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Contract Analysis and Design for Supply Chains with Stochastic Demand

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Abstract

This book is devoted to the analysis and design of supply chain contracts with stochastic demand. The book contains eight chapters, and each chapter is summarized as follows: Chap. 1 provides a comprehensive review of the classical development of supply chain contracts. Chapter 2 examines the effects of demand uncertainty on the applicability of buyback contracts. Chapter 3 conducts a mean-risk analysis for wholesale price contracts, taking into account contracting value risk and risk preferences. Chapter 4 studies the optimization of product service system by franchise fee contracts in the service-oriented manufacturing supply chain with demand information asymmetry. Chapter 5 develops a bidirectional option contract model and explores the optimal contracting decisions and supply chain coordination issue with the bidirectional option. Chapter 6 addresses supply chain options pricing issue, and a value-based pricing scheme is developed for the supply chain options. With a cooperative game theory approach, Chap. 7 explores the issues concerning supply chain contract selection/implementation with the option contract under consideration. Chapter 8 concludes the book and suggests worthy directions for future research.

Keywords: Supply chain management, Supply chain contract, Supply chain coordination, Stochastic demand, Buyback contract, Demand uncertainty, Risk preference, Contract value risk, Wholesale price contract, Product service system, Information asymmetry, Option contract, Spot market, Supply chain options pricing, Option value, Bidirectional option contract, Contract implementation, Game theory

Preface

A supply chain consists of multiple decision-makers who have different risk preferences and incentives. As a result, the optimal supply chain efficiency requires coordination of the actions adopted by the respective supply chain participants. However, it is often difficult to reach supply chain coordination owing to various reasons; typically the actions that are system-wide optimal for the supply chain are often not in the best interests of the individuals, which leads them to have no incentive to do so. To make coordination in supply chains, contracts have to be designed to align the incentives of supply chain members so as to make the individual interest compatible with that of the supply chain system, so that the optimal supply chain efficiency can be achieved. In addition to their employment for channel performance improvement, contracts also serve as important instruments for supply chain members to share risks arising from various uncertainties inherent in the supply chain, such as demand uncertainty, price uncertainty, etc. With the proper assignment of risk among supply chain participants, contracts help to facilitate the supply chain operations in different business environments.

Given the extensive utilization of contracts in supply chains, the issues concerning contract analysis and design are extremely important for supply chain management, and substantial research has been conducted to address the relevant issues over the past years. Despite the abundance of classical research, new research needs to be made in response to new issues emerging with the recent changing business environments, such as the fast-shortening life cycle of product and the increasing globalization of supply chain. Only in this way can we gain a more comprehensive and profound understanding of this important topic.

This book is devoted to addressing issues concerning the analysis and design of supply chain contracts under stochastic demand, with the intention to present new research results on how to apply contracts to improve supply chain management. The book consists of eight chapters, and each chapter is summarized as follows:

In Chap. 1 a comprehensive review is provided for the classical development of supply chain contracts. Given that the literature on supply chain contracts is vast and any categorization of research streams may not be enough to cover all, the review begins with the research under the classical newsvendor model and then covers various extensions. Particular attention is paid to the wholesale price contract, buyback contract, revenue-sharing contract, sales-rebate contract, quantity discount contract, and various flexible supply contracts. Such an organization of the contents aims to make the review follow a clear pattern and better capture the most important features of the past development on supply chain contracts. Besides, the main innovations for the research presented in this book are also summarized in this chapter.

Chapter 2 examines buyback contracts in a supplier-retailer supply chain where the retailer faces a price-dependent downward-sloping demand curve subject to uncertainty. Compared with classical research, a fundamental difference of this research lies in its analytical examination of the effects of demand uncertainty on the applicability of buyback contracts. To this end, the research seeks to characterize the buyback contract model in terms of only demand uncertainty level. With such a new research perspective, some new and interesting findings are obtained for such issues as how demand uncertainty level affects the applicability of buyback practice and how to apply this practice to improve supply chain members' own interests or the supply chain system's efficiency. The research in this chapter demonstrates that the uncertainty level inherent in market demand can be a critical factor influencing the applicability of supply chain contracts, as well as the contract's administrative cost (Cachon 2003) and the uncertainty type (Marvel and Peck 1995).[1]

Given that risk is a pertinent issue in designing supply chain contracts with stochastic demand, Chap. 3 is devoted to developing a mean-risk analysis for the commonly adopted wholesale price contract. The research incorporates contract value risk into the wholesale price contract model. Regarding the contract value risk, it actually relates to the uncertainty in the true value of the contract and arises from various uncertainty sources inherent in the supply chain, such as demand uncertainty, price uncertainty, etc. In addition, given that the supply chain agents with different risk preferences will have different risk attitudes towards the contract value risk, which in turn affects their contracting decisions, the research also considers the degree of supply chain agents' risk-aversion towards the contract value risk. This chapter makes the first attempt to assess the efficiency of wholesale price contracts, incorporating contract value risk and risk preferences attached to it; thereby some interesting managerial and academic insights are generated for supply chain contracts.

Chapter 4 examines franchise fee contracts in the product service system with demand information asymmetry. In this chapter, three types of contracts are devel-

[1]Marvel and Peck (1995) incorporated two types of uncertainties in their model, namely, the uncertainty with respect to product valuation and the uncertainty with respect to the number of customer arrivals. Their studies showed that *only* the valuation uncertainty makes the supplier prefer the wholesale price-only contract, whereas *only* the arrivals uncertainty induces the supplier to offer buyback in its contracts. Their studies reveal that the type of uncertainty can be a significant factor affecting the applicability of supply chain contracts. Cachon (2003) pointed out that the contract's administrative cost may be another critical factor for the applicability of supply chain contracts.

oped to optimize the operations of product service system in the service-oriented manufacturing supply chain. The first is the franchise fee contract under which a two-part tariff including a wholesale price and a franchise fee is provided. The second is the franchise fee with service requirement (FFS) contract under which a service level is specified in addition to a two-part tariff. The third is the franchise fee with centralized service requirement contract which is similar to the FFS contract but that the service level specified is the system-wide optimal solution. This chapter mainly addresses the issues of how to design the contracts by which to assure a credible information sharing across the supply chain and how different these three forms of contracts affect the supply chain. The research can provide managerial insights for optimizing product service system by franchise fee contracts in the service-oriented manufacturing supply chain with demand information asymmetry.

Chapter 5 extends the concept of single directional option to develop a supply contract for a two-echelon manufacturer-retailer supply chain with a bidirectional option for which it may be exercised as either a call option or a put option. With a general demand distribution, the research derives closed-form expressions for the retailer's optimal order strategies under the bidirectional option contract, including the optimal initial order strategy and the optimal option purchasing strategy. The research also analytically examines the feedback effects of the bidirectional option on the retailer's initial order strategy. In addition, a chain-wide perspective is taken to explore how the bidirectional option contract can be designed for supply chain coordination.

Chapter 6 explores the pricing issue for supply chain options. In classical research, this issue is generally considered in the Stackelberg game framework. Such a pricing scheme, however, is usually unacceptable to the follower since the leader always captures all the surplus derived from the options. Being different from the existing literature, this chapter develops a value-based pricing scheme for supply chain options with two modeling scenarios including a single retailer and multiple retailers. The intuition behind such a pricing scheme is to price the option based on the value inherent in the "option right". As shown in the research, such a pricing scheme can assure that each of the contracting partners captures a share of the surplus derived from the options. As a result, the pricing schemes developed in this chapter are more objective and fair and consequently are more likely to be accepted by the contracting partners as compared with those that follow the Stackelberg game approach.

Chapter 7 focuses on exploring issues concerning supply chain contract selection/implementation with the option contracts under consideration. From existing research it is known that various contracts have been developed to attain supply chain coordination and ensure arbitrary allocation of the resulting coordinating profit. However, since the extents to which the individuals involved improve their profits are different with different coordinating contracts, an important issue that remains to be resolved is how to select a coordinating contract that is acceptable for all the contracting partners. In Chap. 7 an effort to address this issue is made with the consideration of option contracts. In this research, the cooperative game approach is taken to consider the supply chain coordination issue with option contracts and to develop the contract negotiation model, taking into account supply chain members' risk preferences and negotiating powers. The negotiation models developed with the option contract can be easily extended to other types of contracts such as the buyback contract, the revenue-sharing contract, the sales-rebate contract, etc. In this sense the research of this chapter presents a theoretical modeling framework for the selection/implementation issue of supply chain contracts.

Finally, as the end, Chap. 8 concludes the book and suggests some worthy directions for future research.

This book is intended for researchers (including graduate students) in supply chain management who have an interest in conducting in-depth studies on supply chain contracts. This book is also intended for business practitioners who would like to seek a better understanding towards supply chain contracts and look for guidance and decision support for the practice of supply chain contracts. To summarize, this book can be useful for both researchers and practitioners in operations management, management science, and business administration.

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Chapter 1
Introduction

1.1 Why Need Contracts in Supply Chain Management

"Supply Chain Management (SCM) deals with the management of material, information, and financial flows in a network consisting of vendors, manufacturers, distributors, and customers" (Anupindi and Bassok 1999a). Since multiple entities are involved in a supply chain, the optimal supply chain efficiency requires coordination of the actions adopted by the individual members in the supply chain. Ideally, this can be attained when the supply chain is managed by a single decision maker who has all information at hand. Such a supply chain is referred to as a centralized or integrated supply chain. However, it is unrealistic to have a centralized supply chain in practice. In contrast to a centralized supply chain, a decentralized supply chain has multiple decision-makers who have different information and incentives. The decentralized supply chains are prevalent in practice, particularly with the development of supply chain globalization and outsourcing. In practice, it is often difficult for a decentralized supply chain to attain coordination without the help of additional mechanisms. There are various reasons why this is the case. First, actions that are optimal for a centralized supply chain are often not in the best interests of the individuals involved, leading them to have no incentive to do so. For instance, decision-makers are often reluctant to share private information regarding cost and demand, hence resulting in a suboptimal supply chain performance. Another typical example is the problem of double marginalization,[1] which was first studied by Spengler (1950) in the economics literature. To attain coordination in supply chains, contracts have to be designed to align the incentives of supply chain members so as

[1]Double marginalization represents a phenomenon that supply chain members only receive a portion of the entire margin of the supply chain system. Therefore, their decisions do not truthfully reflect the system-wide incentive structure. As a result of earning less than the full margin at any given quantity, they will produce less than the channel optimal quantity (Corbett and Tang 1999).

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to make the individual interest aligned with that of the supply chain system, so that the optimal supply chain efficiency can be achieved. This motive of supply chain contracts is called the *channel performance improvement* objective.

Besides the employment for channel performance improvement, supply chain contracts also serve as important instruments for supply chain members to share risks arising from various sources of uncertainty, such as demand, price, product quality, etc. We call this motive of supply chain contracts the *risk-sharing* objective. As an example, let us consider a supplier-retailer supply chain where the retailer orders from the supplier with a wholesale price-only contract and then in turn serves an uncertain market demand. In such a supply chain, the retailer will have to bear all risks associated with the demand uncertainty in end market. However, if the supplier commits to buying back a portion of the items unsold at the end of the selling season, then the supplier will help to share some of the demand uncertainty risk.

In addition to the above two main motives, according to Tsay et al. (1999), contracts also act as instruments to facilitate long-term partnerships by delineating mutual concessions that favor the persistence of business relationship and specifying penalties for non-cooperative behaviors. In practice, the length of time horizon can be a factor that motivates supply chain members to engage in activities that are unfavorable in the short term while have substantial benefits over time. For example, an iron ore supplier might be willing to consign a large portion of its production capacity to a single steel plant even though the spot price of the iron ore in the short term may be more attractive than the contract price. This iron ore supplier's motivation for doing so may be to build a long-term relationship in the hope that this steel plant would be a stable purchaser for many years in the future. In addition to providing a stable business partner, a stable relationship ensured by a contract can reduce transaction costs and allows for greater potential cooperation in the supply chain (Cohen and Agrawal 1999). Another motivation for the use of supply chain contracts is to make the terms of a relationship explicit by making legally concrete the expectation of each party involved in the contract.

1.2 Classification of Supply Chain Contracts

There is a substantial research on supply chain contracts over the past years, and therefore it is difficult to develop a classification that covers all relevant literature. To the best of our knowledge, no commonly accepted taxonomy is available. Anupindi and Bassok (1999a) classified supply chain contracts according to eight contracting clauses, i.e., horizon length, pricing, periodicity of ordering, quantity commitment, flexibility, delivery commitment, quality, and information sharing. Cachon (2003) reviewed the literature of supply chain contracts in terms of complexity of the contract model. According to Krishnan et al. (2004), supply chain contracts can be roughly divided into two types, one are the contracts with a demand-dependent pricing scheme and the other are the contracts with a quantity-dependent pricing scheme. A contract with a demand-dependent pricing scheme is referred to as the one with a linear wholesale price and some conditional *ex post* payment transfer that depends on a resolution of demand uncertainty. Typical examples of this type of contracts include buyback contract, revenue-sharing contract, sales rebate contract, and various flexible supply contracts that depend on demand information updating. A contract with a quantity-dependent pricing scheme is essentially a form of (second-degree) price discrimination, with which issuer of the contract extracts rents from the sales of the first few units, and then decreases the wholesale price to entice buyers to purchase additional units. Examples of such type of contracts include quantity discount contract, two-part pricing (in fact, two-part pricing is also a special case of quantity discount contract).

In view of the fact that the literature on supply chain contracts is vast and any categorization of research streams may not be enough to cover all, we will begin the review with the research under the classical newsvendor model, and then cover various extensions. Particular attention will be paid to the wholesale price contract, buyback contract, revenue-sharing contract, sales rebate contract, quantity discount contract, and various capacity reservation contracts. Such an organization aims to make the review follow a clear pattern and better capture the most important features of the past development of supply chain contracts.

1.3 Various Contracting Forms with Newsvendor Model

With the newsvendor model, there is a two-echelon supply chain consisting of one supplier and one retailer. There is only one selling season with stochastic demand, and a single opportunity for the retailer to replenish order during the horizon. We will describe various forms of the contracts with the newsvendor model. In view of that there are many forms of contracts that have been developed so far, concrete description will be presented for only several typical contracts including the wholesale price contract, buyback contract, revenue-sharing contract, sales rebate contract, quantity discount contract, and various capacity reservation contracts.

With the wholesale price contract, the supplier charges a wholesale price from the retailer for each unit the retailer orders. With the buyback contract, the supplier still charges a wholesale price for each unit the retailer orders, but is committed to buying back all or part of the items unsold by the end of the selling season from the retailer with a predetermined full or partial credit rate. A revenue-sharing contract allows for the supplier to share a predetermined portion of the retailer's selling revenue, in addition to charging a wholesale price for each unit the retailer orders. By a sales rebate contract the supplier charges a wholesale price for each unit the retailer orders, but is committed to giving a predetermined rebate to the retailer for each of the sold quantity that exceeds a predetermined threshold (which may be zero). As to the quantity discount contract, even though there are various variants, it can be divided into two categories, one is to place a price discount on all units ordered, referred to as the all-unit quantity discount contract, and the other is to place a price discount only on the order quantity that is beyond a predetermined threshold, referred to as the incremental quantity discount contract. Roughly speaking, quantity discount contract is designed to align incentives by manipulating the retailer's marginal order cost curve while leaving the retailer's marginal revenue curve unchanged. As to the capacity reservation contract, there are various forms so far. Generally speaking, with this type of contracts an allowance is paid to the supplier from the retailer before the beginning of production season for reserving one unit of the production capacity (probably with a minimum order commitment), which gives the retailer the right to purchase up to the reservation quantity if necessary to satisfy demand during the selling season. Typical examples of such type of contracts include option contract (Barnes-Schuster et al. 2002, Zhao et al. 2010), pay-to-delay contract (Brown and Lee 1998a,b), quantity flexibility contract (Tsay 1999), backup agreement (Eppen and Iyer 1997), etc.

1.4 Review on the Development of Supply Chain Contracts

As discussed above, the review will start from the research with the basic newsvendor model and then cover various extensions. For more surveys of this topic, readers are referred to Whang (1995), Anupindi and Bassok (1999a), Cachon (1999, 2003), Lariviere (1999), Tsay et al. (1999), and Corbett and Tang (1999).

1.4.1 Supply Chain Contracts with Basic Newsvendor Model

The basic newsvendor model is referred to as the newsvendor model with stochastic retail price-independent demand, i.e., retail price is exogenous to the model. This model may be justified when the retail market is so competitive that retailers are essentially price takers or manufacturers may have strong control over the retail price through, e.g., resale price maintenance (Marvel 1985) or minimum advertised price (Kali 1998). This model is simple but rich enough to include some basic issues of SCM, such as the issue concerning supply chain coordination with contracts. With this model coordination requires pushing the retailer to order as much as the channel optimal quantity.

Bresnahan and Reiss (1985) and Cachon (1999) showed that with the basic newsvendor model setting a wholesale price-only contract leads to the retailer holding too little inventory than the channel optimal quantity because of the effect of double marginalization, unless the supplier is willing to price at its marginal cost. Therefore, wholesale price contract alone cannot attain supply chain coordination even if in the basic newsvendor model setting. Therefore, most research of the wholesale price contract is not to address supply chain coordination issue but to analyze its efficiency. Perakis and Roels (2007) measured the efficiency of wholesale price contract with the concept of so-called Price of Anarchy (PoA), which is the ratio of the performance of a centralized system over the worst performance of a decentralized system. It is easy to see that PoA is a concept used to assess the worst-case performance of wholesale price contract. They showed that the PoA of wholesale price contract is at least 1.71 for a two-firm supply chain with a nonnegative demand distribution having the IGFR property.[2] Their result substantiates the inefficiency of wholesale price contract in coordinating the supply chain. For more detailed review of the literature on wholesale price contract please refer to Chap. 3, and for avoiding repetition the details are omitted here.

To remedy the inefficiency resulted from double marginalization effect, various contracting mechanisms have been introduced to the supply chain for pushing the retailer to order at the channel optimal level, resulting in various forms of contracts such as buyback contract, revenue-sharing contract, sales rebate contract, quantity discount contract, and various capacity reservation contracts. Pasternack (1985) was the first to show that buyback contract can attain supply chain coordination in the basic newsvendor model setting. Cachon and Lariviere (2005) demonstrated that buyback and revenue-sharing contracts are theoretically equivalent with the basic newsvendor model in the sense that for any buyback contract, there is a revenue-sharing contract that generates the same cash flow for any demand realization. Hence, revenue sharing contract must also coordinate the supply chain with the basic newsvendor model. For more details of supply chain coordination with contracts including rebate contract, quantity discount contract, and various capacity reservation contracts such as the option contract, readers are referred to Cachon (2003), Dada and Srikanth (1987), Weng (1995), Tsay (1999), Taylor (2002), Kohli and Park (1989), Jueland and Shugan (1983), Altintas et al. (2008), Wang and Liu (2007), Barnes-Schuster et al. (2002), Zhao et al. (2010), and Lee and Rosenblatt (1986), etc.

1.4.2 Supply Chain Contracts with Various Extensions

1.4.2.1 Supply Chain Contracts with Price-Dependent Newsvendor Model

In practice, there are many situations where retailers have some control power over price, and may therefore affect demand by the pricing action. Differentiated from the basic newsvendor model, with such a setting the optimal supply chain performance requires coordination of the retailer's pricing action in addition to its order action. As a result, contracts that can attain channel coordination in the basic newsvendor model do not necessarily ensure coordination in this setting. A key point is contracts that can coordinate the retailer's order action may distort the retailer's pricing action.

[2]For the concept of IGER, please see Lariviere and Porteus (2001) and Lariviere (2006) for more details.

Kandel (1996), Marvel and Peck (1995), and Bernstein and Federgruen (2005) showed that buyback contracts alone cannot attain supply chain coordination in the price-dependent newsvendor model setting. As a result, the research of buyback contracts in this setting generally does not address the supply chain coordination issue, instead, it analyzes the decentralized supply chain in a Stackelberg game framework. Examples include Emmons and Gilbert (1998), Granot and Yin (2005), Song et al. (2008), Padmanabhan and Png (1997, 2004), and Wang (2004). For the sake of avoiding repetition, details of these papers are omitted here and readers are referred to the literature review in Chap. 2. Despite the inefficiency of buyback contracts in coordinating the supply chain with price-dependent demand, Bernstein and Federgruen (2005) showed that if the parameters of buyback contract are made contingent on the retail price chosen, i.e., the parameters of buyback contract are allowed to determine after the retailer commits to a retail price (but before the retailer chooses its order quantity), then in such a way buyback contracts can still coordinate the supply chain with price-dependent demand and arbitrarily allocate the resulting coordinating profit. They called such a buyback contract a price-discount sharing contract, which means that the retailer will gain a lower wholesale price if it discounts the retail price, i.e., the supplier helps the retailer share some of the discounting cost of retail price.

Cachon and Lariviere (2005) showed that in the price-dependent newsvendor model setting, price-discount sharing contract is equivalent to revenue-sharing contract when no penalty cost is considered for lost sales. Therefore, as they demonstrated, revenue-sharing contract can ensure supply chain coordination and allow for arbitrary allocation of the coordinating profit in the price-dependent newsvendor model setting in the absence of penalty cost for lost sales. Otherwise, like the case of buyback contracts, coordination can be attained only when the terms of revenue-sharing contract are allowed to choose after the retailer commits to a retail price. Such dependence vanishes with only a single revenue-sharing contract. Therefore, in the presence of penalty cost for lost sales, there is only a single revenue-sharing contract which can coordinate the supply chain and allow for a single allocation of the resulting coordinating profit (Cachon 2003). Recall that in the basic newsvendor model setting, revenue sharing contracts and buyback contracts are theoretically equivalent. However, this is no longer the case with the price-dependent newsvendor model setting.

Cachon (2003) showed that with sales rebate contracts, the incentive to coordinate the retailer's order quantity leads the retailer to price below the channel optimal price. Hence, like the case of buyback contracts, sales rebate contracts alone cannot attain supply chain coordination in the price-dependent newsvendor setting. However, as remarked by Cachon and Lariviere (2005), if some mechanisms that can provide a counterbalance, such as the buyback mechanism, which can push the retailer to price higher by reducing the retailer's cost for leftover items, hopefully, supply chain coordination may be achieved. As to the quantity discount contract, it adjusts the retailer's marginal order cost curve while leaves the retailer to keep all revenue. As a result, with quantity discount contracts, the incentive to coordinate the retailer's order quantity does not distort the retailer's pricing decision. Hence, quantity discount contracts can attain supply chain coordination and allow for arbitrary allocation of the coordinating profit with the price-dependent newsvendor model setting even if in the presence of a penalty cost for lost sales.

1.4.2.2 Supply Chain Contracts in Effort-Dependent Newsvendor Model

In practice, in addition to exerting pricing action, a retailer can influence demand by some other actions such as costly efforts. For example, the retailer can hire more sales people, increase advertising investment, or give the product a better display location in the store. All these actions are costly. Like the case of price-dependent newsvendor setting, contracts that can coordinate the supply chain in the basic newsvendor model setting may distort the effort level the retailer wishes to exert. It has been shown that with the effort-dependent newsvendor model setting, most types of the contracts mentioned above can no longer coordinate the supply chain because they have distorted the retailer's effort decision when coordinating the retailer's order decision (see, e.g., Taylor 2002, Cachon 2003). Taylor (2002) considered a manufacture-retailer supply chain where a channel rebate is paid from the manufacturer to the retailer based on the sales of the retailer. They showed that supply chain coordination cannot be achieved by a linear rebate scheme as the demand is influenced by the retailer's sales effort. However, a combination of rebate contract with a properly designed target and returns contract can achieve coordination and ensure a win-win outcome. Cachon (2003) showed that an all-unit quantity discount contract can coordinate the supply chain in the effort-dependent demand setting. In fact, the quantity discount contract can be utilized to attain channel coordination in a newsvendor model where the retailer has control power over order quantity, price, and sales effort level. As discussed above, the reason why this is the case is quantity discount contract only adjusts the retailer's order cost curve whereas leaves all revenue to the retailer.

A key problem for channel coordination in the effort-dependent newsvendor model setting is: sales effort benefits both the supplier and the retailer whereas all associated cost is incurred alone by the retailer. An approach to addressing this issue is the strategy of effort-cost sharing, i.e., the supplier helps the retailer to share a portion of the effort cost. For example, the supplier could pay a percentage of the retailer's advertising expenses (see, e.g., Huang and Li 2001, Huang et al. 2002, and Yue et al. 2006) or it could compensate the retailer an allowance for the better display location of its product in the retail store (see, e.g., Wang and Gerchak 2001).

However, several conditions are necessary for the strategy of effort-cost sharing to be feasible in practice: (i) the supplier must be able to observe without much hassle the effort cost incurred by the retailer so that the supplier knows how much to share for the retailer, and (ii) the retailer's effort actions must be verifiable[3] to

[3] A contractual parameter is said to be observable if each member involved in the contract can learn the ultimate realization of this parameter. A contractual parameter is said to be verifiable if an outside enforcer (e.g., the court) can also learn its ultimate realization (Tirole 1988, p. 38).

the court so that the strategy of effort-cost sharing is enforceable. These conditions are met easily in some situations. For example, the supplier can generally observe and verify whether the retailer has purchased advertising in a local newspaper or television station. However, in many cases it may be difficult, especially with the strategy of effort-cost sharing, the retailer generally has an incentive to exaggerate the effort-cost information.

Another approach to addressing the coordination issue in the effort-dependent newsvendor setting is a combined use of several types of contracts that can generate counterbalance in the supply chain. For example, Krishnan et al. (2004) considered a two-echelon supplier-retailer supply chain where the retailer chooses inventory quantity *ex ante* but promotional effort level *ex post*. With such a model they showed that buyback dulls the retailer's promotional incentives and therefore adversely affects the supply chain performance. However, the buyback contract, coupling with an agreement on the sharing of promotional cost (if effort cost is observable), or offering unilateral markdown allowances *ex post* (if demand is observable but not verifiable), or placing additional constraints on the buyback contract terms (if demand is observable and verifiable) can result in coordination. Taylor (2002) showed that a combined use of buyback and sale rebate contracts can attain supply chain coordination in the effort-dependent newsvendor setting. In fact, the chance of returning leftover items with the buyback contract discourages the retailer's effort action while the sales rebate contract encourages it, which is the reason why such a combination can be effective in coordinating the supply chain with effort-dependent demand.

1.4.2.3 Supply Chain Contracts with Competition

This research stream can be summarized into three lines. The first is the research with competition on the downstream side, the second is the research with competition on the upstream side, and the third is the research with competition on both sides including the upstream and the downstream.

Cachon and Lariviere (2005) showed that revenue sharing contract can be utilized to coordinate a supply chain with retailers competing in quantities, e.g., Cournot

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Obviously, only the verifiable parameter can be written into a contract because in the event of any violation against the contractual term a court can intervene effectively. Tirole illustrated the differences between the two types of identification for the parameters involved in a contract in terms of an example as follows: reliable accounting measures may be available only to assess this team's entire performance, but not enough to evaluate the individual members' contributions in this team. However, an insider in this team (e.g., the supervisor of this team) may be able to disentangle these individual contributions whereas an outsider (e.g., a judge) may not. Similarly, in supply chain management a manufacturer may be able to observe the demand realized by a retailer. For example, it has ways to observe customer traffic through the retailer's store. However, it may have no effective way to communicate convincingly with a judge in the court about the demand it has observed. As a result, the demand information observed by the supplier is in fact unsuitable for acting as a piece of evidence in the event of a violation of the retailer against the contractual term.

competitors or competing newsvendors with fixed prices. Deneckere et al. (1997) considered a model where competition occurs over the market clearing price as well as inventory quantity. They showed that competition leads to retailers ordering less than the channel optimal quantity. As a result, with the retail competition, to improve supply chain efficiency, contracts that push retailers to order more can be employed for the supply chain, such as the resale price maintenance, the buyback contract, etc. Padmanabhan and Png (2004) considered a model where there are two retailers and each has an uncertain downward-sloping demand curve that depends on the retail price set by the competitor as well as the retail price set by itself. They showed that buyback benefits the supplier by intensifying the competition at the retail level. For more research of supply chain contracts with competition of the downstream side, please refer to Wang and Gerchak (2001), Cachon (2001), Anupindi and Bassok (1999b), Lippman and McCardle (1997), Savaskan and van Wassenhove (2006), Gerchak and Wang (1994), Deneckere et al. (1996), Bernstein and Federgruen (2003, 2005), Bernstein et al. (2006), Chen et al. (2001), and Dana and Spier (2001).

Cachon and Kok (2010) considered a model where a retailer sells two partially substituting products supplied by two competing manufacturers with a wholesale price contract or a quantity discount contract or a two-part tariff. They showed that the quantity discount contract and the two-part tariff intensify the manufacturers' competition to a larger extent as compared with the wholesale price contract, and this may make the manufacturers worse off while the retailer better off. As a result, the retailer may prefer to the latter two more sophisticated contracts when these contracts are offered by competing manufacturers. These findings are significantly different from the case with competition on the retail side, where quantity discount contract and two-part tariff allow the manufacturer to extract most rents while leave the retailer with only the reservation profit. More research of supply chain contracts with competition on the upstream side can be found in, e.g., Choi (1991), Trivedi (1998), Lee and Staelin (1997), and Gans (2002). As to the research of supply chain contracts with competition from both sides, please refer to the literature on common agency such as Bernheim and Whinston (1985, 1986, 1998), Mathewson and Winter (1987), and O'Brien and Shaffer (1993, 1997). The general result found by this stream of literature is that the manufacturer may prefer exclusive dealing due to reduced competition at the retailer level, even though societal welfare and industry profit may be higher with common agency.

1.4.2.4 Supply Chain Contracts with Information Asymmetry

Most models developed in the supply chain contract literature are blessed with full information, i.e., all members in the supply chain have the same information that is necessary for decision-making. Even though to a good extent this assumption is justified by the availability of the increasingly sophisticated information sharing systems, information asymmetry is still essential for the supply chain in practice. Actually, there are numerous reasons that lead to a member in the supply chain lacking some important piece of information while the other member possessing it, such as lack of the incentives that can induce truthful information sharing across the supply chain, increasing complexity and geographic breadth of modern supply chains, etc.

When information asymmetry exists among supply chain members, the optimal supply chain performance requires effective information sharing among the supply chain members as well as coordination of the individuals' actions. Sometimes, effective information sharing is attained easily for supply chains. For example, if the relevant information is the demand distribution of a product with stationary stochastic demand, then demand forecast can be shared by sharing past or preseasonal sales data (see, e.g., Cachon and Fisher 2000, Chen 1998, Gavirneni et al. 1999, and Lee et al. 2000). However, unfortunately, there are many occasions in which incentives exist to impede effective (truthful) information sharing among the supply chain members. For example, since a retailer will not incur any cost for the build of an increased production capacity, the retailer may inform its supplier of a piece of rosy demand information so as to entice the supplier to prepare more production capacity. This is particularly true when demand forecast is unobservable or unverifiable. As a result, to attain effective (truthful) information sharing in the supply chain, contracts often have to be designed to provide the necessary incentives. This motivation arises an interesting and challenging issue as how to design contracts in supply chains to encourage effective information sharing (see, e.g., Cachon and Lariviere 2001, Özer and Wei 2006, Chen 2005, Li and Zhang 2008, Ha 2001, Corbett et al. 2004, and Corbett and Groote 2000). Cachon and Lariviere (2001) considered the option contract with information asymmetry under two compliance regimes, namely the voluntary compliance regime and the enforced compliance regime.[4] In their model, the manufacturer has an incentive to inflate the demand forecast by which to induce the supplier to build more capacity for a critical component. Being aware of this motive of the manufacturer, the supplier may be suspicious of the forecast informed by the manufacture. The ineffective information sharing between the supplier and the manufacturer tends to lead to a suboptimal

[4]It is intuitively reasonable to assume that the supplier cannot force the retailer to accept more than the retailer has ordered. However, it is not clear that whether the supplier is allowed to deliver less than the retailer has ordered. In practice, there are numerous reasons that lead to a failure of the supplier to satisfy the retailer's full order. Even though some of these reasons may be due to self interest, i.e., the supplier will benefit from doing so, there are many other reasons that are beyond the supplier's control, such as the unforeseen world financial crisis in 2008 and European financial debt crisis in 2011, the supply disruption owing to various natural disasters such as a fire disaster, etc. Hence, a supply chain contract can be operated under the premises of two different compliance regimes, namely the *voluntary compliance regime* and the *forced compliance regime*. The voluntary compliance assumes that the supplier delivers the retailer the quantity that is optimal (but does not exceed the retailer's order quantity) for itself, given the terms of the contract. The forced compliance assumes that the supplier is prohibited from delivering less than the retailer has ordered, i.e., the supplier will always satisfy the retailer's full order. The reason why this is the case can be various, such as any violation may need to incur a prosecution from court for the supplier or a significant impairment of its reputation, etc.

supply chain performance. In the paper they developed the option contracts that allow for sharing demand forecasts credibly in the supply chain under the premises of voluntary and enforced compliance regimes, respectively.

Another interesting issue along with this extension of information asymmetry is to examine the impact (value) of information on decision making and supply chain performance (with contracts) (see, e.g., Ha and Tong 2008, Ha 2001, Corbett et al. 2004, Lee et al. 2000, Raghunathan 2001, Li and Zhang 2008, Li et al. 2005a,b, Cheng and Wu 2005, Wu and Cheng 2008). Ha and Tong (2008) studied the effect of information asymmetry on supply chain performance with two different contract types – contract menus and linear price contract. They considered two manufacturer-retailer supply chains that are identical except that they may have different investment costs for information sharing. Their model is divided into two stages: At the first stage, the manufacturers decide whether to invest in information sharing. At the second stage, the manufacturers offer contracts to their retailers, given the information structure built at the first stage. After that, the retailers engage in Cournot competition. They found that for the case of contract menus, to invest in information sharing is beneficial to each of the two supply chains when the investment costs are low. For the case of linear price contract, however, it is harmful regardless of the investment costs. Their results suggest that the effect of information asymmetry in supply chains depends on the contract type used, and information sharing capability can be a competitive advantage for supply chains. More details for the research with information asymmetry can be found in Chen (2003), which provided an excellent survey of the literature on information sharing in supply chains.

1.4.2.5 Supply Chain Contracts with Flexibility

Under flexible supply contracts, the retailer can enjoy flexibility in order quantity or order time or other dimensions, which allows the retailer to make a final decision in response to the up-to-date market information collected after further observing market signals (such as the sales data in an earlier stage or the sales of related items in the market). The manufacturer can also enjoy the early commitment from the retailer and hence make better planning for capacity and materials. The typical examples of flexible supply contracts include option contract, quantity flexibility contract, backup agreement, pay-to-delay contract, etc. So far substantial research has been developed for various flexible supply contracts, such as Barnes-Schuster et al. (2002), Brown and Lee (1998a,b, 2003), Eppen and Iyer (1997), Donohue (2000), Moinzadeh and Nahmias (2000), Tsay (1999), Li and Kouvelis (1999), Zhao et al. (2010), Milner and Rosenblatt (2002), etc. Li and Kouvelis (1999) developed a time-flexible contract, which allows for a purchase amount over a given period of time without specifying the exact time of the purchase. With such a time-flexible supply contract, they studied the purchase time and order quantity that minimize the expected net present value of the order cost plus the inventory holding cost. Their research demonstrates how time flexibility, quantity flexibility, supplier selection, and risk sharing can effectively reduce the sourcing cost in environments with the price uncertainty.

A close look at the flexible supply contracts reveals that the value of a flexible supply contract closely depends on the quality of market information updating, that is, what we think of as the opportunity to learn about future demand from the demand signal collected at an earlier stage. As a matter of fact, some papers have appeared to theoretically explore the issue concerning the effects of quality of information updating on a flexible supply contract. The typical example is Brown and Lee (2003), which analytically characterized the order quantity decision as a function of the demand signal quality with a futures-options contract. For a more detailed review of the literature on the flexible supply contracts, readers are referred to the literature review in Chaps. 5, 6, and 7, and for avoiding repetition the details are omitted here.

1.4.2.6 Supply Chain Contracts with Spot Trading

In addition to employing contracts, the retailer may also depend on spot procurement for order satisfaction when the spot market is available. In this setting, the spot market and the contract market inevitably interplay and the optimal order decision requires balancing the tradeoff between the two markets. Hence, two fundamental research issues with this contract setting are how the spot market and the contract market affect one another and what is the optimal portfolio of procuring through the spot market and the contract market. Mendelson and Tunca (2007) considered a two-echelon supply chain where a single supplier sells an intermediate good to multiple retailers. In their model, procurement takes place through a combination of bilateral fixed-price contracts and open market trading among all the supply chain participants. They studied how the strategic behaviors of the supply chain members in the spot market affect the fixed-price contract, the supply chain efficiency, etc. Martinez-de-Albeniz and Simchi-Levi (2005) considered a multi-period supply chain environment where procurement takes place depending on the long-term contracts, option contracts, and the spot market. They developed a general framework for supply contracts to analyze and optimize the portfolios of contracts. For more studies of this research stream, please refer to Kouvelis and Lariviere (2000) where an internal market was considered, Lee and Whang (2002) where a second market was considered, and Shen et al. (2011), Wu et al. (2002), Spinler et al. (2003), Wu and Kleindorfer (2005), and Fu et al. (2010).

1.4.2.7 Supply Chain Contracts with Risk Aversion

In the supply chain contract research there is a substantial literature which addressed issue under study by assuming risk-neutrality. However, it is common for supply chain members to be risk averse in practice. When one or more agents in the supply chain are risk averse, the concept of supply chain coordination is different from that in the risk-neutrality case. For example, Gan et al. (2004) defined supply chain coordination with risk aversion in terms of Pareto-optimality criteria, which means no agent can become better off without making someone else worse off and each agent receives at least his reservation profit. This definition is a natural generalization of the standard definition in the risk-neutrality case. With this concept they developed the coordinating contracts for three specific supplier-retailer supply chains: (i) the supplier is risk neutral while the retailer maximizes his expected profit subject to a downside risk constraint; (ii) the supplier and the retailer each maximizes his own mean-variance objective function (Markowitz 1959); and (iii) the supplier and the retailer each maximizes his own expected utility. The research in Gan et al. (2005) showed that a retailer subject to a downside risk constraint may order less than the channel optimal quantity under the wholesale price contract, buyback contract, or revenue-sharing contract. Lau and Lau (1999) considered buyback contracts in a two-echelon supply chain consisting of a monopolistic supplier and a retailer who has the mean-variance objective functions. They studied the optimal buyback contract that maximizes the supplier's utility. Agrawal and Seshadri (2000a) considered a newsvendor model where a risk-averse retailer faces uncertain end market demand and makes the order quantity and selling price decisions with the objective of maximizing its expected utility. They assumed two different ways by which price affects the demand distribution. In the first model they assumed that a change in price affects the scale of the demand distribution. In the second model they assumed a change in price only affects the location of the demand distribution. They showed that comparing with a risk-neutral retailer, a risk-averse retailer in the first model will place a less order and set a higher retail price; whereas in the second model a risk-averse retailer will set a lower retail price. Wu et al. (2010) analytically examined the effects of risk aversion on the manufacturer's optimal ordering decisions with a commitment-option supply contract model. More studies of this research stream can be found in, e.g., Buzacott et al. (2011), Sobel and Turcic (2008), Kohli and Park (1989), Lau (1980), Agrawal and Seshadri (2000b), Tsay (2002), Wu et al. (2006), Shen et al. (2011), and Choi et al. (2008a,b).

1.4.2.8 Supply Chain Contracts with Empirical (Experimental) Research

In addition to theoretical explorations, some empirical (experimental) research has also been developed recently for supply chain contracts. Generally, the roles of empirical (experimental) research of supply chain contracts lie in two aspects, one is to test the theoretical results, and thereby substantiating the theoretical results with empiricism or finding some different insights; the other is to extract new findings from the empirical or experimental data. For example, using a supermarket's scanner data, Ray et al. (2006) showed with an empirical approach that the cost of retail price adjustment may result in an asymmetric pricing phenomenon on the wholesale price: a small change in the wholesale price does not translate into a commensurate retail price change when the retailer incurs a cost associated with the retail price adjustment. Therefore, suppliers will benefit from a small increase in the wholesale price, because doing so will not lead to an increase in the retail price and hence will not lose any customer. On the contrary, suppliers will suffer a loss from a small decrease in the wholesale price, since doing so will not induce a decrease in the retail price and therefore will not increase any sale. However, this is no longer the case when there is a relatively large change in the wholesale price, because in this case the cost incurred by the retailer for the retail price adjustment can be compensated by the increase in the retailer's revenue that is resulted from the corresponding large retail price change. In addition, with an experimental approach, Katok and Wu (2009) tested the performances of three commonly studied supply chain contracts, including the wholesale price contract, the buyback contract, and the revenue-sharing contract, with a two-echelon supply chain comprising one supplier and one retailer. Their experimental results demonstrate that even though the buyback and revenue-sharing contracts improve the supply chain performance relative to the wholesale price contract, the extent of improvement is smaller than the theoretical prediction. Furthermore, recall that in the basic newsvendor model setting, buyback contract and revenue sharing contract are theoretically equivalent (Cachon and Lariviere 2005), however, their experimental test shows that the two contracts generally do not result in the equivalent supply chain performance. More studies of this research stream can be found in, e.g., Becker-Peth et al. (2013), Ho et al. (2014), Kremer and Van Wassenhove (2014), Katok et al. (2014), Kalkanci et al. (2011, 2014), Wu and Chen (2014), and Wu (2013).

1.5 Main Contributions Included in This Book

Despite the abundance of classical research, new research needs to be conducted in response to new issues emerging with the rapidly changing business environment over time. This book will present some new research results on analysis and design of supply chain contracts with stochastic demand. The book consists of eight chapters, and the focal point and potential contributions of each chapter are summarized as follows.

(1) Chapter 2 explores buyback contracts in a supplier-retailer supply chain where the retailer faces a price-dependent downward-sloping demand curve subject to uncertainty. As compared with existing literature, a fundamental contribution of this research is the analytical examination of how uncertainty level inherent in market demand affects the applicability of buyback contracts in supply chain management. To explore the issue under study, the research seeks to characterize the buyback contract model in terms of only demand uncertainty level (DUL). With such a new research perspective, some new results and interesting findings are obtained for buyback contracts. For instance, but not limited to, this research has identified how DUL relates to buyback contract's efficiency and the analytical circumstances under which buyback increases the profit of the supplier, the retailer, or the both and subsequently achieves Pareto-

improvement. With this research, it is demonstrated that DUL is an important dimension affecting the applicability of buyback contracts. This finding can be useful for explaining a phenomenon appearing in industries, that is, in the same business setting, some forms of contracts are exploited more often than another, or the same form of contract is utilized more often in one specific business setting than in another one. For this issue, even though there is never a systematical investigation so far, some factors have been shown to have significant effects on the applicability of supply chain contracts, such as the administrative cost involved in contract (Cachon 2003) and the uncertainty type (Marvel and Peck 1995). With this study, it is shown that DUL is another important factor affecting the applicability of supply chain contracts.

(2) Risk is a pertinent issue for analysis or design of supply chain contracts with stochastic demand. The risk associated with a supply chain contract that is designed based on the concept of expectation (expected profit), in addition to credit risk, includes uncertainty risk in the ultimately realized outcome with the contract even if the contractual terms have been well obeyed by each of the contracting parties. Such risk of a supply chain contract occurs owing to various uncertainties inherent in the supply chain, such as demand uncertainty, price uncertainty, etc, and therefore is qualitatively different from the credit risk. Such risk of a supply chain contract is termed *value risk*. Value risk is obviously an important factor in design or analysis of a supply chain contract. In addition, individuals in supply chains with different risk preferences can have different risk attitudes towards the contract value risk, which in turn affects their contracting decisions. Motivated by these observations, Chap. 3 conducts a mean-risk analysis for the wholesale price contract with a supplier-retailer supply chain facing a stochastic price-dependent downward-sloping demand curve, taking into account contract value risk and degree of the retailer's risk-aversion towards the contract value risk. This study makes the first attempt to assess the efficiency of wholesale price contracts incorporating contract value risk. Formulating the problem under study as a supplier-led Stackelberg game, analytical results are developed in closed form in terms of only the retailer's risk preference parameter, and thereby some interesting managerial and academic insights are obtained. This research provides a new perspective of looking at the performance of a supply chain contract.

(3) Traditional manufacturing simply means production of tangible products, nowadays, however, customers are becoming much more demanding than ever and they impose captious requirements not only on the quality of the product itself but also on the services associated with the product. As a result of this change in the feature of customers, those firms that are capable of providing the service-enhanced products tend to achieve a more satisfactory customer service and hence capture a more market share than those that are only able to offer a pure product. As a result of this trend, the concept of product service system (PSS) becomes prevalent recently. In addition, in practice it is relatively common that the supply chain members have asymmetric information structures, that is, one of the supply chain members lacks some important piece

of information while the other possesses it. There are many reasons leading to this is so (see the literature review ahead). When information asymmetry exists among supply chain members, the optimal supply chain performance requires effective information sharing among the supply chain members as well as coordination of the individuals' actions, which leads to much more complexities for design and analysis of the supply chain contract. As an attempt to optimize the operations with contract for the recently emerging business mode, the PSS, under asymmetric information structure, Chap. 4 conducts a comprehensive study for the problem of how to provide an effective PSS with franchise fee contracts in the service-oriented manufacturing supply chain with demand information asymmetry. It is assumed that the PSS is operated heterogeneously and complementarily, in which the manufacturer provides a basic product while the retailer who possesses the private demand information is responsible for adding the necessary value-added service for the basic product. In this chapter, three types of contracts are developed to help the supply chain members make decisions and to enhance the supply chain operations efficiency. The first is the franchise fee (FF) contract , under which the manufacturer provides a two-part tariff contract including a wholesale price and a franchise fee to influence the retailer's decision and detect its private demand information. The second is the franchise fee with service requirement (FFS) contract, under which the manufacturer specifies the service level required in addition to the two-part tariff. The third is the franchise fee with centralized service requirement (FFCS) contract, which is similar to the FFS contract but that the service level specified by the manufacturer is the system-wide optimal solution. This chapter mainly addresses the issues of how to design the contract to assure a credible information sharing across the supply chain and how different these three forms of contracts affect the decisions and profitabilities of the supply chain members. The analytical studies show that all these three forms of contracts can assure a credible demand information sharing across the supply chain with the FFCS contract achieving the highest channel profit. In addition to analytical studies, numerical experiments are also presented and sensitivity analysis of the service level and profit are also conducted for comparing the performances of these three forms of contracts under different scenarios. This chapter contributes to the literature by developing systematical theoretical results for the optimization of the recently emerging business mode, i.e., the PSS, by franchise fee contracts in the service-oriented manufacturing supply chain with demand information asymmetry.

(4) Classical research of the option contract model has usually focused on the single directional option, namely the call option or the put option, with which order adjustment is allowed only upwards or only downwards. It has been shown that, however, if only upward adjustment is permitted, the option buyer tends to make a conservative order commitment, which leads to an increase on the channel shortage cost, whereas if only downward adjustment is available, the option buyer tends to make an aggressive order commitment that may lead to excess inventory in the supply chain, thus increasing the channel overstocking

cost. Furthermore, the biased orders under the single directional option may exacerbate the bullwhip effect in the supply chain (Wang and Tsao 2006, Lee et al. 1997). Hence, it is not yet in the best interest of the supply chain to allow the option buyer to adjust order only at a single direction. In addition, market may be so volatile that at the juncture to purchase the option the option buyer is not sure of which direction it may need to adjust the order quantity. In this case, bidirectional option allowing for the bidirectional adjustments of order may be more helpful than the single directional option in facilitating the supply chain operations. Motivated by these observations and insights, Chap. 5 extends the concept of single directional option to develop the bidirectional option contract model in which the option may be exercised as a call option and a put option. A two-echelon supply chain comprising one manufacturer and one retailer is considered for the problem under study. With a general demand distribution, the research characterizes the retailer's optimal order strategies including the initial order strategy and the option purchasing strategy in closed-form with the bidirectional option contract. Analytical examination of the feedback effects from introducing bidirectional option in the supply chain is also presented. In addition, a chain-wide perspective is taken to explore how the bidirectional option contract can be designed for supply chain coordination.

(5) In order to employ option contracts in practice, an issue that has to be addressed is to price the option in a reasonable way such that the pricing scheme is acceptable for all the contracting partners. For addressing this issue, an effort is made in Chap. 6 to explore the supply chain options pricing issue. In classical research, this issue is generally considered in the Stackelberg game framework. Such a pricing scheme, however, is usually unacceptable to the follower since it serves only the leader's interest. Being different from the existing literature, in this chapter a value-based pricing scheme is developed for the supply chain options with two model scenarios, i.e., a single retailer and multiple retailers. The intuition behind such a pricing scheme is to price the option based on the value inherent in the "option right". Furthermore, as shown in the research, such a pricing scheme can assure that each of the contracting partners is able to capture a share of the surplus derived from the option. As a result, the pricing schemes developed in this chapter are more objective and fair, and consequently are more likely to be accepted by all the contracting partners as compared with those that follow the Stackelberg game approach.

(6) As reviewed ahead, various forms of contracts have been developed to ensure supply chain coordination and allow arbitrary allocation of the resulting coordinating profit between the contracting partners. Note that it is essentially important for a coordinating contract form to enable arbitrary allocation of the coordinating profit, because this implies that there are always some coordinating contracts (a subset of all the coordinating contracts, possibly there is only a single contract in the subset) under which Pareto-improvement can be attained as compared with the decentralized case. Therefore, this property ensures that the coordinating contract form can be implemented with the satisfaction of individual rationality. However, there is still an issue that remains to be resolved:

since the extents to which individual contracting parties improve in profit are different with different contracts of this subset, it remains unclear how to select one that is acceptable to all the contracting partners from this subset. Obviously, the ultimate outcome will closely depend on risk preferences and negotiation powers of the contracting partners. As an attempt to address this issue, using the case of option contracts, Chap. 7 considers the selection issue of the coordinating option contracts with a cooperative game theory approach, taking into account risk preferences and negotiation powers of the supply chain members. The research developed with the option contract can be easily extended to other types of supply chain contracts that have been well studied in theory and widely adopted in practice, such as the buyback contract, the revenue-sharing contract, etc. In this sense, this research presents a theoretical modeling framework for the selection/implementation issue of supply chain contracts.

Chapter 2
Buyback Contracts with Price-Dependent Demands: Effects of Demand Uncertainty

2.1 Introduction

It is well-known that because of the effect of double marginalization, the wholesale price-only contracts often lead to some impairment in the efficiency of the supply chain facing uncertain end market demand. In order to mitigate this loss of efficiency, numerous other contracting mechanisms have been developed in supply chain management. Typical among these is the buyback mechanism, by which the retailer still pays a wholesale price for each unit ordered, but is allowed to return at the end of the selling season all or part of the unsold items to the supplier with a predetermined full or partial refund per unit. Buyback contracts have been exploited extensively in various retail sectors such as publishing, fashion apparels, computers, and cosmetics (Kandel 1996, Padmanabhan and Png 1995, 1997, Emmons and Gilbert 1998).

It is frequently observed in the retail industry that the retailer only has some knowledge (such as probabilistic knowledge) about the demand but not accurate and full information of the exact demand trend/curve. This situation arises when, e.g., the future (macro) market environment is uncertain (see Vaagen and Wallace (2008) for an illustration). Furthermore, the demand uncertainty level (DUL) often varies across different business settings, as reported by Nahmias and Smith (1994), it is common for the retail industry to observe a variability of from 3 to 300 in the variance-to-mean ratio of demand. Motivated by these observations in industry, in this chapter demand uncertainty is taken into account to explore its effects on the buyback contract with a supplier-retailer supply chain where for future demand, the retailer only knows the respective probabilistic price-dependent demand curve.

The research of this chapter is based on Zhao et al. (2014b).

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To existing research, a fundamental contribution of this chapter is the analytical examination of issue as how uncertainty level inherent in market demand affects the applicability of buyback contracts in supply chain management. For addressing this issue, in the research we seek to characterize the buyback contract model in terms of only demand uncertainty level (DUL). As shown by the studies, such a research perspective does allow us to develop some new analytical results and obtain some interesting and profound findings for buyback contracts. For instance, but not limited to, we have identified how the DUL relates to the buyback's efficiency and the analytical circumstances under which buyback increases the profit of the supplier, the retailer, or both and subsequently achieving Pareto-improvement, in a decentralized supply chain setting. With these explorations, some interesting new findings are obtained for buyback contracts. For example, we find that (i) even though the supply chain's efficiency will change over the DUL with a wholesale price-only contract, it will be maintained constantly with the provision of buyback regardless of the DUL; (ii) in the practice of buyback, buyback issuer should only adjust the buyback price in reaction to different DULs while leave the wholesale price unchanged as that in the corresponding deterministic demand setting; (iii) only in the demand setting with an intermediate level of uncertainty (which is identified quantitatively in Theorem 2.6.1 in the following), buyback provision is beneficial simultaneously for the supplier, the retailer, and the supply chain system, while it is not this case in the other demand setting; and vise versa.

In industry it can be observed that in the same business setting, some forms of contracts are exploited more often than another, or the same form of contracts is utilized more often in one specific business setting than in another one. Why is it this case? Even though we have never seen a systematical investigation of this issue, we note that some factors have been found to have significant effects on the applicability of supply chain contracts. For example, Marvel and Peck (1995) showed that the uncertainty type (they considered the valuation uncertainty and the consumer arrival number uncertainty) is one crucial factor. Cachon (2003) pointed out contract's administrative cost may also be one. With this study, it is demonstrated that DUL is another important dimension affecting the applicability of supply chain contracts. Of course, similar to the limitation existing in most theoretical modeling research, the results and findings derived in this chapter depend to a good extent on the model setup and it may be difficult to generalize them to a general case (which actually constitutes a challenging and significant issue for future research). Despite this acknowledged limitation, we believe that this research has revealed some important analytical closed-form properties of buyback contracts and made a good contribution to the related literature.

The remainder of this chapter is organized as follows: Sect. 2.2 reviews the relevant literature. Section 2.3 formulates the model. Section 2.4 characterizes the supply chain with wholesale price-only contracts. Section 2.5 characterizes the supply chain with buyback provision. Section 2.6 discusses the value of buyback for the respective supply chain members and the system. Section 2.7 examines the efficiency of buyback in coordinating the supply chain. Section 2.8 explores the effects of buyback on the retail price. Section 2.9 concludes the paper. All the proofs of the main results are put in the Appendix.

2.2 Literature Review

To highlight the contributions included in this chapter, the review will only focus on literature that is representative and the most closely related to this chapter (namely the literature to consider buyback contracts). For more detailed review of supply chain contracts, readers are referred to Chap. 1 and the excellent review papers by Lariviere (1999) and Cachon (2003).

The first related research stream examines buyback contracts in the classical price-taking newsvendor setting. Pasternack (1985) appears the first to explore the channel coordination issue with buyback contracts in this setting. He showed that (i) allowing full returns with full credit and allowing no returns are both channel suboptimal and (ii) there exists a continuum of coordinating full-returns policies with partial credit that is independent of the demand distribution in the end market. Furthermore, the resulting coordinating profit can be allocated arbitrarily by a proper choice of the contract terms in the continuum. A commentary of this paper is available in Pasternack (2008). More research of this related stream can be found in, e.g., Donohue (2000), and Tsay (2001), with some additional complexities in the model.

The second related research stream explores buyback contracts in a stochastic price-dependent demand setting. There is no buyback contract that can attain supply chain coordination in this setting (see Bernstein and Federgruen 2005 and Cachon 2003).[1] Hence, research of the buyback contract with this setting generally does not address the coordination issue, instead, it most analyzes the decentralized supply chain in a Stackelberg game framework. Emmons and Gilbert (1998) examined the effects of buyback on supply chain members' profits in a decentralized manufacturer-retailer supply chain with price-dependent multiplicative demand. Granot and Yin (2005) studied buyback in a similar framework. By assuming several types of deterministic demand functions, multiplied by a uniformly distributed random part, they analytically explored the Stackelberg equilibrium, the resulting supply chain members' profits, and the efficiency of buyback contracts. Song et al. (2008) integrated the various demand-specific insights on the buyback contract from Granot and Yin (2005) and other sources, and extended them to develop fairly general structural properties of the optimal buyback contract for price-dependent multiplicative demand setting. Different from the above reviewed papers, in this chapter we explore buyback contracts in a price-dependent additive demand setting. It is worth noting that the main results developed by the above mentioned papers for multiplicative demand setting generally cannot be extended to additive demand setting. As pointed out by Song et al. (2008), none of the major results developed by them with multiplicative demand remains valid for additive demand. Furthermore, according to Granot and Yin (2005), it is more challenging to deal with additive

[1] Note that despite so, a "price-discount sharing" scheme is shown to work for this setting in Bernstein and Federgruen (2005).

demand model than multiplicative demand model. Nevertheless, in this chapter, with an additive form of demand, we are able to develop the analytical results in closed-form and derive the respective insights for buyback contracts, and hence make a contribution to the literature.

The most related research to this chapter is Padmanabhan and Png (1997), which studied buyback in two market environments respectively, one is a competitive retail environment with deterministic demand curves and the other is an uncertain downward-sloping demand curve with no retail competition. For the first market environment, they showed buyback can increase the supplier's profit by intensifying the retail level competition. However, this result was disproved later by Wang (2004). Subsequently, Padmanabhan and Png (2004) returned to this problem and showed that this result holds only in the presence of demand uncertainty. For the second environment, they explored the conditions under which buyback can increase the supplier's profit. The research of this chapter is the most related to their studies of buyback for the second market environment, however, with some fundamental differences as follows: First, they explored buyback by assuming full returns with full credit, which means only one decision variable, namely the wholesale price, is involved in their buyback scheme. Even though this considerably simplifies their model analysis, it imposes a restriction on the strategy space of the supplier and consequently results in a suboptimal outcome (see the discussions following Theorems 2.5.3 and 2.6.1 for more details). Differentiated from them, this chapter considers a full returns scheme with partial or full credit, which involves two decision variables. We argue that such a change is essential, because, as shown by our study, their buyback model leads to a suboptimal outcome for the supplier, and our buyback model strictly dominates theirs from the supplier's perspective (even though our analysis is also much more complicated). Second, they looked at buyback only from the supplier's perspective, whereas this chapter examines buyback from both the supplier's and the retailer's perspectives, with an attempt to analytically explore the business circumstances under which Pareto-improvement can be obtained by using buyback. Third, this chapter seeks to characterize the supply chain with buyback in terms of only the DUL, so as to analytically explore how the uncertainty level embedded in demand affects the applicability of a supply chain contract. However, This is not the case in their studies. In fact, as indicated in Sect. 2.1, such a new research perspective does facilitate developing some interesting new findings for buyback contracts.

The third related research stream, different from the first two streams (including the research of this chapter), integrates another component, i.e., costly effort, in the analysis of buyback contracts by considering a newsvendor with stochastic effort-dependent demand. In such a setting, Taylor (2002) showed that a combinatorial use of buyback and rebate contracts can achieve supply chain coordination. Taylor and Xiao (2009) showed that buyback can be more effective than rebate in encouraging a retailer to improve demand forecasting.

Another different study is the one carried out by Marvel and Peck (1995). By incorporating two types of uncertainties in their model, namely, the uncertainty with respect to product valuation and the uncertainty with respect to number of customer arrivals, they showed that *only* the valuation uncertainty makes the supplier prefer the wholesale price-only contract, whereas *only* the arrivals uncertainty induces the supplier to offer buyback in its contracts. Their studies reveal that the type of uncertainty can be a significant factor influencing the applicability of a supply chain contract. The research of this chapter, with different focal point and model, demonstrates that the DUL can be another critical factor.

2.3 Model Formulation

Consider a two-echelon supply chain consisting of one supplier and one retailer. The retail demand is characterized by a price-dependent downward-sloping demand curve, given by $q = (m - p)/\delta$, where m is a random variable, $\delta > 0$, and q is the product quantity demanded at the retail price p, $0 \leq p \leq m$. Since m/δ is the demand when $p = 0$, it is the maximum potential market size. The model has two time periods: 0 and 1. At time 0, the retailer faces uncertainty in parameter m of the downward-sloping demand curve, which is distributed as follows:

$$m = \begin{cases} m_H & \text{with probability } \alpha, \\ m_L & \text{with probability } 1 - \alpha, \end{cases} \tag{2.3.1}$$

where $m_H > m_L \geq 0$ and $0 \leq \alpha \leq 1$. With this uncertainty, the retailer places its order denoted by Q at a unit wholesale price of w from the supplier at time 0. At time 1 the uncertainty in the demand curve is resolved. In response, the retailer decides to release a quantity q to the end market from the available amount Q ordered at time 0.

It should be pointed out that assumption (2.3.1) is very common in the literature (see, e.g., Burnetas and Ritchken 2005, Padmanabhan and Png 1997). Furthermore, (2.3.1) is equivalent to a linear deterministic demand function with an additive random part that follows a binary distribution, i.e., $q = \overline{m} - p + \epsilon$, where $\overline{m} = \alpha m_H + (1 - \alpha)m_L$ is the maximum potential market size at expectation and ϵ is a random variable characterized by

$$\epsilon = \begin{cases} (1 - \alpha)(m_H - m_L)/\delta \text{ with probability } \alpha, \\ -\alpha(m_H - m_L)/\delta \text{ with probability } 1 - \alpha. \end{cases} \tag{2.3.2}$$

Now, a wholesale price contract is specified with a buyback provision, by which to allow the demand risk to be shared between the channel partners and to mitigate the effect of double marginalization. To be specific, assume that the supplier will buy back all the items not sold by the end of the selling season at a refund of b per unit. In order to avoid the trivial cases, it should be required that $0 \leq b \leq w$. The Stackelberg framework is employed with the supplier as the leader and the retailer as the follower, for which the sequence of events is depicted in Fig. 2.1.

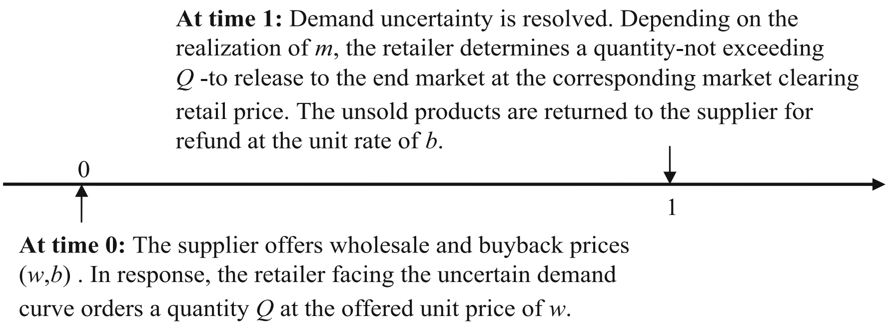

Fig. 2.1 Time line of the wholesale price contract with buyback provision

Based on the model, given the order quantity Q at time 0, the profit of the retailer in pursuing the strategy of releasing q_i $(i = H, L)$ at time 1 is

$$\Pi_i = q_i(m_i - \delta q_i) + b(Q - q_i). \tag{2.3.3}$$

Thus the retailer's problem at time 1 is to choose a quantity $q_i \leq Q$ that maximizes its profit function (2.3.3) to release to the market. At time 0, the retailer needs to decide the order quantity Q, allowing for uncertainty in the demand characterized by (2.3.1). The retailer's optimal order quantity Q at time 0 is obtained by maximizing its expected profit

$$\alpha\hat{\Pi}_H + (1-\alpha)\hat{\Pi}_L - wQ, \tag{2.3.4}$$

where $\hat{\Pi}_i$ is the maximum profit earned by the retailer at time 1 corresponding to the realization of m_i. As for the supplier, it will decide optimally the wholesale and buyback prices in anticipation of the retailer's optimal response. Let c be the supplier's unit product cost.

The purpose of the model is to explore the effects of buyback on the supply chain operations at different DULs. A usual measure of uncertainty level is the standard deviation

$$\begin{aligned}\sigma &= \sqrt{\alpha(m_H - \overline{m})^2 + (1-\alpha)(m_L - \overline{m})^2} \\ &= \sqrt{(1-\alpha)\alpha}(m_H - m_L),\end{aligned} \tag{2.3.5}$$

where

$$\overline{m} = \alpha m_H + (1-\alpha)m_L, \tag{2.3.6}$$

represents the expected maximum potential market size. Therefore, it is reasonable to require that $\overline{m} > c$. From (2.3.5) and (2.3.6), we obtain

$$\begin{cases} m_H = \overline{m} + \sqrt{\frac{1-\alpha}{\alpha}}\sigma, \\ m_L = \overline{m} - \sqrt{\frac{\alpha}{1-\alpha}}\sigma. \end{cases} \tag{2.3.7}$$

In order to explore how the DUL can affect the applicability of buyback, we keep $\overline{m}$ unchanged and allow σ to change in the analysis. It is easy to obtain from (2.3.7) and $m_L \geq 0$ that the maximum value of σ, for given $\overline{m}$, is

$$\overline{\sigma} = \sqrt{\tfrac{1-\alpha}{\alpha}}\overline{m}. \tag{2.3.8}$$

Hence, we confine σ in the range of $[0, \overline{\sigma}]$ in the following discussion.

2.4 Equilibrium with No Buyback

In this section we discuss the equilibrium strategies with the wholesale price-only contract. In particular, we analyze the equilibrium wholesale price and order quantity in terms of the parameter σ which reflects the uncertainty level in demand. An interesting special case is that of $\sigma = 0$, which we discuss first to obtain the benchmark for assessing the effects of the uncertainty level in demand. We denote by $\hat{w}$, the equilibrium wholesale price, and by $\hat{Q}$, the equilibrium order quantity, in the case of the deterministic demand curve given by $p = \overline{m} - \delta q$. Then, it is easy to obtain

$$(\hat{w}, \hat{Q}) = (\frac{\overline{m}+c}{2}, \frac{\overline{m}-c}{4\delta}). \tag{2.4.1}$$

Let $\hat{R}_s$ and $\hat{R}_r$ be the equilibrium profits of the supplier and the retailer respectively, i.e.,

$$(\hat{R}_s, \hat{R}_r) = [\frac{(\overline{m}-c)^2}{8\delta}, \frac{(\overline{m}-c)^2}{16\delta}]. \tag{2.4.2}$$

It is obvious that when $\sigma = 0$, the retailer will order at time 0 just the quantity that is optimal for releasing to the market at time 1. This observation does not carry over to the case of $\sigma > 0$, where the retailer might order a larger quantity in the hope that demand will be high and might have to release a part of the order only if the demand turns out to be low. It is on the case the research focuses next.

The following proceeds to explore the retailer's optimal order quantity $\overline{Q}_w$ at time 0 for a given wholesale price w. This problem can be solved as a two-stage dynamic program. First, to obtain the retailer's optimal release quantity at time 1 for each realization of the demand curve; then, to maximize the retailer's expected profit at time 0, by which to derive the following result.

Lemma 2.4.1. *For a given w, if $w \leq \overline{m}$, then*

$$\overline{Q}_w = \begin{cases} \frac{\alpha\overline{m}-w}{2\alpha\delta} + \frac{\sqrt{1-\alpha}}{2\delta\sqrt{\alpha}}\sigma, & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}w \leq \sigma \leq \overline{\sigma}; \\ \frac{\overline{m}-w}{2\delta}, & \text{if } 0 \leq \sigma < \sqrt{\frac{1-\alpha}{\alpha}}w. \end{cases} \tag{2.4.3}$$

Otherwise, $\overline{Q}_w = 0$ for all σ.

It is seen from Lemma 2.4.1 that the retailer always orders nothing regardless of the DUL if $w > \overline{m}$, which in turn leads to the supplier and the retailer both earning zero profit. For avoiding such uninteresting cases we confine w in the range of $[0, \overline{m}]$ in the following development of the model.

To obtain the equilibrium wholesale price and order quantity $(\overline{w}, \overline{Q})$, we must first find the optimal wholesale price for the supplier to charge in anticipation of the retailer's optimal response. Based on Lemma 2.4.1, the supplier's problem can be formulated as

$$\mathrm{P}_{2.1}: \quad \overline{w} = \arg\max_{w}\{\Pi_1, \Pi_2\}, \tag{2.4.4}$$

where

$$\begin{aligned} \Pi_1 &= \max_{0 \le w \le \sqrt{\frac{\alpha}{1-\alpha}}\sigma} (w-c)\left(\tfrac{\alpha\overline{m}-w}{2\alpha\delta} + \tfrac{\sqrt{1-\alpha}}{2\delta\sqrt{\alpha}}\sigma\right), \\ \Pi_2 &= \max_{\sqrt{\frac{\alpha}{1-\alpha}}\sigma \le w \le \overline{m}} (w-c)\left(\tfrac{\overline{m}-w}{2\delta}\right). \end{aligned} \tag{2.4.5}$$

By solving $\mathrm{P}_{2.1}$, the following result is derived to characterize the equilibrium with no buyback in terms of σ.

Theorem 2.4.2. *With no buyback:*

(i) *The equilibrium wholesale price and order quantity are given by the vector*

$$(\overline{w}, \overline{Q}) = \begin{cases} [\hat{w} - \frac{1-\alpha}{2}(\overline{m} - \sqrt{\frac{\alpha}{1-\alpha}}\sigma),\ \hat{Q} + \frac{1-\alpha}{4\alpha\delta}(\sqrt{\frac{\alpha}{1-\alpha}}\sigma - c)], & \text{if } \rho_1 \le \sigma \le \overline{\sigma}; \\ (\hat{w}, \hat{Q}), & \text{if } 0 \le \sigma < \rho_1, \end{cases} \tag{2.4.6}$$

where $(\hat{w}, \hat{Q})$*, given by* (2.4.1)*, is the equilibrium in the deterministic case of* $\sigma = 0$*, and*

$$\rho_1 = \frac{\sqrt{1-\alpha}}{\alpha + \sqrt{\alpha}}c + \frac{\sqrt{1-\alpha}}{1+\sqrt{\alpha}}\overline{m} < \overline{\sigma}. \tag{2.4.7}$$

(ii) *The quantity that is optimal for the retailer to release at time 1 is given by*

$$\overline{q}_{wH} = \overline{Q} \text{ for all } \sigma \in [0, \overline{\sigma}]. \tag{2.4.8}$$

in the case of the realization of m_H*, and by*

$$\overline{q}_{wL} = \begin{cases} \frac{\overline{m}}{2\delta} - \frac{\sqrt{\alpha}\sigma}{2\delta\sqrt{1-\alpha}}, & \text{if } \rho_1 \le \sigma \le \overline{\sigma}; \\ \overline{Q}, & \text{if } 0 \le \sigma < \rho_1, \end{cases} \tag{2.4.9}$$

in the case of the realization of m_L*.*

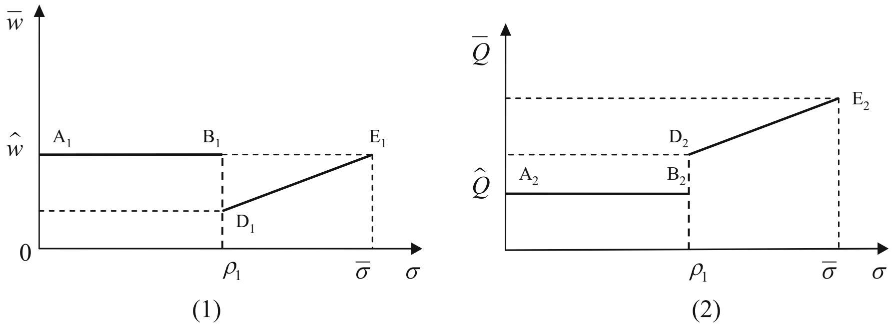

Fig. 2.2 Effects of σ on equilibrium with wholesale price-only contract

Figure 2.2 graphically illustrates how the DUL affects the equilibrium with the wholesale price-only contract. An interesting result revealed in Theorem 2.4.2(i) and Fig. 2.2 is that there exists a threshold DUL, given by ρ_1, under which the equilibrium keeps unchanged as the same to that in the corresponding deterministic demand case, above which the equilibrium wholesale price is always lower than $\hat{w}$ and the equilibrium order quantity is always larger than $\hat{Q}$. A closer look at this result reveals that double marginalization effect is alleviated to some extent when the uncertainty in demand is relatively high. An explanation for this result is as follows: when the DUL is relatively low, the supplier's and the retailer's decisions will not be affected at equilibrium and they still make the decisions as in the deterministic demand case. However, with the DUL increasing to a certain degree, the decision robustness is destroyed and equilibrium will change. Due to that stockout cost increases with the relatively high DUL, and for hedging against the increased stockout cost, it is beneficial for the supplier to decrease the wholesale price and for the retailer to increase the ordering quantity in response, which in turn leads to an alleviation of the double marginalization effect in the channel.

In addition, from Theorem 2.4.2(ii) it is seen that ρ_1 also serves as the threshold DUL, above which the retailer has an incentive to withhold a part of its order from the market at time 1 for a more profitable retail price. From (2.4.8) and (2.4.9), it is easy to obtain the expected quantity for the retailer to withhold from the market at equilibrium, which is given by

$$\overline{N}_w = \begin{cases} [\frac{1-\alpha^2}{4\delta\sqrt{\alpha(1-\alpha)}}]\sigma - \frac{(1-\alpha)(\alpha\overline{m}+c)}{4\alpha\delta}, & \text{if } \rho_1 \le \sigma \le \overline{\sigma}; \\ 0, & \text{if } 0 \le \sigma < \rho_1. \end{cases} \tag{2.4.10}$$

Actually, it can be widely observed in industries for the practice to withhold inventory from the market for an improved operations or/and profitability. For example, in the luxury fashion industry, many high-end luxury fashion brands do not sell all inventory at once after start of the selling season, instead, they withhold some of the inventory. The logic behind doing so is that they don't want the market to feel that there are plenty of inventories for the items which will cheapen the value of the products and hence advocate the strategic consumer behaviors of waiting for discount and mark-down (Tereyağoğlu and Veeraraghavan 2012). In the mass market, many fast fashion companies often also withhold inventory for a very limited product availability at the retail sales floor. The logic with such a strategy lies in that it can encourage consumer impulse purchase and reduce inventory carrying cost at the retail sales floor (Cachon and Swinney 2011, Choi 2013a).

The following proceeds to discuss the expected profits obtained by the retailer and the supplier at equilibrium, which we denote by $\overline{R}_r$ and $\overline{R}_s$, respectively. Then, based on Theorem 2.4.2(i), the following results can be obtained.

Theorem 2.4.3. *With no buyback:*

(i) *The supplier's expected profit at equilibrium is*

$$\overline{R}_s = \begin{cases} \hat{R}_s + \frac{H_1(\sigma)}{8\alpha\delta}, & \text{if } \rho_1 \le \sigma \le \overline{\sigma}; \\ \hat{R}_s, & \text{if } 0 \le \sigma < \rho_1, \end{cases} \tag{2.4.11}$$

where $\hat{R}_s$, given by (2.4.2), is the supplier's profit at equilibrium in the deterministic case of $\sigma = 0$, and

$$H_1(\sigma) = (\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2 - \alpha(\sqrt{1-\alpha}\overline{m} - \sqrt{\alpha}\sigma)^2, \tag{2.4.12}$$

which is non-negative and strictly increasing in σ on $[\rho_1, \overline{\sigma}]$ for arbitrarily given $\alpha \in (0, 1)$ and $\overline{m}$.

(ii) *The retailer's expected profit at equilibrium is*

$$\overline{R}_r = \begin{cases} \hat{R}_r + \frac{H_2(\sigma)}{16\alpha\delta}, & \text{if } \rho_1 \le \sigma \le \overline{\sigma}; \\ \hat{R}_r, & \text{if } 0 \le \sigma < \rho_1, \end{cases} \tag{2.4.13}$$

where $\hat{R}_r$, given by (2.4.2), is the retailer's profit at equilibrium in the deterministic case of $\sigma = 0$, and

$$H_2(\sigma) = (\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2 + 3\alpha(\sqrt{1-\alpha}\overline{m} - \sqrt{\alpha}\sigma)^2, \tag{2.4.14}$$

which is positive for all σ, and strictly decreasing in σ on $[\rho_1, \rho_2]$ but strictly increasing in σ on $[\rho_2, \overline{\sigma}]$ for arbitrarily given $\alpha \in (0, 1)$ and $\overline{m}$, where

$$\rho_2 = \frac{\sqrt{1-\alpha}}{\sqrt{\alpha}(3\alpha+1)}c + \frac{3\sqrt{\alpha(1-\alpha)}}{3\alpha+1}\overline{m} < \overline{\sigma}. \tag{2.4.15}$$

Figure 2.3 graphically illustrates how the expected profits of the supplier and the retailer at equilibrium are affected by the DUL. A surprising result is that the expected profits of both the supplier and the retailer at equilibrium are always larger in a relatively high DUL (i.e., $\rho_1 < \sigma < \overline{\sigma}$) than in a relatively low DUL (i.e., $0 \le \sigma < \rho_1$). In fact, this is just an outcome resulted from the alleviation of

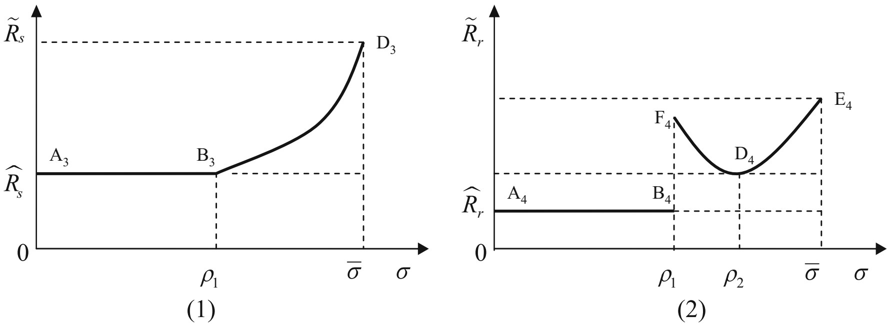

Fig. 2.3 Effects of σ on equilibrium expected profits with wholesale price-only contract

double marginalization effect in the supply chain. To be specific, it can be known from Theorem 2.4.2(i) that, as compared with the case of $0 \le \sigma < \rho_1$, the effect of double marginalization is reduced in the case of $\rho_1 < \sigma < \overline{\sigma}$, and therefore the supply chain efficiency increases from which both the supplier and the retailer benefit.

2.5 Equilibrium with Buyback

In the following sections, we examine the case where the supplier offers the retailer an additional chance of returning all the quantity withheld from the market at a refund of b per unit. We mainly explore four issues in terms of the uncertainty level of demand: (i) the equilibrium buyback contract and order quantity, (ii) the expected profits of the supplier and the retailer at equilibrium, (iii) the value of buyback for the supplier and the retailer respectively, and its efficiency in coordinating the supply chain, and (iv) the effect of buyback on the retail price. With these explorations, we expect to see the effects of buyback and how the uncertainty level of demand influences these effects.

2.5.1 Retailer's Optimal Response with Buyback

We first obtain the optimal response of the retailer, given a buyback contract, say (w, b), where w is the wholesale price and b is the buyback price with $w \ge b \ge 0$. As in the case of no buyback, this problem can be solved by two-stage dynamic programming: First, we analyze the retailer's optimal response at time 1, given the realized demand curve. Then, we analyze the retailer's optimal response at time 0 based on its optimal response at time 1, given the buyback contract (w, b). The solution for this problem is summarized in the following lemma.

Lemma 2.5.1. *Given* (w, b)*, the optimal order quantity of the retailer at time* 0 *is*

$$\tilde{Q}_b = \begin{cases} \frac{\sqrt{1-\alpha}}{2\delta\sqrt{\alpha}}\sigma + \frac{\alpha\overline{m}+(1-\alpha)b-w}{2\alpha\delta}, & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}(w-b) \le \sigma \le \overline{\sigma}; \\ \frac{\overline{m}-w}{2\delta}, & \text{if } 0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}(w-b). \end{cases} \tag{2.5.1}$$

2.5.2 Equilibrium Buyback Contract and Order Quantity

In anticipation of the retailer's optimal response given in Lemma 2.5.1, the supplier decides optimally the wholesale and buyback prices. In particular, from Lemma 2.5.1 and its proof we know that, given $0 \le \sqrt{\frac{1-\alpha}{\alpha}}(w-b) \le \sigma$, the supplier's problem is:

$$\tilde{M}_{s1} = \max_{0 \le \sqrt{\frac{1-\alpha}{\alpha}}(w-b) \le \sigma} E\Pi_{bs1}(w, b), \tag{2.5.2}$$

where

$$\begin{aligned} E\Pi_{bs1}(w, b) &= \alpha(w-c)\left[\frac{\sqrt{1-\alpha}}{2\delta\sqrt{\alpha}}\sigma + \frac{\alpha\overline{m}+(1-\alpha)b-w}{2\alpha\delta}\right] \\ &\quad + (1-\alpha)\left[(w-c)\left(\frac{\sqrt{1-\alpha}}{2\delta\sqrt{\alpha}}\sigma + \frac{\alpha\overline{m}+(1-\alpha)b-w}{2\alpha\delta}\right)\right. \\ &\quad \left. - b\left(\frac{\sqrt{1-\alpha}}{2\delta\sqrt{\alpha}}\sigma + \frac{\alpha\overline{m}+(1-\alpha)b-w}{2\alpha\delta} - \frac{\overline{m}-\sqrt{\frac{\alpha}{1-\alpha}}\sigma-b}{2\delta}\right)\right] \\ &= (w-c)\left[\frac{\sqrt{1-\alpha}}{2\delta\sqrt{\alpha}}\sigma + \frac{\alpha\overline{m}+(1-\alpha)b-w}{2\alpha\delta}\right] \\ &\quad - b(1-\alpha)\left[\frac{\sqrt{1-\alpha}}{2\delta\sqrt{\alpha}}\sigma + \frac{\alpha\overline{m}+(1-\alpha)b-w}{2\alpha\delta} - \frac{\overline{m}-\sqrt{\frac{\alpha}{1-\alpha}}\sigma-b}{2\delta}\right]. \end{aligned} \tag{2.5.3}$$

Given $\sigma < \sqrt{\frac{1-\alpha}{\alpha}}(w-b) \le \sqrt{\frac{1-\alpha}{\alpha}}\overline{m} = \overline{\sigma}$, the supplier's problem is:

$$\tilde{M}_{s2} = \max_{\sigma < \sqrt{\frac{1-\alpha}{\alpha}}(w-b) \le \overline{\sigma}} E\Pi_{bs2}(w, b) = (w-c)\left(\frac{\overline{m}-w}{2\delta}\right). \tag{2.5.4}$$

To summarize, the equilibrium buyback contract denoted by $(\tilde{w}, \tilde{b})$ is determined by solving

$$\text{P}_{2.2}: \quad (\tilde{w}, \tilde{b}) = \arg\max_{(w,b)}\{\tilde{M}_{s1}, \tilde{M}_{s2}\}. \tag{2.5.5}$$

Its solution yields the equilibrium with buyback stated below.

Theorem 2.5.2. *With buyback:*

(i) *The equilibrium contract is*

$$(\tilde{w}, \tilde{b}) = \begin{cases} (\hat{w},\ \frac{\overline{m}}{2} - \sqrt{\frac{\alpha}{4(1-\alpha)}}\sigma), & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \le \sigma \le \overline{\sigma}; \\ (\hat{w}, b^*) \text{ with } 0 \le b^* \le \frac{\overline{m}+c}{2} - \sqrt{\frac{\alpha}{1-\alpha}}\sigma, & \text{if } 0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c, \end{cases} \tag{2.5.6}$$

where $\hat{w} = \frac{\overline{m}+c}{2}$ *is the equilibrium wholesale price in the deterministic case of* $\sigma = 0$.

(ii) *The equilibrium order quantity at time* 0 *is*

$$\tilde{Q}_b = \begin{cases} \hat{Q} + \frac{1-\alpha}{4\alpha\delta}(\sqrt{\frac{\alpha}{1-\alpha}}\sigma - c), & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \le \sigma \le \overline{\sigma}; \\ \hat{Q}, & \text{if } 0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c, \end{cases} \tag{2.5.7}$$

where $\hat{Q} = \frac{\overline{m}-c}{4\delta}$ *is the equilibrium order quantity in the deterministic case of* $\sigma = 0$.

(iii) *The optimal quantity for the retailer to release at time* 1*, when* m_H *is realized, is*

$$\tilde{q}_{bH} = \tilde{Q}_b \text{ for all } \sigma \in [0, \overline{\sigma}], \tag{2.5.8}$$

and, when m_L *is realized, is*

$$\tilde{q}_{bL} = \begin{cases} \frac{\overline{m}}{4\delta} - \frac{\sqrt{\alpha}\sigma}{4\delta\sqrt{1-\alpha}}, & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \le \sigma \le \overline{\sigma}; \\ \tilde{Q}_b, & \text{if } 0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c. \end{cases} \tag{2.5.9}$$

Figure 2.4 graphically illustrates how the equilibrium with buyback provision is affected by the DUL. It is seen from Theorem 2.5.2(i) and Fig. 2.4(1) that with the use of buyback, regardless of the DUL, at equilibrium the supplier does not

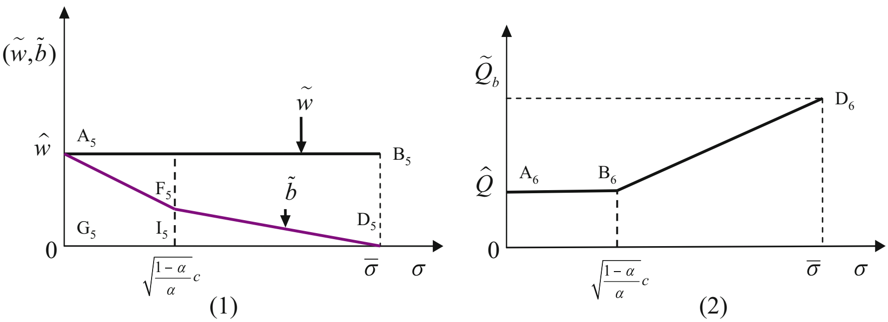

Fig. 2.4 Effects of σ on equilibrium with buyback provision

deviate from the equilibrium wholesale price associated with the corresponding deterministic demand case, and it only needs to adjust the buyback price downwards in reaction to the increased DUL. This is significantly different from the case of no buyback, where the supplier decreases the wholesale price at equilibrium in response to a high DUL (i.e., $\rho_1 < \sigma < \overline{\sigma}$). A closer look at this result reveals that in the negotiation of setting buyback contracts, the supplier should bargain with the retailer only on the buyback price, while leave the wholesale price unchanged as that in the corresponding deterministic demand case. One explanation for this result is that in the case with buyback, it is enough for the supplier to mitigate the effects of different levels of demand uncertainty by adjusting the buyback price only, and there is no necessity to change the wholesale price at equilibrium. Note that $(\hat{w}, 0)$ is always one of the equilibrium buyback contracts when σ increases from 0 to $\sqrt{\frac{1-\alpha}{\alpha}}c$, which means that for the supplier there is no difference between the cases with buyback and with no buyback when the DUL is lower than $\sqrt{\frac{1-\alpha}{\alpha}}c$.

As to the retailer, in response to the equilibrium buyback contract offered by the supplier, its equilibrium order quantity remains unchanged at $\hat{Q}$ until σ reaches $\sqrt{\frac{1-\alpha}{\alpha}}c$. Thereafter, it increases strictly in σ. A counterintuitive result is that the retailer's order quantity increases strictly as the buyback price decreases. A second thought on this result reveals its behind reasonability that it is just a balanced outcome between the effects resulted from a decreased buyback price and an increased DUL. To be specific, on one hand, a decreased buyback price will make the retailer reduce its order quantity; on the other hand, however, an increased DUL will push the retailer to increase its order quantity by which to hedge against the increased demand uncertainty risk. With a balance of the effects from these two aspects, it is seen that the order quantity of the retailer seems to increase as the buyback price decreases.

By comparing Theorems 2.4.2(i) and 2.5.2(ii), It is seen that (1) the equilibrium in the deterministic demand case carries over to some cases of $\sigma > 0$ (i.e., $0 < \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c$), that is, despite having some demand uncertainty in these cases, buyback actually has no role for the supply chain. (2) Only at the intermediate DUL (i.e., $\sqrt{\frac{1-\alpha}{\alpha}}c < \sigma < \rho_1$) buyback provision pushes the retailer to order more, otherwise there is no effect on the order size. This result is somewhat counterintuitive, because it is often claimed that giving an additional buyback provision always induces the retailer to order more (Katok and Wu 2009, p. 1958). In addition, intuitively, the retailer has a stronger incentive to withhold products from the market under a buyback provision, as compared to the case of no buyback. By Theorem 2.5.2(iii), we obtain the expected quantity for the retailer to withhold from the market at equilibrium (in fact, he returns the quantity to the supplier) with buyback as follows:

$$\tilde{N}_b = \begin{cases} [\frac{1-\alpha^2}{4\delta\sqrt{\alpha(1-\alpha)}}]\sigma - \frac{(1-\alpha)(\alpha\overline{m}+c)}{4\alpha\delta} + \frac{1-\alpha}{4\delta}(\overline{m} - \sqrt{\frac{\alpha}{1-\alpha}}\sigma), & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \le \sigma \le \overline{\sigma}; \\ 0, & \text{if } 0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c. \end{cases} \tag{2.5.10}$$

Comparing (2.5.10) with (2.4.10), we see that buyback does encourage the retailer to withhold products from the market for each $\sigma \in [\sqrt{\frac{1-\alpha}{\alpha}}c, \overline{\sigma})$. Furthermore, it is clear that the use of buyback decreases the threshold DUL, above which the retailer has an incentive to withhold products from the market. This gives, once again, the intuitive result that the retailer has a stronger incentive to withhold products from the market when it can return products to the supplier.

Theorem 2.5.3. *With buyback:*

(i) *The supplier's expected profit at equilibrium is*

$$\tilde{R}_s = \begin{cases} \hat{R}_s + \frac{1}{8\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2, & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \le \sigma \le \overline{\sigma}; \\ \hat{R}_s, & \text{if } 0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c, \end{cases} \tag{2.5.11}$$

where $\hat{R}_s$, given by (2.4.2), is the supplier's profit at equilibrium in the deterministic case of $\sigma = 0$.

(ii) *The retailer's expected profit at equilibrium is*

$$\tilde{R}_r = \begin{cases} \hat{R}_r + \frac{1}{16\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2, & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \le \sigma \le \overline{\sigma}; \\ \hat{R}_r, & \text{if } 0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c, \end{cases} \tag{2.5.12}$$

where $\hat{R}_r$, given by (2.4.2), is the retailer's profit at equilibrium in the deterministic case of $\sigma = 0$.

Figure 2.5 graphically illustrates how the expected profits of the supplier and the retailer are affected by the DUL at equilibrium with buyback. We see from Theorem 2.5.3 and Fig. 2.5 that the expected profits of the supplier and the retailer at equilibrium are maintained constantly at $\hat{R}_s$ and $\hat{R}_r$, respectively, until the DUL

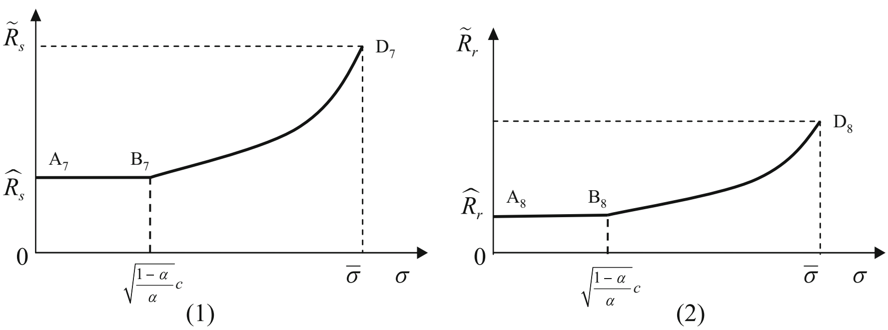

Fig. 2.5 Effects of σ on equilibrium expected profits with buyback provision

reaches the value of $\sqrt{\frac{1-\alpha}{\alpha}}c$. Thereafter, their expected profits increase strictly with the DUL increasing. A closer look at this result reveals its behind driver as follows: At a relatively low DUL (i.e., $0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c$), buyback actually plays no role on the supply chain. With the DUL increasing, however, buyback can facilitate the supply chain operations with remedying to some extent the channel double marginalization effect. Thereby, the supply chain efficiency is enhanced from which both the supplier and the retailer benefit.

Padmanabhan and Png (1997) considered unlimited returns with full credit from the supplier's perspective with a model equivalent to ours. We mentioned in Sect. 2.2 that the buyback scheme considered by them results in a suboptimal outcome for the supplier. With the help of some calculations in the Appendix, we can conclude that the supplier's profit given by Padmanabhan and Png (1997) comes to

$$\hat{R}_s - \frac{4c\sqrt{1-\alpha}}{8\delta\sqrt{\alpha}}\sigma. \tag{2.5.13}$$

By comparing this with (2.5.11), we see that the expected profit obtained by the supplier at equilibrium with the buyback scheme considered in Padmanabhan and Png (1997) is strictly less than that with the buyback scheme considered in our research for all $\sigma > 0$. Hence, their buyback model is strictly dominated by ours from the supplier's perspective.

2.6 Value of Buyback Contract

This section explores the value of buyback for the supplier and the retailer at different DULs. In particular, it is intuitive that for the deterministic case of $\sigma = 0$, the value of buyback is zero for both the supplier and the retailer, because in this case the retailer knows accurately the demand and will therefore order only the quantity that is optimal for releasing to the market. Further discussion in the following shows that this observation even carries over to some cases of $\sigma > 0$.

Denote by V_{sb} and V_{rb} the values of buyback for the supplier and the retailer, respectively. Then, by comparing Theorems 2.4.3 and 2.5.3, we derive directly the following results quantifying V_{sb} and V_{rb} in terms of σ.

Theorem 2.6.1. *The range of uncertainty in the demand for which both the supplier and the retailer benefit from buyback is*

$$\sqrt{\frac{1-\alpha}{\alpha}}c < \sigma < \rho_1. \tag{2.6.1}$$

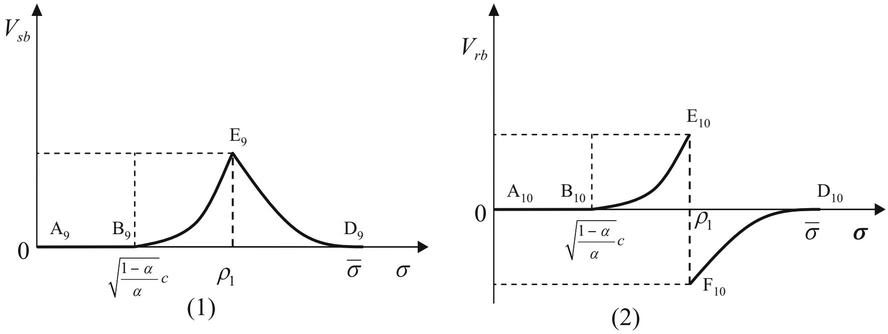

Fig. 2.6 Effects of σ on the value of buyback

In particular,

(i) *The value of buyback for the supplier is*

$$V_{sb} = \begin{cases} \frac{(1-\alpha)}{8\delta}(\overline{m} - \sqrt{\frac{\alpha}{1-\alpha}}\sigma)^2, & \text{if } \rho_1 \le \sigma \le \overline{\sigma}; \\ \frac{1}{8\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2, & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \le \sigma < \rho_1; \\ 0, & \text{if } 0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c. \end{cases} \tag{2.6.2}$$

(ii) *The value of buyback for the retailer is*

$$V_{rb} = \begin{cases} \frac{-3(1-\alpha)}{16\delta}(\overline{m} - \sqrt{\frac{\alpha}{1-\alpha}}\sigma)^2, & \text{if } \rho_1 \le \sigma \le \overline{\sigma}; \\ \frac{1}{16\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2, & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \le \sigma < \rho_1; \\ 0, & \text{if } 0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c. \end{cases} \tag{2.6.3}$$

Figure 2.6 graphically illustrates how the values of buyback are affected by the DUL. It is seen from Theorem 2.6.1 and Fig. 2.6 that the supplier never suffers a loss with the use of buyback regardless of the DUL. Furthermore, it benefits when the DUL is not too low (higher than $\sqrt{\frac{1-\alpha}{\alpha}}c$). However, the retailer benefits from buyback only at an intermediate DUL (i.e., $\sqrt{\frac{1-\alpha}{\alpha}}c < \sigma < \rho_1$). As a result, there exists a range of DUL in which Pareto-improvement can be achieved using buyback. This result reveals the demand circumstances where buyback is acceptable to both the supplier and the retailer in a decentralized setting. As mentioned in Sect. 2.2, Marvel and Peck (1995) demonstrated that the type of uncertainty can be a factor influencing the applicability of supply chain contracts. Cachon (2003) pointed out that, to some extent, the contract's administrative cost may be utilized to explain why a supply chain contract form can be observed in practice over another. Despite our model does not consider the administrative cost and uncertainty type issues, Theorem 2.6.1 indicates that DUL can be another critical factor affecting the applicability of supply chain contracts. To be specific, it can be observed from Theorem 2.6.1 that it is easier for buyback contracts to be applied in the market circumstance with an intermediate DUL (because in which Pareto-improvement can be obtained even under a decentralized setting).

A closer look at this result reveals its explanations as follows: when the DUL is relatively low, it is obvious that buyback actually has no value for both the supplier and the retailer; when the DUL is relatively high, for hedging against high demand unceratainty risk, the supplier, leaving the wholesale price unchanged, has to offer a buyback price that is so low that the retailer becomes worse than that without buyback; when the DUL is intermediate, even though the supplier also tries to capture the channel profit by offering a selfish buyback price, at equilibrium the buyback price is still in a range that can guarantee the retailer receiving some benefits from buyback, and thus Pareto-improvement takes place under buyback.

Padmanabhan and Png (1997) showed that the supplier's expected profit is strictly greater with a returns policy than with a no returns policy if $c = 0$ and a condition equivalent to

$$\sigma < \frac{\sqrt{1-\alpha}}{1+\sqrt{\alpha}}\overline{m} \tag{2.6.4}$$

holds (see the Appendix for the details). From (2.6.2), we see that, if $c = 0$, then (2.6.2) implies the supplier's expected profit at equilibrium is strictly greater with a returns policy than without for each $\sigma \in (0, \overline{\sigma}]$ in our buyback model, where we see that $\overline{\sigma} = \sqrt{\frac{1-\alpha}{\alpha}}\overline{m} > \frac{\sqrt{1-\alpha}}{1+\sqrt{\alpha}}\overline{m}$. Furthermore, the corresponding expected profit received by the supplier at equilibrium also strictly dominates that in the setting of Padmanabhan and Png (1997)'s buyback model by (2.5.11) and (2.5.13). This, together with the discussion following Theorem 2.5.3, further indicates that the buyback scheme considered in Padmanabhan and Png (1997) results in a suboptimal outcome for the supplier.

2.7 Efficiency of Buyback

It is known that buyback contract alone cannot achieve supply chain coordination in the setting of a price-dependent newsvendor, which our model shares. In this section, we examine the efficiency of buyback in coordinating the supply chain. To do this, we first need to derive the system-wide optimal profit for the supply chain. This problem can be solved by taking the supplier and the retailer collectively as a centralized entity. The corresponding results are summarized in the following lemma, where we denote by $\tilde{Q}_c$ the system-wide optimal product quantity and by $\tilde{R}_c$ the system-wide optimal expected profit for the supply chain.

Lemma 2.7.1. (i) *The system-wide optimal decision for the supply chain is*

$$\tilde{Q}_c = \begin{cases} 2[\hat{Q} + \frac{1-\alpha}{4\alpha\delta}(\sqrt{\frac{\alpha}{1-\alpha}}\sigma - c)], & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \le \sigma \le \overline{\sigma}; \\ 2\hat{Q}, & \text{if } 0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c, \end{cases} \tag{2.7.1}$$

where $\hat{Q}$, given by (2.4.1), is the equilibrium order quantity in the deterministic case of $\sigma = 0$.

(ii) *The system-wide optimal expected profit for the supply chain is*

$$\tilde{R}_c = \begin{cases} 4\hat{R}_r + \frac{1}{4\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2, & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \le \sigma \le \overline{\sigma}; \\ 4\hat{R}_r, & \text{if } 0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c, \end{cases} \tag{2.7.2}$$

where $\hat{R}_r$, given by (2.4.2), is the retailer's profit at equilibrium in the deterministic case of $\sigma = 0$.

In what follows we assess the efficiency of buyback when used in our model. To more clearly see the effect of buyback on the system efficiency, we also give the system efficiency with no buyback so that we can compare them. We define here the system efficiency as the ratio of the decentralized system's total expected profit to the system-wide optimal expected profit. The corresponding results are summarized in the following theorem, where we denote by EFF_b the efficiency of the supply chain system with buyback and by EFF_w the efficiency with no buyback.

Theorem 2.7.2. *Compared with the case of no buyback, the efficiency of the supply chain system with buyback is the same when $0 \le \sigma \le \sqrt{\frac{1-\alpha}{\alpha}}c$, is strictly higher when $\sqrt{\frac{1-\alpha}{\alpha}}c < \sigma < \rho_1$, and is strictly lower when $\rho_1 \le \sigma < \overline{\sigma}$. In particular,*

(i) *The efficiency of the supply chain with buyback is*

$$EFF_b = 0.75 \text{ for all } \sigma \in [0, \overline{\sigma}]; \tag{2.7.3}$$

(ii) *The efficiency of the supply chain with no buyback is*

$$EFF_w = \begin{cases} 0.75 + \frac{(\sqrt{1-\alpha}\overline{m} - \sqrt{\alpha}\sigma)^2}{4[(\overline{m}-c)^2 + (\sigma - \sqrt{\frac{1-\alpha}{\alpha}}c)^2]}, & \text{if } \rho_1 \le \sigma \le \overline{\sigma}; \\ 0.75 - \frac{3(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2}{4[\alpha(\overline{m}-c)^2 + (\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2]}, & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \le \sigma < \rho_1; \\ 0.75, & \text{if } 0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c. \end{cases} \tag{2.7.4}$$

Figure 2.7 graphically illustrates how the efficiencies of the supply chain with buyback and with no buyback are affected by the DUL. It is easy to know that for the deterministic case of $\sigma = 0$, the efficiency of the supply chain system is 75 % of the system-wide optimal expected profit. From Theorem 2.7.2 and Fig. 2.7, it is seen

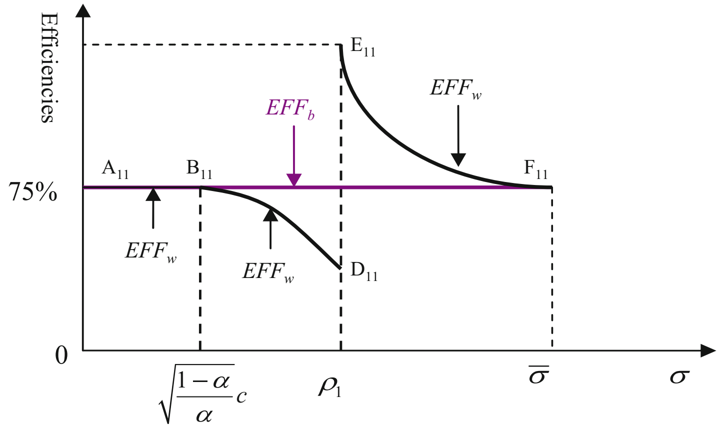

Fig. 2.7 Effects of σ on efficiencies of supply chain with buyback and with no buyback

that the use of buyback can maintain the system efficiency constantly at this level regardless of the DUL, as indicated by the color line segment A_{11}-F_{11} in Fig. 2.7. However, the system efficiency with a wholesale price-only contract can be higher or lower than 75 %, which closely depends on the DUL and changes over different DULs.

Note that even though the system efficiency, 75 %, may only be a result of our specific model setup, it is interesting to find that the system efficiency will maintain constantly at the level of the corresponding deterministic demand case with buyback, while this is not the case with the wholesale price-only contracts, for which the system efficiency changes over different DULs. The behind driver leading to this result is the role of buyback. To be specific, buyback can mitigate the effects of different DULs on the system and thereby maintain the system efficiency as the same to that in the deterministic demand case of $\sigma = 0$. In addition, it is clear by Theorem 2.7.2 that, looking at buyback from the system's perspective, only in the DUL range given by $\sqrt{\frac{1-\alpha}{\alpha}}c < \sigma < \rho_1$ can buyback contracts beat wholesale price-only contracts. In all the other cases, buyback actually plays no role on facilitating the system operations. Moreover, it is even detrimental to the system when $\rho_1 < \sigma < \overline{\sigma}$, the case with a relatively high DUL.

2.8 Effect of Buyback on Retail Price

This section explores the effects of buyback on the end consumers through examining its effects on the expectation and standard deviation (SD) of the retail price at different DULs. The main results are summarized in the following theorem, where we denote by $\overline{p}_w$ and $\overline{\sigma}_p$ the expectation and SD of the retail price with no buyback, and by $\tilde{p}_b$ and $\tilde{\sigma}_p$ the expectation and SD with buyback, respectively.

Theorem 2.8.1. *Compared with the case of no buyback:*

(i) *The expectation of the retail price with buyback is strictly higher when $\rho_1 \leq \sigma < \overline{\sigma}$, is the same when $0 \leq \sigma < \rho_1$. In particular, with buyback, $\tilde{p}_b = \frac{3\overline{m}+c}{4}$ for all σ, and with no buyback,*

$$\overline{p}_w = \begin{cases} \frac{3\overline{m}+c}{4} - \frac{1-\alpha}{4}(\overline{m} - \sqrt{\frac{\alpha}{1-\alpha}}\sigma), & \text{if } \rho_1 \leq \sigma \leq \overline{\sigma}; \\ \frac{3\overline{m}+c}{4}, & \text{if } 0 \leq \sigma < \rho_1. \end{cases} \tag{2.8.1}$$

(ii) *The SD of the retail price with buyback is strictly lower when $\sqrt{\frac{1-\alpha}{\alpha}}c < \sigma < \overline{\sigma}$, is the same when $0 \leq \sigma \leq \sqrt{\frac{1-\alpha}{\alpha}}c$. In particular, with buyback,*

$$\tilde{\sigma}_p = \begin{cases} \frac{3}{4}\sigma + \frac{c}{4}\sqrt{\frac{1-\alpha}{\alpha}}, & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \leq \sigma \leq \overline{\sigma}; \\ \sigma, & \text{if } 0 \leq \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c, \end{cases} \tag{2.8.2}$$

and with no buyback,

$$\overline{\sigma}_p = \begin{cases} \frac{3}{4}\sigma + \frac{c}{4}\sqrt{\frac{1-\alpha}{\alpha}} + \frac{1}{4}\alpha(\overline{\sigma} - \sigma), & \text{if } \rho_1 \leq \sigma \leq \overline{\sigma}; \\ \sigma, & \text{if } 0 \leq \sigma < \rho_1. \end{cases} \tag{2.8.3}$$

Figure 2.8 graphically depicts how the expectation and SD of the retail price are affected by the DUL. From Theorem 2.8.1 and Fig. 2.8, the effects of buyback on the expectation and SD of the retail price can be summarized as follows: (i) increasing the expectation of the retail price while decreasing its SD, i.e., its fluctuation level, at a relatively high DUL (i.e., $\rho_1 < \sigma < \overline{\sigma}$), (ii) keeping the expectation of the retail price unchanged while decreasing its fluctuation level at an intermediate DUL (i.e., $\sqrt{\frac{1-\alpha}{\alpha}}c < \sigma < \rho_1$), and (iii) having no effect on the both at a relatively low DUL (i.e., $0 \leq \sigma \leq \sqrt{\frac{1-\alpha}{\alpha}}c$). Hence, looking at buyback from the end consumers' perspective, the end consumers never benefit from the use of buyback, and in some cases, such as at a relatively high DUL, they suffer a loss in the presence of buyback. In addition, it is seen from Theorem 2.8.1(i), Theorem 2.4.2(i), and Theorem 2.5.2(i) that the retailer will maintain earning a constant margin with buyback regardless of the DUL, while this is not the case with the wholesale price-only contract.

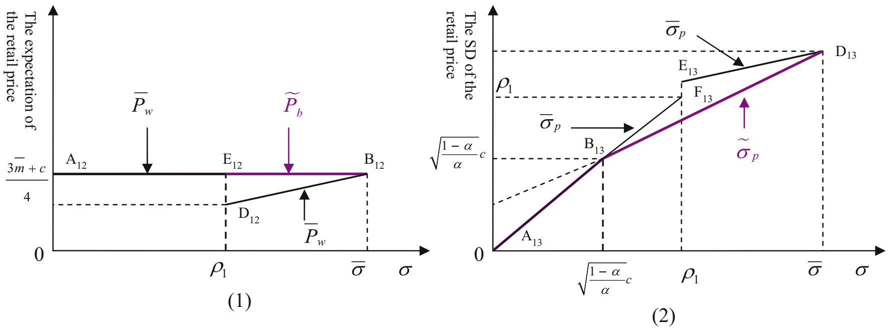

Fig. 2.8 Effects of σ on expectation and SD of the retail price

2.9 Conclusion

This chapter has conducted a study for buyback contracts in terms of the DUL in a two-echelon supply chain facing the end market demand characterized by a price-dependent downward-sloping demand curve subject to uncertainty. In this chapter, we have characterized the buyback contract, the order quantity, and the respective supply chain members' expected profits at equilibrium. We have also examined the value of buyback, its efficiency in coordinating the supply chain, and the expectation and standard deviation of the retail price with buyback. We derive all the results explicitly in terms of the DUL, which is measured by the SD. In addition, we have made detailed comparisons between the scenarios with buyback and no buyback over different DULs, which allow us to see more clearly the effects of buyback and how DUL influences these effects. We summarize the relevant results in Table 2.1.

As a conclusion, we would like to conduct a comparison between our studies and those by Granot and Yin (2005). First, some results in our studies, to some extent, can be viewed as the extensions of the corresponding results in Granot and Yin (2005) to the additive form of demand. For example, our studies, with a special additive form of demand, demonstrate that the wholesale price and the system efficiency with buyback coincide with those in the corresponding deterministic demand case. These results are very consistent with those found by Granot and Yin (2005). Furthermore, it should be noted that we have shown these results over different DULs. By contrast, Granot and Yin (2005) derived them with a uniform distribution over [0,2], i.e., a singly given DUL determined by the uniform distribution over [0,2]. In terms of such a sense, our studies, even though with a special form, have extended these core results in Granot and Yin (2005) to the case of additive form of demand, and thereby substantiated the first conjecture proposed by Granot and Yin (2005) to a good extent by a consideration of the additive form of demand (see Theorem 5.1 and Conjecture 5.3(i) in Granot and Yin (2005)).

In addition to the extended resemblances, our studies also demonstrate that some results developed in Granot and Yin (2005) can not be extended with an additive form of demand. For example, Granot and Yin (2005) have found that for the linear

Table 2.1 Effects of buyback with different DULs

	$0 < \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c$	$\sqrt{\frac{1-\alpha}{\alpha}}c < \sigma < \rho_1$	$\rho_i < \sigma < \overline{\sigma}$
The wholesale price at equilibrium	=	=	+
The order quantity at equilibrium	=	+	=
The supplier's expected profit at equilibrium	=	+	+
The retailer's expected profit at equilibrium	=	+	−
The efficiency of the supply chain system at equilibrium	=	+	−
The expectation of the retail price at equilibrium	=	=	+
The standard deviation of the retail price at equilibrium	=	−	−

expected demand, at equilibrium buyback will benefit all the supply chain members and the system with a multiplicative form of demand as in their studies. However, they have actually derived this result with a given DUL determined by the uniform distribution over [0, 2]. Hence, readers may not help wondering whether this result still holds as the DUL changes (or the PDF for the stochastic part of the demand changes). Different answers are possible for this issue. At least this is the case with the additive form of demand, as shown in our studies with a special additive form of demand, buyback is beneficial simultaneously for the supplier, the retailer, and the system only in a range of the intermediate DULs, and with the other cases this is not the case. For another example, Granot and Yin (2005) have found that for the linear expected demand, the system efficiency with the wholesale price-only contracts can not exceed 75 % (see Proposition 3.6 in Granot and Yin (2005)). However, is it also so with different DULs or PDFs for the stochastic demand? The answer may be yes or no. Actually, our studies have found that for the additive form of demand, depending on different DULs, the system efficiency with the wholesale price-only contracts can be higher or lower than 75 %. In addition, Theorem 2.5.2 in our studies has exhibited a direct deny to Conjecture 5.3(ii) in Granot and Yin (2005) for the additive form of demand.

The above comparison raises some research directions that are worth pursuing in the future. First, whether the results obtained in Granot and Yin (2005) and our paper still hold for more general cases. Second, whether the extensions discussed in the above comparison still hold for more general additive form of demand, such as an extension of our model that the parameter m follows a general distribution. Third, in addition to the formal difference, what are the essences that are driving the resemblances and differences between the results with the two forms of demand model. We leave these as interesting topics for future research.

Appendix: Proofs of the Main Results

Proof of Lemma 2.4.1. We first examine the retailer's optimal strategy at time 1, given his order quantity Q at time 0. When the realized demand curve at time 1 is $p = m_H - \delta q$, we formulate the retailer's problem as $\max\limits_{0\le q\le Q} q(m_H - \delta q)$. By solving this problem, we obtain the corresponding release quantity

$$q_{wH} = \begin{cases} \frac{m_H}{2\delta}, & \text{if } \frac{m_H}{2\delta} \le Q; \\ Q, & \text{otherwise.} \end{cases} \tag{2.9.1}$$

Likewise, when the realization of the demand curve is $p = m_L - \delta q$ at time 1, we formulate the retailer's problem as $\max\limits_{0\le q\le Q} q(m_L - \delta q)$, and obtain the corresponding release quantity

$$q_{wL} = \begin{cases} \frac{m_L}{2\delta}, & \text{if } \frac{m_L}{2\delta} \le Q; \\ Q, & \text{otherwise.} \end{cases} \tag{2.9.2}$$

In the following we analyze the retailer's optimal order quantity at time 0, which we discuss based on three cases of Q:

(i) $0 \le Q \le \frac{m_L}{2\delta}$: Based on (2.9.1) and (2.9.2), we formulate the problem faced by the retailer at time 0 as

$$\text{P}_{2\text{A}.1}: \quad \max_{0\le Q\le \frac{m_L}{2\delta}} E\Pi_{wr1}(Q) = \alpha Q(m_H - \delta Q) + (1-\alpha)Q(m_L - \delta Q) - wQ. \tag{2.9.3}$$

We obtain its solution

$$\overline{Q}_{w1} = \begin{cases} \frac{m_L}{2\delta}, & \text{if } w \le \alpha(m_H - m_L); \\ \frac{\overline{m}-w}{2\delta}, & \text{if } \alpha(m_H - m_L) \le w \le \overline{m}; \\ 0, & \text{otherwise.} \end{cases} \tag{2.9.4}$$

(ii) $\frac{m_L}{2\delta} \le Q \le \frac{m_H}{2\delta}$: Based on (2.9.1) and (2.9.2), we formulate the problem faced by the retailer at time 0 as

$$\text{P}_{2\text{A}.2}: \quad \max_{\frac{m_L}{2\delta}\le Q\le \frac{m_H}{2\delta}} E\Pi_{wr2}(Q) = \alpha Q(m_H - \delta Q) + (1-\alpha)\frac{m_L}{2\delta}(m_L - \delta\frac{m_L}{2\delta}) - wQ. \tag{2.9.5}$$

We obtain its solution

$$\overline{Q}_{w2} = \begin{cases} \frac{\alpha m_H - w}{2\alpha\delta}, & \text{if } w \le \alpha(m_H - m_L); \\ \frac{m_L}{2\delta}, & \text{if } w \ge \alpha(m_H - m_L). \end{cases} \tag{2.9.6}$$

(iii) $Q \geq \frac{m_H}{2\delta}$: Similarly, we formulate the problem faced by the retailer at time 0 as

$$\text{P}_{2\text{A}.3}: \quad \max_{Q \geq \frac{m_H}{2\delta}} E\Pi_{wr3}(Q) = \alpha \tfrac{m_H}{2\delta}(m_H - \delta \tfrac{m_H}{2\delta}) + (1-\alpha)\tfrac{m_L}{2\delta}(m_L - \delta \tfrac{m_L}{2\delta}) - wQ. \tag{2.9.7}$$

We obtain its solution

$$\overline{Q}_{w3} = \frac{m_H}{2\delta}. \tag{2.9.8}$$

We summarize the above results as follows:

(1) When $w \leq \alpha(m_H - m_L)$, the retailer's optimal order quantity at time 0 is determined by

$$\overline{Q}_w = \arg\max\{E\Pi_{wr1}(\tfrac{m_L}{2\delta}),\ E\Pi_{wr2}(\tfrac{\alpha m_H - w}{2\alpha\delta}),\ E\Pi_{wr3}(\tfrac{m_H}{2\delta})\}. \tag{2.9.9}$$

Since

$$\begin{aligned} E\Pi_{wr2}(\tfrac{\alpha m_H - w}{2\alpha\delta}) &\geq E\Pi_{wr2}(\tfrac{m_L}{2\delta}) = E\Pi_{wr1}(\tfrac{m_L}{2\delta}), \\ E\Pi_{wr2}(\tfrac{\alpha m_H - w}{2\alpha\delta}) &\geq E\Pi_{wr2}(\tfrac{m_H}{2\delta}) = E\Pi_{wr3}(\tfrac{m_H}{2\delta}), \end{aligned} \tag{2.9.10}$$

where the inequalities in (2.9.10) become equalities, if and only if, $w = \alpha(m_H - m_L)$, the optimal order quantity in this case is $\overline{Q}_w = \frac{\alpha m_H - w}{2\alpha\delta}$.

(2) When $\alpha(m_H - m_L) \leq w \leq \overline{m}$, the retailer's optimal order quantity at time 0 is determined by

$$\overline{Q}_w = \arg\max\{E\Pi_{wr1}(\tfrac{\overline{m} - w}{2\delta}),\ E\Pi_{wr2}(\tfrac{m_L}{2\delta}),\ E\Pi_{wr3}(\tfrac{m_H}{2\delta})\}. \tag{2.9.11}$$

Since

$$E\Pi_{wr1}(\tfrac{\overline{m} - w}{2\delta}) \geq E\Pi_{wr1}(\tfrac{m_L}{2\delta}) = E\Pi_{wr2}(\tfrac{m_L}{2\delta}) > E\Pi_{wr2}(\tfrac{m_H}{2\delta}) = E\Pi_{wr3}(\tfrac{m_H}{2\delta}), \tag{2.9.12}$$

where the first inequality becomes an equality, if and only if, $w = \alpha(m_H - m_L)$, the optimal quantity in this case is $\overline{Q}_w = \frac{\overline{m} - w}{2\delta}$.

(3) When $w > \overline{m}$, the retailer's optimal order quantity at time 0 is determined by

$$\overline{Q}_w = \arg\max\{E\Pi_{wr1}(0),\ E\Pi_{wr2}(\tfrac{m_L}{2\delta}),\ E\Pi_{wr3}(\tfrac{m_H}{2\delta})\}. \tag{2.9.13}$$

Since

$$E\Pi_{wr1}(0) > E\Pi_{wr1}(\tfrac{m_L}{2\delta}) = E\Pi_{wr2}(\tfrac{m_L}{2\delta}) > E\Pi_{wr2}(\tfrac{m_H}{2\delta}) = E\Pi_{wr3}(\tfrac{m_H}{2\delta}), \tag{2.9.14}$$

the optimal quantity in this case is $\overline{Q}_w = 0$.

To summarize, we obtain

$$\overline{Q}_w = \begin{cases} \frac{\alpha m_H - w}{2\alpha\delta}, & \text{if } w \le \alpha(m_H - m_L); \\ \frac{\overline{m} - w}{2\delta}, & \text{if } \alpha(m_H - m_L) \le w \le \overline{m}; \\ 0, & \text{otherwise.} \end{cases} \tag{2.9.15}$$

In order to see the effects of σ, we express (2.9.15) in terms of σ. To this end, we substitute (2.3.7) into $w \le \alpha(m_H - m_L)$, $\alpha(m_H - m_L) \le w \le \overline{m}$, and $\frac{\alpha m_H - w}{2\alpha\delta}$ to obtain

$$w \le \alpha(m_H - m_L) \Longleftrightarrow w \le \alpha\left(\sqrt{\tfrac{1-\alpha}{\alpha}} + \sqrt{\tfrac{\alpha}{1-\alpha}}\right)\sigma \Longleftrightarrow \sigma \ge \sqrt{\tfrac{1-\alpha}{\alpha}}w, \tag{2.9.16}$$

$$\alpha(m_H - m_L) \le w \le \overline{m} \Longleftrightarrow \sigma \le \sqrt{\tfrac{1-\alpha}{\alpha}}w \ (w \le \overline{m}), \tag{2.9.17}$$

and

$$\frac{\alpha m_H - w}{2\alpha\delta} = \frac{\alpha(\overline{m} + \sqrt{\frac{1-\alpha}{\alpha}}\sigma) - w}{2\alpha\delta} = \frac{\alpha\overline{m} - w}{2\alpha\delta} + \frac{\sqrt{1-\alpha}}{2\delta\sqrt{\alpha}}\sigma. \tag{2.9.18}$$

Substitution of (2.9.16), (2.9.17), and (2.9.18) into (2.9.15), together with (2.3.8), gives Lemma 2.4.1. □

Proof of Theorem 2.4.2. By substituting (2.3.7) into the problem $P_{2.1}$ which is given by (2.4.4), we transform it to

$$P_{2A.4}: \quad \overline{w} = \arg\max_{w}\{\Pi_1, \Pi_2\}, \tag{2.9.19}$$

where

$$\Pi_1 = \max_{0 \le w \le \alpha(m_H - m_L)} E\Pi_{ws1}(w) = (w - c)\left(\tfrac{\alpha m_H - w}{2\alpha\delta}\right), \tag{2.9.20}$$

$$\Pi_2 = \max_{\alpha(m_H - m_L) \le w \le \overline{m}} E\Pi_{ws2}(w) = (w - c)\left(\tfrac{\overline{m} - w}{2\delta}\right). \tag{2.9.21}$$

Solution of Π_1 is

$$\overline{w}_1 = \begin{cases} \frac{\alpha m_H + c}{2}, & \text{if } c \le \alpha m_H - 2\alpha m_L; \\ \alpha(m_H - m_L), & \text{if } c > \alpha m_H - 2\alpha m_L. \end{cases} \tag{2.9.22}$$

Solution of Π_2 is

$$\overline{w}_2 = \begin{cases} \alpha(m_H - m_L), & \text{if } c < \alpha m_H - (1+\alpha)m_L; \\ \frac{\overline{m} + c}{2}, & \text{if } c \ge \alpha m_H - (1+\alpha)m_L. \end{cases} \tag{2.9.23}$$

Summarizing (2.9.22) and (2.9.23), we have

$$\overline{w}=\begin{cases}\frac{\alpha m_H+c}{2}, & \text{if } c<\alpha m_H-(1+\alpha)m_L;\\ \frac{\overline{m}+c}{2}, & \text{if } c>\alpha m_H-2\alpha m_L;\\ \frac{\alpha m_H+c}{2}, & \text{if } \alpha m_H-(1+\alpha)m_L\le c\le \alpha m_H-2\alpha m_L \text{ and}\\ & E\Pi_{ws1}(\frac{\alpha m_H+c}{2})\ge E\Pi_{ws2}(\frac{\overline{m}+c}{2});\\ \frac{\overline{m}+c}{2}, & \text{if } \alpha m_H-(1+\alpha)m_L\le c\le \alpha m_H-2\alpha m_L \text{ and}\\ & E\Pi_{ws1}(\frac{\alpha m_H+c}{2})< E\Pi_{ws2}(\frac{\overline{m}+c}{2}).\end{cases} \quad (2.9.24)$$

Since

$$E\Pi_{ws1}(\tfrac{\alpha m_H+c}{2})=(\tfrac{\alpha m_H+c}{2}-c)(\tfrac{\alpha m_H-\frac{\alpha m_H+c}{2}}{2\alpha\delta})=\tfrac{(\alpha m_H-c)^2}{8\alpha\delta}, \quad (2.9.25)$$

$$E\Pi_{ws2}(\tfrac{\overline{m}+c}{2})=(\tfrac{\overline{m}+c}{2}-c)(\tfrac{\overline{m}-\frac{\overline{m}+c}{2}}{2\delta})=\tfrac{(\overline{m}-c)^2}{8\delta}, \quad (2.9.26)$$

we have

$$\begin{aligned}&E\Pi_{ws1}(\tfrac{\alpha m_H+c}{2})-E\Pi_{ws2}(\tfrac{\overline{m}+c}{2})\\ =&\tfrac{(\alpha m_H-c)^2}{8\alpha\delta}-\tfrac{(\overline{m}-c)^2}{8\delta}\\ =&\tfrac{1-\alpha}{8\alpha\delta}[c^2-2\alpha(m_H-m_L)c+\alpha^2m_H^2-2\alpha^2m_Hm_L-\alpha(1-\alpha)m_L^2].\end{aligned} \quad (2.9.27)$$

Since $\frac{1-\alpha}{8\alpha\delta}>0$, the inequality $E\Pi_{ws1}(\frac{\alpha a_H+c}{2})\le E\Pi_{ws2}(\frac{\overline{m}+c}{2})$ is equivalent to

$$c^2-2\alpha(m_H-m_L)c+\alpha^2m_H^2-2\alpha^2m_Hm_L-\alpha(1-\alpha)m_L^2\le 0. \quad (2.9.28)$$

Solving inequality (2.9.28), we see that $E\Pi_{ws1}(\frac{\alpha m_H+c}{2})\le E\Pi_{ws2}(\frac{\overline{m}+c}{2})$ holds, if and only if, $c\in[\theta_2,\theta_1]$, where $\theta_1=\alpha m_H+(\sqrt{\alpha}-\alpha)m_L$ and $\theta_2=\alpha m_H-(\sqrt{\alpha}+\alpha)m_L$. Since $\alpha m_H-(1+\alpha)m_L\le\theta_2\le\alpha m_H-2\alpha m_L\le\theta_1$, we can simplify (2.9.24) as

$$\overline{w}=\begin{cases}\frac{\alpha m_H+c}{2}, & \text{if } c\le\theta_2;\\ \frac{\overline{m}+c}{2}, & \text{if } c>\theta_2.\end{cases} \quad (2.9.29)$$

By (2.9.29) and (2.9.15), we obtain the equilibrium order quantity of the retailer as

$$\overline{Q}=\begin{cases}\frac{\alpha m_H-c}{4\alpha\delta}, & \text{if } c\le\theta_2;\\ \frac{\overline{m}-c}{4\delta}, & \text{if } c>\theta_2.\end{cases} \quad (2.9.30)$$

In order to see the effects of σ, we express (2.9.29) and (2.9.30) in terms of σ. Hence, we substitute (2.3.7) into the expression of θ_2 to obtain

$$\begin{aligned} c \le \theta_2 &\Longleftrightarrow c \le \alpha[\overline{m} + \sqrt{\tfrac{1-\alpha}{\alpha}}\sigma] - (\sqrt{\alpha}+\alpha)[\overline{m} - \sqrt{\tfrac{\alpha}{1-\alpha}}\sigma] \\ &\Longleftrightarrow c \le -\sqrt{\alpha}\overline{m} + [\sqrt{\alpha(1-\alpha)} + (\alpha+\sqrt{\alpha})\sqrt{\tfrac{\alpha}{1-\alpha}}]\sigma \\ &\Longleftrightarrow \sigma \ge \tfrac{\sqrt{1-\alpha}}{\alpha+\sqrt{\alpha}}c + \tfrac{\sqrt{1-\alpha}}{1+\sqrt{\alpha}}\overline{m}. \end{aligned} \tag{2.9.31}$$

Similarly, we have

$$\tfrac{\alpha m_H + c}{2} = \tfrac{\alpha(\overline{m}+\sqrt{\frac{1-\alpha}{\alpha}}\sigma)+c}{2} = \tfrac{\overline{m}+c}{2} - \tfrac{1-\alpha}{2}(\overline{m} - \sqrt{\tfrac{\alpha}{1-\alpha}}\sigma) = \hat{w} - \tfrac{1-\alpha}{2}(\overline{m} - \sqrt{\tfrac{\alpha}{1-\alpha}}\sigma), \tag{2.9.32}$$

$$\tfrac{\alpha m_H - c}{4\alpha\delta} = \tfrac{\alpha(\overline{m}+\sqrt{\frac{1-\alpha}{\alpha}}\sigma)-c}{4\alpha\delta} = \tfrac{\overline{m}-c}{4\delta} + \tfrac{1-\alpha}{4\alpha\delta}(\sqrt{\tfrac{\alpha}{1-\alpha}}\sigma - c) = \hat{Q} + \tfrac{1-\alpha}{4\alpha\delta}(\sqrt{\tfrac{\alpha}{1-\alpha}}\sigma - c). \tag{2.9.33}$$

Substituting (2.9.31), (2.9.32), and (2.9.33) into (2.9.29) and (2.9.30) and letting $\rho_1 = \frac{\sqrt{1-\alpha}}{\alpha+\sqrt{\alpha}}c + \frac{\sqrt{1-\alpha}}{1+\sqrt{\alpha}}\overline{m}$, together with (2.3.8), we obtain Theorem 2.4.2(i). As for Theorem 2.4.2(ii), it is a direct result of (2.9.30), (2.9.1), and (2.9.2). □

Proof of Theorem 2.4.3. We first show (i). By (2.9.29), (2.9.30), and (2.9.31), we obtain that, if $\rho_1 \le \sigma \le \overline{\sigma}$, then

$$\begin{aligned} \overline{R}_s &= (\overline{w} - c)\overline{Q} \\ &= (\tfrac{\alpha m_H + c}{2} - c)(\tfrac{\alpha m_H - c}{4\alpha\delta}) \\ &= \tfrac{(\alpha m_H - c)^2}{8\alpha\delta} \\ &= \tfrac{(\overline{m}-c)^2}{8\delta} + [\tfrac{(\alpha m_H - c)^2}{8\alpha\delta} - \tfrac{(\overline{m}-c)^2}{8\delta}] \\ &= \tfrac{(\overline{m}-c)^2}{8\delta} + \tfrac{1}{8\alpha\delta}[(\alpha m_H - c)^2 - \alpha(\overline{m}-c)^2] \\ &= \tfrac{(\overline{m}-c)^2}{8\delta} + \tfrac{1}{8\alpha\delta}[\alpha^2(\overline{m}+\sqrt{\tfrac{1-\alpha}{\alpha}}\sigma)^2 - 2\alpha c(\overline{m}+\sqrt{\tfrac{1-\alpha}{\alpha}}\sigma) + (1-\alpha)c^2 - \alpha\overline{m}^2 + 2\alpha\overline{m}c] \\ &= \hat{R}_s + \tfrac{1}{8\alpha\delta}[\alpha(1-\alpha)\sigma^2 + 2\sqrt{\alpha(1-\alpha)}(\alpha\overline{m}-c)\sigma + (1-\alpha)(c^2 - \alpha\overline{m}^2)] \\ &= \hat{R}_s + \tfrac{H_1(\sigma)}{8\alpha\delta}, \end{aligned} \tag{2.9.34}$$

where the sixth equation follows from (2.3.7), $\hat{R}_s = \frac{(\overline{m}-c)^2}{8\delta}$ corresponds to the supplier's profit at equilibrium in the case $\sigma = 0$, and the last equation follows by letting

$$\begin{aligned} H_1(\sigma) &= \alpha(1-\alpha)\sigma^2 + 2\sqrt{\alpha(1-\alpha)}(\alpha\overline{m}-c)\sigma + (1-\alpha)(c^2 - \alpha\overline{m}^2) \\ &= (\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2 - \alpha(\sqrt{1-\alpha}\overline{m} - \sqrt{\alpha}\sigma)^2. \end{aligned} \tag{2.9.35}$$

Putting $H_1(\sigma) = 0$ and solving this equation, we obtain its two solutions for σ as

$$\sigma_1 = \tfrac{\sqrt{1-\alpha}}{\sqrt{\alpha}+\alpha}c + \tfrac{\sqrt{1-\alpha}}{1+\sqrt{\alpha}}\overline{m} = \rho_1, \quad \sigma_2 = \tfrac{1+\sqrt{\alpha}}{\sqrt{\alpha(1-\alpha)}}c - \tfrac{1+\sqrt{\alpha}}{\sqrt{1-\alpha}}\overline{m}. \tag{2.9.36}$$

Since $\rho_1 = \sigma_1 > \sigma_2$ and $\alpha(1-\alpha) > 0$, $H_1(\sigma) \geq 0$ for all $\sigma \in [\rho_1, \overline{\sigma}]$. Again, we obtain by (2.9.35) that, for all $\sigma \in [\rho_1, \overline{\sigma}]$,

$$\begin{aligned}\tfrac{dH_1(\sigma)}{d\sigma} &= 2\alpha(1-\alpha)\sigma + 2\sqrt{\alpha(1-\alpha)}(\alpha\overline{m} - c)\\ &\geq 2\alpha(1-\alpha)\rho_1 + 2\sqrt{\alpha(1-\alpha)}(\alpha\overline{m} - c)\\ &= 2\alpha\sqrt{1-\alpha}(\overline{m} - c) > 0.\end{aligned} \tag{2.9.37}$$

Therefore, $H_1(\sigma)$ strictly increases with σ on $[\rho_1, \overline{\sigma}]$.

If $0 \leq \sigma < \rho_1$, then by (2.9.29), (2.9.30), and (2.9.31), we obtain

$$\overline{R}_s = (\overline{w} - c)\overline{Q} = (\tfrac{\overline{m}+c}{2} - c)(\tfrac{\overline{m}-c}{4\delta}) = \tfrac{(\overline{m}-c)^2}{8\delta} = \hat{R}_s, \tag{2.9.38}$$

where $\hat{R}_s$ corresponds to the supplier's profit at equilibrium in the case $\sigma = 0$.

We proceed to show (ii). Likewise, by (2.9.29), (2.9.30), and (2.9.31), in view of the proofs of Lemma 2.4.1 and Theorem 2.4.2, we obtain that if $\rho_1 \leq \sigma \leq \overline{\sigma}$, then

$$\begin{aligned}\overline{R}_r &= \alpha\overline{Q}(m_H - \delta\overline{Q}) + (1-\alpha)\tfrac{m_L}{2\delta}(m_L - \delta\tfrac{m_L}{2\delta}) - \overline{w}\overline{Q}\\ &= \alpha\tfrac{\alpha m_H - c}{4\alpha\delta}(m_H - \delta\tfrac{\alpha m_H - c}{4\alpha\delta}) + (1-\alpha)\tfrac{m_L}{2\delta}(m_L - \delta\tfrac{m_L}{2\delta}) - (\tfrac{\alpha m_H + c}{2})(\tfrac{\alpha m_H - c}{4\alpha\delta})\\ &= \tfrac{(\alpha m_H - c)^2}{16\alpha\delta} + \tfrac{(1-\alpha)m_L^2}{4\delta}\\ &= \tfrac{1}{2}\overline{R}_s + \tfrac{(1-\alpha)m_L^2}{4\delta}\\ &= \tfrac{1}{2}(\hat{R}_s + \tfrac{H_1(\sigma)}{8\alpha\delta}) + \tfrac{(1-\alpha)m_L^2}{4\delta}\\ &= \hat{R}_r + \tfrac{H_1(\sigma)}{16\alpha\delta} + \tfrac{(1-\alpha)m_L^2}{4\delta}\\ &= \hat{R}_r + \tfrac{1}{16\alpha\delta}[(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2 + 3\alpha(1-\alpha)(\overline{m} - \sqrt{\tfrac{\alpha}{1-\alpha}}\sigma)^2],\\ &= \hat{R}_r + \tfrac{H_2(\sigma)}{16\alpha\delta},\end{aligned} \tag{2.9.39}$$

where the fourth equation follows from the third equation in (2.9.34), the fifth equation follows from the last equation in (2.9.34), the sixth equation follows from (2.4.2), in which $\hat{R}_r = \frac{1}{2}\hat{R}_s$ corresponds to the retailer's profit at equilibrium in the case $\sigma = 0$, the seventh equation follows from (2.9.35) and (2.3.7), and the last equation follows by letting

$$H_2(\sigma) = (\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2 + 3\alpha(1-\alpha)(\overline{m} - \sqrt{\tfrac{\alpha}{1-\alpha}}\sigma)^2. \tag{2.9.40}$$

It is obvious that $H_2(\sigma) > 0$ for any σ. In addition, we obtain from (2.9.40) that for each $\sigma \in [\rho_2, \overline{\sigma}]$, where ρ_2 is given by (2.4.15),

$$\begin{aligned}\tfrac{dH_2(\sigma)}{d\sigma} &= 2\alpha(3\alpha+1)\sigma - 2\sqrt{\alpha(1-\alpha)}(c + 3\alpha\overline{m})\\ &\geq 2\alpha(3\alpha+1)\rho_2 - 2\sqrt{\alpha(1-\alpha)}(c + 3\alpha\overline{m})\\ &= 2\alpha(3\alpha+1)[\tfrac{\sqrt{1-\alpha}}{\sqrt{\alpha}(3\alpha+1)}c + \tfrac{3\sqrt{\alpha(1-\alpha)}}{3\alpha+1}\overline{m}] - 2\sqrt{\alpha(1-\alpha)}(c + 3\alpha\overline{m}) = 0,\end{aligned} \tag{2.9.41}$$

where the equality holds, if and only if, $\sigma = \rho_2$. Therefore, $H_2(\sigma)$ is strictly decreasing in σ on $[\rho_1, \rho_2)$ and strictly increasing in σ on $[\rho_2, \overline{\sigma}]$.

Similarly, it follows in view of the proofs of Lemma 2.4.1 and Theorem 2.4.2 that if $0 \le \sigma < \rho_1$, then

$$\begin{aligned}\overline{R}_r &= \alpha\overline{Q}(m_H - \delta\overline{Q}) + (1-\alpha)\overline{Q}(m_L - \delta\overline{Q}) - \overline{w}\overline{Q}\\ &= \alpha\hat{Q}(m_H - \delta\hat{Q}) + (1-\alpha)\hat{Q}(m_L - \delta\hat{Q}) - \hat{w}\hat{Q}\\ &= \alpha\tfrac{\overline{m}-c}{4\delta}(m_H - \delta\tfrac{\overline{m}-c}{4\delta}) + (1-\alpha)\tfrac{\overline{m}-c}{4\delta}(m_L - \delta\tfrac{\overline{m}-c}{4\delta}) - (\tfrac{\overline{m}+c}{2})(\tfrac{\overline{m}-c}{4\delta})\\ &= \tfrac{(\overline{m}-c)^2}{16\delta} = \hat{R}_r.\end{aligned} \tag{2.9.42}$$

This completes the proof. □

Proof of Lemma 2.5.1. We first derive the retailer's optimal strategy at time 1, given its order, say Q, at time 0.

(i) When the realized demand curve at time 1 is $p = m_H - \delta q$, we formulate the retailer's problem as

$$\mathrm{P}_{2A.5}: \quad \max_{0\le q\le Q}[q(m_H - \delta q) + b(Q-q)], \tag{2.9.43}$$

where q corresponds to the quantity released by the retailer to the market. Solution of $\mathrm{P}_{2A.5}$, denoted by q_{bH}, is:

$$q_{bH} = \begin{cases}\frac{m_H-b}{2\delta}, & \text{if } \frac{m_H-b}{2\delta} \le Q;\\ Q, & \text{otherwise.}\end{cases} \tag{2.9.44}$$

(ii) When the realized demand curve at time 1 is $p = m_L - \delta q$, we formulate the retailer's problem as

$$\mathrm{P}_{2A.6}: \quad \max_{0\le q\le Q}[q(m_L - \delta q) + b(Q-q)]. \tag{2.9.45}$$

Solution of $\mathrm{P}_{2A.6}$, denoted by q_{bL}, is:

$$q_{bL} = \begin{cases}\frac{m_L-b}{2\delta}, & \text{if } \frac{m_L-b}{2\delta} \le Q;\\ Q, & \text{otherwise.}\end{cases} \tag{2.9.46}$$

We discuss the retailer's optimal order quantity at time 0, given three cases of Q.

First, we consider the case where $0 \le Q \le \frac{m_L-b}{2\delta}$. By (2.9.44) and (2.9.46), we formulate the retailer's problem as

$$\begin{aligned}\mathrm{P}_{2A.7}: \quad & \max E\Pi_{br1}(Q) = \alpha Q(m_H - \delta Q) + (1-\alpha)Q(m_L - \delta Q) - wQ\\ & s.t.\ \ 0 \le Q \le \tfrac{m_L-b}{2\delta}.\end{aligned} \tag{2.9.47}$$

Its solution denoted by $\overline{Q}_{b1}$ is:

$$\overline{Q}_{b1} = \begin{cases} \frac{m_L-b}{2\delta}, & \text{if } w-b \leq \alpha(m_H - m_L); \\ \frac{\overline{m}-w}{2\delta}, & \text{otherwise.} \end{cases} \tag{2.9.48}$$

Second, we consider the case where $\frac{m_L-b}{2\delta} \leq Q \leq \frac{m_H-b}{2\delta}$. By (2.9.44) and (2.9.46), we formulate the retailer's problem as

$$\begin{aligned} \mathrm{P}_{2A.8}: \quad & \max E\Pi_{br2}(Q) = \alpha Q(m_H - \delta Q) + (1-\alpha)[\tfrac{m_L-b}{2\delta}(m_L - \delta\tfrac{m_L-b}{2\delta}) \\ & \qquad + b(Q - \tfrac{m_L-b}{2\delta})] - wQ \\ & s.t. \ \tfrac{m_L-b}{2\delta} \leq Q \leq \tfrac{m_H-b}{2\delta}. \end{aligned} \tag{2.9.49}$$

Its solution denoted by $\overline{Q}_{b2}$ is:

$$\overline{Q}_{b2} = \begin{cases} \frac{\alpha m_H + (1-\alpha)b - w}{2\alpha\delta}, & \text{if } w-b \leq \alpha(m_H - m_L); \\ \frac{m_L-b}{2\delta}, & \text{otherwise.} \end{cases} \tag{2.9.50}$$

Finally, we consider the case where $Q \geq \frac{m_H-b}{2\delta}$. By (2.9.44) and (2.9.46), we formulate the retailer's problem as

$$\begin{aligned} \mathrm{P}_{2A.9}: \quad & \max E\Pi_{br3}(Q) = \alpha[\tfrac{m_H-b}{2\delta}(m_H - \delta\tfrac{m_H-b}{2\delta}) + b(Q - \tfrac{m_H-b}{2\delta})] \\ & \qquad + (1-\alpha)[\tfrac{m_L-b}{2\delta}(m_L - \delta\tfrac{m_L-b}{2\delta}) + b(Q - \tfrac{m_L-b}{2\delta})] - wQ \\ & s.t. \ Q \geq \tfrac{m_H-b}{2\delta}. \end{aligned} \tag{2.9.51}$$

It is easy to obtain the optimal solution for this problem as $\overline{Q}_{b3} = \frac{m_H-b}{2\delta}$.

To summarize, we obtain:

(i) If $w - b \leq \alpha(m_H - m_L)$, the retailer's optimal order quantity at time 0 is determined by

$$\begin{aligned} \overline{Q}_b = \arg\max\{ \ & E\Pi_{br1}(\overline{Q}_{b1}),\ E\Pi_{br2}(\overline{Q}_{b2}),\ E\Pi_{br3}(\overline{Q}_{b3}), \\ & \text{where } \overline{Q}_{b1} = \tfrac{m_L-b}{2\delta}, \overline{Q}_{b2} = \tfrac{\alpha m_H + (1-\alpha)b - w}{2\alpha\delta}, \text{and } \overline{Q}_{b3} = \tfrac{m_H-b}{2\delta}\}. \end{aligned} \tag{2.9.52}$$

Since $E\Pi_{br2}(Q)$is strictly concave in Q, we obtain

$$\begin{aligned} E\Pi_{br2}(\tfrac{\alpha m_H + (1-\alpha)b - w}{2\alpha\delta}) \geq E\Pi_{br2}(\tfrac{m_L-b}{2\delta}) = E\Pi_{br1}(\tfrac{m_L-b}{2\delta}), \\ E\Pi_{br2}(\tfrac{\alpha m_H + (1-\alpha)b - w}{2\alpha\delta}) \geq E\Pi_{br2}(\tfrac{m_H-b}{2\delta}) = E\Pi_{br3}(\tfrac{m_H-b}{2\delta}), \end{aligned} \tag{2.9.53}$$

where the inequalities in (2.9.53) become the equalities, if and only if, $w - b = \alpha(m_H - m_L)$. Hence, the optimal order quantity in this case is $\frac{\alpha m_H + (1-\alpha)b - w}{2\alpha\delta}$.

(ii) If $w - b > \alpha(m_H - m_L)$, the optimal order quantity of the retailer at time 0 is determined by

$$\overline{Q}_b = \arg\max\{\ E\Pi_{br1}(\overline{Q}_{b1}),\ E\Pi_{br2}(\overline{Q}_{b2}),\ E\Pi_{br3}(\overline{Q}_{b3}), \text{ where } \overline{Q}_{b1} = \tfrac{\overline{m}-w}{2\delta}, \overline{Q}_{b2} = \tfrac{m_L-b}{2\delta}, \text{ and } \overline{Q}_{b3} = \tfrac{m_H-b}{2\delta}\}. \tag{2.9.54}$$

Likewise, by the strict concavity of $E\Pi_{br1}(Q)$ and $E\Pi_{br2}(Q)$ in Q, we obtain

$$E\Pi_{br1}(\tfrac{\overline{m}-w}{2\delta}) > E\Pi_{br1}(\tfrac{m_L-b}{2\delta}) = E\Pi_{br2}(\tfrac{m_L-b}{2\delta}) > E\Pi_{br2}(\tfrac{m_H-b}{2\delta}) = E\Pi_{br3}(\tfrac{m_H-b}{2\delta}). \tag{2.9.55}$$

Then, the optimal order quantity in this case is $\frac{\overline{m}-w}{2\delta}$.

Hence, we have

$$\tilde{Q}_b = \begin{cases} \frac{\alpha m_H+(1-\alpha)b-w}{2\alpha\delta}, & \text{if } w-b \leq \alpha(m_H-m_L); \\ \frac{\overline{m}-w}{2\delta}, & \text{if } w-b > \alpha(m_H-m_L). \end{cases} \tag{2.9.56}$$

In order to more clearly see the effects of σ, we express (2.9.56) in terms of σ. Hence by (2.3.7), we have

$$w-b \leq \alpha(m_H-m_L) \iff w-b \leq \alpha[\sqrt{\tfrac{1-\alpha}{\alpha}}\sigma + \sqrt{\tfrac{\alpha}{1-\alpha}}\sigma] \iff \sigma \geq \sqrt{\tfrac{1-\alpha}{\alpha}}(w-b), \tag{2.9.57}$$

$$\frac{\alpha m_H+(1-\alpha)b-w}{2\alpha\delta} = \frac{\alpha(\overline{m}+\sqrt{\frac{1-\alpha}{\alpha}}\sigma)+(1-\alpha)b-w}{2\alpha\delta} = \frac{\sqrt{1-\alpha}}{2\delta\sqrt{\alpha}}\sigma + \frac{\alpha\overline{m}+(1-\alpha)b-w}{2\alpha\delta}. \tag{2.9.58}$$

Substitution of (2.9.57) and (2.9.58) into (2.9.56), together with (2.3.8), gives Lemma 2.5.1. □

Proof of Theorem 2.5.2. For ease of calculation, we substitute (2.3.7) into the problem $P_{2.2}$ given by (2.5.5), and transform it to

$$P_{2A.10}: \quad \arg\max\{\tilde{M}_{s1}, \tilde{M}_{s2}\}, \tag{2.9.59}$$

where

$$\begin{aligned} &P_{2A.11}: \quad \tilde{M}_{s1} = \max_{0\leq w-b\leq\alpha(m_H-m_L)} E\Pi_{bs1}(w,b), \\ &E\Pi_{bs1}(w,b) = (w-c)(\frac{\alpha m_H+(1-\alpha)b-w}{2\alpha\delta}) - b(1-\alpha)[\frac{\alpha m_H+(1-\alpha)b-w}{2\alpha\delta} - \frac{m_L-b}{2\delta}], \\ &P_{2A.12}: \quad \tilde{M}_{s2} = \max_{\overline{m}\geq w-b>\alpha(m_H-m_L)} E\Pi_{bs2}(w,b) = (w-c)(\frac{\overline{m}-w}{2\delta}). \end{aligned} \tag{2.9.60}$$

We analyze in the following the solution for the problem $P_{2A.10}$:

(i) We first solve $P_{2A.11}$ by ignoring the constraints in it. Since $E\Pi_{bs1}(w,b)$ is jointly and strictly concave in w and b, the solution for the corresponding unconstrained problem is given by the first-order optimality conditions:

$$\begin{cases} \frac{\partial E\Pi_{bs1}(w,b)}{\partial w} = 0, \\ \frac{\partial E\Pi_{bs1}(w,b)}{\partial b} = 0. \end{cases} \tag{2.9.61}$$

After some algebra, we obtain from (2.9.61) that

$$\begin{cases} w - (1-\alpha)b = \frac{c}{2} + \frac{\alpha}{2} m_H, \\ w - b = \frac{c}{2} + \frac{\alpha}{2}(m_H - m_L), \end{cases} \tag{2.9.62}$$

which gives the solution for the unconstrained problem as

$$(\tilde{w}_1, \tilde{b}_1) = (\frac{\overline{m} + c}{2}, \frac{m_L}{2}). \tag{2.9.63}$$

With the consideration of the constraints in $P_{2A.11}$ and the joint and strict concavity of $E\Pi_{bs1}(w,b)$ in w and b, we obtain the solution of $P_{2A.11}$ as

$$(\tilde{w}_2, \tilde{b}_2) = \begin{cases} (\frac{\overline{m}+c}{2}, \frac{m_L}{2}), & \text{if } c \leq \alpha(m_H - m_L); \\ (\frac{\overline{m}+c}{2}, \frac{m_L + c - \alpha(m_H - m_L)}{2}), & \text{if } \overline{m} \geq c > \alpha(m_H - m_L). \end{cases} \tag{2.9.64}$$

(ii) We proceed to solve $P_{2A.12}$. It is easy to see that $E\Pi_{bs2}(w,b)$ achieves its maximum at $w = \frac{\overline{m}+c}{2}$ and any b. To satisfy the constraints in $P_{2A.12}$, it suffices to set $0 \leq b < \frac{m_L + c - \alpha(m_H - m_L)}{2}$. Hence, the optimal solution for $P_{2A.12}$ is given by

$$(\tilde{w}_3, \tilde{b}_3) = (\frac{\overline{m} + c}{2}, b^*) \text{ with } 0 \leq b^* < \frac{m_L + c - \alpha(m_H - m_L)}{2}. \tag{2.9.65}$$

(iii) In order to obtain the solution of $P_{2A.10}$, we compare $E\Pi_{bs1}(\frac{\overline{m}+c}{2}, \frac{m_L}{2})$ and $E\Pi_{bs1}(\frac{\overline{m}+c}{2}, \frac{m_L + c - \alpha(m_H - m_L)}{2})$ with $E\Pi_{bs2}(\frac{\overline{m}+c}{2}, b^*)$ for each b^* satisfying

$$0 \leq b^* < \frac{m_L + c - \alpha(m_H - m_L)}{2}. \tag{2.9.66}$$

By (2.9.60) and with some algebra, we obtain

$$\begin{aligned} E\Pi_{bs1}(\tfrac{\overline{m}+c}{2}, \tfrac{m_L}{2}) &= \tfrac{(\alpha m_H - c)^2}{8\alpha\delta} + \tfrac{(1-\alpha)m_L^2}{8\delta} \\ &= \tfrac{(\overline{m}-c)^2}{8\delta} + [\tfrac{(\alpha m_H - c)^2}{8\alpha\delta} - \tfrac{(\overline{m}-c)^2}{8\delta}] + \tfrac{(1-\alpha)m_L^2}{8\delta} \\ &= \hat{R}_s + \tfrac{1-\alpha}{8\alpha\delta}[\alpha(m_H - m_L) - c]^2, \end{aligned} \tag{2.9.67}$$

where $\hat{R}_s = \frac{(\overline{m}-c)^2}{8\delta}$ corresponds to the supplier's profit at equilibrium in the case $\sigma = 0$. Similarly, with some algebra, we obtain

$$E\Pi_{bs1}(\tfrac{\overline{m}+c}{2}, \tfrac{m_L+c-\alpha(m_H-m_L)}{2}) = \tfrac{(\overline{m}-c)^2}{8\delta} = \hat{R}_s \tag{2.9.68}$$

and

$$E\Pi_{bs2}(\tfrac{\overline{m}+c}{2}, b^*) = \tfrac{(\overline{m}-c)^2}{8\delta} = \hat{R}_s \text{ for all } 0 \le b^* < \tfrac{m_L+c-\alpha(m_H-m_L)}{2}. \tag{2.9.69}$$

Summarizing (i), (ii), and (iii), we obtain the solution of $P_{2A.10}$, denoted by $(\tilde{w}, \tilde{b})$, as follows:

$$(\tilde{w}, \tilde{b}) = \begin{cases} (\frac{\overline{m}+c}{2}, \frac{m_L}{2}), & \text{if } c < \alpha(m_H - m_L); \\ (\frac{\overline{m}+c}{2}, b^*) \text{ with } 0 \le b^* \le \frac{m_L+c-\alpha(m_H-m_L)}{2}, & \text{if } \overline{m} \ge c > \alpha(m_H - m_L). \end{cases} \tag{2.9.70}$$

In order to see the effects of σ, we express (2.9.70) in terms of σ by substituting (2.3.7) into it. Thereby, we obtain the equivalent form of (2.9.70) that is characterized in terms of σ as follows:

$$(\tilde{w}, \tilde{b}) = \begin{cases} (\frac{\overline{m}+c}{2}, \frac{\overline{m}}{2} - \sqrt{\frac{\alpha}{4(1-\alpha)}}\sigma), & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}\overline{m} \ge \sigma \ge \sqrt{\frac{1-\alpha}{\alpha}}c; \\ (\frac{\overline{m}+c}{2}, b^*) \text{ with } 0 \le b^* \le \frac{\overline{m}+c}{2} - \sqrt{\frac{\alpha}{1-\alpha}}\sigma, & \text{if } 0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c. \end{cases} \tag{2.9.71}$$

Substituting (2.9.70) into the equivalent form (2.9.56) of Lemma 2.5.1, we obtain the equilibrium order quantity of the retailer as

$$\tilde{Q}_b = \begin{cases} \frac{\alpha m_H - c}{4\alpha\delta}, & \text{if } c \le \alpha(m_H - m_L); \\ \frac{\overline{m}-c}{4\delta}, & \text{if } \overline{m} \ge c > \alpha(m_H - m_L). \end{cases} \tag{2.9.72}$$

In a similar way, we obtain the equivalent form of (2.9.72), which is characterized in terms of σ as follows:

$$\tilde{Q}_b = \begin{cases} \frac{\overline{m}-c}{4\delta} + \frac{1-\alpha}{4\alpha\delta}(\sqrt{\frac{\alpha}{1-\alpha}}\sigma - c), & \text{if } \overline{\sigma} \ge \sigma \ge \sqrt{\frac{1-\alpha}{\alpha}}c; \\ \frac{\overline{m}-c}{4\delta}, & \text{if } 0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c. \end{cases} \tag{2.9.73}$$

By (2.9.71) and (2.9.73), we obtain Theorem 2.5.2(i) and (ii). As for Theorem 2.5.2(iii), it is a direct result of (2.9.44), (2.9.46), and (2.9.73). Thus, the proof is completed. □

Proof of Theorem 2.5.3. We first derive the supplier's expected profit, denoted by $\tilde{R}_s$, at equilibrium. By Theorem 2.5.2 and its proof, we have

(i) If $\overline{\sigma} \geq \sigma \geq \rho_1 (> \sqrt{\frac{1-\alpha}{\alpha}}c)$, then $\tilde{R}_s = E\Pi_{bs1}(\frac{\overline{m}+c}{2}, \frac{m_L}{2})$. From (2.9.67), we have

$$\begin{aligned}\tilde{R}_s &= E\Pi_{bs1}(\tfrac{\overline{m}+c}{2}, \tfrac{m_L}{2}) \\ &= \hat{R}_s + \tfrac{1-\alpha}{8\alpha\delta}[\alpha(m_H - m_L) - c]^2 \\ &= \hat{R}_s + \tfrac{1}{8\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2,\end{aligned} \tag{2.9.74}$$

or

$$\begin{aligned}\tilde{R}_s &= E\Pi_{bs1}(\tfrac{\overline{m}+c}{2}, \tfrac{m_L}{2}) \\ &= \tfrac{(\alpha m_H - c)^2}{8\alpha\delta} + \tfrac{(1-\alpha)m_L^2}{8\delta} \\ &= \overline{R}_s + \tfrac{(1-\alpha)m_L^2}{8\delta} \\ &= \overline{R}_s + \tfrac{(1-\alpha)}{8\delta}(\overline{m} - \sqrt{\tfrac{\alpha}{1-\alpha}}\sigma)^2,\end{aligned} \tag{2.9.75}$$

where the third equality in (2.9.75) follows from the third equality in (2.9.34) and the last equality follows from (2.3.7).

(ii) If $\rho_1 > \sigma \geq \sqrt{\frac{1-\alpha}{\alpha}}c$, then by (2.9.74),

$$\begin{aligned}\tilde{R}_s &= \hat{R}_s + \tfrac{1}{8\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2 \\ &= \overline{R}_s + \tfrac{1}{8\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2,\end{aligned} \tag{2.9.76}$$

where the second equality follows from $\hat{R}_s = \overline{R}_s$ for $0 \leq \sigma < \rho_1$ by Theorem 2.4.3.

(iii) If $0 \leq \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c$, then $\tilde{R}_s = E\Pi_{bs1}(\frac{\overline{m}+c}{2}, \frac{m_L+c-\alpha(m_H-m_L)}{2})$ or $E\Pi_{bs2}(\frac{\overline{m}+c}{2}, b^*)$, where $0 \leq b^* < \frac{m_L+c-\alpha(m_H-m_L)}{2}$. From (2.9.68) and (2.9.69), we have

$$\tilde{R}_s = E\Pi_{bs1}(\frac{\overline{m}+c}{2}, \frac{m_L + c - \alpha(m_H - m_L)}{2}) = E\Pi_{bs2}(\frac{\overline{m}+c}{2}, b^*) = \frac{(\overline{m}-c)^2}{8\delta} = \hat{R}_s = \overline{R}_s, \tag{2.9.77}$$

where the last equality also follows from $\hat{R}_s = \overline{R}_s$ for $0 \leq \sigma < \rho_1$ by Theorem 2.4.3.

To summarize, we obtain

$$\tilde{R}_s = \begin{cases} \hat{R}_s + \frac{1}{8\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2 \text{ or } \overline{R}_s + \frac{(1-\alpha)}{8\delta}(\overline{m} - \sqrt{\frac{\alpha}{1-\alpha}}\sigma)^2, & \text{if } \rho_1 \leq \sigma \leq \overline{\sigma}; \\ \hat{R}_s + \frac{1}{8\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2 \text{ or } \overline{R}_s + \frac{1}{8\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2, & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \leq \sigma < \rho_1; \\ \hat{R}_s \text{ or } \overline{R}_s, & \text{if } 0 \leq \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c. \end{cases} \tag{2.9.78}$$

We derive the retailer's expected profit at equilibrium, denoted by $\tilde{R}_r$, in the following.

(i) By Theorem 2.5.2 and in view of the proof of Lemma 2.5.1, we obtain that if $\sqrt{\frac{1-\alpha}{\alpha}}c \leq \sigma \leq \overline{\sigma}$, then

$$\begin{aligned}\tilde{R}_r &= \alpha\tilde{Q}(m_H - \delta\tilde{Q}) + (1-\alpha)[\tfrac{m_L-b}{2\delta}(m_L - \delta\tfrac{m_L-b}{2\delta}) + \tilde{b}(\tilde{Q} - \tfrac{m_L-b}{2\delta})] - \tilde{w}\tilde{Q}\\ &= \alpha\tfrac{\alpha m_H-c}{4\alpha\delta}(m_H - \delta\tfrac{\alpha m_H-c}{4\alpha\delta}) + (1-\alpha)[\tfrac{m_L-b}{2\delta}(m_L - \delta\tfrac{m_L-b}{2\delta})\\ &\quad + \tfrac{m_L}{2}(\tfrac{\alpha m_H-c}{4\alpha\delta} - \tfrac{m_L-b}{2\delta})] - (\tfrac{\overline{m}+c}{2})(\tfrac{\alpha m_H-c}{4\alpha\delta}).\end{aligned} \tag{2.9.79}$$

After some algebra, we have

$$\tilde{R}_r = \frac{(\alpha m_H-c)^2}{16\alpha\delta} + \frac{(1-\alpha)m_L^2}{16\delta}. \tag{2.9.80}$$

By the third equality in (2.9.39), we have that if $\rho_1 \leq \sigma \leq \overline{\sigma}$,

$$\begin{aligned}(2.9.80) &= \overline{R}_r - \tfrac{(1-\alpha)m_L^2}{4\delta} + \tfrac{(1-\alpha)m_L^2}{16\delta}\\ &= \overline{R}_r - \tfrac{3(1-\alpha)m_L^2}{16\delta}\\ &= \overline{R}_r - \tfrac{3(1-\alpha)}{16\delta}(\overline{m} - \sqrt{\tfrac{\alpha}{1-\alpha}}\sigma)^2\\ &= \hat{R}_r + \tfrac{H_2(\sigma)}{16\alpha\delta} - \tfrac{3(1-\alpha)}{16\delta}(\overline{m} - \sqrt{\tfrac{\alpha}{1-\alpha}}\sigma)^2\\ &= \hat{R}_r + \tfrac{1}{16\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2,\end{aligned} \tag{2.9.81}$$

where $\overline{R}_r = \hat{R}_r + \frac{H_2(\sigma)}{16\alpha\delta}$ corresponds to the retailer's expected profit at equilibrium for $\rho_1 \leq \sigma \leq \overline{\sigma}$, $H_2(\sigma)$ is given by (2.4.14), and $\hat{R}_r = \frac{(\overline{m}-c)^2}{16\delta}$ corresponds to the retailer's profit at equilibrium in the case $\sigma = 0$. From the third and the last equalities in (2.9.34), we have that if $\sqrt{\frac{1-\alpha}{\alpha}}c \leq \sigma < \rho_1$, then

$$\begin{aligned}(2.9.80) &= \tfrac{1}{2}[\hat{R}_s + \tfrac{H_1(\sigma)}{8\alpha\delta}] + \tfrac{(1-\alpha)m_L^2}{16\delta}\\ &= \hat{R}_r + \tfrac{H_1(\sigma)}{16\alpha\delta} + \tfrac{(1-\alpha)m_L^2}{16\delta}\\ &= \hat{R}_r + \tfrac{1}{16\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2\\ &= \overline{R}_r + \tfrac{1}{16\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2,\end{aligned} \tag{2.9.82}$$

where $\overline{R}_r$ corresponds to the retailer's expected profit at equilibrium for $\sqrt{\frac{1-\alpha}{\alpha}}c \leq \sigma < \rho_1$, and the last equality follows from Theorem 2.4.3, which indicates that $\overline{R}_r = \hat{R}_r$ for $\sqrt{\frac{1-\alpha}{\alpha}}c \leq \sigma < \rho_1$.

(ii) By Theorem 2.5.2 and in view of the proof of Lemma 2.5.1, we have that if $0 \leq \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c$, then

$$\begin{aligned}\tilde{R}_r &= \alpha\tilde{Q}(m_H - \delta\tilde{Q}) + (1-\alpha)\tilde{Q}(m_L - \delta\tilde{Q}) - \tilde{w}\tilde{Q}\\ &= \alpha\tfrac{\overline{m}-c}{4\delta}(m_H - \delta\tfrac{\overline{m}-c}{4\delta}) + (1-\alpha)\tfrac{\overline{m}-c}{4\delta}(m_L - \delta\tfrac{\overline{m}-c}{4\delta}) - \tfrac{\overline{m}+c}{2}\tfrac{\overline{m}-c}{4\delta}\\ &= \tfrac{(\overline{m}-c)^2}{16\delta} = \hat{R}_r = \overline{R}_r,\end{aligned} \tag{2.9.83}$$

where the last equality follows from Theorem 2.4.3.

To summarize, we obtain

$$\tilde{R}_r = \begin{cases} \hat{R}_r + \frac{1}{16\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2 \text{ or } \overline{R}_r - \frac{3(1-\alpha)}{16\delta}(\overline{m} - \sqrt{\frac{\alpha}{1-\alpha}}\sigma)^2, \text{ if } \sqrt{\frac{1-\alpha}{\alpha}}\overline{m} \geq \sigma \geq \rho_1, \\ \hat{R}_r + \frac{1}{16\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2 \text{ or } \overline{R}_r + \frac{1}{16\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2, \text{ if } \sqrt{\frac{1-\alpha}{\alpha}}c \leq \sigma < \rho_1; \\ \hat{R}_r \text{ or } \overline{R}_r, \quad \text{if } 0 \leq \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c. \end{cases} \tag{2.9.84}$$

In view of (2.9.78) and (2.9.84), the proof is completed. □

Proofs of (2.5.13) **and** (2.6.4). Padmanabhan and Png (1997) showed in their buyback scheme that the supplier's expected profit at equilibrium is given by $\frac{(\overline{\alpha}+\beta c)^2 - 4\beta c\alpha_h}{8\beta}$. Further, they showed that the supplier's expected profit is strictly greater with a returns policy than with a no returns policy if $c = 0$ and $\frac{\alpha_h}{\alpha_l} < \frac{\lambda}{(1-\lambda)^{\frac{1}{2}} - (1-\lambda)}$; refer to Table 3 and Proposition 2(b) in Padmanabhan and Png (1997) respectively, and see therein for the implications of the parameters $\overline{\alpha}, \beta, c, \alpha_h, \alpha_l$, and λ.

By comparing our model with that of Padmanabhan and Png (1997), it is easy to show the validity of the following relations between the notations used in the two models: $\alpha_h \iff \beta m_H$, $\alpha_l \iff \beta m_L$, $\beta \iff 1/\delta$, $\lambda \iff 1-\alpha$, $1-\lambda \iff \alpha$, and $\overline{\alpha} \iff \beta\overline{m}$. Substituting these relations into $\frac{(\overline{\alpha}+\beta c)^2 - 4\beta c\alpha_h}{8\beta}$ yields

$$\frac{(\overline{\alpha}+\beta c)^2 - 4\beta c\alpha_h}{8\beta} \iff \hat{R}_s - \frac{4c\sqrt{1-\alpha}}{8\delta\sqrt{\alpha}}\sigma, \tag{2.9.85}$$

where $\hat{R}_s = \frac{(\overline{m}-c)^2}{8\delta}$. Similarly, substituting these relations into $\frac{\alpha_h}{\alpha_l} < \frac{\lambda}{(1-\lambda)^{\frac{1}{2}} - (1-\lambda)}$, together with (2.3.7) and (2.3.8), we obtain

$$\frac{\alpha_h}{\alpha_l} < \frac{\lambda}{(1-\lambda)^{\frac{1}{2}} - (1-\lambda)} \iff \sigma < \frac{\sqrt{1-\alpha}}{1+\sqrt{\alpha}}\overline{m} \ (< \sqrt{\frac{1-\alpha}{\alpha}}\overline{m} = \overline{\sigma}). \tag{2.9.86}$$

Hence, the proofs are completed. □

Proof of Lemma 2.7.1. Using the marginal production cost c to replace the wholesale price w in the proof of Lemma 2.4.1, we obtain in a similar way the system-wide optimal product quantity for the supply chain as follows:

$$\tilde{Q}_c = \begin{cases} \frac{\alpha m_H - c}{2\alpha\delta}, \text{ if } \sqrt{\frac{1-\alpha}{\alpha}}c \leq \sigma \leq \overline{\sigma}; \\ \frac{\overline{m}-c}{2\delta}, \quad \text{if } 0 \leq \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c. \end{cases} \tag{2.9.87}$$

By substituting (2.3.7) into (2.9.87), we transform it to

$$\tilde{Q}_c = \begin{cases} 2[\hat{Q} + \frac{1-\alpha}{4\alpha\delta}(\sqrt{\frac{\alpha}{1-\alpha}}\sigma - c)], \text{ if } \sqrt{\frac{1-\alpha}{\alpha}}c \leq \sigma \leq \overline{\sigma}; \\ 2\hat{Q}, \quad \text{if } 0 \leq \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c, \end{cases} \tag{2.9.88}$$

where $\hat{Q} = \frac{\overline{m}-c}{4\delta}$ is the equilibrium order quantity in the case $\sigma = 0$.

Setting $w = c$ in (2.9.5) and then substituting (2.9.87) into it, we obtain that if $\sqrt{\frac{1-\alpha}{\alpha}}c \leq \sigma \leq \overline{\sigma}$, then

$$\begin{aligned}
\tilde{R}_c &= \alpha\tilde{Q}_c(m_H - \delta\tilde{Q}_c) + (1-\alpha)\tfrac{m_L}{2\delta}(m_L - \delta\tfrac{m_L}{2\delta}) - c\tilde{Q}_c \\
&= \alpha\tfrac{\alpha m_H - c}{2\alpha\delta}(m_H - \delta\tfrac{\alpha m_H - c}{2\alpha\delta}) + (1-\alpha)\tfrac{m_L}{2\delta}(m_L - \delta\tfrac{m_L}{2\delta}) - c\tfrac{\alpha m_H - c}{2\alpha\delta} \\
&= \tfrac{(\alpha m_H - c)^2}{4\alpha\delta} + \tfrac{(1-\alpha)m_L^2}{4\delta} \\
&= 2\hat{R}_s + \tfrac{H_1(\sigma)}{4\alpha\delta} + \tfrac{(1-\alpha)m_L^2}{4\delta} \\
&= 2\hat{R}_s + \tfrac{1}{4\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2 \\
&= 4\hat{R}_r + \tfrac{1}{4\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2,
\end{aligned} \tag{2.9.89}$$

where the fourth equality follows from the third and the last equations in (2.9.34), $\hat{R}_s = \frac{(\overline{m}-c)^2}{8\delta}$, and $\hat{R}_r = \frac{(\overline{m}-c)^2}{16\delta}$ respectively correspond to the supplier's and the retailer's profits at equilibrium in the case $\sigma = 0$.

Similarly, if $0 \leq \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c$, then by setting $w = c$ in (2.9.3) and substituting (2.9.87) into it, we obtain

$$\begin{aligned}
\tilde{R}_c &= \alpha\tilde{Q}_c(m_H - \delta\tilde{Q}_c) + (1-\alpha)\tilde{Q}_c(m_L - \delta\tilde{Q}_c) - c\tilde{Q}_c \\
&= \alpha\tfrac{\overline{m}-c}{2\delta}(m_H - \delta\tfrac{\overline{m}-c}{2\delta}) + (1-\alpha)\tfrac{\overline{m}-c}{2\delta}(m_L - \delta\tfrac{\overline{m}-c}{2\delta}) - c\tfrac{\overline{m}-c}{2\delta} \\
&= \tfrac{(\overline{m}-c)^2}{4\delta} \\
&= 4\hat{R}_r.
\end{aligned} \tag{2.9.90}$$

This completes the proof. □

Proof of Theorem 2.7.2. It follows from Theorem 2.5.3 that the system's total expected profit with buyback is

$$\tilde{R}_s + \tilde{R}_r = \begin{cases} \hat{R}_s + \hat{R}_r + \frac{3}{16\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2, & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \leq \sigma \leq \overline{\sigma}; \\ \hat{R}_s + \hat{R}_r, & \text{if } 0 \leq \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c. \end{cases} \tag{2.9.91}$$

Since $\hat{R}_s = 2\hat{R}_r$, we can rewrite (2.9.91) as

$$\tilde{R}_s + \tilde{R}_r = \begin{cases} 3\hat{R}_r + \frac{3}{16\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2, & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \leq \sigma \leq \overline{\sigma}; \\ 3\hat{R}_r, & \text{if } 0 \leq \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c. \end{cases} \tag{2.9.92}$$

By comparing (2.9.92) with (2.7.2), we can derive the efficiency of the supply chain system as

$$EFF_b = 0.75 \text{ for each } \sigma \in [0, \overline{\sigma}]. \tag{2.9.93}$$

Similarly, by Theorem 2.4.3, we obtain the supply chain system's total expected profit with no buyback as

$$\overline{R}_s + \overline{R}_r = \begin{cases} 3\hat{R}_r + \frac{H_1(\sigma)}{8\alpha\delta} + \frac{H_2(\sigma)}{16\alpha\delta}, & \text{if } \rho_1 \le \sigma \le \overline{\sigma}; \\ 3\hat{R}_r, & \text{if } 0 \le \sigma < \rho_1, \end{cases} \tag{2.9.94}$$

where $H_1(\sigma)$ is given by (2.4.12) and $H_2(\sigma)$ is given by (2.4.14). With some algebra, we obtain

$$\begin{aligned} \frac{H_1(\sigma)}{8\alpha\delta} + \frac{H_2(\sigma)}{16\alpha\delta} &= \frac{3}{16\alpha\delta}[\alpha(\alpha+3)\sigma^2 - 2\sqrt{\alpha(1-\alpha)}(3c+\alpha\overline{m})\sigma + (1-\alpha)(3c^2+\alpha\overline{m}^2)] \\ &= \frac{3}{16\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2 + \frac{1}{16\delta}(\sqrt{1-\alpha}\overline{m} - \sqrt{\alpha}\sigma)^2. \end{aligned} \tag{2.9.95}$$

Substituting (2.9.95) into (2.9.94) yields

$$\overline{R}_s + \overline{R}_r = \begin{cases} 3\hat{R}_r + \frac{3}{16\alpha\delta}(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2 + \frac{1}{16\delta}(\sqrt{1-\alpha}\overline{m} - \sqrt{\alpha}\sigma)^2, & \text{if } \rho_1 \le \sigma \le \overline{\sigma}; \\ 3\hat{R}_r, & \text{if } 0 \le \sigma < \rho_1. \end{cases} \tag{2.9.96}$$

By comparing (2.9.96) with (2.7.2), we obtain the efficiency of the supply chain system with no buyback as follows:

$$EFF_w = \begin{cases} 0.75 + \frac{(\sqrt{1-\alpha}\overline{m} - \sqrt{\alpha}\sigma)^2}{4[(\overline{m}-c)^2 + (\sigma - \sqrt{\frac{1-\alpha}{\alpha}}c)^2]}, & \text{if } \rho_1 \le \sigma \le \overline{\sigma}; \\ 0.75 - \frac{3(\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2}{4[\alpha(\overline{m}-c)^2 + (\sqrt{\alpha}\sigma - \sqrt{1-\alpha}c)^2]}, & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \le \sigma < \rho_1; \\ 0.75, & \text{if } 0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c. \end{cases} \tag{2.9.97}$$

When we compare (2.9.93) with (2.9.97), we see that with the introduction of buyback, as compared with the case of no buyback, the efficiency of the supply chain system increases strictly when $\sqrt{\frac{1-\alpha}{\alpha}}c < \sigma < \rho_1$, decreases strictly when $\rho_1 \le \sigma < \overline{\sigma}$, and keeps unchanged when $0 \le \sigma \le \sqrt{\frac{1-\alpha}{\alpha}}c$. Thus, the proof is completed. □

Proof of Theorem 2.8.1. From Theorem 2.5.2 and the proof of Lemma 2.5.1, we obtain that for $\sqrt{\frac{1-\alpha}{\alpha}}c \le \sigma \le \overline{\sigma}$,

$$\begin{aligned} \tilde{p}_b &= \alpha(m_H - \delta\tilde{Q}_b) + (1-\alpha)(m_L - \delta\frac{m_L - \tilde{b}}{2\delta}) \\ &= \alpha(m_H - \delta\frac{\alpha m_H - c}{4\alpha\delta}) + (1-\alpha)(m_L - \delta\frac{m_L - \frac{m_L}{2}}{2\delta}) = \frac{3\overline{m}+c}{4}, \end{aligned} \tag{2.9.98}$$

$$\begin{aligned} \tilde{\sigma}_p^2 &= \alpha(m_H - \delta\tilde{Q}_b - \tilde{p}_b)^2 + (1-\alpha)(m_L - \delta\frac{m_L - \tilde{b}}{2\delta} - \tilde{p}_b)^2 \\ &= \alpha(m_H - \delta\frac{\alpha m_H - c}{4\alpha\delta} - \frac{3\overline{m}+c}{4})^2 + (1-\alpha)(m_L - \delta\frac{m_L - \frac{m_L}{2}}{2\delta} - \frac{3\overline{m}+c}{4})^2 \\ &= \frac{1-\alpha}{16\alpha}(3\alpha m_H - 3\alpha m_L + c)^2, \end{aligned} \tag{2.9.99}$$

and for $0 \le \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c$,

$$\begin{aligned}\tilde{p}_b &= \alpha(m_H - \delta\tilde{Q}_b) + (1-\alpha)(m_L - \delta\tilde{Q}_b)\\ &= \alpha(m_H - \delta\tfrac{\overline{m}-c}{4\delta}) + (1-\alpha)(m_L - \delta\tfrac{\overline{m}-c}{4\delta}) = \tfrac{3\overline{m}+c}{4},\end{aligned} \tag{2.9.100}$$

$$\begin{aligned}\tilde{\sigma}_p^2 &= \alpha(m_H - \delta\tilde{Q}_b - \tilde{p}_b)^2 + (1-\alpha)(m_L - \delta\tilde{Q}_b - \tilde{p}_b)^2\\ &= \alpha(m_H - \delta\tfrac{\overline{m}-c}{4\delta} - \tfrac{3\overline{m}+c}{4})^2 + (1-\alpha)(m_L - \delta\tfrac{\overline{m}-c}{4\delta} - \tfrac{3\overline{m}+c}{4})^2\\ &= \alpha(1-\alpha)(m_H - m_L)^2\\ &= \sigma^2.\end{aligned} \tag{2.9.101}$$

Similarly, from Theorem 2.4.2 and the proof of Lemma 2.4.1, we obtain that for $\rho_1 \le \sigma \le \overline{\sigma}$,

$$\begin{aligned}\overline{p}_w &= \alpha(m_H - \delta\tilde{Q}_b) + (1-\alpha)(m_L - \delta\tfrac{m_L}{2\delta})\\ &= \alpha(m_H - \delta\tfrac{\alpha m_H - c}{4\alpha\delta}) + (1-\alpha)(m_L - \delta\tfrac{m_L}{2\delta})\\ &= \tfrac{3\overline{m}+c}{4} - \tfrac{(1-\alpha)m_L}{4},\end{aligned} \tag{2.9.102}$$

$$\begin{aligned}\overline{\sigma}_p^2 &= \alpha(m_H - \delta\tilde{Q}_b - \overline{p}_w)^2 + (1-\alpha)(m_L - \delta\tfrac{m_L}{2\delta} - \overline{p}_w)^2\\ &= \alpha(m_H - \delta\tfrac{\alpha m_H - c}{4\alpha\delta} - \tfrac{3\alpha m_H + 2(1-\alpha)m_L + c}{4})^2 + (1-\alpha)(m_L - \delta\tfrac{m_L}{2\delta} - \tfrac{3\alpha m_H + 2(1-\alpha)m_L + c}{4})^2\\ &= \tfrac{1-\alpha}{16\alpha}(3\alpha m_H - 2\alpha m_L + c)^2,\end{aligned} \tag{2.9.103}$$

and for $0 \le \sigma < \rho_1$,

$$\begin{aligned}\overline{p}_w &= \alpha(m_H - \delta\tilde{Q}_b) + (1-\alpha)(m_L - \delta\tilde{Q}_b)\\ &= \alpha(m_H - \delta\tfrac{\overline{m}-c}{4\delta}) + (1-\alpha)(m_L - \delta\tfrac{\overline{m}-c}{4\delta}) = \tfrac{3\overline{m}+c}{4},\end{aligned} \tag{2.9.104}$$

$$\begin{aligned}\overline{\sigma}_p^2 &= \alpha(m_H - \delta\tilde{Q}_b - \overline{p}_w)^2 + (1-\alpha)(m_L - \delta\tilde{Q}_b - \overline{p}_w)^2\\ &= \alpha(m_H - \delta\tfrac{\overline{m}-c}{4\delta} - \tfrac{3\overline{m}+c}{4})^2 + (1-\alpha)(m_L - \delta\tfrac{\overline{m}-c}{4\delta} - \tfrac{3\overline{m}+c}{4})^2\\ &= \alpha(1-\alpha)(m_H - m_L)^2\\ &= \sigma^2.\end{aligned} \tag{2.9.105}$$

By substituting (2.3.7) into (2.9.99), (2.9.102), (2.9.103) and summarizing, we obtain

$$\tilde{p}_b = \tfrac{3\overline{m}+c}{4} \text{ for all } \sigma, \tag{2.9.106}$$

$$\overline{p}_w = \begin{cases}\tfrac{3\overline{m}+c}{4} - \tfrac{1-\alpha}{4}(\overline{m} - \sqrt{\tfrac{\alpha}{1-\alpha}}\sigma), & \text{if } \rho_1 \le \sigma \le \overline{\sigma};\\ \tfrac{3\overline{m}+c}{4}, & \text{if } 0 \le \sigma < \rho_1.\end{cases} \tag{2.9.107}$$

$$\tilde{\sigma}_p = \begin{cases} \frac{3}{4}\sigma + \frac{c}{4}\sqrt{\frac{1-\alpha}{\alpha}}, & \text{if } \sqrt{\frac{1-\alpha}{\alpha}}c \leq \sigma \leq \overline{\sigma}; \\ \sigma, & \text{if } 0 \leq \sigma < \sqrt{\frac{1-\alpha}{\alpha}}c, \end{cases} \tag{2.9.108}$$

$$\overline{\sigma}_p = \begin{cases} \frac{3}{4}\sigma + \frac{c}{4}\sqrt{\frac{1-\alpha}{\alpha}} + \frac{1}{4}\alpha(\overline{\sigma} - \sigma), & \text{if } \rho_1 \leq \sigma \leq \overline{\sigma}; \\ \sigma, & \text{if } 0 \leq \sigma < \rho_1. \end{cases} \tag{2.9.109}$$

By comparing (2.9.106) with (2.9.107) and (2.9.108) with (2.9.109), we can conclude that compared with the case of no buyback, the expectation of the retail price with buyback is strictly higher when $\rho_1 \leq \sigma < \overline{\sigma}$, is the same when $0 \leq \sigma < \rho_1$, and the standard deviation of the retail price with buyback is strictly lower when $\sqrt{\frac{1-\alpha}{\alpha}}c < \sigma < \overline{\sigma}$, is the same when $0 \leq \sigma \leq \sqrt{\frac{1-\alpha}{\alpha}}c$. Thus, the proof is completed. □

Chapter 3
Mean-Risk Analysis of Wholesale Price Contracts with Stochastic Price-Dependent Demand

3.1 Introduction

Various types of contract have been developed for coordination or proper risk-sharing in the supply chain. Reviewing past research on supply chain contracts, it is seen that most studies have been conducted based on the concept of expectation (e.g., expected profit). For example, various types of supply chain coordinating contracts have been developed based on the idea of properly designing contractual terms so as to make the expected channel profit achieved in a decentralized supply chain just equal to that in a centralized one (see, e.g., Cachon 2003). When a supply chain contract is developed based on the concept of expectation, however, there is uncertainty risk associated with the realized contract value. For instance, let us consider a coordinating supply chain contract that is designed based on the concept of expected profit. Even if all the contractual terms have been well obeyed by the individual contracting parties, the ultimate outcome achieved by this contract may not be the coordinating profit as predicted *ex ante*. Such risk associated with the supply chain contract actually arises from various uncertainties inherent in the supply chain, such as demand uncertainty, price uncertainty, etc. Since it relates to the true value of the contract, in this chapter we call it *value risk* of the supply chain contract. Intuitively, value risk can be an important factor affecting the performance of a supply chain contract. To examine the effects of value risk on supply chain contract, a mean-risk analysis is developed for the wholesale price contract with a supplier-retailer supply chain. In the analysis, in addition to expected outcome achieved with the contract, it also takes into account the uncertainty level associated with the realized outcome under the contract, which generally can be characterized by the standard deviation (SD). In addition, individuals with different

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risk preference structures can have different attitudes towards the value risk of supply chain contract. For instance, the more risk averse an individual is, the more emphasis it will place on the contract value risk when making decisions. Therefore, the research also incorporates the risk attitude into the analysis of wholesale price contracts. To clearly show the effect generated from risk attitude, all the results are characterized in closed form in terms of only the parameter of risk aversion degree.

Therefore, the research allows for assessing the performance of wholesale price contracts incorporating contract value risk and effect of the degree of an individual's risk-aversion. As an initial attempt, the study is conducted with a simple two-echelon supply chain facing a price-dependent downward-sloping demand curve subject to uncertainty characterized by the simple Bernoulli distribution. We believe this is a reasonable balance between modeling the tractability and the essence of the issue concerned and the complexity of the industrial reality. With this research, it is shown that previous research to assess the performance of a supply chain contract in terms of only the concept of expectation can be substantially deficient and value risk of a supply chain contract is a very important factor that affects the supply chain performance. In fact, it is often the case that the contract value risk at coordination tends to be much higher than that at the Stackelberg equilibrium. As demonstrated in the research, the value risk at coordination with a wholesale price contract can even be twice as that at the Stackelberg equilibrium wholesale price contract. This chapter assesses the efficiency of a wholesale price contract incorporating contract value risk, and thereby provides a new perspective for the design or analysis of supply chain contracts. With such a new perspective, some interesting managerial and economical insights are obtained for the wholesale price contract. For example, it is shown that when the retailer's degree of aversion to the contract value risk is above a certain threshold (which is identified quantitatively in Theorem 3.5.2), the wholesale price contract always makes the supply chain earn less than 75 % of the expected channel profit at coordination. Furthermore, this efficiency strictly decreases with the degree of the retailer's risk-aversion, and even it may be as low as 50 % only. At the same time, however, the SD of the channel profit can be only half of the SD of the channel profit at coordination. To see whether the findings under the premise of binary distribution still hold for other more general cases, an extension of the model to the case of uniform distribution is considered with numerical experiment. The experimental results demonstrate that these findings still well hold for this more general case. Of course, similar to the limitation of other modeling research, the derived results and findings are based on the model setup and they cannot be generalized to all business settings. Despite the acknowledged limitation, it should be believed that this research has made a good contribution to the literature by characterizing the wholesale price contract model in term of risk preference structure and presenting an analysis of the efficiency of wholesale price contracts incorporating value risk of the contract. As an initial attempt to assess the performance of a supply chain contract incorporating contract value risk, this study has generated some significant insights into some critical issues concerning supply chain contracts, such as why supply chain contracts that have proved theoretically effective for supply chain coordination actually often do not work well in practice.

The remainder of this chapter is organized as follows. Section 3.2 reviews the related literature. Section 3.3 describes and formulates the model. Section 3.4 examines the equilibrium wholesale pricing and order quantity, and the expectation and SD of the profits achieved by the respective supply chain agents in equilibrium. Section 3.5 explores the EPBE and the SDBE of wholesale price contracts in coordinating the supply chain. Section 3.6 discusses the expectation and SD of the retail price. Section 3.7 considers an extension of the model with numerical experiment, and Sect. 3.8 concludes the paper. All the proofs of the main results are put in the Appendix for clarity.

3.2 Literature Review

There is a substantial amount of research on supply chain contracts. See Chap. 1 and the review papers including Cachon (2003), Tsay et al. (1999), and Anupindi and Bassok (1999a) for reviews of the relevant literature. Majority of the literature addressed the issues under study based on the concept of expectation, see, e.g., Pasternack (1985) for buyback contracts, Bernstein and Federgruen (2005) for price-discount sharing contracts, Cachon and Lariviere (2005) for revenue sharing contracts, Tsay (1999) for quantity flexibility contracts, and Weng (1995) and Altintas et al. (2008) for quantity discount contracts. For simplicity we only review the details of the literature focusing on wholesale price contracts.

Bresnahan and Reiss (1985) and Cachon (1999) demonstrated that wholesale price contracts alone cannot attain supply chain coordination, unless the supplier is willing to price at its marginal cost. Thus, most of the research on wholesale price contracts is not to address the supply chain coordination issue but to analyze its efficiency in affecting the supply chain's performance. Lariviere and Porteus (2001) presented a worst-case analysis for the efficiency of the wholesale price contract in a manufacturer-retailer supply chain by restricting to demand distributions that have an increasing generalized failure rate (IGFR). Their research identified that the coefficient of variation of the demand distribution is a key to the efficiency of the wholesale price contract: The efficiency increases with a decrease in the coefficient of variation of the demand distribution. Perakis and Roels (2007) measured the efficiency of the wholesale price contract based on a concept called Price of Anarchy (PoA), which is the ratio of the performance of a centralized system to the worst performance of a decentralized system. It is easy to see that PoA is a concept used to assess the worse-case performance of wholesale price contracts. They showed that the PoA of wholesale price contracts is at least 1.71 for a two-firm supply chain model with all nonnegative demand distributions that have the IGFR property. Their result substantiates the inefficiency of wholesale price contracts in coordinating the supply chain. Dong and Zhu (2007) considered a two-wholesale-price contract and studied how inventory decision right and ownership are shifted in a supplier-retailer supply chain with changes in the two wholesale prices. Ray et al. (2006) explored a phenomenon called asymmetric wholesale pricing and tested this phenomenon empirically using a supermarket's scanner data. As compared with this research stream, the research of this chapter differs in that it incorporates both contract value risk and degree of the retailer's risk-aversion towards the contract value risk into the analysis of the wholesale price contract.

Another research stream that is particularly relevant to this work is the contract literature that considers risk preferences of the supply chain members. Examples include, e.g., Sobel and Turcic (2008), Kohli and Park (1989), Chen and Federgruen (2000), Choi (2013b), Chiu and Choi (2013), Lau (1980), Agrawal and Seshadri (2000b), Tsay (2002), Martinez-de-Albéniz and Simchi-Levi (2006), Choi et al. (2008a,b), Wu et al. (2010), Lin et al. (2010), and Li et al. (2014). For a more detailed review of this related research stream, readers are referred to Sect. 1.4.2.7 of Chap. 1, and we omit them here for avoiding repetition. As compared with this research stream, the research of this chapter differs fundamentally in the following two aspects: (i) to assess the efficiency of the wholesale price contract in terms of both the expected profit achieved with the contract, referred to as the *expected-profit-based efficiency* (EPBE), and the contract value risk, referred to as the *SD-based efficiency* (SDBE), which provides a new perspective to look at the performance of a supply chain contract. (ii) To characterize the wholesale price contract model analytically in terms of only the risk preference structure of the retailer, and derive all the results in closed-form. With such a research approach, the effects of contract value risk and risk attitude on the wholesale price contract are analytically examined.

3.3 Model Formulation

Consider a two-echelon supply chain consisting of one supplier and one retailer, where the supplier distributes its product via the retailer to the end market. The model runs through time 0 to time 1. At time 0 the retailer faces a stochastic price-dependent downward-sloping demand curve given by

$$I = (a - p)/\delta, \tag{3.3.1}$$

where p is the retail price, I is the market demand at a price of p, $\delta > 0$, and a is a stochastic variable that is characterized by the following Bernoulli distribution with parameter α

$$a = \begin{cases} a_H & \text{with probability } \alpha \\ a_L & \text{with probability } 1-\alpha, \end{cases} \tag{3.3.2}$$

where $a_H > a_L > 0$ and $0 < \alpha < 1$. With the uncertain demand curve given by (3.3.1) and (3.3.2), the retailer places its order denoted by Q at a unit wholesale price w from the supplier at time 0. At time 1, the uncertainty in the demand curve (3.3.1) is resolved. In response to the revealed demand curve, the retailer determines product quantity and retail price to release the product to the market

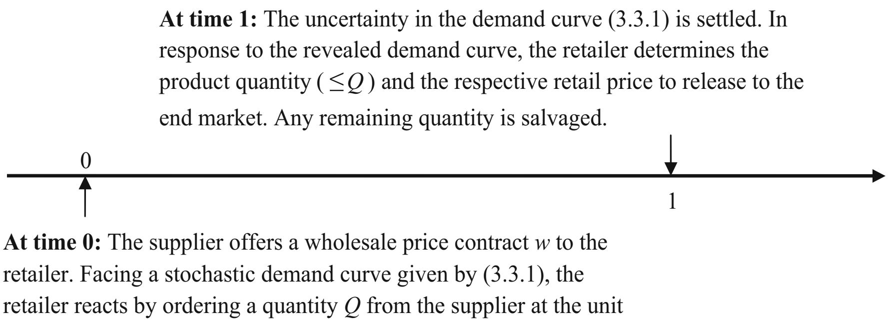

Fig. 3.1 Time line of the model

with the constraints of the order quantity Q, and any remaining quantity is salvaged at a unit price v. Without loss of generality, we normalize v to 0. As to the supplier, it will decide the wholesale price in anticipation of the retailer's response.

Note that since a/δ is the demand at $p = 0$, it is the maximum potential market size. Again, since the demand is zero at the retail price $p = a$, realized a is the maximum retail price that the retailer is able to charge. Hence, $\overline{A} = \alpha a_H + (1-\alpha)a_L$ represents the expected maximum retail price. Let c be the unit production cost of the supplier. It is obviously reasonable to require that $\overline{A} > c$ and $w > c$.

It should be pointed out that assumption (3.3.2) is commonly used in the literature (see, e.g., Burnetas and Ritchken 2005, Padmanabhan and Png 1997). Furthermore, (3.3.2) is equivalent to a linear deterministic demand function with an additive random part that follows a binary distribution (see Chap. 2 for more details). We formulate the problem under study as a supplier-led Stackelberg game, for which the sequence of events is depicted in Fig. 3.1.

In our model, the retailer has to bear all risks associated with the demand uncertainty. Hence, the retailer has to consider two dimensions when making the order decision. One is to increase the expected profit and the other is to decrease the fluctuation level in the profit, which can generally be characterized by the standard deviation (SD) of the profit. For an order strategy given by Q at time 0, the expected profit obtained by the retailer is formulated as

$$E_{wr}(Q) = \alpha \Pi_H(Q) + (1-\alpha)\Pi_L(Q) - wQ, \tag{3.3.3}$$

where $\Pi_H(Q)$ and $\Pi_L(Q)$ are respectively the optimal profits achieved by the retailer at time 1 corresponding to the realization of a_H and a_L for given Q. The corresponding SD of the profit is formulated as

$$SD_{wr}(Q) = \sqrt{\alpha[\Pi_H(Q) - E_{wr}(Q)]^2 + (1-\alpha)[\Pi_L(Q) - E_{wr}(Q)]^2}. \tag{3.3.4}$$

Hence, the retailer's problem at time 0 is to determine an order quantity that is a Pareto-optimal solution for the following double-objective programming problem

$$\text{DP}: \qquad \max_{Q \geq 0}\{E_{wr}(Q), -SD_{wr}(Q)\}. \tag{3.3.5}$$

Probably there are multiple Pareto-optimal solutions for problem DP and which of them will be selected by the retailer depends on the attitude of the retailer towards the two objectives. Suppose that the weights that the retailer places on the two objectives are r_1 and r_2, respectively. With such a risk preference structure the retailer will select one Pareto-optimal solution of problem DP that is optimal to the following single-objective programming problem

$$\mathrm{SP}_{3.1}: \quad \max_{Q \geq 0}\{r_1 E_{wr}(Q) - r_2 SD_{wr}(Q)\}. \tag{3.3.6}$$

Note that $r_1 = 0$ responds to the extreme case where the retailer emphasizes only the objective of minimizing the SD of the profit when making the order decision. Obviously, this will lead to an optimal solution $Q = 0$, which in turn makes the retailer and the supplier both earn zero profit. Therefore, to avoid such uninteresting cases, we require that $r_1 > 0$. Thus, problem $\mathrm{SP}_{3.1}$ is equivalent to the following problem

$$\mathrm{SP}_{3.2}: \quad \max_{Q \geq 0}\{E_{wr}(Q) - \eta SD_{wr}(Q)\}, \tag{3.3.7}$$

where $\eta = r_2/r_1$. As a matter of fact, η essentially serves as a parameter indicating the extent to which the retailer places an emphasis on the SD of the profit: The larger η is, the larger is the weight the retailer places on the SD of the profit, which indicates the more risk averse the retailer is. On the contrary, the more risk averse the retailer is, the larger weight the retailer will place on the SD of the profit, which implies the larger η is. Therefore, we can view η as a parameter to be used to indicate the degree of the retailer's risk-aversion towards the contract risk value. For a retailer with the degree of risk-aversion indicated by η, the optimal order quantity will be the Pareto-optimal solution of problem DP that corresponds to the optimal solution of problem $\mathrm{SP}_{3.2}$.

The purpose of the model is to examine how the contract value risk and the degree of the retailer's risk-aversion affect the use of a wholesale price contract in the supply chain. To this end, the research seeks to characterize the supply chain in terms of only the parameter η, which indicates the degree of the retailer's aversion to the contract value risk.

3.4 Equilibrium Wholesale Pricing and Order Quantity

This section will explore the equilibrium wholesale price and order quantity, and the expectation and SD of the profits achieved by the respective supply chain agents in equilibrium, taking into account value risk of the wholesale price contract and degree of the retailer's risk-aversion towards the contract value risk. The research will first characterize the optimal response of the retailer in terms of its degree of risk-aversion towards the contract value risk. The corresponding results are summarized as in Lemma 3.4.1, where $\overline{Q}_w(\eta)$ denotes the retailer's optimal order quantity for given w and $\overline{Q}_\eta(w)$ denotes the retailer's optimal order quantity for given η.

Lemma 3.4.1. *Given w,*

$$\overline{Q}_w(\eta) = \begin{cases} \frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H-w}{2\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}, & \text{if } \eta \leq \frac{\alpha(a_H-a_L)-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)}; \\ \frac{\overline{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}, & \text{if } \frac{\alpha(a_H-a_L)-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)} < \eta \leq \frac{\overline{A}-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)}; \\ 0, & \text{otherwise.} \end{cases} \tag{3.4.1}$$

Or equivalently, given η,

$$\overline{Q}_\eta(w) = \begin{cases} \frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H-w}{2\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}, & \text{if } w \leq [\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-a_L); \\ \frac{\overline{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}, & \text{if } [\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-a_L) < w \\ & \quad \leq \overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L); \\ 0, & \text{otherwise.} \end{cases} \tag{3.4.2}$$

Since $w > c$, we see from Lemma 3.4.1 that the retailer will order nothing for all w if its degree of risk-aversion towards the contract value risk is larger than $\frac{\overline{A}-c}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)} (\triangleq \eta_{max})$. This in turn leads to the supplier and the retailer both earning zero profit. To avoid such uninteresting cases, we restrict η in the range $[0, \eta_{max}]$ in the following development of the model.

In what follows we examine the equilibrium wholesale price for the supplier to charge based on the retailer's optimal response. By Lemma 3.4.1, when $[\alpha - \eta\sqrt{\alpha(1-\alpha)}](a_H - a_L) \geq c$, i.e., $\eta \leq \frac{\alpha(a_H-a_L)-c}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)} (\triangleq \eta_1)$, we formulate the supplier's problem as

$$\mathrm{P}_{3.1}: \quad \overline{w} = \arg\max_w \{\overline{H}_{3.1}, \overline{H}_{3.2}\}, \tag{3.4.3}$$

where

$$\begin{aligned} \overline{H}_{3.1} &= \max_{c \leq w \leq [\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-a_L)} \left\{\frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H-w}{2\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}\right\}(w-c), \\ \overline{H}_{3.2} &= \max_{[\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-a_L) < w \leq \overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)} \left[\frac{\overline{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}\right](w-c). \end{aligned} \tag{3.4.4}$$

When $[\alpha - \eta\sqrt{\alpha(1-\alpha)}](a_H - a_L) \leq c$, i.e., $\eta \geq \eta_1$, we formulate the supplier's problem as

$$\mathrm{P}_{3.2}: \quad \max_{c \leq w \leq \overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)} \left[\frac{\overline{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}\right](w-c). \tag{3.4.5}$$

Solving problems $P_{3.1}$ and $P_{3.2}$, we obtain the equilibrium wholesale price, which in turn yields the equilibrium order quantity. We summarize these results in Theorem 3.4.2, which characterizes the equilibrium of the supply chain in terms of η.

Theorem 3.4.2. *(i) The equilibrium wholesale price is given by*

$$\overline{w}(\eta) = \begin{cases} \frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H+c}{2}, & \text{if } \eta \le \eta_2; \\ \frac{\bar{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{2}, & \text{if } \eta_2 < \eta \le \eta_{max}, \end{cases} \tag{3.4.6}$$

where

$$\eta_2 = \frac{4(a_H-a_L)[\alpha(a_H-a_L)-c]-2a_L^2-2a_L\sqrt{a_L^2+4c(a_H-a_L)}}{4\sqrt{\alpha(1-\alpha)}(a_H-a_L)^2}. \tag{3.4.7}$$

(ii) The equilibrium order quantity is given by

$$\overline{Q}(\eta) = \begin{cases} \frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H-c}{4\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}, & \text{if } \eta \le \eta_2; \\ \frac{\bar{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)-c}{4\delta}, & \text{if } \eta_2 < \eta \le \eta_{max}. \end{cases} \tag{3.4.8}$$

Figure 3.2 graphically illustrates how the equilibrium wholesale price and order quantity are affected by the retailer's degree of risk-aversion towards the contract value risk. As demonstrated in Fig. 3.2, there is a threshold in the degree of the retailer's risk-aversion, given by η_2, so that as η increases from 0 to η_2, the equilibrium wholesale price decreases linearly with η, as indicated by the line segment A_1–B_1 in Fig. 3.2(1). In response, the equilibrium order quantity decreases strictly along the concave curve indicated by A_2B_2 in Fig. 3.2(2). However, when the retailer's degree of risk-aversion reaches η_2, there is an upward jump in the equilibrium wholesale price. In response, there is a downward jump in the

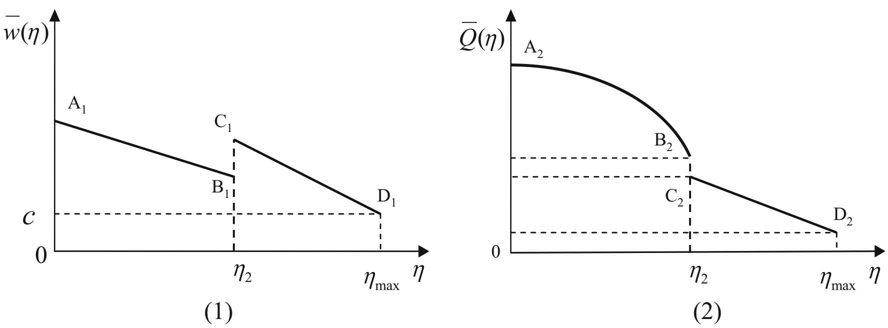

Fig. 3.2 Effects of retailer's risk preference structure on equilibrium

equilibrium order quantity. Thereafter, the equilibrium wholesale price decreases linearly with η until c and in response the equilibrium order quantity also decreases linearly with η, as indicated by the line segments C_1–D_1 in Fig. 3.2(1) and C_2–D_2 in Fig. 3.2(2), respectively.

It is seen from Theorem 3.4.2 that, as illustrated in Fig. 3.2, the equilibrium wholesale price generally decreases with η except an upward jump at the threshold η_2. At the same time, however, the equilibrium order quantity also decreases with η. This is somewhat counterintuitive. However, this is actually a result of the tradeoff between a decrease in the wholesale price and an increase in the retailer's degree of risk-aversion towards the contract value risk. To be specific, a decrease in the wholesale price entices the retailer to order more, whereas an increase in the degree of risk-aversion makes the retailer order less. A balance between the two aspects leads to the equilibrium order quantity decreasing with η.

Theorem 3.4.2 yields Theorem 3.4.3, which characterizes the expectation and the SD of the profits achieved respectively by the retailer and the supplier in equilibrium. Therein, we denote by $E_{wr}(\eta)$ and $SD_{wr}(\eta)$ the expectation and the SD of the retailer's profit, and by $R_{ws}(\eta)$ and $SD_{ws}(\eta)$ the supplier's.

Theorem 3.4.3. *(1) Except a possible discontinuity at η_2, the profit achieved by the supplier in equilibrium strictly decreases with η on $[0, \eta_{max}]$. In particular,*

$$R_{ws}(\eta) = \begin{cases} \frac{[(\alpha-\eta\sqrt{\alpha(1-\alpha)})a_H-c]^2}{8\delta(\alpha-\eta\sqrt{\alpha(1-\alpha)})}, & \text{if } \eta \le \eta_2; \\ \frac{[\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)-c]^2}{8\delta}, & \text{if } \eta_2 < \eta \le \eta_{max}. \end{cases} \tag{3.4.9}$$

$$SD_{ws}(\eta) = 0 \text{ for all } \eta \in [0, \eta_{max}]. \tag{3.4.10}$$

(2) Except a possible discontinuity at η_2, the SD of the profit achieved by the retailer in equilibrium strictly decreases with η on $[0, \eta_{max}]$. In particular,

(i) If $\eta \le \eta_2$,

$$\begin{cases} E_{wr}(\eta) = \frac{(\theta a_H-c)^2}{16\delta\theta} + \frac{(1-\theta)a_L^2}{4\delta} + \eta\sqrt{\alpha(1-\alpha)}[\frac{(\theta a_H-c)(3\theta a_H+c)}{16\delta\theta^2} - \frac{a_L^2}{4\delta}], \\ SD_{wr}(\eta) = \sqrt{\alpha(1-\alpha)}[\frac{(\theta a_H-c)(3\theta a_H+c)}{16\delta\theta^2} - \frac{a_L^2}{4\delta}]. \end{cases} \tag{3.4.11}$$

(ii) If $\eta_2 < \eta \le \eta_{max}$,

$$\begin{cases} E_{wr}(\eta) = \frac{(\overline{B}-c)^2}{16\delta} + \frac{\eta\sqrt{\alpha(1-\alpha)}}{4\delta}(\overline{B}-c)(a_H-a_L), \\ SD_{wr}(\eta) = \frac{\sqrt{\alpha(1-\alpha)}}{4\delta}(\overline{B}-c)(a_H-a_L), \end{cases} \tag{3.4.12}$$

where

$$\theta = \alpha - \eta\sqrt{\alpha(1-\alpha)}, \quad \overline{B} = \overline{A} - \eta\sqrt{\alpha(1-\alpha)}(a_H-a_L). \tag{3.4.13}$$

3.5 Efficiency of Wholesale Price Contract

This section examines the EPBE and the SDBE of the wholesale price contract in coordinating the supply chain. To this end, it is supposed that the supplier "sells" its firm to the retailer. Then for the centralized entity, it needs to determine the optimal production quantity at time 0, facing a stochastic price-dependent downward-sloping demand curve characterized by (3.3.1). The corresponding results are summarized in Lemma 3.5.1, in which $\overline{Q}_c(\eta)$ denotes the optimal production quantity of the centralized entity and $E_c(\eta)$ and $SD_c(\eta)$ denote the expectation and the SD of the channel profit at coordination, respectively.

Lemma 3.5.1. *(1) The optimal production quantity for the centralized entity is given by*

$$\overline{Q}_c(\eta) = \begin{cases} \frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H-c}{2\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}, & \text{if } \eta \le \eta_1; \\ \frac{\overline{A}-c-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}, & \text{if } \eta_1 < \eta \le \eta_{max}. \end{cases} \tag{3.5.1}$$

(2) The expectation and the SD of the channel profit at coordination are given by

(i) If $\eta \le \eta_1$,

$$\begin{cases} E_c(\eta) = \frac{\theta(\theta a_H-c)^2+\theta^2(1-\theta)a_L^2+\eta\sqrt{\alpha(1-\alpha)}[(\theta a_H-c)(\theta a_H+c)-\theta^2 a_L^2]}{4\delta\theta^2}, \\ SD_c(\eta) = \sqrt{\alpha(1-\alpha)}[\frac{(\theta a_H-c)(\theta a_H+c)-\theta^2 a_L^2}{4\delta\theta^2}]. \end{cases} \tag{3.5.2}$$

(ii) If $\eta_1 < \eta \le \eta_{max}$,

$$\begin{cases} E_c(\eta) = \frac{(\overline{B}-c)^2+2\eta\sqrt{\alpha(1-\alpha)}(\overline{B}-c)(a_H-a_L)}{4\delta}, \\ SD_c(\eta) = \frac{\sqrt{\alpha(1-\alpha)}}{2\delta}(\overline{B}-c)(a_H-a_L), \end{cases} \tag{3.5.3}$$

where

$$\theta = \alpha - \eta\sqrt{\alpha(1-\alpha)}, \;\; \overline{B} = \overline{A} - \eta\sqrt{\alpha(1-\alpha)}(a_H-a_L). \tag{3.5.4}$$

The following theorem characterizes the EPBE and the SDBE of wholesale price contracts in coordinating the supply chain. The EPBE is defined as the ratio of the expected channel profit earned in the decentralized supply chain at equilibrium to the expected channel profit achieved in the centralized entity, and define the SDBE as the ratio of the SD of the channel profit achieved in the decentralized supply chain at equilibrium to the SD of the channel profit achieved in the centralized entity.

Theorem 3.5.2. *(i) The expected-profit-based efficiency of wholesale price contracts is given by*

$$EPBE(\eta) = \begin{cases} 75\,\% + \frac{[(1-\alpha)a_L^2 + 2ca_H]\theta^2 - 2c(\alpha a_H + c)\theta + 2\alpha c^2}{4H(\eta)}, & \text{if } \eta \le \eta_2; \\ 75\,\% + \frac{3\theta^2(\overline{B}-c)^2 + 4\theta^2\eta\sqrt{\alpha(1-\alpha)}(\overline{B}-c)(a_H-a_L) - 3H(\eta)}{4H(\eta)}, & \text{if } \eta_2 < \eta \le \eta_1; \\ 75\,\% - \frac{\eta\sqrt{\alpha(1-\alpha)}(\overline{B}-c)(a_H-a_L)}{2[(\overline{B}-c)^2 + 2\eta\sqrt{\alpha(1-\alpha)}(\overline{B}-c)(a_H-a_L)]}, & \text{if } \eta_1 < \eta \le \eta_{max}, \end{cases} \tag{3.5.5}$$

where

$$H(\eta) = \theta(\theta a_H - c)^2 + \theta^2(1-\theta)a_L^2 + \eta\sqrt{\alpha(1-\alpha)}[(\theta a_H - c)(\theta a_H + c) - \theta^2 a_L^2], \tag{3.5.6}$$

$$0 \le \frac{\eta\sqrt{\alpha(1-\alpha)}(\overline{B}-c)(a_H-a_L)}{2[(\overline{B}-c)^2 + 2\eta\sqrt{\alpha(1-\alpha)}(\overline{B}-c)(a_H-a_L)]} \le 25\,\% \text{ for all } 0 \le \eta \le \eta_{max}. \tag{3.5.7}$$

(ii) The SD-based efficiency of wholesale price contracts is given by

$$SDBE(\eta) = \begin{cases} 75\,\% - \frac{2c(\theta a_H - c) + \theta^2 a_L^2}{4[(\theta a_H - c)(\theta a_H + c) - \theta^2 a_L^2]}, & \text{if } \eta \le \eta_2; \\ \frac{a_L}{a_H + a_L} + \frac{\theta^2(a_H^2 - a_L^2)[\theta(a_H - a_L) - c] + c^2 a_L}{[\theta^2(a_H^2 - a_L^2) - c^2](a_H + a_L)}, & \text{if } \eta_2 < \eta \le \eta_1; \\ 50\,\%, & \text{if } \eta_1 < \eta \le \eta_{max}, \end{cases} \tag{3.5.8}$$

where

$$\frac{2c(\theta a_H - c) + \theta^2 a_L^2}{4[(\theta a_H - c)(\theta a_H + c) - \theta^2 a_L^2]} > 0 \text{ for all } \eta \le \eta_2, \tag{3.5.9}$$

$$\frac{\theta^2(a_H^2 - a_L^2)[\theta(a_H - a_L) - c] + c^2 a_L}{[\theta^2(a_H^2 - a_L^2) - c^2](a_H + a_L)} > 0 \text{ for all } \eta_2 < \eta \le \eta_1. \tag{3.5.10}$$

Since $H_3(\eta)$ strictly increases with η on $(\eta_1, \eta_{max}]$ (see the proof of Theorem 3.5.2), where

$$H_3(\eta) = \frac{\eta\sqrt{\alpha(1-\alpha)}(\overline{B}-c)(a_H-a_L)}{2[(\overline{B}-c)^2 + 2\eta\sqrt{\alpha(1-\alpha)}(\overline{B}-c)(a_H-a_L)]}, \tag{3.5.11}$$

it is seen from Theorem 3.5.2 that when the retailer's degree of risk-aversion is larger than η_1, the EPBE of wholesale price contracts is always lower than 75 % of the expected channel profit at coordination. Furthermore, this efficiency strictly decreases with η, whereas it is bounded from below by 50 % (see from (3.5.7)). However, at the same time, the SD of the channel profit is only half of the SD of the channel profit at coordination. When the retailer's degree of risk-aversion is lower than η_1, there are some ambiguities in the changes of the two efficiencies of wholesale price contracts. However, it can be seen clearly from (3.5.8) that when the retailer's degree of risk-aversion is lower than η_2, the SD of the channel profit achieved at equilibrium in the decentralized supply chain decreases by at least 25 % as compared with that at coordination. Besides, it is known from the proof of Theorem 3.5.2 that

$$H_1(\eta)\begin{cases}\text{is positive and strictly decreases with } \eta \text{ on } [0,\eta_3);\\ \text{is nonpositive on } [\eta_3,\ \eta_4];\\ \text{is positive and strictly increases with } \eta \text{ on } (\eta_4,+\infty).\end{cases} \tag{3.5.12}$$

where

$$\begin{aligned} H_1(\eta) &= [(1-\alpha)a_L^2+2ca_H]\theta^2-2c(\alpha a_H+c)\theta+2\alpha c^2,\\ \eta_3 &= \tfrac{\alpha(1-\alpha)a_L^2+(\alpha a_H-c)c-c\sqrt{(\alpha a_H-c)^2-2\alpha(1-\alpha)a_L^2}}{\sqrt{\alpha(1-\alpha)}[(1-\alpha)a_L^2+2ca_H]}>0,\\ \eta_4 &= \tfrac{\alpha(1-\alpha)a_L^2+(\alpha a_H-c)c+c\sqrt{(\alpha a_H-c)^2-2\alpha(1-\alpha)a_L^2}}{\sqrt{\alpha(1-\alpha)}[(1-\alpha)a_L^2+2ca_H]}\geq\eta_3>0. \end{aligned} \tag{3.5.13}$$

Therefore, when the retailer's degree of risk-aversion towards the contract value risk is lower than $\min\{\eta_2,\eta_3,\eta_4\}$, the EPBE of wholesale price contracts strictly decreases with η, whereas it is always above 75 % of the expected channel profit at coordination.

3.6 Expectation and SD of the Retail Price

This section examines the effects of the retailer's degree of risk-aversion towards the contract value risk on the end consumer. To this end, the research seeks to characterize the expectation and SD of the retail price in terms of η.

Theorem 3.6.1. *The expected retail price strictly increases with η on $[0,\eta_{max}]$, and the SD of the retail price strictly increases with η on $[0,\eta_2]$, whereas it remains unchanged on $(\eta_2,\eta_{max}]$. In particular, the expectation and the SD of the retail price are given by*

(i) If $\eta\leq\eta_2$,

$$\begin{cases} E_p(\eta)=\frac{\alpha c}{4\theta}+\frac{3\alpha a_H+2(1-\alpha)a_L}{4},\\ SD_p(\eta)=\sqrt{\alpha(1-\alpha)}\left(\frac{c}{4\theta}+\frac{3a_H-2a_L}{4}\right). \end{cases} \tag{3.6.1}$$

(ii) If $\eta_2<\eta\leq\eta_{max}$,

$$\begin{cases} E_p(\eta)=\frac{3\bar{A}+\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{4},\\ SD_p(\eta)=\sqrt{\alpha(1-\alpha)}(a_H-a_L). \end{cases} \tag{3.6.2}$$

We see from (3.6.1) and (3.6.2) that the end consumer will suffer a loss from an increase in the degree of the retailer's risk-aversion towards the contract value risk. This is particularly true when $\eta\geq\eta_2$.

3.7 Extension with Numerical Experiment

This section will extend the model to the case where a follows a uniform distribution over $[m, n]$. Since a rigorous theoretical analysis is impossible with such an extension, a numerical research is deployed to explore this extended model, by which to check whether the findings with the binary distribution still hold with a uniform distribution.

In the numerical experiment, the parameters are set as follows: $m = 1$, $n = 100$, $\delta = 0.2$, and $c = 4$, the EPBE and SDBE are defined as in Sect. 3.5. The experimental results are presented in Figs. 3.3, 3.4, 3.5, and 3.6. With the experimental results given in Figs. 3.3 and 3.4, it can be seen that the equilibrium wholesale price and order quantity are both decreasing with the retailer's degree of risk aversion. This is very consistent with the findings in the case of binary distribution. In addition, from Figs. 3.5 and 3.6, it can be seen that even though the EPBE of the supply chain decreases as the retailer becomes more risk averse, the SDBE also decreases simultaneously. This is also consistent to the findings in the case of binary distribution. In particular, it can be observed that the EPBE of the supply chain can low to 50 %, at the same time, however, the associated SDBE can low to below 50 %. All these results demonstrate that, to a good extent, the findings under the premise of binary distribution also hold for the more general case of uniform distribution. Actually, an intuitive explanation of these results is: when the retailer gets more risk averse, its order quantity will reduce and hence the retail price will increase, which will exacerbate the double marginalization effect

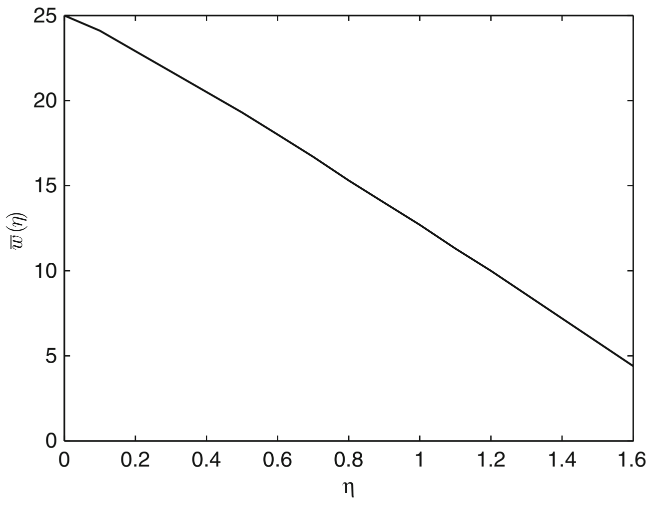

Fig. 3.3 Changes of the equilibrium wholesale price with respect to η

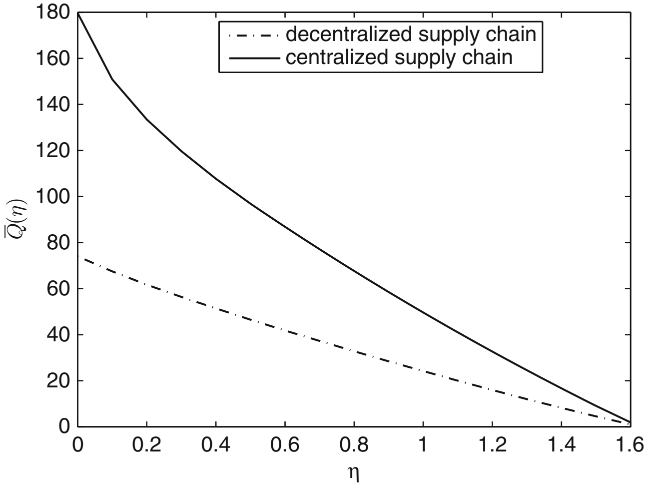

Fig. 3.4 Changes of the equilibrium order quantity with respect to η

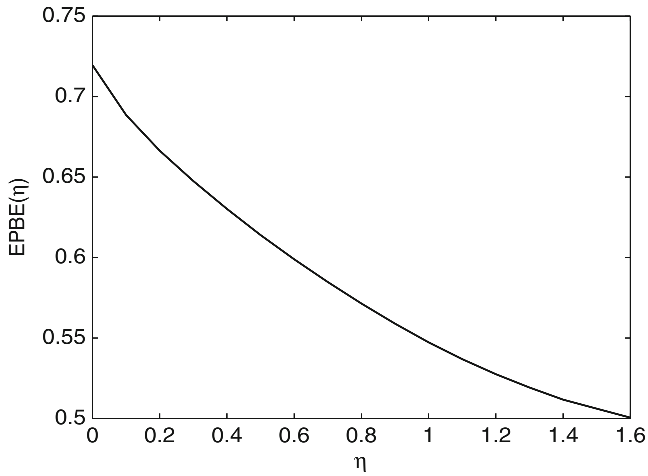

Fig. 3.5 Changes of the EPBE with respect to η

and hence lead to more efficiency loss in the channel. At the same time, however, the risk embedded in profit can be hedged better with the retailer becoming more averse of the risk, which leads to a relatively low SDBE.

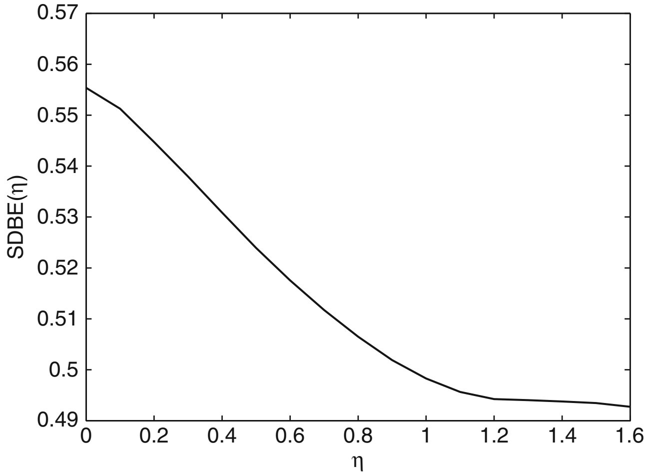

Fig. 3.6 Changes of the SDBE with respect to η

3.8 Conclusion

In this chapter, a mean-risk analysis is developed for the wholesale price contract with a supplier-retailer supply chain facing a stochastic price-dependent downward-sloping demand curve, taking into account value risk of the wholesale price contract and degree of the retailer's risk-aversion towards the contract value risk. This research makes the first attempt to assess the efficiency of the wholesale price contract incorporating contract value risk. The results demonstrate that contract value risk can be an important factor for assessing the performance of a supply chain contract, and therefore is worth taking into consideration in the design or analysis of supply chain contracts. More research needs to be developed for a wider range of supply chain contracts that have been well studied in theory based on the concept of expectation and widely adopted in practice, such as the buyback contract, the revenue-sharing contract, etc., which are left as the interesting topics for future research.

Appendix: Proofs of the Main Results

Proof of Lemma 3.4.1. First, to examine the retailer's optimal strategy at time 1, given its order quantity Q at time 0. When the realized demand curve at time 1 is $p_H = a_H - \delta q$, the retailer's problem can be formulated as $\max_{0\leq q\leq Q} q(a_H - \delta q)$. Solving this problem, the solution, denoted by $\overline{q}_{wH}$, is obtained as follows:

$$\overline{q}_{wH} = \begin{cases} \frac{a_H}{2\delta}, & \text{if } \frac{a_H}{2\delta} \leq Q; \\ Q, & \text{otherwise.} \end{cases} \tag{3.8.1}$$

Likewise, when the realization of the demand curve at time 1 is $p_L = a_L - \delta q$, the retailer's problem can be formulated as $\max_{0 \le q \le Q} q(a_L - \delta q)$. Solving this problem, the solution, denoted by $\overline{q}_{wL}$, is obtained as follows:

$$\overline{q}_{wL} = \begin{cases} \frac{a_L}{2\delta}, & \text{if } \frac{a_L}{2\delta} \le Q; \\ Q, & \text{otherwise.} \end{cases} \tag{3.8.2}$$

The following analyzes the retailer's optimal order quantity at time 0, and this problem can be solved based on three cases of Q:

Case (i) $0 \le Q \le \frac{a_L}{2\delta}$. For this case, by (3.8.1) and (3.8.2), the retailer's expected profit can be obtained as

$$E\Pi_{wr1}(Q) = \alpha Q(a_H - \delta Q) + (1-\alpha)Q(a_L - \delta Q) - wQ, \tag{3.8.3}$$

and the corresponding variance of the profit can be obtained as

$$\begin{aligned} \sigma^2_{wr1}(Q) &= \alpha[Q(a_H - \delta Q) - wQ - E\Pi_{wr1}(Q)]^2 \\ &\quad + (1-\alpha)[Q(a_L - \delta Q) - wQ - E\Pi_{wr1}(Q)]^2 \\ &= \alpha[Q(a_H - \delta Q) - wQ - \alpha Q(a_H - \delta Q) - (1-\alpha)Q(a_L - \delta Q) + wQ]^2 \\ &\quad + (1-\alpha)[Q(a_L - \delta Q) - wQ - \alpha Q(a_H - \delta Q) - (1-\alpha)Q(a_L - \delta Q) + wQ]^2 \\ &= \alpha(1-\alpha)Q^2(a_H - a_L)^2. \end{aligned} \tag{3.8.4}$$

Therefore, the standard deviation (SD) of the retailer's profit is given by

$$\sigma_{wr1}(Q) = \sqrt{\alpha(1-\alpha)}Q(a_H - a_L). \tag{3.8.5}$$

Thus, the problem faced by the retailer at time 0 is formulated as

$$\mathrm{P}_{3A.1}: \quad \max_{0 \le Q \le \frac{a_L}{2\delta}} ED_{wr1}(Q) = E\Pi_{wr1}(Q) - \eta\sigma_{wr1}(Q), \tag{3.8.6}$$

where η indicates the retailer's degree of risk-aversion towards the value risk of the wholesale price contract. Solving problem $\mathrm{P}_{3A.1}$, we obtain the solution denoted by $\overline{Q}_{w1}$ as follows:

$$\overline{Q}_{w1}(\eta) = \begin{cases} \frac{a_L}{2\delta}, & \text{if } \eta \le \frac{\alpha(a_H - a_L) - w}{\sqrt{\alpha(1-\alpha)}(a_H - a_L)}; \\ \frac{\bar{A} - w - \eta\sqrt{\alpha(1-\alpha)}(a_H - a_L)}{2\delta}, & \text{if } \frac{\alpha(a_H - a_L) - w}{\sqrt{\alpha(1-\alpha)}(a_H - a_L)} < \eta \le \frac{\bar{A} - w}{\sqrt{\alpha(1-\alpha)}(a_H - a_L)}; \\ 0, & \text{otherwise.} \end{cases} \tag{3.8.7}$$

Case (ii) $\frac{a_L}{2\delta} \le Q \le \frac{a_H}{2\delta}$. For this case, by (3.8.1) and (3.8.2), the retailer's expected profit can be obtained as

$$E\Pi_{wr2}(Q) = \alpha Q(a_H - \delta Q) + (1-\alpha)\frac{a_L}{2\delta}(a_L - \delta\frac{a_L}{2\delta}) - wQ, \tag{3.8.8}$$

and the corresponding variance of the profit can be obtained as

$$\begin{aligned}\sigma_{wr2}^2(Q) &= \alpha[Q(a_H-\delta Q)-wQ-E\Pi_{wr2}(Q)]^2\\&\quad+(1-\alpha)[\tfrac{a_L}{2\delta}(a_L-\delta\tfrac{a_L}{2\delta})-wQ-E\Pi_{wr2}(Q)]^2\\&=\alpha(1-\alpha)[Q(a_H-\delta Q)-\tfrac{a_L^2}{4\delta}]^2.\end{aligned} \tag{3.8.9}$$

Since $Q(a_H-\delta Q)\geq \frac{a_L}{2\delta}(a_H-\delta\frac{a_L}{2\delta}) > \frac{a_L}{2\delta}(a_L-\delta\frac{a_L}{2\delta}) = \frac{a_L^2}{4\delta}$ for all $\frac{a_L}{2\delta}\leq Q\leq\frac{a_H}{2\delta}$, the SD of the retailer's profit is given by

$$\sigma_{wr2}(Q)=\sqrt{\alpha(1-\alpha)}[Q(a_H-\delta Q)-\tfrac{a_L^2}{4\delta}]. \tag{3.8.10}$$

Thus, the problem faced by the retailer at time 0 can be formulated as

$$\mathrm{P}_{3A.2}:\quad \max_{\frac{a_L}{2\delta}\leq Q\leq\frac{a_H}{2\delta}} ED_{wr2}=E\Pi_{wr2}(Q)-\eta\sigma_{wr2}(Q). \tag{3.8.11}$$

Solving problem $\mathrm{P}_{3A.2}$, the solution, denoted by $\overline{Q}_{w2}$, is obtained as follows:

$$\overline{Q}_{w2}(\eta)=\begin{cases}\frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H-w}{2\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}, & \text{if } \eta\leq\frac{\alpha(a_H-a_L)-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)};\\ \frac{a_L}{2\delta}, & \text{if } \frac{\alpha(a_H-a_L)-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)}<\eta\leq\sqrt{\frac{\alpha}{1-\alpha}};\\ \frac{a_H}{2\delta}, & \text{if } \sqrt{\frac{\alpha}{1-\alpha}}<\eta.\end{cases} \tag{3.8.12}$$

Case (iii) $Q\geq\frac{a_H}{2\delta}$. Similarly, by (3.8.1) and (3.8.2), the retailer's expected profit can be obtained as

$$E\Pi_{wr3}(Q)=\alpha\tfrac{a_H}{2\delta}(a_H-\delta\tfrac{a_H}{2\delta})+(1-\alpha)\tfrac{a_L}{2\delta}(a_L-\delta\tfrac{a_L}{2\delta})-wQ, \tag{3.8.13}$$

and the corresponding variance of the profit can be obtained as

$$\begin{aligned}\sigma_{wr3}^2(Q) &= \alpha[\tfrac{a_H}{2\delta}(a_H-\delta\tfrac{a_H}{2\delta})-wQ-E\Pi_{wr3}(Q)]^2\\&\quad+(1-\alpha)[\tfrac{a_L}{2\delta}(a_L-\delta\tfrac{a_L}{2\delta})-wQ-E\Pi_{wr3}(Q)]^2\\&=\alpha(1-\alpha)(\tfrac{a_H^2-a_L^2}{4\delta})^2.\end{aligned} \tag{3.8.14}$$

Therefore, the SD of the retailer's profit is given by

$$\sigma_{wr3}(Q)=\sqrt{\alpha(1-\alpha)}(\tfrac{a_H^2-a_L^2}{4\delta}). \tag{3.8.15}$$

Thus, the problem faced by the retailer at time 0 can be formulated as

$$\mathrm{P}_{3A.3}:\quad \max_{Q\geq\frac{a_H}{2\delta}} ED_{wr3}=E\Pi_{wr3}(Q)-\eta\sigma_{wr3}(Q). \tag{3.8.16}$$

Solving problem $P_{3A.3}$, the optimal solution, denoted by $\overline{Q}_{w3}$, is obtained as follows:

$$\overline{Q}_{w3}(\eta) = \frac{a_H}{2\delta} \text{ for all } \eta. \tag{3.8.17}$$

Summarizing (3.8.7), (3.8.12), and (3.8.17), we obtain

(1) When $\eta \leq \frac{\alpha(a_H - a_L) - w}{\sqrt{\alpha(1-\alpha)}(a_H - a_L)}$, the optimal quantity for the retailer to order at time 0 is determined by

$$\overline{Q}_w(\eta) = \arg\max_Q \{ED_{wr1}(\tfrac{a_L}{2\delta}),\ ED_{wr2}(\tfrac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H - w}{2\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}),\ ED_{wr3}(\tfrac{a_H}{2\delta})\}. \tag{3.8.18}$$

Since

$$\begin{aligned} &ED_{wr2}(\tfrac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H - w}{2\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}) \geq ED_{wr2}(\tfrac{a_L}{2\delta}) = ED_{wr1}(\tfrac{a_L}{2\delta}), \\ &ED_{wr2}(\tfrac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H - w}{2\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}) \geq ED_{wr2}(\tfrac{a_H}{2\delta}) = ED_{wr3}(\tfrac{a_H}{2\delta}), \end{aligned} \tag{3.8.19}$$

where the inequalities in (3.8.19) become equality, if and only if $\eta = \frac{\alpha(a_H - a_L) - w}{\sqrt{\alpha(1-\alpha)}(a_H - a_L)}$, the optimal order quantity in this case is $\overline{Q}_w(\eta) = \frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H - w}{2\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}$.

(2) Since we do not know the magnitudes of $\frac{\overline{A}-w}{\sqrt{\alpha(1-\alpha)}(a_H - a_L)}$ and $\sqrt{\frac{\alpha}{1-\alpha}}$, the following proceeds the proof by considering two cases as follows:

Case (i) $\sqrt{\frac{\alpha}{1-\alpha}} \leq \frac{\overline{A}-w}{\sqrt{\alpha(1-\alpha)}(a_H - a_L)}$, i.e., $w \leq a_L$. Then by (3.8.7), (3.8.12), and (3.8.17), when $\frac{\alpha(a_H - a_L) - w}{\sqrt{\alpha(1-\alpha)}(a_H - a_L)} < \eta \leq \sqrt{\frac{\alpha}{1-\alpha}}$, the optimal quantity for the retailer to order at time 0 is determined by

$$\overline{Q}_w(\eta) = \arg\max_Q \{ED_{wr1}(\tfrac{\overline{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H - a_L)}{2\delta}),\ ED_{wr2}(\tfrac{a_L}{2\delta}),\ ED_{wr3}(\tfrac{a_H}{2\delta})\}. \tag{3.8.20}$$

Since

$$ED_{wr1}(\tfrac{\overline{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H - a_L)}{2\delta}) > ED_{wr1}(\tfrac{a_L}{2\delta}) = ED_{wr2}(\tfrac{a_L}{2\delta}) \geq ED_{wr2}(\tfrac{a_H}{2\delta}) = ED_{wr3}(\tfrac{a_H}{2\delta}), \tag{3.8.21}$$

the optimal quantity in this case is $\overline{Q}_w(\eta) = \frac{\overline{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H - a_L)}{2\delta}$. When $(\frac{\alpha(a_H - a_L) - w}{\sqrt{\alpha(1-\alpha)}(a_H - a_L)} <) \sqrt{\frac{\alpha}{1-\alpha}} < \eta \leq \frac{\overline{A}-w}{\sqrt{\alpha(1-\alpha)}(a_H - a_L)}$, the optimal quantity for the retailer to order at time 0 is determined by

$$\overline{Q}_w(\eta) = \arg\max_Q \{ED_{wr1}(\tfrac{\overline{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H - a_L)}{2\delta}),\ ED_{wr2}(\tfrac{a_H}{2\delta}),\ ED_{wr3}(\tfrac{a_H}{2\delta})\}. \tag{3.8.22}$$

Since

$$
\begin{aligned}
&ED_{wr1}(\tfrac{\bar{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}) - ED_{wr2}(\tfrac{a_H}{2\delta})(= ED_{wr3}(\tfrac{a_H}{2\delta}))\\
&= \tfrac{[\bar{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)-w]^2}{4\delta} - [\tfrac{\alpha-\eta\sqrt{\alpha(1-\alpha)}}{4\delta}(a_H^2-a_L^2) + \tfrac{1}{4\delta}(a_L^2-2wa_H)]\\
&= \tfrac{1}{4\delta}[((\alpha-\eta\sqrt{\alpha(1-\alpha)})(a_H-a_L)-w)^2 + 2w(a_H-a_L)\\
&\quad -(\alpha-\eta\sqrt{\alpha(1-\alpha)})(a_H-a_L)^2]\\
&> 0 \text{ (note that } \eta > \sqrt{\tfrac{\alpha}{1-\alpha}} \Rightarrow \alpha-\eta\sqrt{\alpha(1-\alpha)} < 0),
\end{aligned}
\tag{3.8.23}
$$

the optimal quantity in this case is $\overline{Q}_w(\eta) = \frac{\bar{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}$.

Case (ii) $\sqrt{\frac{\alpha}{1-\alpha}} > \frac{\bar{A}-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)}$, i.e., $w > a_L$. Then by (3.8.7), (3.8.12), and (3.8.17), when $\frac{\alpha(a_H-a_L)-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)} < \eta \leq \frac{\bar{A}-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)} (< \sqrt{\frac{\alpha}{1-\alpha}})$, the optimal quantity for the retailer to order at time 0 is determined by

$$
\overline{Q}_w(\eta) = \arg\max_Q\{ED_{wr1}(\tfrac{\bar{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}),\ ED_{wr2}(\tfrac{a_L}{2\delta}),\ ED_{wr3}(\tfrac{a_H}{2\delta})\}.
\tag{3.8.24}
$$

Since

$$
ED_{wr1}(\tfrac{\bar{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}) > ED_{wr1}(\tfrac{a_L}{2\delta}) = ED_{wr2}(\tfrac{a_L}{2\delta}) \geq ED_{wr2}(\tfrac{a_H}{2\delta}) = ED_{wr3}(\tfrac{a_H}{2\delta}),
\tag{3.8.25}
$$

the optimal quantity in this case is $\overline{Q}_w(\eta) = \frac{\bar{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}$. When $(\frac{\alpha(a_H-a_L)-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)} <)\frac{\bar{A}-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)} < \eta \leq \sqrt{\frac{\alpha}{1-\alpha}}$, the optimal quantity for the retailer to order at time 0 is determined by

$$
\overline{Q}_w(\eta) = \arg\max_Q\{ED_{wr1}(0),\ ED_{wr2}(\tfrac{a_L}{2\delta}),\ ED_{wr3}(\tfrac{a_H}{2\delta})\}.
\tag{3.8.26}
$$

Since

$$
ED_{wr1}(0) > ED_{wr1}(\tfrac{a_L}{2\delta}) = ED_{wr2}(\tfrac{a_L}{2\delta}) \geq ED_{wr2}(\tfrac{a_H}{2\delta}) = ED_{wr3}(\tfrac{a_H}{2\delta}),
\tag{3.8.27}
$$

the optimal quantity in this case is $\overline{Q}_w(\eta) = 0$.

(3) When $\eta > \max\{\frac{\bar{A}-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)}, \sqrt{\frac{\alpha}{1-\alpha}}\}$, by (3.8.7), (3.8.12), and (3.8.17), the optimal quantity for the retailer to order at time 0 is determined by

$$
\overline{Q}_w(\eta) = \arg\max_Q\{ED_{wr1}(0),\ ED_{wr2}(\tfrac{a_H}{2\delta}),\ ED_{wr3}(\tfrac{a_H}{2\delta})\}.
\tag{3.8.28}
$$

Since

$$
ED_{wr2}(\tfrac{a_H}{2\delta}) = ED_{wr3}(\tfrac{a_H}{2\delta}) < 0 = ED_{wr1}(0),
\tag{3.8.29}
$$

the optimal quantity in this case is $\overline{Q}_w(\eta) = 0$.

Summarizing the above analysis, we obtain that if $w \le a_L$, the optimal quantity for the retailer to order at time 0 is given by

$$\overline{Q}_w(\eta) = \begin{cases} \frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H-w}{2\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}, & \text{if } \eta \le \frac{\alpha(a_H-a_L)-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)}; \\ \frac{\overline{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}, & \text{if } \frac{\alpha(a_H-a_L)-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)} < \eta \le \sqrt{\frac{\alpha}{1-\alpha}}; \\ \frac{\overline{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}, & \text{if } \sqrt{\frac{\alpha}{1-\alpha}} < \eta \le \frac{\overline{A}-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)}; \\ 0, & \text{otherwise;} \end{cases} \tag{3.8.30}$$

if $w > a_L$, the optimal quantity for the retailer to order at time 0 is given by

$$\overline{Q}_w(\eta) = \begin{cases} \frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H-w}{2\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}, & \text{if } \eta \le \frac{\alpha(a_H-a_L)-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)}; \\ \frac{\overline{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}, & \text{if } \frac{\alpha(a_H-a_L)-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)} < \eta \le \frac{\overline{A}-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)}; \\ 0, & \text{if } \frac{\overline{A}-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)} < \eta \le \sqrt{\frac{\alpha}{1-\alpha}}; \\ 0, & \text{otherwise.} \end{cases} \tag{3.8.31}$$

Summarizing (3.8.30) and (3.8.31), we obtain

$$\overline{Q}_w(\eta) = \begin{cases} \frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H-w}{2\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}, & \text{if } \eta \le \frac{\alpha(a_H-a_L)-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)}; \\ \frac{\overline{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}, & \text{if } \frac{\alpha(a_H-a_L)-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)} < \eta \le \frac{\overline{A}-w}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)}; \\ 0, & \text{otherwise.} \end{cases} \tag{3.8.32}$$

Equivalently, we transform (3.8.32) into

$$\overline{Q}_\eta(w) = \begin{cases} \frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H-w}{2\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}, & \text{if } w \le [\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-a_L); \\ \frac{\overline{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}, & \text{if } [\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-a_L) < w \\ & \qquad \le \overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L); \\ 0, & \text{otherwise.} \end{cases} \tag{3.8.33}$$

Hence, the proof is completed. □

Proof of Theorem 3.4.2. Theorem 3.4.2 will be shown based on two cases as follows:

Case (i): $[\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-a_L) \ge c$, i.e., $\eta \le \frac{\alpha(a_H-a_L)-c}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)} (\triangleq \eta_1)$. By Lemma 3.4.1, the supplier's problem in this case can be formulated as

$$\mathrm{P}_{3A.4}: \quad \overline{w} = \arg\max_{w}\{\overline{H}_1, \overline{H}_2\}, \tag{3.8.34}$$

where

$$\overline{H}_1 = \max_{c \le w \le [\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-a_L)} \{\frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H - w}{2\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}\}(w-c),$$
$$\overline{H}_2 = \max_{[\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-a_L) < w \le \overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)} [\frac{\overline{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}](w-c). \tag{3.8.35}$$

Solving problem $\overline{H}_1$, we obtain the solution denoted by $\overline{w}_1(\eta)$ as follows:

$$\overline{w}_1(\eta) = \begin{cases} \frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H+c}{2}, & \text{if } c \le [\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-2a_L); \\ (\alpha-\eta\sqrt{\alpha(1-\alpha)})(a_H-a_L), & \text{if } c > [\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-2a_L). \end{cases} \tag{3.8.36}$$

Solving problem $\overline{H}_2$, the solution, denoted by $\overline{w}_2(\eta)$, is obtained as follows:

$$\overline{w}_2(\eta) = \begin{cases} [\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-a_L), & \text{if } c \le [\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-a_L)-a_L; \\ \frac{\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{2}, & \text{if } c > [\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-a_L)-a_L. \end{cases} \tag{3.8.37}$$

For ease of exposition, we denote

$$c_1 = [\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-a_L)-a_L \text{ and } c_2 = [\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-2a_L). \tag{3.8.38}$$

Since $c_2 \ge c_1$, summarizing (3.8.36) and (3.8.37), it is obtained that

$$\overline{w}(\eta) = \begin{cases} \frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H+c}{2}, & \text{if } c < c_1; \\ \frac{\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{2}, & \text{if } c > c_2; \\ \frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H+c}{2}, & \text{if } c_1 \le c \le c_2 \text{ and} \\ & \overline{H}_1(\frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H+c}{2}) \ge \overline{H}_2(\frac{\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{2}); \\ \frac{\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{2}, & \text{if } c_1 \le c \le c_2 \text{ and} \\ & \overline{H}_1(\frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H+c}{2}) < \overline{H}_2(\frac{\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{2}). \end{cases} \tag{3.8.39}$$

It is easy to obtain

$$\begin{aligned} \overline{H}_1(\frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H+c}{2}) &= (\frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H - \frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H+c}{2}}{2\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]})(\frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H+c}{2}-c) \\ &= \frac{[(\alpha-\eta\sqrt{\alpha(1-\alpha)})a_H-c]^2}{8\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}, \end{aligned} \tag{3.8.40}$$

$$\begin{aligned} &\overline{H}_2(\frac{\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{2}) \\ &= [\frac{\overline{A}-\frac{\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{2}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}](\frac{\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{2}-c) \\ &= \frac{[\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)-c]^2}{8\delta}. \end{aligned} \tag{3.8.41}$$

Therefore,

$$\begin{aligned}
&\overline{H}_1(\tfrac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H+c}{2})-\overline{H}_2(\tfrac{\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{2})\\
&=\tfrac{[(\alpha-\eta\sqrt{\alpha(1-\alpha)})a_H-c]^2}{8\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}-\tfrac{[\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)-c]^2}{8\delta}\\
&=\tfrac{1-[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}{8\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}[c^2-2[\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-a_L)c+[(\alpha-\eta\sqrt{\alpha(1-\alpha)})a_H]^2\\
&\quad-2[\alpha-\eta\sqrt{\alpha(1-\alpha)}]^2a_Ha_L-[\alpha-\eta\sqrt{\alpha(1-\alpha)}][1-(\alpha-\eta\sqrt{\alpha(1-\alpha)})]a_L^2].
\end{aligned} \tag{3.8.42}$$

Since $0<\alpha-\eta\sqrt{\alpha(1-\alpha)}\leq\alpha<1$ for all $0\leq\eta\leq\eta_1$, $\frac{1-[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}{8\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}>0$ for all $0\leq\eta\leq\eta_1$. Therefore, the inequality $\overline{H}_1(\frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H+c}{2})\leq\overline{H}_2(\frac{\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{2})$ is equivalent to

$$\begin{aligned}
&c^2-2[\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-a_L)c+[(\alpha-\eta\sqrt{\alpha(1-\alpha)})a_H]^2\\
&-2[\alpha-\eta\sqrt{\alpha(1-\alpha)}]^2a_Ha_L-[\alpha-\eta\sqrt{\alpha(1-\alpha)}][1-(\alpha-\eta\sqrt{\alpha(1-\alpha)})]a_L^2\leq 0.
\end{aligned} \tag{3.8.43}$$

Solving inequality (3.8.43), we obtain that

$$\overline{H}_1(\tfrac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H+c}{2})\leq\overline{H}_2(\tfrac{\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{2}) \tag{3.8.44}$$

holds iff $c\in[c_3,c_4]$, where

$$\begin{aligned}
c_3&=a_H[\alpha-\eta\sqrt{\alpha(1-\alpha)}]-a_L[\sqrt{\alpha-\eta\sqrt{\alpha(1-\alpha)}}+(\alpha-\eta\sqrt{\alpha(1-\alpha)})],\\
c_4&=a_H[\alpha-\eta\sqrt{\alpha(1-\alpha)}]+a_L(\sqrt{\alpha-\eta\sqrt{\alpha(1-\alpha)}}-[\alpha-\eta\sqrt{\alpha(1-\alpha)}]).
\end{aligned} \tag{3.8.45}$$

Since $c_1\leq c_3\leq c_2\leq c_4$, we can simplify (3.8.39) as

$$\overline{w}_1(\eta)=\begin{cases}\frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H+c}{2}, & \text{if } c\leq c_3;\\ \frac{\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{2}, & \text{if } c>c_3.\end{cases} \tag{3.8.46}$$

In addition,

$$\begin{aligned}
c\leq c_3 &\Longleftrightarrow (a_H-a_L)[\alpha-\eta\sqrt{\alpha(1-\alpha)}]-a_L\sqrt{\alpha-\eta\sqrt{\alpha(1-\alpha)}}-c\geq 0\\
&\Longleftrightarrow \sqrt{\alpha-\eta\sqrt{\alpha(1-\alpha)}}\leq\tfrac{a_L-\sqrt{a_L^2+4c(a_H-a_L)}}{2(a_H-a_L)} \text{ or}\\
&\qquad\ \sqrt{\alpha-\eta\sqrt{\alpha(1-\alpha)}}\geq\tfrac{a_L+\sqrt{a_L^2+4c(a_H-a_L)}}{2(a_H-a_L)}.
\end{aligned} \tag{3.8.47}$$

Since $\sqrt{\alpha-\eta\sqrt{\alpha(1-\alpha)}}>0$ while $\frac{a_L-\sqrt{a_L^2+4c(a_H-a_L)}}{2(a_H-a_L)}<0$,

$$\begin{aligned} c\le c_3 &\Longleftrightarrow \sqrt{\alpha-\eta\sqrt{\alpha(1-\alpha)}}\ge \frac{a_L+\sqrt{a_L^2+4c(a_H-a_L)}}{2(a_H-a_L)} \\ &\Longleftrightarrow \eta\le\eta_2, \end{aligned} \tag{3.8.48}$$

where

$$\eta_2=\frac{4(a_H-a_L)[\alpha(a_H-a_L)-c]-2a_L^2-2a_L\sqrt{a_L^2+4c(a_H-a_L)}}{4\sqrt{\alpha(1-\alpha)}(a_H-a_L)^2}. \tag{3.8.49}$$

It is easy to obtain that

$$\begin{aligned} \eta_1-\eta_2 &= \frac{\alpha(a_H-a_L)-c}{\sqrt{\alpha(1-\alpha)}(a_H-a_L)}-\frac{4(a_H-a_L)[\alpha(a_H-a_L)-c]-2a_L^2-2a_L\sqrt{a_L^2+4c(a_H-a_L)}}{4\sqrt{\alpha(1-\alpha)}(a_H-a_L)^2} \\ &= \frac{2a_L^2+2a_L\sqrt{a_L^2+4c(a_H-a_L)}}{4\sqrt{\alpha(1-\alpha)}(a_H-a_L)^2}>0, \end{aligned} \tag{3.8.50}$$

i.e., $\eta_1>\eta_2$. Hence, the equilibrium wholesale price in the case of $\eta\le\eta_1$ is given by

$$\overline{w}_1(\eta)=\begin{cases}\frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H+c}{2}, & \text{if } \eta\le\eta_2;\\ \frac{\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{2}, & \text{if } \eta_2<\eta\le\eta_1.\end{cases} \tag{3.8.51}$$

Case (ii): $[\alpha-\eta\sqrt{\alpha(1-\alpha)}](a_H-a_L)<c$, i.e., $\eta>\eta_1$. By Lemma 3.4.1, the supplier's problem in the case can be formulated as

$$(\mathrm{P}_{3A.5}):\quad \max_{c\le w\le\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)} \left[\frac{\overline{A}-w-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)}{2\delta}\right](w-c), \tag{3.8.52}$$

Solving problem ($\mathrm{P}_{3A.5}$), we obtain the solution denoted by $\overline{w}_2$ as

$$\overline{w}_2(\eta)=\frac{\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{2} \text{ for all } \eta\in(\eta_1,\eta_{max}]. \tag{3.8.53}$$

Summarizing (3.8.51) and (3.8.53), the equilibrium wholesale price is obtained as

$$\overline{w}(\eta)=\begin{cases}\frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H+c}{2}, & \text{if } \eta\le\eta_2;\\ \frac{\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)+c}{2}, & \text{if } \eta_2<\eta\le\eta_{max}.\end{cases} \tag{3.8.54}$$

Substituting (3.8.54) into (3.8.32), the equilibrium order quantity of the retailer is obtained as

$$\overline{Q}(\eta) = \begin{cases} \frac{[\alpha-\eta\sqrt{\alpha(1-\alpha)}]a_H-c}{4\delta[\alpha-\eta\sqrt{\alpha(1-\alpha)}]}, & \text{if } \eta \leq \eta_2; \\ \frac{\overline{A}-\eta\sqrt{\alpha(1-\alpha)}(a_H-a_L)-c}{4\delta}, & \text{if } \eta_2 < \eta \leq \eta_{max}. \end{cases} \tag{3.8.55}$$

Thus, the proof is completed. □

Proof of Theorem 3.4.3. For ease of exposition, denote $\theta(\eta) = \alpha - \eta\sqrt{\alpha(1-\alpha)}$ and $\overline{B}(\eta) = \overline{A} - \eta\sqrt{\alpha(1-\alpha)}(a_H - a_L)$ in the following. Furthermore, if no confusion, denote $\theta(\eta)$, $\overline{B}(\eta)$, $\overline{w}(\eta)$, and $\overline{Q}(\eta)$ simply by θ, $\overline{B}$, $\overline{w}$, and $\overline{Q}$, respectively. First to show Theorem 3.4.3(1). By (3.8.54) and (3.8.55), it is obtained that if $\eta \leq \eta_2$,

$$\overline{R}_{ws}(\eta) = (\overline{w} - c)\overline{Q} = (\tfrac{\theta a_H+c}{2} - c)(\tfrac{\theta a_H-c}{4\delta\theta}) = \tfrac{(\theta a_H-c)^2}{8\delta\theta}. \tag{3.8.56}$$

It is easy to obtain

$$\frac{d\overline{R}_{ws}(\eta)}{d\eta} = -\frac{\sqrt{\alpha(1-\alpha)}(\theta a_H+c)(\theta a_H-c)}{8\delta\theta^2}. \tag{3.8.57}$$

Obviously, (3.8.57)< 0 for all $\eta \leq \eta_2$. Therefore, $\overline{R}_{ws}(\eta)$ strictly decreases with η for all $\eta \leq \eta_2$. Similarly, if $\eta_2 < \eta \leq \eta_{max}$, by (3.8.54) and (3.8.55), it is obtained that

$$\overline{R}_{ws}(\eta) = (\overline{w} - c)\overline{Q} = (\tfrac{\overline{B}+c}{2} - c)(\tfrac{\overline{B}-c}{4\delta}) = \tfrac{(\overline{B}-c)^2}{8\delta}, \tag{3.8.58}$$

$$\frac{d\overline{R}_{ws}(\eta)}{d\eta} = -\frac{\sqrt{\alpha(1-\alpha)}(a_H-a_L)(\overline{B}-c)}{4\delta}. \tag{3.8.59}$$

Since $\overline{B} > c$ for all $\eta \in (\eta_2, \eta_{max})$, (3.8.59)$< 0$ for all $\eta \in (\eta_2, \eta_{max})$. Therefore, $\overline{R}_{ws}(\eta)$ strictly decreases with η on $(\eta_2, \eta_{max}]$. To summarize, the result given in Theorem 3.4.3(1) is derived.

The following proceeds to show Theorem 3.4.3(2). Likewise, by (3.8.54) and (3.8.55), together with checking the proofs of Lemma 3.4.1 and Theorem 3.4.2, it is obtained that if $\eta \leq \eta_2$,

$$\begin{aligned} E_{wr}(\eta) &= \alpha\overline{Q}(a_H - \delta\overline{Q}) + (1-\alpha)\tfrac{a_L}{2\delta}(a_L - \delta\tfrac{a_L}{2\delta}) - \overline{w}\overline{Q} \\ &= \alpha(\tfrac{\theta a_H-c}{4\delta\theta})(a_H - \delta\tfrac{\theta a_H-c}{4\delta\theta}) + (1-\alpha)\tfrac{a_L}{2\delta}(a_L - \delta\tfrac{a_L}{2\delta}) - (\tfrac{\theta a_H+c}{2})(\tfrac{\theta a_H-c}{4\delta\theta}) \\ &= \tfrac{(\theta a_H-c)^2}{16\delta\theta} + \tfrac{(1-\theta)a_L^2}{4\delta} + \eta\sqrt{\alpha(1-\alpha)}[\tfrac{(\theta a_H-c)(3\theta a_H+c)}{16\delta\theta^2} - \tfrac{a_L^2}{4\delta}], \end{aligned} \tag{3.8.60}$$

$$\begin{aligned} SD_{wr}(\eta) &= \sqrt{\alpha(1-\alpha)}[\overline{Q}(a_H - \delta\overline{Q}) - \tfrac{a_L^2}{4\delta}] \\ &= \sqrt{\alpha(1-\alpha)}[\tfrac{\theta a_H-c}{4\delta\theta}(a_H - \delta\tfrac{\theta a_H-c}{4\delta\theta}) - \tfrac{a_L^2}{4\delta}] \\ &= \sqrt{\alpha(1-\alpha)}[\tfrac{(\theta a_H-c)(3\theta a_H+c)}{16\delta\theta^2} - \tfrac{a_L^2}{4\delta}]. \end{aligned} \tag{3.8.61}$$

Since for all $\eta \leq \eta_2$,

$$\frac{dSD_{wr}(\eta)}{d\eta} = -\frac{\alpha(1-\alpha)(\theta a_H + c)c}{8\delta\theta^3} < 0, \tag{3.8.62}$$

$SD_{wr}(\eta)$ strictly decreases with η for all $\eta \leq \eta_2$. Similarly, if $\eta_2 < \eta \leq \eta_{max}$, it is obtained

$$\begin{aligned} E_{wr}(\eta) &= \alpha\overline{Q}(a_H - \delta\overline{Q}) + (1-\alpha)\overline{Q}(a_L - \delta\overline{Q}) - \overline{w}\overline{Q} \\ &= \alpha\frac{\overline{B}-c}{4\delta}(a_H - \delta\frac{\overline{B}-c}{4\delta}) + (1-\alpha)\frac{\overline{B}-c}{4\delta}(a_L - \delta\frac{\overline{B}-c}{4\delta}) - (\frac{\overline{B}+c}{2})(\frac{\overline{B}-c}{4\delta}) \\ &= \frac{(\overline{B}-c)^2}{16\delta} + \frac{\eta\sqrt{\alpha(1-\alpha)}}{4\delta}(\overline{B}-c)(a_H - a_L), \end{aligned} \tag{3.8.63}$$

$$\begin{aligned} SD_{wr}(\eta) &= \sqrt{\alpha(1-\alpha)}\overline{Q}(a_H - a_L) \\ &= \frac{\sqrt{\alpha(1-\alpha)}}{4\delta}(\overline{B}-c)(a_H - a_L). \end{aligned} \tag{3.8.64}$$

Since

$$\frac{dSD_{wr}(\eta)}{d\eta} = -\frac{\alpha(1-\alpha)}{4\delta}(a_H - a_L)^2 < 0, \tag{3.8.65}$$

$SD_{wr}(\eta)$ strictly decreases with η for all $\eta \in (\eta_2, \eta_{max}]$. Thus, the proof is completed. □

Proof of Lemma 3.5.1. Using the marginal production cost c to replace the wholesale price w in the proof of Lemma 3.4.1, the system-wide optimal production quantity for the centralized entity can be obtained in a similar way as follows:

$$\overline{Q}_c(\eta) = \begin{cases} \frac{\theta a_H - c}{2\delta\theta}, & \text{if } \eta \leq \eta_1; \\ \frac{\overline{B}-c}{2\delta}, & \text{if } \eta_1 < \eta \leq \eta_{max}, \end{cases} \tag{3.8.66}$$

where

$$\theta = \alpha - \eta\sqrt{\alpha(1-\alpha)}, \;\; \overline{B} = \overline{A} - \eta\sqrt{\alpha(1-\alpha)}(a_H - a_L). \tag{3.8.67}$$

Putting $w = c$ in (3.8.8) and then substituting (3.8.66) into it, it is obtained that if $\eta \leq \eta_1$,

$$\begin{aligned} E_c(\eta) &= \alpha\overline{Q}_c(a_H - \delta\overline{Q}_c) + (1-\alpha)\frac{a_L}{2\delta}(a_L - \delta\frac{a_L}{2\delta}) - c\overline{Q}_c \\ &= \alpha\frac{\theta a_H - c}{2\delta\theta}(a_H - \delta\frac{\theta a_H - c}{2\delta\theta}) + (1-\alpha)\frac{a_L}{2\delta}(a_L - \delta\frac{a_L}{2\delta}) - c\frac{\theta a_H - c}{2\delta\theta} \\ &= \frac{\theta(\theta a_H - c)^2 + \theta^2(1-\theta)a_L^2 + \eta\sqrt{\alpha(1-\alpha)}[(\theta a_H - c)(\theta a_H + c) - \theta^2 a_L^2]}{4\delta\theta^2} \\ &= \frac{H(\eta)}{4\delta\theta^2}, \end{aligned} \tag{3.8.68}$$

where, for ease of exposition in the following, it is denoted that

$$\theta(\theta a_H - c)^2 + \theta^2(1-\theta)a_L^2 + \eta\sqrt{\alpha(1-\alpha)}[(\theta a_H - c)(\theta a_H + c) - \theta^2 a_L^2] \triangleq H(\eta). \tag{3.8.69}$$

Substituting (3.8.66) into (3.8.10), it is obtained that

$$\begin{aligned} SD_c(\eta) &= \sqrt{\alpha(1-\alpha)}[\overline{Q}_c(a_H - \delta\overline{Q}_c) - \tfrac{a_L^2}{4\delta}] \\ &= \sqrt{\alpha(1-\alpha)}[\tfrac{\theta a_H - c}{2\delta\theta}(a_H - \delta\tfrac{\theta a_H - c}{2\delta\theta}) - \tfrac{a_L^2}{4\delta}] \\ &= \sqrt{\alpha(1-\alpha)}[\tfrac{(\theta a_H - c)(\theta a_H + c) - \theta^2 a_L^2}{4\delta\theta^2}]. \end{aligned} \tag{3.8.70}$$

Similarly, if $\eta_1 < \eta \le \eta_{max}$, by setting $w = c$ in (3.8.3) and then substituting (3.8.66) into it, we obtain

$$\begin{aligned} E_c(\eta) &= \alpha\overline{Q}_c(a_H - \delta\overline{Q}_c) + (1-\alpha)\overline{Q}_c(a_L - \delta\overline{Q}_c) - c\overline{Q}_c \\ &= \alpha\tfrac{\overline{B}-c}{2\delta}(a_H - \delta\tfrac{\overline{B}-c}{2\delta}) + (1-\alpha)\tfrac{\overline{B}-c}{2\delta}(a_L - \delta\tfrac{\overline{B}-c}{2\delta}) - c\tfrac{\overline{B}-c}{2\delta} \\ &= \tfrac{(\overline{B}-c)^2 + 2\eta\sqrt{\alpha(1-\alpha)}(\overline{B}-c)(a_H - a_L)}{4\delta}. \end{aligned} \tag{3.8.71}$$

Substituting (3.8.66) into (3.8.5), we have

$$\begin{aligned} SD_c(\eta) &= \sqrt{\alpha(1-\alpha)}\overline{Q}_c(a_H - a_L) \\ &= \tfrac{\sqrt{\alpha(1-\alpha)}}{2\delta}(\overline{B} - c)(a_H - a_L). \end{aligned} \tag{3.8.72}$$

To summarize, the proof is completed. □

Proof of Theorem 3.5.2. First to show Theorem 3.5.2(i). It can be obtained by Theorem 3.4.3 that the expected channel profit in the decentralized supply chain is

$$E_{total}(\eta) = \overline{R}_{ws}(\eta) + E_{wr}(\eta) = \begin{cases} \frac{3H(\eta) + \theta^2(1-\theta)a_L^2 - \eta\sqrt{\alpha(1-\alpha)}[2c(\theta a_H - c) + \theta^2 a_L^2]}{16\delta\theta^2}, & \text{if } \eta \le \eta_2; \\ \frac{3(\overline{B}-c)^2 + 4\eta\sqrt{\alpha(1-\alpha)}(\overline{B}-c)(a_H - a_L)}{16\delta}, & \text{if } \eta_2 < \eta \le \eta_{max}, \end{cases} \tag{3.8.73}$$

where

$$\theta = \alpha - \eta\sqrt{\alpha(1-\alpha)}, \quad \overline{B} = \overline{A} - \eta\sqrt{\alpha(1-\alpha)}(a_H - a_L), \tag{3.8.74}$$

$$H(\eta) = \theta(\theta a_H - c)^2 + \theta^2(1-\theta)a_L^2 + \eta\sqrt{\alpha(1-\alpha)}[(\theta a_H - c)(\theta a_H + c) - \theta^2 a_L^2]. \tag{3.8.75}$$

Comparing (3.8.73) with (3.8.68) and (3.8.71), it follows that

$$EFFe(\eta) = \begin{cases} 75\,\% + \frac{\theta^2(1-\theta)a_L^2 - \eta\sqrt{\alpha(1-\alpha)}[2c(\theta a_H - c) + \theta^2 a_L^2]}{4H(\eta)}, & \text{if } \eta \le \eta_2; \\ 75\,\% + \frac{3\theta^2(\overline{B}-c)^2 + 4\theta^2\eta\sqrt{\alpha(1-\alpha)}(\overline{B}-c)(a_H - a_L) - 3H(\eta)}{4H(\eta)}, & \text{if } \eta_2 < \eta \le \eta_1; \\ 75\,\% - \frac{\eta\sqrt{\alpha(1-\alpha)}(\overline{B}-c)(a_H - a_L)}{2[(\overline{B}-c)^2 + 2\eta\sqrt{\alpha(1-\alpha)}(\overline{B}-c)(a_H - a_L)]}, & \text{if} \eta_1 < \eta \le \eta_{max}. \end{cases} \tag{3.8.76}$$

It can be obtained that

$$\begin{aligned} & \theta^2(1-\theta)a_L^2 - \eta\sqrt{\alpha(1-\alpha)}[2c(\theta a_H - c) + \theta^2 a_L^2] \\ = & \ \theta^2(1-\theta)a_L^2 + \theta[2c(\theta a_H - c) + \theta^2 a_L^2] - \alpha[2c(\theta a_H - c) + \theta^2 a_L^2] \\ = & \ [(1-\alpha)a_L^2 + 2ca_H]\theta^2 - 2c(\alpha a_H + c)\theta + 2\alpha c^2. \end{aligned} \tag{3.8.77}$$

Substituting (3.8.77) into (3.8.76) derives (3.5.5) in Theorem 3.5.2(i). For ease of exposition, denote

$$H_1(\eta) = [(1-\alpha)a_L^2 + 2ca_H]\theta^2 - 2c(\alpha a_H + c)\theta + 2\alpha c^2. \tag{3.8.78}$$

Since

$$\frac{d^2 H_1(\eta)}{d\eta^2} = 2\alpha(1-\alpha)[(1-\alpha)a_L^2 + 2ca_H] > 0, \tag{3.8.79}$$

$H_1(\eta)$ is strictly convex in η. Let $H_1(\eta) = 0$. Then if there is no solution for this equation with regard to η, then $H_1(\eta) > 0$ for all $\eta \le \eta_2$; otherwise, solving this equation with regard to η, we obtain its two solutions as

$$\begin{aligned} \eta_3 &= \frac{\alpha(1-\alpha)a_L^2 + (\alpha a_H - c)c - c\sqrt{(\alpha a_H - c)^2 - 2\alpha(1-\alpha)a_L^2}}{\sqrt{\alpha(1-\alpha)}[(1-\alpha)a_L^2 + 2ca_H]}, \\ \eta_4 &= \frac{\alpha(1-\alpha)a_L^2 + (\alpha a_H - c)c + c\sqrt{(\alpha a_H - c)^2 - 2\alpha(1-\alpha)a_L^2}}{\sqrt{\alpha(1-\alpha)}[(1-\alpha)a_L^2 + 2ca_H]}. \end{aligned} \tag{3.8.80}$$

Again, since $H_1(\eta)$ is strictly convex in η and $H_1(0) = (1-\alpha)\alpha^2 a_L^2 > 0$, it follows that $\eta_4 \ge \eta_3 > 0$. Furthermore,

$$H_1(\eta) \begin{cases} \text{is positive and strictly decreases with } \eta \text{ on } [0, \eta_3); \\ \text{is nonpositive on } [\eta_3, \eta_4]; \\ \text{is positive and strictly increases with } \eta \text{ on } (\eta_4, +\infty). \end{cases} \tag{3.8.81}$$

In addition, denote

$$H_2(\eta) = \frac{\eta\sqrt{\alpha(1-\alpha)}(\overline{B}-c)(a_H - a_L)}{2[(\overline{B}-c)^2 + 2\eta\sqrt{\alpha(1-\alpha)}(\overline{B}-c)(a_H - a_L)]}. \tag{3.8.82}$$

We obtain by calculation that for all $\eta_1 < \eta < \eta_{max}$,

$$\frac{dH_2(\eta)}{d\eta} = \frac{\sqrt{\alpha(1-\alpha)}(\overline{B}-c)^3(a_H - a_L) + \eta\alpha(1-\alpha)(\overline{B}-c)^2(a_H - a_L)^2}{2[(\overline{B}-c)^2 + 2\eta\sqrt{\alpha(1-\alpha)}(\overline{B}-c)(a_H - a_L)]^2} > 0. \tag{3.8.83}$$

Therefore, $H_2(\eta)$ strictly increases with η on $(\eta_1, \eta_{max}]$. Furthermore, obviously, $0 \le H_2(\eta) \le 25\,\% = H_2(\eta_{max})$ for all $\eta_1 < \eta \le \eta_{max}$. To summarize, Theorem 3.5.2(i) follows.

The following proceeds to show Theorem 3.5.2(ii). Since the profit obtained by the supplier is deterministic, the SD of the channel profit achieved in the decentralized supply chain is determined by the SD of the retailer's profit. Then, by (3.8.61) and (3.8.64), we have

$$D_{total}(\eta) = \begin{cases} \sqrt{\alpha(1-\alpha)}[\frac{(\theta a_H - c)(3\theta a_H + c) - 4\theta^2 a_L^2}{16\delta\theta^2}], & \text{if } \eta \le \eta_2; \\ \frac{\sqrt{\alpha(1-\alpha)}}{4\delta}(\overline{B} - c)(a_H - a_L), & \text{if } \eta_2 < \eta \le \eta_{max}. \end{cases} \tag{3.8.84}$$

Comparing (3.8.84) with (3.8.70) and (3.8.72), it follows that

$$EFF_{sd}(\eta) = \begin{cases} 75\,\% - \frac{2c(\theta a_H - c) + \theta^2 a_L^2}{4[(\theta a_H - c)(\theta a_H + c) - \theta^2 a_L^2]}, & \text{if } \eta \le \eta_2; \\ \frac{a_L}{a_H + a_L} + \frac{\theta^2(a_H^2 - a_L^2)[\theta(a_H - a_L) - c] + c^2 a_L}{[\theta^2(a_H^2 - a_L^2) - c^2](a_H + a_L)}, & \text{if } \eta_2 < \eta \le \eta_1; \\ 50\,\%, & \text{if } \eta_1 < \eta \le \eta_{max}. \end{cases} \tag{3.8.85}$$

Since $\theta(a_H - a_L) > c$ for all $\eta \le \eta_2 (< \eta_1)$,

$$\frac{2c(\theta a_H - c) + \theta^2 a_L^2}{4[(\theta a_H - c)(\theta a_H + c) - \theta^2 a_L^2]} > 0 \text{ for all } \eta \le \eta_2, \tag{3.8.86}$$

$$\frac{\theta^2(a_H^2 - a_L^2)[\theta(a_H - a_L) - c] + c^2 a_L}{[\theta^2(a_H^2 - a_L^2) - c^2](a_H + a_L)} > 0 \text{ for all } \eta_2 < \eta \le \eta_1. \tag{3.8.87}$$

To summarize, the proof of Theorem 3.5.2 is completed. □

Proof of Theorem 3.6.1. By Theorem 3.4.2, together with checking the proof of Lemma 3.4.1, it is obtained that if $\eta \le \eta_2$,

$$\begin{aligned} E_p(\eta) &= \alpha(a_H - \delta\overline{Q}) + (1-\alpha)(a_L - \delta\tfrac{a_L}{2\delta}) \\ &= \alpha(a_H - \delta\tfrac{\theta a_H - c}{4\delta\theta}) + (1-\alpha)(a_L - \delta\tfrac{a_L}{2\delta}) \\ &= \tfrac{\alpha c}{4\theta} + \tfrac{3\alpha a_H + 2(1-\alpha)a_L}{4}, \end{aligned} \tag{3.8.88}$$

$$\begin{aligned} SD_p(\eta) &= \sqrt{\alpha(a_H - \delta\overline{Q} - E_p(\eta))^2 + (1-\alpha)(a_L - \delta\tfrac{a_L}{2\delta} - E_p(\eta))^2} \\ &= \sqrt{\alpha(a_H - \delta\tfrac{\theta a_H - c}{4\delta\theta} - E_p(\eta))^2 + (1-\alpha)(a_L - \delta\tfrac{a_L}{2\delta} - E_p(\eta))^2} \\ &= \sqrt{\alpha(1-\alpha)}(\tfrac{c}{4\theta} + \tfrac{3a_H - 2a_L}{4}), \end{aligned} \tag{3.8.89}$$

where $\theta = \alpha - \eta\sqrt{\alpha(1-\alpha)}$. It is easy to see that $E_p(\eta)$ and $SD_p(\eta)$ both strictly increase with η for all $\eta \le \eta_2$. Similarly, if $\eta_2 < \eta \le \eta_{max}$,

$$\begin{aligned} E_p(\eta) &= \alpha(a_H - \delta\overline{Q}) + (1-\alpha)(a_L - \delta\overline{Q}) \\ &= \alpha(a_H - \delta\tfrac{\overline{B} - c}{4\delta}) + (1-\alpha)(a_L - \delta\tfrac{\overline{B} - c}{4\delta}) \\ &= \tfrac{3\overline{A} + \eta\sqrt{\alpha(1-\alpha)}(a_H - a_L) + c}{4}. \end{aligned} \tag{3.8.90}$$

Since $\frac{\theta a_H - c}{4\delta\theta} \geq \frac{a_L}{2\delta}$ for all $\eta \leq \eta_2$ whereas $\frac{\overline{B}-c}{4\delta} \leq \frac{a_L}{2\delta}$ for all $\eta_2 < \eta \leq \eta_{max}$, we see from (3.8.88) and (3.8.90) that

$$\frac{\alpha c}{4[(\alpha - \eta_2\sqrt{\alpha(1-\alpha)})]} + \frac{3\alpha a_H + 2(1-\alpha)a_L}{4} \leq \frac{3\overline{A} + \eta_2\sqrt{\alpha(1-\alpha)}(a_H - a_L) + c}{4}. \tag{3.8.91}$$

Besides, we have

$$\begin{aligned} SD_p(\eta) &= \sqrt{\alpha(a_H - \delta\overline{Q} - E_p(\eta))^2 + (1-\alpha)(a_L - \delta\overline{Q} - E_p(\eta))^2} \\ &= \sqrt{\alpha(a_H - \delta\tfrac{\overline{B}-c}{4\delta} - E_p(\eta))^2 + (1-\alpha)(a_L - \delta\tfrac{\overline{B}-c}{4\delta} - E_p(\eta))^2} \\ &= \sqrt{\alpha(1-\alpha)}(a_H - a_L). \end{aligned} \tag{3.8.92}$$

Obviously, $E_p(\eta)$ strictly increases with η on $(\eta_2, \eta_{max}]$ and $SD_p(\eta)$ remains unchanged on $(\eta_2, \eta_{max}]$. To summarize, the proof is completed. □

Chapter 4
Franchise Fee Contracts in Product Service System with Demand Information Asymmetry

4.1 Introduction

Traditionally, manufacturing simply means production of tangible goods. Nowadays, however, customers are becoming much more demanding than ever and they desire for not only the product itself but also the related services. Consequently, the term "service-enhanced products", which refer to those tangible products bundled with an array of intangible services (Lester 1998), has evolved. More manufacturing firms begin recognizing that with providing the service-enhanced products, they can achieve a higher profit than only supplying a pure product. As a result, the concept of product service system (PSS) has become prevalent recently. Under the PSS, functionality of a product is maintained for customers throughout the whole product lifecycle in terms of both the physical product and the related services (Lifset 2000, Mont 2002, Wang and Jiang 2014). Examples of the traditional manufacturing firms which have shifted to be the PSS providers include IBM, GM, Intel, Shanghai Electric Group, and Xi'an Shangu Power Co., Ltd., etc. (Karmakar 2004, Li 2007). These firms not only integrate services into their products but also provide the services throughout the whole product lifecycle. To be more specific, these firms offer engineering, design, and testing skills to customize their products, as well as providing installation, training, and maintenance services. By providing a complete solution for the customer demand, these companies can effectively enhance the customer service level and hence capture a more market share. However, many manufacturers are incapable of providing the whole-of-life product-service package by themselves alone and they therefore need to collaborate with other firms when doing so. The survey conducted by Marceau and Martinez (2002) shows that firms offering services in addition to the physical products are more likely to collaborate

The research of this chapter is based on Xie et al. (2013).

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with other organizations as compared with those that offer no service and only the physical products. This finding justifies the claim that PSS is able to promote collaboration among firms.

This chapter focuses on the PSS in the supply chain that involves manufacturing of a tangible product and provision of the intangible services to fulfill the specific needs of the customers. While different services are linked to the product at different stages of the product lifecycle, this research considers those services that are provided at the final stage of distribution and marketing, which include physical product-based services, such as product transshipments (Alvarez et al. 2013), upgrading (Jiang et al. 2007), and maintenance and repair services (Jiang and Fukuda 2001). Specifically, the research considers a two-echelon supply chain comprising a manufacturer and a retailer. The manufacturer (he) has the main resources for production of the tangible product, such as equipment, plant, workers, etc., and the retailer (she) is a service provider who has marketing expertise and the long-term relationships with customers. By providing some additional services she can attract demands that are not directly accessible to the manufacturer (Gupta 2008). The manufacturer may choose to delegate a number of value-added service decisions to a well-informed retailer who is more capable of tailoring the service schemes to fit her customers. To be specific, the manufacturer sells a basic product first and after that he asks the retailer to add further services to the basic product before and/or after the product is sold to the end customer (Kurata and Nam 2010). Actually, the retailer specializes in providing value-enhancing services for the basic product by which to stimulate the final demand for the PSS.

The operations mode of the PSS has fundamental differences as compared with the traditional manufacturing supply chain. To be specific, with the latter the manufacturer generally assembles the basic components provided by upstream suppliers into a final tangible product and after that the product is passed to the retailer for sale to the end customer. With the PSS, however, the manufacturer assembles the core components into a basic product according to the bill of material (BOM), after that, the retailer supplements value-added services for the basic product. Since the retailer is free in choosing service level for the basic product, a major concern of the manufacturer is how to motivate the retailer to choose a beneficial service level for himself or the system. In addition, on the side of the retailer, due to her proximity to the end market and the marketing expertise, the retailer has more information about the demand than the manufacturer. In the absence of the mechanism for truthful information sharing, the demand information will be private to the retailer and subsequently adverse selection problem arises in the supply chain. Actually, the retailer can claim that high sales are due to her value-added services, while low sales are resulted from the sluggish demand. For solving the manufacturer's concern mentioned above and optimizing the operations of the PSS for the service-oriented manufacturing supply chain, this chapter is devoted into contract design by which the manufacturer can drive the retailer to choose an appropriate service level, as well as assuring a credible demand information sharing across the supply chain.

This chapter considers three types of contracts, namely, the franchise fee (FF) contract, the franchise fee with service requirement (FFS) contract, and the franchise fee with centralized service requirement (FFCS) contract. All these three forms of contracts involve the manufacturer's charging of a franchise fee but the service level requirements with the contract are different. To be specific, under the FF contract, the manufacturer imposes no service level requirement and the retailer can select her individual optimal service level. Under the FFS contract, the manufacturer specifies for the retailer a service level that maximizes his own profit. Under the FFCS contract, the manufacturer imposes a system-wide optimal service level on the retailer. In general, supply and service decisions in a manufacturer-retailer supply chain are made independently, that is, the manufacturer determines his own optimal policy to supply a product and the retailer chooses her own optimal policy to serve the customers. As a result, the optimal decisions made by the two parties may not be consistent with the system-wide optimal decision, leading to that the optimal channel profit cannot be achieved. In order to improve operations efficiency in the supply chain, Goyal (1976) and Landeros and Lyth (1989) develop some models to address the supply chain coordination issue with centralized control. These studies suggest approaches for determining an integrated order and delivery policies, so as to minimize the joint cost incurred by the both parties. It is shown that in the integrated models, one party's gain generated from the integrated policy exceeds the loss of the other. Thus, the net benefit can be shared by the both parties (the manufacturer and the retailer) in the equitable state. Many studies assume that the manufacturer and the retailer have completely symmetric information when making decisions. However, in many situations, some information is private to one party only and the other party has to make decisions with limited availability of these information. In addition, under the FFS and FFCS contracts, the same assumption as Haresh and Yi (2006) is assumed that one party in the supply chain is strong enough to impose a decision on the other. That is, the manufacturer is assumed to have the market power to implement his optimal service level specified by the FFS contract (the system-wide optimal service level specified by the FFCS contract), while the retailer has to commit to the required service level. Even though so, the retailer can still employ her information advantage to influence the manufacturer's contracting decision. Actually, in the presence of demand information asymmetry, the retailer has an incentive to inflate/deflate the true demand information when sharing this piece of information with the manufacturer, with the intent to make the contracting service level better compatible with her own interest.

This chapter mainly addresses the issue of contract design by which the manufacturer can drive the retailer to choose an appropriate service level, as well as assuring a credible demand information sharing across the supply chain. The research begins with the basic setting that the manufacturer allows the privately informed retailer to utilize her superior information to select a contract from a menu of the contracts that stipulate a wholesale price and a franchise fee. The wholesale price $w(\theta)$ and the franchise fee $L(\theta)$ are used to construct a FF contract. Based on the FF contract, the manufacturer can construct the FFS contract by specifying a service level $v(\theta)$. If the manufacturer has the market power to impose the system-wide optimal service level $v^c(\theta)$ on the retailer, then $\{w(\theta), L(\theta), v^c(\theta)\}$ will be a menu of the FFCS contracts. It is shown that a nonlinear franchise fee scheduling to a wholesale price and a service level can induce credible information sharing and simultaneously maximize the manufacturer's profit and ensure the retailer's participation. This process is modeled as follows: the manufacturer commits to a wholesale price w, which is observable with $w \leq p$, where p is a fixed sale price. After that, the retailer chooses a service level v for promoting the sales of the PSS. The analysis is conducted under the framework of the principal-agent theory for which the manufacturer acts as the principal and the retailer acts as the agent who provides value-added services for the product.

The rest of this chapter is organized as follows. Section 4.2 reviews the relevant literature. Section 4.3 describes and formulates the problem under study. Section 4.4 addresses design of the optimal contracts and identifies the structural properties for them. Section 4.5 presents numerical examples for the theoretical studies. Section 4.6 concludes the chapter with comments and a discussion of research extensions.

4.2 Literature Review

This chapter devotes to contract analysis and design for the PSS with asymmetric demand information. There are two streams of literature which are closely related to this research, one is those on PSS and the other is those on contract models in the supply chain.

There has been a substantial interest in the PSS recently and a comprehensive review of the relevant literature can be found in Baines et al. (2007). However, most of the existing literature focuses on the PSS concept, business design, application and case analysis (see, e.g., Brax 2005), and the literature to explore optimization of the PSS is quite seldom. Viswanadham et al. (2005) develop a dynamical model by which to investigate the bullwhip effect resulted from mismatches and delays between the manufacturing and service processes. Kameshwaran et al. (2009) propose a framework for the manufacturing firms to make decisions with respect to product-service bundling and pricing. In this paper, the after-sales repair and maintenance services are considered. Huang et al. (2011) present a comprehensive performance evaluation method for the PSS and develop an efficient algorithm for finding out the optimal solutions of the service selection and composition. All these papers mentioned above devote to design or configuration of the PSS for some specific business contexts and they do not explore the issues concerning operations optimization of the PSS in a general setting.

In the field of supply chain management, most of the studies related to contract theory focus on the issues concerning adverse selection (when one of the parties has private information) or moral hazard (when effort made by one party is unverifiable by the other). Recently, contract theory has also been applied to the business-to-business settings from cross-functional coordination (see, e.g., Whang 1992, Kouvelis and Lariviere 2000, Roels et al. 2010, Heredia et al. 2012, Ai et al. 2012) to supply chain coordination (see, e.g., Lariviere 1999, Corbett 2001, Cachon 2003, Perakis and Roels 2007, Taylor and Plambeck 2007, Shin and Tunca 2010). For instance, Whang (1992) develops a theoretical game model in which an outside contractor is hired to develop software for a buyer. The model considered in this chapter shares some common features with this paper. Specifically, like the retailer in our model, the software developer (as a service provider) in Whang (1992)'s model has private information about the production cost. However, the model considered in this paper differs from ours in that the buyer in the model of this paper cannot influence the service provider by its actions. For research related to supply chains with information asymmetry, Desiraju and Moorthy (1997) study the case of information asymmetry in the supply chain with a price- and service-sensitive demand curve. They show that coordination can be achieved by a requirement of the service performance. Lee et al. (2000) show that it is worthwhile to share demand information when the demand process is related to time. Ai et al. (2012) consider two competing supply chains with each consisting of a single manufacturer and a single retailer and compare the performances of various supply chain contracts under the supply chain competition and demand uncertainty. For research related to joint production in supply chains, Kim (2000) considers a supply chain in which the manufacturer's supports for its supplier's innovation can eventually lead to a reduction of the supply cost, while they do not consider information asymmetry. Roels et al. (2010) conduct an analysis for contracting issues that arise from the collaborative services, such as consulting, financial planning, and information technology outsourcing, in the supply chain that efforts made by the supply chain members are unverifiable to each other.

As seen from the above, there is a vast body of literature exploring the joint production contracts or supply contracts under information asymmetry. The research of this chapter can be viewed as a hybrid that includes some features of both the joint production model (the final product is provided jointly by two parties) and the supply chain model with asymmetric demand information. This chapter mainly addresses the issue concerning design of the contract mechanism for the PSS by conducting a vertical cooperative analysis between the manufacturer and his downstream partner with the information economics and game theories. The main contributions made by this chapter are as follows: First, this chapter develops three forms of contracts, namely, the FF, FFS, and FFCS contracts, for the PSS in the service-oriented manufacturing supply chain. Furthermore, the optimal contract structures are presented for minimizing the losses resulted from information asymmetry and double marginalization effect. Second, this chapter conducts a comprehensive analytical comparison for the FF, FFS, and FFCS contracts.

4.3 The Model

Consider a supply chain comprising a manufacturer and a retailer in which a PSS is adopted. To be specific, the manufacturer produces a basic product at a constant unit cost which, without loss of generality, is normalized to zero. After that, the retailer sells the basic product to end consumers with supplementing additional demand-enhancing value-added services, by which the retailer augments the basic product with additional value. Let v be the additional value added by the retailer, and for the value-added product, it is sold by the retailer at a fixed unit price p. The value-added product is referred as PSS product. In addition, assume that the manufacturer has enough market power and is able to dictate the selling price of the PSS product for the retailer. Let the wholesale price charged by the manufacturer be w and the service level provided by the retailer be v which is determined after the manufacturer sets the wholesale price. Besides, it is assumed that the retailer knows the end market demand better than the manufacturer because of her superior relationship with the customers, her proximity to the market, and her expertise about product sales. In other words, the retailer has private demand information which is unknown to the manufacturer.

Given that linear demand function is widely used in the literature (see, e.g., Lal 1990, Desiraju and Moorthy 1997), a linear demand function is assumed to characterize the demand of the PSS product as follows:

$$D = \theta + \gamma v, \tag{4.3.1}$$

where γv indicates the demand increased by the retailer with an addition of the value-added service v. Without loss of generality, the parameter γ is normalized to 1 herein and then the demand function (4.3.1) reduces to

$$D = \theta + v. \tag{4.3.2}$$

Denote θ as the base demand which is a piece of private demand information that is only known to the retailer. It is quite natural for the manufacturer to wish the retailer to share this piece of important information for his decision-making. The shared information is considered credible only if the retailer does not have any incentive to distort her private demand information. In the absence of a credible information sharing, the manufacturer has only a prior belief towards the base demand and considers the base demand a continuous random variable that takes values over $[0, T]$ with a cumulative distribution function (CDF) of $F(\cdot)$ and a probability density function (PDF) of $f(\cdot)$. These information is the common knowledge to both the manufacturer and the retailer.

Let $c(v)$ be the cost per unit resulted from the services added by the retailer. Given that a linear cost function is quite common in the literature, the function form $c(v) = \rho v$ is assumed in the research, where ρ measures the retailer's cost efficiency when she supplements the services for the basic product.

4.3.1 Centralized Supply Chain

The research begins with considering the first-best solution of the supply chain system. When the manufacturer and the retailer are vertically integrated, neither information asymmetry nor payment exists between them. The integrated supply chain makes the service level decision based on the known demand information. For a given demand information θ, the profit of the centralized supply chain is

$$\pi_T = (p - c)D. \tag{4.3.3}$$

The optimal service level that maximizes (4.3.3) is given by

$$v^c = \frac{p - \rho\theta}{2\rho}. \tag{4.3.4}$$

4.3.2 Wholesale Pricing Contract

In the decentralized supply chain, the objectives of the manufacturer and the retailer are to maximize their respective profits, which are given by (4.3.5) and (4.3.6), respectively.

$$\pi_m = wD = w(\theta + v), \tag{4.3.5}$$

$$\pi_r = (p - w - c)D = (p - w - c)(\theta + v). \tag{4.3.6}$$

If the manufacturer has access to the retailer's private demand information θ, by inferring the best response of the retailer regarding the service level decision, he can suitably choose a wholesale price by which to maximize his profit. In the case with symmetric demand information, he will maximize (4.3.5) by setting the wholesale price to be

$$w^I = \frac{p + \rho\theta}{2}. \tag{4.3.7}$$

And the retailer will maximize (4.3.6) by setting the service level to be

$$v^I = \frac{p - w^I - \rho\theta}{2\rho} = \frac{p}{4\rho} - \frac{3}{4}\theta. \tag{4.3.8}$$

It is observed that if θ is known only to the retailer, the retailer will have an incentive to deflate θ according to (4.3.7). This incentive arises because the wholesale price charged by the manufacturer is increasing in θ. Given such an incentive for the retailer, the manufacturer has to consider the demand information

θ informed by the retailer incredible and hence he will make the wholesale pricing decision only based on his prior belief regarding θ. Such an observation leads to that the manufacturer will maximize his expected profit by setting the wholesale price w as

$$w^a = \frac{p + \rho E(\theta)}{2}, \tag{4.3.9}$$

and in response the retailer will maximize her expected profit by setting the service level as

$$v^a = \frac{p - w^a - \rho\theta}{2\rho} = \frac{p}{4\rho} - \frac{\theta}{2} - \frac{E(\theta)}{4}, \tag{4.3.10}$$

where $E(\theta) = \int_0^T xf(x)dx$. Therefore, under the wholesale price contract w^a, the manufacturer's expected profit, the retailer's expected profit, and the channel's expected profit are given, respectively, as follows:

$$\pi_r^a = (p - w - c)(\theta + v) = (\frac{p}{4} - \frac{\rho E(\theta)}{4} + \frac{\rho\theta}{2})(\frac{p}{4\rho} + \frac{\theta}{2} - \frac{E(\theta)}{4}), \tag{4.3.11}$$

$$\pi_m^a = w(\theta + v) = (\frac{p + \rho E(\theta)}{2})(\frac{p}{4\rho} + \frac{\theta}{2} - \frac{E(\theta)}{4}), \tag{4.3.12}$$

$$\pi_T^a = (p - c)(\theta + v) = (\frac{3p}{4} + \frac{\rho E(\theta)}{4} + \frac{\rho\theta}{2})(\frac{p}{4\rho} + \frac{\theta}{2} - \frac{E(\theta)}{4}). \tag{4.3.13}$$

Comparing the service level decision for the decentralized system with asymmetric demand information, which is given by (4.3.10), with that for the centralized system, which is given by (4.3.4), and that for the decentralized system with symmetric demand information, which is given by (4.3.8), it is seen that the supply chain suffers two sources of inefficiency, one is the lack of a credible demand information sharing and the other is the double marginalization effect. The first source of inefficiency originates from the asymmetric demand information between the retailer and the manufacturer. Actually, it is seen from (4.3.9) that the manufacturer's wholesale price decision w^a is just related to $E(\theta)$ under the asymmetric demand information, and without a credible demand information sharing, the manufacturer cannot adjust the wholesale price to account for the retailer's private information. The inefficiency resulted from asymmetric information can cause losses for the both parties. For example, if the demand is much lower than the expected, the wholesale price will be relatively high and thus the service level will be relatively low, which causes sales loss and profit reduction for both the manufacturer and the retailer. On the other hand, if the demand is higher than the expected, the manufacturer will suffer a loss resulted from the relatively low wholesale price. The best way to deal with this inefficiency is to induce credible information sharing in the supply chain with appropriate mechanisms. The second source of inefficiency is the effect of double marginalization. It is noted that with symmetric demand information, the optimal service level v^I is responsive to θ. Since $w > 0$, v^I is lower than the service level v^c under the centralized system. Hence, the channel profit is lower than that under the centralized system. For the inefficiency resulted from double marginalization effect, a coordinating contract can be used to remedy it effectively (Cachon 2003).

4.4 Contract Design

4.4.1 FF Contract

With the FF contract, a menu of the contract including a wholesale price and a franchise fee is designed to screen the truthful demand information and push the retailer to provide an appropriate service level. The practice of coordinating a supply chain by a franchise fee L from the retailer to the manufacturer can be found in the literature such as Haresh and Yi (2006) and Lal (1990).

Before demand is realized, the manufacturer needs to make two decisions, one is the wholesale price and the other is the franchise fee. For a principle-agent model in which the manufacturer acts as the leader and the retailer acts as the follower, the sequence of event is as follows: (1) The retailer learns the demand information θ, which is known only by her and not by the manufacturer. (2) The manufacturer acquires a prior belief about θ, which is characterized by the CDF, $F(\cdot)$, and the PDF, $f(\cdot)$. (3) The manufacturer designs and offers a menu of the FF contract for the retailer, taking into account adverse selection problem. After that, (4) the retailer chooses an optimal contract that can maximize her profit from the menu of the FF contract. (5) With the chosen FF contract, the retailer makes an optimal service level decision with some service costs incurred and then delivers the final PSS product to the end customers. Finally, (6) a payment is transferred from the retailer to the manufacturer.

Let $\hat{\theta}$ denote the demand information observed by the manufacturer with the contract chosen by the retailer, which is not necessary to be the true demand. Separately, the true demand is denoted as θ. Then, the manufacturer's profit can be formulated as

$$\pi_m(w(\hat{\theta}), L(\hat{\theta}), v, \theta) = w(\hat{\theta})(\theta + v) + L(\hat{\theta}), \tag{4.4.1}$$

and the retailer's profit can be formulated as

$$\pi_r(w(\hat{\theta}), L(\hat{\theta}), v, \theta) = (p - w(\hat{\theta}) - \rho v)(\theta + v) - L(\hat{\theta}). \tag{4.4.2}$$

According to the revelation principle, it is sufficient to focus on the contracts under which the retailer will reveal her private information by the option she has selected.

Therefore, the main problem for the manufacturer is to determine a contract menu, denoted as $\{w(\theta), L(\theta)\}_{\theta \in [0,T]}$, to maximize his expected profit as follows:

$$\int_0^T [\pi_m(w(\theta), L(\theta), \theta, v)]f(\theta)d\theta, \tag{4.4.3}$$

subject to two constraints. The first one is the incentive compatibility (IC) constraint which is to ensure the retailer labeled with information θ only chooses the contract $\{w(\theta), L(\theta)\}$. In other words, with the IC constraint, the retailer can maximize her profit when she truthfully reports θ, which can be formulated as follows:

$$\max_v \pi_r(w(\theta), L(\theta), \theta, v) \geq \max_v \pi_r(w(\hat{\theta}), L(\hat{\theta}), \theta, v) \; \textit{for each} \; \theta \in [0, T]. \tag{4.4.4}$$

The second is the participation constraint (PC) which is to ensure the retailer to have an incentive to participate in the trade. With the participation constraint, the retailer's profit under the PSS is at least as high as that achieved with her outside options, which is denoted as π^r_{min}, that is,

$$\max_v \pi_r(w(\theta), L(\theta), \theta, v) \geq \pi^r_{min} \; \textit{for each} \; \theta \in [0, T]. \tag{4.4.5}$$

Since the retailer makes the service level decision after she has chosen the contract, the research will first solve the optimal v given a specific contract. This problem can be solved by maximizing the retailer's profit $\pi_r(w(\theta), L(\theta), \theta, v)$, which leads to the following result

$$v = \frac{p - w - \rho\theta}{2\rho}. \tag{4.4.6}$$

Substituting this specific expression of v into Eqs. (4.4.3), (4.4.4) and (4.4.5), the manufacturer's problem can be rewritten as

$$\max_{w,L} \int_0^T [\frac{(p - w(\theta) + \rho\theta)w(\theta)}{2\rho} + L(\theta)]f(\theta)d\theta \tag{4.4.7}$$

s.t.

$$IC: \frac{(p - w(\theta) + \rho\theta)^2}{4\rho} - L(\theta) \geq \frac{(p - w(\hat{\theta}) + \rho\theta)^2}{4\rho} - L(\hat{\theta}) \text{ for all } \theta \in [0, T], \tag{4.4.8}$$

$$PC: \frac{(p - w(\theta) + \rho\theta)^2}{4\rho} - L(\theta) \geq \pi^r_{min} \text{ for all } \theta \in [0, T]. \tag{4.4.9}$$

Solving the above problem leads to the following result.

Lemma 4.4.1. *Let $\pi_r(\theta)$ be the maximum profit achieved by the retailer with the private demand information θ. The contract menu $\{w(\theta), L(\theta)\}_{\theta\in[0,T]}$, where $w(\theta)$ is decreasing in θ, satisfies PC and IC if $\pi_r(\theta)$ and $L(\theta)$ are given by*

$$(i)\ \ \pi_r(\theta) = \int_0^\theta \frac{p - w(x) + \rho x}{2} dx + \pi^r_{min}, \tag{4.4.10}$$

$$(ii)\ \ L(\theta) = \frac{(p - w(\theta) + \rho\theta)^2}{4\rho} - \int_0^\theta \frac{p - w(x) + \rho x}{2} dx - \pi^r_{min}. \tag{4.4.11}$$

Proof. Suppose the retailer with θ_i maximizes her profit by revealing $\widehat{\theta_i}$. Since

$$\frac{\partial\ \pi_r(w(\hat{\theta}), L(\hat{\theta}), \theta)}{\partial\theta} = \frac{p - w(\hat{\theta}) + \rho\theta}{2} \geq 0, \tag{4.4.12}$$

it is obtained for $\theta_1 < \theta_2$ that

$$\pi_r(w(\widehat{\theta_1}), L(\widehat{\theta_1}), \theta_1) \leq \pi_r(w(\widehat{\theta_1}), L(\widehat{\theta_1}), \theta_2). \tag{4.4.13}$$

Also, since the retailer with θ_2 maximizes her profit by revealing $\widehat{\theta_2}$, it follows that

$$\pi_r(w(\widehat{\theta_1}), L(\widehat{\theta_1}), \theta_2) \leq \pi_r(w(\widehat{\theta_2}), L(\widehat{\theta_2}), \theta_2). \tag{4.4.14}$$

The IC requires that the retailer achieves a maximum profit when $\widehat{\theta_i} = \theta_i$. Thus,

$$\pi_r(w(\theta_1), L(\theta_1), \theta_1) \leq \pi_r(w(\theta_2), L(\theta_2), \theta_2). \tag{4.4.15}$$

In other words, $\pi_r(w(\theta), L(\theta), \theta)$ is increasing in θ. Hence, PC will be satisfied if $\pi_r(w(0), L(0), 0) \geq \pi^r_{min}$. Furthermore, since π_m is increasing in $L(\theta)$, the manufacturer will increase $L(\theta)$ until the retailer just receives a profit of π^r_{min}. Hence, the PC can be replaced by $PC' : \pi^r_{min} = \pi_r(w(0), L(0), 0)$. The retailer's maximum profit is $\pi_r(\theta) = \max_{\hat{\theta}} \pi_r(w(\hat{\theta}), L(\hat{\theta}), \theta)$. The envelope theorem implies that

$$\frac{d\pi_r(\theta)}{d\theta} = \frac{d\pi_r(w(\hat{\theta}), L(\hat{\theta}), \theta)}{d\theta} \Big|_{\hat{\theta}=\theta} = \frac{p - w(\theta) + \rho\theta}{2}. \tag{4.4.16}$$

Since $\pi^r_{min} = \pi_r(w(0), L(0), 0)$, by integrating the Eq. (4.4.16), the retailer's maximum profit can be expressed as

$$\pi_r(\theta) = \int_0^\theta \frac{p - w(x) + \rho x}{2} dx + \pi^r_{min}. \tag{4.4.17}$$

Next, the retailer's profit can be rewritten as

$$\begin{aligned}\pi_r(w(\hat{\theta}), L(\hat{\theta}), \theta) &= \int_0^\theta \frac{\partial \pi_r(w(\hat{\theta}), L(\hat{\theta}), x)}{\partial x} dx \\ &= \pi_r(w(\hat{\theta}), L(\hat{\theta}), \hat{\theta}) + \int_{\hat{\theta}}^\theta \frac{p - w(\hat{\theta}) + \rho x}{2} dx \\ &= \pi_r(w(\theta), L(\theta), \theta) + \int_{\hat{\theta}}^\theta [\frac{p - w(\hat{\theta}) + \rho x}{2} - \frac{p - w(x) + \rho x}{2}] dx.\end{aligned} \tag{4.4.18}$$

If $\theta > \hat{\theta}$, since $w(\theta)$ is decreasing in θ, the integrand is nonpositive and hence $\pi_r(w(\hat{\theta}), L(\hat{\theta}), \theta) \leq \pi_r(w(\theta), L(\theta), \theta)$. Similarly, if $\theta < \hat{\theta}$, the inequality also holds. Therefore, the condition that $w(\theta)$ is decreasing implies IC holds. In addition, given $\pi_r(\theta)$, the optimal franchise fee can be derived from (4.4.2), by which to obtain (ii). Thus, the proof is completed. □

Under the principal-agent model, the manufacturer wishes the retailer to choose a service level that can maximize the channel (or his) profit. It is seen from Lemma 4.4.1 that to achieve this end, the manufacturer has to offer the retailer with an information rent. Actually, the manufacturer will extract the channel profit with leaving an information rent and a reservation profit to the retailer. Furthermore, it is seen that the information rent is increasing in the retailer's private demand information, that is, the retailer will obtain a higher profit with the higher private demand information.

Theorem 4.4.2. *Let* $\{w^F(\theta), L^F(\theta)\}$ *denote the optimal menu of contracts that solves the manufacturer's problem in (4.4.3) for* $0 \leq \theta \leq T$. *Then,*

$$(i) \quad \begin{cases} w^F(\theta) = p & \text{for } \theta \leq \theta_1, \\ w^F(\theta) = \rho\eta(\theta) & \text{for } \theta \geq \theta_1. \end{cases} \tag{4.4.19}$$

where θ_1 *is the solution of* $\rho\eta(\theta) = p$ *and* $\eta(\theta) = \frac{1 - F(\theta)}{f(\theta)}$.

$$(ii) \quad L^F(\theta) = \frac{(p - w^F(\theta) + \rho\theta)^2}{4\rho} - \int_0^\theta \frac{p - w^F(x) + \rho x}{2} dx - \pi_{min}^r. \tag{4.4.20}$$

Proof. Note that the manufacturer's profit is the total channel profit minus the retailer's profit, the objective function in (4.4.7) can be rewritten as

$$\int_0^T [\frac{(p + \rho\theta)^2 - w^2}{4\rho} - \int_0^\theta \frac{p - w(x) + \rho x}{2} dx] f(\theta) d\theta - \pi_{min}^r. \tag{4.4.21}$$

If $w(\theta)$ is decreasing in θ, the optimal contract can be identified by maximizing (4.4.21). With using the integration by parts, it follows that this problem is equivalent to

$$\max_w \int_0^T [\frac{(p + \rho\theta)^2 - w^2}{4\rho} - \frac{1 - F(\theta)}{f(\theta)} \frac{p - w(\theta) + \rho\theta}{2}] f(\theta) d\theta. \tag{4.4.22}$$

The optimal wholesale price can be determined by the first-order optimization condition for (4.4.22) with respect to w, which is to solve

$$\eta(\theta) - \frac{w(\theta)}{\rho} = 0. \tag{4.4.23}$$

Since the second-order optimization condition of (4.4.22) is $-\frac{1}{\rho} < 0$, the optimal solution of (4.4.22) can be obtained by the first-order optimization condition, which leads to (4.4.19).

To complete the proof, the following proceeds to prove that $w(\theta)$ is decreasing in θ. To this end, differentiating (4.4.23) with respect to θ yields

$$\frac{dw}{d\theta} = \frac{d\eta(\theta)}{d\theta}\rho. \tag{4.4.24}$$

Herein, the hazard rate is assumed to be non-increasing, i.e., $\frac{d\eta(\theta)}{d\theta} \leq 0$, which is actually a very common and reasonable assumption (Bolton and Dewatripont 2005). With this assumption, the decreasing property of $w(\theta)$ in θ follows immediately. With substitutions of $w^F(\theta)$ into $L(\theta)$ and $v(\theta)$ in (4.4.11) and (4.4.6), the optimal contract menu $\{w^F(\theta), L^F(\theta)\}$ and the optimal service level $v^F(\theta)$ provided by the retailer are then obtained. □

Corollary 4.4.3. *$w^F(\theta)$ is decreasing in θ, $L^F(\theta)$ is increasing in θ.*

Proof. From the proof of Theorem 4.4.2, it follows that $w^F(\theta)$ is decreasing in θ. In addition, according to the expression of $L^F(\theta)$ in Theorem 4.4.2, it is obtained that

$$\frac{dL^F(\theta)}{d\theta} = -\frac{(p - w(\theta) + \rho\theta)}{2\rho}\frac{dw(\theta)}{d\theta} \geq 0. \tag{4.4.25}$$

Hence, $L^F(\theta)$ is increasing in θ. Thus, the proof is completed. □

With Corollary 4.4.3 it is derived that for the optimal contract menu, the wholesale price and the franchise fee are complementary to each other; that is, an increase in one means a decrease in the other. Thus, the retailer tends to choose a contract that includes a relatively low wholesale price when the private demand information is relatively high. With such a choice of the contract she can realize a higher demand for the PSS product and capture a higher margin and hence achieve a higher profit. Actually, even though the franchise fee to be paid with the high demand information increases, in balance the retailer is left with more surplus.

Corollary 4.4.4. *Let $\pi_r^F(\theta)$ and $\pi_m^F(\theta)$ denote the retailer's and the manufacturer's profits, respectively, when the optimal contract menu $\{w^F(\theta), L^F(\theta)\}$ is adopted. Then, both $\pi_r^F(\theta)$ and $\pi_m^F(\theta)$ are increasing in θ.*

Proof. From the proof of Theorem 4.4.2, it follows that $\pi_r^F(\theta)$ is increasing in θ. From the expression of $\pi_m^F(\theta)$ in (4.4.7), it is easy to obtain

$$\frac{d\pi_m^F(\theta)}{d\theta} = \frac{w}{2\rho}(\rho - \frac{dw(\theta)}{d\theta}) > 0. \tag{4.4.26}$$

Therefore, $\pi_m^F(\theta)$ is also increasing in θ. Thus, the proof is completed. □

The retailer's profit $\pi_r^F(\theta)$ represents the information rent, which is the payment that the manufacturer has to pay for learning the retailer's private demand information. With Corollary 4.4.4 it is shown that the information rent is increasing in the retailer's private demand information. Since both $\pi_r^F(\theta)$ and $\pi_m^F(\theta)$ are increasing in θ, the manufacturer has an incentive to provide more information rent for a higher θ.

4.4.2 FFS Contract

This section examines the FFS contract under information asymmetry. With the FFS contract, the manufacturer will offer a menu of the contract $\{w(\theta), v(\theta), L(\theta)\}$ that includes a wholesale price $w(\theta)$, a franchise fee $L(\theta)$ to charge the retailer, as well as a required service level $v(\theta)$ for the demand information θ. For the true demand information θ, the profit received by the retailer who announces the demand information to be $\hat{\theta}$ is

$$\pi_r(w(\hat{\theta}), L(\hat{\theta}), v(\hat{\theta}), \theta) = (p - w(\hat{\theta}) - \rho v(\hat{\theta}))(\theta + v(\hat{\theta})) - L(\hat{\theta}). \tag{4.4.27}$$

Accordingly, the manufacturer's profit is

$$\pi_m(w(\hat{\theta}), L(\hat{\theta}), v(\hat{\theta}), \theta) = w(\hat{\theta})(\theta + v(\hat{\theta})) + L(\hat{\theta}). \tag{4.4.28}$$

Hence, the manufacturer's problem can be rewritten as

$$\max_{w,v,L} \int_0^T [w(\theta)(\theta + v(\theta)) + L(\theta)]f(\theta)d\theta \tag{4.4.29}$$

s.t.

$$\begin{aligned} IC : &(p - w(\theta) - \rho v(\theta))(\theta + v(\theta)) - L(\theta) \\ &\geq (p - w(\hat{\theta}) - \rho v(\hat{\theta}))(\theta + v(\hat{\theta})) - L(\hat{\theta}) \text{ for all } \hat{\theta} \in [0, T], \end{aligned} \tag{4.4.30}$$

$$PC : (p - w(\theta) - \rho v(\theta))(\theta + v(\theta)) - L(\theta) \geq \pi_{min}^r \text{ for all } \theta \in [0, T]. \tag{4.4.31}$$

Lemma 4.4.5. *The contract menu $\{w(\theta), v(\theta), L(\theta)\}_{\theta \in [0,T]}$, where $w(\theta)$ and $v(\theta)$ are decreasing in θ, satisfies PC and IC if $\pi_r(\theta)$ and $L(\theta)$ are given by*

$$(i)\ \ \pi_r(\theta) = \int_0^\theta (p - w(x) - \rho v(x))dx + \pi_{min}^r, \tag{4.4.32}$$

$$(ii) \;\; L(\theta) = (p - w(\theta) - \rho v(\theta))(\theta + v(\theta)) - \int_0^\theta (p - w(x) - \rho v(x))dx - \pi_{min}^r. \tag{4.4.33}$$

Given that the proof is similar to Lemma 4.4.1, it is omitted herein.

Note that the manufacturer's profit is the total channel profit minus the retailer's profit, the objective function in (4.4.29) can be rewritten as

$$\int_0^T [(p - \rho v(\theta))(\theta + v(\theta)) - \eta(\theta)(p - w(\theta) - \rho v(\theta))]f(\theta)d\theta. \tag{4.4.34}$$

The optimal service level is determined by solving the first-order optimization condition for (4.4.34) with respect to v, which is equivalent to

$$p + \rho\eta(\theta) - \rho\theta - 2\rho v(\theta) = 0. \tag{4.4.35}$$

Since the second-order optimization condition of (4.4.34) is $-2\rho < 0$, the optimal service level is given by $v^s(\theta) = \frac{p+\rho\eta(\theta)-\rho\theta}{2\rho}$. Since the retailer's information rent is nonnegative for any θ, $\int_0^\theta (p - w(x) - \rho v(x))dx \geq 0$, which shows that the wholesale price should be bounded from above by $\frac{p-\rho\eta(0)}{2}$. With the objective function (4.4.34), it is easy to see that the manufacturer's profit is an increasing function of w. Therefore, the optimal wholesale price will be the upper bound and given by $w^s(\theta) = \frac{p-\rho\eta(0)}{2}$. By substituting $w^s(\theta)$ and $v^s(\theta)$ into $L(\theta)$ in (4.4.33), the optimal contract menu $\{w^s(\theta), v^s(\theta), L^s(\theta)\}$ is obtained and Theorem 4.4.6 summarizes the results.

Theorem 4.4.6. *The optimal menu of contracts $\{w^s(\theta), v^s(\theta), L^s(\theta)\}$ that solves the manufacturer's problem in (4.4.29) are given by*

$$(i) \;\; w^s(\theta) = \frac{p - \rho\eta(0)}{2}, \tag{4.4.36}$$

$$(ii) \;\; v^s(\theta) = \frac{p + \rho\eta(\theta) - \rho\theta}{2\rho}, \tag{4.4.37}$$

$$(iii) \;\; L^s(\theta) = \frac{(\theta + \eta(0) - \eta(\theta))(p + \rho\eta(\theta) + \rho\theta)}{4} - \int_0^\theta \frac{\rho x + \rho\eta(0) - \rho\eta(x)}{2}dx - \pi_{min}^r. \tag{4.4.38}$$

Let $\pi_r^s(\theta)$ and $\pi_m^s(\theta)$ denote the retailer's and the manufacturer's profits under the optimal contract $\{w^s(\theta), v^s(\theta), L^s(\theta)\}$, respectively. It is easy to obtain Corollary 4.4.7 as follows.

Corollary 4.4.7. *$\pi_r^s(\theta)$ and $\pi_m^s(\theta)$ are both increasing in θ.*

4.4.3 FFCS Contract

With the FFCS contract, the retailer has to adopt the centralized service level decision $v^c = \frac{p-\rho\theta}{2\rho}$. Given the contract menu $\{w^{cs}(\theta), v^c(\theta), L^{cs}(\theta)\}$, the retailer chooses a particular contract $\{w^{cs}(\hat{\theta}), v^c(\hat{\theta}), L^{cs}(\hat{\theta})\}$ that can maximize her profit. By doing so, she announces the demand information to be $\hat{\theta}$ and determines the service level to be $v^c = \frac{p-\rho\hat{\theta}}{2\rho}$. Substituting the specific expression of v into the Eq. (4.3.6), the retailer's profit function with the wholesale price-only contract is rewritten as

$$\pi_r = (\frac{p+\rho\hat{\theta}}{2} - w)(\theta + \frac{p-\rho\hat{\theta}}{2\rho}). \tag{4.4.39}$$

Take the first partial derivative of π_r with respect to $\hat{\theta}$, it can be obtained that

$$\frac{\partial \pi_r}{\partial \hat{\theta}} = \frac{w\theta}{2} + \frac{\rho}{2}(\hat{\theta} - \theta). \tag{4.4.40}$$

Therefore, with asymmetric demand information, the retailer has an incentive to inflate her report of $\hat{\theta}$. This incentive arises because the retailer's profit is increasing in her report of $\hat{\theta}$. As a result, the manufacturer will consider the demand information informed by the retailer incredible in this case. In order to screen the true demand information, the manufacturer provides the same contract format as in the FFS contract, which includes a wholesale price $w(\theta)$, a service level requirement $v(\theta)$, and a franchise fee $L(\theta)$ charged from the retailer. According to the revelation principle, it is sufficient to focus on the contracts under which the retailer will reveal her private information by the option she has selected. Thus, the manufacturer observes the retailer's private information $\hat{\theta}$ according to the contract $\{w^{cs}(\hat{\theta}), v^c(\hat{\theta}), L^{cs}(\hat{\theta})\}$ selected by the retailer. Accordingly, the retailer's and the manufacturer's profits are given respectively as follows.

$$\pi_r(w(\hat{\theta}), L(\hat{\theta}), \hat{\theta}, \theta) = (\frac{p+\rho\hat{\theta}}{2} - w(\hat{\theta}))(\theta + \frac{p-\rho\hat{\theta}}{2\rho}) - L(\hat{\theta}), \tag{4.4.41}$$

$$\pi_m(w(\hat{\theta}), L(\hat{\theta}), \hat{\theta}, \theta) = w(\hat{\theta})(\theta + \frac{p-\rho\hat{\theta}}{2\rho}) + L(\hat{\theta}). \tag{4.4.42}$$

The manufacturer's problem can be rewritten as

$$\max_{w,L} \int_0^T [\frac{(p-\rho\theta)w(\theta)}{2\rho} + L(\theta)]f(\theta)d\theta \tag{4.4.43}$$

s.t.

$$IC: (\frac{p+\rho\theta}{2} - w(\theta))(\frac{p+\rho\theta}{2\rho}) - L(\theta)$$
$$\geq (\frac{p+\rho\hat{\theta}}{2} - w(\hat{\theta}))(\theta + \frac{p-\rho\hat{\theta}}{2\rho}) - L(\hat{\theta}) \text{ for all } \hat{\theta} \in [0, T], \tag{4.4.44}$$

$$PC: \ (\frac{p+\rho\theta}{2} - w(\theta))(\frac{p+\rho\theta}{2\rho}) - L(\theta) \geq \pi^r_{min} \text{ for all } \theta \in [0, T]. \tag{4.4.45}$$

Lemma 4.4.8. *The contract menu $\{w(\theta), v(\theta), L(\theta)\}_{\theta\in[0,T]}$, where $w(\theta)$ is decreasing in θ, satisfies PC and IC if $\pi_r(\theta)$ and $L(\theta)$ are given by*

$$(i)\ \ \pi_r(\theta) = \int_0^\theta \frac{p - 2w(x) + \rho x}{2} dx + \pi^r_{min}, \tag{4.4.46}$$

$$(ii)\ \ L(\theta) = \frac{(p+\rho\theta - 2w(\theta))(p+\rho\theta)}{4\rho} - \int_0^\theta \frac{p-2w(x)+\rho x}{2} dx - \pi^r_{min}. \tag{4.4.47}$$

Given that the proof is similar to Lemma 4.4.1, it is omitted herein.

Since the retailer's information rent is non-negative for any θ, it implies that $\int_0^\theta \frac{p-2w(x)+\rho x}{2} dx \geq 0$, which shows that the wholesale price should be $w \leq \frac{p}{2}$. Note that the manufacturer's profit is the total channel profit minus the retailer's profit, the objective function in (4.4.43) can be rewritten as

$$\int_0^T [\frac{(p+\rho\theta)^2}{4\rho} - \eta(\theta)\frac{p-2w(\theta)+\rho\theta}{2}] f(\theta) d\theta. \tag{4.4.48}$$

It is seen from (4.4.48) that the manufacturer's profit is an increasing function of w, and therefore the optimal wholesale price is the upper bound and given by $w^{cs}(\theta) = \frac{p}{2}$. Substituting $w^{cs}(\theta)$ into (4.4.47) yields the optimal contract menu $\{w^{cs}(\theta), v^c(\theta), L^{cs}(\theta)\}$ and the results are summarized in Theorem 4.4.9.

Theorem 4.4.9. *Let $\{w^{cs}(\theta), v^c(\theta), L^{cs}(\theta)\}$ denote the optimal menu of contracts that solves the manufacturer's problem in (4.4.43). Then,*

$$(i)\ \ w^{cs}(\theta) = \frac{p}{2}, \tag{4.4.49}$$

$$(ii)\ \ L^{cs}(\theta) = \frac{\theta(p+\rho\theta)}{4} - \int_0^\theta \frac{\rho x}{2} dx - \pi^r_{min}. \tag{4.4.50}$$

Let $\pi^{cs}_r(\theta)$ and $\pi^{cs}_m(\theta)$ denote the retailer's and the manufacturer's profits under the optimal contract, respectively. It is easy to obtain Corollary 4.4.10 as follows.

Corollary 4.4.10. *$\pi^{cs}_r(\theta)$ and $\pi^{cs}_m(\theta)$ are both increasing in θ.*

4.4.4 Comparison of the FF, FFS, and FFCS Contracts

Table 4.1 summarizes the equilibrium solutions associated with the FF, FFS, and FFCS Contracts under asymmetric demand information. Despite the similarities in the above discussions, there exist some fundamental differences for the FF, FFS, and FFCS contracts. Corollary 4.4.11 summarizes the comparisons with respect to the service level, the retailer's profit, and the manufacturer's profit under these three forms of contracts.

Corollary 4.4.11. *(i) As compared with the other two forms of contracts, under the FFCS contract, the manufacturer achieves the highest profit, the retailer achieves the lowest profit, and the channel achieves the highest profit.*

(ii) If $p \geq \rho\eta(0)$, $\pi_r^F \geq \pi_r^s$, $\pi_m^F \leq \pi_m^s$; *otherwise,* $\pi_r^F < \pi_r^s$, $\pi_m^F > \pi_m^s$.

(iii) The service level under the FFS contract is the highest, followed by that under the FFCS contract. The service level under the FF contract is the lowest with $v^s(\theta) \geq v^{cs}(\theta) \geq v^F(\theta)$.

The main insight derived from the analysis is that the contract performance depends on whether the manufacturer chooses a flexible or rigid contract. With the FF contract, the manufacturer specifies the wholesale price and the franchise fee, but leaves the service level term to be flexible. In other words, the retailer can choose her individual optimal service level. However, with the FFS and FFCS contracts, the manufacturer specifies all contractual terms. The difference between the FFS and FFCS contracts is that the retailer has to take a system-wide optimal service level decision under the FFCS contract.

The FFCS contract, with the centralized service level requirement, ensures the highest profits for both the channel and the manufacturer, whereas leaves the retailer with the least information rent. Therefore, the manufacturer always prefers the FFCS contract regardless of the demand information. For the FFS contract, it provides a more effective mechanism for motivating the service level, hence realizing the highest sales as compared with those associated with the FF and FFCS contracts. In addition, the channel profit under the FFS contract is lower than that under the FFCS contract. The reason behind this result is that the service level under the FFCS contract is just the system-wide optimal solution v^c, while that under the FFS contract is higher than the first-best solution of the system, which leads to some losses in the system efficiency. When the probability of the low demand type (a small θ) is high enough to make the inequality $p \geq \rho\eta(0) = \frac{\rho}{f(0)}$ hold, the manufacturer can achieve a higher profit by using the FFS contract. However, if the probability of the high demand type (a large θ) is relatively high, the manufacturer may achieve a higher profit by using the more flexible FF contract. Thus, a tighter control over the retailer does not always guarantee a higher profit for the manufacturer. If the private demand information θ is likely to be very small, the more rigid FFS contract can assure a higher profit for the manufacturer while a lower profit for the retailer, and vice versa. From the retailer's perspective, a rigid

Table 4.1 Equilibrium solutions under the FF, FFS, and FFCS contracts

	FF contract $\{w^F(\theta), v^F(\theta), L^F(\theta)\}$	FFS contract $\{w^s(\theta), v^s(\theta), L^s(\theta)\}$	FFCS contract $\{w^{cs}(\theta), v^c(\theta), L^{cs}(\theta)\}$
$v(\theta)$	$\frac{p-\rho\theta-w^F(\theta)}{2\rho}$	$\frac{p-\rho\theta+\rho\eta(\theta)}{2\rho}$	$\frac{p-\rho\theta}{2\rho}$
$w(\theta)$	$\begin{cases} w^F(\theta) = p & \text{for } \theta \leq \theta_1, \\ w^F(\theta) = \rho\eta(\theta) & \text{for } \theta \geq \theta_1. \end{cases}$	$\frac{p-\rho\eta(0)}{2}$	$\frac{p}{2}$
$L(\theta)$	$\frac{(p-w^F(\theta)+\rho\theta)^2}{4\rho} - \int_0^\theta \frac{p-w^F(x)+\rho x}{2} dx - \pi^r_{min}$	$\frac{(p+\rho\eta(\theta)+\rho\theta)(\eta(0)-\eta(\theta)+\theta)}{4} - \int_0^\theta \frac{\rho\eta(0)-\rho\eta(x)+\rho x}{2} dx - \pi^r_{min}$	$\frac{\theta(p+\rho\theta)}{4} - \int_0^\theta \frac{\rho x}{2} dx - \pi^r_{min}$
π_r	$\int_0^\theta \frac{p-w^F(x)+\rho x}{2} dx + \pi^r_{min}$	$\int_0^\theta \frac{\rho\eta(0)-\rho\eta(x)+\rho x}{2} dx + \pi^r_{min}$	$\int_0^\theta \frac{\rho x}{2} dx + \pi^r_{min}$
π_m	$\frac{(p+\rho\theta)^2-(w^F(\theta))^2}{4\rho} - \int_0^\theta \frac{p-w^F(x)+\rho x}{2} dx - \pi^r_{min}$	$\frac{(p+\rho\theta)^2-(\rho\eta(\theta))^2}{4\rho} - \int_0^\theta \frac{\rho\eta(0)-\rho\eta(x)+\rho x}{2} dx - \pi^r_{min}$	$\frac{(p+\rho\theta)^2}{4\rho} - \int_0^\theta \frac{\rho x}{2} dx - \pi^r_{min}$
π_T	$\frac{(p+\rho\theta)^2-(w^F(\theta))^2}{4\rho}$	$\frac{(p+\rho\theta)^2-(\rho\eta(\theta))^2}{4\rho}$	$\frac{(p+\rho\theta)^2}{4\rho}$

control is preferred when the probability of a low demand is relatively high, while a flexible control is more desirable when the probability of a high demand is relatively high.

Since the retailer can choose her individual optimal service level with the FF contract, a decrease in the wholesale price will enhance the retailer's marginal profit and thus increase her information rent. As a result, the manufacturer finds it advantageous to increase the wholesale price with a discount of the franchise fee, and vice versa. Hence, the retailer prefers choosing a low wholesale price with a high franchise fee if the private demand information is relatively high. When the retailer can accept more rigid contract terms, the manufacturer will implement the FFS and FFCS contracts. In these cases, an increase in the wholesale price will reduce the retailer's marginal profit, which drives down her information rent. Since the wholesale price cannot exceed $\frac{p-\rho\eta(0)}{2}$ and $\frac{p}{2}$ under the FFS and FFCS contracts, it is optimal for the manufacturer to set the highest wholesale price as $\frac{p-\rho\eta(0)}{2}$ and $\frac{p}{2}$, respectively, under the two contracts. Even though $w(\theta)$ is fixed, the manufacturer can use $\{L(\theta), v(\theta)\}$ to extract more surplus. The impacts of the both contracts on the franchise fee are opposite to those on the service level. As a result, the service level that the retailer needs to commit has to increase when the franchise fee charged from her is reduced.

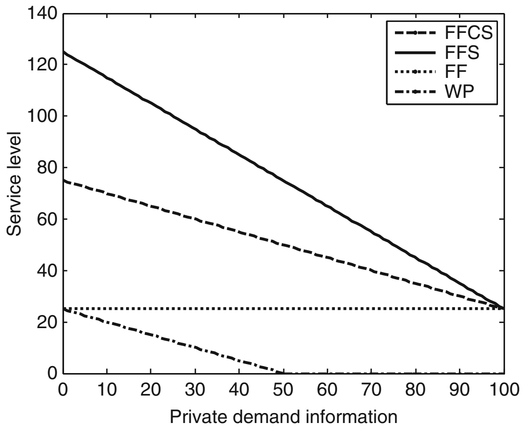
4.5 Numerical Examples

This section examines the contract performances by numerical experiments. The numerical examples serve for two purposes, one is to verify the analytical findings and the other is to gain more insights into the optimal policies and thereby identify more managerial guidelines. In the numerical experiments, all the cost and revenue parameters are assumed to be exogenous and it is set that $p = 30, \rho = 0.2$. In addition, the retailer has the private demand information θ which is unknown to the manufacturer. Even though so, the manufacturer has a prior probability distribution towards the retailer's private demand information, which is assumed to follow a uniform distribution from 0 to 100.

4.5.1 Service Level Comparison

In Fig. 4.1, the optimal service levels under the wholesale price contract, the FF contract, the FFS contract, and the FFCS contract are depicted, respectively. Note first that the optimal service level under any contract is non-increasing in θ, which implies that the retailer with high private demand information has no incentive to increase the investment on the service level. Second, the service levels under the FF, FFS, and FFCS contracts, as functions of the retailer's demand information, are all

Fig. 4.1 Comparisons of service level



higher than that under the wholesale price contract. Further, it is seen from Fig. 4.1 that $v^s \geq v^c \geq v^F \geq v^a$. Since there is no distortion on the service level for the retailer with demand information being $\theta = T$, the service level for this case is just equal to the first-best solution of the system, i.e., $v^s(T) = v^F(T) = v^c(T) = 25$.

In addition, it is further seen that if the manufacturer wishes the retailer to choose a service level that maximizes the channel profit, the manufacturer has to offer the retailer with an information rent. If the manufacturer does not coordinate the retailer's decision by contracting, the retailer will provide a lowest service level because of double marginalization effect and asymmetric information structure.

4.5.2 Profit Comparison

The comparison of profit is clear from Fig. 4.2. To be specific, Fig. 4.2a depicts the channel profits under the respective contracts with asymmetric demand information. It is observed that the wholesale price contract realizes the lowest channel profit as compared with the other contracts. Since the manufacturer can impose the system-wide optimal service level decision on the retailer with the FFCS contract, the supply chain achieves the highest efficiency under this contract. If $p \leq \rho\eta(\theta), w(\theta) = \rho\eta(\theta)$, which leads to that $\pi_T^F = \pi_T^s$. On the other hand, if $p > \rho\eta(\theta), w(\theta) = p$, which leads to that $\pi_T^F \leq \pi_T^s$. Figure 4.2b plots the manufacturer's profits as the functions of the private demand information under the respective contracts. Note that the manufacturer will achieve the lowest profit with the wholesale price contract. In addition, it can be derived that $\pi_M^F > \pi_M^s$ and $\pi_r^F < \pi_r^s$ for $p > \rho\eta(\theta)$. Furthermore, the manufacturer will benefit the most by using the FFCS contract as compared with the FF and FFS contracts. The retailer's profit, as a function of her private demand information, will reduce as the manufacturer exerts more controls on the contractual terms, as the observations from Fig. 4.2c that $\pi_r^F \geq \pi_r^s \geq \pi_r^{cs}$. For the

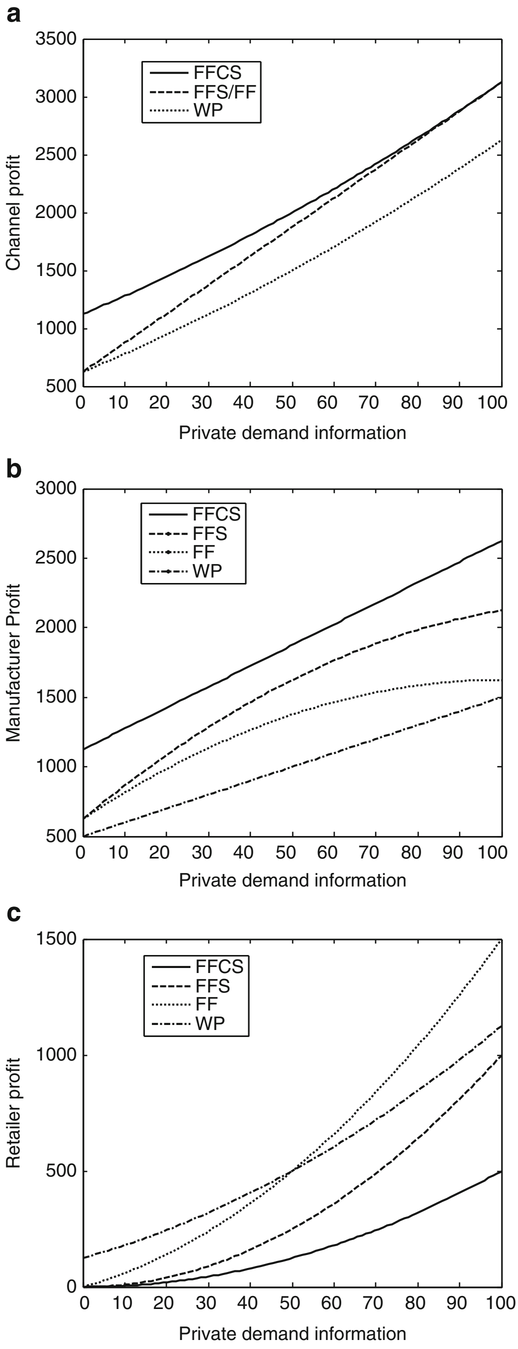


Fig. 4.2 Profit comparisons under different contract menus. (**a**) Channel profit under different demand information. (**b**) Manufacturer's profit under different demand information. (**c**) Retailer's profit under different demand information

retailer with higher demand information, the FF contract will ensure her of a higher information rent and a higher profit than the wholesale price contract. However, for the retailer with lower demand information, the FF contract will provide her with a lower information rent and a higher wholesale price, which makes the retailer's profit lower than that under the wholesale price contract. For all the FF, FFS, and FFCS contracts, Fig. 4.2 shows that the retailer's and the manufacturer's profits both increase with the demand information. In addition, the information rent is increasing in the retailer's private demand information, and it is zero when the private demand information is zero ($\theta = 0$); otherwise, it is always positive.

4.6 Conclusion

In this chapter, a service-oriented manufacturing supply chain that provides customers with not only product but also service is considered. To be specific, this chapter considers a two-echelon supply chain in which a manufacturer provides a basic product and a retailer provides value-added services based on the basic product. In industries, Such a new business mode has become quite prevalent and it is referred as the PSS. To optimize the PSS, three forms of contracts are developed in this chapter, namely, the FF contract, the FFS contract, and the FFCS contract. The research focuses on addressing the optimal contract structures that can ensure a credible information sharing and push the retailer to set an appropriate service level. With the research it is shown that information asymmetry will cause channel inefficiency with the FF and FFS contracts, and this is not the case with the FFCS contract for which the service level and channel profit are just equal to those under the completely symmetric information structure.

This chapter also examines the differences for the FF, FFS, and FFCS contracts and thereby provides managerial guidelines for contract selection. The research shows that FFS contract is the most effective for motivating the retailer's service effort whereas FFCS contract is the most capable of generating the maximum channel profit. Under the FFS and FFCS contracts, the manufacturer designates a service level decision for the retailer. With asymmetric demand information, however, the manufacturer cannot avoid the retailer deviating from the first-best solutions of the supply chain system under the FFS contract. The FFCS contract is designed in a way to make the service level and channel profit just the same to the first-best solutions of the supply chain system. Besides, the retailer's profit is affected by her private demand information, and the higher the demand is, the higher profit she will obtain. With a comparison of the retailer's profits under the respective contracts, it is found that the retailer's profit decreases with the degree of control on the contractual terms. Actually, it is noted that the retailer achieves the lowest profit under the FFCS contract as compared with those under the FF and FFS contracts. In addition, as compared to the FF contract, the FFS contract does not necessarily ensure a higher profit for the manufacturer, and this is so only when the private demand information is relatively low, and vice versa.

The research of this chapter can be extended along with various directions. One is to consider a more complex demand function than the linear type. The other is to consider the case where the private demand information follows a more general probability distribution than the uniform distribution. In addition, it will also be interesting to consider the situation in which the retailer has market power to negotiate the wholesale price with the manufacturer, rather than only a price-taker. Finally, a more complex model in which the manufacturer and the retailer both participate in the value-added services is another interesting issue for future research.

Chapter 5
Coordination of Supply Chains with Bidirectional Option Contracts

5.1 Introduction

The present business environment is full of uncertainties. To compete effectively in such an environment, firms need to develop the capability of responding flexibly to changing market conditions. This is particularly true for firms dealing with perishable products with comparatively long production lead-times and short selling seasons, and subject to high demand uncertainty. In order to hedge against the loss associated with over- and under-ordering, retailers usually have to order less but more frequently from their upstream firms such as manufacturers so that they can well accommodate demand volatility. Such an order policy, however, exerts great pressure on the manufacturers because it requires them to have flexible capacity to cater for the irregular orders, which results in an increase in the manufacturers' costs. As a result, conflicts arise between the channel partners, which inevitably impair channel efficiency.

To address the issue of channel inefficiency, a typical approach is to design incentive contracts that provide the retailer with flexibility to respond to unanticipated demand without burdening the manufacturer. Among all such contract types, the option contract has attracted extensive attention and has been demonstrated to be effective in resolving the channel conflicts described above (Barnes-Schuster et al. 2002, Zhao et al. 2010, Wang and Liu 2007). Usually the option contract can be either the call option contract or the put option contract, which is characterized by two parameters, namely the option price and the exercise price. For the call option contract, the option price is an allowance paid by the retailer to the manufacturer for reserving one unit of the production capacity. The exercise price is the payment by the retailer to the manufacturer for exercising one unit of the call option.

The research of this chapter is based on Zhao et al. (2013b).

Y. Zhao et al., *Contract Analysis and Design for Supply Chains with Stochastic Demand*, International Series in Operations Research & Management Science 234,
DOI 10.1007/978-1-4899-7633-8_5

Hence, with the call option contract, the option buyer has the right to buy a certain quantity of the product at a specified price (the exercise price) on or before an expiry date. For the put option contract, the option price is an allowance paid by the retailer to the manufacturer for cancelling or returning one ordered unit of the product. The exercise price is the refund from the manufacturer to the retailer for exercising one unit of the put option. Hence, with the put option contract, the option buyer has the right to cancel or return some ordered units of the product at a specified refund per unit (the exercise price) on or before an expiry date. Obviously, with the option contract, on the one hand, the retailer acquires order flexibility by paying the option price. On the other hand, the manufacturer receives an early commitment from the retailer so that it can carry out better capacity and material planning, while the allowance received from selling the option justifies its assuming some demand risks from the retailer. Therefore, balancing these two countervailing forces, the option mechanism is beneficial to the channel and the channel partners concerned. This is especially true when the manufacturer is powerful enough and capable of absorbing the risks associated with demand and price uncertainties (Ritchken and Tapiero 1986).

However, research on the option mechanism in the past often focused on the single directional option, namely either the call option or the put option, with which the retailer is allowed to adjust the initial order only upwards or only downwards. Wang and Tsao (2006) showed that the call option contract induces the retailer to reduce the initial order commitment, leading to an increase in the channel shortage cost, while the put option contract prompts the retailer to increase the initial order commitment, leading to an increase in the channel over-stocking cost. Moreover, the biased orders under the single directional option may exacerbate the bullwhip effect in the supply chain. Hence, it is not in the best interest of the channel to allow the retailer to adjust the order only in a single direction. In addition, the market may be so volatile that at the juncture to purchase the option, the retailer is not sure of the direction in which it will need to adjust the order quantity. In this case, the retailer may need to purchase a bidirectional option so that it is able to adjust the order in either direction. Moinzadeh and Nahmias (2000, p. 421) pointed out that a natural extension of the single directional option is the bidirectional option. So far, however, some issues of the bidirectional option have remained unaddressed. For example, how is the retailer's initial order strategy affected with the bidirectional option provision? In addition, how should the bidirectional option contract be set to achieve supply chain coordination? These issues are far more complicated than those of the single directional option because the bidirectional option provides the retailer with the chance of adjusting the order in either direction. In this chapter, analytical explorations of these issues will be presented for the bidirectional option contract. As an initial attempt on these issues, the research will start with the simplest supply chain structure comprising of one manufacturer and one retailer. We believe this should be a reasonable balance between modeling the tractability and the industrial reality and complexity.

The rest of this chapter is organized as follows: Section 5.2 reviews the literature. Section 5.3 constructs the bidirectional options model. Sections 5.4 and 5.5 analyze the bidirectional options model and discuss the supply chain coordination issue with bidirectional options. Section 5.6 concludes the chapter with future research directions. All the proofs of the main results are put in the Appendix.

5.2 Literature Review

This chapter is devoted to the research of bidirectional option contracts. In order to highlight the contribution included in this chapter, we will only review the literature that is particularly related to this work. For more details on supply chain contracts, readers are referred to Chap. 1 and the excellent review papers by Lariviere (1999) and Cachon (2003).

The first research stream related to this paper is the literature on the call option that allows the buyer to adjust the order upwards. Generally, this stream of the literature can be classified into two research lines. The first is to consider the pure option contract, and the other is to consider the mixed market option that operates in the presence of an option contract market and a spot market. For the literature on the pure option contract, most works mainly emphasize the managerial flexibility and economic efficiency derived from the option contract (see, e.g., Eppen and Iyer 1997, Barnes-Schuster et al. 2002, Edlin and Hermalin 2000, Cachon and Lariviere 2001, Brown and Lee 2003, Burnetas and Ritchken 2005, Wang and Liu 2007, Zhao et al. 2010, Buzacott et al. 2011, Hazra and Mahadevan 2009, Jin and Wu 2007, Erkoc and Wu 2005). Eppen and Iyer (1997) considered a backup agreement that is essentially a form of the option contract with a two-stage model. They demonstrated that the backup agreement can have a substantial impact on the expected profits of the supply chain members and the committed order quantity. Barnes-Schuster et al. (2002) also developed a two-stage model to explore the roles of the option contract, including how the option contract provides flexibility in response to market variations and how the option contract achieves supply chain coordination. Edlin and Hermalin (2000) studied the principal-agent problem with options. They explored when the option contract can be used to remedy the holdup effect and when it cannot in the presence of re-negotiation. Hazra and Mahadevan (2009) considered a capacity reservation model in which a buyer reserves capacity in advance from one or multiple suppliers with demand uncertainty. They explored the issues of how much capacity the buyer should reserve and how many suppliers it should select. For the mixed market option literature, the research emphasis has been placed on some issues concerning the interplay between the option contract market and the spot market, such as how the option contract and the spot market affect each other and what the optimal contract decisions (pricing) are when the spot market is available (see, e.g., Wu et al. 2002, Hazra et al. 2002, Spinler et al. 2003, Wu and Kleindorfer 2005, Serel et al. 2001, Norden and Velde 2005, Serel 2007, Inderfurth and Kelle 2011, Wu et al. 2005, Spinler and Huchzermeier 2006). Wu et al. (2002) characterized the seller's optimal option contract bidding and the buyers' optimal contracting strategies in the presence of a stochastic spot market.

Serel et al. (2001) considered a capacity reservation contract where the buyer has the right to receive any desired proportion of the capacity reserved. They explored the buyer's optimal sourcing decisions when both the capacity reservation contract and spot market are available. All the above research including the first and second lines only considers the call option that allows the option buyer to adjust the order upwards only. Our work differs from this research stream in that we consider both the put and call option contracts simultaneously.

The second related research stream is the literature on the put option that allows the option buyer to adjust the order downwards only. Chen and Parlar (2007) considered a newsvendor model in which the newsvendor can purchase the put option to hedge against losses associated with low demand. They examined the value of the put option when the newsvendor is risk averse. A typical category within this research stream is the buyback contract literature. With the buyback contract, the manufacturer promises to reclaim all or part of the leftover inventory from the retailer at a pre-specified price (Pasternack 1985, Padmanabhan and Png 1997). The buyback contract allows the retailer to adjust the order downwards and hence is a type of the put option. Generally, we can classify the buyback contract literature into two lines. The first examines the buyback contract in the classical price-independent newsvendor setting. This line of research generally considers how to set the buyback contract to attain supply chain coordination (see, e.g., Pasternack 1985, Donohue 2000, Yang and Qi 2009). A particularly interesting study is that of Yang and Qi (2009), who developed a general three-step method to find a coordinating contract for a supply chain comprising one supplier and one retailer. They showed that several well-known contract types such as the buyback contract and the revenue-sharing contract can be viewed as applications of their method. The other line of research examines the buyback contract in the stochastic price-dependent demand setting. Since no buyback contract alone can attain supply chain coordination in this demand setting (see Cachon 2003, Bernstein and Federgruen 2005), this line of the research generally does not address the supply chain coordination issue but analyzes the buyback contract in the Stackelberg game framework (see, e.g., Padmanabhan and Png 1997, Emmons and Gilbert 1998, Wang 2004, Padmanabhan and Png 2004, Granot and Yin 2005, Song et al. 2008). Our work differs from this second related research stream in that we assume that the retailer can place an additional order if necessary, as well as being given a chance to return some ordered units of the product.

The research of this chapter is most related to the third research stream that considers bidirectional adjustments over the initial order (see, e.g., Milner and Rosenblatt 2002, Wang and Tsao 2006, Gomez_Padilla and Mishina 2009). Milner and Rosenblatt (2002) considered a two-period supply contract with bidirectional adjustments. They assumed that the buyer determines the orders for two periods at the beginning of the planning horizon. After observing the demand in the first period, the buyer is allowed to adjust the order placed for the second period in either direction with an adjustment penalty cost per unit. They mainly analyzed the buyer's optimal order strategies under such a contract mechanism. Wang and Tsao (2006) developed a single-period two-stage supply contract with the bidirectional option.

Similar to Milner and Rosenblatt (2002), they also considered the bidirectional option contract only from the option buyer's perspective and developed closed-form solutions for the buyer's optimal order strategies with uniformly distributed stochastic demand. Gomez_Padilla and Mishina (2009) analyzed the performance of the bidirectional option contract in the multi-period setting. They used simulation to demonstrate that the bidirectional option contract can benefit the retailer, the supplier, and the supply chain. The research of this chapter differs from these studies mainly in the following three aspects: First, the research derives closed-form expressions for the retailer's optimal order strategies, including the initial order strategy and the option purchasing strategy, with a general demand distribution. Second, the research analytically examines feedback effects of the bidirectional option on the retailer's initial order strategy. A particularly interesting finding is that the convexity and concavity of the demand cumulative distribution function, as well as the contracting cost parameters, can be critical determinants for the effects that the bidirectional option may have on the retailer's initial order strategy. Third, taking a chain-wide perspective to look at the bidirectional option contract, the research develops the distribution-free contracting form that can attain supply chain coordination.

5.3 The Model

Consider a two-echelon supply chain consisting of one manufacturer and one retailer. The manufacturer distributes its product via the retailer to the end market. The product is perishable with a comparatively long production lead-time and a short selling season, and subject to high demand uncertainty. The end market demand can be characterized by a stochastic variable X that follows a strictly increasing cumulative distribution function (CDF) $F(x)$, with $x \geq 0$, and a probability density function (PDF) $f(x)$, with $f(x) = F'(x)$. Let the manufacturer's wholesale price and marginal production cost be w and c, respectively, and the retailer's retailing price be p. Any leftovers by the end of the selling season will be salvaged at price v per unit for both the manufacturer and the retailer. Clearly, to ensure the model is reasonable, we require that $p > w > c > v$.

The bidirectional option contract considered in this chapter is characterized by two parameters, one is the option price, denoted as o, and the other is the exercise price of the bidirectional option, denoted as e. The retailer purchases the option at unit price o, which gives it the right (but not the obligation) to adjust its initial order either upwards or downwards, depending on demand realization at the juncture when the option is exercised. If upward adjustment occurs, the retailer will procure an additional quantity of the product that does not exceed the option quantity at price e per unit. If downward adjustment occurs, the retailer will return some quantity of the product that likewise does not exceed the option quantity at refund e per unit. To be specific, the model is described as follows.

The model captures the activities from the beginning of the production season to the end of the selling season. At the beginning of the production season, the manufacturer offers the retailer a bidirectional option contract, denoted as (o, e), as well as a wholesale price contract, denoted as w. The retailer then places an initial order, denoted as Q, at the unit price w, and purchases an option quantity, denoted as q, at the unit price o. With the option quantity purchased, the retailer can adjust the initial order either upwards or downwards in the selling season, depending on demand realization, in the manner pre-specified by the bidirectional option contract. After that, following the retailer's order strategies, the manufacturer produces the product during the production season. To fulfil the option quantity exercised by the retailer, the manufacturer adopts the make-to-order production policy and commits to producing the product up to $Q+q$. During the selling season, the retailer determines how to exercise the option upon demand realization, subject to the option quantity constraint q, and any unsatisfied demand is lost at penalty cost g per unit.

To avoid the trivial and unreasonable cases, it should be required that $q \leq Q$, $o + v < c, w - o < e < w + o, p + g - e > e - v$, and $p + g - c > c - v$. As a matter of fact, the first constraint avoids the unreasonable scenario that the returned product quantity may exceed the one purchased by the retailer. The second constraint avoids the unreasonable scenario that the manufacturer may arbitrage with the option. The first part of the third constraint avoids the unreasonable scenario that the unit purchase price of the product under the wholesale price contract may be higher than that under the bidirectional option contract, while the second part avoids that the retailer may earn some profit by only exercising the bidirectional option as a put option. The fourth constraint avoids the unreasonable scenario that the retailer prefers to return the product to the manufacturer rather than satisfying the demand. The fifth constraint assures that the opportunity revenue for the supply chain is always larger than the opportunity loss; otherwise, there is no incentive to produce the product.

5.4 Retailer's Optimal Order Strategies and Feedback Effect of Bidirectional Option

At the beginning of production season, the retailer needs to determine the initial order quantity Q and the purchasing option quantity q given a combination (w, o, e) of the wholesale price contracts and the bidirectional option contracts. With the combination (w, o, e), the retailer's expected profit when pursuing a pair of strategies (Q, q) can be expressed as

$$\begin{aligned} E\pi_r(Q, q) = E[&p \min\{Q, X\} + (p - e) \max\{\min\{X - Q, q\}, 0\} \\ &+e \max\{\min\{Q - X, q\}, 0\} + v \max\{Q - q - X, 0\} \\ &-wQ - oq - g \max\{X - Q - q, 0\}]. \end{aligned} \tag{5.4.1}$$

The first term is the sale revenue, the second term is the stringent sale profit by exercising call options. The third term is the stringent refund by exercising put options, the fourth term is the salvage revenue. The fifth and the sixth terms are respectively the costs of purchasing the initial order and the options, and the last term is the shortage cost. Hence, the retailer's problem can be formulated as

$$\begin{aligned} \mathrm{P}_{5.1}: \quad & \max E\pi_r(Q,q) \\ & s.\,t.\;\; Q \geq q \geq 0. \end{aligned} \tag{5.4.2}$$

Denote $\overline{Q}_{or}$ and $\overline{q}_{or}$ as the optimal solutions of problem $\mathrm{P}_{5.1}$. By solving the problem $\mathrm{P}_{5.1}$ the following results are derived.

Theorem 5.4.1. *With a combination (w,o,e) of the wholesale price and bidirectional option contracts, the retailer's optimal order strategies are given by*

$$\begin{cases} \overline{Q}_{or} = \frac{1}{2}[F^{-1}(a) + F^{-1}(b)] \\ \overline{q}_{or} = \frac{1}{2}[F^{-1}(a) - F^{-1}(b)] \end{cases} \tag{5.4.3}$$

if $w > \tilde{c}$, otherwise

$$\begin{cases} \overline{Q}_{or} = \frac{1}{2}[F^{-1}(a) + F^{-1}(b)] \\ \overline{q}_{or} = 0 \end{cases} \tag{5.4.4}$$

where $a = \frac{2p+2g-w-o-e}{2(p+g-e)}$, $b = \frac{o+e-w}{2(e-v)}$, and $\tilde{c} = \frac{(o+2v-e)(p+g)-v(o+e)}{p+g+v-2e}$.

It can be seen from Theorem 5.4.1 that the retailer will only place an initial committed order quantity with the wholesale price contracts and not purchase any quantity of the option if the wholesale price is low to $\tilde{c}$, otherwise the retailer will place an initial order accompanied with a quantity of the bidirectional option.

Given that only the wholesale price contract is available (without the bidirectional option provision), the retailer's optimal order quantity is $\overline{Q}_{wr} = F^{-1}(\frac{p+g-w}{p+g-v})$ (see the Appendix for the proof). It is known that the call option will push the retailer to reduce the initial order quantity whereas the put option will push the retailer to increase the initial order quantity (Wang and Tsao 2006). However, what will happen to the retailer's initial order decision when the bidirectional option is available? This issue is far from trivial because, given the bidirectional option, the retailer has the chance to adjust the initial order in either direction. We also see from the following results that the effects of the only-call option or the only-put option on the retailer's initial order do not necessarily carry over to the case of the bidirectional option.

Theorem 5.4.2. *(i) If $F(x)$ is concave and two-order differentiable, then $\overline{Q}_{wr} < \overline{Q}_{or}$ when $w > \tilde{c}$ and $\frac{p+g-e}{p+g-v} < \frac{1}{2}$ or $w < \tilde{c}$ and $\frac{p+g-e}{p+g-v} > \frac{1}{2}$.*

(ii) If $F(x)$ is convex and two-order differentiable, then $\overline{Q}_{wr} > \overline{Q}_{or}$ when $w > \tilde{c}$ and $\frac{p+g-e}{p+g-v} > \frac{1}{2}$ or $w < \tilde{c}$ and $\frac{p+g-e}{p+g-v} < \frac{1}{2}$.

Hence, the retailer's initial order quantity may increase or decrease given the bidirectional option. A closer look at the bidirectional option reveals the intuition behind this difference between the single directional option, i.e., a call option or a put option, and the bidirectional option. Given the only-call option, the retailer has the chance of placing an additional order if demand realization is above the initial expectation, and has no chance of returning the leftovers to the manufacturer if demand turns out to be low. As a result, it is in the retailer's interest to place a relatively smaller initial order so as to hedge against the loss associated with over-ordering. The reverse happens in the case of the only-put option. Given the bidirectional option, however, the retailer has the chances of both placing a supplementary order and returning the leftovers. If the retailer places a bigger weight on the loss associated with over-ordering, then it will adopt a prudent order policy by reducing the initial order quantity. If the retailer places a bigger weight on the loss associated with under-ordering, then it will adopt an aggressive order policy by increasing the initial order quantity. However, as to on which side the retailer should place a bigger weight, it will depend on the demand distribution and the cost parameters associated with the wholesale price contract and the bidirectional option contract. In essence, Theorem 5.4.2 reveals a profound insight that the convexity and concavity of the demand cumulative distribution function, as well as the cost parameters associated with the respective contracts, can be a critical determinant of the side on which the retailer should place a bigger weight when making the initial order decision, which is an interesting result. With respect to the convexity or concavity of the demand distribution, as a matter of fact, there exists a wide range of cumulative demand distribution functions that satisfy these properties. To be specific, examples of convex demand CDF include the exponential, Pareto, and uniform distributions. The CDF of the demand that follows the beta, gamma, or Weibull distribution is also convex with certain parameters. In addition, examples of concave demand CDF include the beta distribution with certain parameters and the uniform distribution. Table 5.1 summarizes the above examples.

Table 5.1 Examples for convex or concave demand CDFs

Properties of CDFs	Distributions	PDFs	Examples
Convex	Beta	$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$	$\alpha = 1$ and $\beta = 3$
Convex	Chi-square	$\frac{1}{2^{k/2}\Gamma(k/2)}x^{k/2-1}e^{-\frac{x}{2}}$	$k = 1$ or $k = 2$
Convex	Exponential	$\lambda e^{-\lambda x}$	for all $\lambda > 0$
Convex	Pareto	$\frac{\alpha x_m^\alpha}{x^{\alpha+1}}$ for $x \geq x_m$	for all $\alpha > 0$
Convex	Weibull	$\frac{k}{\lambda}(\frac{x}{\lambda})^{k-1}e^{-(x/\lambda)^k}$	$\lambda = 1; k = 0.5$
Convex	Uniform	$\frac{1}{b-a}$ for $x \in [a, b]$	for all $a > b$
Concave	Beta	$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$	$\alpha = 5; \beta = 1$
Concave	Uniform	$\frac{1}{b-a}$ for $x \in [a, b]$	for all $a > b$

Furthermore, the following results can be derived.

Theorem 5.4.3. *Given $w > \tilde{c}$,*

(i) $\overline{Q}_{or}$ *is increasing in e for all* $o \leq p+g-w$ *and decreasing in w, and* $\overline{q}_{or}$ *is decreasing in o.*
(ii) $\overline{Q}_{or} + \overline{q}_{or} = F^{-1}(a)$*, which is decreasing in o and w respectively, increasing in e for all* $o \leq p+g-w$ *whereas decreasing in e for all* $o > p+g-w$.

In fact, the results in Theorem 5.4.3(i) also hold with the single directional option. In addition, by Theorem 5.4.3(ii) and the make-to-order production policy adopted by the manufacturer, we know that given $w > \tilde{c}$, the manufacturer's production quantity is $F^{-1}(a)$, which decreases with both the option price and the wholesale price, and increases with the exercise price when the option price is lower than $p+g-w$, and decreases with the exercise price when the option price is higher than $p+g-w$.

5.5 Supply Chain Coordination with Bidirectional Option

Supply chain coordination means that the system-wide optimal expected profit is achieved for the supply chain.

In the following, the research will explore how the bidirectional option contract, combined with the wholesale price contract, can be set to achieve supply chain coordination. To derive the optimal expected profit of the channel, the supply chain is taken as an entity and then the expected profit of the supply chain system can be formulated as follows:

$$E\pi_s(Q_s) = E[p\min\{Q_s, X\} + v\max\{Q_s - X, 0\} - cQ_s - g\max\{X - Q_s, 0\}], \qquad (5.5.1)$$

where Q_s denotes the production quantity of the supply chain system. The first term is the system's sales revenue, the second term is the salvage revenue, the third term is the production cost, and the last term is the shortage cost. It is easy to show that $E\pi_s(Q_s)$ is concave in Q_s. By the first-order optimality condition, it follows the first-best production quantity of the system as $\overline{Q}_s = F^{-1}(\frac{p+g-c}{p+g-v})$. Note that since the manufacturer adopts the make-to-order production policy and produces the product up to $\overline{Q}_{or} + \overline{q}_{or}$, in order to achieve the optimal expected profit of the channel, it is sufficient for the manufacturer to provide the bidirectional option contract combined with an appropriate wholesale price to push the retailer to pursue a pair of strategies (Q, q) under which their sum is just $\overline{Q}_s$. Based on this observation, the following result is derived.

Theorem 5.5.1. *A combination of the bidirectional option contract and the wholesale price contract that satisfies the following system can attain channel coordination in the supply chain,*

$$\begin{cases} e = \frac{p+g-v}{p+g+v-2c}o + \frac{(p+g)(2v+w-2c)-wv}{p+g+v-2c}, \\ \frac{(c-v)(p+g-w)}{p+g-c} < o < \min\{p+g-w,\ c-v\}, \\ w > \max\{\tilde{c}, c\}, where\ \tilde{c} = \frac{(o+2v-e)(p+g)-v(o+e)}{p+g+v-2e}. \end{cases} \tag{5.5.2}$$

Wang and Liu (2007) and Zhao et al. (2010) considered the supply chain coordination issue with the call option that allows the retailer to adjust the initial order only upwards. Given the call option, they demonstrated that at coordination the exercise price negatively correlates with the option price (see Proposition 2 in Wang and Liu 2007 and Propositions 4 and 5 in Zhao et al. 2010). However, it is seen from Theorem 5.5.1 that given the bidirectional option, at coordination the exercise price positively correlates with the option price. The intuition behind this difference can be explained as follows: Given the call option as considered in Wang and Liu (2007) and Zhao et al. (2010), the retailer is exposed to only the chance of loss from an increase in the exercise price. However, given the bidirectional option, the effects of an increase in the exercise price on the retailer can be the two sides of a coin. One is the adverse effect that the retailer may need to pay a higher unit price for the additional product quantity purchased by exercising the option as a call option. The other is the beneficial effect that the retailer is able to return the leftovers at a higher refund per unit by exercising the option as a put option. In other words, the retailer may both benefit and suffer from an increase in the exercise price of the bidirectional option. As a result, while the contract structure that a higher option price implies a higher exercise price cannot achieve supply chain coordination with the only-call option, it may do so with the bidirectional option. In addition, it should be noted that the system given in Theorem 5.5.1 is distribution free, which implies that the manufacturer can utilize it to coordinate different retailers without knowing their respective demand distributions.

5.6 Conclusion

In this chapter, a supply contract with the bidirectional option is developed for a manufacturer-retailer supply chain. Under the bidirectional option contract, the research derives closed-form expressions for the retailer's optimal order strategies, including the initial order strategy and the option purchasing strategy, with a general demand distribution. The research also analytically examines the feedback effects of the bidirectional option on the retailer's initial order strategy. Note that since the bidirectional option provides the retailer with the chances of both supplementing the initial order and returning the leftovers, it is unclear whether the retailer will increase or decrease the initial order given the bidirectional option. It is also unclear in which scenarios the retailer should adopt a prudent or aggressive initial order policy given the bidirectional option. The study generates interesting insights into these issues, i.e., in addition to the cost parameters of the contract, the convexity and concavity of the demand cumulative distribution function can be a critical determinant of the retailer's decisions to address these issues. In addition, the research develops a distribution-free form of the bidirectional option contract that can attain supply chain coordination.

The findings in this chapter substantiate the claim that there are fundamental differences between the single directional option and the bidirectional option. A limitation of the research is that only a simple two-echelon supply chain structure is considered for the bidirectional option contract. Obviously, an extension of the model to more complex supply chains is worth pursuing in future research. A particularly interesting issue along with this extension is to introduce horizontal and/or vertical competition in the supply chain and explore how such competition affects the effectiveness of the bidirectional option contract. Another possible extension is to incorporate demand information updating in the model, i.e., the retailer can dynamically take advantage of information updates in making decisions.

Appendix: Proofs of the Main Results

Proof of Theorem 5.4.1. With some algebra it is obtained that

$$\begin{aligned} E[p\min\{Q,X\} - wQ - oq] &= p[\textstyle\int_0^Q xdF(x) + \int_Q^{+\infty} QdF(x)] - wQ - oq \\ &= (p-w)Q - p\textstyle\int_0^Q F(x)dx - oq, \end{aligned} \tag{5.6.1}$$

$$\begin{aligned} E[\max\{\min\{X-Q,q\},0\}] &= \textstyle\int_Q^{Q+q}(x-Q)dF(x) + \int_{Q+q}^{+\infty} qdF(x) \\ &= q - \textstyle\int_Q^{Q+q} F(x)dx, \end{aligned} \tag{5.6.2}$$

$$\begin{aligned} E[\max\{\min\{Q-X,q\},0\}] &= \textstyle\int_0^{Q-q} qdF(x) + \int_{Q-q}^{Q}(Q-x)dF(x) \\ &= \textstyle\int_{Q-q}^{Q} F(x)dx, \end{aligned} \tag{5.6.3}$$

$$\begin{aligned} &E[v\max\{Q-q-X,0\} - g\max\{X-Q-q,0\}] \\ &= v\textstyle\int_0^{Q-q}(Q-q-x)dF(x) - g\int_{Q+q}^{+\infty}(x-Q-q)dF(x) \\ &= v\textstyle\int_0^{Q-q} F(x)dx + (Q+q)g - g\mu - g\int_0^{Q+q} F(x)dx, \end{aligned} \tag{5.6.4}$$

where $\mu = \int_0^{+\infty} xdF(x)$, represents the expected demand. Substituting (5.6.1), (5.6.2), (5.6.3), and (5.6.4) into (5.4.1), it follows that

$$\begin{aligned} E\pi_r(Q,q) = (p+g-w)Q &+ (p+g-o-e)q - (p+g)\textstyle\int_0^{Q+q} F(x)dx \\ &+ e\textstyle\int_{Q-q}^{Q+q} F(x)dx + v\int_0^{Q-q} F(x)dx - g\mu. \end{aligned} \tag{5.6.5}$$

To solve the problem $P_{5.1}$, we first ignore the constraints $Q \geq q \geq 0$ to solve the corresponding unconstrained problem, and then check whether the optimal solution of the unconstrained problem satisfies these constraints. With some algebra we obtain

$$[\frac{\partial^2[E\pi_r(Q,q)]}{\partial Q^2}][\frac{\partial^2[E\pi_r(Q,q)]}{\partial q^2}] - [\frac{\partial^2[E\pi_r(Q,q)]}{\partial Q \partial q}]^2 = 4(e-v)(p+g-e)f(Q+q)f(Q-q) > 0, \tag{5.6.6}$$

which, associated with $\frac{\partial^2[E\pi_r(Q,q)]}{\partial Q^2} < 0$, indicates that $E\pi_r(Q,q)$ is jointly strictly concave in Q and q. By the first-order optimality condition, we derive that the optimal solution of the unconstrained problem is given by the following system.

$$\begin{cases} \frac{\partial[E\pi_r(Q,q)]}{\partial Q} = p+g-w-(p+g-e)F(Q+q)-(e-v)F(Q-q)=0, \\ \frac{\partial[E\pi_r(Q,q)]}{\partial q} = p+g-o-e-(p+g-e)F(Q+q)+(e-v)F(Q-q)=0. \end{cases} \tag{5.6.7}$$

Solving (5.6.7), we obtain the optimal solution of the unconstrained problem as

$$\begin{cases} \overline{Q}_{or} = \frac{1}{2}[F^{-1}(a)+F^{-1}(b)] \\ \overline{q}_{or} = \frac{1}{2}[F^{-1}(a)-F^{-1}(b)] \end{cases} \tag{5.6.8}$$

where $a = \frac{2p+2g-w-o-e}{2(p+g-e)} > 0, b = \frac{o+e-w}{2(e-v)} > 0$. It is clear that (5.6.8) satisfies the constraint $Q \geq q$. Therefore, (5.6.8) is also the optimal solution of the constrained problem if $F^{-1}(a) > F^{-1}(b)$, i.e., $a > b$, otherwise the optimal solution of the constrained problem is given by

$$\begin{cases} \overline{Q}_{or} = \frac{1}{2}[F^{-1}(a)+F^{-1}(b)] \\ \overline{q}_{or} = 0. \end{cases} \tag{5.6.9}$$

By $a > b$ we obtain $w > \frac{(o+2v-e)(p+g)-v(o+e)}{p+g+v-2e}$. To summarize, the desired result follows and the proof is completed. □

Proof of the retailer's optimal order strategy with the wholesale price contracts The retailer's problem with the wholesale price contract w is to solve

$$\max_{Q_{wr} \geq 0} E\pi(Q_{wr}) = E[p\min\{Q_{wr}, X\} + v\max\{Q_{wr}-X, 0\} - wQ_{wr} - g\max\{X-Q_{wr}, 0\}]. \tag{5.6.10}$$

With some algebra we obtain from (5.6.10) that

$$\begin{aligned} E\pi(Q_{wr}) &= p\int_0^{Q_{wr}} xdF(x) + p\int_{Q_{wr}}^{+\infty} Q_{wr}dF(x) + v\int_0^{Q_{wr}}(Q_{wr}-x)dF(x) \\ &\quad -wQ_{wr} - g\int_{Q_{wr}}^{+\infty}(x-Q_{wr})dF(x) \\ &= (p+g-w)Q_{wr} - (p+g-v)\int_0^{Q_{wr}} F(x)dx - g\mu, \end{aligned} \tag{5.6.11}$$

where $\mu = \int_0^{+\infty} x dF(x)$. From (5.6.11) we have

$$\frac{dE\pi(Q_{wr})}{dQ_{wr}} = (p + g - w) - (p + g - v)F(Q_{wr}), \tag{5.6.12}$$

$$\frac{d^2E\pi(Q_{wr})}{dQ_{wr}^2} = -(p + g - v)f(Q_{wr}) < 0. \tag{5.6.13}$$

It follows from (5.6.13) that $E\pi(Q_{wr})$ is strictly concave in Q_{wr}, so the first-order optimality condition works. We therefore obtain from (5.6.12) the retailer's optimal order quantity with the wholesale price-only contract w as $\overline{Q}_{wr} = F^{-1}(\frac{p+g-w}{p+g-v})$. Thus the proof is completed. □

Proof of Theorem 5.4.2. With some algebra we obtain

$$\frac{p+g-w}{p+g-v} < \frac{o+e-w}{2(e-v)} = b \Longleftrightarrow w < \tilde{c}, \tag{5.6.14}$$

$$\frac{p+g-w}{p+g-v} < a = \frac{2p+2g-w-o-e}{2(p+g-e)} \Longleftrightarrow w > \tilde{c}, \tag{5.6.15}$$

$$\frac{o+e-w}{2(e-v)} = b < \frac{2p+2g-w-o-e}{2(p+g-e)} = a \Longleftrightarrow w > \tilde{c}, \tag{5.6.16}$$

where $\tilde{c} = \frac{(o+2v-e)(p+g)-v(o+e)}{p+g+v-2e}$. Again, it is easy to obtain

$$\frac{p+g-w}{p+g-v} = \lambda a + (1 - \lambda)b, \quad \text{where } \lambda = \frac{p+g-e}{p+g-v}. \tag{5.6.17}$$

If $F(x)$ is concave and two-order differentiable, then $F''(x) \leq 0$. Hence $F^{-1''}(y) = \frac{-F''(x)}{[f(x)]^3} \geq 0$, where $F^{-1}(y)$ denotes the inverse function of $F(x)$, that is, $F^{-1}(y)$ is convex. Therefore,

$$\begin{aligned} \overline{Q}_{wr} &= F^{-1}(\tfrac{p+g-w}{p+g-v}) = F^{-1}(\lambda a + (1 - \lambda)b) \\ &\leq \lambda F^{-1}(a) + (1 - \lambda)F^{-1}(b). \end{aligned} \tag{5.6.18}$$

If $w > \tilde{c}$, from (5.6.16) we have $F^{-1}(a) > F^{-1}(b)$, which with $\lambda = \frac{p+g-e}{p+g-v} < \frac{1}{2}$ and (5.6.18) together leads to

$$\overline{Q}_{wr} < \tfrac{1}{2}F^{-1}(a) + \tfrac{1}{2}F^{-1}(b) = \overline{Q}_{or}. \tag{5.6.19}$$

If $w < \tilde{c}$, from (5.6.16) we have $F^{-1}(a) < F^{-1}(b)$, which with $\lambda = \frac{p+g-e}{p+g-v} > \frac{1}{2}$ and (5.6.18) together also leads to

$$\overline{Q}_{wr} < \tfrac{1}{2}F^{-1}(a) + \tfrac{1}{2}F^{-1}(b) = \overline{Q}_{or}. \tag{5.6.20}$$

Similarly, if $F(x)$ is convex and two-order differentiable, then $F''(x) \geq 0$. Hence $F^{-1''}(y) = \frac{-F''(x)}{[f(x)]^3} \leq 0$, with which it follows that $F^{-1}(y)$ is concave. Therefore,

$$\begin{aligned}\overline{Q}_{wr} &= F^{-1}(\tfrac{p+g-w}{p+g-v}) = F^{-1}(\lambda a + (1-\lambda)b) \\ &\geq \lambda F^{-1}(a) + (1-\lambda)F^{-1}(b).\end{aligned} \tag{5.6.21}$$

If $w > \tilde{c}$, then $F^{-1}(a) > F^{-1}(b)$, which with $\lambda = \frac{p+g-e}{p+g-v} > \frac{1}{2}$ and (5.6.21) together leads to

$$\overline{Q}_{wr} > \tfrac{1}{2}F^{-1}(a) + \tfrac{1}{2}F^{-1}(b) = \overline{Q}_{or}. \tag{5.6.22}$$

If $w < \tilde{c}$, then $F^{-1}(a) < F^{-1}(b)$, which with $\lambda = \frac{p+g-e}{p+g-v} < \frac{1}{2}$ and (5.6.21) together also leads to

$$\overline{Q}_{wr} > \tfrac{1}{2}F^{-1}(a) + \tfrac{1}{2}F^{-1}(b) = \overline{Q}_{or}. \tag{5.6.23}$$

To summarize, Theorem 5.4.2 follows and the proof is completed. □

Proof of Theorem 5.4.3. By Theorem 5.4.1 we obtain that given $w > \tilde{c}$,

$$\tfrac{d\overline{Q}_{or}}{de} = \tfrac{1}{4}\Big[\tfrac{p+g-w-o}{(p+g-e)^2 f(\overline{Q}_{or}+\overline{q}_{or})} + \tfrac{w-o-v}{(e-v)^2 f(\overline{Q}_{or}-\overline{q}_{or})}\Big] > 0, \tag{5.6.24}$$

$$\tfrac{d\overline{Q}_{or}}{dw} = -\tfrac{1}{4}\Big[\tfrac{1}{(p+g-e)f(\overline{Q}_{or}+\overline{q}_{or})} + \tfrac{1}{(e-v)f(\overline{Q}_{or}-\overline{q}_{or})}\Big] < 0, \tag{5.6.25}$$

$$\tfrac{d\overline{q}_{or}}{do} = -\tfrac{1}{4}\Big[\tfrac{1}{(p+g-e)f(\overline{Q}_{or}+\overline{q}_{or})} + \tfrac{1}{(e-v)f(\overline{Q}_{or}-\overline{q}_{or})}\Big] < 0. \tag{5.6.26}$$

Theorem 5.4.3(i) follows immediately from (5.6.24) to (5.6.26).

In addition, it is obvious that $\overline{Q}_{or} + \overline{q}_{or} = F^{-1}(a)$ given $w > \tilde{c}$. Further, we obtain

$$\tfrac{d(\overline{Q}_{or}+\overline{q}_{or})}{do} = \tfrac{d[F^{-1}(a)]}{do} = -\tfrac{1}{2(p+g-e)f(\overline{Q}_{or}+\overline{q}_{or})} < 0, \tag{5.6.27}$$

$$\tfrac{d(\overline{Q}_{or}+\overline{q}_{or})}{dw} = \tfrac{d[F^{-1}(a)]}{dw} = -\tfrac{1}{2(p+g-e)f(\overline{Q}_{or}+\overline{q}_{or})} < 0, \tag{5.6.28}$$

$$\tfrac{d(\overline{Q}_{or}+\overline{q}_{or})}{de} = \tfrac{d[F^{-1}(a)]}{de} = \tfrac{p+g-w-o}{2(p+g-e)^2 f(\overline{Q}_{or}+\overline{q}_{or})}. \tag{5.6.29}$$

Theorem 5.4.3(ii) follows immediately from (5.6.27) to (5.6.29). Thus the proof is completed. □

Proof of Theorem 5.5.1. The first-best production quantity for the supply chain system is $\overline{Q}_s = F^{-1}(\frac{p+g-c}{p+g-v})$. In order to attain coordination in the supply chain, it is sufficient for the manufacturer to provide the retailer with an appropriate combination of the bidirectional options and wholesale price contracts with which the retailer has an incentive to pursue a pair of order strategies (Q, q) for which the summation $Q + q = \overline{Q}_s$. From Theorem 5.4.1 we know that the retailer's optimal order strategies are given by (5.4.3) if $w > \tilde{c} = \frac{(o+2v-e)(p+g)-v(o+e)}{p+g+v-2e}$. Hence, any bidirectional option contract satisfying the following equation, combined with $w > \tilde{c}$, can provide the retailer with such an incentive,

$$F^{-1}(\tfrac{2p+2g-w-o-e}{2(p+g-e)}) = F^{-1}(\tfrac{p+g-c}{p+g-v}). \tag{5.6.30}$$

By the strict monotonicity of $F(x)$ we have $\frac{2p+2g-w-o-e}{2(p+g-e)} = \frac{p+g-c}{p+g-v}$, which leads to

$$e = \tfrac{p+g-v}{p+g+v-2c}o + \tfrac{(p+g)(2v+w-2c)-wv}{p+g+v-2c}. \tag{5.6.31}$$

Again, in order to avoid unreasonable cases, it is necessary that $w-o < e < w+o$. By

$$e = \tfrac{p+g-v}{p+g+v-2c}o + \tfrac{(p+g)(2v+w-2c)-wv}{p+g+v-2c} > w-o, \tag{5.6.32}$$

we follow $o > \frac{(c-v)(p+g-w)}{p+g-c}$; by

$$e = \tfrac{p+g-v}{p+g+v-2c}o + \tfrac{(p+g)(2v+w-2c)-wv}{p+g+v-2c} < w+o, \tag{5.6.33}$$

we follow $o < p+g-w$. In addition, in order to avoid unreasonable cases it should be assumed that $0 < o < c-v$, which, associated with $\frac{(c-v)(p+g-w)}{p+g-c} < \min\{p+g-w,\ c-v\}$, leads to

$$\tfrac{(c-v)(p+g-w)}{p+g-c} < o < \min\{p+g-w,\ c-v\}. \tag{5.6.34}$$

To summarize, Theorem 5.5.1 follows and the proof is completed. □

Chapter 6
A Value-Based Approach to Option Pricing: The Case of Supply Chain Options

6.1 Introduction

As reviewed in Chaps. 1 and 5, the option contract is a well-observed practice in supply chain management. For example, as reported in Eppen and Iyer (1997), option contracts (such as those in the form of backup agreements) have been extensively utilized in supply chains in the apparel industry. Essentially, an option contract is characterized by two parameters, namely the option price, denoted as o, and the exercise price, denoted as e. The option price is an allowance paid by the retailer to the manufacturer for reserving one unit of the production capacity. The exercise price is the payment by the retailer to the manufacturer for exercising one unit of the option. In addition to employing option contracts, many firms often seek supplies from a spot market, which may be a real-time or close to real-time market, or an online B2B e-marketplace such as the China Bulk Commodity Electronic Exchange Portal (whose website is http://www.chinahhce.com). For instance, HP has taken a portfolio approach with options and a spot market to procuring components such as memory chips so as to reduce risks in procurement-related spending (Billington 2002). To be specific, 50 % of HP's procurement-related spending is incurred in long-term contracts. About 35 % goes to option contracts and the remaining 15 % is left to the spot market, which could be an online electronic exchange or an opportunistic buy (Carbone 2001).

In order to utilize option contracts, an issue that has to be addressed is how to price the option in a reasonable way such that the pricing scheme is well accepted by all the contracting partners. A closer look at the option contract mechanism reveals that, with the option contract, the option buyer (such as a retailer) can make a final order decision in response to demand realization, so it can hedge against the loss

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associated with over- and under-ordering. In addition, since the option contract allows the retailer to purchase a certain quantity of the product at a price agreed today, the retailer also enjoys the benefit of hedging against the risk associated with price fluctuations of the underlying product in the future. As a result, the value inherent in an option contract for the retailer is affected mainly by two factors, one is the stochastic form of the demand for the underlying product, which can be characterized by the cumulative distribution function (CDF) of the demand, and the other is the trend of the price of the underlying product in the future, which can be represented by the CDF of the spot price of the underlying product. Given that the option value is closely affected by these two factors, so is the option price. In fact, it is natural and reasonable to price an option based on the value inherent in the option at expectation. This is particularly true for a risk-neutral environment. Motivated by such an intuitive observation, this chapter develops a value-based (VB) approach to the pricing of supply chain options. The research first addresses this issue under the simplest supply chain structure comprising a single manufacturer and a single retailer, and after that the research extends the model to include multiple retailers.

As a matter of fact, the issue concerning option pricing has attracted substantial attention in the area of finance. Pioneered by Black and Scholes (1973), there is a vast body of literature that takes this issue into account (Wu 2004, Merton 1973, Cox et al. 1979, Merton 1976, Wilmott et al. 1994, Cox and Ross 1976, Smith 1976). However, the option pricing schemes in the context of finance are developed generally based on the price fluctuations of the underlying product and have seldom taken into account the demand for the underlying product. In the case of supply chain options, however, factors that can significantly affect the option value (hence the option price) include the stochastic form of the demand for the underlying product, as well as the price fluctuations of the underlying product. Therefore, the option pricing schemes developed in the context of finance are generally inapplicable to the pricing of supply chain options. Some other literature appears to have taken into account the option pricing issue in the context of supply chains, such as Wu et al. (2002), Spinler et al. (2003), Wu and Kleindorfer (2005), Burnetas and Ritchken (2005), Cachon and Lariviere (2001), Barnes-Schuster et al. (2002), and Wang and Liu (2007). It should be noticed that these existing literature generally addresses the option pricing issue by the Stackelberg game approach. As a result, the option pricing schemes developed by them serve only the leader's interest, which inevitably raises the question: will the pricing schemes with the Stackelberg game theory be well accepted by the follower of the game? The answer is "no" in most cases since the leader always captures all the surplus accrued from the option. Different from the existing literature, this chapter develops an option pricing scheme based on the value inherent in the "option right". As a result, the pricing schemes developed in this chapter are more objective and fair, and consequently are more likely to be accepted by all the contracting partners as compared with those that follow the Stackelberg game approach.

The rest of this chapter is organized as follows: In Sect. 6.2 the model is introduced. In Sect. 6.3 the option pricing scheme is developed for the case of a single retailer. Section 6.4 extends the model to the case of multiple retailers. Section 6.5 concludes the chapter and suggests future research directions. All the proofs of the main results are put in the Appendix for clarity.

6.2 Model Description

Consider a supply chain involving a perishable product, which has a comparatively long production lead-time and a short sale season (Milner and Rosenblatt 2002). Such type of products usually require an early order commitment. The research first considers the option pricing issue for a supply chain comprising a manufacturer and a retailer. After that, it will be extended to consider the model including multiple retailers.

For the case of a single retailer, the manufacturer distributes its product via the retailer to the end market. Let the marginal production cost of the manufacturer be c and the retail price of the retailer be p $(> c)$. The end market demand faced by the retailer is characterized by a random variable X, which has a cumulative distribution function (CDF) $F(x)$ with $x \geq 0$ and a probability density function (PDF) $f(x)$ with $f(x) = F'(x)$. The retailer has two order sources, namely (i) ordering from the manufacturer by signing an option contract in advance at the beginning of the production season and (ii) purchasing directly from a spot market upon demand realization during the sale season. To be specific, at the beginning of the production season, the manufacturer signs an option contract, denoted as (o, e), with the retailer. The retailer then determines the option purchase quantity Q, taking into account the spot purchase opportunity for the underlying product, about which the retailer now only has some probability knowledge. We assume that the spot price of the underlying product at this time is characterized by a random variable W, which follows a CDF $G(w)$ with a support of $[c, p]$. Obviously, if the option contract prices are not attractive, the retailer will reduce the option purchase quantity and rely more on spot purchase for the supplies. During the sale season, the retailer decides the option quantity q $(\leq Q)$ to exercise and the product quantity to purchase from the spot market to satisfy the demand upon demand realization and spot price realization of the underlying product.

For the case of multiple retailers, it is assumed that the manufacturer distributes its product simultaneously via n $(n > 1)$ retailers. The retail prices for all the retailers are p. As in the case of a single retailer, each of the retailers can order by signing an option contract in advance with the manufacturer at the beginning of the production season and spot purchase from the spot market upon demand realization during the sale season. The demand faced by retailer i is characterized by a random variable X_i, which follows a CDF $F_i(x)$ with $x \geq 0$. All the CDFs are assumed to be independent of one another.

The purpose of the model is to explore the worth of the "option right" when a stochastic spot market is available so as to develop some option pricing schemes with good reasonability. It is assumed that the demand and spot price in the spot market are only the result of market equilibrium, and any single manufacturer or retailer by itself is incapable of affecting the spot market. We argue that this assumption is reasonable because the spot market is open to many manufacturers and retailers, such as the B2B e-marketplace, where even "buyers on the spot market often don't know who they are buying from" (Kaplan and Sawhney 2000). A specific example of the spot market is the China Bulk Commodity Electronic Exchange Portal (the website is http://www.chinahhce.com). In addition, to capture the effects of the spot market, the spot market is assumed to be liquid, which means that the retailer can purchase as many of the product as it wants from the spot market to satisfy the demand. We argue that the assumption is also reasonable because (i) as shown by Dong and Liu (2007), with the risk-hedging benefit, a forward (which is the forward option contract here) is still attractive even when a liquid spot market is available, and (ii) an illiquid spot market would only strengthen the main results on the importance of the forward option contract. In addition, the retailer cannot sell its product to the spot market due to reasons such as restricted entrance, etc.

6.3 Option Pricing in the Case of a Single Retailer

The following develops the option pricing scheme for the case of a single retailer. Since the option contract allows the retailer to purchase a certain quantity of the product (which depends on the realized demand and the purchasing option quantity) at the exercise price agreed today, the option value to the retailer actually comes from the benefit of hedging against the risks associated with price and demand fluctuations. A natural and reasonable alternative to represent the option value is the order cost saving with the option, which can be measured by the difference in the retailer's order cost between the cases with and without the option quantity. Based on such an observation, the following result is derived.

Theorem 6.3.1. *Given Q units of the option purchased by the retailer at an exercise price e, the value for the Q units of the option is given by*

$$V_{opt.}(Q) = z(e)[Q - \int_0^Q F(x)dx], \tag{6.3.1}$$

where $z(e) = \int_e^p (w-e)dG(w)$ represents the amount by which the spot price of the underlying product is higher than the exercise price at expectation.

As a matter of fact, in addition to looking at the option value from the perspective of order cost saving, another relatively intuitive and reasonable perspective to look at this issue is the expected profit increment accruing to the retailer from the option quantity. That is, we can use the difference between the expected profits obtained by the retailer with and without the option to measure the option value. By an analysis it can be seen that the option value measured from the perspective of expected profit increment is actually identical to that measured from the perspective of expected order cost saving (see the Appendix for the proof). Hence, (6.3.1) does well measure the option value with intuitive explanations. From (6.3.1), it is seen that the option value is affected by two factors, namely the degree to which the spot price of the underlying product is higher than the exercise price at expectation (which is closely related to how the spot price of the underlying product will move in the future) and the demand distribution for the underlying product.

Despite that the total value for Q units of the option is given by (6.3.1), the marginal value for each unit of the option is different. As a matter of fact, the marginal value for the yth ($\forall y \leq Q$) unit of the option is given by

$$MV_{opt.}(y_{th}) = \frac{dV_{opt.}(Q)}{dQ} \mid_{Q=y} = z(e)[1 - F(y)]. \tag{6.3.2}$$

It is easy to see that the larger y is, the smaller is $1-F(y)$, so the lower is the marginal value. That is, the option has a decreasing marginal value to the retailer. A close look at (6.3.2) reveals the intuition behind it as follows: only when the demand is at least y will the yth unit of the option be exercised, and accordingly the retailer obtains a value of $z(e)$ at expectation from the yth unit of the option.

Using (6.3.2), we can develop an intuitive characterization of the retailer's optimal option purchase strategy. Given an option contract (o, e), by (6.3.2) it can be known that the marginal surplus accruing to the retailer from the yth unit of the option is

$$MV_{opt.}(y_{th}) - o = z(e)[1 - F(y)] - o. \tag{6.3.3}$$

It is clear that the retailer's maximum expected profit is achieved just at the option quantity $\overline{Q}$ that satisfies the following equation

$$MV_{opt.}(\overline{Q}_{th}) = z(e)[1 - F(\overline{Q})] = o, \tag{6.3.4}$$

which leads to the following result.

Theorem 6.3.2. *Given the option contract* (o, e)*, the retailer's optimal option purchase strategy is given by* $\overline{Q} = 0$ *if* $o \geq z(e)$*; otherwise* $\overline{Q} = F^{-1}[\frac{z(e)-o}{z(e)}]$.

An interesting insight gained from (6.3.4) is an intuitive characterization of the retailer's optimal option purchase strategy: the option quantity making the marginal surplus derived from the last unit of the option equal to 0 is optimal for the retailer. In addition, we see from Theorem 6.3.2 that the retailer will withdraw from the option contract market and only count on the spot market for the supplies once the option price reaches $z(e)$.

In the following we discuss the issue of how much the "option right" is worth. From Theorem 6.3.1, it is easy to obtain that given Q units of the option purchased at the exercise price e, the value per unit of the option on average is

$$V_{uop.}(Q) = V_{opt.}(Q)/Q = z(e)[\frac{Q - \int_0^Q F(x)dx}{Q}]. \tag{6.3.5}$$

In a risk-neutral environment, a natural approach for the manufacturer to price the option is by the value per unit of the option at expectation. As a result, looking at this issue from the manufacturer's perspective, the manufacturer can price the option as

$$\overline{o}_m = V_{uop.}(Q) = z(e)[\tfrac{Q-\int_0^Q F(x)dx}{Q}]. \tag{6.3.6}$$

However, looking at the option pricing issue from the retailer's perspective, we see that given Q units of the option purchased with an option contract (o, e), the surplus accruing to the retailer by the Q units of the option is

$$\overline{H}_{r,sur.}(Q) = \int_0^Q [z(e)(1 - F(y)) - o]dy = z(e)[Q - \int_0^Q F(x)dx] - oQ. \tag{6.3.7}$$

Comparing (6.3.6) with (6.3.7), we see that with the pricing scheme (6.3.6), the retailer receives nothing and all the benefit inherent in the option quantity is captured by the manufacturer. This no doubt offers no incentive to the retailer to participate in the option contract market. Therefore, the option pricing scheme (6.3.6) is in fact infeasible in practice.

To offer the retailer an incentive to participate in the option contract market, the manufacturer should allocate an appropriate proportion of the option value to the retailer. We assume that the manufacturer allocates to the retailer a ratio $1-\lambda$ of the value of the unit option and keeps the remaining ratio λ to itself, where $\lambda \in (0, 1)$. With such an allocation rule, the manufacturer can price the option as $\lambda V_{uop.}(Q) = \lambda z(e)[\frac{Q-\int_0^Q F(x)dx}{Q}]$. Thus we have the following result.

Theorem 6.3.3. *(i) An VB approach to option pricing is given by*

$$o_{1,VB} = \lambda z(e)[\tfrac{Q-\int_0^Q F(x)dx}{Q}], \tag{6.3.8}$$

where Q is the purchasing option quantity and λ represents the proportion of the value per unit of the option shared by the manufacturer.

(ii) With the VB option pricing scheme (6.3.8), the retailer captures the following surplus

$$H_{r,sur.}(Q) = (1 - \lambda)z(e)[Q - \int_0^Q F(x)dx]. \tag{6.3.9}$$

To apply (6.3.8), an issue that needs addressing is how to determine the ratio λ. In fact, λ is a result of bargaining between the manufacturer and the retailer. There are various models that can be used to determine λ, such as the Nash bargaining model, the Kalai-Smorodinsky model, and the Eliashberg model (see, e.g., Eliashberg 1986, Kohli and Park 1989). Note that the surplus accruing to the retailer with the pricing scheme (6.3.8) is just the proportion $1 - \lambda$ of the total value generated from the option. The remaining proportion of the option value (i.e., $\lambda z(e)[Q - \int_0^Q F(x)dx]$) is captured by the manufacturer. Hence, the pricing scheme (6.3.8) assures that both the manufacturer and the retailer have an incentive to participate in the option contract market. In addition, since $\frac{Q-\int_0^Q F(x)dx}{Q}$ is decreasing in Q, $o_{1,VB}$ is decreasing in Q, which implies that the more the retailer purchases the option, the lower is the option price. Hence, the VB pricing scheme (6.3.8) essentially applies the idea of quantity discount to the pricing of supply chain options. Furthermore, since $0 < \frac{Q-\int_0^Q F(x)dx}{Q} \leq 1$ for all Q and CDFs, $0 < o_{1,VB} \leq z(e)$, i.e., $o_{1,VB}$ is bounded from above by the amount that the spot price of the underlying product is higher than the exercise price at expectation. From Theorem 6.3.2, it is easy to see that this is necessary for an option pricing scheme to be reasonable because the retailer will purchase no quantity of the option once the option price reaches $z(e)$.

The following proceeds to explore the retailer's optimal option purchasing strategy under the VB option pricing scheme (6.3.8). In addition, since the manufacturer can adjust the retailer's option purchase strategy through setting the parameter λ in (6.3.8), the research also explores how λ should be set to achieve supply chain coordination. These issues are addressed as follows.

Theorem 6.3.4. *(i) Under the VB option pricing scheme (6.3.8), the retailer's optimal option purchase strategy is given by the solution to the following equation*

$$L(Q) = [1 - F(Q)] - \lambda[Q - \textstyle\int_0^Q F(x)dx]/Q = 0. \tag{6.3.10}$$

(ii) The following form of the VB option pricing scheme can achieve channel coordination in the supply chain

$$\hat{o}_{VB} = \hat{\lambda} z(e)[\tfrac{Q-\int_0^Q F(x)dx}{Q}], \tag{6.3.11}$$

where $\hat{\lambda} = \frac{\hat{Q}[1-F(\hat{Q})]}{\hat{Q}-\int_0^{\hat{Q}} F(x)dx}$, $\hat{Q} = F^{-1}(\frac{\overline{w}-c}{\overline{w}})$ *which represents the first-best production quantity of the channel, and* $\overline{w} = \int_c^p w dG(w)$, *which is the expected spot price of the underlying product.*

Despite that there may be multiple solutions to the Eq. (6.3.10), the retailer can choose $\min\{Q \mid Q > 0, L(Q) = 0\}$ as the optimal option purchase quantity. To illustrate the above results, we present an example as follows: suppose the demand follows a uniform distribution over $[a, b]$ with $0 < a < b$. The proportions of the value of the unit option shared by the manufacturer and the retailer from are given by λ and $1-\lambda$ respectively, where $\lambda \in (0, 1)$. Then by Theorem 6.3.3, the manufacturer can price the option based on the value of the option as

$$\tilde{o} = [\lambda z(e)(2b - Q)]/2(b - a). \tag{6.3.12}$$

With the option pricing scheme (6.3.12), the retailer's optimal option purchase quantity can be obtained by the unique solution to the following equation

$$L(Q) = \frac{(\lambda-2)Q-2b\lambda+2b}{2(b-a)} = 0, \tag{6.3.13}$$

which leads to the optimal option purchase quantity for the retailer as $\tilde{Q} = \frac{2b(1-\lambda)}{2-\lambda}$. It is easy to see that $\tilde{Q}$ is decreasing in λ, which implies that the more the manufacturer allocates the option value to the retailer, the more option quantity the retailer will purchase.

6.4 Option Pricing in the Case of Multiple Retailers

In the following the option pricing issue is considered for the supply chain involving multiple retailers. Assume that there are totally n retailers and the demand faced by retailer i follows a distribution $F_i(x)$ with $x \geq 0$. The demands faced by the retailers are independent of one another. All the retailers can procure the product by either signing an option contract in advance with the manufacturer at the beginning of the production season or spot purchase from the spot market upon demand realization during the sale season. Since the demand distributions faced by the retailers are different, we see that the option prices under the pricing scheme (6.3.8) are different for the same quantity of the option, which leads to a violation of the Robinson-Patman Act on antitrust litigation. As a result, the pricing scheme (6.3.8) is inapplicable to the case of multiple retailers and some revisions of it are necessary for this case.

Assume that the manufacturer allocates to retailer i a proportion $1 - \lambda_i$ of the value per unit of the option. In addition, the relative importance of retailer i to the manufacturer can be measured by α_i. Without loss of generality, it is required that $\Sigma_{i=1}^n \alpha_i = 1$. Incorporating the relative importance of the retailers in (6.3.8), a VB option pricing scheme for the case of multiple retailers is developed as follows:

Theorem 6.4.1. *(i) A VB option pricing scheme for the case of n (> 1) retailers is given by*

$$o_{n,VB} = [z(e)/Q]\Sigma_{i=1}^n \alpha_i \lambda_i [Q - \int_0^Q F_i(x)dx], \tag{6.4.1}$$

where Q is the purchasing option quantity, λ_i represents the proportion of the value per unit of the option shared by the manufacturer, and α_i indicates the relative importance of retailer i to the manufacturer.

(ii) With the option pricing scheme (6.4.1), the retailers in the set B will withdraw from the option contract market, where

$$\begin{aligned} B = \{i \in \{1, 2, \ldots, n\} \mid (1 - \alpha_i \lambda_i)[Q - \textstyle\int_0^Q F_i(x)dx] \\ \leq \Sigma_{j=1, j\neq i}^n \alpha_j \lambda_j [Q - \textstyle\int_0^Q F_j(x)dx]\}. \end{aligned} \tag{6.4.2}$$

The remaining retailers will stay in the option contract market and for retailer k ($k \in \{1, 2, \ldots, n\}/B$), the surplus accruing to it is given by

$$H_{rk,sur.}(Q) = z(e)[Q - \int_0^Q F_k(x)dx - \Sigma_{i=1}^n \alpha_i \lambda_i (Q - \int_0^Q F_i(x)dx)], \tag{6.4.3}$$

and the optimal option purchase strategy is given by the solution to the equation $L_k(Q) = 0$*, where*

$$L_k(Q) = [1 - F_k(Q)] - \Sigma_{i=1}^{n} \alpha_i \lambda_i [Q - \int_0^Q F_i(x)dx]/Q. \tag{6.4.4}$$

It can be seen that with the option pricing scheme (6.4.1), the manufacturer charges a uniform price for Q units of the option purchased by different retailers. Thus such a pricing scheme is in compliance with the Robinson-Patman Act. In addition, it is developed essentially based on the value inherent in the option to the retailers. Hence, as in the case of a single retailer, it follows the VB approach. With this pricing scheme, the retailers in the set B earn nothing. As a result, they will withdraw from the option contract market and only count on spot purchase for the supplies. However, the remaining retailers can obtain some surplus from the option and consequently have an incentive to participate in the option contract market, as well as the spot market. Despite that there may be multiple solutions to the equation $L_k(Q) = 0$, retailer k can select $\min\{Q_k \mid Q_k > 0, L_k(Q_k) = 0\}$ as the optimal option purchase quantity. As in the case of a single retailer, we have $0 < o_{n,VB} \leq z(e)$ for all Q and CDFs, which is necessary for an option pricing scheme to be reasonable since a retailer will purchase no quantity of the option once the option price reaches $z(e)$. In fact, an interesting result implied by Theorem 6.4.1 is that when the option price reaches $z(e)$, the option contract market will completely vanish and all the retailers will only count on spot purchase for the supplies. Note that $z(e)$ represents the amount by which the spot price of the underlying product is expected to be higher than the exercise price. A closer look at this result reveals that it actually applies to the case of a supply chain network comprising multiple manufacturers and multiple retailers. The intuition behind this result is that, given an option contract with the exercise price being e, the option price must not exceed the maximum benefit inherent in the unit option (which is just the amount $z(e)$); otherwise, the "option right" has no attractiveness to the option buyer. As a result, $z(e)$ essentially represents the threshold at which the mixed market scenario (i.e., a market in which the option contract market and the spot market co-exist) transits to the pure spot market scenario. To summarize, we have the following result.

Theorem 6.4.2. *When* $o \geq z(e)$*, the option contract market will vanish and only the spot procurement exists.*

6.5 Conclusion

It is reasonable and intuitive that the value inherent in supply chain options is well measured by the expected order cost saving or the expected profit increment accruing to the retailer with the option. In addition, how much the "option right" is worth is determined to a good extent by the value inherent in it. Based on these intuitive observations, this chapter develops an VB approach to the pricing of supply chain options. Since our option pricing schemes are based on the option value to the option buyer, it is objective and fair. Furthermore, under our option pricing schemes, each of the contracting partners can capture a share of the total value accrued from the option. As a result, our option pricing schemes are likely to be well accepted by the contracting partners. Hence, the option pricing schemes that follow the VB approach have some new merits as compared with those that follow the Stackelberg game approach. Of course, there are some limitations in our study, which require further exploration in the future. For example, we calculate the value of the option based on the concept of expectation, which implies that no risk attitude is considered in our study. However, the risk attitudes of supply chain members obviously play an important role in determining the option value, which in turn affects the option pricing. Therefore, research that takes into account the risk preferences of supply chain members is worth pursuing in the future.

Appendix: Proofs of the Main Results

Proof of Theorem 6.3.1. Given Q units of the option purchased by the retailer at an exercise price e, when $w \le e$, the retailer will only use spot purchase for the supplies and abandon all the option quantity. Therefore, the order cost saving accruing to the retailer in this case will be 0. However, when $w > e$, the retailer will receive an order cost saving of $(w-e)$ per unit for all the demand that does not exceed the purchasing option quantity Q. To summarize, the expected order cost saving accruing to the retailer with Q units of the option can be expressed as

$$\int_e^p [\int_0^Q (w-e)xdF(x) + \int_Q^{+\infty}(w-e)QdF(x)]dG(w). \tag{6.5.1}$$

With some algebra, we obtain from (6.5.1) that

$$\int_e^p [\int_0^Q (w-e)xdF(x) + \int_Q^{+\infty}(w-e)QdF(x)]dG(w) = z(e)[Q - \int_0^Q F(x)dx], \tag{6.5.2}$$

where $z(e) = \int_e^p (w-e)dG(w)$. Thus Theorem 6.3.1 follows and the proof is completed. $\square$

Proof of the result that the expected profit increment accruing to the retailer with Q units of the option is given by (6.3.1) With Q units of the option, the retailer can satisfy the demand during the sale season by either exercising a certain quantity of the option or spot purchase from the spot market. As a result, the retailer's expected profit (without excluding the cost of purchasing the option) with Q units of the option can be expressed as

$$E\pi_1(Q) = \int_c^e \int_0^{+\infty} (p-w)xdF(x)dG(w) + \int_e^p \int_0^Q (p-e)xdF(x)dG(w) + \int_e^p \int_Q^{+\infty} [px - eQ - w(x-Q)]dF(x)dG(w). \tag{6.5.3}$$

Without the option provision, the retailer can only count on spot purchase for the supplies upon demand realization during the sale season. As a result, the retailer's expected profit can be expressed as

$$E\pi_2 = \int_c^p \int_0^{+\infty} (p - w)xdF(x)dG(w). \tag{6.5.4}$$

The difference between the expected profits with and without the Q units of the option is given by

$$\begin{aligned} &E\pi_1(Q) - E\pi_2 \\ &= \int_e^p [\int_0^Q (w - e)xdF(x) + \int_Q^{+\infty} (w - e)QdF(x)]dG(w) \\ &= z(e)[Q - \int_0^Q F(x)dx], \end{aligned} \tag{6.5.5}$$

where $z(e) = \int_e^p (w - e)dG(w)$. Thus the desired result follows and the proof is completed. □

Proof of Theorem 6.3.3. It suffices to show Theorem 6.3.3(ii). Since the value of the yth ($\leq Q$) unit of the option is given by

$$MV_{opt.}(y_{th}) = z(e)[1 - F(y)], \tag{6.5.6}$$

the surplus accruing to the retailer from the yth unit of the option is

$$z(e)[1 - F(y)] - o_{1,VB} = z(e)[1 - F(y)] - \lambda z(e)[\tfrac{Q - \int_0^Q F(x)dx}{Q}]. \tag{6.5.7}$$

The total surplus accruing to the retailer from all the Q units of the option is given by

$$\begin{aligned} H_{r,sur.}(Q) &= \int_0^Q [z(e)(1 - F(y)) - \lambda z(e)(\tfrac{Q - \int_0^Q F(x)dx}{Q})]dy \\ &= (1 - \lambda)z(e)[Q - \int_0^Q F(x)dx], \end{aligned} \tag{6.5.8}$$

which is the desired result. Thus the proof is completed. □

Proof of Theorem 6.3.4. For Q units of the option purchased by the retailer at an exercise price e, the value of the Qth of the option is given by

$$MV_{opt.}(Q_{th}) = z(e)[1 - F(Q)]. \tag{6.5.9}$$

Obviously, the retailer achieves the maximum expected profit only at the option quantity that makes the value of the last unit of the option just equal to the marginal cost of purchasing the option. Therefore, the optimal option purchase quantity for the retailer is given by the following equation

$$z(e)[1 - F(Q)] = \lambda z(e)[\tfrac{Q - \int_0^Q F(x)dx}{Q}], \tag{6.5.10}$$

which leads to

$$L(Q) = [1 - F(Q)] - \lambda[Q - \int_0^Q F(x)dx]/Q = 0. \tag{6.5.11}$$

Thus Theorem 6.3.4(i) follows. To show Theorem 6.3.4(ii), we first derive the first-best production quantity in the channel. To this end, we take the supply chain as a centralized entity. Since the spot price w of the product is bounded in the range (c, p), the centralized entity will satisfy the demand by using in-house production prior to purchasing from the spot market; otherwise, the centralized entity never achieves the optimal performance. Hence, the overall profit of the centralized supply chain is given by

$$E\pi_s(Q) = E_W E_X[pX - W\max\{X - Q, 0\} - cQ], \tag{6.5.12}$$

where Q denotes the production quantity of the channel. The first term of (6.5.12) is the sales revenue, the second term is the spot procurement cost, and the last term is the in-house production cost. Furthermore, by (6.5.12) we obtain

$$\begin{aligned} E\pi_s(Q) &= \int_c^p [\int_0^{+\infty} pxdF(x) - w\int_Q^{+\infty}(x - Q)dF(x) - cQ]dG(w) \\ &= (\overline{w} - c)Q + (p - \overline{w})\overline{x} - \overline{w}\int_0^Q F(x)dx, \end{aligned} \tag{6.5.13}$$

where $\overline{w} = \int_c^p wdG(w)$ represents the expected spot price of the underlying product and $\overline{x} = \int_0^{+\infty} xdF(x)$ represents the expected demand. Since $\frac{d^2E\pi_s(Q)}{dQ^2} = -\overline{w}f(Q) < 0$, $E\pi_s(Q)$ is strictly concave in Q. Thus the first-order optimality condition works. Letting $\frac{dE\pi_s(Q)}{dQ} = \overline{w} - c - \overline{w}F(Q) = 0$, we obtain the first-best production quantity of the channel as $\hat{Q} = F^{-1}(\frac{\overline{w}-c}{\overline{w}})$.

Obviously, an option pricing scheme that can provide the retailer with an incentive to purchase as many as $\hat{Q}$ units of the option can be used to coordinate the channel. The option pricing scheme making $\hat{Q}$ the unique solution to (6.5.10) just provides the retailer with such an incentive. Hence, letting $Q = \hat{Q}$ in (6.5.10) leads to

$$z(e)[1 - F(\hat{Q})] = \lambda z(e)[\frac{\hat{Q} - \int_0^{\hat{Q}} F(x)dx}{\hat{Q}}], \tag{6.5.14}$$

from which $\hat{\lambda} = \frac{\hat{Q}[1-F(\hat{Q})]}{\hat{Q}-\int_0^{\hat{Q}} F(x)dx}$. To summarize, the proof of Theorem 6.3.4 is completed. □

Proof of Theorem 6.4.1. It suffices to show Theorem 6.4.1(ii). With the option pricing scheme (6.4.1), it is clear that retailer i will withdraw from the option contract market if

$$[z(e)/Q]\Sigma_{j=1}^n \alpha_j\lambda_j[Q - \int_0^Q F_j(x)dx] \geq [z(e)/Q][Q - \int_0^Q F_i(x)dx]. \tag{6.5.15}$$

With some algebra, we obtain from (6.5.15) that

$$(1-\alpha_i\lambda_i)[Q-\textstyle\int_0^Q F_i(x)dx] \le \Sigma_{j=1,j\neq i}^n \alpha_j\lambda_j[Q-\int_0^Q F_j(x)dx]. \tag{6.5.16}$$

For any retailer k ($k \in \{1,2,\ldots,n\}/B$), it will stay in the option contract market since it is able to derive some surplus under the option pricing scheme (6.4.1). Specifically, the surplus accruing to retailer k can be expressed as

$$\begin{aligned} H_{rk,sur.}(Q) &= \textstyle\int_0^Q [z(e)(1-F_k(y)) - z(e)\Sigma_{i=1}^n \alpha_i\lambda_i(Q-\int_0^Q F_i(x)dx)/Q]dy \\ &= z(e)[Q-\textstyle\int_0^Q F_k(x)dx - \Sigma_{i=1}^n \alpha_i\lambda_i(Q-\int_0^Q F_i(x)dx)]. \end{aligned} \tag{6.5.17}$$

The optimal option purchase strategy for retailer k is given by the solution Q to the following equation

$$[z(e)/Q]\Sigma_{i=1}^n \alpha_i\lambda_i[Q-\textstyle\int_0^Q F_i(x)dx] = z(e)[1-F_k(Q)], \tag{6.5.18}$$

which leads to

$$[1-F_k(Q)] - \Sigma_{i=1}^n \alpha_i\lambda_i[Q-\textstyle\int_0^Q F_i(x)dx]/Q = 0. \tag{6.5.19}$$

Thus the proof is completed by denoting

$$L_k(Q) = [1-F_k(Q)] - \Sigma_{i=1}^n \alpha_i\lambda_i[Q-\textstyle\int_0^Q F_i(x)dx]/Q. \quad \square \tag{6.5.20}$$

Chapter 7
Selection of Supply Chain Contracts: The Case of Option Contracts

7.1 Introduction

Manufacturer-retailer supply chains commonly adopt a wholesale price mechanism in practice. This mechanism, however, has often led manufacturers and retailers to situations of conflicts of interest, resulting in an inefficient supply chain. For instance, due to uncertain market demand and aversion to incurring inventory costs, retailers prefer to order flexibly from manufacturers so as to accommodate fluctuating market demand. On the other hand, in order to hedge against the risks of over- and under-production, manufacturers prefer retailers to place full orders as early as possible. This conflict between the retailer and the manufacturer can lead to actions that are not system-wide optimal, thereby resulting in an inefficient supply chain. Such problems have been encountered by companies such as Mattel, Inc., a toy maker (see, e.g., Barnes-Schuster et al. 2002). Motivated by this observation, we address in this chapter the issue of whether an option mechanism is a viable alternative to achieving an efficient supply chain. The option mechanism is characterized by two parameters, namely the option price o and the exercise price e. The option price, in essence, is an allowance paid by the retailer to the manufacturer for reserving one unit of the production capacity. The exercise price is to be paid by the retailer to the manufacturer for one unit of the product purchased by exercising the option. Option mechanisms have attracted substantial attention not only in the context of financial market, pioneered by Black and Scholes (1973), but also in the area of supply chain management (SCM). In fact, option mechanisms have been extensively applied in various industries such as fashion apparel (Eppen and Iyer 1997), toys (Barnes-Schuster et al. 2002), and electronics (Billington 2002, Carbone 2001).

The research of this chapter is based on Zhao et al. (2010).

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Relationships between manufacturers and retailers in supply chains have dramatically changed in recent years. One such change is evidenced by the fact that partnerships have become prevalent in supply chains. A typical example is the numerous operating modes based on cooperative relations that have been widely adopted in the practice of SCM, such as *vendor managed inventory (VMI), holding cost subsidies*, and *consignment*. These operating modes have significantly improved the overall performance of supply chains. This industry environment motivates us to address the problem under study by way of a cooperative game approach. Again, much of the literature on supply chain coordination has focused on developing incentive contract forms, and did not take into account issues concerning implementation of these contract forms. However, profit allocations differ under different coordinating contracts. The ultimate implementation outcome of a coordinating contract form inevitably depends on supply chain members' individual risk preferences and negotiating powers. Therefore, given that an incentive contract form has been derived, two important questions remain: (i) What coordinating contracts are feasible? (ii) How do supply chain members' individual risk preferences and negotiating powers impact the ultimate outcome?

Supply chain coordination is an important issue in SCM. Various contract types have been designed to align supply chain members' incentives to drive the optimal action in the supply chain. However, it is clear that feasible coordinating contracts are those that leave each party to be at least as well off as that party would be without these contracts. Motivated by this observation, we propose a notion called *the core of a contract set*, which is a subset of the contract set consisting of all the contracts satisfying the coordination requirement. Again, noting that the wholesale price mechanism is prevalently used in practice, we take the profit level under the wholesale price mechanism as a benchmark and derive the core of option contracts by taking a cooperative game approach. We demonstrate that option contracts can coordinate the supply chain and achieve Pareto-improvement. Hence, the manufacturer and the retailer will both have strong incentives to adopt the option mechanism rather than the wholesale price mechanism.

Even though every option contract in the core, compared with the wholesale price mechanism, can coordinate the supply chain with Pareto-improvement, the profit allocations between the retailer and the manufacturer differ for different option contracts in the core. As a result, the retailer and manufacturer will negotiate the option contract from the core to implement. Obviously, the outcome depends heavily on individual members' negotiating powers, as well as their risk preferences. By taking these factors into consideration, we identify the option contract that the retailer and manufacturer both agree to implement according to Nash's bargaining model and Eliashberg's model, and explore analytically the effects of these factors on the ultimate outcome. The remainder of this chapter is organized as follows: Sect. 7.2 reviews the literature. Section 7.3 introduces the model. Section 7.4 analyzes the model. Section 7.5 examines supply chain coordination with option contracts and discusses the related characteristics. Section 7.6 analytically explores the selection of option contracts and the sharing of the extra profit gained from coordination, taking into consideration the effects of supply chain members' risk preferences and negotiating powers. Section 7.7 concludes the chapter and suggests topics for future research. All the proofs are put in the Appendix.

7.2 Literature Review

This chapter is devoted to exploring the selection/implementation issue of supply chain contracts with the option contract under consideration. To highlight the contributions of this chapter, only the literature that is representative and particularly relevant to this work is reviewed.

First, this chapter is closely related to the literature on the use of options in SCM, such as, but not limited to, Barnes-Schuster et al. (2002), Eppen and Iyer (1997), Wu et al. (2002), Cheng et al. (2003), Cachon and Lariviere (2001), Wang and Liu (2007), Spinler et al. (2003), Brown and Lee (2003), Milner and Rosenblatt (2002), and Wang and Tsao (2006). This research stream mainly focuses on operations flexibility and economics efficiency derived from the options. For the detailed review of this related research stream, readers are referred to the relevant sections in Chaps. 5 and 6.

Second, this chapter is closely related to the literature on supply chain coordination with contracts and supply chain contract negotiation. We review only a few representative studies in these areas. Pasternack (1985) showed that coordination between a buyer and a supplier can be achieved with buyback contracts. Cachon and Lariviere (2005) demonstrated that revenue sharing can coordinate a two-echelon supply chain with a price-setting newsvendor. Taylor (2002) considered sales-rebate contracts with sales effort effects. He showed that when demand is affected by the retailer's sales effort, a target rebate and returns contract can achieve channel coordination. The other alternative contracts that have been shown to achieve coordination in supply chains include quantity discounts (see, e.g., Weng 1995) and quantity-flexibility contracts (see, e.g., Tsay 1999). Cachon (2003) gave an excellent survey on supply chain coordination with contracts. Note that numerous contracts can be utilized to achieve supply chain coordination and the subsequent profit allocations differ. Hence, an issue that needs resolving is to identify the contract that will be accepted by all the supply chain members and will subsequently be implemented. This issue has attracted some researchers' attention. Kohli and Park (1989) considered simply the issues concerning the selection of quantity discount contracts using Nash's bargaining model, Kalai-Smorodinsky's model, and Eliashberg's model. Sobel and Turcic (2008) developed a general model for supply chain contract negotiation with risk aversion using Nash's bargaining model. Nagarajan and Sošić (2008) gave a survey on using cooperative bargaining models to allocate profit between supply chain members.

This chapter focuses on the economics efficiency of options and we deploy a cooperative game approach to address the relevant problems. Compared with the literature on option models, the critical difference of the research in this chapter does not lie in the developed option model but lies in the analytical exploration of the issues concerning implementation of option contracts in the supply chain, taking into account supply chain members' risk preferences and negotiating powers. In fact, to more clearly explore these issues, we develop in this chapter a relatively simple option model, as compared with previous ones. Compared with the literature on supply chain coordination with contracts and supply chain contract negotiation, our research of this chapter is different in the following ways. First, we focus on employing the option mechanism to achieve supply chain coordination. Second, more importantly, we analytically derive the results, most of which are in closed-form, on the implementation of the coordinating option contract, including the coordinating option contract that will be accepted by all the supply chain members, the corresponding profit allocations between them, and the effects of their risk references and negotiating powers. It is not this case in the existing literature such as Sobel and Turcic (2008) and Nagarajan and Sošić (2008). Third, different from the existing literature such as Sobel and Turcic (2008) and Nagarajan and Sošić (2008), we incorporate the use of Eliashberg's model, the supply chain members' risk preferences, and their negotiation powers (by a line aggregation rule) into the analysis of the issues concerning implementation of the option contract. We think doing so is important because the implementation outcome of supply chain contracts clearly depends on supply chain members' negotiation powers on the negotiation table, as well as their risk preferences. In addition, our research approach can easily be extended to other types of supply chain contracts popular in SCM research. Of course, we also discuss the limitations of our study. Despite the acknowledged limitations, we believe the research of this chapter has made a good contribution to the literature on the *implementation aspect* of supply chain contracts.

7.3 Model Description

Consider a two-echelon supply chain consisting of a single manufacturer and a single retailer where the manufacturer's product is sold via the retailer to end consumers. Market demand for the product is a stochastic variable X, which follows a strictly increasing distribution function $F(x)$ with $x \geq 0$. We assume the option mechanism is employed to facilitate the production and procurement in the supply chain, which is characterized by two parameters, namely the option price o and the exercise price e. The option price is essentially an allowance paid by the retailer to the manufacturer for reserving one unit of the production capacity at the beginning of the production season. The exercise price is to be paid by the retailer to the manufacturer for one unit of the product purchased by exercising the option after the market demand is realized. In addition, considering the current industry environment in which many retailers are as powerful as, or even more powerful than, their manufacturers, we assume the manufacturer uses the "make-to-order" policy for production. The event sequence of our model can then be described as follows: At the beginning of the production season, the retailer reserves a quantity of the production capacity beforehand from the manufacturer, say Q, at a unit price of o. Using the "make-to-order" production policy, the manufacturer produces Q units of the product by following the quantity reserved by the retailer. During the selling season, according to the realized market demand of the product, the retailer purchases a quantity of the product up to Q units from the manufacturer at the exercise price e to satisfy the demand, and any unsatisfied demand is lost with no penalty cost. Demand information is assumed to be symmetric between the manufacturer and the retailer. Let the marginal production cost of the manufacturer be c and the salvage value per unit of unsold product be v for both the manufacturer and the retailer. Let p be the retailer's retail price. We focus on the reasonable and non-trivial case where $p > c > v$, $0 \leq o < c - v$, and $e > v$. In fact, assuming $o < c - v$ avoids the unreasonable case where the manufacturer is risk-free for its production while assuming $e > v$ avoids the trivial case where the retailer always exercises all the purchased options.

To focus on implementation issue of the option contract, as reviewed in Sect. 7.2, this chapter will consider a relatively simple option model. The primary purpose of this chapter is to explore the implementation issue of the coordinating option contract form, taking into account effects of the supply chain members' risk preferences and negotiating powers.

7.4 Basic Option Contract Model

Since the wholesale price mechanism is commonly used in manufacturer-retailer supply chains in practice, we use the wholesale price mechanism as the benchmark against which we will compare the option mechanism developed in this study. With the wholesale price mechanism, the event sequence is identical to the option model except for the option mechanism, which should be replaced by the wholesale price mechanism. Then, the retailer's expected profit function under the wholesale price mechanism is given by

$$E\Pi_{wr}(Q_{wr}) = E[p\min\{Q_{wr}, X\} - wQ_{wr} + v\max\{Q_{wr} - X, 0\}], \tag{7.4.1}$$

where Q_{wr} is the retailer's order quantity and w is the wholesale price. Obviously, in order to avoid the unreasonable cases, we require $p > w > c$. The first term in (7.4.1) is the retailer's sales revenue, the second term is the order cost, and the third term is the salvage value. Hence, with the wholesale price mechanism, the retailer's problem is to maximize the expected profit function (7.4.1) with respect to Q_{wr}, which yields the following proposition.

Proposition 7.4.1. *With the wholesale price mechanism, the retailer will earn an expected profit of*

$$\pi_{wr} = (p - w)\overline{Q}_{wr} - (p - v)\int_0^{\overline{Q}_{wr}} F(x)dx, \tag{7.4.2}$$

and the manufacturer will earn an expected profit of $\pi_{wm} = (w - c)\overline{Q}_{wr}$, *where* $\overline{Q}_{wr} = F^{-1}(\frac{p-w}{p-v})$.

If the manufacturer does not use the "make-to-order" production policy but plans its production for its own interest, then its optimal production quantity is to maximize the following expected profit function with respect to the production quantity Q_{wm},

$$E\Pi_{wm}(Q_{wm}) = E[w\min\{Q_{wm}, X\} - cQ_{wm} + v\max\{Q_{wm} - X, 0\}]. \quad (7.4.3)$$

Similar to the proof of Proposition 7.4.1, we derive the manufacturer's optimal production quantity as $\overline{Q}_{wm} = F^{-1}(\frac{w-c}{w-v})$. All these results obtained with the wholesale price mechanism will be taken as benchmarks against which we will compare the option mechanism developed in this chapter.

In what follows, we consider the option model. With the option contract mechanism, say (o, e), the retailer's expected profit function is given by

$$E\Pi_{or}(Q_{or}) = E[(p-e)\min\{Q_{or}, X\} - oQ_{or}], \quad (7.4.4)$$

where Q_{or} is the retailer's reserved quantity under the option contract mechanism. The first term in (7.4.4) is the retailer's sales profit and the second term is the allowance payout for the reserved capacity. Similarly, when the manufacturer plans its production quantity for its own interest instead of using the "make-to-order" production policy, its expected profit function is given by

$$E\Pi_{om}(Q_{om}) = E[oQ_{om} + e\min\{Q_{om}, X\} + v\max\{Q_{om} - X, 0\} - cQ_{om}]. \quad (7.4.5)$$

Hence, with the option mechanism, the retailer's problem is to maximize the expected profit function (7.4.4) and the manufacturer's is to maximize (7.4.5), which lead to the following proposition.

Proposition 7.4.2. *(i) Given (o, e), the manufacturer's optimal production quantity is $\overline{Q}_{om} = F^{-1}(\frac{e+o-c}{e-v})$ and the retailer's optimal reserved quantity is $\overline{Q}_{or} = F^{-1}(\frac{p-o-e}{p-e})$.*

(ii) Given $o + e$, $E\Pi_{or}(Q_{or})$ is decreasing in o or increasing in e, whereas $E\Pi_{om}(Q_{om})$ is increasing in o or decreasing in e.

(iii) Only if the option contract satisfies $e = p - \frac{p-v}{c-v}o$ $(o < c - v)$ will the retailer's optimal reserved quantity be just consistent with the manufacturer's optimal production quantity.

By the option mechanism, we know that $o + e$ is the unit price for the quantity of the product purchased by the retailer. From Proposition 7.4.2, we see that, given $o + e$, the higher the retailer is willing to pay for the option price, the higher is the optimal production quantity of the manufacturer. Besides, the retailer is prone to reserve more if it is allowed to pay a lower option price and a higher exercise price later. In addition, the retailer prefers to pay a lower option price and a higher exercise price later. However, the converse is preferred by the manufacturer. These indeed coincide with the intuition from practice. Furthermore, in terms of the option contract that makes the retailer's optimal reserved quantity just consistent with the manufacturer's optimal production quantity, the exercise price is a negative linear function of the option price.

Comparing with the wholesale price mechanism, we have the following proposition:

Proposition 7.4.3. *(i)* $\overline{Q}_{wr} < \overline{Q}_{or}$ *iff* $o < \frac{(p-e)(w-v)}{p-v}$; *(ii)* $\overline{Q}_{wm} < \overline{Q}_{om}$ *iff* $o > \frac{(c-v)(w-e)}{w-v}$.

Proposition 7.4.3 shows that compared with the wholesale price mechanism, the option contract mechanism with an option price being lower than $\frac{(p-e)(w-v)}{p-v}$ will induce the retailer to reserve more, whereas an option price that is higher than $\frac{(c-v)(w-e)}{w-v}$ will push the manufacturer to produce more.

7.5 Coordinating Option Contracts with Pareto-Improvement

To derive the system-wide optimal expected profit for the supply chain, we take the supply chain as a centralized entity. Let $E\Pi_s(Q_s)$ denote the expected profit of the centralized entity when the production quantity is Q_s. With some algebra, we have

$$\begin{aligned} E\Pi_s(Q_s) &= E[p\min\{Q_s, X\} + v\max\{Q_s - X, 0\} - cQ_s] \\ &= p\int_0^{Q_s} xdF(x) + p\int_{Q_s}^{+\infty} Q_s dF(x) + v\int_0^{Q_s}(Q_s - x)dF(x) - cQ_s \\ &= (p-c)Q_s - (p-v)\int_0^{Q_s} F(x)dx. \end{aligned} \tag{7.5.1}$$

Similar to the proof of Proposition 7.4.2, we can show the strict concavity of $E\Pi_s(Q_s)$ with respect to Q_s. Therefore, by the first-order optimality condition, we find that the first-best production quantity for the supply chain is $\overline{Q}_s = F^{-1}(\frac{p-c}{p-v})$, and accordingly the system-wide optimal expected profit, denoted by π_c, is

$$\pi_c = E\Pi_s(\overline{Q}_s) = (p-c)\overline{Q}_s - (p-v)\int_0^{\overline{Q}_s} F(x)dx. \tag{7.5.2}$$

It can be shown that $\overline{Q}_s > \overline{Q}_{wr}$ and $\overline{Q}_s > \overline{Q}_{wm}$ (see the Appendix). We discuss below supply chain coordination with the option contract. Obviously, an option contract that provides the retailer with an incentive to reserve as much as $\overline{Q}_s = F^{-1}(\frac{p-c}{p-v})$ will make the supply chain system achieve the system-wide optimal expected profit. This perspective yields the following theorem.

Theorem 7.5.1. *(i) The system-wide optimal expected profit of the supply chain can be achieved under any option contract* (o, e) *in the following set* M*:*

$$M = \{(o,e) : o = \lambda(c-v), e = (1-\lambda)p + \lambda v, \textit{where } \lambda \in [0,1)\}. \tag{7.5.3}$$

Compared with the wholesale price mechanism, any option contract in M can increase the profit of the supply chain system by

$$\Delta\pi = (p - c)(\overline{Q}_s - \overline{Q}_{wr}) - (p - v)\int_{\overline{Q}_{wr}}^{\overline{Q}_s} F(x)dx. \tag{7.5.4}$$

(ii) Under any option contract (o, e) *in* M*, we have* $\overline{Q}_{or} = \overline{Q}_{om} = \overline{Q}_s$*. Furthermore, with the option contract associated with* λ*, the maximum expected profit received by the retailer, denoted by* $\pi_{or}(\lambda)$*, is given by*

$$\pi_{or}(\lambda) = \lambda\pi_c, \tag{7.5.5}$$

and the maximum expected profit received by the manufacturer, denoted by $\pi_{om}(\lambda)$*, is given by*

$$\pi_{om}(\lambda) = (1 - \lambda)\pi_c, \tag{7.5.6}$$

where π_c*, given by (7.5.2), is the system-wide optimal expected profit of the supply chain.*

The relationship between o and e can be derived from (7.5.3) as

$$e = p - \frac{p - v}{c - v}o, \tag{7.5.7}$$

which has the following implications: (i) the exercise price negatively correlates with the option price, and (ii) an increase in the option price by a unit will induce a decrease in the exercise price by $\frac{p-v}{c-v} > 1$. This coincides with the intuition in practice that the earlier one pays for the product, the lower the price is. From $\overline{Q}_{or} = \overline{Q}_{om} = \overline{Q}_s$, we know that, for any option contract in M, the manufacturer's production quantity in our model is just the optimal one, even under the "make-to-order" production policy, which greatly enhances the robustness of the option contract in implementation. In addition, Theorem 7.5.1 implies that the supply chain system profit can be allocated arbitrarily by the option contracts in M using different λ's $\in [0, 1)$. In fact, we see from (7.5.5) that λ is the fractional split by the retailer of the optimal joint profit of the supply chain system with the option contract associated with λ in M. An option contract in M with a larger λ will make the retailer share more profit, and the converse holds for the manufacturer. In addition, it is worth noting that $\frac{(c-v)(w-e)}{w-v} < o < \frac{(p-e)(w-v)}{p-v}$ for any option contract (o, e) in M (see the Appendix for the proof). This is consistent with Proposition 7.4.3 and we can explain it as follows: with the wholesale price mechanism, the retailer is prone to order less than the system-wide optimal quantity because of the effects of double marginalization, unless the manufacturer is willing to offer a wholesale price lower than the unit production cost (Cachon 2003). By Proposition 7.4.3, it is clear that only the option contract satisfying $\frac{(c-v)(w-e)}{w-v} < o < \frac{(p-e)(w-v)}{p-v}$ will induce the retailer and the manufacturer to reserve and produce as much as the system-wide optimal quantity.

In practice, although there exist some contracts under which the supply chain's system-wide optimal profit can be achieved, some of them may be infeasible because such contracts may not be in each member's interest. Therefore, contracts that can effectively coordinate a supply chain must be designed to be in each member's interest, as well as ensuring the achievement of the system-wide optimal profit. Based on this idea, we propose a notion called *the core of a contract set* and define it as a subset of the contract set that consists of all the contracts fulfilling the coordination requirement.

We proceed to derive the core of the option contract set M below, taking the wholesale price mechanism as the benchmark. First, we see from (7.5.5) that, with the option contract associated with the parameter $\lambda = \frac{\pi_{wr}}{\pi_c}$ in the set M, the retailer will just earn as much as that it will under the wholesale price mechanism. Since $\lambda\pi_c$ strictly increases in λ, only the option contracts in M with λ satisfying $1 > \lambda > \frac{\pi_{wr}}{\pi_c}$ will make the retailer strictly better off than that under the wholesale price mechanism. Any other option contract in M will make the retailer worse off and subsequently is unacceptable to it. We denote $\lambda_{min} = \frac{\pi_{wr}}{\pi_c}$. Similarly, we see from (7.5.6) that, with an option contract associated with $\lambda = \frac{\pi_c - \pi_{wm}}{\pi_c}$ in M, the manufacturer will be just as well off as that it will under the wholesale price mechanism. Again, since $(1-\lambda)\pi_c$ strictly decreases in λ, only the option contracts in M with λ satisfying $0 \leq \lambda < \frac{\pi_c - \pi_{wm}}{\pi_c}$ will make the manufacturer earn more profit than that under the wholesale price mechanism. Any other option contract in M is unacceptable to it because it will make the manufacturer worse off. We denote $\lambda_{max} = \frac{\pi_c - \pi_{wm}}{\pi_c}$. To summarize, hence, only the option contracts in the set M with $\lambda \in [\lambda_{min}, \lambda_{max}]$ will make both the retailer and the manufacturer at least as well off as that under the wholesale price mechanism, and the remaining λ's, even though making the supply chain achieve system-wide optimal profit, should be excluded from M because they are unacceptable by either the retailer or the manufacturer. This discussion leads to the following theorem, which gives the core of the option contract set M.

Theorem 7.5.2. *The core of the option contract set M is given by*

$$N = \{(o,e) : (o,e) \in M,\ \lambda \in [\lambda_{min}, \lambda_{max}]\}, \tag{7.5.8}$$

where

$$\lambda_{min} = \frac{\pi_{wr}}{\pi_c} \text{ and } \lambda_{max} = \frac{\pi_c - \pi_{wm}}{\pi_c}. \tag{7.5.9}$$

Figure 7.1 graphically illustrates the problem of profit allocation between the retailer and the manufacturer under the option mechanism. If there is no coordination, the retailer receives profit π_{wr} and the manufacturer receives profit π_{wm},

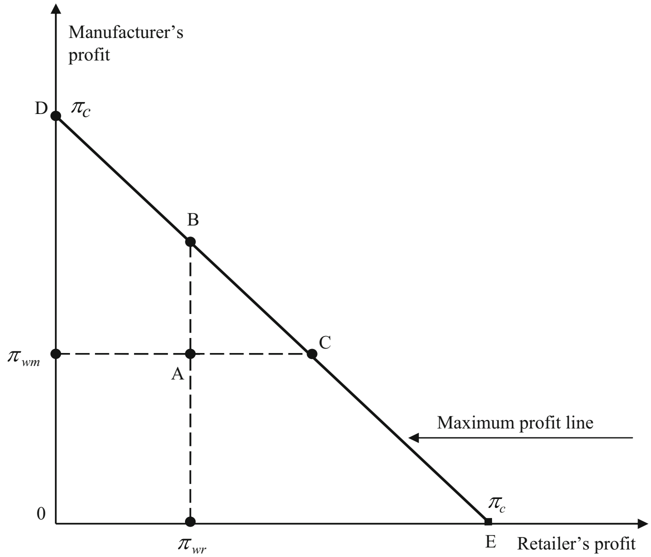

Fig. 7.1 Allocations of the channel profit

represented in Fig. 7.1 as point "A". Except the point "E" (denoted by a square), all the other points on the line segment D-E correspond to the profit allocations under the option contracts given as in M. However, due to the fact that neither the retailer nor the manufacturer is willing to accept less after coordination is achieved than before it is achieved, the likely allocations of the profit will only correspond to the points on the line segment B-C, which correspond to the profit allocations under the option contracts in the core N. The point "B" corresponds to the coordinating option contract under which the manufacturer captures all the additional profit gained from coordination, whereas the retailer just earns as much as before coordination is achieved. The converse is applicable to the point "C". Other points in the line segment B-C correspond to those option contracts under which the retailer and the manufacturer both receive some benefits from coordination.

7.6 Selection of Option Contracts

The discussion at the end of Sect. 7.5 implies that any option contract in the core N can be exploited to coordinate the supply chain with Pareto-improvement, as compared with the wholesale price mechanism. Consequently, the manufacturer and retailer will both have strong incentives to shift to adopting an option contract in N from the wholesale price mechanism. However, for different option contracts in N, the profit allocations between the manufacturer and retailer differ. The option contract with a larger λ is preferred by the retailer and the converse is preferred by the manufacturer. As a result, they will negotiate over which option contract to select from N, or equivalently over selecting λ from $[\lambda_{min}, \lambda_{max}]$. In this section we discuss issues concerning selection of the option contract that the retailer and manufacturer both agree to implement and the corresponding profit allocation.

For ease of exposition, we denote

$$\begin{aligned}\Delta\pi_r(\lambda) &= \pi_{or}(\lambda) - \pi_{wr} = \lambda\pi_c - \pi_{wr},\\ \Delta\pi_m(\lambda) &= \pi_{om}(\lambda) - \pi_{wm} = (1-\lambda)\pi_c - \pi_{wm},\end{aligned} \tag{7.6.1}$$

where $\Delta\pi_r(\lambda)$ and $\Delta\pi_m(\lambda)$ correspond to the additional profits split by the retailer and the manufacturer from $\Delta\pi$, which are their own increased profits from coordination with the option contract associated with λ in M. $\Delta\pi$, given as (7.5.4), corresponds to the total extra profit from coordination. Clearly, $\Delta\pi_r(\lambda)+\Delta\pi_m(\lambda) = \Delta\pi$ for all λ. For simplicity, we will omit the argument λ hereafter and denote $\Delta\pi_r(\lambda)$ and $\Delta\pi_m(\lambda)$ as $\Delta\pi_r$ and $\Delta\pi_m$ respectively.

Since the demand is stochastic, $\Delta\pi$ must be uncertain for any option contract in N. We assume that such uncertainty is represented by the probability distribution of $\Delta\pi$. In order to focus on the determination of the ultimate coordinating option contract and the allocation of the additional profit, we assume that the retailer is consistent with the manufacturer with respect to the probability distribution of $\Delta\pi$. Also, we suppose that both the retailer and the manufacturer have risk preferences towards the profit shared from $\Delta\pi$ and their preferences are represented by von Neumann and Morgenstern's (vN-M) utility functions (von Neumann and Morgenstern 1953) with respect to $(\Delta\pi_r, \Delta\pi_m)$, which can be assessed by their preferences over lotteries involving $(\Delta\pi_r, \Delta\pi_m)$. Let $U_r(\Delta\pi_r, \Delta\pi_m)$ be the retailer's vN-M utility function and $U_m(\Delta\pi_r, \Delta\pi_m)$ is the manufacturer's. For more details about the assessment of utility functions, the reader is referred to, e.g., Fishburn (1970) and Keeney and Raiffa (1976). We also assume that for the supply chain system, there is a system utility function $U_s(\Delta\pi_r, \Delta\pi_m)$ that is based on individual members' utility functions in the supply chain and their bargaining positions, and is determined under the "linear aggregation rule". There are various forms of utility functions, among which the additive form is used extensively in realistic applications of decision analysis. A utility function is said to be additive if it has the following form

$$U_i(\Delta\pi_r, \Delta\pi_m) = \gamma_{i1}U_{i1}(\Delta\pi_r) + \gamma_{i2}U_{i2}(\Delta\pi_m), \tag{7.6.2}$$

where $U_{i1}(\Delta\pi_r)$ is the conditional utility function of member i $(i = r, m)$ for $\Delta\pi_r$ (assumed to be a monotonic and increasing function of $\Delta\pi_r$). Similar implications are applicable to $U_{i2}(\Delta\pi_m)$, and γ_{i1} and γ_{i2} are positive scaling constants. A utility function is additive if and only if $\Delta\pi_r$ and $\Delta\pi_m$ are additively independent, which means that the preferences over $(\Delta\pi_r, \Delta\pi_m)$ only depend on their marginal probability distributions (Keeney and Raiffa 1976). Under the linear aggregation rule, the supply chain system utility function $U_s(\Delta\pi_r, \Delta\pi_m)$ is expressed as

$$U_s(\Delta\pi_r, \Delta\pi_m) = \lambda_r U_r(\Delta\pi_r, \Delta\pi_m) + \lambda_m U_m(\Delta\pi_r, \Delta\pi_m), \tag{7.6.3}$$

where λ_r and λ_m are the aggregation weights that reflect the relative negotiating powers of the retailer and the manufacturer on the bargaining table. Without loss of generality, we suppose $\lambda_r + \lambda_m = 1$. The general additive form of an utility function implies that an individual has preferences not only over his own share but also over his partner's share. However, a reasonable special case is the degenerate additive form, in which an individual only cares about his own share. This form has received extensive attention in the decision science literature (Eliashberg 1986, Raiffa 1968). In our discussion, we focus only on the degenerate form of the individual's utility function. Thus, we simplify the retailer's utility function $U_r(\Delta\pi_r, \Delta\pi_m)$ as $U_r(\Delta\pi_r)$ and the manufacturer's utility function $U_m(\Delta\pi_r, \Delta\pi_m)$ as $U_m(\Delta\pi_m)$. Also, we define the Pratt-Arrow risk aversion functions (Pratt 1964) as follows:

$$R_r(\Delta\pi_r) = -\frac{U_r''(\Delta\pi_r)}{U_r'(\Delta\pi_r)}, \quad R_m(\Delta\pi_m) = -\frac{U_m''(\Delta\pi_m)}{U_m'(\Delta\pi_m)}, \tag{7.6.4}$$

where the single prime indicates the first derivative of U_i $(i = r, m)$ and the double prime denotes the second derivative of U_i (the same are true for these notations below). $R_r(\Delta\pi_r)$ represents the retailer's risk aversion measurement for $\Delta\pi_r$ and $R_m(\Delta\pi_m)$ represents the manufacturer's for $\Delta\pi_m$. In the following sections, taking into account supply chain members' risk preferences and negotiating powers, we will discuss issues concerning implementation of the option contract by Nash's bargaining model and Eliashberg's model, and explore analytically how risk preferences and negotiation powers affect the ultimate implementation outcome. While it is difficult to obtain meaningful insights on these issues by assuming completely abstract utility functions, we consider in the following several concrete types of the utility functions that have been used extensively in the decision science literature (see, e.g., Kohli and Park 1989, Huang and Li 2001, and Huang et al. 2002). Despite this limitation, we are able to obtain some helpful insights on these issues. Furthermore, our approach to address these issues can be easily extended to any other type of the utility function.

7.6.1 The Case of Nash's Bargaining Model

Nash's bargaining model predicts that the option contract that the retailer and manufacturer both agree to implement maximizes the product of each member's utility over their own disagreement point, which, herein, is represented by the profit level under the wholesale price mechanism. We consider below several cases for illustration.

Case 1 Consider a manufacturer-retailer supply chain in which the retailer's utility function is $U_r(\Delta\pi_r) = (\Delta\pi_r)^\alpha$ and the manufacturer's utility function is $U_m(\Delta\pi_m) = (\Delta\pi_m)^\beta$, where $0 < \alpha \leq 1$ and $0 < \beta \leq 1$. By the Pratt-Arrow risk aversion functions, we know that $\frac{-U_r''(\Delta\pi_r)}{U_r'(\Delta\pi_r)} = \frac{1-\alpha}{\Delta\pi_r}$. Therefore, a smaller α

indicates a more risk-averse retailer. The Nash's bargaining solution is obtained by solving the following programming problem:

$$\begin{aligned} \mathrm{P}_{7.1}: \ & \max_{(o,e)} \ U_m(\Delta\pi_m)U_r(\Delta\pi_r) = [\pi_{om}(\lambda) - \pi_{wm}]^{\beta}[\pi_{or}(\lambda) - \pi_{wr}]^{\alpha} \\ & s.\,t. \ \ (o,e) \in N. \end{aligned} \tag{7.6.5}$$

The optimal solution for problem $\mathrm{P}_{7.1}$ is clearly the option contract in N with λ solving the following programming problem:

$$\begin{aligned} \mathrm{P}_{7.2}: \ & \max_{\lambda} U_m(\Delta\pi_m)U_r(\Delta\pi_r) = [(1-\lambda)\pi_c - \pi_{wm}]^{\beta}(\lambda\pi_c - \pi_{wr})^{\alpha} \\ & s.\,t. \ \ \lambda \in [\lambda_{min}, \lambda_{max}], \end{aligned} \tag{7.6.6}$$

where $\lambda_{min} = \frac{\pi_{wr}}{\pi_c}$ and $\lambda_{max} = \frac{\pi_c - \pi_{wm}}{\pi_c}$.

For ease of exposition, we denote

$$H(\lambda) = [(1-\lambda)\pi_c - \pi_{wm}]^{\beta}(\lambda\pi_c - \pi_{wr})^{\alpha}. \tag{7.6.7}$$

With some algebra, we obtain

$$\begin{aligned} \frac{dH(\lambda)}{d\lambda} = & \ \pi_c(\lambda\pi_c - \pi_{wr})^{\alpha-1}[(1-\lambda)\pi_c - \pi_{wm}]^{\beta-1} \\ & [\alpha((1-\lambda)\pi_c - \pi_{wm}) - \beta(\lambda\pi_c - \pi_{wr})], \end{aligned} \tag{7.6.8}$$

$$\begin{aligned} \frac{d^2H(\lambda)}{d\lambda^2} = & \ \beta(\beta-1)\pi_c^2(\lambda\pi_c - \pi_{wr})^{\alpha}[(1-\lambda)\pi_c - \pi_{wm}]^{\beta-2} \\ & -2\alpha\beta\pi_c^2(\lambda\pi_c - \pi_{wr})^{\alpha-1}[(1-\lambda)\pi_c - \pi_{wm}]^{\beta-1} \\ & +\alpha(\alpha-1)\pi_c^2(\lambda\pi_c - \pi_{wr})^{\alpha-2}[(1-\lambda)\pi_c - \pi_{wm}]^{\beta}. \end{aligned} \tag{7.6.9}$$

It is clear that (7.6.9)< 0 for all $\lambda \in (\lambda_{min}, \lambda_{max})$, which means $H(\lambda)$ is strictly concave with respect to λ on $[\lambda_{min}, \lambda_{max}]$. Therefore, by the first-order optimality condition, the optimal solution for problem $\mathrm{P}_{7.2}$ can be obtained by finding λ that satisfies (7.6.8)$=0$ over $[\lambda_{min}, \lambda_{max}]$. Again, it is easy to see that the optimal solution for problem $\mathrm{P}_{7.2}$ cannot be λ_{min} and λ_{max} (because $H(\lambda_{min}) = H(\lambda_{max}) = 0$ and $H(\lambda)$ is strictly concave with respect to λ on $[\lambda_{min}, \lambda_{max}]$). Hence, the optimal solution for problem $\mathrm{P}_{7.2}$ can be obtained by finding λ that satisfies (7.6.8)$=0$ over $(\lambda_{min}, \lambda_{max})$, which gives

$$\alpha((1-\lambda)\pi_c - \pi_{wm}) - \beta(\lambda\pi_c - \pi_{wr}) = 0. \tag{7.6.10}$$

Denote the solution of (7.6.10) as λ^*, then it follows from (7.6.10) that

$$\frac{\lambda^*\pi_c - \pi_{wr}}{(1-\lambda^*)\pi_c - \pi_{wm}} = \frac{\Delta\pi_r(\lambda^*)}{\Delta\pi_m(\lambda^*)} = \frac{\alpha}{\beta}. \tag{7.6.11}$$

Further, we obtain by solving (7.6.11) that

$$\lambda^* = \frac{\beta}{\alpha+\beta}\lambda_{min} + \frac{\alpha}{\alpha+\beta}\lambda_{max}. \tag{7.6.12}$$

Clearly, $\lambda_{min} < \lambda^* < \lambda_{max}$. Therefore, λ^* is the optimal solution of problem $P_{7.2}$. Equation (7.6.11) indicates that, based on Nash's bargaining model, the share split by a member is inversely proportional to its degree of risk aversion. By (7.6.12) and (7.5.3), we obtain that Nash's bargaining model predicts an option contract as

$$(\overline{o}, \overline{e}) = (\lambda^*(c-v), (1-\lambda^*)p + \lambda^* v) \tag{7.6.13}$$

with λ^* given by (7.6.12). Accordingly, the expected profit obtained by the retailer is

$$\pi_{or}(\lambda^*) = \lambda^*\pi_c = (\frac{\beta}{\alpha+\beta}\lambda_{min} + \frac{\alpha}{\alpha+\beta}\lambda_{max})\pi_c = \pi_{wr} + \frac{\alpha}{\alpha+\beta}\Delta\pi, \tag{7.6.14}$$

and the expected profit obtained by the manufacturer is

$$\pi_{om}(\lambda^*) = (1-\lambda^*)\pi_c = \pi_{wm} + \frac{\beta}{\alpha+\beta}\Delta\pi. \tag{7.6.15}$$

Particularly, if $\alpha = \beta$, i.e., the retailer and the manufacturer are equally risk-averse, then $\lambda^* = \frac{\lambda_{min}+\lambda_{max}}{2}$, and accordingly they will split the extra profit $\Delta\pi$ in equal proportions. Of course this is also applicable to the case where $\alpha = \beta = 1$. Therefore, being both risk-neutral can be viewed as a special case of equal risk-aversion. To summarize, we give the following proposition:

Proposition 7.6.1. *Based on Nash's bargaining model, for a manufacturer-retailer supply chain where the retailer's utility function is $U_r(\Delta\pi_r) = (\Delta\pi_r)^\alpha$ $(0 < \alpha \leq 1)$ and the manufacturer's is $U_m(\Delta\pi_m) = (\Delta\pi_m)^\beta$ $(0 < \beta \leq 1)$, the share split by a member is inversely proportional to its degree of risk-aversion, and the option contract predicted is given by*

$$(\overline{o}, \overline{e}) = (\lambda^*(c-v), (1-\lambda^*)p + \lambda^* v), \tag{7.6.16}$$

where $\lambda^* = \frac{\beta}{\alpha+\beta}\lambda_{min} + \frac{\alpha}{\alpha+\beta}\lambda_{max}$. Accordingly, the expected profit obtained by the retailer is $\pi_{or}(\lambda^*) = \pi_{wr} + \frac{\alpha}{\alpha+\beta}\Delta\pi$ and the expected profit obtained by the manufacturer is $\pi_{om}(\lambda^*) = \pi_{wm} + \frac{\beta}{\alpha+\beta}\Delta\pi$. In particular, when the retailer and the manufacturer are equally risk-averse or both are risk-neutral, they will split the extra profit $\Delta\pi$ in equal proportions.

Case 2 Consider a risk-averse retailer with a strictly concave utility function $U_r(\Delta\pi_r)$ and a risk-neutral manufacturer with a utility function $U_m(\Delta\pi_m) = \Delta\pi_m$. We suppose that $U_r(\Delta\pi_r)$ is increasing, twice differentiable, and

$U_r(0) = 0$. Then, similar to Case 1, the Nash's bargaining solution can be obtained by solving the following programming problem:

$$\begin{aligned} P_{7.3}: \ & \max_{\lambda} \ U_r(\Delta\pi_r)U_m(\Delta\pi_m) = U_r(\lambda\pi_c - \pi_{wr})[(1-\lambda)\pi_c - \pi_{wm}] \\ & s.\,t. \ \ \lambda \in [\lambda_{min}, \lambda_{max}]. \end{aligned} \tag{7.6.17}$$

For ease of exposition, we denote

$$M(\lambda) = U_r(\lambda\pi_c - \pi_{wr})[(1-\lambda)\pi_c - \pi_{wm}]. \tag{7.6.18}$$

With some algebra, we obtain from (7.6.18) that

$$\frac{dM(\lambda)}{d\lambda} = \pi_c[U_r'(\lambda\pi_c - \pi_{wr})((1-\lambda)\pi_c - \pi_{wm}) - U_r(\lambda\pi_c - \pi_{wr})], \tag{7.6.19}$$

$$\frac{d^2M(\lambda)}{d\lambda^2} = \pi_c^2[U_r''(\lambda\pi_c - \pi_{wr})((1-\lambda)\pi_c - \pi_{wm}) - 2U_r'(\lambda\pi_c - \pi_{wr})]. \tag{7.6.20}$$

Since $U_r(\Delta\pi_r)$ is increasing and strictly concave, $U_r'(\lambda\pi_c - \pi_{wr}) \geq 0$ and $U_r''(\lambda\pi_c - \pi_{wr}) < 0$ for all $\lambda \in [\lambda_{min}, \lambda_{max}]$, which lead to (7.6.20)$< 0$ for all $\lambda \in [\lambda_{min}, \lambda_{max}]$. Therefore, $M(\lambda)$ is strictly concave on $[\lambda_{min}, \lambda_{max}]$. Again, with some algebra, it follows from (7.6.19) that

$$\begin{aligned} \frac{dM(\lambda)}{d\lambda}\Big|_{\lambda=\lambda_{min}} &= \pi_c[U_r'(\lambda_{min}\pi_c - \pi_{wr})((1-\lambda_{min})\pi_c - \pi_{wm}) - U_r(\lambda_{min}\pi_c - \pi_{wr})] \\ &= \pi_c U_r'(0)\Delta\pi \geq 0, \end{aligned} \tag{7.6.21}$$

$$\begin{aligned} \frac{dM(\lambda)}{d\lambda}\Big|_{\lambda=\lambda_{max}} &= \pi_c[U_r'(\lambda_{max}\pi_c - \pi_{wr})((1-\lambda_{max})\pi_c - \pi_{wm}) - U_r(\lambda_{max}\pi_c - \pi_{wr})] \\ &= -\pi_c U_r(\Delta\pi) < 0. \end{aligned} \tag{7.6.22}$$

By (7.6.21), (7.6.22), and the continuity property, since (7.6.19) strictly decreases in λ on $[\lambda_{min}, \lambda_{max}]$, there exists a unique $\tilde{\lambda} \in [\lambda_{min}, \lambda_{max}]$ such that

$$\frac{dM(\lambda)}{d\lambda}\Big|_{\lambda=\tilde{\lambda}} = \pi_c[U_r'(\tilde{\lambda}\pi_c - \pi_{wr})((1-\tilde{\lambda})\pi_c - \pi_{wm}) - U_r(\tilde{\lambda}\pi_c - \pi_{wr})] = 0. \tag{7.6.23}$$

Obviously, $\tilde{\lambda}$ is the unique optimal solution for problem $P_{7.3}$ because $M(\lambda)$ is strictly concave on $[\lambda_{min}, \lambda_{max}]$. Let

$$G(z) = \pi_c[U_r'(z)z - U_r(z)]. \tag{7.6.24}$$

It is easy to see that $G'(z) = \pi_c U_r''(z)z < 0$ for all $z > 0$, which means $G(z)$ is strictly decreasing in z on $[0, +\infty)$. Again, since $G(0) = 0$, it follows that $G(z) < 0$ for all $z > 0$. Let $\bar{\lambda} = \frac{\lambda_{min}+\lambda_{max}}{2}$. Then we have

$$\begin{aligned} \frac{dM(\lambda)}{d\lambda}\Big|_{\lambda=\bar{\lambda}} &= \pi_c[U_r'(\bar{\lambda}\pi_c - \pi_{wr})((1-\bar{\lambda})\pi_c - \pi_{wm}) - U_r(\bar{\lambda}\pi_c - \pi_{wr})] \\ &= \pi_c[\tfrac{1}{2}\Delta\pi U_r'(\tfrac{1}{2}\Delta\pi) - U_r(\tfrac{1}{2}\Delta\pi)] = G(\tfrac{1}{2}\Delta\pi) < 0. \end{aligned} \tag{7.6.25}$$

From (7.6.20), since $\frac{dM(\lambda)}{d\lambda}$ is strictly decreasing in λ on $[\lambda_{min}, \lambda_{max}]$, it follows from (7.6.23) and (7.6.25) that $\tilde{\lambda} < \overline{\lambda}$. Therefore, for a manufacturer-retailer supply chain in Case 2, Nash's bargaining model predicts that the risk-averse retailer will obtain a smaller share of the extra profit than the risk-neutral manufacturer. Similarly, this is also applicable to the side of the manufacturer. To summarize, we give the following proposition:

Proposition 7.6.2. *Based on Nash's bargaining model, for a manufacturer-retailer supply chain consisting of a risk-averse member with a strictly concave utility $U(t)$ being increasing, twice differentiable, and $U(0) = 0$, and a risk-neutral member, the risk-averse member will obtain a smaller share of the extra profit than the risk-neutral one.*

Example 7.1. Consider a risk-averse retailer with the exponential utility function $U_r(\Delta\pi_r) = 1 - \exp(-\alpha\Delta\pi_r)$ $(\alpha > 0)$, and a risk-neutral manufacturer with the utility function $U_m(\Delta\pi_m) = \Delta\pi_m$. It is clear that $U_r(\Delta\pi_r)$ is strictly concave, increasing, twice differential, and $U_r(0) = 0$. By Proposition 7.6.2, we know that Nash's bargaining model predicts that the risk-averse retailer will obtain a smaller share of the extra profit than the risk-neutral manufacturer, and the option contract that the retailer and manufacturer both agree to implement is given by

$$(\overline{o}, \overline{e}) = (\tilde{\lambda}(c - v), (1 - \tilde{\lambda})p + \tilde{\lambda}v), \tag{7.6.26}$$

where $\tilde{\lambda}$ is the unique solution of (7.6.23).

7.6.2 *The Case of Eliashberg's Model Involving Negotiating Power*

It is worth pointing out that Nash's bargaining model does not take individual members' negotiating powers into account while predicting the outcome, which is a severe deficiency of the model because the selection of a contract clearly depends on supply chain members' negotiating powers. In order to overcome this deficiency, an alternative way is to apply the approach introduced by Eliashberg (1986). Eliashberg's model predicts an option contract that maximizes the group utility function reflecting the joint preferences of the supply chain members. By Eliashberg's model, we can incorporate supply chain members' negotiating powers into the ultimate implementation outcome, with the use of aggregation weights that measure the relative negotiating powers of the supply chain members. We consider below an example for illustration.

Example 7.2. Consider a supply chain consisting of a risk-averse retailer with the exponential utility function $U(\Delta\pi_r) = -\exp(-\alpha\Delta\pi_r)$ and a manufacturer also with the exponential utility function $U(\Delta\pi_m) = -\exp(-\beta\Delta\pi_m)$, where $\alpha, \beta > 0$. By the Pratt-Arrow risk aversion functions, it is easy to know that a larger α or β indicates a more risk-averse member. We suppose the retailer's relative power is measured by r_1 and the manufacturer's by r_2. Without loss of generality, we assume $r_1 + r_2 = 1$. Then, Eliashberg's model predicts an option contract in the core N with λ that solves the following programming problem:

$$\begin{aligned} \mathrm{P}_{7.4}: \ & \max_{\lambda} U_s(\Delta\pi_r, \Delta\pi_m) = -r_1\exp(-\alpha\Delta\pi_r) - r_2\exp(-\beta\Delta\pi_m) \\ & s.\,t. \ \ \lambda \in [\lambda_{min}, \lambda_{max}], \end{aligned} \tag{7.6.27}$$

where $U_s(\Delta\pi_r, \Delta\pi_m)$ is the supply chain system utility function over $(\Delta\pi_r, \Delta\pi_m)$ under the linear aggregation rule. Denote the optimal solution for problem $\mathrm{P}_{5.4}$ as λ^*. Similar to Case 2, we can show the strict concavity of $U_s(\Delta\pi_r, \Delta\pi_m)$ with respect to λ, by which we obtain the following results (we omit the details for the sake of conciseness):

(i) If $\frac{r_2}{r_1} \le \frac{\alpha}{\beta\exp(\alpha\Delta\pi)}$, then the optimal solution $\lambda^* = \lambda_{max}$, the corresponding option contract is given by $(\overline{o}, \overline{e}) = (\lambda_{max}(c - v), (1 - \lambda_{max})p + \lambda_{max}v)$, and the allocation of the extra profit $\Delta\pi$ between the retailer and the manufacturer is given by $(\Delta\pi_r(\lambda_{max}), \Delta\pi_m(\lambda_{max})) = (\Delta\pi, 0)$.

(ii) If $\frac{r_2}{r_1} \ge \frac{\alpha\exp(\beta\Delta\pi)}{\beta}$, then $\lambda^* = \lambda_{min}$, the corresponding option contract is given by $(\overline{o}, \overline{e}) = (\lambda_{min}(c - v), (1 - \lambda_{min})p + \lambda_{min}v)$, and the allocation of the extra profit $\Delta\pi$ is given by $(\Delta\pi_r(\lambda_{min}), \Delta\pi_m(\lambda_{min})) = (0, \Delta\pi)$.

(iii) If $\frac{r_2}{r_1} \in (\frac{\alpha}{\beta\exp(\alpha\Delta\pi)}, \frac{\alpha\exp(\beta\Delta\pi)}{\beta})$, then

$$\lambda^* = \frac{\alpha}{\alpha + \beta}\lambda_{min} + \frac{\beta}{\alpha + \beta}\lambda_{max} - \frac{\ln\frac{\beta r_2}{\alpha r_1}}{\pi_c(\alpha + \beta)}, \tag{7.6.28}$$

the corresponding option contract is given by $(\overline{o}, \overline{e}) = (\lambda^*(c - v), (1 - \lambda^*)p + \lambda^* v)$, where λ^* is given by (7.6.28), and the allocation of the extra profit $\Delta\pi$ is given by

$$\Delta\pi_r(\lambda^*) = \frac{\beta}{\alpha + \beta}\Delta\pi - \frac{\ln\frac{\beta r_2}{\alpha r_1}}{\alpha + \beta}, \qquad \Delta\pi_m(\lambda^*) = \frac{\alpha}{\alpha + \beta}\Delta\pi + \frac{\ln\frac{\beta r_2}{\alpha r_1}}{\alpha + \beta}. \tag{7.6.29}$$

Hence, based on Eliashberg's model, for such a supply chain, the retailer will obtain a share $\frac{\beta}{\alpha+\beta}$ from the extra profit $\Delta\pi$ and the manufacturer will obtain a share $\frac{\alpha}{\alpha+\beta}$. A compensation fee between the retailer and the manufacturer is $\frac{\ln\frac{\beta r_2}{\alpha r_1}}{\alpha+\beta}$, which represents a fee paid by the retailer to the manufacturer if $\frac{r_2}{r_1} \ge \frac{\alpha}{\beta}$; otherwise from the manufacturer to the retailer. Clearly, we see from (7.6.29) that the proportions shared by the retailer and the manufacturer do not depend on their relative power measurements r_1 and r_2 but only depend on their risk aversion measurements α and β. The more risk-averse a member is, the less share it will obtain from the extra profit $\Delta\pi$. As for the compensation fee, we observe that when $\frac{r_2}{r_1} \geq \frac{\alpha}{\beta}$, an increase in r_2 or a decrease in r_1 means an increasing compensation fee from the retailer to the manufacturer. Particularly, when r_2 increases or r_1 decreases to the point where $\frac{r_2}{r_1} \geq \frac{\alpha\exp(\beta\Delta\pi)}{\beta}$, the manufacturer will receive a compensation fee from the retailer that is just equal to $\frac{\beta}{\alpha+\beta}\Delta\pi$. In other words, with an increase in the relative power of the manufacturer with respect to the retailer, it will receive a higher compensation fee from the retailer. When the manufacturer's relative power is high enough with respect to the retailer (e.g., $\frac{r_2}{r_1} > \frac{\alpha\exp(\beta\Delta\pi)}{\beta}$), the result of coordination will be that the manufacturer captures all the extra profit, whereas the retailer receives nothing from coordination. A similar analysis is applicable to the case where $\frac{r_2}{r_1} < \frac{\alpha}{\beta}$. Besides, it is worth noting that when the retailer and the manufacturer are equally risk-averse, i.e., $\alpha = \beta$, they will split the extra profit in equal proportions, and their relative power measurements r_1 and r_2 will be the only factors that decide whether a member receives a positive or negative compensation fee from the other one.

7.7 Conclusion

In view of the current industry environment in which cooperative relations are increasingly becoming prevalent in supply chains, we took a cooperation approach in this chapter to address the coordination issues for manufacturer-retailer supply chains using option contracts. Given that the wholesale price mechanism is the prevalent form used in manufacturer-retailer supply chains in practice, we took the profit level under the wholesale price mechanism as the benchmark against which we compared our developed option contracts. Our study demonstrates that, compared with the wholesale price mechanism, any option contract in the core can coordinate the manufacturer-retailer supply chain with Pareto-improvement. In addition, we also explored analytically the issues concerning implementation of the coordinating option contract form, taking into account supply chain members' risk preferences and negotiating powers. Our study demonstrates that under the option mechanism: (i) an individual's risk preference plays a significant role in the coordination outcome. Specifically, when the retailer and manufacturer are both risk-averse, the more risk-averse a member is, the smaller share of the extra profit it will obtain. When the retailer and the manufacturer are equally risk-averse or both are risk-neutral, they will split the extra profit in equal proportions. (ii) In addition to risk preferences, an individual's negotiating power also has significant effects on the ultimate outcome. The higher a member's relative negotiating power is, the higher the compensation fee it will receive from the other side, and vice versa. Our findings provide a relatively comprehensive insight on how option contracts can be used to coordinate a manufacturer-retailer supply chain.

It is worth noting that for any option contract in the core N, both the manufacturer and the retailer operate under *voluntary compliance*, which means that the manufacturer's optimal production quantity coincides with the retailer's optimal reserved quantity, given the option contract terms (which can be seen from Theorem 7.5.1). Clearly, to a large extent, this property improves the robustness of the option contracts developed in this chapter. Besides, it should also be noted that the option contracts in the core N are "distribution-free", which implies that they can be utilized to coordinate the supply chain without knowing the demand distribution of the retailer. This property renders their implementation easier in practice. In addition, as a remark, we should point out that in our model we neglected the penalty cost for demand that is not satisfied. In fact, we have checked that such an additional consideration has no effect on the results developed in this chapter, other than complicating the notation. In addition, we wish to point out that the buyback contract and the option contract differ fundamentally in their impacts on the behaviors of supply chain members and on the cash flow of the supply chain. For example, in contrast to the buyback contract under which the retailer often places the order after the commencement of the production season, the option contract prompts the retailer to order before the production season begins. Hence, under the option contract, the retailer can take advantage of order quantity flexibility to better accommodate changes in demand. On the other hand, the manufacturer enjoys the retailer's early commitment and can have better capacity and materials planning. However, it is not this case for buyback contracts. As a natural extension of this work, future studies should consider the issues concerning bidirectional option by which the retailer can freely adjust the initial order quantity both upwards and downwards. Some research attempts for this issue have been made by Milner and Rosenblatt (2002), Wang and Tsao (2006), and Zhao et al. (2013a).

Appendix: Proofs of the Main Results

Proof of Proposition 7.4.1. From (7.4.1), the retailer's problem with the wholesale price mechanism is to solve

$$\mathrm{P}_{7\mathrm{A}.1}: \quad \max_{Q_{wr}\geq 0} E\Pi_{wr}(Q_{wr}) = E[p\min\{Q_{wr}, X\} - wQ_{wr} + v\max\{Q_{wr} - X, 0\}]. \tag{7.7.1}$$

With some algebra, it is obtained from (7.7.1) that

$$\begin{aligned} E\Pi_{wr}(Q_{wr}) &= E[p\min\{Q_{wr}, X\} - wQ_{wr} + v\max\{Q_{wr} - X, 0\}] \\ &= p\int_0^{Q_{wr}} xdF(x) + p\int_{Q_{wr}}^{+\infty} Q_{wr}dF(x) - wQ_{wr} + v\int_0^{Q_{wr}} (Q_{wr} - x)dF(x) \\ &= (p - w)Q_{wr} - (p - v)\int_0^{Q_{wr}} F(x)dx. \end{aligned} \tag{7.7.2}$$

Further, from (7.7.2)

$$\frac{dE\Pi_{wr}(Q_{wr})}{dQ_{wr}} = (p-w) - (p-v)F(Q_{wr}), \tag{7.7.3}$$

$$\frac{d^2E\Pi_{wr}(Q_{wr})}{dQ_{wr}^2} = -(p-v)f(Q_{wr}) < 0. \tag{7.7.4}$$

It follows from (7.7.4) that $E\Pi_{wr}(Q_{wr})$ is strictly concave in Q_{wr}, so the first-order optimality condition works. We therefore obtain from (7.7.3) the retailer's optimal order quantity as $\overline{Q}_{wr} = F^{-1}(\frac{p-w}{p-v})$. Substituting $\overline{Q}_{wr}$ into (7.7.2), we see that the retailer's maximum expected profit under the wholesale price mechanism, denoted by π_{wr}, is given by

$$\pi_{wr} = (p-w)\overline{Q}_{wr} - (p-v)\int_0^{\overline{Q}_{wr}} F(x)dx. \tag{7.7.5}$$

Based on the assumptions in our model, since the manufacturer uses the "make-to-order" production policy, which means that it is a quantity-taker, the manufacturer's expected profit under the wholesale price mechanism, denoted by π_{wm}, is given by $\pi_{wm} = (w-c)\overline{Q}_{wr}$ with $\overline{Q}_{wr} = F^{-1}(\frac{p-w}{p-v})$. Thus, the proof is completed. □

Proof of Proposition 7.4.2. (i) From (7.4.4), we see that the retailer's problem with the option mechanism (o, e) is to solve

$$\text{P}_{7\text{A}.2}: \max_{Q_{or}\geq 0} E\Pi_{or}(Q_{or}) = E[(p-e)\min\{Q_{or}, X\} - oQ_{or}]. \tag{7.7.6}$$

With some algebra, we obtain

$$\begin{aligned} E\Pi_{or}(Q_{or}) &= (p-e)\int_0^{Q_{or}} xdF(x) + (p-e)\int_{Q_{or}}^{+\infty} Q_{or}dF(x) - oQ_{or} \\ &= (p-o-e)Q_{or} - (p-e)\int_0^{Q_{or}} F(x)dx. \end{aligned} \tag{7.7.7}$$

From (7.7.7), we obtain

$$\frac{dE\Pi_{or}(Q_{or})}{dQ_{or}} = (p-o-e) - (p-e)F(Q_{or}), \tag{7.7.8}$$

$$\frac{d^2E\Pi_{or}(Q_{or})}{dQ_{or}^2} = -(p-e)f(Q_{or}) < 0. \tag{7.7.9}$$

It follows from (7.7.9) that $E\Pi_{or}(Q_{or})$ is strictly concave in Q_{or}, so the first-order optimality condition works. We therefore obtain from (7.7.8) that the retailer's optimal reserve quantity is $\overline{Q}_{or} = F^{-1}(\frac{p-o-e}{p-e})$. Similarly, from (7.4.5), we know the manufacturer's optimal production quantity with the option contract mechanism is to solve

$$\text{P}_{7\text{A}.3}: \max_{Q_{om}\geq 0} E\Pi_{om}(Q_{om}) = E[oQ_{om} + e\min\{Q_{om}, X\} + v\max\{Q_{om}-X, 0\} - cQ_{om}]. \tag{7.7.10}$$

With some algebra, we obtain

$$E\Pi_{om}(Q_{om}) = (e - c + o)Q_{om} - (e - v)\int_0^{Q_{om}} F(x)dx. \tag{7.7.11}$$

In a similar way, we can show the strict concavity of $E\Pi_{om}(Q_{om})$ in Q_{om}. Therefore, by the first-order optimality condition, the manufacturer's optimal production quantity is obtained as $\overline{Q}_{om} = F^{-1}(\frac{e+o-c}{e-v})$.

(ii) Given $o + e$, from (7.7.7) and (7.7.11), it is clear that $E\Pi_{or}(Q_{or})$ is decreasing in o or increasing in e, and $E\Pi_{om}(Q_{om})$ is increasing in o or decreasing in e.

(iii) Since the retailer's optimal reserve quantity is $\overline{Q}_{or} = F^{-1}(\frac{p-o-e}{p-e})$ and the manufacturer's optimal production quantity is $\overline{Q}_{om} = F^{-1}(\frac{e+o-c}{e-v})$, only if $F^{-1}(\frac{p-o-e}{p-e}) = F^{-1}(\frac{e+o-c}{e-v})$, i.e., $e = p - \frac{p-v}{c-v}o$, will the retailer's optimal reserve quantity be just consistent with the manufacturer's optimal production quantity. Again, in view of the assumptions on the model parameters, we require that $o < c - v$. Thus, the proof is completed. □

Proof of Proposition 7.4.3. $\overline{Q}_{wr} < \overline{Q}_{or}$ iff

$$F^{-1}(\frac{p-w}{p-v}) < F^{-1}(\frac{p-o-e}{p-e}), \tag{7.7.12}$$

which is equivalent to $o < \frac{(p-e)(w-v)}{p-v}$, thus establishing (i). So is (ii) established in a similar way. □

Proof of the relations $\overline{Q}_s > \overline{Q}_{wr}$ and $\overline{Q}_s > \overline{Q}_{wm}$. It is obvious that $\overline{Q}_s > \overline{Q}_{wr}$. Again, since

$$\frac{p-c}{p-v} - \frac{w-c}{w-v} = \frac{(p-w)(c-v)}{(p-v)(w-v)} > 0, \tag{7.7.13}$$

and $F(x)$ is strictly increasing on $[0, +\infty)$, it follows that

$$\overline{Q}_s = F^{-1}(\frac{p-c}{p-v}) > F^{-1}(\frac{w-c}{w-v}) = \overline{Q}_{wm}. \tag{7.7.14}$$

Hence, the proof is completed. □

Proof of Theorem 7.5.1. By (7.7.7), we know that under the option contract (o, e), the retailer's expected profit function is given by

$$E\Pi_{or}(Q_{or}) = (p - o - e)Q_{or} - (p - e)\int_0^{Q_{or}} F(x)dx, \tag{7.7.15}$$

where Q_{or} denotes the retailer's reservation quantity with the option contract. By (7.5.1), the supply chain system's profit function is given by

$$E\Pi_s(Q_s) = (p - c)Q_s - (p - v)\int_0^{Q_s} F(x)dx. \tag{7.7.16}$$

where Q_s denotes the production quantity of the supply chain system. Let

$$\begin{cases} p - o - e = \lambda(p - c), \\ p - e = \lambda(p - v). \end{cases} \tag{7.7.17}$$

By the assumption that $e > v$, we require $\lambda \in [0, 1)$. Substituting (7.7.17) into (7.7.15), we obtain

$$E\Pi_{or}(Q_{or}) = \lambda[(p - c)Q_{or} - (p - v)\int_0^{Q_{or}} F(x)dx]. \tag{7.7.18}$$

Comparing (7.7.18) with (7.7.16), we know that any option contract (o, e) satisfying (7.7.17) will push the retailer to reserve as much as $\overline{Q}_s$, i.e., $\overline{Q}_{or} = \overline{Q}_s$, where $\overline{Q}_s = F^{-1}(\frac{p-c}{p-v})$ corresponds to the system-wide optimal production quantity for the supply chain. Again, since the manufacture adopts the "make-to-order" production policy, any option contract (o, e) satisfying (7.7.17) will make the supply chain system achieve the maximum expected profit for the channel. Taking o and e as variables and solving (7.7.17), we obtain the following option contract set, denoted as M,

$$M = \{(o, e) : o = \lambda(c - v), e = (1 - \lambda)p + \lambda v, \lambda \in [0, 1)\}. \tag{7.7.19}$$

Substituting $o = \lambda(c - v)$ and $e = (1 - \lambda)p + \lambda v$ into $\overline{Q}_{om} = F^{-1}(\frac{e+o-c}{e-v})$ leads to $\overline{Q}_{om} = F^{-1}(\frac{p-c}{p-v}) = \overline{Q}_s$. Thus, with any option contract (o, e) in M, we have $\overline{Q}_{or} = \overline{Q}_{om} = \overline{Q}_s$. Again, substituting $o = \lambda(c-v), e = (1-\lambda)p+\lambda v$ into (7.7.11), together with (7.7.18), we see that under the option contract (o, e) in the set M associated with the parameter λ, the retailer's expected profit, denoted as $\pi_{or}(\lambda)$, is given by

$$\begin{aligned} \pi_{or}(\lambda) &= E\Pi_{or}(\overline{Q}_{or}) = \lambda[(p - c)\overline{Q}_{or} - (p - v)\int_0^{\overline{Q}_{or}} F(x)dx] \\ &= \lambda[(p - c)\overline{Q}_s - (p - v)\int_0^{\overline{Q}_s} F(x)dx] \\ &= \lambda E\Pi_s(\overline{Q}_s) = \lambda\pi_c, \end{aligned} \tag{7.7.20}$$

where $\pi_c = E\Pi_s(\overline{Q}_s)$ denotes the system-wide optimal profit, and the manufacturer's expected profit, denoted as $\pi_{om}(\lambda)$, is given by

$$\begin{aligned} \pi_{om}(\lambda) &= E\Pi_{om}(\overline{Q}_{om}) = (e - c + o)\overline{Q}_{om} - (e - v)\int_0^{\overline{Q}_{om}} F(x)dx \\ &= (1 - \lambda)[(p - c)\overline{Q}_s - (p - v)\int_0^{\overline{Q}_s} F(x)dx] \\ &= (1 - \lambda)E\Pi_s(\overline{Q}_s) = (1 - \lambda)\pi_c. \end{aligned} \tag{7.7.21}$$

By Proposition 7.4.1, we obtain that with the wholesale price mechanism, the entire supply chain's total profit is

$$\pi_{wr} + \pi_{wm} = (p - c)\overline{Q}_{wr} - (p - v)\int_0^{\overline{Q}_{wr}} F(x)dx. \tag{7.7.22}$$

Hence, with any option contract (o, e) in M, the increased profit of the supply chain system is

$$\begin{aligned}\Delta\pi &= \pi_c - (\pi_{wr} + \pi_{wm}) \\ &= (p-c)(\overline{Q}_s - \overline{Q}_{wr}) - (p-v)\int_{\overline{Q}_{wr}}^{\overline{Q}_s} F(x)dx.\end{aligned} \tag{7.7.23}$$

To summarize, Theorem 7.5.1 follows. □

Proof of the inequalities $\frac{(c-v)(w-e)}{w-v} < o < \frac{(p-e)(w-v)}{p-v}$. Let (o, e) be any option contract in the set M, i.e.,

$$o = \lambda(c-v) \text{ and } e = (1-\lambda)p + \lambda v \text{ for some } \lambda \in [0, 1). \tag{7.7.24}$$

Substituting $e = (1-\lambda)p + \lambda v$ into $\frac{(p-e)(w-v)}{p-v}$ and $\frac{(c-v)(w-e)}{w-v}$ respectively, we obtain

$$\frac{(p-e)(w-v)}{p-v} = \lambda(w-v) > \lambda(c-v) = o, \tag{7.7.25}$$

$$\frac{(c-v)(w-e)}{w-v} = \frac{(c-v)[\lambda(p-v)+w-p]}{w-v}, \tag{7.7.26}$$

where the inequality in (7.7.25) follows by $w > c$. Again, with some algebra, we obtain

$$\lambda(w-v) - \lambda(p-v) - (w-p) = (1-\lambda)(p-w) > 0. \tag{7.7.27}$$

Hence

$$\lambda(w-v) > \lambda(p-v) + (w-p), \tag{7.7.28}$$

which, together with (7.7.26), leads to

$$\frac{(c-v)(w-e)}{w-v} < \frac{\lambda(c-v)(w-v)}{w-v} = \lambda(c-v) = o. \tag{7.7.29}$$

Thus, the desired result follows. □

Chapter 8
Conclusions and Future Research Directions

In this book, systematical research has been presented for several key issues of the supply chain contract. To conclude, Chap. 2 analytically examined the effects of demand uncertainty on the applicability of buyback contracts. This research has demonstrated that DUL can be an important factor affecting the applicability of supply chain contracts. Chapter 3 conducted a mean-risk analysis for wholesale price contracts, taking into account contracting value risk and the risk preferences attached to it. This research has provided a new perspective of looking at the performance of a supply chain contract. Chapter 4 examined franchise fee contracts in the product service system with demand information asymmetry. This research has conducted a comprehensive study for the problem of how to provide effective product service system (PSS) with franchise fee contracts in the service-oriented manufacturing supply chain with demand information asymmetry. Chapter 5 extended the concept of single directional option to develop the bidirectional option contract model. With a general demand distribution, the optimal contracting decisions with bidirectional option have been formulated. Analytical examination of the feedback effects from introducing bidirectional option in the supply chain has been presented. Distribution-free bidirectional option contracts have also been developed for supply chain coordination. Chapter 6 was devoted to addressing the supply chain options pricing issue. In this research, a value-based pricing scheme has been developed for supply chain options with two model scenarios, i.e., a single retailer and multiple retailers. Finally, taking into account supply chain members' risk preferences and negotiating powers, a cooperative game theory approach was taken in Chap. 7 to explore the supply chain coordination issue with option contracts. This research has developed a theoretical modeling framework for the selection/implementation issue of supply chain contracts.

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Despite significant progress has been made for the research of supply chain contracts with this book, there are still some issues (topics) that remain further explorations in the future:

(1) An acknowledged limitation for the theoretical modeling research of supply chain contracts is that the results and findings derived are most based on the model setup, typically the demand setting, and generally cannot be generalized to the general cases. In order to improve the applicability of the theoretical modeling research of supply chain contracts, it is worthwhile in the future to explore some structural properties that are to a good extent independent of the specific model settings or applicable to a relatively wide range of business settings for supply chain contracts. Recalling that Chap. 2 of this book explored the buyback contracts with a price-dependent downward-sloping demand curve subject to uncertainty that is characterized by a binary distribution. In this research an interesting result has been found that with buyback, the supplier need not change the equilibrium wholesale price associated with the corresponding deterministic demand case, and only needs to adjust the buyback price in response to the demand uncertainty of different levels. A particularly interesting research along with this result is to explore whether it still holds for the cases with more general demand distribution. With such a further research, some structural properties that are independent of the specific demand settings may be developed for the buyback contracts.

(2) Even though it is very common to explore supply chain contracts with a simple two-echelon supply chain, the more realistic and essential research is to examine supply chain contracts in a more complex supply chain network. Hence, it is interesting to extend the research with a simple two-echelon supply chain to explore supply chain contracts in a network comprising multiple suppliers and multiple retailers. One interesting issue along with this extension is to introduce vertical or/and horizontal competition in the model, thereby incorporating the effects of competition in the analysis and design of supply chain contracts. In addition, in view of the fact that individuals behave differently in anticipation of a long-term relationship as compared with that in anticipation of a short-term relationship, supply chain members may behave differently when contracting with repetition. This may lead to different contractual structure in optimality as compared with those under the premise of one-shot interaction. As a result, research that takes multi-stage contract models into account is worth substantial attention in the future.

(3) As reviewed in Chap. 1, the concept of supply chain coordination becomes different once the supply chain members are allowed to have different risk preference structures. In fact, decision made by an individual is usually a result of interaction of multiple incentives. Hence, it is not enough to examine supply chain contracts by simply assuming risk-neutrality. Even though so far some papers have made some successful attempts towards addressing the issues of such type, more research taking into account more complex risk preference structures should be pursued in the future. For example, in Chap. 3, with the

wholesale price contract under consideration, it has been demonstrated that the contracting value risk and the risk attitude attached to it can be two critical factors in the analysis or design of supply chain contracts. As a result, more research that takes these two factors into account should be pursued in the future for a wider range of contract types.

(4) A value-based pricing approach has been developed in Chap. 6 for the supply chain options. In this research, the option value is calculated based on the concept of expectation, which implies that only risk-neutrality is considered. However, supply chain members tend to have different risk preferences and obviously their risk preference structures can have substantial effects in determining the option value, which in turn affects the option pricing. Actually, in addition to risk preferences, there are more critical factors that contribute to the option value, such as the uncertainty level inherent in market demand, the quality of demand information updating, as well as the trend of spot price of the underlying product. Hence, further research of the supply chain options pricing that incorporates all these factors is worth pursuing in the future.

(5) Despite the availability of the increasingly sophisticated information sharing systems, information asymmetry remains an essential feature of real relationships in the supply chain. Therefore, as the research of Chap. 4, more research of supply chain contracts that assumes information asymmetry in the model should be pursued in the future. One of the difficulties to incorporate information asymmetry in the supply chain contract research is that it requires a qualitatively more complex information structure, e.g., it generally requires assuming the retailer's briefs about the supplier's briefs, and so on, which makes the theoretical analysis substantially more difficult and often intractable.

(6) In industry it can be observed that at the same business setting, some forms of contracts are exploited more often than another, or the same form of contracts is utilized more often in one specific business setting than in another one. For example, it can be observed that the buyback and revenue-sharing contracts generally are adopted more commonly in some fashion industries, which is characterized by a relatively high demand uncertainty risk, such as apparel fashion, electronics, and cosmetics, as compared with the other industries. Why is this the case? Is this because the two types of contracts are more effective for supply chains of these industries to manage the high demand uncertainty risk? if yes, why? Even though the research of Chap. 2 in this book can provide some valuable insights into these issues, there is an absence of a convincing systematical research about them, which constitutes another worthy topic for the future research.

(7) In addition to theoretical examination, empirical (experimental) research needs to be conducted in the future so as to test or complement the theoretically developed results for supply chain contracts. The reasons why empirical research is necessary for supply chain contracts are that some problems in supply chain contract research are theoretically intractable while an empirical research approach can be effective in tackling them; or that the existing theoretical results remain further tests by empiricism, and probably with the

empirical tests some more or different managerial insights are found. For example, even though theoretical research has shown that buyback contracts and revenue-sharing contracts are mathematically equivalent with the basic newsvendor model setting (Cachon and Lariviere 2005), an empirical research with laboratory data in Katok and Wu (2009) finds that in experiment they both are not equivalent. Also, recalling that in Chap. 7 of this book, a theoretical modeling research has been developed for the selection issues of coordinating option contracts. It may be more interesting for this chapter to further provide an empirical analysis using data from relevant industries or laboratory to test whether the option contract selected and the corresponding profit allocation are consistent with the theoretical predictions.

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