DESIGN OF CONCRETE STRUCTURES

Design Aids for Eurocode 2

BETONVERENIGING
THE CONCRETE SOCIETY
DEUTSCHER BETON-VEREIN E.V.

Design aids for Eurocode 2, part 1 (ENV 1992-1-1)

E & FN SPON
An Imprint of Chapman & Hall

Also available as a printed book
see title verso for ISBN details

Design aids for EC2

JOIN US ON THE INTERNET VIA WWW, GOPHER, FTP OR EMAIL:

www:	http://www.thomson.com	A service of I(T)P
GOPHER:	gopher.thomson.com	
FTP:	ftp.thomson.com	
EMAIL:	findit@kiosk.thomson.com	

Design aids for EC2

Design of concrete structures

Design aids for ENV 1992–1–1 Eurocode 2, part 1

Betonvereniging
The Concrete Society

Deutscher Beton-Verein

E & FN SPON

An Imprint of Chapman & Hall

London · Weinheim · New York · Tokyo · Melbourne · Madras

Published by E & FN Spon, an imprint of
Chapman & Hall, 2–6 Boundary Row, London SE1 8HN, UK

Chapman & Hall, 2–6 Boundary Row, London SE1 8HN, UK

Chapman & Hall GmbH, Pappelallee 3, 69469 Weinheim, Germany

Chapman & Hall USA, 115 Fifth Avenue, New York, NY 10003, USA

Chapman & Hall Japan, ITP-Japan, Kyowa Building, 3F, 2–2–1 Hirakawacho,
Chiyoda-ku, Tokyo 102, Japan

DA Book (Aust.) Pty Ltd, 648 Whitehorse Road, Mitcham 3132, Victoria, Australia

Chapman & Hall India, R.Seshadri, 32 Second Main Road, CIT East, Madras 600 035, India

First edition 1997

This edition published in the Taylor & Francis e-Library, 2005.

"To purchase your own copy of this or any of Taylor & Francis or Routledge's
collection of thousands of eBooks please go to www.eBookstore.tandf.co.uk."

© 1997 Betonvereniging, The Concrete Society and Deutscher Beton-Verein

ISBN 0-203-47639-5 Master e-book ISBN

ISBN 0-203-78463-4 (Adobe eReader Format)
ISBN 0 419 21190 X (Print Edition)

Apart from any fair dealing for the purposes of research or private study, or criticism or review, as permitted under the UK Copyright Designs and Patents Act, 1988, this publication may not be reproduced, stored, or transmitted, in any form or by any means, without the prior permission in writing of the publishers, or in the case of reprographic reproduction only in accordance with the terms of the licences issued by the Copyright Licensing Agency in the UK, or in accordance with the terms of licences issued by the appropriate Reproduction Rights Organization outside the UK. Enquiries concerning reproduction outside the terms stated here should be sent to the publishers at the London address printed on this page.

The publisher and the authors make no representation, express or implied, with regard to the accuracy of the information contained in this book and cannot accept any legal responsibility or liability for any errors or omissions that may be made.

A catalogue record for this book is available from the British Library

Publisher's Note This book has been prepared from camera ready copy provided by Betonvereniging, The Concrete Society and Deutscher Beton-Verein E.V.

Contents

Preface
The European concrete standards in practice

The German, UK and Netherlands Concrete Societies are working together on a SPRINT project for the development of supporting tools for use with the European Structural Concrete Code. The project is in three parts essentially covering:

1. An investigation of what tools the industry needs and prefers to enable it to work with the new code.
2. The development of preferred tools.
3. Publication and dissemination of the tools developed and consideration of the possible development of further aids to the use of the code.

In the first phase, the societies questioned a wide range of practitioners about their needs and preferences for design tools. It was found that, although there is considerable interest in developing information systems through computer processes, the immediate need and preference was for a traditional "hard copy" Technical Document containing information, guidance and examples of the use of the Code.

In response, the societies concentrated efforts in the second phase into the production of such a document, which this now is. During the development of the material, an important meeting was held in Amsterdam in October 1994 when the societies were able to present draft material for examination and comment and to seek views on the direction of their work. Discussion at this meeting confirmed the earlier analysis of the industry's immediate needs and interest in the development of other information systems for the future. Comments made on the draft at and after the meeting were subsequently considered by the societies and, where appropriate, material was modified or added.

The publication of this document marks the completion of the second phase and forms part of the final phase which will concentrate on the dissemination of the information in this document. This last phase will also involve a further examination of other methods to highlight the material that has been prepared and to consider how other tools and systems may be developed to aid industry.

Finally, it must be stressed that this document is not an alternative to the European Structural Concrete Code. It is an aid to use in conjunction with the Code to help designers in their work.

March 1996

1 General information

Dr.-Ing. H.-U.Litzner, Wiesbaden: Chairman of CEN/TC250/SC2

1.1 Construction products directive and European harmonized standards for concrete structures

The European construction market was officially established in January 1993. This means that in this market, as in other areas of the economy, goods, services, people and capital are able to move freely within the European Union (EU). An important instrument in this connection is the "Construction products directive" [1], adopted by the EU-Commission in December 1988. This directive sets out the conditions under which a construction product (e.g. cement, ready-mixed concrete, reinforcement, precast element) can be imported and exported and used for its intended purposes without impediment in EU countries. This directive has been integrated into the national legislation of most EU Member States.

"Technical specifications"—i.e. harmonized European standards, or, where these are lacking, European technical approvals —are necessary for the practical application of this directive. Figure 1.1 shows the European code of practice system for concrete structures that is currently being elaborated at different levels on the basis of the Directive. This standards system will quantify requirements for concept, design, detailing and execution of structures.

According to Article 6 of the directive, a construction product may move freely within the EU provided it meets certain basic requirements. These criteria, denoted in the Directive as "Essential requirements", primarily relate, however, to the structure into which the construction product is to be incorporated. The "Essential requirements" concern:

- mechanical resistance and stability
- safety in case of fire
- hygiene, health and the environment
- safety in use
- protection against noise
- energy economy and heat retention.

This establishes the framework for further consideration.

The "Essential requirements" are only qualitatively described in the directive text. Further European documents are needed for practical application. These include the so-called "Interpretative documents", in which the essential requirements are defined, the previously mentioned "Technical specifications" (European harmonized standards and European guidelines for technical approval), as well as regulations for the positive assessment of the conformity of a construction product ("Certification").

1.2 Future European code of practice for concrete structures

On the basis of provisional mandates of the EU, a code of practice for concrete structures is being established by the European Committee for Standardization (abbreviated CEN) which, in the longer term, will replace national standards. Its structure is comparable to that of existing national standards systems (Figure 1.1).

It comprises:

- a safety concept relevant to any type of construction (ENV 1991–1);
- Eurocode 1 concerning actions on structures (including traffic loads in ENV 1991–3);
- codes of practice for design and execution of structures;
- construction material standards (concrete, reinforcement, prestressing steel);
- standards for the testing of construction materials (ISO or CEN standards).

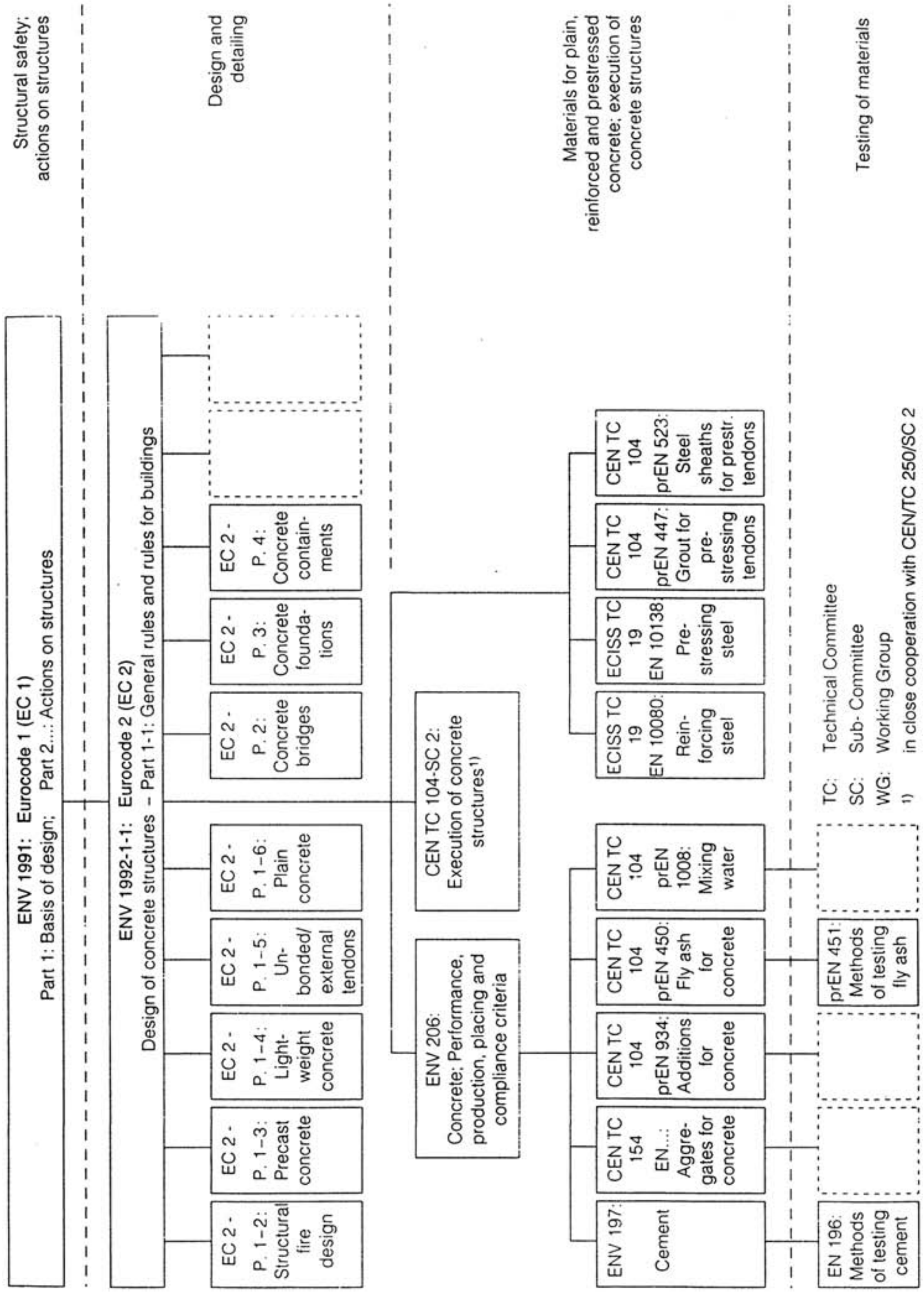


Figure 1.1 Structure of the future European harmonized standards for concrete.

From this it becomes clear that the future European standards for concrete structures are aimed at the "essential requirements", particularly at the mechanical resistance and stability, structural fire design and safety in use, whereby the initially mentioned requirement also incorporates criteria regarding durability. This objective is also expressed in the foreword to Eurocode 2 [2] which states, among other things, the following:

"0.1 Objectives of the Eurocodes

(1) The Structural Eurocodes comprise a group of standards for the structural and geotechnical design of buildings and civil engineering works.
(2) They are intended to serve as reference documents for the following purposes:

(a) As a means to prove compliance of building and civil engineering works with the essential requirements of the Construction Products Directive (CPD)
(b) As a framework for drawing up harmonized technical specifications for construction products.

(3) They cover execution control only to the extent that is necessary to indicate the quality of the construction products, and the standard of the workmanship, needed to comply with the assumptions of the design rules.
(4) Until the necessary set of harmonized technical specifications for products and for methods of testing their performance is available, some of the Structural Eurocodes cover some of these aspects in informative annexes."

"0.2 Background to the Eurocode programme

(1) The Commission of the European Communities (CEC) initiated the work of establishing a set of harmonized technical rules for the design of building and civil engineering works which would initially serve as an alternative to the different rules in force in the various Member States and would ultimately replace them. These technical rules became known as the 'Structural Eurocodes'.
(2) In 1990, after consulting their respective Member States, the CEC transferred work of further development, issue and updates of the Structural Eurocodes to CEN and the EFTA Secretariat agreed to support the CEN work.
(3) CEN Technical Committee CEN/TC250 is responsible for all Structural Eurocodes."

Paragraph 0.1 (2)(b) quoted above applies in particular to precast structural elements for which the CEN Technical Committee (TC) 229 is currently elaborating product standards in accordance with the 1988 Directive. These products include, for example, prestressed concrete hollow slabs and factory produced concrete masts and piles. As far as possible, the design concept is based on Eurocode 2 [2].

1.3
Safety concept relevant to any type of construction material

The outlines of the safety concept for any type of construction material in the Eurocodes are defined in the interpretative document "Mechanical resistance and stability". [3] Based on this, ENV 1991–1 [4] explains how the satisfaction of these "Essential requirements" in accordance with the Construction products directive [1] may be verified and provides as models the ultimate limit states concept as well as serviceability limit states.

The ultimate limit states concern the danger potential associated with collapse of the structure or other forms of structural failure. Among other criteria, these include the loss of global equilibrium (transformation into a mechanism, sliding, overturning), the failure or a state before failure of parts of the structure (failure of cross-section, states of deformation, exceeding the bearing capacity), loss of stability (buckling, lateral buckling of slender beams, local buckling of plates) as well as material fatigue.

These ultimate limit states are modelled mathematically in EC2. In its chapter 4.3, the ultimate limit states are distinguished as:

4.3.1 ultimate limit states for bending and longitudinal force;
4.3.2 ultimate limit states for shear;
4.3.3 ultimate limit states for torsion;
4.3.4 ultimate limit states of punching;
4.3.5 ultimate limit states induced by structural deformation (buckling).

The serviceability limit states in EC2 correspond to a structural state beyond which the specified service requirements are no longer met. The corresponding models in its chapter 4.4 are:

4.4.2 limit states of cracking;
4.4.3 limit states of deformation;

as well as excessive stresses in the concrete, reinforcing or prestressing steel under serviceability conditions, which likewise can adversely affect proper functioning of a member (section 4.4.1).

1.4
Eurocode 2 for the design and execution of concrete structures

1.4.1
General

Eurocode 2 “Design of concrete structures; Part 1–1: General rules and rules for buildings” was issued as European Prestandard ENV 1992–1–1 [2] by the European Committee for Standardisation (CEN). There is no obligation to implement this Prestandard into national standard systems or to withdraw conflicting national standards.

Consequently, the first parts of the future European system of harmonized standards for concrete structures (Figure 1.1) are available in the form of ENV 1992–1–1 (EC2) and the Prestandard ENV 206 for concrete technology. The gaps, which are due to the current lack of further ENV standards, e.g. covering constituent materials for concrete, reinforcement, prestressing steel, quality control, are covered by National Application Documents (NAD). This is to enable the provisional application of the new European standards as recommended by the EU. Approval (“notification”) as a technical building regulation (guideline) by the relevant supervisory authorities has been carried out in most Member States.

1.4.2
Contents of Eurocode 2: principles and application rules: indicative numerical values

The design concept of EC2 does not differentiate between prestressed and non-prestressed structural members. Likewise, no distinction is made between full, limited or partial prestressing.

EC2 is divided into “Principles” and “Application rules”. “Principles” comprise verbally defined general requirements (e.g. regarding structural safety), to which no alternative is permitted. On the whole, these are definitions and obvious requirements which can be adopted by all EU countries. The “Application rules” are generally recognized rules (for example detailing rules) that follow the “Principles” and satisfy their requirements.

It is permissible to use alternative design rules provided that it is shown that these rules accord with the relevant “Principles” and that they are at least equivalent to those in EC2. Similar questions regarding methods have yet to be resolved. However, the principle of interchangeability of rules is generally anchored in the national codes of practice. A further characteristic of EC2 is the so-called “indicative” values, i.e. figures given as an indication (e.g. the partial factors of safety) and identified in the text by a “box”.

During an interim period, at least, they can be determined nationally by the individual EU countries. Where necessary, such modifications are given in special cases in the National Application Documents (NAD) during provisional application of EC2.

1.4.3
Essential requirements for design and execution

The essential requirements in chapter 2.1 of EC2 for design and construction stipulate among other things:

“P(1) A structure shall be designed and constructed in such a way that:
- with acceptable probability, it will remain fit for the use for which it is required, having due regard to its intended life and its cost, and
- with appropriate degrees of reliability, it will sustain all actions and influences likely to occur during execution and use and have adequate durability in relation to maintenance costs.”

“P(2) A structure shall also be designed in such a way that it will not be damaged by events like explosions, impact or consequence of human errors, to an extent disproportionate to the original cause…”

“P(4) The above requirements shall be met by the choice of suitable materials, by appropriate design and detailing and by specifying control procedures for production, construction and use as relevant to the particular project.”

With these requirements the overall framework is clearly defined into which the subsequent EC2 chapters 2.2 to 2.5 and 3 to 7 fit with their technical content (Table 1.1). Worthy of note is the fact that the durability requirement ranks high. This was one of the main reasons for the drafting of chapter 4.1 “Durability requirements” which, in the form of a sort of “checklist”, specifies the essential parameters which are to be seen in connection with durability. Attention is also drawn here to the CEN standard ENV 206 which includes important requirements for the choice of constituent materials for concrete and for the composition of concrete.

Table 1.1 Contents of Eurocode 2

Chapter	Title
1	Introduction
2	Basis of design
2.1	Fundamental requirements
2.2	Definitions and classifications
2.3	Design requirements
2.4	Durability
2.5	Analysis
3	Material properties
4	Section and member design
4.1	Durability requirements
4.2	Design data
4.3	Ultimate limit states
4.4	Serviceability limit states
5	Detailing provisions
6	Construction and workmanship
7	Quality control

1.5 References

1. The Council of the European Communities: Council Directive of 21 December 1988 on the approximation of laws, regulations and administrative provisions of the Member States relating to construction products (89/106/EEC).
2. ENV 1992–1–1: 1991: Eurocode 2: Design of Concrete Structures. Part 1: General Rules and Rules for Buildings; European Prestandard. December 1991.
3. Commission of the European Communities: Interpretative Document for the Essential Requirement No. 1—Mechanical Resistance and Stability. Last version complete, July 1993.
4. ENV 1991–1-Eurocode 1: Basis of design and actions on structures. Part 1: Basis of design. Edition 1994.

2
Main symbols used in EC2

A_c	Total cross-sectional area of a concrete section
A_{cl}	Maximum area corresponding geometrically to A_{co}, and having the same centre of gravity
A_{co}	Loaded area
$A_{ct,ext}$	Area of concrete external to stirrups
$A_{c.eff}$	Effective area of concrete in tension
A_k	Area enclosed within the centre-line of the idealized thin-walled cross-section including inner hollow areas
A_{ct}	Area of concrete within the tension zone
A_p	Area of a prestressing tendon or tendons
A_s	Area of reinforcement within the tension zone
A_{s2}	Area of reinforcement in the compression zone at the ultimate limit state
A_{sf}	Area of reinforcement across the flange of a flanged beam
$A_{s,min}$	Minimum area of longitudinal tensile reinforcement
$A_{s,prov}$	Area of steel provided
$A_{s,req}$	Area of steel required
$A_{s,surf}$	Area of surface reinforcement
A_{st}	Area of additional transverse reinforcement parallel to the lower face
A_{sv}	Area of additional transverse reinforcement perpendicular to the lower face
A_{sw}	Cross-sectional area of shear reinforcement
E_{cd}	Design value of the secant modulus of elasticity
$E_{c(t)}$	Tangent modulus of elasticity of normal weight concrete at a stress of σ_c=0 and at time t
$E_{c(28)}$	Tangent modulus of elasticity of normal weight concrete at a stress of σ_c=0 and at 28 days
E_{cm}	Secant modulus of elasticity of normal weight concrete
$E_{c,nom}$	Either the mean value of E_{cm} **or** The corresponding design value E_{cd}
$E_{d,dst}$	Design effects of destabilising actions
$E_{d,stb}$	Design effects of stabilising actions
E_s	Modulus of elasticity of reinforcement or prestressing steel
F_c	Force due to the compression block at a critical section at the ultimate limit state
ΔF_d	Variation of the longitudinal force acting in a section of flange within distance a_c
F_{px}	Ultimate resisting force provided by the prestressing tendons in a cracked anchorage zone
$F_{sd,sup}$	Design support reaction
F_s	Force in the tension reinforcement at a critical section at the ultimate limit state
F_s	Tensile force in longitudinal reinforcement
F_v	Vertical force acting on a corbel
$G_{d,inf}$	Lower design value of a permanent action
$G_{d,sup}$	Upper design value of a permanent action
G_{ind}	Indirect permanent action
$G_{k,inf}$	Lower characteristic value of a permanent action
$G_{k,sup}$	Upper characteristic value of a permanent action
$G_{k,j}$	Characteristic values of permanent actions
H_c	Horizontal force acting at the bearing on a corbel
H_{fd}	Additional horizontal force to be considered in the design of horizontal structural elements, when taking account of imperfections
ΔH_j	Increase in the horizontal force acting on the floor of a frame structure, due to imperfections

ΔM_{Sd} Reduction in the design support moment for continuous beams or slabs, due to the support reaction $F_{sd,sup}$, when the support provides no restraint to rotation
I_b Moment of inertia (gross section) of a beam
I_c Second moment of area of a concrete section
I_{col} Moment of inertia (gross section) of a column
$J(t, t_o)$ Creep function at time t
K_1 Reduction factor for the calculation of the second order eccentricity e_2
K_2 Coefficient, taking account of decrease in curvature ($1/r$) due to increasing axial force
M_{Rd} Design resisting moment
M_{sd} Design value of the applied internal bending moment
M_{sd1} First order applied moment
N_{pd} Prestressing force corresponding to initial value without losses
N_{Rd} Resisting design axial compression force
N_{sd} Design value of the applied axial force (tension or compression)
N_{ud} Design ultimate capacity of the section subjected to axial load only
$P_{m,t}$ Mean value of the prestressing force at time t, at any point distance x along the member
P_o Initial force at the active end of the tendon immediately after stressing
Q_{ind} Indirect variable action
$Q_{k,1}$ Characteristic value of one of the variable actions
$Q_{k,i}$ Characteristic values of the other variable actions
T_{Sd} Design value of the applied torsional moment
V_{ccd} Force component in the compression zone, parallel to V_{od}, of elements with variable depth
V_{cd} Design shear contribution of the concrete section
V_{od} Design shear force in the section, uncorrected for effects of variable section depth
V_{pd} Force component due to inclined prestressing tendons
V_{Rd1} Design shear resistance of a section in elements without shear reinforcement
V_{Rd2} Maximum design shear force that can be carried without web failure
$V_{rd2,red}$ Reduced value of V_{Rd2}, due to axial force
V_{Rd3} Design shear resistance of a section, in elements with shear reinforcement
V_{Rds} Total resistance to flexural and punching shear
V_{Sd} Design value of the applied shear force at the ultimate limit state
V_{td} Force component in the tensile zone, parallel to V_{od}, in elements with variable depth
V_{wd} Contribution of shear reinforcement
a Horizontal clear distance between two parallel laps
a_1 Horizontal displacement of the envelope line of the tensile force (shift rule)
a_c Distance between the point of application of the applied vertical load and the face of the supporting member (corbel design)
a_d Design values of geometrical date
a_{nom} Nominal value of geometrical data
a_v Distance between points of zero and maximum moment
b Overall width of a cross-section
or Actual flange width in a T or L beam
or Lateral concrete cover in the plane of a lap
b_{eff} Effective flange width of a T or L beam
b_{sup} Breadth of a support
b_t Mean width of a beam in tension zone
b_w Width of the web on T, I or L beams
$b_{w,nom}$ Nominal web thickness
c Minimum concrete cover
d Effective depth of a cross-section
d_{crit} Distance of critical section for punching shear from the centroid of a column
d_g Largest nominal maximum aggregate size

e_2 Second order eccentricity

e_a	Additional eccentricity covering the effects of geometrical imperfections
e_e	Equivalent eccentricity
e_o	First order eccentricity
e_{o1}, e_{o2}	Values of the first order eccentricity of the axial load at the ends of the member, denoted so that $\lvert e_{o1} \rvert \quad \lvert e_{o2} \rvert$
e_{tot}	Total eccentricity
e_y	Eccentricity in the y-direction
e_z	Eccentricity in the z-direction
f_{bd}	Design value for ultimate bond stress
f_c	Compressive strength of concrete
f_{cd}	Design value of concrete cylinder compressive strength
f_{ck}	Characteristic compressive cylinder strength of concrete at 28 days
f_{cm}	Mean value of concrete cylinder compressive strength
$f_{ct.eff}$	The tensile strength of the concrete effective at the time when cracks are expected
f_{ctk}	Characteristic axial tensile strength of concrete
$f_{ctk\ 0.05}$	Lower characteristic tensile strength (5% fractile)
$f_{ctk\ 0.95}$	Upper characteristic tensile strength (95% fractile)
f_{ctm}	Mean value of axial tensile strength of concrete
f_p	Tensile strength of prestressing steel
f_{pk}	Characteristic tensile strength of prestressing steel
$f_{p0.1}$	0.1% proof stress of prestressing steel
$f_{p0.1k}$	Characteristic 0.1% proof-stress of prestressing steel
$f_{0.2k}$	Characteristic 0.2% proof-stress of reinforcement
f_t	Tensile strength of reinforcement
f_{tk}	Characteristic tensile strength of reinforcement
f_y	Yield strength of reinforcement
f_{yd}	Design yield strength of reinforcement
f_{yk}	Characteristic yield strength of reinforcement
f_{ywd}	Design yield strength of shear reinforcement
h	Overall depth of a cross-section
h_c	Overall depth of a corbel at the face of the supporting member
h_f	Overall depth of a flange in T or L beams
h_H	Depth of an enlarged column head
k	Coefficient which allows for the effects of non-uniform self-equilibrating stresses
k_c	Stress distribution coefficient
k_1	Coefficient to take account of the influence of the bond properties of bar on the crack spacing
k_2	Coefficient to take account of the influence of the form of the strain distribution on the crack spacing
k_A or k_B	Coefficients describing the rigidity of restraint at the column ends
l	Length **or** Span **or** Total height of a structure in metres
l_{col}	Height of column measured between idealized centres of restraint
l_{eff}	Effective span of beams and slabs
l_n	Clear distance between the faces of the supports
l_0	Length of span(s) between points of zero moment
l_{ot}	Length of a compression flange measured between lateral supports
l_b	Basic anchorage length for reinforcement
$l_{b,min}$	Minimum anchorage length
$l_{b,net}$	Required anchorage length
l_{ba}	Anchorage length over which the ultimate tendon force (F_{pu}) in pre-tensioned members if fully transmitted to the concrete
l_{bp}	Transmission length, over which the prestressing force from a pre-tensioned tendon is fully transmitted to the concrete
l_{bpd}	Design value for transmission length
l_{bpo}	Length of a neutralized zone at the ends of pre-tensioned members, in the case of sudden release

$l_{p.eff}$	Dispersion length, over which the concrete stresses gradually disperse to a linear distribution across the section (effective transfer)
l_s	Necessary lap length
$l_{s,min}$	Minimum lap length
l_x, l_y	Spans between columns on the *x*- and *y*-directions respectively
m_{Sdx}, m_{Sdy}	Minimum design bending moments in the *x*- and *y*-directions respectively
n	Number of transverse bars along anchorage length
	or Number of vertical continuous members acting together
n_1	Number of layers with bars anchored at the same point
n_2	Number of bars anchored in each layer
n_b	Number of bars in a bundle
p	Mean transverse pressure (N/mm^2) over the anchorage length
$1/r$	Curvature at the critical section at the base of a model column
s	Spacing of stirrups
s_1	Spacing of longitudinal wires in a welded mesh fabric, or in surface reinforcement
s_f	Spacing of reinforcing bars across the flange of flanged beams
s_{max}	Maximum longitudinal spacing of successive series of stirrups
s_{rm}	Average final crack spacing
s_t	Spacing of transverse wires in a welded mesh fabric or in surface reinforcement
s	Snow load according to Eurocode 1
t	Thickness of a supporting element
u_k	Circumference of area $\boldsymbol{A_k}$
u	Perimeter of critical section for punching shear
	or Perimeter of concrete cross-section
v_{Rd1}	Design shear resistance per unit length of the critical perimeter, for a slab without shear reinforcement
v_{Rd2}	Maximum design shear resistance per unit length of the critical perimeter, for a slab with shear reinforcement
v_{Rd3}	Design shear resistance per unit length of the critical perimeter, for a slab with shear reinforcement
v_{Sd}	Shear force per unit length along critical section
w_k	Design or characteristic crack width
x	Neutral axis depth
z	Lever arm of internal forces
Z_{cp}	Distance between the centre of gravity of the concrete section and the tendons

α	Reduction factor for concrete compressive strength
	or Angle of the shear reinforcement with the longitudinal reinforcement (main steel) **or** E_s/E_{cm}
α_a	A coefficient for determining the effectiveness of anchorages
α_1	Coefficient for effectiveness of laps
α_2	Coefficient for the calculation of the lap length of welded mesh fabrics
β	Coefficient taking account of the effects of eccentricity of load
	or Coefficient relating the average crack width to the design width
	or l_o/l_{col}
	or Shear force enhancement coefficient
β_1	Coefficient taking account of the influence of the bond properties of bar on the average strain
β_2	Coefficient taking account of the influence of the duration of the loading or of repeated loading on the average strain
β_b	Coefficient relating transmission length of prestressing tendons to concrete strength
γ_A	Partial safety factor for accidental actions *A*
γ_c	Partial safety factor for concrete material properties
$\gamma_{G,inf}$	Partial safety factor for permanent actions, in calculating the lower design values
$\gamma_{G,sup}$	Partial safety factor for permanent actions, in calculating the upper design values
γ_{GA}, $\gamma_{GA,j}$	Partial safety factor for permanent actions, for accidental design situations
$\gamma_{G,j}$	Partial safety factor for any permanent action *j*
γ_M	Partial safety factor for a material property, taking account of uncertainties in the material property itself and in the design model used
γ_P	Partial safety factor for actions associated with prestressing, *P*

$\gamma_{Q,i}$	Partial safety factor for any variable action *i*
$\gamma_{Q,1}$	Partial safety factor for the basic most unfavourable variable action
γ_s	Partial safety factor for the properties of reinforcement or prestressing steel
Δa	Change made to nominal geometrical data for particular design purposes (e.g. assessment of effects of imperfections)
δ	Ratio of redistributed moment to the moment before redistribution
ε_c	Compressive strain in the concrete
ε_{c1}	Compressive strain in the concrete at the peak stress f_c
ε_{cu}	Ultimate compressive strain in the concrete
ε_{sm}	Strain in the reinforcement taking account of tension stiffening
ε_{s1}	Strain in tension reinforcement, for section analysis
ε_{s2}	Strain in compression reinforcement, for section analysis
ε_{pm}	Steel strain corresponding to $P_{m,t}$
ε_{yd}	Design yield strain of the steel reinforcement
θ	Angle between the concrete struts and the longitudinal axis **or** Sum of angular displacement over a distance *x* (irrespective of direction or sign)
λ	Slenderness ratio
λ_{crit}	Critical slenderness ratio
μ	Coefficient of friction between the tendons and their ducts
v	Angle of inclination of a structure, assumed in assessing the effects of imperfections **or** Efficiency factor **or** Coefficient relating the average design compressive stress in struts to the design value of concrete compressive strength (f_{cd})
ρ_1	Equivalent longitudinal reinforcement ratio
ρ_{1x}	Longitudinal reinforcement ratio in *x*-direction
ρ_{1y}	Longitudinal reinforcement ratio in *y*-direction
ρ_r	Effective reinforcement ratio

ρ_w	Reinforcement ratio for shear reinforcement
σ_c	Compressive stress in the concrete
σ_{cu}	Compressive stress in the concrete at the ultimate compressive strain
σ_{cg}	Stress in the concrete adjacent to the tendons, due to self-weight and any other permanent actions
σ_{cpo}	Initial stress in the concrete adjacent to the tendons, due to prestress
$\sigma_{o,max}$	Maximum stress applied to a tendon
σ_{pmo}	Stress in the tendon immediately after stressing or transfer
σ_{pgo}	Initial stress in the tendons due to prestress and permanent actions
σ_s	Stress in the tension reinforcement calculated on the basis of a cracked section
σ_{sr}	Stress in the tension reinforcement calculated on the basis of a cracked section under conditions of loading leading to formation of the first crack
τ_{Rd}	Basic shear strength of members without shear reinforcement
$\varphi(\infty, t_o)$	Final value of creep coefficient
ø	Diameter of a reinforcing bar or of a prestressing duct
$ø_n$	Equivalent diameter of a bundle of reinforcing bars
$ø_s$	Adjusted maximum bar diameter
$ø^*_s$	Unadjusted maximum bar diameter (Table 4.11)
ψ	Factors defining representative values of variable actions
ψ_0	Used for combination values
ψ_1	Used for frequent values
ψ_2	Used for quasi-permanent values

3
Overview of flow charts

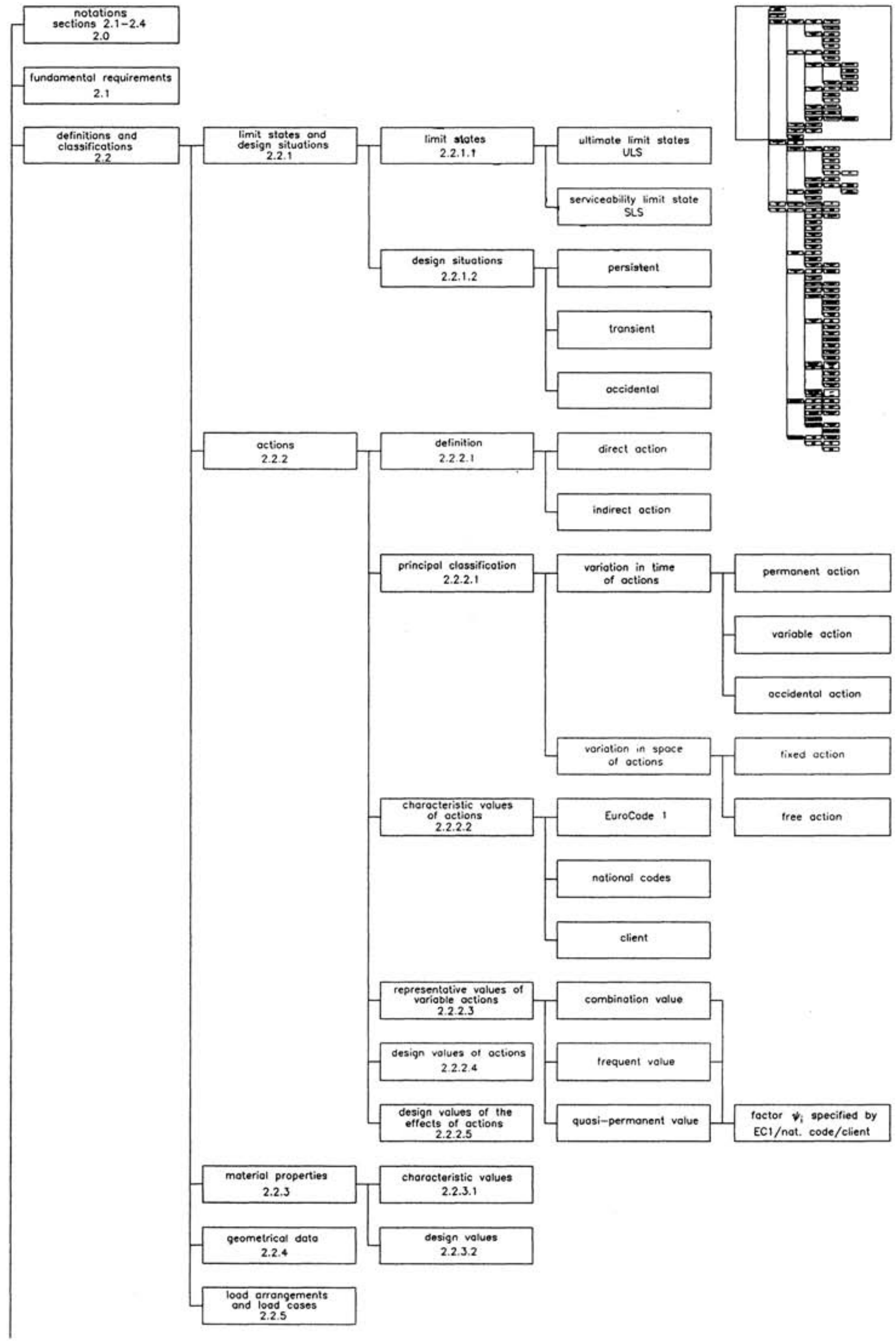
The flow charts function as a guide through Eurocode 2. The cross-references used in the flow charts therefore refer to Eurocode 2.

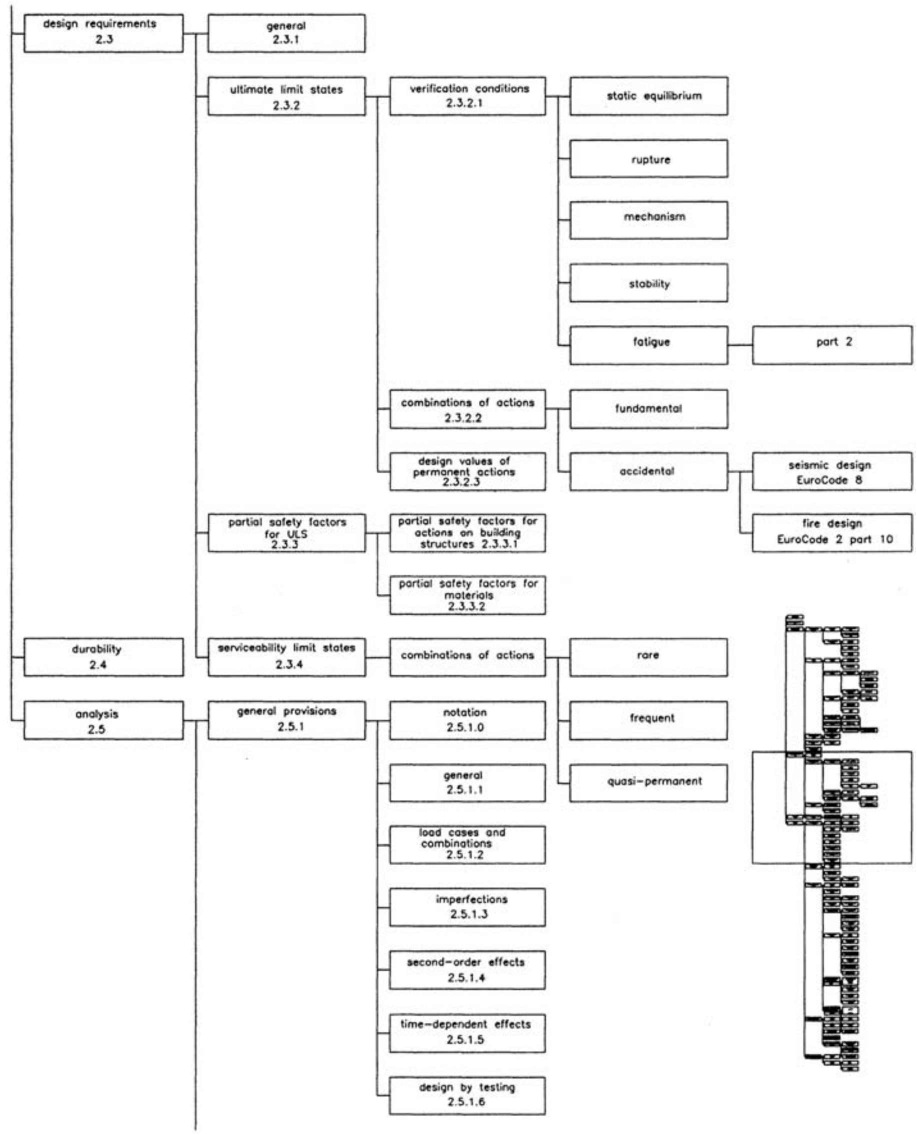
There are three main levels of flow charts.

Level 1	Basis of design		2.
	Flow chart 3.0	Overview	
Level 2	Section and member design		4.
	Flow chart 3.0.1	General	
	Flow chart 3.0.2	Ultimate limit states (ULS)	
	Flow chart 3.0.3	Serviceability limit states (SLS)	
Level 3	Detailed calculations and detailing provisions		4.
Level 3.1	ULS		4.3
Level 3.1.1	Bending		4.3.1
	Flow chart 3.1.1.1	Bending and longitudinal force	
Level 3.1.2	Shear		4.3.2
	Flow chart 3.1.2.1	Design method	
	Flow chart 3.1.2.2	Elements with shear reinforcement	
Level 3.1.3	Torsion		4.3.3
	Flow chart 3.1.3.1	Pure torsion	
	Flow chart 3.1.3.2	Torsion, combined effects of actions	
	Flow chart 3.1.3.3	Torsion and flexure	
	Flow chart 3.1.3.4	Torsion and shear	
Level 3.1.4	Punching		4.3.4
	Flow chart 3.1.4.1	Punching	
	Flow chart 3.1.4.2	Punching shear reinforcement	
Level 3.1.5	Buckling		4.3.5
	Flow chart 3.1.5.1	General guide	
	Flow chart 3.1.5.2	Structure as a whole	
	Flow chart 3.1.5.3	Isolated columns	
Level 3.2	SLS		4.4
Level 3.2.1	Stresses		4.4.1
	Flow chart 3.2.1.1	Limitation of stresses	
Level 3.2.2	Cracking		4.4.2
	Flow chart 3.2.2.1	Minimum reinforcement	
	Flow chart 3.2.2.2	With or without calculation	
Level 3.2.3	Deformations		4.4.3
	Flow chart 3.2.3.1	Deformation without calculation	
	Flow chart 3.2.3.2	Deformation by calculation	
Level 3.3	Detailing		5.
Level 3.3.1	Anchorage		5.2.3
	Flow chart 3.3.1.1	General	
Level 3.3.2	Splices		5.2.4
	Flow chart 3.3.2.1	Splices for bars or wires	
	Flow chart 3.3.2.2	Splices for welded mesh fabrics	

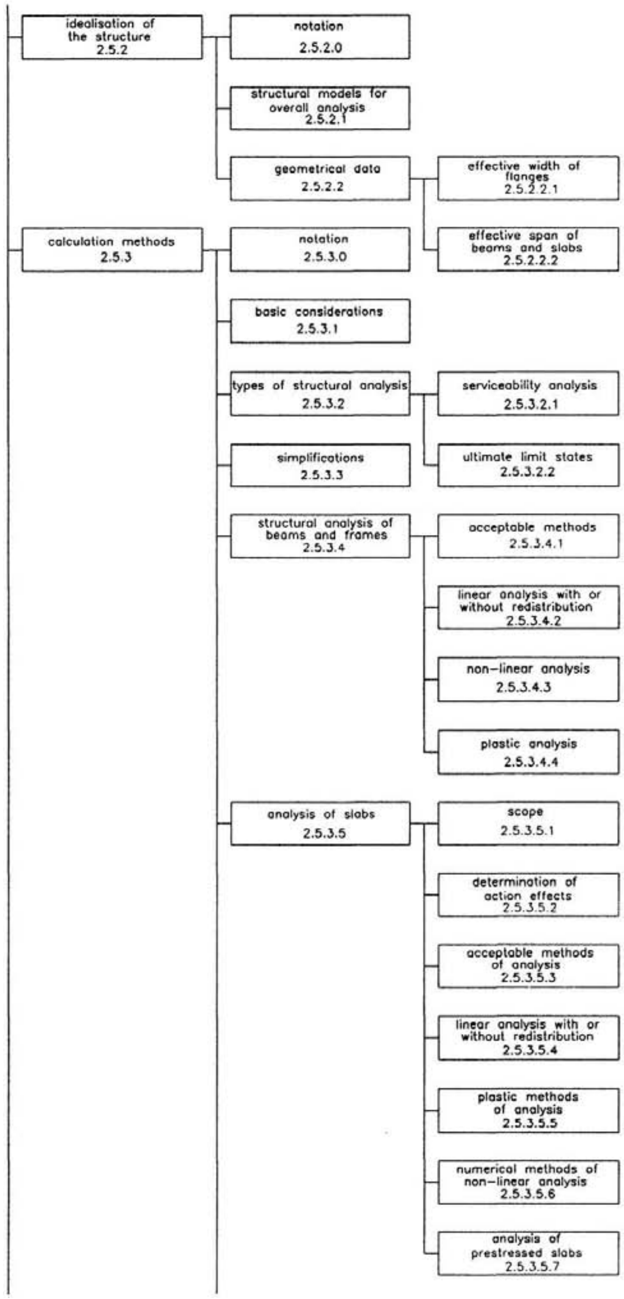
Flowchart 3.0

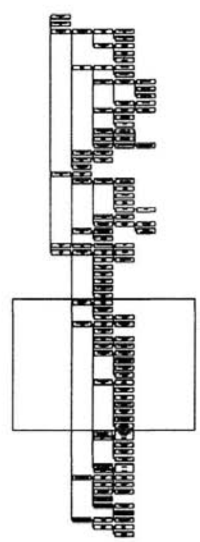
Basis of design: overview

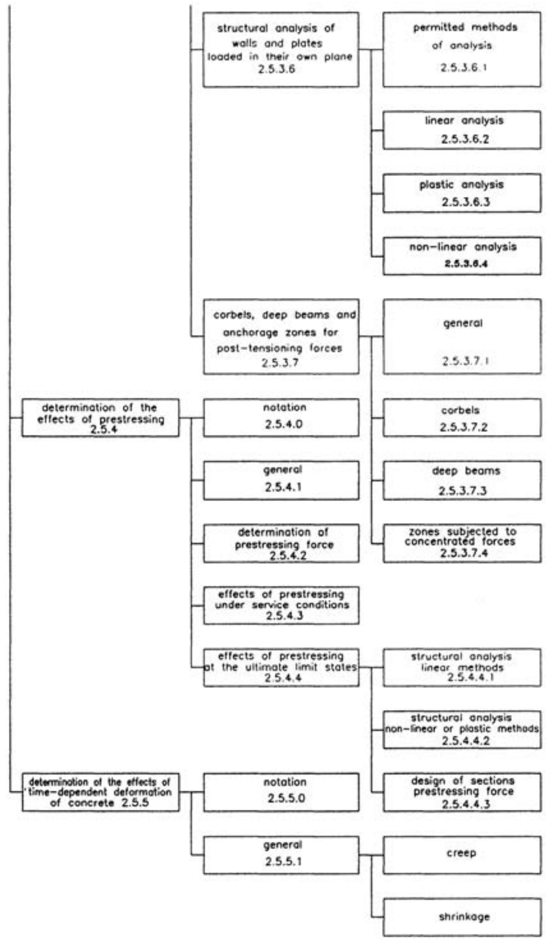


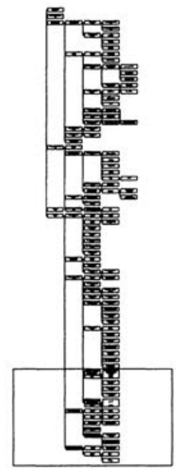


design requirements 2.3
general 2.3.1
ultimate limit states 2.3.2
verification conditions 2.3.2.1
static equilibrium
rupture
mechanism
stability
fatigue
part 2
combinations of actions 2.3.2.2
fundamental
design values of permanent actions 2.3.2.3
accidental
seismic design EuroCode 8
fire design EuroCode 2 part 10
partial safety factors for ULS 2.3.3
partial safety factors for actions on building structures 2.3.3.1
partial safety factors for materials 2.3.3.2
durability 2.4
serviceability limit states 2.3.4
combinations of actions
rare
frequent
quasi-permanent
analysis 2.5
general provisions 2.5.1
notation 2.5.1.0
general 2.5.1.1
load cases and combinations 2.5.1.2
imperfections 2.5.1.3
second-order effects 2.5.1.4
time-dependent effects 2.5.1.5
design by testing 2.5.1.6


idealisation of the structure 2.5.2
notation 2.5.2.0
structural models for overall analysis 2.5.2.1
geometrical data 2.5.2.2
effective width of flanges 2.5.2.2.1
effective span of beams and slabs 2.5.2.2.2
calculation methods 2.5.3
notation 2.5.3.0
basic considerations 2.5.3.1
types of structural analysis 2.5.3.2
serviceability analysis 2.5.3.2.1
ultimate limit states 2.5.3.2.2
simplifications 2.5.3.3
structural analysis of beams and frames 2.5.3.4
acceptable methods 2.5.3.4.1
linear analysis with or without redistribution 2.5.3.4.2
non-linear analysis 2.5.3.4.3
plastic analysis 2.5.3.4.4
analysis of slabs 2.5.3.5
scope 2.5.3.5.1
determination of action effects 2.5.3.5.2
acceptable methods of analysis 2.5.3.5.3
linear analysis with or without redistribution 2.5.3.5.4
plastic methods of analysis 2.5.3.5.5
numerical methods of non-linear analysis 2.5.3.5.6
analysis of prestressed slabs 2.5.3.5.7

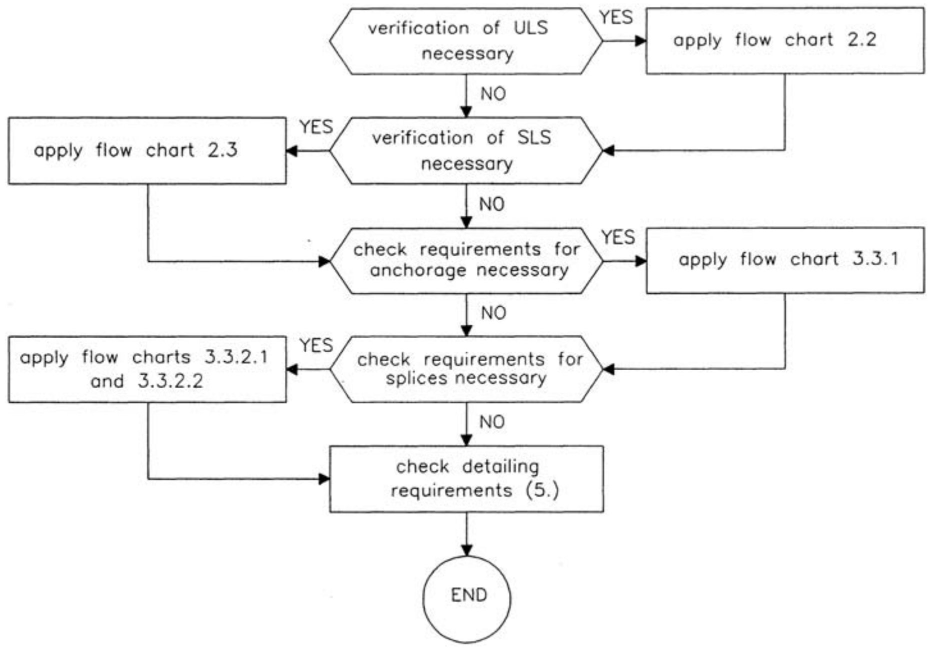



structural analysis of walls and plates loaded in their own plane 2.5.3.6
permitted methods of analysis 2.5.3.6.1
linear analysis 2.5.3.6.2
plastic analysis 2.5.3.6.3
non-linear analysis 2.5.3.6.4
corbels, deep beams and anchorage zones for post-tensioning forces 2.5.3.7
general 2.5.3.7.1
determination of the effects of prestressing 2.5.4
notation 2.5.4.0
corbels 2.5.3.7.2
general 2.5.4.1
deep beams 2.5.3.7.3
determination of prestressing force 2.5.4.2
zones subjected to concentrated forces 2.5.3.7.4
effects of prestressing under service conditions 2.5.4.3
effects of prestressing at the ultimate limit states 2.5.4.4
structural analysis linear methods 2.5.4.4.1
structural analysis non-linear or plastic methods 2.5.4.4.2
determination of the effects of time-dependent deformation of concrete 2.5.5
notation 2.5.5.0
design of sections prestressing force 2.5.4.4.3
general 2.5.5.1
creep
shrinkage



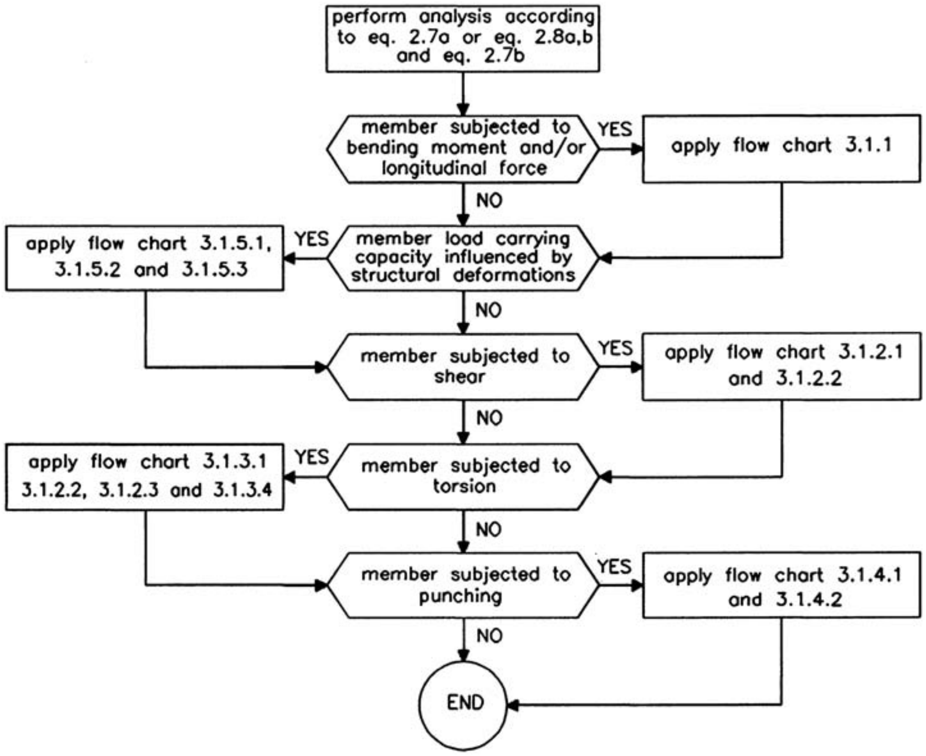
Flow chart 3.0.1

Section and member design: general



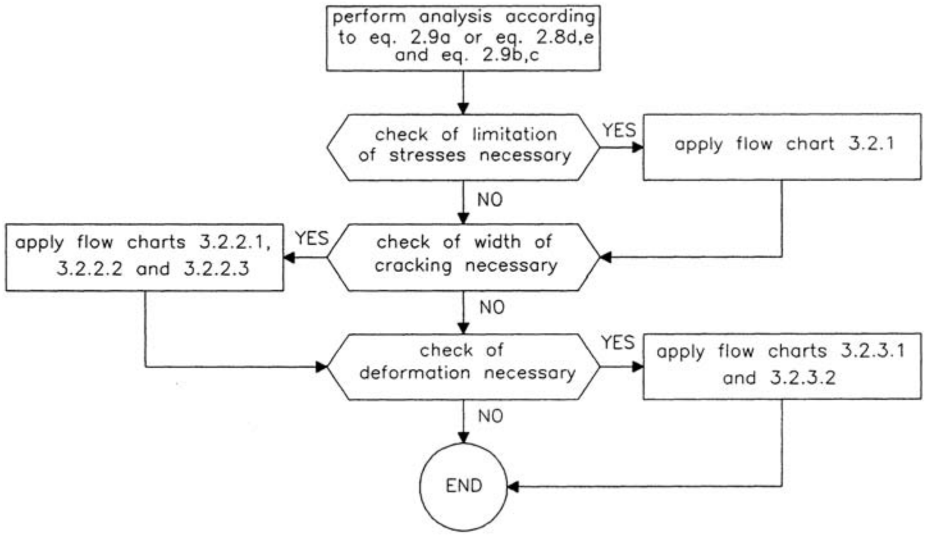
Flow chart 3.0.2

Section and member design: ultimate limit state (ULS)



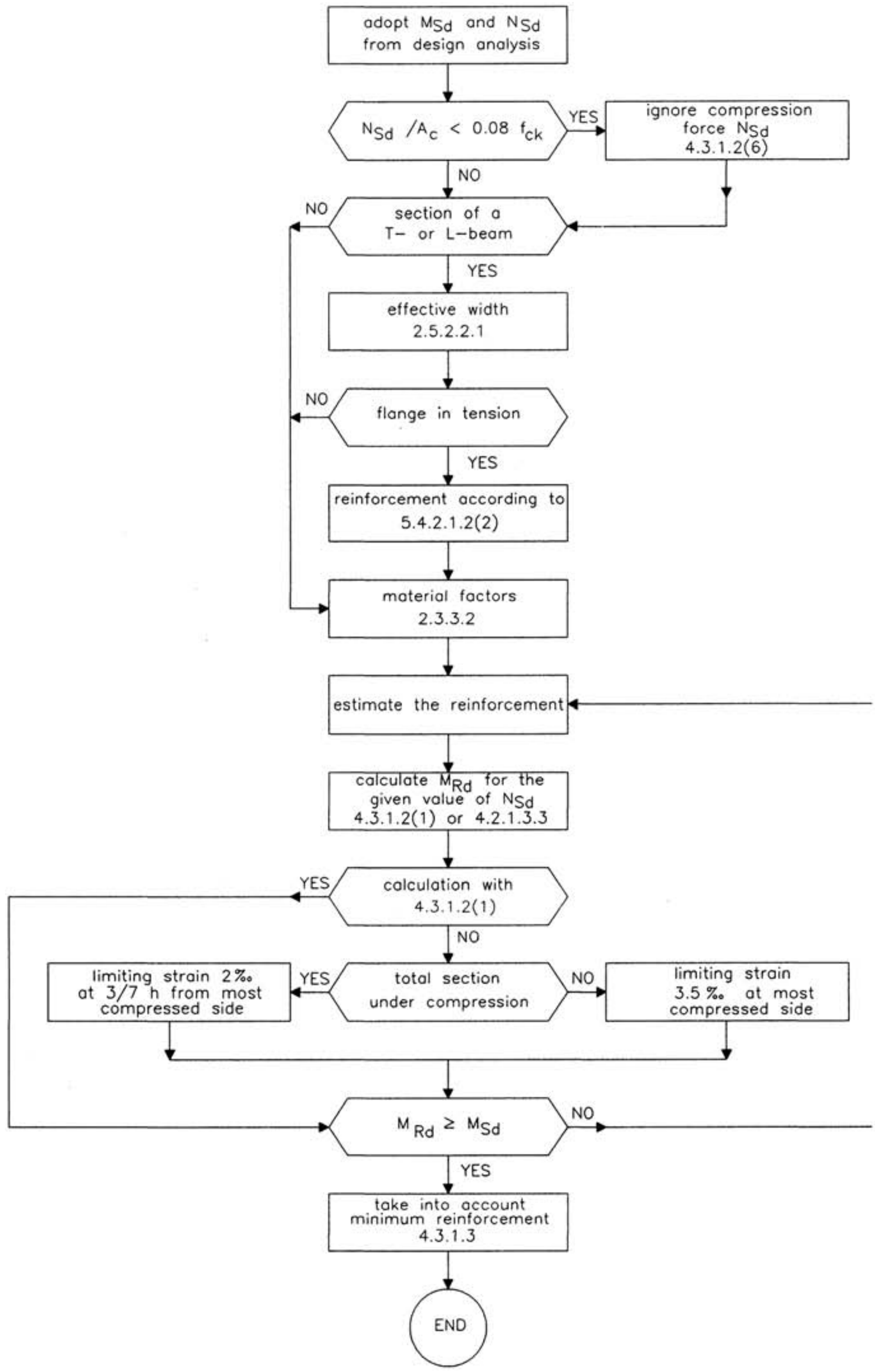
Flow chart 3.0.3

Section and member design: serviceability limit state (SLS)



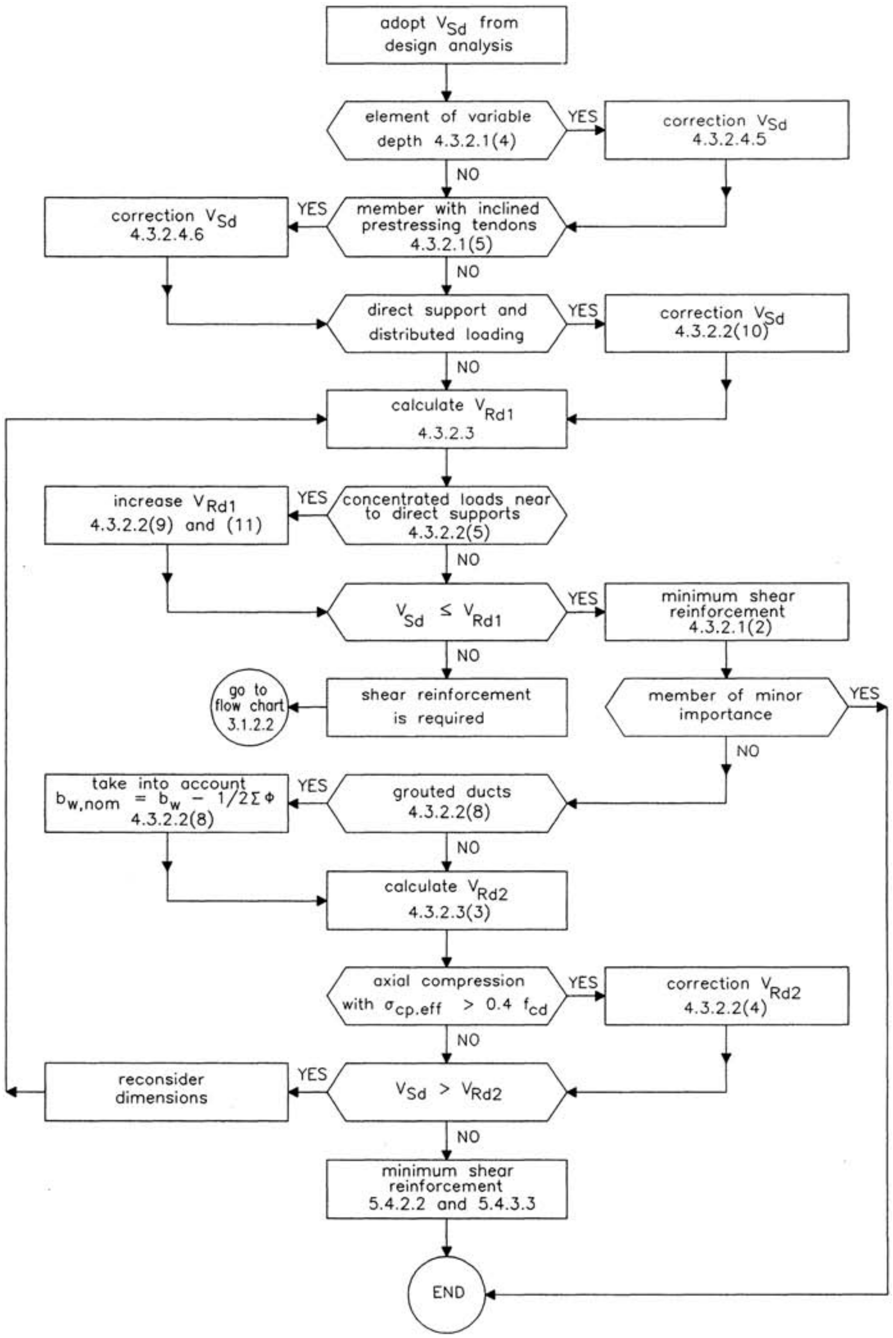
Flow chart 3.1.1.1

Bending: bending and longitudinal force



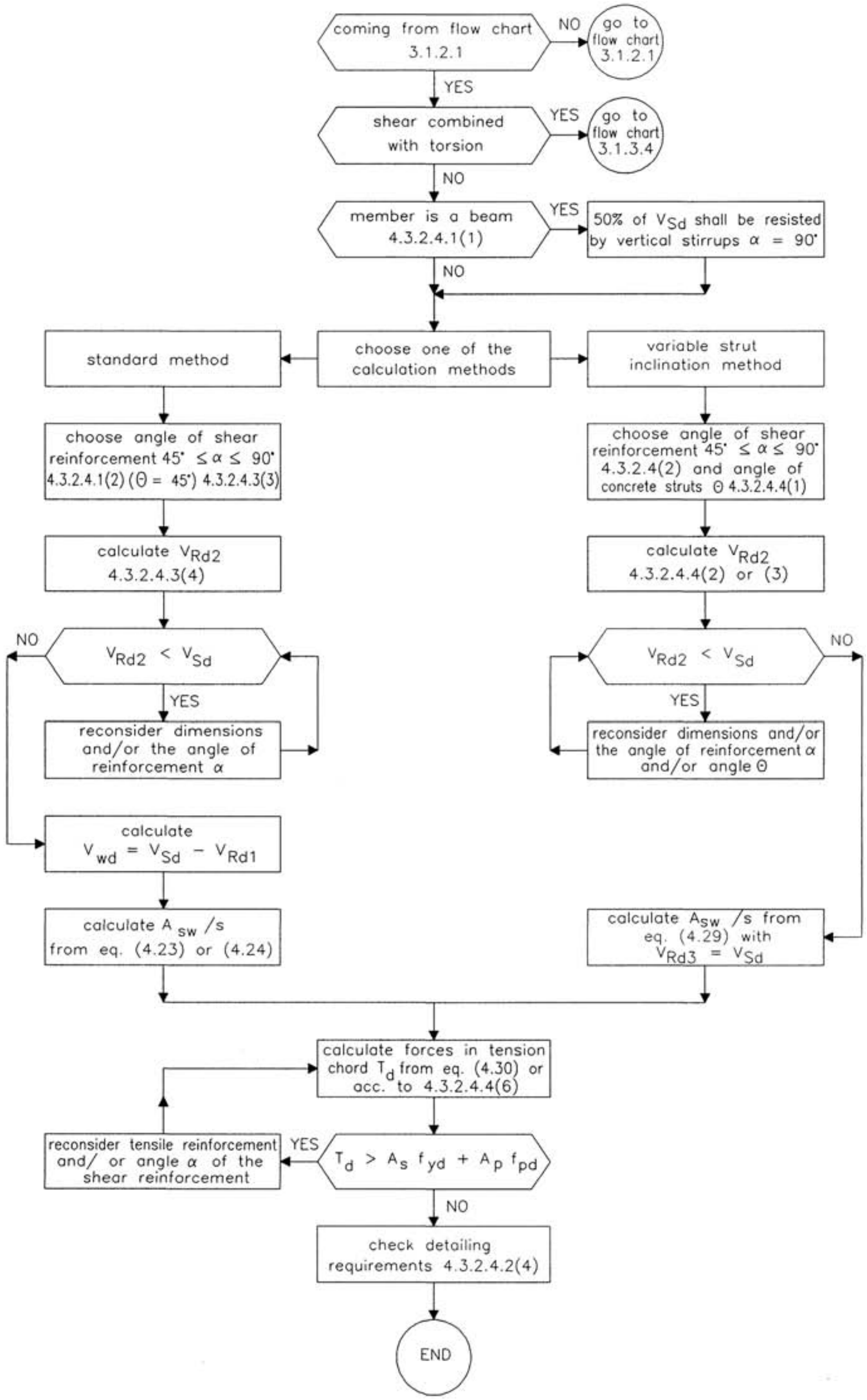
Flow chart 3.1.2.1

Shear: design method



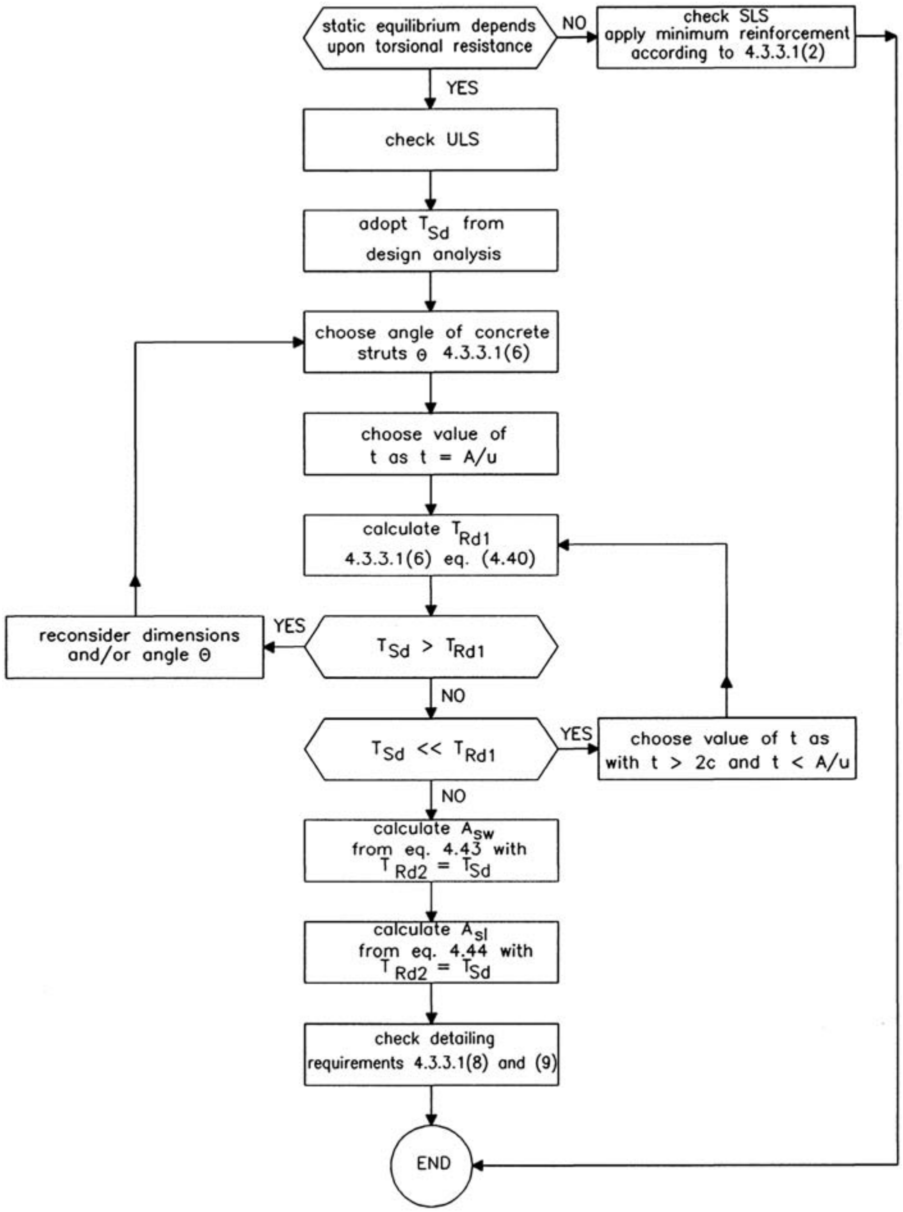
Flow chart 3.1.2.2

Shear: elements with shear reinforcement



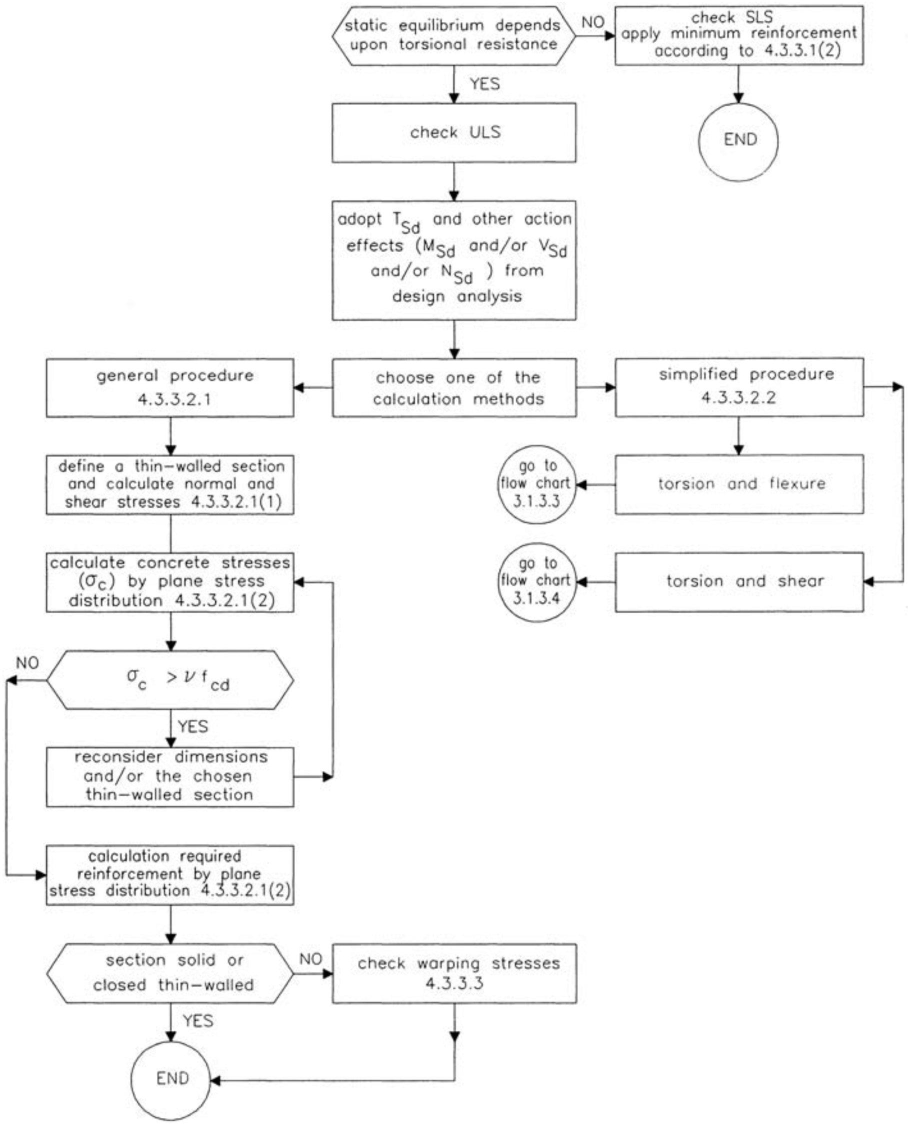
Flow chart 3.1.3.1

Torsion: pure torsion



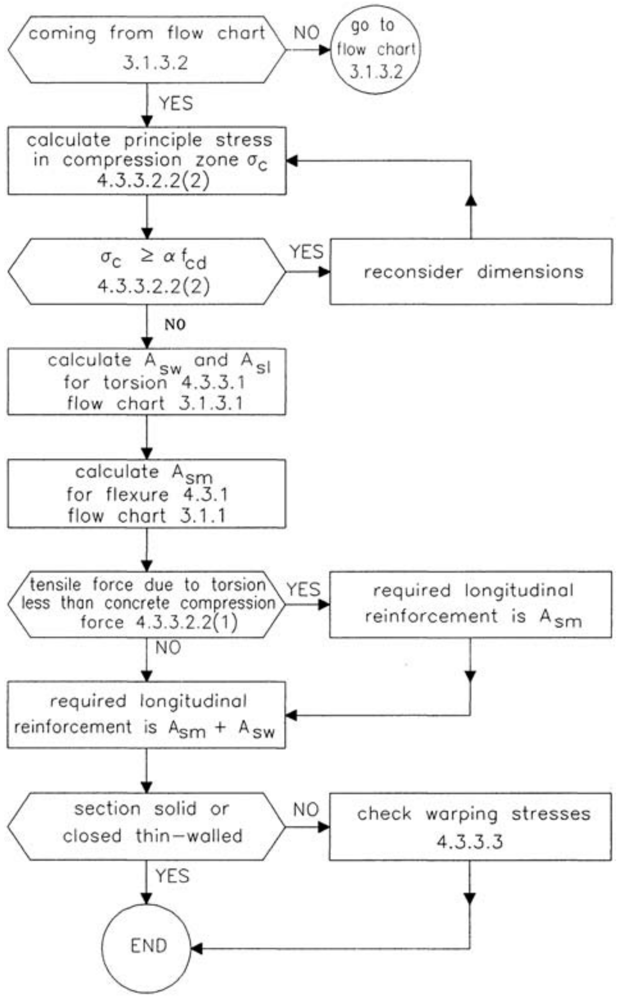
Flow chart 3.1.3.2

Torsion: torsion, combined effects of action



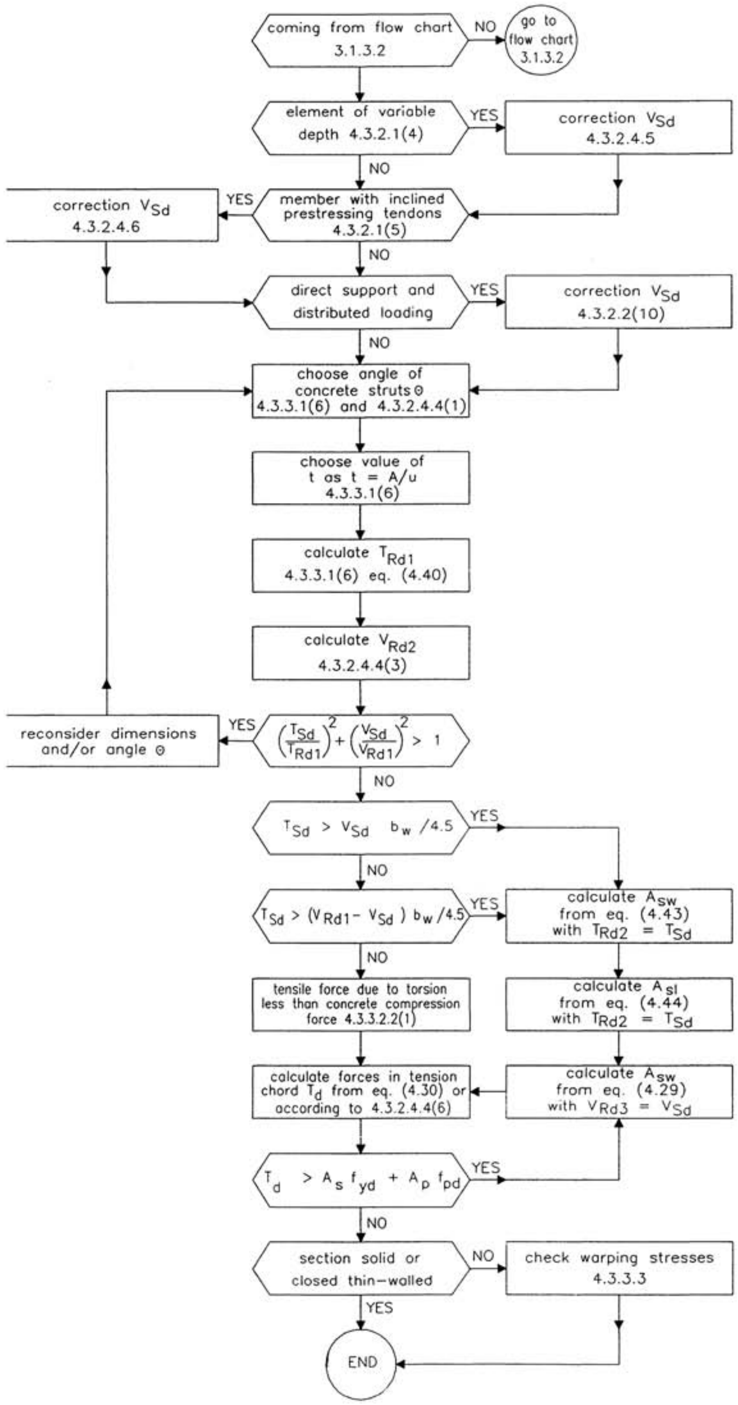
Flow chart 3.1.3.3

Torsion: torsion and flexure



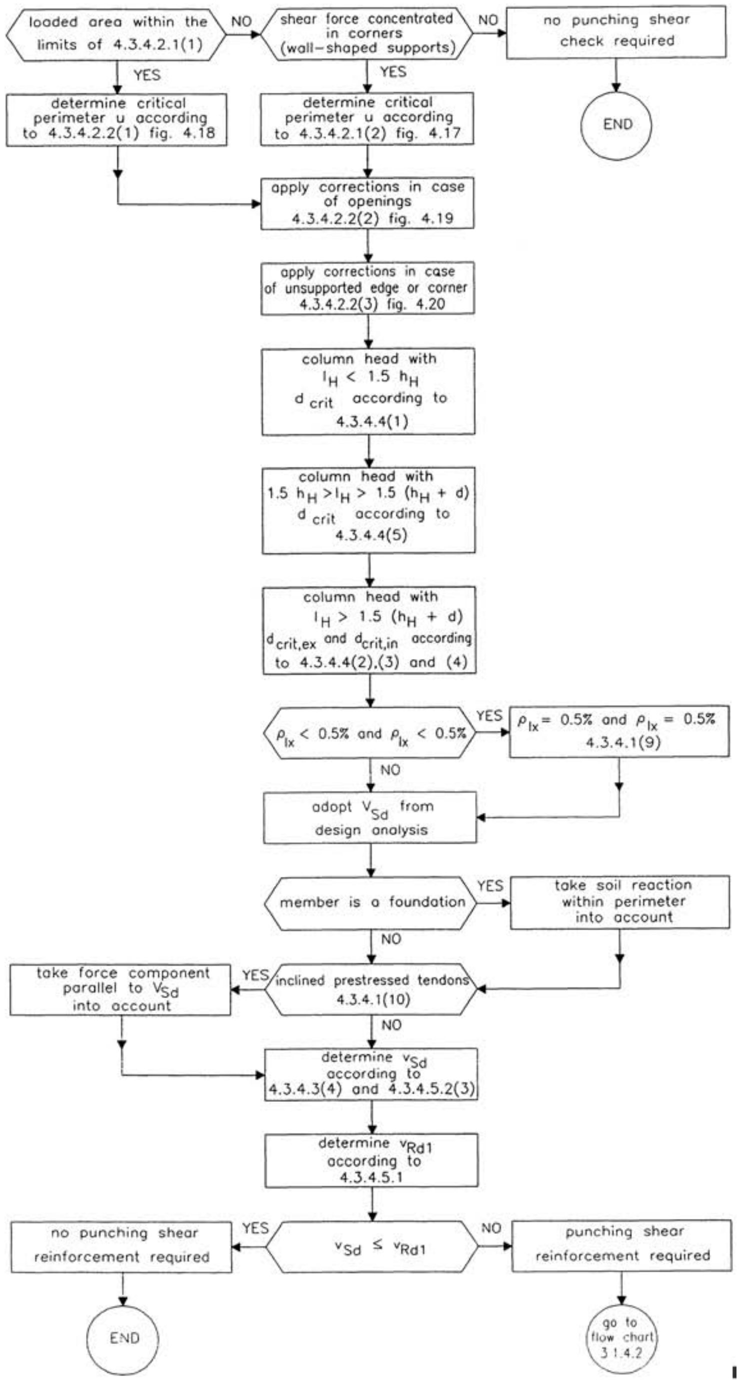
Flow chart 3.1.3.4

Torsion: torsion and shear



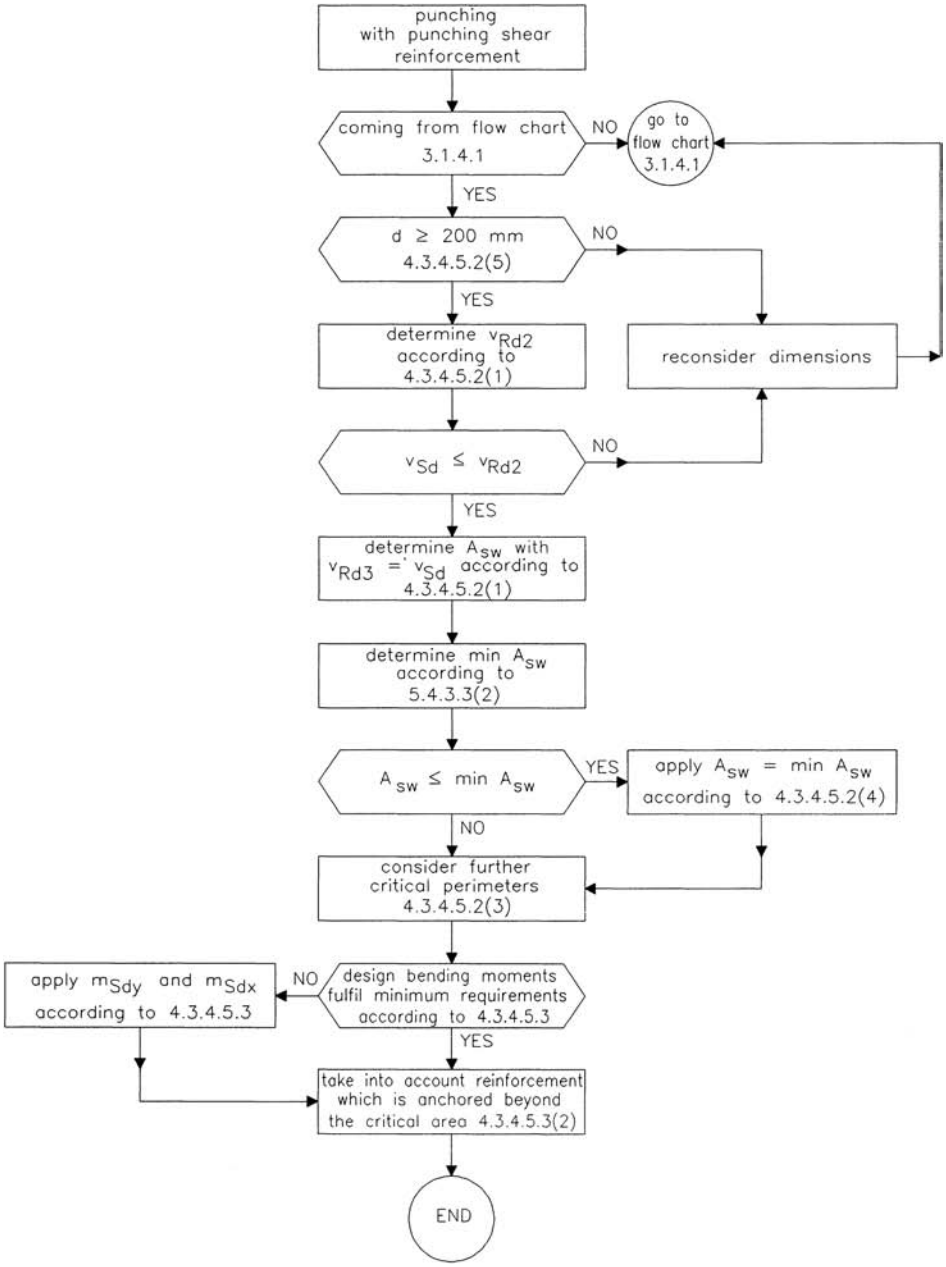
Flow chart 3.1.4.1

Punching: punching



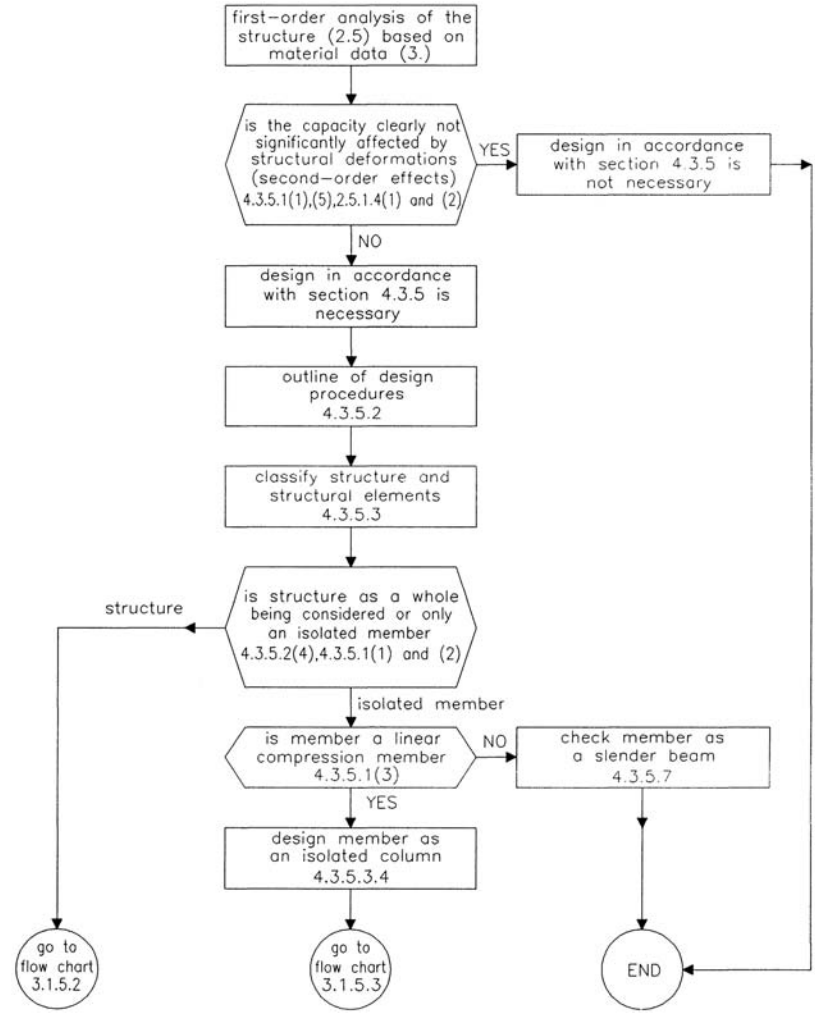
Flow chart 3.1.4.2

Punching: punching shear reinforcement



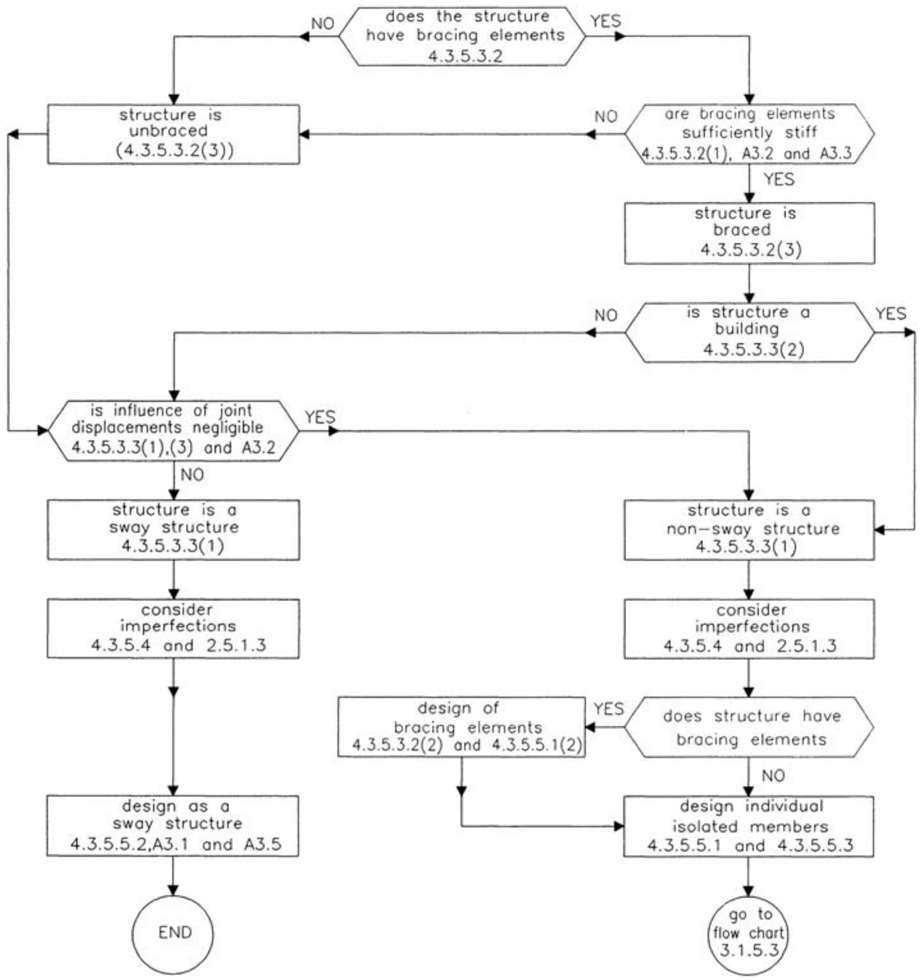
Flow chart 3.1.5.1

Buckling: general guide



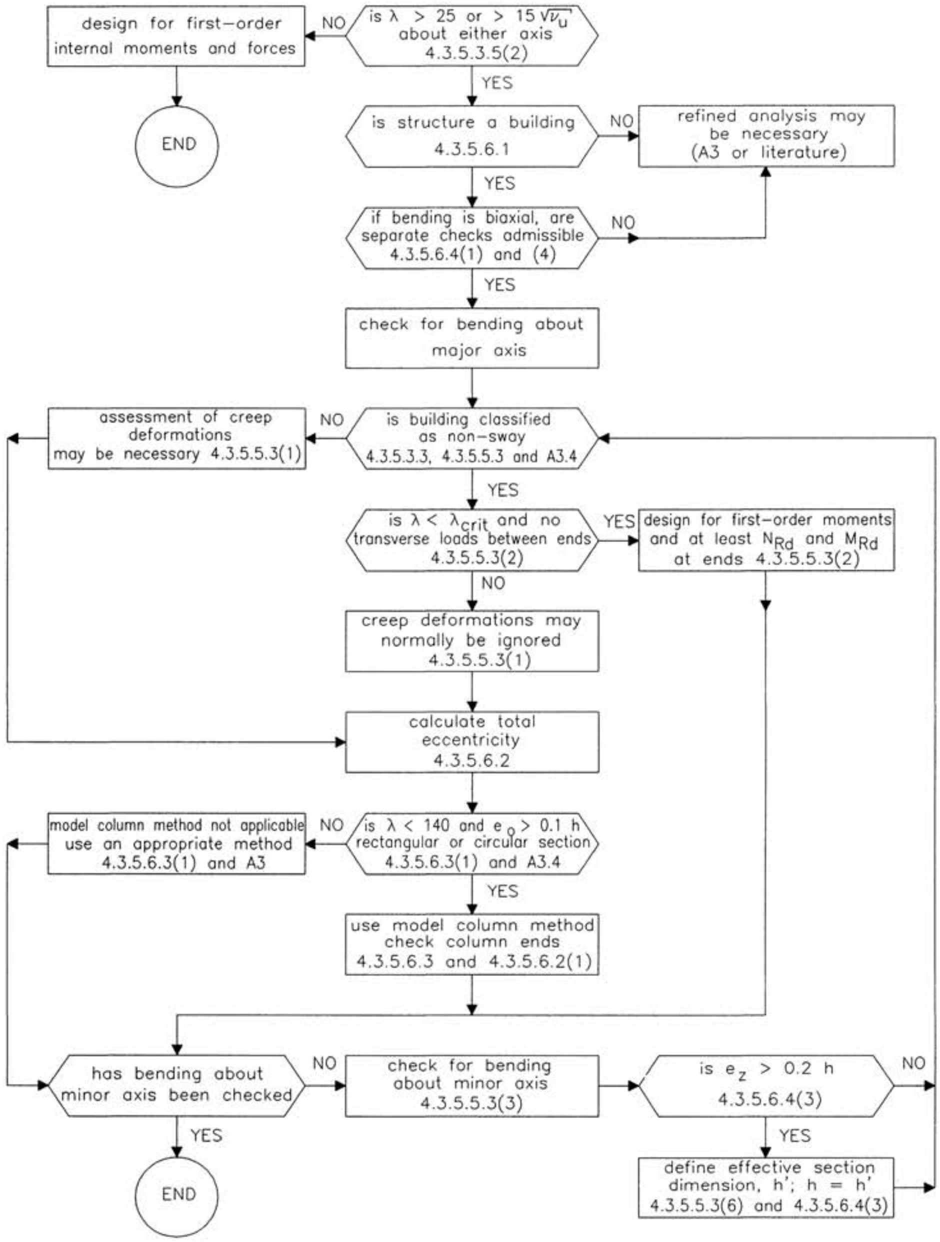
Flow chart 3.1.5.2

Buckling: structure as a whole



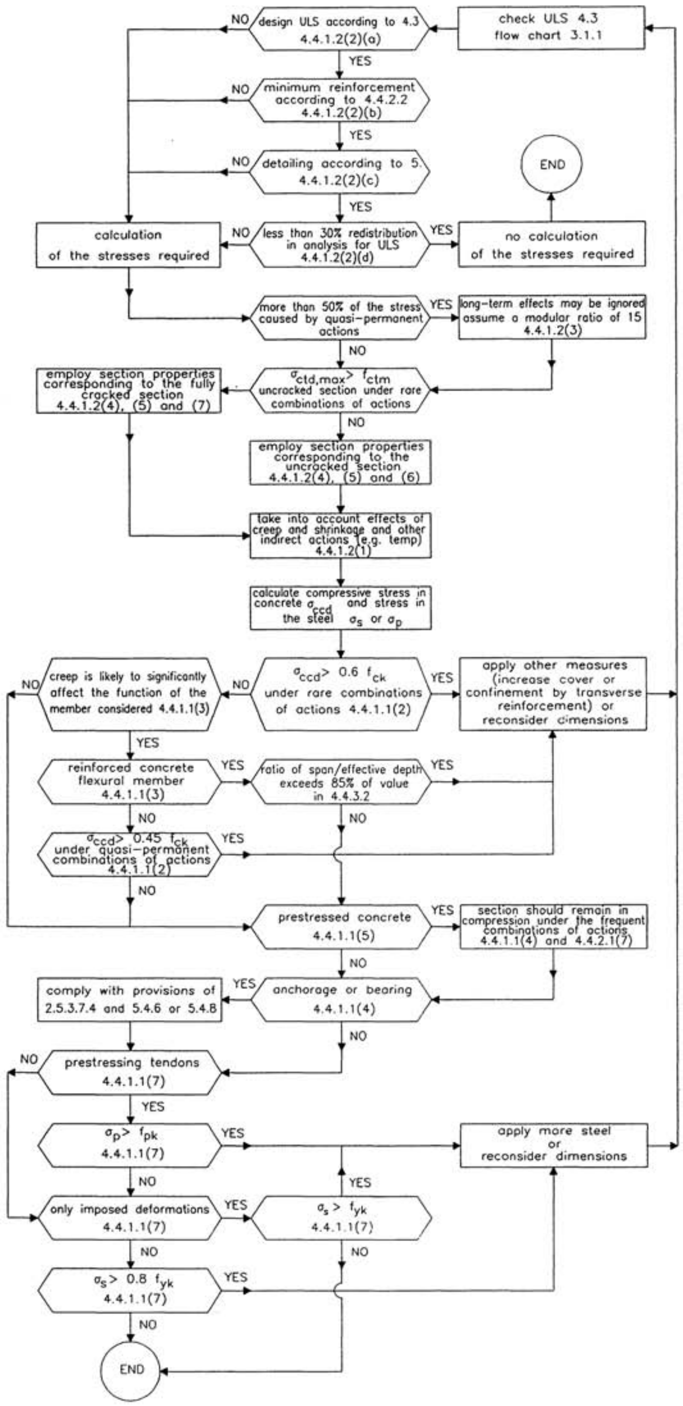
Flow chart 3.1.5.3

Buckling: isolated columns



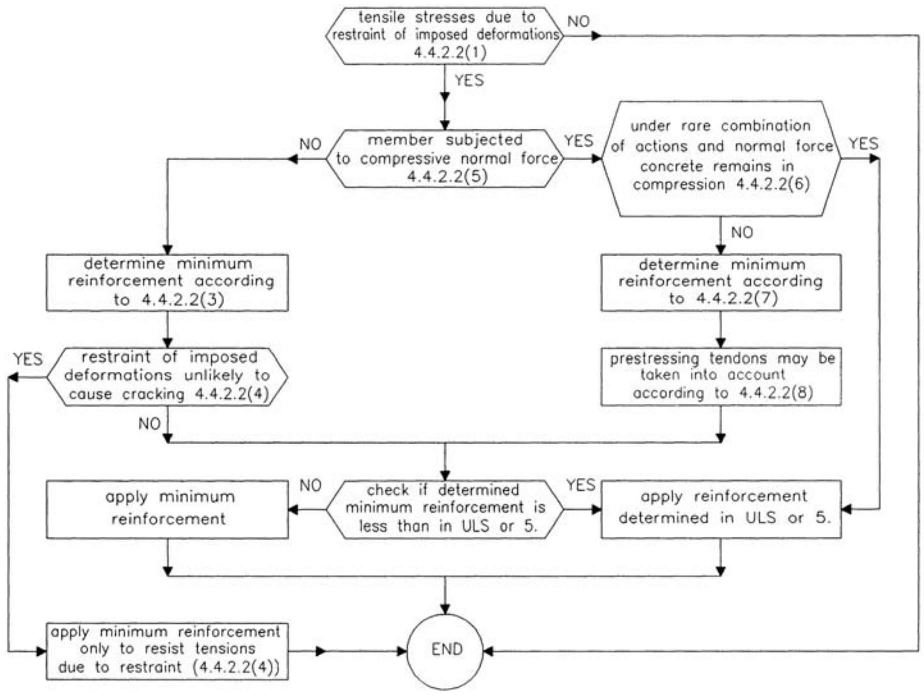
Flow chart 3.2.1.1.

Stresses: limitation of stresses



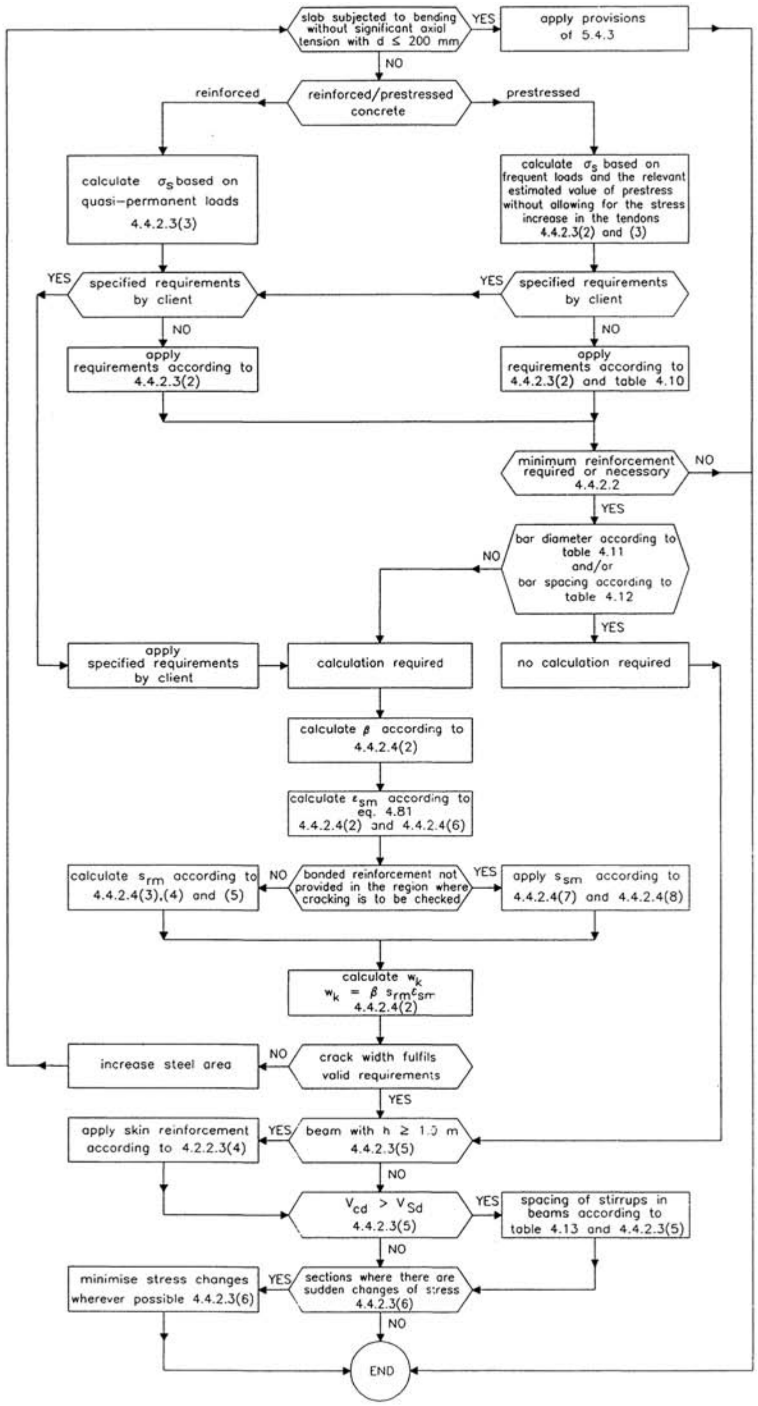
Flow chart 3.2.2.1

Cracking: minimum reinforcement



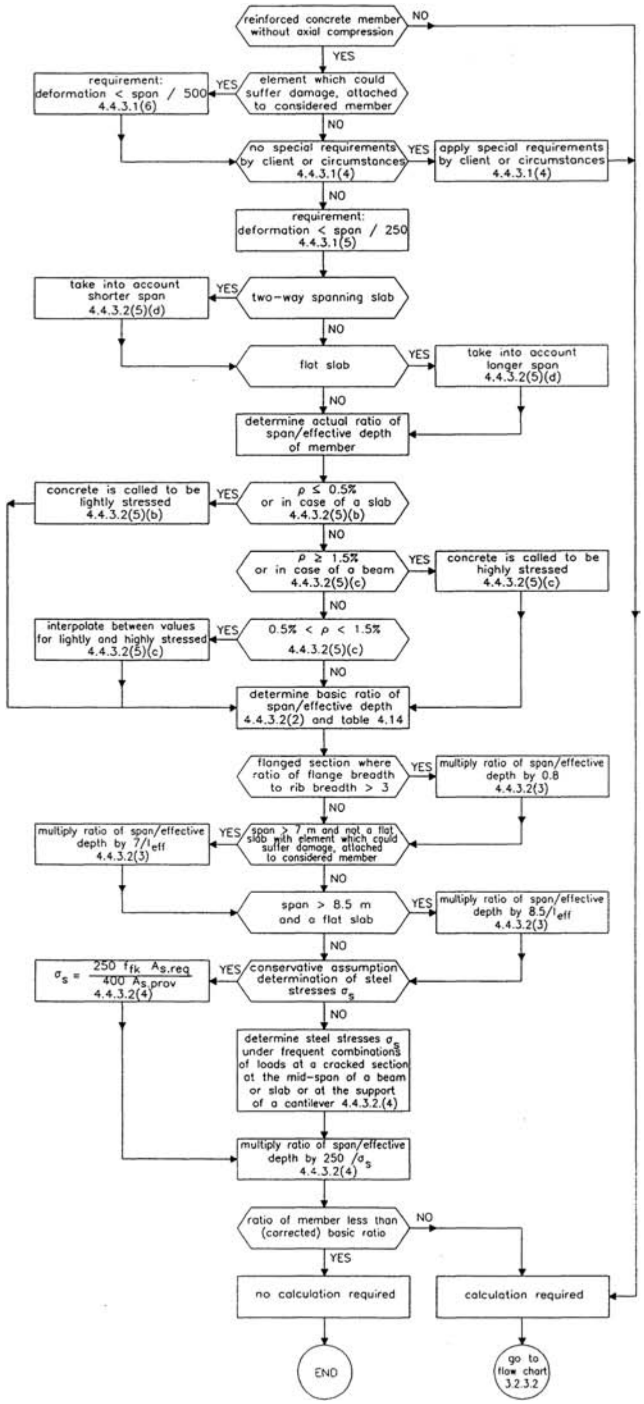
Flow chart 3.2.2.2

Cracking: with or without calculation



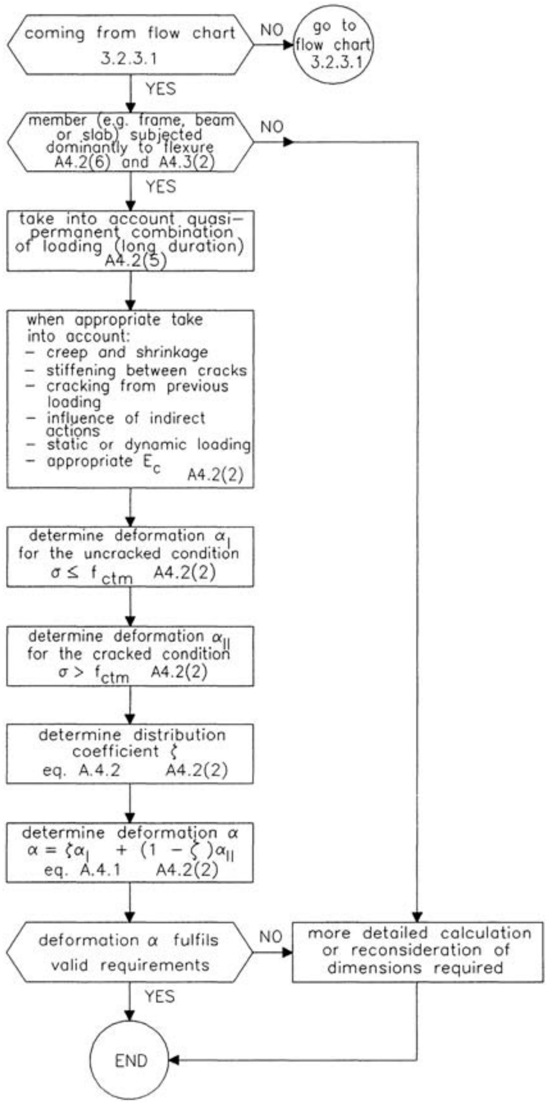
Flow chart 3.2.3.1

Deformation: deformation without calculation



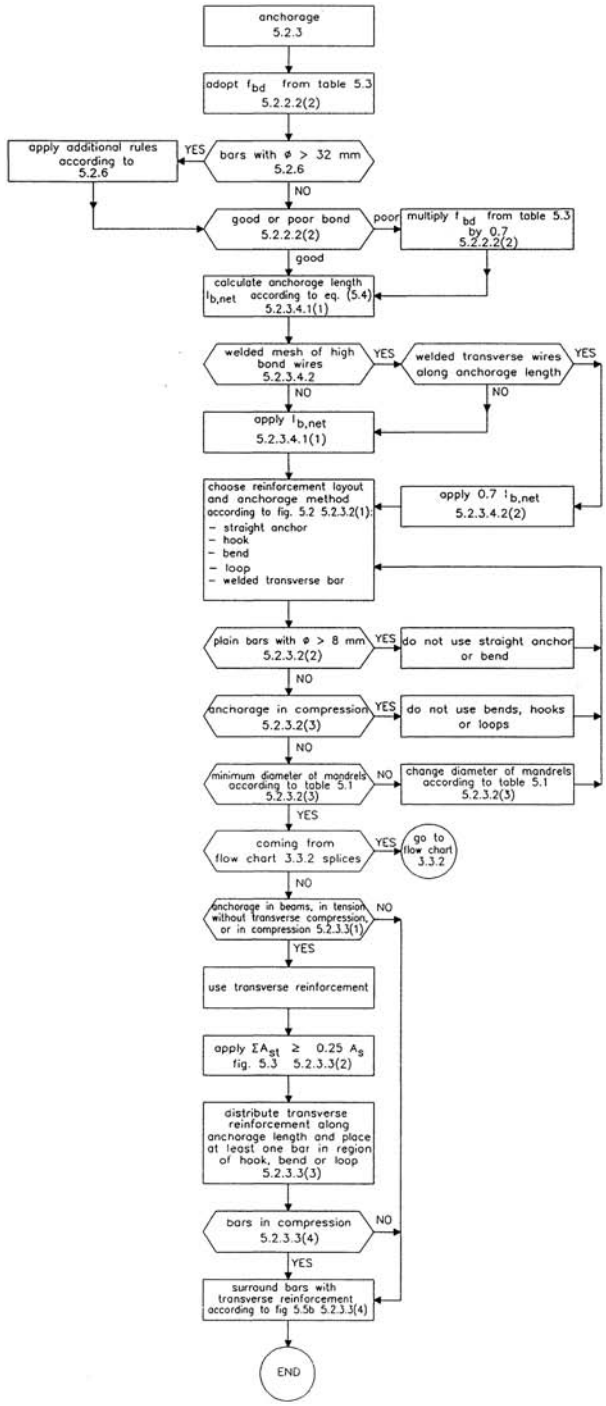
Flow chart 3.2.3.2

Deformation: deformation by calculation



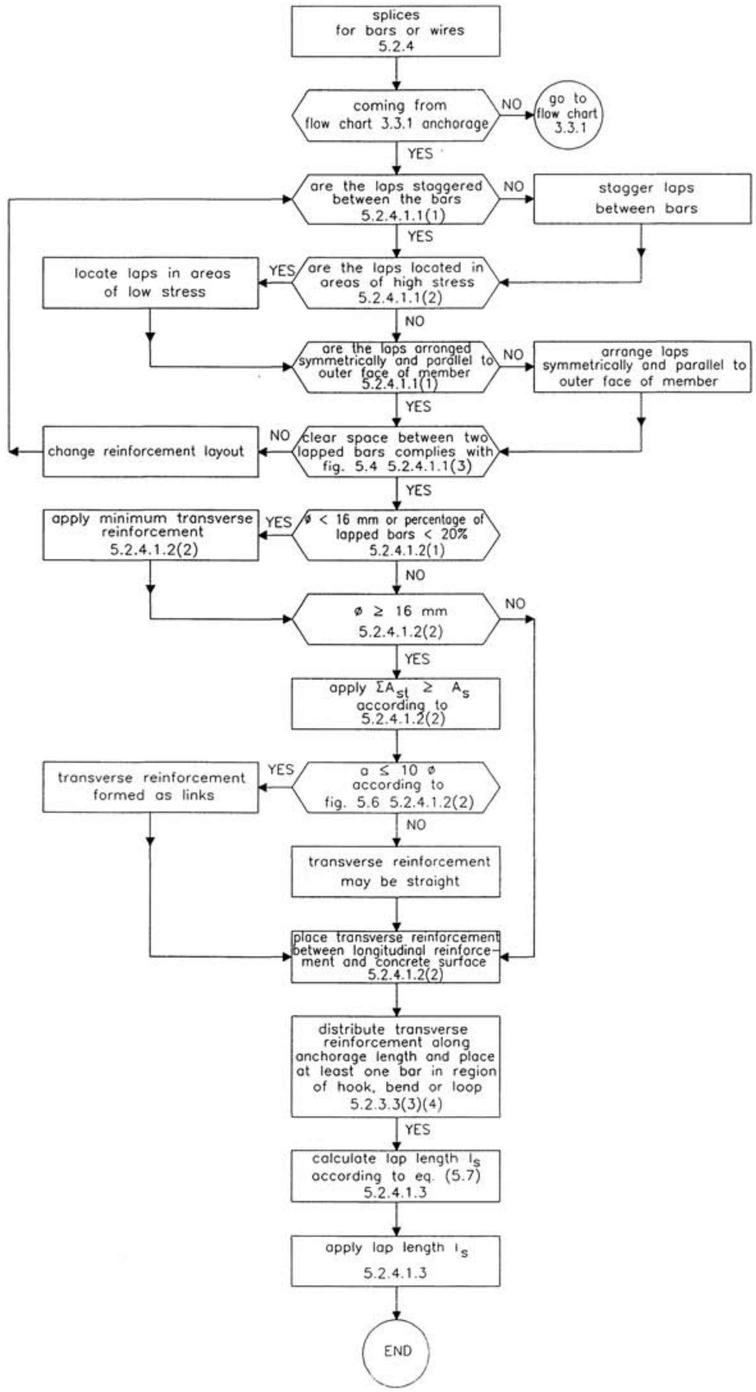
Flow chart 3.3.1.1

Anchorage: general



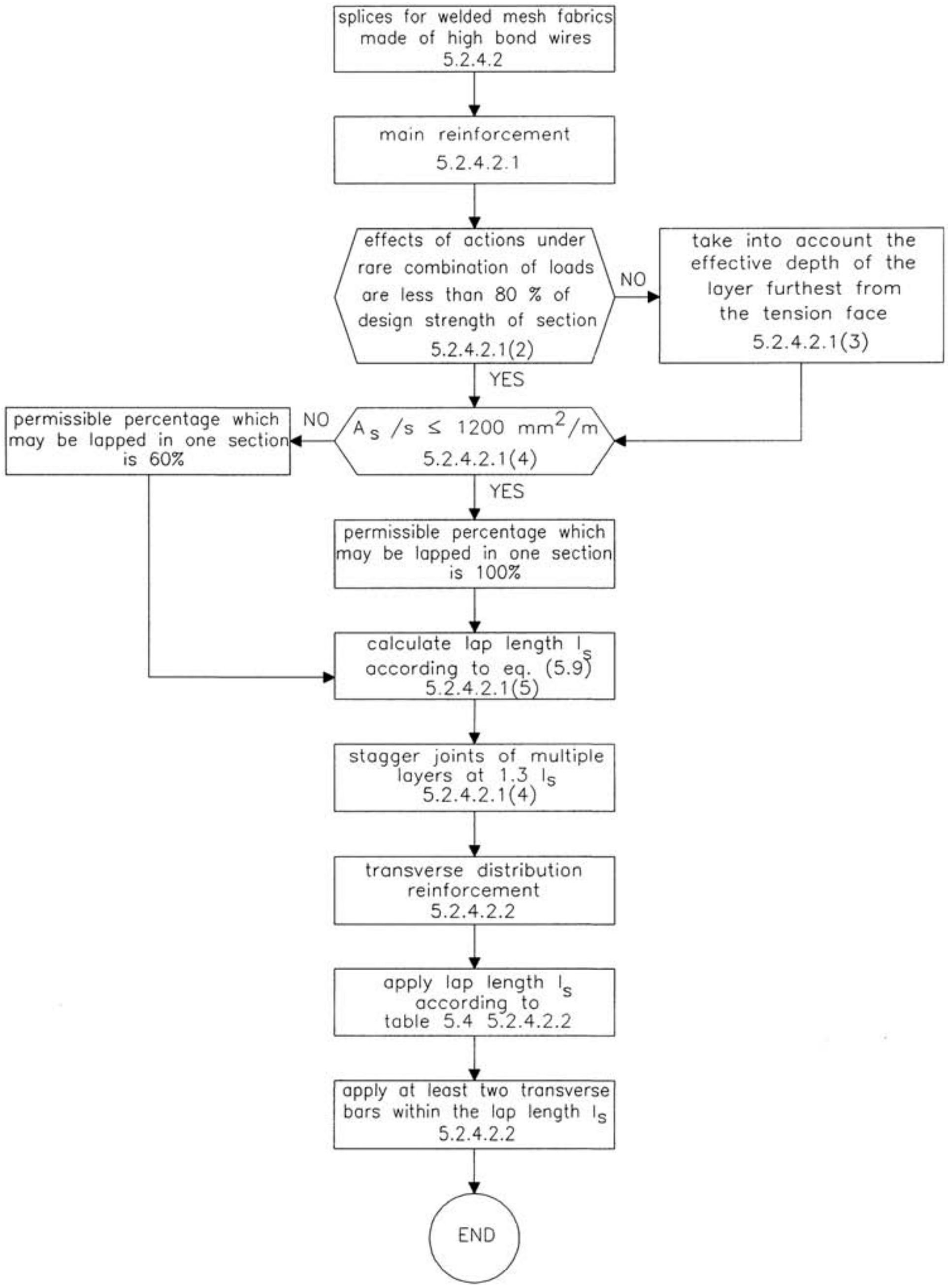
Flow chart 3.3.2.1

Splices: splices for bars or wires



Flow chart 3.3.2.2

Splices: splices for welded mesh fabrics



4
Design requirements

Throughout the following, the numbers on the right refer to relevant clauses of EC2 and square brackets - [ ] - in these references refer to relevant formulae in EC2. Please note, however, that square brackets in text indicate boxed values in the appropriate NAD.

4.1
Combinations of actions

Ultimate limit states 2.3.2.2

Fundamental combinations

$$\sum(\gamma_{G,j}G_{k,j}) + \gamma_{Q,1}Q_{k,1} + \sum_{i>1}(\gamma_{Q,i}\psi_{0,i}Q_{k,i})$$ [2.7(a)]

Accidental combinations

$$\sum(\gamma_{GA,j}G_{k,j}) + A_d + \psi_{1,1}Q_{k,1} + \sum_{i>1}(\psi_{2,i}Q_{k,i})$$ [2.7(b)]

$G_{k,j}$=characteristic values of permanent actions
$Q_{k,1}$ = characteristic value of one of the variable actions
$Q_{k,i}$ = characteristic values of the other variable actions
A_d = design value (specified value) of the accidental actions
$\gamma_{G,j}$ = partial safety factors for any permanent action j
$\gamma_{GA,j}$ as $\gamma_{G,j}$ but for accidental design situations
$\gamma_{Q,i}$ = partial safety factors for any variable action i
ψ_0, ψ_1 ψ_2 combination coefficients to determine the combination, frequent and quasi-permanent values of variable actions

In expressions [2.7(a)] and [2.7(b)], prestressing shall be introduced where relevant.

Simplified method for fundamental combinations 2.3.3.1(8)

One variable action

$$\sum(\gamma_{G,j}G_{k,j}) + [1.5]\, Q_{k,1}$$ [2.8(a)]

Two or more variable actions

$$\sum(\gamma_{G,j}G_{k,j}) + [1.35]\sum_{i\geq 1}Q_{k,i}$$ [2.8(b)]

whichever gives the larger value

For the boxed values, apply the values given in the appropriate NAD.

Serviceability limit states 2.3.4

Rare combinations

$$\sum G_{k,j}\ (+P) + Q_{k,1} + \sum_{i>1}(\psi_{0,i}Q_{k,i})$$ [2.9(a)]

Frequent combinations

$$\sum G_{k,j}\ (+P) + \psi_{1,1}Q_{k,1} + \sum_{i>1}(\psi_{2,i}Q_{k,i})$$ [2.9(b)]

Quasi-permanent combinations

$$\sum G_{k,j}\ (+P) + \sum_{i\geq 1}(\psi_{2,i}Q_{k,i})$$ [2.9(c)]

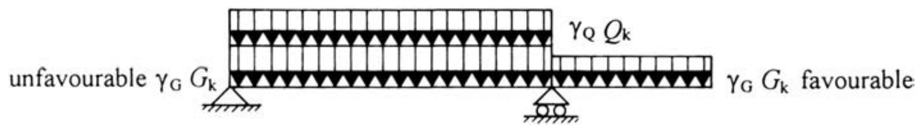


Figure 4.1 Maximum (positive) bending moment in middle of span and maximum shear at bearings of span.

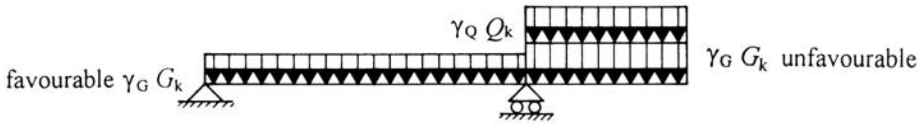


Figure 4.2 Minimum (positive or negative) bending moment in middle of span and maximum (negative) bending moment and maximum shear at bearing of cantilever.

P=prestressing force

Simplified method for rare combinations 2.3.4(6)

One variable action

$$\sum G_{k,j} + Q_{k,1} \qquad [2.9(d)]$$

Two or more variable actions

$$\sum G_{k,j} + 0.9\sum_{i\geq 1} Q_{k,i} \qquad [2.9(e)]$$

whichever gives the larger value.

Permanent actions

Where the results of a verification may be very sensitive to variations of the magnitude of a permanent action from place to place in the structure, the unfavourable and the favourable parts of this action shall be considered as individual actions in ULS (2.3.2.3(3)).

For beams and slabs in buildings with cantilevers subjected to dominantly uniformly distributed loads, this requirement leads to the following decisive combinations of actions (see Figures 4.1 and 4.2):

For continuous beams and slabs in buildings without cantilevers subjected to dominantly uniformly distributed loads, it will generally be sufficient to consider only the two load cases in ULS (2.5.1.2(4)): alternate spans carrying the design variable and permanent loads ($\gamma_Q Q_k+\gamma_G G_k$), other spans carrying only the design permanent load ($\gamma_G G_k$) (2.5.1.2(4)(a)) (see Figure 4.3); any two adjacent spans carrying the design variable and permanent loads ($\gamma_Q Q_k+\gamma_G G_k$), other spans carrying only the design permanent load ($\gamma_G G_k$) (2.5.1.2(4)(b)) (see Figure 4.4).

4.2 Categories and values of imposed loads

Categories of imposed loads (Eurocode 1, part 2.1 (ENV 1991–2–1))

Areas of dwelling, offices, etc.

Category A Areas for domestic and residential activities,

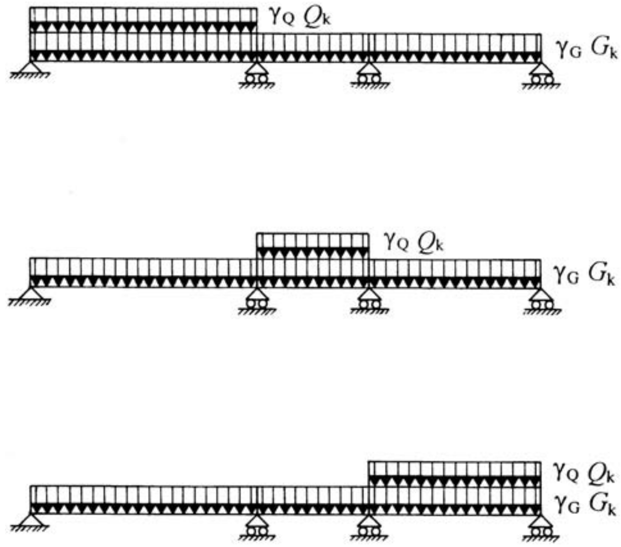


Figure 4.3 Alternate spans carrying the design variable load.

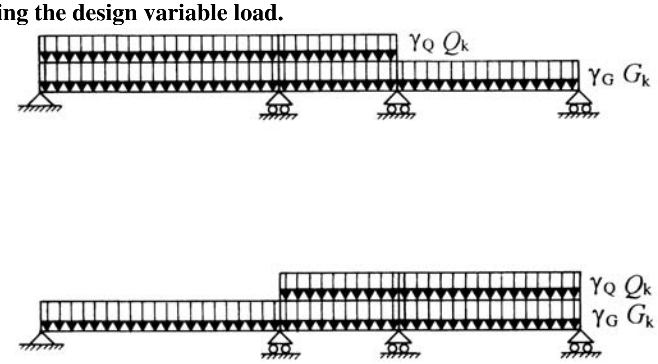


Figure 4.4 Two adjacent spans carrying the design variable load.

e.g. rooms in residential buildings and houses; rooms and wards in hospitals; bedrooms in hotels and hostels; kitchens and toilets.

Category B Office areas

Category C Areas where people may congregate (with the exception of areas defined under categories A, B, D and E)

- C1 areas with tables, etc.
  e.g. areas in schools, cafés, restaurants, dining halls, reading rooms, receptions, etc.
- C2 areas with fixed seats,
  e.g. areas in churches, theatres or cinemas, conference rooms, lecture halls, assembly halls, waiting rooms, etc.
- C3 obstacle-free areas for moving people,
  e.g. areas in museums, exhibition rooms, and access areas in public and administration buildings, hotels, etc.
- C4 areas with possible physical activities,
  e.g. dance halls, gymnasiums, stages, etc.
- C5 areas susceptible to overcrowding,
  e.g. in buildings for public events like concert halls, sports halls including stands, terraces and access areas, etc.

Category D Shopping areas

- D1 areas in general retail shops
- D2 areas in department stores,
  e.g. areas in warehouses, stationery and office stores, etc.

Category E Areas susceptible to accumulation of goods, including access areas

Areas for storage including libraries. The loads defined in Table 4.1 with values of imposed loads shall be taken as minimum loads unless more appropriate loads are defined for the specific case

Garage and vehicle traffic areas

Category F Traffic and parking areas for light vehicles (30 kN total weight and 8 seats excluding driver)
Category G Traffic and parking areas for medium-weight vehicles (>30 kN, 60 kN total weight, on two axles)

Areas for storage and industrial activities

Roofs

Category H Roofs not accessible except for normal maintenance, repair and cleaning
Category I Roofs accessible with occupancy according to categories A-G
Category K Roofs accessible for special services

Values of imposed loads

Table 4.1 Values of imposed loads (Eurocode 1, part 2.1 (ENV 1991–2–1))

Loaded areas			q_k (kN/m^2)	Q_k (kN)
Areas of dwellings, offices, etc.				
Category A	general		2.0	2.0
	stairs		3.0	2.0
	balconies		4.0	2.0
Category B			3.0	2.0
Category C	C1		3.0	4.0
	C2		4.0	4.0
	C3		5.0	4.0
	C4		5.0	7.0
	C5		5.0	4.0
Category D	D1		5.0	4.0
	D2		5.0	7.0
Category E			6.0	7.0
Garage and vehicle traffic areas				
Category F			2.0	10
Category G			5.0	45
Areas for storage and industrial activities			to be specified	to be specified
Roofs				
Category H	roof slope:	<20°	0.75*	1.5
		>40°	0.00*	1.5
Category I			according to categories A-G	according to categories A-G
Category K			to be specified	to be specified

* For roof slopes between 20° and 40°, q_k may be determined by linear interpolation

4.3
ψ factors

Table 4.2 ψ factors (Eurocode 1, part 2.1 (ENV 1991–2–1))

Loaded areas	ψ_0	ψ_1	ψ_2
Areas of dwelling, offices, etc.			
Category A	0.7	0.5	0.3
Category B	0.7	0.5	0.3

Loaded areas	ψ_0	ψ_1	ψ_2
Category C	0.7	0.7	0.6
Category D	0.7	0.7	0.6
Category E	1.0	0.9	0.8
Garage and vehicle traffic areas			
Category F	0.7	0.7	0.6
Category G	0.7	0.5	0.3
Areas for storage and industrial activities	to be specified	to be specified	to be specified
Roofs			
Category H	0.0	0.0	0.0
Category I	according to categories A-G	according to categories A-G	according to categories A-G
Category K	to be specified	to be specified	to be specified

4.4
Partial safety factors for actions

Table 4.3 Partial safety factors for actions (Eurocode 1, part 1 (ENV 1991–1: 1993))

Case[(1)]	**Action**	**Symbol**	**Situations**
P/T	**A**		
Case A Loss of static equilibrium; strength of structural material or ground insignificant (see 9.4.1)			
Permanent actions: self-weight of structural and non-structural components, permanent actions caused by ground-water and free water			
- unfavourable	$\gamma_{Gsup}{}^{(2,4)}$	[1.10]	[1.00]
- favourable	$\gamma_{Ginf}{}^{(2,4)}$	[0.90]	[1.00]
Variable actions			
- unfavourable	γ_Q	[1.50]	[1.00]
Accidental actions	γ_A		[1.00]
Case B[(5)] Failure of structure or structural elements, including those of the footing, piles, basement walls, etc., governed by strength of structural materials (see 9.4.1)	Permanent actions[(6)] (see above)		
- unfavourable	$\gamma_{Gsup}{}^{(3,4)}$	[1.35]	[1.00]
- favourable	$\gamma_{Ginf}{}^{(3,4)}$	[1.00]	[1.00]
Variable actions			
- unfavourable	γ_Q	[1.50]	[1.00]
Accidental actions	γ_A		[1.00]
Case C[(5)] Failure in the ground			
Permanent actions (see above)			
- unfavourable	$\gamma_{Gsup}{}^{4)}$	[1.00]	[1.00]
- favourable	$\gamma_{Ginf}{}^{4)}$	[1.00]	[1.00]
Variable actions			
- unfavourable	γ_Q	[1.30]	[1.00]
Accidental actions	γ_A		[1.00]
P: Persistent situation	T: Transient situation	A: Accidental situation	

NOTES
1. The design should be separately verified for each case A, B and C as relevant.
2. In this verification, the characteristic value of the unfavourable part of the permanent action is multiplied by the factor 1.1 and the favourable part by 0.9. More refined rules are given in ENV 1993 and ENV 1994.

Case[1]	Action	Symbol	Situations
P/T	A		

3. In this verification, the characteristic values of all permanent actions from one source are multiplied by 1.35 if the total effect of the resulting action is unfavourable and by 1.0 if the total effect of the resulting action is favourable.
4. When the limit state is sensitive to variations of permanent actions, the upper and lower characteristic values of these actions should be taken according to 4.2 (3).
5. For cases B and C, the design ground properties may be different: see ENV 1997–1–1.
6. Instead of using γ_G (1.35) and γ_Q (1.50) for lateral earth pressure actions, the design ground properties may be introduced in accordance with ENV 1997 and a model factor γ_{Sd} applied.

For the boxed values, apply the values given in the appropriate NAD.

Table 4.4 Partial safety factors for actions (Eurocode 2, part 1 (ENV 1992–1–1: 1991))

	Permanent actions (γ_G)	**Variable actions (γ_Q)**		**Prestressing (γ_P)**
One with its characteristic value	Others with their combination value			
Favourable effect	[1.00]	-	-	[0.9] or [1.0]
Unfavourable effect	[1.35]	[1.50]	[1.50]	[1.2] or [1.0]

4.5 Partial safety factors for materials

Table 4.5 Partial safety factors for materials (Eurocode 2, part 1 (ENV 1992–1–1: 1991))

Combination	**Concrete (γ_c)**	**Steel reinforcement or prestressing tendons (γ_s)**
Fundamental	[1.50]	[1.15]
Accidental (except earthquakes)	[1.30]	[1.00]

For the boxed values, apply the values given in the appropriate NAD.

5
Calculation methods

5.1
Flat slabs

5.1.1
Introduction

Slabs are classified as flat slabs when they transfer loads to columns directly without any beam supports. Slabs may be solid or coffered (ribbed in two directions). Unlike two-way spanning slabs, flat slabs can fail by yield lines in either of the two orthogonal directions. Flat slabs should therefore be designed to carry the total load on the panel in each direction.

EC2 does not provide any specific guidance for the analysis of the flat slabs. The methods given are based on common practice in a number of countries in Europe. General methods of analysis include: (a) equivalent frame method; (b) use of simplified coefficients; (c) yield-line analysis; and (d) grillage analysis.

5.1.2
Equivalent frame method

The structure is divided in two orthogonal directions into frames consisting of columns and strips of slab acting as "beams". The width of the slab to be used as "beams" is determined as follows:

For vertical loading,

when $ly<2lx$,

width in x-direction	=	$0.5(l_{x1}+l_{x2})$
width in y-direction	=	$0.5(l_{x1}+l_{y2})$

when $l_y>2l_x$,

width in x-direction	=	$0.5\ (l_{x1}+l_{x2})$
width in y-direction	=	$(l_{x1}+l_{x2})$

In these expressions, l_x and l_y are the shorter and longer spans respectively and l_{x1} and l_{x2} refer to the lengths of adjacent spans in x-direction. The stiffness of the "beams" for analysis should be based on the widths calculated above. When the loading is horizontal, the stiffness used in analysis should be taken as half that derived for vertical loading, to allow for uncertainties associated with the slab-column joints.

Analysis

A braced structure may be analysed using any of the standard linear elastic methods such as moment distribution method. The structure may be analysed as a whole or split into sub-frames consisting of the slab at any one level and the columns. The remote ends of the columns are normally treated as fixed unless they are obviously not.

Lateral distribution of moments

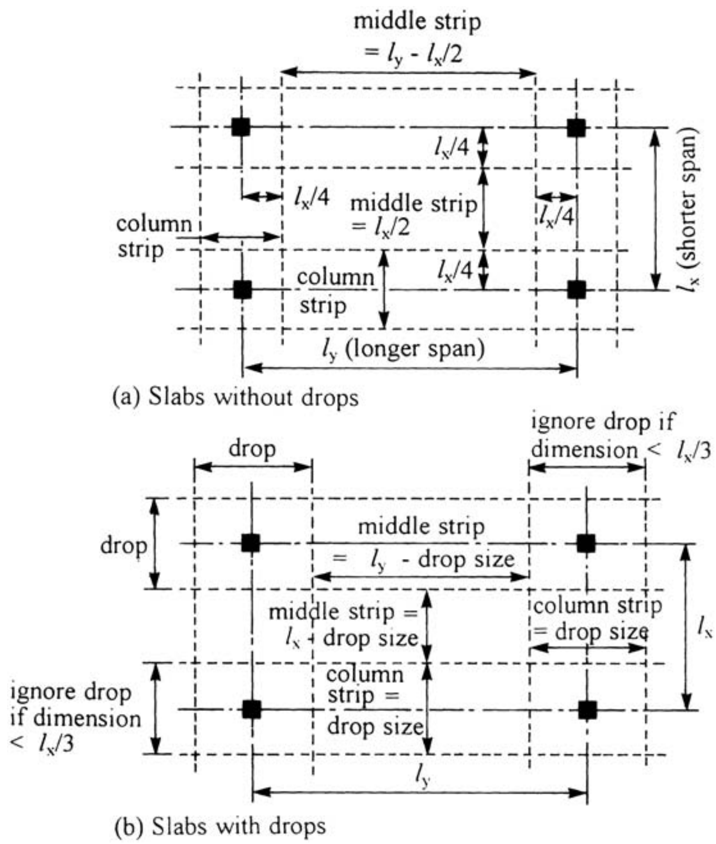


Figure 5.1: Division of slab into strips.

The slab should be divided into column and middle strips as shown in Figure 5.1. The slab bending moments obtained from analysis should be apportioned across the width of the slab as follows:

	Column strip	**Middle strip**
Negative moments	75%	25%
Positive moments	55%	45%

These figures are percentages of the total positive or negative moments obtained in analysis. Where the width of the column strip is taken as equal to that of a drop and thereby the width of the middle strip is increased, the design moments to be resisted by the middle strip should be increased in proportion to the increased width. The design moments in the column strip may be reduced accordingly.

Moment transfer at edge columns

The effective width to the slab through which moments are transferred between the edge (or corner) columns and slab should be calculated as shown in Figure 5.2. The maximum moment that can be transferred to the column is

$M_{max}=0.167b_e d^2 f_{ck}$	for concrete grades C35/45 or less;
$M_{max}=0.136b_e d^2 f_{ck}$	for concrete grades C40/50 or greater.

The structure should be sized so that M_{max} is at least 50% of the moment obtained from an elastic analysis.

When the bending moment at the outer support obtained from the analysis exceeds M_{max}, the moment at this support should be limited to M_{max} and the moment in the span should be increased accordingly.

5.1.3 Use of simplified coefficients

Bending moments using the coefficients given below may be used for flat slabs where:

(a) the structure consists of at least three spans; and

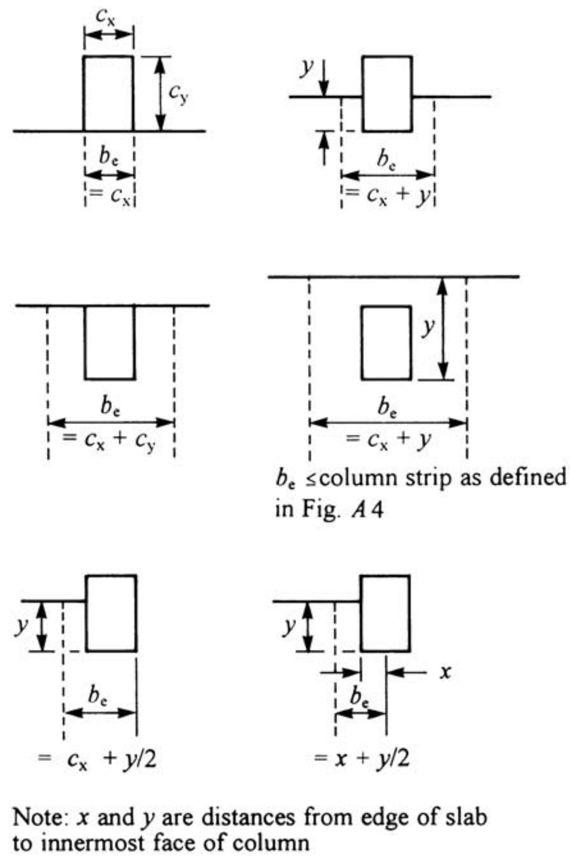


Figure 5.2

(b) the ratio of the longest to the shortest span does not exceed 1.2; and
(c) the loading is predominantly uniformly distributed

At outer support	Near middle of end span	At first interior support	At middle of interior spans	At interior supports
0	0.09*Fl*	0.11*Fl*	0.07*Fl*	0.10*Fl*

NOTES

l is the effective span. *F* is the total ultimate load on the span=$1.35G_k+1.5Q_k$. No redistribution should be carried out on the moments.

5.1.4 Reinforcement

Reinforcement should be sufficient to resist the minimum bending moment specified in Table 4.9 of EC2. The reinforcement required in each column and middle strip should be distributed uniformly. In slabs without drops, the reinforcement required to resist the negative moment in the column strips should be placed with 66% of the reinforcement within the middle half of the strip.

5.2 Strut-and-tie models

Strut-and-tie models may be used for structural analysis, where the assumption of linear strain distribution through the structure is not valid. This powerful plastic method is useful in a number of instances, including anchorage zones of prestressed members, members with holes, pile caps, deep beams and beam-column junctions. Typical models are shown in Figure 5.3.

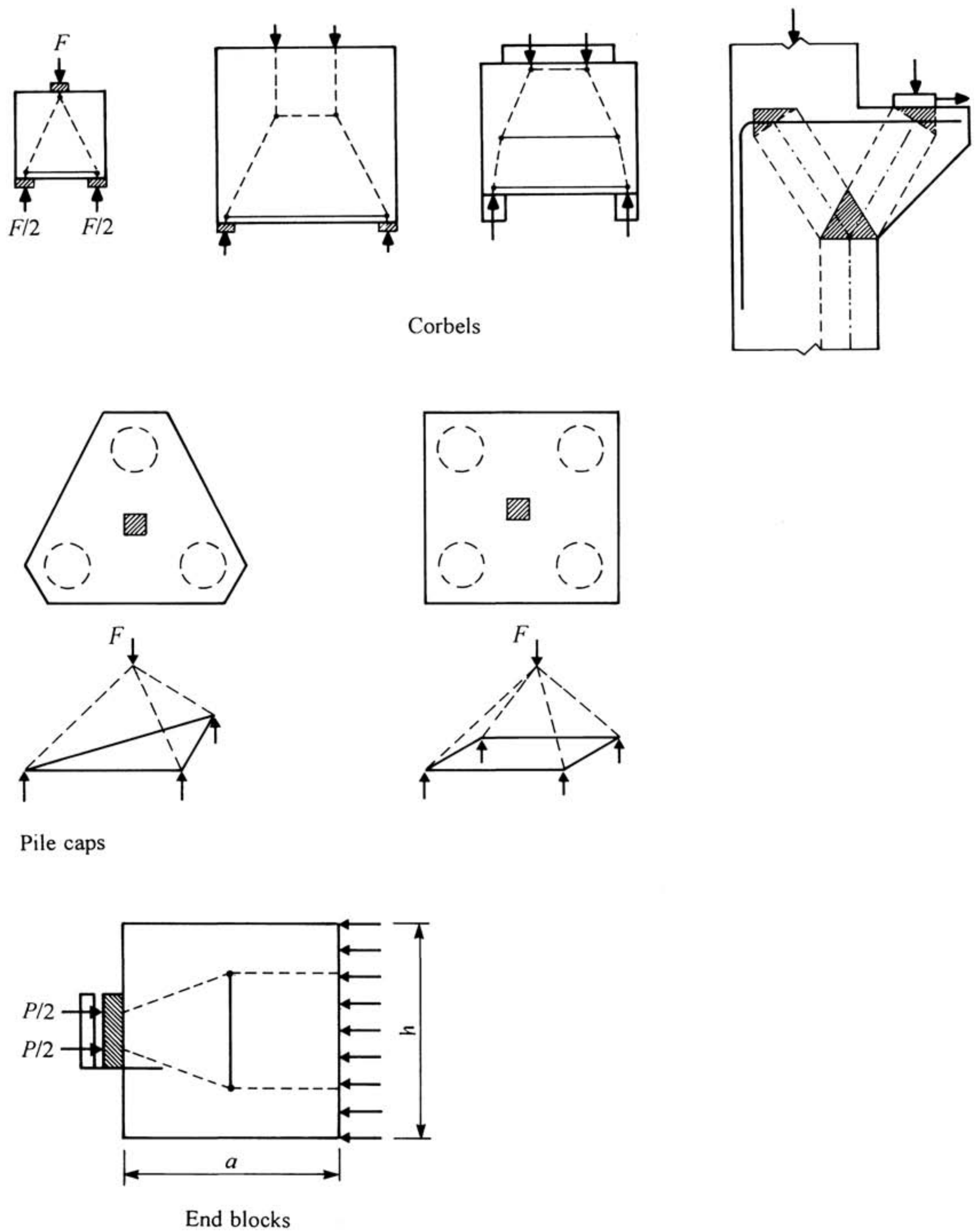


Figure 5.3 Typical strut-and-tie models.

The structure is divided into struts (concrete) and ties (reinforcement bars). The model should reflect closely the elastic stress trajectories. In general, the angle between the struts and ties should not be less than 30°. Internal stresses are calculated so that equilibrium with external loads is achieved.

Limiting permissible stresses are as follows.

Reinforcement ties	f_{yd}
Struts under uniaxial stress	$0.6f_{cd}$
Struts under triaxial stress	$1.0f_{cd}$

6
Material properties

6.1
Concrete

Material properties of concrete (Eurocode 2, part 1 (ENV 1992–1–1: 1993))

Strength class (N/mm^2)	f_{ck}	f_{cm} (1)	f_{cd}	$\alpha f_{ck}/\gamma_c$ (2)	f_{ctm}	$f_{ctk\ 0.05}$	$f_{ctk\ 0.95}$	τ_{Rd}	E_{cm} (1)	E_{cd} (1)	ε_{cu} (1) (‰)	ε_{cu} (2) (‰)
C12/15	12	20	8.0	6.4	1.6	1.1	2.0	0.18	26000	17300	3.6	3.5
C16/20	16	24	10.7	9.1	1.9	1.3	2.5	0.22	27500	18300	3.5	3.5
C20/25	20	28	13.3	11.3	2.2	1.5	2.9	0.26	29000	19300	3.4	3.5
C25/30	25	33	16.7	14.2	2.6	1.8	3.3	0.30	30500	20300	3.3	3.5
C30/37	30	38	20.0	17.0	2.9	2.0	3.8	0.34	32000	21300	3.2	3.5
C35/45	35	43	23.3	19.8	3.2	2.2	4.2	0.37	33500	22300	3.1	3.5
C40/50	40	48	26.7	22.7	3.5	2.5	4.6	0.41	35000	23300	3.0	3.5
C45/55	45	53	30.0	25.5	3.8	2.7	4.9	0.44	36000	24000	2.9	3.5
C50/60	50	58	33.3	28.3	4.1	2.9	5.3	0.48	37000	24700	2.8	3.5

NOTES
1. Structural analysis of sections with a rectangular compression zone; take into account f_{cm} and E_{cm} or f_{cd} and E_{cd}
2. Cross-section design

f_{ck}	=	characteristic compressive cylinder strength of concrete at 28 days in N/mm^2
f_{cm}	=	mean value of compressive cylinder strength of concrete at 28 days in N/mm^2
	=	f_{ck}+[8](N/mm2)
f_{cd}	=	design value of compressive cylinder strength of concrete at 28 days in N/mm^2
	=	f_{ck}/γ_c where γ_c=partial safety factor for concrete=[1.5]; if γ_c 1.5, multiply by 1.5/γ_c
$\alpha \frac{f_{ck}}{\gamma_c}$	=	reduced design compressive cylinder strength of concrete at 28 days in N/mm^2
α	=	coefficient taking account of long-term effects on the compressive cylinder strength of concrete and of unfavourable effects resulting from the way the load is applied
	=	[0.85]; if α 0.85, multiply by α/0.85
γ_c	=	[1.5]; if γ_c 1.5, multiply by 1.5/γ_c
f_{ctm}	=	mean value of the axial tensile strength of concrete at 28 days in N/mm^2
$f_{ctk\ 0.05}$	=	lower characteristic axial tensile strength (5%-fractile) of concrete at 28 days in N/mm^2
	=	0.7f_{ctm}
$f_{ctk\ 0.95}$	=	upper characteristic axial tensile strength (95%-fractile) of concrete at 28 days in N/mm^2
	=	1.3f_{ctm}
τ_{Rd}	=	basic design shear strength of concrete at 28 days in N/mm^2= $\frac{025 f_{ctk\ 0.05}}{\gamma_c}$ with γ_c=[1.5]; if γ_c 1.5, multiply by 1.5/ γ_c
E_{cm}	=	mean value of secant modulus of elasticity of concrete in N/mm^2
	=	$9.5*10^3(f_{ck}+8)^{1/3}$
E_{cd}	=	design value of secant modulus of elasticity of concrete in N/mm^2=E_{cd}/γ_c with γ_c=[1.5]; if γ_c 1.5, multiply by 1. 5/γ_c
ε_{cu}	=	ultimate compressive strain in the concrete in ‰

For the boxed values, apply the values given in the appropriate NAD.

6.2
Reinforcing steel

Material properties of reinforcing steel (Eurocode 2, part 1 (ENV 1992–1–1: 1993) and ENV 10080: 1994)

Steel name	f_{tk} (N/mm^2)	f_{td} (N/mm^2)	f_{yk} (N/mm^2)	f_{yd} (N/mm^2)	ε_{uk} (%)
B500A	525	455	500	435	2.5[(1)]
B500B	540	470	500	435	5.0

NOTES
1. 2.0% for bars with d=5.0 and 5.5mm, where d is diameter of bar in mm

f_{tk} = characteristic tensile strength of reinforcing steel in N/mm^2
f_{td} = design tensile strength of reinforcing steel in N/mm2 f_{tk}=γ_c
γ_s = partial safety factor for reinforcing steel=[1.15]; if γ_s 1.15, multiply by 1.15/γ_s
f_{yk} = characteristic yield stress of reinforcing steel in N/mm^2
f_{yd} = design yield stress of reinforcing steel in N/mm^2 f_{yk}/γ_s with γ_s=[1.15]; if γ_s 1.15, multiply by 1.15/γ_s
$f_{0.2k}$ = characteristic 0.2% proof-stress of reinforcing steel in N/mm^2
$f_{0.2d}$ = design 0.2% proof-stress of reinforcing steel in N/mm^2=$f_{0.2k}/\gamma_s$
ε_{uk} = characteristic elongation of reinforcing steel at maximum load in %
$(f_t/f_y)_k$ = characteristic ratio of tensile strength to yield stress
E_s = modulus of elasticity of reinforcing steel E_s=2*10^5 N/mm^2
Density=7850 kg/m^3.
Coefficient of thermal expansion=10^{-5}/°C

Bond characteristics

Ribbed bars: resulting in high bond action (as specified in EN 10080)
Plain, smooth bars: resulting in low bond action

Ductility characteristics

High ductility:	ε_{uk}>[5.0]%	and	$(f_t/f_y)_k$>[1.08]
Normal ductility:	ε_{uk}>[2.5] %	and	$(f_t/f_y)_k$>[1.05]

For the boxed values, apply the values given in the appropriate NAD.

6.3
Prestressing steel

Material properties of prestressing steel (Eurocode 2, part 1 (ENV 1992–1–1:1993) and ENV 10138: 1994)

Wires

Steel name	f_{pk} (N/mm2)	f_{pd} (N/mm2)	$f_{p0.1k}$ (N/mm^2)	$f_{p0.1d}$ (N/mm^2)	E_s (N/mm^2)	ε_{uk} (%)
Y1860C	1860	1620	1600	1390	205000	3.5
Y1770C	1770	1540	1520	1320	205000	3.5
Y1670C	1670	1450	1440	1250	205000	3.5
Y1570C	1570	1370	1300	1130	205000	3.5

Strands

Steel name	f_{pk} (N/mm^2)	f_{pd} (N/mm^2)	$f_{p0.1k}$ (N/mm^2)	$f_{p0.1d}$ (N/mm^2)	E (N/mm^2)	ε_{uk} (%)
Y2060S	2060	1790	1770	1540	195000	3.5

Steel name	f_{pk} (N/mm^2)	f_{pd} (N/mm^2)	$f_{p0.1k}$ (N/mm^2)	$f_{p0.1d}$ (N/mm^2)	E (N/mm^2)	ε_{uk} (%)
Y1960S	1960	1700	1680	1460	195000	3.5
Y1860S	1860	1620	1600	1639	195000	3.5
Y1770S	1770	1540	1520	1250	195000	3.5

Bars

Steel name	f_{pk} (N/mm^2)	f_{pd} (N/mm^2)	$f_{p0.1k}$ (N/mm^2)	$f_{p0.1d}$ (N/mm^2)	E_s (N/mm^2)	ε_{uk} (%)
Y1030	1030	900	830	720	205000	4.0
Y1100	1100	960	900	780	205000	4.0
Y1230	1230	1070	1080	940	205000	4.0

f_{pk} = characteristic tensile strength of prestressing steel in N/mm^2
f_{pd} = design tensile strength of prestressing steel in N/mm^2 $= f_{pk}/\gamma_s$
γ_s = partial safety factor for prestressing steel=[1.15]; if γ_s 1.15, multiply by 1.15/γ_s
$f_{p0.1k}$ = characteristic 0.1% proof-stress of prestressing steel in N/mm^2
$f_{p0.1d}$ = design 0.1% proof-stress of prestressing steel in N/mm^2=$f_{p0.1}/\gamma_s$ with γ_s=[1.15]; if γ_s 1.15, multiply by 1.15/γ_s
ε_{uk} = characteristic elongation of prestressing steel at maximum load in %
E_s = modulus of elasticity of reinforcement E_s=2 * 10^5 N/mm^2 (taken into account in stress-strain diagram)
Density=7850 kg/m^3
Coefficient of thermal expansion=10^{-5}/°C

Classes of relaxation

Class 1: for wires and strands, high relaxation
Class 2: for wires and strands, low relaxation
Class 3: for bars

For the boxed values, apply the values given in the appropriate NAD.

7
Basic design

Table 7.1 Exposure classes

Exposure class		**Examples of environmental conditions**
1 Dry environment		Interior of dwellings or offices
2 Humid environment	(a) Without frost	Interior of buildings with high humidity, e.g. laundries Exterior components Components in non-aggressive soil and/or water
	(b) With frost	Exterior components exposed to frost Components in non-aggressive soil and/or water and exposed to frost Interior components where the humidity is high and exposed to frost
3 Humid environment with frost and de-icing agents		Interior and exterior components exposed to frost and de-icing agents
4 Seawater environment	(a) Without frost	Components completely or partially submerged in seawater or in the splash zone Components in saturated salt air (coastal area)
	(b) With frost	Components partially submerged in seawater or in the splash zone and exposed to frost Components in saturated salt air and exposed to frost
The following classes may occur alone or in combination with the above		
5 Aggressive chemical environment[2]	(a)	Slightly aggressive chemical environment (gas, liquid or solid) Aggressive industrial atmosphere
	(b)	Moderately aggressive chemical environment (gas, liquid or solid)
	(c)	Highly aggressive chemical environment (gas, liquid or solid)

NOTES

1. This exposure class is valid as long as, during construction, the structure or some of its components are not exposed to more severe conditions over a prolonged period
2. Chemically aggressive environments are classified in ISO 9690. The following exposure conditions may be used:

Exposure class 5a: ISO classification A1G, A1L, A1S
Exposure class 5b: ISO classification A2G, A2L, A2S
Exposure class 5c: ISO classification A3G, A3L, A3S

Table 7.2 Minimum cover requirements for normal weight concrete

		Exposure class according to Table 7.1								
		1	2a	2b	3	4a	4b	5a	5b	5c
Minimum cover (mm)	Reinforcement	15	20	25	40	40	40	25	30	40
Prestressing steel		25	30	35	50	50	50	35	40	50

NOTES

1. For slab elements, a reduction of 5 mm may be made for exposure classes 2–5.
2. A reduction of 5 mm may be made where concrete of strength class C40/50 and above is used for reinforced concrete in exposure classes 2a–5b and for prestressed concrete in exposure classes 1–5b. However, the minimum cover should never be less than that for class 1.
3. For exposure class 5c, a protective barrier should be used to prevent direct contact with aggressive media.

Table 7.3 Durability requirements related to environmental exposure

	Exposure class								
	1	2a	2b	3	4a	4b	5a	5b	5c[1]
Maximum w/c ratio for[2]									
Plain concrete	-	0.70							
Reinforced concrete	0.65	0.60	0.55	0.50	0.55	0.50	0.55	0.50	0.45
Prestressed concrete	0.60	0.60							
Minimum cement content[2] *(kg/m^3) for*									
Plain concrete	150	200	300				200		
Reinforced concrete	260	280	280	300	300	300	280	300	300
Prestressed concrete	300	300	300				300		
Minimum air content of fresh concrete (%) for nominal maximum aggregate size of[3]									
32 mm	-	-	4[4]	4[4]	-	4[4]	-	-	-
16 mm	-	-	5	5	-	5	-	-	-
8mm	-	-	6	6	-	6	-	-	-
Frost-resistant aggregates[6]	-	-	Yes	Yes	-	Yes	-	-	-
Impermeable concrete according to clause 7. 3.1.5	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Types of cement for plain and reinforced concrete according to EN 197							Sulfate-resisting cement[5] >500 mg/kg in water or >3000 mg/kg in soil		

NOTES

These w/c ratios and cement contents are based on cements for which there is considerable experience in many countries. However, at the time this pre-standard was drafted, experience with some of the cements standardized in EN 197 was limited to local climates in some countries. Therefore, during the life of this prestandard, particularly for exposure classes 2b, 3 and 4b, the choice of cement type and composition should follow the national standards and regulations locally in force. Alternatively, cement CEI may be used generally for prestressed concrete. Other types may be used if experience of them is available and the application is permitted by the national standards or local regulations.

1. In addition, the concrete shall be protected against direct contact with aggressive media by coatings unless such protection is considered unnecessary. 2. For minimum cement content and maximum w/c ratio in this pre-standard, only cement listed in 4.1 shall be taken into account. When pozzolanic or latent hydraulic additions are added to the mix, national standards or regulations locally

Exposure class								
1	2a	2b	3	4a	4b	5a	5b	5c[(1)]

in force may state whether, and how, the minimum or maximum values may be modified. 3. With a spacing factor of the air-entrained void system <20 mm measured on the hardened concrete. 4. Applicable where the degree of saturation is high for prolonged periods of time. Other values may apply if the concrete is tested and documented to have adequate frost resistance according to the national standards or regulations locally in force. 5. The sulfate resistance of the cement shall be judged on the basis of national standards or regulations locally in force. 6. Assessed against the national standards or regulations locally in force.

Table 7.4 Strength classes to satisfy maximum water/cement ratio requirements

Strength class of cement	Water/cement ratio				
0.65	0.60	0.55	0.50	0.45	
CE 32.5	C20/25	C25/30	C30/37	C35/45	C40/50
CE 42.5	C25/30	C30/37	C35/45	C40/50	C45/55

7.5 Prestressed concrete

7.5.1 Material properties

Concrete grades should be chosen to satisfy durability requirements for particular exposure conditions. In any case, the strength class for post-tensioned work should not be less than C25/30 and for pre-tensioned work not less than C30/37.

A bilinear stress-strain diagram with a horizontal branch with a value of $(0.9f_{pk})/\gamma_s$ may be used for prestressing steel, where f_{pk} is the characteristic strength of prestressing steel and γ_s=1.15.

7.5.2 Minimum number of tendons

In isolated statically determinate members a minimum number of prestressing bars/wires/ tendons, as shown below, should be provided.

Type	Minimum number
Individual bars and wires	3
Bars and wires forming a strand or tendon	7
Tendons except strands	3

7.5.3 Initial prestressing force

The maximum tendon force is given by $P_o=A_p\ (0.8f_{pk})$ or $A_p\ (0.9f_{p0.1k})$ whichever is less, where $f_{p0.1k}$ is the characteristic 0.1 % proof-stress of the prestressing steel and A_p is the area of prestressing steel.

The prestressing force applied to the concrete immediately after tensioning (in pre-tensioned work) or after transfer (in post-tensioned work) is given by $P_{m,o}=A_p\ (0.75f_{pk})$ or $A_p\ (0.85f_{p0.1,k})$, whichever is less.

The force applied to the concrete should be calculated allowing for losses caused by: (a) friction (if applicable), short-term relaxation and elastic shortening for pre-tensioned members; and (b) duct friction, anchorage slip and elastic shortening for post-tensioned members.

7.5.4 Loss of prestress

(a) The mean effective prestressing force $P_{m,t}$, is the force at the active end of the tendon less the relevant losses, which should be calculated on the basis of experience and data relating to the materials and methods used. As a result, only the types of losses to be taken into account are indicated below. National practice conforming to the National Application Document should be adopted.

(b) The losses to be considered are anchorage slip, elastic shortening, friction in ducts, creep of concrete, shrinkage of concrete and relaxation of steel.

7.5.5
Anchorage

(a) Pre-tensioned members

The transmission length is given by $l_{bp}=\beta_b\phi$, when ϕ is the nominal size of the tendon and β_b is obtained, in the absence of other more accurate data, from the Table below.

The design value of lbp should be taken as either $0.8l_{bp}$ or $1.2l_{bp}$ whichever is more critical. The length over which the stresses across the section of concrete gradually disperse to a linear distribution may be taken as

$$l_{p,eff} = \sqrt{(l_{bpd}^2 + d^2)}$$

If the principal tensile stress at the ultimate limit state does not exceed $0.7f_{ctm}$ the anchorage is considered satisfactory. If not, the following should be satisfied.

$[(M_{sd}/z)+(V_{sd}/2)]\ \ (X/l_{bpd})\ P_o\ \ A_{p0.1k}/1.15$, where X is the distance of a section from the support.

(b) Post-tensioned members

The bearing stress behind anchorage plates caused by the force $A_p f_{ck}$ should not exceed

$$f_{Rd,u} = 0.67 f_{ck}\sqrt{A_{c1}/A_{co}} \leq 2.2 f_{ck}$$

where A_{c1} is the maximum area having the same centre of gravity and shape as the loaded area A_{co}, which it is possible to inscribe within the total area of member A_c.

Tensile stresses caused by the concentrated forces should be assessed by strut-and-tie model or other appropriate idealization and the anchorage zone should be reinforced accordingly.

Table 7.5 Minimum dimensions for fire resistance of rectangular or circular reinforced (normal weight) concrete columns

Standard fire resistance

Column width *b*/axis distance *a* (both in mm)

Column exposed on more than one side	Column exposed on one side	
R 30	150/10	100/10
R 60	200/10	120/10
R 90	240/35	140/10
R 120	280/40	160/45
R 180	360/50	200/60
R 240	450/50	300/60

NOTE
The ratio of the design effect of actions in the fire to the cold resistance of the structural element is assumed to be 0.7.

Table 7.6 Minimum dimensions for fire resistance of load-bearing reinforced (normal weight concrete walls made with siliceous aggregate

Standard fire resistance	Wall thickness/axis distance (both in mm)	
Exposed on one side	**Exposed on two sides**	
REI 30	120/10	120/10
REI 60	130/10	140/10
REI 90	140/25	170/25
REI 120	160/35	220/35
REI 180	210/55	300/55
REI 240	270/70	360/70

NOTE
The ratio of the design effect of actions in the fire to the cold resistance of the structural element is assumed to be 0.7.

Table 7.7 Minimum dimensions for fire resistance of simply supported reinforced concrete (normal weight) beams

Standard fire resistance (mm)	**Possible combinations of the average axis distance *a* and the beam width *b* (both in mm)**				**Web thickness b_w of I-beams (mm)**
R 30	*a*=25 *b*=80	*a*=15 *b*=120	*a*=10 *b*=160	*a*=10 *b*=200	80
R 60	*a*=40 *b*=120	*a*=35 *b*=160	*a*=30 *b*=200	*a*=25 *b*=300	100
R 90	*a*=55 *b*=150	*a*=45 *b*=200	*a*=45 *b*=250	*a*=35 *b*=400	100
R 120	*a*=65 *b*=200	*a*=55 *b*=240	*a*=50 *b*=300	*a*=45 *b*=500	120
R 180	*a*=80 *b*=240	*a*=70 *b*=300	*a*=65 *b*=400	*a*=60 *b*=600	140
R 240	*a*=90 *b*=280	*a*=80 *b*=350	*a*=75 *b*=500	*a*=70 *b*=700	160
a_{st}=*a*+10 mm (see note below)			a_{st}=*a* (see note below)		

a_{st}=increased axis distance of the outermost bar (tendon, wire) from the side surface of the cross-section, where steel is in a single layer

NOTES

1. For prestressed members, the axis distances should be increased by 10 mm for prestressing bars and by 15 mm for wires or strands.
2. The table applies to beams exposed to fire on three sides.
3. For beams exposed to fire on all four sides, the height should at least equal the minimum dimension b_{min} in the table for the required fire resistance and its cross-sectional area should be at least $2b_{min}^2$.
4. The minimum axis distance to any individual bars should not be less than that required for R 30 in the table nor less than half the average axis distance.

Table 7.8 Minimum dimensions for fire resistance of continuous reinforced concrete (normal weight) beams

Standard fire resistance (mm)	**Possible combinations of the average axis distance *a* and the beam width *b* (both in mm)**			**Web thickness b_w of I-beams (mm)**
R 30	*a*=12 *b*=80		*a*=20 *b*=200	80
R 60	*a*=25 *b*=120	*a*=12 *b*=200	*a*=25 *b*=300	100
R 90	*a*=35 *b*=150	*a*=45 *b*=250	*a*=25 *b*=400	100
R 120	*a*=45 *b*=200	*a*=35 *b*=300	*a*=35 *b*=500	120
R 180	*a*=50 *b*=240		*a*=50 *b*=600	140
R 240	*a*=60 *b*=280		*a*=60 *b*=700	160
a_{st}=*a*+10 mm (see note below)			a_{st}=*a* (see note below)	

a_{st}=increased axis distance of the outermost bar (tendon, wire) from the side surface of the cross-section, where steel is in a single layer

NOTES

1. For prestressed members, the axis distances should be increased by 10 mm for prestressing bars and by 5 mm for wires or strands.
2. The table applies to beams exposed to fire on three sides.
3. For beams exposed to fire on all four sides, the height should at least equal the minimum dimension b_{min} in the table for the required fire resistance and its cross-sectional area should be at least $2b_{min}^2$.
4. The minimum axis distance to any individual bars should not be less than that required for R 30 in the table nor less than half the average axis distance.
5. For R 90 and above, the top reinforcement over each intermediate support should extend at least $0.3l_{eff}$ from the centre of support, where the effective span l_{eff}>4 metres and l_{eff}/h>20, *h* being the beam depth. In other cases, this minimum may be reduced to $0.15l_{eff}$.
6. If the above detailing requirement is not met and the moment redistribution in the analysis exceeds 15%, each span of the continuous beam should be assessed as a simply supported beam.
7. In a continuous I-beam, b_w should not be less than *b* for a distance of 2*h* from an intermediate support unless a check for explosive spalling is carried out.
8. In two-span I-beam systems with no rotational restraint at the end, with predominantly concentrated loading with M_{sd}/V_{sd} between 2.5 and 3, and with $V_{sd}>{}^{2/3}V_{rd2}$, the minimum width of the beam web between the concentrated loads should be: 220 mm for R 120. 400 mm for R 180 and 600 mm for R 240.

Table 7.9 Minimum dimensions for fire resistance for solid (normal weight) reinforced concrete slabs spanning one and two ways

Standard fire resistance	**Slab thickness h_s (mm)**	**Average axis distance span *a* (mm)**		
One way	**Two way**			
$l_y/l_x<1.5$	$1.5<l_y/l_x<2$			
REI 30	60	10	10	10
REI 60	80	20	10	15
REI 90	100	30	15	20
REI 120	120	40	20	25

Standard fire resistance	Slab thickness h_s (mm)	Average axis distance span a (mm)		
One way	**Two way**			
$l_y/l_x<1.5$	$1.5<l_y/l_x<2$			
REI 180	150	55	30	40
REI 240	175	65	40	50

l_x and l_y are the spans of a two-way slab (two directions at right-angles) where l_y is the longer span

NOTES

1. For prestressed members, the axis distances should be increased by 10 mm for prestressing bars and by 15 mm for wires or strands.
2. The minimum cover to any bar should not be less than half the average axis distance.
3. The table values of axis distance for two-way slabs apply to slabs supported on all four edges. For all other support conditions, the values for one-way slabs should be used.
4. The table values of slab thickness and cover for two-way slabs with $l_y/l_x<1.5$ should be used.
5. For R 90 and above, the top reinforcement over each intermediate support should extend at least $0.3l_{eff}$ from the centre of support, where the effective span $l_{eff}>4$ metres and $l_{eff}/h>20$, h being the beam depth. In other cases, this minimum may be reduced to $0.15l_{eff}$.
6. If the above detailing requirement is not met and the moment redistribution in the analysis exceeds 15%, each span of the continuous slab should be assessed as a simply supported slab.
7. Minimum top reinforcement of $0.005/A_c$ should be used over intermediate supports when the reinforcement has "normal" ductility, when there is not rotational restraint at ends of two-span slabs, and when transverse redistribution of load effects cannot occur.

Table 7.10 Minimum dimensions for fire resistance of reinforced and prestressed (normal weight) concrete slabs

Standard fire resistance	Slab thickness h_s (mm), excluding finishes	Axis distance a (mm)
REI 30	150	10
REI 60	200	15
REI 90	200	25
REI 120	200	35
REI 180	200	45
REI 240	200	50

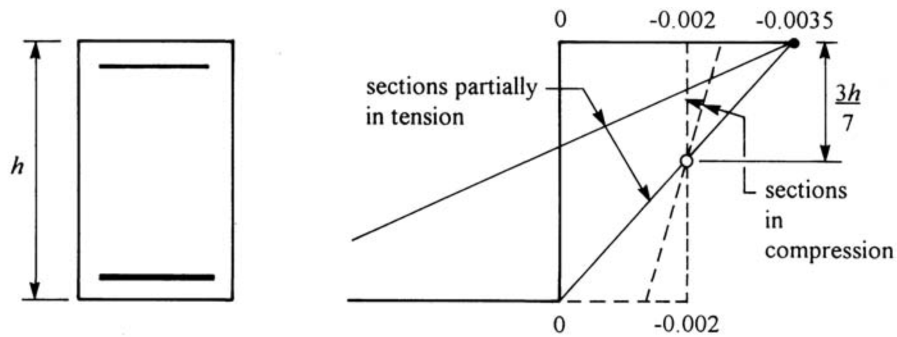
NOTES

1. For prestressed members, the axis distances should be increased by 10 mm for prestressing bars and by 15 mm for wires or strands.
2. It is assumed that the moment redistribution in this analysis does not exceed 15%. If it does exceed 15%, the axis distances in this table should be replaced by those for one-way slabs.
3. Over intermediate supports in each direction, at least 20% of the total top reinforcement calculated for cold design should extend over the full span, in the column strips.

8
Bending and longitudinal force

8.1
Conditions at failure

Figure 8.1 (taken from 4.11 in EC2) shows the strain conditions assumed at the ultimate limit state for reinforced concrete.



For cross-sections not fully in compression, the concrete is assumed to fail in compression when the strain reaches 0.0035. The strain in the tension reinforcement need not be limited where a horizontal top branch is assumed for the reinforcement stress-strain curve.

For cross-sections that are completely in compression, the strain is limited to 0.002 at a height of 3/7h from the most compressed face.

The strains in the reinforcement at ultimate are given by the formulae in Table 8.1.

Table 8.1 Strains in reinforcement at ultimate

(A) x h

Compression reinforcement

$$\epsilon_s' = \frac{0.0035}{x}(x-d')$$

Reinforcement near tension or least compressed face

(B) $x>h$

Reinforcement near most compressed face

$$\epsilon_s = -\frac{0.002}{(x-3h/7)}(x-d')$$

Reinforcement near least compressed face

$$\epsilon_s = \frac{0.002}{(x-3h/7)}(d-x)$$

In general, it is satisfactory to assume that the reinforcement near to the most compressed face is yielding but there are cases when this may not be so. Table 8.2 sets out the conditions for the reinforcement to be yielding, assuming a bilinear stress-strain diagram.

Table 8.2 Conditions for yield of reinforcement

(A) $x \;\; h$

Compression steel

$$f_{yd} \le 700(1 - d'/x) \qquad \text{or} \qquad \frac{d'}{x} \le 1 - f_{yd}/700$$

Tension steel

$$\frac{x}{d} \le \frac{1}{f_{yd}/700 + 1}$$

(B) $x>h$

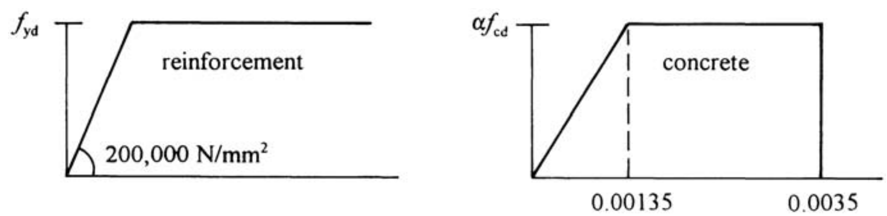
Compression steel

$$\frac{(1 - d'/x)}{(1 - 3h/7x)} > \frac{f_{yd}}{900} \qquad \text{or} \qquad \frac{x}{h} \le \frac{\left(\frac{3f_{yd}}{2800} - d'/h\right)}{\left(\frac{f_{yd}}{400} - 1\right)}$$

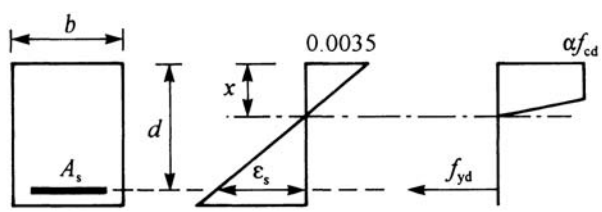
8.2 Design of rectangular sections subject to flexure only

I Derivation of equations

Stress-strain curves for reinforcement and concrete.



Conditions in section at ultimate in a singly reinforced section.



The limits to x/d will ensure that stress in steel is at yield. The average stress in compression zone is $0.807f_{cd}$. Distance from compression face to centre of concrete compression is $0.411x$.

Equilibrium of axial forces gives:

$$\frac{x}{d} = \frac{1.239A_s f_{yd}}{\alpha f\ .bd}$$

Defining $\omega = \frac{A_s f_{yd}}{\alpha f_{cd} bd}$

$$\frac{x}{d} = 1.239\omega \qquad \text{I}$$

The lever arm, z, is given by:

$$\frac{z}{d} = 1 - 0.411\left(\frac{x}{d}\right) \qquad \text{IIa}$$

or

$$\frac{z}{d} = 1 - 0.5092\omega \qquad \text{IIb}$$

The moment is given by:

$M = A_s f_{yd} z$

hence $\frac{M}{bd^2 \alpha f_{cd}} = \omega(1 - 0.5092\omega)$ IIIa

Defining

$m = \frac{M}{bd^2 \alpha f_{cd}}$ and solving for ω gives:

$\omega = (1 - \sqrt{1 - 2.0368m})/1.0184$ IIIb

or, approximately, $\omega = 1 - \sqrt{1 - 2m}$

Equation 2.17 in Eurocode 2 can be rewritten to give:

for $f_{cu} \leq 35$ $\left(\frac{x}{d}\right)_{lim} = (\delta - 0.44)/1.25$ IVa

for $f_{cu} > 35$ $\left(\frac{x}{d}\right)_{lim} = (\delta - 0.56)/1.25$ IVb

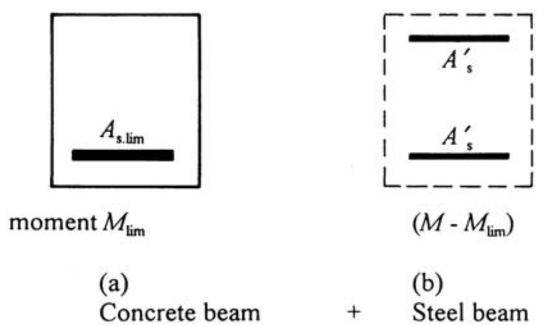
From I

$\omega_{lim} = 0.807\left(\frac{x}{d}\right)_{lim}$ V

From III(a) and II(a)

$m_{lim} = \left(\frac{x}{d}\right)_{lim}\left(0.411\left(\frac{x}{d}\right)_{lim}\right)$ VI

If $m > m_{lim}$, compression steel is needed to maintain the neutral axis at the limiting value. The moment capacity can then be calculated by assuming two superimposed sections.



The steel area required in the 'steel beam' is given by:

$A_s = \frac{(M - M_{lim})}{f_{yd}(d - d')}$ VIIa

(Assuming reinforcement in compression is yielding)

or $\omega = \frac{A_s f_{yd}}{\alpha f_{cd}} = \frac{(m - m')}{(1 - d'/d)}$ VIIb

The area of steel required for the 'concrete beam' is given by equation V.

Hence, total areas of reinforcement are given by:

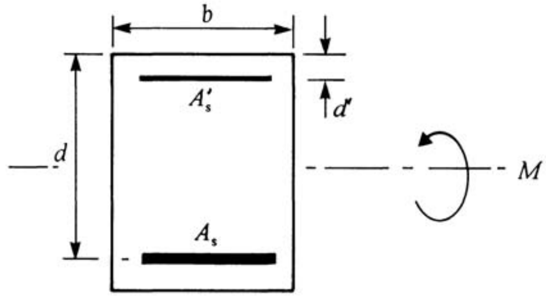
$\omega = \frac{(m - m')}{(1 - d'/d)}$

and

$\omega = \omega_{lim} + \omega'$ VIII

The procedure for using these equations directly for calculating reinforcement areas is summarized below in Table 8.3.

Table 8.3 Design of rectangular beams



1. Calculate $m = \frac{M}{bd^2 \alpha f_{cd}}$
2. Calculate $\left(\frac{x}{d}\right)_{lim}$:

   if $f_{cu} \le 35$, $\left(\frac{x}{d}\right)_{lim} = (\delta - 0.44)/1.25$

   if $f_{cu} > 35$, $\left(\frac{x}{d}\right)_{lim} = (\delta - 0.56)/1.25$
3. Calculate $M_{lim} = \left(\frac{x}{d}\right)_{lim} (1 - 0.411\left(\frac{x}{d}\right)_{lim})$
4. If $m < m_{lim}$. simply reinforced beam will suffice

   $$\omega = 1 - \sqrt{1 - 2m}$$

   Hence calculate A_s - END
5. If $m > m_{lim}$

   $$\omega' = \frac{(m - m_{lim})}{(1 - d'/d)}$$

   $$\omega = 0.807\left(\frac{x}{d}\right)_{lim} + \omega'$$

   Hence calculate A_s and A_s -END

Design Tables

The equations can be presented as design tables as shown below.

Table 8.4 gives values of *x/d* and *ω* for singly reinforced beams as a function of *m*.

Table 8.4 Values of *x/d* and ω for singly reinforced beams

$\frac{M}{bd^2\alpha f_{cd}}$	$\frac{A_s f_{yd}}{bd\alpha f_{cd}}$	$\frac{x}{d}$	$\frac{M}{bd^2\alpha f_{cd}}$	$\frac{A_s f_{yd}}{bd\alpha f_{cd}}$	$\frac{x}{d}$
0.01	0.010	0.012	0.17	0.188	0.233
0.02	0.020	0.025	0.18	0.200	0.248
0.03	0.030	0.038	0.19	0.213	0.264
0.04	0.041	0.052	0.2	0.226	0.280
0.05	0.051	0.064	0.21	0.239	0.296
0.06	0.062	0.077	0.22	0.252	0.313
0.07	0.073	0.090	0.23	0.266	0.330
0.08	0.084	0.104	0.24	0.280	0.347
0.09	0.095	0.117	0.25	0.294	0.364
0.1	0.106	0.131	0.26	0.308	0.382
0.11	0.117	0.145	0.27	0.323	0.400
0.12	0.128	0.159	0.28	0.338	0.419
0.13	0.140	0.173	0.29	0.354	0.438
0.14	0.152	0.188	0.3	0.370	0.458
0.15	0.164	0.203	0.31	0.386	0.478
0.16	0.176	0.218	0.32	0.402	0.499

Table 8.5 gives $\left(\frac{x}{d}\right)_{lim}$, ω_{lim} and m_{lim} as a function of the amount of redistribution.

Table 8.5 Limiting values of $\left(\frac{x}{d}\right)$, $\frac{M}{bd^2\alpha f_{ck}}$ and $\frac{A_s f_{yd}}{bd\alpha f_{ck}}$

Percentage redistribution	δ	$\left(\frac{x}{d}\right)_{lim}$		$\left(\frac{M}{bd^2\alpha f_{ck}}\right)_{lim}$		$\left(\frac{A_s f_{yd}}{bd\alpha f_{ck}}\right)_{lim}$	
		f_{ck} 35	f_{ck}>35	f_{ck} 35	f_{ck}>35	f_{ck} 35	f_{ck}>35
0	1.00	0.448	0.352	0.295	0.243	0.362	0.284
5	0.95	0.408	0.312	0.274	0.220	0.329	0.252
10	0.90	0.368	0.272	0.252	0.195	0.267	0.220
15	0.85	0.328	0.232	0.229	0.169	0.265	0.187
20	0.80	0.288	0.192	0.205	0.143	0.232	0.155
25	0.75	0.248	0.152	0.180	0.115	0.200	0.123
30	0.70	0.208	0.112	0.154	0.086	0.168	0.090

Tables 8.4 and 8.5 can be used to streamline the procedure set out in Table 8.3.

Flanged beams

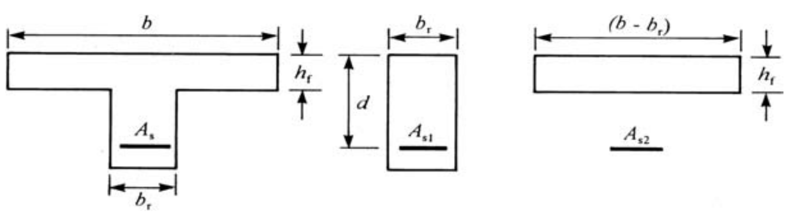
Since concrete in tension is ignored, the design of a flanged beam is identical to that for a rectangular beam provided that the neutral axis at failure lies within the flange.

Thus the procedure for design can be:

1. Follow steps 1 to 4 in Table 8.3 using the overall flange breadth as *b*.
2. Calculate $\left(\frac{x}{d}\right) = \omega/0.807 \not> \left(\frac{x}{d}\right)_{lim}$

   If $\left(\frac{x}{d}\right) < \left(\frac{h_f}{d}\right)$, design is OK. This will normally be the case.

   If $\left(\frac{x}{d}\right) > \left(\frac{h_f}{d}\right)$, then further equations need to be derived. This can most easily be achieved by considering the base to be made up of two parts as shown below:



It will be assumed that the neutral axis is large enough for the whole flange to be at a stress of αf_{cd}.

Hence, by equilibrium,

$$A_{s2} = (b - b_r)h_f \alpha f_{cd} / f_{yd} \qquad \text{IX}$$

$$M_2 = A_{s2} f_{yd}(d - h_f/2) \qquad \text{X}$$

The steel area required for the rectangular rib can now be obtained by using Table 8.3 to assess the reinforcement area needed for a rectangular beam of breadth b_r to support a moment of $M_1=(M-M_2)$.

Although very unlikely to be exceeded, the limiting moment for a flanged beam where $(x/d)_{lim}$ exceeds (h_f/d) is given by:

$$M_{lim} = M_{1.lim} + b_{hf}\alpha f_{cd}(d - h_f/2)$$

$$\left[\left(\frac{x}{d}\right)_{lim}\left[1 - 0.0411\left(\frac{x}{d}\right)_{lim}\right]b_r d^2 + bh_f\left[d - \left(\frac{h_f}{z}\right)\right]\right]\alpha f_{cd}$$

The required steel areas can then be calculated using Equations VIIIa, XI, X and V.

The procedure for the design of flanged sections is summarized in Table 8.6.

Table 8.6 Design of flanged sections for flexure

1. Calculate $m = \frac{M}{bd\alpha f_{cd}}$
2. Follow Table 8.3 to obtain ω. Calculate $\frac{x}{d} = \frac{\omega}{0.807} \ngtr \left(\frac{x}{d}\right)_{lim}$

   If $\left(\frac{x}{d}\right) \leq \left(\frac{h_f}{d}\right)$, calculate A_s from ω (END)
3. If $\left(\frac{x}{d}\right) > \left(\frac{h_f}{d}\right)$

   Calculate $A_{s2} = (b - b_r)h_f \alpha f_{cd} / f_{yd}$
   $M_2 = A_{s2} f_{yd} (d - h_f/2)$
4. Use Table 8.3 to calculate steel areas for rectangular sections of breadth b_r to resist moment of $(M–M_2)$.
5. Areas of steel=sum of those obtained from steps 3 and 4.

Minimum reinforcement

There are two provisions defining minimum areas of flexural steel. These are:

(a) minimum for crack control 4.4.2.2.
(b) overall minimum 5.4.2.1.1.

The formula in 4.4.2.2 is:

$$A_s \geq A_s \geq k_c k f_{ct.eff} A_{ct} / \sigma_s.$$

where, for bending, k_c=0.4

$f_{ct.eff}$ is suggested as 3, k is 0.8 for sections with depths not greater than 300 mm and 0.5 for sections deeper than 800 mm, σ_s may be taken as f_{yk}. A_{ct}, thearea of concrete in the tension zone immediately before cracking, will be $bh/2$ for rectangular sections and an approximate value for flanged beams could be taken as 0.75 $b_t h$ where b_t is the breadth of the tension zone. If h is assumed to be 1.15d, the above equation thus reduces to:

for rectangular beams	h 300mm	$0.55bd/f_{yk}$
	h 800mm	$0.34bd/f_{yk}$
for flanged beams	h 300mm	$0.83b_t d/f_{yk}$
	h 800mm	$0.55b_t d/f_{yk}$

Interpolation is permitted for depths between 300 and 800 mm.

Clause 5.4.2.1.1 gives:

$$A_s \geq \frac{0.6 b_t d}{f_{yk}} \geq 0.0015 b_t d$$

Assuming f_{vk}>400, 0.0015$b_t d$ will govern.

It will be seen, in any case, that the rule in 5.4.2.1.1 will always govern except for shallow flanged beams and, for commonly used reinforcement, the limit of 0.0015 $b_t d$ will be the controlling factor in 5.4.2.1.1. The following general rule therefore seems adequate for normal beams.

Table 8.7: Minimum tension reinforcement

If	f_{yk}=500N/mm2
or	f_{yk}<500 N/mm^2 and beam is either rectangular or flanged with h<700 mm
then	A_s 0.001 $b_t d$
else	$A_s \geq 0.083 - \frac{(h-300)}{1786} \frac{b_t d}{f_{yk}} \geq 0.0015 b_t d$

8.5
Design charts for columns (combined axial and bending)

The following 59 charts are provided for the design of symmetrically reinforced rectangular columns and circular columns.

The charts provided are:

Charts 8.1 to 8.12: Charts for uniaxial bending of rectangular sections.

Charts are drawn for ratios of d_1/h of 0.05, 0.1, 0.15 and 0.2 for three different arrangements of reinforcement as follows.

Charts 8.1 to 8.4: Columns with the reinforcement concentrated along the edges parallel to the axis of bending.

Charts 8.5 to 8.8: Columns with the reinforcement distributed along the edges perpendicular to the axis of bending. Three bars in each face are assumed: near the corners and at the centre of the face. This represents the worst case.

Charts 8.9 to 8.12: Columns with the reinforcement distributed along all sides. Bars are assumed at the corners and at the centre of the sides.

Charts 8.13 to 8.19: Charts for circular columns. The reinforcement is assumed to consist of six bars uniformly spaced round the perimeter. This will be slightly conservative for larger numbers of bars. Curves are drawn for values of $\frac{A_s f_{yd}}{h^2 \alpha f_{cd}}$ from 0 to 1.0.

Charts 8.20 to 8.59: Charts for bi-axial bending of symmetrically reinforced rectangular columns.

The bi-axial charts are used as follows:

(1) Calculate $m_x = \frac{M_x}{bh^2 \alpha f_{cd}}$

and $m_y = \frac{M_y}{hb^2 \alpha f_{cd}}$

(2) M_{max}=greater of M_x or M_y
M_{min}=lesser of M_x or M_y
Calculate M_{min}/M_{max}

(3) Calculate $\frac{N_{sd}}{bh \alpha f_{cd}}$

(4) Select most appropriate chart for the reinforcement arrangement, d'/h and M_{min}/M_{max} and read off $\frac{A_{sd} f_{yd}}{bh \alpha f_{cd}}$
Interpolation between charts may be necessary.

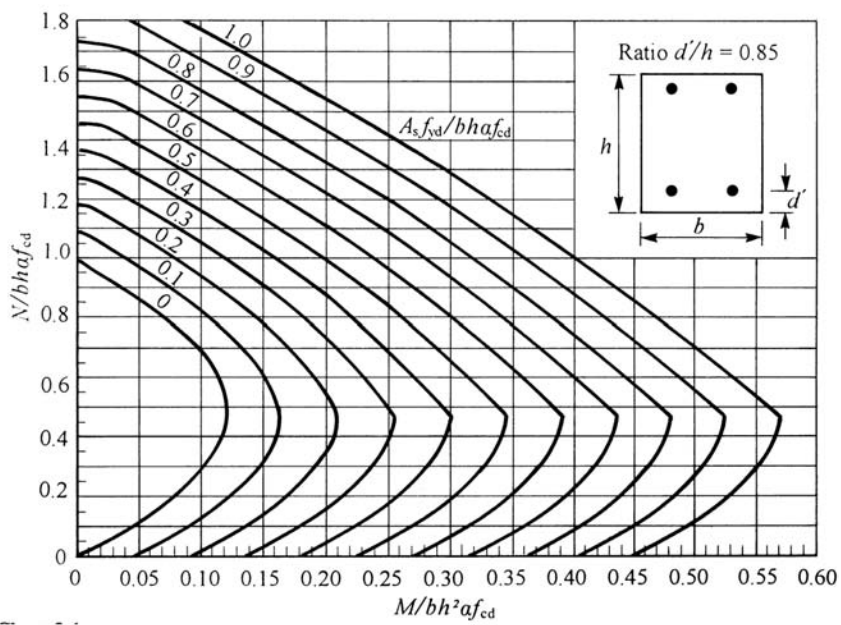


Chart 8.1

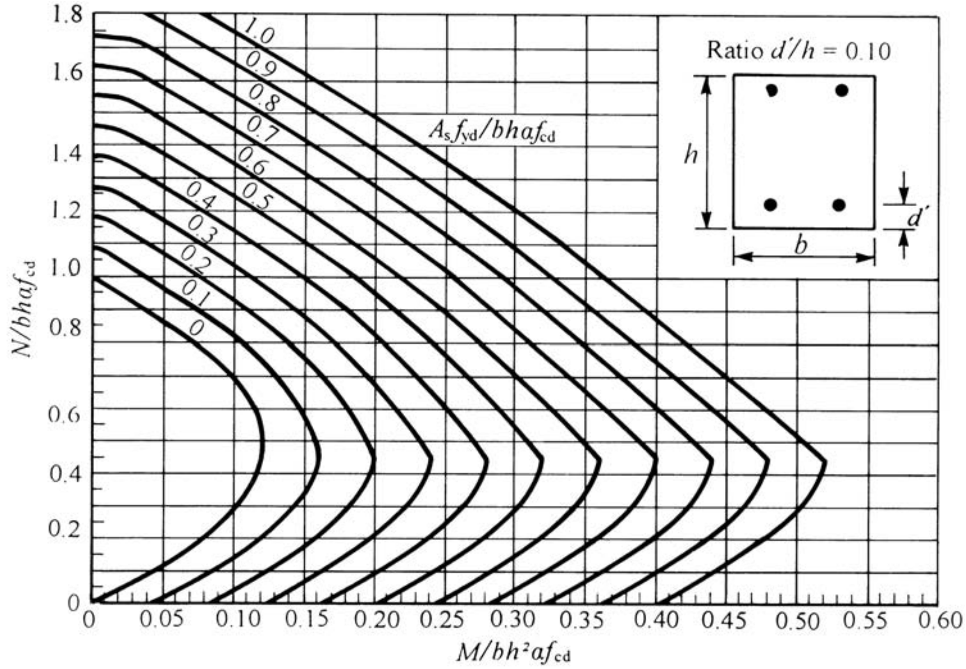


Chart 8.2

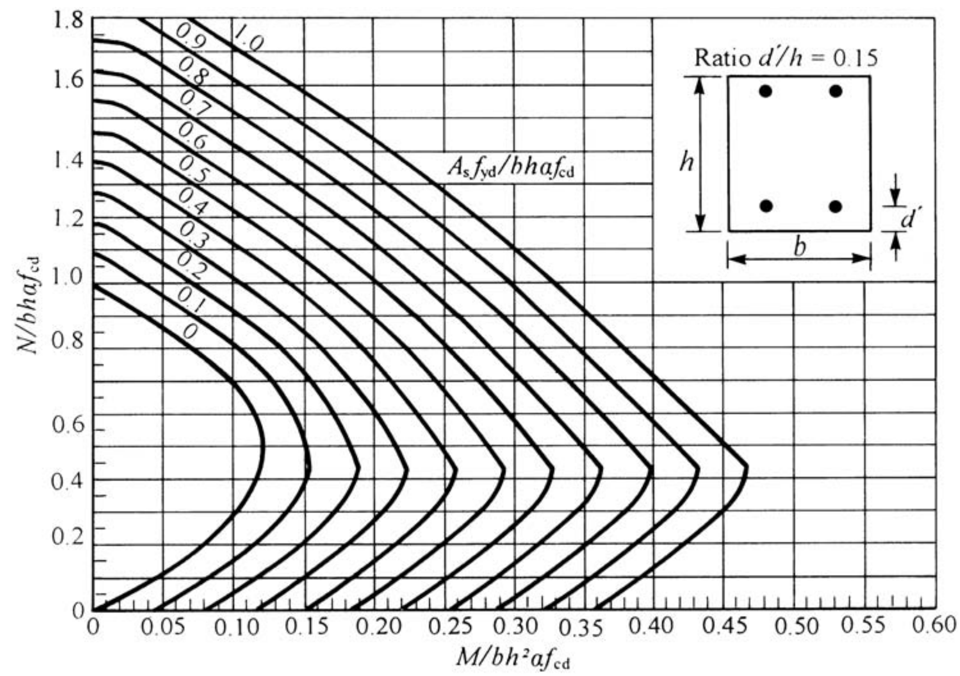


Chart 8.3

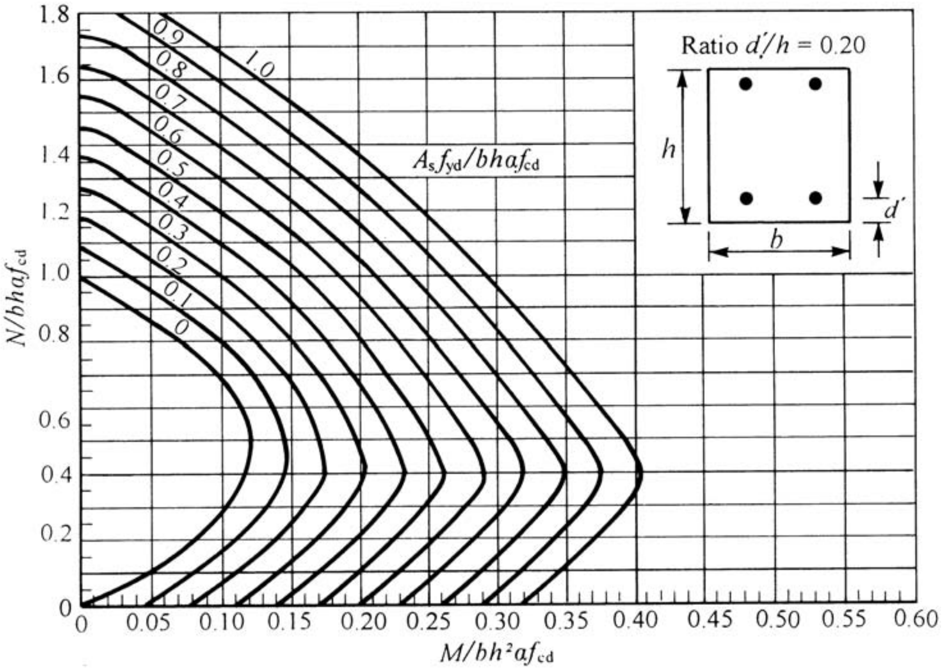


Chart 8.4

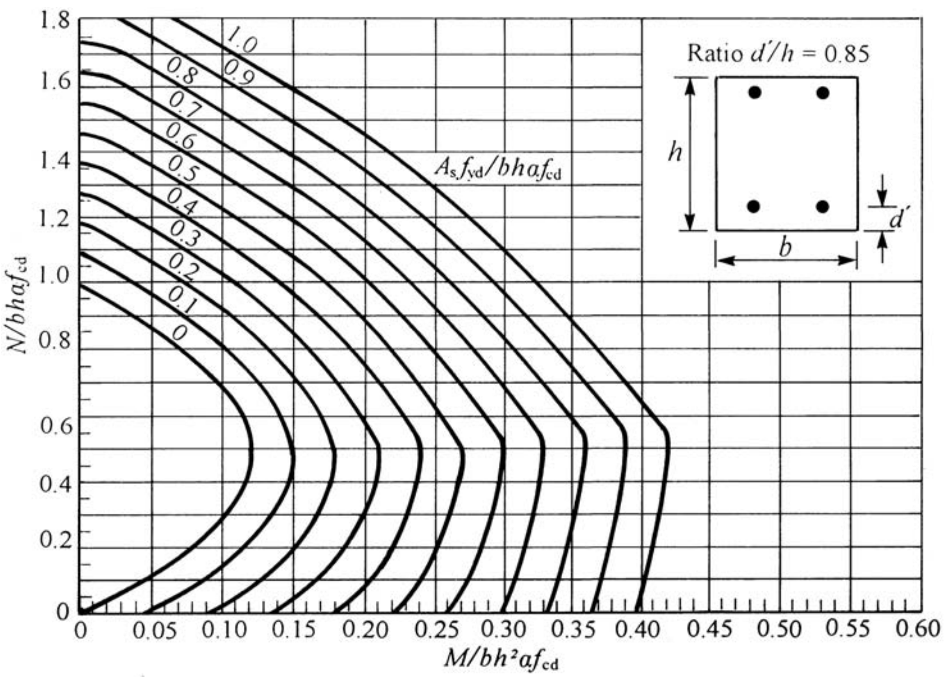


Chart 8.5

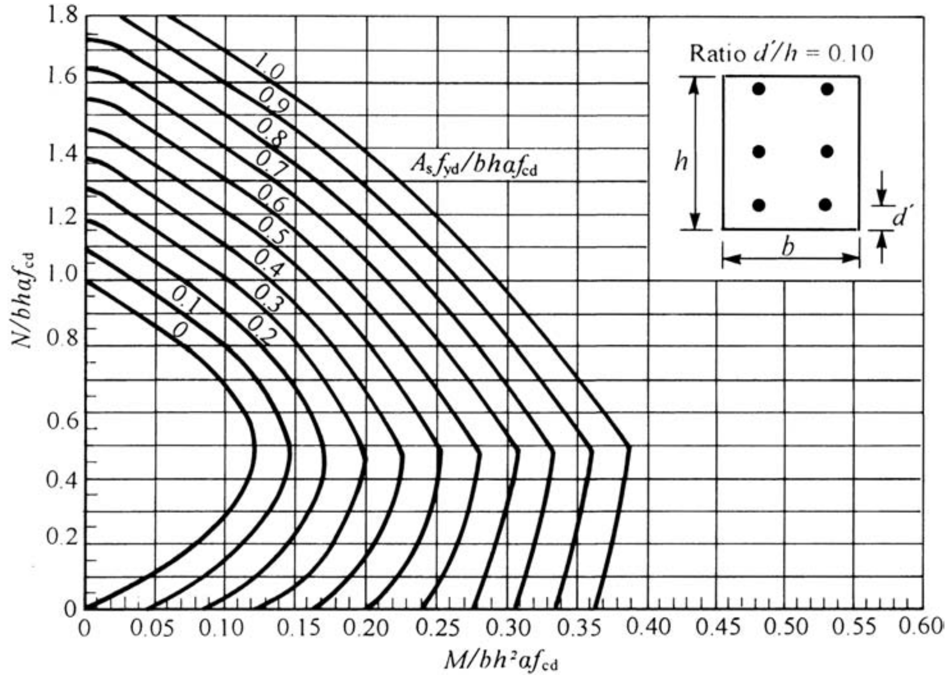


Chart 8.6

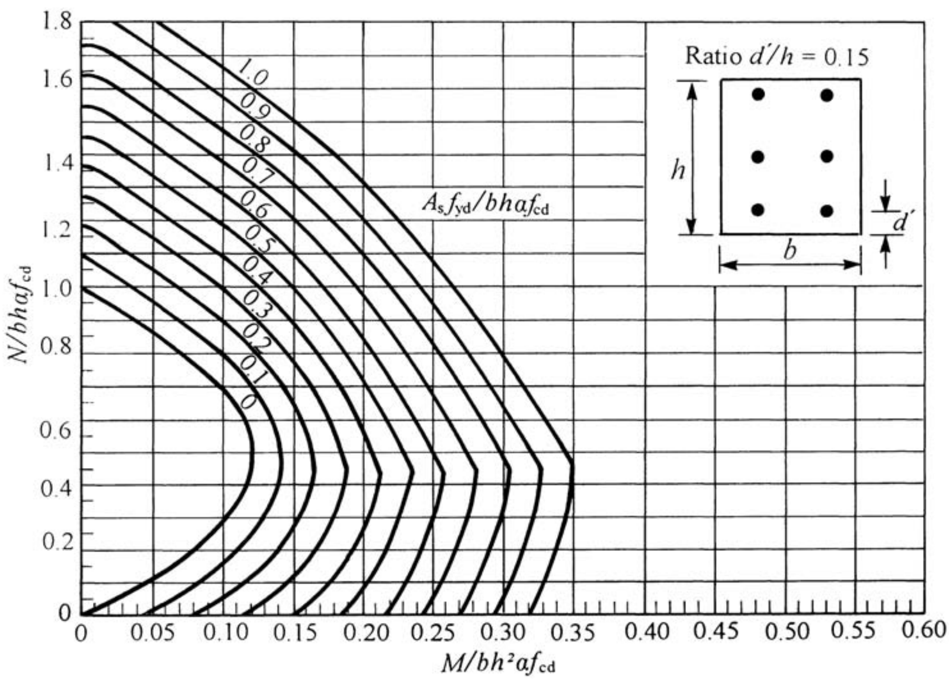


Chart 8.7

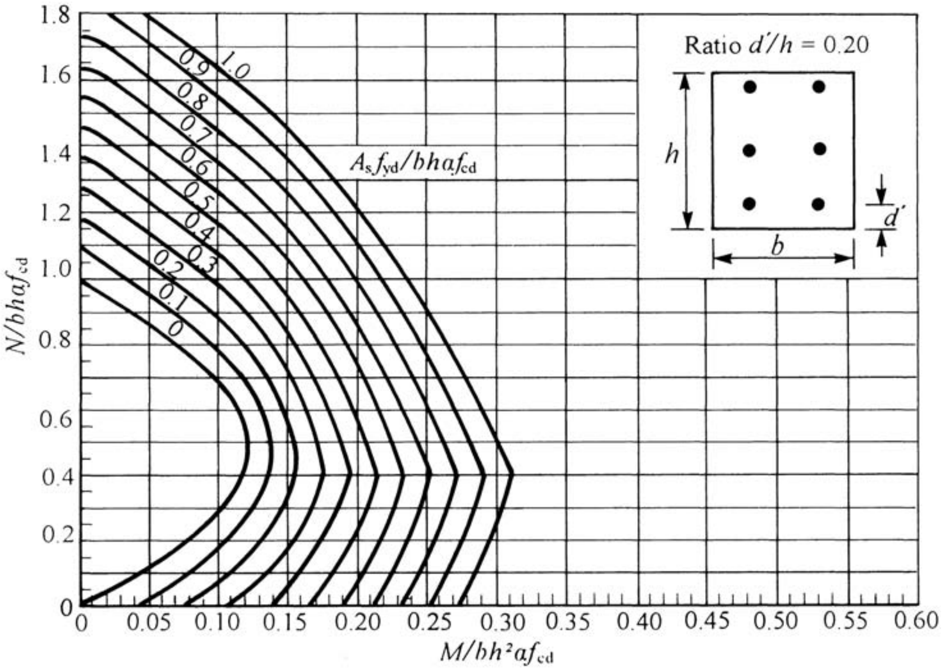


Chart 8.8

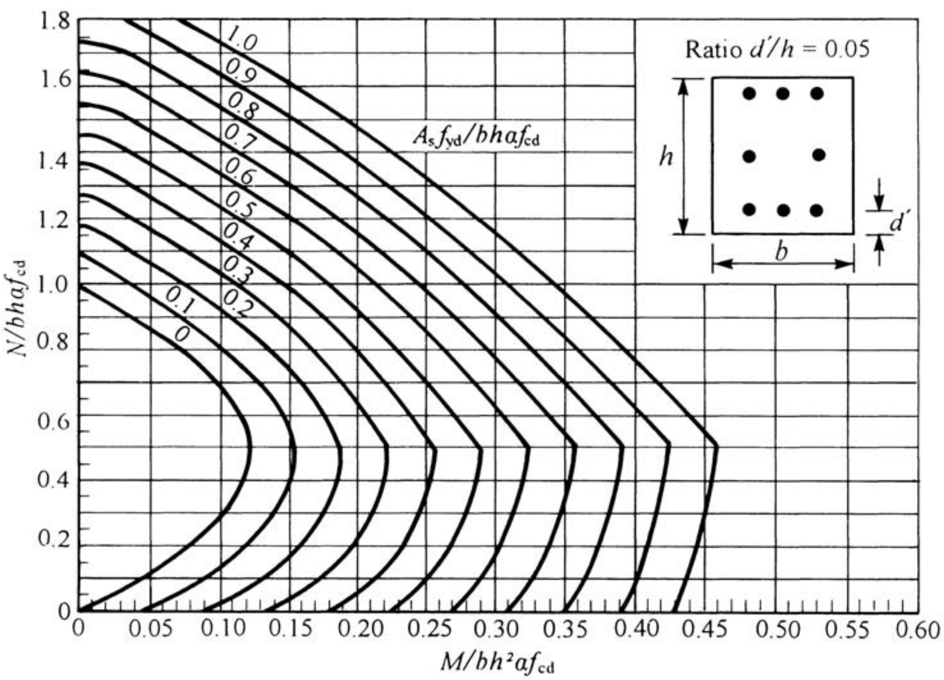


Chart 8.9

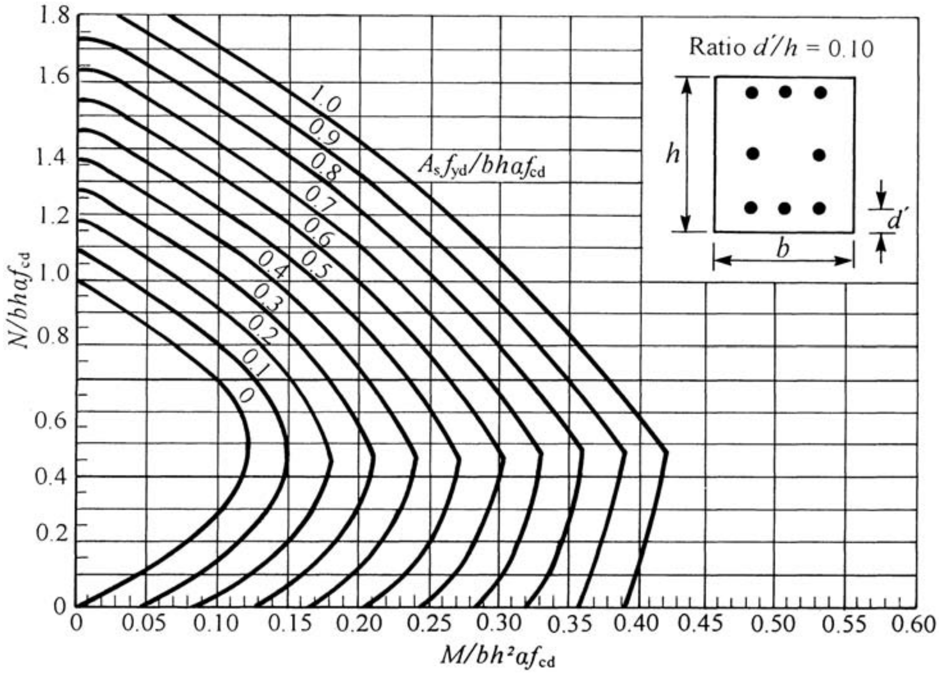


Chart 8.10

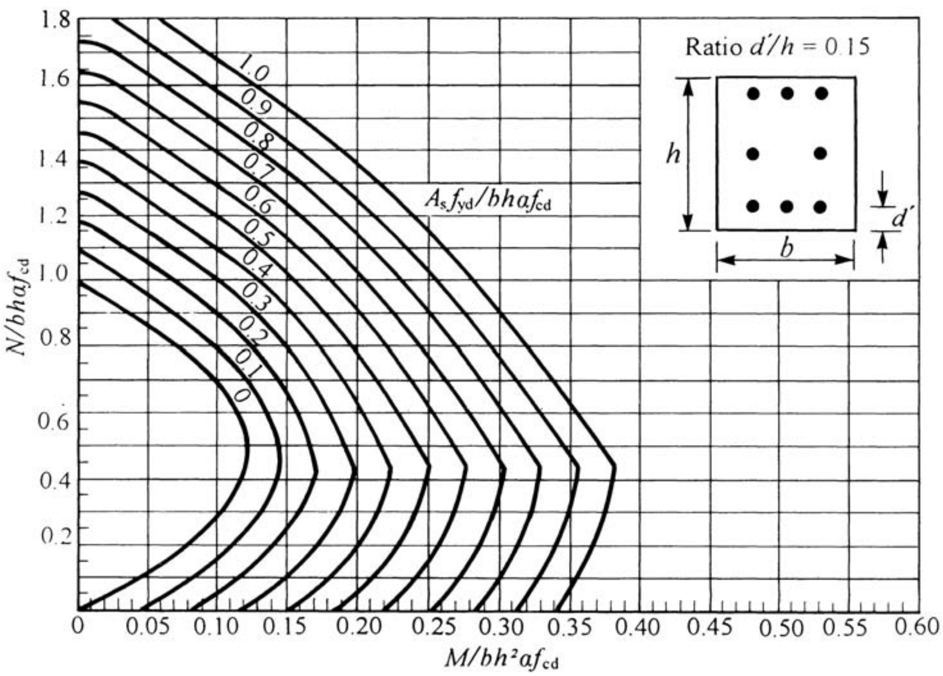


Chart 8.11

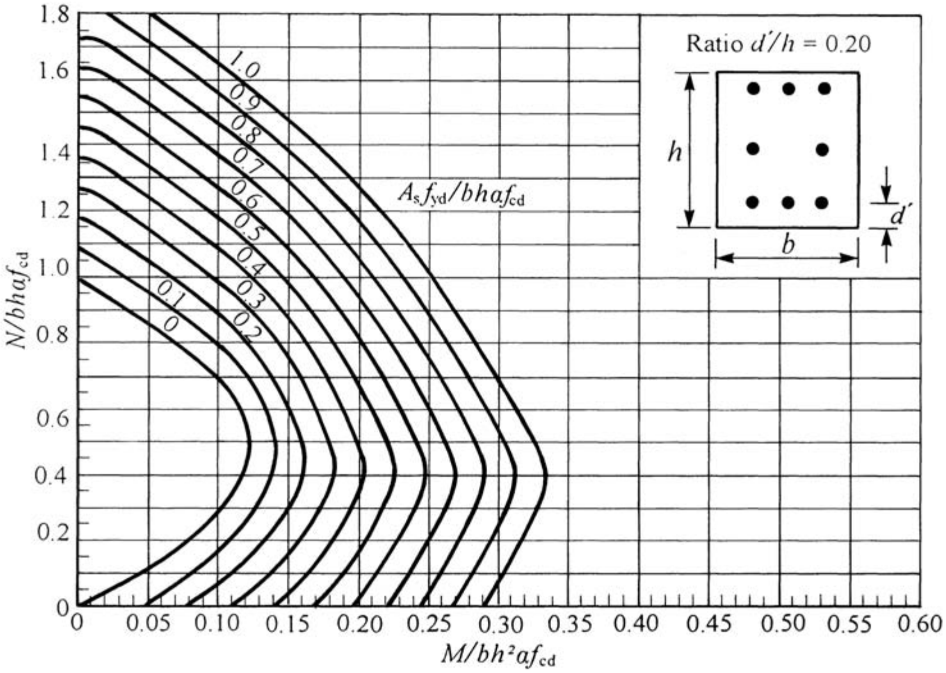


Chart 8.12

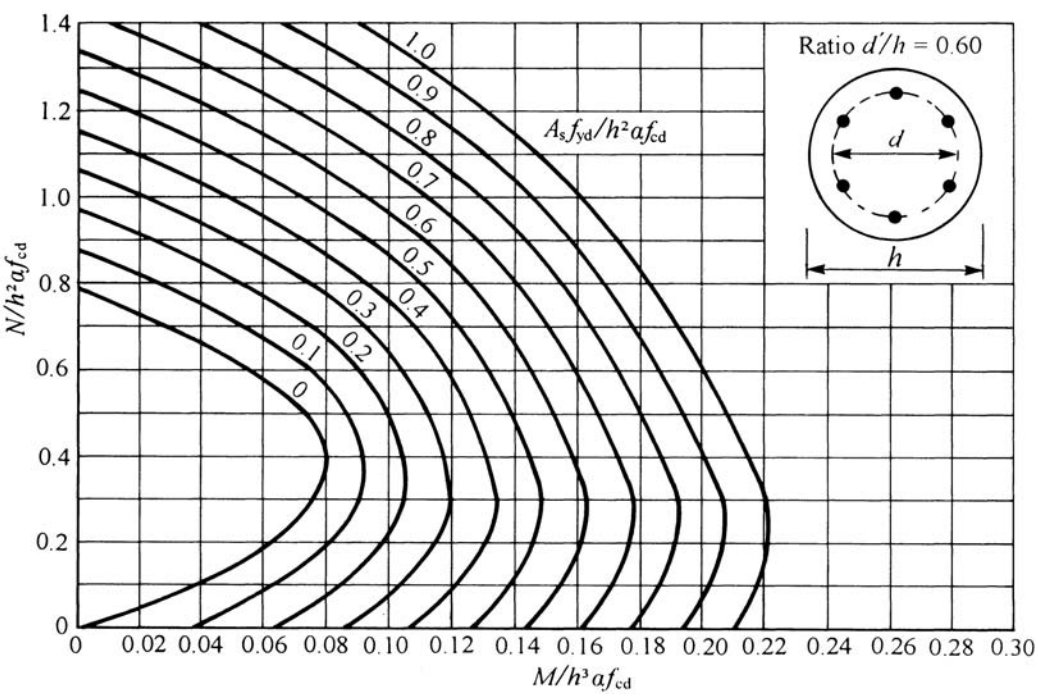


Chart 8.13

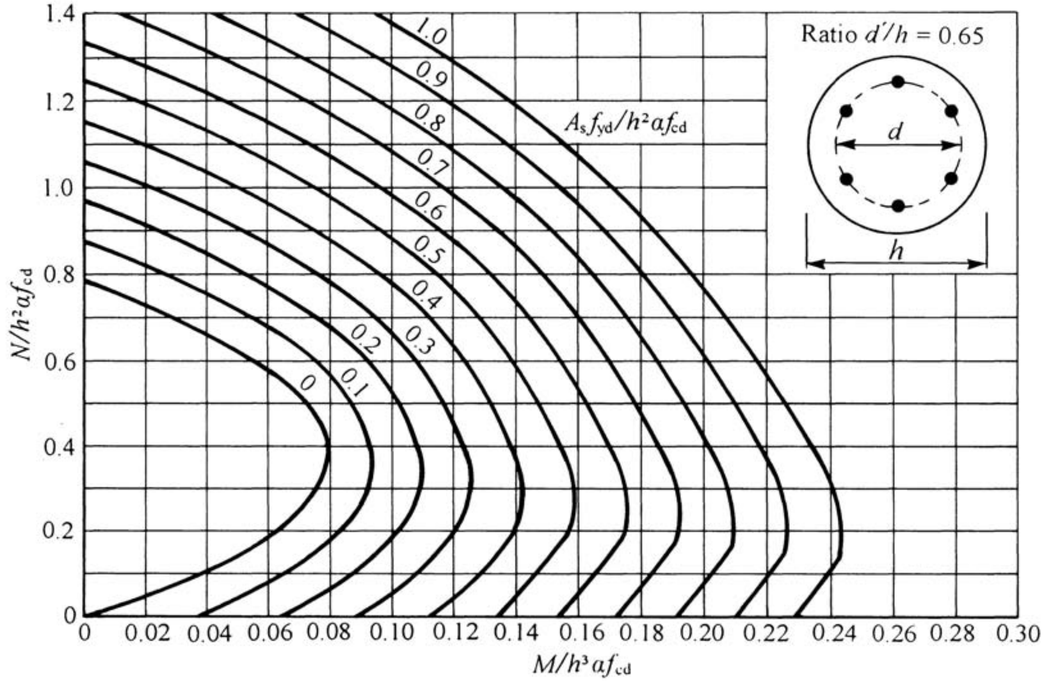


Chart 8.14

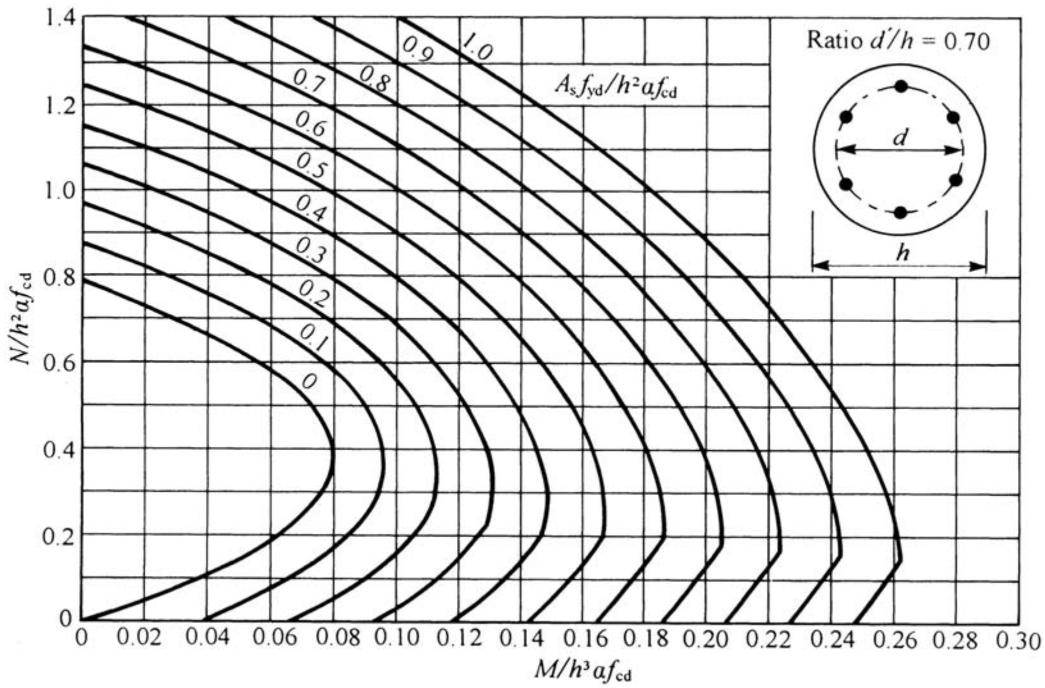


Chart 8.15

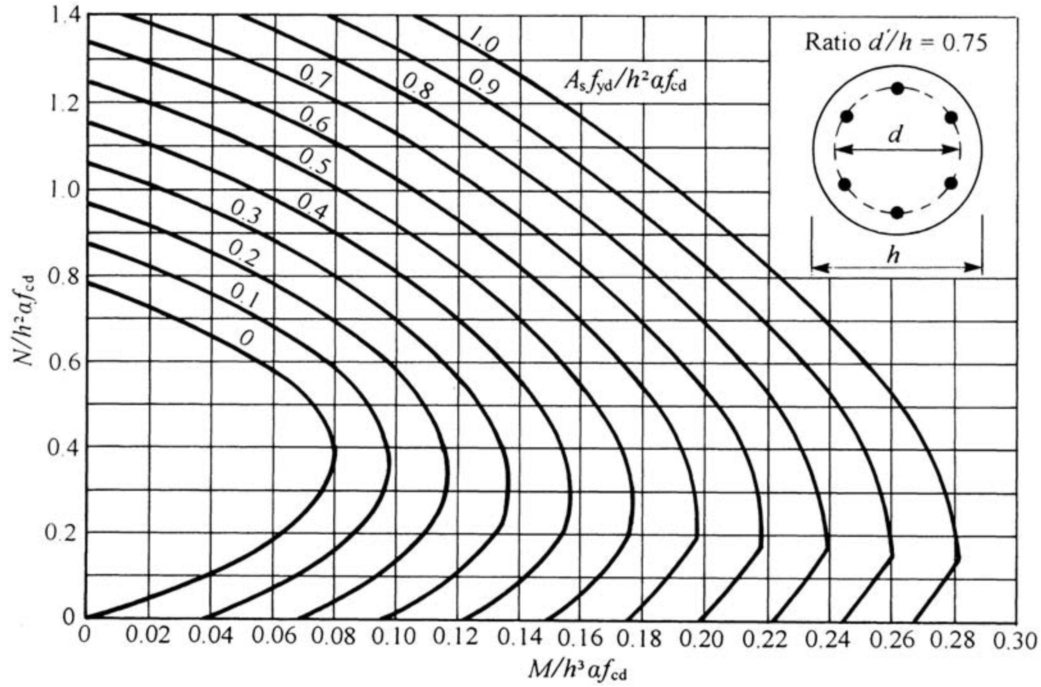


Chart 8.16

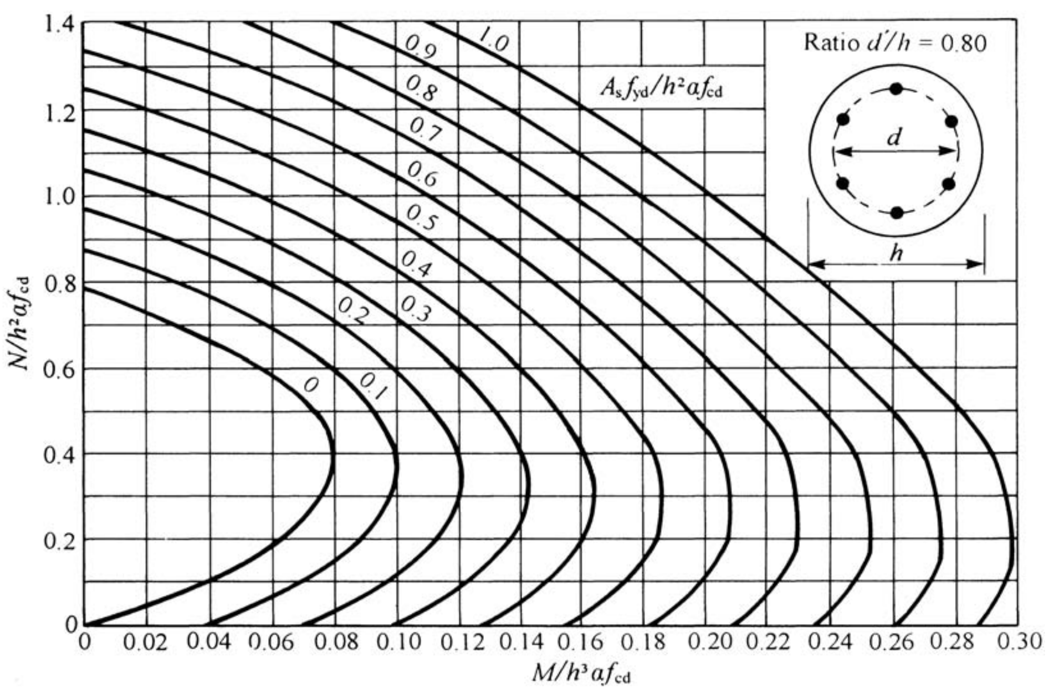


Chart 8.17

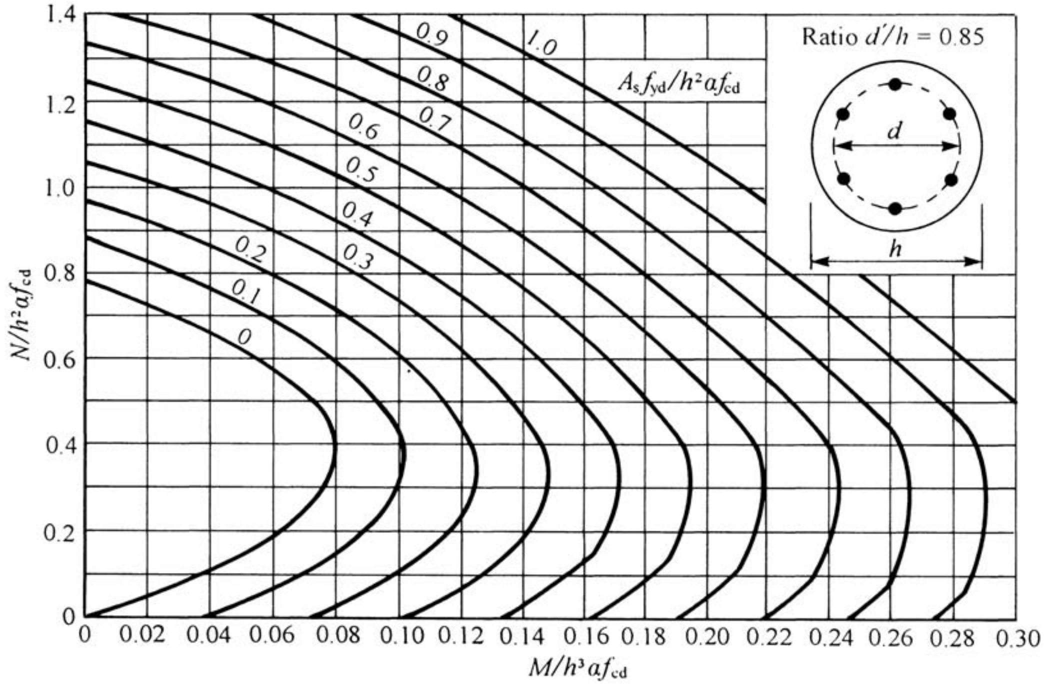


Chart 8.18

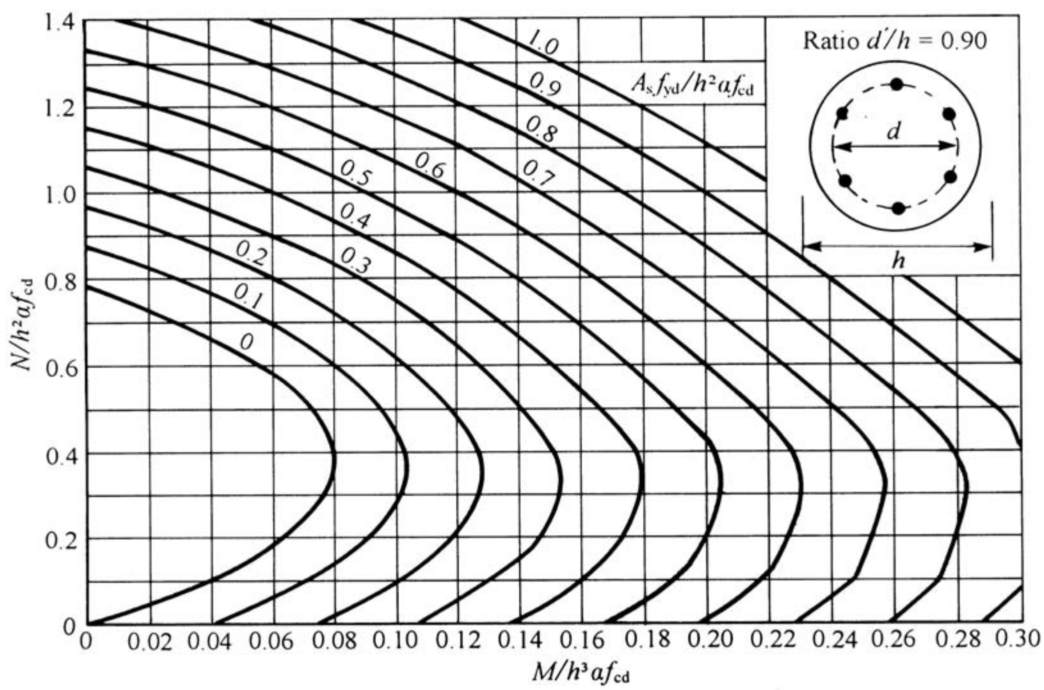


Chart 8.19

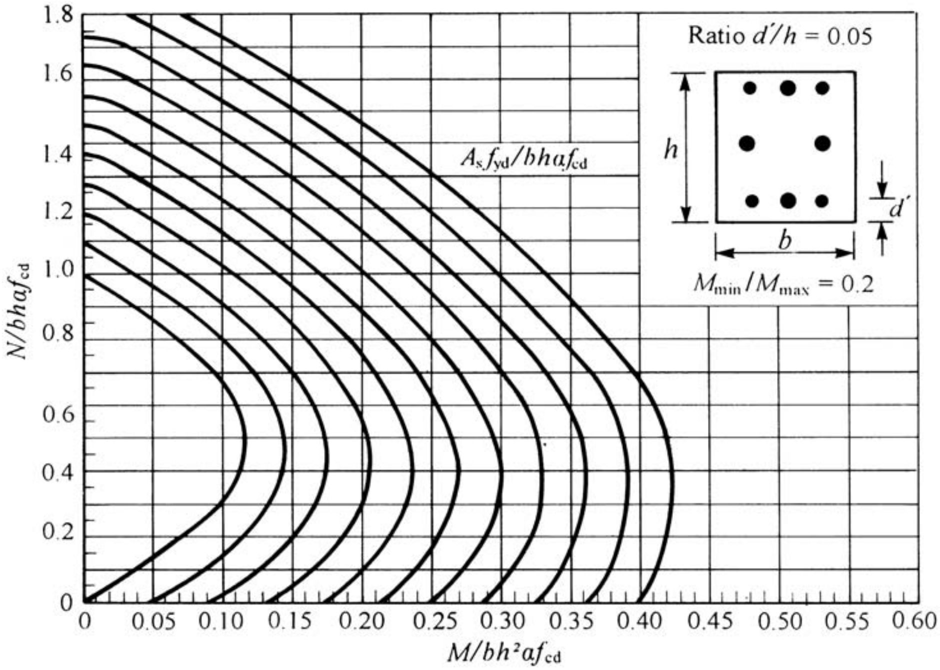


Chart 8.20

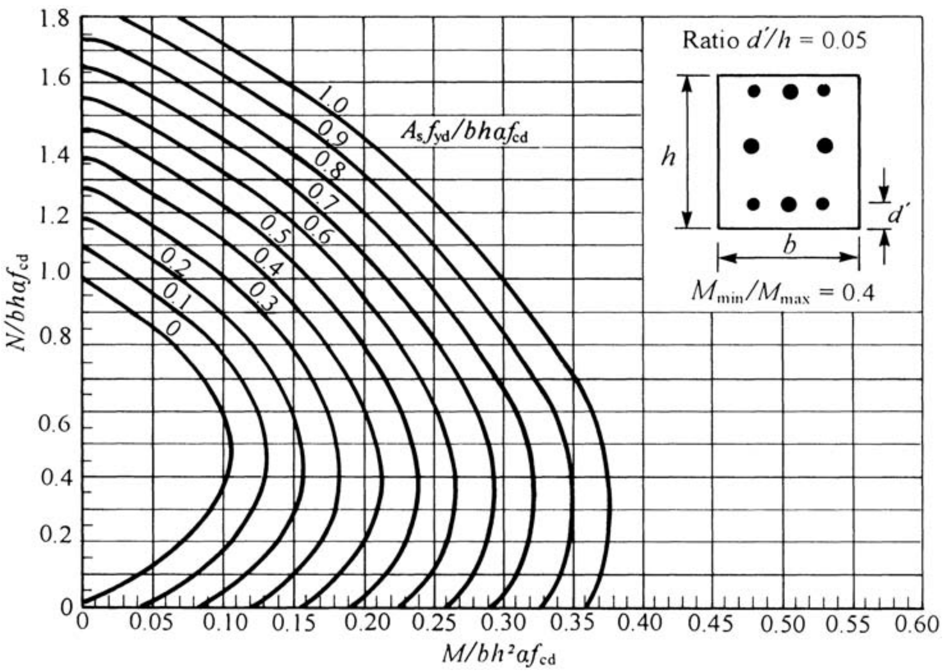


Chart 8.21

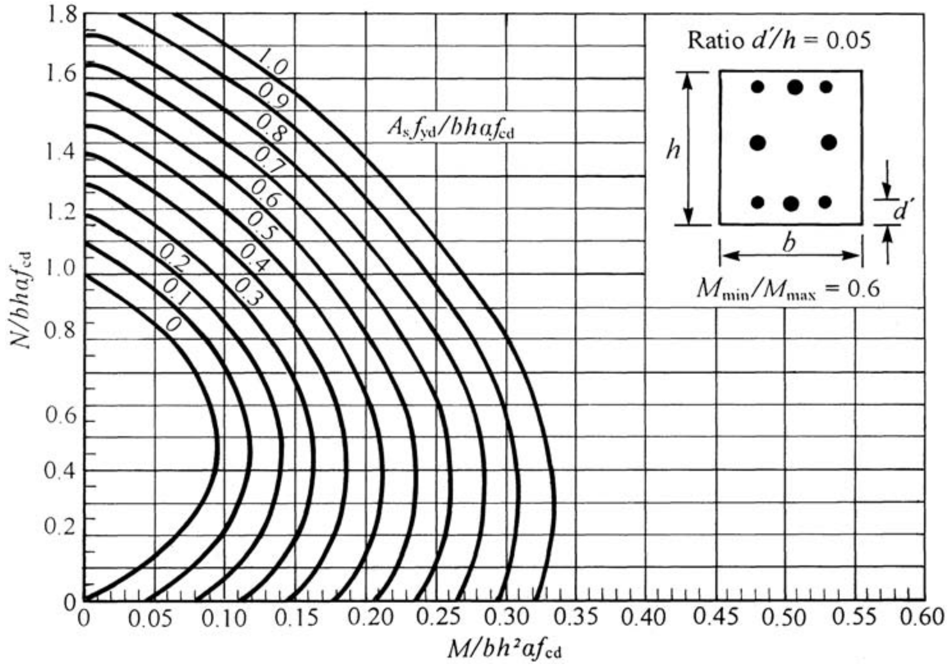


Chart 8.22

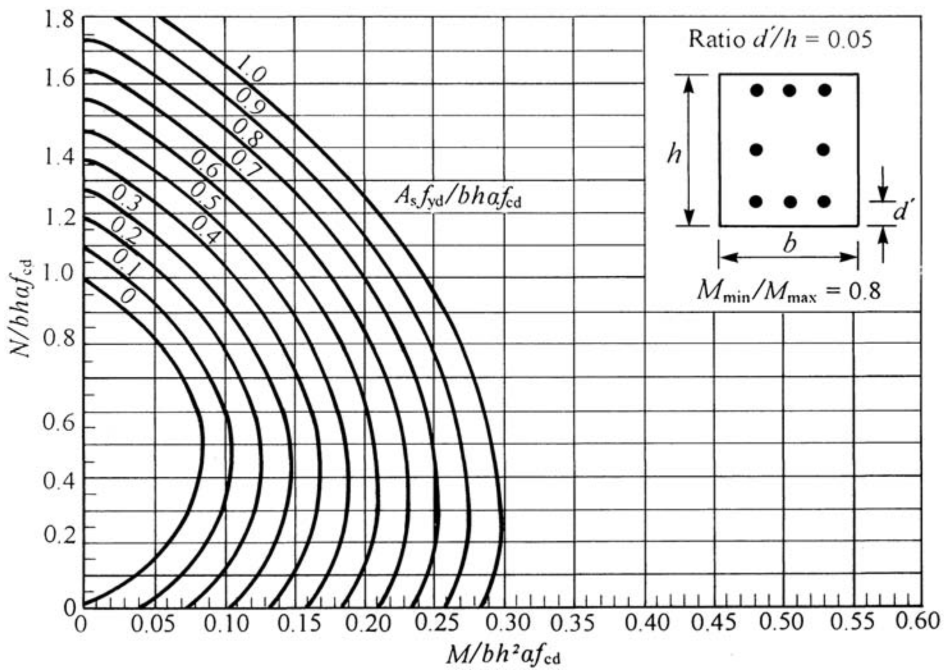


Chart 8.23

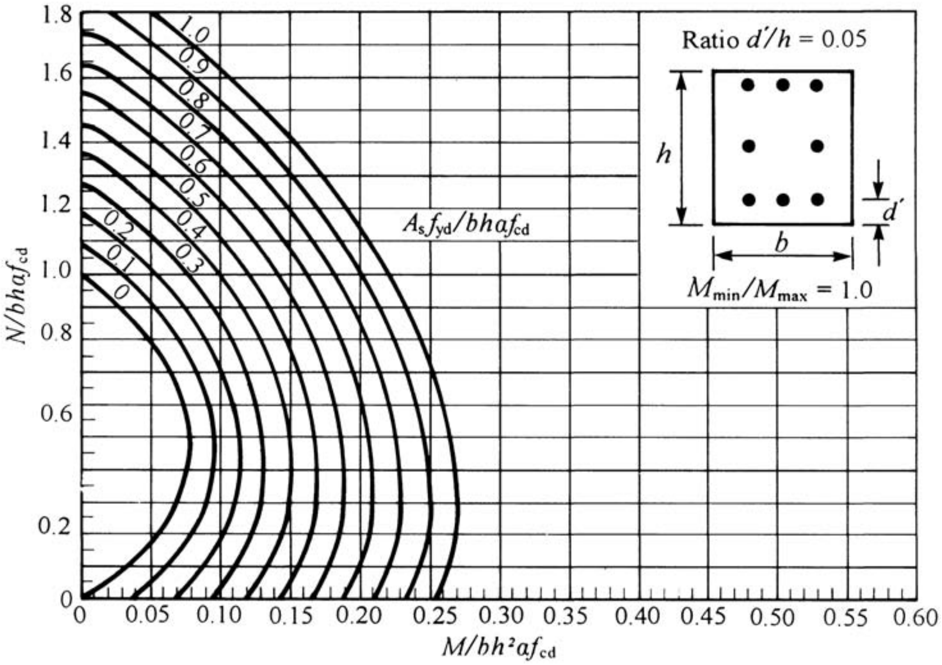


Chart 8.24

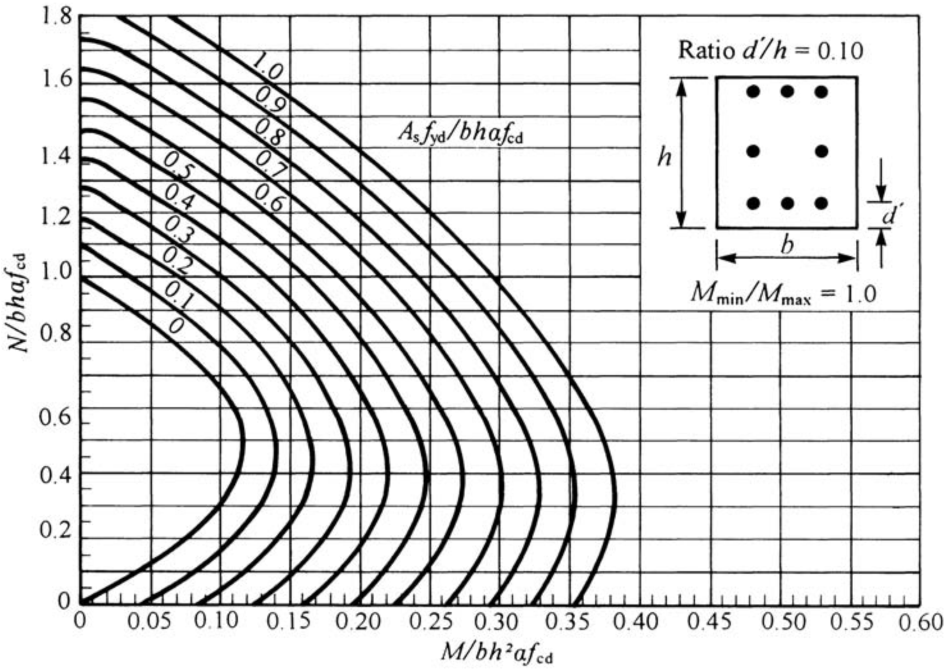


Chart 8.25

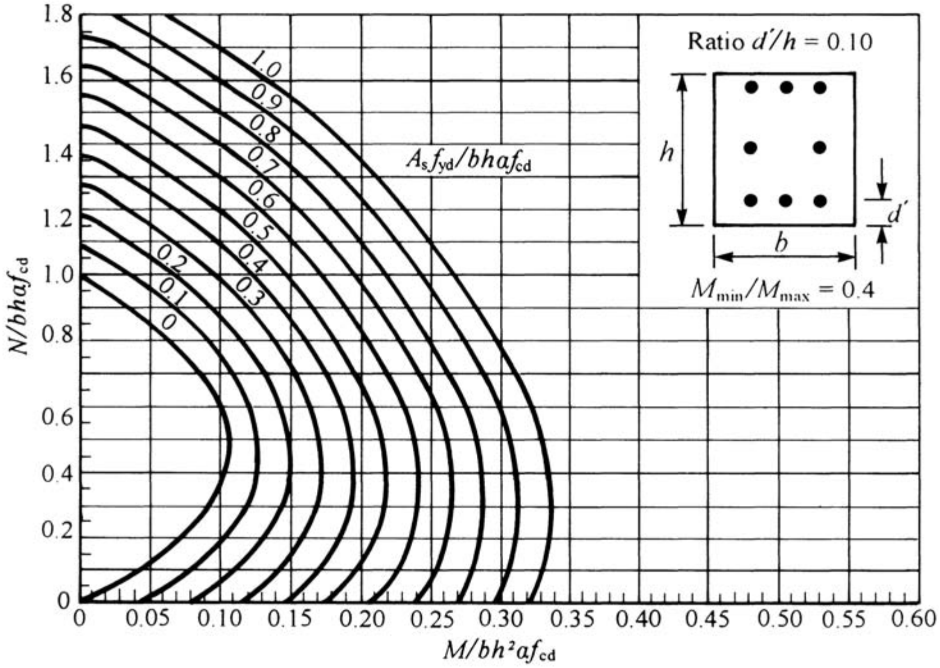


Chart 8.26

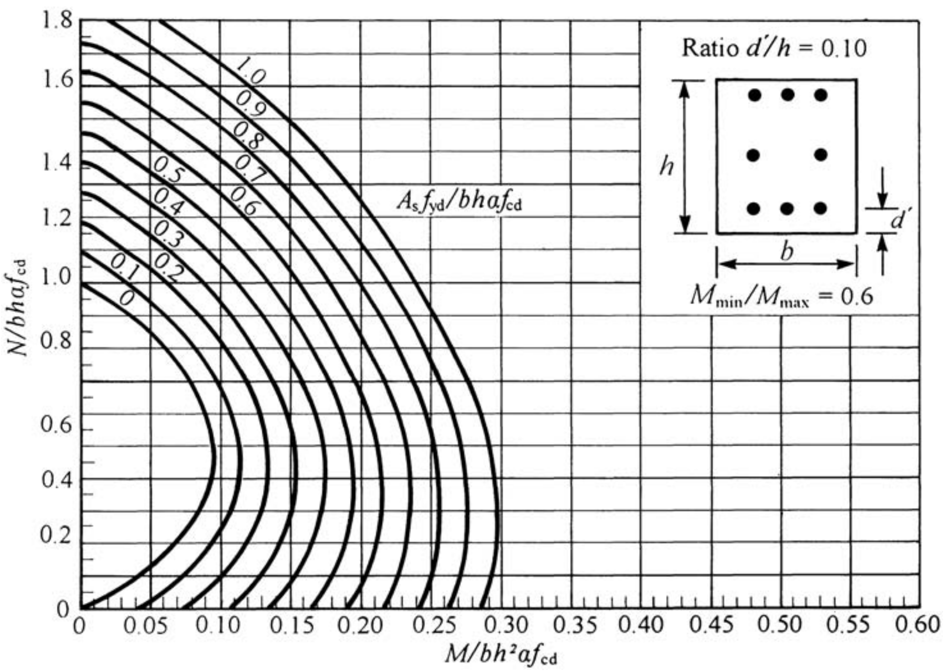


Chart 8.27

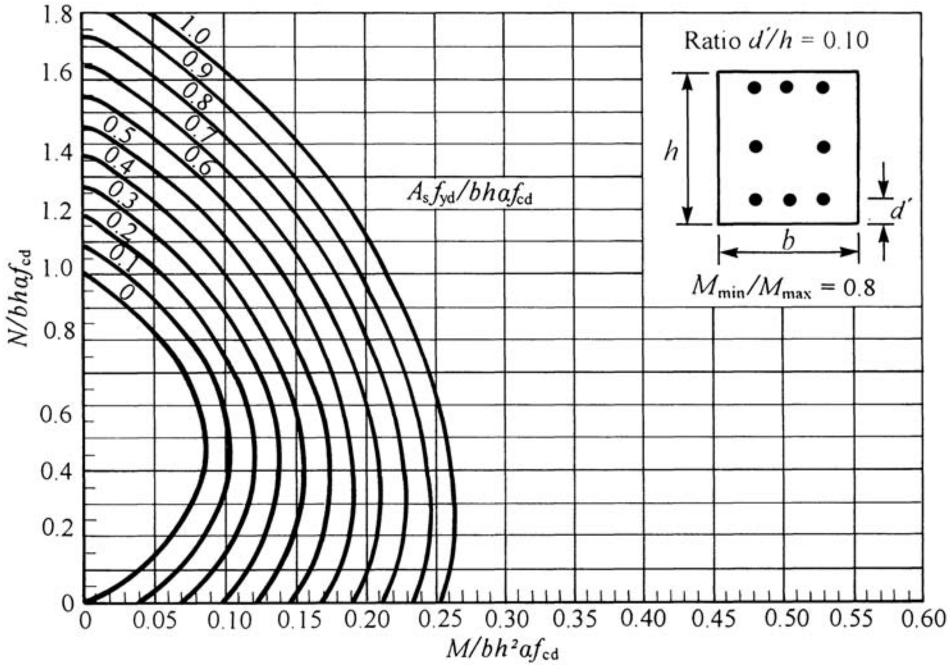


Chart 8.28

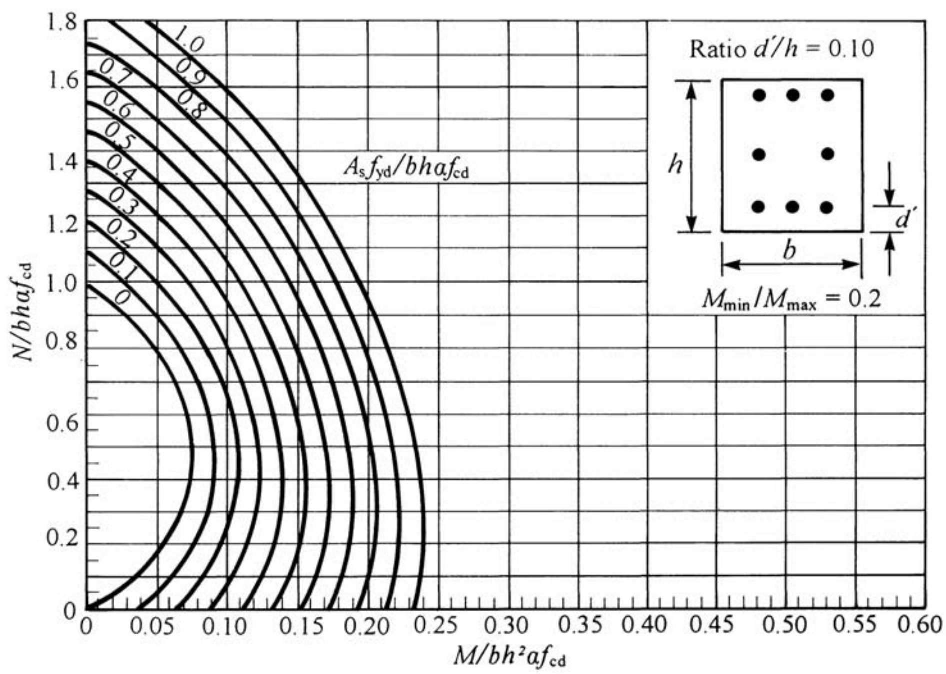


Chart 8.29

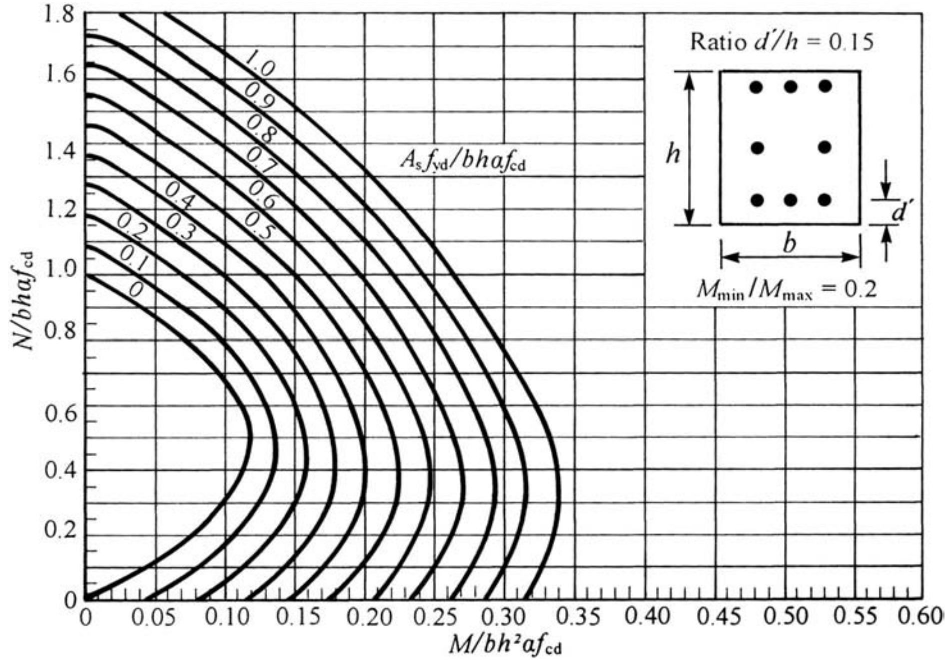


Chart 8.30

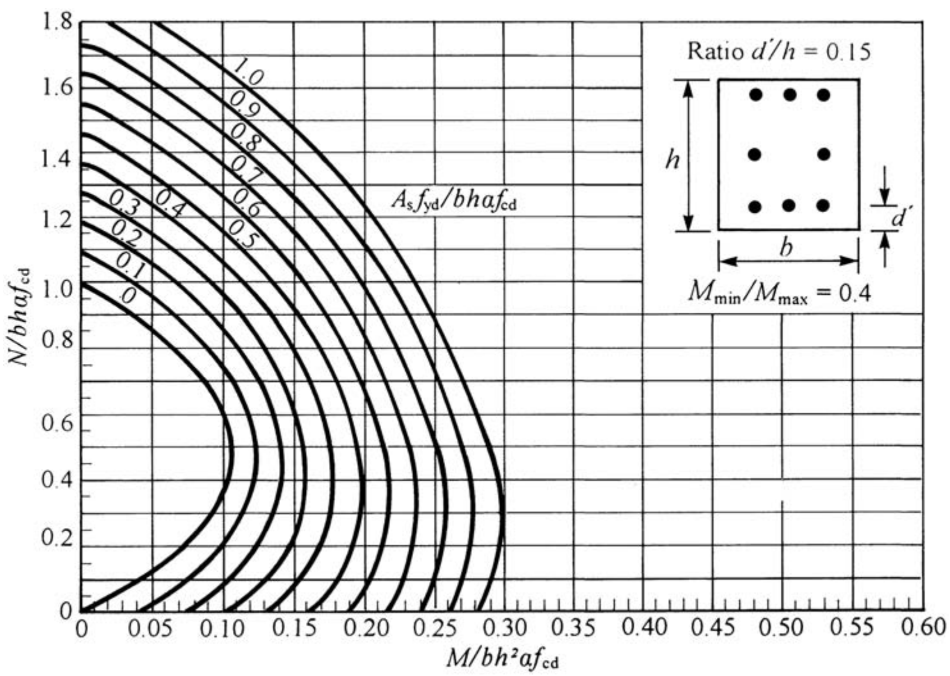


Chart 8.31

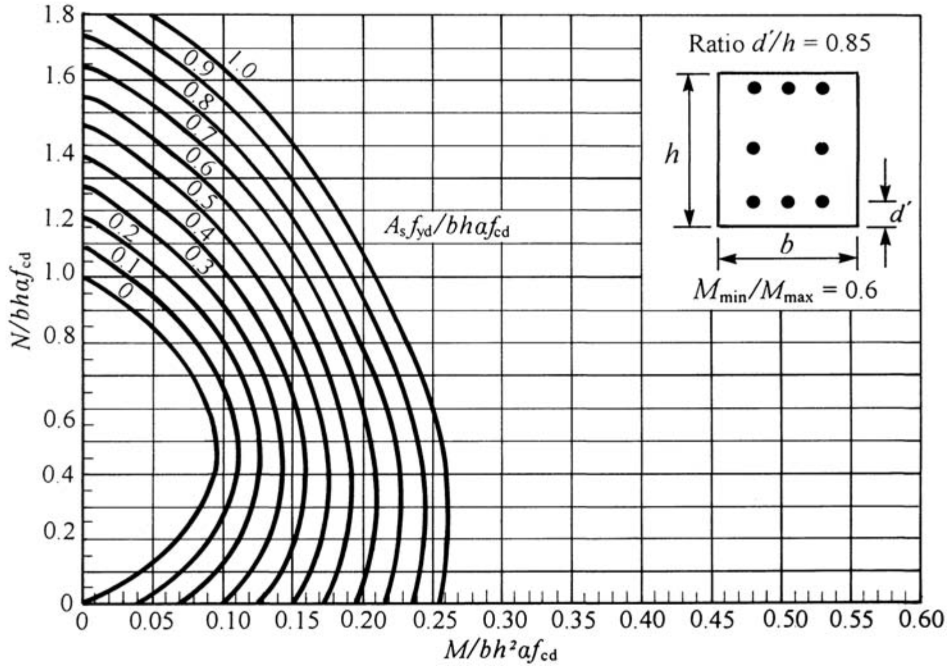


Chart 8.32

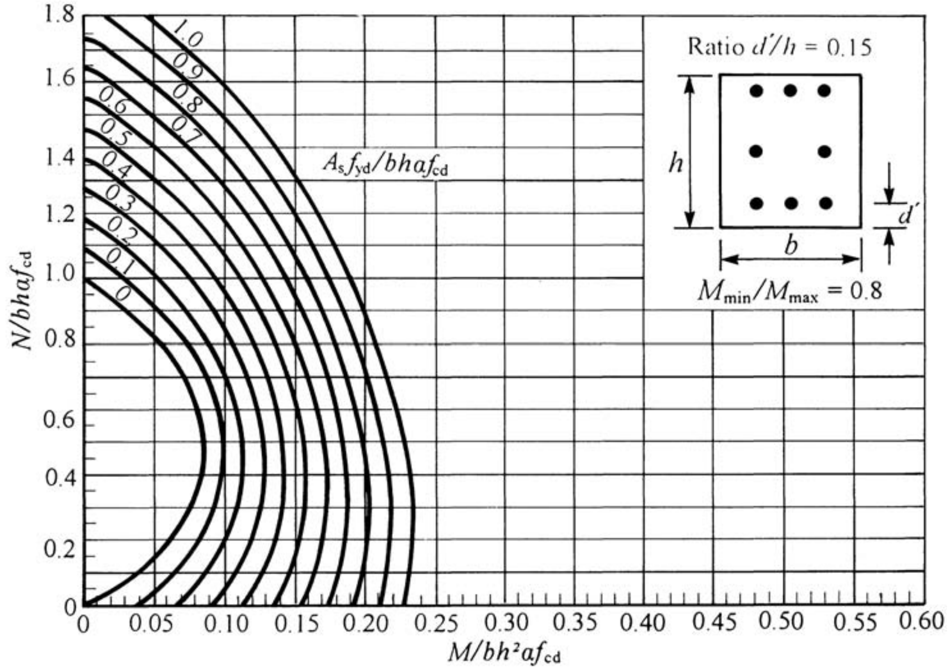


Chart 8.33

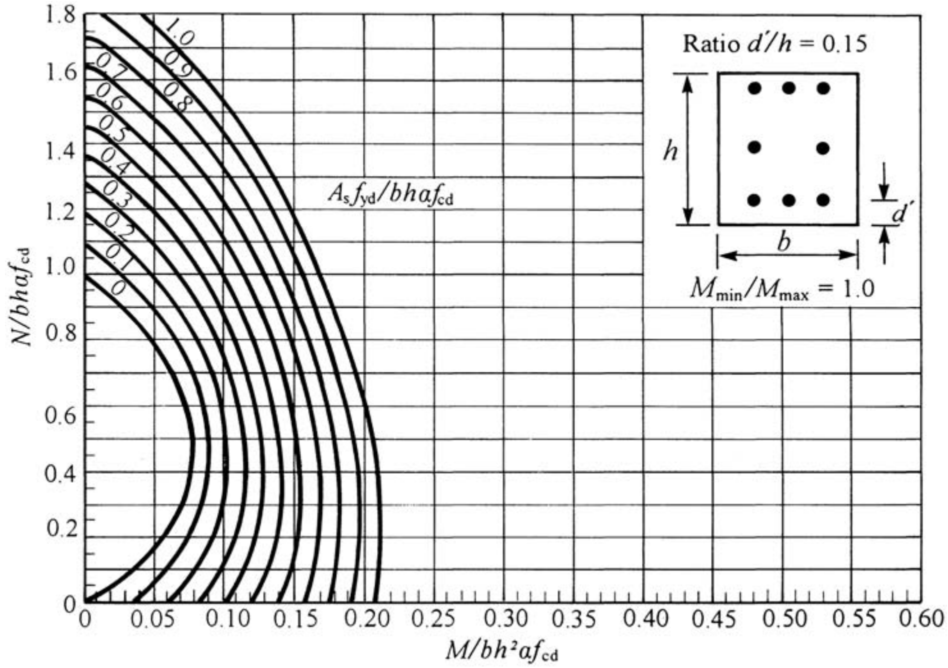


Chart 8.34

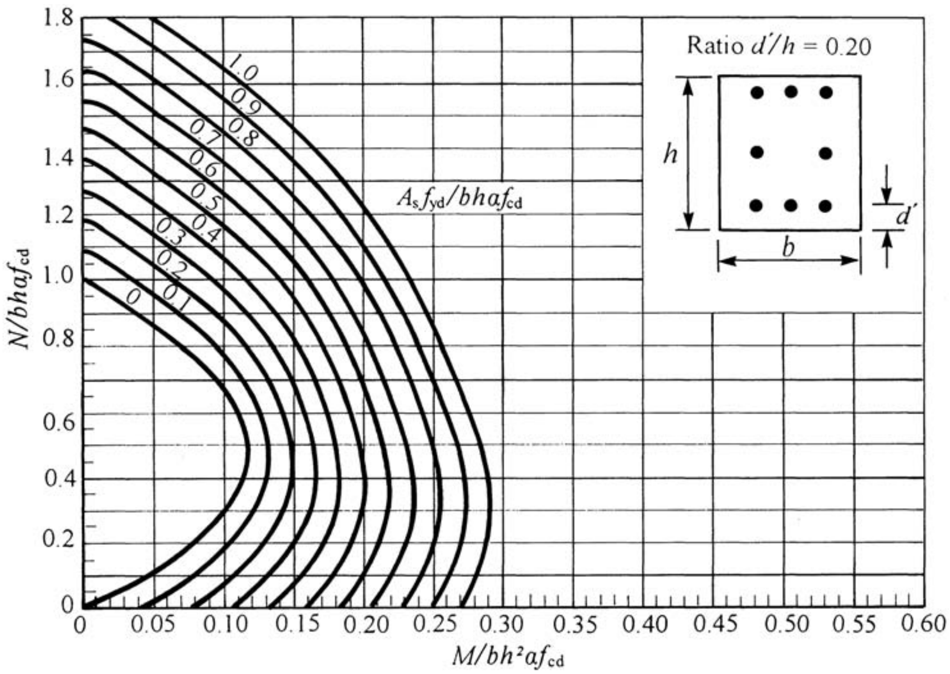


Chart 8.35

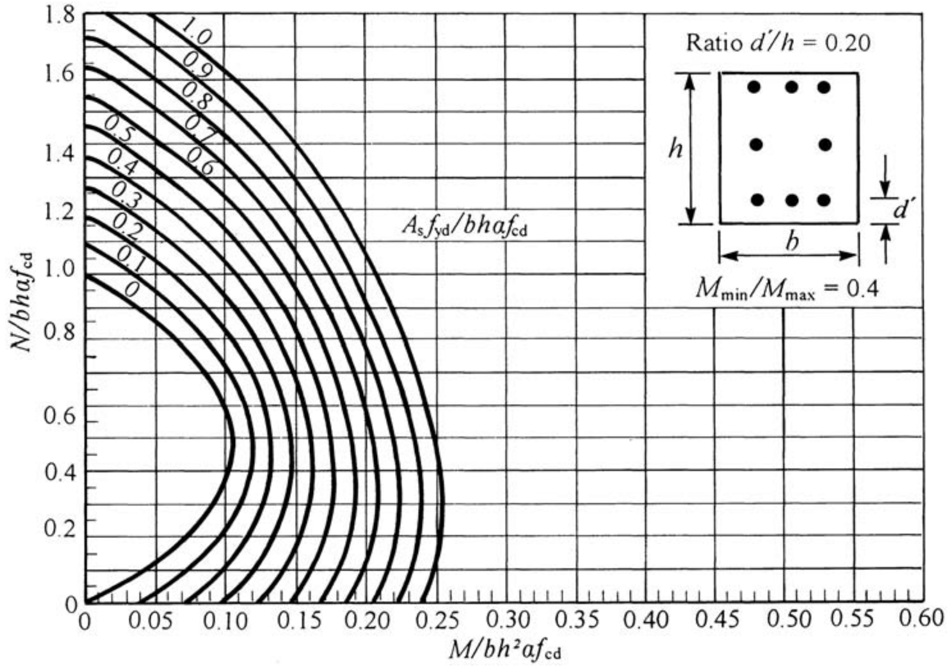


Chart 8.36

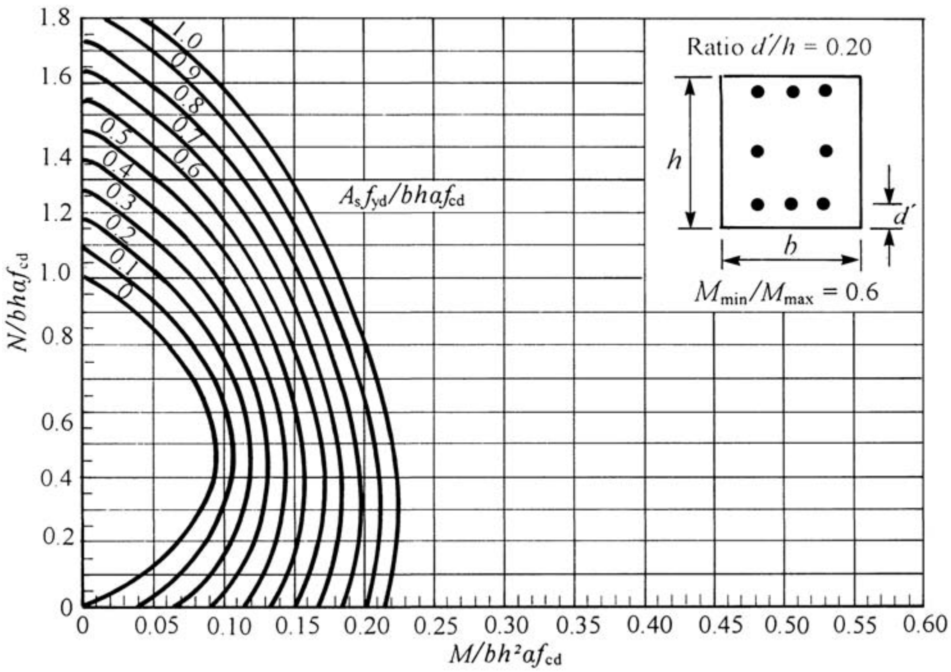


Chart 8.37

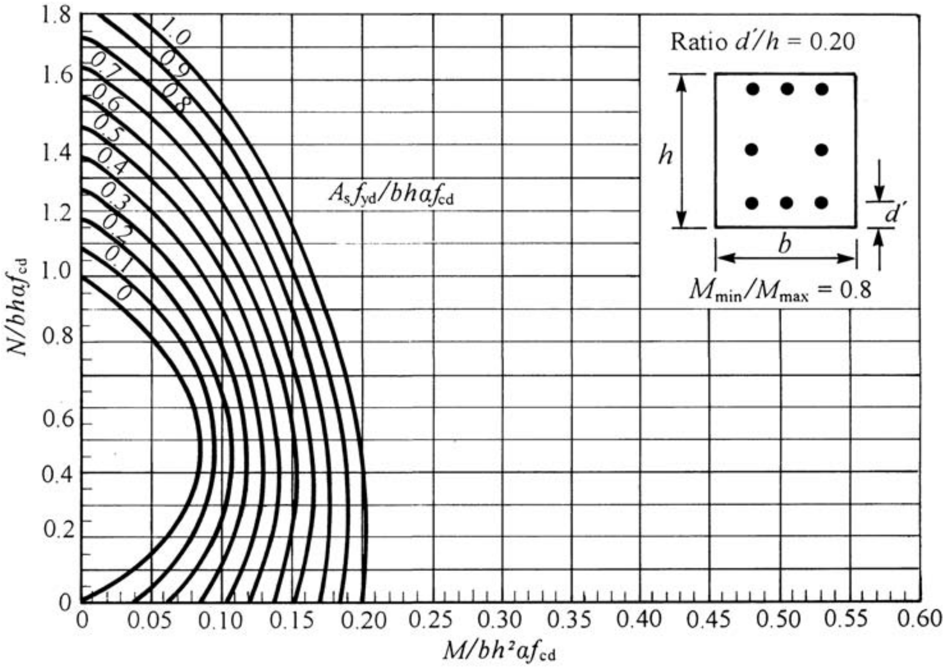


Chart 8.38

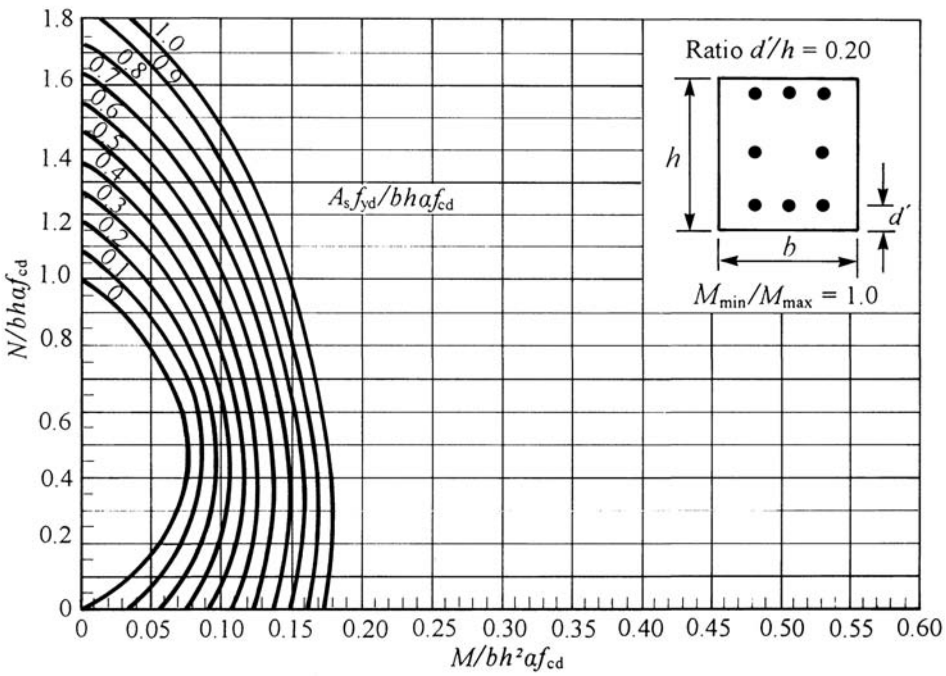


Chart 8.39

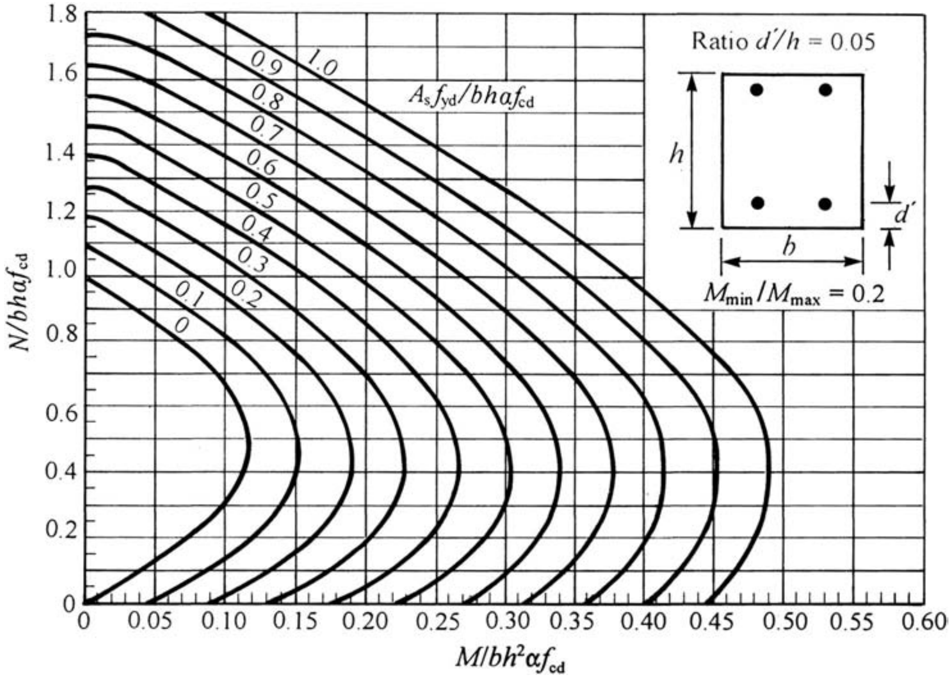


Chart 8.40

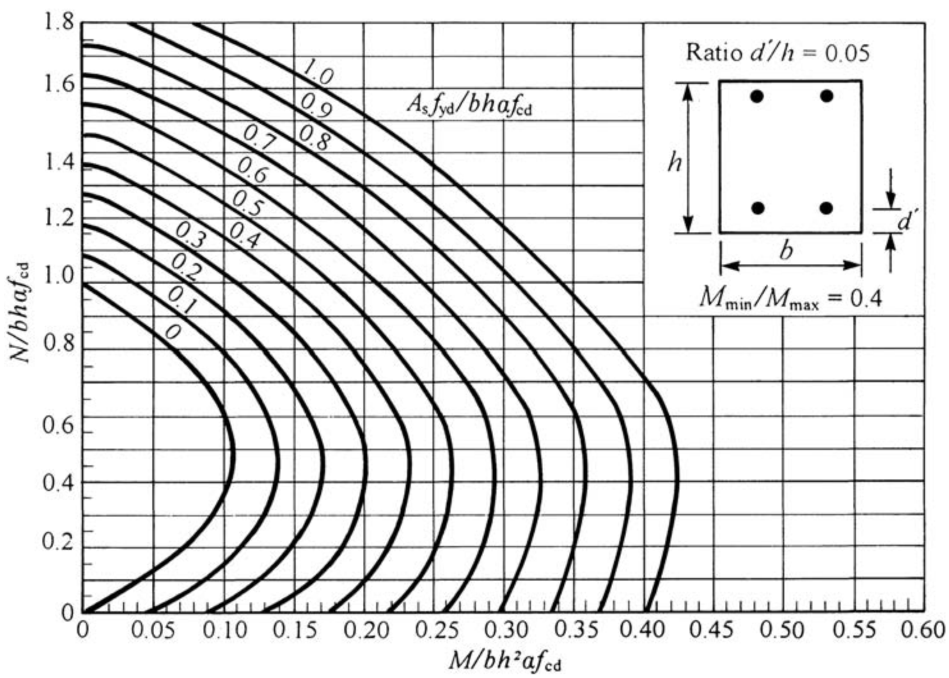


Chart 8.41

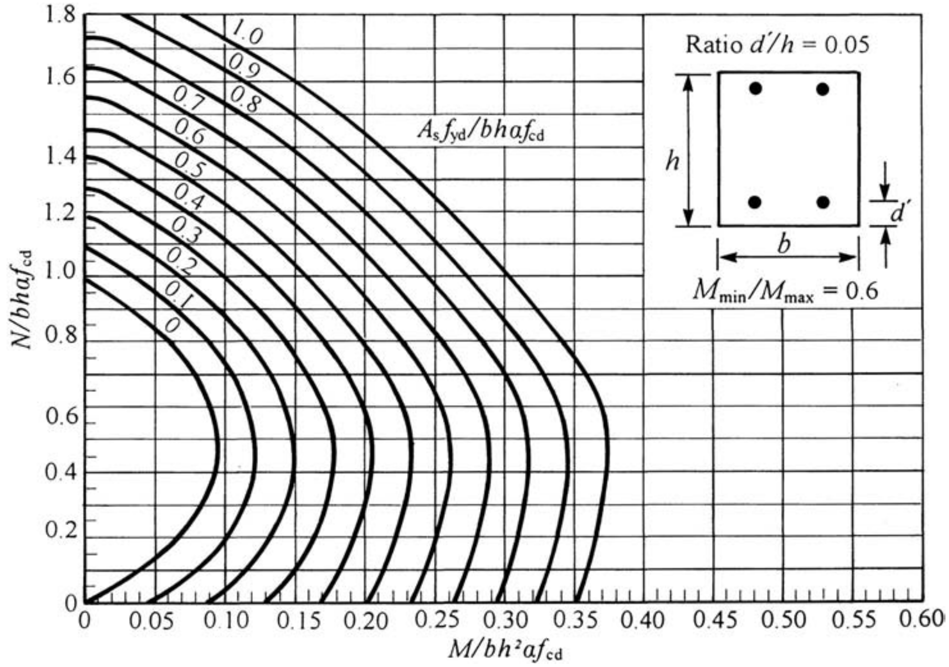


Chart 8.42

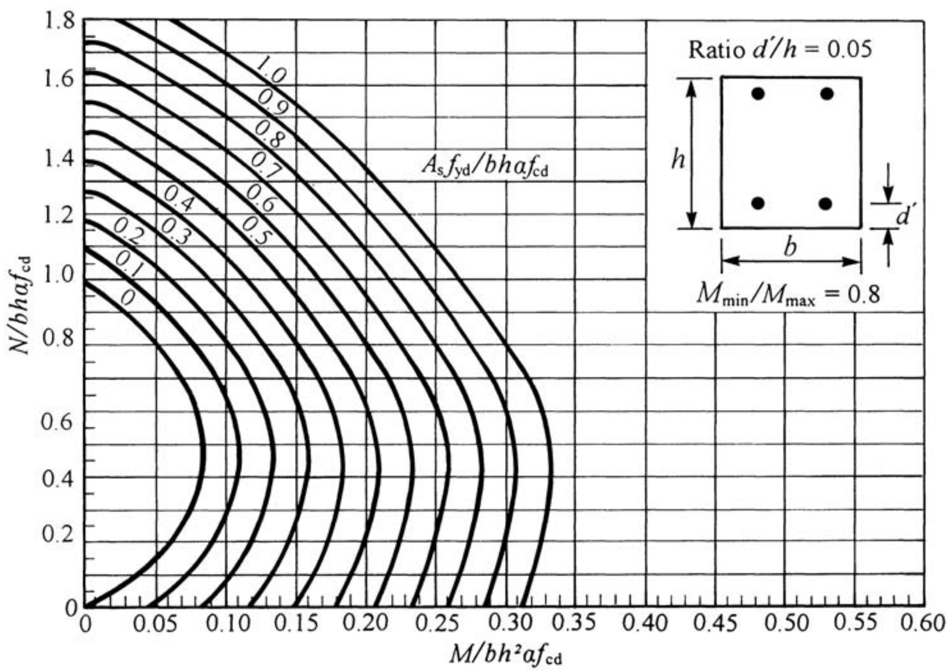


Chart 8.43

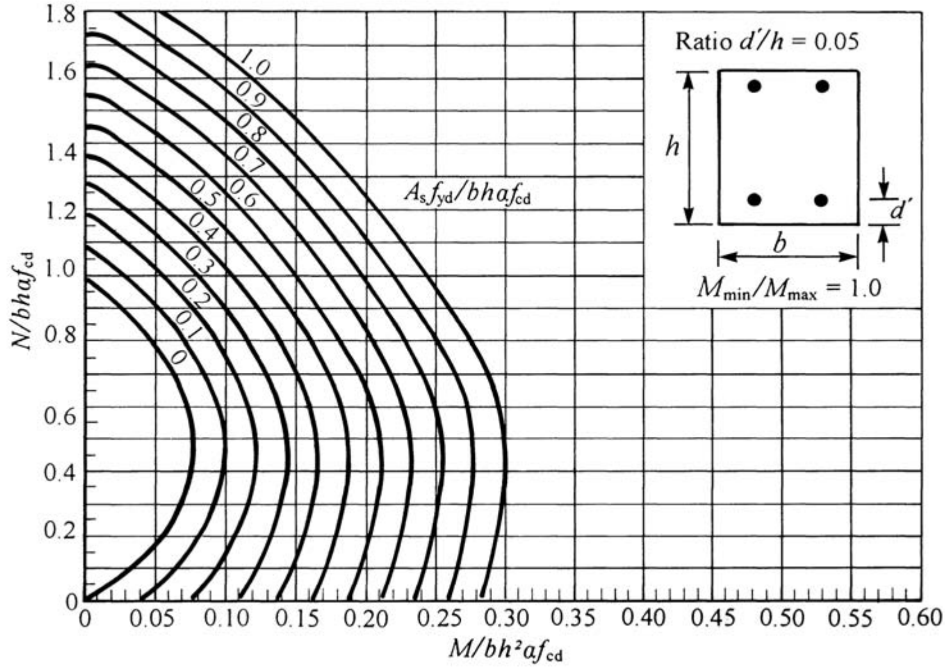


Chart 8.44

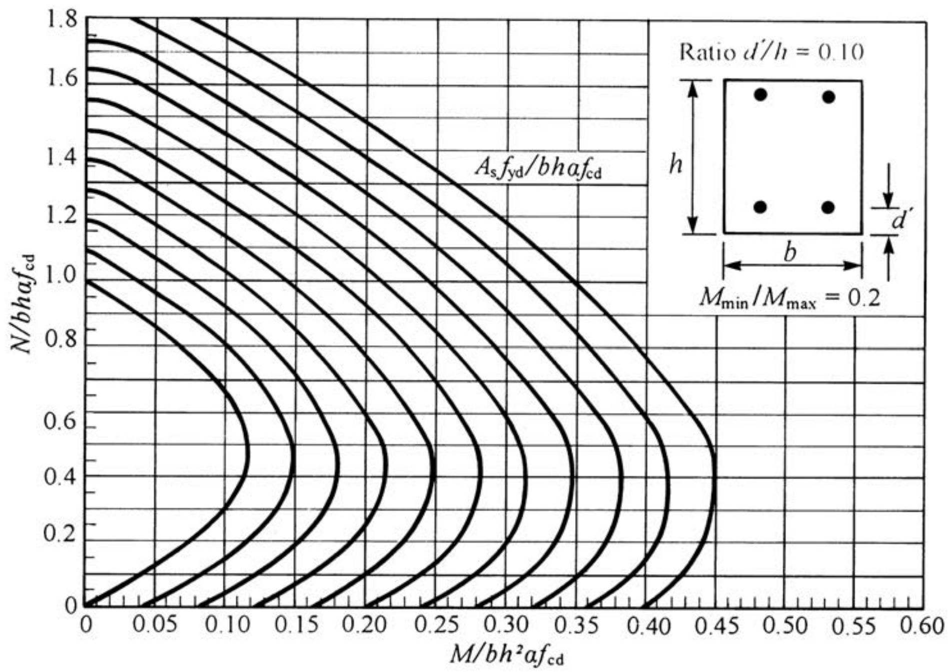


Chart 8.45

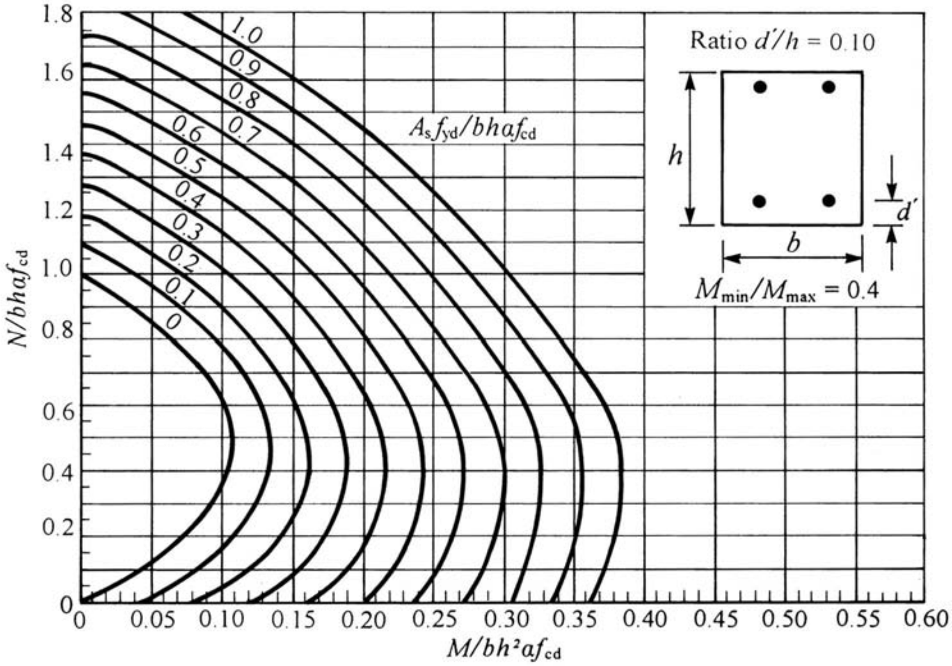


Chart 8.46

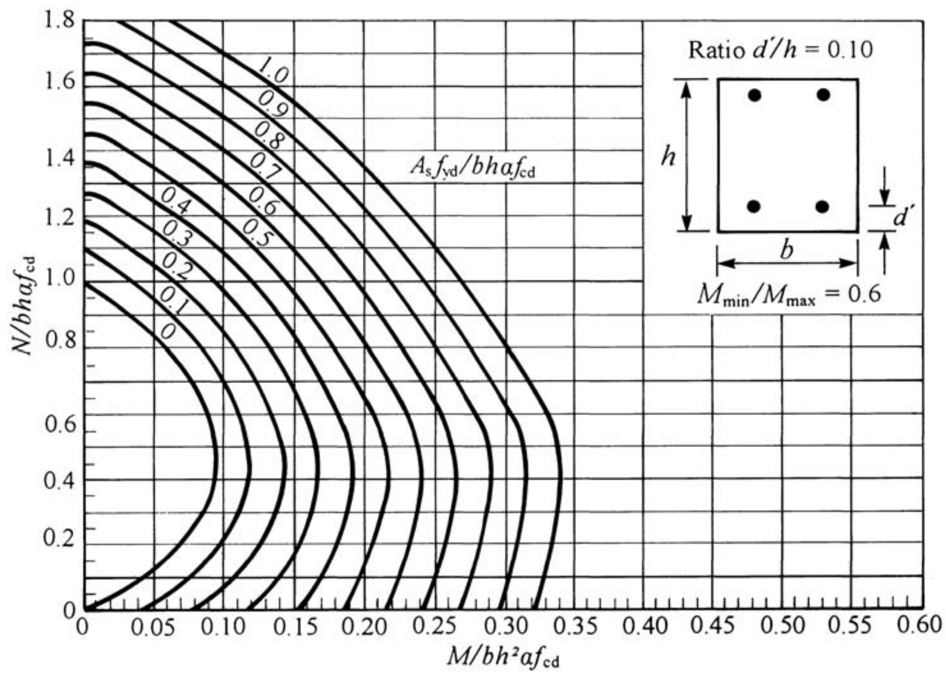


Chart 8.47

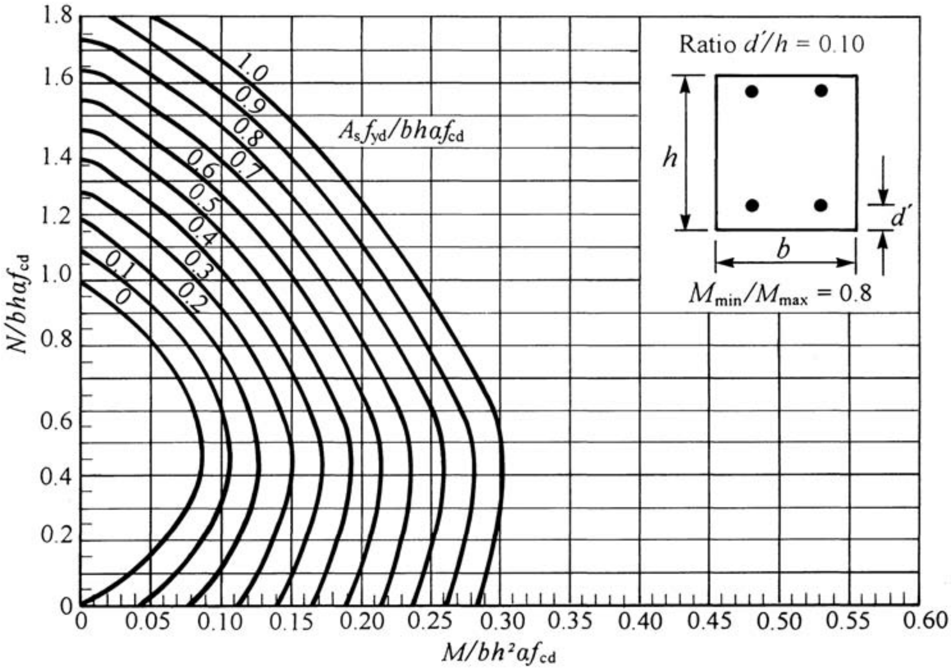


Chart 8.48

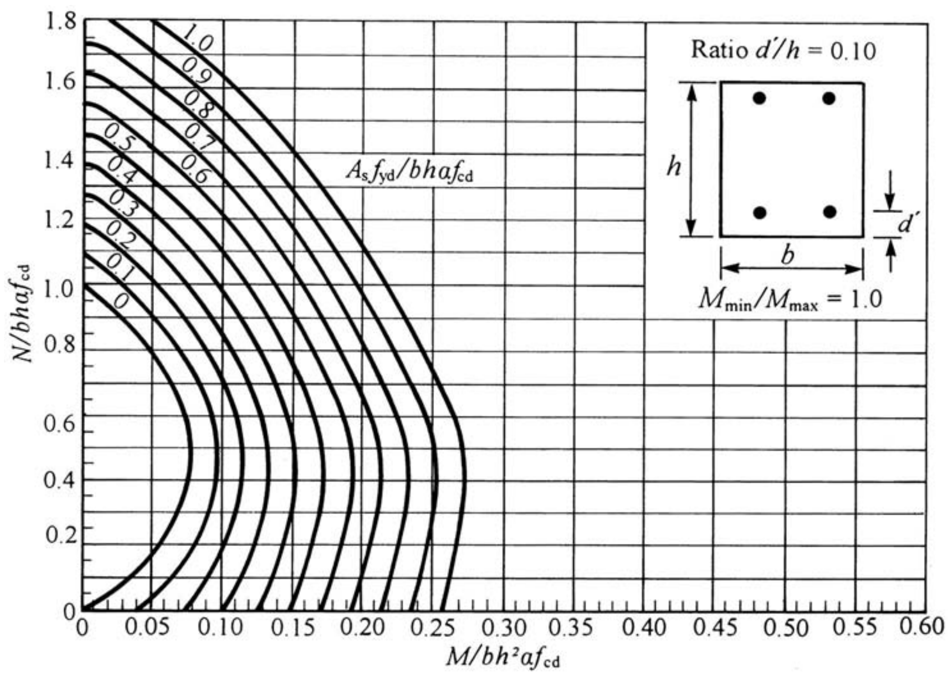


Chart 8.49

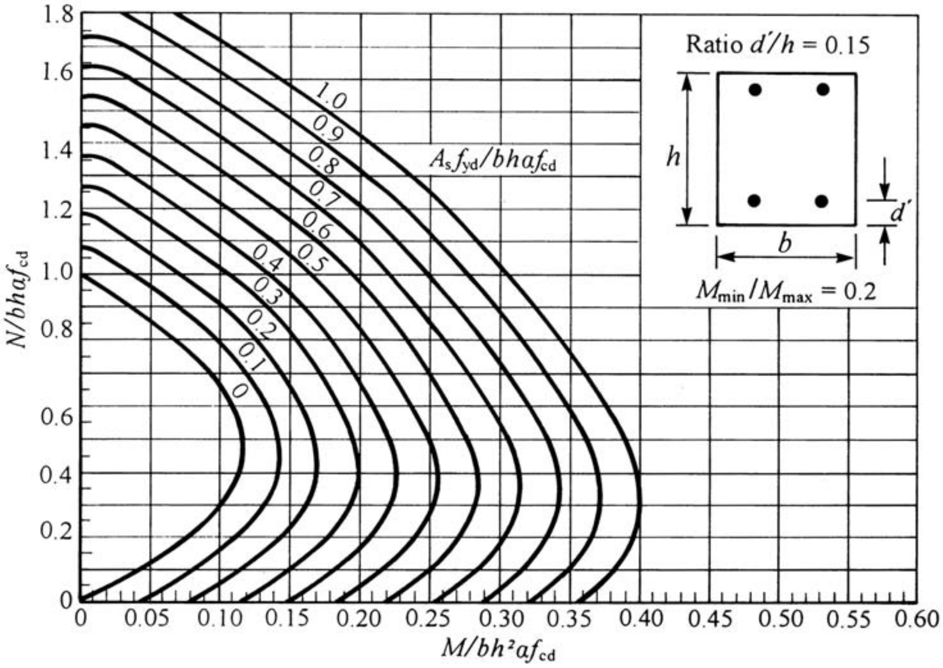


Chart 8.50

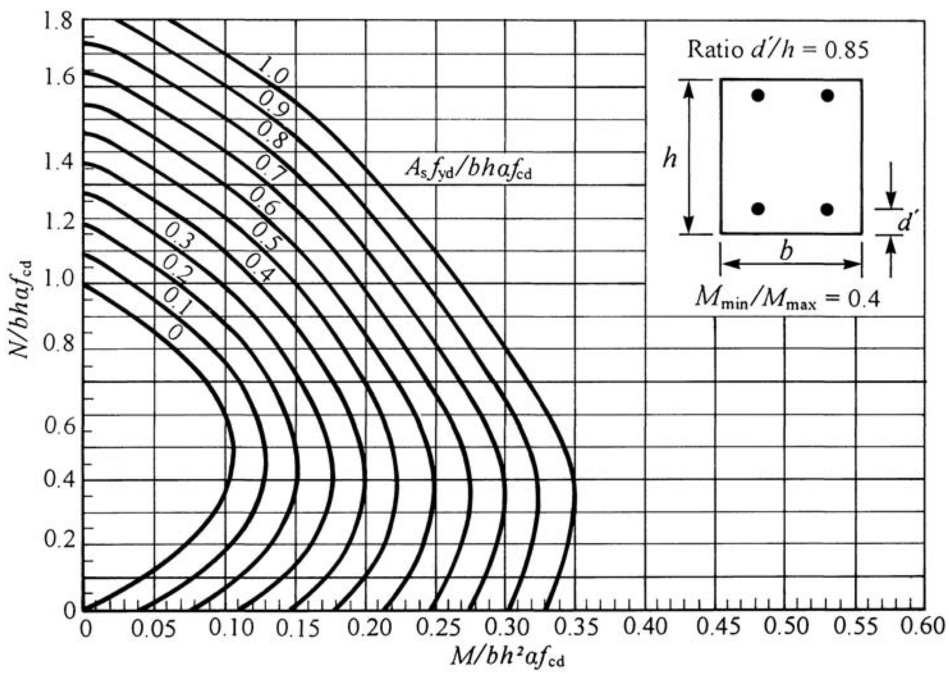


Chart 8.51

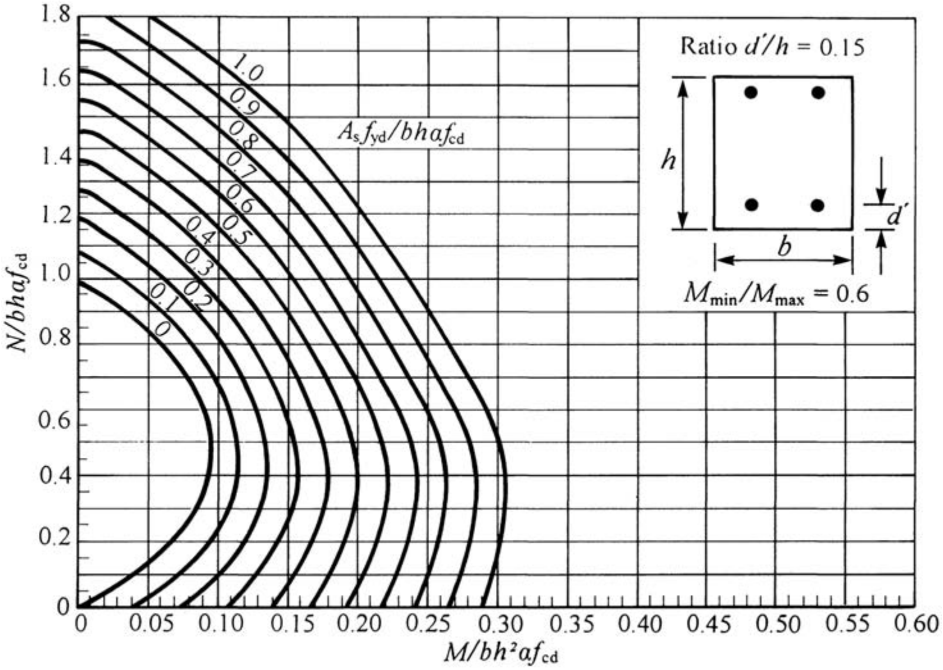


Chart 8.52

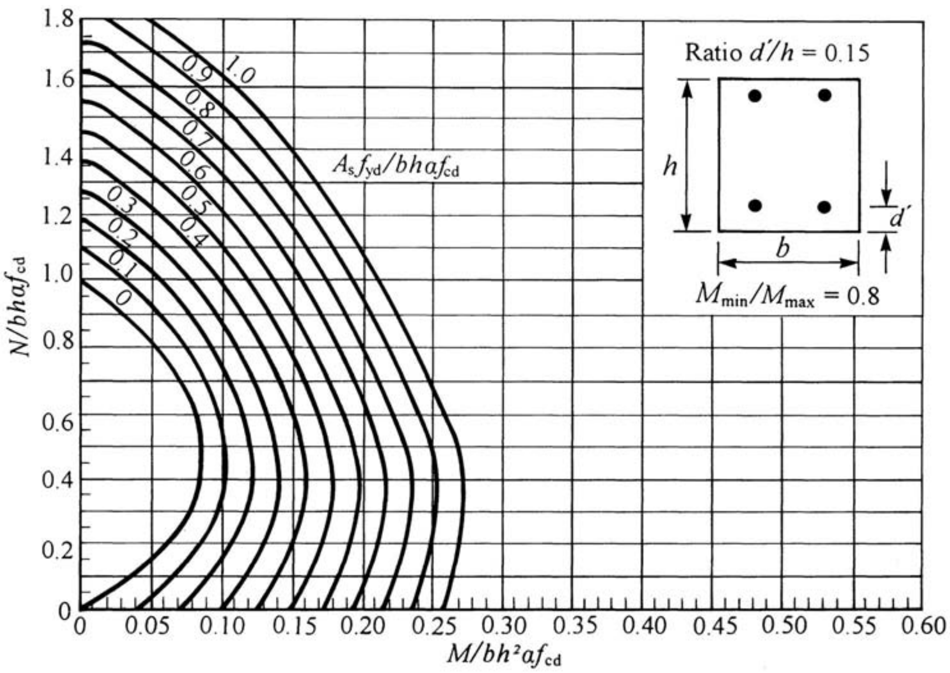


Chart 8.53

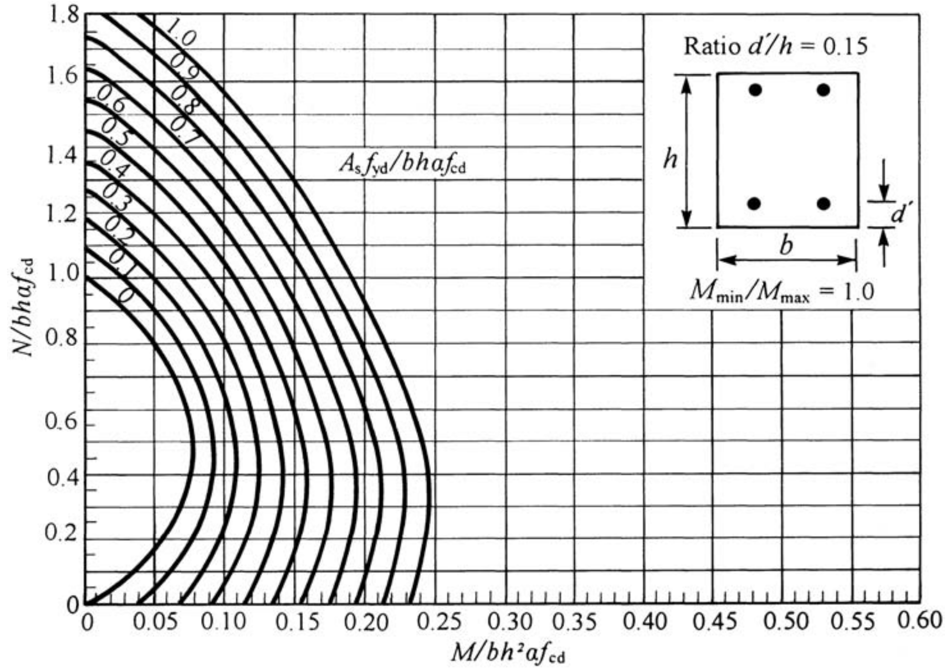


Chart 8.54

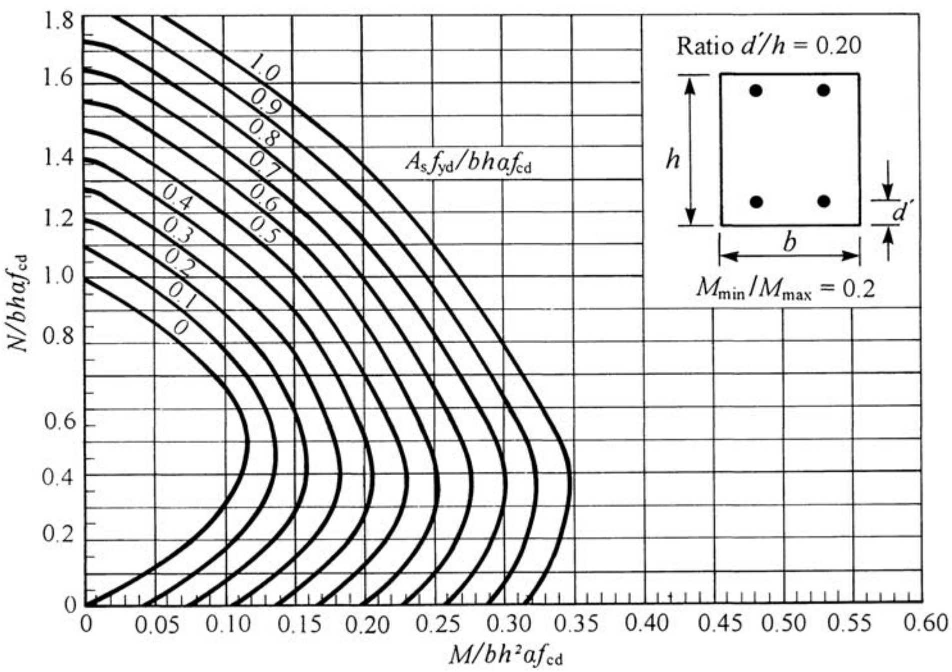


Chart 8.55

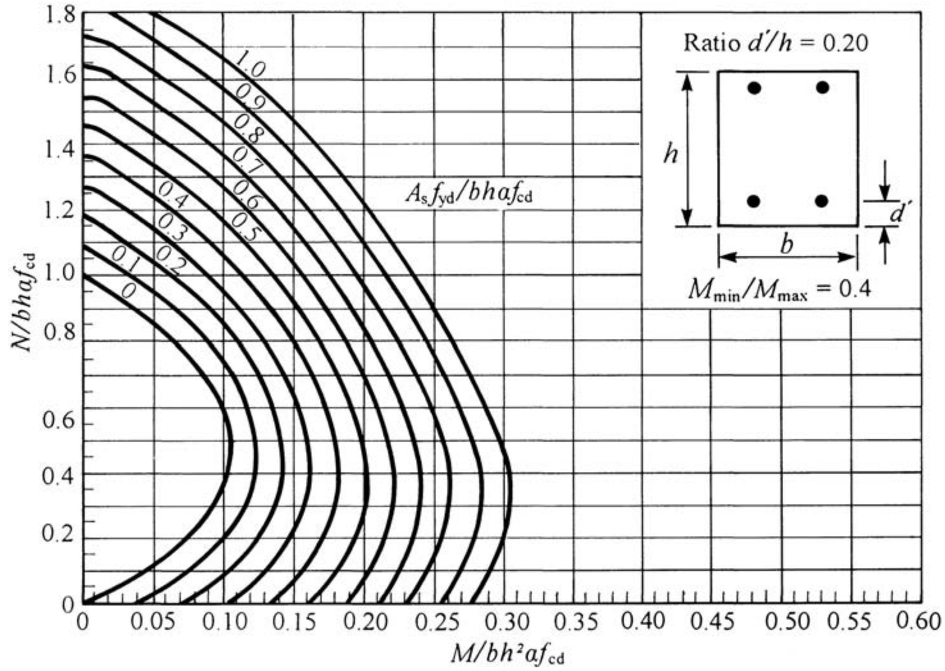


Chart 8.56

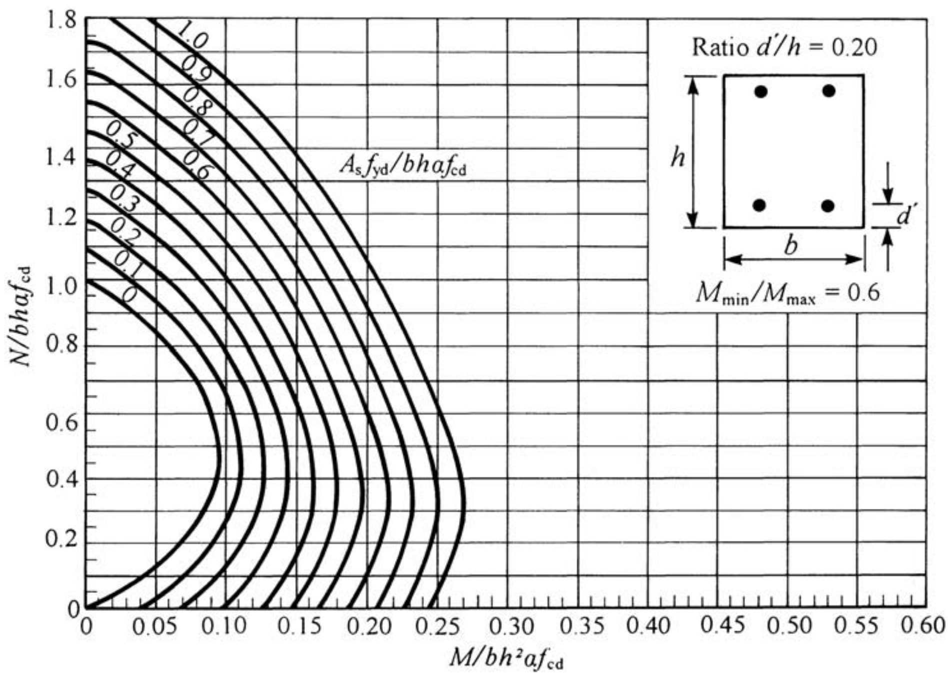


Chart 8.57

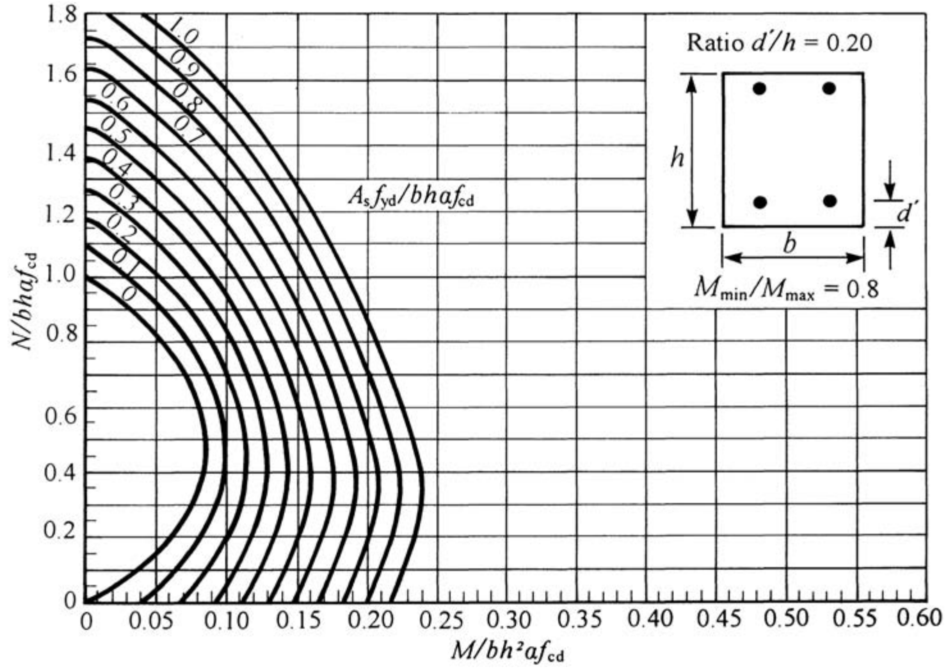


Chart 8.58

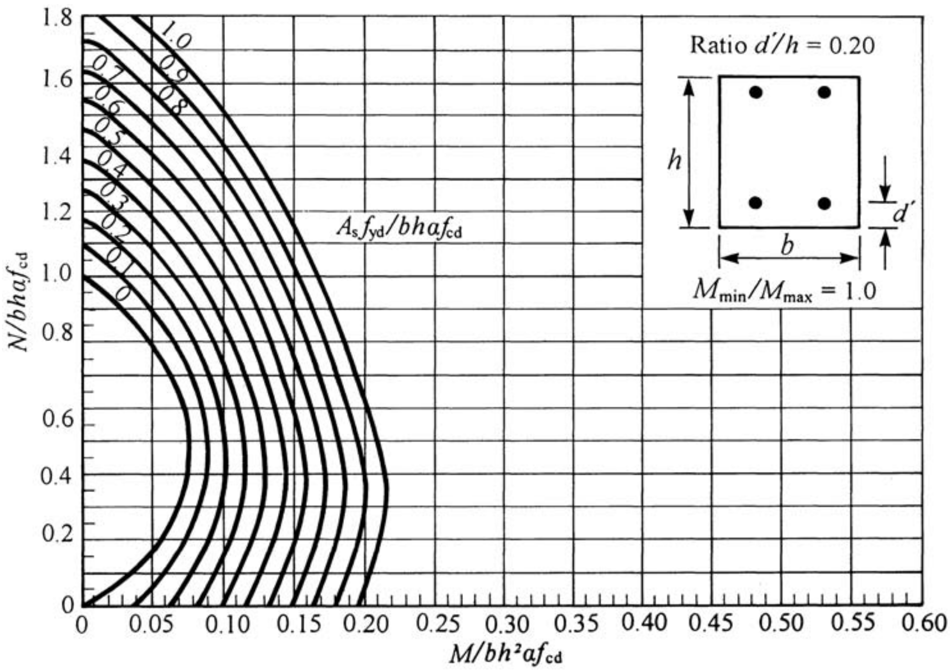


Chart 8.59

9
Shear and torsion

9.1
Shear

9.1.1
General

Elements without shear reinforcement 4.3.2.3

Requirement:

$$V_{Sd} \leq V_{rd1} \quad \rightarrow \quad \frac{V_{Sd}}{b_w d} \leq \frac{V_{Rd1}}{b_w d}$$ 4.3.2.2(2)

V_{Sd}	design shear force
V_{Rd1}	design shear resistance of the member without shear reinforcement
b_w	minimum width of the web
d	effective depth

$$\frac{V_{Rd1}}{b_w d} = \beta (\tau_{Rd}\, k\, (1.2 + 40 \rho_1) + 0.15 \sigma_{cp})$$ according to Table 9.1.2 below

Elements with shear reinforcement 4.3.2.4

Standard method 4.3.2.4.3

Requirements:

$$V_{sd} \leq V_{Rd3}$$ 4.3.2.2(3)

with

$$V_{Rd3} = V_{Rd1} + V_{wd} \quad \rightarrow \quad \frac{V_{Sd}}{d} \leq \frac{V_{Rd1}}{d} + \frac{V_{wd}}{d}$$ [4.22]

and

$$V_{Sd} \leq V_{Rd2} \quad \rightarrow \quad \frac{V_{Sd}}{b_w d} \leq \frac{V_{Rd2}}{b_w d}$$ 4.3.2.2(4)

V_{Rd3}	design shear resistance of the member with shear reinforcement
V_{wd}	contribution of the shear reinforcement
V_{Rd2}	maximum design shear force that can be carried without crushing of the notional concrete compressive struts

$$\frac{V_{wd}}{d} = \frac{A_{sw}}{s}\, 0.9\, \frac{f_{ywk}}{\gamma_s}\, (1 + \cot\alpha)\, \sin\alpha$$

according to Table 9.1.5 below

$$\frac{V_{Rd2}}{b_w d} = \tfrac{1}{2}\, \upsilon\, \frac{f_{ck}}{\gamma_c}\, 0.9\, (1 + \cot\alpha)$$

according to Table 9.1.3a below

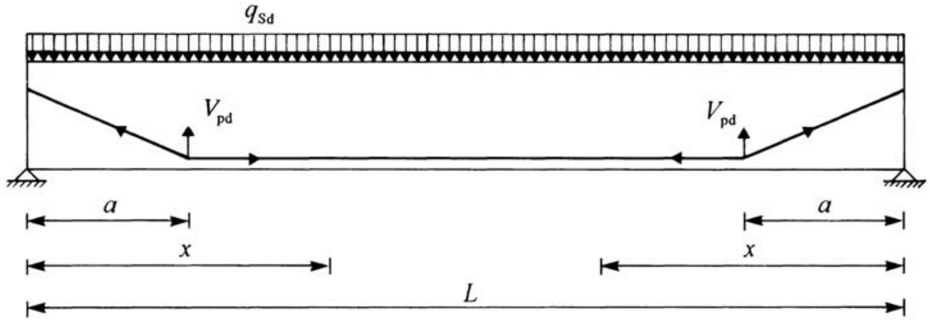


Figure 9.1 Force component (V_{pd}) of the inclined prestressed tendons, parallel to the design shear force in the section (V_{od}).

If the effective average stress in the concrete ($\sigma_{cp.eff}$) is more than 40% of the design value of the compressive cylinder strength of concrete (f_{cd}), V_{Rd2} should be reduced in accordance with the following equation:

$$V_{Rd2.red} = 1.67\ V_{Rd2} \left(1 - \frac{\sigma_{cp.eff}}{f_{ck}}\ \gamma_c\right) \leq V_{Rd2}$$ according to Table 9.1.4 below

$V_{Rd2.red}$ reduced maximal design shear force that can be carried without crushing of the notional concrete compressive struts

Variable strut inclination method 4.3.2.4.4 below

Requirements:

$$V_{Sd} \leq V_{Rd3} \quad \rightarrow \quad \frac{V_{Sd}}{d} \leq \frac{V_{Rd3}}{d}$$ 4.3.2.2(3)

and

$$V_{Sd} \leq V_{Rd2} \quad \rightarrow \quad \frac{V_{Sd}}{b_w d} \leq \frac{V_{Rd2}}{b_w d}$$ 4.3.2.2(4)

$$\frac{V_{wd}}{d} = \frac{A_{sw}}{s}\ \frac{z}{d}\ \frac{f_{ywk}}{\gamma_s}\ (1 + \cot\alpha)\ \sin\alpha\ k_\theta$$

according to Table 9.1.5 below

with $k_\theta = \dfrac{\cot\theta + \cot\alpha}{1 + \cot\alpha}$

$$\frac{V_{Rd2}}{b_w d} = \frac{\upsilon\ \dfrac{f_{ck}}{\gamma_c}\ \dfrac{z}{d}\ (\cot\theta + \cot\alpha)}{(1 + \cot^2\theta)}$$ according to Table 9.1.3b below

If the effective average stress in the concrete ($\sigma_{cp.eff}$) is more than 40% of the design value of the compressive cylinder strength of concrete (f_{cd}), V_{Rd2} should be reduced in accordance with the following equation (4.3.2.2(4)):

$$V_{Rd2.red} = 1.67\ V_{Rd2} \left(1 - \frac{\sigma_{cp.eff}}{f_{ck}}\ \gamma_c\right) \leq V_{Rd2}$$

according to Table 9.1.4 below

For members with inclined prestressing tendons, V_{Sd} is given by:

$$V_{Sd} = V_{od} - V_{pd}$$ [4.32]

V_{od} design shear force in the section

V_{pd} force component of the inclined prestressed tendons, parallel to V_{od} (see Figure 9.1)

Apply the detailing requirements according to 4.3.2.4(4)

For the notation for members subjected to shear, see Figure 9.2.

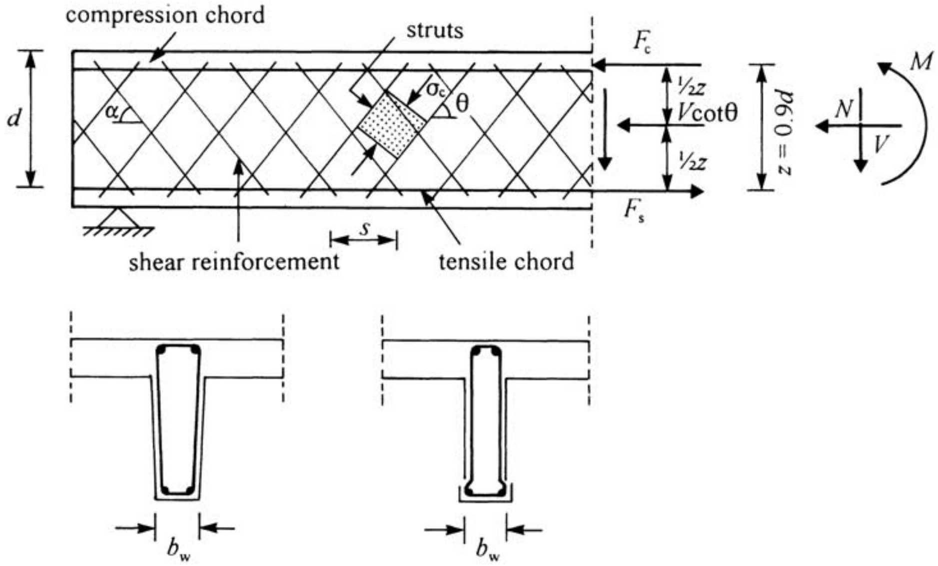


Figure 9.2 Notation for members subjected to shear

9.1.2 $\frac{V_{Rd1}}{b_w d}$

[4.17 and 4.18]

Table 9.1

$100\rho_1$ (%)	**$V_{Rd1}/b_w d$ (with β=1.0; γ=1.5; k=1.0; σ_{cp}=0 N/mm^2) in N/mm^2 per concrete class**								
C12/15	**C16/20**	**C20/25**	**C25/30**	**C30/37**	**C35/45**	**C40/50**	**C45/55**	**C50/60**	
0.0	0.216	0.264	0.312	0.360	0.408	0.444	0.492	0.528	0.576
0.1	0.223	0.273	0.322	0.372	0.422	0.459	0.508	0.546	0.595
0.2	0.230	0.282	0.333	0.384	0.435	0.474	0.525	0.563	0.614
0.3	0.238	0.290	0.343	0.396	0.449	0.488	0.541	0.581	0.634
0.4	0.245	0.299	0.354	0.408	0.462	0.503	0.558	0.598	0.653
0.5	0.252	0.308	0.364	0.420	0.476	0.518	0.574	0.616	0.672
0.6	0.259	0.317	0.374	0.432	0.490	0.533	0.590	0.634	0.691
0.7	0.266	0.326	0.385	0.444	0.503	0.548	0.607	0.651	0.710
0.8	0.274	0.334	0.395	0.456	0.517	0.562	0.623	0.669	0.730
0.9	0.281	0.343	0.406	0.468	0.530	0.577	0.640	0.686	0.749
1.0	0.288	0.352	0.416	0.480	0.544	0.592	0.656	0.704	0.768
1.1	0.295	0.361	0.426	0.492	0.558	0.607	0.672	0.722	0.787
1.2	0.302	0.370	0.437	0.504	0.571	0.622	0.689	0.739	0.806
1.3	0.310	0.378	0.447	0.516	0.585	0.636	0.705	0.757	0.826
1.4	0.317	0.387	0.458	0.528	0.598	0.651	0.722	0.774	0.845
1.5	0.324	0.396	0.468	0.540	0.612	0.666	0.738	0.792	0.864
1.6	0.331	0.405	0.478	0.552	0.626	0.681	0.754	0.810	0.883
1.7	0.338	0.414	0.489	0.564	0.639	0.696	0.771	0.827	0.902
1.8	0.346	0.422	0.499	0.576	0.653	0.710	0.787	0.845	0.922
1.9	0.353	0.431	0.510	0.588	0.666	0.725	0.804	0.862	0.941
2.0	0.360	0.440	0.520	0.600	0.680	0.740	0.820	0.880	0.960
	$\frac{V_{Rd1}}{b_w d} = \beta(\tau_{Rd}k(1.2+40\rho_1)+0.15\sigma_{cp})$ (with $\beta=1.0$; $\gamma=1.5$; $k=1.0$; $\sigma_{cp}=0$ N/mm^2) in N/mm^2								
τ_{Rd} (N/mm^2)	0.18	0.22	0.26	0.30	0.34	0.37	0.41	0.44	0.48

• If the distance x of a concentrated load is less than 2.5d from the face of the support, multiply by β=2.5d/x 5 to determine the design shear resistance of the member without shear reinforcement for the concentrated load

$100\rho_1$ (%)	**$V_{Rd1}/b_w d$ (with β=1.0; γ=1.5; k=1.0; σ_{cp}=0 N/mm^2) in N/mm^2 per concrete class**							
C12/15	**C16/20**	**C20/25**	**C25/30**	**C30/37**	**C35/45**	**C40/50**	**C45/55**	**C50/60**

- If γ_c 1.5, multiply by $1.5/\gamma_c$
- If d<0.6 m, multiply by k=1.6–d (d in metres)
- If 100 ρ_1>2.0%, take $100\rho_1$=2.0% into account
- If σ_{cp} 0 N/mm^2, add β*$0.15\sigma_{cp}$ (compression positive)

9.1.3a Standard method $\frac{V_{Rd2}}{b_w d}$ [4.19, 4.20 and 4.25]

Table 9.2

α (degrees)	$\frac{V_{Rd2}}{b_w d}$ (with γ_c=1.5) in N/mm^2 per concrete class								
C12/15	C 16/20	C20/25	C25/30	C30/37	C35/45	C40/50	C45/55	C50/60	
90	2.30	2.98	3.60	4.31	4.95	5.51	6.00	6.75	7.50
85	2.51	3.24	3.92	4.69	5.38	5.99	6.52	7.34	8.16
80	2.71	3.50	4.23	5.07	5.82	6.48	7.06	7.94	8.82
75	2.92	3.77	4.56	5.47	6.28	6.99	7.61	8.56	9.51
70	3.14	4.06	4.91	5.88	6.75	7.52	8.18	9.21	10.23
65	3.38	4.36	5.28	6.32	7.26	8.08	8.80	9.90	11.00
60	3.63	4.69	5.68	6.80	7.81	8.70	9.46	10.65	11.83
55	3.92	5.06	6.12	7.33	8.42	9.37	10.20	11.48	12.75
50	4.24	5.47	6.62	7.93	9.10	10.14	11.03	12.41	13.79
45	4.61	5.95	7.20	8.63	9.90	11.03	12.00	13.50	15.00

$$\frac{V_{Rd2}}{b_w d} = \tfrac{1}{2}\,\upsilon\,\frac{f_{ck}}{\gamma_c}\,0.9\,(1+\cot\alpha) \qquad \text{(with } \upsilon = 0.7 - \frac{f_{ck}}{200} \nless 0.5 \text{ } (f_{ck} \text{ in N/mm}^2) \text{ and } \gamma_c = 1.5) \text{ in N/mm}^2$$

For sections without designed shear reinforcement, α=90° should be taken [4.19].

- If γ_c 1.5, multiply by $1.5/\gamma_c$

9.1.3b Variable strut inclination method $\frac{V_{Rd2}}{b_w d}$ [4.26 and 4.28]

Table 9.3

α (degrees)	**θ (degrees)**	$\frac{V_{Rd2}}{b_w d}$ (with γ_c = 1.5 and $\frac{z}{d}$ = 0.9) in N/mm^2 per concrete class								
C12/15	C16/20	C20/25	C25/30	C30/37	C35/45	C40/50	C45/55	C50/60		
90	68	1.60	2.07	2.50	3.00	3.44	3.83	4.17	4.69	5.21
60	2.00	2.58	3.12	3.73	4.29	4.77	5.20	5.85	6.50	
45	2.30	2.98	3.60	4.31	4.95	5.51	6.00	6.75	7.50	
30	2.00	2.58	3.12	3.73	4.29	4.77	5.20	5.85	6.50	
22	1.60	2.07	2.50	3.00	3.44	3.83	4.17	4.69	5.21	
75	68	2.66	3.44	4.16	4.98	5.72	6.37	6.93	7.80	8.67
60	2.92	3.77	4.56	5.47	6.28	6.99	7.61	8.56	9.51	
45	2.92	3.77	4.56	5.47	6.28	6.99	7.61	8.56	9.51	
30	2.30	2.98	3.60	4.31	4.95	5.51	6.00	6.75	7.50	
22	1.77	2.29	2.77	3.32	3.81	4.24	4.62	5.20	5.77	
60	68	3.89	5.02	6.07	7.28	8.35	9.30	10.12	11.39	12.65
60	3.99	5.15	6.24	7.47	8.57	9.55	10.39	11.69	12.99	
45	3.63	4.69	5.68	6.80	7.81	8.70	9.46	10.65	11.83	
30	2.66	3.44	4.16	4.98	5.72	6.37	6.93	7.79	8.66	
22	1.97	2.55	3.08	3.69	4.24	4.72	5.14	5.78	6.43	

α (degrees)	θ (degrees)	$\frac{V_{Rd2}}{b_w d}$ (with γ_c = 1.5 and $\frac{z}{d}$ = 0.9) in N/mm² per concrete class								
		C12/15	C16/20	C20/25	C25/30	C30/37	C35/45	C40/50	C45/55	C50/60
45	68	5.56	7.18	8.69	10.41	11.95	13.31	14.48	16.29	18.10
	60	5.45	7.04	8.52	10.20	11.71	13.04	14.20	15.97	17.75
	45	4.61	5.95	7.20	8.63	9.90	11.03	12.00	13.50	15.00
	30	3.15	4.07	4.92	5.89	6.76	7.53	8.20	9.22	10.25
	22	2.25	2.90	3.51	4.21	4.83	5.38	5.85	6.58	7.31

$$\frac{V_{Rd2}}{b_w d} = \frac{\upsilon \frac{f_{ck}}{\gamma_c} \frac{z}{d} (\cot\theta + \cot\alpha)}{(1 + \cot^2\theta)}$$ (with $\upsilon = 0.7 - \frac{f_{ck}}{200} \nless 0.5$ (f_{ck} in N/mm²); $\gamma_c = 1.5$ and $\frac{z}{d} = 0.9$) in N/mm²

- If γ_c 1.5, multiply by $1.5/\gamma_c$
- If z 0.9d, multiply by $z/(0.9d)$

9.1.4 $\frac{V_{Rd2.red}}{V_{Rd2}}$

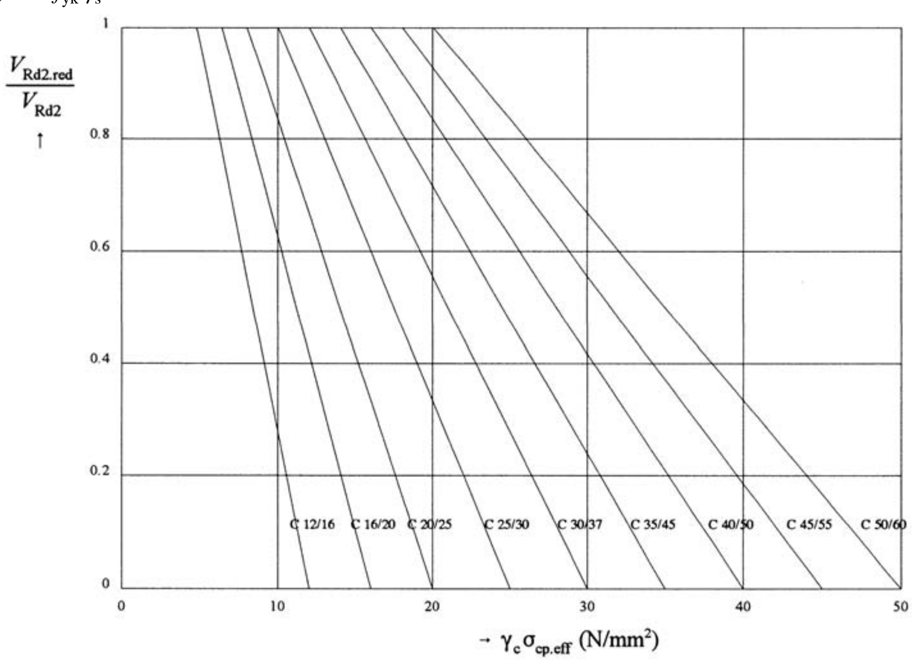
[4.15 and 4.16]

Table 9.4

$$\frac{V_{Rd2.red}}{V_{Rd2}} = 1.67\left(1 - \frac{\sigma_{cp.eff}}{f_{ck}}\gamma_c\right) \leq 1 \quad [4.15]$$

$$\sigma_{cp.eff} = \frac{N_{Sd}}{A_c} - \frac{A_{s2} f_{yk}}{A_c \gamma_s} \quad [4.16]$$

- If f_{yk}/γ_s>400 N/mm², take f_{yk}/γ_s=400 N/mm² into account



9.1.5 $\frac{V_{wd}}{d}$ and $\frac{V_{Rd3}}{d}$

[4.24 and 4.29]

Table 9.5

Stirrups with two legs		$\frac{A_{sw}}{s}$ (mm²/m)	$\frac{V_{wd}}{d}$ **in kN/m for α =**				**Stirrups with two legs**		$\frac{A_{sw}}{s}$ (mm²/m)	$\frac{V_{wd}}{d}$ **in kN/m for α =**			
			90°	**75°**	**60°**	**45°**				**90°**	**75°**	**60°**	**45°**
ø 5	100	393	154	188	210	217	ø 10	100	1571	615	753	840	870
	150	262	103	126	140	145		150	1047	410	502	560	580
	200	196	77	94	105	109		200	785	307	377	420	435
	250	157	61	75	84	87		250	628	246	301	336	348
	300	131	51	63	70	72		300	524	205	251	280	290
ø 6	100	565	221	271	302	313	ø 12	100	2262	886	1085	1210	1252
	150	377	148	181	202	209		150	1508	590	723	806	835
	200	283	111	136	151	157		200	1131	443	542	605	626
	250	226	88	108	121	125		250	905	354	434	484	501
	300	188	74	90	101	104		300	754	295	362	403	417
ø 8	100	1005	393	482	538	557	ø 16	100	4021	1,574	1928	2151	2226
	150	670	262	321	358	371		150	2681	1,050	1285	1434	1484
	200	503	197	241	269	278		200	2011	787	964	1075	1113
	250	402	157	193	215	223		250	1608	630	771	860	891
	300	335	131	161	179	186		300	1340	525	643	717	742

• If θ=45° **Standard method**

$$\frac{V_{wd}}{d} = \frac{A_{sw}}{s}\,0.9\,\frac{f_{ywk}}{\gamma_s}\,(1 + \cot\alpha)\sin\alpha \qquad (\text{with } \gamma_s = 1.15 \text{ and } f_{ywk} = 500\ \text{N/mm}^2) \text{ in N/mm}^2$$

• If γ_s 1.15 multiply by $1.15/\gamma_s$

• If f_{ywk} 500 N/mm², multiply by $f_{ywk}/500$

• If θ 45° **Variable strut inclination method**

$$\frac{V_{wd}}{d} = \frac{A_{sw}}{s}\,\frac{z}{d}\,\frac{f_{ywk}}{\gamma_s}\,(1 + \cot\alpha)\sin\alpha\, k_\theta \qquad (\text{with } \frac{z}{d} = 0.9)$$

k_θ according to:

θ	α	k_θ	θ	α	k_θ	θ	α	k_θ	θ	α	k_θ
68	90	0.404	60	90	0.577	30	90	1.732	22	90	2.475
	75	0.530		75	0.667		75	1.577		75	2.163
	60	0.622		60	0.732		60	1.464		60	1.935
	45	0.702		45	0.789		45	1.366		45	1.738

$$k_\theta = \frac{\cot\theta + \cot\alpha}{1 + \cot\alpha}$$

• If z 0.9d, multiply by $z/(0.9d)$

The upper part of the Table represents the values V_{wd}/d according to equation 4.24
The values in the upper part multiplied by k_θ represent V_{Rd3}/d according to equation 4.29

9.2
Torsion

9.2.1
General

Requirements

$$T_{Sd} \le T_{Rd1} \qquad \rightarrow \qquad \frac{T_{Sd}}{h^3} \le \frac{T_{Rd1}}{h^3} \qquad [4.38]$$

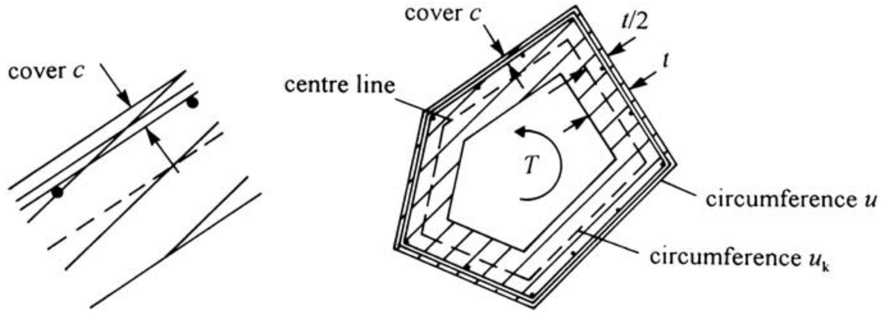


Figure 9.3 Notation used in relation to torsion

and

$$T_{Sd} \le T_{Rd2} \quad \rightarrow \quad \frac{T_{Sd}}{h^2} \le \frac{T_{Rd2}}{h^2}$$

or [4.39]

$$\rightarrow \quad \frac{T_{Sd}}{h^3} \le \frac{T_{Rd2}}{h^3}$$

T_{Sd} design torsional moment

T_{Rd1} maximum torsional moment that can be resisted by the compressive struts in the concrete

T_{Rd2} maximum torsional moment that can be resisted by the torsion reinforcement

$$\frac{T_{Rd1}}{h^3} = \upsilon\, f_{cd}\, \frac{t}{h}\left(\frac{b}{h} - \frac{t}{h}\right)\left(1 - \frac{t}{h}\right) k_{\theta,1}$$

according to Table 9.2.2 below

with $k_{\theta,1} = \dfrac{2}{\cot\theta + \tan\theta}$

$$\frac{T_{Rd2}}{h^2} = 2\left(\frac{b}{h} - \frac{t}{h}\right)\left(1 - \frac{t}{h}\right)\frac{f_{ywk}}{\gamma_s}\,\frac{A_{sw}}{s}\, k_{\theta,2}$$

according to Table 9.2.3 below

with $k_{\theta,2=}\cot\theta$

$$\frac{T_{Rd2}}{h^3} = \frac{A_{sl}}{bh}\,\frac{f_{ylk}}{\gamma_s}\,\frac{\frac{b}{h}\left(\frac{b}{h} - \frac{t}{h}\right)\left(1 - \frac{t}{h}\right)}{\frac{b}{h} - 2\frac{t}{h} + 1}\, k_{\theta,3}$$

according to Table 9.2.4 below

with $k_{\theta,3} = \dfrac{1}{\cot\theta}$

For the notations used in relation to torsion, see Figure 9.3.

9.2.2 $\frac{T_{Rd1}}{h^3}$

[4.40]

Table 9.6

$\frac{b}{h}$	$\frac{t}{h}$	$\frac{T_{Rd1}}{h^3}$ **in N/mm² per concrete class**								
C12/15	C 16/20	C20/25	C25/30	C30/37	C35/45	C40/50	C45/55	C50/60		
1.0	0.250	0.504	0.651	0.788	0.943	1.083	1.206	1.313	1.477	1.641
	0.200	0.459	0.593	0.717	0.859	0.986	1.098	1.195	1.344	1.493
	0.150	0.388	0.502	0.607	0.727	0.834	0.929	1.012	1.138	1.264
	0.100	0.290	0.375	0.454	0.543	0.624	0.695	0.756	0.851	0.945
	0.050	0.162	0.209	0.253	0.303	0.347	0.387	0.421	0.474	0.526

$\frac{b}{h}$	$\frac{t}{h}$	$\frac{T_{Rd1}}{h^3}$ **in N/mm² per concrete class**								
		C12/15	C 16/20	C20/25	C25/30	C30/37	C35/45	C40/50	C45/55	C50/60
0.8	0.222	0.358	0.462	0.559	0.670	0.769	0.856	0.932	1.049	1.165
	0.200	0.344	0.444	0.538	0.644	0.739	0.823	0.896	1.008	1.120
	0.150	0.297	0.384	0.464	0.556	0.638	0.711	0.774	0.870	0.967
	0.100	0.226	0.292	0.353	0.423	0.485	0.540	0.588	0.662	0.735
	0.050	0.128	0.165	0.200	0.239	0.274	0.305	0.333	0.374	0.416
0.6	0.188	0.225	0.291	0.352	0.422	0.484	0.539	0.587	0.660	0.733
	0.150	0.206	0.266	0.321	0.385	0.442	0.492	0.536	0.602	0.669
	0.100	0.161	0.208	0.252	0.302	0.347	0.386	0.420	0.473	0.525
	0.050	0.094	0.121	0.146	0.175	0.201	0.224	0.244	0.274	0.305
0.4	0.143	0.113	0.146	0.176	0.211	0.242	0.270	0.294	0.331	0.367
	0.100	0.097	0.125	0.151	0.181	0.208	0.232	0.252	0.284	0.315
	0.050	0.060	0.077	0.093	0.112	0.128	0.143	0.155	0.175	0.194
0.2	0.083	0.032	0.041	0.050	0.060	0.069	0.076	0.083	0.094	0.104
	0.050	0.026	0.033	0.040	0.048	0.055	0.061	0.067	0.075	0.083

$$\frac{T_{Rd1}}{h^3} = \upsilon f_{cd} \frac{t}{h}\left(\frac{b}{h} - \frac{t}{h}\right)\left(1 - \frac{t}{h}\right) k_{\theta,1} \quad \text{with } \upsilon = 0.7\left(0.7\ \frac{f_{ck}}{200}\right) \nless 0.35\ (f_{ck} \text{ in N/mm}^2)\ ;\ \gamma_c = 1.5;\ \theta = 45°$$

and so $k_{\theta,1}$=1.0) in N/mm²

For *t/h* any value meeting the requirements *t A/u, t* ≯ the actual wall thickness of a hollow section, and *t* ≮ *2c* may be chosen. The maximum value *T=A/u* is given as the maximum value in the Table for each value of *b/h.*

- If γ_c 1.5, multiply by $1.5/\gamma_c$
- If θ 45°, multiply by $k_{\theta,1}$ according to:

θ (degrees)		68	65	60	55	50	45
22	25		30	35	40	45	
$k_{\theta,1}$		0.69	0.77	0.87	0.94	0.98	1.00

$k_{\theta,1}$=2/(cotθ+tan θ)

9.2.3a $\frac{T_{Rd2}}{h^2}$ **[4.43]**

Table 9.7

$\frac{b}{h}$	$\frac{t}{h}$	$\frac{T_{Rd2}}{h^2}$ **in kN/m per configuration of stirrups with one leg**									
		Stirrups ø 5-*s*					Stirrups ø 6-*s*				
		100	150	200	250	300	100	150	200	250	300
		A_{sw}/s (mm²/m)					A_{sw}/s (mm²/m)				
		196	131	98	79	65	283	188	141	113	94
1.0	0.250	96.1	64.1	48.0	38.4	32.0	138.4	92.2	69.2	55.3	46.1
	0.200	109.3	72.9	54.7	43.7	36.4	157.4	105.0	78.7	63.0	52.5
	0.150	123.4	82.3	61.7	49.4	41.1	177.7	118.5	88.9	71.1	59.2
	0.100	138.4	92.2	69.2	55.3	46.1	199.2	132.8	99.6	79.7	66.4
	0.050	154.2	102.8	77.1	61.7	51.4	222.0	148.0	111.0	88.8	74.0
0.8	0.222	76.8	51.2	38.4	30.7	25.6	110.5	73.7	55.3	44.2	36.8
	0.200	82.0	54.7	41.0	32.8	27.3	118.1	78.7	59.0	47.2	39.4
	0.150	94.4	62.9	47.2	37.8	31.5	135.9	90.6	68.0	54.4	45.3
	0.100	107.6	71.7	53.8	43.0	35.9	155.0	103.3	77.5	62.0	51.7

$\frac{b}{h}$	$\frac{t}{h}$	$\frac{T_{Rd2}}{h^2}$ **in kN/m per configuration of stirrups with one leg**									
		Stirrups ø 5-*s*					Stirrups ø 6-*s*				
		100	150	200	250	300	100	150	200	250	300
		A_{sw}/s (mm^2/m)					A_{sw}/s (mm^2/m)				
		196	131	98	79	65	283	188	141	113	94
	0.050	121.7	81.1	60.9	48.7	40.6	175.3	116.8	87.6	70.1	58.4
0.6	0.188	57.3	38.2	28.6	22.9	19.1	82.4	55.0	41.2	33.0	27.5
	0.150	65.3	43.6	32.7	26.1	21.8	94.1	62.7	47.0	37.6	31.4
	0.100	76.9	51.2	38.4	30.7	25.6	110.7	73.8	55.3	44.3	36.9
	0.050	89.3	59.5	44.6	35.7	29.8	128.5	85.7	64.3	51.4	42.8
0.4	0.143	37.7	25.1	18.8	15.1	12.6	54.2	36.1	27.1	21.7	18.1
	0.100	46.1	30.7	23.1	18.4	15.4	66.4	44.3	33.2	26.6	22.1
	0.050	56.8	37.9	28.4	22.7	18.9	81.8	54.5	40.9	32.7	27.3
0.2	0.083	18.3	12.2	9.1	7.3	6.1	26.3	17.5	13.2	10.5	8.8
	0.050	24.3	16.2	12.2	9.7	8.1	35.1	23.4	17.5	14.0	11.7

$$\frac{T_{Rd2}}{h^2} = 2\left(\frac{b}{h} - \frac{t}{h}\right)\left(1 - \frac{t}{h}\right)\frac{f_{ywk}}{\gamma_s}\frac{A_{sw}}{s}k_{\theta,2} \quad \text{(with } f_{ywk} = 0.5 \text{ kN/mm}^2;\ \gamma_s = 1.15;\ \theta = 45° \text{ and so } k_{\theta,2} = 1.0\text{) in kN/m}$$

- If γ_s 1.15, multiply by 1.15/γ_s.
- If f_{ywk} 0.5 kN/mm^2, multiply by f_{ywk}/0.5
- If θ 45°, multiply by $k_{\theta,2}$=cotθ

For *t/h* and θ, the same values as in Table 9.6 should be used.

9.2.3b $\frac{T_{Rd2}}{h^2}$ **[4.43]**

Table 9.8

$\frac{b}{h}$	$\frac{t}{h}$	$\frac{T_{Rd2}}{h^2}$ **in kN/m per configuration of stirrups with one leg**									
		Stirrups ø 8-*s*					Stirrups ø 10-*s*				
		100	150	200	250	300	100	150	200	250	300
		A_{sw}/s (mm^2/m)					A_{sw}/s (mm^2/m)				
		503	335	251	201	168	785	524	393	314	262
1.0	0.250	246.0	164.0	123.0	98.4	82.0	384.4	256.2	192.2	153.7	128.1
	0.200	279.9	186.6	139.9	112.0	93.3	437.3	291.5	218.7	174.9	145.8
	0.150	316.0	210.6	158.0	126.4	105.3	493.7	329.1	246.8	197.5	164.6
	0.100	354.2	236.1	177.1	141.7	118.1	553.5	369.0	276.7	221.4	184.5
	0.050	394.7	263.1	197.3	157.9	131.6	616.7	411.1	308.3	246.7	205.6
0.8	0.222	196.5	131.0	98.3	78.6	65.5	307.1	204.7	153.5	122.8	102.4
	0.200	209.9	139.9	105.0	84.0	70.0	328.0	218.7	164.0	131.2	109.3
	0.150	241.6	161.1	120.8	96.6	80.5	377.5	251.7	188.8	151.0	125.8
	0.100	275.5	183.7	137.8	110.2	91.8	430.5	287.0	215.2	172.2	143.5
	0.050	311.6	207.7	155.8	124.6	103.9	486.8	324.6	243.4	194.7	162.3
0.6	0.188	146.6	97.7	73.3	58.6	48.9	229.0	152.7	114.5	91.6	76.3
	0.150	167.3	111.5	83.6	66.9	55.8	261.4	174.2	130.7	104.5	87.1
	0.100	196.8	131.2	98.4	78.7	65.6	307.5	205.0	153.7	123.0	102.5
	0.050	228.5	152.3	114.2	91.4	76.2	357.0	238.0	178.5	142.8	119.0
0.4	0.143	96.4	64.3	48.2	38.6	32.1	150.6	100.4	75.3	60.2	50.2
	0.100	118.1	78.7	59.0	47.2	39.4	184.5	123.0	92.2	73.8	61.5

$\frac{b}{h}$	$\frac{t}{h}$	$\frac{T_{Rd2}}{h^2}$ in kN/m per configuration of stirrups with one leg									
Stirrups ø 8-*s*					Stirrups ø 10-*s*						
100	150	200	250	300	100	150	200	250	300		
A_{sw}/s (mm^2/m)					A_{sw}/s (mm^2/m)						
503	335	251	201	168	785	524	393	314	262		
	0.050	145.4	96.9	72.7	58.2	48.5	227.2	151.5	113.6	90.9	75.7
0.2	0.083	46.8	31.2	23.4	18.7	15.6	73.1	48.7	36.5	29.2	24.4
	0.050	62.3	41.5	31.2	24.9	20.8	97.4	64.9	48.7	38.9	32.5

$$\frac{T_{Rd2}}{h^2} = 2\left(\frac{b}{h}-\frac{t}{h}\right)\left(1-\frac{t}{h}\right)\frac{f_{ywk}}{\gamma_s}\frac{A_{sw}}{s}k_{\theta,2}$$ (with $f_{ywk} = 0.5$ kN/mm^2; $\gamma_s = 1.15$; $\theta = 45°$ and so $k_{\theta,2} = 1.0$)` in kN/m

- If γ_s 1.15, multiply by 1.15/γ_s
- If f_{ywk} 0.5 kN/mm^2, multiply by f_{ywk}/0.5
- If θ 45°, multiply by $k_{\theta,2}$=cotθ

For *t/h* and θ, the same values as used in Table 9.6 should be used.

9.2.3c

$$\frac{T_{Rd2}}{h^2} \qquad [4.43]$$

Table 9.9

$\frac{b}{h}$	$\frac{t}{h}$	$\frac{T_{Rd2}}{h^2}$ in kN/m per configuration of stirrups with one leg									
Stirrups ø 12-*s*					Stirrups ø 16-s						
100	150	200	250	300	100	150	200	250	300		
A_{sw}/s in mm^2/m					A_{sw}/s in mm^2/m						
1131	754	565	452	377	2011	1340	1005	804	670		
1.0	0.250	553.5	369.0	276.7	221.4	184.5	983.9	656.0	492.0	393.6	328.0
	0.200	629.7	419.8	314.9	251.9	209.9	1119.5	746.3	559.8	447.8	373.2
	0.150	710.9	473.9	355.5	284.4	237.0	1263.8	842.6	631.9	505.5	421.3
	0.100	797.0	531.3	398.5	318.8	265.7	1416.9	944.6	708.4	566.8	472.3
	0.050	888.0	592.0	444.0	355.2	296.0	1578.7	1052.5	789.3	631.5	526.2
0.8	0.222	442.2	294.8	221.1	176.9	147.4	786.1	524.1	393.0	314.4	262.0
	0.200	472.3	314.9	236.1	188.9	157.4	839.6	559.8	419.8	335.9	279.9
	0.150	543.6	362.4	271.8	217.5	181.2	966.5	644.3	483.2	386.6	322.2
	0.100	619.9	413.3	309.9	248.0	206.6	1102.0	734.7	551.0	440.8	367.3
	0.050	701.1	467.4	350.5	280.4	233.7	1246.3	830.9	623.2	498.5	415.4
0.6	0.188	329.8	219.9	164.9	131.9	109.9	586.3	390.8	293.1	234.5	195.4
	0.150	376.4	250.9	188.2	150.5	125.5	669.1	446.1	334.5	267.6	223.0
	0.100	442.8	295.2	221.4	177.1	147.6	787.2	524.8	393.6	314.9	262.4
	0.050	514.1	342.7	257.1	205.6	171.4	914.0	609.3	457.0	365.6	304.7
0.4	0.143	216.9	144.6	108.4	86.7	72.3	385.5	257.0	192.8	154.2	128.5
	0.100	265.7	177.1	132.8	106.3	88.6	472.3	314.9	236.1	188.9	157.4
	0.050	327.2	218.1	163.6	130.9	109.1	581.6	387.7	290.8	232.6	193.9
0.2	0.083	105.2	70.2	52.6	42.1	35.1	187.1	124.7	93.5	74.8	62.4
	0.050	140.2	93.5	70.1	56.1	46.7	249.3	166.2	124.6	99.7	83.1

$$\frac{T_{Rd2}}{h^2} = 2\left(\frac{b}{h}-\frac{t}{h}\right)\left(1-\frac{t}{h}\right)\frac{f_{ywk}}{\gamma_s}\frac{A_{sw}}{s}k_{\theta,2}$$ (with $f_{ywk} = 0.5$ kN/mm^2; $\gamma_s = 1.15$; $\theta = 45°$ and so $k_{\theta,2} = 1.0$) in kN/m

- If γ_s 1.15, multiply by 1.15/γ_s
- If f_{ywk} 0.5 kN/mm^2, multiply by f_{ywk}/0.5

$\frac{b}{h}$	$\frac{t}{h}$	$\frac{T_{Rd2}}{h^2}$ **in kN/m per configuration of stirrups with one leg**							
Stirrups ø 12-*s*					Stirrups ø 16-s				
100	150	200	250	300	100	150	200	250	300
A_{sw}/s in mm^2/m					A_{sw}/s in mm^2/m				
1131	754	565	452	377	2011	1340	1005	804	670

• If θ 45°, multiply by $k_{\theta,2}$=cotθ

For *t/h* and θ, the same values as in Table 9.6 should be used.

9.2.4 $\frac{T_{Rd2}}{h^3}$ **[4.43]**

Table 9.10

$\frac{b}{h}$	$\frac{t}{h}$	$\frac{T_{Rd2}}{h^3}$ (N/mm^2) $\frac{100A_{sl}}{bh}$ (%) 0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2	max
1.0	0.250	0.163	0.326	0.489	0.653	0.816	0.979	1.305	1.631	1.958	1.641
	0.200	0.174	0.348	0.522	0.696	0.870	1.044	1.392	1.740		1.493
	0.150	0.185	0.370	0.555	0.740	0.924	1.109	1.479			1.264
	0.100	0.196	0.392	0.587	0.783	0.979					0.945
	0.050	0.207	0.413	0.620							0.526
0.8	0.222	0.115	0.231	0.346	0.461	0.577	0.692	0.923	1.154	1.384	1.165
	0.200	0.119	0.239	0.358	0.477	0.597	0.716	0.955	1.193		1.120
	0.150	0.128	0.256	0.385	0.513	0.641	0.769	1.025			0.967
	0.100	0.137	0.274	0.411	0.548	0.685	0.822				0.735
	0.050	0.146	0.292	0.438							0.416
0.6	0.188	0.071	0.143	0.214	0.286	0.357	0.428	0.571	0.714	0.857	0.733
	0.150	0.077	0.154	0.230	0.307	0.384	0.461	0.614	0.768		0.669
	0.100	0.084	0.168	0.252	0.336	0.419	0.503	0.671			0.525
	0.050	0.091	0.182	0.273	0.364						0.305
0.4	0.143	0.034	0.069	0.103	0.138	0.172	0.207	0.275	0.344	0.413	0.367
	0.100	0.039	0.078	0.117	0.157	0.196	0.235	0.313	0.392		0.315
	0.050	0.045	0.089	0.134	0.178	0.223					0.194
0.2	0.083	0.009	0.018	0.027	0.036	0.045	0.054	0.072	0.090	0.108	0.104
	0.050	0.011	0.023	0.034	0.045	0.056	0.068	0.090			0.083

$$\frac{T_{Rd2}}{h^3} = \frac{A_{sl}}{bh}\frac{f_{ylk}}{\gamma_s}\frac{\frac{b}{h}\left(\frac{b}{h}-\frac{t}{h}\right)\left(1-\frac{t}{h}\right)}{\frac{b}{h}-2\frac{t}{h}+1}k_{\theta,3}$$

(with f_{ylk} = 500 N/mm^2; γ_s = 1.15; θ = 45° and so $k_{\theta,3}$ = 1.0) in N/mm^2

• If γ_s 1.15, multiply by 1.15/γ_s
• If f_{ylk} 500, N/mm^2 multiply by f_{ylk}/500
• If θ 45°, multiply by $k_{\theta,3}$=1/cotθ

For *t/h* and θ, the same values as in Table 9.6 should be used.

9.3 Combination of torsion and shear

Torsion and shear 4.3.3.2.2(3)

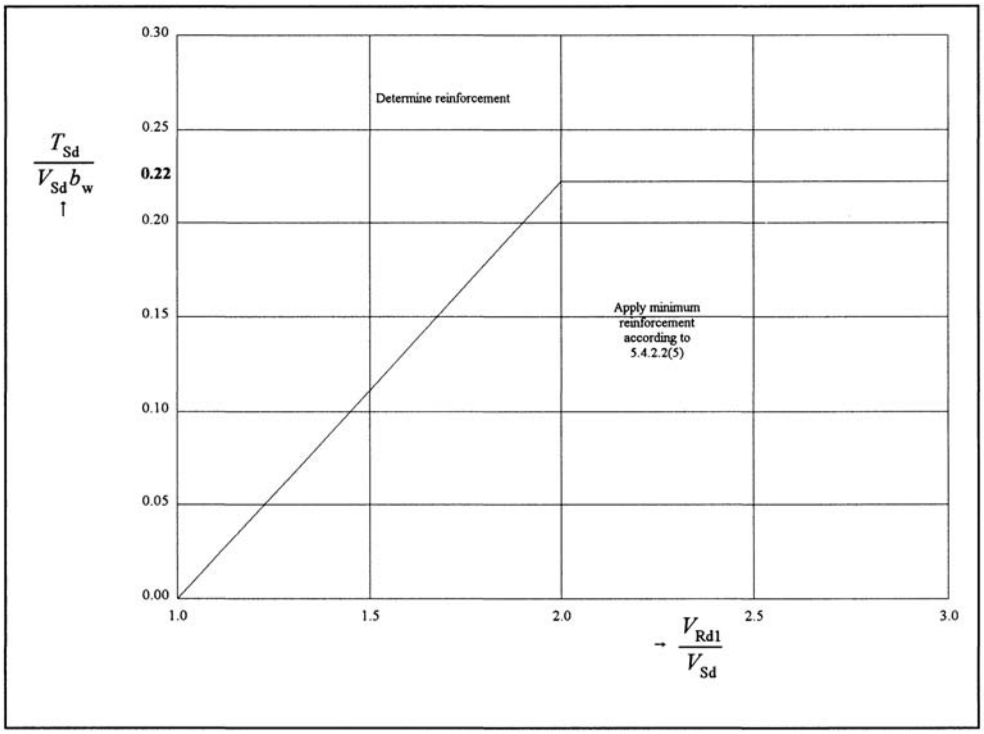


Figure 9.3

Determine T_{Rd1} according to Table 9.6, V_{Rd2} according to Table 9.7, 9.8, 9.9 or 9.10 and check whether the following condition is satisfied:

$$\left(\frac{T_{Sd}}{T_{Rd1}}\right)^2 + \left(\frac{V_{Sd}}{V_{Rd2}}\right)^2 \leq 1 \qquad [4.47]$$

Torsion and shear for solid, approximately rectangular sections 4.3.3.2.2(5)

$$\frac{T_{Sd}}{V_{Sd} b_w} \leq \frac{1}{4.5} \quad \text{for} \quad \frac{V_{Rd1}}{V_{Sd}} > 2 \qquad [4.48]$$

$$\frac{T_{Sd}}{V_{Sd} b_w} = \frac{1}{4.5}\left(\frac{V_{Rd1}}{V_{Sd}} - 1\right) \quad \text{for} \quad 1 \leq \frac{V_{Rd1}}{V_{Sd}} \leq 2 \qquad [4.49]$$

10
Punching

10.1
General

Punching 4.3.4

Slabs without punching shear reinforcement 4.3.4.5.1

Requirement:

$$v_{Sd} \leq v_{Rd1} \quad \rightarrow \quad \frac{v_{Sd}}{d} \leq \frac{v_{Rd1}}{d}$$ 4.3.4.3(2)

with:

$$\frac{v_{Sd}}{d} = \frac{V_{Sd}\,\beta}{d\,u} = \frac{\frac{V_{Sd}}{d^2}\,\beta}{\frac{u}{d}}$$

according to Table 10.2a or 10.2b

$$\frac{v_{Rd1}}{d} = \tau_{Rd}\;k\;(1.2\;+\;40\rho_1)$$

according to Table 10.3

Slabs with punching shear reinforcement 4.3.4.5.2

Requirements:

$$v_{Sd} \leq v_{Rd3} \quad \rightarrow \quad \frac{v_{Sd}}{d} \leq \frac{v_{Rd3}}{d}$$ 4.3.4.3(3)

and

$$v_{Sd} \leq [1.6]\;v_{Rd1} \quad \rightarrow \quad \frac{v_{Sd}}{d} \leq [1.6]\;\frac{v_{Rd1}}{d}$$ [4.57]

with:

$$\frac{v_{Sd}}{d} = \frac{V_{Sd}\,\beta}{d\,u} = \frac{\frac{V_{Sd}}{d^2}\,\beta}{\frac{u}{d}}$$

according to Table 10.2a or 10.2b

$$\frac{v_{Rd1}}{d} = \tau_{Rd}\,k\,(1.2\;+\;40\rho_1)$$

according to Table 8.3

$$\frac{v_{Rd3}}{d} = \frac{v_{Rd1}}{d} + \frac{\sum A_{sw} f_{ywk} \sin\,\alpha}{u\,d\,\gamma_s}$$

according to Table 10.4a or 10.4b

Apply minimum punching shear reinforcement by taking [60%] of the appropriate value of Table 5.5 (EC2). 4.3.4.5.2(4)

For the boxed values, apply the values given in the appropriate NAD.

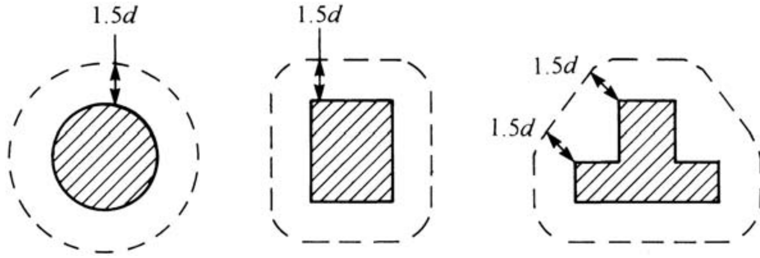


Figure 10.1 Critical perimeter round loaded areas located away from an unsupported edge.

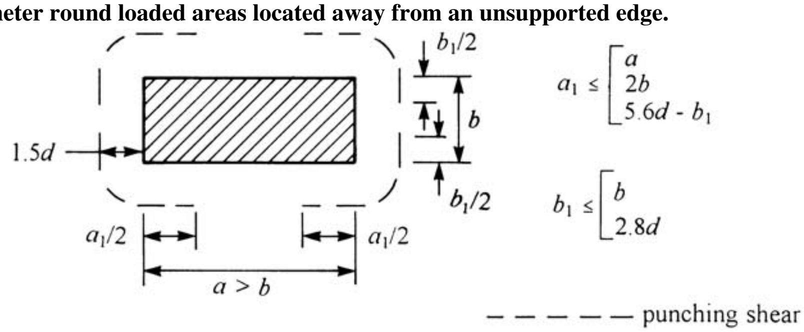


Figure 10.2 Application of punching provisions in non-standard cases.

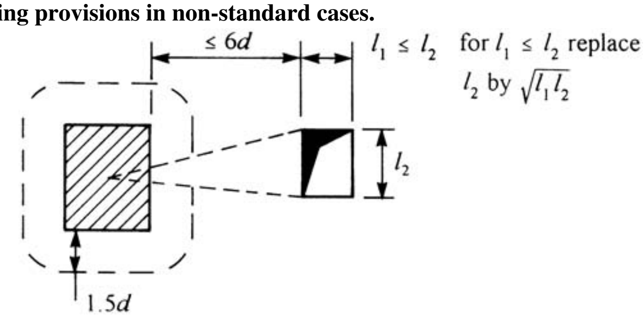


Figure 10.3 Critical perimeter near an opening.

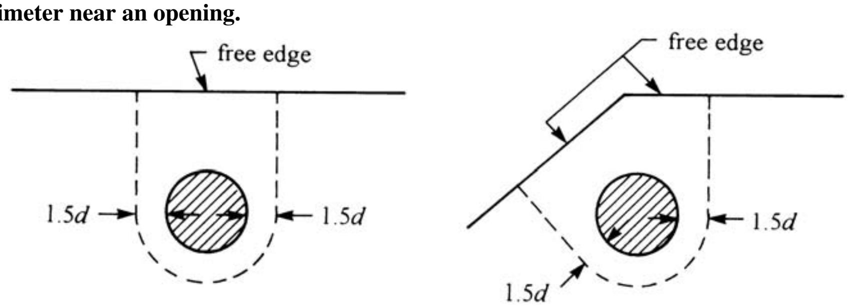


Figure 10.4 Critical sections near unsupported edges.

Loaded area, critical perimeter and critical section 4.3.4.2.1–4

The critical perimeter is defined as a perimeter surrounding the perimeter of the loaded area at a defined distance of 1.5d (Figures 10.1 and 10.2).

For a circular loaded area with diameter a, the perimeter of the loaded area is πa.

The critical perimeter for a circular loaded area located away from unsupported edges is:

$$u = \pi(a + 3d) \quad \rightarrow \quad u/d = \pi(a/d + 3)$$

Limiting value: $a/d \ngtr 3.5$

For a rectangular loaded area with dimensions a and b the perimeter of the loaded area is $2(a+b)$.

The critical perimeter for a rectangular loaded area located away from unsupported edges is:

$$u = 2(a + b) + 3\pi d \quad \rightarrow \quad u/d = 2(a/d + b/d) + 3\pi$$

Limiting values: $a/d + b/d \ngtr 5.5$ and $a/b \ngtr 2$

For openings, determine the critical perimeter according to Figure 10.3.

For loaded areas near or on an unsupported edge or corner, determine the critical perimeter according to Figure 10.4.

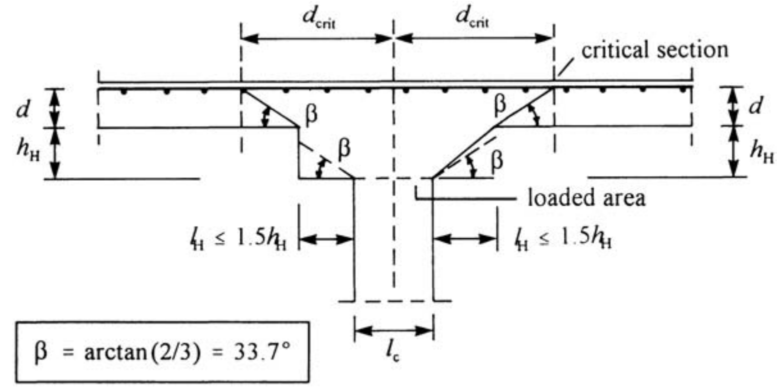


Figure 10.5 Slab with column heads where l_H 1.5h_H.

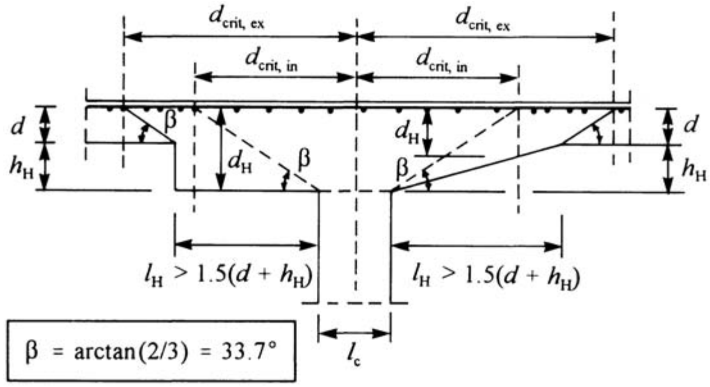


Figure 10.6 Slabs with enlarged column head where $l_H>1.5(d+h_H)$.

For slabs with column heads where $l_H<1.5h_H$, determine critical sections according to Figure 10.5.

For slabs with enlarged column head where $l_H>1.5(d+h_H)$, determine critical sections according to Figure 10.6.

For column heads where $1.5h_H<l_H<1.5(d+h_H)$, the distance from the centroid of the column to the critical section may be taken as:

$$d_{crit} = 1.5l_H + 1.5l_c$$

Coefficient β 4.3.4.3(4)

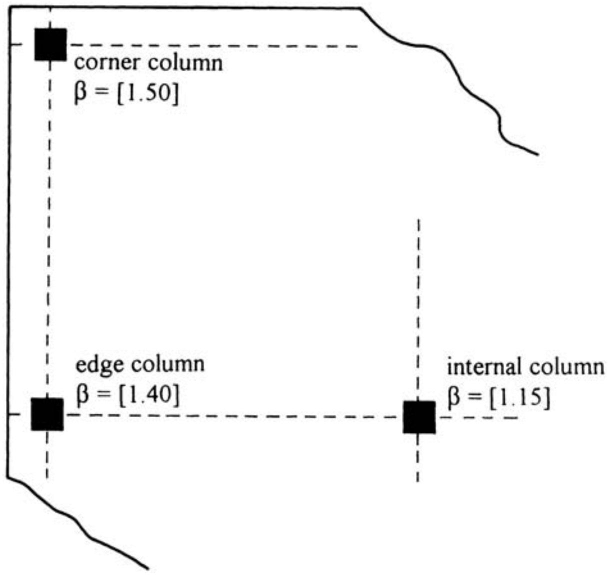


Figure 10.7 Approximate values for β.

β is a coefficient which takes account of the effects of eccentricity of loading. In cases where no eccentricity of loading is possible, β may be taken as 1.0. In other cases, the values given in Figure 10.7 may be adopted (4.3.4.3(4)).
For the boxed values, apply the values given in the appropriate NAD.

Minimum design moments 4.3.4.5.3

Design slab for minimum bending moments per unit width, m_{Sdx} and m_{Sdy} in the x- and y-direction, unless structural analysis leads to higher values according to:

$$m_{Sdx} \text{ (or } m_{Sdy}) \geq nV_{Sd} \qquad [4.25]$$

Take n into account according to Table 10.1 and Figure 10.8.

Position of column	n for m_{Sdx}			n for m_{Sdy}		
Top	Bottom	Effective width	Top	Bottom	Effective width	
Internal column	–0.125	0	0.30 l_y	–0.125	0	0.3 l_x
Edge columns, edge of slab parallel to x-axis	–0.250	0	0.15 l_y	–0.125	+0.125	(per m)
Edge columns, edge of slab parallel to y-axis	–0.125	+0.125	(per m)	–0.250	0	0.15 l_x
Corner column	–0.500	0	(per m)	+0.500	–0.500	(per m)

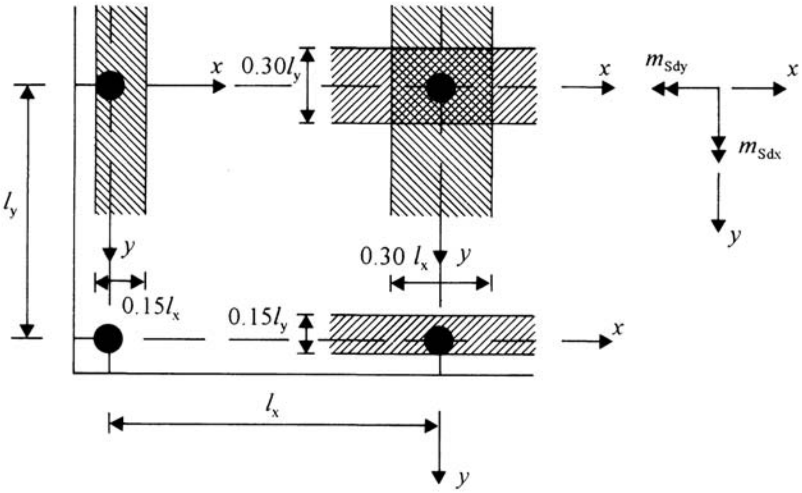
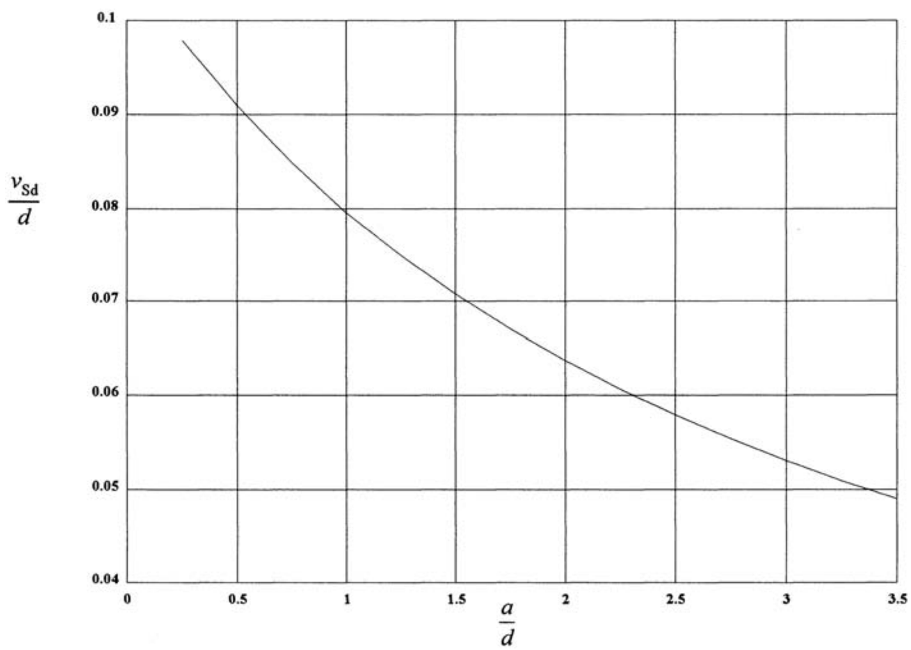


Figure 10.8 Bending moments m_{Sdx} and m_{Sdy} in slab-column joints subjected to eccentric loading, and effective width for resisting these moments.

$\frac{v_{Sd}}{d}$ 10.2a for circular loaded areas [4.50]

Table 10.2

$\frac{v_{Sd}}{d}$ in N/mm² for circular loaded areas



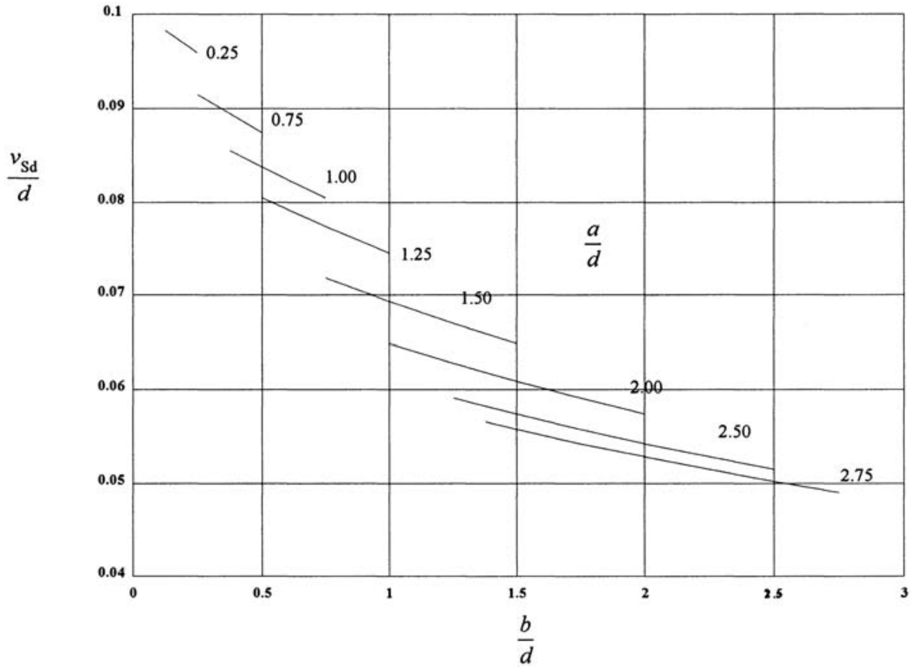
$$\frac{v_{Sd}}{d} = \frac{\frac{V_{Sd}}{d^2}\beta}{\pi\left(\frac{a}{d} + 3\right)} \text{ with } \frac{V_{Sd}}{d^2} = 1.0 \text{ N/mm}^2 \text{ and } \beta = 1.0$$

- If $\frac{V_{Sd}}{d^2} \neq 1.0$ N/mm², multiply by $\frac{V_{Sd}}{d^2}$
- If β 1.0, multiply by β according to Figure 10.7

$\frac{v_{Sd}}{d}$ 10.2b for rectangular loaded areas [4.50]

Table 10.3

$\frac{v_{Sd}}{d}$ in N/mm² for rectangular loaded areas



$$\frac{v_{Sd}}{d} = \frac{\frac{V_{Sd}}{d^2}\beta}{2\left(\frac{a}{d} + \frac{b}{d}\right) + 3\pi} \text{ with } \frac{V_{Sd}}{d^2} = 1.0 \text{ N/mm}^2 \text{ and } \beta = 1.0$$

- If $\frac{V_{Sd}}{d^2} \neq 1.0$ If N/mm², multiply by $\frac{V_{Sd}}{d^2}$
- If β 1.0, multiply by β according to Figure 10.7

$\frac{v_{Rd1}}{d}$ 10.3 [4.56]

Table 10.4

$100\rho_1$ (%)	**$\frac{v_{Rd1}}{d}$ (with γ_c=1.5 and k=1.0) in N/mm² per concrete class**								
C12/15	C 16/20	C20/25	C25/30	C30/37	C35/45	C40/50	C45/55	C50/60	
0.5	0.252	0.308	0.364	0.420	0.476	0.518	0.574	0.616	0.672
0.6	0.259	0.317	0.374	0.432	0.490	0.533	0.590	0.634	0.691
0.7	0.266	0.326	0.385	0.444	0.503	0.548	0.607	0.651	0.710
0.8	0.274	0.334	0.395	0.456	0.517	0.562	0.623	0.669	0.730
0.9	0.281	0.343	0.406	0.468	0.530	0.577	0.640	0.686	0.749
1.0	0.288	0.352	0.416	0.480	0.544	0.592	0.656	0.704	0.768
1.1	0.295	0.361	0.426	0.492	0.558	0.607	0.672	0.722	0.787
1.2	0.302	0.370	0.437	0.504	0.571	0.622	0.689	0.739	0.806
1.3	0.310	0.378	0.447	0.516	0.585	0.636	0.705	0.757	0.826
1.4	0.317	0.387	0.458	0.528	0.598	0.651	0.722	0.774	0.845
1.5	0.324	0.396	0.468	0.540	0.612	0.666	0.738	0.792	0.864

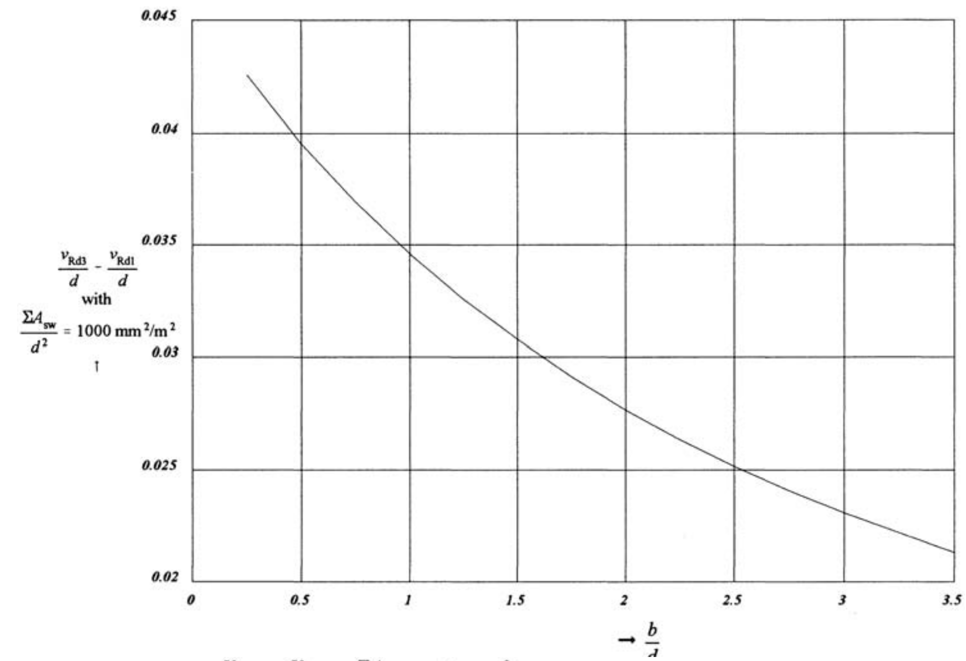
$100\rho_1$ (%)	$\frac{v_{Rd1}}{d}$ (with γ_c=1.5 and k=1.0) in N/mm² per concrete class								
C12/15	C 16/20	C20/25	C25/30	C30/37	C35/45	C40/50	C45/55	C50/60	
	$\frac{v_{Rd1}}{d} = \tau_{Rd} k(1.2 + 40\rho_1)$ (with $\gamma_c = 1.5$ and $k = 1.0$) in N/mm²								
τ_{Rd} (N/mm²)	0.18	0.22	0.26	0.30	0.34	0.37	0.41	0.44	0.48

- If γ_c 1.5, multiply by $1.5/\gamma_c$
- If d<0.6 m, multiply by k=1.6−d where d is in metres
- If $100\rho_1$<0.5%, with $\rho_1 = \sqrt{\rho_{1x} + \rho_{1y}}$ apply $100\rho_1$=0.5%
- If $100\rho_1$>1.5 %, with $\rho_1 = \sqrt{\rho_{1x} + \rho_{1y}}$, take $100\rho_1$=1.5% into account

10.4a $\frac{v_{Rd3}}{d} - \frac{v_{Rd1}}{d}$ circular loaded areas [4.58]

Table 10.5

$\frac{v_{Rd3}}{d} - \frac{v_{Rd1}}{d}$ in N/mm² for circular loaded areas



$$\frac{V_{Rd3}}{d} - \frac{V_{Rd1}}{d} = \frac{\Sigma A_{sw}}{d^2} \frac{1}{\pi\left(\frac{a}{d} + 3\right)} \frac{f_{ywk}}{\gamma_s} \sin\alpha$$

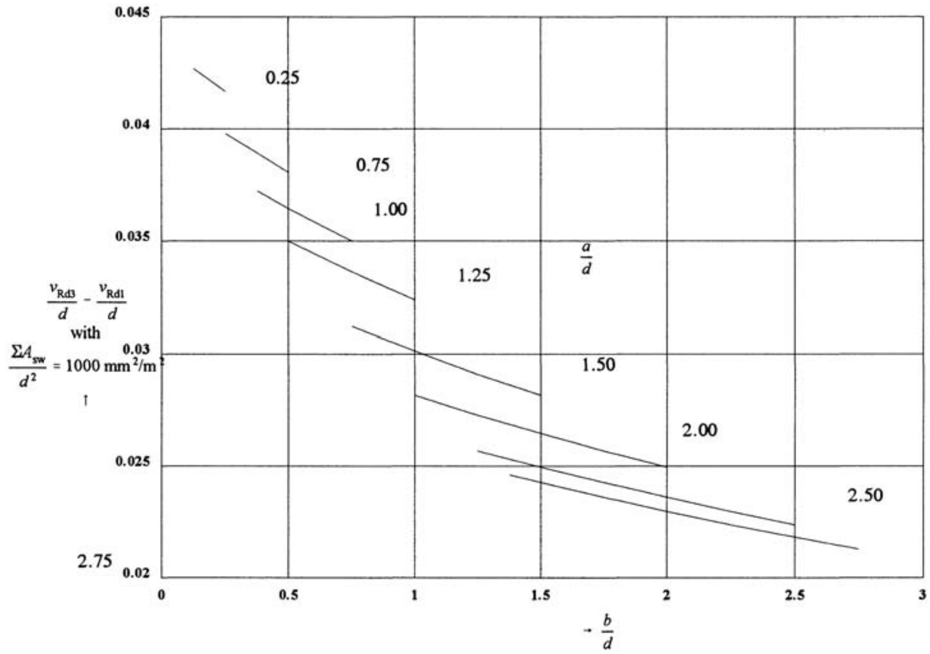
(with $\frac{\Sigma A_{sw}}{d^2}$ = 1000 mm²/m²; γ = 1.15; f_{ywk} = 500 N/mm²; $\alpha = 90°$)

- If $\frac{\Sigma A_{sw}}{d^2} \neq 1000$ mm²/m², multiply by $\frac{\Sigma A_{sw}}{1000\,d^2}$
- If γ_s 1.15, multiply by $1.15/\gamma_s$
- If f_{ywk} 500, multiply by $f_{ywk}/500$
- If α 90°, multiply by sin α

10.4b $\frac{v_{Rd3}}{d} - \frac{v_{Rd1}}{d}$ rectangular loaded areas [4.58]

Table 10.6

$\frac{v_{Rd3}}{d} - \frac{v_{Rd1}}{d}$ in N/mm² for rectangular loaded areas



$$\frac{V_{Rd3}}{d} - \frac{V_{Rd1}}{d} = \frac{\Sigma A_{sw}}{d^2} \frac{1}{2\left(\frac{a}{d} + \frac{b}{d}\right) + 3\pi} \frac{f_{ywk}}{\gamma_s} \sin\alpha$$

(with $\frac{\Sigma A_{sw}}{d^2} \neq 1000\ mm^2/m^2$; γ_s 1.15; f_{ywk} 500 N/mm² and α 90°)

- If $\frac{\Sigma A_{sw}}{d^2}$ 1000mm2/m2, multiply by $\frac{A_{sw}}{1000} d^2$
- If γ_s 1.15, multiply by $1.15/\gamma_s$
- If f_{ywk} 500, multiply by $f_{ywk}/500$
- If α 90°, multiply by $\sin\alpha$

11
Elements with second order effects

11.1
Determination of effective length of columns

The effective length of a column depends on the stiffness of the column relative to the stiffness of the structure connected to either end of the column. The effective length may be estimated from the relation:

$$l_{eff} = \beta\, l_{col}$$

where β may be obtained from Figure 11.1.

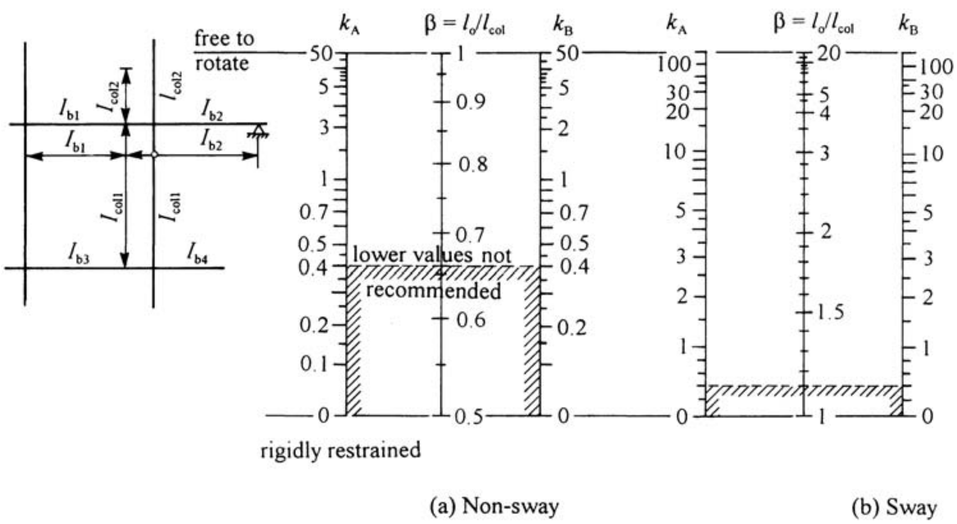


Figure 11.1: Nomogram for assessing effective lengths.

coefficients K_A and K_B denote the rigidity of restraint at the column ends:

$$\frac{\Sigma E_{cm}\, I_{col}/l_{col}}{\Sigma E_{cm}\, \alpha I_b / l_{eff}}$$

where

E_{cm}	=	modulus of elasticity of the concrete
I_{col}, I_b	=	moment of inertia (gross section) of the column or beam respectively
l_{col}	=	height of the column measured between centres of restraint
l_{eff}	=	effective span of the beam
α	=	factor taking into account the conditions of restraint of the beam at the opposite end:
	=	1.0 opposite end elastically or rigidly restrained
	=	0.5 opposite end free to rotate
	=	0 for a cantilever beam

Alternatively, for columns in braced frames, the effective height for framed structures may be taken as the lesser of:

$$l_e = l_o[0.7 + 0.05(\alpha_{c,1} + \alpha_{c,2})] < l_o$$
$$l_e = l_o[0.85 + 0.05\alpha_{c,min}] < l_o$$

The effective height for unbraced framed structures may be taken as the lesser of:

$$l_e = l_o[1.0 + 0.15(\alpha_{c,1} + \alpha_{c,2})]$$
$$l_e = l_o[2.0 + 0.3\alpha_{c,min}]$$

where

l_e	=	effective height of a column in the plane of bending considered
l_o	=	height between end restraints
$\alpha_{c,1}$	=	ratio of the sum of the column stiffnesses to the sum of the beam stiffnesses at the lower end of a column
$\alpha_{c,2}$	=	ratio of the sum of the column stiffnesses to the sum of the beam stiffnesses at the upper end of a column
$\alpha_{c,min}$	=	lesser of α_{c1} and α_{c2}

Where creep may significantly affect the performance of a member (e.g. where members are not well restrained at the ends by monolithic connections), this can be allowed for by increasing the effective length by a factor:

$$\left(1 + \frac{M_{qp}}{M_{sd}}\right)^{½}$$

where
M_{qp} is the moment under the quasi-permanent load
M_{sd} is the design first order moment.

Table 11.1 Simplified assessment of β for non-sway frames

(A) Assess K for each end of column using the following method:
- (i) K=0.5
- (ii) If there is a column continuing beyond the joint, $K=K*2$
- (iii) If there is a beam on only one side of the joint, $K=K*2$
- (iv) If the span of the beam is more than twice the height of the columns, $K=K*1.5$
- (v) If the beams or slabs framing into the column are shallower than the column dimension, $K=K*2$
- (vi) If the joint nominally carries no moment (e.g. connection with a pad footing), K=10

(B) Obtain β from the following:

***K* for lower joint**	***K* for upper joint**							
	0.5	0.75	1.0	1.5	2	3	10	PIN
0.5	0.69	0.70	0.74	0.75	0.77	0.8	0.81	0.84
0.75	0.70	0.74	0.75	0.77	0.80	0.81	0.84	0.85
1.0	0.74	0.75	0.77	0.80	0.81	0.84	0.85	0.86
1.5	0.75	0.77	0.80	0.81	0.84	0.85	0.86	0.90
2	0.77	0.80	0.81	0.84	0.85	0.86	0.90	0.92
3	0.80	0.81	0.84	0.85	0.86	0.90	0.92	0.95
10	0.81	0.84	0.85	0.86	0.90	0.92	0.95	0.98
PIN	0.84	0.85	0.86	0.90	0.92	0.95	0.98	1.00

Table 11.2 Model column method for isolated non-sway columns

If $\lambda = \lambda_{crit}$, $N_{Rd} = N_{sd}$
$M_{Rd} = N_{sd}h/20$

If $\lambda > \lambda_{crit}$, $N_{Rd} = N_{sd}$
$M_{Rd} = N_{sd}e_{tot} < N_{sd}h/20$
$< N_{sd}e_{0.2}$

e_{tot}	=	$e_o+e_a+e_2$
e_o	=	first order eccentricity
	=	$0.4e_{0.1}+0.6e_{0,2}<0.4e_{0.2}$
e_a	=	accidental eccentricity

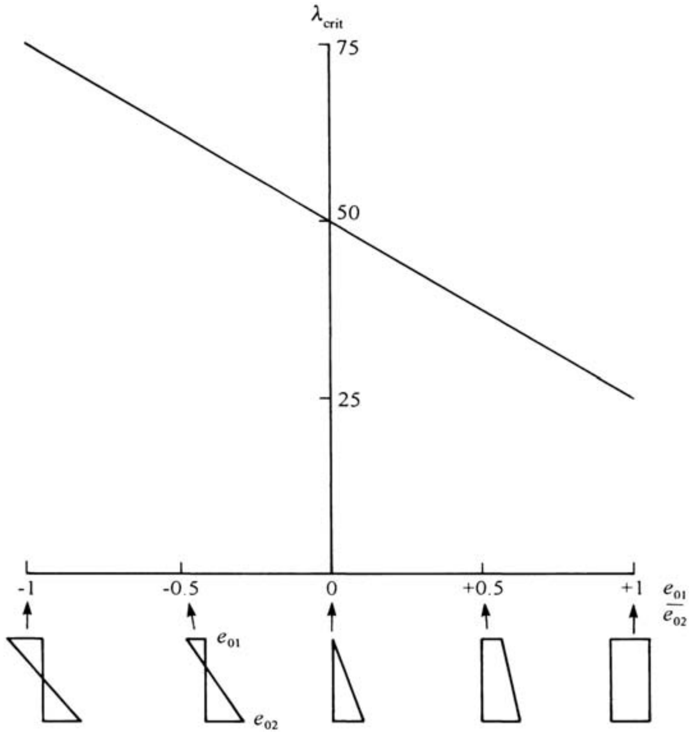


Figure 11.2 Critical slenderness ratio for isolated column.

	=	$vl_o/2$
$\nu = \dfrac{\alpha_n}{\sqrt{100l}} \geq \dfrac{l}{200}$		
		$\alpha_n = \sqrt{(1 + l/n)/2}$
l	=	total height of structure in metres
n	=	number of vertical elements acting together
e_2	=	second order eccentricity
	=	$\dfrac{k_1 k_2 l_o^2 f_{yd}}{90{,}000}$

$$k_1 = \frac{\lambda}{20} - 0.75 \text{ for } 15 < \lambda \leq 35$$

$$= 1 \text{ for } \lambda > 35$$

$$k_2 = (N_{ud} - N_{sd})/(N_{ud} - N_{bal}) \leq 1$$

Table 11.3 Detailing requirements for columns (EC2 Clause 5.4.1)

Minimum dimensions:

200 mm	vertical columns, cast in-situ
140 mm	precast columns cast horizontally

Minimum area of longitudinal reinforcement

$$A_{s.min} = \frac{0.15N_{sd}}{f_{yd}} < 0.003A_c$$

Maximum area of longitudinal reinforcement

$$A_{s.max} = 0.08A_c$$

Transverse reinforcement:

Minimum diameter of links: $\dfrac{\text{maximum longitudinal bar size}}{4} < 6$ mm

Maximum spacing: the smallest of:

- 12 times minimum diameter of longitudinal bars
- the least dimension of the column
- 300mm

The resulting maximum spacing should be multiplied by 0.6

- in sections immediately above or below a beam or slab over a height equal to the larger dimension of the column
- near lapped joints where the size of the longitudinal bars exceed 14 mm

12
Control of cracking

It should be clearly understood that there are many causes of cracking and that only certain of these lead to cracks that will be controlled by the provisions of chapter 4.4.2 of EC2. Chapter 4.4.2 is concerned with cracks that form in hardened concrete either from restrained imposed deformations, such as shrinkage or early thermal movements, or from the effects of loads.

The fundamental principle behind the provisions of the code is as follows. Crack control is only possible where spread cracking can occur (i.e. the tensile strain is accommodated in multiple cracks, or a crack accommodates only tensile strains that arise near the crack). For this to occur, there must be sufficient reinforcement in the section to ensure that the reinforcement does not yield on first cracking. The rules for minimum reinforcement areas in 4.4.2.2 are aimed at ensuring that this requirement is met. Provided this minimum is present, crack widths can normally be controlled by simple detailing rules.

Table 12.1 Minimum areas of reinforcement

$$A_s \geq K_c K f_{ct.eff} A_c/\sigma_s$$

where:

A_c	=	the area of concrete in tension immediately before the formation of the first crack
$f_{ct.eff}$	=	the tensile strength of the concrete effective at the time when the cracks first form. Except where the cracks can be guaranteed to form at an early age, it is suggested that the value chosen should not be less than 3 N/mm^2
σ_s	=	the stress in the reinforcement, which may be taken as the yield strength of the reinforcement
K	=	a coefficient that takes account of the effects of non-linear stress distribution. See Table 12.2 for values for K
K_c	=	a coefficient taking account of the form of loading causing the cracks. See Table 12.2 for values of K_c

Table 12.2 Values of *K* and *K*$_c$

(1) Values of K:

(a) Extrinsic, or external deformations imposed on a member: K=1.0

(b) Internal deformations (e.g. restrained shrinkage or temperature change):

for members with least dimension 300 K=0.8

for members with least dimension 800 K=0.5

Interpolation may be used between these values

(2) Values of K_c

(a) Pure tension K_c=1.0

(b) Pure flexure: K_c=0.4

(c) Section in compression with zero stress at least compressed fibre (under rare load combination) K_c=0

(d) Sections where the neutral axis depth calculated on the basis of a cracked section under the cracking load is less than the lesser of h/2 or 500 mm: K_c=0

(e) Box sections

Webs: K_c=0.4

Tension chords: K_c=0.8

(f) Parts of sections in tension distant from main reinforcement

$0.5<K_c<1.0$

To help interpolation between (a), (b), (c), Fig. 12.1 may be used. Checking the crack width requires (a) crack width criteria and (b) an estimate of the stress in the reinforcement under the quasi-permanent load. The criteria are given in Table 12.3.

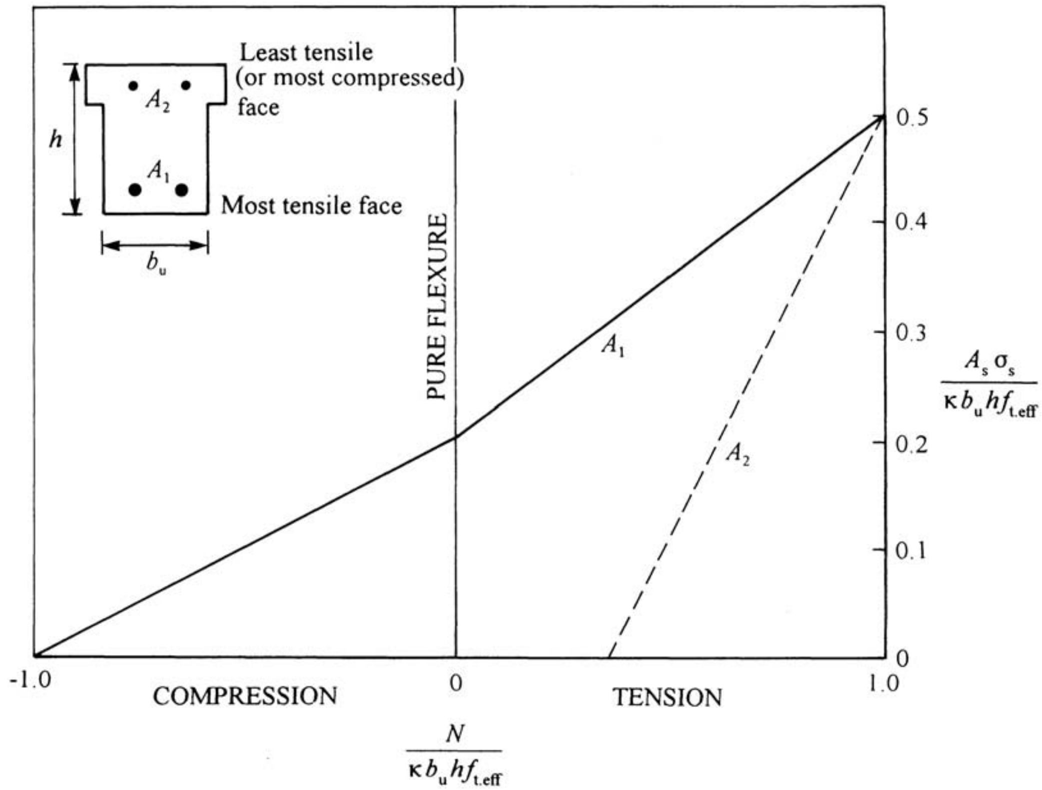
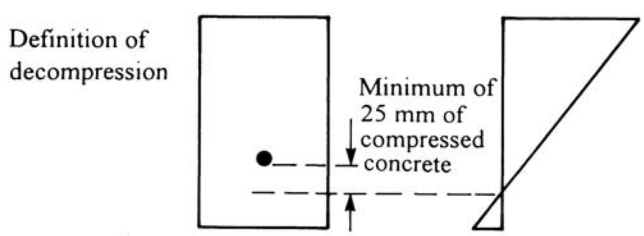


Figure 12.1 Calculation of minimum reinforcement areas.

Table 12.3: Crack width criteria

(1) Reinforced concrete: 0.3 m. If acceptable, a greater value may be used in exposure Class 1.
(2) Prestressed members.

Exposure class	Design crack width, w_k, under the frequent load combination (mm)	
	Post-tensioned	Pre-tensioned
1	0.2	0.2
2	0.2	Decompression
3	Decompression or coating of the tendons and w_k=0.2	
4		



The steel stress may be calculated on the basis of a cracked section under the quasi-permanent load. Creep may be allowed for by taking the modular ratio as 15. Table 12.4 and Figures 12.2 and 12.3 may be used to estimate the properties of a cracked section.

Alternatively, an approximate estimate of the stress may be obtained for reinforced concrete using the formula:

$$\sigma_s \frac{f_{yk}\delta}{\gamma_s}\frac{M_{qp}}{M_{sd}}\frac{A_{s.req}}{A_{s.prov}}$$

where

δ = the ratio of the design ultimate moment after redistribution to the elastically calculated value under the ultimate loads

γ_s = the partial safety factor on the reinforcement (i.e. 1.15)
M_{qp} = the moment under the quasi-permanent load

M_{sd} = the design ultimate load
$A_{s.req}$ = the reinforcement area required for the ultimate limit state
$A_{s.prov}$ = the area of tension reinforcement provided

M_{qp}/M_{sd} may be taken approximately as N_{qp}/N_{sd} where N_{qp}, and N_{ad} are, respectively, the quasi-permanent and design ultimate loads on the member.

Where the stress in the reinforcement is dominantly due to imposed deformations, the value of σ_s used in Table 12.1 should be adopted.

Crack control may be achieved either by satisfying the provisions of either Table 12.5 or Table 12.6 or by direct calculation of crack widths. This is covered in Table 12.7.

Table 12.4 Neutral axis depths and moments of inertia for flanged beams (a) with *hf/d*=0.2

αp	*br/b*=1		*br/b*=0.5		*br/b*=0.4		*br/b*=0.3		*br/b*=0.2	
	x/d	*I/bd*³	*x/d*	*I/bd*³	*x/d*	*I/bd*³	*x/d*	*I/bd*³	*x/d*	*I/bd*³
0.02	0.181	0.015								
0.03	0.217	0.022	0.217	0.022	0.217	0.022	0.217	0.022	0.217	0.022
0.04	0.246	0.028	0.248	0.028	0.248	0.028	0.249	0.028	0.249	0.028
0.05	0.270	0.033	0.274	0.033	0.275	0.033	0.276	0.033	0.278	0.033
0.06	0.292	0.038	0.298	0.038	0.300	0.038	0.302	0.038	0.304	0.038
0.07	0.311	0.043	0.320	0.043	0.322	0.043	0.325	0.043	0.327	0.043
0.08	0.328	0.048	0.340	0.047	0.343	0.047	0.346	0.047	0.349	0.047
0.09	0.344	0.052	0.358	0.052	0.361	0.052	0.365	0.051	0.369	0.051
0.10	0.358	0.057	0.375	0.056	0.379	0.056	0.383	0.055	0.388	0.055
0.11	0.372	0.061	0.390	0.060	0.395	0.059	0.400	0.059	0.406	0.059
0.12	0.384	0.064	0.405	0.063	0.410	0.063	0.416	0.063	0.422	0.062
0.13	0.396	0.068	0.418	0.067	0.424	0.066	0.430	0.066	0.437	0.065
0.14	0.407	0.072	0.431	0.070	0.437	0.070	0.444	0.069	0.452	0.069
0.15	0.418	0.075	0.443	0.073	0.450	0.073	0.457	0.072	0.466	0.071
0.16	0.428	0.078	0.455	0.076	0.462	0.076	0.470	0.075	0.478	0.074
0.17	0.437	0.082	0.466	0.079	0.473	0.078	0.481	0.078	0.491	0.077
0.18	0.446	0.085	0.476	0.082	0.484	0.081	0.493	0.080	0.502	0.079
0.19	0.455	0.088	0.486	0.085	0.494	0.084	0.503	0.083	0.513	0.082
0.20	0.463	0.091	0.495	0.087	0.504	0.086	0.513	0.085	0.524	0.084
0.21	0.471	0.094	0.504	0.090	0.513	0.089	0.523	0.088	0.534	0.086
0.22	0.479	0.096	0.513	0.092	0.522	0.091	0.532	0.090	0.543	0.089
0.23	0.486	0.099	0.521	0.094	0.531	0.093	0.541	0.092	0.552	0.091
0.24	0.493	0.102	0.529	0.097	0.539	0.095	0.549	0.094	0.561	0.093

Table 12.4 Neutral axis depths and moments of inertia for flanged beams (b) with *hf/d*=0.3

αp	*br/b*=1		*br/b*=0.5		*br/b*=0.4		*br/b*=0.3		*br/b*=0.2	
	x/d	*I/bd*³	*x/d*	*I/bd*³	*x/d*	*I/bd*³	*x/d*	*I/bd*³	*x/d*	*I/bd*³
0.02	0.181	0.015								
0.03	0.217	0.022								
0.04	0.246	0.028								
0.05	0.270	0.033								
0.06	0.292	0.038								
0.07	0.311	0.043	0.311	0.043	0.311	0.043	0.311	0.043	0.311	0.043
0.08	0.328	0.048	0.328	0.048	0.329	0.048	0.329	0.048	0.329	0.048

αp	br/b=1		br/b=0.5		br/b=0.4		br/b=0.3		br/b=0.2	
	x/d	I/bd³	x/d	I/bd³	x/d	I/bd³	x/d	I/bd³	x/d	I/bd³
0.09	0.344	0.052	0.345	0.052	0.345	0.052	0.345	0.052	0.346	0.052
0.10	0.358	0.057	0.360	0.056	0.361	0.056	0.361	0.056	0.362	0.056
0.11	0.372	0.061	0.375	0.060	0.375	0.060	0.376	0.060	0.377	0.060
0.12	0.384	0.064	0.388	0.064	0.389	0.064	0.390	0.064	0.391	0.064
0.13	0.396	0.068	0.401	0.068	0.402	0.068	0.403	0.068	0.404	0.068
0.14	0.407	0.072	0.413	0.071	0.414	0.071	0.416	0.071	0.417	0.071
0.15	0.418	0.075	0.425	0.075	0.426	0.075	0.428	0.075	0.430	0.075
0.16	0.428	0.078	0.436	0.078	0.437	0.078	0.439	0.078	0.441	0.078
0.17	0.437	0.082	0.446	0.081	0.448	0.081	0.450	0.081	0.452	0.081
0.18	0.446	0.085	0.456	0.084	0.458	0.084	0.461	0.084	0.463	0.084
0.19	0.455	0.088	0.466	0.087	0.468	0.087	0.471	0.087	0.473	0.087
0.20	0.463	0.091	0.475	0.090	0.477	0.090	0.480	0.090	0.483	0.089
0.21	0.471	0.094	0.483	0.093	0.486	0.092	0.489	0.092	0.493	0.092
0.22	0.479	0.096	0.492	0.095	0.495	0.095	0.498	0.095	0.502	0.095
0.23	0.486	0.099	0.500	0.098	0.503	0.098	0.507	0.097	0.511	0.097
0.24	0.493	0.102	0.508	0.100	0.511	0.100	0.515	0.100	0.519	0.099

Table 12.4 Neutral axis depths and moments of inertia for flanged beams (c) with *hf/d*=0.4

αp	br/b=1		br/b=0.5		br/b=0.4		br/b=0.3		br/b=0.2	
	x/d	I/bd³	x/d	I/bd³	x/d	I/bd³	x/d	I/bd³	x/d	I/bd³
0.02	0.181	0.015								
0.03	0.217	0.022								
0.04	0.246	0.028								
0.05	0.270	0.033								
0.06	0.292	0.038								
0.07	0.311	0.043								
0.08	0.328	0.048								
0.09	0.344	0.052								
0.10	0.358	0.057								
0.11	0.372	0.061								
0.12	0.384	0.064								
0.13	0.396	0.068								
0.14	0.407	0.072	0.407	0.072	0.407	0.072	0.407	0.072	0.407	0.072
0.15	0.418	0.075	0.418	0.075	0.418	0.075	0.418	0.075	0.418	0.075
0.16	0.428	0.078	0.428	0.078	0.428	0.078	0.428	0.078	0.428	0.078
0.17	0.437	0.082	0.438	0.082	0.438	0.082	0.438	0.082	0.438	0.082
0.18	0.446	0.085	0.447	0.085	0.447	0.085	0.448	0.085	0.448	0.085
0.19	0.455	0.088	0.456	0.088	0.457	0.088	0.457	0.088	0.457	0.088
0.20	0.463	0.091	0.465	0.091	0.465	0.091	0.466	0.091	0.466	0.091
0.21	0.471	0.094	0.473	0.094	0.474	0.094	0.474	0.094	0.474	0.093
0.22	0.479	0.096	0.481	0.096	0.482	0.096	0.482	0.096	0.483	0.096
0.23	0.486	0.099	0.489	0.099	0.490	0.099	0.490	0.099	0.491	0.099
0.24	0.493	0.102	0.496	0.101	0.497	0.101	0.498	0.101	0.498	0.101

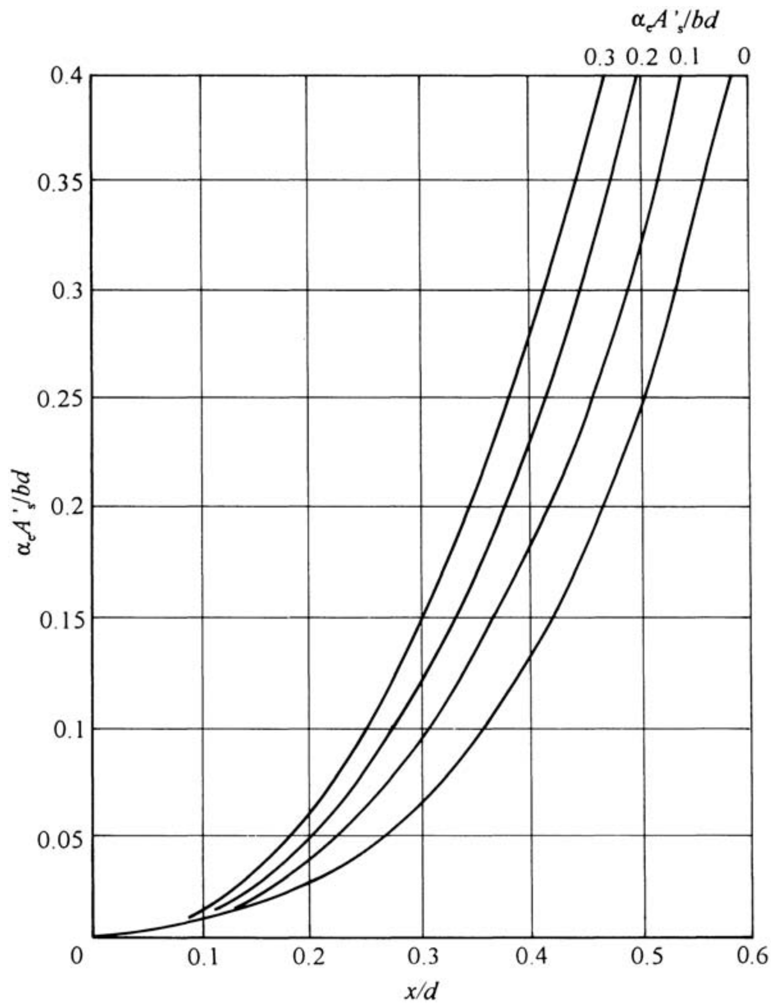


Figure 12.2 Neutral axis depths for rectangular sections.

Table 12.5 Maximum bar diameters

Steel stress (MPa)	**Maximum bar size (mm)**	
Reinforced sections	Prestressed sections	
160	32	25
200	25	16
240	20	12
280	16	8
320	12	6
360	10	5
400	8	4
450	6	

For reinforced concrete, the maximum bar diameter may be modified as follows

$\phi_s = \phi_s^* f_{ctm}/(2.5)h/[10(h-d)] \geq \phi_s^*(f_{ctm}/2.5)$ for restraint cracking

$\phi_s = \phi_s^* \dfrac{h}{10(h-d)} \geq \phi_s^*$ for load-induced cracking

where:

ϕ_s	=	the adjusted maximum bar diameter
ϕ_s^*	=	the maximum bar size in Table 12.5
h	=	the overall depth of the section

Table 12.6 Maximum bar spacings for high bond bars

Steel stress (MPa)	**Maximum bar spacing (mm)**		
Pure flexure	Pure tension	Prestressed sections (bending)	
160	300	200	200
200	250	150	150
240	200	125	100
280	150	75	50
320	100	-	-
360	50	-	-

Table 12.7 Crack w

$$w = \beta\left(50 + 0.25k_1k_2\left(\frac{ø}{\rho_r}\right)\right)\varepsilon_{sm}$$

Design crack width,

β	=	coefficient relating the maximum crack spacing to the average value.
	=	1.7 for load-induced cracking and for restraint cracking in members with a minimum dimension greater than 800 mm
	=	1.3 for sections with a minimum dimension less than 300 mm. Intermediate values may be interpolated
ø	=	bar size in mm. For a mixture of bar sizes in a section, take the average
k_1	=	a coefficient that takes account of the bond properties of the bars; k_1=0.8 for high bond bars and 1.6 for plain bars. In the case of imposed deformations, k_1 should be replaced by k_1, k, with k being in accordance with Table 12.2
k_2	=	a coefficient that takes account of the form of the strain distribution
	=	0.5 for bending and 1.0 for pure tension

For cases of eccentric tension or for local areas, intermediate values of k_2 should be used which can be calculated from the relation:

$$k_2 = \frac{\varepsilon_1 + \varepsilon_2}{2\varepsilon_1}$$

where ε_1 is the greater and ε_2 the lesser tensile strain at the boundaries of the section considered, assessed on the basis of a cracked section

ρ_r = the effective reinforcement ratio, $A_s/A_{c.eff}$, where Ac is the area of reinforcement contained within the effective tension area $A_{c.eff}$

The effective tension area is generally the area of concrete surrounding the tension reinforcement of depth equal to 2.5 times the distance from the tension face of the section to the centroid of the reinforcement (see Figure 12.4). For slabs,

ε_{sm} is the mean strain allowing for the effects of tension stiffening, shrinkage, etc. under the relevant load combinations, and may be calculated from the relation:

$$\varepsilon_{sm} = \frac{\sigma_s}{E_s}\left[1 - \beta_1\beta_2\left(\frac{\sigma_{sr}}{\sigma_s}\right)^2\right]$$

where

σ_s	=	the stress in the tension reinforcement calculated on the basis of a cracked section
σ_{sr}	=	the stress in the tension reinforcement calculated on the basis of a cracked section under the loading conditions causing first cracking
β_1	=	a coefficient that takes account of the bond properties of the bars
	=	1.0 for high bond bars
	=	0.5 for plain bars
β_2	=	a coefficient that takes account of the duration of the loading or of repeated loading
	=	1.0 for a single, short-term load
	=	0.5 for a sustained load or for many cycles of repeated loading

For members subjected only to intrinsic imposed deformations, σ_s may be taken as equal to σ_{sr}.

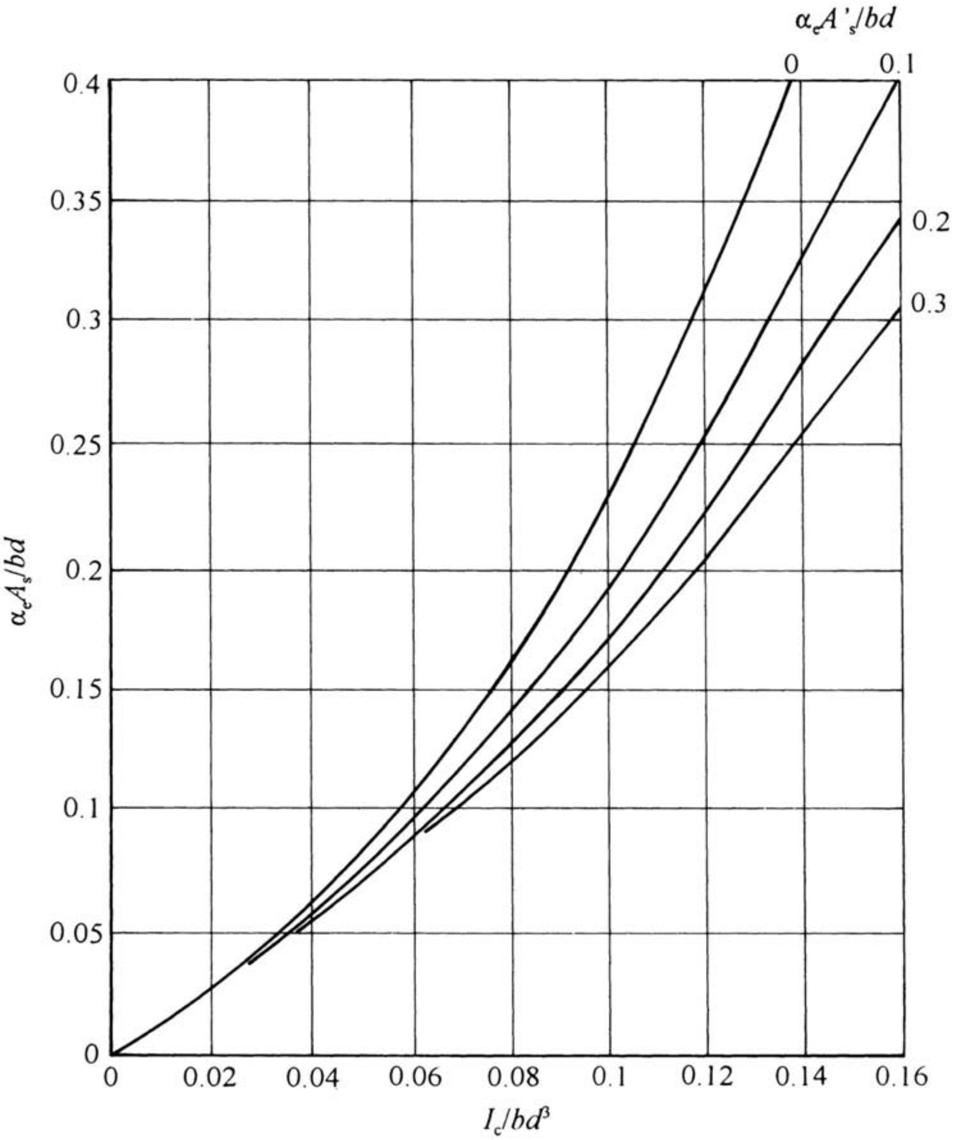


Figure 12.3 Second moments of area of rectangular sections based on a cracked transformed section

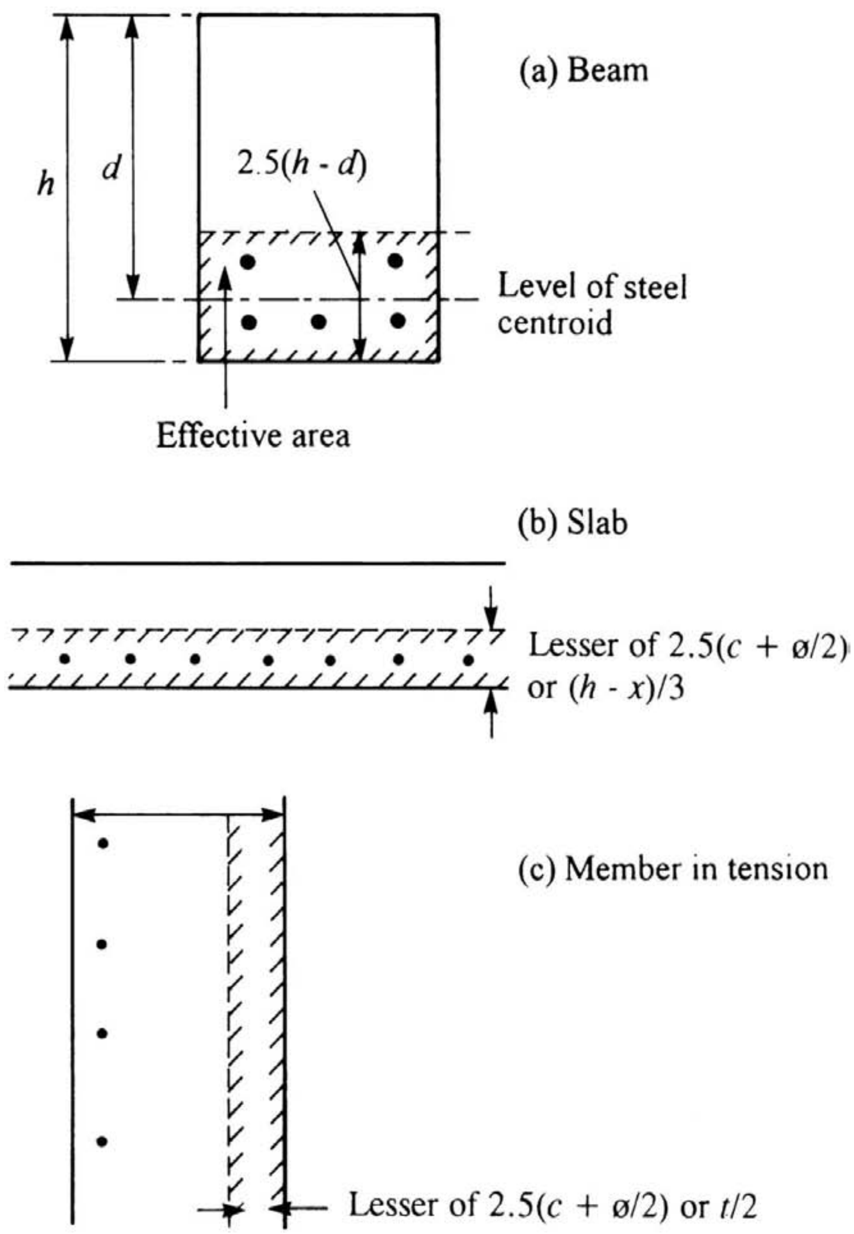


Figure 12.4 Effective area for a typical case.

13
Deflections

13.1
General

Eurocode 2 assumes that deflections will generally be checked using span/effective depth ratios, though a calculation method is also given (in Appendix 4). Owing particularly to uncertainties about the likely tensile strength of the concrete, calculation of deflection in the design stage for reinforced concrete members is likely to be very approximate. Hence direct calculation, rather than use of simple checks, is generally inappropriate.

Limits to deflection should be considered in the light of the intended function of the structure and the nature of finishes and partitions. The limits given in the code are intended only as guidance. They are: (1) Limit to overall total deflection: span/250; (2) Limit to deflection after construction of partitions and finishes where these are susceptible to damage: span/500

13.2
Ratios of span to effective depth

The span/effective depth ratios should generally ensure that these limits are met. The ratios depend upon: the nature of the structural system; the stress in the tension reinforcement; the reinforcement ratio; the geometry of the section (whether flanged or rectangular).

Figure 13.1 gives permissible ratios on the assumption that f_{yk} is 500 N/mm^2 and hence that the service stress at the critical section is approximately 250 N/mm^2. The values in Figure 13.1 should be adjusted according to those in Table 13.1. The critical section for assessing the reinforcement ratio and the steel stress is at mid-span for all members but cantilevers where the support section is used. For two-way spanning slabs supported on beams on all sides, the span/effective depth ratios should be based on the shorter span. For flat slabs, the longer span should be used.

Table 13.1 Adjustment factors for span/effective depth ratios

(1)	Different levels of stress in tension reinforcement	
	Multiply ratios by $250/f_s$	
f_s	=	stress under quasi-permanent load. This may be estimated approximately from:
f_s	=	$\frac{f_{yk}}{\gamma_s}\frac{M_{qp}}{M_{sd}}\frac{A_{s.req}}{A_{s.prov}}\frac{1}{\delta}$
where		
γ_s	=	partial safety factor for reinforcement
M_{qp}	=	moment at critical section under the quasi-permanent load
M_{sd}	=	design ultimate load
$A_{s.req}$	=	area of tension reinforcement required at critical section
$A_{s.prov}$	=	area of tension reinforcement provided
δ	=	ratio of design moment after redistribution to the elastically calculated moment
(2)	Flanged beams where b_t/b<0.3	
	Multiply ratios by 0.8	
(3)	Long spans	
(a)	Members other than flat slabs with spans >7 m	
	Multiply by 7/span	

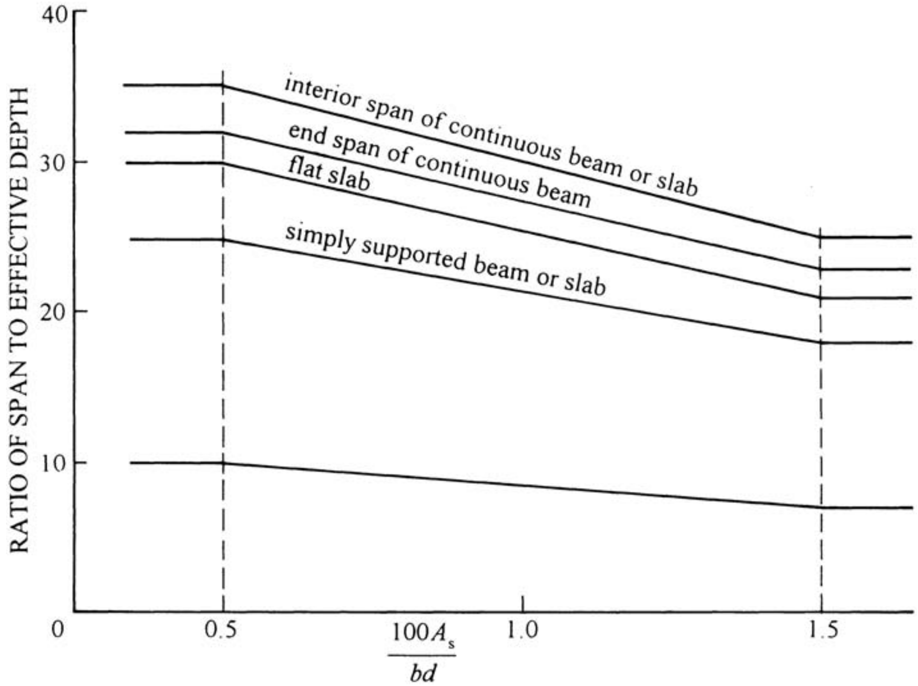


Figure 13.1 Permissible ratios of span to effective depth.

(b)	Flat slabs with spans >8.5 m
	Multiply by 8.5/span
(c)	Other deflection limits: for total deflections other than span/250
	Multiply by 250/*k* where new deflection limit is span/*k*

13.3
Calculation of deflection

There are two ways of approaching the calculation of deflections: one rigorous, the other more approximate. In the more rigorous approach, the curvature is calculated at a reasonable number of sections along the beam and then the deflection is calculated by numerical double integration.

The curvature may be calculated from:

$$\frac{1}{r} = \xi(1/r)_{II} + (1 - \xi)(1/r)_{I}$$

where

$(1/r)_I$ = the curvature calculated assuming the section is uncracked

$(1/r)_{II}$ = the curvature calculated assuming the section to be fully cracked

ξ = a distribution factor = $-\beta_1\beta_2\left(\frac{\sigma_{sr}}{\sigma_s}\right)^2$ where

β_1 = a coefficient that takes account of the bond properties of the bars
= 1 for high bond bars
= 0.5 for plain bars

β_2 = a coefficient that takes account of the duration of the loading or of repeated loading
= 1 for a single short-term loading
= 0.5 for sustained loads or many cycles of repeated loading

σ_s = the stress in the tension steel calculated on the basis of a cracked section

σ_{sr} = the stress in the tension steel calculated on the basis of a cracked section under the loading which will just cause cracking at the section being considered (Note: σ_s/σ_{sr} can be replaced by M/M_{cr} for flexure or N/N_{cr} for pure tension.)

ξ = zero for uncracked sections

The curvature may be calculated from the relation:

$\frac{1}{r} = \frac{M}{EI_I}$ or $\frac{M}{EI_{II}}$ as appropriate or, for cracked sections, from the relation:

$$\frac{1}{r} = \frac{\varepsilon_{sII}}{(d - x)}$$

where ε_{sII} is the strain in the reinforcement calculated on the basis of a cracked section. Values of I_{II} can be obtained from Table 12.4 or Figure 12.3.

The second method is to use the relation:

$$a = \xi a_I + (1 - \xi) a_{II}$$

where

a	=	the deflection
a_I	=	the deflection calculated on the basis of an uncracked section
a_{II}	=	the deflection calculated on the basis of a cracked section

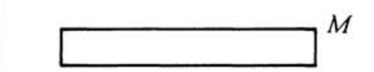
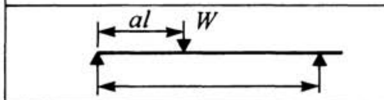
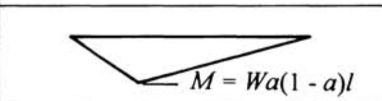
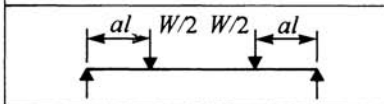
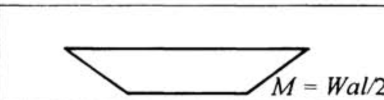
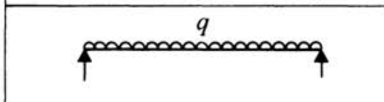
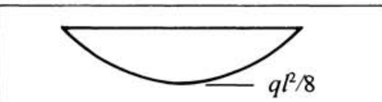
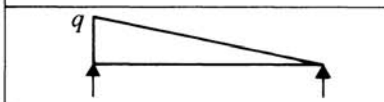
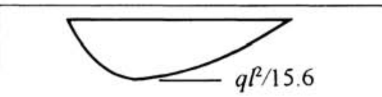
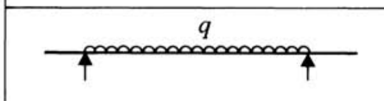
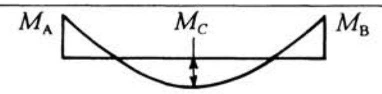
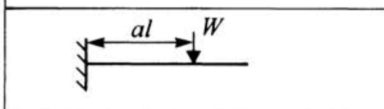
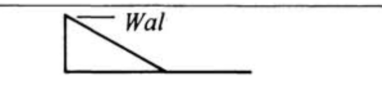
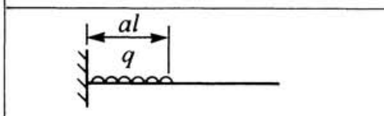
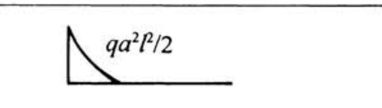
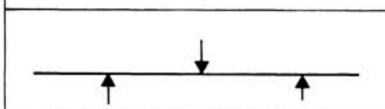
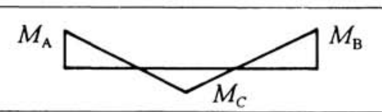
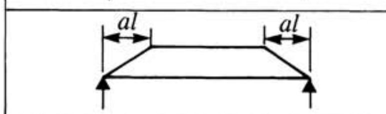
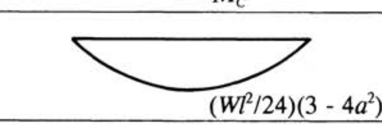
Standard elastic formulae may be used for obtaining a_I and a_{II}, using the appropriate values of II and III.

The calculation of a_I and aI$_I$ may be obtained from the relation:

$$a = \frac{kL^2M}{EI}$$

where k is a coefficient that depends on the shape of the bending moment diagram. Values for k are given in Table 13.2. taken from the UK code, BS8110, Part 2.

Table 13.2 Values of κ for various bending moment diagrams

Loading	Bending moment diagram	κ
		0.125
		$\frac{3 - 4a^2}{48(1 - a)}$ if $a = \frac{1}{2}$, $\kappa = \frac{1}{12}$
		0.0625
		$0.125 - \frac{a^2}{6}$
		0.104
		0.102
		$\kappa = 0.104(1 - \frac{\beta}{10})$ $\beta = \frac{M_A + M_B}{M_C}$
		end deflection = $\frac{a(3 - a)}{6}$ load at end κ = 0.333
		$\frac{a(4 - a)}{12}$ if $a = l$, $\kappa = 0.25$
		$\kappa = 0.083(1 - \frac{\beta}{4})$ $\beta = \frac{M_A + M_B}{M_C}$
		$\frac{1}{80}\frac{(5 - 4a^2)^2}{3 - 4a^2}$

14
Detailing

14.1
Bond conditions

The bond conditions affect the anchorage and lap lengths. Good and poor bond conditions are illustrated in Figure 14.1.

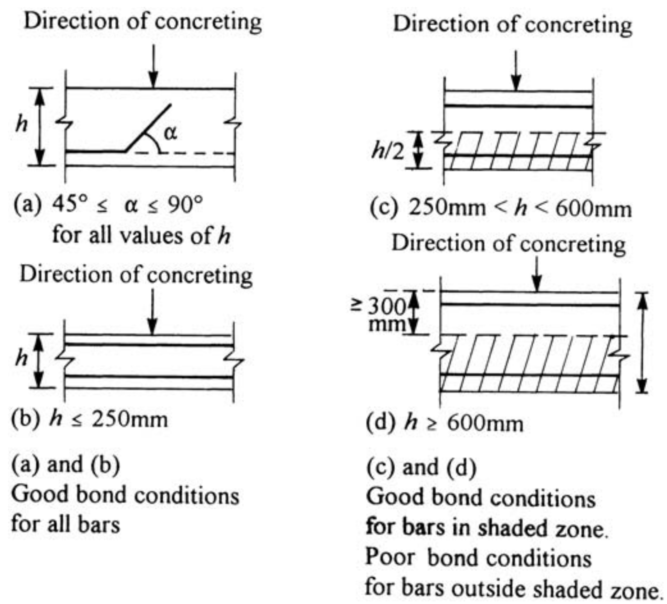


Figure 14.1 Good bond conditions.

14.2
Anchorage and lap lengths

Anchorage and lap lengths should be obtained from Table 14.1 for high-bond bars and Table 14.2 for weld mesh fabric made with high-bond bars.

14.3
Transverse reinforcement

(a) Anchorage zones

Transverse reinforcement should be provided for all anchorages in compression. In the absence of transverse compression caused by support reactions, transverse reinforcement should also be provided for anchorage in tension.

The minimum total area of transverse reinforcement required within the anchorage zone is 25% of the area of the anchored bar.

The transverse reinforcement should be evenly distributed in tension anchorages and concentrated at the ends of compression anchorages.

(b) Laps

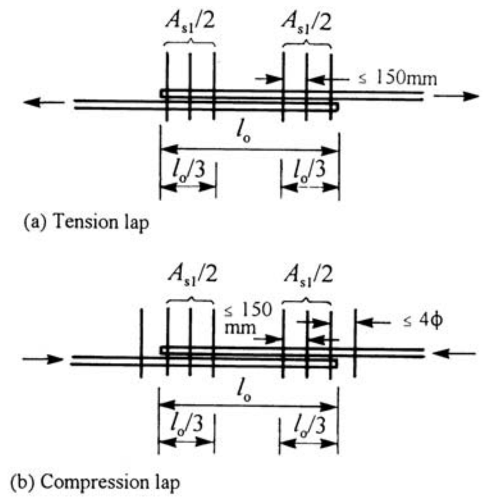


Figure 14.2 Transverse reinforcement at laps.

No special transverse reinforcement is required if the size of bars lapped is less than 16 mm or fewer than 20% of the bars in the section are lapped. When required, the transverse reinforcement should be placed as shown in Figure 14.2.

14.4 Curtailment of bars in flexural members

When a bar is curtailed in a flexural member, it should be anchored beyond the point when it is no longer required, for a length of $l_{b.net}$ or d, whichever is the greater.

In determining the location when a bar is no longer required, force in bars should be calculated taking into account (a) the bending moment and (b) the effect of truss modal for resisting shear.

A practical method for curtailment is as follows:

(a) Determine where the bar can be curtailed based on bending moment alone; and
(b) Anchor this bar beyond this location for a distance $l_{b,net}+a_1$, where a_1=0.45d for beams and 1.0d for slabs.

This procedure is diagrammatically illustrated in Figure 14.3.

At simply supported ends, the bars should be anchored beyond the line of contact between the member and its support by

0.67 $l_{b.net}$ at a direct support and
1.00 $l_{b.net}$ at an indirect support.

This requirement is illustrated in Figure 14.4.

Table 14.1 Anchorage and lap lengths as multiples of bar size: high bond bars f_{yk}=500 N/mm²

Concrete strength (N/mm²)						
	f_{ck}	20	25	30	35	40
	f_{cu}	25	30	37	45	50
Anchorage straight bars compression and tension		48	40	37	33	29
Anchorage - curved bars[4] tension		34	28	26	23	21
Laps - compression - tension [5]		48	40	37	33	29
Laps - tension [6]		67	57	52	46	41
Laps - tension [7]		96	80	74	65	59

NOTES:

General

1. For bars with f_{yk} other than 500 N/mm², the values should be multiplied by (f_{yk}/500).

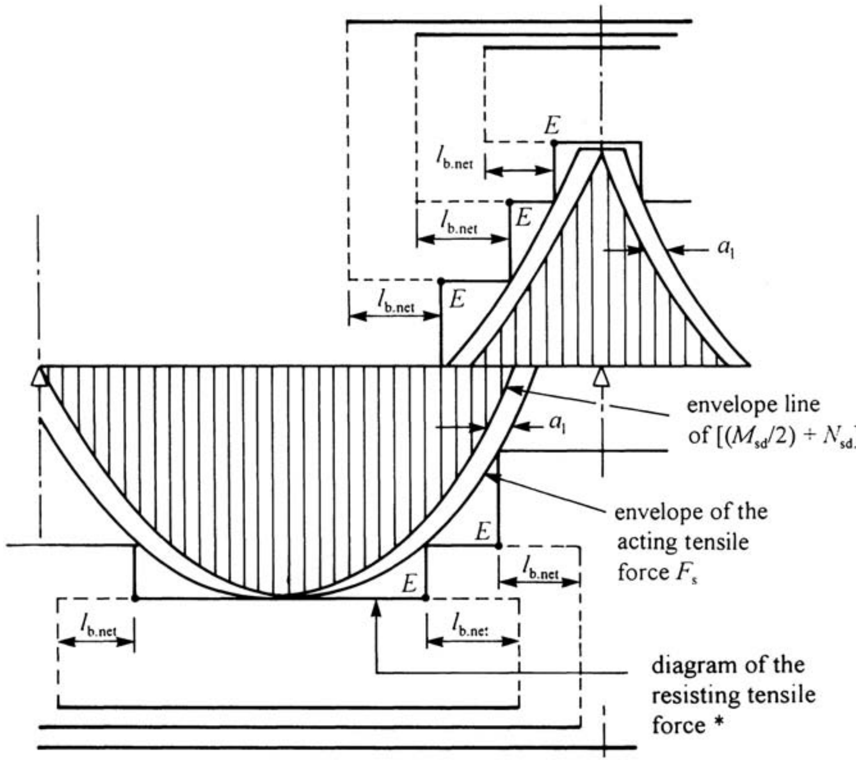


Figure 14.3 Illustration of 'shift-rule' for curtailment of bars. (* It is also permitted to use a diagram in which the resisting tensile force progressively decreases along the length $l_{b,net}$.)

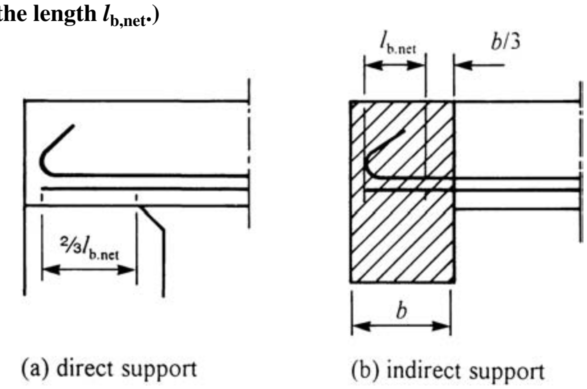


Figure 14.4 Anchorage of bottom reinforcement on end supports.

2. The values in the Table apply to (a) good bond conditions (see Fig. 14.1) and (b) bar size 32.
3. For poor bond conditions (see Figure 14.1), the Table values should be divided by 0.7.
4. For bar size>32, the values should be divided by [(132 -ø)/100], where ø is the bar diameter in mm.

Specific conditions

5. In the anchorage region, cover perpendicular to the plane of curvature should be at least 3ø.
6. Proportion of bars lapped at the section <30% and clear spacing between bars 10ø and side cover to the outer bar 5ø.
7. Proportion of bars lapped at the section >30% or clear spacing between bars <10ø or side cover to the outer bar <5ø.
8. Proportion of bars lapped at the section >30% and clear spacing between bars <10ø or side cover to the outer bar <5ø.

Table 14.2 Anchorage and lap lengths as multiples of bar size. Welded mesh fabric made with high-bond bars with f_{yk}=500 N/mm²

Concrete strength f_{ck} (N/mm²)	20	25	30	35	40

Basic anchorage and lap lengths (mm)	48	40	37	33	29

Notes:

1. For bars with f_{yk} other than 500 N/mm^2, the values should be multiplied by (f_{yk}/500).
2. Where welded transverse bars are present in the anchorage zone, the Table values for anchorage may be multiplied by 0.7.
3. The values given in the Table apply to good bond conditions and to bar sizes 32mm.
4. For poor bond conditions, the values should be divided by 0.7.
5. For bar sizes >32mm, the values should be divided by [(132–ø)/100], where ø is the diameter of the bar in mm.
6. The Table values should be multiplied by the following factors corresponding to the different *(A_s/S)* values. A_s is the area of the main reinforcement (mm^2) bar and *S* is the spacing of the bars forming the main reinforcement (m).

A_s/S	480	680	880	1080	1280
Multiplier	1.00	1.25	1.50	1.75	2.00

15
Numerical examples designed to ENV 1992–1–1

15.1
Introduction

Three types of building have been designed to Eurocode 2 (ENV 1992–1–1). Criteria for the choice of the buildings were:

- the type of structural members
- magnitude of vertical (imposed) loads
- character of the imposed loads (i.e. static or dynamic)
- the ultimate limit states to be considered (e.g. punching, fatigue).

The objectives of these calculations were to demonstrate the applicability of Eurocode 2 in practice. The main conclusion of these calculations therefore is that no basic difficulties have been observed when applying the new European Prestandard in a practical design process.

15.2
References

ENV 1991–1:	Eurocode 1: Basis of design and actions on structures. Part 1: Basis of design. Edition 1994.	
ENV 1991–2–1:	Eurocode 1: Basis of design and actions on structures. Part 2.1: Densities, self-weight and imposed loads. Final draft April 1993.	
ENV 1991–2–3: Eurocode 1:	Basis of design and actions on structures. Part 2.3: Snow loads. Final draft April 1993.	
ENV 1991–2–4: Eurocode 1:	Basis of design and actions on structures. Part 2.4: Wind loads. Final draft April 1993.	
ENV 1992–1:	Eurocode 2: Design of concrete structures. Part 1: General rules and rules for buildings. Edition 1991.	EC2
ENV 1992–1–2:	Eurocode 2: Design of concrete structures. Part 1–2: Structural fire design. Draft August 1994.	EC2–1.2
pr ENV 1992–2:	Eurocode 2: Design of concrete structures. Part 2: Concrete bridges. Draft June 1995.	EC2–2
ENV 10 080:	Steels for the reinforcement of concrete; Weldable ribbed reinforcing steel grade B500; Technical delivery conditions for bars, coils and welded fabrics. Final draft April 1994.	
ENV 206:	Concrete production, placing and compliance criteria. Edition 1990.	
DIN 15 018:	Cranes; Principles for steel structures, stress analysis. Part 1. Edition November 1984.	

Abbreviation

EC1–1
EC1–2.1
EC1–2.3
EC1–2.4

ENV 10 080
ENV 206

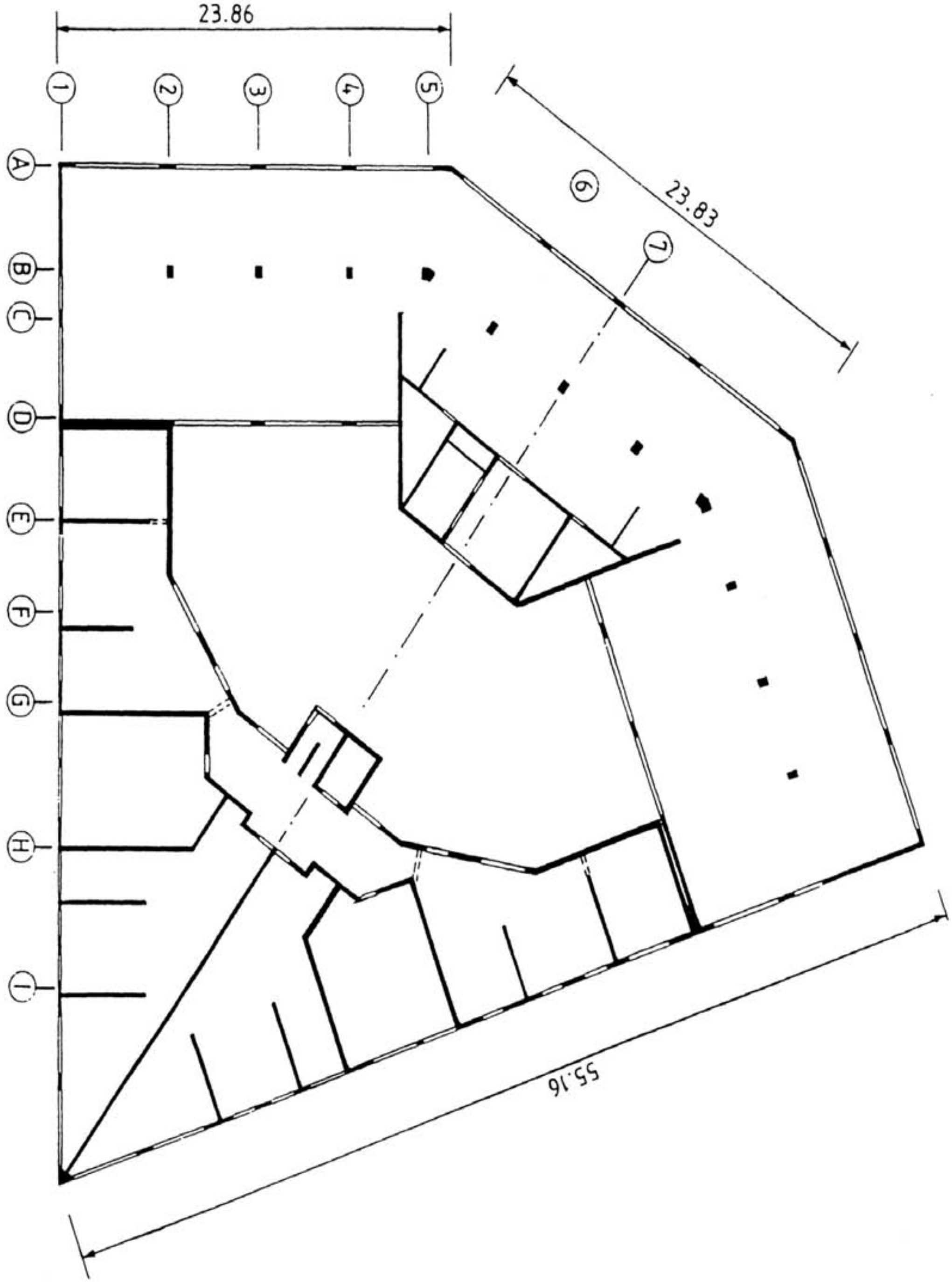
15.2 References

1. Litzner, H.-U.: *Design of Concrete Structures to ENV 1992-Eurocode 2. Concrete Structures-Euro-Design Handbook.* 1st volume 1994/1996. Berlin: Ernst & Sohn 1994.
2. Deutscher Ausschuß für Stahlbeton (DAfStb): *Bemessungshilfsmittel zu Eurocode 2 Teil 1 (DIN V ENV 1992 Teil 1–1, Ausgabe 06. 92). 2. ergänzte Auflage. Heft 425 der DAfStb-Schriftenreihe.* Berlin, Köln: Beuth Verlag GmbH 1992.
3. British Cement Association: *Worked examples for the design of concrete buildings.* Crowthorne: British Cement Association 1994.
4. Deutscher Beton-Verein E.V.: *Beispiele zur Bemessung von Betontragwerken nach EC2.* Wiesbaden, Berlin: Bauverlag GmbH 1994.
5. Dieterle, H.: *Zur Bemessung quadratischer Stützenfundamente aus Stahlbeton unter zentrischer Belastung mit Hilfe von Bemessungsdiagrammen.* Heft 387 der DAfStb-Schriftenreihe 1987.
6. British Cement Association: *Concise Eurocode for the design of concrete buildings.* Crowthorne, 1993.
7. Betonvereniging: *GTB Deel 2. Grafieken en Tabellen voor Beton.* Gouda 1992.

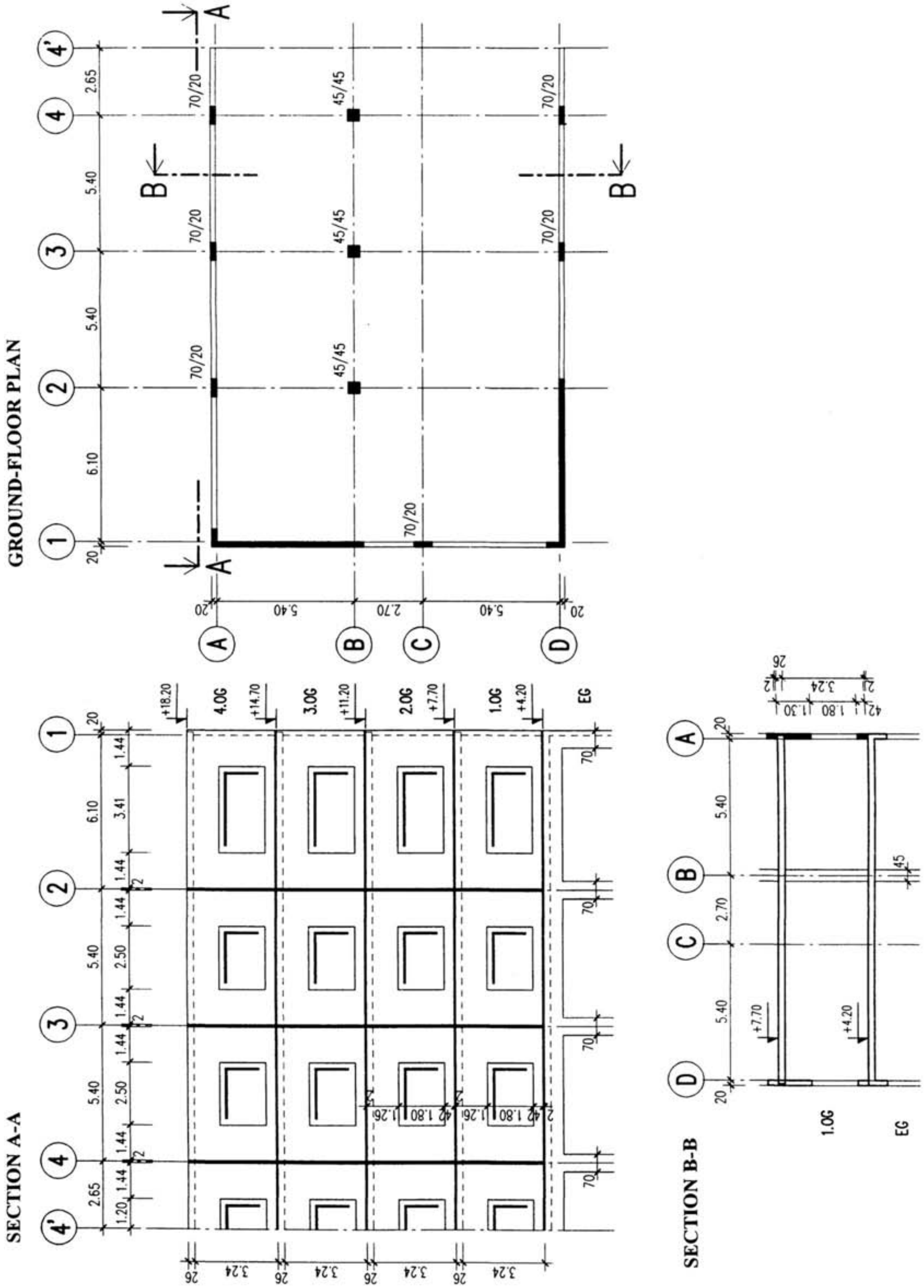
15.3
Calculation for an office building

15.3.1
Floor plan, structural details and basic data

15.3.1.1
Floor plan of an office building



15.3.1.2
Structural details of an office building



15.3.1.3
Basic data of structure, materials and loading

Intended use:	Office block			
Fire resistance:	1 hour for all elements			
Loading (excluding self-weight of structure):				
Flat slab:	- imposed:	Q_k	=	3kN/m^2
	- finished:	$G_{k,2}$	=	1.25kN/m^2
Category B				
	- partitions:	$G_{k,3}$	=	1.25kN/m^2
Combination factors:				
Frequent actions:		ψ_1	=	0.5
Quasi-permanent actions:		ψ_2	=	0.3
Exposure classes:				
Flat slab:				
Internal columns:	Class 1 (indoors)			
Façade elements:	Class 2b (humid environment with frost)			
Block foundation:	Class 5a (slightly aggressive chemical environment)			
Subsoil conditions:				
Sand, gravel	Allowable pressure 300 kN/m^2			
Materials:				
Concrete grade	C 30/37			
Steel grade	B500			
Self-weight of concrete	25 kN/m^3			

Reference

see floor plan

EC2–1.2, 1.3

EC1–2.1

EC1–2.1

EC1–1, Table 9.3, Category B

EC2, Table 4.1

from soil investigation

EC2, Table 3.1; ENV 206, Table 3 and Table 20; ENV 10 080; EC1–2.1

15.3.2
Calculation of a flat slab

15.3.2.1
Actions

Self-weight of slab: 0.26*25		=	6.50 kN/m^2
Finishes			1.25kN/m^2
Partitions			1.25 kN/m2
Permanent actions:	G_k	=	9.00 kN/m^2
Imposed load:	Q_k	=	3.00 kN/m^2
Design values of actions at the ultimate limit states:			
$\gamma_G G_k+\gamma_Q Q_k$=1.35*9.0+1.5*3.0		=	16.65 kN/m^2
Design values of actions at the serviceability limit states:			
Rare combination of actions:		=	12.00 kN/m^2
Frequent combination:			
$G_k+\psi_1 Q_k$=9.00+0.5*3.00		=	10.50 kN/m^2
Quasi-permanent combination:			
$G_k+\psi_2 Q_k$=9.00+0.3*3.00		=	9.90 kN/m^2

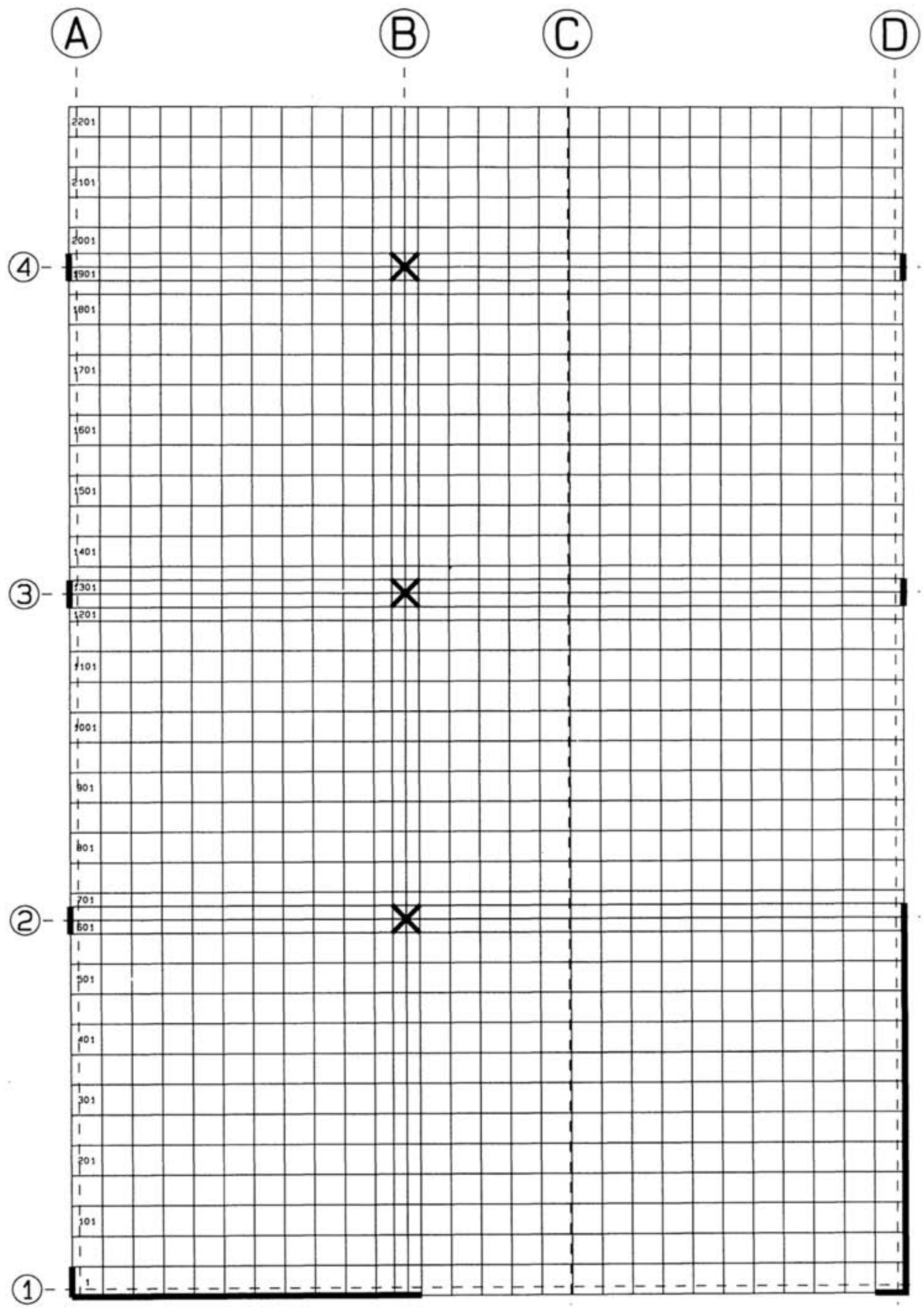
Reference

h=0.26 m

EC2, Equation (2.7a), fundamental combination

EC2, 2.3.4

15.3.2.2
Structural model at the ultimate limit states (finite element grid)



15.3.2.3
Design values of bending moments (example)

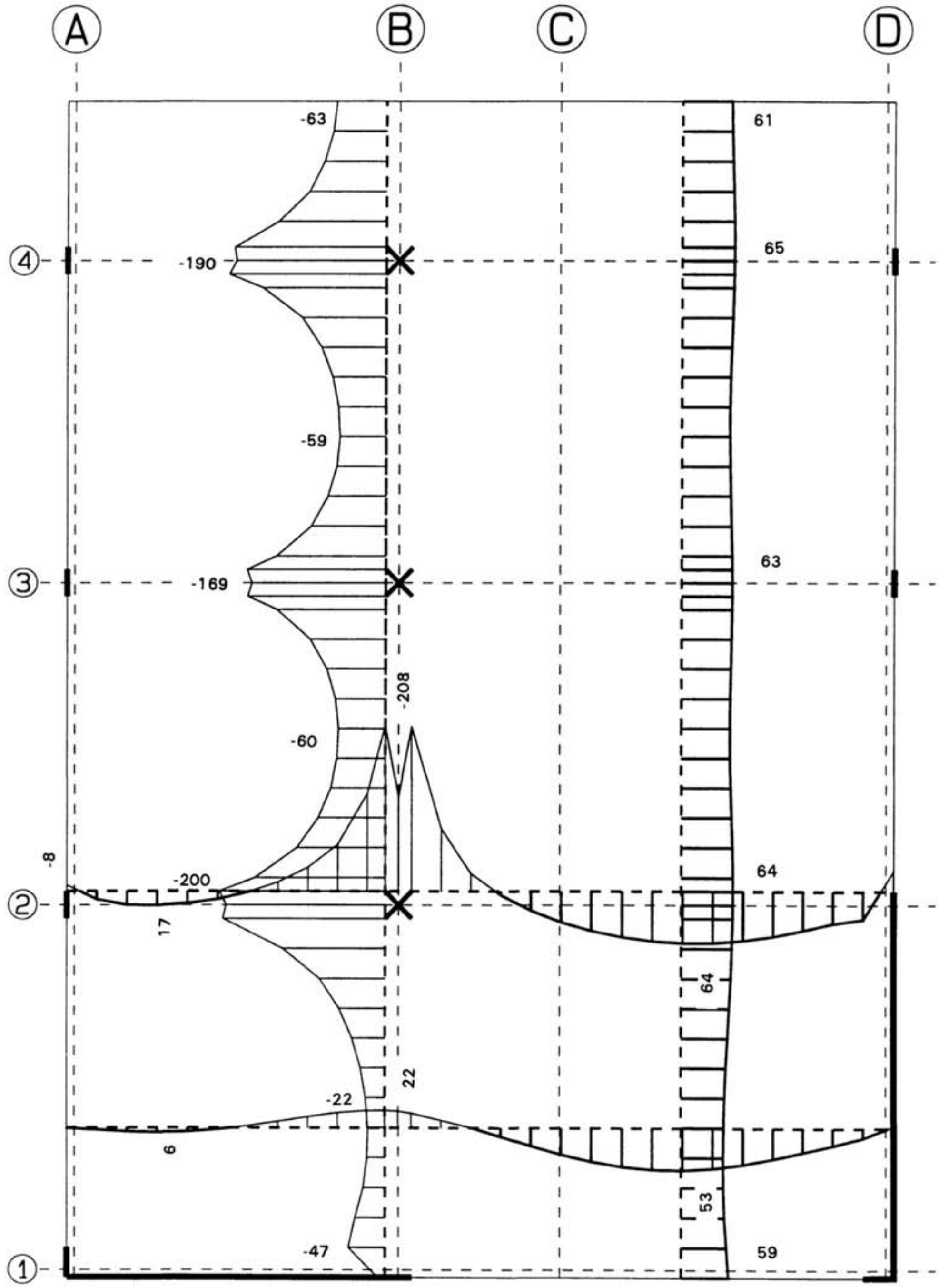


Table 15.1: Design bending moments at the ultimate limit states

Direction	Location	Section	Kind of moment	Moment (kNm/m)	Mean value of moment (kNm/m)
		left of axis B		–184.59	
x	B/2	centre of axis B	support: min m_{Sd}	–175.07	–180.72
		right of axis B		–182.50	
		left of axis B		63.26	
x	B-D/2	centre of axis B	span: max m_{Sd}	64.02	63.85
		right of axis B		64.26	

Direction	Location	Section	Kind of moment	Moment (kNm/m)	Mean value of moment (kNm/m)
		top of axis 2		–208.26	
y	B/2	centre of axis 2	support: m_{Sd}	–200.02	–204.16
		bottom of axis 2		–204.21	
		top of axis 2		93.14	
y	B/1–2	centre of axis 2	span: m_{Sd}	93.00	92.95
		bottom of axis 2		92.71	

15.3.2.4
Design of bending at the ultimate limit states

Table 15.2: Design for bending

Direction	x		y	
Axis	B/1–3		2/A-D	
Location	Support	Span	Support	Span
m_{Sd} (kN/m)	180.72	63.85	204.16	92.95
$d(m)$	0.219	0.224	0.233	0.235
μ_{Sds}	0.188	0.063	0.188	0.084
ω	0.2163	0.0657	0.2163	0.0888
ξ	0.3142	0.1069	0.3142	0.1332
$A_{s,\ req}$ (mm^2/m)	$21.78*10^2$	$6.80*10^2$	$23.17*10^2$	$9.59*10^2$
Selected B 500B(S)	2*ø 14–14.0		2*ø 14–14.0	
Selected B 500A(M)		2*ø 7.0–100		2*ø 8.0–100
$A_{s,prov}$ (mm^2/m)	$22.00*10^2$	$7.70*10^2$	$23.68*10^2$	$10.05*10^2$

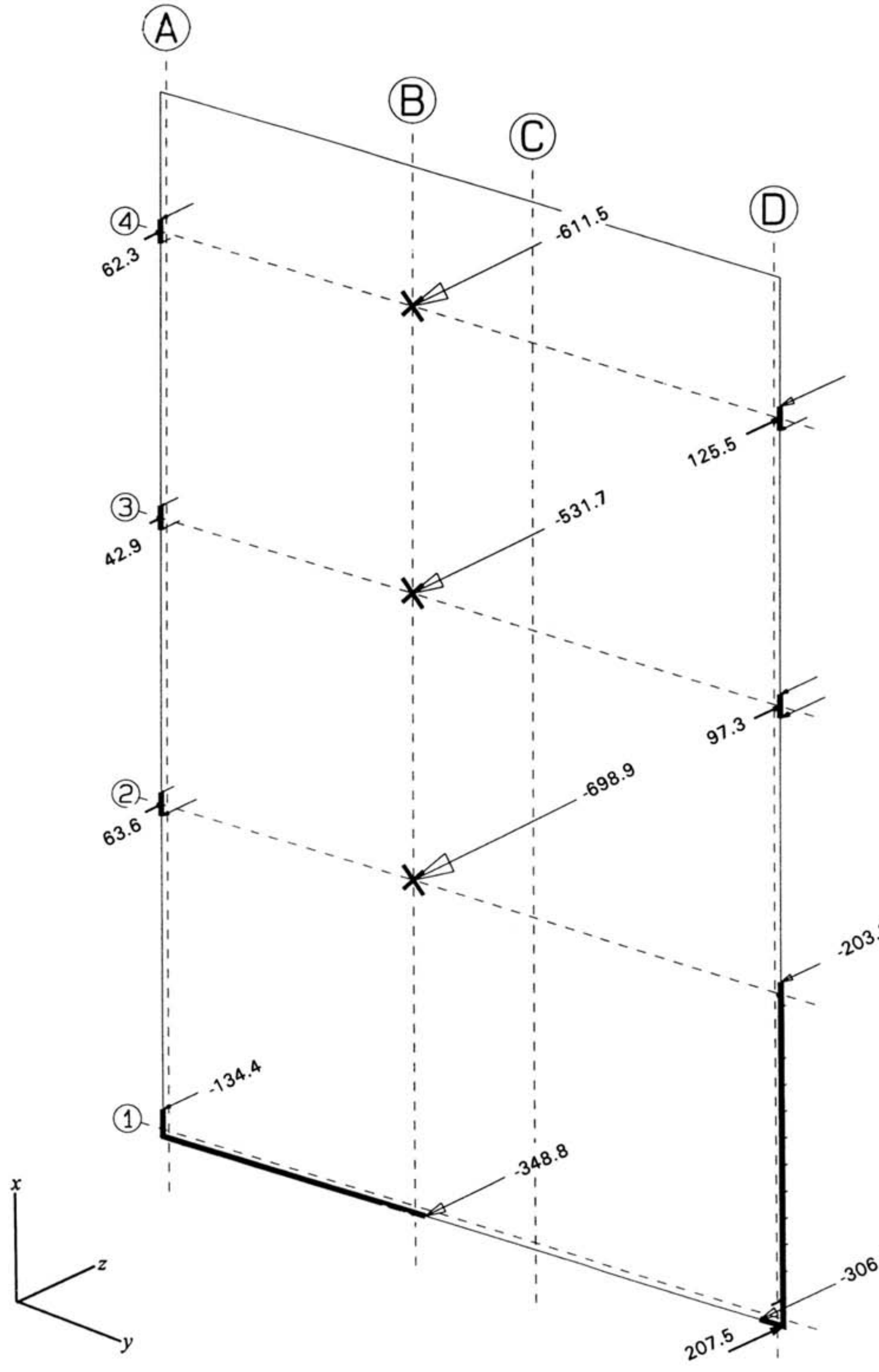
f_{cd} = 30/1.5 = 20N/mm^2
f_{yd} = 500/1.15 = 435N/mm^2

Calculation for supports

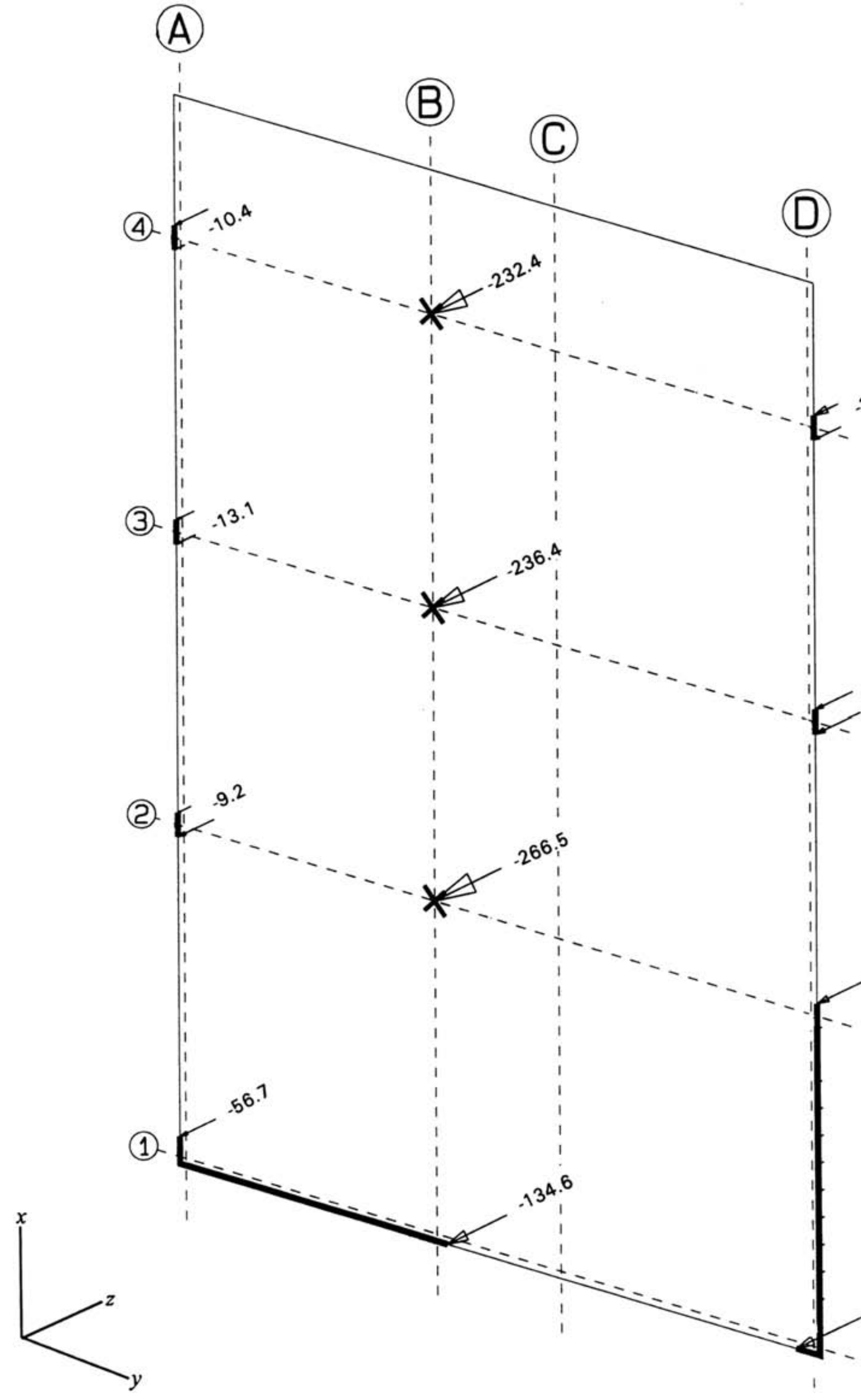
d_y = h–(min c+Δ/h)–ø/2
= 0.260–(0.015+0.005)–0.014/2 = 0.233 m
d_x = d_y–ø=0.233–0.014 = 0.219 m
(S): Reinforcing bars
(M): Welded mesh fabric

15.3.2.5
Ultimate limit state for punching shear

Shear forces due to permanent actions



Shear forces due to variable actions



Design for punching shear in axis B/2

V_{sd}	=	698.9+266.5	=	966 kN
d_m	=	(0.233+0.219)/2	=	0.226 m see Table 2
Critical perimeter				
u	=	4*0.45+2π*1.5*0.226	=	3.93m
Acting shear force				
v_{Sd}	=	966*1.15/3.93	=	283 kN/m

Shear resistance of slabs without shear reinforcement

ρ_1		0.01		
V_{Rd1}	=	$0.34*(1.6–0.226)*(1.2+40*0.01)*0.226*10^3$		
				= 169 kN/m
				< v_{Sd}
$1.6*v_{Rd1}$	=	1.6*169	= 270 kN/m	<v_{Sd}

ρ_1 must be increased

Required ρ_1=[283/(1.6*0.34*1.374*0.226* 10^3)–1.2]/40 = 1.19%

Calculation of shear reinforcement

$v_{Sd}–v_{Rd1}$	=	283–283/1.6	=	107 kN/m
$A^s{}_w$	=	$107*10^{3*}3.93/(435*\sin 60°)$	=	$11.2*10^2$ mm^2

Selected four bent-up bars ø 14 mm provided 4.2.1.5.3=12.2*10^2 mm^2

Minimum shear reinforcement

$0.6*\min\rho_w$	=	0.6*0.11	=	0.066%
The critical area minus the loaded area	=			
	=	$4*0.45*1.5*0.226+\pi*(1.5*0.226)^2$	=	0.97 m^2
$A_{sw,min}$	=	$0.066*10^{-2}*0.97*10^6/\sin 60°$	=	$7.9*10^2$ mm^2
				<$12.2*10^2$ mm^2

Reference

EC2, 4.3.4
see distribution of shear forces
EC2, 4.3.4.2.2
EC2, Eq.(4.50) for internal columns
EC2, 4.3.4.5.1 see Table 2; >0.5%
EC2, Eq.(4.56)
<1.5% Table 2: ρ_{1x}=1.19*21.9*10* =26.0*10^2mm^2/m ρ_{1y}=1.19*23.3*10^2 =27.7*10^2 mm^2/m
EC2, Eq. (4.58)
α=60°
EC2, 4.3.4.5.2(4)
EC2, Table 5.5

Minimum design moment

$m_{Sd,min}$=–966*0.125	=	©121 kNm/m
	<	m_{Sd}

15.3.2.6
Limitation of deflections

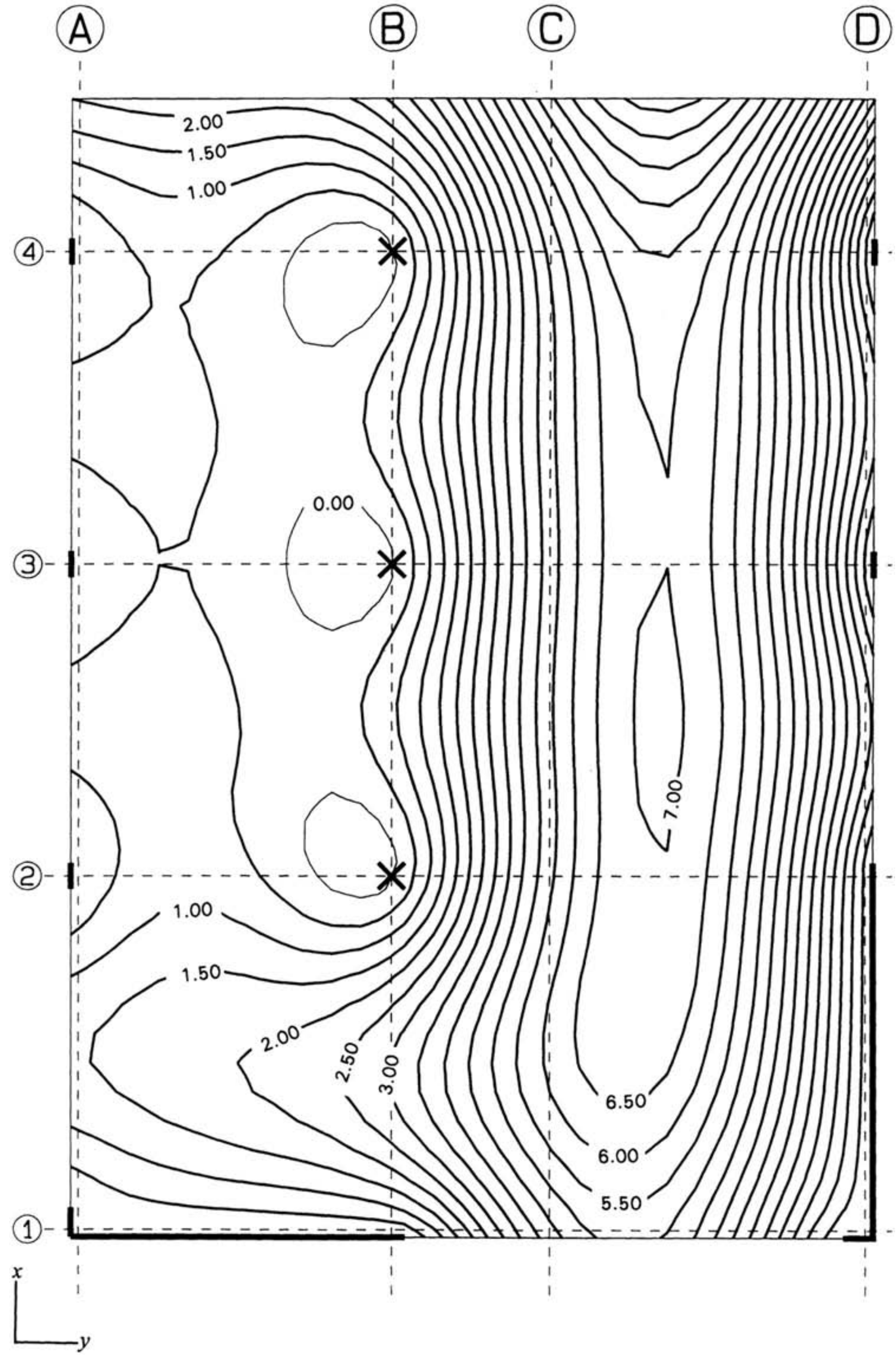
It is assumed that, with regard to deflections under quasi-permanent actions, a limiting value of 25 mm was agreed with the client. The deflection diagram for cracked cross-section shows that this requirement is met between axes 1 and 4. The deflection of the cantilever slab accounting for creep deformations is about 34 mm.

Therefore, in order to ensure proper functioning and appearance of the structure, precamber of the cantilever slab of 10 mm is suggested.

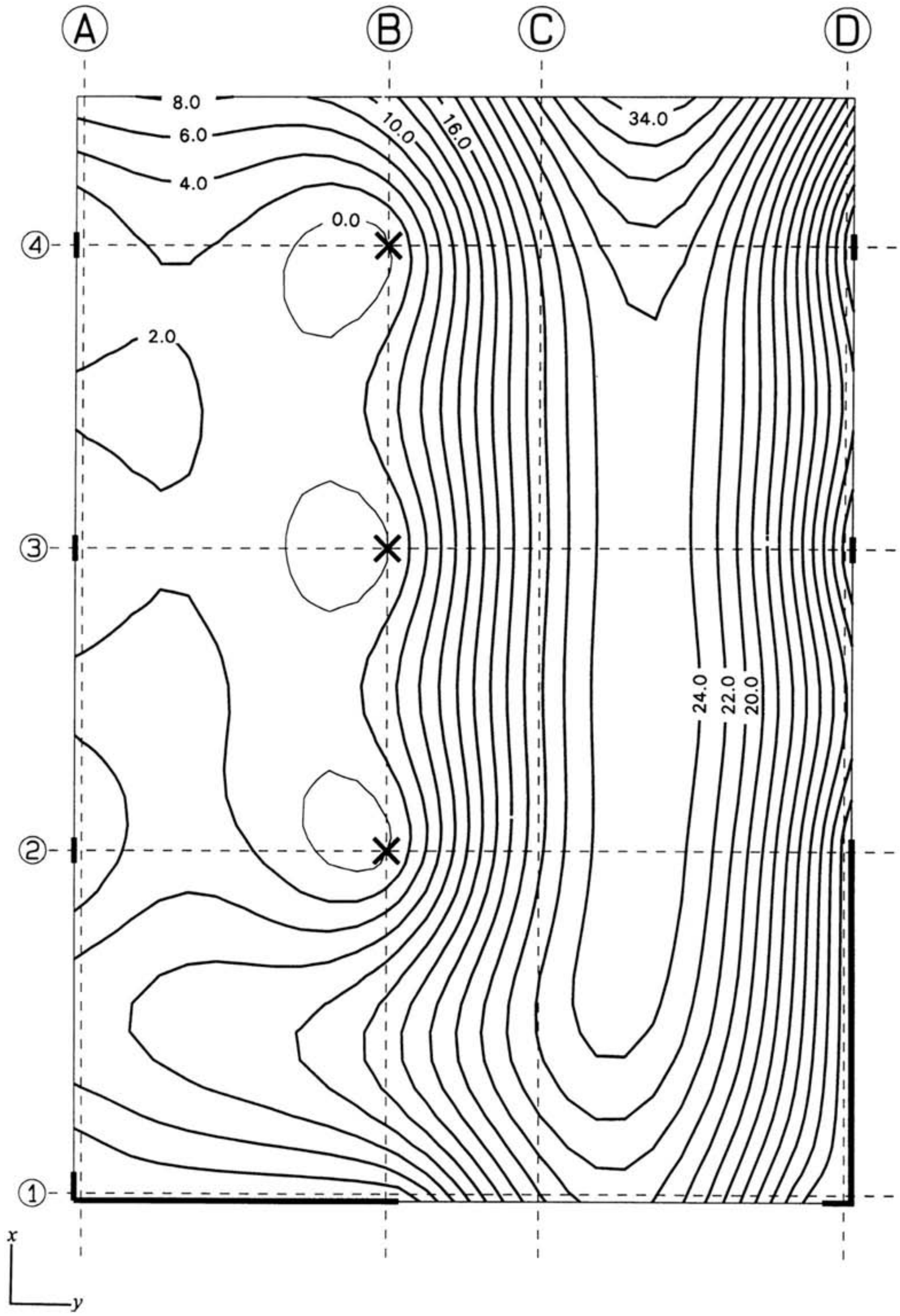
Reference

EC2, 4.3.4.5.3
for internal columns see Table 15.1 above
EC2, 4.4.3
EC2, 4.4.3.1P(2)
see following deflection Figures EC2, Eq.(A.4.3)

Deformations of flat slab due to quasi-permanent actions, uncracked cross-sections assumed

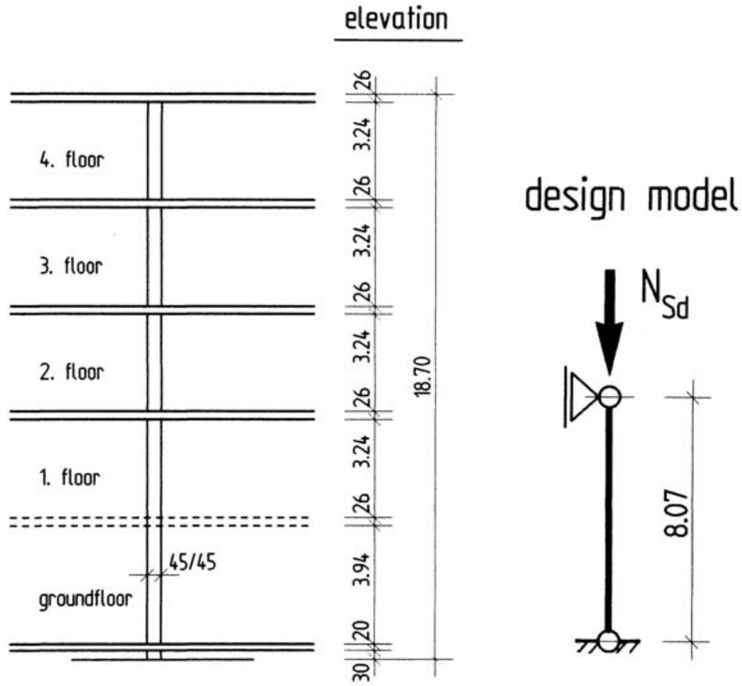


Deformations of flat slab due to quasi-permanent actions, cracked cross-sections assumed



15.3.3
Internal column

Design model of the column



The column in the ground floor/first floor in axis B/2 will be designed to EC2. The structural model is shown in the Figure above. The column is analysed on the assumption that the adjacent slab and block foundation provide no rotational restraint.

Design value of the axial force N_{sd}:

On the roof, a uniformly distributed snow load is assumed:

s	=	0.9 kN/m^2

The combination factor for this load is taken as:

ψ_0	=	1.0

N_{sd}=–[4*698.9+3*266.5+266.5*0.9/3.0+0.45^2*18.7*25]

–3800 kN

Reference

see 15.3.2.5 above EC2, 2.5.3.3(3)

see 15.3.2.5 above

EC1–2.3

conservative assumption

see 15.3.2.5, Figures of shear forces

Design of the column

f_{cd}	=	20 N/mm^2				
f_{yd}	=	435 N/mm^2				

Creep deformations are neglected.

Additional eccentricity e_a:

e_a	=	l_0/400	=	8.07/400	=	0.02m
e_a/h	=	0.02/0.45			=	0.05
l_o/h	=	8.07/0.45			=	18
v_u	=	–3.8/(0.45^2*20)			=	–1.0

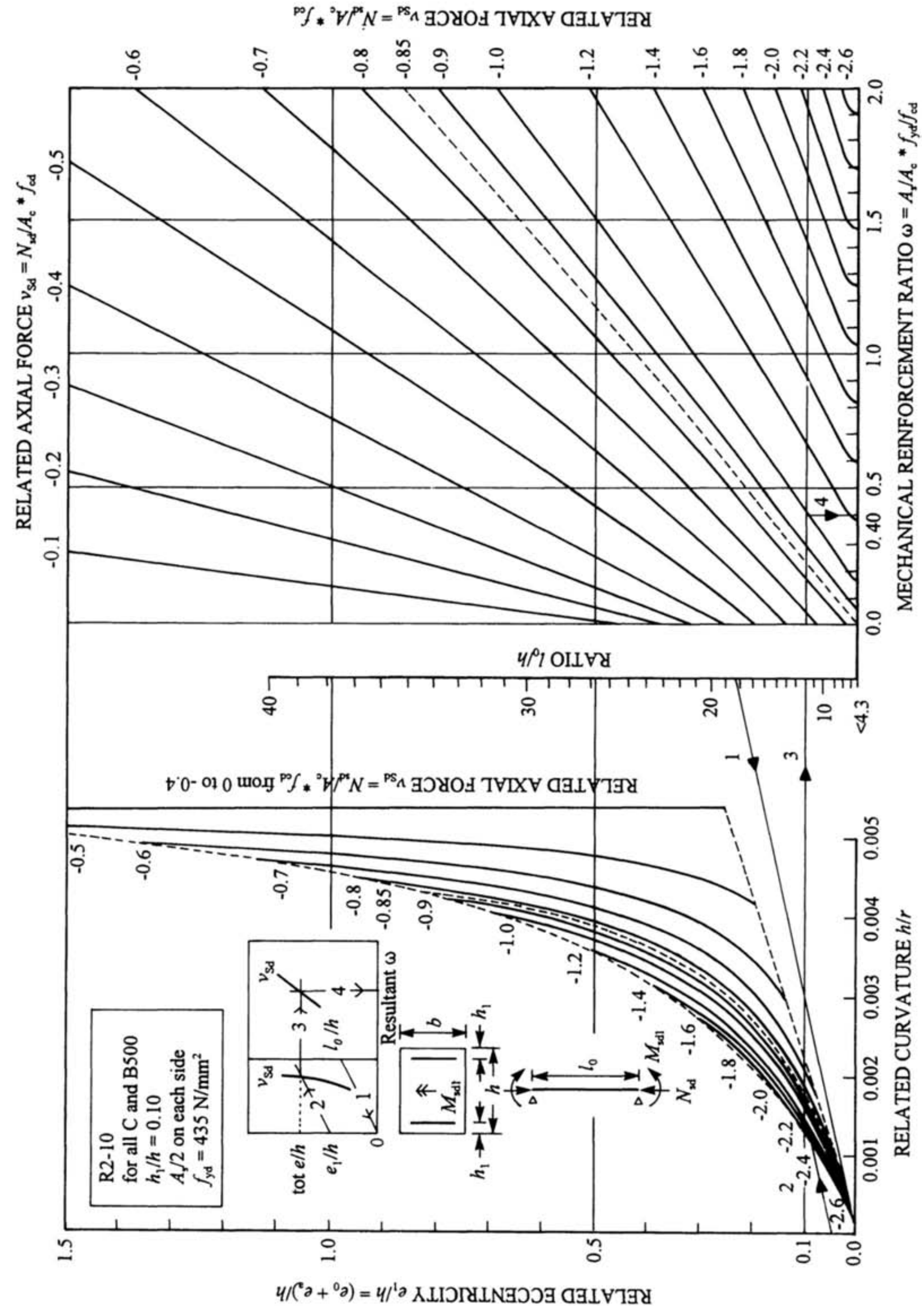
From the design diagram, ω is taken as:

ω	=	0.40				
$A_{s,tot}$	=	0.40*450^2*20/435			=	37.3*10^2 mm^2

Selected eight bars ø 25 mm $A_{s,prov}$=39.3*10^{2}mm2 Links ø 8 mm–300 mm

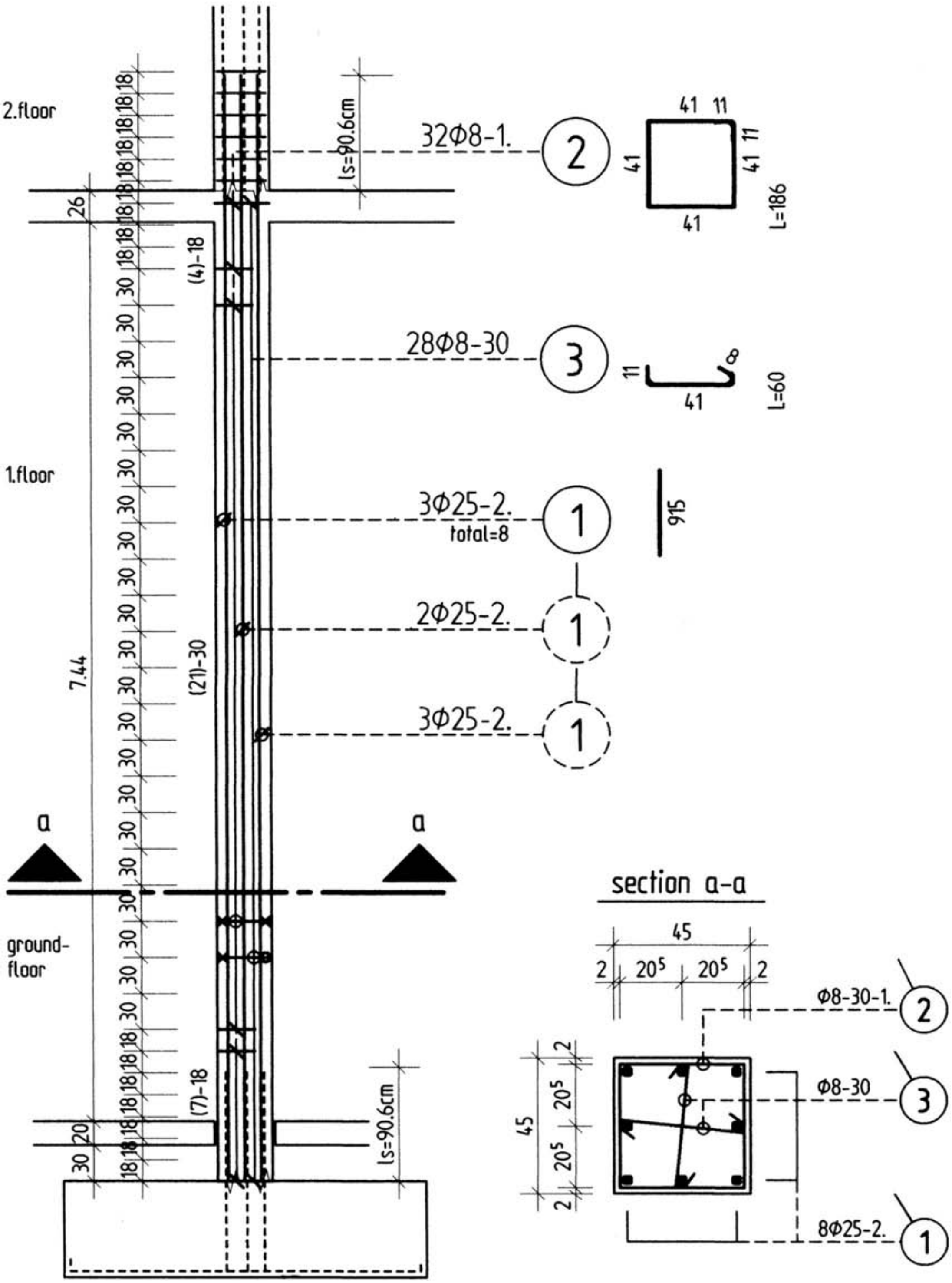
Reference

using the Figures in [2] C 30/37
EC2, A.3.4(9)
EC2, 4.3.5.4(3)
EC2, 5.4.1.2.1(2)
$A_{s.min}$ not relevant here
Design diagram for the column



Reference [2], diagram R2–05

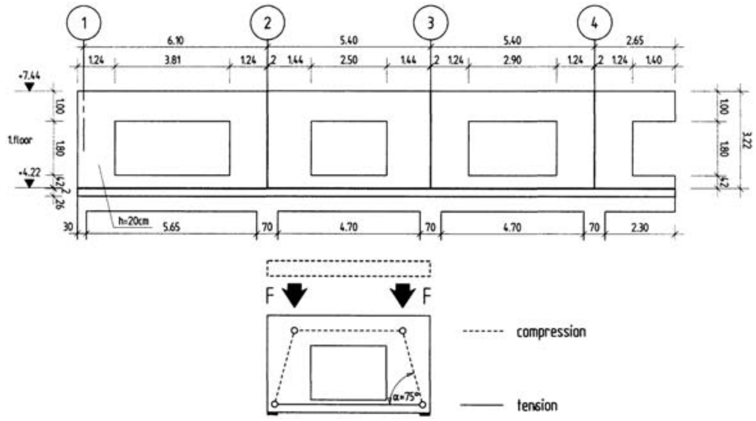
Detailing of reinforcement



15.3.4 Facade element

The facade of the building consists of precast elements (see Figure below). As an example, the element between axis 2 and 3 will be designed to EC2. As model, strut and ties are used.

For simplification, the maximum shear forces, between axis 1 and 2 are considered



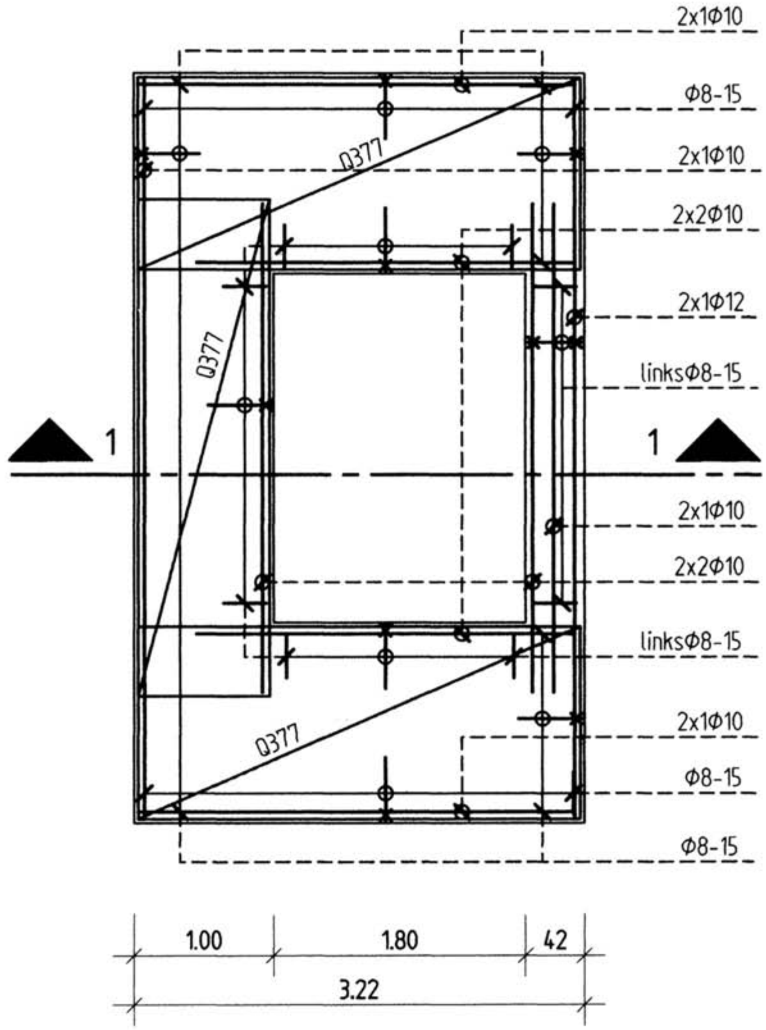
max F	=	135+70.6	=	206 kN
Maximum tie force:				
max T	=	max F cos α		
	=	206 cos 75°	=	79 kN
$A_{s,req}$	=	$79*10^3/435$	=	1.8*1
$A_{s,prov}$	=	2 ø 12+2 ø 10	=	$3.8*10^2$ mm^2

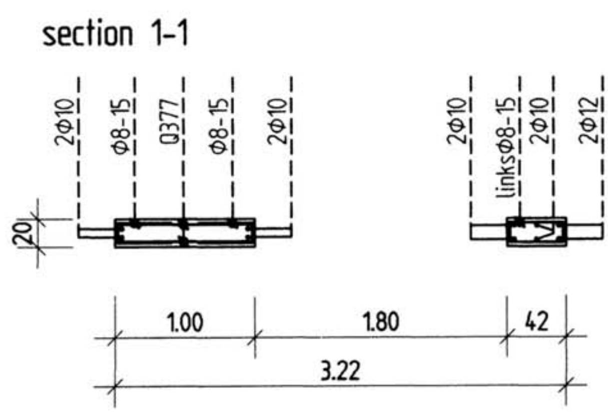
Reference

see 15.3.2.5, Figures for shear forces are considered

see details of reinforcement

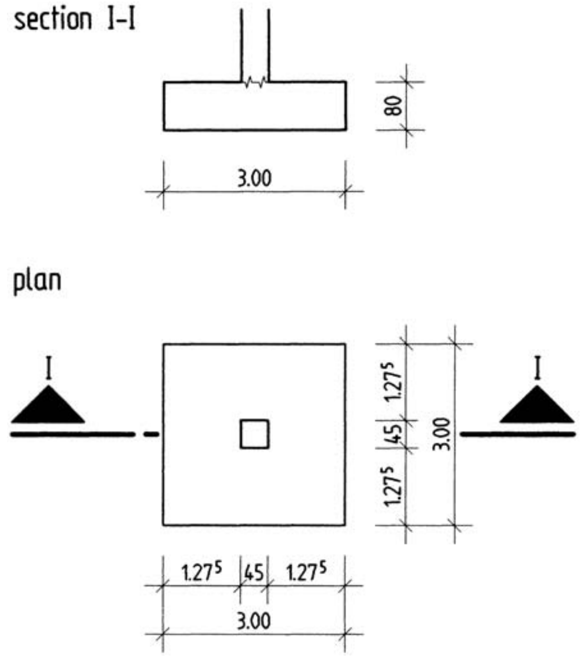
Reinforcement details





15.3.5 Block foundation

It is assumed that the foundation is subjected to an axial force N_{Sd} only which acts in the centre of gravity of the foundation slab. The axial force N_{Sd} results from the internal column in axis B/2 and is given by



N_{Sd} = = –3800 kN

Design value of bending moment:

M_{sd} = $N_{Sd}\, a(1–h_{col}/a)^2/8$

= $3800*3.0*(1–0.45/3.0)^2/8$ = 1030 kNm

Reference

see 15.3.3 above
page 97 in [5]

Design for bending

Effective depth:

d_x = h_f–(min $c+\Delta h+ø_x/2$)

= 0.800–(0.040+0.010+0.012/2) = 0.744 m

d_y = d_x–ø=0.744–0.012 = 0.732 m

μ_{Sds} = $1.030/(3.0*0.732^2*20)$ = 0.032

ω = = 0.033

$A_{s,req}$ = 0.033*3000*732*20/435 = $34*10^2$ mm^2

Provided in both directions:

$A_{s,prov}$=36 ø 12=40.7*10^2 mm^2

Design for punching shear

d_m = (0.744+0.732)/2 = 0.738

Distance of the critical perimeter from the face of the column

s = $1.5*d_m$=1.5*0.738 = 1.10m

Length of critical perimeter

u = 4*0.45+2π*1.10 = 8.71m

Mean value of ground pressure due to

σ_s = N_{sd}/a^2=3800/9.00 = 422kN/m2

Area within critical perimeter

= 0.45^2+4*0.45*1.10+π*1.102 = 6.0m^2

Critical force to be resisted

V_{Sd}	=	3800–6.0*422	=	1268 kN
v_{Sd}	=	1268/6.0	=	212kN/m

Design shear resistance of slabs without punching shear reinforcement:

ρ_1	=	40.7/(300*73.8)	=	18%
k	=	1.6–0.738	=	0.862
			<	1.0
τ_{Rd}	=		=	0.34 N/mm^2
v_{Rd1}	=	1.0*0.34*(1.2+40*0.0018)*0.738*10^3		
			=	319 kN/m
			>	v_{Sd}

Reference

EC2, 4.3.1

EC2, 4.1.3.3(9): min c=40 mm assumption: ø=12 mm [1], Table 7.1b

spacing see reinforcement details below

EC2, 4.3.4

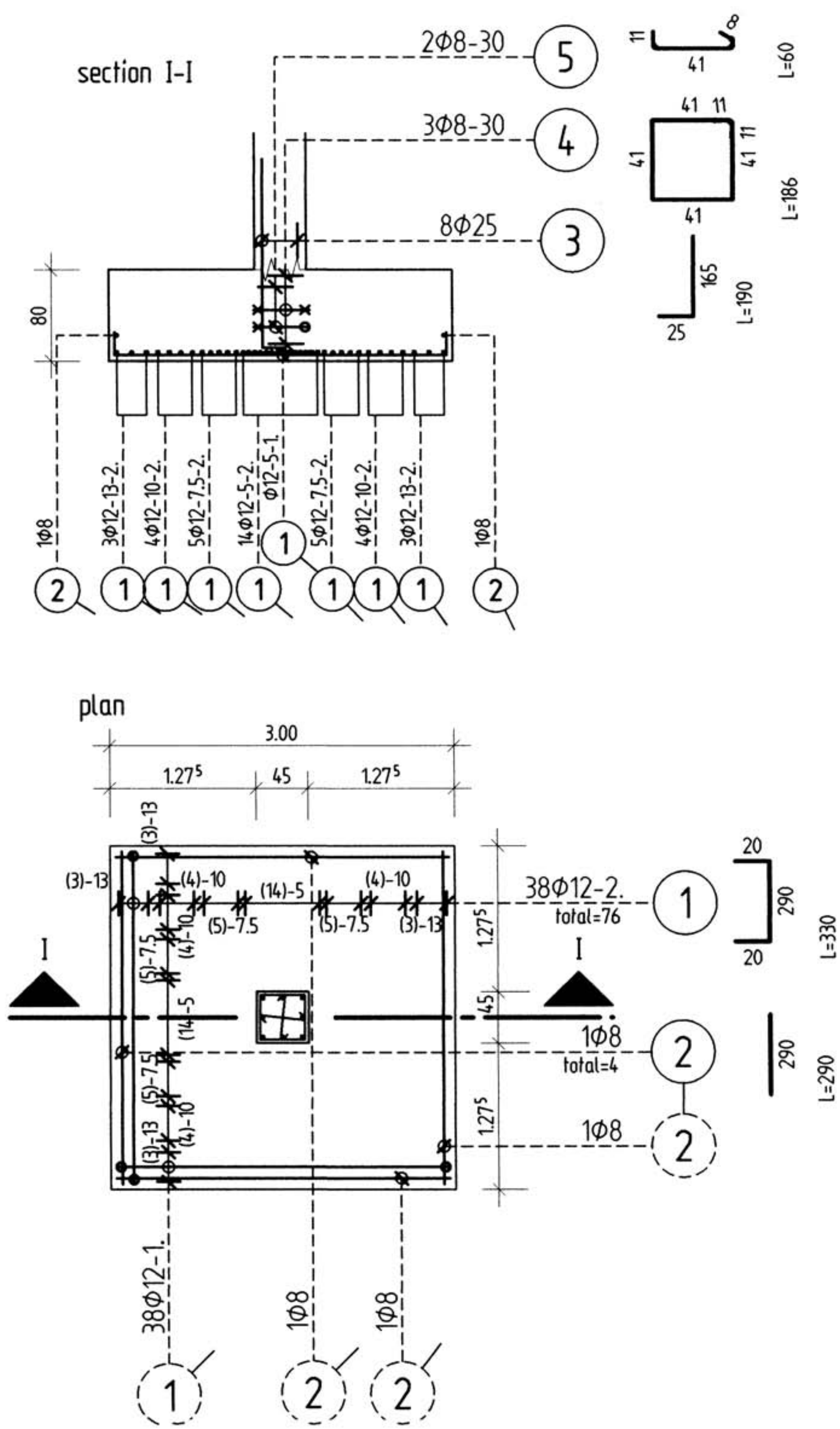
EC2, 4.3.4.2.2

EC2, 4.3.4.1(5)

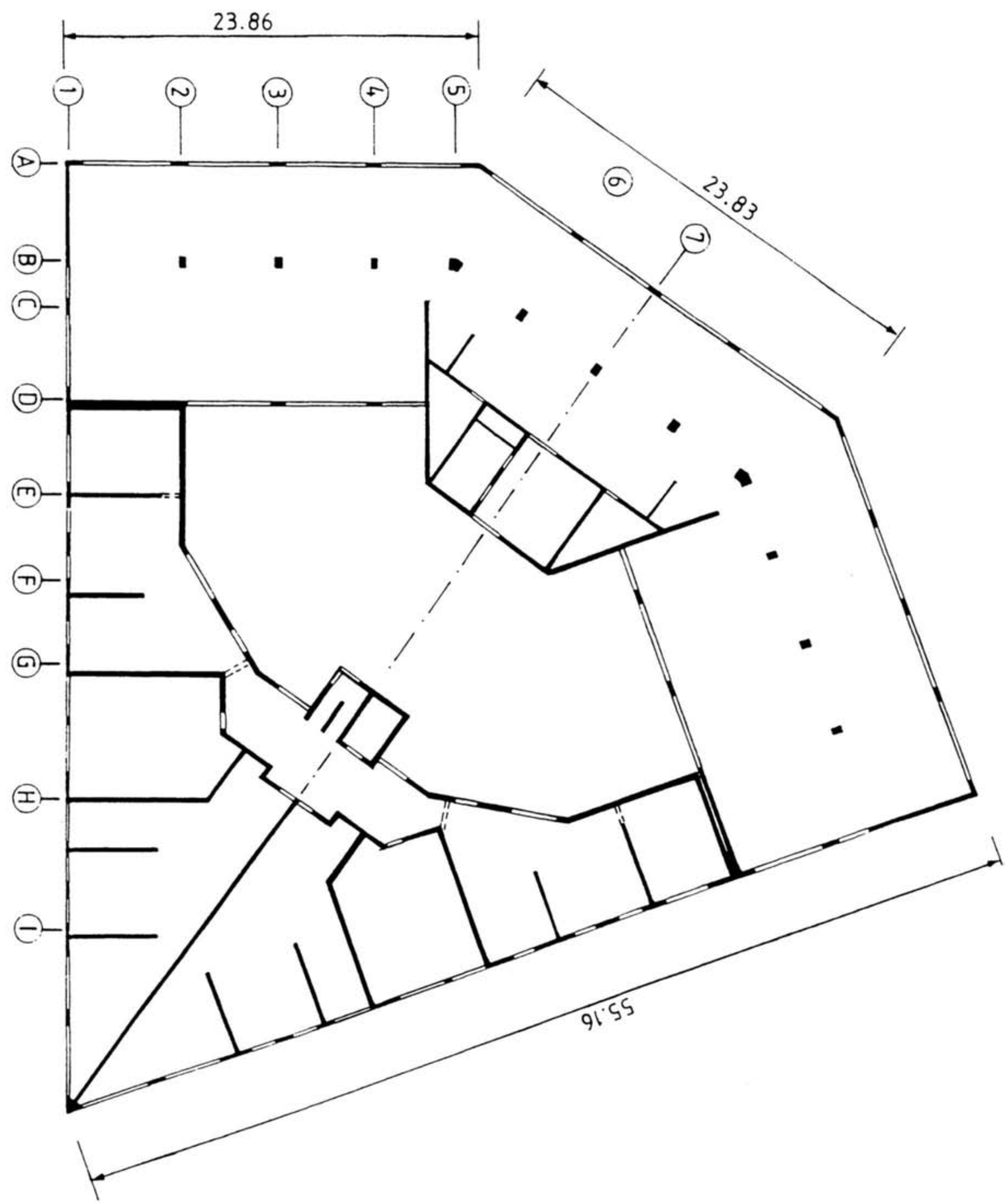
EC2, Eq.(4.56)

for C 30/37

Reinforcement details


section I-I
2Φ8-30
5
3Φ8-30
4
8Φ25
3
L=60
L=186
L=190
80
1Φ8
3Φ12-13-2.
4Φ12-10-2.
5Φ12-7.5-2.
14Φ12-5-2.
Φ12-5-1
5Φ12-7.5-2.
4Φ12-10-2.
3Φ12-13-2.
1Φ8
plan
3.00
1.27⁵
45
1.27⁵
38Φ12-2.
total=76
L=330
1Φ8
total=4
L=290
1Φ8
38Φ12-1.
1Φ8
1Φ8

15.4
Calculation for a residential building



15.4.1.2
Basic data of structure, materials and loading

Intended use:	Residential building
Fire resistance:	1 hour for all elements

Loading (excluding self-weight of structure):

Continuous slab:	-imposed: 2.0 kN/m^2 -finishes: 1.5 kN/m^2

Combination factors:
Serviceability limit states are not considered
Exposure classes:
Class 1 (indoor) for all members
Materials:

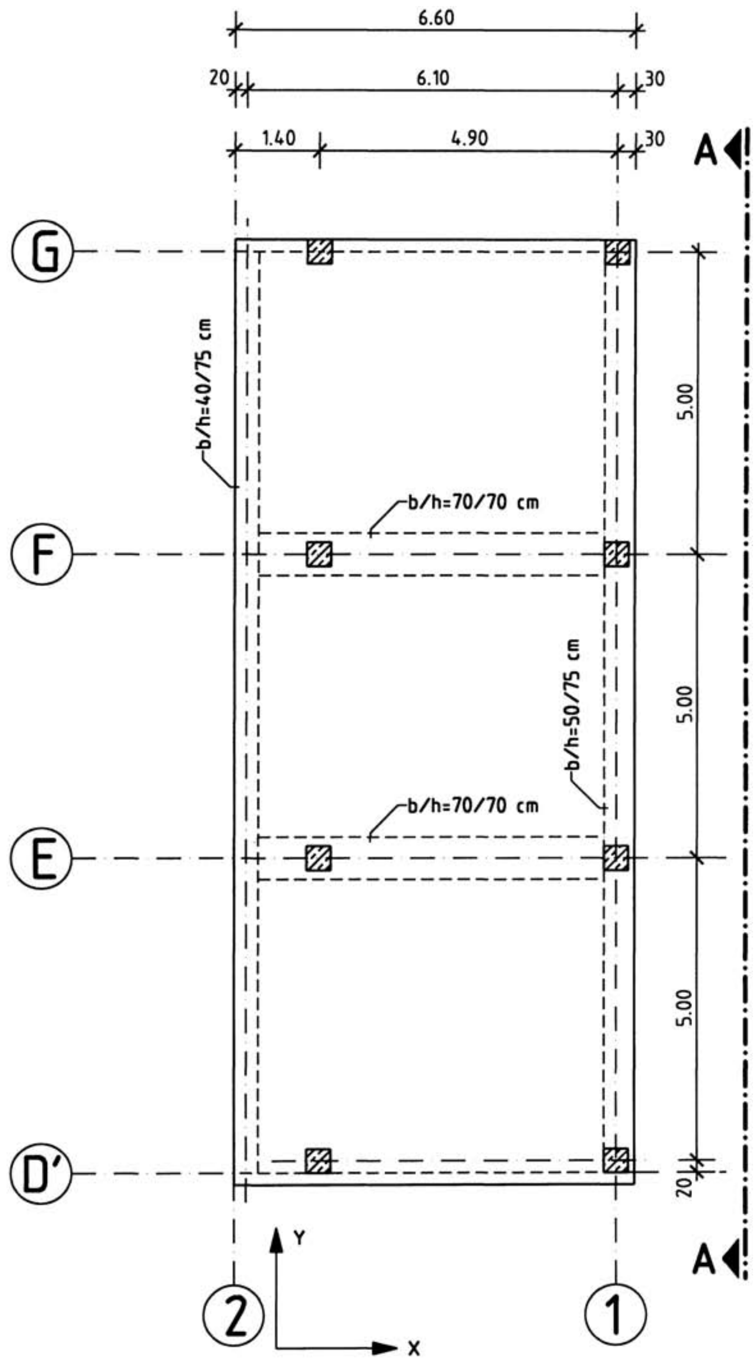
Concrete grade	C 30/37
Steel grade	B500
Self-weight of concrete	25 kN/m^3

Reference
see floor plan
EC2–1.2, 1.3
EC1–2.1 for Category A EC1–2.1
EC2, Table 4.1
EC2, Table 3.1
ENV 10 080
EC1–2.1

15.4.2
Continuous slab (end span)

15.4.2.1
Floor span and idealization of the structure

Floor plan of the continuous slab



Idealization of structure:

The end span of a two-way continuous slab is designed to Eurocode 2.

The effective spans are given as:

l_x	=	6.10+0.2/3+0.3/3	=	6.27m
l_y	=		=	5.00m

15.4.2.2
Limitation of deflections

Assumptions:

- admissible deflection is given by $l_y/250$
- σ_s 250 N/mm^2 in service conditions
- ρ_1 0.5%, i.e. concrete is considered as lightly stressed.

From Table 4.14 in Eurocode 2 with ø assumed as 10 mm:

l_y/d	=		=	32
d_{req}	=	5.00/32	=	0.156 m
h_{req}	=	d_{req}+nom c+ø/2=0.156+0.025+0.005	=	0.186 m
Selected:		h	=	0.25 m

15.4.2.3 Actions

Self-weight of slab: 0.25*25		=	6.25 kN/m^2
Finishes		=	1.50 kN/m2
Permanent load	G_k	=	7.75 kN/m^2
Imposed load	Q_k	=	2.0 kN/m^2

15.4.2.4 Structural analysis

In the present example, only the ultimate limit states are considered. The slab is analysed using the simplified yield-line method in [6], A3.2. This method is based on the following assumptions.

- high ductility reinforcement is used
- at the ultimate limit state for bending, the ratio x/d 0.25
- the spans in any one direction are approximately the same
- the loadings on the adjacent panels are approximately the same.

Loading on the panels:

$\gamma_G G_k$	=	1.35*7.75	=	10.50 kN/m^2
$\gamma_Q Q_k$	=	1.5*2.00	=	3.00 kN/m^2
				13.50 kN/m^2

Reference

EC2, 2.5.2.1
EC2, Eq. (2.15)
a_i in axis 1 and 2
EC2, 4.4.3
EC2, 4.4.3.1 and 4.4.3.2
ø assumed as 10 mm
EC2, 2.5
EC2, 2.5.3.2.2
EC2, 3.2.4.2
[6], A3.2(1)
l_x=5.0 m for all span conditions

Design moment over the continuous edge (l_y/l_x=6.27/5.0=1.25):

m_0	=	13.50*5.0^2	=	337.5 kN
$m_{S,dx}$	=	–337.5*0.0735	=	–24.81 kNm/m

Design span moment in x-direction:

$m_{Sd,x}$	=	337.5*0.055	=	+18.56 kNm/m

Span moment in y-direction:

$m_{Sd,y}$	=	337.5*0.044	=	14.85 kNm/m
Maximum shear force:				
V_{sd}	=	0.52*13.5*5.0	=	35.1 kN/m

15.4.2.5 Design at ultimate limit states for bending and axial force

f_{cd}	=	30/1.5	=	20 N/mm^2
f_{yd}	=	500/1.15	=	435 N/mm^2
d_x	=	0.25–0.03	=	0.22 m
Design of the continuous edge:				
μ_{Sds}	=	$24.81*10^{-3}/(1.0*0.22^2*20)$	=	0.026
$A_{s,req}$	=	0.027*103*220*20/435	=	$2.73*10^2$ mm^2/m
x/d	=	0.067	<	0.25

Selected: Welded mesh with twin bars ø 6.0 mm

Steel B 500 B - R 377 2 * ø 6.0 - 150
$A_{s,prov}$=3.77*10^2 mm^2/m

Design for the span moments:
x-direction:

μ_{Sds}	=	$18.56*10^{-3}/(1.0*0.22^2*20)$	=	0.019
$A_{s,req}$	=	0.020*103*220*20/435	=	$2.02*10^2$ mm^2/m

y-direction:

μ_{Sds}	=	$14.85*10^{-3}/(1.0*0.21^2*20)$	=	0.017
$A_{s,req}$	=	$0.018*10^3*210*20/435$	=	$1.74*10^2$ mm^2/m

Selected in *x*-direction:
Welded mesh with twin bars ø 5.5 mm

Steel B 500 B - R 317 2 * ø 5.5 - 150
$A_{s,prov}$=3.17*10^3 mm^2/m
$A_{s,prov}$=0.64*10^2 mm^2/m

Reference

[6], A3.2(1), and Table A2 three edges discontinuous, one edge continuous
[6], Eq. (A5)
three edges discontinuous, one edge continuous [6], A3.2. (2)
EC2, 4.3.1
see 15.4.2.2 above
Table 7.1 (b) in [1]
see 15.4.2.7 and 15.4.2.8.1 below
Table7.1(b) in [1]
see 15.4.2.7 and 15.4.2.8.1 below
in *x*-direction
in *y*-direction
Additional span reinforcement in *y*-direction:
Selected: Welded mesh fabric with bars ø 7.0

Steel B 500 B - R 257 ø 7.0 - 150

$A_{s,prov}$=2.57*10^2 mm^2/m
$A_{s,prov}$=0.64*10^2 mm^2/m

Total reinforcement in *y*-direction:

$A_{s,prov}$	=	0.64*102+2.57*10^2	=	3.21*10^2 mm^2/m

15.4.2.6
Design for shear

τ_{Rd}	=		=	3.21*10^2 mm^2/m
ρ_1	=	3.77/(100*22)	=	0.17%
k	=	1.6–0.22	=	1.38
V_{Rd1}	=	0.34*1.38*(1.2+40*0.0017)*0.22*10^3		
			=	130 kN/m
			>	V_{sd}

Design shear resistance of compression struts:

V_{Rd2}	=	0.5*0.575*20*0.9*0.22*10^3	=	1138 kN/m
			>	V_{sd}

15.4.2.7
Minimum reinforcement for crack control

A_s	=	$k_c k f_{ct,eff} A_{ct}/\sigma_s$		
where				
k_c	=	0.4 for bending		
k	=	0.8 for h 300 mm		
$f_{ct,eff}$	=	3.0 N/mm^2		
A_{ct}	=	0.25/2*1.0	=	0.125 m^2
σ_s	=	400 N/mm^2		
A_s	=	0.4*0.8*3*0.125*10^6/400	=	3.0*10^2 mm^2/m
$A_{s,prov}$		3.21*10^2 mm^2/m	>	3.0*10^2 mm^2/m

15.4.2.8
Detailing of reinforcement

15.4.2.8.1
Minimum reinforcement areas for the avoidance of brittle failure

$A_{s,min}$	=	0.0015*220*10^3	=	3.3*10^2 mm^2/m
$A_{s,prov}$	=	3.17*10^2+0.64*102	=	3.81*10^2 mm^2/m

15.4.2.8.2
Basic anchorage length

l_b	=	0.25 ø f_{yd}/f_{bd} or		
l_b	=	0.25 ø f_{yd}/f_{bd}		
f_{bd}	=	=		3.0 N/mm^2

Reference
in *x*-direction

in *y*-direction
EC2, 4.3.2
EC2, Table 4.8 for C30/37
EC2, Eq. (4.18)
EC2, Eq. (4.19)
EC2, 4.4.2.2
EC2, Eq. (4.78)
from EC2, Table 4.11, column 2, for ø 8 mm
EC2, 5
EC2, 5.4.2.1.1
EC2, Eq. (5.14) in *x*-direction
EC2, 5.2.2.2
EC2, Eq. (5.3) for twin bars
EC2, Table 5.3, for C30/37

Location		ø (mm)	$ø_n$ (mm)	l_b (mm)
Support		6.0	8.5	310
Span	*x*	5.5	7.8	290
y	4.5	-	170	

15.4.2.8.3
Anchorage at the discontinuous edges

F_s	=	$V_{Sd}a_1/d+N_{sd}$		
a_1	=	d		
F_s	=	35.1*1.0+0	=	35.1 kN/m
$A_{s,req}$	=	35.1*103/435	=	$0.81*10^2$ mm^2/m

Required anchorage length:

$l_{b,net}$	=	0.7*290*0.81/3.17	=	52 mm

Minimum values:

$l_{b,min}$	=	0.3*290	=	87 mm
	=	10ø=10*5.5=10*5.5	=	55 mm
	=		=	100 mm

Anchorage length:

$l_{b,anch}$	=	2/3*100	=	70 mm

15.4.2.8.4
Anchorage at the continuous edges

$l_{b,anch}$	=	10ø=10*7.8	=	80 mm

15.4.2.8.5
Lap lengths of mesh fabrics in *y*-direction

l_s	=	$\alpha_2\, l_b\, A_{s,req}/A_{s,prov}$		
α_2	=	0.4+64/800	=	0.48
			<	1.00
l_s	=	1.0*170*10	=	170 mm
$l_{s,min}$	=	0.3*1.0*170	=	51 mm
	=	s_t	=	150 mm
	=		=	200 mm

Reference

for mesh fabric R 317

EC2, 5.4.3.2.1(5)

EC2, Eq. (5.15)

EC2, 5.4.3.2.1(1)

EC2, 5.2.3.4.1

EC2, Eq. (5.4), and 5.2.3.4.2(2)

The largest value of $l_{b,min}$ should be used

EC2, 5.4.2.1.4(3)

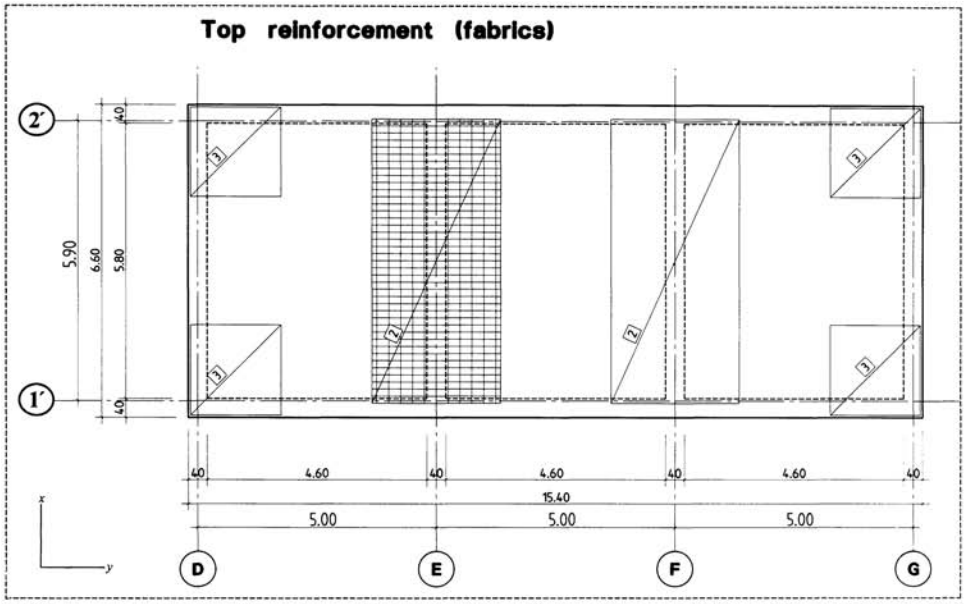
EC2, 5.4.3.2.1(5)

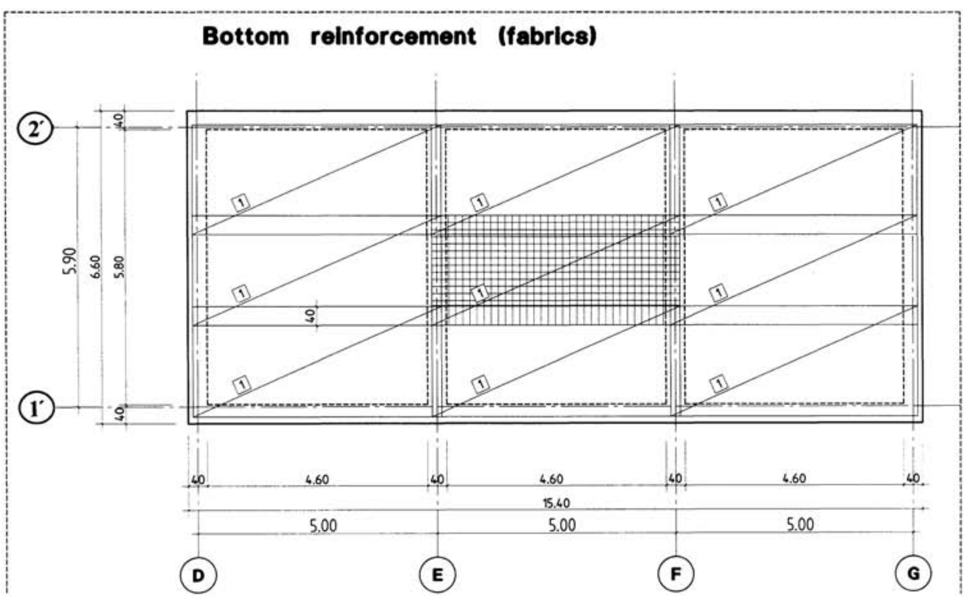
Lapping of mesh fabrics R 317 with bar diameter ø 4.5

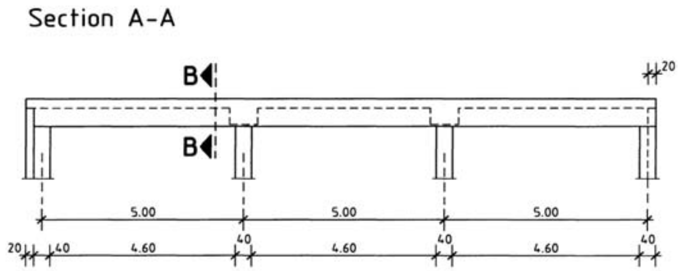
EC2, Eq. (5.9)

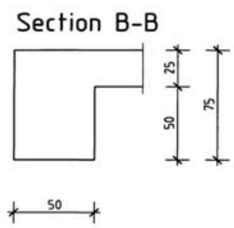
The largest value of $l_{s,min}$ be used

Detailing of reinforcement









15.4.3
Continuous edge beam (end span)

15.4.3.1
Structural system

15.4.3.2
Actions

The beam is subjected to the following actions (see sketch below):

(a) **permanent actions**

- self-weight, $G_{k,1}$
- self-weight of parapets, $G_{k,2}$
- self-weight of supported slab, $G_{k,3}$
- self-weight of supported facade elements, $G_{k,4}$ to $G_{k,9}$
- concentrated forces due to permanent load, $G_{k,10}$ and $G_{k,11}$

(b) **variable actions**

- imposed load of the adjacent slab, $Q_{k,1}$
- variable actions transmitted by the facade elements, $Qk_{,2}$ to $Q_{k,7}$
- concentrated variable loads, $Qk_{,8}$ and $Q_{k,9}$.

Reference

see floor plan below

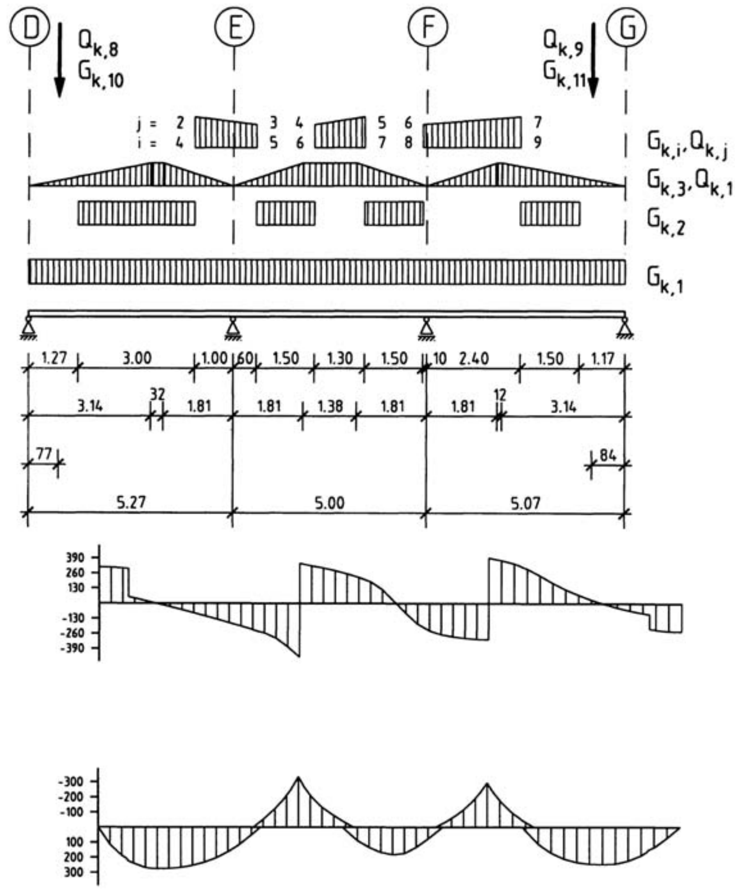
see 15.4.2 above

In the following it is assumed that neither the permanent nor the variable actions are dependent upon each other.

Table 15.3: Permanent, $G_{k,j}$, and variable actions, $Q_{k,j}$, acting on the beam

Action		Magnitude of the actions (kN/m; kN)										
subscript i=		1	2	3	4	5	6	7	8	9	10	11
subscript j=		-	-	1	2	3	4	5	6	7	8	9
$G_{k,i}$	γ_o=1.0	6.25	41.20	24.32	198.60	19.40	105.50	154.10	44.60	109.00	152.86	86.70

Action		Magnitude of the actions (kN/m; kN)										
subscript i=		1	2	3	4	5	6	7	8	9	10	11
subscript j=		-	-	1	2	3	4	5	6	7	8	9
γ_o=1.35	8.44	55.62	32.84	268.11	26.19	208.04	208.04	60.12	147.15	206.36	117.05	
$Q_{k,j}$	γ_Q=1.0	-	-	6.28	20.60	2.00	16.50	24.00	3.40	8.40	18.36	–3.90
γ_Q=1.5	-	-	9.42	30.90	3.00	24.75	36.00	5.10	12.60	27.54	–5.85	



15.4.3.3
Structural analysis

(a) Linear analysis without redistribution

The action effects resulting from a linear analysis without redistribution are summarized below.

Reference

EC2, 2.5.3.4.2

Schematic shear and moment diagram

Support reactions

Support	Support reaction (kN) due to			
$G_{k,i}$	max $Q_{k,j}$	min $Q_{k,j}$	$G_{k,i}$+$Q_{k,j}$	
D	276.05	310.43	271.71	306.10
E	717.47	807.24	711.78	801.55
F	668.51	749.30	660.05	740.84
G	212.23	226.48	202.75	217.00

Bending moments and shear forces in spans 1 to 3

Span	x (m)	max V_{Sd} (kN)	min V_{Sd} (kN)	max MSd (kNm)	min M_{Sd}
	0.00	310.43	271.71	0.00	0.00
	0.77				205.91
	1.32	51.09	42.51	268.41	233.61
1	1.91			283.85	
	2.64	–63.90	–73.58	259.60	218.08
	3.95	–188.03	–209.11	79.70	44.81
	5.27	–425.06	–472.84	–332.74	–383.43
	0.00	334.40	286.72	–332.74	–383.43
	1.25	221.59	182.91	–23.95	–58.02
2	2.50	42.99	25.87	157.08	97.69
	2.65			159.91	100.12
	3.75	–198.25	–242.77	8.30	–28.24
	5.00	–293.13	–342.25	–323.75	–374.64
	0.00	407.05	366.92	–323.77	–374.66
3	2.86			265.26	
	4.23				166.29
	5.07	–202.76	–226.49	–0.04	-

Reference

shear diagram

moment diagram

(b) Linear analysis with redistribution

The cross-section over support E will be designed for the design bending moment

M_{sd} = = –333kNm

This corresponds to a distribution factor δ of

δ = 332.74/383.43 = 0.867

15.4.3.4
Design of span 1 for bending

Design data:

C 30/37 f_{cd} = 20 N/mm^2

B500 B f_{yd} = 435N/mm^2

effective depth d = 0.71 m

Design of the cross-section of support E:

b_w = = 0.50m

μ_{Sds} = 0.333/(0.50*0.71^2*20) = 0.066

ω = = 0.070

x/d = = 0.139

$A_{s,req}$ = 0.070*500*710*20/435 = 11.5*10^2 mm^2

δ_{perm} = 0.44+1.25*0.139 = 0.62

< 0.867

Selected 4 ø 20; $A_{s,prov}$=12.56*10^2 mm^2

Design for maximum span moment:

effective width

b_{eff} = 0.5+0.1*0.85*5.27 = 0.95 m

M_{Sd} = 284 kNm

μ_{SdS}	=	0.284/(0.95*0.712*20)	=	0.03
ω	=		=	0.031
$A_{s,req}$	=	0.031*950*710*20/435	=	$9.61*10^2$ mm^2

Selected 2 ø 25; $A_{s,prov}$=9.81*10^2 mm^2

Reference
EC2, 2.5.3.4.2 see Table above
EC2, 2.5.3.4.2(3)
for bar diameter ø 25
[1], Table 7. 1a
permissible coefficient δ
EC2, 2.5.2.2.1 for an L-beam see Table above
[1], Table 7.1a

15.4.3.5
Design for shear

max V_{sd}	=		=	473 kN

Design shear at the distance d from the face of the support:

V_{Sd}		max V_{Sd}–d $G_{k,1}$		
	=	473–0.71*8.44	=	467 kN

The variable strut-inclination method is used; assumption:

cot θ	=		=	1.25
v	=	0.7–30/200	=	0.55
V_{Rd2}	=	0.50*0.9*0.71*0.55*20/2.05*10^3	=	1714 kN
			>	V_{Sd}
$(A_{sw}/s)_{req}$	=	467*10^3/(0.9*0.71*435*1.25)	=	13.44*10^2 mm^2/m

Selected stirrups ø 12 - spacing s=150 mm

$(A_{sw}/s)_{prov}$	=		=	15.07*10^2 mm^2/m
maximum spacing:				
V_{sd}/V_{Rd2}	=	467/1714	=	0.273
s_{max}	=		=	300 mm
			>	150 mm
$(A_{sw}/s)_{min}$	=	0.0011*500*1*1000	=	5.5*10^2 mm^2/m

15.4.3.6
Control of cracking

Cracking is controlled by limiting the bar diameter ø. The steel stress σ_s is estimated as

σ_s	=	$f_{yd}A_{s,req}/A_{s,prov}$ $(1/\gamma_F)$		
	=	435*9.61/9.81*(1/1.5)	=	280 N/mm^2

From Table 4.11 in EC2 for reinforced concrete:

$ø^*_s$	=		=	16 mm
$ø_s$	=	16*71/(10*4)	=	28 mm
			>	25 mm

15.4.3.7
Detailing of reinforcement

Basic anchorage length

l_b	=	$0.25\quad 2*20*10^{-3}*435/2.8$	=	1.10 m
l_b	=	$0.25*25*10^{-3}*435/2.8$	=	0.97 m

Reference

EC2, 4.3.2
see Table above
see diagram of actions above; the opposite formula is a conservative assumption
EC2, 4.3.2.4.4
θ 40°
EC2, Eq. (4.26)
α=90°
EC2, 5.4.2.2(7)
EC2, Table 5.5
EC2, 4.4.2
EC2, Table 4.11
in span
EC2, 5
for ø=20 mm
for ø=25 mm
Anchorage of bottom reinforcement

- intermediate support

$l_{b,net}$	=	$10*25*10^{-3}$	=	0.25 m

- end support

V_{Sd}	=		=	311 kN
α_1	=	0.9*0.71*1.25/2	=	0.40 m
F_s	=	311*0.4/0.71	=	175 kN
$A_{s,req}$	=	$175*10^3/435$	=	$4.0*10^2$ mm^2
$l_{b,net}$	=	1.0*0.97*4.0/9.81	=	0.396 m
2/3 $l_{b,net}$	=	2/3*0.396	=	0.26 m

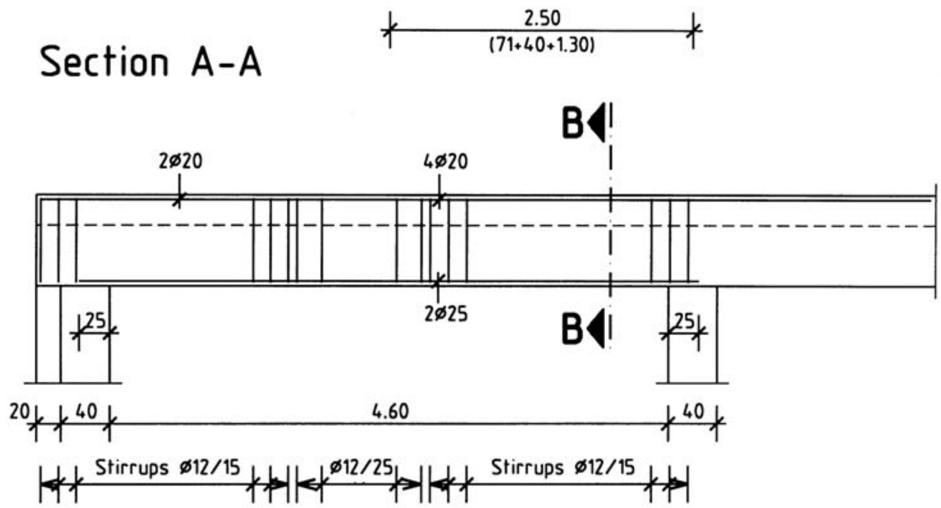
Anchorage of the top reinforcement

$l_{b,net}$	=	0.3*1.10	=	0.33 m
or	=	d	=	0.71 m

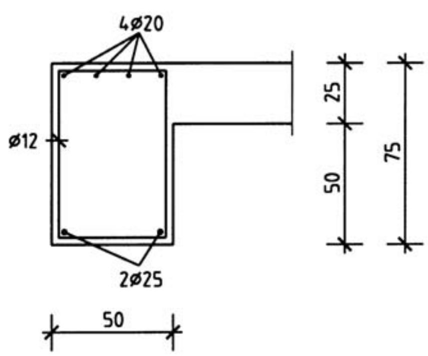
Reference

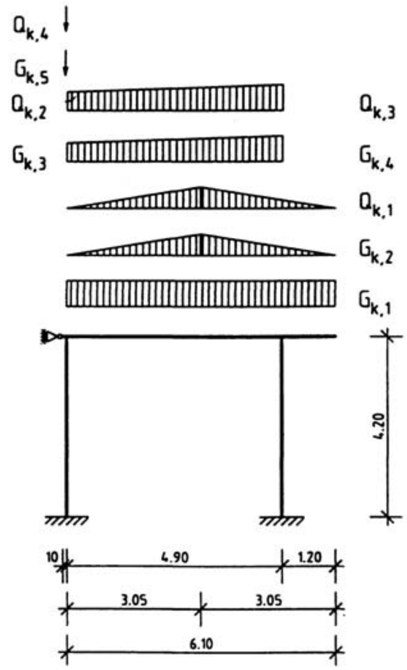
EC2, 5.4.2.1.5
see Table above
EC2, 5.4.2.1.3(1)
EC2, 5.4.2.1.4(2)
EC2, Eq. (5.4) for straight bars
EC2, Eq. (5.5)
EC2, 5.4.2.1.3(2)
Detailing of reinforcement

Section A-A



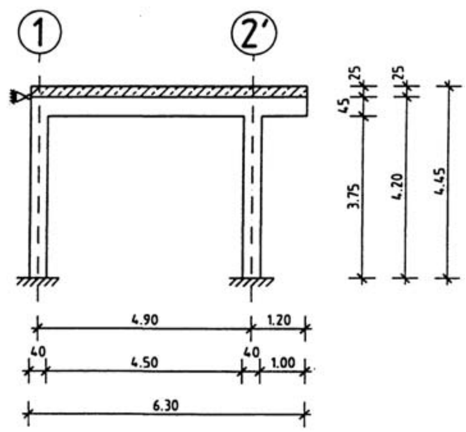
Section B-B





15.4.4
Braced transverse frame in axis E

15.4.4.1
Structural system; cross-sectional dimensions



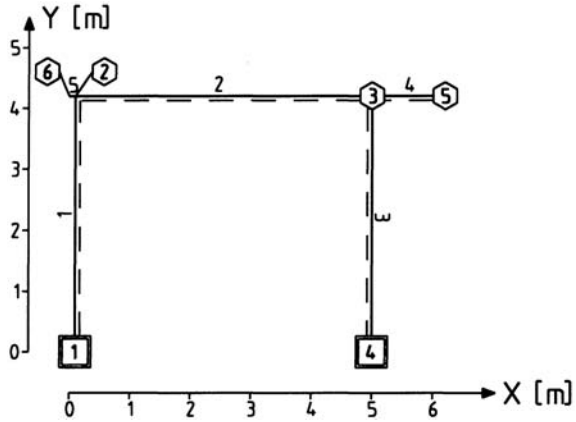
15.4.4.2
Actions

The frame is subjected to the following actions (see Figure above):

(a) **permanent actions**

- self-weight of beam, $G_{k,1}$
- self-weight of supported slab, $G_{k,2}$

Reference
see floor plan in 15.4.2.1



- self-weight of supported slab, $G_{k,3}$; $G_{k,4}$
- support reaction of continuous beam, $G_{k,5}$.

(b) **variable actions**

- imposed load of supported slab, $Q_{k,1}$
- imposed load of supported slab, $Q_{k,2}$,; $Q_{k,3}$
- support reaction of continuous beam, $Q_{k,4}$.

Action		**Magnitude of the actions (kN/m; kN)**				
subscript i=		1	2	3	4	5
subscript j=		-	1	2	3	4
$G_{k,i}$	γ_G=1.0	7.85	28.1	30.0	227.0	540.4
γ_G=1.35	10.60	37.9	40.5	306.5	729.5	
$Q_{k,j}$	γ_Q=1.0	-	7.3	3.0	70.0	63.2
γ_Q=1.5	-	10.9	4.5	105.0	94.8	

It is assumed that all permanent actions and all imposed loads act simultaneously.

15.4.4.3 Structural analysis

For the purposes of structural analysis, the frame is subdivided into elements and nodes as shown below.

Reference

see 15.4.3 above

EC2, 2.5

Node	**Coordinates**		**Support conditions**		
***x* (m)**	***y* (m)**	***x***	***y***	***m***	
1	0.10	0.00	1	1	1
2	0.10	4.20	0	0	0
3	5.00	4.20	0	0	0
4	5.00	0.00	1	1	1
5	6.20	4.20	0	0	0
6	0.00	4.20	0	0	0

x: free in *x*-direction

y: free in *y*-direction

m:	free rotation
1:	no freedom
0:	freedom

Element	**Defined by node**		**A**
left	**right**	**(m^2)**	
1	1	2	0.160
2	2	3	0.677
3	3	4	0.160
4	3	5	0.677
5	2	6	0.677

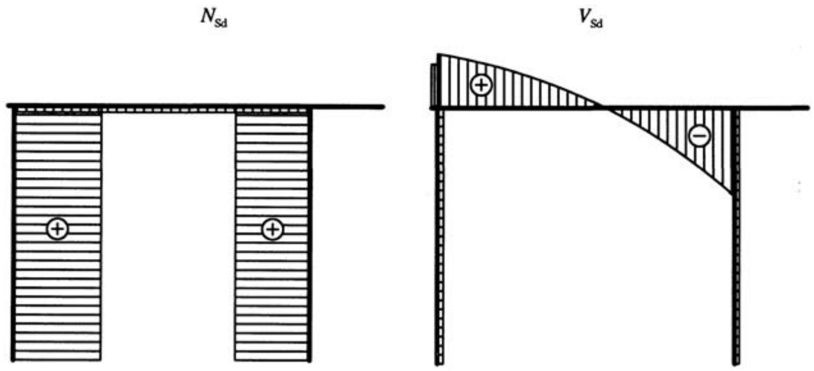
Reference

Action effects due to permanent actions

Element No.	***x* (m)**	**N_{Sd} (kN)**	**V_{Sd} (kN)**	**M_{Sd} (kNm)**
1	0.00	–1146.09	–39.60	–15.58
0.70	–1146.09	–39.60	–43.30	
1.40	–1146.09	–39.60	–71.02	
2.10	–1146.09	–39.60	–98.73	
2.80	–1146.09	–39.60	–126.45	
3.50	–1146.09	–39.60	–154.17	
4.20	–1146.09	–39.60	–181.88	
2	0.00	–39.60	415.55	–254.88
0.65	–39.60	368.58		
0.82	–39.60	351.52	61.50	
1.63	–39.60	243.13	307.13	
2.45	–39.60	90.11	446.29	
2.85	–39.60		464.46	
3.27	–39.60	–106.85	442.29	
4.08	–39.60	–335.93	263.53	
4.70	–39.60	–526.38		
4.90	–39.60	–593.22	–114.13	
3	0.00	–614.92	39.60	–102.90
0.70	–614.92	39.60	–75.18	
1.40	–614.92	39.60	–47.46	
2.10	–614.92	39.60	–19.75	
2.60	–614.92	39.60		
2.80	–614.92	39.60	7.97	
3.50	–614.92	39.60	35.69	
4.20	–614.92	39.60	63.40	
4	0.00	0.00	21.70	–11.23
0.20	0.00	16.84	–7.38	
0.40	0.00	12.48	–4.46	
0.60	0.00	8.61	–2.36	
0.80	0.00	5.25	–0.98	
1.00	0.00	2.37	–0.23	
1.20	0.00			
5	0.00	0.00	–730.53	73.00
0.02	0.00	–730.35	60.58	

Action effects due to permanent actions

Element No.	***x* (m)**	**N_{Sd} (kN)**	**V_{Sd} (kN)**	**M_{Sd} (kNm)**
0.03	0.00	–730.18	48.90	
0.05	0.00	–730.00	36.49	
0.07	0.00	–729.82	24.08	
0.08	0.00	–729.65	12.40	
0.10	0.00	–729.47		



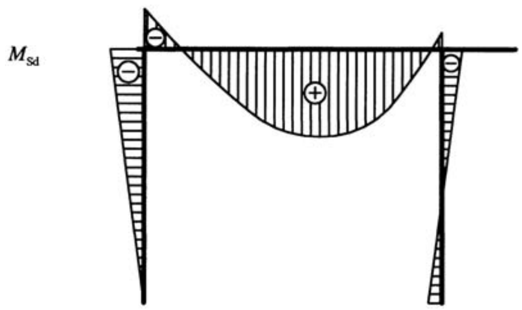
Qualitative presentation of action effects due to permanent actions
Reference

Action effects due to imposed loads

Element	***x* (m)**	**N_{Sd} (kN)**	**V_{Sd} (kN)**	**M_{Sd} (kNm)**
1	0.00	–208.12	–12.32	–8.34
0.70	–208.12	–12.32	–16.97	
1.40	–208.12	–12.32	–25.59	
2.10	–208.12	–12.32	–34.21	
2.80	–208.12	–12.32	–42.84	
3.50	–208.12	–12.32	–51.46	
4.20	–208.12	–12.32	–60.09	
2	0.00	–12.32	113.35	–69.56
0.65	–12.32	105.73		
0.82	–12.32	101.64	19.35	
1.63	–12.32	73.91	92.07	
2.45	–12.32	30.08	135.64	
2.85	–12.32	142.39		
3.27	–12.32	–29.64	136.86	
4.08	–12.32	–101.87	83.97	
4.70	–12.32	–165.44		
4.90	–12.32	–185.48	–32.64	
3	0.00	–188.04	12.32	–31.62
0.70	–188.04	12.32	–23.00	
1.40	–188.04	12.32	–14.37	
2.10	–188.04	12.32	–5.75	
2.60	–188.04	12.32		
2.80	–188.04	12.32	2.88	
3.50	–188.04	12.32	11.50	
4.20	–188.04	12.32	20.12	
4	0.00	0.00	2.56	–1.02
0.20	0.00	1.78	–0.59	

Action effects due to imposed loads

Element	***x*** **(m)**	N_{Sd} **(kN)**	V_{Sd} **(kN)**	M_{Sd} **(kNm)**
0.40	0.00	1.14	–0.30	
0.60	0.00	0.64	–0.13	
0.80	0.00	0.28	–0.04	
1.00	0.00	0.07	0.00	
1.20	0.00			
5	0.00	0.00	–94.77	9.48
0.02	0.00	–94.77	7.87	
0.03	0.00	–94.77	6.35	
0.05	0.00	–94.77	4.74	
0.07	0.00	–94.77	3.13	
0.08	0.00	–94.77	1.61	
0.10	0.00	–94.77		



Reference

15.4.4.4
Design for the ultimate limit states

15.4.4.4.1
Basic data

Concrete C 30/37	f_{cd}	=	20 N/mm^2
Steel B 500	f_{yd}	=	435 N/mm^2

15.4.4.4.2
Design of the beam for the ultimate limit states of bending and longitudinal force

(a) Design value of the acting bending moment M_{Sd} in node 2

M_{Sd}	=	–254.88–69.56	=	–324.44 kNm
Bending moment at the face of the support:				
M'_{Sd}	=	–324.44+(415.55+113.35)*0.2	=	–219 kNm
d	=		=	0.645 m
μ_{Sds}	=	0.219/(0.7*0.645^2*20)	=	0.038
ω	=		=	0.04
$A_{s,req}$	=	0.04*700*645*20/435	=	8.30*10^2 mm^2

Selected 5 ø 16; $A_{s,prov}$=10.1*10^2 mm^2

(b) Design in mid-span

M_{sd}	=	464.46+142.39	=	607 kNm
d	=		=	0.645 m
Effective width for a T-beam:				
l_0	=	0.7*4.90	=	3.43 m
b_{eff}	=	0.7+0.2*3.43	=	1.39
μ_{Sds}	=	$0.607/(1.39*0.645^2*20)$	=	0.053
ω	=		=	0.056
x/d	=		=	0.13
x	=	0.13*0.645	=	0.084 m
			<	0.25 m
$A_{s,req}$	=	0.056*1390*645*20/435	=	$23.1*10^2$ mm^2

Selected 5 ø 20; $A_{s,prov}$=25.5*10^2 mm^2

15.4.4.3
Design of the beam for shear

max V_{Sd}	=	593.22+185.48	=	778.7 kN

For the design, the variable strut inclination method is used.
Design shear force at the distance *d* from the face of the support:

V'_{Sd}		max V_{sd}–(0.2+d) ($G_{k,1}$+$G_{k,4}$+$Q_{k,3}$)		
	=	778.7–0.845(10.6+260+87)	=	477 kN
cot θ	=	1.25; α=90°		

Reference
EC2, 4.3
see 15.4.1 above
Element No. 2
see Table in 15.4.4.3 above
EC2, 2.5.3.3(5)
[1], Table 7.1a
EC2, 2.5.2.2.1
[1], Table 7.1a
EC2, 4.3.2
EC2, 4.3.2.4.4
$G_{k,4}$; $Q_{k,3}$ coordinates at x=0.845 m [1]

A_{sw}/s	=	$477*10^3/(0.9*0.645*435*1.25)$	=	$15.1*10^2$ mm^2/m
V_{Rd2}	=	$0.7*0.9*0.645*0.55*20/2.05*10^3$	=	2180 kN
			>	V'_{Sd}
V'_{Sd}/V'_{Rd2}	=	477/2180	=	0.22
Maximum longitudinal spacing of stirrups:				
max s_w	=	0.6*645	=	387 mm
			>	300 mm
Maximum transverse spacing of legs:				
max $s_{w,t}$	=		=	300 mm

Selected shear links with four legs ø 12 - 300 mm
$(A_{sw}/S)_{prov}$=15.08*10^2mm^2/m

In mid-span:

Shear links with two legs ø 12 - 300 mm
$(A_{sw}/S)_{prov}$=7.54*10^2 mm^2/m

ρ_w	=	7.54*10^{-4}/(0.70*1.0*1.0)	=	0.0011
			=	min ρ_w

15.4.4.4.4
Design for the ultimate limit states induced by structural deformations (buckling)

In this example, only element No. 1 is designed to EC2.

Design action effects:

N_{sd}	=	–1146.09–208.12	=	–1354 kN
Bending moment in node 1:				
$M_{Sd,1}$	=	–15.58–8.34	=	–24 kNm
Bending moment in node 2:				
$M_{Sd,2}$	=	–181.88–60.09	=	–242 kNm

Cross-sectional dimensions: *b/h*=400/400 mm
Slenderness ratio in the plane of the frame:

ß		0.7		
l_0	=	0.7*4.20	=	2.94 m
λ	=	2.94/(0.289*0.40)	=	25.5
λ_{ilm}	=	15/ ν_u		
ν_u	=	1.354/(0.42*20)	=	0.423
λ_{lim}	=	15/ (0.423)	=	23.0
λ_{crit}	=	25*(2–0.018/0.179)	=	47.5

Check for second order effects is not necessary.

M_{Rd}	=	N_{Sd}*h/20	=1354*0.4/20	=	27.1 kNm
				<	242 kNm

Reference

EC2, Eq. (4.27)
EC2, 5.4.2.2(7)
EC2, 5.4.2.2(9)
EC2, Table 5.5, for C 30/37 and B 500
EC2, 4.3.5
see 15.4.4.3 above
e_0=0.018 m
e_0=0.179 m
EC2, 4.3.5.3.5; in the transverse direction, buckling is prevented by structural members
EC2, 4.3.5.3.5(2)
not relevant here EC2, Eq. (4.62)
EC2, Eq. (4.64)

d	=		=	0.355 m
Design of the column in node 2 using the tables in [2]				
ν_{Sd}	=	–1.354/(0.16*20)	=	–0.423
μ_{sd}	=	0.242/(0.16*0.4*20)	=	0.20
ω	=	0.28		
$A_{s,tot}$	=	0.28*400^2*20/435	=	20.6*10^2 mm^2

Selected 2*5=10 ø 16
$A_{s,prov}$=20.1*10^2 mm^2

In element 3, 4 ø 16 are provided on each side.

15.4.4.5
Detailing of reinforcement

15.4.4.5.1
Columns

Bar diameters provided:		ø	=	16 mm
			>	12 mm
Minimum reinforcement areas:				
min A_s	=	0.15*1354*103/435	=	4.7*10^2 mm^2
or	=	0.003*400^2	=	4.8*10^2 mm^2
			<	20.1*10^2 mm^2
Transverse reinforcement (links)		$ø_w$	=	10 mm
			>	6 mm
Spacing:				
$s_{w,max}$	=	12*16	=	192 mm
0.6$s_{w,max}$	=	0.6*192	=	115 mm

15.4.4.5.2
Beam

Minimum reinforcement area to avoid brittle failure:

$A_{s,min}$	=	0.0015*700*645	=	6.8*10^2 mm^2
				< $A_{s,prov}$

Anchorage of bottom reinforcement

l_b	=	10 ø=10*25*10^{-3}	=	0.25 m

Basic anchorage length of bars with ø=16 mm

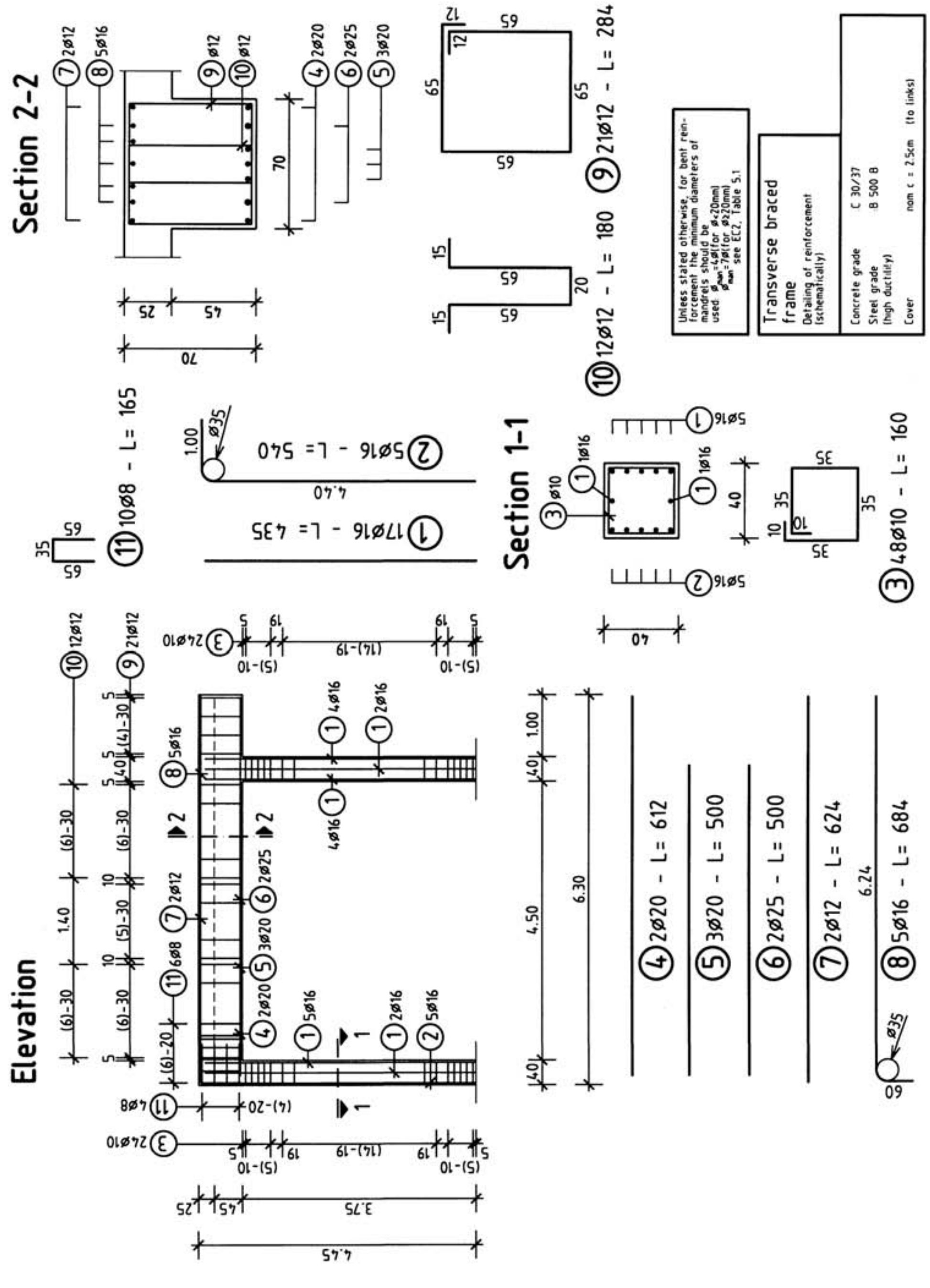
l_b	=	0.25* 2*16*10^{-3}*435/3.0	=	0.82 m

Lap length of the bars ø 16 in node 2:

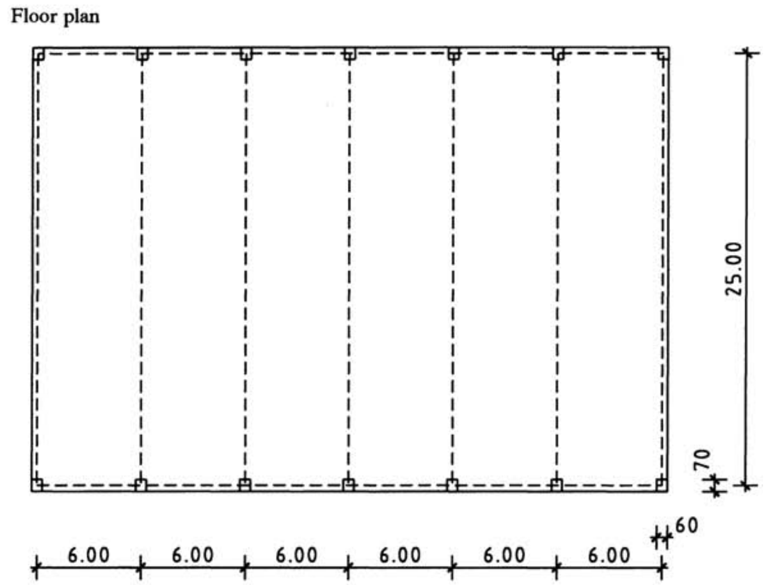
l_s	=	α_1 $l_{b,net}$		
	=	2.0*1.0*0.82*8.30/10.1	=	1.36 m
$l_{s,min}$	=	0.3*1.0*2.0*0.82	=	0.50 m

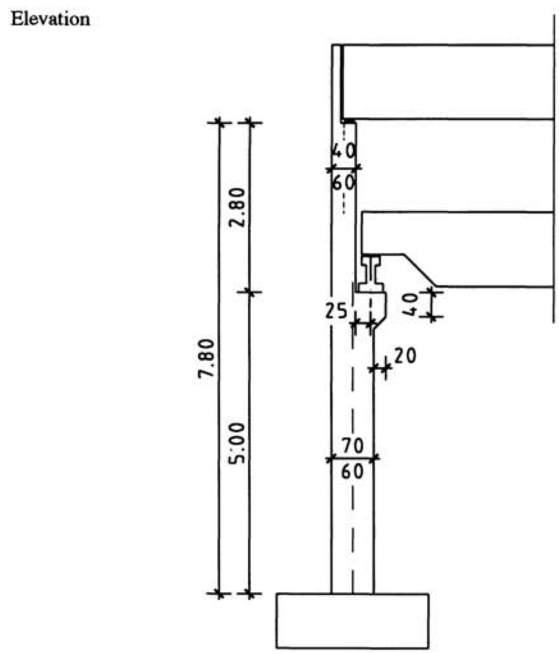
Reference
assumption
[2], page 64, Table 6.4 b:
elements 1 and 3
EC2, 5.4.1.2.1
EC2, 5.4.1.2.1(2)
EC2, 5.4.1.2.2(1)
EC2,5.4.1.2.2(3), (4) relevant here
EC2, Eq. (5.14)
supports in nodes 2 and 3 are considered as restrained
EC2, Eq. (5.3)
EC2, Table 5.3, for poor bond conditions
EC2, 5.2.4.1.3(1)
EC2, Eq.(5.7)
EC2, 5.2.4.1.3(1)
EC2, Eq. (5.8)

Calculation for a residential building



15.5.1
Floor plan; elevation



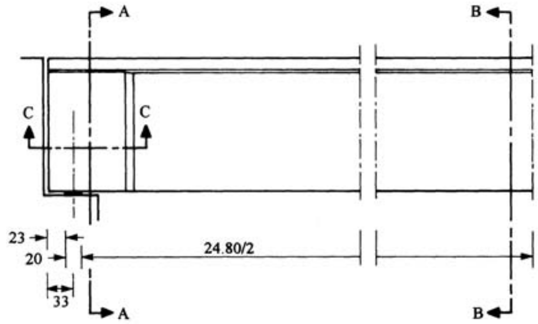


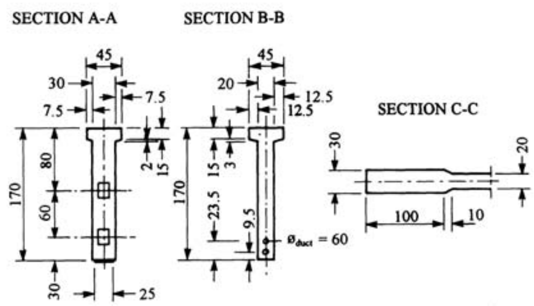
Reference

15.5.2
Calculation of prestressed concrete beam

15.5.2.1
Basic data

Structural system; cross-sectional dimensions
Elevation
Exposure class:
Class 1 (indoor conditions)





Materials:

Concrete grade	C 35/45
Steel grade	B 500

Tendons:			
7-wire strands	$f_{p0.1,k}/f_{pk}$	=	1500/1770 N/mm^2
Modulus of elasticity	E_s	=	200000 N/mm^2
Relaxation class 2			
Diameter of sheathing	$ø_{duct}$	=	60mm
Cross-sectional area:	A_p	=	7.0*10^2 mm^2

Reference

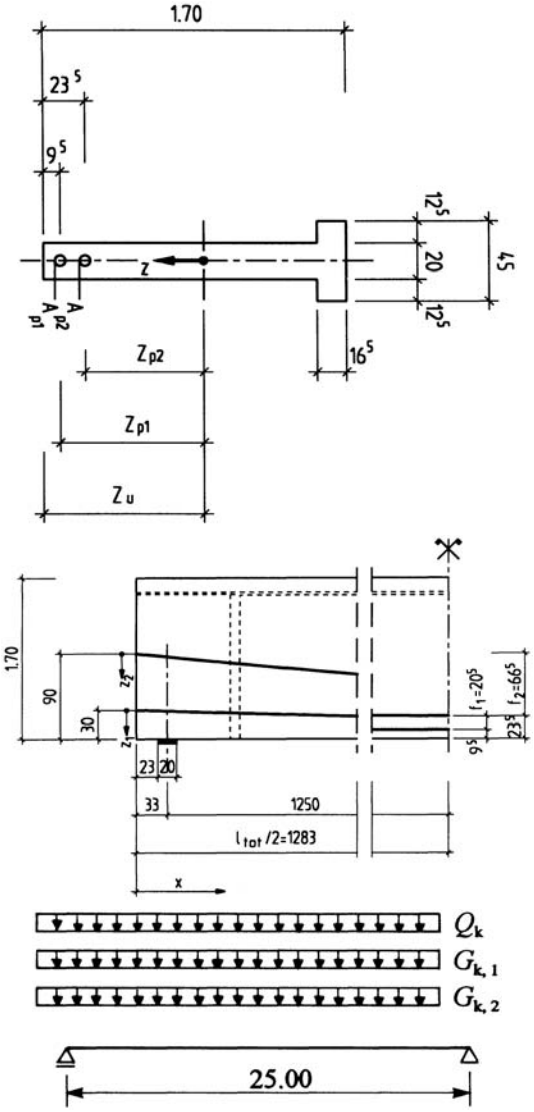
EC2, Table 4.1
EC2, Table 3.1 ENV 10 080
EC2, 4.2.3.4.1(2)

Coefficient of friction:	μ	=	0.22
Anchorage slip	Δl_{sl}	=	3.0 mm
Unintentional displacement	k	=	0.005
Cover to reinforcement:			
- links:	nom c_w	=	25 mm
- tendons:	nom c_p	=	65 mm

Geometric data of the beam in mid-span section:

$ø_{duct}$	=		=	60 mm
A_{p1}	=	A_{p2}	=	7.0*10^2 mm^2
α_e	=	200000/33500	=	5.97

Cross-section	**A_c; A_{c1} (m^2)**	**I_c; I_{c1} (m^4)**	**Z_u (m)**	**Z_{p1} (m)**	**Z_{p2} (m)**
A_c	0.381	0.104	0.933	0.838	0.698



Cross-section	A_c; A_{c1} (m^2)	I_c; I_{c1} (m^4)	Z_u (m)	Z_{p1} (m)	Z_{p2} (m)
$A_{c,net}$	0.376	0.100	0.945	0.850	0.710
A_{ci}	0.406	0.122	0.927	0.832	0.692

Reference

EC2, 4.1.3.3

modular ratio

Tendon profile

Description of the tendon profile:

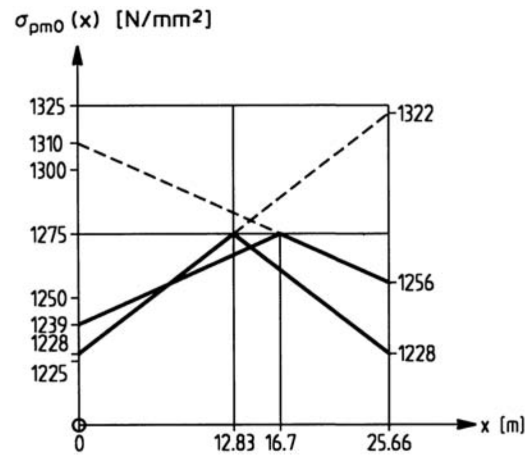
Tendon 1:

$Z_1(x)$ = $4*0.205*[x/l_{tot}-(x/l_{tot})^2]$

Tendon 2:

$Z_2(x)$ = $4*0.665*[x/l_{tot}-(x/l_{tot})2]$

15.5.2.2
Actions



$G_{k,1}$	=	=	9.5 kN/m
$G_{k,2}$	=	=	10.0 kN/m
Q_k	=	=	4.8 kN/m

15.5.2.3
Action effects due to $G_{k,1}$, $G_{k,2}$ and Q_k

max M_{sd}	=	[1.35*19.5+1.5*4.8]*25.02/8	=	2620 kNm
max V_{sd}	=	[1.35*19.5+1.5*4.8]*25.0/2	=	420 kN

Reference

l_{tot}=25.66 m
self-weight of beam
roofing
snow

15.5.2.4
Action effects due to prestress

Stresses σ_{pm0} in the tendons at t=0 allowing for friction, anchorage slip and unintentional angular displacement
Action effects N_P, M_p and V_P due to prestressing at the serviceability limit states

Location	**Action effects at**					
***t*=0**			***t*=**			
N_p (kN)	**M_p (kNm)**	**V_p (kN)**	**N_p(kN)**	**M_p (kNm)**	**V_p (kN)**	
Left support	–1727.2	–483.6	–117.3	–1452.8	–406.8	–98.6
Mid-span	–1779.4	–1387.9	0	–1505.0	–1146.8	0
Right support	–1738.8	–486.9	–118.1	–1464.4	–410.1	–99.4

Reference

15.5.2.5
Design for the ultimate limit states for bending and longitudinal force

(a) Material data; design values of material strength

Concrete	C 35/45			f_{ck}	=	35/1.5 N/mm^2
f_{cd}	=	f_{ck}/γ_c	=	35/1.5	=	23.33 N/mm^2

Reinforcing steel	B 500			f_{yk}	=	500 N/mm^2
f_{yd}	=	f_{yk}/γ_s	=	500/1.15	=	435 N/mm^2
Prestressing steel	1500/1770			f_{pk}	=	1770 N/mm^2
f_{pd}	=	$0.9f_{pk}/\gamma_s$	=	0.9*1770/1.15	=	1385 N/mm^2

(b) Design at mid-span

max M_{Sd} = = 2620 kNm

Effective depth at mid-span:

d_m = 1.70–[(4*2.0/26.0)*4.1+(2*2.0/26.0)*7.7 +(14.0/26.0)*16.5]*10^{-2} = 1.58 m

Related bending moment:

μ_{Sds} = max M_{Sd} /($b_f d^2{}_m f_{cd}$)=2.620/(0.45*1.582*35/1.5) = 0.10

with

h_f/d = 0.165/1.58 0.1

b_f/b_w = 45/20 = 2.25

The mechanical reinforcement ratio ω is given as:

ω = 108/1000 = 0.108

$A_{s,req}$ = $(1/f_{yd})(\omega b_f\, d_m f_{cd}-A_p\sigma_{pd})$

where

σ_{pd} = $(\gamma_p\, \epsilon_{pm} + \Delta\epsilon_p)\, E_p \le f_{pd}$ = 1385 N/mm^2

A trial calculation has shown that

$$(\gamma_p * \epsilon_{pm} + \Delta\epsilon_p) > \epsilon_{p0.1k},$$

so that

σ_{pd} = f_{pd} = 1385 N/mm^2

$A_{s,req}$ = (1/435)(0.108*0.45 *1.58*23.33–14.0*10^{-4}*1385)10^4

< 0

i.e., for the resistance of max M_{Sd}, no reinforcement is necessary.

Reference

EC2, 4.3.1

see 15.5.2.3 above

[2], p. 59

[2], p. 59, Table 6.3a

Minimum reinforcement area required to avoid brittle failure:

$A_{s,min}$ = 0.0015*200*1580 = 4.74*10^2 mm^2

Selected reinforcing steel B 500 6 bars ø 16; $A_{s,prov}$=12*10^2 mm^2

(c) Check of the pre-compressed tensile zone

It needs to be checked that the resistance of the pre-compressed tensile zone subjected to the combination of the permanent load $G_{k,1}$ and prestress is not exceeded.

Design value of bending moment due to $G_{k,1}$:

$M_{Sd,G}$ = $\gamma_G G_{k,1} l^2{}_{eff}/8$=1.0*9.5*25.02/8 = 742 kNm

Characteristic value of prestress:

P_k =$\gamma_p P_{m0}$ =1.0*1780 = 1780 kN

Bending moment due to prestress:

M_k = $\gamma_p M_p$=–1.0*1388 = –1388 kNm

The cross-section in mid-span needs to be designed for the combination of

N_{Sd} = $-\gamma_P/P_k$ = –1780 kN

M_{Sd} = $M_{Sd,G}+\gamma_p M_k$=742–1388 = –646 kNm

Distance z_s of the reinforcement in the flange from the centre of gravity:

z_s = $h-z_u-h_f/2$=1.70–0.945–0.165/2 = 0.67 m

d	=	$h-h_f/2$=1.70–0.165/2	=	1.60 m
μ_{Sds}	=	(0.646–1780*0.67)/(0.2*1.602*23.33)	=	0.15
			<	435 N/mm^2

ω	=	0.167;	σ_{sd}	=	f_{yd}	=	435 N/mm^2
$A_{s,req}$	=	(1/435) (0.167*0.2*1.6*23.33–1.78)10^4				<	0

No reinforcement in the flange is necessary.

15.5.2.6
Design for shear

Design value of the applied shear force:

V_{Sd} = $V_{od}-V_{pd}$

Design value V'_{od} at a distance d from the face of the support:

V'_{od}	=	$(\gamma_G G_k+\gamma_Q Q_k)(l_{eff}/2-a_L/2-d)$	=	
	=	(1.35*19.5+1.5*4.8) (12.5–0.1–1.65)=		361kN

Force component V_{pd} due to the inclined tendons:

V_{pd} = $\gamma_p \sigma p_{m,t} A_p \tan \alpha_i$

Reference

EC2, 5.4.2.1.1(1)
see 15.5.2.4 above
no compression reinforcement necessary [1], Table 7.1
EC2, 4.3.2.4.6
EC2, Eq. (4.32)
EC2, 4.3.2.2(10)
The stress σ=$p_{m,}$ for t= was calculated as:

$\sigma p_{m,}$ = = 1040 N/mm^2

tan α_i is given by:
- for tendons 1:

tan α_1 = 4*0.205[1/25.66–2 (1.65+0.43)/25.66^2] = 0.0265

- for tendons 2:

tan α_2	=	4*0.665[1/25.66–2(2.08)/25.66^2]	=	0.0868
V_{pd}	=	0.9*1040*7*10^{-1} (0.0268+0.0868)	=	74 kN
V_{Sd}	=	361–74	=	287 kN

Selected stirrups ø 8–200
A_{sw}/s_w=5.0*10^2 mm^2/m

Design shear resistance V_{Rd3} using the variable strut inclination method and assuming α=90° and cot θ=1.25:

V_{Rd3} = 5.0*102*0.9*1.65*435*1.25*10^{-3} = 403.7 kN

Design shear resistance of the compression struts

$b_{w,net}$	=	$b_w – ø_{duct}/2=0.20–0.006/2$	=	0.17 m
v	=	0.7–35/200	=	0.525
V_{Rd2}	=	$0.17*0.9*1.65*0.525*23.33/2.05*10^3$	=	1508 kN

Minimum shear reinforcement:

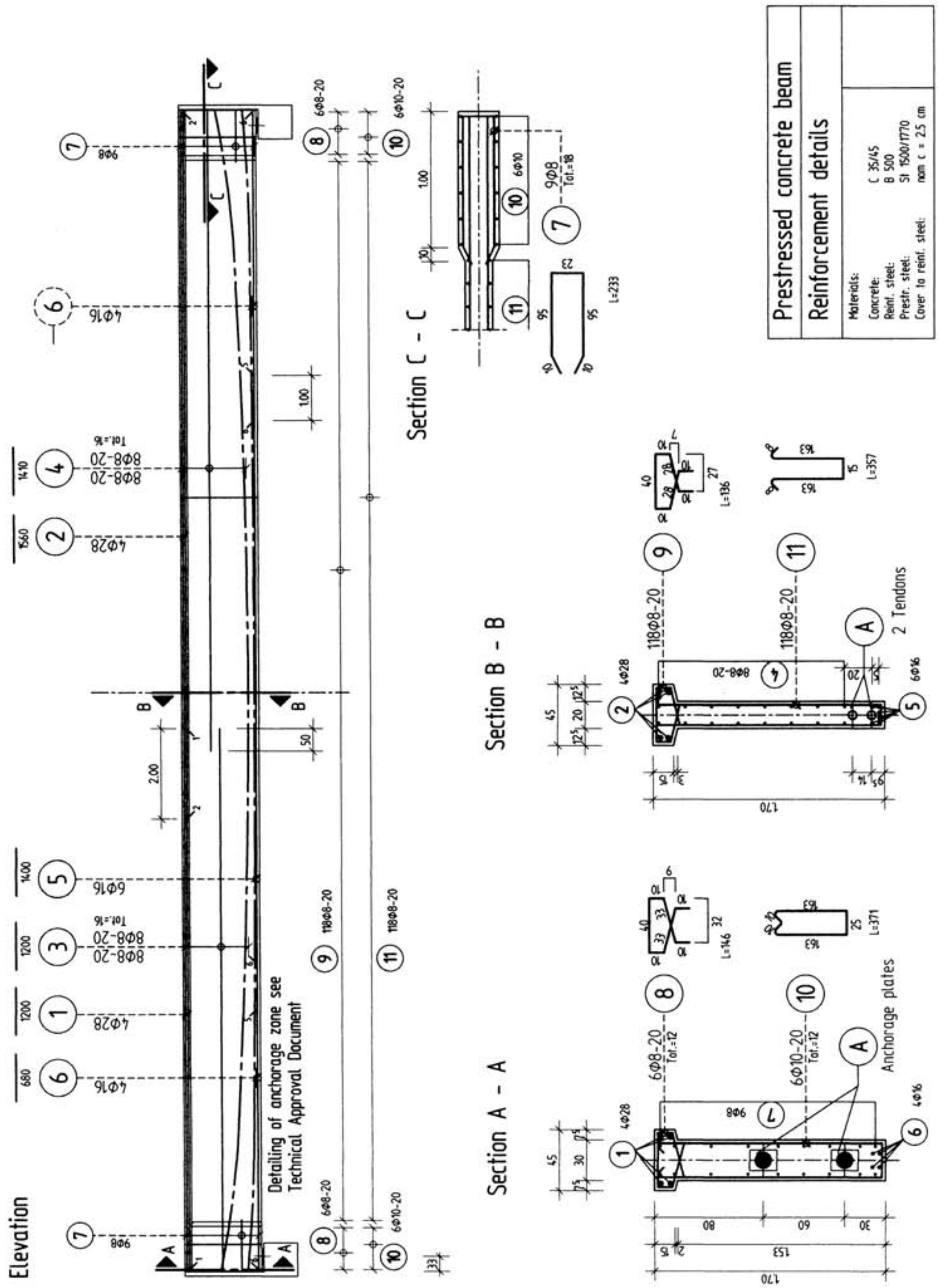
$(A_{sw}/s_w)_{min}$	=	0.0011*200*1000	=	$2.2*10^2$ mm^2/m

maximum longitudinal spacing $s_{w,max}$:

V_{sd}/V_{Rd2}	=	287/1508	=	0.19
			<	0.20
$s_{w,max}$	=		=	300 mm

Reference

see 15.5.2.1 above
EC2, 4.3.2.4.4
EC2, Eq. (4.26)
EC2, 5.4.2.2(5)
EC2, Eq. (5.16)
EC2, 5.4.2.2(7)
EC2, Eq. (5.17)
Detailing of reinforcement



15.5.3
Calculation of edge column subjected to crane-induced actions

15.5.3.1
Basic data and design value of actions

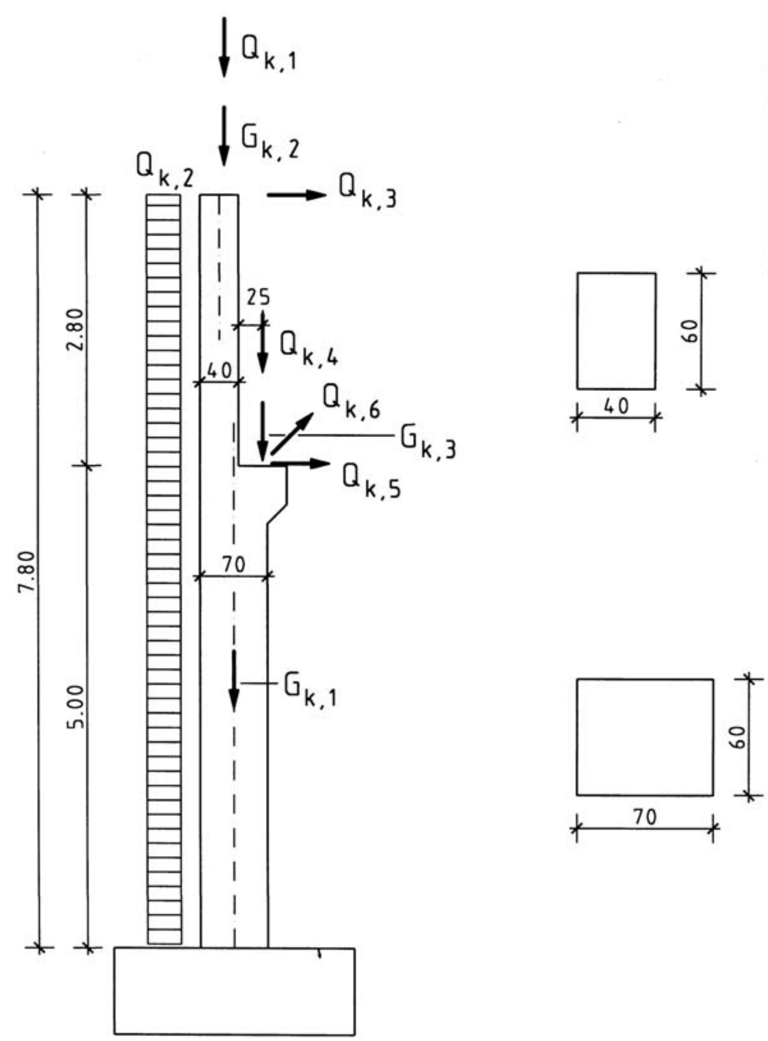
Structural system; cross-sectional dimensions

Elevation

Reference

i.e. fatigue verification to EC2–2 is performed

Exposure class:



Class 2a (humid environment without frost)

Materials:

Concrete grade C 45/55

For the second order analysis of the column (see 15.5.3.2 below), the general stress-strain diagram ace. to Figure 4.1 in EC2 will be used. For the design of the cross-section, the parabolic-rectangular diagram will be applied.

Steel grade B 500

For structural analysis and the design of cross-sections, the bi-linear diagram with a horizontal top branch will be used.

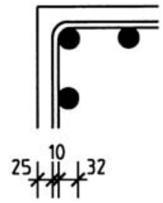
Cover to reinforcement (stirrups)

min c_w = = 20 mm
nom c_w = = 25 mm

Actions

Permanent actions (self-weight)

$G_{k,1}$ = 25.0 kN/m^3
$G_{k,2}$ = 244.0 kN (prestressed beam)



$G_{k,3}$ = 42.0 kN (crane girder)

Crane-induced variable actions

$Q_{k,1v}$	=	551.0 kN	(vertical)
$Q_{k,1t}$	=	114.0 kN	(transverse action)
$Q_{k,1b}$	=	±70.0 kN	(braking force)

Variable actions except crane-induced actions

$Q_{k,2}$	=	60.0 kN	(snow)
$Q_{k,3}$	=	3.6 kN/m	(wind)
$Q_{k,4}$	=	16.0 kN	(sliding force)

Combination coefficients

- for crane-induced actions	ψ_{0c}	=	1.0
- for snow	ψ_{0s}	=	0.6
- for wind	ψ_{0w}	=	0.6
- for sliding force	ψ_{0sl}	=	0.6

15.5.3.2
Design values of actions

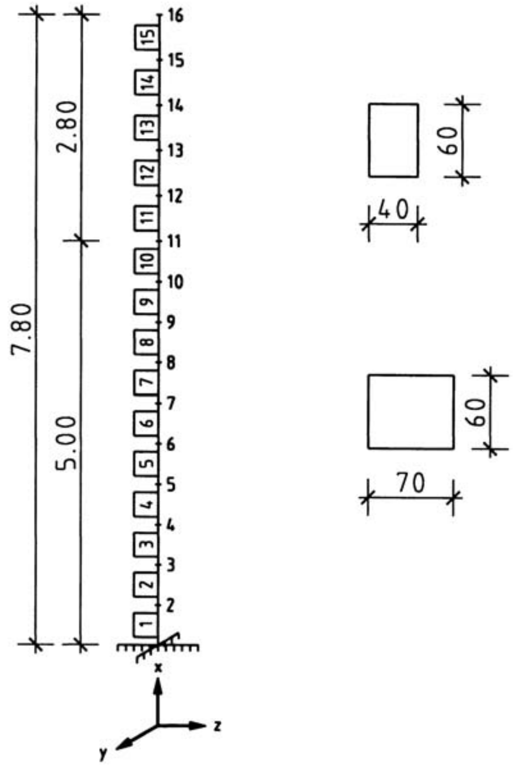
(a) Permanent actions (γ_G=1.35)

$\gamma_G G_{k,1}$	=	1.35*25.0	=	33.8 kN/m^2
$\gamma_G G_{k,2}$	=	1.35*244.0	=	329.4 kN
$\gamma_G G_{k,3}$	=	1.35*42.0	=	56.7 kN

Reference

EC2, Table 4.1
EC2, Table 3.1
EC2, 4.2.1.3.3(a), (5)
EC2, Eq. 4.2
ENV 10080
EC2, 4.2.2.3.2
EC2, Fig. 4.5
EC2, Table 4.2, for exposure class 2a
see 15.5.2.1 above
EC1–1, 9.4.4
see 15.5.2.1 above
EC2, 2.2.2.4
The combination with γ_G=1.0 is not relevant in this example
(b) Variable actions (γ_Q=1.50)

$\gamma_Q Q_{k,1v}$	=	1.5*551.0	=	826.5 kN
$\gamma_Q Q_{k,1t}$	=	1.5*114.0	=	171.0 kN



$\gamma_Q Q_{k,1b}$	=	±1.5*70.0	=	54.0 kN
$\gamma_Q \psi_{0.2} Q_{k,2}$	=	1.5*0.6*60.0	=	±105.0 kN
$\gamma_Q \psi_{0.3} Q_{k,3}$	=	1.5*0.6*3.6	=	3.24 kN/m
$\gamma_Q \psi_{0.4} Q_{k,4}$	=	1.5*0.6*16.0	=	14.4 kN

(c) Fundamental combination of actions

$$\begin{aligned}\Sigma(\gamma_{G,j}\, G_{k,j}) + \gamma_{Q,1}\, Q_{k,1} + \Sigma(\gamma_{Q,i}\, \psi_{0,i}\, Q_{k,i}) = {} & 1.35(G_{k,1} + G_{k,2} + G_{k,3}) \\ & + 1.5(Q_{k,1v} + Q_{k,1t} + Q_{k,1b}) \\ & + 1.5 * 0.6(Q_{k,2} + Q_{k,3} + Q_{k,4})\end{aligned}$$

15.5.3.3
Design of the column for the ultimate limit states induced by structural deformations

(a) General

For the design of the column at the ultimate limit states induced by structural deformations, a rigorous computer-based second-order analysis is carried out. The design model is shown below. In this program, the steel reinforcement, $A_{s,req,}$ required in the individual cross-sections is calculated automatically.

Reference

vertical crane load transverse action braking force snow wind sliding force

EC2, Eq. (2.7a); the accidental combination of actions is not considered

Crane-induced actions are the main variable actions.

EC2, 4.3.5

see 15.5.3.3 (e) below

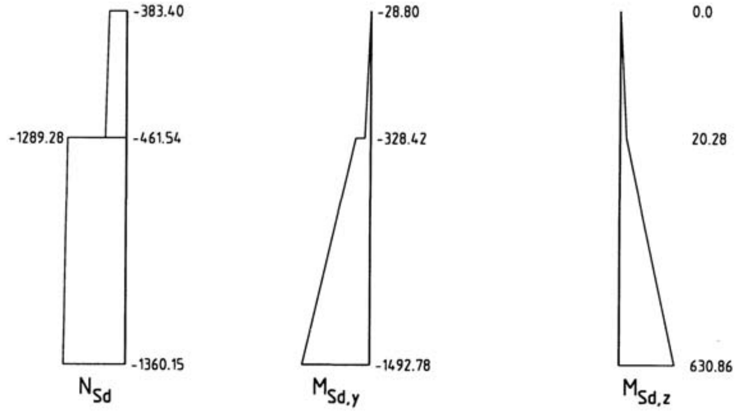
(b) Imperfections

For structural analysis, an inclination of

v = 1/200

in the direction of the theoretical failure plane is assumed.

(c) Creep



Allowance for creep deformations is made by using the simplified method and Appendix 3 proposed by QUAST in [2], i.e. to multiply the second order eccentricities e_2 by a coefficient

$$f_{\psi}=(1+M_{Sd,c}/M_{Sd})$$

where

$M_{Sd,c}$ is the factored bending moment due to quasi-permanent actions
M_{Sd} is the bending moment due to the relevant combination of permanent and variable actions

(d) Design actions in the nodes of the design model

Node	$F_{Sd,x}$ (kN)	$F_{Sd,y}$ (kN)	$F_{Sd,z}$ (kN)	$M_{Sd,y}$ (kNm)
16	14.4	0	383.4	–28.8
11	171.0	–105.0	883.2	–204.0

Wind:

$$q_{Sd,x}=3.24 \text{ kN/m}$$

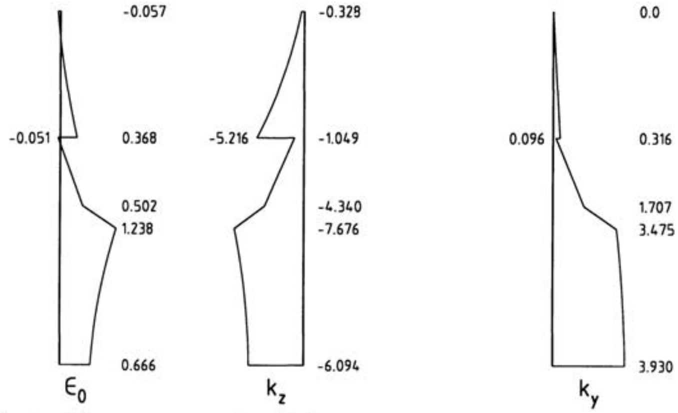
Rotation due to imperfections and creep

v_x = –4.24*10–3
v_y = –7.57*10–3

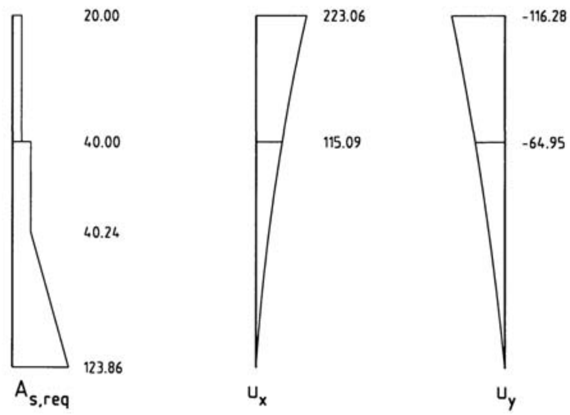
(e) Summary of design results Internal forces and moments

Reference

EC2, 4.3.5.4 and 2.5.1.3, Eq. (2.10)
introduced by iteration
EC2, 4.3.5.5.3
[2], p. 85, 9.4.3



Required reinforcement areas, $A_{s,req}$; displacements



Deformation and curvature

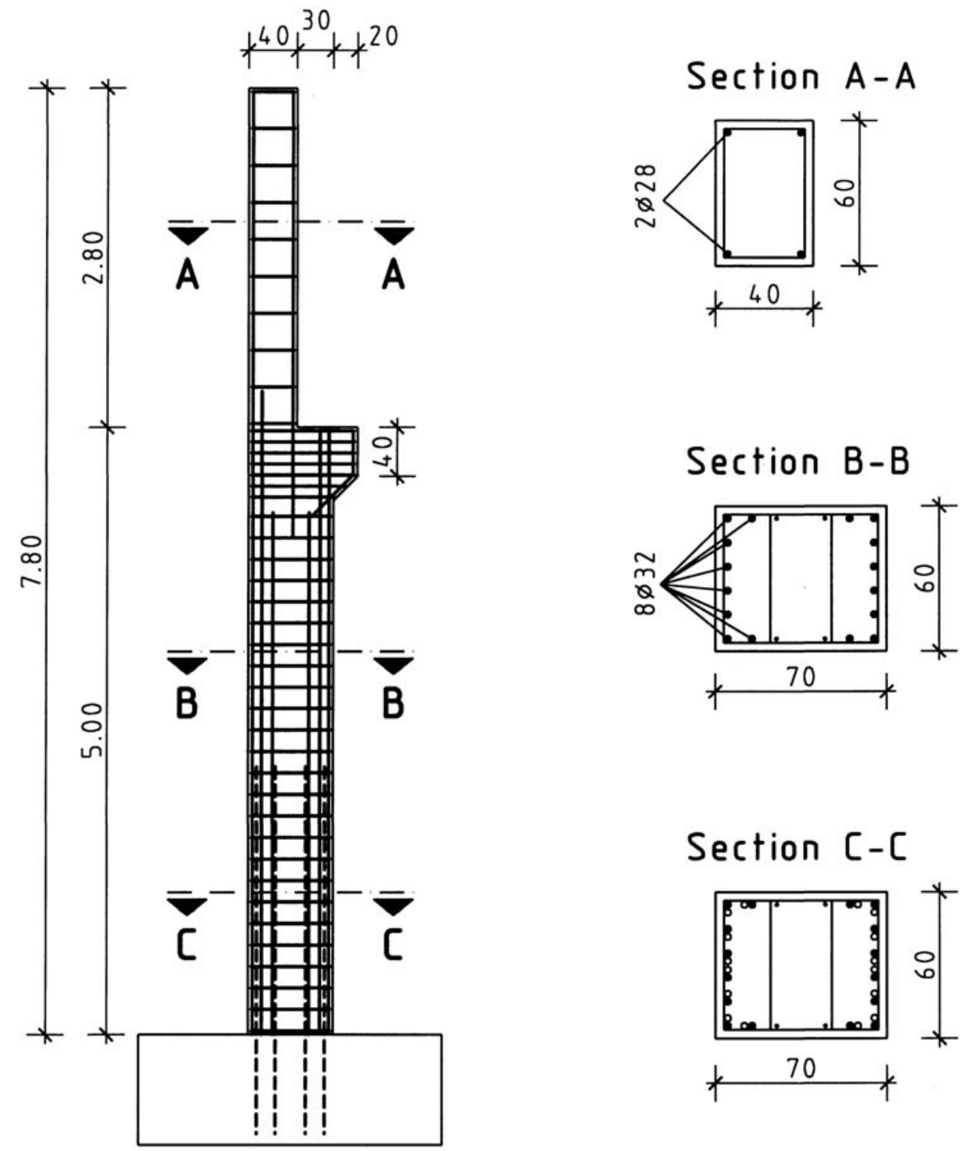
15.5.3.4
Design of the column; detailing of reinforcement

Required reinforcement area, $A_{s,req}$ at the restrained cross-section:

$A_{s,req}$ = = $123.86*10^2$ mm^2

Selected steel B 500 2*8=16 ø 32
$A_{s,prov=}128.68*10^2$mm^2

Detailing of reinforcement:
see Figure below
Reference
$A_{s,req}$ in mm^2*10^2
see 15.5.3.3 (e), above
EC2,5
Detailing of reinforcement
Elevation



15.5.3.5
Ultimate limit state of fatigue

15.5.3.5.1
General

The edge column is subjected to crane-induced actions. It needs therefore to be checked for the ultimate limit state for fatigue. In this ultimate limit state, see 15.5.3.0 above, it shall be verified that

$$D_{sd} = \sum_{i} (n_i/N_i) \leq 1$$

where

D_{Sd} is the design value of the fatigue damage factor calculated using the PALMGREN-MINER summation

n_i denotes the number of acting stress cycles associated with the stress range for steel and the actual stress levels for concrete

N_i denotes the number of resisting stress cycles

For the above verification, the stress calculation shall be based on the assumption of cracked cross-sections neglecting the tensile stength of concrete but satisfying compatibility of strains.

The fatigue strength of reinforcing steel and concrete are given by EC2–2, 4.3.7.8 and 4.3.7.9 respectively.

15.5.3.5.2 Combination of actions

In the present example, fatigue verification will be performed under the frequent combination of actions using the partial safety factors

γ_F	=	1.0	for actions
γ_{Sd}	=	1.0	for model uncertainties
$\gamma_{c,fat}$	=	1.5	for concrete
$\gamma_{s,fat}$	=	1.15	for reinforcing steel

Therefore the relevant combination of actions is given by:

$$E_d = \sum (1.0 * G_{k,j}) + 1.0 * \psi_{1,1} (Q_{k,1v} + Q_{k,1b}) + \sum (1.0 * \psi_{2,i} Q_{k,i})$$

where

$Q_{k,1b}$ = the component of the braking force $Q_{k,1b}$ that is relevant for fatigue verification. In this example, it is assumed that

$Q_{k,1b}$=±59.7 kN

For the verification, the following combination coefficients $\psi_{1,1}$ and $\psi_{2,i}$ are asssumed:

$\psi_{1,1}$	=	1.0	(for crane-induced actions)
$\psi_{2,2}$	=	0	(for snow loads)
$\psi_{2,3}$	=	0	(for wind)
$\psi_{2,i}$	=	0	(for all other variable actions $Q_{k,i}$)

Design values of actions:

(a) Vertical actions

$G_{d,j}$	=	25.0+244.0+42.0	=	311.0 kN
$Q_{d,1v}$	=		=	551.0 kN

Reference

EC2–2, 4.3.7

EC2–2, 4.3.7.5

EC2–2, 4.3.7.3

EC2–2, 4.3.7.2

see 15.5.3.1 above i=1t, 2, 3, 4

see 15.5.3.1 above

(b) Horizontal actions

$Qd_{,1b}$	=	=	±59.7kN
$Q_{d,4}$	=	=	16.0 kN

15.5.3.5.3 Damage factor D_{Sd}

For the calculation of the damage factor D_{Sd}, the spectrum of actions S2 in DIN 15 018 Part 1 is assumed. This approach is based on a linear relationship between actions and stresses assuming cracked cross-sections.

15.5.3.5.4
Calculation of the stress range $\Delta\sigma_s$

A trial calculation shows that the most unfavourable stress range $\Delta\sigma_s$ occurs at the coordinate x=3.0 m above the level of restraint. A rigorous second order analysis leads to the following stresses:

$\sigma_{s,max}$	=		=	+112.2 N/mm2
$\sigma_{s,min}$	=		=	–24.3 N/mm^2
$\Delta\sigma_s$	=	112.2–(–24.3)	=	136.5 N/mm^2

Since both tensile and compressive stresses in the reinforcing steel occur, fatigue verification is necessary.

15.5.3.5.5
Calculation of the stress range $\Delta\sigma_c$

The extreme concrete stresses occur at the level of restraint (i.e. x=0). A rigorous second order analysis leads to he following values:

$\sigma_{c,max}$	=		=	–11.3 N/mm2
$\sigma_{c,min}$	=		=	–2.9 N/mm^2
$\Delta\sigma_c$	=	11.3–2.9	=	8.4 N/mm^2

15.5.3.5.6
Verification of the fatigue strength of the reinforcing steel

The fatigue requirement for reinforcing steel will be met if the following expression is satisfied:

$$\gamma_F \, \gamma_{Sd} \, \Delta\sigma_{s,equ} \leq \Delta\sigma_{Rsk}(N^*)/\gamma_{s,fat}$$

where

$\Delta\sigma_{Rsk}(N^*)$	=	stress range at N^* cycles from the appropriate S-N lines
$\Delta\sigma_{s,equ}$	=	the damage equivalent stress range which is the stress range of a constant stress spectrum with N^*=10^6 stress cycles which results in the same damage as the spectrum of stress ranges caused by flowing traffic loads
γ_F	=	1.0
γ_{Sd}	=	1.0
$\gamma_{s,fat}$	=	1.15

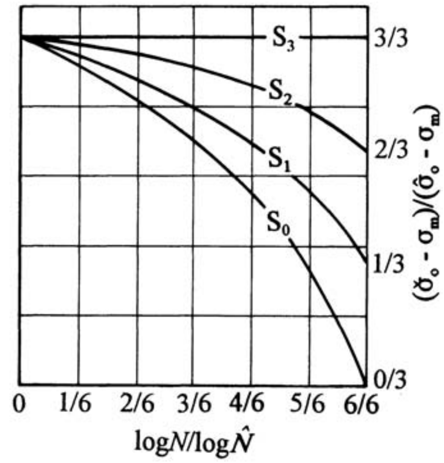
For bars with diameter, ø > 25 mm, the stress range, $\Delta\sigma_{,Rsk}$, at N^*=10^6 cycles is given as:

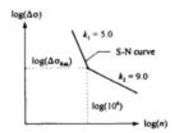
$\Delta\sigma_{Rsk}(N^*)$	=	=	195 N/mm^2

Reference

for reinforcing steel
Tension Compression
EC2–2, 4.3.7.1
for concrete
Compression
Compression
EC2–2, 4.3.7.5
see 15.5.3.4.2 above
EC2–2, 4.3.7.8
The shape of the relevant S-N curve is defined by the coefficients

k_1	=	5.0
k_2	=	9.0





For N+N^* cycles, the damage equivalent stress range $\Delta\sigma_{s,equ}$ is given by:

$$\Delta\sigma_{s,equ} = \alpha_p \kappa \Delta\sigma_s$$

For the calculation of the coefficients α_p and κ, the following assumptions have been used:

- spectrum of actions S2 according to DIN 15 018 Part 1
- $\Delta\sigma_s$=136.5 N/mm^2
- number of cycles: n

design lifetime: 50 years
working time: 10 hours/per day
one cycle/minute

n = 50*365*10*60*1 = 1.1*10^7

From the α_p-diagram below:

α_p = = 1.0

Coefficient A:

A = $\gamma_{Sd}\Delta\sigma_{s,max}\,\gamma_s/\Delta\sigma_{Rsk}$
= 1.0*136.5*1.15/195 = 0.81
< 1.0

From the κ-diagram below:

A = = 1.0

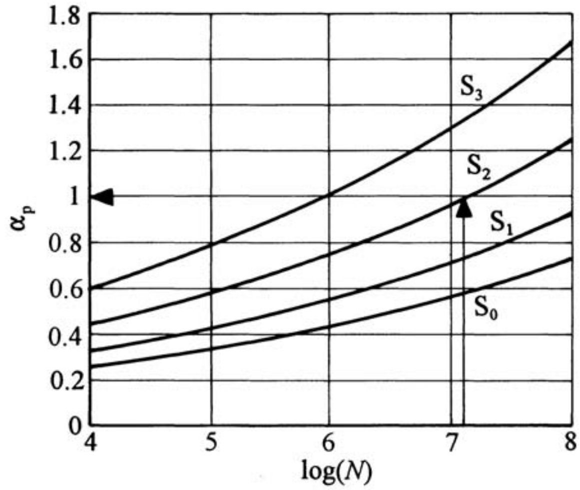
Thus:

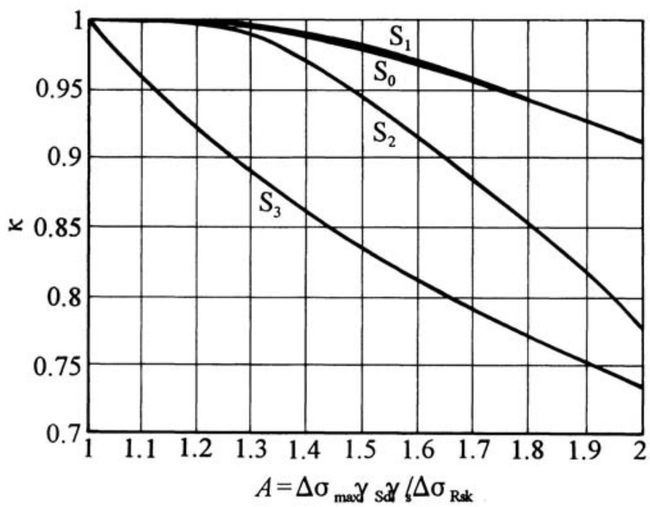
$\Delta\sigma_{s,equ}$ = 1.0*1.0*136.5<195/1.15 = 169.5 N/mm^2

Requirements for reinforcing steel are met.

Reference

see 15.5.3.4.3 above
see 15.5.3.4.4 above
Annex A gives more details on the fatigue verification
Diagrams for reinforcing steel





15.5.3.5.7
Verification of the fatigue strength of concrete

The design fatigue strength of concrete is given by the S-N curve according to:

$$\log(N) = 14\ \frac{1 - S_{cd,max}}{\sqrt{1 - R}}$$

$$S_{cd,max} = |\sigma_{c,max}| \frac{1}{f_{cd,fat}}$$

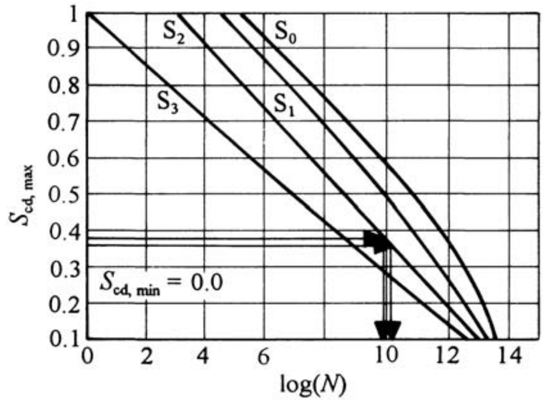
$$S_{cd,min} = |\sigma_{c,min}| \frac{1}{f_{cd,fat}}$$

$$R = \frac{S_{cd,min}}{S_{cd,max}}$$

$$f_{cd,fat} = \beta_{cc}(t)\ \frac{f_{ck}}{\gamma_c}$$

$$\beta_{cc} = \exp\left(s\left[1 - \sqrt{\frac{28}{t_0/t_1}}\right]\right)$$

where



N = number of cycles to failure

$\beta_{cc}(t_0)$ = coefficient which depends on the age of concrete t_0 in days when fatigue loading starts. If no information is available β_{cc} can be taken as 1.0

t_1 = 1 day

In the present case, the basic data are given by:

Concrete strength class	C 45/55:	f_{yk}	=	45 N/mm^2
$f_{cd,fat}$	=	45/1.5	=	30 N/mm^2
$S_{cd,max}$	=	11.3/30	=	0.38
$S_{cd,min}$	=	2.9/30	=	0.10
				0

from the diagram below:

N	=	$7.9*10^9$	>	$1.1*10^7$

Reference

EC2–2, 4.3.7.4

see 15.5.3.4.5 above

ANNEX A

15.6
Guidance for the calculation of the equivalent stress range $\Delta\sigma_{s,equ}$ for reinforcing steel and of the S-N curve for concrete in compression using the single load level method

15.6.1
Reinforcing steel

Design value of the fatigue damage D_d, using the PALMGREN-MINER summation:

$$D_d = \sum \frac{n_i}{N_i} \qquad (1)$$

where

n_i denotes the number of acting stress cycles associated with the stress range for steel and the actual stress levels for concrete

N_i denotes the number of resisting stress cycles

The shape of the S-N curve is given by:

$$\Delta\sigma^{k_j} N^* = \Delta\sigma_i^{k_j} N_i ; k_j = 1 \text{ or } 2 \qquad (2)$$

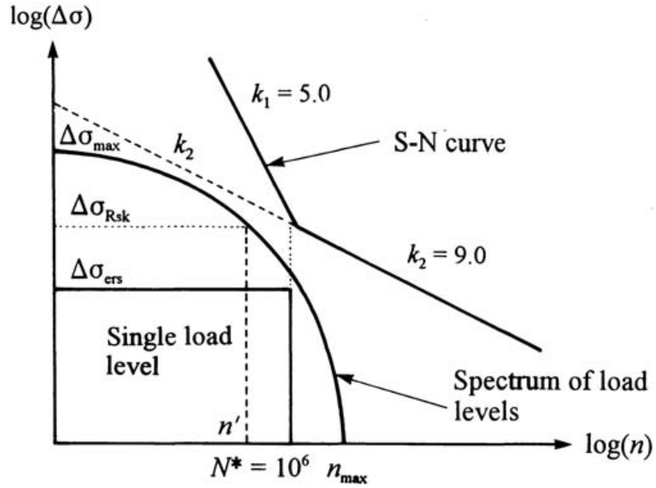


Figure A1 Graphical presentation of the design concept for reinforcing steel.

$$\Delta\sigma^{*} = \frac{\Delta\sigma_{Rsk}}{\gamma_s}$$

$$\Delta\sigma_i = \Delta\sigma_{max}\ \eta\ \gamma_{Sd} \tag{3}$$

where

η	=	coefficient describing the spectrum of load levels
$\Delta\sigma_{max}$	=	maximum acting stress range
γ_s, γ_{Sd}	=	partial safety factors

Equation 2 may be written as:

$$N_i = \left(\frac{\Delta\sigma^{*}}{\Delta\sigma_i}\right)^{k_j} N^{*} \tag{4}$$

Equations 1 and 4 lead to the following expression for E_d:

$$D_d = \sum_{0}^{n'}\left[\frac{n_i}{\left(\frac{\Delta\sigma^{*}}{\Delta\sigma_i}\right)^{k_1} N^{*}}\right] + \sum_{n'}^{n_{max}}\left[\frac{n_i}{\left(\frac{\Delta\sigma^{*}}{\Delta\sigma_i}\right)^{k_2} N^{*}}\right] \tag{5}$$

$$D_d = \sum_{0}^{n'}\left[\frac{n_i}{\left(\frac{\Delta\sigma^{*}}{\Delta\sigma_i}\right)^{k_1} N^{*}}\right] + \sum_{0}^{n_{max}}\left[\frac{n_i}{\left(\frac{\Delta\sigma^{*}}{\Delta\sigma_i}\right)^{k_2} N^{*}}\right] - \sum_{0}^{n'}\left[\frac{n_i}{\left(\frac{\Delta\sigma^{*}}{\Delta\sigma_i}\right)^{k_2} N^{*}}\right] \tag{6}$$

or

$$D_d = \frac{1}{N^{*}}\left(\frac{1}{\Delta\sigma^{*}}\right)^{k_2}\left[\sum_{0}^{n_{max}}\left(\Delta\sigma_i^{k_2} n_i\right) + \sigma^{*(k_2-k_1)}\sum_{0}^{n'}\left(\Delta\sigma_i^{k_1} n_i\right) - \sum_{0}^{n'}\left(\Delta\sigma_i^{k_2} n_i\right)\right] \tag{7}$$

An equivalent single load level with N* cycles shall satisfy the condition:

$$D_d = D_{equ} = \frac{N^{*}}{N_{equ}} \tag{8}$$

Using equation 2 for the S-N curve and equation 7 for the equivalent damage factor, the equivalent steel stress $\Delta\sigma_{s,equ}$ may be calculated as:

S-N curve

$$\Delta\sigma^{*k_2} N^{*} = \Delta\sigma_{s,equ}^{\ \ k_2} N_{s,equ} \tag{9}$$

Equivalent number of cycles

$$N_{s,equ} = \left(\frac{\Delta\sigma^{*}}{\Delta\sigma_{s,equ}}\right)^{k_2} N^{*} \tag{10}$$

Equivalent damage factor

$$D_d = D_{s,equ} = \left[\frac{\Delta\sigma_{s,equ}}{\Delta\sigma^*}\right]^{k_2} \quad (11)$$

Equivalent steel stress

$$\Delta\sigma_{s,equ} = \Delta\sigma^* \sqrt[k_2]{D_d} \quad (12)$$

From equation 7, it follows that

$$\Delta\sigma_{s,equ} = \Delta\sigma^* \sqrt[k_2]{\frac{1}{N^*}\left(\frac{1}{\Delta\sigma^*}\right)^{k_2}\left[\sum_0^{n_{max}}\left(\Delta\sigma_i^{k_2} n_i\right) + \sigma^{*(k_2-k_1)}\sum_0^{n'}\left(\Delta\sigma_i^{k_1} n_i\right) - \sum_0^{n'}\left(\Delta\sigma_i^{k_2} n_i\right)\right]} \quad (13)$$

The equivalent steel stress $\Delta\sigma_{s,equ}$ may be expressed by:

$$\Delta\sigma_{s,equ} = \Delta\sigma_{max}\,\gamma_{Sd}\,\alpha_p\,\kappa \quad (14)$$

The coefficients α_p and κ are defined as:

$$\alpha_p = \sqrt[k_2]{\frac{1}{N*}\sum_0^{n_{max}}\eta_i^{k_2} n_i} \quad (15)$$

and

$$\kappa = \sqrt[k_2]{1 + \frac{\Delta\sigma^{*(k_2-k_1)}\sum_0^{n'}\left(\Delta\sigma_i^{k_1} n_i\right)}{\sum_0^{n_{max}}\left(\Delta\sigma_i^{k_2} n_i\right)} - \frac{\sum_0^{n'}\left(\Delta\sigma_i^{k_2} n_i\right)}{\sum_0^{n_{max}}\left(\Delta\sigma_i^{k_2} n_i\right)}} \quad (16)$$

Format for fatigue verification:

$$\Delta\sigma_{s,equ} \le \frac{\Delta\sigma_{Rsk}}{\gamma_s}$$

$$\gamma_{Sd}\,\Delta\sigma_{max}\,\alpha_p\,\kappa \le \frac{\Delta\sigma_{Rsk}}{\gamma_s} \quad (17)$$

The coefficients α_p and κ may be taken from Figure A2.

15.6.2 Concrete

The fatigue verification of concrete is analogous to that for steel reinforcement. However, there are differences. The fatigue requirements under cyclic loading will be met if the required lifetime (number of cycles, n_{max} is less than or equal to the number of cycles to failure (N_{equ}). In addition, the simplified S-N function given by equation 18 is used.

S-N curve of concrete:

$$\log(N) = 14\,\frac{1 - S_{cd,max}}{\sqrt{1 - R}} \quad (18)$$

The calculation of the coefficients in equation 18 is based on EC2–2 (pr ENV 1992–2):

S-N curve of concrete, i.e. equation 1:

$$D_d = \sum\frac{n_i}{N_i} \quad (1)$$

Equivalent damage factor, D_{equ}:

$$D_d \le D_{equ} = \frac{n_{max}}{N_{equ}} \quad (19)$$

Equivalent spectrum of load levels:

N_{equ} is calculated on the assumption that D_{sd} =1.0 for a given spectrum of load levels and for given parameters $S_{cd,max}$ and R of the relevant S-N curve for concrete.

$$D_d \le 1$$

Verification format:

$$n_{max} \le N_{equ} \quad (20)$$

N_{equ} should be taken from Figure A3.

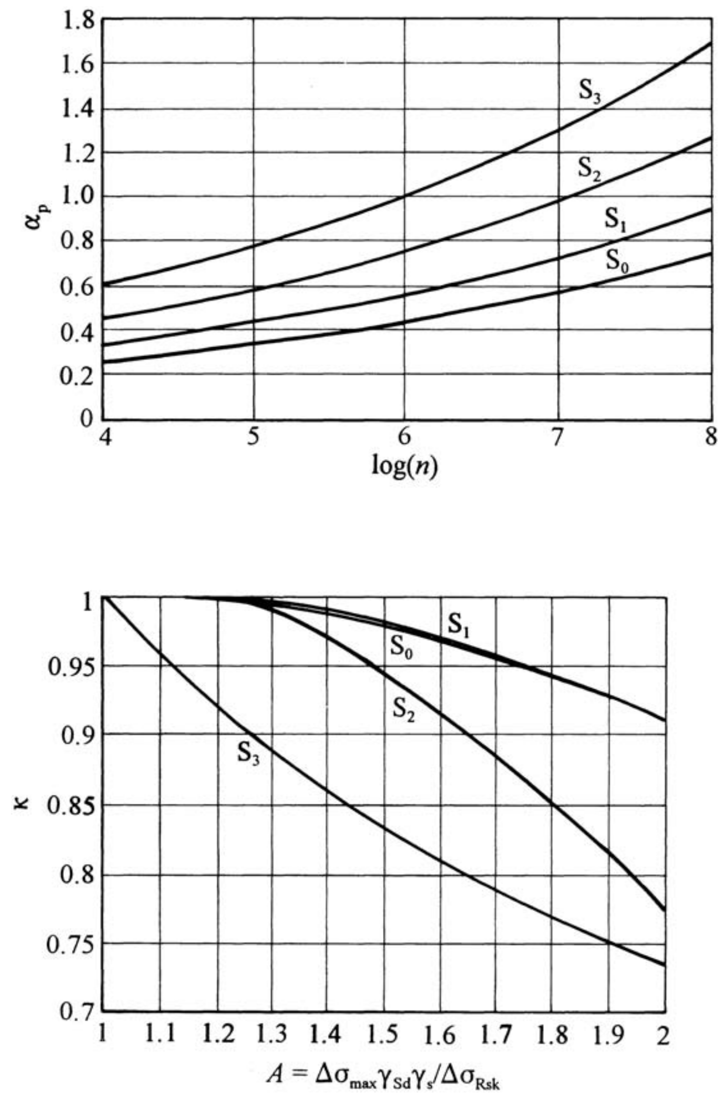


Figure A2 Coefficients for reinforcing steel, (a) α_p (b) κ.

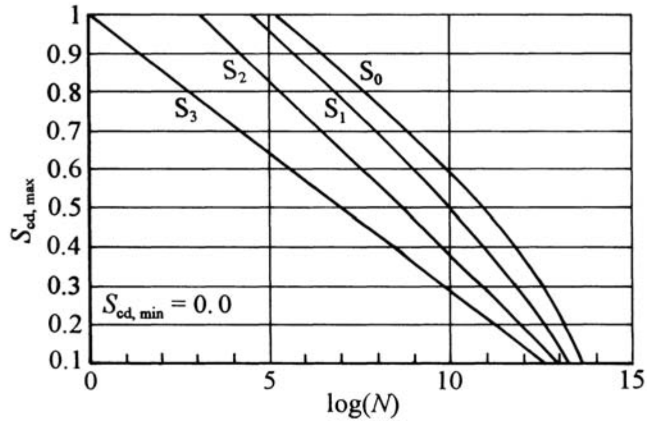
ANNEX B

15.7
Design of purpose-made fabrics

In the present design examples, purpose-made fabrics as defined in ENV 10 080 *Steels for the reinforcement of concrete; Weldable ribbed reinforcing steel B 500* have been chosen. The graphical representation is shown in Figure B1 for the top reinforcement of a continuous slab and in Figure B2 for the respective bottom reinforcement.

Each individual fabric is characterized by a position number, i.e. ① to ③ in Figures B1 and B2. Their characteristics are described graphically in Figures B3 to B5 by means of the diameters and spacing of both the longitudinal and transverse bars. The total number of bars and their lengths lead to the total weight of the fabric.

The presentation of the fabrics corresponds to ISO 3766–1977(E) *Building and civil engineering drawings—Symbols for concrete reinforcement,* particularly clause 2.3.1. Each fabric is characterized by a rectangular frame (see, for example, Figure B2), the diagonal connected to the position number denoting the direction of the main bars. In Figure B2, the lap length (s_t=400 mm) of the transverse bars is also defined.


$S_{cd,max}$
$S_{cd,min} = 0.0$
S_0
S_1
S_2
S_3
log(N)

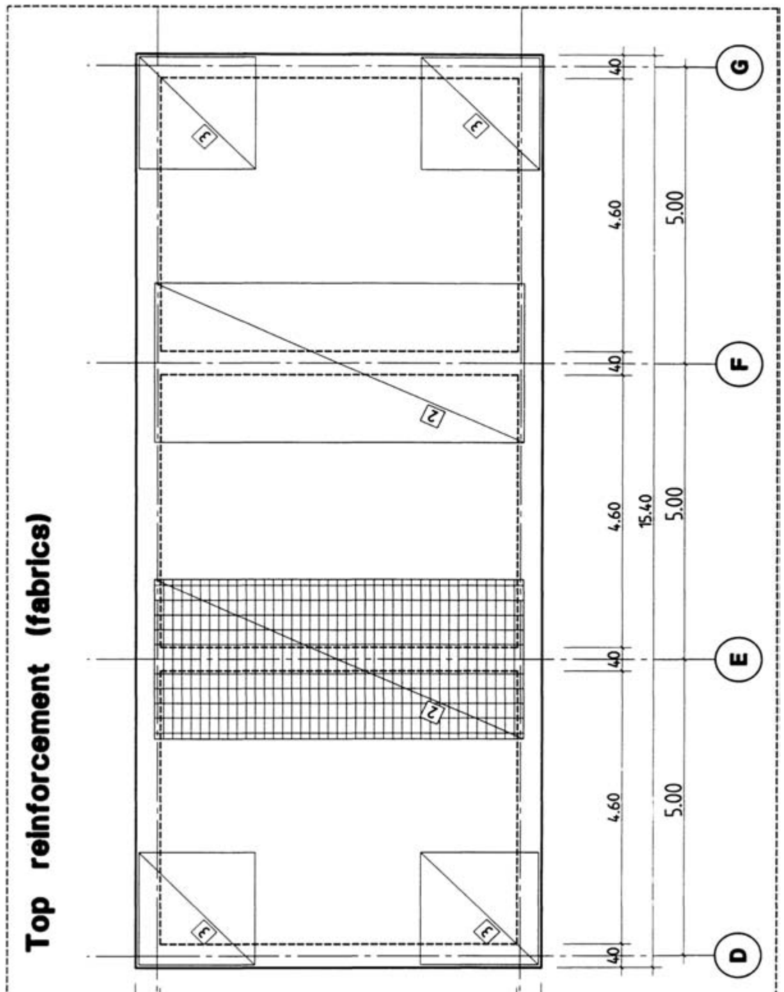

Top reinforcement (fabrics)
G
F
E
D
12
13
15.40
5.00
4.60
40
5.90
6.60
5.80

Figure A3 Relationship between N_{equ} and $S_{c,max}$ for different values of $S_{cd,min}$.

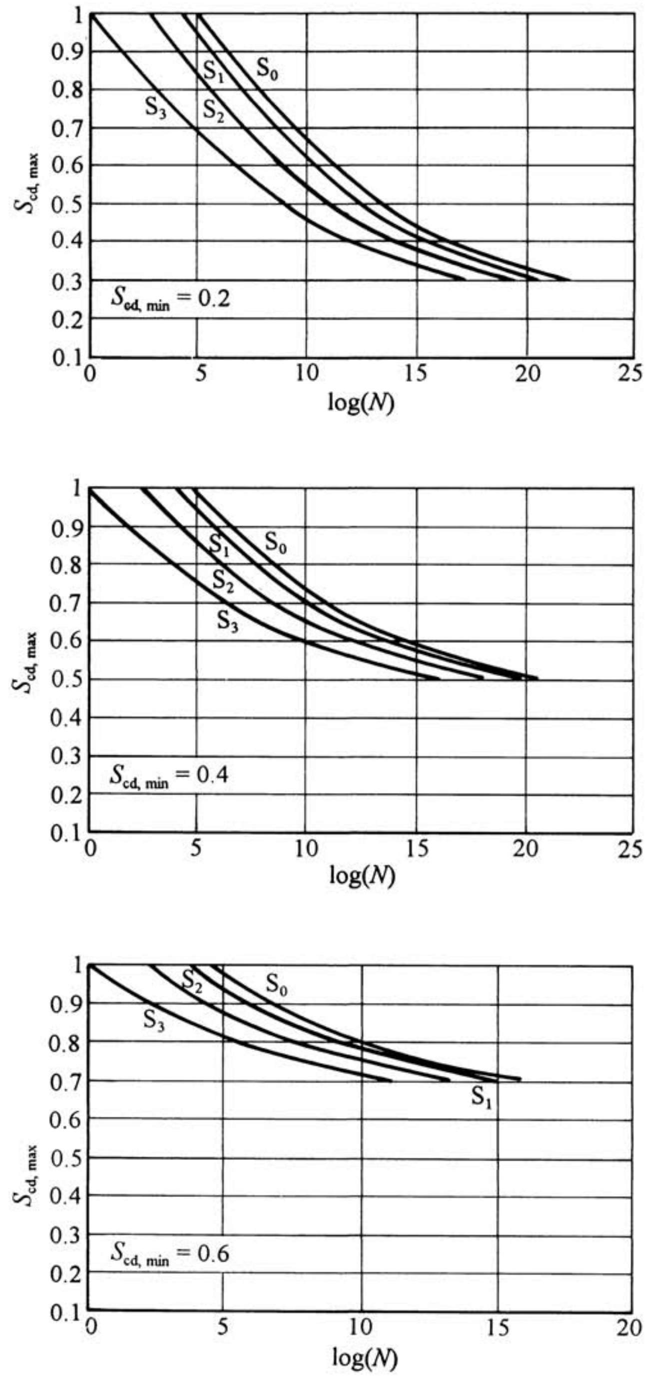


Figure B1

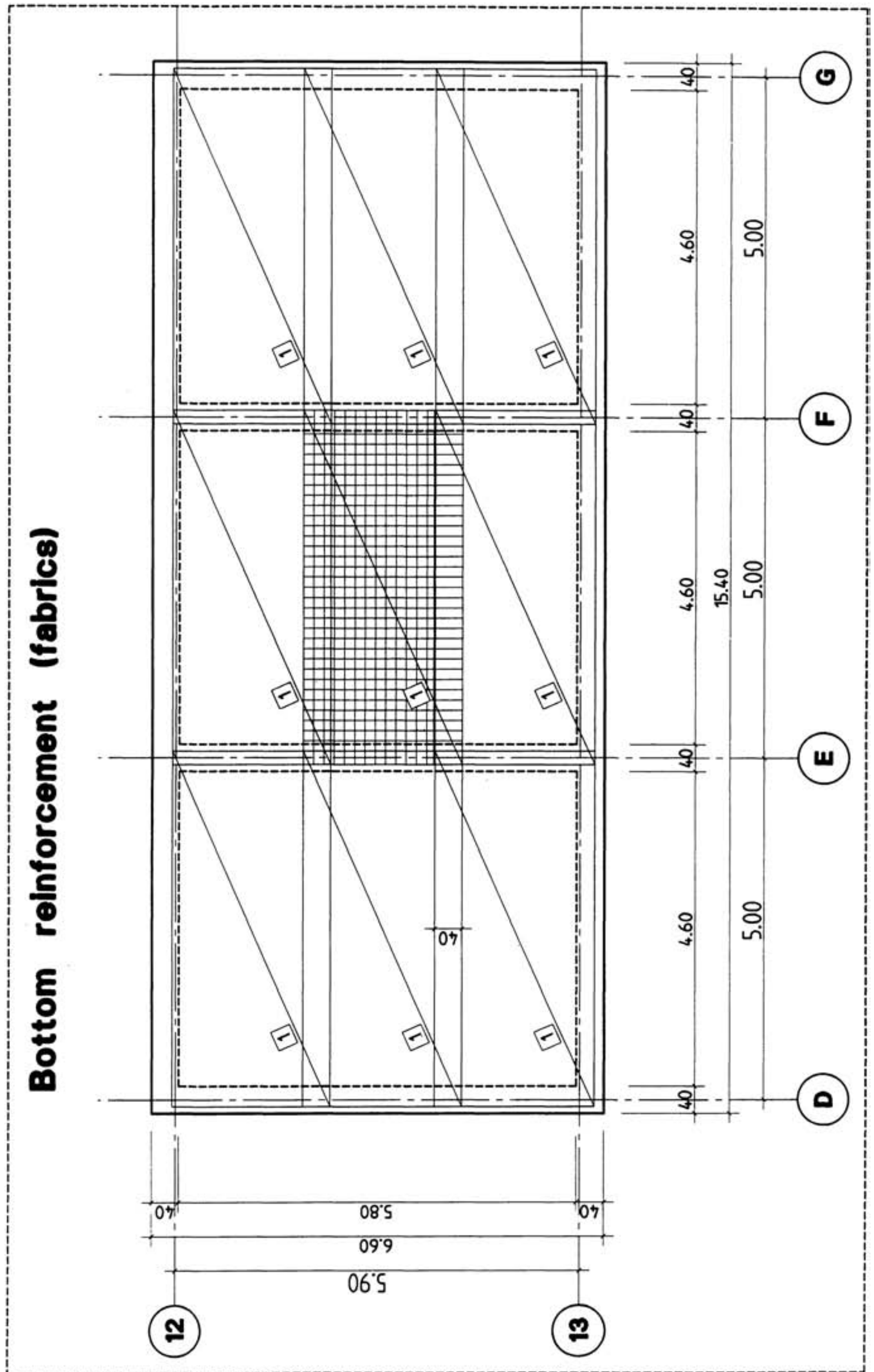
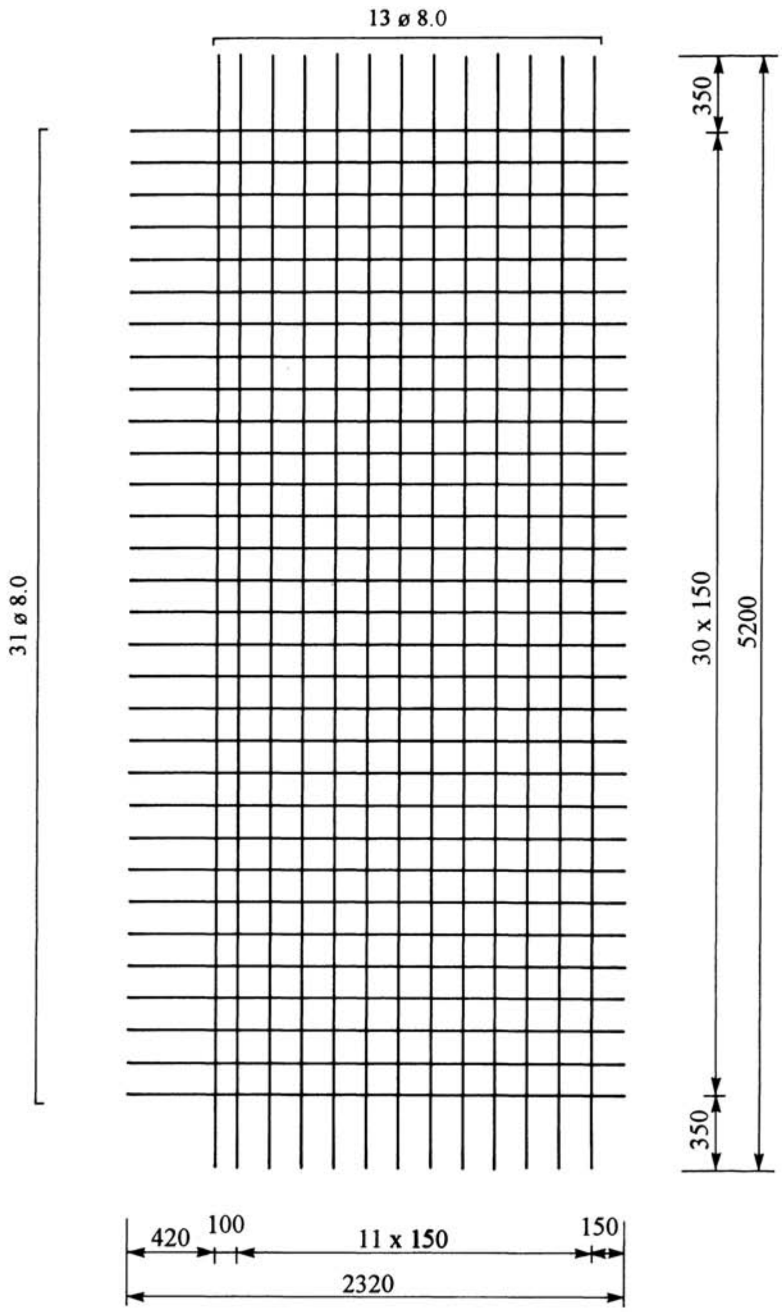
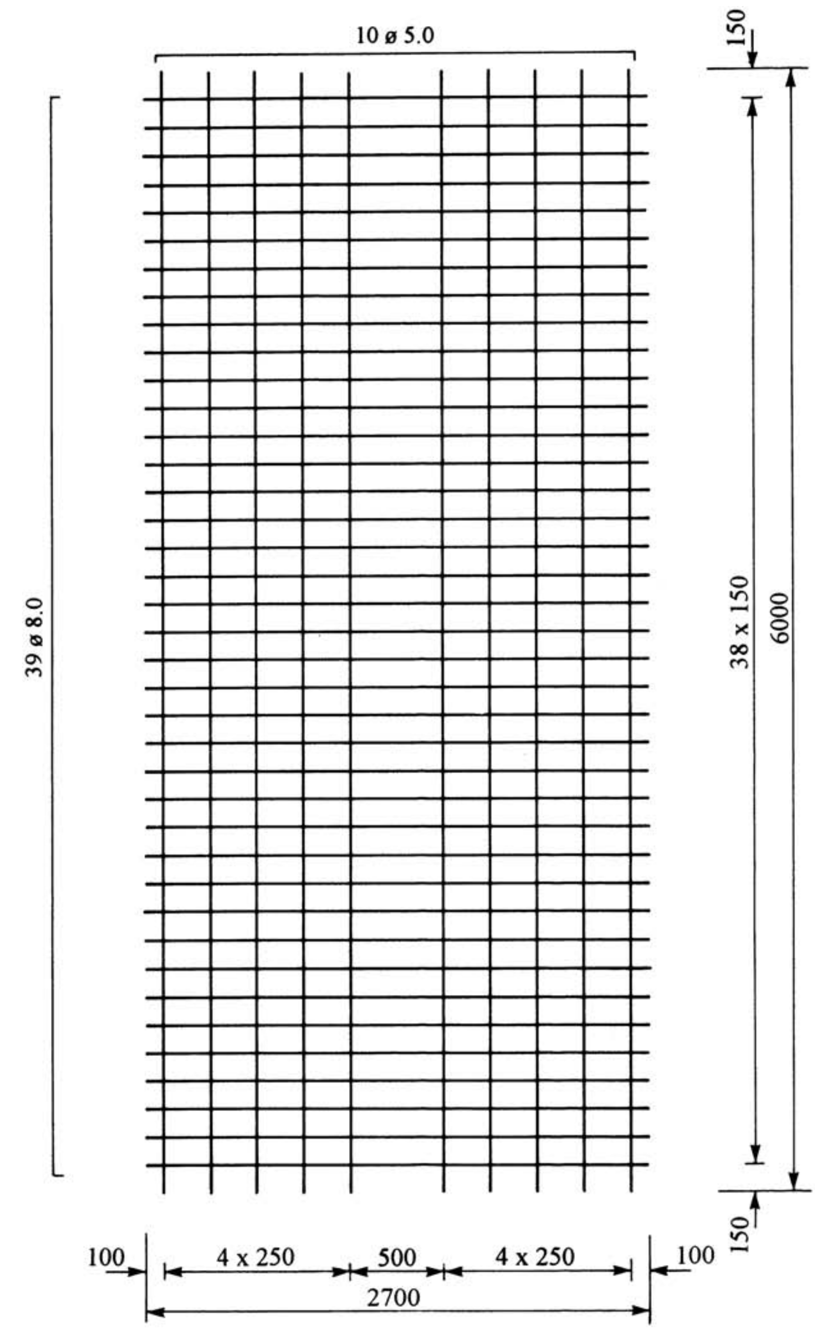

Bottom reinforcement (fabrics)
D
E
F
G
12
13
40
4.60
5.00
15.40
5.80
6.60
5.90
1

Figure B2



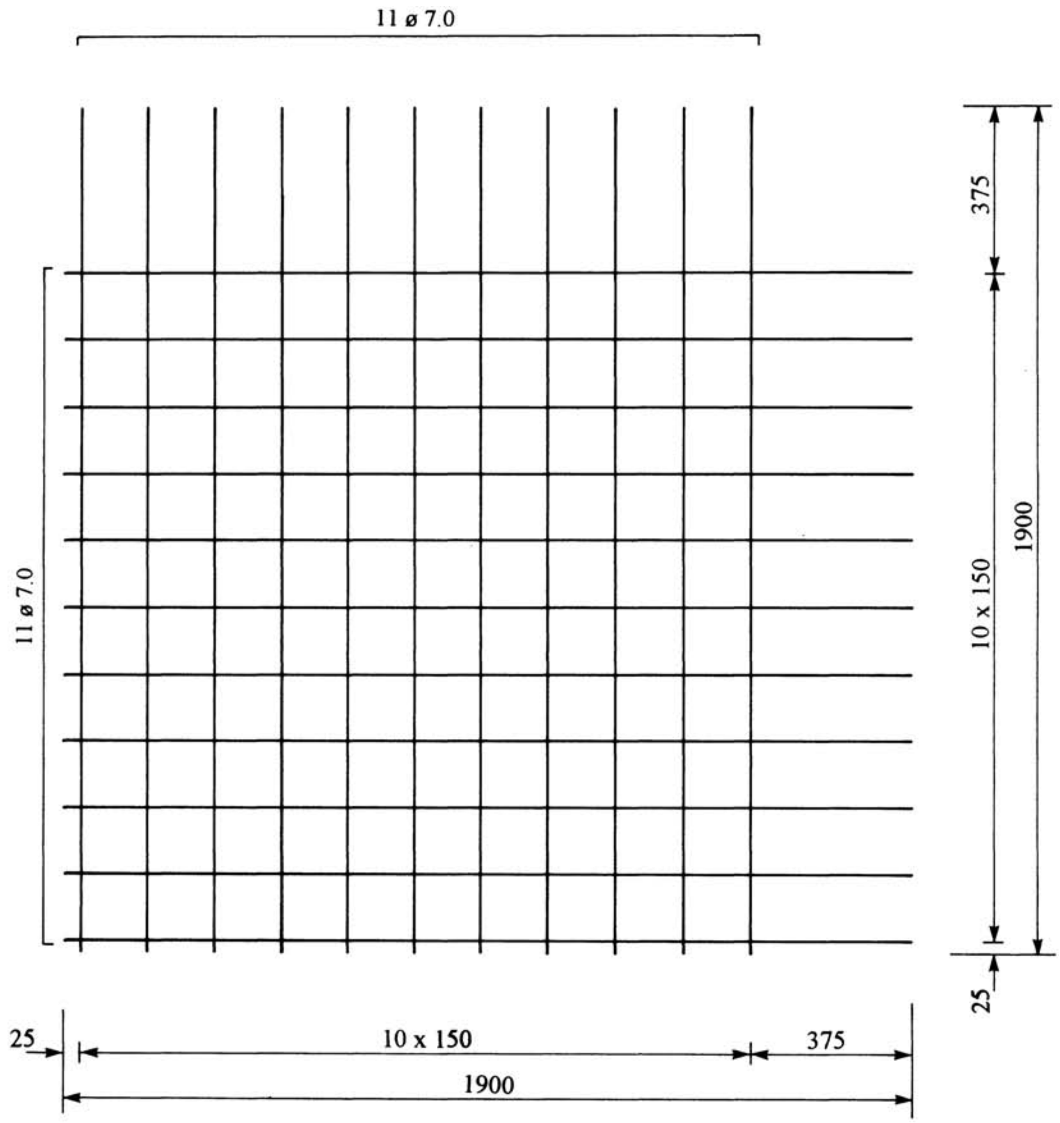
Longitudinal bars 13 ø 8.0 x 5200 = 26.702 **kg**
Transverse bars 31 ø 8.0 x 2320 = 28.408 kg
Total weight 58.110 kg

Figure B3



Longitudinal bars 10 ø 5.0 x 6000 = 9.240 **kg**
Transverse bars 39 ø 8.0 x 2700 = 41.594 kg
Total weight 50.834 kg

Figure B4



Longitudinal bars 11 ø 7.0 x 1900 = 6.312 kg
Transverse bars 11 ø 7.0 x 1900 = 6.312 kg
Total weight 12.624 kg

Figure B5

Index