external walls. The splendid ceiling from that temple, now in the Louvre, showing their contemporary view of the constellations, Figure 64, is much later than that of Figure 63.

At left in the figure is a seated cow, usually representative of Hathor, with the star name Sirius overlaid by museum curators; by this late stage, Hathor had fused with Isis. The figure of a king, Osiris perhaps, is shown below her, bottom left, wearing the white crown of Upper Egypt, and holding a staff; this is evidently not the image of Orion the Hunter, although the curators have marked Orion at that location on the map to indicate the location of today's zodiacal constellation. Next to the king is an unnamed bull, in broadly the right position for Taurus.

Curators have overlaid the positions of today's constellations, Ursa Minor and Ursa Major, but they do not correspond to bears on the map. The large inverted figure of a


Figure 65. Ursa Major and Ursa Minor as the Bull of Heaven?
Ursa Major might represent the Bull, while Ursa Minor might represent its genitalia


Figure 66. Hieroglyph for Bull
rampant hippopotamus is in broadly in the position of today's constellation, Draco. Even at this relatively late stage, some 2,500 years after the $4^{\text {th }}$ Dynasty pyramid were built, the ancient Egyptians evidently did not use the same constellations as we do today.

## Identifying the Enduring Bull of the Sky

Many of the utterances refer to bulls. The king is "the enduring bull of the Wild Bulls," suggesting that he takes his place, perhaps, in the form of a bull. Alternatively, the passage might imply that the king's bull-like seminal power should endure.
"...may a stairway to the Netherworld be set up for you to the place where Orion is, may the Bull of the Sky take you by the hand, may you eat the food of the gods."

Utterance 610, Pyramid Texts
This utterance suggests that "the Bull of the Sky" is already in the heavens when the king's spirit arrives.

What can be made of these, and many other, similar, utterances? Figure 65 shows today's Ursa Major and Ursa Minor, believed to be associated during the Old Kingdom with a bull image. We do not know just which, if any, stars made up the Egyptian "Bull of the Sky", but it does not take too much imagination to see a possible animal figure in Ursa Major and the bull's reproductive organs in Ursa Minor. Were this perception valid then Kocab would be one of the gonads, with Pherkad as the other.

In this context, it is interesting to note that the hieroglyph for "bull" is as shown in Figure 66. It is transliterated as "ka", identifying the word for bull with the word for life essence. The hieroglyph consists of the $k a$ symbol, two arms raised in the classic position of praise, surmounting (presumably) a bull's genitalia and with a picture of a bull as determinative.

When looking towards the celestial pole for representations of a bull among the stars, it may be that we should be looking, not for a bull-shape, but for the hieroglyph "writ large in the heavens."

Figure 67 shows some of the options. Ursa Minor alone, at the bottom of the figure, could represent the bull, since it appears to show the bull's genitalia. On the other hand, Ursa Minor might represent the bull together with the stars picked out by the dotted line, since together they appear to recreate the hieroglyph, less the determinative. Finally, Ursa Minor might represent the Bull together with the stars
picked out by the dashed line. We know the latter as the Plough, the Big Dipper, or Charlie's Haywain, but to the ancient Egyptians, working from the viewpoint of their cattle culture, it could easily look like another set of genitalia, this time pointing to the right.


Figure 67. Enduring Bull of the Sky?
All of this is pure speculation, of course; stars can be joined up to imagine just about any shape.

## Navigating the Skies by the Bright Stars

Since time immemorial, people have found their way around the stars using bright stars and their patterns to lead them to other stars. We have already seen the three stars in Orion's Belt used as pointers to Sirius, and many will be familiar with using the pointers in the Plough to locate Polaris, today's Pole Star. It is highly likely that the ancient Egyptians did the same.
"Your third is Isis/Sirius pure of thrones and it is she who will guide you both (i.e. Osiris and Pharaoh) on the goodly roads which are in the sky in the Field of Rushes (Milky Way)"

Pyramid Texts, see panel on page 76
The conventional explanation for this curious passage is that it addresses the pharaoh and Osiris, so that Isis then counts as the third character. That may be the simple, and correct, explanation, although other utterances refer to "your fourth" and even "your fifth". As Figure 68 shows, another interpretation is possible, and may seem more plausible, since it addresses the whole utterance, rather than just a part.


Figure 68. Navigating by the Bright Stars
If one uses Ursa Minor as a pointer, with Kocab as the tail-feathers and Polaris as the arrowhead, then there are only 2 bright stars on the resulting line before reaching Sirius. So Sirius is the third bright star. Notice that the line from Kocab to Sirius passes through the Milky Way - the Field of Rushes. Notice also that a line from Alnitak/Osiris also passes through the Milky Way.

It is possible, and not unreasonable, to interpret the utterance as describing a route, a journey, or even the passage of celestial seminal fluid, across the heavens from the Kocab in the Enduring Bull of the Sky (i.e. the dead pharaoh's spirit) to Isis/Sirius.

Overall, the passage may be interpreted as describing a potential journey or transmission from the Bull of the Sky constellation in the north, representing the seminal power of the dead pharaoh-as-Osiris, to Sirius in the southwest, representing Isis. This has the hallmark of a fertility ritual, and would be consistent with the supposition that Kocab represents one of the Bull's testes. The dead pharaoh-as-Osiris would then become the seminal force of the fertility ritual, ensuring continued Inundations and fertility for Egypt for all time.

## Symmetry and Sequential Integers in Architecture

## Integer Succession

Previous chapters showed how pyramid slopes were controlled in terms of opposite divided by adjacent sides, as in a right angle triangle. We have further seen how there was a tendency for "opposite over adjacent" to be made up of two successive or sequential integers, e.g. 4:3, 3:2.

The use of sequential integers to determine dimensions appears throughout Old Kingdom architecture. In addition, there appears to have been an aversion to 45 degrees, or one-to-one, slopes. Converting 45 degrees gives a slope of exactly 7 seked, a seemingly convenient integer, or proportions of 1:1. Taken together, sequential integers and an aversion to 45 degrees, suggest that theological or magical considerations, rather than architectural ones, influenced the choice of slopes.

## Queen's chamber

The Queens Chamber in Khufu's Great Pyramid, Figure 69, affords a good example. Measured in metres, it seems unexceptional. Measured in royal cubits, however, it turns out to be some $10 \times 11 \times 12$ cubits.


So, the dimensions are made up from 3 sequential integers. It might be supposed that this choice was simply for engineering convenience, and to be sure, the room has pleasing proportions.
However, the Egyptian* architects had to undertake some serious calculations - or model developments - to set the peak of the pitched roof at 12 cubits.
The choice of three sequential integers for the Queen's Chamber is unlikely to have occurred by chance.

## The Angle $26.5^{\circ}$

The angle $26.5^{\circ}$ appears widely throughout Old Kingdom architecture. It is used for the up or down slope of passages in Khufu's, Khafre's and Unas' pyramids amongst many others. On the face of it, this angle is a peculiar choice.
Figure 70 shows how simply drawing a diagonal across two abutted squares forms the angle. The rectangle formed by these two squares is an example of


[^33]sequential integers, of course, with sides of one and two.

## The Angle 21 ${ }^{\circ} 41$ '

Khafre's Pyramid has two entrances on the north face. The steeper descends at $26^{\circ} 52^{\prime}$, the shallower at $21^{\circ} 41^{\prime}$. How is this shallower slope formed?
Figure 71 suggests the probable answer. If the 2square figure is extended on the long side from two to two and one half, then the


Figure 71. The Angle $21^{\circ} 41^{\prime}$ resultant diagonal is $21^{\circ} 48^{\prime}$. Although not precisely Khafre's passage slope, this is sufficiently close to suggest that this was, indeed, the method of slope derivation and control. If so, this is both an example of sequential integer and of unitary fractions, as seen in the slopes of the pyramids themselves.

## King's Chamber Dimensions

The King's Chamber floor plan is a rectangle, 10 cubits by 20 cubits - see Figure 72. Being a floor plan, this arrangement cannot be concerned with slope angles; there is none involved. The implication must be that the two abutted squares, while explaining the $26.5^{\circ}$ passage slopes, have deeper significance concerned, perhaps, with the symmetry and duality of two identical squares. The diagonal creates two identical triangles, so there is duality as well as symmetry, which may have been deemed significant too ${ }^{*}$.
Unlike the Queen's Chamber, with its pitched roof, the King's Chamber is roofed with horizontal stone beams. The height of King's chamber is the semi-diagonal of floor, i.e. 11.18 Royal cubits. This evidently provided the builders with a simple way to set and measure wall height around this relatively large chamber. An accurately flat top to the walls would have been vital in the even distribution of down thrust from the superstructure. Above the chamber are the so-called


Figure 72. Floor Plan of King's Chamber relieving chambers - see Figure

[^34]32 above - which may have theological and magical, as well as constructional, purposes; these chambers are so heavy that ensuring a flat, level surface to the top of the King's Chamber wall would have been paramount.

## Purpose of the Shafts in the Great Pyramid

The dimensions of the Great Pyramid and of the chambers within it have been the subject of much study and speculation, some of it wild. If enough measurements are taken of any complex structure, relationships can surely be found between some of them. Such isolated relationships are not evidential, however. Some consistency of pattern is needed to posit a rational theory of relationships, and preferably across many structures rather than in just one. This presents particular problems for any unique features, since crosschecking is then impractical.
The four shafts emanating from the Queen's and King's chamber in the Great Pyramid (on page 42 and Figure 30), lie somewhere between the limits: they are unique, in the sense that only the Great Pyramid has them; on the other hand, there are 4 , all different, which suggests that a single explanation might be found such that it encompassed all the differences.

## Ventilation Theory

As we have seen, the shafts were originally thought to be for ventilation, although this was scarcely credible: some shaft openings were originally covered with stone; moreover, the shafts pursue their individual inclines through many layers of stone, necessitating construction effort beyond that appropriate for simple ventilation.

In April 1993, a young German, Rudolf Gantenbrink was employed to clear the shaft in the south wall of the Queen's Chamber. The shaft proved to be about 8 in. by 8 in. and some 200 feet long. Instead of opening out at the surface of the Pyramid, he found a small marble portcullis door, with the remnants of two copper handles still attached. Clearly, this shaft had never been intended for ventilation.

## Starshaft Theory

Robert Bauva ${ }^{38}$ has championed the notion that the four shafts point at the dawn transits of four stars, Figure 73. Today, the four shafts do not point to any stars of note. Winding back the clock to $\sim 2500 \mathrm{BC}$, allowing for precession and proper motion, 4 stars culminated ${ }^{\dagger}$, one over each shaft, as shown in the figure.
The shaft in the Queen's Chamber pointed in the general direction of the dawn transit of Sirius. The shaft in the King's Chamber pointed in the direction of Alnitak. As we have seen, these two stars appear to have been associated with Isis and Osiris respectively, so apparently providing some kind explanation for both the south facing shafts.

[^35]

Sirius at that time exhibited a dawn transit when due South on $11^{\text {th }}$ September each year. This is 42 weeks ${ }^{*}$ before the dawn rising, Figure 54, on $6^{\text {th }}$ July each year, i.e. similar to the human gestation period. It may have been for this reason that Sirius became associated with Isis as the mother of Horus. The Inundation would then, not unreasonably, have been associated with Isis' annual "breaking of the waters."

Similarly, it could be argued that the King's Chamber shaft (KC (S)) pointed at Alnitak, hence at Osiris, and so the pyramid was in some way connecting Osiris with Isis. Perhaps Osiris would need to inseminate Isis at some point in an annual ritual.

This idea was, and is, conceptually charming and romantic, but it did not explain the rôle and purpose of Gantenbrink's Chamber, however. Neither did it explain horizontal sections at the start of each shaft, which precluded their ever being used for viewing stars. It was also evident, as noted in the figure, that the two southern shafts pointed accurately at their respective stars during quite widely separated times.

The Queen's Chamber northern shaft, QC (N), emerges from the chamber horizontally and then bends around the underside of the Grand Gallery as it rises, pointing in the general direction of Kocab. It does not reach the outside of the pyramid.

Kocab was an interesting star: it seemed to be involved in finding north, and so in orienting the pyramid accurately; it also shared its dawn transit with the dawn rising of Alnitak; and, it might conceivably have been involved as a source of celestial semen in a stellar fertility ritual, as was mooted in "Navigating the Skies by the Bright Stars" above.

Finally, the shaft KC (N) pointed broadly at Thuban, the then Pole Star. This star had a potential rôle as indicated in the Pyramid Texts, as the Great Mooring Post, i.e. the

[^36]one fixed point in the cosmos to which a rope or rope ladder could be attached, enabling the pharaoh's spirit to climb to heaven.

So, it seemed that, except for one or two issues, the enigma was solved: the four shafts pointed at the dawn transits of four stars which had important rôles in the ancient celestial fertility rituals designed to ensure the Inundation for each year and for all time.


Figure 74 Khufu's Shafts and Annual Dawn Transit Dates, 2575BC
Figure 74 shows a view of the internal structure of Khufu's Pyramid, annotated with the various dates of dawn transits. Notice that the shafts pointing from the chambers towards the north emerge from their respective chambers with a short horizontal section, before canting upwards: they are then curved around the Grand Gallery. At about 8 inches square internally, all of the shafts are too small to permit human access.
Table 4 shows the various dawn transit dates drawn together with suggestions for what might have been the associated event in an annual calendar of rituals and ceremonies*. The dates vary slightly from previous transit dates, these having been

[^37]Table 4. Putative Annual Ceremonies, ~2450BC

| Date | Star | Event | Ceremony | Shaft/Place |
| :---: | :---: | :---: | :---: | :---: |
| $7{ }^{\text {th }}$ June | Osiris/Alnitak | HR | Osiris appears | Giza, looking East and $\mathrm{QC}(\mathrm{N})$ |
|  | Khufu/Kocab | C | Khufu transmogrifies into Osiris-replay of change from Horus to Osiris on death | QC(N) |
| $25^{\text {th }}$ Aug | Osiris/Alnitak | C | Osiris/Alnitak matures | KC(S) |
| $12^{\text {th }}$ Sept | Isis/Sirius | C | Isis matures. Osiris,Khufu symbolically inseminate Isis. | QC(S) |
| $5^{\text {dh }}$ July | Isis/Sirius | HR | 9 months later, Isis' waters break as the Inundation. Egyptian New Year begins | Giza, looking East |

- Heliacal Risings (HR) are new, initiating events.
- Culminations (C) are maturing events
- Thuban $(\mathrm{KC}(\mathrm{N}))$ is stationary tethering pole in rotating sphere of stars
- Whole is an annual fertility rite to maintain the fertility of Egypt for ever
calculated in 2450 BC as opposed to 2575 BC , the somewhat arbitrary date used before. Over 125 years, none of the dates changes by more than one day.


## Rituals and Ceremonies

There is no proof that the people or priests of Khufu's reign held ceremonies on the dates and for the purposes indicated in Table 4, page 94 below. There is, however, at least a trail of circumstantial evidence.

Using worship, spells, lustrations and incantations as an energy source, the plan could have been to project Khufu-Osiris's $k a$ to the star Kocab, thereafter to be KhufuOsiris's Imperishable celestial home. Thus Khufu-Osiris would become the seminal source in Egypt's Enduring Bull, and would inseminate Isis annually, ensuring Inundation for all time.

The boats found buried by the Great Pyramid suggest that the Pharaoh's spirit was intended to return, and to cruise up and down the Nile, perhaps visiting and looking after his people. Pharaoh's $k a$ would also need two boats to travel across the sky, Mandjet by day and Mesektet by night.

Were that speculation valid, it would suggest that the ancient Egyptian people were audacious on a scale heretofore unsuspected. Khufu's Pyramid complex may have
been, not just a transmogrification and resurrection machine ${ }^{*}$, but a "psychic transporter", too.
Khufu's Pyramid is unique in having the four shafts. The trail of circumstantial evidence for other pyramids would not support such a speculation, but would go only as far as their being resurrection machines. The pharaohs' spirits too would be able to leave their pyramid homes and wander Egypt in spirit form. They too might reach the stars, but the means was not explicit. Perhaps the $b a$ would fly there. Perhaps the $k a$ would climb the rope ladder to the tether point at Thuban.
Such ideas are fundamental archetypes of belief ${ }^{\dagger}$, as Jung ${ }^{39}$ described them. These include implicit belief in resurrection and an afterlife. Comparing ancient Egyptian ideas of the afterlife with present day notions, it seems that the Egyptians displayed many, if not all, of the afterlife ideas and beliefs that abound today ${ }^{\ddagger}$, including reincarnation, heaven as some kind of perfect place, an underworld inhabited with monsters, and many more.


[^38]One other speculative idea concerns the curious niche found in the east wall of the Queen's Chamber of Khufu's Great Pyramid. See Figure 75. This is a composite figure, with the statuette of Khufu, in reality only a few inches high, superimposed on the niche, which is much taller than a man.

The statuette is the only authenticated figure of Khufu. There is, however, a magnificent statue of Khafre, his son, see Figure 38. This life-size masterpiece was found in Khafre's Valley temple, and would have fitted neatly into a corbelled niche such as that found in the Queen's Chamber of Khufu's Pyramid. There is no equivalent in Khafre's own pyramid.

There is no evidence for the purpose of the corbelled niche. However, in the spirit of openly declared speculation, it is possible to reconstruct the spiritual activities that the ancient Egyptians may have fancied would happen.


Figure 76. The Spirit of Osiris Activates Khufu
The outline of a ritual belief that follows, Figure 76, is based on the simultaneous heliacal rising of Alnitak and the dawn transit of Kocab, already described.

1. Osiris' spirit ( $k a$ ? ) emanates from the dawn sky, with the heliacal rising, $8^{\text {th }}$ June each year
2. Having traversed the covered causeway, the spirit enters the pyramid through the false door in the mortuary temple provided for the purpose
3. Osiris' ka enters the Queen's Chamber and activates the hawk* on the back of Khufu's (missing) statue, initiating the ritual transmogrification into KhufuOsiris

[^39]4. The $k a$ Khufu-Osiris exits using the QC (N) shaft, with its bend, to navigate around the Grand Gallery, and shoots directly towards Kocab, which obligingly culminates at just that moment.
This description, although fabricated around very few facts, nonetheless manages to offer some explanations. The QC (N) shaft may be near the east wall of the Queen's Chamber to facilitate the ka Khufu Osiris on his journey.
The false door on the west wall of the mortuary temple would be used continually by the spirit Khufu-Osiris, initially each year to activate the transmogrification, and subsequently as the spirit went back and forth between stars and Egypt.

On the other hand, the description does not account for all the hard evidence. Why would the spirit Khufu-Osiris choose the northern shaft over the southern shaft, which would, on another date, point to Sirius? Were there, perhaps, more ceremonies? Once activated by Osiris, could the spirit Khufu-Osiris perhaps traverse the QC (S) shaft on $11^{\text {th }}$ September and ritually inseminate Isis-Sirius?

In similar vein, why would the spirit of Osiris, having entered through the false door, choose to go to the Queen's Chamber; why not the King's Chamber? One speculative response to that question is that, with the plug stones removed to plug the entrance passage, the Grand Gallery would provide a rather splendid avenue of approach from the lower Queen's Chamber to the upper King's Chamber.

## Limitations in the Starshaft Theory

If the four shafts in the Great Pyramid pointed at particular stars in antiquity, they did not do so accurately, at least not all at the same time, Figure 73 on page 92.
The effects of precession over time can be plotted, Graph 4. The graph shows two lines, plotted over some 600 years around the time of the pyramid age: the horizontal line is the (fixed) pointing angle of the $\mathrm{KC}(\mathrm{N})$ shaft; the climbing line is the elevation above the horizon of Thuban, taken at the time of its dawn transit, as it varies due to precession.
The two lines cross at 2350 BC , shown by the vertical dotted line, indicating that the shaft in the King's Chamber pointed directly at Thuban around that year. The best estimate for the building of the pyramid is around 2575BC, some 225 years earlier. This discrepancy could suggest several things: that the estimated construction date is wrong; there was a slight misalignment in sighting; that building control suggested some more convenient proportion; the year of some (then) future festival; or, absolutely nothing at all.
Note that, in about 2800 BC, Thuban had a transit elevation of $30^{\circ}$, at which time it coincided with the celestial pole, as seen from Giza. It is possible that astronomers of the time, having observed Thuban at the pole, then took it customarily to be their pole star, or Mooring Post, even after it had wandered away.

[^40]

Graph 4. Precessional Wander of Thuban's Dawn Transit

Table 5. Elevation Angles of Stars, ~2575BC

| Star | Elevation <br> from Giza | Shaft <br> Angle | Difference <br> in degrees |
| :--- | :---: | :---: | :---: |
| Alnitak | $44.6^{\circ}$ | KC (S) <br> $\sim 45^{\circ}$ | $\sim 0.4^{\circ}$ |
| Sirius | 38.93 | QC (S) <br> $\sim 39.5^{\circ}$ | $\sim 0.57^{\circ}$ |
| Thuban | $31.2^{\circ}$ | KC (N) <br> $\sim 32.47^{\circ}$ | $\sim 1.27^{\circ}$ |
| Kocab | $39.8^{\circ}$ | QC (S) <br> $\sim 39^{\circ}$ | $\sim 0.8^{\circ}$ |

Table 5 shows the pointing errors between the shafts and the stars with which they are linked during the supposed time of construction of the shafts in the Great Pyramid, $\sim 2575$ BC. These errors are significant when compared with the accuracies of other slopes routinely achieved by the pyramid builders - assuming that the shafts were, indeed, intended to point at the nominated stars.
Instead of looking at the supposed sighting errors on a particular date, an alternative viewpoint is to see in which year each shaft pointed accurately at its respective star's transit as altered by precession. The four sets of calculations showing when the shafts pointed directly at their supposed target stars are drawn together in Graph 5.

The graph includes error bars, suggesting that, while three of the dates are compatible, that for Alnitak may have a different basis. In addition, the generally agreed latest

date for the completion of the pyramid was about 2566, shown as a dotted line, since it appeared to have been completed by the time of Khufu's death, which is generally calculated to that year.
So, there is a problem. Were the builders uncharacteristically slapdash, failing to construct the shafts accurately? Or, were the shafts never intended to point at particular stars? Is this whole, intriguing storyline really false? Did the shafts, perhaps, have some other purpose altogether?

## The Stone Amulet Theory

The evidence of Gantenbrink's Chamber has damaged the Starshaft Theory. Coupled with the atypical poor pointing accuracy, there must be a strong suggestion that the shafts were not intended to point at particular stars, and that the theory has been based on near-coincidences.

The flaws in the Starshaft theory dictate a search for a better explanation. Knowing that the ancient Egyptians used proportions instead of angles when measuring slopes suggests examining the numbering scheme used for the shafts, in much the same way as numbering schemes were evidently used for the pyramid slopes themselves.

## Is there a pattern in the Shaft Angles/Slopes

Table 6 shows a table of results using a simple, trial and error approach to determining the observed shaft slopes (not the angles to any stars) using proportions, Egyptian style.

The manner in which the table was produced is simple, if painstaking. Knowing the angle of each shaft, it is simple to derive the corresponding tangent. The columns marked opposite and adjacent are then determined by trial and error to give the closest fit to the respective tangent. This suggests the actual numbers that the ancient Egyptians might have used. The final column calculates the percentage error between the ratio and the tangent for each angle, assuming throughout that the given shaft angle measurements are free of error (which cannot be precisely correct.)

Table 6. Patterns in Khufu's Shafts

| Shaft | Angle(d) | Tan(Angle) | Opposite | Adjacent | Ratio | Error\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| KC(S) | 45 | 1.0000 | 7 | 7 | 1.0000 | 0.0000 |
| KC(N) | 32.4667 | 0.6363 | 7 | 11 | 0.6364 | 0.0173 |
| QC(S) | 39.5 | 0.8243 | 14 | 17 | 0.8235 | -0.0979 |
| QC(N) | 39 | 0.8098 | 17 | 21 | 0.8095 | -0.0321 |

The first row is simple, as the angle is $45^{\circ}$. Tangent $=1$, opposite/adjacent $=1$, error $0 \%$. Subsequent rows are not so easy. The accuracy of the results is, however, startling. From the table, the greatest error of $0.0979 \%$ equates to an accuracy of 1 in 1000. Had one, or even two, of these calculations given such accuracy, the results would have been suspect. For all four to be so accurate, does indeed suggest that the ancient Egyptians used simple ratios to determine the slopes of the four shafts. Moreover, it suggests that they may have used the actual numbers in the opposite and adjacent columns. Looking at those numbers in the table, it seems that they are mutually connected. The number ' 7 ' and multiples of 7 are evident.

## KC (N) and the Pyramid Slope

Figure 77 shows a chart on which have been drawn the proportions corresponding to $\mathrm{KC}(\mathrm{N})$ and to Khufu's Great Pyramid itself.


Figure 77. KC (N) and the Pyramid Slope

Using proportions, instead of angles, results in a chart with which the ancient Egyptian architects might have been familiar, supposing them to use apparatus like that of Figure 59.

Interpreting the chart requires trying to think like an ancient Egyptian. The pyramid has a slope of $14: 11$. What would be half that slope? Using proportions, $7: 11$ would be half the slope - no other conclusion would be reasonable.

Reformulating the measurements in the ancient Egyptians unit of slope, the seked, reinforces this conclusion. A slope of 7:11 would have been measured as precisely 11 seked. The slope of the Khufu's Pyramid, measured $51 / 2$ seked. So, measured in
seked, the two slopes are in the ratio, $2: 1$. This endows the $\mathrm{KC}(\mathrm{N})$ shaft with perfect symmetry: it is exactly half way between the horizon and Khufu's Pyramid slope.
So, there is a simple, logical progression that explains why Khufu's Pyramid had the slope $14: 11$, and it had nothing to do with any discredited theories based on $\pi$. Instead, it was tied to the Pole Star as seen from Giza, determined to be about elevation 7:11.

## QC Shafts



Charting the two shafts from the Queen's Chamber, QC (N) and QC (S) respectively, Figure 78, gives a quite different result. The proportions deduced in Table 6 are used. Examining the near identical slopes (there is only $0.5^{\circ}$ difference), it is difficult to avoid observing that there is a connection between the chosen proportions: for QC $(\mathrm{N})$, the slope is $17: 21$; for $\mathrm{QC}(\mathrm{S})$ the slope is $14: 17$. Seventeen, a prime number, occurs in both proportions; and 14 and 21, are, of course multiples of 7.
By themselves, these observations may be curious, but have little significance.

## Looking at the Overall Pattern

Figure 79 shows the results of plotting all the slopes on an "Opposite-Adjacent" chart. It shows that the various slopes seem, prima facie, to be linked. This visually reinforces the observation from the table that the number ' 7 ' seemed to occur frequently in the proportions*.

[^41]The visual representation shows how the various slopes form a symmetric zigzag pattern, which may have appealed to any ancient observers. Notice, too, how the pyramid's slope joins the two pairs of King's Chamber and Queen's Chamber slopes.


Figure 79. Slope and Proportion Pattern
The rôle of the number 14 also becomes pivotal, appearing overtly on the $y$-axis, but generating the zigzag on the $x$-axis by having points at +3 and -3 set symmetrically about it.

## Two Primes Interplay

What can we reasonably deduce from the above? It would be easy to be seduced by numbers and statistics, and there are only 5 pairs of proportions to support any assertions, since Khufu's Pyramid is unique in having the four shafts. It is not feasible, therefore, to invoke other examples to support any theories.

Ancient mathematicians are known to have had interests in prime numbers. This interest persists right up to the present day. Primes have special significance because they cannot be factored, which seems to endow them in some people's minds with ideas of purity or mystery.
Looking at the proportions above, it is possible, even straightforward, to see a progression formed from only two prime numbers, 3 and 7, as follows:

$$
\begin{aligned}
7 & =1 \times 7 \\
11 & =2 \times 7-3 \\
14 & =2 \times 7 \\
17 & =2 \times 7+3 \\
21 & =3 \times 7
\end{aligned}
$$

> Note that 11 and 17 are also primes, and that the series of numbers is symmetrical about 14 , with 11 and 17 being $14 \pm 3$, and 7 and 21 being $14 \pm 7$

The forming of such a progression could be simple coincidence, of course, but it is just the kind of prime number and symmetry "magic" that ancient Egyptians mathematicians might have loved. Moreover 7 is a number that, through the ages, has been endowed with magical properties. We have 7 days in our week reputedly because of Babylonian belief in the magic of 7. For the ancient Egyptians, 7 was the number of (10-day) weeks for mummification, and the number of (10-day) weeks from the dawn rising to the dawn transit of Sirius/Sothis/Isis". The rare inclination of 1:1 measures 7 seked, using the ancient Egyptian scale. There were 7 palms in the Royal cubit. Seven knots tied in a piece of cloth and attached to a patient's toe would cure his ailments ${ }^{40}$. Evidently, the ancient Egyptians were very interested, even obsessed, with the number 7 .

Adding to all that the great accuracy with which the ratios correspond to shaft angles and there is a prima facie case for deducing that the various shafts were set to a sequence of proportions based on selected prime numbers. Further, that they were anchored to the then Pole Star, and that the slope of the Khufu's Pyramid was also anchored to the Pole Star, being exactly twice that elevation when measured using proportions. We shall call this the Stone Amulet Theory, as it suggests that the whole pyramid was designed as an amulet to protect the pharaoh and assure his resurrection.

## The Stone Amulet Concept

It would not have been unnatural for the chief architect, Hemon, to invoke magical support. We already know that the internal features of the pyramid that he built have been carefully placed: the Queen's Chamber, as we have seen, is constructed according to successive integer proportions, is directly below the tip of the pyramid, and is on the east-west centre line; the King's Chamber is in the $2: 1$ floor plan ratio; the passages also rise and fall using the $2: 1$ ratio.

Now the Stone Amulet Theory proposes that the shafts display the ratios of symmetrical prime numbers, also with magic intent.
All internal structures within the Great Pyramid occur in a single, north-south plane or septum through the pyramid centre. To the east of this vertical plane is


Figure 80. Southern Shaft Opening, Queen's Chamber, Great Pyramid.

The opening is at about head height for an average man.

[^42]the world of the living: to the west is the afterlife.
The Stone Amulet Theory proposes that the shafts mark out the precise point of the vertical plane separating these two worlds, that they form a magical "screen", separating the living from the dead, and that they magically protect the $k a$ of the dead pharaoh.
It is for this reason that the northern shafts emerging from both the King's and the Queen's Chamber into the underside of the Grand Gallery; the shafts marked the centre line of the magic "screen", and so they had to emerge at that point. It is also for this reason that the entrance to the King's Chamber is to the east of the plane, while the sarcophagus is sited against the western wall.

The Stone Amulet concept is illustrated in the artist's impression, Figure 81. The view is from the east end of the King's Chamber, the area of the living, looking towards the sarcophagus at the western end, the area of the dead. In between, emanating from the shafts, is an invisible screen separating the two worlds, powered the magic prime numbers, and protecting the $k a$ of the pharaoh which has direct access to Osiris in the South and the Imperishable Gods in the North. A similar screen divides the Queen's Chamber.


Figure 81. Stone Amulet Concept, King's Chamber

The concept of the shafts as part of a vast stone amulet is, at least, founded on a sound basis in measurement and relationship between the shafts. But there has to be more. The shafts are hollow, suggesting that some medium is meant to be contained or to flow within them, or perhaps something is meant to pass through them, yet the shafts from the Queen's Chamber, QC (N) and QC (S), Figure 80, were
closed at both ${ }^{*}$ ends. They could not have been intended for sighting, for listening, for ventilation, or for anything of a practical nature. Their purpose, then, could only have been ritual or magical.

The shafts in the King's chamber penetrate the masonry to emerge at the outside; presumably, they also once penetrated the casing stones, no longer in place. These shafts could have had some practical purpose such as listening, either from the outside or the inside. Conceivably, lustrations could have been poured into the openings from the outside, although this would have involved quite a feat of climbing once the smooth Tura limestone casing was in place.

It seems more reasonable to suppose, however, that the purpose of all four shafts was broadly the same, so that the upper shafts, $\mathrm{KC}(\mathrm{S})$ and $\mathrm{KC}(\mathrm{N})$ would be for ritual/magical purposes, too. Since they were open to the sky, there may have been a sky or star connection for these two shafts.
We can speculate as follows:
The King's Chamber was permanently "open" to the Imperishable Stars and celestial pole. $K C(N)$, with its slope of 11 seked, also brought the power of perfect symmetry into the chamber as it divided the pyramid's 5-1/2 seked slope in two.
The Chamber was also "open" at the sacred slope of 1:1 to Alnitak, ( $K C(S)$ ) which, as Osiris, gave magical connections(and perfect symmetry) to the afterlife every day throughout the year ${ }^{\dagger}$. The shafts in the King's Chamber therefore provided complete, uninterrupted access from the King's Chamber with its sarcophagus to the celestial pole in the Imperishable Stars, and to Osiris in Orion ${ }^{*}$, and back again.

The Queen's Chamber was permanently connected with (the contents of) Gantenbrink's Chamber, $(Q C(S))$, and with whatever might be at the end of the QC (N) shaft. The shafts provided passages linking the hidden chamber(s) with the Queen's Chamber. The Grand Gallery, sloping at a magic 26.5 degrees, interconnected the two principal chambers.
The shafts in both chambers marked the precise transition from the living world, east, to the afterlife, west. Crossing that line would have invoked the magic protection - or wrath - of the Amulet. The horizontal sections in each shaft indicate that each represented a conventional passageway, just like those entering all three main chambers, which also had horizontal sections at the point of entry. The small cross sections of the shafts, too narrow for any person, indicate that they were intended as passages for the ka of the

[^43]pharaoh. Interactions through the various shafts were assured and enabled by the prime number magic connecting the four shafts.

The whole was a living, magical resurrection machine, able to project the pharaoh to the stars and to bring him back again, and able to protect his ka on its regular passages to and from the stars. His ka was also able to spend time in his pyramid, using the various chambers for contemplation and to await auspicious moments for travel to the stars or into Egypt. The Queen's Chamber may have served as a serdab, or kiosk, like that of Djoser, Figure 19. Prayers and lustrations of the dedicated priesthood maintained the whole pyramid in operation, supposedly forever.

The Stone Amulet Theory stands up to scrutiny on accuracy grounds. It finds no difficulty in accommodating Gantenbrink's Chamber, suggesting instead (as symmetry would require) that there could be another chamber at the end of QC (N), both 'secret' chambers acting as amulets. It was common practice to incorporate many amulets* into the mummy of a pharaoh; why not into his pyramid?

Speculation is always entertaining and theories can never be proved, of course only disproved. The only hard evidence is that of the pyramid remains, the arcane Pyramid Texts, together with scant knowledge of Old Kingdom mathematics and astronomy gleaned from papyri that, although written later, suggest that they were copied from originals contemporary with the pyramid builders. The pyramids in general and the Great Pyramid in particular undoubtedly still hold many surprises in store.

[^44]
## PART 2

## BUILDING <br> PYRAMIDS

## Chapter 4. Pyramid Logistics

Herodotus was told that 100,000 men worked in gangs for 3 months at a time building Khufu's Great Pyramid. However, he was told this some 2,000 years after the event along with other information that clearly raised doubts in his mind:
"There is a notice in Egyptian script on the pyramid about how much was spent on radishes, onions, and garlic for the labourers, and if my memory serves me well, the translator reading the notice to me said that the total cost was sixteen hundred talents* of silver. If this is so, how much more must have been spent, in all likelihood, on iron for the tools, and on food and clothing for the work force, considering how much time, as I mentioned, was spent building the pyramid.
"And then, I suppose, there was the not inconsiderable amount of time spent quarrying the stone and bringing it to the site and excavating the underground chambers ${ }^{41}$."

The priests informing Herodotus clearly had limited knowledge of Old Kingdom practices; it seems unlikely that money had been invented, or that there were iron tools. As to the number of men involved, 100,000 seems to be a large workforce for the time. It does not seem unreasonable to confirm, or challenge, the manpower total using calculations based on simple physics and human biology.

## Try a little Science

It is possible to calculate the work needed to raise Khufu's pyramid using straightforward physics, Appendix 1 to this chapter, page 123. It is also possible to estimate each pyramid worker's output, based on (then) diet, believed to be $\sim 2500$ kilocalories per day, Appendix 2 to this chapter, page 125.
Dividing the total work required by the work output of each worker per day gives the number of "worker days" needed to raise the pyramid. Knowing the time that it took to build the pyramid, it is possible to estimate how many men it took. It is even possible to work out the farm acreage needed to grow food to support the workers!
Physics calculations omit many unknowns, so they are bound to provide an underestimate. Consequently, they cannot prove, but may disprove, some of the many theories. On the other hand, they may help to confirm calculations/predictions made using other approaches.

## The Pyramid Calculator

A more convenient and effective way to undertake the calculations, other than the formulaic method use in Appendix 1, is to develop and use a dynamic simulator, one that measures the amount of work needed to raise the pyramid one tier at a time; this provides time dependant results, showing how effort was needed and how long it took to reach each tier level Once constructed, such a "pyramid calculator" can be set for any pyramid's dimension.

[^45]

Figure 82. The Pyramid Calculator
The control panel of the Pyramid Calculator*, Figure 82, reveals many of the factors that must be considered when analysing pyramid building parameters and logistics. The graph at the top of the figure, which can be selected to show many different parameters, shows in this instance the time taken to build Khufu's Pyramid, tier-bytier, when the manpower employed is constant year on year.

Various conditions and parameters can be set before each run of the calculator. The figure shows two switch panels, one at centre, the other bottom right. The switch selections on the latter shows that the graph has accounted for Quarrying Blocks, Sliding Blocks (from the quarry to the base of the pyramid) and Stone Dressing: it has not accounted for the Temenos Wall and Courtyard, the Mortuary and Valley Temples, the Causeway, and the Queens' and $k a$ Pyramids. The central panel shows that the method of raising stone up the pyramid is presumed to be by ramp, rather than by a lifting method that did not involve sliding friction. It is also possible to omit the pyramid itself, so that the work associated with individual items, such as the Causeway, can be examined on their own.

[^46]Bottom left is a panel where the pyramid dimensions can be entered; those shown correspond to the Great Pyramid, but any set can be inserted according the pyramid under investigation.
At left are slider controls for: causeway build-time; work per man, per day; calorific input (i.e. total food energy per day); food apportionment (the proportion of total food set aside for the full-time workers, as opposed to the corvée); coefficient of friction when sliding wooden sledges over wooden battens; the man days per stone block needed for dressing; and finally, the number of men in the core team, the full-time workers.

Other panels give instantaneous numerical values of parameters such as tier weight, tier volume, tier height, numbers of full time workers, numbers in the corvée, etc.

Using the Pyramid Calculator eases the complexity and duration of conventional calculations, allows a wide variety of calculations to be executed for any one pyramid, and enables comparisons to be drawn readily across many pyramids. Following sections show results from using the Pyramid Calculator.

## Using the Pyramid Calculator

## How much Stone?

Initial calculations address only the pyramid, Akhet Khufu, Khufu's Horizon. They do not include the ramp, Temenos wall, queen's pyramids, valley and mortuary temples, etc., which will be considered later.


The mass of stone used in the construction of the Great Pyramid can be calculated from its volume and the density of the limestone rock from which it is largely formed. The total mass is some 6.5 million tonnes. The Pyramid Calculator shows how this is
accumulated tier-on-tier, Graph 6, which gives the same figure at tier $200^{*}$. Using the Calculator also shows how this mass is accumulated, so, for instance, by Tier 60, the accumulated mass is 4.2 million tonnes, and by Tier 101 (half the total number of tiers) the accumulated mass is 5.6 million tonnes, i.e. $86 \%$ of total. The shape results from successive tiers being smaller in volume and mass than their predecessors as the pyramid builds up.
Such vast amounts dictate that the bulk of stone for any pyramid be obtained from the vicinity of the pyramid site, to limit the effort of transporting from quarry to site. Quarries are found close to their respective pyramids; the limestone quarry for the Great Pyramid is some 300 m distant.

Pyramid builders also employed granite for structural features, because it is much stronger than limestone; it was generally obtained from quarries in Aswan, Figure 83, up river to the south, and brought down by boat. Casing stones were formed from Tura Limestone, brought across the flooded Nile by boat from the Tura Mountains to the east of Giza. The vast bulk of the stone was limestone from the local quarries, however.


Figure 83. Granite Quarry, Aswan

## How many Men?

Calculations to determine the number of men must consider the corvée. Part time men worked for only 3 months per year during the Inundation when, as farmers, they could not work their land. The ratio of full time to part time men is a variable affecting the peak numbers of men in any year. Fewer workers operating all year round necessitate more part-time workers during the Inundation, boosting the peak numbers of men employed. Initial calculations assume a build time of 20 years.

[^47]

It is possible that each pyramid maintained a constant, steady workforce over the decades of its building. Indeed, this may have been the practice for some pyramids. It presents a problem, however, revealed in Graph 7. Calculating the amount of work needed to raise each tier shows that, for the pyramid itself, there is a peak at $1 / 3^{\text {rd }}$ the height of the pyramid, in this example at tier 68. This is true whether or not a ramp is used to raise stone up the pyramid.
The peak comes about as follows. Lower down in the pyramid, there are more stones per tier, but each stone has to be raised through only a relatively small vertical distance above the ground level. Towards the top, although each stone has to be raised through a larger vertical distance, there are many fewer stones per tier. The greatest amount of work lies in between these extremes, at $1 / 3^{\text {rd }}$ height. The peculiar peak at the highest tiers derives from work needed to for packing and casing stones to finish the exterior of the pyramid.
So, a pharaoh watching his pyramid being built by a constant workforce would see progress apparently slow as time passed, as their constant effort raised fewer tiers with the passing of time. Perhaps it is for this reason that several pyramids were abandoned, approximately $1 / 3^{\text {rd }}$ built.

As the graph indicates, adding in the work involved in quarrying and sliding the stone from the quarry up to the pyramid site exacerbates the situation, heavily front-loading the work profile. For these reasons, it would be much more likely that a powerful pharaoh, such as Khufu, would draw upon labour resources to accelerate the process, especially during the earlier, labour intensive stages.

Graph 7 shows that simply looking at the building process tier by tier reveals much of what had to occur during the building process. The calculator can examine a wide range of variables and can invoke sensitivity checks to see if the results are sensible and reasonable.


Graph 8 puts the basic physics of Graph 7 into some context. Herodotus was told ${ }^{42}$ that, rather than ramps, the builders used "appliances* made out of short pieces of wood". The graph shows the 20 -year period and the number of workers who might have been employed either using ramps or "appliances." Ramps necessitate work to overcome sliding friction, and hence require more men than an undefined lifting method, which could be virtually frictionless in principle.


## Graph 9. Work Associated with Other Necropolis Features

There was much more to building a pyramid complex than just the pyramid, see Graph 9 . Many pyramid had a causeway leading from the valley temple by the Nile to the mortuary temple abutting the pyramid. Before these temples were built, the causeway would be used as a haul-way, for dragging stones and other resources from Nile boats up to the pyramid site. Eventually, the causeway would be enclosed in

[^48]stone, with carvings on the inner walls, and would act as a processional way. The pyramid complex included some or all of: valley and mortuary temples; Queens Pyramids and $k a$ pyramid; and a Temenos wall surrounding the site/ pyramid, plus paved courtyards within it.
There is uncertainty about the causeway, as indicated in Graph 9. Khufu's causeway took 10 years to build:
"They (the priests) said it took ten years of hard labour for the people to construct the causeway along which they hauled the blocks of stone, which I would think involved not much less work than building the pyramid, since the road is 5 stades* long, ten fathoms ${ }^{\dagger}$ wide and eight fathoms high at its highest point, and is made of polished stone with figures carved on it.
"So they spent ten years over this road and the underground rooms which Cheops (Khufu) had constructed as his sepulchral chambers in the hill on which the pyramids stand, which he turned into an island by bringing water from the Nile there along a canal. The actual pyramid took 20 years to build ${ }^{43}$."
Was the causeway completed before any pyramid building was started, or did the two constructions run side by side? The calculator provides for the total estimated work to be concentrated or spread out.


Graph 10. Great Pyramid Complex - Annual Manpower Estimates
Graph 10 shows the result of including the work associated with constructing the overall complex. The calculator estimates that at peak, some 26,000 men may have been employed. It is important to remember that the number of men employed at any

[^49]time depends on the ratio of full time to part time workers. More full time workers would reduce the total, which includes a high proportion of part time workers.

The peak number of men also depends on the time scale for building the causeway: extending the period would smooth out the hump at the start associated with its construction.
Omissions include: work on the boat pits; work on associated tombs of nobles or workers; time and work on any carving or engraving; and many more...There are also some major unknowns, upon which the calculated values depend, including the following.

First, the man-days needed to quarry and dress a stone: an assumption of 10 days average is incorporated in the Pyramid Calculator, on the basis that, although many of the visible stones are finely dressed and would have taken longer, others would have been finished more roughly.

Second, the coefficient of friction between wooden sledges and wooden battens was assumed for the model to average 0.1 . One of the few values that is generally accepted for this uncertain coefficient is that between two wooden surfaces, coefficient of friction $=0.2$. It is believed that the ancient Egyptians lubricated the runners of sledges used for hauling stone, so a value lower than 0.2 has been used. Results are sensitive to this value, however

Third, the work output per man was assumed to be 0.625 MJ per day, see Appendix 2. This is highly dependent not only upon diet, but also upon organization and method, both unknown. Results are particularly sensitive to this value, which pervades everything.


To overcome the limitations imposed by these three indeterminate parameters, the Pyramid Calculator simulation was run 100 times; during each run, values for all three parameters were randomly selected from a statistical distribution about their mean values. Individual runs were then accumulated and the statistics calculated, Graph 11.

The graph suggests that the mean time to build the Great Pyramid of Khufu was more likely to be $\sim 25$ years, but that there is significant uncertainty. One combination of variables even produced an estimate of 183 years, which may be discounted: there was a marked reluctance for successors to spend much effort on finishing their preceding pharaoh's pyramids, let alone take 6 generations to complete.

## Time to Build the Red Pyramid

There is an opportunity to checkout the results predicted by the pyramid calculator. Rainer Stadelmann of the German Archaeological Institute discovered graffiti on stones at different tier levels on the Red Pyramid at Dahshur, which showed how long it had taken to reach particular heights. It was clear that 6 layers or tiers had been completed by Year 2, 15 metres by Year 4, and that the whole pyramid took 17 years.


Graph 12. Red Pyramid Build Rate - Theory Vs. Practice
If these are plotted against the output from the dynamic simulator, set to include building the whole Red Pyramid complex, including quarrying and dressing stone, mortuary and valley temples, etc., then the correlation is good, Graph 12. The results do not prove that the calculator is correct, but serve to provide a modicum of confidence.

## Logistics and Feeding the Workforce

Because the calculator works from the food input to each worker, it also suggests information about the amounts of that food. Calculations for the Great Pyramid suggest that $\sim 1750$ acres of emmer wheat and barley would need to be farmed at peak to support the workforce with both bread and beer staples. (Man in solar-based monsoon agriculture, to which ancient Egypt with its Inundation was not dissimilar, produces $\sim 80 \mathrm{kcal} / \mathrm{m}^{2}$ per day for half of year ${ }^{44}$.) Beer was drunk because the alcohol tended to kill infections carried in the river water.

Other foods ${ }^{*}$ included dates, honey (there was no sugar), fish , radishes, leeks, garlic, onions, figs, pomegranates, cucumbers, sweet melons, watermelons, milk \& milk products. Calculated statistics include, at peak building rate, the following.


Figure 84. Bored Man Waits for Bread to Prove, $4^{\text {th }}$ Dynasty, Cairo Museum
The amount of bread consumed, Graph 13 , was up to 7.5 tonnes, $30 \mathrm{~m}^{3}$ per day. This


[^50]required $\sim 70$ bread mixing and proving vessels being used twice per day, assuming a vessel volume of $\sim 0.8 \mathrm{~m}^{3}$, estimated from tomb paintings and statuettes. After proving, the dough would be divided into convenient shapes for baking individual loaves - see Figure 84, which shows a man nodding off to sleep while he waits for dough to prove. Figure 85 shows a royal bakery, with workers busy side-by-side, kneading and rolling. Some 50 men would have been needed to carry grain from dock to bakery, assuming $\sim 50 \mathrm{~kg}$ load per man and 3 trips per day.


Figure 85. Model of a Royal Bakery, British Museum
Similarly, making beer would have necessitated $\sim 20$ tonnes water per day, occupying $20 \mathrm{~m}^{3}$. Some 21 vats, would have been needed, assuming each vat brewed up to 3000 litres, and took 3 days to brew. The amount of water/beers suggests some $\sim 135$ men carrying water from dock, assuming $\sim 50 \mathrm{~kg}$ load and 3 trips per day per man.


Graph 14. Grain Boat and Stone Boat Deliveries per day - Great Pyramid, Proportional Manpower

Backtracking further, the calculator indicates that there would be grain boats delivering every $3 / 4$ of a day, i.e. 3 boats over 4 days, and there would have been $\sim 93$ stone delivery boats per day at peak, Graph 14.
It is even possible to estimate the acreage of emmer wheat that would have been cultivated each year, making reasonable assumptions about crop yields ${ }^{45}$, Graph 15. This wheat would have been used both in the making of bread and in the brewing of beer.

The workers were probably paid principally in food and clothing, so flax would also have been grown for linen. Additionally, the workers had, as outlined above, a wide and varied diet, so that food of many different varieties would have been grown and garnered up and down the Nile.
Herodotus discovered several interesting aspects of Egyptian practices:
"...they gather their crops with less effort than anyone else in the world, including the rest of Egypt*. They do not work at breaking the land up into furrows with a plough, they do not have to wield hoes or carry out any of the other crop-farming tasks that everyone else does. Instead, the river rises of its own accord and irrigates their fields, and when the water has receded again each of them sows seed in his own field and sends pigs into it to tread the seed down. Once this has been done he has only to wait for harvest time... ${ }^{46 \text { " }}$


Graph 15. Acres of Emmer Wheat to be Cultivated, Tier by Tier
"Other people live off barley and ordinary wheat, but Egyptians regard it as demeaning to make those grains one's staple diet; their staple is hulled wheat, or 'emmer' a it is sometimes known...47,"
"Each of them (priests) is also provided with a generous daily allowance of beef and goose-meat, and their wine is donated as well. They are not allowed to eat fish. The Egyptians do not cultivate beans in their country at all ${ }^{48}$."
Since fish remains have been found aplenty at Giza, see footnote on page 118, it must be assumed either that practices had changed, or that fish eating was banned only for priests.

Such calculated statistics are inevitably subject to error and interpretation. Surprisingly, perhaps, they give results that are consistent with estimates of manpower reached by other means, such as organizational statistics. The nature

[^51]of the calculations suggests that the values given above would be minima, and that particular values could have been significantly higher in practice. Nonetheless, the direct approach relating energy in food to work needed to raise a pyramid complex proves useful. The approach is especially apposite for pyramid building where there was no machinery, no draft animals, and only human muscle power.

## Appendix 1 to Chapter 6: Work Needed to Raise Khufu's Pyramid



Figure 86. Work and Energy Parameters - Khufu's Great Pyramid.
Figure 86 shows the approach to calculating the work needed in raising Khufu's Pyramid. The quarry where the bulk of the limestone was quarried is some 350 m from the pyramid, it is 30 m deep, and stone has to be raised up a $6^{\circ}$ incline to reach the pyramid base.
At that point there are choices, depending on whether the use of building ramps is assumed or not. The almost universal assumption that ramps were used has problems, and there were alternative building methods. It is necessary to make calculations for, basically, two methods as a minimum: the ramp method, which involves sliding friction, and any other method that raises stone vertically, and may not invoke friction. The amount of work should then, in principle, lie between these two limits.

So, the total work needed to raise Khufu's Pyramid:
$=$ work to raise stone from quarry + work to slide stone to pyramid + work to raise stone

## Equation 1 Raise quarry stone to ground level

$(22 / 14)^{2} \mathrm{H}^{3}$. $\mathrm{\rho}$.g.D $/ 3=1.91 \times 10^{12} \mathrm{~J}$
Equation 2 Slide stones to plateau
$(22 / 14)^{2} H^{3}$.l. ..g. $\{\sin \theta+\mu \cos \theta\} / 3=6.75 \times 10^{12} \mathrm{~J}$
Equation 3 Raise Khufu above plateau
$(22 / 14)^{2} \mathrm{H}^{4}$. $\mathrm{\rho} . \mathrm{g} .\{1+\mu \cot \phi\} / 12=5.648 \times 10^{12} \mathrm{~J}$

## TOTAL Work Done $=1.4 \times 10^{13} \mathrm{~J}$

Where:
H is height ( m )
1 is sliding distance (m)
$\rho$ is density $\left(\mathrm{kgm}^{-3}\right)$

$$
\begin{aligned}
& \mathrm{g}=9.81 \mathrm{~ms}^{-2} \\
& \mu \text { is coefficient of friction } \\
& \theta \text { is slope angle from quarry to pyramid } \\
& \phi \text { is the angle of slope for a ramp on the pyramid, if any } \\
& \mathrm{J} \text { is Joules of energy/work }
\end{aligned}
$$

To calculate the work to raise Akhet Khufu above the plateau without sliding friction, set the coefficient of friction, $\mu$, equal to zero in Equation 3.

## Appendix 2 to Chapter 6. Work Output Per Ancient Egyptian Builder

It is difficult to define the work output per ancient Egyptian man. Much of human food intake goes to maintaining Basal Metabolic Rate (BMR), the rest energy needed to keep the body working and support the immune system. Work output over and above BMR depends on how much of each working day applies to active work. For pyramid building, this would mean how much of the time was dedicated to raising stone, and how much was standing around, organizing, moving, etc.

## Research in Present Day India

Present day research ${ }^{49}$ in parts of India relates work output to calorific intake:
"In Katavi village in India where we examined the food energy intakes and actual productive energy expenditures of eight males ... for ten months in 1982.... The male average daily intakes were 2430 Calories ... The average male weight was 48.6 kilograms. The average ...male BMR was 0.88 Calories per minute ...
... The males worked an average of 8.14 hours...The male average work output was 1183 Calories."

This output equates to 170 W , or 5 MJ per man per working day for a 2430 Calorie diet.

## UK MAFF Values

An alternative source of nutritional value is the UK Ministry of Agriculture Fisheries and Food ${ }^{50}$. - see Table 7. The Ministry observes:
"Basal metabolic rate.. rate at which a person uses energy to maintain basic body functions - around 4.6 kJ ( 1.1 kcal ) every minute for adults...The BMR accounts on average for about three quarters of an individual's energy needs."

Table 7. Energy Utilization Data

| Type of Activity | $\mathbf{k J / m i n}$ | $\mathbf{k C / m i n}$ |
| :--- | :---: | :---: |
| Light (most domestic work, golf, lorry driving, <br> carpentry, bricklaying) | $10-20$ | $2.5-4.9$ |
| Moderate (gardening, tennis, dancing, jogging, <br> cycling up to 20km per hour, digging) | $21-30$ | $5.0-7.4$ |
| Strenuous (coal mining, cross-country running, <br> football, swimming [crawl]) | $>30$ | $>7.5$ |
| Source: Ministry of Agriculture, Fisheries and Food. Manual of Nutrition, |  |  |
| London, HMSO, 1992 |  |  |

From the table, raising pyramids would count as strenuous. From the figure in the appropriate column, $30 \mathrm{~kJ} / \mathrm{min}$, the basal metabolic rate must be subtracted, as follows:

$$
30-4.6 \mathrm{~kJ} / \mathrm{min}=423 \mathrm{~W}
$$

This work rate cannot be maintained for long. During, say, 10 hours, energy would be applied in bursts - perhaps 10 bursts of 20 minutes for instance, but depending on the type of work. This "bursty" output, typical of males, rather than females, equates to 70 W averaged over a day, or 5MJ of "burst work" per day. Surprisingly, since both people and environment are different, the result is similar to that from the Indian village.

## Treadmill Experiments

Less reliable, but included as a cross check, are typical results from treadmill experiments on adult males in the UK on a typical diet of $\geq 3500$ Calories:

Weight 70kg, Speed $160 \mathrm{~m} / \mathrm{min}$, Inclination Angle $=10^{\circ}$
Vertical distance $=160 \mathrm{~m}$, Time $=10 \mathrm{~min}$
Total work $=11,200 \mathrm{kgm}\left(\mathrm{x} 9.81 \mathrm{~ms}^{-2}\right)$
$=183 \mathrm{~W}$ for 10 minutes
Cannot be maintained, muscle recovery needed
Maintainable repetition rate $=\mathrm{c} .5$ times per day (?)
Then accumulated "burst work" $=5.27 \mathrm{MJ}$ per day
These results do, indeed, confirm an approximate figure for strenuous, "bursty" work output per male adult, accumulated one working day, as being $\sim 5 \mathrm{MJ}$ per day.
Summarising, the energy intake per male adult ancient Egyptian is assumed to be some 2500 Calories per day $=\sim 10.5 \mathrm{MJ}$. The productive work potential averaged over working day $=\sim 5 \mathrm{MJ}$ per day. Not all of that potential is employed in raising stone, however:

- Quarrying, cutting quoins, etc., would employ some $50 \%$ of the available work
- Organizational factors-standing around, climbing up and down without stone, etc. would consume a further $50 \%$ (minimum!)
- ...Which leaves $\sim 1.25 \mathrm{MJ}$ per day for sliding/raising stone
- Applying this energy in bursts, such as would arise when raising stones, allows/requires the use of the Maximum Power Transfer Theorem (i.e. maximum rate of doing work coincides with $50 \%$ efficiency), once again halving the work done on the stone - the other half being expended as heat within the man's body.
- Which finally leaves $\sim 0.625 \mathrm{MJ}$ per man per day imparted to "burst work" of sliding/raising stone

This calculation of the work applied to raising stone is, at best, simplified. Further calculations based this figure should be subjected to sensitivity analyses.

## Chapter 5. Pyramid Construction

## True Pyramid Design



Figure 87. Archetypal Old Kingdom Pyramid Design
The ancient Egyptians were undoubted masters of stone building. The pyramids that they built were very much more than stones piled on stones; they had an elegant internal architecture. Figure 87 shows the structure of a nominal pyramid. No single pyramid conforms precisely to this configuration, but the figure shows most typical features.
A burial chamber, or chambers, was often dug into the bedrock, although they were sometimes constructed in the masonry. One or more passages lead to the burial chamber, generally from the north face of the pyramid above ground level. Portcullis doors protected the access via the passageway to the chamber(s). Portcullis doors often occur in threes. There was an internal "stepped pyramid" composed from horizontal tiers of limestone core blocks. Stones at each "face" between the slanting step columns were well finished, but between the faces, stones may be more roughly finished. Accurately cut and finished packing blocks filled in the steps. Finally, casing blocks, often of fine, white Tura Limestone, sealed the outside of the pyramid, with a pyramidion for a capstone.
The construction results in a number of concentric interfaces between the steps: in effect, each step is constructed like an inverted, rectangular cup. This has the effect of diverting the down thrust of the massive masonry away from the top of the burial chamber; it also spreads the load from the pyramid more evenly over the levelled foundations. Finally, and not unimportantly, it allows the bulk of the masonry to be finished roughly and hence quickly; without this feature, constructing most pyramids would have been impracticable using the hand tools available.

## Some Methods and Tools

Little is known about the tools that the ancient Egyptians used to build the pyramids. What is known, seems to be largely negative: they had no levers*', pulleys or block and tackle; they had no iron tools; copper was just becoming available, but its use to make cutting tools at this early time is uncertain; the wheel had yet to be invented ${ }^{\dagger}$, so presumably there were no rollers. They did have excellent ropes however, up to 5 inches in diameter.

The remarkable accuracy with which they laid out and constructed buildings and pyramids is indicative of great care and capability. It is possible, therefore, to discount many potential measurement and construction methods on the grounds that they simply would not have been accurate enough.

For example, the ancient Egyptians were very accurate over long distances when constructing tunnels rising or falling at 26.5 degrees; as we have seen, this is the angle formed across the diagonals of two butted squares.


Double Square


Figure 88. A Working Platform for Constructing 26.5 degree Passages.
A method that would have worked, Figure 88, uses a platform formed from the double square, cut across the diagonal and then rearranged to form the structure on the right. If the platform is maintained horizontal, using perhaps the flame from lamps reflected in a tray of water or oil to act as a level, then the rising edges of the structure must be at 26.5 degrees. Was this the method that was used? We do not know, but the apparatus/jig would have afforded the observed accuracy.

## Building Slopes at $51^{\circ} 50^{\prime} \mathbf{4 0 \prime \prime}$ - i.e. Khufu's Pyramid Slope

One of the problems facing pyramid builders was the control of slope angle as the pyramid building progressed. This is not quite as simple as it may seem, since the thickness of successive courses of stone was not identical. Instead, courses tended to

[^52]become shallower with height while, of course, the slope must be maintained. Figure 89 shows how this control might have been achieved, using a simple tool. This particular version would have been suitable for Khufu's Pyramid: other pyramids would require a similar tool jigged to their particular slope*.


Figure 89. Set Square for Aligning to Khufu's Pyramid Slope
The tool, which would be greater in height than that of a man would consist of a stout wooden frame, dimensions proportioned as shown, with a plumb bob to show when the device was vertical in both the ' $x-y$ ' and ' $y-z$ ' planes, i.e. clockwise and in and out of the paper. The device would be laid against the corners of stones, to ensure that they were set at the proper slope. Notice that it makes no difference what the thickness of a stone might be, so long as it has a square face which is brought up to just touch the device.

Egyptian builders were known to have used plumbs and set squares so, although this precise tool has not been found, it is highly likely that it, or something very like it, was in use. The survival of such tools from the Old Kingdom is unlikely, as they would have been made from wood and prone to decay.

## Found in QC (N) Shaft

When the north seeking star-shaft in the Queen's Chamber was broken into, three objects were found that must have been there since construction: a diorite (hard granite) ball - believed to be a mason's hammer; a length of wood, which might have a surveyor's rod, level or ruler - the piece was broken; and a small copper implement.

It is interesting to speculate as to why they were left. It could have been an accident or carelessness, of course, although that would not be in keeping with the builders' evident high standards elsewhere. Could they, perhaps, have been left as a "signature"

[^53]of the three main artisan skills, much as today builders write their name in concrete foundations, or leave a simple time capsule for later generations to find? Were they intended as kit of tools for the Pharaoh's $k a$ to complete the construction of QC (S) shaft or some chamber within the bulk of the masonry?


Figure 90. Diorite Hammer and Copper Device found in QC (S) - Collage British Museum

The small, copper device is of curious shape, with "forked horns" and two rivets on its small shaft: see Figure 90, which shows the diorite ball hand hammer, together with two views of the device. The rivets, clearly visible in the lower picture, suggest that the device was designed to be fixed flat against a thin lathe or shaft, with the horns then protruding above the support. The pattern of corrosion suggests the copper was impure. The "horns" were constructed with significant accuracy, however. What could be the purpose of such a device?
The Romans, some 2, 500 years later would use a similar device on the end of a pole for handling ropes and cords. The "horns" would be used to pick up cord, while the recess between the horns would serve to hold a cord in place on top of a vertical stick. The device shown in the figure does not appear to be particularly robust, however, and would seem more concerned with precision.
Several such devices could have been used to set up sighting lines, and one might have even been fitted on top of the pole in Figure 59, although that is speculation mounted on speculation. It would certainly have been possible, however, to set up an elevation line, using one or more of these tools like the sights on a rifle
It is possible to simulate the effects of using the tool to sight a star at dusk or at night. Figure 91 shows one frame from such a simulation, in which the light from a star is seen reflected off the surface of the tool, assuming that surface to have originally been burnished..


Figure 91. Star-sight Simulation

Such a tool, if indeed that were one of its uses, would assist in the determination of the precise point of star culmination. As the light from the star moves towards the centre of the fork, the reflections from both "horns" tends to equalise, reduce in length and finally virtually disappear when precisely in the "valley". The tool could therefore be used on any one night to find due north or due south, and on successive nights to find the date of the dawn transit.
There is, however, insufficient evidence to propose the star-sight hypothesis too seriously. Other possibilities for the horned tool are equally likely - or unlikely. Figure 92 shows two further options. At left is modern Egyptian corkscrew handle (Cairo, Hilton), which appears to be almost identical in both size and shape. Could the curious device have been part of a tool for opening containers?

At right is the handle of a medieval dagger made of fine Damascus steel Mughal, India, AD1585. Again, the shape is very similar to that of the tool in Figure 90, once the pommel is removed, even to the slight asymmetry.

In the first example, the purpose of the tool might be to ensure that the pharaoh had food or drink


Figure 92. Competing ideas for the "horned" tool. Photograph at right adapted from Scientific American 284,1 January 2001 on his journey to the after life. In the second, the purpose might have been to arm him, to deal with threats on the journey, or to deal with snakes and scorpions.

One factor not to be overlooked is its copper construction, in the late Stone Age. Copper was rare; it evidently had ceremonial uses, judging by the portcullis door at the top of the QC (S) passage.
Overall, fascinating although the tool may be, it raises many more questions than answers.

## Area of a circle

The Rhind Papyrus shows that the ancient Egyptians knew how to measure the area of a circle, using the method of squares, which works as follows.


Figure 93. Introducing the Method of Squares
Figure 93 shows two identical circles. The circle at left appears inside a square. The right hand circle has a square inside it. The left hand square's area is larger than that of the circle, while the right hand square's area is less than the circle's. There must, therefore, exist a square - smaller than the left square, but larger than the right square - that is exactly the same area as that of the circle.


Figure 94. Finding the Area of a Circle using the Method of Squares

Figure 94 shows a circle scribed on a matrix of squares, 18 x 18. There are two squares, in effect; the outer darker tiles and the inner lighter square.
It turns out that the area of a circle is given by the inner square of tiles, i.e. by a square of side $8 / 9 \mathrm{x}$ the circle diameter. As the figure shows by the highlighted areas at top left, the circle area less than the inner square equals the circle area greater than the inner square. Therefore, the circle area equals the area of the inner square.
So, the ancient Egyptians discovered
that, for a circle:

$$
\text { Area }=(8 / 9 \text { Diameter })^{2}
$$

i.e. take $1 / 9$ th off diameter and square the result. The method is about $0.6 \%$ accurate, which is sufficient for most practical purposes.
The ancient Egyptians evidently had a pragmatic way of calculating the area of a circle, and hence of the volumes of cones, truncated cones, etc.

## Constructing Shafts

Jean Kerisel, President of the France Egypt Association, believes that the shafts in Khufu's Pyramid were constructed from prefabricated stone blocks, cut to the required shape perhaps in a special quarry. The possible method can be seen in Figure 95 , which shows, with the lower bed stones cut to the right angle and the upper stones cut with a groove to create the shaft.


Figure 95. Prefabricating Shafts
Hundreds of these inclined and grooved pairs of stones would have been quarried and shaped. Additionally, the curves in the two northern shafts had to be formed - not too difficult provided the work was done on a bed already inclined at the appropriate angle, enabling the mason to cut the curve while working horizontally.

When fitting the shafts, the inclined stones would have been complemented by stones above them cut to the same incline, but revolved by $180^{\circ}$ about the vertical to give a horizontal upper surface, so that the superstructure could be continued as horizontal layers. Using simple proportions and standard tools would have facilitated managing and controlling the accuracy of this work. There would have been 4 sets of such tools to go with the four sets of proportions.
In summary, the architect and masons were faced with a significant and unprecedented challenge to design and construct shafts of fixed slopes in a layered horizontal construction without compromising the strength of the structure and without crushing the shafts as the superstructure was erected above them. They rose to the challenge magnificently, yet using the simplest of methods.

## Constructing Burial Chambers

Figure 96 illustrates a typical burial chamber construction process. First, top left, two inter connected chambers were dug out from the bedrock, together with a passage, including grooves for portcullis doors. Second, bottom left, the burial chamber was filled with sand, and the granite sarcophagus dragged on to the sand pile. The sand
was then removed through the interconnected chamber and passage, lowering the sarcophagus to the chamber floor.


The chamber was refilled with sand, this time raised up above ground level; the first layer of capstones was dragged into place on the sand pile, and locked into place with lateral stone slabs. The sand was removed, leaving the view at bottom right. At some point, the walls were carved with Pyramid Texts ${ }^{*}$, top right, and stars were carved into the underside of the roofing slabs. It may be that carving was undertaken only once the chamber was covered, to prevent filling and emptying of sand from damaging the fine work.
By the time two or 3 tiers of the pyramid were raised, the burial chamber, the antechamber and its entrance passage way were concealed and the portcullis doors were suspended in the masonry, awaiting their time to drop into place and seal the chambers.

## Levelling the Foundations

Each pyramid was dependant on good foundations. Some, such as the Bent Pyramid at Dahshur, had been built on unsuitable ground. The pyramids at Giza, however, were based on solid limestone strata.

[^54]

The method used for levelling the pyramid site, Figure 97, comprised four stages. First, a channel was dug around the proposed base perimeter. Second, the channel was filled with water, and the sides of the channel marked off at the height of the water; this served as a level*. Third, the ground inside the channels was levelled off to the mark. Fourth, the first course of stones was laid.

It was not essential to level the whole site, provided the periphery was accurately level. If the central area contained a mound, the ancient Egyptian architects were adept at incorporating that mound into the structure. Cutting flats and steps into the mound to represent the various tiers made intelligent use of the intrusion, which they may have wished to incorporate and preserve as symbolic of the Mound of Creation.

## Pyramid Architecture

The generally accepted sequence of major activities in the construction of a simple pyramid is illustrated in Figure 98. First, the site is levelled, top left. Then the pyramid is constructed in a series of tiers with the courses of stones running horizontally, top right. The Great Pyramid had 202 such tiers. Once the inner stepped pyramid is complete, packing stones are added from the ground up, bottom left. Finally, the casing stones, made of fine Tura limestone, are set in place, this time from the top down, starting with the pyramidion on the peak - bottom right. As the figure

[^55]

Figure 98. Pyramid Construction Sequence
shows, this pyramidion may have been covered with electrum, a mixture of silver and gold.

Where, as in the case of many $4^{\text {th }}$ Dynasty pyramids, the masonry contains several chambers and passages, the simple view expressed in the figure will need to be modified. The sequence does not indicate how ramps would be employed; models and experiments show difficulties using ramps at corners and near the top. An alternative approach to finishing the pyramid, raising stone tier by tier, presents a convenient and simple way for finishing off the pyramid structure.

The schematic of Figure 99 shows how the potentially difficult task of finishing off the outer layers of the pyramid could be addressed using rocker methods (q.v.), which lift stones up all four sides in parallel without sliding. The figure shows a partly completed pyramid, with the core tiers still visible through the packing and outercasing stones. The simplest approach would be to fit packing stones on the corners, and to fit quoins from bottom to top. Packing and casing stones would then be raised to the appropriate tier using the still-exposed tier steps.
Once the four corners and the top had been completed, packing of each tier would start at the highest exposed level. Casing stones would then be fitted, also from the top down, working from the corners into the centre. The tier below that being cased would act as a platform for those placing, fitting and dressing the casing stones. Packing and casing would proceed downwards tier-by-tier, covering the entrance, and finishing at centre bottom of each side, all four of which would be worked in parallel. Paving around the pyramid would then cover the very bottom of the lowest casing stones to provide a completely sealed, pristine white pyramid.


Figure 99. Finishing Packing and Casing using Rocker Methods

## Engineering the Pyramid

It is uncertain how the Great Pyramid was engineered. It is widely assumed that the ancient Egyptians used ramps to haul the stone up from the quarries and then used more ramps to haul the stone up to the required level on the pyramid. Such ramps are illustrated in books and, in long shot, on films. While there is evidence of a ramp from the quarry to the base of the Great Pyramid, there is less certainty about ramps on the Pyramid itself.
Herodotus was told ${ }^{51}$ quite specifically how the Great Pyramid was constructed:
"The pyramid was built like a flight of stairs (others use the image of staggered battlements or altar steps). When that first stage of the construction process was over, they used appliances made out of short pieces of wood to lift the remaining blocks of stone up the sides. First they would raise a block of stone from the ground on to the first tier, and when the block had been raised to that point, it was put on to a different device which was positioned on the first level and from there it was hauled up to the second level on another device. Either there were the same numbers of devices as there were tiers, or alternatively, if the device was a single manageable unit, they transferred the same one from level to level once they had removed the stone from it. I have mentioned two alternative methods, because that is exactly how the information was given to me. Anyway, they finished off the topmost parts of
the pyramid first, then the ones just under it, and ended with the ground levels and the lowest ones."

Over the years, investigators have either ignored what Herodotus was told, or have chosen to disbelieve it. In either case, the prevailing view is that one or more ramps were used and that there were no "appliances made out of short pieces of wood", or "contrivances made out of short timbers", as the phrase is sometimes translated.
Nonetheless, according to the account that Herodotus extracted*, there were no ramps.

## The Evidence in the Stones

One way to investigate how the Great Pyramid was built, and to see if Herodotus was correct or not, is to explore the dynamics of setting stone in place. For instance, knowing how long it took to raise the pyramid, and its overall mass, it is straightforward to calculate the rate of setting stone in place over the 20 or so years of construction as 89 tonnes per hour average. Long-term averages, as we have already seen, can conceal short-term peaks; using the Pyramid Calculator introduced in an earlier chapter, which analyses the building process tier by tier, can reveal the latter. Very high build rates would militate against the use of ramps, since they would create queues and hold-ups.

## The Workforce

It is not clear, either, whether there was a constant workforce each year, or whether large numbers of workers were drafted in at the start of the project, in proportion to


[^56]the amount of work outstanding. (In either case, there may have been an annual corvée, but it could have recruited more men at the start of the construction.) Using the Pyramid Calculator enables both situations to be assessed.
The effects of the alternative manning situations are shown in Graph 16. A constant workforce, year in year out, would take longer to set the stone in the lower tiers, simply because there is a lot more stone to set per tier lower down. The graph shows the first tier taking nearly 70 days, then time gradually reducing until at Tier 181 it takes just over 2 days. The sudden increase from 181 to the end accounts for setting the packing and casing stones.

The horizontal line, on the other hand, shows the amount of time taken to raise each tier if the manpower is recruited in proportion to the work outstanding. In this event, each tier takes 36 days to set in place, and involves many more men for the lower tiers, with numbers gradually reducing towards the upper tiers. (It is simple to verify the 36 -day figure. 20 years is $\sim 7,300$ days. Dividing some 201 tiers equally between 7,300 gives $\sim 36$ days per tier.)


Knowing how long it took to set each tier in place, and knowing the mass of each tier in tonnes, allows us to calculate the rate of setting stones in tonnes per day or tonnes per hour. Graph 17 shows the results of the calculations for both the fixed annual workforce and the proportional annual workforce. When employing the proportional workforce, the mean rate of setting stone is much higher at the start because it involves many more men at the start - effectively "front-loading" the work to make significant early progress.

The situation switches above Tier 100, with the fixed annual workforce setting more stone in place at the higher tiers. Note from the graph the drop in setting rate at Tier 181 due again to work on packing and casing stones. This effect does not appear on the proportional workforce line, since more workers would be recruited to deal with the problems as they arose, so maintaining a smooth line to the end.

No matter how the workforce is recruited, the same total tonnage must be set in place by the end of the supposed 20 -year building period.


Graph 18 shows the situation. The Mean Rate, Proportional Manpower line is the same as that in Graph 17. The Cumulative Rate, Proportional Manpower line shows how the mass of the Pyramid accumulated, tier by tier. The final value of 89 tonnes per hour, reached by dividing the total mass of the Pyramid by the time it took to build in hours, is often quoted as the rate of setting stone for the Great Pyramid. However, as the graph shows, this is a deeply misleading figure. If, for instance, during the building process, one was to calculate the cumulative (or running) mean up to Tier 41, say, it would have averaged, from the graph, $\sim 217$ tonnes per hour.
Using Proportional Manpower increases the rate of setting stones in the early, lower part of the Pyramid. Graph 19 shows the equivalent figures when employing Constant Manpower year-on-year.
The cumulative rate must, as before, equal 89 tonnes per hour by the end of the building time, after some 20 years. However, the shape of the cumulative line is much flatter than that of Graph 18 , owing to the lower rate of setting stone at the start when employing fewer men, and the consequent need to maintain high rates well into the upper tiers.

In the absence of direct evidence, it is difficult to determine which, if either, of the manning strategies outlined above was used on the Great Pyramid. However, a powerful ruler like Khufu would have wanted to see progress, and may have become impatient at the length of time it was taking to raise his Pyramid. It is not unlikely, then, that more men would have been recruited at the start, particularly as the lower tiers required more work and, perhaps, less skill that the higher tiers with their chambers, shafts and galleries.
A simple solution would have been to have a fixed, core team of experienced builders who worked all year round, supplemented each year by a corvée which would recruit

as many men as could be usefully used in that particular year, not only for the Pyramid, but for the causeway, temples, stone quarrying and finishing, Queen's and $k a$ pyramids, etc.

## Ramps and Ramp Theories

Although there is some question about the use of ramps to build Khufu's Great Pyramid, with its massive stones, ramps were undoubtedly used elsewhere, and may have been employed on other $4^{\text {th }}$ and $5^{\text {th }}$ Dynasty pyramids.

## Desert Ramps

There are several ways, Figure 100 on page 143, in which ramps might have been configured and employed. Top left is a ramp, going out into the desert. Ramps have a maximum sensible slope, above which it would be too difficult to haul stone on them. Experiments and information from the Anastasi Papyrus indicate that $8^{\circ}$ is about the upper limit. So, as the pyramid begins to grow, the "desert" ramp must go further and further into the desert. Before very long, the desert ramp would become more massive than the pyramid it is supplying with stone. Graph 20 shows the situation.

As a pyramid is built, its volume naturally rises, but the rate of volume rise falls off nearing the top as the cross sectional area reduces with height. For a ramp leading out into the desert, the situation differs. As the graph shows, the ramp volume initially accelerates as the truncated pyramid grows - hence the upward turning graph-line.
For the Great Pyramid, the volume of a desert ramp equalled that of the truncated pyramid when the latter reached some 58 m height; such ramps would surely be deemed uneconomic and impractical well before this height was reached. As the graph shows, using an 8 -degree ramp leading straight from the desert to the very top of the Great Pyramid would involve a ramp of well over twice the volume of the completed pyramid. After building, of course, this massive structure would have to be


Graph 20. Volumes of Truncated Pyramid and Notional Desert Ramp.
(Figures for Great Pyramid)
removed and disposed of. Since there is no real evidence of the remains of massive ramps, it must be assumed that desert ramps, if they were used at all, served only the lower courses and tiers of the pyramid.

## Helical and Zigzag Ramps

A small stepped pyramid is shown top right, Figure 100. In addition to two straight ramps approaching from the desert, there are helical ramps and zigzag ramps. A helical ramp would allow workers to climb gradually along one face, then higher still along the next face, and so on. A zigzag ramp would allow workers to climb back and forth along the same face. Up to four of either of these ramp types could be accommodated, one per corner and one per face respectively.
As the figure shows, there is an incompatibility between ramps from the desert and the other types. The rising peripheral ramp effectively blocks access from the desert ramp except at each corner. Note also the substantial corners needed for peripheral and zigzag ramps to provide room for manoeuvring stones on sledges. Further, as the length of each tier side reduces with height, the tier thickness would also have to reduce, or else the ramp, limited to $8^{\circ}$ slope, would not be able to reach the following tier. In fact, tier thickness does reduce as the pyramid rises, perhaps to prove the ramp theory correct, or perhaps simply because it is more difficult to get large stones up so high. All of these factors militate against maintaining a high average rate of laying stone.

A ramp and sledge are shown bottom right. Ramps would have been composed largely of limestone chippings with wooden inserts. The inserts were not rollers, but would be lubricated with water, fat or lime to reduce sliding friction.


## Ramp Capacity

All is not as straightforward with the ramp theory as artists' impressions might suggest. Graph 17, Graph 18 and Graph 19 above showed the times to raise tiers and the rates of setting stone in tonnes per hour. Each stone is thought to have had a mass of some 2.5 tonnes on average, so we can estimate the number of stones being set per hour as the tonnes per hour / 2.5 .
The single lane capacity of a ramp can be estimated, rather like estimating the capacity of a lane on a motorway or turnpike. Each stone would be hauled up an incline on a sledge by a team of some 25 men, most pulling on ropes, one or two levering and lubricating, navigating awkward corners on the way. A reasonable estimate for the length of such a team, with ropes and sledge is about 33 metres. It is also reasonable to assume that the mean rate of such teams passing a fixed point on a ramp is unlikely to exceed some 2 per hour on a continuous basis*. Putting these figures together indicates a single path on a rising ramp accommodating less than, or

[^57]
equal to 5 tonnes per hour, i.e. 2 stones, each of 2.5 tonnes*.
Another important factor to consider is the corvée. During the months of the Inundation the numbers of workers would swell considerably, by a factor of at least 4 . That means that the rate of laying stone during the Inundation would also rise by a similar factor, in the worst case exceeding 1000 tonnes per hour ${ }^{\dagger}$ on the lowest tiers.
Graph 21 shows the results of putting these various factors together. The graph shows the number of parallel paths that would be needed to maintain the necessary rates of setting stone, tier-by-tier for the Great Pyramid. The alternative manning strategies are shown, each with a corvée during the Inundation months: constant annual manpower and proportional annual manpower.
The number of parallel paths that would have been needed on a ramp to maintain the essential rate of setting stone is very large. Even if there were, say, four helical ramps, one from each corner, the number of parallel paths per ramp would still be very large. We are talking not so much about simple ramps, more about multilane highways screwing their way up into the sky!

For example, a 4-ramp strategy would probably see 3 ramps for raising stones and one ramp for bringing workers and empty sledges down again. At Tier 68, some one third of the height, this means some 33 parallel paths for each of 3 rising ramps or, with constant workforce, some 22 parallel paths for each of 3 rising ramps.
At Tier 100, halfway, both manning strategies would necessitate the same number of

[^58]ramp paths, i.e. $\sim 53$. Using the 4 -ramp strategy, that would necessitate some 18 parallel paths for each of the three climbing ramps, i.e. $53 / 3$. Beyond that height, the constant annual manning strategy starts to show its weakness, with 38 parallel paths still needed at Tier 181.

Not only must ramps carry phenomenal loads, they must also be continually extended and maintained. It is difficult to conceive any construction ramp design that would successfully meet such phenomenal requirements.


Knowing approximate dimensions of a putative helical or zigzag ramp enables the calculation of ramp volume. Graph 22 shows the result, which is necessarily a broad estimate, since the way the ramp was built - if indeed a ramp was built at all - is not known. Comparing this graph with that of Graph 20, it can be seen that the helical or zigzag ramp occupies significantly less volume than the desert ramp

It is possible to combine the desert ramp and the helical ramp approaches. Looking at the respective graphs suggests that the desert ramp might have been used on all four sides up to about Tier 25. After that, 4 helical or zigzag ramps might have been used, from each corner or across each face. As work neared the top, the distance between successive turns of the helical ramp would get closer and closer, until they mutually interfered. So, higher up the number of ramps would necessarily reduce to one. And finally, experiments have shown that ramps are unhelpful near the top, where manoeuvring room is at a premium.

Looking at the overall figures and graphs, and particularly at the number of parallel paths that would have been necessary to accommodate the phenomenal building rates, the ever-popular ramp theory seems barely credible in practical engineering terms. But is there any alternative?

## "Rocking" Methods

If Herodotus' report, page 137, is taken as factual and added to what archaeologists believe was the contemporary knowledge of ancient Egyptian technology, then the means by which the Pyramids could have been built are severely constrained. Nonetheless, it is possible to conceive of methods for raising very large stones indeed that comply precisely with what Herodotus was told, and which would have been well within the technological capabilities of the builders.

## Rocking



Figure 102. Raising Stones by Rocking
Figure 102 shows one possible method - delightful in its simplicity. The stone to be raised is manoeuvred over two wedges, butted together as shown in the figure to create two adjacent fulcrums. The stone is then rocked to one side and a wooden plank is fitted underneath one wedge - upper right picture. Men would stand on top of the stone and walk along it from end to end to make it rock. By virtue of the wedge shape, the stone could not "over-rock" and topple.
Rocking the stone to the other end would allow another plank to be fitted, this time twice as thick as the first - lower left picture. This process would then repeat using the same thickness cribs until the stone had been raised to the desired height. Finally, the stone would be swivelled horizontally off the wedges and on to a waiting platform - not shown. This method would work even for the massive, 75 -tonne stones of Khufu's Relieving chambers - see Figure 32 on page 44 - when the use of sledges and ramps seems quite incredible. (What kind of wooden sledge would sustain a 75-
tonne granite beam* while it was hauled up the Causeway and then up $2 / 3$ rds of Khufu's Pyramid?)
The rocking method fits Herodotus' description precisely: the wedges would be made either of hard wood ${ }^{\dagger}$, or stone.

## Rocking Statistics

Figure 103 shows the vital statistics of stone rocking. The figure shows a block resting on two fulcrums, which in turn sit on stacks of short wooden planks. The wedge-shaped fulcrums, or pivots, are arranged so that the stone is balanced between them, but can be tipped clockwise or anticlockwise by a weight at either end.
The tilting weight would be supplied by several "stone walkers," men who walked from one end of the stone to the other and back again, tilting the stone first one way and then the other to allow the planks of wood to be inserted. The figure shows the clockwise and anticlockwise moments, which would be equal at the point of balance for the left-hand fulcrum. The angle on the wedge, which limits how much the block may tilt, is some 10 degrees off the horizontal at maximum.


For a 50 -tonne block of granite, 6 metres in length, with 3/4 average weight stone walkers, the value of $X$ is some 15 cm ., and the "lift per tilt" is of the order of 5 cm ; this determines the thickness of each plank. The lift per tilt may seem small, but once the team of men got into a rhythm, one tilt every 2 minutes would seem to be readily attainable. For a straight lift, one tilt per 2 minutes means that the 50 -tonne stone

[^59]could be raised vertically through 1 meter in 40 minutes.
Once the stone block had been raised to the height of the next tier, it would be swivelled on to the next set of pivot stones, which would already be set in place. This would be a more delicate job, requiring the stone walkers to balance the stone while it was swivelled first one way and then the other. Ropes would assist the crew to apply the necessary sideways forces.
Allowing 1 hour to transfer from one tier to the next, a 50 -tonne stone could be raised through 100 metres ( $\sim 140$ tiers for the Great Pyramid) in some 20-25 ten-hour days. If that sounds a long time, remember that 10-15 fifty-tonne stones could be raised per side at the same time, all in 20-25 days. Overall, this method is simple, sound and, of course, consistent with Herodotus*.

## Rocker Sledges

A second method, that also fits Herodotus' description, and which makes use of rocker sledges would work as shown in the next figure. While it would not be practicable for massive, 75 -tonne stone beams, it would accommodate the more usual limestone blocks and beams with comfort.

Figure 104 shows the method. At top left, the stone is mounted on a stout rocker sledge, perhaps using the rocking method described above to get the stone in place. At


Figure 104. Raising Stones Using Rocker Sledges

[^60]top right, the sledge is tipped using manpower on top of the stone walking from end to end, and a plank is placed under the free end of the sledge. The process now proceeds as before, rocking the sledge from side to side and forming stacks under each side as the rocking gives opportunity. Bottom left shows the second level of planks being completed. Bottom right, all the planks are in place and the sledge is swivelled sideways on to the waiting platform. The sledge would be manhandled using strong ropes and many men pulling in unison. The whole process is robust and rhythmic. Inexperienced builders could easily get "into the swing" and little training would be needed.

The rocker sledge is evidently consistent with Herodotus; both rocker methods described above fit Herodotus' description, page 137, precisely. There would have been several sets of stacks on each tier level, and on all four sides at once.
The rocker itself could easily be made with the tools of the time, and would be reusable and repairable many times. Note the mortise and tenon construction, requiring no metallic parts.
Rocking methods are not without their concerns. The pressure of heavy stones and sledges on the wooden planks would cause compression problems in the wood, and there is also the issue of off-centre weights causing the stacks to distort like a vertical concertina. Such problems could have been addressed in several ways:
The wooden cribs could be stacked alternately with the wood grain running parallel to, and orthogonal to, the rocking plane, and could have been bound to hold the grain in place. The planks could themselves have been wedge-shaped, with the slightly thicker edge towards the centre, anticipating the crushing effect of the stone. Ultimately, the planks could have been made, not from wood, but from accurately cut stone.

## Parallel working opportunities



Figure 105. Parallel Teams Raising Stones

A major advantage of the rocking methods shown above is that they facilitate parallel working - see Figure 105. Top left shows stones being dragged from the quarries by three notional teams working in parallel - the teams are, of course, invisible. Top right, six teams are at work, three raising stone from ground to first tier and three raising stones from first to second tiers. Bottom left shows the next stage, with three teams raising stone from second to third tiers while three teams swivel and drag stones over the surface of the first tier. Finally, bottom right shows a perspective view with some 20 teams working on all four sides of the developing pyramid. Note that these pictures, taken from two virtual reality sequences, are not to scale. In practice it would have been possible to have many more teams working in parallel, particularly in lower tiers.


Figure 106 shows how parallel working might have been organized. There would have been two gangs of 1000 men per gang, i.e. 250 per side. A team was called a zaa, or a phyle in Greek. One gang would raise stones while the other gang fetched stone from quarry, or hauled it up from the river using the causeway.

There would have been $\sim 10$ teams per side* with $\sim 25$ men per team. In support of the team idea, some vertical seams have been found on Menkaure's pyramids, which could be evidence of team margin.
Ten teams per side makes 40 teams altogether, with each team able to raise $\sim 2$ tonnes per hour: Experiments show this to be a steep but feasible target. Each team would work, say, 2-hour shifts during heat of day.
2 gangs $=2,000$ men $x$ two shifts $=4000$ men. Plus food, water, tools, ropes, quarry masons, stone dressers, ramps, sledge makers...

So, the widely accepted ramp theory is insecure. Not only are there many problems with building ramps, queuing on ramps, and extending and maintaining ramps, but also the stone rocking alternatives are much simpler and allow many more teams to work in parallel. Only by using rocking methods and attacking the Great Pyramid from all sides at once does the task of fitting stone at rates of up to 1000 -tonnes per hour even begin to be feasible.

[^61]We may never know the whole story, but the burden of evidence seems to be shifting away from ramps and more in favour of Herodotus...Moreover, there is nothing mysterious or magical about the way in which the ancient Egyptians built the pyramids. The architects were careful, precise, methodical and experienced. They used materials and the technology to hand in a simple way to produce excellent results. Keys to their success included tenacity of purpose, fine craftsmanship*, patience in execution, and the ability to command, organize and direct a large and enthusiastic workforce.

[^62]
## Chapter 6. Systems Engineering-Egyptian Style

## Without Reductionism

Large or complex problems, issues or tasks are tackled today by breaking them into smaller, easier parts, solving each part and then bringing the part solutions together to solve the whole. This process, called reductionism, is at the heart of current thinking, and was codified by René Descartes during the European Renaissance. It seems to have been conceived initially by the Greeks, some 2000 years after the Pyramids. Reductionism is an artifice, a method or a technique. It gives an approximate answer for relatively simple problems. It has been remarkably successful: some would say that it provided the foundation for the Industrial Age.
The method is, however, flawed; it depends on the notion that it is possible to envisage the whole by looking at the parts, which may be true for simple systems only. Clearly, it is not possible to envisage a tornado by looking at the atoms of hydrogen and oxygen from which it is formed, nor human intellect from an examination of neurons and dendrites. For such complex systems, there are many interactions between the parts, which change the way the whole behaves, but which reductionism overlooks in the interests of simplicity.
Systems engineering was formalized in the middle of the $20^{\text {th }}$ Century with the objective of overcoming the deficiencies of reductionism by taking an overall "systems view" of problems, issues, tasks, organizations, etc. While reductionism breaks things apart to see how they work, systems engineering brings things together to synthesize some whole. A contemporary view might suggest that reduction and synthesis are complementary.
Reductionism is still firmly embedded in most people's thinking today; we are taught to reduce at our mother's knee, and throughout our schooling. To most of us, there is no other way. That is one of the reasons why it is important to try and understand how the ancient Egyptians went about planning, organizing and building their pyramid complexes. As reductionism had not yet been invented, it is reasonable to assume that the ancient Egyptians did not approach their tasks in a reductionist frame of mind. Yet, they managed to build pyramids the like of which we could not even contemplate building today. Is it conceivable that today's reductionist approaches, far from helping us, may be inhibiting our ability to build bigger, better, more complex and successful systems?
Mercifully, the ancient Egyptians were spared the ancient Greeks, Henry Ford and Taylorism; their ideas would take some 4,500 years to emerge fully! For the ancient Egyptians, it seems their instinct was to build-so "big" was good, "takes many years" was fine, "cost" was no problem. Importantly, as Snefru showed, they had not yet been spoilt by conscious fear of failure and risk*.
That is not to say that the ancient Egyptians operated without goals. On the contrary, they seemed to fixate on the goal of providing the Pharaoh with his eternal resting place in time for the occasion of his death.

[^63]
## Planning the Great Pyramid of Khufu



Figure 107 shows notionally how we might approach the task of creating a pyramid complex without recourse to Cartesian Reduction:

1. Establish Customer and User Wants and Needs. The ancient Egyptian architects would not have used today's jargon, of course, but they would have either been aware of, or have sought views from, both Pharaoh and priests about the forthcoming pyramid complex design. More likely still, the architects would have been one with the Pharaoh's "administration" and his priestly followers. Hemon, the architect of the great Pyramid is thought to have been a relative of Khufu, possibly his cousin ${ }^{52}$. Hemon was both Prince and vizier, as well as architect
2. Conceive Future Vision with Customer. Then as now, this would be a key activity. It would not do for the Pharaoh to be expecting one thing, while the architect was conceiving something else. Creating a joint Future Vision, then as now, probably involves some form of model or representation, since words are a poor medium for describing objects, products, services, systems or pyramids
3. Establish the Emergent Properties ${ }^{53}$ of the Future Vision . It is doubtful whether the ancient Egyptians would have expressed the idea of emergent properties, those properties of a whole system that "emerge both from the parts and from their mutual interactions"." Nonetheless, the ancient Egyptians would have been aware that the way in which parts of the complex were laid out, related, interconnected, covered in, etc., would create impressions on the "customers and users." Many of the emergent properties would have been obvious, too: size, colour, orientation, proximity and security.
4. Conceive strategies to achieve requisite emergent properties. Having determined the emergent properties, which are outcomes from the building and construction processes, it would then be necessary to develop strategies to

[^64]achieve those emergent properties. For instance, if the final colour of the pyramid were to be white (a visible emergent property) then one strategy for achieving this would be to finish the pyramid with white Tura limestone, conveniently situated on the opposite bank of the Nile near Giza. A strategy is a way of achieving the emergent property. (A strategy is evident when there is more than one way of achieving the requisite emergent property, offering choice between them.) There were undoubtedly other sources of white stone, but none so convenient and easily worked as limestone from the Tura mountains
5. Select, Correlate, Resource and Pursue Effective Strategies to Realize Requisite Emergent Properties. Item 4 generates a number and variety of strategies. These have to be put into a sensible plan, since some must take place before others, some may run in parallel with others and some will need to be reconciled with others. In particular, it would have been important to pursue effective strategies, and these would include manpower-efficient strategies. For instance, using nearby quarries for the vast bulk of the stone in the pyramids is a cost-effective strategy. In fact, electing to build the pyramids on a plateau near the river that had a plentiful supply of limestone was also a cost-effective strategy. Resourcing would have been important, too. Bringing in manpower resources early to build the Causeway, for instance, would have been seen as useful, since it would facilitate the flow of food, water and stone via the River Nile to support the main building programme
6. Hence, Realize Future Vision. Holding to the resourced plan from 5 should result in 6 , given time
So, it is possible, even rather straightforward, to plan and manage the building of a pyramid complex without reduction, although we do not do it that way today. Did the ancient Egyptians do it this way? We cannot be sure, but the speed with which they raised their pyramids suggests that they were both efficient and effective in working on the whole complex. Besides, there is a fundamental sequence of activities that must have been pursued for the pyramids to be constructed successfully. Vary from that fundamental sequence, and they not have been built, or would have taken too long, or would have fallen down...

## The Pyramid Project Bar Chart

Some form of forward planning, forecasting and control system would have been necessary to co-ordinate building activities within the complex. If the ancient Egyptian architects did not use a time-bar chart, they must have had something similar, or at least carried similar ideas in their minds - and that would have been difficult, for such a vast project. Graph 23 shows a modern bar, or Gantt, chart, which might be used to outline the sequence of activities and durations for a construction project, in this case the Great Pyramid.
The chart is consistent with the sequences used in the Pyramid Calculator (on page 111.) At left are the major activities that are to be undertaken, and the length of each bar indicates the anticipated duration of the corresponding activity. Planners must know the rate of build, in order to estimate durations: for the Pyramid, this means that the duration is dependent upon having sufficient, experienced manpower to achieve the goal. If the manpower needed for each task in each year were summed at the

bottom of the chart, then planners would know in advance how many men were needed for the annual corvée.

From the bar chart, it can be seen that the passage down to the underground chamber can be started once the site has been "pegged out" and before levelling is complete. The Causeway is shown as taking 10 years to build, in deference to Herodotus. The Valley Temple is shown as taking some 15 years; work would be spasmodic during this period as work on the Valley Temple would be continually interrupted by unloading cargo boats carrying food, water, stone, workers and visiting dignitaries. The Mortuary Temple is left until the end: it butts against the side of the pyramid, and could not be finished until that area of the pyramid was complete.

In the case of the Great Pyramid, the surrounding Temenos wall enclosed a paved area with polished marble floors. This would have to be kept until last, so as not to damage the marble. Similarly, covering in the Causeway would be left until last. The internal walls were carved with hieroglyphs and images; these would have been kept from general view, and use of the Causeway for hauling would have been stopped to prevent damage to the paving.
The Queens' Pyramids and the $k a$ Pyramid would have been left to the end to minimize damage to the Temenos wall and polished paving. Finally, and logically, packing the Pyramid to fill in the steps, and then encasing the packed pyramid in Tura limestone must come at the end.

## Behaviour Diagram

A modern Behaviour Diagram for designing and planning the Great Pyramid is shown in Figure 108. The ancient Egyptians would not have used such a scheme, but they would have had to go through all the activities and processes to achieve the end result - the pyramid complex.


Figure 108. Behaviour Diagram for Designing and Planning the Great Pyramid

The Behaviour Diagram works as follows: Start with the central sequence of activities from top to bottom. Note that this is compatible with the generic plan shown at Figure 107. Then, for each central activity in turn, work from left to right. At left is the input (source material, data, or other resource) needed to make the activity work. At right is the output or product that results from the activity. Observe that the products at the right relate to each other, in that each leads naturally to the next

Note that the Behaviour Diagram assumes implicitly that Hemon, the architect of the Great Pyramid, did not start with a blank papyrus. He already knew of, and had first hand experience with, Snefru's pyramids, so that the design of Khufu's Pyramid evolved from Snefru's Red Pyramid. Hemon's major concerns would have been the many additional features needed for the Great Pyramid. As the Behaviour Diagram shows, these new requirements would have come from theological advance and ceremonial, or ritual, evolution.

## Interpreting Khufu's Requirement

Pharaoh Khufu undoubtedly had definite views about his final resting place, his Akhet Khufu. Significantly, the slope was to be $7: 51 / 2$, or 5.5 seked. This slope had not been used since the pyramid at Meidum ${ }^{*}$ (see Figure 23 on page 35 above for the Meidum Pyramid, and Graph 3 on page 60 above for the various pyramid slopes.)

Other essential features of the pyramid complex would have included facilities to: preserve Khufu's $k a$; enable Khufu/Kocab to orchestrate annual rebirth rituals

[^65]eternally; facilitate the transmogrification of Khufu from Horus in life to Osiris after physical death of the body; maintain a constant flow of ministrations by priests; support major annual ceremonies on due dates as the major power sources maintaining Khufu in the afterlife; and to prevent any disturbance of the mortal remains, necessary for the reuniting of the $b a$ and the $k a$.

## Emergent Properties from Khufu's Vision

Table 8 shows more explicit detail for Step 4 of the systems engineering process illustrated in Figure 107 on page 154. This is the crucial, creative step in which strategies are conceived and chosen that, when effected, should result in the emergent properties of the full pyramid complex.
In the table, emergent properties have been classified as: Aesthetic - those emergent properties concerned with perceptions, e.g. of beauty, awe, splendour, numinousness; Behavioural, emergent properties concerned with reaction to stimulus; Temporal, emergent properties which vary over time, or have a time variant context; Functional, emergent properties which describe activities or capabilities for "doing"; or Physical, tangible emergent properties such as size, weight, orientation, colour
When looking at ancient ruins of temples, or pyramids, we can easily fail to see them as the people who conceived and worshipped there saw them. We may not envisage the priests, the guards, the officials and dignitaries, the pomp and ceremony, the music, dancing and feasting. We may not imagine the royal barge, carrying the dead Pharaoh, or the dark, enclosed Causeway leading from Valley to Mortuary temple lit only by daylight filtering through small and occasional clerestories.
The ancient Egyptian designers must have imagined all of these factors, and many more, for the Giza Pyramids to have been built and to have been the successful sites for worship that, for a time, they evidently were. In looking at the table, it is interesting to note just how many of the emergent properties are neither physical nor functional; today, so many of our designs are only physical and functional - a byproduct of Cartesian Reduction.

Table 8. Designing the Pyramid Complex Akhet Khufu

| Emergent Properties | Strategies and Principal Activities to Achieve Emergent Properties |
| :---: | :---: |
| 1. Numinous open-air temples for worship of Kings and regular religious ceremonies <br> aesthetic, temporal, behavioural | - Observe mythological stars, record passage, angles, choose scales on Earth, locate site with respect to Nile and horizon, survey potential sites, select Giza <br> - Set up observatory at Giza. Record star angles from Giza. Note effects of precession over several decades <br> - Mark cardinal directions. <br> - Mark and record heliacal risings and culminations |


|  | -Conceive dark passages and <br> chambers in polished stone, <br> orientated to solar and stellar <br> directions to augment |
| :--- | :--- |
|  | numinousness |
|  | - Include statues and carvings of |
| deities to heighten numinousness |  |$|$



## Pyramid Models

Preceding paragraphs have mentioned models on several occasions. It is difficult to conceive any way in which the ancient Egyptians could have undertaken a project as complicated and enormous as the Great Pyramid without first building one or more models of the whole, or parts, of the pyramid. After all, it would be grossly unacceptable to construct the whole edifice, only to find that sliding plug stones would not slide, portcullis doors would not drop, or the Grand Gallery collapsed under the weight of the superstructure.

It would be equally unacceptable to raise the pyramid as far as the level of the Queen's Chamber, only to find that there was no way of fitting the many passages together: the base of the Grand Gallery rising at $26.5^{\circ}$; the passage from the entrance also rising at $26.5^{\circ}$; the horizontal passage into the Queen's Chamber; the escape passage for the builders; and the two shafts emerging from the Queens Chamber into the masonry at different and very particular inclinations. Figure 109 gives some small idea of the 3-dimensional complexity, with the entrance to the Queen's Chamber nestling between the ramps at the foot of the Grand Gallery.

Examining the implementation of any great building reveals that models were involved. Some models become famous in their own right. There is a model of St Paul's Cathedral in London, for instance, which was used by the architect Sir Christopher Wren to gain approval for his design, but which bears little resemblance to the final structure. In that case, the model was used as a political device, to gain approval from sceptical councillors. Generally, models are used to sell ideas to prospective clients, to work out how complex parts are meant to fit together or
operate, or simply to visualize what the whole system will look like - the Future Vision.

In the case of the Great Pyramid, there might have been several models, some to see how particular parts might work, others to visualize the whole, and perhaps one to manage the project. Today's project managers would look with trepidation on the task of managing the Great Pyramid project, and with good reason. Some 2.3 million blocks were to be laid, each weighing on average 2.5 tonnes, but some as much as 75 tonnes.

One way that the ancient Egyptians might have managed such a complex and vast project was through the use of a scale model*. The process would work as follows:

1. Build a scale model of the complete pyramid from scaled bricks,
2. Shape and mark each model brick with its position, orientation, material, etc., as the pyramid model is built up

3. Demonstrate to Pharaoh, seek his approval
4. Disassemble the model and store the model bricks course-by-course, tier-bytier
5. When ready, start the construction project at ground level using the bricks from the first course as tokens to instruct the quarry master
6. As stones are produced from the quarry, reassemble the model brick-by-brick, course-by-course, in step with real pyramid construction
7. Pay the builders and the quarrymen using the reassembled bricks as a tally of achievement
8. Use the model to brief Pharaoh on design, progress, problems and ideas
[^66]
## Model Building 26.5 ${ }^{\circ}$ Shafts



Figure 110. Building a Model of a $26.5^{\circ}$ Passage
A simple model-building kit, Figure 110, might have been used to represent a shaft or passage inclined at $26.5^{\circ}$, as are the passages in the Great Pyramid.
A back panel consists of two squares, butted end on to form a rectangle with diagonal at $26.5^{\circ}$ ( 14 seked.) A wooden shaft is fixed against the back panel, on the diagonal line. Bricks are then fitted, made from bone or, if they are to be shaped, from baked Nile mud. Once fitted, the bricks are marked with their position in $x, y$ and possibly $z-$ axes, their dimensions and probably some suitable prayers or incantations. Building such simple models would be an exercise for student architects and builders.

## Sliding Plug Stones

Some models, particularly those representing function and activity, are best made full scale and from representative materials.
Figure 111 shows a full-scale model of sliding plug stones. Each of the pictures shows a sectioned passage, sloped at $26.5^{\circ}$, made from limestone of the sort that would be proposed for use in the real pyramid. (In practice, the complete passage would have been used: the sectioned passage is shown for illustration only.) Top left are four granite plug stones, shown in their vertical position, notionally as stored in the Grand Gallery. Top right, one of the stones has been entered into the passage and has slid a third of the way down; the other stones are shown as being manhandled into position to be entered in their turn.

Bottom left, the first plug stone has reached the bottom and has blocked the exit. Bottom right, two more stones have struck the first, which has been neither shattered, nor moved by the impacts; the fourth and final stone is about to strike...

This model raises many more questions than it answers. How are the stones manhandled into place to be entered into the passage? How many men are involved (they all have to escape once the work is finished.) How well do the stones slide? Is there a chance of a blockage? What can be done to prevent a blockage, which would
be a disaster in the final stages of sealing the pyramid? Can the sliding stones be lubricated, to reduce friction? With what would they be lubricated: water, sand, lime? If they are lubricated, do they then hit the bottom stone too fast and too hard?


Figure 111. Full-scale Model of Sliding Plug Stones
How is the first stone stopped in just the right place and so that following stones do not shunt it out into the lower passage? (In fact, it seems that the builders made the bottom of the passage slightly narrower, so that the first stone jammed in just the right place ${ }^{54}$.)

Do successive stones shatter on impact? And would it matter if they did, since they would be constrained in just the right position anyway?

## Constructing a Grand Gallery

The Grand Gallery of the Great Pyramid is deservedly considered the masterpiece of the mason's art of any age. Natural awe at the magnificence of its execution leads inexorably to curiosity about how it might have been constructed.
The Grand Gallery is corbelled, and there are well-founded rules for constructing corbelled roofs. Figure 112 shows the situation. A number of stones is laid one on another, but slightly displaced laterally. A second column, mirroring the first, would be placed alongside so that the two would create an arch. But there is a potential problem. The very next stone laid on the pile will cause the pile to topple.

The reason concerns the centre of gravity of the pile. Each successive stone moves the centre of gravity of the pile to the right. Adding the next stone would move the centre of gravity to the right of the edge of the bottom stone, making the whole stack topple.

The purpose of corbelling in pyramids is to create a void (chamber) between the corbelled walls, diverting the enormous down thrust from the masonry above, and effectively sharing it between the corbelled walls.


Figure 112. Rules for Corbelling
These walls can only transmit that thrust effectively if they keep well away from the limiting condition shown in the figure. If they are even close to the toppling point, they concentrate the down thrust along the inner edges of the corbelling masonry, risking cracking and crumbling.
Figure 113 shows the situation inside the Grand Gallery, looking upwards from the bottom towards the King's Chamber at the top. The corbels are made up from massive stones in some 9 layers (depending upon what counts as a layer), each inset by only a few centimetres from the one below.
Provided the corbel stones have adequate width, in the terms illustrated in Figure 112, then there is no risk of toppling or stress failure. The proof, of course, has been in the eating: the Grand Gallery was built with safety margins to stand the test of time.


Figure 113. Grand Gallery showing Corbels

We can infer from the corbelling rules and from the picture that the hidden depth of the corbels stones cannot be slight. The stone must be at least square in cross-section. They may need to be oblong in cross section, however, since they presumably have to key into the surrounding courses of masonry that the Grand Gallery penetrates.
How this keying is achieved is one of the many puzzles surrounding the construction of the Grand Gallery.


Figure 114. Grand Gallery Virtual Model

Figure 114 shows a frame from a virtual reality flythrough simulation of the Grand Gallery. The corbelling can be seen both inside and outside. The outside is, of course, supposition, since nobody knows quite how the exterior is formed and finished.
Exterior courses of stone are horizontal, while the corbels rise at $26.5^{\circ}$. It might be possible to leave a space between the horizontal stones and the corbels, but that would create problems as the courses of stone rose, since the gap would enlarge as the
courses rose up the outside of the Grand Gallery.
The alternative is to shape and fit each external course of stone in 3 dimensions to the exterior of the corbel. In such an event, no 2 stones would be alike. This would be an awesome task, but it seems to be the most likely, even inevitable, consequence of incorporating a tilted, corbelled gallery. Without such shaping and fitting, the down thrust from the masonry above the Grand Gallery would not be spread evenly, with risks to the lower structure, including the Queen's Chamber and its two shafts.
Figure 115 starts a sequence of virtual reality frames, showing how the building of the Grand Gallery might have been approached. Nobody knows how it was really done, of course, but the act of building the dynamic simulation reveals that some features must be done before or after others, and that some of the features visible today can be explained as deriving from the construction techniques.
Figure 115 shows the Great


Figure 115. Starting to Build the Grand Gallery Pyramid at the level of construction where the entrance to the Queen's Chamber, the start of The Grand Gallery, and the rising passage from the entrance, just visible at left, all come together. Although the Queen's Chamber, at right, would not have been constructed in full at this stage, it is shown in situ to indicate how the Chamber and its entrance


Figure 116. Grand Gallery VR Construction Sequence
passage relate to other features. The first stage of a $26.5^{\circ}$ ramp is shown on the far side of the entrance passage.

Figure 116 shows four frames taken from the virtual reality sequence for building the Grand Gallery. Top left, both ramps have been built, and the first of the large corbel stone blocks have been set in place. Note that the masonry has been built up beside the two ramps; this would be necessary to raise the corbels sufficiently so that they could be swivelled on to the ramps. The gap between the two ramps contains the passage to the Queen's Chamber, off picture at right.
Top right shows two courses in place and further build up of surrounding masonry. Each course of corbels rises at $26.5^{\circ}$ on its ramp. The higher stones would be raised up on higher masonry, swivelled into place and then slid gently down to butt against the lower corbel blocks. The lowest column of corbels would require buffers, not shown, to prevent them sliding off their respective ramps.
Bottom left is a front view showing the Queen's Chamber passage leading between the two ramps into the Queen's Chamber at the back. Blue sky is visible beyond. Compare with Figure 113, where the construction ramps are also clearly visible, with slots cut in them at intervals, twenty-eight on each side. Bottom right shows the masonry being built up and the roof being added to create a horizontal tier above the cap corbel.
Several important details were missed out of the virtual reality sequence. The absence of buffer stones has been mentioned. The shafts from the north and south walls of the Queen's Chamber have been omitted for clarity. Remembering the curve of the northseeking shaft underneath and around the side of the Grand Gallery, their inclusion would have presented a significant challenge. Additionally, the plug stones, which
would eventually seal the entrance passage, had to be fitted into the Grand Gallery at some stage before it was closed in.
Once the Pharaoh died and had been mummified, his ceremonial interment may have involved the royal mummy, suitably clad and encased, being taken up the Grand Gallery to be installed in the red granite sarcophagus. (The sarcophagus, being slightly wider than the entrance to the King's Chamber, would have been installed during building, just after the Grand Gallery had been enclosed.) This raises an issue: how could the Pharaoh's mummy and the cortege pass up the Grand Gallery if it was full of plug stones?

Ludwig Borchardt (1863-1938), the noted German Egyptologist, decided that the 28 slots cut each side into the ramps were made to hold wooden uprights, supporting a wooden platform. He calculated that such a framework would be able to hold the plug stones above the cortege. Afterwards, the stones would be let down on to the ramps, the framework presumably removed down the entrance passage, and the plug stones slid down the passage. Finally, the masons would have escaped through the passage designed for that purpose.

The way in which the ancient Egyptians designed and constructed their pyramids was truly remarkable. There is evidently much still to understand about their methods, and their approach some 4,500 years ago to what we call systems engineering today stands as a spectacular, challenging, example.

## Chapter 7. Naissance and Renaissance

## Past Pyramids

To suppose that the pyramid shape occurred in one place and that people then carried that idea to the four corners of the earth would be fanciful. The general pyramid shape has arisen so often, in so many different parts of the world, that it is more reasonable to suppose that the general shape of the pyramid occurs naturally and readily in the minds of monument designers and builders.

After all, if the only materials to hand for building were stone, earth and sand, then the only stable, large-scale monument that could be built was a pile or a mound. Then, the choice of shapes would be limited: cubes were out, since the sides fell away; cones were possible, and so were pyramids, but in either case the sides could not afford to be too steep or material would slide off. Stepped structures built in layers would work, too. To create a really tall structure, it was probably simpler and more practical to work in stages, building a modest pile to start and then building more on top ${ }^{*}$, and so on.

The pyramid shape has become a global icon or motif. It represents more in the minds of people than the structure of a burial mound; perhaps it always has. Even the ancient Egyptians were not immune, Figure 117; ceremonial dress often included a pyramid-shaped "skirt" protruding out in front of the wearer. The figure shows a sculpture from Cairo Museum, and there is an identical one in the $5^{\text {th }}$ dynasty Tomb of $\mathrm{Ti}, \sim 2400 \mathrm{BC}$. The same style of skirt is to be found on statues of Tutankhamun, 1334-1325BC, a millennium later. The precise implications of this unique skirt shape are uncertain, but it would be perverse to suggest that it was not analogous to a pyramid.
The pyramid appears on US currency, as a motif of longevity and dependability. Organizations, particularly military ones, organize their hierarchies along pyramid lines, with one leader having, say, 3 subordinates each of whom has 3 subordinates, and so on. When drawn out on paper, such organizations look distinctly "pyramid-shaped." Pyramid selling employs the motif to describe the practice in which one person sells to, say, five others, each of whom sells to another five, and so on; again, if the connective structure were drawn out on paper, it would look distinctly


Figure 117. Unassigned Old Kingdom Statue with "Pyramid" Skirt, Cairo Museum pyramid-shaped.

[^67]Pyramids appear today on top of square cross-section structures; obelisks, skyscrapers, bungalows and even church towers. Pyramid-shaped buildings are used commercially as elaborate greenhouses, gymnasia and corporate headquarters ${ }^{55}$. It seems the natural thing to do. But then, Jung would probably have suggested that the pyramid is an archetypal shape, and so its occurrence in our everyday thoughts and actions is just that: natural

Geometrically, there are two types of pyramid, those with a square base and those with a triangular base. Euclidian pyramids, then, are characterized by straight edges and either four or five plane faces.

The pyramids of Giza fit neatly with the geometrical definition: the Bent Pyramid of Dahshur does not. Many other structures around the world might best be described as only "pyramidal" in that they have rounded corners, flat tops, curved faces, layers, etc. There is a propensity in the contemporary world to describe a shape as a "pyramid", perhaps to make it more interesting or mysterious. This is particularly true of South American and Central American "pyramids" which, while being substantial structures in their own right, may be only vaguely pyramidal. It is, perhaps, doing them a disservice to call them pyramids: many have their very own, distinctive shapes; moreover their purposes appear to be quite different from those of the ancient Egyptians.

Ever since humans buried their dead and sought to safeguard the remains, the burial spot has been marked and covered with stones. Cairns are to be found the world over, and some of them are distinctly pyramidal ${ }^{*}$, though not always on the same scale as the Pyramids of Giza. However some mounds are both very ancient and most impressively large.

## Silbury Hill

To suggest that the first pyramid structure appeared in England may seem bizarre. Silbury Hill, in Wiltshire, England, was built at the same time as, and using techniques not dissimilar to those of Djoser's Stepped Pyramid. In one particular, however, it was quite different: it was conical; that is, it used the one archetypal shape that could be considered as alternative to the pyramid.


Figure 118. The Construction of Silbury Hill, c.2660BC

[^68]Silbury Hill is the tallest man-made mound in Europe and, until the pyramids of ancient Egypt were built, may have been, briefly, the tallest in the world. It is located just south of Avebury village, the site off a major megalithic monument.
Silbury Hill is approximately 130 feet ( 40 m .) high, with a flat top. It has a circular base with a circumference of 1640 feet and is built up over 12 million cubic feet ( 339,600 cubic $m$.) of chalk and earth, covering 5 acres (2 ha). Peculiarly, Silbury Hill occupies a low-lying site and does not protrude significantly above the horizon, except when viewed from West Kennet Long Barrow, a Neolithic burial site on the brow of a nearby hill.
Silbury Hill was built in three stages, the first begun around 2,660BC The last phase comprised the building of six concentric steps or terraces of chalk, left-hand image of Figure 118, which were eventually covered with chalk rubble, flints, gravel and finally soil, to form a cone-shaped mound, right hand image of Figure 118. Each of the six steps was concealed within the overall profile of the mound, except the last one at the top, which was left as a terrace or ledge about 17 feet ( 5 m .) below the summit. This terrace is clearly visible today on the eastern side of the mound, Figure 119 , but is less distinct from the west. The chalk was largely extracted from a 30 -foot deep quarry ditch beside the mound, and the excavated area around the mound formed a moat: it seems reasonable to suppose that this was intentional.
Recently, archaeologists found the tips of antlers used by the builders during the latter parts of construction, and the Oxford University radiocarbon unit dated these to about 2490-2340BC, with $95 \%$ certainty of accuracy.


Figure 119. Silbury Hill Today from the Southeast
The building programme therefore extended over some 250 years. Over that span of time, the builders were evidently able to command a large workforce, suggesting
sophisticated organization and control. Unfortunately, they left no graphic or pictorial evidence.

The purpose of Silbury Hill is unknown. There is no burial site or cavity within the mound, and it is not known to cover any underground chamber entrance. Any supposed association with Avebury or West Kennet Long Barrow is speculative.
Like Djoser's Stepped Pyramid, Silbury was built in three phases. Remarkably, both monuments were started at just about the same time. Both monuments are stepped. Both comprise six steps. In each case, ashlars were used to form an outer edge to the steps, with inner stones packed inside this "skin". Both monuments make use of stone material immediately to hand.
The white chalk from which Silbury Hill's rings were formed would have been reflected in the water of the surrounding moat, creating a magnificent view from the surrounding hills. It may be that, like Djoser's Pyramid, the creation of spectacle was one of the driving forces behind construction. At night, Figure 120, the rings would show up white in the moonlight, while the moat would reflect starlight and moonlight, the whole presenting an numinous, ethereal view.


Figure 120. Silbury by Moonlight - Simulation
With a slope in the region of 35 -degrees (10-seked), Silbury Hill was much shallower than Djoser's Stepped Pyramid with its 5-1/2 seked slope. Why the white chalk steps were filled in at some stage to create the truncated cone is unknown. However, in so doing, the builders unconsciously emulated the pattern of the Egyptian pyramids, with their smooth exteriors concealing a stepped structure.

The similarities between Silbury Hill and the Egyptian Pyramids are not suggestive of any cultural exchange or migration; the use of the alternative archetypal shape is too singular to suggest that. Instead, the progression from small mound, to larger mound, to stepped structure, to smooth structure, emerges as an archetypal developmental
process. That such parallel processes should occur at the same time in independent environments ${ }^{*}$ recalls Jung's proposition of the human collective unconscious ${ }^{56}$.

## Ancient Pyramidal Structures

There appears to be a large number of pyramids in China, of which little is known in the western world. The "Great White Pyramid," in the Qin Ling Shan Mountains, about 100 km southwest of the city of Xi'an, is reported to be some 300 metres tall Tentative estimates of age are some 4,500 years; if correct, this would time the construction as parallel with that of the Egyptian Pyramids of the Old Kingdom ${ }^{\dagger}$.


Figure 121. The Great Temple at Chichen-Itza, ~AD1000. Simulation

The many pyramidal monuments in the Americas have recently been "joined" by six large, mound-shaped ceremonial platforms at Caral in Peru dating from 4,000 years ago. These mounds precede all others classified so far in the Americas, and they appear at this early stage to bear shapes characteristic of later mounds, suggesting that the later mounds recalled the patterns of Caral ${ }^{8}$.
The site, located in the Supe Valley about 14 miles from the Pacific Ocean, becomes the oldest-known urban centre in the Americas. Caral, and several sites nearby, may be of comparable age, making them all, curiously, similar in date to the Old Kingdom pyramids of Egypt.

[^69]There have in the past been many suggestions that the pyramidal mounds and temples of the Aztecs and Incas were inspired, through some cultural link, by those of the Old Kingdom in Egypt. That supposed connection has always been unlikely, but the recent classification of Caral confounds the suggestion altogether. The pyramidal mounds and temples of the Americas emerge from an environment and culture that is almost as old as that of ancient Egypt, but which was quite independent and substantive in its own right.

The pyramidal form in the Americas elaborated over time. The Great Temple at Chichen-Itza in Mexico, Figure 121, built around AD1000, has a truncated stepped pyramid foundation with relatively very large stairways and superstructure. To describe the resulting structure as a pyramid falls short of the mark.


Figure 122. Middle Kingdom Pyramid of Amenemhat III, 1842-1797BC

Egypt is famous for its Old Kingdom pyramids, but the practice of building pyramids and the pyramid motif survives in Egypt to this day. Pyramids were built during the Middle Kingdom, Figure 122, but the construction was poor and they did not last - at least, not in pristine condition.


Figure 123. Mountain Overlooking the Valley of the Kings.

Pyramids took on a different form in the New Kingdom. The mountain overlooking the Valley of the Kings was viewed as a pyramid "presiding" over the valley, Figure 123 , and was even worshiped as a god in its own right. More generally, however, they were small and more pointed, although they were used in much the same way, to mark the entrance to a tomb.


Figure 124. Frieze from the Egyptian Gallery, British Museum
The Book of the Dead ${ }^{57}$ shows a small pyramid, illustrated in Figure 124, as a backdrop to the Opening of the Mouth Ceremony. The significance of this small pyramid becomes evident in Figure 125, which shows a scene on the hillside of the Valley of the Workers at Deir el-Medina, near Luxor. At top left is a small pyramid, very similar to that in Figure 124, built by one of the valley workers to mark the entrance to his tomb.


Figure 125. Pyramid Tomb Entrance, Valley of the Workers, New Kingdom

The workers in the Valley of the Kings had the means and the opportunity to build their own tombs. Unconstrained by the artistic canon, which limited their work in the Valley when working on royal tombs, they could give vent to their creative urges when working for their personal eternity. That some of them chose to create pyramids is indicative of the enduring influence of the pyramid form in their minds.

TThere was a late outburst of pyramid building in the Sudan by the people of Kush. The $25^{\text {th }}$ Dynasty of Ancient Egypt, the so-called Nubian/Kushite Dynasty, lasted from 747 - 656BC. The Nubian pharaohs revived royal burials under pyramids, at 3 sites in particular: El Kurru, Nuri and Meroe. Altogether, the Nubians built twice as many pyramids, 180, over a period of 1000 years, than the Egyptians had during their earlier 1000-year period. Nubian pyramids were shaped more like the new Kingdom pyramid shown in Figure 125, however.


Figure 126. The Mappa Mundi, by Richard de Bello, Hereford Cathedral, England, ~AD1290

Following their intense interest in mathematics, the ancient Greeks categorized a unique set of five shapes, the so-called Platonic Solids, including the tetrahedron* All Platonic solids satisfy the following rules: they are convex (ball-shaped), they have equivalent faces and sides that are all the same length, and each face is a single kind of regular polygon. The tetrahedron is a four-sided figure with each side being an identical triangle: the base is therefore triangular, rather than square like Egyptian pyramids.
Despite their search for ascendant mathematical purity, the Greeks did build rectilinear pyramids; some sixteen or more are known, all very small and many dilapidated. Those at Hellenikon and Ligourion are the best known, but there has been little research, and the purpose of the Greek pyramids remains obscure. Dating of the mortar of the Hellenikon pyramid reportedly dated it to 2720BC, earlier than Djoser's Step Pyramid.

## Middle Ages and Renaissance Pyramids

The pyramid motif was evident during the Middle Ages and the Renaissance. The Mappa Mundi in Hereford Cathedral, Figure 126, the largest, most detailed and most perfectly preserved medieval map in the world, shows the Sphinx with the Giza Pyramids as Joseph's Barns or granaries - as they were believed to be at the time.
The map is believed to be based on a $4^{\text {th }}$ Century AD Roman map. The perimeter represents the Earth, with East at the top. Right at the top is the island of Paradise. Jerusalem, the Holy City, is dead centre. The upper half represents Asia, and Palestine is disproportionately large both to represent its importance and to permit mythical and allegorical figures to be incorporated. Above the central point, the Tower of Babel is conspicuously marked. The pyramids and the Sphinx are to the right of Jerusalem; they stand on the bank of the Nile, which is shown with a Delta region.


Figure 127. Kircher's Pyramids

[^70]Egyptian pyramids were depicted by Sébastien Munster (1488 - 1552) in his famous Cosmography, as very tall, thin structures, and later by Athanasius Kircher (1602-1680) in Edipus Aegyptiacus. Kircher showed the Great Pyramid, Figure 127,with a cutaway section showing scattered structures inside at ground level; these may have represented sarcophagi. He also showed the stepped pyramid, although with a pyramidion, and the Bent Pyramid.

One of the most unexpected appearances of the pyramids in renaissance art must be due to Lorenzo Ghiberti of Florence. His most


Figure 128. Noah's Ark*, Gates of Paradise, Lorenzo Ghiberti, 1378-1455, Museo dell'Opera del Duomo, Florence famous work must surely be the bronze doors of the Cathedral Baptistry, and in particular the east doors, called the Gates of Paradise. One of the panels, Figure 128, concerns Noah's Ark and, unexpectedly, the ark is represented as a large pyramid. The shape of the pyramid coincides with those of the $4^{\text {th }}$ Dynasty at Giza.


Chigi Chapel, Santa Maria del Popolo, Rome. Designed by Raphael, finished by Giovanni Lorenzo Bernini, 1598-1680


Tomb of Archduchess Maria Christina of Austria by Antonio Canova, 1798-1805, Church of the Augustinians, Vienna

Figure 129. Renaissance Pyramid Tombs
Pyramid images from Greek and, particularly, Roman, culture re-emerged throughout the European Renaissance, particularly in the tomb designs of the rich and famous, Figure 129. The figure shows the Raphael's Chigi Chapel pyramid in Rome at left, and Canova's pyramid forming part of the tomb for the Archduchess Maria Christina of Austria at right. In this latter, particularly, there is a strong echo of the New

Kingdom tomb entrance, Figure 125. In both examples, the pyramids are more pointed than those of the Old Kingdom and are much more in line with New Kingdom pyramid slopes.


Figure 130. Memorial of the Unknown Soldier in Nasr City, Cairo

## Contemporary Pyramids

The pyramid motif is alive in Egypt today. The impressive Memorial of the Unknown Soldier, Figure 130, is not a pyramid as such, but is remarkably inspired by the pyramid shape. It is possible to "see" many different pyramids within it.


Figure 131. Egyptian Street Art.
Unsurprisingly, the Giza Pyramids pervade modern Egypt. Figure 131 shows a picture sewn by a street artist for tourists from pieces of cloth. Peculiarly, the three pyramids have doors, recalling the ideas of Joseph's granaries. The picture is not meant to
represent an old view however; the donkey rider wears Arab headgear, and there were no camels ${ }^{*}$ in Egypt during the pyramid age.

Perhaps the most remarkable contemporary pyramids are those of the Chinese-born American architect, Ieoh Ming Pei, Figure 132


Figure 132. Pei's Pyramids at the Louvre
The figure shows the well-known glass pyramids covering the entrance to the Louvre Museum in Paris, a daring juxtaposition of the old and the new styles of building. Or, is it so very daring? Perhaps the readiness with which the new style has been accepted is because it appears in the pyramid form, and so appeals to some archetype within us.


[^71]It is possible to overlook the smaller of the Pei pyramids; as can be seen, Figure 133, their construction differs


Figure 134. Pei Pyramid from Underneath, showing Construction
The construction of the large pyramid is clearly shown from underneath, Figure 134.
The structure is held in tension by a mesh of translucent struts forming overlapping hexagons, invisible from the outside and unobtrusive from the inside. It is this mesh, a neat variation on the geodesic dome structure, which maintains the planar surface of each face. The whole creates an underground space that could otherwise be claustrophobic, but which has a feeling of spaciousness and light: notice the sun shining on members of the public as they move freely in the large, airy underground space created by the Pei Pyramid.
Egyptian Obelisks* appear in Rome, London and Paris, and there is an obelisk structure in Washington DC, although it is not Egyptian in origin. The obelisk in Paris stood originally outside the Temple at Luxor, one of a pair erected by Rameses II; the other still stands there. The pyramidion at the top is covered with gold, Figure 135; originally it may have been covered with electrum.


Figure 135. Obelisk, Place de la Concorde, Paris

[^72]

Figure 136. Glass Pyramid Atop Canary Wharf, London
The pyramidion is ubiquitous as a way of finishing off a square cross-section tower or erection. Some pyramidions become recognized as pyramids in their own right. The top of the tower in Canary Wharf, in London's Dockland, Figure 136, is a typical example. Lit up at night, the glass pyramid can be seen for tens of miles at ground level, and from hundreds of miles when airborne.


St. Thomas Becket Church Tower, $\mathbf{1 3}^{\text {th }}$ Century, Salisbury, England


Supermarket Clock Tower, $20^{\text {th }}$ Century, Salisbury, England

Figure 137. Finishing Off Square Towers, England
At a more mundane level, pyramidions abound, it seems, wherever architects can give vent to their creativity. Towers are frequently capped with pyramids, right hand of Figure 137, which shows a modern supermarket clock tower capped with a pyramidion. This is not as obvious and natural as it might seem, however; a common practice is to have a ridged, rather than pointed, roof. In previous ages, the practice was quite different again: the left hand photograph is of the Church of St. Thomas Becket, separated from the supermarket by some 150 m and seven centuries; then, the
solution was a hexagonally based turret to finish off a square tower. Fashions may change, but the pyramid shape keeps returning.

## Future Pyramids?

As man's present population explosion continues, and as we spread concrete inexorably across our landscape, it will eventually become both necessary and preferable to make use of the $2 / 3$ rds of the globe that are covered by water. The signs are already there, with round-the-world cruises, underwater hotels and sea-going oilrigs that submerge to evade bad weather.
Figure 138 shows a possible future seascape, in which a habitat group rises to the surface after a storm. The submersible habitat comprises 5 spheres, including the one from which the view is seen. At the left is a farm sphere, with fields, trees, insects, biosphere and fresh water from reverse osmosis, using the weight of the sphere as a driving force. Second from left is a living sphere, with houses, halls, theatres, hospitals, arenas, etc., indeed all the features of a robust modern/future society of perhaps $40-50,000$ people. The structure inside each sphere is pyramidal to ensure stability and receive most light.

Far right is a seawater extraction plant, recycling and manufacturing facility. The


Figure 138. Pyramids of the Future?
habitat set is entirely self sufficient, extracting metals and minerals from the seawater, synthesizing drugs, materials and structures. It also recycles all waste from the habitat set. The manufacturing plant is sufficient to re-create any or all of the spheres from within its own capabilities, without any support from outside the habitat set.

Finally, second from the right is a habitat devoted to creativity, activity and industry. The whole habitat set is part of a number of global supply chains, which receive, supply, and add value to goods. The habitat set is not just a holiday camp - although it can be a holiday resort, too. It is part of a global community of such habitat sets, creating and developing new technologies, new ideas and new ways of living. But, using as its principal structural base, an idea that is as old, it seems, as man - the pyramid.

## Index

## A

Abu Roash, 46, 48, 50, 55, 62
Abusir, 10, 30, 55, 58, 61, 96
Abydos, 9, 22, 24, 26, 31, 53, 74, 83
afterlife, $14,17,19,24,26,39,48,84,95,104$, 105, 156
akh, 16
Akhet Khufu, v, 39, 56, 155
Alnilam, 73
Alnitak, x, 53, 54, 73, 78, 79, 88, 91, 92, 96, 98, 104
Amenemhat III, xi, 172
antechamber, 37, 48, 134
Anubis, 14, 15, 84
appliances, 114, 137, 138
Archaic Period, 9, 22
archetypal shape, 168,170
archetypes, 12, 13, 95
architecture, $1,31,34,53,62,88,89,127,157$
ashlars, $33,48,63,143,170$
Aswan, ix, x, 3, 5, 18, 30, 49, 53, 112, 157
Aswan Dam, 5, 30
Athanasius Kircher, 176
Atum, 19, 74
Avebury, 169, 170

## B

ba, 16, 24, 95, 156
Basal Metabolic Rate, 125
Bauval, 53, 54, 91, 187, 189
beer, 21, 77, 117, 119, 120
Behaviour Diagram, vii, x, 154, 155
belief system, 16, 17, 19
beliefs, xiv, $8,9,16,17,19,95$
Bent Pyramid, ix, 36, 37, 38, 39, 47, 55, 58, 59, 60, 134, 146, 168, 176
Bes, 19
Betelgeuse, 73, 74
Black Land, 2
boats, 2, 21, 24, 25, 76, 94, 114, 120, 146, 154
Book of the Dead, 15, 173, 189
bread, 11, 21, 77, 117, 118, 120
British Museum, x, xi, 60, 83, 119, 130, 173, 187, 189
builders, xiv, $10,21,42,43,60,63,66,69,90$, $99,112,114,129,140,145,148,158,159$, 160, 161, 167, 169, 170
Bull of the Sky, vi, x, 86, 87, 88
bulls, 86
bureaucracy, 10
burial chamber, $31,37,38,39,48,50,51,52$, 53, 63, 65, 127, 133, 134

## C

Cairo Museum, ix, x, 2, 8, 23, 32, 33, 51, 118, 167
Canary Wharf, 180
canon, 59, 174
capstones, 44, 134
Caral, 171, 172
cardinal points, $32,65,79,82$
casing stones, $37,40,47,104,113,135,136$, 139
cathedral, 36, 157
causeway, $25,30,39,96,111,114,115,116$, 141, 150
celestial pole, 30, 71, 72, 73, 76, 79, 86, 97, 104
central shaft construction, 52
chambers, $21,31,36,37,38,39,41,42,43$, $44,45,48,51,52,53,65,66,67,68,71,90$, $91,93,105,109,115,127,133,134,136$, 140, 146, 156, 157, 159
Cheops, 12, 115
Chichen-Itza, xi, 171, 172
Chigi Chapel, 176
circumpolar stars, 24, 30, 32, 39, 84
cities of the dead, 17
coefficient of friction, 111, 116, 124
collective unconscious, 12, 13, 171
Constant Manpower, 140, 141
constellations, 53, 73, 84, 85
contrivances, 114, 138
copper, 91, 128, 129, 130, 131
corbelled, 36, 37, 39, 42, 96, 161, 163; corbel, 37; corbels, 38
corvée, $9,11,12,21,111,112,139,140,143$, 144, 154
creation mounds, 65
culture, v , xiv, $1,4,9,20,65,82,83,87,171$, 172, 176

D
Dahshur, ix, 30, 36, 38, 47, 54, 55, 64, 117, 134, 168
dawn rising, 77, 78, 92, 103
dawn transit, 78, 91, 92, 93, 96, 97, 103, 131
deities, 18, 24, 156
Delta, 3, 4, 7, 120, 175
Descartes, 151
desert, 2, 7, 14, 15, 57, 141, 142, 145
Divine King, 14
Djedefre, v, 23, 46, 48, 50, 51, 52, 55, 61, 62
Djedkare-Isesi, 60
Djoser, ix, 22, 23, 30, 31, 32, 33, 34, 39, 46, $50,51,52,54,55,61,64,105,168,170$, 175
down thrust, $44,45,66,90,127,161,162,163$
duality, $14,15,65,90$

## E

electrum, 136, 179
Elephantine, 3, 18, 30, 53
emergent properties, 152, 156
emmer, 77, 117, 120
enduring Bull, 83
enthusiasm, 21
escape tunnel, 43
Ethiopia, 1

## F

farmers, 2, 5, 9, 21, 112
Field of Rushes, 75, 87, 88
fish, 20, 21, 118, 120
fixed annual workforce, 139
fractions, 59, 60, 61, 79, 82, 90
Freud, 20
Future Vision, 152, 153, 159

## G

Gantenbrink, 43, 91, 92, 99, 105
Gates of Paradise, 176
Geb, 19, 76
Ggantija, ix, 66, 67
Giza Plateau, 12, 60
gods, $8,14,16,17,18,19,23,76,83,86$
Governors of Kush, 18
Grand Gallery, vii, x, 37, 42, 43, 92, 93, 97, $104,105,158,160,161,162,163,164,165$
Great Mooring-post, 76, 104
Great Pyramid, v, vi, vii, ix, x, xiii, 12, 23, 25, 29, 30, 34, 37, 39, 40, 41, 42, 43, 44, 45, 47, $51,52,56,61,62,65,71,88,91,94,96,97$, $98,99,100,103,105,109,111,112,115$, $116,117,118,119,123,135,137,138,139$, $140,141,142,143,144,147,150,152,153$, $154,155,158,159,160,161,163,176$
Great Pyramid of Khufu, vii, 12, 37, 117, 152
Green Man, 15

## H

Hagar Q'im, 67, 68
Hall of Judgement, 15, 16
Hathor, 19, 50, 84, 85
heliacal, 4, 77, 96, 156
Heliopolis, 9, 19, 46
Hemon, xiii, 34, 45, 103, 152, 155
Heqaib, 18
Herodotus, 1, 3, 7, 12, 25, 109, 114, 120, 137, $138,145,146,148,150,154,189$
horizon, $4,33,49,55,56,58,71,73,76,77$, $78,80,97,101,156,169$
Horus, ix, 8, 15, 19, 22, 23, 24, 51, 72, 74, 78, 84, 92, 97, 156, 157
human gestation period, 92

## I

idealism, 21
Imhotep, 31, 60
Imperishable Stars, v, ix, 49, 72, 73, 76, 78, 83, 104
individuation, 20, 21
infrastructure, 22, 27
innocence, 21
interment, 24, 25, 37, 45, 157, 165

Inundation, v , ix, xiv, $1,2,3,4,5,6,9,11,16$, $21,23,25,56,57,77,78,92,93,94,112$, $117,143,144,157$
irrigation, 1, 2, 4, 11
Isis, 14, 15, 19, 54, 74, 79, 85, 87, 88, 91, 92, 94, 97, 103

## J

job trees, 11
Joseph's Barns, 175
Jung, 12, 13, 16, 20, 95, 168, 171, 189
justice, 10, 15

## K

ka, iii, 16, 24, 25, 31, 37, 39, 45, 64, 76, 86, $94,95,96,97,104,105,110,115,130,141$, 154, 155
Khaba, 23, 34
Khafre, v, ix, 30, 40, 46, 47, 48, 50, 51, 52, 53, $58,59,60,89,90,96$
Khasekhemwy, ix, 22, 23
Khufu, v, vi, vii, ix, x, xiii, 12, 21, 23, 24, 29, $30,34,39,40,41,42,46,47,48,49,50,51$, $52,53,56,58,59,60,61,62,63,88,89,92$, $93,94,95,96,97,99,100,101,102,103$, $109,110,111,113,114,115,116,123,124$, $128,129,133,140,141,146,152,155,156$, 159, 178
King as Shepherd, 14
King's Chamber, vi, 42, 43, 44, 45, 90, 91, 92, 97, 102, 103, 104, 162, 165
Kocab, x, 78, 79, 80, 86, 88, 92, 94, 96, 97, 98, 155

## L

Lamarckism, 13
logistics, 21, 22, 110
Lorenzo Ghiberti of Florence, 176
Lower Egypt, ix, 1, 5, 7, 8, 9, 30

## M

Ma'at, 15
Mandjet, 94
Manetho, 9
manning strategies, 140, 144
Mappa Mundi, xi, 174, 175
mastabas, $9,17,31,39,55$
Meidum, ix, 30, 35, 36, 37, 39, 48, 55, 61, 62, 64, 155
Memorial of the Unknown Soldier, 177
Memphis, ix, 1, 9, 10, 23, 27, 30, 32, 35, 55, 56, 57, 58, 120
Menes, 9
Menkaure, v, ix, 21, 30, 48, 49, 50, 51, 53, 57, 58, 150
Mesektet, 94
Milky Way, 53, 75, 87, 88
Mintaka, 73
Mizar, x, 79, 80
Mnajdra, ix, 68
models, xiii, xiv, 136, 158, 159, 160
Morning Star, 76, 77
mortuary temple, $30,35,39,40,49,57,65,96$, 97, 114
Mound of Creation, ix, 66, 135

## $N$

Narmer, ix, 8, 9, 22
nascent, 20, 21, 32, 34
necropolis, $9,18,19,30,53,54,55,56$
Neferirkare, 9,58
Nephthys, 19
New Year, 4
Nile, v, ix, xiv, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, $14,15,16,18,19,24,25,30,39,41,53,56$, $57,58,65,94,112,114,115,120,153,156$, 160, 175
Nilometer, ix, 3
Niuserre, 58, 61
nomarchs, $9,10,18,26$
nomes, $9,10,11,26$
Nubian pyramids, 174
Nut, 19

## O

obelisk, 179
obelisks, 168
Old Kingdom, ix, x, 2, 5, 9, 10, 12, 20, 23, 24, $25,26,27,30,35,51,53,56,61,62,65,71$, $74,77,79,81,82,86,88,89,105,109,127$, $129,167,171,172,177,187$
Opening of the Mouth, 173
organization, $9,11,21,22,34,95,116,170$
Orion, v, ix, 49, 53, 73, 74, 75, 76, 77, 78, 85, 86, 87, 104, 187, 189
Orion's Belt, 49, 53, 73, 78, 87
Osirian legends, 24, 73
Osiris, ix, x, 14, 15, 19, 24, 49, 54, 58, 65, 72, $74,75,76,78,79,85,87,88,91,92,94,96$, 97, 104, 156, 157

## P

packing stones, $37,38,40,47,135,136$
Palermo Stone, 2, 3, 9, 10, 189
papyrus, $10,11,60,61,155$
parallel working, 149, 150
Pei Pyramid, 179
Pei pyramids, 179
Pepi II, 18
phallus, $14,15,74$
Pherkad, 86
pitched roof, $44,45,52,65,89,90$
placenta, 14, 84
Platonic Solids, 175
Plough, 87
plug stone, 43,160
Polaris, 71, 87, 88
Pole Star, v, 71, 76, 87, 92, 101, 103
portcullis, $37,43,48,50,65,91,131,133$, $134,157,158,159$
precession, 71, 79, 91, 97, 98, 156
priesthood, 76, 105
proportional annual workforce, 139
Proportional Manpower, 118, 119, 140
prototype, 37
pyramid builders, xiv, 12, 17, 98, 105, 128
pyramid building, $12,23,27,51,53,62,110$, $115,121,125,128,174$
Pyramid Calculator, vi, $x, 109,110,111,112$, $116,138,139,153$
pyramid complex, 24, 25, 31, 52, 114, 121, $152,153,154,155,156,157$
pyramid construction, xiii, xiv, $2,34,39,63$, 159
pyramid slopes, $59,61,62,88,99,155,177$
Pyramid Texts, v, ix, 18, 19, 52, 53, 63, 71, 72, $73,74,75,76,77,78,83,86,87,92,105$, 134, 189
pyramidion, 127, 135, 176, 179, 180
$\boldsymbol{Q}$
Queen's Chamber, 41, 42, 43, 45, 89, 90, 91, 92, 96, 97, 101, 103, 104, 105, 129, 158, 163
Queen's Pyramids, 39, 49, 57
quoins, 47, 126, 136

## R

Ra, 19, 22, 24
ramp theory, 142, 143, 145, 150
ramps, $45,114,123,136,137,138,141,142$, $144,145,146,150,158,164,165$
Raneferef, 10, 58, 96
rectilinear, 14, 68, 175
Red Land, 2
Red Pyramid, vi, ix, 38, 39, 54, 55, 64, 117, 155
reductionism, 20, 151
Relieving Chambers, ix, 43, 44, 45
Renaissance, vii, xi, 1, 151, 167, 175, 176
resurrection, 15, 19, 24, 56, 63, 77, 93, 95, 103, 105
resurrection machine, 24
Rhind Papyrus, 60, 61, 131
ritual, 25, 39, 45, 47, 88, 92, 96, 104, 155, 157
robbery, 39
rocker methods, 136, 148
rocker sledge, 148
rôle of the pharaoh, 22
ropes, $11,72,128,130,143,148,150$
royal cubit, 3,59

## S

Sah, 74
Sahure, 58
Saqqara, ix, 23, 30, 31, 35, 50, 51, 54, 55, 56, 63, 74, 134
sarcophagus, $24,25,37,39,43,45,50,104$, 133, 165
scale model, 159, 160
scribes, 9,10
Sébastien Munster, 176
security, 64, 152, 159
seked, 59, 60, 61, 88, 100, 103, 155, 160, 170
Sekhemkhet, 23, 34
sequential integers, $88,89,90$
serdab, 32, 39, 105
Seth, ix, 14, 15, 19, 22, 23
shoaling, 20, 21
Shu, 19
Silbury Hill, vii, xi, 168, 169, 170
Sirius, ix, 4, 53, 71, 73, 74, 75, 76, 77, 78, 79,
$85,87,88,91,92,97,98,103$
slave labour, 12
sledge, 142, 143, 146, 148, 150
sledges, 111, 116, 142, 144, 146, 148, 149
Snefru, v, ix, 23, 34, 35, 36, 38, 39, 41, 43, 53, $54,58,59,60,61,62,146,151,155$
society, $1,10,12,15,17,20,21,26,32,62$, 181
Sothis. See Sirius
St. Thomas Becket, 180
standard formula, 53
stepped pyramid, $34,35,37,46,55,64,127$, 135, 142, 172, 176
Stone Amulet Theory, vi, 99, 103, 104, 105
stone rocking, 147, 150
stone walkers, 147
successive integers, 60,82
sun temples, 24,52
supermarket clock tower, 180
symmetry, 60, 65, 79, 90, 101, 103, 105
systems engineering, xiii, 151, 156, 165

## $T$

Tarxien Temple, 68
technology, xiii, 4, 12, 145, 150
Tefnut, 19
Temenos, 31, 39, 110, 111, 115, 154
terrain simulation, 56

Thuban, 71, 72, 76, 92, 95, 97, 98
Tomb of Archduchess Maria Christina of Austria, 176
totem, 14, 50, 84
transmogrification, 78, 95, 96, 97, 156
transport, 2, 4, 25
trial and error, 34, 59, 99, 100
true pyramid, $35,38,49,61,65$
Tura, 47, 64, 104, 112, 127, 135, 153, 154, 157

## $\boldsymbol{U}$

Unas, ix, 51, 52, 74, 75, 89
underground chamber, $41,42,43,45,46,52$, 154, 170
unitary fraction, 60
unitary fractions, 60, 82
Upper Egypt, ix, 5, 6, 8, 9, 14, 31, 51, 85
Ursa Major, x, 73, 79, 84, 85, 86
Ursa Minor, x, 73, 78, 84, 85, 86, 88
Userkaf, 51

## V

Valley of the Kings, xi, 172, 173, 174
Valley of the Workers, xi, 173
valley temple, $25,30,39,47,114$
ventilation, $42,45,91,104$
virtual reality, xiii, 43, 149, 163, 164
viziers, $9,10,22$
W
West Kennet Long Barrow, 169, 170
World View, 16

## Z

zoomorphs, 14
П
$\pi, v, 61,62,101$

## Bibliography

- Aldred, Cyril, 1974, Egypt to the End of the Old Kingdom, Thames and Hudson
- Aldred, Cyril, 1984, The Egyptians, Thames and Hudson
- Barocas, Claudio, 1972, Monuments of Civilization, Egypt, Readers Digest
- Bauval, Robert \&Gilbert, Adrian, 1994, The Orion Mystery, Heinemann
- BBC TV, 1994, Gateway to the Stars
- Clayton, Peter A., Chronicles of the Pharaohs, Thames and Hudson
- Hart, George, 1991, Pharaohs and Pyramids, Herbert Press
- Jenkins, Nancy, 1980, The Boat Beneath the Pyramid, Thames and Hudson
- Macauley, David, 1975, Pyramid, Collins
- Mertz, Barbara, 1992, Temples, Tombs and Hieroglyphs, O’Mara
- Murray, Margeret A., 1949, The Splendour that was Egypt, Sidwick and Jackson
- Posener, Georges (Ed), 1962, A Dictionary of Egyptian Civilization, Methuen
- Rice, Michael, 1990, Egypt's Making, Routledge
- Vercoutter, Jean, 1992, The Search for Ancient Egypt, Thames and Hudson
- Lehner, Mark, 1997, The Complete Pyramids, Thames and Hudson
- Davies, Vivian and Friedman, Renée, 1998, Egypt, British Museum Press


## References

${ }^{1}$ Waterfield, R., 1998, Herodotus the Histories, Book 2(5), Oxford University Press
${ }^{2}$ Ibid, Book 2(12)
${ }^{3}$ St. John, Michael, 1999, The Palermo Stone -A Mathematical View , Museum Bookshop Press, London
${ }^{4}$ Waterfield, op. cit. Book 2(4)
${ }^{5}$ Rodenback, M, 1991, Egypt from the Air, Thames \& Hudson Ltd., London
${ }^{6}$ Waterfield, op. cit. Book 2(95)
${ }^{7}$ Manley, Bill, 1996, Historical Atlas of Ancient Egypt, Penguin
${ }^{8}$ Waterfield, op. cit. Book 2(5)
${ }^{9}$ Davies, V and Friedman, R., 1998, Egypt, British Museum Press
${ }^{10}$ Rice, Michael, 1990, Egypt's Making, Guild Publishing, London
${ }^{11}$ Campbell, Joseph, Ed., 1971, The Portable Jung, Penguin Books
${ }^{12}$ Ibid, pp. 59-60
${ }^{13}$ Faulkner, R.O., 1985, The Ancient Egyptian Book of the Dead, University of Texas Press
${ }^{14}$ Ibid
${ }^{15}$ Manley, op. cit.
${ }^{16}$ Faulkner, R.O., 1969, The Ancient Egyptian Pyramid Texts , Oxford University Press
${ }^{17}$ Campbell, op. cit.
${ }^{18}$ Rice, op. cit.
${ }^{19}$ Clayton, Peter A., 1994, The Chronicles of the Pharaohs, Thames and Hudson Ltd, London
${ }^{20}$ Faulkner, R.O., 1969, The Ancient Egyptian Pyramid Texts , Oxford University Press
${ }^{21}$ Waterfield, op. cit. Book 2(124)
${ }^{22}$ Lehner, M., 1997, The Complete Pyramids, Thames and Hudson Ltd., London
${ }^{23}$ Mendelssohn, Kurt, 1975, The Riddle of the Pyramids , Thames and Hudson Ltd., London
${ }^{24}$ Davies, V and Friedman, R., op.cit.
${ }^{25}$ Ibid, p. 74 .
${ }^{26}$ Bauval, Robert \& Gilbert, Adrian, 1994, The Orion Mystery, William Heinemann Ltd
${ }^{27}$ Murray, M.A., 1972, The Splendour that was Egypt, Sidgewick \& Jackson, London ${ }^{28}$ Ibid
${ }^{29}$ Robins, Gay \& Shute, Charles, 1987, The Rhind Mathematical Papyrus, British Museum Press
${ }^{30}$ Mendelssohn, op. cit.
${ }^{31}$ Butter, Rachel, 1999, Kilmartin, Kilmartin House Trust
${ }^{32}$ Zammit, Sir Themistocles, The Prehistoric Temples of Malta and Gozo , S. Masterson
${ }^{33}$ Rudgley, Richard, 2000, Secrets of the Stone Age, Century, London
${ }^{34}$ Faulkner, R.O., 1969, The Ancient Egyptian Pyramid Texts , Oxford University Press
${ }^{35}$ Davies, V and Friedman, R., op. cit., pp 36-38
${ }^{36}$ Faulkner, R.O., 1969, The Ancient Egyptian Pyramid Texts , Oxford University Press
${ }^{37}$ Macaulay, David, 1975, Pyramid, Collins
${ }^{38}$ Bauval, op. cit.
${ }^{39}$ Campbell, op.cit.
${ }^{40}$ Griffiths, F.U. and Thompson, H., (Ed), 1974, The Leyden Magical Papyrus , pp. 33-5, Dover Publications, New York
${ }^{41}$ Waterfield, op.cit. Book 2(145)
${ }^{42}$ Ibid, Book 2(145)
${ }^{43}$ Ibid, Book 2(145)
${ }^{44}$ Odum, Howard T, 1971, Environment, Power and Society , p136, John Wiley \& Sons, Inc.
${ }^{45}$ Ibid
${ }^{46}$ Waterfield, op.cit. Book 2(14)
${ }^{47}$ Ibid, Book 2(36)
${ }^{48}$ Ibid, Book 2(37)
${ }^{49}$ Edmundson, W.C. and S.A. Edmundson. 1988. Food intake and energy expenditure of male and female farmers in an impoverished Indian village. British Journal of Nutrition. 60(3): 236-243.
${ }^{50}$ Ministry of Agriculture, Fisheries and Food, 1992, Manual of Nutrition, HMSO, London.
${ }^{51}$ Waterfield, op.cit. Book 2(125)
${ }^{52}$ Aldred, Cyril, 1992, The Egyptians, Thames and Hudson Ltd., London
${ }^{53}$ Hitchins, Derek K., 1992, Putting Systems to Work, John Wiley \& Sons, Chichester
${ }^{54}$ Cottrell, Leonard, 1955, The Mountains of the Pharaoh , The Scientific Book Club, London
${ }^{55}$ Lehner, op.cit. pp.240-243
${ }^{56}$ Campbell, op.cit.pp. pp.59-69
${ }^{57}$ Faulkner, R.O., 1972, The Ancient Egyptian Book of the Dead , p.54, University of Texas Press


[^0]:    * The Palermo Stone , an important primary source of Old Kingdom information and numerical data, comprises seven fragments, the largest being in Palermo, hence the title. Five smaller pieces are in the Cairo Museum, and one piece is in the Petrie Collection at University College, London.
    ${ }^{\dagger}$ Unfortunately, the fragments do not provide a complete record: there are gaps. However, events such as the construction of ships, taking a census, granting tax exemption, and donating land to temples are recorded, along with the regnal year of the pharaoh in which they occurred.
    \# The height of the Inundation was not measured in metres, of course, but in cubits, palms and fingers. The records used 7 palms to a cubit and 4 fingers to one palm. Fractions of fingers were used, giving good accuracy of measurement. The span was also used, where 1 span is half of 1 cubit.

[^1]:    * The royal cubit, in use at the time, measured some 52.3 cm .

[^2]:    * The method presumes flat terrain, since hills in either the direction of the rising star or the rising Sun would upset parity. The Delta was the flattest area, so it seems likely that heliacal risings were observed principally in the Delta region.

[^3]:    * This type of mud-built, reinforced corner structure is still in use today in the Yemen.

[^4]:    * The photograph shows a copy of the famous palette, $\sim 3150 \mathrm{BC}$, in Cairo Museum . The recess created by the intertwined necks of the serpopards was for mixing kohl, used to outline and protect eyes.
    ${ }^{\dagger}$ These images could be representations of historical events, but could equally be propaganda, depicting what Narmer proposed would happen, as opposed to what had happened.

[^5]:    * Heliopolis is located in the suburbs of present day Cairo. It was referred to as On in the Bible.
    ${ }^{\dagger}$ Pharaoh translates as "big house". Using the term "pharaoh" might originally have been not dissimilar to referring to the US president as "he of the White House."
    ${ }^{*}$ Mastaba is an Arabic word meaning "bench". It refers to the shape of ancient burial mounds, which were rectangular, steep-sided structures with flattened tops

[^6]:    * Found by Miroslav Verner of the Czech Mission to Abusir
    ${ }^{\dagger}$ The cult of the Pharaoh Raneferef was served for some 200 years by some 200 priests in 5 shifts of 40 priests, providing food for the dead pharaoh by day and watching the stars by night. Records were kept of duty rosters, with meticulous inventories of furnishings, stockpiles and cult objects.

[^7]:    * The Greeks had a penchant for changing the name of everyone and everything, including all the names of places and pharaohs. Khufu is the ancient Egyptian name of the pharaoh of the Great Pyramid: Cheops is the Greek name.
    ${ }^{\dagger}$ In any event, any supposed mystery race would still have had to develop pyramid-building skills. Such alternate theories simply relocate problems; they do nothing to solve them.

[^8]:    * Lamarckism proposes that organisms develop and evolve by the effort of adapting to new conditions. This is a quite different view from that of Darwin.

[^9]:    *Some Roman Caesars believed this of themselves. Kings of France appeared to believe it, too
    ${ }^{\dagger}$ During WWII, the dock area in the east end of London was heavily bombed, while Buckingham Palace in central London escaped. Finally a bomb hit the Palace, upon which Queen Elizabeth famously remarked: 'Now I can look the East-Enders in the eye." Although not of the people, she felt herself responsible for the people, exhibiting the archetypal behaviour of the King as Shepherd.

[^10]:    * Osiris is often depicted with green skin, the colour of vegetation, to indicate his rôle in the resurrection/regeneration of life. This tradition of the Green Man survives to this day, and can be seen in England in present day mummery of the legend of St George and the Dragon, and in the many public houses called "The Greene Man".
    ${ }^{\dagger}$ Societies exhibit this duality today. Political parties are either left or right. Situations are either black or white. People are either "with you" or "against you." Such statements are not true, of course, but people in general tend to look at situations and perceive them as opposing pairs The north south divide seems to be universal, even to this day. In virtually every country, large or small, there is a north-south divide. The US had the Unionists Vs. the Confederates. UK has Scotland vs. England. Scotland has Highlands vs. Lowlands. Wales has North Wales vs. South Wales. England has northerners vs. southerners. And so on. Curiously, there is virtually no substantial example of an east-west split...

[^11]:    * It is interesting to speculate on the origins of these spirits. The $B a$ could have originated in dreams of flying which many people experience. The $A k h$ is reminiscent of the light which people report during near-death experiences. The $K a$ resonates with modern ideas of an enduring soul.

[^12]:    * Gunter Dreyer of the German Archaeological Institute has recently discovered seal impressions of King Djoser at Khasekhemwy's burial site at Abydos, indicating that Djoser buried Khasekhemwy, his stepfather. This suggests that Djoser was the direct successor of Khasekhemwy, and that the list may need revised at the end of the $2^{\text {nd }}$ and start of the $3^{\text {rd }}$ Dynasties.
    ${ }^{\dagger}$ It is often possible to see pharaonic titles even in Old Testament characters. So, Abraham, for example, can be read as "Ib-ra-him" which, were it hieroglyphics, might be read as "priest of the heart of Ra." This would even make sense, as the ancient Egyptians believed the intellect was located in the heart, so making Ib-ra-him mean "priest who knows the thoughts of god". Did the name Abraham originate in hieroglyphs, to be adopted later by the Hebrews? Or is that just coincidence?

[^13]:    * The serekh was a symbol of the king, often bearing his name. Later, the cartouche would replace it in general use.

[^14]:    * Taking the king's mummy to the pyramid temple does not preclude its being interred elsewhere. A subsequent journey from pyramid temple to burial site might have been viewed as creating a guiding pathway, giving the pharaoh's $k a$ a mental route map which he could then follow whenever needed.

[^15]:    * Imhotep may have been responsible for the first mathematics. He was later worshipped as a god of architecture and medicine by the Egyptians, and was assimilated as the god of medicine, Asclepius, by the Greeks.

[^16]:    * The pyramid pictured in the complex is not to scale; Khufu 's pyramid is enormous and would overshadow other features if shown to scale in this artist's impression.

[^17]:    * The title is a misnomer; there are, after all, three Queens Pyramids on the Eastern perimeter of the Pyramid.

[^18]:    * The down thrust from the Relieving Chambers transmits through the spacers on to the edge of the beams roofing the King's Chamber, directly on to the chamber walls, avoiding the central span of the beams. In this sense, the chambers relieve, but the capstones above Campbell's Chamber would suffice on their own.

[^19]:    * One suggestion for the choice of pyramid shape is that it emulates the Sun breaking through cloud. The photograph enables readers to make up their own minds about that theory.
    ${ }^{\dagger}$ If Nebka's Unfinished Pyramid was attempted before Menkaure 's, failure to complete it, coupled with a natural desire to locate with his father and grandfather, may have persuaded Menkaure to choose Giza even with the limited space available.

[^20]:    * Medieval cathedrals in Europe were also built using proportions, not angles.
    ${ }^{\dagger}$ This incidentally makes anachronistic nonsense of claims that the Giza Pyramids were purposely built on latitude 30 N . There is no evidence of either an archaic spherical Earth model or spherical celestial model, and even had one existed, there is no reason to suggest that such spheres would be divided into $360^{\circ}$. While the Babylonians may have used 360 divisions in a circle, Egyptian mathematics at this time would have supported either a decimal or binary numbering system, e.g. either 100 or 64 units in a quadrant or circle.
    ${ }^{*}$ To calculate slopes in seked, take Adjacent/Opposite*7. So, for Khufu 's Pyramid, 11/14*7 $=5-1 / 2$ seked.
    ${ }^{\S}$ The ancient Egyptians used unitary fractions , fractions with unity in the numerator, for preference. They also avoided using equal fractions in a sum. So $2 / 7$ could not be written as $1 / 7+1 / 7$. Instead, $2 / 7$

[^21]:    * $\pi$, or $p i$, is the Greek letter used to signify the ratio of the circumference of a circle to its diameter. It is commonly used today to measure the area of circles, the volume of spheres, etc.

[^22]:    * The popular "staircase to the stars" hypothesis seems to have a fatal flaw. If each pyramid were supposed to provide a staircase, why would the ancient Egyptians fill in the steps to form a smooth pyramid? Surely, this would prevent the pharaoh's $k a$ from climbing?

[^23]:    * The real movement of stars is called "proper motion".
    ${ }^{\dagger}$ Neither is Polaris precisely at the celestial pole today.

[^24]:    * This utterance hedges its bets, with three different ways of reaching the sky.
    ${ }^{\dagger}$ This utterance, on the other hand, is firmly tied to the rope ladder theory.

[^25]:    * I.e. is visible
    ${ }^{\dagger}$ Curiously, Sirius is a double star, with one bright element and a much fainter companion, invisible to the naked eye. Could the faint companion have been visible during the Old Kingdom, leading to the idea of the star as Isis bearing her child Horus?

[^26]:    * Photograph at left is illuminated by torchlight to show the cartouche containing the hare, part of the name Wenis (Unas)
    ${ }^{\dagger}$ One reason for believing them to be older than the $4{ }^{\text {th }}$ Dynasty, at least in part.
    ${ }^{\ddagger}$ Possible interpretations of this particular utterance will be explored in following sections.

[^27]:    * Ornament of a young child.
    ${ }^{\dagger}$ Montju is believed to be a star and from the context it is in the northern sky, but it has not been identified.

[^28]:    * Curiously, a similar shift is observed in the eastward orientation of early English churches.
    ${ }^{\dagger}$ I.e. it can be seen as a double star by anyone with normal vision

[^29]:    * There is no hard evidence of such a tool; the existence of such a tool, or something like it, is a reasonable deduction, however.

[^30]:    * The figure is from a New Kingdom wall painting, rather than Old Kingdom
    . Nonetheless is exemplifies the strength and persistence of the ancient cattle culture in Egypt.

[^31]:    * In this instance the mace was round, while later maces would be shaped.

[^32]:    * Note the serekh top centre, with the hand for D and the wavy line for N , making up the name DeN. Horus stands above the serekh, while to the right is a totem with Anubis facing what has been identified as a placenta. It is believed that the pharaoh's placenta was mummified at birth and may have had some significance as a "place-holder" for the pharaoh in the afterlife.

[^33]:    * The ancient Greeks attributed magical powers to the ancient Egyptians, and some Egyptians were vaunted as great magicians. Numbers, proportions and dimensions may have composed a significant part of this magical lore.

[^34]:    * Much later, the Athenians of ancient Greece would build temples with floor plans set on a $\sqrt{ } 5: 1$ ratio. It may be that they chose this ratio because the floor diagonal of a $2: 1$ floor is in the ratio $\sqrt{ } 5$, i.e. $\sqrt{ }\left(1^{2}+2^{2}\right)$, a number considered interesting because it had no precisely calculable value. Whether or not the Athenians acquired this notion from the ancient Egyptians is not evident.

[^35]:    * A horizontal shaft would have served admirably for ventilation , especially one facing north towards the prevailing wind. Such a shaft would not have had to maintain accurate slope while penetrating scores of pyramid tiers.
    ${ }^{\dagger}$ Culminations or transits always occur north or south by definition. The Great Pyramid shafts point north and south. Since precession makes each star's transit elevation rise and fall, it follows that many stars will line up with the shafts at some time over the processional cycle.

[^36]:    * Today, $95 \%$ of pregnancies take between 38 and 42 weeks.

[^37]:    * In case the idea of such annual festivals and rituals should seem strange, we should perhaps remember that Christian cathedrals are laid out in shape of a cross, facing East-West, with the altar at Eastern end. The life of Christ is compressed into four months from Christmas-birth, until Easter-death and resurrection Christian rituals include symbolically drinking the blood, and eating the body of Christ, which practice had early Christians labelled as cannibals

    Similarly in respect of dates, Easter occurs on the first Sunday after the first full moon occurring after the vernal equinox. This dating arrangement was established in AD 325, at Nicea in Asia Minor. Easter was originally Eostre, which combines East with Aurora, meaning Eastern Dawn. Easter coincides with pagan spring fertility rituals, which the early Christian church simply took over.

[^38]:    * The term machine would be valid in this instance - "any apparatus or organization that applies power to create movement".
    $\dagger$ We can sympathize today. We talk about heaven as "up there" and hell as "down there" with no idea as to how a soul might make the journey. Mediums in a trance "hear" voices "from the other side". Nobody asks "other side of what," nor how the voice gets through whatever is "in between".
    * One of the most fascinating aspects of studying the Egyptians is the opportunity to see our own psyches on display, as it were.

[^39]:    * As the collage Figure 38 shows, top left, Khafre 's statue included a hawk on the pharaoh's neck. Raneferef's statue, found in his mortuary temple at Abusir, similarly had a hawk on the neck. The hawk

[^40]:    is conventionally seen as Horus, and as a sign of kingship. However, there are statues of prone royal bodies with hawks on the ground by each shoulder, and a hawk on the pharaoh's genitals, head facing the pharaoh's, wings spread forward and down in the so-called "mantling" position; the precise meaning of these icons is uncertain. In view of the other royal statues, it is not unreasonable to expect there to have been life size statues of Khufu, and at least one with a hawk at the neck. Why only one statuette of Khufu remains today is a mystery; as one of the most powerful rulers of the $4^{\text {th }}$ Dynasty, more would have been expected.

[^41]:    * The slope of $\mathrm{KC}(\mathrm{S})$ has been presented as opposite $/$ adjacent $=7 / 7$ to accommodate this predilection; it would normally be presented as $1 / 1$ today.

[^42]:    * $6^{\text {th }}$ July to $11^{\text {th }}$ September approximately.

[^43]:    * The shafts entrances in the Queen's Chamber were originally covered with stone, and were chiselled open in AD1872 by Waynman Dixon.
    ${ }^{\dagger}$ The ancient Egyptians would have been aware that Alnitak, like every other star, transited due south at the same elevation every day throughout the year. On only one day of the year did this transit coincide with dawn: for the rest of the year, it could occur at any time, even during the days when Osiris was in the Netherworld of faded stars.
    * Utterance 466, introduced above on page 76 now begins to make more sense: "O King, you are this great star, companion of Orion, who traverses the sky with Orion, who navigates the Netherworld with Osiris... the Great Mooring-post cries out to you as to Osiris..."

[^44]:    * Over 100 amulets have been found in the wrappings of some royal mummies.

[^45]:    * An ancient unit of weight and of money, between 25 and 38 kg of gold or silver

[^46]:    * This version uses the STELLA ${ }^{\text {TM }}$ simulation and modelling package by High Performance Systems, Inc

[^47]:    * This graph, along with others from the Pyramid Calculator , is not a continuous line. Instead it is formed from 201 individual dots, each corresponding to a tier.

[^48]:    *Some translations say: "...contrivances made of short timbers."

[^49]:    * The stade was an ancient Greek measure equivalent to 184 metres
    ${ }^{\dagger}$ The fathom was a measure set by the reach between outstretched arms; now it is equal to 6 feet, or 1.8 metres.

[^50]:    * Recent findings at Giza by Mark Lehner, Visiting Assistant Professor at the Oriental Institute, University of Chicago, suggest that fish was plentiful and, surprisingly, that the workers' diets were rich in meat.
    ${ }^{\dagger}$ Proportional manpower assumes that the number of men recruited to work on the pyramid each year, rather than being a fixed number year-on-year, is proportional to the outstanding work for that year.

[^51]:    *Said of Egypt north of Memphis, i.e. the Nile Delta area.

[^52]:    * Even the shaduf, the long, counterbalanced pole used for lifting buckets of water out of the river, had not yet been invented.
    ${ }^{\dagger}$ Potters may have spun their clay when making pots, but this does not seem to have translated into the idea of wheels or rollers for general use.

[^53]:    * A similar tool, set to the appropriate slope, would have been used to set the internal sloping columns, Figure 87.

[^54]:    * The hieroglyphs shown in the figure are from the pyramid of Teti at Saqqara.

[^55]:    * The water levelling method works well provided there is no wind. A breeze will tend to "pile" the water up slightly at the down wind end of a long channel or gully.

[^56]:    * Some modern scholars scoff at Herodotus, since his work appears to be littered with errors. In recent years, there has been a growing acceptance that he was right more often than not. In the case of the Pyramids, his is the only account of their construction, although he acquired it some 2000 years after the Pyramids were built. In any country other than ancient Egypt, such dated reports might be disregarded. Egypt was not like other countries, however, and the handing down of information from generation to generation was assiduous.

[^57]:    * The figures presented here are very approximate indeed. It would be virtually impossible to run a representative trial, with a number of seasoned teams continually hauling stones on sledges up a representative ramp, made from the right materials, with the right frequency of corners, during the three summer months on the Giza plateau. Instead, in a "thought experiment", teams were seen to be hauling and resting, getting stuck on corners, wearing ruts in the wooden battens, queuing, etc. Computer simulation indicates that, if it took $\sim 20$ minutes to haul a sledge up the ramp across each pyramid face, and $\sim 20$ minutes to negotiate each corner, then the single lane capacity would be $\sim 2$ stones per hour. Counter intuitively, ramp repair delays of a few hours had little effect on capacity. The figures presented in the text are felt to be reasonable to optimistic, but in need of confirmation.

[^58]:    * The average mass of stones is widely assumed as 2.5-tonnes: between ashlars, stones could have been large, small, irregular, or even rubble packed with sand. Short of dismantling parts of the Great Pyramid, it is not possible to know. From a simple physics viewpoint, provided the structure was solid, the same mass was raised through the same height regardless.
    ${ }^{\dagger}$ This figure derives from Graph 17, simply by multiplying the Mean Rate, Proportional manpower, by four. Such a high rate of setting stone might seem impossible to achieve. However, it would be feasible provided a number of stones had been previously quarried and stockpiled near or around the Great Pyramid

[^59]:    *As a rough guide, a single-decker bus weighs about 9 - 10 tonnes. A 75 -tonne stone beam would weigh as much as 7 or 8 buses, although it would be much more compact.
    ${ }^{\dagger}$ Objections that there was no wood at that time in Egypt are invalid; although local wood may have been in short supply, cedar wood was imported from Lebanon in large quantities. Snefru's Bent Pyramid is shored with massive timbers of cedar. Khufu's boats were also built using cedars of Lebanon, evidently with no expense spared.

[^60]:    * Old illustrations of the method for raising stone that Herodotus recounted show men working levers and ropes. Even had the ancient Egyptians had levers, these would not have been strong enough to raise 50-75-tonne stones

[^61]:    * For the first tier, there could have been very many teams as there was no climb - just an expanse to fill with stones. For mid-range tiers, there could have been, say, 40 teams working from the bottom up part way, plus another set of 40 taking over above them, and so on, i.e. "rings" of teams working up the side of the growing pyramid. This parallel working method allows the labour force to be organized in a variety of efficient and effective ways.

[^62]:    * A good craftsman will be satisfied with nothing less than the best, and will be his own harshest judge. Craftsmanship is largely discounted today, in favour of speed and low cost - one reason, perhaps, why we could not contemplate building a pyramid.

[^63]:    * Today some industries are obsessed with risk and risk management, to the point that they may create a modest, impoverished and ineffective system, or perhaps never build at all.

[^64]:    * Today, we might talk about emergent properties , capabilities and behaviours, meaning not only static properties, but also ultimate performance, capacity, reactions to stimulus and many other features to be found in complex interactive systems.

[^65]:    * Perhaps Khufu was trying to emulate his father, Snefru 's feats, only bigger and better; this would not be untypical for an eldest son. We shall probably never know.

[^66]:    *When finished, such a model would need to be destroyed or concealed to safeguard pyramid secrets: it would contain all the secret passages, chambers, and security devices, just as in the real pyramid. For those same reasons, it could become a revered object - it would contain the magic proportions, ratios, inclinations and configurations, too. In this case, it may have been considered necessary to conceal it. One obvious place to conceal the model of Akhet Khufu would be in that Pyramid. So, one possibility for what lies behind the portcullis door at the top of the QC (S) shaft is a model of the pyramid, or rather what is left of the model.

[^67]:    * Height seems to have been important in making a statement: the higher the better. Psychologists looking at the spires of gothic cathedrals built during the Middle Ages have suggested phallic symbolism, but the earlier pyramid form belies this notion.

[^68]:    *The Cairn of Barnenez in Germany, for example, is a stepped structure covering a burial, and dates back to $\sim 4000 \mathrm{BC}$.

[^69]:    * Alternatively, it might suggest that humanity, having radiated from a common root stock in Africa, has some form of cultural clock that triggers widely separated cultures to behave in the same way at broadly the same epoch.
    ${ }^{\dagger}$ Mesopotamian Ziggurats, often described as pyramidal, were layered structures with staircases. They appear to date from about $2,100 \mathrm{BC}$, so their precursors, too, could turn out to be contemporaneous with the Old Kingdom
    ${ }^{\ddagger}$ Report in the journal Science, April 27, 2001
    ${ }^{8}$ It is unlikely that such a complex and sophisticated site as Caral sprang into being without precursors, yet to be found, but which could take the origins of the culture back another 500 years or more.

[^70]:    * The others were the octahedron, the icosahedron, the dodecahedron, and the cube. Four of the five regular solids, according to Plato, represented the four elements, while the dodecahedron represented the universe as a whole. The tetrahedron, for instance, represented fire.

[^71]:    * The first record of camels in Egypt is during the $9{ }^{\text {th }}$ Century BC; this is based on C ${ }_{14}$ dating of camel dung. Hollywood depictions of Pharaoh Khufu's camel legions are fiction.

[^72]:    * Curiously, obelisk is from the Greek word meaning "spit".

